FUTURE MATHEMATICS IN A TI-83 GRAPHING CALCULATOR ENVIRONMENT

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Abstract

Reformed mathematics curricula are demanding increased use of graphing calculator technology. This report describes the background, need for, and evolution of a set of activities supporting implementation of graphing calculator technology in high school. Associated with the manual is a five-day professional development institute project. The manual is a self-contained body of notes and linked exercises describing TI-83 application throughout secondary school mathematics. It assumes no prior TI-83 knowledge. It is designed to accommodate personal instruction. Full solutions and extensive explanations are provided. A facilitator package including a complete description of managing such an institute is appended. It includes a glossary of terms and is fully referenced. Teaching tips, which provide alternate strategies and enhancements, are also included.

Future Mathematics in a TI-83

Graphing Calculator Environment

Since 1989, beginning with the National Council of Teachers of Mathematics' (NCTM) first *Standards* document, mathematics education has been the subject of considerable scrutiny and growth. NCTM's *Curriculum and Evaluation Standards* (*The Standards*) have provoked reform efforts internationally and thrust mathematics as a subject, and the teaching and learning of mathematics in general, into an arena that has grown to include political and public interest.

A catalyst of this reform is the graphing calculator (GC). Reform efforts are embracing the notion that learning experiences must involve tabular, symbolic, and graphical multi-representations of mathematics. The interrelated nature of multi-representational mathematics provides a balance that is essential to gaining a more dynamic sense of mathematics in the real world. The graphing calculator is a tool particularly suited to that task.

Recently, the Atlantic Canadian provinces formed the Atlantic Provinces Education Foundation (APEF). "In 1993 work began on the development of common curricula in specific core programs" (APEF, 1997b, p. 2). Secondary school mathematics in particular is undergoing a re-structuring on a scale exceeding any mathematics reform in these provinces' history. "The philosophy and outcomes of the Atlantic Canada mathematics curriculum are based firmly on those articulated by the National Council of Teachers of Mathematics (NCTM) in its *Curriculum and Evaluation Standards for School Mathematics* (1989)" (APEF, 1997b, p. 1). A major focus is the re-examination of technology and the relationship it has with the teaching and learning of mathematics.

The APEF mathematics curriculum has affirmed a place for technology, and in particular for the graphing calculator, in instruction. "The new technology not only has made calculations and graphing easier, it has changed the very nature of the problems and the methods mathematicians use to investigate them. ... The visualization approach offered through the use of graphing utilities such as the graphing calculator affords more students greater access to mathematics" (APEF, 1997a, ii). Clearly, GCs are an important technological support to mathematics teaching under such a curriculum effort.

Current professional development experiences and materials that introduce and/or support GCs are usually the product of a GC manufacturer. While valuable and necessary as learning tools, there are some concerns associated with them. Such events are usually constructed to serve a broad audience or to highlight specific powers of a machine. Therefore, materials may not closely match local content. Materials that poorly match local content, and/or are not grounded in local curriculum, can leave participants struggling to form a meaningful framework on which to construct their own skills with the machine.

Another concern lies with developing materials that rely on the capable assistance of an experienced facilitator for interpretation and mastery. This may stem from a need to construct a professional development package that holds as many examples of uses of the machine as possible while minimizing copy volume. This can leave participants with a resource that cannot effectively support the participant outside of the inservice environment. It may also hinder secondary professional development as participants may lack enough confidence with that resource to share their learnings with colleagues.

From a local perspective, there seems to be no experience available to a Newfoundland mathematics teacher that integrates GCs throughout the new APEF secondary mathematics curriculum in a manner overcoming the limitations mentioned. Nor does an organized implementation model, designed to foster teachers' growth with this technology, appear to exist within the provincial Department of Education. Hence, though graphing calculators have become more common in the hands of some students and teachers due to course requirements (such as the AP courses of The College Board) or personal interest, no broad base of expertise, consistent utilization model, or support network has evolved. This has made use of graphing calculators a largely unsupported endeavour in mathematics instruction in Newfoundland.

To address these concerns a project has been constructed which supports secondary mathematics teachers' professional development through providing an enhanced GC learning experience in a graphing calculator environment (GCE). A GCE treats the graphing calculator as an available and essential tool similar to a text, writing instrument, straightedge, and paper. It is no less or more important and should receive no more or less attention. A GCE can be fostered by the approach taken to problems. It facilitates a multi-representational approach to the learning of mathematics for the student where the graphical validates the tabular, which validates the symbolic. This is the balanced approach of a GCE. The GCE of a classroom is created through the style of learning experience created, coupled with the nature of the topics, and the type of materials accessed.

The project is built as a five-day professional development institute for secondary mathematics teachers, and is designed to enable a current non-GC-user or novice-GC-user to acquire operating skills and teaching techniques using the TI-83 within reformed curriculum and curricular approaches. It uses existing local and APEF mathematics curricula to ensure immediate usefulness and serve an impending need. It is also structured to be of substantial use as a resource outside of the inservice environment. How, then, have GCs ignited such an effort?

GC Brief History

The invention of the GC by Casio in 1985 (Waits & Demana, 1995) opened an array of possibilities for the teaching and learning of mathematics. The GC allows an enhanced environment in which to study mathematics in a patterning way, an exploratory way, and a fun way; while balancing the algebraic, tabular, and graphical representations in a manner not previously possible, in one small, utilitarian package.

GC power has grown rapidly. In 1986, Hewlett-Packard produced one of the first graphing calculators with symbolic algebra capability: the HP-28S (Hewlett-Packard, 1996). Texas Instruments (TI) entered the market with the TI-81 in 1990 (Texas Instruments, 1995). TI has recently produced the first and only calculator combining fully interactive graphics with symbolic algebra capability (including calculus), a powerful pseudo-BASIC programming language, dynamic and interactive geometry, a data and matrix editor, and a text editor. In 1998, TI presented *Flash Technology* in some new models – a means by which the operating system can be upgraded, or software installed, by connecting to the TI website.

Most models available today are equipped with a graphing and statistics editor, a calculus menu, extensive programmability, drawing capabilities, matrix operations, and much more. Some models offer financial packages, text editors, algebra tools, and dynamic geometry software. These units are available for between \$75.00 and \$150.00 with the upper end capabilities mentioned available at \$300.00. A GC costs less than the pieces of software needed to match its integrated power, and far less than the cost of the personal computer required to run that software.

Accessory devices enable TI GCs to collect empirical data anywhere the calculator can go. The TI CBL or *Calculator Based Laboratory* (a unit about the same size as the TI-83) can measure pH, voltage, motion, and many other physical phenomena using different probes. The TI-83 can store collected data into lists; can view the data through several statistical plot styles; and can perform various regression analyses.

With computer linking software, data collected by TI GCs can be uploaded to a computer for storage, printing, or for further analysis with other software; or data can be sent to others via email or the Internet. In fact, it is the flexibility, power, ease of use, and continuing evolution of the TI family of GCs that made TI the manufacturer of choice for this project. The particular model used will be the TI-83, which is widely held out as affordable, best suited to secondary mathematics, and the most user-friendly GC available.

The research pace of manufacturers continues to escalate as mathematics reform, and reformed attitudes toward technologies supporting the learning of mathematics, solidifies in the minds of Departments of Education, school boards, teachers, and other stakeholders. Though computers are now common in the learning environment, it is the affordability and applicability of GCs that seems to have assured their niche in mathematics education. Aside from portability and price, what are some of the tools the TI-83 possesses that make teaching with it better?

It is important in the teaching of mathematics that the symbolic not exist in isolation of the graphical or the numeric; that the graphical and tabular support the algebraic. The TI-83 supports four relation graphing styles: functions; parametric plots; polar plots; and, sequence plots. It manages table views side by side with graphical plots and symbolism. All can be viewed as separate entities or interrelated ones. For example, as a plot is traced on the graph screen, the symbolic relation is simultaneously displayed. At the same time the table values update to reflect the trace position shown on the plot. As students become more versed in the interrelated nature of mathematics using GC technology, they may more freely speculate on how the plot may look and why. They may wonder how a problem may be solved using the graphical representation or the table or ask questions about the algebra based on an observation made from the graph.

Statistical calculation and analysis involving one and two variables under many regression models is easy with the TI-83. Raw data may be plotted independently in varied plot styles with regression curves overlaid. The concrete visual of plotting the curve of best fit from regression analysis through the data and getting a visual sense of correlation, can aid in the delivery of the concept.

Graphing patterns among classes of functions may be easily established and the student need not wait for manual calculations to be done before plots are drawn. This relieves arithmetic drudgery, where arithmetic proficiency was not the goal, and frees the student to examine relationships.

Graph analysis may take place that may not otherwise have been possible symbolically. For example, in a senior math course using the assistance of the TI83, a student could use the calculus options and the solver to explore roots of functions, slopes of secants or tangents at specific points on functions, and intersections of complicated functions, without having to necessarily understand symbolic calculus. While some teachers may be uncomfortable with this sort of approach, it is not necessarily a disadvantage. It could be an enlightening approach, as students could become familiar with the terminology and some underlying concepts of calculus before having to wrestle with the symbolism. Explorations using technology could act as a foreshadowing and support the analysis that students may be given the algebra tools to do in the near future. In this way, exploration of concepts underpinning calculus, studied using graphing calculator technology, can enhance the study of calculus as a whole. Teachers seeking to best utilize graphing calculators into instruction may have to reconsider the way they approach topics for the educational experience. They will have to consider altering the testing experience to include more analysis questions as plotting a simple quadratic and identifying the vertex, for example, is a simple exercise now. Depth of treatment may be altered as students and teachers explore the power of the machines.

Though curricular materials frame, support, and often delineate delivery, it will be the teacher's responsibility to ensure that the learning environment is as enabling as possible. This now means inclusion of GC technology. How successful have teachers been in that endeavour?

Supporting Studies

Quesada and Maxwell (1994) conducted one of the defining studies encapsulating many aspects of the issue. This study followed others, most notably the Ohio State University Calculator and Computer Pre-calculus Project (also known as the C^2PC Project). The Quesada and Maxwell study began in the fall of 1990 – a time when graphing calculators had been around for about 5 years. This could be considered sufficient time for many educators and students to become familiar with the technology, as well as time for manufacturers to have evolved a second generation of more user-friendly machines.

Scores among the study groups which had graphing calculators incorporated into instruction and used by each student were consistently higher than control groups. This seems to demonstrate support for incorporating graphing calculators into the learning environment. One interesting aspect of this study was the identification of factors that might have caused students using graphing calculator to have higher scores.

Quesada and Maxwell incorporated a survey into their study to help determine the students' impressions of the value of the graphing calculator as a learning tool. The students perceived that they did more exploration, and that the graphing calculator helped them understand course concepts. In addition, the students responded to open-ended questions and indicated three main aspects of graphing calculator assisted learning that reflected the positive influence of the technology (many educators and students might assert the same feelings today). That using graphing calculators "...(i) facilitates understanding, (ii) provides the ability to check answers, and (iii) saves time on tedious calculations" (p. 212).

Additional student comments shed light on the appeal of graphing calculators, and indeed may illustrate why some researchers are having difficulty accessing just what it is about this technology that seems to encourage exploration of concepts and improve some performance. For example, according to Quesada and Maxwell, students find themselves thinking graphically about problems before trying to solve them algebraically – a new experience for many of them.

The study also suggests that although some significant research has been conducted, there seems to be some ambiguity as to whether the graphing calculator enhances performance, learning, or both in specific course offerings. In addition, there is insufficient empirical evidence representing the effect of this technology upon the college level student. This is not surprising as we consider "...using graphing calculators, though common in high school mathematics classes, is still somewhat rare in many college algebra or calculus classes" (Jones, p. 232).

The tone of the concluding discussion of this study begs the question of whether a lack of familiarity with the technology influenced the structure of the study, hence causing difficulty in clarifying causal connections between graphing calculator use and performance, or learning, or both. It may well be that researchers have not yet reconciled how to completely address the effects of the technology, and have concentrated primarily on performance variations for empirical data. There may be factors associated with the graphing calculator that are as yet unexamined and therefore not quantified.

One of the earliest studies examining the effects of calculators on test results was conducted in Missouri in 1987. This study was a response to research which suggested that pencil and paper computational proficiency does not promote problem solving ability, and to a challenge to respond to the realities of the availability of technology to students and teachers. The Missouri Mastery and Achievement Tests (also known as the MMAT) was administered for the first time allowing the use of calculators.

There were significant differences in levels of performance among each grade. Those with calculators consistently performed better than those without calculators. The more complex the task however, the closer the results of the calculator and non-calculator groups were (Long, Reys, & Osterlind, 1989). This study earns recognition as a pioneering work in examining use of calculators in mathematics testing.

Extensive institutes and projects such as the Oklahoma Graphing Calculator Project (also begun in 1990) have focused upon putting technology into teachers' and students' hands and providing training (Stenglein, 1996). It seems, though, that there is still a paucity of machines in many classrooms. Though significant efforts are being made to encourage the integration of this technology, graphing calculators are not yet generally mandated as a supplied instructional tool for teachers. It is still common that if a teacher possesses a graphing calculator it was a personal purchase. Until a broader base of graphing-calculator-active curricula, text support, and school purchasing begins to foster development of a base of instructional expertise, it may be difficult to structure questions that truly discern calculator and graphing calculator influences on learning. There may be debate on *when* calculators are best applied to the learning of mathematics but *whether* calculators belong seems to have been clarified. The position of NCTM, as articulated in *The Standards*, is certainly clear.

Curricula Shifts

Dubinsky (1995) poses the following question (from a student's point of view) about calculus: "So why do we need calculus if we have all this technology? I mean, if you want me to find relative maximum values of a function, I'll just draw the graph, zoom in on the humps, and get as close as you want"(p.146). This perspective does little to recognize the limits a viewing window and limited resolution forces on analysis. A student relying on a GC only to solve a problem like this may struggle. That is, the view afforded is limited and conclusions likewise limited.

It also fails to serve an underlying principle of mathematics. That is, it fails to see the forest and sees only one tree – and it must be a short tree to have such a limited view of even the trees nearby! Mathematics is about connections. The algebraic connects with the graphical connects with the geometrical. No one tool or approach can reveal many complexities if used in isolation of all others. It is a narrow view that sees only the task at hand and only seeks one tool to address it. There is little chance of creativity, innovation, or balance here. A grasp of symbolism, technology, and the various representations of relationships, are all necessary to truly appreciate and understand mathematics in a graphing calculator environment.

This may not remain the case as newer computer/calculators may solve equations and display all solutions in exact form. The question now is begged: "Why do it with pencil and paper when I can do it with this tool, be correct, and explain how I did it?"

The NCTM *Standards* examined four areas of instructional practices: curriculum content; teaching methods; technology; and assessment methods. Within curriculum content, the structure of number systems, functions, discrete mathematics, and probability and statistics were to receive greater emphasis, and factoring was to receive less emphasis. Factoring is a traditionalist's dream…it is an elegant and predictable waltz with technique. It is an example of a curricular topic whose treatment and/or presence must be re-thought.

Many aspects of our curriculum rely on the traditional skills such as factoring since the tools simply did not exist to affordably equip secondary students to attack complex problems having *un-nice* solutions. That is not to say that those expressions that factor and used to be so militantly protected as essential to high school mathematics are to be disregarded! On the contrary, they should be illustrated as the rare jewels they are in a sea of un-nice objects – they factor. Very few do, and that in itself makes them unique, interesting, and worthy of study. Hence, there is the risk of losing underpinnings, connections, and aesthetic qualities if graphical tools are applied while symbolism is sacrificed.

Franklin Demana and Bert Waits have made significant contributions to the development of the TI family of graphing calculators and have acted as pioneers in developing textbooks and supporting manuals that incorporate graphing calculators into instruction. They co-authored one of the first comprehensive graphing-calculator-active textbooks on pre-calculus mathematics (Demana & Waits, 1990) and continue to be leaders in authoring calculus texts which are graphing-calculator-active. "A computer-graphing-based approach provides the underpinning needed for successful algebraic experiences, reduces the need for contrived problems, and gives students a general and powerful problem solving tool" (Waits & Demana, 1989, 331). This confident position, though not widely shared initially, now reflects the position of many reformists.

Waits and Demana (1995) go further to suggest that curricula should be re-examined with a view to identifying which pencil and paper skills, processes, etc. should now be considered obsolete and better executed using the calculator/computer. This statement implies a significant undertaking. It will force teachers to deeply re-examine content, and seriously review curricula and assessment. Explaining the significance of relationships as opposed to simply executing operations; explaining and describing what has been learned instead of stating a result; these are becoming issues of some importance in mathematics classrooms.

A re-thinking of perspective for the elements of our traditional curriculum is now occurring, such that realistic problems are presented and solved while allowing the beauty of those problems whose solutions are particularly elegant to be appreciated. A graphing calculator environment can enhance this examination. What must be reiterated however, is that no single tool should be used in isolation. Graphs are a representation of tabular data and are a visual confirmation of algebraic relationships. Each tool should justify the other. Ideally, to efficiently accomplish a task, all available tools should be accessible. If a tool seems extraneous to a problem, or could not currently be applied in a productive manner, then it could be put aside. At least, if the available tools are accessible, choices as to applicability can be left to the learner. It may well be that our students are quite capable of making those decisions.

The College Board is a well-respected educational body in the United States whose position with respect to calculators in general perhaps has shifted more and faster than any other agency. In May 1993, the College Board permitted the use of scientific calculators on parts of the Advanced Placement (AP) Calculus AB and BC examinations for the first time. This was a significant evolution of policy for this forty-year-old non-profit organization dedicated to providing collegiate-level courses to the exceptionally able high school student. The exams were cited as *calculator-active*. This was the first recognized use of the term.

In May 1995, Calculus AB and BC were made *graphing-calculator-active* (College Board, 1995). GCs were necessary for some parts of some questions on the examination and assumed incorporated into instruction. These courses remain graphing-calculator-active (College Board, 1997) and graphing calculators remain a vital, required tool for instruction and analysis for students and teachers.

Calculus AB and BC have enjoyed phenomenal growth in enrollment. In May 1995, approximately 120,000 students worldwide wrote an AP examination in mathematics alone (Piccolino, 1996). These courses, having been developed under the auspices of some of the most credible and hallowed universities and colleges in the United States, are widely recognized as having high standards of curriculum development and evaluation.

In May 1997, the College Board offered an examination in statistics for the first time. Students were required to bring a graphing calculator with statistical capabilities. Exploring, estimating and modeling are also a part of NCTM's reform thrusts. The syllabus focuses on exploring data and finding ways to simplify data descriptions. "It reflects the recommendations contained within the statistics standard for grades 9-12 in the National Council of Teachers of Mathematics' (or NCTM) Curriculum and Evaluation Standards..." (Piccolino, 376).

It is clear that graphing calculator technology is being embraced by the College Board in a reformist environment supporting *The Standards*. These courses offer what might be described as a GCE in the delivery of a challenging curriculum examined under high standards. The rapid evolution of policy respecting calculator use in instruction and evaluation by the College Board has mirrored availability of new generations of graphing-calculator-active textbooks and graphing-calculator-active courses. This reflects a re-thinking of mathematics curricular emphases and delivery models spurred by *The Standards*.

Calculators have been accepted in the secondary school learning environment for some time. The extent to which teachers have actively incorporated scientific calculators into instructional techniques and lessons has gone relatively unexamined though allowance in class and test settings is common. Questions as to how each teacher is making best use of all resources and best delivering revised curricula will likely provoke an atmosphere of change, reflection, and increased research throughout the next decade.

Does this mean a graphing calculator environment is suitable or unsuitable for our classrooms? Insight gained from research on calculator and graphing calculator use seems to support the medium as a positive influence on learning. The true gauging of its effects may be somewhat imprecise, but the direction of influence is clear. Graphing calculators positively influence the learning experience. Continued research further delineating specifics of GC impact is also needed.

Teaching Tensions

In a time of global communication, enhanced environmental awareness, and apparent harmony in many world quarters, attention has focused on interaction and information exchange. Technology is exploding upon all aspects of society and educators are gaining particular attention due to their responsibility for preparing youth for such a dynamic workplace. At the same time society is demanding a dynamic educational environment, the current teaching force in North America is aging as is the population as a whole. Many mathematics teachers have been schooled in an environment supporting "teaching by telling" where a strong commitment to teaching in the way they were taught promotes a positive sense of efficacy, and affects ability to alter underlying assumptions about mathematics curricula and instruction (Smith, 1996). Hence, dynamic curricula supported by dynamic tools can provoke nervousness as well as be inherently exciting.

A powerful analysis of how emphases found in *The Standards* has impacted educational institutions and teachers can be found in Garet and Mills (1995) as they examine the effect of *The Standards* as they were being implemented in the Chicago area in 1991. As has been noted, one need only look at the Atlantic Provinces Educational Foundation's mathematics curriculum to see the impact *The Standards* have had on local curriculum development.

Recent technological evolution has been rapid yet many of our teaching methods remain traditional. Students may be better able to embrace technology in the learning environment than teachers creating new tensions in the student-teacher dynamic. The graphing calculator directly affects the breadth and depth of topics that a teacher and student may explore and the manner in which it is explored. It is an example of a technological tool that is helping to drive curricular reform and reform educational thinking. Never before has such an affordable tool had such potential impact on teaching and learning. The influence of graphing calculator technology on the teaching and learning climate has become unavoidable as many students and school boards embark on purchasing programs to help modernize the delivery of mathematics curriculum in schools. This brings pressures to bear on teachers as they adapt methods and approaches to the teaching of mathematics in this environment.

Jones (1995) suggests there are professional tensions which influence teachers' ability to effect personal change; for example, the conflict between teaching mathematics as they were taught vs. the manner supported by the mathematics reform movement. Other tensions include: increases in documenting and implementing new assessment techniques; increases in accountability for student performance; and, increasing demands to incorporate technology in the teaching and learning environment.

Smith (1996) asserts that a teacher's view of mathematics directly affects the way mathematics is taught, and often it is in the manner that teacher was taught. The comfort engendered by exercising the trusted and familiar view of the classroom increases teacher efficacy. When operating in an environment subject to the tensions suggested, where one is expected to re-examine virtually every aspect of curricula, delivery, and interaction, teachers may feel confusion, fear of loss of control, and feel concern about their ability to provide mathematical leadership.

"This active view of learning mathematics substantially changes what teachers must do to enable learning. They no longer present the content through clear demonstrations; they must instead create the conditions that will allow students to take their own effective actions" (Smith, p. 393). This is a classroom of a significantly different nature than the "teaching by telling", self-affirming, comfortable teacher-as-authority model many teachers were trained in. The resulting flux can result in discomfort and questioning of one's own ability to effect positive change in the classroom until successful adaptation on the part of teacher and learner has occurred.

An additional concern and one certainly affected by reform is assessment of learning and performance in a technological environment. Students learn to value that which earns a grade (Wilson, 1994). This provokes far-reaching questions as revised assessment models must also struggle with formal evaluation schema designed to reflect what is seen as important for students. If explanation is an emphasis in a GCE, then it must be assessed in a valid and reliable manner and formally expressed for reporting purposes. This is a challenge throughout reform.

Without doubt, becoming a better mathematics teacher involves a continuous re-conceptualization of what constitutes good teaching. This is perhaps the biggest challenge of teaching. One way to assist this mission is to identify and reward teachers within systems who seem to have successfully negotiated the demands of reform amid the tensions influencing teachers and teaching. These teachers will have done more than become skilled with a piece of technology. They will have become leaders in reforming the learning environment and will manifest efficacy. They will be the curriculum and delivery leaders of tomorrow.

A graphing calculator environment holds promise as an engine helping to maintain the speed of reform. It is difficult for teachers to avoid the influence of a tool that has become pervasive in recent literature and the subject of vigorous market jostling on the part of manufacturers. No tool offering the power and portability of newer graphing calculators has ever entered a classroom. The personal computer and word-processor suggests an analogy that seems appropriate as a means to afford perspective on some aspects of the influence of a GCE.

Until relatively recently, draft documents were comfortable written by hand – perhaps that is still the case. As technology progressed to typewriters, presentation improved. The typewriter likely did not remove any aspect of the thinking that contributed to the product. Considering today's technology, the word-processor, we find built-in options for spell checking, grammar checking, and word replacement. Has its effect on the user been substantially different than the typewriter? Is concern about effects resulting from use a fundamental and ongoing concern and enough to limit how it is used? These questions might be more closely related to the question of calculator influence than is realized.

The word-processor and the graphing calculator serve entirely different functions of course. The graphing calculator is much more attuned to student interaction with data than creation of prose; but underlying its widespread support is the tenet that mathematical skills seem to improve as a result of the use of graphing calculators.

In an educational setting, the fear may arise that individuals who have difficulty keeping up may fall by the intellectual wayside. This is prompted by the fact that the pace of technology of the past twenty years or so is unprecedented in history. Work ethic alone will not determine economic survival in the twenty-first century. It may be that even those who traditionally thought of themselves as reasonably intelligent members of an earlier society may feel overwhelmed by the pace of change. A natural question, then, would be whether the few are driving the many, and whether survival of the fit in this society is provoking uncertainty as the fit of the past may not be the fit of the future.

These are serious implications. Those most directly involved in guiding our youth through education must be given all the support possible as they strive to implement changes associated with *The Standards* in this rapidly evolving technological environment. To do less would be to invite disillusionment in teachers and ensure a greater delay in realizing the fruits of reform.

It is time for mathematics educators to become excited about this era. Opportunities exist here to broaden the scope and power of analysis within math expertise such that more students than ever can interact with issues and problems in the real world that surrounds them.

This power is unprecedented. Formerly, only those capable of reveling in the mechanics of algebra had risen to a level where complex analysis could take place. There will always be those who seek to intimately understand the mathematics of our world in such a manner, but it should be recognized that the evolution of GCs has opened much more of the fabric of mathematics up to interested learners. A novel concept indeed – that more people might enjoy the patterning, the graphical, and the visual aspects of mathematics through the insights afforded by GC technology. The time is now ripe to support teachers in the transition into a balanced approach to teaching with technology.

APEF, and reform efforts internationally, are embracing the notion that perspectives on the multi-representational nature of mathematics must be enhanced to provide a more dynamic sense of mathematics in the real world. The graphing calculator seems the best tool to do that in classrooms. This project seeks to support teachers' professional growth through a balanced perspective of mathematics learning experiences utilizing technology by creating a graphing calculator environment within which they can re-visit the learning of mathematics.

This project was conceived from a desire to allay some long-standing and nagging concerns felt by the author regarding GC inservices as a whole. Through delivering inservices on using GCs for almost a decade, it became quite clear that (a) there never seemed to be enough time allotted for the experience, (b) materials designed to support a reformed view of the teaching of mathematics seemed out of context with local content leaving participants interested in the material but unconvinced they should move toward larger scale classroom use of the GC, and (c) materials often were exercises in *doing through immersion*, requiring a facilitator's guidance, instead of exercises containing explanations which are enhanced by a facilitator yet could serve as a readable resource outside of the inservice.

The time concern seemed not always to stem from contact time with a facilitator. The author has facilitated sessions varying from 1 hour to 5 days in duration and the comment was always made. It seemed to stem from the speed of introduction due to the immersion approach taken.

The local content concern seems now a moot point since reforming curriculum, while on the horizon for quite some time, has finally filtered down to classroom implementation. Teachers now will be managing a mandated curriculum focus that embraces a GCE, and other technologies, toward making mathematics more meaningful and relevant. Materials that even a few years ago did not appear to match existing curriculum or curricular demands, suddenly becomes matched very well. These same materials now may be viewed as essential preparatory work.

Teachers, boards, and Departments of Education in Newfoundland, and in Atlantic Canada, are rapidly moving to supplement that technology component of their schools to support this curricular shift, and are looking for appropriate inservice materials The design of materials used in GC inservices has not yet evolved however. The concern that materials often were exercises in *doing through immersion*, requiring a facilitator's guidance, instead of exercises containing explanations which are enhanced by a facilitator yet could serve as a readable resource outside of the inservice, still exists. There are, of course, materials out there – materials developed by major GC manufacturers and some publishing companies, as well as materials developed by passionate teachers. They bring their own concerns (as mentioned earlier).

The decision to create an inservice manual supporting an institute (or five-day minimum inservice event) arose from the most successful of the inservice experiences of the author – a five-day summer institute. The preference is always to give as much inservice time as needed. But in an already cramped school year, or perhaps as a summer offering, five days seems a reasonable timeline. So, the development of this project (hence called the institute or the manual) titled "Future Mathematics in a TI-83 Graphing Calculator Environment" began.

Method

To deliver on the concerns expressed in the directions proposed first required a study of supporting documents. These included APEF's *Foundation for the Atlantic Canada Mathematics Curriculum*, and the NCTM *Standards*. Then, copies of the APEF *Mathematics Curriculum - Teachers Guidelines* [DRAFT] for all three secondary levels, as well as textbook draft material for level one, were gathered. This was done to ensure the most up-to-date and complete local curriculum material was woven throughout.

Materials from past GC inservices were examined. Research was conducted in the form of conversations with Newfoundland teachers and board coordinators who have attended GC inservices. Meetings were held with the Program Specialist - Mathematics, and the Director of Curriculum for Newfoundland, to ensure the direction of materials and implementation model would be consistent with Newfoundland's interpretation of APEF mathematics. After considerable thought, drafting of a skeleton inservice was initiated and evolution of a distinct inservice operating under new curriculum principles began.

Some essential points of departure for the project rapidly became clear.

- The first one lies in the approach to using the machine. The initial focus of this manual is on operating the TI-83 as a scientific calculator only. The intent is to familiarize the participant with the menu structure and operating system through the common background experience of using a scientific calculator.
- A second difference is in the physical structure of the material. The manual is really divided into three major sections each with its own mandate and approach. Each section has two main parts. The first contains *Notes* and the second contains exercises called *Engagement Masters*. Such a structure is not known by the author to exist elsewhere and should serve the learner well.
- The *Notes* describe the intent and approach of the section. They include point form descriptions of the goals, and the skills covered by each *Engagement Master*. These point form pages are numbered to match the *Engagement Master* numbering, and the checklist nature of the skills listing serves to make finding an exercise with a

particular skill in it easy. The *Engagement Masters* are the activities that create the learning experiences.

- Another essential difference that distinguishes this manual is the cross-referencing of skills. The *Engagement Masters* are sequenced and do presume coverage of previous masters. However, cross-references and quick refresher style comments are provided to assist retention. In addition, if another useful operation on the machine behaves in a manner similar to the one under study, and is not addressed as a separate exercise, it is alluded to and perhaps lightly detailed. Extension ideas are also included, as are ideas for alternate assessment.
- Two appendices, referenced through some *Engagement Masters*, present topics that are not normally contained in secondary curriculum. These topics add another dimension to delivery of other contained topics.
- The quantity of explanation used throughout is substantial. This includes extensive keystroke detailing, inclusion of many screenshots, and management of common error conditions. Successful teaching strategies witnessed or used by the author are integrated throughout where appropriate as *Teaching Tips*.
- A glossary of commonly confused terms or terms whose understanding is essential immediately follows the *Backword* – a section which serves as a conclusion to the manual.
- The complete sphere of materials supporting the TI-83 and its use are discussed. These include supporting publications and electronic discussion groups. Accessory products are described. More importantly, their use is explained and well detailed.

These include the Calculator Based Laboratory (CBL) and TI-Graph Link. The CBL allows the calculator to be used to collect empirical data from field or lab experiments greatly enhancing cross-disciplinary teaching. TI-Graph Link is a computer connectivity package that permits data exchange between a computer and TI GCs.

- The manual is built for expansion as teachers' experiences with new curriculum may place new demands on this type of inservice. So, binder style margins are used and there is a gap in the numbering system between the third major section and the *Backword.*
- Facilitator and registration packages are included as appendices. The facilitator package describes the use of the manual for institute delivery. It explains how the manual is built, describes its use, and includes a complete outline and suggested daily timeline. There is information about setting up and conducting of such an institute including ordering calculators, preparing the site, things to do in advance, etc. It also contains a one-page synopsis that can be used as an interest-gathering device in advance of the offering of such an institute.
- The registration package contains a covering letter with date and time information, introductory information about the institute, and a survey designed to gather information about the entering skill level with GCs of participants. The intention is that this information be sent to participants in advance of the institute as a complete description of the experience. The return of the survey is to serve as a confirmation of registration (recall that the one-pager may serve as the initial registration impetus). The registration package is also available on floppy disk to allow for emailing.

As these materials and ideas were evolving, Mathematics Department Heads from three major high schools in the St. John's, NF area were asked to review sections of materials in early April 1998. They were directed to criticize any part in any manner for approach, language, level of difficulty, or need. Excellent feedback was given – especially as related to language – and many refinements were made from this valuable scrutiny. Need seemed a moot point as all persons involved saw this effort as essential to implementation of the new curriculum, and were pleased to have seen some of that curriculum integrated into a "living" professional development product.

Once the exercises and approach began to take final form, mathematics departments from two schools, who had earlier proofed materials, were approached and offered a short inservice session using activities from the manual. This served as a trial run for some of the materials. Arrangements were made to obtain a class set of calculators and these sessions occurred in mid-May 1998. Feedback was good. A concern was that not enough time was available to gain the experience that the participants wanted. This was because the sessions were necessarily short (a morning or afternoon only) and nowhere near the five days suggested as necessary. However, praise was given to the nature of the materials. Description and explanation was rated as excellent. These teachers were glad to be able to take away materials that they could read and learn with at their own pace at home. Generally the comment was that this approach and resource was badly needed and timely.

A third inservice was offered to the Avalon East School Board in St. John's, NF. It was billed as an introduction to a GCE utilizing the TI-83 and would illustrate some of the skills and approaches that would support the APEF mathematics curriculum scheduled to be implemented at secondary school level one in September 1999. Approximately 30 Mathematics Department Heads or their delegates registered to attend a morning session in late June 1998. These sessions were intended to assist the teachers involved, and served to gather further feedback as to the approach and structure of the project.

The result is a manual can stand as an institute manual for inservice purposes, and can stand as a readable resource outside of an inservice setting. It is written by a teacher for teachers who wish to make the most of an existing underutilized technology now made essential under reformed curriculum. It can serve as an adaptable and integrated package for inservice and contains all necessary information to be used as such.

Discussion

Recommendations, which serve as indications of the expectations upon which the success of this inservice endeavour is predicated, must be made clear. One such recommendation stems from an action by a major manufacturer that occurred in 1995. Hewlett-Packard, in an initiative to place their new HP-38G graphing calculator in the hands of teachers, offered a free evaluation model of that calculator to any legitimate teacher who called a special telephone number. In Newfoundland at least, this translated into a number of free units but not a lot of retail sales (if calculators in classrooms is any indication). In the experience of the author, the quickest and surest way to encourage student and teacher use of a GC is to place a model in the school that teachers can use in instruction. That is, place an overhead GC (the TI product is called a Viewscreen) in secondary schools in the province, and support its placement with inservice such as this institute.

Without doubt, improvement of this scope demands professional development time. Most teachers in Newfoundland have little or no experience with GCs and many are nervous about shifting curriculum and practice. This is clear from the author's conversations with teachers from across the province. Therefore, teachers will have to be assisted in growing from non-user to classroom user with the GC during school year 1998-1999 and beyond. It is difficult to understand how one is to discuss and incorporate the latest technology into teaching, with any credibility, without sanctioned and supported professional development lending it validity.

Another recommendation involves identifying the *bright lights* in teaching, and fostering the sharing of expertise they have gained. This could serve as a validation of the type of good work that is occurring in schools. It can help avoid more teachers re-inventing the wheel as a networking of strategies can occur. It can serve to realize the extent to which Newfoundland has resources and expertise in many, many areas of technology.

On a final note, this institute is an altered perspective on teaching that is dynamic and embraces GC technology as an essential tool in mathematics teaching. It fosters new perspectives on the inter-relatedness of the various forms and representations mathematics can take. It is embedded with APEF mathematics, and reflects the NCTM *Standards* lending it a broader curricular base than Atlantic Canada, and more aptly reflecting its title.

Many teachers may now re-visit what it feels like to be a student. This institute provides opportunities for teachers to experience mathematics from a student's point of view, and demonstrates the benefits of a lifelong learning perspective as it relates to technology. It reflects teaching styles that support teaching in an atmosphere of reform. It provides teachers with examples of mathematics teaching under a GCE.

A lasting goal of this institute is to foster a network of support and expertise as teachers transition from existing to revised curriculum under a graphing calculator environment. As a small assist to networking, the institute package includes a page designed to facilitate the recording of individual contact information from the institute.

After participating in this institute, teachers should feel their skills have been broadened and enhanced, and they will feel the joy of learning and using a technology that is fun, powerful, and immediately applicable under a reformed curricular umbrella.

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FUTURE MATHEMATICS

IN A

TI-83

GRAPHING CALCULATOR

ENVIRONMENT

Institute Manual

Created by

Paul Gosse
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Memorial University of
Newfoundland

NAME & EMAIL	SCHOOL ADDRESS	PHONE NUMBERS	
@		*School* Ph.: Fax:	*Home* Ph.: Fax:
@		*School* Ph.: Fax:	*Home* Ph.: Fax:
@		*School* Ph.: Fax:	*Home* Ph.: Fax:
@		*School* Ph.: Fax:	*Home* Ph.: Fax:

My Favourite Contacts From: **Future Mathematics In A TI-83 Graphing Calculator Environment** Date:

NAME & EMAIL	SCHOOL ADDRESS	PHONE NUMBERS	
@		*School* Ph.: Fax:	*Home* Ph.: Fax:
@		*School* Ph.: Fax:	*Home* Ph.: Fax:
@		*School* Ph.: Fax:	*Home* Ph.: Fax:
@		*School* Ph.: Fax:	*Home* Ph.: Fax:

My Favourite Contacts From: **Future Mathematics In A TI-83 Graphing Calculator Environment** Date:

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A final comment–all known sources and influences have been credited in this work and the author takes full responsibility for any errors or omissions. Images of Texas Instruments Products, and images associated with or copyright by Texas Instruments, are used with permission.

FUTURE MATHEMATICS CURRICULA

IN A

TI-83 GRAPHING CALCULATOR ENVIRONMENT

Foreword

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Foreword

Once again, welcome. All participants should have received an introductory commentary containing information about guiding principles, a graphing calculator environment, graphing calculator manuals (and this manual), other relevant notes, and a confirmation of registration survey. For the convenience of anyone who may not have received that package, it is found as an appendix titled "Registration Package" at the back of this manual.

In addition, this experience is designed to support networking among teachers. The contact sheet (*2 copies) found immediately under the "Institute Manual" cover is placed to make contacting others just that little bit easier.

It is the author's belief that skill with a tool like the graphing calculator requires significant attention being paid initially to the structure and characteristics of the machine. Unless some frame of understanding of structure and function is built, before the numerous capabilities of the TI-83 (or other similar product) are illustrated, no real skeleton exists to hang all the information on.

There are two concerns here. First, recollections naturally fade. This is unavoidable. This manual strives to create a working structure and foundation for the TI-83 that provides strong anchors for learning. The stronger the initial

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framework, the more durable the retention and the more easily knowledge can be refreshed after the fact. Second, some inservice supports may not translate well outside of the inservice environment. That is, a facilitator is needed to make use of materials. This manual has been built to serve as a stand-alone learning tool as well as an inservice tool. Much thought has been put into the sequencing of topics and the connections made so as to enhance readability and make this material useful as a resource entirely on its own - with or without available expertise. Of course, a facilitator can greatly enhance the experience and can provide valuable opportunities for questioning. A facilitator can also tailor the experience to suit the unique needs of participants.

This institute reserves time up front to illustrate the layout, structure, and basic operating principles of the TI-83. This is done through helping participants operate the TI-83 firstly as a scientific calculator. Most operating conventions are established on this more familiar ground. This should help lend a positive outlook to the remainder of the experience.

Every attempt has been made to ensure accuracy of keystrokes and faithfulness to the TI Graphing Calculator Guidebook and TI-83 itself in terms of accuracy of comment and instruction. Should any discrepancy exist between this

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manual and TI material, the TI material should be taken as the reference. Should this hopefully rare instance arise, please inform your facilitator and/or the author (see the *Registration Package* for contact information). Also note that any references made to the TI-83 Manual are to the *TI-83 Graphing Calculator Guidebook*.

Participants have the potential to grow as resource persons for their school, district, and region. Through utilizing the TI-83 in a graphing calculator environment, mathematics and science teaching can be re-vitalized, strengthened, and excitingly broadened.

What makes the approach of this manual different? It is not a collection of activities only that require adjunct expertise to ensure success. In fact, it could stand-alone as a *"TI-83 for Beginners"*, but it is much more than that.

It contains most of what teachers will use everyday under current and future curricular thrusts as written by an experienced teacher. It is written in a detailed yet conversational manner to ensure readability outside of an institute forum. It contains suggestions about exploring and finding other ways to do things, while illustrating what the author believes to be the most user-friendly and convenient approaches.

It is interlaced with *Teaching Tips* and learning experience suggestions, and some TI-83 Graphing Calculator Guidebook references are provided where

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especially helpful. It is designed to be eminently readable. It strives to support the *NCTM Standards*. It offers insights and perspectives designed foster growth.

Some topics here are discussed in a manner that differs from traditional approaches. An effort was made to link the traditional or classic approaches many teachers may be comfortable with, with a more data-driven, exploratory, modeling approach. In fact, the section titled *Basic Training* deliberately bridges the two and periodically slips into a traditional style.

Finally (the last bit of philosophy), some topics can no longer exist under old teaching paradigms and remain viable. They may even be rendered obsolete by technologies that are becoming common in schools. Newer curricular emphases will certainly drive shifts in teaching/learning models-in-action in our schools. Without the support of technology, what would be a simple activity – a fun activity - a tremendous opportunity to see a web of interrelationships - becomes lost in the inefficiency of manual calculations or chalk and talk.

There is tremendous opportunity for growth in our personal views and approaches to teaching. This institute seeks to ignite some of the energy, vision, and paradigm shifts of a reformed curriculum in local classrooms through an enhanced look at a graphing calculator environment.

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Contents

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Contents

Boot Camp	Boot Camp Introduction: Basic data management on the homescreen and in the statistic/list editor. Graphing illustrated and enhanced with tables and other tools. Intersections, optimization, and inequalities also covered.	**Boot Camp Notes** **2-1 to 2-10**
Boot Camp Engagement Masters	1. Making A List And Checking It Twice To manage list data on the homescreen and in the stats/list editor [2-1-1 to 2-1-8] 2. Operations With Lists To perform simple list arithmetic and sort lists [2-2-1 to 2-2-6] 3. Powers of 2 To link lists using formulas, and create scatter and xy-line plots [2-3-1 to 2-3-12] 4. A Linear Relation To explore a two variable discrete data set reflecting a linear relationship [2-4-1 to 2-4-14] 5. Working With Linear Relation To apply the TI-83 graphing toolset to a linear relation [2-5-1 to 2-5-19] 6. Polynomial Graphing To broaden the application of graphing tools using a quadratic relation [2-6-1 to 2-6-12] 7. Inequlaities - Shaded and Otherwise To illustrate and solve inequalities using shading and logical tests [2-7-1 to 2-7-5]	**Boot Camp Engagement Masters** **2-1-1 to 2-7-5**

In The Service	In The Service Introduction: A more data-driven section with realistic applications plus some unfamiliar and familiar topics such as the normal distribution, matrices, and optimization. Strong links between algebra and plots can be found in "Poly Wants A Plot" and "Transforming Plots".	**In The Service Notes** **3-1 to 3-14**
In The Service Engagement Masters	1. Coin Collectors To plot and analyze one variable data [3-1-1 to 3-1-20] 2. The Ruler Drop To plot and analyze one variable data using a frequency table [3-2-1 to 3-2-4] 3. Ping Pong, Golf, Tennis Anyone? To apply two linear regression models to bi-variate data [3-3-1 to 3-3-12] 4. Is This Normal? To use and illustrate the Normal Distribution for sample data [3-4-1 to 3-4-11] 5. No Free Parking! To illustrate a real-life application of step functions [3-5-1 to 3-5-4] 6. Matrices: Systems and Networks To solve systems using matrices, and provide connections among inverse, identity, and determinant [3-6-1 to 3-6-16] 7. Strung Out To model and optimize a rectangular enclosure problem [3-7-1 to 3-7-10] 8. Ploy Wants A Plot! To explore some polynomial graphing patterns [3-8-1 to 3-8-11] 9. Transforming Plots Using the table view to examine transformed functions [3-9-1 to 3-9-5]	**In The Service Engagement Masters** **3-1-1 to 3-9-5**

Appendices		**Appendices**
		9-1-1 to 9-6-14

1. Facilitator Package

   [9-1-1 to 9-1-8]

   - Manual Structure and Institute Description
   - Essential Advanced Planning
   - Essential "To-Do List"
   - Other Planning Information including: assumptions about the calculators; physical arrangements; and contact information

2. Registration Package

   [9-2-1 to *** #s of elements are independent of manual system - total = 24p.]

   - One-Pager
   - Covering Letter Template

   Registration Package - Part I [Outline]

   - Welcome
   - Principles and Structure
   - A Graphing Calculator Environment
   - Timeline
   - A Word About Manuals
   - Final Notes and Contact Info.

   Registration Package - Part II [Survey]

   - Registration Confirmation Survey

3. Friendly Windows

   [9-3-1 to 9-3-5]

   - Setting viewing windows with *nice* pixel steps
   - Friendly Window Magic Numbers for most TI-8* and 92 GCs

TIMELINE

Institute Day	Suggested Master Coverage
Day 1	Welcome Basic Training Masters #1 to 6 Contact Sheet noted
Day 2	Reflection and Discussion of last day's activities Boot Camp Masters #1 to 4 TI Appendix as recommended reading
Day 3	Reflection and Discussion of last day's activities Boot Camp Masters #5 to 7 In The Service Masters #1 to 2
Day 4	Reflection and Discussion of last day's activities In The Service Masters #3 to 6 Institute Evaluation to take home
Day 5	Reflection and Discussion of last day's activities In The Service Masters # 7 to 9 Institute Evaluation collected prior to participant departure

Note: *Any given institute, run with this manual as its main resource, may or may not utilize all activities. That would depend upon audience interest and comfort, available time, and other exercises or resources available. This product was designed to be useable outside of an inservice environment also for that reason.*

FUTURE MATHEMATICS CURRICULA

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TI-83 GRAPHING CALCULATOR ENVIRONMENT

Basic Training

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BASIC TRAINING

General Introduction

This manual has three main sections: *Basic Training*, *Boot Camp*; and *In The Service*. A word about the reference to using Armed Forces' ranks as section titles seems appropriate.

Basic Training and *Boot Camp*, I'm told, are synonymous terms as far as the Armed Forces are concerned. Categories associated with that type of rank advancement seem to fit this context very well. In fact, *Basic Training* and *Boot Camp* together make up a set of skills that are indisputably essential to enhancing teaching and learning under this technological environment. *In The Service* is intended to address everyday skills and topics, embedded in reformed curriculum, in a graphing calculator environment.

The *Contents* section of course outlines the elements of the manual, but it also provides a comment about each main section's notes; the goals for each main section's activity masters; and the main points of each appendix, as a quick reference.

As suggested by the *Contents* statement above, the three main sections in the manual each have two parts. The first part contains *Notes* that serve as a brief

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introduction to the activities to follow. It also contains *Notes Masters* that serve as numbered checklists of skills and topics covered by the activities to follow. The second part contains the activities for that section called *Engagement Masters*.

The *Notes Masters* and the *Engagement Masters* are commonly numbered and partner each other. As mentioned, the *Notes Masters* serve as a checklist of skills covered in each master, and provides a synopsis of the activities. These checklists can serve as quick locators for a skill you want to practice or a master you want to review. The *Engagement Masters* are the engines, the sequenced and guided learning activities, of whole manual.

The *Notes* and the *Engagements* for each section have space reserved for personal note-taking. Please use these spaces as you see fit. Any comments on the activities-concerns-strengths of the manual (or institute itself) are welcome and could be documented in this space also. This could aid in the evaluation process of the Institute.

The *Appendices*, found at the back of this manual, contain valuable material that supplement topics in the masters, provide support information regarding the use of this manual, and provide information regarding the support services offered by TI.

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Any term appearing in **bold print** (upon first use only) indicates that the term appears in the glossary (some calculator commands will be bolded also). Other terms the author considers valuable, or that are commonly questioned, appear in the glossary as well. There is a complete *Table of Functions and Instructions* on pages A2 - A38 and a complete *Index* on pages Index 1 - Index 13 at the back of the TI-83 Manual which provide a very complete resource bank of terms.

If you are using this manual as a personal resource, you may want to read appendix 1 "Facilitator Package" to gain a better idea of how this material delivery was intended to be structured. Or, you may simply want to work through each section's masters. In any case, become acquainted with the *Contents* listing and look for those activities that most meet your needs (working through from the earliest to the last to preserve sequenced elements if necessary).

Basic Training Introduction

Basic Training is designed to describe and make familiar the physical and operating structure of the TI-83, without delving deeply into raw power or becoming lost in the many strengths of the TI-83.

The *Notes* section begins with a comment on the screen, picks up with the physical layout of the keypad, and goes on (and on) about how key sequences are

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written in this manual. The other main sections do not have this type of extensive discussion and detail in the *Notes* section. This was done to provide background practice, in a conversational tone, even prior to jumping into the first *Engagement Master.*

You should activate the keys as the manual talks about them to begin to determine where keys are and what the menu structure is about. Some tips are included, as is some operating advice. You may not recall all of this and that's perfectly fine. What is intended here is that you will recall that Basic Training had a bit that talked about the screen, the physical layout, menus, and key sequences should you ever need to check on something.

Screen

The liquid crystal display (or LCD) is a grid of 94 by 62 **pixels** and is commonly referred to as your viewing **window**. It will display activity for whatever your current application is. For example, if you are using the **graphing editor**, then a window unique to that editor is displayed. If you are accessing an operation under a **menu**, then the menu path is offered until an end choice is made. Much like *Windows* on a computer, whatever your active application or menu is, that is the view you get.

The main window, outside of any editor, is called the **homescreen**. This is where normal calculations take place. You can also use a **split window** on the

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homescreen to view two things at once. This will be illustrated in an activity and in an appendix but will not be a thrust of most work here.

Occasionally, it will be necessary to adjust the **contrast** of the TI-83 LCD screen. Like most TI-graphing calculators, this is done by pressing 2nd and holding ▲ (the up cursor key) until the display darkens sufficiently. Similarly, the display is lightened by pressing 2nd and pressing and holding ▼ until the desired level of lightness is reached. An alternative to holding the ▲ or ▼ cursor key after pressing 2nd, is to press the key once to cause an increase/decrease of a single contrast increment. To increase/decrease another increment requires the repeated pressing of 2nd and ▲ or ▼ however. If at any time the calculator display cannot be seen even though the calculator is on, try increasing the contrast by the method above to correct the problem.

Keypad Layout

The keypad can be examined in three main sections. If we number the rows from the top down we get 10 rows. Row 1 holds the graphing **editor** keys. Rows 2 & 3 hold mainly the editing and cursor keys (plus link, statistics, and list menu keys). Row 4 holds mainly the advanced function keys. And, rows 5 to 10 hold mainly the scientific calculator keys.

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Key Functions - First, Second, and Alpha

Most keys have three main uses... their first, second, and alpha functions. The second and alpha keys, and the corresponding function associated with them, are colour coded: Yellow = second function; and, Green = alpha function. The first function of each key is printed on the key in White.

The physical body of the each key is colour-coded for quick location. They are: Blue = graphing editor, cursor movement, basic operation keys + - * /, and ENTER; Grey = number keys; Yellow = 2nd key; Green = Alpha key; and, Black = all remaining keys.

As a simple example of key use, the ON key is in the bottom left corner of the keypad. It has one other function... to turn the calculator off. OFF is its second or shifted function. Therefore, to turn the calculator off, the second function key (top left yellow key) would have to be pressed before the key labeled ON is pressed. The manner in which key sequences are written throughout this manual from this point on is described under *Keystrokes* coming up.

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Menus

The TI-83 makes extensive use of menus. These are cascading lists of choices, sometimes under several categories, which allows less cluttering of the keypad. When you choose a menu by pressing a key, the screen temporarily changes to show you options under that menu. Once you make an end choice, the screen reverts back to wherever you were prior to entering that menu.

For example, enter 4 / 6 on the homescreen but do not press the ENTER key (see figure 1). To force this result to appear as a reduced fraction, we could access the ▸ **Frac** command which is under the MATH menu. If we now press the MATH key, that menu opens for us and your screen should appear as in figure 2. Note that there are four sub-menus: MATH, NUM, CPX, and PRB. The first sub-menu is active and the

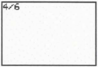

figure 1

figure 2

figure 3

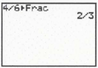

figure 4

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choice we want is the first choice in the MATH sub-menu.Therefore, making this choice is very easy.

To choose the hi-lited option (which is the ▸ **Frac** command) simply press 1. This was an end menu choice and your TI-83 now reverts to the homescreen which should now appear as in figure 3. To complete the calculation on the homescreen, press ENTER (see figure 4). This gives an early illustration of how menus function onboard the TI-83.

Pressing a menu key provides immediate access to that menu. That is, you can jump directly from one menu to another. It is very much like *Windows* on a computer but without the mouse (so far).

Keystrokes

Firstly, you should note that if you end up in a different menu than you intended, or wish to quit a menu at any time, 2nd MODE [QUIT] brings you to the homescreen at all times, and CLEAR will get you out of a menu window and will return you to the active editor or homescreen...this is how we *Quit and Go Home* (see

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Quitting A Menu in the glossary for further information). Already you can see some of how key sequences are written by the ways keys and borders have appeared!

Writing keystrokes (key sequences) in this manual is managed as follows:

➢ Menu use as a first function is indicated by the key name usually wrapped with a border as follows: MATH

Note: Turn the calculator on and find the MATH key again. Press it to see your options. Can you see the $\sqrt[3]{\ }$(choice? Cube root takes one **argument** following the root symbol so we must paste the cube root menu choice to the homescreen first. This is done by pressing the number key associated with the menu choice or by using the up or down cursor key to move to the desired choice and pressing the ENTER key to make the choice. Choose $\sqrt[3]{\ }$(.

Enter a numerical argument for cube root and press ENTER.

Can you think of another way to calculate a cube root? Try it!

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- Menu use as a second (yellow) function is indicated by its first function, preceded by 2^{nd}, and followed immediately by the activated menu enclosed in square brackets as follows:

2^{nd} MATH [TEST]...accesses the TEST menu...2^{nd} function of the MATH key.

> Note: Press 2^{nd} MATH [TEST] now and look at your options under the TEST menu. You can jump from one menu to another directly at any time by pressing another menu key. Sub-menus can be accessed by using the left and right cursor keys when more than one menu **header** appears.
>
> The Alpha function of a key behaves in the same way as the 2^{nd} function but activates the green options above keys. For example, Alpha ENTER [SOLVE] would activate the Solve command (in the correct context).

- Menus under other menus will be consecutively listed and enclosed in square brackets. For example: MATH [NUM] or 2^{nd} X,T,θ,n [LINK][RECEIVE]

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...accesses the NUM sub-menu under MATH, and the RECEIVE sub-menu under LINK a 2nd function of the X,T,θ,n key.

> Note: A sub-menu that is a first choice under a menu key may not always be written. For example, ▸ **Frac** is actually in the MATH [MATH] menu. Since the [MATH] sub-menu is the immediate default under the MATH key, the ▸ **Frac** choice will more succinctly be written as MATH ▸ **Frac**.

- <u>Operations as first or second functions</u> are indicated in the same way as a menu. For example, SIN or 2nd SIN [SIN $^{-1}$].
- <u>Operations under a menu</u> have the menu path written as above but the final function will be written without bracket wrapping and will be bolded. For example, under the MATH menu is another menu NUM. The first operation choice under NUM is **abs (**. This would show as the following keystrokes: MATH [NUM] **abs (**. Though glossary terms are also bolded upon first use, context should clearly distinguish them from operations such as **abs (**.

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Recall Important Note: If you are in the wrong menu, or wish to quit a menu at any time, 2nd MODE [QUIT] brings you to the homescreen at all times, and CLEAR will get you out of a menu window and will return you to the **active editor** or homescreen.

➢ Cursor Keys will be written as follows: ▲ ▼ ◄ ►

Note: Cursor keys can be used for quick scrolling within a menu. Using Alpha ▲ or Alpha ▼ will scroll a menu a page at a time. Menus also wrap… so if a menu continues off the bottom of a screen, you can immediately use ▲ to pick up the list at the bottom or ▼ when on the bottom to pick up the top item. Similarly, ◄ and ► for left and right menu header wrapping. Try this with the MATH menu now as see how it works!

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For quick reference in finding a particular menu location you may have forgotten, a complete **menu map** can be found in the TI-83 Manual.

Cursor Style

Cursor style varies. Much like the cursor on a computer varies in appearance if you are inserting text, deleting text, etc., so does the TI-83 cursor.

Press CLEAR or 2^{nd} MODE [QUIT] [CLEAR] to return to the homescreen and blank it. Your homescreen should now display a regular overwriting input cursor ... a solid 5 by 7 pixel blinking block (see figure 5).

Pressing 2^{nd} activates a backlit *up* arrow within this block indicating a second or shifted function can now be engaged. Pressing 2^{nd} on your TI-83 should cause the homescreen to appear as in figure 6.

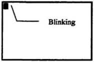

figure 5

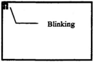

figure 6

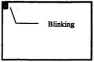

figure 7

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Pressing Alpha activates a backlit letter A indicating an Alpha function can now be engaged. Pressing Alpha now should cause your homescreen to appear as in figure 7.

2nd and Alpha toggle on and off. Pressing once turns them on... pressing again turns them off.

Cursor style also varies when in inserting characters (see the *Insert and Delete Functions* section of *Basic Training*) and with actions within some editors.

Insert and Delete Functions

DEL deletes the single character the cursor is on. If that character is a function name, then the entire name disappears (a function name is treated as one character).

Press a multi-digit number of your choice. The active line you see is a **command line.** Cursor back to a point within your number. Press DEL. Notice the character the cursor was on has disappeared.

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2[nd] DEL [INS] causes the cursor to change to a blinking base-line. Figures 8a and 8b illustrate the number 1234567 with the cursor moved to the 4. Then the insert option was activated. The cursor flashed between the number and the number with the 4 replaced by a short base-line (see figure 9).

figure 8a

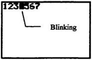

figure 8b

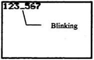

figure 9

Press some numbers now and you should see entries fill to the right of the insertion point. Insert toggles on and off or you can disengage it by simply moving the cursor.

<u>An editing shortcut:</u> Pressing 2[nd] ▶ will bring you to the rightmost character in a command line or field. Similarly, 2[nd] ◀ will bring you to the leftmost character. As with many aspects of the TI-83, hints like this one work in most editors and not just on the homescreen. Experiment!

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Keypad Reserved Variables

2nd [7], 2nd [8], and 2nd [9] hold three reserved **sequence** names (u, v, and w) designed for quick keypad access (Note: there is a complete list of the variable rules and capacities of the TI-83 on page 1-13, and other specific variable notes at other locations, in the TI-83 Manual).

2nd [1] through 2nd [6] hold six reserved **list** names ($L_1 - L_6$) designed for quick keypad access. It should be noted that the reserved sequence and list names above are <u>only</u> available by direct keypad access. This is unlike most operations and characters onboard the TI-83.

Letters A-Z and θ are Alpha characters beginning at row 4 key 1 and ending at row 9 key 4. These can be used to name 27 variable locations reserved for storage of numbers, or to input characters for other uses. These are the only variable locations reserved for real number storage though numbers can be stored in other ways (for example as strings) by creative users.

2nd [(-)] [ANS] is a reserved memory location containing the value of the last executed calculation performed on the homescreen. It is useful as an iterative

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engine and very useful in programming. Press 2 + 3 ENTER. Now subtract one by pressing − 1 ENTER. Notice how Ans printed automatically. This allows chain calculations to be performed easily.

2nd ENTER [ENTRY] is a reserved memory location containing the last keyed entry put on the command line on the homescreen in its entirety. Repeatedly pressing 2nd ENTER [ENTRY] cycles through the previous entries to a maximum of 128 bytes worth of memory at which point it cycles back to the most recent entry. Try it! This can be very useful for re-visiting a calculation and re-executing it, or for grabbing a recent result.

Other Keypad Notes

, is the comma key. It is necessary for separating successive arguments in multi-argument or optional argument functions.

(and) are regular grouping parentheses.

2nd ({ and 2nd) } are list **delimiter** braces.

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2nd [x] [and 2nd [−]] are matrix delimiter brackets.

2nd [0] [CATALOG] contains a list of functions and characters onboard the TI-83, a few of which are not accessible any other way, and is extremely useful for finding a command or function whose menu location is forgotten. The **catalog** list can be quick-searched by pressing the key whose alpha function contains the first letter of the command you wish to access.

2nd [+] [MEM] activates the memory manager menu. All deletions, memory checks, and re-sets occur through this menu. Careful! Actions are not undoable. This is especially important for deletions. Always ensure you move carefully in the memory manager to avoid unintended losses. If in doubt, backup your individual data, or the whole calculator, using the TI-Graph Link software and cables to your PC. Information about TI-Graph Link software and other accessories can be found in the appendix titled "TI Calcs./Access./Support".

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Remainder of the Manual

Basic Training now continues with *Notes Masters*, followed by their *Engagement Masters*, which create the experiences designed to evolve basic competencies and familiarity with the TI-83 and its classroom applications.

Feel free to explore the calculator and refer to any of the information presented thus far and share questions and ideas with other participants or note them so you can share them with others. Remember, this manual was designed to be a readable reference supporting the TI-83. It strives to create an atmosphere of exploration and growth. All feedback is valued.

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BASIC TRAINING NOTES MASTER #1

THE TI-83 AS A SCIENTIFIC CALCULATOR

Goals:
- ✓ To explore keypad layout and menu structure

Description:
- ✓ Clearing the homescreen
- ✓ **ERR: SYNTAX** message

A traditional worksheet format practices:

- ✓ 4 basic operations, exponents and roots, absolute value, exponentials, values of trigonometric functions, LCM, permutations and combinations
- ✓ Keystrokes exhaustively detailed
- ✓ Focus on MATH menu operations
- ✓ Ans and Entry variables discussed
- ✓ Ins and Del functions used
- ✓ Summary questions posed

Basic Training Notes Master #2

Setting Global Modes On The Ti-83

Goals:

- ✓ To understand the effects of changing global mode settings on calculations and display

Description:

- ✓ Norm. ↔ Sci
- ✓ Quit and Go Home
- ✓ Real ↔ Complex
- ✓ Re-setting to factory defaults
- ✓ Sci. ↔ Eng. ↔ Norm
- ✓ Setting default angle measurement units (and over-riding the default on demand)
- ✓ Horizontal split screen briefly explored
- ✓ Summary questions posed

BASIC TRAINING NOTES MASTER #3

Storing and Retrieving Real #s With The TI-83

Goals:

- ✓ To store and retrieve values using real number variable locations

Description:

- ✓ Identification of real number variable location names
- ✓ Store and recall result of computation on homescreen
- ✓ Global nature of variables discussed
- ✓ Creating a counter using a compound command line
- ✓ Solving an equation using iteration with the variable Ans
- ✓ Summary questions posed

BASIC TRAINING NOTES MASTER #4

To Seq Or Not To Seq

Goals:

- ✓ To explore a recursive sequence using Ans on the homescreen and in the Sequence Editor

Description:

- ✓ Iteration on the homescreen
- ✓ Fixing displayed decimal places vs. truncation
- ✓ Global mode set to **Seq**
- ✓ Sequence generated in sequence editor
- ✓ Sequence in table view
- ✓ Setting a window manually based on the table view
- ✓ Sequence in graph view
- ✓ Tracing a sequence graph
- ✓ **Extra** - Storing to sequence variables u, v, or w from the homescreen
- ✓ Storing to function variables Y_n from the homescreen

BASIC TRAINING NOTES MASTER #5

Fib On Who?

Goals:
- ✓ To create and analyze a recursive sequence having two initial terms

Description:
- ✓ Creating a two initial term sequence in the sequence editor using a list
- ✓ View the sequence as a table
- ✓ Set table input from **Auto** to **Ask**
- ✓ View elements of sequence by keying the term number/position in directly

BASIC TRAINING NOTES MASTER #6

½ Way There

Goals:

- ✓ To create and analyze a non-recursive sequence

Description:

- ✓ Iteration modeled using Ans on homescreen
- ✓ Force results above to appear as fraction
- ✓ Derive explicit expression for sequence
- ✓ Model sequence with sequence editor
- ✓ Set TblStart and ΔTbl appropriately and view the table of values
- ✓ Set window based upon table results
- ✓ View sequence as a traceable graph
- ✓ *Teaching Tip* – Population Modeling Reference

FUTURE MATHEMATICS CURRICULA

IN A

TI-83 GRAPHING CALCULATOR ENVIRONMENT

Basic Training Engagement Masters

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Basic Training Engagement Master #1

The TI-83 as a scientific calculator

First a note about the negation and enter keys. The negation key **(-)**, Row 10 Key 4, must be used for inputting a negative number. Subtraction requires two arguments. If the subtraction key is used where negation was intended, you will likely get an syntax error message. That will be discussed shortly. The **ENTER** key, Row 10 Key 5, executes a command line or accepts input. It functions much like the "equals" key on a non-graphing calculator.

Let's start by getting the homescreen up and clearing it. Press 2nd **MODE** [QUIT] and then press **CLEAR**. You should now have a blank screen. This tells the TI-83 to quit any active application or editor and return to the homescreen, and then clear the homescreen. If your screen is not completely empty, press **CLEAR** again.

Notes/Comments/Ideas

A very common message you will see the TI-83 display is: ERR:SYNTAX, 1: Quit 2: Goto . Trust me, it *will* happen ☺. Let's create our first computation and illustrate that message. Press **2 +** . This line is incomplete since addition requires two arguments. An active line like this, which is waiting to be executed, is called a command line. Pressing ENTER causes the mentioned error message to appear.

When a command line with an error in it and is executed, the calculator will say ERR:SYNTAX 1: Quit 2: Goto. If you've already experimented with the calculator you might have seen this. To have the calculator point to where it "thinks" the error/omission is press 2. To quit that command line and return to a new command line, press 1 or CLEAR. If you haven't experimented yet, don't worry. This will be re-stated.

For simplicity, the four basic operations and numbers will not always be bordered and only two decimal places will be used for answers in this Engagement Master.

Attempt the following 23 questions now. You may wish to cover the solutions over or to work through them noting solution strokes as you go. All keystrokes are illustrated for you and answers are provided.

Notes/Comments/Ideas

Compute:

1. 45×3	45 x 3 ENTER	135
2. $5 \div -3$	5 / (-) 3 ENTER	- 1.67
3. Reduce $24/9$	24 / 9 MATH ▸ **Frac** ENTER	8/3

Note:
[1] ▸ **Frac** is in the MATH menu.
[2] A menu choice can be made by using the cursor to get to the choice and pressing ENTER, or by pressing the number associated with that choice. ▸ **Frac** is chosen by pressing ENTER with the cursor on that choice, or by pressing **1** while under the MATH menu.

* Experiment with other MATH menu commands!

4. π^2	2nd ^ [π] x^2 ENTER or	9.87
	2nd ^ [π] ^ 2 ENTER	
5. π^3	2nd ^ [π] ^ 3 ENTER	31.01

Notes/Comments/Ideas

6. $\pi^{\sqrt{2}}$	2nd [^] [π] [^] 2nd [x^2] [√] 2 [ENTER]	5.05
7. $\pi^{\sqrt{2}+1}$	2nd [^] [π] [^] [(] 2nd [x^2] [√] 2 [)] +1 [ENTER]	15.86

Note:
No closing brackets are necessary where syntax is valid and no other arguments follow.

8. $\sqrt{9}$	2nd [x^2] [√] 9 [ENTER] or 9 [^] [(] 1 / 2 [ENTER]	3
9. $\sqrt[3]{9}$	[MATH] $\sqrt[3]{}$(9 [ENTER] or 9 [^] [(] 1 / 3 [ENTER]	2.08

Notes/Comments/Ideas

10. $\sqrt[5]{9}$	5 MATH $\sqrt[x]{\ }$(9 ENTER or 9 ^ (1 / 5 ENTER	1.55
11. $\sqrt{-4}$	2nd x^2 [√] (-) 4 ENTER	2i

Note:
Did you get a non-real result error?
This result can be avoided. Hint: See *Basic Training* Engagement Master # 2: "Setting Global Modes on the TI-83" under the heading *Changing the number system.*
We explore this further but for now you should quit this question by pressing 2nd MODE [Quit] or CLEAR or by pressing 1.

12. $\lvert -3 \rvert$	MATH [NUM] abs ((-) 3 ENTER	3
13. e^2	2nd LN [e^x] 2 ENTER	7.39

Can you find other ways to do this?

Notes/Comments/Ideas

14. e^{-1}	2nd LN [e^x] (-) 1 ENTER	0.37

Can you find other ways to do this?

15. $2e$	2 2nd ÷ [e] ENTER	5.44

Note:
No closing brackets are necessary where syntax is valid and no other arguments follow.

16. $5000(1.0875)^2$	5000 (1.0875) ^ 2 ENTER	5913.28

Can you find other ways? Hint: x^2 key

17. $5000(1.0875)^3$	5000 (1.0875) ^ 3 ENTER	6430.69

Can you find other ways? Hint: ^{3} is in the MATH menu. Since the expression is essentially the same as the last one, try 2nd ENTER [ENTRY] and edit the expression using 2nd DEL INS as necessary. Use this for the next question also.

Notes/Comments/Ideas

18. $5000(1.0875)^{3.5}$	5000 (1.0875) ^ 3.5 ENTER	6706.14
	Can you see how this type of procedure, using 2nd ENTER [Entry], could be used to solve a problem by successive guessing?	
19. *Cos*(2)	Cos 2 ENTER	- 0.42
	You get the result shown if your calculator is set to radians. If your default is in degrees, you can override the default as follows (similarly for radians):	
	cos 2 2nd MATRIX [ANGLE] r ENTER	
20. $Tan^{-1}(1)$	2nd tan [tan $^{-1}$] 1 ENTER	0.79
	Your answer depends upon your angle measurement default.	
21. Find the least common multiple of 3 and 8.	MATH [NUM] **lcm(** 3, 8 ENTER	24
	You may need to scroll the MATH menu to find **lcm(** as it is item # 8.	

Notes/Comments/Ideas

22. How many 5 card hands are possible with a 52 card deck.	52 **MATH** [PRB] **nPr** 5 **ENTER** Note:that the syntax of the nPr and nCr commands is similar to the n^{th} root command.	311875200
23. Repeat #22 without duplication.	52 **MATH** [PRB] **nCr** 5 **ENTER**	2598960

Some final questions

- Do you feel comfortable so far with key locations?
- Do you feel comfortable operating with TI-83 menus?
- Can you quit a menu?
- Can you clear an entry line and homescreen?
- Can you use insert and delete to edit an entry?
- Can you use Ans and ENTRY?

Notes/Comments/Ideas

Basic Training Engagement Master #2

Setting Global Modes on the TI-83

Press MODE. There are eight rows of mode choices. Current defaults are the backlit choices as shown in the first screenshot on the right.

Rows 1 & 2 affect display style. ▶ to **Sci** and press ENTER. Pressing ENTER activates the choice and it becomes backlit as shown in the next screenshot on the right. You could now cursor down to other rows and make similar changes but first let's explore the effect of the one we just made.

2nd MODE [QUIT] will be a common sequence we will use and will be referred to as "Quit and go home" since it returns you to the home screen. So, "Quit and go home" leaving the **Sci** choice active. Note that as soon as a choice is made by pressing ENTER, it is active and remains a default setting. *There is no such thing as an undo key on the* TI-83.

Notes/Comments/Ideas

Press 235 ENTER. It should appear in scientific notation automatically as will every entry now in every editor. Now go back to the mode screen and re-set this default to **Normal**. The number of **decimal places** displayed operates in a similar way by selecting FLOAT or by electing the number of decimals you wish displayed. Change that setting and repeat the activity.

A caution for setting the display to a fixed decimal display is that students may equate the *trimmed* displayed value with the actual value. This bears cautioning and is probably a good reason not to indiscriminately fix decimal places for the display notwithstanding the similar restriction of any fixed number of decimals for display or calculation purposes.

Row 3 sets the default angle measurement choice. We will leave it in **Radian** mode. As you saw in the first Engagement Master "The TI-83 As A Scientific Calculator", this setting can be overridden by manually assigning measurement units.

Row 4 sets the grapher function/relation default and will be discussed in the *Boot Camp* section of the Manual.

Row 5 sets whether plotted pixels are connected to smooth the appearance of a graph or not.

Notes/Comments/Ideas

Row 6 sets whether selected graphs are plotted in order of appearance in the graphing editor or all at the same time.

Row 7 sets the number system used for calculations.

Row 8 sets the homescreen to a full window, horizontal split window, or vertically split graph-table window. Screen splitting is explored in detail a little later. For now, let's explore display style, angle measure units, and number system setting together with a brief look at a split screen.

Changing the Number System

Set your number system to **Real**. It was the default probably.

Compute the square root of negative 4 as you were asked to in the first Engagement Master. You get a non-real result error message.

Set your number system to **a + bi** and repeat the exercise. What happened?

Re-set back to **Real**.

Enter a complex number from the keypad using 2nd [.] [i] and operate with it. Apparently the TI-83 can work with non-real numbers if given one, but it will not

Notes/Comments/Ideas

return a non-real answer unless its number system is set to something other than real.

Re-setting to factory defaults

This is useful if you want to quickly ensure a uniform operating environment for a number of users or if you don't know what else to do to return to the original default calculator settings.

Press 2nd [+] [MEM]. You are presented with five rows of choices. Row 1 provides a **RAM check** which will detail how memory is used and how much is free. Row 2 provides a deletion window for all onboard data. These deletions cannot be undone. Row 3 clears the 2nd [ENTER] [ENTRY] list which is useful in programs. Row 4 clears the contents of all lists but does not delete the list name. Again this is useful in programs. Row 5 is the one we want.

Again, 2nd [+] [MEM] **Reset. All Memory** clears all contents of memory and re-sets factory defaults. Do not do this now. It's roughly equivalent to formatting a hard drive on a computer and will wipe out any and all programs, data, etc. **Defaults** simply returns the calculator to its operating defaults without deleting

Notes/Comments/Ideas

anything. Choose **Defaults** then choose **Reset**. To illustrate one effect of this choice, return to the global modes menu and notice that all settings occur in the left column. These are the factory defaults.

Perform each of the following:

1. Convert 123456 to scientific notation	MODE **Sci** CLEAR 123456 ENTER Note: Don't forget to press ENTER to activate the **Sci** choice or to change any mode setting. CLEAR exits the MODE menu. You could also "Quit and go home".	1.23456E5
2. Convert 123456 to engineering notation	MODE **Eng** CLEAR ENTER	123.456E3

Notes/Comments/Ideas

3. Convert your answer in 2 to normal notation	MODE **Normal** CLEAR ENTER Note how pressing ENTER without making any new command line entry told the calculator to use the last answer ANS variable though Ans didn't appear.	123456
4. Enter 1.234E4 and have the calculator convert to normal notation	1234 2nd , [EE] 4 ENTER Note that this will convert the entry to normal notation if the last question was completed.	12340000
5. Calculate tan (2)	tan 2 ENTER default should be radians.	-2.19

Notes/Comments/Ideas

6. Calculate tan (45°)	tan 45 2nd MATRIX [ANGLE] ° ENTER Note: this overrides the radian setting. An option is to set MODE to degrees first.	1.00
7. Calculate $\tan^{-1}(1)$	2nd tan [$\tan^{-1}$] 1 ENTER Note: this value is linked to the default angle units.	0.79
8. Set screen to a horizontal split and calculate the absolute value of (3 – 4i)	MODE **Horiz** MATH [NUM] **abs(** 3 + 4 2nd . [i] ENTER Note: you can directly exit a menu by directly entering another menu. Also note how a horizontal split screen looks. Calculations that would normally occur on the homescreen are on the lower half of the	5 *on lower half of screen only*

Notes/Comments/Ideas

screen. If any plots were active, they would appear on the top half of the screen. Re-set the screen to **Full**.

Other effects of MODE settings will be discussed under specific editors if appropriate.

- Can you change global mode settings comfortably?
- Can you manually choose angle measure units?
- Can you change to a complex number system?
- Can you re-set the calculator to its defaults?

Notes/Comments/Ideas

Basic Training Engagement Master #3

Storing and Retrieving Real #s with the TI-83

Real numbers can be stored to 27 single letter named locations: A to Z and θ.

Storing a Real Number for Near Term Use

1. Compute 45 x 3 and store it to variable A.	45 x 3 STO→ Alpha MATH [A] ENTER Note that the value of variable A was displayed upon storage. If the contents of a memory location have a value, it is the value that is displayed and used.	135
2. Recall the contents of variable A.	2nd STO→ [RCL] Alpha MATH [A] ENTER ENTER	135

Notes/Comments/Ideas

3. Enter A + 1 directly from the keypad. Alpha MATH [A] + 1 136

The Global Nature of Variables

Suppose the variable location A now holds a number. It will continue to hold that number until another number is assigned there, or until a re-set or deletion action is taken. It is not wise to count on a variable's contents remaining the same if you are running programs. Often these variable locations are needed as storage locations by programs so contents are transient.

Also, the variables X, T, θ, n (the active letter depends upon the graph relation setup in the Global Mode menu) are also global and will be updated upon action within the graphing editor. *Therefore it is unwise to use these as holding locations for any other values.* This can be an advantage! As you trace a graph, you can "Quit and go home" and immediately store the X value (which is updated from the editor and may represent an intersection point for example) to another location, say A, for future use.

Notes/Comments/Ideas

Let's view the current contents of variable X on the homescreen and save to another variable. There are several ways to do this, but the quickest is to grab X from the **X, T, θ, n** key and press **ENTER**. Store this value to A by pressing **STO→** [A]. Notice that the Alpha **MATH** notation was not written here before the [A]. A slow reduction of keystrokes will occur for practiced, repeated actions.

Suppose you were in the graphing editor and had found an intersection point whose x-value you wanted to save and use. You realize that as soon as you move your tracing cursor or calculate another intersection etc., that value of X will be updated. That's why "Quit and go home" and X **STO→** [variable name] can be a valuable technique. It is true for programming also.

A Simple Counter & Compound Command Line

This will illustrate a simple programming technique that can be very useful. We will use 'C' as our counter variable.

Let's see how many times $\sqrt{\ }$ must be pressed to arrive at 1 given that we start using the number 2.

Notes/Comments/Ideas

First initialize our counter at 0:	0 STO→ Alpha PRGM [C]
Then, place a colon to create a compound command line	Alpha . [:]
We store our seed number to variable A for use in our next algorithm and press ENTER to execute these two commands.	2 STO→ Alpha MATH [A] ENTER

Now we set up our continuing algorithm:

Here we will shorten keystrokes again. Alpha PRGM [C] will now be written as C. In fact, from now on, all keystrokes will be reduced unless it's a first use.

Increment our counter by 1 each time, find the square root of A, and replaces the current contents of A with that calculated root.	C + 1 STO→ C : √ A STO→ A ENTER

Notes/Comments/Ideas

Pressing ENTER again, without beginning a new command line, causes the last command to be re-executed. So, if we press ENTER again, C increases by 1, and the square root of A replaces the current contents of A. The value displayed after each pressing of ENTER, is the value of the last executed item in the previous command line.

Keep pressing ENTER now until the display shows the whole number 1 for the first time. Then...

Press C ENTER to view the contents of C which should be the number of times the ENTER key was pressed.	Display should show 31 times!

Why did we use variable A in this exercise instead of the variable ANS?

Another Iteration

1. Solve the equation: $x = \sqrt{x+8}$. First take a guess, say 5.	5 ENTER Which puts 5 in variable ANS initially.

Notes/Comments/Ideas

Now set up the iterative expression.

2nd $\boxed{x^2}$ [$\sqrt{\ }$ ( ] 2nd $\boxed{(-)}$ [ANS] + 8

$\boxed{)}$ $\boxed{\text{ENTER}}$

It is important to recognize here that each execution displays a value, and **that** *value becomes the current value of variable ANS.*

That's why comparing the two most recent results actually is showing ANS and [$\sqrt{\ }$] (ANS + 8) as if it were solving $x = [\sqrt{\ }\ ]\ (x + 8)$.

After four executions of this line performed by pressing ENTER three more times, we arrive at a fairly constant answer... 3.37.

Another Iteration Reference:

Texas Instruments (1996a). CT³ - uncredited exercise.

Notes/Comments/Ideas

- Can you store a result to a real number variable from the homescreen?
- Can you recall the value a variable holds from the homescreen?
- Can you store the value held in one variable location to another location?
- Can you use the variable ANS in a chain calculation?

Notes/Comments/Ideas

Basic Training Engagement Master #4

To Seq Or Not To Seq

This master is designed to illustrate simple sequences on the homescreen initially, and then in the table and graph views. This is our first real rendezvous with these editors. Rather than delve too deeply into graphing and tables, a light treatment is given here. More detail about graphs and tables is given in *Boot Camp*.

Suppose a tree farm has 4000 tree seedlings in inventory. After much thought, it has been decided to harvest 20% of the trees this year and to plant 1000 more for future supply. This should leave more than is harvested. The question is, would a plan such as this be sustainable were it done yearly i.e. 20% harvested with 1000 new plants (ignoring for now, disease, mortality, etc.)?

Various Solutions Using The TI-83:

We can model this on the homescreen using Ans. Try this yourself before examining the approach shown.

Notes/Comments/Ideas

4000 ENTER	Stores 4000 to Ans
0.80 x 2nd (-) [Ans]	Would return 80% of the popⁿ to Ans upon execution
+1000	Would return 80% of the popⁿ plus 1000 to Ans upon execution
ENTER	Returns 80% of the popⁿ plus 1000 to Ans
ENTER	Repeats previous command line and so re-executes: 0.80 x 2nd (-) [Ans] +1000 ENTER

Keep Pressing ENTER until you believe you have enough evidence to answer the question.

Notes/Comments/Ideas

As you may have noticed, the population stabilizes at about 5000 but our screen rapidly becomes cluttered with longer and longer decimals. How can we clean this view up?

There are two ways typically chosen: fixing the number of decimals displayed; and, truncating or chopping off the decimal altogether. Fixing decimal places affects only the displayed number and not the calculated number. In this case, that is very significant as we probably should not consider parts of trees in this problem. Therefore, truncating seems more appropriate.

Decimals can be truncated using the **iPart (** command in the MATH [NUM] menu. This is not the same as the **int(** command in the same menu which returns the greatest integer less than or equal to *x*. So, if we revise our iterative expression to **iPart (** 0.80*Ans) +1000 and run it again, we should have a more accurate model and a cleaner display to boot. This is an example of a recursive sequence (most iterative processes which use Ans can be handled more dynamically by the Sequence Editor).

First, we must set the global mode from **Func**, or whatever it may currently be set to, to **Seq**. We do this by pressing MODE **Seq** (remember to select **Seq** by

Notes/Comments/Ideas

pressing ENTER). This sets the graphing and table editors to accommodate sequences.

This is our first journey into the world of editors. The **y=** key is the inroad to the graphing editor which alternately handles functions, parametric relations, polar relations, and sequences as set by the MODE key. If we view the **y=** or graphing editor now by pressing that key, you will notice the how the editor has set itself up for sequences (see figure 1).

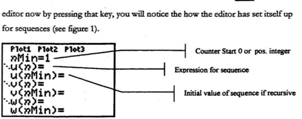


figure 1

Set **nMin** to 1 and **u (nMin)** to **iPart** (0.80* u (n – 1)) +1000 and u (**nMin**) = 4000 by using the number keys and either cursoring to other fields or pressing ENTER to go to other fields. The following illustrates the necessary keystrokes:

Notes/Comments/Ideas

1 ENTER	Puts 1 in nMin and moves cursor to next field.
MATH [NUM] **iPart** (0.80 x 2nd 7 [u] (X,T,θ,n – 1)) + 1000 ENTER	Puts expression in sequence **u**. Note that the X,T,θ,n returns *n* as the global mode is currently set to sequence.
4000 ENTER	Puts initial value of 4000 as u (nMin) i.e we have 4000 seedlings on our farm initially.

It is worth noting here that the sequence variables u, v, and w can only be accessed from the keypad as is the case with list names $L_1 - L_6$. Your screen should appear as in figure 2.

figure 2

Let's view this sequence as a table but we have some setting up to do. Press 2nd WINDOW [Tblset] and set TblStart to 1 and ΔTbl to 1. Then press 2nd GRAPH [Table]. Your screen should appear as in figure 3.

Notes/Comments/Ideas

Scroll down to see other years. You could also set ΔTbl to 5 or 10 to cover time more quickly.

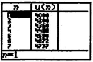


figure 3

So a reasonable domain start and domain step provides a table which generates range values easily. These can be used to establish a suitable window for a graph view.

Let's set a graph view that seems reasonable. Press WINDOW. Set the parameters as in figure 4 and 5. These parameters are discussed in detail a little later.

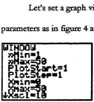


figure 4

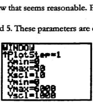


figure 5

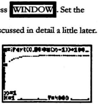


figure 6

Notice the down arrow in the bottom left corner of figure 4. It indicates that the choices on this screen scroll down i.e. continue, using your cursor keys. When you scroll to the first line currently off screen, the top line scrolls off (indicated by an up arrow in the top left corner).

Notes/Comments/Ideas

See if you can anticipate what each parameter in the above windows does! It is important to note here that the n -related parameters are used for calculation in this mode. The x and y parameters are used only for display purposes i.e. axes markings.

Press TRACE and your screen should appear as in figure 6. Try moving around using the cursor keys after you press TRACE. We can see that this sequence appears to be approaching a *limit* and is traceable. You now have an indication of how the TRACE key works. This will also be explained more in an upcoming master. Values can be keyed directly into the trace also. Try pressing 25 ENTER while in the trace screen. What happened? Could you use this for interpolation/extrapolation? What have you learned so far about the table and graphing screens?

Extra

We can also store expressions to variables such as u, v, and w from the homescreen. This requires the expression be enclosed in quotation marks using ALPHA + " before the store key is pressed as follows:

Notes/Comments/Ideas

"iPart (0.80*u (n – 1)) + 1000 " **STO →** 2nd **7** **[u]** **ENTER** (see figure 7). This is the same as keying it directly into the sequence editor. This syntax is discussed further later since, if you were to store data to a program (which we will do), it becomes necessary.

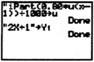


figure 7

Storing a function to a Y_n variable while on the homescreen is identical (see figure 7). Note that the global mode was set back to **Func.** This allowed the **X,T,θ,n** key to give X instead of n. Y_1 was pasted using **VARS** [Y-VARS] Y_1.

The syntax for doing this as shown on the homescreen is identical to storing functions and sequences to those reserved variables using program code.

This will come up often in your own practice as a nice thing to know how to do. It is well worth highlighting!

<u>Reference:</u>

Texas Instruments (1996b). *Getting started: forest and trees.* TI-83 Graphing Calculator Guidebook, p. 6-2.

Notes/Comments/Ideas

Basic Traning Engagement Master #5

Fib On Who?

The Fibonacci Sequence pops up in all sorts of places and is an example of a recursive sequence. This sequence begins with the two terms 1 and 1. Other terms are established by adding the previous two terms. That is,

$$u(n) = u(n-1) + u(n-2).$$

If you are following the *Engagement Master* order, your global mode is still set as **Sequence**. If it is not, refer to *Basic Training Engagement Master* "To Seq Or Not To Seq" and set to **Sequence**. Now go to the **y =** screen and set it for the Fibonacci sequence as in figure 1 below:

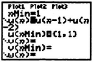


figure 1

A quick help here is to remember that **u** can only be obtained by pressing 2nd **7** and that the sequence variable *n* is available by pressing the **X, T, θ, *n*** key. Additionally, the list brackets are 2nd **(** { and 2nd **)** }.

Notes/Comments/Ideas

Notice that the **u (nMin)** setting contains a list. This is because **u (nMin)**, the initial u-value, actually requires two values here. Therefore, it is given as a list of two values **{1, 1}** such that u (n – 1) and u (n – 2) are both defined and we don't get an error message.

To view this as a table, set Tbl Start at **1** and ΔTbl at **1** (again, see "To Seq Or Not To Seq" to review setting up a table). This will give sequence values in consecutive order. This table, if viewed, would resemble the table we saw in the last Master.

We can key specific values in directly using table view. To do this, change the Tblset setting from **Auto** to **Ask**. Press [Tblset]. Change **Indpnt** from **Auto** to **Ask**. Remember to cursor to the choice you want and press ENTER to select it.

Now press 2nd GRAPH [Table]. Now you should see a blank table waiting for you to input values as in figure 2.

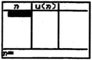


figure 2

Notes/Comments/Ideas

Now any value of interest can be keyed in directly. For example, the 10^{th} term of the Fibonacci Sequence can be established by simply pressing 10 ENTER. You screen should appear as in figure 3.

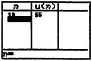


figure 3

Explore the Fibonacci Sequence through the table and graph skills you've now gained.

Notes/Comments/Ideas

Basic Training Engagement Master #6

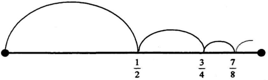
½ Way There

This master explores a non-recursive sequence. You will notice that keystrokes that have become familiar will continue to be slowly be reduced in documentation. For example, 2nd (-) [Ans] will now be shown only as [Ans]. This will commonly be the case with the four basic operation symbols, and operations like ENTER, TblSet, Table, and GRAPH.

Suppose we have a unique grasshopper. This grasshopper is unique because it can only ever jump ½ way toward its destination! The grasshopper spots some food and anxiously strives to reach it by jumping ½ way there, ½ way there, ½ way there.... Does the grasshopper ever make it?

Notes/Comments/Ideas

Solutions Using The TI-83:



A line drawing can illustrate this in a helpful way. It is especially useful to notice the distance that remains after each jump because half of that is what the next jump will achieve. For example, assume the grasshopper is at position 0 or leftmost above and is trying to get to position 1 or rightmost above. So, at 0 the grasshopper will achieve ½ of 1 [the distance currently remaining between the hopper and the food] = ½ . Now, ½ of the original distance remains and half of that is the next achievement i.e. ½ of ½ = ¼ for an accumulated total of ½ + ¼ = ¾. On the third jump, ¼ of the distance remains half of which is 1/8 for an accumulating total of 7/8...and so on.

Notes/Comments/Ideas

Of course, many of us can tell what the sequence is now – as can many students! In the interests of flexing our iterative prowess, we will explore this with the full power of the sequence editor; but first, let's try the homescreen.

On the homescreen, store 0 to Ans by pressing 0 ENTER. We will add our jump distances to Ans as we go. Hopper is at 0 and wants to be at 1. So, the distance in front of hopper is (1 – 0) of which Hopper can make half. Therefore, we want the expression that determines the remaining distance to 1, takes half of it, and adds it to our distance achieved so far.

Since Ans is Hopper's current position, then (1 – Ans) is the remaining distance to the food. 0.5*(1 - Ans) is the next jump distance. Ans + 0.5* (1 – Ans) is the expression which adds the next jump distance to the current position of Hopper. It is the expression we want. Computing this yields a screen like figure 1.

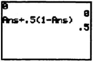


figure 1

Continued execution of this algorithm results in the decimals we would associate with ½, ¾, 7/8, and so on. To have these appear as fractions and make the resulting pattern more obvious requires the ▶ **frac** command in the MATH menu.

Notes/Comments/Ideas

Revise the command line by pressing 2nd ENTER [ENTRY] and editing it to read Ans + 0.5* (1 – Ans) ▸ **frac** and repeat the executions above. Press ENTER enough times such that you are satisfied the expression seems to be approaching a limiting value of 1 (we already knew that 1 was a natural limit for the problem as the grasshopper set out loftily to get there). This results in a screen like figure 2.

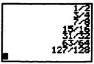


figure 2

Can you create an explicit expression to generate the general or n^{th} term?

Notes/Comments/Ideas

One expression that does this is $\frac{2^n - 1}{2^n}$. Let's use the sequence editor to model this. Set your global mode screen to **Seq** if not already set that way. Press **y =** and clear any sequences present using your cursor and the **CLEAR** key but leave nMin at 1. Set u (n) = (2^(n) – 1)/ 2^n as in figure 3.

figure 3

Setting TblStart to 1 and ΔTbl to 1 then pressing [Table] gives figure 4.

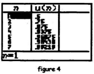


figure 4

Setting a reasonable window, such as the one shown in figure 5a and continued in figure 5b, results in a trace like figure 6.

figure 5a

figure 5b

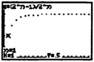


figure 6

Notes/Comments/Ideas

This is strong evidence that this sequence has limit 1. The real question is "Does the grasshopper know that?"

Teaching Tip - Population Modeling

Using two sequences, say u and v, could allow for interesting population modeling (see TI-83 Graphing Calculator Guidebook p.6-14 for a good sample of this). Because successive values are calculated discretely, sequences can be useful for simultaneous comparisons of expressions on the same domain.

Notes/Comments/Ideas

FUTURE MATHEMATICS CURRICULA

IN A

TI-83 GRAPHING CALCULATOR ENVIRONMENT

Boot Camp

Created by

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M. Ed. Candidate
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Boot Camp

Boot Camp Introduction

Boot Camp is designed to bridge traditional curriculum and future curriculum, while building a solid foundation for developing a graphing calculator environment.

A major focus of reformed curriculum involves linking mathematics and real phenomena. This may be done through written exercises, or through conducting experiments and using data to create mathematical models. Hence, facility with lists and the **Statistics/List Editor** becomes essential.

Exercises built upon data collection are more the focus of *In The Service*. *Boot Camp* introduces and develops the skills needed for managing empirical data without requiring actual collection. *Boot Camp* builds upon symbolic expressions initially, but departs from traditional treatments thereafter in an attempt foster extension of topics.

We begin with *Engagements* devoted to enhancing facility with lists and the Statistics/List editor. These masters conclude with a class lesson called the "Powers of 2" which introduces creating scatter and xy-line plots.

Personal Notes

Then a master is presented which does a bit of both; continuing the statistics theme and evolving to linear relations. Familiar topics of tables and graphs are addressed in a more familiar manner and are extended to include finding zeros, using the solver, computing slope, and finding intersections.

The next master discusses polynomial graphing through a quadratic relation, and includes: determining a suitable window from the range shown in the table; restricting a domain for table and plotting purposes; optimization (finding max./min. points); and, testing a function and its inverse graphically for functionality. The masters end with a look at inequalities and the several ways the TI-83 can be useful in their study including shading and logical tests.

The sequence of these Masters is intended to permit an innovative course in the approach taken to graphing. For those who have used graphing utilities before, all the typical graphing tools are illustrated. There is a definite bridging of data to symbolism, and there are some bridges built that link tools thought married to calculus with grade ten mathematics. Functional topics such as the vertical line test are addressed, and logical testing plays a significant role here and should be thought of as a foundation topic to further study.

Personal Notes

BOOT CAMP NOTES MASTER #1

Making A List And Checking It Twice

Goals:

- ✓ To manage list data on the homescreen and in the stats/list editor

Description:

- ✓ Uses L_1- L_6
- ✓ Create a list on the homescreen
- ✓ Store a list to a reserved list variable name
- ✓ View a list in stats/list editor
- ✓ Edit a list on homescreen using **Rcl** command
- ✓ Edit a list element in stats/list editor spreadsheet view
- ✓ Clearing a list in the stats/list editor
- ✓ Creating a list in the stats/list editor
- ✓ Summary questions posed

BOOT CAMP NOTES MASTER #2

Operations With Lists

Goals:

- ✓ To perform simple list arithmetic and sort lists

Description:

- ✓ Create two lists on the homescreen
- ✓ Perform simple arithmetic with lists
- ✓ Grabbing a list element
- ✓ Brief discussion of **Dim Error** message
- ✓ Sorting a list in ascending order
- ✓ Paired sorting of related lists
- ✓ Exploring the [LIST] [MATH] menu including maximum, minimum, mean, median, sum, product, standard deviation, and variance
- ✓ Summary questions posed

BOOT CAMP NOTES MASTER #3

Powers of 2

Goals:

- ✓ To link lists using formulas, and create scatter and xy-line plots

Description:

- ✓ A short *Powers of 2* lesson on the homescreen
- ✓ Clearing a list in the stats/list editor
- ✓ Linking a formula in the stats/list editor
- ✓ Unlinking a formula in the stats/list editor
- ✓ Sort of paired data
- ✓ Define a scatterplot in the Stat Plot editor
- ✓ View a scatterplot
- ✓ Manually set a viewing window - window parameters described
- ✓ **ZoomStat** command applied
- ✓ Free-cursor coordinates. vs. trace coordinates
- ✓ Formatting a graph window - **GridOn, GridOff**
- ✓ View an xy-line plot
- ✓ Summary questions posed

Boot Camp Notes Master #4

A Linear Relation

Goals:
- ✓ To explore a two variable discrete data set reflecting a linear relationship

Description:
- ✓ Tape & CD Problem posed
- ✓ STAT [EDIT] **SetUpEditor** command
- ✓ [LIST] [OPS] **Seq(** command
- ✓ Scatterplot created – [List] [Names] menu used
- ✓ XY-line graph created and traced
- ✓ Bridge to continuous relation through posing a similar problem
- ✓ TI-83 plotting order
- ✓ Graph style and selection status indicators noted
- ✓ **ZoomStd** introduced
- ✓ Creating Y_1 = function
- ✓ Switching between active plots
- ✓ Turning a stat plot on/off in the graphing editor
- ✓ Tracing a function
- ✓ Turning all stats plots on/off -**PlotsOn, PlotsOff**
- ✓ Deleting List Names
- ✓ Summary questions posed

Boot Camp Notes Master #5

Working With Linear Relations

Goals:

- ✓ To apply the TI-83 graphing toolset to a linear relation

Description:

- ✓ Clearing the y = list
- ✓ Setting up a table view of function
- ✓ Using a table to zoom in and zoom out
- ✓ Table view of function
- ✓ Table view discussed and extension questions posed
- ✓ **ZStandard** applied
- ✓ **Zoom In/Out** applied
- ✓ ZOOM [MEMORY] **SetFactors...** command
- ✓ Y-intercept found using evaluation feature of Trace
- ✓ **ZBox** - Zoom box command applied
- ✓ [CALC] menu illustrated
- ✓ X-intercept found using the [CALC] **zero** command
- ✓ MATH **Solver** command applied to X-intercept
- ✓ **dy/dx** and a discussion of slope

Description (c'tued):

- ✓ *Teaching Tip* - Mental Arithmetic Using Tables
- ✓ *Teaching Tip* - End Behaviour Using Tables (including finding zeros and other values)
- ✓ **A Dalliance With Intersect** - solving equations by graphing
- ✓ Cost-Revenue Activity - for independent work
- ✓ Summary questions posed

BOOT CAMP NOTES MASTER #6

Polynomial Graphing

Goals:

- ✓ To broaden the application of graphing tools using a quadratic relation

Description:

- ✓ Create a quadratic relation using a physics formula
- ✓ Set a realistic domain and view relation with a table
- ✓ *Teaching Tip* – Tables and Ranges
- ✓ Manually set a graphing window based upon table view
- ✓ Plot a quadratic function - algebraic domain vs. real domain
- ✓ 9 screenshots shown - story requested
- ✓ Using logical tests to restrict a domain i.e. the MATH [TEST] menu
- ✓ *Teaching Tip* - Piece-wise Defined Functions
- ✓ Finding The Maximum - [CALC] **maximum** command
- ✓ *Teaching Tip* - A Test For Functionality including the illustration and application of DRAW **Vertical**

BOOT CAMP NOTES MASTER #7

Inequalities – Shaded And Otherwise

Goals:

- ✓ To illustrate and solve inequalities using shading and logical tests

Description:

- ✓ Changing plot style options to shade above and shade below a function
- ✓ Plot shaded regions identifying overlap
- ✓ Create solution functions to the inequality using logical tests without changing plot style
- ✓ Plot solution functions
- ✓ [DRAW] **Shade (** command

FUTURE MATHEMATICS CURRICULA

IN A

TI-83 GRAPHING CALCULATOR ENVIRONMENT

Boot Camp

Engagement Masters

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Boot Camp Engagement Master #1

Making a List and Checking it Twice

Since this is about lists, let's begin with a list!

- A list is simply a collection of numbers. It may be linked to another list, as in a relation, or may stand alone as data.
- A list may be keyed in number by number, or may be generated by a command or a function.
- A list may be viewed on the homescreen, or by the spreadsheet view of the Statistics/List Editor.
- Since some familiarity with the structure of the TI-83 has likely been gained by now, complete keystroke illustrations will slowly be reduced. Screenshots and complete keystroke steps will be given for new or more complex procedures.

The Statistics/Lists Editor may at various times be referred to as the stats/list editor or the stats editor...an author quirk ☺

Notes/Comments/Ideas

Forming a List:

We will now create a list to hold the data: 25, 16, 9, 4, 1. Remember, { and } are list delimiters or containers and mark the beginning and ending of a list.

2nd **(** [{] 25 **,** 16 **,** 9 **,** 4 **,** 1 2nd **)** [}] **ENTER** (see figure 1). Store this list to reserved list name L_1 by pressing **STO→** 2nd **1** [L_1] **ENTER** (see figure 2).

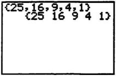

figure 1

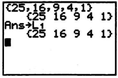

figure 2

Note: the statistics spreadsheet view should already be set up to view and edit the reserved lists $L_1 - L_6$. It can view and edit any list but must be told specifically if others are to be used. If

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your view does not match figure 3 in a moment, press STAT ***SetUpEditor*** *and* ENTER. *Then return to the stats editor.*

We just viewed L_1 on the homescreen. Now let's view L_1 in the Statistics editor. Press STAT [EDIT] **Edit** (see figure 3). We also see other reserved list names as column headers. They are probably empty right now.

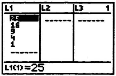

figure 3

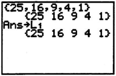

figure 4

Quit and go home. The homescreen probably still looks like figure 4. Now, recall and edit L_1 by pressing 2nd STO→ [RCL] 2nd 1 L_1 ENTER ...the contents of L_1 should show as a command line. Using your cursor and Ins and Del skills, change the 16 to a 100. When you're done, press ENTER to accept the changes. The changes you made are saved as soon as you press ENTER and will

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show whenever you view L_1 on the homescreen, in the Stats editor, or anywhere else. Go back to the Stats editor and view L_1 and see if your changes are there.

We can also edit L_1 in the Stats editor. Go to the Stats editor now by pressing STAT [EDIT] Edit. The changed L_1 should be there, the cursor should be on the first element, and your screen should look like figure 5.

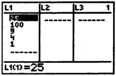

figure 5

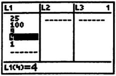

figure 6

Cursor down the column. Notice how each entry becomes backlit, and how the list name, element number, and element value show on an entry line at the bottom of the screen. Pressing ENTER while on an entry makes that entry editable on a command line which shows at the bottom of the screen. Try it!

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Change the 4 to a 121 and press ENTER. Now the list should read as in figure 7. Notice the edited entry has changed and the current choice moved down one. Once you press ENTER, the cursor should move down one position.

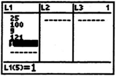

figure 7

Important Note:

At the top of this screen view (see figure 7) you can see the list names in a row. This is called the header row and the list names called headers. L_1, L_2, and L_3 are the headers in view. The number 1 on the right means simply that the first list, of those currently appearing in the editor view, is the active list

Cursor up to the header of list 1. Notice how the header is now backlit and how the contents of list 1 now appear on a command line at the bottom of the screen. Similar to just cursoring down elements, we are currently just viewing

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contents. If we press ENTER though, we make the whole list editable at the bottom of the screen. Try it!

We can use this insight to quickly edit several values in a list, or to clear a list. Go back to the header of list 1 and make it editable. We could edit this list now by using our cursor keys, and our **Ins Del** skills, pressing ENTER to accept the edit. We will now clear the list by pressing CLEAR. Press ENTER to accept the new list contents i.e. in this case it accepts a cleared list. List 1 is now cleared.

A list can also be created from scratch in this editor. Go to the list 1 header again, make it editable by pressing ENTER, use { and } to contain a list of numbers of your choice as we did on the homescreen earlier, then press ENTER to accept it. Make a note of your revised values.

Let's re-visit the homescreen just to be sure L_1, as you most recently revised it, views the same there (check it against your note). Remember that we do not need to use the recall feature to view a variable's contents. Just pressing 2nd 1 [L_1] ENTER will place the current contents of L_1 on the homescreen. You should get a screen like figure 8 (in this case, the contents of list 1 were simply re-created after

Notes/Comments/Ideas

clearing). Your list 1 data should present the same data whether viewed in the Stats/List editor, on the homescreen, or through any other means.

Recalling the discussion in Basic Training Engagement Masters: "Setting Global Modes in the TI-83" and "Storing and Retrieving Real #s with the TI-83", *all variable locations contain information as it was last posted there regardless of where you are onboard the machine. Therefore, where you view or edit L_1, say, is irrelevant. It's the same location... just stored to, or viewed, through multiple means.*

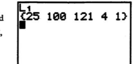

figure 8

A reminder that from now on, direct entry of a keypad variable will be shortened to the entry itself for the common key sequences we've worked with so far. For example, instead of having to say: press 2nd 1 [L_1] , the indication will just be: press L_1 . Where a key sequence is new or not often used, full keystrokes will be illustrated.

Notes/Comments/Ideas

- Can you create, recall, and edit a list on the homescreen?
- Can you store a list to $L_1 - L_6$ on the homescreen?
- Can you view an existing $L_1 - L_6$ in the Stats/List editor?
- Can you create, view, and edit a list in the Stats/List editor?

Notes/Comments/Ideas

Boot Camp Engagement Master #2

Operations With Lists

Think of lists as data. What are some of the things we commonly do with data? Sort it? Sort it as pairs, as in last name-first name, or in larger dependent orders? Multiply the data by 1.15 to calculate our HST? Average some of it and use different factors on different averages to create cumulative marks?

The following examples illustrate how to perform some simple list operations.

Adding Lists and Numbers

Using your recent skills, make two lists as follows: $L_1 = \{5, 4, 3, 2, 1\}$ and $L_2 = \{2, 4, 6, 8, 10\}$. Now add 3 to all elements of L_1. On the homescreen, press L_1 + 3 ENTER. Your screen should look like figure 1 below.

Now subtract 3 from the list by just pressing – 3 ENTER. Your screen should look like figure 2. Subtract L_1 from L_2 (see figure 3). *Notice how if lists are operated on together, individual elements pair and operate with the element in corresponding position.*

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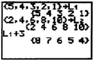

figure 1

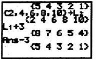

figure 2

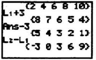

figure 3

We could illustrate the first four multiples of a number by taking the number desired and multiplying it by the list $\{1, 2, 3, 4\}$. Try generating the first four multiples of 9 this way.

Grabbing a list element illustrates a method of naming elements for functions, lists, arrays, etc. that can be very useful. Much like the way we would indicate the evaluation of a function at a given value by writing $f(3)$, an element of an existing list can be grabbed by pressing L_1 (5). This returns the value of the fifth element in L_1.

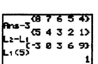

figure 4

Currently, $L_1 = \{5, 4, 3, 2, 1\}$ as in figure 2 (if you completed the last action successfully). Pressing L_1 (5) now should give a screen like figure 4. It returns 1 since

Notes/Comments/Ideas

it is the fifth element of L_1. The same technique can be used to grab elements of matrices, and evaluate specific functions for a given value.

To perform operations using two or more lists together, requires that the list be equally dimensioned (i.e. the same length). Otherwise you get a **Dim Error** representing items of disagreeing dimensions. You should note that currently L_1 and L_2 are the same size. This is necessary for our L_1 and L_2 calculation and for our paired sorting command (coming up) to work properly, and, is necessary for linking formulas if more than one list is involved eg. L_3 = "$L_1 + L_2$".

Sorting a Single List

Sorting, like other list commands, is found in the LIST menu. View the main LIST menu now by pressing 2nd STAT [LIST] (*Note: from now on, the indication will just be* [LIST]). Cursor to [OPS]. Notice that the first operation under [OPS] is **SortA(** ... meaning sort ascending. This operation can take one argument or more where the argument is a list ('cause lists are used to hold raw number data we love to sort!).

Now we sort L_1 in ascending order, and then view L_1. [LIST] [OPS] **SortA(** L_1 ENTER L_1 ENTER. Note that some keystrokes are omitted now (for

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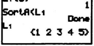

figure 5

example, 2nd **1** wasn't written for L_1). Your homescreen should appear as in figure 5 .

This could have been done more efficiently by using a compound command line (you may recall having done this earlier). Can you create a compound command line that sorts L_1 in ascending order and views it with one press of ENTER?

Sorting Paired Data

Recall that L_1 currently holds {1, 2, 3, 4, 5} since it was sorted, and L_2 still holds {2, 4, 6, 8, 10} from a while ago. Let's view them in the Stats editor. You screen should look like figure 6.

Together they could represent the pairs (1, 2) (2, 4) (3, 6) (4, 8) and (5, 10). If we wanted to view these related pairs but sorted with the first elements in *descending* order, we could do the following: [LIST] [OPS] **SortD(** L_1 , L_2 **ENTER**. Notice how you automatically got bumped out to the homescreen since that's the only place this command can run right now. The list order is important because the first one is

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the sort order key, and the other is the related data. Your homescreen should now show a "Done", message and look like figure 7.

If we view the list now in the Stats editor they should be sorted and appear as in figure 8.

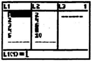

figure 6

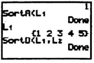

figure 7

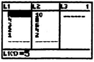

figure 8

Look at the other things under the [LIST] [OPS] menu. A practical menu also is the [LIST] [MATH] menu where you can find functions for the smallest, biggest, mean, median, elements of a list and much more.

Let's see if the syntax of the **mean** (under [LIST] [MATH] is intuitive. Try it by attempting to calculate the average of your current L_1. Actually, facility with all the [LIST] [MATH] commands is useful. Sooo…

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...use the [LIST] [MATH] menu to calculate the maximum, minimum, mean, median, sum, product, standard deviation, and variance of a single list. What happens if multiple list names are supplied as arguments for these operations? Try it!

The TI-83 quite comfortably handles many operations within discrete mathematics. To test your own level of comfort, consider the following:

- Can you perform simple arithmetic with a list?
- Can you sort a single list or two paired lists?
- Can you apply the [LIST] [MATH] operations to one or more lists?

Notes/Comments/Ideas

Boot Camp Engagement Master #3

Powers of 2

A Class Lesson – Exponent Patterns

$2^3 =$
$2^2 =$
$2^1 =$
and so on

Typically, a teacher may write the following table on the board and prompt students to explore what they think is the pattern in the powers of 2. We can use our homescreen to do that.

2 ^ 3 **ENTER** 2 ^ 2 **ENTER** 2 ^ 1 **ENTER**

2 ^ 0 **ENTER** 2 ^ - 1 **ENTER**

Notice how the values are scrolling off the top of the screen (see figures 1a-c)

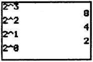

figure 1a

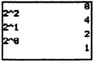

figure 1b

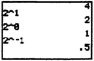

figure 1c

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Ouch... 2 ^ (- 1) displays as a decimal! We may be just as well off with the board! *There's another way too though. Use a list.!*

On the homescreen, try 2 ^ { 3, 2, 1} ENTER. Have students relate the answer list values to the input list values. You can see that <u>a list can be used just about anyplace a numerical argument can go</u> including in an editor.

Try 2 ^ { 0, - 1, - 2} ENTER. *Hint: If you don't want decimals here, try to remember a command that forces fractional answers and incorporate it into your command.*

There's another way to view the powers of two, without having to re-key the expression or create an overly long list, using the Stats/List editor (referred to interchangeably as the Stats or Stats/List Editor).

Open the Stats editor and clear any lists that appear. To do this, go to the header lines, press ENTER to make them editable, press CLEAR, and then ENTER to accept the cleared list. There are other efficient ways to clear lists if more lists are involved (see **Clearing a List**). Put the values 3, 2, and 1 in L_1. We will now make L_2 produce the values of 2 ^ L_1.

Make the header of L_2 editable. You screen should appear as in figure 2.

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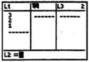

figure 2

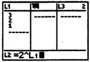

figure 3

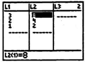

figure 4

Enter the formula you want L_2 to execute directly. *Note: We are entering a formula and not list elements, so we don't use { }*. Press 2 ^ L_1 as in figure 3. Then press ENTER to accept this for list 2 (see figure 4).

Go to the header of L_2 again. Notice that the formula doesn't show… only the contents show! That's because of the way we entered it. Similar to a homescreen calculation, we performed a calculation so only the results were displayed.

We can force the TI-83 to act somewhat like a spreadsheet and generate values in L_2 upon placing a new value in L_1. This requires the same formula as we just used but we force re-calculation by telling the TI-83 we want to link L_2 to L_1 using a **linked formula** as follows:

Make the header for L_2 editable again. Clear the present contents. Now press Alpha + ["] before entering the same formula as before i.e. 2 ^ L_1 (see figure 5).

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You don't need to close the " as no arguments follow it. Press ENTER. Your screen should now look like figure 6.

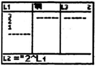

figure 5

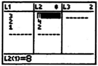

figure 6

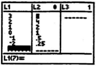

figure 7

Notice how the results are displayed as they were before but now the header appears differently. Notice that the header now shows a dot. This indicates the linked nature of a formula associated with that list name.

The formula can be now viewed by cursoring up to the header. You may recall that when we used 2 ^ L_1, without linking it as a formula, cursoring to the header showed only the contents of that list. No evidence of the formula remains after it is executed unless it is a linked formula.

Lists can also be created, edited, and have formulas attached to them from the homescreen (see TI-83 Manual section 11-7, 8). The same syntax applied to

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storing to a list from the homescreen as we saw as an EXTRA late in the *Basic Training Engagement Master* "To Seq Or Not To Seq".

To unlink a formula, cursor to the header of the linked column, press ENTER to make it editable, CLEAR to clear the link, and ENTER to accept the cleared link. Note that the values generated by the previously linked formula remain. To clear those as well, simply clear the list as we did earlier in this master (header-ENTER-CLEAR-ENTER).

Continue the "Powers of 2" exploration by entering other values of interest in L_1 and seeing L_2 provide the results as you go. *Note: The author is unaware of how to force this editor to display fractions (see figure 7). Should you discover a way, let me know* ☺

Utilizing the Stats/List editor in this way can be valuable when exploring models in which data is used to create results. These results can be viewed as we just did, or can be plotted. Plotting data lists against each other is essential in statistics.

We will now create a scatterplot and an xy-line plot to view the powers of 2. First, sort the data in ascending order by L_1 (see the *Boot Camp Engagement Master* "Operations With Lists" to review this). The lists should now be paired such that (- 2, 0.25) (- 1, 0.5) ... represent the paired members of the lists from top to bottom in the stats editor.

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We now need the **y =** key but there are aspects of this key that should be made clear first. The two uses of this key are the activation of the stat plot editor, and the activation of the function editor. Stat plot setup is accomplished through the 2nd **y =** [Stat Plot] editor. The **y =** or function editor allows us to set up functional, parametric, polar, or sequence graphing depending upon the graphing mode the calculator is set to. Both editors share the graph screen, and a stat plot's on/off status can be toggled from within the **y =** editor. Otherwise, they are distinct editors.

Define your stat plot by activating the stat plot editor._ 2nd **y =** [Stat Plot]. You should now see a menu with 5 choices. The first three are essentially the same. They allow you to define up to 3 active stat plots. Choices 4 & 5 turn all or selected plots on/off from the homescreen or a program. You should see a screen resembling figure 8 (there may be some slight differences in details but nothing of any concern). Then choose **1:** which should give you a screen like figure 9.

Notes/Comments/Ideas

figure 8

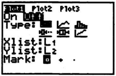

figure 9

Use your cursor keys to move and press ENTER to select a choice. Now select **On**. There are now 6 types of plot styles to choose from: scatterplot, xy-line plot, bar graph, box plot, modified box plot, and normal probability line plot. Choose the scatterplot and make Xlist and Ylist $L_1$and L_2 respectively using the keypad. Choose a mark style you are happy with. Press GRAPH.

Is this a view you expected? What happened?

We can view the screen parameters responsible for this particular window. Press WINDOW . Change the parameters to those shown in figure 10.

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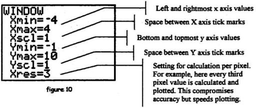

figure 10

This is how we manually choose a window and establishes the notion that any view is arbitrary. It's like any window… you only can see what happens to be viewable in that window at that time; you can revise your view by re-positioning yourself; and you can never see everything. Choosing windows manually becomes especially important with bar graphs for defining the frequency cutoff points. However, a very handy Zoom menu choice sets up virtually every other statistics plot nicely.

Press ZOOM. The ZOOM menu offers many window options. Most are obvious and will be explored in future masters. For now, we apply only [ZoomStat] by pressing ZOOM [ZoomStat]. Notice that the graph window is immediately re-

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plotted and displayed (see figure 11). This happens when any zoom instruction is executed.

Note: Don't use ZoomStat blindly! This is an example of how technology can assist teaching. For example, once ZoomStat is applied and the data is viewed, look at the window settings and decide whether they truly reflect the data, how good a window it really is, and whether you want to keep it. Critical thinking can be fruitfully applied here.

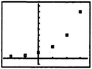


figure 11

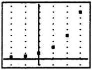


figure 12

figure 13

Now, move your cursor around the window by pressing the cursor keys and see what happens. Try to free-cursor (i.e. move using the cursor keys in the graph screen without first pressing TRACE) directly over the point (- 2, 0.25). Can you do it? What do you notice? Now, trace the plot. Press TRACE. Now, cursor around again and note the differences in the TRACE movement vs. the free-cursor movement. Discuss these with others.

Notes/Comments/Ideas

The long string of decimal places which normally appear as a result of cursor movement in a graph screen are sometimes a distracter for students and can occasionally be inconvenient for other purposes. A valuable teaching tip has been included in the appendix called *"Friendly Windows"*. It illustrates how to create windows with nice decimal pixel steps so the decimal places which present while tracing are not cumbersome. There is also a command in the ZOOM menu called ZDecimal which actually creates a *"Friendly Window"* for you.

Let's play a little with the format of our view. Press 2nd ZOOM [Format]. This small editor allows us to personalize the graph window display format. For example, in the 2nd ZOOM [FORMAT] menu, activate **GridOn** and **CoordOff** (remember, you have to cursor to these choices and press ENTER to select them). Return to your plot by pressing GRAPH and see figure 12 (note that pressing GRAPH causes an immediate switch to the graph view). Move your cursor freely again without pressing TRACE. What changed? Try tracing now. What do you notice that is different than before? Explore the other formatting options. They apply to the graph window regardless of what is plotted.

Notes/Comments/Ideas

We happen to know that the powers of 2 aren't discrete. Let's revise our plot to an xy-line plot to make it "look" continuous.

[Stat Plot] [Plot 1] and change the plot type to xy-line. Press GRAPH (see figure 13). Try tracing again. Are there any surprises in the way the graph looks and the way the trace behaves? Note any interesting conclusions. Stat Plot is of course limited to discrete data. Continuous relationships are best managed though the table and function graph editors though this activity illustrates that explorations can certainly occur in the Stat Plot mode.

It should be clear that even a discussion of limits, exponential growth, etc. can be generated with this fairly simple exercise.

Notes/Comments/Ideas

- Can you link lists through formulas?
- Can you create a scatterplot plot from data?
- Can you create an xy-line plot from data?
- Can you have the TI-83 refine or choose a view for a plot?
- Can you manually choose a view for a plot?
- Can you trace a plot?
- Can you format a plot?

Notes/Comments/Ideas

Boot Camp Engagement Master #4

A Linear Relation

This engagement is designed to take advantage of the Stats/list editor skills gained in the last master. We use the Stats/list editor to hold data, and use the graphing editor to view the data as a scatterplot or an xy-line plot. The opportunity again arises to create a linked formula. In this case, linked formula skill is essential and is used as a bridge into the topic of function graphing.

The master that follows this explores all aspects of function graphing in a more traditional style.

Suppose you have a carrying case that has six spots which can fit either a CD or a Cassette Tape but not both. For example, if you choose to take one CD, you could take up to five cassette tapes. What are the possible combinations of CDs and Tapes the case could carry for us? Could we represent the list of possibilities as a set of ordered pairs:

Ordered Pairs:

Notes/Comments/Ideas

Classify this data discrete or continuous. ____________________

Hint...Can you put 5 ½ CDs in there? ...π CDs in there?

Write a formula relating these two quantities. ____________________

Hint... Use names suggestive of the data in your formula if you like.

Now let's use the stats/list editor and create our own list names to represent this data. We now explore how to name our own lists other than with L_1 etc.; how to setup the editor with only the list names we want; and how to use the **seq (** command to save us having to type 1,2,3,4, etc. all the time.

Let's use the list names **Tapes** and **CDs** to represent our data. If you've chosen other names, then substitute those where Tapes and CDs are used here. If we use the ordered pairs (Tapes, CDs) as a convention, then the ordered pairs we have are: { (0 , 6) (1 , 5) (2 , 4) (3 , 3) (4 , 2) (5 , 1) (6, 0) }.

There's a wonderful command called **SetUpEditor** that can be very useful for tailoring the stats/list editor headers. It can set up the editor with up to 20 list names that you choose (it creates them if they haven't been already created) in the order you choose. It can also easily re-set the stats/list editor to display $L_1 - L_6$ again

Notes/Comments/Ideas

whenever you want. The only restriction is the list names must follow list name rules – basically a maximum of 5 characters and can't start with a number.

STAT [EDIT] **SetUpEditor** TAPES, CDS ENTER

Grabbing the SetUpEditor command puts you on the homescreen.

You'll want to use 2nd ALPHA [A-Lock] to ease the letter typing.

Don't forget to use a comma to separate the two arguments i.e. the list names.

You should see a **Done** message on your homescreen.

You should now see the following homescreen (see figure 1). [Stat] [Edit] should show two new headers, with one spare empty header which is always available, as you named them in the Stat Editor View (see figure 2).

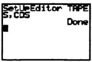


figure 1

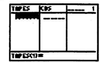


figure 2

Notes/Comments/Ideas

Notes:

1. The stats/list editor always reserves one spare column for use.
2. Lists that are created this way appear in the stats/list editor and the list names menu but have dimension zero (i.e. no length) until you put data into them (i.e. we can't yet use these names in calculations or linked formulas as dimension errors will occur).
3. SetUpEditor creates new list names that each take up memory and can quickly clutter up your list names menu. Use this naming style to aid work only when necessary. Most times, $L_1 - L_6$ function perfectly well if you keep track of what represents what.
4. We will delete the list names we create here at the end of this activity. *Note:* to save this data for future loading without having the list names show up anywhere until needed, see Teaching Tip – Saving List Data As A Program (*In the Service Engagement Master* "Coin Collectors").
5. For future reference only, lists can be inserted or deleted from the stats/list editor view by cursoring to a header and pressing Ins or Del. Using Del doesn't delete the list from memory, only from this view.

Notes/Comments/Ideas

Don't do this now unless you want to re-create the headers we have, but you should explore this as a personal activity using the TI-83 Manual.

6. The SetUpEditor command used on the homescreen (with no arguments) returns the stats/list editor to its defaults (i.e. $L_1 - L_6$ as headers holding empty lists). This is useful for returning to the default stats/list editor view quickly or to recover from an accidental deletion of the variables L_1 - L_6 in the memory manager.

Let's use the stats/list view to put the data list { 0, 1, 2, 3, 4, 5, 6 } into TAPES and the data list { 6, 5, 4, 3, 2, 1 } into CDS. We introduce the sequence command as a great way to quickly generate a sequence of values. **Seq(** is generally best used if the sequenced data list is longer than say 8 or 10 values. Otherwise, typing them in is probably just as efficient.

In the stats/list view, cursor up to the TAPES header and press ENTER to make it editable (see figure 3).

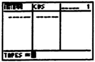


figure 3

Notes/Comments/Ideas

In the active command line shown by the blinking cursor, enter the **Seq(** command as follows: [LIST] [OPS] **seq** (x, x, 0, 6, 1) see figures 4 and 5. Note that the last parameter is optional i.e. the increment/step for a sequence defaults to 1 so it is only really necessary to include a value if the step is other than 1.

Press ENTER and you should have a screen like figure 6. **Seq (** command parameters are described in the diagram immediately following figure 6.

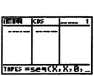


figure 4

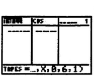


figure 5

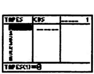


figure 6

Notes/Comments/Ideas

The Sequence Command

[LIST] [OPS]**Seq(** x , x , 0 , 6 , 1)

Expression to generate sequence

Variable used in expression

Increment/Step value for variable

Ending value of variable

Starting value of variable

Can you use the **Seq (** command to create the data in a similar way for the CDs list? In any case, you should create two lists as in figure 7.

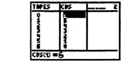


figure 7

figure 8

figure 9

Notes/Comments/Ideas

Create a scatterplot by using 2nd **y =** [Stat Plot] turning plot 1 on, setting it as a scatterplot, and setting the Xlist as TAPES and Ylist as CDS. These names should be grabbed from the [LIST] [NAMES] menu. We do this by cursoring to the Xlist field and pressing [LIST] [Names] and selecting the name we want by pressing ENTER (the only names likely present right now are the two names we have created - L_1 - L_6 do not show here and are only accessible from the keypad). The selected name is then pasted into the Xlist position. Do the same for the Ylist field. Then choose a mark for your scatterplot and you should have a screen like figure 8. Now use ZoomStat (recall: **ZOOM** **9:**) to view this in a reasonable plot window (see figure 9).

Important Note: *List names are most easily pasted into fields by using the [LIST] [NAMES] menu. That menu puts the name with correct syntax in place at all times (there are other ways such as typing but refer to your manual for syntax restrictions or ask your facilitator).*

Though this data is discrete, we can clearly see that the path it presents is straight as in a line. Therefore, this is called a linear relation. The formula you were asked to construct earlier probably looked something like this: CDS = 6 – TAPES.

Notes/Comments/Ideas

This is an example of a linear formula. We can get an idea of a real linear graph if we change our plot style to xy-line instead of scatterplot to view our data (see figure 10). The problem is that, while this resembles a linear relation, the data is discrete. Try tracing it and you'll see what we mean (Note that this type of difference was noted in the "Powers of 2" master).

figure 10

What if our problem had been "Our case can hold 6 kg of stuff. There are two kinds of stuff. What are the possibilities for our case's contents?" Note that this doesn't change the tracing problem but illustrates the need for a continuous plot!

Now we will explore the formula applied here in a more general way. That is, we will use the function graphing editor to plot a continuous version of CDS = 6 – TAPES.

Press $\boxed{y=}$. If there are any functions in the y = list, use your cursor to go there and then press CLEAR to blank the entry. Your screen should look like figure 11 when all functions are cleared.

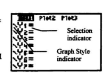


figure 11

Notes/Comments/Ideas

Note that Plot 1 is backlit (see figure 11). This actually indicates that plot 1 is active and will plot whenever a graph is displayed (it was left active from our last plot i.e. we didn't turn it off). In fact, the plot order for any graphing activity is stat plots first, then other active plots in the graphing editor.

The two indicator marks (see figure 11) surrounding the Y_1, Y_2, etc. entries show graph style on the left (there are 7 different graph styles), and whether that plot is active (i.e. selected for graphing) by the lighted status of the = sign on the right. Before we enter the functional version of our stat data formula, let's visit a standard viewing window for functions.

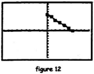


figure 12

Press ZOOM [ZoomStd] or ZOOM 6: .

You should see your stat plot in a 10 by 10 window (see figure 12). You may wish to re-visit the "Powers of 2" master for a discussion of window settings and what each window field controls. We will explore the remainder of the ZOOM choices in the "Introduction To Polynomial Graphing" master coming up shortly in this section.

Let's translate our stats/list formula to a function. Let the answer for # of CDS be represented by Y, and the initial input number, the number of TAPES we have, be X. Then our formula CDS = 6 – TAPES becomes Y = 6 – X.

Notes/Comments/Ideas

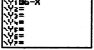


figure 13

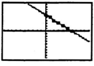


figure 14

Enter $6 - X$ in the position held by Y_1. If you are still on your graph window, press y= to return to the graphing editor. Cursor to Y_1 and place your cursor immediately to the right of the = sign. Key in 6 – X,T,θ,n which puts X there while in function mode (recall that this key cycles with the graph setting as controlled from the MODE screen). You should see a screen like figure 13.

Notice in figure 13 that the = sign for Y_1 is now backlit. This shows that both the statplot **Plot 1** and the function $\mathbf{Y_1}$ will plot when GRAPH is pressed.

Press GRAPH. The linear function $y = 6 - x$ graphs right through our stat plot data (see figure 14). Let's trace now and see if anything interesting results.

Pressing Trace immediately puts an active cursor on the first plot i.e. the active stat plot and shows this by giving the plot number and the lists involved (see figure 15). If we use the up or down cursor while in trace mode, we cause **switching between active plots.**

Notes/Comments/Ideas

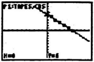


figure 15

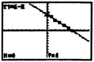


figure 16

Press ▲ and watch how the display changes (see figure 16). You should now see the next active plot which in this case was our function. Notice how the window shows us which function is active and what the relation is. Trace around using ▲ or ▼ to switch between active plots discussing interesting discoveries with your neighbour. Change the window using ZoomStat, or manually choosing a window, to improve the view.

figure 17

Now turn off your stat plot and trace just the function plot. Press y=. Cursor up to the Plot 1 header (much as we did in the stats/list header stuff). Once on a header or plot name, pressing ENTER alternately cycles its plot status on/off. When backlit it is on, when not backlit it is off. Press ENTER until it is not backlit (see figure 17). Note that the = sign by Y_1 is still backlit. In a similar way, its plot on/off status is changed by cursoring to the = and pressing ENTER.

Notes/Comments/Ideas

Pressing GRAPH now plots only the backlit function. Try tracing now. Can you switch between plots as before? Why, why not? Turning stat plots off is important if you don't want them cluttering up your function plots.

Tip: You may recall that in the Stat plot menu, one option is **PlotsOn** and another is **PlotsOff**. Try executing them and see what happens! Similarly, turning off your function plots is important if you don't want them plotting over or through the stat plots you are examining. This is one way of easily managing what does and doesn't plot and is especially valuable in a program.

The TI-83 has the capability to generate its own best guess at a linear relation (which best fits data such as this) and tell us the functional version of it. We can even plot both and examine exactly how accurate a fit we get. That is called calculating and plotting a Regression Line (or Using A Regression Model). That is a major focus in the *In The Service* section of this Manual.

To avoid cluttering of extra list names we delete TAPES and CDS. 2nd + [MEM] **[Delete] [List]** and cursor to **CDS**, press ENTER to delete it. Repeat the same steps for **TAPES**. If you have trouble, then quit and go home for now and ask for clarification at the end of the activity.

Notes/Comments/Ideas

- ❖ Can you set up the stats/list editor using list names other than L_1 - L_6?
- ❖ Can you enter a function in the graphing editor
- ❖ Can you plot a function in a standard viewing window?
- ❖ Can you switch between plots while tracing?
- ❖ Can you turn a plot's status on/off?
- ❖ Can you delete lists once done with them?

Notes/Comments/Ideas

Boot Camp Engagement Master #5

Working With Linear Relations

This engagement is intended to add some *serious* graphing tools to a repertoire! The approach is familiar but some innovative applications of tables and the [CALC] menu provide food for thought. We begin by providing a symbolic relation. Note that your calculator should be in function mode. If it is not, then use MODE and choose **Func** using your cursor and pressing ENTER. Let's explore everything we can about the simple linear relation: y = 2x + 1.

To view a table of pairs generated from this relation press y= , then CLEAR to blank out any functions in the y = list that aren't empty, and enter 2x +1 using the X, T, θ, n key. This creates a function in the y = list. The table editor can only display relations present in the y = editor.

Notes/Comments/Ideas

Now, we define our table. Press 2nd WINDOW [Tblset]. You should see a screen like figure 1 though values present may vary. We'll choose our values now. For a domain embracing the origin we could choose TblStart = - 3 and Δ Tbl = 1. This sets our table to start at – 3 and count up in steps of 1. This is a lot like the sequence command we saw before and is just as flexible. A step of – 1 for example would count down. A step of 100 counts up with a zoomed out view, and 0.01 a zoomed in view (we take advantage of this later). Leave the Indpnt and Depend fields set to Auto for now (see figure 2). To view our table, we now need only press 2nd GRAPH [Table] (see figure 3).

figure 1

figure 2

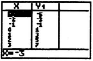


figure 3

Notes/Comments/Ideas

We can see that the selected or active function(s) show as headers in this view. If more than one function were in the y = list, but only some of them were selected for graphing, then only those selected functions appear in the table view too.

The Y = headers are editable just like the stats/list in the spreadsheet view. Values calculated for the domain values defined in the TblSet editor are displayed. By cursoring up and down the X column, all pairs under the conditions defined through TblSet can be viewed and the data scroll. Cursor up and down the Y column and discuss how this action differs from the X column.

Several valuable student explorations can be made using only the table view.

- For equal X-steps in the domain, what do you notice about the corresponding Y-step? (this can be done by comparing successive values or simply asking about any patterns in the table)
- Look at the X change for any two points and compare it to the Y change. What do you notice? (This is different because the notion of equivalent ratios can be explored).
- Where does this plot cross the Y-axis? Justify your answer.
- Where does this plot cross the X-axis? Justify your answer.

Notes/Comments/Ideas

Let's have the TI-83 plot our relation in a standard viewing window. Press ZOOM [ZStandard] or ZOOM 6. This causes any active stat plot and any active function etc. to be plotted (Remember, if any stat plot is active, it can be turned off in the y = editor by cursoring up to its indicator and pressing ENTER until it is not backlit). You should see a screen like figure 4.

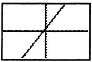


figure 4

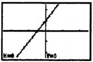


figure 5

It's a little hard to determine the intercepts in this window.

Let's Zoom in. Press ZOOM [Zoom In]. You see a graph window with nothing seemingly happening. If you look closely, you'll see the center of your window (in this case the origin) is flashing. It's waiting for you to indicate which location you would like to serve as the centre for your zoomed window. To accept the origin as your new zoomed window centre, press ENTER (see figure 5) or cursor to another point preferred for the centre of a shifted and zoomed window and then press ENTER. Upon examination, the Y-intercept appears to be 1.

Notes/Comments/Ideas

If this window is not to your liking either, then you can manually set one. You can also set the Zoom factor to a smaller or larger number (the default is 4* on both axes) using **ZOOM** [MEMORY] **SetFactors.** A nice window here is – 5 by 5 on each axis with tick marks set at 1 unit apart. Set your window to reflect this (see figure 6) and then prepare to trace the graph. You can also explore other aspects of the zoom menu though they will be discussed in more detail in other Masters.

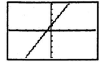


figure 6

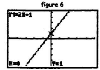


figure 7

Press **TRACE** (see figure 7). Cursor around and note any interesting aspects of the display.

Find the Y-intercept directly by keying in 0. Watch how the display changes and accepts your input.

How can we find the X-intercept ?

Initial tracing tells us it is between – 0.426 and – 0.532. We could zoom in and trace repeatedly until we were satisfied we were close enough. Note that this

Notes/Comments/Ideas

Zoom-Trace technique can be used to solve intersections, find zeros, establish function values, etc.

We could explore the **ZBox** command and quickly isolate our area of interest by pressing ZOOM **ZBox**. Your display should now have a flashing cursor that is awaiting two inputs from you. Imagine a box around an area you wish to zoom in on. Cursor to one corner of said box. Press ENTER. Then cursor to the diagonally opposite corner of same said box. Press ENTER. Now your display is automatically replotted with your boxed area serving as the new full screen. Try tracing again and you should see a significant improvement in determining the X-intercept.

We could also use the evaluative feature of trace and key in better and better guesses as X-values until we got close enough.

We could simply solve $0 = 2x + 4$ manually (which in this case is the most efficient).

We could use the TblSet and Table views to zoom in by using smaller ΔTbl values and refining TblStart until accuracy was satisfactory.

Notes/Comments/Ideas

We could use the [CALC] menu. Press 2nd TRACE [CALC] (see figure 8). Wow! This menu packs a bit of punch! Calculus anyone?

figure 8

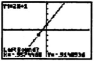


figure 9a

There are choices similar to this in the MATH menu but those are more for use on the homescreen. These are designed to be more plot-friendly. Press **2** or **zero**. You are now prompted for a left bound. Cursor to the left of what appears to be the zero and accept that left bound by pressing ENTER. You are then similarly prompted for a right bound, and then an estimate of the zero. Figures 9a-c illustrate this process and figure 9 d illustrates the result.

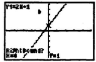


figure 9b

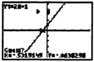


figure 9c

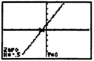


figure 9d

Notes/Comments/Ideas

The other [CALC] menu functions: **minimum**, **maximum**, and **intersect** work in a very similar way with accompanying prompts.

We could use the **Solver** command in the MATH menu. Press MATH **0** or MATH **Solver.** Cursor up to the header in this window to make it editable. The equation we want is $0 = Y_1$. If anything other than that appears right now, press CLEAR. Now, you should have a screen like figure 10 a.

figure 10a

figure 10b

figure 10c

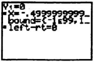


figure 10d

Notes/Comments/Ideas

If there is a function in the Y_1 field in the graphing editor, it will probably present itself as a default as in figure 10b (unless the solver has been recently used with another equation). If Y_1 is not present in the solver, we will need to place Y_1 there on the right of the = sign. As with list names, there is a list of function names where you <u>must</u> go to grab function names like Y_1 etc. It's a bit of a pain but that's it. Remember, when accessing a menu from an editor, the editor remembers you were there and pastes your menu choice back into that editor.

Go to VARS [Y–VARS][Function] **Y_1** (don't forget to press ENTER)

which pastes Y_1 in the field we just left. It is a bit cumbersome but you quickly get as used to it as you did to ZoomStat as ZOOM **9**. Your screen should now match figure 10b.

Press ENTER or use your cursor to move to the X = field which now awaits your estimate of the solution as input (see figure 10c). It is worth noting here that the prompted value here is your last cursor X-value. So, tracing near the zero <u>before</u> using the solver gives you a good guess immediately. The **Solve** operation is the Alpha function of ENTER.

Notes/Comments/Ideas

Our last step here is Alpha ENTER [Solve]. The result is figure 10d. Discuss the meaning of the fields with partners or your facilitator.

dy/dx And A Discussion Of Slope

$\frac{dy}{dx}$ offers an interesting look at slope here. This ratio could be described as a slope of two points very close together on either side of a point you can pick. So, the slope that $\frac{dy}{dx}$ returns is like looking at the slope of a very short segment of the graph. We saw in the table, that the X-change Y-change ratio seemed constant at 2. Does our $\frac{dy}{dx}$ operation give us that too?

$\frac{dy}{dx}$ applied to a graph. Press [CALC] [$\frac{dy}{dx}$]. You can cursor to a point of interest on a graph as in a trace, or you can key in an X-value directly. Determining the slope at - 2 will be illustrated. When $\frac{dy}{dx}$ is pressed, an interactive graph screen is shown (see figure 11a). We enter our value of interest (or cursor to it if our window setting make this possible). Your screen should now appear as in figure 11b. Press

Notes/Comments/Ideas

ENTER to accept it and notice that the grapher illustrates your point and shows the value of the short slope we talked about (see figure 11 c).

By repeating this, the constant slope property of a line can be reinforced and, with a bit of flexibility, the variable slope of other familiar curves can easily be explored without any need for calculus or even the mention of calculus. The [DRAW] menu offers the opportunity to draw non-traceable tangents to curves to further enhance such explorations for capable groups.

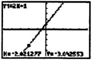


figure 11a

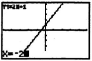


figure 11b

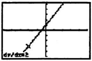


figure 11c

Notes/Comments/Ideas

Teaching Tip – Mental Arithmetic Using tables

Changing your [TblSet] **Indpnt** field to **Ask** leaves the X column blank. It waits for you to choose input values. So, a teaching strategy here could be to have students write the relation down as a column header on paper and manually compute Y-values for -values suggested by you or by students. A nice value that's always easy to compute is X = 0 and, of course, its significance can be readily established, or linked via an earlier trace, to the Y-intercept. Other values can follow. This is good mental exercise.

Here's how you do it. Go to [TblSet] and set **Indpnt** to **Ask** (remember to press Enter so that's it is backlit). Now go to [Table]. You should see a screen like figure 12a.

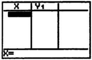


figure 12a

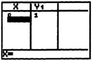


figure 12b

Notes/Comments/Ideas

Notice that the X cell entry is blacked out... it is awaiting your input. You could ask, "If x is 0, what does this relation return?" After the responses, press 0 ENTER (see figure 12b).

It's that easy.

Notes/Comments/Ideas

Teaching Tip - End Behaviour Using Tables

This table option is also excellent for mentally exploring what a relation does over a given domain before actually viewing a table or plotting. For example, "As x gets really big, what do you think will happen to our answers?" A teacher could entertain guesses and then entertain some really big domain values as determined by your perceptive audience.

[Table] is very flexible and very powerful for studying end behaviour, and other aspects of the graph, since massive changes in scale are easily inputted and viewed. Key them in after discussion and continue exploring. The same exercise would work for any domain value exploration – big, small, close to some number, ...you get the idea. Some such exercises are illustrated below.

The rough equivalence of the expressions ax^n and $ax^n + bx^{n-1} + \ldots + k$, as $x \to \pm\infty$ can be reinforced also through tables.

For example, consider $2x^3$ versus $2x^3 + 3x - 5$. Viewed in a table, these relations reveal differing values. With TblStart at 1 and ΔTbl at 1 we get the table of figure 13. Zooming out however reveals startling agreement in the magnitude of the answers (see figure 14a-c). While this is a little misleading, it can foster a valuable

Notes/Comments/Ideas

discussion on rates of growth, limits, and classes of algebraic objects. An interesting diversion now, could include asking the same question of other functions such as rational functions, square root functions, sine, etc.

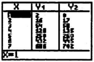


figure 13

figure 14a

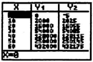


figure 14b

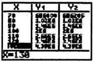


figure 14c

Finding zeros with tables can be done through the Auto Ask feature with some lead information. For example, if you already know approximately where a zero is (or, for that matter, a maximum point or other point of interest), you could refine guesses toward the zero, until desired accuracy is obtained, while leaving a record of the guesses made. In fact, this could represent the beginning of a competition among partners in a class!

Notes/Comments/Ideas

Another small tip is that the headers for the table feature are also editable like the stats/list view was. A side effect of this editability is that a quick reference back to the relation being calculated can be made from within the table view– this is especially useful while using the table to compare range values of more than one relation. To view a relation used as a table header, simply cursor to the header. To quickly change any aspect of the relation, cursor to its header, press ENTER to make it editable, and enter your change. Press ENTER again to accept it. Try it!

Reference:

Texas Instruments (1996a). *CT³ - Canadian teachers teaching with technology* [manual]. Dallas, TX: Author, p.4-6 section 4.1.3.

Notes/Comments/Ideas

A Dalliance With Intersect

Solve by graphing: $4x-3=5$. This problem can be done several ways, but the opportunity exists here to illustrate the intersection command in the Calculate menu. Set $Y_1 = 4x - 3$ and $Y_2 = 5$. Plot them both in a standard window i.e. [ZOOM] [ZStandard]. We can see there is one intersection visible and upon tracing seems close to 2. We can determine that in it is 2 if we use the evaluate feature of trace and happen to input 2 as we get 5 exactly. However, the intersection command doesn't assume a nice answer (though this one is nice).

Press 2nd TRACE [CALC] **intersect** and follow the prompts as described. To select the first curve, use ▲ or ▼ to cycle through the plots and press ENTER when the first curve's relation shows in the top left corner or when you are clearly on your first curve.

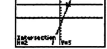


figure 15

Do the same for the second curve prompt. Then trace to the apparent intersection point when prompted. Press ENTER to execute (see figure 15).

It's that easy. We could also use the difference function and use [CALC] **zero** or the **solver** as we did in an earlier master.

Notes/Comments/Ideas

Cost-Revenue Activity

A game costs $5000 to develop and $10 per game to manufacture. The selling price is planned to be $19.99 per game.

- Create a cost table for production based upon making 1000 games.
- Create a revenue table for sales of this game.
- Choose a common domain and range from you table learnings and set a viewing window for these two functions.
- Plot the graphs of the cost and the revenue data.
- Under these conditions, will those plots intersect?
- What do the different regions of the xy-plane that the lines define represent?

Notes/Comments/Ideas

For a Linear Relation...

- Can you create a table, with parameters of your choice, for a function?
- Can you create a function graph?
- Can you trace a function graph?
- Can you find the Y-intercept in several ways?
- Can you determine the zero(s) / find the X-intercept(s) in several ways?
- Can you compute dy/dx at a given point?
- Can you determine intersection(s) in several ways?

Notes/Comments/Ideas

Boot Camp Engagement Master #6

Polynomial Graphing

An opportunity exists here to utilize the **CBL** to gather empirical data, and apply the quadratic regression power of the TI-83 to that CBL data. The CBL is discussed in detail in the appendix titled "TI Calcs./Accessories/Support.

A physics formula describing position above the earth given an initial height of h_0 and an initial velocity of v_0 could be: $H = -\frac{1}{2}at^2 + v_0t + h_0$.

Assuming a = about 9.8 m/s², gives us: $H = -4.9\,t^2 + v_0t + h_0$, where H is height above a zero reference in m, t is elapsed time in sec., and v is initial vertical velocity in m/s. *Note that the domain of this relation is t for time. The TI-83 uses X for functions so we replace the t variable with X for now. Parametric equations offer another point of view for this kind of problem (see appendix titled "Parametric Equations and Functions").*

A platform diver is about to perform a dive. The platform is 3m high and initial velocity off the platform is 3 m/s.

Write the relation applying to this dive. ______________________

Notes/Comments/Ideas

Determine reasonable values for the domain t for this relation for a table view (for example, is $t = -5$ seconds under consideration?).

View the results of this relation under your reasonable domain in a table. Note that these skills have been practiced several times now and are not repeated here. The focus of this master is on graphing tools thus far un- or under-utilized.

Notes/Comments/Ideas

Teaching Tip - Tables and Ranges

A reasonable domain is often a natural discovery upon thinking about the problem. The table view on a given domain yields accurate range values for your domain and some sense of the scope of the resulting range for that relation.

Therefore, the use of [Table] on a relevant domain should be a regular part of a student's exploration of a new relation, and should precede any plot of that relation.

The values viewed in the table should be used to set up an initial viewing window for the plot, and the table and graph should always validate each other. Comparing the two is always fertile ground for learning.

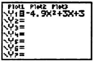
Symbolic analysis of a relation (including the tools of calculus) yields insight into global behaviours but can be labourious. Clearly technology has its advantages but cannot exist in isolation. *The graphical supports and supplements the tabular which supports and supplements the symbolic.*

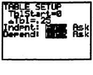
Notes/Comments/Ideas

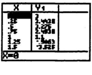
Let's choose a viewing window for this relation based upon the table results. Plot the relation in this window and consider the accuracy of your plot versus the real path of the diver.

After completing the exercise above, several considerations arise: domains and viewing windows aren't always nice numbers; neither are ranges; *a table is a good way to get a feel for the real domain and range of any relation*; and, a grapher plots a relation wherever it is defined, over the full real number system unless told otherwise.

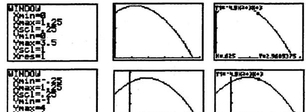
Before we discuss restricted domains for graphing, finding a maximum, finding a zero, the significance of the Y-intercept; testing functionality of a graph, and testing one-to-oneness of a graph, a walk through this problem produced the following result as viewed by this 9-screenshot story...can you re-create the story behind the *plot*?







Notes/Comments/Ideas



Why is the last graph more representative of the relation, though less representative of the dive?

Could we keep the better trace window of the second graph somehow yet adjust our domain to show only the diver's real path?

Restricting A Domain

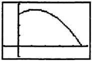
As the grapher plots, it begins at the leftmost x pixel value and calculates the relation for all pixels until it reaches the rightmost (as determined by the window parameters). Hence, if your window view includes values outside of the domain you want to view (the real domain in this case), they get plotted anyway.

Notes/Comments/Ideas

Logical tests like x > 0 return a value of 1 when true and 0 when false. We all know what we get when we multiply or divide something by 1, and we all know what we get when we multiply or divide something by 0. So, if we take a relation, say *f*, and we enter *f* / (x > 0) then, as the x value updates (as the grapher plots from left to right) the relation evaluates as *f* / 0 (which is undefined and doesn't plot) until x passes zero when it evaluates to *f* / 1 (which is *f* anyway and plots as we'd expect). That's it.

If two restrictions are required, divide by both tests. Using the inequality symbols found in the 2nd MATH [TEST] menu we create the function required.

The following two screenshots illustrate how to restrict the last relation to the domain 0 < x < 1.15 (i.e. from 0 until the diver hits the water at about 1.15s - found using [CALC] **zero**). Immediately following the next *Teaching Tip*, we determine the greatest height this diver reaches also through the [CALC] menu.



Notes/Comments/Ideas

Finding the Maximum

To find the maximum height our diver reaches, we use the **maximum** command. [CALC] **maximum,** followed by bound choices and an estimate, yields the following screenshot story (you should arrive at the same maximum value even with slightly different bounds)...

So our diver reaches a maximum height of about 3.46m after about 0.31s in flight and splashes down after about 1.15s.

Notes/Comments/Ideas

Teaching Tip – A Test For Functionality

The 2nd PRGM [DRAW] menu offers many options for adding non-traceable elements to a plot. This can aid clarity as well as be very functional.

For example, DRAW **Vertical,** when accessed from within the graphing editor, allows you to drag a vertical line across the screen using ▸ and ◂ and drop it at any location you want by pressing ENTER. Of course, we need not drop it right away! Dragging it across the plot illustrates the vertical line test for functionality!

Dropping the line on areas failing the vertical line test is a good concrete visual reinforcement for this topic. The horizontal line test for one-to-oneness (i.e. the functionality of the graph of the inverse of a relation) can be performed in the same manner using **Horizontal** instead of **Vertical.**

This next sequence of shots shows that our last relation would pass the vertical line test on the domain we chose:

Notes/Comments/Ideas

Notes/Comments/Ideas

Teaching Tip – Piece-wise Defined Functions

A similar technique using logical tests may be used for graphing piece-wise defined functions. Though not mentioned a moment ago, multiplying by 1 and 0 would work in a very similar way as dividing for restricting a domain.

Dividing a function by 0 makes it undefined there; hence it doesn't plot where you don't want it to. Perfect.

Multiplying it by 0 however, makes the function evaluate to 0 where you prefer it not be defined; hence it does plot...it plots y = 0! It doesn't look like it plotted anything though because the axes effectively hide it. Want proof? Turn the axes off via ZOOM [FORMAT] **AxesOff** and check again.

This is deceptive in a way and generally is a technique that this author prefers not to use.

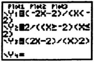
On the other hand, the multiplication by zero trick, used to *zero out* a function over a part of the X-axis, does have its advantages. For one thing, it's easier to key in. For another, it sets an easier convention for graphing piece-wise functions for teaching purposes. I'm still not happy about it ☹. But, here goes.

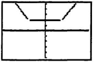
Notes/Comments/Ideas

Suppose we want to graph the piece-wise defined function:

$$\begin{cases} -2x-2, & x<-2 \\ 2, & -2 \le x \le 2. \\ 2x-2, & x>2 \end{cases}$$

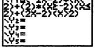
One way to do this is to put each piece in the y= list as a separate function and restrict the domain of each, as we did a moment ago with the quadratic function, using division to make functions undefined where necessary. This is illustrated in two screenshots shown below:

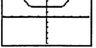




Another way, and a common alternative, is to create a composite function through addition where components evaluate to 0 wherever they are not defined. In this case we could create: $(-2x-2)(x<-2)+(2)(-2 \le x \le 2)+(2x-2)(x>2)$. This is illustrated in two screenshots shown below:

Notes/Comments/Ideas





There are other variations on this technique, but essentially that's it. It is worth noting that creating a composite function through addition where components are undefined fails as a technique! That is, the equation $(-2x-2)\,/\,(x<-2)+(2)\,/\,(-2\le x\le 2)+(2x-2)\,/\,(x>2)$ will not create the function we want–in fact, the calculator freezes and doesn't like being asked to do such things! With a little thought, you should be able to figure out why.

You may have noticed two slight gaps in the first graph shot. With a little thought you should be able to conjecture about why that happened. You may even be able to correct it through refining some values. As a hint, check out the appendix titled "Friendly Windows" for a reason some values *disappear*.

Notes/Comments/Ideas

Boot Camp Engagement Master #7

Inequalities - Shaded and Otherwise

Inequalities are easily illustrated on the TI-83 in two ways: [1]as shaded regions; and, [2] as a plot over only the solution domain.

Illustrating solution regions to multiple inequalities can be achieved by observing overlapping areas of shading. Alternatively, by using logical tests on functions, one can create a composite unshaded plot reflecting the domain of the solution.

Using shading styles, illustrate and solve the inequality defined by:

$y > 2x + 1$ and $y < -2x^2$.

By using the shading style indicators referred to previously, we enter the edges as functions and assign each with and appropriate shading style. We then use the overlap, and knowledge of the nature of the edges, to determine the solution region.

Notes/Comments/Ideas

Set $Y_1 = 2x - 1$ and $Y_2 = -2x^2 + 5$ in the graphing editor as in figure 1. Then, cursor to the position immediately to the left of the Y symbol. This is the graph style indicator (this was noted and illustrated in *Boot Camp Engagement Master #4* "A Linear Relation"). Pressing ENTER repeatedly cycles through the seven graph styles. This has been explored already except for the two styles in use here. Set the Y_1 style to shade above and Y_2 to shade below by pressing ENTER until the 4th and 3rd style choices in the graphing style list of choices appear in the graphing style indicator location as shown in figure 2.

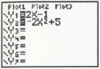


figure 1

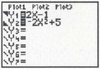


figure 2

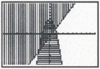


figure 3

A decent window can be established by using ZStandard here. Press ZStandard - recall that the plot is created immediately upon choosing any ZOOM option. Observe the graph (see figure 3). The region of overlap indicates the solution region, and the actual domain values involved can be determined using zoom and trace or other intersection options.

Notes/Comments/Ideas

Another option for solving inequalities is to use logical tests.

Revise the functions to plot with the default line plot style (visible in figure 5). Create a new function, Y_3 that tests the inequality you wish to solve - in this case, upon some thought, $Y_2 > Y_1$ is the region in question. Set Y_3 as shown in figure 5 - using these keystrokes when in the Y_3 position: VARS [Y-Vars] [Function] Y_2 2nd MATH [TEST] > VARS [Y-Vars] [Function] Y_1 and ENTER. Note that you should only have the first three of the functions shown in figure 5 in your graphing editor at this point, and each of those should be selected i.e. have their equal sign backlit - unlike figure 5.

If we plot the first three of these functions, we will find the two plots we expect, and a horizontal line segment value 1 near the origin (see figure 4). The test

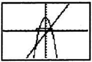


figure 4

figure 5

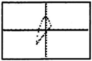


figure 6

$Y_2 > Y_1$ tests as true where the segment appears and so evaluates to 1. Therefore, the value 1 is plotted. Elsewhere it evaluates to 0, which is also plotted, but may not be

Notes/Comments/Ideas

noticed it since it coincides with the axis. This is how we can view the domain of an inequality's solution.

An enhancement on this is to cause the two original functions to be subject to the test forcing a plot of the solution graph only. Create $Y_4 = Y_3 * Y_1$ and $Y_5 = Y_3 * Y_2$ using the VARS menu as before to paste functions names where needed. Then turn all plots off except for the two functions just created recall - this is done by cursoring to the = sign and pressing ENTER such that the = sign is not backlit). This disables plotting of that function as discussed earlier. Your screen should now match figure 5 exactly. Pressing GRAPH results in figure 6. This clearly shows the domain of the solution region though is not nearly as visual as the overlapping shading. Note that figures 5 and 6 show the inequalities used throughout this exercise but the plot status indicators (which show only plots Y_4 and Y_5 turned on) agree only with the final plot discussion currently ongoing.

A final note for illustrative purposes is the use of the **Shade (** command found in the [DRAW] menu. The shade command takes two arguments and has two optional arguments as follows: **Shade(Lower curve, Upper curve [, Left X-Value, Right X-Value, Pattern, Patres])**. Lower and Upper can be function names or

Notes/Comments/Ideas

expressions. Left and Right are domain restrictions (if desired). Pattern is a number from 1 to 4 representing different shading styles. Patres is a number from 1 to 8 representing shading resolution from shading of every pixel (#1) to every eighth pixel (#8). For further information, see TI-83 Manual 8-10.

Reference:

Texas Instruments (1996b). *Graphing Inequalities.* TI-83 Graphing Calculator Guidebook, p. 17-5.

Notes/Comments/Ideas

FUTURE MATHEMATICS CURRICULA

IN A

TI-83 GRAPHING CALCULATOR ENVIRONMENT

In The Service

Created by

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M. Ed. Candidate
Memorial University of
Newfoundland

In The Service

In The Service Introduction

Congratulations! You have successfully completed your *Basic Training* and survived *Boot Camp*. You are now *In The Service*.

This section of the manual focuses upon exploring relationships found in data – generated empirically or constructed. Reformed mathematics curricula has a greater emphasis on data-driven and reality-based mathematics. As a result, the skills of the past for mathematics teaching and learning may not all survive as the skills of the present - let alone the future.

The nature of this more data driven environment pre-supposes some facility with a technology that can manage data in a friendly way. Data sorting, data viewing through table and plot, and regression modeling all become essential.

The skills and facilities presented in these masters are applied to one-variable and bi-variate data sets. *In the Service* uses data that is collected and/or generated by participants and does not go too far afield. This should help make the activities more interesting and perhaps insightful.

Personal Notes

The nature of the data, with the relevant master name in quotation marks, include: measuring a population ("Coin Collectors" & "Ruler Reaction Time"); sampling ("Is This Normal?"); experimentation ("Ping Pong, Golf And Tennis Anyone?"); and, modeling ("Strung Out"). Linear and quadratic regressions are plotted and compared to the data, so aspects of polynomial curve analysis arise. Correlation is discussed as is the Normal Curve.

Some common, but perhaps not so familiar topics, are examined including: step functions ("No Free Parking!"); and, matrices ("Systems and Networks"). And, some familiar topics are visited (though not always in traditional ways). These include: polynomial graphing and factoring ("Poly Wants A Plot"); and, graphing patterns a.k.a. shifting and scaling ("Transforming Plots").

In this section, participants should now get a better sense of what a graphing calculator environment might be like as using the TI-83 evolves into second nature. It is always there and its use becomes a matter of course. It is also likely that the treatment of topics like using graphs or tables to reinforce equivalence might spark other insights along similar lines.

The ease with which the TI-83 may be used to manage statistics, assist in quantifying trends, and facilitate exploration, may actually free you from any

Personal Notes

reticence you may have associated with statistics. It may free you to explore aspects of curves that you may have needed serious symbolism, or a hefty analysis program, to do before. With a powerful tool that is so easy to use, more of the curious learner in all of us may creep out!

Personal Notes

IN THE SERVICE NOTES MASTER #1

Coin Collectors

Goals:

- ✓ To plot and analyze one variable data

Description:

- ✓ Collection of value of coins possessed by participants distinguished by gender
- ✓ Three important list naming notes provided
- ✓ Combining two lists using the **augment(** command
- ✓ Define bar graph of combined list
- ✓ View bar graph with **ZoomStat** command
- ✓ Refine bar width (and window if nec.) manually
- ✓ Outliers requested
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- ✓ View component lists as boxplots and compare to each other
- ✓ View all three Boxplots simultaneously and compare discussing high, low, median, mean, dispersion, etc. of each list, and the effect of each component list on the combined list plot
- ✓ Perform one variable analysis on each list and discuss results

Description (c'tued):

- ✓ Extension questions posed
- ✓ *Teaching Tip* – List Name Housekeeping
- ✓ *Teaching Tip* – Saving List Data As A Program
- ✓ *Teaching Tip* – Editing and Copying Programs
- ✓ *Teaching Tip* – Sharing Data With Other TI-83s

IN THE SERVICE NOTES MASTER #2

The Ruler Drop

Goals:
- ✓ To plot and analyze one variable data using a frequency table

Description:
- ✓ Experiment with repeated trials performed
- ✓ Frequency table generated
- ✓ Store data and frequencies to lists
- ✓ View boxplot
- ✓ Perform one variable analysis using **1-Var Stats** command
- ✓ Extension questions posed

In The Service Notes Master #3

Ping Pong, Golf, Tennis Anyone?

Goals:

- ✓ To apply two linear regression models to bi-variate data

Description:

- ✓ Gather bi-variate data
- ✓ SetUpEditor applied with four list names
- ✓ Create scatterplots with different mark styles simultaneously
- ✓ Median-Median regression(s) performed
- ✓ Regression equation, **RegEq** command, pasted into the graphing editor and plotted - discuss fit with data
- ✓ Least Squares Line regression(s) performed
- ✓ Interpolation and extrapolation using table
- ✓ Extension questions posed and include **DiagnosticsOn** and **DiagnosticsOff** commands
- ✓ *Teaching Tip* – Varying Plot Style
- ✓ *Teaching Tip* – Residuals
- ✓ *Teaching Tip* – Regression Eq. Compared to Data

IN THE SERVICE NOTES MASTER #4

Is This Normal?

Goals:

- ✓ To use and illustrate the Normal Probability Distribution for sample data

Description:

- ✓ Sample data from a normal population stored to L_1
- ✓ 95% confidence interval determined for mean of population based upon sample using **Tinterval...**
- ✓ Percentile Rank for sample data value determined using **normalcdf (**
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- ✓ Normal Distribution function plotted for this population
- ✓ **ZoomFit** command utilized and discussed
- ✓ **normalpdf (** and **invNorm (** discussed
- ✓ *Teaching Tip* - A Roll Of The Dice

IN THE SERVICE NOTES MASTER #5

No Free Parking!

Goals:

- ✓ To illustrate a real-life application of step functions

Description:

- ✓ Three parking lots offering different fees presented
- ✓ Each cost modeled symbolically as a function in the graphing editor using the MATH [NUM] **int (** command
- ✓ Table parameters set to suit problem
- ✓ Cost functions examined with table view
- ✓ Best buy identified with table view
- ✓ Cost functions viewed as plots
- ✓ Best buy identified with graph view
- ✓ Continuity of plots discussed
- ✓ Plot mode changed from **Connected** to **Dot**

IN THE SERVICE NOTES MASTER #6

Matrices: Systems and Networks

Goals:

- ✓ To solve systems using matrices, and provide connections among inverse, identity, and determinant

Description:

- ✓ Keying in matrices on the homescreen
- ✓ Matrix operations on the homescreen
- ✓ Matrices in the matrix editor
- ✓ Two common error conditions - **ERR: UNDEFINED** and **ERR: INVALID DIM**
- ✓ Solving a System of Equations using $A^{-1} * C$
- ✓ *Teaching Tip* - Identities & Inverse Matrices
- ✓ Solving a System of Equations using reduced row-echelon form i.e. the **rref (** command
- ✓ Network/Digraph Population model represented using matrices
- ✓ Matrices used to predict successive applications of the model

IN THE SERVICE NOTES MASTER #7

Strung Out

Goals:

- ✓ To model and optimize a rectangular enclosure problem

Description:

- ✓ Scale model a rectangular enclosure problem having a fixed perimeter
- ✓ Create eight variations noting which rectangles appeared to enclose greatest area
- ✓ Stats/List editor set to contain width, length, and area data
- ✓ Linked formula created to compute LEN from WID
- ✓ Linked formula created to compute AREA from WID and LEN
- ✓ Maximum area identified from AREA formula results
- ✓ Scattergram of AREA vs. WID plotted
- ✓ Regression models on TI-83 listed
- ✓ Quadratic regression performed on scatterplot
- ✓ Regression equation stored to Y_1
- ✓ Values for continuous regression equation viewed through TABLE, and **table zooming** used to estimate the maximum

Description (c'tued):

- ✓ Regression equation plotted
- ✓ [CALC] **maximum** command used to find the maximum AREA from regression model
- ✓ *Teaching Tip* – Degree and Plot Style
- ✓ *Teaching Tip* – SPF 15, 30, ...? (Quadratic regression question using "time to burn" data)

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Poly Wants A Plot !

Goals:

- ✓ To explore some polynomial graphing patterns

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- ✓ Determination of zeros, expansion, and degree of factored polynomials required
- ✓ Conclusions based on above requested
- ✓ Viewing the graphs of selected expressions
- ✓ *Teaching Tip* – Factoring and Equivalence
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IN THE SERVICE NOTES MASTER #9

Transforming Plots

Goals:

- ✓ Using the table view to examine transformed functions

Description:

- ✓ Square Root x viewed in table
- ✓ Error messages in table questioned
- ✓ Square Root x graph requested
- ✓ Square Root x shifted and scaled tables presented
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FUTURE MATHEMATICS CURRICULA

IN A

TI-83 GRAPHING CALCULATOR ENVIRONMENT

In The Service

Created by

Paul Gosse
M. Ed. Candidate
Memorial University of Newfoundland

In The Service

In The Service Introduction

Congratulations! You have successfully completed your *Basic Training* and survived *Boot Camp*. You are now *In The Service*.

This section of the manual focuses upon exploring relationships found in data – generated empirically or constructed. Reformed mathematics curricula has a greater emphasis on data-driven and reality-based mathematics. As a result, the skills of the past for mathematics teaching and learning may not all survive as the skills of the present - let alone the future.

The nature of this more data driven environment pre-supposes some facility with a technology that can manage data in a friendly way. Data sorting, data viewing through table and plot, and regression modeling all become essential.

The skills and facilities presented in these masters are applied to one-variable and bi-variate data sets. *In the Service* uses data that is collected and/or generated by participants and does not go too far afield. This should help make the activities more interesting and perhaps insightful.

Personal Notes

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IN A

TI-83 GRAPHING CALCULATOR ENVIRONMENT

In The Service Engagement Masters

Paul Gosse
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In The Service Engagement Master #1

Coin Collectors

This engagement collects one variable data and then compares two subgroups of the data with each other and with the whole. Since the data for this activity does not yet exist, no presumptions are made and so no window shots or parameter values are shown. However, most of the commands and skills used have been practiced and should be familiar.

Please take a moment to total the value of the coins you have on your person and record it in the *Notes* space below. Data for the whole group will be collected as two separate lists, Males and Females, for now. *Your facilitator will do this, one by one, around the room.*

We will store this data now using only the homescreen. You may recall that the SetUpEditor function was used to create named lists earlier, and we then used the stats/list editor to input the data. We will use SetUpEditor again later, but there is another method for storing values into a list which some may find preferable.

Notes/Comments/Ideas

Create a list on your homescreen mirroring the male coin data keyed in by your facilitator. That is { ...data ...}.

Store it to a list named MALE. STO→ MALE. Three notes follow below. They will help clarify some common questions about how the TI-83 manages storing to list names (the same would be true for strings & matrices).

List Naming Note #1:

Because the object we are storing to the variable MALE is a list, the TI-83 automatically creates and designates MALE as a list name.

Another way to name a list is to use the $_L$ prefix found in the LIST [OPS] menu immediately before the rest of the name e.g. $_L$**MALE**. This prefix is occasionally necessary and you should read about it in the TI Manual p. 11-16.

List Naming Note #2:

If you use the LIST [NAMES] menu for pasting names, and use {...data...} STO→ [Listname] (as we did before these notes started), or use SetUpEditor to create your lists, you'll seldom need the $_L$ prefix.

Notes/Comments/Ideas

List Naming Note #3:

When we view a list in the stats/list editor the $_L$ prefix doesn't appear. That's because the <u>only</u> valid input in that editor is a list.

If you recall a list on the homescreen however, the $_L$ prefix shows to indicate the nature of that variable. This is because MALES, say, could represent implied multiplication as in M*A*L*E*S, or could be a string variable, a program name, ... you get the idea.

Prefixes present (are necessary) in fields (situations) where more than one data type is valid.

IMPORTANT:

It's always a good idea to grab list names, etc. from their respective NAMES menus. Doing that places the correct prefix/syntax in place at all times.

Repeat the process for the female data naming your list FEM.

We could have collected all data in one pile but there's a really nice command that concatenates (joins) two lists into one allowing us to pool male and female data while keeping the two original lists intact.

Notes/Comments/Ideas

From the homescreen, we will create our combined list and store it to a new list named TOTAL. Press LIST [OPS] **augment(** $_L$MALE , $_L$FEM) STO→ TOTAL. The LIST [NAMES] menu is used to grab/paste the list names MALE and FEM in that command line.

> Note that the $_L$ appears automatically since we're on the homescreen. Since many variable types are valid on the homescreen, the prefix shows to identify the nature of the variables present.
>
> Note also that from this point on, the use of the LIST [NAMES] menu should be assumed when a list name is pasted to a location unless otherwise indicated.

We have now created three new lists. Let's view the full data list and create a bargraph/histogram.

Go to the [STAT PLOT] editor and turn all plots off. Define STAT PLOT 1 as ON, as a Bar Graph, with Xlist: TOTAL, and leave Freq:1 as it is (we explore frequency list data in a upcoming master). Press ZoomStat and see what you get.

Usually ZoomStat is our friend. But in the case of bar graphs, where bar widths are frequency ranges, it is most likely we will want to manually adjust this

Notes/Comments/Ideas

window. Go to WINDOW. If you'd like a slightly wider or narrower X axis, adjust Xmin and Xmax. Similarly for height, adjust Ymin and Ymax.

The really important window setting here though is Xscl. This controls bar width. Set it to reflect the bin size you prefer for collating the data. For example, if you want bins representing $2.00 steps or increments, and Xmin is $ 0.00, then your bar widths effectively translate to $ 0.00 - $ 1.99, $ 2.00 - $ 3.99, and so on. Your stat plot counts the data dumped into each bin and plots the frequency.

Does anything in this graph surprise you? Try tracing the graph. Note any interesting aspects of the graph. Is your data skewed? Are there any **Outliers**?

Increase Ymax a little bit now to make room for a comparative boxplot of the same data. Define plot 2 as a boxplot with TOTAL as your Xlist. Press GRAPH and you should see both your bar graph and box plot. Note that these plots show distinctly different things.

Trace your boxplot and your bar graph and reconcile what they are both showing you (remember, you can switch plots by using the ▲ or ▼ keys while tracing). Use your graphs to estimate the mean and median of the data by cursoring to where the middle appears to be. Discuss the spread of the data (i.e. its dispersion).

Notes/Comments/Ideas

Now we will view male and female data separately. Re-define plot 1 as a boxplot and turn it OFF for a moment. Re-define the plot 2 Xlist to MALE and leave it as a boxplot. Define plot 3 with Xlist FEM and set it as a boxplot. We should now have all three data sets (TOTAL, MALE, and FEM) in plots 1, 2, and 3 respectively. We choose boxplots for a comparative view of all three sets as it is easier to view three boxplots than three overlaid bar graphs!

View your stat plots. MALE and FEM should appear (remember we turned plot 1 off for a moment). Discuss the differences, outliers, if any, and possible reasons for the nature of the data (this can be fun!).

Turn plot 1 ON again and reconcile the effects of MALE and FEM on the TOTAL.

Note: At no time during this activity was the stat/list editor needed. As with the y= editor and the Graph versus Table window, you don't need to view function data in a table to create a function graph. Neither do you need to view the list data in a table (i.e. the stats/list editor) to view a resulting plot.

We will now perform one variable data analysis on all three lists. If you wanted to, you could grab the individual analysis commands from the LIST

Notes/Comments/Ideas

[MATH] menu and apply them one-by-one. A better and more convenient one variable analysis command, which yields most common one variable statistics, is STAT [CALC] **1-Var Stats.**

Run this command now on the TOTAL list putting the list name after the command as an argument. Scrolling the results should yield mean, data sum, data squares sum, sample and population standard deviation, number of data entries, the minimum and maximum entries, and, the first and third quartiles with the median. That's a fairly comprehensive list of analysis data generated by just one command.

Repeat that analysis with MALE and FEM, and compare the results to the analysis of TOTAL.

Consider the following questions for discussion:

- If the facilitator gave everyone a $2.00 coin, what results of the analysis and plot would change?
- If one person found $ 15.00 in coins on the floor, what results of the analysis and plot would change?
- If Xscl was halved or doubled in stat plot, what effect would this have on the graph? Is that a useful effect?

Notes/Comments/Ideas

- How can a boxplot or histogram be used to help learn about standard deviation?

Reference:

This activity is loosely based upon the 3.1.1 One Variable Data: Counting Change from the following reference.

Texas Instruments. (1996a). *CT³ - Canadian teachers teaching with technology* [manual]. Dallas, TX: Author, 3-3 to 3-9.

Notes/Comments/Ideas

Teaching Tip – List Name Housekeeping

If you choose to name your lists for your own sanity or for presentation purposes, you may find your LIST [NAMES] menu becoming *very* cluttered. Memory space is used for each list name and its contents too.

Clearing list contents can be achieved by erasing the contents of all lists using 2nd + [MEM] **ClrAllLists**, or by erasing contents of single lists with the STAT [Edit] **ClrList** command with needed list names separated by commas as arguments. This still leaves the list names themselves in memory though.

To clear list names from memory there is no option the author is aware of other than using the memory manager and deliberately deleting them one by one. The good news is it's *very* quick to do. 2nd + [MEM] [Delete…] **List…** then cursor ▲ or ▼ to the name you wish deleted and press ENTER to delete it.

Be careful! Once a name is deleted, the name disappears from the page and the next entry down assumes the cursor. So, if you press ENTER twice by mistake, you'll lose two list names and their contents. This action cannot be undone so be

Notes/Comments/Ideas

sure you're deleting the lists you want. All permanent deletions are done in the same manner and require the same care.

If you mistakenly delete L_1, L_2, L_3, etc. and wonder if you'll ever get them back, then you're only recourse is to admit this to the proper authorities! No, actually you can re-establish those by either storing to L_1 etc. from the keypad, or by running SetUpEditor (with no arguments) from the STAT menu. This restores L_1, L_2, L_3, etc. to the stats/list editor view. Though it appears as if they are deleted, and in a way they are, they are easily restored to former glory with no consequences other than the loss of any data contained in them at the time.

Notes/Comments/Ideas

Teaching Tip – Saving List Data As A Program

There are two options for saving list data for recall or presentation later. One, don't clear or delete the list and simply leave it under the current list name onboard. A problem arises here if the name is generic and may again be useful e.g. MALE, L_1. So you may wish to store the data under a more unique name if it is to be left onboard.

To copy a list's contents to a new list name: 2nd STO→ [RCL] old_list_name ENTER. This places the oldlistname contents in an active command line. Now store to your new list name by pressing STO→ new_list_name. Try it!

The other way to save data, while not worrying about name re-use, is through a program. Relaxxx. This is not flow chart or C++ stuff. Watch....

We create a new program by pressing PRGM [NEW] [Create New]. Pick a name (up to 8 characters). Press ENTER. So far so good? The program has been created and now exists. As with other aspects of the TI-83, it is saved directly upon entry. Of course, it has no code in it yet so it doesn't do anything. Let's remedy that.

Notes/Comments/Ideas

A program is simply a series of commands that are executed when the program is run. The command we want executed is the storage of our current data values to a specific list name(s). That's it. One command…one line of program code.

Important Note: this *Teaching Tip* assumes your data is already stored under a known list name or names, and that you wish to store the values contained in that list or lists for future use. If your data is not stored in a list name, then the data itself must be put in the line of the program immediately before the store command.

First recall the data you want to preserve. While in any editor, program editor included, a recall or grabbing of a name or data, simply pastes that data in the position you were just at in the editor. Press 2nd STO→ [RCL] list_name. As an example, let's use MALE. Press [RCL] MALE. You should now just see the data that was stored in the variable MALE flow into the program line.

Press STO→ (which pastes that command into the program line) and enter the list name you want the data to be stored to later. We'll use MALE. The list name you use here will be created if it doesn't exist at the time the program. If it does exist at the time the program is run, then data in that list will be overwritten when the program is run. That's why it is always a good idea to use specific list names, that are

Notes/Comments/Ideas

not too generic in nature, to avoid confusion and loss of onboard data through overwriting. That's it. Let's quit the program editor by quitting and going home.

When this program is run, that line will execute placing that data into that list name leaving you ready to use it. In the meantime, you could now delete that list entirely and not miss it (provided you don't delete the program too). You re-create it by running the program. That is, the running of the program executes each line of the program as if you keyed it in directly then.

We can run the program by pressing PRGM [EXEC] program_name (that is, cursor to the name and press ENTER or press the number associated with the program name). This pastes the program name variable with the prgm prefix as a command on the homescreen. Pressing ENTER now will execute the command hence run the program. Try it!

To use this technique with more than one list of data, simply complete the first line as described above, then press ENTER. This creates another line ready to accept code (i.e. commands) in this program. Create the same type of line as before. That is, {.... Next Data or list name....} STO→ listname. Create as many lines as necessary. When finished, just quit and go home (remember, changes are

Notes/Comments/Ideas

automatically saved). When the program is run, each line is executed in order and all lists are re-created.

<u>Reference</u>:

This *Teaching Tip* is based upon the following reference.

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Notes/Comments/Ideas

Teaching Tip – Editing and Copying Programs

Since you now know how to create a new program and enter code as a program line, it is useful to see how to edit a program or create a copy of a program.

To edit the program we just created, press PRGM [EDIT] and choose the program name by cursor or number. Let's suppose the name was ALF. Unlike the running of a program which first requires the program command be placed on the homescreen, and then requires pressing ENTER to run execute it, the program editor opens the instant you select the program name. You can now use your cursor, and your Ins and Del skills, to change any line you wish.

A neat shortcut for moving quickly through screens of code at a time (should your programming skills grow, this will save a lot of time) is to press ALPHA ▲ or ALPHA ▼ while in the editor.

To make a copy of a program, first create a new program with another name. Then, move to the first line of the new program and press 2nd STO→ [RCL]. You should now have an active command line immediately to the right of a **Rcl** operator which is waiting for you to tell it what to recall. Since we want to recall a program,

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we need a way to access that program name. Getting to the program name is not quite as simple as going to the NAME menu and picking it as we would with a list or matrix. However, it's not too bad.

To get to the program name while Rcl is active press PRGM. There are now three headers showing...CTL, I/O, and EXEC. These are not the same headers as we saw when we pressed PRGM before. This is because we are not on the homescreen; we are in the program editor (this will be explained in more detail shortly). While it seems odd, the one we want is EXEC. So, go to [EXEC] and choose the name ALF. That is, we just recalled the name of the executable program ALF into the new program and all the code in ALF code pops neatly into place effectively creating a copy.

Why would you want to do this? Suppose you are learning to program, or are editing a program, and you have parts of it working. You want to experiment with changing some lines or whatever. By making a copy of your current program, you have effectively created a backup. So, you can experiment without worrying about losing anything.

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The best way to backup a program, and for that matter, write and edit a program, is to use TI-Graph Link (see appendix titled "TI Calcs./Access./Support" for a thorough discussion of TI-Graph Link).

Part of me doesn't want to suggest this... you'll see why...but it is important to know and may be of use.

Never, never underestimate the creativity of a student - especially a student who knows anything about programming. Here's why...program code is saved in a readable format. That's why during editing you can still see the commands as you entered them. The first time a program is run, the TI-83 compiles it into its machine language and you'll notice a scrolling LCD bar in the top right corner of the screen as it is doing this. This makes the program execute faster. If the program is ever edited, it compiles again on first running. So what does this have to do with the above comment?

Suppose a program is written never intending it to run. Aaaaaah.... Though not designed this way nor for this use, a program opened in the program editor functions as a nice text storage device. It could be used to write notes, formulas, phone numbers, whatever. It isn't particularly readable or searchable, but for small tasks or really important motivating factors like tests, it can be useful.

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Another way to accomplish storage of information through programming is to get to know how the PRGM [I/O] **DISP** and **Output (** commands work (*note that while in the program editor, the PRGM key presents program editing menus instead of the normal EXEC, EDIT, and NEW menus*).

PRGM [I/O] **DISP** and **Output (** place text on the screen when a program is run. You could, for example, have programs named SinLaw, CosLaw, etc., where notes print to the screen as the program is run. Using the PRGM [CTL] **Pause** command can even cause the notes to print a screen at a time with each press of ENTER displaying the next screen.

In fairness, that's what programs are supposed to do…enable the running of a series of commands to achieve a task that we don't want to do manually over and over. The real power of programming is not to do this sort of stuff however. It's in writing code to compute the solution to a sine law problem with appropriate prompts; or to change marks to per cents and give the pass rate; or to drive a CBL (see appendix titled "TI Calcs./Access./Support"). By experimenting a little with the

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ideas posed here though, you may find that programming is not as hard as you thought and actually provides even more power to your TI-83!

And, as a friend of mine keeps saying, "I don't have to know how to do it, as long as I know someone who does". So, if you have a student or teacher who appears to be good at this, give them a task you'd like turned into a program on the TI-83! Maybe they have programs (or games?) you want. They can easily be transmitted between calculators too (see next *Teaching Tip*). Happy programming ☺

Notes/Comments/Ideas

Teaching Tip – Sharing Data With Other TI-83s

There is a really useful trick for setting everybody up the same way. Write a program that places data you want to share into the lists with names you specify. Send that program to other machines with the link cable and have others run the program. Now, everyone has exactly the same data, under exactly the same list names, in minutes!

A simpler option that's quicker (but requires you to keep the list data outside of a program in a list name) is to use your link cable and transmit the list(s) directly to other machines. If your partners can follow instructions, this works really well too.

If you really want to be creative, you could include the SetUpEditor command in the program. This would even customize the stats/list table so you could view data, not only under the same list names, but also, in the same table with the same headings. With a little thinking, you could probably figure out how to link formulas and have new information generate once your data is in… all done automatically when a program is run.

You have now gotten a sense of how easy simple programming is. With a little familiarization with loops and logical operators, much more is possible.

Notes/Comments/Ideas

In The Service Engagement Master #2

The Ruler Drop

Materials:

1. Ruler
2. Pencil

You and a partner will be asked to collect individual data reflecting your reaction times. You will record your results in a frequency table and then view both your and your partner's data simultaneously as a boxplot discussing your experiment along each step.

Repeat this activity switching roles with your partner once your data is collected.

Lay your arm flat along a desk or table edge so that your hand extends freely over the edge. Your partner will hold a ruler so that the 0 mark is between your thumb and index finger. Your partner will then release the ruler and you will grab it

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between your fingers as quickly as you can. Then make a note of spot you grabbed the ruler to the nearest centimeter.

Record your data in the table supplied. Repeat this 11 more times for a total of 12 trials. Take the single highest and lowest of the data and scratch them. You should now have 10 measurements that reflect your reaction time. Do a frequency count for repeated data. It should have less than 10 entries unless each distance was different.

Use the other part of the table for your partner.

DATA 1		FREQ TABLE 1		DATA 2		FREQ TABLE 2	
		Data	Freq.			Data	Freq.

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For ease, we will use $L_1 - L_4$ to store this frequency list data. Store your distances in L_1 with the frequencies in L_2. Similarly, your partner's in L_3 with frequencies in L_4.

Define plot 1 as your data boxplot. Set Xlist to L_1. Notice that in defining the plot, there is a field for Freq. Previously we left this field at its default of 1 meaning that each piece of data in our Xlist occurred once (i.e. every value was an individual data point in the Xlist). Now, we have chosen to record our data differently and have to set the name of the list representing our frequency list. Set Freq as L_2.

Similarly, define plot 2 as your partner's data boxplot and use ZoomStat to view both plots. So, who was quicker in general? Are there differences in dispersion? Are there other differences? Can they be explained?

Perform one variable analysis on each data set and discuss the above again. Note: The command **1-Var Stats** is set up for two arguments. The first is the data list. The second, separated from the first by a comma, is the frequency list (if any) associated with the data list. The frequency list is an optional argument and, if omitted, is set at 1 the same as the default when we defined plots in the stat plot editor. To perform 1-Var Stats analysis on L_1 with L_2 as the freq. list we use: STAT

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[CALC] **1-Var Stats** L_1 $\boxed{,}$ L_2 ENTER. Each one variable statistics analysis must be done separately.

Consider the following questions for discussion:

- ❖ Can you create a stat plot using data recorded in a frequency table?
- ❖ Can you perform one variable analysis using data recorded in a frequency table?

Notes/Comments/Ideas

In The Service Engagement Master #3

Ping Pong, Golf, or Tennis Anyone?

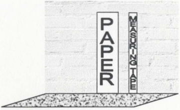
Materials:

1. Adhesive tape
2. About 2.5 meters of cash resister style paper roll
3. Measuring tape
4. Ping pong ball, a golf ball, and a tennis ball
5. Pencil

In this activity, we will generate data based upon the height three different balls bounce as compared to the height of their release point. This activity should practice identifying a trend in data, and applying median-median line, and least squares line regression models.

Notes/Comments/Ideas

Set up your drop as shown on the right:



In a moment, we will begin the ball drops, but first the drops are described. We conduct three separate drops from each of several pre-determined heights. For each drop, have a partner mark the distance from the floor to the bottom of the ball at its greatest bounce height on the paper. Use the average of the three bounces to represent the bounce data value from that drop height for that ball. That average should be recorded in the supplied table.

As an example, from a release point 200cm above the floor, suppose a ping pong ball bounced 120cm, 125cm, and 115cm respectively for each of three drops. Then 120cm (the average) is recorded under Ping Pong Ball as the 200cm Bounce Height entry.

The supplied table for recording data follows:

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Complete the table below using **centimeters** for bounce height:

Drop Height in cms	Ping Pong Ball Bounce Height	Golf Ball Bounce Height	Tennis Ball Bounce Height
200			
175			
150			
125			
100			
75			
50			
25			

Run **SetUpEditor** HGHT, PING, GOLF, TENN. Go to the stat/list editor and record your data (or store it to those lists from the homescreen). That is, store {200, 175, 150, 125, 100, 75, 50, 25} to HGHT and the generated data to their respective lists corresponding to the heights that generated them.

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Turn plot 1 on and define it as a scatterplot, with Xlist = HGHT, Ylist = PING, and mark style as the first style there (there are three). Do the same for plots 2 and 3 but substitute GOLF and TENN for the Ylist, choosing a different mark for each.

Press [ZOOM] [ZoomStat] and you should have data from all three balls plotted as scatterplots on the same screen. It is not necessary to plot all three lots of data like this but this activity illustrates that up to three plot styles/data lists can easily be viewed for comparison.

Use [STAT] [CALC] **Med-Med** HGHT, PING to calculate the Median-Median Line regression values for the ping pong ball. We want to paste this equation in Y_1 so we can plot it and trace it. To do this, press y =. Cursor to Y_1. Now we'll retrieve this regression equation and paste it here. Press VARS [Statistics...] [EQ] **RegEQ** ENTER. This should have pasted an equation of the form (ax+b), with the a & b values we saw a minute ago when the regression model was run, into the Y_1 field. Press GRAPH to plot this equation through our data. How well does this model visually fit our data?

Notes/Comments/Ideas

Use [STAT] [CALC] **LinReg(ax+b)** HGHT, PING to calculate the Least Squares regression values for the ping pong ball. We want to paste this equation in Y_1 so we can plot it and trace it. To do this, press y=. Cursor to Y_1. Now we'll retrieve this regression equation and paste it here. Press VARS [Statistics...] [EQ] **RegEQ** ENTER. This should have pasted an equation of the form (ax+b), with the a & b values we saw a minute ago when the regression model was run, into the Y_1 field. Press GRAPH to plot this equation through our data. How well does this model visually fit our data?

If closer comparison between Median-Median and Least Squares Lines is desired, one of them could be pasted into Y_2 allowing both regression equations to be plotted and/or viewed in a table together.

Repeat this procedure to plot the other two regressions on the same screen substituting GOLF and TENN respectively and putting those equations in Y_2 and Y_3 respectively.

There is a teaching tip presented shortly which illustrates varying plot styles. It could be useful in an activity such as this and is certainly useful in the classroom setting to distinguish one plot from another. You may wish to try varying plot styles

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for the Median-Median and Least Squares lines done a moment ago to see how useful it can be, or explore it individually during a future activity. In any case, the three regression equations for each of the three data pools should be in Y_1, Y_2, and Y_3 respectively.

These y= plots are trace-able and table-able. Therefore, they can allow us to interpolate and extrapolate with ease. To interpolate or extrapolate on the graph screen, simply press TRACE and enter the X-value you wish evaluated. If it is within your view it will be hi-lighted. If not, the plot Y-value will just be displayed.

To interpolate or extrapolate in a table, set up your table in TblSet with reasonable TblStart and ΔTbl and scroll away refining your view but adjusting TblStart and ΔTbl as necessary. Or, in Table Setup, set Indpnt to **Ask**. This allows you to simply key in the value of interest and it will generate answer values for all selected relations which currently represent all three of your regression lines.

Consider the following questions for discussion:

- Predict the bounce height for all three balls if dropped from 2.5m.
- Predict the bounce height for all three balls if dropped from 1m.

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- Predict the bounce height for all three balls if dropped from 10m.
- How reliable are our models under the conditions in the three questions above?
- Are the slopes of our regression lines the same? Why, why not?
- What do the slopes represent?
- Are the Y-intercepts of our regression lines the same? Why, why not?
- What do the Y-intercepts represent?
- Can you think of a way to determine the best line out of all three regressions?
- Execute the **DiagnosticsOn** command from the CATALOG and run the 1-Var Analysis again.
- What do you have now that you didn't have before?
- To turn off this option, execute the **DiagnosticsOff** command from the catalog.

Notes/Comments/Ideas

Reference:

Ping Pong, Golf, Tennis Anyone? is based upon the generic ball drop experiment of early high school physics classes. Heights chosen and approach are based upon 2.1 Collecting, Graphing and Analyzing Data 3: The Ball Drop from the following reference though it has been significantly enhanced.

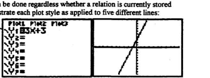
Texas Instruments. (1996a). *CT³ - Canadian teachers teaching with technology* [manual]. Dallas, TX: Author, 2-1.

Notes/Comments/Ideas

Teaching Tip – Varying Plot Style

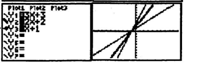
When plotting several relations at once, especially using data of a similar nature, it is helpful to vary the plot styles. There are seven plot styles available of which five are really for this purpose. The current plot style for any relation is indicated by the symbol immediately to the left of Y in the Y= editor. The style is changed by cursoring to this indicator and cycling through its choices by pressing ENTER. This can be done regardless whether a relation is currently stored there. The following screen shots illustrate each plot style as applied to five different lines:

Plot style is regular line



Plot style is regular line
Plot style is thick line

Plot style 1 is regular line
Plot style 2 is thick line
Plot style 5 is regular line with path led by a circle which then disappears



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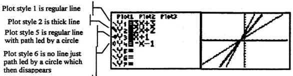

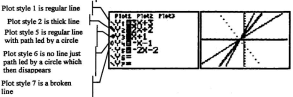

As you may have noted, plot styles 3 & 4 weren't mentioned. Those styles shade above and below the selected plot and so would not be useful here. Also, Plot styles 5 & 6 are not of much use here either as 5 leaves a line the same as style 1 and style 6 leaves no plot at all. That doesn't mean they have no uses (we'll see that soon), but they are of no real use here.

Therefore, we really have three plot styles that are different enough to allow comparisons... styles 1, 2 and 7... regular line, thick line, and broken line. Coincidentally, that's exactly the number of distinct plot styles we need here.

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Teaching Tip – Residuals

Each time a regression is run, a list of residuals (differences between actual values and computed values from the regression model) is stored to a list named Resid. This list is a good way to compare regression fit to data and can be very useful.

If we use [LIST] [MATH] **Sum (** Resid we can show that these differences sum to zero. The magnitude of the values may be more useful however. This can be done by taking the absolute value of Resid, or squaring the entries in Resid and then taking the square root which can be a foundation for establishing groundwork for the Least Squares Line.

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Teaching Tip – Regression Eq. Compared To Data

Another way to compare actual vs. computed values under a regression model is to link a formula to a list that computes the regression values for the independent variable in the stats/list editor.

Let's suppose a regression model has been run on a data list whose independent variable data values were called HGHT, and that the regression equation is in Y_1.

A list could be created called REGR. Linking a formula such as REGR = "Y_1 (HGHT)" would produce a list of values using the regression equation in Y_1 and place those values in the list REGR for comparison.

Residuals could then be calculated by subtracting the two columns as could the squares of the residuals etc....

In fact, here we can see another way to utilize the graphing and the stats/list editors. Suppose you have a non-uniform discrete data and have a model you wish to have those values subject to for the purposes of creating needed output.

Put the function in one of the y = fields say, Y_1. We could now accomplish the task by using the TblSet command, setting Auto to Ask, and keying in the independent variable data values one by one.
It can also be done by putting the independent variable values in a list say, L_1. If we now set $L_2 = Y_1(L_1)$, we will generate output data in L_2 for all of the L_1 data at once. If further input were desired, link the formulas as follows: L_2 = "$Y_1(L_1)$" which allows for generation of L_2 data of existing L_1 values as well as subsequently keyed ones.

Notes/Comments/Ideas

In The Service Engagement Master #4

Is This Normal?

This master will illustrate the T - Test on a sample of a population thought to be normal. It includes finding a confidence interval for the mean, determining the percentile rank of a piece of data, determining the data value that would represent a given percentile rank, and drawing a normal probability curve for a given mean and standard deviation. Note that a Z - Test could be performed in exactly the same way.

Suppose we wish to determine how long a line a pen can write until it runs dry. We normally assume such populations to be normally distributed.

Suppose a very small (but good) manufacturer of specialty pens culls a random data sample representing 10% of the population of one production. Upon being tested, that sample yields the following line lengths:

1651m, 1734m, 1577m, 1601m, 1688m, 1733m, 1614m, 1596m, 1665m, 1647m. Enter this data into L_1.

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Let's further assume that this is our first run of these specialty pens, so we do not yet know the standard deviation of this population. Therefore, we should perform a T - Test to determine a confidence interval for the mean of the full population. That is, we can use our sample, and our sense that it is likely normally distributed, to say with some confidence that the population mean is around some number. This either makes us happy or sends the pens back to engineering!

STAT [TESTS] contains the menu item **Tinterval...** (menu item #8). This function determines a confidence interval for the mean of a data list. It can use either the data itself or the summary data (i.e. mean and standard deviation) if it has already been computed.

Let's determine a 95% confidence interval for the mean of the population of pens that the above sample came from.

Press STAT [TESTS] **Tinterval....** Now we are faced with editor choices. These choices will reflect the last settings of this editor and may contain old data (your screen will be similar to figure 1a or 1b).

We will now set up for our data. Since we are entering data as a list, and not entering the summary statistics for that list, we make sure `Data` is selected in the

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first line (remember, to select a menu item with unnumbered fields, cursor to the choice and press ENTER - your screen should now match figure 1a). To use our raw data stored in L_1, each occurring with frequency one, and with a 95% confidence interval, set your fields as in figure 2 leaving Freq: 1.

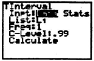

figure 1a

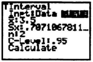

figure 1b

If we had a frequency list for our data, we would use the list name as the Freq: argument.

Move your cursor to CALCULATE and press ENTER. This yields the screen shown as figure 3.

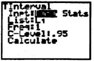

figure 2

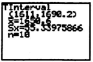

figure 3

Notes/Comments/Ideas

This screen is saying that the interval 1611 to 1690.2 is the requested 95% confidence interval for the population mean, given that the sample mean is 1650.6, that the sample standard deviation is about 55.3, and that the sample size is 10.

If we already had performed one variable analysis on this data and knew the sample mean and standard deviation, we could have chosen `Stats` instead of `Data` for the Inpt: field shown in figure 1, and simply entered the summary data. Try it!

One of the pens in the sample writes 1688m. Assuming a normally distributed population, at what percentile rank would that pen fall?

To determine this, we need the cumulative area of probability under the normal curve from the left, ending at the value in question, under the sample conditions. 2nd VARS [DISTR] **normalcdf (** is the normal cumulative distribution function.

Press 2nd VARS [DISTR] **normalcdf (** 0, 1688, 1650.6, 55.3 ENTER

returns a value of about 0.75. That is, the interval from 0 to 1688, for a sample with mean 1650.6 and standard deviation 55.3, accumulates 0.75 of the area under the normal curve for the population in question. Hence, 1688 is at the 75^{th} percentile. Note that 0 is not a required leftmost value here. In fact, any value 3 deviations or

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more to the left of the mean would have produced a comparable result but it is possible to have a faulty pen which does not write at all.

If we know the percentile rank we are interested in, can we determine the data value needed to represent that percentile?

To determine this, we use **invNorm (** which computes the inverse cumulative normal distribution function. This function yields a value that accumulates the required percent of area under the normal distribution from left (beginning at 0) to right (ending at the yielded value). Instead of accumulating the area for a given interval, it takes an area and gives the right endpoint of the interval required to produce it.

[DISTR] **invNorm (** 0.95, 1650.6, 55.3 ENTER returns a result of 1741.56. That is, 0.95 of the area under a normal curve is accumulated under a normal curve from the leftmost position to 1741.56 in a sample with mean 1650.6 and standard deviation 55.3. We can use this method to double check our last calculation. Running [DISTR] **invNorm (** 0.75, 1650.6, 55.3 ENTER returns 1687.899 or about 1688.

Can we draw the normal distribution for a given mean and standard deviation?

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The command [DISTR] **normalpdf (** returns the normal probability density function at a specified *x*-value. That is, it returns the values whose accumulating area function yields percentiles associated with the normal distribution. To plot a normal curve then, we must use this function in a **y=** field and choose an appropriate window for our data.

In the **y=** editor, paste **normalpdf (** and its arguments **x, 1650.6, 55.3)** in Y_1. This means the normal probability density function will be calculated and the result plotted for each *x*-value in the window (i.e. each pixel step as the plotter sweeps from Xmin to Xmax) using a population having mean 1650.6 and standard deviation 55.3.

We will draw the curve in a moment, but first some comments about graphing windows and the normal distribution.

Viewing this curve requires extra special attention to window settings. By far the best choice, as is mentioned in the TI-83 Manual, is to use a domain that suits your sample data, and then choosing ZoomFit from the ZOOM menu to create a decent view. ZoomFit basically leaves the chosen domain intact, and sizes a window which fits the Ymax and Ymin resulting from the chosen relation.

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Tracing the curve gives a sense of why the window parameters can be so tricky here. After all, we are talking about probabilities that are established from the accumulated area function. This is quite different than what many students are used to. It may, however, be a really nice exploration for calculus students who have begun to study the integral as an accumulating area function.

Once practiced with what these window parameters typically are, and once experienced with the notion of a skewed curve, manual window setting can become more comfortable. ZoomFit works very well for these situations however and will be the author's choice for some time to come!.

Let's get on with drawing our curve!

Set your domain to cover an *x*-spread of three deviations roughly (i.e. 3*55.3) above and below the mean 1650.6. So, one domain, which is reasonable, might be1450 to 1850 with xScl set at 50. Then press ZoomFit, which should yield a screen like figure 4. Another command you may now wish to explore is [DISTR] [DRAW] **ShadeNorm** (. [DISTR] [DRAW] menu choices function like the regular DRAW menu choices (i.e. draws untraceable objects on a graph screen) but has tools specific to statistics-based graphs.

figure 4

Notes/Comments/Ideas

It is worthwhile to note here that this is the normal curve that **normalcdf (** used earlier to return an accumulated area of 75% of the total for the data value 1688 in our sample.

Normal curves and probability distributions offer many opportunities for further study. Students could be asked to determine the differences between particular tests are and their uses in determining confidence intervals. Or perhaps the theory behind some of the everyday confidence intervals heard on media (such as "...is correct within 5 points 19 times out of 20.") could be researched.

Notes/Comments/Ideas

Teaching Tip – A Roll Of The Dice

Probabilities are everywhere in real life and abound in many educational resources. A lot of day-to-day probabilities have been determined through experience. For example, the probability that a 30 year old male, non-smoker will die in any given subsequent year is calculated based upon known death rates for that type of person, in that part of the country, under those conditions,.... These probabilities are used to determine insurability risks and set life insurance rates.

Probabilities of other events are not so morbid. Some may even be said to occur at random...like winning the lottery. The TI-83 has a tool onboard that can assist in simulating such events. It has random number generators that, with just a little adaptation, can produce random results within established bounds.

The MATH [PRB] menu has 7 choices: **rand**; **nPr**; **nCr**; **!** ; **randInt** (; **randNorm** (; and **randBin** (. These functions generate: a random number equal to or between 0 and 1; the number of permutations of a collection of items taken *n* at a time; the number of combinations of a collection of items taken *n* at a time; the factorial of a number; a random integer within a specified range; a random real number from a specified normal distribution; and, a random real number from a specified binomial distribution. There is also an operation in the MATRX [MATH] menu that generates a random matrix.

Each of the "rand" prefixed choices has an optional parameter for number of trials. We will use **rand** and **randInt (** in the following examples. Note that the full menu path to these commands will no longer be repeated during this *Teaching Tip*.

To model the rolling of a fair dice, we need to create a list of integers, in the range of 1 to 6, that occur randomly. The simplest way to do this is to use **randInt**.

Press **randInt (1 , 6)** ENTER. You should see an integer somewhere between and including 1 and 6. Keep pressing ENTER and you get more.

Notes/Comments/Ideas

Suppose you knew you wanted to run 10 trials. Press **randInt (1 , 6 , 10)** . The result is a list of 10 random integers between or including 1 and 6. That's a pretty quick and easy way to simulate 10 rolls of a dice.

The command **rand** is a little different and offers other advantages. **rand** generates a random number ≥ 0 and ≤ 1 to fourteen decimal places. The only optional argument for **rand** is the number of trials. So, **rand (3)** would create a list of 3 random numbers.

It is a little easier to create random numbers of more interesting characteristics than integers using the **rand** command.

Let's use **rand** to create random numbers with particular characteristics. Suppose we wanted a random number between and including 30.0 and 40.0. From our definition of **rand**, it seems that **rand + 30** puts us in the ballpark but would give us 14 decimal places to contend with!

The MATH [NUM] **round (** command comes to our rescue! **round (** takes two arguments...the value in question and the number of decimal places to a maximum of 9.

round (rand + 30, 1) ENTER creates a number in the range desired. Remember, these functions can take lists as arguments too, and can have the number of trials specified.

What do you think **round (rand (2)** + 30, 1 ENTER will do?

How about **randInt ({ 1 , 6} , {5 , 10}** ?

*Here are a few simulations to acquaint you further with the **randInt** command.*

Pair of dice:

randInt ({ 1 , 1} , { 6 , 6} ...list elements pair in order. So, this command generates a two random integer list where the first integer is between or including 1 to 6, and so is the second.

Notes/Comments/Ideas

Pair of dice repeated trails:

The **randInt (** command above will not accept the normal optional argument in the third position for number of trials when lists are used as arguments for the bounds of the integers.

The simplest way to run repeated trials is to repeatedly press ENTER on the command line above. A simple program for-next or similar loop using the **augment (** command could collect all the trial results for you if you feel like experimenting with programming commands.

Sum of pair of dice:

sum (randInt ({ 1 , 1} , { 6 , 6} ... **sum (** is taken from the LIST [MATH] menu and sums the list generated from the example above. Same problem happens here for repeated trials. Same solution works too. Unless you count the method below....

Summing a pair of "crappy" dice:

It's the same thing as above really. It is just another way around the problem.

randInt (1 , 6 , 10) + **randInt (1 , 6 , 10)**. Since **randInt (1 , 6 , 10)** generates a list of 10 integers from 1 to 6, and adding lists adds the paired elements, the result here will be a list of 10 sums of random pairs of integers of which each integer is between or including 1 to 6.

Notes/Comments/Ideas

In The Service Engagement Master #5

No Free Parking!

Upon arriving in a new city, you decide rent a car to get around. You have a meeting downtown tomorrow, so you take a drive down there today to be sure you'll know how to find the place. While driving you notice that there aren't any parking meters around because of the narrow streets, but there are three parking lots nearby with signs out for parking.

One company charges $3.00 to enter and $0.20 per hour. Another company charges only $1.50 to enter but $0.40 per hour. A third company charges no entry fee but charges $0.75 per hour. We would like to model these cost structures to decide which is best if we must be there for an 8-hour business day.

We note that this type of function does not count parts of hours parked. A function on the TI-83 which does this is **int (** i.e. the greatest integer function. Another note is that domain values less than 0 make no sense (in fact, less than 1 makes no sense either but it is useful to examine the domain value 0 for another reason).

Notes/Comments/Ideas

Let's set up a table to view whole hours from 0 up in steps of 1. Press [TblSet] and set TblStart = 0 with ΔTbl = 1. Let's also set up the x-values of our viewing window to reflect this. Press WINDOW setting Xmin = 0, Xmax = 12 (which more than covers our day), Xscl = 1 to reflect 1 hour increments, and Xres at 1 since we will need that resolution later.

Now let's enter our three scenarios (see figure 1). View the table results (see figures 2 and 3). Note that scrolling ▸ showed us Y_3, and scrolling ▾ moves us past 6 hours though that isn't illustrated..

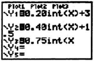


figure 1

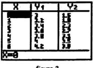


figure 2

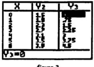


figure 3

For parking of up to 6 hours, company #2 seems best. Using our table we can make the decision on the best buy but there's more to be learned about such scenarios. Let's graph the results.

Notes/Comments/Ideas

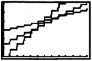


figure 4

We can see from our table that the parking time we are particularly interested in can result in bills of up to $6.00. Therefore, we set Ymin = 0, Ymax = 6, and Yscl = 1. Press Graph (see figure 4). The graphs appear very rough, and appear continuous. Why is this? Are the functions in fact continuous? Explain.

To correct the continuous appearance, we set our global graphing mode from **Connected** to **Dot**. This will force the grapher to only display the pixels for which the function values were actually calculated, instead of those same pixels connected in an effort to make the graph appear continuous (though connected pixels are useful as a means to compensate for the lack of screen pixel density for plotting continuous relations). For a related discussion to this pixel problem, see the appendix titled "Friendly Windows".

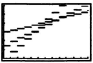


figure 5

Repeating the graphing command now shows more distinct steps and illustrates clearly the discontinuous nature of these functions (see figure 5). Examination may be enhanced by using the different plot styles discussed in "Ping Pong, Golf, Tennis Anyone?" in

Notes/Comments/Ideas

this section.

It may also be useful to consider the ZOOM menu if the intersection of the graphs, or value of the graphs near the eight hour mark, remains uncertain leaving the best buy hard to determine. In this case however, the TABLE option is probably the easiest to manage through specifying TblSet, TblStart, and ΔTbl appropriately.

Consider the following questions for discussion:

- Why does the third plot have no Y-intercept?
- At roughly what value are all three companies close in cost?
- Which company is definitely the cheapest for short runs?
- How are these functions' parameters similar to linear functions?
- What other common events create distinct units of increase like this?

Notes/Comments/Ideas

In The Service Engagement Master #6

Matrices: Systems and Networks

This section will not specifically discuss the nature and defining conditions of matrices. Basic operations and some interesting applications will be illustrated and will utilize addition, multiplication, and inverses of matrices. A list of many of the available matrix operations available on the TI-83 is given later in this master. For any course where much work is to be done with matrices, section 10-14 in the TI-83 Graphing Calculator Guidebook should be considered essential reading.

The TI-83 has ten reserved matrix variable locations A, B, ..., I, J. These locations can store rectangular arrays of real numbers to a maximum of 99 rows or columns depending upon available memory. These matrix names can only be accessed through the MATRIX [NAMES] menu.

Matrices on the Homescreen

Matrix delimiters are [and] which are the shifted functions of the multiplication and subtraction keys respectively. To begin a matrix we press [. Each

Notes/Comments/Ideas

row is begun and ended using **[** and **]** with commas separating entries within that row. Simply beginning a new row separates rows. To end a matrix we press **]**.

For example, to enter the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, we press **[[1, 2] [3, 4]]** ENTER.

Your screen should look like figure 1. As you can see, and as we have seen already with lists, we can operate on matrices on the home screen without having to name the matrix i.e. store it to a specific variable location. Unlike lists, but like the graphing editor's Y_n entries, we can only store matrices to the reserved variable names. We cannot create user-named matrices, nor can we key in the matrix names of reserved variables using the keyboard. The TI-83 will only recognize the matrix names [A], [B], …, [J] as grabbed from the MATRIX [NAMES] menu.

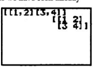


figure 1

Matrices can be operated on by keying the operation directly. For example, the matrix sum $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ could be keyed in directly as follows: [[1,2] [3, 4]] + [[5, 6] [7, 8]] ENTER yielding a result of: $\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$ as in figure 2.

Notes/Comments/Ideas

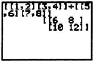


figure 2

Note: if you are keying in several operations on similarly dimensioned matrices where only the values differ, it would probably be easier to use the 2nd ENTRY feature to paste the previous entry line and simply edit in the new values.

Matrices in the Matrix Editor

figure 3

The matrix editor looks a lot like the stats/list editor initially. It is set up with only the reserved names. Unlike the stats/list editor however, it <u>cannot</u> be customized. The matrix editor is accessed by pressing MATRIX choosing [EDIT] (see figure 3) and selecting the matrix you wish to edit (do not select a name yet). Please note that only the matrix names appear on the illustrated screen at present. If any of the reserved

Notes/Comments/Ideas

matrices onboard your calculator have been previously defined and not deleted, their dimensions, as they currently exist, will show next to the name.

Let's select [A]. We now have the matrix editor open with matrix [A] ready to be edited (see figure 4). Your cursor should be on the first 1 next to the matrix name (the default size for any previously undefined matrix is 1 by 1). The TI-83 is waiting for you to dimension the matrix into the preferred number of rows and columns.

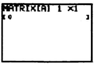


figure 4

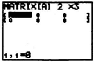


figure 5

Let's define [A] as 2 by 3 i.e. 2 rows by 3 columns. Press 2 and ENTER or ▸ . Then press 3 and ENTER or ▸ or ▾. An entry is accepted by pressing ENTER or by cursoring to another field. Your cursor should now be on the first element of matrix [A] (see figure 5). This is the matrix editor view. To edit an element, cursor to it and press ENTER to activate the editing command line, or simply enter the new element value directly. In the event that you activate an element for editing but change your mind, press CLEAR and ENTER to restore the element as it was before.

Notes/Comments/Ideas

We will define matrix [A] to hold the elements $\begin{bmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \end{bmatrix}$ (see figure 6). We have now defined reserved matrix name [A] to hold $\begin{bmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \end{bmatrix}$. We can view [A] in the matrix editor, as now, or we can view it on the homescreen. To paste any defined matrix name on the homescreen for use in an operation, use the MATRIX [NAMES] menu, cursor to the name, and press ENTER. To view the contents, we simply paste the name and press ENTER. [A] viewed on the homescreen should look like figure 7.

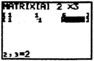


figure 6

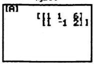


figure 7

Many operations with matrices are available with the TI-83 including:

- Negating
- Powers
- Transposing
- Grabbing an individual element
- Finding the inverse

Notes/Comments/Ideas

- Adding, subtracting, and multiplying
- Rounding
- Using relational operators
- Truncating (by taking the integer part only or the fractional part only)
- Finding the identity
- Evaluating the determinant
- Filling matrices with chosen elements or with random values
- Augmenting
- Converting a matrix to a list and a list to a matrix
- Row operations such as addition, row-echelon form, and reduced row-echelon form.

The operations available to [A] as mentioned above are available for all matrices created by the TI-83. Recall however, that the only matrix names that will be recognized, are those names pasted from the MATRIX [NAMES] menu. Other

Notes/Comments/Ideas

matrices that are to be operated on must be keyed directly on the homescreen or created through programming.

We can operate on [A] as we would any other quantity now that it is defined and dimensioned. If we had attempted to operate with the matrix named [A] before we had defined it, we would have gotten an **ERR: UNDEFINED** condition. If we had attempted to operate on a defined matrix whose dimesnions did not agree with the operation involved, we would have gotten an **ERR: INVALID DIM** condition.

Solving A System Of Equations

Solving the system: $\begin{cases} x+y=6 \\ x-y=2 \end{cases}$ is a very simple matter given algebraic techniques. So is its solution using the TI-83. Such a system results in three matrices: $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $C = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ such that $A * X = C$. The solving of such a situation can be accomplished by recognizing that $X = A^{-1} * C$.

A^{-1} can be found directly if desired. To find A^{-1}, enter A as a matrix and press $\boxed{x^{-1}}$. Note that the power negative 1 (keyed as ^ - 1) is not defined here for matrices and will produce an error message. Inverses of matrices <u>must</u> be done using

Notes/Comments/Ideas

the x^{-1} key. Let's find A^{-1} using the homescreen. Figure 8 illustrates the matrix A above with the inverse operation applied. The second entry shown in figure 8 illustrates the use of the ▸ **Frac** command on the same operation. We could now simply multiply Ans by C for the desired result (see figure 9). This wasn't necessary though it does illustrate how to view the inverse of a matrix.

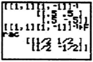


figure 8

Of course we could have just keyed A^{-1} C to begin with (see figure 10). Note that implied multiplication is understood. We could also solve this system using the matrix editor to create matrices A and C, and then keying in A^{-1} C using those names from the MATRIX [NAMES] menu.

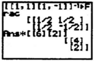


figure 9

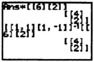


figure 10

Notes/Comments/Ideas

Teaching Tip - Identities & Inverse Matrices

When solving systems of equations, multiplying the constant matrix by the inverse of the coefficient matrix is a valuable approach. There is an opportunity here to discuss whether an inverse will always exist and how it is found.

If a coefficient matrix has no inverse you will get an error condition on the TI-83. This means then, that the system either has no solution, or perhaps has many solutions i.e. is inconsistent or dependent.

To determine if the equations in a system are in fact the same equation, one could use the row multiplication operations in the MATRX [MATH] menu.

For example, taking the quotient of the first equation's x coefficient to the second equation's x coefficient yields their ratio. Multiplying each row of the second equation by this ratio, ***row (** works well for this, will quickly reveal which of the two possibilities were present by illustrating whether or not the second equation was simply a constant multiple of the first.

Here we have the opening for exploring the connection between an identity matrix and the determinant of a matrix arising directly out of a need to solve a problem whose variables are under a series of constraints.

An identity matrix [I] is defined as a matrix that does not alter another matrix under the operation of multiplication i.e. [A] * [I] = [A]. The 2 by 2 identity matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ This extends to the notion of inverse i.e. [I] = $[A]^{-1}$* [A]. So, we can set up an equation now to explore finding an inverse.

Notes/Comments/Ideas

For a matrix such as $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, the inverse matrix would have to satisfy the equation representing

[matrix]*[inverse] = [identity]: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ... yielding equations: $\begin{cases} a+2c=1 \\ b+2d=0 \\ 3a+4c=0 \\ 3a+4c=1 \end{cases}$.

These systems can be solved (we now know how to apply the TI-83 to this task) and yields:

$\begin{cases} a=-2 \\ b=1 \\ c=1.5 \\ d=-0.5 \end{cases}$ i.e. the inverse matrix is: $\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$.

The determinant and inverse matrix operations on the TI-83, together with a series of chosen matrices (2 by 2 are probably most easily managed), could provide a valuable exploration into whether a connection appears between a matrix whose inverse produces an error (i.e. whose inverse does not exist), and the value of its determinant. Defining determinant, and its use in establishing the existence of an inverse matrix, could be food for further class exploration or for a portfolio piece for students. The following reference provides valuable insight into this type of question.

Reference:

Murdoch, J., Kamischke, E., & Kamischke, E. (1996). *Advanced algebra through data exploration: A graphing calculator approach* (Prelim. Ed.). Berkeley, CA: Key Curriculum Press, 420-7, 440-1.

Notes/Comments/Ideas

Another way, and probably the quickest way, to solve a system is to use the reduced row-echelon form operation available in the MATRIX [MATH] menu. To take advantage of this operation, we create a new matrix containing all coefficients of the system i.e. $\begin{bmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \end{bmatrix}$. You may recall that we created this matrix earlier and it probably still resides as the reserved matrix [A] contents.

Again, view [A] on the homescreen by pasting its name and pressing ENTER (see figure 6 from earlier). Paster the **rref(** command from the MATRIX [MATH] menu to the homescreen, and then paste the name [A] by grabbing it from the MATRIX [NAMES] menu (see figure 11). Press ENTER to execute the operation (see figure 12). We can clearly see the solution generated by **rref (** as: $1x + 0y = 4$ and $0x + 1y = 2$. That's pretty easy too!

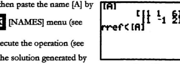


figure 11

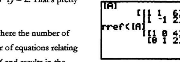


figure 12

Systems of equations where the number of variables is less that the number of equations relating them, can be solved using **rref (** and results in the

Notes/Comments/Ideas

limiting equations on the solution values.

Simple Network/Digraph

Another application of matrices involves simple networks/digraphs as illustrated by this example adapted from Murdoch et al. and appearing in the CT[3] Manual (1996). Much of the language is taken verbatim from *CT[3] - Canadian teachers teaching with technology* [manual] p. 2-8 to 2-9, but has been adapted with extensive explanation.

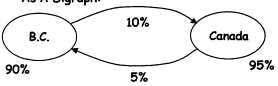
Suppose that in Canada there are 2 million people who live in British Columbia and 22 million who live outside B.C. Each year 10% of the population of B.C moves to another province in Canada. The other 90% remain in B.C. Also, each year 5% of the total Canadian population outside B.C. moves to B.C., while the other 95% remains in Canada outside B.C.

Assuming that the total population of Canada stays at 24 million and the year-to-year transition rates remain the same, find the number of people in B.C. (a) next year (b) 2 years from now (c) 3 years from now.

Two views of this scenario are digraph and matrix.

Notes/Comments/Ideas

As A Digraph:



As A Matrix:

The following table structures our transition rates:

Next Year

		B.C	Canada
Now	B.C.	0.90	0.10
	Canada	0.05	0.95

Notes/Comments/Ideas

We can easily calculate next year's B.C population by applying these rates to current populations. 90% of B.C.ers stay and 5% of Canadians move in...so, 2*0.90 + 22*0.05= 2.9 million people. By repeating this calculation, and tracking both populations, we could answer the questions posed...but that's tedious!

We can create two matrices which represent the quantities needed. Let the current population matrix be [B.C. , Canada]. We know the transition matrix (it's in the table). Therefore, $[B.C. \quad Canada] * \begin{bmatrix} 0.90 & 0.10 \\ 0.05 & 0.95 \end{bmatrix}$ yields

$[B.C. * 0.90 + Canada * 0.05 \quad B.C. * 0.10 + Canada * 0.95]$ representing the next year's populations of B.C. and Canada as elements respectively. This is becomes much clearer when values are used.

Let the initial population matrix be $P_0 = [2 \quad 22]$ and the transition matrix be $T = \begin{bmatrix} 0.90 & 0.10 \\ 0.05 & 0.95 \end{bmatrix}$. Then, the following summarizes population changes over the next three years:

$P_1 = \quad P_0 * T$

$P_2 = \quad P_1 * T = \quad (P_0 * T) * T = \quad P_0 * T^2$

Notes/Comments/Ideas

$$P_3 = \quad P_2 * T = \quad (P_0 * T^2) * T = \quad P_0 * T^3$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$P_n = \quad P_{n-1} * T = \quad (P_0 * T^{n-1}) * T = \quad P_0 * T^n$$

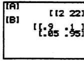


figure 12

We can use our knowledge of matrix operations with the TI-83 to perform the calculations needed for year 3. Store the initial population to matrix [A] and the transition rates to matrix [B] using the matrix editor (see figure 12 where [A] and [B] have been printed on the homescreen for reference). We confirm our setup by re-calculating the population for year 1 which we recall to be 2.9 million (see figure 13 and recall the first element is B.C's population and the second element is Canada's population).

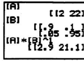


figure 13

Therefore, year 3 is determined in a similar way (see figure 14) and is approximately 4.3 million. Applications of this method are limited only by the complexity of the digraphs and available information.

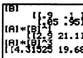


figure 14

Notes/Comments/Ideas

Reference:

Murdoch, J., Kamischke, E., & Kamischke, E. (1996). *Advanced algebra through data exploration: A graphing calculator approach* (Prelim. Ed.). Berkeley, CA: Key Curriculum Press, 389-390.

Texas Instruments. (1996a). *CT³ - Canadian teachers teaching with technology* [manual]. Dallas, TX: Author, 2-8 to 2-9.

Notes/Comments/Ideas

In The Service Engagement Master #7

Strung Out

Materials:

1. 30 cm of string
2. 4 Push Pins
3. Small square of cardboard
4. Pencil

This is a typical problem that has been found in mathematics courses for a very long time. The gain here is in the physical modeling, and in developing a better understanding of relations through establishing the formulas linking length to width, and area to width under a perimeter condition.

Suppose you wish to make a rectangular enclosure for your dog. You price material and figure you can afford 30m of fencing.

Notes/Comments/Ideas

What is the biggest rectangle you can make out of that fencing for your dog to enjoy?

Let's create a scale model of this problem with pins and string, and establish some reasonable possibilities for the dimensions. Use this data, and your TI-83, to determine the maximum sized rectangle possible.

Use the string and push pins to form 8 different rectangles on your cardboard. For convenience, use whole centimeters for widths (you don't really have to but it makes this exploration easier).

Record width and length in the table provided as you go while visually examining the changes in areas afforded by each rectangle. After doing one or two, try to predict the length you get given a width you choose.

Width	Length

Once all eight rectangles have been constructed and measured, make a note of which of your dimensions seemed to visually produce the biggest rectangle.

Notes/Comments/Ideas

Using the SetUpEditor command, we create a table in the stats/list editor with headers WID, LEN, and AREA. We do this as follows: STAT **SetUpEditor** WID, LEN, AREA ENTER. You may wish to use Alpha lock to help in typing the names and recall that names have to be five characters or less – hence the abbreviation. This creates the headers but the lists have no dimensions yet (so if we tried linking any formulas right now, we would get a **dim** error).

Go to the stats/list editor and enter your recorded widths under the WID header (for illustrative purposes in this exercise, the widths 1, 2, 3, 4, 5, 6, 7, 8 will be used).

Were you able to predict the length for a given width as you constructed some of your rectangles? If so, write on the line below how you would calculate the length if you were given a width. If not, discuss it with a partner or explore some more.

Use the relation you just created to make a linked formula for the header LEN so that Lengths are computed automatically. They should yield the same lengths that you found experimentally in your handwritten table. The formula should

Notes/Comments/Ideas

appear on your stats/list editor command line as shown in figure 1. LEN should appear as in figure 2 after we press ENTER to accept the linked formula.

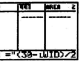


figure 1

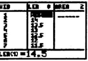


figure 2

You were asked to note which of your constructed rectangles appeared the biggest. Create a linked formula to the header AREA that takes the values from WID and LEN and computes AREA automatically. Your screen should look like figure 3, and then like figure 4 when ENTER is pressed. (remember, your data may be different) Which of *your* computed rectangles actually produced the largest calculated area?

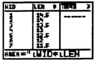


figure 3

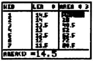


figure 4

Notes/Comments/Ideas

Create a scatterplot that shows Area vs. Width. What shape does it appear to have? ____

There are many regression models available on the TI-83 including:

- Median-Median Line
- Least Squares Line (ax + b)
- Quadratic
- Cubic
- Quartic
- Least Squares Line (b + ax)
- Natural Logarithm
- Natural Exponential
- Power
- Logistic
- Sine

Which of these regression models should we apply?

Notes/Comments/Ideas

Since the data seems to follow a quadratic path, we will apply the quadratic regression model and paste the resulting regression equation into Y_1. We do this as follows:

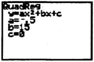


figure 5

STAT [CALC] **QuadReg** WID, AREA ENTER

yielding a screen like figure 5. Then go to the y= editor at the Y_1 position and press VARS [Statistics...][EQ] **RegEQ** ENTER to paste the regression equation to Y_1.

Now use TABLE to view our area values for the continuous relation in Y_1. Adjust your table with using TblStart and ΔTbl in the Tblset menu until you can determine which area is the maximum. Table zooming in and out has been mentioned several times, but it is not usually the most efficient way to determine optimal points of interest.

Now plot the regression equation through the data as you did with the Least Squares and Median-Median lines in the last Master. What options are available to

Notes/Comments/Ideas

determine the maximum from the graph screen (recall our exploration with polynomial graphing in *Boot Camp*)? Hint...re-visit the [CALC] menu.

You should now be able to apply the [CALC] menu **maximum** command, or use successive uses of ZoomIn or Zbox together with TRACE to answer the previous question as all have been illustrated earlier (see *Boot Camp Engagement Master* "Working With Linear Relations").

An interesting activity here would be to apply the slope tool $\frac{dy}{dx}$ at the value obtained from **maximum**, or from any future value yielded by **fmax** or **fmin.** From our earlier work with $\frac{dy}{dx}$, this should be a smooth transition. It could easily be extended to include a discussion of where the area was increasing and decreasing as a function!

This Master has gone from an experimental model to the math model to the optimal choice. Many simulations are possible using these techniques with some scaling of the real world dimensions.

Other aspects could be brought into this situation too. It could be posed as a cost minimization problem requiring a certain area be enclosed with other

Notes/Comments/Ideas

restrictions such as a post every 3m, a fixed cost for a gate, labour costs, particular shape of backyard, etc.

Notes/Comments/Ideas

Teaching Tip - Degree and Plot Style

Opportunities exist in this type of question to explore the algebraic underpinnings of first and second degree expressions.

If we use only our experimental data of width vs. length and created a scatterplot, we would see a linear relation.

This could easily be symbolized with or without the linear regression power of the TI-83 being applied. Recalling our LEN formula linked to WID, we might conclude that a first degree equation seems to equal a linear plot i.e. a line.

If we take the Area vs. Width scatterplot, we see a quadratic path.

If we take the symbolism of our linked formula for AREA, that is WID * LEN (substituting the Length in terms of Width that we just did), we see a second degree equation. Hence, it seems that second degree equations yield a quadratic plot i.e. a parabola.

This is a natural extension of this experimental data.

Notes/Comments/Ideas

TEACHING TIP - SPF 15, 30, ...?

This is a nice activity that brings mathematics into everyday living. It is taken directly from the C-T^3 *Manual (as noted below) and is especially good as an extension from the Boot Camp Engagement Master "A Linear Relation" where we first discussed discrete vs. continuous mathematics. This activity was not included earlier as it involves creating a quadratic regression model. Please note that the 24-hour clock will likely be needed to avoid misunderstanding the time of day here.*

The data shown was published in the Mesa (Arizona) Tribune on August 8,1993. It gives the amount of time needed to redden untanned Caucasian skin at different times of the day based upon the predicted weather for that day.

TIME	9am	10am	11am	Noon	1pm	2pm	3pm	4pm
MINUTES	34	20	15	13	14	18	32	60

Plot this data as a scatterplot and consider the following:

1. What model does this data appear to fit?
2. Find an equation that best fits the data.
3. What is an appropriate domain for your equation?
4. Use your equation to interpolate the time of day at which skin will burn in 30 minutes.
5. Would our equation be different if we were in New York? Montreal? St. John's, NF?
6. Can you use your equation and your transforming plots skill (this is addressed by this manual in *In The Service Engagement Master* "Transforming Plots") to rough out a graph of UV intensity through the same day?

Reference:

Texas Instruments. (1996a). *CT*^{3} *- Canadian teachers teaching with technology* [manual]. Dallas, TX: Author, 2-11.

Notes/Comments/Ideas

In The Service Engagement Master #8

Ploy Wants A Plot

There are two very significant *Teaching Tips* included in this master - "Factoring and Equivalence" and "Saving A View : Graph Databases And PICs". These could easily be entire masters on their own but they go a little beyond what is necessary as day-to-day teaching. They have been given unusual attention (by this very mention) since there are ideas and powerful features of the TI-83 in there that should not be missed. That is not to say that they are basic skills...they are not. But, they frame what is possible immediately beyond the typical without being in the realm of the power-user.

Polynomial graphing patterns can be fun if graphing technology is utilized. The following is a walk through a lesson that builds upon the notion of zeros of factored expressions to generate more expressions with increasing numbers of zeros and increasing degree. The lesson is incomplete since only a representative sampling will be used here for reasons of space. It also does not presume to illustrate the patterns in polynomial graphs. Teachers know that material. It is designed to be a

Notes/Comments/Ideas

brief and alternative look at how the TI-83 could be used to enhance presentation of algebra and graphical connections.

Expressions can take many values. For example, the expression $2x$ would take the value 4 if x was 2, - 6 if x was -3, 1 if x was ½, and so on.

Using the table supplied to try to find the particular x-value(s) that makes the expression evaluate to 0. Then, multiply out the expression to determine an equivalent alternate form or its expansion. Then, identify the degree of the expression/expansion.

Notes/Comments/Ideas

Expression	**Zero (s)**	**Multiplied Out Expansion**	**Degree**
2			
(x)			
$(x + 2)$			
$(x - 1/3)$			
$(2x - 1)$			
$(2x + 3)(x - 5)$			
$(x - 3)^2$			
$(5x + 2)(x - 3)^2$			
$(x)(x + 2)(3x - 5)(x - 2)$			

What conclusions can you draw about the nature of the expressions above and the number of zeros, and the degree of the expression and the expansion?

Notes/Comments/Ideas

Using your TI-83, key the first expression in for Y_1 and view the graph in a standard viewing window. What is its plot like? How many zeros does it have? Trace it. What do you notice?

Key x (the second expression) into Y_2 and repeat the above procedures. What is its plot like? How many zeros does it have? Trace it. What do you notice?

Do the same with $(x + 2)$.

Each of these expressions were linear in look i.e. $(ax + b)$ style. Each of the expressions so far had just one linear component.

Let's try $(2x + 3)(x - 5)$. This has two linear components in a product – we call this product *factored* form. Use your calculator and explore the same questions with this expression.

Let's explore the expanded version. If our expanded version and our factored version are equivalent (i.e. good algebra), then what would you expect their two plots to be like? We could plot them both and see. A graphing and table approach to equivalence of expressions is now addressed through a *Teaching Tip*. This activity resumes immediately thereafter.

Notes/Comments/Ideas

Teaching Tip - Factoring And Equivalence

At any point when students are establishing equivalence of expressions, the TI-83's table or graph powers can be applied.

For example, the expression $(2x + 3)(x - 5)$ expands to $2x^2 - 7x - 15$.

Plot the first expression using the default plot style. Plot the second using the animated cursor – plot style 5 (leaves a line) or 6 (just traces the path). If they plot directly over one another, they are likely equivalent. More work would need to be done to avoid students believing that matching on some small window is grounds for earth-shattering equivalence, but it is a useful blending of algebra and graphical representations.

The same activity could be performed with tables. As long as the two plots are active, their tables can be viewed. If the results are identical for the domain values chosen, they are likely equivalent.

This is a quick confirmation of any factoring they perform or of any more complex algebraic manipulations.

A natural transfer of the same principle could be investigating Trigonometric Identities.

<u>Reference</u>:

This *Teaching Tip* is based upon the following reference.

Texas Instruments. (1996a). CT^3 - *Canadian teachers teaching with technology* [manual]. Dallas, TX: Author, 4-27, 4-29, 5-2.

Notes/Comments/Ideas

OK. We have established a link between zeros and expressions. Oh really? Try plotting $(x-3)^2$ or its expansion $x^2 - 6x + 9$. What is different here?

Here are some second degree expressions. Use their graphs to determine a possible factoring for each:

$x^2 - x - 6$ $\quad$ $x^2 - 10x + 25$ $\quad$ $x^2 + 6x$ $\quad$ x^2

Use a procedure similar to the above to draw some conclusions about whether the following expressions factor noting any interesting characteristics which present themselves:

$x^2 - x + 6$ $\quad$ $x^2 + 1$ $\quad$ $x^2 - x + 1$

Clearly, there are some interesting relationships going on between the graphs of second degree expressions and the algebra.

Let's explore some other aspects of the expressions we had earlier. The expression $(2x + 3)(x - 5)$ has zeros $-3/2$ and 5.

Notes/Comments/Ideas

Use a combination of factored symbolism and graphing to address the following questions about $(2x + 3)(x - 5)$.

If we multiplied it by -1, would that change the zeros? Would that change the graph? What if we multiplied it by -2? Would that change the zeros? Would that change the graph? What if we multiplied it by $+2$. Would that change the zeros? Would that change the graph? Draw a conclusion about the effect of constant multiplication on a graph.

What if we multiplied $(2x + 3)(x - 5)$ by x. Would that change the zeros? Would that change the graph? What if we multiplied it by $2x$. Would that change the zeros? Would that change the graph? What if we multiplied it by $-3x$. Would that change the zeros? Would that change the graph? Hmmmm.

Multiplying by anything with another x in it changes the basic shape, the degree, and the number of possible zeros. Multiplying by a constant changes the steepness and can reverse the direction if the constant is negative.

Draw a conclusion about general shape, possible zeros, direction, and any other aspects you can using symbolism and graphing for the following expressions:

$x(x - 3)$	$-2(x + 1)(x - 5)$	$3(2x - 5)^2$
$x(x - 3)(x + 3)$	$-2x(x - 3)(x + 3)$	$-x^3$

Notes/Comments/Ideas

Teaching Tip – Saving A View:

Graph Databases and Pics

Suppose you'd like to save a particular graph window view. Maybe you'd like to share that with a group of users also. 2nd PRGM [DRAW] [STO] has four options for storing and recalling pictures using Pic variables (uneditable drawings which are actually graph screen captures), and graph databases using GDB variables (storage of a series of graph settings including window variables).

A really useful teaching tool is the Graph Database or GDB. Storing GDBs retains all aspects of that graphing window for retrieval at a later time. That is, all window values are saved, all plot details are saved, and when retrieved, this GDB bumps out the present settings and restores your graph views just as you stored them.

<u>Suppose it was time to do a lesson on the effect of a in the familiar story $a\,x^2$.</u>

A nice GDB to have stored would be x^2, $-x^2$, $0.5x^2$, $-0.5x^2$, $2x^2$, $-2x^2$, $5x^2$, $-5x^2$ with only x^2 as an active plot for now, with the standard viewing window. So, enter all this stuff, turn all the plots off except Y_1, Press ZStandard. The plot should just show $y = x^2$ with the others lurking underneath.

[DRAW] [STO] **Store GDB** ENTER will cause all those settings to be saved. You must choose a GDB # though (from 0 to 9) which is a bit of a drag for remembering. Now you are ready to run your lesson. Continue doing whatever it was you were doing and, when the time is right [DRAW] [STO] **RecallGDB** # comes to the rescue and is ready to be performed. This is how a sequenced graph lesson, planned at home, could easily be replayed in class.

Notes/Comments/Ideas

Though not mentioned above, the coefficients for the x^2 functions (or any preferred coefficient set for such a patterning activity) could just as easily have been collected in one list. If that list were named, say as L_1, then L_1 would be all that you would need in front of x^2 to cause the family of functions to graph.

That is, while in the graphing editor, $\{1, 2, 3, -1, -2, -3\}x^2$ would plot each of x^2, $2x^2$, $3x^2$, etc. in the order the coefficients appear in the list. If this list were in L_1, we need only key in $L_1 * x^2$ for the same effect. Then we could apply the same coefficient pattern to another functions, say $\sqrt{x}$. We could even place the now familiar parameters in various other places to study their effect.

An occasionally useful strategy is storing functions to the graphing editor from the homescreen. To do this we enclose the string in double quotes and store it to the Y_n variable desired. For example:

ALPHA + " 2x + 1 ALPHA + " STO→ VARS [Y-VARS] [Function] Y_1 ENTER

...places the relation 2x + 1 in Y_1. If you check the graphing editor, it will be there, exactly where you told it to go. Though this seemed like a lot of keying, you get very used to these locations if you use them a lot. Not only that, you can now use 2[nd] ENTRY to repeat the line editing in your next function. The real reason this was mentioned, is that this is the syntax for storing to functions from within programs.

Another strategy that is very useful when plotting more than one function on the same screen is how to pause during graphing. This hint is repeated, and detailed a little more actually, in an appendix titled "Parametric Eq.s Together With Ftns". To pause a graphing action, press ENTER. To start it again press ENTER again. To interrupt a graphing action entirely, press and briefly hold ON. To re-graph in this case requires the pressing of GRAPH.

Meanwhile, back to GDBs...

Notes/Comments/Ideas

A neat trick is to recall a GDB # as a line of code in a program as we did earlier in the *Teaching Tip* - "Saving Data As A Program". As you create a graph database for a particular class, you could put the recall GDB command in a program named for that demonstration right away. The advantage is that program naming is more flexible and suggestive than GDB naming.

The author is not aware of a comment out command so as to note within a program's code what a line or loop does by including a comment line. It would be helpful if this were included in the TI-83 programming language in future.

You can overlay other pictures or text on a graph using another neat trick... placing PICS on your screen. A PIC is effectively a screen capture.

To be particularly effective, a series of PICS could be run together on a single base graph (or on a partner GGB) to enhance it in stages. Another enjoyable use of PICS is to have several graphs displayed, have one PIC recalled which asks a question about the screen, and then have another PIC recalled which identifies a response. First, we discuss the tools used to draw on a graph to create our PICS.

The [DRAW] menu can allow enhancement of graph screens by such tools as: circle, pen, text, line, DrawInv, etc. You could create a graph (or a GDB) and use those tools to write solutions, mark points, whatever on the graph(s).

Then you could save that screenshot(s) with [DRAW] [STO] **StorePic #** . Now your graph (or GDB) can be recalled, your questioning of your group could be done, and then your solution picture with the tags you made can magically superimpose itself while looking quite professional.

A helpful hint here is to name you PIC number to match the GDB number. That isn't always possible if several PICS are used on one graph or GDB however.

Note that the table settings are not saved with a GDB. Only the functions, selection status, plot style, and window settings are saved.

Notes/Comments/Ideas

This may seem like a lot of work. When combined with the ability to save GDBs and PICs on a PC through TI-Graph Link Software, the gains can be very creative teaching...especially if you consider that such a series of graphs could be transmitted to a student's TI-83 for exploration.

To continue this theme, another idea that some may find interesting is that window settings, function names, all the stuff a user can normally edit, are stored to reserved variable names. These names can be grabbed from either the VARS menu or the Catalog. Translation... a program could include commands to store or recall GDBs or PICS, store one or more functions to the graphing editor, or set up a viewing window(s).

This is similar to recalling a GDB but effectively allows the controlling of parts of it at a time through directly addressing the system variables. A whole series of graph-window sessions could be displayed by the running of such a program. This could easily be transmitted to another calculator via the link cable and could serve as a mini-lesson or exploration.

The TI-85/6 Graphing Calculators allow you to choose a name GDBs and PICS. That's a much more practical way to keep track of GDBs for class purposes. The only problem is that stat plot stuff isn't included in a GDB for either the TI-82/3 or the 85/6 and there is no such thing yet as a statistics database or (SGB).

A final note for this section, as is mentioned in the "Parametric Eq.s Together With Ftns" appendix, is that the setting for how coordinates are displayed on the bottom of the graph screen during a trace can be changed from the default rectangular to polar. This affects the graph trace screen only and not the table view.

To do this, press 2nd ZOOM and select **PolarGC** instead of the default **RectGC**.

Try tracing a function now and see the difference! This is a good bridge into an alternate system of coordinatizing a plane. It would be good to switch coordinate systems like this as an exploration using an established and familiar function whose **RectGC** are known.

Notes/Comments/Ideas

In The Service Engagement Master #9

Transforming Plots

It is not the intention here to present a series of pictures detailing what varying specific parameters does to specific functions. Teachers know that material. What is intended is the illustration of the incorporation of a TABLE view into the discussion; and, the building of grounds for a more comprehensive foundation for establishing the effects of parameter changes on the values functions take, and on their graphs.

First, we should set our viewing window to 10 by 10 for the examples we will use. So, ZOOM [ZoomStd] for a 10 by 10 window. Set TblSet to start at -3 and increment in steps of 1. The screenshots in this Master may not match your screens immediately. For clarity, the cursor was moved to a Y_n header so as to highlight the function associated with that header in the command line at the bottom of the screen.

Notes/Comments/Ideas

Create $Y_1 = \sqrt{x}$ in the graphing editor. Most of us know what the domain of this relation is but let's see if the TI-83 does! View the table with [TABLE]. Figure 1 illustrates how the TI-83 responded to our requests. Discuss the significance of the ERROR messages.

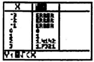


figure 1

GRAPH this relation and examine its shape and location.

We will now use Y_2 repeatedly to examine the effects of transforming our relation by changing some of the parameters.

For example, what happen s if we replace $\sqrt{x}$ with $\sqrt{x}+1$? Put $\sqrt{x}+1$ in Y_2 and let's see what is does to our results. Figure 2 illustrates both results as a table. Hmmm. Square root x doesn't seem to like negatives in both cases and our answers are predictably one more in the case of Y_2 for the same x-values.

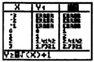


figure 2

Notes/Comments/Ideas

State the first three ordered pairs visible in the table for $y=\sqrt{x}$ and $y=\sqrt{x}+1$. Discuss the effect and non-effect of this change. Graph them to see the effect. Reconcile the relation changes with the table and graph.

Try $\sqrt{x}+3$ and $\sqrt{x}-2$ and see if results are as you expect.

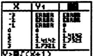


figure 3

Let's try changing something else. Replace Y_2 with $\sqrt{x+1}$ and let's see what that does to our answers (see figure 3). State the first three ordered pairs visible in the table for $y=\sqrt{x}$ and $y=\sqrt{x+1}$. Discuss the effect and non-effect of this change. Graph them to see the effect. Reconcile the relation changes with the table and graph.

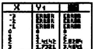


figure 4

Let's try changing something else. Replace Y_2 with $2\sqrt{x}$ and let's see what that does to our table answers (see figure 4). Discuss as before and state a hypothesis for such a change. Graph them to see the effect. Reconcile the relation changes with the table and graph. *Using table in this manner illustrates the power it has for comparing answers and for examining domain.*

Notes/Comments/Ideas

Replace Y_2 with $-2\sqrt{x}$ and let's see what that does to our answers (see figure 5). Oh! Discuss as before and state a hypothesis regarding such a change. Graph them to see the effect. Reconcile the relation changes with the table and graph.


figure 5

Let's combine strategies. Predict the effect of changing x to $x+1$ under the root and multiplying by 3 outside. Predict the effect of changing x to $x-3$ under the root and multiplying -2 outside the root. Try it!

Let's try changing something else. Replace Y_2 with $\sqrt{2x}$ and let's see what that does to our answers (see figure 6). Discuss as before and state a hypothesis regarding such a change. Graph them to see the effect. Reconcile the relation changes with the table and graph.


figure 6

Replace Y_2 with $\sqrt{-2x}$ and let's see what that does to our answers (see figure 7). This one is more interesting! Discuss as before and state a hypothesis regarding such a change.


figure 7

Notes/Comments/Ideas

Let's combine strategies. Predict the effect of changing x to $x + 1$ under the root and also multiplying by 3 inside i.e. 3 $(x + 1)$ under the root. Predict the effect of changing x to $x + 1$ under the root and also multiplying -3 inside the root i.e. 3 $(x + 1)$. Try multiplying outside and inside with varying numbers! A similar approach could be taken for the rational function $\frac{1}{x}$ and its partners

$\frac{1}{x}+1, \frac{1}{x}-2, \frac{1}{x+3}, \frac{2}{x}, \frac{-2}{x}, \frac{1}{2x}, and\ \frac{1}{-2x}$ and combinations similar to earlier examples.

Notes/Comments/Ideas

FUTURE MATHEMATICS CURRICULA

IN A

TI-83 GRAPHING CALCULATOR ENVIRONMENT

Backword

Created by

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Backword

So,.... You've reached the end! I thought I had reached the end once too only to discover that the highway of life seems a lot like a rubber Mobius Band — lots of times it looks familiar only things have flexed around on me. Nothing stays the same.

The graphing calculator, like any technology, only feels new for a little while. Technologies quickly evolve or are replaced. The good news is that the replacements are usually easier to pick up and run with and are all a product of their histories.

Technologies such as the GC, invigorate the relationship between empirical data and symbolic mathematics. Through an accessory such as the CBL, interdisciplinary teaching becomes enhanced and far more than the math teacher using physics formulas on the board. It may well become the physics teacher and the math teacher generating and using math hand in hand through the same instruments.

A new term could be coined here for technologies like the graphing calculator that are so intertwined with teaching. I have stumbled upon this spelling and term because I keep misspelling technologies on my keyboard. I think all technologies employed in altering the perspectives of others for educational purposes should be treated as "Teachnologies"... the theory being that we should

Notes

never lose sight of the struggles and previous experiences a learner carries while grappling with new technologies in a learning situation.

This material may become dated fairly quickly. That's fine. The whole motivation for this effort was to make the employment of a current technology easier for teachers. The technology happened to be one I have enjoyed growing with for some time as a part of my teaching. If there was one message I could conclude with, that supercedes other points made in this document, it would be this...

We have all experienced the joy of discovery. It is a feeling the very young experience all the time, the teen-ager experiences a little less often, the young adult experiences after the fact, and the adult experiences....ahh...I forget! We tend to pick our spots a little more carefully I guess.

One of the greatest benefits of teaching is that it keeps you in touch with the young. Paradigms are shifting around them all the time so we cannot help but shift a little too. Think of the last time you were walking down a street with an adult non-teaching friend and saw a group of kids involved in something. Perhaps your friend became nervous or was uncertain about walking near them. You may have been too...but chances are you saw kids reflecting the spirit you see in school; the spirit you had when you were there.

Notes

Teaching changes us from others who do not deal with the young, yet some of us feel obligated to limit or restrain our own joy of discovery... the very pleasure we recognize and cultivate in educating children!

That is the feeling this effort attempted to ignite through its breadth of explanation and shameless teacher support. I hope it enabled each of its readers to re-emerge as learners, reveling in re-joining a journey of discovery, and embracing a youthful outlook with the wisdom of all our experience. That's what I believe makes great teachers. Boy, have I got work to do! 8 🕮 8

Paul W. Gosse, St. John's, NF　　　　August 03, 1998

Notes

FUTURE MATHEMATICS CURRICULA

IN A

TI-83 GRAPHING CALCULATOR ENVIRONMENT

Evaluation

Created by

Paul Gosse
M. Ed. Candidate
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Site: ______________________ **Date:** ______________________

______________________ **Facilitator:** ______________________

Please take a moment to complete an honest and constructive assessment of this inservice. There are questions specific to the facilitator, materials, and site. All comments are welcome.

Thank-you for participating in this institute.

Please score responses in this section from 1 to 5 as indicated by the legend.

Facilitator

Legend

1 = unacceptable 2 = poor 3 = fair 4 = good 5 = excellent

The facilitator's...

Consideration of audience	1	2	3	4	5
Pace	1	2	3	4	5
Attitude and enthusiasm	1	2	3	4	5
Ability to communicate	1	2	3	4	5
Knowledge of materials	1	2	3	4	5
Skill with calculator	1	2	3	4	5

OTHER COMMENTS ______________________

Material

The material's...

User-friendliness / Readability	1	2	3	4	5
Quality of print	1	2	3	4	5
Clarity of explanation	1	2	3	4	5
Provokes new teaching ideas	1	2	3	4	5
Type and depth of activities	1	2	3	4	5
Quality as a resource / reference	1	2	3	4	5

OTHER COMMENTS ______________________

Site

The site's...

General suitability	1	2	3	4	5
Comfort / ambience	1	2	3	4	5
Amenities {refreshments, Service}	1	2	3	4	5

OTHER COMMENTS ______________________

Concluding Section

Please complete the following sentences with the first thoughts that jump to mind or circle *n/a*.

During the first day of the inservice I felt ______________________

By mid-week I felt ______________________

Today I feel ______________________

The thing that pleased me most about the institute *n/a* ______________________

The thing that disappointed me most about the institute *n/a* ______________________

The most valuable thing I will take away from the institute *n/a* ______________________

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My view of GC technology has changed in that n/a

My view of teaching has changed in that n/a

With respect to value for money, I would rate this institute

With respect to overall organization, I would rate this institute

Compared to other GC institutes I have attended, I would rate this institute

Compared to other PD opportunities, I would rate this institute

The thing about this institute I would definitely changes is n/a

The thing about this institute I would definitely not change is n/a

Other Comments:

Thank-you

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Glossary

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Glossary Of Selected Terms

There is also glossary in the TI-83 Graphing Calculator Guidebook.

Active Editor	If an editor key is pressed (eg. Y=) then that editor (graphing in this case) is said to be active. It replaces the homescreen until another editor is activated, or until it is "quit". Menu choices made while in an active editor paste to the cursor location in that editor.
Argument	Functions generally require arguments i.e. values upon which to operate. For example, 2 + 3 illustrate an operation + that requires two arguments 2 & 3 here. The operation cube root requires one argument etc.
Catalog	The catalog is a listing of most commands, symbols, operations, etc. onboard the TI-83. Some commands that are not commonly used are available only in the catalog. The catalog is especially useful if you've forgotten the location of a command or for pasting special characters into programs. You can quickly go to commands beginning with a particular letter by pressing the key whose alpha function contains the first letter of the command you want. The alpha scrolling shortcut of a menu page at a time does not work here.
CBL - Calculator based laboratory	A handheld device, similar in size to the TI-8* calculator, that connects to the TI-8* via a calculator to calculator cable and that accepts probes for the purpose of collecting experimental data. The CBL is driven by programs available from TI, http://www.ti.com/calc/docs/cbl.htm, or from Vernier Software, http://www.vernier.com/cbl/progs.html, and stores data onboard the attached calculator. Some probes currently available from TI or Vernier software include: Current and Voltage, Temperature, pH, Barometric Pressure, Heart Rate, Sound Microphone, Relative Humidity, Magnetic Fields, Force, Thermocouple, Accelerometer, Calorimeter, and Motion. Using **TI-Graph Link** software, this data can be transferred to a PC or Mac for storage or other use.
Command Line	The active entry line on the homescreen.
Contrast	Contrast is increased by pressing 2nd and ▲ repeatedly or by pressing 2nd and holding ▲ until the desired level of darkness is reached. Contrast is decreased in a similar way except ▼ is used instead of ▲ until the display is lightened to the degree desired..

Decimal Places	Decimal places displayed is set through the MODE menu and presents the number rounded to the number of places specified. This is for display purposes only and does not affect internal accuracy. This setting governs all display and editor screens.
Delimiters	Enclosing symbols marking the beginning and/or ending of a group for a particular purpose. For example, in {1,2,3}. The TI-83 knows that { is a left delimiter for a list i.e. a list has been indicated to have begun. The elements are separated with by a comma in correct syntax, and the end of the list is signaled by the right delimiter }.
Dim Mismatch Error	Indicates objects being operated on must be of certain dimensions and currently are not. As an example, if two lists are to be added they must be of equal length. If they are not, this error presents.
Editor	An editor is an operating environment that replaces the homescreen while active and has an editing capacity. It does not simply offer end menu choices. There are several editors onboard the TI-83...they are: Graphing and Table; Statistics; Matrix; Program; and, Finance. These operate as separate environments outside the homescreen and are interactive.
Graph DataBase	Named storage location, which retains all parameters, associated with the graphing editor. Extremely useful for recalling a particular series of graphs or a particular window setting quickly.
Graphing Editor	Accessed by pressing the **Y=** key. This editor contains ten fields for function/parametric/polar/sequence plots and sets their plot style and status. It can also set stat plot status.
Header	A header is a generic term for a label presented on top of a column. The word header is used mostly in connection with the Stats/List editor spreadsheet view and the Graphing Editor Table view.
Homescreen	A screen outside of any editor on which normal calculations are displayed. You can return to the homescreen by pressing 2nd **QUIT** if in an editor, or by pressing **CLEAR** if in a menu outside of an editor.
Linked Formula	A formula can link lists. That is, the linked list can be connected through a formula to create new data in the list utilizing the formula. This is done by editing the header located listname in the Stats/List editor and enclosing the formula in quotes.
List	A list is a collection of data delimited by braces and separated by commas. Operations act on a list as if it was a single object except when two or more lists are operated on together in which case their lengths must be equal. A sample list could look like {5, - 2, 3}.
List (clearing a list)	A list can be cleared by using the ClrList command in the

	STAT [EDIT] menu. Syntax is ClrList [listname] with multiple listnames separated by commas. ClrAllLists in the MEM [MEMORY] menu reduces all onboard list dimensions to zero, saving space, but does not delete the listname from memory.
Logical Tests	MATH [TESTS] menu offers logical tests which evaluate to 1 if true and 0 if false. This is very useful for controlling branching in programs or domains in graphing.
Menu	A series of commands collected under one name. Menus can be stacked (see **sub-menus**). Choices are made by using the cursor keys to highlight the desired item, or pressing the menu item number indicated, and pressing ENTER to accept it which pastes the choice to the homescreen or the active editor.
Menu Map	A complete menu map for the TI-83 can be found on page A-39 to A-48 of the Tables and Reference Information section of the TI-83 Manual.
Outliers	Values that are more than 1.5 *(the inter-quartile range) above the upper quartile or below the lower quartile. If you find an outlier in data, determining the reason for its existence can be important to truly understanding the data. The lower quartile, median, and upper quartile divide data into four parts of approximately the same number of data points in each part. Inter-quartile range is the upper quartile minus the first quartile a.k.a. the range between the 75th and 25th percentile.
Piece-wise Graphing	Using the idea of **restricted domains**, we can create a piece-wise function. For example: (fin 1)*(domain test 1) + (fin 2)*(domain test 2) + ... + (fin *n*)*(domain test *n*) ...creates a function which evaluates to (fin *n*) *(1) where that test is true and (fin *n*)*(0) where that test is false. Provided the domains in the tests are independent, the true pieces plot on a their test domain while the other parts sum to 0.
Pixel	A screen dot. 5828 (94*62) of them in a rectangular matrix form the screen of the TI-83.
Plots (switching between)	While tracing plots, the active plot can be switched by using the ▲ or ▼ cursor key. The active plot is numbered in the top right corner, and the active plot expression is illustrated in the top left corner.
Quitting a Menu	Any menu or editor can be quit by pressing 2nd MODE [QUIT]. This brings you immediately to the homescreen. If you simply want to exit a menu from within an editor, but not leave

	the editor, then press CLEAR. Any menu can be immediately jumped to by simply activating it. Any current menu is simply left though the active editor remains active.
RAM Check	The TI-83 has about 32K of user RAM. The amount currently in use can be found by 2nd + [MEM] **Check RAM.** This is useful prior to undertaking data gathering with a CBL or loading programs to ensure adequate memory is available. It also indicates where memory is being consumed and can direct you quickly to what may need to be deleted or moved to a computer to free some RAM.
Restricted Domains	A function may be plotted on a restricted domain by * or / by the appropriate logical test. Recall that a **logical test** evaluates to 1 where the test is true and 0 where the test is false. For example: $(2x + 1)/(x > 0)$ plots $(2x + 1) / (1)$ where $x > 0$ and plots $(2x + 1) / (0)$ everywhere else. Since the latter is undefined wherever $x > 0$ is false, we only get $(2x + 1) / (1) = 2x + 1$ plotted.
Scrolling	Scrolling a screen can be done by pressing the up or down cursor keys one item at a time; by pressing and holding the up or down cursor keys to continue scrolling; or by using Alpha up or Alpha down to scroll a page at a time.
Sequence	A collection of distinct values related by a rule, or a command in the LIST [OPS] menu. For example, the command SEQ(X,2X,1,10,1) uses the first X as a counter which goes from 1 to 10 in steps of 1. The second argument is the value of the X^{th} term in the sequence as related to X i.e. value is twice the counter. The sequence generated is {2, 4, ..., 18, 20}
Split Screen/Window	The homescreen viewing window can be split in two ways. Under the global Mode menu, there are choices FULL; HORIZ; and, G-T. HORIZ creates a horizontal split. The top half displays the active graph(s). The bottom half will assume activities that would normally be performed or accessed from the homescreen. Note, menus and editors can temporarily occupy full screen display as choices are being made. G-T is a left-right split which specifically reserves the left side of the screen for the active graph(s) and reserves the right side for a corresponding, updateable, table.
Standard Viewing Window	Accessed through the ZOOM [ZStandard] option, it sets the window parameters to 10⁻ 10 on each axis with tick marks showing at 1 unit spacing. **Xres** is set to 1.
Statistics/List Editor	A spreadsheet style window for viewing lists of data accessed through the STAT [Edit] menu. New data can be generated

	through **linked formulas**.
Sub-menus	Menus within menus indicated by more than one **header**
Switching Between Active Plots	While in TRACE mode on a graphing screen, switching between active plots is done by pressing the ▲ or ▼ cursor. This is very useful for comparing values taken by several relations for the same domain value.
TI-Graph Link Software	Software for the C or Mac that allows communication between the computer and the graphing calculator. It also allows the grabbing of screenshots, the backing up of a calculator's memory, and the collecting and editing of any other memory object onboard the calculator.
Window	The current view. Whatever the display is showing. It can also refer to a set of Xmax, Xmin, Xscl, Ymax, Ymin, Yscl values dictating a particular view. See also, Graph DataBase.
Xmax	The numerical value of all next to rightmost pixels (the right most pixel is reserved for the status indicator). This value is altered whenever WINDOW values are changed either manually or through executing a ZOOM.
Xmin	The numerical value of all leftmost pixels. This value is altered whenever WINDOW values are changed either manually or through executing a ZOOM.
Xres	Window setting that controls the number of available pixels whose y-values are computed as the plotting algorithm sweeps across the screen. An Xres of 1 means each domain pixel has a corresponding y-value computed. An Xres of 3 means every third domain pixel has a corresponding y-value computed. This speeds plotting but leaves a "rougher" looking plot.
Ymax	The numerical value of all uppermost pixels (the uppermost pixel is reserved for the status indicator).. This value is altered whenever WINDOW values are changed either manually or through executing a ZOOM.
Ymin	The numerical value of all lowermost pixels. This value is altered whenever WINDOW values are changed either manually or through executing a ZOOM.
ZStandard	ZOOM [ZStandard] sets a 10 by 10 viewing window with tick marks at 1 unit spacing and **Xres** = 1.

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Appendices

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APPENDIX #1 - FACILITATOR PACKAGE

This section of the manual contains instructions and other planning information to assist the facilitator in delivering this manual in an institute setting.

Instructions

The following is an abbreviated section list in order of appearance in the manual (scrolling from left to right):

Acknowledgements	Foreword	Contents	Basic Training Notes
Basic Training Engagement Masters	Boot Camp Notes	Boot Camp Engagement Masters	In The Service Notes
In The Service Engagement Masters	Backword	Institute Evaluation	Glossary
References	Appendices		

The first material a facilitator should read in preparing to use this manual as an inservice is the appendix titled "Registration Package". This appendix contains a one-pager — a one-page information sheet that could be used to gather initial interest in the delivery of this institute — and, contains all the material describing the

Notes/Comments/Ideas

institute that those participants should receive in advance (this is discussed in greater detail in a moment). This is valuable for the participant and gives the facilitator a sense of what the participants would be expecting.

Next the *Foreword*, and the "General Introduction" section (at the very beginning of *Basic Training*) should be examined. This manual has three main sections called *Basic Training*, *Boot Camp*, and *In The Service*. There are notes and activities in each section as detailed in the "General Introduction". The "General Introduction" frames all the materials.

The detailed *Contents* section delineates the entire manual and includes a suggested *5-Day Timeline* for delivery (the Timeline is also included in the Registration Package for information purposes).

This information should describe the tenor and structure of the manual, and should allow the development of a suitable institute.

What to Plan in Advance

The least a facilitator can expect to manage is the provision of a sufficient number of calculators, a viewscreen, and perhaps posters and overhead masters to illustrate other points as sessions are running.

Notes/Comments/Ideas

A class set of graphing calculators (normally 30) or a CBL Kit (normally 15) come with a viewscreen model, a poster, an overhead transparency of the model of calculator used, several calculator-to-calculator cables, and spare batteries. These sets must be booked a minimum of one month in advance. This is further explained in the appendix titled "TI Calcs./Access./Support" under *Workshop Loan Program*.

There is a mail out as a part of the registration process built in to this manual. The second appendix, which was mentioned as necessary reading already, contains an unnumbered covering letter (a welcome letter with provisions left to insert location, date, and time of day information regarding the institute), and two following parts each under a separate cover. The first is the description referred to above. The second part contains a survey to gather data about participants' entrance skills.

It is intended that each participant be mailed/faxed/emailed a copy of the contents of the Registration Package, enough in advance of the institute, so as to allow completion and return of the survey. This survey also serves as a confirmation of institute numbers. The letter both following parts of the *Registration Package* , and the one-pager referred to earlier, are all included on a floppy disk to facilitate customization, and to allow for emailing.

Notes/Comments/Ideas

The calculators will arrive a few days before the inservice date, and will contain a cover letter naming a contact person and a return waybill (do not lose it!). For return, the calculators must be re-packed as they arrived, the case secured with the provided tie-down, and sent immediately following the last session day using the provided waybill. Assistance in arranging this can be gotten through the Workshop Loan Program (see heading titled *Workshop Loan Program* in the appendix titled "TI Calcs./Accessories/Support").

The facilitator should also ensure that suitable overhead projectors and screens are available at the site. Higher intensity overheads work best, especially in brightly lit rooms, and it is often helpful to have two…one for the viewscreen palette, and one for any transparencies you may wish to use or create.

What Not to Forget to Do!

Visit the site a few days in advance and decide on a physical arrangement for the first session. During that session, you may want to re-evaluate or re-arrange the room.

Check your class set immediately upon its arrival to count the number of units, and assess their working state. You may want to "clean" them up or load certain lists/programs/etc. prior to the institute. Pay special attention to the

Notes/Comments/Ideas

viewscreen model as if is the one you will be using for demonstration purposes. If there are any concerns about the equipment, the person referenced on the cover letter that arrived in the calculator case should be contacted.

Make sure you have a few pushpins or some poster adhesive in case you want to use the provided poster. A few transparencies and markers could help in responding to particular examples someone may give (they could be written down and displayed so others could refer to them while the viewscreen is used to illustrate an answer). A laser pointer is also very helpful and avoids disturbing the image by interrupting its transmission to a screen. Of course, all of these points are suggestions and may or may not be utilized at any given institute.

Always preview every *Engagement Master* you intend to use in advance. This is in case any errors have crept in, and if any masters were omitted, some pre-supposed skills in later masters may not have been covered.

Conduct an evaluation! There is an evaluation instrument immediately following the *Backword* that should be administered on the morning of the last day for receipt later in the day. These evaluations should be carefully considered for future evolution of the materials as well as for the delivery. It may be helpful to make copies of the evaluation instrument! It may be that participants would like to keep

Notes/Comments/Ideas

the blank copy in their manuals for other purposes, so it may be a good idea to have some copies on hand.

The above, together with the following other suggestions, should provide a reasonable framework for this professional development institute.

Other Planning Information

Calculators

An assumption of the manual is that nothing is loaded on the calculator and that it is set to its defaults with all editors empty. All GCs in use should be set to their defaults in advance, and be ready to use. If CBL programs are to be used as an enhancement, ensure that they are also loaded on the calculators in advance or that loading programs becomes a part of the institute experience.

Main Section Features

The manual is a binder style. This allows space for significant elaboration on concepts through illustrations and conversational language. It also permits easier editing and future expansion.

It consists of three main sections called *Basic Training*, *Boot Camp*, and *In The Service*. Each of these sections has an introductory part called, for example, *Basic*

Notes/Comments/Ideas

Training Notes, and an activities part called, for example, *Basic Training Engagement Masters*.

The *Notes* sections have discussions of the general approach for that main section, and has summary checklists of skills covered in each of the activities. Both sections have space reserved for *Personal Notes* or *Notes/Comments/Ideas* for participants to take advantage of. The facilitator is encouraged to use the NOTES sections of the manual for constructive criticism or delivery notes.

The *Engagement Masters* usually have questions designed initially to confirm a given skill level has been reached. These questions move toward extension as the manual progresses.

Physical Arrangement

Groups of 4 or so persons arranged in pods works well for graphing calculator inservices. Those experienced with the TI-83 should disperse among the pods and serve as a support to others.

Learning Models

There is great opportunity here to employ cooperative learning, mentoring, and many other learning environment enhancements.

Notes/Comments/Ideas

Reflection Time

It is very important that ample reflection and exploration time be allotted throughout this experience (this is also mentioned in the *Timeline* section of the manual). This should be as creative and enjoyable an experience as possible. Remember, most of the participants will likely not have had any experience with this technology before, and will gain from ample reflection time and a supportive environment.

Additions / Deletions

Any additions, deletions or other possible improvements to this experience can be forwarded to the author at: <pgosse@stemnet.nf.ca> or via surface mail to: Paul Gosse, 291 Frecker Drive, St. John's, NF, A1E 5T8.

Notes/Comments/Ideas

Appendix #2 – Registration Package

First, this appendix contains a one-page information sheet (under a separate cover) that may be used for initial interest gathering.

The remainder of this appendix constitutes the registration mail out that all participants should have received prior to attending a professional development institute using this manual. The registration mail out includes: a cover letter template; a series of introductory comments and principles (under a separate cover); and a confirmation of registration survey (under a separate cover) designed to provide the facilitator with specific information to help tailor the delivery of the institute to the individuals present.

Please note that since this appendix (excluding the one-pager) is to be used as a mail out in advance of the institute, its function is really outside of the manual or institute setting. Therefore, each of the parts mentioned above under cover is numbered independently. In addition, the survey is numbered in a different style to separate it from other materials even more. This is to allow easier administration outside of the mail out use (should it be necessary). That is, this appendix (excluding this particular page) is <u>not</u> numbered in the running style of the manual.

Notes

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One-Pager

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Future Mathematics In A TI-83 Graphing Calculator Environment

... is a 5-Day institute that has been developed by a teacher experienced in the use of graphing calculators. The author has owned various models of graphing calculators since 1988. He has delivered inservice in this field since 1991 for audiences such as: individual schools and boards; NF Dept. of Education; NLTA Math SIC AGM; NCTM; Texas Instruments' at C-T^3; APICS, Summer PD Institutes, and others.

The project option for completion of a M.Ed. [Teaching and Learning Mathematics] has provided the opportunity to create an institute manual containing notes, 22 fully detailed activities with solutions, and a collection of teaching tips to support teachers delivering a new curriculum. The TI-83 has been chosen for delivery as it has the best combination of power and user-friendliness in the current education calculator market.

The institute begins with a keypad tour and a walk through the function of the TI-83 as a scientific calculator only. Changing global mode settings is illustrated as is storing and retrieving numbers to variables. Sequences are also modeled and plotted. Some basic work with lists and data is included and participants learn to construct statistical plots, learn about viewing windows, and learn how to link formulas making the TI-83 behave a bit like a spreadsheet.

Then, discrete linear data vs. continuous symbolism is explored via function graphing. A traditional quadratic function is explored including finding its zero(s), X and Y-intercept, and max/min values. Additional topics here include piece-wise graphing, the vertical and horizontal line test, dy/dx and the constant slope property of a line, finding intersections several ways, and using tables to evolve a sense of range. The full graphing toolset is illustrated as is solving inequalities.

Data gathering and statistical analysis now becomes a focus. Distribution of data, linear (including Median-Median and Least Squares) and quadratic regression modeling is explored. The Normal Distribution is applied to sample data and is used to discuss percentile ranks. The accumulating area nature of the normal curve and the construction of a normal curve for specified data are illustrated. Real life examples of step functions, using matrices for intersections and networking applications, modeling enclosure optimization, and connections between factoring and polynomial graphs are other interesting topics that are explored. Finally, examining transformed functions using a table and graph view is illustrated.

Additional topics such as friendly windows, parametric graphing, and a full set of TI resources can be found as appendices. Linking your TI-83 with other TI-83s and/or a PC, and more are included as *Teaching Tips* throughout the institute manual.

The author has applied the following theme throughout this work: "How can I make a graphing calculator experience that doesn't feel like the blur of immersion; one that talks to participants instead of at them; one that leaves them with a not-too-technical but absolutely thorough resource they can read and learn from off-site back in their schools; and, one that tries to share valuable teaching tips in appropriate settings?"

The result is this institute. It is hoped that the institute and the resources together will fertilize the expertise we have in our schools to provide a network of support in this technology as we begin to grow a graphing calculator environment in our schools. *Please note that topics, or order of topics, may be subject to change.*

FUTURE MATHEMATICS IN A TI-83 GRAPHING CALCULATOR ENVIRONMENT

[Click **here** and type today's date]

Dear Sir or Madam:

Thank-you for applying to *Future Mathematics In A TI-83 Graphing Calculator Environment Institute.* This professional development opportunity will be:

- of [Click **here** and type length of inservice] duration
- located at [Click **here** and type complete address of location]
- beginning at [Click **here** and type starting time each day] and ending at about [Click **here** and type ending time each day] each day

Morning and afternoon refreshments will be provided but participants will be responsible for lunch.

Registration Surveys found in this package may be returned to the facilitator via:

- surface mail or fax at: address shown on the footer of this letter

Should you require any additional information, please contact me at the address shown or by:

- email at: [Click **here** and type email address]

I look forward to seeing you there!

Sincerely,

[Click **here** and type your name]
Facilitator

FUTURE MATHEMATICS CURRICULA

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TI-83 GRAPHING CALCULATOR ENVIRONMENT

Registration Package - Part I [Outline]

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TABLE OF CONTENTS

Personal Notes

Welcome

You have been tentatively registered as a participant in a *Future Mathematics Curricula In A TI-83 Graphing Calculator Environment* Institute. This experience is designed to improve your comfort in using graphing calculator technology in your teaching. The particular graphing calculator used in this institute will be the Texas Instruments' TI-83 graphing calculator (or GC). The whole institute is designed to be delivered over 5-days and will include many activities and discussions. Variations may allow for greater or lesser time.

This "Registration Package" contains two parts. The first is outlined through the "Contents" section of the manual. The second, under a separate cover, is a survey. Please confirm your registration by completing this survey and forwarding it (mail/fax) to the address supplied in the cover letter attached to this package.

This institute is <u>not</u> a demonstration of how to use everything you've ever heard or dreamed could be done with the TI-83! This institute was built:

1) ...to enhance and support teaching and learning under revised mathematics curricula; and,

Personal Notes

2) ...to create an environment within which GC technology (a technology particularly suited to revised curricular demands) is not overlaid, but is infused.

Principles & Structure

First and foremost is the global principle of providing information in a manner that the participant can immediately use, and can continue to use and share months later.

The binder-style manual contains notes, activities (called *Engagement Masters*), a glossary, references, and appendices that it is hoped will be clear, friendly, concise, accessible, and transferable. The manual is conversational, and documents in detail the activities a facilitator using this manual could work through.

The objective of the manual is to serve as a ready reference and, in partnership with your TI-83 Manual, to foster real growth in teaching with this technology.

This institute will include material from traditional mathematics curriculum to provide continuity and immediate grounding, as well as material supporting revised curriculum initiatives that reflect the new direction mathematics teaching and learning is embarking upon. Reflection and extra practice time is built in and is seen

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as essential to the experience. In addition, avenues for personal growth are suggested.

Inservice events occasionally assume that teachers somehow learn in a different or faster way than others; or, that teachers learn with less support and training time than other learners in the private sector. This institute will endeavour to deal with that in a proper and professional manner such that each participant will re-visit excitement in learning, at a comfortable pace, with challenges and reflection opportunities appropriate to your own level.

It is the intention of this institute to provide a set of comprehensive activities built with a conversational tone. It includes clear illustrations of keystrokes and screens and aims to evolve basic competencies with the TI-83 while utilizing a delivery model reflecting a graphing calculator environment.

A Graphing Calculator Environment

You will explore your own learning of mathematics here under a *graphing calculator environment* (or GCE). What is a GCE?

The graphing calculator, for the purposes of this experience (and hopefully for many other mathematics experiences for you from now on), is treated as an

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available and essential tool similar to a text, writing instrument, straightedge, and paper. It is no less or more important and should receive no more or less attention. That, essentially, is a GCE.

From the perspective of this institute, a GCE is illustrated by the approach taken to problems. A multi-representational approach to the learning of mathematics is facilitated by embracing a GCE. In a GCE, the graphical validates the tabular which validates the symbolic. This is the balanced approach of this inservice. *No one representation will dominate as a point of departure for a lesson.* Since we are learning how to use the TI-83 however, the technology receives the emphasis.

The GCE of your own classroom is created through the style of learning experience you create, coupled with the nature of the topics, and the type of materials accessed. A GCE will play an important role in the new direction mathematics curriculum has taken.

It should be noted here that your communication skills, mathematics skills, and skill level in any given area, are not the skills of your students. They do not have your symbolic skill, your mathematics background, or your insight. Neither do they have your biases. They will utilize a different tool set in their learning of mathematics than many of us did - and that tool set will include the graphing calculator. We must adapt our methods and how we utilize our tools to encompass all of their tools.

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Many participants of this institute will not have accessed a GC for classroom teaching. Therefore, for now, there is the problem of creating an atmosphere at this institute within which the machine is treated as a comfortable tool of second nature. It is hoped that this preliminary commentary, together with the first two sections of the manual (*Basic Training* and *Boot Camp*), will prepare participants and foster an embracing of the technology such that a sense of what a GCE can be for each of us can evolve more naturally.

TIMELINE

Institute Day	Suggested Master Coverage
Day 1	Welcome Basic Training Masters #1 to 6 Contact Sheet noted
Day 2	Reflection and Discussion of last day's activities Boot Camp Masters #1 to 4 TI Appendix as recommended reading

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Day 3	Reflection and Discussion of last day's activities
	Boot Camp Masters #5 to 7
	In The Service Masters #1 to 2
Day 4	Reflection and Discussion of last day's activities
	In The Service Masters #3 to 6
	Institute Evaluation to take home
Day 5	Reflection and Discussion of last day's activities
	In The Service Masters # 7 to 9
	Institute Evaluation collected prior to participant departure

Note: *Any given institute, run with this manual as its main resource, may or may not utilize all activities. That would depend upon audience interest and comfort, available time, and other exercises or resources available. This product was designed to be useable outside of an inservice environment also for that reason.*

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A Word About Manuals

GC manuals are not written to be read in order. That is, most provide an "operational basics" section up front, and the remainder is usually application based. In fact, the manuals themselves usually state this. For example, "[t]his manual describes how to use the TI-83 Graphing Calculator. Getting Started is an overview of TI-83 features. Chapter 1 describes how the TI-83 operates. Other chapters describe various interactive features" (Texas Instruments, 1996b).

This institute offers a compromise between that approach and a strictly sequenced one. The first section focuses on providing a description of the physical layout and operating conventions of the TI-83. This is done through a familiar avenue - that of utilizing the power of a scientific calculator. In *Basic Training*, (the first working section of the manual), we explore the buttons, structure, and scientific calculator applications of the TI-83. There we lay groundwork such that the curricular sections to follow should not be too much like learning by immersion while feeling like submersion!

Once *Basic Training* is accomplished, most other activities can be explored and mastered. However, if you jump from section to section without having completed all the activities, there are a few things you may notice. As you will find

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noted at the beginning of *Basic Training*, new or important terms appearing in the glossary are bolded only upon first appearance. This flag might be missed for some glossary entries that recur in a subsequent section. Check the glossary or the TI-83 Graphing Calculator Guidebook (a.k.a. the TI-83 Manual) for any term whose use you are uncertain about. Key-sequencing illustrations deliberately fade as the manual progresses, which may leave a reader pondering menu/command location from time to time. And finally, not all recurring skills have their first illustration in the manual referenced - though many do. Therefore, some familiarity with skills gained through previous exercises is pre-supposed throughout the manual.

Exercises are built as *Engagements* - that is, learning occurs best when we are truly *engaged* in the process. They are called *Engagement Masters* as they serve a purpose beyond simple blackline masters; have solutions and discussion integrated throughout; yet may be taken out of the manual as separate entities (i.e. masters) for independent work.

There are *Notes* pages in each manual section which immediately precede and partner with the *Engagement Masters*. The *Notes* provide relevant discussion, name the goals of each exercise to come, and provide a description of the skills/menus/commands (in checklist form) utilized in each partnered *Engagement Master*.

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The *Engagement Masters* provide problem experiences and solutions. They reflect current mathematics courses, and also use impending curriculum as a driver. There is an effort at sequencing these activities/masters so as to progressively lighten keystroke notations, provide some referential flow, and create bridges between traditional treatments and new ventures into the learning of mathematics. This is a substantial departure from most calculator manuals though, in fairness, that is not what those manuals were built to do.

Final Notes

If you have a TI-83 please bring it, and your calculator to calculator link cable, to the institute. Though TI-83 calculators will be provided, familiarity with your own machine may aid in retention and enhance comfort with the technology. Other Texas Instrument graphing calculators such as the TI-82, 85, 86, and 92 have similar operating styles. Of course, the menu structures and some syntax will vary, and the TI-83 will be the institute instrument; but if you are already familiar with a particular TI machine, you may still find this institute useful. If you are unsure, you could always bring it, work with the supplied TI-83, and decide on-site whether or not you wish to use your own calculator.

Personal Notes

You may have noticed a space reserved for **Personal Notes** at the bottom of each page here, and in the main sections of the manual (except for the content checklist pages). This may be identified as a deficit in some similar materials. This space is provided for ***notes on the fly, evaluative comments***, or to serve as a scratchpad. Make notes anywhere you wish so that as many questions as possible may be dealt with. The *Engagement Masters* have a similar space reserved called **Notes/Comments/Ideas**. That space may be used for evaluation of the activities undertaken, or used also for notes.

The *Engagement Masters* also include some *Teaching Tips*...tips found to be valuable by the author in classroom settings. The nature of these tips is listed in the *Notes* sections of the manual in the content checklist for each activity.

These activities are by no means intended as an exhaustive list of TI-83 power or teaching tips. If everything were included it would be too large to do in a week - and you would certainly be a "Super Teacher"! On the other hand, many teaching tips are discovered as people find a need to do something different and many are published in journals such as *Eightysomething* (from Texas Instruments which is also online) and *The Mathematics Teacher* (from the National Council of Teachers of Mathematics - NCTM). You will evolve your own skills and techniques. *Please note that an extensive collection of Texas Instruments products, services, and general*

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information may be found in an appendix titled TI Calcs./Access./Support. Virtually all current TI graphing calculator products, accessories, services, PD opportunities, publications, and much more is in there.

Evaluation of this institute manual and delivery will also be ongoing. There is an evaluation instrument included in the binder following the *Backword*. It is designed to be completed at the end of the institute. Please take the time to complete this important measure of the experience and help inform the facilitator and/or the author. Every delivery of this institute should be better than the last with your help.

It is also hoped that, as new topics evolve within new mathematics curricula, new components will grow out of that and be infused into this institute. In the interim, comments and criticisms can be forwarded to the author through email or surface mail at the addresses shown below, or, through your institute facilitator.

pgosse@stemnet.nf.ca	291 Frecker Drive St. John's, NF A1E 5T8

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It is hoped this experience will prove enjoyable. If you have any concerns or questions about this introductory information or the institute itself, please contact the facilitator of your session or the author.

The "Registration Survey" follows immediately under a separate cover. Once again, please complete it and forward it now to the institute facilitator as directed in the covering letter.

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FUTURE MATHEMATICS CURRICULA

IN A

TI-83 GRAPHING CALCULATOR ENVIRONMENT

Registration Package - Part II [Survey]

Created by

Paul Gosse
M. Ed. Candidate
Memorial University of Newfoundland

FUTURE MATHEMATICS CURRICULA IN A TI-83 GRAPHING CALCULATOR ENVIRONMENT

Welcome!

A place has been reserved for you at a delivery of the above institute to be held at:

[______________________]beginning at [________]from [______________].
Location Time Dates

Please confirm your registration by completing this survey carefully and returning it to the address supplied with this package. This will assist the facilitator and developer of this institute to improve and tailor the institute more closely to your needs.

General Information

Name (last, first):	____________	School/Group:	____________
Home Address:	____________ Street	Work Address:	____________ Street
	City or Town Province		City or Town Province
Home Phone:	Postal Code	Work Phone:	Postal Code
Home Fax:	____________	Work Fax:	____________
Email:	____________	Years Experience:	____________

Courses normally taught

- ❑ Practical Stream ❑ Academic or Advanced Stream ❑ Advanced Placement

Why did you register for this institute? ❑ Interest ❑ Board Request ❑ Dept./Ministry Request
❑ Other: ____________

Have you ever attended a graphing calculator institute or inservice? ❑ Yes ❑ No

If so, Offered by whom: ____________
Offered when: ____________
Using what calculator: ____________

FUTURE MATHEMATICS CURRICULA IN A TI-83 GRAPHING CALCULATOR ENVIRONMENT

Do you feel this institute is a necessary PD opportunity? ☐ Yes ☐ No

If so, why? If not, why not? ______________________

Please use this space list other institutes/inservices you are aware of that may also fill this need? ______________________

Scientific Calculator Information

Do you have access to a scientific calculator? ☐ Yes ☐ No

If so, what brand: ☐ Casio ☐ HP ☐ TI ☐ Sharp ☐ Other: ______________________

Are you comfortable using a basic scientific calculator? ☐ Yes ☐ No

Graphing Calculator Information

Do you have access to a graphing calculator? ☐ Yes ☐ No

If so, what brand: ☐ Casio ☐ HP ☐ TI ☐ Sharp ☐ Other: ______________________

Have you ever used a graphing calculator? ☐ Yes ☐ No

If so, which best describes your current skill level and comfort with graphing calculators:

☐ None ☐ Novice ☐ User ☐ Skilled

If you own a TI graphing calculator, please indicate which model:

☐ TI-80/1 ☐ TI-82/3 ☐ TI-85/6 ☐ TI-92

Does your Mathematics Department…

… own a viewscreen or overhead graphing calculator? ☐ Yes ☐ No

If so, what brand: ☐ Casio ☐ HP ☐ TI ☐ Sharp ☐ Other: ______________________

… intend to obtain an overhead graphing calculator? ❑ Yes ❑ No

If so, what brand: ❑ Casio ❑ HP ❑ TI ❑ Sharp
❑ Other: ______________________

… own a class set of graphing calculators? ❑ Yes ❑ No

If so, what brand: ❑ Casio ❑ HP ❑ TI ❑ Sharp
❑ Other: ______________________

… intend to obtain a class set of graphing calculators? ❑ Yes ❑ No

If so, what brand: ❑ Casio ❑ HP ❑ TI ❑ Sharp
❑ Other: ______________________

Have *you* ever used an overhead graphing calculator? ❑ Yes ❑ No

If so, what brand: ❑ Casio ❑ HP ❑ TI ❑ Sharp
❑ Other: ______________________

Have *you* ever used a class set of graphing calculators? ❑ Yes ❑ No

If so, what brand: ❑ Casio ❑ HP ❑ TI ❑ Sharp
❑ Other: ______________________

Please share your current feelings about graphing calculator technology:

For personal use: ______________________

For student use: ______________________

For instructional use: ______________________

In testing situations: ______________________

FUTURE MATHEMATICS CURRICULA IN A TI-83 GRAPHING CALCULATOR ENVIRONMENT

Computer Information

Do you own a computer?	❑ Yes	❑ No
Do you use a computer at work?	❑ Yes	❑ No

If you answered yes to either of the last two questions, please indicate which of the following categories of software you commonly use:

❑ Word-processing ❑ Spreadsheet ❑ Html Editor ❑ Graphing/Mathematics

Have you installed software?	❑ Yes	❑ No
Have you surfed the Internet?	❑ Yes	❑ No
If you answered yes to the last question, have you downloaded files from the Internet?	❑ Yes	❑ No
Do you have a computer in your classroom?	❑ Yes	❑ No
If so, do you use it for		
… teacher driven demonstration/research	❑ Yes	❑ No
… administrative tasks	❑ Yes	❑ No
… student driven demonstration/research	❑ Yes	❑ No

General Comments

What would make this the best possible experience possible for you?

What learnings do you hope to gain from this institute?

Other Valuable Information About Current Graphing Calculators:

Some graphing calculators share some of the following capabilities with the TI-83:

- are detachable from the overhead LCD display unit so lessons can be developed elsewhere and displayed in class using the same calculator
- can communicate with another calculator of the same model series through an I/O port
- can communicate with a PC or Mac using software and a serial or PS/2 cable for backup or data exchange
- can grab any onboard data including collected data, other lists, programs, or screenshots for pasting into a word-processor, spreadsheet, or other software
- can be used with a portable laboratory data-gathering device (such as the CBL and CBR) for conducting experiments as you would with Vernier software, probes, and a computer; and, has programs and probes available directly from Vernier
- a class set of 25-30 units costs about the same as two personal computers
- is a part of a volume purchasing program.(with 45 proofs of purchase of TI-8* calculators, you can earn a free Viewscreen Model from the TI-8* family)

Closing Comments

This institute is not sponsored by Texas Instruments or any other agent. The contents are the creation of the author except where otherwise acknowledged and credited. Every effort has been made to ensure accuracy. Should any errors be suspected or found, please inform your facilitator or the author. The author assumes no initial or subsequent liability, for any use or misuse of the information contained in the Manual titled "Future Mathematics Curricula In A TI-83 Graphing Calculator Environment". The Manual should serve as a learner's guide and a ready reference in using the TI-83.

Thank you for taking the time to complete this survey. Please forward it now as indicated in the accompanying letter.

Appendix #3 – Friendly Windows

Friendly windows are a mechanism for avoiding what I call *the pixel problem*. When a function is graphed and then traced, the cursor moves along the function in X-steps of one pixel and presents the X and Y-coordinate values on the bottom of the window (if this is turned on). The standard window on the TI-83, found by pressing ZStandard, is [- 10, 10] [- 10, 10]. So, Xmax – Xmin is 20. There are 94 pixel intervals along the X-axis in this window yielding a pixel X-step of about 0.2127659574....

Each X-step in a trace of any function graphed in this window will display a similarly long number of decimal places for the X-coordinate. As suggested in the above calculation, the value of this X-step, and each pixel, can be found by looking at the width of the window (Xmax – Xmin) and dividing that value by the number of pixel intervals (94) or by (46) in G-T Mode. That is, $\Delta x = \textit{pixel step} = \left(\frac{x_{max} - x_{min}}{94}\right)$.

ZDecimal is an option that forces each X-step to be 0.1 and represents one of two built-in "friendly windows" (the other is ZInteger with X-steps of 1). With a little bit of other creativity you can actually find many good friendly windows using

Notes/Comments/Ideas

ZDecimal and Zoom In and Zoom Out. Try this after completing the rest of this appendix!

The TI-83 screen matrix actually has 95 pixels along the X-axis and 63 along the Y. These result in pixel intervals of 94 and 62. So, it is 94 and 62 that are our key numbers for calculation of *nice* X and Y steps (only X-steps are illustrated here, as this is the most common task). Therefore, we use 94 to determine *magic* numbers for your (Xmax – Xmin) difference to be. Any (Xmax – Xmin) that is a nice multiple or fraction of 94, the divisor (# of pixel intervals) in determining the X-pixel-step, will provide a "friendly window" with an X-step equaling that multiple or fraction factor.

For example, Xmin = - 4.7 and Xmax = 4.7 gives (Xmax – Xmin) = 9.4 yielding X-step = 0.1 i.e. your ZDecimal default. This is clear since the difference, (Xmax – Xmin) is one tenth of 94 i.e. 9.4. We can shift this window by adding an equal amount to Xmax and Xmin...say 0.3 yielding Xmin = - 4.4 and Xmax = 5. This also gives (Xmax – Xmin) = 9.4 yielding X-step = 0.1.

To make X-step = 0.5, simply make (Xmax – Xmin) one half of 94 instead of one tenth of 94 i.e. (Xmax – Xmin) = 47. So, a friendly window with X-step = 0.5 straddling the origin is Xmin = - 23.5 and Xmax = 23.5. To shift our friendly window, add or subtract an equal amount to each of Xmin and Xmax.

Notes/Comments/Ideas

An alternative way to locate a "friendly window" is to choose your **Xmin** (or **Xmax**) and simply add (or subtract) the factor of 94 you want to be your X-step. For example, if an X-step of 10 were desired for a window that started at **Xmin** = 0, then we make **Xmax** = (**Xmin** + 10*94) = 940. In general, choosing **Xmin** and X-step and setting **Xmax** = (**Xmin** + X-step*94) creates a "friendly window".

With a little exploration you could determine your favourite "Friendly Windows" and even store them as Graph DataBases (GBDs) for quick recall.

An important note here has to do with tracing functions in "unfriendly windows". Due to *the pixel problem*, tracing rarely results in an integral X-pixel value or a pixel value such as 0.333…. As a result, the likelihood of a trace directly revealing a zero of the type of relation typically found in secondary school texts, is low. This is because the pixel step can often cause the calculator to skip over (i.e. not to "land" on) the value in question.

This can of course be gotten around by applying "friendly windows". If we change the viewing window slightly from the standard window using - 8.8 to 10 on the X-axis, we get (Xmax - Xmin) 18.4 and X-step = 18.8/94 = 0.2. Any relation traced in this window will be evaluated at - 8.8, - 8.6, …, 8.8, 10. We will likely hit a typical zero value with a trace using these or other similar values. If we attempt to trace off screen, the calculator will pan for you. Try it!

Notes/Comments/Ideas

It may well be that graphs viewed in such a window will be far easier to analyze on first blush. This is not to disempower all other abilities of the student utilizing the TI-83. It is simply presented as an example of how limiting a window can be, and as an example of why we should always keep a broad perspective and various methods of analysis at our disposal.

The following table summarizes "Friendly Window Magic #s" for the TI-8* series of graphing calculators:

Graphing Calculator	Rows By Columns of Screen Pixels	Friendly Window X-Axis Magic # (i.e. # of pixel intervals)
TI-80	63 by 47	62
TI-81	96 by 64	95
TI-82	95 by 63	94
TI-85	127 by 63	126
TI-86	127 by 63	126
TI-92	239 by 103	238

Notes/Comments/Ideas

An excellent activity, also found in this reference, is to have students consider the algebraic representation of a given relation, analyze its critical behaviours, and then determine a suitable "Friendly Window" designed to provide a reasonable view encompassing all aspects of interest. This type of exercise encourages a more holistic sense of technology's place in the study of relations.

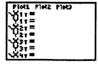
Reference:

"Friendly Windows" is attributed to Stuart Moskowitz, Mathematics Dept., Humboldt State University, Arcata, CA, 95521, <smoskow@cello.gina.calstate.edu> as found in:

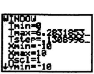
Texas Instruments. (1996a). *CT^3 - Canadian teachers teaching with technology* [manual]. Dallas, TX: Author, A-2.

Notes/Comments/Ideas

Appendix #4 – Parametric Eqs. Together w/ Ftns.



Setting the TI-83 up to graph parametric equations is simply a matter of setting the global graphing mode to **Par.** To set the TI-83 this way press MODE **Par.** Accessing the graphing editor shows the immediate effects of this setting as illustrated.

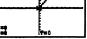


The usual function style has been replaced by X_{1T} and Y_{1T}. This allows for both coordinates to be calculated separately as functions of a separate parameter T. Press ZOOM [ZStandard] then WINDOW. We get a view of how this works in the second screenshot shown. We have a counter variable T (illustrated by Tmin, Tmax, and Tstep), followed by the normal window parameters Xmin, Xmax, etc. As before, Xmin, Xmax, etc. are the axes settings for display purposes but now are no longer the settings for calculation purposes. This appears new but actually was also the case when we first plotted sequences in one of the very first masters.

Notes/Comments/Ideas

Plotting a Function in Parametric Mode

To plot a normal function using parametric graphing, set the X coordinate equal to *t*. This causes the relation to behave as if the X-coordinate was being substituted and used to directly calculate the Y-coordinate (though both actually depend upon T).



Therefore, to plot say $y = 2x$, we could set $X_{1T} = T$, and $Y_{1T} = 2T$. Enter these two expressions now and trace the graph. Note that all three values show in the trace...T, X, and Y. Also note that T, our counter, began at 0 in this case, so we see our plot beginning at the origin. Changing the Tmin value using the WINDOW menu can alter this to suit the window chosen.

Important Note: As mentioned, Xmin, Xmax, etc. are the axes settings for display purposes but now are no longer the settings for calculation purposes. What does this mean and is it a problem? It means that the window settings under the $\boxed{\text{WINDOW}}$ key control the view. They determine the Xmin, Xmax (thus determining Xstep), and Xscl — similarly the Y axes settings). While in **Func**

Notes/Comments/Ideas

graphing mode, they control the plot since X is the independent variable. Other graphing modes use other parameters to control the plot — though in the case of parametric plots, X is indirectly involved. This is not usually a problem but is something to be aware of. For example, when tracing, directly inputting a trace value assigns that value not to X, but to the parameter. The solver is affected in a similar way.

If the T vs. X discrepancy bothers you greatly, a neat trick to reconcile things a bit (this is especially useful if graphing functions) is to chose your window settings appropriately first, then cursor up to the Tmin field and place Xmin there. Here's how... Cursor to Tmin, press VARS [VARS] [Window] [X/Y] **Xmin.** This pastes the reserved variable Xmin into the Tmin field. We have just forced our counter to reflect our window — at least for that setting. Repeat for Tmax pasting Xmax there.

For Tstep, we have a choice. If we want our counter to increment by the same amount as our display pixel steps (see appendix titled "Friendly Windows" for a detailed explanation of this), then set Tstep equal to Xscl. If not, then choose your Tstep. As you pasted the X variables into the T fields you may have noticed that the names didn't paste — only the values did. As with all variables on the TI-83, if they have a value it is the value that is placed where the variable is used. This is important

Notes/Comments/Ideas

since changing the window settings will not automatically cause your T settings to change. To update the T settings would require repetition of the earlier steps we did. *Hint*: this is really easy to program. The only lines would be:

: Xmin [STO→] Tmin

: Xmax [STO→] Tmax

: Xscl [STO→] Tstep (if desired)

Running this program after choosing the window values would perform the steps we did above automatically. For more information on simple programming, see *In The Service Engagement Master* "Coin Collectors" *Teaching Tips* "Saving List Data As A Program" and "Editing and Copying Programs".

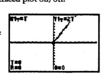
Setting Rectangular vs. Polar Trace Coordinates

As was noted once before, we can actually change the nature of the displayed graphing coordinates from rectangular to polar independent of the global graphing editor setting. 2nd [ZOOM] [FORMAT]

Notes/Comments/Ideas

produces the menu in the screenshot shown on the right.

This menu controls characteristics of the graph display. It has five lines of choices. The first is discussed shortly. The others turn free-cursor (non-trace) coordinates on/off; grid marks at Xscl and Yscl points throughout the screen on/off; axes lines on/off (useful for examining hidden behaviours near the axes); axes labels on/off; and, displaying the expression of the traced plot on/off.

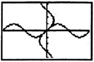


Currently, **RectGC** is the coordinate display style. If we cursor to **PolarGC,** press ENTER to activate it, and return to our trace, we find the screen displaying position using polar coordinates.

Illustrating Inverses

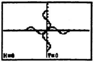
Parametric equations are very useful for plotting inverses. For example, to plot the sine graph and its inverse on the same set of axes, we enter the expressions as shown. These expressions highlight the inverse nature of the two plots we are using as the xy-pairs have their coordinates interchanged.

Notes/Comments/Ideas



We choose a window using ZTrig. This sets Xmin = - 2π, Xmax = 2π, Xscl = π/2, Ymin = - 4, Ymax = 4, and Yscl = 1. We must set up the counter independently. For now, set Tmin = - 2π, Tmax = 2π, and Tstep = π/10 or less. This will cause our counter to plot on our entire window and not only to the right of the origin as it did in the very first plot in this section. Press GRAPH. You should see the screenshot on the right.

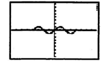
We can create a better view now by zooming out now by a factor of 2. The default zoom factor setting is 4, so we will probably need to change the factor on the calculator. Zoom factors can be set using the ZOOM [MEMORY] menu under **SetFactors...** factors set at 2 work well for most plots. Pressing **Zoom Out** after changing the factors to 2 produces the shot on the right. Notice that since we didn't alter the Tmin counter, the plot did not fill the window.



Notes/Comments/Ideas

Mini Teaching Tip - Pausing During Graphing

As you watched these graphs plot, you may have thought how useful it would be for class purposes to hold the first graph up there and then plot the second after discussing what the inverse of sine would look like.



To pause any graphing operation while in progress, press ENTER. You will see a frozen looking LCD bar in the top right corner of the screen indicating that the action has been paused (see screenshot above).

To re-start graphing, press ENTER again. The screenshot shown, has the plot paused immediately after the first plot completed but before the second began.

Notes/Comments/Ideas

Mini Teaching Tip - Interrupting Graphing or Program Execution

If a graph is not plotting as you wanted and you see that as it is being plotted, or if a program isn't running as you want and you wish to interrupt it, pressing and briefly holding ON will interrupt the action.

For graphing, such an interrupt has no consequences associated with it. To re-start a graphing action, make and needed changes and press GRAPH again.

Be careful if interrupting a program however! Interrupting a program brings a Quit or Goto message. It assumes you interrupted the program to correct a coding problem and, if you select Goto, will bring you to the specific line of code that was being run at the time you pressed ON. Remember that any changes you make from within the program editor, intentional or not, are saved immediately. To simply quit the running program without making any changes, choose Quit.

So why would you want to interrupt a running program? If you are debugging a program, you will want to go to the line of code causing a particular problem. Interrupting a running program this way brings you immediately to the line of code that was running at the time of the interruption.

Notes/Comments/Ideas

Functionality of Inverse Plots

The drawing tools DRAW **Horizontal** and DRAW **Vertical** can be a very concrete visual illustration of HLT and VLT when applied to a graph. Use of these tools was discussed in *Boot Camp Engagement Master* "Polynomial Graphing" *Teaching Tip* "A Test For Functionality". We can drag and/or drop a vertical line over each of these plots to discuss the functional nature of each. We can also discuss how a vertical line and horizontal line dragged over the original plot can address functionality of both the plot and its inverse.

For a little creativity, this example could be customized to ask students how the first plot could be **Boxed** (i.e. using ZOOM **Box**) such that the inverse of the boxed part would also be a function. **Box** is used here used solely as an illustrative tool, but it would redefine the window if completed as a command. Then, the window would be altered accordingly and both could be plotted on the cropped window (hence the restricted domain). This is an excellent exploration of functions and inverses via another graphing system.

Modeling Motion with Parametric Equations

This excellent activity can be found in the TI-83 Graphing Calculator Guidebook, p.4-2.

Notes/Comments/Ideas

The physics formulas: $t \cdot v_0 \cdot \cos\theta$ and $t \cdot v_0 \cdot \sin\theta$ can be used to evaluate the horizontal and vertical component vectors of a ball's motion. We can plot the path of such a ball using parametric equations. Using the animated plot style (discussed earlier) we can simultaneously view the magnitude of the component vectors as circles (from the animated plot styles) moving along the axes. This is a very easy and visual activity which integrates physics with mathematics.

Notes/Comments/Ideas

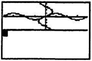
APPENDIX #5 – SPLIT SCREEN USES

Split Screens - A First Look

The global mode menu offers three screen settings: **Full, Horiz,** and **G-T**. We have been working nearly exclusively so far with the full screen view.

The horizontal split screen dedicates the top of the window as a graph screen, but the bottom may be set to an abbreviated view of the homescreen, y = editor, stats/list editor, window editor, or table editor. As you choose menus or editors, the appropriate split screen becomes active.

This split screen has some nice uses in programming (say for displaying a graphical logo at the top while displaying text at the bottom). It also allows the user to make a quick switch from studying a graph, to making a calculation or checking something outside of the graph window (while keeping an image of the graph available). Here is a screenshot of our first look at sine and its inverse with an active blank homescreen on the bottom.

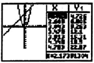


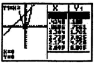
The vertical split screen is dedicated to viewing a graph and its table…hence the command name **G-T**. Unlike a horizontal split, which governs all views at all

Notes/Comments/Ideas

times, the vertical split screen is only activated when either GRAPH or TABLE is pressed. It is a lot more in line with how a graph and a table would appear in a text or on board, so it is a nice way to examine both on one screen. Any action not associated with graph or table causes the calculator to revert back to a normal full screen view.

As an example, let's switch back to function graphing mode and use a **G-T** split screen to view the two functions: x^2 and $(x + 6)$ in a standard viewing window. Initially, your screen should resemble the first screenshot shown below. Pressing TRACE should place a trace cursor in the center of our window (the origin is the center of the zoom standard window) as in the second screenshot shown below. Notice how the traced function's expression and trace coordinates appear, and how the table updates to reflect your trace points. Move around and see what happens!





You should see that the highlighted points in your table match exactly the trace cursor coordinates.

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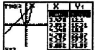
An interesting note here is that in this view, the headers of the table are not editable like they have been before, and we can only see one Y_n header at a time. This is not the case with the horizontal split which gives reduced size but fully functional windows.



Since we are in trace mode, trace to where the intersection points appear to be. They are near 3 & - 2. To help determine if 3 is an intersection point, key in 3 directly. The screenshot to the right shows that Y_1 yields 9 when X = 3.

Alas! A "clash in values"! When we key in 3 directly, the graph side clearly shows that it has evaluated the function at the requested spot and illustrates about where it is. The table however, generates Y-values based upon the actual pixel values only - though it goes as close to the x-value 3 as it can. Recall that user-inputted values are only accepted if **Auto** is changed to **Ask** in the TABLE. Doing that allows accurate table values but the trace point is not linked to the table so the trace isn't updated.

Basically, the only action causing simultaneous updating of both the graph and the table occurs with a graph trace. To make particular values appear on the graph (and therefore in the table) during a trace, requires a friendly window (see appendix titled "Friendly Windows"). The vertical split uses 46 pixel steps across the

Notes/Comments/Ideas

X-axis and 50 pixel steps across the Y-axis for the graph screen. This is also an excellent illustration of the *skipping a value* problem referred to in the appendix "Friendly Windows".

To determine the value of Y_2 at the same X-value of 3, requires the same procedure as doing it on a full screen trace. After entering 3 a moment ago and seeing Y_1 evaluate to 9, we must now press ▲ or ▼ to switch plots (the graph, and table header and values update), and press 3 ENTER again to confirm that Y_2 also yields 9 at $X = 3$.

It is worth noting also that the [CALC] menu works in the same way here as it did on a full screen. For example, if we activate **Intersect** we get the following screen and the same prompts thereafter as we did earlier.

The Unit Circle and Split Screens

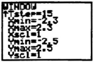
This is an excellent use of split screens and is adapted from the TI-83 Graphing Calculator Guidebook, p.9-2.

Parametric equations are excellent for viewing some conics - in this case the circle. Letting: $X_{1T} = \cos(t)$ and $Y_{1T} = \sin(t)$ provides the graph of the circle radius

Notes/Comments/Ideas

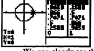
1 yielding cosine and sine as the traced coordinate values. Because we are using a parameter *t* however, we must set our parameter to progress through the angles we have in mind.

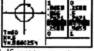
For simplicity, let's choose degrees and a *t*-step of 15 °. Go to the global mode menu and choose **Par** once again and choose **Degree**. Then set your window as shown in the screenshots below:



This *t* - step will hit all of our special angles as they occur once around a circle. Press GRAPH. You may notice that the graph seems fine, but that the table may not be that of the new relation. That is because we have not really activated the TABLE feature yet and it has remained inactive. Pressing TRACE would cause it to update to reflect traced coordinates. Pressing TABLE would also cause it to update as a separate editor. Press TRACE now and you should have the first screen shown below:

Notes/Comments/Ideas





We can clearly see the decimal equivalents of one half, root two over two, etc. reflecting our trigonometric ratios of 30 °, 45 °, etc. appearing in the table. Tracing the graph shows *t* as the angle in question with the trigonometric ratios showing as coordinates. The screenshot on the right above illustrates the cosine and sine ratios associated with 60 °. Trace around and find other special angle ratios!

Notice how clearly this table of selected angles (in steps of 15 ° remember) illustrates the related nature of cosine and sine. As one goes up the other appears to go down. Values appear to be inverted in some way. A little more tracing shows that signs are different in different quadrants. Finding all patterns in this graph and table example can be a very valuable exploration for students and an excellent topic for a short assignment or as a portfolio entry!

This plot, and all its accompanying parameters, may be saved as a GDB for later recall or exploration in the same manner discussed in *In The Service Engagement Master* "Ploy Wants A Plot" *Teaching Tip* "Saving A View: Graph Databases and PICS". Your students may even think of doing this to help them remember unit circle values!

Notes/Comments/Ideas

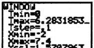
Unwrapping the Sine Function

This exercise is taken from the TI-83 Graphing Calculator Guidebook p. 17-10.

Revert back to a full screen view for a moment. An interesting supplementary exercise in the same style as our previous graphs in this section is observing the ***unwrapping*** of the sine function as the unit circle is plotting.

It's best if these two curves are plotted simultaneously. Set graphing to full screen, simultaneous plotting, and units to degrees by pressing MODE **Full Simul** and **Radian.** We remain in parametric graphing mode. Leave X_{1T} = *cos* (T) and Y_{1T} = *sin* (T) and define X_{2T} = T and Y_{2T} = *sin* (T).

Since we have changed to radian measure, we need to update our window variables for a reasonable view. Set values as shown.



Note that Tmax = 2π and Xscl = $\pi/2$.

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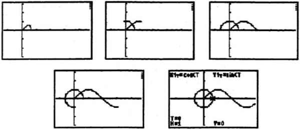
Pressing **TRACE** now will automatically cause the graphs of the unit circle and the sine function to plot (recall they will plot simultaneously). We can pause and re-start their plotting by repeatedly pressing ENTER if desired. The following 5-screenshot story illustrates the plots evolving:



Other trigonometric functions can be unwrapped in a similar manner. The simultaneous graphing option is a good one for showing a direct relationship between graphs such as this.

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Appendix #6 - TI Calc.s-Accessories-Support

TI Graphing Calculator Brief History

Please note that website addresses are inherently unstable though commercial ones such as these are generally reliable. All addresses used in this appendix were accurate as of July 19, 1998.

The TI-8* series of graphing calculators have provided power and ease of use since the TI-81 hit the market in 1990. It was followed quickly by the TI-82 and the TI-85. Since that time, the 83 has evolved from the 82, the TI-92 leapt forward starting a series all to itself, the 86 has evolved from the 85, and the TI-80 represented a first entry into the middle school. The 83 and 86 are the most powerful, and arguably the most popular, of this original series. *Please note that the images below were obtained from the TI website and do not represent true relative size.*

TI-83

TI-86

TI-92

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Recently several new models with dramatic enhancements have been introduced. The TI-73 is another powerful graphing calculator entry into the middle school/junior high level. The TI-89 is an enhanced model that resembles the 86 in look but really shares a lot of the power and features of the 92. And, the TI-92 has been enhanced through re-engineering a "Plus Module". The Plus Module means that a different plug-in module containing its operating system is available and upgradeable through "Flash Technology" (description forthcoming). Otherwise, the 92 Plus Module calculator appears very similar to the 92.

TI-73

TI-89

A complete history of TI calculators may be found online at: *http://www.ti.com/calc/docs/calchist.htm* and the main graphing calculator page, with brief descriptions and links to all the above models and more, can be found at: *http://www.ti.com/calc/docs/graph.htm*.

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Common Features

Inter-Calculator Communication

All of these calculators may be connected to another of the same model via an I/O port, using a calculator to calculator cable supplied with the unit (this cable can be seen connecting the CBL to a calculator later in this appendix). Through this port (which is an earphone style jack located on the bottom center of the unit), any object on one calculator may be sent using the LINK menu to another calculator of the same model.

In cases where an immediate predecessor model exists, such as with the 83 as a re-engineered 82 or the 86 as a re-engineered 85, the newer model will accept most transmitted objects from the predecessor. As the newer models often have an altered and enhanced operating system, transmitting from a newer model to its predecessor may or may not work depending upon what it is you are transmitting. Information about this circumstance may be found in both the TI-83 and TI-86 Graphing Calculator Guidebook.

Viewscreen a.k.a. Overhead Model

All of these calculators are available as a Viewscreen model which allows for overhead displaying of work. A Viewscreen model consists of a special calculator and

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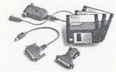
an LCD palette. The palette has a special cable which connects to a dedicated port on the top of the calculator (this viewscreen port is only on the viewscreen calculator). This detachable cable (older models were not detachable) allows the calculator to be separated from the palette and reconnected later for demonstration purposes.

TI-Graph Link



All of these calculators can connect with a computer through TI-Graph Link. The software part of this package is available at no charge on the net, or can be ordered through TI. The other part of TI-Graph Link is a special cable to connect the calculator to a computer.

The basic cable supplied with TI-Graph Link has an I/O port connector on one end and a parallel port connector on the other. The TI-Graph Link cable, an assortment of necessary adapters (to allow connection from the cable's parallel connector to an available com port on your computer), and the necessary software on floppy disks, is available as a package from TI (see graphic above). If you have the

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means to download files from the net, and have a halfway decent computer store in your neighbourhood, then the basic cable may be ordered alone.

Once installed, this software is easy to use. It has a menu and mouse driven graphical user interface that runs on Macintosh and Windows. It has a decent online help file, and a manual available in .pdf format (this requires Adobe Acrobat Reader software, version 3.0 or higher, available free at: *http://www.adobe.com/acrobat/*) from the TI website (address supplied momentarily). The manual is not overly detailed but the online help files are reasonably good.

The power of this software is that the world is opened up to the calculator user. Some examples of the flexibility that TI-Graph Link provides are as follows:

- Programs/data available on the web can now be downloaded to your computer/disk and transmitted to your calculator (and from there shared through the I/O port cable to others)
- Programs/data can be made available on the web for use by others (for example: if students have the software, they can access programs/data as above from a distance)
- Programs/data can be created on the computer with all available commands as if you were creating the program using the calculator

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(though programs cannot run on the computer) - there is even a simulated keypad to help in locating menus and functions

- Programs/data can be opened for editing, downloading, or printing
- Data can be copied and pasted into another computer program (for example: if you are using a data analysis program other than the tools of the TI-8*, but you like the portability of the calculator and CBL for data gathering purposes, then this is a valuable feature for you)
- CBL programs can be customized and/or created using the CBL menu (explore the help feature and CBL menu items to create a fictional program and learn a lot about what the lines in a CBL program actually do)
- Programs/data can easily be emailed
- Screenshots can be grabbed (and modified through other software) for placing in tests, worksheets, on a bulletin board, or on the web
- Object you wish to protect from erasure onboard your calculator (such as GDBs or programs which run lessons or perform frequently used tasks) can be backed up or stored to your computer/disk

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- The entire calculator can be backed up to the computer and restored through TI-Graph Link if needed
- If lack of memory becomes a problem on your calculator (for example, your CBL programs are now getting numerous!) you can free memory by moving files to your computer/disk - note:...CBL is described shortly in an upcoming section of this appendix

For more information about TI-Graph Link, including downloading it and a Condensed Guidebook in .pdf format, see: *http://www.ti.com/calc/docs/link.htm.*

New Model Technology

Texas Instruments have recently unveiled an advance in graphing calculator systems called "Flash Technology" which is utilized in the newest models: the TI-73; TI-89; and, TI-92 Plus Module. They allow the upgrading of your hardware platform by upgrading software (or by obtaining new software as it becomes available for your calculator), and customizing of your calculator to specific curriculum. You can read more about:

- Flash Technology at: *http://www.ti.com/calc/docs/flash.htm*
- TI-73 at: *http://www.ti.com/calc/docs/73.htm*

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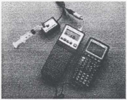
- TI-89 at: *http://www.ti.com/calc/docs/89.htm*
- TI-92 Plus Module at: *http://www.ti.com/calc/docs/92plus.htm*

Accessories

Calculator Based Lab (CBL)



The CBL is a data-gathering tool which uses probes to gather such experimental physical data as voltage, temperature, pH, dissolved oxygen, and much, much more. The CBL unit connects to the I/O port on the calculator. The probes connect to ports on the CBL itself.

The type of probe connected and the sampling commands are all controlled by programs which, when run on the calculator, send and get information from the CBL. So, the programs are the drivers for the CBL and retrieve, store, and display the data (according to the program code) on the calculator.

The good news about CBL programs is that all the work is done for you, they are very easy to use, and they run well.

Notes/Comments/Ideas

The bad news is if you want to change units of measurement, sampling rates, viewing windows for graphs, etc. you will have to venture into the land of the programmer and change a parameter somewhere. These things can be done however and information about this may be found in CBL program experiment books, the online help for TI-Graph Link, or from some of the T^3 materials (see Texas Instruments (1996a) p. A-16).

Since altering a CBL driver means altering a program, the TI-Graph Link software becomes invaluable. As mentioned earlier, TI-Graph Link has a CBL menu and online help which detail what many needed commands actually do. TI-Graph Link also allows uploading, viewing, editing, downloading and printing of code which can be very useful for spotting the code you need to address. In fact, if you utilize programming at all in any aspect of your work with a TI graphing calculator, you need TI-Graph Link.

Many CBL programs are available online as are some experiments (though you may need to acquire other probes to do other experiments). Of course, to get these programs from the web (or from floppy disks) into a calculator requires TI-Graph Link (or a link cable and another calculator that already has them onboard!).

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The CBL comes with: a workbook of experiments; floppy disks which contain the needed calculator programs in Mac and Windows formats; voltage, temperature, and light probes; batteries; an AC adaptor; and, a carrying case.

TI's main CBL page is found at: *http://www.ti.com/calc/docs/cbl.htm*, and more of TI's CBL programs may be found at: *ftp.ti.com/pub/graph-ti/cbl/programs/*.

<u>Calculator Based Ranger (CBR)</u>

The CBR is a motion detector or motion probe that connects to the CBL allowing acceleration due to gravity experiments and more. For information about the CBR see:

http://www.ti.com/calc/docs/cbr.htm.

<u>CBL Resources</u>

Additional resources in the form of TI guidebooks are also available. Information about the "CBL System - Experiment Workbook", a primary source for experiments and programs, is available at: *http://www.ti.com/calc/docs/cblwb1.htm*. Science teachers may want to check out "Exploring Physics and Math with the CBL System" at: *http://www.ti.com/calc/docs/cblwb2.htm*. And, "Real-World Math with the CBL System" can be visited at: *http://www.ti.com/calc/docs/cblwb3.htm*.

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Vernier Software

Vernier, a name many physics, biology, and chemistry teachers will recognize, also has materials, probes, programs and experiments available for the CBL. Vernier is online at: *http://www.vernier.com* and their CBL page may be found at: http://www.vernier.com/cbl/progs.html.

Publications

Eightysomething

"Eightysomething" is a newsletter for TI-8* Graphing Calculator users. It is available through surface mail. The electronic version may be found at: *http://www.ti.com/calc/docs/80xthing.htm* where you can download issues (past and present) in .pdf format.

It's About T.I.M.E. (Technology In Math Education)

Is a newsletter directed at supporting K-8 teachers as they incorporate TI technology into curriculum. It is available through surface mail. The electronic version may be found at: *http://www.ti.com/calc/docs/time.htm.*

TI-Cares

Notes/Comments/Ideas

TI-Cares Educational Support Programs for TI-Products is available at: *http://www.ti.com/calc/docs/ticares.htm* and holds information for teachers using any TI technology in their classrooms.

Professional Development

There are many inservice efforts made through Texas Instruments and through boards and schools interested in supporting TI products.

T³ (Teachers Teaching with Technology)

These are workshops offered all over Canada and the United States each year to further teachers' abilities with TI products. Information about T³ can be found at: *http://www.ti.com/calc/docs/t3.htm,* and look especially for those offered in Canada...request them even!

Workshop Loan Program

There is a Workshop Loan Program where teachers can avail of a viewscreen model and a class set of calculators or a CBL kit at no cost for the purposes of conducting an inservice. Calculators are normally sent to arrive a few days prior to and must be returned immediately following the event. Ordering must be done a minimum of 3 to 4 weeks in advance to ensure availability.

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There is even a grant structure that provides opportunity to defray other costs associated with a TI inservice. Information about these opportunities can be found at: *http://www.ti.com/calc/docs/loan.htm*.

Product Purchasing

Texas Instruments' products are available at selected retailers in Canada and through some publishing organizations. Some of these merchants may be found at: *http://www.ti.com/calc/docs/dealers.htm* or at: *http://www.ti.com/calc/docs/buygraph.htm*.

Volume Purchase Program

Many teachers have become aware that saving proofs of purchase or POPs can be quite productive. For 45 POPs of TI-8* Graphing Calculators you can get a Viewscreen TI-8* model free for your school. More information about this program can be found at: *http://www.ti.com/calc/docs/vpp.htm*.

Miscellaneous Stuff

In addition to all the above, TI hosts an extensive website of educational resources, other product information and accessories, customer support and services, a calculator program archive, and much more. For example, there are online discussion groups on just about very matter associated with TI graphing calculators. Here you will find questions posed and answered, or perhaps more questions posed.

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Novel applications of the technology are there, and games creep in there too! Even if you've lost a guidebook, or want a look at what the other models can really do, TI hasn't forsaken you. You can download most guidebooks at: *http://www.ti.com/calc/docs/guides.htm.*

Is there anything left? Oh yes... information about the many ways to contact TI for support, questions, or ordering can be found at: *http://www.ti.com/calc/docs/communicate.htm*, or by calling 1-800-TI-CARES and...no I don't work for TI.

Notes/Comments/Ideas