EFFECTIVENESS OF REACTIVE POWER SOURCES FOR POWER SYSTEM PERFORMANCE ENHANCEMENT

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Effectiveness of Reactive Power Sources for Power System Performance Enhancement

By

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Abstract

Increased load demand can severely deteriorate the performance of a power system. Building new generation and transmission facilities are not easy due to economic and environmental constraints. As a result, power systems are operated close to their limits. The probability of blackouts during contingencies is high when the power system is operating under stressed conditions. The power system should be operated in such a way that the voltage limits and thermal limits of equipments are not violated. In addition, the possibilities of voltage collapse and voltage stability problems must be considered.

Reactive power planning is an important aspect of power system planning and operation when a power system is highly stressed. The most important requirement of a power system for maintaining desired performance is the existence of a sufficient amount of reactive power reserve in the proper location of the power system. Installing devices that supply reactive power (like capacitors, static VAR compensator etc.) can enable the power system to be operated closer to their limits and thus make it possible to get the 'best' from the existing resources. Optimal Power Flow (OPF) is used to optimize the power system performance.

This thesis shows the effectiveness of reactive power supply for the performance enhancement of power systems. The problems considered are related to maintaining acceptable voltage profile and ensuring adequate voltage stability margin during a

contingency is within an acceptable level. Case studies are presented throughout the thesis to show the effectiveness of reactive power supply. These techniques are well known when considering a specific problem individually. However, there remains a major challenge to determine the best locations and controls for reactive power discuses to provide maximum benefit to the power system as a while during normal operation as well as during contingencies.

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List of Abbreviations and Symbols

IP	:Integer Programming
LP	:Linear Programming
NLP	:Nonlinear Programming
OPF	:Optimal Power Flow
p.u.	:Per Unit
Q	:Reactive Power
QP	:Quadratic Programming
SVC	:Static VAR Capacitor
VD	:Voltage Deviation
SQP	:Sequential Quadratic Programming
Y	:Bus Admittance Matrix
$/hr	:Dollar per Hour
MW	:Mega Watt
MVAr	:Mega Volt Ampere Reactive
MVA	:Mega Volt Ampere
VAR	:Volt-Ampere Reactive

Chapter 1

Introduction

1.0 Background of the Research

Insufficient reactive power supply can result in voltage collapse, which has been one of the reasons for some major blackouts. For example, the US-Canada Power System Outage Task Force states in its report that insufficient reactive power was an issue in the August 2003 blackout, and recommended strengthening the reactive power and voltage control practice in all North American Electric Reliability Council Regions [1].

The traditional solution was to install new costly transmission lines to meet the constant increase in power demand that are often faced with environmental restrictions and economic infeasibility. Planning of reactive power compensation has changed the way utility industry handles the increased load and extremely low voltages. Local reactive power compensation that will allow a power system to safely manage a load increase is to allocate shunt reactive compensation devices at locations throughout a system to provide sufficient local reactive power to system loads. This mitigates reactive power that must be produced by generators and transferred over the transmission infrastructure. By providing reactive power locally, load voltages can be regulated; voltage stability can be enhanced; and transmission system can be used more effectively as the negative effects of reactive power transmission are significantly reduced [2].

This thesis presents a conventional optimization algorithm to minimize the fuel cost, minimize the transmission loss, and minimize the voltage deviation once a time in a way that meets the limitations of the equipments and other operating constraints. The goal is achieved by proper adjustment of generators' power output to support a particular load demand at the lowest possible fuel cost or transmission loss. This optimization problem is known as the optimal power flow (OPF) problem. To solve the optimization problem, a number of conventional optimization techniques have been researched. These include Non-linear Programming (NLP), Quadratic Programming (QP), and Linear Programming (LP). Though these techniques have been successfully applied for solving the OPF problem, some difficulties are still associated with them. One of the difficulties is the multimodal characteristic of the problems to be handled.

1.1 Objectives of the Research

Reactive power planning is a sub-problem of optimal power flow (OPF), which has objectives of improving the system voltage profile, reducing operation cost, and minimizing the system transmission loss. This is achieved through redistribution of reactive power in the power system through optimal settings of generator terminal voltages, reactive power outputs, and output of other compensation devices such as capacitors, static Volt-Ampere Reactive (VAR) compensator, synchronous condensers and so on. The objective behind the present study is to explore the impact of the reactive power compensation devices on the system voltage profile and active power transmission loss by solving each OPF objective. The principle goals of this research are summarized as follows:

1. Review the applications of the optimization techniques in the power system engineering field.
2. Discuss the constrained optimization methods for power system economic dispatch and optimal power flow.
3. Recognize the negative impacts of the remote transmission of reactive power and how reactive compensation can mitigate them.
4. Describe the general formulation of the OPF and perform case studies that demonstrate the effectiveness of reactive power supply for the performance enhancement of power systems.
5. Investigate the effect of reactive power compensation on power system voltage stability.

1.2 Organization of the Thesis

Chapter 2 focuses on the fundamental concept of reactive power compensation. The difficulties of remote reactive power transmission are described and illustrated using a 2-Bus case study. The concept and benefit of shunt reactive power compensation techniques are discussed.

Chapter 3 presents the important background of optimization problems, which covers its definition, classification, conditions and algorithm. The classification, theories and features of general optimization techniques are briefly presented. Case studies discuss and illustrate classic economic dispatch problem by distinguishing it from a conventional power system optimization problem called optimal power flow (OPF). A brief review of OPF along with a case study on the 7-Bus power system is presented.

Chapter 4 starts with an observation of power system response when systems are overloaded. The single objective optimization method is applied to solve each objective of OPF during systems' stressed scenario. The additional reactive power supply is applied to perform a comparison of power system response. A stressed 6-Bus power system and a stressed 26-Bus power systemare used to perform these case studies and to illustrate the effectiveness of reactive power compensation on maintaining acceptable voltage profile.

Chapter 5 extends reactive power compensation study on voltage stability analysis. A 5-Bus system and a stressed IEEE 39-Bus system are employed to illustrate the effectiveness of reactive power compensation on ensuring adequate voltage stability margin.

Chapter 6 recaps and highlights the key contributions of the research presented in this thesis along with suggestions for future work.

Chapter 2

Reactive Power Transmission and Compensation

2.0 Introduction

The remote generation and transmission of reactive power from load demand has a strong negative impact on power system operations. This chapter presents an overview of the challenges associated with the transmission of reactive power over a power system network. It will first give a brief overview of reactive power in power systems in section 2.1. The derivation of generated, transmitted and consumed reactive power will be presented in section 2.2 based on an elementary transmission system model. Section 2.3 will give a fundamental understanding of shunt reactive power compensation and how it can be used to mitigate the negative effects of remote reactive power transmission. Section 2.4 concludes this chapter.

2.1 Reactive Power in Power Systems

It is recognized that the management of reactive power is fundamental to power systems [3]:

- The Source of Reactive Power

  "Power" refers to the energy-related quantities flowing in the distribution network. Instantaneously, power is the product of voltage and current. When voltage and current are not in phase, there are two components which is real or active power that is measured in Watts. Reactive power, referred to as imaginary number, is measured in VAR. The combination is complex power or apparent power. The term "power" normally refers to active power.

- The Need of Reactive Power

Reactive power is required to maintain the voltage to deliver active power (Watts) through transmission lines. Motor loads and other loads require reactive power to convert the flow of electrons into useful work. When there is not enough reactive power, the voltage sages down and it is not possible to push the power demanded by loads through the lines.

- The Importance of Reactive Power

Reactive power refers to circulating power in the grid that does not do useful work. It results from energy storage elements in the power grid (mainly inductors and capacitors). It also has a strong effect on system voltages. Besides that, it must have balance in the grid to prevent voltage problems. Lastly, reactive power levels have an effect on voltage.

- Reactive Power Limitation

Reactive Power does not travel very far. It is usually necessary to produce it close to the location where it is needed. A supplier or source close to the location of the need is in a much better position to provide reactive power versus one that is located far from the location of the need. Reactive power supplies are closely tied to the ability to deliver real or active power.

- Implementation of Reactive Power Control

Reactive power injections regulate and control voltage to a desired nominal value at the location of the injection. Reactive power control effects tend to be localized.

2.2 Power Transmission Using Elementary Models

The need for reactive power compensation of a transmission line can be seen by taking an elementary model shown in Figure 2.1.

Figure 2.1 Model of the Single Transmission Line

Figure 2.1 shows the simplified model of a power transmission system. Two buses are connected by a transmission line which is assumed lossless and represented by the reactance X. $E_s\angle\delta$ and $V_r\angle 0$ represent the sending end and receiving end voltages with voltage phase angle δ between the two. P_r and Q_r represent the active power and reactive power at the load bus. The current in the transmission line is given by [2]:

$$I = \frac{E_s cos\delta + jE_s sin\delta - V_r}{jX} \tag{2.1}$$

The apparent power at the receiving end can be calculated as:

$$S_r = P_r + jQ_r = V_r I^* = V_r\left[\frac{E_s cos\delta + jE_s sin\delta - V_r}{jX}\right]^* = \frac{E_s V_r}{X} sin\delta + j\left[\frac{E_s V_r cos\delta - V_r^2}{X}\right] \tag{2.2}$$

The active power and reactive power atthe load bus are given by:

$$P_r = \frac{E_s V_r}{X} sin\delta = P_{max} sin\delta, \quad Q_r = \frac{E_s V_r cos\delta - V_r^2}{X} \tag{2.3}$$

Similarly, the active power and reactive power at generator bus are given by:

$$P_s = \frac{E_s V_r}{X} sin\delta = P_{max} sin\delta, \quad Q_s = \frac{E_s^2 - E_s V_r cos\delta}{X} \tag{2.4}$$

Equations (2.1) through (2.4) indicate that the current flow, as well as the active and reactive power can be regulated by controlling the voltage magnitudes, phase angles and line impedance of the transmission system. From Equations (2.3) and (2.4), P_r and P_s have the same expression under the assumption of the lossless transmission line; the active power flow will reach the maximum when the phase angle δ is 90^o. In practice, a small angle, usually kept below 45^o, is used to keep the system stable from the transient and dynamic oscillations [14]. For small angles, use $cos\theta \cong 1$. $Q_r$and Q_s can be expressed as:

$$Q_r = \frac{V_r(E_s - V_r)}{X}, \quad Q_s = \frac{E_s(E_s - V_r)}{X} \tag{2.5}$$

Equation (2.5) states that reactive power transmission depends mainly on voltage magnitudes and flows from the highest voltage to the lowest voltage. Q and V are closely coupled. To better understand how the voltage has an impact on the reactive power, Q_s and Q_r can be calculated using Equations for an angle of 30^o. A substantial voltage gradient of 10% is between the two ends. E_s can be 1.00 p.u. and V_r can be 0.9 p.u..

$$Q_s = \frac{E_s^2 - E_s V_r cos\delta}{X} = \frac{0.22}{X}, \quad Q_r = \frac{V_r(E_s - V_r)}{X} = -\frac{0.03}{X} \tag{2.6}$$

The amount of reactive power goes into the transmission line; none of it comes out from the line at the receiving end. The negative value of the Q_r means that the transmission line demands $\frac{0.03}{X}$ from the load end.The transmission line becomes a drain on the transmission system. The reactive loss through the transmission line is the sum of the reactive power going into the line,

$\frac{0.25}{X}$ p.u.. Similar to the minimization of real power loss, reactive power loss should also be minimized for the sake of economic consideration. The reactive power loss is given by:

$$Q_{loss} = I^2 X = \bar{I} \cdot \bar{I}^* X = \left(\frac{P - jQ}{\bar{V}^*}\right)\left(\frac{P + jQ}{\bar{V}}\right) X = \frac{P^2 + Q^2}{V^2} X \tag{2.7}$$

The derivation shows that it is possible to minimize reactive power loss by keeping the load voltage high. The following two case studies are analyzed in PowerWorld Simulator.

2.2.1 Case Study of Reactive Power Transmission

Case 1 considers the lossless two bus system with base values (500 MVA and 735 KV) shown in Figure 2.2. It is given the values of 0.15 p.u. for the reactance of the transmission lines, and 2.5 p.u. for the shunt changing of the transmission lines with a customer load demand (1230 MW+j404 MVAr). It is assumed that the voltage at the generator bus is 1.0 per unit at zero degrees phase angle.

Figure 2.2 One Line Diagram of the 2-Bus Power System

The load demand (1230 MW+j404 MVAr) is varied gradually by each step *k* of 0.1 starting from 0 to 1.0. The corresponding receiving end voltages and transferred powerare shown in Table 2.1.

Table 2.1 Active Power Transfer vs. Receiving End Voltage

k	power (MW)	reactive power (MVAR)	Vr(p.u.)
0	0	0	1.23
0.1	123	40.4.	1.22
0.2	246	80.8	1.2
0.3	369	121.2	1.19
0.4	492	161.6	1.17
0.5	615	202	1.15
0.6	738	242.4	1.13
0.7	861	282.8	1.1
0.8	984	323.2	1.07
0.9	1107	363.6	1.04
1	1230	404	0.99

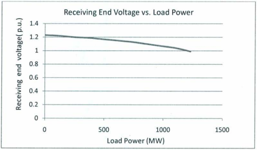

Figure 2.3 The Receiving End Voltage vs. the Transferred Power

With a slow increase in the system demand, a voltage characteristic at the load bus looks like the graph in Figure 2.3. PowerWorld Simulation [15] results demonstrate that as load increases, the load voltage drops.

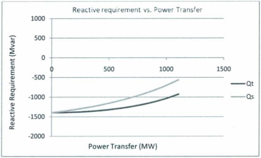

Figure 2.4 Reactive Consumption vs. Real Power Transmission

As the load gradually increases, the reactive power absorbed by transmission line (the black line) and reactive power generated by the generator (the grey line) are demonstrated in Figure 2.4.It indicates that the reactive power generated by the generator cannot supply the reactive power consumed by the transmission line. This shortage of reactive power supply is getting more severe as more load demand is required. The remote supply and transmission of reactive power from the generator to the load is difficult; the load voltage cannot be kept close to the nominal voltage 1.00 p.u.. At the pre-set maximum power transfer of 1230 MW, the generator can only supply half of the reactive power consumed by the transmission line. The reactive power absorbed by the transmission line starts to increase rapidly as the voltage at the load bus begins to drop. Figure 2.4 also reveals that if the reactive power support reaches the limit, the system will approach the maximum loading point or voltage collapse point. Thus, the increased load demand eventually leads to a shortage of reactive power and declined voltage. This phenomenon can also be seen from the plot of the voltage at the load end versus the power transferred. The plots in Figure 2.3 are referred to as the PV curve, which will be introduced in the later of the thesis.

The reason that the load bus voltage level decreases as load demand increases is that the transmission reactance has its own reactive power requirement for carrying load power demand. This reactive requirement comes in the form of I^2X losses, which depresses system voltages. In order to enhance the dropped voltage magnitude at the load while the system is still able to meet the load demand, the shortage of reactive power needs to be compensated.

Local reactive power compensation is a convenient and common method to control reactive power flow to meet the desired load voltage level. Most compensation devices come in the form of switching inductor or capacitor banks that are installed in parallel to various load centers throughout a power system. Their purpose is to supply or absorb reactive power to loads such that the generation and transmission systems are unburdened by load reactive power demand. While there are many forms of reactive power compensation devices such as shunt capacitors, synchronous generators, synchronous condensers or static reactive power generators with respect to loads, this thesis will focus strictly on capacitors and static VAR capacitors (SVC). The effectiveness of reactive power compensation on voltage variation is investigated in the following case study.

2.3 Shunt Compensation for Reactive Power Flow Control

Reactive power compensation is the common practice to stabilize and support the voltage in power systems and its provision was transitionally considered to be part of the duties of the system dispatching active power. The thesis proposes the provision of reactive power by the load that enables them to inject reactive current to support the voltage locally. As shown in the previous section, reactive power transmission has a negative impact on many aspects of power system operations. Without the proper control of reactive power, a power system can be forced to operate in ways that threaten the systems voltage and its efficiency. The major objectives that the control of reactive power must satisfy to achieve reliable and efficient power system operation are [16]:

- Bus voltages should be within an acceptable limit to ensure that all equipments connected to the buses are operating in the conditions for which they were designed.
- Reactive power flow is minimized to reduce both active and reactive losses over transmission systems. This will ensure existing transmission infrastructure is utilized more efficiently.
- Increase power system stability by utilizing the transmission system more effectively.

Shunt reactive power compensation is a convenient and common method to control reactive power flow to meet the mentioned objectives. Voltage and reactive power based on local operation of the voltage and reactive power control equipment are investigated in further detail in this section.

2.3.1 Case Study of Shunt Compensation

Case 2 assumes that reactive power compensation is provided at the load bus as shown in the diagram in Figure 2.5. The fictitious generator supplies reactive support at the load to keep the load voltage fixed at 1.00 p.u.; synchronous machines are indicated at both ends. The transmission line has an equivalent reactance 0.15 p.u., and shunt charging 2.5 p.u..

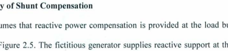

Figure 2.5 A Fictitious Generator Used to Supply Reactive Power

In order to stress the load bus, the load demand is gradually increased by the small step *k* of 0.1, from the full load 1230 MW to 3444 MW. PowerWorld Simulator determines the reactive power demanded at the load while meeting the load demand and ensuring voltage at the desired nominal level (1.00 p.u.) The phase angle difference between these two buses δ is also studied. The simulation results for each load level is recorded in Table 2.2.

Table 2.2 Power System Performance vs.Reactive Power Compensation

k	real power (MW)	reactive power (MVAR)	compensated (MVAR)	Delta (degrees)
1	1230	404	14	21.654
1.1	1353	444	106	23.948
1.2	1476	485	205	26.283
1.3	1599	525.43	309	28.666
1.4	1722	565.85	420	31.104
1.5	1845	606.267	538	33.607
1.6	1968	646.6848	665	36.185
1.7	2091	687.1	800	38.851
1.8	2214	727.52	944	41.621
1.9	2337	767.94	1099	44.515
2	2460	808.356	1267	47.561
2.1	2583	848.77	1450	50.796
2.2	2706	889.19	1651	54.272
2.3	2829	929.61	1875	58.071
2.4	2952	970	2130	62.326
2.5	3075	1010.445	2432	67.297
2.6	3198	1050.4	2820	73.637
2.7	3321	1090.8	3511	85.043
2.8	3444	1131.2	4022	90

It is seen in Figure 2.6 that the compensated reactive power is dramatically increased to meet the gradually increased load demand and to maintain the voltage magnitude at a constant level of 1.00 p.u.. The plot of reactive power requirement versus power transfer graphically appears as below.

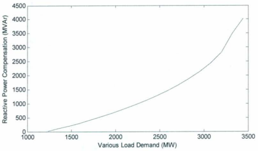

Figure 2.6 Reactive Power Compensation vs. Power Transfer

The plot indicates initially no reactive power injection at the full load. To stabilize the constant load voltage, reactive power injection is correspondingly required to support the increased power transfer in the system. The maximum 3444 MW can be carried by the system when the extra 4022 MVAr reactive power is locally demanded. Further, the 2890.8 MVAr reactive power produced by the generator is injected into the system to ease the transmission line requirements for transport of the required active power as well as to reduce the generator's reactive power output. The effect of this local injection of reactive power keeps load bus voltage constant as 1.00 p.u. and the increased active load demand can be met without any dramastic changes in current generator and line configurations. Thus, reactive compensation has a direct economic and voltage stability benefit to a transmission system. Additionally, the first extra Qc support the system to carry the extra 50% active power; the second extra 500 MVAr only enables the system to transfer 30% more active power. The reactive power injection does not linearly scale in proportion to active power transfer.

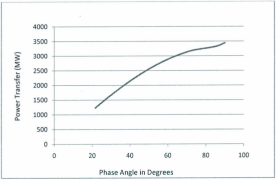

Figure 2.6 Power Transfer vs. Power Angle

The observation shows that the transmitted power can be significantly increased, the peak point shifts from 20^o to 90^o, and the maximum power transfer occurs at 90^o of power angle. Both derivation and numerical cases revealed the fact that reactive power is difficult to transmit across large power angles even with substantial voltage magnitude gradients. High angles are due to long lines and high real power transfer.

The main purpose of reactive power support is to increase system active power transfer capability, while at the same time maintaining a certain desired voltage profile if it is assumed that only power transfer capability and voltage stability are a concern. However, the sources of reactive power compensation are diverse. Shunt capacitors are the simplest and most widely used form of reactive power compensation due to the low implementation and equipment cost; it is considered to be an efficient method to guarantee the performance of overloaded power systems.

2.4 Summary

The chapter has discussed the basic foundations on the difficulties of reactive power transmission. The study presented showed that transmitting reactive power through transmission lines affects power system performance. It is necessary to produce it close to the location where it is needed. It was also shown that system bus voltages through a system are negatively impacted by remote reactive power transmission. A simple 2-Bus power system was used to illustrate the concepts regarding reactive power.

An effective and highly used approach to increase the efficiency of a power system was described. The use of local reactive power compensation devices that can greatly increase the ability of a power system to meet a wide variety of load demands while ensuring the system works within a specified voltage profile. This is an attractive option to power system planners as the costs associated with it are significantly lower than installing new transmission or generation systems to satisfy the increasing power demand.

Chapter 3

Application of Optimization Methods in Power Systems

3.0 Introduction

Optimization is the process of determining the best results or methods from a set of alternatives under certain given circumstances. It has been wildly applied to the engineering field to define economical reliable, secure, efficient systems as well as to devise plans and procedures to improve the operation of the existing systems. For example, the real and reactive power provided by the generators can be adjusted within certain limits to meet the desired load demand with minimum fuel cost. This is also known as Optimal Power Flow (OPF). It can be achieved by minimizing the objective function, which the total fuel cost of the generating units, subject to the constraints that the sum of the powers generated must equal to the sum of the transmission loss and the power consumed by the load. Economic dispatch is a special case of OPF, which neglects the transmission limits.

This chapter defines the concept of engineering optimization problems and some associated applications. Section 3.1 presents a general review of optimization methods. Section 3.2 focuses on the economic dispatch. It starts with introducing the theory of economic dispatch and typical economic dispatch problems. The economic dispatch of generation for minimization of the total operating cost neglecting and including transmission loss is presented and discussed. Section 3.3 gives a fundamental understanding of OPF problems. The formulation and conventional algorithms of OPF are introduced. OPF problems for minimizing the fuel cost and minimizing the transmission loss are demonstrated through the 7-Bus Power System. A conclusion of this chapter is given in section 3.4.

3.1 Optimization Formulation of Problems

3.1.1 Statement of an Optimization Problem

Optimization is defined as the process of finding the conditions that give the maximum or minimum value of a function. In the simplest cases, optimization means solving problems in which one seeks to minimize or maximize an objective function by systematically choosing the values of variables within certain constraints [27-29]. Mathematical programming techniques are useful in finding the maximum or minimum of a function of several variables under a prescribed set of constraints. An optimization or a mathematical programming problem can be stated as follows.

Find
$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, whichoptimize f(x) \tag{3.1}$$

Subject to the constraints

$$g_j(x) \leq 0, j = 1,2, \dots, m \tag{3.2}$$

$$h_k(x) = 0, k = 1,2, \dots, p \tag{3.3}$$

$$x_i^{max} \leq x_i \leq x_i^{min}, i = 1,2, \dots, n \tag{3.4}$$

Where

x is an n-dimensional vector called the design vector.

$f(x)$ is termed the objective function.

$g_j(x)$ and $h_k(x)$ are known as inequality and equality constraints.

x_i^{max} and x_i^{min} are an upper and lower bound.

The number of variables n and the number of constraints m and/or p need not be related in any way. The problem stated in Equation (3.1) is called a constrained optimization problem. Some optimization problems that do not involve any constraints are called unconstrained optimization problems.

3.1.1.1 Design Vector

Any engineering system or component is defined by a set of quantities some of which are viewed as variables during the design process. Certain quantities are usually fixed at the outset and these are called pre-assigned parameters. All the other quantities are treated as variables in the design process and are called design or decision variables x_i. The design variables are collectively represented as a design vector x [27]. The goal is to find the x value that satisfies some criterion. If there are several values of x that satisfy the criterion, then $x_1, x_2, \ldots, x_n$ are used to distinguish them [28].

3.1.1.2 Design Constraints

In many practical problems, the design variables cannot be chosen arbitrarily. They have to satisfy certain specified requirements. The restrictions that must be satisfied to produce an acceptable design are collectively called design constraints [27]. In a power system, constraints that represent limitations on the behavior or performance of the system are termed behavior or functional constraints. Optimization problems applied in power operation systems will be illustrated in the optimal power flow problems.

3.1.1.3 Objective Function

The conventional design procedures aim at finding an acceptable or adequate design which satisfies the requirements of the problems. The purpose of optimization is to choose the best one of the many acceptable designs available. Thus a criterion has to be chosen for comparing the different alternative acceptable designs and for selecting the best one. The criterion with respect to which the design is optimized, when expressed as a function of the design variables, is known as the objective function. The choice of the objective function is governed by the nature of problem [27, 29].

3.1.2 Classification of Optimization Problems

Optimization problems can be classified in several ways based on different criterion, such as the existence of constraints, the nature of the design variables, the physical structure of the problem, the nature of the equations involved, the permissible values of the design valuables and so on. From the computational point of view, the classification based on the nature of the equations involved is extremely useful, since there are many specific methods available for the efficient solution of a particular class of problems [27]. Classification of optimization problems is based on the nature of expressions for the objective functions and constraints. According to this classification, optimization problems can be classified as linear, nonlinear, and quadratic programming problems [28].

- **Linear**

If the objective function and all the constraints are linear functions of the design variables, the mathematical programming problem is called a linear programming (LP) problem. A linear program is often stated in the following standard form:

Find $$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \text{ which optimize } f(x) = \sum_{i=1}^{n} c_i x_i \quad (3.5)$$

Subject $$\sum_{i=1}^{n} a_{ij} x_i = b_j, j = 1, 2, \ldots, m \quad (3.6)$$

$$x_i \geq 0, i = 1, 2, \ldots, n \quad (3.7)$$

Where c_i, a_{ij}, and b_j are constants.

- **Quadratic**

A Quadratic programming problem is a nonlinear programming problem with a quadratic objective function and linear constraints. It is usually formulated as follows:

Find $$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \text{ which optimize } f(x) = c + \sum_{i=1}^{n} q_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} Q_{ij} x_i x_j \quad (3.8)$$

Subject $$\sum_{i=1}^{n} a_{ij} x_i = b_j, j = 1, 2, \ldots, m \quad (3.9)$$

$$x_i \geq 0, i = 1, 2, \ldots, n \quad (3.10)$$

Where c, q_i, Q_{ij}, a_{ij} and b_j are constants.

- **Nonlinear**

If any of the functions among the objective and constraint functions is nonlinear, the problem is called a nonlinear programming (NLP) problem. This is the most general programming problem and all other problems can be considered as special cases of the NLP problem.

3.1.3 Application of Optimization in Power Systems

This section briefly summaries the application of optimization techniques in the power system.

- Linear and quadratic programming methods are used to solve power systems problems with regards to optimal power flow, load flow, reactive power planning and active and reactive power dispatch [20, 39].
- Nonlinear programming method has been applied to various areas of power system for optimal power flow, security constrained optimal power flow and hydrothermal scheduling [19, 51, 52].
- Integer and Mixed-Integer Programming method is employed to solve power system problems with regards to optimal reactive power planning, power system planning, unit commitment, and generation scheduling [52].
- Dynamic Programming method has been applied to various areas of power systems such as reactive power control, transmission planning and unit commitment [52].

3.2 Optimal Dispatch of Generation

Usually the generating stations are located hundreds of kilometers away from load centers and their fuel cost is different. Also, under normal operating conditions, the generation capacity is more than the total load demand and loss. Thus, there are many options for scheduling and planning generation [18]. Economical Dispatch (ED) is the most common and simple way to optimize generation of the electricity without considering the voltage deviation and transmission limits.

3.2.1 Operating Cost of a Thermal Plant

The majority of generators in power systems are of three types – nuclear, hydro, and fossil (coal, oil, and gases). Nuclear plants tend to be operated at constant output levels and hydro plants have essentially no variable operating cost. Thus, the components of cost that fall under the category of dispatching procedures are the cost of the fuel burnt in the fossil plants [19]. The factors influencing power generation at minimum cost are operating efficiencies of generators, fuel cost, and transmission loss. The most efficient generator in the system does not guarantee minimum cost as it may be located in an area where fuel cost is high. Also, if the plant is located far from the load center, transmission loss may be considerably higher, which causes the plant to operate in an uneconomic fashion. Therefore, the problem is to determine the generation of different plants, such that the total operating cost is minimized. Various situations are discussed to illustrate how the operating cost plays an important role in the economic scheduling.

In all practice cases, the fuel cost of generator ith fossil plant can be expressed in a quadratic function form of real power generation [18, 20, 24].

$$C_i = a_i + b_i P_i + c_i P_i^2 \tag{3.11}$$

Where

a, b and c are cost coefficients, and i is the total number of generating units.

3.2.2 Economic Dispatch Neglecting loss and No Generator Limits

The simplest economic dispatch problem is the case when transmission line loss is neglected. The model in Figure 3.1 assumes that the system has only one bus and all generation and loads are connected to it.

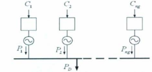

Figure 3.1 Plants Connected to a Common Bus (taken from [22])

Since transmission loss is neglected, the total demand P_D is the sum of all generation. A cost function C_i is assumed to be known for each plant. The problem is to find the real power generation for each plant such that the total production cost is minimized. The minimum total production cost is defined by the objective function [20]:

$$C_t = \sum_{i=1}^{n_g} C_i = \sum_{i=1}^{n} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \quad (3.12)$$

Subject to the constraint

$$\sum_{i=1}^{n_g} P_i = P_D \quad (3.13)$$

Where

C_t is total production cost.

C_i is the production cost of ith plant.

P_i is the generation of ith plant.

P_D is the total load demand.

n_g is the total number of dispatchable generating plants.

A typical approach is to augment the constraints into objective function by using the Lagrange multipliers.

$$\pounds = C_t + \lambda(P_D - \sum_{i=1}^{n_g} P_i) \tag{3.14}$$

The minimum of this unconstrained function is found at the point where the partials of the function to its variables are zero.

$$\frac{\partial \pounds}{\partial P_i} = 0 \tag{3.15}$$

$$\frac{\partial \pounds}{\partial \lambda} = 0 \tag{3.16}$$

The first condition given by Equation (3.15), results in

$$\frac{\partial C_t}{\partial P_i} = \lambda(0 - 1)$$

$$\frac{\partial C_t}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda$$

Therefore, the condition for optimum dispatch is

$$\frac{dC_i}{dP_i} = \lambda i = 1, \ldots, n_g \tag{3.17}$$

The second condition given by Equation (3.16), results in

$$\sum_{i=1}^{n_g} P_i = P_D \tag{3.18}$$

Equation (3.18) is the equality constraint that was to be imposed. In conclusion, when loss is neglected with no generator limits, for most economic operations, all plants must operate at equal incremental production cost while satisfying the equality constraint given by Equation (3.18).

3.2.2.1 Economic Dispatch Case Study 1

For the 7-Bus Power System, the fuel cost for each plant in B7FLAT power system [15]:

$$C_1=761.94+2.04*(7.62*P_1+0.0013*P_1^2)$$

$$C_2=831.84+2.061*(7.52*P_2+0.00136*P_2^2)$$

$$C_3=530.03+2.093*(7.84*P_3+0.00134*P_3^2)$$

$$C_4=831.92+2.139*(7.57*P_4+0.00131*P_4^2)$$

$$C_5=500.08+2.574*(7.77*P_5+0.00194*P_5^2)$$

According to Equation (3.17), the minimum total operation cost occur when

$$\frac{dC_1}{dP_1}=\frac{dC_2}{dP_2}=\frac{dC_3}{dP_3}=\frac{dC_4}{dP_4}=\frac{dC_5}{dP_5}=\lambda$$

Incremental production cost for each plant

$$15.5448+0.005304P_1=\lambda$$

$$15.49872+0.00560592P_2=\lambda$$

$$16.40912+0.00560924P_3=\lambda$$

$$16.19223+0.00560418P_4=\lambda$$

$$19.99998+0.00998712P_5=\lambda$$

$$P_1+P_2+P_3+P_4+P_5=765.6$$

In matrix form

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0.0053 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0.0056 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0.0056 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0.0056 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0.01 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ -\lambda \end{bmatrix} = \begin{bmatrix} 765.6 \\ -15.5448 \\ -15.4987 \\ -16.4091 \\ -16.1922 \\ -20.0000 \end{bmatrix}$$

Solutions,

$$P_1=337.1492(\text{MW})$$

P_2=327.2111(MW)

P_3=164.7138(MW)

P_4=203.5640(MW)

P_5=-267.0380(MW)

λ=17.3330($/MWhr)

The power output of any generator should not exceed its rating nor should it be below that which is necessary for stable boiler operation. The calculated negative power output of generation plant 5 is against the real power generation situation. Thus, the generations are restricted to be within given minimum and maximum limits.

3.2.3 Economic Dispatch Including Loss and Generator Limits

When transmission distances are very small and load density is very high transmission loss may be neglected and the optimal dispatch of generation is achieved with all plants operating at equal incremental production cost. However, the common practice for including the effect of transmission loss is to express the total transmission loss as a quadratic function of the generation power outputs. The quadratic form can be stated as [20, 21, 26]

$$P_L = \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} P_i B_{ij} P_j + \sum_{i=1}^{n_g} B_{0i} P_i + B_{00} \tag{3.19}$$

Where B_{ij} is loss coefficients or B-coefficients, which are assumed constant.

Using the Lagrange multiplier and adding additional terms to include the inequality constraints, the equation can be generated

$$\mathcal{L} = C_t + \lambda\left(P_D + P_L - \sum_{i=1}^{n_g} P_i\right) + \sum_{i=1}^{n_g} \mu_{i(max)}(P_i - P_{i(max)}) + \sum_{i=1}^{n_g} \mu_{i(min)}(P_i - P_{i(min)}) \tag{3.20}$$

If the constraint is not violated, its associated μ variable is zero and the corresponding term does not exist, which means $\mu_{i(min)}$ =0 when $P_i > P_{i(min)}$; $\mu_{i(max)}$=0 when $P_i < P_{i(max)}$. The constraint only becomes active when violated. The minimum of this unconstrained function is found at the point where the partials of the function to its variables are zero.

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0 \quad (3.21)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad (3.22)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{i(min)}} = P_i - P_{i(min)} = 0 \quad (3.23)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{i(max)}} = P_i - P_{i(max)} = 0 \quad (3.24)$$

The first condition, given by Equation (3.21), results in

$$\frac{\partial C_t}{\partial P_i} + \lambda\left(0 + \frac{\partial P_L}{\partial P_i} - 1\right) = 0$$

Since

$$C_t = C_1 + C_2 + C_3 \cdots + C_{n_g}$$

Then

$$\frac{\partial C_t}{\partial P_i} = \frac{dC_i}{dP_i}$$

Therefore, the condition for optimum dispatch is

$$\frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda i = 1, \dots, n_g \quad (3.25)$$

The term $\frac{\partial P_L}{\partial P_i}$ is known as the incremental transmission loss.

The second condition, given by Equation (3.22), results in

$$\sum_{i=1}^{n_g} P_i = P_D + P_L \quad (3.26)$$

Equation (3.26) is the equality constraint that was to be imposed. Equation (3.25) is rearranged as

$$\left(\frac{1}{1-\frac{\partial P_L}{\partial P_i}}\right)\frac{dC_i}{dP_i} = \lambda i = 1, \dots, n_g \tag{3.27}$$

$$L_i = \frac{1}{1-\frac{\partial P_L}{\partial P_i}} \tag{3.28}$$

The effect of transmission loss is to introduce a penalty factor L_i of plant i. Equation (3.25) shows that the minimum cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for all plants.

The incremental production cost is given by Equation (3.11), and the incremental transmission loss is obtained from the loss formula Equation (3.19), which yields

$$\frac{\partial P_L}{\partial P_i} = 2\sum_{j=1}^{n_g} B_{ij}P_j + B_{0i} \tag{3.29}$$

Substituting the expression for the incremental production cost and the incremental transmission loss in Equation (3.25) results in

$$\beta_i + 2\gamma_i P_i + 2\lambda\sum_{j=1}^{n_g} B_{ij}P_j + B_{0i}\lambda = \lambda \text{or} \left(\frac{\gamma_i}{\lambda} + B_{ij}\right)P_i + \sum_{\substack{j=2 \\ j\neq i}}^{n_g} B_{ij}P_j = \frac{1}{2}\left(1 - B_{0i} - \frac{\beta_i}{\lambda}\right) \tag{3.30}$$

Extending Equation (3.21) to all plants results in the following linear equations in matrix form

$$\begin{bmatrix} \frac{\gamma_1}{\lambda} + B_{11} & B_{12} & \cdots & B_{1n_g} \\ B_{21} & \frac{\gamma_2}{\lambda} + B_{22} & \cdots & B_{2n_g} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n_g1} & B_{n_g2} & \cdots & \frac{\gamma_{in_g}}{\lambda} + B_{1n_gn_g} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 - B_{01} - \frac{\beta_1}{\lambda} \\ 1 - B_{02} - \frac{\beta_2}{\lambda} \\ \vdots \\ 1 - B_{0n_g} - \frac{\beta_{n_g}}{\lambda} \end{bmatrix} \tag{3.31}$$

or in short term

$$EP = D \tag{3.32}$$

To find the optimal dispatch for an estimated value of $\lambda^{(1)}$, the simultaneous linear equation given by Equation (3.32) is solved. The iterative process is continued using the gradient method. To do this, from Equation (3.31), P_i at the kth iteration is expressed as

$$P_i^{(k)} = \frac{\lambda^{(k)}(1-B_{0i})-\beta_i-2\lambda^{(k)}\sum_{j\neq i}B_{ij}P_j^{(k)}}{2(\gamma_i+\lambda^{(k)}B_{ii})} \tag{3.33}$$

Substituting for P_i from Equation (3.32) in Equation (3.26) results in

$$\sum_{i=1}^{n_g} \frac{\lambda^{(k)}(1-B_{0i})-\beta_i-2\lambda^{(k)}\sum_{j\neq i}B_{ij}P_j^{(k)}}{2(\gamma_i+\lambda^{(k)}B_{ii})} = P_D + P_L^{(k)} \tag{3.34}$$

or

$$f(\lambda)^{(k)} = P_D + P_L^{(k)} \tag{3.35}$$

Expanding the left-hand side of the above equation in Taylor's series about an operating point$(\lambda)^{(k)}$, and neglecting the higher-order terms results in

$$f(\lambda)^{(k)} + \left(\frac{df(\lambda)}{d\lambda}\right)^{(k)} \Delta\lambda^{(k)} = P_D + P_L^{(k)} \tag{3.36}$$

Or

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}} = \frac{\Delta P^{(k)}}{\sum\left(\frac{dP_i}{d\lambda}\right)^{(k)}} \tag{3.37}$$

Where

$$\sum_{i=1}^{n_g}\left(\frac{\partial P_i}{\partial\lambda}\right)^{(k)} = \sum_{i=1}^{n_g} \frac{\gamma_i(1-B_{0i})+B_{ii}\beta_i-2\gamma_i\sum_{j\neq i}B_{ij}P_j^{(k)}}{2(\gamma_i+\lambda^{(k)}B_{ii})^2} \tag{3.38}$$

Therefore,

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)} \tag{3.39}$$

Where

$$\Delta P^{(k)} = P_D + P_L^{(k)} - \sum_{i=1}^{n_g} P_i^{(k)} \tag{3.40}$$

The process is continued until $\Delta P^{(k)}$ is less than a specified accuracy. If an approximate loss formula expressed by

$$P_L = \sum_{i=1}^{n_g} B_{ii}P_i^2 \tag{3.41}$$

$B_{ij} = 0$, $B_{00} = 0$ and solution of the simultaneous equation given by Equation (3.33) reduces to the following simple expression

$$P_i^{(k)} = \frac{\lambda^{(k)} - \beta_i}{2(\gamma_i + \lambda^{(k)} B_{ii})} \tag{3.42}$$

and Equation (3.38) reduces to

$$\sum_{i=1}^{n_g} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(k)} = \sum_{i=1}^{n_g} \frac{\gamma_i + B_{ii}\beta_i}{2(\gamma_i + \lambda^{(k)} B_{ii})^2} \tag{3.43}$$

3.2.3.1 Economic Dispatch Case Study 2

Consider the 6-Bus power system to illustrate optimal dispatch of generation considering loss and generator limits [20]. The fuel cost in $/h of three thermal plants of a power system are:

$$C_1 = 200 + 7.0P_1 + 0.008P_2^2\,(\$/h)$$

$$C_2 = 180 + 6.3P_2 + 0.009P_2^2\,(\$/h)$$

$$C_3 = 140 + 6.8P_3 + 0.007P_3^2\ (\$/h)$$

P_1, P_2, and P_3 are in MW. Plant outputs are subject to the following limits as:

$$10(MW) \leq P_1 \leq 85(MW)$$

$$10(MW) \leq P_2 \leq 80(MW)$$

$$10(MW) \leq P_3 \leq 70(MW)$$

Assume the real power loss is given by the simplified expression as:

$$P_{L(pu)} = 0.0218P_{1(pu)}^2 + 0.0228P_{2(pu)}^2 + 0.0179P_{3(pu)}^2$$

The loss coefficients are specified in per unit on a 100 MVA base. The optimal dispatch of generation is determined when the total system load is 150 MW.

In the cost function P_i is expressed in MW. Therefore, the real power loss in terms of MW generation is

$$P_L = \left[0.0218\left(\frac{P_1}{100}\right)^2 + 0.0228\left(\frac{P_2}{100}\right)^2 + 0.0179\left(\frac{P_3}{100}\right)^2\right] \times 100(MW)$$

$$= 0.000218P_1^2 + 0.000228P_2^2 + 0.000179P_3^2(MW)$$

For the numerical solution using the gradient method, assume the initial value of $\lambda^{(1)} = 8.0$.

From coordination equations, given by Equation (3.33),

$$P_1^{(1)} = \frac{8.0 - 7.0}{2(0.008 + 8.0 \times 0.000218)} = 51.3136\ (MW)$$

$$P_2^{(1)} = \frac{8.0 - 6.3}{2(0.009 + 8.0 \times 0.000228)} = 78.5292\ (MW)$$

$$P_3^{(1)} = \frac{8.0 - 6.8}{2(0.007 + 8.0 \times 0.000179)} = 71.1575\ (MW)$$

The real power loss is

$$P_L{}^{(1)} = 0.000218(51.3136)^2 + 0.000228(78.5292)^2 + 0.000179(71.1575)^2 = 2.886$$

Since $P_D = 150\ MW$, the error $\Delta P^{(1)}$ from (3.31) is

$$\Delta P^{(1)} = 150 + 2.8864 - (51.3136 + 78.5292 + 71.1575) = -48.1139$$

From Equation (3.34)

$$\sum_{i=1}^{3}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(1)} = \frac{0.008 + 0.000218 \times 7.0}{2(0.008 + 8.0 \times 0.000218)^2} + \frac{0.009 + 0.000228 \times 6.3}{2(0.009 + 8.0 \times 0.000228)^2}$$

$$+ \frac{0.007 + 0.000179 \times 6.8}{2(0.007 + 8.0 \times 0.000179)^2} = 152.4924$$

From Equation (3.28)

$$\Delta\lambda^{(1)} = \frac{-48.1139}{152.4924} = -0.31552$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 8.0 - 0.31552 = 7.6845$$

Continuing the process, for the second iteration,

$$P_1^{(2)} = \frac{7.6845 - 7.0}{2(0.008 + 7.6845 \times 0.000218)} = 35.3728(MW)$$

$$P_2^{(2)} = \frac{7.6845 - 6.3}{2(0.009 + 7.6845 \times 0.000228)} = 64.3821(MW)$$

$$P_3^{(2)} = \frac{7.6845 - 6.8}{2(0.007 + 7.6845 \times 0.000179)} = 52.8015(MW)$$

$$P_L{}^{(2)} = 0.000218(35.3728)^2 + 0.000228(64.3821)^2 + 0.000179(52.8015)^2 = 1.717$$

Since $P_D = 150\ MW$, the error $\Delta P^{(2)}$ from Equation (3.31) is

$$\Delta P^{(2)} = 150 + 1.7169 - (35.3728 + 64.3821 + 52.8015) = -0.8395$$

From Equation (3.34)

$$\sum_{i=1}^{3} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(2)} = \frac{0.008 + 0.000218 \times 7.0}{2(0.008 + 7.684 \times 0.000218)^2} + \frac{0.009 + 0.000228 \times 6.3}{2(0.009 + 7.684 \times 0.000228)^2}$$

$$+ \frac{0.007 + 0.000179 \times 6.8}{2(0.007 + 7.6845 \times 0.000179)^2} = 154.588$$

From Equation (3.28)

$$\Delta\lambda^{(2)} = \frac{-0.8395}{154.588} = -0.005431$$

Therefore, the new value of λ is

$$\lambda^{(3)} = 7.6845 - 0.005431 = 7.679$$

For the third iteration,

$$P_1^{(3)} = \frac{7.679 - 7.0}{2(0.008 + 7.679 \times 0.000218)} = 35.0965(MW)$$

$$P_2^{(3)} = \frac{7.679 - 6.3}{2(0.009 + 7.679 \times 0.000228)} = 64.1369(MW)$$

$$P_3^{(3)} = \frac{7.679 - 6.8}{2(0.007 + 7.679 \times 0.000179)} = 52.4834(MW)$$

The real power loss is

$$P_L{}^{(3)} = 0.000218(35.0965)^2 + 0.000228(64.1369)^2 + 0.000179(52.4834)^2 = 1.699$$

Since $P_D = 150\ MW$, the error $\Delta P^{(3)}$ from Equation (3.31) is

$$\Delta P^{(3)} = 150 + 1.6995 - (35.0965 + 64.1369 + 52.4834) = -0.01742$$

From Equation (3.34)

$$\sum_{i=1}^{3} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(3)} = \frac{0.008 + 0.000218 \times 7.0}{2(0.008 + 7.679 \times 0.000218)^2} + \frac{0.009 + 0.000228 \times 6.3}{2(0.009 + 7.679 \times 0.000228)^2}$$

$$+ \frac{0.007 + 0.000179 \times 6.8}{2(0.007 + 7.679 \times 0.000179)^2} = 184.625$$

From Equation (3.28)

$$\Delta\lambda^{(3)} = \frac{-0.01742}{184.624} = -0.0001127$$

Therefore, the new value of λ is

$$\lambda^{(4)} = 7.679 - 0.0001127 = 7.6789$$

Since $\Delta\lambda^{(3)}$, is small the equality constraint is met in four iterations, and the optimal dispatch for $\lambda = 7.6789$ are

$$P_1^{(4)} = \frac{7.6789 - 7.0}{2(0.008 + 7.679 \times 0.000218)} = 35.0907(MW)$$

$$P_2^{(4)} = \frac{7.6789 - 6.3}{2(0.009 + 7.679 \times 0.000228)} = 64.131(MW)$$

$$P_3^{(4)} = \frac{7.6789 - 6.8}{2(0.007 + 7.679 \times 0.000179)} = 52.4767(MW)$$

The real power loss is

$$P_L{}^{(4)} = 0.000218(35.0907)^2 + 0.000228(64.1317)^2 + 0.000179(52.4767)^2 = 1.699$$

The total fuel cost is

$$C_t = 200 + 7.0(35.0907) + 0.008(35.0907)^2 + 180 + 6.3(64.1317) + 0.009(64.1317)^2$$
$$+ 140 + 6.8(52.4767) + 0.007(52.4767)^2 = 1{,}592.65(\$/hr)$$

Matlab Optimization Tool box [15] isused to simplify the calculation, determine real power output of each generator unit, and the total production cost in order to minimize total cost.

3.3 Optimal Power Flow

Economic dispatch is a special case of the OPF, which neglects the transmission limits. In practical utilities, power systems are subject to the constraints that the sum of the power generated must equal to the sum of the transmission loss and the power consumed by the load. Under normal operating conditions, the generation capacity is more than the total load demand and loss. Thus, there are many options for scheduling and planning generation [18]. This means that the real and reactive power provided by the generators can be adjusted within certain limits to meet the desired load demand with minimum fuel cost. This is also known as OPF. OPF is an extension of the conventional economic dispatch to determine the optimal settings for control variables while respecting various constraints [18, 27]. OPF provides a useful support to the operatorto overcome many difficulties in the real time control and operation planning of power systems [28-30]. It optimizes the generation while enforcing the transmission line, which overcomes the drawback of the economic dispatch solution.

3.3.1 Mathematical Formulation of Optimal Power Flow Problems

OPF problems can be formed as a set of nonlinear equations, which can be stated as [6, 18]:

$$\text{Find the vectors } x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

Which minimizing or maximizing:

$$f_1(x,u), f_2(x,u), \dots f_n(x,u) \tag{3.44}$$

Subject to:

$$g(x,u) = 0 \tag{3.45}$$

$$h(x,u) \leq 0 \tag{3.46}$$

$$(x,u)^{min} \leq (x,u) \leq (x,u)^{max} \tag{3.47}$$

It can be described as minimizing the general objective function $f_n(x,u)$ while satisfying the constraints $g(x,u) = 0$ and $h(x,u) \leq 0$, where $g(x,u)$ represents nonlinear equality constraints (power flow equations) and $h(x,u)$ is nonlinear inequality constraints (transmission line limits) on the vector x and u. The vector x contains the dependent variables including bus voltage magnitudes and phase angles and the power output of generators designed for bus voltage control. The vector x also includes fix parameters, such as the slack bus angle, non-controlled generators' active power output and reactive power output, non-controlled load on fixed voltage, line parameters and so on. The vector consists of control variables involving [6, 55]:

- Active and reactive power generation
- Phase shifter angles
- Load MW and MVAR (load shedding)
- DC transmission line flows
- Control voltage settings
- LTC transformer tap setting
- Line switching

Typical goal of OPF problems are minimization of the total fuel cost, minimization of the active power loss, and minimization of the load voltage deviation.

3.3.2 Objective Functions

The OPF problems capture both technique and economic aspects of power operation system. The optimization problem of power system performance can be examined by solving the following objective functions.

- Minimization of the transmission loss: the active power transmission loss can be translated into the difference between the generated power and the consumed power.
- Minimization of the total fuel cost: the most efficient and low cost operation of a power system is determined by dispatching the available electricity generation resources to supply the load on the system.
- Minimization of the load voltage deviation: the quality of power service is directly related to the difference between the actual voltage profile and a desired voltage profile.

3.3.2.1 Active Power Loss

The active power loss is also known as the transmission loss, which is a form of wasted power and must be minimized for economic purposes. Transmission loss minimization can be expressed as minimization of difference between active power outputs of generators and active power load demands. The loss expression also can be expressed as voltage polar forms. The mathematical formulation for transmission loss minimization is adopted by the more straightforward expression as the difference between generation power and load demands, as shown below [3, 38]:

$$P_{loss}(\text{x, u}) = \sum_{i=1}^{N_G} P_{Gi}(\text{x, u}) - \sum_{i=1}^{N_L} P_{Li} \tag{3.48}$$

Where

P_{Gi} is the active power output of the generator at the ith bus.

P_{Li} is the active power consumed at the ith load.

Gi is the generator at the ith bus.

Li is the load at the ith bus.

N_G is the number of the generator buses.

N_L is the number of the load buses.

3.3.2.2 Total Fuel Cost

The total fuel cost of system generator units is derived from the fuel cost of thermal plants, which can be expressed as:

$$F = \sum_{i=1}^{N_G} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \tag{3.49}$$

Where

a_i, b_i and c_i are the cost coefficients of the ith generator.

P_{Gi} is the active power output of the generator at ith bus.

N_G is the number of the generator buses.

3.3.2.3 Voltage Deviation

Voltage deviation (VD) refers to the deviation of load bus voltages from their nominal value (1.00 p.u.). Reactive power transfer is highly dependent on system bus voltage levels (as discussed in Chapter 2). By keeping load bus voltages close to their nominal values, less reactive power will be transferred to each load bus in the system. This has the effect of reducing line currents which also reduces the power loss I^2R. The I^2R loss as a form of wasted power has a strong financial impact. Additionally, power systems that have their load bus voltages close to their nominal values are more resilient to voltage instability scenarios due to unforeseen contingencies such as a line outage. The calculation of the load bus voltage deviation, $V_{dev,}$ used in this thesis is given by the following expression [4, 40]:

$$V_{dev} = \sum_{i=1}^{N_L} (V_i - V_i^*)^2 \tag{3.50}$$

Where

V_i is the actual voltage magnitude at the ith bus.

V_i^* is the desired voltage magnitude at the ith bus.

N_L is the number of load buses contained in the system.

3.3.3 Control Variables

Control variables are usually independent variables in the OPF problems, which have an impact on the system performance and objective functions, such as active and reactive power outputs of generation units and generator. Dependent variables known as state variables include load voltage magnitudes and phase angles.

3.3.4 Operational Constraints

Constraints contained within the OPF are put in place to ensure that the solutions obtained by solving the OPF are feasible for practical power system operations. This section discusses typical operational constraints used in the OPF. System operating constraints include equality and inequality constraints.

3.3.4.1 Equality Constraints

The equality constraints are represented by power flow equations seen in Equation (3.51) and (3.52). These equations define the physical link between scheduled generation and load demand and cannot be violated as they define the conditions of state variables for a given system operation point.

$$P_{Gi} - P_{Li} - \sum_{j=1}^{Nbus} |\overline{V}_i||\overline{V}_j||Y_{ij}|cos(\theta_{ij} - \delta_i + \delta_j) = 0 \quad i = 1, \cdots, N \tag{3.51}$$

$$Q_{Gi} - Q_{Li} + Q_{ci} + \sum_{j=1}^{Nbus} |\overline{V}_i||\overline{V}_j||Y_{ij}|sin(\theta_{ij} - \delta_i + \delta_j) = 0 \quad i = 1, \cdots, N_L \tag{3.52}$$

Where

P_{Gi} and Q_{Gi} are the active and reactive power produced by the generator power at the ith bus.

P_{Li} and Q_{Li} are the active and reactive power demand at the ith bus.

N is the number of buses except the slack bus.

N_G is the number of the generator buses.

N_L is the number of the load buses.

Y_{ij} is the admittance of the ijth transmission line.

$|\overline{V}_i|$ is the voltage magnitudes of the generator.

$|\overline{V}_j|$ is the voltage magnitudes of the load.

The equality constraints are included using power flow equations. The output power of generators and generator terminal bus voltages are the control variables and are self-restricted by the non-linear optimization algorithm.

3.3.4.2 Inequality Constraints

Inequality constraints define the tolerable limit on both state variables and equipment usage. Load bus voltages, reactive power generation of generator and line flow limit are state variables, whose limits are satisfied by adding penalty terms in the objective function. These constraints are stated as:

- Generator limits

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad (3.53)$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \quad (3.54)$$

- Voltage limits

$$|V_{Gi}| = Constant \quad (3.55)$$

$$|V_{Li}^{min}| \leq |V_{Li}| \leq |V_{Li}^{max}| \quad (3.56)$$

- Transmission line limits

$$S_{ij} \leq S_{Tmax} \forall ij; i \neq j \quad (3.57)$$

Where

P_{Gi}^{min} and P_{Gi}^{max} are the minimum and maximum active power output of the ith generation unit.

Q_{Gi}^{min} and Q_{Gi}^{max} are the minimum and maximum reactive power output of the ith generation unit.

V_{Li}^{min} and V_{Li}^{max} are lower and upper bounds on the voltage magnitude at the ith load bus.

S_{Tmax} is the maximum transmitted apparent power through transmission lines.

Bus voltage magnitudes must be held between certain ranges to ensure that equipment is operating under design specifications. Allowable bus voltage levels depend on the normal voltages that are applied to the bus. As an example, a typical tolerable voltage range for a 138kV is within $\pm5\%$ of this value, while buses with 345 kV and over should be within $\pm10\%$ [14].

Equation (3.56) clarifies the upper and lower bounds on the voltage magnitude within the acceptable range. Equation (3.53) and (3.54) define the upper and lower limits of active and reactive power outputs, which are design characteristics of thermal generators, and directly taken from their capability curves to ensure that the system is operated safely. Equation (3.57) defines power transmission capability of the lines.

3.3.5 The Solution of Optimal Power Flow

Optimal power flow algorithms are designed to find an AC power flow solution which optimizes a performance function, such as fuel cost or network loss, while at the same time enforcing the loading limits imposed by the system equipment, such as voltage and transmission loading limits. For example, when system loss are minimized, an optimal schedule of generator active power outputs, transformer tap settings and controllable voltage settings are determined which produce the minimum operating costs while at the same avoiding any violations [56].

OPF problems can be mathematically formed as nonlinear constrained optimization problems. System size and the number of unknown variables significantly affect the difficulty of solving OPF. As the size of the system increasing, solving OPF problem is more difficult.

Matlab Optimization Toolbox [43] is used to implement OPF minimizing loss algorithm. The command 'fmincon' is used to call and solve constrained nonlinear functions in the main program. Objective functions and constraints equations are written in different 'm' files to be the function files. PowerWorld Simulator [15] is used for studying cost minimization.

3.3.6 Optimal Power Flow: Cost Minimization

Minimizing generation cost is to reduce the total fuel cost, which is primarily an operational planning problem. The objective is to minimize the fuel cost. Based on previous research of OPF problems, the minimizing generation cost problem can be mathematically formed as:

$$\text{Find the vectors } P_G = \begin{bmatrix} P_{G1} \\ P_{G2} \\ \vdots \\ P_{Gn} \end{bmatrix}, V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m \end{bmatrix}, \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_m \end{bmatrix} \tag{3.58}$$

Which minimizing:

$$f(P_G, V, \theta) = \sum_{i=1}^{N_G} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \tag{3.59}$$

Where

$f(P_G, V, \theta)$ is the total fuel cost in dollar per hour ($/hr).

n is the number of generator buses.

m is the number of buses.

Equation (3.58) establishes the objective function. Equation (3.51) and (3.52) defines the equality constraints (power flow equations). Equations (3.53) – (3.57) are inequality constraints, which clarify the upper and lower bounds on generating units, voltage magnitudes and transmission lines. Active power outputs of dispatchable generation units are control variables need to be solved to achieve the optimal operation of the minimum fuel cost.

The case study is discussed to illustrate the constrained optimization method for OPF. The study uses the Optimization Toolbox available in Matlab. For some aspects of the studies, PowerWorld Simulator is also used. The fuel costs of all the generating units are represented using cubic cost models.

3.3.6.1 Case Study of Cost Minimization

The goal of OPF for the 7-Bus system is to minimize the total fuel cost while adhering to power flow equations, specified branch flow through transmission lines, bus voltage magnitudes, slack generator active power limits. It contains 5 generators, 5 loads, 7 buses and 11 transmission lines, and bus 7 is the slack bus. The limits, fuel cost coefficients and the system parameters are found in Appendix A. Figure 3.2 shows the one line diagram of the 7-Bus power system. The total loads are 759.4 MW and 130 MVAr. For the base case, the total fuel cost is $16,939/hr and the transmission loss is 8.6 MW.

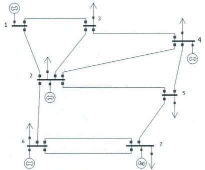

Figure 3.2 One Line Diagram of the 7-Bus Power System

PowerWorld Simulator is applied to achieve the main goal of minimizing the generation cost. Table 3.1 summarizes the results of OPF. Power and loss are in MW, voltage is in per unit (p.u.) and cost is in $/hr. The total hourly cost is $16,371/hr and the loss is 10.6 MW. The economic dispatch results are also obtained. Table 3.1 illustrates that the economic dispatch has the lowest cost and the base case has the highest cost. With the dispatch pattern obtained by economic dispatch, transmission line 2-5 and line 4-5 (in Figure 3.2) are violated by 124% and 189%, respectively. The cost of OPF is less than that of the base case, but it is more expensive than that of economic dispatch. With the operation pattern obtained from OPF, none of transmission lines is overloaded. The power flow on Line 2-5 and line 4-5 reaches the maximum limits. The bus voltage magnitudes maintain a reliable level at approximate 1 p.u.. The total fuel cost in economic dispatch is lower than that in OPF, because OPF considers the impact of the transmission system.

Table 3.1 OPF Cost Minimization of the 7-Bus Power System

	Number	Name	Base case	Economic Dispatch	OPF
Generation (MW)	1	Bus 1	102	196	126
	2	Bus 2	170	288	230
	3	Bus 4	95	128	71
	4	Bus 6	200	164	291
	5	Bus 7	201	0	52
Total Generation (MW)			760	776	770
Total Load (MW)			759.4	759.4	759.4
Total Loss (MW)			8.6	16.6	10.6
Total Hourly Cost ($/hr)			16,939	16,226	16,371

3.3.7 Optimal Power Flow: Loss Minimization

Minimizing the power loss is to minimize the transmission loss, which is another primary application of OPF. The expression for the overall transmission loss accumulated in a power system is defined in Equation (3.48). The transmission loss minimization problem can be mathematically formulated as:

$$\text{Find the vectors } P_G = \begin{bmatrix} P_{G1} \\ P_{G2} \\ \vdots \\ P_{Gn} \end{bmatrix}, V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m \end{bmatrix}, \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_m \end{bmatrix} \quad (3.60)$$

Which minimizing:

$$f(P_G, V, \theta) = \sum_{i=1}^{N_G} P_{Gi}(\text{x}, \text{u}) - \sum_{i=1}^{N_L} P_{Li} \tag{3.61}$$

The equality constraints and inequality constraints are as same as those used in the total fuel cost minimization problem.

3.3.7.1 Case Study of Loss Minimization

Case study 3.3.7.1 repeats the same power system examined in case study 3.3.6.1, except that the objective is to minimize the transmission loss. The base case of the 7-Bus power system is displayed in Figure 3.2. The goal is to minimizing generator fuel cost while adhering to power flow equations, specified branch flow through transmission lines, bus voltage magnitudes, generator power limits. The limits, fuel cost coefficients and the system parameters are found in Appendix A. The total loads are 760 MW and 130 MVAr. For the base case, the transmission loss is 7.9 MW and the total fuel cost is $16, 939/hr.

The formulation OPF regarding minimizing loss for the 7-Bus system is established using Equations (3.60) – (3.61). The formulation of OPF is constrained by Equations (3.53) – (3.57). Matlab Optimization Toolbox is applied to provide the scheme of the generating units. PowerWorld Simulator is further used to achieve the main goal (minimizing the transmission loss).

Table 3.2 summarizes the results of OPF with considering the transmission line limits. The total transmission loss is reduced to 3.25 MW with hourly cost $17,150/hr. Table 3.2 illustrates that the operation pattern obtained from OPF with minimum loss has the lowest loss, but it is the most expensive cost. The bus voltage magnitudes maintain a reliable level (approximate 1.00 p.u.) and vice versa. OPF with minimum cost has the lowest cost, but trades off the highest transmission loss.

Table 3.2 OPF Loss Minimization of the 7-Bus Power System

	Number	Name	Base case	OPF (Minimizing Loss)	OPF (Minimizing Cost)
Generation (MW)	1	Bus 1	102	100	126
	2	Bus 2	170	150	230
	3	Bus 4	95	109	71
	4	Bus 6	200	150	291
	5	Bus 7	201	254	52
Total Loss (MW)			8.6	3.25	10.6
Total Hourly Cost ($/hr)			16,939	17,150	16,371

The goal of OPF is to provide the electric utility with suggestions to optimize the current power system state online with respect to various objectives under various constraints. Most general formulation of OPF is a single objective, large scale, non-convex optimization problem. It can be achieved by minimizing or maximizing the general objective functions while satisfying the constraints. The specified variables are real and reactive power at load buses, power outputs of generating units and voltage magnitudes at generation buses, and voltages and angles at slack buses. In mathematical terms, OPF problems can be formed as a set of nonlinear equations. Typical goals of OPF problems under normal power system operations are illustrated in the previous two case studies of the total fuel cost minimization and the transmission loss minimization. Minimization of the voltage deviation as another OPF optimization goal especially is discussed in the stressed systems.

3.4 Conclusion

This chapter provides an overview of key economic dispatch concepts along with the benefits of using it for solving optimal dispatch problems. The transmission loss, the operating efficiency of generators and fuel cost are major factors influencing optimal dispatch of power generation. By using the economic dispatch, the generators' power output can be varied within certain limits to support a particular load demand at the lowest possible fuel cost. Economic dispatch has one significant shortcoming. It ignores the limits imposed by the devices in the transmission system. With the worldwide trend toward deregulation of the electric utility industry, the transmission system is becoming increasingly constrained. OPF provides the solution for the concerns of economic dispatch. OPF is a functionally combined power flow with economic dispatch to redispatch generating units while enforcing the transmission lines. A seen from the case studies, the fuel cost of a power system at OPF setting is higher than that of a power system at economic dispatch setting. This is because that the transmission line limits and other limits may be violated in economic dispatch.

The study presented in this chapter shows that the nonlinear programming based optimization techniques can handle OPF efficiently. For heavily loaded power systems, the additional reactive power supply will be considered to deal with the system violation due to increased load demand. The study concentrates benefits obtained from the additional reactive power supply rather than the cost of additional reactive power supply to the system.

Chapter 4

Reactive Power Compensation and Voltage Profile

4.0 Introduction

Chapter 2 has illustrated that local reactive power compensation is an efficient approach to maintain a proper voltage profile with no need to change the power system's infrastructure. A proper voltage profile helps power systems to avoid many failures due to voltage instability [33-37]. Providing reactive power sources at all the load centers can be expensive. This chapter shows that by providing reactive power at selected locations, the overall voltage profile can be maintained at an acceptable level.

Optimal Power Flow (OPF) is a well established technique implemented by utilities. The common objectives of minimization of the total fuel cost, minimization of the transmission loss and minimization of load voltage deviation have been formulated in Chapter 3. The inequality constraints ensure that the limits (like generating units' capability, transmission line ratings, bus voltage) are not violated. The equality constraints are the basic power flow equations that are normally expressed in terms of bus voltage magnitude and angles as well as system bus admittance matrix. The equality constraints ensure that the net complex power is zero at all buses. The use of reactive power compensation devices has become a practical solution in controlling the flow of reactive power and to increase the reactive power reserves of the system. In most instances these devices are looked as a solution to increase system voltage and decrease active power loss over a stressed system. The stressed system is caused by a prediction of load growth that is known to potentially violate voltage constraints. Although OPF can achieve the objectives of loss or cost minimization, the stressed load leads to an unexpected low voltage profile. The additional reactive power supply is able to bring the low voltage profile back to an acceptable voltage level.

The optimal placement of reactive sources throughout a power system is not a simple task. As there are no widely accepted tools to plan for reactive power installation, many planning procedures resort to a trial and error approach in order to determine the best site locations and allocation of reactive power devices to meet a variety of objectives and constraints [3]. In this thesis, the locations for placing new reactive power sources are based on voltage observation; critical load buses (load voltage less than 0.95 p.u.) are evenly considered the candidates for the placement of reactive power supply. The additional reactive power supply is employed in each of selected load buses separately and independently under stressed systems.

Section 4.1presents an overview of two sample power systems and the tools used for the different studies. In section 4.2 by applying the additional reactive power supply in each stressed system, a voltage profile comparison will be performed between the OPF solution and the solution of the additional reactive power supply; the numerical comparison is used to explore the effectiveness of reactive power supply on maintaining an acceptable voltage profile. The conclusion is given in section 4.3.

4.1 Test Power Systems and Tools for Studies

Two sample power systems are used for different case studies presented in this chapter. The 6-Bus power system is the example in [23]. The 26-Bus power system is the example from [20]. The single line diagrams of the 6-Bus power systemand the 26-Bus power system are shown in Figure 4.1 and Figure 4.2 respectively. As an alternative to real life voltage collapse situations, a low voltage profile is required to perform case studies in insufficient reactive power systems. In order to stress the normal operation systems, the loads and generations (except the slack bus) are scaled up by 1.5 uniformly, turning the system network as an adequate candidate for reactive power compensation. The distinctive load demand and generator schedule of two stressed systems used for case studies have been listed in Table 4.1 and Table 4.2.

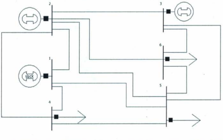

Figure 4.1 One Line Diagram of the 6- Bus Power System "taken from [23]"

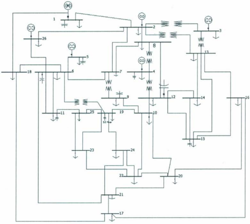

Figure 4.2 One Line Diagram of the 26-Bus Power System (taken from [20])

Table 4.1 Operational Constraints for the Stressed 6-Bus System

Generator Reactive Power Limits (MVAr)			
Bus	1	2	3
Q_g^{max}	150	150	150
Q_g^{min}	0	0	-15
Generator Active Power Limits (MW)			
Bus	1	2	3
P_g^{max}	300	225	270
P_g^{min}	75	56.25	67.5
Load Bus Voltage Limits (p.u.)			
V_{bus}^{max}		V_{bus}^{min}	
1.00		0.95	
Total Load Demand			
Active Power Load (MW)		Reactive Power Load (MVAr)	
315		315	

Table 4.2 Operational Constraints for the Stressed 26-Bus System

Generator Reactive Power Limits (MVAr)						
Bus	1	2	3	4	5	26
Q_g^{max}	390	375	225	120	240	75
Q_g^{min}	375	60	60	37.5	60	22.5
Generator Active Power Limits (MW)						
Bus	1	2	3	4	5	26
P_g^{max}	750	300	450	225	300	180
P_g^{min}	150	75	120	75	75	75
Load Bus Voltage Limits (p.u.)						
V_{bus}^{max}			V_{bus}^{min}			
1.00			0.95			
Total Load Demand						
Active Power Load (MW)			Reactive Power Load (MVAr)			
1894.5			955.5			

Matlab and PowerWorld Simulator are the main tools used for the different case studies. These tools are fairly easy to use and PowerWorld Simulator provides very useful features for power system studies. Cost minimization can be directly implemented via PowerWorld Simulator. The codification of loss minimization studies has been implemented in Matlab Optimization Tool Box available [15]. The command 'fmincon' is used to call and solve constrained nonlinear functions in the main program. Objective function and constraint equations are written in different 'm' files for the function files. PowerWorld Simulator is again used for verification of optimal results obtained from Matlab. In the following sections, details about the case studies and results are given. Along with this, a discussion on the obtained results is also provided.

4.2 Case Studies

The following important assumptions are made, with respect to performing the each objective in OPF and supplying the additional reactive power source:

- OPF solution is the present position with the stressed load, without the additional reactive power supply.
- Additional reactive power supply is the new position, with respect to OPF solution and the stressed load situation can be released by this additional reactive power supply.
- For brevity of presentation, only 50% overloading scenario is considered in different power operation systems to demonstrate the effectiveness of reactive power compensation on voltage profile enhancement. However, in practice, multiple unstressed and stressed conditions should be studied.
- A predefined power generation schedule will be accordingly scaled up by 50% to meet the 50% increased load demand. The only variability to the total generator active power output is from the system's slack bus.
- The insufficient reactive power support in stressed systems leads to the low voltage profiles which attempts to cause voltage collapse and system instability.
- Load buses with low voltages will be considered as candidate buses (critical buses) for installation of reactive power compensation devices.
- The generators associated power operation systems in this thesis are all thermal plants, thus the operation cost is considered only the fuel cost.
- The study concentrates benefits obtained from the additional reactive power supply rather than the cost of additional reactive power supply to the system.

4.2.1 Case Study 1: the Stressed 6-Bus Power System

In this section, the case study is performed where the goal of optimization is to minimize the total active power loss, total fuel cost, and load bus voltage deviation for a scheduled load demand. These objectives were discussed in detail in Chapter 3. The objective functions will be treated independently instead of augmenting the objective together.

The stressed 6-Bus power system (shown in Figure 4.1) consists of 3 generators, 3 loads, and 11 transmission lines. Note that bus 1 is the slack bus. All system parameters along with the initial load demand and generation schedule are available in Appendix B using an apparent power base of 100 MVA. Before beginning with the planning of a reactive power supply scheme, it is important to first verify that the system cannot be operated with the increased load demand. A base case power flow is performed to get an idea of the severity of the constraint violations. For the base case power flow, Table 4.3 lists the important system response of this stressed 6-Bus system. Power and transmission loss are in mega watt (MW), Voltage is in per unit (p.u.), and Cost is in dollars per hour ($/hr). The total hourly cost is $4,669/hr and the loss is 21.46 MW. It should be noted that even after increasing the generator scheme by 1.5 to meet the increased load demand, the voltage violations of load buses are still apparent; all load bus voltage magnitudes are below the normal operation voltage (0.95 p.u.). In order to enhance the voltage profile and remove the voltage violation, all three load buses are considered as possible locations for the additional reactive power supply. The stressed system is reconstructed by applying additional reactive power supplyat the load centers. The system responses of each objective in this stressed 6-Bus systemare discussed sequentially in subsection as follows.

Table 4.3 Base Case Summary for the Stressed 6-Bus System

Stressed 6-Bus system	Base Case	Values
Generation (MW)	P1	176.98
	P2	75
	P3	90
Voltage (p.u.)	V4	0.93313
	V5	0.91112
	V6	0.93969
Transmission loss (MW)	Ploss	26.98
Operation cost ($/hr)	F	4,744

4.2.1.1 Minimum Loss

This section performs the objective of minimizing transmission power loss, while adhering to power flow equations and other system and equipment constrains defined in section 4.1.3. The OPF solution minimizing the total loss is obtained by PowerWorld Simulator. Matlab optimization tool box is used to implement the scheme to minimize the total transmission loss. The results from Matlab are used in PowerWorld Simulator for further power system studies. The optimization results for the transmission loss are presented in Table 4.4. OPF not only helps the system to reduce the total power transmission loss, but also to lower the operation cost; however the voltage violation still exists. These results show the need to add local reactive power support to the overloaded system, allowing the stressed system to operate within desired voltage specifications.

Table 4.4 Summary of Minimum Loss for the Stressed 6-Bus System

Minimum Loss		Base case	OPF solution	Additional VAR supply
Generation (MW)	P1	176.98	70.37	104.83
	P2	75	88.72	116.84
	P3	90	173.41	105.32
Voltage (p.u.)	V4	0.93313	0.9391	1.00
	V5	0.91112	0.9178	1.00
	V6	0.93969	0.9485	1.00
Transmission loss (MW)	Ploss	26.98	17.45	11.84
Operation cost($/hr)	F	4,744	4,589	4,485

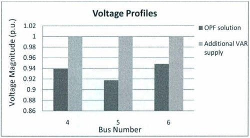

Figure 4.3 Voltage Profiles of the 6-Bus Power System (Minimum Loss)

The reactive power supply is allocated on the basis that each of the system's load significant amount of reactive power that can be met with local reactive power support. The system response after the additional reactive power supply is also listed in Table 4.4 and the allocated reactive power support at each load bus is listed in Table 4.5. The size of additional reactive power supply obtained is more than sufficient to meet load reactive power demand as each load demand; meanwhile, the load voltage profile can be enhanced. Figure 4.3 provides an overview of the voltage profile during OPF of loss minimization and the additional reactive power supply.

The additional reactive power support in the stressed system perfectly brings the voltage magnitudes back to the nominal level, which releases the load's stress and reduces the risk of power system instability occurring in the stressed system. Meanwhile, the transmission loss can be further minimized, and operation cost can be further reduced as well.

Table 4.5 VAR Allocation of Minimum Loss for the Stressed 6-Bus System

Bus number	Reactive power (MVAr)
Bus 4	76.23
Bus 5	91.12
Bus 6	65.83

4.2.1.2 Minimum Cost

The same equality and inequality constraints of loss minimization are used to minimize the total fuel cost for the same specified loading conditionin the stressed 6-Bus power system. All the generating units are assumed to be thermal with the fuel cost expressed as a cubic function of the output of the generating units. Each thermal generator features its own the generation capacity with respect to the cost coefficients shown in Table 4.6.

Table 4.6 Cost Coefficients of the 6-Bus Thermal Plants

Generators	From (MW)	To (MW)	Cost coefficients		
			a	B	c
1	50	200	213.1	11.669	0.00533
2	37.5	150	200.0	10.333	0.00889
3	45	185	240.0	10.833	0.00741

The scheme to minimize the total transmission cost can be directly determined by OPF function of PowerWorld Simulator. Table 4.7 summarizes the OPF results of minimizing cost and the results with the additional reactive power supply at load buses. With the additional reactive power supply, the system operation cost can be optimized to $4,441/hr; more importantly, the undesirable low voltage magnitudes at load buses are enhanced to 1.00 p.u.. The significant enhancement of the load voltages can be clearly illustrated in Figure 4.4.

Table 4.7 Summary of Minimum Cost for the Stressed 6-Bus System

Minimum Cost		Base case	OPF solution	Additional VAR supply
Generation (MW)	P1	176.98	79.46	84.5
	P2	75	150	124.37
	P3	90	115.99	114.82
Voltage (p.u.)	V4	0.93313	0.93995	1.00
	V5	0.91112	0.91643	1.00
	V6	0.93969	0.94567	1.00
Transmission loss (MW)	Ploss	26.98	18.47	8.82
Operation cost($/hr)	F	4,744	4,564	4,441

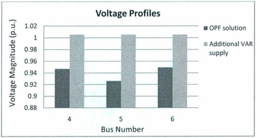

Figure 4.4 Voltage Profiles of the 6-Bus Power System (Minimum Cost)

Compared to the OPF results, the additional reactive power supply can achieve more additional reduction in total active power loss, providing upto 52% more reduction. Although additional reactive power supply has offered only 2.7% additional reduction in cost, this little improvement would be significant in terms of MW loss reduction and the revenue saving per annum. The allocation of reactive power compensation has been specified in Table 4.8 along with the minimization of thermal fuel cost during OPF. The optimal reactive power support is sufficient to enhance the unacceptable load voltage profile.

Table 4.8 VAR Allocation of Minimum Cost for the Stressed 6-Bus System

Bus number	Reactive power (MVAr)
Bus 4	71.84
Bus 5	89.66
Bus 6	62.75

The previous two sections show that by providing reactive power at selected locations, the overall load voltage profile can be improved, which ensures the stressed system to operate without any violation; meanwhile, the transmission loss can be minimized. Reduction in transmission loss turns into the direct economic benefit, which partially compensates the cost of reactive power supply.

4.2.1.3 Minimum Voltage Deviation

Voltage deviation minimization is coded in Matlab Optimization Tool box according to optimization objective function in Equation (3.50). The optimization results shown in Tables 4.9 indicate that the voltage deviation minimization is able to keep the load voltage magnitudes closer to their nominal values. The overall voltage profile enhancement is illustrated in Figure 4.5.

Table 4.9 Base Case Summary for the Stressed 6-Bus System

Minimum Voltage Deviation		Base case	OPF solution
Generation (MW)	P1	176.98	79.46
	P2	75	150
	P3	90	115.99
Voltage (p.u.)	V4	0.93313	0.9773
	V5	0.91112	0.9617
	V6	0.93969	0.9787
Transmission loss (MW)	Ploss	26.98	19.02
Operation cost ($/hr)	F	4,744	4,634

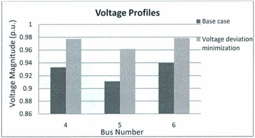

Figure 4.5 Voltage Profiles of the 6-Bus Power System (Minimum Voltage Deviation)

Similarly to the effect of supplying additional reactive power support at stressed load, minimizing voltage violation can also enhance the load voltage profile without injecting reactive power in the load. In addition to relieving the voltage violation, voltage deviation minimization also reduces the transmission loss and lowers the operation cost. Technically, it is an efficient method to maintain the desired load voltage without compensating extra reactive power to the increased load demand; nevertheless, the practical application may be infeasible due to the difficult implementation.

4.2.2 Case Study 2: the Stressed 26-Bus Power System

In this section, the case study is performed where the goal of optimization is to minimize the real power transmission loss, total fuel cost and load bus voltage deviation. These three objectives are identical to the objectives used in the stressed 6-Bus case study. The 26-Bus power system in Figure 4.2 consists of 26 buses, 46 transmission lines, 6 generators, 9 shunt capacitors and 23 loads. The system generators are located at bus 1, 2, 3, 4, 5 and 26. Bus 1 is the slack bus.

All system parameters along with the initial load demand and generation schedule are attached in Appendix C using an apparent power base of 100 MVA. Table 4.10 gives rescheduled parameter values used as the base case for this case study. The base case power flow presented here is performed to get an idea of the severity of the operational voltage violation. Table 4.10 summarizes the system response for this base case power flow. Due to the stressed load demand a significant portion of the system's constraints are being violated. Many of the load bus voltages are well below the specified voltage level 0.95 p.u.. In the following subsections, the detailed discussion of each OPF objective will be given. A discussion on the results obtained by OPF and the additional reactive power will be provided.

Table 4.10 Base Case Summary for the Stressed 26-Bus System

Stressed 26-Bus system	Base Case	Values
Generation (MW)	P1	1092.46
	P2	118.50
	P3	30
	P4	150
	P5	450
	P26	90
Transmission loss (MW)	Ploss	36.46
Operation cost ($/hr)	Cop	28,191

4.2.2.1 Minimum Loss

For the purpose of this test case, Matlab Optimization Tool box is used to determine the optimal scheme of generator units to minimize the total transmission loss. The results from Matlab are used in PowerWorld Simulator for further power system studies. From the OPF solution for the stressed 26-Bus power system, it is noticed that the voltages at some load buses are extremely low, especially from bus 20 to bus 25. By providing additional reactive power supply at these critical buses, it is possible to improve the voltage profile. Table 4.11 presents a summary of different power responses with respect to the base case, OPF loss minimization and the additional reactive power. Figure 4.6 illustrates that the voltage profile has been enhanced once the additional reactive power support is provided at the selected load buses.

Table 4.11 Summary of Minimum Loss for the Stressed 26-Bus System

Minimum Loss		Base case	OPF solution		Additional VAR supply (PowerWorld)
			(MATLB)	(PowerWorld)	
Generation (MW)	P1	1092.46	716.9499	718.48	713.99
	P2	118.50	252.4517	252.4517	74.9711
	P3	30	326.2747	326.2747	119.9711
	P4	150	225.000	225.000	104.3585
	P5	450	273.5672	273.5672	74.9711
	P26	90	128.5505	128.5505	180.0289
Transmission loss (MW)	Ploss	36.46	28.30	29.82	25.33
Operation cost ($/hr)	F	28,191	25,817	25,762	25,698

Table 4.12 VAR Allocation of Minimum Loss for the Stressed 26-Bus System

Bus number	Reactive power (MVAr)
Bus 21	44.92
Bus 22	81.17
Bus 23	14.72
Bus 24	52.72
Bus 25	26.88

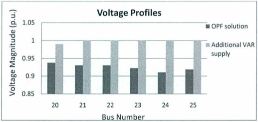

Figure 4.6 Voltage Profiles of the 26-Bus Power System (Minimum Loss)

4.2.2.2 Minimum Cost

Similarly to minimizing the total fuel cost for the stressed 6-Bus system, all the generating units are assumed to be thermal plants with the fuel cost expressed as a cubic function of the output of the generating units. The generator units of 26-Bus system are featured in Table 4.13. For the stressed 26-Bus power system, the OPF solution of minimizing the total fuel cost is directly obtained by PowerWorld Simulator. The total fuel cost is decreased by $2,429/hr with OPF simulation. It is noticed that the voltage violations at certain load buses (from bus 20 to bus 25) still exist. By providing additional reactive power at those buses, the voltage profile can be improved. Table 4.14 summarizes the system response with respect to the base case, OPF and the additional local reactive power supply. The voltage enhancement due to the sufficient reactive power support at the stressed load can be clearly demonstrated in Figure 4.7.

Table 4.13 Cost Coefficients of the 26-Bus Thermal Plants

Generators	From (MW)	To (MW)	Cost coefficients		
			a	b	c
1	100	500	240	7	0.007
2	50	200	200	10	0.0095
3	80	300	220	8.5	0.009
4	50	150	200	11	0.009
5	50	200	220	10.5	0.008
26	50	120	190	12.0	0.0075

Table 4.14 Summary of Minimum Cost for the Stressed 26-Bus System

Minimum Cost		Base case	Case 1 (PowerWorld)	Case 2 (PowerWorld)
Generation (MW)	P1	1092.46	749.99	749.27
	P2	118.50	168.55	167.84
	P3	30	170.37	169.65
	P4	150	200.05	199.34
	P5	450	500.05	499.34
	P26	90	140.05	139.34
Transmission loss (MW)	Ploss	36.46	34.58	30.29
Operation cost ($/hr)	F	28,191	24,843	24,766

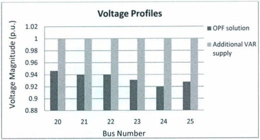

Figure 4.7 Voltage Profiles of the 26-Bus Power System (Minimum Cost)

Table 4.15 VAR Allocation of Minimum Cost for the Stressed 26-Bus System

Bus number	Reactive power (MVAr)
Bus 20	102.37
Bus 21	44.96
Bus 22	48.64
Bus 23	18.25
Bus 24	54.6
Bus 25	31.3

It is apparent that OPF simulation can help the system reduce the transmission loss and the total fuel cost, but it cannot prevent the voltage profile to avoid the unexpected low level. By injecting the additional reactive power into the buses with low voltage magnitudes, the transmission loss and fuel cost can be further minimized. This test case again confirms that the local reactive power supply can optimize the performance of stressed systems, in both technical and economic aspects.

By placing the reactive power supply at predefined load buses, OPF has been run to determine the optimal setting of control variables for both cost minimization and loss minimization objectives under the same heavily loading system. With the help of reactive power supply, the voltage profile can be dramatically enhanced; meanwhile, the system can be operated in the normal operation mode. The voltages are no longer violated, and the load is no longer suffering from the overloading stress.

4.2.2.3 Minimum Voltage Deviation

Instead of feeding additional reactive power supply to critical load buses, Matlab Optimization Toolbox handles the voltage improvement problem with objective function of optimizing voltage deviation. The simulation results are available in Table 4.16. The optimization algorithm for voltage deviation minimization can keep load bus voltages close to their nominal values, and at the same time meeting all the constraints. During the minimization of voltage deviation, the voltage profile is within the acceptable voltage range shown in Figure 4.8; however, the transmission loss becomes more severe and hourly operation cost is higher. Technically, it is an efficient method to maintain the desired load voltage because no additional reactive power is invested in the power system; nevertheless, the practical application may be infeasible with the consideration of the extremely high annual operational cost and increased transmission loss. Figure 4.8 illustrates the voltage difference before and after voltage deviation minimization.

Table 4.16 Summary of Minimum Voltage Deviation for the Stressed 26-Bus System

Voltage deviation minimization		Base case	Voltage deviation (Matlab)
Generation (MW)	P1	1092.46	1520
	P2	118.50	75
	P3	30	120
	P4	150	75
	P5	450	75
	P26	90	75
Transmission loss (MW)	Ploss	37.54	40.95
Operation cost ($/hr)	F	28,215	32,600

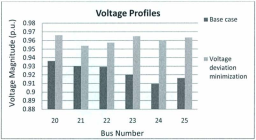

Figure 4.8 Voltage Profiles of the 26-Bus Power System (Minimum Voltage Deviation)

4.3 Conclusion

Reactive power compensation is introduced to overloaded power systems for sake of maintaining the desired voltage profile along with different optimization objectives. The idea behind reactive power compensation is to spot optimum bus location for reactive power sources and improve the voltage profile due to the increase in the load. To simplify the optimization algorithm, all load buses with low voltage magnitudes (less than 0.95 p.u.) are assumed as possible candidates for reactive power installation. Conventional OPF is demonstrated on the stressed 6-Bus and 26-Bus power systems with promising results. The results confirm the capability of reactive power compensation to enhance the overall load voltage profile and show its effectiveness and superiority.

The essential advantages of reactive power compensation have been illustrated during case studies on the stressed 6-Bus and the stressed 26-Bus systems. It is shown that the objective of voltage deviation minimization can improve the voltage profile, reduce the power transmission loss, and decrease the total fuel cost by certain degree. The following conclusions can be made.

1) In this work, load buses having top priority ranked according to their voltage magnitudes corresponding to the load suffering from the stress, which can be selected for optimal placement of reactive power sources.
2) Conventional OPF under the stressed power systems can reduce the total power loss and the total fuel cost in stressed systems, but it cannot improve the voltage profile. It may improve the voltage in some cases if it is included in the inequality constraints.
3) With the additional reactive power supply, additional reduction in both the total active power loss and the fuel cost can be effectively achieved; the voltage profile is also improved.
4) The little improvement of hourly cost would be significant in terms of MW loss reduction and the revenue saving per annum.
5) Without the method of additional reactive power supply at certain load buses, a flatter voltage profile can also be achieved by minimizing voltage deviation of the critical load buses.

The feasible operation of reactive power compensation has been emphasized as observing the voltage profile to ensure that the stressed system is acceptable for the normal and postoverloaded conditions. Although reactive power compensation generally provides plenty of benefits to system operation; the heavy use of reactive power compensation sources may lead to economic infeasibility. Cost considerations generally limit the extent to which these benefits can be applied. The expense of reactive power should cover the capital investments, in the form of installing capacitors at load buses, and marginal cost of reactive power generation. The reactive power price, reactive power supply size and its placement problems need be further researched to ensure that both investment and operating costs are recovered in a manner equitable to utility and to the customers.

The additional reactive power supply requires the planner to perform many power flow studies while varying reactive power compensation settings and other pertinent system controls in order to ensure that the planned installations meet desired operation requirements. The trial and error method is cumbersome and does not guarantee that the proposed solution is optimal. A further research of the optimal allocation of reactive power compensation (for example, an investigation of multi-objective optimization problems for the reactive power planning and system stability problem) is a challenging task.

Chapter 5

Voltage Stability Margin and Reactive Power Compensation

5.0 Introduction

With the increased loading of transmission lines, voltage stability has become a very critical issue for most power system planners and operators. Voltage stability refers to the ability of a power system to maintain steady voltage at all buses in the system after being subjected to a disturbance from a given initial operating condition [44]. Case studies in Chapter 4 have illustrated that inadequate local reactive power supply is characterized by continuous and slow reduction of the voltage magnitude at one or more load, which occurs when the system is heavily loaded. A possible result of voltage instability is the loss of load in an area, or tripping of transmission lines and other element by their protective systems leading to cascading outages. Local reactive power compensation is a solution to the system instability problem and it is able to maintain a desired load voltage profile under the overloading system phenomenon. However, voltage profile is a poor indicator of proximity to the limits of stability under normal or post-contingency power system. Voltage stability margin analysis supplements this deficiency and determines how close the system is away from instability limits. Thus, the additional reactive power supply has become essential.

In general, the analysis of voltage stability of a given power system should cover the examination of following aspects:

- To show the voltage collapse point of buses in the system
- To study the maximum transfer of power between buses before voltage collapse point
- To size the reactive power compensation devices required at relevant buses to improve voltage stability margin and prevent voltage collapse
- To study the influence of reactive power compensation devices on the maximum transfer power to meet increased load demand

The voltage stability analysis can be achieved using several different techniques. Basically they can be divided into methods that give an indication (index [45] or margin [46]) of the proximity of voltage instability and methods that analyze with more detail the mechanism of voltage instability [44]. For this study, voltage stability margin is analyzed by means of PV curve analysis [20]. PV curves provide a graphic hint of how close the system is away to the voltage stability limit under specific operation conditions.

The chapter is organized as follows: section 5.1 presents the basic concept of voltage stability margin. The introduction of PV curve analysis is illustrated using a 2-Bus power system. Section 5.3 presents two sample power systems and the tool applied for case studies. Section 5.4 proposes two different reactive power compensation devices to illustrate the effectiveness of VAR sources in improving the voltage stability margin. Section 5.5 provides concluding remarks.

5.1 Voltage Stability Margin

Voltage stability margin in this thesis is defined as the amount of additional load in a specific pattern of load increase that would cause voltage instability [52]. Contingencies such as unexpected component outages in an electric power system often reduce the voltage stability margin. Computation of voltage stability margin is an option to keep adequate voltage stability on system operation and planning, which is defined as the distance between the actual operation point and the point of collapse, measured in megawatts or percentage of the base case loading.

In this study, voltage stability margin is obtained by implementing PoweWorld adds-on PV-curve function [15]. PV curves are constructed by considering load increase for all the system load buses in a proportional way to the base case loading. System generation level is also increased (in proportion to the base case injections) in order to match the increased load. The set of equilibrium points obtained by solving the power flow problem at each load defines the PV curve. An example of PV curve construction is demonstrated using a 2-Bus system in section 5.2.

5.2 PV Curves

The PV curve is illustrated for a 2-Bus power system shown in Figure 5.1. This may be considered as a model of a generating station connected to a load center through the parallel transmission lines. The generating station voltage is kept at 1.0 p.u.. For different loadings the voltage at 2-Bus is calculated (note that there are two solutions for the voltage). In an actual power system, the load voltage will be equal to the higher voltage. The two solutions for the voltage come together at the nose of the PV curve.

Figure 5.1 One Line Diagram of the 2-Bus Power System

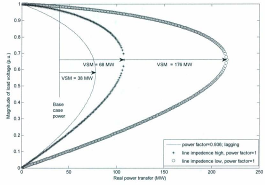

Figure 5.2 PV Curves for Three Different Cases

Figure 5.2 shows PV curves for the 2-Bus power system. Three different operating conditions are considered: load power factor unity; load power factor lagging; load power factor unity, but line impedance high simulating the effect of a line outage. In all three cases, the voltage stability margins are different. The convergence points of the graph are referred as the voltage stability limits and indicate the points where voltage collapse is imminent. The upper part of the curve corresponds to the normal operational state of the system. The lower part represents the state that required high current and low voltage and are not operationally acceptable. Voltage instability margin is mainly associated with reactive power imbalance. Unity load power factor is equivalent to no reactive power load demand; lagging load power factor is equivalent to some reactive power load demand. The operating system with lagging load power factor has the smallest voltage stability margin (38 MW); when the load power factor becomes unity, the voltage stability margin is increased to be 176 MW; the unexpected high transmission line impedance deteriorates the voltage stability margin dramatically; the voltage stability margin now is reduced to be 68 MW. Thus, the voltage stability margin highly depends on the reactive power at load buses, which can be expressed in terms of load power factor. Hence, the way to improve the voltage stability margin is to reduce the reactive power load or add additional reactive power prior to reaching the point of voltage collapse. Numerous reactive power devices have been considered to supply reactive power. Static VAR capacitors (SVC) and capacitors are used in the present work.

5.3 Test Power Systems and Tools for Studies

Two sample power systems are used for the different case studies presented in this chapter. The single line diagram of the 5-Bus power system [18] is shown in Figure 5.3. The initial parameter is available in Appendix D. The goal of the base case study for 5-Bus power system is to indicate the effectiveness of reactive power supply on the voltage stability margin under the normal power operation system. The load and generation schemes remain the same. The single line diagram of the 39-Bus power system [48] is shown in Figure 5.4. The initial parameter is available in Appendix E. The goal of this system study is to illustrate how the reactive power supply improves the voltage stability margin when the system is overloaded. For the study presented in the stressed 39-Bus system, the load and generation (except in slack bus) are scaled by 1.4 uniformly. The main features of both power systems are summarized in Table 5.1. PowerWorld add-on function, PV curve is the main tool used for determining the voltage stability margin.

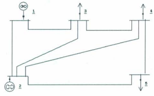

Figure 5.3 One Line Diagram of the 5-Bus Power System

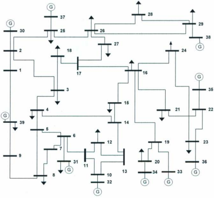

Figure 5.4 One Line Diagram for the 39-Bus Power System

Table 5.1 Summary of the 5-Bus and Stressed 39-Bus Power System

Power Systems	Load Demand		Generation Schedule	
	Active power (MW)	Reactive power (MVAr)	Active power (MW)	Reactive power (MVAr)
5 Bus	145.0	30.0	148.9	11.5
39 Bus	8,608.74	1,972.46	8,706.1	3,047.5

5.4 Case Studies

Usually, placing adequate reactive power support at the appropriate location improves voltage instability margin. There are many reactive power compensation devices adapted by the utilities for this purpose, each of which has its own characteristics and limitations. Constant reactive power supply devices (SVC, STATCOM etc.) and conventional capacitors are the most commonly reactive power compensation devices. Capacitors and constant reactive power supply devices are considered one at a time in two sample power systems to compare their effects on voltage stability margin improvement. Reactive power output of capacitors is proportional to the square of the voltage magnitude, which makes the provided reactive power decrease rapidly when voltage decreases, thus reducing its stability. To investigate their effects on voltage stability margin in detail, the 5-Bus and stressed 39-Bus power systems are examined along with the discussion and comparison. SVC represents the constant reactive power device in the following case studies. The selected bus locations for considering additional VAR sources are the critical load buses (voltage magnitude is less than 0.95 p.u. in OPF with cost minimization).

5.4.1 Case Study 1: the 5-Bus Power System

In order to illustrate the effectiveness of local reactive power supply on the voltage stability margin, the case study starts with applying an additional 55 MVAr reactive power supply at bus 5 with a normal 5-Bus power system. Since the voltage profile is not violated under the system normal condition, bus 5 is arbitrarily selected for observing the effectiveness of the additional reactive power supply on the power system. Figure 5.5 provides the results of this study. The margin is lower (547.7 MW) when capacitors are used to supply reactive power. This is due to the fact that the reactive power supplied by capacitors is reduced as the voltage drops due to increase in the load. The margin is higher (578.7 MW) when constant reach power (SVC) is supplied at the load bus 5.

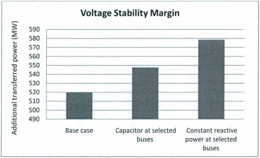

Figure 5.5 Comparison of Voltage Stability Margins for the 5-Bus System

The study shows that the local reactive power supply improves voltage stability margin; switching in more reactive power helps the system to deliver more active power under the normal operation system. SVC is modeled by using a generator with constant reactive power output.

5.4.2 Case Study 2: the Stressed 39-Bus Power System

In this section, the effectiveness of reactive power supply on the voltage stability margin is studied on 40% overloaded 39-Bus power system. Table 5.2 lists the bus violations during OPF of the fuel cost minimization. These listed load buses are the possible location of considering reactive power sources. Capacitors and SVC (constant reactive power supply) are examined at these selected buses on the stressed 39-Bus system one at a time. PowerWorld Simulator determines the allocation of reactive power supply among these load buses. It is noted that not all load buses are involved in reactive power supply. A snap shot of the maximum power transfer in Figure 5.6 indicates that reactive power compensation would be able to enhance the voltage stability margin; especially constant reactive power supply dramatically improves the voltage stability margin. Figure 5.6 also reveals that 4,784 MW of additional power can be transferred when constant reactive power sources are connected to selected load buses of the 39-Bus power system. The scheme of the local reactive power compensation shown in Table 5.2 is obtained by implementing OPF cost minimization in PowerWorld Simulator.

Table 5.2 Summary of VAR Allocation at Selected Buses

Bus No.	4	5	6	7	8	12	13	14	15
Before VAR	0.92965	0.94062	0.94595	0.93035	0.92956	0.92729	0.94987	0.93821	0.93571
VAR (MVAr)	235.75	-	-	84.77	174.42	68.15	-	-	247.38

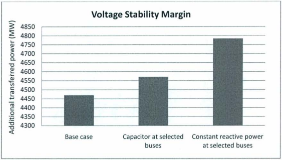

Figure 5.6 Comparison of Voltage Stability Margins for the Stressed 39-Bus System

Table 5.3 provides a summary of this study. The margin is lower when capacitors are used to supply reactive power. This is due to the fact that the reactive power supplied by capacitors is reduced as the voltage drops due to increase in the load. The margin is improved when SVC (constant reactive power supply) is supplied at the load buses. An additional 4,784 MW power can be transferred when constant reactive power supply is connected to the selected load buses.

Table 5.3 Summary of Voltage Stability Margin Analysis

Three different cases	Maximum allowable power (MW)	Minimum VAR compensation (MVAR)	Voltage Stability margin (MW)
Base case	12,669.9	-	4,470
Cap	12,758.0	225	4,571
SVC	12,963.4	845	4,784

In addition to the voltage stability margin, PV curves also provide extra information of the reactive power effect on the power system performance, such as voltage profile. Consider the example of PV curve analysis at bus 8 shown in Figure 5.7. At the beginning that system experiences light load, the voltage profile of this bus with SVC and capacitors are almost the same. When the load of the system is increased, the effect of SVC in improving the voltage is more adequate than capacitors. Both SVC and capacitors significantly affect the shape of the PV curve, which improves the critical point without masking the nose point by only shifting out the PV curve; however, SVC provides a better voltage profile at the voltage collapse point.

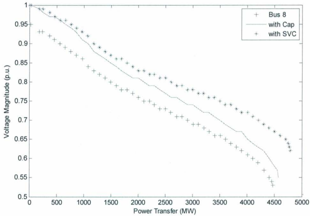

Figure 5.7 PV Curves of Bus 8 (the 39-Bus Power System)

5.5 Conclusion

Voltage instability is mainly associated with reactive power imbalance. The loadability of a bus in the power system depends on the reactive power support that the bus can receive from the system. As the system approaches the maximum loading point or voltage collapse point, both real and reactive power loss increase rapidly. Therefore, reactive power supports have to be local and adequate. The power flow transfer slowly occurs in the power system that eventually leads to a shortage of reactive power and declining voltage. This phenomenon can be graphically described in PV curves. PV curve analysis is widely used in the industry for investigating voltage stability problems. As the power transfer increases, the voltage at the load end decreases. Eventually, the critical (nose) point, the power at which the system reactive power is short in supply, is reached where any further increase in active power transfer will lead to very rapid decrease in voltage magnitudes. Before reaching the critical point, the large voltage drop due to heavy reactive power loss can be observed. The only way to save the system from voltage collapse is to reduce the reactive power load or add additional reactive power prior to reaching the point of voltage collapse.

The study presented in this study has not considered the cost of reactive power at the best location to install these devices considering voltage stability margin. Model analysis [45] and voltage stability margin index [46] are two of the methods to identify weak location in power systems and implement suitable devices (SVC, STATCOM et.) to improve voltage stability margin.

Chapter 6

Conclusion and Future Work

6.0 Recap of the Thesis

As a result of the expanded electricity demand, power systems are operated close their limits. Power systems should be operated in such a way that voltage limits and thermal limits of equipments are not being violated. Building new generation and transmission facilities are not easy due to economic and environmental constraints. Reactive power compensation can enable power systems to be operated closer to their limits and thus make it possible to maximize the use of the existing resources. A dilemma with reactive power is that a sufficient quantity of it is needed to provide the load and loss in the power system, but having too much reactive power flowing around in the power system causes excessive heating and undesirable voltage drop. A solution to this problem is to provide reactive power sources exactly at the location where it is consumed. This thesis shows the effectiveness of reactive power compensation on the system performance, especially on maintaining acceptable voltage profile and adequate voltage stability margin.

The solution of the economic dispatch by the equal incremental cost method was a precursor of Optimal Power Flow (OPF). Economic dispatch, however, only considers real power generations, and represents the electrical networks by a single equality constraint (the power balance equation). The OPF method functionally combines the economic dispatch with power flow studies, which features practical applications in power systems.

The conventional OPF method is used to achieve the economic system operation. However, reliable and secure aspects of power operation systems face challenging when the system is overloaded. The results of the work presented in the thesis show that the additional reactive power can be effectively applied in power transmission systems to solve the problems of load growth, voltage regulation and voltage stability. Many case studies are presented throughout the thesis using sample power systems to show the effectiveness of reactive power sources for power system performance enhancement.

6.1 Summary of the Research and Contribution of the Thesis

The main contribution of this thesis can be summarized as follows:

1. A complete analysis on the negative effects of reactive power transmission and the use of reactive power compensation devices to mitigate these effects.
2. Conventional optimization methods including their application to power system is investigated.
3. A detailed study on OPF problems, including economic dispatch, OPF objectives and required operational constraints are studied for different objectives. Matlab Optimization Toolbox and PowerWorld Simulator are used to handle the case studies of OPF problems.

4. An investigation into the effect of reactive power compensation on the voltage profile in stressed power systems.
5. An overview of the voltage stability analysis based on the PV-curve method, exploring the effect of reactive power compensation devices on voltage stability margin.
6. Publication of a technical paper [54] related to this thesis.

6.2 Recommendations for Future Work

This thesis has considered three key problems for power systems: economy, transmission loss and security. Following three areas are proposed for future research in regard to the reactive power compensation problem.

1. An investigation of multi-objective optimization problems for the reactive power planning and system stability problem.
2. A further research for the cost of the reactive power sources, feasibility of their location as well as the effect of contingencies must be considered.

This thesis has presented the effectiveness of reactive power supply on power system performance. Case studies are presented to show the performance improvement in terms of voltage profile and voltage stability margin. However, those two problems are focused individually. An integrated approach to deciding VAR sources based on optimization methods will be useful. The final selection of specific reactive power supply devices should based on a complete technical and economic analysis [49, 50]. Deciding the optimal location and control of reactive power sources to provide maximum benefits to the power as a whole during normal operation and under potential contingencies in a challenging are of research [51, 52].

Reactive power planning is shown to be an exceedingly difficult optimization problem as its formulation is multi-objective, partially discrete, non-linear, highly constrained and of large scale. As power systems rely on reactive compensation as a means to overcome operational constraint violations due to increased load demand, tools that rise above the limitations of classical optimization techniques must be developed in order to allocate compensation in an optimal way. In further research, a multi-objective reactive power planning problem is addressed along with three objective functions together. A single objective optimization algorithm only provides a unique optimal solution; there is no guarantee that the solution obtained from one objective is the one for another objective. Additionally, multi-objective optimization techniques need be researched to reveal the relationships among these objectives.

Although multi-objective optimization algorithms are proposed to optimize reactive power planning problems, there remains a major challenge to determine the best locations and controls for reactive power devices to provide maximum benefit to the power system as a whole during normal operation and contingent circumstance [28, 53]. Besides, the cost of the reactive power sources, feasibility of their location as well as the effect of contingencies must also be considered.

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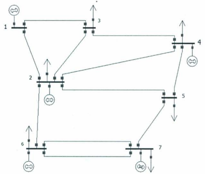
Appendix A: 7-Bus Power System Data

Appendix A contains the information about the 7-Bus Power system [15] discussed in the thesis. The one line diagram is shown in FigureA.1. The generations, loads and generation fuel cost coefficients are presented in TableA.1, A.2, andA.3respectively.



FigureA.1 One Line Diagram of the 7-Bus Power System

Table A. 1 Generation Schedule and Generator Limits for the 7-Bus Power System

Bus	Real Power Generation (MW)	Maximum Real Power Generation (MW)	Minimum Real Power Generator (MW)
1	102	400	100
2	170	500	150
4	95	200	50
6	200	500	150
7	201	600	0

Table A. 2 Load Demand for the 7-Bus Power System

Bus	Real Power Load (MW)	Reactive Power Load (MVAR)
2	40	20
3	110	40
4	80	30
5	130	40
6	200	0
7	200	0

Table A.3 Generator Fuel Cost Coefficients for the 7-Bus Power System

Bus	a	*b*	c
1	373.5	7.62	0.002
2	403.61	7.52	0.0014
4	253.24	7.84	0.0013
6	388.93	7.57	0.0013
7	194.28	7.77	0.0019

The total fuel cost: $C_{Gi} = a_i + b_i P_i + c_i P_i^2$, i represents the ith generator.

Appendix B: 6-Bus Power System Data

Appendix B contains the information about the 6-Bus Power system [23] discussed in the thesis. The one line diagram is shown in Figure B.1. The generations, loads and generation fuel cost coefficients are presented in Table B.1, B.2 and B.3 respectively.

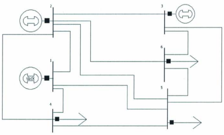


Figure B.1 One Line Diagram of the 5-Bus Power System

Table B.1: Initial Generation Schedule and Generator Limits for the 6-Bus System

Bus	Voltage magnitude (p.u)	Output Active Power (MW)	Maximum Active Power Generation (MW)	Minimum Active Power Generation (MW)	Maximum Reactive Power Generation (MVAr)	Minimum Reactive Power Generation (MVAr)
1	1.05	-	200	50	100	0
2	1.05	50	150	37.5	100	0
3	1.07	60	180	45	100	-10

Table B.2: Initial Active and Reactive Load Demand for the 6-Bus System

Bus	Active Power Load (MW)	Reactive Power Load (MVAr)
4	70	70
5	70	70
6	70	70

Table B.3 Generator Fuel Cost Coefficients for the 6-Bus Power System

Bus	a	*b*	c
1	213.1	11.67	0.0053
2	200	10.33	0.0089
3	240	10.83	0.0074

The total fuel cost: $C_{Gi} = a_i + b_i P_i + c_i P_i^2$, i represents the ith generator.

Appendix C: 26-Bus Power System Data

Appendix C gives the pertinent information about the 26-Bus Power system [20] discussed in the thesis. The one line diagram is shown in Figure C.1.Thegenerations, loads and generation fuel cost coefficients are presented in Table C.1, C.2, and C.3 respectively.

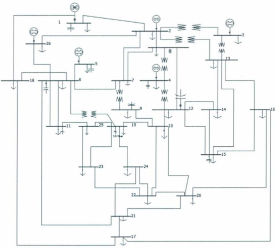


Figure C.1 One Line Diagram of the 26-Bus Power System

Table C. 1 Generation Schedule and Generator Limits for the 26-Bus Power System

Bus	Real Power Generation (MW)	Maximum Real Power Generation (MW)	Minimum Real Power Generator (MW)
1	472.44	500	100
2	50	200	50
3	15	300	80
4	75	150	50
5	225	200	50
26	119.23	120	50

Table C. 2 Load Demand for the 26-Bus Power System

Bus	Real Power Load (MW)	Reactive Power Load (MVAR)
1	38.25	30.75
2	16.5	11.25
3	48	37.5
4	18.75	14.25
5	37.5	22.5
6	57	21.75
7	0	0
8	0	0
9	66.75	37.5
10	0	0
11	18,75	11.25
12	66.75	36
13	23.25	11.25
14	18	9
15	52.5	23.25
16	41.25	20.25
17	58.5	28.5
18	114.75	50.25
19	56.25	11.25
20	36	20.25
21	34.5	17.25
22	33.75	16.5
23	18.75	9
24	40.5	20.25
25	21	9.75
26	30	15

Table C.3 Generator Fuel Cost Coefficients for the 26-Bus Power System

Bus	a	*b*	c
1	240	7	0.007
2	200	10	0.0095
3	220	8.5	0.009
4	200	11	0.009
5	220	10.5	0.008
26	190	12	0.0075

The total fuel cost: $C_{Gi} = a_i + b_i P_i + c_i P_i^2$, i represents the ith generator.

Appendix D:5-Bus Power System Data

Appendix D contains the information about the 5-Bus Power system [39] discussed in the thesis. The one line diagram is shown in Fig. D.1. Thegenerations, loads and generation fuel cost coefficients are presented in Table D.1, D.2and D.3 respectively.

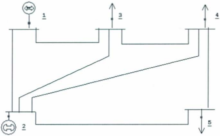


Figure D.1 One Line Diagram of the 5-Bus Power System

Table D. 1 Generation Schedule and Generator Limits for the 5-Bus Power System

Bus	Real Power Generation (MW)	Maximum Real Power Generation (MW)	Minimum Real Power Generator (MW)
1	58.79	400	-9900
2	120	500	-9900
3	60	1000	-9900

Table D.2 Load Demand for the 5-Bus Power System

Bus	Real Power Load (MW)	Reactive Power Load (MVAR)
2	19.6	9.8
3	19.6	4.7
4	49	29.4
5	58.8	39.2

Table D.3 Generator Fuel Cost Coefficients for the 5-Bus Power System

Bus	a	b	c
1	373.5	10	0.016
2	403.6	8	0.018
3	253.2	12	0.018

The total fuel cost: $C_{Gi} = a_i + b_i P_i + c_i P_i^2$, i represents the ith generator.

Appendix E: IEEE 39-Bus Power System Data

Appendix E gives the pertinent information about the IEEE 39-Bus Power system [59] discussed in the thesis. The one line diagram is shown in Figure E.1. The generations, loads and generation fuel cost coefficients are presented in Tables E.1, E.2and E.3 respectively.

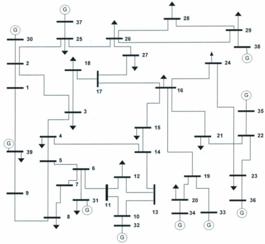


Figure E.1 One Line Diagram of the IEEE 39-Bus Power System

Table E.1 Generation Schedule and Generator Limits for the IEEE 39-Bus Power System

Bus	Real Power Generation (MW)	Maximum Real Power Generation (MW)	Minimum Real Power Generator (MW)
30	340	350	50
31	658.8	650	50
32	735.4	800	50
33	540	750	50
34	600	650	50
35	670	750	50
36	550	750	50
37	600	750	50
38	890.3	900	50
39	1052.2	1200	50

Table E.2 Load Demand for the IEEE 39-Bus Power System

Bus	Real Power Load (MW)	Reactive Power Load (MVAR)
3	322	2.4
4	500	184
7	233.8	84
8	522	176
12	8.5	88
15	320	153
16	329	32.3
18	158	30
20	680	103
21	274	115
23	247.5	84.6
24	308.6	-92.2
25	224	47.2
26	139	17
27	281	75.5
28	206	27.6
29	283.5	26.9
31	9.2	4.6
39	1104	250

Table E.3 Generator Fuel Cost Coefficients for the IEEE 39-Bus Power System

Bus	a	*b*	c
30	0	6.9	0.0193
31	0	3.7	0.0111
32	0	2.8	0.0104
33	0	4.7	0.0088
34	0	2.8	0.0128
35	0	3.7	0.0094
36	0	4.8	0.0099
37	0	3.6	0.0113
38	0	3.7	0.0071
39	0	3.9	0.0064

The total fuel cost: $C_{Gi} = a_i + b_iP_i + c_iP_i^2$, i represents the ith generator.