NEAR WELL BORE STREAMLINE SIMULATION

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Near Well Bore Streamline Simulation

by

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Abstract

This thesis presents a methodology for generating and analyzing streamlines in reservoirs near wellbores using a finite difference method and the Pollock method. The proposed methodology has been applied in 2D using Cartesian and Polar coordinate systems.

This proposed method demonstrates how all potential geological and mechanical factors affect streamline behaviour and analyze their effect on route of stream path lines movement. By using this method, stream path lines can be visualized for any given boundary conditions. Subject to the basic assumptions of incompressible fluid and avoidance of gravity effects, there is an opportunity to simplify numerical equations for streamline simulation and retrieve more effective results. Fluids are moved along the natural streamline grids.

Permeability, well conditions and other process factors such as pressure, velocity and time of travel dictate the route and direction of streamlines. The advantages of this method include flow visualization, more efficiency and computational speed, and the ability to add of additional boundary conditions in various numbers of grid blocks. In particular, this method offers streamline visualization in inter-well connectivity.
Streamline simulation is increasingly employed by geoscientists and engineers in the oil and gas industry to model the flow of fluids in reservoirs. Flow simulation can run on fine-grid, high-resolution models in reasonable times on off the shelf hardware. In addition, streamline simulation identifies fluid flow paths between sinks and sources. This visual and quantifiable information allows geoscientists to examine model connectivity, as well as drainage and irrigation zones associated with producers and injectors. Also, it helps reservoir engineers analyze streamline migrations and predict targets for efficient drilling.

The Pollock’s method generally is used for an orthogonal grid cell, however in reality most of the reservoirs near wellbore do not adjust in Cartesian modeling shape. Thus, the next advancement in this research explored ways to find stream path lines according to the Pollock method but with assuming Polar coordinates instead of Cartesian coordinates, when wells’ location is in the center of the rings and all streamlines come from different directions and collect in the center of the circle.

My contribution in this study includes the MATLAB implementation of the following steps:

- Developing specific finite difference grid blocks according to given reservoir parameters
- Implementing numerical pressure solvers in both Cartesian and Polar coordinates
• Finding time of flight for streamline movement in grid blocks according to published methods

• Visualizing streamlines in 2D by finding streamline entrance and exit points in each grid cell and connecting all points by line segments.

Most of the mathematical formulations are taken from available literature and other documents. This methodology is demonstrated through the use of three case studies for both Cartesian and polar coordinates. The research includes two main topics: Streamline simulation in Cartesian and Polar coordinates. Each topic has two main parts:

• Theoretical part; in this part, all numerical and mathematical modeling is described, using finite difference methods for pressure solving.

• MATLAB code programming; in this part, a MATLAB program is written according to theoretical equations.

The MATLAB code is included in an appendix. This MATLAB code can visualize all streamlines with different boundary conditions, various process and geological factors in different reservoir areas. The graphs which are resulted from this numerical
method can provide valuable information for engineers to have better understanding of fluid flow in the reservoir.

Regarding research novelty, streamline simulation in near well bore regions using radial geometry has not been done before. Numerical methods are effective tools to optimize time and costs in oil and gas projects and reduce uncertainties.

Finally this research has significant potential applications in the future to improve fluid flow modeling near well bores. The numerical method discussed in this study can be extended to three dimensional coordinates. In this study fluid has been assumed to be incompressible. The method can be used with compressible fluids in future, which is closer to reality.
Acknowledgement

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Table of Content

Abstract.................................................................I
Acknowledgement..................................................V
Table of Contents..................................................VII
List of Figures and Illustrations...............................XIII
List of Symbols, Abbreviation and Nomenclature...............XV

Chapter I
Introduction

1-1 Problem Identification...........................................3
1-2 Well Productivity Modeling....................................8
1-3 Objectives of the Research.....................................10
1-4 Thesis Organization............................................10

Chapter II
Literature Review

2-1 What is Streamline Modeling..................................13
2-2 Variation of Streamline Modeling.............................20
   2-2-1 Eulerian- Lagrangian Methods............................20
Table of Content (Cont.)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-2-2 Particle Tracking</td>
<td>21</td>
</tr>
<tr>
<td>2-2-3 Streamline vs. Streamtube Modeling</td>
<td>22</td>
</tr>
<tr>
<td>2-3 Periodic Updating of Streamlines</td>
<td>25</td>
</tr>
<tr>
<td>2-4 Numerical Modeling Streamlines Simulation</td>
<td>26</td>
</tr>
<tr>
<td>2-5 Gravity</td>
<td>26</td>
</tr>
<tr>
<td>2-6 Compressible Flow</td>
<td>27</td>
</tr>
</tbody>
</table>

Chapter III

The Pollock Method for Streamline Generation

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-1 Mass Conservation</td>
<td>29</td>
</tr>
<tr>
<td>3-2 Time of Flight</td>
<td>30</td>
</tr>
<tr>
<td>3-3 The Streamline Generation Procedure in Cartesian Coordinate</td>
<td>33</td>
</tr>
<tr>
<td>3-3-1 Flow Modeling</td>
<td>33</td>
</tr>
</tbody>
</table>

Chapter IV

Pressure Solution in Cartesian Geometries

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-1 Implementation Numerical Pressure Solvers in Cartesian Coordinate</td>
<td>47</td>
</tr>
<tr>
<td>4-2 The Laplace Equation In Cartesian Coordinates</td>
<td>47</td>
</tr>
<tr>
<td>4-2-1 Isotropic Medium</td>
<td>53</td>
</tr>
</tbody>
</table>
# Table of Content (Cont.)

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-2-2</td>
<td>Pressure Solving at the Boundary of Impermeable Layers</td>
<td>53</td>
</tr>
<tr>
<td>4-2-3</td>
<td>Pressure Solving at the Boundary of Two Different Layers</td>
<td>54</td>
</tr>
<tr>
<td>4-3</td>
<td>Velocity</td>
<td>57</td>
</tr>
</tbody>
</table>

## Chapter V

### Pressure Solution in Radial Geometries

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-1</td>
<td>Polar Coordinate System</td>
<td>60</td>
</tr>
<tr>
<td>5-2</td>
<td>Gradient in Polar Coordinate</td>
<td>62</td>
</tr>
<tr>
<td>5-2-1</td>
<td>Gradient in $r$ Direction</td>
<td>62</td>
</tr>
<tr>
<td>5-2-2</td>
<td>Gradient in $\theta$ Direction</td>
<td>63</td>
</tr>
<tr>
<td>5-3</td>
<td>Velocity in Polar Direction</td>
<td>64</td>
</tr>
<tr>
<td>5-4</td>
<td>Darcy’s Law</td>
<td>65</td>
</tr>
<tr>
<td>5-4-1</td>
<td>Velocity in $r$ Direction</td>
<td>66</td>
</tr>
<tr>
<td>5-4-2</td>
<td>Velocity in $\theta$ Direction</td>
<td>66</td>
</tr>
<tr>
<td>5-5</td>
<td>Permeability in Polar Coordinate</td>
<td>67</td>
</tr>
<tr>
<td>5-6</td>
<td>The Laplace Equation in Polar Coordinates</td>
<td>68</td>
</tr>
<tr>
<td>5-6-1</td>
<td>Laplacian in Isotropic Medium Case</td>
<td>69</td>
</tr>
<tr>
<td>5-6-2</td>
<td>Laplacian in Homogenous Medium Case</td>
<td>69</td>
</tr>
<tr>
<td>5-6-3</td>
<td>Laplacian in Homogenous and Isotropic Case</td>
<td>69</td>
</tr>
<tr>
<td>5-7</td>
<td>General Laplacian in 3D Polar Coordinate</td>
<td>70</td>
</tr>
<tr>
<td>Table of Content (Cont.)</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>5-8 The General Steps in Streamline Calculations in a Radial Geometry</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>5-8-1 Velocity Formula Assumption in $\theta$ Direction</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>5-8-2 Velocity Formula Assumption in $r$ Direction</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>5-8-3 Finding Time of Flight for Streamline Movement in Grid Blocks</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>5-9 Discretization</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>5-10 Radius Equation</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>5-11 Flow rate Equation</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>5-12 Permeability Equation</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>5-12-1 Radial Mobility</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>5-12-2 Tangential Mobility</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>5-12-3 Theta Mobility</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>5-13 Face Velocity Equation in Polar Coordinate System</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>5-13-1 Descretized Face Velocity Equation in $r$ Direction</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>5-13-2 Face Velocity Formula in $r$ Direction in Isotropic Case</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>5-13-3 Descretized Face Velocity Equation in $\theta$ Direction</td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>5-13-4 Face Velocity Formula in $\theta$ Direction in Isotropic Case</td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>5-14 Discritization of the general Lapacian equation</td>
<td>84</td>
<td></td>
</tr>
</tbody>
</table>
Table of Content (Cont.)

Chapter VI
Case Studies

6-1 Summary of Previous Chapters.................................................................93
6-2 Case Studies in Cartesian Geometries.......................................................97
  6-2-1 Homogenous and Isotropic medium..................................................97
  6-2-2 Low Permeability Region in the Middle of a Specific Reservoir..........103
  6-2-3 Assuming a Well in the corner of the Region.................................110
6-3 Case Studies in Radial Geometries.......................................................116
  6-3-1 Isotropic and Homogenous Medium................................................116
  6-3-2 Low Permeability Region in the Middle of a Specific Reservoir .......121

Chapter VII
Conclusions

7-1 Summary....................................................................................................127
7-2 Research Novelty.......................................................................................128
7-3 Recommendations for Future Research..................................................130

References.....................................................................................................132

Appendix.........................................................................................................137
# List of Figures and Illustrations

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1:</td>
<td>Exit Point (Calculated)</td>
<td>3</td>
</tr>
<tr>
<td>1.2:</td>
<td>Velocity across the Four Faces of Grid Cell</td>
<td>4</td>
</tr>
<tr>
<td>1.3:</td>
<td>Entrance and Exit Point</td>
<td>5</td>
</tr>
<tr>
<td>1.4:</td>
<td>Considering Grid Cell</td>
<td>6</td>
</tr>
<tr>
<td>1.5:</td>
<td>Wells in Fields</td>
<td>8</td>
</tr>
<tr>
<td>1.6:</td>
<td>Numerical and Actual Well Trajectory</td>
<td>9</td>
</tr>
<tr>
<td>2.1:</td>
<td>Streamtube- Schematic Figure</td>
<td>22</td>
</tr>
<tr>
<td>3.1:</td>
<td>Particle Travel Through Streamline</td>
<td>30</td>
</tr>
<tr>
<td>3.2:</td>
<td>Explanation of Fluid Flow in Each Side of Cell</td>
<td>34</td>
</tr>
<tr>
<td>3.3:</td>
<td>A Brief Demonstration of Particle Movement</td>
<td>41</td>
</tr>
<tr>
<td>3.4:</td>
<td>Schematic of a Grid cell and fluid flow in $x$ Direction</td>
<td>42</td>
</tr>
<tr>
<td>3.5:</td>
<td>Schematic of the Grid Cell and Time of Travel in $x$ and $y$ Directions</td>
<td>43</td>
</tr>
<tr>
<td>3.6:</td>
<td>Schematic of the Grid Cell and Exit Coordinate Possibilities</td>
<td>44</td>
</tr>
<tr>
<td>3.7:</td>
<td>Flow Chart- Streamline Modeling</td>
<td>45</td>
</tr>
<tr>
<td>4.1:</td>
<td>Pressure Distribution in a Sample Grid Blocks</td>
<td>49</td>
</tr>
<tr>
<td>4.2:</td>
<td>Different Case Studies for Solving Pressure distribution</td>
<td>52</td>
</tr>
<tr>
<td>5.1:</td>
<td>Radial Geometry</td>
<td>60</td>
</tr>
<tr>
<td>Figure (Cont.)</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>5.2: Radial Grid</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>6.1: Theoretical Expectation in First Case Study in Cartesian Coordinate</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>System</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.2: Permeability Quiver for 50 by 50 Grid Cells for First Case Study in Cartesian Coordinate System</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>6.3: Pressure Contour in 50 by 50 Meshes for First Case Study in Cartesian Coordinate System</td>
<td>101</td>
<td></td>
</tr>
<tr>
<td>6.4: Streamline Behavior in 100 by 100 Gird Blocks for First Case Study in Cartesian Coordinate System</td>
<td>102</td>
<td></td>
</tr>
<tr>
<td>6.5: Theoretical Expectation in Second Case Study in Cartesian Coordinate System..</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.6: Permeability Quiver for 50 by 50 Grid Cells for Second Case Study in Cartesian Coordinate System</td>
<td>107</td>
<td></td>
</tr>
<tr>
<td>6.7: Pressure Contour for 50 by 50 Grid Cells for Second Case Study in Cartesian Coordinate System</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>6.8: Streamline Simulation in 100 by100 Grid Cells for Second Case Study in Cartesian Coordinate System</td>
<td>109</td>
<td></td>
</tr>
<tr>
<td>6.9: Theoretical Expectation in Third Case Study in Cartesian Coordinate System..</td>
<td>112</td>
<td></td>
</tr>
<tr>
<td>System</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.10: Permeability Quiver for 50 by 50 Grid Cells for Third Case Study in Cartesian Coordinate System</td>
<td>113</td>
<td></td>
</tr>
</tbody>
</table>
6.11: Pressure Contour for Assumed 50 by 50 Grid Cells in Cartesian Coordinate System .................................................................114

6.12: Streamline Simulation for 100 by 100 Grid Cells for Third Case Study in Cartesian Coordinate System ........................................115

6.13: Theoretical Expectation for First Case Study in Radial Geometry ...............118

6.14: Pressure Contour in Polar Coordinate for First Case Study in Radial Geometry .................................................................119

6.15: Streamline Simulation in Polar Coordinate for First Case Study in Radial Geometry .................................................................120

6.16: Theoretical Expectation for Second Case Study in Radial Geometry .............123

6.17: Pressure Contour in Polar Coordinate for Second Case Study in Radial Geometry .................................................................125

6.18: Streamline Simulation in Polar Coordinate for Second Case Study in Radial Geometry .................................................................126
List of Symbols, Abbreviation and Nomenclature

Abbreviations:

EOS equation of state
RC reservoir condition

Greek Symbols

\( \alpha \) Liquid volume fraction or liquid holdup
\( \beta \) Water volume fraction
\( \mu \) Viscosity
\( \overline{\rho} \) Average two phase density
\( \rho \) Density

Symbols

\( A \) Cross-sectional area
\( D \) Diameter
\( D_h \) Hydraulic diameter
\( f \) Friction factor
\( g \) Gravitational acceleration
\( K \) Absolute permeability
\( L \) Length
\( m \) Mass flux
\( N \) Number of finite difference grid cell segments
\( p \) Pressure
\( Q \) Flow rate
\( q \) Volumetric Flux
\( r \) Radius
\( r_w \) Well bore radius
\( r_e \) Well outer radius
\( t \) Time
\( u \) Darcy's velocity or volumetric flux
\( V \) Volume
\( v \) Velocity
Subscript

\( I \)  \hspace{1em} \text{In-flow}
\( i, j, k \)  \hspace{1em} \text{Index for phase or components}
\( o \)  \hspace{1em} \text{Oil phase}
\( p \)  \hspace{1em} \text{Particle}
\( res \)  \hspace{1em} \text{Reservoir or reservoir condition}
\( x \)  \hspace{1em} \text{\( x \)-Direction}
\( y \)  \hspace{1em} \text{\( y \)-Direction}
\( z \)  \hspace{1em} \text{\( z \)-Direction}
Chapter I

Introduction

Background

Streamline simulation of displacement processes in oil reservoirs gained industrial attention during the 1980s and 1990s, when the method was implemented as a 3D procedure (Bratvedt et al. (1993)). Originating from the more complicated and less applicable 3D front tracing method (Bratvedt et al. (1993) and Glimm et al. (1983)), it offered the opportunity of incorporating operator splitting techniques to include the effects of fluid compressibility, diffusion and gravity effects.

As the front tracking methods were for purely hyperbolic systems only, such operator splitting techniques used a three step strategy to solve parabolic problems:

First, the 3D pressure equation is solved and streamlines generated according to the pressure field.
Secondly, the fluids are moved along the computed 1D streamlines over a time step ignoring gravity.

Thirdly, a gravity correction step is made where the fluids of different densities are segregating vertically.

The success of this streamline approach lies in the fact that operating splitting is mathematically rigorous, together with the fact that fluid transport along streamlines does not need to be based on the assumptions of zero diffusion and incompressibility.

Furthermore, streamlines can be updated at any time, for example if new wells are drilled. In fact, streamlines may be updated on each time step in which case a streamline method reduces to an adaptive grid finite difference method. When streamlines are updated on each time step the computational advantage, in terms of CPU timesavings, is lost.

In spite of the successful application of operator splitting techniques, streamline simulation has limitations and can never replace conventional simulation techniques. First of all, this is because the splitting of a pressure equation from the viscous/diffusive part of the problem is not accurate if the fluids are highly compressible and/or when gravity effects dominate over viscous transport mechanisms. Therefore, streamline simulation is mostly used for water flooding in oil reservoirs and less for gas injection and recovery of heavy oil.
Another limitation of the streamline approach is that the numerical method used for streamline generation (the Pollock method) tends to be numerically unstable near well bores. Therefore, detailed streamline modeling of near-well flow is absent from the published literature. The work presented here focuses on this aspect of streamline modeling.

1-1 Problem Identification

The traditional Pollock method for streamline generation is for Cartesian geometries. It is a numerical procedure, which takes a streamline entrance point in a grid block as input and calculates the exit point based on the pressure in the grid block itself and the pressures in the adjacent grid blocks. This information is available from a priori solution of the pressure equation (Laplacian). Figure 1.1 depicts the procedure.

![Diagram](image)

**Figure 1.1: Exit Point (Calculated)**
Once the exit point is calculated, a straight line-draw is made to connect the exit and entrance point to constitute a segment of a large-scale streamline.

The Pollock method is described in detail in Chapter III. Here, it suffices to identify the lack of accuracy inherent in the method as applied to near well situations. Referring to Figures 1.1 and Figure 1.2, the fluid velocity across the faces of the grid block can be approximated:

The velocity, $V_1$, across the upper face is calculated from Darcy’s Law using upscaled permeabilities and the pressures $P_1$ and $P_0$ in Figure 1.1. Similarly, the velocities $V_2$, $V_3$ and $V_4$ across the other faces are calculated (Figure 1.2).

Figure 1.2: Velocity Across the Four Faces of Grid Cell
Each of the velocities $V_i$ are assumed to be constant over the face $i$, however, the horizontal $x$ and vertical $y$ velocities are assumed to vary linearly inside the grid block, i.e. it is assumed that:

$$
V_h(x) = ax + b \\
V_v(y) = cy + d
$$

(1-1)

where the constants $a$, $b$ and $c$, $d$ are chosen to match the known velocities $V_1$, $V_3$ and $V_2$, $V_4$ respectively.

An obvious limitation of this procedure is that it does not allow an entrance and an exit point to be located on the same face (Figure 1.3). Therefore, grid blocks must be small to capture this effect. In near-well bore situations, a radial geometry mitigates this disadvantage since grid blocks in a cylindrical grid are small near the well bore.

![Figure 1.3: Entrance and Exit Point](image)

Such situations can occur close to well bores, for example, when fluids have to converge into perforations in the presence of small but significant heterogeneities.
Another problem with the Pollock method is that for near-well flow, the flow velocity does not vary linearly as linear flow geometries as shown in equation (1-1). Therefore, consider a cylindrical grid, as shown in Figure 1.4. As the fluids approach the well bore, the velocity increases, however not linearly.

![Cylindrical Grid Cells](image)

**Figure 1.4: Cylindrical Grid Cells**

This follows from the fact that the pressure decreases logarithmically towards the well bore (assuming purely radial):

\[(1-2) \quad P(r) = a \ln r + b\]

According to Darcy’s Law:

\[(1-3) \quad V = \text{Const} \frac{\partial P}{\partial r} = \text{Const} \frac{a}{r}\]
When we also allow angular flow in Figure 1.4, we must allow fluids to “leak” into the neighboring blocks in the angular direction. Instead of equation (1-3), we therefore assume;

\[ V(r) = \frac{a}{r} + b \]  

(1-4)

where the constants \(a\) and \(b\) are chosen to match the known radial velocities over the faces of the block in the \(r\)-direction.

In the angular direction, the assumption of linear change is adequate:

\[ V(\theta) = c\theta + d \]  

(1-5)

where again the constant \(c\) and \(d\) are determined from the known face velocities. The primary objective of this work is therefore to implement the modified Pollock method in Polar coordinates to better adapt to the near-well geometry.

The streamline generation method discussed above is based on knowing the pressure distribution a priori. This is obtained by solving the Laplacian. In this work, we are primarily focusing on the method as applied to the near well region, i.e. we need to solve the Laplacian in a cylindrical geometry. In reservoirs, we can not impose an assumption of homogenous and isotropic medium.

It is a fact that the general (anisotropic) Laplacian in a cylindrical geometry frequently is misformulated in the petroleum literature. This is because a non-tensorial formulation is highly desirable for simplicity reasons. Therefore, the concept of angular direction permeability is often used. However, this entity does not exist in
an anisotropic medium and consequently a full tensorial formulation cannot be avoided. A secondary objective of this work is therefore to rigorously derive the appropriate Laplacian for near well streamline modeling. This is presented in Chapter VI.

1-2 Well Productivity Modeling

In conventional reservoir models, the wells are represented as sources/sinks within individual grid blocks. The well segment penetrating a grid block is assumed to be aligned with the block boundaries (faces). This was an adequate approach before 1990 where all wells in fields were vertical (Figure 1.5).

Figure 1.5: Wells in Fields
However, during the 90s, formidable technology developments were made within drilling and completion of advanced wells specifically lateral wells, and drilling wells of 2000-3000 m long production sections became common practice. The traditional well representation for such wells can be completely inadequate as shown in Figure 1.6.

![Figure 1.6: Numerical and Actual Well Trajectory](image)

Clearly, representation of advanced wells in reservoir simulation therefore needs considerable improvements. By using streamline modeling near well bores, the details of the flow pattern can be captured and the well productivity can be accurately
calculated by integrating flow along streamlines. These local productivity models can subsequently be incorporated into standard simulator models.

The work presented here is the first step to achieve the ultimate goal of this research: To develop accurate, local productivity models, which incorporate near well heterogeneities, anisotropy and well completion details for complex well trajectories.

1-3 Objectives of the Research

The objectives of this work are:

1- To rigorously derive and describe the equations for 2D streamline simulation in both Cartesian and cylindrical geometries

2- To implement streamline simulation in two dimensions in Cartesian coordinate and Polar coordinate systems.

3- Develop MATLAB programs for both Cartesian and Polar coordinates for streamlines simulation near well bores.

4- To investigate flow behavior using the streamline models in heterogeneous isotropic reservoirs with different reservoir conditions and well conditions.

5- To investigate effects of various boundary conditions on streamline behaviour.

6- To provide recommendations on further development of the models.
1-4 Thesis Organization

This research attempts to provide a methodology for evaluating streamline behaviour near well bores by finding stream path lines in the reservoir. A comprehensive model is developed based on locating the entry and exit points of each streamline in each given grid cell using the Pollock method for Cartesian coordinate system and a modified Pollock method for cylindrical geometries.

This thesis is divided into seven chapters. The first chapter provides an introduction to the thesis, background information, objectives of research, scope of study and thesis organization.

Chapter II presents literature review, previous background works and researches on streamline simulation. Then, variation of streamline modeling is reviewed.

In Chapter III, there is a comprehensive explanation of the Pollock method. Finite difference grid cells and the structure of the grid blocks are introduced. In addition, fluid flow modeling is introduced. In addition, formulation and equation series for computation of flow modeling are described.
Chapter IV presents the pressure solution in Cartesian coordinates. This chapter provides calculation process for finding the pressure distribution across the given reservoir by solving Laplacian equation.

Chapter V presents solution for the pressure distribution in radial coordinates and shows step-by-step calculations to develop a MATLAB code to simulate streamlines in reservoirs in radial coordinate system.

Chapter VI presents case studies in both Cartesian and Polar coordinate system. Three case studies in Cartesian coordinates and two case studies in Polar coordinates with various boundary conditions are presented.

The thesis is summarized and concluded in Chapter VII where research novelty and recommendations for future research are discussed.
Chapter II

Literature Review

The current popularity of streamline simulation should more aptly be termed resurgence, given that streamlines have been in the literature since the paper by Muskat et al. (1934). The method has received repeated attention since then, and over the last 70 years different ideas and applications have been published about streamline simulation as discussed in this chapter, e.g. Thiele (2001).

2-1 What is Streamline Modeling?

Streamlines, pathlines and streaklines are convenient tools for describing and visualizing flow given by an external flow velocity field \( \vec{V} \):

\[
(2-1) \quad \vec{V} = [V_x, V_y, V_z]
\]
Streamlines are a family of curves $S$ that are instantaneously tangent to the velocity vector $\vec{V}$ at every point;

\begin{equation}
\frac{d\vec{s}}{dt} = \vec{V}
\end{equation}

Streamlines can be traced for any vector field, although the most common is that $\vec{V}$ represents a velocity obtained from the solution of a set of flow equations. For incompressible flow, streamlines defined at a single instant do not intersect and cannot begin or end inside the medium except at singularities (sources and sinks). Streamtubes are regions bounded by streamlines. Because streamlines are tangent to the velocity field, a fluid that is inside a streamtube must remain within the same streamtube.

A pathline $x(t)$ is the trajectory traced out by an imaginary massless particle following the flow of the fluid from a given starting point,

\begin{equation}
\frac{dx}{dt} = \vec{V}
\end{equation}

\begin{align*}
x(t_0) &= x_0
\end{align*}

A streakline is the locus at a given instance of the positions of all fluid particles that have gone through a fixed spatial point in the past. In steady state flow streamlines, streaklines and pathlines coincide. In an unsteady flow they can be different. In this work we consider only steady state flow. Streamline simulation is a technique that predicts multi-phase displacements along the streamlines generated from numerical solutions to the Laplace equation. The technique decouples computation of saturation variation from the computation of pressure variation in time and space. Using a finite
difference method, the initial steady state pressure field is computed based on spatial variations in mobility, and is updated in response to significant time-dependent changes in mobility. The flow velocity field is then computed from the pressure field, and streamlines are traced based on the underlying velocity field. Streamlines originate at the injectors (source) and culminate at producers (sink). Once the streamline paths are determined, displacement processes are computed along the streamlines using 1-D, analytical or numerical models (Thiele (1994), Batycky (1997)) and ResAssist (http://www.res-assist.com/).

The integration of equation (2-3) to obtain particle paths and/or travel times is known as particle tracking, for which there exists extensive literature. The particle tracking literature is primarily concerned with problems where the velocity field is only known at a finite set of points, either measured or calculated from a flow model, and interpolation is needed to integrate pathlines. In computational fluid dynamics, particle tracking has been used for visualization (Kipfer et al. (2003), Knight et al. (1996), Sadarjoen et al. (1997), Shirayama (1993) and Strid et al. (1989)).

Velocity interpolation in control-volume mixed finite-element methods is a related subject to particle tracking and has been considered in some papers such as Naff et al. (2002). Within groundwater flow simulation, particle tracking is used to model contaminant transport (Oliveira et al. (1998), Cordes et al. (1992), Schafer-Perini et al. (1991) and Shafer (1987)).
The integration of equation (2-3) for visualization, is usually done numerically using a Runge–Kutta type solver, whereas in groundwater flow, semi-analytical integration is the most common. In some research, streamline tracking is a subset of particle tracking, since streamlines may be computed by particle tracking if the streamline parameters are introduced as an artificial time variable for which the instantaneous velocity field $\vec{V}$ is steady.

Most researchers consider streamline tracing in the context of streamline simulation of flow in hydrocarbon reservoirs (Batycky (1997), Bratvedt et al. (1993) and King et al. (1998)). In some cases, the fluid velocity $\vec{V}$ is typically given as the numerical solution of a set of flow equations for $\vec{V}$ and the fluid pressure $p$ of the form;

\[
\begin{align*}
\frac{\partial p}{\partial t} + \nabla \cdot \vec{V} &= b \\
\vec{V} &= -a(\vec{x}) \nabla p
\end{align*}
\]

The two equations are commonly referred to as the pressure equation and Darcy’s law, respectively. The corresponding discrete velocity approximation depends on the numerical method:

- For finite-difference methods, the pressure is usually computed at cell centers, and fluxes can be obtained at cell edges by application of a discrete form of Darcy’s law (Zheng et al. (2002)).
- For finite-element methods, the numerical solution gives a continuously defined pressure approximation as the sum of the basis functions for all elements weighted by the corresponding node values. Although a continuously defined velocity can be
obtained from Darcy’s law, a better strategy where continuous fluxes are obtained at cell edges, is given in some literatures (Durlofsky (1994) and Kinzelbach et al. (1992)).

- Mixed finite-element methods solve for velocity and pressure simultaneously, resulting in a more accurate velocity field than finite difference and standard finite element. The continuously defined velocity is given by the degrees of freedom at the edges and the corresponding basis functions (Kaasschieter (1995) and Durlofsky (1994)).

- Finite-volume methods include multi-point flux approximations and control-volume finite-element methods (Verma et al. (1997), Aavatsmark (2002) and Edwards (2002)).

In these methods fluxes are computed at cell edges. In other words, a continuously defined velocity field is obtained only for the mixed finite-element method. For the other methods one must use an interpolation scheme to determine the velocity from the discrete fluxes at the cell edges.

In reservoir simulation and groundwater flow, the predominant way of computing streamlines is by use of a semi-analytical technique. In semi analytical methods, the interpolation of the velocity is simple enough that analytical integration is possible within each grid cell. As an example, let us consider the popular Pollock method, (Pollock (1988)) which is described in more detail in Chapter III. Given an entry point of a streamline into a grid cell, Pollock’s method starts by mapping the grid cell
onto the unit square (or unit cube in 3D). Each component of the velocity field is then approximated in reference space by a linear function, in which case the streamline path in each direction is given as an exponential function of the travel time. To trace the streamline, Pollock’s method determines the travel time through the grid block as the minimum time to exit in each spatial direction, which is given by a logarithmic expression. Then the travel time is used to compute the exit point and the exit point is mapped back into physical space to give the entry point into the next cell, and so on. In this method, incompressible fluids are assumed and gravity effects are ignored. Thus, severe limitations are associated with this methodology. However, as will be discussed later, these assumptions can both be relaxed using operator splitting and streamlines can then be approximated.

Flow phenomena are governed by Partial Differential Equations (PDEs) involving functions of space and time. Most single-phase flow models in engineering are partial differential equations of second order and are often linear, while multi-phase flow models are highly non-linear.

A general second order linear partial differential equation in two Cartesian variables can be written as;

$$(2-5) \quad A(x,y) \frac{\partial^2 u}{\partial x^2} + B(x,y) \frac{\partial^2 u}{\partial x \partial y} + C(x,y) \frac{\partial^2 u}{\partial y^2} = f \left( x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \right)$$

Three main types arise, based on the value of the discriminant, $D$, $D = B^2 - 4AC$.

Hyperbolic, wherever $(x, y)$ is such that $D > 0$;
Parabolic, wherever \((x, y)\) is such that \(D = 0\); 

Elliptic, wherever \((x, y)\) is such that \(D < 0\).

The complete solution of a PDE requires additional information, in the form of initial conditions (values of the dependent variable and its first partial derivatives at \(t = 0\)), boundary conditions (values of the dependent variable on the boundary of the domain) or some combination of these conditions.

Streamlines exist only for elliptic problems when the following conditions are satisfied:

- Single phase flow
- Incompressible fluid
- No gravity (Horizontal flow)
- Zero dispersion

However, reasonable approximations to other situations have been prepared which will be discussed below.
2-2 Variation of Streamline Modeling

2-2-1 Eulerian-Lagrangian Methods

For multi-phase flow, the governing PDEs are highly non-linear in the unknown pressure and saturations. Some researchers used Eulerian-Lagrangian methods in a sequential approach, first solving a pressure equation and subsequently a saturation equation for two-phase immiscible flow (Dahle et al. (1990), Dahle et al. (1992), Dawson (1991), Douglas et al. (1997) and Espedal et al. (1999)).

The Eulerian-Lagrangian methods are widely used to observe and analyze fluid flows, either by observing the trajectories of specific fluid parcels which yields what is commonly termed a Lagrangian representation, or by observing the fluid velocity at fixed positions which yields an Eulerian representation. Lagrangian methods are often the most efficient way to sample a fluid flow and the physical conservation laws are inherently Lagrangian since they apply to moving fluid volumes rather than to the fluid that happens to be present at some fixed point in space. Nevertheless, the Lagrangian equations of motion applied to a three dimensional continuum are quite difficult in most applications. Therefore, almost all theories in fluid mechanics are developed within the Eulerian system. Lagrangian and Eulerian concepts and methods are used side-by-side in many investigations.
The Eulerian-Lagrangian methods require a tracking algorithm to trace a streamline from the starting point until the exit boundaries. The Lagrangian method calculates streamlines locally around specific group points on a fixed grid. Thus to advect the solution along the local streamlines, front-tracking is used. In the Eulerian method the advected solution is projected back onto a fixed grid cell and then used as initial condition for the equation (Ask, et al. (2000)).

2-2-2 Particle Tracking Technique

A popular method for tracing streamline in ground water is the particle tracking technique. Particle tracking techniques are important methods for petroleum reservoir and ground water aquifers. Normally, the velocity for each grid block is found prior to generation of streamlines and the streamline computation is done by semi-analytical methods (Pollock (1988), Goode (1990), Lu (1994), Datta-Gupta et al. (1995), Russel et al. (2000)), or less efficiently by ODE-solvers such as Runge-Kutta methods (Ask et al. (2000)).

The paper by Pollock (1988), (described later in more detail), was a breakthrough for this method. Since the velocity field is obtained numerically, the method can rigorously handle arbitrary geometries, rate imbalance and areal heterogeneities, (Datta-Gupta et al. (1994)). In addition, streamline methods used on tracer tests can help to find the preferred flow paths and include reservoir heterogeneities data that generally does not appear in a conventional pressure transient test. In addition, the
particle tracking technique is designed to trace particles of the fluid flow field where
the flow velocity is calculated at a limited number of locations. Given a velocity
field, a particle is traced one element by one element until either a boundary is
encountered or the available time is completely consumed (Cheng et al. (1998)).

2-2-3 Streamline VS. Streamtube Modeling

A region bounded by streamlines is called a streamtube. In a steady flow, because the
streamlines are tangent to the flow velocity, the fluid that is inside a stream tube must
remain forever within that same stream tube.

Imagine a set of streamlines starting at points that form a closed loop, (Figure 2.1).
These streamlines form a tube with impermeable walls since the walls of the tube are
made up of streamlines. By definition, in the absence of dispersion and gravity there
can be no flow normal to a streamline. This tube is called a streamtube.

![Closed loop](image)

Figure 2.1: Streamtube- Schematic Figure

Streamline and streamtube technology, to a large extent, have been driven by the fact
that heterogeneity controls recovery factors for most fields. This realization caused
the derivation of more complex geological models. After Muskat et al. (1930) research about streamline modeling, streamtube technology was developed in the 1960s, (Higgins, et al. (1962), Leblanc, et al. (1971)). Two dimensional streamtube models were initially available for homogenous permeability regular flow patterns, such as five-spot patterns. Streamtube models were later generated for irregular well positions and areally heterogeneous reservoirs. Streamtube models only allow constant well rates and positions. Gravity effects, and therefore the vertical sweep efficiency are not accounted for. Thus, streamtube modeling often cannot allow for infill drilling or shut in of wells and streamlines are fixed in space.

From mass conservation, we see that for a steady flow, the mass-flow rate is constant along a streamtube. In a constant density flow, therefore, the cross-sectional area of the streamtube gives information on the local velocity. In the fast flow region, the streamlines come closer together, and the area between them decreases. Since the fluid density is constant, the velocity must increase according to the principle of mass conservation. For constant density flow, wherever the area between streamlines decreases, the velocity increases. This is similar to what happens with constant-density flow through a duct or a pipe; if the area decreases, the velocity increases so that the volume flow rate is maintained constant (volume flow rate out must equal volume flow rate in by continuity).
Before the current technology of streamline simulation developed in 1990s, several methods had been proposed. Muskat et al. (1930) considered streamline for 2D flow modeling. Fay et al. (1951) brought in "time of flight" for finding movement along streamlines in 2 dimensions. Higgins et al. (1962) used streamtube method and solved a 1D conservation equation in each streamtube.

The difference of streamline with streamtube simulation using the time of flight approach is that the flow can be determined by integrating along a streamline rather than first defining the streamtubes.

Using streamlines for solving the problem started from the 90s and became more popular in oil and gas research areas, (Datta-Gupta et al. (1995), Bratvedt et al. (1996), Hewett et al. (1997) and King et al. (1998)), and now streamlines based flow simulation is accepted as an effective and complementary technology to more traditional flow modeling approaches such as finite difference method (FD).

Batycky (1997) presented the development and application of a three-dimensional, two phases streamline simulator that is applied to field scale multi-well problems. A streamline simulator for the pressure field and for the saturation distribution consists of two factors:
1-stream path line tracking and
2-modeling streamline solution in 1D

A streamline model uses an IMPES approach i.e. implicit in pressure and explicit in saturation, which is computationally more effective than a fully implicit approach. As long as there is more than one phase, the equations are nonlinear. This is the main advantage of streamline models compared to conventional reservoir models.

2-3 Periodic Updating of Streamlines

Assuming a fixed steamtube/ fixed streamline assumption was one of the disadvantages of the streamlines method during 70’s and 80’s. Martin et al. (1979) and Renard (1990) considered changing streamlines and since the middle of 90’s this assumption has not been used anymore (Thiele et al. (1996) and Batycky et al. (1997)). The main application of considering changing streamlines was for problems with changing well conditions and gravity. To define changing streamlines the problem was assumed steady state at each fixed time interval before being updated. This method worked well for mobility induced nonlinear case studies, but mapping analytical, self-similar hyperbolic solutions would not allow solving systems with changing well conditions and gravity (Theile et al. (1995), Theile et al. (1997), Theile (2001)). Another important factor required was the ability to solve transport problems with generalized initial conditions along each streamlines (Batycky et al. (1997)).
Then streamline changing could be modeled while guaranteeing the fluid transportation in correct direction.

2-4 Numerical Modeling for Streamlines Simulation

Bommer et al. (1979) introduced numerical 1D solution for streamlines to solve a Uranium leaching problem. They used fixed streamline assumption for the numerical solution. Batycky et al. (1997), combined changing 3D streamlines with 1D numerical solution in time-of-flight (TOF)-space. This combination of the ideas had significant effects on using streamline-based simulation in reality where the streamlines change because of mobility differences and changing well conditions. Therefore, streamlines could adjust their initial conditions with the new location. i.e. the conditions existing at the end of the previous timestep. Using 1D numerical solution also made it possible to consider any 1D solution along streamlines, including complex compositional displacements or contaminant migration displacements, (Crane et al. (2000) and Thiele et al. (1997), Thiele (2001)).

2-5 Gravity

Gravity is another factor that needed to be considered in calculations. The velocity vector is the sum of the phase vectors; however, the phase vectors are not parallel
when gravity exists. Bratvedt et al. (1996) presented a solution using the concept of operator splitting, an idea that had been used before in front tracking, (Glimm et al (1983), Bratvedt et al (1992)). This method solves the material balance equations in two steps:

A "convective step" along streamlines and a "gravity step" along gravity lines. In the second step, fluids are segregated in the vertical direction with regards to their phase densities.

2-6 Compressible Flow

The assumption of incompressible fluid was used for all streamlines and streamtubes in the past. Using this assumption helps to simplify equations and calculations in streamline simulation. Other assumptions introduced together with incompressible flow are:

1) Streamlines must start in a source and end in a sink

2) Flow rate along each streamline is constant. However, there is no incompressible flow in reality.

In compressible flow, streamlines can start from or end in any grid block (e.g. because of pressure change) but in an incompressible flow, each grid block has constant rate of flow regardless of pressure increase or decrease. When compressible flow moves through grid blocks, the flow rate can change because of pressure
difference or other physical factors in one grid block. It means each grid block in a compressible flow can play the role of being sink or a source and change the volume of the flow.
Chapter III

The Pollock Method for Streamline Generation*

In this chapter, we first give detailed description of the Pollock method in 2D computational grids as presented in Pollock (1988). Furthermore, we will extend the streamline simulation method to more general flow problems including gravity, compressibility and time dependency. Finally, we will modify the Pollock method for radial geometries as also discussed in the introduction.

3-1 Mass conservation

The general mass conservation (continuity) equation for a single fluid in a porous medium is:

\[ \frac{\partial}{\partial t} (\phi \rho) + \nabla \cdot (\rho \mathbf{v}) = \text{Sources} \]  

(3-1)

*) From Pollock, (1998)
where $\rho$ is fluid density and $\vec{v}$ is volumetric flux. If we assume incompressible fluid and rock, this reduces to:

$$\nabla.(\vec{v}) = 0$$

at any source free point in the medium. Equation (3-2) is the elliptic (time independent) Laplace equation which can be solved for pressure at any location in the medium. Once pressure distribution is known, the velocity field can be calculated from Darcy’s Law, which is the base for streamline generation.

### 3.2 Time of Flight

The variable called Time Of Flight (TOF) was first introduced in Milligan et al. (1951). Physically, it is the time required for a particle that travels along a streamline to reach a location $\xi$ measured curvilinearly along the streamline at $\xi$ from the beginning of the streamline, Fig 3.1.

![Particle Travel Through Streamline](image-url)

**Figure 3.1: Particle Travel Through Streamline**
Once the streamlines have been determined from equation (3-2) as explained in section 3-1, the time of flight (TOF) can be determined from:

\[ t = \int_{A}^{B} \frac{\phi}{u} d\xi \]

where \( u \) is the length of the volumetric flux vector; \( u = \|u\| \).

In multi-phase flow in porous media, the conservation equation for a fluid component \( i \) is:

\[ \frac{\partial}{\partial t} (\rho_i \phi S_i) + \nabla \cdot (\rho_i \overrightarrow{u_i}) = \text{Sources of } i \]

where \( S_i \) is the fluid saturation. If we assume incompressible fluids, the equation (3-3) transforms equation (3-4) into a one-dimensional conservation law:

\[ \frac{\partial S_i}{\partial t} + \frac{\partial F_i}{\partial \tau} = 0 \]

where \( F_i \) is the fractional flow of component \( i \). This explains the benefit of streamline simulation over conventional methods: equation (3-5) is one-dimensional and can be solved along individual streamlines using time of flight.

In other words, the streamline method has reduced the 3D flow problem to:

1) Solution of 3D Laplacian
2) Integration of equation (3-5) along one-dimensional streamlines.

It is emphasized that this simplification assumes incompressibility, negligible influence of gravity (all fluids have the same density) and incompressible rock. We will discuss this in more detail later.
Another advantage of the new model of streamline simulation is the consideration of three-dimensional modeling rather than two dimensions. In 2D modeling, the vertical dimension is ignored.

As mentioned the advantage of Pollock’s method is competently tracing streamlines in 3D in terms of time of flight (TOF) using a simple analytical calculation method. Darcy’s law is used to find velocities across boundaries of grid blocks. It is assumed that the flux is known from the pressure solution and Darcy’s law. The exit point of a streamline is calculated. This exit point is the entrance point for the next cell. Assuming linear velocity in each direction, time of flight can be calculated for each cell.

Pollock’s method is used for an orthogonal grid, however in reality most reservoirs do not adjust in a Cartesian modeling shape. Thus, the next advancement in the research explored ways to find stream path lines according to the Pollock method but with assuming isotropic transformation that transforms corner point geometry grids (CPG) into unit cubes, (Prevost et al. (2001) or Cordes et al (1994)). Therefore, the Pollock method can also be used for irregular grids.
3-3 The Streamline Generation Procedure in Cartesian Coordinates

3-3-1 Flow Modeling

In finite difference method, finding the velocity at every point is based on an interpolation scheme. The velocity vector is determined at each node of a finite difference grid in the modeled reservoir. In other methods, such as analytical methods, the velocity vector can be found through analytical solutions at every point.

Three kinds of interpolations for this issue may be considered:

A- Step function of interpolation
B- Simple linear interpolation
C- Multi linear interpolation

In -A- the velocity components are constant between grid cells and will only change in the nodes. In -B- the velocity direction changes linearly through the grid cell and independently from other directions. In -C- velocities in three directions are dependent on all three-velocity vectors; therefore each velocity in each direction shall be computed through the other two directions’ velocity vectors.

Simple linear interpolation, -B- is used in this research for finding velocity vectors.
From equation (3-1), the mass conservation theory in steady state single fluid incompressible flow can be written as:

$$(3-6) \quad \frac{\partial (\phi V_x)}{\partial x} + \frac{\partial (\phi V_y)}{\partial y} + \frac{\partial (\phi V_z)}{\partial z} = W$$

$V_x, V_y$ and $V_z$: Average linear velocity components

$\phi$: Porosity

$W$: Volume rate of water created (Source term) or consumed per unit bulk volume

Figure 3.2 shows one cell and the incoming and outgoing flows diagonally from each of the six faces of the cell. As it is shown in Figure 3.2, $x_1, x_2, y_1, y_2, z_1$ and $z_2$ are the coordinates defining the faces of the cell. The average velocity element for each face is found by the volume flow rate divided by cross sectional area for each side and the porosity of the material in the cell.
\[ V_{x1} = \frac{Q_{x1}}{(\phi \Delta y \Delta z)} \]
\[ V_{x2} = \frac{Q_{x2}}{(\phi \Delta y \Delta z)} \]
\[ V_{y1} = \frac{Q_{y1}}{(\phi \Delta x \Delta z)} \]
\[ V_{y2} = \frac{Q_{y2}}{(\phi \Delta x \Delta z)} \]
\[ V_{z1} = \frac{Q_{z1}}{(\phi \Delta z \Delta y)} \]
\[ V_{z2} = \frac{Q_{z2}}{(\phi \Delta z \Delta y)} \]

\(Q\): Volume flow rate across a cell face

\(\Delta x, \Delta y, \Delta z\): Length of cell faces

\(\phi\): Porosity

### 3-3-2 Discretized Mass Balance Equation

Comparing the above two equations (3-6) & (3-7), a discrete form of mass conservation equation can be written:

\[ (3-8) \quad \frac{(\phi V_{x2} - nV_{x1})}{\Delta x} + \frac{(\phi V_{y2} - nV_{y1})}{\Delta y} + \frac{(\phi V_{z2} - nV_{z1})}{\Delta z} = \frac{Q}{\Delta x \Delta y \Delta z} \]

In addition, Darcy's law can be written for each side cell. Substitution of Darcy's law for each of the flow terms in equation (3-8) results in a set of algebraic equation
expressed in terms of pressures at nodes located at the cell centers, (McDonald and Harbaugh, (1984)).

The solution of those equations gives the value of flow rate in each node. When the flow rates for each node were calculated, the intercellular flow rates can be computed from Darcy’s law using the value of pressure at the nodes points, (Pollock (1988)).

In order to achieve the main target of finding the path lines, the calculation process should be set up to find the elements of velocity vectors at each point in the reservoir.

As it was mentioned before, the main goal is to locate the path lines by using simple linear interpolation. The components of velocity vectors obtained with simple linear interpolation are given by:

\[
\begin{align*}
V_x &= A_x (x - x_i) + V_{x1} \\
V_y &= A_y (y - y_i) + V_{y1} \\
V_z &= A_z (z - z_i) + V_{z1}
\end{align*}
\]

(3-9)

where \([A_x, A_y, A_z]\) is velocity gradient within a cell, i.e.

\[
\begin{align*}
A_x &= \frac{(V_{x2} - V_{x1})}{\Delta x} \\
A_y &= \frac{(V_{y2} - V_{y1})}{\Delta y} \\
A_z &= \frac{(V_{z2} - V_{z1})}{\Delta z}
\end{align*}
\]

(3-10)
To elaborate, when linear velocity elements shown in equation (3-9) are substituted into equation (3-6), the three derivatives of equation (3-6) will be constant. They are the same as the three terms on the left side of equation (3-8).

Accordingly, linear interpolation of the six cells faces velocity components results in a velocity vector field within the cell that automatically satisfies equation (3-8) at every point inside the cell. This is correct provided that any internal sources or sinks are assumed to be uniformly distributed within the cell, (Pollock (1988)).

The change rate of particle movement can be found as:

\[
(3-11) \quad \left( \frac{dV}{dt} \right)_p = \left( \frac{dV}{dx} \right)_p \left( \frac{dx}{dt} \right)_p
\]

where subscript \( p \) is used for “particle”;

\[
\left( \frac{dx}{dt} \right)_p = \text{Velocity}
\]

The velocity vector in \( x \)-direction is:

\[
(3-12) \quad V_x = \left( \frac{dx}{dt} \right)_p
\]

Also, from equation (3-9) the velocity gradient can be found as follows:

\[
(3-13) \quad \left( \frac{dV_x}{dx} \right) = A_x
\]

Consequently, according to equations (3-11), (3-12) and (3-13):
\[
\begin{align*}
\left( \frac{dV_x}{dt} \right)_p &= A_x V_{xp} \\
\left( \frac{dV_y}{dt} \right)_p &= A_y V_{yp} \\
\left( \frac{dV_z}{dt} \right)_p &= A_z V_{zp}
\end{align*}
\]

(3-14)

\[V_{xp} \text{ and } V_{yp} \text{ are the velocity of particle } p \text{ in } x \text{ and } y \text{ direction at } (x_p, y_p) \text{ location.}\]

Thus equation (3-14), becomes:

\[
\left( \frac{1}{V_{xp}} \right) dV_{xp} = A_x dt
\]

(3-15)

\[
\left( \frac{1}{V_{yp}} \right) dV_{yp} = A_y dt
\]

\[
\left( \frac{1}{V_{zp}} \right) dV_{zp} = A_z dt
\]

Integrating equation (3-15) between \( t_1 \) and \( t_2 \):

\[
\int_{t_1}^{t_2} \left( \frac{1}{V_{xp}} \right) dV_{xp} = \int_{t_1}^{t_2} A_x dt \Rightarrow \ln \left( \frac{V_{xp}(t_2)}{V_{xp}(t_1)} \right) = A_x \Delta t
\]

(3-16)

\[
\int_{t_1}^{t_2} \left( \frac{1}{V_{yp}} \right) dV_{yp} = \int_{t_1}^{t_2} A_y dt \Rightarrow \ln \left( \frac{V_{yp}(t_2)}{V_{yp}(t_1)} \right) = A_y \Delta t
\]

\[
\int_{t_1}^{t_2} \left( \frac{1}{V_{zp}} \right) dV_{zp} = \int_{t_1}^{t_2} A_z dt \Rightarrow \ln \left( \frac{V_{zp}(t_2)}{V_{zp}(t_1)} \right) = A_z \Delta t
\]

where \( \Delta t = t_2 - t_1 \).
To find exit point coordinates, \( x_p(t_2) \), \( y_p(t_2) \) and \( z_p(t_2) \) the below procedure should be followed for each direction:

For \( x \) direction according to equation series (3-9):

\[
V_{xp}(t_2) = A_x (x_p(t_2) - x_i) + V_{x1} \\
\]

\[
(3-17) \quad x_p(t_2) = \frac{1}{A_x} (V_{xp}(t_2) - V_{x1}) + x_i
\]

\( V_x \) can be found from equation series (3-16):

\[
\int_{t_1}^{t_2} \left( \frac{1}{V_{xp}} \right) \, dv_{xp} = \int_{t_1}^{t_2} A_x \, dt \Rightarrow \ln \left( \frac{V_{xp}}{V_{xp}(t_1)} \right) = A_x \Delta t
\]

\[
(3-18) \quad \ln \left( \frac{V_{xp}(t_2)}{V_{xp}(t_1)} \right) = A_x \Delta t
\]

\[
(3-19) \quad V_{xp}(t_2) = V_{xp}(t_1)(\exp(A_x \Delta t))
\]

Therefore from equations (3-17) and (3-19):

\[
x_p(t_2) = \frac{1}{A_x} (V_{xp}(t_2) - V_{x1}) + x_i
\]

\[
V_{xp}(t_2) = V_{xp}(t_1)(\exp(A_x \Delta t))
\]

\[
\Rightarrow \quad x_p(t_2) = \left( \frac{1}{A_x} \right) [V_{xp}(t_1)(\exp(A_x \Delta t)) - V_{x1}] + x_i
\]
Repeating the above procedure for \( y \) and \( z \) directions results in:

\[
x_p(t_2) = x_1 + \left( \frac{1}{A_x} \right) \left[ V_{x_p}(t_1) \exp(A_x \Delta t) - V_{x_1} \right]
\]

(3-20) \[
y_p(t_2) = y_1 + \left( \frac{1}{A_y} \right) \left[ V_{y_p}(t_1) \exp(A_y \Delta t) - V_{y_1} \right]
\]

\[
z_p(t_2) = z_1 + \left( \frac{1}{A_z} \right) \left[ V_{z_p}(t_1) \exp(A_z \Delta t) - V_{z_1} \right]
\]

Therefore, if the velocity at \( t_1 \) is known, the location of the particle at time \( t_2 \) can be easily found from equation (3-20). This model can be used as an algorithm for finding each path line location in one grid cell and repeating the calculations again for the following next grid cells.

In Figure 3.2 one grid cell is shown with all velocity vectors. \( V_{x_1} \) and \( V_{x_2} \) velocities at the \( x_1 \) and \( x_2 \) faces, respectively. In Figure 3.2, \( V_{x_1}, V_{x_2}, V_{y_1} \) and \( V_{y_2} \) are assumed to be positive.
(x_p, y_p, z_p): Particle coordinates in three dimensions at the entrance point

(x_e, y_e, z_e): Particle coordinates at the exit point

Δt_x: Streamline particle time of travel in x direction

Δt_y: Streamline particle time of travel in y direction

V_{x1}: Velocity of Streamline in x direction

V_{y2}: Velocity of Streamline in y direction

Figure 3.3: A Brief Demonstration of Particle Movement, (Pollock (1988))

As mentioned, to find x and y location for each streamline particle for every grid blocks equation series (3-20) is used.

By connecting all entrance and exit points, fluid flow is visualized as streamlines.
As explained before, no particle disappears inside cells. Furthermore, no sinks or sources are assumed to be inside grid cells. Sources or sinks can only be located on the boundaries.

Let \((x_p, y_p)\) be streamline particle coordinate at the entrance point of the first grid cell and \(V_{xp}\) be the particle velocity in \(x\) direction. Hence \(V_{xp}\) can be calculated from equation (3-9). In equation (3-9), \(V_x\) is the same as \(V_{x2}\) at face \(x_2\) (Figure 3.4).

![Figure 3.4: Schematic of a Grid cell and Fluid Flow in \(x\) Direction](image)

After calculating \(V_{xp}\), the time difference in \(x\) and \(y\) direction can be easily found from equation (3-18) and equation (3-21) which are resulted from equation (3-16). For \(y\) direction results in:

\[
(3-21) \quad \frac{1}{A_y} \ln \left[ \frac{V_{y2}}{V_{yp}} \right] = \Delta t_y
\]
After finding \( \Delta t_x \) and \( \Delta t_y \) the time of flight in a grid cell is:

\[
\Delta t_c = \text{Min} (\Delta t_x, \Delta t_y)
\]

Hence, according to above calculation process, this specifies the next target of the particle to enter the next grid block. If \( \Delta t_y \) is less than \( \Delta t_x \), the particle reaches face \( y_2 \) first, leaves the cell across from the face \( y_2 \) and enters grid cell \((i+1, j)\). Similarly, if \( \Delta t_x < \Delta t_y \), the particle enters cell \((i,j+1)\).

If \( \Delta t_x = \Delta t_y \) it can be said that the particle goes exactly through the corner of the cell and will enter to the cell \((i+1, j+1)\).
Figure 3.6: Schematic of the Grid Cell and Exit Coordinate Possibilities

We next determine how to find the exit location of particle $P(x_e, y_e)$ using equation (3-20):

Thus the exit time can be found easily through $t_e = t_p + \Delta t_e$.

This computation series procedure can be reiterated until the mentioned particle reaches the boundary outlet. Thus, this kind of iteration shall be generated as one specific algorithm as is shown in Figure 3.7.

According to Figure 3.7, this progression of computation for steady state flow can be
used for more than two dimensions \((x, y)\).

Figure 3.7: Flow Chart- Streamline Modeling (Pollock (1998)).
Chapter IV

Pressure Solution in Cartesian Geometry

This chapter deals with oil streamline numerical modeling in Cartesian coordinates. Each component of the velocity field is approximated in reference space by a linear function, in which case the streamline path in each direction is given as an exponential function of the travel time. To trace the streamline, Pollock’s method determines the travel time through the grid block as the minimum time to exit in each spatial direction, which is given by a logarithmic expression. Then the travel time is used to compute the exit point and the exit point is mapped back into physical space to give the entry point into the next cell, and so on.

Note: As an essential assumption in this method, incompressible fluids are assumed and gravity effects are ignored.
4-1 Implementation of Numerical Pressure Solvers in Cartesian Coordinates

At first, mapping the specific finite difference grid blocks onto the unit square (or unit cube in 3D) should be provided for the given reservoir. There is no limitation for choosing the number of grid blocks in this method. It could be different according to the accuracy needed in a study and time available. The Laplace equation is implemented as a numerical pressure solver in Cartesian coordinates. Thus, according to the Laplacian solver one specific pressure is obtained for each grid cell. The following sub-sections show how to find the pressure distribution over a finite difference grid.

4-2 The Laplace Equation In Cartesian Coordinates

The following equation come from material balanced in a source free medium.

\[(4-1) \quad \nabla \tilde{u} = 0\]

where \(\tilde{u}\) is volume flux in a source free medium.

For the sake of clarity, we give a simplistic description of the numerical procedure for solving the Laplacian. The description is a compromise between accuracy and
simplicity, and for cases with small permeability contrasts between grid blocks, it is reasonably accurate. However, it is emphasized that when solving the general Laplacian adequate upscaling procedures must be used, i.e. arithmetic average of permeabilities for flow parallel to layers and harmonic average for flow perpendicular to layers. For flow in radial geometries, which is the main focus in this work, we will give a rigorous description of the solution of the Laplacian in chapter V. Therefore, it suffices to give a simplistic approach for Cartesian geometries.

The numerical formulation for solving pressure distribution in finite difference grid cells starts with using a simple form of the Laplace equation in two dimensions. Incompressible fluid is assumed, i.e. $\mu$ and $\rho$ are constant. This determines the pressure at any point given an appropriate set of boundary conditions.

\[(4-2)\]
\[\nabla.(K\nabla p) = 0\]

\[\Rightarrow\]

\[(4-3)\]
\[\frac{\partial}{\partial x}[K_x(\frac{\partial p}{\partial x})] + \frac{\partial}{\partial y}[K_y(\frac{\partial p}{\partial y})] = 0\]

If the medium is homogenous then:

\[(4-4)\]
\[K_x(\frac{\partial^2 p}{\partial x^2}) + K_y(\frac{\partial^2 p}{\partial y^2}) = 0\]

$K_x$: Permeability in $x$ direction

$K_y$: Permeability in $y$ direction
$p$: Pressure

![Diagram of pressure distribution in a sample grid blocks](image)

**Figure 4.1: Pressure Distribution in a Sample Grid Blocks, (Das (1997))**

According to Darcy's law:

\begin{equation}
Q = -\frac{KA}{\mu} \frac{p_1 - p_2}{L}
\end{equation}

Dividing both sides of the equation by the area and using more general notation leads to:

\begin{equation}
q = -\frac{K}{\mu} \nabla p
\end{equation}
where \( q \) is the flux (discharge per unit area, with units of length per time, m/s), \( K \) is permeability (m\(^2\)), \( \mu \) is viscosity (Pa.s) and \( \nabla p \) is the dimensionless pressure gradient vector (Pa/m).

In Figure 4.1, \( p_0, p_1, p_2, p_3 \) and \( p_4 \) are pressures at points 0, 1, 2, 3 and 4 respectively and \( \Delta x \) and \( \Delta y \) are length and width of each grid block. At the middle point, \( p_0 \) can be found through the following calculations using finite difference method.

The following equations can be written for the rate of flow for all four points through the channels shown in Figure 4.1, according to equation (4-6);

\[
q_{1-0} = \frac{K_s}{\mu} p_1 - p_0 \Delta y \Delta x \quad \text{from point 1 to point 0}
\]

(4-7) \[
q_{0-3} = \frac{K_s}{\mu} p_0 - p_3 \Delta y \Delta x \quad \text{from point 0 to point 3}
\]

\[
q_{2-0} = \frac{K_s}{\mu} p_2 - p_0 \Delta x \Delta y \quad \text{from point 2 to point 0}
\]

\[
q_{0-4} = \frac{K_s}{\mu} p_0 - p_4 \Delta x \Delta y \quad \text{from point 0 to point 4}
\]

Here, \( w \) is the width of the model perpendicular to \( x, y \)-directions.
In Figure 4.1, since the total rate of flow into point \( \theta \) is equal to the total rate of flow out of point \( \theta \), \( q_{in} - q_{out} = 0 \)

hence,

\[(4-8) \quad (q_{1-0} + q_{2-0}) - (q_{0-3} + q_{0-4}) = 0\]

Substituting equation (4-7) into equation (4-8), we get:

\[
\frac{K_x}{\mu} \frac{p_1 - p_0}{\Delta x} \Delta y + \frac{K_y}{\mu} \frac{p_2 - p_0}{\Delta y} \Delta x - \frac{K_x}{\mu} \frac{p_0 - p_3}{\Delta x} \Delta y - \frac{K_y}{\mu} \frac{p_0 - p_4}{\Delta y} \Delta x = 0
\]

\[\Rightarrow \]

\[
\frac{K_x}{\Delta x} \frac{\Delta y}{\Delta x} (p_1) - \frac{K_x}{\Delta x} \frac{\Delta y}{\Delta y} (p_0) + \frac{K_y}{\Delta y} \frac{\Delta x}{\Delta y} (p_2) - \frac{K_y}{\Delta y} \frac{\Delta x}{\Delta x} (p_0) + \frac{K_x}{\Delta x} \frac{\Delta y}{\Delta y} (p_0) - \frac{K_x}{\Delta x} \frac{\Delta x}{\Delta y} (p_0) + \frac{K_y}{\Delta y} \frac{\Delta x}{\Delta y} (p_4) = 0
\]

\[\Rightarrow \]

\[
\frac{K_x}{\Delta x} \frac{\Delta y}{\Delta x} (p_1) + \frac{K_x}{\Delta x} \frac{\Delta y}{\Delta y} (p_3) + \frac{K_y}{\Delta y} \frac{\Delta x}{\Delta y} (p_2) + \frac{K_y}{\Delta y} \frac{\Delta x}{\Delta x} (p_4) = 2\left(\frac{K_x}{\Delta x} + \frac{K_y}{\Delta y}\right) p_0
\]

\[\Rightarrow \]

\[
(4-9) \quad \frac{K_x}{\Delta x} \frac{\Delta y}{\Delta x} (p_1 + p_3) + \frac{K_y}{\Delta y} \frac{\Delta x}{\Delta y} (p_2 + p_4) = 2\left(\frac{K_x}{\Delta x} + \frac{K_y}{\Delta y}\right) p_0
\]

Multiply equation (4-9) by \((\Delta y)(\Delta x)\) gives;
(4-10) \[ \frac{K_x \Delta y^2 (p_1 + p_3) + K_y \Delta x^2 (p_2 + p_4)}{2(K_x \Delta y^2 + K_y \Delta x^2)} = p_0 \]

Equation (4-10) can be used to define a linear system of equations in the unknown pressures. The pressure at the middle point, \( p_0 \), will be found from equation (4-10), where \( K_x \) and \( K_y \) are constant.

Figure 4.2: Different Case Studies for Solving Pressure Distribution, (Das (1997))
4-2-1 Isotropic Medium

If the medium is isotropic (e.g. Figure 4.2a), the permeabilities in directions $x$ and $y$ are equal.

If $K_x = K_y = K$ and $\Delta x = \Delta y$, equation (4-10) simplifies to

$$p_0 = \frac{(\Delta x)^2 K_x (p_1 + p_2 + p_3 + p_4)}{4((\Delta x)^2 K_x)} = \frac{1}{4} (p_1 + p_3 + p_2 + p_4)$$

4-2-2 Pressure Solving at the Boundary of Impermeable Layers

If the point 0 is located on the boundary of a pervious and impervious layer as shown in Figure 4.2b, equation (4-10) must be modified as follows:

$$q_{1-0} = \frac{K_x}{\mu} \frac{p_1 - p_0 \Delta y}{\Delta x}$$

$$q_{0-3} = \frac{K_x}{\mu} \frac{p_0 - p_3 \Delta y}{\Delta x}$$

$$q_{0-2} = \frac{K_y}{\mu} \frac{p_0 - p_2 \Delta x}{\Delta y}$$

For continuity of flow,

$$q_{1-0} - q_{0-2} - q_{0-3} = 0$$

Substituting equation (4-12) into equation (4-13):
Assuming $\Delta y = \Delta x$, and $K_x = K_y$, it leads to the following equation:

\[ \Delta y = \Delta x \Rightarrow p_0 = \frac{1}{4} (p_1 + p_2 + 2p_2) \]

**4-2-3 Pressure Solving at the Boundary of Two Different Layers**

Equation (4-15) is valid for grid cells in homogenous soils. However, in real fields, each reservoir consists of various layers with different permeabilities. When flow region includes different reservoir layers with different permeabilities (e.g. Figure 4.2c), equation (4-15) must be modified accordingly.

For the case of flow in $x$ direction in Figure 4.2c, the flow region is located equally in layers 1 and 2 with coefficients of permeability of $K_1$ and $K_2$, respectively. According to upscaling laws, when flow direction is parallel to the boundary between two soil layers equivalent coefficient of permeability at $x$ direction can be used as:

\[ K_x = \frac{K_1 + K_2}{2} \]

For the case of flow in $y$- direction in Figure 4.2.c, if we replace soil 2 with soil 1, and use $p_4'$ instead of $p_4$ for the pressure at point 4, for the velocity to remain the same,
\[
\frac{K_1}{\Delta y}p_4' - p_0 = \frac{K_2}{\Delta y}p_4 - p_0
\]

(4-17) \quad or

\[
p_4' = \frac{K_2}{K_1}(p_4 - p_0) + p_0
\]

Thus equation (4-4) can be written as:

\[
K_x \frac{\partial^2 p}{\partial x^2} + K_y \frac{\partial^2 p}{\partial y^2} = 0
\]

\[
\Rightarrow
\]

\[
\frac{K_1 + K_2}{2} \left( \frac{p_1 + p_3 - 2p_0}{(\Delta x)^2} \right) + \frac{K_1}{(\Delta y)^2} \left( p_2 + p_4' - 2p_0 \right) = 0
\]

\[
\Rightarrow
\]

\[
\frac{K_1 + K_2}{2(\Delta x)^2} (p_1 + p_3 - 2p_0) + \frac{K_1}{(\Delta y)^2} (p_2 + p_4' - 2p_0) = 0
\]

using \( p_4' \) from equation (4-17):

\[
\frac{K_1 + K_2}{2(\Delta x)^2} (p_1 + p_3 - 2p_0) + \frac{K_1}{(\Delta y)^2} \frac{K_2}{K_1} (p_4 - p_0) + p_0 - 2p_0 = 0
\]

\[
\Rightarrow
\]

\[
\frac{K_1 + K_2}{2(\Delta x)^2} (p_1 + p_3 - \frac{K_1 + K_2}{(\Delta x)^2} (p_0) + \frac{K_1}{(\Delta y)^2} (p_2) + \frac{K_2}{(\Delta y)^2} (p_4) - \frac{K_1}{(\Delta y)^2} (p_0) - \frac{K_1}{(\Delta y)^2} (p_0) = 0
\]
\[
\frac{K_1 + K_2}{2(\Delta x)^2} (p_1 + p_3) + \frac{K_1}{(\Delta y)^2} (p_2) + \frac{K_2}{(\Delta y)^2} (p_4) = \left( \frac{K_1}{(\Delta x)^2} + \frac{K_2}{(\Delta y)^2} \right) p_0
\]

Multiplying by \(2\Delta x^2 \Delta y^2\):

\[
(\Delta y)^2 (K_1 + K_2) (p_1 + p_3) + 2K_1 (\Delta x)^2 (p_2) + 2K_2 (\Delta x)^2 (p_4) = 2(K_1 + K_2) (\Delta y^2 + \Delta x^2) p_0
\]

Solving for \(p_0\), the general equation for this case would be:

\[
(4-18) \quad p_0 = \frac{(\Delta y)^2 (K_1 + K_2) (p_1 + p_3) + 2K_1 (\Delta x)^2 (p_2) + 2K_2 (\Delta x)^2 (p_4)}{2(K_1 + K_2) (\Delta y^2 + \Delta x^2)}
\]

If taking \(\Delta y = \Delta x\):

\[
p_0 = \frac{(K_1 + K_2) (p_1 + p_3) + 2K_1 (p_2) + 2K_2 (p_4)}{4(K_1 + K_2)}
\]

\[
(4-19) \quad p_0 = \frac{1}{4} (p_1 + p_3) + \frac{K_1}{2(K_1 + K_2)} (p_2) + \frac{K_2}{2(K_1 + K_2)} (p_4)
\]

Therefore having the relevant equation according to the grid cell and boundary conditions in the assumed finite difference grid, the pressure distribution for the whole block will be found.
4-3 Velocity

After finding pressures for all grid cells, the volumetric flux can be found according to Darcy’s law, for each grid cell.

\[ Q = K \frac{A}{\rho g} \left( \frac{p_1 - p_2}{L} + \rho g \right) \]

\[ (4-20) \Rightarrow \]

\[ Q = K \frac{A}{\mu} \left( \frac{p_1 - p_2}{L} + \rho g \right) \]

where:

\( \mu \) : Fluid viscosity

\( \rho \) : Fluid density

\( K \) : Permeability of the soil and

\( A \) : Cross section of the cell

\[ (4-21) \quad u = \frac{Q}{A} \Rightarrow u = \frac{K}{\mu} \left( \frac{p_1 - p_2}{L} + \rho g \right) \]

The differential form of Darcy’s Law is:

\[ (4-22) \quad u = -\frac{K}{\mu} \left( \frac{\partial p}{\partial x} + \rho g \sin \theta \right) \]
Here, \( L \) is flow direction and \( \theta \) is the angle between the positive \( x \) - axis and positive flow direction measured counter clockwise. Therefore, according to all pressures derived from the Laplacian and Darcy’s Law, the entrance velocity and exit velocity will be found for each grid cell.

After substituting all relevant coefficients in Darcy’s Law, volumetric flux or Darcy’s velocity in \( x \) direction will be found either as entrance velocity in the grid or exit velocity from the grid cell. Consequently after finding pressure and velocity for each grid cell, time of travel in \( x \) direction will be found easily according to equations (3-15) and (3-16).

For \( y \) direction, all the above calculations process is repeated. Then, according to equation (3-16), the time of travel for \( y \) direction will be found as well. In the next step, the time of flight is calculated according to equation (3-22).

Finally, all required coefficients are substituted in equation (3-20) thus \( x_e \) and \( y_e \) as exit point for the streamline particle in one grid block will be found.

These calculation series can be reiterated until the assumed particle reaches to the boundary. Hence all \( x_e \) and \( y_e \) (exit coordinates) are assumed the entrance points for the next grid cell and the calculation procedure can be repeated.
After all, from the beginning until the last boundary place, there are bench-marks for each grid cell as entrance points and exit points for streamline flow. Easily they can connect together like a chain and produce one prolonged line that shows one streamline in the given reservoir. It visualizes streamline behavior according to various geological and process factors and also physical boundary conditions.
Chapter V

Pressure solution in Radial Geometries*

5-1 Polar Coordinate System

Let $x, y$ and $z$ be the Cartesian coordinate system aligned with the principal permeability directions with permabilities $K_x, K_y$ and $K_z$ respectively. Assume a well is parallel to the z-axis. Consider a cylindrical cross section perpendicular to the well, Figure 5.1.

Figure 5.1: Radial Geometry

*) From T. Johansen, (2009)
The Polar representation of point \((x, y)\), \((r, \theta)\) is:

\[
\begin{align*}
x &= r \cos \theta \\
y &= r \sin \theta \\
r &= \sqrt{x^2 + y^2} \\
\theta &= \arctan\left(\frac{y}{x}\right)
\end{align*}
\]

(5-1)

The gradient of a vector in Cartesian coordinates is:

\[
\nabla = \left[ \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right]
\]

(5-2)

Using the chain rule for differentiation:

\[
\begin{align*}
\frac{\partial}{\partial x} &= \frac{\partial}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial}{\partial \theta} \frac{\partial \theta}{\partial x} \\
\frac{\partial}{\partial y} &= \frac{\partial}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial}{\partial \theta} \frac{\partial \theta}{\partial y}
\end{align*}
\]

(5-3)

According to equation (5-1), in \(x\) direction, it is calculated:

\[
x = r \cos \theta \Rightarrow \\
y = r \sin \theta \Rightarrow \\
r = \sqrt{x^2 + y^2} \Rightarrow \frac{\partial r}{\partial x} = \frac{2x}{2\sqrt{x^2 + y^2}} = \frac{x}{r} = \frac{r \cos \theta}{r} = \cos \theta
\]

(5-4)

\[
\frac{\partial r}{\partial y} = \frac{2y}{2\sqrt{x^2 + y^2}} = \frac{y}{r} = \frac{r \sin \theta}{r} = \sin \theta
\]

Consequently, the same process for \(y\) direction should be done:

\[
\frac{\partial \theta}{\partial x} = -\frac{1}{r} \sin \theta
\]

(5-5)
\[
\frac{\partial \theta}{\partial y} = \frac{1}{r} \cos \theta
\]

\[
\frac{\partial}{\partial x} = \frac{\partial}{\partial r} \cos \theta - \frac{1}{r} \sin \theta \frac{\partial}{\partial \theta}
\]

\[
\frac{\partial}{\partial y} = \frac{\partial}{\partial r} \sin \theta + \frac{1}{r} \cos \theta \frac{\partial}{\partial \theta}
\]

5-2 Gradient in Polar Coordinate

Hence, the gradient in Polar coordinates becomes:

\[
\nabla = \left[ \frac{\partial}{\partial r} \cos \theta - \frac{1}{r} \sin \theta \frac{\partial}{\partial \theta}, \frac{\partial}{\partial r} \sin \theta + \frac{1}{r} \cos \theta \frac{\partial}{\partial \theta} \right]
\]

The components of gradient in the direction of the unit vectors \( \vec{e}_r, \vec{e}_\theta \) in Figure 5.1, respectively, are:

\[
\vec{e}_r = [\cos \theta, \sin \theta]
\]

\[
\vec{e}_\theta = [-\sin \theta, \cos \theta]
\]

5-2-1 Gradient in \( r \) Direction

Therefore the gradient in \( r \) direction is:

\[
\nabla_r = \vec{e}_r \cdot \nabla = [\cos \theta, \sin \theta] \left[ \frac{\partial}{\partial r} \cos \theta - \frac{1}{r} \sin \theta \frac{\partial}{\partial \theta}, \frac{\partial}{\partial r} \sin \theta + \frac{1}{r} \cos \theta \frac{\partial}{\partial \theta} \right]
\]

\[
\nabla_r = \vec{e}_r \cdot \nabla = \left[ \frac{\partial}{\partial r} \cos^2 \theta - \frac{1}{r} \sin \theta \cos \theta \frac{\partial}{\partial \theta} + \frac{\partial}{\partial r} \sin^2 \theta + \frac{1}{r} \sin \theta \cos \theta \frac{\partial}{\partial \theta} \right]
\]

62
The divergence of a vector \( \mathbf{u} \) in Cartesian coordinates is:

\[
(5-12) \quad \text{Div} \mathbf{u} = \nabla \cdot \mathbf{u} = \left[ \frac{\partial u_x}{\partial x}, \frac{\partial u_y}{\partial y} \right]
\]
5-3 Velocity in Polar Direction

Let \( u_r, u_\theta \) be the coordinate of \( \vec{u} \) in the coordinate system \( e_r, e_\theta \),

\[
\vec{u} = [u_r, u_\theta]
\]

Note that:

\[
\begin{align*}
\vec{e}_r &= [\cos \theta, \sin \theta] \Rightarrow \frac{\partial \vec{e}_r}{\partial \theta} = [-\sin \theta, \cos \theta] = \vec{e}_\theta \\
\frac{\partial \vec{e}_r}{\partial r} &= 0
\end{align*}
\]

\[
\begin{align*}
\vec{e}_\theta &= [-\sin \theta, \cos \theta] \Rightarrow \frac{\partial \vec{e}_\theta}{\partial \theta} = [-\cos \theta, -\sin \theta] = -\vec{e}_r \\
\frac{\partial \vec{e}_\theta}{\partial r} &= 0
\end{align*}
\]

Therefore, the velocity in Polar coordinate are:

\[
\begin{align*}
u_r &= \vec{e}_r \cdot \vec{u} \\
u_\theta &= \vec{e}_\theta \cdot \vec{u}
\end{align*}
\]

Therefore, the divergence in Polar coordinates will be:

\[
\nabla_{r, \theta} \cdot \vec{u} = \left[ \frac{\partial}{\partial r}, \frac{1}{r} \frac{\partial}{\partial \theta} \right] \cdot \vec{u}
\]

\[
\begin{align*}
\nabla_{r, \theta} \cdot \vec{u} &= \left[ \frac{\partial}{\partial r}, \frac{1}{r} \frac{\partial}{\partial \theta} \right] \cdot \vec{u} = \left[ \vec{e}_r \left( \frac{\partial}{\partial r} \right) + e_\theta \frac{1}{r} \frac{\partial}{\partial \theta} \right] \cdot \left[ u_r \vec{e}_r + u_\theta \vec{e}_\theta \right]
\end{align*}
\]

\[
\begin{align*}
\nabla_{r, \theta} \cdot \vec{u} &= \left[ -\frac{\partial}{\partial r} (u_r, \vec{e}_r) + e_r \frac{\partial}{\partial r} (u_\theta \vec{e}_\theta) + e_\theta \frac{1}{r} \frac{\partial}{\partial \theta} (u_r, \vec{e}_r) + e_\theta \frac{1}{r} \frac{\partial}{\partial \theta} (u_\theta \vec{e}_\theta) \right]
\end{align*}
\]
\[ \nabla_{r,\theta} \cdot \mathbf{u} = e_r \frac{\partial u_r}{\partial r} + e_{r,\theta} \frac{\partial u_r}{\partial \theta} + e_\theta \frac{\partial u_\theta}{\partial r} + e_{\theta,\theta} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial e_r}{\partial r} + \frac{1}{r} \frac{\partial e_r}{\partial \theta} + e_r \frac{\partial e_r}{\partial r} + e_{r,\theta} \frac{1}{r} \frac{\partial e_r}{\partial \theta} + e_\theta \frac{\partial e_\theta}{\partial r} + e_{\theta,\theta} \frac{1}{r} \frac{\partial e_\theta}{\partial \theta} + \frac{1}{r} \frac{\partial e_\theta}{\partial \theta} \]

Hence, according to equation (5-14), the above equation (5-15) can be written as:

\[ \nabla_{r,\theta} \cdot \mathbf{u} = \frac{\partial u_r}{\partial r} + 0 + 0 + 0 + \frac{u_r}{r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} - 0 = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} = \frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} \]

which gives:

\[ (5-17) \quad \nabla_{r,\theta} \cdot \mathbf{u} = \frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{\partial (u_\theta)}{\partial \theta} \]

### 5-4 Darcy’s Law

Darcy’s Law in Cartesian coordinates using principal permeability directions is:

\[ (5-18) \quad \mathbf{u} = -\frac{1}{\mu} K \nabla p \]

Where since coordinate axes are parallel to the principal permeability directions:

\[ (5-19) \quad K = \begin{pmatrix} k_x & 0 \\ 0 & k_y \end{pmatrix} \]

The explicit expressions for the flux vector components \( u_r \) and \( u_\theta \) are as follows:

According to equations (5-14), (5-15) and (5-18):

\[ (5-20) \quad u_r = e_r \mathbf{u} = -\frac{1}{\mu} \cos \theta, \sin \theta \left[ K_x \frac{\partial p}{\partial x}, K_y \frac{\partial p}{\partial y} \right] \]

65
According to equation (5-3):

\[
\begin{align*}
\frac{\partial}{\partial x} &= \frac{\partial}{\partial r} \cos \theta + \frac{\partial}{\partial \theta} \frac{\partial \theta}{\partial x} \\
\frac{\partial}{\partial y} &= \frac{\partial}{\partial r} \sin \theta + \frac{\partial}{\partial \theta} \frac{\partial \theta}{\partial y}
\end{align*}
\Rightarrow
\begin{align*}
\frac{\partial}{\partial x} &= \frac{\partial}{\partial r} \cos \theta - \frac{1}{r} \sin \theta \frac{\partial}{\partial \theta} \\
\frac{\partial}{\partial y} &= \frac{\partial}{\partial r} \sin \theta + \frac{1}{r} \cos \theta \frac{\partial}{\partial \theta}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial p}{\partial x} &= \frac{\partial p}{\partial r} \cos \theta - \frac{1}{r} \sin \theta \frac{\partial p}{\partial \theta} \\
\frac{\partial p}{\partial y} &= \frac{\partial p}{\partial r} \sin \theta + \frac{1}{r} \cos \theta \frac{\partial p}{\partial \theta}
\end{align*}
\]

5-4-1 Velocity in \( r \) direction

The velocity in \( r \) direction can be obtained from following equations:

\[
u_r = -\frac{1}{\mu} \left[ \cos \theta, \sin \theta \right] \left[ \frac{\partial p}{\partial r} \times K_x \cos \theta - \frac{\partial p}{\partial \theta} \times K_x \sin \theta, \frac{\partial p}{\partial r} \times K_y \sin \theta + \frac{\partial p}{\partial \theta} \times K_y \cos \theta \right]
\]

\[
\Rightarrow
nu_r = -\frac{1}{\mu} \left[ \frac{\partial p}{\partial r} \times K_x \cos^2 \theta - \frac{\partial p}{\partial \theta} \times K_x \cos \theta \sin \theta \frac{\partial p}{\partial \theta} \times r + \frac{\partial p}{\partial r} \times K_y \sin^2 \theta + \frac{\partial p}{\partial \theta} \times K_y \sin \theta \cos \theta \right]
\]

(5-22) \[nu_r = -\frac{1}{\mu} \left[ \frac{\partial p}{\partial r} (K_x \cos^2 \theta + K_y \sin^2 \theta) + \frac{1}{r} \frac{\partial p}{\partial \theta} \times \sin \theta \cos \theta (K_y - K_x) \right]
\]

5-4-2 Velocity in \( \theta \) Direction

The velocity in \( \theta \) direction can be obtained from below equations:
\[ u_\theta = e_\theta \cdot u = -\frac{1}{\mu} \left[ -\sin \theta, \cos \theta \right] \left[ K_x \frac{\partial p}{\partial x}, K_y \frac{\partial p}{\partial y} \right] \]

\[ u_\theta = e_\theta \cdot u = -\frac{1}{\mu} \left[ -\sin \theta, \cos \theta \right] \left[ \frac{\partial p}{\partial r} \times K_x \cos \theta - \frac{\partial p}{\partial \theta} \times K_x \sin \theta \times \frac{K_x \sin \theta}{r}, \frac{\partial p}{\partial r} \times K_y \sin \theta + \frac{\partial p}{\partial \theta} \times K_y \cos \theta \times \frac{K_y \cos \theta}{r} \right] \]

\[ u_\theta = -\frac{1}{\mu} \left[ -\frac{\partial p}{\partial r} \times K_x \sin \theta \cos \theta + \frac{\partial p}{\partial \theta} \times \frac{K_x \sin^2 \theta}{r} + \frac{\partial p}{\partial r} \times K_y \cos \theta \sin \theta \times \frac{K_y \cos \theta}{r} + \frac{\partial p}{\partial \theta} \times \frac{K_y \cos^2 \theta}{r} \right] \]

\[ u_\theta = -\frac{1}{\mu} \left[ \frac{\partial p}{\partial r} \times (K_y \cos \theta \sin \theta - K_x \sin \theta \cos \theta) + \frac{\partial p}{\partial \theta} \left( \frac{K_x \sin^2 \theta}{r} + \frac{K_y \cos^2 \theta}{r} \right) \right] \]

\[ \Rightarrow \]

(5-23) \[ u_\theta = -\frac{1}{\mu} \left[ \frac{\partial p}{\partial r} \times \cos \theta \sin \theta (K_y - K_x) + \frac{1}{r} \frac{\partial p}{\partial \theta} (K_x \sin^2 \theta + K_y \cos^2 \theta) \right] \]

### 5.5 Permeability represented in Polar Coordinates

Introducing the following derived quantities:

\[ K_r = K_x \cos^2 \theta + K_y \sin^2 \theta \]

(5-24) \[ K_\theta = K_x \sin^2 \theta + K_y \cos^2 \theta \]

\[ K_\theta = (K_y - K_x) \sin \theta \cos \theta \]

According to the above equation series, equations (5-22), (5-23) can be written as:

(5-25) \[ u_r = -\frac{1}{\mu} \left[ \frac{\partial p}{\partial r} (K_r \cos^2 \theta + K_\theta \sin^2 \theta) + \frac{1}{r} \frac{\partial p}{\partial \theta} \times \sin \theta \cos \theta (K_r - K_\theta) \right] = -\frac{1}{\mu} \left( K_r \frac{\partial p}{\partial r} + \frac{1}{r} K_\theta \frac{\partial p}{\partial \theta} \right) \]

\[ u_\theta = -\frac{1}{\mu} \left[ \frac{\partial p}{\partial r} \times \cos \theta \sin \theta (K_y - K_x) + \frac{1}{r} \frac{\partial p}{\partial \theta} (K_x \sin^2 \theta + K_y \cos^2 \theta) \right] = -\frac{1}{\mu} \left( K_r \frac{\partial p}{\partial r} + \frac{1}{r} K_\theta \frac{\partial p}{\partial \theta} \right) \]

67
Therefore:

\[
\begin{align*}
    u_r &= -\frac{1}{\mu} (K_r \frac{\partial p}{\partial r} + \frac{1}{r} K_\theta \frac{\partial p}{\partial \theta}) \\
    u_\theta &= -\frac{1}{\mu} (K_\theta \frac{\partial p}{\partial r} + \frac{1}{r} K_r \frac{\partial p}{\partial \theta})
\end{align*}
\]

(5-26)

\Rightarrow u = [u_r, u_\theta] = -\frac{1}{\mu} \begin{pmatrix} K_r & K_\theta \\ K_\theta & K_r \end{pmatrix} \begin{pmatrix} \frac{\partial p}{\partial r} \\ \frac{1}{r} \frac{\partial p}{\partial \theta} \end{pmatrix}

Recalling equation (5-18), it turns into the following equation:

(5-27)

\[ \ddot{\mathbf{u}} = -\frac{1}{\mu} \mathbf{K} \cdot \nabla \mathbf{u} = -\frac{1}{\mu} \begin{pmatrix} K_r & K_\theta \\ K_\theta & K_r \end{pmatrix} \begin{pmatrix} \frac{\partial p}{\partial r} \\ \frac{1}{r} \frac{\partial p}{\partial \theta} \end{pmatrix} \]

This is Darcy's law in Polar coordinates.

Note that \( K_r \) and \( K_\theta \) are directional permeabilities in the radial and tangential directions, respectively.

### 5-6 The Laplace Equation in Polar Coordinates

The Laplacian in Polar coordinates is:

\[ \nabla_{r,\theta} \ddot{\mathbf{u}} = 0 \]

Using equations (5-15), (5-25) and (5-26);

(5-28) \[ \nabla_{r,\theta} \ddot{\mathbf{u}} = \frac{1}{r} \frac{\partial}{\partial r} (ru_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (u_\theta) = \frac{1}{r} \frac{\partial}{\partial r} (rK_r \frac{\partial p}{\partial r} + K_\theta \frac{\partial p}{\partial \theta}) + \frac{1}{r} \frac{\partial}{\partial \theta} (K_\theta \frac{\partial p}{\partial r} + \frac{1}{r} K_r \frac{\partial p}{\partial \theta}) \]

Hence, the general Laplacian in Polar coordinates becomes:
For an isotropic medium, $K_r = K_i = K$ and $K_\theta = 0$ for this case, the Laplacian becomes:

$$\frac{1}{r} \frac{\partial}{\partial r} (r K_r \frac{\partial p}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial r} (K_\theta \frac{\partial p}{\partial r}) + \frac{1}{r^2} \frac{\partial}{\partial \theta} (K_\theta \frac{\partial p}{\partial \theta}) = 0$$

This is the general Laplacian in Polar coordinates for the case when the Cartesian coordinate system aligns with the principal permeability direction.

### 5-6-1 Laplacian in Isotropic Medium Case

For an isotropic medium, $K_r = K_i = K$ and $K_\theta = 0$ for this case, the Laplacian becomes:

$$\frac{1}{r} \frac{\partial}{\partial r} (r K_r \frac{\partial p}{\partial r}) + \frac{1}{r^2} \frac{\partial}{\partial \theta} (K_\theta \frac{\partial p}{\partial \theta}) = 0$$

### 5-6-2 Laplacian in Homogenous Medium Case

For a homogeneous medium, permeabilities in equation (5-29) can be placed outside differentiation:

$$K_r \frac{\partial^2 p}{\partial r^2} + \frac{K_r}{r} \frac{\partial p}{\partial r} + \frac{2K_\theta}{r} \frac{\partial^2 p}{\partial r \partial \theta} + \frac{K_\theta}{r^2} \frac{\partial^2 p}{\partial \theta^2} = 0$$

### 5-6-3 Laplacian in Homogenous and Isotropic Case

For the homogenous and isotropic case, equation (5-29) reduces to:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} = 0$$
5-7 General Laplacian in 3D Polar Coordinate System

In 3D case studies, it is straightforward to add the z direction to the 2D description since the z direction is a principle permeability direction. Using the unit vectors,

\[
\vec{e}_r = [\cos \theta, \sin \theta, 0]
\]
\[
\vec{e}_\theta = [-\sin \theta, \cos \theta, 0]
\]
\[
\vec{e}_z = [0, 0, 1]
\]

Darcy's law will be:

\[
\vec{u} = -\frac{1}{\mu} \begin{pmatrix} K_r & K_\theta & 0 \\ K_\theta & K_t & 0 \\ 0 & 0 & K_z \end{pmatrix} \begin{pmatrix} \frac{\partial p}{\partial r} \\ \frac{1}{r} \frac{\partial p}{\partial \theta} \\ \frac{\partial p}{\partial z} \end{pmatrix}
\]

Thus the general Laplacian would be:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left(r K_r \frac{\partial p}{\partial r}\right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left(K_\theta \frac{\partial p}{\partial \theta}\right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(K_\theta \frac{\partial p}{\partial \theta}\right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(K_t \frac{\partial p}{\partial \theta}\right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial p}{\partial z}\right) = 0
\]

5-8 The General Procedures in Streamline Calculations in a Radial Geometry

The velocities \(u_r\) and \(u_\theta\) given by equation (5-26) are used in the Pollock method to generate streamlines. These velocities can be approximated once the Laplacian is solved for pressure on a radial grid.
According to Figure 5.2, a grid block \((i, j)\) is considered and an entrance point \(E_n\) for the streamline is assumed. The goal is to find the exit point, which must be located on one of the four faces of the grid blocks. Thus, after finding the pressure distribution throughout the grid blocks, the velocities \(u_r\) and \(u_\theta\) on each of the faces of grid \((i, j)\) are calculated. They are estimated using the pressures in the nodes shown in Figure 5.2. In addition, correct upscaling procedures for permeability must be used in the calculation procedure. In order to calculate the exit point, the velocity field inside of \((i, j)\) is needed as well.

5-8-1 Velocity Formula Assumption in \(\theta\) Direction

The general formula of velocity in the \(\theta\) direction is assumed linear, and is given as:

\[
(5-36) \quad u_\theta(\theta) = a\theta + b
\]
The constants $a$ and $b$ are determined to match the known velocities on the faces of grid $(i, j)$.

\begin{equation}
5-37 \quad a = \frac{u_{\theta_2}(\theta) - u_{\theta_1}(\theta)}{(\theta_2 - \theta_1)}
\end{equation}

\begin{equation}
5-38 \quad b = u_{\theta_1}(\theta) - \theta_1 \left( \frac{u_{\theta_2}(\theta) - u_{\theta_1}(\theta)}{(\theta_2 - \theta_1)} \right)
\end{equation}

5-8-2 Velocity Formula Assumption in $r$ Direction

General formula of velocity in the $r$ direction will be:

\begin{equation}
5-39 \quad u_r(r) = \frac{a}{r} + b
\end{equation}

The constants $a$ and $b$ are found according to known boundary conditions which are coming from case study substituted in equation (5-39);

\begin{equation}
5-40 \quad a = \frac{u_{r_2}(r) - u_{r_1}(r)}{\frac{1}{r_2} - \frac{1}{r_1}}
\end{equation}

\begin{equation}
5-41 \quad b = u_{r_1}(r) - \frac{1}{r_1} \left( \frac{u_{r_2}(r) - u_{r_1}(r)}{\frac{1}{r_2} - \frac{1}{r_1}} \right)
\end{equation}

This is consistent with a radial flow situation with an angular “leak off”.

This approach is novel and differs from the Cartesian case, as it is a non-linear relationship. Consequently, the formulas for exit point and time of flight calculation are also novel. These formulas are derived next.
5-8-3 Finding Time of Flight for Streamline Movement in Grid Blocks

Travel Time in \( r \) Direction

In Figure 5.2, the time needed for a particle to travel with the velocity \( u_r(r) \) from the entrance point to the opposite angular face is:

\[
t_r = \int_{r_{en}}^{r_{ex}} \frac{dr}{a + br} = \frac{1}{b} \int_{r_{en}}^{r_{ex}} \frac{brdr}{a + br} = \frac{1}{b} \left[ \int_{r_{en}}^{r_{ex}} \frac{(a + br)dr}{a + br} - \int_{r_{en}}^{r_{ex}} \frac{adr}{a + br} \right] =
\]

\[
\Rightarrow
\]

\[
t_r = \frac{1}{b} \left[ \int_{r_{en}}^{r_{ex}} \frac{dr}{a + br} - \int_{r_{en}}^{r_{ex}} \frac{adr}{a + br} \right] = \frac{1}{b^2} \left[ \log(b(r_{ex}) - \log(a + b(r_{ex}))) - \log(b(r_{en}) - \log(a + b(r_{en}))) \right] =
\]

\[
\Rightarrow
\]

\[
t_r = \frac{1}{b^2} \left[ b(r_{ex} - r_{en}) - a[\log(a + b(r_{ex})) - \log(a + b(r_{en}))] \right] = \frac{1}{b^2} \left[ b(r_{ex} - r_{en}) - a\log(\frac{a + b(r_{ex})}{a + b(r_{en})}) \right]
\]

\[
(5-42) \quad t_r = \frac{1}{b^2} (b(r_{ex} - r_{en}) - a(\log(\frac{a + b(r_{ex})}{a + b(r_{en})})))
\]

\( a \) and \( b \) can be substituted from equations (5-40) and (5-41), to the \( t_r \) equation (5-42).
Travel Time in $\theta$ Direction

Similarly, the time needed for the particle to travel with velocity $u_{\theta}(\theta)$ from the entrance point to the opposite face in radial direction is:

$$t_{\theta} = \int_{\theta_{en}}^{\theta_{ex}} \frac{d\theta}{a\theta + b} = \left[ \frac{\log(a\theta + b)}{a} \right]_{\theta_{en}}^{\theta_{ex}} = \left( \frac{\log(a\theta_{ex} + b)}{a} - \frac{\log(a\theta_{en} + b)}{a} \right) = \frac{1}{a} \log\left( \frac{a\theta_{ex} + b}{a\theta_{en} + b} \right)$$

$$\Rightarrow$$

(5-43) $$t_{\theta} = \frac{1}{a} \log\left( \frac{a\theta_{ex} + b}{a\theta_{en} + b} \right)$$

$a$ and $b$ can be substituted from equation (5-37) and equation (5-38), to the $t_{\theta}$ equation (5-43).

Time of Flight

The time for the particle to travel from the entrance to the exit face is then,

(5-44) $$t = \text{Min}(t_r, t_{\theta})$$

This is the time of flight as discussed before. The following equations determine $r_{\text{exit}}$ and $\theta_{\text{exit}}$:

(5-45) $$r_{\text{exit}}(i, j) = r_{\text{particle}}(i + \frac{1}{2}, j) - (u_{r_{\text{particle}}}(i + \frac{1}{2}, j) \times T_{\text{min}})$$

(5-46) $$\theta_{\text{exit}}(i, j) = \text{Min}(\theta_{\text{particle}}(i, j) - [\left( \frac{1}{r(i + \frac{1}{2}, j)} \right) \times T_{\text{min}}], \pi)$$
5-9 Discretization

In constructing the grid in Figure 5.2 the nodes are chosen such that the pressure drop between nodes in ring number $i$ and ring number $(i+1)$ is the same for all $i = 1, 2, 3, ..., N$ for the special case when the cylinder is homogenous and isotropic.

5-10 Radius Equation

Grid blocks are ring fragments as shown in Figure 5.2. The relation between the radius of consecutive rings is:

$$r_{i+1} = r_i M = r_w M^{i+1}$$

where $M$ is constant and $r_0 = r_w =$ well bore radius.

If $r_e$ is the drainage radius,

$$r_N = r_e = r_w M^N \Rightarrow M = \left(\frac{r_e}{r_w}\right)^\frac{1}{N}$$

Therefore:

$$r_{i+1} = r_w \left(\frac{r_e}{r_w}\right)^\frac{i}{N}$$
5-11 Flow Rate Equation

The flow rate between block $i$ and $(i+1)$ can be calculated using Darcy’s law.

\[ q_{i+1/2} = \frac{2\pi Kr_{i+1/2} \Delta z}{\mu} \frac{dp}{dr} = \frac{2\pi Kr_{i+1/2} \Delta z}{\mu} \frac{p_{i+1} - p_i}{r_{i+1} - r_{i}} \]

where $\Delta z$ is the length of the well segment. Integrating equation (5-48) we get:

\[ q_{i+1/2} = \frac{2\pi K \Delta z}{\mu} \frac{(p_{i+1} - p_i)}{\ln(\frac{r_{i+1}}{r_i})} \]

Here, $r_{i+1/2}$ is the radius of the ring between blocks $i$ and $(i+1)$. Using these two expressions for $q_{i+1/2}$ we find:

\[ r_{i+1/2} = \frac{(r_{i+1} - r_i)}{\ln(\frac{r_{i+1}}{r_i})} \]

A second consistent definition of $r_{i+1/2}$ is obtained if we use:

\[ q_{i+1/2} = \frac{2\pi K \Delta z}{\mu} \frac{(p_{i+1} - p_{i+1/2})}{\ln(\frac{r_{i+1}}{r_i})} = \frac{2\pi K \Delta z}{\mu} \frac{(p_{i+1/2} - p_i)}{\ln(\frac{r_{i+1/2}}{r_i})} \]

Demanding $p_{i+1} - p_{i+1/2} = p_{i+1/2} - p_i$ we find:

\[ r_{i+1/2} = \sqrt{r_i r_{i+1}} \]
5-12 Permeability Equation

5-12-1 Radial Mobility

Permeability is considered to be a function of the radius:

\[ K = K(r) \]

This is reasonable because well bore damage caused by mud invasion is constant in the angular direction and various radially.

Then flow rate equation can be presented as:

\[ Q = \frac{2\pi h}{\mu} \int_{r_e}^{r_w} \frac{dr}{K(r)r}(P_e - P_w) \]

Consequently, following formula is used for permeability:

\[ \frac{\log(r_e) - \log(r_w)}{r_w} = \int_{r_e}^{r_w} \frac{dr}{K(r)r} = \sum_{i=1}^{N} \frac{1}{K_i} \log\left(\frac{r_i}{r_{i-1}}\right) \]

According to equation (5-52) and using transmissibility \( \lambda \), i.e. \( \lambda = \frac{K}{\mu} \)

The upscaled radial transmissibility will be:

\[ \lambda_{r+1/2} = \frac{\ln\left(\frac{r_{i+1}}{r_i}\right)}{\frac{1}{\lambda_i} \ln\left(\frac{r_{i+1/2}}{r_{i+1}}\right) + \frac{1}{\lambda_{r+1}} \ln\left(\frac{r_{i+1}}{r_{i+1/2}}\right)} \]

77
\[
\lambda_{i-1/2}^r = \frac{\ln\left(\frac{r_i}{r_{i-1}}\right)}{\frac{1}{\lambda_{i}^r} \ln\left(\frac{r_i}{r_{i-1/2}}\right) + \frac{1}{\lambda_{i-1}^r} \ln\left(\frac{r_{i-1}}{r_{i-1/2}}\right)}
\]

5-12-2 Tangential Transmissibility

If the sector angle in Figure 5.2 is \( \Delta \theta \), the upscaled tangential mobility is:

\[
\lambda_{j+1/2}^t = \frac{\lambda_{j}^t \lambda_{j+1}^t}{\lambda_{j}^t + \lambda_{j+1}^t}
\]

\[
\lambda_{j-1/2}^t = \frac{\lambda_{j}^t \lambda_{j-1}^t}{\lambda_{j}^t + \lambda_{j-1}^t}
\]

5-12-3 Theta Mobility

For the upscaled value \( \lambda_{i+1/2}^\theta \), \( \lambda_{j+1/2}^\theta \) in the third and second term of equation (5-35), respectively, the harmonic average similar to equation (5-55) should be used, i.e.

\[
\lambda_{i+1/2}^\theta = \frac{\ln\left(\frac{r_{i+1}}{r_i}\right)}{\frac{1}{\lambda_{i}^\theta} \ln\left(\frac{r_{i+1}}{r_{i+1/2}}\right) + \frac{1}{\lambda_{i+1}^\theta} \ln\left(\frac{r_{i+1}}{r_{i+1/2}}\right)}
\]

\[
\lambda_{i-1/2}^\theta = \frac{\ln\left(\frac{r_{i-1}}{r_i}\right)}{\frac{1}{\lambda_{i}^\theta} \ln\left(\frac{r_{i-1}}{r_{i-1/2}}\right) + \frac{1}{\lambda_{i-1}^\theta} \ln\left(\frac{r_{i-1}}{r_{i-1/2}}\right)}
\]

The second term arithmetic average will be:
\[ \lambda_{j}^{\theta} = \frac{2 \times \lambda_{j}^{\theta} \lambda_{j+1}^{\theta}}{\frac{\lambda_{j}^{\theta}}{\lambda_{j+1}^{\theta}} + \frac{\lambda_{j}^{\theta}}{\lambda_{j+1}^{\theta}}} \]

\[ \lambda_{j-1}^{\theta} = \frac{2 \times \lambda_{j}^{\theta} \lambda_{j-1}^{\theta}}{\frac{\lambda_{j}^{\theta}}{\lambda_{j-1}^{\theta}} + \frac{\lambda_{j}^{\theta}}{\lambda_{j-1}^{\theta}}} \]

5-13 Face Velocity Equation in Polar Coordinate System

The face velocities between block \((i, j)\) and the neighbouring block is calculated according to the following equation series:

5-13-1 Discretized Face Velocity Equations in \(r\) Direction

\[ u'_{i+1/2,j} = \lambda_{i+1/2,j}^{r} \left( \frac{p_{i+1,j} - p_{i,j}}{r_{i+1} - r_{i}} \right) + \frac{1}{2r_{i+1/2}} \left[ \lambda_{i,j+1/2}^{\theta} \left( \frac{p_{i,j+1} - p_{i,j}}{\Delta \theta} \right) + \lambda_{i,j-1/2}^{\theta} \left( \frac{p_{i,j} - p_{i,j-1}}{\Delta \theta} \right) \right] \]

\[ u'_{i-1/2,j} = \lambda_{i-1/2,j}^{r} \left( \frac{p_{i,j} - p_{i-1,j}}{r_{i} - r_{i-1}} \right) + \frac{1}{2r_{i-1/2}} \left[ \lambda_{i,j+1/2}^{\theta} \left( \frac{p_{i,j+1} - p_{i,j}}{\Delta \theta} \right) + \lambda_{i,j-1/2}^{\theta} \left( \frac{p_{i,j} - p_{i,j-1}}{\Delta \theta} \right) \right] \]

5-13-2 Face Velocity Formula in \(r\) Direction in Isotropic Case

For an isotropic medium:

\[ K_{\theta} = 0 \]
Therefore,

\[ \lambda^0 = 0 \]

Thus equations (5-62) and (5-63) are converted to:

\begin{align*}
(5-64) \quad u'_{i+1/2,j} &= \lambda'_{i+1/2,j} \frac{(p_{i+1,j} - p_{i,j})}{r_{i+1} - r_i} \\
(5-65) \quad u'_{i-1/2,j} &= \lambda'_{i-1/2,j} \frac{(p_{i,j} - p_{i-1,j})}{r_i - r_{i-1}}
\end{align*}

In equations (5-64) and (5-65) \( \lambda' \) can be substituted from equations (5-54) and (5-55):

\begin{align*}
(5-66) \quad u'_{i+1/2,j} &= \frac{\ln(r_{i+1})}{r_i} \frac{(p_{i+1,j} - p_{i,j})}{r_{i+1} - r_i} \\
\lambda'_{i+1/2} &= \frac{1}{\lambda'_{i}} \frac{\ln(r_{i+1/2})}{r_i} + \frac{1}{\lambda'_{i+1}} \frac{\ln(r_{i+1})}{r_{i+1/2}} \\
\Rightarrow \\
u'_{i+1/2,j} &= \frac{\ln(r_{i+1})}{r_i} \frac{(p_{i+1,j} - p_{i,j})}{r_{i+1} - r_i} \\
\Rightarrow \\
\left( \frac{1}{r_{i+1} - r_i} \times \frac{\ln(r_{i+1})}{r_i} \right) \times (p_{i+1,j}) - \left( \frac{1}{r_{i+1} - r_i} \times \frac{\ln(r_{i+1})}{r_i} \right) \times (p_{i,j}) &= u'_{i+1/2,j}
\end{align*}
When \( r_{i+1/2} \) is multiplied by \( u'_{i+1/2,j} \), the result will be:

\[
\begin{align*}
\mathcal{L}_i^r \times \left( \frac{\ln(r_{i+1})}{r_i} - \left( \frac{r_{i+1/2}}{r_i} \times \frac{\ln(r_{i+1/2})}{r_{i+1}} + \frac{1}{\mathcal{L}_i^r} \ln(r_{i+1}) \right) \right) \times (p_{i,j})
\end{align*}
\]

(5-67)

\[
\begin{align*}
+ \left( \frac{r_{i+1/2}}{r_i} \times \frac{\ln(r_{i+1/2})}{r_{i+1}} + \frac{1}{\mathcal{L}_i^r} \ln(r_{i+1}) \right) \times (p_{i+1,j}) = r_{i+1/2} u'_{i+1/2,j}
\end{align*}
\]

Now in this step, \( u'_{i-1/2,j} \) formula will be found;

For \( u'_{i-1/2,j} \):

\[
\begin{align*}
u'_{i-1/2,j} &= \mathcal{L}_i^r \frac{(p_{i,j} - p_{i-1,j})}{r_i - r_{i-1}} \\
\Rightarrow \\
\mathcal{L}_i^r &= \frac{\ln(r_{i-1})}{r_{i-1}} - \frac{1}{\mathcal{L}_i^r} \ln(r_{i-1/2}) + \ln(r_{i-1}) \\
\Rightarrow \\
u'_{i-1/2,j} &= \left( \frac{\ln(r_{i-1})}{r_{i-1}} - \frac{1}{\mathcal{L}_i^r} \ln(r_{i-1/2}) + \ln(r_{i-1}) \right) \times (p_{i,j} - p_{i-1,j}) \\
\Rightarrow \\
(5-68) \\
&= \left( \frac{1}{r_i - r_{i-1}} \times \frac{\ln(r_{i-1})}{r_{i-1}} \right) \times (p_{i,j}) - \left( \frac{1}{r_{i+1} - r_{i-1}} \times \frac{\ln(r_{i+1/2})}{r_{i+1}} \right) \times (p_{i-1,j}) = u'_{i-1/2,j}
\end{align*}
\]
When \( r_{i-1/2} \) is multiplied by \( u^r_{i-1/2,j} \), it will be:

\[
(5-69) \quad \left[ \frac{r_{i-1/2}}{r_i - r_{i-1}} \times \frac{\ln(r_{i-1/2})}{r_{i-1}} \right] \times \left( \frac{\ln(r_i)}{r_{i-1/2}} \right) \times \left( \frac{1}{\lambda^r_i} + \frac{1}{\lambda^r_{i-1}} \right) = r_{i-1/2} u^r_{i-1/2,j}
\]

5-13-3 Descritized Face Velocity Equation in \( \theta \) Direction

\[
(5-70) \quad u^\theta_{i,j+1/2} = \lambda^\theta_{i+1/2,j} \frac{(p_{i+1,j} - p_{i,j})}{2(r_{i+1} - r_i)} + \lambda^\theta_{i-1/2,j} \frac{(p_{i,j} - p_{i-1,j})}{2(r_i - r_{i-1})} + \frac{1}{r_i} \lambda^\theta_{i,j+1/2} \frac{(p_{i,j+1} - p_{i,j})}{\Delta \theta}
\]

\[
(5-71) \quad u^\theta_{i,j-1/2} = \lambda^\theta_{i+1/2,j} \frac{(p_{i+1,j} - p_{i,j})}{2(r_{i+1} - r_i)} + \lambda^\theta_{i-1/2,j} \frac{(p_{i,j} - p_{i-1,j})}{2(r_i - r_{i-1})} + \frac{1}{r_i} \lambda^\theta_{i,j-1/2} \frac{(p_{i,j} - p_{i,j-1})}{\Delta \theta}
\]

5-13-4 Face Velocity Formula in \( \theta \) Direction in Isotropic Case

For an isotropic medium:

\( K_\theta = 0 \)

Therefore

\( \lambda^\theta = 0 \)

Thus equations (5-70) and (5-71) will be reduced to:
\begin{align*}
(5-72) \quad u^\theta_{i,j+1/2} &= \frac{1}{r_i} \lambda^i_{i,j+1/2} \frac{(p_{i,j+1} - p_{i,j})}{\Delta\theta} \\
(5-73) \quad u^\theta_{i,j-1/2} &= \frac{1}{r_i} \lambda^i_{i,j-1/2} \frac{(p_{i,j} - p_{i,j-1})}{\Delta\theta}
\end{align*}

\lambda^i_{j+1/2} \text{ could be substituted from equations (5-56) and (5-57), thus for } u^\theta:\n
\begin{align*}
&\lambda^i_{j+1/2} = \frac{\lambda^i_{j} \lambda^i_{j+1}}{\lambda^i_{j} + \lambda^i_{j+1}} \\
\Rightarrow 
&\begin{align*}
(5-74) \quad u^\theta_{i,j+1/2} &= \frac{1}{r_i} \lambda^i_{j} \frac{(p_{i,j+1} - p_{i,j})}{\Delta\theta} = \left(\frac{1}{\Delta\theta} \frac{\lambda^i_{j} \lambda^i_{j+1}}{r_i \lambda^i_{j} + \lambda^i_{j+1}}\right) (p_{i,j+1}) - \left(\frac{1}{\Delta\theta} \frac{\lambda^i_{j} \lambda^i_{j+1}}{r_i \lambda^i_{j} + \lambda^i_{j+1}}\right) (p_{i,j})
\end{align*}
\end{align*}

When \( \frac{1}{r_i} \frac{1}{\Delta\theta} \) is multiplied by \( u^\theta_{i,j+1/2} \), it results in:

\begin{align*}
(5-75) \quad \left[ \left(\frac{1}{\Delta\theta^2} \frac{1}{r_i^2} \frac{\lambda^i_{j} \lambda^i_{j+1}}{\lambda^i_{j} + \lambda^i_{j+1}}\right) \right] \times (p_{i,j}) + \left(\frac{1}{\Delta\theta^2} \frac{1}{r_i^2} \frac{\lambda^i_{j} \lambda^i_{j+1}}{\lambda^i_{j} + \lambda^i_{j+1}}\right) (p_{i,j+1}) = \frac{1}{r_i} \frac{1}{\Delta\theta} u^\theta_{i,j+1/2}
\end{align*}

Now \( u^\theta_{i,j-1/2} \), can be found.

From equation (5-57), \( \lambda^i_{j-1/2} \) can be substituted in equation (5-76).

Thus for \( u^\theta \),

\begin{align*}
(5-76) \quad u^\theta_{i,j-1/2} &= \frac{1}{r_i} \lambda^i_{i,j-1/2} \frac{(p_{i,j} - p_{i,j-1})}{\Delta\theta}
\end{align*}
\[
\lambda_{j-1/2} = \frac{\lambda_{j-1} \lambda_j}{\lambda_{j} + \lambda_{j-1}}
\]

\[
\Rightarrow
\]

\[
u_{i,j-1/2}^\theta = \frac{1}{r_i} \left( \frac{\lambda_{j-1} \lambda_j}{\lambda_j + \lambda_{j-1}} \right) \frac{(p_{i,j} - p_{i,j-1})}{\Delta \theta} = \left( \frac{1}{\Delta \theta} \frac{\lambda_{j-1} \lambda_j}{r_i \Delta \theta \lambda_j + \lambda_{j-1}} \right) (p_{i,j}) - \left( \frac{1}{\Delta \theta} \frac{\lambda_{j-1} \lambda_j}{r_i \Delta \theta \lambda_j + \lambda_{j-1}} \right) (p_{i,j-1})
\]

When \( \frac{1}{r_i} \frac{1}{\Delta \theta} \) is multiplied by \( u_{i,j-1/2}^\theta \), it will be:

\[
(5-77)
\]

\[
\left[ \left( \frac{1}{\Delta \theta^2} \frac{1}{r_i^2 \lambda_j + \lambda_{j-1}} \right) \right] (p_{i,j}) - \left( \frac{1}{\Delta \theta^2} \frac{1}{r_i^2 \lambda_j + \lambda_{j-1}} \right) (p_{i,j-1}) = \frac{1}{r_i} \frac{1}{\Delta \theta} u_{i,j-1/2}^\theta
\]

5-14 Discrimization of The General Laplacian Equation

According to equation (5-29), the discretization of the general Laplacian can be written as follows:

\[
(5-78)
\]

\[
\frac{1}{r_i (r_{i+1/2} - r_{i-1/2})} \left[ r_{i+1/2} u_{i+1/2,j} - r_{i-1/2} u_{i-1/2,j} \right] + \frac{1}{r_i} \frac{1}{\Delta \theta} (u_{i,j+1/2}^\theta - u_{i,j-1/2}^\theta) = 0
\]

Therefore, in equation (5-78) all the previous known factors can be substituted according to their specific equations as previously explained in this chapter. Consequently, face velocity equation will be found according to the equations (5-67) and (5-69):
\[ r_{i+1/2} u^r_{i+1/2,j} - r_{i-1/2} u^r_{i-1/2,j} = \]
\[ \left[ \left( \frac{r_{i+1/2}}{r_i} - \ln \left( \frac{r_{i+1/2}}{r_i} \right) \right) - \left( \frac{r_{i-1/2}}{r_{i-1}} - \ln \left( \frac{r_{i-1/2}}{r_{i-1}} \right) \right) \right] \times (p_{i,j}) \]
\[ + \left( \frac{r_{i+1/2}}{r_i} - \ln \left( \frac{r_{i+1/2}}{r_i} \right) \right) \times (p_{i+1,j}) \]
\[ + \left( \frac{r_{i-1/2}}{r_{i-1}} - \ln \left( \frac{r_{i-1/2}}{r_{i-1}} \right) \right) \times (p_{i-1,j}) = r_{i+1/2} u^r_{i+1/2,j} - r_{i-1/2} u^r_{i-1/2,j} \]

(5.79)

For equations (5.75) and (5.77):

\[ \frac{1}{r_i} \frac{1}{\Delta \theta} (u^\theta_{i,j+1/2} - u^\theta_{i,j-1/2}) = \]
\[ \Rightarrow \]
\[ \left[ \left( \frac{1}{\Delta \theta^2} \frac{\lambda^r_{i,j+1} \lambda^r_{i,j}}{r^2} \right) - \left( \frac{1}{\Delta \theta^2} \frac{\lambda^r_{i,j} \lambda^r_{i,j-1}}{r^2} \right) \right] \times (p_{i,j}) + \left( \frac{1}{\Delta \theta^2} \frac{\lambda^r_{i,j} \lambda^r_{i,j-1}}{r^2} \right) (p_{i,j-1}) \]
\[ + \left( \frac{1}{\Delta \theta^2} \frac{\lambda^r_{i,j+1} \lambda^r_{i,j}}{r^2} \right) (p_{i+1,j}) = \]
\[ \Rightarrow \]

(5.80) \[ \frac{1}{r_i} \frac{1}{\Delta \theta} (u^\theta_{i,j+1/2} - u^\theta_{i,j-1/2}) = \frac{1}{r_i} \frac{1}{\Delta \theta^2} \left[ \frac{\lambda^r_{i,j+1} \lambda^r_{i,j}}{r^2} \right] \times (p_{i,j}) + \left( \frac{\lambda^r_{i,j} \lambda^r_{i,j-1}}{r^2} \right) (p_{i,j-1}) + \left( \frac{\lambda^r_{i,j+1} \lambda^r_{i,j}}{r^2} \right) (p_{i+1,j}) \]
Substituting equations (5-79) and (5-80) into equation (5-78):

\[
(5-78) \quad \frac{1}{\Delta \theta} \left[ \frac{1}{r_i(r_{i+1/2} - r_{i-1/2})} \left( r_{i+1/2} u_{i+1/2,j} - r_{i-1/2} u_{i-1/2,j} \right) + \frac{1}{r_i} \right] \left( u_{i+1/2,j} - u_{i-1/2,j} \right) = 0
\]

\[
\Rightarrow
\]

\[
\left[ -\frac{1}{r_i(r_{i+1/2} - r_{i-1/2})} \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) \times \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) \right. \\
\left. \frac{1}{r_i} \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) + \frac{1}{\lambda_{i+1}^r} \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) + \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) \right) \times (p_{i,j})
\]

\[
-\frac{1}{\Delta \theta^2} \frac{1}{r_i^2} \left( \frac{\lambda_j^i \lambda_{j+1}^i}{\lambda_j^i + \lambda_{j+1}^i} \right)
\]

\[
\left[ + \left( \frac{1}{r_i(r_{i+1/2} - r_{i-1/2})} \left( \frac{r_{i+1/2}}{r_{i+1/2}} \right) \times \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) \right. \\
\left. \frac{1}{r_i} \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) + \frac{1}{\lambda_{i+1}^r} \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) + \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) \right) \times (p_{i+1,j})
\]

\[
\left. + \frac{1}{r_i(r_{i+1/2} - r_{i-1/2})} \left( \frac{r_{i-1/2}}{r_{i-1/2}} \right) \times \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) + \frac{1}{\lambda_{i+1}^r} \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) + \ln \left( \frac{r_{i+1/2}}{r_{i-1/2}} \right) \right) \times (p_{i-1,j})
\]

\[
\left. + \left( \frac{1}{\Delta \theta^2} \frac{1}{r_i^2} \frac{\lambda_j^i \lambda_{j+1}^i}{\lambda_j^i + \lambda_{j+1}^i} \right) (p_{i,j-1}) + \left( \frac{1}{\Delta \theta^2} \frac{1}{r_i^2} \frac{\lambda_j^i \lambda_{j+1}^i}{\lambda_j^i + \lambda_{j+1}^i} \right) (p_{i,j+1}) = 0
\]

(5-81)
Equation (5-82) can be written similar to equation (5-50) for a neighboring grid block:

\[(5-50) \quad r_{i+1/2} = \frac{(r_{i+1} - r_i)}{\ln\left(\frac{r_{i+1}}{r_i}\right)}\]

\[(5-82) \quad r_{i-1/2} = \frac{(r_{i} - r_{i-1})}{\ln\left(\frac{r_{i}}{r_{i-1}}\right)}\]

Substituting equations (5-50) and (5-82) in equation (5-81) results in:

\[
\left[ \frac{1}{r_i(r_{i+1/2} - r_{i-1/2})} \left( \frac{r_{i+1/2}}{r_{i+1}} - \frac{r_{i-1/2}}{r_{i-1}} \right) \times \frac{1}{\ln\left(\frac{r_{i+1}}{r_i}\right)} \cdot \frac{1}{\lambda_i} \ln\left(\frac{r_{i+1/2}}{r_i}\right) + \frac{1}{\lambda_{i+1}} \ln\left(\frac{r_{i+1}}{r_{i+1/2}}\right) \right] \times (p_{i,j})
- \frac{1}{\Delta \theta^2 r_i^2} \left( \frac{\lambda_i^j \lambda_{i+1}^j}{\lambda_i^j + \lambda_{i+1}^j} + \frac{\lambda_i^j \lambda_{i-1}^j}{\lambda_i^j + \lambda_{i-1}^j} \right)
\]

\[
+ \left( \frac{1}{r_i(r_{i+1/2} - r_{i-1/2})} \left( \frac{r_{i+1/2}}{r_{i+1}} - \frac{r_{i-1/2}}{r_{i-1}} \right) \times \frac{1}{\ln\left(\frac{r_{i+1}}{r_i}\right)} \cdot \frac{1}{\lambda_i} \ln\left(\frac{r_{i+1/2}}{r_i}\right) + \frac{1}{\lambda_{i+1}} \ln\left(\frac{r_{i+1}}{r_{i+1/2}}\right) \right) \times (p_{i+1,j})
+ \left( \frac{1}{2r_i^2 \Delta \theta} \left( \frac{1}{r_{i+1} - r_i} - \frac{1}{r_i - r_{i-1}} \right) \left( \frac{1}{\lambda_i} \ln\left(\frac{r_{i+1/2}}{r_i}\right) + \frac{1}{\lambda_{i+1}} \ln\left(\frac{r_{i+1}}{r_{i+1/2}}\right) \right) \right)
\]
\[ + \frac{1}{r_i(r_{i+1/2} - r_{i-1/2})} \left( \frac{r_{i-1/2}}{(r_i - r_{i-1})} \ln \left( \frac{r_i}{r_{i-1}} \right) \right) \times (p_{i-1,j}) + \\
\frac{1}{r_i} \ln \left( \frac{r_i}{r_{i+1/2}} \right) + \frac{1}{r_{i-1}} \ln \left( \frac{r_{i-1}}{r_{i+1/2}} \right) \times (p_{i+1,j}) = 0 \]

(5-83) \[ \frac{1}{\Delta \theta^2 r_i^2} \left( \frac{\lambda'_i \lambda'_j}{\lambda'_i + \lambda'_j} \right) \times (p_{i,j}) + \frac{1}{\Delta \theta^2 r_i^2} \left( \frac{\lambda'_i \lambda'_{j+1}}{\lambda'_i + \lambda'_{j+1}} \right) \times (p_{i,j+1}) = 0 \]

\[ \Rightarrow \]

Multiplying by \( \Delta \theta^2 r_i^2 \):

\[ \left[ \begin{array}{c}
- \frac{\Delta \theta^2 r_i}{(r_{i+1/2} - r_{i-1/2})} \times \frac{1}{\lambda'_i} \ln \left( \frac{r_{i+1/2}}{r_i} \right) + \frac{1}{\lambda'_{i+1}} \ln \left( \frac{r_{i+1}}{r_{i+1/2}} \right) \\
- \frac{\Delta \theta^2 r_i}{(r_{i+1/2} - r_{i-1/2})} \times \frac{1}{\lambda'_i} \ln \left( \frac{r_i}{r_{i-1/2}} \right) + \frac{1}{\lambda'_{i+1}} \ln \left( \frac{r_{i+1}}{r_{i-1/2}} \right) \\
- \frac{\lambda'_i \lambda'_{j+1}}{\lambda'_i + \lambda'_{j+1}} 
\end{array} \right] \times (p_{i,j}) = 0 \]

\[ + \left[ \begin{array}{c}
\frac{\Delta \theta^2 r_i}{(r_{i+1/2} - r_{i-1/2})} \times \frac{1}{\lambda'_i} \ln \left( \frac{r_{i+1/2}}{r_i} \right) + \frac{1}{\lambda'_{i+1}} \ln \left( \frac{r_{i+1}}{r_{i+1/2}} \right) \\
\frac{1}{(r_{i+1} - r_i)} - \frac{1}{(r_{i+1} - r_i)} \left( \frac{1}{\lambda'_i} \ln \left( \frac{r_{i+1/2}}{r_i} \right) + \frac{1}{\lambda'_{i+1}} \ln \left( \frac{r_{i+1}}{r_{i+1/2}} \right) \right) 
\end{array} \right] \times (p_{i+1,j}) = 0 \]

(5-84) \[ \frac{\Delta \theta^2 r_i}{(r_{i+1/2} - r_{i-1/2})} \times \frac{1}{\lambda'_i} \ln \left( \frac{r_i}{r_{i-1/2}} \right) + \frac{1}{\lambda'_{i+1}} \ln \left( \frac{r_i}{r_{i+1/2}} \right) \times (p_{i-1,j}) + \\
\frac{\lambda'_i \lambda'_{j+1}}{\lambda'_i + \lambda'_{j+1}} (p_{i,j}) + \frac{\lambda'_i \lambda'_{j+1}}{\lambda'_i + \lambda'_{j+1}} (p_{i,j+1}) = 0 \]

The above equation (5-84) can be written as:
(5-85) \[ a(i, j)p_{i,j} + b(i, j)p_{i,j+1} + c(i, j)p_{i,j-1} + d(i, j)p_{i-1,j} + e(i, j)p_{i+1,j} = 0 \]

For \( i = 2, \ldots, N; \ j = 1, \ldots, M \)

For the first ring, close to the inner boundary condition and near the well bore, equation (5-85) becomes:

(5-86) \[ a(1, j)p_{1,j} + b(1, j)p_{1,j+1} + c(1, j)p_{1,j-1} + e(1, j)p_{2,j} = -d(1, j)p_w \]

For \( j = 1, \ldots, M \)

For the last ring, close to the outer boundary condition:

(5-87) \[ a(N, j)p_{N,j} + b(N, j)p_{N,j+1} + c(N, j)p_{N,j-1} + d(N, j)p_{N-1,j} = -e(N, j)p_e \]

For \( j = 1, \ldots, M \)

where \( M \) is the number of sectors and \( N \) is the number of the rings.

The boundary condition on the inner boundary is \( P_{0,j} = P_w \) and on the outer boundary is \( P_{N+1,j} = P_e \).

According to equation (5-85) all coefficients \( a, b, c, d \) and \( e \) can be found as:

(5-88) \[ a(i, j) = -\frac{\Delta \theta r_j}{(r_{i+1/2} - r_{i-1/2})} \left( \frac{1}{\lambda_{r_{i+1/2}}} \ln(\frac{r_{i+1/2}}{r_i}) + \frac{1}{\lambda_{r_{i-1/2}}} \ln(\frac{r_{i-1/2}}{r_i}) + \frac{1}{\lambda_{r_{i+1}}} \ln(\frac{r_{i+1}}{r_i}) + \frac{1}{\lambda_{r_{i-1}}} \ln(\frac{r_{i-1}}{r_i}) \right) - \left( \frac{\lambda_{r_{i+1/2}}}{\lambda_{r_{i+1}}} + \frac{\lambda_{r_{i+1/2}}}{\lambda_{r_{i-1}}} \right) \]
The equations series (5-88) to (5-92) can be simplified using $\lambda'_{r+1/2}$ and $\lambda'_{r-1/2}$ as:

\[
\begin{align*}
(5-93) \quad \lambda'_{r+1/2} &= \frac{\ln\left(\frac{r_{i+1}}{r_i}\right)}{\frac{1}{\lambda'_{i+1/2}} \ln\left(\frac{r_{i+1/2}}{r_i}\right) + \frac{1}{\lambda'_{i+1}} \ln\left(\frac{r_{i+1/2}}{r_{i+1}}\right)} \\
&= \frac{1}{\lambda'_{i+1/2} \ln\left(\frac{r_{i+1/2}}{r_i}\right) + \lambda'_{i+1} \ln\left(\frac{r_{i+1/2}}{r_{i+1}}\right)} \\
(5-94) \quad \lambda'_{r-1/2} &= \frac{\ln\left(\frac{r_{i-1}}{r_{i-1/2}}\right)}{\frac{1}{\lambda'_{i-1/2}} \ln\left(\frac{r_{i-1/2}}{r_{i-1}}\right) + \frac{1}{\lambda'_{i-1}} \ln\left(\frac{r_{i-1/2}}{r_{i-1}}\right)} \\
&= \frac{1}{\lambda'_{i-1/2} \ln\left(\frac{r_{i-1/2}}{r_{i-1}}\right) + \lambda'_{i-1} \ln\left(\frac{r_{i-1/2}}{r_{i-1}}\right)}
\end{align*}
\]

Finally equation series (5-88) to (5-92) will convert to:

\[
(5-95) \quad a(i, j) = -\frac{\Delta \theta^2 r_i}{(r_{i+1/2} - r_{i-1/2})} \left(\frac{\lambda'_{r+1/2}}{r_{i+1/2}} + \lambda'_{r-1/2}\right) = \left(\frac{\lambda'_{r+1/2}}{r_{i+1/2}} + \frac{\lambda'_{r-1/2}}{r_{i-1/2}}\right) - \left(\frac{\lambda'_{i+1}}{r_{i+1}} + \frac{\lambda'_{i-1}}{r_{i-1}}\right)
\]
\[
 \begin{align*}
 b(i, j) &= \left( \frac{\frac{\lambda_j^r \lambda_j^t}{\lambda_j^r + \lambda_j^t}}{\lambda_j^t + \lambda_j^t} \right) \\
 c(i, j) &= \left( \frac{\frac{\lambda_j^r \lambda_{j-1}^t}{\lambda_j^r + \lambda_{j-1}^t}}{\lambda_j^t + \lambda_{j-1}^t} \right) \\
 d(i, j) &= \frac{\Delta \theta^2 r_i}{(r_{i+1/2} - r_{i-1/2})} \times \frac{\lambda_j^{r_{i+1/2}}}{\ln\left(\frac{r_i}{r_{i-1}}\right)} \\
 e(i, j) &= \left( \frac{\Delta \theta^2 r_i}{(r_{i+1/2} - r_{i-1/2})} \times \frac{\lambda_j^{r_{i+1/2}}}{\ln\left(\frac{r_i}{r_{i-1}}\right)} \right)
\end{align*}
\]

Note that \( r_0 = r_o \) and \( r_{n+1} = r_e \).

In the \( \theta \) direction \( p_{i,M+1} = p_{i,1} \) and \( p_{i,0} = p_{i,M} \).

Consequently, according to equation (5-84) and knowing equation series (5-95) to (5-99) as coefficient of \( p_{i,j} \), the unknown vector could be found in different ways using the MATLAB code.

After solving the Laplacian and finding the pressure for each grid block in a Polar grid according to the previous equation series and writing the appropriate MATLAB code for this step, velocities for each grid block in both directions \( r \) and \( \theta \) will be found.

According to equation series (5-62) and (5-63) for \( r \) direction, and equation series (5-70) and (5-71) for \( \theta \) direction, the appropriate pressure can be substituted for the
assumed grid block in these equations and the relevant velocity for that cell can be found.

The velocity distribution for all finite difference grid cells in both $r$ and $\theta$ directions can now be from the pressure distribution.

The next step is to determine streamline time of travel for both directions in one grid cell. Time of travel in $r$ direction is found according to equation (5-42). Parameters $a$ and $b$ are found from the velocity equations, (equations (5-39), (5-40) and (5-41)). Similarly for $\theta$ direction, time of travel is found according to equation (5-43) using velocity equation in $\theta$ direction and finding $a$ and $b$ according to equation (5-37) and (5-38).

Consequently, when time of travel for each grid cell in $r$ and $\theta$ direction are found, then the time of flight for the streamlines’ particle is fixed according to equation (5-44).

After finding time of flight for the streamline at each cell, next exit location, $r_{\text{exit}}$ and $\theta_{\text{exit}}$, can be found according to equations (5-45) and (5-46). This calculation process shall be repeated step by step for the next grid cell and at the end the streamlines’ route can be visualized in Polar coordinates. Figure 3.7 shows the calculation sequence in a flow chart.
Chapter VI

Case Studies

6-1 Summary of Previous Chapters

As mentioned in previous chapters, this research includes streamline simulation in Cartesian and Polar coordinates. The solution for each coordinate system comprises of two main parts: Theoretical part and MATLAB code programming.

The goal of the theoretical part is to develop a new method for generating and analyzing streamlines in reservoirs near well bores using a finite difference method and the Pollock method in each coordinate system.

In Cartesian coordinate system, as it is mentioned in chapter III and chapter IV, the pressure distribution is found by solving the Laplace equation using finite difference method, (details of the pressure solution are described in chapter IV). Then, one
streamline particle is assumed at the entrance point of one grid block at the outer boundaries. Exit coordinate of streamline’s particle for the grid block can be found according to the Pollock method, equation series (3-6) to (3-20). Hence, the new exit point will be the entrance point for the new grid block. This calculation procedure can be reiterated for next grids.

This procedure continues until the next boundary condition (e.g. the well bore) is encountered. At the final step, all these entrance and exit points connect together and constitute the streamline.

The same procedure can be used for new entrance points at the boundary to produce more streamline paths. By using this method streamline behavior and fluid flow, all across the reservoir can be simulated in a Cartesian coordinate system.

In a Polar coordinate system, the relevant equations for pressure distribution are developed for isotropic medium case according to chapter V. Then, equations for velocity and time of travel in \( r \) and \( \theta \) directions are developed, (equation series (5-35) to (5-46)). In the third step, radial, tangential and theta transmissibility equations are developed and substituted in Darcy’s equation and face velocity equations, (equation series (5-47) to (5-77)). Consequently, all these known factors are substituted in general discritized Laplacian equation, equation (5-78) and equation (5-84).
After solving the Laplacian and finding the pressure for every grid block in a Polar coordinate system, the velocities for each grid block in both \( r \) and \( \theta \) directions are found, (equation series (5-62) to (5-71)). The pressure for the assumed grid block, which has been calculated before, can be substituted in these equations and the velocity for that cell can be found.

At the next step, time of travel in \( r \) direction, (the time needed for a particle to travel with the velocity \( u_r(r) \) from the entrance point to the opposite angular face) and time of travel in \( \theta \) direction, (the time needed for the particle to travel with velocity \( u_\theta(\theta) \) from the entrance point to the opposite face in radial direction) are calculated. Consequently time of flight, (the minimum of time of travel in \( r \) and \( \theta \) directions) for streamlines in every cell is found from equation series (5-38) to (5-42). Subsequently next exit location can be found via these equations.

For other grid cells the same calculations could be repeated step by step and at the end, the streamlines’ path can be visualized in the Polar coordinate system according to the specified boundary conditions.

The second main part of this research is programming and developing streamline simulation code using MATLAB. In the theoretical part of the research, all relevant equations for modeling are developed. In this part, the MATLAB code is developed according to all theoretical equations. Some examples are shown in this chapter and details of the MATLAB code are presented in the Appendix. The developed
MATLAB code has strong ability to visualize all streamlines with different boundary conditions and heterogeneous reservoirs.

Two separate types of MATLAB codes are developed for streamline simulation in Cartesian and Polar coordinate systems. The code developed in the Cartesian coordinate system includes two main parts: main file and subroutines. Permeability, pressure and velocity for each node are found by subroutines and other calculations are conducted in the main file.

The MATLAB code developed in the Polar coordinate system includes main route and two subroutines. All the calculations are done in main route and the subroutines do plotting.

Various outputs are resulted from the codes depending on the case studies and requirements. The main output is a visualization of streamlines for different boundary conditions.

Five case studies are presented in this thesis. The numerical results are compared with theoretical expectation.
6-2 Case Studies in Cartesian Geometries

6-2-1 Homogenous and Isotropic Medium

In this case study (Figure 6.1), the inflow boundary is the vertical boundary at the left. A no-flow boundary condition is assumed at both horizontal boundaries. The inflow boundary has higher pressure than the exit boundary on the right side. This case study is assumed to be in a homogenous medium.

Basic Assumptions

1- Isotropic and homogenous medium for the given reservoir
2- No-flow boundary condition for horizontal boundaries

Permeability Distribution

\[ K_x(i, j) = 0.2 \times 10^{-14} m^2 \]
\[ K_y(i, j) = 0.2 \times 10^{-14} m^2 \]

\((i, j)\): Grid cells numbering

Grid Cells Numbering (N, M)

The problem is solved for two cases of 100 by 100 \((M=N=100)\) and 50 by 50 \((M=N=50)\) grid cells.

\(N\) is the number of grid cell(s) in horizontal direction.
$M$ is the number of grid cell(s) in vertical direction.

**Grid Cell Length (L)**

Dimensions of each grid cell in $x$ and $y$ directions are one meter.

$\Delta x = 1.0 \ m$

$\Delta y = 1.0 \ m$

The relevant total length of reservoir is equal to the number of grid cells in each direction multiplied by the grid cell length:

$L_x = 100 \times 1 \ m = 100m$

$L_y = 100 \times 1 \ m = 100m \Rightarrow 100m \times 100m$

or

$L_x = 50 \times 1 \ m = 50m$

$L_y = 50 \times 1 \ m = 50m \Rightarrow 50m \times 50m$

**Boundary Conditions**

**Pressure**

At the entry points of the inflow boundary:

$P(i,1) = 1 \ bar$

and on the exit boundary:

$P(i,N) = 0.01 \ bar$

**Viscosity**

$\mu = 0.0005 \ Pa.s$

**Porosity**

$\phi = 0.2$
Physical Expectation

\[ K_y = K_x = \text{Const.} \]

\[ \Delta y = 1.0 \text{ m} \]
\[ \Delta x = 1.0 \text{ m} \]

No-flow through from both horizontal boundaries

Constant
Higher pressure

Constant
Low Pressure

Figure 6.1: Theoretical Expectation for First Case Study in Cartesian
Coordinate System

For numerical calculations, in isotropic medium, pressures for every grid cells are found according to equations (4-11) to (4-19) and are substituted in equation (4-20). According to equation (4-20), the differential pressure has direct effect on velocity. Since, streamlines’ time of travel is a function of velocity, and then from equation (3-20), the exit location can be found. Therefore, it is expected to see smooth and straight streamlines with no curvature or vertical component of displacement.
MATLAB Program Result

For details of MATLAB code, refer to Appendix.

Figure 6.2: Permeability Quiver for 50 by 50 Grid Cells for First Case

Study in Cartesian Coordinate System

As it is shown in Figure 6.2, $K_x$ and $K_y$, permeabilities in $x$ and $y$ directions have the same values and the quiver of permeability is the same for all grid cells.
Figure 6.3: Pressure Contour in 50 by 50 Meshes for First Case Study in Cartesian coordinate system

Figure 6.3, shows pressure decreases linearly in the $x$ -direction and is constant in $y$ – direction.
Figure 6.4 shows streamline behavior in 100 by 100 grids according to the previous assumptions. As shown in the Figure 6.4, all streamlines are horizontal from high pressure to low pressure boundary condition with no vertical movements. There is one streamline per vertical grid cell.
6-2-2 Low Permeability Region in the Middle of an otherwise homogeneous
Reservoir

In this case study the streamline behaviour is analyzed in an isotropic, heterogeneous
reservoir. A low permeability region is located in the central part of reservoir and the
background permeability has a higher value.

Basic Assumption

1- No-flow through horizontal boundaries
2- Heterogeneous and Isotropic medium

Permeability Distribution

In this case study different permeabilities are assumed in different locations.

Low Permeability Region

\[ K_x(i, j) = 0.1 \times 10^{-20} \text{ m}^2 \]
\[ K_y(i, j) = 0.1 \times 10^{-20} \text{ m}^2 \]

Background Permeability

\[ K_x(i, j) = 0.2 \times 10^{-14} \text{ m}^2 \]
\[ K_y(i, j) = 0.2 \times 10^{-14} \text{ m}^2 \]

\((i, j)\) : Grid cells numbering
Grid Cells Numbering \((N, M)\)

The problem is solved for two cases of 100 by 100 \((M=N=100)\) or 50 by 50 \((M=N=50)\) grid cells.

\(N\) is the number of grid cell(s) in horizontal direction.

\(M\) is the number of grid cell(s) in vertical direction.

Grid Cell Length \((L)\)

Horizontal and vertical dimensions of each grid cell are one meter.

\[\Delta x = 1.0 \ m\]

\[\Delta y = 1.0 \ m\]

Total Length

The relevant total length of reservoir is equal to:

\[L_x = 100 \times 1 \ m = 100 \ m\]
\[L_y = 100 \times 1 \ m = 100 \ m\]

\[\Rightarrow 100m \times 100m\]

or

\[L_x = 50 \times 1 \ m = 50 \ m\]
\[L_y = 50 \times 1 \ m = 50 \ m\]

\[\Rightarrow 50m \times 50m\]

Boundary Condition

At the entry points of the inflow boundary:

\[P(i, 1) = 1 \ \text{bar}\]

and on the exit boundary:
\[ P(i, N) = 0.01 \text{ bar} \]

\( N \) is the number of grid cell(s) in horizontal rows.

**Viscosity**

\( \mu = 0.0005 \text{ Pa.s} \)

**Porosity**

\( \phi = 0.2 \)

**Physical Expectation**

\[ \Delta x = 1.0 \text{ m} \]

\[ \Delta y = 1.0 \text{ m} \]

No-flow through from both horizontal boundaries

**Figure 6.5: Theoretical Expectation for Second Case Study in Cartesian Coordinate System**
A constant high pressure in the inflow boundary on the left side of the reservoir and a constant lower pressure on the outflow boundary are assumed. A low permeability region is assumed at the middle of the reservoir, and other parts have constant higher permeability. The Pressure distribution is calculated according to equations (4-10), (4-11) and (4-18). In one grid cell streamlines exit point is found according to equation (4-6). When there is lower permeability, lower velocities will be found. In the equation (3-16), with low velocity, a larger time of travel will be found. In other words, reaching to the next grid cell that has lower permeability takes more time in the low permeability region. Hence, according to equation (3-22), minimum time of travel is the actual time of flight, which means that streamlines' particle prefers to pass through higher permeability region and move around low permeability regions. Therefore, according to the given boundary conditions, it is expected to see streamlines bypass the low permeability region, (Figure 6.5).
MATLAB Program Result

For details of MATLAB code, refer to Appendix.

Figure 6.6: Permeability Quiver for 50 by 50 Grid Cells for Second Case Study in Cartesian Coordinate System
Figure 6.7: Pressure Contour for 50 by 50 Grid Cells for Second Case Study in Cartesian coordinate system

As it is shown in the Figure 6.7, pressure contours in the low permeability area are closer which is because of higher-pressure gradient when permeability is lower.
Figure 6.8: Streamline Simulation in 100 by 100 Grid Cells for Second Case

Study in Cartesian coordinate system

Figure 6.8, shows the output of numerical simulation where streamlines tend to move around the low permeability region and by pass it. There is a minor asymmetry in the streamlines in the middle of the reservoir which is supposed to be in low permeability area. The asymmetry comes from a numerical error in MATLAB script that could not be tracked.
6-2-3 Assuming a Well in the Corner of the Region

In this case study, (Figure 6.9), the inflow boundary is the vertical boundary on the left hand side. In the exit boundary, on the right side of the reservoir, a well is assumed with lower pressures at the upper corner and the rest of the right hand side vertical boundary is assumed as no-flow boundary. The no-flow boundary condition is assumed at both horizontal boundaries. This case study is assumed to be in an isotropic, homogeneous reservoir.

Basic Assumption

1- Isotropic and homogeneous medium

Permeability Distribution

\[ K_x(i, j) = 0.2 \times 10^{-14} m^2 \]
\[ K_y(i, j) = 0.2 \times 10^{-14} m^2 \]

\((i, j)\) : Grid cells numbering

Grid Cells Numbering \((N, M)\)

The problem is solved for two cases of 100 by 100 \((M=N=100)\) or 50 by 50 \((M=N=50)\) grid cells.

\(N\) is the number of grid cell(s) in horizontal direction.

\(M\) is the number of grid cell(s) in vertical direction.
**Grid Cell Length**

Horizontal and vertical dimensions of each grid cell are equal to one meter.

\[ \Delta x = 1.0 \text{ m}, \quad \Delta y = 1.0 \text{ m} \]

**Total Length**

The relevant total length of reservoir is:

\[ L_x = 100 \times 1m = 100m \]
\[ L_y = 100 \times 1m = 100m \quad \Rightarrow \quad 100m \times 100m \]

or

\[ L_x = 50 \times 1m = 50m \]
\[ L_y = 50 \times 1m = 50m \quad \Rightarrow \quad 50m \times 50m \]

**Boundary Condition**

At the inflow boundary:

\[ P(i,1) = 1 \text{ bar} \]

and on the exit boundary:

\[ P(T,N) = 0.001 \text{ bar} \]

\( T \) is the number of grid cells in vertical rows in well bore region.

\[ \frac{7M}{10} \leq T \leq M \quad (M \text{ is the total number of grid cells in vertical rows}) \]

**Viscosity**

\[ \mu = 0.0005 \text{ Pa.s} \]

**Porosity**

\[ \phi = 0.2 \]
Figure 6.9: Theoretical Expectation in Third Case Study in Cartesian Coordinate System

According to equation (3-6), larger differential pressures result in larger velocities. Then, based on equation (3-16), time of travel in the direction with larger differential pressure is less than the other direction and consequently streamlines' particle tend to go in the direction with larger differential pressure.

Therefore, according to the boundary conditions and assumptions, it is expected to see streamlines moving toward the upper, right corner and collect in the sink, (well) area.
MATLAB Program Result

For details of MATLAB code, refer to the Appendix.

![Permeability Quiver for 50 by 50 grid blocks for Third case study](image)

**Figure 6.10: Permeability Quiver for 50 by 50 Grid Cells for Third Case Study in Cartesian Coordinate System**

As it is shown in the figure 6.10, \( K_x \) and \( K_y \) in both directions have same values and the quiver of both \( K_x \) and \( K_y \) has the same direction in all grid cells.
Figure 6.11: Pressure Contour for Assumed 50 by 50 Grid Cells in Cartesian Coordinate System

Figure 6.11 shows pressure decreasing from high-pressure boundary at the left to lower pressure boundary at the well area.
Figure 6.12: Streamline Simulation for 100 by 100 Grid Cells for Third Case

Study in Cartesian coordinate system

As it is shown in Figure 6.12, streamlines smoothly tend to go toward the well area and collect in the lower pressure boundary (sink).
6-3 Case Studies in Radial Geometries

6-3-1 Isotropic and Homogeneous Medium

In this case study, (Figure 6.13), higher pressure is assumed at the outer boundary (inflow) and lower pressure is assumed in the center (well). Isotropic, homogeneous medium is assumed for the given reservoir.

Basic Assumption

1- Isotropic homogeneous medium
2- Constant pressure at the boundaries

Permeability Distribution

In this case, permeability is constant.

\[ K_{ij} = 2 \times 10^{-12} \text{ m}^2 \]

Boundary Condition

Pressure

Pressure at the center of the ring, (well);

\[ P_w = 0.2 \text{ bar} \]

Pressure at the inflow boundary;

\[ P_e = 1.0 \text{ bar} \]
$P_w$: Well pressure

$P_e$: Pressure at the outer boundary

Radius

$r_w = 0.2 \text{ m}$

$r_e = 1.10 \text{ m}$

Viscosity

$\mu = 0.001 \text{ Pa.s}$

Porosity

$\phi = 0.2$

Grid Cells Numbering

Number of the rings ($N$) = 10

Number of sectors ($M$) = 60

Grid Cells Measurements

The angle for each sector is $\frac{2\pi}{60}$

The radius, $r$ for the rings increases exponentially

Physical Expectation
The pressure distribution is found according to equation series (5-88) to (5-91). Then, these found pressures are substituted in equation series (5-64), (5-65), (5-72) and (5-73) and velocities in $r$ and $\theta$-directions are found. Radial and tangential transmissibility are found according to equation series (5-54) to (5-57). Note that theta transmissibility is not used in this case because of isotropic case. After finding time of flight according to equations (5-42), (5-43) and (5-44), the results are substituted in equation (5-45) and (5-46) and $r$ and $\theta$ for exit coordinates are found. With high pressure at the outer boundary and lower pressure in the center of the rings, it is expected to see streamlines being radial, gathering in the low-pressure area (well).
MATLAB Program Result

For details of MATLAB code, refer to the Appendix.

Figure 6.14: Pressure Contour in Polar Coordinates System for the First Case Study in Radial Geometry

Pressure distribution is shown in Figure 6.14, decreasing from high pressure at the outer boundary to lower pressure in the center.
Figure 6.15: Streamline Simulation in Polar Coordinate for the First Case Study in Radial Geometry System

As it is shown in Figure 6.15, all streamlines move toward the center of the rings radially.
6-3-2 Low Permeability Region in the Middle of an otherwise homogeneous, isotropic Reservoir

In this case study the streamline behaviour is analyzed in an isotropic, heterogenous reservoir in Polar coordinate system. A low permeability region is located in the reservoir and the background permeability has a higher value.

**Basic Assumption**

1- Heterogeneous, Isotropic medium

2- Constant pressure at the boundaries

**Permeability Distribution**

1- Permeability in low permeability region;

\[ K_r(i, j) = 0.1 \times 10^{-16} \, m^2 \]

2- Permeability in other areas:

\[ K_r(i, j) = 2 \times 10^{-12} \, m^2 \]

\((i, j)\) : Grid Cells numbering in Polar coordinate system in \(r\) and \(\theta\) direction
Boundary Conditions

Pressure

Pressure at the center of the ring, (well);

\[ P_w = 0.2 \text{ bar} \]

Pressure at the inflow boundary;

\[ P_e = 1.0 \text{ bar} \]

\( P_w \): Well pressure

\( P_e \): Pressure at the outer boundary

Radius

\[ r_w = 0.2 \text{ m} \]

\[ r_e = 100.0 \text{ m} \]

Viscosity

\[ \mu = 0.001 \text{ Pa.s} \]

Porosity

\[ \phi = 0.2 \]

Grid Cells Numbering

Number of the rings \((N) = 80\)
Number of sectors \((M) = 80\)

**Grid Cells Measurements**

The angle for each sector is \(\frac{2\pi}{80}\)

The radius, \(r\) for different rings increases exponentially.

**Physical Expectation**

![Diagram of a circular pattern with labeled areas: Low permeability area, High Pressure area, Source, Lower Pressure, well.]

**Figure 6.16: Theoretical Expectation for the Second Case Study in Radial Geometry**

When there is low permeability region in the grid cells, then radial and tangential transmissibility have smaller values according to equations (5-54) to (5-57). Therefore velocities in \(r\) direction have smaller values compared to \(\theta\) direction,
based on equation series (5-64), (5-65), (5-72) and (5-73). Then, it takes more time for streamlines' particle (time of travel) to reach to the other side of the grid cell and according to equation (5-44) time of flight will be equal to the time of travel in $\theta$ direction. Consequently, streamlines' particle changes its path to in the $\theta$ direction and gets around the lower permeability region.

**MATLAB Program Result**

For details of MATLAB code, refer to the Appendix.

Pressure distribution is shown in Figure 6.17. Generally, pressure contours are decreasing from high pressure at the outer boundary to lower pressure in the center. As it is presented in the graph, heterogeneity causes deformed contour shapes because of higher pressure gradient at low permeability area.

When low permeability region is assumed in the middle of the reservoir, as is shown in Figure 6.18, streamlines try to move around the low permeability region. In figure 6.18 the main goal has been capturing the path of streamlines near the low permeability region.

There is a minor asymmetry in the streamlines on the other side of the reservoir that comes from a numerical error in MATLAB script that could not be tracked. The conclusion then is that there is a minor programming error which adds noise on top of
otherwise sound solutions. The behavior shown in the figures would not have been possible had there been a major modeling error.

As mentioned before the shape of streamlines in the low permeability region conforms to what we expect from theory.

Figure 6.17: Pressure contour in Polar Coordinate for the second Case Study in Radial Geometry
Figure 6.18: Streamline Simulation in Polar Coordinate for Second Case Study in Radial Geometry
Chapter VII

Research Novelty, Conclusions and Recommendations

7-1 Summary and Conclusions

This research presents a methodology in Cartesian and Polar coordinates for generating and analyzing streamlines in reservoirs near wellbores in finite difference grids using the Pollock method.

Given an entry point of a streamline into a grid cell, Pollock’s method starts by mapping the grid cell onto the unit square. Each component of the velocity field is then approximated in reference space by a linear function, in which case the streamline path in each direction is given as an exponential function of the travel time. To trace the streamline, Pollock’s method determines the travel time through the grid block as the minimum time to exit each spatial direction, which is given by a logarithmic expression. Then the travel time is used to compute the exit point. The
exit point is mapped back into physical space to give the entry point of the next cell, and so on.

This method demonstrates how streamlines behave when mechanical and geological factors change in the reservoir and effects on route of stream path lines. By using this proposed method, stream path lines can be visualized for any given boundary conditions.

This method has the strong ability for considering large finite difference grids with thousands of grid blocks and various potential fluid flow mechanical factors, such as pressure, velocity and viscosity and also various geological factors. All these processes are working for both Cartesian and Polar geometries with two different variables \((x, y)\) and \((r, \theta)\), respectively.

In this research, five case studies have been examined to ensure acceptable and stable results when applied to real physical examples.

**7-2 Research Novelty**

To our knowledge, streamline simulation in near well bore regions using radial geometry has not been done before.
Numerical analytical tools are regularly used in the oil and gas industry as means to establish the optimal procedure to maximize the efficiency of the project in computation time, less error calculation and in a cost effective manner. Several researches show streamline-based simulation is more powerful and time-efficient than older modeling methods. Some of the benefits of streamline method in calculation process are as follows (Thiele 2001):

- **Flow Visualization:**

Flow visualization is one of the interesting issues for academic and industries because it shows flow path lines and the geometric condition of wells. Using flow visualization allows for the opportunity to follow streamline behaviour in a reservoir and gives the ability to analyze well conditions and reservoir modeling with respect to streamline movements.

- **Full Field Modeling**

When there is a problem in a reservoir, most of the time, considering a small model in a limited area to solve a problem in that location will not give real results because of all interaction and effects that come from outer boundaries; thus, the best way to solve the problem is modeling the entire reservoir. When all parts of the reservoir are considered in calculation, all the influences and cell communications will also be considered.
• **Efficiency and Computational Speed**

Streamline simulation method has more ability to simplify computation and is more efficient and faster in approaching the result compared with more conventional calculation methods.

• **Flow Physics- Starting With the Simplest Model**

In streamline simulation, the problem, solving can start with the simplest model, which is the fastest. Then progressing flow physics and model complexities can be added.

7-3 Recommendations for Future Research

This research developed a comprehensive approach to simulate streamlines near a well bore in a reservoir. With further research, the proposed method has great potential to improve upon the initial assumptions, which will lead to more accurate and practical results.

7-3-1 In this research, oil streamline is simulated in two dimensions in Cartesian coordinate in $x$ and $y$ direction. This research can be modified in three dimensions, adding $z$ dimension to model streamline behaviour in reservoirs.
7-3-2 In this research, isotropic medium is assumed. The next step could be refined by choosing an anisotropic medium for the reservoir.

7-3-3 In this research, fluid is assumed to be incompressible; therefore, further research that uses compressible flows in a case study close to real conditions is recommended.
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Appendix

Source Code for Streamline Simulation

This appendix includes five parts, which is described in following pages.

This MATLAB code is developed originally by MARJAN HASHEM.
end

Xp(1,1)=0;
Yp(1,1)=0;
Xpp=Xp(1,1);
Ypp=Yp(1,1);
Vxp(1,1)=Vx(1,1);
Vyp(1,1)=Vy(1,1);
i=1;
j=1;

while ((Xpp<(X-1) & (Ypp)<(Y+1)), (i<(X-3)))

location

if (rr==1);
break
end

lll;
if (Ye(i,j) >= 0 & Xe(i,j) >= 0)

gg=(Xe(i,j)-X(j));

if (gg>=0.98 & gg<1.1)

777;
Xp(i,j+1)=Xe(i,j);
Yp(i,j+1)=Ye(i,j);
Xpp=Xp(i,j+1);
Ypp=Yp(i,j+1);
i=i;
j=j+1;
else

5555;
Xp(i+1,j)=Xe(i,j);
Yp(i+1,j)=Ye(i,j);
Xpp=Xp(i+1,j);
Ypp=Yp(i+1,j);
i=i+1;
j=j;
end

rr=0;
else

8787;
rr=1;
break
end

end

fprintf(fid2,' Xe Ye Ax Vxp Ay Vyp /l\n\n');

permeabilities

fprintf(fid2,'c');

end

end

fprintf(fid2,' Xe Ye Ax Vxp Ay Vyp /l\n\n');

plotFinal

hold on

Xp=zeros(X,Y);
Yp=zeros(X,Y);

Vxp=zeros(X,Y);
Vyp=zeros(X,Y);

for m=2:

for b=1:X+1

x(b)=(b-1);
end

for t=1:Y+1

y(t)=(t-1);

M=0.5*10^-3;
velocities

end
Xp(m,1)=0;
Yp(m,1)=(m-1);
Xpp=Xp(m,1);
Ypp=Yp(m,1);
Vxp(m,1)=Vx(m,1);
Vyp(m,1)=Vy(m,1);
i=m;
j=1;

while ((Xpp<(X-1)) & (Ypp<Y) & (i<(Y+1)) & (j<(X-1)))

location;

% sign
if (rr==1);
break
end

if (Ye(i,j)>=0 & Xe(i,j)>=0)

gg=(Xe(i,j)-X(j));

if (gg>=0.98 & gg<=1.1)

Xp(i,j+1)=Xe(i,j);
Yp(i,j+1)=Ye(i,j);
Xpp=Xp(i,j+1);
Ypp=Yp(i,j+1);
i=i+1;
j=j+1;
else

55555;
Xp(i+1,j)=Xe(i,j);
Yp(i+1,j)=Ye(i,j);
Xpp=Xp(i+1,j);
Ypp=Yp(i+1,j);
i=i+1;
j=j;
else

rr=0;

end

rr=1;
break
end

x(j);

fprintf(fid2,' Xe Ye Ax Vxp Ay Vyp %L
\n');
fprintf(fid2,' %f %f %f %f %f %e\n','Xe,Ye,Ax,Vxp,Ay,Vyp);
plotFinalforYdirection
hold on
Xp=zeros(X,Y);
Yp=zeros(X,Y);
end

for m=Y+1;
Xp(m,1)=0;
Yp(m,1)=(m-1);
Xpp=Xp(m,1);
Ypp=Yp(m,1);
Vxp(m,1)=Vx(m-1,1);
Vyp(m,1)=Vy(m,1);
i=m;

% i=14
j=1;

while ((Xpp<(X-1)) & (Ypp<Y) & (i<(Y+1)) & (j<(X-1)))

location;

% sign
if (rr==1);
break
end

if (Ye(i,j)>=0 & Xe(i,j)>=0)

gg=(Xe(i,j)-X(j));

if (gg>=0.98 & gg<=1.1)

Xp(i,j+1)=Xe(i,j);
Yp(i,j+1)=Ye(i,j);
Xpp=Xp(i,j+1);
Ypp=Yp(i,j+1);
i=i+1;
j=j+1;
else

55555;
Xp(i+1,j)=Xe(i,j);
Yp(i+1,j)=Ye(i,j);
Xpp=Xp(i+1,j);
Ypp=Yp(i+1,j);
i=i+1;
j=j;
end

rr=0;

end

rr=1;
break
end

x(j);

if (rr==1);
break
end

fprintf(fid2,' Xe Ye Ax Vxp Ay Vyp %L\n\n');
fprintf(fid2,' %f %f %f %f %f %e\n','Xe,Ye,Ax,Vxp,Ay,Vyp);
plotFinalforYdirection
end

139
% Streamline Simulation Near Well Bore
% By MARIAN HASHEM

% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARIAN HASHEM

% First Case Study in Cartesian Coordinate
% Subroutine for finding Permeability

function permeabilities
global X Y
global Kx Ky
global P
global VxVy
global XeYe
global maxerr maxr errormatrix
global x y
global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr
global Ax Ay Vxp Vyp

for i=1:Y
for j=1:X
Kx(i,j)=(0.2*(10^-2));
Ky(i,j)=(0.2*(10^-2));
end
end
Kx Ky

% Streamline Simulation Near Well Bore
% By MARIAN HASHEM

% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARIAN HASHEM

% First Case Study in Cartesian Coordinate
% Subroutine for finding the Velocity

function velocities
global X Y
global Kx Ky
global P
global VxVy
global XeYe
global maxerr maxr errormatrix
global x y
global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr
global Ax Ay Vxp Vyp

M=(0.5*10^-3);
Vx=zeros(Y+1,X+1);
Vy=zeros(Y+2,X);
for i=1:(Y+1)
Vx(i,1)=(0.005) ;
end
for i=2:Y+1
for j=1:X
Vx(i-1,j)=(-1)*((Kx(i-1,j)*P(i-1,j)-P(i-1,j-1))/M');
end
end
for i=Y+1
for j=1:(X+1)
Vx(i,j)=Vx(i-1,j);
end
end

for i=2:Y
for j=2:X+1
Vy(i,j)=(-1)*((Ky(i-1,j-1)*P(i-1,j)-P(i-1,j))/M');
end
end
Vy

Vx
Vy
quiver(Vx,Vy)
hold on

% Streamline Simulation Near Well Bore
% By MARIAN HASHEM

% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARIAN HASHEM

% First Case Study in Cartesian Coordinate
% Subroutine for finding the pressure distribution all across the grid blocks

function pressure
global X Y
global Kx Ky
global P
global VxVy
global XeYe
global maxerr maxr errormatrix
global x y
global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr
global Ax Ay Vxp Vyp
P=zeros(Y,X);
for i=1:Y
if P(i,1)=100000;
for i=1:Y
P(i,1)=100000;
end
maxr=1;
errormatrix=zeros(X,Y);
iteration=0;
while maxr>maxerr
    for i=1:Y
        for j=2:X-1
            tempval=P(i,j);
x=1;
y=1;
if i==1
    P(i,j)=((P(i,j)-
        1)+P(i,j+1)+2*P(i+1,j))/4;
else if i==Y
    P(i,j)=((P(i,j)-
        1)+P(i,j+1)+2*P(i-1,j))/4;
else
    P(i,j)=((P(i,j-1)+P(i,j-
        1)+P(i+1,j)+P(i-1,j))/4);
end
end
end
maxr=max(max(errormatrix));
iteration=iteration+1;
end

for i=1:Y
    for j=1:X
        P(i,j);
    end
end

contourf(P,50,'DisplayName',
    'PRESSURE'); colormap autumn;
figure
iteration;

% Streamline Simulation Near Well Bore
% By MARJAN HASHEM
% Developed MATLAB Program for Streamline
% This Code developed originally by MARJAN
% First Case Study in Cartesian Coordinate
% Subroutine for finding the exit coordinate of the
function location
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxr errormatrix
global x y
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr

% fid2 = fopen('any.name.dat','w');

% input('first model')
disp('first model')
9876
if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i+1,j)>0 & Vx(i+1,j+1)>0 & 
    Vy(i,j)>0 & Vy(i,j+1)>0 & Vy(i+1,j)>0 & Vy(i+1,j+1)>0 & 
    Vxp(i,j)>0 & Vxp(i,j+1)>0 & Vxp(i+1,j)>0 & Vxp(i+1,j+1)>0 & 
    Vyp(i,j)>0 & Vyp(i,j+1)>0 & Vyp(i+1,j)>0 & Vyp(i+1,j+1)>0 & 
    x(i,j)>0 & x(i,j+1)>0 & x(i+1,j)>0 & x(i+1,j+1)>0 & 
    y(i,j)>0 & y(i,j+1)>0 & y(i+1,j)>0 & y(i+1,j+1)>0 &
    i>0 & j>0 & i<Y & j<X & i<Y & 
    x(i,j)>0 & x(i,j+1)>0 & x(i+1,j)>0 & x(i+1,j+1)>0 & 
    y(i,j)>0 & y(i,j+1)>0 & y(i+1,j)>0 & y(i+1,j+1)>0 &)
    disp(' 2-1 ')
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
else if i==Y
    Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i-1,j))/4;
else
    Ax(i,j)=[(Vx(i,j-1)+Vx(i,j-
        1)+Vx(i+1,j)+Vx(i-1,j))/4];
end
end
end
end
end
disp(' 2-1 ')
Ax(i,j)=[(Vx(i,j)-
        1)+Vx(i,j+1)+2*Vx(i+1,j))/4;
\[
\begin{align*}
X_e(i,j) &= (T_m(i,j))^* \\
V_x(i,j) &= \frac{1}{(V_y(i,j))^*} (V_y(i,j))^* x(i,j) \\
Y_e(i,j) &= \frac{1}{(A_y(i,j))^*} (V_y(i,j))^* y(i,j) \\
X_e(i,j) &= y(i,j) \\
Y_e(i,j) &= \end{align*}
\]

Code:

```matlab
% (Second model)'
if (V_y(i,j) > 0 & V_x(i,j+1) > 0 & V_y(i+1,j) > 0)
    disp('6-1-1')
    if V_y(i,j) < 0
        A_y(i,j) = (V_y(i,j))
        V_x(i,j) = (X(i,j) - X(i,j)) + (V_y(i,j)) + (Y(i,j))
        if Ty(i,j) < 0
            Ty(i,j) = Ty(i,j) (+1) \\
        end \\
        if Tx(i,j) < 0
            Tx(i,j) = Tx(i,j) (+1) \\
        end \\
        Ty(i,j) = (1/A_y(i,j))^* (V_y(i,j))^* x(i,j) \\
        Y_e(i,j) = (V_y(i,j))^* y(i,j)
    end \\
else if (V_x(i,j) > 0 & V_x(i,j+1) > 0 & V_y(i+1,j) < 0)
    disp('6-1-2')
    if V_x(i,j) < 0
        A_x(i,j) = (V_x(i,j))
        V_y(i,j) = (Y(i,j) - Y(i,j)) + (V_x(i,j)) + (X(i,j))
        if Ty(i,j) < 0
            Ty(i,j) = Ty(i,j) (+1) \\
        end \\
        if Tx(i,j) < 0
            Tx(i,j) = Tx(i,j) (+1) \\
        end \\
        Ty(i,j) = (1/A_x(i,j))^* (V_x(i,j))^* x(i,j) \\
        Y_e(i,j) = (V_x(i,j))^* y(i,j)
    end \\
else if (V_x(i,j) > 0 & V_x(i,j+1) > 0 & V_y(i+1,j) = 0)
    disp('5-1-1')
    if V_x(i,j) < 0
        A_x(i,j) = (V_x(i,j))
        V_y(i,j) = (Y(i,j) - Y(i,j)) + (V_x(i,j)) + (X(i,j))
        if Ty(i,j) < 0
            Ty(i,j) = Ty(i,j) (+1) \\
        end 
    end \\
    if Tx(i,j) < 0
        Tx(i,j) = Tx(i,j) (+1) \\
    end \\
    Ty(i,j) = (1/A_x(i,j))^* (V_x(i,j))^* x(i,j) \\
    Y_e(i,j) = (V_x(i,j))^* y(i,j)
end
```

142
disp('^-6-1-2')
% Ax(i,j)=(Vx(i,j)+1)-
Vx(i,j)/(x(j)+1-x(j))
Vxp(i,j)=(Ax(i,j)\*x^i(j)-
x(j))+Vx(i,j);
% Tx(i,j)=(1/Ax(i,j))^log
(Vx(i,j)+1)/Vxp(i,j));
% Tx(i,j)=(x^j)+
Xp(i,j)/Vx(i,j);
% Ay(i,j)=(Vy(i,j+1)-
Vy(i,j)/(y(i)+1-y(i)))
% Vyp(i,j)=Ay(i,j)\*x^i(j)-
% Ty(i,j)=(1/Ay(i,j))^log
(Vy(i+1,j)+Vyp(i,j));
if Ty(i,j)<0
Tx(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
if Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*
Vxp(i,j)+Vx(i,j);
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
if Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*
Vxp(i,j)+Vx(i,j);
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
if Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*
Vxp(i,j)+Vx(i,j);
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
else if (Vx(i,j)+1)\*Vxp(i,j) &
Vy(i,j)<0 &
Vy(i+1,j)\*Vxp(i,j) &
Vx(i,j)<0 &
Vy(i,j)<Vy(i+1,j)
% disp('^-6-1-1')
Ax(i,j)=(Vx(i,j)+1)-
Vx(i,j)/(x(j)+1-x(j))
Vxp(i,j)=(Ax(i,j)\*x^i(j)-
x(j))+Vx(i,j);
% Tx(i,j)=(1/Ax(i,j))^log
(Vx(i,j)+1)/Vxp(i,j));
% Tx(i,j)=(x^j)+
Xp(i,j)/Vx(i,j);
% Ay(i,j)=(Vy(i,j+1)-
Vy(i,j)/(y(i)+1-y(i)))
% Vyp(i,j)=Ay(i,j)\*x^i(j)-
% Ty(i,j)=(1/Ay(i,j))^log
(Vy(i+1,j)+Vyp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
if Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*
Vxp(i,j)+Vx(i,j);
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j))
else if (Vx(i,j)+1)\*Vxp(i,j) &
Vy(i,j)<0 &
Vy(i+1,j)\*Vxp(i,j) &
Vx(i,j)<0 &
Vy(i,j)<Vy(i+1,j)
% disp('^-6-1-1')
Ax(i,j)=(Vx(i,j)+1)-
Vx(i,j)/(x(j)+1-x(j))
Vxp(i,j)=(Ax(i,j)\*x^i(j)-
x(j))+Vx(i,j);
% Tx(i,j)=(1/Ax(i,j))^log
(Vx(i,j)+1)/Vxp(i,j));
% Tx(i,j)=(x^j)+
Xp(i,j)/Vx(i,j);
% Ay(i,j)=(Vy(i,j+1)-
Vy(i,j)/(y(i)+1-y(i)))
% Vyp(i,j)=Ay(i,j)\*x^i(j)-
% Ty(i,j)=(1/Ay(i,j))^log
(Vy(i+1,j)+Vyp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
if Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*
Vxp(i,j)+Vx(i,j);
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
else if (Vx(i,j)+1)\*Vxp(i,j) &
Vy(i,j)<0 &
Vy(i+1,j)\*Vxp(i,j) &
Vx(i,j)<0 &
Vy(i,j)<Vy(i+1,j)
% disp('^-6-1-1')
Ax(i,j)=(Vx(i,j)+1)-
Vx(i,j)/(x(j)+1-x(j))
Vxp(i,j)=(Ax(i,j)\*x^i(j)-
x(j))+Vx(i,j);
% Tx(i,j)=(1/Ax(i,j))^log
(Vx(i,j)+1)/Vxp(i,j));
% Tx(i,j)=(x^j)+
Xp(i,j)/Vx(i,j);
% Ay(i,j)=(Vy(i,j+1)-
Vy(i,j)/(y(i)+1-y(i)))
% Vyp(i,j)=Ay(i,j)\*x^i(j)-
% Ty(i,j)=(1/Ay(i,j))^log
(Vy(i+1,j)+Vyp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
if Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*
Vxp(i,j)+Vx(i,j);
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
else if (Vx(i,j)+1)\*Vxp(i,j) &
Vy(i,j)<0 &
Vy(i+1,j)\*Vxp(i,j) &
Vx(i,j)<0 &
Vy(i,j)<Vy(i+1,j)
% disp('^-6-1-1')
Ax(i,j)=(Vx(i,j)+1)-
Vx(i,j)/(x(j)+1-x(j))
Vxp(i,j)=(Ax(i,j)\*x^i(j)-
x(j))+Vx(i,j);
% Tx(i,j)=(1/Ax(i,j))^log
(Vx(i,j)+1)/Vxp(i,j));
% Tx(i,j)=(x^j)+
Xp(i,j)/Vx(i,j);
% Ay(i,j)=(Vy(i,j+1)-
Vy(i,j)/(y(i)+1-y(i)))
% Vyp(i,j)=Ay(i,j)\*x^i(j)-
% Ty(i,j)=(1/Ay(i,j))^log
(Vy(i+1,j)+Vyp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
if Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*
Vxp(i,j)+Vx(i,j);
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
% Ye(i,j)=x^j+1;
% Ye(i,j)=(1/Ay(i,j))^log
(Vy(i,j)+1)/Vyp(i,j));
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i));
Vyp(i,j)=Ay(i,j)*(Yp(i,j)-y(i))+Vy(i,j); %
Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)/Vyp(i,j));

if Ty(i,j)<0
   Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
   Tx(i,j)=Tx(i,j)*(-1);
end

Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*x(j);
Xvp(i,j)=x(j);

Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)))*Tm(i,j))-Vy(i,j))+Vy(i,j);
Xe(i,j);

Ye(i,j);

% - third model

elseif (Vx(i,j)>0 & Vx(i+1,j+1)>0 &
Vx(i,j)==Vx(i+1,j+1) &Vy(i,j)==0 &
Vy(i+1,j)>0)
   disp(' - 10-1')

Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j);
Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); %
Ty(i,j)=(1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j));
Tx(i,j)=(x(j+1)-Xp(i,j))/Vx(i,j); Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i));
Vyp(i,j)=Ay(i,j)*(Yp(i,j)-y(i))+Vy(i,j); %
Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)/Vyp(i,j));
Ty(i,j)=Ty(i,j)*(-1);

if Ty(i,j)<0
   Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
   Tx(i,j)=Tx(i,j)*(-1);
end

Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*x(j);
Xvp(i,j)=x(j);

Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)))*Tm(i,j))-Vy(i,j));
Xe(i,j);

Ye(i,j);

elseif (Vx(i,j)>0 & Vx(i+1,j+1)>0 &
Vx(i,j)==Vx(i+1,j+1) &Vy(i,j)==0 &
Vy(i+1,j)>0)
   disp(' - 13-1')

Ax(i,j)=(Vx(i,j+1)-Vx(i+1,j+1))/x(j)-x(j);
Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); %
Ty(i,j)=(1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j));
Tx(i,j)=(x(j+1)-Xp(i,j))/Vx(i,j); Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i));
Vyp(i,j)=Ay(i,j)*(Yp(i,j)-y(i))+Vy(i,j); %
Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)/Vyp(i,j));
Ty(i,j)=Ty(i,j)*(-1);

if Ty(i,j)<0
   Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
   Tx(i,j)=Tx(i,j)*(-1);
end

Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*x(j);
Xvp(i,j)=x(j);

Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)))*Tm(i,j))-Vy(i,j));
Xe(i,j);

Ye(i,j);

elseif (Vx(i,j)>0 & Vx(i+1,j+1)>0 &
Vx(i,j)==Vx(i+1,j+1) &Vy(i,j)==0 &
Vy(i+1,j)>0)
   disp(' - 12-1')
   Ax(i,j)=(Vx(i,j+1)-Vx(i+1,j+1))/x(j)-x(j);
   Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); %
   Ty(i,j)=(1/Ax(i,j))*log
   (Vx(i,j+1)/Vxp(i,j));
   Tx(i,j)=(x(j+1)-Xp(i,j))/Vx(i,j); Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i));
   Vyp(i,j)=Ay(i,j)*(Yp(i,j)-y(i))+Vy(i,j); %
   Ty(i,j)=(1/Ay(i,j))*log
   (Vy(i+1,j)/Vyp(i,j));
   Ty(i,j)=Ty(i,j)*(-1);

   if Ty(i,j)<0
      Ty(i,j)=Ty(i,j)*(-1);
   end
   if Tx(i,j)<0
      Tx(i,j)=Tx(i,j)*(-1);
   end

   Tm(i,j)=min (Tx(i,j),Ty(i,j));
   Xe(i,j)=Tm(i,j)*x(j);
   Xvp(i,j)=x(j);

   Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)))*Tm(i,j))-Vy(i,j));
   Xe(i,j);

   Ye(i,j);

else
   disp(' - second model')

% - first model

elseif (Vx(i,j)>0 & Vx(i+1,j+1)>0 &
Vx(i,j)==Vx(i+1,j+1) &Vy(i,j)==0 &
Vy(i+1,j)>0)
   disp(' - 12-1')
   Ax(i,j)=(Vx(i,j+1)-Vx(i+1,j+1))/x(j)-x(j);
   Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); %
   Ty(i,j)=(1/Ax(i,j))*log
   (Vx(i,j+1)/Vxp(i,j));
   Tx(i,j)=(x(j+1)-Xp(i,j))/Vx(i,j); Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i));
   Vyp(i,j)=Ay(i,j)*(Yp(i,j)-y(i))+Vy(i,j); %
   Ty(i,j)=(1/Ay(i,j))*log
   (Vy(i+1,j)/Vyp(i,j));
   Ty(i,j)=Ty(i,j)*(-1);

   if Ty(i,j)<0
      Ty(i,j)=Ty(i,j)*(-1);
   end
   if Tx(i,j)<0
      Tx(i,j)=Tx(i,j)*(-1);
   end

   Tm(i,j)=min (Tx(i,j),Ty(i,j));
   Xe(i,j)=Tm(i,j)*x(j);
   Xvp(i,j)=x(j);

   Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)))*Tm(i,j))-Vy(i,j));
   Xe(i,j);

   Ye(i,j);

end
\% Ty(i,j)=(i/Ay(i,j))*log
\{V\{i+1,j\}/V\{i,j\}\};
Ty(i,j)=y(i+1) -

Yp(i,j)/V\{i,j\\};
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=-Tx(i,j)*(-1);
end
Tm(i,j) =min (Tx(i,j),Ty(i,j));

Xe(i,j)=((1/Ax(i,j))*((V\{p\{i,j\}\}*exp(Ax(i,j)*Tm(i,j)))-V\{x\{i,j\}\}))*y(i);
Ye(i,j)=(Tm(i,j)*

V\{p\{i,j\}\})*y(i);
Xe(i,j);
Ye(i,j);

elseif (Vx(i,j)>0 & Vx(i,j+1)>0 &
Vx(i,j+1)>0 & Vx(i,j)>Vx(i,j+1) & Vx(i,j+1)>0 & Vx(i,j+2)>Vx(i,j+1) & Vx(i,j+1)>Vx(i,j+2))
disp(' -3-2')
Ax(i,j)=(Vx(i,j+1) -
Vx(i,j))/(x(j+1)-x(j));
Vxp(i,j)=[Ax(i,j)*x(j)];
Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j));
Ay(i,j)=Vx(i,j+1) -
Vx(i,j)/(y(i+1)-y(i));
Yxp(i,j)=[Ay(i,j)*y(i+1)];
Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)/Vxp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=-Tx(i,j)*(-1);
end
Tm(i,j)=min (Tx(i,j),Ty(i,j));

Xe(i,j)=((1/Ax(i,j))*((V\{p\{i,j\}\}*exp(Ax(i,j)*Tm(i,j)))-V\{x\{i,j\}\}))*y(i);
Ye(i,j)=[Ay(i,j)*log
(Vx(i,j+1)/Vxp(i,j))];
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=-Tx(i,j)*(-1);
end
Tm(i,j)=min (Tx(i,j),Ty(i,j));

Xe(i,j)=((1/Ax(i,j))*((V\{p\{i,j\}\}*exp(Ax(i,j)*Tm(i,j)))-V\{x\{i,j\}\}))*y(i);
Ye(i,j)=[Ay(i,j)*log
(Vx(i,j+1)/Vxp(i,j))];
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=-Tx(i,j)*(-1);
end
Tm(i,j)=min (Tx(i,j),Ty(i,j));
% second model
if \( (Vx(i,j) > 0 \& \& \ Vx(i,j+1) > 0 \& \& \ Vx(i,j) \geq Vx(i,j+1) \& \& \ Vy(i,j) < 0 \& \& \ Vy(i+1,j) > 0) \)
    \[
    \operatorname{disp}(\text{'-6-2'}); \\
    A_x(i,j) = (Vx(i,j+1) - Vx(i,j))/\sqrt{(x(i+1)+x(i))}; \\
    Vx(i,j) = (Ax(i,j)^*\{x(i+1) - x(i)\}) + Vx(i,j); \\
    Tx(i,j) = (1/Ax(i,j) + Vx(i,j))/\log(\{Vx(i,j+1)/Vx(i,j)\}); \\
    \]
    \[
    \begin{align*}
    Ay(i,j) &= (Vy(i+1,j) - Vy(i,j))/\{y(i+1) - y(i)\}; \\
    Vp(i,j) &= (Ay(i,j)^*\{x(i+1) - x(i)\}) + Vy(i,j); \\
    Ty(i,j) &= (1/Ay(i,j) + Vp(i,j))/\log(\{Vp(i+1,j)/Vp(i,j)\}); \\
    \end{align*}
    \]
    \[
    \text{if } Ty(i,j) < 0 \\
    Tx(i,j) = -Ty(i,j)^{-1}; \\
    \]
    \[
    \text{if } Vp(i,j) < 0 \\
    \operatorname{disp}(\text{'-6-2-1'}); \\
    Yx(i,j) = y(i); \\
    Xe(i,j); \\
    Ye(i,j); \\
    \]
    \[
    \text{else } Vp(i,j) > 0 \\
    \operatorname{disp}(\text{'-6-2-2'}); \\
    Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/\sqrt{(x(i+1)+x(i))}; \\
    Vx(i,j) = (Ax(i,j)^*\{x(i+1) - x(i)\}) + Vx(i,j); \\
    Tx(i,j) = (1/Ax(i,j) + Vx(i,j))/\log(\{Vx(i,j+1)/Vx(i,j)\}); \\
    \]
    \[
    \begin{align*}
    Ay(i,j) &= (Vy(i+1,j) - Vy(i,j))/\{y(i+1) - y(i)\}; \\
    Vp(i,j) &= (Ay(i,j)^*\{x(i+1) - x(i)\}) + Vp(i,j); \\
    Ty(i,j) &= (1/Ay(i,j) + Vp(i,j))/\log(\{Vp(i+1,j)/Vp(i,j)\}); \\
    \end{align*}
    \]
    \[
    \text{if } Tx(i,j) < 0 \\
    Tx(i,j) = -Tx(i,j)^{-1}; \\
    \]
    \[
    \text{if } Vx(i,j) > 0 \& \& \ Vx(i,j+1) > 0 \& \& \ Vx(i,j) \leq Vx(i,j+1) \& \& \ Vy(i,j) < 0 \& \& \ Vy(i+1,j) < Vy(i+1,j+1) \)
    \operatorname{disp}(\text{'-9-2'}); \\
    A_x(i,j) = (Vx(i,j+1) - Vx(i,j))/\sqrt{(x(i+1)+x(i))}; \\
    Vx(i,j) = (Ax(i,j)^*\{x(i+1) - x(i)\}) + Vx(i,j); \\
    Tx(i,j) = (1/Ax(i,j) + Vx(i,j))/\log(\{Vx(i,j+1)/Vx(i,j)\}); \\
    \]
    \[
    \begin{align*}
    Ay(i,j) &= (Vy(i+1,j) - Vy(i,j))/\{y(i+1) - y(i)\}; \\
    Vp(i,j) &= (Ay(i,j)^*\{x(i+1) - x(i)\}) + Vp(i,j); \\
    Ty(i,j) &= (1/Ay(i,j) + Vp(i,j))/\log(\{Vp(i+1,j)/Vp(i,j)\}); \\
    \end{align*}
    \]
Ty(i,j) = (y(i)-
Yp(i,j)) / Vy(i,j);
if Ty(i,j) < 0
  Ty(i,j) = -Ty(i,j)*(-1);
end
if Tx(i,j) < 0
  Tx(i,j) = -Tx(i,j)*(-1);
end
Tm(i,j) = min (Tx(i,j), Ty(i,j));
Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j)) * Tm(i,j)) - Vx(i,j)) * x(j);
Ye(i,j) = (1/Ay(i,j)) * Yp(i,j) * y(i);
Xe(i,j) = Xe(i,j) + Xe(i,j);  
Ye(i,j) = Ye(i,j) + Ye(i,j);

elseif (Vx(i,j) > 0 & Vx(i,j+1) > 0 & Vx(i,j) > Vx(i,j+1) & Vy(i,j) < 0 & Vy(i+1,j) < 0)
disp(' -10-2');
Ax(i,j) = (Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j);
Tx(i,j) = 1 / Ax(i,j) * log(Vx(i,j+1) / Vxp(i,j));
Ay(i,j) = (Vy(i+1,j) - Vy(i,j)) / (y(i+1,j) - y(i));
Vyp(i,j) = (Ay(i,j) * Yp(i,j) - y(i)) + Vy(i,j);
Ty(i,j) = (y(i) - Yp(i,j)) / Vy(i,j);
if Ty(i,j) < 0
  Ty(i,j) = -Ty(i,j)*(-1);
end
if Tx(i,j) < 0
  Tx(i,j) = -Tx(i,j)*(-1);
end
Tm(i,j) = Tx(i,j);
Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j)) * Tm(i,j)) - Vx(i,j)) * x(j);
Ye(i,j) = (1/Ay(i,j)) * Yp(i,j) * y(i);
Xe(i,j) = Xe(i,j) + Xe(i,j);  
Ye(i,j) = Ye(i,j) + Ye(i,j);

% - third model
elseif (Vx(i,j) > 0 & Vx(i,j+1) > 0 & Vx(i,j) > Vx(i,j+1) & Vy(i,j) == 0 & Vy(i+1,j) == 0)
disp(' -11-2');
Ax(i,j) = (Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j);
Tx(i,j) = 1 / Ax(i,j) * log(Vx(i,j+1) / Vxp(i,j));
Ay(i,j) = (Vy(i+1,j) - Vy(i,j)) / (y(i+1,j) - y(i));
Vyp(i,j) = (Ay(i,j) * Yp(i,j) - y(i)) + Vy(i,j);
Ty(i,j) = (y(i) - Yp(i,j)) / Vy(i,j);
if Ty(i,j) < 0
  Ty(i,j) = -Ty(i,j)*(-1);
end
if Tx(i,j) < 0
  Tx(i,j) = -Tx(i,j)*(-1);
end
Tm(i,j) = min (Tx(i,j), Ty(i,j));
Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j)) * Tm(i,j)) - Vx(i,j)) * x(j);
Ye(i,j) = (1/Ay(i,j)) * Yp(i,j) * y(i);
Xe(i,j) = Xe(i,j) + Xe(i,j);  
Ye(i,j) = Ye(i,j) + Ye(i,j);
disp(’ - THIRD model’)

elseif (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)<Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)>0 & Vy(i,j+1)>0 & Vy(i+1,j+1)>0)

disp(’ - 3-3’)

Ax(i,j)=[Vx(i,j+1)-Vx(i,j)]/[(x[j+1]-x[j])]
Vxp(i,j)=[Ax(i,j)*Xp(i,j)-x(j)]*Vx(i,j);
Tx(i,j)=(1/Ax(i,j))*log(Vx(i,j+1)/Vx(i,j));
Ay(i,j)=[Vy(i+1,j)-Vy(i,j)]/[(y[i+1,j]-y[i,j])];
Vyp(i,j)=[Ay(i,j)*Yp(i,j)-y(j)]*Vy(i,j);

Xe(i,j)=(1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))+Vx(i,j))/x(j));
Ye(i,j)=(Tm(i,j)*Vxp(i,j))/y(j); Xe(i,j), Ye(i,j);

elseif (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)<Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)>0 & Vy(i,j+1)>0 & Vy(i+1,j+1)>0)

disp(’ - 4-3’)

Ax(i,j)=[Vx(i,j+1)-Vx(i,j)]/[(x[j+1]-x[j])]
Vxp(i,j)=[Ax(i,j)*Xp(i,j)-x(j)]*Vx(i,j);
Tx(i,j)=(1/Ax(i,j))*log(Vx(i,j+1)/Vx(i,j));
Ay(i,j)=[Vy(i+1,j)-Vy(i,j)]/[(y[i+1,j]-y[i,j])];
Vyp(i,j)=[Ay(i,j)*Yp(i,j)-y(j)]*Vy(i,j);

Xe(i,j)=(1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j))+Vy(i,j))/y(j));
Ye(i,j)=(Tm(i,j)*Vyp(i,j))/x(j); Xe(i,j), Ye(i,j);

elseif (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)<Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)>0 & Vy(i,j+1)>0 & Vy(i+1,j+1)>0)

disp(’ - 5-3’)

Ax(i,j)=[Vx(i,j+1)-Vx(i,j)]/[(x[j+1]-x[j])];
Vxp(i,j)=[Ax(i,j)*Xp(i,j)-x(j)]*Vx(i,j);
Tx(i,j)=(1/Ax(i,j))*log(Vx(i,j+1)/Vx(i,j));
Ay(i,j)=[Vy(i+1,j)-Vy(i,j)]/[(y[i+1,j]-y[i,j])];
Vyp(i,j)=[Ay(i,j)*Yp(i,j)-y(j)]*Vy(i,j);

Xe(i,j)=(1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j))+Vy(i,j))/y(j));
Ye(i,j)=(Tm(i,j)*Vyp(i,j))/x(j); Xe(i,j), Ye(i,j);

else

Ax(i,j)=[Vx(i,j+1)-Vx(i,j)];
Vxp(i,j)=[Ax(i,j)*Xp(i,j)-x(j)]*Vx(i,j);
Tx(i,j)=(1/Ax(i,j))*log(Vx(i,j+1)/Vx(i,j));
Ay(i,j)=[Vy(i+1,j)-Vy(i,j)];
Vyp(i,j)=[Ay(i,j)*Yp(i,j)-y(j)];

Xe(i,j)=(1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j))+Vy(i,j))/y(j));
Ye(i,j)=(Tm(i,j)*Vyp(i,j));

end

if Tx(i,j)<0
   Tx(i,j) = Tx(i,j)*(-1);
end
end

if Ty(i,j)<0
   Ty(i,j) = Ty(i,j)*(-1);
end

end

if Tm(i,j)<0
   Tm(i,j) = Tm(i,j)*(-1);
end

end

Xe(i,j)=(1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))/Vx(i,j));
Ye(i,j)=((Tm(i,j)*Vxp(i,j))/y(j)); Xe(i,j), Ye(i,j);

elseif (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)<Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)>0 & Vy(i,j+1)>0 & Vy(i+1,j+1)>0)

disp(’ - 3-3’)

Ax(i,j)=[Vx(i,j+1)-Vx(i,j)];
Vxp(i,j)=[Ax(i,j)*Xp(i,j)-x(j)]*Vx(i,j);
Tx(i,j)=(1/Ax(i,j))*log(Vx(i,j+1)/Vx(i,j));
Ay(i,j)=[Vy(i+1,j)-Vy(i,j)];
Vyp(i,j)=[Ay(i,j)*Yp(i,j)-y(j)];

Xe(i,j)=(1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j))+Vy(i,j))/y(j));
Ye(i,j)=(Tm(i,j)*Vyp(i,j));

end

end
Ay(i,j) = (Vy(i+1,j) - Vy(i,j)) / (y(i+1) - y(i));
Vyp(i,j) = Ay(i,j) * (Vy(i,j) - y(i)) + Vy(i,j);
Ty(i,j) = (y(i+1) - y(i)) / (Vy(i,j) - Vy(i+1,j));
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = min (Tx(i,j), Ty(i,j));

Xe(i,j) = (1 / Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) / x(j);% Ye(i,j) = (Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) / y(i);% Vyp(i,j) = Ay(i,j) * (Vy(i,j) - y(i)) + Vy(i,j);% if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = min (Tx(i,j), Ty(i,j));

Ax(i,j) = (Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (xp(i,j) - x(j))) + Vx(i,j);
Tx(i,j) = 1 / Ax(i,j) * log (Vx(i,j+1) / Vxp(i,j));
Ay(i,j) = (Vy(i+1,j) - Vy(i,j)) / (y(i+1) - y(i));
Vyp(i,j) = Ay(i,j) * (Vy(i,j) - y(i)) + Vy(i,j);% Ty(i,j) = (1 / Ay(i,j)) * log (Vy(i+1,j) / Vyp(i,j));
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = min (Tx(i,j), Ty(i,j));

Xe(i,j) = (1 / Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) / x(j);% Ye(i,j) = (Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) / y(i);% Vyp(i,j) = Ay(i,j) * (Vy(i,j) - y(i)) + Vy(i,j);% if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = min (Tx(i,j), Ty(i,j));

Xe(i,j) = (1 / Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) / x(j);% Ye(i,j) = (Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) / y(i);% Vyp(i,j) = Ay(i,j) * (Vy(i,j) - y(i)) + Vy(i,j);% if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = min (Tx(i,j), Ty(i,j));

% - third model
% else if (Vx(i,j) > 0 & Vx(i+1,j) > 0 & Vx(i,j+1) > 0 & Vy(i,j) == 0 & Vy(i+1,j) == 0) disp(' error');

Ax(i,j) = (Vx(i+1,j) - Vx(i,j)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (xp(i,j) - x(j))) + Vx(i,j);
Tx(i,j) = 1 / Ax(i,j) * log (Vx(i+1,j) / Vxp(i,j));

\begin{verbatim}
Tm(i,j) = Tx(i,j);
Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) + x(j);
Ye(i,j) = y(i);
Xe(i,j);
Ye(i,j);

disp(' - FORTH model')
  % first model
  elseif (Vx(i,j) > 0 & Vx(i,j+1)) < 0 &
    Vy(i,j) > 0 & Vy(i+1,j) < 0 &
    Vy(i,j) == Vy(i+1,j)
      disp(' - 1-4')
      Ay(i,j) = Vx(i,j) - Vx(i+1,j)
      Vy(i,j) / (y(i+1) - y(i))
      Vxp(i,j) = Ay(i,j) * (Vy(i,j) -
        y(i)) + Vy(i,j)
      Ty(i,j) = (1/Ay(i,j)) * log
        (Vy(i+1,j) / Vy(i,j))
      Ty(i,j) = y(i+1) -
        Vy(i,j) / Ty(i,j);
      if Ty(i,j) < 0
        Ty(i,j) = Ty(i,j) * (-1);
      end
      Tm(i,j) = Ty(i,j);
      Xe(i,j) = x(j);
      Ye(i,j) = ((Tm(i,j)) * Vxp(i,j)) * y(i);
      Xe(i,j);
      Ye(i,j);

  elseif (Vx(i,j) > 0 & Vx(i,j+1)) < 0 &
    Vy(i,j) > 0 & Vy(i+1,j) > 0 &
    Vy(i,j) < Vy(i+1,j)
      disp(' - 2-4')
      Ay(i,j) = (Vx(i+1,j) -
        Vx(i,j)) / (y(i+1) - y(i))
      Vyp(i,j) = (Ay(i,j)) * (Vy(i,j) -
        y(i)) + Vy(i,j)
      Ty(i,j) = (1/Ay(i,j)) * log
        (Vy(i+1,j) / Vy(i,j))
      if Ty(i,j) < 0
        Ty(i,j) = Ty(i,j) * (-1);
      end
      Tm(i,j) = Ty(i,j);
      Xe(i,j) = x(j);
      Ye(i,j) = ((Tm(i,j)) * Vxp(i,j)) * y(i);
      Xe(i,j);
      Ye(i,j);

  elseif (Vx(i,j) > 0 & Vx(i,j+1)) < 0 &
    Vy(i,j) > 0 & Vy(i+1,j) < 0 &
    Vy(i,j) < Vy(i+1,j)
      disp(' - 3-4')
      Ay(i,j) = (Vx(i+1,j) -
        Vx(i,j)) / (y(i+1) - y(i))
      Vyp(i,j) = (Ay(i,j)) * (Vy(i,j) -
        y(i)) + Vy(i,j)
      Ty(i,j) = (1/Ay(i,j)) * log
        (Vy(i+1,j) / Vy(i,j))
      Yp(i,j) = y(i);
      Xe(i,j) = x(j);
      Ye(i,j) = ((Tm(i,j)) * Vxp(i,j)) * y(i);
      Xe(i,j);
      Ye(i,j);

  elseif (Vx(i,j) > 0 & Vx(i,j+1)) < 0 &
    Vy(i,j) < Vy(i+1,j) > 0 &
    Vy(i,j) < Vy(i+1,j)
      disp(' - 5-4')
      Ay(i,j) = (Vx(i+1,j) -
        Vx(i,j)) / (y(i+1) - y(i))
      Vyp(i,j) = (Ay(i,j)) * (Vy(i,j) -
        y(i)) + Vy(i,j)
      Ty(i,j) = (y(i) - Vy(i,j)) / Vy(i,j);
      if Ty(i,j) < 0
        Ty(i,j) = Ty(i,j) * (-1);
      end
      Tm(i,j) = Ty(i,j);
      Xe(i,j) = x(j);
      Ye(i,j) = ((Tm(i,j)) * Vxp(i,j)) * y(i);
      Xe(i,j);
      Ye(i,j);

  elseif (Vx(i,j) > 0 & Vx(i,j+1)) < 0 &
    Vy(i,j) < Vy(i+1,j) > 0 &
    Vy(i,j) < Vy(i+1,j)
      disp(' - 6-4')
      Ay(i,j) = (Vx(i+1,j) -
        Vx(i,j)) / (y(i+1) - y(i))
      Vyp(i,j) = (Ay(i,j)) * (Vy(i,j) -
        y(i)) + Vy(i,j)
      Ty(i,j) = y(i);
      Xe(i,j) = x(j);
      if Vyp(i,j) < 0
        disp(' - 6-4-1')
        Yp(i,j) = y(i);
        Xe(i,j);
        Ye(i,j);
        else Vyp(i,j) > 0
          disp(' - 6-4-2')
          Xe(i,j) = x(j);
          Yp(i,j) = y(i+1);
          Xe(i,j);
          Ye(i,j);
          end
      end

end
\end{verbatim}
% Ty(i,j) = (1/Ay(i,j))*log
(Vy(i+1,j))/Vy(i,j); Ty(i,j) = (y(i)-
Yp(i,j))/Vy(i,j);
if Ty(i,j)<0
Ty(i,j) = Ty(i,j)*(-1);
end

Tm(i,j) = Ty(i,j);
Xe(i,j) = X(j);
Ye(i,j) = (1/Ay(i,j))*((Vy(i,j)*exp(Ay(i,j)*
Tm(i,j)))-Vy(i,j))*y(i);
Xe(i,j);
Ye(i,j);
elseif ((Vx(i,j)>0 & Vx(i,j+1)<0 &
Vy(i,j)<0 & Vy(i+1,j)< Vy(i+1,j))
\ disp('8-4'))

Ay(i,j) = (Vy(i+1,j)-
Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j) = Ay(i,j)*(Yp(i,j)-
y(i))+Vy(i,j);
Ty(i,j) = (1/Ay(i,j))*log
(Vy(i+1,j)/Vy(i,j));
if Ty(i,j)<0
Ty(i,j) = Ty(i,j)*(-1);
end

Tm(i,j) = Ty(i,j);
Xe(i,j) = X(j);
Ye(i,j) = (1/Ay(i,j))*((Vy(i,j)*exp(Ay(i,j)*
Tm(i,j)))-Vy(i,j))*y(i);
Xe(i,j);
Ye(i,j);
elseif (Vx(i,j)>0 & Vx(i,j+1)<0 &
Vy(i,j)<0 & Vy(i+1,j)< Vy(i+1,j))
\ disp('9-4')

Ay(i,j) = (Vy(i+1,j)-
Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j) = Ay(i,j)*(Yp(i,j)-
y(i))+Vy(i,j);
Ty(i,j) = (1/Ay(i,j))*log
(Vy(i+1,j)/Vy(i,j));
if Ty(i,j)<0
Ty(i,j) = Ty(i,j)*(-1);
end

Tm(i,j) = Ty(i,j);
Xe(i,j) = X(j);
Ye(i,j) = (1/Ay(i,j))*((Vy(i,j)*exp(Ay(i,j)*
Tm(i,j)))-Vy(i,j))*y(i);
Xe(i,j);
Ye(i,j);
elseif (Vx(i,j)>0 & Vx(i,j+1)<0 &
Vy(i,j)<0 & Vy(i+1,j)< Vy(i+1,j))
\ disp('10-4')

Ay(i,j) = (Vy(i+1,j)-
Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j) = Ay(i,j)*(Yp(i,j)-
y(i))+Vy(i,j);
\begin{verbatim}
% X(i,j)=x(j)+1
Xp(i,j)=x(j+1)

Ay(i,j)=(Vx(i+1,j)-Vx(i,j))/y(i+1)-y(i)
Vyp(i,j)=(Ay(i,j)*Vx(i,j)-y(i))*Vx(i,j)

if Ty(i,j)<0
   Ty(i,j)=Ty(i,j)*(-1)
end
if Tx(i,j)<0
   Tx(i,j)=Tx(i,j)*(-1)
end

TM(i,j)=min (Tx(i,j),Ty(i,j))

Xe(i,j)=(TM(i,j)*X(i,j))/x(j)

Vx(i,j)=Ax(i,j)*Vx(i,j)
Vy(i,j)=Ay(i,j)*Vy(i,j)

if Vx(i,j)>0 & Vx(i,j+1)==0 &
   Vy(i,j)>0 & Vy(i,j+1)>0 &
   Vy(i,j+1)>0
   disp('2-5')
   Ax(i,j)=(Vx(i+1,j)-Vx(i,j))/x(j+1)-x(j)
   Vxp(i,j)=(Ax(i,j)*Vx(i,j)-x(j))*Vx(i,j)
   Vxp(i,j)=Ax(i,j)*Vx(i,j)
   Ye(i,j)=(Ax(i,j)*Vx(i,j)-y(i))*Vx(i,j)
   Ty(i,j)=(1/Ay(i,j))*log(1+Vx(i+1,j)/Vx(i,j))
   if Ty(i,j)<0
      Ty(i,j)=Ty(i,j)*(-1)
   end
   if Tx(i,j)<0
      Tx(i,j)=Tx(i,j)*(-1)
   end
   TM(i,j)=min (Tx(i,j),Ty(i,j))
   Xe(i,j)=(TM(i,j)*X(i,j))/x(j)
end

else
   disp('1-4')
   X(i,j)=Ax(i,j)*X(i,j)
   Vxp(i,j)=Ax(i,j)*Vx(i,j)
   Ye(i,j)=(Ax(i,j)*Vx(i,j)-y(i))*Vx(i,j)
   Ay(i,j)=(Vx(i+1,j)-Vx(i,j))/y(i+1)-y(i)
   Vyp(i,j)=(Ay(i,j)*Vy(i,j)-x(j))*Vx(i,j)
   if Ty(i,j)<0
      Ty(i,j)=Ty(i,j)*(-1)
   end
   if Tx(i,j)<0
      Tx(i,j)=Tx(i,j)*(-1)
   end
   TM(i,j)=min (Tx(i,j),Ty(i,j))
   Xe(i,j)=(TM(i,j)*X(i,j))/x(j)
end

if Vx(i,j)>0 & Vx(i,j+1)==0 &
   Vy(i,j)>0 & Vy(i,j+1)==0 &
   Vy(i,j+1)<0
   disp('4-5')
   disp('I dont know')
rr=1
end
else
   disp('5-5')
   disp('I dont know')
rr=1
end

% - second model
else
   disp('3-5')
   Ax(i,j)=(Vx(i+1,j)-Vx(i,j))/x(j+1)-x(j)
   Vxp(i,j)=(Ax(i,j)*X(i,j)-x(j))*Vx(i,j)
   Xe(i,j)=(Ax(i,j)*Vx(i,j)-y(i))*Vx(i,j)
   Ay(i,j)=(Vx(i+1,j)-Vx(i,j))/y(i+1)-y(i)
   Vyp(i,j)=(Ay(i,j)*Vy(i,j)-x(j))*Vx(i,j)
   if Ty(i,j)<0
      Ty(i,j)=Ty(i,j)*(-1)
   end
   if Tx(i,j)<0
      Tx(i,j)=Tx(i,j)*(-1)
   end
   TM(i,j)=min (Tx(i,j),Ty(i,j))
   Xe(i,j)=(TM(i,j)*X(i,j))/x(j)
end

end
\end{verbatim}
Ye(i,j) = 0 & Vx(i,j+1) = 0 & Vy(i,j) < 0 &Vy(i+1,j) < 0 & Vy(i,j) > Vy(i+1,j)
    disp('8-5')
    Ax(i,j) = (Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j))
    Vxp(i,j) = (Ax(i,j) * (xp(i,j) - x(j))) + Vx(i,j)
    %
    Ty(i,j) = (1/Ay(i,j)) * log(Vy(i,j+1) / Vxp(i,j));
    Tx(i,j) = (x(j+1) - x(j)) / (Ay(i,j)) * log(Vy(i,j+1) / Vxp(i,j));
    Xp(i,j) = Vxp(i,j) / Vx(i,j);
    if Tx(i,j) < 0
        Tx(i,j) = Tx(i,j) * (-1);
    end
    X(i,j) = X(i,j) + Xp(i,j); + x(j);
    end
    Vxp(i,j) = (Ax(i,j) * (Vx(i,j+1) - Vx(i,j)) / x(j)) + x(j);
    Ye(i,j) = (Vx(i,j+1) - Vx(i,j)) * exp(Ax(i,j)) * exp(Ax(i,j) * (Vx(i,j+1) - Vx(i,j)) / x(j));
    %
    Vx(i,j) = (1/Ax(i,j)) * (Vxp(i,j)) + x(j);
    %
    Ye(i,j) = ((1/Ay(i,j)) * log(Vy(i,j+1) / Vy(i,j)) + y(i);
    if Vxp(i,j) < 0
        disp('6-5-1')
        Ye(i,j) = y(i);
        Xe(i,j) = x(j);
        Ye(i,j); + x(j);
    else
        Vxp(i,j) > 0
        disp('6-5-2')
        Ax(i,j) = Vx(i,j+1) - Vx(i,j) / (x(j+1) - x(j));
        Vxp(i,j) = (Ax(i,j) * (xp(i,j) - x(j))) + Vx(i,j);
        %
        Tx(i,j) = (1/Ax(i,j)) * log(Vx(i,j+1) / Vxp(i,j));
        Xp(i,j) = Vxp(i,j) / Vx(i,j);
        if Tx(i,j) < 0
            Xp(i,j) = Tx(i,j) * (-1);
            end
            Xe(i,j) = (TM(i,j) - X(i,j)) + x(j);
            Vxp(i,j) = (Ax(i,j) * (Vx(i,j+1) - Vx(i,j)) / x(j)) + x(j);
            Ye(i,j) = (Vx(i,j+1) - Vx(i,j)) * exp(Ax(i,j)) * exp(Ax(i,j) * (Vx(i,j+1) - Vx(i,j)) / x(j));
            %
            Ax(i,j) = (Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
            Vxp(i,j) = (Ax(i,j) * (xp(i,j) - x(j))) + Vx(i,j);
            %
            Tx(i,j) = (1/Ax(i,j)) * log(Vx(i,j+1) / Vxp(i,j));
            Xp(i,j) = Vxp(i,j) / Vx(i,j);
            if Tx(i,j) < 0
                Xp(i,j) = Tx(i,j) * (-1);
                end
                Xe(i,j) = (TM(i,j) - X(i,j)) + x(j);
                Vxp(i,j) = (Ax(i,j) * (Vx(i,j+1) - Vx(i,j)) / x(j)) + x(j);
                Ye(i,j) = (Vx(i,j+1) - Vx(i,j)) * exp(Ax(i,j)) * exp(Ax(i,j) * (Vx(i,j+1) - Vx(i,j)) / x(j));
                %
                Ax(i,j) = (Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
                Vxp(i,j) = (Ax(i,j) * (xp(i,j) - x(j))) + Vx(i,j);
                %
                Tx(i,j) = (1/Ax(i,j)) * log(Vx(i,j+1) / Vxp(i,j));
                Xp(i,j) = Vxp(i,j) / Vx(i,j);
                if Tx(i,j) < 0
                    Xp(i,j) = Tx(i,j) * (-1);
                    end
                    Xe(i,j) = (TM(i,j) - X(i,j)) + x(j);
                    Vxp(i,j) = (Ax(i,j) * (Vx(i,j+1) - Vx(i,j)) / x(j)) + x(j);
                    Ye(i,j) = (Vx(i,j+1) - Vx(i,j)) * exp(Ax(i,j)) * exp(Ax(i,j) * (Vx(i,j+1) - Vx(i,j)) / x(j));
                    %
                    Ax(i,j) = (Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
                    Vxp(i,j) = (Ax(i,j) * (xp(i,j) - x(j))) + Vx(i,j);
                    %
                    Tx(i,j) = (1/Ax(i,j)) * log(Vx(i,j+1) / Vxp(i,j));
                    Xp(i,j) = Vxp(i,j) / Vx(i,j);
                    if Tx(i,j) < 0
                        Xp(i,j) = Tx(i,j) * (-1);
                        end
                        Xe(i,j) = (TM(i,j) - X(i,j)) + x(j);
                        Vxp(i,j) = (Ax(i,j) * (Vx(i,j+1) - Vx(i,j)) / x(j)) + x(j);
                        Ye(i,j) = (Vx(i,j+1) - Vx(i,j)) * exp(Ax(i,j)) * exp(Ax(i,j) * (Vx(i,j+1) - Vx(i,j)) / x(j));
Ye(i,j) = (TM(i,j) * Vyp(i,j)) * y(i);
Xe(i,j);
Ye(i,j);
else if (Vx(i,j)>0 & Vx(i+1,j)>0 & Vy(i,j)<0 & Vy(i+1,j)<0)
    disp('10-5')
    A(i,j) = 1/(Vx(i,j)+Vx(i+1,j));
    Vxp(i,j) = (A(i,j) * Vxp(i,j) - x(i))) / Vx(i,j);
    if Tx(i,j)<0
        Tx(i,j) = (Vx(i+1,j) - x(i)) / x(i);
    else
        Tx(i,j) = (Vx(i,j) - x(i)) / x(i);
    end
end

% third model
else if (Vx(i,j)>0 & Vx(i+1,j)>0 & Vy(i,j)<0 & Vy(i+1,j)<0)
    disp('11-5')
    A(i,j) = 1/(Vx(i,j)+Vx(i+1,j));
    Vxp(i,j) = (A(i,j) * Vxp(i,j) - x(i))) / Vx(i,j);
    if Tx(i,j)<0
        Tx(i,j) = (Vx(i+1,j) - x(i)) / x(i);
    else
        Tx(i,j) = (Vx(i,j) - x(i)) / x(i);
    end
end

% sixth model
else if (Vx(i,j)<0 & Vx(i+1,j)<0 & Vy(i,j)>0 & Vy(i+1,j)>0)
    disp('13-5')
    % Y(i,j) = Y(i+1,j) -
    Xp(i,j) = (Vx(i,j) - x(i)) / x(i);
    if Ty(i,j)<0
        Ty(i,j) = (Vy(i+1,j) - y(i)) / y(i);
    else
        Ty(i,j) = (Vy(i,j) - y(i)) / y(i);
    end
end

% fourth model
else if (Vx(i,j)<0 & Vx(i+1,j)<0 & Vy(i,j)<0 & Vy(i+1,j)<0)
    disp('12-5')
    A(i,j) = 1/(Vx(i,j)+Vx(i+1,j));
    Vxp(i,j) = (A(i,j) * Vxp(i,j) - x(i))) / Vx(i,j);
    if Tx(i,j)<0
        Tx(i,j) = (Vx(i+1,j) - x(i)) / x(i);
    else
        Tx(i,j) = (Vx(i,j) - x(i)) / x(i);
    end
end
if ( Vxp[i,j] > 0)
    disp( '-1-6-1' )
    Ay(i,j) = (Vy(i+1,j) - 
        Vy(i,j) / (y(i+1) - y(i)) ;
    Vyp(i,j) = (Ay(i,j) * (Vp(i,j) - 
        y(i))) + Vy(i,j) ;
    if Ty(i,j) < 0
        Ty(i,j) = Ty(i,j) * (-1) ;
    end
    Tm(i,j) = Ty(i,j) ;
    Xe(i,j) = x(j+1) ;
    Ye(i,j) = (Tm(i,j) * 
    Vxp(i,j)) * y(i) ;
    Xe(i,j) ;
    Ye(i,j) ;
    else if ( Vxp[i,j] > 0 )
        disp( '-1-6-2' )
        Ay(i,j) = (Vy(i+1,j) - 
            Vy(i,j) / (y(i+1) - y(i)) ;
        Vyp(i,j) = (Ay(i,j) * (Vp(i,j) - 
            y(i))) + Vy(i,j) ;
        Ty(i,j) = (1/Ay(i,j)) * log
            (Vyp(i,j) / Vyp(i,j)) ;
        if Ty(i,j) < 0
            Ty(i,j) = Ty(i,j) * (-1) ;
        end
        Tm(i,j) = Ty(i,j) ;
        Xe(i,j) = x(j+1) ;
        Ye(i,j) = (Tm(i,j) * 
        Vxp(i,j)) * y(i) ;
    Xe(i,j) ;
    Ye(i,j) ;
    else if ( Vxp[i,j] < 0 )
        disp( '-3-6-1' )
        Ax(i,j) = (Vx(i+1,j) - 
            Vx(i,j) / (x(i+1) - x(i)) ;
        Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - 
            x(j))) + Vx(i,j) ;
        Ay(i,j) = (Vy(i+1,j) - 
            Vy(i,j) / (y(i+1) - y(i)) ;
        Vyp(i,j) = (Ay(i,j) * (Vp(i,j) - 
            y(i))) + Vy(i,j) ;
        Ty(i,j) = (1/Ay(i,j)) * log
            (Vyp(i,j) / Vyp(i,j)) ;
        if Ty(i,j) < 0
            Ty(i,j) = Ty(i,j) * (-1) ;
        end
        Tm(i,j) = Ty(i,j) ;
        if ( Vxp[i,j] < 0 )
            disp( '-3-6-1' )
            Xe(i,j) = x(j) ;
    Ye(i,j) = (Tm(i,j) * 
    Vxp(i,j)) * y(i) ;
    Xe(i,j) ;
    Ye(i,j) ;
    else if ( Vxp[i,j] > 0 )
        disp( '-3-6-2' )
        Ay(i,j) = (Vy(i+1,j) - 
            Vy(i,j) / (y(i+1) - y(i)) ;
        Vyp(i,j) = (Ay(i,j) * (Vp(i,j) - 
            y(i))) + Vy(i,j) ;
        Ty(i,j) = (1/Ay(i,j)) * log
            (Vyp(i,j) / Vyp(i,j)) ;
        if Ty(i,j) < 0
            Ty(i,j) = Ty(i,j) * (-1) ;
        end
        Tm(i,j) = Ty(i,j) ;
        if ( Vxp[i,j] > 0 )
            disp( '-3-6-2' )
            Xe(i,j) = x(j) ;
    Ye(i,j) = (Tm(i,j) * 
    Vxp(i,j)) * y(i) ;
    Xe(i,j) ;
    Ye(i,j) ;
    else if ( Vxp[i,j] < 0 )
        disp( '-4-6-1' )
        Ax(i,j) = (Vx(i+1,j) - 
            Vx(i,j) / (x(i+1) - x(i)) ;
        Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - 
            x(j))) + Vx(i,j) ;
    Xe(i,j) ;
    Ye(i,j) ;
    else if ( Vxp[i,j] > 0 )
        disp( '-4-6-1' )
        Ax(i,j) = (Vx(i+1,j) - 
            Vx(i,j) / (x(i+1) - x(i)) ;
        Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - 
            x(j))) + Vx(i,j) ;
    Xe(i,j) ;
    Ye(i,j) ;
if \( \text{Vxp}(i,j) < 0 \) 
\hspace{1cm} \text{disp}('-4-6-1') 
\hspace{1cm} \text{Xe}(i,j) = x(j); 
\hspace{1cm} \text{Ye}(i,j) = y(i); 
\hspace{1cm} \text{Xe}(i,j); 
\hspace{1cm} \text{Ye}(i,j); 
\hspace{1cm} \text{Ye}(i,j); 
\hspace{1cm} \text{end} 

elseif \( \text{Vxp}(i,j) > 0 \) 
\hspace{1cm} \text{disp}('-8-6-2') 
\hspace{1cm} \text{Axi}(j) = \text{Vxp}(i,j); 
\hspace{1cm} \text{Ax}(i,j) = \text{Ax}(i,j) + \text{Vxp}(i,j); 
\hspace{1cm} \text{Xp}(i,j) = \text{Ax}(i,j) * x(j); 
\hspace{1cm} \text{Ye}(i,j) = y(i); 
\hspace{1cm} \text{Xe}(i,j); 
\hspace{1cm} \text{Ye}(i,j); 
\hspace{1cm} \text{Ye}(i,j); 
\hspace{1cm} \text{end} 

elseif \{ \text{Vx}(i,j) < 0 \ & \ \text{Vx}(i,j+1) > 0 \ & \ \text{Vy}(i,j) > 0 \ & \ \text{Vy}(i+1,j) = 0 \} 
\hspace{1cm} \text{disp}('-5-6-1') 
\hspace{1cm} \text{Ax}(i,j) = \text{Vx}(i,j+1) - \text{Vx}(i,j); 
\hspace{1cm} \text{Vxp}(i,j) = \text{Ax}(i,j) * (x(j+1) - x(j)); 
\hspace{1cm} \text{if} \ ( \text{Vxp}(i,j) < 0 \) 
\hspace{2cm} \text{disp}('-5-6-1') 
\hspace{2cm} \text{end} 

elseif \( \text{Vx}(i,j) > 0 \ & \ \text{Vx}(i,j+1) > 0 \ & \ \text{Vy}(i,j) > 0 \ & \ \text{Vy}(i+1,j) < 0 \) 
\hspace{1cm} \text{disp}('-7-6-1') 
\hspace{1cm} \text{if} \ \text{Vxp}(i,j) < 0 
\hspace{2cm} \text{disp}('-7-6-1') 
\hspace{2cm} \text{Ty}(i,j) = (1 / \text{Ax}(i,j)) * \text{log} \ (\text{Vx}(i,j+1)) / \text{Vxp}(i,j); 
\hspace{2cm} \text{Ye}(i,j) = (\text{Vy}(i+1,j) - \text{Vy}(i,j)) / \text{Vyp}(i,j); 
\hspace{2cm} \text{Ty}(i,j) = (\text{y}(i+1) - \text{y}(i)) / \text{Vy}(i,j); 
\hspace{2cm} \text{if} \ \text{Ty}(i,j) < 0 
\hspace{3cm} \text{Ty}(i,j) = \text{Ty}(i,j)^{-1}; 
\hspace{2cm} \text{end} 
\hspace{1cm} \text{end} 
\hspace{1cm} \text{end} 

elseif \( \text{Vx}(i,j) < 0 \ & \ \text{Vx}(i,j+1) > 0 \ & \ \text{Vy}(i,j) < 0 \ & \ \text{Vy}(i+1,j) < 0 \ & \ \text{Vy}(i+1,j) < 0 \) 
\hspace{1cm} \text{disp}('-7-6-1') 
\hspace{1cm} \text{if} \ \text{Vxp}(i,j) < 0 
\hspace{2cm} \text{disp}('-7-6-1') 
\hspace{2cm} \text{Ty}(i,j) = (1 / \text{Ax}(i,j)) * \text{log} \ (\text{Vy}(i+1,j)) / \text{Vyp}(i,j); 
\hspace{2cm} \text{Ty}(i,j) = (\text{y}(i+1) - \text{y}(i)) / \text{Vy}(i,j); 
\hspace{2cm} \text{if} \ \text{Ty}(i,j) < 0 
\hspace{3cm} \text{Ty}(i,j) = \text{Ty}(i,j)^{-1}; 
\hspace{2cm} \text{end} 
\hspace{1cm} \text{end} 
\hspace{1cm} \text{end} 

% - second model 
if \( \text{Vx}(i,j) < 0 \ & \ \text{Vx}(i,j+1) > 0 \) 
\hspace{1cm} \text{disp}('-8-6') 
\hspace{1cm} \text{if} \ \text{Vxp}(i,j) < 0 
\hspace{2cm} \text{disp}('-8-6') 
\hspace{2cm} \text{Ye}(i,j) = y(i); 
\hspace{2cm} \text{else} \ \text{Vyp}(i,j) > 0 
\hspace{3cm} \text{disp}('-8-6') 
\hspace{3cm} \text{Ye}(i,j) = y(i+1); 
\hspace{2cm} \text{end} 
\hspace{1cm} \text{end} 
\hspace{1cm} \text{end}
if Vxp(i,j)<0
    disp('"-8-6-1"
    Ay(i,j)= (Vy(i+1,j)-
    Vy(i,j))/(y(i+1)-y(i));
    Vyp(i,j)=Ay(i,j)*(Yp(i,j)-
    y(i))+Vy(i,j);
    Ty(i,j)=1/Ay(i,j)*log
    (Vy(i+1,j)/Vyp(i,j));
    if Ty(i,j)<0
        Ty(i,j)=Ty(i,j)*(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(j);

    Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*
    Tm(i,j))))-Vy(i,j))*y(i);
    Xe(i,j);
    Ye(i,j);
else
    Vxp(i,j)>0
    disp('"-8-6-2"
    Ay(i,j)= (Vy(i+1,j)-
    Vy(i,j))/(y(i+1)-y(i));
    Vyp(i,j)=Ay(i,j)*(Yp(i,j)-
    y(i))+Vy(i,j);
    Ty(i,j)=1/Ay(i,j)*log
    (Vy(i+1,j)/Vyp(i,j));
    if Ty(i,j)<0
        Ty(i,j)=Ty(i,j)*(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(j+1);

    Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*
    Tm(i,j))))-Vy(i,j))*y(i);
    Xe(i,j);
    Ye(i,j);
else
    (Vx(i,j)<0 & Vx(i,j+1)>0 &
    Vy(i,j)<0 & Vy(i+1,j)<0 & Vy(i,j)==
    Vy(i+1,j))
    disp('"-9-6"
    if Vxp(i,j)<0
        disp('"-9-6-1"
        Ay(i,j)= (Vy(i+1,j)-
        Vy(i,j))/(y(i+1)-y(i));
        Vyp(i,j)=Ay(i,j)*(Yp(i,j)-
        y(i))+Vy(i,j);
        Ty(i,j)=(y(i)-
        Yp(i,j))/Vyp(i,j);
        if Ty(i,j)<0
            Ty(i,j)=Ty(i,j)*(-1);
        end
        Tm(i,j)=Ty(i,j);
        Xe(i,j)=x(j);
        Ye(i,j)=Tm(i,j)*
        Vyp(i,j)+y(i);
        Xe(i,j);
        Ye(i,j);
else
    Vxp(i,j)>0
        disp('"-9-6-2"
        Ay(i,j)= (Vy(i+1,j)-
        Vy(i,j))/(y(i+1)-y(i));
        Vyp(i,j)=Ay(i,j)*(Yp(i,j)-
        y(i))+Vy(i,j);
        Ty(i,j)=1/Ay(i,j)*log
        (Vy(i+1,j)/Vyp(i,j));
        if Ty(i,j)<0
            Ty(i,j)=Ty(i,j)*(-1);
        end
        Tm(i,j)=Ty(i,j);
        Xe(i,j)=x(j+1);
% third model
  elseif (Vx(i,j)<0 & Vx(i,j+1)>0 &Vy(i,j)==0 & Vy(i+1,j)>0)
    disp(' -11-6-6')
    Ax(i,j)=(Vx(i,j+1)-
    Vx(i,j))/x(j+1)-x(j));
    Vxp(i,j)=Ax(i,j)*Xp(i,j)-
    x(j));+Vx(i,j);
    if Vxp(i,j)<0
      disp(' -11-6-1')
      Ay(i,j)=Vy(i+1,j)-
      Vyi(i,j)/yi(i)-y(i));
      Vyp(i,j)=Ay(i,j)*Yp(i,j)-
      y(i));+Vy(i,j);
      Ty(i,j)=(Ay(i,j)*log
      (Vy(i+1,j)/Vy(i,j)));
      if Ty(i,j)<0
        Ty(i,j) =Ty(i,j)*(-1);
      end
      Tm(i,j)=Ty(i,j);
      Xe(i,j)=x(j);
      Ye(i,j);
    else Vxp(i,j)>0
      disp(' -11-6-2')
      Ay(i,j)=Vy(i+1,j)-
      Vyi(i,j)/yi(i)-y(i));
      Vyp(i,j)=Ay(i,j)*Yp(i,j)-
      y(i));+Vy(i,j);
      Ty(i,j)=(Ay(i,j)*log
      (Vy(i+1,j)/Vy(i,j)));
      if Ty(i,j)<0
        Ty(i,j) =Ty(i,j)*(-1);
      end
      Tm(i,j)=Ty(i,j);
      Xe(i,j)=x(j+1);
      Ye(i,j);
    end
    Ye(i,j);
  elseif (Vx(i,j)<0 &
  Vx(i,j+1)>0 & Vy(i,j)==0 & Vy(i+1,j)==0)
    disp(' -13-6')
    Ax(i,j)=(Vx(i,j+1)-
    Vx(i,j))/x(j+1)-x(j));
    Vxp(i,j)=Ax(i,j)*Xp(i,j)-
    x(j));+Vx(i,j);
    if Vxp(i,j)<0
      disp(' -12-6-1')
      Tx(i,j)=(-1/Ax(i,j))*log
      (Vx(i,j+1)/Vxp(i,j))
      if Ty(i,j)<0
        Ty(i,j) =Ty(i,j)*(-1);
      end
      Tm(i,j)=Ty(i,j);
      Xe(i,j)=x(j+1);
      Ye(i,j);
    else Vxp(i,j)>0
      disp(' -13-6-2')
      Ax(i,j)=(Vx(i,j+1)-
      Vx(i,j))/x(j+1)-x(j));
      Vxp(i,j)=Ax(i,j)*Xp(i,j)-
      x(j));+Vx(i,j);
      if Vxp(i,j)<0
        disp(' -12-6-1')
        Ay(i,j)=Vy(i+1,j)-
        Vyi(i,j)/yi(i)-y(i));
        Vyp(i,j)=Ay(i,j)*Yp(i,j)-
        y(i));+Vy(i,j);
        Ty(i,j)=(Ay(i,j)*log
        (Vy(i+1,j)/Vy(i,j)));
        if Ty(i,j)<0
          Ty(i,j) =Ty(i,j)*(-1);
        end
        Tm(i,j)=Ty(i,j);
        Xe(i,j)=x(j);
        Ye(i,j);
      end
      Ye(i,j);
end

% first model

Vxp(i,j)=Vxp(i,j+1)
Vxp(i,j)=Vxp(i,j)

Ax(i,j)=Ax(i,j+1)
Ax(i,j)=Ax(i,j+1)

Xp(i,j)=Xp(i,j)
Xp(i,j)=Xp(i,j)

Ay(i,j)=Ay(i,j+1)
Ay(i,j)=Ay(i,j+1)

Vy(i,j)=Vy(i,j+1)
Vy(i,j)=Vy(i,j+1)

Tm(i,j)=Tm(i,j+1)
Tm(i,j)=Tm(i,j+1)

Xe(i,j)=x(j+1)
Xe(i,j)=x(j+1)

Ye(i,j)=Ye(i,j+1)
Ye(i,j)=Ye(i,j+1)
else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j)==Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)>0 & Vy(i+1,j)<Vy(i,j))
disp('4-7')
Ax(i,j)=(Vx(i,j+1)-Vx(i,j))
Vx(i,j)/[(x[j+1]-x[j])]
Vxp(i,j)=[Ax(i,j)*(x[p(i,j)]-
% dx(j)])+Vx(i,j)]
% Tx(i,j)=1/Ax(i,j)*log
(Vx(i,j+1)/Vxp(i,j))
Tx(i,j)=(x[j+1]-x[j])
% Ay(i,j)=Vy(i+1,j)-
Vy(i,j)/[(y[i+1]-y[i])]
Vyp(i,j)=[Ay(i,j)*(y[p(i,j)]-
% dy(i)])+Vy(i,j)]
% Ty(i,j)=1/Ay(i,j)*log
(Vy(i+1,j)/Vyp(i,j))
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1)
end
if Tx(i,j)<0
 Tx(i,j)=Tx(i,j)*(-1)
end
tm(i,j)=min(Tx(i,j),Ty(i,j))
Xe(i,j)=Xm(i,j)*
Vxp(i,j)+X(j)
Ye(i,j)=Ym(i,j)+
Vxp(i,j)+Y(j)
Xe(i,j)
Ye(i,j)

else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j)==Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)>0 & Vy(i+1,j)<Vy(i,j))
disp('4-7')
Ax(i,j)=(Vx(i,j+1)-Vx(i,j))
Vx(i,j)/[(x[j+1]-x[j])]
Vxp(i,j)=[Ax(i,j)*(x[p(i,j)]-
% dx(j)])+Vx(i,j)]
% Tx(i,j)=1/Ax(i,j)*log
(Vx(i,j+1)/Vxp(i,j))
Tx(i,j)=(x[j+1]-x[j])
% Ay(i,j)=Vy(i+1,j)-
Vy(i,j)/[(y[i+1]-y[i])]
Vyp(i,j)=[Ay(i,j)*(y[p(i,j)]-
% dy(i)])+Vy(i,j)]
% Ty(i,j)=1/Ay(i,j)*log
(Vy(i+1,j)/Vyp(i,j))
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1)
end
if Tx(i,j)<0,
 Tx(i,j)=Tx(i,j)*(-1)
end
tm(i,j)=min(Tx(i,j),Ty(i,j))
Xe(i,j)=Xm(i,j)*
Vxp(i,j)+X(j)
Ye(i,j)=Ym(i,j)+
Vxp(i,j)+Y(j)
Xe(i,j)
Ye(i,j)

else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j)==Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)>0 & Vy(i+1,j)<Vy(i,j))
disp('4-7')
Ax(i,j)=(Vx(i,j+1)-Vx(i,j))
Vx(i,j)/[(x[j+1]-x[j])]
Vxp(i,j)=[Ax(i,j)*(x[p(i,j)]-
% dx(j)])+Vx(i,j)]
% Tx(i,j)=1/Ax(i,j)*log
(Vx(i,j+1)/Vxp(i,j))
Tx(i,j)=(x[j+1]-x[j])
% Ay(i,j)=Vy(i+1,j)-
Vy(i,j)/[(y[i+1]-y[i])]
Vyp(i,j)=[Ay(i,j)*(y[p(i,j)]-
% dy(i)])+Vy(i,j)]
% Ty(i,j)=1/Ay(i,j)*log
(Vy(i+1,j)/Vyp(i,j))
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1)
end
if Tx(i,j)<0,
 Tx(i,j)=Tx(i,j)*(-1)
end
tm(i,j)=min(Tx(i,j),Ty(i,j))
Xe(i,j)=Xm(i,j)*
Vxp(i,j)+X(j)
Ye(i,j)=Ym(i,j)+
Vxp(i,j)+Y(j)
Xe(i,j)
Ye(i,j)

else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j)==Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)>0 & Vy(i+1,j)<Vy(i,j))
disp('4-7')
Ax(i,j)=(Vx(i,j+1)-Vx(i,j))
Vx(i,j)/[(x[j+1]-x[j])]
Vxp(i,j)=[Ax(i,j)*(x[p(i,j)]-
% dx(j)])+Vx(i,j)]
% Tx(i,j)=1/Ax(i,j)*log
(Vx(i,j+1)/Vxp(i,j))
Tx(i,j)=(x[j+1]-x[j])
% Ay(i,j)=Vy(i+1,j)-
Vy(i,j)/[(y[i+1]-y[i])]
Vyp(i,j)=[Ay(i,j)*(y[p(i,j)]-
% dy(i)])+Vy(i,j)]
% Ty(i,j)=1/Ay(i,j)*log
(Vy(i+1,j)/Vyp(i,j))
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1)
end
if Tx(i,j)<0,
 Tx(i,j)=Tx(i,j)*(-1)
end
tm(i,j)=min(Tx(i,j),Ty(i,j))
Xe(i,j)=Xm(i,j)*
Vxp(i,j)+X(j)
Ye(i,j)=Ym(i,j)+
Vxp(i,j)+Y(j)
Xe(i,j)
Ye(i,j)
\[
Ty(i,j) = (y(i) - Yp(i,j))/Vy(i,j);
\]
\[
\text{if } Ty(i,j) < 0 \quad Ty(i,j) = Ty(i,j)*(-1);
\]
end
\[
\text{if } Tx(i,j) < 0 \quad Tx(i,j) = Tx(i,j)*(-1);
\]
end
\[
Tm(i,j) = \text{min} \{Tx(i,j),Ty(i,j)\};
\]
\[
Vxp(i,j) = x(j);
\]
\[
Vyp(i,j) = (1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))/Vy(i,j)) + y(i);
\]
\[
Xe(i,j) = x(j);
\]
\[
Ye(i,j) = y(i);
\]
\[
\text{if } Ty(i,j) < 0 \quad Ty(i,j) = Ty(i,j)*(-1);
\]
end
\[
\text{if } Tx(i,j) < 0 \quad Tx(i,j) = Tx(i,j)*(-1);
\]
end
\[
\text{if } (Vx(i,j) < 0 \& \& Vx(i,j+1) < 0) \& \& Vy(i,j) < 0 \& \& Vy(i,j+1) < 0
\]
\[
disp(' -7-7 ');
\]
\[
Ax(i,j) = (Vx(i,j) + 1)*Vx(i,j)/((Vx(i,j) + 1) - Vx(i,j));
\]
\[
Vxp(i,j) = Ax(i,j)*(Xp(i,j) - x(j));
\]
\[
Tx(i,j) = (1/Ax(i,j))*log((Vx(i,j+1)/Vxp(i,j));
\]
\[
Tm(i,j) = (Tx(i,j) - Xp(i,j))/Vxp(i,j);
\]
\[
Ay(i,j) = (Vy(i,j+1) - Vy(i,j))/y(i);
\]
\[
Vxp(i,j) = Ax(i,j)*(Vyp(i,j) - y(i));
\]
\[
Vxp(i,j) = Vxp(i,j); \quad Vxp(i,j) = x(j);
\]
\[
Ye(i,j) = (1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))/Vy(i,j)) + y(i);
\]
\[
Xe(i,j) = x(j);
\]
\[
Ye(i,j) = y(i);
\]
\[
\text{else } Vyp(i,j) > 0 \quad disp(' -6-7-2 ');
\]
\[
Ax(i,j) = (Vx(i,j) + 1)*Vx(i,j)/((Vx(i,j) + 1) - x(j));
\]
\[
Vxp(i,j) = Ax(i,j)*(Xp(i,j) - x(j));
\]
\[
Tx(i,j) = (1/Ax(i,j))*log((Vx(i,j+1)/Vxp(i,j));
\]
\[
Tm(i,j) = (Tx(i,j) - Xp(i,j))/Vxp(i,j);
\]
\[
Ay(i,j) = (Vy(i,j+1) - Vy(i,j))/y(i);
\]
\[
Vxp(i,j) = Ax(i,j)*(Vyp(i,j) - y(i));
\]
\[
Ty(i,j) = Ty(i,j)*(-1);
\]
end
\[
\text{if } Ty(i,j) < 0 \quad Ty(i,j) = Ty(i,j)*(-1);
\]
end
\[
\text{if } Tx(i,j) < 0 \quad Tx(i,j) = Tx(i,j)*(-1);
\]
end
\[
\text{if } (Vx(i,j) < 0 \& \& Vx(i,j+1) < 0 \& \& Vy(i,j) < 0 \& \& Vy(i,j+1) < 0
\]
\[
disp(' -7-7 ');
\]
\[
Ax(i,j) = (Vx(i,j) + 1)*Vx(i,j)/((Vx(i,j) + 1) - Vx(i,j));
\]
\[
Vxp(i,j) = Ax(i,j)*(Xp(i,j) - x(j));
\]
\[
Tx(i,j) = (1/Ax(i,j))*log((Vx(i,j+1)/Vxp(i,j));
\]
\[
Tm(i,j) = (Tx(i,j) - Xp(i,j))/Vxp(i,j);
\]
\[
Ay(i,j) = (Vy(i,j+1) - Vy(i,j))/y(i);
\]
\[
Vxp(i,j) = Ax(i,j)*(Vyp(i,j) - y(i));
\]
\[
Ty(i,j) = Ty(i,j)*(-1);
\]
end
\[
\text{if } Ty(i,j) < 0 \quad Ty(i,j) = Ty(i,j)*(-1);
\]
end
\[
\text{if } Tx(i,j) < 0 \quad Tx(i,j) = Tx(i,j)*(-1);
\]
end
\[
Tm(i,j) = \text{min} \{Tx(i,j),Ty(i,j)\};
\]
\[
Xe(i,j) = x(j);
\]
\[
Ye(i,j) = y(i);
\]
\[
\text{else } Vyp(i,j) > 0 \quad disp(' -6-7-2 ');
\]
\[
Ax(i,j) = (Vx(i,j) + 1)*Vx(i,j)/((Vx(i,j) + 1) - x(j));
\]
\[
Vxp(i,j) = Ax(i,j)*(Xp(i,j) - x(j));
\]
\[
Tx(i,j) = (1/Ax(i,j))*log((Vx(i,j+1)/Vxp(i,j));
\]
\[
Tm(i,j) = (Tx(i,j) - Xp(i,j))/Vxp(i,j);
\]
Ye(i,j) = ((1/Ay(i,j)) * (Vy(i,j) * exp(Ay(i,j) * Tm(i,j)) - Vy(i,j))) + y(i,j);
    Xe(i,j); 
    Ye(i,j);

elseif (Vx(i,j) < 0 & Vx(i,j+1) < 0 & Vy(i,j+1) > 0 & Vy(i,j) < 0)
...

elseif (Vx(i,j) < 0 & Vx(i,j+1) > 0 & Vy(i,j+1) == 0 & Vx(i,j) < 0)

% third model
elseif (Vx(i,j) < 0 & Vx(i,j+1) < 0 & Vy(i,j+1) >= 0 & Vy(i,j) < 0)
...
else
  (Vx(i,j)<0 &
   Vx(i,j+1)>Vx(i,j+1) &
   Vy(i,j)>0 &
   Vy(i+1,j)==0)
  disp(' -6-8-2 ')
  Ax(i,j)=(Vx(i,j+1)-
            Vx(i,j))/(x[i+1]-x[i]);
  Vp[i,j]=Ax(i,j)*xp[i,j]-
            Vx(i,j); Ax(i,j)=Ax(i,j)*xp[i,j];
  Tx(i,j)=1/Ax(i,j)*log
          (Vx(i,j+1)/Vx(i,j));
  Ty(i,j)=(y[i+1]-y[i])/Vy(i,j);
  Vyp[i,j]=Ay(i,j)*y[i]-
            Vyp[i,j];
  Tx(i,j+1)=1/Ay(i,j)*log
            (Vy[i+1,j]/Vy[i,j]);
  Ty(i,j)=(y[i+1]-y[i])/Vy(i,j);
  Vyp[i,j+1]=Ay(i,j)*y[i]-
            Vyp[i,j+1];
  if Tx(i,j)<0
    Tx(i,j)=Tx(i,j)*(-1); end
  if Tx(i,j)>0
    Tx(i,j)=Tx(i,j); end
  else
    (Vx(i,j)<0 &
     Vx(i,j+1)<0 &
     Vx(i,j)>Vx(i,j+1) &
     Vy(i,j)<0 &
     Vy(i+1,j)<0 &
     Vy(i,j+1)>0 &
     Vy(i+1,j+1)<0)
    disp(' -6-8-1 ')
    Ax(i,j)=(Vx(i,j)-
            Vx(i,j+1))/(x[i+1]-x[i]);
    Vp[i,j]=Ax(i,j)*xp[i,j]-
            Vx[i,j];
    Tx(i,j)=1/Ax(i,j)*log
            (Vx(i,j)/Vx[i,j+1]);
    Ty(i,j)=(y[i+1]-y[i])/Vy[i,j];
    Vyp[i,j]=Ay[i,j]*y[i]-
            Vyp[i,j];
    if Vy[i,j]<0
      Vy[i,j]=Vy[i,j]*(-1);
    elseif (Vy[i,j]<0 &
              Vy[i,j+1]<0 &
              Vy[i,j]>Vy[i,j+1] &
              Vy[i+1,j]<0 &
              Vy[i+1,j]>Vy[i,j+1] &
              Vy[i,j+1]<0 &
              Vy[i+1,j+1]<0)
      disp(' -8-8 ')
      Ax(i,j)=(Vx[i,j+1]-
                Vx(i,j))/(x[i+1]-x[i]);
      Vp[i,j]=Ax[i,j]*xp[i,j]-
                Vx[i,j];
      Tx[i,j]=1/Ax[i,j]*log
                (Vx[i,j+1]/Vx[i,j]);
      Ty[i,j]=(y[i+1]-y[i])/Vy[i,j];
      Vyp[i,j]=Ay[i,j]*y[i]-
                Vyp[i,j];
      if Vy[i,j]<0
        Vy[i,j]=Vy[i,j]*(-1); end
      if Tx[i,j]<0
        Tx[i,j]=Tx[i,j]*(-1); end
    end
  end
end

Tx(i,j)=Tx(i,j)*(-1); end

Xe(i,j)=(1/Ax(i,j))*((Vxp[i,j]*exp(Ax(i,j)*
                        Tm(i,j))))-Vx(i,j));
Xe(i,j)=Xe(i,j)*x[i];
Ye(i,j)=y[i];
Xe(i,j)=Xe(i,j)*x[i];
Ye(i,j)=y[i];

else
  Vyp[i,j]>0
  disp(' -6-8-2 ')
  Ax(i,j)=(Vx(i,j+1)-
            Vx(i,j))/(x[i+1]-x[i]);
  Vxp[i,j]=Ax(i,j)*(xp[i,j]-
                        x[i]));
  Tx(i,j)=1/Ax(i,j)*log
          (Vx(i,j+1)/Vxp[i,j]);
  if Tx(i,j)<0
    Tx(i,j)=Tx(i,j)*(-1); end
  if Tx(i,j)>0
    Tx(i,j)=Tx(i,j); end
  else
    (Vx(i,j)<0 &
     Vx(i,j+1)<0 &
     Vx(i,j)>Vx(i,j+1) &
     Vy(i,j)>0 &
     Vy(i+1,j)<0 &
     Vy(i,j+1)<0 &
     Vy(i+1,j+1)<0)
    disp(' -8-8 ')
Ax(i,j) = (Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j)]
Vxp(i,j) = (Ax(i,j)*Xp(i,j)-x(j)])
Vx(i,j) = Vxp(i,j);
Tx(i,j) = (1/Ax(i,j))^log(Vx(i+1,j)/Vxp(i,j));
Ay(i,j) = (Vyi(i,j)-Vyi(j-1)/yi(i)-yi(j-1))
Vyi(i,j) = (Ay(i,j)*Yp(i,j)-y(j)])
Vpi(i,j) = (Ay(i,j)*Yp(i,j)-y(j)])+Vy(i,j);
Ty(i,j) = (1/Ay(i,j))^log(Vy(i+1,j)/Vyi(i,j))
if Ty(i,j)<0
Ty(i,j) = Ty(i,j)-(-1);
end
if Tx(i,j)<0
Tx(i,j) = Tx(i,j)-(-1);
end
Tm(i,j) = min(Tx(i,j),Ty(i,j));
Xe(i,j) = (1/Ay(i,j))*((Vxp(i,j)*exp(Ax(i,j)+Tm(i,j)))-Vx(i,j)])+x(j);
Ye(i,j) = (1/Ax(i,j))*((Vxp(i,j)*exp(Ay(i,j)+Tm(i,j)))-Vy(i,j)])+y(i);
else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j+1)>0 & Vy(i+1,j)<0 &Vy(i+1,j+1)=0 &Vy(i+1,j)>0)
disp(' - 8th model')
Ax(i,j) = (Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j])
Vxp(i,j) = (Ax(i,j)*Xp(i,j)-x(j)])
Vx(i,j) = Vxp(i,j);
Ay(i,j) = (Vyi(i,j)-Vyi(j-1)/yi(i)-yi(j-1))
Vyi(i,j) = (Ay(i,j)*Yp(i,j)-y(j)])
Vpi(i,j) = (Ay(i,j)*Yp(i,j)-y(j)])+Vy(i,j);
Ty(i,j) = (1/Ay(i,j))^log(Vy(i+1,j)/Vyi(i,j))
if Ty(i,j)<0
Ty(i,j) = Ty(i,j)-(-1);
end
if Tx(i,j)<0
Tx(i,j) = Tx(i,j)-(-1);
end
Tm(i,j) = min(Tx(i,j),Ty(i,j));
Xe(i,j) = (1/Ay(i,j))*((Vxp(i,j)*exp(Ay(i,j)+Tm(i,j)))-Vx(i,j)])+x(j);
Ye(i,j) = (1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)+Tm(i,j)))-Vy(i,j)])+y(i);
else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j+1)>0 & Vy(i+1,j)<0 &Vy(i+1,j+1)=0 &Vy(i+1,j)>0)
disp(' - 10th model')
Ax(i,j) = (Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j])
Vxp(i,j) = (Ax(i,j)*Xp(i,j)-x(j)])
Vx(i,j) = Vxp(i,j);
Ay(i,j) = (Vyi(i,j)-Vyi(j-1)/yi(i)-yi(j-1))
Vyi(i,j) = (Ay(i,j)*Yp(i,j)-y(j)])
Vpi(i,j) = (Ay(i,j)*Yp(i,j)-y(j)])+Vy(i,j);
Ty(i,j) = (1/Ay(i,j))^log(Vy(i+1,j)/Vyi(i,j))
if Ty(i,j)<0
Ty(i,j) = Ty(i,j)-(-1);
end
if Tx(i,j)<0
Tx(i,j) = Tx(i,j)-(-1);
end
Tm(i,j) = min(Tx(i,j),Ty(i,j));
Xe(i,j) = (1/Ay(i,j))*((Vxp(i,j)*exp(Ay(i,j)+Tm(i,j)))-Vx(i,j)])+x(j);
Ye(i,j) = (1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)+Tm(i,j)))-Vy(i,j)])+y(i);
else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j+1)>0 & Vy(i+1,j)<0 &Vy(i+1,j+1)=0 &Vy(i+1,j)>0)

165
\( \text{TM}(i,j) = \min (T_x(i,j), T_y(i,j)) \)

\[ \text{Xe}(i,j) = \min(1/\text{Ax}(i,j), (V_x(i,j) \times \text{Ax}(i,j) \times \text{TM}(i,j))) - V_x(i,j)) + x(j) \]

\[ \text{Ye}(i,j) = \min(1/\text{Ay}(i,j), (V_y(i,j) \times \text{Ay}(i,j) \times \text{TM}(i,j))) - V_y(i,j)) + y(i) \]

\( \text{Xe}(i,j); \text{Ye}(i,j); \text{Ax}(i,j); \text{Ay}(i,j); \text{Vx}(i,j); \text{Vy}(i,j) \)

\( \text{disp}('1-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('4-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('5-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('9-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('1-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('4-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('5-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('9-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('1-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('4-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('5-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( \text{disp}('9-9') \)

\[ \text{Ax}(i,j) = (V_x(i,j+1) - V_x(i,j)) / (x(j+1) - x(j)) \]

\[ \text{Vx}(i,j) = (V_x(i,j) - V_x(i,j-1)) / (x(j) - x(j-1)) \]

\( 167 \)
end
    if T(i,j)<0
        T(i,j) = T(i,j)*(-1);
    end
    Tm(i,j) = min (Tx(i,j),Ty(i,j));

Xe(i,j) = (1/Ax(i,j))*((Vx(1,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j);
Ye(i,j) = (1/Ay(i,j))*((Vy(1,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(i);;

else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i,j+1)<0)
    disp(' 10-9')

Ax(i,j) = (Vx(i,j)-Vy(i,j)*x(j))/(Vx(i,j+1)-Vy(i,j+1)*x(j+1));
Vx(i,j) = (Ax(i,j)*Xp(i,j)-x(j))/Vx(i,j*);

if Ty(i,j)<0
    Ty(i,j) = Ty(i,j)*(-1);
end

if T(i,j)<0
    T(i,j) = T(i,j)*(-1);
end
Tm(i,j) = min (Tm(i,j),Ty(i,j));

Xe(i,j) = (1/Ax(i,j))*((Vx(1,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j);
Ye(i,j) = (1/Ay(i,j))*((Vy(1,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(i);;

else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i,j+1)<0)
    disp(' 8-9')

Ax(i,j) = (Vx(i,j)-Vy(i,j)*x(j))/(Vx(i,j+1)-Vy(i,j+1)*x(j+1));
Vx(i,j) = (Ax(i,j)*Xp(i,j)-x(j))/Vx(i,j*);

if Ty(i,j)<0
    Ty(i,j) = Ty(i,j)*(-1);
end

if T(i,j)<0
    T(i,j) = T(i,j)*(-1);
end
Tm(i,j) = min (Tm(i,j),Ty(i,j));

Xe(i,j) = (1/Ax(i,j))*((Vx(1,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j);
Ye(i,j) = (1/Ay(i,j))*((Vy(1,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(i);;

% - third model
else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i,j+1)<0)
    disp(' -11-9')

Ax(i,j) = (Vx(i,j+1)-Vy(i,j)*x(j))/(Vx(i,j+1)-Vy(i,j+1)*x(j+1));
Vx(i,j) = (Ax(i,j)*Xp(i,j)-x(j))/Vx(i,j*);

if Ty(i,j)<0
    Ty(i,j) = Ty(i,j)*(-1);
end

if T(i,j)<0
    T(i,j) = T(i,j)*(-1);
end
Tm(i,j) = min (Tm(i,j),Ty(i,j));

Xe(i,j) = (1/Ax(i,j))*((Vx(1,j)*exp(Ax(1,j)*Tm(i,j)))-Vx(i,j))*x(j);
Ye(i,j) = (1/Ay(i,j))*((Vy(1,j)*exp(Ay(1,j)*Tm(i,j)))-Vy(i,j))*y(i);;
% first model

elseif (Vx(i,j)<0 & Vx(i,j+1)>=0 & Vx(i+1,j)>0 & Vx(i+1,j+1)<0 &
Vy(i+1,j)>0 & Vy(i+1,j)>0 &
Vy(i+1,j)=Vy(i+1,j))
    disp(' - tenth model')
end

disp(' - tenth model')
% - first model

169
else if (Vx(i,j)<0 && Vx(i,j+1)>=0 & Vy(i,j)>0 && Vy(i,j+1)<0)
    disp(' Enter 100')
    Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/|x(j+1)-x(j)|;
    Vxp(i,j)=(Ax(i,j)*VX(i,j)-
X(j))+Vx(i,j);
    Ye(i,j)=(Vx(i,j)+Vx(i,j)*
exp[(Ax(i,j)*|Vy(i,j)-
y(i)|)])/Vy(i,j);
    Ty(i,j)=Ty(i,j)-
Vx(i,j)/Ax(i,j);
    if Ty(i,j)<0
        Ty(i,j)=Ty(i,j)+(-1);
    end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)+(-1);
    end
    Tm(i,j)=min(Tx(i,j),Ty(i,j));
end
else if (Vx(i,j)>0 && Vx(i,j+1)<0 & Vy(i,j)<0 && Vy(i,j+1)>0)
    disp('
 Enter 6-10')
    Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/|x(j+1)-x(j)|;
    Vxp(i,j)=(Ax(i,j)*Vx(i,j)-
x(j))+Vx(i,j);
    Tx(i,j)=(1/Ax(i,j))log
(Vx(i,j)+Vx(i,j)*
exp[(Ax(i,j)*|Vy(i,j)-
y(i)|)])/Vy(i,j);
    Ye(i,j)=Ye(i,j)+
Vy(i,j)/Ax(i,j);
    if Vy(i,j)<0
        Vy(i,j)=Vy(i,j)+(-1);
    end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)+(-1);
    end
    Tm(i,j)=min(Tx(i,j),Ty(i,j));
end
elseif (Vx(i,j)<0 & Vx(i,j+1)>=0 & Vy(i,j)<0 & Vy(i,j+1)>0)
    disp(' Enter -5-10')
    Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/|x(j+1)-x(j)|;
    Vxp(i,j)=(Ax(i,j)*Vx(i,j)-
x(j))+Vx(i,j);
    Tx(i,j)=(1/Ax(i,j))log
(Vx(i,j)+Vx(i,j)*
exp[(Ax(i,j)*|Vy(i,j)-
y(i)|)])/Vy(i,j);
    Ye(i,j)=Ye(i,j)+
Vy(i,j)/Ax(i,j);
    if Vy(i,j)<0
        Vy(i,j)=Vy(i,j)+(-1);
    end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)+(-1);
    end
    Tm(i,j)=Tx(i,j);
end
elseif (Vx(i,j)>0 & Vx(i,j+1)<0 & Vy(i,j)>0 & Vy(i,j+1)<0)
    disp(' Enter 6-10')
    Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/|x(j+1)-x(j)|;
    Vxp(i,j)=(Ax(i,j)*Vx(i,j)-
x(j))+Vx(i,j);
    Tx(i,j)=(1/Ax(i,j))log
(Vx(i,j)+Vx(i,j)*
exp[(Ax(i,j)*|Vy(i,j)-
y(i)|)])/Vy(i,j);
    Ye(i,j)=Ye(i,j)+
Vy(i,j)/Ax(i,j);
    if Vy(i,j)<0
        Vy(i,j)=Vy(i,j)+(-1);
    end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)+(-1);
    end
    Tm(i,j)=Ty(i,j);
end
elseif Vx(i,j)>0
    disp(' Enter -6-10')
    Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/|x(j+1)-x(j)|;
    Vxp(i,j)=(Ax(i,j)*Vx(i,j)-
x(j))+Vx(i,j);
    Tx(i,j)=(1/Ax(i,j))log
(Vx(i,j)+Vx(i,j)*
exp[(Ax(i,j)*|Vy(i,j)-
y(i)|)])/Vy(i,j);
    Ye(i,j)=Ye(i,j)+
Vy(i,j)/Ax(i,j);
    if Vy(i,j)<0
        Vy(i,j)=Vy(i,j)+(-1);
    end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)+(-1);
    end
    Tm(i,j)=Ty(i,j);
end
elseif Vx(i,j)<0
    disp(' Enter -6-10')
    Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/|x(j+1)-x(j)|;
    Vxp(i,j)=(Ax(i,j)*Vx(i,j)-
x(j))+Vx(i,j);
    Tx(i,j)=(1/Ax(i,j))log
(Vx(i,j)+Vx(i,j)*
exp[(Ax(i,j)*|Vy(i,j)-
y(i)|)])/Vy(i,j);
    Ye(i,j)=Ye(i,j)+
Vy(i,j)/Ax(i,j);
    if Vy(i,j)<0
        Vy(i,j)=Vy(i,j)+(-1);
    end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)+(-1);
    end
    Tm(i,j)=Ty(i,j);
end
else
    disp(' Enter -6-10')
end

Xe(i,j)=(1/Ax(i,j))*((Vxp(i,j)*exp[Ax(i,j)*
Tm(i,j)])-Vx(i,j))
Xe(i,j)+Vx(i,j);
Ye(i,j)+(1/Ax(i,j))*((Vxp(i,j)*exp[Ay(i,j)*
Tm(i,j)])-Vy(i,j))
Ye(i,j)+Vy(i,j);

170
\% \hspace{1cm} T(i,j) = (1/Ax(i,j)) \star \log
\hspace{1cm} (V(x(i,j+1)/V(x(i,j))
\hspace{1cm} T(x(i,j)) = ((x(i,j) -
\hspace{1cm} Xp(x(i,j))/V(x(i,j)));
\hspace{1cm} -1)

\hspace{1cm} if T(x(i,j)) < 0
\hspace{1cm} T(x(i,j)) = T(x(i,j)) \star (-1);
\hspace{1cm} end
\hspace{1cm} Tm(i,j) = T(x(i,j));

\hspace{1cm} Xe(i,j) = (((1/Ax(i,j)) \star ((V(x(i,j)) \star exp(Ax(i,j)) \star Tm(i,j))) \star V(x(i,j)))) \star x(i,j));
\hspace{1cm} Ye(i,j) = y(i,j+1);
\hspace{1cm} Xe(i,j);
\hspace{1cm} Ye(i,j);
\hspace{1cm} end

\hspace{1cm} elseif V(x(i,j)) < 0 &
\hspace{1cm} V(x(i,j+1)) = 0 & V(y(i,j)) = 0 & V(y(i+1,j)) = 0
\hspace{1cm} & V(y(i,j)) > V(y(i+1,j)
\hspace{1cm}) disp('9-10:');
\hspace{1cm} Ax(i,j) = (V(x(i,j+1)) -
\hspace{1cm} V(x(i,j)))/(x(j+1) - x(j));
\hspace{1cm} Vxp(i,j) = (Ax(i,j) \star x(j));
\hspace{1cm} Tx(i,j) = (1/Ax(i,j)) \star \log
\hspace{1cm} [V(x(i,j+1))/V(x(i,j))]
\hspace{1cm} Xp(i,j) = [V(x(i,j))];
\hspace{1cm} Ay(i,j) = (Ty(i,j) -
\hspace{1cm} Vp(i,j))/(y(i+1) - y(i));
\hspace{1cm} Vyp(i,j) = (Ay(i,j) \star (Ty(i,j) -
\hspace{1cm} y(i))); Vyp(i,j) = Vyp(i,j);
\hspace{1cm} Ty(i,j) = (1/Ay(i,j)) \star \log
\hspace{1cm} [V(y(i+1,j))/V(y(i,j))]
\hspace{1cm} if Ty(i,j) < 0
\hspace{1cm} Ty(i,j) = Ty(i,j) \star (-1);
\hspace{1cm} end
\hspace{1cm} if Tx(i,j) < 0
\hspace{1cm} Tx(i,j) = Tx(i,j) \star (-1);
\hspace{1cm} end
\hspace{1cm} Tm(i,j) = min (Tx(i,j), Ty(i,j));

\hspace{1cm} Xe(i,j) = (((1/Ax(i,j)) \star ((V(x(i,j)) \star exp(Ax(i,j)) \star Tm(i,j))) \star V(x(i,j)))) \star x(i,j));
\hspace{1cm} Ye(i,j) = (((1/Ay(i,j)) \star ((V(y(i,j)) \star exp(Ay(i,j)) \star Tm(i,j))) \star V(y(i,j)))) \star y(i,j));
\hspace{1cm} Xe(i,j);
\hspace{1cm} Ye(i,j);
\hspace{1cm} elseif (V(x(i,j)) < 0 &
\hspace{1cm} V(x(i,j+1)) = 0 & V(y(i,j)) = 0 & V(y(i+1,j)) = 0
\hspace{1cm} & V(y(i,j)) < V(y(i+1,j)
\hspace{1cm}) disp('9-10:');
\hspace{1cm} Ax(i,j) = (V(x(i,j+1)) -
\hspace{1cm} V(x(i,j)))/(x(j+1) - x(j));
\hspace{1cm} Vxp(i,j) = (Ax(i,j) \star x(j));
\hspace{1cm} Tx(i,j) = (1/Ax(i,j)) \star \log
\hspace{1cm} [V(x(i,j+1))/V(x(i,j))]
\hspace{1cm} Xp(i,j) = [V(x(i,j))];
\hspace{1cm} Ay(i,j) = (Ty(i,j) -
\hspace{1cm} Vp(i,j))/(y(i+1) - y(i));
\hspace{1cm} Vyp(i,j) = (Ay(i,j) \star (Ty(i,j) -
\hspace{1cm} y(i))); Vyp(i,j) = Vyp(i,j);
\hspace{1cm} Ty(i,j) = (1/Ay(i,j)) \star \log
\hspace{1cm} [V(y(i+1,j))/V(y(i,j))]
\hspace{1cm} if Ty(i,j) < 0
\hspace{1cm} Ty(i,j) = Ty(i,j) \star (-1);
\hspace{1cm} end
\hspace{1cm} if Tx(i,j) < 0
\hspace{1cm} Tx(i,j) = Tx(i,j) \star (-1);
\hspace{1cm} end
\hspace{1cm} Tm(i,j) = min (Tx(i,j), Ty(i,j));

\hspace{1cm} Xe(i,j) = (((1/Ax(i,j)) \star ((V(x(i,j)) \star exp(Ax(i,j)) \star Tm(i,j))) \star V(x(i,j)))) \star x(i,j));
\hspace{1cm} Y(i,j) = (((1/Ay(i,j)) \star ((V(y(i,j)) \star exp(Ay(i,j)) \star Tm(i,j))) \star V(y(i,j)))) \star y(i,j));
\hspace{1cm} Xe(i,j);
\hspace{1cm} Ye(i,j);
\hspace{1cm} elseif (V(x(i,j)) < 0 &
\hspace{1cm} V(x(i,j+1)) = 0 & V(y(i,j)) = 0 & V(y(i+1,j)) = 0
\hspace{1cm} & V(y(i,j)) < V(y(i+1,j)
\hspace{1cm}) disp('9-10:');
\hspace{1cm} Ax(i,j) = (V(x(i,j+1)) -
\hspace{1cm} V(x(i,j)))/(x(j+1) - x(j));
\hspace{1cm} Vxp(i,j) = (Ax(i,j) \star x(j));
\hspace{1cm} Tx(i,j) = (1/Ax(i,j)) \star \log
\hspace{1cm} [V(x(i,j+1))/V(x(i,j))]
\hspace{1cm} Xp(i,j) = [V(x(i,j))];
\hspace{1cm} Ay(i,j) = (Ty(i,j) -
\hspace{1cm} Vp(i,j))/(y(i+1) - y(i));
\hspace{1cm} Vyp(i,j) = (Ay(i,j) \star (Ty(i,j) -
\hspace{1cm} y(i))); Vyp(i,j) = Vyp(i,j);
\hspace{1cm} Ty(i,j) = (1/Ay(i,j)) \star \log
\hspace{1cm} [V(y(i+1,j))/V(y(i,j))]
\hspace{1cm} if Ty(i,j) < 0
\hspace{1cm} Ty(i,j) = Ty(i,j) \star (-1);
\hspace{1cm} end
\hspace{1cm} if Tx(i,j) < 0
\hspace{1cm} Tx(i,j) = Tx(i,j) \star (-1);
\hspace{1cm} end
\hspace{1cm} Tm(i,j) = min (Tx(i,j), Ty(i,j));

\hspace{1cm} Xe(i,j) = (((1/Ax(i,j)) \star ((V(x(i,j)) \star exp(Ax(i,j)) \star Tm(i,j))) \star V(x(i,j)))) \star x(i,j));
\hspace{1cm} Y(i,j) = (((1/Ay(i,j)) \star ((V(y(i,j)) \star exp(Ay(i,j)) \star Tm(i,j))) \star V(y(i,j)))) \star y(i,j));
\hspace{1cm} Xe(i,j);
\hspace{1cm} Ye(i,j);
\hspace{1cm} elseif (V(x(i,j)) < 0 &
\hspace{1cm} V(x(i,j+1)) = 0 & V(y(i,j)) = 0 & V(y(i+1,j)) = 0
\hspace{1cm} & V(y(i,j)) < V(y(i+1,j)
\hspace{1cm}) disp('9-10:');
\hspace{1cm} Ax(i,j) = (V(x(i,j+1)) -
\hspace{1cm} V(x(i,j)))/(x(j+1) - x(j));
\hspace{1cm} Vxp(i,j) = (Ax(i,j) \star x(j));
\hspace{1cm} Tx(i,j) = (1/Ax(i,j)) \star \log
\hspace{1cm} [V(x(i,j+1))/V(x(i,j))]
\hspace{1cm} Xp(i,j) = [V(x(i,j))];
\hspace{1cm} Ay(i,j) = (Ty(i,j) -
\hspace{1cm} Vp(i,j))/(y(i+1) - y(i));
\hspace{1cm} Vyp(i,j) = (Ay(i,j) \star (Ty(i,j) -
\hspace{1cm} y(i))); Vyp(i,j) = Vyp(i,j);
\hspace{1cm} Ty(i,j) = (1/Ay(i,j)) \star \log
\hspace{1cm} [V(y(i+1,j))/V(y(i,j))]
\hspace{1cm} if Ty(i,j) < 0
\hspace{1cm} Ty(i,j) = Ty(i,j) \star (-1);
\hspace{1cm} end
\hspace{1cm} if Tx(i,j) < 0
\hspace{1cm} Tx(i,j) = Tx(i,j) \star (-1);
\hspace{1cm} end
\hspace{1cm} Tm(i,j) = min (Tx(i,j), Ty(i,j));

\hspace{1cm} Xe(i,j) = (((1/Ax(i,j)) \star ((V(x(i,j)) \star exp(Ax(i,j)) \star Tm(i,j))) \star V(x(i,j)))) \star x(i,j));
\hspace{1cm} Y(i,j) = (((1/Ay(i,j)) \star ((V(y(i,j)) \star exp(Ay(i,j)) \star Tm(i,j))) \star V(y(i,j)))) \star y(i,j));
\hspace{1cm} Xe(i,j);
\hspace{1cm} Ye(i,j);
if Tx(i,j)<0
  Tx(i,j) = T[i,j] \times -1;
end

Tm(i,j) = min (T[i,j], Ty(i,j));

Xe(i,j) = ((1/Ax(i,j))*(Vx(i,j) \times exp(Ax(i,j)*Tm(i,j))))+x[i];
Ye(i,j) = ((1/Ay(i,j))*(Vy(i,j) \times exp(Ay(i,j)*Tm(i,j))))+y[i];

if Vx(i,j)<0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i,j+1)<0
  disp('11-10');

  Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/((y[i+1]-y[i])-(x[i+1]-x[i]));
  Vxp(i,j) = (Ax(i,j) \times (x[i+1]-x[i]));
  % Tx(i,j) = (1/Ax(i,j)) \times log(\frac{Vx(i,j+1)}{Vxp(i,j)});
  % Xp(i,j) = (x[i+1]-x[i]);
  % Ay(i,j) = (Vy(i,j+1)-Vy(i,j))/((y[i+1]-y[i])-(x[i+1]-x[i]));
  % Vyp(i,j) = (Ay(i,j) \times (y[i+1]-y[i]));
  % Ty(i,j) = (1/Ay(i,j)) \times log(\frac{Vy(i,j+1)}{Vyp(i,j)});
  % Ty(i,j) = (y[i+1]-y[i]);
  if Ty(i,j)<0
    Ty(i,j) = Ty(i,j) \times -1;
  end
  if Tx(i,j)<0
    Tx(i,j) = Tx(i,j) \times -1;
  end
  Tm(i,j) = min (T[i,j], Ty(i,j));
end

Xe(i,j) = ((1/Ax(i,j))*(Vx(i,j) \times exp(Ax(i,j)*Tm(i,j))))-x[i];
Ye(i,j) = ((1/Ay(i,j))*(Vy(i,j) \times exp(Ay(i,j)*Tm(i,j))))-y[i];

if Vx(i,j)<0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i,j+1)<0
  disp('12-10');

  Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/((y[i+1]-y[i])-(x[i+1]-x[i]));
  Vxp(i,j) = (Ax(i,j) \times (x[i+1]-x[i]));
  % Tx(i,j) = (1/Ax(i,j)) \times log(\frac{Vx(i,j+1)}{Vxp(i,j)});
  % Xp(i,j) = (x[i+1]-x[i]);
  % Ay(i,j) = (Vy(i,j+1)-Vy(i,j))/((y[i+1]-y[i])-(x[i+1]-x[i]));
  % Vyp(i,j) = (Ay(i,j) \times (y[i+1]-y[i]));
  % Ty(i,j) = (1/Ay(i,j)) \times log(\frac{Vy(i,j+1)}{Vyp(i,j)});
  % Ty(i,j) = (x[i+1]-x[i]);
  if Ty(i,j)<0
    Ty(i,j) = Ty(i,j) \times -1;
  end
  if Tx(i,j)<0
    Tx(i,j) = Tx(i,j) \times -1;
  end
  Tm(i,j) = min (T[i,j], Ty(i,j));
end

else if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vy(i,j)>0 & Vy(i,j+1)>0)
  disp('11-11');

  Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/((y[i+1]-y[i])-(x[i+1]-x[i]));
  Vxp(i,j) = (Ax(i,j) \times (x[i+1]-x[i]));
  % Tx(i,j) = (1/Ax(i,j)) \times log(\frac{Vx(i,j+1)}{Vxp(i,j)});
  % Xp(i,j) = (x[i+1]-x[i]);
  % Ay(i,j) = (Vy(i,j+1)-Vy(i,j))/((y[i+1]-y[i])-(x[i+1]-x[i]));
  % Vyp(i,j) = (Ay(i,j) \times (y[i+1]-y[i]));
  % Ty(i,j) = (1/Ay(i,j)) \times log(\frac{Vy(i,j+1)}{Vyp(i,j)});
  % Ty(i,j) = (y[i+1]-y[i]);
  if Ty(i,j)<0
    Ty(i,j) = Ty(i,j) \times -1;
  end
  if Tx(i,j)<0
    Tx(i,j) = Tx(i,j) \times -1;
  end
  Tm(i,j) = min (T[i,j], Ty(i,j));
end

else if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vy(i,j)<0 & Vy(i,j+1)<0)
  disp('12-11');

  Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/((y[i+1]-y[i])-(x[i+1]-x[i]));
  Vxp(i,j) = (Ax(i,j) \times (x[i+1]-x[i]));
  % Tx(i,j) = (1/Ax(i,j)) \times log(\frac{Vx(i,j+1)}{Vxp(i,j)});
  % Xp(i,j) = (x[i+1]-x[i]);
  % Ay(i,j) = (Vy(i,j+1)-Vy(i,j))/((y[i+1]-y[i])-(x[i+1]-x[i]));
  % Vyp(i,j) = (Ay(i,j) \times (y[i+1]-y[i]));
  % Ty(i,j) = (1/Ay(i,j)) \times log(\frac{Vy(i,j+1)}{Vyp(i,j)});
  % Ty(i,j) = (y[i+1]-y[i]);
  if Ty(i,j)<0
    Ty(i,j) = Ty(i,j) \times -1;
  end
  if Tx(i,j)<0
    Tx(i,j) = Tx(i,j) \times -1;
  end
  Tm(i,j) = min (T[i,j], Ty(i,j));
end

else
  disp('eleventh model');

  Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/((y[i+1]-y[i])-(x[i+1]-x[i]));
  Vxp(i,j) = (Ax(i,j) \times (x[i+1]-x[i]));
  % Tx(i,j) = (1/Ax(i,j)) \times log(\frac{Vx(i,j+1)}{Vxp(i,j)});
  % Xp(i,j) = (x[i+1]-x[i]);
  % Ay(i,j) = (Vy(i,j+1)-Vy(i,j))/((y[i+1]-y[i])-(x[i+1]-x[i]));
  % Vyp(i,j) = (Ay(i,j) \times (y[i+1]-y[i]));
  % Ty(i,j) = (1/Ay(i,j)) \times log(\frac{Vy(i,j+1)}{Vyp(i,j)});
  % Ty(i,j) = (x[i+1]-x[i]);
  if Ty(i,j)<0
    Ty(i,j) = Ty(i,j) \times -1;
  end
  if Tx(i,j)<0
    Tx(i,j) = Tx(i,j) \times -1;
  end
  Tm(i,j) = min (T[i,j], Ty(i,j));
end
\[ T_{m(i,j)} = \min \{ T_x(i,j), T_y(i,j) \}; \]
\[ X_e(i,j) = \frac{1}{A_x(i,j)}\cdot \left( V_x(i,j) - V_{x(i,j)} \right) \cdot \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right); \]
\[ Y_e(i,j) = \frac{1}{A_y(i,j)}\cdot \left( V_y(i,j) - V_{y(i,j)} \right) \cdot \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right); \]

**else** if \( V_x(i,j) < 0 \) \& \( V_x(i,j+1) > 0 \) \& \( V_y(i,j) > 0 \) \& \( V_y(i,j+1) > 0 \)
\[ a_x(i,j) = V_x(i,j+1) - V_x(i,j); \]
\[ a_y(i,j) = V_y(i,j+1) - V_y(i,j); \]
\[ X_e(i,j) = \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right) \cdot \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right); \]
\[ Y_e(i,j) = \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right) \cdot \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right); \]

**else** if \( V_y(i,j) < 0 \) \& \( V_y(i,j+1) > 0 \) \& \( V_x(i,j) > 0 \) \& \( V_x(i,j+1) > 0 \)
\[ a_x(i,j) = V_x(i,j+1) - V_x(i,j); \]
\[ a_y(i,j) = V_y(i,j+1) - V_y(i,j); \]
\[ X_e(i,j) = \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right) \cdot \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right); \]
\[ Y_e(i,j) = \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right) \cdot \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right); \]

**else** if \( V_x(i,j) = 0 \) \& \( V_x(i,j+1) > 0 \) \& \( V_y(i,j) = 0 \) \& \( V_y(i,j+1) < 0 \)
\[ a_x(i,j) = V_x(i,j+1) - V_x(i,j); \]
\[ a_y(i,j) = V_y(i,j+1) - V_y(i,j); \]
\[ X_e(i,j) = \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right) \cdot \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right); \]
\[ Y_e(i,j) = \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right) \cdot \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right); \]

**else** if \( V_y(i,j) = 0 \) \& \( V_y(i,j+1) > 0 \) \& \( V_x(i,j) < 0 \) \& \( V_x(i,j+1) < 0 \)
\[ a_x(i,j) = V_x(i,j+1) - V_x(i,j); \]
\[ a_y(i,j) = V_y(i,j+1) - V_y(i,j); \]
\[ X_e(i,j) = \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right) \cdot \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right); \]
\[ Y_e(i,j) = \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right) \cdot \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right); \]

**else** if \( V_x(i,j) = 0 \) \& \( V_x(i,j+1) = 0 \) \& \( V_y(i,j) > 0 \) \& \( V_y(i,j+1) = 0 \)
\[ a_x(i,j) = V_x(i,j+1) - V_x(i,j); \]
\[ a_y(i,j) = V_y(i,j+1) - V_y(i,j); \]
\[ X_e(i,j) = \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right) \cdot \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right); \]
\[ Y_e(i,j) = \frac{1}{A_y(i,j)} \cdot \left( V_y(i,j) - V_{y(i,j)} \right) \cdot \frac{1}{A_x(i,j)} \cdot \left( V_x(i,j) - V_{x(i,j)} \right); \]
```matlab
Vyp(i,j) = (Ay(i,j) * (Yp(i,j) - y(i))) * Vy(i,j);
Ty(i,j) = (1/Ay(i,j)) * log((Vy(i+1,j) / Vyp(i,j)) - Ty(i+1,j));
if Ty(i,j) < 0
    Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
    Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = min(Tx(i,j), Ty(i,j));
Xe(i,j) = ((1/Ax(i,j)) * (Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j);
Ye(i,j) = (Tm(i,j) * Yp(i,j)) * y(i);

else
    if (Vy(i,j) == 0 || Vx(i+1,j) > 0 && Vy(i+1,j) == 0)
        disp('10-11')
        Ax(i,j) = (Vx(i,j) - Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
        Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) / Vx(i,j);
        Ty(i,j) = (1/Ax(i,j)) * log((Vx(i,j+1) / Vxp(i,j))
            - Tx(i,j) - x(j+1) - Xp(i,j) / Vxp(i,j));
        Ay(i,j) = (Vy(i,j) - Vx(i,j) - Vxp(i,j)) / (y(i+1) - y(i));
        Yp(i,j) = (Ay(i,j) * (Yp(i,j) - y(i))) / Vx(i,j);
        Ty(i,j) = -(Ty(i,j) * (-1));
        if Ty(i,j) < 0
            Ty(i,j) = Ty(i,j) * (-1);
        end
        if Tx(i,j) < 0
            Tx(i,j) = Tx(i,j) * (-1);
        end
        Tm(i,j) = min(Tx(i,j), Ty(i,j));
        Xe(i,j) = ((1/Ax(i,j)) * (Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j);
        Ye(i,j) = (Tm(i,j) * Yp(i,j)) * y(i);
        Ye(i,j) =

        else
            if (Vx(i,j) == 0 || Vx(i,j+1) > 0 && Vy(i+1,j) == 0)
                disp('12-11')
                Ax(i,j) = (Vx(i,j) - Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
                Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) / Vx(i,j);
                Ty(i,j) = (1/Ax(i,j)) * log((Vx(i,j+1) / Vxp(i,j))
                    - Tx(i,j) - x(j+1) - Xp(i,j) / Vxp(i,j));
                Ay(i,j) = (Vy(i,j) - Vx(i,j) - Vxp(i,j)) / (y(i+1) - y(i));
                Yp(i,j) = (Ay(i,j) * (Yp(i,j) - y(i))) / Vx(i,j);
                Ty(i,j) = -(Ty(i,j) * (-1));
                if Ty(i,j) < 0
                    Ty(i,j) = Ty(i,j) * (-1);
                end
                if Tx(i,j) < 0
                    Tx(i,j) = Tx(i,j) * (-1);
                end
                Tm(i,j) = min(Tx(i,j), Ty(i,j));
                Xe(i,j) = ((1/Ax(i,j)) * (Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j);
                Ye(i,j) = (Tm(i,j) * Yp(i,j)) * y(i);
        end
end
end

% third model
else
    if (Vx(i,j) == 0 || Vx(i,j+1) > 0 && Vy(i+1,j) == 0)
        disp('11-11')
        Ax(i,j) = (Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
        Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) / Vx(i,j);
        Ty(i,j) = (1/Ax(i,j)) * log((Vx(i+1,j) / Vxp(i,j))
            - Ty(i+1,j) - Ty(i,j));
        if Ty(i,j) < 0
            Ty(i,j) = Ty(i,j) * (-1);
        end
        if Tx(i,j) < 0
            Tx(i,j) = Tx(i,j) * (-1);
        end
        Tm(i,j) = min(Tx(i,j), Ty(i,j));
        Xe(i,j) = ((1/Ax(i,j)) * (Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j);
        Ye(i,j) = (Tm(i,j) * Yp(i,j)) * y(i);
        Ye(i,j) =

end

% - third model
else
    if (Vx(i,j) == 0 || Vx(i,j+1) > 0 && Vy(i+1,j) == 0)
        disp('10-11')
        Ax(i,j) = (Vx(i,j) - Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
        Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) / Vx(i,j);
        Ty(i,j) = (1/Ax(i,j)) * log((Vx(i+1,j) / Vxp(i,j))
            - Ty(i+1,j) - Ty(i,j));
        if Ty(i,j) < 0
            Ty(i,j) = Ty(i,j) * (-1);
        end
        if Tx(i,j) < 0
            Tx(i,j) = Tx(i,j) * (-1);
        end
        Tm(i,j) = min(Tx(i,j), Ty(i,j));
        Xe(i,j) = ((1/Ax(i,j)) * (Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j);
        Ye(i,j) = (Tm(i,j) * Yp(i,j)) * y(i);
        Ye(i,j) =

end
end
```

Ye(i,j); elseif (Vx(i,j)==0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i+1,j)>0 & Vx(i,j)<Vy(i+1,j)) disp(' -3-12') Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j)); Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); % Tx(i,j)=[1/Ax(i,j)]*log(Vx(i,j+1)/Vxp(i,j)) Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j)); A Vy(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)-y(i)); Vyp(i,j)=(Ay(i,j)*(Yp(i,j)-y(i)))/Vy(i,j); % Ty(i,j)=[1/Ay(i,j)]*log(Vy(i+1,j)/Vyp(i,j)) Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)-y(i)); Vyp(i,j)=(Ay(i,j)*(Yp(i,j)-y(i)))/Vy(i,j); % if Ty(i,j)<0 Ty(i,j)=Ty(i,j)*(-1); end if Tx(i,j)<0 Tx(i,j)=Tx(i,j)*(-1); end X(i,j)=min(Tx(i,j),Ty(i,j)); ye(i,j)=(1/Ax(i,j))*(Vxp(i,j)*exp(Ax(i,j)*X(i,j)))-Vx(i,j)); ye(i,j)=(1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)*Y(i,j)))-Vy(i,j)); Vxp(i,j)=y(i); Vyp(i,j)=y(i); X(i,j); Y(i,j); else (Vx(i,j)==0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i+1,j)>0 & Vx(i,j)<Vy(i+1,j)) disp(' -2-12') Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j)); Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); % Tx(i,j)=[1/Ax(i,j)]*log(Vx(i,j+1)/Vxp(i,j)) Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j)); Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); % if Ty(i,j)<0 Ty(i,j)=Ty(i,j)*(-1); end if Tx(i,j)<0 Tx(i,j)=Tx(i,j)*(-1); end X(i,j)=min(Tx(i,j),Ty(i,j)); ye(i,j)=(1/Ax(i,j))*(Vxp(i,j)*exp(Ax(i,j)*X(i,j)))-Vx(i,j)); ye(i,j)=(1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)*Y(i,j)))-Vy(i,j)); Vxp(i,j)=y(i); Vyp(i,j)=y(i); X(i,j); Y(i,j); elseif (Vx(i,j)==0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i+1,j)>0 & Vx(i,j)<Vy(i+1,j)) disp(' -4-12') Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j)); Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); % Tx(i,j)=[1/Ax(i,j)]*log(Vx(i,j+1)/Vxp(i,j)) Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j)); Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); % if Ty(i,j)<0 Ty(i,j)=Ty(i,j)*(-1); end if Tx(i,j)<0 Tx(i,j)=Tx(i,j)*(-1); end X(i,j)=min(Tx(i,j),Ty(i,j)); ye(i,j)=(1/Ax(i,j))*(Vxp(i,j)*exp(Ax(i,j)*X(i,j)))-Vx(i,j)); ye(i,j)=(1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)*Y(i,j)))-Vy(i,j)); Vxp(i,j)=y(i); Vyp(i,j)=y(i); X(i,j); Y(i,j); elseif (Vx(i,j)==0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i+1,j)>0 & Vx(i,j)<Vy(i+1,j)) disp(' -5-12') Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j)); Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); % Tx(i,j)=[1/Ax(i,j)]*log(Vx(i,j+1)/Vxp(i,j)) Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j)); Vxp(i,j)=(Ax(i,j)*(Xp(i,j)-x(j)))/Vx(i,j); % if Ty(i,j)<0 Ty(i,j)=Ty(i,j)*(-1); end if Tx(i,j)<0 Tx(i,j)=Tx(i,j)*(-1); end X(i,j)=min(Tx(i,j),Ty(i,j)); ye(i,j)=(1/Ax(i,j))*(Vxp(i,j)*exp(Ax(i,j)*X(i,j)))-Vx(i,j)); ye(i,j)=(1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)*Y(i,j)))-Vy(i,j)); Vxp(i,j)=y(i); Vyp(i,j)=y(i); X(i,j); Y(i,j);
\[ Ay(i,j) = (Vy(i+1,j) - Vy(i,j)) / (y(i+1) - y(i)), \]
\[ Vxp(i,j) = (Ay(i,j) * (Yp(i,j) - y(i))) + Vy(i,j); \]
\[ Tm(i,j) = t(i,j) / (1 + Ay(i,j) * log(Vy(i+1,j) / Vxp(i,j))) \]

\[ Ty(i,j) = (y(i+1) - Yp(i,j)) / Vy(i,j); \]
\[ % \]
\[ Xe(i,j) = x(i,j) = (Xa(i,j) + Xp(i,j)) / 2; \]
\[ Ye(i,j) = y(i,j) + Vyp(i,j); \]

%  - second model
else if Vx(i,j) == 0 & Vx(i+1,j) < 0 & Vy(i+1,j) > 0
disp('6-12-1')
\[
Ax(i,j) = (Vx(i+1,j) - Vx(i,j)) / (x(i+1) - x(i));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(i))) + Vx(i,j);
\]
% \[
Ty(i,j) = (y(i+1) - Yp(i,j)) / Vy(i,j); \]
\[
% \]
\[
Xe(i,j) = x(i,j) = (Xa(i,j) + Xp(i,j)) / 2; \]
\[
Ye(i,j) = y(i,j) + Vyp(i,j); \]
\[ if Vyp(i,j) < 0 \]
\[ disp('6-12-1') \]
\[ if Tx(i,j) < 0 \]
\[ Tx(i,j) = Tx(i,j) * (-1); \]
end
\[ Tm(i,j) = Ty(i,j) * (-1); \]
end
\[ Xe(i,j) = (1/Ax(i,j)) * (Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) / Vx(i,j); \]
\[ Ye(i,j) = y(i,j); \]

else Vyp(i,j) > 0
disp('6-12-2')
\[
Ax(i,j) = (Vx(i+1,j) - Vx(i,j)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j);
\]
\[ % \]
\[
Ty(i,j) = (y(i+1) - Yp(i,j)) / Vy(i,j); \]
\[
% \]
\[
Xe(i,j) = x(i,j) = (Xa(i,j) + Xp(i,j)) / 2; \]
\[
Ye(i,j) = y(i,j) + Vyp(i,j); \]
\[ if Vyp(i,j) < 0 \]
\[ disp('6-12-2') \]
\[ if Tx(i,j) < 0 \]
\[ Tx(i,j) = Tx(i,j) * (-1); \]
end
\[ Tm(i,j) = Ty(i,j) * (-1); \]
Ye(i,j)=(1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))) - Vy(i,j))*y(i);
    Xe(i,j);
    Ye(i,j);
}

else if (Vx(i,j)<0 &Vy(i,j)<0 & Vy(i+1,j)<0 & Vx(i+1,j)<0)
    disp('9-12')
    Ax(i,j)=(X(i,j)+1)-
    Vx(i,j);  
    Vy(i,j);  
    Vxp(i,j)=[(Ax(i,j)*Xp(i,j)-
    x(j))]*Vx(i,j);
    %
    Tx(i,j)=[1/Ax(i,j)]*log
    Yv(i,j+1)/Vxp(i,j));
    X(i,j)=((Vx(i,j)-
    Yv(i,j))/(y(i)-1));
    Yv(i,j)=[(Ax(i,j)*exp(Ax(i,j)-
    y(i)))*Vx(i,j)];
    %
    Ty(i,j)=[1/((Ax(i,j)*log
    Vv(i,j)))]*y(i);
    Xe(i,j);
    Ye(i,j);
}

else if (Vx(i,j)<0 &Vy(i,j)<0 & Vy(i+1,j)==0)
    disp('10-12')
    Ax(i,j)=(X(i,j)-1)-
    Vx(i,j);  
    Vxp(i,j)=[(Ax(i,j)*Xp(i,j)-
    x(j))]*Vx(i,j);
    %
    Tx(i,j)=[1/Ax(i,j)]*log
    Yv(i,j+1)/Vxp(i,j));
    X(i,j)=((Vx(i,j)-
    Yv(i,j))/(y(i)+1));
    Yv(i,j)=[(Ax(i,j)*exp(Ax(i,j)-
    y(i)))*Vx(i,j)];
    %
    Ty(i,j)=[1/((Ax(i,j)*log
    Yp(i,j)))]*y(i);
    Xe(i,j);
    Ye(i,j);
}

else if (Vx(i,j)==0 &Vy(i,j)<0 & Vy(i+1,j)<0)
    disp('11-12')
    Ax(i,j)=(X(i,j)+1)-
    Vx(i,j);  
    Vxp(i,j)=[(Ax(i,j)*Xp(i,j)-
    x(j))]*Vx(i,j);
    %
    Tx(i,j)=[1/Ax(i,j)]*log
    (Vx(i+1,j)/Vxp(i,j));
    Ty(i,j)=[(y(i)-1));
    if Ty(i,j)<0
        Ty(i,j)=Ty(i,j)-(-1);
    end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)-(-1);
    end
    Tm(i,j)=min (Tx(i,j),Ty(i,j));
    Xe(i,j)=((1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))))-Vy(i,j)))*x(j);
\[
\begin{align*}
Tx(i,j) &= Tx(i,j)^{-1}; \\
end \quad \\
Tm(i,j) &= \min \left( Tx(i,j), Ty(i,j) \right); \\
Xe(i,j) &= \left( (1/AX(i,j)) \cdot (XVP(i,j) \cdot \exp(AX(i,j)) \cdot TM(i,j)) \right) \cdot Vx(i,j); \\
Ye(i,j) &= \left( (1/Ay(i,j)) \cdot (YVP(i,j) \cdot \exp(Ay(i,j)) \cdot TM(i,j)) \right) \cdotVy(i,j); \\
Xe(i,j); \\
Ye(i,j); \\
else \quad \\
Vx(i,j+1) &< 0 \quad \text{or} \quadVy(i+1,j) &< 0 \\
\quad \quad \text{disp}('13-12'); \\
\quad \quad \text{Ax(i,j)} &= (Vx(i,j+1) - Vx(i,j))/(x[i+1] - x[i]); \\
\quad \quad \text{Vxp(i,j)} &= (Ax(i,j) \cdot (Xp[i,j] - x[i])); \\
\quad \quad \text{Tm(i,j) &=} \left( (1/AX(i,j)) \cdot \log \right) \\
\quad \quad \text{Vxp(i,j) / Vx(i,j+1)); if}\ Tm(i,j) < 0 \\
\quad \quad \quad \quad \quad \quad \quad \quad \text{disp}('13-12'); \\
\quad \quad \quad \quad \quad \quad \quad \quad \text{Ax(i,j)} &= (Vx(i,j+1) - Vx(i,j))/(x[i+1] - x[i]); \\
end \quad \\
Xe(i,j) &= \left( (1/AX(i,j)) \cdot (XVP(i,j) \cdot \exp(AX(i,j)) \cdot TM(i,j)) \right) \cdot Vx(i,j); \\
Ye(i,j) &= (Ay(i,j) \cdot (YVP(i,j) \cdot \exp(Ay(i,j)) \cdot TM(i,j)) \right) \cdotVy(i,j); \\
Xe(i,j); \\
Ye(i,j); \\
\end{align*}
\]
disp('I don't know')

rr=1

else
disp('new model')
    Vx(i,j)
    Vx(i,j+1)
    Vy(i,j)
    Vy(i+1,j)
end

S755823
fprintf(fid2,' Xe Ye A x
Vxp Ay Vyp /\n\n');
fprintf(fid2,'%f %f %e %e %e
\n',Xe,Ye,Ax,Vxp,Ay,Vyp);
end

% Streamline Simulation Near Well Bore
% By MARJAN HASHEM

% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM

% First Case Study in Cartesian Coordinate
% Subroutine for drawing the streamline

% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM

% First Case Study in Cartesian Coordinate
% Subroutine for drawing streamline in "x" Direction

function sign
global X Y
global Ks Ks
global P
global Vx Vy
global Xe Ye
global maxerr maxr errormatrix
global X Y
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr

if (Ye(i,j)>0 & Xe(i,j)>0)
    gg=|Xe(i,j)-X(i)|;
    if (gg>0.98 & & gg<1.1)
        ???
        Xp(i,j+1)=Xe(i,j);
        Yp(i,j+1)=Ye(i,j);
        Xpp=Xp{i,j+1};
        Ypp=Yp(i,j+1); i=i-1
        j=j+1
    else
        5555
        Xp(i+1,j)=Xe(i,j);
        Yp(i+1,j)=Ye(i,j); Xpp=Xp{i+1,j};
        Ypp=Yp{i+1,j}; i=i+1
        j=j

end
rr=-0
else
    Yp(i,j)=Ye(i,j);
    Xp(i,j)=Xe(i,j);
    rr=1;

end

Second Case Study in Cartesian Coordinate

% Streamline Simulation Near Well Bore
% By MARJAN HASHEM
% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM

% Second Case Study in Cartesian Coordinate
% Main route

clc;
clear all;
close all;
format long e

Y = input('Number of Nodes in Y ');
X = input('Number of Nodes in X ');

% Boundary values can be set along the corner or in the middle
reply = input('Corner or Middle boundary conditions C/M [C]: ', 's');

%Maximum Error abs(x2-x1)
maxerr= input('Desired Maximum Error [.00001]: ','
if isempty(reply)
    reply = 'C';
end
if isempty(maxerr)
    maxerr = .00001;
end

global X Y
global Ks Ks
global P
global Vx Vy
global Xe Ye
global maxerr maxr errormatrix
global X Y
global Vxp Vyp
global Xp Yp
global Vxp Vyp
global gg ff
global i j
global rr
global Ax Ay Vxp Vyp
global Ki K2
global A B
fprintf(fid2,' Xe Ye Ax Vxp Ay Vyp /L

permeabilities
reply=lower(reply);
if reply=='c'
    pressure
else
    for i=floor(X/3);floor(2*X/3)
        P(i,1)=(2*i1);
    end
end

M=0.5*10^-3;
velocities
for b=1:X+1
    x(b)=(b-1);
end
for t=1:Y+1
    y(t)=(t-1);
end
Xp=zeros(X,Y);
Yp=zeros(X,Y);

Vxp=zeros(X,Y);
Vyp=zeros(X,Y);

for m=2:Y

Xp(m,1)=0;
Yp(m,1)=(m-1);
Xpp=Xp(m,1);
Ypp=Yp(m,1);
Vxp(m,1)=Vx(m,1);
Vyp(m,1)=Vy(m,1);
i=m;

j=1;

while ((Xpp<(X-1)) & (Ypp<Y) & (i<(Y)) & (j<(X-1)))

location ;
fprintf(fid2,'%f %f %e %e %e
',Xe,Ye,Ax,Vx,Av,Vyp);
if rr==1;
break
end

if (Ye(i,j)==0 & Xe(i,j)==0)

Xpp
Ypp
Xe(i,j)
Ye(i,j)
x(j)
y(i)
i

j
if (abs(Ye(i,j)-Ypp)>1.25
    disp('benvis')
    if (Ye(i,j)-Ypp)<0
        Ye(i,j)=(fix(Ypp)-1)
    elseif (Ye(i,j)-Ypp)>0
        Ye(i,j)=(fix(Ypp)+1)
    else
    end
end

if (abs(Xe(i,j)-Xpp)>1.25
disp('benvis')
if (Xe(i,j)-Xpp)<0
   Xe(i,j)=(fix(Xpp)-1)
elseif (Xe(i,j)-Xpp)>0
   Xe(i,j)=(fix(Xpp)+1)
else
end
end

if (Xe(i,j)-(fix(Xe(i,j))))>0

B=Xe(i,j)+1
elseif ((Xe(i,j)-(fix(Xe(i,j))))<0.9 & (Xe(i,j)-(fix(Xe(i,j))))<1

B=(round(Xe(i,j)))+1
else

B=(fix(Xe(i,j)))+1
end

if (Ye(i,j)-(fix(Ye(i,j))))==0

A=Ye(i,j)+1
elseif ((Ye(i,j)-(fix(Ye(i,j))))>0.9 & (Ye(i,j)-(fix(Ye(i,j))))<1

A=(round(Ye(i,j)))+1
elseif ((Ye(i,j)-(fix(Ye(i,j))))<0.01

A=(round(Ye(i,j)))+1
else

A=(fix(Ye(i,j)))+1
end

A
Xp(A,B)=Xe(i,j);
Yp(A,B)=Ye(i,j);
Xpp=Xp(A,B);
Ypp=Yp(A,B);
i=A;
j=B;
rr==0;
break
end

x(j);

plotFinalforXdirection
hold on
Xp=zeros(X,Y);
ifp=zeros(X,Y);
function velocities
global X Y
global Kx Ky
global P
global Vx Vy
global XeYe
global maxerr maxr errormatrix
global x y
global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg ff
global i j
global rr
global Ax Ay Vxp Vyp

global K1 K2
global A B

for i=1:Y
    for j=1:X
        Kx(i,j)=(0.2*10^-2);
        Ky(i,j)=(0.2*10^-2);
    end
end

for i=floor((Y)/3):floor((2*Y)/3)
    for j=floor((X)/3):floor((2*X)/3)
        Kx(i,j)=0.1*10^-16;
        Ky(i,j)=0.1*10^-16;
    end
end

K1=0.1*10^-16;
K2=0.2*10^-2;

Kx;
Ky;
function location
global X Y
global Kx Ky
global P
global VX VX
global Xe Ye
global maxerr maxerr errormatrix
global x y
global Xp Yp
global gg ff
global l j
global Ax Ay Vxp Vyp
global K1 K2
global A B

% input2 = fopen('any name.dat', 'w');

% disp('first model')
% disp('first model')
if (Vx(i,j) > 0 & Vx(i,j+1) > 0 & Vx(i,j)=Vx(i,j+1) &Vy(i,j) > 0 &Vy(i+1,j)=Vy(i+1,j))
disp('1-1');
Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/(x[j+1]-x[j]);
Vxp(i,j)=Ax(i,j)*[Xp(i,j)-
x[j]]+Vx(i,j);
Tx(i,j)=(x[j+1]-
Xp(i,j))/Vx(i,j);
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/(y[i+1]-y[i]);
Vyp(i,j)=(Ay(i,j)*[Yp(i,j)-
y[i]])+Vy(i,j);
Ty(i,j)=(y[i+1]-
Yp(i,j))/Vy(i,j);
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
Tm(i,j)=min(Tx(i,j),Ty(i,j));

elseif (Vx(i,j)>0 & Vx(i,j+1)>0 &
Vx(i,j)=Vx(i,j+1) &Vy(i,j) > 0 &
Vy(i+1,j)>0 &Vy(i,j)>Vy(i+1,j))
disp('2-1');
Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/-(x[j+1]-x[j]);
Vxp(i,j)=[Ax(i,j)*[Xp(i,j)-
x[j]]+Vx(i,j];
Tx(i,j)=(x[j+1]-
Xp(i,j))/Vx(i,j);
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/y[i+1]-y[i]);
Vyp(i,j)=(Ay(i,j)*[Yp(i,j)-
y[i]])+Vy(i,j);
Ty(i,j)=1/(Ay(i,j))*log
(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
Tm(i,j)=min(Tx(i,j),Ty(i,j));

elseif (Vx(i,j)>0 & Vx(i,j+1)>0 &
Vx(i,j)=Vx(i,j+1) &Vy(i,j) > 0 &
Vy(i+1,j)>0 &Vy(i,j)>Vy(i+1,j))
disp('3-1');
Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/-(x[j+1]-x[j]);
Vxp(i,j)=[Ax(i,j)*[Xp(i,j)-
x[j]]+Vx(i,j];
Tx(i,j)=(x[j+1]-
Xp(i,j))/Vx(i,j);
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/y[i+1]-y[i]);
Vyp(i,j)=(Ay(i,j)*[Yp(i,j)-
y[i]])+Vy(i,j);
Ty(i,j)=1/(Ay(i,j))*log
(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
Tm(i,j)=min(Tx(i,j),Ty(i,j));

Xe(i,j)=Tm(i,j)*
Vxp(i,j)+x[i,j];
Ye(i,j)=Tm(i,j)*
Vxp(i,j)+y[i,j];

else

Xe(i,j)=(1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j);
Ye(i,j)=Tm(i,j);
else if ( \( Vx(i,j) > 0 \) \&\& \( Vx(i,j+1) > 0 \) \&\& \( Vy(i,j) > 0 \) \&\& \( Vy(i+1,j) > 0 \) ) \{

disp('4-1');
Ax(i,j) = (Vx(i,j) - Vx(i,j+1)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j);
%
Tx(i,j) = (1/Ax(i,j)) * log(Vx(i,j+1)/Vxp(i,j));
Tx(i,j) = (x(j+1) - x(j)) / Vxp(i,j);
% 
Ty(i,j) = Vxp(i,j) * (Ty(i,j) - y(i));
if Ty(i,j) < 0
    Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
    Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = Tx(i,j); 
Xe(i,j) = (Tm(i,j) * Vxp(i,j)) / x(j);
Ye(i,j) = y(i) + Xe(i,j);
Ye(i,j) = y(i);
else if ( \( Vx(i,j) > 0 \) \&\& \( Vx(i,j+1) > 0 \) \&\& \( Vy(i,j) > 0 \) \&\& \( Vy(i+1,j) > 0 \) ) \{

disp('-5-1');
Ax(i,j) = (Vx(i,j) - Vx(i,j+1)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j);
%
Tx(i,j) = (1/Ax(i,j)) * log(Vx(i,j+1)/Vxp(i,j));
Tx(i,j) = (x(j+1) - x(j)) / Vxp(i,j);
Ay(i,j) = (Vy(i+1,j) - Vy(i,j)) / (y(i+1) - y(i));
Vyp(i,j) = (Ay(i,j) * (Yp(i,j) - y(i))) + Vy(i,j);
Ty(i,j) = y(i) - Yp(i,j) + Vy(i,j); 
if Ty(i,j) < 0
    Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
    Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = Tx(i,j); 
Xe(i,j) = (Tm(i,j) * Vxp(i,j)) / x(j);
Ye(i,j) = y(i) + Xe(i,j);
Ye(i,j) = y(i);
else 
    Vyp(i,j) > 0 
    disp('-6-1-2');
    Vxp(i,j) = (Ax(i,j) * Xp(i,j)) / (x(j)) + Vx(i,j);
    Xe(i,j) = (Tm(i,j) * Vxp(i,j)) / x(j);
    if Tx(i,j) < 0 
        Tx(i,j) = Tx(i,j) * (-1);
    end 
    Tm(i,j) = Tx(i,j); 
    Xe(i,j) = (Tm(i,j) * Vxp(i,j)) / x(j);
    Ye(i,j) = y(i) + Xe(i,j);
    Ye(i,j) = y(i);
end
else if ( \( Vx(i,j) > 0 \) \&\& \( Vx(i,j+1) > 0 \) \&\& \( Vy(i,j) > 0 \) \&\& \( Vy(i+1,j) > 0 \) ) \{

disp('-7-1');
Ax(i,j) = (Vx(i,j) - Vx(i,j+1)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j);
%
Tx(i,j) = (1/Ax(i,j)) * log(Vx(i,j+1)/Vxp(i,j));
Tx(i,j) = (x(j+1) - x(j)) / Vxp(i,j);
Ay(i,j) = (Vy(i+1,j) - Vy(i,j)) / (y(i+1) - y(i));
Vyp(i,j) = (Ay(i,j) * (Yp(i,j) - y(i))) + Vy(i,j);
Ty(i,j) = y(i) - Yp(i,j) + Vy(i,j); 
if Ty(i,j) < 0
    Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
    Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = Tx(i,j); 
Xe(i,j) = (Tm(i,j) * Vxp(i,j)) / x(j);
Ye(i,j) = y(i) + Xe(i,j);
Ye(i,j) = y(i);
else
    Vyp(i,j) > 0 
    disp('-6-1-1');
    Vxp(i,j) = (Ax(i,j) * Xp(i,j)) / (x(j)) + Vx(i,j);
    X(p(i,j)) = (Ax(i,j) * Xp(i,j)) / (x(j)) + Vx(i,j);
    Xe(i,j) = (Tm(i,j) * Vxp(i,j)) / x(j);
    if Tx(i,j) < 0 
        Tx(i,j) = Tx(i,j) * (-1);
    end 
    Tm(i,j) = Tx(i,j); 
    Xe(i,j) = (Tm(i,j) * Vxp(i,j)) / x(j);
    Ye(i,j) = y(i) + Xe(i,j);
    Ye(i,j) = y(i);
end
end
\[
Xe(i,j) = (Tm(i,j))^* x(i,j);
\]
\[
Ye(i,j) = ((1/Ay(i,j))*(Vyp(i,j))*exp(Ay(i,j)*Ty(i,j)+Vyp(i,j)) + Xe(i,j)) * Y(i,j);
\]
\[
Xe(i,j) = Tm(i,j) * Ty(i,j);
\]
\[
Vxp(i,j) = (Ax(i,j) * Xp(i,j) + Ax(i,j) + Xe(i,j));
\]
\[
Ty(i,j) = Ty(i,j) * (-1);
\]
\[
Tm(i,j) = min(Tx(i,j), Ty(i,j));
\]
\[
Vxp(i,j) = (Ax(i,j) * Xp(i,j) + Ax(i,j) + Xe(i,j));
\]
\[
Ax(i,j) = (Xp(i,j) + Xe(i,j)) * (1/Ay(i,j) * exp(Ay(i,j)*Ty(i,j) + Y(i,j)) + Xe(i,j));
\]
\[
Tx(i,j) = Tx(i,j) * (-1);
\]
\[
Tm(i,j) = min(Tx(i,j), Ty(i,j));
\]
\[
Vxp(i,j) = (Ax(i,j) * Xp(i,j) + Ax(i,j) + Xe(i,j));
\]
\[
Ye(i,j) = (1/Ay(i,j) * (Vyp(i,j)*exp(Ay(i,j)*Ty(i,j))) + Y(i,j));
\]
\[
Xe(i,j) = Tm(i,j) * Ty(i,j);
\]
\[
Vxp(i,j) = (Ax(i,j) * Xp(i,j) + Ax(i,j) + Xe(i,j));
\]
\[
Ax(i,j) = (Xp(i,j) + Xe(i,j)) * (1/Ay(i,j) * exp(Ay(i,j)*Ty(i,j) + Y(i,j)) + Xe(i,j));
\]
\[
Tx(i,j) = Tx(i,j) * (-1);
\]
\[
Tm(i,j) = min(Tx(i,j), Ty(i,j));
\]
\[
Vxp(i,j) = (Ax(i,j) * Xp(i,j) + Ax(i,j) + Xe(i,j));
\]
\[
Ax(i,j) = (Xp(i,j) + Xe(i,j)) * (1/Ay(i,j) * exp(Ay(i,j)*Ty(i,j) + Y(i,j)) + Xe(i,j));
\]
\[
Tx(i,j) = Tx(i,j) * (-1);
\]
\[
Tm(i,j) = min(Tx(i,j), Ty(i,j));
\]
\[
Vxp(i,j) = (Ax(i,j) * Xp(i,j) + Ax(i,j) + Xe(i,j));
\]
\[
Ax(i,j) = (Xp(i,j) + Xe(i,j)) * (1/Ay(i,j) * exp(Ay(i,j)*Ty(i,j) + Y(i,j)) + Xe(i,j));
\]
Ye(i,j)=((1/Ay(i,j))*((Yx(i,j))*exp(Ay(i,j)*Tx(i,j))-Vy(i,j)))+y(i);
Xe(i,j);
Ye(i,j);

elseif (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)==Vx(i,j+1) & Vx(i,j)>0 & Vy(i+1,j)>0 & Vy(i,j)==Vy(i+1,j))
disp('-1-2')
Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j));
Vxp(i,j)=((Ax(i,j)*x(j)+Vx(i,j);)
Tx(i,j)=(1/Ax(i,j))*log((Vx(i,j+1)/Vxp(i,j));
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)=(Ay(i,j)*(Yp(i,j)-y(i)))+Vy(i,j);
Ty(i,j)=(1/Ay(i,j))*log((Vy(i+1,j)/Vyp(i,j));
Ty(i,j)=(1-1); if Ty(i,j)<0
else
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
Tm(i,j)=min(Tx(i,j),Ty(i,j));
Xe(i,j)=(1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tx(i,j))-Vy(i,j)))+x(j);
Ye(i,j)=(Tm(i,j)*Vxp(i,j));
Ye(i,j);

elseif (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)==Vx(i,j+1) & Vx(i,j)>0 & Vy(i+1,j)>0 & Vy(i,j)==Vy(i+1,j))
disp('-1-3')
Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j));
Vxp(i,j)=((Ax(i,j)*x(j)+Vx(i,j);)
Tx(i,j)=(1/Ax(i,j))*log((Vx(i,j+1)/Vxp(i,j));
Xp(i,j)=((1/Ay(i,j))*((Yx(i,j))*exp(Ay(i,j)*Tx(i,j))-Vy(i,j)))+x(j);
if Tx(i,j)<0
end
Tm(i,j)=Tx(i,j)*(-1);
end
Xe(i,j)=(1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tx(i,j))-Vy(i,j)))+x(j);
Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Ty(i,j))-Vx(i,j)))+y(i);
Xe(i,j);
Ye(i,j);
end
Tm(i,j)=Tx(i,j);
Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))))-Vx(i,j));
Ye(i,j)=y(i+1);
Xe(i,j); Ye(i,j);
ext else if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j+1)>=Vy(i,j)<0 & Vy(i+1,j)<0 & Vy(i+1,j)<Vy(i+1,j) )
% disp('-7-2')
Ax(i,j)=(Vx(i,j)-Vx(i,j+1)-
Vx(i,j))/(x(j+1)-x(j));
Vxp(i,j)=((1/Ax(i,j))*(Xp(i,j)-
X(j)))+Vx(i,j);
Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j)+Vy(i,j));
Ay(i,j)=(Vy(i+1,j)-
Vx(i,j))/(y(i+1)-y(i));
Vyp(i,j)=(1/Ay(i,j))*Vy(i,j); Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)+Vy(i,j));
if Ty(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
% disp('-9-2')
Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/(x(j+1)-x(j));
Vxp(i,j)=((1/Ax(i,j))*(Xp(i,j)-
X(j)))+Vx(i,j);
Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j)+Vy(i,j));
Ay(i,j)=(Vy(i+1,j)-
Vx(i,j))/(y(i+1)-y(i));
Vyp(i,j)=(1/Ay(i,j))*Vy(i,j); Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)+Vy(i,j));
if Ty(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
% disp('-8-2')
Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/(x(j+1)-x(j));
Vxp(i,j)=((1/Ax(i,j))*(Xp(i,j)-
X(j)))+Vx(i,j);
Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j)+Vy(i,j));
Ay(i,j)=(Vy(i+1,j)-
Vx(i,j))/(y(i+1)-y(i));
Xe(i,j); Ye(i,j);
ext else if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j+1)<Vy(i,j)<0 & Vy(i+1,j)<Vy(i+1,j) )
% disp('-10-2')
Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/(x(j+1)-x(j));
Vxp(i,j)=((1/Ax(i,j))*(Xp(i,j)-
X(j)))+Vx(i,j);
Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j)+Vy(i,j));
Ay(i,j)=(Vy(i+1,j)-
Vx(i,j))/(y(i+1)-y(i));
Vyp(i,j)=(1/Ay(i,j))*Vy(i,j); Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)+Vy(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
Tm(i,j)=min (Tm(i,j),Ty(i,j));
Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))))-Vx(i,j));
Ye(i,j)=y(i+1);
Xe(i,j); Ye(i,j);
ex end
Tm(i,j)=min (Tm(i,j),Ty(i,j));
Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))))-Vx(i,j));
Ye(i,j)=y(i+1);
Xe(i,j); Ye(i,j);
e end if Vx(i,j)<0
Xe(i,j)+(Vx(i,j)+Vx(i,j+1))/2
Tx(i,j)+Vx(i,j)/2
x(j)+Vx(i,j)/2
Xe(i,j); Ye(i,j);
end
Xe(i,j); Ye(i,j);
e else if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j+1)<Vy(i,j)<0 & Vy(i+1,j)<Vy(i+1,j) )
% disp('-9-2')
Ax(i,j)=(Vx(i,j)-Vx(i,j+1)-
Vx(i,j))/(x(j+1)-x(j));
Vxp(i,j)=((1/Ax(i,j))*(Xp(i,j)-
X(j)))+Vx(i,j);
Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j)+Vy(i,j));
Ay(i,j)=(Vy(i+1,j)-
Vx(i,j))/(y(i+1)-y(i));
Vyp(i,j)=(1/Ay(i,j))*Vy(i,j); Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)+Vy(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Xe(i,j); Ye(i,j);
\begin{verbatim}
T_{x(i,j)} = \frac{1}{A_x(i,j)} \log(V_{x(i,j)} / V_{x\text{avg}(i,j)});
A_y(i,j) = (V_y(i+1,j) - V_y(i,j)) / V_y(i,j);
V_{y\text{avg}(i,j)} = (A_y(i,j) \times V_y(i,j)) + V_{y\text{avg}(i,j)};
\% T_y(i,j) = \frac{1}{A_y(i,j)} \log([V_{y(i+1,j)} / V_{y(i,j)}])
T_{y(i,j)} = ([y(i+1) - y(i)] / V_{y\text{avg}(i,j)}) + T_{y(i,j)};
\% Tx(i,j) = Tx(i,j)*(1 - 1 / T_{x(i,j)});
Ty(i,j) = Ty(i,j)*(1 - 1 / T_{y(i,j)});
\end{verbatim}
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j)) * Vx(i,j));
T(x, i,j) = (1/Ax(i,j)) * log
(Vx(i,j) * Vxp(i,j));
Ay(i,j) = (Vy(i+1,j) -Vy(i,j)) / y(i+1) - y(i);
Vyp(i,j) = (Ay(i,j) * (Yp(i,j) - y(i))) * Vyp(i,j);
Ty(i,j) = (1/Ay(i,j)) * log
(Vy(i+1,j) / Vyp(i,j));
Ty(i,j) = (y(i+1) - y(i)) * Vyp(i,j);
end if Ty(i,j) < 0

Ty(i,j) = Ty(i,j) * (-1);
end if Tx(i,j) < 0

Tx(i,j) = Tx(i,j) * (-1);
end if Tm(i,j) = min (Tx(i,j), Ty(i,j));

Xe(i,j) = ((1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j))) * x(j);
Ye(i,j) = (Tm(i,j) - Xv(i,j)) / Vx(i,j);
Vxp(i,j) * y(i);
Xe(i,j); Ye(i,j); Vxp(i,j);

else if (Vx(i,j) > 0 & Vx(i,j+1) > 0 & Vx(i,j) < Vx(i,j+1) & Vy(i,j) > 0 & Vy(i+1,j) > 0 & Vy(i,j) > Vy(i+1,j))
% disp('7-3')
Ax(i,j) = (Vx(i,j) - Vx(i,j+1)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) / Vx(i,j); 

% disp('4-3')
Ax(i,j) = (Vx(i,j) - Vx(i,j+1)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) / Vx(i,j);

Xe(i,j) = ((1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j))) * x(j);
Ye(i,j) = (Tm(i,j) - Xv(i,j)) / Vx(i,j);
Vxp(i,j) * y(i);
Xe(i,j); Ye(i,j); Vxp(i,j);

else if (Vx(i,j) > 0 & Vx(i,j+1) > 0 & Vx(i,j) < Vx(i,j+1) & Vy(i,j) > 0 & Vy(i+1,j) > 0 & Vy(i,j) > Vy(i+1,j))
% disp('7-3')
Ax(i,j) = (Vx(i,j) - Vx(i,j+1)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) / Vx(i,j); 

% disp('4-3')
Ax(i,j) = (Vx(i,j) - Vx(i,j+1)) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) / Vx(i,j);

Xe(i,j) = ((1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j))) * x(j);
Ye(i,j) = (Tm(i,j) - Xv(i,j)) / Vx(i,j);
Vxp(i,j) * y(i);
Xe(i,j); Ye(i,j); Vxp(i,j);
end
if Tx(i,j)<0
    Tx(i,j) =Tx(i,j)*(-1);
end

Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=(1/Ax(i,j))*(Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)))+x(j);
Ye(i,j)=(1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-y(i,j);
Xe(i,j); Ye(i,j);

% - second model
else if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)<Vx(i,j+1) & Vy(i,j)<0 & Vy(i+1,j)>0 )
    disp('6-3')
    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(i+1)+x(i));
    Vxp(i,j)=(Ax(i,j)*Xp(i,j)-x(i))+Vx(i,j);
    Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j));
    Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)+y(i));
    Vyp(i,j)=(Ay(i,j)*Yp(i,j)-y(i))+Vy(i,j);
    Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)/Vyp(i,j));
    if Tx(i,j)<0
        Tx(i,j) =Tx(i,j)*(-1);
    end
    Tm(i,j)=Tx(i,j);
Xe(i,j)=(1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)))+x(j);
if Vyp(i,j)<0
    disp('6-3')
else Vyp(i,j)>0
    disp('6-3')
end

% - second model
else if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)<Vx(i,j+1) & Vy(i,j)<0 & Vy(i+1,j)>0 )
    disp('7-3')
    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(i+1)+x(i));
    Vxp(i,j)=(Ax(i,j)*Xp(i,j)-x(i))+Vx(i,j);
    Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j));
    Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)+y(i));
    Vyp(i,j)=(Ay(i,j)*Yp(i,j)-y(i))+Vy(i,j);
    Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)/Vyp(i,j));
    if Tx(i,j)<0
        Tx(i,j) =Tx(i,j)*(-1);
    end
    if Ty(i,j)<0
        Ty(i,j) =Ty(i,j)*(-1);
    end
    Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)=(1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)))+x(j);
if Vyp(i,j)<0
    disp('7-3')
else Vyp(i,j)>0
    disp('7-3')
end


\( Tm(i,j) = \min \) (\( Tx(i,j) \), \( Ty(i,j) \))

\( Xe(i,j) = (1/Ax(i,j)) * (Vxp(i,j) * \exp(Ax(i,j) * Tm(i,j))) * x(j) \)

\( Ye(i,j) = (1/Ay(i,j)) * (Vyp(i,j) * \exp(Ay(i,j) * Tm(i,j))) * y(i) \)

\( Xe(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( Ye(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( \) \( \) \( \) \( \)

\( Xe(i,j) = (1/Ax(i,j)) * (Vxp(i,j) * \exp(Ax(i,j) * Tm(i,j))) * x(j) \)

\( Ye(i,j) = (1/Ay(i,j)) * (Vyp(i,j) * \exp(Ay(i,j) * Tm(i,j))) * y(i) \)

\( Xe(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( Ye(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( \) \( \) \( \) \( \)

\( Xe(i,j) = (1/Ax(i,j)) * (Vxp(i,j) * \exp(Ax(i,j) * Tm(i,j))) * x(j) \)

\( Ye(i,j) = (1/Ay(i,j)) * (Vyp(i,j) * \exp(Ay(i,j) * Tm(i,j))) * y(i) \)

\( Xe(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( Ye(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( \) \( \) \( \) \( \)

\( Xe(i,j) = (1/Ax(i,j)) * (Vxp(i,j) * \exp(Ax(i,j) * Tm(i,j))) * x(j) \)

\( Ye(i,j) = (1/Ay(i,j)) * (Vyp(i,j) * \exp(Ay(i,j) * Tm(i,j))) * y(i) \)

\( Xe(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( Ye(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( \) \( \) \( \) \( \)

\( Xe(i,j) = (1/Ax(i,j)) * (Vxp(i,j) * \exp(Ax(i,j) * Tm(i,j))) * x(j) \)

\( Ye(i,j) = (1/Ay(i,j)) * (Vyp(i,j) * \exp(Ay(i,j) * Tm(i,j))) * y(i) \)

\( Xe(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( Ye(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( \) \( \) \( \) \( \)

\( Xe(i,j) = (1/Ax(i,j)) * (Vxp(i,j) * \exp(Ax(i,j) * Tm(i,j))) * x(j) \)

\( Ye(i,j) = (1/Ay(i,j)) * (Vyp(i,j) * \exp(Ay(i,j) * Tm(i,j))) * y(i) \)

\( Xe(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( Ye(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( \) \( \) \( \) \( \)

\( Xe(i,j) = (1/Ax(i,j)) * (Vxp(i,j) * \exp(Ax(i,j) * Tm(i,j))) * x(j) \)

\( Ye(i,j) = (1/Ay(i,j)) * (Vyp(i,j) * \exp(Ay(i,j) * Tm(i,j))) * y(i) \)

\( Xe(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)

\( Ye(i,j) = \) \( Ty(i,j) = \) \( Xy(i,j) \)
Vyp(i,j)= Ay(i,j) * (Yp(i,j) - y(i)) + Vy(i,j) %
Ty(i,j)=(1/Ay(i,j))* log (Vy(i+1,j) /Vyp(i,j));
Ty(i,j)= (y(i+1)-Yp(i,j))/Vy(i+1,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
Tm(i,j)=min (Tx(i,j),Ty(i,j));

Xe(i,j)=((1/Ax(i,j))* (Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j));
Ye(i,j)=((1/Ay(i,j))* (Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(i));
Xe(i,j); Ye(i,j);
else (Vx(i,j)>0 & Vx(i,j)<Vx(i,j+1) &
Vy(i,j)>0 & Vy(i,j+1)>0 &
% disp('13-3')
Ay(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j);
Vxp(i,j)=(Ax(i,j)*Xp(i,j)-
Vx(i,j));
Tx(i,j)= (1/Ax(i,j))*log (Vx(i,j+1)/Vxp(i,j));
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
end

Xe(i,j)=((1/Ax(i,j))* (Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j));
Ye(i,j)=y(i));
Ye(i,j);
% disp(' FORTH model')
% = first model
else (Vx(i,j)>0 & Vx(i,j+1)<0 &
Vy(i,j)>0 & Vy(i,j+1)>0 &
Vy(i,j)=Vy(i+1,j))
% disp('1-4')
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-
Vyp(i,j)=(Ay(i,j)* (Yp(i,j)-
y(i)));Vy(i,j);
Ty(i,j)=y(i+1)-
Vp(i,j)/Vy(i,j); %
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=x(j);
Ye(i,j)=Tm(i,j)*Xp(i,j)+
Vxp(i,j)*y(i));
Xe(i,j); Ye(i,j);
% disp('I dont know')
444
555
return

else (Vx(i,j)>0 &
Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i,j+1)<0 &
Vy(i,j)=Vy(i+1,j))
% disp('5-6')
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/y(i+1)-
Vyp(i,j)=(Ay(i,j)* (Yp(i,j)-
y(i)));Vy(i,j);
Ty(i,j)=(1/Ay(i,j))*log (Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=x(j);
Ye(i,j)=Tm(i,j)*Xp(i,j)+
Vxp(i,j)*y(i));
Xe(i,j); Ye(i,j);
\[
\begin{align*}
Y(i,j) &= (1/Ay(i,j))^\ast \exp[Ay(i,j) \ast Tm(i,j) - Vy(i,j)] \ast y(i); \\
Xe(i,j); \\
Ye(i,j); \\
\end{align*}
\]

\% - second model

\begin{align*}
\text{else} \quad \{Vx(i,j) > 0 \& Vx(i,j+1) < 0 \& Vy(i,j) < 0 \& Vy(i+1,j) > 0\} \\
\text{\% disp(' -6-4')}
\end{align*}

\begin{align*}
Ay(i,j) &= (Vy(i+1,j) - Vy(i,j))/(y(i+1) - y(i)); \\
Vp(i,j) &= (Ay(i,j) \ast (Yp(i,j) - y(i)))/Vy(i,j); \\
Xe(i,j) &= x(j); \\
\text{if} \quad Vp(i,j) < 0 \\
\text{\% disp(' -6-4-1')}
\end{align*}

\begin{align*}
Ye(i,j) &= y(i); \\
Xe(i,j); \\
Ye(i,j); \\
\text{else} \quad Vp(i,j) > 0 \\
\text{\% disp(' -6-4-2')}
\end{align*}

\begin{align*}
Xe(i,j) &= x(j); \\
Ye(i,j) &= y(i+1); \\
Xe(i,j); \\
Ye(i,j); \\
\text{end}
\end{align*}

\begin{align*}
\text{else} \quad \{Vx(i,j) > 0 \& Vx(i,j+1) < 0 \& Vx(i+1,j) < 0 \& Vy(i+1,j) > 0\} \\
\text{\% disp(' -10-4')}
\end{align*}

\begin{align*}
Ay(i,j) &= (Vy(i+1,j) - Vy(i,j))/(y(i+1) - y(i)); \\
Vp(i,j) &= (Ay(i,j) \ast (Yp(i,j) - y(i)))/Vy(i,j); \\
Ty(i,j) &= (1/Ay(i,j))^\ast \log [Vy(i+1,j) / Vy(i,j)]; \\
\text{if} \quad Ty(i,j) < 0 \\
Ty(i,j) &= Ty(i,j)^{-1}; \\
\end{align*}

\begin{align*}
Tm(i,j) &= Ty(i,j); \\
Xe(i,j) &= x(j); \\
Ye(i,j); \\
\end{align*}

\begin{align*}
\text{else} \quad \{Vx(i,j) > 0 \& Vx(i+1,j) < 0 \& Vx(i,j+1) < 0 \& Vx(i+1,j+1) > 0\} \\
\text{\% disp(' -8-4')}
\end{align*}

\begin{align*}
Ax(i,j) &= (Vx(i,j+1) - Vx(i,j))/x(j+1) - x(j)); \\
Ay(i,j) &= (Vy(i+1,j) - Vy(i,j))/(y(i+1) - y(i)); \\
Vp(i,j) &= (Ay(i,j) \ast (Yp(i,j) - y(i)))/Vy(i,j); \\
Y(i,j); \\
\end{align*}

\begin{align*}
\text{else} \quad \{Vx(i,j) > 0 \& Vx(i+1,j) < 0 \& Vx(i,j+1) < 0 \& Vx(i+1,j+1) > 0\} \\
\text{\% disp(' -11-4')}
\end{align*}

\begin{align*}
3333 \\
Xe(i,j) &= x(j+1); \\
Ye(i,j) &= y(i); \\
\end{align*}

222

\text{rr} = 1
```matlab
elseif (Vx(i,j)>0 & Vx(i,j+1)<0 &Vy(i,j+1)>0 & Vy(i+1,j)<0 )
   disp('I dont know')
end

rr=1

elseif (Vx(i,j)>0 & Vx(i,j+1)==0 & Vy(i,j)>0 & Vy(i+1,j)>0 &Vy(i,j)==Vy(i+1,j))
   disp('-15-5')
   Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j));
   Vxp(i,j)=(Ax(i,j)*x(j+1)-x(j)))/Vx(i,j);%
   Ty(i,j)=(1/2*(Vx(i,j)+Vx(i,j+1)))*log(Vx(i,j+1)/Vx(i,j));
   Xe(i,j)=(1/2*(Tx(i,j)+Tx(i,j+1)))/Ax(i,j)*Vx(i,j));
   Tm(i,j)=min(Tx(i,j),Ty(i,j));
   Xe(i,j)=(1/2*(Ax(i,j)+Ax(i,j+1)))*log(Vx(i,j+1)/Vx(i,j));
   Vxp(i,j)=x(j+1)-x(j));
   Yp(i,j)/Vx(i,j);%
   Xe(i,j)=(1/2*(Ax(i,j)+Ax(i,j+1)))*log(Vx(i,j+1)/Vx(i,j))
   Tx(i,j)=Tm(i,j);%
   Xe(i,j)=(1/2*(Ax(i,j)+Ax(i,j+1)))*log(Vx(i,j+1)/Vx(i,j));
   Tm(i,j)=Tm(i,j);%
   Xe(i,j)=(1/2*(Ax(i,j)+Ax(i,j+1)))*log(Vx(i,j+1)/Vx(i,j));
   Vxp(i,j)=x(j));
   Xe(i,j)=x(j));
end
```

\[ \begin{align*}
\text{if } T(i,j) < 0 & \quad & T(i,j) = T(i,j) \times (-1); \\
\text{end} & & Xe(i,j) = (Tm(i,j) * Vxp(i,j) + x(j)); \\
\text{Xv(i,j)} &= (x(j) - \sqrt{x(i,j)}) * \exp(Ax(i,j) * Tm(i,j) - Vx(i,j)); \\
\text{Tm(i,j)} &= (1/Ax(i,j)) * \exp(Ax(i,j) * Tm(i,j) - Vx(i,j)); \\
\text{else} & & \text{Yv(i,j)} = \text{Yv(i,j)} + \text{Yp(i,j)};
\end{align*} \]
\[
\begin{align*}
Y_v(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) \\
X_v(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) \\
X_e(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) \\
X_p(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i)
\end{align*}
\]

else if \((V_x(i,j) \cdot x(i,j)) \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) < 0 \)
\[
\begin{align*}
Y_v(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) \\
X_v(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) \\
X_e(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) \\
X_p(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i)
\end{align*}
\]

else if \((V_x(i,j) \cdot x(i,j)) \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) < 0 \)
\[
\begin{align*}
Y_v(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) \\
X_v(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) \\
X_e(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i) \\
X_p(i,j) & = \frac{1}{A(i,j)} \cdot \{V_y(i,j) \cdot \exp(A(i,j) \cdot T(i,j)) \} \cdot y(i)
\end{align*}
\]
\% Ty(i,j) = (1/Ay(i,j))*log
(Vy(i+1,j) / Vy(i,j));
Ty(i,j) = (y(i+1) - 
Vy(i,j))/Vy(i,j); 
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j)*(-1); 
end 
if Tx(i,j) < 0
Tx(i,j) = Tx(i,j)*(-1); 
end 
Tm(i,j) = min(Tx(i,j),Ty(i,j)); 
\% Xe(i,j) = (Tm(i,j) * Vxp(i,j) + X[i]);
Xe(i,j) = (1/AX(i,j))*(Vxp(i,j)*exp(Ax(i,j)) - Tm(i,j)*x[i]);
Ye(i,j) = (1/AY(i,j))*((Vyp(i,j)*exp(Ay(i,j)) - Tm(i,j))*y[i]);
Xe(i,j); 
Ye(i,j); 
else 
if Vx(i,j) > 0 
Vx(i+1,j) = 0 & Vy(i,j) = 0 & Vy(i+1,j) = 0 
\% disp('-13-5')
\% disp('I dont know')

rr = 1
\% elseif (Vx(i,j) < 0 & Vx(i,j+1) > 0 & 
Vy(i,j) > 0 & Vy(i,j+1) > 0 & 
Vy(i,j) = Vy(i+1,j); 
\% disp('-1-6')
if ( Vxp(i,j) < 0 
\% disp('-1-6-1')
Ay(i,j) = (Vy(i+1,j) - Vy(i,j))/(y(i+1) - y(i)); 
Vyp(i,j) = (Ay(i,j) - (Yp(i,j) - 
Y(i))/Vyp(i,j); 
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j)*(-1); 
end 
Tm(i,j) = Ty(i,j); 
Xe(i,j) = x[t]; 
Ye(i,j) = (Tm(i,j) * Vxp(i,j) + X[i]);
Xe(i,j); 
Ye(i,j); 
else ( Vxp(i,j) > 0 
\% disp('-2-6-2')
Ay(i,j) = (Vy(i+1,j) - Vy(i,j))/(y(i+1) - y(i)); 
Vyp(i,j) = (Ay(i,j) - (Yp(i,j) - 
Y(i))/Vyp(i,j); 
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j)*(-1); 
end 
Tm(i,j) = Ty(i,j); 
Xe(i,j) = x[t+1];
Ye(i,j) = (Tm(i,j) * Vxp(i,j) + X[i]);
Xe(i,j); 
Ye(i,j); 
else
Xe(i,j) = x[j+1]; 
Ye(i,j) = (Tm(i,j) * Vxp(i,j) + X[i]);
Xe(i,j); 
Ye(i,j); 
end 
elseif (Vx(i,j) < 0 & Vx(i,j+1) > 0 & 
Vy(i,j) > 0 & Vy(i+1,j) < 0 & 
Vy(i+1,j) > Vy(i,j); 
\% disp('-2-6-1')
if ( Vxp(i,j) < 0
\% disp('-2-6-1')
Ay(i,j) = (Vy(i+1,j) - 
Vy(i,j))/(y(i+1) - y(i)); 
Vyp(i,j) = (Ay(i,j) - (Yp(i,j) - 
Y(i))/Vyp(i,j); 
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j)*(-1); 
end 
Tm(i,j) = Ty(i,j); 
Xe(i,j) = x[t];
Ye(i,j) = (Tm(i,j) * Vxp(i,j) + X[i]);
Xe(i,j); 
Ye(i,j); 
end 
elseif (Vx(i,j) < 0 & Vx(i,j+1) > 0 & 
Vy(i,j) < Vy(i+1,j); 
\% disp('-3-6')
if ( Vxp(i,j) < 0
\% disp('-3-6-1')
Ax(i,j) = (Vy(i+1,j) - 
Vy(i,j))/(x[i+1] - x[i]); 
Vxp(i,j) = Ax(i,j) * (Xp(i,j) - 
X[i])/Vxp(i,j); 
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j)*(-1); 
end 
Tm(i,j) = Ty(i,j); 
Xe(i,j) = x[t+1];
Ye(i,j) = (Tm(i,j) * Vxp(i,j) + X[i]);
Xe(i,j); 
Ye(i,j); 
end 
elseif (Vx(i,j) < 0 & Vx(i,j+1) > 0 & 
Vy(i,j) > Vy(i+1,j); 
\% disp('-3-6-1')
Ax(i,j) = (Vy(i+1,j) - 
Vy(i,j))/(x[i+1] - x[i]); 
Vxp(i,j) = Ax(i,j) * (Xp(i,j) - 
X[i])/Vxp(i,j); 
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j)*(-1); 
end 
Tm(i,j) = Ty(i,j); 
Xe(i,j) = x[t];
Ye(i,j) = (Tm(i,j) * Vxp(i,j) + X[i]);
Xe(i,j); 
Ye(i,j); 
elseif (Vx(i,j) < 0 & Vx(i,j+1) > 0 & 
Vy(i,j) > Vy(i+1,j); 
\% disp('-3-6-1')
Ax(i,j) = (Vy(i+1,j) - 
Vy(i,j))/(x[i+1] - x[i]); 
Vxp(i,j) = Ax(i,j) * (Xp(i,j) - 
X[i])/Vxp(i,j); 
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j)*(-1); 
end 
Tm(i,j) = Ty(i,j); 
Xe(i,j) = x[t];
Ye(i,j) = (Tm(i,j) * Vxp(i,j) + X[i]);
Xe(i,j); 
Ye(i,j); 
end
\[
Ay(i,j) = (Vy(i+1,j) - Vyi(i,j))/(y(i+1) - y(i));
Vy(i,j) = Ay(i,j) \cdot (y(i+1) - y(i)) + Vy(i,j);
Ty(i,j) = (1/Ay(i,j)) \cdot \log((Vy(i+1,j)/Vy(i,j))); 
\]
if \(Ty(i,j) < 0\)
\[
Ty(i,j) = Ty(i,j) \cdot (-1); 
\]
end

\[
Tm(i,j) = Ty(i,j);
Xe(i,j) = x(j+1);
Ye(i,j) = \begin{cases} 
(1/Ay(i,j)) \cdot \exp(Ay(i,j) \cdot Tm(i,j)) \cdot Vy(i,j) + y(i); & \text{if } Vx(i,j) > 0 \\
Vy(i,j) + x(j); & \text{else if } Vx(i,j) < 0 \end{cases}
\]
else
\[
Vx(i,j) < 0 \& Vx(i,j+1) > 0 \\
Vy(i,j) < 0 \& Vy(i+1,j) > 0
\]
\[
\% \text{ disp(} ' -8-6' \)
if \(Vx(i,j) < 0\)
\[
Ay(i,j) = (Vy(i+1,j) - Vyi(i,j))/(y(i+1) - y(i));
Vy(i,j) = Ay(i,j) \cdot (y(i+1) - y(i)) + Vy(i,j);
Ty(i,j) = (1/Ay(i,j)) \cdot \log((Vy(i+1,j)/Vy(i,j))); 
\]
if \(Ty(i,j) < 0\)
\[
Ty(i,j) = Ty(i,j) \cdot (-1); 
\]
end

\[
Tm(i,j) = Ty(i,j);
Xe(i,j) = x(j+1);
Ye(i,j) = \begin{cases} 
(1/Ay(i,j)) \cdot \exp(Ay(i,j) \cdot Tm(i,j)) \cdot Vy(i,j) + y(i); & \text{if } Vx(i,j) > 0 \\
Vy(i,j) + x(j); & \text{else if } Vx(i,j) < 0 \end{cases}
\]
eelse
\[
Vx(i,j) > 0 \& Vx(i,j+1) > 0 \\
Vy(i,j) < 0 \& Vy(i+1,j) < 0 \\
Vy(i,j) = 0
\]
\[
\% \text{ disp(} ' -9-6-2' \)
Ay(i,j) = (Vy(i+1,j) - Vyi(i,j))/(y(i+1) - y(i));
Vy(i,j) = Ay(i,j) \cdot (y(i+1) - y(i)) + Vy(i,j);
Ty(i,j) = (1/Ay(i,j)) \cdot \log((Vy(i+1,j)/Vy(i,j))); 
\]
if \(Ty(i,j) < 0\)
\[
Ty(i,j) = Ty(i,j) \cdot (-1); 
\]
end

\[
Tm(i,j) = Ty(i,j);
Xe(i,j) = x(j+1);
Ye(i,j) = [Tm(i,j) + Vx(i,j)] \cdot (y(i+1) - y(i));
\]
else
\[
Vx(i,j) < 0 \& Vx(i,j+1) = 0 \\
Vy(i,j) < 0 \& Vy(i+1,j) = 0
\]
\[
\% \text{ disp(} ' -10-6' \)
Ay(i,j) = (Vy(i+1,j) - Vyi(i,j))/(x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) \cdot (xp(i,j) - x(j)) + Vy(i,j)); 
\]
if \(Vxp(i,j) < 0\)
\[
\% \text{ disp(} '-10-6-1' \)
Ay(i,j) = (Vy(i+1,j) - Vyi(i,j))/(y(i+1) - y(i));
Vyp(i,j) = Ay(i,j) \cdot (y(i+1) - y(i)) + Vyp(i,j);
Ty(i,j) = (1/Ay(i,j)) \cdot \log((Vyi(i,j) + Vyp(i,j))); 
\]
if \(Ty(i,j) < 0\)
\[
Ty(i,j) = Ty(i,j) \cdot (-1); 
\]
end

\[
Tm(i,j) = Ty(i,j);
Xe(i,j) = x(j); 
Ye(i,j) = \begin{cases} 
(1/Ay(i,j)) \cdot \exp(Ay(i,j) \cdot Tm(i,j)) \cdot Vy(i,j) + y(i); & \text{if } Vx(i,j) > 0 \\
Vy(i,j) + x(j); & \text{else if } Vx(i,j) < 0 \end{cases}
\]
eelse
\[
Vx(i,j) < 0 \& Vx(i,j+1) > 0 \\
Vy(i,j) < 0 \& Vy(i+1,j) > 0 \\
Vy(i,j+1) > 0
\]
\[
\% \text{ disp(} ' -9-6 ' \)
if \(Vxp(i,j) < 0\)
\[
\% \text{ disp(} '-9-6-1' \)
Ye(i,j) = (1/Ay(i,j)) \cdot \exp(Ay(i,j) \cdot Tm(i,j)) \cdot Vy(i,j) + y(i); 
Xe(i,j) = x(j+1);
Ye(i,j) ; 
end
}
Ye(i,j);
else Vxp(i,j)>0
    disp('10-6-2')
    Ay(i,j)=(Vy(i+1,j)-
    Vy(i,j))/(y(i+1)-y(i));
    Vxp(i,j)=Ay(i,j)*[Vp(i,j)-
    y(i)]+Vy(i,j);
    Ty(i,j)=(1/Ay(i,j))*log
    (Vy(i+1,j)/Vyp(i,j));
    Ty(i,j)=(y(i)-
    Vyp(i,j)/Vy(i,j));
    if Ty(i,j)<0
        Ty(i,j) =Ty(i,j)*(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(i+1);
end
Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(1,j)*
    Tm(i,j))))-Vy(i,j))*y(i);
Xe(i,j);
Ye(i,j);
end

%  - third model
elseif (Vx(i,j)<0 & Vx(i,j+1)>0 & Vx(i,j+2)>0) & Vy(i,j)>0 & Vy(i+1,j)>0 &
    disp('1-16')
    Ax(i,j)=(Vx(i,j+1)-
    Vx(i,j))/(x(i+1)-x(i));
    Vxp(i,j)=Ax(i,j)*[Xp(i,j)-
    x(i)]+Vx(i,j);
    if Vxp(i,j)<0
        disp('1-16-1')
        Ax(i,j)=(Vy(i+1,j)-
    Vy(i,j))/(y(i+1)-y(i));
    Vyp(i,j)=Ay(i,j)*[Yp(i,j)-
    y(i)]+Vy(i,j);
    Ty(i,j)=(1/Ay(i,j))*log
    (Vy(i+1,j)/Vyp(i,j));
    if Ty(i,j)<0
        Ty(i,j) =Ty(i,j)*(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(i);
end
%  Vx(i,j)>0 & Vy(i,j)>0 &
Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(1,j)*
    Tm(i,j))))-Vy(i,j))*y(i);
Xe(i,j);
Ye(i,j);
else Vxp(i,j)<0
    disp('1-16-2')
    Ay(i,j)=(Vy(i+1,j)-
    Vy(i,j))/(y(i+1)-y(i));
    Vyp(i,j)=Ay(i,j)*[Yp(i,j)-
    y(i)]+Vy(i,j);
    Ty(i,j)=(1/Ay(i,j))*log
    (Vy(i+1,j)/Vyp(i,j));
    if Ty(i,j)<0
        Ty(i,j) =Ty(i,j)*(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(i+1);
end
Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(1,j)*
    Tm(i,j))))-Vy(i,j))*y(i);
Xe(i,j);
Ye(i,j);
else Vxp(i,j)>0
    disp('1-16-2')
    Ay(i,j)=(Vy(i+1,j)-
    Vy(i,j))/(y(i+1)-y(i));
    Vyp(i,j)=Ay(i,j)*[Yp(i,j)-
    y(i)]+Vy(i,j);
    Ty(i,j)=(1/Ay(i,j))*log
    (Vy(i+1,j)/Vyp(i,j));
    if Ty(i,j)<0
        Ty(i,j) =Ty(i,j)*(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(i);
Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j)) + Vx(i,j);
if Vxp(i,j) <= 0
% disp(' 13-6-1')
Xe(i,j) = x(j);
Ye(i,j) = y(i);
else
Vxp(i,j) > 0
% disp('13-6-2')
Xe(i,j) = x(j) + 1;
Ye(i,j) = y(i) + 1;
end

% disp('-seventh model')
% first model
elseif (Vx(i,j) <= 0 & Vx(i,j+1) <= 0 &
Vv(i,j) == Vx(i,j+1) & Vy(i,j) > 0 &
Vy(i,j+1) > 0 & Vy(i,j+2) == Vy(i,j+1))
% disp('13-7-1')
Ax(i,j) = Vx(i,j+1) - Vx(i,j) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Vp(i,j) - x(j))) + Vx(i,j);
% disp('13-7-2')
Ty(i,j) = (1/Ax(i,j)) * log(Vx(i,j+1) / Vp(i,j));
Txi(j) = Ty(i,j) / (y(i+1) - y(i));
Vyi(j) = (Ay(i,j) * (Vp(i,j) - y(i))) + Vy(i,j);
% disp('13-7-3')
Ty(i,j) = (1/Ay(i,j)) * log(Vy(i,j+1) / Vp(i,j));
Ypi(j) = (Ay(i,j) * (Vp(i,j) - y(i))) +Vy(i,j);
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) + 1;
end
end
else
Vxp(i,j) = x(j) + 1;
Vxp(i,j) = Ty(i,j) + 1;
end

% disp(' 4-7-1')
Ax(i,j) = Vx(i,j+1) - Vx(i,j) / (x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j);
% disp(' 4-7-2')
Vx(i,j) = (Ax(i,j) * Vp(i,j) - x(j)) / Ay(i,j);
Vxp(i,j) = (Ax(i,j) * Vp(i,j) - x(j)) / Ay(i,j);

Tm(i,j) = min (Tx(i,j), Ty(i,j));
Xe(i,j) = Tm(i,j) * Xp(i,j) + x(j);
Ye(i,j) = Tm(i,j) * Yp(i,j) + y(i);
Ye(i,j)=y(i); 

Xe(i,j); 
Ye(i,j);

elseif (Vx(i,j)<0 & 
Vx(i,j+1)<0 & Vx(i,j)==Vx(i,j+1) & 
Vy(i,j)<0 & Vy(i+j+1)==0 ) 
% disp('5-7') 
Ax(i,j)=(Vx(i,j+1)- 
Vx(i,j))/[(x(j+1)-x(j)); 
Vxp(i,j)= (Ax(i,j)* (Xp(i,j)- 
x(j)))+Vx(i,j); 

% Tm(i,j)=-(1/Ax(i,j))*log 
(Vx(i,j+1)/Vxp(i,j)) 
Tx(i,j)=[(x(j)- 
Xp(i,j))/Vxp(i,j)); 

Ay(i,j)=[Vy(i+1,j)- 
Vy(i,j)]; 
Vyp(i,j)= (Ny(i,j)* (Yp(i,j)- 
y(i))); 

Ty(i,j)=(y(i)-Yp(i,j))/Vy(i,j); 

if Ty(i,j)<0 
Ty(i,j)=Ty(i,j)*(-1); 
end 

if Tx(i,j)<0 
Tx(i,j)=Tx(i,j)*(-1); 
end 

Tm(i,j)=min (Tx(i,j),Ty(i,j)); 
Xe(i,j)=[Tm(i,j)* 
Vxp(i,j)]+x(j); 

Ye(i,j)=((1/Ay(i,j)))* ((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j))))+y(i); 

Xe(i,j) 
Ye(i,j); 

elseif (Vx(i,j)<0 & 
Vx(i,j)<0 & Vx(i,j+1)<0 & 
Vy(i+1,j)>0 ) 
% disp('6-7') 
Ax(i,j)=(Vx(i,j+1)- 
Vx(i,j))/[(x(j+1)-x(j)); 
Vxp(i,j)= (Ax(i,j)* (Xp(i,j)- 
x(j)))+Vx(i,j); 

% Tm(i,j)=-(1/Ax(i,j))*log 
(Vx(i,j+1)/Vxp(i,j)) 
Tx(i,j)=[(x(j)- 
Xp(i,j))/Vxp(i,j)); 

Ay(i,j)=[Vy(i+1,j)- 
Vy(i,j)]; 
Vyp(i,j)= (Ny(i,j)* (Yp(i,j)- 
y(i))); 

Ty(i,j)=(y(i)-Yp(i,j))/Vy(i,j); 

if Vyp(i,j)<0 
% disp('6-7') 

if Ty(i,j)<0 
Ty(i,j)=Ty(i,j)*(-1); 
end 

if Tx(i,j)<0 
Tx(i,j)=Tx(i,j)*(-1); 
end 

Tm(i,j)=Tx(i,j); 
Xe(i,j)=[Tm(i,j)* 
Vxp(i,j)]+x(j); 

Ye(i,j)=((1/Ay(i,j)))* ((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j))))+y(i); 

Xe(i,j) 
Ye(i,j);
if Ty(i,j)<0  
Ty(i,j) = Ty(i,j)*(-1);  
end  
if Tx(i,j)<0  
Tx(i,j) = Tx(i,j)*(-1);  
end  
Tm(i,j) = min (Tx(i,j),Ty(i,j));  
Xe(i,j) = (Tm(i,j)*  
Vxp(i,j)) + x(j);  
Ye(i,j) = ((1/Ay(i,j)) * (Vxp(i,j)*exp(Ay(i,j) * Tm(i,j)) - Vy(i,j)) + y(i);  
Xe(i,j);  
Ye(i,j);  
else if (Vx(i,j)<0 & Vx(i,j+1)<0 &  
& Vx(i,j)==Vx(i,j+1) & Vy(i,j)==0 &  
Vy(i,j+1)<0 & Vy(i,j)==Vy(i,j+1))  
% disp(' -9-7')  
Ax(i,j)=Vx(i,j)+1 - 
Vx(i,j)/x(j)+1-x(j));  
Vxp(i,j)=Ax(i,j)*Xp(i,j) 
/ (x(j)+1-x(j));  
Ay(i,j)=Vy(i,j)+1 - 
Vy(i,j)/y(j)+1-y(j);  
Vyp(i,j)=Ay(i,j)*Yp(i,j) 
/ (y(j)+1-y(j));  
Ty(i,j) = (y(j)- 
Yp(i,j))/Vy(i,j);  
if Ty(i,j)<0  
Ty(i,j) = Ty(i,j)*(-1);  
end  
if Tx(i,j)<0  
Tx(i,j) = Tx(i,j)*(-1);  
end  
Tm(i,j) = min (Tx(i,j),Ty(i,j));  
Xe(i,j) = (Tm(i,j)*  
Vxp(i,j)) + x(j);  
Ye(i,j) = (Tm(i,j)*  
Vyp(i,j)) + y(i);  
Xe(i,j);  
Ye(i,j);  
else if (Vx(i,j)<0 &  
& Vx(i,j+1)<0 & Vy(i,j)==Vx(i,j+1) &  
& Vx(i,j)==Vx(i,j+1) & Vy(i,j)==0 &  
Vy(i,j+1)<0 & Vy(i,j)==Vy(i,j+1)  
% disp(' -11-7')  
Ax(i,j)=(Vx(i,j)+1 - 
Vx(i,j)/x(j)+1-x(j));  
Vxp(i,j)=Ax(i,j)*Xp(i,j) 
/ (x(j)+1-x(j));  
% Tx(i,j)=(1/Ax(i,j))*log  
(Vx(i,j)+1-Vx(i,j))/Vxp(i,j);  
Vxp(i,j)=(x(j)- 
Xp(i,j)/Vxp(i,j);  
Ay(i,j)=(Vy(i,j)+1 - 
Vy(i,j)/y(j)+1-y(j);  
Vyp(i,j)=(Ay(i,j)*Yp(i,j) 
/ (y(j)+1-y(j));  
Ty(i,j) = (y(j)- 
Yp(i,j))/Vy(i,j);  
if Ty(i,j)<0  
Ty(i,j) = Ty(i,j)*(-1);  
end  
if Tx(i,j)<0  
Tx(i,j) = Tx(i,j)*(-1);  
end  
Tm(i,j) = min (Tx(i,j),Ty(i,j));  
Xe(i,j) = (Tm(i,j)*  
Vxp(i,j)) + x(j);  
Ye(i,j) = (Tm(i,j)*  
Vyp(i,j)) + y(i);  
Xe(i,j);  
Ye(i,j);  
else if (Vx(i,j)<0 & Vx(i,j+1)<0 &  
& Vx(i,j+1)<0 & Vy(i,j)==Vx(i,j+1) &  
& Vx(i,j)==Vx(i,j+1) & Vy(i,j)==0 &  
Vy(i,j+1)<0 & Vy(i,j)==Vy(i,j+1)  
% disp(' -12-7')  
Ax(i,j)=(Vx(i,j)+1 - 
Vx(i,j)/x(j)+1-x(j));  
Vxp(i,j)=Ax(i,j)*Xp(i,j) 
/ (x(j)+1-x(j));  
% Tx(i,j)=(1/Ax(i,j))*log  
(Vx(i,j)+1-Vx(i,j))/Vxp(i,j);  
T(x(i,j))=(x(j)- 
Xp(i,j)/Vxp(i,j);  
Ay(i,j)=(Vy(i,j)+1 - 
Vy(i,j)/y(j)+1-y(j);  
Vyp(i,j)=(Ay(i,j)*Yp(i,j) 
/ (y(j)+1-y(j));  
Ty(i,j) = (y(j)- 
Yp(i,j))/Vy(i,j);  
if Ty(i,j)<0  
Ty(i,j) = Ty(i,j)*(-1);  
end  
if Tx(i,j)<0  
Tx(i,j) = Tx(i,j)*(-1);  
end  
Tm(i,j) = min (Tx(i,j),Ty(i,j));  
Xe(i,j) = (Tm(i,j)*  
Vxp(i,j)) + x(j);  
Ye(i,j) = (Tm(i,j)*  
Vyp(i,j)) + y(i);  
Xe(i,j);  
Ye(i,j);
else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j)>Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)<0 )
    disp(' -4-8-1')
    Ax(i,j) = (Vx(i+1,j) - Vx(i,j))/x(j+i+1)-x(j); 
    Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j))/Vx(i,j); 
    Tx(i,j) = (1/4)/(Ax(i,j))*log(Vx(i,j)+1)/Vxp(i,j);
    if Tx(i,j)<0
    Ta(i,j) = Tx(i,j)*(1-1);
    end
    Xe(i,j) = ((1/4)/(Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*y(i)); 
    Ye(i,j) = y(i);
    Xe(i,j);
    Ye(i,j);

else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j)>Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)<0 )
    disp(' +4-8-1')
    Ax(i,j) = (Vx(i+1,j) - Vx(i,j))/x(j+i+1)-x(j); 
    Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j))/Vx(i,j); 
    Tx(i,j) = (1/4)/(Ax(i,j))*log(Vx(i,j)+1)/Vxp(i,j); 
    if Tx(i,j)<0
    Ta(i,j) = Tx(i,j)*(1-1);
    end
    Xe(i,j) = ((1/4)/(Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*y(i)); 
    Ye(i,j) = y(i);
    Xe(i,j);
    Ye(i,j);

end

if Vyp(i,j)<0 % disp(' -6-8-1')
    Ty(i,j) = (1/4)/(Ax(i,j)) * log(Vy(i+1,j)/Vyp(i,j));
    if Ty(i,j)<0
    Tt(i,j) = Ty(i,j)*(-1);
    end
    Xe(i,j) = ((1/4)/(Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)))*y(i)); 
    Ye(i,j) = y(i);
    Xe(i,j);
    Ye(i,j);

else
    Vyp(i,j)>0 % disp(' +6-8-2')
    Ax(i,j) = (Vx(i+1,j) - Vx(i,j))/x(j+i+1)-x(j); 
    Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j))/Vx(i,j); 
    Tx(i,j) = (1/4)/(Ax(i,j))*log(Vx(i,j)+1)/Vxp(i,j); 
    if Tx(i,j)<0
    Ta(i,j) = Tx(i,j)*(-1);
    end
    Xe(i,j) = ((1/4)/(Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)))*y(i)); 
    Ye(i,j) = y(i+1);
    Xe(i,j);
    Ye(i,j);
end

end

Xe(i,j) = ((1/4)/(Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*y(i)); 
Ye(i,j) = y(i);
Xe(i,j);
Ye(i,j);

elseif (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j)>Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)<0 & Vy(i+1,j+1)<0 & Vy(i+1,j+1)>0 )
    disp(' -6-8-3')
    Ax(i,j) = (Vx(i+1,j) - Vx(i,j))/x(j+i+1)-x(j); 
    Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j))/Vx(i,j); 
    Tx(i,j) = (1/4)/(Ax(i,j))*log(Vx(i,j)+1)/Vxp(i,j); 
    if Tx(i,j)<0
    Ta(i,j) = Tx(i,j)*(-1);
    end
    Xe(i,j) = ((1/4)/(Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*y(i)); 
    Ye(i,j) = y(i); 
    Xe(i,j);
    Ye(i,j);

else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j)>Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)<0 & Vy(i+1,j+1)<0 & Vy(i+1,j)+1)<0 & Vy(i+2,j+1)>0 )
    disp(' +6-8-4')
    Ax(i,j) = (Vx(i+1,j) - Vx(i,j))/x(j+i+1)-x(j); 
    Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j))/Vx(i,j); 
    Tx(i,j) = (1/4)/(Ax(i,j))*log(Vx(i,j)+1)/Vxp(i,j); 
    if Tx(i,j)<0
    Ta(i,j) = Tx(i,j)*(-1);
    end
    Xe(i,j) = ((1/4)/(Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*y(i)); 
    Ye(i,j) = y(i+1);
    Xe(i,j);
    Ye(i,j);

elseif (Vx(i,j)<0 & Vx(i,j)+1)<0 & Vx(i,j)>Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)<0 & Vy(i+1,j+1)<0 & Vy(i+1,j+1)>0 )
    disp(' -7-8-1')
    Ax(i,j) = (Vx(i+1,j) - Vx(i,j))/x(j+i+1)-x(j); 
    Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j))/Vx(i,j); 
    Tx(i,j) = (1/4)/(Ax(i,j))*log(Vx(i,j)+1)/Vxp(i,j); 
    if Tx(i,j)<0
    Ta(i,j) = Tx(i,j)*(-1);
    end
    Xe(i,j) = ((1/4)/(Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*y(i)); 
    Ye(i,j) = y(i+1);
    Xe(i,j);
    Ye(i,j);

else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j)>Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)<0 & Vy(i+1,j+1)<0 & Vy(i+1,j+1)>0 )
    disp(' +7-8-2')
    Ax(i,j) = (Vx(i+1,j) - Vx(i,j))/x(j+i+1)-x(j); 
    Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j))/Vx(i,j); 
    Tx(i,j) = (1/4)/(Ax(i,j))*log(Vx(i,j)+1)/Vxp(i,j); 
    if Tx(i,j)<0
    Ta(i,j) = Tx(i,j)*(-1);
    end
    Xe(i,j) = ((1/4)/(Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*y(i)); 
    Ye(i,j) = y(i); 
    Xe(i,j);
    Ye(i,j);

end

Xe(i,j) = ((1/4)/(Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*y(i)); 
Ye(i,j) = y(i);
Xe(i,j);
Ye(i,j);
else if \((Vx(i,j)) < 0 \& Vx(i,j+1) > 0\) 
& \(Vx(i,j+1) > Vx(i,j) \& Vy(i,j+1) > 0 \& Vy(i,j+1) > Vy(i,j)\) 
\(\% \text{ disp(' } -8-8\text{'})\)

\[
Ax(i,j) = (Vx(i,j+1) - Vx(i,j)) / (x[j+1] - x[j]);
\]

\[
Vxp(i,j) = (Ax(i,j) \times (Xp(i,j) - x[j])) + Vx(i,j);
\]

\[
Tx(i,j) = (1 / Ax(i,j)) \times \log\left(\frac{Vx(i,j+1)}{Vx(i,j)}\right);
\]

\[
Ay(i,j) = (Vy(i+1,j) - Vy(i,j)) / (y[i+1] - y[i]);
\]

\[
Vyp(i,j) = (Ay(i,j) \times (Yp(i,j) - y[i])) + Vy(i,j);
\]

\[
Ty(i,j) = (1 / Ay(i,j)) \times \log\left(\frac{Vy(i+1,j)}{Vy(i,j)}\right);
\]

\[
\{Vy(i+1,j)/Vy(i,j)\};
\]

if \(Ty(i,j) < 0\) 
\(Ty(i,j) = Ty(i,j)*(-1)\);
end
if \(Tx(i,j) < 0\) 
\(Tx(i,j) = Tx(i,j)*(-1)\);
end
\(Tm(i,j) = \min\{Tx(i,j), Ty(i,j)\}\);

\[
Xe(i,j) = ((1 / Ax(i,j)) \times \{Vxp(i,j) \times \exp(Ax(i,j) \times Tm(i,j))\}) - Vx(i,j);\]

\[
Xe(i,j) = ((1 / Ay(i,j)) \times \{Vyp(i,j) \times \exp(Ay(i,j) \times Tm(i,j))\}) - Vy(i,j);\]

\[
Ye(i,j) = ((1 / Ay(i,j)) \times \{Vyp(i,j) \times \exp(Ay(i,j) \times Tm(i,j))\}) - Vy(i,j);\]

\[
Ye(i,j) = ((1 / Ay(i,j)) \times \{Vyp(i,j) \times \exp(Ay(i,j) \times Tm(i,j))\}) - Vy(i,j);\]

\[
Ye(i,j) = ((1 / Ay(i,j)) \times \{Vyp(i,j) \times \exp(Ay(i,j) \times Tm(i,j))\}) - Vy(i,j);\]

\[
Ye(i,j) = ((1 / Ay(i,j)) \times \{Vyp(i,j) \times \exp(Ay(i,j) \times Tm(i,j))\}) - Vy(i,j);\]
```plaintext
else (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j)>Vx(i+1,j+1) & Vy(i,j)< Vy(i+1,j) & Vy(i,j+1)<Vy(i+1,j)) & (Vy(i+1,j)<0 & Vx(i,j)<Vx(i+1,j+1) & Vx(i,j+1)<0)
  % disp(' -12-8')
  Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j)+x(j);
  Vxp(i,j)=Ax(i,j)*(Xp(i,j)-x(j));
  Vxp(i,j+1)=Ax(i,j+1)*(Xp(i,j+1)-x(j));
  Tx(i,j)=1/(Ax(i,j))*log((Vx(i,j+1)-Vxp(i,j))/x(j));
  Ay(i,j+1)=Vy(i,j+1)-Vy(i,j);
  Vyp(i,j)=Ay(i,j+1)*(Yp(i,j+1)-y(j))
  %
  Ty(i,j)=1/(Ay(i,j))*log((Vy(i+1,j)-Vyp(i,j))/y(j));
  Ty(i,j+1)=(y(i+1)-y(i))/y(i+1)
  end
end

```

```matlab
Tm(i,j) = min (Tx(i,j),Ty(i,j));

Xe(i,j) = ((1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))) - Vx(i,j)))*x(j);
Ye(i,j) = ((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j))) -Vy(i,j)))*y(i);

else if (Vx(i,j) < 0 & Vx(i,j+1) < 0 & Vy(i,j+1) < 0 & Vx(i,j+1) < 0 & 

% disp('-4-9')
Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j));
Tx(i,j) = (1/Ax(i,j))*log(Vx(i,j+1)/Vxp(i,j));

if Tx(i,j) < 0
    Tx(i,j) = Tx(i,j)*(-1);
end
Tm(i,j) = Tx(i,j);

Xe(i,j) = ((1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))) - Vx(i,j))/x(j);
Ye(i,j) = ((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))) - Vy(i,j))/y(j);

% else Vxp(i,j) > 0 & Vx(i,j+1) < 0 & Vy(i,j+1) < 0 & 
% disp('-5-9')
Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j));
Tx(i,j) = (1/Ax(i,j))*log(Vx(i,j+1)/Vxp(i,j));

if Tx(i,j) < 0
    Tx(i,j) = Tx(i,j)*(-1);
end
Tm(i,j) = Tx(i,j);

Xe(i,j) = ((1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))) - Vx(i,j))/x(j);
Ye(i,j) = ((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))) - Vy(i,j))/y(j);

% end if

% second model
Xe(i,j) = ((1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))) - Vx(i,j))/x(j);
Ye(i,j) = ((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))) - Vy(i,j))/y(j);

end
```

% disp('-6-9')

Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j));
Tx(i,j) = (1/Ax(i,j))*log(Vx(i,j+1)/Vxp(i,j));

if Tx(i,j) < 0
    Tx(i,j) = Tx(i,j)*(-1);
end
Tm(i,j) = Tx(i,j);

Xe(i,j) = ((1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))) - Vx(i,j))/x(j);
Ye(i,j) = ((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))) - Vy(i,j))/y(j);

end

else if Vx(i,j+1) < 0 & Vx(i,j+1) < 0 & Vy(i,j+1) < 0 & Vy(i,j+1) < 0 & 
% disp('-7-9')
Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j)*Xp(i,j) - x(j));
Tx(i,j) = (1/Ax(i,j))*log(Vx(i,j+1)/Vxp(i,j));

if Tx(i,j) < 0
    Tx(i,j) = Tx(i,j)*(-1);
end
Tm(i,j) = min (Tx(i,j),Ty(i,j));

Xe(i,j) = ((1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))) - Vx(i,j))/x(j);
Ye(i,j) = ((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))) - Vy(i,j))/y(j);

end
end
```

```
Txi,j = Txi,j(-1)
end
Tm(i,j) = \text{min}(Txi,j, Ty(i,j))

Xe(i,j) = ((\frac{1}{Ax(i,j)})^2 * (Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j)

Ye(i,j) = (\frac{1}{Ay(i,j)})^2 * (Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) - Vy(i,j) * y(i)

Xe(i,j)
Ye(i,j)

else
(Vx(i,j) < 0 & Vx(i,j) + 1 < 0 & Vx(i,j) - 1 < 0 & Vy(i,j) < 0 & Vy(i,j) + 1 > 0) \% disp('10-9')

Ax(i,j) = (Vx(i,j) + 1 - Vx(i,j)) / (x(j) + 1 - x(j))
Vxp(i,j) = Ax(i,j) * (x(j) + 1 - x(j))
Txi,j = 1 / (Ax(i,j) * log(Vx(i,j) + 1 / Vxp(i,j))
Ay(i,j) = Vy(i,j) / y(i)
Vyp(i,j) = Ay(i,j) * (y(i) + 1 - y(i))
Ty(i,j) = Ty(i,j + 1, Ty(i,j))

if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) - 1
end
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) - 1
end
Tm(i,j) = \text{min}(Txi,j, Ty(i,j))

Xe(i,j) = ((\frac{1}{Ax(i,j)})^2 * (Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j)

Ye(i,j) = (\frac{1}{Ay(i,j)})^2 * (Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) - Vy(i,j) * y(i)

Xe(i,j)
Ye(i,j)

if Vx(i,j) < 0 & Vx(i,j) + 1 < 0 & Vx(i,j) - 1 < 0 & Vy(i,j) < 0 & Vy(i,j) + 1 > 0
\% disp('11-9')

Ax(i,j) = (Vx(i,j) + 1 - Vx(i,j)) / (x(j) + 1 - x(j))
Vxp(i,j) = Ax(i,j) * (x(j) + 1 - x(j))
Txi,j = 1 / (Ax(i,j) * log(Vx(i,j) + 1 / Vxp(i,j))
Ay(i,j) = Vy(i,j) / y(i)
Vyp(i,j) = Ay(i,j) * (y(i) + 1 - y(i))
Ty(i,j) = Ty(i,j + 1, Ty(i,j))

if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) - 1
end
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) - 1
end
Tm(i,j) = \text{min}(Txi,j, Ty(i,j))

Xe(i,j) = ((\frac{1}{Ax(i,j)})^2 * (Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j)

Ye(i,j)

\% - third model
else
(Vx(i,j) < 0 & Vx(i,j) + 1 < 0 & Vx(i,j) - 1 < 0 & Vy(i,j) < 0 & Vy(i,j) + 1 > 0
\% disp('11-9')

Ax(i,j) = (Vx(i,j) + 1 - Vx(i,j)) / (x(j) + 1 - x(j))
Vxp(i,j) = Ax(i,j) * (x(j) + 1 - x(j))
Txi,j = 1 / (Ax(i,j) * log(Vx(i,j) + 1 / Vxp(i,j))
Ay(i,j) = Vy(i,j) / y(i)
Vyp(i,j) = Ay(i,j) * (y(i) + 1 - y(i))
Ty(i,j) = Ty(i,j + 1, Ty(i,j))

if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) - 1
end
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) - 1
end
Tm(i,j) = \text{min}(Txi,j, Ty(i,j))

Xe(i,j) = ((\frac{1}{Ax(i,j)})^2 * (Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j)

Ye(i,j)
Xe(i,j)=((1/AX(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Txm(i,j)))))/Vx(i,j));
Ye(i,j)=((1/AY(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tym(i,j))))/Vy(i,j));

else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j+1)<0 & Vy(i,j+1)<0 & Vy(i,j+1)<0 & Vy(i,j+1)<0)
  disp(' -10-9')
  Ax(i,j)=(-1/AX(i,j))^log
  Ay(i,j)=(-1/AY(i,j))^log
  Xe(i,j)=(-1/(Ax(i,j))^y(i,j));
  Yp(i,j)=(-1/(Ay(i,j))^y(i,j));
end

elseif (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j+1)<0 & Vy(i,j+1)<0 & Vy(i,j+1)<0 & Vy(i,j+1)<0)
  disp(' -1-9')
  Ax(i,j)=(-1/AX(i,j))^log
  Ay(i,j)=(-1/AY(i,j))^log
  Xe(i,j)=(-1/(Ax(i,j))^y(i,j));
  Yp(i,j)=(-1/(Ay(i,j))^y(i,j));
end

else if (Vx(i,j)<0 & Vx(i,j+1)<0 & Vx(i,j+1)<0 & Vy(i,j+1)<0 & Vy(i,j+1)<0 & Vy(i,j+1)<0)
  disp(' -10-10')
  Ax(i,j)=(-1/AX(i,j))^log
  Ay(i,j)=(-1/AY(i,j))^log
  Xe(i,j)=(-1/(Ax(i,j))^y(i,j));
  Yp(i,j)=(-1/(Ay(i,j))^y(i,j));
end

end

end
\[
\begin{align*}
V_x(i,j) &= (Ax(i,j) * Xp(i,j) - x(j)) + Vx(i,j); \\
T_x(i,j) &= (1/Ax(i,j)) * \log \frac{Vx(i,j+1)}{Vx(i,j)}; \\
V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
\end{align*}
\]

\[
\begin{align*}
V_x(i,j) &= (Ax(i,j) * Xp(i,j) - x(j)) + Vx(i,j); \\
T_x(i,j) &= (1/Ax(i,j)) * \log \frac{Vx(i,j+1)}{Vx(i,j)}; \\
V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
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T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
\end{align*}
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\begin{align*}
V_x(i,j) &= (Ax(i,j) * Xp(i,j) - x(j)) + Vx(i,j); \\
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V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
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\begin{align*}
V_x(i,j) &= (Ax(i,j) * Xp(i,j) - x(j)) + Vx(i,j); \\
T_x(i,j) &= (1/Ax(i,j)) * \log \frac{Vx(i,j+1)}{Vx(i,j)}; \\
V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
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V_x(i,j) &= (Ax(i,j) * Xp(i,j) - x(j)) + Vx(i,j); \\
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V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
\end{align*}
\]

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\begin{align*}
V_x(i,j) &= (Ax(i,j) * Xp(i,j) - x(j)) + Vx(i,j); \\
T_x(i,j) &= (1/Ax(i,j)) * \log \frac{Vx(i,j+1)}{Vx(i,j)}; \\
V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
\end{align*}
\]

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\begin{align*}
V_x(i,j) &= (Ax(i,j) * Xp(i,j) - x(j)) + Vx(i,j); \\
T_x(i,j) &= (1/Ax(i,j)) * \log \frac{Vx(i,j+1)}{Vx(i,j)}; \\
V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
\end{align*}
\]

\[
\begin{align*}
V_x(i,j) &= (Ax(i,j) * Xp(i,j) - x(j)) + Vx(i,j); \\
T_x(i,j) &= (1/Ax(i,j)) * \log \frac{Vx(i,j+1)}{Vx(i,j)}; \\
V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
\end{align*}
\]

\[
\begin{align*}
V_x(i,j) &= (Ax(i,j) * Xp(i,j) - x(j)) + Vx(i,j); \\
T_x(i,j) &= (1/Ax(i,j)) * \log \frac{Vx(i,j+1)}{Vx(i,j)}; \\
V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
\end{align*}
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V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
\end{align*}
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\begin{align*}
V_x(i,j) &= (Ax(i,j) * Xp(i,j) - x(j)) + Vx(i,j); \\
T_x(i,j) &= (1/Ax(i,j)) * \log \frac{Vx(i,j+1)}{Vx(i,j)}; \\
V_y(i,j) &= (Ay(i,j) * Yp(i,j) - y(j)) + Vy(i,j); \\
T_y(i,j) &= (1/Ay(i,j)) * \log \frac{V_y(i,j+1)}{V_y(i,j)}. \\
\end{align*}
\]
```plaintext
end

Tm(i,j)=min(Tx(i,j),Ty(i,j));

Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)^exp(Ax(i,j)*Tm(i,j)))-Vx(i,j));
Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)^exp(Ay(i,j)*Tm(i,j)))-Vy(i,j));

else if ((Vx(i,j)<0 & Vx(i,j+1)==0 & Vy(i,j)<0 & Vy(i,j+1)<0 & Vy(i,j)>Vy(i+1,j)) & disp('9-10'))

Ax(i,j)=(Vx(i,j)+Vx(i,j+1))/2;
Ay(i,j)=(Vy(i,j)+Vy(i,j+1))/2;

% disp('9-10')

Xp(i,j)=Ax(i,j)*Vx(i,j);
Yp(i,j)=Ay(i,j)*Vy(i,j);

% disp('9-10')

Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)))*log(Vx(i,j+1)/Vxp(i,j));
Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)))*log(Vy(i,j+1)/Vyp(i,j));
if Ty(i,j)<0
Tm(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tm(i,j)=Tx(i,j)*(-1);
end

Tm(i,j)=min(Tx(i,j),Ty(i,j));

Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)))*log(Vx(i,j+1)/Vxp(i,j));
Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)))*log(Vy(i,j+1)/Vyp(i,j));
if (Vx(i,j)<0 & Vx(i,j+1)==0 & Vy(i,j)<0 & Vy(i,j+1)<0 & Vy(i,j)>Vy(i+1,j)) & disp('9-10')

Ax(i,j)=(Vx(i,j)+Vx(i,j+1))/2;
Ay(i,j)=(Vy(i,j)+Vy(i,j+1))/2;

Xp(i,j)=Ax(i,j)*Vx(i,j);
Yp(i,j)=Ay(i,j)*Vy(i,j);

Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)))*log(Vx(i,j+1)/Vxp(i,j));
Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)))*log(Vy(i,j+1)/Vyp(i,j));
if Ty(i,j)<0
Tm(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tm(i,j)=Tx(i,j)*(-1);
end

Tm(i,j)=min(Tx(i,j),Ty(i,j));

Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)))*log(Vx(i,j+1)/Vxp(i,j));
Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)))*log(Vy(i,j+1)/Vyp(i,j));
if (Vx(i,j)<0 & Vx(i,j+1)==0 & Vy(i,j)<0 & Vy(i,j+1)<0 & Vy(i,j)>Vy(i+1,j)) & disp('9-10')

Ax(i,j)=(Vx(i,j)+Vx(i,j+1))/2;
Ay(i,j)=(Vy(i,j)+Vy(i,j+1))/2;

Xp(i,j)=Ax(i,j)*Vx(i,j);
Yp(i,j)=Ay(i,j)*Vy(i,j);

Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)))*log(Vx(i,j+1)/Vxp(i,j));
Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)))*log(Vy(i,j+1)/Vyp(i,j));
if Ty(i,j)<0
Tm(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tm(i,j)=Tx(i,j)*(-1);
end
```
end
    Tm(i,j)=min (Tx(i,j),Ty(i,j));
    Xe(i,j)=((1/Ax(i,j)))*(Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j);
    Ye(i,j)=((1/Ay(i,j)))*(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(j);
    Xe(i,j);
    Ye(i,j);

end % - third model
elseif (Vx(i,j)<0 &
    Vx(i,j+1)=0 & Vy(i,j+1)>0)
    disp('-11-10')

    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j));
    Vxp(i,j)=Ax(i,j)*(x(j+1)-x(j))+Vx(i,j);
    % Tx(i,j)=(1/Ax(i,j))*log(Vx(i,j+1)/Vxp(i,j));
    Tx(i,j)=x(j+1)-x(j);
    % Ty(i,j)=(1/Ay(i,j))*log(Vy(i,j+1)/Vyp(i,j));
    Ty(i,j)=y(j+1)-y(j);
    Yp(i,j)/Vy(i,j+1));
    if Ty(i,j)<0
      Ty(i,j)=Ty(i,j)*(-1);
    end
    if Tx(i,j)<0
      Tx(i,j)=Tx(i,j)*(-1);
    end
    Tm(i,j)=min (Tx(i,j),Ty(i,j));
    Xe(i,j)=((1/Ax(i,j)))*(Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j);
    Ye(i,j)=((1/Ay(i,j)))*(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(j);
    Xe(i,j);
    Ye(i,j);
elseif (Vx(i,j)<0 &
    Vx(i,j+1)=0 & Vy(i,j+1)>0 & Vy(i,j)>0)
    disp('-11-10')

    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j));
    Vxp(i,j)=Ax(i,j)*(x(j+1)-x(j))+Vx(i,j);
    % Tx(i,j)=(1/Ax(i,j))*log(Vx(i,j+1)/Vxp(i,j));
    Tx(i,j)=x(j+1)-x(j);
    % Ty(i,j)=(1/Ay(i,j))*log(Vy(i,j+1)/Vyp(i,j));
    Ty(i,j)=y(j+1)-y(j);
    Yp(i,j)/Vy(i,j+1));
    if Ty(i,j)<0
      Ty(i,j)=Ty(i,j)*(-1);
    end
    if Tx(i,j)<0
      Tx(i,j)=Tx(i,j)*(-1);
    end
    Tm(i,j)=min (Tx(i,j),Ty(i,j));
    Xe(i,j)=((1/Ax(i,j)))*(Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j);
    Ye(i,j)=((1/Ay(i,j)))*(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(j);
    Xe(i,j);
    Ye(i,j);
end

elseif (Vx(i,j)<0 &
    Vx(i,j+1)>0 & Vy(i,j+1)>0 & Vy(i,j)>0)
    disp('-11-11')

    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j));
    Vxp(i,j)=(Ax(i,j)*(x(j+1)-x(j))+Vx(i,j);
    % Tx(i,j)=(1/Ax(i,j))*log(Vx(i,j+1)/Vxp(i,j));
    Tx(i,j)=x(j+1)-x(j);
    % Ty(i,j)=(1/Ay(i,j))*log(Vy(i,j+1)/Vyp(i,j));
    Ty(i,j)=y(j+1)-y(j);
    Yp(i,j)/Vy(i,j+1));
    if Ty(i,j)<0
      Ty(i,j)=Ty(i,j)*(-1);
    end
    if Tx(i,j)<0
      Tx(i,j)=Tx(i,j)*(-1);
    end
    Tm(i,j)=min (Tx(i,j),Ty(i,j));
    Xe(i,j)=((1/Ax(i,j)))*(Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j);
    Ye(i,j)=((1/Ay(i,j)))*(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(j);
    Xe(i,j);
    Ye(i,j);
end
\[ Tm(i,j) = \min \{ Tx(i,j), Ty(i,j) \}; \]

\[ Xe(i,j) = \frac{1}{Ax(i,j)} \times (Vxp(i,j) \times \exp(Ax(i,j)) - Xv(i,j)) \times x(j); \]

\[ Ye(i,j) = \frac{1}{Ay(i,j)} \times (Vyp(i,j) \times \exp(Ay(i,j)) - Yv(i,j)) \times y(j); \]

\[ Vxp(i,j) \times y(i); \]

\[ Xe(i,j); Ye(i,j); \]

\[ \text{else if } (Vx(i,j) < 0 \text{ and } Vx(i,j+1) > 0 \text{ and } Vy(i,j) < 0 \text{ and } Vy(i,j+1) > 0) \]

\[ \% \text{ disp}('-2-1'); \]

\[ Ax(i,j) = (Vx(i,j) - x(j)); \]

\[ Vxp(i,j) = (Ax(i,j) \times (Xp(i,j) - x(j))) \times Vx(i,j); \]

\[ \% \text{ Tx(i,j) = } (1/Ax(i,j)) \times \log(Vx(i,j+1)/Vxp(i,j)); \]

\[ Xv(i,j) = (Xp(i,j) - x(j)); \]

\[ \text{if } Ty(i,j) < 0 \]

\[ Ty(i,j) = Ty(i,j) \times (-1); \]

\[ \text{end if } Tx(i,j) < 0 \]

\[ Tx(i,j) = Tx(i,j) \times (-1); \]

\[ \text{end \ Tm(i,j) = } \min \{ Tx(i,j), Ty(i,j) \}; \]

\[ Xe(i,j) = \frac{1}{Ax(i,j)} \times Vxp(i,j) \times \exp(Ax(i,j)); \]

\[ Ye(i,j) = \frac{1}{Ay(i,j)} \times Vyp(i,j) \times \exp(Ay(i,j)); \]

\[ \text{else if } (Vx(i,j) < 0 \text{ and } Vx(i,j+1) > 0 \text{ and } Vy(i,j) < 0 \text{ and } Vy(i,j+1) > 0) \]

\[ \% \text{ disp}('-3-2'); \]

\[ Ax(i,j) = (Vx(i,j) - x(j)); \]

\[ Vxp(i,j) = (Ax(i,j) \times (Xp(i,j) - x(j))) \times Vx(i,j); \]

\[ Xv(i,j) = (Xp(i,j) - x(j)); \]

\[ \% \text{ Tx(i,j) = } (1/Ax(i,j)) \times \log(Vx(i,j+1)/Vxp(i,j)); \]

\[ Xv(i,j) = (Xp(i,j) - x(j)); \]

\[ \text{if } Ty(i,j) < 0 \]

\[ Ty(i,j) = Ty(i,j) \times (-1); \]

\[ \text{end if } Tx(i,j) < 0 \]

\[ Tx(i,j) = Tx(i,j) \times (-1); \]

\[ \text{end \ Tm(i,j) = } \min \{ Tx(i,j), Ty(i,j) \}; \]

\[ Xe(i,j) = \frac{1}{Ax(i,j)} \times (Vxp(i,j) \times \exp(Ax(i,j)) - Xv(i,j)) \times x(j); \]

\[ Ye(i,j) = \frac{1}{Ay(i,j)} \times (Vyp(i,j) \times \exp(Ay(i,j)) - Yv(i,j)) \times y(j); \]

\[ Vxp(i,j) \times y(i); \]

\[ Xe(i,j); Ye(i,j); \]

\[ \text{else if } (Vx(i,j) < 0 \text{ and } Vx(i,j+1) > 0 \text{ and } Vy(i,j) < 0 \text{ and } Vy(i,j+1) > 0) \]

\[ \% \text{ disp}('-5-1'); \]

\[ \% \text{ disp}('-1'); \]

\[ Ax(i,j) = (Vx(i,j) - x(j)); \]

\[ Vxp(i,j) = (Ax(i,j) \times (Xp(i,j) - x(j))) \times Vx(i,j); \]

\[ Xv(i,j) = (Xp(i,j) - x(j)); \]

\[ \% \text{ Tx(i,j) = } (1/Ax(i,j)) \times \log(Vx(i,j+1)/Vxp(i,j)); \]

\[ Xv(i,j) = (Xp(i,j) - x(j)); \]

\[ \text{if } Ty(i,j) < 0 \]

\[ Ty(i,j) = Ty(i,j) \times (-1); \]

\[ \text{end if } Tx(i,j) < 0 \]

\[ Tx(i,j) = Tx(i,j) \times (-1); \]

\[ \text{end \ Tm(i,j) = } \min \{ Tx(i,j), Ty(i,j) \}; \]

\[ Xe(i,j) = \frac{1}{Ax(i,j)} \times (Vxp(i,j) \times \exp(Ax(i,j)) - Xv(i,j)) \times x(j); \]

\[ Ye(i,j) = \frac{1}{Ay(i,j)} \times (Vyp(i,j) \times \exp(Ay(i,j)) - Yv(i,j)) \times y(j); \]

\[ Vxp(i,j) \times y(i); \]

\[ Xe(i,j); Ye(i,j); \]

\[ \% \text{- second model} \]

217
elseif (Vx(i,j)==0 & Vx(i,j+1)>0 & Vy(i,j)<0 & Vy(i+1,j)>0 )
  disp('  6-11-1')
  Ax(i,j)=[Vx(i,j+1)-Vx(i,j)]/(x(j+1)-x(j));
  Vxp(i,j)=Ax(i,j)*(x(j+1)-x(j))'+Vx(i,j);
  %
  Tx(i,j)=1/Ax(i,j)*log(Vx(i,j+1)/Vxp(i,j))
  Tm(i,j)=Tx(i,j)-Tx(i,j-1);
  Xe(i,j)=[(1/Ax(i,j))*([Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))]-Vx(i,j))]*x(j);%
  Xe(i,j)=Xe(i,j)-x(j+1);
  Ye(i,j)=[(1/Ay(i,j))*([Vxp(i,j)*exp(Ay(i,j)*Tm(i,j))]-Vy(i,j))]*y(j);
  Yp(i,j)=Yp(i,j)+Vy(i,j)-y(j);
  if Vyp(i,j)<0
    disp('  8-11-2')
    Ax(i,j)=[Vx(i,j+1)-Vx(i,j)]/(x(j+1)-x(j));
    Vxp(i,j)=Ax(i,j)*(x(j+1)-x(j))'+Vx(i,j);
    %
    Tx(i,j)=1/Ax(i,j)*log(Vx(i,j+1)/Vxp(i,j));
    if Tx(i,j)<0
      Tx(i,j)=Tx(i,j)-1;
    end
    Tm(i,j)=Tx(i,j);%
    Xe(i,j)=[(1/Ax(i,j))*([Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))]-Vx(i,j))]*x(j);
    %
    Xe(i,j)=x(j+1);
    Ye(i,j)=[(1/Ay(i,j))*([Vxp(i,j)*exp(Ay(i,j)*Tm(i,j))]-Vy(i,j))]*y(j);
    Yp(i,j)=Yp(i,j)+Vy(i,j)-y(j);%
    elseif (Vx(i,j)==0 & Vx(i,j+1)>0 & Vy(i,j)<0 & Vy(i+1,j)<0 & Vy(i,j)< Vy(i+1,j))
      disp('  7-11-1')
      Ax(i,j)=[Vx(i,j+1)-Vx(i,j)]/(x(j+1)-x(j));
      Vxp(i,j)=Ax(i,j)*(x(j+1)-x(j))'+Vx(i,j);
      %
      Tx(i,j)=1/Ax(i,j)*log(Vx(i,j+1)/Vxp(i,j))
      Tm(i,j)=Tx(i,j)-Tx(i,j-1);
      Xe(i,j)=[(1/Ax(i,j))*([Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))]-Vx(i,j))]*x(j);%
      Xe(i,j)=y(j+1);
      Ye(i,j)=[(1/Ay(i,j))*([Vxp(i,j)*exp(Ay(i,j)*Tm(i,j))]-Vy(i,j))]*y(j);
      Yp(i,j)=Yp(i,j)+Vy(i,j)-y(j);%
    end
  end
  Xe(i,j)=[(1/Ax(i,j))*([Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))]-Vx(i,j))]*x(j);
  Yp(i,j)=Yp(i,j)+Vy(i,j)-y(j);
  if Vyp(i,j)<0
    disp('  7-11-1')
    Ax(i,j)=[Vx(i,j+1)-Vx(i,j)]/(x(j+1)-x(j));
    Vxp(i,j)=Ax(i,j)*(x(j+1)-x(j))'+Vx(i,j);
    %
    Tx(i,j)=1/Ax(i,j)*log(Vx(i,j+1)/Vxp(i,j));
    Tm(i,j)=Tx(i,j)-Tx(i,j-1);
    Xe(i,j)=[(1/Ax(i,j))*([Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))]-Vx(i,j))]*x(j);%
    Xe(i,j)=y(j+1);
    Ye(i,j)=[(1/Ay(i,j))*([Vxp(i,j)*exp(Ay(i,j)*Tm(i,j))]-Vy(i,j))]*y(j);
    Yp(i,j)=Yp(i,j)+Vy(i,j)-y(j);%
  elseif (Vx(i,j)==0 & Vx(i,j+1)>0 & Vy(i,j)<0 & Vy(i+1,j)<0 & Vy(i,j)> Vy(i+1,j))
    disp('  7-11-1')
    Ax(i,j)=[Vx(i,j+1)-Vx(i,j)]/(x(j+1)-x(j));
    Vxp(i,j)=Ax(i,j)*(x(j+1)-x(j))'+Vx(i,j);
    %
    Tx(i,j)=1/Ax(i,j)*log(Vx(i,j+1)/Vxp(i,j))
    Tm(i,j)=Tx(i,j)-Tx(i,j-1);
    Xe(i,j)=[(1/Ax(i,j))*([Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))]-Vx(i,j))]*x(j);%
    Xe(i,j)=y(j+1);
    Ye(i,j)=[(1/Ay(i,j))*([Vxp(i,j)*exp(Ay(i,j)*Tm(i,j))]-Vy(i,j))]*y(j);
    Yp(i,j)=Yp(i,j)+Vy(i,j)-y(j);%
  end
end
218
%  Ty(i,j)=(1/Ay(i,j))*log
  (Vy(i+1,j)/Vy(i,j))
  Ty(i,j)=((y(i)-
  Yp(i,j))/Vy(i,j))
  if Ty(i,j)<0
    Ty(i,j)=Ty(i,j)*(-1);
  end
  if Tx(i,j)<0
    Tx(i,j)=Tx(i,j)*(-1);
  end
  Tm(i,j)=min (Tx(i,j),Ty(i,j));

  Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)*exp(Ax{
    i,j)* Tm(i,j)))-Vx(i,j)))*x(j);
  Ye(i,j)=Tm(i,j)*
  Vyp(i,j))*y(i);
  Xe(i,j); Ye(i,j);

elseif (Vx(i,j)==0 &
  Vx(i,j+1)>0 & Vy(i,j)<0 & Vy(i+1,j)==0)
  % disp(' -10-11 ')
    Ax(i,j)=-Vx(i,j+1)-
    Vx(i,j)/x(j+1)-x(j));
  Vxp(i,j) = (Ax(i,j))*(Xp(i,j)-
    x(j)))
  %  Xp(i,j) = (1/Ax(i,j))*log
  [Vx(i,j+1)/Vxp(i,j)])
  Xp(i,j)=x(j+1)-
  Xp(i,j)/Vx(i,j+1);
  Ay(i,j)=Vy(i+1,j)-
  Vy(i,j)/y(i+1)-y(i));
  Vyp(i,j) = (Ay(i,j))*(Yp(i,j)-
    y(i)))
  %  Yp(i,j) = (1/Ay(i,j))*log
  [Vy(i+1,j)/Vyp(i,j)])
  Yp(i,j)=(y(i)-
  Yp(i,j)/Vyp(i,j));
  if Ty(i,j)<0
    Ty(i,j)=Ty(i,j)*(-1);
  end
  if Tx(i,j)<0
    Tx(i,j)=Tx(i,j)*(-1);
  end
  Tm(i,j)=min (Tx(i,j),Ty(i,j));

  Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)*exp(Ax{
    i,j)* Tm(i,j)))-Vx(i,j)))*x(j);
  Ye(i,j)=Tm(i,j)*
  Vyp(i,j))*y(i);
  Xe(i,j); Ye(i,j);

else     % third model

  elseif (Vx(i,j)==0 &
    Vx(i,j+1)>0 & Vy(i,j)==0 & Vy(i+1,j)>0)
  % disp(' -11-11 ')
    Ax(i,j)=Vx(i,j+1)-
    Vx(i,j)/x(j+1)-x(j));
  Vxp(i,j) = (Ax(i,j))*(Xp(i,j)-
    x(j)))
  %  Xp(i,j) = (1/Ax(i,j))*log
  [Vx(i,j+1)/Vxp(i,j)])
  Xp(i,j)=x(j+1)-
  Xp(i,j)/Vx(i,j+1);
  Ay(i,j)=Vy(i+1,j)-
  Vy(i,j)/y(i+1)-y(i));
  Vyp(i,j) = (Ay(i,j))*(Yp(i,j)-
    y(i)))
  %  Yp(i,j) = (1/Ay(i,j))*log
  [Vy(i+1,j)/Vyp(i,j)])
  Yp(i,j)=(y(i)+
  Yp(i,j)/Vyp(i,j));
  if Ty(i,j)<0
    Ty(i,j)=Ty(i,j)*(-1);
  end
  if Tx(i,j)<0
    Tx(i,j)=Tx(i,j)*(-1);
  end
  Tm(i,j)=min (Tx(i,j),Ty(i,j));

  Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)*exp(Ax{
    i,j)* Tm(i,j)))-Vx(i,j)))*x(j);
  Ye(i,j)=Tm(i,j)*
  Vyp(i,j))*y(i);
  Xe(i,j); Ye(i,j);

  % - TWELVE MODEL '

  if Vx(i+1,j)==0 &
    Vx(i,j+1)>0 & Vy(i,j)==0 & Vy(i+1,j)==0)
  % disp(' -13-11 ')
  % disp(' I dont know')

  rr=1

  % disp(' -TWELVE MODEL')
else if (Vx(i,j) == 0 & Vx(i,j+1) < 0 & Vx(i+1,j) > 0 &
         Vx(i,j) == Vx(i+1,j) + 1)
    if (Ty(i,j) < 0)
        Ty(i,j) = Ty(i,j) - 1;
    end
    if (Ty(i,j) < 0)
        Tx(i,j) = Tx(i,j) - 1;
    end
    Tm(i,j) = min (Tx(i,j), Ty(i,j));

    Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j))) * Xv(i,j) + Xj);
    Ye(i,j) = (1/Ay(i,j)) * (Vy(i,j) * exp(Ay(i,j))) * Yv(i,j) + Yj;

    elseif (Vx(i,j) == 0 & Vx(i,j+1) < 0 & Vx(i+1,j) > 0 &
            Vx(i,j) == Vy(i+1,j) + 1)
    % disp(' -12')
    Ax(i,j) = (Vx(i,j+1) -
            Vx(i,j)) / (x(i+1,j) - x(i,j));
    Vxp(i,j) = (Ax(i,j) * (Xp(i,j) -
            x(i,j))) + Xp(i,j);
    %
    Tx(i,j) = (1/Ax(i,j)) * log
            (Vx(i,j+1) / Vxp(i,j));
    Tx(i,j) = (x(i+1,j) -
            Vx(i,j)) / (y(i+1,j) - y(i,j));
    Vyp(i,j) = (Ay(i,j) * (Yp(i,j) -
            y(i,j))) + Yp(i,j);
    Ty(i,j) = (1/Ay(i,j)) * log
            (Vy(i+1,j) / Vyp(i,j));
    if (Ty(i,j) < 0)
        Ty(i,j) = Ty(i,j) - 1;
    end
    if (Tx(i,j) < 0)
        Tx(i,j) = Tx(i,j) - 1;
    end
    Tm(i,j) = min (Tx(i,j), Ty(i,j));

    Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j))) * Xv(i,j) + Xj);
    Ye(i,j) = (1/Ay(i,j)) * (Vy(i,j) * exp(Ay(i,j))) * Yv(i,j) + Yj;

    elseif (Vx(i,j+1) == 0 & Vy(i+1,j) == 0 & Vx(i+1,j) < 0 &
            Vx(i,j) == Vy(i+1,j) + 1)
    % disp(' -5')
    Ax(i,j) = (Vx(i+1,j) -
            Vx(i,j)) / (x(i+1,j) - x(i,j));
    Vxp(i,j) = (Ax(i,j) * (Xp(i,j) -
            x(i,j))) + Xp(i,j);
    %
    Tx(i,j) = (1/Ax(i,j)) * log
            (Vx(i,j+1) / Vxp(i,j));
    Tx(i,j) = (y(i+1,j) -
            Vx(i,j)) / (y(i+1,j) - y(i,j));
    Vyp(i,j) = (Ay(i,j) * (Yp(i,j) -
            y(i,j))) + Yp(i,j);
    Ty(i,j) = (1/Ay(i,j)) * log
            (Vy(i+1,j) / Vyp(i,j));
    if (Ty(i,j) < 0)
        Ty(i,j) = Ty(i,j) - 1;
    end
    if (Tx(i,j) < 0)
        Tx(i,j) = Tx(i,j) - 1;
    end
    Tm(i,j) = min (Tx(i,j), Ty(i,j));

    Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j))) * Xv(i,j) + Xj);
    Ye(i,j) = (1/Ay(i,j)) * (Vy(i,j) * exp(Ay(i,j))) * Yv(i,j) + Yj;

    elseif (Vx(i,j) == 0 & Vx(i,j+1) < 0 & Vx(i+1,j) > 0 &
            Vx(i,j) == Vx(i+1,j) + 1)
    % disp(' -3')
    Ax(i,j) = (Vx(i,j+1) -
            Vx(i,j)) / (x(i+1,j) - x(i,j));
    Vxp(i,j) = (Ax(i,j) * (Xp(i,j) -
            x(i,j))) + Xp(i,j);
    %
    Tx(i,j) = (1/Ax(i,j)) * log
            (Vx(i,j+1) / Vxp(i,j));
    Tx(i,j) = (x(i+1,j) -
            Vx(i,j)) / (y(i+1,j) - y(i,j));
    Vyp(i,j) = (Ay(i,j) * (Yp(i,j) -
            y(i,j))) + Yp(i,j);
    Ty(i,j) = (1/Ay(i,j)) * log
            (Vy(i+1,j) / Vyp(i,j));
    if (Ty(i,j) < 0)
        Ty(i,j) = Ty(i,j) - 1;
    end
    if (Tx(i,j) < 0)
        Tx(i,j) = Tx(i,j) - 1;
    end
    Tm(i,j) = min (Tx(i,j), Ty(i,j));

    Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j))) * Xv(i,j) + Xj);
    Ye(i,j) = (1/Ay(i,j)) * (Vy(i,j) * exp(Ay(i,j))) * Yv(i,j) + Yj;

    elseif (Vx(i,j+1) == 0 & Vy(i+1,j) == 0 & Vy(i+1,j) < 0 &
            Vx(i,j) == Vy(i+1,j) + 1)
    % disp(' -12')
    Ax(i,j) = (Vx(i,j+1) -
            Vx(i,j)) / (x(i+1,j) - x(i,j));
    Vxp(i,j) = (Ax(i,j) * (Xp(i,j) -
            x(i,j))) + Xp(i,j);
    %
    Tx(i,j) = (1/Ax(i,j)) * log
            (Vx(i,j+1) / Vxp(i,j));
    Tx(i,j) = (y(i+1,j) -
            Vx(i,j)) / (y(i+1,j) - y(i,j));
    Vyp(i,j) = (Ay(i,j) * (Yp(i,j) -
            y(i,j))) + Yp(i,j);
    Ty(i,j) = (1/Ay(i,j)) * log
            (Vy(i+1,j) / Vyp(i,j));
    if (Ty(i,j) < 0)
        Ty(i,j) = Ty(i,j) - 1;
    end
    if (Tx(i,j) < 0)
        Tx(i,j) = Tx(i,j) - 1;
    end
    Tm(i,j) = min (Tx(i,j), Ty(i,j));

    Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j))) * Xv(i,j) + Xj);
    Ye(i,j) = (1/Ay(i,j)) * (Vy(i,j) * exp(Ay(i,j))) * Yv(i,j) + Yj;

    elseif (Vx(i,j) == 0 & Vx(i,j+1) < 0 &
            Vx(i,j) == Vx(i+1,j) + 1)
    % disp(' -4')
    Ax(i,j) = (Vx(i,j+1) -
            Vx(i,j)) / (x(i+1,j) - x(i,j));
    Vxp(i,j) = (Ax(i,j) * (Xp(i,j) -
            x(i,j))) + Xp(i,j);
    %
    Tx(i,j) = (1/Ax(i,j)) * log
            (Vx(i,j+1) / Vxp(i,j));
    Tx(i,j) = (x(i+1,j) -
            Vx(i,j)) / (y(i+1,j) - y(i,j));
    Vyp(i,j) = (Ay(i,j) * (Yp(i,j) -
            y(i,j))) + Yp(i,j);
    Ty(i,j) = (1/Ay(i,j)) * log
            (Vy(i+1,j) / Vyp(i,j));
    if (Ty(i,j) < 0)
        Ty(i,j) = Ty(i,j) - 1;
    end
    if (Tx(i,j) < 0)
        Tx(i,j) = Tx(i,j) - 1;
    end
    Tm(i,j) = min (Tx(i,j), Ty(i,j));

    Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j))) * Xv(i,j) + Xj);
    Ye(i,j) = (1/Ay(i,j)) * (Vy(i,j) * exp(Ay(i,j))) * Yv(i,j) + Yj;

    elseif (Vx(i,j+1) == 0 & Vy(i+1,j) == 0 &
            Vx(i,j) == Vy(i+1,j) + 1)
    % disp(' -5')
    Ax(i,j) = (Vx(i,j+1) -
            Vx(i,j)) / (x(i+1,j) - x(i,j));
    Vxp(i,j) = (Ax(i,j) * (Xp(i,j) -
            x(i,j))) + Xp(i,j);
    %
    Tx(i,j) = (1/Ax(i,j)) * log
            (Vx(i,j+1) / Vxp(i,j));
    Tx(i,j) = (y(i+1,j) -
            Vx(i,j)) / (y(i+1,j) - y(i,j));
    Vyp(i,j) = (Ay(i,j) * (Yp(i,j) -
            y(i,j))) + Yp(i,j);
    Ty(i,j) = (1/Ay(i,j)) * log
            (Vy(i+1,j) / Vyp(i,j));
    if (Ty(i,j) < 0)
        Ty(i,j) = Ty(i,j) - 1;
    end
    if (Tx(i,j) < 0)
        Tx(i,j) = Tx(i,j) - 1;
    end
    Tm(i,j) = min (Tx(i,j), Ty(i,j));

    Xe(i,j) = (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j))) * Xv(i,j) + Xj);
    Ye(i,j) = (1/Ay(i,j)) * (Vy(i,j) * exp(Ay(i,j))) * Yv(i,j) + Yj;
Ay[i,j]=\(Vy[i+1,j]-Vy[i,j]\) / (y[i+1]-y[i]);

Vy[i,j]=Ay[i,j]* (Yp[i,j]-y[i]) + Vy[i,j];

% Ty[i,j]=1/Ay[i,j] * log
(Vy[i+1,j]/Vyp[i,j])
Ty[i,j]=(1/Ay[i,j] - Yp[i,j]) / Vy[i,j];
if Ty[i,j] < 0
Ty[i,j] = Ty[i,j] * (-1);
end
if Tx[i,j] < 0
Tx[i,j] = Tx[i,j] * (-1);
end
Tm[i,j]=min (Tx[i,j],Ty[i,j]);

Ye[i,j]=(1/Ay[i,j])*((Vyp[i,j]*exp(Ax[i,j] * Tm[i,j]))-Vx[i,j])*x[j];

Ye[i,j]=((1/Ay[i,j])*(Vyp[i,j]*exp(Ay[i,j]*Tm[i,j]))-Vy[i,j])*y[l];
endif
endif

e - second model
if Vx[i,j]<0 & Vx[i+1,j]<0 & Vy[i,j]<0 & Vy[i+1,j]<0 & disp('6-12')

Ax[i,j]=Vx[i,j]+1-
Vx[i,j]/(x[j+1]-x[j]);

Vxp[i,j]=Ax[i,j]*Vx[i,j]-
x[j]);

% Tx[i,j]=(1/Ax[i,j])* log
(Vx[i,j+1]/Vxp[i,j])
Tx[i,j]=x[j+1]-
Xp[i,j])/Vx[i,j+1];

Ay[i,j]=(Vy[i+1,j]-
Vy[i,j])/(y[i+1]-y[i]);

Vyp[i,j]=(Ax[i,j]*Vx[i,j]-(Ax[i,j]+1)*y[l] + Vy[i,j]);
if Vyp[i,j] < 0
% disp('6-12-1')
if Tx[i,j] < 0
Tx[i,j] = Tx[i,j] * (-1);
end
Tm[i,j]=Tx[i,j];

Xe[i,j]=(1/Ax[i,j])*((Vxp[i,j]*exp(Ax[i,j]*Tm[i,j]))-Vx[i,j])*x[j];

Ye[i,j]=y[i];
else
Vyp[i,j]>0
% disp('6-12-2')

Ax[i,j]=Vx[i,j]+1-
Vx[i,j]/(x[j+1]-x[j]);

Vxp[i,j]=Ax[i,j]*Vx[i,j]-
x[j]);

Tx[i,j]=(1/Ax[i,j]) * log
(Vx[i,j+1]/Vxp[i,j])
if Tx[i,j] < 0
Tx[i,j] = Tx[i,j] * (-1);
end
Tm[i,j]=Tx[i,j];

Xe[i,j]=(1/Ax[i,j])*((Vxp[i,j]*exp(Ax[i,j]*Tm[i,j]))-Vx[i,j])*x[j];

Ye[i,j]=y[i];
else
Vyp[i,j]>0
% disp('6-12-2')

Ax[i,j]=Vx[i,j]+1-
Vx[i,j]/(x[j+1]-x[j]);

Vxp[i,j]=Ax[i,j]*Vx[i,j]-
x[j]);

Tx[i,j]=(1/Ax[i,j]) * log
(Vx[i,j+1]/Vxp[i,j])
if Tx[i,j] < 0
Tx[i,j] = Tx[i,j] * (-1);
end
Tm[i,j]=Tx[i,j];

Xe[i,j]=(1/Ax[i,j])*((Vxp[i,j]*exp(Ax[i,j]*Tm[i,j]))-Vx[i,j])*x[j];

Ye[i,j]=y[i];
endif
else if (Vx(i,j)==0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vx(i+1,j)<0 & Vy(i+1,j)==0 & Vx(i+1,j+1)<0 & 
Vy(i+1,j)==0) 
\% disp('third case') 
\% disp('3') 
Ax(i,j)=(Vx(i,j)+1-Vx(i+1,j))/x(i+1)-x(i); 
Vx(i,j)=[Ax(i,j)*Vx(i,j)-Vx(i,j)+X(i+1,j)+Vx(i,j)]; 
Ty(i,j)="(Ax(i,j)*Vx(i,j)-Vx(i,j)+X(i+1,j)+Vx(i,j))"; 
Vy(i,j)="(Ax(i,j)*Vx(i,j)-Vx(i,j)+X(i+1,j)+Vx(i,j))"; 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
Tm(i,j)="min(Tx(i,j),Ty(i,j))"; 
Xe(i,j)="((Ax(i,j)+Vx(i,j)+X(i+1,j)+Vx(i,j))/x(i+1)-x(i))"; 
Vyp(i,j)="(Ax(i,j)*Vx(i,j)-Vx(i,j)+X(i+1,j)+Vx(i,j))"; 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
Tm(i,j)="min(Tx(i,j),Ty(i,j))"; 
Xe(i,j)="((Ax(i,j)+Vx(i,j)+X(i+1,j)+Vx(i,j))/x(i+1)-x(i))"; 
Vyp(i,j)="(Ax(i,j)*Vx(i,j)-Vx(i,j)+X(i+1,j)+Vx(i,j))"; 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
Tm(i,j)="min(Tx(i,j),Ty(i,j))"; 
Xe(i,j)="((Ax(i,j)+Vx(i,j)+X(i+1,j)+Vx(i,j))/x(i+1)-x(i))"; 
Vyp(i,j)="(Ax(i,j)*Vx(i,j)-Vx(i,j)+X(i+1,j)+Vx(i,j))"; 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
Tm(i,j)="min(Tx(i,j),Ty(i,j))"; 
Xe(i,j)="((Ax(i,j)+Vx(i,j)+X(i+1,j)+Vx(i,j))/x(i+1)-x(i))"; 
Vyp(i,j)="(Ax(i,j)*Vx(i,j)-Vx(i,j)+X(i+1,j)+Vx(i,j))"; 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
Tm(i,j)="min(Tx(i,j),Ty(i,j))"; 
Xe(i,j)="((Ax(i,j)+Vx(i,j)+X(i+1,j)+Vx(i,j))/x(i+1)-x(i))"; 
Vyp(i,j)="(Ax(i,j)*Vx(i,j)-Vx(i,j)+X(i+1,j)+Vx(i,j))"; 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
Tm(i,j)="min(Tx(i,j),Ty(i,j))"; 
Xe(i,j)="((Ax(i,j)+Vx(i,j)+X(i+1,j)+Vx(i,j))/x(i+1)-x(i))"; 
Vyp(i,j)="(Ax(i,j)*Vx(i,j)-Vx(i,j)+X(i+1,j)+Vx(i,j))"; 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
if Ty(i,j)<0 
Ty(i,j)="Ty(i,j)*(-1)"; 
end 
Tm(i,j)="min(Tx(i,j),Ty(i,j))";
Ye(i,j)=(1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)))-Ty(i,j)) + y(i);
Xe(i,j);
Ye(i,j);
else if (Vx(i,j)==0 & Vx(i,j+1)<0 &Vy(i,j)==0 & Vy(i+1,j)==0 & Vx(i,j+1)<0 & Vy(i,j)==0 & Vy(i+1,j)==0 &Vy(i+1,j+1)<0)
% disp(' - thirteenth model')
elseif (Vx(i,j)==0 & Vx(i,j+1)==0 & Vy(i,j)<0 & Vy(i+1,j)>0 &Vy(i,j)==Vy(i+1,j))
% disp(' - i-13')
ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i));
yp(i,j)=(Ay(i,j)*((Yp(i,j)-y(i)))/Vyp(i,j))
% if Ty(i,j)<0
Ty(i,j)=Ty(i,j)-1;
end
Tm(i,j)=Ty(i,j);
Xe(i,j);
Ye(i,j);
elseif (Vx(i,j)==0 & Vx(i,j+1)==0 & Vy(i,j)==0 & Vy(i+1,j)==0 &Vx(i,j+1)<0 & Vy(i,j)>0 & Vx(i+1,j+1)<0 & Vy(i,j+1)==0 & Vy(i+1,j)<0)
% disp(' - Idont know')
rr=1
elseif (Vx(i,j)==0 & Vx(i,j+1)==0 & Vy(i,j)<0 & Vy(i+1,j)==0)
% disp(' - 5-13')
% disp(' i dont know')
rr=1
% - second model
elseif (Vx(i,j+1)==0 & Vx(i,j+1)<0 &Vy(i+1,j)==0 &Vy(i+1,j)<0)
% disp(' - 6-13 ')
ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)-y(i));
yp(i,j)=(Ay(i,j)*((Yp(i,j)-y(i)))/Vyp(i,j))
% if Vyp(i,j)<0
% disp(' - 6-13-1')
Xe(i,j)=x(i);
Ye(i,j);
else Vyp(i,j)>0
% disp(' - 6-13-2')
Xe(i,j)=x(i);
Ye(i,j)=y(i+1);
else Ne(i,j);
end
```
else if (Vx(i,j)==0 & Vx(i+1,j)==0 & Vy(i,j)<0 & Vy(i+1,j)<0 &Vy[i+1,j]> Vy[i+1,j] )
% disp(' -7-13')
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i));
Vyp(i,j)=(Ay(i,j)*{(Yp(i,j)-y(i))}+Vy(i,j);)
Ty(i,j)=(1/Ay(i,j))*log
{Vy[i+1,j]/Vyp(i,j)};
if Ty(i,j)<0
Ty(i,j)=-Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=X(i);
Ye(i,j)=((1/Ay(i,j))*{(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j)}+y(i))
Xe(i,j);
Ye(i,j);
else if (Vx(i,j)==0 & Vx(i+1,j)==0 & Vy(i,j)<0 & Vy(i+1,j)<0 &Vy[i+1,j]< Vy[i+1,j] )
% disp(' -9-13')
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i));
Vyp(i,j)=(Ay(i,j)*{(Yp(i,j)-y(i))}+Vy(i,j);)
Ty(i,j)=(1/Ay(i,j))*log
{Vy[i+1,j]/Vyp(i,j)};
if Ty(i,j)<0
Ty(i,j)=-Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=X(i);
Ye(i,j)=((1/Ay(i,j))*{(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j)}+y(i))
Xe(i,j);
Ye(i,j);
else if (Vx(i,j)==0 & Vx(i+1,j)==0 & Vy(i,j)==0 & Vy(i+1,j)<0 &Vy[i+1,j]< Vy[i+1,j] )
% disp(' -12-13')
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i));
Vyp(i,j)=(Ay(i,j)*{(Yp(i,j)-y(i))}+Vy(i,j);)
Ty(i,j)=(1/Ay(i,j))*log
{Vy[i+1,j]/Vyp(i,j)};
if Ty(i,j)<0
Ty(i,j)=-Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=X(i);
Ye(i,j)=((1/Ay(i,j))*{(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j)}+y(i))
Xe(i,j);
Ye(i,j);
else if (Vx(i,j)==0 & Vx(i+1,j)==0 & Vy(i,j)==0 &Vy(i+1,j)==0 &Vy[i+1,j]== Vy[i+1,j] )
% disp(' -10-13')
Ay(i,j)=(Vy[i+1,j]-Vy(i,j))/y[i+1]-y(i));
```
function pressure
global X X Y global
global Kx Ky global
global Vx Vy global
global Xe Ye global
global maxerr maxr errormatrix
global x y
global Xp Xp Yp
global Vxp Vyp
global gg ff
global i j
global rr
global Ax Ay Vxp Vyp
global K1 K2
global A B
P=zeros(X,Y);

maxr=1;
errormatrix=zeros(X,Y);
iteration=0;
while maxr>maxerr
  for i=1:Y
    for j=2:X-1
      tempval=P(i,j);
x=1;
y=1;

if i==1
  P(i,j)=((P(i,j)-1)+P(i,j+1)+2*P(i+1,j))/4;
else if i==Y
  P(i,j)=((P(i,j)-1)+P(i,j+1)+2*P(i-1,j))/4;
else if i==fix(Y/3) & j<fix((2*X)/3) & j>fix((X)/3)
  P(i,j)=(1/4)*P(i,j+1)+((2*K1)/(K1+K2))*P(i+1,j)+P(i,j-1)+((2*K2)/(K1+K2))*P(i-1,j));
  end
else if i==fix(Y/3) & j<fix((2*X)/3) & j>fix((X)/3)
  P(i,j)=(1/4)*P(i,j+1)+((2*K1)/(K1+K2))*P(i+1,j)+((2*K2)/(K1+K2))*P(i,j-1));
  end
else if j==fix((2*X)/3) & i>fix((2*Y)/3)
  if i==fix(Y/3) & i<fix((2*X)/3)
    P(i,j)=(1/4)*P(i,j+1)+((2*K1)/(K1+K2))*P(i+1,j)+((2*K2)/(K1+K2))*P(i,j-1)+P(i,j+1)+P(i+1,j+1)/4;
  else
    P(i,j)=((P(i,j)-1)+P(i,j+1)+2*P(i-1,j))/4;
  end
else if i==fix(Y/3) & j>fix((2*X)/3) & j<fix((X)/3)
  P(i,j)=(1/4)*P(i,j+1)+((2*K1)/(K1+K2))*P(i+1,j)+((2*K2)/(K1+K2))*P(i,j-1)+P(i,j+1)+P(i+1,j+1)/4;
  end
else
  errormatrix(i,j)=abs(P(i,j)-tempval);
  end
  maxr=max(max(errormatrix))
  iteration=iteration+1;
  end

225
global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr
global Ax Ay Vxp Vyp
global K1 K2
temp = find(Xp==0);
xx = Xp(temp);
yy = Yp(temp);
xx = [xx];
yy = [yy];
plot(xx,yy,'*-')
grid on
axis equal
axis square

% Streamline Simulation Near Well Bore
% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM
% Second Case Study in Cartesian Coordinate
% Subroutine for drawing the Streamline

global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxr errormatrix
global x y
global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg ff
global i j
global rr
global Ax Ay Vxp Vyp
global K1 K2
temp = find(Xp);
xx = Xp(temp);
yy = Yp(temp);
xx = [xx];
yy = [yy];
plot(xx,yy,'*-')
grid on
axis equal
axis square

% Streamline Simulation Near Well Bore
% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM
% Second Case Study in Cartesian Coordinate
% Subroutine for drawing the Streamline in "y" Direction
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxr errormatrix
global x y
global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg ff
global i j
global rr
global Ax Ay Vxp Vyp
global K1 K2
global A B
temp = find(Yp==0);
xx = Xp(temp);
yy = Yp(temp);
xx = [xx];
yy = [yy];
plot(xx,yy,'*-')
grid on
axis equal
axis square

% Streamline Simulation Near Well Bore
% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM
% Second Case Study in Cartesian Coordinate
% Subroutine for drawing the Streamline in "x" Direction
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxr errormatrix
global x y
% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASEM

% Third Case Study in Cartesian Coordinate
% Main route
clc;
clear all;
close all;
format long e

Y = input('Number of Nodes in Y ');%Boundary values can be set along the corner or in the middle
X = input('Number of Nodes in X ');

%Maximum Error abs(x2-x1)
maxerr = input('Desired Maximum Error [0.00001]:');
if isempty(reply)
    reply = 'C';
end
if isempty(maxerr)
    maxerr = 0.00001;
end
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxr errmatrix
global x y
global Xp Yp
global Vxp Vyp
global gg ff
global i j
global rr
global Ax Ay Vxp Vyp
global K1 K2
global A B

permeabilities

reply=lower(reply);
if reply=='c'
    pressure
else
    for i=floor(X/3):floor(2*X/3)
        P[i,1]=(2*i);
    end
end

M=(0.5*10^-3);
velocities

for b=1:X+1
    x(b)=(b-1);
end
for t=1:Y+1
    y(t)=(t-1);
end
Xp=zeros(X,Y);

Vxp=zeros(X,Y);
Vyp=zeros(X,Y);

123456789
for m=2:Y
    Xp(m,1)=0;
    Yp(m,1)=(m-1);
    Xpp=Xp(m,1);
    Ypp=Yp(m,1);
    Vxp(m,1)=Vx(m,1);
    Vyp(m,1)=Vy(m,1);
    i=m;
    j=1;
    while (Xpp<X-1) & (Ypp<Y)
        & (i<i-1) & (j<i-1)
        & (Ypp<Y)
    location;
    888
    if rr==1;
        999;
        break
    end
1111
if (Ye(i,j)>0 & Xe(i,j)>0)
    1111
    Xpp
    Ypp
    Xe(i,j)
    Ye(i,j)
    x(j)
    y(i)
    i
    j
    if (Xe(i,j)-(fix(Xe(i,j))))==0
        1231
        B=Xe(i,j)+1
        elseif ((Xe(i,j)-(fix(Xe(i,j))))>0.9 & (Xe(i,j)-(fix(Xe(i,j))))<0.01)
            4561
            B=round(Xe(i,j))+1
            elseif ((Xe(i,j)-(fix(Xe(i,j))))>0 & (Xe(i,j)-(fix(Xe(i,j))))<0.01)
                4562
                B=round(Xe(i,j))+1
                else
                    B=(fix(Xe(i,j)))+1
                    end
                    B
            if (Ye(i,j)-(fix(Ye(i,j))))==0
                1232
                A=Ye(i,j)+1
                elseif ((Ye(i,j)-(fix(Ye(i,j))))>0.9 & (Ye(i,j)-(fix(Ye(i,j))))<1)
                    4562
                    A=round(Ye(i,j))+1
                    elseif ((Ye(i,j)-(fix(Ye(i,j))))>0 & (Ye(i,j)-(fix(Ye(i,j))))<0.01)
end
Kx
Ky

% Streamline Simulation Near Well
Bore
% By MARJAN HASHEM

% Developed MATLAB Program for
Streamline Simulation
% This Code developed originally by
MARJAN HASHEM

% Third Case Study in Cartesian
Coordinate
% Subroutine for finding the
velocity
function velocities
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxerr errormatrix
global xy global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr
global Ax Ay Vxp Vyp

M=(0.5*10^-3);
Vx=zeros(Y+1,X+1);
Vy=zeros(Y+2,X);
for i=1:(Y+1)
    Vx(i,1)=0.005;
end
for i=2:Y+1
    for j=2:X
        Vx(i,j)=((-1)*(Kx(i-1,j-1)*(P(i-1,j)-F(i-1,j-1)))/(M*X));
    end
end
for i=Y+1
    for j=1:(X+1)
        Vx(i,j)=Vx(i-1,j);
    end
end
for i=2:Y
    for j=2:X+1
        Vy(i,j-1)=((-1)*(Ky(i-1,j-1)*(P(i,j-1)-F(i-1,j-1)))/(M*Y));
    end
end

% Streamline Simulation Near Well
Bore
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% Third Case Study in Cartesian
Coordinate
% Subroutine for finding the
velocity
function velocities
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxerr errormatrix
global xy global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr
global Ax Ay Vxp Vyp

M=(0.5*10^-3);
Vx=zeros(Y+1,X+1);
Vy=zeros(Y+2,X);
for i=1:(Y+1)
    Vx(i,1)=0.005;
end
for i=2:Y+1
    for j=2:X
        Vx(i,j)=((-1)*(Kx(i-1,j-1)*(P(i-1,j)-F(i-1,j-1)))/(M*X));
    end
end
for i=Y+1
    for j=1:(X+1)
        Vx(i,j)=Vx(i-1,j);
    end
end
for i=2:Y
    for j=2:X+1
        Vy(i,j-1)=((-1)*(Ky(i-1,j-1)*(P(i,j-1)-F(i-1,j-1)))/(M*Y));
    end
end

% Streamline Simulation Near Well
Bore
% By MARJAN HASHEM

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% Third Case Study in Cartesian
Coordinate
% Subroutine for finding the
velocity
function velocities
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxerr errormatrix
global xy global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr
global Ax Ay Vxp Vyp

M=(0.5*10^-3);
Vx=zeros(Y+1,X+1);
Vy=zeros(Y+2,X);
for i=1:(Y+1)
    Vx(i,1)=0.005;
end
for i=2:Y+1
    for j=2:X
        Vx(i,j)=((-1)*(Kx(i-1,j-1)*(P(i-1,j)-F(i-1,j-1)))/(M*X));
    end
end
for i=Y+1
    for j=1:(X+1)
        Vx(i,j)=Vx(i-1,j);
    end
end
for i=2:Y
    for j=2:X+1
        Vy(i,j-1)=((-1)*(Ky(i-1,j-1)*(P(i,j-1)-F(i-1,j-1)))/(M*Y));
    end
end

% Streamline Simulation Near Well
Bore
% By MARJAN HASHEM

% Developed MATLAB Program for
Streamline Simulation
% This Code developed originally by
MARJAN HASHEM

% Third Case Study in Cartesian
Coordinate
% Subroutine for finding the
velocity
function velocities
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxerr errormatrix
global xy global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr
global Ax Ay Vxp Vyp

M=(0.5*10^-3);
Vx=zeros(Y+1,X+1);
Vy=zeros(Y+2,X);
for i=1:(Y+1)
    Vx(i,1)=0.005;
end
for i=2:Y+1
    for j=2:X
        Vx(i,j)=((-1)*(Kx(i-1,j-1)*(P(i-1,j)-F(i-1,j-1)))/(M*X));
    end
end
for i=Y+1
    for j=1:(X+1)
        Vx(i,j)=Vx(i-1,j);
    end
end
for i=2:Y
    for j=2:X+1
        Vy(i,j-1)=((-1)*(Ky(i-1,j-1)*(P(i,j-1)-F(i-1,j-1)))/(M*Y));
    end
end

% Streamline Simulation Near Well
Bore
% By MARJAN HASHEM

% Developed MATLAB Program for
Streamline Simulation
% This Code developed originally by
MARJAN HASHEM

% Third Case Study in Cartesian
Coordinate
% Subroutine for finding the
velocity
function velocities
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxerr errormatrix
global xy global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr
global Ax Ay Vxp Vyp

M=(0.5*10^-3);
Vx=zeros(Y+1,X+1);
Vy=zeros(Y+2,X);
for i=1:(Y+1)
    Vx(i,1)=0.005;
end
for i=2:Y+1
    for j=2:X
        Vx(i,j)=((-1)*(Kx(i-1,j-1)*(P(i-1,j)-F(i-1,j-1)))/(M*X));
    end
end
for i=Y+1
    for j=1:(X+1)
        Vx(i,j)=Vx(i-1,j);
    end
end
for i=2:Y
    for j=2:X+1
        Vy(i,j-1)=((-1)*(Ky(i-1,j-1)*(P(i,j-1)-F(i-1,j-1)))/(M*Y));
    end
end

% Streamline Simulation Near Well
Bore
% By MARJAN HASHEM

% Developed MATLAB Program for
Streamline Simulation
% This Code developed originally by
MARJAN HASHEM

% Third Case Study in Cartesian
Coordinate
% Subroutine for finding the
velocity
function velocities
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxerr errormatrix
global xy global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg
global i j
global rr
global Ax Ay Vxp Vyp

M=(0.5*10^-3);
Vx=zeros(Y+1,X+1);
Vy=zeros(Y+2,X);
for i=1:(Y+1)
    Vx(i,1)=0.005;
end
for i=2:Y+1
    for j=2:X
        Vx(i,j)=((-1)*(Kx(i-1,j-1)*(P(i-1,j)-F(i-1,j-1)))/(M*X));
    end
end
for i=Y+1
    for j=1:(X+1)
        Vx(i,j)=Vx(i-1,j);
    end
end
for i=2:Y
    for j=2:X+1
        Vy(i,j-1)=((-1)*(Ky(i-1,j-1)*(P(i,j-1)-F(i-1,j-1)))/(M*Y));
    end
end
end
end
Vx;
Vy;

% Streamline Simulation Near Well Bore
% By MARJAN HASHEM

% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM

% Third Case Study in Cartesian Coordinate
% Subroutine for Finding Pressure distribution all across the grid blocks function pressure
global X Y
global Kx Ky
global P
global Vx Vy
global Xe Ye
global maxerr maxerr errormatrix
global x y
global Xpp Ypp
global Xp Yp
global Vxp Vyp
global gg ff
global i j
global rr
global Ax Ay Vxp Vyp
global K1 K2
global A B
P=zeros(Y,X);
for i=1:Y
    P(i,1)=100000;
    for i=(fix((7*Y)/10)):((fix((9*Y)/10))
        P(i,X)=100;
    end
end

maxr=1;
errormatrix=zeros(X,Y);
iteration=0;
while maxr>maxerr
    for i=1:Y
        for j=2:X
            tempval=P(i,j);
            x=i;
            y=j;
            if (i==1 & j<=(X-1))
                P(i,j)=((P(i,j-1)+P(i,j)+2*P(i+1,j))/4);
            elseif (j==Y & j<=(X-1))
                P(i,j)=((P(i,j)+P(i,j)+2*P(i-1,j))/4);
            elseif (j==X & i==1)
                P(i,j)=((P(i-1,j)+P(i,j))/2);
            elseif (j==X & i==Y)
                P(i,j)=((P(i-1,j)+P(i,j-1))/2);
            elseif (j==X & i>1 &
i>=(fix((7*Y)/10))+1 & i<Y)
                P(i,j)=((P(i-1,j)+P(i,j)+2*P(i,j-1))/4);
            elseif (j==X &
i>=(fix((7*Y)/10)) &
i<=(fix((9*Y)/10)))
                P(i,j)=100;
            else
                P(i,j)=100;
            end
        end
    end
end
errormatrix(i,j)=abs(P(i,j)-tempval);
end
maxr=max(max(errormatrix));
iteration=iteration+1;
end
for i=1:Y
    for j=1:X
        P(i,j);
    end
end
contourf(P,50, 'DisplayName', 'PRESSURE');colormap autumn;
figure
iteration
p
function location
    global X Y
    global x y
    global Xp Yp
    global i j
    global Tm
    global A B

    % tid2 = fopen('any name.dat', 'w');
    disp('first model')
    9876
    if (Vx(i,j)>0 & Vx(i,j+1)>0 &
        Vx(i,j)>Vx(i,j+1) & Vy(i,j)>0 &
        Vy(i+1,j)>0 & Vy(i,j+1)>Vy(i+1,j))
        disp(' 1-1 ')
    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/((x(j+1)-x(j)));
    Vxp(i,j) = (Ax(i,j))*(Xp(i,j)-x(j));
    Tx(i,j) = (Ty(i,j)-Ty(i+1,j))/((y(i)-y(i+1)));
    Vyp(i,j) = (Ay(i,j))*(Yp(i,j)-y(j));
    end
    if Ty(i,j)<0
        1317
        Ty(i,j) = Ty(i,j)*(-1);
    end
    end
    end

    % disp('3-1 ')
    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/((x(j+1)-x(j)));
    Vxp(i,j) = (Ax(i,j))*(Xp(i,j)-x(j));
    Tx(i,j) = (Ty(i,j)-Ty(i+1,j))/((y(i)-y(i+1)));
    Vyp(i,j) = (Ay(i,j))*(Yp(i,j)-y(j));
    end
    if Ty(i,j)<0
        1317
        Ty(i,j) = Ty(i,j)*(-1);
    end
    end
    end
    end

    % disp('4-1 ')
    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/((x(j+1)-x(j)));
    Vxp(i,j) = (Ax(i,j))*(Xp(i,j)-x(j));
    Tx(i,j) = (Ty(i,j)-Ty(i+1,j))/((y(i)-y(i+1)));
    Vyp(i,j) = (Ay(i,j))*(Yp(i,j)-y(j));
    end
    if Ty(i,j)<0
        1317
        Ty(i,j) = Ty(i,j)*(-1);
    end
    end
    end
    end

    Tm(i,j) = min (Tx(i,j),Ty(i,j));
    Xe(i,j)=(Tm(i,j))*
    Vxp(i,j)*x(j);
    Yei(i,j)=(Tm(i,j)*
    Vxp(i,j)+y(i);
    Xe(i,j);
    Yei(i,j);
Ax(i,j) = Vx(i,j+1) - Vx(i,j)/x(j+1) - x(j)
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j)))/Vx(i,j);
T = (Ax(i,j)) * log(Vx(i,j+1)/x(j)) / Vxp(i,j);
if Ty(i,j) <= 0
1317
Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
1318
Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = Tx(i,j);
Xe(i,j) = Tm(i,j) * Vxp(i,j) * x(j);
Ye(i,j) = y(i) + Xe(i,j);
Ye(i,j) = Y;블
else if (Vx(i,j) > 0 & Vx(i,j) > 0 & Vx(i,j+1) = 0)
1319
disp('5-1')
Ax(i,j) = Vx(i,j+1) - Vx(i,j)/x(j+1) - x(j);
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j)))/Vx(i,j);
if Tx(i,j) < 0
1318
Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = Tx(i,j);
Xe(i,j) = Tm(i,j) * Vxp(i,j) * x(j);
Ye(i,j) = y(i) + Xe(i,j);
Ye(i,j) = Y;블
else if (Vx(i,j) > 0 & Vx(i,j) > 0 & Vx(i,j+1) = 0)
1319
disp('5-1')
Ax(i,j) = Vx(i,j+1) - Vx(i,j)/x(j+1) - x(j);
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j)))/Vx(i,j);
if Tx(i,j) < 0
1318
Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = Tx(i,j);
Xe(i,j) = Tm(i,j) * Vxp(i,j) * x(j);
Ye(i,j) = y(i) + Xe(i,j);
Ye(i,j) = Y;블
else if (Vx(i,j) > 0 & Vx(i,j) > 0 & Vx(i,j+1) = 0)
1319
disp('5-1')
Ax(i,j) = Vx(i,j+1) - Vx(i,j)/x(j+1) - x(j);
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j)))/Vx(i,j);
if Tx(i,j) < 0
1318
Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = Tx(i,j);
Xe(i,j) = Tm(i,j) * Vxp(i,j) * x(j);
Ye(i,j) = y(i) + Xe(i,j);
Ye(i,j) = Y;블
else if (Vx(i,j) > 0 & Vx(i,j) > 0 & Vx(i,j+1) = 0)
1319
disp('5-1')
Ax(i,j) = Vx(i,j+1) - Vx(i,j)/x(j+1) - x(j);
Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j)))/Vx(i,j);
if Tx(i,j) < 0
1318
Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = Tx(i,j);
Xe(i,j) = Tm(i,j) * Vxp(i,j) * x(j);
Ye(i,j) = y(i) + Xe(i,j);
Ye(i,j) = Y;бл
end
Tm(i,j)=min(Tx(i,j), Ty(i,j));
Xe(i,j)=(Tm(i,j))*
Vxp(i,j)*x(j);

Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Ty(i,j)))*y(i);
Xe(i,j);
Ye(i,j);
else if (Vx(i,j)>0 & Vx(i,j+1)>0 &
Vx(i,j)>=Vx(i,j+1) & Vy(i,j)<0 &
Vy(i+1,j)<0 & Vy(i,j)<Vy(i+1,j))
disp('-S-1')

Ax(i,j)=-(Vx(i,j)+1)-
Vx(i,j)/(x(j+1)-x(j));
Vxp(i,j)= (Ax(i,j)*x(j));

x(j)+1)+Vx(i,j); %
Tx(i,j)=(1/Ax(i,j))*x(j)+
(Vx(i,j+1)/Vxp(i,j));

Xp(i,j)=(Vx(i,j+1)-
Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)=(Ay(i,j)*Vy(i,j)-

Ty(i,j)=(x(j+1)-
(Vy(i+1,j)+Vyp(i,j));

if (Ty(i,j)<0
1317
Ty(i,j) = Ty(i,j)*(-1);
end
if (Tx(i,j)<0
1318
Ty(i,j) = Tx(i,j)*(-1);
end
Tm(i,j)=min(Tx(i,j), Ty(i,j));
Xe(i,j)=(Tm(i,j)*
Vxp(i,j)*x(j);

Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Ty(i,j)))*y(i);
Xe(i,j);
Ye(i,j);
else if (Vx(i,j)>0 & Vx(i,j+1)>0 &
Vx(i,j)>=Vx(i,j+1) & Vy(i,j)<0 &
Vy(i+1,j)<0 & Vy(i,j)<Vy(i+1,j))
disp('-S-1')

Ax(i,j)=(Vx(i,j)+1)-
Vx(i,j)/(x(j+1)-x(j));
Vxp(i,j)= (Ax(i,j)*x(j));

x(j)+1)+Vx(i,j); %
Tx(i,j)=(1/Ax(i,j))*x(j)+
(Vx(i,j+1)/Vxp(i,j));

Xp(i,j)=(Vx(i,j+1)-
Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)=(Ay(i,j)*Vy(i,j)-

Ty(i,j)=(x(j+1)-
(Vy(i+1,j)+Vyp(i,j));

if (Ty(i,j)<0
1317
Ty(i,j) = Ty(i,j)*(-1);
end
if (Tx(i,j)<0
1318
Ty(i,j) = Tx(i,j)*(-1);
end
Tm(i,j)=min(Tx(i,j), Ty(i,j));
Xe(i,j)=(Tm(i,j)*
Vxp(i,j)*x(j);

Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Ty(i,j)))*y(i);
Xe(i,j);
Ye(i,j);
else if (Vx(i,j)>0 & Vx(i,j+1)>0 &
Vx(i,j)>=Vx(i,j+1) & Vy(i,j)<0 &
Vy(i+1,j)<0 & Vy(i,j)<Vy(i+1,j))
disp('-S-1')

Ax(i,j)=(Vx(i,j)+1)-
Vx(i,j)/(x(j+1)-x(j));
Vxp(i,j)= (Ax(i,j)*x(j));

x(j)+1)+Vx(i,j); %
Tx(i,j)=(1/Ax(i,j))*x(j)+
(Vx(i,j+1)/Vxp(i,j));

Xp(i,j)=(Vx(i,j+1)-
Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)=(Ay(i,j)*Vy(i,j)-

Ty(i,j)=(x(j+1)-
(Vy(i+1,j)+Vyp(i,j));

if (Ty(i,j)<0
1317
Ty(i,j) = Ty(i,j)*(-1);
end
if (Tx(i,j)<0
1318
Ty(i,j) = Tx(i,j)*(-1);
end
Tm(i,j)=min(Tx(i,j), Ty(i,j));
Xe(i,j)=(Tm(i,j)*
Vxp(i,j)*x(j);

Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Ty(i,j)))*y(i);
Xe(i,j);
Ye(i,j);
1317 Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
1318 Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = min (Tx(i,j), Ty(i,j));
Xe(i,j) = (Tm(i,j) * Vxp(i,j)) * x(j);
Ye(i,j) = ((1/Ay(i,j)) * (Vyp(i,j) * exp(Ay(i,j) * Tm(i,j)) - Vy(i,j))) * y(i);
Xe(i,j);
Ye(i,j);

elseif (Vx(i,j) > 0 & Vx(i,j+1) > 0 & Vy(i,j) > Vy(i,j+1) > 0 & Vy(i+1,j) < 0 & Vy(i+1,j+1) < 0 &
1319 disp('1-2')
Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/x(j+1) - x(j));
Vxp(i,j) = (Ax(i,j) * (Vxp(i,j) - X(i,j)))/x(j);
Vyp(i,j) = (Ay(i,j) * (Vyp(i,j) - Y(i,j)))/y(j);
Ty(i,j) = (1/Ay(i,j)) * log(Vy(i+1,j)/Vy(i,j));
if Ty(i,j) < 0
1317 Ty(i,j) = Ty(i,j) * (-1);
end
if Tx(i,j) < 0
1318 Tx(i,j) = Tx(i,j) * (-1);
end
Tm(i,j) = min (Tx(i,j), Ty(i,j));
Xe(i,j) = (Tm(i,j) * Vxp(i,j)) * x(j);
Ye(i,j) = ((1/Ay(i,j)) * (Vyp(i,j) * exp(Ay(i,j) * Tm(i,j)) - Vy(i,j))) * y(i);
Tm(i,j)=\min (Tx(i,j),Ty(i,j));
Xe(i,j)=(1/(1Ax(i,j)))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j);
Ye(i,j)=(1/(1Ay(i,j)))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(j); 
Xe(i,j); 
Ye(i,j);
else if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j+1)>Vx(i,j) & Vy(i,j)>0 & Vy(i,j+1)> Vy(i,j)) then
   disp('3-2');
   Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j)-x(j));
   Vxp(i,j)=(Ax(i,j)*Vxp(i,j)-x(j));
   Tx(i,j)=(1/Ax(i,j))*log
   (Vx(i,j)+Vxp(i,j));
   Ay(i,j)=(Vy(i,j+1)-Vy(i,j))/y(j+1)-y(j));
   Vyp(i,j)=(Ay(i,j)*Vy(i,j)-y(j));
   Ty(i,j)=(1/Ay(i,j))*log
   (Vy(i,j)+Vyp(i,j));
   if Tx(i,j)<0 then
      1317
      Tx(i,j)=Tx(i,j)-1;
      if Ty(i,j)<0 then
         1317
         Ty(i,j)=Ty(i,j)-1;
      end
      Tm(i,j)=\min (Tx(i,j),Ty(i,j));
   end
   Xe(i,j)=(1/(1Ax(i,j)))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j);
   Ye(i,j);
   else if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)>Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)>Vy(i,j)) then
      disp('4-2');
      Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j));
      Vxp(i,j)=(Ax(i,j)*Vxp(i,j)-x(j));
      Tx(i,j)=(1/Ax(i,j))*log
      (Vx(i,j)+Vxp(i,j));
      if Tx(i,j)<0 then
         1318
         Tx(i,j)=Tx(i,j)-1;
      end
      Tm(i,j)=Tx(i,j); 
      Xe(i,j)=(1/(1Ax(i,j)))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(j);
      Ye(i,j);
      else if (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)>Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)>Vy(i,j)) then
         disp('5-2');
         Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j));
         Vxp(i,j)=(Ax(i,j)*Vxp(i,j)-x(j));
         Tx(i,j)=(1/Ax(i,j))*log
         (Vx(i,j)+Vxp(i,j));
         if Tx(i,j)<0 then
            1318
            Tx(i,j)=Tx(i,j)-1;
         end
         Xe(i,j); 
         Ye(i,j);
         else Vyp(i,j)>0 then
            disp('6-2');
            Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j));
            Vxp(i,j)=(Ax(i,j)*Vxp(i,j)-x(j));
            Tx(i,j)=(1/Ax(i,j))*log
            (Vx(i,j)+Vxp(i,j));
            if Tx(i,j)<0 then
               1318
               Tx(i,j)=Tx(i,j)-1;
            end
            Xe(i,j);
            Ye(i,j); 
            end
         end
   end
end
% Ty(i,j)=(y(i+1)-Yp(i,j))/Vy(i,j)) if Tx(i,j)<0
1318
Tx(i,j)=Tx(i,j)*(-1);
end
if Ty(i,j)<0
1317
Ty(i,j)=Ty(i,j)*(-1);
end

Tm(i,j)=min (Tx(i,j),Ty(i,j));

Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)+exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(i,j);
Ye(i,j)=y(i+1); Ax(i,j)=(Vx(i,j)+exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)

Xe(i,j)=Xe(i,j)*x(i,j);
Ye(i,j)=Ye(i,j); Ax(i,j)=(Vx(i,j)+exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)

Xe(i,j)=Xe(i,j); Ax(i,j)=(Vx(i,j)+exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)

else if (Vx(i,j)>0 & Vx(i,j)+1)>0 & Vx(i,j)>Vx(i,j)+1 & Vy(i,j)<0 &Vy(i,j+1)< Vy(i,j)+Vy(i,j+1) & disp('-E-2')
Ax(i,j)=(Vx(i,j)+1-Vx(i,j))/x(i,j); Vxp(i,j)=(Ax(i,j)*x(i,j)+Vx(i,j));

if Tx(i,j)<0
1318
Tx(i,j)=Tx(i,j)*(-1);
end
if Ty(i,j)<0
1317
Ty(i,j)=Ty(i,j)*(-1);
end

Tm(i,j)=min (Tx(i,j),Ty(i,j));

Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)+exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(i,j);
Ye(i,j)=y(i+1); Ax(i,j)=(Vx(i,j)+exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)

Xe(i,j)=Xe(i,j)*x(i,j);
Ye(i,j)=Ye(i,j); Ax(i,j)=(Vx(i,j)+exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)

else if (Vx(i,j)>0 & Vx(i,j)+1)>0 & Vx(i,j)>Vx(i,j)+1 & Vy(i,j)<0 &Vy(i,j+1)< Vy(i,j)+Vy(i,j+1) & disp('-E-2')
Ax(i,j)=(Vx(i,j)+1-Vx(i,j))/x(i,j); Vxp(i,j)=(Ax(i,j)*x(i,j)+Vx(i,j));

if Tx(i,j)<0
1318
Tx(i,j)=Tx(i,j)*(-1);
end
if Ty(i,j)<0
1317
Ty(i,j)=Ty(i,j)*(-1);
end

Tm(i,j)=min (Tx(i,j),Ty(i,j));

Xe(i,j)=((1/Ax(i,j))*(Vxp(i,j)+exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(i,j);
Ye(i,j)=y(i+1); Ax(i,j)=(Vx(i,j)+exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)

Xe(i,j)=Xe(i,j)*x(i,j);
Ye(i,j)=Ye(i,j); Ax(i,j)=(Vx(i,j)+exp(Ax(i,j)*Tm(i,j)))-Vx(i,j)

else if (Vx(i,j)>0 & Vx(i,j)+1)>0 & Vx(i,j)>Vx(i,j)+1 & Vy(i,j)<0 &Vy(i,j+1)< Vy(i,j)+Vy(i,j+1) & disp('-E-2')
Ax(i,j)=(Vx(i,j)+1-Vx(i,j))/x(i,j); Vxp(i,j)=(Ax(i,j)*x(i,j)+Vx(i,j));
elseif (Vy(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)>Vx(i,j+1) & Vy(i,j)>=0 & Vy(i+1,j)<0)
    disp(' -12-2')
    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j));
    Vxp(i,j) = (Ax(i,j)*(Xp(i,j)-x(j)))/(Vx(i,j)) - Tx(i,j);
    Ty(i,j) =Ty(i,j)*(-1);
end

% - third model
elseif (Vx(i,j)>0 & Vx(i,j+1)>0 & Vy(i+1,j)>0 & Vy(i+1,j+1)<0)
    disp('[11-2]

    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j));
    Vxp(i,j) = (Ax(i,j)*(Xp(i,j)-x(j)))/(Vx(i,j));
    Vp(i,j) = (Vy(i+1,j)-y(i)) + (1/Ay(i,j))*exp(Ay(i,j)*Ty(i,j))/Vyp(i,j);
    Xp(i,j) = (Ax(i,j)*Tx(i,j))/Vxp(i,j);
    Tx(i,j) = Tx(i,j)*(-1);
end

% - first model
elseif (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)<Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)<0)
    disp(' -13-2')
    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j));
    Vxp(i,j) = (Ax(i,j)*(Xp(i,j)-x(j)))/(Vx(i,j));
    Vp(i,j) = (Vy(i+1,j)-y(i)) + (1/Ay(i,j))*exp(Ay(i,j)*Ty(i,j))/Vyp(i,j);
    Xp(i,j) = (Ax(i,j)*Tx(i,j))/Vxp(i,j);
    Tx(i,j) = Tx(i,j)*(-1);
end

% end

% - first model
elseif (Vx(i,j)>0 & Vx(i,j+1)>0 & Vx(i,j)<Vx(i,j+1) & Vy(i,j)>0 & Vy(i+1,j)<0)
    disp(' -13-2')
    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(j+1)-x(j));
    Vxp(i,j) = (Ax(i,j)*(Xp(i,j)-x(j)))/(Vx(i,j));
    Vp(i,j) = (Vy(i+1,j)-y(i)) + (1/Ay(i,j))*exp(Ay(i,j)*Ty(i,j))/Vyp(i,j);
    Xp(i,j) = (Ax(i,j)*Tx(i,j))/Vxp(i,j);
    Tx(i,j) = Tx(i,j)*(-1);
end

% end

% disp(' - THIRD model')
\begin{align*}
\text{Ay}(i,j) &= (Vy(i+1,j) - Vy(i,j))/(y(i+1) - y(i));
\text{Vyp}(i,j) &= \text{Ay}(i,j) \ast (Vy(i,j) - y(i)) \ast Vy(i,j); \\
\% \quad \text{Ty}(i,j) &= 1 / \text{Ay}(i,j) \ast \log \left(\frac{Vx(i,j) + Vyp(i,j)}{Vx(i,j)}\right); \\
\text{Ty}(i,j) &= \left(y(i) - Vyp(i,j)\right) /Vy(i,j); \\
\text{if} \quad \text{Ty}(i,j) < 0 & \\
1317 & \text{Ty}(i,j) = \text{Ty}(i,j) \ast (-1); \\
\text{end} \\
\text{end} \\
\text{end} \\
\begin{align*}
\text{Xe}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vxp(i,j) \ast \exp(\text{Ax}(i,j) \ast \text{Xe}(i,j))) \ast Vxp(i,j); \\
\% \quad \text{Ye}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vyp(i,j) \ast \exp(\text{Ay}(i,j) \ast \text{Xe}(i,j))) \ast Vy(i,j); \\
\text{Xe}(i,j) &; \\
\text{Ye}(i,j) &;
\end{align*}
\quad \% \quad \text{ - second model} \\
\begin{align*}
\text{else} \quad \text{if} \quad (Vx(i,j) > 0 \quad \text{&} \quad Vx(i,j+1) > 0 \quad \text{&} \quad Vy(i,j) < 0 \quad \text{&} \quad Vy(i+1,j) > 0) \\
1318 & \text{Ax}(i,j) = (Vx(i,j)) / (x(i+1) - x(i)); \\
\text{Vxp}(i,j) &= \left(\text{Ax}(i,j) \ast (x(i,j) - x(j))\right) \ast Vx(i,j); \\
\text{Tx}(i,j) &= 1 / \text{Ax}(i,j) \ast \log \left(\frac{Vx(i,j)}{Vy(i,j) - y(i)}\right); \\
\text{Vyp}(i,j) &= (Ax(i,j) \ast (y(i+1) - y(i))) \ast Vy(i,j); \\
\text{Y}(i,j) &= (1 / \text{Ax}(i,j)) \ast \log \left(\frac{Vy(i+1,j)}{Vx(i,j)}\right); \\
\text{if} \quad \text{Tx}(i,j) < 0 & \\
1318 & \text{Tx}(i,j) = \text{Tx}(i,j) \ast (-1); \\
\text{end} \\
\text{end} \\
\text{end} \\
\begin{align*}
\text{Xe}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vxp(i,j) \ast \exp(\text{Ax}(i,j) \ast \text{Xe}(i,j))) \ast Vxp(i,j); \\
\% \quad \text{Ye}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vyp(i,j) \ast \exp(\text{Ay}(i,j) \ast \text{Xe}(i,j))) \ast Vy(i,j); \\
\text{Xe}(i,j) &; \\
\text{Ye}(i,j) &;
\end{align*}
\quad \% \quad \text{if} \quad \text{Tx}(i,j) > 0 \\
\begin{align*}
\text{Tm}(i,j) &= \text{Tx}(i,j); \\
\text{Xe}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vxp(i,j) \ast \exp(\text{Ax}(i,j) \ast \text{Tm}(i,j))) \ast Vxp(i,j); \\
\% \quad \text{Ye}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vyp(i,j) \ast \exp(\text{Ay}(i,j) \ast \text{Tm}(i,j))) \ast Vy(i,j); \\
\text{Xe}(i,j) &; \\
\text{Ye}(i,j) &;
\end{align*}
\quad \% \quad \text{else} \quad \text{if} \quad (Vx(i,j) > 0 \quad \text{&} \quad Vx(i,j+1) > 0 \quad \text{&} \quad Vy(i,j) < 0 \quad \text{&} \quad Vy(i+1,j) > 0) \\
\begin{align*}
\text{disp}(' -3 - ') \\
\text{Ax}(i,j) &= (Vx(i,j)) / y(i+1) - x(i); \\
\text{Vxp}(i,j) &= \left(\text{Ax}(i,j) \ast (x(i,j) - x(j))\right) \ast Vx(i,j); \\
\text{Ay}(i,j) &= [Vx(i+1,j) - \\
\text{Vy}(i,j)] / (y(i+1) - y(i)); \\
\text{Vyp}(i,j) &= (Ax(i,j) \ast (y(i+1) - y(i))) \ast Vy(i,j); \\
\% \quad \text{Ty}(i,j) &= (1 / \text{Ax}(i,j)) \ast \log \left(\frac{Vy(i+1,j)}{Vx(i,j)}\right); \\
\text{if} \quad \text{Tx}(i,j) < 0 & \\
1318 & \text{Tx}(i,j) = \text{Tx}(i,j) \ast (-1); \\
\text{end} \\
\text{end} \\
\text{end} \\
\begin{align*}
\text{Xe}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vxp(i,j) \ast \exp(\text{Ax}(i,j) \ast \text{Xe}(i,j))) \ast Vxp(i,j); \\
\% \quad \text{Ye}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vyp(i,j) \ast \exp(\text{Ay}(i,j) \ast \text{Xe}(i,j))) \ast Vy(i,j); \\
\text{Xe}(i,j) &; \\
\text{Ye}(i,j) &;
\end{align*}
\quad \% \quad \text{else} \quad \text{if} \quad \text{ Tx}(i,j) > 0 \\
\begin{align*}
\text{Tm}(i,j) &= \text{Tx}(i,j); \\
\text{Xe}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vxp(i,j) \ast \exp(\text{Ax}(i,j) \ast \text{Tm}(i,j))) \ast Vxp(i,j); \\
\% \quad \text{Ye}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vyp(i,j) \ast \exp(\text{Ay}(i,j) \ast \text{Tm}(i,j))) \ast Vy(i,j); \\
\text{Xe}(i,j) &; \\
\text{Ye}(i,j) &;
\end{align*}
\quad \% \quad \text{disp}(' -3 - ') \\
\text{Ax}(i,j) &= (Vx(i,j)) / (x(i+1) - x(i)); \\
\text{Vxp}(i,j) &= \left(\text{Ax}(i,j) \ast (x(i,j) - x(j))\right) \ast Vx(i,j); \\
\text{Tx}(i,j) &= 1 / \text{Ax}(i,j) \ast \log \left(\frac{Vx(i,j)}{Vy(i,j) - y(i)}\right); \\
\text{Vyp}(i,j) &= (Ax(i,j) \ast (y(i+1) - y(i))) \ast Vy(i,j); \\
\% \quad \text{Ty}(i,j) &= (1 / \text{Ax}(i,j)) \ast \log \left(\frac{Vy(i+1,j)}{Vx(i,j)}\right); \\
\text{if} \quad \text{Tx}(i,j) < 0 & \\
1318 & \text{Tx}(i,j) = \text{Tx}(i,j) \ast (-1); \\
\text{end} \\
\text{end} \\
\text{end} \\
\begin{align*}
\text{Xe}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vxp(i,j) \ast \exp(\text{Ax}(i,j) \ast \text{Xe}(i,j))) \ast Vxp(i,j); \\
\% \quad \text{Ye}(i,j) &= (1 / \text{Ax}(i,j)) \ast (Vyp(i,j) \ast \exp(\text{Ay}(i,j) \ast \text{Xe}(i,j))) \ast Vy(i,j); \\
\text{Xe}(i,j) &; \\
\text{Ye}(i,j) &;
\end{align*}
\quad \% \quad \text{end}
\[ Vxp(i,j) = \left( Ax(i,j) \right) \cdot \left( Xp(i,j) - x(j) \right) + Vx(i,j); \]
\[ Tx(i,j) = \left( 1/Ax(i,j) \right) \cdot \log \left( Vxp(i,j) / Vxp(i,j) \right); \]
\[ Ay(i,j) = \left( Vy(i,j) - y(i) \right) / \left( y(i) - y(i) \right); \]
\[Vy(i,j) = \left( Ay(i,j) \right) \cdot \left( Yp(i,j) - y(i) \right) + Vy(i,j); \]
\[Ty(i,j) = \left( 1/Ay(i,j) \right) \cdot \log \left( Vy(i,j) / Vy(i,j) \right); \]
\[ Xe(i,j) = \left( 1/Ax(i,j) \right) \cdot \left( Vxp(i,j) \right) \cdot \exp \left( Ax(i,j) \cdot Tm(i,j) \right) \cdot x(j); \]
\[ Ye(i,j) = \left( 1/Ay(i,j) \right) \cdot \left( Vy(i,j) \right) \cdot \exp \left( Ay(i,j) \cdot Ty(i,j) \right) \cdot y(i); \]
\[ Tm(i,j) = \min \left( Tm(i,j), Ty(i,j) \right); \]
\[ Ax(i,j) = \left( 1/Ax(i,j) \right) \cdot \left( Vxp(i,j) \right) \cdot \exp \left( Ax(i,j) \cdot Tm(i,j) \right) \cdot x(j); \]
\[ Ay(i,j) = \left( 1/Ay(i,j) \right) \cdot \left( Vy(i,j) \right) \cdot \exp \left( Ay(i,j) \cdot Ty(i,j) \right) \cdot y(i); \]
\[ Tm(i,j) = \min \left( Tm(i,j), Ty(i,j) \right); \]
\[ Tx(i,j) = \left( 1/Ax(i,j) \right) \cdot \left( Vxp(i,j) \right) \cdot \exp \left( Ax(i,j) \cdot Tm(i,j) \right) \cdot x(j); \]
\[ Ty(i,j) = \left( 1/Ay(i,j) \right) \cdot \left( Vy(i,j) \right) \cdot \exp \left( Ay(i,j) \cdot Ty(i,j) \right) \cdot y(i); \]
\[ Tm(i,j) = \min \left( Tm(i,j), Ty(i,j) \right); \]
\[ Ax(i,j) = \left( 1/Ax(i,j) \right) \cdot \left( Vxp(i,j) \right) \cdot \exp \left( Ax(i,j) \cdot Tm(i,j) \right) \cdot x(j); \]
\[ Ay(i,j) = \left( 1/Ay(i,j) \right) \cdot \left( Vy(i,j) \right) \cdot \exp \left( Ay(i,j) \cdot Ty(i,j) \right) \cdot y(i); \]
\[ Tm(i,j) = \min \left( Tm(i,j), Ty(i,j) \right); \]
\[ Ax(i,j) = \left( 1/Ax(i,j) \right) \cdot \left( Vxp(i,j) \right) \cdot \exp \left( Ax(i,j) \cdot Tm(i,j) \right) \cdot x(j); \]
\[ Ay(i,j) = \left( 1/Ay(i,j) \right) \cdot \left( Vy(i,j) \right) \cdot \exp \left( Ay(i,j) \cdot Ty(i,j) \right) \cdot y(i); \]
\[ Tm(i,j) = \min \left( Tm(i,j), Ty(i,j) \right); \]
\[ Ax(i,j) = \left( 1/Ax(i,j) \right) \cdot \left( Vxp(i,j) \right) \cdot \exp \left( Ax(i,j) \cdot Tm(i,j) \right) \cdot x(j); \]
\[ Ay(i,j) = \left( 1/Ay(i,j) \right) \cdot \left( Vy(i,j) \right) \cdot \exp \left( Ay(i,j) \cdot Ty(i,j) \right) \cdot y(i); \]
\[ Tm(i,j) = \min \left( Tm(i,j), Ty(i,j) \right); \]
Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j))\*exp(Ay(i,j))\*Ty(i,j)))^y(i);  
Xe(i,j); 
Ye(i,j); 
elseif (Vxi(i,j)>0 \& Vyi(i,j)<0 \& Vyi(i+1,j)>0) 
\quad disp('5-4') 
\quad Ay(i,j)=(Vyi(i+1,j)-
\quad Vyi(i,j))/((Vyi(i+1,j)-y(i))\*Vyi(i,j)); 
\quad Vyp(i,j)=(Ay(i,j)\*Ty(i,j)-y(i))\*Vyi(i,j); 
\quad \% Ty(i,j)=(1/Ay(i,j))^log 
\quad (Vyi(i+1,j)/Vyi(i,j)); 
\quad if Ty(i,j)<0 
\quad Ty(i,j)=-Ty(i,j);(-1); 
\quad end 
\quad Tm(i,j)=Ty(i,j); 
\quad Xe(i,j)=x(j); 
\quad Ye(i,j); 
\% - second model 
elseif (Vxi(i,j)>0 \& Vxi(i,j+1)<0 
\& Vyi(i,j)<0 \& Vyi(i+1,j)>0) 
\quad disp('6-4') 
\quad Ay(i,j)=(Vyi(i+1,j)-
\quad Vyi(i,j))/((Vyi(i+1,j)-y(i))\*Vyi(i,j)); 
\quad Vyp(i,j)=(Ay(i,j)\*Ty(i,j)-y(i))\*Vyi(i,j); 
\quad if Vyp(i,j)<0 
\quad disp('6-4-1') 
\quad Ye(i,j)=y(i); 
\quad Xe(i,j); 
\quad Ye(i,j); 
\quad else Vyp(i,j)<0
\quad disp('6-4-2') 
\quad Xe(i,j)=x(j); 
\quad Ye(i,j)=y(i+1); 
\quad Xe(i,j); 
\quad Ye(i,j); 
\quad end 
\quad elseif (Vxi(i,j)>0 \& 
\quad Vxi(i,j+1)<0 \& 
\quad Vy(i,j)<0 \& Vy(i+1,j)<0 \& 
\quad Vy(i,j)>Vy(i+1,j) 
\quad disp('7-4') 
\quad Ay(i,j)=(Vy(i+1,j)-
\quad Vy(i,j))/((Vy(i+1,j)-y(i))\*Vy(i,j)); 
\quad Vyp(i,j)=(Ay(i,j)\*Ty(i,j)-
\quad y(i))\*Vy(i,j); 
\quad Ty(i,j)=(1/Ay(i,j))^log 
\quad (Vy(i+1,j)/Vy(i,j)); 
\quad if Ty(i,j)<0 
\quad Ty(i,j)=-Ty(i,j);(-1); 
\quad end 
\quad Tm(i,j)=Ty(i,j); 
\quad Xe(i,j)=x(j); 
\quad Ye(i,j); 
\% Axi(i,j)=(Vy(i+1,j)-Vyi(i,j))/(x(j)-
\quad x(i)); 
\quad Ay(i,j)=(Vy(i+1,j)-
\quad Vy(i,j))/(y(i+1)-y(i)); 
\quad Vyp(i,j)=(Ay(i,j)*Ty(i,j)-
\quad y(i))\*Vy(i,j); 
\quad if Ty(i,j)<0 
\quad Ty(i,j)=Ty(i,j);(-1); 
\quad end 
\quad Tm(i,j)=Ty(i,j); 
\quad Xe(i,j)=x(j); 
\quad Ye(i,j); 
\% - second model 
elseif (Vxi(i,j)>0 \& Vxi(i,j+1)<0 
\& Vy(i,j)<0 \& Vy(i+1,j)<0 \& Vy(i,j)==Vy(i+1,j) 
\quad disp('9-4') 
\quad Ay(i,j)=(Vy(i+1,j)-
\quad Vy(i,j))/(y(i+1)-y(i)); 
\quad Vyp(i,j)=(Ay(i,j)*Ty(i,j)-
\quad y(i))\*Vy(i,j); 
\quad if Ty(i,j)<0 
\quad Ty(i,j)=Ty(i,j);(-1); 
\quad end 
\quad Tm(i,j)=Ty(i,j); 
\quad Xe(i,j)=x(j); 
\quad Ye(i,j); 
\% Vyp(i,j)=y(i);
\quad Xe(i,j); 
\quad Ye(i,j); 
\% - second model 
elseif (Vy(i,j)>0 \& 
\quad Vy(i,j+1)<0 \& 
\quad Vy(i,j)<Vy(i+1,j) 
\quad disp('10-4') 
\quad Ay(i,j)=(Vy(i+1,j)-
\quad Vy(i,j))/(y(i+1)-y(i)); 
\quad Vyp(i,j)=(Ay(i,j)*Ty(i,j)-
\quad y(i))\*Vy(i,j); 
\quad if Ty(i,j)<0 
\quad Ty(i,j)=Ty(i,j);(-1); 
\quad end 
\quad Tm(i,j)=Ty(i,j); 
\quad Xe(i,j)=x(j); 
\quad Ye(i,j); 
\% Xe(i,j)=x(j); 
\quad Ye(i,j);
\( X_e(i,j) = x(j); \)
\[ Y_e(i,j) = \left( \frac{1}{A_y(i,j)} \right) \exp(A_y(i,j)) \exp(T_m(i,j)) \exp(V_y(i,j)) \exp(y(i)); \]
\( X_e(i,j); \)
\( Y_e(i,j); \)

```matlab
% - third model
elseif (Vx(i,j) > 0 & Vx(i,j+1) < 0 & Vy(i,j) == 0 & Vy(i+1,j) > 0)
disp('11-4')
3333
X_e(i,j) = x(j+1);
Y_e(i,j) = y(i);

% X_e(i,j);
% Y_e(i,j);
rr = 1

elseif (Vx(i,j) > 0 & Vx(i,j+1) < 0 & Vy(i,j) == 0 & Vy(i+1,j) < 0)
disp('12-4')
A_y(i,j) = \left( \frac{V_y(i) - y(i)}{V_y(i,j)} \right)\exp(A_y(i,j) - y(i)) + V_y(i,j);

% Ty(i,j) = (1/A_y(i,j)) * log
(V_y(i+1,j) + V_y(i,j));
Ty(i,j) = (1/A_y(i,j)) * log
(V_y(i+1,j) + V_y(i,j));
if Ty(i,j) < 0
Ty(i,j) = Ty(i,j) * (-1);
end
T_m(i,j) = Ty(i,j) * (-1);
elseif (Vx(i,j) > 0 & Vx(i,j+1) < 0 & Vy(i,j) < 0 & Vy(i+1,j) == 0)
disp('13-4')
disp('I dont know')
rr = 1

% fifth model
elseif (Vx(i,j) > 0 & Vx(i,j+1) == 0 & Vy(i,j) == 0 & Vy(i+1,j) > 0 & Vy(i,j) == Vy(i+1,j) > 0 & Vx(i,j) = Vy(i,j+1) == 0 & Vx(i,j+1) == 0)
disp('15-4')
A_x(i,j) = \left( V_x(i,j+1) - x(j) \right) + V_x(i,j);

% Tx(i,j) = (1/A_x(i,j)) * log
(V_x(i,j+1) + V_x(i,j));
Tx(i,j) = (1/A_x(i,j)) * log
(V_x(i,j+1) + V_x(i,j));
Ay(i,j) = \left( V_y(i+1,j) - y(i) \right) + V_y(i,j);

% Vx(i,j) + V_y(i,j);
Ay(i,j)=(Vy(i+1,j)-Vyp(i,j))/(y(i+1)-y(i));
Vy(i,j)=Ay(i,j)*(Yp(i,j)-y(i))+Vyp(i,j);
Ty(i,j)=(1/Ay(i,j))*Log(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<=0
    Ty(i,j)=Ty(i,j)*(-1);
endif
if Tx(i,j)<0
    Tx(i,j)=Tx(i,j)*(-1);
endif
Tm(i,j)=min(Tx(i,j),Ty(i,j));
% Xe(i,j)=(Tm(i,j)*
Vxp(i,j))*x(j);

Xe(i,j)=((1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vxp(i,j)))*x(j);
Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vyp(i,j))*y(i);
Ye(i,j)=x(j);
else (Vx(i,j),Vy(i,j))<0 & Vy(i,j+1)=0 &Vy(i,j+1)>0 &Vy(i+1,j)<0
    disp('-4-5')
    disp('I dont know')
    rr=1
else (Vx(i,j),Vy(i,j))<0 & Vy(i,j+1)=0 &Vy(i,j)>0 &Vy(i+1,j)>=0
    disp('-5-5')
    disp('I dont know')
    rr=1
else if (Vx(i,j),Vy(i,j))<0 & Vy(i,j+1)=0 &Vy(i,j)<0 &Vy(i+1,j)>0
    disp('-6-5')
    Ax(i,j)=(Vx(i,j)+1-Vx(i,j))/x(j)+1;
    Vxp(i,j)=(Ax(i,j)*x(j));
    Tx(i,j)=1/(Ax(i,j)+1)*log(Vx(i,j)+1/Vxp(i,j)));
    Tx(i,j)=(x(j)+1);
    Xp(i,j)/Vxp(i,j); Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i);
    Vyp(i,j)=(Ay(i,j)+y(i));
    Ty(i,j)=(1/(Ax(i,j)+1)*log(Vy(i+1,j)/Vyp(i,j)));
    if Ty(i,j)<0
        Ty(i,j)=Ty(i,j)*(-1);
    endif
    Tm(i,j)=Tx(i,j)
    Xe(i,j)=(Tm(i,j)*
Vxp(i,j))*x(j);
    Xe(i,j)=(1/Ax(i,j))*((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vxp(i,j)))*x(j);
    if Vyp(i,j)<0
        disp('-6-5-1')
        Ye(i,j)=y(i);
elseif (Vx[i,j] > 0 & Vx[i+1,j] == 0 & Vy[i,j] == 0 & Vy[i+1,j] < 0) 
  disp(' -12-5')
  Ax[i,j]=(Vx[i,j]+1-
  Vx[i,j])/(x[i+1]-x[i]);
  Vxp[i,j]=Ax[i,j]*x[j];
  Vxp[i,j]=Vxp[i,j]+Vxp[i,j]*(Vx[i,j]-
  x[j])+Vx[i,j];
  % Tx[i,j]=(1/Ax[i,j])*log
  % (Vx(i+1)/Vxp[i,j]);
  % Tx[i,j]=(x(j+1)-
  Xp[i,j]/Vx[i,j]);
  Ay[i,j]=Vy[i+1,j]-
 Vy[i,j]/(y[i+1]-y[i]);
  Yp[i,j]=(Ay[i,j]*y[i+1]-
  y[i])+Vy[i,j];
  % Ty(i,j)=(1/Ay(i,j))*log
  % (Vy(i+1)/Vyp[i,j]);
  % Ty(i,j)=(y(i+1)-
  Yp[i,j]/Vy[i,j]);
  if Ty[i,j]<0
    Ty[i,j]=-Ty[i,j]*(-1);
  end
  if Tx[i,j]<0
    Tx[i,j]=-Tx[i,j]*(-1);
  end
  Vxp[i,j]=x[j];
  Xe[i,j]=(Tx[i,j]*
  Xe[i,j]+(1/Ax[i,j])*Vxp[i,j]*exp(Ax[i,j]*
  Vxp[i,j]));
  Vxp[i,j]=Xe[i,j];
  Ye[i,j]= Vy[i+1,j]-
 Vy[i,j]*exp(Ay[i,j]*
  Vyp[i,j]);
  if Vx[i,j]<0 & Vx[i+1,j]==0 &
  Vx[i+1,j]==0 
  disp('-13-5')
  disp('I dont know')
end

rr=1

disp(' -sixth model')

elseif (Vx[i,j] < 0 & Vx[i+1,j] > 0 &
Vy[i,j] > 0 & Vy[i+1,j] > 0 &
Vy[i+1,j] == 0 &
Vx[i,j] < 0) 
  disp('-1-6-6')
  if ( Vxp[i,j]<0) 
    disp('-1-6-1')
  Ay[i,j]=(Vy[i+1,j]-
  Vy[i,j])/(y[i+1]-y[i]);
  Vyp[i,j]=(Ay[i,j]*y[i+1]-
  y[i])+Vy[i,j];
  % Ty(i,j)=(1/Ay(i,j))*log
  % (Vy(i+1)/Vyp[i,j]);
  % Ty(i,j)=(y(i+1)-
  Yp[i,j]/Vy[i,j]);
  if Ty[i,j]<0
    Ty[i,j]=-Ty[i,j]*(-1);
  end
  TM[i,j]=Ty[i,j];
  Xe[i,j]=x[j];

Ye[i,j]=((1/Ay(i,j))*Vyp[i,j]*exp( Ay(i,j)*
  Vyp[i,j]));
  Xe[i,j];
  Ye[i,j];
  elseif ( Vxp[i,j] < 0) 
  disp('-1-6-2')
  Ay[i,j]=(Vy[i+1,j]-
  Vy[i,j])/(y[i+1]-y[i]);
  Vyp[i,j]=(Ay[i,j]*y[i+1]-
  y[i])+Vy[i,j];
  % Ty(i,j)=(1/Ay(i,j))*log
  % (Vy(i+1)/Vyp[i,j]);
  % Ty(i,j)=(y(i+1)-
  Yp[i,j]/Vy[i,j]);
  if Ty[i,j]<0
    Ty[i,j]=-Ty[i,j]*(-1);
  end
  TM[i,j]=Ty[i,j];
  Xe[i,j]=x[j];

Ye[i,j]=((1/Ay(i,j))*Vyp[i,j]*exp( Ay(i,j)*
  Vyp[i,j]));
  Xe[i,j];
  Ye[i,j];
  end

if (Vx[i,j]<0 &
Vy[i,j] > 0 &
Vx[i+1,j] > 0 &
Vy[i+1,j] > 0) 
  disp('-2-6')
  if ( Vxp[i,j]<0) 
    disp('-2-6-1')
  Ay[i,j]=(Vy[i+1,j]-
  Vy[i,j])/(y[i+1]-y[i]);
  Vyp[i,j]=(Ay[i,j]*y[i+1]-
  y[i])+Vy[i,j];
  % Ty(i,j)=(1/Ay(i,j))*log
  % (Vy(i+1)/Vyp[i,j]);
  % Ty(i,j)=(y(i+1)-
  Yp[i,j]/Vy[i,j]);
  if Ty[i,j]<0
    Ty[i,j]=-Ty[i,j]*(-1);
  end
  TM[i,j]=Ty[i,j];
  Xe[i,j]=x[j];

Ye[i,j]=((1/Ay(i,j))*Vyp[i,j]*exp( Ay(i,j)*
  Vyp[i,j]));
  Xe[i,j];
  Ye[i,j];
  elseif ( Vxp[i,j] > 0) 
  disp('-2-6-2')
  Ay[i,j]=(Vy[i+1,j]-
  Vy[i,j])/(y[i+1]-y[i]);
  Vyp[i,j]=(Ay[i,j]*y[i+1]-
  y[i])+Vy[i,j];
  % Ty(i,j)=(1/Ay(i,j))*log
  % (Vy(i+1)/Vyp[i,j]);
  % Ty(i,j)=(y(i+1)-
  Yp[i,j]/Vy[i,j]);
  if Ty[i,j]<0
    Ty[i,j]=-Ty[i,j]*(-1);
  end
  TM[i,j]=Ty[i,j];
  Xe[i,j]=x[j];

Ye[i,j]=((1/Ay(i,j))*Vyp[i,j]*exp( Ay(i,j)*
  Vyp[i,j]));
  Xe[i,j];
  Ye[i,j];

end
disp('3-6')
if ( Vxp(i,j)<0 )
disp('3-6-1')
Ax(i,j)=(Vx(i,j)+1)-
Vx(i,j))/(x(j)+1-x(j));
Vxp(i,j) = (Ax(i,j)*x(j))-
Xp(i,j)+Vx(i,j);
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/(y(i)+1-y(i));
Vyp(i,j) = (Ay(i,j)*y(j))-
y(j)+Vy(i,j);
Ty(i,j)=(1/Ay(i,j))'*log
(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j) =Ty(i,j)'(-1);
end
Tm(i,j)=Ty(i,j);
if ( Vxp(i,j)<0 )
    disp('3-6-1')
    Xe(i,j)=x(j);
end
Ye(i,j)=((1/Ay(i,j))'*{(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j)})'*y(i);
Xe(i,j);
Ye(i,j);
else
    disp('3-6-2')
    Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/(y(i)+1-y(i));
    Vyp(i,j) = (Ay(i,j)*y(j))-
y(j)+Vy(i,j);
    Ty(i,j)=(1/Ay(i,j))'*log
(Vy(i+1,j)/Vyp(i,j));
    if Ty(i,j)<0
        Ty(i,j) =Ty(i,j)'(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(j);
end
Ye(i,j)=((1/Ay(i,j))'*{(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j)})'*y(i);
Xe(i,j);
Ye(i,j);
else ( Vxp(i,j)<0 & Vx(i,j)+1>0 & Vy(i,j)>0 & Vy(i,j+1)<0 )
disp('4-6')
Ax(i,j)=(Vx(i,j)+1)-
Vx(i,j))/(x(j)+1-x(j));
Vxp(i,j) = (Ax(i,j)*x(j))-
Xp(i,j)+Vx(i,j);
if ( Vxp(i,j)<0 )
disp('4-6-1')
Xe(i,j)=x(j);
Ye(i,j)=y(i);
Xe(i,j);
Ye(i,j);
else ( Vxp(i,j)<0 )
disp('4-6-2')
Xe(i,j)=x(j+1);
Ye(i,j)=y(i);
Xe(i,j);
Ye(i,j);
end
if ( Vxp(i,j)<0 )
disp('4-6-3')
Xe(i,j)=x(j);
else ( Vxp(i,j)<0 )
disp('4-6-4')
Xe(i,j)=x(j+1);
end
else ( Vxp(i,j)<0 & Vx(i,j)+1>0 & Vy(i,j)<0 & Vy(i,j+1)<0 & Vy(i,j)>Vy(i,j+1) )
disp('7-6')
if Vxp(i,j)<0
else ( Vxp(i,j)<0 & Vx(i,j)+1>0 & Vy(i,j)<0 & Vy(i,j+1)<0 & Vy(i,j)>Vy(i,j+1) )
disp('5-6')
Ax(i,j)=(Vx(i,j)+1)-
Vx(i,j))/(x(j)+1-x(j));
Vxp(i,j) = (Ax(i,j)*x(j))-
Xp(i,j)+Vx(i,j);
if ( Vxp(i,j)<0 )
disp('5-6-1')
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/(y(i)+1-y(i));
Vyp(i,j) = (Ay(i,j)*y(j))-
y(j)+Vy(i,j);
Ty(i,j)=(1/Ay(i,j))'*log
(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j) =Ty(i,j)'(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=x(j);
end
Ye(i,j)=((1/Ay(i,j))'*{(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j)})'*y(i);
Xe(i,j);
Ye(i,j);
else ( Vxp(i,j)<0 )
disp('5-6-2')
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/(y(i)+1-y(i));
Vyp(i,j) = (Ay(i,j)*y(j))-
y(j)+Vy(i,j);
Ty(i,j)=(1/Ay(i,j))'*log
(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j) =Ty(i,j)'(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=x(j+1);
end
Ye(i,j)=((1/Ay(i,j))'*{(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j)})'*y(i);
Xe(i,j);
Ye(i,j);
end
else ( Vxp(i,j)<0 & Vx(i,j)+1>0 & Vy(i,j)<0 & Vy(i,j+1)<0 )
disp('6-6')
if Vyp(i,j)<0
disp('6-6-1')
Ye(i,j)=y(i);
else Vyp(i,j)>0
disp('6-6-2')
Ye(i,j)=y(i+1);
end
if Vxp(i,j)<0
disp('6-6-3')
Xe(i,j)=x(j+1);
else Vxp(i,j)>0
disp('6-6-4')
Xe(i,j)=x(j);
end
else ( Vxp(i,j)<0 & Vx(i,j)+1>0 & Vy(i,j)<0 & Vy(i,j+1)<0 & Vy(i,j)>Vy(i,j+1) )
disp('7-6')
if Vxp(i,j)<0
246
disp(' -7-6-1')
Ay(1,j)=(Vy(i+1,j)-
Vy(i,j))/y(i+1)-y(i));
Vyp(1,j)=Ay(1,j)*(Yp(i,j)-
y(i));
Vy(i,j)=Ty(i,j)+log
(Vy(i+1,j)/Vyp(1,j));
if Ty(i,j)<0
Ty(i,j) =Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=x(i+1);

Ye(i,j)=((1/Ay(1,j))*exp[Ay(i,j)*
Tm(i,j)])- Vy(i,j)+y(i));
Xe(i,j);
Ye(i,j);
else Vxp(i,j)<0 disp(' -7-6-2')
Ay(1,j)=(Vy(i+1,j)-
Vy(i,j))/y(i+1)-y(i));
Vyp(1,j)=Ay(1,j)*(Yp(i,j)-
y(i));
Vy(i,j)=Ty(i,j)+log
(Vy(i+1,j)/Vyp(1,j));
if Ty(i,j)<0
Ty(i,j) =Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=x(i+1);

Ye(i,j)=((1/Ay(1,j))*exp[Ay(i,j)*
Tm(i,j)])- Vy(i,j)+y(i));
Xe(i,j);
Ye(i,j);
end
elseif (Vx(i,j)<0 & Vx(i,j+1)>0 &
Vy(i,j)<0 & Vy(i+1,j)<0 & Vy(i,j)==
Vy(i+1,j) )
disp(' -9-6')
if Vxp(i,j)<0 disp(' -9-6-1')
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/y(i+1)-y(i));
Vyp(i,j)=Ay(i,j)*(Yp(i,j)-
y(i));
Ty(i,j)=1/Ay(i,j)*log
(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j) =Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=x(i+1);

Ye(i,j)=((1/Ay(i,j))*exp[Ay(i,j)*
Tm(i,j)])- Vy(i,j)+y(i));
Xe(i,j);
Ye(i,j);
end
elseif (Vx(i,j)<0 & Vx(i,j+1)>0 &
Vy(i,j)<0 & Vy(i+1,j)<0 & Vy(i,j)==
Vy(i+1,j) )
disp(' -9-6-2')
if Vxp(i,j)<0 disp(' -9-6-2')
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/y(i+1)-y(i));
Vyp(i,j)=Ay(i,j)*(Yp(i,j)-
y(i));
Ty(i,j)=1/Ay(i,j)*log
(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j) =Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=x(i+1);

Ye(i,j)=((1/Ay(i,j))*exp[Ay(i,j)*
Tm(i,j)])- Vy(i,j)+y(i));
Xe(i,j);
Ye(i,j);
else Vxp(i,j)>0 disp(' -10-6')
Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(i+1)-
x(i));
Vxp(i,j) =Ax(i,j)*(Xp(i,j)-
x(i));
end
else Vxp(i,j)<0 disp(' -8-6')
if Vxp(i,j)<0 disp(' -8-6-1')
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/y(i+1)-y(i));
Vyp(i,j)=Ay(i,j)*(Yp(i,j)-
y(i));
Ty(i,j)=1/Ay(i,j)*log
(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j) =Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=x(i+1);

Ye(i,j)=((1/Ay(i,j))*exp[Ay(i,j)*
Tm(i,j)])- Vy(i,j)+y(i));
Xe(i,j);
Ye(i,j);
else Vxp(i,j)>0 disp(' -8-6-2')
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/y(i+1)-y(i));
Vyp(i,j)=Ay(i,j)*(Yp(i,j)-
y(i));
Ty(i,j)=1/Ay(i,j)*log
(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j) =Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
Xe(i,j)=x(i+1);

Ye(i,j)=((1/Ay(i,j))*exp[Ay(i,j)*
Tm(i,j)])- Vy(i,j)+y(i));
Xe(i,j);
Ye(i,j);
end
if Vxp(i,j)<0 
    disp('(-10-6-1')
    Ay(i,j)=Vxp(i,j)-Vy(i,j)/y(i+1)-y(j);
    Vyp(i,j)=Ay(i,j)*(Vp(i,j)-y(i))/Vy(i,j);
    Ty(i,j)=(-y(i)-Vp(i,j)+Vy(i,j));
    if Ty(i,j)<0
        Ty(i,j)=-Ty(i,j)*(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(j);
    Ye(i,j)=((1/Ay(i,j))*((Vxp(i,j)*exp(Ay(i,j)*Tm(i,j))-Vy(i,j))+y(i));
    Xe(i,j);
    Ye(i,j);
else Vxp(i,j)>0 
    disp('(-10-6-2')
    Ay(i,j)=Vxp(i,j)-Vy(i,j)/y(i+1)-y(j);
    Vyp(i,j)=Ay(i,j)*(Yp(i,j)-y(i))/Vy(i,j);
    Ty(i,j)=(-y(i)-Yp(i,j)/Vy(i,j));
    if Ty(i,j)<0
        Ty(i,j)=-Ty(i,j)*(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(j+1):
    Ye(i,j)=((1/Ay(i,j))*((Vxp(i,j)*exp(Ay(i,j)*Tm(i,j))-Yp(i,j)))+y(i));
    Xe(i,j);
    Ye(i,j);
end

% third model
elseif (Vx(i,j)<0 & Vx(i,j+1)>0 & Vy(i,j)==0 & Vy(i+1,j)>0 )
disp('(-12-6')
    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j);
    Vxp(i,j) = (Ax(i,j))*(Xp(i,j)-x(j))/Vx(i,j);
    if Vxp(i,j)<0 
        disp('(-11-6-1')
        Ay(i,j)=Vxp(i,j)-Vy(i,j)/y(i+1)-y(j);
        Vyp(i,j)=Ay(i,j)*(Yp(i,j)-y(i))/Vy(i,j);
        Ty(i,j)=(-y(i)-Vp(i,j)+Vy(i,j));
        (Vy(i+1,j)+Vp(i,j));
        Ty(i,j)=(-y(i+1)-Yp(i,j)/Vy(i,j)+Ty(i,j));
    if Ty(i,j)<0
        Ty(i,j)=-Ty(i,j)*(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(j+1):
    Ye(i,j)
end
Ye(i,j)=((1/Ay(i,j)))*((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j))-Vx(i,j)+y(i));
    Xe(i,j);
    Ye(i,j);
else Vxp(i,j)>0 
    disp('(-12-6-2')
    Ay(i,j)=Vxp(i,j)-Vy(i,j)/y(i+1)-y(j);
    Vyp(i,j)=Ay(i,j)*(Yp(i,j)-y(i))/Vy(i,j);
    Ty(i,j)=(-y(i)-Yp(i,j)/Vy(i,j));
    if Ty(i,j)<0
        Ty(i,j)=-Ty(i,j)*(-1);
    end
    Tm(i,j)=Ty(i,j);
    Xe(i,j)=x(j+1):
    Ye(i,j);
end
Ye(i,j)=((1/Ay(i,j))*exp(Ay(i,j)*Tm(i,j)))-Vx(i,j)+y(i));
    Xe(i,j);
    Ye(i,j);
Ye(i,j)=((1/Ay(i,j))*exp(Ay(i,j)*
Xe(i,j)-Vy(i,j)))+y(1);
Xe(i,j);
Ye(i,j);
end

elseif (Vx(i,j)<0 &&
Vx(i,j+1)>0 && Vy(i,j)=0 && Vy(i,j+1)==0)
disp('13-6:1')

Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/((x(j+1)-x(j));
Vxp(i,j)=(Ax(i,j))*(Xp(i,j)-
x(j))+Vx(i,j);
if Vxp(i,j)<0
disp('-13-6-1')

% Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j));
Xe(i,j)-x(j);
Ye(i,j)=y(1);
Xe(i,j);
Ye(i,j);
else Vxp(i,j)>0
disp('-13-6-2')

Xe(i,j)=x(j+1);
Ye(i,j)=y(1);
Xe(i,j);
Ye(i,j);
end

disp('1-seventh model')

% first model
elseif (Vx(i,j)<0 && Vx(i,j+1)<0 &&
Vx(i,j)=Vx(i,j+1) && Vy(i,j)>0 &&
Vy(i+1,j)=0 && Vy(i,j+1)==0)
disp('-1:7')

Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/((x(j+1)-x(j));
Vxp(i,j)=(Ax(i,j))*(Xp(i,j)-
x(j))+Vx(i,j);

% Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j));
Xe(i,j)=x(j);
Ye(i,j)=y(1);
Xe(i,j);
Ye(i,j);

% Ty(i,j)=(1/Ay(i,j))*log
(Vy(i,j+1)/Vyp(i,j));
Ty(i,j)=y(i+1);
Ye(i,j)=y(1);
Yp(i,j)=((Ay(i,j))*(Yp(i,j)-
y(i)));%1)
% Ty(i,j)=(1/Ay(i,j))*log
(Vy(i,j+1)/Vyp(i,j));
Ty(i,j)=y(i+1);
Ye(i,j)=y(1);
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)-1;
end
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)-1;
end

end

end

end

475
elseif \((Vx(i,j)<0 \& Vx(i,j+1)>0 \& Vx(i,j)==Vx(i,j+1)\& Vy(i,j)>0 \& Vy(i+1,j)==0\) disp('4-7')
    \(Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j)\);
    \(Vxp(i,j)=Ax(i,j)*(Vxp(i,j)-x(j))\)+Vxp(i,j);
    \(Tx(i,j)=(1/Ax(i,j))*\log(Vx(i,j+1)/Vxp(i,j))\);
    \(Ty(i,j)=(y(i+1)-y(i))\);
    \(Vyp(i,j)=Ay(i,j)*(Vyp(i,j)-y(i))\)+Vyp(i,j);
    \(Ty(i,j)=(1/Ay(i,j))*\log(Vy(i,j+1)/Vyp(i,j))\);
    \(Ty(i,j)=Ty(i,j)+y(i)\);
    \(Ye(i,j)=y(i)\);
    \(Ye(i,j)=y(i)\);

elseif \((Vx(i,j)<0 \& Vx(i,j+1)>0 \& Vx(i,j)==Vx(i,j+1)\& Vy(i,j)>0 \& Vy(i+1,j)==0\) disp('5-7')
    \(Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j)\);
    \(Vxp(i,j)=Ax(i,j)*(Vxp(i,j)-x(j))\)+Vxp(i,j);
    \(Tx(i,j)=(1/Ax(i,j))*\log(Vx(i,j+1)/Vxp(i,j))\);
    \(Ty(i,j)=(y(i+1)-y(i))\);
    \(Vyp(i,j)=Ay(i,j)*(Vyp(i,j)-y(i))\)+Vyp(i,j);
    \(Ty(i,j)=(1/Ay(i,j))*\log(Vy(i,j+1)/Vyp(i,j))\);
    \(Ty(i,j)=Ty(i,j)+y(i)\);
    \(Ye(i,j)=y(i)\);
    \(Ye(i,j)=y(i)\);
    \(Ye(i,j)=y(i)\);

elseif \((Vx(i,j)<0 \& Vx(i,j+1)>0 \& Vx(i,j)==Vx(i,j+1)\& Vy(i,j)>0 \& Vy(i+1,j)>0\) disp('6-7')
    \(Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j)\);
    \(Vxp(i,j)=Ax(i,j)*(Vxp(i,j)-x(j))\)+Vxp(i,j);
    \(Tx(i,j)=(1/Ax(i,j))*\log(Vx(i,j+1)/Vxp(i,j))\);
    \(Ty(i,j)=(y(i+1)-y(i))\);
    \(Vyp(i,j)=Ay(i,j)*(Vyp(i,j)-y(i))\)+Vyp(i,j);
    \(Ty(i,j)=Ty(i,j)+y(i)\);
    \(Ye(i,j)=y(i)\);
    \(Ye(i,j)=y(i)\);
    \(Ye(i,j)=y(i)\);

else
    \(Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/x(j+1)-x(j)\);
    \(Vxp(i,j)=Ax(i,j)*(Vxp(i,j)-x(j))\)+Vxp(i,j);
    \(Tx(i,j)=(1/Ax(i,j))*\log(Vx(i,j+1)/Vxp(i,j))\);
    \(Ty(i,j)=(y(i+1)-y(i))\);
    \(Vyp(i,j)=Ay(i,j)*(Vyp(i,j)-y(i))\)+Vyp(i,j);
    \(Ty(i,j)=Ty(i,j)+y(i)\);
    \(Ye(i,j)=y(i)\);
    \(Ye(i,j)=y(i)\);
    \(Ye(i,j)=y(i)\);

end

end
Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)/Vyp(i+1,j));
if Ty(i,j)<0
Ty(i,j)=-Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=-Tx(i,j)*(-1);
end
Tm(i,j)=min(Tx(i,j),Ty(i,j));
Xe(i,j)=Tm(i,j)*Vxp(i,j)+x(j);
Ye(i,j)=((1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(i);
Xe(i,j);
Ye(i,j);
elseif (Vx(i,j)<0 & Vx(i,j+1)<0 & Vy(i+1,j)<0 & Vy(i+1,j)<0 & Vy(i,j)<Vy(i+1,j))
disp(' -8-? ');
Ax(i,j)=(Vx(i,j)-1)/Vy(i+1,j)-x(j);
Vxp(i,j)=(Ax(i,j)*Xp(i,j));
Tx(i,j)=((x(j)-Xp(i,j))/Vy(i,j));
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)=((Ay(i,j)*Yp(i,j)-y(i)))/Vy(i,j);
Xe(i,j)=Tm(i,j)*Vxp(i,j)+x(j);
Ye(i,j)=((1/Ay(i,j))*exp(Ay(i,j)*Tm(i,j)))-Vy(i,j))*y(i);
Xe(i,j);
Ye(i,j);
elseif (Vx(i,j)<0 & Vx(i,j+1)<0 & Vy(i+1,j)<0 & Vy(i+1,j)<0)
disp(' -9-? ');
Ax(i,j)=(Vx(i,j)-1)/Vy(i+1,j)-x(j);
Vxp(i,j)=(Ax(i,j)*Xp(i,j));
Tx(i,j)=((x(j)-Xp(i,j))/Vy(i,j));
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)=((Ay(i,j)*Yp(i,j)-y(i)))/Vy(i,j);
Xe(i,j)=Tm(i,j)*Vxp(i,j)+x(j);
Ye(i,j);
elseif (Vx(i,j)<0 & Vx(i,j+1)<0 & Vy(i+1,j)<0 & Vy(i+1,j)<0)
disp(' -10-7 ');
Ax(i,j)=(Vx(i,j)-1)/Vy(i+1,j)-x(j);
Vxp(i,j)=(Ax(i,j)*Xp(i,j));
Tx(i,j)=((x(j)-Xp(i,j))/Vy(i,j));
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)=((Ay(i,j)*Yp(i,j)-y(i)))/Vy(i,j);
Xe(i,j)=Tm(i,j)*Vxp(i,j)+x(j);
Ye(i,j);
elseif (Vx(i,j)<0 & Vx(i,j+1)<0 & Vy(i+1,j)<0 & Vy(i+1,j)<0)
disp(' -11-7 ');
Ax(i,j)=(Vx(i,j)-1)/Vy(i+1,j)-x(j);
Vxp(i,j)=(Ax(i,j)*Xp(i,j));
Tx(i,j)=((x(j)-Xp(i,j))/Vy(i,j));
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)=((Ay(i,j)*Yp(i,j)-y(i)))/Vy(i,j);
Xe(i,j)=Tm(i,j)*Vxp(i,j)+x(j);
Ye(i,j);
% Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)/Vy(i,j))
Ty(i,j)=y(i+1)-
Yp(i,j))/Vy(i+1,j);
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)= (Tm(i,j)*
Vxp(i,j))/Xp(i,j);
Ye(i,j)=((1/Ay(i,j)) * ((Vyp(i,j)*exp(Ay(i,j)*Tm(i,j))))-Vy(i,j))+y(i);
Xe(i,j);
Ye(i,j);

elseif (Vx(i,j)<0 & Vx(i,j+1)<0 &
Vx(i,j)=Vx(i,j+1) & Vy(i,j)=0 &
Vy(i+1,j)<0)
disp(’-12-7’);
Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/(x(i+1)-x(i));
Vxp(i,j)=-(Ax(i,j)*(Xp(i,j)-
x(i)))+Vx(i,j);
% Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j))
Tx(i,j)= (x(i)-
Xp(i,j))/Vxp(i,j);
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)=(Ay(i,j)*(Yp(i,j)-
y(i)))+Vy(i,j);%
Ty(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)/Vyp(i,j))
Ty(i,j)= (y(i+1)-
Yp(i,j))/Vyp(i,j);
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)= (1/(1/Ax(i,j)) * ((Vxp(i,j)*exp(Ax(i,j)*
Tm(i,j))))*x(i)));
Ye(i,j)= (1/(1/Ay(i,j)) * ((Vyp(i,j)*exp(Ay(i,j)*
Tm(i,j))))*y(i));
end

else (Vx(i,j)<0 & Vx(i,j+1)<0 &
Vx(i,j)=Vx(i,j+1) & Vy(i,j)>0 &
Vy(i+1,j)>0 & Vy(i,j)=Vy(i+1,j))
disp(’-1-8’);
Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/(x(i+1)-x(i));
Vxp(i,j)= (Ax(i,j)*(Xp(i,j)-
x(i)))+Vx(i,j);
% Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j))
Tx(i,j)= (x(i)-
Xp(i,j))/Vxp(i,j);
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)= (Ay(i,j)*(Yp(i,j)-
y(i)))+Vy(i,j);
% Ty(i,j)=(1/(1/Ay(i,j)) * log
(Vy(i+1,j)/Vyp(i,j))
Ty(i,j)= (y(i+1)-
Yp(i,j))/Vyp(i,j);
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)= (1/(1/Ax(i,j)) * ((Vxp(i,j)*exp(Ax(i,j)*
Tm(i,j))))*x(i)));
Ye(i,j)= (1/(1/Ay(i,j)) * ((Vyp(i,j)*exp(Ay(i,j)*
Tm(i,j))))*y(i));
end

elseif (Vx(i,j)<0 & Vx(i,j+1)<0 &
Vx(i,j)=Vx(i,j+1) & Vy(i,j)>0 &
Vy(i+1,j)>0 & Vy(i,j)=Vy(i+1,j))
disp(’-2-8’);
Ax(i,j)=(Vx(i,j+1)-
Vx(i,j))/(x(i+1)-x(i));
Vxp(i,j)= (Ax(i,j)*(Xp(i,j)-
x(i)))+Vx(i,j);
% Tx(i,j)=(1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j))
Tx(i,j)= (1/Ax(i,j))*log
(Vx(i,j+1)/Vxp(i,j))
Ay(i,j)=(Vy(i+1,j)-
Vy(i,j))/(y(i+1)-y(i));
Vyp(i,j)= (Ay(i,j)*(Yp(i,j)-
y(i)))+Vy(i,j);
% Ty(i,j)=(1/(1/Ay(i,j)) * log
(Vy(i+1,j)/Vyp(i,j))
Ty(i,j)= (y(i+1)-
Yp(i,j))/Vyp(i,j);
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
if Tx(i,j)<0
Tx(i,j)=Tx(i,j)*(-1);
end
Tm(i,j)=min (Tx(i,j),Ty(i,j));
Xe(i,j)= (1/(1/Ax(i,j)) * ((Vxp(i,j)*exp(Ax(i,j)*
Tm(i,j))))*x(i)));
Ye(i,j)= (1/(1/Ay(i,j)) * ((Vyp(i,j)*exp(Ay(i,j)*
Tm(i,j))))*y(i));
end
\[
\begin{align*}
Ye(i,j) &= (1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)*Ty(i,j)) + Vy(i,j)) + y(i)); \\
Xe(i,j) &= Tm(i,j) - Ty(i,j); \\
Ye(i,j) &= \text{else if } Vx(i,j) < 0 \& Vx(i,j+1) < 0 \& Vy(i,j+1) > 0 \& Vy(i,j+1) < Vy(i,j); \\
& \quad \text{disp}('3-8'); \\
& \quad Ax(i,j) = [Vx(i,j+1) - Vx(i,j)] / (x(j+1) - x(j)); \\
& \quad Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j); \\
& \quad Tx(i,j) = 1 - Ax(i,j) * \log(Vx(i,j+1)/Vxp(i,j)); \\
& \quad Ay(i,j) = (Vy(i,j+1) - Vy(i,j)) / (y(i+1) - y(i)); \\
& \quad Vyp(i,j) = Ay(i,j) * (Yp(i,j) - y(i)) + Vy(i,j); \\
& \quad Ty(i,j) = 1 - Ay(i,j) * \log(Vy(i,j+1)/Vyp(i,j)); \\
& \quad \text{if } Ty(i,j) < 0 \\
& \quad \quad Ty(i,j) = Ty(i,j) - 1; \\
& \quad \text{if } Tx(i,j) < 0 \\
& \quad \quad Tx(i,j) = Tx(i,j) - 1; \\
& \quad \text{end} \\
& \quad \text{end;} \\
& \quad \text{Tm(i,j) = min(Tx(i,j), Ty(i,j));} \\
Xe(i,j) &= (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j)); \\
Ye(i,j) &= (1/Ay(i,j)) * ((Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) - Vy(i,j)) * y(j)); \\
Xe(i,j) &= Tm(i,j) - Ty(i,j); \\
Ye(i,j) &= \text{else if } Vx(i,j) < 0 \& Vx(i,j+1) < 0 \& Vy(i,j+1) > 0 \& Vy(i,j+1) < Vy(i,j); \\
& \quad \text{disp}('4-8'); \\
& \quad Ax(i,j) = [Vx(i,j+1) - Vx(i,j)] / (x(j+1) - x(j)); \\
& \quad Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j); \\
& \quad Tx(i,j) = 1 - Ax(i,j) * \log(Vx(i,j+1)/Vxp(i,j)); \\
& \quad \text{if } Tx(i,j) < 0 \\
& \quad \quad Tx(i,j) = Tx(i,j) - 1; \\
& \quad \text{end} \\
& \quad \text{Tm(i,j) = Tx(i,j);} \\
Xe(i,j) &= (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j)); \\
Ye(i,j) &= (1/Ay(i,j)) * ((Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) - Vy(i,j)) * y(j)); \\
Xe(i,j) &= Tm(i,j) - Ty(i,j); \\
Ye(i,j) &= \text{else if } Vx(i,j) < 0 \& Vx(i,j+1) < 0 \& Vy(i,j+1) > 0 \& Vy(i,j+1) < Vy(i,j); \\
& \quad \text{disp}('5-8'); \\
& \quad Ax(i,j) = [Vx(i,j+1) - Vx(i,j)] / (x(j+1) - x(j)); \\
& \quad Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j); \\
& \quad Tx(i,j) = 1 - Ax(i,j) * \log(Vx(i,j+1)/Vxp(i,j)); \\
& \quad \text{if } Tx(i,j) < 0 \\
& \quad \quad Tx(i,j) = Tx(i,j) - 1; \\
& \quad \text{end} \\
& \quad \text{Tm(i,j) = Tx(i,j);} \\
Xe(i,j) &= (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j)); \\
Ye(i,j) &= (1/Ay(i,j)) * ((Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) - Vy(i,j)) * y(j)); \\
Xe(i,j) &= Tm(i,j) - Ty(i,j); \\
Ye(i,j) &= \text{else if } Vx(i,j) < 0 \& Vx(i,j+1) < 0 \&Vy(i,j+1) > 0 \& Vy(i,j+1) < Vy(i,j); \\
& \quad \text{disp}('6-8'); \\
& \quad Ax(i,j) = [Vx(i,j+1) - Vx(i,j)] / (x(j+1) - x(j)); \\
& \quad Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j); \\
& \quad Tx(i,j) = 1 - Ax(i,j) * \log(Vx(i,j+1)/Vxp(i,j)); \\
& \quad \text{if } Tx(i,j) < 0 \\
& \quad \quad Tx(i,j) = Tx(i,j) - 1; \\
& \quad \text{end} \\
& \quad \text{Tm(i,j) = Tx(i,j);} \\
Xe(i,j) &= (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * Tm(i,j))) - Vx(i,j)) * x(j)); \\
Ye(i,j) &= (1/Ay(i,j)) * ((Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) - Vy(i,j)) * y(j)); \\
Xe(i,j) &= Tm(i,j) - Ty(i,j); \\
Ye(i,j) &= \text{else if } Vx(i,j) > 0 \& Vx(i,j+1) > 0 \&Vy(i,j+1) > 0 \& Vy(i,j+1) < Vy(i,j); \\
& \quad \text{disp('6-8-1');} \\
& \quad Ty(i,j) = 1 - Ay(i,j) * \log(Vy(i,j+1)/Vyp(i,j)); \\
& \quad \text{if } Vxp(i,j) < 0 \\
& \quad \quad Tm(i,j) = Tm(i,j) - 1; \\
& \quad \text{else} \\
& \quad \quad Vyp(i,j) > 0 \\
& \quad \quad Vxp(i,j) = (Ax(i,j) * (Xp(i,j) - x(j))) + Vx(i,j); \\
& \quad \quad Tx(i,j) = 1 - Ax(i,j) * \log(Vx(i,j+1)/Vxp(i,j)); \\
& \quad \quad \text{if } Tx(i,j) < 0 \\
& \quad \quad \quad Tx(i,j) = Tx(i,j) - 1; \\
& \quad \quad \text{end} \\
& \quad \quad \text{Tm(i,j) = Tx(i,j);} \\
Xe(i,j) &= (1/Ax(i,j)) * ((Vxp(i,j) * exp(Ax(i,j) * (Tm(i,j) - Vx(i,j)))) * x(j)); \\
Ye(i,j) &= (1/Ay(i,j)) * ((Vyp(i,j) * exp(Ay(i,j) * (Tm(i,j) - Vy(i,j)))) * y(j)); \\
Xe(i,j) &= Tm(i,j) - Ty(i,j); \\
Ye(i,j) &= \text{end;}
\end{align*}
\]
else if (Vx[i,j] < 0 & Vx[i,j+1] > Vx[i,j+1]) & Vx[i,j] < 0 & Vy[i+1,j] < 0 & Vy[i,j] > 0
    Vx[i,j] = (Vx[i,j] + 1)
    Vy[i,j] = (Vy[i,j] + 1)

else if (Vx[i,j] > 0 & Vx[i,j+1] < 0)
    Vx[i,j+1] = (Vx[i,j] - 1)
    Vy[i,j] = (Vy[i,j] - 1)

else if (Vx[i,j] < 0 & Vx[i,j+1] < 0)
    Vx[i,j] = (Vx[i,j] + 1)
    Vy[i,j] = (Vy[i,j] + 1)

Xe[i,j] = Ax[i,j]*exp(Ax[i,j]*Xp[i,j]-x[j]) + Vx[i,j]*Vy[i,j] + Ty[i,j] + Vxp[i,j] + Vy[i,j]

else if (Vx[i,j] < 0 & Vx[i,j+1] < 0)
    Vx[i,j] = (Vx[i,j] + 1)
    Vy[i,j] = (Vy[i,j] + 1)

else if (Vx[i,j] > 0 & Vx[i,j+1] > 0)
    Vx[i,j] = (Vx[i,j] - 1)
    Vy[i,j] = (Vy[i,j] - 1)

Xe[i,j] = Ax[i,j]*exp(Ax[i,j]*Xp[i,j]-x[j]) + Vx[i,j]*Vy[i,j] + Ty[i,j] + Vxp[i,j] + Vy[i,j]

else
    Vx[i,j] = (Vx[i,j] + 1)
    Vy[i,j] = (Vy[i,j] + 1)

Xe[i,j] = Ax[i,j]*exp(Ax[i,j]*Xp[i,j]-x[j]) + Vx[i,j]*Vy[i,j] + Ty[i,j] + Vxp[i,j] + Vy[i,j]
else
    \[ V\{i,j\} > 0 \]
    \[ \text{disp}(' -6.9-2') \]
    \[ A\{i,j\} = (Vx\{i,j\}+1) \times (Xp\{i,j\} - x\{j\}) \times (Vx\{i,j\} - x\{j\}) \times V\{x\{j\}\} \]"
\begin{verbatim}
Tm(i,j) = min (Tx(i,j),Ty(i,j));
Xe(i,j) = (1/Ax(i,j))*(Vxp(i,j)*exp(Ax(i,j)*Tm(i,j)) - Vx(i,j))*x(j);
Ye(i,j) = (1/Ay(i,j))*(Vyp(i,j)*exp(Ay(i,j)*Tm(i,j)) - Vy(i,j))*y(i);

\textbf{-- third model}
\begin{verbatim}
if (Vx(i,j) < 0 & Vx(i,j+1) < 0 &Vy(i,j+1) > 0)
    disp('-11-9')
    Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/(x(j+1) - x(j));
    Vxp(i,j) = (Ax(i,j) + (Xp(i,j) - x(j)))*Vx(i,j);
    Ty(i,j) = (1/Ay(i,j))*log(Vx(i,j+1)/Vx(i,j));
    Vyp(i,j) = (Ay(i,j) + (Yp(i,j) - y(i)))*Vy(i,j);
    Ty(i,j) = (1/Ay(i,j))*log(Vy(i+1,j)/Vyp(i,j));
    Ty(i,j) = Ty(i,j)*(-1);
end
if Tx(i,j) < 0
    Tx(i,j) = Tx(i,j)*(-1);
end
end
end
\end{verbatim}

\textbf{-- first model}
\begin{verbatim}
if (Vx(i,j) < 0 & Vx(i,j+1) < 0 &Vy(i,j+1) > 0 &
     Vy(i,j) == 0 & Vy(i+1,j) == 0)
    disp('-13-9')
    Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/(x(j+1) - x(j));
    Vxp(i,j) = (Ax(i,j) + (Xp(i,j) - x(j)))*Vx(i,j);
    Ty(i,j) = (1/Ay(i,j))*log(Vx(i,j+1)/Vxp(i,j));
    if Tx(i,j) < 0
        Tx(i,j) = Tx(i,j)*(-1);
    end
    end
end
end
\end{verbatim}

% disp('-tenth model')
% -- first model
\begin{verbatim}
if (Vx(i,j) < 0 & Vx(i,j+1) < 0 &Vy(i,j+1) > 0 &
     Vy(i,j) == 0 & Vy(i+1,j) == 0)
    disp('-13-9')
    Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/(x(j+1) - x(j));
    Vxp(i,j) = (Ax(i,j) + (Xp(i,j) - x(j)))*Vx(i,j);
    Ty(i,j) = (1/Ay(i,j))*log(Vx(i,j+1)/Vxp(i,j));
    if Tx(i,j) < 0
        Tx(i,j) = Tx(i,j)*(-1);
    end
    end
end
end
\end{verbatim}
\end{verbatim}
\end{verbatim}

258
%second model
else if (Vx(i,j)<0 & Vx(i+1,j)==0 & Vv(i,j)<0 & Vv(i+1,j)>0)
    disp(' -6-10')

    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(i+1)-x(i));
    Vx(i,j)=(Ax(i,j)*(Vx(i,j)-x(i)))+Vx(i,j);
    %
    Tx(i,j)=(1/Ax(i,j))*log
    (Vx(i+1,j)/Vx(i,j));

    Tx(i,j)=(x(i)-Xp(i,j))/Vx(i,j);
    Ay(i,j)=Vy(i+1,j)-Vy(i,j)/(y(i+1)-y(i));
    Vy(i,j)=(Ay(i,j)*(Yp(i,j)-y(i)))+Vy(i,j);
    if Vy(i,j)<0
        disp(' -6-10-1')
        Ty(i,j)=(1/Ay(i,j))*log
        (Vy(i+1,j)/Vy(i,j));

        if Ty(i,j)<0
            Tx(i,j)=Tx(i,j)*(-1);
        end
        Tm(i,j)=Tx(i,j);

    end

    Xe(i,j)=(1/(Ax(i,j)))*((Vx(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(i);
    Ye(i,j)=y(i);
    Xe(i,j);
    Ye(i,j);

else Vy(i,j)>0
    disp(' -6-10-2')

    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(i+1)-x(i));
    Vx(i,j)=(Ax(i,j)*(Xp(i,j)-x(i)))+Vx(i,j);
    %
    Tx(i,j)=(x(i)-Xp(i,j))/Vx(i,j)*(-1);
    %
    Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)-y(i));
    Vy(i,j)=(Ay(i,j)*(Yp(i,j)-y(i)))+Vy(i,j);
    %
    Ty(i,j)=(1/Ay(i,j))*log
    (Vy(i+1,j)/Vy(i,j));

    if Ty(i,j)<0
        Tx(i,j)=Tx(i,j)*(-1);
    end
    Tm(i,j)=Tx(i,j);

    Xe(i,j)=(1/(Ax(i,j)))*((Vx(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(i);
    Ye(i,j)=y(i);
    Xe(i,j);
    Ye(i,j);

    end

end

Tx(i,j)=((x(j)-Xp(i,j))/Vx(i,j));
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/(y(i+1)-y(i));
Vy(i,j)=(Ay(i,j)*(Yp(i,j)-y(i)))+Vy(i,j);
Tm(i,j)=(1/Ay(i,j))*log
(Vy(i+1,j)/Vy(i,j));
    if Ty(i,j)<0
        Ty(i,j)=Ty(i,j)*(-1);
    end
    Tm(i,j)=min(Tm(i,j),Ty(i,j));

Xe(i,j)=(1/(Ax(i,j)))*((Vx(i,j)*exp(Ax(i,j)*Tm(i,j)))-Vx(i,j))*x(i);
Ye(i,j)=y(i);
Xe(i,j);
Ye(i,j);

else (Vx(i,j)<0 & Vx(i,j+1)==0 & Vv(i,j)<0 & Vv(i+1,j)>0)
    disp(' -7-10')

    Ax(i,j)=(Vx(i,j+1)-Vx(i,j))/(x(i+1)-x(i));
    Vx(i,j)=(Ax(i,j)*(Xp(i,j)-x(i)))+Vx(i,j);
    %
    Tx(i,j)=(Ax(i,j)*(Vx(i,j)-x(i)))+Vx(i,j);
    %
    Tx(i,j)=(1/(Ax(i,j)))*log
    (Vx(i,j+1)/Vx(i,j));

end
Ay(i,j) = (Vy(i+1,j) - Vy(i,j))/(y(i+1)-y(i));
Vy(i,j) = Ay(i,j) * (Yp(i,j) - y(i));
Ty(i,j) = Ty(i,j) * log((Vx(i+1,j)/Vx(i,j)) - (Vx(i,j)/Vx(i-1,j))).

X(i,j) = (1/Ax(i,j))^((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))) - X(j));
Y(i,j) = (1/Ax(i,j))^((Vxp(i,j)*exp(Ax(i,j)*Tm(i,j))) - Y(j));

elseif (Vx(i,j)<0 & Vx(i,j+1)>=0 & Vy(i,j)<0 & Vy(i+1,j)<0)
    disp('-10-10')
    Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/x(i+1)-x(i);
    Vxp(i,j) = Ax(i,j) * (Xp(i,j) - X(j));
    Ty(i,j) = (1/Ax(i,j))^log((Vx(i,j+1)/Vx(i,j)) - (Xp(i,j)/Vx(i,j)));
    Ay(i,j) = (Vy(i+1,j) - Vy(i,j))/y(i+1)-y(i);
    Vyp(i,j) = Ay(i,j) * (Yp(i,j) - Y(j));
    Tm(i,j) = min(Tx(i,j),Ty(i,j));

elseif (Vx(i,j)<0 & Vx(i,j+1)<0 & Vy(i,j)<0 & Vy(i+1,j)<0)
    disp('-13-13')
    Ax(i,j) = (Vx(i,j+1) - Vx(i,j))/x(i+1)-x(i);
    Vxp(i,j) = Ax(i,j) * (Xp(i,j) - X(j));
    Ty(i,j) = (1/Ax(i,j))^log((Vx(i,j+1)/Vx(i,j)) - (Xp(i,j)/Vx(i,j)));
    Ay(i,j) = (Vy(i+1,j) - Vy(i,j))/y(i+1)-y(i);
    Vyp(i,j) = Ay(i,j) * (Yp(i,j) - Y(j));
    Tm(i,j) = min(Tx(i,j),Ty(i,j));

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\[
A(x,i,j) = (Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
\]
\[
Vxp(i,j) = \frac{x(i,j) * (x(i,j) - x(j)))}{Vx(i,j)};
\]
\[
Tx(i,j) = (1/Ax(i,j)) * \log (Vx(i,j+1) / Vxp(i,j));
\]
\[
Tm(i,j) = max(Tx(i,j), Ty(i,j));
\]
\[
Xe(i,j) = (Vxp(i,j)^2 + x(j))^2 + Vxp(i,j) x(j);
\]
\[
Ye(i,j) = Vxp(i,j);\]
\[
Tm(i,j) = \text{min}(Tx(i,j), Ty(i,j));
\]
\[
Xe(i,j) = (1/Ax(i,j)) * (Vxp(i,j)^2 + x(j))^2 + Vxp(i,j) x(j);
\]
\[
Ye(i,j) = Vxp(i,j);\]
\[
Tm(i,j) = \text{min}(Tx(i,j), Ty(i,j));\]
\[
Xe(i,j) = (1/Ax(i,j)) * (Vxp(i,j)^2 + x(j))^2 + Vxp(i,j) x(j);
\]
\[
Ye(i,j) = Vxp(i,j);\]
\[
Tm(i,j) = \text{min}(Tx(i,j), Ty(i,j));\]
if Tx(i,j)<0
  Tx(i,j) = Tx(i,j) * (-1);
end
  Tm(i,j) = Tx(i,j);

  Xe(i,j) = (L/Ax(i,j)) * (Vxp(i,j) * exp(Ax(i,j)) * Tm(i,j)) - Vx(i,j) * y(x);
  Ye(i,j) = y(i);
  Xe(i,j);
  Ye(i,j);

else if (Vx(i,j)==0 & Vx(i,j+1)!=0 & Vy(i,j)!0 & Vy(i+1,j)==0)
  disp(' -5-1-1');
  disp('I dont know');
  Ax(i,j) = (Vx(i,j+1) - Vx(i,j)) / (x(j+1) - x(j));
  Vxp(i,j) = (Ax(i,j) * Xp(i,j) - Xp(i,j)) + Vx(i,j);
  Tx(i,j) = (L/Ax(i,j)) * log((Vx(i,j+1) / Vxp(i,j)) - (Ax(i,j) * Xp(i,j) - Xp(i,j)) + Vx(i,j)) / (y(i+1) - y(i));
  Vy(i,j) = (Ay(i,j) * (yp(i,j) - y(i))) / Vy(i,j);
  Ty(i,j) = y(i+1);
  Yp(i,j) / Vy(i,j);
  if Ty(i,j)<0
    Ty(i,j) = Ty(i,j) * (-1);
  end
  if Tx(i,j)<0
    Tx(i,j) = Tx(i,j) * (-1);
  end
  end
  Tm(i,j) = min (Tx(i,j), Ty(i,j));

  Xe(i,j) = (L/Ax(i,j)) * (Vxp(i,j) * exp(Ax(i,j)) * Tm(i,j)) - Vx(i,j) * y(x);
  Ye(i,j) = (L/Ay(i,j)) * (yp(i,j) - y(i)) / Vy(i,j);

% - second model
else if (Vx(i,j)==0 & Vx(i,j+1)!=0 & Vy(i,j)<0 & Vy(i+1,j)>0)
  disp(' -6-1-1');
  % Ty(i,j) = (1/Ay(i,j)) * log(Vy(i+1,j) / Vyp(i,j))
  if Tx(i,j)<0
    Tx(i,j) = Tx(i,j) * (-1);
  end
  if Tx(i,j)<0
    Tx(i,j) = Tx(i,j) * (-1);
  end
  if Ty(i,j)<0
    Ty(i,j) = Ty(i,j) * (-1);
  end
  end
  Tm(i,j) = min (Tx(i,j), Ty(i,j));

  Xe(i,j) = (L/Ax(i,j)) * (Vxp(i,j) * exp(Ax(i,j)) * Tm(i,j)) - Vx(i,j) * y(x);
  Ye(i,j) = (L/Ay(i,j)) * (yp(i,j) - y(i)) / Vy(i,j);

end
```matlab
Xe(i,j)=-(Vx(i,j)*exp(Ax(i,j)))+(Vxp(i,j)*exp(Ax(i,j)))*Tm(i,j)-Vx(i,j)*y(i,j);
Ye(i,j)=-(Vy(i,j)*exp(Ay(i,j)))(Vyp(i,j)*exp(Ay(i,j)))*Tm(i,j)-Vx(i,j)*y(i,j);
Xe(i,j); Ye(i,j);
else if (Vx(i,j)==0 & Vx(i,j+1)>0 & Vy(i,j)==0 & Vy(i,j+1)<0)
    disp(' -12-11')
    Ax(i,j)=(Vx(i,j)+1-
    Vx(i,j))/((x(i+1)-x(i)));
    Vxp(i,j)=A(i,j)*Xp(i,j)-
    x(j)+Vx(i,j);
    % Tx(i,j)=1/Ax(i,j)*y(i,j);
    if (Vy(i,j)<=0)
        Ty(i,j)=Ty(i,j)*(-1);
    end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)*(-1);
    end
    Tm(i,j)=min (Tx(i,j), Ty(i,j));
Xe(i,j)=-(1/Ax(i,j))*((Vx(i,j)*exp(Ax i,j)))*Tm(i,j))-Vx(i,j)*y(i,j);
Ye(i,j)=-(1/Ay(i,j))*((Vy(i,j)*exp(Ay i,j)))*Tm(i,j))-Vx(i,j)*y(i,j);
else if (Vx(i,j)==0 & Vx(i,j+1)<0 & Vy(i,j)>0 & Vy(i,j+1)<0)
    disp(' --13-11')
    disp(' I dont know')
end
```

% Ty(i,j)=1/Ay(i,j)*log
(Vy(i+1,j))/Vy(i,j)
Ty(i,j)=(y(i+1)-
Vy(i,j))/Vy(i,j);
if Ty(i,j)<0
    Ty(i,j)=Ty(i,j)*(-1);
    end
Xe(i,j)=-(1/Ax(i,j))*((Vx(i,j)*exp(Ax i,j)))*Tm(i,j))-Vx(i,j)*y(i,j);
Ye(i,j)=-(1/Ay(i,j))*((Vy(i,j)*exp(Ay i,j)))*Tm(i,j))-Vx(i,j)*y(i,j);
```
\%  \\
%  \begin{align*}
%  \text{T} & = \left( \frac{1}{\text{Ax}(i,j)} \right) \log \\
%  & \left( \frac{\text{Vx}(i,j+1)}{\text{Vx}(i,j)} \right) \left( \frac{\text{Xp}(i,j)}{\text{x}(j)} \right) \\
%  & \left( \frac{\text{Ty}(i,j)}{\text{Ty}(i,j+1)} \right) \\
%  \end{align*}
%  \end{document}
else if (Vx(i,j)==0 & Vx(i,j+1)<0 &Vy(i,j)==0 & Vy(i+1,j)>0)
    disp('11-12')
    Ax(i,j)=(Vx(i,j)+1-Vx(i,j))/x(j)+x(j));
    Vxp(i,j)=(Ax(i,j)*Xp(i,j)-x(j));
    %
    Tx(i,j)=(1/((Ax(i,j)))*log
    (Vx(i,j)+1)/Vxp(i,j));
    Tx(i,j)=Xp(i,j)+((x(j)+1)-
    Xp(i,j))/Vxp(i,j)+1;
    Ay(i,j)=(Vy(i+1,j)-
    Yv(i,j))/y(j)+y(j));
    Vyp(i,j)=(Ay(i,j)*Yp(i,j)-
    y(j))/Vyp(i,j);
    Ty(i,j)=(1/(Ay(i,j)))*log
    (Vy(i+1,j)+Vyp(i,j));
    if Ty(i,j)<0
        Ty(i,j)=Ty(i,j)*(-1);
        end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)*(-1);
    end
    Tm(i,j)=min(Tx(i,j),Ty(i,j));

    Xe(i,j)=(1/(Ax(i,j)))*((Vxp(i,j)*exp(Ax(i,j)+
    Tm(i,j))))-Vx(i,j))*x(j);
    Ye(i,j)=(1/(Ay(i,j)))*((Vyp(i,j)*exp(Ay(i,j)+
    Tm(i,j))))*y(j);
    Xe(i,j); Ye(i,j);

else if (Vx(i,j)==0 & Vx(i,j+1)<0 &Vy(i,j)==0 & Vy(i+1,j)>0)
    disp('13-12')
    Ax(i,j)=(Vx(i,j)+1-Vx(i,j))/x(j)+x(j));
    Vxp(i,j)=(Ax(i,j)*Xp(i,j)-x(j));
    %
    Tx(i,j)=(1/((Ax(i,j)))*log
    (Vx(i,j)+1)/Vxp(i,j));
    Tx(i,j)=Xp(i,j)+((x(j)+1)-
    Xp(i,j))/Vxp(i,j)+1;
    Ay(i,j)=(Vy(i+1,j)-
    Yv(i,j))/y(j)+y(j));
    Vyp(i,j)=(Ay(i,j)*Yp(i,j)-
    y(j))/Vyp(i,j);
    Ty(i,j)=(1/(Ay(i,j)))*log
    (Vy(i+1,j)+Vyp(i,j));
    if Ty(i,j)<0
        Ty(i,j)=Ty(i,j)*(-1);
        end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)*(-1);
    end
    Tm(i,j)=min(Tx(i,j),Ty(i,j));

    Xe(i,j)=(1/(Ax(i,j)))*((Vxp(i,j)*exp(Ax(i,j)+
    Tm(i,j))))-Vx(i,j))*x(j);
    Ye(i,j)=(1/(Ay(i,j)))*((Vyp(i,j)*exp(Ay(i,j)+
    Tm(i,j))))*y(j);
    Xe(i,j); Ye(i,j);
    disp('thirteenth model')

else if (Vx(i,j)==0 & Vx(i,j+1)<0 &Vy(i,j)>0 & Vy(i+1,j)>0)
    disp('1-13')
    Ay(i,j)=(Vy(i+1,j)-
    Vy(i,j))/y(i)+y(i));
    Vyp(i,j)=(Ay(i,j)*Yp(i,j)-
    y(i))/Vyp(i,j);
    Ty(i,j)=(1/(Ay(i,j)))*log
    (Vy(i+1,j)+Vyp(i,j));
    if Ty(i,j)<0
        Ty(i,j)=Ty(i,j)*(-1);
        end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)*(-1);
    end
    Tm(i,j)=min(Tx(i,j),Ty(i,j));

    Xe(i,j)=(1/(Ax(i,j)))*((Vxp(i,j)*exp(Ax(i,j)+
    Tm(i,j))))-Vx(i,j))*x(j);
    Ye(i,j)=(1/(Ay(i,j)))*((Vyp(i,j)*exp(Ay(i,j)+
    Tm(i,j))))*y(j);
    Xe(i,j); Ye(i,j);

else if (Vx(i,j)==0 & Vx(i,j+1)==0 & Vy(i,j)>0 & Vy(i+1,j)>0)
    disp('2-13')
    Ay(i,j)=(Vy(i+1,j)-
    Vy(i,j))/y(i)+y(i));
    Vyp(i,j)=(Ay(i,j)*Yp(i,j)-
    y(i))/Vyp(i,j);
    Ty(i,j)=(1/(Ay(i,j)))*log
    (Vy(i+1,j)+Vyp(i,j));
    if Ty(i,j)<0
        Ty(i,j)=Ty(i,j)*(-1);
        end
    if Tx(i,j)<0
        Tx(i,j)=Tx(i,j)*(-1);
    end
    Tm(i,j)=min(Tx(i,j),Ty(i,j));

    Xe(i,j)=(1/(Ax(i,j)))*((Vxp(i,j)*exp(Ax(i,j)+
    Tm(i,j))))-Vx(i,j))*x(j);
    Ye(i,j)=(1/(Ay(i,j)))*((Vyp(i,j)*exp(Ay(i,j)+
    Tm(i,j))))*y(j);
    Xe(i,j); Ye(i,j);

end

268
Ye(i,j)=((1/Ay(i,j))*((Vyp(i,j)*exp(Ay(i,j)))*exp(Tm(i,j)))-Vy(i,j)))+y(i);
x(i,j); {elseif (Vx(i,j)==0 & Vx(i,j+1)==0 & Vy(i,j)>0 & Vy(i,j+1)<0 & Vy(i,j)<Vy(i+1,j))}
disp('3-13');
Ay(i,j)=Vy(i+1,j)-Vy(i,j)/y(i+1)-y(i);
Vyp(i,j)=Ay(i,j)*(Yp(i,j)-y(i))+Vy(i,j);
Ty(i,j)=1/Ay(i,j)*log(Vy(i+1,j)/Vyp(i,j));
if Ty(i,j)<0
Ty(i,j)=Ty(i,j)*(-1);
end
Tm(i,j)=Ty(i,j);
x(i,j)=x(j);
Ye(i,j)=Ty(i,j);{elseif (Vx(i,j)==0 & Vx(i,j+1)==0 & Vy(i,j)>0 & Vy(i,j+1)<0 & Vy(i,j)<Vy(i+1,j))}
disp('4-13');
disp('I dont know');
rr=1
elseif (Vx(i,j)==0 & Vx(i,j+1)==0 & Vy(i,j)<0 & Vy(i,j+1)<0 & Vy(i,j)<Vy(i+1,j))
disp('5-13');
disp('I dont know');
rr=1
elseif (Vx(i,j)==0 & Vx(i,j+1)==0 & Vy(i,j)<0 & Vy(i,j+1)<0 & Vy(i,j)<Vy(i+1,j))
disp('6-13');
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i);
Vyp(i,j)=(Ay(i,j)*(Yp(i,j)-y(i))+Vy(i,j))if Vyp(i,j)<0
disp('-6-13-1');
x(i,j)=x(j);
y(i,j)=y(i);
x(i,j);
y(i,j);
else Vyp(i,j)<0
disp('-6-13-2');
x(i,j);
end
elseif (Vx(i,j)==0 & Vx(i,j+1)==0 & Vy(i,j)<0 & Vy(i,j+1)<0 & Vy(i,j)<Vy(i+1,j))
disp('7-13');
Ay(i,j)=(Vy(i+1,j)-Vy(i,j))/y(i+1)-y(i);
Ty(i,j) = Ty(i,j) * (-1);
end
Tm(i,j) = Ty(i,j);
Xe(i,j) = x(i);
Ye(i,j) = ((1/Ay(i,j)) * (Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) - Vy(i,j)) + y(i);
Xe(i,j);
Y(i,j);
Ye(i,j);

elseif (Vx(i,j) == 0 & Vx(i,j+1) == 0 & Vy(i,j) == 0 & Vy(i,j+1) > 0)
    disp(' -11-13 ');
    Ay(i,j) = (Vx(i+1,j) - Vy(i,j))/((y(i+1) - y(i)));
    Vyp(i,j) = (Ay(i,j) * (Vp(i,j) - y(i))) + Vy(i,j);
    % Ty(i,j) = log((Vx(i+1,j)/Vx(i,j)))
    % Ty(i,j) = (y(i+1) - Yp(i,j))/Vy(i,j);
    if Ty(i,j) < 0
        Ty(i,j) = Ty(i,j) * (-1);
    end
    Tm(i,j) = Ty(i,j);
    Xe(i,j) = x(i);
    Ye(i,j) = ((1/Ay(i,j)) * (Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) - Vy(i,j)) + y(i);
    Xe(i,j);
    Ye(i,j);
elseif (Vx(i,j) == 0 & Vx(i,j+1) == 0 & Vy(i,j) == 0 & Vy(i,j+1) < 0)
    disp(' -12-13 ');
    Ay(i,j) = (Vx(i+1,j) - Vy(i,j))/((y(i+1) - y(i)));
    Vyp(i,j) = (Ay(i,j) * (Vp(i,j) - y(i))) + Vy(i,j);
    % Ty(i,j) = log((Vx(i+1,j)/Vx(i,j)))
    % Ty(i,j) = (y(i+1) - Yp(i,j))/Vy(i,j);
    if Ty(i,j) < 0
        Ty(i,j) = Ty(i,j) * (-1);
    end
    Tm(i,j) = Ty(i,j);
    Xe(i,j) = x(i);
    Ye(i,j) = ((1/Ay(i,j)) * (Vyp(i,j) * exp(Ay(i,j) * Tm(i,j))) - Vy(i,j)) + y(i);
    Xe(i,j);
    Ye(i,j);
end

rr = 1

disp(' new model ');

Vx(i,j);
Vx(i,j+1);
Vy(i,j);
Vy(i,j+1);

end

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fprintf(fid2, ' Xe Ye Ax Vxp Ay Vyp /\n\n');
fprintf(fid2, ' if  % if  % % % % % % % % Xe Ye, Ax, Vxp, Ay, Vyp );
end

% Streamline Simulation Near Well Bore
% By MARJAN HASHEM

% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM

% Third Case Study in Cartesian Coordinate
% Subroutine for drawing streamline in "x" direction
global X Y
global Kx Ky
global P
global Vx Vy
global xe ye
global maxerr maxr errmatrix
global x y
global Xp Yp
global Vxp Vyp
global gg
if global
    global rr
    global Ax Ay Vxp Vyp
    temp = find(Xp==0);
    xx = Xp(temp)';
    yy = Yp(temp)';
    xx = [xx];
    yy = [yy];
    plot(xx,yy, '-');
    grid on
    axis equal
    axis square
% Streamline Simulation Near Well Bore
% By MARJAN HASHEM

% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM

% Third Case Study in Cartesian Coordinate
% Subroutine for drawing streamline in "y" direction
global X Y
global Kx Ky
global P
global Vx Vy
global xe ye
global maxerr maxr errmatrix
global x y
global Xp Yp
global Xx Yy
global Xx Yy
global Ax Ay Vx Vy
global Xp Yp

% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM

% Third Case Study in Cartesian Coordinate
% Subroutine for drawing streamline

global X Y
global Xe Ye

% temp = find(Yp==0);
temp1 = find(Xp);
temp = Xp(temp);
X = Xp(temp);
Y = Yp(temp);
Xx = X(x);
Yy = Y(y);
plot(X,Y,'*')
grid on
axis equal
axis square

% Streamline Simulation Near Well Bore
% By MARJAN HASHEM

% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM

% First Case Study in Polar Coordinate
% Main route
clc;
clear all;
Close all;
format long e
global radius p T r
N=10;
M=60;
w=1.10;
s=0.2;
n=(w/se);
r(1)=rw;
Mu=(10^-3);
theta=(2*pi)/M;
Pw=20000;
Pe=100000;

for j=1:M+1
for i=1:N+1
    if i=(round(N/2))
        Kr(i,j)=(2*(10^-12));
    else
        Kr(i,j)=(2*(10^-12));
    end
end
end

for i=1:N
    m=i/(N);
k=(n)^m;
r(i+1)=rw*k;
end

for i=1:N
    q=i+0.5;
p=q-0.5;
RP(p)=(r(i+1)-r(i))/(log(r(i+1)/r(i))); end

RMP=ruise plus 0.5
for i=1:N
    C=i-0.5;
    D=C+0.5;
    RMMP=(r(i)-r(i-1))/(log(r(i)/r(i-1))); end

RMP pause
for i=1:N

q=i+0.5;
p=q-0.5;
for j=1:M

\%landar(i+0.5,j)=landaR(p,j)
landaR[p,j]=((log(r(i+1)/r(i)))/((1/landar[i,j])*(log(RP(i)/r(i))))+(1/landar[i,j])*(log(r(i+1)/RP(p)))))
\%
landar(p)=((ln(r(i+1)/r(i)))/((1/landar(i+1)))*(ln(r(i+0.5)/r(i)))+(1/landar(i+1))*(ln(r(i+1)/r(i+0.5))))
end
end
landaR

for i=2:N
C=1-0.5;
D=C+0.5;
for j=1:M

\%landar(i+0.5,j)=landaR(p,j)
landaRM(D,j)=((log(r(i)/r(i-1)))/((1/landar[i,j])*(log(r(i)/RM(D))))+(1/landar[i,j])*(log(RM(D)/r(i-1))))
\%
landar(p)=((ln(r(i+1)/r(i)))/((1/landar(i)))*(ln(r(i+0.5)/r(i)))+(1/landar(i))*(ln(r(i+1)/r(i+0.5))))
end
end
landaRM
pause

for i=1:N
for j=1:M
landat(i,j)=(2*10^(-9));
end
end
landat

for i=1:N
for j=1:M
u=j+0.5;
v=u-0.5;
if j=M
landaTeta(i,j)=((landatetaj)*landatetha(i))/(landatetaj+landatetha(i));
else
landaTeta(i,j)=((landatetaj)*landatetha(j+1))/(landatetaj+landatetha(j+1));
end
end
landaTeta

pause

for i=2:N
for j=1:M
q=i+0.5;
p=q-0.5;
landaTeta[p,j]=((log(r(i+1)/r(i)))/((1/landatetaj)*(log(RP(p)/r(i))))+(1/landatetaj)*(log(r(i+1)/RP(p)))))
\%
landatetaj=(ln(r(i+1)/r(i)))/((1/landatetaj)*(ln(r(i+0.5)/r(i)))+(1/landatetaj)*(ln(r(i+1)/r(i+0.5))))
end
end
landaTeta

for i=1:N
for j=1:M

\%a(i,j)=((tetha^2)*r(i)/(RP(p)-RM(D)))*((landaR(p,j))/log(r(i+1)/r(i)))+(landaRM(D,j)/log(r(i)/r(i-1))))
end
end
landaT

for i=1:N+1
for j=1:M+1
landatetetha(i,j)=(2*10^(-9));
end
end
landatetetha

for i=1:M

\%landata RM(D, j) = (((log(r(i)/r(i-1)))/((1/landata[i,j])*(log(RP(i)/r(i))))+(1/landata[i,j])*(log(r(i+1)/RP(p)))))
\%
landata[i,j] = ((ln(r(i+1)/r(i)))/((1/landata[i+1]))*(ln(r(i+0.5)/r(i)))+(1/landata[i+1])*ln(r(i+1)/r(i+0.5)))
end
end
landata

for i=1:2
for j=1:2
u=j+0.5;
v=u-0.5;
if j=M
landataTeta[i,v]=((landatetaj)*landatetha(i))/(landatetaj+landatetha(i));
else
landaTeta[i,v]=((landatetaj)*landatetha(j+1))/(landatetaj+landatetha(j+1));
end
end
end
landaTeta

pause

for i=2:2
for j=2:2
q=i+0.5;
p=q-0.5;
landaTeta[p,j]=((log(r(i+1)/r(i)))/((1/landatetaj)*(log(RP(p)/r(i))))+(1/landatetaj)*(log(r(i+1)/RP(p)))))
\%
landatetaj=(ln(r(i+1)/r(i)))/((1/landatetaj)*(ln(r(i+0.5)/r(i)))+(1/landatetaj)*(ln(r(i+1)/r(i+0.5))))
end
end
landaTeta

for i=1:2
for j=1:2

\%a(i,j)=((tetha^2)*r(i)/(RP(p)-RM(D)))*((landaR(p,j))/log(r(i+1)/r(i)))+(landaRM(D,j)/log(r(i)/r(i-1))))
end
end
landaT

for i=1:N+1
for j=1:M+1
landatetetha(i,j)=(2*10^(-9));
end
end
landatetetha

else
else

\%a(i,j)=((tetha^2)*r(i)/(RP(p)-RM(D)))*((landaR(p,j))/log(r(i+1)/r(i)))+(landaRM(D,j)/log(r(i)/r(i-1))))
end
end
landaT
a(i,j) = (((tetha)^2)*r(i))/((RP(p)-RM(D)))*((LandarP(p,j))/log(r(i)+1/r(i-1)))+((LandarRM(D,j))/log(r(i)/r(i-1)))-(((Landar(j))^2)*Landar(j+1)/(Landar(j)+Landar(j-1)))/((Landar(j)+Landar(j-1)))

% pause
end

for i=1:N
    for j=1:M
        q = i+0.5;
        p = q-0.5;
        C = i-0.5;
        D = C+0.5;
        if j=1
            b(i,j) = ((Landar(j)*Landar(1))/(Landar(j)+Landar(1)));
        else
            b(i,j) = ((Landar(j)*Landar(j+1))/(Landar(j)+Landar(j+1)));
        end
        end
    end
    pause
for i=1:N
    for j=1:M
        q = i+0.5;
        p = q-0.5;
        C = i-0.5;
        D = C+0.5;
        if j=1
            c(i,j) = ((Landar(j)*Landar(M))/(Landar(j)+Landar(M)));
        else
            c(i,j) = ((Landar(j)*Landar(j-1))/(Landar(j)+Landar(j-1)));
        end
    end
    pause
for i=2:N
    for j=1:M
        q = i+0.5;
        p = q-0.5;
        C = i-0.5;
        D = C+0.5;
        d(i,j) = (((tetha)^2)*r(i))/((RP(p)-RM(D)))*((LandarRM(D,j))/log(r(i)/r(i-1)));
    end
end
pause
for i=2:N
    for j=1:M
        q = i+0.5;
        p = q-0.5;
        C = i-0.5;
        D = C+0.5;
        e(i,j) = (((tetha)^2)*r(i))/((RP(p)-RM(D)))*((LandarP(p,j))/log(r(i+1)/r(i)))
        % e(i,j) = (((tetha)^2)*r(i))/((RP(p)-RM(D)))*((LandarP(p,j))/log(r(i+1)/r(i)))
        % e(i,j) = (((tetha)^2)*r(i))/((RP(p)-RM(D)))*((LandarP(p,j))/log(r(i+1)/r(i)))
        % e(i,j) = (((tetha)^2)*r(i))/((RP(p)-RM(D)))*((LandarP(p,j))/log(r(i+1)/r(i)))
        % e(i,j) = (((tetha)^2)*r(i))/((RP(p)-RM(D)))*((LandarP(p,j))/log(r(i+1)/r(i)))
    end
end
pause
% a(i,j) = ...
    (((tetha)^2)*r(i))/((RP(p)-r(i-0.5)))*((Landar(i+0.5)/ln(r(i+1)/r(i)))+((Landar(i+0.5)/ln(r(i-1)/r(i))))
    (((Landar(j)*Landar(j-1))/(Landar(j)+Landar(j-1)))+(Landar(j)*Landar(j-1))/(Landar(j)+Landar(j-1)));

% b(i,j) = ...
    ((Landar(j)*Landar(j+1))/(Landar(j)+Landar(j+1)));
% c(i,j) = ...
    ((Landar(j)*Landar(j-1))/(Landar(j)+Landar(j-1)));
% d(i,j) = ...
    (((tetha)^2)*r(i))/((r(i+0.5))/ln(r(i+0.5)/r(i)));
% e(i,j) = ...
    (((tetha)^2)*r(i))/((r(i+0.5))/ln(r(i+0.5)/r(i)))
% Z = zeros((N-1)*M,(N-1)*M);
% i=2
for j=1:M
    r1 = j;
    s1 = j;
    Z(r1,s1) = a(i,j);
    if j=M
        r2 = j;
        s2 = 1;
        Z(r2,s2) = b(i,j);
    else
        r2 = j;
        s2 = j+1;
        Z(r2,s2) = b(i,j);
    end
    if j=1
        r3 = j;
        s3 = M;
        Z(r3,s3) = c(i,j);
    else
        r2 = j;
        s2 = j+1;
        Z(r2,s2) = b(i,j);
    end
end

% my alternative when in last series of equation pl,j=1 change to pN,j-1
% A = zeros((N-1)*M,(N-1)*M)
% set up for loops
end
for i=3:N-1
for j=1:M
r1=M*(i-2)+j;
s1=(i-1)+M*(j-2);
Z(r1,s1)=a(i,j);
if j==M
r2=M*(i-2)+j;
s2=(i)+(j-1)+M*(j-2);
Z(r2,s2)=b(i,j);
else
r2=M*(i-2)+j;
s2=(j)+(i-1)+M*(j-2);
Z(r2,s2)=b(i,j);
end
if j==1
r3=M*(i-2)+j;
s3=M*(i-2)+M;
Z(r3,s3)=c(i,j);
else
r3=M*(i-2)+j;
s3=M*(i-2)+(j-1);
Z(r3,s3)=c(i,j);
end
r4=M*(i-2)+j;
s4=M*(i-3)+j;
Z(r4,s4)=d(i,j);
end
end
end

% set up for loops
V=zeros(((N-1)*M),1);
i=2
for j=1:M
r1=j;
s1=1;
V(r1,s1)=-d(i,j)*Pw;
end
i=N
for j=1:M
r1=((N-2)*M)+j;
s1=1;
V(r1,s1)=-e(i,j)*Pw;
end
V
P=zeros((N-1)*M,1);
% (V'
end
P=P/Pw
k=zeros(N,M);
for i=1:(N-1)
for j=1:M
if i==1
j;
k(i,j)=Q(i,j);
else
i;
if j
k(i,j)=Q((i-1)*M)+j)
end
end
k
for i=1:N;
for j=1:M
k(i,j)=Fe;
end
end
contour (k,30);
contour (k,30, 'DisplayName', 'PRESSURE'); colormap autumn;
pause
% urP(i,j)=landaRP(p,j)*P(i+1,j)-P(i,j))/(r(i+1)-r(i))
% ur(p,j)=(landaRP(p,j)*(p(i+1,j))-P(i,j))/(r(i+1)-
Developed MATLAB Program
This Code developed originally by MARJAN HASHEM
First Case Study in Polar Coordinate
Subroutine for drawing the streamline

plot(final2)
end
for i=1:N+1
(RP(i)+RP(i))/2
end
% % %
Tm(i,j)=min
(Ttheta(i,j),Tt(i,j));

Re(i,j)=((1/At(i,j))*((Vt(i,j)*Te(i,j)+
At(i,j)*Tm(i,j))/2);

%% Streamline Simulation Near Well Bore
% By MARJAN HASHEM
% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM
% First Case Study in Polar Coordinate
% Subroutine for drawing the streamline
global radius p T
% temp = find(Xp==0);
x(i,j)=radius p(i,j)*cos(T(j));
y(i,j)=radius p(i,j)*sin(T(j));
xx = [ x ];
yy = [ y ];
plot(xx,yy,'--');
grid on
axis equal
axis square

%% Streamline Simulation Near Well Bore
% By MARJAN HASHEM
% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM
% First Case Study in Polar Coordinate
% Subroutine for drawing the streamline
global radius p T
x(i,j)=radius p(i,j)*cos(T(j));
y(i,j)=radius p(i,j)*sin(T(j));
xx = [ x ];

yy = [y;];
plot(xx,yy,'*-')
grid on
axis equal
axis square

% Second Case Study in Polar Coordinate
% Streamline Simulation Near Well Bore
% By MARJAN HASHEM
% Developed MATLAB Program for Streamline Simulation
% This Code developed originally by MARJAN HASHEM
% Second Case Study in Polar Coordinate
% Main route
clc;
clear all;
close all;
format long e
global radius TETHA N M j T
global urp TMIN utethav RP urpp
utethavpp
global radius TETHAe TETHAp
global TETHA utethav TETHA
global radiush TETHA
global urp TMIN utethav N M i j x y
global radiush TETHAe TETHAp r utethav
deltathaeexit checkthesign
global radius TETHA Tr1 Tr TTETHA
TTETHA1 radiusiue Tetha
thetamoredniaz checkthesize

N=60;
re=100;
rw=0.2;
n=(re/rw);
M=60;

n=(re/rw);
for j=1:M
r(i,j)=rw;
end;
Mu=(10^-(-3));
tetha=(2*pi)/M;
Pw=(0.5*10000000);
Pe=100000000;
Kr=zeros(N+1,M+1);
1
for j=1:M+1
for i=1:N+1
if i<=(round(N/2))
Kr(i,j)=(0.5*(10^-(-12)));
landar(i,j)=(Kr(i,j)/Mu);
else
Kr(i,j)=(0.5*(10^-(-12)));
landar(i,j)=(Kr(i,j)/Mu);
end
end
end

for j=fix((3*M)/10):fix((7*M)/10)
for i=fix((5*N)/10):fix((6*N)/10)

Kr(i,j)=(0.5*(10^-(-17)));
landar(i,j)=(Kr(i,j)/Mu);
end

end
Kr;
landar;

3
for i=1:N
for j=1:N+1
m=i/(N);
k=(n)^m;
r(i+1,j)=rw*k;
end
end
T;
4
for i=1:N
for j=1:M
q=i+0.5;
p=q-0.5;
RP(p,j)=((r(i+1,j)-
- r(i,j))/log(r(i+1,j)/r(i,j)));
end
end
RP;
5
for i=2:N
for j=1:M
C=1-0.5;
D=C+0.5;
RM(D,j)=((r(i,j)-r(i-1,j))/
- log(r(i,j)/r(i-1,j)))
end
end
RM;
6
pause
for i=1:N
q=i+0.5;
p=q-0.5;
for j=1:M

landaRP(p,j)=((log(r(i+1,j)/r(i,j)))/(1/
- landar(i,j))*(log(RP(p,j)/r(i,j))))+
- ((1/landar(i+1,j))*(log(r(i+1,j)/RP(p,j))));
end
end
landaRP;
7
for i=2:N
C=1-0.5;
D=C+0.5;
for j=1:M

landaRM(D,j)=((log(r(i,j)/r(i-1,j)))/(1/
- landar(i,j))*(log(r(i,j)/RM
(D,j)))+((1/landar(i-1,j))*(log(RM(D,j)/r(i-1,j))));
end
end
landaRM;
8
```plaintext
q = i + 0.5;
p = q - 0.5;
C = i - 0.5;
D = C + 0.5;
if j = M
b(i, j) = ((landat(i, j) * landat(i, 1)) / (landat(i, j) + landat(i, 1)));
else
b(i, j) = ((landat(i, j) * landat(i, j + 1)) / (landat(i, j) + landat(i, j + 1)));
end
end
b;
15
% pause
for i = 1:N
for j = 1:M
q = i - 0.5;
p = q - 0.5;
C = i - 0.5;
D = C + 0.5;
if j = 1
c(i, j) = ((landat(i, j) * landat(i, M)) / (landat(i, j) + landat(i, M)));
else
c(i, j) = ((landat(i, j) * landat(i, j - 1)) / (landat(i, j) + landat(i, j - 1)));
end
end
end
c;
16
% pause
for i = 2:N
for j = 1:M
q = i + 0.5;
p = q - 0.5;
C = i - 0.5;
D = C + 0.5;
d(i, j) = (((+2) * r(i, j)) / (log(r(i, j)) / (r(i - 1, j)))));
end
d;
17
% pause
for i = 2:N
for j = 1:M
q = i + 0.5;
p = q - 0.5;
C = i - 0.5;
D = C + 0.5;
e(i, j) = (((+2) * r(i, j)) / (log(r(i + 1, j)) / (r(i, j)))));
end
e;
18
Z = zeros((N-1) * M, (N-1) * M);
i = 2
for j = 1:M
r1 = j;
s1 = j;
Z(r1, s1) = a(i, j);
if j = M
r2 = j;
s2 = 1;
Z(r2, s2) = b(i, j);
else
r2 = j;
s2 = j + 1;
Z(r2, s2) = b(i, j);
end
if j = 1
r3 = j;
s3 = M;
Z(r3, s3) = c(i, j);
else
r3 = j;
s3 = j - 1;
Z(r3, s3) = c(i, j);
end
r4 = j;
s4 = M * (i - 1) + j;
Z(r4, s4) = e(i, j);
end
19
for i = 3:N-1
for j = 1:M
r1 = M * (i - 2) + j;
s1 = j + M * (i - 2);
Z(r1, s1) = a(i, j);
if j = M
r2 = M * (i - 2) + j;
s2 = (j + M * (i - 2));
Z(r2, s2) = b(i, j);
else
r2 = M * (i - 2) + j;
s2 = (j + 1) + M * (i - 2);
Z(r2, s2) = b(i, j);
end
if j = 1
r3 = M * (i - 2) + j;
s3 = M * (i - 2) + M;
Z(r3, s3) = c(i, j);
else
r3 = M * (i - 2) + j;
s3 = M * (i - 2) + j - 1;
Z(r3, s3) = c(i, j);
end
r4 = M * (i - 2) + j;
s4 = M * (i - 3) + j;
Z(r4, s4) = d(i, j);
r5 = M * (i - 2) + j;
s5 = M * (i - 1) + j;
Z(r5, s5) = e(i, j);
end
end
20
i = N;
for j = 1:M
r1 = M * (i - 2) + j;
```
s1=M*(i-2)+j;
Z(r1,s1)=a(i,j);
if j=M
r2=M*(i-2)+j;
s2=M*(i-2)+1;
Z(r2,s2)=b(i,j);
else
r2=M*(i-2)+j;
s2=M*(i-2)+(j+1);
Z(r2,s2)=b(i,j);
end
if j=1
r3=M*(i-2)+j;
s3=M*(i-2)+(M);
Z(r3,s3)=c(i,j);
else
r3=M*(i-2)+j;
s3=M*(i-2)+(j-1);
Z(r3,s3)=c(i,j);
end
r4=M*(i-2)+j;
s4=M*(i-3)+j;
Z(r4,s4)=d(i,j);
end

V=zeros(((N-1)*M,1));
i=2
for j=1:M
r1=j;
s1=1;
V(r1,s1)=-(d(i,j)*Pw);
end
i=1
for j=1:M
r1=(N-2)*M+j;
s1=1;
V(r1,s1)=-(e(i,j)*Pe);
end
V;
P=zeros((N-1)*M,1);
\% (V=inv(A))*P
Q=(Z\V);
P=Q/Pw;
k=zeros(N,M);
for i=1:(N-1)
for j=1:M
if i=1
i;
k(i,j)=Q(1,1);
else
i;
j;
end
end
k;
26
for i=N
for j=1:M
k(i,j)=Pe;
end
end
27
contourf(k,30,'DisplayName','PRESSURE');colormap autumn;
for i=1:N-1
q=i+0.5;
p=q-0.5;
for j=1:M
if i=(N)
1357;
i;
j;
landaRP(p,j);
2.5;
k(i,j);
3.5;
Pe;
3.5;
(PE-k(i,j));
3.75;
r(i+1);
4.5;
r(i);
5.5;
urp(p,j)=(landaRP(p,j)*((Pe-k(i,j))/(r(i+1,j)-r(i,j))));
else
1976;
i;
j;
urp(p,j)=(landaRP(p,j)*((k(i+1,j)-k(i,j))/(r(i+1,j)-r(i,j))));
end
end
urp;
28
for i=2:N
for j=1:M
C=i-0.5;
D=C+0.5;
urD(D,j)=(landaRM(D,j)*((k(i,j)-k(i-1,j))/(r(i,j)-r(i-1,j))));
end
urD;
29
for i=1:N
for j=1:M
u=j+0.5;
v=u-0.5;
if \( j = M \)

\[
\text{utethav}(i,v) = \frac{1}{r(i,j)} \cdot (\text{landaTp}(i,j) \cdot \left( k(i,1) - k(i,j) \right) / \thetaetah))
\]

else

\[
\text{utethav}(i,v) = \frac{1}{r(i,j)} \cdot (\text{landaTd}(i,j) \cdot (k(i,j) - k(i,M)) / \thetaetah))
\]

end

end

end

utethav;

30

for \( i=1:N \)

for \( j=1:M \)

\( u = u + 0.5 \);  

\( w = w + 0.5 \);  

if \( j = 1 \)

\[
\text{utethav}(i,w) = \frac{1}{r(i,j)} \cdot (\text{landaTd}(i,j) \cdot (k(i,j) - k(i,M)) / \thetaetah))
\]

else

\[
\text{utethav}(i,w) = \frac{1}{r(i,j)} \cdot (\text{landaTd}(i,j) \cdot (k(i,j) - k(i,j-1)) / \thetaetah))
\]

end

end

end

utethav;

31

% URP

for \( i=1:N-1 \)

for \( j=1:M \)

\( \text{RPF}(i,j) = \text{RF}(i,j) \);  

end

end

% RPF

32

% pause

\( \text{theta}(2*\pi/M) \)

for \( i=1:N \)

for \( j=1:M \)

\( t(i,j) = (((2*\pi) \cdot (j)) / M) \)  

end

end

\( t; \)

33

for \( i=1:N \)

for \( j=1 \)

\( T(i,j) = 0 \);  

end

end

for \( i=1:N \)

for \( j=2:M+1 \)

\( T(i,j) = t(i,j-1) \);  

end

end

% T =

% \( T=[0 \ T] \)

34

K=zeros(N,M+1);

for \( i=1:N \)

for \( j=1:M \)

\( K(i,j) = k(i,j) \);

end

end

for \( i=1:N \)

\( K(i,M+1) = k(i,1) \);  

end

k;

35

for \( i=1:N \)

for \( j=1:M+1 \)

\( f(i,j) = r(i,j) \cdot \cos(T(i,j)) \);  

\( g(i,j) = r(i,j) \cdot \sin(T(i,j)) \);  

end

end

f;

g;

35.5

countourf(f,g,K,20),colormap autumn

figure

36

37

for \( j=1:M \)

for \( i=N \)

\( \text{radiusp}(i,j) = \text{RF}(i,j) \);  

end

end

38

i=N-1;

j=1;

1970

% j=1

\( \text{TETHA}(i,1) = 0 \);  

\( \text{radiuspp} = \text{radiusp}(i,1) \);  

\( \text{TETHAp} = \text{TETHA}(i,1) \);  

\( \text{utethavpp}(i,j) = \text{utethav}(i,j) \);  

% pause;

\( \text{urpp}(i,1) = \text{urp}(i,j) \);  

while \( \text{radiusp}(i,j) > (\text{RF}(2,1)) \)

1971

location

12345

radius;

TETHAe;

if \( \text{TMIN}(i,j) = \text{Tr}(i,j) \)

1983

\( \text{radiusp}(i-1,j) = \text{radius}(i,j) \);  

\( \text{TETHA}(i-1,j) = \text{TETHAe}(i,j) \);  

\( \text{TETHAp} = \text{TETHAp}(i-1,j) \);  

\( \text{radiuspp} = \text{radiusp}(i-1,j) \);  

i=i-1;

j=j;

elseif \( \text{TMIN}(i,j) = \text{TTETHA}(i,j) \)

if deltatethaexit(i,j)<0

if \( j=1 \)

\( \text{radiusp}(i,M) = \text{radius}(i,j) \);  

\( \text{TETHA}(i,M) = \text{TETHAe}(i,j) \);  

\( \text{TETHAp} = \text{TETHAp}(i,j) \);  

\( \text{radiuspp} = \text{radiusp}(i,j) \);  

\( j=M \);  

i=i-1;

else

\( \text{radiusp}(i,j-1) = \text{radius}(i,j) \);  

end

end

281
elseif TMIN(i,j)==TERTHA(i,j)
    if deltathetaexit(i,j)<0
        if j=1
            radiusp(i,M)=radiusi(i,j);
            TETHAp(i,M)=TETHAe(i,j);
            TETHApp=TETHAp(i,M);
            radiuspp=radiusi(i,M);
            j=M;
            i=1;
        else
            radiusp(i,j-1)=radiusi(i,j);
            TETHAp(i,j-1)=TETHAe(i,j);
            TETHApp=TETHAp(i,j-1);
            radiuspp=radiusi(i,j-1);
            j=j-1;
            i=1;
        end
    else
        radiusp(i,j+1)=radiusi(i,j);
        TETHAp(i,j+1)=TETHAe(i,j);
        TETHApp=TETHAp(i,j+1);
        radiuspp=radiusi(i,j+1);
        j=j+1;
        i=1;
    end
    else
        fprintf('nemidoonam'
    end
end

2000
    urpp=zeros(N-1,M);
    utethavp=zeros(N-1,M);
    radius=zeron(N-1,M);
    TETHA=zeros(N-1,M);
    radiuspp=zeron(N-1,M);
    TETHApp=zeron(N-1,M);

1980
    for j=2:M
        i=N-1
        j;
        radiusp[i,j]=RP(i,j);
        radiuspp=radius[i,j];
        TETHAp[i,j]=T(i,j);
        TETHApp=TETHAp[i,j];
        utethavp[i,j]=utethav(1,j);
        urpp[i,j]=urp(i,j);
        while ((radiusp[i,j]>RP(2,1))&(TETHApp<=2*pi))
            1981
                locationnew
                radiuse;
                TETHAe;
                1982
                    if TMIN(i,j)==Tr(i,j)
                        1983
                            radiusp(i-1,j)=radiusi(i,j);
                            TETHA(i-1,j)=TETHAe(i,j);
                            TETHAp=TETHA(i-1,j);
                            radiuspp=radius(i-1,j);
                            i=1-1;
                            j=j;
                        else
                            radiusp(i,j+1)=radiusi(i,j);
                            TETHA(i,j+1)=TETHAe(i,j);
                            TETHAp=TETHA(i,j+1);
                            radiuspp=radius(i,j+1);
                            j=j+1;
                            i=1;
                        end
                    else
                        radiusp(i,j-1)=radiusi(i,j);
                        TETHA(i,j-1)=TETHAe(i,j);
                        TETHAp=TETHA(i,j-1);
                        radiuspp=radius(i,j-1);
                        j=j-1;
                        i=1;
                    end
                end
            end
        end
    end
else
    fprintf('nemidoonam'
end

end

1388
3010
end

global radius TETHAe

for j=1:M
    for i=1:N-1
        j;
        i;
        hash(i,j)=radiusi(i,j)*cos(TETHAe(i,j));
bash(i, j) = radius_e(i, j) \times \sin(TETH Ae(i, j));

    plot(hash, bash, '-')
    grid on
    axis equal
    axis square

end
end