QUANTITATIVE RISK ANALYSIS IN AN UNCERTAIN AND DYNAMIC ENVIRONMENT

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BY

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ABSTRACT

Quantitative risk analysis (QRA) is an integral and essential part of risk analysis, which quantifies the risk of any unwanted events in industrial process facilities. However, the application of QRA in the industrial process facility is still limited. One major barrier is handling uncertainties while performing QRA using available techniques. Other important weaknesses include unrealistic assumptions and the absence of a dynamic aspect in QRA. These weaknesses undermine the credibility and utility of the output results from QRA.

Fault Tree Analysis (FTA) and Event Tree Analysis (ETA) are two common and important techniques of QRA for evaluating the likelihoods of unwanted occurrences. Traditionally, both techniques impose two major assumptions to simplify the analysis. The first assumption is related to the likelihood values of input events, and the second assumption is concerned about interdependence of events (for ETA) or basic-events (for FTA). FTA and ETA both use crisp probabilities; however, to deal with uncertainties, the probability distributions of likelihoods of input events can be assumed. These probability distributions as well as the crisp probabilities are often hard to come by, and even if available, they are subjected to different types of uncertainties including incompleteness (partial ignorance) and imprecision. Furthermore, both FTA and ETA assume that events (or basic-events) are independent. In practice, these assumptions are often unrealistic and introduce *data* and *model* uncertainties while performing FTA and ETA.

Bow-tie analysis has recently gained popularity as another important technique for QRA. It can combine both FTA and ETA techniques and describe the total accident scenarios for an unwanted event, also called a critical event (CE), in a common diagram with two parts: the first corresponds to a fault tree defining possible causes leading to the CE and the second represents an event tree to reach possible consequences of the CE. Unfortunately, in spite of having this feature, the application of bow-tie analysis in QRA is still limited to a graphical representation of causes and consequences for the unwanted event.

To overcome the challenges of QRA, this research explores uncertainty handling approaches for analyzing the fault tree and event tree, which further extends to bow-tie analysis for developing a generic framework utilizing different techniques for QRA. First, fuzzy- and evidence theory- based approaches have been developed to express the uncertainties related to *data* and *model* inadequacy of input events (events or basic events) in FTA, ETA and Bow-tie analysis. Second, an updating inference comprised of another two approaches, fuzzy-bayesian and IAE (integrity of available evidence) approaches, has been developed to integrate the dynamic aspect in QRA. In addition to these approaches, a sensitivity analysis method has also been developed for bow-tie analysis to identify the important risk contributors and evaluate corresponding risk reduction.

Applications of the developed frameworks, approaches and updating inferences have been explored in four different illustrative examples. The first example is the event tree analysis of an "LPG release" where the likelihoods of different outcomes of the event tree are determined in an uncertain data environment. In the second example, two separate sub-examples, i.e., "fault tree of a runaway reaction and "event tree of an LPG release" are considered to describe the utility of the developed approaches in case of *data* and *model* uncertainties. The third example discusses the application of the developed framework and approaches for bow-tie analysis of the BP Texas city accident. In the final example, updating approaches have been used in the bow-tie analysis of an offshore oil & gas process facility. In these examples, the likelihood of occurrence has been estimated for the unwanted event, critical event and outcome events, and the important risk contributors have been also determined. The analysis of these results helps to perform a systematic QRA in uncertain and dynamic conditions, and to measure the risk and likely losses associated with an unwanted occurrence for industrial process facilities.

Keywords: Quantitative risk analysis (QRA), uncertainty, interdependence, likelihoods, fault tree analysis (FTA), event tree analysis (ETA), fuzzy set, evidence theory, Bow-tie, and updating

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LIST OF SYMBOLS

C_d	Dependency coefficient
E	New evidence
E_i	Events as input events
D	Percentage deviation
k	Degree of conflict
$m(p_i)/m(c_i)$	Belief mass or basic probability assignment
m_{1-m}	1 to *m* numbers of experts' knowledge
n	Number of events
N	Total number of random samples
P	Power set
Pi	Probability of events (i =1, 2,...n)
$\tilde{P}_i$	Fuzzy representation of event probability
P_{OR}	"OR" gate operation
P_{AND}	"AND" gate operation
μ_p	Degree of membership for event probability
α	Membership function at a specific level
α_E	weighting parameter for prior knowledge
β_E	weighting parameter for posterior knowledge
Ω	Frame of discernment
Φ	Null set
$\cap$	Symbol for intersection
$\subseteq$	Symbol for subsets
BE_i	Basic Events as input events
IE_i	Input events for bow-tie analysis
OE_i	Outcome events
Subscript (L)	Lower value of a TFN
Subscript (m)	Most likely value of a TFN
Subscript (U)	Upper value of a TFN

LIST OF ABBREVIATIONS

G	Gate	VP	Very probable
H	High	RI	Rather improbable
I	Independent	RP	Rather probable
L	Low	SA	Sensitivity Analysis
M	Moderate	T, F	True, False
M^-	Negatively Moderate	S, F	Success, Failure
P	Perfect dependency	*bpa*	Basic probability assignment
P-	Opposite dependency	FOD	Frame of discernment
R	Risk reduction	FTA	Fault Tree Analysis
S	Strong	ETA	Even Tree Analysis
S^-	Negatively Strong	MCS	Monte Carlo Simulation
W	Weak	PDF	Probability Density Function
W^-	Negatively Weak	QRA	Quantitative Risk Analysis
CE	Critical Event	TFN	Triangular Fuzzy Number
DS	Dempster & Shafer	ZFN	Trapezoidal Fuzzy Number
HI	Highly improbable	*Bel,Pl*	Belief, Plausibility
HP	Highly probable	SIL	Safety Integrity Level
MH	Moderately High	FMEA	Failure Mode Effect Analysis
IE	Input Events	HAZOP	Hazard and Operability study
ML	Moderately Low	LOPA	Layer of Protection Analysis
OEs	Outcome events	ALARP	As Low As Reasonably Practicable
RE	Rank correlation coefficient		
RH	Rather High		
RI	Rather improbable		
TE	Top event		
VH	Very High		
VI	Very improbable		
VL	Very Low		

CHAPTER 1

Introduction

1.1 Risk analysis of industrial facility

An industrial process facility comprises a number of interacting elements termed as sub-systems, smallest-subsystems and components that in unison assist the system to perform its main purpose. A sample flow diagram of a diesel hydro-water flare system is provided in Figure 1.1 that demonstrates the interaction between the different elements in a typical industrial facility. Depending on the type of services, the facility (system) can be a nuclear plant, oil & gas facility, chemical plant, aerospace industry, manufacturing facility or other industrial facility. Among these, the vulnerabilities of nuclear, oil & gas and chemical plants can be significantly higher since during the period of operation, these plants usually deal with a large inventory of hazardous materials such as radioactive, flammable hydrocarbon, toxic and fugitive chemical compounds (Crowl and Louvar, 2002). Moreover, most of the time, the process area of these industries is highly congested due to complex piping systems, reactors, and other subsystems, including high and low-pressure compression, separation, storage, blending, and mixing units, and necessary components such as 1,2,3-way valves, relief valves, flanges, gauges, sensors, and so on. These operating conditions can be highly vulnerable, and an occurrence of a single event such as fugitive emissions, toxic releases, or a valve leakage may escalate different adverse consequences and entail major losses to the facility (Modarres, 2006).

The consequences of such occurrences are often severe and exceedingly damaging and destructive to people, environment, economy and normal operating condition of the facility.

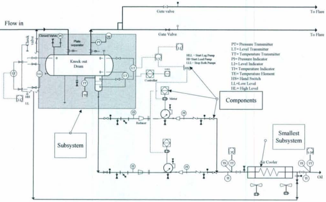

Figure 1.1: Process flow diagram of a typical industrial facility

Colloquially any unwanted or undesired occurrence in the facility is termed as an incident. Hazards generally refer to those events that have the potential to cause an incident or accident. An accident is a resulting outcome of an occurrence of a single incident or multiple incidents or events. Risk analysis is widely recognized as a systematic process to model the probable accident scenarios for the industrial facility and quantify the losses and consequences in a measurement of risk (Daneshkhah, 2004). It has now become a common term which has various implications and is usually defined as a combination of the likelihood of occurrence of an unwanted event (accident) and its consequences. Alternatively, it can also be defined with the following explanations:

Kaplan and Garrick (1981) define "risk as a set of scenarios (occurrences), each of which has a probability (likelihood) and consequences".

Kumamoto and Henley (1996) define "risk as collections of likelihoods and likely occurrences".

AIChE (2000) defines "risk as a combination of probability of the occurrence and its consequences".

Crowl and Louvar (2002) define "risk as a probability of a hazard resulting in an accident".

Ayyub (2003) defines "risk as a characteristic of an uncertain future and is neither a characteristic of the present nor past. It results from a hazardous event or sequence of hazardous events referred to as causes and if it occurs, results in different adverse consequences".

Bedford and Cook (2001) define "risk with two particular elements: hazard (a source of danger) and uncertainty (quantified by probability).

Risk involved in a potential accident or incident is evaluated based on systematic analysis which usually comprises a number of steps including a detailed qualitative and quantitative evaluation (Modarres, 2006; Markowski *et al.*, 2009). A detailed risk analysis is always designed to answer three fundamental questions about an occurrence in a facility: (1) what can happen and why? (2) what are the likelihoods?, and (3) what are the consequences? (Modarres, 2006). Four major steps, namely: hazard identifications, consequence assessment, likelihood assessments and risk characterization have to be conducted in a comprehensive risk analysis in order to get the answers to these questions (Ferdous, 2006). Figure 1.2 provides the logical connection between these steps for a risk analysis. In risk analysis, the first step, hazard identification, identifies the potential events or hazards that cause an accident or incident to happen. The second step, consequence assessment, defines the possible outcomes along with the measurement of degree of negative effects observed due to such outcomes. The third step, likelihood assessment, provides an assessment of expected frequency (rate of occurrence) or probability (chance of occurrence) of occurrence of an accident as well as outcome events. The final step, risk characterization evaluates the risk associated with an accident as a function of its consequence and probability (or frequency) of occurrences, and prioritizes the major sources of risk.

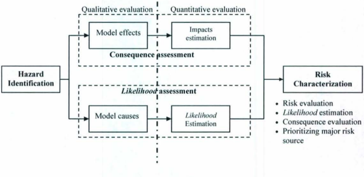

Figure 1.2: Steps in risk analysis (Ferdous, 2006)

1.2 Significance of risk analysis

An industrial facility can never be completely safe, and cannot be totally risk free. However, an appropriate risk analysis improves the degree of inherent safety and ensures the maintenance of a risk level that is *as low as reasonably practicable* (ALARP). A study of HSE (1996) and Mansfield et al. (1996) revealed that around 80% of industrial accidents start from major or minor incidents during a process operation. The potential source of these incidents includes riser or process leaks, fire, explosion, pipeline rupture, vessel rupture, chemical release, or design faults of a facility (Pula, 2005, Ferdous 2006). Rapid industrialization can be a threat for increasing these sources of risky incidents and moreover, their inadequate control also increases the probability of occurrence of industrial accidents. These are reflected in a few industrial accident examples that have occurred in the last few decades, such as the Flixborough, England accident, which cost the lives of 28 people, the whole plant and many injuries; and the Bhopal India accident, which killed more than 2000 civilians and injured over 20,000 (Crowl and Louvar, 2002). A massive explosion in Pasadena, Texas on Oct. 23, 1989, resulted in 23 fatalities, 314 injuries, and capital loss of over $715 million (Lees, 1996). On March 23, 2005, the Isomerization unit explosions of British Petroleum, Texas City, killed 15 people and injured over 170 persons (BP, 2005; Mogford, 2005, CSB, 2007). In a recent accident, on August 10, 2008, heavy explosions at the Sunrise propane storage facility, Toronto, caused 2 peoples' death, and 1000 people were evacuated (CBC,2008). The investigation team of BP's Texas city explosion has revealed that the inappropriate level indicator design of the Raffinate Splitter was one of the main contributors to this accident

(Mogford, 2005, CSB, 2007). Similarly, the history of all previous industrial accidents clearly identifies that most of these accidents occurred due to improper identification of risk contributors and correlations of these contributors with an accident. However, the catastrophic accidents mentioned above rarely happen in an industrial facility, but minor incidents commonly occur on a day to day basis and result in many occupational injuries and illnesses, and cost billions of dollars every year. An effective risk analysis and safety management strategy can easily restrict and mitigate the occurrence of these types of accidents for industrial facilities.

People today are very aware of industrial risks, and there is pressure to develop a systematic methodology for estimating financial and environmental risks. The ultimate goal of any risk management plan is to control risk as well as to prevent the major or minor incidents that occur in process facilities on a daily basis. In 2000, the American Institute of Chemical Engineers (AIChE, 2000) developed guidelines for quantitative risk analysis strategy for the chemical process industry. Now all developed countries follow specific guidelines for industrial safety to maintain risk below a desirable level (such as ALARP). Crowl and Louvar (2002) mentioned that more than 50 federal regulations of developed countries are directly related to process safety. A few safety management organizations such as the Occupational Safety and Health Administration (OSHA), the Process Safety Management (PSM), the Environmental Protection Agency (EPA) and the Risk Management Program (RMP) have generally worked to introduce the mitigation of industrial risk into the SIL (Safety Integrity Level) or LOPA (Layer of Protection Analysis) for federal or state regulations (Ferdous, 2006).

1.3 Risk analysis methodology

Risk analysis can be qualitative and quantitative. It estimates and predicts the risk associated with unwanted events, measures societal risk, individual risk, potential loss of life, probability of an accident, and reliability of a system. Qualitative evaluation is usually performed at each stage of system development to identify the possible hazards with relevant causes. Most of the traditional qualitative evaluation methods, e.g. HAZOP (Hazard and Operability Study), Functional Hazard Analysis, Safety Review, Checklist Analysis, Relative Ranking, "What-if" Analysis, Preliminary Hazard Analysis (PHA), and Failure Modes and Effects Analysis, are descriptive and generally used for identifying possible system hazards (Wang, 2004; Modarres, 2006). Normally these methods are used in preparation for consequence analysis or failure frequency analysis modeling of the risk analysis process, and also when a more detailed study is not required (Hauptmanns, 1988; Lees, 1996, 2005). After identifying the possible hazard scenarios of a system, the principal task of risk analysis is to determine the logical causes and consequences for the identified hazard scenarios and to evaluate the risk in a quantitative manner for the unwanted events.

Quantitative risk analysis (QRA) for a process system can either be deterministic or probabilistic (Wang, 2004, Ferdous 2006, 2009). The deterministic methods focus on consequence assessment (such as worst-case scenario analysis), while the probabilistic approaches consider both frequency and consequence. The probabilistic approach of QRA evaluates risk for an industrial facility in terms of its numerical evaluation of consequences and frequencies of an accident or an incident. Probabilistic data and information about the possible hazard scenarios of an accident are the main required parameters of probabilistic QRA. The final outcome of QRA is a numerical evaluation of the overall facility in terms of calculating the probability of occurrences of potential hazards and their contributions towards risk.

A variety of techniques including Fault Tree Analysis, (FTA), Event Tree Analysis (ETA), Cause-Consequence Analysis (CCA), Human Reliability Analysis (HRA) and the latest technique, "Bow-tie" analysis have been used in QRA to perform risk analysis (CCPS, 1992; Lees, 1996, Badreddine and Amor, 2010). This thesis focuses on improvements in the evaluation strategy of fault tree, event tree and bow-tie diagrams for quantitative treatment of risk analysis. Brief overviews of FTA, ETA and bow-tie analysis are presented in different sections and chapters of this thesis. Some fundamentals about FTA ETA and bow-tie analysis techniques have been described in the following sections. The evaluation and analysis strategy of these techniques is discussed in Chapter 2.

1.4 FTA, ETA and Bow-tie analysis

FTA, ETA and bow-tie analysis are diagrammatic methods and extensively used for investigating the potential risk of events, especially where process safety and risk management is a major concern (Kumamoto and Henley, 1996; CMPT, 1999; Crowl and Louvar, 2000; Lees, 2005; Modarres, 2006; Badreddine and Amor, 2010). An event tree construction starts with an unwanted event, such as an initiating event, and works forwards to its consequences; whereas a fault tree starts with an unwanted event (top-event) and works backwards to its causes (Haasl, 1965; Vesely *et al.*, 1981; Hauptmanns, 1980, 1988; AIChE , 2000; Andrews and Dunnett 2000). In the bow-tie diagram, the initiating event and unwanted event are tied to a single common event, and the causes and consequences of such an event are presented on the left and right sides of the diagram (Cockshoti, 2005; Chevreau *et al.*, 2006; Duijm, 2009; Markowski *et al.*, 2009; Badreddine and Amor, 2010). The quantitative evaluation of ETA estimates the likelihood (frequency or probability of occurrence) of possible outcomes for the initiating event. On the other hand, FTA quantitatively measures the likelihood (probability of occurrence) of the unwanted event, as well as the contribution of different causes to that event. Like FTA and ETA, bow-tie analysis estimates the likelihood of occurrence of outcome events in an integrated way with the development of a logical relationship among the causes and consequences of an occurrence in the industrial facility (Markowski *et al.*, 2009; Badreddine and Amor, 2010). In QRA, the following basic terminologies are used to perform FTA, ETA and bow-tie analysis in the risk evaluation process.

1.4.1 Basic terminology

Initiating event: Any unwanted, unexpected or undesired event (e.g., system or equipment failure, human error or a process upset, toxic or flammable release) refers to the initiating event for the event tree.

Events: The events following the initiating event are termed as precursor events, or sometimes termed only as the events for the event tree (e.g., ignition, explosion, release drifting).

Outcome events: The possible effects, scenarios or outcomes of an initiating event, are known as outcome events (e.g., fireball, vapour cloud, explosions).

Top-event: The unwanted event that is placed at the top in a fault tree, and further analyzed to find the basic causes, is known as the top-event.

Basic-event: The basic causes that are not further developed or defined are known as basic-events (e.g., equipment or components failure, human failure, external event). It represents the basic causes for the fault tree.

Intermediate Events: An event in the fault tree that can be further developed by basic-events is known as an intermediate event.

Critical events: The initiating and unwanted event is commonly termed as a critical event in the bow-tie analysis.

Input events: Bow-tie analysis uses a common term "input events" to describe the causes and consequences for a critical event.

For simplicity, instead of using basic-event and event for FTA and ETA, respectively, henceforth in the text the common term "event" is used, unless stated otherwise.

1.4.2 Challenges in FTA, ETA and Bow-tie analysis

ETA uses the combination of events and their probability to evaluate frequency or probability of occurrence of possible outcome events following the initiating event, whereas FTA uses the sequence and the probability of basic-events to estimate probability of a top-event. In bow-tie analysis, the probability of corresponding input events in the fault tree and event tree part are employed to determine the probability of the critical event and outcome events, as well as the contribution of input events leading to a critical event and outcome events.

Common techniques in QRA often make two major assumptions in order to simplify the risk evaluation strategy of the industrial facility. First, the probability of occurrence for the basic-events, events or input events is assumed to be crisp and precisely known (Vesely *et al.*, 1981; CMPT, 1999; Sadiq *et al.*, 2008). Secondly, the interdependencies among all kinds of input events in FTA, ETA or bow-tie are independent (CMPT, 1999; Lee, 2005, Modarres, 2006; Sadiq *et al.*, 2008). In practice, because of variant failure modes, design faults, poor understanding of failure mechanisms, as well as the vagueness of system phenomena, it is often difficult, if not impossible, to acquire precise probability data for the industrial components (Sawyer and Rao, 1994; Lin and Wang,1997; Wu, 2004; Yuhua and Datao, 2005). Sometimes it is not even easy to accumulate the data at all for every component. Further, particularly for an industrial process facility, it is not necessarily true that the relationships among the events are independent (Ferson *et al.*, 2004; Sadiq *et al.*, 2008).

1.5 Scope of research

The scope of the present research involves resolving the above mentioned challenges to carry out a reliable QRA. It includes:

i. Relaxing the assumptions related to the assignment of crisp likelihood and relationships in different techniques of QRA. The first assumption is related to the *data* uncertainty, while the second assumption is related to the *dependency or* model uncertainty. These assumptions limit the application of QRA to only two specific conditions: firstly, when 'enough' data about a component's failure or event's occurrence are available, and secondly, when the subsystems and components act independently in an industrial system.

ii. Relaxing the traditional assumptions in the bow-tie analysis. Bow-tie analysis inherits the assumptions of FTA and ETA. These include the event's independence and *data* uncertainty.

iii. Introducing the dynamic aspect in QRA. The traditional QRA -either using bow-tie or FTA and ETA is unable to update the risk with time as new evidence or information becomes available.

iv. Introducing fuzzy and evidence theory based formulations to handle uncertainties in QRA. The traditional QRA is often challenged with subjective and incomplete information, leading to an unreliable risk estimate. Fuzzy set theory helps to overcome subjective uncertainty in the information, whereas evidence theory helps to overcome incompleteness in the information. Thus, use of these formulations enhances overall reliability of the risk estimate.

1.6 Research objectives

The overall objective of the research is twofold; first, to address different kinds of uncertainties in quantitative risk analysis, and second, to conduct dynamic risk analysis. In order to achieve this, this research explores the methodologies and approaches for characterization of uncertainties, making use of expert knowledge for the missing data, and incorporating the dynamic aspect.

More specifically, the research has the following objectives:

[1] to develop a quantitative framework for addressing the uncertainty issues in FTA and ETA. This includes:

- development of a fuzzy-based approach and evidence theory-based approach to deal with the *data* uncertainty.
- development of empirical equations to define interdependent relationships among the events or basic-events during analysis in order to address the *model* or *dependency* uncertainty.

[2] to develop a framework for bow-tie analysis, which includes

- development of a qualitative framework for constructing a bow-tie diagram to represent structural linkages among causes and consequences of an occurrence.
- development of a quantitative framework for analyzing the bow-tie under different uncertainties.
- development of a systematic sensitivity analysis approach to predict and identify the most important input events as risk contributors.

[3] to develop an updating inference for revising and improving earlier analysis with better confidence by incorporating new industrial data or information into the analysis.

[4] to demonstrate the utility of the developed approaches and methodologies in industrial application through illustrative examples or case studies.

Prototype computer codes are programmed in the Excel and MATLAB environment to demonstrate the applicability of the developed approaches using illustrative examples. An electronic appendix with all developed simulation codes and analysis results is added at the end of this thesis. The ultimate goal of this research is to develop a standard QRA tool through a computer package and to guide decision makers toward more formal and more robust analysis to prevent minor incidents and major accidents, and reduce risk in an industrial facility.

1.7 Thesis overview

A manuscript based thesis has been written to describe the entire work of the developed research. It combines four manuscripts in four different chapters (i.e., Chapters 3, 4, 5 and 6) following the thesis writing guidelines approved by Memorial University of Newfoundland. The logical connections between the four chapters are provided, as the first two manuscripts, published in two different journals, individually assist to achieve the first objective of the thesis; and the last two manuscripts, submitted to two different journals, separately help to achieve the second and third objectives of the thesis. The case studies described in each manuscript individually assist to achieve the fourth objective of the thesis. The organizational structure of the entire thesis is shown in Figure 1.3 and the overview of different chapters is discussed hereafter:

Chapter 1 introduces a broad overview of risk analysis, its methodologies, their significance and the current practices. Basic definitions and assumptions for traditional techniques of QRA are also discussed. Finally, the current challenges in QRA are discussed and the research objectives are laid out.

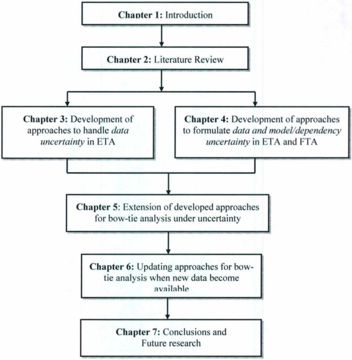

Figure 1.3: Thesis organization

Chapter 2 provides a discussion of uncertainty related issues in the context of the techniques used in QRA, especially FTA, ETA and bow-tie analysis. The recent literature reviews on available techniques and methods are also discussed.

Chapter 3, Chapter 4, Chapter 5 and Chapter 6 comprise four different research papers which individually explore the frameworks, methodologies, and approaches to handle uncertainty, and integrate the dynamic aspects in FTA, ETA and bow-tie analysis. Two of these papers, have already been published and others have been submitted for publication in international journals.

Research paper 1

Handling data uncertainties in event tree analysis (2009). Process Safety and Environment Protection, 87(5):pp. 283–292.

Research paper 2

Fault and Event Tree analyses for process systems risk analysis: uncertainty handling formulations (2011). Risk analysis: an international journal, 31(1): pp.86-107.

Research paper 3

Analyzing system safety and risks under uncertainty using a bow-tie diagram: an innovative approach. Process Safety and Environment Protection (submitted for a journal publication, November, 2010).

Research paper 4

Handling and updating uncertain information in bow-tie analysis. Journal of Loss Prevention in the Process Industries (accepted).

Chapter 7 provides the summary and conclusions, and describes the originality of the research. In addition, recommendations for future research are provided.

CHAPTER 2

Literature Review

2.1 Introduction

The literature review is divided into three sections. The first section covers the discussion of different steps involved in traditional FTA, ETA and bow-tie analysis in relation to performing QRA for an industrial facility. The second section discusses the types of uncertainty involved at various stages of QRA using FTA, ETA and bow-tie analysis. Various formulations to handle uncertainty are also described. Finally pros and cons of different uncertainty formulations are reviewed.

2.2 FTA ETA and Bow-tie analysis

FTA, ETA and bow-tie analysis have been extensively used as important techniques of QRA for developing graphical relationships of different causes, consequences and unwanted events that may lead to accidents in the industrial facility (AIChE, 2000; Ferdous 2006, Kalantarnia, 2009). These techniques help to minimize risk associated with these accidents. Fault tree and event tree develop graphical models of causation and consequences for the unwanted events (AIChE, 2000; Modarres, 2006), whereas the bow-tie analysis goes one step further and develops an integrated logical structure from causes to consequences (Cockshott, 2005; Markowski *et al.*, 2009). The fundamentals to develop and perform these techniques for industrial facilities are discussed in the following sections.

2.2.1 FTA technique

Haasl *et al.* (1965) proposed the FTA technique and applied it to a wide variety of problems including industrial safety and reliability assessment. Since then the application of FTA has proliferated in every sector, especially where safety and risk analysis of process systems are major concerns. FTA technique comprises the following steps:

1. *Fault tree development*: A fault tree builds graphical relationships among the events and an unwanted event using logic gates. The unwanted event, termed a 'top-event', is placed at the apex of the tree. Toxic chemical or flammable gas release, fire, explosion, component rupture and malfunction are a few examples of a top-event. Beginning with the top-event, the events and the intermediate events are hierarchically placed at different levels until the required level of detail is reached. The interactions between the top-event and the other events (e.g., basic-events, intermediate events) are usually expressed using the "AND" or "OR" gate (Veseley *et. al.*, 1981). The events are placed at the bottom of the tree, and the intermediate events, which can be further developed using the combinations of events or gate events, are placed in between. A simplified fault tree diagram of a reactor shut down system is shown in Figure 2.1 (AIChE, 2000).

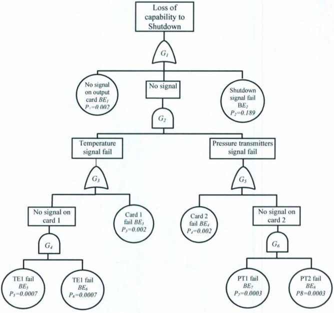

Figure 2.1: Fault tree diagram of a reactor shutdown system

2. *Qualitative evaluation*: This identifies failure modes and weakest links in a fault tree. The failure mode refers to the minimal cutsets (MCSs), which are a combination of basic events (BE), and shows the shortest pathway that leads to the *top-event*. Top-down approach and bottom-up approach are two simplified algorithms generally preferred to determine the MCSs for a simple fault tree (Hauptmanns, 1988; Kumamoto and Henley, 1996; Bedford and Cook, 2001). The MCSs using the top-down algorithm for the fault tree diagram are shown in Table 2.1.

Table 2.1: Minimal cutsets (MCSs) for the fault tree

Cut Sets (*C*)	**MCSs**			
C_1	BE_1			
C_2	BE_2			
C_3	BE_3	BE_4		
C_4	BE_3	BE_7	BE_8	
C_5	BE_4	BE_5	BE_6	
C_6	BE_5	BE_6	BE_7	BE_8

3. Quantitative evaluation: *Quantitative evaluation*: Traditionally, crisp probability values are used to determine the probability of the top-event based on the structure of the fault tree from bottom to top-event (Lawely, 1980; AIChE, 2000). Equations 2.1 and 2.2 are used to evaluate the "OR" and "AND" gate operations, respectively. For the fault tree shown in Figure 2.1, the top-event probability (P_{TOP}) is estimated to be 0.191. In addition, the quantitative evaluation also helps to rank MCSs for a fault tree (Veseley *et al.*, 1981.

$$P_{OR} = 1 - \prod_{i=1}^{n}(1 - P_{BE_i}) \tag{2.1}$$

$$P_{AND} = \prod_{i=1}^{n} P_{BE_i} \tag{2.2}$$

2.2.2 ETA technique

Process systems in nuclear and chemical industries use ETA to evaluate the effectiveness of installed protective systems and to determine the possible effects in case of failure (Ramzan *et al.,* 2007). Rausmussen (1975) and Arendt (1986) used ETA in pre-incident and post-incident applications for the process facility. The following steps are usually used to perform ETA in process systems:

1. *Event tree development*: Contrary to the fault tree, event tree construction starts with the initiating event and proceeds until it reaches the final consequences. It is simpler than the fault tree, since instead of using logic gates, the initiating event uses dichotomy (Principle of Excluded Middle) i.e., success/ failure, true/ false or yes/no, to propagate the events' consequences in different branches of the tree (AIChE, 2000; Lees, 2005). An example of an event tree diagram for a flammable gas release is shown in Figure 2.2.

2. *Qualitative Evaluation*: The individual paths that are followed by the different branches identify the possible outcome events for a particular initiating event. For the initiating event, the qualitative evaluation categorizes the credible consequence as a precursor event at different branch points and the possible effect as an outcome event at the end point of the event tree. This evaluation helps to recognize the additional safety systems requirement for a process facility to achieve lower, targeted likelihood of occurrence of an untoward event. The qualitative analysis for the flammable gas release event tree is summarized in Table 2.2.

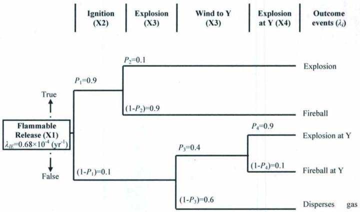

Figure 2.2: Event tree for flammable gas release

Table 2.2: Qualitative analysis of flammable gas release event tree

Events Name	Name
Initiating event	Flammable gas release from a gas process unit
Precursor events	Ignition Explosion Release drifting by wind to Y Explosion at Y
Outcome event	Explosion Fireball Explosion at Y Fireball at Y Gas dispersion from release point

3. *Quantitative evaluation*: Quantitative evaluation estimates the frequency of an outcome event and ranks the consequence severity of outcome events for an event tree. Like the FTA, the deterministic approach in ETA also uses crisp data for the events' probability (precursor events) to calculate the frequency of outcome events using Equation 2.3. Based on probabilities assigned in Figure 2.2, the frequency of outcome events is calculated (Table 2.3).

$$\lambda_i = \lambda_{IE} \times \prod_{i=1}^{n} P_{E_i} \tag{2.3}$$

Table 2.3: Outcome events' frequency of flammable gas release event tree

Outcome Event	Frequency
Explosion	6.10E-06
Fireball	5.50E-05
Explosion at Y	2.40E-06
Fireball at Y	2.70E-07
Gas dispersion from release point	4.10E-06

2.2.3 Bow-tie analysis technique

Since the early nineties, bow-tie analysis has become a well accepted technique, especially when the Royal Dutch/Shell Group developed it for the Piper Alpha disaster (RPS, 2009). Currently, this technique has been used as a constructive risk management tool in many industrial facilities (Dianous and Fiévez, 2006; Duijm, 2009; Badreddine and Amor, 2010). The interest in using the bow–tie concept is increasing daily since the unwanted consequences of an initiated accident can be pictorially analyzed from the root causes of such an occurrence. A brief review of the bow-tie technique is provided below.

1. *Bow-tie development*: The bow-tie diagram is developed for a critical event. A complete scenario from basic reasons to probable outcomes of the critical event is structured in two parts of the diagram. The left side of the diagram represents basic causes of occurrence whereas the right side represents the possible consequences. A sample graphical structure of the bow-tie diagram is presented in Figure 2.3.

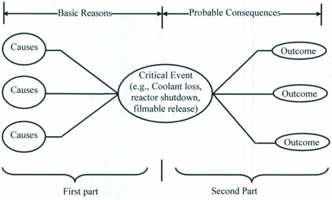

Figure 2.3: Structure of a "Bow-tie" diagram

2. *Evaluation:* A Fault tree model is used to analyze the left part of the bow-tie diagram, which basically describes the various parallel and sequential combinations of faults, failures and errors (causes) resulting in the occurrence of a critical (top) event. In order to represent the possible consequences, the event tree model is used to analyze the right part of the diagram (Markowski *et al.*, 2009; Badreddine and Amor, 2010). Once the bow-tie has been constructed, the quantitative evaluation is subsequently carried out with the equations and operations used in FTA and ETA. Assuming the

independence of basic causes, the MCSs based evaluation uses Equations 2.4-2.6 to perform bow-tie analysis (Markowski *et al.*, 2009).

$$P_{CE} = 1 - \prod_{i=1}^{NC} (1 - C_i) \tag{2.4}$$

$$C_{1,2,3 \ldots\ldots\ldots NC} = 1 - \prod_{i=1}^{n} (1 - P_{BE_i}) \tag{2.5}$$

$$P_{OE_i} = P_{CE} \times \prod_{i=1}^{m} P_{E_i} \tag{2.6}$$

In the above equations NC is the total number of MCSs (C) and m is total number of outcome events (OE) of the bow-tie.

2.3 Uncertainty in QRA

2.3.1 Types of uncertainties

FTA, ETA and Bow-tie analysis are important techniques used to perform QRA. The credibility of these techniques is extremely important. They traditionally assume all input variables (e.g., basic events, events or input events) are crisp or deterministic and don't consider interdependencies among variables. The point estimate of risk can be quite conservative (precautionary principle) (Hammonds *et al.*, 1994). In practice, an industrial facility has a large number of components, sub-systems, systems and control mechanisms which may lead to uncertainties in the prediction of outcome events, and they are represented as different end events for the bow-tie diagram. Similarly, the top-event or the critical event in a fault tree may occur due to large numbers of combinations of failure modes and components involving two or more events. Therefore, the likelihoods of occurrence of the critical event or outcome events may randomly change according to the behaviour of process components or the nature of unwanted events. Moreover, especially in the early design stage of process systems, when statistical data for the events are not available, the experts' knowledge or experience is often used alternatively.

Uncertainty is such an unavoidable and inevitable term in risk analysis that it often challenges the credibility and utility of output results from QRA (Abrahamsson, 2002). Without an appropriate definition and classification of uncertainties involved in different stages of QRA, the practical use of the output results in absolute terms becomes limited. Broadly, uncertainties are classified as two types, *aleatory* (or stochastic) and *epistemic* (or knowledge-based) uncertainty (Apostolakis, 1990; Thacker *et al.*, 2003; Helton, 2004; Daneshkhah, 2004; Ayyub and Klir, 2006). The most important distinction between these two types of uncertainty is that *aleatory* uncertainty means the objective or stochastic uncertainty which may occur due to the natural variation or randomness or inherent variability of the system (Agarwal *et al.*, 2004). *Aleatory* uncertainty is irreducible (Abrahamsson, 2002). *Epistemic* uncertainty, on the other hand, refers to subjective or knowledge-based uncertainty, that may arise due to incompleteness and imprecision (Baraldi and Zio, 2008). *Epistemic* uncertainty can be reduced by collecting more data and knowledge (Abrahamsson, 2002). Since the likelihoods and the interdependence among the input events are often missing and depend on experts' judgments, both *aleatory* and *epistemic* uncertainty can appear in the FTA, ETA and bow-tie analysis.

2.3.2 Uncertainty-based formulations

Characterization, representation, propagation, and interpretation are the key factors to formulate the uncertainty for QRA (Hammonds *et al.*, 1994, Ferdous 2009). The uncertainty formulation assists the risk analysis to propagate and analyze the uncertainty, and estimates the effect of data error in the final result (likelihood of a critical event and output events). Several techniques have been developed to formulate the uncertainty for risk analysis, which are summarized in Table 2.4 (Wilcox and Ayyub, 2003). Some of these, especially evidence theory, have not been tested much on FTA, ETA and bow-tie analysis (Ferdous *et al.*, 2009b, 2011). The main focus of this study is to utilize fuzzy set theory and evidence theory for addressing and handling the uncertainties in FTA, ETA and bow-tie analysis.

Monte Carlo Simulation (MCS), based on probability theory has been used extensively in characterizing the *aleatory* uncertainties (Suresh *et al.*, 1996, Vose, 2008). This technique sometimes extends to "higher order MCS" for addressing both types of uncertainties in QRA (Baraldi and Zio, 2008). The outer loop of "higher order MCS" generates random samples to address *epistemic* uncertainty, whereas the inner loop generates random samples to characterize *aleatory uncertainty* for the uncertain input parameters (Rao *et al.*, 2007). Besides probability theory, fuzzy sets and evidence theory have recently been used in many engineering applications, especially where expert knowledge is preferred as an alternative to define the input parameters (Cheng, 2000; Sentz and Ferson, 2002; Wilcox and Ayyub, 2003; Bae *et al.*, 2004; Agarwal *et al.*, 2004; Ayyub and Klir , 2006 ; Ferdous *et al.*, 2006, 2009).

Table 2.4: Uncertainty types and formulations

Types	Nature	Techniques
Aleatory	Stochastic, Objective, Irreducible, Random	Probability theory Evidence theory (random sets)
Epistemic	Imprecise, Incomplete, Ambiguous, Ignorant, Inconsistent, Vague	Fuzzy set theory Evidence theory (random sets) Info-gap theory p-boxes

2.4 Uncertainty analysis in QRA

In a comprehensive risk analysis, the *aleatory* and *epistemic* uncertainty can be further classified into three more different sub-categories, which are introduced at different stages of the analysis (Markowski *et al.*, 2009). According to the sources and natures of the uncertainty, three sub-categories include *data* uncertainty, *model* uncertainty and *quality* uncertainty (Abrahamsson, 2002; Markowski *et al.*, 2009). Table 2.5 provides detailed descriptions of these three categories of uncertainty. *Quality* uncertainty is sometimes defined as *completeness* uncertainty and is usually introduced due to the incomplete and incomprehensive evaluation of hazards. The *data* and *model* uncertainties are respectively known as *parameter* and *dependency* uncertainty, which arise due to insufficient or missing data and consideration of invalid or unrealistic assumptions (e.g., independent). A recursive effort is usually required when performing the HAZOP, HAZID, and FMEA to reduce *quality* uncertainty in risk analysis (Skelton, 1997; AIChE, 2000; Crowl and Louvar, 2002). At this stage, a point needs to be cleared; that the reduction or minimization of *quality* uncertainty for risk analysis is not an important concern for this thesis. *Data* and *model* uncertainty in FTA, ETA and Bow-tie analysis are two major concerns in this study. Several formulations and techniques to deal with these types of uncertainties have been developed so far, which are discussed below in the following sections.

Table 2.5: Source of uncertainty in risk analysis

Steps	Objectives	Techniques	Category of uncertainty		
			Completeness	Modeling	Parameter
Hazard Identification	Identify the possible hazards, develop logic structure of representative accident scenarios (RAS).	HAZOP, PHA, FMEA, Fault Tree and Event Tree	Inability to identify all contributions to risk and all RAS	Wrong interaction between different risk contributors and variables	Imprecision or vagueness in characteristic properties of contributors and variables
Consequence Assessment	Define the possible outcomes, Measure degree of adverse impact on health, property and environmental	Consequence Models	Incorrectness in identification of all types of the consequences as well as of all interactions among consequences	Improper, imprecise and inadequate models for source terms, dispersion and physical effects	Lack or inadequacy or vagueness in values for model variables
Likelihood Assessment	Determine the probability or frequency of RAS	FTA, ETA, bow-tie analysis	Wrong selection of events, safety function and number of accident outcome cases	Wrong analysis and assumptions in FTA, ETA and bow-tie analysis	Limited or unavailable data for components failure rates, events occurrence and interdependent relationships
Risk Characterization	Risk indexes, risk ranking or risk category	Risk matrix, SIL, LOPA	Limited assumptions in external conditions, and incorrect interpretation of results	Inadequacy in selection of appropriate risk measures as well as risk acceptance criteria	Insufficient and limited data on weather conditions, ignition sources and population

2.4.1 Data uncertainty

The failure and the occurrence probabilities of input events in FTA, ETA and bow-tie analysis are difficult to measure and accuracy in their estimates is often questionable because 'enough' data are often hard to acquire. The probability and fuzzy set theories have been used in the last few decades to overcome the situation effectively. Though the techniques are capable of addressing random or subjective uncertainty in a limited context, these are unable to handle the interdependent relationships, which may exist to any extent between the basic-events, events or input events.

2.4.1.1 Probability theory

Probability theory is the most common technique. To avoid the mathematical complexity in the analytical methods of probability theory, Monte Carlo Simulation has preferably been used to address uncertainties due to randomness in the estimates of input parameters (e.g., events probability) (Hammonds *et al.*, 1994; Abrahamsson, 2002; Wilcox and Ayyub, 2003; Vose, 2008). Ordinary MCS uses three basic steps: i) define the probability density function (PDF) for uncertain parameters, ii) generate the random sample from the selected PDF, and iii) use the generated random sample in the model to produce the PDF for the output (Hammonds *et al.*, 1994;Vose, 2008). In Figure 2.4, the uncertainty analysis using MCS is schematically described.

Hauptmanns (1988) provided an MCS methodology for fault tree analysis. Kumamoto and Henley (1996) also demonstrated a few examples of uncertainty analysis in FTA using MCS. Similarly, Suresh *et al.* (1996) addressed the data uncertainties (event probability) and analyzed the fault tree using MCS. Baraldi and Zio (2008) demonstrated the use of MCS in ETA.

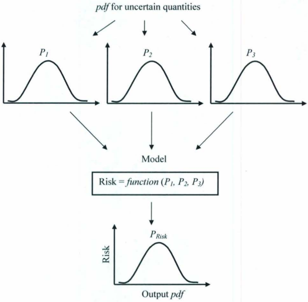

Figure 2.4: Uncertainty analysis using MCS

2.4.1.2 Fuzzy set theory

Zadeh (1965) first introduced the concept of fuzzy sets and since then thousands of papers and books have been published to describe its application. Among them, Ross (1995, 2004), and Ayyub and Klir (2006) especially elaborated the discussion of fuzzy set theory for engineering applications. Other works that include Kenarangui (1991); Rivera *et al.* (1999); Huang *et al.* (2001), and Wilcox and Ayyub (2003) also attempted to exploit fuzzy set theory in ETA. They used fuzzy numbers to express the event's probability and used the *extension principle* to determine the frequency of the outcome events. Tanaka *et al.* (1983), Misra and Weber (1990), Singer (1990), Sawer and Rao (1994), Suresh *et al.* (1996), and Wilcox and Ayyub (2003) used fuzzy set theory to define the probability of events and analyze the fault tree using fuzzy arithmetic operations.

Cockshoti (2005), Dianous and Fiévez (2006), and Duijm (2009) described and developed the probabilistic model for bow-tie analysis. This model helps to mitigate and define and mitigate the pathways of an accident occurrence by evaluating the likelihoods in a crisp boundary. Markowski *et al.* (2009) attempted to exploit fuzzy logic for the bow-tie analysis, which is limited to only capturing subjective uncertainty, and unable to characterize uncertainty due to inconsistent, incomplete and conflicting data as well as the interdependence of input events in QRA techniques. Baderddine and Amor (2010) proposed a probabilistic dynamic model for bow-tie analysis to study the impact of different input events of bow-tie to limit the occurrence of the top-event (so called critical event) and also to reduce the severity of its consequences in a more realistic and dynamic manner.

Fuzzy set theory is able to address the uncertainties that are induced due to subjective and qualitative expert judgments. The imprecision (vagueness) in the estimate is expressed using a fuzzy number, which can have a triangular or trapezoidal membership function. The fuzzy numbers are used in fuzzy arithmetic operations to propagate uncertainties and obtain the fuzzy number for an output event. Uncertainty analysis using the fuzzy numbers for an output event $\tilde{P}_{Risk}$ following the Risk = *function* ($\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$) is shown in Figure 2.5.

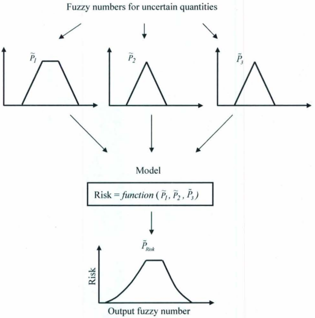

Figure 2.5: Fuzzy set theory for uncertainty formulation

2.4.1.3 Evidence theory

Besides probability theory and fuzzy set theory, evidence theory has been used in risk analysis (Guth, 1991; Liu *et al.*, 2005; Ayyub and Klir, 2006). The motivation for the development of this theory was to characterize the uncertainty caused by partial ignorance, knowledge deficiency or inconsistency about a system provided by different experts (Sadiq *et al.*, 2006; Wang *et al.*, 2006). Unlike traditional probability theory, evidence theory considers the subjective probabilities assigned by an expert as evidence and allocates them to the corresponding subsets of a power set. The unassigned mass due to unknown information is assigned as a mass for ignorance subset (as opposed to the Bayesian approach that distributes missing evidence in remaining disjointed subsets). The important features of evidence theory are:

- Individual beliefs from different sources can be expressed through the probability mass function that may bear incompleteness from partial to full ignorance,
- A belief interval (a boundary of probability estimation) can be obtained for each uncertain parameter, and
- Bias from a specific source can be avoided and conflicts among different sources can be resolved through a belief structure (Sentz and Ferson, 2002).

Evidence theory generalizes classical probability theory through a belief interval constructed by assigning upper and lower bounds for probabilities (Guth, 1991). It uses four basic constituents: *frame of discernment (FOD)*; *basic probability assignment* (*bpa*); *Belief* measure (*Bel*), and *Plausibility* measure (*Pl*) to characterize the quality of uncertainty, such as probability of basic-events, events or input events (Sadiq *et al.*, 2006). The theory also includes reasoning based on the rule of combination of degrees of belief according to different evidence.

For a given FOD, *(Ω)* in Figure 2.6, *bpa (mass)* is distributed over the set of all possible subsets of *Ω*: the power set of *Ω* and written 2^{Ω}. The unassigned mass, calculated by 1- *m (p) -m (¬p)*, is assigned to the belief mass for the ignorance subset.

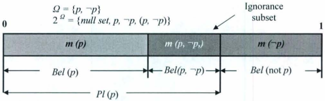

Figure 2.6: Formulation of uncertainty using evidence theory

2.4.1.4 Comparison of different theories

The pros and cons of different uncertainty formulations are summarized in Table 2.6. However, the traditional method is highly desired in FTA, ETA and bow-tie analysis since the analysis complexity, input data requirement, and analysis time are minimum for this method and the method is also well accepted (AIChE, 2000; Abrahamsson, 2002). The traditional method is incapable of handling any kind of data uncertainty, which most of the time provides an unreliable analysis for FTA/ETA/Bow-tie analysis (Yang and Suzuki, 1995; Abrahamsson, 2002). Probability theory is the most common method to address random uncertainties (Vose, 2008; Ren et.al., 2009). However, this requires sufficient empirical information to derive the PDFs for the inputs (Hammonds *et al.*, 1994; Wilcox and Ayyub, 2003; Abrahamsson, 2002; Chojnacki, 2005, Ferdous, 2009). Moreover, the classical MCS framework cannot differentiate random and subjective uncertainties in the uncertainty analysis (Berztiss, 2001; Abrahamsson, 2002). Using fuzzy set theory and evidence theory, uncertainty analysis can be performed with subjectively assigned fuzzy numbers and basic probability assignments (*bpas*) by the experts (Wilcox and Ayyub, 2003, Ferdous, 2009). The fuzzy numbers are sufficient to address the subjective uncertainty, when the empirical information is sparse or completely unavailable for the uncertain parameters (Chojnacki, 2005; Ren *et al.*, 2009; and Ferdous, 2006, 2009). Unlike probability and fuzzy set theory, the *bpa* in evidence theory is appropriate to represent uncertainty associated with ignorance and incompleteness of expert knowledge, and able to generalize the overall uncertainty in a belief interval (Bae *et al.*, 2004; Chojnacki, 2005). In some cases, the fuzzy arithmetic and evidence theory-based formulations are still not well-defined, which often limits their acceptability in risk analysis.

Table 2.6: Comparison of different theories

Characteristics	**Traditional**	**Probability theory**	**Fuzzy set theory**	**Evidence theory**
Analysis complexity	*1	3	2	2
Data requirement	1	1	2	2
Handling data uncertainty due to subjectivity	3	3	1	2
Handling data uncertainty due to incomplete and inconsistent information	3	3	2	1
propagating different uncertainties	3	2	2	2
Simplicity in the interpretation of results	1	2	3	3
Data aggregation	3	3	2	1
Analysis time	1	3	2	2
Theory acceptance	1	2	3	3

*3: Least desired; 2: Moderately desired; 1: Highly desired

2.4.2 Dependency uncertainty

The independence assumption might be convenient, but it is not always realistic for FTA, ETA and bow-tie analysis in QRA (Ferson, *et. al.*, 2004). Vesely *et al.* (1981) showed several examples of FTA where the events are not generally independent. The dependency among the different events may be positively or negatively correlated. In order to define various kinds of event interdependencies, Ferson *et al.* (2004) used the Frank copula, whereas Li (2007) proposed a dependency factor based fuzzy approach to address the dependency uncertainty. Pearson correlation is used in Frank copula that describes the full range of dependencies i.e., from perfect dependence to opposite dependence (Sadiq *et al.*, 2008). Li's method uses fuzzy numbers to define the dependency factor among the events (Li, 2007).

2.5 Updating risk analysis

The dynamic aspect in risk analysis is a fairly new concept and has become an integral part of quantitative risk analysis. This special feature provides an inference in QRA to update the analysis recursively considering the knowledge or data of an occurrence in the industrial facility as a function of time (Kalantarnia *et al.*, 2009 and 2010; Yang *et al.*, 2010). Updating risk analysis basically refers to this dynamic feature of risk analysis which has the ability to revise the likelihoods assessment of QRA when new expert knowledge or new data become available (Yang *et al.*, 2010). The probability of basic-events, events and input events in FTA, ETA and bow-tie analysis dynamically changes every time since the failure of components and subsystems in an industrial facility can occur randomly, and the accident escalation factors change frequently (Kalantarnia *et al.*, 2009 and 2010). In a real time risk analysis, the probabilities for the input events in different QRA techniques must be updated with available new knowledge. The resulting analysis attained after using the updated probabilities is referred to as posterior risk analysis. Bayes' theorem in Equation 2.7 uses the traditional probability theory for continually updating whenever new data accumulate (Sandar and Badoux, 1991; Bedford and Cook 2001; Modarres, 2006; Vose, 2008; Yang *et al.*, 2010). The common difficulty of the Bayes' theorem lies in the normalizing constant, the denominator of Equation 2.7, which is required to be integrated over a valid domain of the uncertain parameters being, estimated (Vose, 2008). Selecting a prior conjugate for a given PDF allows the simplification of characterizing the resulting posterior distribution, without the necessity of performing any integrations or complicated mathematics (Fink, 1997; Ferson 2005). However, this limits the flexibility of implementing Bayes' theorem for using other kinds of PDFs that are excluded from conjugate prior families (Fink, 1997; Ferson 2005).

$$f(p/d) = \frac{h(p)l(d/p)}{\int_{-\infty}^{\infty} h(p)l(d/p)dp} \qquad (2.7)$$

where, p is the uncertain parameter of interest, $h(p)$ is a continuous prior PDF and $l(d/p)$ is the likelihood function based on real time data d.

2.6 Proposed frameworks

The "higher order MCS" can be used to deal with both *aleatory* and *epistemic* uncertainties separately (not discussed in this study) only when sufficient data are available to distinguish both types of uncertainties using PDFs. However, it may be possible to characterize PDF for *aleatory* uncertainty, but characterizing PDFs for *epistemic* uncertainty is not a trivial task (Baraldi and Zio, 2008). Fuzzy sets and evidence theory can deal with these limitations. In Chapters 3 the fundamentals of these two theories are presented. The on-going research aims to integrate these two theories (fuzzy set theory and evidence theory) to charaterize different kinds of uncertainties in FTA and ETA for the process system. To deal with *data* uncertainty, fuzzy set theory is employed to deal with linguistic/subjective uncertainties while evidence theory is used to handle ignorance, incompleteness and inconsistency in expert knowledge. It also proposes a dependency coefficient to deal with *dependency* uncertainty in FTA, ETA and bow-tie analysis. Further, updating inferences integrated with fuzzy and evidence theory are used to explore the dynamic aspect in FTA, ETA and bow-tie analysis.

Four frameworks, i.e., ETA with uncertainty, FTA and ETA with uncertainty, Bow-tie analysis, and updating risk estimate in bow-tie analysis have been developed in the following four chapters to develop uncertainty and dynamic risk analysis-based methodology for QRA. Each chapter has been published as a paper and comprises a specific task as shown in Figure 2.7. The contents of the chapters are focused on the development methodology and approaches of for the mentioned techniques of QRA to accomplish the objectives of the thesis.

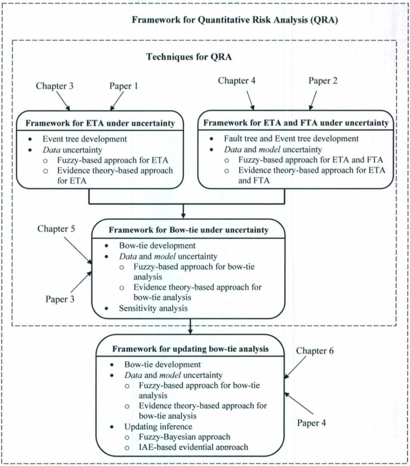

Figure 2.7: Positions of the papers 1-4 for methodology and approaches development

References

Abrahamsson, M. (2002). *Uncertainty in quantitative risk analysis - characterization and methods of treatmen,* Fire Safety Engineering and Systems Safety.

Agarwal, H., Renaud, J. E., Preston, E. L., and Padmanabhan, D. (2004). Uncertainty quantification using evidence theory in multidisciplinary design optimization. *Reliability Engineering and System Safety, 85*(1-3), 281. doi: 10.1016/j.ress.2004.03.017

AIChE. (2000). *Guidelines for chemical process quantitative risk analysis* (2nd Edition ed.). New York, USA: Center for Chemical Process Safety/AIChE.

Andrews, J. D., and Dunnett, S. J. (2000). Event-tree analysis using binary decision diagrams. *Reliability, IEEE Transactions on, 49*(2), 230-238.

Apostolakis, G. (1990). The concept of probability in safety assessments of technological systems. *Science, 250*(4986), 1359-1364. doi:10.1126/science.2255906

Arendt, J. S. (1986). Determining heater retrofits through risk assessment. *Plant/Operations Progress, 5*(4), 228-231. doi:10.1002/prsb.720050410

Ayyub B. M, and Klir G. J. (2006). *Uncertainty modeling and analysis in engineering and the sciences*. Boca Raton, FL 33487-2742, US: Chapman & Hall. doi:10.1201/9781420011456.

Ayyub, B. M. (2003). *Risk analysis in engineering and economics*. New York, USA: Chapman and Hall/CRC.

Badreddine, A., and Amor, N. B. (2010). A dynamic barriers implementation in bayesian-based bow tie diagrams for risk analysis. *Computer Systems and Applications, ACS/IEEE International Conference on,* 1-8.

Bae, H., Grandhi, R. V., and Canfield, R. A. (2004). An approximation approach for uncertainty quantification using evidence theory. *Reliability Engineering & System Safety, 86*(3), 215. doi: 10.1016/j.ress.2004.01.011"

Baraldi, P., and Zio, E. (2008). A combined monte carlo and possibilistic approach to uncertainty propagation in event tree analysis. *Risk Anal, 28*(5), 1309-1326.

Bedford, T., and Cooke, R. (2001). *Probabilistic risk analysis: Foundations and methods* (1st ed.) Cambridge University Press.

Berztiss, A. T. (2001). Uncertainty management. In *Handbook of Software engineering and knowledge engineering* () World Scientific.

BP (2005). *Fatal accident investigation report.* (Investigation) BP. Texas city

CBC (2008).Ontario to review propane safety in wake of Toronto explosion. CBC, Canada.

Cheng , Y. (2000). *Uncertainties in fault tree analysis. Tamkang Journal of Science and Engineering, 3*(1), 23-29.

Chevreau, F. R., Wybo, J. L., and Cauchois, D. (2006). Organizing learning processes on risks by using the bow-tie representation. *Journal of Hazardous Materials, 130*(3), 276. doi:DOI: 10.1016/j.jhazmat.2005.07.018.

Chojnacki, E., Mercat-Rommens, C., and Baudrit, C. (2005). Influence of mathematical modeling of knowledge. application to the transfer of radionuclides in the environment. *Presented at Workshop on the Evaluation of Uncertainties in Relation to Severe Accidents and Level II Probabilistic Safety Analysis,* Cadarache, France.

Cockshott, J. E. (2005). Probability bow-ties: A transparent risk management tool. *Process Safety and Environmental Protection, 83*(4), 307. doi:DOI: 10.1205/psep.04380.

Crowl, D. A., and Louvar, J. F. (2001). *Chemical process safety, fundamentals with applications* (2nd ed.). Upper Saddle River, New Jersey, USA: Prentice Hall PTR.

CSB (2007). *Investigation report: Refinery explosion and fire.* (Investigation Report No. 2005-04-I-Tx). Texas City, Texas: U.S. Chemical Safety and Hazard Investigation Board.

Daneshkhah, A. (2004). *Uncertainty in probabilistic risk assessment: A review.* Unpublished manuscript.

Dianous, de V., and Fiévez, C. (2006). ARAMIS project: A more explicit demonstration of risk control through the use of bow–tie diagrams and the evaluation of safety barrier performance. *Journal of Hazardous Materials, 130*(3), 220-233. doi: 10.1016/j.jhazmat.2005.07.010

Duijm, N. J. (2009). Safety-barrier diagrams as a safety management tool. *Reliability Engineering and System Safety, 94*(2), 332-341. doi: 10.1016/j.ress.2008.03.031.

Durga Rao, K., Kushwaha, H. S., Verma, A. K., and Srividya, A. (2007). Quantification of epistemic and aleatory uncertainties in level-1 probabilistic safety assessment studies. *Reliability Engineering & System Safety, 92*(7), 947-956. doi:10.1016/j.ress.2006.07.002.

Ferson, S. (2006).*Bayesian methods in risk assessment.* (Technical report No. RAMAS). France.

Ferdous, R. (2006), "Methodology for Computer Aided Fuzzy Fault Tree Analysis", *Thesis Submitted To Memorial University of Newfoundland, Canada, in Partial Fulfillment of the Requirements for the Degree of Master of Engineering.*

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., and Veitch, B., (2009a). Methodology for computer aided fuzzy fault tree analysis. *Process Safety and Environment Protection, 87*(4), 217–226

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., and Veitch, B., (2009b). "Handling Data Uncertainties in Event Tree Analysis". *Process Safety and Environment Protection*, 87(5), 283–292.

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., and Veitch, B., (2011). "Fault and Event Tree Analyses for Process Systems Risk Analysis: Uncertainty Handling Formulations". Risk Analysis, 31(1), 86-107

Ferson, S., Hajagos, J., Berleant, D., Zhang, J., Tucker, W. T., Ginzburg, L., and Oberkampf, W. (2004). *Dependence in dempster-shafer theory and probability bounds analysis**. US: Sandia National Laboratories.

Fink, D.*A compendium of conjugate priors.*Tech. Rep., 1995. Retrieved from http://www.people.cornell.edu/pages/df36/CONJINTRnew\%20TEX.pdf

Guth, M. A. S. (1991). A probabilistic foundation for vagueness and imprecision in fault-tree analysis. *Reliability, IEEE Transactions on, 40*(5), 563-571.

Haasl, F. D. (1965). Advanced concepts in fault tree analysis. *System Safety Symposium,* Boeing Company, Seattle, Washington.

Hammonds S. J., Hoffman O. F. and Bartell M. S. (1994). *An Introductory Guide to Uncertainty Analysis in Environmental and Health Risk Assessment.* (Technical report No. ES/ER/TM-35/R1).SENES Oak Ridge, Inc. Oak Ridge, Tennessee.

Hauptmanns, U. (1980). Fault tree analysis of a proposed ethylene vaporization unit. *Industrial & Engineering Chemistry Fundamentals, 19*(3), 300-309. doi:10.1021/i160075a012.

Hauptmanns, U. (1988). Fault tree analysis for process industries engineering risk and hazard assessment. In *Engineering risk and hazard assessment* (pp. 21-59). Florida,US: CRC Press Inc.

Helton, J. C. (2004). Alternative representations of epistemic uncertainty. *Special Issue of Reliability Engineering and System Safety, 85*(1-3).

HSE. (1996). *Offshore accident and incident statistics report.* (Technical No. OTO96.954).Health and Safety Executive.

Huang, D., Chen, T., and Wang, M. J. (2001). A fuzzy set approach for event tree analysis. *Fuzzy Sets and Systems, 118*(1), 153-165. doi: 10.1016/S0165-0114(98)00288-7.

Kalantarnia, M., Khan, F., and Hawboldt, K. (2009). Dynamic risk assessment using failure assessment and bayesian theory. *Journal of Loss Prevention in the Process Industries, 22*(5), 600-606. doi:DOI: 10.1016/j.jlp.2009.04.006.

Kalantarnia, M., Khan, F., and Hawboldt, K. (2010). Modelling of BP texas city refinery accident using dynamic risk assessment approach. *Process Safety and Environmental Protection, 88*(3), 191-199. doi:DOI: 10.1016/j.psep.2010.01.004

Kaplan, S., and Garrick, B. (1981). On the quantitative definition of risk. *Risk Analysis, 1*(1), 11-27. doi:10.1111/j.1539-6924.1981.tb01350.x.

Kenarangui, R. (1991). Event-tree analysis by fuzzy probability. *IEEE Transactions on Reliability, 40*(1), 120-124. doi:10.1109/24.75348.

Khan, F. I., and Abbasi, S. A. (2001). Risk analysis of a typical chemical industry using ORA procedure. *Journal of Loss Prevention in the Process Industries, 14*(1), 43. doi:DOI: 10.1016/S0950-4230(00)00006-1.

Kumamoto, H., and Henley, J. E. (1996). *Probablistic risk assessment and management for engineers and scientists* (2nd ed.) Wiley-IEEE Press.

Lawley, H. G. (1980). Safety technology in the chemical industry: A problem in hazard analysis with solution. *Reliability Engineering, 1*(2), 89-113. doi: 10.1016/0143-8174(80)90002-5.

Lees, F. (1996). *Loss prevention in the process industries* (2nd ed.). Butterworth-Heinemann, Oxford.

Lees, F. P. (2005). In Mannan S., O'Connor M. K. (Eds.), *Loss prevention in the process industries* (3rd ed.) Elsevier.

Li, H. (2007). *Hierarchical risk assessment of water supply systems.* (Unpublished Doctor of Philosophy). Loughborough University, UK.

Lin, C., and Wang, M. J. (1997). Hybrid fault tree analysis using fuzzy sets. *Reliability Engineering and System Safety, 58*(3), 205-213. doi:DOI: 10.1016/S0951-8320(97)00072-0.

Liu, Y., Chen, Y., Gao, F., and Jiang, G. (2005). Risk evaluation using evidence reasoning theory. *Machine Learning and Cybernetics, 5*(1), 2855-2860. doi:10.1109/ICMLC.2005.1527429

Mansfield, D. P., Kletz, T. A., and Al-Hassn, T. (1996). Optimizing safety by inherent offshore platform design. *Proceedings of 1996 OMAE Conference on Safety and Reliability, -Volume II).*

Markowski, A. S., Mannan, M. S., and Bigoszewska, A. (2009). Fuzzy logic for process safety analysis. *Journal of Loss Prevention in the Process Industries, 22*(6), 695-702. doi:DOI: 10.1016/j.jlp.2008.11.011.

Misra, K. B., and Weber, G. G. (1989). A new method for fuzzy fault tree analysis. *Microelectronics Reliability, 29*(2), 195-216. doi:DOI: 10.1016/0026-2714(89)90568-4.

Modarres, M. (2006). *Risk analysis in engineering techniques, tools and trends*. Bocca Raton, Florida, U.S.A.: Taylor and Francis.

Mogford, J. (2005). Fatal accident investigation report: Isomerization Unit Explosion. (Investigation report: Texas City, Texas, USA.

Pula, C. R. (2005). *An integrated system for fire and explosion consequence analysis of offshore process facilities* (Unpublished M.Eng.). Memorial university of Newfoundland, St John's, NL, Canada.

Ramzan, N., Compart, F., and Witt, W. (2007). Application of extended hazop and event-tree analysis for investigating operational failures and safety optimization of distillation column unit. *Process Safety Progress, 26*(3), 248-257. doi:10.1002/prs.10202.

Rasmussen, N. C. (1975). *Reactor safety study: An assessment of accident risk in US nuclear power plants.* (Technical Report No. WASH- 1400, NUREG 751014).U.S. Nuclear Regulatory Commission.

Ren, J., Jenkinson, I., Wang, J., Xu, D. L., and Yang, J. B. (2009). An offshore risk analysis method using fuzzy bayesian network. *Journal of Offshore Mechanics and Arctic Engineering, 131*(4), 041101. doi:10.1115/1.3124123.

RPS (2009). BowTieXP Introductory Slide Show. RPS Energy, Perth: Western Australia http://www.bowtiexp.com.au/bowtiexp.libraryB.asp.

Ross, J. T. (1995). *Fuzzy logic with engineering applications*. New York, USA: McGraw-Hill.

Ross, J. T. (2004). *Fuzzy logic with engineering applications* (2nd ed.). West Sussex, England.: John Wiley and Sons, Ltd,.

Sadiq, R., Saint-Martin, E., and Kleiner, Y. (2008). Predicting risk of water quality failures in distribution networks under uncertainties using fault-tree analysis. *Urban Water Journal, 4*(5), 287.

Sadiq, R., Najjaran, H., and Kleiner, Y. (2006). Investigating evidential reasoning for the interpretation of microbial water quality in a distribution network. *Stochastic Environmental Research and Risk Assessment, 21*(1), 63-73.

Sander, P., and Badoux, R. (1991). *Bayesian methods in reliability* Kluwer Academic Pub.

Sawyer, J. P., and Rao, S. S. (1994). Fault tree analysis of fuzzy mechanical systems. *Microelectronics Reliability, 34*(4), 653-667.

Sentz, K., and Ferson, S. (2002). *Combination of evidence in dempster-shafer theory*. (Technical US Department of Energy: Sandia National Laboratories.

Singer, D. (1990). A fuzzy set approach to fault tree and reliability analysis. *Fuzzy Sets and Systems, 34*(2), 145-155. doi:DOI: 10.1016/0165-0114(90)90154-X.

Skelton, B. (1997). *Process safety analysis - an introduction*. Warwickshire, UK: IChemE.

Spouge, J. (1999). *A guide to quantitative risk assessment for offshore installations*. London, UK,: CMPT Publication 99/100.

Suresh, P. V., Babar, A. K., and Raj, V. V. (1996). Uncertainty in fault tree analysis: A fuzzy approach. *Fuzzy Sets and Systems, 83*(2), 135-141. doi:DOI: 10.1016/0165-0114(95)00386-X.

Thacker, B., and Huyse, L. (2003). Probabilistic assessment on the basis of interval data. *44th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference*, Norfolk, Virginia.

Vesely, W. E., Goldberg, F. F., Roberts, N. H., and Haasl, F. D. (1981). *Fault tree handbook*. Washington, DC, USA: U.S. Nuclear Regulatory Commission.

Vose, D. (2008). *Risk Analysis: A Quantitative Guide*. New York, USA: John Wiley & Sons, Ltd., West Sussex, England.

Wang, Y. (2004). *Development of a computer-aided fault tree synthesis methodology for quantitative risk analysis in the chemical process industry*. (Doctor of Philosophy, Texas AandM University)., 1-164.

Wang, Y., Yang, J., Xu, D., and Chin, K. (2007). On the combination and normalization of interval-valued belief structures. *Information Sciences, 177*(5), 1230-1247.

Wilcox, R. C., and Ayyub, B. M. (2003). Uncertainty modeling of data and uncertainty propagation for risk studies. *Uncertainty Modeling and Analysis, International Symposium on,* 184.

Wu, H. (2004). Fuzzy reliability estimation using bayesian approach. *Computers and Industrial Engineering, 46*(3), 467-493. doi:DOI: 10.1016/j.cie.2004.01.009.

Yang, Z., Suzuki, K., Shimada, Y., and Sayama, H. (1995). Fuzzy fault diagnostic system based on fault tree analysis. *Fuzzy Systems, International Conference on Fuzzy Systems and the Second International Fuzzy Engineering Symposium, 1*, 165-170. doi:10.1109/FUZZY.1995.409676.

Yang, X., Rogers, W. J., and Mannan, M. S. (2010). Uncertainty reduction for improved mishap probability prediction: Application to level control of distillation unit. *Journal of Loss Prevention in the Process Industries, 23*(1), 149-156. doi:DOI: 10.1016/j.jlp.2009.07.006.

Yuhua, D., and Datao, Y. (2005). Estimation of failure probability of oil and gas transmission pipelines by fuzzy fault tree analysis. *Journal of Loss Prevention in the Process Industries, 18*(2), 83-88. doi:DOI: 10.1016/j.jlp.2004.12.003.

Zadeh, L. (1965). Fuzzy sets. *Fuzzy Sets. Information and Control, 8*, 338-353.

CHAPTER 3

Handling Data Uncertainties in Event Tree Analysis

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Preface

The manuscript developed for this chapter provides an extensive review of different types and sources of uncertainties, and theories to handle the uncertainties for ETA. Based on the review, a quantitative framework with two different approaches has been developed for handling *data* uncertainty in ETA. A version of this manuscript has been published in the *Journal of Process Safety and Environmental Protection.*

The co-authors, Drs Khan, Sadiq, Amyotte and Veitch, motivated the principal author, Refaul Ferdous, to develop the research on the entitled topic and helped him to conceptualize the techniques and theories available for this topic. The principal author conducted an extensive literature review and developed the overall concepts and framework, and identified the limitations and challenges in current techniques. In addition, he also carried out a case study to demonstrate the utility of the developed approaches and framework in an industrial example, and wrote a manuscript for this topic. The co-authors reviewed the approaches and manuscript, and provided the necessary suggestions and comments for the manuscript.

Abstract

Event Tree Analysis (ETA) is an established risk analysis technique to assess likelihood (in a probabilistic context) of an accident. The objective data available to estimate the likelihood is often missing (or sparse), and even if available, is subjected to incompleteness (partial ignorance) and imprecision (vagueness). Without addressing incompleteness and imprecision in the available data, ETA and subsequent risk analysis give a false impression of precision and correctness that undermines the overall credibility of the process. This paper explores two approaches to address data uncertainties, namely, fuzzy sets and evidence theory, and compares the results with Monte Carlo simulations. A fuzzy-based approach is used for handling imprecision and subjectivity, whereas evidence theory is used for handling inconsistent, incomplete and conflicting data. Application of these approaches in ETA is demonstrated using a example of an LPG release near a processing facility.

Keywords: Data uncertainties, fuzzy-based approach, evidence theory, event tree analysis, and Monte Carlo simulations.

3.1 Introduction

Event Tree Analysis (ETA) represents a logic combination of various events that may follow from an initiating event (e.g., an accident event such as LPG release). The initiating event of the tree uses dichotomous conditions, i.e., success/ failure (true/false or yes/no) to propagate the event consequence in different branches of the tree (AIChE, 2000; Lees, 2005). Each individual path that is followed by the different branches eventually identifies the possible outcome events via developing an event-consequence model. In risk analysis, the event-consequence model and the outcome events are successively used in pre-incident application, to examine the incident precursors and post-incident application, and to identify the possible hazards (outcome events) for an accidental event (CMPT, 1999; AIChE, 2000).

Qualitative analysis in an event tree identifies the possible outcome events of an initiating event, whereas quantitative analysis estimates the outcome event probability or frequency (likelihood) for the tree. Traditionally, quantitative analysis of an event tree uses crisp probabilities of events to estimate the outcome event probability or frequency (Kenarangui, 1991; Lees, 2005; Ferdous, 2006). In practice, it is difficult and expensive to obtain precise estimates of event probability because in a majority of cases these estimates are the result of an expert's limited knowledge, incomplete information, poor quality data or imperfect interpretation of a failure mechanism. These unavoidable issues impart uncertainties in the ETA and make the entire risk analysis process less credible for decision-making.

In a general taxonomy of uncertainty, *aleatory* and *epistemic* uncertainties are the major classes (Thacker *et al.*, 2003; Ayyub *et al.*, 2006). Aleatory uncertainty accounts for natural variation or randomness in the behavior of a system and in the case of data availability, probability-based approaches are found to be the best choice (Agarwal *et al.*, 2004). On the other hand, epistemic uncertainty accounts for ambiguity and vagueness that arises due to incompleteness and imprecision. To describe uncertainties in input data (i.e., event likelihood) and propagate them through ETA, probability-based approaches such as Monte Carlo simulations (MCS) have been traditionally used (Bae *et al.*, 2004). This approach requires sufficient empirical information to derive probability density functions (PDFs) of the input data, which are generally not available (Wilcox *et al.*, 2003). As an alternative to objective data, expert knowledge/judgment is used, especially when the data collection is either difficult or very expensive (Rosqvist, 2003).

Expert judgments are qualitative/linguistic in nature and may suffer from inconsistency if lack of consensus among various experts arises. The classical probabilistic framework is not very effective to deal with vague or incomplete/ inconsistent concepts (Druschel *et al.*, 2006). Abrahamsson (2002), Thacker *et al.* (2003) and Wilcox *et al.* (2003) discussed methods to handle uncertainties in expert judgment and to interpret them for the purpose of conducting risk analysis. Fuzzy sets and evidence theory have proven effective and efficient in handling these types of uncertainties (Cheng, 2000; Sentz *et al.*, 2002; Wilcox *et al.*, 2003; Bae *et al.*, 2004; Agarwal *et al.*, 2004; Ayyub *et al.*, 2006).

The main focus of this paper is to describe different types of uncertainties in ETA using approaches such as fuzzy set theory and evidence theory, where the former is employed to deal with linguistic/subjective uncertainties of event probabilities, and the latter is used to handle incomplete/partial ignorance of expert knowledge (Figure 3.1). To demonstrate the applicability of these approaches, a case study for LPG release is revisited (Lees, 2005).

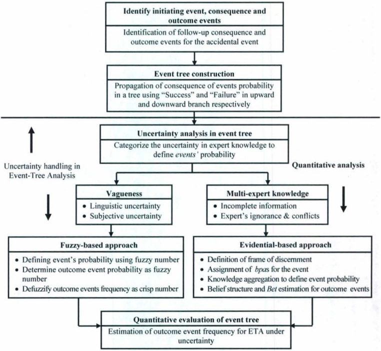

Figure 3.1: Framework for ETA under uncertainty

3.2 Fuzzy set theory

Zadeh (1965) first introduced *fuzzy sets* in his pioneering work, where he argued that probability alone is insufficient to represent all types of uncertainties because it lacked the ability to model human conceptualizations of the real world. Fuzzy-based approaches introduce robustness into systems by allowing a certain amount of imprecision to exist, thus paving the way to represent human linguistic terms as fuzzy sets, hedges, predicates and quantifiers (Rivera *et al.*, 1999). During the last ~45 years the success of fuzzy-based systems has led to their general acceptance in various engineering disciplines.

Fuzzy logic provides a language with syntax and semantics to translate qualitative knowledge/judgments into numerical reasoning. In many engineering problems, the information about the probabilities of various risk items is vaguely known or assessed. The term *computing with words* has been introduced Zadeh (1996) to explain the notion of reasoning linguistically rather than with numerical quantities.

Fuzzy-based approaches help in addressing deficiencies inherent in binary logic. They effectively deal with imprecision that arises due to subjectivity/vagueness, and are helpful to propagate uncertainties throughout the risk analysis and decision-making process. Fuzzy-based approaches are a generalized form of interval analysis used to address uncertain or imprecise information. A fuzzy number describes the relationship between an uncertain quantity p (e.g., event probability) and a membership function μ, which ranges between 0 and 1. A fuzzy set is an extension of the traditional set theory (in which p is either a member of set P or not) so that p can be a member of set P with a certain degree of membership μ. Any shape of a fuzzy number is possible, but the selected shape should be justified by available information (if it is normal, bounded and convex). Generally, triangular or trapezoidal fuzzy numbers (TFN or ZFN) are used for representing linguistic variables (Kenarangui, 1991; Rivera and Baron, 1999).

The following sub-sections describe the steps to analyze an event tree using fuzzy set theory. In the proposed approach, the subjective judgment of event probability is assumed linguistic and described using a TFN. The fuzzy probabilities of initiating are then used to estimate the outcome event probability that is also estimated as a fuzzy number. The fuzzy-based approach used for ETA comprises the following three steps:

1. define event probability using TFNs,
2. determine outcome event probability as a TFN, and
3. defuzzify outcome event frequency as a crisp number (point estimate)

3.2.1 Define event probability using TFNs (fuzzy numbers)

Experts prefer to use linguistic expressions (such as *likely*, *probable*, *improbable*) rather than numerical expressions to justify the *probability* of an event (Ayyub *et al.*, 2006). An expert's linguistic judgment is assigned a TFN. A typical TFN for an uncertain quantity (e.g., event probability) is shown in Figure 3.2. The TFN is a vector (p_L, p_m, p_R) that represents the minimum, most likely and maximum values of event probability, whereas the α-cut level is a degree of membership μ_P. For a TFN, nested intervals $\tilde{P}_\alpha$ can be generated by incrementally changing the α-cut levels as follows:

$$\widetilde{P}_\alpha = \{\alpha_i, p_{Li}, p_{Ri}\} \qquad i = 1, 2, \ldots, n \tag{3.1}$$

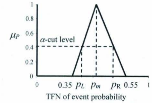

Figure 3.2: TFN to represent event probability

The present study used eight qualitative grades represented by TFNs (Figure 3.3) to express the linguistic probabilities. The eight grades are *Highly improbable* (HI), *Very improbable* (VI), *Rather improbable* (RI), *Improbable* (I), *Probable* (P), *Rather probable* (RP), *Very probable* (VP), and *Highly probable* (HP).

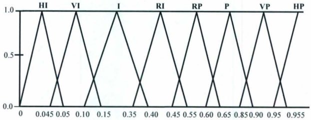

Figure 3.3: Mapping linguistic variables on fuzzy scale

3.2.2 Determine outcome event probability as a TFN (fuzzy number)

Membership function (μ_P) $\in[0, 1]$ of a TFN represents uncertainty in the event probability (Li, 2007). The α-cuts are used to determine fuzzy intervals (i.e., nested intervals in a fuzzy number) with a membership grade (μ_P) greater or equal to the α-cut value (Wilcox *et al.*, 2003). In a TFN, the membership function uses the following relationship to determine the interval at the α-cut level:

$$\widetilde{P}_{\alpha} = [p_L + \alpha(p_m - p_L), p_R - \alpha(p_R - p_m)] \tag{3.2}$$

Fuzzy arithmetic operations are used to determine the outcome event probability, which is based on the *extension principle* (Rivera *et al.*, 1999). An alternative method α-cut formulation is also used in fuzzy arithmetic for simplifying the analysis (Lai *et al.*, 1993; Siler *et al.*, 2005; Li, 2007). In ETA, the membership function (μ_P) representing the degree of uncertainty in event probability can either be the same or different for the events in a specific path. This study uses two methods, the *random α-cut* and *predefined α-cut*, to describe these situations for ETA. The former method uses the *extension principle* and the later method uses α-cut formulation to calculate the outcome event probability. ETA essentially requires two operations, multiplication and addition, to calculate the outcome event probability (Rivera *et al.*, 1999). For event probabilities P_1 and P_2 (represented by two TFNs), the fuzzy arithmetic operations of these two methods are described in Table 3.1.

Table 3.1: Fuzzy arithmetic for event tree analysis

Fuzzy arithmetic	Operation	Equations
Extension principle	$\widetilde{P}_1 \times \widetilde{P}_2$	$[(p_i \times p_j, \min\{\mu_{P_1}(p_i), \mu_{P_2}(p_j)\})], \quad p_i \in P_1, p_j \in P_2$
	$\widetilde{P}_1 + \widetilde{P}_2$	$[(p_i + p_j, \min\{\mu_{P_1}(p_i), \mu_{P_2}(p_j)\})], \quad p_i \in P_1, p_j \in P_2$
α-cut formulation	$\widetilde{P}_{1\alpha} \times \widetilde{P}_{2\alpha}$	$[p_{1L} \times p_{2L}, p_{1R} \times p_{2R}]$
	$\widetilde{P}_{1\alpha} + \widetilde{P}_{2\alpha}$	$[p_{1L} + p_{2L}, p_{1R} + p_{2R}]$

3.2.3 Defuzzify outcome event frequency as a crisp number (point estimate)

Defuzzification transforms a fuzzy number into a crisp value (Klir *et al.*, 2001). Many defuzzification methods are available in the literature (e.g., Klir *et al.*, 2001; Ross, 2004). The weighted average method is a computationally efficient method (Ross, 2004; Khan *et al.*, 2005). The following equation is used for defuzzification of outcome event probability or frequency.

$$P_{out} = \frac{\sum \left[\mu_P(\widetilde{P}) \cdot \widetilde{P}\right]}{\sum \mu_P(\widetilde{P})} \tag{3.3}$$

3.3 Evidence theory (Evidential reasoning)

Multiple expert (multi-expert) knowledge can provide more reliable information for an observation (e.g., an event probability) than a single expert. The knowledge and ignorance cannot be absolute, are socially constructed and negotiated (Ayyub, 2001), and often suffer from incompleteness and conflict. These uncertainties in knowledge acquisition can be minimized through a proper aggregation process that leads to consensus and an agreement in multi-experts knowledge.

Event tree analysis takes into account the degree of ignorance and degree of disagreement (conflicts), while aggregating expert knowledge from multiple sources. A Bayesian approach and evidence theory are widely known in risk analysis for this purpose and play an important role in the management of uncertainties, especially where multi-expert knowledge is desired in a decision-making process (Yang *et al.*, 2004). The Bayesian approach is based on probability theory; it aggregates data without differentiating aleatory and epistemic uncertainties. Moreover, it requires priori information which sometimes limits its application to updating existing information (Sadiq *et al.*, 2006). Therefore, when the ignorance or conflicts are significantly high, a Bayesian approach may not properly aggregate multi-expert knowledge. Evidence theory addresses these issues effectively and is able to combine multi-expert knowledge by taking into account ignorance and conflicts through a belief structure (Lefevre *et. al.*, 2002; Bae *et al.*, 2004; Sadiq *et al.*, 2006).

3.3.1 Fundamentals

Evidence theory was first proposed by Dempster (1967, 1968) and later extended by Shafer (1976). This theory is also called Dempster-Shafer Theory (DST) (Sentz *et al.*, 2002; Li, 2007). DST uses three basic parameters, i.e., *basic probability assignment* (*bpa*), *Belief* measure (*Bel*), and *Plausibility* measure (*Pl*) to characterize the uncertainty in a belief structure (Cheng, 2000; Lefevre *et al.*, 2002; Bae *et al.*, 2004). The belief structure represents a continuous interval [*belief, plausibility*] in which true probability may lie. A narrow belief structure indicates more precise probabilities. The main contribution of DST is a combination rule to aggregate multi-expert knowledge according to their individual degrees of belief.

In evidence theory, *frame of discernment* Ω is defined as a set of mutually exclusive elements that allow having a total of $2^{|\Omega|}$ subsets in a power set (*P*), where $|\Omega|$ is the *cardinality* of a *frame of discernment*. For example, if $\Omega = \{T, F\}$, then the power set (*P*) includes four subsets, i.e., $\{\Phi$ (a null set), $\{T\}$, $\{F\}$, and $\{T, F\}\}$, as the cardinality is two. The following discussion builds the fundamentals of DST that are used in this study.

The *basic probability assignment* (*bpa*), sometimes known as belief mass, is denoted by $m(p_i)$. The *bpa* represents the proportion of knowledge to every subset (p_i) of power set (P) such that the sum of the proportion is 1. The *focal elements*, i.e., $p_i \subseteq P$ with $m(p_i) > 0$, collectively represent the acquired knowledge from expert elicitation. The *bpa* can be characterized by the following equations:

$$m(p_i) \rightarrow [0,1] \;\; ; m(\Phi) = 0 \;\; ; \sum_{p_i \subseteq P} m(p_i) = 1 \tag{3.4}$$

The *belief* (*Bel*) measure, sometimes termed as lower bound for a set p_i, is defined as the sum of all the *bpas* of the proper subsets p_k of the set of interest p_i, i.e., $p_k \subseteq p_i$. The relation between *bpa* and *belief measure* is written as:

$$Bel(p_i) = \sum_{p_k \subseteq p_i} m(p_k) \tag{3.5}$$

The upper bound i.e., the *plausibility (Pl)* measure for a set p_i is the summation of *bpas* of the sets p_k that intersect with the set of interest, p_i i.e., $p_k \cap p_i \neq \Phi$. Therefore, the relation can be written as:

$$Pl(p_i) = \sum_{p_k \cap p_i \neq \Phi} m(p_k) \tag{3.6}$$

3.3.2 Rule of combination – making inferences

The knowledge obtained from multiple experts requires aggregation to be used for useful ETA. The combination rules allow aggregating the individual beliefs of multi-experts. The most common combination rule was first proposed by Dempster & Shafer (DS), which is also known as the DS combination rule. Many modifications of the DS rule of combination have been reported. The most common modifications include Yager, Smets, Inagaki, Dubois and Prade, Zhang, Murphy, and more recently Dezert and Smarandache (Sadiq *et al.*, 2006). Detailed discussions on these rules can be found in Dezert and Smarandache (2004).

In this study, DS and Yager combination rules are discussed in detail and compared in the LPG event tree case study. To combine multi-expert knowledge, combination rules use the following orthogonal sum (Equation 3.7).

$$m_{1-n} = m_1 \oplus m_2 \oplus m_3 \oplus \ldots\ldots\ldots\ldots\ldots\ldots \oplus m_n \tag{3.7}$$

where the symbol ⊕ represents operator of combination.

1. DS combination rule:

The DS combination rule uses a normalizing factor (1-*k*) to develop an agreement among the acquired knowledge from multiple sources, and ignore all conflicting evidence through *normalization.* Assuming that the knowledge sources are independent, this combination rule uses AND-type operators (product) (Sadiq *et al.*, 2006). For example, if the m_1 (p_a) and m_2 (p_b) are two sets of evidence for the same event collected from two independent sources, the DS combination rule uses the following relation to combine the evidence.

$$[m_1 \oplus m_2](p_i) = \begin{cases} 0 & for\ p_i = \Phi \\ \dfrac{\sum_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b)}{1-k} & for\ p_i \neq \Phi \end{cases} \tag{3.8}$$

In the above equation, $m_{1\text{-}2}(p_i)$ denotes the combined knowledge of two experts for an event, and k measures the *degree of conflict* between the two experts, which is determined by the factor:

$$k = \sum_{p_a \cap p_b = \Phi} m_1(p_a) m_2(p_b)$$

2. Yager combination rule:

Zadeh (1984) pointed out that the DS combination rule yields counterintuitive results and exhibits the numerical instability if conflict is large among the sources (Sentz et al., 2002). To resolve this issue, Yager (1987) proposed an extension of the combination rule. The modified combination rule is similar to the DS combination rule except that it assigns conflicting mass to be part of ignorance Ω instead of normalization. However, in no (or less) conflicting cases, the Yager combination rule (Equation 3.9) exhibits similar results as the DS combination rule.

$$[m_l \oplus m_2](p_i) = \begin{cases} 0 & \text{for } p_i = \Phi \\ \sum_{p_a \cap p_b = p_i} m_l(p_a) \times m_2(p_b) & \text{for } p_i \neq \Omega \\ \sum_{p_a \cap p_b = p_i} m_l(p_a) \times m_2(p_b) + k & \text{for } p_i = \Omega \end{cases} \quad (3.9)$$

In ETA, different experts can provide the probability of an event, and each expert uses his/her belief or knowledge to justify the assessment that may be incomplete and be in conflict with the others. In an evidential reasoning framework, partial ignorance refers to assigning probability mass to *frame of discernment*, i.e., {T, F}. The conflict among the sources is handled through combination rules as discussed above. The following sub-sections describe the steps to analyze an event tree using evidential theory.

3.3.3 Definition of frame of discernment

Traditionally the outcomes of event trees are dichotomous, i.e., {T} and {F}. Therefore the *frame of discernment* Ω is {T, F} that leads to four subsets in a power set (*P*) that includes {Φ, {T}, {F}, {T, F}}.

3.3.4 Assignment of *bpas* for the event

The *bpas* or belief mass for each individual event is acquired from the different sources. Explicitly, the assigned *bpas* represents the degree of expert belief for each subset, and implicitly, it represents the total evidence to clarify the event probability. For example, an expert may report that the occurrence probability of an event is 80% true and 10% false. Mathematically, this can be written as $m(\{T\}) = 0.8$, $m(\{F\}) = 0.1$ and $m(\{T, F\}) = 0.1$, because, $m(\{T, F\}) = 1 - m(\{T\}) - m(\{F\})$.

3.3.5 Knowledge aggregation to define event probability

The redundant knowledge from different sources is aggregated using either DS (Equation 3.8) or Yager (Equation 3.9) combination rules. Unlike the DS combination rule, the Yager combination rule does not rely on non-conflicting evidence [i.e., (1-*k*)] to normalize the joint evidence (Sadiq *et al.*, 2006). Thus, for a high conflict case (i.e., higher *k* value), the Yager combination rule gives more stable and robust results than the DS combination rule. In the case of a higher degree of conflict (*k*), the Yager rule of combination is preferred.

Now, consider another expert report of the same event probability with $m(\{T\}) = 0.6$, $m(\{F\}) = 0.3$ and $m(\{T, F\}) = 0.1$. These two independent assessments for the same event can be combined using the DS and Yager combination rules (Table 3.2). The belief structure for the true probability of the event obtained by DS and Yager combination rules are [0.89, 0.9] and [0.62, 0.93], respectively.

Table 3.2: Evidence combination for ignition source probability

m_1 \ m_2		{T} 0.6	{F} 0.3	{T, F} 0.1
{T}	0.8	{T}=0.48	Φ= 0.24	{T}=0.08
{F}	0.1	Φ= 0.06	{F}=0.03	{F}=0.01
{T, F}	0.1	{T}=0.06	{F}=0.03	{T, F}=0.01
k		0.3		
$\sum_{p_a \cap p_b = p_i} m_1(p_a) m_2(p_b)$		0.62	0.07	0.01
$m_{1\text{-}2}$(DS)		0.89	0.1	0.014
$m_{1\text{-}2}$(Yager)		0.62	0.07	0.31

3.3.6 Belief structure and "*Bet*" estimation for outcome events

The interval obtained from the *belief* and *plausibility* measures gives the belief structure of expert knowledge. The belief structure takes into account the ignorance and conflicts in multi-expert knowledge and provides a range for the event probability. *"Bet"* estimate gives a point estimate in belief structure (similar to defuzzification), which can be estimated by the following equation.

$$bet(P) = \sum_{P \subseteq p_i} \frac{m(p_i)}{|p_i|} \tag{3.10}$$

where $|p_i|$ is the cardinality (number of elements) in the set pi. In the continuation of the previous example, the "bet" estimate for the true probability obtained from the DS rule combination can be calculated as:

$$bet(P) = \frac{m(\{T\})}{1} + \frac{m(\{T,F\})}{2} = \frac{0.89}{1} + \frac{0.014}{2} = 0.897$$

The denominators "1" and "2" represent the cardinality in the respective subsets.

3.4 LPG release - an example of event tree analysis

LPG is a highly flammable gas. Any significant amount of LPG release may lead to fire and explosion in the presence of an ignition source. To demonstrate the proposed approaches, a case study of LPG release at a Detergent Alkylate Plant (DAP), which was earlier reported by Lees (2005), is re-visited (Figure 3.4). This study revised the event tree and constructed the tree started from an initiating event of a large LPG release. The released LPG on ignition may cause either an explosion or fireball in the vicinity of the release point. The explosion causes the vapor cloud as an outcome. If there is no ignition, then the release drifts towards the DAP and may cause a delayed explosion at the DAP, which also causes another vapor cloud depending on the wind direction. From the LPG release point, the LPG vapor may drift in some other direction if there is no explosion at the DAP. Four events are identified: ignition, explosion, wind to DAP, and a delayed explosion at DAP. It was assumed that these events are mutually exclusive, and the event probabilities are propagated into the different branches of the tree. Each branch generates a path that may lead to a specific outcome event.

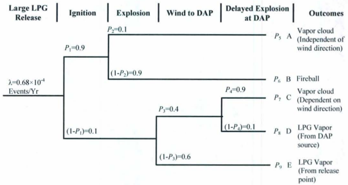

Figure 3.4: Event tree for LPG release

For the case study of LPG release in the vicinity of the DAP, five possible outcome events were identified. Assuming the events are independent; the probability of a path or an outcome event is calculated by multiplying the probabilities associated with this path. Equation 3.11 is a general equation to calculate the outcome event frequencies. λ_i in this equation denotes the frequency for the initiating event and outcome events.

$$\lambda_i = \lambda \times \prod_{i=1}^{n} P_i \qquad (3.11)$$

In addition to the proposed approaches (fuzzy-based and evidence theory), Monte Carlo simulations (traditional uncertainty analysis) and a deterministic approach (traditional crisp analysis) were also performed for ETA of LPG release.

3.4.1 Deterministic approach

This traditional approach provides a quick analysis and uses crisp probabilities in each branch or path of the event tree. It uses Equation 3.11 to calculate the outcome event frequency for the event tree. Based on assigned probabilities (Figure 3.4) the outcome event frequencies for LPG release are calculated, which are crisp numbers (Table 3.3).

Table 3.3: Outcome event frequency of the LPG release event tree

Outcome Event	Frequency (λ)	Events/year
A	λ_5	6.10E-06
B	λ_6	5.50E-05
C	λ_7	2.40E-06
D	λ_8	2.70E-07
E	λ_9	4.10E-06

3.4.2 MCS-based approach

Monte Carlo Simulation (MCS) is one of the most common techniques for probability-based uncertainty analysis (Abrahamsson, 2002). It is based on random sampling from predefined PDFs (in our study, triangular shape PDFs are used similar to TFNs). We used 5000 iterations to obtain PDFs of the outcome events. The frequencies for the outcome events were calculated using Equation 3.11. The 90% confidence intervals for all outcome events of the LPG event tree are summarized in Table 3.4. This approach assumes that uncertainties arise only due to randomness in the occurrence of events.

Table 3.4: Outcome event frequency by MCS-based approach

Outcome Event	90% confidence interval		Median value (at 50%)
	Lower Bound	Upper Bound	
A	1.958E-06	1.024E-05	6.100E-06
B	4.935E-05	6.090E-05	5.512E-05
C	7.353E-07	4.151E-06	2.443E-06
D	3.057E-10	5.467E-07	2.736E-07
E	1.300E-06	6.850E-06	4.080E-06

3.4.3 Fuzzy-based approach

The revised event tree with fuzzy linguistic variables is illustrated in Figure 3.5. This approach uses the two different methods, namely, *predefined α-cut* and *random α-cut* to perform fuzzy arithmetic.

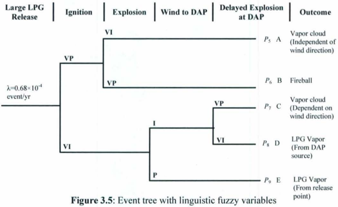

Figure 3.5: Event tree with linguistic fuzzy variables

3.4.3.1 Predefined α-cut

In this method, a preferred α-cut level (i.e., a pre-defined membership function) is maintained through all the events. The probabilities for the outcome events are then estimated using α-cut based fuzzy formulation (Table 3.1). For example, the path leading to the outcome event "A", shown in Figure 3.5, is followed by two events. The probabilities of these two events are linguistically expressed and assumed to be "Very Improbable" and "Very Probable". These two variables are assigned TFNs (based on Figure 3.3), and then the TFN for outcome event "A" is calculated. The TFNs of the input events and outcome event "A" for the LPG release event tree are shown in Figure 3.6. At a specific α-cut level, the TFN for the outcome events is defuzzified to obtain the crisp probability for the event. Table 3.5 provides the defuzzified frequencies of outcome events for the LPG release event tree.

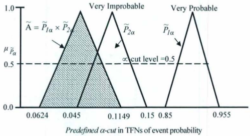

Figure 3.6: Outcome event probability for "A"

Table 3.5: Defuzzified outcome event frequency of LPG release event tree

α-cut level	Defuzzified outcome events frequency (events/yr)				
	A	B	C	D	E
0.10	6.135E-06	5.554E-05	2.284E-06	3.027E-07	5.406E-06
0.30	6.075E-06	5.548E-05	2.095E-06	2.577E-07	5.235E-06
0.50	6.030E-06	5.543E-05	1.915E-06	2.217E-07	5.106E-06
0.70	6.000E-06	5.540E-05	1.743E-06	1.932E-07	5.021E-06
0.90	5.985E-06	5.539E-05	1.577E-06	1.709E-07	4.978E-06
1.00	5.984E-06	5.539E-05	1.496E-06	1.616E-07	4.973E-06

3.4.3.2 Random α-cut

In this method, the membership functions (μ_P) for the events in a path are changed randomly (as with MCS). The outcome event probability that is followed by this path is calculated using fuzzy arithmetic. An example for this case is shown in Figure 3.7. In the same way, the outcome event frequencies for the event tree are estimated using Equation 3.11. For demonstration purposes, the fuzzy interval for the outcome events "B" is shown in Figure 3.8.

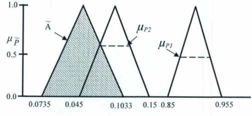

Figure 3.7: Outcome event probability for "A"

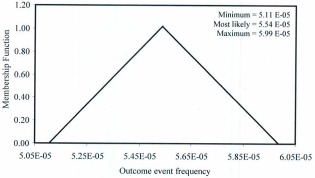

Figure 3.8: Fuzzy intervals for out come event "B"

3.4.4 Evidence theory-based approach

Table 3.6 provides evidence obtained from two unbiased and independent experts. The belief structure for the outcome events of LPG release is provided in Table 3.7. The belief structures obtained for the outcome event "B" by using both combination rules are plotted in Figure 3.9. Figure 3.9 shows that for the same outcome event "B", the Yager combination rule provides a larger belief structure for the outcome event "B" than the DS combination rule.

Table 3.6: Different expert's knowledge for events

Events	m_1			m_2		
	{T}	{F}	{T, F}	{T}	{F}	{T, F}
Ignition	0.8	0.1	0.1	0.6	0.3	0.1
Explosion	0.1	0.8	0.1	0.05	0.8	0.15
Wind to DAP	0.4	0.5	0.1	0.5	0.4	0.1
Ignition Explosion at DAP	0.85	0.1	0.05	0.8	0.1	0.1

Table 3.7: Belief structure for the outcome events

Outcome Event		Outcome event frequency (event/yr)			
		DS		Yager	
		Belief	Plausibility	Belief	Plausibility
A-	HF release	1.71E-06	2.78E-06	1.05E-06	1.01E-05
B-	Fireball	5.75E-05	5.95E-05	3.54E-05	6.17E-05
C-	HF release drifting north-east	3.22E-06	3.83E-06	1.11E-06	1.79E-05
D-	Drifting Cloud	1.00E-07	1.42E-07	3.45E-08	3.58E-06
E-	Drifting Cloud	3.34E-06	3.95E-06	1.38E-06	1.83E-05

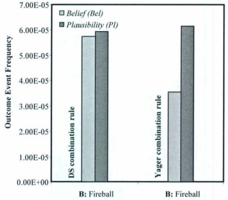

Figure 3.9: Belief structure of outcome event "B

3.5 Summary and conclusions

Uncertainty in ETA arises due to subjectivity, incompleteness (partial ignorance) or inconsistency in acquired knowledge of event probabilities. The proposed framework (Figure 3.1) uses two different approaches, fuzzy-based and evidence theory, to address different types of uncertainties that are not generally addressed explicitly using the available approaches. The traditional approach for ETA is deterministic and does not consider any kind of uncertainty in the analysis. If the PDFs are 'reasonably known', MCS can be the best approach to estimate and propagate uncertainties, especially 2D-MCS which can deal with aleatory and epistemic uncertainties separately (not discussed in this study). Risk analysis generally requires expert knowledge, as PDFs and crisp estimates of event probabilities are unknown or partially known. Neither the deterministic approach nor the MCS-based approach effectively deals with this kind of uncertainty.

In the fuzzy-based approach, the TFNs are assigned to linguistic and subjective judgment of expert knowledge using a membership function μ_P. The *predefined α-cut* method uses intervals based on a predefined membership level in performing fuzzy arithmetic, whereas *random α-cut* uses fuzzy intervals based on random selection of memberships for the events in a specific path.

In the evidence theory- based approach, the *bpas* are assigned to define the degree of ignorance and belief of expert knowledge to clarify event probabilities. The incomplete and inconsistent *baps* from multiple sources are combined by using combination rules of evidence theory. The Yager combination rule yields more robust results in the context of having high conflicts in the sources. Consequently, this rule provides more appropriate results for ETA under uncertainty, leading to lower values for the *belief measure* and higher values for the *plausibility measure* compared to the DS combination rule.

A comparative view of different approaches used to obtain the frequency for the outcome event "B" is shown in Table 3.8. The *percentage deviation* (D) in the results is estimated using a "base value of events probability". The "base value of events probability" refers to the probability of events that do not include any deviation while analyzing the event tree for LPG release. For example, if 10% deviation is introduced in the initiating event probability (LPG release event tree) in the case of the deterministic approach, approximately 9 % deviation in the frequency of the outcome event "B" is observed. In contrast, the fuzzy-based approach gives more robust results, i.e., ~0.003 % deviation for the same (10%) deviation in initiating event probability. The MCS-based approach yields ~0.8 % deviation for the same scenario. The evidence theory-based approach yields ~6 % deviation in estimating frequency for the same event. It is emphasized, however, that evidence theory accounts for expert ignorance in defining the event probability, which cannot be dealt with using the other approaches.

Table 3.8: Estimated deviation in the final results by different approaches

Approaches	**Frequency of outcome event "B"**				**D (% Deviation)**
	Fuzzy interval /Belief structure (with 10% deviation)		**Defuzzification/ "*bet*" estimation/Mean**		
	Left/Belief/ Lower Bound	**Right/Plausibility/ Upper Bound**	**Estimated with 10% deviation**	**Estimated with no deviation**	
Fuzzy-based	5.47E-05	5.60E-05	*5.54E-05	*5.53E-05	0.003%
Evidence theory-based	4.96E-05	2.34E-04	*1.42E-04	*1.50E-04	6.05%
MCS-based	5.50E-05	5.57E-05	*5.53E-05	*5.49E-05	0.80%
Deterministic			5.01E-05	5.51E-05	9.0%

* *Defuzzification for the fuzzy-based approach, the bet estimation for the evidence theory-based approach and mean for the MCS-based approach are used to estimate the outcome event frequency for "B".*

Two aspects of the proposed approaches could be further explored in the future. First, the assumption of 'independence' among events is often unrealistic, and can be handled using 'fuzzy measures' or extensions of the DS rule of combinations. Second, the possibility of dealing with subjectivity (using fuzzy-based approach) and incompleteness (evidence theory) as a single formulation (hybrid soft computing methods, e.g., Fuzzy-Dempster-Shafer) can assist in developing a more generic framework for ETA.

References

Abrahamsson, M. (2002), "Uncertainty in Quantitative Risk Analysis –Characterization and Methods of Treatment" *Report # 1024(ISSN: 1402-3504), Department of Fire Safety Engineering, Lund University, Sweden.*

Agarwal, H., Renaud, E. J., Preston, L.E. and Padmanabhan, D. (2004), "Uncertainty quantification using evidence theory in multidisciplinary design optimization", *Reliability Engineering and System Safety 85 (2004) 281–294, Publisher Elsevier Ltd.*

AIChE, (2000), "Guidelines for chemical process quantitative risk analysis", *Second Edition, New York: AIChE.*

Ayyub, B. (2001), "A Practical Guide on conducting expert-opinion elicitation of probabilities and consequences for corps facilities", *Prepared for U.S. Army Corps of Engineers Institute for Water Resources Alexandria, VA 22315-3868.*

Ayyub, B. and Klir, J. G. (2006), "Uncertainty Modeling and Analysis in Engineering and the Sciences", *Published by Chapman & Hall/CRC.*

Bae, H., Grandhi, V. R. and Canfield, A. R. (2004), "An approximation approach for uncertainty quantification using evidence theory", *Reliability Engineering and System Safety 86: 215–225.*

Cheng, Y. (2000), "Uncertainties in Fault Tree Analysis", *Tamkang Journal of Science and Engineering, 3(1), 23-29.*

CMPT, (1999), "A Guide to Quantitative Risk Assessment for Offshore Installations", The Centre of Marine And Petroleum Technology, UK.

Dezert, J. and Smarandache, F. (2004), "Presentation of DSmT. Chapter 1in advances and applications of DSmT for information fusion (collected works)", American Research Press, Rehoboth, 3–35.

Druschel, R. B., Ozbek, M. and Pinder, G. (2006), "Application of Dempster-Shafer theory to hydraulic conductivity", *CMWR – XVI, Conference Program, Copenhagen, Denmark, V, 31-33.*

Ferdous, R. (2006), "Methodology for Computer Aided Fuzzy Fault Tree Analysis", *Thesis Submitted To Memorial University of Newfoundland, Canada, in Partial Fulfillment of the Requirements for the Degree of Master of Engineering.*

Kenarangui, R. (1991), "Event-Tree Analysis by Fuzzy Probability", *IEEE Transactions on Reliability, 40(1):12-124.*

Khan, I F. and Sadiq R. (2005), "Risk-Based Prioritization of Air Pollution Monitoring Using Fuzzy Synthetic Evaluation Technique" *Environmental Monitoring and Assessment 105: 261–283, Springer.*

Klir, J. G. and Yuan, B. (2001), "Fuzzy Sets and Fuzzy logic Theory and Applications", *Prentice, Hall of India Private Ltd.*

Lai, F. S., Shenoi, S. and Fan, T.L. (1993), "Fuzzy fault tree analysis theory and applications", *Engineering Risk and Hazard Assessment, CRC Press Inc., Florida, V1, 117-137.*

Lefevre, E., Colot, O. and Vannoorenberghe, P. (2002), "Belief function combination and conflict management", *Information Fusion 3 (2002) 149–162, Elsevier Science.*

Li, H. (2007), "Hierarchical Risk Assessment of Water Supply Systems", *Submitted for the Degree of Doctor of Philosophy, Loughborough University, UK.*

Lees, F. P. (2005), "Loss prevention in the process industries", *Third Edition, 1, London: Butterworths, P-9/05-9/122.*

Rivera, S.S. and Barón, J.H. (1999), "Using Fuzzy Arithmetic in Containment Event Trees", *International Conference on Probabilistic Safety Assessment- PSA 99, Washington, USA, 22-25.*

Rosqvist, T. (2003), "On use of experts judgment in the quantification of risk assessment", *Dissertation for the degree of Doctor of Technology, Helsinki University of Technology, Espoo, Finland.*

Ross J. T. (2004),"Fuzzy Logic with Engineering Applications" *John Wiley & Sons, Ltd, West Sussex, England*

Sadiq, R., Najjaran, H. and Kleiner, Y. (2006), "Investigating evidential reasoning for the interpretation of microbial water quality in a distribution network", *Stoch. Environ. Res Risk Assess (2006) 21: 63–73.*

Sentz, K. and Ferson, S. (2002), "Combination of evidence in Dempster–Shafer theory", *SAND 2002–0835.*

Siler, W. and Buckley, J. J. (2005), "Fuzzy Expert Systems and Fuzzy Reasoning", *John Wiley & S*ons, Inc.., Hoboken, New Jersey.

CHAPTER 4

Fault and Event Tree Analyses for Process Systems Risk Analysis: Uncertainty Handling Formulations

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Preface

In the first few sections of the manuscript, the traditional assumptions and techniques along with the associated uncertainty issues in ETA and FTA are discussed. The subsequent sections describe the development of proposed approaches to overcome the current limitations and uncertainty issues for FTA and ETA. A version of this manuscript has already been published in the *Journal of Risk Analysis*

All authors worked as a team in developing the research and manuscript for this chapter. The principal author conceptualized the problem based on an extensive literature review, and developed the framework and approaches for ETA and FTA with the help of the other team members. The application of the developed approaches has also been illustrated by the principal author through two separate industrial examples.

The co-authors, Dr(s) Khan, Sadiq, Amyotte and Veitch, supervised and critically reviewed the approaches and their application to the process facility. They also provided valuable comments and corrections to improve the quality of the manuscript.

Abstract

Quantitative risk analysis (QRA) is a systematic approach for evaluating likelihood, consequences, and risk of adverse events. QRA based on Event (ETA) and Fault Tree Analyses (FTA) employs two basic assumptions. The first assumption is related to likelihood values of input events, and the second assumption is regarding interdependence among the events (for ETA) or basic-events (for FTA). Traditionally FTA and ETA both use crisp probabilities; however, to deal with uncertainties, the probability distributions of input event likelihoods are assumed. These probability distributions are often hard to come by and even if available, they are subject to incompleteness (partial ignorance) and imprecision. Furthermore, both FTA and ETA assume that events (or basic-events) are independent. In practice, these two assumptions are often unrealistic.

This article focuses on handling uncertainty in a QRA framework of a process system. Fuzzy set theory and evidence theory are used to describe the uncertainties in the input event likelihoods. A method based on a dependency coefficient is used to express interdependencies of events (or basic-events) in ETA and FTA. To demonstrate the approach, two case studies are discussed.

Keywords: Quantitative risk analysis (QRA), uncertainty, interdependence, likelihoods, fault tree analysis (FTA) and event tree analysis (ETA).

4.1 Introduction

Process systems in chemical engineering are infamous for fugitive emissions, toxic releases, fire and explosions, and operation disruptions. These incidents have considerable potential to cause an accident and incur environmental and property damage, economic loss, sickness, injury or death of workers in the vicinity. QRA is a systematic approach that integrates quantitative information about an incident and provides detailed analysis that helps to minimize the likelihood of occurrence and reduces its adverse consequences. QRA for process systems is a difficult task as the failures of components and the consequences of an incident are randomly varied from process to process. Further, for a process system comprised of thousands of components and steps, it is difficult to acquire the quantitative information for all components (Ferdous *et al.*, 2009a). Finally, the interdependencies of various components are not known and are generally assumed to be independent for the purpose of simplicity.

Event Tree Analysis (ETA) and Fault tree Analysis (FTA) are two distinct methods for QRA that develop a logical relationship among the events leading to an accident and estimate the risk associated with the accident. The term "event" is frequently used in place of the term "accident" in the analyses of fault trees and event trees for QRA (Spouge, 1999). ETA is a technique used to describe the consequences of an event (initiating event) and estimate the likelihoods (frequency) of possible outcomes of the event. FTA represents basic causes of occurrence of an unwanted event and estimates the likelihood (probability) as well as the contribution of different causes leading to the unwanted event. In FTA, the basic causes are termed basic events, and the unwanted event is called the top event (Haasl, 1965; Vesely et al., 1981; Hauptmanns, 1980, 1988). Kumamoto and Henley (1996) provide a detailed description of fault tree development and analysis for a process system.

In the event tree, the unwanted event is named as an initiating event, and the follow-up consequences are termed as events or safety barriers (AIChE , 2000). The ETA represents the dichotomous conditions (e.g., success/ failure, true/ false or yes/no) of the initiating until the subsequent events lead to the final outcome events (AIChE, 2000; Andrews and Dunnett, 2000, Ferdous *et al.,* 2009b). AIChE (2000) and Lees (2005) provide a detailed procedure for constructing and analyzing the ETA for a process system.

Event and fault trees help to conduct the QRA for process systems based on two major assumptions (Spouge, 1999). Firstly, the likelihood of events or basic-events is assumed to be exact and precisely known, which is not very often true due to inherent uncertainties in data collection and defining the relationships of events or basic-events Sadiq *et al.,* 2008, Ferdous *et al.,* 2009a). Moreover, because of variant failure modes, design faults, poor understanding of failure mechanisms, as well as the vagueness of system phenomena, it is often difficult to predict the acquired probability of basic-events/events precisely (Yuhua and Datao, 2005). Secondly, the interdependencies of events or basic-events in an event tree or fault tree are assumed to be independent, which is often an inaccurate assumption (Ferson *et al.,* 2004). These two assumptions indeed misrepresent the actual process system behaviors and impart two different types of the uncertainty, namely *data uncertainty* and *dependency uncertainty*, while performing the

QRA using FTA and ETA. In an attempt to circumvent the *data uncertainty* in risk analysis, a number of research works have been developed by Tanaka *et al.* (1983); Misra and Weber (1990); Singer (1990); Kenarangui (1991); Sawyer and Rao (1994); Suresh *et al.* (1996); Rivera and Barón (1999); Huang *et al.* (2001); Wilcox and Ayyub, (2003); Yuhua and Datao (2005) and Ferdous *et al.* (2009a, 2009b) to facilitate the accommodation of expert judgment/ knowledge in quantification of the likelihood of the basic-events/events for QRA. Sadiq *et al.* (2008), Ferson *et al.* (2004) and Li (2007) proposed methods to describe the *dependency uncertainty* among the basic-events/ events.

Fuzzy-based and evidence theory-based formulations have been proposed and developed to address data and dependency uncertainties in FTA and ETA. The interdependencies among the events (or basic-events) are described by incorporating a dependency coefficient into the fuzzy- and evidence theory-based formulations for FTA/ ETA. Expert judgment/ knowledge can be used to quantify the unknown or partially known likelihood and dependency coefficient of the events (or basic-events).

4.2 Fault and event tree analyses in process systems

The traditional fault and event trees can be analyzed either deterministically or probabilistically. The deterministic approach uses the crisp probability of events (or basic-events) and determines the probability of the top-event and the frequency of outcome events in the fault and event trees, respectively. The probabilistic approach treats the crisp probability as a random variable and describes uncertainty using probability density functions (PDF) (Sursh *et al.,* 1996; Wilcox and Ayyub, 2003; and Ferdous *et al.,* 2009b). Traditionally, the probabilistic approach uses Monte Carlo

Simulation (MCS) to address the random uncertainty in the inputs (i.e., probability of basic-events or events) and propagate the uncertainty for the outputs (Abrahamsson, 2002). The PDFs for the inputs can be derived from historical information, but are often rare especially when the process system is comprised of thousands of components (Kenarangui, 1991)

With an assumption that the events (or basic-events) are independent, deterministic and probabilistic approaches use the equations in Table 4.1 to analyze the fault and event trees. P_i denotes the probability of i^{th} (i =1, 2, 3...n) events (or basic-events), P_{OR} and P_{AND} respectively denotes the "OR" and "AND" gate operations, and λ_i denotes the frequency for the initiating event and the outcome events.

Table 4.1: Equations uses in traditional FTA and ETA

QRA method	Equation
ETA	$\lambda_i = \lambda \times \prod_{i=1}^{n} P_i$
FTA	$P_{OR} = 1 - \prod_{i=1}^{n} (1 - P_i)$
	$P_{AND} = \prod_{i=1}^{n} P_i$

Two examples - an event tree for "LPG release" (Figure 4.1) and a fault tree for "Runaway reaction" (Figure 4.2) - are considered to illustrate the use of deterministic and probabilistic approaches in QRA for the process system. The event and fault trees for these two examples were earlier studied respectively in Lees (2005) and Skelton (1997). The deterministic approach provides a quick analysis if the probabilities are known accurately (Ferdous *et al.*, 2009b). Based on assigned probabilities (Figure 4.1 and Table

4.2), the frequency of outcome events for "LPG release event tree" and the probability of top-event for "Runaway reaction fault tree" are calculated as crisp values (Table 4.3). In the probabilistic approach, triangular PDFs are assumed to perform MCS (N = 5000 iterations) and the PDFs for the outcome events' frequency and the top-event probability are determined based on this assumption. The 90% confidence intervals for the outcome events of the "LPG event tree" and top-event of "Runaway reaction fault tree" are summarized in Table 4.4 and Figure 4.3, respectively.

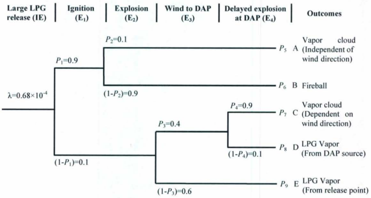

Figure 4.1: Event tree for LPG release

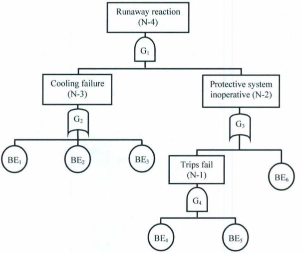

Figure 4.2: Fault tree for runaway reaction in a reactor

Table 4.2: Basic-events causing the runaway reaction

Symbol	Basic-event	Probability of basic-event
BE_1	Pump Fails	0.2
BE_2	Line Block	0.01
BE_3	No Cooling Water	0.1
BE_4	Low Coolant Flow	0.01
BE_5	High Temp	0.01
BE_6	Dump Valve Fails	0.001

Table 4.3: Deterministic results for FTA and ETA

LPG release event tree	Frequency of outcome events (events/yr)	A	B	C	D	E
		6.1E-06	5.5E-05	2.4E-06	2.7E-07	4.1E-06
Runaway reaction fault tree	Probability of top-event	P_{Top} = 3.16E-04				

Table 4.4: Frequency determination of outcome events using MCS

Outcome events	**90% Confidence Interval**		**Median**
	Lower bound (5th percentile)	**Upper bound (95th percentile)**	**(50th percentile)**
A	1.958E-06	1.024E-05	6.100E-06
B	4.935E-05	6.090E-05	5.512E-05
C	7.353E-07	4.151E-06	2.443E-06
D	3.057E-10	5.467E-07	2.736E-07
E	1.300E-06	6.850E-06	4.080E-06

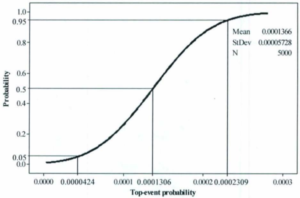

Figure 4.3: 90% confidence interval for top-event probability

4.3 Uncertainty in FTA and ETA

FTA and ETA require probability data of events (or basic-events) as inputs to conduct a comprehensive QRA for a process system. Since most of the time the crisp data as well as PDFs are rarely available for all events and basic-events, expert's judgment/ knowledge are often employed as an alternative to the objective data (Yuhua and Datao, 2005). Two types of uncertainties, namely *aleatory and epistemic uncertainties*, are usually addressed while using the expert's knowledge in QRA Thacker and Huyse, 2003; Ayyub and Klir, 2006; Ferdous *et al.*, 2009b). *Aleatory uncertainty* is a natural variation, randomness or heterogeneity of a physical system. It can be well described using probabilistic methods if enough experimental data are available to support the analysis

(Agarwal *et al.*, 2004). *Epistemic uncertainty* means ambiguity and vagueness, ignorance, knowledge deficiency, or imprecision in system behaviors.

In QRA, it is important to characterize, represent, and propagate the uncertainty accurately in order to get a reliable analysis. However, when the input PDFs are 'reasonably known', MCS can be used to estimate and propagate the uncertainties, especially two dimensional MCS which can effectively deal with both *aleatory and epistemic uncertainties* (not discussed here) (Baraldi and Zio, 2008). If knowledge is limited for definition of the PDFs, probabilistic approaches might not be the best choice to handle the uncertainty in QRA (Druschel *et al.*, 2006). In addition, the independence assumption of events (or basic-events) might be convenient to simplify the FTA or ETA, however it is not always true for all cases (Ferson *et al.*, 2004). This assumption in fact is adding other kind of uncertainty, i.e., the *dependency uncertainty*, during the analyses. Vesely *et al.* (1981) shows several cases of FTA in where the independent assumptions of basic-events are not valid.

Fuzzy set and evidence theories have recently been used in many engineering applications where expert knowledge is employed as an alternative to crisp data or PDFs (Sadiq *et al.* 2008, Wilcox and Ayyub, 2003; Bae *et al.*, 2004; Agarwal *et al.*, 2004; Ayyub and Klir, 2006). Fuzzy set theory is used to address the subjectivity in expert judgment. Whereas, the evidence theory is more promptly employed in handling the uncertainty arise due to ignorance, conflict and incomplete information. In addition to describe the *dependency uncertainty* among the basic-events in FTA, Ferson *et al.* (2004) described the Frank copula and Frechet's limit For known dependency, the Pearson correlation in Frank copula describes the full range of dependencies; i.e., from perfect dependence to opposite dependence (Sadiq *et al.*, 2008). Li (2007) proposed a dependency factor based fuzzy approach to address the dependencies in performing risk analysis. Li (2007) uses fuzzy numbers to define the dependency factor among basic-events.

In this article, the probabilities of events (or basic-events) and their dependency coefficients are treated as fuzzy numbers or *bpas*, which are derived through expert knowledge. Fuzzy set and evidence theories along with dependency coefficient are used to explore the *data* and *dependency uncertainty* in ETA/ FTA. The fuzzy numbers in fuzzy set theory describe linguistic and subjective uncertainty while *bpas* in evidence theory are used to handle ignorance, incompleteness and inconsistency in expert knowledge. A generic framework is shown in Figure 4.4 illustrating the use of fuzzy set theory and evidence theory to handle two different kinds of *uncertainties* in FTA and ETA. The following sections describe the fuzzy set theory and the evidence theory with respect to handling uncertainties.

4.4 Fuzzy set theory

Zadeh (1965) introduced fuzzy sets that have recently been applied where probability theory alone was found insufficient to represent all types of uncertainties. Fuzzy set theory is flexible in describing linguistic terms as fuzzy sets, hedges, predicates and quantifiers (Khan and Sadiq, 2005). Fuzzy set theory is an extension of traditional set theory, which represents imprecise values as fuzzy numbers and characterizes the uncertainty using a continuous membership function (μ).

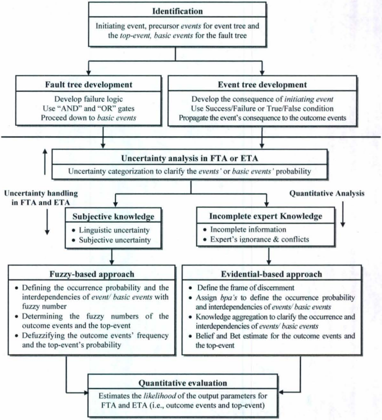

Figure 4.4: Framework for FTA and ETA under uncertainty

4.4.1 Fundamentals

Fuzzy numbers are used to describe the vagueness and subjectivity in expert judgment through a relationship between the uncertain quantity p (e.g., event or basic event probability) and a membership function μ that may range between 0 and 1. Any shape of a fuzzy number is possible, but the selected shape should be justified by available information (but it should be normal, bounded and convex). Generally, triangular or trapezoidal fuzzy numbers (TFN or ZFN) are used for representing linguistic variables (Kenarangui, 1991; El-Iraki and Odoom, 1998; Rivera and Barón, 1999; Cheng, 2000). In this study, we used triangular fuzzy numbers (TFN) in which the fuzzy intervals are derived using α-cuts. Figure 4.5 shows a TFN in which fuzzy intervals are estimated using Equation 4.1. The values p_L, p_m, and p_R below represent the minimum, most likely and maximum values, respectively, in an interval $\widetilde{P}_\alpha$.

$$\widetilde{P}_\alpha = \left[p_L + \alpha(p_m - p_L), p_R - \alpha(p_R - p_m)\right] \quad (4.1)$$

Fuzzy set theory uses the fuzzy arithmetic operations based on α-cut formulation to manipulate fuzzy numbers (Lai *et al.*, 1993; Siler and Buckley, 2005; Li, 2007). Traditional fuzzy arithmetic operations assume that the events (or basic-events) are independent and use equations in Table 4.5 for fault tree and event tree analyses (e.g., Tanaka *et al.*, 1983; Lai *et al.*, 1993; Misra and Weber, 1990; Rivera and Barón, 1999; Kenarangui, 1991; Singer, 1990; Sawyer and Rao, 1994; Suresh *et al.*,1996 and Wilcox and Ayyub, 2003).

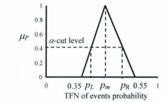

Figure 4.5: TFN to represent the probability of events (or basic-events)

Table 4.5: Traditional α-cut based fuzzy arithmetic operations

Method	Operation	α-cut formulation
	Frequency estimation	$\lambda_i = \lambda \times \prod_{i=1}^{n} (p_{iL}^{\alpha}, p_{iR}^{\alpha})$
ETA	$\widetilde{P}_1 \times \widetilde{P}_2$	$p_L^{\alpha} = \prod_{i=1}^{n} p_{iL}^{\alpha}\,; p_R^{\alpha} = \prod_{i=1}^{n} p_{iR}^{\alpha}$
	$\widetilde{P}_1 + \widetilde{P}_2$	$p_L^{\alpha} = \sum_{i=1}^{n} p_{iL}^{\alpha}\,; p_R^{\alpha} = \sum_{i=1}^{n} p_{iR}^{\alpha}$
FTA	"OR" gate	$p_L^{\alpha} = 1 - \prod_{i=1}^{n} (1 - p_{iL}^{\alpha})\,; p_R^{\alpha} = 1 - \prod_{i=1}^{n} (1 - p_{iR}^{\alpha})$
	"AND" gate	$p_L^{\alpha} = \prod_{i=1}^{n} p_{iL}^{\alpha}\,; p_R^{\alpha} = \prod_{i=1}^{n} p_{iR}^{\alpha}$

4.4.2 Fuzzy-based approach for FTA/ETA

In the proposed fuzzy-based approach, the probability of events (or basic-events) can be defined linguistically and described using TFN. The interdependence of events (or basic-events) is defined linearly using a dependency coefficient (C_d) that can also be described using a TFN. Fuzzy probability and dependency coefficients are used to determine the probability of top-event and the frequency of outcome events in fuzzy terms. The fuzzy-based approach is comprised of the following three steps:

1. Definition of input probability and dependency coefficient using TFN
2. Determination of likelihood of outcome events (ETA) and top-event (FTA) as a TFN
3. Defuzzification

4.4.2.1 Definition of input probability and dependency coefficient using TFN

Experts are more comfortable using linguistic expression rather than numerical judgment when they are asked to define an uncertain quantity like the probability of occurrence of events (or basic-events) and dependency coefficients (Ayyub and Klir, 2006). In order to capture these linguistic expressions, eight linguistic grades are defined in the proposed approach (Figure 4.6). It include: *Very Highly* (VH), *Very Low* (VL), *Moderately High* (MH), *Moderately Low* (ML), *Low* (L), *Moderate* (M), *High* (H), *Rather High* (RH). Theses grades can be used to assign the probability of events (or basic-events) for ETA (or FTA).

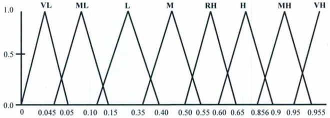

Figure 4.6: Mapping linguistic grades for FTA and ETA

As mentioned earlier, the traditional methods of FTA and ETA assume that the events in an event tree and the basic-events in a fault tree are independent. However, in practice, the interdependencies among the events (or basic-events) could be ranged from *perfectly dependent* to *oppositely dependent*. A scalar quantity ∈ [+1, -1] may describes the dependency between two events, where the scalar quantity +1 refers to *perfect dependence* and -1 refers to *opposite dependence* (Ferson *et al.*,2004). More specifically the positive dependence belongs to an interval [0, +1], whereas the negative dependence belongs to an interval [-1, 0]. However, various levels of dependency are possible in between the events (or basic-events). The current work explores only the positive dependence of events (or basic-events) at each node in FTA (or ETA). Six linguistic grades are used in this study to describe the different levels of interdependencies among the events and basic-events that include: *Perfectly Dependent* (P), *Very Strong* (VS), *Strong* (S), *Weak* (W), *Very Weak* (VW) and *Independent* (I). The left bound (C_{dL}) and the right bound (C_{dR}) in Table 4.6 are representing the TFNs boundary for the dependency coefficients.

Table 4.6: Scale to categorize the interdependence among the basic-events/events

Linguistic grade	Description	Minimum (C_{dL})	Maximum bound (C_{dU})
P	Perfect dependence between the events	1.000	1.000
VS	Very strong dependence , but not fully dependent	0.800	0.995
S	Strong dependence, but not too strong	0.450	0.850
W	Weak dependence, but not too weak	0.150	0.500
VW	Very weak dependence, but not fully independent	0.005	0.200
I	Perfect independence between the events	0.000	0.000

4.4.2.2 Determination of likelihood of outcome event and top-event as a TFN

The dependency coefficient C_d defines the dependence of the events (or basic-events) at each node of a fault and event tree (Table 4.6). The modified fuzzy arithmetic with the empirical relation for FTA and ETA are described in Table 4.7, where $C_d \approx 1$ refers to perfect dependence and $C_d \approx 0$ refers to complete independence among the event (or basic-events).

Table 4.7: Modified α-cut based fuzzy arithmetic operations

Method	Operation	α-cut formulation
ETA	Frequency estimation	$\lambda_i = \lambda \times \prod_{i=1}^{n} (p_{iL}^{\alpha}, p_{iR}^{\alpha})$
	$\tilde{P}_1 \times \tilde{P}_2$	$p_L^{\alpha} = \left[1-(1-C_{dL}^{\alpha})\times(1-p_{1L}^{\alpha})\right]\times p_{2L}^{\alpha}$ $p_R^{\alpha} = \left[1-(1-C_{dR}^{\alpha})\times(1-p_{1R}^{\alpha})\right]\times p_{2R}^{\alpha}$
FTA	$\tilde{P}_1$"OR"$\tilde{P}_2$	$p_L^{\alpha} = \left\{1-(1-p_{1L}^{\alpha})\times\left[1-(1-C_{dL}^{\alpha})\times p_{2L}^{\alpha}\right]\right\}$ $p_R^{\alpha} = \left\{1-(1-p_{1R}^{\alpha})\times\left[1-(1-C_{dR}^{\alpha})\times p_{2R}^{\alpha}\right]\right\}$
	$\tilde{P}_1$"AND"$\tilde{P}_2$	$p_L^{\alpha} = \left\{\left[1-(1-C_{dL}^{\alpha})\times(1-p_{1L}^{\alpha})\right]\times p_{2L}^{\alpha}\right\}$ $p_R^{\alpha} = \left\{\left[1-(1-C_{dR}^{\alpha})\times(1-p_{1R}^{\alpha})\right]\times p_{2R}^{\alpha}\right\}$

4.4.2.3 Defuzzification

Defuzzification transforms the fuzzy number into a crisp value (Klir and Yuan, 2001). The crisp value is useful in evaluating the rank of outcome events' frequency for ETA and calculating the contribution of basic-events leading to the top-event FTA A numbers of defuzzification methods including max membership principle, centroid method, weighted average method, mean max membership, center of sums, center of largest area and first (or last) of maxima, are available in the literature (Klir and Yuan, 2001; Ross, 1995, 2004). The weighted average method is comparatively easy and computationally efficient to implement (Ross, 2004; Khan and Sadiq, 2005). The following equation for the weighted average method is used to defuzzify the obtained fuzzy numbers for the event tree and fault tree outputs (Ross, 2004).

$$P_{out} = \frac{\sum \left[\mu_P(\tilde{P}) \cdot \tilde{P}\right]}{\sum \mu_P(\tilde{P})} \tag{4.2}$$

4.5 Evidence theory (evidential reasoning)

Multi-source knowledge can provide more reliable information about the probability of events (or basic-event) than a single source. Knowledge can never be absolute as it is socially constructed and negotiated and often suffers incompleteness and conflict (Ayyub, 2001). Evidence theory has alternatively been used in many applications, especially when the uncertainty is due to ignorance and incomplete knowledge (Sadiq *et al.*, 2006; Wang *et al.*, 2006). The main advantages of evidence theory are:

1. individual belief, including complete ignorance, can be assigned,
2. an interval probability can be obtained for each uncertain parameter, and
3. multi-source information can be combined that helps to avoid bias due to some specific source (Sentz and Ferson, 2002).

4.5.1 Fundamentals

Evidence theory was first proposed by Dempster and later extended by Shafer. This theory is also known as Dempster-Shafer Theory (DST) (Sentz and Ferson, 2002; Li, 2007). DST uses three basic parameters, i.e., *basic probability assignment* (*bpa*), *Belief measure* (*Bel*), and *Plausibility measure* (*Pl*) to characterize the uncertainty in a belief structure (Cheng, 2000; Lefevre *et al.*, 2002; Bae *et al.*, 2004). The belief structure represents a continuous interval [*belief*, *plausibility*] for the uncertain quantities in which the true probability may lie. Narrow belief structures are representative of more precise probabilities. The main contribution of DST is a scheme for the aggregation of multi-source knowledge based on individual degrees of belief.

4.5.1.1 Frame of discernment

Frame of discernment (FOD) Ω is defined as a set of mutually exclusive elements that allows having a total of $2^{|\Omega|}$ subsets in a power set (P), where $|\Omega|$ is the *cardinality* of a FOD. For example, if $\Omega = \{T, F\}$, then the power set (P) includes four subsets, i.e., $\{\Phi$ (a null set), {T}, {F}, and {T, F}}, as the cardinality is two.

4.5.1.2 Basic probability assignment

The basic probability assignment (*bpa*), also known as belief mass, is denoted by $m(p_i)$. The *bpa* represents the portion of total knowledge assigned to the proposition of the power set (P) such that the sum of the proposition is 1. The focal elements, i.e.,

$p_i \subseteq P$ with $m(p_i) > 0$ collectively represent the evidence. The *bpa* can be characterized by the following equation:

$$m(p_i) \rightarrow [0,1] \ ; m(\Phi) \rightarrow 0 \ ; \sum_{p_i \subseteq P} m(p_i) = 1 \tag{4.3}$$

For example, suppose an expert reports that the occurrence probability of an event in ETA is 80% true and 10% false. For this example, the *baps* of every subset of $m(p_i)$ can be written as $m(\mathrm{T}) = 0.8$, and $m(\mathrm{F}) = 0.1$. The unassigned *bpa* is referred to the set $m(\Omega) = m(\mathrm{T, F}) = 0.1$.This is because, the unassigned *bpa* is taken as ignorance, which is usually represented by the subset $m\{\Omega\}$(Sadiq *et al.*, 2006).

4.5.1.3 Belief measure

The *Belief (Bel) measure*, sometimes termed as the lower bound for a set p_i, is defined as the sum of all the *bpas* of the proper subsets p_k of the set of interest p_i, i.e., $p_k \subseteq p_i$. The relationship between *bpa* and *Belief measure* is written as:

$$Bel(p_i) = \sum_{p_k \subseteq p_i} m(p_k) \tag{4.4}$$

The *Belief measures* in the above example are given by:

$Bel(\mathrm{T}) = m(\mathrm{T}) = 0.8$; $Bel(\mathrm{F}) = m(\mathrm{F}) = 0.1$ and $\mathrm{Bel(T, F) = m(T) + m(F) + m(T, F) = 1.0}$

4.5.1.4 Plausibility measure

The upper bound, i.e., the *Plausibility (Pl) measure* for a set p_i is the summation of *bpas* of the sets p_k that intersect with the set of interest p_i, i.e., $p_k \cap p_i \neq \Phi$. Therefore, the relationship can be written as:

$$Pl(p_i) = \sum_{p_k \cap p_i \neq \Phi} m(p_k) \qquad (4.5)$$

The Plausibility measures for the above example are given by:

$Pl(T) = m(T) + m(T, F) = 0.8 + .01 = 0.9$;

$Pl(F) = m(F) + m(T, F) = 0.1 + 0.1 = 0.2$; and $Pl(T, F) = 1.0$

4.5.1.5 Rule of combination for inference

The combination rules allow aggregating the individual beliefs of experts and provide a combined belief structure. DS combination rule is the fundamentals for all combination rules. Many modifications of the DS rule of combination have been reported. The most common modifications include those by Yager, Smets, Inagaki, Dubois and Prade, Zhang, Murphy, and more recently by Dezert and Smarandache (Sadiq *et al.*, 2006). Detailed discussions on these rules can be found in Dezert and Smarandache (2004). In the current study, DS and Yager combination rules are discussed in detail and used in developing the evidence theory-based approach for FTA and ETA.

DS combination rule: The DS combination rule uses a normalizing factor (1-k) to develop an agreement among the acquired knowledge from multiple sources, and ignores all conflicting evidence through *normalization.* Assuming that the knowledge sources are independent, this combination rule uses the AND-type operator (product) for aggregation (Sadiq *et al.*, 2006). For example, if the m_1 (p_a) and m_2 (p_b) are two sets of evidence for the same event collected from two different experts, the DS combination rule uses the following relation to combine the evidence.

$$m_{1\text{-}2}(p_i) = \begin{cases} 0 & \text{for } p_i = \Phi \\ \dfrac{\sum_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b)}{1-k} & \text{for } p_i \neq \Phi \end{cases} \tag{4.6}$$

In the above equation, $m_{1\text{-}2}(p_i)$ denotes the combined knowledge of two experts for the event, and k measures the *degree of conflict* between the two experts, which is determined by the factor k.

$$k = \sum_{p_a \cap p_b = \Phi} m_1(p_a) \times m_2(p_b) \tag{4.7}$$

Yager combination rule: Zadeh (1984) pointed out that the DS combination rule yields counterintuitive results and exhibits the numerical instability if conflict is large among the sources (Sentz and Ferson , 2002). To resolve this issue, Yager (1987) proposed an extension, which is similar to the DS combination rule except that it does not allow normalization of joint evidence with the normalizing factor (1-k). The total degree of conflict (k) is assigned to be part of ignorance Ω (Sadiq *et al.*, 2006). However, in a non- (or less) conflicting case, the Yager combination rule exhibits similar results as the DS combination rule. For high conflict cases (i.e., higher k value), it provides more stable and robust results than the DS combination rule (Ferdous *et al.*, 2009b).

$$
m_{1\text{-}2}(p_i) = \begin{cases} 0 & \text{for } p_i = \Phi \\ \sum\limits_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b) & \text{for } p_i \neq \Omega \\ \sum\limits_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b) + k & \text{for } p_i = \Omega \end{cases} \quad (4.8)
$$

In the above example, if we assume another expert reports new evidence for the same event: m ({T}) = 0.6, m ({F}) = 0.3 and m ({T, F}) = 0.1. Both bodies of evidence are combined using DS and Yager combination rules. The aggregation of the knowledge is performed using Equations 4.6 and 4.8. Equations 4.3 and 4.4 are used to obtain the combined belief structure of the event (Table 4.8).

Table 4.8: Modified α-cut based fuzzy arithmetic operations

m_1 \ m_2		**{T}** 0.60	**{F}** 0.30	**{T, F}** 0.10
{T}	0.80	{T}= 0.48	{Φ}= 0.24	{T}= 0.08
{F}	0.10	{Φ}= 0.06	{F}= 0.03	{F}= 0.01
{T, F}	0.10	{T}= 0.06	{F}= 0.03	{T, F}= 0.01
	k	0.30		
$\sum\limits_{p_a \cap p_b = p_i} m_1(p_a) m_2(p_b)$		0.62	0.07	0.01
$m_{1\text{-}2}$(DS)		0.89	0.1	0.014
$m_{1\text{-}2}$(Yager)		0.62	0.07	0.31

Rules of combination	**Belief Structure**			
	***Bel* (T)**	***Pl* (T)**	***Bel* (F)**	***Pl* (F)**
DS rule	0.89	0.90	0.10	0.11
Yager rule	0.62	0.93	0.07	0.38

4.5.2 Evidence theory-based approach for FTA/ ETA

Expert knowledge is used to define the probability of occurrence and dependency coefficient of events (or basic-events). Each expert may have their own belief or knowledge that may be incomplete and that may be in conflict with the others. In an evidential reasoning framework, the ignorance in an evidence is assigned to a subset $m(\Omega)$. The conflict among the sources is dealt with using combination rules as discussed above. The following sections describe the steps of the evidence theory-based approach to analyze the event tree/fault tree under uncertainties.

4.5.2.1 Definition of frame of discernments

Three different FODs for three uncertain quantities in FTA and ETA including the probability of events, the probability of basic-events and the dependency coefficient (C_d) are used to acquire the belief masses from different experts. The subsets for the FODs are generated based on the cardinality of each FOD (Ω).

Traditionally, the consequences of events in event tree analysis are dichotomous, i.e., {T} and {F}. Therefore the FOD for ETA is defined as Ω {T, F} that leads to four subsets in a power set (P) that includes {Φ, {T}, {F}, {T, F}}.

The operational state of a system is usually defined on the basis of evaluating the success (S) or failure (F) state of basic components (Vesely *et al.*, 1981; Hauptmanns, 1980, 1988). Hence, the occurrence probability of a basic-event in FTA can be described using the FOD Ω ={S, F}. As the cardinality is two for the FOD, the power set (P) of each event is comprised of four subsets that includes {Φ, {S}, {F}, {S, F}}.

Six qualitative grades of dependency are categorized in current study to describe interdependences through dependency coefficients for FTA or ETA. The notations of these grades are: Independent (I); Very Weak (M); Weak (W); Strong (S); Very Strong (VS); and Perfect dependence (P). The FOD for this case consists of six cardinal elements which is represented by Ω = {P, VS, S W, VW, I}.

4.5.2.2 Assignment of bpas to basic-events/events

The *bpa*s or belief masses for the events (or basic-events) and the dependency coefficients (C_d) are elicited using the expert's knowledge. Assuming that the knowledge sources are independent, the *bpas* are assigned to particular subsets of each FOD. However, for the dependency coefficient, experts knowledge are collected only for the subsets {P}, {VS}, {S}, {W}, {VW}, {I}, and {Ω}. The *bpa*s for each subset individually represent the degree of belief of each expert, and implicitly, it represents the total evidences that support the probability of occurrence of an event (or a basic-event) and a dependency coefficient (C_d).

4.5.2.3 Belief structure and Bet estimation

The combination rules allow merging the knowledge from different sources as coherent evidence. These rules help to account ignorance into the knowledge and resolve conflicts among the sources. The DS (Equation 4.6) or Yager (Equation 4.8) combination rules are used in the current study to aggregate collected knowledge from different sources. Equations 4.4 and 4.5 are then used to derive the *belief* and *plausibility* measure for the probability and dependency coefficients of events (or basic-events) .The *belief* and *plausibility* measure for six kinds of dependencies (in each node of FTA or ETA) are normalized to attain a generalized belief structure. Information in Table 4.6, which represents the *belief* and *plausibility* for each kind of dependency, is used for normalizing the belief structure of dependency coefficient for each node. Subsequently, equations shown in Table 4.9 are used to estimate the likelihoods of outcome events and top-event for the ETA and FTA, respectively.

"Bet" provides a point estimate in belief structure (similar to defuzzification), which is often used to represent the crisp value of the final events. It is estimated based on the following equation:

$$Bet(P) = \sum_{p_i \subseteq P} \frac{m(p_i)}{|p_i|} \tag{4.9}$$

where, where, $|p_i|$ is the *cardinality* in the set p_i. For the example, the *"Bet"* estimate for the belief structure obtained using the DS combination rule is calculated as.

$$Bet(P_T) = \frac{m(T)}{1} + \frac{m(T,F)}{2} = \frac{0.89}{1} + \frac{0.01}{2} = 0.895$$

The denominators "1" and "2" represent the cardinality in the respective subsets.

Table 4.9: Equations to analyze the event and fault trees

Method	Operation	Formulation
ETA	Frequency estimation	$\lambda_i = \lambda \times \prod_{i=1}^{n} \left[Bel(P_i), Pl(P_i) \right]$
	$P_1 \times P_2$	$Bel(P_{out}) = \left[1 - \{1 - Bel(C_d)\} \times \{1 - Bel(P_1)\} \right] \times Bel(P_2)$
		$Pl(P_{out}) = \left[1 - \{1 - Pl(C_d)\} \times \{1 - Pl(P_1)\} \right] \times Pl(P_2)$
FTA	P_1 "OR" P_2	$Bel(P_{out}) = 1 - \{1 - Bel(P_1)\} \times \left[1 - \{1 - Bel(C_d)\} \times Bel(P_2) \right]$
		$Pl(P_{out}) = 1 - \{1 - Pl(P_1)\} \times \left[1 - \{1 - Pl(C_d)\} \times Pl(P_2) \right]$
	P_1 "AND" P_2	$Bel(P_{out}) = \left[1 - \{1 - Bel(C_d)\} \times \{1 - Bel(P_1)\} \right] \times Bel(P_2)$
		$Pl(P_{out}) = \left[1 - \{1 - Pl(C_d)\} \times \{1 - Pl(P_1)\} \right] \times Pl(P_2)$

4.6 Application of developed approaches

The same examples discussed earlier in Section 4.2 are studied in detail here using both fuzzy-based and evidence theory-based approaches.

4.6.1 LPG release - event tree analysis

4.6.1.1 Fuzzy-based approach

The revised event tree with fuzzy probabilities is illustrated in Figure 4.7. In the fuzzy-based approach, the probability of events (or basic-events) and their dependency coefficients (C_d) are defined using TFNs. The frequency for the outcome events are then estimated using the α-cut based fuzzy formulations developed in Table 4.7. For example, the path leading to the outcome event "A" is followed by the two events. The probability and the coefficient of dependency (C_d) of these two events are linguistically expressed, which are respectively assumed to be "Moderately Low (ML)", "Moderately High (MH)" and "Strong (S)". The assigned linguistic expressions for these two events are converted into TFNs (based on Figure 4.6 and Table 4.6). The TFN for the outcome event "A" (shown in Figure 4.8) is derived using the empirical equations described in Table 4.7. Using numerous trials for event dependency at each node of the LPG event tree, the uncertainty ranges (i.e., fuzzy interval) for the outcome event "A" are estimated (shown in Figure 4.9). It can be observed that the uncertainty ranges are varied according to the change of event dependency at each node of the event tree.

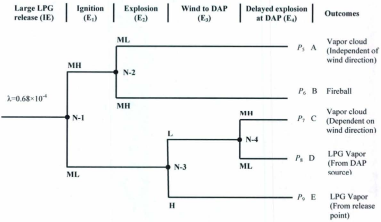

Figure 4.7: Event tree with fuzzy linguistic grades

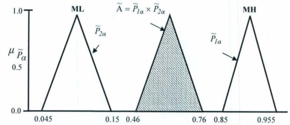

Figure 4.8: TFNs of outcome event "A" with "Strong" dependency

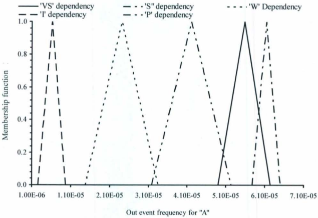

Figure 4.9: Uncertainty in outcome event's frequency "A"

4.6.1.2 Evidence theory-based approach

To demonstrate the evidence theory-based approach, experts' knowledge from two unbiased and independent sources is considered for determining the probability as well as the dependency coefficients of events for ETA. The elicited knowledge from the sources is shown in Tables 4.10 (a) and 4.10 (b).

Table 4.10(a): Experts' knowledge on the probability of events

Symbol	Events' name	Expert 1 (m_1)			Expert 2 (m_2)		
		{T}	{F}	{T, F}	{T}	{F}	{T, F}
E_1	Ignition	0.80	0.10	0.10	0.60	0.30	0.10
E_2	Explosion	0.10	0.80	0.10	0.05	0.80	0.15
E_3	Wind to DAP	0.40	0.50	0.10	0.50	0.40	0.10
E_4	Ignition Explosion at DAP	0.85	0.10	0.05	0.80	0.10	0.10

(b): Experts' knowledge on interdependence of events at different nodes

Number of Experts	Node (N)	{P}	{VS}	{S}	{W}	{VW}	{I}	{Ω}
Expert 1 (m_1)	N-1	0.15	0.00	0.30	0.10	0.00	0.00	0.45
	N-2	0.00	0.30	0.20	0.00	0.10	0.00	0.40
	N-3	0.40	0.00	0.20	0.00	0.00	0.20	0.20
	N-4	0.50	0.20	0.00	0.00	0.00	0.20	0.10
Expert 2 (m_2)	N-1	0.30	0.00	0.20	0.15	0.00	0.00	0.35
	N-2	0.20	0.30	0.00	0.00	0.00	0.15	0.35
	N-3	0.00	0.20	0.40	0.00	0.20	0.00	0.20
	N-4	0.00	0.30	0.40	0.00	0.20	0.00	0.10

DS and Yager combination rules are used to aggregate and determine the belief structures of probability and dependency coefficients of events for the ETA. Table 4.11 lists the belief structures of events and dependency coefficients for the LPG release event tree (Figure 4.1). These belief structures and the equations in Table 4.9 are used to derive the belief structures for the outcome events of LPG release. Two different kinds of dependence, i.e., independent and dependent are considered while estimating the belief structures for the outcome events. The results are presented in Table 4.12. An order of magnitude difference is observed in the "*Bet*" estimation for the outcome event "E". This difference signifies the importance of defining the dependency relationships in ETA.

Table 4.11: Belief structures for the probability and interdependence of events

Reference in the event tree		DS rule of combination		Yager rule of combination	
		Bel	***Pl***	***Bel***	***Pl***
E_1	T[+]	0.8857	0.9000	0.6200	0.9300
	F	0.1000	0.1143	0.0700	0.3800
E_2	T	0.0284	0.0455	0.0250	0.1600
	F	0.9545	0.9716	0.8400	0.9750
E_3	T	0.6970	0.7273	0.4600	0.8200
	F	0.2727	0.3030	0.1800	0.5400
E_4	T	0.9641	0.9701	0.8050	0.9750
	F	0.0299	0.0359	0.0250	0.1950
*N-1		0.2549	0.7450	0.1605	0.8395
N-2		0.2755	0.7245	0.1526	0.8474
N-3		0.3150	0.6849	0.0769	0.9230
N-4		0.4013	0.5987	0.0512	0.9488

***Normalization of belief structure at N-1 for DS rule of combination**

		P	VS	S	W	VW	I
N-1	*Bel*	0.305	0.000	0.334	0.154	0.000	0.000
	Pl	0.511	0.207	0.541	0.361	0.207	0.207

$$Bel(C_d) \quad \frac{(0.305\times1+0\times0.80+0.334\times0.45+0.154\times0.15+0\times0.005+0\times0)}{(0.305+0\times0.80+0.334\times0.45+0.154\times0.15+0\times0.005)+(0.551+0.207\times0.995+0.541\times0.85+0.361\times0.5+0.207\times0.2)}=0.255$$

$$Pl(C_d) \quad \frac{(0.511\times1+0.207\times0.995+0.541\times0.85+0.361\times0.5+0.207\times0.2+0\times0.207)}{(0.305+0\times0.80+0.334\times0.45+0.154\times0.15+0\times0.005)+(0.551+0.207\times0.995+0.541\times0.85+0.361\times0.5+0.207\times0.2)}=0.745$$

[+]T-*True and* F-*False*

Table 4.12: Outcome events frequency for two kind of interdependence of events

Outcome events	Interdependence of events					
	Independent			Dependent		
	Belief structures (Yager-rule of combination)		*Bet*	Belief structures (Yager-rule of combination)		*Bet*
	Bel	*Pl*		*Bel*	*Pl*	
A	1.054E-06	1.012E-05	*5.586E-06	1.153E-06	1.076E-05	5.958E-06
B	3.541E-05	6.166E-05	4.854E-05	3.873E-05	6.559E-05	5.216E-05
C	1.763E-06	2.066E-05	1.121E-05	6.186E-06	6.556E-05	3.587E-05
D	5.474E-08	4.132E-06	2.093E-06	1.921E-07	1.311E-05	6.652E-06
E	8.568E-07	1.395E-05	7.405E-06	1.733E-06	3.497E-05	1.835E-05

*Belief structure of outcome event "A" is [1.054E-06, 1.012E-05]. So, $m(T)$= 1.054E-06, and $m(T,F)$ = 9.064E-06

$$Bet(\text{A}) = \frac{m(T)}{1} + \frac{m(T,F)}{2} = \frac{1.054\text{E}-06}{1} + \frac{9.064\text{E}-06}{2} = 5.586\text{E}-06$$

The difference of using the DS and Yager combination rules is shown in Figure 4.10. In the figure, different kinds of dependencies are labeled on the x-axis. The *belief* and *plausibility* measure for each kind of dependency is plotted on the y-axis. The minimum and maximum values presented in Table 4.6 are considered as the belief structure of dependency coefficient for each kind of dependency. The shaded areas in Figure 4.10 represent the *belief* and *plausibility* measures for the outcome event "A". These areas show that the Yager combination rule measures a large belief structure in comparison to the DS combination rule. Hence, an interpretation can be made that the Yager combination rule yields more conservative results (i.e., a larger belief structure) in the context of existing high conflicts in the sources.

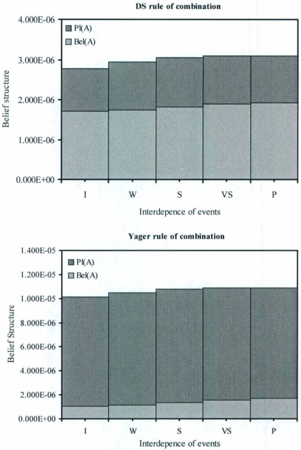

Figure 4.10: Belief structure representing the frequency for outcome event "A"

4.6.2 Runaway reaction - fault tree analysis

4.6.2.1 Fuzzy-based approach

To demonstrate the fuzzy-based approach for FTA, the probability of basic-events and their dependencies are defined using expert linguistic expressions. The linguistic expressions are converted into TFNs. The linguistic expressions and the corresponding TFNs are given in Table 4.13. A total of seven different trials and the fuzzy arithmetic operations (described in Table 4.7) are used to evaluate the TFN for the top-event. The trials are categorized based on different assumptions of dependencies at each node of the fault tree. The TFNs of the top-event for the different trials are shown in Figure 4.11. In trial 7, when perfect dependencies are assumed, the top-event probability bears the maximum uncertainty. Contrary to trial 1, when the events are assumed independent, the top-event probability bears the smallest uncertainty.

Table 4.13: Expert's knowledge on the probability of basic-events

Event	Linguistic variable	TFNs
BE_1	L	(0.1,0.25,0.4)
BE_2	VL	(0,0.025,0.05)
BE_3	ML	(0.045,0.0975,0.15)
BE_4	VL	(0,0.025,0.05)
BE_5	VL	(0,0.025,0.05)
BE_6	VL	(0,0.025,0.05)

Dependency of basic-events in different Nodes (N)					TFNs of top-event probability
Trials (T)	N-1	N-2	N-3	N-4	(P_L, P_m, P_R)
1	I	I	I	I	(0, 0.013, 0.027)
2	VW	VW	VW	VW	(0, 0.061, 0.122)
3	W	W	W	W	(0, 0.122, 0.244)
4	VS	S	W	W	(0, 0.121, 0.243)
5	S	S	S	S	(0, 0.179, 0.359)
6	VS	VS	VS	VS	(0, 0.199, 0.399)
7	P	P	P	P	(0, 0.200, 0.400)

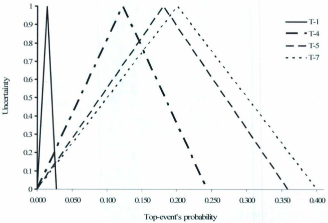

Figure 4.11: Uncertainty representation for top-event using different trials

4.6.2.2 Evidence theory-based approach

The fault tree for the runaway reaction as shown in Figure 4.2 is studied to demonstrate the application of evidence theory-based approach in FTA. The probability of basic-events and the dependency coefficients for the fault tree are obtained from two independent sources. Tables 4.10 (b) and 4.14 show the experts' knowledge for defining the probability and dependency coefficients of basic-events for the FTA.

Table 4.14: Multi-source knowledge for the probability of basic-events

Basic-event	Expert 1 (m_1)			Expert 2 (m_2)		
	{F}	{*S}	{SF}	{F}	{S}	{SF}
BE_1	0.150	0.750	0.100	0.250	0.650	0.100
BE_2	0.020	0.800	0.180	0.015	0.900	0.085
BE_3	0.200	0.700	0.100	0.100	0.800	0.100
BE_4	0.015	0.950	0.035	0.025	0.950	0.025
BE_5	0.015	0.900	0.085	0.010	0.980	0.010
BE_6	0.002	0.950	0.048	0.001	0.940	0.059

*S - *Success* and F - *Failure*

DS and Yager combination rules are used to aggregate the knowledge and estimate the belief structures for the basic events and dependency coefficients. The belief structure of the top-event is then calculated by using the equations in Table 4.9. Table 4.15 shows the belief structure and the *"Bet"* estimate of the top-event for two combination rules. A total of seven trials are performed using different assumptions of interdependence between the basic-events. The belief structure for each kind of dependence is defined in Table 4.6. Table 4.15 indicates that the uncertainty in calculating the belief structure and *"Bet"* estimate varies accordingly with the change of interdependence at different nodes.

Table 4.15: Belief structures and "*Bet*" estimations of top-event for different trails

Trials (T)	Dependency of basic-events at different nodes				Belief structure of top-event's probability					
					DS rule			Yager rule		
	N-1	N-2	N-3	N-4	*Bel*	*Pl*	*Bet*	*Bel*	*Pl*	*Bet*
1	I	I	I	I	3.440E-05	6.234E-04	3.289E-04	2.579E-05	3.942E-03	1.984E-03
2	VW	VW	VW	VW	3.595E-05	3.865E-02	1.934E-02	2.721E-05	1.133E-01	5.666E-02
3	W	W	W	W	9.650E-05	8.294E-02	4.152E-02	8.555E-05	2.441E-01	1.221E-01
4	VS	S	W	W	1.998E-04	8.292E-02	4.156E-02	1.772E-04	2.483E-01	1.242E-01
5	S	S	S	S	2.506E-04	1.151E-01	5.768E-02	2.440E-04	3.458E-01	1.730E-01
6	VS	VS	VS	VS	3.167E-04	1.222E-01	6.126E-02	3.160E-04	3.718E-01	1.860E-01
7	*P	P	P	P	2.000E-04	1.224E-01	6.130E-02	2.000E-04	3.725E-01	1.864E-01

*P-*Perfect dependence*, VS - *Very Strong*, S-*Strong*, W - *Weak*, VW - *Very Weak*, and I - *Independent*

4.7 Uncertainty-based formulations for fault and event tree analyses: a comparison

The level of uncertainty associated with a system is proportional to its complexity, which arises as a result of vaguely known relationships among various entities, and randomness in the mechanisms governing the domain. Sadiq *et al.* (2009) described complex systems such as environmental, socio-political, engineering, or economic systems, which involve human interventions, and where vast arrays of inputs and outputs could not all possibly be captured analytically or controlled in any conventional sense. Moreover, relationships between causes and effects in these systems are often not well understood but can be expressed empirically. Typical complex systems consist of numerous interacting factors or concepts. These systems are highly non-linear in behavior and the combined effects of contributing factors are often sub-additive or super-additive. The modeling of complex dynamic systems requires methods that combine human knowledge and experience as well as expert judgment. When significant historical data exist, model-free methods such as artificial neural networks (ANN) can provide insights into cause-effect relationships and uncertainties through learning from data (Ross, 2004). But, if historical data are scarce and/or available information is ambiguous and imprecise, soft computing techniques can provide an appropriate framework to handle such relationships and uncertainties. Such techniques include probabilistic and evidential reasoning (Dempster-Shafer theory), fuzzy logic and evolutionary algorithms (Sadiq *et al.*, 2009). Table 4.16 provides a qualitative comparison between five soft computing techniques including artificial neural networks (ANN), decision trees (DT), fuzzy rule-based models (FRBM), Bayesian networks (BN) and cognitive maps/ fuzzy cognitive maps (CM/ FCM). Central to this comparison is an assessment of how each technique treats inherent uncertainties and its ability to handle interacting factors that encompass issues specific to engineering systems (Sadiq *et al.*, 2009).

Qualitative and quantitative comparisons have been performed in this section to investigate the features and uncertainty handling abilities of different tools and the proposed approaches for FTA and ETA. The qualitative comparison presented in Table 4.17 illustrates that most of the tools such as Relex V7.7 (2003), RAM Commander 7.7 (2009) and PROFAT (1999) are unable to handle *dependency uncertainty*. Except for PROFAT, the other tools cannot handle subjective uncertainty in the fault and event trees for a system. PROFAT (1999) is a fuzzy based tool that can handle subjective uncertainty; however, it fails to account for *epistemic uncertainty* owing to ignorance or incompleteness of an expert's knowledge.

Table 4.16: Comparison of various techniques for complex systems (Sadiq *et al.*, 2009)

Attributes	Soft computing techniques				
	Decision tree	**Fuzzy rule-based models**	**Artificial neural networks**	**Bayesian networks**	**Cognitive maps/fuzzy cognitive maps**
Network capability	N[1]	L[2]	N	H[3]	VH[4]
Ability to express causality	H	M	N	H	VH
Formulation transparency	H	H	N[5]	H	VH
Ease in model development	H	M	M	M	VH
Ability to model complex systems	M	H	VH	H	VH
Ability to handle qualitative inputs	H	H	N	H	VH
Scalability and modularity	VL	L	VL[6]	H	VH[7]
Data requirements	H	L	VH	M	L[8]
Difficulty in modification	VH	H	M	L	N
Interpretability of results	VH	VH	VH	VH	H
Learning/training capability	H	M[9]	VH[10]	H[11]	H[12]
Time required for simulation	L	L	H	L	L
Maturity of science	VH	H	H	VH	M
Ability to handle dynamic data	L	H	H	H	M
Examples of hybrid models (ability to combine with other approaches)	H	VH[13]	VH[13]	H	H[14]

Ratings: N = No or Negligible; VL = very low; L = low; M = medium; H = high; VH = very high
1 Structure is hierarchical
2 Dimensionality is a major problem and formulation becomes complicated for network systems
3 Can manage networks but cannot handle feedback loops, therefore referred to as directed acyclic graphs (DAG)
4 Can handle feedback loops
5 Generally referred to as black box models
6 ANN needs to be retrained for new set of conditions
7 Very easy to expand, because algorithm is in the form of matrix algebra
8 Minimal data requirement, because causal relationships are generally soft in nature
9 Clustering techniques, e.g., Fuzzy C-means
10 Algorithms, e.g., Hebbian learning
11 Algorithms, e.g., evolutionary algorithms and Markov chain Monte Carlo
12 Training algorithms are available which have been successful in training ANNs
13 Examples are available in the literature to develop models using hybrid techniques, e.g., neuro-fuzzy models
14 Has a potential to be used with other soft techniques

Table 4.17: Qualitative comparisons of proposed approach with available FTA/ETA tools

Uncertainty	[1]Relax V7.7 (2003)	[1]RAM commander 7.7 (2009)	PROFAT (1999)	Proposed approach
Subjective (fuzzy-based)	NC[2]	NC	C[3]	C
Incompleteness (evidence based)	NC	NC	NC	C
Dependency	NC	NC	NC	C

[1]A commercial software, [2] not considered and [3] considered

Another type of uncertainty arises due to lack of information on dependencies among events. Traditional fault tree analysis uses a default assumption of "independence" among the risk events to determine the joint probability (risk) of a parent event. This assumption simplifies the analysis, but may not be very realistic. The relationship between risk events may be positively or negatively correlated (or independent). In the case of two independent events X and Y, the joint probability of their conjunction is a simply a product of their individual probabilities (Ferson *et al.*, 2004; Sadiq *et al.*, 2008). There exist many different methods to express correlation (dependence) but the Frank model (copula) is the most common.

Simple dependency coefficient based empirical relations [similar to Li's (2007) approach] embedded within the proposed approach can concurrently handle the *dependency uncertainty* in fault and event tree analyses. The proposed approach successfully accounts for the subjective uncertainty using a membership function and evaluates the uncertainty range as a fuzzy interval. The evidence theory based-approach can describe the *epistemic* and *aleatory uncertainties* in experts' knowledge using *bpa* and is able to provide a measure of uncertainty using belief structures.

Relax V7.7 (2003) and RAM Commander 7.7 (2009) are two useful tools for reliability and safety engineering. The probability of top-event for the "Runaway reaction fault tree" and the frequency of outcome events for the "LPG release event tree" have been analyzed using these tools for the same input (Figure 4.1 and Table 4.2). Results (Table 4.18) show that by introducing 10% uncertainty into the input data, these two tools accumulated about 19 % and 9 % of uncertainty on the calculated top-event's probability and outcome event's frequency of "B". The original input data (Figure 4.1 and Table 4.2) are reduced by 10% to introduce the uncertainty into the analysis. The traditional (probabilistic) methods used predefined PDFs to describe the uncertainty in the input data (i.e., the probability of basic events or events in FTA or ETA). When the crisp data or the PDFs for the input data are not known or limited (a very common situation in process systems), the FTA or ETA are highly dependent on expert knowledge. In these situations traditional tools and probabilistic approaches are not helpful. This makes the FTA/ ETA less credible. Both fuzzy set theory and evidence theory are not limited by availability of detailed data. The results using both approaches are presented in Table 4.18. An expert knowledge and assumption of independence among events (or basic events) are used in calculating the top-event probability and outcome events frequency. In fuzzy-based approach, the uncertainty is assigned using the membership function. The TFNs corresponding to 90% membership are considered as input data for the analysis. In evidence theory-based approach the uncertainty is allocated through the unassigned mass (as ignorance) of the power set. For the 10% uncertainty in the basic event probabilities, the evidence theory-based approach estimates about 9% and 8% uncertainties in the response, i.e., $[2.55\times10^{-4}, 3.16\times10^{-4}]$ and $[4.46\times10^{-5}, 5.63\times10^{-5}]$, respectively. Similarly, in fuzzy-based approach measures less than 1% uncertainty results in the response (top-event's probability as well as outcome event's frequency "B".) with corresponding fuzzy intervals of $[7.38\times10^{-3}, 1.01\times10^{-2}]$ and $[5.47\times10^{-5}, 5.60\times10^{-5}]$.

Table 4.18: Quantitative comparison of FTA/ETA tools

Determination of probability of top-event for "Runaway reaction fault tree"							
Commercial packages				**Fuzzy-based Defuzzified value**		**Evidence theory- based *Bet* estimation**	
Relex V7.7 (2003)		**RAM commander 7.7 (2009)**					
No Uncertainty	10% Uncertainty	No Uncertainty	10% Uncertainty	No Uncertainty	10% Uncertainty	No Uncertainty	10% Uncertainty
3.16E-04	2.55E-04	3.41E-04	2.74E-04	8.71E-03	8.76E-03	3.16E-04	2.86E-04

Determination of frequency of outcome events for LPG release						
Outcome events	**Relex V7.7 (2003), RAM commander 7.7 (2009)**		**Fuzzy-based Defuzzified value**		**Evidence theory-based *Bet* estimation**	
	No Uncertainty	10% Uncertainty	No Uncertainty	10% Uncertainty	No Uncertainty	10% Uncertainty
A	6.12E-06	4.95E-06	5.98E-06	5.99E-06	6.12E-06	8.36E-06
B	5.51E-05	5.01E-05	5.54E-05	5.54E-05	5.51E-05	5.05E-05
C	2.45E-06	3.76E-06	1.50E-06	1.58E-06	2.45E-06	3.60E-06
D	2.72E-07	8.84E-07	1.62E-07	1.71E-07	2.72E-07	6.64E-07
E	4.08E-06	8.27E-06	4.97E-06	4.98E-06	4.08E-06	5.79E-06

4.8 Results and discussion

Two types of uncertainty, namely *data* and *dependency uncertainty*, were explored. Expert knowledge in terms of fuzzy linguistic grades and *bpas* was used instead of assigning the likelihood and interdependencies of basic-events/events as crisp probabilities for FTA/ETA. The dependency coefficient in each node of the fault tree and event tree addressed the *dependency uncertainty* and described the relationships among the basic-events/events. Fuzzy linguistic grades were assigned to TFNs and α-cut based fuzzy empirical relations of the fuzzy-based approach were used to handle the linguistic and subjective uncertainty in expert knowledge. For multi-source knowledge, the incomplete and inconsistent *bap*s were combined by using combination rules. The dependency coefficients in evidence theory-based empirical relations were used to describe the *dependency uncertainty* and analyze the fault tree and event tree under uncertainty due to inconsistent, incomplete and partial ignorance of multi-source knowledge.

The developed approaches were applied to two case studies: "LPG release event tree" and "Runaway reaction fault tree". The interdependencies among the events (or basic-events) were varied in each node of the fault tree or event tree. The impacts of the interdependencies were observed so as to understand the effects of the dependencies of events (or basic-events) in FTA/ETA for process systems. For two dependence cases of basic-events/events, Independent and perfectly dependent, the output results for the FTA and ETA are provided in Tables 4.19 and 4.20, respectively. It can be observed in the first three rows of Table 4.19 that the results remain almost the same. However when dependency was considered (fourth row in Table 4.19), the results varied by an order of magnitude. This highlights the importance of dependencies in ETA.

Table 4.19: Summary of ETA results

Approach		Dependency of events	Frequency of outcome event "A"
Deterministic approach		Independent	6.12E-06
MCS-based approach	90% confidence interval	Independent	(1.96E-05, 1.02E-04)
	Median		6.10E-06
Fuzzy-based approach	Fuzzy interval	Independent	(2.60E-06, 9.74E-06)
	Defuzzified value		6.17E-06
Fuzzy-based approach	Fuzzy interval	Perfectly dependent	(5.78E-05, 6.49E-05)
	Defuzzified value		6.14E-05

The results in Table 4.20 are inconsistent mainly because of different types of uncertainties modeled in the different approaches. The perfectly dependent case in FTA determines the probability range for the top-event as [0, 0.400], which is a maximum in comparison to the independent case for representing the uncertainty. It can also be observed in Table 4.20 that when the basic-events are perfectly dependent, the point estimate (defuzzified value) of the top-event exhibits a higher ordered magnitude in comparison to the deterministic approach and MCS-based approach. This confirms the significance of including the dependencies of the basic-events in FTA. Similar observations of using the evidence theory–based approach for FTA/ETA (Tables 4.12 and XV) confirm that a reliable and robust result cannot be attained without considering the interdependencies of events (or basic-events).

Table 4.20: Summary of FTA results

Approach		Dependency of basic-events	Top-event's probability (P_{Top})
Deterministic approach		Independent	3.16E-04
MCS-based approach	90% Confidence interval	Independent	(4.24E-05, 2.31E-04)
	Median		1.30E-04
Fuzzy-based approach	Fuzzy interval	Independent	(0, 2.70E-02)
	Defuzzified value		1.35E-02
Fuzzy-based approach	Fuzzy interval	Perfectly Dependent	(0, 4.00E-01)
	Defuzzified value		2.00E-01

4.9 Conclusions

FTA and ETA are two fairly established techniques; however, the uncertainty in defining the probabilities and the relationships of events (or basic-events) can lead to questionable results for QRA. The traditional approaches require the known probability and the independence assumption of events (or basic-events), which are rare and often unrealistic for process systems. Two different approaches to handle these types of uncertainties in FTA and ETA are derived in this study by combining expert knowledge with fuzzy set theory and evidence theory. The application of these approaches to two different case studies shows the proposed approaches are more robust to handle the uncertainty in QRA for the process systems in the following ways.

- Fuzzy-based approach and evidence theory-based approach properly address the uncertainties in expert knowledge and analyze the event trees or fault trees associated with different kinds of uncertainties in expert knowledge.

- Introduction of dependency coefficient in the fuzzy- and evidence theory-based approaches describes interdependencies among the events (or basic-events) in a fault tree/event tree.
- The proposed approaches can be applied to FTA/ETA for any process systems that have *data* and *dependency uncertainties.*
- Including the negative dependencies of events (or basic-events), and combining the subjectivity (using fuzzy-based approach) and incompleteness (evidence theory) into a single approach, e.g., Fuzzy-Dempster-Shafer, may offer additional future improvement to the approaches developed here.

References

Abrahamsson, M. (2002). *Uncertainty in Quantitative Risk Analysis –Characterization and Methods of Treatment.* Department of Fire Safety Engineering, Lund University, Sweden.

Agarwal, H., Renaud, E. J., Preston, L.E. and Padmanabhan, D. (2004). *Uncertainty quantification using evidence theory in multidisciplinary design optimization.* Reliability Engineering and System Safety, Elsevier Ltd, 85, 281–294.

American Institute of Chemical Engineers (AIChE), (2000). *Guidelines for chemical process quantitative risk analysis.* Second Edition, New York: AIChE.

Andrews, D. J. and Dunnett J. S. (2000). *Event-Tree Analysis Using Binary Decision Diagrams.* IEEE Transactions on Reliability, 49(2), 230-238.

Ayyub, B. (2001). *A Practical Guide on conducting expert-opinion elicitation of probabilities and consequences for corps facilities*. Prepared for U.S. Army Corps of Engineers Institute for Water Resources, Alexandria.

Ayyub, B. and Klir, J. G. (2006). *Uncertainty Modeling and Analysis in Engineering and the Sciences*. Published by Chapman & Hall/CRC.

Bae, H., Grandhi, V. R. and Canfield, A. R. (2004). *An approximation approach for uncertainty quantification using evidence theory.* Reliability Engineering and System Safety, 86, 215–225.

Baraldi, P. and Zio, E. (2008). *A Combined Monte Carlo and Possibilistic Approach to Uncertainty Propagation in Event Tree Analysis.* Risk Analysis, 28(4), 1-17.

Cheng, Y. (2000). *Uncertainties in Fault Tree Analysis.* Tamkang Journal of Science and Engineering, 3(1), 23-29.

Dezert, J. and Smarandache, F. (2004). *Presentation of DSmT. Chapter 1in advances and applications of DSmT for information fusion (collected works).* American Research Press, Rehoboth, 3–35.

Druschel, R.B., Ozbek, M. and Pinder, G. (2006). *Application of Dempster-Shafer theory to hydraulic conductivity.* CMWR – XVI, Conference Program, Copenhagen, Denmark, 5, 31-33.

El-Iraki, A. and Odoom, E.R. (1998). *Fuzzy probist reliability assessment of repairable systems.* Fuzzy Information Processing Society - NAFIPS, Conference of the north America, Pensacola Beach, FL, USA, 96-100,

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., and Veitch, B., (2009a). *Methodology for computer aided fuzzy fault tree analysis.* Process Safety and Environment Protection, 87 (4), 217–226.

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., and Veitch, B., (2009b). *Handling Data Uncertainties in Event Tree Analysis.* Process Safety and Environment Protection, 87 (5), 283-292

Ferson, S., Hajagos J., Berleant D., Zhang J., Tucker W. T., Ginzburg L. and Oberkampf W. (2004). *Dependence in Dempster-Shafer Theory and Probability Bounds Analysis.* US: Sandia National Laboratories.

Hassal, D.F. (1965). *Advanced concepts in fault tree analysis.* System Safety Symposium, Seattle, Boeing Company, 8-9.

Hauptmanns, U. (1980). *Fault tree analysis of a proposed ethylene vaporization unit.* Industrial Engineering Chemical Fundamentals, 19,300- 309.

Hauptmanns, U. (1988). *Fault tree analysis for process industries engineering risk and hazard assessment.* Engineering Risk and Hazard Assessment, CRC Press Inc., Florida, 1, 21-59.

Henley, E.J. and Kumamoto, H. (1996). *Probabilistic risk assessment and management for engineers and scientists,* Second Edition, New York: IEE Press

Huang, D., Chen, T. and Wang, J. M. (2001). *A fuzzy set approach for event tree analysis.* Fuzzy Sets and Systems, 118, 153-165.

Kenarangui, R. (1991). *Event-Tree Analysis by Fuzzy Probability.* IEEE Transactions on Reliability, 40(1), 12-124.

Khan, I. F. and Abbasi S.A. (1999). *PROFAT: A user friendly system for probabilistic fault tree analysis, Process Safety Progress*, 18(1), 42-49.

Khan, I F. and Sadiq R. (2005). *Risk-Based Prioritization of Air Pollution Monitoring Using Fuzzy Synthetic Evaluation Technique.* Environmental Monitoring and Assessment, Springer, 105, 261–283.

Klir, J.G. and Yuan, B. (2001). *Fuzzy Sets and Fuzzy logic Theory and Application.* Prentice, Hall of India Private Ltd.

Lai, F.S., Shenoi, S. and Fan, T.L. (1993). *Fuzzy fault tree analysis theory and applications.* Engineering Risk and Hazard Assessment, CRC Press Inc., Florida, 1, 117-137.

Lefevre, E., Colot, O. and Vannoorenberghe, P. (2002). *Belief function combination and conflict management.* Information Fusion, Elsevier Science, 3,149–162.

Li, H. (2007). *Hierarchical Risk Assessment of Water Supply Systems*. Submitted for the Degree of Doctor of Philosophy, Loughborough University, UK.

Lees, F. P. (2005). *Loss prevention in the process industries*. Third Edition, 1, London: Butterworths, 9/05-9/122.

Misra, B. K. and Weber, G. G. (1990). *Use of Fuzzy set theory for level-1 studies in probabilistic risk assessment.* Fuzzy Sets and system, 37,139-160.

RAMS. (2009).Reliability, Availability, Maintainability and Safety Software, ALD Ltd. (Advanced Logistics Development).

Relex (2003). Reliability Software V7.7, Relex Software Corporation.

Rivera, S.S. and Barón, J.H. (1999). *Using Fuzzy Arithmetic in Containment Event Trees.* International Conference on Probabilistic Safety Assessment- PSA, Washington, USA, 22-25.

Ross, T. (1995). *Fuzzy logic with engineering applications*, McGraw-Hill, New York.

Ross J. T. (2004). *Fuzzy Logic with Engineering Applications.* John Wiley & Sons, Ltd, West Sussex, England.

Sadiq, R., Najjaran, H. and Kleiner, Y. (2006). *Investigating evidential reasoning for the interpretation of microbial water quality in a distribution network.* Stoch. Environ. Res Risk Assess 21, 63–73.

Sadiq, R, Saint-Martin E. and Kleiner Y. (2008). *Predicting risk of water quality failures in distribution networks under uncertainties using fault-tree analysis.* Urban Water Journal, 5(4), 287-304.

Sadiq, R. Kleiner, Y., and Rajani, B. (2009). Q-WARP proof-of-concept model to predict water quality failures in distribution pipe networks, Water Research Foundation, Denver, CO, USA.

Sawyer, P.J. and Rao, S.S. (1994). *Fault tree analysis of Fuzzy Mechanical system.* Microelectronics and Reliability, 34(4), 653-667.

Sentz, K. and Ferson, S. (2002). *Combination of evidence in Dempster–Shafer theory.* Sandia National Laboratories, US.

Siler, W. and Buckley, J.J. (2005). *Fuzzy Expert Systems and Fuzzy Reasoning.* John Wiley & Sons, Inc., Hoboken, New Jersey.

Singer, D. (1990). *A fuzzy approach to fault tree and reliability analysis.* Fuzzy Sets and Systems, 34,145-155.

Skelton, B. (1997). *Process Safety Analysis: an introduction.* Institution of Chemical Engineers.

Spouge J. A. (1999). *Guide to Quantitative Risk Assessment for Offshore Installations. Aberdeen*, UK: CMPT publication.

Suresh, V.P., Babar, A. K. and Raj, V. V. (1996). *Uncertainty in Fault tree analysis: A fuzzy approach.* Fuzzy Sets and Systems, 83,135-141.

Tanaka, H., Fan, T. L., Lai, F. S & Toughi, K. (1983). *Fault tree analysis by fuzzy probability.* IEEE Transactions on reliability, 32(5), 455-457.

Thacker, B. and Huyse, L. (2003), *Probabilistic assessment on the basis of interval data.* Structural Dynamics, and Materials Conference, Published by the American Institute of Aeronautics and Astronautics Inc.

Vesely, W.E., Goldberg, F. F., Roberts, N. H. and Haasl, D. F. (1981). *Fault tree handbook.* U.S. Nuclear Regulatory Commission, NUREG-0492, Washington, DC.

Wang, Y., Yang, J., Xu D. and Chin, K. (2006). *On the combination and normalization of interval-valued belief structures.* J. of Information Sciences 17, 1230–1247.

Wilcox, C. R. and Ayyub, M. B. (2003). *Uncertainty Modeling of Data and Uncertainty Propagation for Risk Studies.* IEEE Proceedings on Uncertainty Modeling and Analysis, 184- 191.

Yager, R. (1987). *On the Dempster-Shafer Framework and New Combination Rules.* Information Sciences, 41: 93-137.

Yuhua, D. and Datao, Y. (2005). *Estimation of failure probability of oil and gas transmission pipelines by Fuzzy fault tree analysis.* Journal of Loss prevention in the process industries, 18, 83-88.

Zadeh, L.A. (1965). Fuzzy sets. Information and Control, 8, 338–353.

Zadeh, L.A. (1984). Review of Books: A Mathematical Theory of Evidence. The AI Magazine, 5(3), 81–83.

CHAPTER 5

Analyzing System Safety and Risks under Uncertainty using a Bow-tie Diagram: an Innovative Approach

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Preface

The developed manuscript for this chapter provides a detailed description of bow-tie analysis including its construction and evaluation procedure for industrial facilities. A version of this manuscript has been submitted to the *Journal of Process Safety and Environmental Protection* for possible publication.

The principal author and the co-authors worked together to develop the research and manuscript for this chapter. The co-authors provided directions and recommendations to develop the framework and approaches for bow-tie analysis.

The principal author designed a case study to demonstrate the applicability of the developed framework and approaches. He developed the Matlab code to simulate the case study and generate the results for interpretation. The co-authors monitored the progress, and investigated and reviewed the output results. The principal author prepared an initial draft of the manuscript, which was later consecutively revised and improved based on the suggested comments and corrections by the co-authors.

Abstract

A bow-tie diagram combines a fault tree and an event tree to represent the risk control parameters on a common platform for mitigating an accident. Quantitative analysis of a bow-tie is still a major challenge since it follows the traditional assumptions of fault and event tree analyses. The assumptions consider the crisp probabilities and "independent" relationships for the input events. The crisp probabilities for the input events are often missing or hard to come by, which introduces *data uncertainty*. And, only the assumption of "independence" introduces *model uncertainty*. Elicitation of expert's knowledge for the missing data may provide an alternative; however, such knowledge incorporates uncertainties and may undermine the credibility of risk analysis.

This paper attempts to accommodate the expert's knowledge to overcome missing data and incorporate fuzzy set and evidence theory to assess the *uncertainties*. Further, dependency coefficient-based fuzzy and evidence theory approaches have been developed to address the *model uncertainty* for bow-tie analysis. In addition, a method of sensitivity analysis is proposed to predict the most contributing input events in the bow-tie analysis. To demonstrate the utility of the approaches in industrial application, a bow-tie diagram of the BP Texas City accident is developed and analyzed.

Keywords: Quantitative risk analysis (QRA), uncertainty, interdependence, likelihoods, fault tree analysis (FTA) and event tree analysis (ETA).

5.1 Introduction

"Accident" is the term often used for the occurrence of a single event or a sequence of events that cause undesired consequences. These undesired consequences may be environmental damage, property damage, economic loss, sickness, injury or death. "Risk" is a function of a set of scenario(s), likelihood of occurrence (f) and the consequences themselves (c) (Kaplan and Garrick, 1981; AIChE, 2000).

$$\text{Risk} = g\,(s,\ c,\ f)$$

Risk analysis is a systematic approach that gathers and integrates qualitative and quantitative information of potential causes, consequences, and likelihoods of adverse events. Likelihood of an event refers to a quantitative measurement of occurrence, which is expressed either as frequency or probability of occurrence. Fault tree analysis (FTA) and event tree analysis (ETA) are two well established techniques in performing risk analysis for a system. From a risk analysis perspective, a fault tree develops a graphical model for a particular system through exploring the logical relationship between the causes and occurrence of an undesired event, typically termed as basic events, and a top event (Vesely et al., 1981; Hauptmanns, 1980, 1988). It uses the likelihoods of basic events as input event data and determines the likelihood of the top event. The event tree constructs a graphical model of consequences considering the undesired event as an initiating event and identifies possible outcome events at the end (Lees, 2005). The initiating event propagates through a number of intermediate consequences, which are termed as events. Each event represents a barrier to escalate the consequences of the initiating event until the final outcome events are identified (AIChE, 2000). Like FTA, ETA also considers the likelihoods of events and initiating event as input event data and estimates the likelihoods for the outcome events. Traditional FTA and ETA assume the input events (probability) data are "precisely" known and the independence of the input events (i.e., basic events and events) are independent (CMPT, 1999; Sadiq et al., 2008; Ferdous et al., 2009b, Ferdous et al., 2010). However, these assumptions are often unrealistic and lead to erroneous conclusions and defy the purpose of risk analysis (Ferson et al., 2004; Sadiq et al., 2008; Ferdous et al., 2009b; Markowski et al., 2009, Ferdous et al., 2010).

FTA and ETA distinctly investigate the causes and the consequences of an undesired event for a system. A bow-tie diagram is a combined concept of risk analysis that integrates a fault tree and an event tree on the left and right side of the diagram to represent the risk control parameters such as causes, threats (hazards) and consequences, on a common platform for mitigating an accident. The quantitative analysis of a bow-tie diagram determines the likelihoods of the undesired event as well as the outcome events. Cockshoti (2005), Chevreau et al. (2006), Dianous and Fiévez (2006), and Duijm (2009) describe the procedure of bow-tie analysis in detail. However, they did not consider the associated uncertainties in quantitative evaluation. In the last few years, the bow-tie method has gained acceptance as a credible risk and safety management tool because of the following advantages.

- provides a graphical representation of accident scenarios,
- provides explicit linkages between the causes and the potential outcomes,
- connects possible outcome events with the undesired event and basic events,

- provides guidance throughout, stating from basic causes to the final consequences, and
- provides systematic help in performing comprehensive risk analysis and safety assessment

The common objective of any safety assessment and risk analysis technique is to assure that a process or a system is designed and operated to meet "accepted risk" or a "threshold" criterion such as ALARP (Skelton, 1997; Markowski et al., 2009). These techniques follow several systematic steps: hazard analysis, consequence analysis, likelihood assessment and risk estimation (AIChE, 2000). In each step different approaches may be used, that collectively guide towards estimating the risk, safety and reliability of a system. FTA and ETA individually assist the risk and safety assessment by providing a qualitative hazard analysis and a detail quantitative assessment of likelihood (CMPT, 1999). However, uncertainties hinder FTA and ETA in performing meaningful quantitative analyses. Characterization, representation, and propagation of uncertainties are important and also vital for bow-tie analysis, since the credibility of the analysis fundamentally depends on the FTA and ETA.

Uncertainty is inherent and unavoidable in performing risk analysis since it belongs to the physical variability of a system response and also to the lack of knowledge about the system (Markowski et al., 2009). In general taxonomy, the uncertainty due to natural variation or random behavior of a system is named *aleatory* uncertainty, whereas the uncertainty due to lack of knowledge or incompleteness is termed *epistemic* uncertainty (Bae et al., 2004, Ferdous et al., 2010). These two types of uncertainty can be introduced from any of the three different sources represented in Figure 5.1 (Henley and Kumamoto, 1996; AIChE, 2000; Fredous, 2006). According to Figure 5.1, the sources of uncertainty can be classified as *data uncertainty*, *model uncertainty* and *quality uncertainty*. *Quality uncertainty* refers to the complete and comprehensive evaluation of hazards, including the identification and description of their relationships in developing the fault and event tree. Recursive effort and the implementation of HAZOP, HAZID, and FMEA can reduce this kind of uncertainty for risk analysis (Skelton, 1997; AIChE, 2000; Crowl and Louvar, 2002). It should be noted that the current paper does not address this type of uncertainty while analyzing the bow-tie method. The main objective of this paper is to develop a generic framework for bow-tie analysis under uncertainties, which includes exploiting appropriate techniques to handle *data uncertainty* and introducing the interdependence of input events to explore *model uncertainty*. In addition, a method for sensitivity analysis has been proposed to identify the most important input events and measure the risk for the corresponding events in bow-tie analysis.

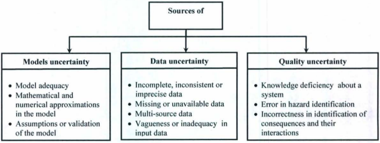


Figure 5.1: Sources of uncertainty (Ferdous, 2006)

5.2 Bow-Tie analysis

Bow-tie analysis is an integrated probabilistic technique that analyzes accident scenarios in terms of assessing the probability and pathways of occurrences (Duijm, 2009). It is intended to prevent, control and mitigate undesired events through development of a logical relationship between the causes and consequences of an undesired event (Dianous and Fiévez, 2006). The fundamentals of bow-tie analysis are described in the following sub-sections.

5.2.1 Basic elements

A bow-tie diagram comprises five basic elements. Figure 5.2 shows the relationships among these elements.

- Causes: The causes are the fundamental reasons that result in failures, malfunctions, faults, or human error at a component level. These reasons are termed basic events (BE).
- Fault Tree (FT): FT graphically represents the path of causation leading to an undesired event. The undesired event is the top event and the interactions of different causes are described using basic events, intermediate events and logic gates.
- Critical Event (CE): In a bow-tie diagram, the top-event of a FT is the initiating event for an ET. This event is called a critical event in the bow-tie.
- Event Tree (ET): ET sequences the possible consequences of the CE considering a dichotomous barrier (i.e., success/failure, true/false, or yes/no) of safety function

(e.g., alarm, automatic shutdown) or accident escalation factor (e.g., ignition, explosion, dispersion).

- Outcome events (OE): The final consequences resulting from systematic propagation of a CE through the barriers are named outcome events.

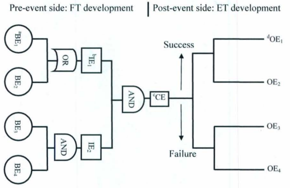


Figure 5.2: Elements of a "Bow-tie" diagram
([a]BE-Basic Events; [b]IE-Intermediate Events; [c]CE- Critical Event; and [d]OE-Outcome Events)

5.2.2 Construction

The construction of a bow-tie diagram follows the same basic rules as required in development of FT and ET. The FT is placed on the left side of the diagram; it starts with the critical event (i.e., top event) and diverges until the basic or intermediate causes are described in terms of basic events with the use of logic gates (e.g., AND and OR gates). The right side of the bow-tie diagram corresponds to ET development, which begins from the critical event as the initiating event and follows the sequences of events (consequences) to reach the outcome events. Based on coupled FT and ET, all causes and consequences related to a critical event are clearly and jointly identified on the bow-tie diagram (Figure 5.2). Many researchers including Cockshoti (2005), Chevreau et al. (2006), Dianous and Fiévez (2006), and Markowski et al. (2009) and Duijm (2009) have illustrated the following basic rules for bow-tie construction:

- Output of a Fault Tree (i.e., top event) is the starting point (i.e., initiating event) for the Event Tree.
- FT and ET are linked to a common critical event.
- Typical causes are identified and placed on the *pre-event side* (left side of diagram).
- Credible scenarios and outcomes are depicted on the *post-event side* (right side of diagram).
- All branches from the pre-*event side* converge towards the critical event and the branches on the *post-event* side diverge until all possible outcome events are identified.

5.2.3 Analysis

Once the bow-tie diagram is constructed, quantitative analyses can be performed following the traditional assumptions and mathematical operations (Table 5.1) for FTA and ETA. Hassal (1965), Veseley et al. (1981), Henley and Kumamoto (1996), AIChE (2000) and Ferdous et al. (2006, 2009b, 2010) describe the traditional conjunction operation for "OR" gates and the intersection operation for "AND" gates for FTA and ETA. The quantitative evaluation to determine the likelihoods of the top event and outcome events for FTA or ETA is often challenging and highly dependent on the quality of knowledge about the system and availability of precise data such as probability and interdependence of input events. The precise probability values of input events are rather scarce and are either typically missing or difficult to acquire (Pan and Yun, 1997).

Table 5.1: Equations used in traditional bow-tie analysis

Approach	Operation	Equation
ETA	Intersection	$P_{OE} = \prod_{i=1}^{n} P_i$
FTA	Conjunction	$P_{OR} = 1 - \prod_{i=1}^{n} (1 - P_i)$
	Intersection	$P_{AND} = \prod_{i=1}^{n} P_i$

5.3 Bow-Tie analysis under uncertainty

Data and *model uncertainty* are common and generally unavoidable. In a majority of cases, the likelihoods of input events are often missing or limited, and lead to *data uncertainty* (Sadiq et al., 2008; Ferdous et al., 2009a, 2009b, 2010). On the other hand, deficiencies in addressing the interdependence of input events in formulation of the conjunction and intersection operations introduce *model uncertainty*. Bow-tie analysis combines the operations of FTA and ETA and determines the likelihood of a critical event as well as the outcome events. Hence, any unaddressed uncertainties in FTA and ETA eventually propagate to the final estimation of bow-tie analysis. A number of theories including probability theory, fuzzy set theory and evidence theory have been proposed to describe uncertainties in risk analysis (Abrahamsson, 2002; Sentz and Ferson; 2002, Wilcox and Ayyub , 2003; Ferdous et al., 2009a, 2009b,2010). Monte Carlo Simulation (MCS) is one of the most popular and common techniques in probability theory (Abrahamsson, 2002; Wilcox and Ayyub, 2003). MCS is a sampling technique that requires probability density functions that are either derived from historical data or are assumed. However, these probability density functions are difficult to obtain (Wilcox and Ayyub, 2003). Expert judgment/knowledge is often employed as an alternative source of objective data to avoid *data uncertainty* in ETA and FTA. This elicited judgment/knowledge may be subjected to imprecision, vagueness, incompleteness and inconsistency (Ayyub and Klir, 2006; Ferdous et al., 2009b, 2010). In an attempt to circumvent these types of *uncertainties* in ETA and FTA, many researchers including Tanaka et al.(1983), Misra and Weber (1990), Singer (1990), Kenarangui (1991), Sawyer and Rao (1994), Suresh et al. (1996), Rivera et al.(1999), Huang et al.(2001), Wilcox and Ayyub (2003), and Ferdous et al. (2009a, 2009b,2010) have explored different methodologies. Markowski et al. (2009) specifically developed a fuzzy-based approach for bow-tie analysis; however, this approach is not capable of capturing uncertainty due to ignorance, incompleteness and inconsistency in the knowledge. Further, this approach was unable to characterize *model uncertainty* that arises due to the assumption of independence among the input events in FTA or ETA.

A generic framework for bow-tie analysis has been proposed in Figure 5.3 that can handle data and model uncertainties in risk analysis. Two different approaches, fuzzy-based and evidence theory-based are developed and used in the framework to address the different kinds of uncertainties in bow-tie analysis. Fuzzy-based approach is used to address the uncertainty due to vagueness, imprecision and subjectivity in an expert's knowledge, whereas evidence theory is used for handling inconsistent, incomplete and conflicting evidence elicited from the different experts. To describe the interdependence of input events, a dependency coefficient (C_d) has been adopted and embedded in both approaches.

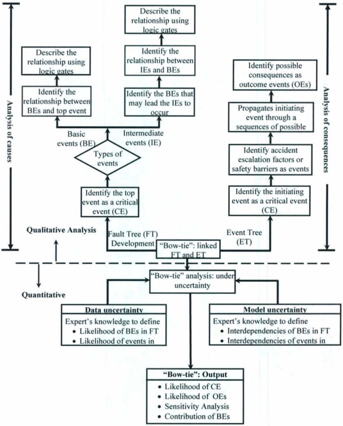


Figure 5.3: Proposed framework for Bow-tie analysis

A dependency coefficient (C_d) within the range of scalar quantity $\in [+1, -1]$ may describe the possible kind of interdependencies among the input events. The scalar quantity $C_d = +1$ refers to perfect dependence, $C_d = 0$ refers to independence, and $C_d = -1$ refers to the existence of opposite dependence among the input events (Ferson et al., 2004; Li, 2007). The fundamentals and details of the proposed approaches are subsequently described in the following sub-sections.

5.3.1 Fundamentals

The basics of fuzzy set theory and evidence theory are discussed in this section.

5.3.1.1 Fuzzy set theory

Zadeh (1965) first introduced fuzzy set theory in his pioneering work, where he argued that traditional probability theory alone is insufficient to represent all types of uncertainties because it lacked the ability to model human conceptualizations that may occur in practice. Fuzzy set theory is able to capture subjective and vague uncertainty and can be viewed as an extension of traditional set theory (Ferdous et al., 2009b, 2010). It provides a language with syntax and semantics to translate qualitative knowledge/judgments into numerical reasoning. For many engineering applications including safety assessment and risk analysis, fuzzy set theory is now a well-accepted and established technique, especially with respect to handling vagueness. Ross (1995, 2004) and Ayyub and Klir (2006) elaborate on the foundations and arithmetical operations of this technique for engineering applications.

Fuzzy set theory uses fuzzy numbers to exploit the numerical relationship between an uncertain quantity p (e.g., basic events, or event probability) and a membership function, which ranges between 0 and 1. A fuzzy number can be formed by any normal, bounded and convex function, e.g., triangular, trapezoidal and Gaussian shapes. However, the selection of a function essentially depends on the variable characteristics, available information and expert's opinion. Triangular or trapezoidal fuzzy numbers (TFN or ZFN) are commonly preferred due to their simplicity. In the current paper, TFNs are used to quantify the subjective and vague uncertainty in an expert's knowledge. For example, a TFN (Figure 5.4) is the simplest possible shape that can express the uncertainty in the likelihood estimates of input events and dependency coefficients for interdependence. A TFN is a vector ($p_{L,}$ p_m, p_U) that can be represented by a lower boundary, most likely value, and upper boundary. The α-cut in a TFN represents the degree of membership or confidence about the uncertainty in a quantity.

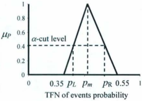


Figure 5.4: TFN representing the uncertainty of likelihood of an input

5.3.1.2 Evidence theory

Evidence theory evolved during the 1970s through the combined effort of Dempster and Shafer (Yang and Kim, 2006; Sentz and Ferson, 2002). The motivation behind the development of this theory was to characterize the uncertainty caused by partial ignorance, knowledge deficiency or inconsistency about a system by the experts (Sentz and Ferson, 2002, Sadiq et al., 2006; Wang et al., 2006). Unlike traditional probability theory, evidence theory considers the subjective probabilities assigned by an expert as evidence and allocates them into the corresponding subsets of a power set. The unassigned probability due to missing information is assigned to the ignorance subset (as opposed to the Bayesian approach that distributes missing evidence in remaining disjoint subsets) (Sentz and Ferson, 2002; Sadiq et al., 2006). Sentz and Ferson (2002) have described the following advantages of evidence theory.

- individual belief can be expressed by probability mass function that may bear incompleteness from partial to full ignorance,
- a belief interval (similar to interval probabilities) can be obtained for each uncertain parameter, and
- bias from a specific source can be avoided and conflicts among different sources can be resolved through a belief structure.

Evidence theory uses three basic measures - *basic probability assignment* (*bpa*), *Belief* (*Bel*), and *Plausibility* measure (*Pl*) - to characterize the uncertainty in a belief structure (Cheng, 2000; Lefevre et al., 2002; Bae et al., 2004, Ferdous et al., 2010). The belief structure is a continuous interval [*belief*, *plausibility*] in which a true probability may lie. A narrow belief structure indicates more precise probabilities. The evidence theory also provides reasoning-based combination rules, which allow the aggregation of different beliefs provided by different experts (Sadiq et al., 2006, Ferdous et al., 2010).

Evidence theory characterizes the uncertainty in a parameter (e.g., likelihood of basic event, event and dependency coefficient) with a definition of *frame of discernment* (FOD). The FOD is a set of mutually exclusive elements that allows having a total of $2^{|\Omega|}$ subsets in a power set (*P*), where $|\Omega|$ is the *cardinality* of the set. For example, two cardinal elements, True (T) and False (F), can be represented by a FOD Ω = {T, F} and may contain four subsets, i.e., {Φ(a null set), {T}, {F}, and {T, F}}. The last subset, {T, F}, accounts for the ignorance of an expert's knowledge which arises due to incomplete and lacking information about a system. The following equations in evidence theory are generally used to develop a computable methodology using the expert's knowledge.

- *bpa*: The basic probability assignment (*bpa*) refers to the subjective probability for a proposition, and is denoted by m(pi). It provides the supporting evidence for each subset of a power set.

$$m(p_i) \rightarrow [0,1] \;;\; m(\Phi) \rightarrow 0 \;;\; \sum_{p_i \subseteq P} m(p_i) = 1 \tag{5.1}$$

- *Bel*: *Belief measure* (*Bel*) represents the lower bound belief for a set p_i and is defined as the sum of all the *bpas* proper subsets p_k of the set of interest p_i, i.e., $p_k \subseteq p_i$.

$$Bel(p_i) = \sum_{p_k \subseteq p_i} m(p_k) \tag{5.2}$$

- *Pl*: *Plausibility* measure (*Pl*) represents the upper bound belief for a set p_i and is the summation of *bpas* of the sets p_k that intersect with the set of interest p_i, i.e., $p_k \cap p_i \neq \Phi$.

$$Pl(p_i) = \sum_{p_k \cap p_i \neq \Phi} m(p_k) \tag{5.3}$$

5.3.2 Application of uncertainty approaches in bow-tie analysis

Both the fuzzy-based and evidence theory-based approaches have been considered to handle *data and model uncertainties* in bow-tie analysis. The likelihoods of the input events and their interdependency relationships are defined by fuzzy numbers or *bpas* . Expert knowledge from a single or multiple sources is employed to elicit the fuzzy numbers or *bpas*. A dependency coefficient (C_d) has been introduced to describe the interdependence of input events (basic events and events) in the bow-tie. The fuzzy numbers address the linguistic and subjective uncertainty whereas *bpas* in evidence theory explores the uncertainty due to incompleteness and inconsistency in the expert's knowledge. The following two sections elaborate the stepwise methodology development of fuzzy-based and evidence theory-based approaches.

5.3.2.1 Fuzzy-based approach

In the proposed fuzzy-based approach, the likelihoods of input events are defined linguistically using TFNs. The interdependence of input events is defined linearly using a dependency coefficient (C_d) that can also be derived using the TFN. The fuzzy-based approach is comprised of the following four steps:

1. Fuzzy numbers to define likelihoods of input events: Experts are more comfortable using linguistic expression rather than numerical judgment when they are asked to define an uncertain quantity like the likelihoods of input events or dependency coefficients (Ayyub and Klir, 2006). In order to capture these linguistic expressions, eight linguistic grades have been proposed to define the likelihoods of input events (Figure 5.5). However, the lower and upper boundary of TFNs for each kind of

linguistic grade can be varied according to the definition of a system. The proposed grades are *Very High* (VH), *Very Low* (VL), *Moderately High* (MH), *Moderately Low* (ML), *Low* (L), *Moderate* (M), *High* (H), *Rather High* (RH). The likelihood or probability of input events for the bow-tie can be assigned using these grades.

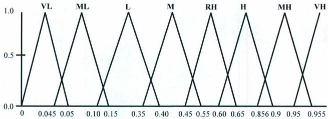


Figure 5.5: Mapping linguistic grades on fuzzy scale

In practice, the interdependence of input events (i.e., the events or basic events) can lie anywhere in the range from *perfect dependence* to *opposite dependence* (Ferdous et al., 2010). The positive dependence belongs to the interval [0, +1], whereas the negative dependence belongs to the interval [-1, 0] (Ferdous et al., 2010). Five linguistic grades are introduced in this study to describe the five types of positive dependence of input events that include: *Perfect Dependence (P), Strong (S), Moderate (M), Weak (W) and Independent (I).* A similar linguistic grade with a negative sign defines the negative dependence for the input events. The dependency coefficient (C_d) categorizes the different kinds of dependence with a numerical range bounded with the lower (C_{dL}) and upper (C_{dU}) values. The ranges in Figure 5.6 are considered in constructing the TFNs for the dependency coefficients.

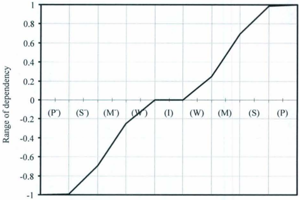


Figure 5.6: Lower (C_{dL}) and Upper (C_{dU}) bounds for each kind of dependency

2. Aggregation of fuzzy numbers: Aggregation provides an agreement among the conflicted knowledge provided by different experts (Lin and Wang, 1997). Wagholiar (2007) summarized a number of aggregation operations including minimum, maximum, arithmetic mean, median, quasi-arithmetic mean, symmetric sum and t-norm for aggregating the fuzzy numbers. The weighted average method is the most common method, which allows the aggregation according to prior weights on the arguments. The weighted average equation for aggregating m experts' knowledge in fuzzy numbers can be defined as:

$$P_i = \frac{\sum_{j=1}^{m} w_j P_{i,j}}{\sum_{k=1}^{m} w_j} \qquad i = 1,2,3,.......n \qquad (5.4)$$

where P_{ij} is the fuzzy number of uncertain input event i elicited from expert j, n is the number of input events, m is the number of experts, w_j is a weighting factor assigned for expert j and P_i is the aggregated fuzzy number. The same equation can also be used in aggregating the fuzzy numbers of dependency coefficients provided by m experts.

3. Determination of likelihood of critical event and outcome events: Fuzzy arithmetical operations are required to calculate the likelihood of a critical event and the outcome events for bow-tie analysis. The dependency coefficient-based fuzzy arithmetic operations have been developed and proposed for the bow-tie analysis. Table 5.2 summarizes the modified fuzzy arithmetic with relative equations.

Table 5.2: Modified fuzzy arithmetic operations

Operation	Evaluation	Formulation
Intersection	Likelihood of outcome events (OE)	$P_{OE} = \prod_{i=1}^{n} (p_{iL}^{\alpha}, p_{iR}^{\alpha})$
	$\widetilde{P}_1 \times \widetilde{P}_2$	$^{*}P_L^{\alpha} = \left[1-(1 \pm C_{dL}^{\alpha}) \times (1 - p_{1L}^{\alpha})\right] \times p_{2L}^{\alpha}$ $P_R^{\alpha} = \left[1-(1 \pm C_{dR}^{\alpha}) \times (1 - p_{1R}^{\alpha})\right] \times p_{2R}^{\alpha}$
Conjunction	$\widetilde{P}_1$ "OR" $\widetilde{P}_2$	$P_L^{\alpha} = \left\{1-(1 - p_{1L}^{\alpha}) \times \left[1-(1 \pm C_{dL}^{\alpha}) \times p_{2L}^{\alpha}\right]\right\}$ $P_R^{\alpha} = \left\{1-(1 - p_{1R}^{\alpha}) \times \left[1-(1 \pm C_{dR}^{\alpha}) \times p_{2R}^{\alpha}\right]\right\}$
Intersection	$\widetilde{P}_1$ "AND" $\widetilde{P}_2$	$P_L^{\alpha} = \left\{\left[1-(1 \pm C_{dL}^{\alpha}) \times (1 - p_{1L}^{\alpha})\right] \times p_{2L}^{\alpha}\right\}$ $P_R^{\alpha} = \left\{\left[1-(1 \pm C_{dR}^{\alpha}) \times (1 - p_{1R}^{\alpha})\right] \times p_{2R}^{\alpha}\right\}$

*"+" is applied for negative dependence and "-" is applied for positive dependence

4. Defuzzification: Defuzzification transforms the fuzzy numbers into a crisp value (Klir and Yuan, 2001). The crisp value is useful in determining the ranks of likelihood of outcome events and calculating the contribution of basic events leading to the critical event and outcome events in bow-tie analysis. A number of defuzzification methods including max membership principle, centroid method, weighted average method, mean max membership, center of sums, center of largest area and first (or last) of maxima, are available (Klir and Yuan, 2001; Ross, 2004). The weighted average method is comparatively easy and computationally efficient. This method is used to defuzzify the fuzzy numbers for the bow-tie analysis (Ross, 2004; Khan and Sadiq, 2005).

$$P_{out} = \frac{\sum \mu_P(\tilde{P}) \cdot \tilde{P}}{\sum \mu_P(\tilde{P})} \qquad (5.5)$$

*5.3.2.2 **Evidence theory-based approach***

Different experts may have different beliefs that may be incomplete and conflict with each other. Evidential reasoning can address the incompleteness, inconsistency and ignorance in the experts' knowledge. The theory allocates the missing *bpa* to the ignorance subset, i.e., $m(\Omega)$ and deals with the conflicts among the sources by employing combination rules (Ferdous et al., 2010). The following sections describe the steps of the evidence theory-based approach for bow-tie analysis.

1. Definition of frame of discernments: Three different FODs for three different uncertain parameters (i.e., likelihood of events and basic events, and dependency coefficient (C_d)) are defined to acquire evidences as *bpas* from the expert's

knowledge. The subsets for each kind of FOD are generated based on their cardinality in the FOD (*Ω*).

Traditionally, the consequence of an event is dichotomous and considers the binary situations, i.e., True (T) or False (F), Yes (Y) or No (N) and Success (S) or Failure (F), to propagate the consequences for identifying the outcome events. Therefore, the FOD for an event can be defined as Ω {S, F} that leads to four subsets in a power set (*P*) that includes $\{\Phi, \{S\}, \{F\}, \{S, F\}\}$.

The operational state of a system is usually defined on the basis of evaluating the success (S) or failure (F) state of basic components (Vesely et.al., 1981). The basic components termed as basic events can be described with the FOD $\Omega = \{S, F\}$ (Hauptmanns, 1980, 1988). As the cardinality is two for this FOD, the power set of each basic event is comprised of four subsets that include $\{\Phi, \{S\}, \{F\}, \{S, F\}\}$.

Nine qualitative grades are categorized in the current study to describe positive and negative dependence of input events for bow-tie analysis. The notation of these grades are: *Opposite dependence* (P^-)*; Negatively Strong* (S^-)*; Negatively Moderate* (M^-)*; Negatively Weak* (W^-)*; Independent (I); Strong (S); Moderate (M); Weak (W); and Perfect dependence (P)* . The FOD for this case consists of nine cardinal elements which can be represented by $\Omega = \{P^-, S^-, M^-, W^-, I, S, M, W, P\}$.

2. Determination of *bpas*: The experts' knowledge has been used to acquire the *bpas* or belief masses to define the likelihoods of the input events and dependency coefficients. Assuming that the knowledge sources are independent, the *bpas* are assigned to particular subsets of each FOD. However, to define the dependency

coefficient, expert knowledge is collected only for the subsets {P-}, {S-}, {M-}, {W-}, {S}, {M}, {W}, {P}, and {Ω}. The *bpas* in each subset individually represents the degree of belief for each expert, and implicitly represents the total evidence that supports the definition of likelihoods of input events (i.e., events or basic events) and dependency coefficients (Cd).

3. Combination of knowledge: The combination rules in evidence theory allow aggregation of different degrees of belief from different expert's knowledge and provide a combined belief structure (Ferdous et al., 2010). The Dempster and Shafer (DS) rule is the fundamental combination rule developed in evidence theory. A number of modifications of the DS rule on the basis of minimization and normalization of conflicts among the different sources have been reported (Sentz and Ferson, 2002; Sadiq et. al., 2006). The most common modifications include those by Yager, Smets, Inagaki, Dubois and Prade, Zhang, Murphy, and more recently by Dezert and Smarandache (Sadiq et al., 2006). Detailed discussion and comparisons of these rules can be found in Dezert and Smarandache (2004). To address two extreme cases of conflictions, high-conflict and non-conflict issues in the experts' knowledge, DS and Yager combination rules are used in this study. The details of these two rules are given below.

    a. *DS rule of combination:* DS combination rule uses a normalizing factor (1-*k*) to develop an agreement among the acquired knowledge from multiple sources, and completely ignores the conflicting evidence through *normalization.* The combination rule uses the AND-type operator (product) for aggregating knowledge from independent sources (Sadiq et al., 2006). For example, if the m_1 (p_a) and m_2 (p_b) are two sets of evidence for the same event collected from two different experts, the DS combination rule uses the following relation to combine the evidence.

$$m_{1\text{-}2}(p_i) = \begin{cases} 0 & \text{for } p_i = \Phi \\ \dfrac{\sum_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b)}{1-k} & \text{for } p_i \neq \Phi \end{cases} \tag{5.6}$$

In the above equation, $m_{1\text{-}2}$ (p_i) denotes the combined knowledge of two experts for the event, and k measures the *degree of conflict* between the two experts, which is determined as:

$$k = \sum_{p_a \cap p_b = \Phi} m_1(p_a) \times m_2(p_b) \tag{5.7}$$

b. Yager rule of combination: Zadeh (1984) pointed out that the DS combination rule yields counterintuitive results and exhibits numerical instability if the conflict among the sources is large (Sentz and Ferson, 2002). To resolve this issue, Yager (1987) proposed an extension, which is similar to the DS combination rule except that it does not allow normalization of joint evidence with the normalizing factor (1-k). The total degree of conflict (k) is assigned to the ignorance subset (Sadiq et al., 2006). However, in a non- (or less) conflicting case, the Yager combination rule exhibits similar results as the DS combination rule. For high-conflict cases (i.e., higher k value), it provides comparatively more stable and robust results than the DS combination rule (Ferdous et al., 2009b, 2010).

$$m_{I\text{-}2}(p_i)=\begin{cases} 0 & for\ p_i=\Phi \\ \sum\limits_{p_a\cap p_b=p_i} m_I(p_a)\times m_2(p_b) & for\ p_i\neq\Omega \\ \sum\limits_{p_a\cap p_b=p_i} m_I(p_a)\times m_2(p_b)+k & for\ p_i=\Omega \end{cases} \quad (5.8)$$

4. Belief structure and Bet estimation: Belief structures for the uncertain parameters including input events and dependency coefficients are derived using the assigned *bpas*, combination rules, and equations of Bel and Pl measures. In order to attain a generalized belief structure, the belief and plausibility measures for coefficients are normalized. The ranges depicted in Figure 5.6 and the belief structures of each kind of dependence are employed for normalizing the final belief structure. Equation 5.9 refers to the normalization technique that is used to determine the belief structure of the dependency coefficient for the input events. Table 5.3 is then applied to calculate the likelihoods of a critical event and the outcome events for the bow-tie analysis.

$$\begin{aligned} Bel(C_d)&=\frac{\sum\limits_{i=1}^{9} Bel(C_{d_i})\times C_{dL_i}}{\sum\limits_{i=1}^{9} Bel(C_{d_i})\times C_{dL_i}+\sum\limits_{i=1}^{9} Pl(C_{d_i})\times C_{dU_i}} \\ Pl(C_d)&=\frac{\sum\limits_{i=1}^{9} Pl(C_{d_i})\times C_{dU_i}}{\sum\limits_{i=1}^{9} Bel(C_{d_i})\times C_{dL_i}+\sum\limits_{i=1}^{9} Pl(C_{d_i})\times C_{dU_i}} \end{aligned} \quad (5.9)$$

where, $Bel(C_{di})$ and $Pl(C_{di})$ are the belief and plausibility measures for each kind of dependency (e,g, *P, S, M,I*); C_{dLi} and C_{dUi} are the lower and upper bounds for each kind of dependency depicted in Figure 5.6 (e.g., for S, $C_{dLi} = 0.7$ and $C_{dUi}=0.995$) *"Bet"* represents a point estimation based on belief structure (similar to defuzzification). It can be determined using the following equation:

$$Bet(P) = \sum_{p_i \subseteq P} \frac{m(p_i)}{|p_i|} \tag{5.10}$$

where, $|p_i|$ is the *cardinality* in the set p_i

Table 5.3: Dependency coefficient based equations

Operation	Evaluation	Formulation
Intersection	Likelihood of outcome events (OE)	$P_{OE} = \prod_{i=1}^{n} \left[Bel(P_i), Pl(P_i)\right]$
	$\tilde{P}_1 \times \tilde{P}_2$	$^{*}Bel(P_{out}) = \left[1 - \{1 \pm Bel(C_d)\} \times \{1 - Bel(P_1)\}\right] \times Bel(P_2)$
		$Pl(P_{out}) = \left[1 - \{1 \pm Pl(C_d)\} \times \{1 - Pl(P_1)\}\right] \times Pl(P_2)$
Conjunction	$\tilde{P}_1$"OR"$\tilde{P}_2$	$Bel(P_{out}) = 1 - \{1 - Bel(P_1)\} \times \left[1 - \{1 \pm Bel(C_d)\} \times Bel(P_2)\right]$
		$Pl(P_{out}) = 1 - \{(1 - Pl(P_1)\} \times \left[1 - \{1 \pm Pl(C_d)\} \times Pl(P_2)\right]$
Intersection	$\tilde{P}_1$"AND"$\tilde{P}_2$	$Bel(P_{out}) = \left[1 - \{1 \pm Bel(C_d)\} \times \{1 - Bel(P_1)\}\right] \times Bel(P_2)$
		$Pl(P_{out}) = \left[1 - \{1 \pm Pl(C_d)\} \times \{1 - Pl(P_1)\}\right] \times Pl(P_2)$

*"+" is applied for negative dependence and "-" is applied for positive dependence

5.3.2.3 Sensitivity analysis

Likelihood assessments in bow-tie analysis provide a numerical approximation of occurrence of the critical event and outcome events without identifying the most significant contributing input events (Ferdous et al., 2009a). Sensitivity analysis (SA) is a systematic approach that can provide a quantitative evaluation to identify the weakest links and better design alternatives of a system, as well as the important sources of variability and uncertainty in the risk analysis (Contini et al., 2000; EPA, 2001; Sadiq, 2001).

SA can be performed using analytical, statistical and graphical methods (Frey and Patil, 2002). Frey and Patil (2002) discuss and review the advantages and disadvantages of each method. The statistical method for SA allows the variation of one or more input events at a time and measures the contributions of each input event on the output event. The analytical method evaluates the sensitivity of an input event while other input events remain constant. The graphical method provides a visual representation of contributions of each input event to an output event. The proposed SA method for bow-tie analysis is comprised of the following two steps:

i. Contribution of input events: Determination of the *correlation coefficient* is the initial step to calculate the contribution of each input event in causing the output events. The traditional statistical method undermines the correlation coefficient if random values of input and output are clustered together (Sadiq, 2001). Spearmen's rank correlation coefficients offer an alternative to avoid such situations (Sadiq, 2001). In this method, the random values are generated from the defined distributions and ranked after sorting the values in ascending order. The calculated output events also need to be ranked in the same way. By definition, the rank correlation coefficient may vary from +1 and -1, and can be determined using the following equation (Lohman et al., 2000).

$$RE_i = \frac{\sum_{l=1}^{N}\left(I_{i,l} - \bar{I}_l\right)\left(O_l - \bar{O}\right)}{\sqrt{\sum_{l=1}^{N}\left(I_{i,l} - \bar{I}_l\right)^2 \sum_{l=1}^{N}\left(O_l - \bar{O}\right)^2}} \qquad i = 1,2,3,.......n \qquad (5.11)$$

where, RE_i refers to the rank correlation coefficients, N to the total number of random values, $I_{i,l}$ *and* O_l denote the ranks of input and output events, respectively, and $\bar{I}_{i,l}$ and $\bar{O}i_{,l}$ represent the mean rank of $I_{i,l}$ and O_l.

The rank correlation coefficients are squared and normalized to 100% in order to estimate the percent contribution of input events leading to an output event (Maxwell and Kastenberg, 1999). A graphical plot, typically named as a tornado plot, can then be drawn to represent the relationships of the input events causing the output events.

ii. Risk reduction: Risk reduction provides a numerical estimation of deducing risk in the output events if the likelihoods of the contributing input events are reduced to a certain level. It is a difficult task to identify the most important input events for large and complex system in order to mitigate the overall system risk. The tornado plot, which graphically represents the correlation of the input events to an output event, is integrated to enhance this task for bow-tie analysis. As the proposed approaches for bow-tie analysis provide interval estimation for the output events (i.e., the critical event and the outcome events), the risk reduction in this case cannot be estimated as a point value (Suresh et al., 1996). The present work proposes an interval based estimation (Equation 5.12) to measure the percentage of risk reduction in the corresponding output event. This equation is developed by following the basic principle of Birnbaum importance measures, which estimate the importance using the difference between the unavailability of a system including and excluding the contributed input events in the calculation (Suresh et al., 1996). Tanaka et al. (1983) and Lai et al. (1993) also use a similar equation to measure the improvement index of each input event in fuzzy measures

$$R_i(O,O_i)=\sum_{b=1}^{r}(O^L-O_i^L)+(O^U-O_i^U) \tag{5.12}$$

where R_i is the risk reduction in an output event, O refers to the likelihood of the output event while the occurrences of all input events are considered, O_i denotes the likelihood of the output event while the likelihood of the input event i is reduced to a certain level, and b refers to the number of values in an interval. For example, the TFN uses three values, p_L, p_m, p_U, (Figure 5.4), and a belief structure exploits two values to represent the uncertainty.

5.4 Explosion at BP Texas city refinery: an illustrative example

On March 23, 2005, a massive explosion and fire erupted in the BP refinery, located 30 miles southwest of Houston in Texas City, Texas. This accident caused fifteen fatalities and injured over 180 people (CSB 2007, 2008). BP (2005) and CSB (2007) have published a detailed investigation report of the accident. The fire and explosion occurred in the refinery during restart of the ISOM unit, as shown in Figure 5.7, and involved the Raffinate splitter, Blowdown drum and stack as a part of daily operation (CSB, 2007 and

2008). Khan and Amyotte (2007), and CSB (2007) present a detailed process description and quantitative risk assessment study. As noticed in CSB (2007, 2008), the explosions occurred due to a significant release of high flammable hydrocarbon from the blowdown drum and stack, which did not have a flare system. The released hydrocarbon immediately formed a vapor cloud and exploded in the presence of a suspected ignition source of an idling diesel pickup truck located about 25 ft away from the blowdown drum (CSB, 2007 and 2008). Considering the highly flammable hydrocarbon release as a critical event, a bow-tie diagram for the BP accident has been constructed in Figure 5.8. Table 5.4 gives the identified causes as basic events and consequences. Two references, CSB (2008) and Yang et al. (2010), have been used to derive the information in Table 5.4. The proposed uncertainty-based bow-tie analysis was performed to analyze the risk of the possible outcomes of the BP accident. The implementation of the proposed bow-tie analysis will provide an opportunity to reinvestigate the events and possible pre-events to such accidents in the future.

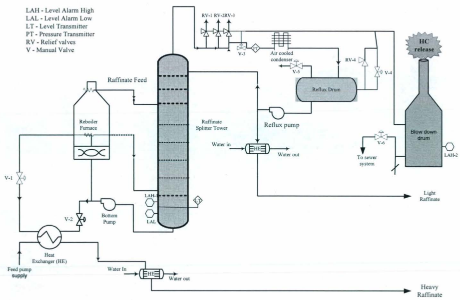


Figure 5.7: Hydrocarbon release from ISOM unit at BP accident

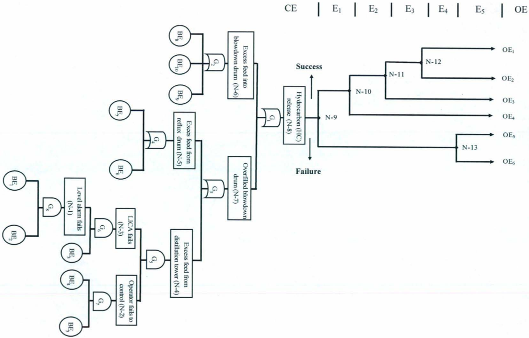


Figure 5.8: "Bow-tie" diagram for BP Texas City refinery accident

Table 5.4: Identified causes and consequences for BP Texas City refinery accident

Bow-tie reference	Basic events	Bow-tie reference	Events	Bow-tie reference	Outcome events
BE_1	LAH-1 fails	E_1	Vapor Clouds	OE_1	Vapor Cloud Explosion (VCE)
BE_2	LAL-2 fails	E_2	Drifting Vapor Clouds	OE_2	Fire
BE_3	LT fails reading low	E_3	Ignition	OE_3	Flammable HC Vapor Cloud
BE_4	Low flow alarm ignored	E_4	VCE / Fire	OE_4	HC Vapor Cloud over ISOM unit
BE_5	Temperature alarms ignored	E_5	Post-explosion fire	OE_5	Pool fire
BE_6	RV-6 fails to close			OE_6	Pool of HC
BE_7	Pump fails				
BE_8	RV-1,2,3 (Relief valves) fail to close				
BE_9	V-6 fails to open				
BE_{10}	LAH-3 fails				

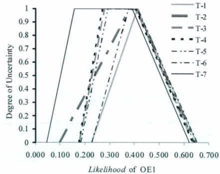
5.4.1 Fuzzy-based approach

Elicited knowledge from two experts was used to define the likelihoods of input events for the bow-tie analysis (Table 5.5). Equal weights were assigned to both experts and expert aggregated values were estimated as shown in Table 5.5. To calculate the likelihood of the critical event and outcome events, fuzzy arithmetic operations described in Table 5.2 were applied. Seven different trials based on different interdependence assumptions for the input events at nodes N-3, N-4 and N-8 were performed while estimating the likelihoods. In Table 5.6, the TFN of the dependency coefficient for a single trial and the executed operations in each node are presented. For the different trials, the uncertainties in the estimates of the likelihood of the CE (critical events) and OE_1 (outcome event 1) were measured and are shown in Figure 5.9. It is obvious that the interdependence of input events has a strong influence over the measurement of uncertainties for the output events (e.g., CE or OEs). In trial 7, when perfect dependences are assumed, the likelihood estimates for the CE bear the maximum uncertainty. Contrary to trial 1, when the input events are assumed as independent, the likelihood of CE bears the smallest uncertainty.

Table 5.5: Expert knowledge in fuzzy scale for the input events of Bow-tie

Input Events	Linguistic grades		Likelihood as TFN (p_L, p_m, p_U)		Aggregated TFNs (p_L, p_m, p_U)
	Expert 1	Expert 2	Expert 1	Expert 2	
BE_1	M	RH	(0.350,0.450,0.550)	(0.450,0.550,0.650)	(0.400,0.500,0.600)
BE_2	M	L	(0.350,0.450,0.550)	(0.100,0.250,0.400)	(0.225,0.350,0.475)
BE_3	M	H	(0.350,0.450,0.550)	(0.600,0.750,0.900)	(0.475,0.600,0.725)
BE_4	RH	L	(0.450,0.550,0.650)	(0.100,0.250,0.400)	(0.275,0.400,0.525)
BE_5	L	RH	(0.100,0.250,0.400)	(0.450,0.550,0.650)	(0.275,0.400,0.525)
BE_6	L	VL	(0.100,0.250,0.400)	(0.000,0.025,0.050)	(0.050,0.138,0.225)
BE_7	VL	ML	(0.000,0.025,0.050)	(0.045,0.097, 0.15)	(0.023,0.061,0.100)
BE_8	ML	L	(0.045,0.097,0.150)	(0.100,0.250,0.400)	(0.073,0.174,0.275)
BE_9	ML	L	(0.045,0.097,0.150)	(0.100,0.250,0.400)	(0.073,0.174,0.275)
BE_{10}	M	RH	(0.350,0.450,0.550)	(0.450,0.550,0.650)	(0.400,0.500,0.600)
E_1	MH	VH	(0.850,0.902,0.955)	(0.950,0.975,1.000)	(0.900,0.939,0.978)
E_2	VH	H	(0.950,0.975,1.000)	(0.600,0.750,0.900)	(0.775,0.863,0.950)
E_3	VH	MH	(0.950,0.975,1.000)	(0.850,0.902,0.955)	(0.900,0.939,0.978)
E_4	H	RH	(0.600,0.750,0.900)	(0.450,0.550,0.650)	(0.525,0.650,0.775)
E_5	MH	RH	(0.850,0.902,0.955)	(0.450,0.550,0.650)	(0.650,0.726,0.803)

Interdependences of input events in assigned Nodes (N)				Fuzzy Numbers of OE_1
Trials (T)	N-3	N-4	N-8	
1	I	I	I	(0.228,0.413,0.413,0.650)
2	S^-	S^-	S^-	(0.098,0.382,0.411,0.648)
3	M	M	M	(0.177,0.278,0.406,0.637)
4	S	W^-	M	(0.176,0.270,0.404,0.635)
5	S	S	M	(0.183,0.293,0.408,0.642)
6	S	W	W	(0.227,0.376,0.412,0.654)
7	P	P	P	(0.047,0.158,0.392,0.622)



Interdependences of input events in assigned Nodes (N)				Fuzzy Numbers of CE
Trials (T)	N-3	N-4	N-8	
1	I	I	I	(0.691,0.836,0.836,0.924)
2	S^-	S^-	S^-	(0.298,0.388,0.803,0.884)
3	M	M	M	(0.538,0.563,0.821,0.906)
4	S	W^-	M	(0.533,0.547,0.818,0.903)
5	S	S	M	(0.555,0.592,0.826,0.912)
6	S	W	W	(0.689,0.762,0.834,0.930)
7	P	P	P	(0.142,0.320,0.875,0.944)

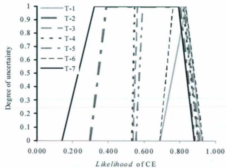


Figure 5.9: Uncertainty representation for OE_1 and CE for different trials

Table 5.6: Dependency of input events (trial 3)

Nodes	Operation	Linguistic grades Expert-1	Dependency coefficient as TFN (C_{dL},C_{dm},C_{dR})
N-1	Intersection	I	(0.000,0.000,0.000)
N-2	Intersection	I	(0.000,0.000,0.000)
N-3	Intersection	VS	(0.800,0.898,0.995)
N-4	Intersection	S	(0.450,0.650,0.850)
N-5	Conjunction	I	(0.000,0.000,0.000)
N-6	Conjunction	Refer to Table 5.6(a)	(0.000,0.000,0.000)
N-7	Conjunction	I	(0.000,0.000,0.000)
N-8	Conjunction	W	(0.150,0.325,0.500)
N-9	Intersection	I	(0.000,0.000,0.000)
N-10	Intersection	I	(0.000,0.000,0.000)
N-11	Intersection	I	(0.000,0.000,0.000)
N-12	Intersection	I	(0.000,0.000,0.000)
N-13	Intersection	I	(0.000,0.000,0.000)

Table 5.6(a): Dependency matrix of input events at N-6

	BE_8	BE_9	BE_{10}
BE_8		I	I
BE_9	I		I
BE_{10}	I	I	

5.4.2 Evidence theory-based approach

In order to demonstrate the evidence theory-based approach, two unbiased and independent experts were engaged to define the likelihoods as well as the dependency coefficients of the input events. Tables 5.7 and 5.8 provide the expert knowledge for these two parameters.

Table 5.7: Expert knowledge on the likelihood of input events

Input Events	Expert 1 (m_1)			Expert 2 (m_2)		
	{S}	{F}	{S, F}	{S}	{F}	{S, F}
BE_1	0.300	0.500	0.200	0.300	0.210	0.490
BE_2	0.200	0.330	0.470	0.200	0.433	0.367
BE_3	0.450	0.250	0.300	0.400	0.350	0.250
BE_4	0.240	0.370	0.390	0.240	0.370	0.390
BE_5	0.310	0.430	0.260	0.310	0.430	0.260
BE_6	0.020	0.650	0.330	0.015	0.115	0.870
BE_7	0.027	0.685	0.288	0.027	0.069	0.905
BE_8	0.073	0.450	0.477	0.063	0.650	0.287
BE_9	0.065	0.650	0.285	0.043	0.550	0.407
BE_{10}	0.300	0.500	0.200	0.300	0.210	0.490
E_1	0.800	0.100	0.100	0.600	0.300	0.100
E_2	0.500	0.140	0.360	0.600	0.250	0.150
E_3	0.650	0.100	0.250	0.550	0.200	0.250
E_4	0.300	0.300	0.400	0.500	0.200	0.300
E_5	0.450	0.150	0.400	0.500	0.250	0.250

Table 5.8: Expert knowledge for the dependency coefficient at different nodes

Experts (Ex)	Node (N)	Possible kind of dependency									
		$\{P^-\}$	$\{S^-\}$	$\{M^-\}$	$\{W^-\}$	{I}	{W}	{M}	{S}	{P}	{Ω}
Ex -1 (m_1)	N-3	0.000	0.250	0.100	0.100	0.100	0.150	0.100	0.140	0.000	0.060
	N-4	0.000	0.150	0.120	0.150	0.200	0.100	0.100	0.140	0.000	0.040
	N-8	0.000	0.000	0.200	0.200	0.100	0.150	0.240	0.100	0.000	0.010
Ex -2 (m_2)	N-3	0.000	0.000	0.200	0.150	0.050	0.150	0.300	0.000	0.000	0.150
	N-4	0.000	0.270	0.130	0.100	0.050	0.080	0.200	0.100	0.000	0.070
	N-8	0.000	0.150	0.190	0.150	0.000	0.150	0.100	0.140	0.000	0.120

[*] P^- - Opposite dependence, S^-- Negatively Strong, M^- - Negatively Moderate, W^-- Negatively Weak, I - Independent, S - Strong, M - Moderate, W- Weak, P - Perfect dependence and Ω-Ignorance subset

The experts' knowledge was aggregated using the DS and Yager combination rules. After aggregation, the final belief structure of dependency coefficients for nodes N-3, N-4 and N-8 were determined by using Equation 5.9. For the others nodes, the interdependence among input events were considered to be independent. Table 5.9 illustrates the belief structures for the input events and dependency coefficients for nodes N-3, N-4 and N-8. Equations in Table 5.3 were then used to derive the belief structures of likelihood of the critical event and outcome events for the bow-tie analysis

Table 5.9: Belief structures of input events and dependency coefficients

Input Events or Nodes (N)	DS rule of combination		Yager rule of combination	
	Bel	*Pl*	*Bel*	*Pl*
*BE1	0.377	0.502	0.297	0.608
BE2	0.245	0.448	0.207	0.533
BE3	0.556	0.657	0.413	0.745
BE4	0.298	0.483	0.245	0.575
BE5	0.351	0.443	0.257	0.592
BE6	0.023	0.314	0.023	0.322
BE7	0.034	0.300	0.033	0.314
BE8	0.060	0.208	0.056	0.268
BE9	0.044	0.168	0.042	0.221
BE10	0.377	0.502	0.297	0.608
E1	0.100	0.114	0.070	0.380
E2	0.185	0.253	0.146	0.409
E3	0.117	0.193	0.095	0.343
E4	0.291	0.443	0.230	0.560
E5	0.215	0.339	0.175	0.463
N-3	-0.075	0.074	-0.007	0.105
N-4	-0.126	0.017	-0.008	0.103
N-8	-0.051	0.112	-0.004	0.109

* The Belief structures for the failure (F) of input events are shown in the table

Table 5.10 presents the results of estimated likelihoods of the critical event (CE) and outcome events for the bow-tie diagram shown in Figure 5.8. For the different types of dependencies, the variations of uncertainty in the bet estimates were measured and are summarized in Table 5.11. An observation can be made from Table 5.11 that the *bet* estimation of the critical event varies significantly with the change of interdependence assumptions for the input events.

Table 5.10: Likelihood of critical event (CE) and outcome events for the Bow-tie

Bow-tie Reference	Name of outcome events	DS rule of combination		Yager rule of combination	
		Bel	***Pl***	***Bel***	***Pl***
CE	Hydrocarbon release	0.457	0.829	0.398	0.895
OE_1	Vapor Cloud Explosion (VCE)	0.136	0.381	0.042	0.495
OE_2	Fire	0.071	0.238	0.022	0.360
OE_3	Flammable HC Vapor Cloud	0.035	0.117	0.014	0.243
OE_4	HC Vapor Cloud over ISOM unit	0.075	0.189	0.036	0.340
OE_5	Pool-fire	0.030	0.074	0.015	0.281
OE_6	Pool of HC	0.010	0.032	0.005	0.157

Table 5.11: Likelihood of critical event (CE) for different kinds of dependencies

Interdependences of input events in assigned Nodes (N)				***Bet* estimation of CE**	
Trials (T)	N-3	N-4	N-8	DS rule of combination	Yager rule of combination
1	I	I	I	*0.661	0.652
2	S-	S-	S-	0.531	0.534
3	M	M	M	0.570	0.572
4	W	W	W	0.643	0.639
5	S	S	S	0.529	0.532
6	P	P	P	0.489	0.497

*Belief structure of critical event for independent case s [0.475, 0.847].

So, m (F) = 0.475, and m(S, F) = 0.375

$$Bet(\text{CE}) = \frac{m(F)}{1} + \frac{m(S,F)}{2} = \frac{0.475}{1} + \frac{0.375}{2} = 0.661$$

The difference in using the DS and Yager combination rules for estimation of the likelihood of outcome event (OE_1) is plotted in Figure 5.10. For the same outcome event, the shaded area indicates that the Yager combination rule provides a large belief structure in comparison to the DS combination rule. Therefore, an interpretation can be made that the Yager combination rule yields more conservative results (i.e., a larger belief structure) in the context of existing high conflicts in the sources of knowledge.

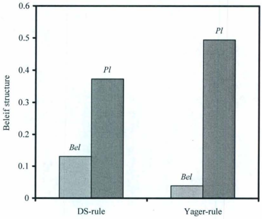


Figure 5.10: Belief structure to represent the likelihood of OE_1 (VCE)

5.5 Results and discussion

CSB (2007) investigated a number of causes and consequences for the BP accident at Texas City. In Table 5.4, some important causes and consequences have been identified as input events for the BP accident bow-tie analysis. The investigation report identified the interdependence relationships of the mechanical component failures and the operator failures as important factors causing the failure of the ISOM unit at BP. Since the likelihoods and the interdependence of most of the input events are unknown for the accident, conducting bow-tie analysis in such uncertain conditions is challenging. Fuzzy-based and evidence theory-based approaches have therefore been carried out to analyze the bow-tie under such uncertain conditions. The demonstration of these two approaches in bow-tie analysis has been described in the previous section.

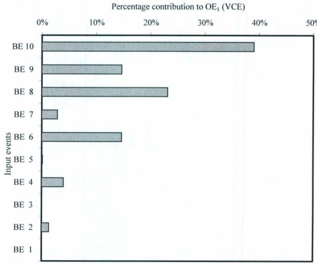
Two types of uncertainties namely, *data* and *model uncertainty,* are explored while analyzing the bow-tie for the BP accident. Elicitation of experts' knowledge and their aggregation are used to minimize the *data uncertainty* while defining the likelihoods or dependency coefficients for the input events. The dependency coefficients are assigned to address the *model uncertainty* and describe the interdependence of input events for the bow-tie analysis.

For example, to address the interdependence of components for the ISOM unit in Figure 5.8, different types of dependency at N-3, N-4 and N-8 were assigned in the corresponding basic events while calculating the likelihood of hydrocarbon release (CE) and outcome events (OEs) for the BP accident (Table 5.4). The output results are depicted in Table 5.10 and Figure 5.9. A significant variation is observed in the estimates of likelihoods for the critical event as well as outcome events while the interdependence at N-3, N-4, and N-8 is varied. For example, see trial 7 in Figure 5.9; here perfect dependence is assigned in the nodes, and the likelihood estimates of the outcome events as well as the critical event bear the maximum uncertainty. This is contrary to trial 1, when the input events are assumed to be independent and the likelihood estimates of these events bear the smallest uncertainty. Similar observations are noted in Table 5.11; i.e., about 24% (Yager rule) variation is observed in the *bet* estimation of the critical event while the interdependence of input events is varied from independent to perfect dependence.

The tornado plot in Figure 5.11 highlights the failure of LAH-3, RV-1, 2, 3, 6 and V-6 as the most significant contributing input events causing the occurrence of OE_1, the vapor cloud explosion. Independent relationships among the input events and a thousand trials were used to perform sensitivity analysis for the bow-tie. The results are provided in Table 5.12 and illustrate that 41% risk can possibly be reduced for the OE_1 (using fuzzy-based approach) if the likelihood of the input event LHL-3 is reduced by about 20%.

Table 5.12: Risk reduction on OE_1 for the most contributed input events

Most contributed input events: Symbol	Most contributed input events: Name	Likelihood measure		Original Likelihood	20 % devalued the likelihood	Risk reduction per % devalued
BE6	RV-6 fails to close	Fuzzy numbers (p_L, p_m, p_U)		(0.050,0.138,0.225)	(0.040,0.110,0.180)	3.39%
		Belief structure [*Bel,Pl*]	DS rules	[0.023,0.33]	[0.018,0.251]	3.57%
			Yager rules	[0.023 0.322]	[0.018,0.257]	2.65%
BE8	RV-1,2,3 fails to close	Fuzzy numbers		(0.073,0.174,0.275)	(0.058,0.139,0.220)	4.54%
		Belief structure	DS rules	[0.060,0.208]	[0.048,0.167]	2.85%
			Yager rules	[0.056,0.268]	[0.044,0.215]	2.31%
BE9	Pump fails	Fuzzy numbers		(0.073,0.174,0.275)	(0.058,0.139,0.220)	4.54%
		Belief structure	DS rules	[0.044,0.168]	[0.035,0.135]	2.14%
			Yager rules	[0.042,0.221]	[0.033,0.177]	1.76%
BE10	LAH-3 fails	Fuzzy numbers		(0.400,0.500,0.600)	(0.320,0.400,0.480)	41.13%
		Belief structure	DS rules	[0.377,0.502]	[0.302,0.402]	16.54%
			Yager rules	[0.297,0.608]	[0.238,0.486]	10.88%



	BE 1	BE 2	BE 3	BE 4	BE 5	BE 6	BE 7	BE 8	BE 9	BE 10
Series1	0.00%	1.34%	0.00%	4.03%	0.17%	14.69	2.88%	23.14	14.69	39.07

Figure 5.11: Tornado plot for OE_1

A comparison of the proposed and traditional approaches was performed based on handling uncertainty in the input events. Table 5.5 provides the basic data for carrying out the comparisons. Equations in Table 5.1 are used to estimate the likelihood of outcome event 1 (OE_1) using the traditional approach. To check the error propagation, the interdependence of input events (i.e., basic events or events) is assumed to be independent and the *percentage deviation* (D) for the OE_1 is measured with 20% introduction of uncertainty in the basic input data. In the fuzzy-based approach, the uncertainty is assigned using the membership function and the TFNs corresponding to 80% membership grade are considered as input-event data. The evidence theory-based approach allocates the uncertainty in terms of *bpa* for the unassigned mass to the power set. The analysis results are shown in Table 5.13, which shows that with 20% uncertainty in the input-event data, 65% deviation is obtained while estimating the likelihood of OE_1 using the traditional approach. The fuzzy- and evidence theory-based approaches measured almost 0.25% and 9% deviation for the same outcome event.

Table 5.13: Error propagation for different approaches

Approaches	Likelihood of VCE (OE_1) *Defuzzified value/ Bet / Deterministic estimation		D (Percentage Deviation)
	Estimated with 20% uncertainty	**Estimated with no uncertainty**	
Fuzzy-based	0.413	0.412	0.24%
Evidence theory-based	0.328	0.360	8.88%
Traditional	0.126	0.360	65.00%

* Defuzzified estimation for the fuzzy-based approach, the Bet measure for the evidence theory-based approach and deterministic estimation for traditional are used to estimate the likelihood of OE_1

5.6 Conclusions

Bow-tie analysis is a relatively new tool for safety assessment and risk analysis of a system. Uncertainties in input data and model adequacy for bow-tie analysis are still a major concern and may mislead the decision-making process. To address the uncertainty as well as mitigate the risk, fuzzy-based and evidence theory-based approaches along with a sensitivity analysis technique were developed for bow-tie analysis. The proposed approaches accommodate the following features that permit conducting risk analysis for any systems under uncertainty.

- Knowledge acquisition offers an alternative to overcome missing data and lack of information about a system. The proposed fuzzy- and evidence theory-based approaches can accommodate experts' knowledge and facilitate risk analysis under situations of missing data and existing relationships among the input events. The aggregation rules and combination rules embedded within these approaches minimize uncertainty by providing consensus knowledge.
- Special treatment procedures are required to explore different types of inherent uncertainties in the experts' knowledge. The fuzzy-based approach can properly address the subjective uncertainty and the evidence theory-based approach can appropriately address the uncertainty due to ignorance and inconsistency associated in the expert's knowledge.
- Introduction of a dependency coefficient in the fuzzy- and evidence theory-based approaches can explore the different kinds of interdependence among input events and addresses the model uncertainty for bow-tie analysis.
- The proposed approaches can apply to safety and risk analysis of any systems that are encountered with data and model uncertainty.
- Sensitivity analysis can identify the most significant contributing input events for the output events in bow-tie analysis and provide an evaluation to mitigate the percentage of risk reduction for a system.

- The developed approaches can handle the uncertainty and minimize error accumulation in likelihood estimation of output events.

Updating the likelihoods and/or the interdependencies of input events with newly arrived information is another important aspect of obtaining credible outputs from risk analysis. Integration of a Bayesian updating mechanism can be considered as a future extension of the developed approaches.

References

Abrahamsson, M. (2002). *Uncertainty in Quantitative Risk Analysis –Characterization and Methods of Treatment.* Department of Fire Safety Engineering, Lund University, Sweden.

American Institute of Chemical Engineers (AIChE). (2000). *Guidelines for chemical process quantitative risk analysis* (2nd ed.) New York: AIChE.

Ayyub, B. and Klir, J. G. (2006). Uncertainty Modeling and Analysis in Engineering and the Sciences. Published by Chapman & Hall/CRC.

Bae, H., Grandhi, V. R. and Canfield, A. R. (2004). An approximation approach for uncertainty quantification using evidence theory. Reliability Engineering and System Safety, 86, 215–225.

BP (2005). Fatal accident investigation report, final report, Texas City. Retrieved from http://www.bp.com/liveassets/bp_internet/globalbp/STAGING/global_assets/downloads/T/texas_city_investigation_report.pdf.

Cheng, Y. (2000). *Uncertainties in Fault Tree Analysis.* Tamkang Journal of Science and Engineering, 3(1), 23-29.

CMPT, (1999). A Guide to Quantitative Risk Assessment for Offshore Installation. The Centre of Marine and Petroleum Technology, UK.

Chevreau, F.R., Wybo, J.L., and Cauchois, D. (2006). Organizing learning processes on risks by using the bow-tie representation. *Journal of Hazardous Materials 130* (3) 276–283.

Cockshott, J. E. (2005). Probability bow-ties: A Transparent Risk Management Tool. *Process Safety and Environmental Protection, 83*(B4), 307–316.

Contini, S., Scheer, S. and Wilikens, M. (2000). Sensitivity Analysis for System Design Improvement. Proceedings of the 2000 International Conference on Dependable Systems and Networks, 243-248, New York, USA.

Crowl D.A., Louvar J.F. (2002). Chemical Process Safety, Fundamentals with Applications. 2nd edition, Prentice Hall , New York.

CSB. (March 2007). Investigation report: refinery explosion and fire, BP Texas city incident final investigation report.

CSB. (August 2008). *Anatomy of a Disaster,* Safety videos 2005-2008.

Dezert, J. and Smarandache, F. (2004). Presentation of DSmT. Chapter 1in advances and applications of DSmT for information fusion (collected works). American Research Press, Rehoboth, 3–35.

Dianous, V. and Fiévez, C. (2006). ARAMIS project: A more explicit demonstration of risk control through the use of bow–tie diagrams and the evaluation of safety barrier performance. *Journal of Hazardous Materials, 130* (3), 220–233.

Duijm, N. J. (2009). Safety-barrier diagrams as a safety management tool. *Reliability Engineering and System Safety 94*(2), 332– 341.

EPA. (2001). Risk Assessment Guidance for Superfund, Sensitivity analysis: how do we know what's important? , Volume 3, Part A, Appendix A, *U. S. Environmental Protection Agency,* Washington D.C. EPA 540-R-02-002.

Ferdous, R., (2006). Methodology for Computer Aided Fuzzy Fault Tree Analysis. Submitted for the Degree of Master of Enginerring, Memorial University of Newfoundland, Canada.

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., and Veitch, B., (2009a). Methodology for computer aided fuzzy fault tree analysis. *Process Safety and Environment Protection, 87*(4), 217–226.

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., and Veitch, B., (2009b). Handling Data Uncertainties in Event Tree Analysis. *Process Safety and Environment Protection* 87(5), 283–292.

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., and Veitch, B., (2010). Fault and Event Tree Analyses for Process Systems Risk Analysis: Uncertainty Handling Formulations. Article in-press, Journal of Risk analysis.

Ferson, S., Hajagos J., Berleant D., Zhang J., Tucker W. T., Ginzburg L. and Oberkampf W. (2004). *Dependence in Dempster-Shafer Theory and Probability Bounds Analysis*. US: Sandia National Laboratories.

Frey, H. C. and Patil, S. R. (2002). Identification and Review of Sensitivity Analysis Methods, *An international journal of Risk analysis, 22*(3), 553-578.

Haasal, D.F. (1965). Advanced concepts in fault tree analysis. *System Safety Symposium*, Seattle, Boeing Company, 8-9.

Hauptmanns, U. (1980). Fault tree analysis of a proposed ethylene vaporization unit. *Industrial Engineering Chemical Fundamentals,* 19,300- 309.

Hauptmanns, U. (1988). Fault tree analysis for process industries engineering risk and hazard assessment. *Engineering Risk and Hazard Assessment,* 1, 21-59, CRC Press Inc., Florida.

Henley, E.J. and Kumamoto, H. (1996). *Probabilistic risk assessment and management for engineers and scientists,* Second Edition, New York: IEE Press.

Huang, D., Chen, T. and Wang, J. M. (2001). A fuzzy set approach for event tree analysis. *Fuzzy Sets and Systems,* 118, 153-165.

Kaplan, S. and Garrick, B.J. (1981). On the quantitative definition of risk. *Risk Analysis. 1* (1), 11-27.

Kenarangui, R. (1991). Event-Tree Analysis by Fuzzy Probability. IEEE Transactions on Reliability, 40(1), 12-124.

Khan, I. F. and Amyotte, P.R. (2007). Modeling of BP Texas City refinery incident. *Journal of Loss Prevention in the Process Industries 20* (4-6) 387–395.

Khan, I. F. and Sadiq R. (2005). Risk-Based Prioritization of Air Pollution Monitoring Using Fuzzy Synthetic Evaluation Technique. *Environmental Monitoring and Assessment*, 105, 261–283.

Klir, J.G. and Yuan, B. (2001). *Fuzzy Sets and Fuzzy logic Theory and Application.* Prentice, Hall of India Private Ltd.

Lai, F.S., Shenoi, S. and Fan, T.L. (1993). Fuzzy fault tree analysis theory and applications. *Engineering Risk and Hazard Assessment*, 1, 117-137, CRC Press Inc., Florida.

Lefevre, E., Colot, O. and Vannoorenberghe, P. (2002). Belief function combination and conflict management. Information Fusion, 3,149–162.

Lees, F. P. (2005). *Loss prevention in the process industries*. Third Edition, 1, London: Butterworths, 9/05-9/122.

Lin, C. and Wang M. J. (1997). Hybrid fault tree analysis using fuzzy sets. *Reliability Engineering and System Safety, 58,* 205-213.

Li, H. (2007). *Hierarchical Risk Assessment of Water Supply Systems*. Submitted for the Degree of Doctor of Philosophy, Loughborough University, UK.

Lohman, K., Pai, P., Seigneur, C. and Levin L. (2000).Sensitivity analysis of mercury human exposure. *The science of the total environment, 259* (1-3), 3-11.

Maxwell, R. M. and Kastenberg, W. E. (1999). Stochastic Environmental Risk Analysis: An Integrated Methodology for predicting cancer risk from contaminated Groundwater. *Stochastic Environmental Research and Risk assessment, 13*(1-2), 27-47.

Misra, B. K. and Weber, G. G. (1990). Use of Fuzzy set theory for level-1 studies in probabilistic risk assessment. *Fuzzy Sets and system,* 37,139-160.

Markowski, A. S., Mannan, M. S. and Bigoszewska, A. (2009). Fuzzy logic for process safety analysis. *Journal of Loss Prevention in the Process Industries 22* (6), 695–702.

H. Pan and Y.W. Yun (1997). Fault tree analysis with fuzzy gates, *Computers Industrial Engineering* 33 (3–4), 569–572.

Rivera, S.S. and Barón, J.H. (1999). *Using* Fuzzy Arithmetic in Containment Event Trees. *International Conference on Probabilistic Safety Assessment- PSA,* Washington, USA, 22-25.

Ross, T. (1995). *Fuzzy logic with engineering applications*, McGraw-Hill, New York.

Ross J. T. (2004). *Fuzzy Logic with Engineering Applications*. John Wiley & Sons, Ltd, West Sussex, England.

Sadiq, R. (2001). Drilling waste discharges in the marine environment: A risk based decision methodology. Memorial University of Newfoundland, Canada.

Sadiq, R., Najjaran, H. and Kleiner, Y. (2006). Investigating evidential reasoning for the interpretation of microbial water quality in a distribution network. *Journal of Stochastic Environmental Research and Risk Assessment, 21*(1), 63–73.

Sadiq, R, Saint-Martin E. and Kleiner Y. (2008). Predicting risk of water quality failures in distribution networks under uncertainties using fault-tree analysis. *Urban Water Journal, 5*(4), 287-304.

Sawyer, P.J. and Rao, S.S. (1994). Fault tree analysis of Fuzzy Mechanical system. *Microelectronics and Reliability, 34*(4), 653-667.

Sentz, K. and Ferson, S. (2002). *Combination of evidence in Dempster–Shafer theory*. Sandia National Laboratories, US.

Singer, D. (1990). A fuzzy approach to fault tree and reliability analysis. *Fuzzy Sets and Systems*, 34,145-155.

Skelton, B. (1997). *Process Safety Analysis: an introduction.* Institution of Chemical Engineers.

Suresh, V.P., Babar, A. K. and Raj, V. V. (1996). Uncertainty in Fault tree analysis: A fuzzy approach. *Fuzzy Sets and Systems*, 83,135-141.

Tanaka, H., Fan, T. L., Lai, F. S., and Toughi, K. (1983). Fault tree analysis by fuzzy probability. *IEEE Transactions on reliability, 32*(5), 455-457.

Vesely, W.E., Goldberg, F. F., Roberts, N. H., and Haasl, D. F. (1981). *Fault tree handbook.* Washington, DC: U.S. Nuclear Regulatory Commission.

Wagholiar S. A. (2007). Acquisition of fuzzy measures in multicriteria decision making using similarity-based reasoning. Griffith University, Gold Coast, Australia.

Wang, Y., Yang, J., Xu D. and Chin, K. (2006). On the combination and normalization of interval-valued belief structures. *Journal of Information Sciences,* 17, 1230–1247.

Wilcox, C. R. and Ayyub, M. B. (2003). Uncertainty Modeling of Data and Uncertainty Propagation for Risk Studies. *IEEE Proceedings on Uncertainty Modeling and Analysis,* 184- 191.

Yager, R. (1987). On the Dempster-Shafer Framework and New Combination Rules. *Information Sciences*, 41,93-137.

Yang, B., and Kim K. J. (2006). Application of Dempster–Shafer theory in fault diagnosis of induction motors using vibration and current signals. *Mechanical Systems and Signal Processing, 20* (2), 403–420.

Yang, X., Rogers W. J. and Mannan, M. S.(2010) . Uncertainty reduction for improved mishap probability prediction: Application to level control of distillation unit. *Journal of Loss Prevention in the Process Industries, 23* (1), 149–156.

Zadeh, L.A. (1965). Fuzzy sets. Information and Control, 8, 338–353.

Zadeh, L.A. (1984). Review of Books: A Mathematical Theory of Evidence. The AI Magazine, 5(3), 81–83.

CHAPTER 6

Handling and Updating Uncertain Information in Bow-tie Analysis

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Preface

The chapter presents a manuscript which developed a methodology for characterizing uncertainty, aggregating expert knowledge, and updating prior knowledge for a bow-tie analysis. A version of this manuscript has been submitted to the *Journal of loss Prevention in Process Industries* for possible publication.

The principal author formulated the approaches for developing the methodology and designed a case study for describing the utility of the methodology. The co-authors supervised the methodology development, reviewed the technical aspects and investigated the output results of the case study. They also provided the essential corrections and guidelines to improve the quality of the manuscript.

Abstract

Bow-tie analysis is a fairly new concept in risk assessment that can describe the relationships among different risk control parameters, such as causes, hazards and consequences to mitigate the likelihood of occurrence of unwanted events in an industrial system. It also facilitates the performance of quantitative risk analysis for an unwanted event providing a detailed investigation starting from basic causes to final consequences. The credibility of quantitative evaluation of the bow-tie is still a major concern since uncertainty, due to limited or missing data, often restricts the performance of analysis. The utilization of expert knowledge often provides an alternative for such a situation. However, it comes at the cost of possible uncertainties related to incompleteness (partial ignorance), imprecision (subjectivity), and lack of consensus (if multiple expert judgments are used). Further, if the bow-tie analysis is not flexible enough to incorporate new knowledge or evidence, it may undermine the purpose of risk assessment.

Fuzzy set and evidence theory are capable of characterizing the uncertainty associated with expert knowledge. To minimize the overall uncertainty, fusing the knowledge of multiple experts and updating prior knowledge with new evidence are equally important in addition to addressing the uncertainties in the knowledge. This paper proposes a methodology to characterize the uncertainties, aggregate knowledge and update prior knowledge or evidence, if new data become available for the bow-tie analysis. A case study comprising a bow-tie for a typical offshore process facility has also been developed to describe the utility of this methodology in an industrial environment.

Keywords: Uncertainty, bow-tie, expert knowledge, updating, fuzzy sets and evidence theory.

6.1 Introduction

Risk and safety assessment is a systematic and scientific way to predict and prevent the occurrence of an accident in an industrial system (Khan and Abbasi, 2001). A number of qualitative and quantitative techniques including HAZOP analysis, Fault Tree Analysis (FTA) and Event Tree Analysis (ETA) have been used for risk assessment (Khan and Abbasi, 1998). However, all of these techniques share a common objective, which is to provide an assurance that a process or a system is designed and operated under an "accepted risk" or a "threshold" criterion such as ALARP (As Low As Reasonably Practicable) (Skelton, 1997; Markowski et al., 2009). A systematic risk assessment technique follows four basic steps: hazard analysis, consequence analysis, likelihood assessment and risk estimation (AIChE, 2000). In each step, different techniques mentioned earlier may be used, which collectively guide toward estimating risk and ensuring system safety. FTA and ETA are two well established techniques that individually assist the risk and safety assessment by providing both a qualitative analysis of hazards identification and a detailed quantitative evaluation of likelihood assessment for undesired events (Spouge ,1999; Crowl and Louvar, 2002; Modarres, 2006).

FTA provides a graphical relationship between the undesired event and basic causes of such an occurrence (Hassal, 1965; Vesely *et al.*, 1981; Hauptmanns, 1980, 1988; Kumamoto and Henley, 1996). The undesired event and basic causes in FTA are typically termed as a top-event and basic events, respectively. Unlike FTA, ETA is a graphical model of consequences that considers the unwanted event as an initiating event and constructs a binary tree for probable consequences with nodes representing a set of success or failure states (AIChE, 2000; Huang *et al.*, 2001; Lees, 2005; Modarres, 2006). The follow-up consequences of the initiating event in ETA are usually termed as events or safety barriers, and the events generated in the end states are known as outcome events (AIChE , 2000). Both techniques use the probability of (e.g. failure or success) basic events and events as quantitative inputs and determine the probability of occurrence for the top-event as well as outcome events for likelihood assessments (Crowl and Louvar, 2002; Modarres, 2006). Bow-tie is a combined concept that integrates both techniques at a common platform, considering the top-event and initiating event as linked to a common event called a critical event (Cockshott, 2005, Chevreau *et al.* 2006, Dianous and Fiévez 2006, Duijm 2009, Markowski *et al.*, 2009, and Ferdous *et al.*, 2010). A sample schematic of a bow-tie diagram is given in Figure 6.1. Like FTA and ETA, bow-tie analysis also uses the probability of failure of basic events as input events in the FTA site and the probability of occurrence (either failure or success) of events as input events on the ETA site for evaluating the likelihood of critical and outcome events (Markowski *et al.*, 2009, and Ferdous *et al.*, 2010). Ferdous *et al.* (2010) provide a detailed description of the advantages, construction and analysis strategy of the bow-tie methodology as a safety and risk assessment tool for industrial systems.

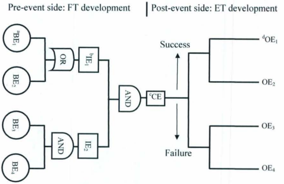

Figure 6.1: Elements of a bow-tie diagram
([a]BE-Basic Event; [b]IE-Intermediate Event; [c]CE- Critical Event; and [d]OE-Outcome Event)

For quantitative bow-tie analysis, the probabilities of input events are required to be known either as precise crisp data or defined probability density functions (PDFs), if uncertainty needs to be considered (Markowski *et al.*, 2009; Ferdous *et al.*, 2010). The crisp data or PDFs are often difficult to come by and even if these are available, precision of this data has many inherent uncertainty issues, such as variant failure modes, design faults, poor understanding of failure mechanisms, as well as the vagueness of system phenomena (Ayyub, 1991; Sawyer and Rao, 1994; Yuhua and Datao, 2005; Wu, 2006, Sadiq *et al.*, 2008; Ferdous *et al.*, 2009a, 2009b, 2011). Since in a majority of cases, crisp data as well as PDFs are rarely available, elicitation of expert knowledge is often employed as an alternative to the acquisition of objective data (Ngyun, 1987; Ayyub, 2001; Yuhua and Datao, 2005; HE *et al.*, 2007).

Uncertainty is inherently unavoidable since it belongs to the physical variability of a system and data unavailability about the system resulting from lack of knowledge or limited information (Ayyub, 1991; Markowski *et al.*, 2009). The uncertainty due to natural variation or randomized behaviour of a physical system is called *aleatory uncertainty*, whereas the uncertainty due to lack of knowledge or incompleteness is termed *epistemic uncertainty* (Bae *et al.*, 2004). Fuzzy sets and evidence theory have been proven to be effective and efficient at handling these types of uncertainties in expert knowledge-based analysis (Bouchon-Meunier *et al.*, 1999; Fagin and Halpern, 1991; Cheng, 2000; Sentz *et al.*, 2002; Wilcox *et al.*, 2003; Boudraa *et al.*, 2004; Bae *et al.*, 2004; Agarwal *et al.*, 2004; Ayyub *et al.*, 2006). However, these theories alone are not capable of updating the likelihoods assessment when a new expert judgement becomes available.

Ferdous *et al.* (2010) developed a framework utilizing fuzzy set and evidence theory to resolve the uncertainties due to employment of expert knowledge in defining the likelihood and interdependence of input events for bow-tie analysis. This framework is intended only for addressing the data and model uncertainty, which are subjected to missing data and interdependent relationships among the input events. It is unable to update the risk estimate of the critical and outcome events in bow-tie analysis if new knowledge or information about an input event is discovered. The current paper is mainly focused on the particular methodology development of implementing an updating mechanism along with the characterization of uncertainty and aggregation of multiple experts' knowledge for bow-tie analysis. The developed methodology helps to address the uncertainty, which occurs in likelihoods assessment and more importantly, updates the analysis recursively if any new knowledge or information is available.

6.2 Risk analysis under uncertainty

Incorporation of expert judgments can help in conducting knowledge-based risk analysis for a complex system. This is especially useful when quantitative information such as the probability of input events is missing or limited (Clemen and Winkler, 1999; Rosqvist, 2003; Ferdous *et al.,* 2009b, 2011). Unfortunately, expert knowledge is often incomplete, inconsistent, vague, or imprecise. This introduces uncertainty in risk analysis (Misra and Weber, 1989; Yuhua and Datao, 2005). In order to recognize this kind of uncertainty and to effectively consider its implications for risk analysis, several formal techniques have been developed (Wilcox and Ayyub, 2003). These techniques can be applied in any quantitative risk analysis model such as fault tree, event tree and bow-tie for uncertainty evaluation. The employment of these techniques is usually categorized based on the type and nature of uncertainty as stated in Table 6.1. Probability theory based Monte Carlo Simulation (MCS) is the most popular among these techniques for conducting uncertainty evaluation (Abrahamsson, 2002; Wilcox and Ayyub, 2003). This sampling based technique requires known PDFs, which are generated from historical data, and is unable to properly address the uncertainty if the knowledge is highly subjective, vague, incomplete or inconsistent (Wilcox and Ayyub, 2003; Druschel *et al*., 2006).

Three different aspects: i) characterization of uncertainty, ii) aggregation of multiple experts knowledge if any, and iii) updating the likelihood with new knowledge, must be considered while formulating the uncertainty of a comprehensive risk analysis, especially when the risk and safety criteria are evaluated based on utilization of expert knowledge. The first aspect, characterization of uncertainty, is essential for categorizing the nature of uncertainty inherited in expert knowledge. Fuzzy numbers in fuzzy set theory and basic probability assignments (*bpas*) in evidence theory are usually employed to address such types of uncertainties. The second aspect, aggregation, is necessary for building a compromise between conflicting data when a lack of consensus arises among the different experts (Lin and Wang, 1997). Dezert and Smarandache (2004) summarized a number of combination rules for evidence theory and Wagholiar (2007) described aggregation techniques for fuzzy set theory that allow fusion of knowledge from different sources. The final aspect, updating, is introduced for incorporating new knowledge with the prior knowledge to obtain an updated likelihood assessment for the analysis. This provides an inference in risk analysis by making a bond between prior knowledge and new knowledge. For each updating, the updated knowledge of the input events is recursively used as new inputs in the risk analysis model (e.g., bow-tie) to attain a revised estimation for likelihood assessment (Freson, 2005).

Table 6.1: Uncertainty categories and theories

Type	Nature	Theory
Aleatory uncertainty	Stochastic, Objective, Irreducible, Random	Probability theory and Evidence theory
Epistemic uncertainty	Imprecise, Incomplete, Ambiguous, Ignorance, Inconsistent, Vague	Possibility theory, Fuzzy set theory and Evidence theory

6.3 Methodology for uncertainty management

The uncertainty-based approaches for ETA, FTA and bow-tie analysis have already been developed (Ferdous *et al*., 2010; 2011). The current work is an extension of the previous developments. In this paper, we attempt to combine the three important aspects of uncertainty management: a) characterization of uncertainty, b) aggregation of multiple expert knowledge, and c) updating prior knowledge for risk analysis. The paper discusses the methodology development for bow-tie analysis, which also encompasses FTA and ETA. In the first step, characterization of uncertainty is developed to address the different kinds of uncertainty in the expert knowledge. Aggregation of knowledge is performed to merge the knowledge from different experts. The updating is integrated for revising the prior knowledge when new information becomes available. The framework developed in Figure 6.2 provides the relationship among the three steps of the proposed methodology. Detailed descriptions for each step are discussed in the following sections.

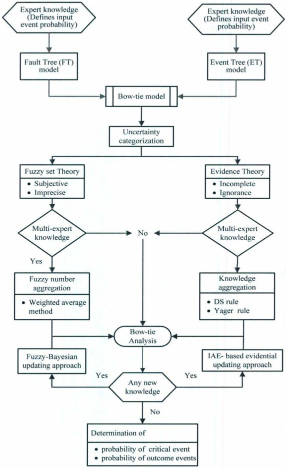

Figure 6.2: Framework for updating risk estimate in bow-tie analysis

6.3.1 Characterization of uncertainty

Expert knowledge offers a better alternative when crisp probability or the PDFs for the input events are not accurately available. Therefore, uncertainty characterization is important in bow-tie analysis as expert knowledge is never absolute and may include different types of uncertainty (Bouchon-Meunier *et al.*, 1999; Ayyub, 2001). To effectively minimize uncertainty, the technique for uncertainty formulation needs to be explored in accordance with the nature of uncertainty existing in the objective data. In the proposed methodology, fuzzy set theory is explored to deal with uncertainty due to vagueness, imprecision and subjectivity in the expert knowledge. Evidence theory is employed to handle uncertainty due to ignorance, incompleteness, and conflicting evidence (Ferdous, 2009b; 2011). The fundamentals of these theories and uncertainty characterization are described in the following sub-sections.

6.3.1.1 Fuzzy Set Theory

Zadeh (1965) first introduced fuzzy set theory in his pioneering work, where he argued that traditional probability theory alone is insufficient to characterize all types of uncertainty associated with human conceptualizations of the real world. Fuzzy set theory is specially designed to provide a language with syntax and semantics to translate qualitative knowledge/judgments into numerical reasoning and to capture subjective and vague uncertainty (Tanaka *et al.*, 1983; Weber, 1994; Abrhamson, 2002; Wu, 2006). Ross (1995; 2004) and Ayyub & Klir (2006) described the foundation and arithmetic operations of fuzzy set theory and its implications for engineering systems for characterization, representation and evaluation of uncertainty in risk analysis.

Fuzzy number: Fuzzy set theory uses fuzzy numbers to capture the imprecision or vagueness in expert assessments (Lin and Wang, 1997). The membership function of a fuzzy number exploits the numerical relationship for an uncertain quantity p (e.g., probability of input events) ranging between 0 and 1 (Sawyer and Rao, 1994). Any type of membership function including normal, bounded and convex functions, e.g., triangular, trapezoidal and Gaussian shapes, can be considered for the formation of a fuzzy number. However, the selection of a function essentially depends on the variable characterization and available information. In the current paper, a TFN is used to quantify subjectivity in the expert knowledge. A TFN can be described by a vector (pL, pm, pU) that represents the lower boundary, most likely value, and upper boundary. The α-cut for a TFN represents the degree of membership of p_f in the set P. The membership function of a TFN can be described as:

$$\mu_P(p_f) = \begin{cases} \dfrac{p_f - p_L}{p_m - p_L} & p_L \le p_f \le p_m \\ \dfrac{p_U - p_f}{p_U - p_m} & p_m \le p_f \le p_U \\ 0 & otherwise \end{cases} \tag{6.1}$$

Risk analysis often articulates expert knowledge/judgment in terms of linguistic variables such as *very high*, *high*, *very low*, *low*, etc. (Ayyub,1991; Wu,2006, Sadiq *et al.*, 2007). Ayyub and Klir, (2006) have provided a chart to define the lower and upper boundary for such variables. Considering the most likely value as an average of these two boundaries, TFNs can be used to represent these types of linguistic variables (Lee, 1996;

Lin And Wang, 1997; Sadiq *et al.*, 2008). For example, eight linguistic variables, e.g., *Very High* (VH), *Very Low* (VL), *Moderately High* (MH), *Moderately Low* (ML), *Low* (L), *Moderate* (M), *High* (H), *Rather* (R), have been proposed in present study to describe expert knowledge for defining the probability of input events. The TFNs of these variables are represented in Figure 6.3 and as an example, the membership functions for *Low (L)*, *Moderate (M)* and *High (H)* are illustrated below:

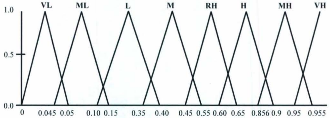

Figure 6.3: Mapping linguistic grades on fuzzy scale

$$\mu_L(p_f)=\begin{cases}\frac{1}{0.15}\times(p_f-0.1) & 0.1\le p_f\le 0.25\\ 1-\frac{1}{0.15}\times(p_f-0.25) & 0.25\le p_f\le 0.4\\ 0 & otherwise\end{cases} \tag{6.2}$$

$$\mu_M(p_f)=\begin{cases}\frac{1}{0.10}\times(p_f-0.35) & 0.35\le p_f\le 0.45\\ 1-\frac{1}{0.10}\times(p_f-0.45) & 0.45\le p_f\le 0.55\\ 0 & otherwise\end{cases} \tag{6.3}$$

$$\mu_H(p_f) = \begin{cases} \frac{1}{0.15} \times (p_f - 0.60) & 0.6 \le p_f \le 0.75 \\ 1 - \frac{1}{0.15} \times (p_f - 0.75) & 0.75 \le p_f \le 0.9 \\ 0 & otherwise \end{cases} \tag{6.4}$$

The fuzzy boundaries of a TFN (i.e., lower and upper boundary) may also be determined from the point of most likely value and error factors (EF) if the rigid fuzzy scale, developed in Figure 6.3, is unable to map the subjective uncertainty of an expert (Huang, 2001). Error factors represent the degree of imprecision associated with experts' knowledge. The magnitude of error factors is often reported along with the most likely value in the literature (Liang and Wang (1993); Huang (2001). Liang and Wang (1993); Suresh *et al.* (1996) and Huang (2001) proposed the equations to determine the fuzzy boundaries of a TFN. The equations also have flexibility to consider the error factors based on direct expert judgment. Equations 6.5a and 6.5b have been derived for two different conditions (i.e., most likely value less than 0.5 or greater than or equal to 0.5) in this study to construct the TFN. Khan and Abbassi (1999) and Ferdous *et al.* (2009a) derived similar equations for trapezoidal fuzzy numbers (ZFN). As an example, the TFN representing an imprecise probability of an input event around "0.2 (most likely probability)" is illustrated in Figure 6.4 and the membership function for this fuzzy number can be derived as Equation 6.6:

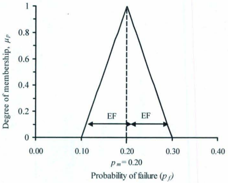

Figure 6.4: TFN represented with error factor (EF)

$$p_l = p_m \times 0.5 \qquad 0 \le p_m < 0.50 \qquad (6.5a)$$
$$p_u = p_m \times 1.5$$

$$p_l = \frac{3p_m - 1}{2} \qquad 0.5 \le p_m < 1.0 \qquad (6.5b)$$
$$p_u = \frac{p_m + 1}{2}$$

$$\mu_p(p_f) = \begin{cases} \frac{1}{0.10} \times (p_f - 0.10) & 0.10 \le p_f \le 0.20 \\ 1 - \frac{1}{0.10} \times (p_f - 0.20) & 0.20 \le p_f \le 0.30 \\ 0 & otherwise \end{cases} \qquad (6.6)$$

6.3.1.2 Evidence theory

The theory of evidence evolved during the 1970s with the joint effort of Dempster and Shafer (Yang and Kim, 2006; Sentz and Ferson, 2002). This theory enables characterization of uncertainty due to partial ignorance or knowledge deficiency in expert judgment (Sentz and Ferson, 2002; Kulasekere *et al.*, 2004; Sadiq *et al.*, 2006; Wang *et al.*, 2006). Unlike traditional probability theory, evidence theory allows the allocation of subjective probabilities, supporting the evidence of expert belief, in corresponding subsets of a power set. The unassigned probability (ignorance) is distributed to an ignorance subset, as opposed to the Bayesian approach, that distributes missing evidence in remaining disjoint subsets (Sentz and Ferson, 2002; Sadiq *et al.*, 2006).

Basic probability assignment (bpa): Evidence theory characterizes uncertainty starting with a definition of frame of discernment (FOD). The FOD represents a set of mutually exclusive elements that allows a total of $2|\Omega|$ subsets in a power set (P), where $|\Omega|$ is the cardinality of the set. The functional state of an input event can be classified in two states: success (S) or failure (F); available or unavailable (Vesely *et al.*, 1981; Stamatelatos, 2002). Therefore, the FOD to characterize the uncertainty of the input event for bow-tie analysis can be defined as Ω {S, F} that leads to four subsets in a power set (P), including {Φ , {S}, {F}, {S, F}}.

In evidence theory, the basic probability assignment (*bpa*), denoted by $m(p_i)$, is used to distribute the probability provided by the expert for each subset belonging to the power set, *P* (Druschel *et al.*, 2006). The unassigned *bpa*, i.e., $m(\Omega)$ =1- m(S) - m(F) accounts for the ignorance or incomplete information in the expert knowledge (Sadiq *et al.*, 2006; Ferdous *et al.*, 2009b).

A belief structure in evidence theory is used to generalize the total uncertainty in an interval bounded by belief (*Bel*) and plausibility (*Pl*) measures. *Bel* (*P*) represents the lower bound of a belief that measures the minimal support for a particular subset, *p*. *Pl*(*P*) represents the upper bound of the belief that determines the maximal support for the subset, *p*. The belief structure for an uncertain parameter like likelihood of input events can be characterized by the following *bpa* function.

$$m(p_i) \rightarrow [0,1] \qquad where,\ m(\Phi) \rightarrow 0 \ and \sum_{p_i \subseteq P} m(p_i) = 1 \tag{6.7}$$

The *Bel* and *Pl* measures for the belief structure can be determined by the following equations:

$$Bel(p_i) = \sum_{p_k \subseteq p_i} m(p_k) \tag{6.8}$$

$$Pl(p_i) = \sum_{p_k \cap p_i \neq \Phi} m(p_k) \tag{6.9}$$

6.3.2 Aggregation of multiple experts knowledge

Knowledge can never be absolute as it is socially constructed and negotiated (Ayyub, 2001). It often suffers from inconsistency since different experts may have different perceptions that may be incomplete and conflict with each other. However, knowledge from multiple experts always provides a better approximation than knowledge from a single expert. In order to incorporate different experts' knowledge in risk analysis, the knowledge from different sources needs to be aggregated before performing the bow-tie analysis (Huang *et al.*, 2001). The following two sub-sections provide the methods of aggregation of fuzzy numbers or *bpas* to define the probability of input events in bow-tie analysis.

6.3.2.1 Fuzzy numbers aggregation

Aggregation provides a mutual agreement and minimizes the conflict among the different sources (Lin and Wang, 1997). A number of methods, e.g., max-min, arithmetic averaging, quasi-arithmetic means, weighted average method, fuzzy Delphi method, symmetric sum and t-norm, are available to aggregate multiple experts' knowledge in the form of fuzzy numbers (Huang *et al.*, 2001; Sadiq *et al.*, 2007; Wagholiar, 2007). The weighted average method is the simplest method allowing aggregation according to prior weights of the arguments. It uses the following equation for aggregating *m* experts' knowledge.

$$P_i = \frac{\sum_{j=1}^{m} w_j P_{i,j}}{\sum_{j=1}^{m} w_j} \qquad i = 1,2,3,\ldots..n \tag{6.10}$$

where P_{ij} is the linguistic expression of uncertain input event i elicited from expert j, n is the number of input events, m is the number of experts, w_j is a weighting factor corresponding to expert j and P_i is the aggregated fuzzy number. For equally weighted knowledge, the weighted average method gives a similar estimation to the arithmetic averaging method.

6.3.2.2 Knowledge aggregation

For identical FODs, the combination rules in evidence theory allow one to aggregate different knowledge from different sources and provide the combined belief structure (Premaratn *et al.*, 2003; Ferson *et al.*, 2004; Sadiq *et al.*, 2007). The Dempster and Shafer (DS) rule is the most fundamental of all the combination rules developed. However, a number of modifications to the DS rule have been executed based on minimization and normalization of conflicts among sources (Sentz and Ferson, 2002; Sadiq *et al.*, 2006). The most common modifications include those by Yager, Smets, Inagaki, Dubois and Prade, Zhang, Murphy, and more recently by Dezert and Smarandache (Sadiq *et al.*, 2006). Detailed discussions and comparisons of these rules can be found in Dezert and Smarandache (2004). In the current study, to address two extreme cases of conflicts i.e., high-conflict and non-conflict issues in experts' knowledge, the DS and Yager combination rules have been used for the purpose of knowledge aggregation. The details of these two rules are given below.

DS rule of combination: The DS combination rule uses a normalizing factor $(1-k)$ to develop an agreement among the acquired knowledge from multiple sources, and completely ignores the conflicting evidence through normalization (Ferson *et al.*, 2004; Sadiq *et al.*, 2007). The combination rule uses the AND-type operator (product) for aggregating knowledge from independent sources (Sadiq *et al.*, 2006). For example, if the $m_1(p_a)$ and m_2 (p_b) are two sets of knowledge for an input event collected from two different experts, the DS combination rule uses the following equation for aggregation.

$$m_{1\text{-}2}(p_i) = \begin{cases} 0 & \text{for } p_i = \Phi \\ \dfrac{\sum\limits_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b)}{1-k} & \text{for } p_i \neq \Phi \end{cases} \tag{6.11}$$

In the above equation, $m_{1\text{-}2}$ (p_i) denotes the combined knowledge of two experts for the event, and k measures the *degree of conflict* between the two experts, which is determined as:

$$k = \sum_{p_a \cap p_b = \Phi} m_1(p_a) \times m_2(p_b) \tag{6.12}$$

Yager rule of combination: Zadeh (1984) pointed out that the DS combination rule may yield counterintuitive results, and exhibits numerical instability if the conflict among the sources is large (Sentz and Ferson, 2002). To resolve this issue, Yager (1987) proposed an extension in Equation 6.13, which is similar to the DS combination rule except that it does not allow normalization of joint evidence with the normalizing factor (1-k). The total degree of conflict (k) is assigned to the ignorance subset (Sadiq *et al.*, 2006). However, in a non- (or less) conflicting case, the Yager combination rule exhibits similar results to the DS combination rule. For high conflict cases (i.e., higher k value), it provides comparatively more stable and robust results than the DS combination rule (Ferdous *et al.*, 2009b).

$$m_{1\text{-}2}(p_i) = \begin{cases} 0 & \text{for } p_i = \Phi \\ \sum\limits_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b) & \text{for } p_i \neq \Omega \\ \sum\limits_{p_a \cap p_b = p_i} m_1(p_a) \times m_2(p_b) + k & \text{for } p_i = \Omega \end{cases} \tag{6.13}$$

6.3.3 Updating prior knowledge

Conditioning is the basic operator for the updating process in probability theory. It provides a recursive way to update prior knowledge conditional to given knowledge (Fagin and Halpern, 1991; Moral and Campos, 1991; Premaratne *et al.*, 2009). Moral and Campos (1991) distinguished the difference between combination and conditioning as combination is a process of merging input of two or more sources of information, whereas conditioning is a restriction of an piece of information that is utilized while the prior knowledge is updated with an another verified new information. The classical probability framework alone is not sufficient for updating the prior probability with incoming knowledge (Nygan, 1987; Chou and Yuan, 1993; Ferson, 2005). Bayes' theorem described in Equation 6.14 provides such an inference for accumulating and updating knowledge based on new given information (Ferson, 2005). Since expert knowledge is often scaled as fuzzy numbers, the integration of Bayes' theorem with fuzzy set theory is essential to update uncertain information in risk analysis. Chou and Yuan (1993), Taheri and Behboodian (2001) and Wu (2006) proposed applications of the fuzzy-Bayesian method in hypotheses testing and structural reliability. In the current work, the fuzzy-Bayesian method is extended for bow-tie analysis considering that the likelihood of input events is not defined precisely by the experts. Unlike the fuzzy-Bayesian approach, conditional notation within the context of evidence theory supports the updating process of a prior mass or belief based on a given proposition (Premaratne *et al.*, 2003; Kulasekere *et al.*, 2004). This also allows updating the *Bel* and *Pl* measures for an input based on the conditional probability (Fagin and Halpern, 1991).

$$P(IE_i / E) = \frac{P(E / IE_i) \times P(IE_i)}{P(E)} \qquad i = 1,2,3,.....n \qquad (6.14)$$

where, IE_i is the i^{th} uncertain input event, for which likelihood is defined as prior knowledge $P(IE_i)$. $P\ (IE_i/E)$ is the posterior knowledge of the input event given new expert knowledge E, and $P\ (E/IE_i)$ the conditional probability for the event following a defined PDF. The denominator of the above equation is called the normalization factor, which can be calculated by the law of total probability (Chou and Yuan, 1993; Ferson, 2005). However, the computation of the normalization factor depends on the aspect of implementation of Bayes' theorem. For bow-tie analysis, it can be calculated using Equation 6.14a, since the likelihood of components in FTA or barriers in ETA (commonly termed as input events in bow-tie analysis) are evaluated on the basis of the conditional success or failure state of the event.

$$P(E) = P(E / IE_s)P(IE_s) + P(E / IE_F)P(IE_F) \qquad (6.14a)$$

6.3.3.1 Fuzzy–Bayesian approach

A fuzzy-Bayesian approach can be used to compute the posterior or updated probability incorporating new subjective knowledge into prior information. The Bernoulli-equation (Equation 6.15) in probability theory is unable to address subjective, imprecise, or vague uncertainty, since the random variable P_f used to describe the likelihood of an input event IE_I may not be exactly unity (event IE_I occurs) or zero (event IE_I does not occur) (Chou and Yuan, 1993). Moreover, the Bayes' theorem given in Equation 6.14 does not consider such fuzziness in the input data (Itoh and itagaki, 1989). The fuzzy-Bayesian approach referenced by Itoh and Itagaki, (1989); Chou and Yuan, (1993); and Carausu and Vulpe, (2001) is appropriate when the likelihood of input events in bow-tie analysis is defined through fuzzy numbers. The proposed fuzzy-Bayesian approach for updating the prior knowledge as well as for computing the posterior probabilities of bow-tie analysis are described in the following discussions.

$$P(IE) = \delta(p_f)P(p_f = 0) + \delta(p_f - 1)P(p_f = 1) \qquad (6.15)$$

where, δ() is the dirac delta function, and P_f is the random variable. In fuzzy measure, p_f is considered as a fuzzy number and represented by a membership function. As an example, if an expert says the probability of an input event IE_I is "Low", then the membership function for this event can be expressed by Equation 6.3. For a continuous fuzzy number, the dirac delta function in Equation 6.15 can be written as Equation 6.16 (Chou and Yuan, 1993).

$$P(IE) = \int_{p_f \in IE} \mu_{IE}(p_f) g(p_f) \qquad (6.16)$$

where, $\mu_{IE}(p_f)$ is the fuzzy number corresponding to the failure probability of input event IE, and $g(p_f)$ is the defined PDF for p_f. The conditional probability in Bayes' theorem can accordingly be revised for fuzzy measure as Equation 6.17.

$$P(E / IE_i) = \int_{p_f \in IE} \mu_{E_i}(p_f) g_{p_f / IE}(p_f) \qquad (6.17)$$

The substitution of Equations 6.16 and 6.17 in Equation 6.14 yields Equation 6.18, that eventually computes the posterior probability for input event IE_i based on the given new expert knowledge E.

$$P(IE_i / E) = \frac{\left[\int_{p_f \in IE} \mu_{E_i}(p_f) g_{p_f/IE}(p_f) df\right] \times P(IE_i)}{P(E)} \tag{6.18}$$

where,

$$P(E) = \left[\int_{p_s \in IE} \mu_{E_i}(p_s) g_{p_s/IE}(p_s)\right] \times P(IE_s) + \left[\int_{p_f \in IE} \mu_{E_i}(p_f) g_{p_f/IE}(p_f)\right] \times P(IE_f)$$

In fuzzy arithmetic, the complement (i.e. probability of success) of the failure probability is determined by Equation 6.19.

$$P(IE_s) == [1 - \widetilde{IE}_f] = [1 - p^{\alpha}_{Lf}, 1 - p^{\alpha}_{mf}, 1 - p^{\alpha}_{Uf}] \tag{6.19}$$

where, $P(IE_s)$ is the complementary probability of IE, p^{α}_{Lf}, p^{α}_{mf} and p^{α}_{Uf} are the left , most-likely and upper values respectively representing the failure probability (IE_f) as TFN.

Experts' knowledge is used to assign the probability of occurrence for the input events in bow-tie analysis. Figure 6.3 is used for constructing the membership functions if the probability values are defined using linguistic variables (e.g., VH, VL, MH, etc.). The probability values defined with an error factor such as "about 0.20", "about 0.15", "about 0.30", etc., are expressed with the membership functions developed using Equations 6.5a and 6.5b. The conditional PDF representing the likelihood of occurrence of an event is usually derived from a set of historical data and fitted to a particular distribution such as exponential, weibull, normal, lognormal, etc., (Ebling, 1997;

Stamatelatos, 2002). In normal operating conditions, exponential distribution is commonly preferred since in that region the likelihood of occurrence of an event follows a constant trend (Ebling, 1997; Crowl and Louvar, 2002, Stamatelatos, 2002) The developed fuzzy-Bayesian approach for bow-tie analysis utilizes exponential distribution (as shown in Figure 6.5) as the conditional PDF function to update prior knowledge whenever new expert knowledge is obtained to define the probability occurrence of input events.

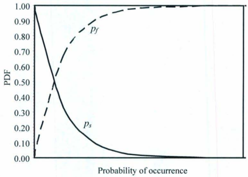

Figure 6.5: Exponential PDF representing sate of input events

6.3.3.2 IAE-based evidential updating

Initial knowledge is typically very buggy, incomplete, and weak (Richardson and Domingos, 2003). The belief structure, representing the range of uncertainty, recursively requires updating with incoming knowledge or evidence in order to create convergence to a true range. Evidence updating strategy conditional to given evidence (knowledge) is proposed in the current study based on the premise (similar to Kulasekere *et al.*, 2004) that updated belief conditional to given evidence is taken to be a linear combination of the originally assigned belief and the conditional belief. Fagin and Halpern (1991) derived the conditional measures by narrating the continual notations in the inner and outer measures to DS notation. Moral and Campos (1991); Premaratne *et al.* (2003, 2009); and Kulasekere *et al.* (2004) explored a number of the expressions to determine the conditional measure and update the belief structure (i.e, [*Bel(IE)*, *Pl(IE)*]. Equation 6.20 is one of the expressions that promptly uses evidence theory to measure the conditional belief.

$$Bel(IE_i / E) = \frac{Bel(IE_i \cap E)}{Bel(IE_i \cap E) + Pl(E - IE_i)} \qquad (6.20)$$

where, *Bel (IE_i(E)* is the conditioning of input event probability IE_i with respect to new evidence E . $\overline{E}$ represents the complementary event of *E*. In a similar manner, the counterpart of Equation 6.20 measures the conditional plausibility. For updating the prior belief and plausibility of an input event of *IE*, the linear combination of *Bel(IE_i) and Bel (IE_i(E)* yields Equations 6.21 and 6.22. The updated probability of input events derived using these equations is then finally used to revise the risk estimate of bow-tie analysis.

$$Bel_E(IE_i) = \alpha_E Bel(IE_i) + \beta_E Bel(IE_i / E) \qquad (6.21)$$

$$Pl_E(IE_i) = \alpha_E Pl(IE_i) + \beta_E Pl(IE_i / E) \qquad (6.22)$$

where, α_E and β_E refer to the weighting parameters, dependent on the conditioning proposition of E. $Bel_E(IE_i)$ and $Pl_E(IE_i)$ denote the updated belief and plausibility measure conditional to E. The summation of the weighting parameters has to be unity, since $m_E(\text{IE}) \mapsto [0,1]$ provided $m_E(\phi) = 0$.

The weighting parameters in the above equations basically measure the inertia of the prior evidence for the updating process. Premaratne *et al.* (2003) and Kulasekere *et al.* (2004) validated the conditioning based updating strategy referred in Equations 6.19 and 6.20 by providing different appealing properties. Several strategies to measure weighting parameters have been reported by Kulasekere *et al.* (2004). In particular, it is a reasonable assumption that the updated belief measure can never be more than the updated plausibility measure, i.e., $Bel_E\,(IE_i) \leq Pl\,(IE_i)$) (Kulasekere *et al.*, 2004). The IAE-based (integrity of available evidence) strategy in Equation 6.23 calculates the weighting parameters that allow the increment of $Bel_E\,(IE_i)$ maximum *to* $Pl\,(IE_i)$.

$$\alpha_E = \begin{cases} \dfrac{1 - Pl(E)}{1 - Bel(E)} & for\ Bel(E) \leq Pl(E) < 1 \\ arbitary\ in\ [0,1] & Bel(E) = Pl(E) = 1 \end{cases} \tag{6.23}$$

6.4 Application of proposed methodology to bow-tie analysis

Fuzzy numbers and *bpas* in the proposed methodology help to characterize the uncertainty associated with expert knowledge for bow-tie analysis. Based on this characterization, either the fuzzy weight average method or combination rules can be employed to unite the knowledge from multiple sources if there are any. Ferdous *et al.* (2010) derived the intersection and conjunction operations to perform bow-tie analysis under different uncertainties with respect to expert knowledge. These operations are also capable of addressing uncertainty regarding the interdependent relationship among input events. For independent cases, these operations can be simplified as the equations depicted in Tables 6.2 and 6.3.

Table 6.2: Fuzzy arithmetic operations for bow-tie analysis

Operation	Evaluation	Formulation	
Intersection	Likelihood of outcome events (OE)	$P_{OE} = \prod_{i=1}^{n} (p_{iL}^{\alpha}, p_{iR}^{\alpha})$	$i = 1,2,3,.....n$
	$\widetilde{P}_1 \times \widetilde{P}_2 \times ... \widetilde{P}_n$	$P_L^{\alpha} = \prod_{i=1}^{n} p_{iL}^{\alpha}$ $P_R^{\alpha} = \prod_{i=1}^{n} p_{iR}^{\alpha}$	$i = 1,2,3,.....n$
Conjunction	$\widetilde{P}_1 \cup \widetilde{P}_2 \cup ..... \cup \widetilde{P}_n$	$P_L^{\alpha} = 1 - \prod_{i=1}^{n} (1 - p_{iL}^{\alpha})$ $P_R^{\alpha} = 1 - \prod_{i=1}^{n} (1 - p_{iR}^{\alpha})$	1,2,3,.....n
Intersection	$\widetilde{P}_1 \cap \widetilde{P}_2 \cap ..... \cap \widetilde{P}_n$	$P_L^{\alpha} = \prod_{i=1}^{n} p_{iL}^{\alpha}$ $P_R^{\alpha} = \prod_{i=1}^{n} p_{iR}^{\alpha}$	1,2,3,.....n

Table 6.3: Evidence reasoning operations for bow-tie analysis

Operation	Evaluation	Formulation	
Intersection	Likelihood of outcome events (OE)	$P_{OE}=\prod_{i=1}^{n}[Bel(P_i), Pl(P_i)]$	
	$\widetilde{P}_1 \times \widetilde{P}_2 \times ... \widetilde{P}_n$	$Bel(P_{out})=\prod_{i=1}^{n} Bel(P_i)$ $Pl(P_{out})=\prod_{i=1}^{n} Pl(P_i)$	$i = 1,2,3,....n$
Conjunction	$\widetilde{P}_1 \cup \widetilde{P}_2 \cup ..... \cup \widetilde{P}_n$	$Bel(P_{out})=1-\prod_{i=1}^{n}[1-Bel(P_i)]$ $Pl(P_{out})=1-\prod_{i=1}^{n}[1-Pl(P_i)]$	$i = 1,2,3,....n$
Intersection	$\widetilde{P}_1 \cap \widetilde{P}_2 \cap ..... \cap \widetilde{P}_n$	$Bel(P_{out})=\prod_{i=1}^{n} Bel(P_i)$ $Pl(P_{out})=\prod_{i=1}^{n} Pl(P_i)$	$i = 1,2,3,....n$

A bow-tie for a typical offshore oil and gas process facility, shown in Figure 6.6, has been developed to demonstrate the utility of the proposed methodology in industrial applications. On an offshore oil and gas process facility, gas leakage is a common issue; this incident may subsequently lead to different credible accidents such as vapor cloud explosion (VCE), fire, explosion and BLEVE. Khan *et al.* (2002) proposed a risk-based safety design and assessment method for offshore facilities to mitigate the risk of such accidents for different process units. They also provided a detailed process description and identified a number of possible causes as basic events that directly or indirectly enhance the occurrence of credible accidents in an offshore facility. Table 6.4

summarizes some of the possible causes as input events for the bow-tie development. In addition to possible causes, other input events are listed in Table 6.4 to describe the likely consequences of a gas leak occurrence on an offshore facility. The developed bow-tie diagram for the facility is illustrated in Figure 6.7. In Figure 6.7, the leakage from the facility is considered as a critical event and the causes and consequences of such an incident are depicted as input events. The models for characterization, aggregation and updating uncertainty in risk estimates are applied to the bow-tie to determine the likelihood of possible outcomes. Different uncertain conditions including the use of expert knowledge for missing data, knowledge from multiple experts, and new knowledge for input events are considered while performing the bow-tie analysis.

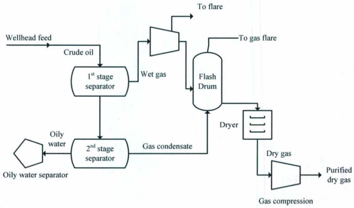

Figure 6.6: Process flow diagram of a typical offshore facility

Table 6.4: Identified causes and consequences for offshore facility

Input event	Bow-tie reference	Description
Basic event	BE_1	Leak from joints
	BE_2	Leak from main pipeline
	BE_3	Leak from joints
	BE_4	Leak from main pipeline
	BE_5	Leak from vessel
	BE_6	Leak from fracture joints and crack
	BE_7	Leak from pipe connections
	BE_8	Leak from safety valves
	BE_9	Leak from release valves
	BE_{10}	Leak from control valves
Event	E_1	Vapor cloud
	E_2	Ignition
	E_3	Drifting vapor cloud
	E_4	Fire in other units
Outcome event	CE	Leakage from unit
	OE_1	Vapor cloud explosion (VCE)
	OE_2	VCE flowed by fire
	OE_3	Fire
	OE_4	Dispersed vapor cloud
	OE_5	VCE fired by other units
	OE_6	Vapor cloud over the unit

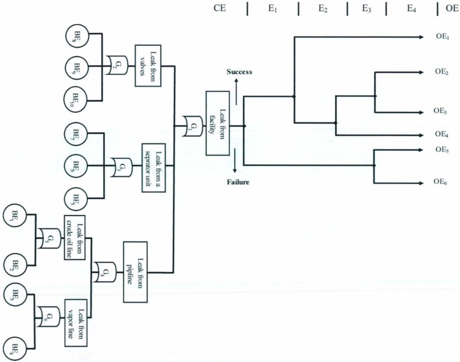

Figure 6.7: "Bow-tie" diagram for the offshore process facility

6.5 Results and analysis

Two different kinds of knowledge (subjective and incomplete) from two different sources were considered while performing the bow-tie analysis. The expert knowledge for the input events is presented in Tables 6.5 and 6.6. The uncertainty due to subjectivity was addressed by fuzzy numbers and aggregated using the weighted average method by assigning equal weights for both experts. The *bpas* in evidence theory considered the incomplete knowledge as ignorance while characterizing the uncertainty and distributing it as an unassigned mass to the power set. DS and Yegar combination rules were applied while combining the two different experts' knowledge for the input events.

Table 6.5: Expert knowledge in fuzzy scale for the input events of bow-tie

Input Events	State {F or S}	Linguistic grades		Likelihood as TFN $((p_L, p_m, p_U)$		Aggregated TFN $((p_L, p_m, p_U)$
		Expert 1	Expert 2	Expert 1	Expert 2	
BE_1	F	VL	"0.03"	(0.000,0.025,0.050)	(0.015,0.030,0.045)	(0.011,0.035,0.059)
BE_2	F	ML	VL	(0.045,0.098,0.150)	(0.000,0.025,0.050)	(0.023,0.061,0.100)
BE_3	F	*"0.035"	VL	(0.018,0.035,0.053)	(0.000,0.025,0.050)	(0.008,0.028,0.048)
BE_4	F	"0.065"	ML	(0.033,0.050,0.098)	(0.045,0.098,0.150)	(0.035,0.074,0.113)
BE_5	F	VL	ML	(0.000,0.025,0.050)	(0.045,0.098,0.150)	(0.023,0.061,0.100)
BE_6	F	"0.10"	VL	(0.050,0.100,0.150)	(0.000,0.025,0.050)	(0.025,0.063,0.100)
BE_7	F	ML	L	(0.045,0.098,0.150)	(0.100,0.250,0.400)	(0.073,0.174,0.275)
BE_8	F	"0.04"	VL	(0.020,0.040,0.060)	(0.000,0.025,0.050)	(0.018,0.037,0.055)
BE_9	F	VL	"0.045"	(0.000,0.025,0.050)	(0.023,0.045,0.068)	(0.010,0.033,0.055)
BE_{10}	F	"0.055"	VL	(0.028,0.055,0.083)	(0.000,0.025,0.050)	(0.014,0.040,0.066)
E_1	S	MH	H	(0.850,0.902,0.955)	(0.600,0.750,0.900)	(0.900,0.939,0.978)
E_2	S	H	"0.9"	(0.600,0.750,0.900)	(0.850,0.900,0.950)	(0.725,0.826,0.928)
E_3	S	H	"0.8"	(0.600,0.750,0.900)	(0.700,0.800,0.900)	(0.550,0.663,0.775)
E_4	S	MH	H	(0.850,0.902,0.955)	(0.600,0.750,0.900)	(0.725,0.826,0.928)

* Values in quotation mark refer to the fuzzy numbers with error factors

Table 6.6: Expert knowledge on the likelihood of input events

Input Event	Expert 1 (m_1)			Expert 2 (m_2)		
	{S}	{F}	{S, F}	{S}	{F}	{S, F}
BE_1	0.800	0.050	0.150	0.850	0.043	0.107
BE_2	0.900	0.025	0.075	0.800	0.070	0.130
BE_3	0.850	0.030	0.120	0.750	0.065	0.185
BE_4	0.670	0.068	0.262	0.730	0.045	0.225
BE_5	0.650	0.065	0.285	0.750	0.050	0.200
BE_6	0.800	0.100	0.100	0.600	0.140	0.260
BE_7	0.650	0.100	0.250	0.700	0.150	0.150
BE_8	0.750	0.050	0.200	0.780	0.035	0.185
BE_9	0.850	0.025	0.125	0.780	0.100	0.120
BE_{10}	0.850	0.080	0.070	0.650	0.095	0.255
E_1	0.870	0.100	0.030	0.780	0.150	0.070
E_2	0.650	0.200	0.150	0.850	0.100	0.050
E_3	0.600	0.300	0.100	0.700	0.200	0.100
E_4	0.750	0.150	0.100	0.650	0.200	0.150

The aggregated knowledge illustrated in Tables 6.5 and 6.7 was employed to determine and evaluate the likelihoods of different outcomes for the offshore facility. The results are presented in Table 6.8 and Figure 6.8. In Table 6.8, with the available prior knowledge, the DS combination rule estimated the belief structures of leak occurrence as [0.290−0.501] for the offshore facility, and VCE as the most likely consequence which measured the highest probability of occurrence as [0.261-0.456]. For the same critical event and outcome event, the Yager combination rule estimated a large belief structure in comparison to the DS combination rule. Therefore, it can be easily interpreted that the Yager combination rule yields more conservative results (i.e., a larger belief structure) in the context of existing high conflicts among the sources.

Table 6.7: Belief structures of input events

Input Event	DS rule of combination				Yager rule of combination			
	Bel		*Pl*		*Bel*		*Pl*	
	S	*F*	*S*	*F*	*S*	*F*	*S*	*F*
*BE_1	0.9675	0.0151	0.9849	0.0325	0.8931	0.0140	0.9861	0.1069
BE_2	0.9782	0.0112	0.9888	0.0218	0.8970	0.0103	0.9898	0.1030
BE_3	0.9593	0.0166	0.9834	0.0407	0.8848	0.0153	0.9847	0.1153
BE_4	0.9032	0.0328	0.9672	0.0968	0.8311	0.0302	0.9699	0.1689
BE_5	0.9048	0.0332	0.9668	0.0952	0.8313	0.0305	0.9695	0.1688
BE_6	0.9034	0.0652	0.9348	0.0966	0.7480	0.0540	0.9460	0.2520
BE_7	0.8739	0.0811	0.9189	0.1261	0.7275	0.0675	0.9325	0.2725
BE_8	0.9412	0.0193	0.9807	0.0588	0.8798	0.0180	0.9820	0.1203
BE_9	0.9632	0.0201	0.9799	0.0369	0.8625	0.0180	0.9820	0.1375
BE_{10}	0.9395	0.0400	0.9601	0.0605	0.8148	0.0347	0.9654	0.1853
E_1	0.9639	0.0335	0.9665	0.0361	0.7629	0.0265	0.9735	0.2371
E_2	0.9314	0.0588	0.9412	0.0686	0.7125	0.0450	0.9550	0.2875
E_3	0.8209	0.1642	0.8358	0.1791	0.5500	0.1100	0.8900	0.4500
E_4	0.8837	0.0963	0.9037	0.1163	0.6650	0.0725	0.9275	0.3350

Table 6.8: Likelihood of critical event and outcome events

Bow-tie Reference	Name of outcome event	DS rule of combination		Yager rule of combination	
		Bel	*Pl*	*Bel*	*Pl*
CE	Leakage from the unit	0.2902	0.5012	0.2580	0.8353
OE_1	Vapor Cloud Explosion (VCE)	0.2605	0.4559	0.1402	0.7765
OE_2	VCE flowed by Fire	0.0119	0.0251	0.0032	0.1930
OE_3	Fire	0.0013	0.0032	0.0004	0.0697
OE_4	Dispersed Vapor Cloud	0.0027	0.006	0.0010	0.1052
OE_5	VCE fired by other units	0.0086	0.0164	0.0045	0.1837
OE_6	VC over the unit	0.0009	0.0021	0.0005	0.0663

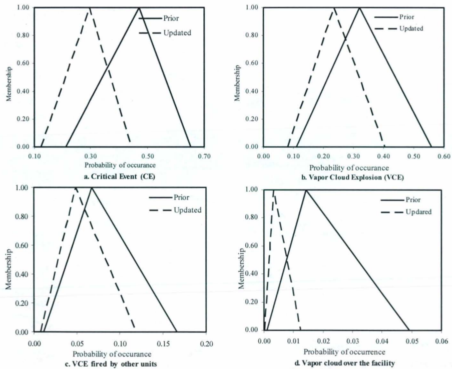

Figure 6.8: Fuzzy numbers representing likelihood of CE and outcome events

The likelihood of the different outcome events for the bow-tie analysis depends on the failure probability of the critical event, as well as the probability of occurrence of subsequent events. Any new knowledge or evidence incorporated with prior information of the input events in bow-tie analysis may provide a different likelihood assessment for the critical event and outcome events. The updating mechanism developed in this study is able to capture the new knowledge and provide updated likelihood for the input events, critical event, and outcome events. As a continuation of bow-tie analysis for the offshore facility, new knowledge for a few selected input events is considered in Table 6.9. The developed fuzzy-Bayesian and IAE-based evidential updating approaches estimated the new probability for these events and provided the updated values as shown in Table 6.10. The bow-tie for the offshore facility is reevaluated based on these updated probabilities that provide a revised estimation (depicted in Figure 6.8 and Table 6.11) for leak occurrence and the outcome events. In Figure 6.8, both the prior and updated fuzzy numbers of leak occurrence and the likelihood of outcome events are illustrated. Figure 6.8 shows that the VCE is the most likely consequence, and the prior most likely value of VCE (measured in a fuzzy number) exhibits 28% deviation when revaluated with the updated knowledge. Table 6.11 represents the updated belief structures for the critical event and outcome events for the bow-tie of the offshore facility. In Table 6.11, it can be observed that the belief estimation, incorporated with the new knowledge, for the outcome event OE_6 is updated almost 22% in comparison to the value calculated in Table 6.8.

Table 6.9: New knowledge for selected input events

Input Events	State	New Expert grade		Belief Structure	
		Linguistic	TFN (p_L, p_m, p_U)	*Bel*	*Pl*
BE_2	*F	"0.02"	(0.010,0.020,0.030)	0.020	0.045
BE_6	F	ML	(0.045,0.098,0.150)	0.090	0.115
BE_7	F	"0.20"	(0.100,0.200,0.300)	0.070	0.150
BE_9	F	ML	(0.045,0.098,0.150)	0.043	0.085
BE_{10}	F	"0.045"	(0.023,0.045,0.068)	0.068	0.160
E_2	S	MH	(0.850,0.902,0.955)	0.856	0.950
E_4	S	"0.85"	(0.775,0.850,0.925)	0.750	0.880

* F- failure state of input events and S- success state of input events

Table 6.10: Updated knowledge for the selected input events

Input Event	TFN (p_L, p_m, p_U)	Belief Structure			
		DS combination rule		Yager combination rule	
		Bel	*Pl*	*Bel*	*Pl*
BE_2	(0.001,0.003,0.006)	0.0110	0.0225	0.2545	0.841
BE_6	(0.005,0.015,0.027)	0.0649	0.0972	0.1665	0.6919
BE_7	(0.032,0.078,0.124)	0.0777	0.1348	0.0025	0.2475
BE_9	(0.002,0.008,0.016)	0.0197	0.0383	0.0002	0.1128
BE_{10}	(0.001,0.005,0.009)	0.0378	0.0673	0.0006	0.1485
E_2	(0.926,0.954,0.981)	0.9334	0.9393	0.0053	0.1681
E_4	(0.650,0.775,0.900)	0.8852	0.9024	0.0004	0.0766

Table 6.11: Updated likelihood for critical event and outcome events

Bow-tie Reference	Name of outcome event	DS rule of combination		Yager rule of combination	
		Bel	*Pl*	*Bel*	*Pl*
CE	Leakage from the unit	0.2853	0.5111	0.2545	0.841
OE_1	Vapor Cloud Explosion (VCE)	0.2567	0.464	0.1665	0.6919
OE_2	VCE flowed by Fire	0.0069	0.0511	0.0025	0.2475
OE_3	Fire	0.0006	0.0094	0.0002	0.1128
OE_4	Dispersed Vapor Cloud	0.0016	0.0121	0.0006	0.1485
OE_5	VCE fired by other units	0.0085	0.0167	0.0053	0.1681
OE_6	Vapor cloud over the unit	0.0007	0.0031	0.0004	0.0766

In order to investigate the nature of the updating approach, the updating of input events was performed fifteen times using the fuzzy-Bayesian approach. The trend of uncertainty range for each update was estimated and observed while evaluating the likelihood of the critical event and outcome events. In each instance of updating, a few arbitrary input events were considered and the prior probability of these events was updated with random new knowledge. The uncertainty range for each update was measured by accounting for the difference in fuzzy boundaries of TFN and plotted in Figure 6.9. The decreasing trends of the uncertainty range for the critical event (CE) and outcome events (OE) in Figure 6.9 clearly show that the uncertainty in the final estimate decreases when the number of updates increases for the input events.

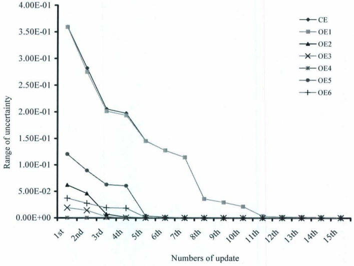

Figure 6.9: Trends of uncertainty range for different number of updates

6.6 Summary and conclusions

Bow-tie analysis is a tool for predicting and analyzing safety and risk for industrial systems. It integrates two well-established techniques (i.e., FTA and ETA) for quantitative risk assessment, it provides an explicit view starting from basic causes to the final consequences of accident scenarios, and it connects possible outcomes of accident scenarios with the critical event and the input events to perform a systematic and comprehensive risk analysis and safety assessment. The quantitative analysis of a bow-tie still has difficulty in estimating the precise occurrence probability of a critical event as well as outcome events, as the probability of occurrence for input events are often missing and estimated using expert knowledge. Expert knowledge is habitually subjected to the uncertainty of incompleteness (partial ignorance) and imprecision (vagueness). The inherent uncertainties (i.e., missing data, natural uncertainties in the expert data, multiple sources of expert data, and incoming knowledge) create challenges to improving the credibility of bow-tie analysis.

A methodology that integrates the characterization of uncertainty, aggregation of different experts' data and updating prior knowledge is developed in the current paper to enhance and improve the overall performance of a bow-tie analysis. The application of this methodology has been demonstrated in bow-tie analysis on a typical offshore process facility. From the analysis, it has been observed that the likelihoods of a critical event and outcome events were computed in a range of values that generalize the total uncertainty associated with expert knowledge. Moreover, incorporating new knowledge or evidence to the input events yields an updated value and provides revised likelihood estimates for a critical event and outcome events. Finally, the developed methodology accommodates the following features which are useful in conducting a systematic risk assessment:

1. Supporting the expert-knowledge elicitation process as a heuristic option for obtaining and updating uncertain information in bow-tie analysis.
2. Accounting for different kinds of uncertainties in expert data while performing likelihood assessment for bow-tie analysis.

3. Facilitating the aggregation and rules of combination techniques to minimize existing conflicts and data inconsistency in different sources of knowledge.
4. Providing compatibility to update the analysis recursively whenever new knowledge becomes available for likelihood assessment in bow-tie analysis.

In the future, this work will be extended towards introducing a similar type of updating approach for describing the interdependent relationships among input events. The different types of conditional PDFs such as weibull, lognormal, normal and others may also be considered in this future extension to explore a more robust fuzzy-Bayesian updating approach for bow-tie analysis

References

Abrahamsson, M. (2002). *Uncertainty in quantitative risk analysis - characterisation and methods of treatment [elektronisk resurs]* Fire Safety Engineering and Systems Safety.

Agarwal, H., Renaud, J. E., Preston, E. L., & Padmanabhan, D. (2004). Uncertainty quantification using evidence theory in multidisciplinary design optimization. *Reliability Engineering & System Safety, 85*(1-3), 281.

American Institute of Chemical Engineers (AIChE). (2000). *Guidelines for chemical process quantitative risk analysis (2nd edition)* Center for Chemical Process Safety/AIChE.

Ayyub Bilal M., & Klir George J. (2006). *Uncertainty modeling and analysis in engineering and the sciences*. Boca Raton, FL 33487-2742, US: Chapman & Hall.

Ayyub, B. M. (2001). *A practical guide on conducting expert-opinion elicitation of probabilities and consequences for corps facilities.* (Technical report no. IWR Report 01-R-01). Alexandria, VA 22315-3868, US: U.S. Army Corps of Engineers. (Expert-Opinion).

Ayyub, B. M. (1991). Systems framework for fuzzy sets in civil engineering. *Fuzzy Sets Syst., 40*(3), 491-508.

Bae, H., Grandhi, R. V., & Canfield, R. A. (2004). An approximation approach for uncertainty quantification using evidence theory. *Reliability Engineering & System Safety, 86*(3), 215.

Bouchon-Meunier, B., Dubois, D., Godo, L., & Prade, H. (1999). Fuzzy sets and possibility theory in approximate and plausible reasoning. In *Handbook of fuzzy sets and possibility theory* (J. Bezdek, D. Dubois, H. Prade, eds. ed., pp. 15-190). Norwell, MA: Kluwer: Kluwer Academic Pub.

Boudraa, A., Bentabet, A., Salzenstein, F., & Guillon, L. (2004). Dempster-shafer's basic probability assignment based on fuzzy membership functions. *Electronic Letters on Computer Vision and Image Analysis, 4*(1), 1-10.

Carausu, A., & Vulpe, A. (2001). Updating fuzzy models for seismic risk assessment. *16th International Conference on Structural Mechanics in Reactor Technology,* Washington, DC ,USA , *16.*

Cheng , Y. (2000). *Uncertainties in fault tree analysis. Tamkang Journal of Science and Engineering, 3*(1), 23-29.

Chevreau, F. R., Wybo, J. L., & Cauchois, D. (2006). Organizing learning processes on risks by using the bow-tie representation. *Journal of Hazardous Materials, 130*(3), 276.

Chou, K. C., & Yuan, J. (1993). Fuzzy-bayesian approach to reliability of existing structures. *Journal of Structural Engineering, 119*(11), 3276-3290.

Clemen, R. T., & Winkler, R. L. (1999). Combining probability distributions from experts in risk analysis. *Risk Analysis, 19*(2), 187-203.

Cockshott, J. E. (2005). Probability bow-ties: A transparent risk management tool. *Process Safety and Environmental Protection, 83*(4), 307.

Crowl, D. A., & Louvar, J. F. (2001). *Chemical process safety, fundamentals with applications* (2nd ed.). Upper Saddle River, New Jersey, USA: Prentice Hall PTR.

Dezert, J., & Smarandache, F. (2004). Presentation of DSmT. In F. Smarandache, & J. Dezert (Eds.), *Advances and applications of DSmT for information fusion* (pp. 1-32). Rehoboth, USA: American Research Press.

Dianous, V., & Fiévez, C. (2006). ARAMIS project: A more explicit demonstration of risk control through the use of bow–tie diagrams and the evaluation of safety barrier performance. *Journal of Hazardous Materials, 130*(3), 220-233

Druschel, R. B., Ozbek, M., & Pinder, G. (2006). Application of dempster-shafer theory to hydraulic conductivity. Copenhagen, Denmark., *5*(31) 31-36.

Duijm, N. J. (2009). Safety-barrier diagrams as a safety management tool. *Reliability Engineering & System Safety, 94*(2), 332-341.

Ebeling, C. (1997). *An introduction to reliability and maintainability engineering.* Boston, MA: McGraw-Hill.

Fagin, R., & Halpern, J. Y. (1991). A new approach to updating beliefs. Paper presented at the *Proceedings of the Sixth Annual Conference on Uncertainty in Artificial Intelligence,* 347-374.

Ferdous, R., Khan, F., Veitch, B., & Amyotte, P. R. (2009a). Methodology for computer aided fuzzy fault tree analysis. *Process Safety and Environmental Protection, 87*(4), 217.

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., & Veitch, B. (2009b). Handling data uncertainties in event tree analysis. *Process Safety and Environmental Protection, 87*(5), 283.

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., & Veitch, B. (2010). Analyzing system safety and risks under uncertainty using a bow-tie diagram: An innovative approach. *Process Safety and Environment Protection (Submitted)*.

Ferdous, R., Khan, F., Sadiq, R., Amyotte, P., & Veitch, B. (2011). Fault and event tree analyses for process systems risk analysis: Uncertainty handling formulations. *Risk Analysis, Risk Analysis, 31(1),* 86–107.

Ferson, S. (2006). *Bayesian methods in risk assessment.* (Technical report No. RAMAS). France.

Ferson, S., Hajagos, J., Berleant, D., Zhang, J., Tucker, W. T., Ginzburg, L., & Oberkampf, W. (2004). *Dependence in dempster-shafer theory and probability bounds analysis**. US: Sandia National Laboratories..

Haasl, F. D. (1965). Advanced concepts in fault tree analysis. *System Safety Symposium,* Boeing Company, Seattle, Washington.

Hauptmanns, U. (1980). Fault tree analysis of a proposed ethylene vaporization unit. *Industrial & Engineering Chemistry Fundamentals, 19*(3), 300-309.

Hauptmanns, U. (1988). Fault tree analysis for process industries engineering risk and hazard assessment. In *Engineering risk and hazard assessment* (pp. 21-59). Florida, US: CRC Press Inc.

He, L., Hong-Zhong Huang, & Zuo, M. J. (2007). Fault tree analysis based on fuzzy logic. Paper presented at the *Reliability and Maintainability Symposium, 2007. RAMS '07. Annual,* 77-82.

Huang, D., Chen, T., & Wang, M. J. (2001). A fuzzy set approach for event tree analysis. *Fuzzy Sets and Systems, 118*(1), 153-165.

Itoh, S., & Itagaki, H. (1989). Application of fuzzy-bayesian analysis to structural reliability. In A. Ang, M. Shinozuka & G. Schueller (Eds.), *Structural safety & reliability* (pp. 1771-1774). New York, USA: ASCE.

Khan, F. I., & Abbasi, S. A. (1998). Techniques and methodologies for risk analysis in chemical process industries. *Journal of Loss Prevention in the Process Industries, 11*(4), 261-277. doi:10.1016/S0950-4230(97)00051-X

Khan, F. I., & Abbasi, S. A. (1999). PROFAT: A user friendly system for probabilistic fault tree analysis. *Process Safety Progress, 18*(1), 42-49.

Khan, F. I., & Abbasi, S. A. (2001). Risk analysis of a typical chemical industry using ORA procedure. *Journal of Loss Prevention in the Process Industries, 14*(1), 43-59.

Khan, F. I., Sadiq, R., & Husain, T. (2002). Risk-based process safety assessment and control measures design for offshore process facilities. *Journal of Hazardous Materials, 94*(1), 1-36.

Kulasekere, E. C., Premaratne, K., Dewasurendra, D. A., Shyu, M. -., & Bauer, P. H. (2004). Conditioning and updating evidence. *International Journal of Approximate Reasoning, 36*(1), 75-108.

Kumamoto, H., & Henley, J. E. (1996). *Probablistic risk assessment and management for engineers and scientists* (2nd ed.) Wiley-IEEE Press.

Lees, F. P. (2005). In Mannan S., O'Connor M. K. (Eds.), *Loss prevention in the process industries* (3rd ed.) Elsevier.

Liang, G., & Wang, M. J. (1993). Evaluating human reliability using fuzzy relation. *Microelectronics and Reliability, 33*(1), 63-80.

Lin, C., & Wang, M. J. (1997). Hybrid fault tree analysis using fuzzy sets. *Reliability Engineering & System Safety, 58*(3), 205-213.

Markowski, A. S., Mannan, M. S., & Bigoszewska, A. (2009). Fuzzy logic for process safety analysis. *Journal of Loss Prevention in the Process Industries, 22*(6), 695-702.

Mattr, M. R., & Domingos, P. (2003). Learning with knowledge from multiple experts. Paper presented at the *International Conference on Machine Learning,* 624-631.

Misra, K. B., & Weber, G. G. (1989). A new method for fuzzy fault tree analysis. *Microelectronics Reliability, 29*(2), 195-216.

Modarres, M. (2006). *Risk analysis in engineering techniques, tools and trends*. Bocca Raton, Florida, U.S.A.: Taylor & Francis.

Moral, S., & De Campos, L. (1991). Updating uncertain information.*521*, 58-67. *Uncertainty in Knowledge Bases (Proc. of the 3rd Inter. Conf. on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU'90), Paris, July 1990)*, Verlag, Berlin, 58–67.

Nguyen, H. T. (1987). On modeling of expert knowledge and admissibility of uncertainly measures. *Mathematical Modelling, 8*, 222-226.

Premaratne, K., Dewasurendra, D. A., & Bauer, P. H. (2003). Evidence updating in a heterogeneous sensor environment. Paper presented at the *Circuits and Systems, Proceedings of the 2003 International Symposium on, 4* IV-824-IV-827 vol.4.

Premaratne, K., Murthi, M. N., Jinsong Zhang, Scheutz, M., & Bauer, P. H. (2009). A dempster-shafer theoretic conditional approach to evidence updating for fusion of hard and soft data. Paper presented at the *Information Fusion, 2009. FUSION '09. 12th International Conference on,* 2122-2129

Rosqvist, T. (2003). *On the use of expert judgment in the qualification of risk assessment.* (Unpublished Doctor of Tech. dissertation). VTT, Espoo, Finland.

Ross, J. T. (1995). *Fuzzy logic with engineering applications.* New York, USA: McGraw-Hill.

Ross, J. T. (2004). *Fuzzy logic with engineering applications* (2nd ed.). West Sussex, England: John Wiley & Sons, Ltd,

Sadiq, R., Saint-Martin, E., & Kleiner, Y. (2008). Predicting risk of water quality failures in distribution networks under uncertainties using fault-tree analysis. *Urban Water Journal, 4*(5), 287.

Sadiq, R., Kleiner, Y., & Rajani, B. (2007). Water quality failures in distribution Networks—Risk analysis using fuzzy logic and evidential reasoning. *Risk Analysis, 27*(5), 1381-1394.

Sadiq, R., Najjaran, H., & Kleiner, Y. (2006). Investigating evidential reasoning for the interpretation of microbial water quality in a distribution network. *Stochastic Environmental Research and Risk Assessment, 21*(1), 63-73.

Sawyer, J. P., & Rao, S. S. (1994). Fault tree analysis of fuzzy mechanical systems. *Microelectronics Reliability, 34*(4), 653-667. doi: 10.1016/0026-2714(94)90030-2

Sentz, K., & Ferson, S. (2002). *Combination of evidence in dempster-shafer theory*. (Technical US Department of Energy: Sandia National Laboratories.

Skelton, B. (1997). *Process safety analysis - an introduction*. Warwickshire, UK: IChemE.

Spouge, J. (1999). *A guide to quantitative risk assessment for offshore installations*. London, UK,: CMPT Publication 99/100.

Stamatelatos, M. (2002). *Probabilistic risk assessment procedures guide for NASA manageress and practitioners*. (Technical Report) Washington, DC, USA: Office of Safety and Mission Assurance, NASA Headquarters.

Suresh, P. V., Babar, A. K., & Raj, V. V. (1996). Uncertainty in fault tree analysis: A fuzzy approach. *Fuzzy Sets and Systems, 83*(2), 135-141. Taheri, S. M., & Behboodian, J. (2001). A bayesian approach to fuzzy hypotheses testing. *Fuzzy Sets and Systems, 123*(1), 39-48.

Tanaka, H., Fan, L. T., Lai, F. S., & Toguchi, K. (1983). Fault-tree analysis by fuzzy probability. *IEEE Transactions on Reliability, 32*(5), 453-457.

Vesely, W. E., Goldberg, F. F., Roberts, N. H., & Haasl, F. D. (1981). *Fault tree handbook*. Washington, DC, USA: U.S. Nuclear Regulatory Commission.

Wagholiar, S. A. (2007). *Acquisition of fuzzy measures in multicriteria decision making using similarity-based reasoning*. (Unpublished Doctor of Philosophy). Griffith University, Gold Coast, Australia.

Wang, Y., Yang, J., Xu, D., & Chin, K. (2007). On the combination and normalization of interval-valued belief structures. *Information Sciences, 177*(5), 1230-1247.

Weber, D. P. (1994). Fuzzy fault tree analysis. *Proceedings of the Third IEEE Conference on Fuzzy Systems,* Orlando, FL , USA. *, 3,* 1899-1904.

Wilcox, R. C., & Ayyub, B. M. (2003). Uncertainty modeling of data and uncertainty propagation for risk studies. *Uncertainty Modeling and Analysis, International Symposium on, 0,* 184.

Wu, H. (2006). Fuzzy bayesian system reliability assessment based on exponential distribution. *Applied Mathematical Modelling, 30*(6), 509-530.

Yager, R. R. (1987). On the dempster-shafer framework and new combination rules. *Information Sciences, 41*(2), 93-137.

Yang, B., & Kim, K. J. (2006). Application of Dempster–Shafer theory in fault diagnosis of induction motors using vibration and current signals. *Mechanical Systems and Signal Processing, 20*(2), 403-420.

Yuhua, D., & Datao, Y. (2005). Estimation of failure probability of oil and gas transmission pipelines by fuzzy fault tree analysis. *Journal of Loss Prevention in the Process Industries, 18*(2), 83-88.

Zadeh, L. (1965). Fuzzy sets. *Fuzzy Sets. Information and Control, 8*, 338-353.

Zadeh, L., (1984). Review of books: A mathematical theory of evidence. *The AI Magazine, 5*(3), 81-83.

CHAPTER 7

Conclusions and Future Research

FTA, ETA and Bow-tie analysis are important techniques for QRA to evaluate, and predict occurrence of accidents for industrial process facilities. Uncertainties are unacceptable and unavoidable, and often undermine the overall purpose of QRA. The uncertainties in QRA are manifested due to insufficient or limited data, unrealistic assumptions, and lack of dynamic nature of risk estimates. Comprehensive frameworks and approaches thus still need to be developed for QRA which can incorporate expert knowledge as an alternative to limited or missing data, characterize and propagate uncertainties, aggregate multiple-source knowledge, and integrate dynamic aspects into risk calculations.

7.1 Summary and conclusions

Literature review (Chapter 2), highlights the limitations associated with the existing QRA methods for industrial process facilities. Most of the previous studies were specific, either following the traditional assumptions or unable to address different types of uncertainties. Furthermore, they were developed for specific methods or approaches, which thus limit their applicability, if a different kind of uncertainty or new knowledge or data become available. The hurdles for developing a comprehensive risk analysis framework include: i) development of appropriate approaches to formulate different kinds of uncertainties, and ii) integration of appropriate method to enhance dynamic qualities in risk estimates.

This study adopted two different theories, namely fuzzy set and evidence theory, to develop uncertainty-based formulations for FTA, ETA and Bow-tie analysis, and updating approaches to integrate dynamic aspect in QRA. The present research is mainly focused on the following objectives: (1) development of a quantitative framework for handling different types of uncertainty in FTA and ETA, (2) development of a comprehensive framework for bow-tie analysis including uncertainty, (3) integration and development of updating inference for incorporating dynamic aspect in the risk analysis, and (4) applications of developed methodologies and approaches in different case studies. Keeping these objectives in perspective, the following conclusions have been achieved.

1. Two types of uncertainty, namely *data* and *dependency* uncertainty, were identified while analyzing a fault tree and event tree following the traditional assumptions. A quantitative framework based on fuzzy and evidence theory was proposed in Chapters 3 and 4 to handle these kinds of uncertainties in FTA and ETA. This framework utilizes the expert knowledge to overcome the *data uncertainty* in FTA and ETA. The dependency coefficient in each node of the fault tree and event tree was used to address the *dependency* uncertainty and to describe the interdependence of basic events/events. In the fuzzy-based approach, the probabilities of events, as well as the dependency coefficient (C_d) of events were defined linguistically using the fuzzy scale comprised of TFNs. The vagueness and subjectivity of the expert knowledge were described employing the extended α-cut based fuzzy empirical equations during analysis with the fault tree and event tree. In the evidence theory-based approach, the *bpa*s were assigned based on expert knowledge. The incomplete and inconsistent

*bap*s from multiple sources were combined using the DS and Yager combination rules. The evidence theory-based empirical relations were then used to address uncertainty related to multiple experts' knowledge and the interdependence of basic events/events. Of the two combination rules, the Yager combination rule provided more reliable aggregation in the context of having high conflicting information in the multiple sources. Consequently, this rule yielded more appropriate results for FTA/ETA with uncertainty, leading to a lower value in the *belief* measure and a higher value in the *plausibility* measure compared to the DS combination rule. The developed approaches are flexible to accommodate the expert knowledge and to handle a wide range of uncertainties associated with the knowledge in case of missing data. These approaches are unique and allow the description of six different levels of interdependence among the basic-events/events.

2. In Chapter 5, fuzzy and evidence theory-based approaches were extended to develop a comprehensive framework (i.e., qualitative and quantitative) for performing bow-tie analysis under *data* and *model* (dependency) uncertainty. First the weighted average method in the fuzzy-based approach was adopted to aggregate the fuzzy numbers assigned by different experts. Second, the developed empirical relations in fuzzy and evidence theory based approaches were modified and extended to address both positive and negative dependence. Finally, sensitivity analysis (SA) comprising two steps were proposed to identify the most contributing input events and estimate the risk reduction for the corresponding events for bow-tie analysis. The developed

framework can also provides a qualitative guideline to construct a bow-tie diagram for any unwanted events stating from the basic causes to its final consequences.

3. Two updating approaches, namely Fuzzy–Bayesian and IAE-based evidential approaches were developed in chapter 6 to incorporate the dynamic aspects and update the prior knowledge for bow-tie analysis. In both updating approaches, first information is considered as prior knowledge. The fuzzy–Bayesian approach uses the TFNs to describe the subjectivity for the prior knowledge, and employs the exponential distribution as a conditional PDF for updating the prior knowledge. On the other hand, the IAE-based evidential updating approach computes the belief interval, comprised of the *belief* and *plausibility* measures, of the posterior knowledge based on the conditional ratio measured from prior *belief* and *plausibility* measures. The *bpa* in the evidence theory-based approach initially characterizes the uncertainty due to incompleteness, deficiency and inconsistency in the knowledge, and measures the belief interval of the prior knowledge. The two updating inferences along with the uncertainty based formulations, i.e., fuzzy and evidence theory based approaches are useful to perform likelihood *assessment* in an uncertain and dynamic environment for risk analysis. The approaches are capable to incorporate new knowledge with prior knowledge and provide revised probability estimation for bow-tie analysis. The application of these approaches can also be extended in developing a real time risk analysis profile for the industrial facility by calculating the new likelihoods for each time when new information becomes available.

4. Applications of developed frameworks, approaches and updated inferences have been demonstrated in four case studies and described in Chapters 3, 4, 5 and 6. Concluding remarks and a short overview of each case study are described below.

   a) Chapter 3: the utility of the developed framework and approaches for ETA was demonstrated in the study of "LPG release at a Detergent Alkylate Plant (DAP)". An event tree model and analysis for the LPG release was reconstructed and reevaluated in order to compare the error robustness of traditional techniques and developed approaches in *data* uncertainty. It was thereby observed that, for 10% error in the probability of the initiating event (LPG release event tree), the deterministic approach exhibited approximately 9% deviation in the frequency estimation of the outcome event "B" for the LPG release event tree. In contrast, the fuzzy-based approach gave more robust results, i.e., ~0.003% deviation for the same percentage of error in the initiating event. The MCS-based approach yielded ~0.8% deviation, while the evidence theory-based approach calculated ~6% deviation for the same scenario. It is emphasized, however, that evidence theory takes into account the ignorance of expert knowledge while defining the probability of events, which the other approaches cannot deal with.

   b) Chapter 4: the second case study, with two separate sub-examples (event tree for "LPG release"; fault tree for "Runaway reaction"), was demonstrated and analyzed to illustrate the compatibility of developed approaches for ETA and FTA instead of traditional techniques. Besides checking the error robustness of the developed approaches, a detailed comparative study for different techniques of FTA and ETA was performed. Two additional comparisons in handling *dependency* uncertainty with the available and developed approaches were also performed for different assumptions of interdependence in FTA and ETA. For two dependence cases, i.e., independent and perfectly dependent, the output results of FTA and ETA were examined and compared for the different approaches. The comparisons of the "LPG release" event tree example revealed that all approaches including the developed approaches provided similar results when the independence assumption was considered. However, when perfect dependence was employed, a higher order of magnitude was estimated while calculating the probability of outcome events using the developed approaches. A similar observation was found for perfect dependence of basic-events for the FTA of "Runaway reaction." These two observations confirmed that relaxing the dependency assumption introduces significant errors in the output results, and the traditional approaches are not capable to address this type of uncertainty. Therefore, the developed approaches are more comprehensive and extensive than the traditional approaches, which provide a reliable and robust result in the situation of *data* and *model* uncertainty for FTA and ETA.

c) Chapter 5: for the third case study, a bow-tie diagram of the BP Texas city accident was constructed following the developed framework and analyzed using the developed approaches. It was observed from the case study that, while the interdependence of input events varied from independence to perfect dependence, the uncertainty measured in the probability of the critical event (CE) and output events (OEs) ranged from minimum to maximum uncertainty. A conclusion was drawn that the interdependence has a strong influence over the measurement of uncertainties in the likelihood (probability) estimates of OEs. To check the robustness of handling data uncertainty, a comparative analysis was performed using the developed and traditional approaches for the bow-tie. In the comparison, the same BP case study was carried out and the error propagation for each approach was observed for a specific outcome event, "OE_1". Analysis of this comparison revealed that introduction of 20% uncertainty in the input-event data lead to 65% deviation in the likelihood estimates of OE_1 while employing the traditional approach. The fuzzy and evidence theory based approaches measured almost 0.25% and 9% deviation for the same OE. Aside from this comparison, a tornado plot was developed using the proposed SA method for identifying the correlations of input events leading to the occurrence of OE_1. The demonstration of SA method in the case study also helped to conclude that a significant percentage of risk of occurrence of OEs could be mitigated if the likelihood for the highest contributing input events may be reduced to a desired percentage.

d) Chapter 6: the last case study was illustrated on an offshore oil & gas processing facility to describe the utility of the developed and updated approaches for bow-tie analysis. The updating approaches were demonstrated only for the bow-tie application. They can also be encompassed with the FTA and ETA. Knowledge from two different sources, along with the subjectivity and incompleteness uncertainty, was considered while performing bow-tie analysis for the case study. The fuzzy weighted average method with the assignment of equal weights on both sources and DS and

Yegar combination rules were applied to aggregate the knowledge for input events. The corresponding probability for the CE (gas leakage) and the OEs was determined using combined knowledge and developed approaches. To demonstrate the applicability and validity of the updated approaches, a few arbitrary input events of the bow-tie were considered and the prior probabilities of these events were updated with some random new knowledge. The trend of the uncertainty range for each update was estimated and observed while evaluating the probability of CE and OEs. The decreasing trends of uncertainty range for CE and OEs confirmed that the uncertainty in the final estimate decreases when the number of updates increases for the input events. The updating inference is useful to enhance the dynamic nature and performance of QRA by adding new knowledge or industrial data to the prior analysis and improving the earlier analysis with heightened confidence. These approaches are also useful for performing a real time analysis using the updated information and rectifying the likelihood assessments of input events and OEs that may escalate to an accident.

Finally, the general conclusion of the developed approaches can be described using Table 7.1. Table 7.1 provides the comparisons of different approaches in perspective of handling and updating the analysis of fault tree, event tree or bow-tie for QRA. It is evident from the table that, for the most part, the proposed approaches is more advanced than the traditional methods.

Table 7.1: Different approaches for FTA/ETA/Bow-tie analysis

Approaches	Input data	Assumptions	Uncertainty handling & Updating
Traditional approach	Use crisp value	• Assigned values are exact and precise. • Basic events /events /input events are independent.	• Incapable of describing uncertainty • Unable to update prior analysis
Traditional MCS based approach	Use PDFs	• PDFs are known and well defined. • Basic events /events /input events are independent	• Only the random uncertainties are properly handled. The other types of uncertainties cannot be described. • Unable to update prior analysis
Proposed approaches	Use TFNs or *bpas*	• TFNs are elicited using expert knowledge • Interdependence of basic events /events /input events can be ranged from perfect to opposite dependence	• Data uncertainty along with *aleatory* and *epistemic* uncertainty, and *model* (or *dependency*) uncertainty can properly be addressed. • Able to update the prior analysis whenever new knowledge becomes available.

7.2 Originality of thesis

The main contribution of this thesis is twofold. First, two different approaches are being embedded in the developed frameworks for ETA, FTA and bow-tie analysis to handle *data* and *dependency (model)* uncertainty in QRA. In addition to these approaches, a sensitivity analysis method has also been developed for identifying the important risk contributors and providing an evaluation of possible risk reduction for bow-tie analysis. Second, to incorporate the dynamic aspects in QRA, two updating mechanisms are integrated with the developed approaches. The originality and significance of the thesis is further described with the following features, which include:

- Supporting the elicitation process of expert knowledge to overcome the *data* uncertainty issues in FTA, ETA and bow-tie analysis.
- Adopting a dependency coefficient to describe a wide range of interdependence for addressing *model* uncertainty in FTA, ETA and bow-tie analysis.
- Enhancing the process to identify the important risk contributors and mitigate risk for industrial facilities.
- Providing the compatibility for QRA in updating and rectifying the analysis recursively whenever new knowledge becomes available.
- Promoting the applicability of QRA for any industrial facilities that endure *data* and *model* (or *dependency*) uncertainties.

7.3 Future research

Based on this research following recommendations for future work can be made:

7.3.1 New frameworks

i. The present study encourages the use of the implication of expert knowledge as an alternative option to limited or missing data. A conceptual framework that describes the procedural steps of the knowledge elicitation process requires to be developed to maintain the quality and credibility of knowledge. Knowledge from multiple sources provides more reliable predictions about an uncertain parameter. Hence, a graphical format to support the knowledge elicitation process, and a scoring or voting system to facilitate the prioritization of knowledge, need to be integrated into the framework. These integrations will basically help to calculate the assignment of weights for each expert, and perform an interactive risk analysis more specifically for a particular system.

ii. This research intended to formulate the uncertainty for FTA, ETA and Bow-tie analysis, and enhance the performance of risk analysis in an uncertain and dynamic environment. However, risk is defined as a function of both consequence and frequency. FTA, ETA and Bow-tie analysis are normally used to estimate the probability of concerned incidents and events. An effort is still required to develop a framework that will integrate both consequence and frequency estimation for an unwanted event and provide an overall risk estimation.

7.3.2 Improvement in the developed approaches

i. The present research used triangular distribution to address random uncertainty in the MCS-based approach for FTA and ETA. In future research, the other types of distributions including exponential, weibull, normal and lognormal (commonly preferred in modeling the failure data for the basic components or occurrence of an event) can also be considered to perform a more comprehensive comparison among the different uncertainty-based approaches for FTA and ETA.

ii. Two types of uncertainty, *data* and *model* (or dependency) uncertainty, were considered in this study to explore the uncertainty-based approaches for the FTA, ETA and bow-tie. Another kind of uncertainty, which may be defined as *structural* or *completeness* or *quality* uncertainty subjected to the incorrectness and inappropriateness of structuring a fault tree or bow-tie for an unwanted event, can also be considered in future research.

iii. The developed uncertainty based formulations in fuzzy and evidential approaches was explored only for the "AND" and "OR" logic gates of FTA and bow-tie analysis. In future research, these formulations can be further extended towards developing the formulations for the other types of gates, such as, "Exclusive OR", "PRIORITY AND", "INHABIT", which will partly help to model the *structural* uncertainty while performing risk analysis using FTA and Bow-tie analysis.

iv. Two distinct approaches, namely fuzzy- and evidence theory-based approaches, were developed in this study to handle subjective and incompleteness uncertainty. In future, both kinds of uncertainties can be considered together using hybrid soft computing methods such as Fuzzy-Dempster-Shafer is required.

v. An inference for updating the prior knowledge of interdependence is required to be developed in future in order to comprehend the applicability of developed approaches for FTA, ETA and bow-tie analysis.

vi. Different conditional PDFs such as weibull, lognormal, normal may also be considered to explore a more robust fuzzy-Bayesian updating approach.

vii. Analytical Hierarchy Process (AHP) can be considered in future to determine weight of expert's knowledge in aggregating the fuzzy numbers from different experts.

Electronic Appendix

Please find the attached CD.

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