DIRECT SECANT ESTIMATION OF LIMIT BEHAVIOUR OF FRAMED STRUCTURES

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by

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Abstract

A direct secant method used to predict the plastic limit load of framed structures is presented in this thesis. Instead of using classical techniques, it utilizes two or more purely elastic analyses to predict the limit load. Secant rigidity of structures is modified based on the result of the first purely elastic analysis. Iterative reanalyses are performed until convergence is reached. By selecting the peak bending moment, potential plastic hinges are listed. The result can be used to predict the plastic limit load as well as the collapse mechanism. The limit load calculated by the direct secant method is compared to the solution of other traditional analyses, where applicable. Generally, the direct secant method is an attractive alternative for evaluating the limit load of framed structures. The results are a significant improvement over traditional methods which are illustrated in the thesis. Similar idea is applied on frame stability analysis. After the first purely elastic analysis, two preliminary and empirical methods for analyzing stability are suggested. Factor C and factor β are investigated to evaluate the critical load. This thesis investigates large deflection by using similar idea inspired from the direct secant method used for analyzing frame stability. Factor η is investigated and used to analyzing the large deflection.

The method is executed by ANSYS software using APDL routines. The problems solved include: Portal frames, Two-bay Single-storey frame, Two-bay Two-storey frame as well as Multistorey Frame Subject to Concentrated and Distributed Loads. The results from the above analyses compare with other traditional methods closely,

and the error is no more than 5%, thus demonstrating the usefulness of the direct secant method.

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List of Symbols

A	a variation parameter
	area of member cross-section
b	shorter side of cross-section
B	a variation parameter
C	factor used to predict elastic buckling load
C_1, C_2	arbitrary constants of the general solution of Euler equation
d	displacement
d_{arb}	arbitrary displacement
d_{mpc}	displacement calculated from bending moment, total compressive force and factor C
d_{peak}	peak displacement
E	young's (elastic) modulus
E_i	secant modulus in *i* iteration
E_0	young's modulus of original material
E_S	modified young's modulus
EI	flexural rigidity of beams and columns
EI_{old}	flexural rigidity of beams and columns of original material
EI_{new}	modified flexural rigidity of beams and columns
$f(s_{ij}), f(s_{ij}^0)$	Von Mises yield function
f_y	yield stress of the material

F	externally applied force
F_i	a set of forces
$F(s_{ij}^0, \sigma^0, m^0, \mu^0, \phi^0)$	function associated with Mura's function
h	longer side of cross-section
I	moment of inertia
I_{old}	moment of inertia of the original cross-section
I_{new}	modified moment of inertia
i	iteration number
k	an arbitrary constant
	citical value for octahedral shearing
K	stiffness matrix
L	length of beams and columns
L_{OD}	length of OD in Fig. 3.4
L_1	the horizontal length of beam in Fig. 3.1
L_2	the vertical length of beam in Fig. 3.2
m^o m'	upper and lower bound multipliers
m_1^0 m_2^0	upper bound multiplier corresponding to applied load
m	structural property determined by Q
m_∂	proposed lower bound multiplier
M	bending moment
M_1	bending moment resulted from applied force P in Fig. 3.4
M_{d0}	bending moment at the location of zero displacement
M_L	limit bending moment
$M_{disp\text{-}peak}$	the bending moment at the location of peak displacement

M_{max}	the Maximum Bending Moment in Reanalysis
M_{min}	the Minimum Bending Moment in Reanalysis
M_0	bending moment from an initial analysis
$M_{0\text{-}max}$	the maximum bending moment in the original analysis
M_P	plastic bending moment
M_{pi}	plastic bending moment for each element
M_{peak}	peak bending moment
$M_{peak\text{-}ave}$	peak average moment
n	an arbitrary constants
	number of peak bending moments
P	an element in a set
	applied load
$P_{compr.total}$	total compressive force in the columns
P_{lim}	limit load
Q	structural property
	applied load
Q_e	effective generalized stress
Q_L	structural properties corresponding to limit load
q	modulus adjustment index
S	*a* set
SI	stress intensity
S_M	code allowable stress intensity
U	strain energy
V	volume of a component or a structure

	potential energy
	shear force
V_R	references volume of a component or a structure
V_T	total volume of a component or a Structure
w	intensity of distributed load
W_E	work done by the external applied forces
W_i	*a* set of work
x, y, z	co-ordinates axes
Z	section modulus
μ	a constant proportionality
μ^0	plastic flow parameter
δ	deflection
ε	elastic strain
ε_{arb}	arbitrary strain in the modulus of elasticity softening process
ε_R	reference strain for reference stress
σ	elastic stress
σ^n	stress determined by the material and external load
σ_1	maximum principal stress
σ_2 and σ_3	principal stress
σ_{arb}	arbitrary stress in the modulus of elasticity softening process
$(\sigma_e)_{r\text{-}node}$	r-node equivalent stress
$(\sigma_e^0)_M$	maximum equivalent stress

σ_{ei}	Von Mises equivalent stress of the ith element
σ_{ek}^{0}	equivalent stress for any element
σ_u and σ_l	upper and lower stresses
$\bar{\sigma}_n$	combined r node stress
σ_R	reference stress
σ_{ni}	R-node Peak Stress
σ_{ref}	reference stress based on the theorem of nesting surface
σ_v	Von Mises Stress
σ_x, σ_y and σ_z	component of stress in a local coordinate system
σ_y	yield stress (Equation 2-3 only)
β	factor used to predict elastic buckling load
γ, γ_1 and γ_2	scaling factors
λ	load factor
λ_b	load factor for beam mechanism
λ_s	load factor for sway mechanism
θ	follow-up angle on the GLOSS diagram
	angle of rotation
δ_d	arbitrary displacement
τ_{xy}、 τ_{yz} and τ_{xz}	shearing stresses
ϕ	curvature
ϕ^0	a point function defined in conjunction with yield criterion

φ	degree of multi-axiality and follow-up
η	the location along the height of the cross-section
ΔV_k	volume of k th element
ΔM	the difference between M_{dcr} and M_0

Subscripts

arb	arbitrary
e	Von Mises Equivalence
ij	tensorial indices
lim	limit
y	yield

Acronyms and Abbreviations

APDL	Aansys Parametric Design Language
CSA	Canadian Standards Association
FEA	Finite Element Analysis
Gloss	Generalized Local Stress and Strain
R-NODE	Redistribution Node
UDL	Uniformly Distributed Load

Chapter 1

Introduction

1.1 General

Structural engineers consider as their primary goal a safe and economical design. One of the main issues in such a design process is the identification of all the potential failure modes of structures and the associated maximum capacities. Designers need to devise strategies and techniques for avoiding collapse and saving cost while implementing the design. During the design process, they are expected to use fast and accurate methods to estimate the limit load carrying capacities. Therefore, such methods are of significant interest to current researchers.

Traditional approaches for the direct estimation of these maximum load capacities are developed on the basis of the two bounding theorems, viz., the upper bound theorem and the lower bound theorem as well as approximate step-by-step iterative formulation.

There are various techniques used for assessing "actual" limit load such as theoretical ('closed form') methods based on minimizing or maximizing the upper or lower bounds respectively, the finite element nonlinear analysis and various other 'robust' methods. Theoretical analysis is easy to apply for simple structures but is not practical for complex structures. Even if their application is feasible, for most situations the procedures are tedious.

With the development of computer techniques, engineers are able to carry out complicated nonlinear analyses utilizing desktop computers in conjunction with finite element analysis (FEA). The FEA has been successfully applied in a great many fields. Finite element nonlinear analyses can be used to obtain the limit load for elaborate and quite complex structural problems. If the procedure is applied with great care and properly verified, the results of nonlinear FEA can be considered to be accurate for practical purposes. However, it costs a great deal of manual effort in terms of care and verification as well as in computer resources (in spite of the cheap computing power available today, the discretization of practical problems is increasing in complexity steadily). To guarantee the accuracy, many iterations are needed. The control of convergence is sometimes difficult to handle. It requires engineers to possess a very strong background in nonlinear FEA and much practical experience to detect and avoid numerical difficulties. Even so, we need an independednt verification mechanism for the final results. Theoretical closed form solutions are obviously unavailable for complex structures to act as verifiers. In view of this, a resonably 'accurate', easy and fast method to solve the limit load problems is a major asset. 'Robust methods' try to fall in to this category.

Robust methods use one or more linear analyses to solve nonlinear problems. Therefore, they are faster like elastic analysis and avoid the drawbacks that nonlinear analysis methods have, such as incremental iteration problems, numerical instability and convergence difficulties. Using the 'robust methods', researchers can hope to evaluate limit load capacities in a direct manner and solve problems that traditionally depend on nonlinear analyses. The word 'robust' implies that the analysis can withstand being overly sensitive to numerical and other difficulties while at the same time being reasonably accurate. The 'robust' nature allows these methods to be potentially applied with an extensive scope including different structural shapes, boundary conditions and loading types. It would be advantageous to develop such robust methods for elementary design of components, plastic limit capacity calculation, estimation of critical loads for buckling, etc. It should be noted that for the purpose of the present study, 'limit load' implies the maximum load capacity of the structure or component for a given set of properties and load patterns. It is not the statistical limit load (obtained by using load factors on the service loads) that the structures are designed for –as in limit states design and LRFD.

In Canada, steel structures are designed as per CAN/CSA-S16-01 [2005] and other standards. The AISC LRFD [2005] specification is used in USA for steel design. Usually, structural limit state design is performed using factored loads and their effects on individual members using elastic analysis. This ignores the potential strength that exists beyond the initial reaching of member capacity at any particular location which implies that there is no reserve strength left when one member is considered to "fail" at any particular location. Plastic analysis methods are needed for including the redistribution effects and estimating the ultimate or 'true' limit loads of structures. Although iterative plastic analysis methods are permitted by the codes, they are rarely used in practice. One of the main reasons for this non-usage is the unavailability of simple to use techniques. On the other hand, linear elastic FEA is extensively used in routine design. In that sense, 'robust methods' that use simple linear analyses are a welcome addition to existing methods.

Two main factors influence limit loads of steel structures: plasticity and buckling. Almost all the strength failure modes should include these effects. Therefore, it is necessary to focus on plasticity and buckling when assessing the strength of steel structures.

The current study focuses on investigating the use of direct secant modifications to member properties to estimate the limit loads of framed structures subject to plasticity and stability effects. An attempt will be made to predict large deflection behaviour of beams, etc., using these techniques as well. The method used is adapted from Adluri [1999] and Bolar & Adluri, [2006]. It is inspired by existing techniques such as the plastic hinge methods [Neal, 1977], the r-node method [Seshadri, 1997], and nonlinear FEA [Bathe, 1996], etc.

1.2 Objectivities

1. Adapt the direct secant method to framed structures and implement it in ANSYS software using APDL routines to carry out the estimation of limit loads due to plasticity.
2. Attempt to use the method to investigate critical loads due to elastic buckling.
3. Investigate the use of the method to analyze the large deflection of simple beams.
4. Compare the analyses of the direct secant procedures with those obtained from other methods and establish the applicability.

1.3 Organization of the Thesis

The organization of the thesis is briefly described bellow:

Chapter 1 gives general background, objectives the present study, etc.

Chapter 2 gives literature review. Some of the basic knowledge related to limit loads and stability is described. Next, traditional robust methods, such as R-Node method, m_{α} method are reviewed. Lastly the principle of direct secant methods is generally discussed. Some of the relevant literature review is included in later chapters, as appropriate.

Chapter 3 focuses on the application of direct secant method to plastic limit load estimation. Review of plastic hinge methods is first given. The basic concepts such as plastic hinges and plastic collapse mechanisms are reviewed. The application of robust direct secant methods on for plastic analysis is then introduced. The limit load of framed structures caused by plastic yielding is obtained for several test cases.

Chapter 4 describes the use of direct secant methods in solving buckling problems. Basic theory of elastic buckling is quickly reviewed. Portal frames with fixed supports subject to vertical and lateral forces are investigated. A proportionality factor is suggested to linearize the procedure. Critical loads for elastic buckling of different portal frames are solved.

Chapter 5 introduces application to large deflection analysis of simple beams. Direct secant method is used for approximately analyzing the large deflection of the beams (and sway of portal frames).

Chapter 6 quickly summarizes the thesis and outlines the main conclusions for the thesis.

Appendices give typical inputs for analysis using ANSYS software.

Chapter 2
Literature Review

2.1 *Buckling and Structural Stability*

Buckling is an instability phenomenon that results in sudden failure without much warning. When relatively long members of structures are subjected to axial compressive forces which are large enough, the members will suddenly suffer large lateral deflection leading to dramatic failure. In practice, design codes derive formulas for columns with imperfections and slight initial load eccentricities that will be subject to large lateral deflections at specific critical loads.

Buckling load is referred to the solution resulted from imperfect columns which exist all over the world. The concept is used in actual cases, such as experiments. Another concept, called critical load, is known as the solution calculated from the perfect column. There is no "absolutely straight column" in the real world, so the concept of critical load is regarded as a theoretical value, and it is based on mathematical model.

2.1.1 Stable Equilibrium

If the elastic structure is applied a small enough external disturbances, it reacts simply by vibration about its original state, the equilibrium is said to be stable. In other words, although the small disturbances cause the vibration of the structure, the structure is able to maintain the original state after the vibration disappears.

2.1.2 Unstable Equilibrium

If the elastic structure is not able to maintain the original state after it is applied the disturbance, the equilibrium is unstable. It will disturb the position of each point and tend to diverge from the changed equilibrium state.

2.1.3 Neutral Equilibrium

The boundary between the stable equilibrium and unstable equilibrium is called as neutral equilibrium. If the structure undergoes both stable equilibrium and unstable equilibrium, the external reason which makes this process is called as "critical", such as "critical load, critical moment and critical displacement"

2.2 *Principle of Stationary Potential Energy*

1 For any arbitrary displacement, the particle is in equilibrium if the total work done by all the forces acting on the particle is equal to zero.

Consider that a set of forces F_i act on a small particle, and it undergoes an arbitrary displacement δ_d. During the displacement, each force acting on this particle will do a set of work W_i, so the total work done by these forces is:

$$W = \sum_{i=1}^{n} W_i = (\sum_{i=1}^{n} F_i)\delta d = F_1\delta d + F_2\delta d + \cdots F_i\delta d \tag{2-1}$$

If the particle is in equilibrium, the total forces acted on the particle are zero, which means that the total work done by these forces must be also zero. This results in the solution that the virtual work must be zero if the particle is in equilibrium.

2. For any arbitrary displacement, an elastic body is in equilibrium if the virtual

work done by the external forces plus the virtual work done by the internal forces is equal to zero. The total virtual work done by the forces can be divided into two parts: the virtual work done by the external and the virtual work done by the internal forces. Based on the previous theory above, if the elastic body is in equilibrium, the total virtual work must be zero. Therefore, the virtual work done by the external forces can be considered as a group and correspondingly, the work done by the internal forces can also be considered as another group. So it can be expressed as:

$$\delta W_E + \delta W_I = 0 \tag{2-2}$$

3. For any small displacement, the elastic structure is in equilibrium if no change occur in the total potential energy of the system. The strain energy change stored in the structures is equal in magnitude and opposite in sign to internal virtual work $\delta Wi = -\delta U$. The total potential energy consists of the strain energy and the potential energy due to external forces. If the structure is in equilibrium, the increment of total potential energy must be zero. If a structure has an infinite number of degrees of freedoms, equilibrium must be established only under the condition that the total potential energy does not change for any possible changes in the displacement of the system. Therefore, the structure has only a single degree of freedom, equilibrium must be established by requiring that no change occurs in one displacement .

$$\delta(U+V) = \frac{d(U+V)}{dx}\delta x \tag{2-3}$$

Since the δx is arbitrary, the increment of potential energy is equal to zero when

$$\frac{d(U+V)}{dx}\delta x = 0 \tag{2-4}$$

2.3 Basic Approaches for Critical Load

Based on the theory of equilibrium, critical load can be calculated in two ways. The first approach answers the question that at which load the neutral equilibrium is possible. It is not necessary to check whether the structure is stable or not. Instead, one has only to establish the equation to find the critical load in order that the total potential energy is equal to zero. By requiring $\delta(U+V)=0$, critical load can be found. The second approach is to determine the load at which the changing from stable equilibrium to unstable equilibrium is possible. It deals with the load at which the neutral equilibrium occurs. In other words, the critical load makes the neutral equilibrium appear. As the boundary of the stable equilibrium and unstable equilibrium, neutral equilibrium can be determined by finding the load at which the second variation of the total potential energy changes from positive to negative accomplished by

$$\delta^2(U+V)=0 \tag{2-5}$$

Although both of these approaches can be used to find the critical load, they stay in different level. The first approach can allow elastic deformations of structures before the buckling occurs, which means that the first approach pertains to "static level". Therefore, the first approach is usually referred to *static approach* .The second approach is said to obtain the boundary between the stable equilibrium and unstable equilibrium, and it can allow relatively small free vibration of the structures, so the second approach pertains to "dynamic level". This approach is called as *dynamic approach.*

2.4 Upper Bound Theorem and Lower Bound Theorem

2.4.1 Upper bound Theorem

In mathematics, an upper bound of a set *S* means an element P which is greater or equal to every element in the set *S*. In structural analysis, the upper bound method is defined as: For certain frames which are subjected to a set of load factor λ, every load factor λ calculated from every possible failure mechanism must be greater than or equal to the load factor λ calculated from the collapse mechanism .This concept is also called as "Kinematic Theorem".

2.4.2 Lower Bound Theorem

An lower bound of a set *S* , accordingly, means an element *P* which is less than or equal to every element in the set *S*. In structural analysis, the lower bound method is defined as: If the load factor λ is both safe and statically admissible for certain frames, the value of load factor λ must be less than to equal to the load factor λ corresponding to the collapse mechanism. It is also called as" Static Theorem".

2.5 Critical Load of Columns with Various Supports

2.5.1 Ideal Column

The issues involving structural stability are complex. The behaviour of an ideal column is well known and is reviewed below. The following assumptions are made for ideal column:

1. The column is perfectly straight.

2. Loads are applied along centroidal axis.
3. Material follows Hooke's Law and is homogeneous.
4. Column bends and bucks in a single plane.
5. Deformations of the columns are small enough.

2.5.2 Euler Equation

A hinge-hinged column is axially loaded, which is shown in figure.2.1. The internal moment at any locations with deflection *y* is

$$M = -EI\frac{d^2y}{dx^2} \tag{2-6}$$

The externally applied moment is *Py*. Equating these two expressions:

$$EI\frac{d^2y}{dx^2} + Py = 0 \tag{2-7}$$

This is homogeneous, linear, and second-order differential equation with constant coefficients. It can be solved using methods of differential equations. The general solution of equation 2-7 is:

$$y = C_1e^{ikx} + C_2e^{-ikx} \tag{2-8}$$

Where $k = \frac{P}{EI}$.

Using the relation :

$$e^{\pm ikx} = \cos kx \pm i\sin kx \tag{2-9}$$

The general solution is written in the form:

$$y = A\sin kx + B\cos kx \tag{2-10}$$

In order to calculate the value of A and B, we need to give the boundary condition of hinged-hinged column:

$$x = 0,\ y = 0 \quad (2\text{-}11)$$

$$x = l,\ y = 0 \quad (2\text{-}12)$$

After substitute these two equations, one can obtain the solution:

$$B = 0 \quad (2\text{-}13)$$

$$A \sin kl = 0 \quad (2\text{-}14)$$

A is not allowed to be zero because P can be any value under this condition. Since $P = \frac{\pi^2 EI}{l^2}$, P is the critical load needed to be obtained, $\sin kl$ is thereby required to be zero. Then,

$$kl = n\pi \quad (2\text{-}15)$$

Substitute the expressions into the equation 2-10 leads to

$$P = \frac{n^2 \pi^2 EI}{l^2} \quad (2\text{-}16)$$

P is obtained by setting n=1:

$$P = \frac{\pi^2 EI}{l^2} \quad (2\text{-}17)$$

This is the Euler load. It is the maximum load for the column capacity and it is on the verge of neutral neutral equilibrium. In other words, Euler load is the transition from stable to unstable equilibrium. The column keeps straight until external applied loads reach the Euler load. At Euler load, the column suddenly bows out and the lateral deflection becomes extremely large.

2.5.3 Critical Load of Columns with Various Supports

The table below depicts critical load of column with different boundary conditions.

Boundary Condition	Hinged-hinged	Fixed-fixed	Hinged-fixed	Fixed-free
Critical Load	$\frac{\pi^2 EI}{L^2}$	$\frac{\pi^2 EI}{(0.5L)^2}$	$\frac{\pi^2 EI}{(0.7L)^2}$	$\frac{\pi^2 EI}{(2L)^2}$

2.6 *Limit Load and Limit State*

2.6.1 Limit Load

Limit load P_l is the maximum load which structures can take in service. It is equal to the product of load factor λ and external applied load P: $P_l = \lambda P$. It is known that limit load is proportional to load factor, which implies if the external applied load is constant, load factor can be used to predict the limit load.

2.6.2 Limit State

Basically, limit load gives the structures the limit state. Limit state in fact is the state when the factor loads reach the limit loads. It namely means the final state of the structures subjected to increasing load. Generally, Limit state design includes two types: the first is called *strength limit state* and the other is called *serviceability limit state*. Strength limit state is concerned with the situations of load capacity such as collapse mechanisms due to plastic hinges and as instability because of buckling. On the other hand, serviceability deals with the situations of the unacceptable serviceability. The examples of the unacceptable serviceability involve deflection and corrosion.

The *strength limit state* design is governed by the limit load design which is carried out by the factor loads. When the factor loads reach the limit load, the structures are not allowed to have any potential strength to resist the factor load. In other words, if the structures are given the loads which are stronger than the limit loads, the structures will collapse. The collapse mechanism because of plastic hinges can be successfully solved by the secant method. Based on the modified geometrical properties, the reanalysis can predict the limit state and calculate the limit loads. The results generated by secant method are better than 95% or more. Instability including the effect of buckling, however, has not been properly solved by the secant method.

2.7 *Elastic Perfectly- plastic Model*

2.7.1 Stress-strain Relation of Mild Steel

It is necessary to introduce the stress-strain relation of mild steel. The material is widely used in the construction of structures. The relation between stress and strain for mild steel in tension is leaner in the elastic region until the stresses caused by factor loads reach the upper yield strength. This is shown in Fig. 2.2. The point *a* represents the upper yield strength location. As the factor loads keep increasing, the stress will suddenly drop down to the lower yield stress location *b*. The increasing external applied factor loads cause the increase of the strain which is in the region called *strain hardening range*. This is shown in region *bc*. The maximum stress is reached at the point *c*, beyond which a neck forms and then the stress decrease until ruptures happen at *d*.

2.7.2 Elastic Perfectly- plastic Model

Engineers showed much interest in the yield line *oa* , which is described in Fig. 2.3. Strain scale is enlarged to give a more legible diagram. The slope of the first elastic line is known as Young's modulus. However, it is not easy to find the real stress and strain curve of mild steel near the yield point, because inevitable eccentricities of loads will generate other significant bending stress. Morrison in 1939 concluded that the yield point, proportional limit of elastic line and elastic limit were coincident and that the stress-strain curve in compression is the same as the one in tension until strain-hardening happens. [R. C. Hibbeler. , 2005] .

The upper yield is not usually exhibited by some material, and the upper yield stress can not influence plastic moments. Therefore, the elastic perfectly plastic relation for stress-strain is often identified as the neglect strain-hardening. σ_y is defined as the yield strength, at which the external applied loads can not make the stress increase anymore. The stresses keep constant while the strains keep increasing until instability occurs. This is termed the *ideal plastic relation* or *elastic perfectly plastic model*.

2.8 *Failure Criterion*

2.8.1 Maximum Principle Stress Criterion

When the maximum principal stress reaches the uniaxial yield strength, yieldding will occur..

$$\sigma_1 \le \sigma_y \tag{2-18}$$

2.8.2 Von Mises Yield Criterion

The Von Mises stress is expressed as:

$$\sigma_v = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}} \tag{2-19}$$

in which σ_1, σ_2 and σ_3 are the principal stress in three directions. In one dimension case, this stress becomes uniaxial stress. If the expression is based on a local coordinate system, it is written as:

$$\sigma_v = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_x-\sigma_z)^2+(\sigma_y-\sigma_z)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2)} \tag{2-20}$$

Plastic yield occurs when the von-mises stress or equivalent stress reaches the yield strength. Von-mises stress can be used to predict the failure of ductile tearing.

The von-mises yield criterion is based on the concept of maximum distortion strain energy. It states that failure occurs when the energy of distortion reaches the same energy for yield or failure in uniaxial tension. Mathematically, it is expressed as:

$$\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2\right] \le \sigma_y^2 \tag{2-21}$$

In the two dimensional situation, one of the principal stresses is zero: $\sigma_3 = 0$ and the Von-mises yield criterion reduces to the expression:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma^2 \leq \sigma_y^2 \quad (2\text{-}22)$$

We can also interpret the von-mises yield criterion in term of octahedral shearing stress. When octahedral shearing stress reaches the criterion which is defined by the critical value , the material yields.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 6k^2 \quad (2\text{-}23)$$

Where k is the critical value and it means the yield stress in the case of pure shear. If the loading is uniaxial, the above equation can be reduced to:

$$k = \frac{f_y}{3} \quad (2\text{-}24)$$

2.9 *Nonlinear Analysis of Structures*

In the finite element technique, the relation between load and displacement is given by:

$$F = Kd \quad (2\text{-}25)$$

in which F is the externally applied load matrix, K is the stiffness matrix and d is the displacement matrix. Whether the analysis of structures is linear or nonlinear depends on the stiffness matrix. If the stiffness matrix is constant during the entire analysis, it is called linear analysis (Fig. 2.4), otherwise it is nonlinear (Fig. 2.5). Linear analysis implies stiffness matrix is not changing. Nonlinear analysis, in contrast, leads to changing stiffness matrix. In other words, the fundamental characteristic of structural nonlinear analysis is a changing stiffness matrix.

It is known that stiffness is related to Young's modulus, moment of inertia and the length of members. Any changing of them occurs among them leads to nonlinear behavior of structures. The changing of Young's modulus leads to *material nonlinearities*. *Geometric nonlinearities* results from the changing of geometric properties, such as moment of inertia and length of members. Another kind of nonlinearities is caused by the *changing status* which includes the changing of loads, contact forms, or other external reasons.

2.10 Limit Load Prediction

Traditional methods of analyzing the limit load utilize the upper and lower bound theorem. If high degree indeterminacy structures are involved, the methods are much more time consuming and tedious and they are not always practicable for complicated structures. Therefore, quick and accurate methods are needed.

Over the years, FEA has been successfully applied to structure analyses. For elastic analyses, it is very popular because of its universality and generality. There are almost no restrictions about the types and the shapes of structures if proper elements and techniques are chosen. The analyses can be static or dynamic. However, if the structures behave inelastically, linear analyses can not work and nonlinear analyses are needed. FEA is able to deal with the nonlinear problems but it has drawbacks. First, because FEA uses iterative elastic calculations to do the nonlinear analyses, a great amount of computer resources and time are needed. Second, convergence criteria are defined for the collapse mechanisms, so numerical difficulties may appear. Many factors can influence the results, such as the types of element, the way of meshing, load step control and convergence criterion. Little changing from the above factors may produce significant differences in the solution. Therefore, it needs analysts to have much work experience and expertise. Researchers are thereby encouraged to develop simpler methods and simplified software for practical application.

The techniques used for nonlinear analyses generally can be divided into two categories: they are tangent and secant stiffness method. One typical tangent stiffness method is Newton-Raphson technique. Secant methods include the direct secant method and the incremental secant method. Robust methods are one of the direct secant methods. Various Robust methods are developed to analyze the inelastic effect based on elastic theories

The robust methods are powerful techniques of analyzing the limit load. They are more popular and attractive than finite element nonlinear analysis when used for complex structures. This chapter describes a set of efficient techniques used for analyzing the limit load. Instead of using finite element nonlinear analysis, these methods use several linear analyses to predict the limit load caused by plasticity, creep or buckling. First, this implies that the time of iterations is reduced to a very small number for inelastic problems, especially for complicated structures. Because fewer iterations are carried out, robust methods save more time than traditional methods. Second, nonlinear analysis sometimes has to deal with the difficulty of convergence for obtaining a solution, and many mathematical techniques are utilized to define certain criteria for convergence and at the same time, complicated computer programs are written to make sure finite element nonlinear analysis can work in computers. Robust method, however, has the merit of easy convergence. One does not need to give complicated convergence criterion and can obtain a final solution with simple calculations. Third, another important significance of robust methods is that they can be used as a criterion to evaluate the result of any nonlinear analysis. Because they are relatively quick and accurate, one can judge the result generated from traditional methods right or wrong by using robust methods. For example, people can use ANSYS to do the nonlinear analyses, but no theoretical solutions are given to judge the result of ANSYS. Robust methods give good theoretical estimation, so that engineers can check their result by using them.

2.10.1 Reference Stress Method

It was recognized that in a creeping beam, stresses at certain skeletal points kept constant. Soderberg (1941) in his experiment first found that some stresses kept constant in a softened system. He calculated the multi-axial creep deformation and observed that even though the system is widely softened and stresses are redistributed in a large range due to softening, there are still some skeletal stresses which always remained constant. Later, the reference stress for pressure vessels was obtained by him. Schulte (1960) observed that in a creeping beam, there were two points in the cross section at which stresses never changed and at the same time, he estimated the deflections of their points. Marriott (1963) and Leckic (1964) observed that some stresses of certain points which undergo the transient creep kept constant with time. Those points were defined as skeletal points. Sim (1971) introduced the analytical technique for reference stress. Since the reference stress is independent of the creep, Sim reasoned as the creep exponent approaches infinity, the stress distribution is similar to the distribution in the perfectly plastic model when plastic hinges form in certain cross-sections. Therefore, the stresses of infinity exponent creep are analogous to the yield stress, and thus the reference stress is expressed as:

$$\sigma_R = (\frac{P}{P_L})\sigma_y \tag{2-26}$$

where P is the applied load , P_l is the limit load and σ_y is the yield stress.

2.10.2 Partial Elastic Modulus Modification

Marriot (1988) developed a technique used to identify the stress redistribution generated from post yield stress. This method includes performing a sequence of elastic analyses and the lower bound theorem. If the stresses caused by the factored loads are greater than the yield stress, the stress redistribution due to the inelastic effect needs to be considered. The modulus of each element should be modified according to the equation: $E_i = E_{i-1}\frac{S_M}{SI}$, where SI is the maximum equivalent stress calculated from the previous iteration , S_M is the arbitrary stress, and S_M is the code allowable stress.

Several iterations are then run based on this modification. Since it should be less than the code allowable stress, the maximum equivalent stress, after several iterations, decreases until it converges to a value which is less than the code allowable value. The equivalent stress calculated from the previous calculation should be statically admissible, so the factored load thus can be considered as a lower bound of the limit load.

This reduced modulus method is aimed to find the maximum equivalent stress for factored loads rather than find the limit load. Thus the reduced modulus can not ensure that the converged stress is always less than the allowable stress. It should be noted that only specified portions of structures undergo the modified moduli, therefore this method does not totally describe the stress redistribution during plastic collapse.

Further work has been extended by Seshadri (1991) and his co-workers (Fernando,1992; Mangalaramanan,1997). The R-Node method has been developed by them to predict the limit load. R-Node stress and repeated elastic moduli modification are used in the R-Node method.

2.10.3 Gloss R-Node Method

Based on the theory above, the Gloss R-Node Method was introduced by Seshadri in 1991. "Gloss" is an acronym for "Generalised Local Stress Strain" and "R-Node" is known as the "Redistrubuted Node". This method is used to evaluate the approximate limit load for both plastic nonlinearity and material nonlinearity. It is a robust and effective technique based on two elastic finite element analyses which can determine the R-Node locations. R-Nodes are introduced as the load controlled locations in the structures, and some of the R-Nodes peaks, acting as the possible plastic hinge locations, finally predict the plastic collapse mechanism and the limit load is thus obtained.

2.10.4 Gloss Method

There are two types of controlled stress: load controlled stress and deformation controlled stress. Load controlled stresses result from static determinate actions. They are caused by the structures to keep the static equilibrium when the structures are subject to external applied forces and moments. Deformation controlled stress, however, occurs in the structures as the result of the statically indeterminate actions. Once the structures undergo the plasticity or creep, the statically indeterminate stress redistribution happens at most portions of the structures except at certain locations, which are known as R-Node locations. The structures under consideration can be divided into two regions: local region and remainder region. The local regions of the structure undergo inelastic deformation, such as plasticity and creep. The remainder regions exhibit elastic deformation.

The principle of the Gloss Method is to utilize elastic analyses to calculate inelasticity. The inelastic stress redistribution due to plasticity or creep can be analyzed by following uniaxial stress relaxation. In an elastic perfectly plastic model, stresses relax to yield stress due to plasticity. If deformation control governs, it will keep the strain a constant value which can be determined by requiring θ to be equal to zero. To ensure the effect of both plasticity and deformation control, the modulus of this pseudo elastically stressed element is modified by the expression:

$$E_S = E_0 \frac{\sigma_{arb}}{\sigma_{ei}} \tag{2-27}$$

E_S is the new modulus for each element, E_0 is the original Young's modulus , σ_{arb} is an arbitrary stress value and σ_{ei} is the equivalent stress for each element. After modifying the modulus, a second analysis is then run. It was suggested that this reduced modulus method predict the limit load with enough accuracy.

Figure.2.6 presents the Gloss diagram. Line OAC is the elastic perfectly plastic curve for stress and strain. Line OAB is the pseudo elastic line on which the first elastic analysis is based. Point B ($\sigma_{arb}, \varepsilon_{arb}$) is the pseudo point. Deformation Control is performed from the point B. The slope of line OCE is the new slope which is modified from the original slope of line OAB. The second elastic analysis is performed by using the new slope of the line OCE, which is called the secant modulus.

2.10.5 R-Node Method

Seshadri in 1991 introduced an approximate method used to determine the limit loads based on two elastic analyses. The method, known as the Gloss R-Node method, is inspired by the reference stress method and modulus modification. When structures encounter inelasticity, such as plasticity and creep, stress redistribution will occur. Most portions of the structures undergo stress redistributions except at statically determined locations, which are load controlled portions. Those locations are known as R-Node locations. R-Nodes always maintain the same stress level though stress redistributions occur in most portions of the cross sections. In other words, those statically indeterminate stresses undergo redistribution while no distribution happens at R-Nodes, which are the load controlled locations. Therefore, the stresses at R-Nodes are proportional to the external loads without influence of materials. If two different external loads are applied on structures satisfying the stable equilibrium, two stress distributions will intersect at the R-Nodes, the principle of which can be utilized to locate R-Nodes [Seshadri. R. , 1991].

Imagine that a beam with rectangular cross section is subject to pure bending. It is shown in Fig. 2.7. The relation between stress and strain is expressed as :

$$\varepsilon = E\sigma^{n} \qquad (2\text{-}28)$$

in which E and n are determined by the material and external loads. If the cross section behaves elastically, $n = 1$ and if the cross section is totally plastic, $n = \infty$. The intersection of these two lines is the location of the R-Node and it is recognized that all the stress redistributions pass through the same nodes (Mangalaramanan and Seshadri,1997).

This method suggests that except at R-Node locations, all the stresses redistribute due to plasticity within components or structures. In the elastic perfectly plastic model, the relationship between the reference stress and the R-Node stress is given by:

$$(\sigma_e)_{r-node} = \mu\sigma_R \qquad (2\text{-}29)$$

Since the induced stresses are proportional to the factored loads or load combinations, this relationship can be given by :

$$(\sigma_e)_{r-node} = \gamma_1 P \qquad (2\text{-}30)$$

$$(\sigma_e)_{r-node} = \gamma_2 < P, M > \qquad (2\text{-}31)$$

where γ is the scaling factor determined by loading, material, and geometrical properties. In an elastic perfectly plastic model, when the induced stress reaches the yield stress, the factored loads will become the limit loads. Therefore, this relationship can be expressed as:

$$(\sigma)_y = \gamma_1 P_L \qquad (2\text{-}32)$$

$$(\sigma)_y = \gamma_2 < P_L, M_L > \tag{2-33}$$

Combine equation (2-13) and (2-15) :

$$P_L = \left[\frac{\sigma_y}{(\sigma_e)_{r-node}}\right] P \tag{2-34}$$

$$< P, M >= \left[\frac{\sigma_y}{(\sigma_e)_{r-node}}\right] < P, M > \tag{2-35}$$

Where $\left[\frac{\sigma_y}{(\sigma_e)_{r-node}}\right]$ is identified as the load factor.

The R-Node method can be used to analyze the limit load of mechanical components and structures in the following steps:

1. A linear elastic analysis is performed by the factored loads, which can be greater than or less than the actual limit loads of the structures. This analysis is the pure elastic analysis without any limitation for the structures, such as yield stress and buckling. Stability is not under consideration for this analysis.
2. The modulus of each element is modified by the equation: $E_S = E_0 \frac{\sigma_{arb}}{\sigma_{ei}}$. σ_{arb} as mentioned before, is an arbitrary nonzero value. According to this modification, the second elastic analysis is then carried out.
3. Two elastic analyses are performed and they result in two elastic lines which act as the basis of the follow-up angle θ. The locations with $\theta = 90^0$ are identified as the R-Node locations. Actually, R-Node means no redistributions occur for the stress, so if the intersections of these two analyses can be obtained, R-Node location can naturally be found. The stresses at the R-Node locations are known as R-Node stress.

4. A plot of R-Node peak stress identifies certain locations within the structures. Theses locations imply that as the externally applied load increases, the cross section of the peak stress locations will become totally plastic faster than ambient cross sections. In other words, the peak stress locations form plastic hinges faster than the neighbor points.
5. R-Node stresses are the results of load control. The limit load thus can be calculated when R-Nodes stress reach the yield stress.

$$\overline{\sigma_n} = \frac{\sum_{i=1}^{n} \sigma_{ni}}{N} \tag{2-36}$$

Where σ_y is the yield stress and $\overline{\sigma_n}$ is the peak average stress.

Compared with other inelastic methods, R-Node method gives relatively simple procedures and conservative results. It is successfully applied on two dimensional situations. For three dimensional structures, it is suggested that a R-Node stress surface should be used to determine the peak R-Node locations [Seshadri. R. , 1997].

2.10.6 The m-α Method

An improved limit load estimate technique inspired from Mura's variational formulation is known as the m-a method (Seshadri and Mangalaramanan 1997). Based on the solution of Mura's variational formulation, the limit load is achieved by leapfrogging on a basis of two linear elastic analyses which result in the upper and lower bound multipliers: m^0 and m'. Similar to the R-Node method, the m-a method modifies the initial elastic moduli for each element in order that stresses can redistribute about load controlled locations. After the second linear elastic analysis, an upper bound multiplier m^0 which satisfies the theorem of nesting surface (presented in 2.10.7) is determined based on the inelastic action of the structures. Compared with the m_θ obtained from the total volume of structures, the one estimated based on this new upper bound theorem is more conservative and advanced. The main procedure used for obtaining m_θ is outlined as the following procedure:

1. The first elastic analysis is performed to analyze the stress distribution which is utilized for modulus modification.
2. According to the modification equation

$$E_S = \left[E_0 \frac{\sigma_{arb}}{\sigma_{ei}} \right]^q \tag{2-37}$$

all the element moduli of the structures are modified, in which E_0 is the Young's modulus , σ_{arb} is an arbitrary stress , σ_{ei} is the equivalent stress and q is the modulus adjustment (usually chosen as 1)

3. The second linear elastic analysis is carried out based on the new modified moduli and a new equivalent stress distribution is evaluated.
4. Calculate the energy dissipation of each element in the prescribed structures and estimate the upper bound m^0 for each element. Two conservative analyses give two multipliers denoted as m_1^0 and m_2^0. Plotting these two curves gives the intersection which is identified as the reference volume where the theorem of nest surface is satisfied (Fig. 2.8). The upper bound multiplier is thus obtained.
5. The lower bound is given by the equation:

$$m' = \frac{2m^0\sigma_y^2}{\sigma_y^2 + (m^0)^2(\sigma_e^0)_M^2} m_\theta \tag{2-38}$$

m' m^0 and $(\sigma_e^0)_M$ are all functions of the iteration variable. Based on iterative calculations, the multiplier m_0 in the end can result in good estimation of the reference volume.

2.10.7 Nesting Surface Theorem

Calladine and Drucker in 1962 introduced the "theorem of nesting surface", which is used to estimate power law creep. Boyle in 1982 redefined the theorem which is used to simplify the analysis of stress in complex structures. The average energy dissipation rate is utilized as the expression of the dissipation rate of structures under multiple loading.

$$\sigma_R \varepsilon_R V = \int_V \sigma_{ij} \varepsilon_{ij} dV \tag{2-39}$$

The material of structures is given by the equation:

$$\varepsilon = E\sigma^n \tag{2-40}$$

Using equivalent stress and strain, the average energy dissipation can be expressed by:

$$\sigma_R^{n+1} V = \int_V \sigma_e^{n+1} dV \tag{2-41}$$

Therefore, the reference stress can be written as :

$$\sigma_R = Q_e(\sigma_{ij}) = F_n(\sigma_{ij}) = \left[\frac{1}{V}\int_V \sigma_e^{n+1} dV\right]^{\frac{1}{n+1}} \tag{2-42}$$

The function is strictly monotonic with the component n. When n=1, the structures behave elastically and the function is at lower boundary. When $n \to \infty$, the function is perfectly plastic and the function is bounded above. At this time, Q_e is considered as a constant value. Thus if an arbitrary stress having a hypersurface Q_e=constant , it must "nest" inside the region between the upper and lower boundary. The reference stress defines stress space with two boundaries or two surfaces. When n=1, it is bounded outside and when $n \to \infty$, it is bounded inside.

$$Q_e \mid_{n=1} \leq Q_e \leq \lim_{n \to \infty} Q_e \quad (2\text{-}43)$$

2.10.8 Extended Lower Bounded Theorem

Mura and Lee in 1965 introduced a lower bound theorem used for estimating limit loads of structures subject to tensile loads. This theorem is based on variational principle and gives good evaluation of limit load. However, real structures are more complicated because they are not only under tension. So a generic approach is necessary. Seshadri and Mangalaramanan in 1997 proposed a method which combined the elastic modification and lower bound theorem. The evaluation of limit load is on the basis of elastic stress distribution.

Mura and Lee demonstrated that if the function

$$F = m^0 - \int_V \mu^0 \left[f(s_{ij}^0) + (\phi^0)^2 \right] dV \quad (2\text{-}44)$$

where

$$f(s_{ij}^0) = \frac{1}{2} s_{ij}^0 s_{ij}^0 + \frac{1}{3} \sigma_y^2 \quad (2\text{-}45)$$

is stationary, the factors m^0, μ^0 and ϕ^0 can be determined. This leads directly to the following three equations:

$$\frac{\partial F}{\partial m^0} = 0 \quad \frac{\partial F}{\partial \mu^0} = 0 \quad \frac{\partial F}{\partial \phi^0} = 0 \quad (2\text{-}46)$$

Evaluating leads to

$$\phi^0 = 0 \quad (2\text{-}47)$$

$$m^0 = \frac{\sigma_y \sqrt{V}}{\sqrt{\sum_{k=1}^{n} (\sigma_{ek}^0)^2 \Delta V_k}} \qquad (2\text{-}48)$$

Comparing the expression for m^0 with the one proposed by Calladine and Drucker in 1961 and Boyle in 1982 which is obtained from the reference stress equations, it is given that:

$$m^0 = \frac{\sigma_y}{\sigma_R} \qquad (2\text{-}49)$$

Therefore, monotonic increasing reference stress will lead to the decrease in the value of m^0. Since it shows a lower bound of the reference stress when $n \to \infty$, the m^0 is thus a upper bound multiplier when n=1.

Mura in 1965 proposed the lower bound theorem:

$$m' = \frac{m^0}{1 + \max\left\{ f(s_{ij}^0) + (\phi^0)^2 \right\} / 2k^2} \le m \qquad (2\text{-}50)$$

This equation can be simplified by substituting equation (2-45) as:

$$m' = \frac{2m^0 \sigma_y^2}{\sigma_y^2 + (m^0)^2 (\sigma_e^0)_M^2} \le m \qquad (2\text{-}51)$$

in which $(\sigma_e^0)_M$ is the maximum equivalent stress for certain factored load. The limit load can thus be estimated by the equation:

$$P_{\lim} = m' P \qquad (2\text{-}52)$$

According to the upper and lower bound theorem and the solution obtained above, the limit load bound is evaluated by :

$$m' \le m \le m_2^0 \qquad (2\text{-}53)$$

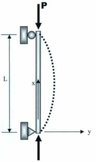


Fig. 2.1 A Simply Supported Beam subject to Axial Forces

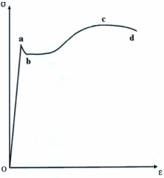


Fig. 2.2 Stress-strain Curve of Mild Steel [Neal, 1977]

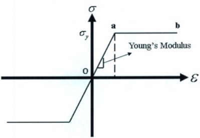


Fig. 2.3 Elastic Perfectly- plastic Stress and Strain Relation

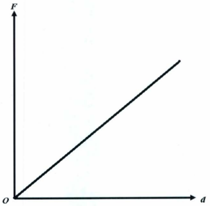


Fig. 2.4 Linear Relation between Load and Displacement

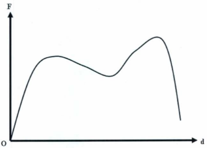


Fig. 2.5 Nonlinear Relation between Load and Displacement

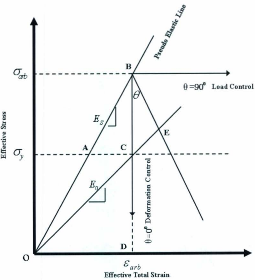


Fig. 2.6 Gloss Diagram (Sheshadri and Fernando, 1992)

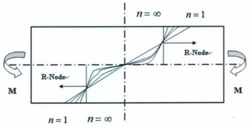


Fig. 2.7 R-Node in Rectangular Cross-Section

(Sheshadri and Fernando, 1992)

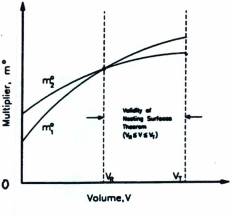


Fig .2.8 m-α method: Calculation of Reference Volume

[Seshadri and Mangalaramanan, 1997]

Chapter 3

Plastic Limit Load Estimation

3.1 Introduction

Structures subject to relatively small loads behave elastically. When larger loads are applied, they exhibit plastic behavior. In frames and frame like structures, plastic zones and eventually plastic hinges are formed with increasing loads. This leads to the redistribution of stresses. A plastic collapse mechanism takes place if enough plastic hinges are formed. This procedure can be analyzed by traditional plastic analysis techniques. Neal [1977] developed a method using minimization of work done to determine the final plastic collapse mechanism. This method can give good estimation of limit loads for simple structures. For more complicated structures, it becomes tedious. One has to determine the collapse mechanism by comparing all the independent and combined mechanisms, especially when the plastic hinge locations are not obvious at the outset, e.g., frames with unevenly distributed loads. The method also completely ignores the effect of lateral displacements. The alternative is to carry out full scale nonlinear finite element analyses which require complete discretization of the entire cross-section throughout the frame. The mesh needs to be especially fine near the locations of plastic hinges. Although the computational time needed for very fine meshes is no longer an overriding issue, complete discretization of the frames needed to capture the progressive plastification of the cross-sections at several locations in the frame is tedious work requiring initial input and extensive cross-checking. Simplified techniques therefore become attractive. Such methods should be progressively more advanced and yet be simple, accurate and robust. The r-node method [Seshadri, 1997; Seshadri & Fernando, 1992] is one such technique as outlined in the previous chapter. The Direct Secant Method is a new technique which is inspired by both the traditional plastic analysis and the r-node method [Adluri, 1999a]. In this method, based on the solutions of the first elastic analysis, rigidities are modified and are used to carry out a second analysis. The results of the second elastic analysis can be utilized to predict the limit loads of structures.

This chapter first illustrates the principles of traditional plastic analysis. The concepts of plastic moment and collapse mechanism are reviewed. Three basic collapse mechanisms, viz., beam mechanism, sway mechanism and combined mechanism are explained. After listing the inspirations of traditional plastic method (will be presented in 3.4) and the R-Node method, this chapter explains the main steps of the direct secant method. In the end, verification examples are shown.

3.2 Plastic Analysis Review

The material nonlinear analyses of structures have been investigated by researchers for many years. It is well known that the redistribution of stresses caused by plasticity can generate extra strength which can be used to design the structures beyond the simple 'elastic limit'. Kazinczy [1914] was one of the early researchers to report experiments on investigating the collapse of a steel beam. He found that the collapse of this fixed ended beam occurred when the cross-sections at the middle and the end become fully plastic. The moment which causes the cross-section to become fully plastic is called the plastic moment. When a cross-section yields fully, it can rotate like a hinge. Normal hinges in structures are not able to take any moments. However, the hinge caused by a plastic moment, known as *the plastic hinge*, can resist the plastic moment but would not have any further moment resistance. The formation of plastic hinges can lead to the collapse of structures. As loads increase, enough plastic hinges form leading to the collapse of the structure under consideration. Just prior to the collapse, the state of the structure, including the locations of plastic hinges, etc., is called the collapse mechanism. The collapse mechanism is assumed to be the final state of the structures [Neal, 1977]. The loads leading to the collapse mechanism are termed as limit loads for the purpose of the current study. The limit loads depend upon the load configuration as well as the structure's properties, geometry, boundary conditions, etc. Thus, for a given structure, there can be several limit loads depending upon the type of loading patterns. For a given pattern, there is a unique limit load. The limit load is usually identified by a multiplying factor applied to increase the nominal load to the limit load. This factor is called as the load factor in the current study. The determination of the load factor and the associated failure mechanism is the purpose of limit analysis and design.

Structural design, most often, ignores the redistribution effects that occur beyond the elastic limit (except some cases, such as seismic design). The results of elastic analyses are used to design the member cross-sections in a plastic sense. This generally implies that as soon as the most critical location reaches its limit, the structure is unusable even though the most critical location can support redistribution thus significantly improving the limit load. This extra capacity usually occurs in structures that are highly indeterminate. The purpose of the plastic design is thereby to find the loads which cause the failure by successive formation of plastic hinges in a mechanism failure pattern. For the current study, it is assumed as per normal convention, that the effects of combined moment and shear are not significant. It should be noted however, that this is not a necessary condition for the method being studied. Even if the shear deformations dominate, the method can be adapted.

The plastic moment capacity, M_P, of any given cross-section can be calculated using geometrical properties and the yield stress. For the rectangular cross-section, the plastic moment capacity is,

$$M_P = Z\sigma_y = \frac{1}{4}bd^2\sigma_y \tag{3-1}$$

where, Z is the plastic section modulus, b is the width and d is the height of the cross-section.

In general, the plastic modulus can be obtained using

$$Z = \int_h b(\eta)\, d\eta \tag{3-2}$$

where, the cross-section width can change with the location η along the height. The traditional method to determine the limit load of frames had been to use the upper bound theorem, equating the work done by the load to the work absorbed in the plastic hinges for a series of possible mechanisms and selecting the least limit load factor. If the mechanisms depend on plastic hinge locations that are continuously changeable, a simple minimization process is carried out. This is illustrated in several standard references such as Neal [1977].

3.2.1 Mechanisms of Failure for Frames

For frames, three possible mechanism types will be shown in this section. Fig. 3. 1 shows a *beam mechanism* where a portal frame is subject to a vertical force in the middle of the beam and the failure of the frame is essentially through the failure of the beam as shown. The end hinges can be located either in the beam itself or at the ends of the column. As the load increases from a completely elastic state of stress, stress at the extreme fibers of the middle of the beam first reaches yield. An increasing load then makes the cross-section in the middle become totally plastic and turns it into a plastic hinge. A redistribution of any further increase will prompt the end of the beam to start yielding and eventually to the formation of two additional hinges. Since three hinges in a line imply instability, the structures will then reach its limit load. Equating the work done by the applied factored load (P λ) for a given rotation (θ) of the beam segment, with the internal work done due to the rotation of the cross-sections at the plastic hinges (thus ignoring the small amount contributed by the elastic deformation of the remaining structure), we can obtain the load factor λ_b for the beam mechanism as,

$$(\lambda_b P)\left(\frac{L_1}{2}\theta\right) = 4(M_P)(\theta) \tag{3-3}$$

The deflection in this case is assumed to be small so that the displacement under the load is directly calculated using the rotation of the beam.

$$\lambda_b = \frac{8M_P}{PL_1} \tag{3-4}$$

Following a similar procedure, the *sway mechanism* is shown in Fig. 3.2. Imagine that a portal frame subject to a lateral force which can cause the lateral deflection. The collapse mechanism include 4 hinges for instability. If the cross-section capacities are all equal to M_P, equating the externally applied work and internally absorbed work gives,

$$(\lambda_s F)(L_2\theta) = 4(M_P)(\theta) \quad \Rightarrow \quad \lambda_s = \frac{4M_P}{FL_2} \tag{3-5}$$

Both *beam and sway mechanism* in the above are assumed to be independent which do not influence each other. The limit load factor (λ) generated from these mechanisms follows the upper bound theorem.

$$\lambda \leq \min(\lambda_b, \lambda_s) \tag{3-6}$$

The examples given are simple portal frames. In a real plastic analysis, most structures encountered have complicated geometries. Thus the combination of those independent mechanisms is a way to solve plastic problems. As an illustration (Fig. 3.3), if the portal frame is subject to both the lateral and vertical force, we can see that most rotations in the combination of the two mechanisms exist, while two disappear. This is because the rotation sign at these locations in the beam mechanism is opposite to the one at the same locations in the sway mechanism. The net gives,

$$\lambda P\left(\frac{L_1}{2}\theta\right) + \lambda F(L_2\theta) = 4(\theta)(M_P) + (2\theta)(M_P) \tag{3-7}$$

The external work is equal to the sum of that in the beam mechanism plus that in the *sway mechanism*. This implies that the patterns of work done by the applied loads are independent and do not influence each other. On other hand, the internal maximum work absorbed is not simply equal to the sum of the work done by the two independent mechanisms. Therefore, the load factor calculated based on plastic hinge cancellation may be smaller than that for each independent mechanism, [Neal, 1977].

The method outlined above gives good estimates of the limit load for frame type structures. However, the procedure is tedious for real structures. The method requires us to locate all the hinge cancellations and calculate the exact maximum internal work, which is not easy to handle especially if the frame has loads distributed in an uneven manner. The method is also not easily amenable for computer implementation. The method also completely ignores the effect of column sway on the column moments due to vertical loads (interaction between the sway forces and vertical forces leading to moment amplification), which, in some cases may reduce the load factor to a certain extent. However, the method gives excellent results in most cases and is used here for comparison purposes.

3.2.2 Plastic Design of Structures

Plastic analysis is of significance for designing steel and other kinds of structures that have redundancies and can withstand extra deformation beyond the initial yield. If the collapse mechanism of structures does not include a sway mechanism or does not include local and overall stability effects, the plastic analysis is called "first order plastic analysis", such as in the case of a fixed-fixed beam subject to vertical loads failing in a beam mechanism. If geometric nonlinearity is to be included in the inelastic analysis, it is known as "second order plastic analysis".

For steel structures, the deformation beyond initial yield depends upon the cross-section classification. For example the CISC [Canadian Institute of Steel Construction] classifies steel sections into four categories. "Class 1" steel sections permit both plastic hinge development and increased rotations facilitating stress redistribution to other locations after the formation of initial plastic hinge at a critical location. The moment-curvature relationship is "ductile" as represented by the bold line in Fig. 3.4. "Class 2" steel sections allow successive plastic hinge formation but can not withstand the effect of large rotations after that. Classes 3 and 4 do not permit full plastification of the cross-sections. Therefore, all steel frames that are designed with class 1 sections and some redundancies (statical indeterminacy), will have strength beyond the normal estimates due to stress redistribution.

As long as the material has ductile behaviour, direct secant method can be used for analyzing the limit load in the similar way. In concrete structures, beams that have reinforcements below the "balance" limit (as is almost always the case) are known to exhibit ductile behaviour similar to that shown in Fig. 3. 4 by the bold line. For all such cases, the stress redistribution increases the capacity of the frame.

3.3 Alternative Methods

Since the computer implementation of the traditional plastic hinge method is cumbersome, it did not find practical utility. The method also ignores the effect of sway on the column moments as mentioned above and leads to what is usually called as "rigid plasticity". On the other hand, full scale nonlinear analysis of practical steel or concrete frames is even more involved in terms of effort and the need to verify whether a proper nonlinear analysis is carried out. Hence, it is not employed on a routine basis. Some of these difficulties can be overcome through the development of alternative methods. Such methods need to be simple, robust and reliable. These alternative methods can be extensions of the currently available methods. One such procedure which is inspired by the traditional plastic hinge analysis [Neal, 1977] as well as the r-node method [Seshadri, 1997] is described and implemented in the present chapter.

3.4 *Inspirations from Plastic Hinge and R-Node Analysis*

In the section below, the main inspirations for the development of the method based on direct secant modifications are described [Adluri, 1999 & 2001, Bolar and Adluri, 2001]. The points also include some of the main assumptions.

1. After the first yield stress takes place, structural members begin to lose their stiffness, gradually becoming members with negligible tangent stiffness. The secant stiffness represented by the slope of the line joining the origin to the current point also changes as the load increases.

2. The distribution of moments and other stress resultants are to be determined by the relative stiffness of structural elements. The changing of the stiffness of certain parts of structural elements can lead to the changing of stress resultants also. As noted by Adluri [2001], the "exact" secant stiffness, if known, will give the "exact" results with one single elastic analysis.

3. There are three types of plastic failure: complete collapse, partial-collapse and over-complete collapse. Complete collapse and over-complete collapse are achieved by removing the redundancies until structures become determinate.

Partial collapse happens when part of the structures is determinate. The changing of indeterminate structures to determinate structures is achieved by reducing the redundancies through the progressive formation plastic hinges.

4. At failure, the moments at plastic hinge locations are all equal to the plastic moment M_P of the respective sections. The section behaviour is assumed to be "ductile" where some transition zone might be present as well.

5. The r-node method is based on the understanding that the stress from the first elastic analysis can be used to obtain the secant modulus. This stress changes continuously at each point in the structure. Thus, there will be a different secant modulus for each point along the depth of the section as well as along the length of the members. Therefore, the discretization generally results in a 3-D model. To obtain the r-nodes, one needs to do two elastic analyses by modifying the modulus of each point. The direct secant method to be used in the present work is also based on a similar concept. Instead of modifying the modulus, the method modifies the rigidity. This results in 1-D mesh for frames as opposed to a 3-D mesh for the r-node method.

6. In the R-Node method, the concepts of load and displacement control are used to find the limit load. In the GLOSS diagram, the following up angle θ is equal to 90^0 at the time of collapse. Similar concepts are applied in the Direct Secant Method.

7. In the r-node method, the final limit load is achieved by increasing the average peak stress to the yield stress while increasing the applied load by the same proportion. The analogy of two-bar model is used for this purpose. The proportionality factor denotes the limit load factor. Similar concepts are used by

the Direct Secant Method.

8. Although the R-Node method can not be applied to determine the large deflection or stability of a structure, the method can be used as an inspiration for further development of the Direct Secant Method towards these goals.

These inspirations from traditional plastic hinge analysis method and the R-Node method have been used to enhance the methods leading to the "Direct Secant Method". The method has previously been described by Adluri [1999, 2001]. Bolar & Adluri [2006] used it to obtain the limit loads of various plate structures. They utilized modified geometrical properties and elastic analyses to estimate the collapse state of plates. In the present work, this method has been used to obtain collapse loads of frame structures and to study the stability and large deflection effects of frames and beams. The following sections describe the procedure and illustrate with examples.

3.5 The Direct Secant Method

Adluri [1999] proposed a method called "Direct Secant Method". It is inspired by the existing methods to find the limit loads using simple elastic analyses –mainly, the R-Node family of techniques. In this method, a purely elastic analysis is first carried out without the consideration of yield stress. Based on the results calculated from the first elastic analysis, the rigidity is modified. When the rigidity is modified, the cross-section is changed at each point and hence the structure becomes highly non-uniform along the length of the members. The modified structure is analyzed again with the same load, the same supports but the new rigidity. After the second elastic analysis, the peak moments and other relevant stress resultants are obtained.

These moments (or stress resultants) may have nothing to do with the corresponding the peak values from the first analysis. These peak values can be used to predict the collapse mechanism and the limit load. This technique gives a very good estimation of the limit load. The basic steps and principles are illustrated below:

1. A purely elastic linear analysis is first carried out. In the analysis, stability is not under consideration. Global or local collapse due to plasticity and buckling never happens in this analysis. By using FEA, the bending moment at any location can be easily obtained. The first elastic analysis is of significance for investigating the distribution of bending moments.

2. The results from the first purely elastic analysis can be used to modify the rigidities of the structure to be analyzed. Adluri [1999] modified the moment of inertia of the beam based on the bending moments.

$$I_{new} = \frac{1}{|M_0(x)|} I_{old} \tag{3-8}$$

The proportionality constant can be chosen to be any nonzero value. To avoid numerical difficulty, Adluri [1999] used the maximum bending moment as the proportionality constant.

$$I_{new}(x) = \left| \frac{M_{0-\max}}{M_0(x)} \right|^q I_{old}(x) \tag{3-9}$$

Usually, the exponent q can be taken between 1 and 2. In his analysis, he took 1 as the exponent.

If the Young's modulus is kept constant, from this equation we can find that the modification stiffens the flexural rigidities of all the elements of the structure. Because $M_{0-\max}$ is not less than $M(x)$, $I_{new}(x)$ is greater or equal to $I_{old}(x)$. However, the significance of the modification is not like this. As mentioned earlier in this chapter, the distribution of bending moments does not simply depend on the absolute flexural rigidities, but on the relatively flexural rigidities. This modification also changes the relative flexural rigidities of structures. Because $M_{0-\max}$ is a constant value, the elements which have relatively greater bending moment will have relatively less flexural rigidities after modification. The element at the maximum bending moment location will have the minimum relative rigidities, and vice versa. Therefore, the modification implies that in the first elastic analysis, the rigidities of the elements that have relatively greater bending moment are softened and stiffened rigidities are given to the elements which have the relatively less bending moments. The modification actually means "harmony". The elements which are weaker when resisting the bending moment are given stronger abilities and the reduced abilities are applied on the stronger elements. Therefore, the modification can be understood as: The elements of the structure are given the same abilities to resist the bending moment. In other words, the elements of structure have the same opportunities to reach the same bending moment. Of course, this same bending moment can be the plastic moment of cross-sections.

Note that the significance of the modification mentioned above is under the condition that the structure is of the same size of cross-sections. All the elements of the structure have the same cross-section so that they can have the same moment of inertia and the same plastic moment. If the cross-sections are not the same between each element, such as if the beams are stronger than the columns, then the "harmony" cannot be simply achieved in this way. This will be mentioned later in this chapter.

3. The second elastic analysis is executed on the basis of the modification in step 2. By using FEA, the distribution of bending moments can be easily obtained. There must be some elements whose moments are greater than the ones of neighboring elements. Those elements in the bending moment diagram are of peak moments. These elements having peak moments "go" faster than neighboring elements, which means although the modification of rigidities gives all the elements the same opportunities to reach the plastic moment, the peak moment elements first reach the plastic moments. Therefore, they are at possible locations of plastic hinges. The peak moment locations are considered as the potential locations of plastic hinges. As mentioned above in this chapter, the plastic collapse occurs because enough plastic hinges form a collapse mechanism. The bending moment at plastic hinges locations at collapse are equal to the plastic moment. Thus, locating the plastic hinge locations are of importance for determining the limit load.

In the first elastic analysis, the maximum bending stress location is the first location of a plastic hinge. After the formation of the first plastic hinge, the stress will redistribute. The other peak moments in the first elastic analysis are thereby not at the locations of potential plastic hinges. In the second analysis, the modification of moment of inertia is based on the bending moment distribution from the first analysis. The elements are modified to have the same opportunities of resisting the applied load. The same opportunities of all the elements lead to all the potential locations of plastic hinges appearing at the same time. This explains why all the locations of peak moments in the second analysis are considered as the potential plastic moments.

Potential locations of plastic hinges are not the exact locations of plastic hinges contributing to the final collapse. So we need to select some of them to form the collapse mechanism. The selection of potential location of plastic hinges can be done in several ways. This will be illustrated in the later section of this chapter.

4. After selecting the locations of plastic hinges, limit load can be calculated by the following equation:

$$\frac{P_{\lim}}{P} = \frac{M_P}{M_{peak-ave}} \tag{3-10}$$

where P is the externally applied vector load, and can have any non-zero value. M_p is the plastic moment, and it is determined by the geometrical properties of the cross-section. $M_{peak-ave}$ is the peak average bending moment in the second analysis. At the locations of these peak bending moments, plastic hinges form and they contribute the plastic collapse.

In the R-Node method, the limit load obtained in the R-Node method depends on the load factor that is equal to the ratio of the yield stress and the peak average stresses.

$$P_{\lim} = \left[\frac{\sigma_y}{(\sigma_e)_{r-node}}\right] P \tag{3-11}$$

To better understand the significance of this load factor, we can modify the equation into:

$$\frac{P_{\lim}}{P} = \left[\frac{\sigma_y}{(\sigma_e)_{r-node}}\right] \qquad (3\text{-}12)$$

3.6 Plastic Limit Load of Non-uniform Structures

Just prior to plastic collapse, the factored load becomes the limit load and the stresses at certain locations are equal to the yield stress. Therefore, the numerators at both sides of the equations are the properties just prior to collapse. They are in the same state, which is called the "collapse state"; at the locations of the denominators, P is the externally applied load and $(\sigma_e)_{r-node}$ is the reference stress resulting from the load P. Thereby they are also corresponding, and they are also in the same state, which is called the "normal state". The load factor actually is the ratio of properties in these two states.

In the R-Node method, the properties are the bending stresses. In the concept of dimension,

$$\frac{load}{stress} = area \qquad (3\text{-}13)$$

and load thus is proportional to the stress. Area is the proportional factor and is never changed in the analysis. Therefore, the exponent of the ratio of these two stresses is equal to one. In equation (3-12), $\sigma_y/(\sigma_e)_{r-node}$ is considered as the load factor.

We also can rewrite equation (3-10) into

$$\lambda = \frac{P_{\lim}}{P} = \frac{M_P}{M_{peak-ave}} \tag{3-14}$$

Where λ is the load factor. Similar to the situation of the R-Node method, $M_P / M_{peak-ave}$ can be viewed as the load factor. The numerators at both sides of the equation are in the same state of "collapse state" and the denominators are in the "normal state". In the concept of dimension:

$$\frac{moment}{load} = length \tag{3-15}$$

and the length is not changed in the modification of the second analysis. Therefore, the exponent of the ratio of these two moments is equal to 1.

The total procedure can be described in Fig. 3.4. Line OA represents the first pseudo-elastic analysis with the slope EI_0. Because it is the relationship between the moment and the curvature, the slope is the rigidity. The moment line OA does not involve the limitation, so the moment can be higher or lower than the plastic moment. The moment is assumed higher. The following behavior of the line depends on the technique used in the analysis. If the structures under consideration is load controlled which implies that it is determinate prior to collapse, the subsequent behavior of the line will be horizontal, φ=90^0. If it is displacement controlled, the line will go down and φ=0^0. In Direct Secant Method analysis to be shown in the following section, the line is expected to go down and intersect the theoretical line OB at the location C. The line AC goes downward directly intersect the X axis at the point D. A new slope is obtained based on the old slope.

Adluri [1999] obtained the new slope and the new secant rigidity is expressed as:

$$EI_{old} = \frac{M_1}{L_{OD}} \tag{3-16}$$

If the structure has geometrical nonlinear properties, the Yong's Modulus on both sides of equation (3-16) can be cancelled. Then the moment of inertia is modified base upon the equation:

$$EI_{new} = \frac{M_p}{L_{OD}} \tag{3-17}$$

where q=1 [Adluri,1999]. In fact, this is the modification that modifies the relative moment of inertia of structures. The plastic moment M_p is a constant value that does not influence the modification of the relative moment of inertia of structures. Therefore, the proportionality factor can be used any nonzero value. Another analysis is done with the new slope. Iterative analyses are carried out until convergence occurs. This is represented by the curve line AB. If the line AD does not go downward, it can go between the horizontal line and vertical line, and $0^0 \le \varphi \le 90^0$. At this time, q is not equal to 1, and it can be other values. No matter how much q is chosen, the iterative calculations are performed until convergence happens.

The members of the structure mentioned earlier are of the same cross-sections. If the yield stress is constant, the same cross-sections will lead to the same moments of inertia and the same plastic moments. However, in the real cases, the cross-sections of the structural members are not always the same. It implies that the situation of structural members with different plastic moments needs to be included. If cross-sections are not the same, for solving non-uniform structures, the technique of Robust Method will be different from the previous one.

The main principle of the direct secant method is that: all the elements are given the same abilities to resist the bending moment and they are thus given the same opportunities to reach the same moment, and it can be the plastic moment. After the analysis based on the modification, one needs to find which elements relatively first reach the peak moments. If the cross-sections are not the same, the opportunities given for each element are different. This is because the same opportunities are given based on the modification which results from two factors: moment of inertia and plastic moment.

For the uniform structures mentioned earlier, the modification is given by the equation

$$I_{new} = \frac{1}{|M_0(x)|} I_{old} \tag{3-18}$$

The distribution of moments $M_0(x)$ is determined by the relative moment of inertia in the first analysis. The proportionality factor can be any nonzero value. From equation (3-18), the plastic moment can not be seen. This is because our purpose is to modify the relative moment of inertia but not the absolute values, and the structures are expected to reach the same moment---plastic moments, there is no need to give other proportional to factors instead of one. In fact the plastic moment can be put in the numerator:

$$I_{new} = \frac{M_p}{|M_0(x)|} I_{old} \tag{3-19}$$

This will not influence the result because the relative moments are not changed.

However, if the structural members have different cross-sections, the situation is different. With different plastic moments between each member, structures are expected to reach different values after the modification. To satisfy the purpose of "the same opportunities", we should adjust the modification by

$$I_{new} = \frac{M_{p_i}}{|M_0(x)|} I_{old} \tag{3-20}$$

where M_{p_i} are the plastic moments of cross-sections. The bending moment $M_0(x)$ should correspond to the plastic moment. In equation (3-20), the plastic moments M_{p_i} is not a constant value and should correspond to the moment calculated from the first analysis $M_0(x)$. After the modification, the structural member are of the same abilities to resist the moments and thus of the same opportunities to reach the same moments.

Finally, the load factor is calculated by the peak average ratio of the plastic and the moment after the first analysis.

$$\lambda = \frac{P_{\lim}}{P} = \frac{1}{n} \frac{1}{\sum_{i=1}^{n} \frac{M_{peak-ave}}{M_{Pi}}} \tag{3-21}$$

Therefore, the procedures for analyzing non-uniform structures are illustrated below:

1. The first elastic analysis is carried out based on the original geometrical properties.
2. Modify the cross-section geometrical properties by using equation (3-20).
3. The second analysis is executed on the basis of the modification.
4. In the bending moment diagram of the second analysis, peak moments are considered as the potential locations of plastic hinges. Some of the peak moments are selected to form the collapse mechanism.
5. The load factor is calculated from the expression:

$$\lambda = \frac{P_{\lim}}{P} = \frac{1}{n} \frac{1}{\sum_{i=1}^{n} \frac{M_{i-peak-ave}}{M_{Pi}}} \tag{3-22}$$

For uniform structures, M_{P_i} is constant, and if it is equal to M_P, the equation can be simplified as:

$$\lambda = \frac{P_{\lim}}{P} = \frac{1}{n} \frac{1}{\frac{\sum_{i=1}^{n} M_{i-peak-ave}}{M_P}} = \frac{1}{n} \frac{M_P}{\sum_{i=1}^{n} M_{i-peak-ave}} \tag{3-23}$$

This is the same as equation (3-14) mentioned above for the uniform structures.

Two factors can determine the plastic moments: one is the yield strength and the other is the section modulus resulting from the cross-section properties. The analysis mentioned above only includes the consideration of different cross-sections but without involving different yield strength. However, if more than one type of material is used, the yield strength will be different. This is common in the real constructions of structures. Therefore, the Robust Method including changing yield strength should be proposed in further work.

A non-uniform portal frame is investigated. The structural members are of different cross-sections with the same strength. This is shown in Fig. 3.16. The frame can be subject to a lateral force that results in a sway mechanism, subject to a vertical force that results in the beam mechanism and subject to both of them that result in a combine mechanism. Three cases of non-uniform portal frame are studied: the beam is stronger than the columns, the columns are stronger than the beam and the beam is stronger than one column and weaker than the other one. Because the cross-sections are different between structural members, the moments of inertia are different. Three moments of inertia are given and they are applied to the structural members separately depending on the cases. The results from the direct secant method are compared with the theoretical values from plastic analysis. Load factors are used as the results.

3.7 *Comparison between Direct Secant Method and R-Node Method*

3.7.1 Modification

Direct secant method: The modification is based upon changing the rigidities of structures. The rigidities of structures include cross-section geometrical properties and material properties. The modification of either of them can successfully modify the rigidities of structures. This implies two advantages: first, in the computer program, engineers can select any properties they wish to modify, so the limitation of the computer program can be avoided. For example, modifying the moment of inertia in ANSYS is much easier than modifying the Young's modulus when APDL is used. Second, the rigidities are modified along the structural members, but for certain locations, the rigidities are of constant value. The new rigidities can be modified based on the old rigidities except at the location of hinges. The disadvantage is that: at the original hinge locations, because a hinge cannot take any moments, the moments at these locations are theoretically equal to zero. In the direct secant method, very tiny moments are used instead of zero. Therefore, the new rigidities are huge but not infinite.

R-Node method: The modification is based on changing the Young's modulus of structural cross-sections. In the R-Node method, one has to modify the Young's modulus without any other choices. This generates difficulties for certain FEA software. Also, the modification is based on equation (2-13), and the equivalent stresses of each cross-section σ_{ei} may be equal to zero at certain locations of cross-section. Numerical difficulties are then encountered. Even if we can successfully modify Young's modulus along the cross-sections, at the top and bottom of each cross-section, we still can not give accurate modification. In FEA, the changing of stress for each element inside the cross-sections is not continuous. Although smaller sizes of elements are given, difference still exists and the time of calculation becomes longer. The R-Node method also can not deal with the situation of the hinges in truss structures. At the locations of hinges, no bending moment leads to no stress. Therefore, the modification has no meaning.

3.7.2 Element Type

The direct secant method uses member level elements, in this case, beam elements to model the frames. The direct secant modification is for the entire cross-section and depends on stress resultants such as moments and shears and does not directly depend on stress itself. As opposed to this, the R-Node method uses solid elements since the modification is based on local stress.

3.7.3 Failure mode and yield criterion

Direct secant method: Many failure modes now have been successfully solved by the direct secant method, such as plastic collapse and buckling. Any yield criterion can be taken by this method.

R-Node method: It can solve the plastic collapse. Until now, there has been no other extension for the limit load calculation. The R-Node method is now only used for a single yield criterion. New techniques for this method need to be developed.

3.7.4 Limit load

Direct secant method: Limit load is calculated by calculating the peak average moments. The collapse mechanism can be determined by this method. R-Node method: Limit load is determined by the yield stress. Collapse mechanism can not be determined by this method.

3.8 Test Cases and Examples

The method described above using direct secant modification is applied to several frame type structures to predict their limit loads. The examples include portal frames, multi-bay and multi-storey frames with different loadings and arrangements. The loads include concentrated and distributed forces, vertical and sway forces. Three methods are used to find the limit loads and compare the results, viz., traditional plastic hinge analysis (as described by Neal [1977]), full scale nonlinear analysis using FEA and the direct secant method presented in the current chapter. It must be noted that the R-Node method [Seshadri, 1977], which forms part of the inspiration for the current work gives very close results to that given by the direct secant method albeit with significantly greater extra effort (both programming and computational). For comparison purposes, the FEA nonlinear analysis is taken as the base instead of the plastic hinge analysis. This is so since the FEA nonlinear analysis can include the effect of sway on column moments where as the plastic hinge analysis ignores it. Besides, the plastic hinge analysis is not easy to apply for multistory frames with mixed loading conditions. Finite element nonlinear analysis has no such limitations.

3.8.1 Finite Element Modeling

For the purpose of analysis the structures have been modeled using the finite element software ANSYS [2006]. This was used for both the nonlinear analysis and the Direct Secant Method. Traditional elastic analysis using FEA is well known [e.g., Logan, 2005] and is not be explained here in detail.

The direct secant analysis used in the current chapter has been implemented in ANSYS using beam elements (BEAM3). As mentioned above, this facilitates the modification of the secant rigidity at the level of the entire cross-section. The modification is based on stress resultants such as moments and is not based on the actual stress itself which changes continuously along the height of each cross-section. After the initial analysis results are obtained, the modifications to the rigidity (in this case, to the moment of inertia of each beam element) have been implemented using APDL routines within ANSYS. Sample routines for the modifications are given in the Appendices at the end of the thesis. They are based on the earlier such modifications implemented by Bolar and Adluri [2006] who did the same for plate structures.

For the nonlinear analysis, the structures were modeled using different options available in ANSYS (for comparison purposes and convenience). The nonlinear analyses have been carried out using beam elements BEAM3. In each case, mesh convergence studies have been carried out. The results for each element type have been tested with known theoretical results for test cases.

Several test cases are shown in Figs. 3.5.1 to 3.16.1. In the figures, the properties shown next to the figures are all non-dimensional. The lengths, loads, etc., are all the ratios of a corresponding base unit of value 1.

Normally, load factors are greater than 1 in the real cases. In this work, the loads were chosen such that the load factors are less than 1 to facilitate the work of FEA nonlinear analyses using a single load step.

3.8.2 The Description of the Test Cases and Examples

3.8.2.1 Uniform Frames

The cases tested by FEA nonlinear analysis and the direct secant method include: beam mechanism, sway mechanism, and combine mechanism as well as several relatively complicated structures subject to concentrated forces and distributed forces.

Table 3.1 shows the results of the frames presented in Figs. 3.5.1 to 3.15.1 analyzed by the direct secant method. The magnitudes of the peak bending moments in the reanalyses of the direct secant method are listed in the table. By using the results in table 3.1, the limit loads in terms of load factors analyzed by the direct secant method are shown in table 3.1.1. Also, the same frames are analyzed by the traditional plastic analysis (except structures in Figs. 3.14 to 3.15) and full scale nonlinear analysis. The results obtained by two methods in terms of load factors are also presented in table.3.1.1. The frames shown in Fig. 3.14.1 and Fig. 3.15.1 are relatively complicated, and it is not easy to apply the traditional plastic method to them. Therefore, only the direct secant method is used to analyze the limit load for the two structures. FEA nonlinear analysis is taken the base for comparison.

In table 3.1.1, the results of FEA nonlinear analysis are used to compare with the results of both the traditional plastic analysis and the direct secant method. It is assumed that if the results of FEA nonlinear analysis are closed to the ones of traditional plastic analysis, the results of FEA nonlinear analysis is considered reliable and can be used to compare with the direct secant method. In the case of relatively complicated structures, traditional plastic analysis seems tedious and sometimes is not feasible. FEA nonlinear analysis in this case is used to be the only base for comparison. In table 3.1.1, the difference of the comparisons is within 3.5%. It verifies that the results of the direct secant method and acceptable and reliable.

Fig.3.5.1 presents a portal frame subject to a concentrated force in the middle of the beam. It is used to test the beam mechanism. Fig. 3.5.2 and Fig.3.5.3 presents the bending moment distribution of nonlinear analysis and the distribution of reanalysis in direct secant method. In Fig. 3.6.1, sway mechanism is tested. Fig. 3.7.1 shows the test of combine mechanism.

A multi-bay structure, which is subject to vertical and lateral forces, has been analyzed by Neal [1977]. To obtain the limit load, the forces are applied independently and each corresponding collapse mechanisms are analyzed by the plastic analysis. All the independent collapse mechanisms are combined respectively to obtain the minimum load factor which is the limit load factor. The same procedure is followed by the analysis of the direct secant method. From Fig. 3.8.1 to Fig. 3.11.1, all the independent and combined mechanisms are tested by FEA nonlinear analysis and direct secant method. Finally, the collapse mechanism obtained is the combine mechanism shown Fig. 3.11.1, and the limit load factor in this case is 0.711.

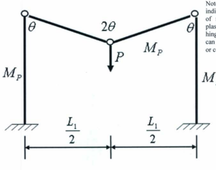
Fig. 3.14.1 presents the test of the partial collapse of a frame. The collapse mechanism is the beam mechanism. The plastic hinge locations are marked in Fig.3.14.3. Fig.3.15.1 shows a more complicated frame subject to lateral uniformly distributed forces and vertical concentrated forces. The collapse mechanism obtained is the sway mechanism on the first storey of the frame. Ten plastic hinge locations are marked in Fig. 3.15.3.

3.8.2.2 Non-uniform Frames

A portal frame with non-uniform geometrical properties subject to a later force and a vertical force is shown in Fig. 3.16.1. The limit load prediction is performed by the direct secant method and compared with FEA nonlinear analysis. Three cases including different properties of beams and columns are given. For each case, three basic collapse mechanisms (beam, sway and combine mechanisms) are analyzed. Tables 3.2.1 to 3.4.1 show the structural geometrical properties. The results obtained from FEA nonlinear analysis and the direct secant analysis are presented in tables 3.2.2 to 3.4.2. The direct secant method gives good estimation and the difference is with 3%.

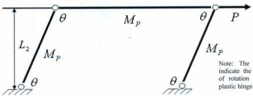
Table 3.1 Re-analysis of Peak Moments Using Direct Secant Estimation for the Different Structures shown in Figs. 3.5.1 to 3.15.1

Location	Fig. 3.5.1	Fig. 3.6.1	Fig. 3.7.1	Fig. 3.8.1	Fig. 3.9.1	Fig. 3.10.1	Fig. 3.11.1	Fig 3.12.1	Fig. 3.13.1	Fig. 3.14.1	Fig. 3.15.1
①	36265	76230	80627	25214	40651	34632	38284	36886	38995	95116	9.52E+05
②	43735	67780	65213	30455	36546	29764	42612	33448	40607	88920	8.67E+05
③	36265	67771	79014	33876	30257	32914	42565	39482	52542	187170	8.74E+05
④		76219	78919			36478	38409	35190	37565		8.56E+05
⑤						34652	41274	43097	41037		8.56E+05
⑥						29781	33719	38526	40012		8.20E+05
⑦								33891	39045		14.24E+05
⑧											14.20E+05
⑨											14.02E+05
⑩											8.33E+05



Note: The angles indicate the angles of rotation of the plastic hinge. The hinges at the joints can be in the beam or column

Fig. 3.1 Beam Mechanism for a Portal Frame



Note: The angles indicate the angles of rotation of the plastic hinge.

Fig. 3.2 Sway Mechanism for a Portal Frame

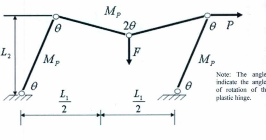


Fig. 3.3 Combined Failure Mechanism for a Portal Frame

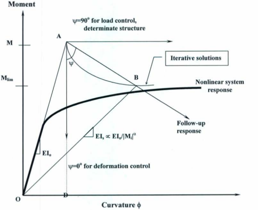
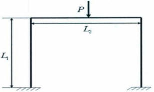


Fig. 3.4 Direct Secant Estimation of Plastic Limit Load [Adluri, 1999]



P(N)	800
L_1(mm)	400
L_2(mm)	400
A(mm^2)	50
I(mm^4)	266. 67
σ_y(MPa)	300
E(N/mm^2)	200,000

Fig. 3.5.1 Portal Frame Subject to Concentrated Vertical Force

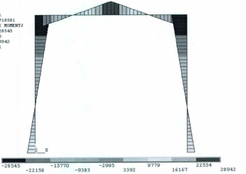


Fig. 3.5.2 Nonlinear Analysis Results for the Portal Frame in Fig. 3.5.1

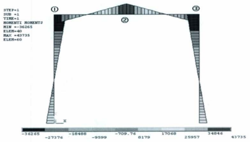
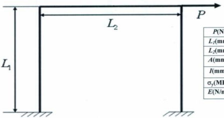


Fig. 3.5.3 Direct Secant Method Results from Reanalysis

for the Problem in Fig. 3.5.1



P(N)	720
L_1(mm)	400
L_2(mm)	400
A(mm^2)	50
I(mm^4)	266. 67
σ_y(MPa)	30000
E(N/m^2)	20000

Fig. 3.6.1 Portal Frame Subject to Lateral Force

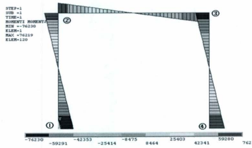
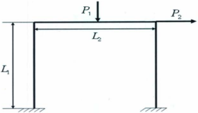


Fig. 3.6.2 Direct Secant Results from Reanalysis for the Problem in Fig. 3.6.1



P_1(N)	800
P_2(N)	720
L_1(mm)	400
L_2(mm)	400
A(mm^2)	50
I(mm^4)	266. 67
σ_y(MPa)	30000
E(N/mm^2)	200,000

Fig. 3.7.1 Portal Frame Subject to Both Lateral and Vertical Forces

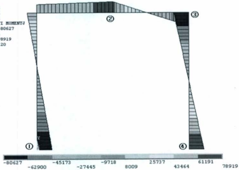
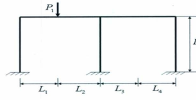


Fig. 3.7.2 Direct Secant Results from Reanalysis for the Problem in Fig. 3.7.1



P_1(N)	600
L_1(mm)	200
L_2(mm)	200
L_3(mm)	200
L_4(mm)	200
L_5(mm)	400
A(mm^2)	50
I(mm^4)	266.67
σ_y(MPa)	300
E(N/m^2)	200,000

Fig. 3.8.1 Two-bay Single-storey Frame -A

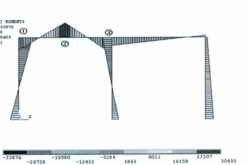
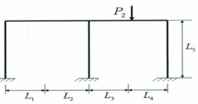


Fig. 3.8.2 Direct Secant Results from Reanalysis for the Problem in Fig. 3.8.1



P_2(N)	720
L_1(mm)	200
L_2(mm)	200
L_3(mm)	200
L_4(mm)	200
L_5(mm)	400
A(mm^2)	50
I(mm^4)	266. 67
σ_y(MPa)	300
E(N/mm^2)	200,000

Fig. 3.9.1 Two-bay Single-storey Frame -B

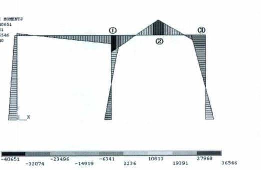
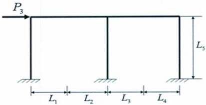


Fig. 3.9.2 Direct Secant Results from Reanalysis for the Problem in Fig. 3.9.1



P_3(N)	500
L_1(mm)	200
L_2(mm)	200
L_3(mm)	200
L_4(mm)	200
L_5(mm)	400
A(mm^2)	50
I(mm^4)	266. 67
σ_y(MPa)	300
E(N/mm^2)	200,000

Fig. 3.10.1 Two-bay Single-storey Frame -C

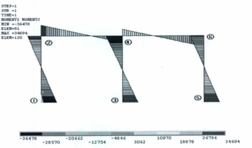
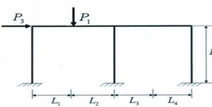


Fig. 3.10.2 Direct Secant Results from Reanalysis for the Problem in Fig. 3.10.1



P_1(N)	600
P_3(N)	500
L_1(mm)	200
L_2(mm)	200
L_3(mm)	200
L_4(mm)	200
L_5(mm)	400
A(mm^2)	50
I(mm^4)	266. 67
σ_y(MPa)	300
E(N/mm^2)	200,000

Fig. 3.11.1 Two-bay Single-storey Frame -D

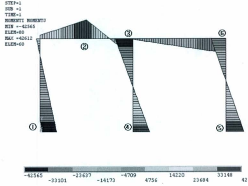
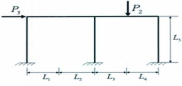


Fig. 3.11.2 Direct Secant Results from Reanalysis for the Problem in Fig. 3.11.1



P_2(N)	720
P_3(N)	500
L_1(mm)	200
L_2(mm)	200
L_3(mm)	200
L_4(mm)	200
L_5(mm)	400
A(mm^2)	50
I(mm^4)	266. 67
σ_y(MPa)	300
E(N/mm^2)	200,000

Fig. 3.12.1 Two-bay Single-storey Frame -E

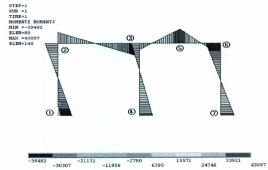
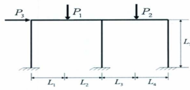


Fig. 3.12.2 Direct Secant Results from Reanalysis for the Problem in Fig. 3.12.1



P_1(N)	600
P_2(N)	720
P_3(mm)	500
L_1(mm)	200
L_2(mm)	200
L_3(mm)	200
L_4(mm)	200
L_5(mm)	400
A(mm^2)	50
I(mm^4)	266. 67
σ_y(Mpa)	300
E(N/mm^2)	200000

Fig. 3.13.1 Two-bay Single-storey Frame -F

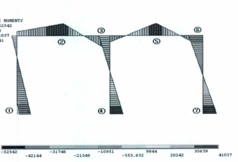
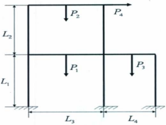


Fig. 3.13.2 Direct Secant Results from Reanalysis for the Problem in Fig. 3.13.1



P_1(N)	4000
P_2(N)	4000
P_3(N)	2500
P_4(N)	2500
L_1(mm)	150
L_2(mm)	150
L_3(mm)	120
L_4(mm)	75
A(mm^2)	50
I(mm^4)	266. 67
σ_y(MPa)	300
E(N/mm^2)	200,000

Fig. 3.14.1 Two-bay Two-storey Frame

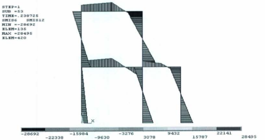


Fig. 3.14.2 Nonlinear Analysis Results for the Problem in Fig. 3.14.1

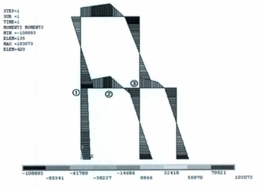
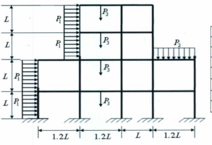


Fig. 3.14.3 Direct Secant Results from Reanalysis for the Problem in Fig. 3.14.1



Properties:

L_1 (mm)	150
P_1(N)	10
P_2(N)	15
P_3(N)	10
A(mm^2)	50
I(mm^4)	266. 67
σ_y(MPa)	300
E(N/mm^2)	200,000

Fig. 3.15.1 Multistoried Frame Subject to Concentrated and Distributed Loads

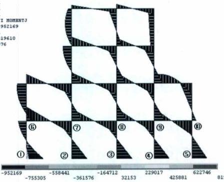
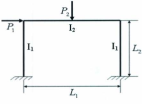


Fig. 3.15.2 Direct Secant Results from Reanalysis for the Problem in Fig. 3.15.1

Non-uniform Portal Frames

Case1: Beam is stronger than both the columns.



Properties:

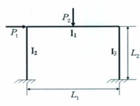
P_1(N)	250000
P_2(N)	250000
L_1(mm)	1200
L_2(mm)	800
I_1(mm^4)	900000
I_2(mm^4)	4166667
I_3(mm^4)	11433333
A_1(mm^2)	3000
A_2(mm^2)	5000
A_3(mm^2)	7000
σ_y(MPa)	300
E(N/mm^2)	200,000

Fig. 3.16.1 Non-Uniform Portal Frame Subject to Lateral and Vertical Forces for Case 1

Table 3.2 Load Factors for the problem in the Fig. 3.16.1

	Direct secant method	Nonlinear Analysis	Error
Beam Mechanism	0.792	0.812	2.46%
Sway Mechanism	0.316	0.316	0%
Combined Mechanism	0.432	0.436	0.92%

Case 2: Columns are stronger than the beam



Properties:

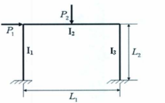
P_1(N)	250000
P_2(N)	250000
L_1(mm)	1200
L_2(mm)	800
I_1(mm^4)	900000
I_2(mm^4)	4166667
I_3(mm^4)	11433333
A_1(mm^2)	3000
A_2(mm^2)	5000
A_3(mm^2)	7000
σ_y(MPa)	300
E(N/mm^2)	200,000

Fig. 3.16.2 Non-Uniform Portal Frame Subject to Lateral and Vertical Forces for Case 2

Table 3.3 Load Factors for the problem in the Fig. 3.16.2

	Direct secant method	Nonlinear Analysis	Error
Beam Mechanism	0.420	0.416	0.96%
Sway Mechanism	0.804	0.828	2.89%
Combined Mechanism	0.552	0.536	2.99%

Case 3: Beam is stronger than one column and weaker than the other



Properties:

P_1(N)	250000
P_2(N)	250000
L_1(mm)	1200
L_2(mm)	800
I_1(mm^4)	900000
I_2(mm^4)	4166667
I_3(mm^4)	11433333
A_1(mm^2)	3000
A_2(mm^2)	5000
A_3(mm^2)	7000
σ_y(MPa)	300
E(N/mm^2)	200,000

Fig. 3.16.3 Non-Uniform Portal Frame Subject to Lateral and Vertical Forces for Case 3

Table 3.4 Load Factors for the problem in the Fig. 3.16.3

	Direct secant method	Nonlinear Analysis	Error
Beam Mechanism	0.980	1.004	2.39%
Sway Mechanism	0.804	0.816	1.47%
Combined Mechanism	0.744	0.764	2.62%

Chapter 4

Estimations for Frame Stability

4.1 Introduction

Stability is a major design consideration for structural columns, frames and allied structures subject to compression or compression and bending in some combination. The elastic stability theory for columns is well known and is extensively taught as part of engineering curriculum (see, e.g., Hibbeler [2008]). Some of it is reviewed in Chapter 2. The theoretical bases for stability theory are found in several well known references, e.g., Timoshenko and Woinowsky-Krieger [1975]. Stability of frames is explored by others such as Chajes [1993], etc. Using the finite element analysis for frame stability has gained popularity. The procedures are described by many authors (e.g., Chen and Lui [1991]). Using the finite element method usually involves full nonlinear analysis with varying degree of adjustments or adaptations for joint flexibility and other issues.

In this chapter, these routes are bypassed in favour of a simple attempt to explore the possibility of obtaining the critical load capacities for frame stability by using algorithms based on methods in Chapter 3. It must be clearly noted that this is only a rudimentary attempt and hence theoretical rigor for full investigation is beyond its scope.

4.2 *Observations from Elastic Buckling Theory*

We can gather several observations from the theory of elastic buckling that are useful for the present study.

1. If a perfectly straight column member is applied a small concentric axial force, it will have only a small compressive displacement along the column and the column remains straight. The column at this time is in stable equilibrium. Lateral displacement caused by a disturbing lateral force (or moment) at this state will vanish when the disturbing force is removed and the column still keeps its straightness. As the axial load increases, a small lateral disturbing force can cause increasingly larger lateral deflections. These will suddenly cause a huge lateral deflection at a particularly high enough axial force. At this stage, the lateral deflection of the column can not recover to original value even if the disturbing force is removed. The column at this time is in an unstable state. The axial force that causes this is very specific and is called as critical (or "buckling") load.

2. One of the key observations from elastic buckling analysis is that the flexural stiffness that resists bending is progressively weakened with the increase in the axial force in the column. The stiffness eventually reaches negligible levels when the axial force reaches the critical (or buckling) load. It is postulated that this loss of stiffness may be simulated to some extent using ideas from secant stiffness modification from Adluri [2001] and used in Chapter 3.

3. The same observations as above apply to columns that are part of a frame. Such frames will also have "buckling" loads the same way that columns do. This is the case whether the frames permit or prevent side sway. These frames can have

different types of connections between beams and columns. The column stability will depend upon the effective end restraint. The beams and their connections will act as partial end restraints to the columns thus modifying the effective length of the columns and their buckling capacities.

4. For frames, the lateral deflection in columns may be produced by axial forces, lateral forces and/or bending moments. We can call the deflection caused by lateral forces and direct bending moments as the primary deflection while the deflection produced by axial forces as the secondary deflection. This "secondary" deflection in fact is more significant for the current chapter.

5. Fig. 4.1 shows the diagram of a segment of an isolated column. If we take the moment equilibrium,

$$Qdx + Pdy + M - (M + \frac{dM}{dx}dx) = 0$$

$$Q = \frac{dM}{dx} - P\frac{dy}{dx} \qquad (4\text{-}1)$$

If the lateral force Q is ignored,

$$dM = Pdy \qquad (4\text{-}2)$$

Therefore, if lateral force effects are removed, the moment increment in the column is equal to the product of axial force and displacement increment at that location.

6. The lateral forces such as Q in Fig. 4.1 cause lateral displacements. The displacement patterns due to these loads may not be the same as the displacement

patterns at the time of column or frame buckling. Therefore, the lateral loads or moments may have the max displacement at one location while the buckling effect may have the maximum displacement at a different location.

7. Just prior to buckling, the maximum displacement of each column can be considered to be the critical displacement. This implies that if any displacement exceeds the critical displacement, failure will occur. We reiterate that as mentioned above, when buckling occurs, the huge displacement location is not exactly the same as the critical displacement location. However, these displacements are most likely to be close to each other. This can be shown in the reanalysis of robust secant method, which will be discussed later.

4.3 Inspiration from Robust Plastic Limit Load Analysis

In addition to the observations above, we can use some of the ideas gained in Chapter 3 to try and simulate frame buckling.

1. We will assume that the frame buckling for this situation does not follow the classical mode of "bifurcation" or a sudden change in status. Rather, the buckling is a transition from more or less linear elastic deformation (involving both axial and lateral displacements of members) to a relatively very large set of lateral displacements due to instability. The magnitude of this "large" displacement is not of as much importance as the fact that it took place at all. This is somewhat similar to the fact that at plastic limit loads, the curvature of the beam at critical segments reaches very large values (its magnitude is not important –rather that it took place at all).

2. The relationship between applied loads and the displacement at critical location is somewhat analogous to that between moment and curvature of the critical cross-section in Chapter 3. Both the relationships have initially linearly increasing portions which turn to large horizontal portions (with little increase in effective load). An approximate load-displacement curve for buckling is shown in Fig. 4.2. It is of course neglecting a realistic post buckling drop in the buckling load and consequent changes in the structure behaviour. It also neglects the nonlinear transition that usually precedes the onset of buckling. The figure (although highly idealized) is similar to the moment-curvature relationship from Chapter 3 (Fig. 3.2).

3. The plastic limit load estimation of Chapter 3 relies on a secant modification of the rigidity to simulate the redistribution of bending moments caused by the onset of yielding. The bending moment re-distribution relies on the relative secant rigidity of different parts of the frame. Similarly, *we can postulate* that the stability of frames is influenced by "effective secant stiffness." Just as in Chapter 3, the secant modification here can try and give all the parts of the frame an equal "opportunity" to buckle. Schematically, this modification is shown in Fig. 4.2.

   It must be noted that the word "secant stiffness" is used loosely here. It is not exactly the same as that used in iterative modified Newton schemes for nonlinear matrix analysis.

4. Just as we obtain locations of peak redistributed moments in Chapter 3, we can try to obtain locations of peak displacements in the structure whose stiffness is modified by the scheme shown in Fig. 4.2. These peak displacements can

potentially be indicative of frame instability.

4.4 Robust "Secant" Analysis Trials for Elastic Buckling

Using the observations above, we can attempt to investigate the elastic buckling of frames. *We repeat that this is a very preliminary idea and needs to be thoroughly analyzed before any further development.*

It is postulated that frames exhibit "buckling" behaviour when a certain critical displacement from simple elastic analysis exceeds a certain limit.

In the following, empirical procedures are proposed for estimating the critical load for a column member in a frame.

1. For a set pattern of applied loading <*P, M*> and an arbitrary load multiplication factor λ, we carry out an initial elastic analysis. This gives us the general displacement pattern for the structure which we assume increases monotonically with load multiplication factor until one or more members or the frame as a whole decides to buckle. As mentioned earlier, even at buckling, the displacement is *not* assumed to "bifurcate". It rather increases significantly at some points thus preventing further possibility of increase in load multiplication factor.

2. In Fig. 4.2, the line OA represents the linear elastic relationship between applied load and displacement. It is obtained based upon the initial stiffness (slope) K_1. As the load increases, the stiffness remains relatively constant while the corresponding displacement increases proportionately. This continues until a critical displacement d_{cr} is reached at point A. The tangent stiffness becomes

effectively very low thereafter. (We are, as mentioned above, neglecting a transition zone near point A). Let us assume that the load applied is F_1 and that it produces a displacement d_1 based on initial stiffness K_1. In order to simulate buckling, we can modify the stiffness of the segment using the secant line shown in Fig. 4.2 (similar to that used in Chapter 3).

$$I_{new} = \frac{d_{arb}}{d} I_{old} \tag{4-3}$$

where, I is the moment of inertia for the segment under consideration. The value of d_{arb} is used since the value of d_{cr} is not actually known and since any reasonable value will serve the purpose of secant adjustment of stiffness. This is inline with the procedure from Adluri [1999] and that in Chapter 3. Note that we are modifying moment of inertia which is proportional to the stiffness of the member. The new moment of inertia tries to increase the stiffness of those areas which produced less displacement and relatively "reduces" the stiffness of those areas that produced more displacement thus encouraging all segments towards "buckling."

Re-analysis of the frame is performed on the basis of the modification above. The locations of peak displacements are obtained. These locations might have shifted from the corresponding ones in the previous analysis (analogous to the situation in Chapter 3).

3. After re-analysis, the critical load can be calculated by one of the following two *empirical methods*. The first method obtains the load factor corresponding to critical load from the ratio of two average displacements. The second method uses the ratio of moment integral and displacement integral. Both the methods

are illustrated below:

4.4.1 Method 1 for Critical Loads

Let $P_{compr.total}$ be the total compressive force in the columns, $M_{disp\text{-}peak}$ be the bending moment at the location of peak displacement and C be a factor depending on the type of frame. Compute an "equivalent" displacement d_{mpc} as below. There may be more than one peak locations. Note that the subscript "mpc" is not to be confused with the multi-point constraint used in several commercial software packages such as ANSYS and ABAQUS.

$$d_{mpc} = C\frac{M_{disp-peak}}{P_{compr\ total}} \quad (4\text{-}4)$$

The load factor λ for the critical load P_{cr} is obtained from the peak displacements,

$$\mathbf{P_{cr}} = \mathbf{P}\,\lambda, \quad \lambda = \frac{1}{n}\frac{\sum_{i=1}^{n} d_{mpc}}{\sum_{i=1}^{n} d_{peak}}. \quad (4\text{-}5)$$

where, d_{peak} is the peak displacement in the reanalysis.

The factor C depends on the type of frame irrespective of the actual physical property values. We do not yet have the necessary theoretical development to determine the factor exactly. Pending the development of the theory, it has been decided to empirically estimate the factor using finite element geometric nonlinear analysis. The factor C is likely to be a function of certain key non-dimensional properties of the frame. However, for the present study, it has been assumed to be a constant.

Fig. 4.3 shows results for the factor C for portal frames with different aspect ratios, where, L_1 is the length of column and L_2 is the length of the beam. The maximum value of C is 0.817 when $\frac{L_2}{L_1}=1$ and the minimum value of C is 0.461 when $\frac{L_2}{L_1}=2$. The curve shows little difference when $\frac{L_2}{L_1}\leq 1$ and it floats around a constant value. It implies that a constant value can be used to cover the changing of this portion of the curve. This constant value is selected to be 0.72.

Fig. 4.4 shows the comparison between the load factor λ for nonlinear analysis and that for robust secant analysis when C=0.72. In the figure, beams are shorter than columns ($\frac{L_2}{L_1}\leq 1$). The solid line denotes engenvalue analysis and the dashed line denotes robust secant analysis. It must be pointed out that the factor C is being *calibrated using the "exact" analysis with the expectation that such factors will be investigated in further work and procedures for finding them will be developed without recourse to the back-calibration used here.*

4.4.2 Method 2 for Critical Loads

In empirical method 2, after reanalysis, the critical load is calculated as below:

1. Identify the locations of peak displacements and the nearby locations of zero displacements. Find the bending moments and displacements at these locations.

2. Calculate

$$\Delta M = M_{dcr} - M_0 \text{ and } \Delta d = d_{cr} - d_0 \qquad (4\text{-}6)$$

where, d_{cr} is the peak displacement in reanalysis, d_0 is the zero displacement, M_{dcr} is bending moment at the peak displacement location and M_0 is the bending moment at zero displacement location.

3. Estimate the critical load by using the equation

$$P_{cr} = \beta \left| \frac{\Delta M}{\Delta d} \right|, \tag{4-7}$$

The factor β is analogous to the factor C in Method 1. It is calibrated using the same nonlinear analysis as used in Method 1.

Fig. 4.5 investigates the factor β for portal frames subject to a vertical force on the beam, where, L_1 is the length of columns and L_2 is the length of beam. The maximum value of β is 0.2581 for $L_2/L_1=1$ and the minimum value of β is 0.1378 when $L_2/L_1=2$. The curve shows little difference when $L_2/L_1<=1$ and it floats around a constant value. It implies that a constant value can be used to cover the range $L_2/L_1<=1$. This constant value is selected to be 0.22.

Fig. 4.6 shows the comparison of the load factor between geometrical nonlinear analysis and robust secant analysis Method 2. The solid line in the figure represents the theoretical load factor for a portal frame with different aspect ratios. The dashed line denotes the curve for robust secant analysis when β=0.22. These two lines are close and do not show much difference when $0.277 \le \frac{L_2}{L_1} \le 1$. The maximum error shown in the figure is 9%.

4.4.3 Comparison Between Method 1 and Method 2

It is obvious that Fig. 4.3 and Fig.4.5, show general trends that are proportional to each other. The value d_{peak} in Method 1 is equal to Δd in Method 2. The value M_{dpeak} in Method 1 is proportional to ΔM in Method 2 because $\Delta M = M_{dpeak} - M_{d0}$. The difference between the factor β and the factor C comes from the difference between M_{dpeak} and ΔM.

Although Method 1 and Method 2 are obviously closely related, the intent of ΔM in Method 2 is not the same as the value of M_{peak} in Method 1. The use of ΔM was meant to represent the total area of bending moment diagram between the peak location and the zero location thus indicating "tripping" per unit displacement that causes buckling. Since the use of an integral is somewhat cumbersome at this stage, it was decided that a simple "change of value" will be used instead. This needs further investigation.

It must also be pointed out that we used the factors C and β as constants (for the range of aspect ratios selected for the present study). They are not actually constants as can be seen from Figs. 4.3 and 4.5. In fact for $L_2 > L_1$, these factors seem to be linearly changing. *This needs further investigation and theoretical development.*

4.5 Results

Several portal frame problems have been analyzed using empirical Methods 1 and 2 described above as can be seen from the points represented on Figs. 4.3-6.

The results of some of those analyses are presented in Figs. 4.7 to 4.13. For the analyses, the following physical data was used (Table.4.1). Although the data was needed to be given to ANSYS in order to obtain numerical results, the ideas presented can easily be shown to be non-dimensional in their nature.

Table 4.1 The Physical Data Used in Analyzing the Structures in Figs. 4.7 to 4.13

P	**20,000**
L_1	**1200**
L_2	**800**
L_3	**400**
b	**8**
h	**10**
E	**200,000**

The analyses include portal frame with one load, two loads and three loads on the beam as well as one lateral (horizontal) force parallel to the beam to simulate the side sway due to wind. The frames cover different aspect ratios.

As can be seen from the results presented, the errors from the proposed analyses are reasonably small in comparison with the results of geometric nonlinear analysis. It must be noted that the geometric nonlinear analysis results are ever so slightly different from those in Figs. 4.3-6 since those use the matrix eigen value analysis. However the difference is within tolerable margins. Typical input files for ANSYS analyses are included in the Appendices.

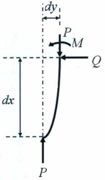


Fig. 4.1 Segment of an Isolated Column

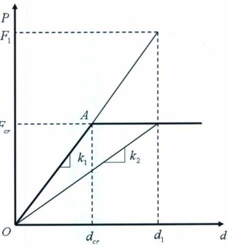


Fig. 4.2 Direct Secant Modification for Stiffness

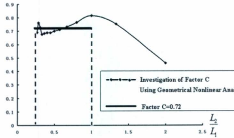


Fig. 4.3 Factor C for Portal Frames with Fixed Ends in Method 1

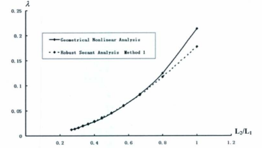


Fig. 4.4 Comparison between Geometrical Nonlinear Analysis and Robust Secant Method 1 for C=0.72

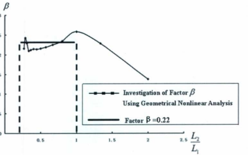


Fig. 4.5 Factor β for Portal Frames with Fixed Ends in Method 2

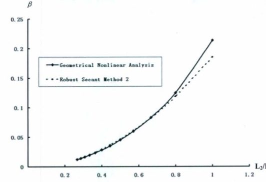
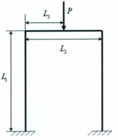
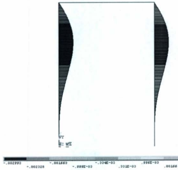


Fig. 4.6 Comparison between Geometrical Nonlinear Analysis and Robust Secant Method for β =0.22



Portal Frame Case 1	
PL_1^2/EI	21.6
L_2/L_1	0.667
L_3/L_2	0.5
C	0.72

Fig. 4.7a Portal Frame with Concentrated Force on the Beam Case 1 (L2 < L1)



d_{peak}/L_1 (10^6)	2.49
M_{peak}/PL_1 (10^6)	2.83
P_{cr} load factor (Geometric Nonlinear Anal.)	0. 830
P_{cr} load factor (Robust Secant method.)	0. 811
Error	2.3%

Fig. 4.7b Displacement Distribution in Reanalysis for the Portal Frame in Case1

(negative for the left column and positive for the right column)

Portal Frame Case 2:

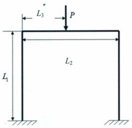


Fig. 4.8a Portal Frame with Concentrated Force on the Beam Case 2 (L2 = L1)

Properties

$\frac{PL_1^2}{EI}$	$\frac{L_2}{L_1}$	$\frac{L_3}{L_2}$
96	**0.5**	**0.5**

Result from Reanalysis

$\frac{d_{peak}}{L_1}$	$\frac{M_{peak}}{PL_1}$
8.41e-5	**2.08e-5**

Estimate of Critical Load

C	**Geometrical Nonlinear Analysis**	**Robust Secant Load factor**	**error**
0.72	**0.2135**	**0.1776**	**16.8%**

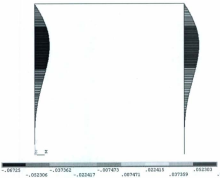


Fig. 4.8b Displacement Distribution in Reanalysis for the Portal Frame in Case2

(negative for the left column and positive for the right column)

Portal Frame Case 3:

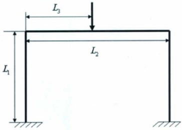


Fig.4.9a Portal Frame with Concentrated Force on the Beam Case 3 (L2 > L1)

Properties

$\frac{PL_1^2}{EI}$	$\frac{L_2}{L_1}$	$\frac{L_3}{L_2}$
54	1.33	0.5

Result from Reanalysis

$\frac{d_{peak}}{L_1}$	$\frac{M_{peak}}{PL_1}$
1.99e-4	8.92e-5

Estimate of Critical Load

C	Theoretical Load factor	Robust Secant Load factor	error
0.72	0.3372	0.3226	4.3%

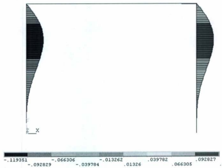


Fig. 4.9b Displacement Distribution in Reanalysis for the Portal Frame in Case3

(negative for the left column and positive for the right column)

Portal Frame Case 4

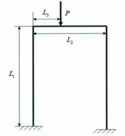


Fig. 4.10a Portal Frame with Concentrated Force on the Beam Case 4 (L2 < L1)

Properties

$\frac{PL_1^2}{EI}$	$\frac{L_2}{L_1}$	$\frac{L_3}{L_2}$
4.8	**0.5**	**0.33**

Result from Reanalysis

peak displacement①	① $\frac{d_{peak}}{L_1}$	① $\frac{M_{peak}}{PL_1}$
	3.89e-4	**8.88e-4**
peak displacement②	② $\frac{d_{peak}}{L_1}$	② $\frac{M_{peak}}{PL_1}$
	4.21e-3	**1.58e-3**

Estimate of Critical Load

C	Theoretical Load factor	Robust Secant Load factor	error
0.72	0.3707	0.3911	5.2%

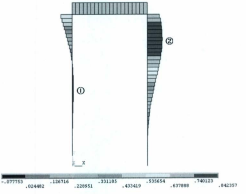


Fig. 4.10b Displacement Distribution in Reanalysis for Portal Frame in Case4

(negative for the left column and positive for the right column)

Portal Frame Case 5

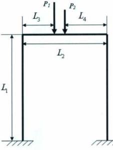


Fig. 4.11a Portal Frame with Two Forces on the Beam -Case 5 (L2 < L1)

Properties

$\frac{P_1L_1^2}{EI}$	$\frac{P_2L_1^2}{EI}$	$\frac{L_2}{L_1}$	$\frac{L_3}{L_2}$	$\frac{L_4}{L_2}$
38.4	28.8	0.75	0.3	0.5

Result from Reanalysis

peak displacement①	① $\frac{d_{peak}}{L_1}$	① $\frac{M_{peak}}{PL_1}$
	3.89e-4	8.88e-4
peak displacement②	② $\frac{d_{peak}}{L_1}$	② $\frac{M_{peak}}{PL_1}$
	4.21e-3	1.58e-3

Estimate of Critical Load

C	Theoretical Load factor	Robust Secant Load factor	error
0.72	0.2824	0.2858	1.2%

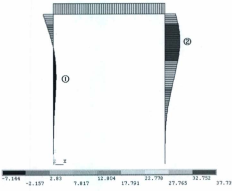


Fig. 4.11b Displacement Distribution in Reanalysis for Portal Frame in Case5

(negative for the left column and positive for the right column)

Portal Frame Case 6

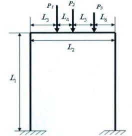


Fig. 4.12a Portal Frame with Three Forces on the Beam –Case 6 (L2 < L1)

Properties

$\frac{P_1L_1^2}{EI}$	$\frac{P_2L_1^2}{EI}$	$\frac{P_3L_1^2}{EI}$	$\frac{L_2}{L_1}$	$\frac{L_3}{L_2}$	$\frac{L_4}{L_2}$	$\frac{L_5}{L_2}$	$\frac{L_6}{L_2}$
15.36	11.52	9.22	0.63	0.3	0.2	0.3	0.2

Result from Reanalysis

peak displacement①	① $\frac{d_{peak}}{L_1}$	① $\frac{M_{peak}}{PL_1}$
	8.18e-3	**6.85e-3**
peak displacement②	② $\frac{d_{peak}}{L_1}$	② $\frac{M_{peak}}{PL_1}$
	1.24e-2	**7.82e-3**

Estimate of Critical Load

C	Theoretical Load factor	Robust Secant Load factor	error
0.72	0.49	0.52	5.2%

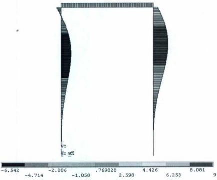


Fig. 4.12b Displacement Distribution in Reanalysis for Portal Frame in Case 6

(negative for the left column and positive for the right column)

Portal Frame Case 7

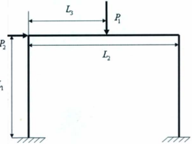


Fig. 4.13a Portal Frame with a Force on the Beam and a lateral force -Case 7

Properties

$\frac{P_1 L_1^2}{EI}$	$\frac{P_2 L_1^2}{EI}$	$\frac{L_2}{L_1}$	$\frac{L_3}{L_2}$
38.4	9.6	1.5	0.5

Result from Reanalysis

$\frac{d_{peak}}{L_1}$	$\frac{M_{peak}}{PL_1}$
0.21	0.11

Estimate of Critical Load

C	Theoretical Load factor	Robust Secant Load factor	error
0.72	0.3247	0.3011	7.3%

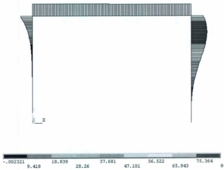


Fig. 4.13b Displacement Distribution in Reanalysis for Portal Frame in Case 7

(negative for the left column and positive for the right column)

Chapter 5

Estimation of Large Deflections in Beams

5.1 Introduction

The study of large deflections is a vast and complicated subject. It has many applications and is used directly and indirectly in several design situations. A very preliminary attempt at estimating large deflections of beams or beam type structures using inspirations from direct secant analysis is made in this chapter.

Total deflection is the vector sum of the deflection in all directions. There are many indicators that can be used to identify if the problem of interest needs large deflection considerations, for example, if the value of the rotation θ of beams cannot be easily replaced with $\sin\theta$ or vice versa. In a design situation, typically, a deflection in excess of "Span/120" is considered exceeding small deflection limit. When large deflection takes place, structural members deviate significantly from their original positions. This deviation is considered to be large enough so that the equations formed on the original geometry are no longer valid for a good estimate of the behaviour.

In this chapter, the large deflection of beams is investigated using Euler-Bernoulli beam theory, linear and nonlinear FEA and robust secant analysis used in earlier chapters. Three cases of beam, viz., cantilever beam, simply supported beam with a roller and simply supported beam without rollers.

5.2 Euler-Bernoulli Beam

5.2.1 Linear Theory

Elementary Euler-Bernoulli beam equation is well known and is widely taught in engineering curriculum (see, e.g., Hibbeler [2008]). The basic assumption is that the beam section that is originally plane remains plane after bending. The popularly known form of the equation assumes that the beam is isotropic. It relates the applied load and the deflection caused by the load. With reference to beams of the kind from Fig. 5.1, the equation is expressed as:

$$\frac{d^2}{dx^2}(EI\frac{d^2y}{dx^2}) = w, \quad EI\frac{d^2y}{dx^2} = M(x), \quad I = \int\int y^2 \cdot dy \cdot dz \tag{5-1}$$

where, E is the material modulus, I is the moment of inertia of the cross section at any x, y(x) is the deflection at any point x, M(x) is the bending moment, and w(x) is the intensity of the distributed load at x. The equations are derived using the following assumptions:

1. Beams have small deflections. The stresses, except bending stresses, are negligible. Beams behave elastically.

$$x = -\theta = -\frac{dy}{dx} = -\sin\theta \tag{5-2}$$

2. The stress resultants Bending moment and Shear force are given by,

$$M(x) = \int\int y \cdot \sigma(x, y) \cdot dy \cdot dz \tag{5-3}$$

$$V(x) = \int\int \sigma_{xy} \cdot (x, y) \cdot dy \cdot dz \qquad (5\text{-}4)$$

Since the main assumption is that plane cross-sections remain plane after deformation, the shear effect on the deflection is not included.

5.2.2 Modified Euler-Bernoulli Equation

If beams experience large deflection, the linear beam theory illustrated above is not entirely correct since the magnitude of θ is no longer the same as $\frac{dy}{dx} = \sin\theta = \tan\theta$.

A cantilever beam subject to a concentrated force at the free end is shown in Fig. 5.1. The vertical force P causes vertical displacement d_y, horizontal displacement d_x and rotation θ of the beam. The total deflection d is the vector sum of the horizontal displacement d_x and vertical displacement d_x. $EI\frac{d\theta}{dx} = M(x)$ is used to evaluate internal bending moment (note that $\theta \neq \frac{dy}{dx}$ in the case of large deflection). External bending moment $M(x)$ is not equal to $P(L-x)$ but to $P(L\text{-}x\text{-}\delta_x)$, where δ_x is the shortening of length in the horizontal direction.

The modified form of Euler-Bernoulli equation is then given by

$$EI\frac{v''}{\left[1+(v')^2\right]^{3/2}} = M(x) \qquad (5\text{-}5)$$

In the above, the curvature term is no longer $v'' = d^2y/dx^2$. Instead, it became $v''/\left[1+(v')^2\right]^{3/2}$. The bending moment $M(x)$ on the other side of the equation should include the effect of changing length.

5.3 Scope of the Analysis

The beams analyzed in this chapter have deflections that range from 5% to nearly 50% of the beam length. It is assumed that the concentrated forces applied on the beams maintain their original direction, even if beam orientation is altered significantly due to large deflection. It must be noted that the calculation is for "total" deflection and not simply the vertical deflection. The following assumptions are applicable for the investigation:

1. The material is isotropic and homogeneous
2. The material is relatively soft. Material failure or plasticity does not govern the deflections.
3. The calculation is for "total" deflection and not simply the vertical deflection.
4. Shear effect is negligible. Only bending stresses and axial stresses govern the deflection. Axial stresses may develop if the cross-Section rotation is significant.
5. Cross-sections maintain their original shapes.
6. Lateral buckling is not considered.

The total deflection is a vector sum of vertical and horizontal deflections, as mentioned earlier. The horizontal deflections are influenced by change in length as well as the curvature of the beam. For simple cases, the change in length can be shown to be

$$\delta_{axial} = \int_0^x \frac{Pv'}{AE} dx \tag{5-6}$$

For a cantilever, the integral evaluates to

$$\delta_{axial} = \frac{Pv(x)}{AE} = \frac{P}{AE}\frac{PL^2}{6EI}\left[1 - 3\frac{x}{L} - \left(1 - \frac{x}{L}\right)^2\right] \tag{5-7}$$

Similarly, we can find the horizontal displacement u(x) due to curvature of the beam using,

$$u_{curvature} = \int \frac{dx}{\sqrt{1 + (v')^2}} \tag{5-8}$$

All these equations (e.g., eq. 5.2.5) can be solved using well known numerical methods such as Runge-Kutta (Order 4), etc., in combination with "shooting" algorithms as demonstrated by Adluri [2009]. Application of such methods becomes complicated if the deflections are being computed for more elaborate cases such as frames. In the following, finite element models have been used to run linear, nonlinear and robust algorithms to determine total deflections of beam structures.

5.4 The Direct Secant Technique

The direct secant technique presented earlier by Adluri [2001], Bolar and Adluri [2005], etc., has been used in Chapter 3 for plastic limit load estimation. Similar ideas are employed in Chapter 4 in a preliminary effort at estimating buckling loads of frames.

Fig. 5.2 describes the basic idea of secant analysis. The initial linear analysis predicts stress and strain based on the original elastic modulus. If the stress exceeds a set value (such as the yield limit), the material modulus is altered using a secant line. A reanalysis is carried out and the process in continued iteratively till convergence.

The secant modulus is updated iteratively as

$$E_{i+1} = \frac{\sigma_y}{\sigma_{ei}} E_i \,. \tag{5-9}$$

The direct secant analysis of Adluri [2001] may not change the modulus. Instead, it may change any item that is directly proportional to secant stiffness such as moment of inertia, rigidity, or even the stiffness matrix itself [Laha and Adluri, 2005]. In that sense, secant stiffness for the next iteration can be expressed as

$$K_1 = \eta K_0 \,, \tag{5-10}$$

where, K_0 is the original stiffness, K_1 is the new stiffness and η is the modification factor.

The analysis input files for ANSYS are given in the Appendices.

5.4.1 Cantilever Beam

A cantilever beam subject to a concentrated load at the free end is shown in Fig. 5.1. This beam is analyzed for linear and nonlinear cases. The algorithms used in Chapter 4 have been employed for obtaining the robust secant analysis results. The results are shown in Fig. 5.3. The plots are non-dimensionalized for the maximum deflection that we can expect at the tip of a cantilever of we use the simple linear theory (the slope of the results of the liner theory on this graph should be 1:1). The robust analysis results are fairly close to those from full nonlinear analysis. Up to a deflection of about 25% of the length of beam, there is little error. The error starts to increase when the deflection increases beyond half the length of the beam. It is deemed that this is probably the most that we can expect in a normal structural application even for relatively "soft" materials. It is to be noted that the deflection includes the shortening of the lever arm as well as the tensile component of the applied load at large deflections.

Fig. 5.4 shows the comparison between the nonlinear results, linear results and a simplified algorithm which considers the secant modification η in Eq. 5.4.2 to be a constant value of 1.05. From Figs. 5.3 and 5.4, we can find that the simplified procedure virtually gives the same results as those from detailed secant analysis. The good fit seems to be mainly because the nonlinear results are relatively linear even at high displacements. Also, somewhat unexpectedly, the nonlinear deflection is smaller than that from linear theory

5.4.2 Simply Supported Beam with a Roller Support

A simply supported beam with a roller support on one end is shown in Fig. 5.5. A concentrated force is applied at the mid span of the beam. The roller on the simply supported beam, as is clear, only permits the horizontal displacement and restricts the vertical movement. When large deflection occurs at the mid span, the total displacement is influenced by both the vertical deflection and horizontal movement of the beam.

The (normalized) results are shown in Fig. 5.6. For this case, unlike the cantilever case, the nonlinear deflection is larger than that from linear theory. The secant analysis approximates the nonlinear result fairly closely.

For this case also, a simplified approach was attempted similar to that used in the case of cantilever. *The factor η was found to be 0.95.* The results for this simplified approach are virtually identical to those from secant analysis. At this stage, it is unknown why the simplified approach works except for the obvious reason that the deflection, even in case of nonlinear analysis, is almost linear and hence a uniform reduction of stiffness (through the use of moment of inertia) might somehow work. We need to remember that the deflection being considered is the total deflection and not simply the usual vertical deflection.

5.4.3 Simply Supported Beam without Roller Support

A simply supported beam without the roller support is shown in Fig. 5.7. The beam is subject to a concentrated force at the mid span. Because there is no roller at the support, the horizontal displacement is constricted. The concentrated force at the mid span only leads to the vertical deflection.

Fig. 5.8 shows the (normalized) results of the analysis. As in the case of the previous beams, the secant analysis results are acceptable. For this case also the simplified technique was tried. A value of η=0.95 gives good results. Again, we are simply calibrating the value of η by using nonlinear analysis. We do not yet have the theoretical basis for the use of uniform secant modification.

A comparison of the results for simply supported beam with and without rollers is shown in Fig. 5.9. The difference is obviously not very much. However, this difference must be seen in light of the results for linear analysis shown in the same graph.

5.4.4 Portal Frame with Lateral Load

A simple portal frame with a lateral load at the beam level is shown in Fig. 5.10. It has the same height as the width. Both beam and column have the same properties. The large deflection analysis and secant analysis results are shown in Fig. 5.11. As can be seen, the secant analysis gives reasonably close result to that from the nonlinear analysis.

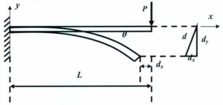


Fig. 5.1 Total Deflection of a Cantilever Beam

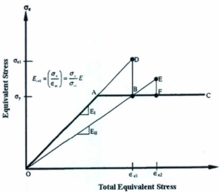


Fig. 5.2 Basic Secant Analysis

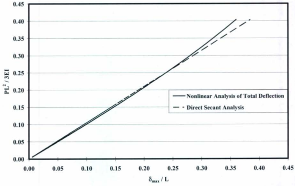


Fig. 5.3 Secant Analysis for Cantilever Beam

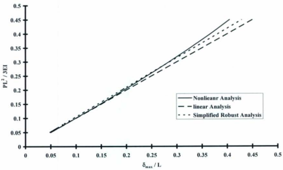


Fig. 5.4 Nonlinear Behaviour of a Cantilever Beam

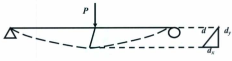


Fig. 5.5 Total Large Deflection of a Simply Supported Beam with One Roller

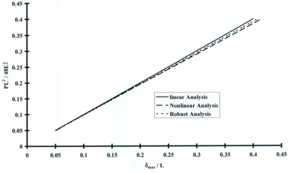


Fig. 5.6 Behaviour of a Simply Supported Beam with One Roller

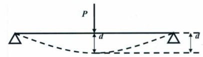


Fig. 5.7 Total Large Deflection of a Simply Supported Beam without Roller

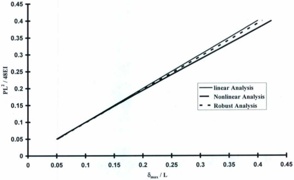


Fig. 5.8 Simply Supported Beam without Roller

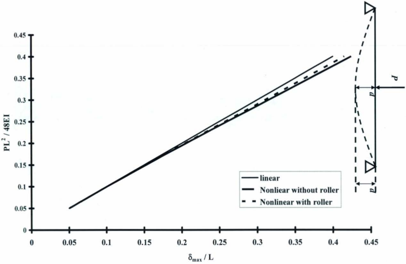


Fig.5.9 Comparison between Simply Supported Beams with and without a Roller

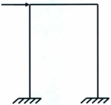


Fig.5.10 Portal Frame subject to a Lateral Force

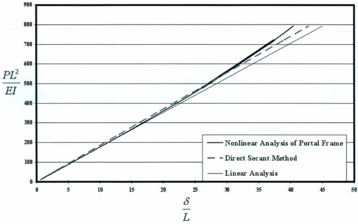


Fig. 5.11 Portal Frame Behavior

Chapter 6
Conclusions

6.1 Introduction

Limit load estimates are very useful for many engineering applications -both in design and analysis type problems. There has always been a need for robust methods for plastic limit load analysis from the point of view of numerical stability and effort. Robust limit load analysis has gained considerable attention over the past several years. Available robust methods adopt secant modulus modification as a means to cause redistribution in an elastic structure thereby producing limit behavior. The most significant among these methods are the r-node method, elastic compensation method and the m_α family of methods. All of these use the von Mises yield criterion to define an effective stress. This effective stress is used to obtain an estimate of secant modulus. The r-node method involves identification of r-node peaks to obtain limit loads. Such identification might require considerable judgment in some cases. The elastic compensation method is based on a maximum stress value. Because of numerical local errors, it can sometimes be difficult to properly identify the failure mechanism and the consequent limit load. The m_α family of methods have better theoretical basis but is more involved than the other methods and is quite conservative in many cases and unconservative in several other situations –especially for bending type problems. All of these modulus modification methods need stress level modifications and consequent discretization requirements that are very elaborate. The present thesis made use of a robust method which has several features of the above mentioned robust techniques for the estimation of limit loads along with additional advantages. The method generalizes the advantages of the existing robust methods so that it can be applied for any yield criterion and any finite element type [Adluri, 1999, 2001]. It has previously been shown to work quite well for beam (frame) and plate type structures by Adluri, Bolar and others. The criteria can be in terms of stresses or generalized forces such as moments and shears. The elements can be beam or plate/shell types. The generalization uses scaled yield criteria and is at least as accurate or better than the other existing methods. It is easier (and cheaper) to apply since any type of finite element can be used. The use of this technique has been demonstrated in the present work for a variety of beam and frame type problems in predicting plastic limit loads, elastic buckling loads and large deflections.

The method has a good theoretical basis for plastic limit loads. However, the theory for buckling and large deflections needs considerably more work in order to be firmly established.

6.2 Summary

Chapter 2 of the thesis gives an over view of the limit theorems, buckling, large deflections, etc. It reviews the material on the current robust methods such as the r-node technique, m_α technique, etc.

Chapter 3 reviews the methods used by Adluri and associates to estimate plastic limit loads for beam and plate type structures. The methods are used to predict frame collapse mechanisms and their limit loads. The basic procedure is taken from Adluri [1999, 2001] and is summarized below:

1. Carry out the initial elastic analysis based on original geometrical properties.

2. Modify the cross-section properties using

$$I_{new} = \frac{M_{P_i}}{|M_0(x)|} I_{old}, \tag{6-1}$$

   where M_{P_i} are the plastic moment capacities of the cross-sections. The bending moment $M_0(x)$ correspond to the elastic results from initial analysis.

3. A second elastic analysis is carried out based on this modification.

4. From the second analysis, the peak bending moments are considered as the locations of plastic hinges. Not all of these locations may be needed to form a mechanism for collapse. Sufficient combinations of these hinge locations are selected to form all possible hinge mechanisms. Usually, there are only a few such possible mechanisms and it is fairly easy to select these mechanisms.

5. For each of the selected mechanisms, the load factor for plastic collapse is calculated from the expression:

$$\lambda = \frac{P_L}{P} = \frac{1}{n} \frac{1}{\sum_{i=1}^{n} \frac{M_{peak-ave}}{M_{P_i}}} \tag{6-2}$$

   For uniform structures, M_{P_i} is constant. Therefore the equation can be simplified as:

$$\lambda = \frac{P_L}{P} = \frac{M_P}{M_{peak-ave}} \tag{6-3}$$

   This is the same as expression as that from Adluri [2001].

6. The lowest of these load factors amongst all possible combinations for hinge mechanisms is the applicable plastic limit load factor.

The results of the above procedure have been compared against the full nonlinear analysis results for many types of frames including portal frames, one-storey frames with multiple bays, and multistory frames.

Chapter 4 applies two adapted versions of the above secant technique to predict the buckling loads of elastic frames.

1 It is assumed that the frame buckling does not follow the classical mode of "bifurcation" or a sudden change in status. Rather, the buckling is a transition from more or less linear elastic deformation (involving both axial and lateral displacements of members) to a relatively very large set of lateral nonlinear displacements due to instability. The magnitude of this "large" displacement is not of as much importance as the fact that it took place at all. This is similar to the fact that at plastic limit loads, the curvature of the beam at critical segments reaches very large values (its magnitude is not important –rather that it took place at all).

2 The relationship between applied loads and the displacement at critical location is somewhat analogous to that between moment and curvature of the critical cross-section in Chapter 3. Both the relationships have initially linearly increasing portions which turn to large horizontal portions (with little increase in effective load).

3 Just as we obtain locations of peak redistributed moments in Chapter 3, we obtain locations of peak displacements in the structure whose stiffness is modified by the

secant scheme. These peak displacements can potentially be indicative of frame instability.

4 Two alternative empirical methods have been used in this chapter. Method 1 computes peak displacements as given below:

$$d_{mpc} = C\frac{M_{disp-peak}}{P_{compr\ total}} \tag{6-4}$$

The load factor λ for the critical load P_{cr} is obtained from the peak displacements,

$$\mathbf{P_{cr}} = \mathbf{P}\,\lambda, \lambda = \frac{1}{n}\frac{\sum_{i=1}^{n} d_{mpc}}{\sum_{i=1}^{n} d_{peak}} \tag{6-5}$$

where, d_{peak} is the peak displacement in the reanalysis.

The factor C depends on the type of frame. We do not yet have the necessary theoretical development to determine this factor exactly. Pending the development of the theory, it has been decided to empirically estimate the factor using finite element geometric nonlinear analysis.

In empirical method 2, after reanalysis, the critical load is calculated as below:

1 Identify the locations of peak displacements and the nearby locations of zero displacements. Find the bending moments and displacements at these locations.

2 Calculate

$$\Delta M = M_{dcr} - M_0 \text{ and } \Delta d = d_{cr} - d_0 \tag{6-6}$$

where, d_{cr} is the peak displacement in reanalysis, d_0 is the zero displacement, M_{dcr} is bending moment at the peak displacement location and M_0 is the bending moment at zero displacement location.

3 Estimate the critical load by using the equation

$$P_{cr} = \beta\left|\frac{\Delta M}{\Delta d}\right| \tag{6-7}$$

The factor β is analogous to the factor C in Method 1. It is calibrated using the same nonlinear analysis as used in Method 1.

The methods have been used to estimate the buckling loads for several different portal frames. Further work is needed to establish the theoretical basis for these methods and to apply them (or improved versions of them) for more complicated frames.

Chapter 5 deals with large deflections of beams and frames. The same techniques as in Chapter 3 are used to predict the large deflections of beams up to a value of nearly 50% of the span length. The results are encouraging. A modified form of the secant technique where the modification is uniform throughout the length also seems to give quite acceptable results. However, as in the case of Chapter 4, more work is needed to establish the theoretical basis for the methods.

The analyses in the thesis are carried out using ANSYS software. Typical input files for all different types of analyses and the APDL routines required are provided in the Appendices.

6.3 Conclusions

There were several studies in the current thesis. Some of the main conclusions are gathered below.

6.3.1 Plastic Limit Loads Estimation

1. The best conclusion of the plastic collapse study of the frames in Chapter 3 is that the method works very nicely and had a sound theoretical basis. The errors compared to the full nonlinear analyses are well within acceptable margins.

2. The method works equally well for frames with non-uniform cross-sections, portal frame, single storey multi-bay frames, and multistorey frames. They include beam mechanism failures and sway mechanism failures as well as combined failure mechanisms. The studies very clearly confirm what has been suggested or claimed by earlier studies. This is perhaps the best contribution of the present thesis.

6.3.2 Buckling

1. The load-displacement relationship in a buckling problem is analogous to the relationship between moment and curvature in a plastic limit load estimation problem.
2. We can modify moment of inertia (in lieu of the stiffness) based on the lateral displacement profiles in order to simulate critical displacement patterns at buckling.
3. Two empirical methods have been examined in Chapter 4 to predict the

buckling load capacities of portal frames. Factors C and β have been calibrated for use in Eq. 4.42 and eq. 4.4.5. These factors are indicative of certain theoretical algorithms for buckling loads. The results using these seem to be fairly good for a range of portal frames and loadings. However, their theoretical basis needs further study.

6.3.3 Large Deflection Analysis

1. For beam like structures studied in this thesis, the linear analysis deflections could be either larger or smaller than those predicted by nonlinear analysis. For cantilever, nonlinear analysis gives less deflection than the linear analysis. For other beams and frames, it is the other way around.

2. For the examples studied, linear and nonlinear analyses do not differ much till the deflection reaches about 20% of the span. Even after that, the difference is not very large till the deflection exceeds 50% of the span.

3. The direct secant approach gives reasonably good estimate of the nonlinear deflections. However, many more examples need to be studied to confirm this observation. This is because it can be argued that even linear analysis is fairly close to the results of nonlinear analysis.

4. A uniform secant modification for the entire beam (instead of deflection based modification that gives a non-uniform beam cross-section) seems to give results that are almost identical to the full secant analysis. The uniform modification factor used for moment of inertia has a narrow range: $0.95 < \eta < 1.05$. If linear deflection is greater than total nonlinear deflection,

$\eta > 1.0$. If linear deflection is smaller than total nonlinear deflection, $\eta < 1.0$. This needs to be studied further and theoretical basis established in order for it to be of practical use.

6.4 Recommendations for Further Work

In the present thesis, the research involved limit load estimation due to plastic collapse, critical load estimation due to elastic buckling and estimation of elastic large deflections in beams. The work is limited to certain types of structures. Much work needs to be carried out to establish the validity of these approaches. Chapter 3 is perhaps with the most theoretical justification as given by Adluri [2001]. Chapter 4 and 5 need considerable further research to establish their theoretical basis. Further work is recommended to the following content:

1. Improve the automatic identification of plastic hinge mechanisms.
2. Extend the robust secant estimation of Chapter 4 to include material nonlinear analysis.
3. Extend the robust secant estimation of buckling to more complicated structures.
4. Factors C and β are used to calculate the critical load. The meaning of the factors needs to be firmly established and made available for different situations.
5. Large deflection estimation needs to be extended to more complicated structures with different boundary conditions and load combinations.

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APPENDICES

Appendix A includes all the first purely elastic analyses in robust secant methods in this research. The macros used for modifying secant rigidity are all presented in Appendix B. Appendix A and appendix B are used together to consist of the entire robust secant analysis. Appendix C involves all the FEA nonlinear analyses in this research. The results of robust secant analysis obtained from Appendix A and B are compared with the nonlinear results from Appendix C.

APPENDIX A

A.1.1 A UNIFORM PORTAL FRAME SUBJECT TO A LATERAL FORCE AND A VERTICAL FORCE

```
! PLASTIC ANALYSIS USING MODIFIED SECANT RIGIDITY
/ PREP7                              ! ENTER PREPORCESSOR

ET, 1,beam3                          !USE BEAM 3 ELEMENT
*SET,EM, 200E3                       !YOUNG'S MODULUS
*SET,A, 50                           !AREA
*SET, I, 266.67                      ! MOMENT OF INERTIA
*SET, H, 8                           ! HEIGHT
*SET,LZ,10                           ! MESH SIZE
*SET,P1, 8e2                         ! VERTICAL FORCE
*SET,P2, 7.2e2                       ! LATERAL FORCE

R,1,A,I,H                            !INPUT AREA, MOMENT OF
                                     ! INERTIA AND HEIGHT
MP,EX, 1, 200e3                      !INPUT  YOUNG'S MODULUS
MP, NUXY,1 ,0                        !INPUT PASSION'S RATION

K,1, 0, 0                            !DEFINITION OF EYPOINTS
K,2,0,400
K,3,400,400
K,4,400,0

L,1, 2
    !DEFINITION OF LINES
L,2, 3
L,3,4

LESIZE, ALL, LZ                      !DEFINITON  OF   MESH SIZE
LMESH, ALL                           !MESH LINES

NSEL,S,LOC,X,200
F,ALL,FY,-P1                         !APPLY THE VERTICAL FORCE

NSEL,S,LOC,X,0
NSEL,R,LOC,Y,400
F,ALL,FX,P2                          !APPLY THE LATERAL FORCE
```

```
NSEL,S,LOC,y,0          !APPLY BOUNDARY CONDITION
D,ALL,ALL
NSEL,ALL

/SOLU
ANTYPE,0                ! DEFINE STATIC ANALYSIS
SAVE
SOLVE
/INPUT MACRO1
```

A.1.2 A TWO BAY AND ONE STOREY FRAME SUBJECT TO TWO VERTICAL FORCES AND ONE LATERAL FORCE

```
! PLASTIC ANALYSIS USING MODIFIED SECANT RIGIDITY
/PREP7                  ! ENTER PREPORCESSOR
ET, 1, beam3            ! USE BEAM 3 ELEMENT

*SET, EM ,200E3         ! YOUNG'S MODULUS
*SET, A, 50             ! AREA
*SET, I, 266.67         ! MOMENT OF INERTIA
*SET, H, 8              ! HEIGHT
*SET,LZ,10              ! MESH SIZE
*SET,P1, e2             ! VERTICAL FORCE
*SET,P2,7.2e2           ! VERTICAL FORCE
*SET,P3 ,5e2            ! LATERAL FORCE

R,1,A,I,H               ! INPUT AREA,MOMENT INITIAL
MP,EX,1,200e3           ! INPUT YOUNG'S MODULUS
MP,NUXY, 1, 0           ! INPUT PASSION'S RATION

K,1,0,0                 ! DEFINITION OF KEYPOINTS
K,2,0,400
K,3,400,400
K,4,400,0
K,5,800,400
K,6,800,0
L,1,2                   ! DEFINITION OF LINES
L,2,3
L,3,4
L,3,5
L,5,6

LESIZE, ALL, 10         ! DEFINITON OF MESH SIZE
LMESH, ALL              ! MESH LINES

NSEL, S, LOC, X, 200    ! APPLY THE VERTICAL FORCE
```

```
F, ALL, FY,-P1

NSEL,S,LOC,X,600                  ! APPLY THE VERTICAL FORCE
F,ALL,FY,-P2

NSEL,S,LOC,X,800                  ! APPLY THE LATERAL FORCE
NSEL,R,LOC,Y,400
F,ALL,Fx,P3

NSEL,S,LOC,y,0                    !APPLY BOUNDARY CONDITIONS
D,ALL,ALL
NSEL,ALL

/SOLU
ANTYPE,0                          !DEFINE STATIC ANALYSIS
SOLVE
FINISH
/INPUT MACRO1
```

A.1.3 A UNIFORM FRAME SUBJECT TO THREE VERTICAL FORCES AND ONE LATERAL FORCE

! PLASTIC ANALYSIS USING MODIFIED SECANT RIGIDITY

```
/PREP7                            ! ENTER PREPORCESSOR
ET, 1,BEAM3                       ! USE BEAM 3 ELEMENT

*SET, EM ,200E3                   !YOUNG'S MODULUS
*SET, A, 50                       ! AREA
*SET, I, 266.67                   ! MOMENT OF INERTIA
*SET, H, 8                        ! HEIGHT
*SET,L, 150                       ! LENGTH
*SET,LZ,2                         ! MESH SIZE
*SET,P1, 4e3                      ! FORCES
*SET,P2,2.5e3

R,1,A,I,H                         ! INPUT AREA,MOMENT INITIAL
MP,EX,1,200e3
MP,NUXY,1,0                       ! INPUT PASSION'S RATION

K,1,0,0                           ! DEFINITION OF KEYPOINTS
K,2,0,L
K,3,0.8*L,L
K,4,0.8*L,0

K,5,0,2*L
K,6,0.8*L,2*L
```

```
K,7,1.3*L,L
K,8,1.3*L,0

L,1,2                              ! DEFINITION OF LINES
L,2,3
L,3,4
L,2,5
L,5,6
L,6,3
L,3,7
L,7,8

LESIZE,ALL,LZ                      ! DEFINITON OF MESH SIZE
LMESH,ALL
/SOLU

NSEL,S,LOC,X,0.4*L                 ! APPLY THE VERTICAL FORCE
NSEL,R,LOC,Y,2*L
F,ALL,FY,-P1

NSEL,S,LOC,X,1.05*L                !APPLY THE VERTICAL FORCE
NSEL,R,LOC,Y,L
F,ALL,FY,-P2

NSEL,S,LOC,X, 0                    ! APPLY THE LATERAL FORCE
NSEL,R,LOC,Y,2*L
F,ALL,Fx,P2

NSEL,ALL

NSEL,S,LOC,y,0                     ! APPLY BOUNDARY CONDITIONS
D,ALL,ALL
NSEL,ALL

/SOLU
ANTYPE,0                           ! DEFINE STATIC ANALSIS
SOLVE
FINISH
/INPUT MACRO1
```

A.1.4 A UNIFORM BIG FRAME SUBJECT TO UNIFORMLY LATERAL DISTRIBUTED FORCES AND VERTICAL CONCENTRATED FORCES

```
! PLASTIC ANALYSIS USING MODIFIED SECANT RIGIDITY
/PREP7                          ! ENTER PREPORCESSOR
ET, 1,BEAM3                     !USE BEAM 3 ELEMENT

*SET, EM ,200E3                 !YOUNG'S MODULUS
*SET, A, 50                     ! AREA
*SET,I,266.67                   ! MOMENT OF INERTIA
*SET, H,8                       ! HEIGHT
*SET,L,150                      ! LENGTH
*SET,LZ, 2                      ! MESH SIZE
*SET, P1, 1e2                   ! FORCES
*SET,P2, 1.5e2                  ! FORCES
*SET,P3,1e2                     ! FORCES

R,1,A,I,H                       ! INPUT AREA,MOMENT INITIAL
MP,EX,1,200e3
MP,NUXY,1,0                     ! INPUT PASSION'S RATION

K,1,0,0                         ! DEFINITION OF KEYPOINTS
K,2,1.2*L,0
K,3,2.4*L,0
K,4,3.4*L,0
K,5,4.4*L,0

K,6,0,L
K,7,1.2*L,L
K,8,2.4*L,L
K,9,3.4*L,L
K,10,4.4*L,L

K,11,0,2*L
K,12,1.2*L,2*L
K,13,2.4*L,2*L
K,14,3.4*L,2*L
K,15,4.4*L,2*L

K,16,1.2*L,3*L
K,17,2.4*L,3*L
K,18,3.4*L,3*L

K,19,1.2*L,4*L
K,20,2.4*L,4*L
```

```
K,21,3.4*L,4*L

L,1,6                               !DEFINITION OF LINES
L,2,7
L,3,8
L,4,9
L,5,10
L,6,7
L,7,8
L,8,9
L,9,10

L,6,11
L,7,12
L,8,13
L,9,14
L,10,15
L,11,12
L,12,13
L,13,14
L,14,15

L,12,16
L,13,17
L,14,18
L,16,17
L,17,18

L,16,19
L,17,20
L,18,21
L,19,20
L,20,21


LESIZE, ALL,LZ                      ! DEFINITON OF MESH SIZE
LMESH, ALL                          ! MESH LINES
/SOLU

ESEL,S,ELEM,,1,75                   ! APPLY THE LATERAL
FORCE
SFBEAM,ALL,1,PRES,P1

ESEL,S,ELEM,,706,780
SFBEAM,ALL,1,PRES,P1

ESEL,S,ELEM,,1411,1485
SFBEAM,ALL,1,PRES,P1
```

```
ESEL,S,ELEM,,1801,1875
SFBEAM,ALL,1,PRES,P1

ESEL,S,ELEM,,1336,1410
SFBEAM,ALL,1,PRES,P2

ESEL,ALL,ALL

NSEL,S,LOC,X,1.8*L                  !APPLY THE VERTICAL FORCE
NSEL,R,LOC,Y,L
F,ALL,FY,-P3

NSEL,S,LOC,X,1.8*L
NSEL,R,LOC,Y,2*L
F,ALL,FY,-P3

NSEL,S,LOC,X,1.8*L
NSEL,R,LOC,Y,3*L
F,ALL,FY,-P3

NSEL,S,LOC,X,1.8*L
NSEL,R,LOC,Y,4*L
F,ALL,FY,-P3

NSEL,S,LOC,y,0
D,ALL,ALL
NSEL,ALL
ANTYPE,0
OUTRES,ALL,ALL
SAVE
NSEL,ALL

NSEL,S,LOC,y,0                      ! APPLY BOUNDARY CONDITIONS
D,ALL,ALL
NSEL,ALL

/SOLU
ANTYPE,0                            ! DEFINE STATIC ANALYSIS
SOLVE
FINISH
/INPUT MACRO1
```

A.1.5 A NON-UNIFORM PORTAL FRAME SUBJECT TO A LATERAL FORCE AND A VERTICAL FORCE

```
! PLASTIC ANALYSIS USING MODIFIED SECANT RIGIDITY
```

```
/PREP7                          ! ENTER PREPORCESSOR
ET, 1,BEAM3                     ! USE BEAM 3 ELEMENT

*SET, EM ,200E3                 !YOUNG'S MODULUS
*SET,L,800

*SET,A1,3000                    ! DEFINE DIFFERENT
                                ! GEOMETRICAL PROPERTIES

*SET,I1,9e5
*SET,H1,60

*SET,A2, 5000
*SET,I2, 4.17e6
 *SET,H2,100

*SET,A3,7000
*SET,I3,1.143e7
*SET,H3,140

*SET,LZ,5                       ! MESH SIZE
*SET,P1,1E6
*SET,P2,1E6

R,1,A1,I1,H1                    ! INPUT AREA,MOMENT INITIAL
R,2,A2,I2,H2
R,3,A3,I3,H3
UIMP,1, EX,,,EM                 !INPUT YOUNG'S MODULUS
MP,NUXY,1,0                     !INPUT PASSION'S RATION

K,1,0,0                         !DEFINITION OF KEYPOINTS
K,2,0,L
K,3,3/2*L,L
K,4,3/2*L,0
L,1,2                           !DEFINITION OF LINES
L,2,3
L,3,4

LSEL,S,,,1                      !SELECT LINE 1
LATT,,2                         !APPLY   REALCONSTANT   2   TO
LINE 1
LSEL,S,,,2                      !SELECT LINE 2
LATT,,1                         !APPLY   REALCONSTANT   1   TO
LINE 2
LSEL,S,,,3                      !SELECT LINE 3
LATT,,3                         !APPLY   REALCONSTANT   3   TO
LINE 3
LESIZE, ALL,LZ                  ! DEFINITON OF MESH SIZE
```

```
LMESH,ALL                    ! MESH LINES
/SOLU
NSEL,S,LOC,X,0               ! APPLY THE LATERAL FORCE
NSEL,R,LOC,Y,L
F,ALL,FX,P1

NSEL,S,LOC,X,3/4*L           ! APPLY THE VERTICAL FORCE
NSEL,R,LOC,Y,L
F,ALL,FY,-P2

NSEL,S,LOC,y,0               !APPLY THE
                             !BOUNDARY CONDITIONS
D,ALL,ALL
NSEL,ALL
ANTYPE,0                     ! DEFINE STATIC ANALYSIS
SOLVE
FINISH
/INPUT MACRO1
```

A.2.1 A UNIFORM PORTAL FRAME SUBJECT TO THREE VERTICAL FORCES ON THE BEAM

```
! BUCKLING ANALYSIS USING MODIFIED SECANT RIGIDITY
/PREP7
ET,1,BEAM3                   !USE BEAM 3 LEMENT

*SET,EM,200E3                !YOUNG'S MODULUS
*SET,L1, 600                 !LENGTH DEFINITION
*SET,L2, 800

*SET,A, 50                   !AREA DEFINITION
*SET, I,416.67               !MOMENT       OF       INERTIA
DEFINITION
*SET,H, 10                   !HEIGHT DEFINITION
*SET,LZ, 5                   !MESH SIZE DEFINITION
*SET,P1,2.0E4                !FORCE DEFINITION
*SET,P2,1.5E4
*SET,P2,1.2E4

R,1,A,I,H                    ! INPUT AREA,MOMENT INITIAL
MP,EX,1, 200e3               !INPUT YOUNG'S MODULUS
TB, BKIN                     !DEFINE BILINEAR MATERIAL
TBDATA, 1, 300,0             !DEFINE YIELD STRESS AND
                             !THE SLOPE AFTER YIEDL STRESS
                             !THE SLOPE AFTER YIEDL STRESS
k,1,0,0                      ! DEFINITION OF KEYPOINTS
k,2,0,L1
```

```
k,3,L2,L1
K,4,L2,0

L,1,2                          !DEFINITION OF LINES
L,2,3
L,3,4

LESIZE,ALL,LZ                  !MESH LINES
LMESH,ALL
/SOLU

NSEL,S ,LOC,Y ,0               ! APPLY THE
                               !BOUNDARY CONDITIONS
D,ALL,ALL

NSEL,S,LOC, X, 0.3 *L2         ! APPLY THE VERTICAL FORCES
NSEL,R,LOC, Y,L1
F,ALL,FY,-P1

NSEL,S,LOC,X,0.5*L2
NSEL,R,LOC,Y,L1
F,ALL,FY,-P2

NSEL,S,LOC,X,0.8 *L2
NSEL,R,LOC,Y,L1
F,ALL,FY,-P3

NSEL, ALL
ANTYPE,0                       ! DEFINE STATIC ANALYSIS
SOLVE
FINISH
/INPUT MACRO2
```

A.2.2 A UNIFORM PORTAL FRAME SUBJECT TO A VERTICAL FORCE AND A LATERAL FORCE

```
! BUCKLING ANALYSIS USING MODIFIED SECANT RIGIDITY
/PREP7
ET,1,BEAM3                     !USE BEAM 3 LEMENT

*SET,EM,200E3                  !YOUNG'S MODULUS
*SET,L1, 600                   !LENGTH DEFINITION
*SET,L2, 800

*SET,A, 80                     !AREA DEFINITION
*SET, I, 666.67                !MOMENT     OF     INERTIA
DEFINITION
```

```
*SET,H, 10                      !HEIGHT DEFINITION
*SET,LZ, 5                      !MESH SIZE DEFINITION
*SET,P1,2.0E4                   !FORCE DEFINITION
*SET,P2,0.5E4

R,1,A,I,H                       !INPUT AREA,MOMENT INITIAL
MP,EX,1, 200e3                  !INPUT YOUNG'S MODULUS
TB, BKIN                        !DEFINE BILINEAR MATERIAL
TBDATA, 1, 300,0                !DEFINE YIELD STRESS AND
                                !THE SLOPE AFTER YIEDL STRESS
                                !THE SLOPE AFTER YIEDL STRESS
k,1,0,0                         ! DEFINITION OF KEYPOINTS
k,2,0,L1
k,3,L2,L1
K,4,L2,0

L,1,2                           !DEFINITION OF LINES
L,2,3
L,3,4

LESIZE,ALL,LZ                   ! MESH LINES
LMESH,ALL
/SOLU

NSEL,S ,LOC,Y ,0                ! APPLY THE
                                !BOUNDARY CONDITIONS
D,ALL,ALL

NSEL,S,LOC, X, 0.5 *L2          !APPLY THE VERTICAL FORCE
NSEL,R,LOC, Y,L1
F,ALL,FY,-P1

NSEL,S,LOC,X,0                  ! APPLY THE LATERAL FORCE
NSEL,R,LOC,Y,L1
F,ALL,FX, P2

NSEL, ALL
ANTYPE,0                        ! DEFINE STATIC ANALYSIS
SOLVE
FINISH
/INPUT MACRO2
```

A.3.1 A CANTILEVER BEAM SUBJECT TO A CONCENTRATED FORCE AT THE FREE END

! LARGE DEFLECTION ANALYSIS USING MODIFIED ROBUST SECANT METHOD

```
/PREP7
*SET,EM,200E3                 !USE BEAM 3 LEMENT

*SET,A1, 0.01                 !AREA
*SET,I1,8.33e-6               !MOMEMNT OF INITIAL
*SET,H1,0.1                   !HEIGH
*SET,L1,1                     !LENGTH
*SET,LZ, 1/50                 !MESH SIZE
*SET,P,(32)*1e2               !FORCE

ET,1,BEAM3                    !USE BEAM 3 ELEEMNT
R,1,A1,I1,H1                  !INPUT GEOMETRICAL PROPERTIES
MP,EX,1,200e3                 !INPUT YOUNG'S MODULUS
MP,NUXY,1,0                   !INPUT PASSION'S RATION

K,1,0,0                       ! DEFINITION OF KEYPOINTS
K,2,L1,0
L,1,2                         !DEFINITON OF LINES
LESIZE,ALL, LZ                !DEFINITION OF MESH SIZE
LMESH,ALL                     !MESH ALL THE LINES

NSEL,S,LOC,X,0                !APPLY THE BOUNDARY CONDITIONS
NSEL,R,LOC,Y,0
D,ALL,ALL

NSEL,S,LOC,X, L1              !APPLY THE VERTICALLOAD
NSEL,R,LOC,Y,0
F,ALL,FY,-P

NSEL,ALL
ANTYPE,0                      ! DEFINE STATIC ANALYSIS
SAVE
/SOLU
SOLVE
FINISH
/INPUT MACRO3
```

A.3.2 A SIMPLY SUPPORTED BEAM WITH ONE ROLLER SUPPORT SUBJECT TO A CONCENTRATED FORCE IN THE MIDDLE

```
! LARGE DEFLECTION ANALYSIS USING MODIFIED ROBUST SECANT METHOD

/PREP7
```

```
*SET,EM,200E3                    !USE BEAM 3 LEMENT

*SET,A1, 0.01                    !AREA IN MM2
*SET,I1, 8.33e-6                 !MOMEMNT OF INITIAL IN MM4
*SET,H1,0.1                      !HEIGH IN MM
*SET,L1,1                        !LENGTH IN MM
*SET,LZ, 1/50                    !MESH SIZE IN MM
*SET,P,(32)*1e2                  !FORCE IN NEWTON

ET,1BEAM3                        !USE BEAM 3 ELEEMNT
R,1,A1,I1,H1                     !INPUT GEOMETRICAL PROPERTIES
MP,EX,1,200e3                    !INPUT YOUNG'S MODULUS
MP,NUXY,1,0                      !INPUT PASSION'S RATION

K,1,0,0                          ! DEFINITION OF KEYPOINTS
K,2,L1,0
L,1,2                            !DEFINITON OF LINES
LESIZE,ALL, LZ                   !DEFINITION OF MESH SIZE
LMESH,ALL                        !MESH ALL THE LINES

NSEL,S,LOC,X,0                   !APPLY THE BOUNDARY CONDTIONS
NSEL,R,LOC,Y,0
D,ALL,UX
D,ALL,UY

NSEL,S,LOC,X,L1
NSEL,R,LOC,Y,0
D,ALL,UY

NSEL,S,LOC,X,0.5*L1              !APPLY THE VERTICAL FORCE
NSEL,R,LOC,Y,0
F,ALL,FY,-P
NSEL,ALL
ANTYPE,0                         !DEFINE STATIC ANALYSIS
/SOLU
SOLVE
FINISH
/INPUT MACRO4
```

A.3.3 A SIMPLY SUPPORTED BEAM WITHOUT ROLLER SUPPORT SUBJECT TO A CONCENTRATED FORCE IN THE MIDDLE

! LARGE DEFLECTION ANALYSIS USING MODIFIED ROBUST SECANT METHOD

```
/PREP7
*SET,EM,200E3                 !USE BEAM 3 LEMENT

*SET,A1, 0.01                 !AREA
*SET,I1, 8.33e-6              !MOMEMNT OF INITIAL IN MM^4
*SET,H1,0.1                   !HEIGH
*SET,L1,1                     !LENGTH
*SET,LZ, 1/50                 !MESH SIZE
*SET,P,(32)*1e2               !FORCE

ET,1BEAM3                     !USE BEAM 3 ELEEMNT
R,1,A1,I1,H1                  !INPUT GEOMETRICAL PROPERTIES
MP,EX,1,200e3                 !INPUT YOUNG'S MODULUS
MP,NUXY,1,0                   !INPUT PASSION'S RATION

K,1,0,0                       !DEFINITION OF KEYPOINTS
K,2,L1,0
L,1,2                         !DEFINITON OF LINES
LESIZE,ALL, LZ                !DEFINITION OF MESH SIZE
LMESH,ALL                     !MESH ALL THE LINES

NSEL,S,LOC,X,0                !APPLY THE BOUNDARY CONDTIONS
NSEL,R,LOC,Y,0
D,ALL,UX
D,ALL,UY

NSEL,S,LOC,X,L1
NSEL,R,LOC,Y,0
D,ALL,UY
D,ALL,UY

NSEL,S,LOC,X,0.5*L1           !APPLY THE VERTICAL FORCE
NSEL,R,LOC,Y,0
F,ALL,FY,-P
NSEL,ALL
ANTYPE,0                      !DEFINE STATIC ANALYSIS
/SOLU
SOLVE
FINISH
/INPUT MACRO4
```

APPENDIX B

B.1 MACRO1 FOR MODIFYING SECANT RIGIDITY IN PLASTIC ANALYSIS

```
*GET,SZ,ELEM,0,COUNT                  !OBTAINING THE
                                      !NUMBER OF ELEMENT
/POST1
*DIM,COL1,ARRAY,SZ,1                  ! DEFINE ARRAY PARAMETERS
*DIM,COL2,ARRAY,SZ,1
*DIM,COL3,ARRAY,SZ,1
*DIM,COL4,ARRAY,1,1
*DIM,COL5,ARRAY,SZ,1
*DIM,COL6,ARRAY,SZ,1
*DIM,COL7,ARRAY,SZ,1
*DIM,COL8,ARRAY,1,1

ETABLE,MI,SMISC,6                     !DEFINE THE NAMES OF
                                      !BENDING MOMENT
ETABLE,MJ,SMISC,12

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! GETTING THE BENDING MOMENT OF EACH
ELEMENT AND INPUT THEM INTO PARRAYS

*DO,KK,1,SZ
*GET,MI,ELEM,KK,ETAB,MI               !GETTING THE
                                      !BENDING MOMENT OF
*GET,MJ,ELEM,KK,ETAB,MJ               !EACH ELEMENT

*VFILL,COL1(KK),DATA,MI               !PUTTING THEM INTO
PARRAYS
*VFILL,COL2(KK),DATA,MJ               !PUTTING THEM INTO
PARRAYS
*SET,COL3(KK),(COL1(KK)+COL2(KK))/2
*ENDDO
*VSCFUN,COL4(1),MAX,COL1(1)  !GETTING THE MAXIMUM BENDING
                             !MOMENT
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!MODIFYING THE PROPETIES

*CFOPEN,MODIFY1
*DIM,IMODIFY,ARRAY,SZ,1
*DIM,HMODIFY,ARRAY,SZ,1
*DO,JJ,1,SZ,1
*GET,MI,ELEM,JJ,ETAB,MI
```

```
*GET,MJ,ELEM,JJ,ETAB,MJ
*VFILL,COL5(JJ),DATA,MI
*VFILL,COL6(JJ),DATA,MJ
*SET,COL7(JJ),(COL5(JJ)+COL6(JJ))/2

*SET,IMODIFY(JJ),(abs(COL4(1)/COL7(JJ)))*I     !MODIFY MOMENT OF
                                               !INITIAL
*SET,HMODIFY(JJ),(12*IMODIFY(JJ)/A)**(1/2)     !MODIFY HEIGHT

*CFWRITE,R,JJ,A,IMODIFY(JJ),HMODIFY(JJ)        !INPUT MODIFIED
                                               !PROPERTIES
*CFWRITE,REAL,JJ
*CFWRITE,EMODIF,JJ
*ENDDO
*CFCLOS

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!REANALYZE THE MODLE
/PREP7
RESUME
*USE,MODIFY1
FINISH
/SOLU
SAVE
SOLVE
```

B.2 MACRO2 FOR MODIFYING SECANT RIGIDITY IN BUCKLING ANALYSIS

```
*GET,SZ,ELEM,0,COUNT                    !OBTAINING THE
                                        !NUMBER OF ELEMENT
/POST1
*DIM,COL1,ARRAY,SZ,1                    !DEFINE ARRAY PARAMETERS
*DIM,COL2,ARRAY,SZ,1
*DIM,COL3,ARRAY,SZ,1

ETABLE,UX,U,X                           !DEFINE THE NAME OF
                                        !LATERAL DISPLACEMENT

*DO,KK,1,SZ
*GET,UX,ELEM,KK,ETAB,UX
*VFILL,COL1(KK),DATA,UX
*ENDDO
*VSCFUN,COL2 (1),MAX,COL1(1)            !GETTING THE MAXIMUM
```

```
                                        !DISPLACEMENT

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!MODIFYING THE PROPETIES

*CFOPEN,MODIFY1
*DIM,IMODIFY,ARRAY,SZ,1
*DIM,HMODIFY,ARRAY,SZ,1
*DO,JJ,1,SZ,1
*GET,UX,ELEM,JJ,ETAB,UX
*VFILL,COL3 (JJ),DATA,UX

*SET,IMODIFY(JJ),(abs(COL2 (1)/COL3 (JJ)))*I   !MODIFY MOMENT OF
                                                ! INERTIA

*SET,HMODIFY(JJ),(12*IMODIFY(JJ)/A)**(1/2)   !MODIFY HEIGHT

*CFWRITE,R,JJ,A,IMODIFY(JJ),HMODIFY(JJ)   !INPUT MODIFIED
                                          !PROPERTIES

*CFWRITE,REAL,JJ
*CFWRITE,EMODIF,JJ
*ENDDO

*CFCLOS
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!REANALYZE THE MODLE
/PREP7
RESUME
*USE,MODIFY1
FINISH
/SOLU
SAVE
NSEL,ALL
SOLVE
```

B.3 MACRO3 FOR MODIFYING SECANT RIGIDITY IN LARGE DEFLECTION ANALYSIS

```
! ANALYSIS FOR CANTILEVER BEAMS

*GET,SZ,ELEM,0,COUNT            !OBTAINING THE NUMBER OF
ELEMENT
/POST1
*DIM,COL1,ARRAY,SZ,1            ! DEFINE ARRAY PARAMETERS
ETABLE,UY,U,Y                   !DEFINE THE NAME OF
                                !LATERAL DISPLACEMENT

*DO,KK,1,SZ
```

```
*GET,UY,ELEM,KK,ETAB,UY
*VFILL,COL1(KK),DATA,UY
*ENDDO
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!MODIFYING THE PROPETIES OF THE BEAM

*CFOPEN,MODIFY1
*DIM,IMODIFY,ARRAY,SZ,1
*DIM,HMODIFY,ARRAY,SZ,1

*DO,JJ,1,SZ,1
*GET,UY,ELEM,JJ,ETAB,UY
*VFILL,COL1 (JJ),DATA,UY

*SET,IMODIFY(JJ),1.05*I1            !MODIFY MOMENT OF INITIAL
                                    !MODIFICATION FACTOR IS 1.05

*SET,HMODIFY(JJ),(12*IMODIFY(JJ)/A1)**(1/2)  !MODIFY HEIGHT
*CFWRITE,R,JJ,A1,IMODIFY(JJ),HMODIFY(JJ)
*CFWRITE,REAL,JJ
*CFWRITE,EMODIF,JJ
*ENDDO

*CFCLOS
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!REANALYZE THE MODLE
/PREP7
RESUME
*USE,MODIFY1
FINISH
/SOLU
SOLVE
```

B.4 MACRO4 FOR MODIFYING SECANT RIGIDITY IN LARGE DEFLECTION ANALYSIS

```
! ANALYSIS FOR SIMPLY SUPPORTED BEAMS
*GET,SZ,ELEM,0,COUNT     !OBTAINING THE NUMBER OF ELEMENT
/POST1
*DIM,COL1,ARRAY,SZ,1          ! DEFINE ARRAY PARAMETERS
ETABLE,UY,U,Y                 !DEFINE THE NAME OF
                              !LATERAL DISPLACEMENT

*DO,KK,1,SZ

*GET,UY,ELEM,KK,ETAB,UY
*VFILL,COL1(KK),DATA,UY
*ENDDO
```

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!MODIFYING THE PROPETIES OF THE BEAM

*CFOPEN,MODIFY1
*DIM,IMODIFY,ARRAY,SZ,1
*DIM,HMODIFY,ARRAY,SZ,1
*DO,JJ,1,SZ,1
*GET,UY,ELEM,JJ,ETAB,UY
*VFILL,COL1 (JJ),DATA,UY

*SET,IMODIFY(JJ),0.95*I1                !MODIFY MOMENT OF INITIAL
                                        !MODIFICATION FACTOR IS 0.95

*SET,HMODIFY(JJ),(12*IMODIFY(JJ)/A1)**(1/2)  !MODIFY HEIGHT
*CFWRITE,R,JJ,A1,IMODIFY(JJ),HMODIFY(JJ)
*CFWRITE,REAL,JJ
*CFWRITE,EMODIF,JJ
*ENDDO

*CFCLOS
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!REANALYZE THE MODLE
/PREP7
RESUME
*USE,MODIFY1
FINISH
/SOLU
SOLVE
```

APPENDIX C

C.1.1 A UNIFORM PORTAL FRAME SUBJECT TO A LATERAL FORCE AND A VERTICAL FORCE

```
!PLASTIC ANALSIS USING GEOMETICAL NONLINEAR METHOD
/PREP7                          ! ENTER PREPORCESSOR

ET,1,BEAM 23                    !USE BEAM23 ELEMENT

*SET,EM,200E3                   !YOUNG'S MODULUS
*SET,A,50                       !AREA IN MM^2
*SET,I, 266.67                  ! MOMENT OF INITIAL IN MM^4
*SET,H,8                        ! HEIGHT IN MM
*SET,LZ, 2                      ! MESH SIZE IN MM
*SET,P1,8e2                     ! LATERAL FORCE
*SET,P2, 7.2e2                  ! VERTICAL FORCE

R,1,A,I, H                      ! INPUT AREA,MOMENT INITIAL
MP,EX,1, 200e3                  !INPUT YOUNG'S MODULUS

TB, MISO                        !DEFINE MUTILINEAR MATERIAL
                                !DEFINE DIFFERENT STRESS AND
TBPT,,210 /(200e3),210          !STRAIN POINTS
TBPT,,1.44E-03,230
TBPT,,2.08E-03,250
TBPT,,3.38E-03,270
TBPT,,7.25 E-03,290
TBPT,,1.5E-02,300
TBPT,, 3.0E-02 ,300

MP, NUXY ,1,0                   !DEFINE PASSION'S RATIO

K,1,0,0                         ! DEFINITION OF KEYPOINTS
K,2,0,400
K,3,400,400
K,4,400,0

L,1,2                           !DEFINITION OF LINES
L,2,3
L,3,4
```

```
LESIZE,ALL, 10                    ! DEFINITON OF MESH SIZE
LMESH, ALL                        ! MESH LINES

NSEL,S,LOC,X,200                  ! APPLY THE LATERAL FORCE
F,ALL,FY,-P1

NSEL,S,LOC,X,0                    ! APPLY THE VERTICAL FORCE
NSEL,R,LOC,Y,400
F,ALL,FX,P2

NSEL,S,LOC,y,0                    ! APPLY BOUNDARY CONDITION
D,ALL,ALL
NSEL,ALL

/SOLU
ANTYPE,0                          !DEFINE STATIC ANALYSIS
OUTRES,ALL,ALL

DELTIM,0.01,0.001,0.01            !DEFINE TIME SIZE STEP
LNSRCH,ON                         !USE LNSRCH TEHCNIQUE
NCNV, 1
SOLVE
```

C.1.2 A TWO BAY AND ONE STOREY FRAME SUBJECT TO TWO VERTICAL FORCES AND ONE LATERAL FORCE

```
!PLASTIC ANALSIS USING GEOMETICAL NONLINEAR METHOD
/PREP7                            ! ENTER PREPORCESSOR

ET,1,BEAM 23                      !USE BEAM23 ELEMENT

*SET,EM,200E3                     !YOUNG'S MODULUS IN
*SET,A,50                         !AREA
*SET,I , 266.67                   ! MOMENT OF INERTIA
*SET, H, 8                        ! HEIGHT
*SET, LZ,10                       ! MESH SIZE
*SET,P1,6e2                       ! VERTICAL FORCE
*SET,P2,7.2e2                     ! VERTICAL FORCE
*SET,P3 ,5e2                      ! LATERAL FORCE

R,1,A,I, H                        ! INPUT AREA,MOMENT INITIAL
MP,EX,1, 200e3                    !INPUT YOUNG'S MODULUS
TB, MISO                          !DEFINE MUTILINEAR MATERIAL
                                  !DEFINE DIFFERENT STRESS AND
TBPT,,210 /(200e3),210            !STRAIN POINTS
TBPT,,1.44E-03,230
```

```
TBPT,,2.08E-03,250
TBPT,,3.38E-03,270
TBPT,,7.25 E-03,290
TBPT,,1.5E-02,300
TBPT,, 3.0E-02 ,300

MP, NUXY ,1,0                    !DEFINE PASSION'S RATIO

K,1,0,0                          !DEFINITION OF KEYPOINTS
K,2,0,400
K,3,400,400
K,4,400,0
K,5,800,400
K,6,800,0

L,1,2                            !DEFINITION OF LINES
L,2,3
L,3,4
L,3,5
L,5,6

LESIZE,ALL, 10                   !DEFINITON OF MESH SIZE
LMESH, ALL                       !MESH LINES

NSEL,S,LOC,X,200                 !APPLY THE VERTICAL FORCE
F,ALL,FY,-P1

NSEL,S,LOC,X,600                 !APPLY THE VERTICAL FORCE
F,ALL,FY,-P2

NSEL,S,LOC,X,800                 !APPLY THE LATERAL FORCE
NSEL,R,LOC,Y,400
F,ALL,Fx,P3

NSEL,S,LOC,y,0                   !APPLY BOUNDARY CONDITION
D,ALL,ALL
NSEL,ALL

/SOLU
ANTYPE,0                         !DEFINE STATIC ANALYSIS
OUTRES,ALL,ALL
DELTIM,0.01,0.001,0.01           !DEFINE TIME SIZE STEP
LNSRCH,ON                        !USE LNSRCH TEHCNIQUE
NCNV, 1
SOLVE
```

C.1.3 A UNIFORM FRAME SUBJECT THREE VERTICAL FORCES AND ONE LATERAL FORCE

!PLASTIC ANALSIS USING GEOMETICAL NONLINEAR METHOD

```
/PREP7                          ! ENTER PREPORCESSOR

ET,1,BEAM 23                    !USE BEAM23 ELEMENT

*SET,EM,200E3                   !YOUNG'S MODULUS
*SET,A,50                       !AREA
*SET,I ,6.25*8*8*8/12           ! MOMENT OF INITIAL
*SET, H,  8                     ! HEIGHT
*SET,L,150                      ! LENGTH
*SET,LZ, 2                      ! MESH SIZE
*SET, P1, 4e3                   ! FORCES
*SET,P2,2.5e3

R,1, A,I, H                     !INPUT AREA,MOMENT INITIAL
MP,EX,1,  200e3                 !INPUT YOUNG'S MODULUS
TB, MISO                        !DEFINE MUTILINEAR MATERIAL
                                !DEFINE DIFFERENT STRESS AND
TBPT,,210 /(200e3),210          !STRAIN POINTS
TBPT,,1.44E-03,230
TBPT,,2.08E-03,250
TBPT,,3.38E-03,270
TBPT,,7.25 E-03,290
TBPT,,1.5E-02,300
TBPT,, 3.0E-02 ,300

MP, NUXY ,1,0                   !DEFINE PASSION'S RATIO
K,1,0,0                         !DEFINITION OF KEYPOINTS
K,2,0,L
K,3,0.8*L,L
K,4,0.8*L,0

K,5,0,2*L
K,6,0.8*L,2*L
K,7,1.3*L,L
K,8,1.3*L,0

L,1,2                           !DEFINITION OF LINES
L,2,3
L,3,4
L,2,5
L,5,6
L,6,3
```

```
L,3,7
L,7,8

LESIZE,ALL,LZ                    ! DEFINITON OF MESH SIZE
LMESH, ALL                       ! MESH LINES
/SOLU

NSEL,S,LOC,X,0.4*L               ! APPLY THE VERTICAL FORCE
NSEL,R,LOC,Y,L
F,ALL,FY,-P1

NSEL,S,LOC,X,0.4*L
NSEL,R,LOC,Y,2*L
F,ALL,FY,-P1

NSEL,S,LOC,X,1.05*L
NSEL,R,LOC,Y,L
F,ALL,FY,-P2

NSEL,S,LOC,X,0                   ! APPLY THE LATERAL FORCE
NSEL,R,LOC,Y,2*L
F,ALL,Fx,P2

NSEL,ALL

NSEL,S,LOC,y,0                   ! APPLY BOUNDARY CONDITION
D,ALL,ALL
NSEL,ALL

/SOLU
ANTYPE,0                         ! DEFINE STATIC ANALYSIS
OUTRES,ALL,ALL
DELTIM,0.01,0.001,0.01           !DEFINE TIME SIZE STEP
LNSRCH,ON                        !USE LNSRCH TEHCNIQUE
NCNV, 1
SOLVE
```

C.1.4 A UNIFORM BIG FRAME SUBJECT TO UNIFORMLY LATERAL DISTRIBUTED FORCES AND VERTICAL CONCENTRATED FORCES

```
!PLASTIC ANALSIS USING GEOMETICAL NONLINEAR METHOD
```

```
/PREP7                          ! ENTER PREPORCESSOR

ET,1,BEAM 23                    !USE BEAM23 ELEMENT

*SET,EM,200E3                   !YOUNG'S MODULUS
*SET,A,50                       !AREA IN MM2
*SET,I ,6.25*8*8*8/12           !MOMENT OF INITIAL
*SET, H, 8                      !HEIGHT
*SET,L,150                      !LENGTH
*SET,LZ, 2                      !MESH SIZE
*SET, P1, 1e2                   !FORCES
*SET,P2, 1.5e2                  !FORCES
*SET,P3,1e2                     !FORCES

R,1, A,I, H                     ! INPUT AREA,MOMENT INITIAL
MP, EX,1, 200e3                 !INPUT YOUNG'S MODULUS
TB, MISO                        !DEFINE MUTILINEAR MATERIAL
                                !DEFINE DIFFERENT STRESS AND
TBPT,,210 /(200e3),210          !STRAIN POINTS
TBPT,,1.44E-03,230
TBPT,,2.08E-03,250
TBPT,,3.38E-03,270
TBPT,,7.25 E-03,290
TBPT,,1.5E-02,300
TBPT,, 3.0E-02 ,300

MP, NUXY ,1,0                   !DEFINE PASSION'S RATIO
K,1,0,0                         !DEFINITION OF KEYPOINTS
K,2,1.2*L,0
K,3,2.4*L,0
K,4,3.4*L,0
K,5,4.4*L,0

K,6,0,L
K,7,1.2*L,L
K,8,2.4*L,L
K,9,3.4*L,L
K,10,4.4*L,L

K,11,0,2*L
K,12,1.2*L,2*L
K,13,2.4*L,2*L
K,14,3.4*L,2*L
K,15,4.4*L,2*L

K,16,1.2*L,3*L
K,17,2.4*L,3*L
K,18,3.4*L,3*L
```

```
K,19,1.2*L,4*L
K,20,2.4*L,4*L
K,21,3.4*L,4*L

L,1,6                                   !DEFINITION OF LINES
L,2,7
L,3,8
L,4,9
L,5,10
L,6,7
L,7,8
L,8,9
L,9,10

L,6,11
L,7,12
L,8,13
L,9,14
L,10,15
L,11,12
L,12,13
L,13,14
L,14,15

L,12,16
L,13,17
L,14,18
L,16,17
L,17,18

L,16,19
L,17,20
L,18,21
L,19,20
L,20,21


LESIZE,ALL,LZ
LMESH,ALL
/SOLU

ESEL,S,ELEM,,1,75                       !APPLY THE LATERAL FORCE
SFBEAM,ALL,1,PRES,P1

ESEL,S,ELEM,,706,780
SFBEAM,ALL,1,PRES,P1
```

```
ESEL,S,ELEM,,1411,1485
SFBEAM,ALL,1,PRES,P1

ESEL,S,ELEM,,1801,1875
SFBEAM,ALL,1,PRES,P1

ESEL,S,ELEM,,1336,1410
SFBEAM,ALL,1,PRES,P2

ESEL,ALL,ALL

NSEL,S,LOC,X,1.8*L
NSEL,R,LOC,Y,L
F,ALL,FY,-P3                            ! APPLY THE VERTICAL FORCE

NSEL,S,LOC,X,1.8*L
NSEL,R,LOC,Y,2*L
F,ALL,FY,-P3

NSEL,S,LOC,X,1.8*L
NSEL,R,LOC,Y,3*L
F,ALL,FY,-P3

NSEL,S,LOC,X,1.8*L
NSEL,R,LOC,Y,4*L
F,ALL,FY,-P3

NSEL,S,LOC,y,0
D,ALL,ALL
NSEL,ALL
ANTYPE,0
OUTRES,ALL,ALL
SAVE
NSEL,ALL
/SOLU
ANTYPE,0                                ! DEFINE STATIC ANALYSIS
OUTRES,ALL,ALL
DELTIM,0.01,0.001,0.01                  !DEFINE TIME SIZE STEP
LNSRCH,ON                               !USE LNSRCH TEHCNIQUE
NCNV, 1
SOLVE
```

C.1.5 A NON-UNIFORM PORTAL FRAME SUBJECT TO A LATERAL FORCE AND A VERITICAL FORCE

!PLASTIC ANALSIS USING GEOMETICAL NONLINEAR METHOD

```
/PREP7                              ! ENTER PREPORCESSOR
ET, 1,BEAM23                        !USE BEAM 23 ELEMENT

*SET, EM ,200E3                     !YOUNG'S MODULUS IN N/MM2
*SET,L,800

*SET,A1,3000                        !DEFINE DIFFERENT GEOMETRICAL
                                    !PROPERTIES
*SET,I1,50*60*60*60/12
*SET,H1,60

*SET,A2,5000
*SET,I2,50*100*100*100/12
*SET,H2,100

*SET,A3,7000
*SET,I3,50*140*140*140/12
*SET,H3,140

*SET,LZ,5                           !MESH SIZE IN MM
*SET,P1,1E6
*SET,P2,1E6


R,1,A1,I1,H1                        !INPUT AREA,MOMENT INITIAL
R,2,A2,I2,H2
R,3,A3,I3,H3
MP,EX,1, 200e3                      !INPUT YOUNG'S MODULUS
TB, MISO                            !DEFINE MUTILINEAR MATERIAL
                                    !DEFINE DIFFERENT STRESS AND
TBPT,,210 /(200e3),210              !STRAIN POINTS
TBPT,,1.44E-03,230
TBPT,,2.08E-03,250
TBPT,,3.38E-03,270
TBPT,,7.25 E-03,290
TBPT,,1.5E-02,300
TBPT,, 3.0E-02 ,300


MP, NUXY ,1,0                       !DEFINE PASSION'S RATIO
K,1,0,0                             ! DEFINITION OF KEYPOINTS
K,2,0,L
K,3,3/2*L,L
K,4,3/2*L,0
L,1,2                               !DEFINITION OF LINES
L,2,3
L,3,4

LSEL,S,,,1                          !SELECT LINE 1
```

```
LATT,,2                          !APPLY REALCONSTANT 2 TO
LINE 1
LSEL,S,,,2                       !SELECT LINE 2
LATT,,1                          !APPLY REALCONSTANT 1 TO
LINE 2
LSEL,S,,,3                       !SELECT LINE 3
LATT,,3                          !APPLY REALCONSTANT 3 TO
LINE 3
LSEL, ALL
LESIZE, ALL,LZ                   ! DEFINITON OF MESH SIZE

LMESH, ALL                       ! MESH LINES
/SOLU
NSEL,S,LOC,X,0                   ! APPLY THE LATERAL FORCE
NSEL,R,LOC,Y,L
F,ALL,FX,P1

NSEL,S,LOC,X,3/4*L               ! APPLY THE VERTICAL FORCE
NSEL,R,LOC,Y,L
F,ALL,FY,-P2

NSEL,S,LOC,y,0                   ! APPLY THE BOUNDARY
CONDITION
D,ALL,ALL
SAVE
NSEL,ALL

/SOLU
ANTYPE,0                         ! DEFINE STATIC ANALYSIS
OUTRES,ALL,ALL
DELTIM,0.01,0.001,0.01           !DEFINE TIME SIZE STEP
LNSRCH,ON                        !USE LNSRCH TEHCNIQUE
NCNV, 1
SOLVE
```

C.2.1 A UNIFORM PORTAL FRAME SUBJECT TO THREE VERTICAL FORCES ON THE BEAM

!BUCLING ANALSIS USING GEOMETICAL NONLINEAR METHOD

```
/PREP7
ET,1,BEAM23                      !USE BEAM 23 LEMENT

*SET,EM,200E3                    !YOUNG'S MODULUS
*SET,L1, 600                     !LENGTH
```

```
*SET,L2, 800

*SET,A, 80                      !AREA
*SET, I, 8*10*10*10/12          !MOMENT OF INITIAL
*SET,H, 10                      !HEIGHT
*SET,LZ, 5                      ! MESH SIZE
*SET,P1,2.0E4                   !FORCE IN NEWTON
*SET,P2,1.5E4
*SET,P2,1.2E4

R,1,A,I,H                       !    INPUT      AREA,MOMENT
INITIAL
UIMP,1, EX,,,EM                 !INPUT YOUNG'S MODULUS
MP,NUXY,1,0                     !INPUT PASSION'S RATION

k,1,0,0                         !DEFINITION OF KEYPOINTS
k,2,0,L1
k,3,L2,L1
K,4,L2,0

L,1,2                           !DEFINITION OF LINES
L,2,3
L,3,4

LESIZE,ALL,LZ                   ! MESH LINES
LMESH,ALL
/SOLU

NSEL,S ,LOC,Y ,0                !APPLY THE
                                !BOUNDARY CONDITIONS
D,ALL,ALL

NSEL,S,LOC, X, 0.3 *L2          !    APPLY    THE    VERTICAL
FORCES
NSEL,R,LOC, Y,L1
F,ALL,FY,-P1

NSEL,S,LOC,X,0.5*L2
NSEL,R,LOC,Y,L1
F,ALL,FY,-P2

NSEL,S,LOC,X,0.8 *L2
NSEL,R,LOC,Y,L1
F,ALL,FY,-P3

NSEL, ALL
ANTYPE,0                        !DEFINE STATIC ANALYSIS
OUTRES, ALL, ALL
NLGEOM, ON                      !LARGE                DEFLECTION
CALCULATION
```

```
DELTIM, 0.001,0.0001,0.001          !DEFINE TIME SIZE STEP
SOLVE
```

C.2.2 A UNIFORM PORTAL FRAME SUBJECT TO A VERTICAL FORCE AND A LATERAL FORCE

!BUCLING ANALSIS USING GEOMETICAL NONLINEAR METHOD

```
/PREP7
ET,1,BEAM23                         !USE BEAM 23 LEMENT

*SET,EM,200E3                       !YOUNG'S MODULUS IN N/MM^2
*SET,L1, 600                        !LENGTH IN MM
*SET,L2, 800

*SET,A, 80                          !AREA IN MM^2
*SET, I, 8*10*10*10/12              !MOMENT OF INITIAL IN MM^4
*SET,H, 10                          !HEIGHT IN MM
*SET,LZ, 5                          !MESH SIZE IN MM
*SET,P1,2.0E4                       !FORCE IN NEWTON
*SET,P2,1.5E4
*SET,P2,1.2E4

R,1,A,I,H                           ! INPUT AREA,MOMENT INITIAL
UIMP,1, EX,,,EM                     !INPUT YOUNG'S MODULUS
MP,NUXY,1,0                         !INPUT PASSION'S RATION

k,1,0,0                             ! DEFINITION OF KEYPOINTS
k,2,0,L1
k,3,L2,L1
K,4,L2,0

L,1,2                               !DEFINITION OF LINES
L,2,3
L,3,4

LESIZE,ALL,LZ                       ! MESH LINES
LMESH,ALL
/SOLU

NSEL,S ,LOC,Y ,0                    ! APPLY THE BOUNDARY CONDITIONS
D,ALL,ALL

NSEL,S,LOC, X, 0.5 *L2              ! APPLY THE VERTICAL FORCES
NSEL,R,LOC, Y,L1
F,ALL,FY,-P1
```

```
NSEL,S,LOC,X, 0
NSEL,R,LOC,Y,L1
F,ALL,FY,P2

NSEL, ALL
ANTYPE,0                          ! DEFINE STATIC ANALYSIS
OUTRES, ALL, ALL
NLGEOM, ON                        ! LARGE DEFLECTION CALCULATION
DELTIM, 0.001,0.0001,0.001        !DEFINE TIME SIZE STEP
SOLVE
```

C.3.1 A CANTILEVER BEAM SUBJECT TO A CONCENTRATED FORCE AT THE FREE END

```
!LARGE DEFLECTION ANALYSIS USING GEOMETRICAL NONLINEAR ANALYSIS
/PREP7
*SET,EM,200E3                     !USE BEAM 3 LEMENT

*SET,A, 8550                      !AREA IN MM^2
*SET,I, 1.04E+08                  ! MOMEMNT OF INITIAL IN MM^4
*SET,H,257                        !HEIGH IN MM
*SET,L1,6000                      !LENGTH IN MM
*SET,LZ, L1/50                    ! MESH SIZE IN MM
*SET,P,(32)*1e2                   !FORCE IN NEWTON

ET,1,BEAM 3                       !USE BEAM3 ELEMENT
R,1,A,I,H                         !INPUT GEOMETRICAL PROPERTIES
MP,EX,1,200e3                     !INPUT YOUNG'S MODULUS
MP,NUXY,1,0                       !INPUT PASSION'S RATIO

K,1,0,0                           !DEFINITION OF KEYPOINTS
K,2,L1,0
L,1,2                             !DEFINITION OF LINES

LESIZE,ALL,LZ                     !MESH LINES
LMESH,ALL

NSEL,S,LOC,X,0                    !APPLY THE BOUNDARY CONDITIONS
NSEL,R,LOC,Y,0
D,ALL,ALL

NSEL,S,LOC,X, L1                  !APPLY THE VERTICAL LOAD
NSEL,R,LOC,Y,0
F,ALL,FY,-P
NSEL,ALL
```

```
/SOLU
ANTYPE,0                          !DEFINE STATIC ANALYSIS
OUTRES,ALL,ALL
NLGEOM, ON                        !LARGE DEFLECTION CALCULATION
DELTIM, 0.001,0.0001,0.001        !DEFINE TIME SIZE STEP
SOLVE
```

C.3.2 A SIMPLY SUPPORTED BEAM WITH ONE ROLLER SUPPORT SUBJECT TO A CONCENTRATED FORCE IN THE MIDDLE

```
! LARGE DEFLECTION ANALYSIS USING GEOMETRICAL
NONLINEAR ANALYSIS
/PREP7
*SET,EM,200E3                     !USE BEAM 3 LEMENT

*SET,A, 8550                      !AREA IN MM2
*SET,I, 1.04E+08                  ! MOMEMNT OF INITIAL IN MM4
*SET,H,257                        !HEIGH IN MM
*SET,L1,6000                      !LENGTH IN MM
*SET,LZ, L1/50                    ! MESH SIZE IN MM
*SET,P,(32)*1e2                   !FORCE IN NEWTON

ET,1,BEAM 3                       !USE BEAM3 ELEMENT
R,1,A,I,H                         !INPUT GEOMETRICAL PROPERTIES
MP,EX,1,200e3                     !INPUT YOUNG'S MODULUS
MP,NUXY,1,0                       !INPUT PASSION'S RATIO

K,1,0,0                           !DEFINITION OF KEYPOINTS
K,2,L1,0
L,1,2                             !DEFINITION OF LINES

LESIZE,ALL,LZ                     !MESH LINES
LMESH,ALL

NSEL,S,LOC,X,0                    !APPLY THE BOUNDARY CONDTIONS
NSEL,R,LOC,Y,0
D,ALL,UX
D,ALL,UY

NSEL,S,LOC,X,L1
NSEL,R,LOC,Y,0
D,ALL,UY

NSEL,S,LOC,X,0.5*L1               !APPLY THE VERTICAL FORCE
```

```
NSEL,R,LOC,Y,0
F,ALL,FY,-P
NSEL,ALL

/SOLU
ANTYPE,0                            ! DEFINE STATIC ANALYSIS
OUTRES, ALL, ALL
NLGEOM, ON                          !LARGE DEFLECTION CALCULATION
DELTIM, 0.001,0.0001,0.001          !DEFINE TIME SIZE STEP
SOLVE
```

C.3.3 A SIMPLY SUPPORTED BEAM WITHOUT ROLLER SUPPORT SUBJECT TO A CONCENTRATED FORCE IN THE MIDDLE

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! LARGE DEFLECTION ANALYSIS USING GEOMETRICAL
NONLINEAR ANALYSIS
/PREP7
*SET,EM,200E3                       !USE BEAM 3 LEMENT

*SET,A, 8550                        !AREA IN MM2
*SET,I, 1.04E+08                    ! MOMEMNT OF INITIAL IN MM4
*SET,H,257                          !HEIGH IN MM
*SET,L1,6000                        !LENGTH IN MM
*SET,LZ, L1/50                      !MESH SIZE IN MM
*SET,P,(32)*1e2                     !FORCE IN NEWTON

ET,1,BEAM 3                         !USE BEAM3 ELEMENT
R,1,A,I,H                           !INPUT GEOMETRICAL PROPERTIES
MP,EX,1,200e3                       !INPUT YOUNG'S MODULUS
MP,NUXY,1,0                         !INPUT PASSION'S RATIO

K,1,0,0                             !DEFINITION OF KEYPOINTS
K,2,L1,0
L,1,2                               !DEFINITION OF LINES

LESIZE,ALL,LZ                       !MESH LINES
LMESH,ALL

NSEL,S,LOC,X,0                      !APPLY THE BOUNDARY CONDTIONS
NSEL,R,LOC,Y,0
D,ALL,UX
D,ALL,UY

NSEL,S,LOC,X,L1
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NSEL,R,LOC,Y,0
D,ALL,UY
D,ALL,UY

NSEL,S,LOC,X,0.5*L1               !APPLY THE VERTICAL FORCE
NSEL,R,LOC,Y,0
F,ALL,FY,-P
NSEL,ALL

/SOLU
ANTYPE,0                          !DEFINE STATIC ANALYSIS
OUTRES, ALL, ALL
NLGEOM, ON                        !LARGE DEFLECTION CALCULATION
DELTIM, 0.001,0.0001,0.001        !DEFINE TIME SIZE STEP
SOLVE
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