PERFORMANCE ANALYSIS OF COOPERATIVE-DIVERSITY NETWORKS WITH DIFFERENT RELAYING AND COMBINING TECHNIQUES

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Performance Analysis of Cooperative-Diversity Networks With Different Relaying and Combining Techniques

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Abstract

Cooperative-diversity networks technology is a promising solution to the high data-rate coverage required in future wireless communications systems. There are two main advantages of this technology: the low transmit RF power requirements and the spatial diversity gain. Different cooperation diversity protocols have been proposed for wireless networks. The basic idea is that in addition to the direct transmission from the transmitter to the receiver, there are other nodes, which can be used to enhance the diversity by relaying the source signal to the destination. This thesis investigates the performance of cooperative diversity networks using various relaying and cooperative techniques.

This thesis addresses five cooperative diversity cases. In the first case, we analyze the error and outage performance of cooperative diversity networks using amplify-and-forward and adaptive decode-and-forward relaying over independent non-identical flat Nakagami-*m* fading channels. We derive closed-form expressions for the error and outage probabilities and analyze their dependence on the channel parameters. In adaptive decode-and-forward relaying, among M relays that can participate, only C relays ($C \leq M$), with good channels to the source, decode and forward (retransmit) the source information to the destination. In amplify-and-forward all the M relays can participate in resending the source signal to the destinations. In both techniques, the destination combines the direct and the indirect signals using the maximum ratio combining (MRC) technique. Results reveal that fixed protocols for amplify and forward increase the performance unlike decode-and-forward which need an adaptive protocol to improve the performance.

In the first case we assume that the destination has perfect knowledge of the channel state information (CSI) of all links. While in some scenarios the CSI can be acquired using the pilot symbols or training sequences, it may not be possible in some systems, particularly with fast fading channels. In the second case we relax this condition by assuming that no CSI is needed at the relays and the destination using differential equal gain combining (EGC) in cooperative diversity networks for both techniques amplify-and-forward and

decode-and-forward. In this case, we derive exact closed-form expressions for the average bit error rate and outage probability. Furthermore, we find the SNR moments, the average signal-to-noise ratio (SNR) and the amount of fading. Numerical results show that the differential EGC can benefit from the path-loss reduction and outperform the traditional multiple-input single output (MISO). Also, numerical results show that the performance of the differential EGC is comparable to the maximum ratio combining (MRC) performance.

In the previous two cases, all the relay nodes relay the source signal using orthogonal channels (time slots, carriers or codes) to avoid cochannel interference. Hence, for a regular cooperative diversity network with M relays, we need $M + 1$ channels (one for the direct link and M for the M indirect links). This means that the number of required channels increases linearly with the number of relays. In the third case, we investigate the performance of the best-relay selection scheme where the "best" relay only participates in the relaying. Therefore, only two channels are needed in this case (one for the direct link and the other one for the best indirect link) regardless of the number of relays (M). We show that the best-relay selection not only reduces the amount of required resources but also can maintain a full diversity order (which is achievable by the regular multiple-relay cooperative diversity system but with much more amount of resources).

Another method to save the channel resources, incremental relaying technique, is examined in the fourth case. In incremental relaying technique we restrict the relaying process to the bad channel conditions only. Incremental relaying cooperative relaying networks exploit limited feedback from the destination terminal, e.g., a single bit indicating the success or failure of the direct transmission. If the destination provides a negative acknowledgment via feedback, the relay retransmits in an attempt to exploit spatial diversity by combining the signals that the destination receives from the source and the relay. Closed-form expressions for the bit error rate and the outage probability are determined. Results show that the incremental relaying cooperative diversity can achieve the maximum possible diversity, compared to the regular cooperative diversity networks, with higher channel utilization (channel capacity).

In the last case, we propose and analyze a novel relaying technique that combines incremental relaying with the best relay selection. We will derive exact closed-form expressions for the error probability and outage probability for this algorithm. The main advantage of this algorithm is that we use the channel resources in a controlled manner to always enhance the spectral efficiency of the cooperative networks.

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Finally, I would like to dedicate this work to my parents and my wife for their love and support during this journey.

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List of Symbols

γ	Instantaneous Signal-To-Noise Ratio Per Symbol
$\mathbf{E}(\bullet)$	Statistical Average Operator
E_s	Signal Transmitted Energy Per Symbol
h	General Fading Amplitude
$\bar{\gamma}$	Average Signal-To-Noise Ratio Per Symbol
$f_X(x)$	Probability Density Function Of The Random Variable X
m	Message Signal
$\max(x, y)$	The Maximum of x and y
S	Source Node
D	Destination Node
R_i	ith Relay Node
$h_{S,D}$	Fading Amplitude Between the Source and the Destination
h_{S,R_i}	Fading Amplitude Between the Source and the ith Relay
$h_{R_i,D}$	Fading Amplitude Between the ith Relay and the Destination
$x(t)$	Transmitted Source Signal
$y_{S,D}(t)$	The Received Signal From the Source at the Destination
$y_{S,R_i}(t)$	The Received Signal From the Source at the ith Relay
$y_{R_i,D}(t)$	The Received Signal From the Source at the ith Relay
$n_{S,D}(t)$	AWGN Term Between the Source and the Destination
$n_{S,R_i}(t)$	AWGN Term Between the Source and the ith Relay
$n_{R_i,D}(t)$	AWGN Term Between the ith Relay and the Destination
$x_r(t)$	The Regenerated Signal From the ith Relay
G_i	The i^{th} Relaying Gain

N_0	Power Spectral Density of the AWGN Channel
$d_{i,j}$	The Distance Between Terminal i and j.
α	The Power Exponent Path
$\gamma_{S,D}$	The Instantaneous SNR Between the Source and the Destination
γ_{S,R_i}	The Instantaneous SNR Between the Source and the ith Relay
$\gamma_{R_i,D}$	The Instantaneous SNR Between the ith Relay and the Destination
$M_X(x)$	The Moment Generating Function of a random variable X
$m_{S,D}$	Nakagami-*m* Fading Parameter of $h_{S,D}$
m_{S,R_i}	Nakagami-*m* Fading Parameter of h_{S,R_i}
$m_{R_i,D}$	Nakagami-*m* Fading Parameter of $h_{R_i,D}$
$\Gamma(\bullet,\bullet)$	The Incomplete Gamma Function
$\Gamma(\bullet)$	The Gamma Function
${}_2F_1(\bullet,\bullet;\bullet;\bullet)$	The Gauss' Hypergeometric Function
$P(e)$	The Average Error Probability
$Q(\bullet)$	Q-Function
$\delta(\bullet)$	Delta Function
R	Spectral Efficiency Bits/Sec/Hz
M	Total Number Of Relays
P_{out}	Outage probability
M	Total Number Of Relays
$\hat{m}$	Estimated Message Signal
$\text{erfc}(\bullet)$	erfc-Function
$\min(x,y)$	The Minimum Of x and y

List of Abbreviations

ARQ	Automatic Repeat Request
AWGN	Additive White Gaussian Noise
BDPSK	Binary Differential Phase Shift Keying
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CDF	Cumulative Density Function
CDM	Code Division Multiplexing
CDMA	Code Division Multiple Access
CRC	Cyclic Redundancy Check
CSI	Channel State Information
dB	Decibel
EGC	Equal Gain Combining
FDM	Frequency Division Multiplexing
GHz	Giga Hertz
LAN	Local Area Network
LOS	Line of Sight
MDPSK	M-ary Differential Phase Shift Keying
MPSK	M-ary Phase Shift Keying
MQAM	M-ary Quadrature Shift Keying
MGF	Moment Generating Function
MIMO	Multiple-Input-Multiple-Output
MISO	Multiple-Input-Single-Output
MRC	Maximum Ratio Combining

PDF	Probability Density Function
QPSK	Quadrature Phase Shift Keying
SER	Symbol Error Rate
SNR	Signal-to-Noise Ratio
TDM	Time division Multiplexing

Chapter 1

Introduction

The high demand on the growing number of wireless applications has made significant development of wireless communication networks. This is especially true in the case of cellular voice and data networks, and more recently in ad-hoc data networks for wireless computer, home, and personal networking. Radio hardware and wireless services are getting more efficient and cost effective as system designers better understand the channel environment and multi-user communications in general. Furthermore technological advances in integrated circuits and radio-frequency electronics increasingly allow for more sophisticated signal processing and channel coding algorithms. However, compared to classical wireless networks, it seems we are only in the early phases to understand the fundamental performance limits of the multi-hop wireless networks and practical ways for approaching these limits. Moreover, given their impact on society as well as other technologies, wireless communications and networking remain important areas of research.

In wireless networks, signal fading arising from multi-path propagation is a particularly severe form of distortion that can be mitigated through the use of diversity. This diversity can be achieved by the transmission of redundant signals over essentially independent channel realizations in conjunction with suitable receiver combining to reduce the channel distortions. Space or multiple-antenna diversity techniques are particularly attractive as they can be readily combined with other forms of diversity, e.g., time and frequency diversity, and still offer dramatic performance gains when other forms of diversity are unavailable.

While systems that use the spatial diversity offered by antenna arrays are attractive due to their appealing performance, their successful use requires the integration of multiple antenna elements at terminals, and, to take full benefit, uncorrelated channels from each of them. In scenarios where these conditions cannot be met (for example due to physical limitation of the mobile terminal), cooperative diversity networks provide an alternative by "distributing" the antennas among terminals.

Cooperative diversity networks technology, also known as user cooperation diversity, virtual antenna arrays, coded cooperation, and distributed coding, is a promising solution for future cellular and ad hoc wireless communications systems in order to achieve broader coverage and to mitigate wireless channel impairments without the need to use high power at the transmitter. In cooperative diversity networks, several terminals form a kind of coalition to assist each other with the transmission of their messages. In general, cooperative diversity networks have a source node broadcast a message to a number of cooperative relays, which in turn resend a processed version to the intended destination node. The destination node combines the signal received from the relays, possibly also taking into account the source's direct transmitted signal. Cooperative diversity exploits two fundamentals features of wireless medium: its broadcast nature and its ability to achieve diversity through independent channels.

1.1 General Background

Radio wave propagation through wireless channels is a complicated phenomenon characterized by various effects, such as multi-path and shadowing. A precise mathematical description of this phenomenon is either unknown or too complex for practical communication systems analyses. However, considerable efforts have been devoted to the statistical modeling of these different effects. The result is a range of relatively simple and accurate statistical models for fading channels which depends on the propagation environments and the underlying communication scenario.

When a signal experiences fading during transmissions, both its envelop and phase fluctuate over time. For coherent modulations, the fading effects on the phase can severely degrade performance unless measures are taken to compensate for them at the receiver. Most often, analyses of systems employing coherent modulations assume the phase effects due to fading are perfectly corrected at the receiver. For non-coherent modulations, phase information is not needed at the receiver and therefore the phase variation due to fading does not affect the performance. Hence, performance analyses for both coherent and non-coherent modulations over fading channels requires only knowledge of the fading envelop statistics and that case will be considered in this thesis.

Furthermore, for slow fading channel, wherein the fading is at least constant over the duration of the symbol time, the fading envelop random process can be represented by a random variable over the symbol time.

When fading affects wireless communication systems, the received carrier amplitude is scaled by the fading amplitude h, where h is a random variable with probability density function (PDF) $f_h(h)$, which is dependent on the nature of the radio propagation environment. After passing through the fading channel, the signal is corrupted at the receiver by additive white Gaussian noise (AWGN), which is typically assumed to be statistically independent of the fading amplitude h and characterized by power spectral density N_0 (W/Hz). Equivalently, the received instantaneous power is scaled by h^2. Thus we define the instantaneous signal-to-noise ratio (SNR) per symbol by $\gamma = h^2 E_s/N_0$ and the average SNR per symbol as $\bar{\gamma} = h^2 E_s/N_0$, where $\mathbf{E}(\bullet)$ is the statistical average operator and E_s is the signal transmitted energy per symbol. Moreover, the PDF of γ is obtained by introducing a change of variable in the expression for the fading PDF $f_h(h)$ of h, yielding [23]

$$f_h(\gamma) = \frac{f_h\left(\sqrt{\mathbf{E}\left(h^2\right)\gamma/\bar{\gamma}}\right)}{2\sqrt{\gamma\bar{\gamma}/\mathbf{E}\left(h^2\right)}}. \tag{1.1}$$

Depending on the nature of the radio propagation environment, there are different models describing the statistical behavior of the multi path fading envelop.

1. **Rayleigh Model**. The Rayleigh distribution is frequently used to model the multi-path fading with no direct line-of-sight (LOS) path. In this case the channel fading amplitude h is distributed as

$$f_h(h) = \frac{2h}{\mathbf{E}(h^2)} \exp\left(-\frac{h^2}{\mathbf{E}(h^2)}\right) \tag{1.2}$$

and hence, following (1.1), the instantaneous SNR of the channel, γ is distributed according to the exponential distribution given by

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \tag{1.3}$$

2. **Nakagami-m Model**. The Nakagami-m distribution is more general than Rayleigh distribution and often gives the best fit to land mobile and indoor mobile multi-path propagation, as well as scintillating ionospheric radio links. The Nakagami-m fading model was initially proposed because it matched empirical results for short ionospheric propagation. The Nakagami distribution or the Nakagami-m distribution is a probability distribution related to the gamma distribution. This more general fading distribution was developed such that its parameters can be adjusted to fit a variety of empirical measurements. The Nakagami-m PDF is in essence a central chi-square distribution given by

$$f_h(h) = \frac{2m^m}{(\mathbf{E}(h^2))^m \Gamma(m)} h^{2m-1} \exp\left(-\frac{m}{\mathbf{E}(h^2)} h^2\right) \tag{1.4}$$

where m is the Nakagami-m fading parameter which ranges from $1/2$ to ∞ and $\Gamma(\bullet)$

is the Gamma function [23]. The PDF of the instantaneous SNR (γ) can be written as

$$f_\gamma(\gamma) = \frac{m^m}{\bar{\gamma}^m \Gamma(m)} \gamma^{m-1} \exp\left(-\frac{m}{\bar{\gamma}}\gamma\right) \tag{1.5}$$

The performance of the wireless communication channel is severely affected by the fading channel unlike the wired channels which are affected only by AWGN. Figure 1.1 shows the error performance for AWGN channel and wireless channel for binary phase shift keying (BPSK). Figure 1.1 shows that with a reasonabl SNR, an AWGN channel provides fairly good performance. The Rayleigh channel (wireless channel) provides very poor performance, and hence an error control code becomes more desirable or we need a very strong power to compensate this high error rate.

The efforts to compensate for the errors and distortion introduced by multi-path fading fall into three categories: forward error correction, adaptive equalization, and diversity technique. In practical mobile communication systems, techniques from all three categories are combined to combat the error rate encountered and we will focus in our thesis on the diversity technique.

Diversity is a powerful communication technique that provides significant improvement for the wireless link quality with little added cost. Diversity exploits the random nature of radio propagation by exploiting the independence of the fading parameters within a communication system. A simple example can explain the diversity concept: If one radio path undergoes a deep fade, another independent path may have a strong signal, so the transmitted signal can still be correctly received. The popular diversity methods are listed as follows:

- **Frequency diversity**

  Frequency diversity is implemented by transmitting information on more than one carrier frequency. The rationale is that frequency separated by more than the coherence bandwidth of the channel will be uncorrelated and will not experience the same fades.

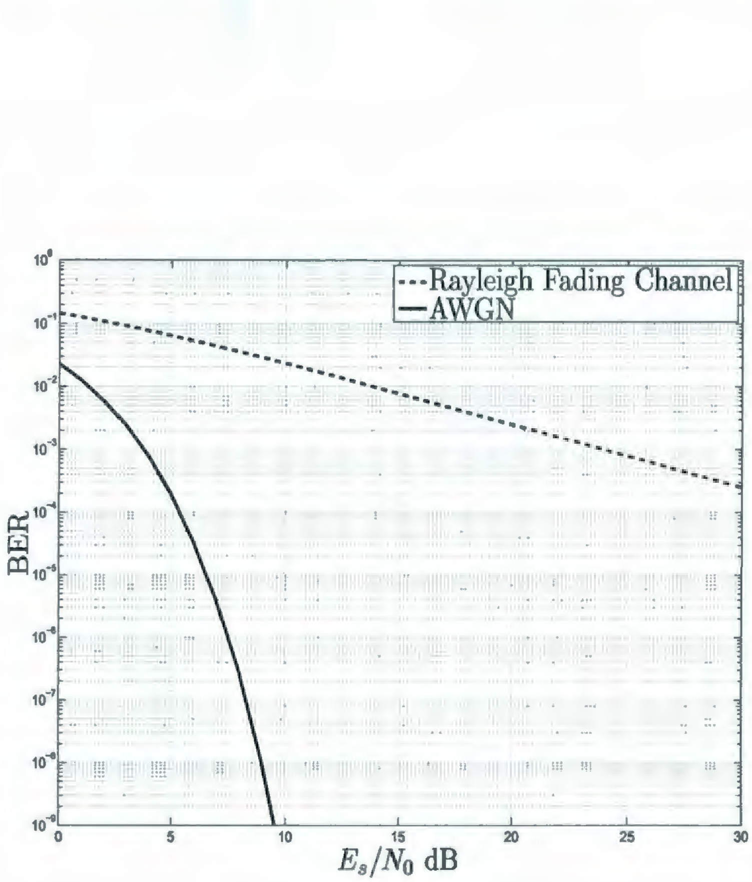

Figure 1.1: Theoretical BER for BPSK under AWGN and Rayleigh fading communication channels.

- **Time diversity**

  Time diversity repeatedly transmits information at time spacing that exceed the coherence time of the radio channel, such that multiple repetitions of the signal will be received with independent fading conditions, thereby providing diversity.

- **Space diversity**

  Space diversity is a very popular diversity technique, due to the fact that the signals received from spatially separated antennas would have essentially uncorrelated envelops for antenna separations of one half wavelength or more.

- **Space-time (space-frequency) diversity**

  Multiple-input-multiple-output (MIMO) systems employing multiple transmit and receive antennas will play a significant role in the development of future broadband wireless communications. By taking diversity of the larger number of propagation paths between the transmitter and receiver antennas, the detrimental effects of channel fading can be significantly reduced. It has been shown that MIMO systems offer a large potential capacity increase compared to single antenna systems. To exploit this diversity, a considerable number of MIMO modulation and coding methods, known as space-time codes (STC), have been proposed.

- **Cooperative diversity**

  Relaying, i.e., the use of intermediate nodes to help transmission from a source to a destination, has been used to enhance the coverage area and to relax the link budget. The idea is to split the distance between a source and a destination node into several hops; the nonlinear relation between propagation loss and distance helps in reducing the end-to-end attenuation and thus in relaxing link budgets. While such conventional relaying has long been known for ad hoc networks, it was only until recently that these concepts have received interest for cellular networks.

  Cooperative diversity goes one step further. By combining the transmissions from various nodes, one can explicitly exploit two benefits that are inherently offered by relaying systems. First, one can make use of the broadcast nature of the wireless medium: a signal transmitted by a node propagates not only to the intended final destination, but it can be received at multiple nodes. Second, viewing the individual nodes of relaying systems as distributed antennas leads to regarding cooperative diversity networks as a generalization of multiple-antenna systems. The related advantages, namely spatial diversity, spatial multiplexing, and power saving, are well-known. In this sense, cooperative diversity brings together the worlds of conventional relaying and MIMO systems.

1.2 Elements of Cooperative Diversity Networks

A cooperative diversity network exploits the broadcast nature of the wireless medium and allows mobile terminals to jointly transmit information through relaying. As illustrated in Figure 1.2 an information source (S) and a destination (D) communicate over a channel with a fading coefficient $h_{S,D}$. A relay terminal (R) participates in this link providing the destination with a second copy of the original signal through fading coefficients $h_{S,R}$ and $h_{R,D}$. Communication takes place in two phases as dictated by the inability of the relay to transmit and receive simultaneously at the same frequency. In the first phase, the source sends its signal. Both the relay and the destination receive faded noisy versions of this signal. The relay suitably processes those received signals, and transmits signals of its own so as to increase the capacity and/or improve reliability of end-to-end transmissions between the source and destination terminals.

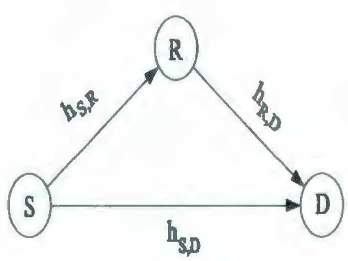

Figure 1.2: Illustration of a cooperative diversity network.

The cooperative diversity network in Figure 1.2 can be generalized easily to more than one relay. Cooperative diversity networks leverage the spatial diversity available when multiple transmissions experience fading and/or shadowing that is essentially independent. For example, if the source signal experiences a deep fade at the destination, there remains a significant chance that it can effectively communicate with the destination via one or more of the available relays. The main fundamental features of the cooperative diversity networks can be summarized as follows:

- **Spatial diversity:** The main advantage of cooperative diversity networks is the spatial diversity that is achieved by antennas that are distributed among different terminals rather than being placed closely to each other at a single station. Essentially, one can assume that the different paths between the network elements are mutually uncorrelated, so that the strong information-theoretic gains [1,2] are not reduced by correlated propagation.

- **Path-loss reduction:** The benefit of the reduced path-loss results from the path-loss non linearity. By dividing a long distance link into shorter distances, the total path-loss can be reduced, thus decreasing transmission power and reducing interferences. As an example, placing a relay between source and destination (as in Figure 1.3) in a normalized propagation scenario with a path-loss exponent α requires a transmission power of $P_{relay} = r^{\alpha} + (1-r)^{\alpha}$ in the two-hop relay case to achieve unit received power at the relay and the destination. In contrast, direct transmission needs a transmission power of P_{direct}. It can be seen that, the power savings amount to 9 dB for $r = 1/2$ and $\alpha = 4$.

- **Security:** Cooperative diversity networks route user information via another user's terminal, thus inherently calling for appropriate security measures to be considered. While security and billing issues are beyond the scope of our thesis, it should be noted here that there are many unsolved problems in this area.

- **Complexity:** The terminal and the protocol complexity increases in networks that allow relaying components.

1.3 Classification of Protocols

Relaying can be performed in different ways depending on the processing that the relay uses to handle the received signal from the source. Here we summarize some of commonly used relaying techniques:

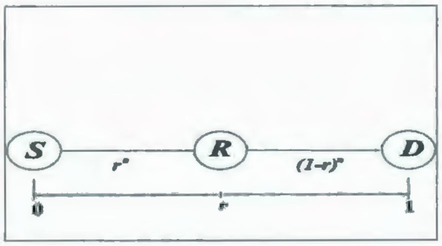

Figure 1.3: Path-loss reduction enabled by a relay placed between source and destination.

- **Amplify-and-Forward:** For amplify-and-forward, relays simply amplify what they receive subject to their power constraint. The retransmitted signal includes the amplified signal as well as the amplified noise. Although amplify-and-forward protocols are simple in nature, they may be difficult to implement practically because it requires either the storage of large amounts of analog data (time division multiplexing) or complicated and expensive transceiver structures (frequency division multiplexing) [3]. Amplify-and-forward schemes are also known as "analog relaying" or "non-regenerative relaying".

- **Decode-and-Forward:** For decode-and-forward, (also referred to as "digital relaying" or "regenerative relaying") relays detect (and probably decode) the received signals and re-encode the information into their transmit signals. This detection, decoding and re-sending process often corresponds to a non-linear transformation of the received signals. Although decoding at the relays has the advantages of reducing the impact of receiver noise they introduce a danger of error propagation that may occur if the relay incorrectly detects a message and retransmits erroneous information to the destination.

- **Fixed versus Adaptive Relaying:** As we might expect, fixed decode-and-forward is limited by direct transmission between the source and relay. However, if the fading coefficients are known to the appropriate receivers, $h_{S,R}$ (the fading coefficient for the channel between the source and the relay, see Figure 1.2) can be measured to high accuracy by the relaying terminals; thus, they can adapt their transmission format according to the realized value of $h_{S,R}$. This observation suggests the following class of adaptive relaying algorithms. If $h^2_{S,R}$ falls below a certain threshold, the relay refrains from resending another copy of the source message and in that case the source can simply continue its transmission to the destination, in the form of repetition or more powerful codes (increasing complexity at the source terminal). If $h^2_{S,R}$ lies above the threshold, the relay forwards what it received from the source, using decode-and-forward technique, in an attempt to achieve the diversity gain. Another technique to achieve adaptive relaying uses simple error detection codes at the relays to ensure that the relay will resend a fresh error-free data to the destination. Since most of the wireless networks are using such codes, hence there is no complexity to be added to the whole system.

- **Best Relay Selection Scheme:** In the best-relay selection scheme the "best" relay only participates in the relaying. Therefore, two channels only are needed in this case (one for the direct link and the other one for the best indirect link) regardless of the number of relays. The best-relay selection not only reduces the amount of required resources but also can maintain a full diversity order.

- **Incremental Relaying:** Fixed and adaptive relaying can make inefficient use of the spectral efficiency of the channel, practically for high data rates, because the terminal relays repeat all the time [4]. Incremental relaying protocols exploit some sort of feedback from the destination terminal, e.g., a single bit indicating the success or failure of the direct transmission, and this can be considered as extensions of incremental redundancy, or hybrid automatic-repeat-request (*HARQ*), to the relay context. In *HARQ*, the source retransmits if the destination provides a negative acknowledgment via feedback. In incremental relaying, the relay retransmits, instead of the source, in an attempt to exploit spatial diversity.

  As an example, consider the following scenario using feedback and amplify-and-forward transmission. Firstly, the source transmits its information to the destination. The destination indicates success or failure by broadcasting a single bit of feedback to the source and relay, and we assume that it will be detected free of error by at least the relay. If the source-destination signal-to-noise ratio (SNR) is sufficiently high the feedback indicates success of the direct transmission, and the relay does nothing. If the source-destination SNR is not sufficiently high for successful direct transmission the feedback requests that the relay amplify-and-forward what it received from the source. In the latter case, the destination may try to combine the two transmissions. Protocols of this form make more efficient use of the spectral efficiency of the channel, because they repeat rarely and only when necessary. The main drawback of this protocol is that it needs a feedback message thereby increasing the complexity and the overhead of the protocol [4].

1.4 Related Work and Problem Motivations

This section provides a survey of the existing literature about the cooperative diversity networks. Results of cooperative diversity can be classified into two categories, namely information-theoretic and communication-theoretic. The information-theoretic results are concerned with the capacity of the cooperative diversity networks, while the communication-theoretic results are more concerned with the reliability of cooperative diversity networks with some particular modulation or coding schemes.

The first interest in the relay channel was the work of van der Meulen [3,5]. Cover *et al.*[6] determined the capacity for the physically degraded relay channel, where coding is performed in an incremental manner [6,7]. While these publications focused on the three-terminal case, a more general approach was taken by Gastpar *et al.* [8,9], who establish performance bounds by examining the situation in which a single source-destination pair is assisted by a network of relay terminals.

Explicit cooperation for the mutual benefit of neighboring nodes was considered by

Sendonaris *et al.* [10–12]. They showed that cooperative diversity increases the channel capacity over the non-cooperative transmission for ergodic fading. Also in [10–12], the authors showed that cooperative diversity improves the outage performance for non-ergodic fading and decreases the sensitivity of the achievable data rate to the variations of the channels. The main drawback is that this work by assuming the channel state information (CSI) available at the transmitter requires considerable modifications to the existing hardware and software of the transmitter and receiver terminals.

Laneman, Tse and Wornell [4] assumed no CSI available at the source and proposed the analysis of cooperative diversity protocols under the framework of diversity-multiplexing tradeoffs. Their basic setup included a source, a destination and a relay. Both analog and digital relaying were considered. Subsequently, the diversity-multiplexing tradeoff of cooperative diversity protocols with multiple relays was studied in [13, 14]. While ref. [13] considered the case of orthogonal transmission between the source and relays, ref. [14] considered the case where the source and relays could transmit simultaneously. It was shown in [14] that by relaxing the orthogonality constraint, a considerable improvement in performance could be achieved, albeit at a higher complexity at the decoder. These approaches were however information theoretic in nature and the design of practical codes that approach these limits was left for further investigation. Such a code design is difficult in practice and an open area of research. While space-time codes for the Multiple Input Multiple Output (MIMO) link do exist [15] (where the antennas belong to the same terminal), more work is needed to use such algorithms in the cooperative-diversity networks, where antennas belong to different terminals distributed in space. The relay channel is fundamentally different from the traditional MIMO since information is not a priori known to the cooperating relays, but rather needs to be communicated over noisy links. Moreover, the number of participating antennas is not fixed since it depends on how many relay terminals participate and how many of them are indeed useful in relaying the information transmitted from the source. For example, for relays that decode and forward, it is necessary to decode successfully before retransmitting. For relays that amplify and forward, it is important to have a good received SNR, otherwise they would forward mostly their own noise. Therefore, the number of participating antennas in cooperative diversity schemes is in general random, and space-time coding invented for fixed number of antennas should be appropriately modified. In short, providing practical space-time codes for the cooperative diversity channel is fundamentally different than space-time coding for the MIMO and is still an open and challenging area of research [40].

Most of the cooperative diversity publications are related to the information-theoretic fundamentals, either in the sense of Shannon, ergodic, or outage capacity. Boyer *et al.* [16] were to our knowledge the first who studied the error probability in Rayleigh fading channel, when they introduced the concept of multi-hop diversity, where each relay combines the signals received from all of the previous transmissions. This kind of spatial diversity is especially applicable in multi-hop ad-hoc networks. The authors in [16] assumed that an error at any intermediate relay results in an error at the final destination, and through this assumption they derived upper bounds on the probability of error performance of the system.

In [17], the authors derived the average symbol error probability for amplify-and-forward cooperative diversity networks in a high SNR regime. The resulting expressions are general as they hold for an arbitrary number of branches, arbitrary number of hops per branch, and various channel fading models provided that the probability density functions (PDF) of the fading coefficients is nonzero at zero instantaneous SNR (which is true for the widely used Rayleigh and Ricean models but it is not true for Nakagami-m distribution). The simplicity of the proposed work provides valuable insights to the performance of cooperative relaying networks and suggests means of optimizing them.

What is missing in the literature is to examine the performance analysis of cooperative diversity networks in more general fading channel like Nakagami-m fading channel. The reason of that shortage is the difficulty to determine the statistics of the SNR at the destination. Also, to our knowledge there is no attempt in the literature to study more efficient cooperative diversity techniques in communication theoretic perspective even for simple Rayleigh fading channels. In this thesis, as the main contribution, we are aim to fill this gap by analyzing performance metrics for cooperative diversity networks such as error rate and outage probability. Also, this thesis analyzes the performance of different cooperative techniques at the relays. These techniques can be classified to into simple techniques like amplify-and-forward and decode-and-forward and more complex techniques like incremental and selection cooperative techniques.

1.5 Problem Statement and Thesis Organization

This thesis tries to examine and introduce a complete analytical analysis for different cooperating techniques that can be used to enhance both the error performance and spectral efficiency of the cooperative diversity network. How to increase the diversity gains without sacrificing the spectral efficiency is the main challenge that faces the cooperative diversity networks. Focusing on this issue, the organization of this thesis is as follows:

1. In Chapter 2, the performance of cooperative diversity networks under more general fading channels like Nakagami-m fading for both techniques amplify-and-forward and decode-and-forward has been investigated. This study can reveal the performance of cooperative diversity under various severity levels of fading channels. In this analysis we assume that state information is available at the relays and the destination.

2. The performance of cooperative diversity networks for both techniques amplify-and-forward and amplify-and-forward under Nakagami-m fading channels but the channel state information is not available at both the relays and the destination will be investigated in Chapter 3. We relax this condition by using differential equal gain combining (EGC) for both schemes amplify-and-forward and decode-and-forward at the destination. This relaxation can significantly reduce the complexity of the receivers.

3. The performance of best-relay selection cooperative-diversity networks in which the best relay only is willing to resend another copy of the source signal will be considered in Chapter 4. The main advantage of this method is to reduce the amount of used channel resources for the same diversity order. This study will be under Rayleigh fading channel for both techniques amplify-and-forward and decode-and-forward.

4. The performance of more efficient cooperative-diversity networks, in such network, the relays will resend only when it is necessary will be considered. This need of resending will be determined at the destination. In this way, we restrict the relaying process to necessary conditions. This technique is known as incremental relaying [13]. This investigation can be found in Chapter 5.

5. A novel algorithm that achieves the spatial diversity gain without sacrificing the spectral efficiency will be introduced in Chapter 6. In this algorithm we are going to combine the best-relay selection and incremental relaying in one algorithm. In this case the utmost benefits can be gained in terms of spectral efficiency and spatial diversity.

1.6 Main Contributions

The main contributions of this thesis are as follows:

1. We introduce a tight upper bound expression on the equivalent SNR at the destination for amplify-and-forward relaying scheme over independent non-identical Nakagami-m fading channels.

2. For the amplify-and-forward regular cooperative diversity scheme we derive novel closed form expressions for the PDF, cumulative distribution function (CDF) and moment generating function (MGF) of the tight approximate model of the SNR at the destination over independent non-identical Nakagami-m fading channels. An upper

bound closed-form expression of the error probability is derived. Outage probability is introduced using the MGF of the equivalent SNR at the destination.

3. Exact closed-form expressions for the error probability and outage probability for the regular adaptive decode-and-forward relaying scheme on Nakagami-m fading channel are introduced.

4. For the differential equal gain combining (EGC) cooperative diversity scheme, we derive for both techniques, amplify-and-forward and decode-and-forward, the equivalent PDF and MGF of the SNR at the destination over independent non-identical Nakagami-m fading channels. The derived MGF is used to find the error probability and the outage probability. Furthermore, an upper bound closed form expression of the error probability is derived for both relaying schemes.

5. A complete performance analysis for the best-relay scheme using both amplify-and-forward and adaptive decode-and-forward is introduced over independent non-identical Rayleigh fading channels. In particular, closed form expressions for the error probability, outage probability, and Shannon capacity are derived. Furthermore, closed form expressions of the PDF, CDF, and MGF of the equivalent SNR at the destination for both relaying schemes are derived. Also, the moments, amount of fading and other statistics of the equivalent SNR at the destination are introduced in closed form expressions. Moreover, asymptotic error performance behavior is derived for both relaying schemes.

6. Error probability and outage probability are derived in closed form expressions for the incremental relaying technique for both relaying schemes over independent non-identical Rayleigh fading channels.

7. Finally, a novel algorithm for the cooperative diversity network is introduced. The error probability and outage probability over independent non-identical Rayleigh fading channels have been derived in closed form expressions.

1.7 Published and Submitted Work

1. Salama Ikki and Mohamed H. Ahmed , "Performance analysis of wireless cooperative networks over Nakagami-*m* fading channels," *IEEE Communaction Letter*, vol. 11, no. 4, pp. 334 - 336, April, 2007.

2. Salama Ikki and Mohamed H. Ahmed , "Performance of Multiple-Relay Cooperative Diversity Systems with Best-Relay Selection over Rayleigh Fading Channels," *EURASIP Journal on Signal Processing,* Special Issue on Cooperative Wireless Communications, May, 2008.

3. Salama Ikki and Mohamed H. Ahmed , "Performance of Cooperative Diversity Using Equal Gain Combining (EGC) over Nakagami-*m* Fading Channels," *IEEE Trans. on Wrls. Commun.*, To appear, 2008.

4. Salama Ikki and Mohamed H. Ahmed , "Closed-From Expressions of Error and Outage Probabilities for multi-hop path-selection Amplify-and-Forward Wireless Relaying Technique," Submitted to *IEEE Trans. on Commun.*.

5. Salama Ikki and Mohamed H. Ahmed , "Performance Analysis of the Adaptive Decode-and-Forward Cooperative Diversity Networks with the best relay selection Scheme," Submitted to *IEEE Trans. on Commun.*.

6. Salama Ikki and Mohamed H. Ahmed , "Performance Analysis of Cooperative Diversity Using Equal Gain Combining (EGC) Technique over Rayleigh Fading Channels," *IEEE International Conference on Communications,* ICC '07, p. 5336-5341, June 2007.

7. Salama Ikki and Mohamed H. Ahmed , "Performance Study of Multi-Hop Transmission Systems over Weibull Fading Channels," *First International Conference on New Technologies, Mobility and Security, (NTMS'07)*, Paris, France, May 2007.

8. Salama Ikki and Mohamed H. Ahmed , "Performance of Decode-and-Forward Cooperative Diversity Networks Over Nakagami-m Fading Channels," *IEEE Global*

Telecommunications Conference, GLOBECOM '07, p. 4328-4333, Nov. 2007.

9. Salama Ikki and Mohamed H. Ahmed , "Performance Analysis of Dual-Hop Relaying Communications Over Generalized Gamma Fading Channels," *IEEE Global Telecommunications Conference,* GLOBECOM '07, p. 3888-3893 , Nov. 2007.

10. Salama Ikki and Mohamed H. Ahmed , "Performance Analysis of Incremental Relaying Cooperative Diversity Networks over Rayleigh Fading Channels," *IEEE Wireless Communications and Networking Conference,* WCNC '08, p. 1311-1315, April, 2008.

Chapter 2

Performance Analysis of Regular Wireless Cooperative Diversity Networks over Nakagami-m Fading Channels

It is widely known that Nakagami-m distribution is a generalized distribution which can be used to model different fading environments. Nakagami-m distribution has the advantage of including the Rayleigh and the one-sided Gaussian distribution as special cases. Furthermore, it can be used to model fading conditions that are less or more severe than those modeled by the Rayleigh distribution. A large body of literature has been devoted to the study of cooperative diversity networks over Rayleigh fading channels. However, to our knowledge there exist few studies for cooperative diversity over Nakagami-m fading channels [18–20].

Hence, in this chapter we are going to study the performance analysis in terms of error probability and outage probability for both techniques, amplify-and-forward and adaptive decode-and-forward, respectively.

2.1 General Background

2.1.1 Amplify and Forward

In [17], the authors derived the asymptotic average symbol error probability (SER) for amplify-and-forward cooperative-diversity networks. The resulting expressions derived in [17] (using the bounding approach) are general for any type of fading distributions provided the PDF of zero instantaneous SNR is not zero, which is not available in the Nakagami-m fading channel environment.

In [18,19] multi-hop relaying transmission (not cooperative diversity network) over Nakagami-m fading channels has been studied. The error rate is determined in [18] using a single integration of the conditional error probability $P(e|SNR)$ times the PDF of the SNR. Hence, it is not straightforward to extend the work in [18] to the case of cooperative diversity with M relay nodes (as shown in Figure 2.1) because we have to use $M+1$ integrations in this case. Furthermore, their lower error bound is not tight enough at medium and high SNR (see Figures 3 and 4 in [18]). In [19], the MGF of the SNR is used to find the error rate. Hence, it is straightforward to extend this work to the case of cooperative diversity with M relay nodes. However, the derived MGF in [19] is limited to the case of identical Nakagami-m fading channels and dual hop only.

In [20] the authors studied the performance of amplify-and-forward cooperative-diversity networks over independent non-identical Nakagami-m fading channels and they found the MGF of the total SNR, but their error probability bound is not tight enough at medium and high SNR (see Figure 2 in [20]).

2.1.2 Decode and Forward

Performance analysis of adaptive decode-and-forward cooperative diversity networks yielded many interesting results including information theoretic metrics, outage probability, and average error probability expressions over Rayleigh-fading channels [16,21]. The authors in [21] have presented an overview of cooperative diversity networks and compared their performance with that of direct transmission and normal relaying networks. In [24–26], the authors have determined the outage probability of adaptive DF cooperative networks with identical and non-identical Rayleigh fading channels. In particular, the authors in [24] have determined an upper bound for the outage probability while the authors in [25] determined a closed-form expression for the outage probability. Furthermore, in [26], the authors found the exact form expression of the error probability under independent non-identical Rayleigh fading channels.

In [27–30], the authors examined selective-relaying schemes based on the SNR to minimize the end-to-end bit error rate (BER) in cooperative digital relaying systems using BPSK modulation. Furthermore, in [27–30], different models based on the availability of different sets of instantaneous and average SNR information at the relay are studied. For each model, the optimal strategy to minimize the BER has been investigated. Approximations for the optimal threshold values that minimize the BER and the resulting performance are derived analytically for BPSK modulation.

All previous publications are devoted to study the cooperative diversity networks over Rayleigh-fading channels. In [31] the authors found the outage probability for adaptive decode-and-forward over identical and independent Nakagami-m fading channels. In particular, the authors in [31] found the outage probability in terms of infinite series and using very advanced special functions. To the best of our knowledge, the error performance, outage probability and average channel capacity of multiple relays adaptive decode-and-forward under Nakagami-m scheme have not been addressed in the literature yet.

2.2 System Model

As shown in Figure 2.1, a source node (S) and a destination node (D) communicate over a channel with a flat Nakagami-m fading coefficient $h_{S,D}$. A number of cooperating nodes (R_i, $i = 1, 2, \ldots, M$) relay the signal to provide the destination with multiple copies of the original signal. The channel coefficients between the source S and R_i (h_{S,R_i}) and between

R_i and D ($h_{R_i,D}$) are also flat Nakagami-m fading coefficients. In addition, $h_{S,D}$, h_{S,R_i} and $h_{R_i,D}$ are mutually-independent and non-identical. We also assume here, without loss of generality that all additive white Gaussian noise (AWGN) terms have zero mean and equal variance (N_0). Here we assume all the transmissions occur through orthogonal channels using time division multiplexing (TDM), frequency division multiplexing (FDM) or code division multiplexing (CDM).

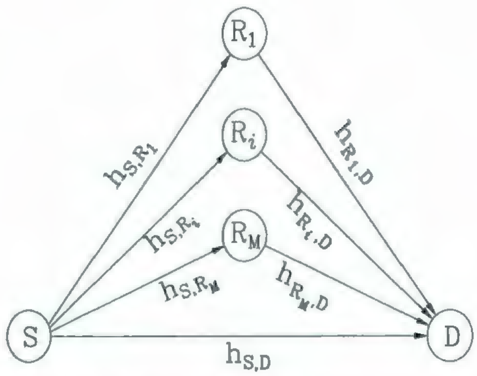

Figure 2.1: Illustration of a multi branch cooperative diversity network.

Mathematically speaking, the received signal from the source at the destination ($y_{S\to D}(t)$) and at the i^{th} relay ($y_{S\to R_i}(t)$) can be written as

$$\begin{aligned} y_{S\to D}(t) &= \sqrt{E_s} h_{S,D} x(t) + n_{S\to D}(t) \\ y_{S\to R_i}(t) &= \sqrt{E_s} h_{S,R_i} x(t) + n_{S\to R_i}(t) \end{aligned} \tag{2.1}$$

where E_s is the transmitted signal energy, $x(t)$ is a transmitted symbol signal with unit energy and $n_{S\to D}(t)$ and $n_{S\to R_i}(t)$ are the AWGN terms. The relays will process the received signal $y_{S\to R_i}(t)$, generate a signal $x_{r_i}(t)$ and transmit it to the destination. The received signal at the destination from the relay is given by

$$y_{R_i\to D}(t) = \sqrt{E_s} h_{R_i,D} x_{r_i}(t) + n_{R_i\to D}(t) \tag{2.2}$$

where $n_{R_i \to D}(t)$ is the AWGN term. We can distinguish between amplify-and-forward scheme and decode-and-forward scheme according to the method the relays use to generate $x_{r_i}(t)$ as follows.

2.2.1 Amplify-and-Forward

For amplify-and-forward scheme, the relays amplify the received signal from the source $y_{S \to R_i}(t)$ and relay the information by transmitting

$$x_{r_i}(t) = G_i y_{S \to R_i}(t) \tag{2.3}$$

where G_i is the relaying gain. In this chapter we assume channel state information (CSI) is available at the relays and the destination[1]. To maintain constant energy output from the relay, equal to the original transmitted energy E_s, the amplification factor G_i can be written as

$$G_i = \sqrt{\frac{1}{E_s h_{S,R_i}^2 + N_0}} \tag{2.4}$$

Hence, the destination can combine $M + 1$ copies of the source signal using the maximum ratio combining technique (MRC).

Finally, the analysis for the general case multi-hops multi-branches amplify-and-forward cooperative diversity networks can be found in Appendix (B).

2.2.2 Decode-and-Forward

For decode-and-forward, the relays decode the transmitted signal from the source. We define the decoding set (C) as the set of relays with the ability to fully decode the source message correctly [2]. In practice, most of the nodes in the wireless networks have this

[1]We will relax this assumption in the following Chapter

[2]Belonging to the decoding set can be determined by using CRC codes or any simple detection codes.

facility).[3] That is, a relay node is said to belong to the decoding set provided that the channel between the source and the relay node is sufficiently good to allow for successful decoding. Then, the destination combines the direct and the indirect links using MRC.

Finally, to capture the effect of the path-loss on the performance we use the model, which is commonly discussed in the literature (e.g., [17–20]), where $\mathbf{E}\left(h_{S,R_i}^2\right) = (d_{S,D}/d_{S,R_i})^\alpha$, $\mathbf{E}\left(h_{R_i,D}^2\right) = (d_{S,D}/d_{R_i,D})^\alpha$ and $\mathbf{E}\left(h_{S,D}^2\right) = 1$ where $d_{i,j}$ is the distance between terminal i and j and α is the power exponent path.

2.3 Error Performance Analysis

In this section, we derive closed-form expressions for the error probability of the amplify-and-forward and adaptive decode-and-forward relaying over independent non-identical Nakagami-m fading channels. In MRC, the received signals from each of the independent paths are first co-phased and then weighted by their respective signal amplitude-to-noise power ratios prior to being combined.

2.3.1 Amplify-and-Forward

With amplify-and-forward, the equivalent SNR at the destination node can be written as [17–20] (See Appendix A for derivation)

$$\gamma_{equ,AF} = \gamma_{S,d} + \sum_{i=1}^{M} \frac{\gamma_{S,R_i}\gamma_{R_i,D}}{\gamma_{S,R_i} + \gamma_{R_i,D} + 1} \tag{2.5}$$

where $\gamma_{S,R_i} = h_{S,R_i}^2 E_s/N_0$ is the instantaneous SNR between S and R_i, $\gamma_{R_i,D} = h_{R_i,D}^2 E_s/N_0$ is the instantaneous SNR between R_i and D, and $\gamma_{S,D} = h_{S,D}^2 E_s/N_0$ is the instantaneous SNR between S and D.

We derive in what follows a tight lower bound on the average error probability. The total SNR for cooperative diversity networks can be approximated by its upper bound (γ_b)

[3]Complete analysis for adaptive decode-and-forward using SNR threshold at the relays over independent non-identical Nakagami-m fading channels can be found in Appendix C

as follows

$$\gamma_{equ,AF} \leq \gamma_b = \gamma_{S,D} + \sum_{i=1}^{M} \gamma_i \tag{2.6}$$

where $\gamma_i = \min\left(\gamma_{S,R_i}, \gamma_{R_i,D}\right)$. The approximate SNR value in (2.6) is analytically more tractable than the exact value in (2.5); and as a result, this facilitates the derivation of the SNR statistics (Cumulative distribution function (CDF), PDF, and MGF). Assuming the independency of γ_{S,R_i}, $\gamma_{R_i,D}$ and ,$\gamma_{S,D}$ the MGF of γ_b can be written as

$$M_{\gamma_b}(s) = M_{\gamma_{S,D}}(s) \prod_{i=1}^{M} M_{\gamma_i}(s) \tag{2.7}$$

where $M_{\gamma_{S,D}}(s)$ and $M_{\gamma_i}(s)$ are the MGF of $\gamma_{S,D}$ and γ_i, respectively. Using the definition of the MGF as $M_X(s) = E\left(\exp\left(-sX\right)\right)$ and the assumption that $h_{S,D}$ is modeled by Nakagami-m distribution, it can be shown that

$$M_{\gamma_{S,D}}(s) = \left(1 + s\frac{\bar{\gamma}_{S,D}}{m_{S,D}}\right)^{-m_{S,D}} \tag{2.8}$$

where $m_{S,D}$ is the Nakagami-m fading parameter of $h_{S,D}$ and $\bar{\gamma}_{S,D} = E\left(h_{S,D}^2 E_s/N_0\right)$ (average SNR between S and D). In order to find $M_{\gamma_i}(s)$, we find the CDF of γ_i as follows

$$\begin{aligned} F_{\gamma_i}(\gamma) &= 1 - P\left(\gamma_{S,R_i} > \gamma\right) P\left(\gamma_{S,R_i} > \gamma\right) \\ &= 1 - \frac{1}{\Gamma\left(m_{S,R_i}\right)\Gamma\left(m_{R_i,D}\right)} \Gamma\left(m_{S,R_i}, \frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}\gamma\right) \Gamma\left(m_{R_i,D}, \frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\gamma\right) \end{aligned} \tag{2.9}$$

where $\Gamma(\bullet, \bullet)$ is the incomplete gamma function [22], $\Gamma(\bullet)$ is the gamma function [22], $\bar{\gamma}_{S,R_i} = E\left(h_{S,R_i}^2 E_s/N_0\right)$, $\bar{\gamma}_{R_i,D} = E\left(h_{R_i,D}^2 E_s/N_0\right)$ are the average SNR between S and R_i and R_i and D, respectively and m_{S,R_i} and $m_{R_i,D}$ are the Nakagami-m fading parameters of h_{S,R_i} and $h_{R_i,D}$, respectively. Thus, the PDF can be found by taking the derivative of (2.9) with respect to γ, yielding

$$
\begin{aligned}
f_{\gamma_i}(\gamma) &= \frac{1}{\Gamma\left(m_{S,R_i}\right)\Gamma\left(m_{R_i,D}\right)} \\
&\times\left[\left(\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}\right)^{m_{S,R_i}}\gamma^{m_{S,R_i}-1}\exp\left(-\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}\gamma\right)\Gamma\left(m_{R_i,D},\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\gamma\right)\right. \\
&+\left.\left(\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right)^{m_{R_i,D}}\gamma^{m_{R_i,D}-1}\exp\left(-\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\gamma\right)\Gamma\left(m_{S,R_i},\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}\gamma\right)\right]
\end{aligned}
\tag{2.10}
$$

Finally, $M_{\gamma_i}(s)$ can be calculated with the help of [22] as

$$
\begin{aligned}
M_{\gamma_i}(s)=&\left(\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}\right)^{m_{S,R_i}}\left(\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right)^{m_{R_i,D}}\frac{\Gamma\left(m_{S,R_i}+m_{R_i,D}\right)}{\Gamma\left(m_{S,R_i}\right)\Gamma\left(m_{R_i,D}\right)}\frac{1}{\left(\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}+\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}+s\right)^{m_{S,R_i}+m_{R_i,D}}} \\
\times&\left[\frac{1}{m_{S,R_i}}{}_2F_1\left(1,m_{S,R_i}+m_{R_i,D},m_{S,R_i}+1;\frac{\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}+s}{\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}+\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}+s}\right)\right. \\
+&\left.\frac{1}{m_{R_i,D}}{}_2F_1\left(1,m_{R_i,D}+m_{S,R_i},m_{R_i,D}+1;\frac{\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}+s}{\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}+\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}+s}\right)\right]
\end{aligned}
\tag{2.11}
$$

where ${}_2F_1(\bullet,\bullet;\bullet;\bullet)$ is the Gauss' hypergeometric function defined in [22] as

$$
{}_2F_1(a,b;c;z)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1\frac{t^{b-1}(1-t)^{c-b-1}}{(1-tz)^a}dt \tag{2.12}
$$

If $m_{S,R_i}=m_{R_i,D}=m_i$ and $\bar{\gamma}_{S,R_i}=\bar{\gamma}_{R_i,D}=\rho_i$ (Identical channels) it can be shown that (2.11) greatly simplifies to the following compact form

$$
M_{\gamma_i}(s)=\left(\frac{m_i}{\rho_i}\right)^{2m_i}\frac{\Gamma(2m_i)}{m_i\Gamma^2(m_i)}\frac{2}{\left(\frac{2m_i}{\rho_i}+s\right)^{2m_i}}{}_2F_1\left(1,2m_i;m_i+1;\frac{\frac{m_i}{\rho_i}+s}{\frac{2m_i}{\rho_i}+s}\right) \tag{2.13}
$$

By substituting (2.8) and (2.11) (or 2.13) into (2.7) we obtain a closed-form expression of

$M_{\gamma_b}(s)$ as

$$\begin{aligned} M_{\gamma_b}(s) &= \left(1+\frac{\bar{\gamma}_{S,D}}{m_{S,D}}s\right)^{-m_{S,D}} \prod_{i=1}^{M}\left(\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}\right)^{m_{S,R_i}}\left(\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right)^{m_{R_i,D}} \\ &\times \frac{\Gamma\left(m_{S,R_i}+m_{R_i,D}\right)}{\Gamma\left(m_{S,R_i}\right)\Gamma\left(m_{R_i,D}\right)} \frac{1}{\left(\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}+\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}+s\right)^{m_{S,R_i}+m_{R_i,D}}} \\ &\times \left[\frac{1}{m_{S,R_i}}{}_2F_1\left(1, m_{S,R_i}+m_{R_i,D}, m_{S,R_i}+1; \frac{\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}+s}{\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}+\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}+s}\right)\right. \\ &+ \left.\frac{1}{m_{R_i,D}}{}_2F_1\left(1, m_{R_i,D}+m_{S,R_i}, m_{R_i,D}+1; \frac{\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}+s}{\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}+\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}+s}\right)\right] \end{aligned} \tag{2.14}$$

Using $M_{\gamma_b}(s)$, the error rate can be evaluated for a wide variety of M-ary modulations (such as M-ary phase-shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM)) [23]. For instance, the average symbol error rate (SER) for M-PSK can be written as [23]

$$P(e) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} M_{\gamma_b}\left(\frac{g_{PSK}}{\sin^2(\theta)}\right)d\theta \tag{2.15}$$

where $g_{PSK} = \sin^2(\pi/M)$. The error probability given in (2.15) can be evaluated using a single finite integral and can be done with simple numerical integration techniques. Furthermore (2.15) can be upper bounded by a simple form as [23]

$$P(e) \leq \left(1-\frac{1}{M}\right) M_{\gamma_b}(g_{PSK}) \tag{2.16}$$

2.3.2 Decode-and-Forward

In the case of the adaptive decode-and-forward, the exact equivalent SNR at the destination node can be written as

$$\gamma_{equ,DF} = \gamma_{S,D} + \sum_{i\in C}\gamma_{R_i,D} \tag{2.17}$$

The average error probability is computed by determining the probability density function (PDF) of $\gamma_{equ,DF}$ and then averaging the conditional error probability, $P\left(e|\gamma_{equ,DF}\right)$, over this PDF. Hence, $P(e)$ is given by

$$P(e) = \int_0^\infty P\left(e|\gamma_{equ,DF}\right) f_{\gamma_{equ,DF}}\left(\gamma_{equ,DF}\right) d\gamma_{equ,DF} \tag{2.18}$$

where $f_{\gamma_{equ,DF}}\left(\gamma_{equ,DF}\right)$ is the PDF of the output SNR of the combiner. Note that for several Gray bit-mapped constellations employed in practical systems, $P\left(e|\gamma_{equ,DF}\right)$ takes the form of $Q\left(\sqrt{\beta\gamma_{equ,DF}}\right)$, where $Q(x)$ is the Q-function defined as $Q\left(x\right) = \left(1/\sqrt{2\pi}\right)\int_x^\infty \exp\left(-t^2/2\right) dt$ and β is a constant that depends on the modulation type (BPSK:$P\left(e|\gamma_{equ,DF}\right) = Q\left(\sqrt{2\gamma_{equ,DF}}\right)$, QPSK: $P\left(e|\gamma_{equ,DF}\right) = Q\left(\sqrt{2\gamma_{equ,DF}}\right)$ and in the case of square/rectangular M-QAM, $P\left(e|\gamma_{equ,DF}\right)$ can be written as a finite weighted sum of $Q\left(\sqrt{\beta\gamma_{equ,DF}}\right)$ terms). Hence, the error probability conditioned on a specific SNR at the combiner output can be written as

$$P\left(e|\gamma_{equ,DF}\right) = Q\left(\sqrt{\beta\gamma_{equ,DF}}\right) \tag{2.19}$$

Substituting (2.19) into (2.18), we can rewrite the average error probability as

$$P(e) = \int_0^\infty Q\left(\sqrt{\beta\gamma_{equ,DF}}\right) f_{\gamma_{equ,DF}}\left(\gamma_{equ,DF}\right) d\gamma_{equ,DF} \tag{2.20}$$

Actually, due to the difficulty of finding the PDF of $\gamma_{equ,DF}$ given in (2.17), it will be difficult to find a closed form expression for the error probability of the adaptive decode-and-forward cooperative networks especially over non-identical fading channels [24–26]. To bypass this difficulty we invoke the technique described in [25, 26] where the cooperative diversity network in Figure 2.1 can be visualized as a communication system that has effectively $M+1$ paths between the source and destination. Let the path number $M+1$ represent the S to D direct link and path i represent the $S \rightarrow R_i \rightarrow D$ indirect (cascaded) link, where $i = 1, \cdots, M$. The SNR at the destination can be represented as a random variable ξ_i that will take account of both, the source to the ith relay link and the ith relay to destination link. Therefore, PDF of ξ_i can be given by

$$\begin{aligned} f_{\xi_i}(x) &= f_{\xi_i|R_i \text{ Decodes Incorrectly}}(x)\Pr(R_i \text{ Decodes Incorrectly}) \\ &+ f_{\xi_i|R_i \text{ Decodes Correctly}}(x)\Pr(R_i \text{ Decodes Correctly}) \end{aligned} \tag{2.21}$$

Thus, the probability that R_i decodes incorrectly can be given as

$$B_i = \int_0^\infty Q\left(\sqrt{\beta\gamma}\right) f_{\gamma_{S,R_i}}(\gamma)\, d\gamma \tag{2.22}$$

For integer and non-integer values of m_{S,R_i} this integral can be evaluated as

$$B_i = \begin{cases} \frac{1}{2}\left[1 - \sqrt{\frac{\beta\bar{\gamma}_{S,R_i}/2}{m_{S,R_i}+\beta\bar{\gamma}_{S,R_i}/2}} \sum_{k=0}^{m_{S,R_i}} \binom{2k}{k} \left(\frac{1}{4} - \frac{\beta\bar{\gamma}_{S,R_i}/2}{4m_{S,R_i}+2\beta\bar{\gamma}_{S,R_i}}\right)^k\right], & \text{Integer } m_{S,R_i}; \\ \frac{\Gamma(m_{S,R_i}+1/2)}{\Gamma(m_{S,R_i}+1)\sqrt{2\pi}} \frac{\sqrt{\beta\bar{\gamma}_{S,R_i}/2m_{S,R_i}}}{(1+\beta\bar{\gamma}_{S,R_i}/2m_{S,R_i})^{m_{S,R_i}+1/2}} & \\ \times {}_2F_1\left(1, m_{S,R_i}+1/2, m_{S,R_i}+1; \frac{m_{S,R_i}}{m_{S,R_i}+\beta\bar{\gamma}_{S,R_i}/2}\right), & \text{Non-integer } m_{S,R_i} \end{cases} \tag{2.23}$$

When R_i decodes incorrectly, the received SNR at the destination by R_i will be 0 (since there is no retransmission from R_i because in this case the relay will be off and does not belong to the decoding set C). Therefore, the conditional PDF $f_{\xi_i|R_i \text{ Decodes Incorrectly}}(x)$ can be written as $f_{\xi_i|R_i \text{ Decodes Incorrectly}}(x) = \delta(x)$. The probability that the ith link decodes the source message correctly is $1 - B_i$. Since the destination in this case will receive another copy of the source signal by R_i with a SNR $\gamma_{R_i,D}$ (because in this case the relay will be on and belongs to the decoding set C), the conditional PDF $f_{\xi_i|R_i \text{ Decodes Correctly}}(x)$ can be written as

$$f_{\xi_i|R_i \text{ Decodes Correctly}}(x) = f_{\gamma_{R_i,D}}(x) = \left(\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right)^{m_{R_i,D}} \frac{\gamma^{m_{R_i,D}-1}}{\Gamma(m_{R_i,D})} \exp\left(-\gamma\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right) \tag{2.24}$$

Therefore, the PDF of ξ_i can be expressed as

$$f_{\xi_i}(x) = B_i\delta(x) + (1 - B_i)\left(\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right)^{m_{R_i,D}} \frac{\gamma^{m_{R_i,D}-1}}{\Gamma(m_{R_i,D})} \exp\left(-\gamma\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right) \tag{2.25}$$

The unconditional PDF of the SNR of the ith link at the destination given in (2.25) represents the ith cascaded link from S to D and accounts for the possible incorrect detection of the source message as well as the fading on the ith relay to destination link. Using this PDF representation, the total equivalent SNR at the destination from all links can be rewritten as[4]

$$\gamma_{equ,DF} = \sum_{i=1}^{M+1} \xi_i \tag{2.26}$$

where $\xi_{M+1} = \gamma_{S,D}$. We should note that $B_{M+1} = 0$ since ξ_{M+1} dose not represent a relay and the PDF of ξ_{M+1} is given as: $f_{\xi_{M+1}}(\gamma) = \left(\frac{m_{S,D}}{\bar{\gamma}_{S,D}}\right)^{m_{S,D}} \frac{\gamma^{m_{S,D}-1}}{\Gamma(m_{S,D})} \exp\left(-\gamma\frac{m_{S,D}}{\bar{\gamma}_{S,D}}\right)$. Note that the expressions of $\gamma_{equ,DF}$ in (2.26) and (2.17) are equal. However, the expression given in (2.26) is analytically more tractable than the expression given in (2.17). As a result, this facilitates the derivation of the SNR statistics (PDF and MGF). By finding the MGF of $\gamma_{equ,DF}$, we can use this MGF to calculate the error probability. Since ξ_i's are assumed to be independent, the MGF of $\gamma_{equ,DF}$ is given by

$$M_{\gamma_{equ,DF}}(s) = \prod_{i=1}^{M+1} M_i(s) \tag{2.27}$$

where $M_{M+1}(s)$ is the MGF of ξ_{M+1} and it is given in (2.8), while $M_i(s), i = 1, \cdots, M$ is the MGF of ξ_i and it can be written as

$$M_{\xi_i}(s) = B_i + (1 - B_i) M_{R_i,D}(s) \tag{2.28}$$

[4]Note that the $(M+1)$th path is the direct path from S to D

where $M_{R_i,D}(s) = \left(1 + s\frac{\bar{\gamma}_{R_i,D}}{m_{R_i,D}}\right)^{-m_{R_i,D}}$. The error probability can be found directly from the MGF of $\gamma_{equ,DF}$ as [23]

$$P(e) = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} M_{\gamma_{equ,DF}}\left(\frac{\beta}{\sin^2(\theta)}\right) d\theta \tag{2.29}$$

As in the amplify-and-forward scheme, we need to evaluate this finite single integral. Furthermore, (2.29) can be upper bounded by a simple form: $P(e) \le M_{\gamma_{equ,DF}}(\beta)$ [23, p. 275].

2.4 Outage Probability Analysis

In this section, we derive expressions for the outage probability of the adaptive decode-and-forward and amplify-and-forward relaying over independent non-identical Nakagami-m fading channels. The mutual information between the source and destination for the adaptive decode-and-forward and the amplify-and-forward, respectively can be written as

$$\begin{aligned} I_{AF} &= \frac{1}{M+1}\log_2\left(1+\gamma_{equ,AF}\right) \\ I_{DF} &= \frac{1}{M+1}\log_2\left(1+\gamma_{equ,DF}\right) \end{aligned} \tag{2.30}$$

The reason of the $\frac{1}{M+1}$ factor is that we need $M+1$ time slots (or orthogonal channels) for transmitting the data. From the total probability law we can write the outage probability as

$$P_{out} = \begin{cases} \Pr\left(I_{AF} \le R\right) = \Pr\left(\gamma_{equ,AF} \le 2^{(M+1)R} - 1\right), & \text{Amplify-and-forward} \\ \Pr\left(I_{DF} \le R\right) = \Pr\left(\gamma_{equ,DF} \le 2^{(M+1)R} - 1\right), & \text{Decode-and-forward} \end{cases} \tag{2.31}$$

where R is some fixed spectral efficiency determined by the designer depending on the application. In order to find P_{out}, we have to find the CDF of $\gamma_{equ,AF}$ and $\gamma_{equ,DF}$, which can be found directly by using the MGFs as

$$P_{out} = \begin{cases} \left[\Im^{-1}\left(\frac{M_{\gamma_{equ,AF}}(s)}{s}\right)\right]_{2^{(M+1)R}-1}, & \text{Amplify-and-forward} \\ \left[\Im^{-1}\left(\frac{M_{\gamma_{equ,AF}}(s)}{s}\right)\right]_{2^{(M+1)R}-1}, & \text{Decode-and-forward} \end{cases} \tag{2.32}$$

where $\Im^{-1}(\bullet)$ denotes the inverse Laplace transform. The inverse Laplace transform can be done analytically or using simple numerical techniques as in [32, 33].

2.5 Numerical Results and Simulations

In this section, we show numerical results of the BER and P_{out} for binary phase shift keying (BPSK) modulation. We plot the performance curves in terms of average BER and outage probability versus E_s/N_0 dB. We also show the results of the computer simulations for verification.

Figure 2.2 shows the BER performance of the amplify-and-forward at different values for a number of cooperating nodes (M). It is clear that the BER bound derived above is tight enough especially for moderate and high E_s/N_0 values. For instance, for $M = 3$ and $E_s/N_0 = 5$ dB, the exact BER (from simulation) is equal to 1×10^{-3}, while our lower bound (from (2.15) is equal to 0.9×10^{-3}. This trend (the tightness of our bound at medium and high E_s/N_0 values) is valid at different values of M as shown in Figure 2.2. From Figure 2.2 we can also notice, as expected, that the number of cooperating relays (M) has a strong impact on the performance enhancement and the achieved diversity order[5]. This effect is clearly seen that as M increases the error performance significantly improves.

Figure 2.3 shows the error probabilities for the adaptive decode-and-forward when $M = 0, \cdots, 3$. Observe that the results obtained using the closed-form expression of (2.29) and the simulation results are in excellent agreement. Also, it can be noted that increasing

[5]If the error rate is plotted versus the SNR on a log-log scale the diversity order can be interpreted as the slope of the so-obtained curve whereas the virtual array gain corresponds to the horizontal position of the curve.

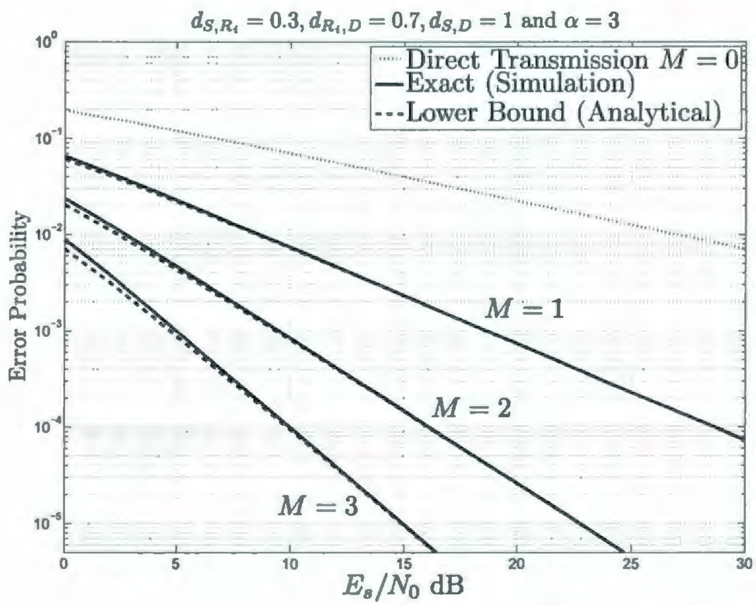

Figure 2.2: Error Probability for amplify-and-forward wireless cooperative diversity networks for $m_{S,D} = 0.5$, $m_{S,R_i} = 0.5$, and $m_{R_i,D} = 1$.

the number of relays always improves the error probability for all the region of SNR and obviously improve the diversity order. It can also be seen from Figures 2.2 and 2.3 that adaptive decode-and-forward slightly outperforms the amplify-and-forward scheme. This behavior is due to that in the adaptive decode-and-forward scheme, the relays will resend a new fresh correct data to the destination while in the amplify-and-forward scheme the relays will send a noisy amplified version of the source signal to the destination. Note that also the complexity at the relays will increase in the adaptive decode-and-forward scheme since we need to use adaptive method which is not needed in the amplify-and-forward scheme leads to reduce the transceiver complexity at the relays.

Figure 2.4 illustrates the outage probability performance determined from (2.31) for amplify-and-forward scheme. It is clear again that the difference between the exact (from simulation) and analytical results (lower bound) for P_{out} is small for medium and high values of E_s/N_0. For example, at SNR = 15 dB, the exact $P_{out} = 4 \times 10^{-2}$ and the approximate

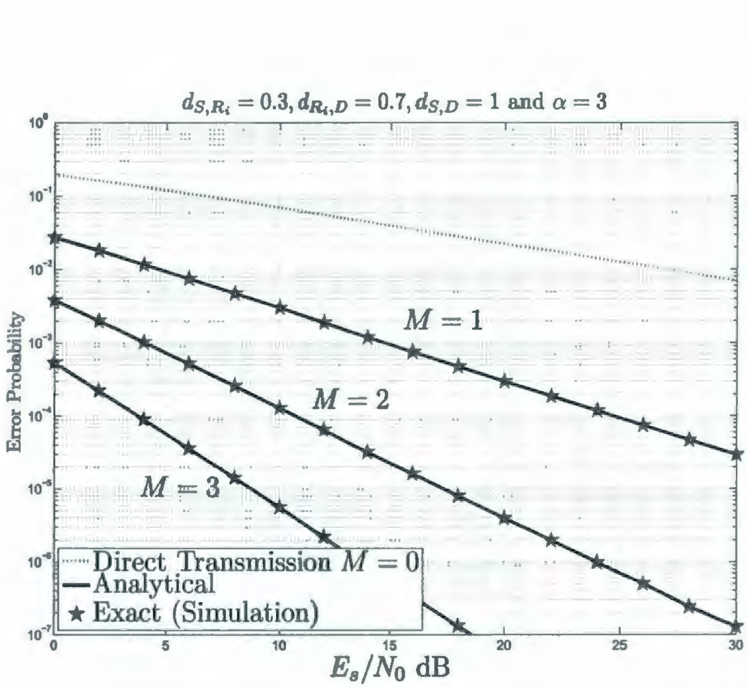

Figure 2.3: Error Probability for decode-and-forward wireless cooperative diversity networks for $m_{S,D} = 0.5$, $m_{S,R_i} = 0.5$, and $m_{R_i,D} = 1$.

$P_{out} = 2 \times 10^{-2}$ and more accurate for lower values of M. It should be noted that from Figures 2.2 and 2.4 the tightness of our mathematical model given in (2.6) for amplify-and-forward improves as E_s/N_0 increases; however, the proposed lower bound (for BER and P_{out}) slightly loses its tightness at low E_s/N_0 values particularly when M increases. This is due to the fact that the accuracy of total SNR approximation (in (2.6)) improves as E_s/N_0 increases.

Figure 2.5 shows the outage probability when the adaptive decode-and-forward is used. Observe the excellent matching between results of the closed-form expression derived in (2.32) and the simulation results. Figures 2.4 and 2.5 show that the cooperative diversity networks for both schemes (amplify-and-forward and decode-and-forward) improves the outage performance at high SNR only but can worsen the outage performance at low SNR. Actually, the critical number of relays that improves the outage probability depends on the value of SNR. The reason of this behavior is that there are two factors that

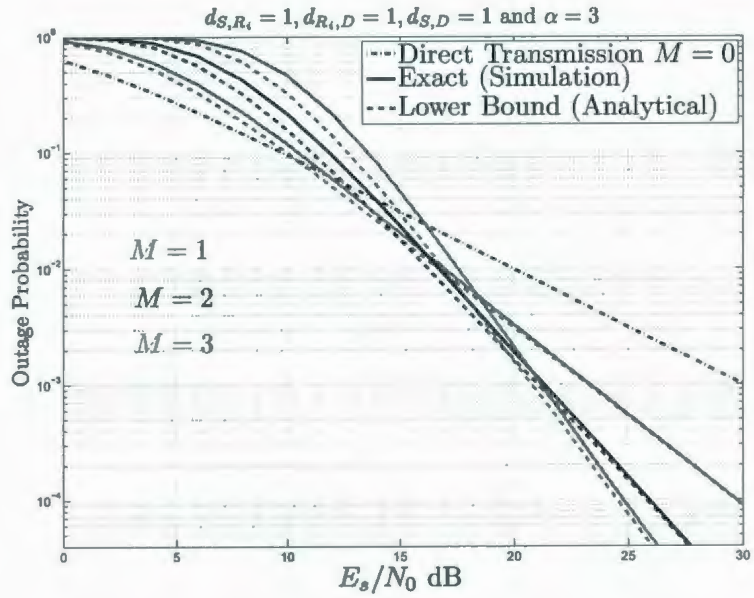

Figure 2.4: Outage Probability for amplify-and-forward wireless cooperative diversity networks for $m_{S,D} = 1$, $m_{S,R_i} = 0.5$, and $m_{R_i,D} = 1$.

affect the outage probability. These two factors are the number of channels and the output SNR. For the low SNR region, the number of channels needed (number of relays M) has the most dominant effect. In particular, this degradation increases as M increases. This problem can be avoided by reducing the number of participated relays as will be seen later in Chapter 4.

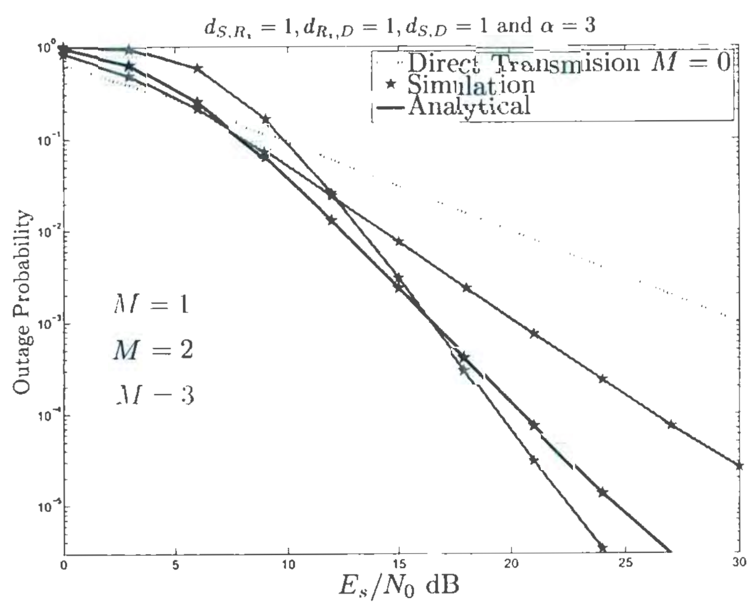

Figure 2.5: Outage Probability for decode-and-forward wireless cooperative diversity networks for $m_{S,D} = 1$, $m_{S,R_i} = 0.5$, and $m_{R_i,D} = 1$.

Chapter 3

Performance Analysis of Differential EGC Wireless Cooperative Diversity Networks over Nakagami-m Fading Channels

The analysis of proposed schemes in the last chapter assumed that the destination and relay nodes have perfect knowledge of the channel state information (CSI) of all transmission links. While in some scenarios, e.g., the slow fading channels, the CSI is likely to be acquired by the use of pilot symbols, it might be infeasible to track the CSI in fast fading channels. In [34], the authors studied the outage and the error performance of the dual-hop systems (not cooperative diversity networks) equipped with non regenerative blind relays (i.e. relays with fixed gain) over Rayleigh fading channels. In the same paper, the authors proposed a specific fixed gain relay, called semi-blind, that benefits from the knowledge of the first hop average fading power. The main advantage of the fixed gain introduced in [34] is that the relay and destination nodes do not need to estimate the CSI of the involved channels, and as a result, this reduces the complexity of the receivers.

Non-coherent cooperative diversity has been proposed in [35] for the decode-and-forward protocol employing frequency shift keying modulation (FSK). The authors in [36] examined the error performance for Binary Differential Phase Shift Keying (BDPSK) and found closed-form expression for error probability using the amplify-and-forward scheme.

In this chapter we derive exact closed-form expressions for the BER and P_{out} for cooperative diversity networks with amplify-and-forward and decode-and-forward schemes over Nakagami-m fading channel using the differential equal gain combining (EGC) technique. The main advantage of this scheme is that it does not need CSI of the transmission links at the relays and destination nodes. This feature reduces the design and implementation complexity. We obtain a closed form expression for the MGF which is used to study the end-to-end performance of the cooperative diversity networks in terms of error probability for Differential M-ary Phase Shift Keying (DMPSK), and outage probability. Simulations are used to validate the analytical results.

3.1 System Model

As in Chapter 2, , we consider a multi-node cooperative wireless network with a source (S), a destination (D) and M relays (R_1, R_2,..., R_i,..., R_M), as shown in Figure 3.1. However unlike Chapter 2, it is assumed that all the fading coefficients are not known at the transmitters, relays, or receivers. Also, the fading coefficients are assumed to be quasi-static so that these coefficients are fixed for each symbol transmission. Without any loss of generality, we assume that all the additive white Gaussian noise (AWGN) terms associated with all links ($S \rightarrow D$, $S \rightarrow R_i$ and $R_i \rightarrow D$) have equal variance (N_0). Finally, we assume that all the signals are transmitted through orthogonal channels (time-slots, carriers or codes).

Suppose that DMPSK modulation is used, i.e., the information is conveyed in the phase difference between two consecutive symbols; then the modulated information at the source can be described as $v_m = exp(j\phi_m)$ where $\{\phi_m\}_{m=0}^{L-1}$ is a set of L information bases. In the case of MDPSK, ϕ_m can be specified as $\phi_m = 2\pi m/L$ where $m = 0, 1, \ldots, L-1$. The source

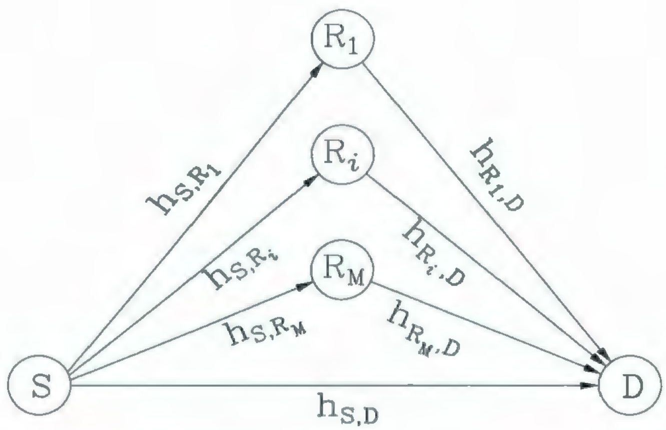

Figure 3.1: Illustration of a multi branch cooperative diversity network.

differentially encodes the information symbol as [37,38]

$$s(n) = v_m s(n-1) \tag{3.1}$$

where n is the time index, and $s(n)$ is the differentially encoded symbol with unit energy to be transmitted at time n. Then, the source transmits $s(n)$ with energy E_s to the destination and the relays. The corresponding received signals at the destination and the i^{th} relay $(i = 1, 2, \ldots, M)$ can be expressed as

$$\begin{aligned} y_{S,D}(n) &= \sqrt{E_s} h_{S,D} s(n) + z_{S,D}(n) \\ y_{S,R_i}(n) &= \sqrt{E_s} h_{S,R_i} s(n) + z_{S,R_i}(n) \end{aligned} \tag{3.2}$$

where $h_{S,D}$ and h_{S,R_i} represent the channel fading coefficients from the source to the destination and from the source to the i^{th} relay, respectively. The terms $z_{S,D}(n)$ and $z_{S,R_i}(n)$ are the additive white Gaussian noise at the destination and the i^{th} relay, respectively. Each relay amplifies or decodes the received signal in (3.2) and forwards it to the destination with a transmitting energy E_s. Accordingly, the received signal at the destination from the ith relay depends on the type of scheme that has been used at the i^{th} relay.

3.1.1 Amplify-and-Forward

In the amplify-and-forward scheme, the received signal at the destination from the *ith* relay is given by

$$y_{R_i,D}(n) = \sqrt{\frac{E_s}{N_0 + E_s \bar{h}^2_{S,R_i}}} h_{R_i,D} y_{S,R_i}(n) + z_{R_i,D}(n) \tag{3.3}$$

where $h_{R_i,D}$ is the channel coefficient from the i^{th} relay to the destination, $z_{R_i,D}(n)$ is the additive noise at the destination and $\bar{h}^2_{S,R_i} = E\left(h^2_{S,R_i}\right)$, where $E(\bullet)$ is the statistical average operator. The choice of this gain $\sqrt{E_s/(N_0 + E_s \bar{h}^2_{S,R_i})}$ aims to set the average output energy from the i^{th} relay at E_s.

The gain is chosen so that the relay does not require the instantaneous channel state information. In fact, the transmitted energy at the relay is normalized by $N_0 + \bar{h}^2_{S,R_i}$, which implies that only the squared mean value of the channel fading coefficient between the source and relay ($\bar{h}^2_{S,R_i}$) and N_0 are required at R_i. In practice, such information can be obtained through long term averaging of the received signal power.

At the destination, the received signals from the source and from the relays are combined together, and then the combined output is differentially decoded. A suitable combining method for multi-channel systems with differential modulation is the equal-gain combining (EGC) [23]. However, the EGC technique is suitable for balanced diversity systems where all diversity branches have identical noise power [23]. This is not the case for amplify-and-forward cooperative diversity, where the direct and relay links have different noise power. A good approach is to normalize $y_{S\to D}(n)$ and $y_{R_i\to D}(n)$ using their own noise power before combining (similar to the normalization process in the Maximum Ratio Combining (MRC) technique). Doing so and after some straightforward manipulations, we obtain the following combined signal

$$y_{AF} = a_s y^*_{S,D}(n-1) y_{S,D}(n) + \sum_{i=1}^{M} a_i y^*_{R_i,D}(n-1) y_{R_i,D}(n) \tag{3.4}$$

where a_s and a_i are the optimum combining weights in terms of received SNR at the destination given by $a_s = 1/N_0$ and $a_i = \left(N_0 + E_s\bar{h}^2_{S,R_i}\right) / \left(N_0^2 + E_s h^2_{R_i,D} N_0 + E_s\bar{h}^2_{S,R_i} N_0\right)$ [37,38]. Unfortunately, a_i is incompatible with the differential detection approach since it requires knowledge of the instantaneous $R_i \rightarrow D$ link ($h_{R_i,D}$). Hence, a sub-optimal combiner that approximates $h_{R_i,D}$ by its mean $\bar{h}_{R_i,D}$ is introduced in [24,37,38]. Therefore, the sub-optimal combining weights will be $\hat{a}_i = \left(N_0 + E_s\bar{h}^2_{S,R_i}\right) / \left(N_0^2 + E_s\bar{h}^2_{R_i,D} N_0 + E_s\bar{h}^2_{S,R_i} N_0\right)$ and $a_s = 1/N_0$. Without acquiring perfect channel state information, the combined signal (3.4) is differentially decoded by using the detection rule $\hat{m} = arg\ \max_{m=0,1,\ldots,L} Re\left\{v_m^* y_{AF}\right\}$ [39].

Finally, it should be noted here that the equivalent output SNR from the differential EGC combiner can be written as [23]

$$\begin{aligned}\gamma_{AF} &= \gamma_{S,D} + \sum_{i=1}^{M} \frac{\gamma_{S,R_i}\gamma_{R_i,D}}{\bar{\gamma}_{S,R_i} + \gamma_{R_i,D} + 1} \\ &= \gamma_{S,D} + \sum_{i=1}^{M} \gamma_{S,R_i,D}\end{aligned} \tag{3.5}$$

where $\gamma_{S,D} = E_s h^2_{S,D}/N_0$, $\gamma_{S,R_i} = E_s h^2_{S,R_i}/N_0$, $\gamma_{R_i,D} = E_s h^2_{R_i,D}/N_0$ and $\bar{\gamma}_{S,R_i} = E_s\bar{h}^2_{S,R_i}/N_0$.

3.1.2 Decode-and-forward

In the adaptive decode-and-forward the relays differentially decode the transmitted symbol from the source. Two consecutive received signals, $y_{S,R_i}(n)$ and $y_{S,R_i}(n-1)$, are required to recover the transmitted information at each symbol period. By assuming that the channel coefficient h_{S,R_i} is almost constant over two symbol periods, the relay differentially decodes based on the decision rule

$$\hat{m} = \arg \max_{m=0,1,\ldots,L} Re\left\{\left(v_m y_{S,R_i}(n-1)\right)^* y_{S,R_i}(n)\right\} \tag{3.6}$$

We define the decoding set (C) as the set of relays with the ability to fully decode the source message correctly. That is, the relay node is said to belong to the decoding set provided that the channel between the source and the relay node is sufficiently good to allow for successful decoding. Then, the destination combines the direct and the indirect links using differential EGC.

In standard differential detection, successful differential decoding requires that the encoder differentially encodes each information symbol with the previously transmitted symbol. In this way, if the information symbols are sent every time slot, then the information symbol to be transmitted at time (n) is differentially encoded with the transmitted symbol at time ($n-1$). However, in the differential adaptive decode-and-forward scheme, the information symbols at the ith relay are transmitted only if they are correctly decoded. Therefore, the transmission time of the previously transmitted symbol can be any time before the current time. We denote such previous transmission time as ($n - \kappa_i$) for κ_i, i.e., $n - \kappa_i$ is the latest time that the ith relay correctly decodes the symbol before time n. In order to perform successful differential decoding, the ith relay needs to store the transmitted symbol at time $n - \kappa_i$. Note that needing a memory at the relay will not increase the complexity of system compared to the standard differential system. The only difference is that the memory in this scheme stores the transmitted symbol at time $n - \kappa_i$ instead of time $n - 1$ as does the standard differential scheme. The differentially re-encoded signal at the relay can be written as

$$r_i(n) = v_m s(n - \kappa_i) \tag{3.7}$$

where $r_i(n)$ is the differentially encoded symbol at the relay at time n. It should be noted from (3.1) and (3.7) that the differentially encoded symbols at the relays and at the source convey the same information symbol v_m.

Finally, at the destination, the received signal from the source and that from the relays (that are in the decoding set C) are combined together, and then the combined output is jointly differentially decoded. Note that, for successfully decoding, the differential detector at the destination requires to store the previously received signal for each relay on the decoding set C. With an assumption that the channel coefficients stay almost constant for several symbol periods, the signal in the memory can be used for efficient differential decoding at the destination. Based on the multi-channel differential detection in [23], the combined signal before being differentially decoded is

$$y_{DF} = a_s y^*_{S,D}(n-1) y_{S,D}(n) + \sum_{i \in C} a_i y^*_{R_i,D}(n-\kappa_i) y_{R_i,D}(n) \tag{3.8}$$

where a_s and a_i are the combining weights given by $a_s = a_i = 1/N_0$ and $n - \kappa_i$ represents the time index of the latest signal in memory, i.e., is the most recent received signal from the ith relay. Note that a_s and a_i, maximize SNR of the combiner output when the destination is able to differentially decode the signals from both source and relays. Based on the combined signal in (3.8), the decoder at the destination jointly differentially decodes the transmitted information symbol by using the following decision rule:

$$\hat{m} = \arg \max_{m=0,1,\ldots,L} Re\left\{v^*_m y_{DF}\right\} \tag{3.9}$$

Finally, it should be noted here that the equivalent output SNR from the differential EGC combiner can be written as [23]

$$\gamma_{DF} = \gamma_{S,D} + \sum_{i \in C} \gamma_{R_i,D} \tag{3.10}$$

3.2 Error Performance Analysis

In this section, we derive closed-form expressions for the error probability of the amplify-and-forward and decode-and-forward relaying over independent non-identical Nakagami-m fading channels.

3.2.1 Amplify-and-Forward

Despite the suitability of $\hat{a}_i$ to the differential detection, the analytical solution in this case is involved [24,37,38]. Hence, we will use a_i in the analytical solution. However, we will analyze the performance of the differential EGC cooperative diversity using $\hat{a}_i$ by simulation. It will be seen later that the performance analysis by simulation (using $\hat{a}_i$) are very close to our analytical performance analysis (using a_i).

Using the optimum combining weights, the instantaneous SNR at the combiner output is given by [38]

$$\begin{aligned}\gamma_{AF} &= \gamma_{S,D} + \sum_{i=1}^{M} \frac{\gamma_{S,R_i}\gamma_{R_i,D}}{\gamma_{S,R_i} + \gamma_{R_i,D} + 1} \\ &= \gamma_{S,D} + \sum_{i=1}^{M} \gamma_{S,R_i,D}\end{aligned} \tag{3.11}$$

For a given SNR γ_{AF} in (3.11), the unconditional BER expression for M-channel diversity receptions can be expressed as [23]

$$P(e) = \frac{1}{2^{2M+2}} \int_{-\pi}^{\pi} \frac{\psi(\phi)}{1 + 2\beta sin(\phi) + \beta^2} M_{\gamma_{S,D}}(\lambda) \prod_{i=1}^{M} M_{\gamma_{S,R_i,D}}(\lambda) d\phi \tag{3.12}$$

where $M_{\gamma_{S,D}}(s) = \mathbf{E}(e^{-s\gamma_{S,D}})$ and $M_{\gamma_{S,R_i,D}}(s) = \mathbf{E}(e^{-s\gamma_{S,R_i,D}})$ are the MGF of $\gamma_{S,D}$ and $\gamma_{S,R_i,D}$, respectively, $\lambda = (a^2 + b^2 + 2ab\sin\phi)/2$, and $0^+ \leq (a/b) = \beta \leq 1$ where a and b are constants that depend on the modulation size, specifically, $a = 10^{-3}$ and $b = \sqrt{2}$ for DBPSK modulation, while $a = \sqrt{2 - \sqrt{2}}$ and $b = \sqrt{2 + \sqrt{2}}$ for DQPSK modulation [23]. The values for larger modulation sizes can be obtained by using the result in [23]. Finally, $\psi(\phi) = \sum_{i=1}^{M+1} \binom{2M+1}{M+1-i} \left[(\beta^{-i+1} - \beta^{i+1}) \cos\left((i-1)(\phi + \frac{\pi}{2})\right) - (\beta^{-i+2} - \beta^{i}) \cos\left(i(\phi + \frac{\pi}{2})\right)\right]$, where $\binom{2M+1}{M+1-i} = \frac{(2M+1)!}{(M+1-i)!(M+i)!}$.

Since $\gamma_{S,D}$ is chi-square distributed, it can easily be shown that the MGF of $\gamma_{S,D}$ is expressed as

$$M_{\gamma_{S,D}}(s) = \left(1 + \frac{\bar{\gamma}_{S,D}}{m_{S,D}} s\right)^{-m_{S,D}} \tag{3.13}$$

where $m_{S,D}$ is the fading parameter of $\gamma_{S,D}$ and $\bar{\gamma}_{S,D} = \bar{h}_{S,D}^2 E_s/N_0$ is the average SNR between the source and the destination. In order to get a closed-form expression of the MGF of $\gamma_{S,R_i,D}$, we assume in this paper that the fading parameter (m_{S,R_i}) between the source and relay takes only integer values; then, the approximate results for performance analysis for non-integer values of m_{S,R_i} can be obtained using interpolation of the final results obtained for integer values. Now, we need to determine the PDF of which can be derived as (see the appendix D):

$$f_{\gamma_{S,R_i,D}}(\gamma) = \frac{2\left(\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}\right)^{m_{S,R_i}}\left(\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right)^{m_{R_i,D}}}{\gamma^{1-m_{S,R_i}}\Gamma\left(m_{S,R_i}\right)\Gamma\left(m_{R_i,D}\right)} \exp\left(\frac{-m_{S,R_i}\gamma}{\bar{\gamma}_{S,R_i}}\right)\sum_{k=0}^{m_{S,R_i}} \frac{m_{S,R_i}!\left(\bar{\gamma}_{S,R_i}+1\right)^k}{(m_{S,R_i}-k)!k!}$$
$$\times\left(\frac{m_{S,R_i}\bar{\gamma}_{R_i,D}\gamma\left(1+\bar{\gamma}_{S,R_i}\right)}{m_{R_i,D}\bar{\gamma}_{S,R_i}}\right)^{(m_{R_i,D}-k)/2} K_{M_{R_i,D}-k}\left(2\sqrt{\frac{\gamma m_{S,R_i}m_{R_i,D}\left(1+\bar{\gamma}_{S,R_i}\right)}{\bar{\gamma}_{S,R_i}\bar{\gamma}_{R_i,D}}}\right) \quad (3.14)$$

where $m_{Ri,D}$ is the fading parameter between the relay R_i and the destination, $\bar{\gamma}_{R_i,D} = \bar{h}_{R_i,D}^2 E_s/N_0$ is the average SNR between the i^{th} relay and the destination and $K_v(.)$ denotes the v^{th}– order modified Bessel function of the second kind. Then, the MGF of $\gamma_{S,R_i,D}$ with the help of [22, eq. 6.643.3], can be expressed as

$$M_{\gamma_{S,R_i,D}}(s) = \frac{\left(m_{S,R_i}/\bar{\gamma}_{S,R_i}\right)^{m_{S,R_i}}}{\left(m_{R_i,D}/\bar{\gamma}_{R_i,D}\right)^{-m_{R_i,D}}\Gamma\left(m_{Ri,D}\right)} \exp\left(\frac{m_{S,R_i}m_{R_i,D}\left(1+\bar{\gamma}_{S,R_i}\right)}{2\bar{\gamma}_{S,R_i}\bar{\gamma}_{R_i,D}\left(s+m_{S,R_i}/\bar{\gamma}_{s,R_i}\right)}\right)$$
$$\times\sum_{k=0}^{m_{S,R_i}} \frac{m_{S,R_i}!\left(1+\bar{\gamma}_{S,R_i}\right)^k}{(m_{S,R_i}-k)!k!}\left(\frac{\left(1+\bar{\gamma}_{S,R_i}\right)m_{S,R_i}\bar{\gamma}_{R_i,D}}{\bar{\gamma}_{S,R_i}m_{R_i,D}}\right)^{m_{R_i,D}/2-k/2}$$
$$\times\frac{\Gamma\left(m_{S,R_i}+m_{R_i,D}-k\right)}{\sqrt{m_{S,R_i}m_{R_i,D}\left(1+\bar{\gamma}_{S,R_i}\right)/\bar{\gamma}_{S,R_i}\bar{\gamma}_{R_i,D}}}\left(s+m_{S,R_i}/\bar{\gamma}_{s,R_i}\right)^{\frac{\left(k+1-m_{R_i,D}-2m_{S,R_i}\right)}{2}}$$
$$\times W_{P,Q}\left(\frac{m_{S,R_i}m_{R_i,D}\left(1+\bar{\gamma}_{S,R_i}\right)}{\bar{\gamma}_{S,R_i}\bar{\gamma}_{R_i,D}\left(s+m_{S,R_i}/\bar{\gamma}_{s,R_i}\right)}\right) \quad (3.15)$$

where $P = \left(1+k-2m_{S,R_i}-m_{R_i,D}\right)/2$, $Q = \left(m_{R_i,D}-k\right)/2$ and $W_{P,Q}(\bullet)$ is Whittaker function defined in [22, eq. 9.222.1].

By substituting (3.15) and (3.13) into (3.12), we can determine the unconditional BER. While the expression found in (3.12) is exact, it still requires numerical evaluation of the integral. By using the same method in [23, eq. 9.113] and after some algebraic manipulation, a closed form upper bound on the error performance can be written as

$$
\begin{aligned}
P(e) \leq & \frac{1}{2^{2M+1}} \sum_{i=1}^{M+1} \frac{(2M+1)!}{(M+1-i)!(M+i)!} \\
& \times \left[\left(\frac{3}{2} + \frac{1-\beta^{i-1}}{\pi\beta^{i-1}(1-\beta)} \right) M_{\gamma_{S,D}}(\zeta) \prod_{k=1}^{M} M_{\gamma_{S,R_k,D}}(\zeta) \right. \\
& \left. - \left(\frac{1}{2} + \frac{1-\beta^{i-1}}{\pi\beta^{i-1}(1-\beta)} \right) M_{\gamma_{S,D}}(\upsilon) \prod_{k=1}^{M} M_{\gamma_{S,R_k,D}}(\upsilon) \right]
\end{aligned}
\tag{3.16}
$$

where $\zeta = b^2(1-\beta)^2/2$ and $\upsilon = b^2(1+\beta)^2/2$. Finally, for a special case (Dual hop relay only) where there is only one relaying path available (assume it is the first path) and the direct path is not available, the probability of error can be given as: $P(e) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} M_{\gamma_{S,R_i,D}}\left(\frac{\sin^2(\pi/M)}{1+\sqrt{1-\sin^2(\pi/M)\cos(\phi)}}\right) d\phi$ and it easily can be upper bounded as $P(e) = \frac{M-1}{M} M_{\gamma_{S,R_i,D}}\left(\frac{\sin^2(\pi/M)}{1+\sqrt{1-\sin^2(\pi/M)}}\right)$.

3.2.2 Decode-and-Forward

Since the decoding set C is not constant and varied from time to time, it would be difficult to use equations (3.6) and (3.8) to find a closed form expression for the error probability of the adaptive decode-and-forward cooperative networks especially over non-identical fading channels. To bypass this difficulty the cooperative diversity network in Figure 3.1 can be visualized as a system that has effectively $M+1$ paths between the source and destination. Let the path i represent the $S \to R_i \to D$ indirect (cascaded) link, where $i = 1, \cdots, M$. the equivalent output SNR at the destination can be represented as a random variable ξ_i that will take account of both the source to the ith relay link and the ith relay to destination link. Therefore, ξ_i has PDF as

$$
\begin{aligned}
f_{\xi_i}(x) &= f_{\xi_i|R_i \text{ Decodes Incorrectly}}(x) \Pr(R_i \text{ Decodes Incorrectly}) \\
&+ f_{\xi_i|R_i \text{ Decodes Correctly}}(x) \Pr(R_i \text{ Decodes Correctly})
\end{aligned}
\tag{3.17}
$$

The probability that the link will be non-active (off) is equivalent to the error probability at R_i which can be written as [23]

$$A_i = \frac{1}{\pi}\int_0^{(L-1)\pi/L} M_{\gamma_{S,R_i}}\left(\frac{g_{PSK}}{1+\sqrt{1-q_{PSK}}\cos(\phi)}\right) d\phi \tag{3.18}$$

where $g_{PSK} = \sin^2(\pi/L)$. It is known that the MGF of γ_{S,R_i} can be written as $\left(1+s\frac{\bar{\gamma}_{S,R_i}}{m_{S,R_i}}\right)^{-m_{S,R_i}}$. For the special case of Binary DPSK wherein $g_{PSK} = 1$ and $L = 2$ we can write

$$A_i = \frac{1}{2}\left(\frac{m_{S,R_i}}{m_{S,R_i}+\bar{\gamma}_{S,R_i}}\right)^{m_{S,R_i}} \tag{3.19}$$

When R_i decodes incorrectly, the received SNR at the destination by R_i will be 0 (since there is no retransmission from R_i because in this case the relay will be off and does not belong to the decoding set C). Therefore, the conditional PDF $f_{\xi_i|R_i \text{ Decodes Incorrectly}}(x)$ can be written as $f_{\xi_i|R_i \text{ Decodes Incorrectly}}(x) = \delta(x)$. The probability that the ith link decodes the source message correctly is $1 - A_i$. Since the destination in this case will receive another copy of the source signal by R_i with a SNR $\gamma_{R_i,D}$ (because in this case the relay will be on and belongs to the decoding set C), the conditional PDF $f_{\xi_i|R_i \text{ Decodes Correctly}}(x)$ can be written as

$$f_{\xi_i|R_i \text{ Decodes Correctly}}(x) = f_{\gamma_{R_i,D}}(x) = \left(\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right)^{m_{R_i,D}} \frac{\gamma^{m_{R_i,D}-1}}{\Gamma(m_{R_i,D})}\exp\left(-\gamma\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right) \tag{3.20}$$

Therefore, the PDF of ξ_i can be expressed as

$$f_{\xi_i}(x) = A_i\delta(x) + (1-A_i)\left(\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right)^{m_{R_i,D}} \frac{\gamma^{m_{R_i,D}-1}}{\Gamma(m_{R_i,D})}\exp\left(-\gamma\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right) \tag{3.21}$$

Now, the equivalent output SNR from the differential EGC combiner can be rewritten as

$$\gamma_{DF} = \gamma_{S,D} + \sum_{i=1}^{M} \xi_i \tag{3.22}$$

Note that the expression of γ_{DF} in (3.22) is equal to the expression given in (3.10). However, (3.22) is more analytically tractable than (3.10) and as a result, this facilitates the derivation of the SNR statistics (MGF).

If we can find the MGF of γ_{DF}, we can use this MGF to calculate the error probability. Since all the links are assumed to be independent; the MGF of γ_{DF} is given by

$$M_{\gamma_{DF}}(s) = M_{\gamma_{S,D}}(s) \prod_{i=1}^{M} M_i(s) \tag{3.23}$$

where $M_{\gamma_{S,D}}(s)$ is the MGF between S and D path where it is given in (3.13) and $M_i(s), i = 1, \cdots, M,$ is the MGF of the ith indirect link variable ξ_i and it can be easily derived from (3.17) as

$$M_i(s) = A_i + (1 - A_i) M_{\gamma_{R_i,D}}(s) \tag{3.24}$$

where $M_{\gamma_{R_i,D}}(s) = \left(\frac{1}{1+s\bar{\gamma}_{R_i,D}/m_{R_i,D}}\right)^{m_{R_i,D}}$ is the MGF between R_i and D path. Finally, the error probability can be written as:

$$P(e) = \frac{1}{2^{2M+2}} \int_{-\pi}^{\pi} \frac{\psi(\phi)}{1 + 2\beta sin(\phi) + \beta^2} M_{\gamma_{S,D}}(\lambda) \prod_{i=1}^{M} M_{\gamma_i}(\lambda) d\phi \tag{3.25}$$

and can be approximated as

$$P(e) \leq \frac{1}{2^{2M+1}} \sum_{i=1}^{M+1} \frac{(2M+1)!}{(M+1-i)!(M+i)!}$$
$$\times \left[\left(\frac{3}{2} + \frac{1-\beta^{i-1}}{\pi\beta^{i-1}(1-\beta)} \right) M_{\gamma_{S,D}}(\zeta) \prod_{k=1}^{M} M_{\gamma_i}(\zeta) - \left(\frac{1}{2} + \frac{1-\beta^{i-1}}{\pi\beta^{i-1}(1-\beta)} \right) M_{\gamma_{S,D}}(\upsilon) \prod_{k=1}^{M} M_{\gamma_i}(\upsilon) \right] \tag{3.26}$$

where $\zeta = b^2(1-\beta)^2/2$ and $\upsilon = b^2(1+\beta)^2/2$.

3.3 Outage Probability Analysis

As in Chapter 2 the outage probability can be written as

$$\begin{aligned} I_{AF} &= \frac{1}{M+1}\log_2\left(1+\gamma_{AF}\right) \\ I_{DF} &= \frac{1}{M+1}\log_2\left(1+\gamma_{DF}\right) \end{aligned} \tag{3.27}$$

The reason of the $\frac{1}{M+1}$ factor is that we need $M+1$ time slots (or orthogonal channels) for transmitting the data. From the total probability law we can write the outage probability as

$$P_{out} = \begin{cases} \Pr\left(I_{AF} \leq R\right) = \Pr\left(\gamma_{AF} \leq 2^{(M+1)R}-1\right), & \text{Amplify-and-forward} \\ \Pr\left(I_{DF} \leq R\right) = \Pr\left(\gamma_{DF} \leq 2^{(M+1)R}-1\right), & \text{Decode-and-forward} \end{cases} \tag{3.28}$$

where R is some fixed spectral efficiency determined by the designer depending on the application. In order to find P_{out}, we have to find the CDF of γ_{AF} and γ_{DF}, which can be found directly by using the MGFs as

$$P_{out} = \begin{cases} \left[\Im^{-1}\left(\frac{M_{\gamma_{AF}}(s)}{s}\right)\right]_{2^{(M+1)R}-1}, & \text{Amplify-and-forward} \\ \left[\Im^{-1}\left(\frac{M_{\gamma_{DF}}(s)}{s}\right)\right]_{2^{(M+1)R}-1}, & \text{Decode-and-forward} \end{cases} \tag{3.29}$$

where $\Im^{-1}(\bullet)$ denotes the inverse Laplace transform. The inverse Laplace transform can be done analytically or using simple numerical techniques as in [32, 33].

3.4 Numerical Results and Simulations

In this section, we show the numerical results of the BER and outage probability for DBPSK modulation. We plot the performance curves of the average BER and outage probability versus the SNR of the transmitted signal (E_s/N_0 dB). Our analytical results are compared with Mont-Carlo simulation results for verification. We compare the cooperative diversity network with a conventional non-cooperative system that involves direct transmission only from the source to the destination. For fair comparison, we set the sum of the transmitted energy from the source and the relay nodes of the cooperative diversity system equal to that of the non-cooperative system.

3.4.1 Amplify-and-Forward

Figure 3.2 shows the average BER of differential EGC scheme (using a single relay) obtained via analysis using the optimal combining weights (a_i) and by simulation using the sub-optimal combining weights ($\hat{a}_i$) for $m_{S,R_1} = m_{R_1,D} = 1, 2$, and 3 and $\bar{h}^2_{S,D} = \bar{h}^2_{S,R_1} = \bar{h}^2_{R_1,D} = 1$ along with the average BER of the non-cooperative differential BPSK ($m_{S,D} = 1$). Overall, it can be seen that the analytical results (using the sub-optimal weights) are very close to the Mont-Carlo simulation results of (with sub-optimal combining weights as well as with optimum combining weights). This validates the analytical results and shows that the use of the sub-optimal weights does not have significant impact on the performance. Also, it is evident that the differential EGC cooperative scheme outperforms the non-cooperative differential BPSK. Clearly, and as expected, the diversity order increases as the fading parameter increases. For example, the diversity order increase from 1 in case of direct link to 2 for $m_{S,R_1} = m_{R_1,D} = m_{S,D} = 1$, and to 2.8 approximately for $m_{S,R_1} = m_{R_1,D} = 2$ and $m_{S,D} = 1$.

Figure 3.3 compares the error performance of the differential EGC cooperative diversity with that of the optimal MRC cooperative diversity and conventional MISO system (without relaying nodes and with two transmit antennas at S and a single receive antenna at D).

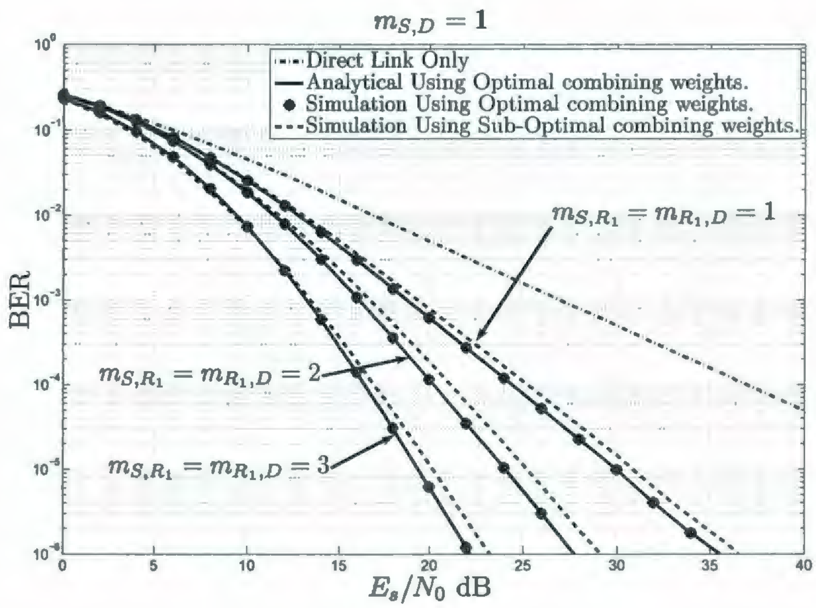

Figure 3.2: Average BER in Nakagami-m fading channel for differential amplify-and-forward cooperative diversity networks.

A single relay node is used in the cooperative diversity networks. An asymmetric network geometry is examined where R is located along the straight line connecting S and D where $d_{S,R} = 0.6$ and $d_{R,D} = 0.4$. The length of the direct ($S - D$) link is normalized to 1 and the path-loss exponent α is chosen to be equal to 3. Also, we assume here that the total transmit energy from the two antennas for the MISO system equals E_s (where each antenna is given energy $E_s/2$) and for the cooperative diversity the total transmit energy for both the source and the relay equals E_s (where the source and the relay have $E_s/2$ for each). As expected, the MRC outperforms the differential EGC system. However, the differential EGC has less complexity at both the relays and the destination since CSI is not required. This reduced complexity is very appealing especially in fast fading channel environments.

The other interesting result is that differential EGC system outperforms the conventional MISO system. This is due to the fact that differential EGC system benefits from the path loss reduction which gives differential EGC system more gain than that in the MISO system. It can also be seen that the benefit of path loss reduction increases as the channel gets better because the effect of the path loss reduction will be more dominant than the diversity effect. Furthermore, MISO system, obviously, requires multiple antennas at the source, and this is not feasible for small size terminals such as in mobile and WLAN networks. On the other hand, it should be noted that the transmission rate of the cooperative diversity system is equal to $1/(M+1)$ of that of the MISO system capacity (for the same SNR and total bandwidth). This rate reduction is one of the drawbacks of cooperative diversity systems.

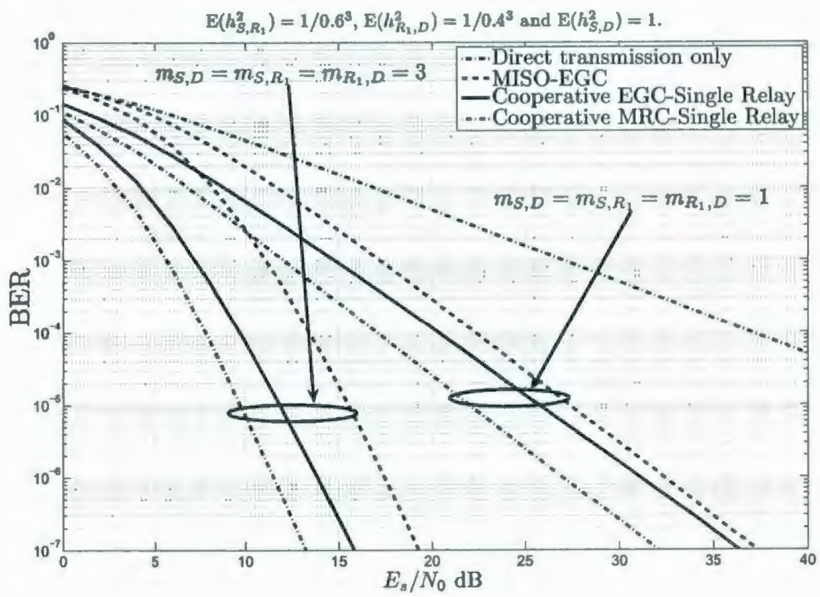

Figure 3.3: Comparison of BER performance for different type of spatial diversity techniques. $d_{S,R} = 0.6$ and $d_{R,d} = 0.4$ and $d_{S,D} = 1$.

Figure 3.4 shows the BER performance at different values of the number of cooperating nodes ($M = 0, 1, 2,$ and 3) for $m_{S,D} = 0.5$, $m_{S,R_i} = 1$, and $m_{R_1,D} = 0.5$ and $\bar{h}^2_{S,D} = \bar{h}^2_{S,R_i} = \bar{h}^2_{R_i,D} = 1$. Again, it is clear that the analytical results based on the optimal weights are very close to the simulation results based on the sub-optimal ones. For instance, for $M = 3$ and $E_s/N_0 = 25$ dB, the BER from simulation is equal to 1×10^{-4}, while the BER from analytical solution is equal to 1.6×10^{-4}. This trend is valid for different values of M as shown in Figure 3.4. From Figure 3.4, we can also notice that the number of cooperating relays (M) has a strong impact on the BER performance enhancement and the achieved diversity order. For example the diversity order of the direct link only ($M = 0$) is approximately 0.5 while the diversity order of the combined signal is 1, 1.57 and 2 for $M = 1$, 2 and 3, respectively.

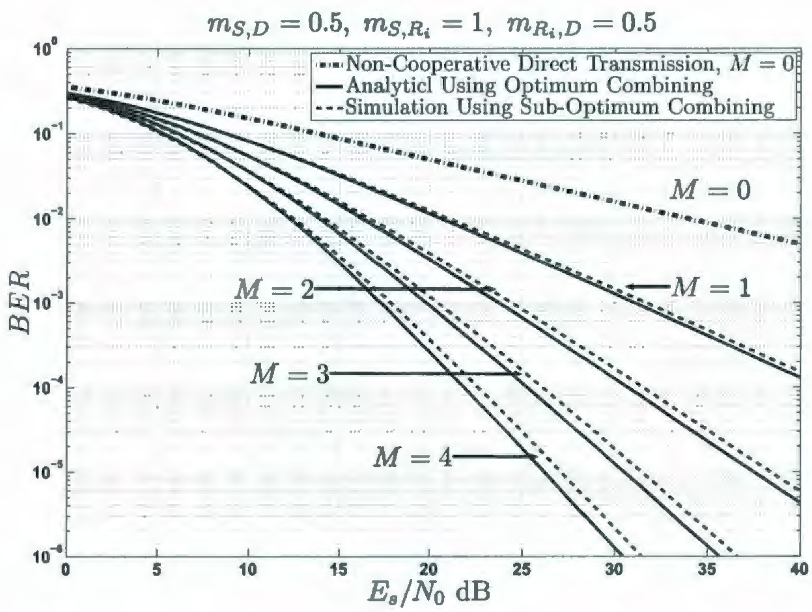

Figure 3.4: BER performance of EGC technique for different number of participating nodes ($M = 0, \cdots, 4$).

Even though the power allocation problem is not the main focus in this thesis and left for future research investigations; here we have an interesting result should be mentioned for single cooperative diversity. It is not difficult to see that the above relay selection problem is equivalent to the following power allocation problem: suppose that the internodes distances are identical but the total transmission power from the source and relay(s) is fixed, how we should split the power among the source and relay nodes to optimize the BER performance? To show the impact of power allocation, we invoke the model used in [42] by letting $0 < \epsilon < 1$ be the power allocation factor that controls power allocation for a single relay as follows: $\bar{\gamma}_{S,D} = \bar{\gamma}_{S,R_i} = \epsilon E_s/N_0$ and $\bar{\gamma}_{R_i,D} = (1-\epsilon)E_s/N_0$. In effect, $\epsilon > 0.5$ means more power allocated to the source than to the relay, and vice versa. Equivalently, $\epsilon > 0.5$ also corresponds to the case when the relay is closer to the source than to the destination as in the relay selection problem.

Using the analytical results we show in Figure 3.5 the average BER of differential amplify-and-forward as a function of ϵ when $E_s/N_0 = 10, 20$, and 30 dB. It can be seen that for high SNR = 30 dB, the error for a wide range of power allocation is very small (approximately equal). For example, the error range for $0.3 < \epsilon < 0.9$ is between 2×10^{-5} and 1×10^{-5} only and this range decreases as the SNR decreases. Also, it can be noticed that the equal power allocation between the source and the relay ($\epsilon = 0.5$) gives the best error performance (or very close to the best performance) at different SNR values, which can considerably simplify the power allocation problem.

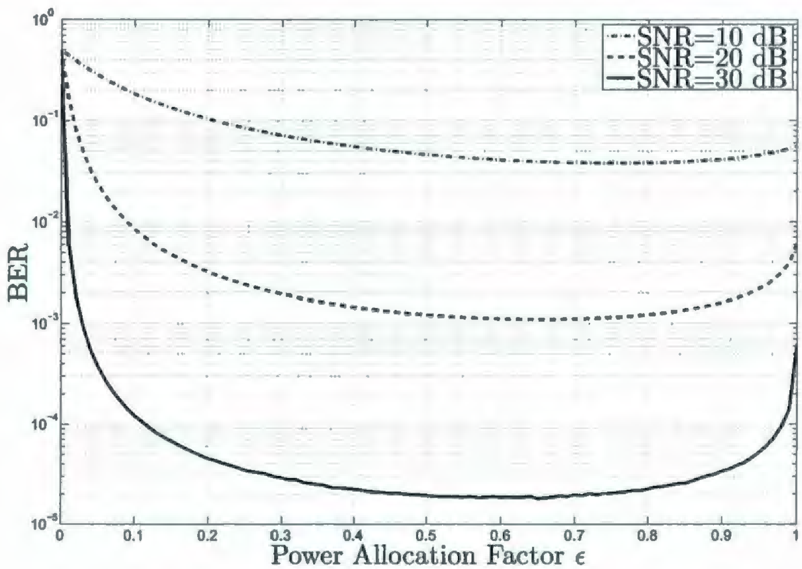

Figure 3.5: Average BER for the Power allocation factor.

3.4.2 Decode-and-Forward

Figure 3.6 shows the error probabilities when 1, 2 and 3 relay nodes are used. Observe that the results obtained using the closed-form expression derived in this chapter and the simulation results are in excellent agreement. Also as in amplify-and-forward increasing the number of relays always improves the error probability for all regions of SNR and obliviously improves the diversity order. We can see that the differential cooperative scheme achieves higher diversity orders as M increases. For example, at an error probability of 10^{-4}, we observe about $2-3$ dB gain as M increases from 2 to 3.

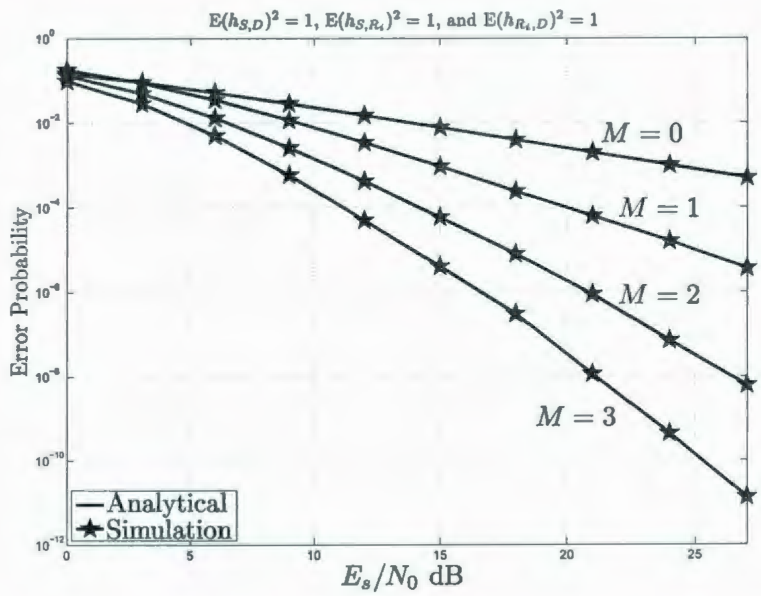

Figure 3.6: Error probability of adaptive DF differential EGC with $M = 0, \cdots, 3$.

Figure 3.7 shows the outage probability for differential adaptive decode-and-forward when 1, 2 and 3 relay nodes are used. In Figure 3.7, we have chosen $R = 1$ bit/sec/Hz. Again observe the exact match between the simulation and the analytical results, pointing out the validity of our proposed analysis. Figure 3.7 shows that the differential adaptive decode-and-forward EGC cooperative diversity networks improves the outage performance at high SNR only but can worsen the outage performance at low SNR. Hence, the critical number of relays that improves the outage probability depends on the value of SNR. It is worthy to mention here that this behavior of the outage probability can be extended to amplify-and-forward scheme.

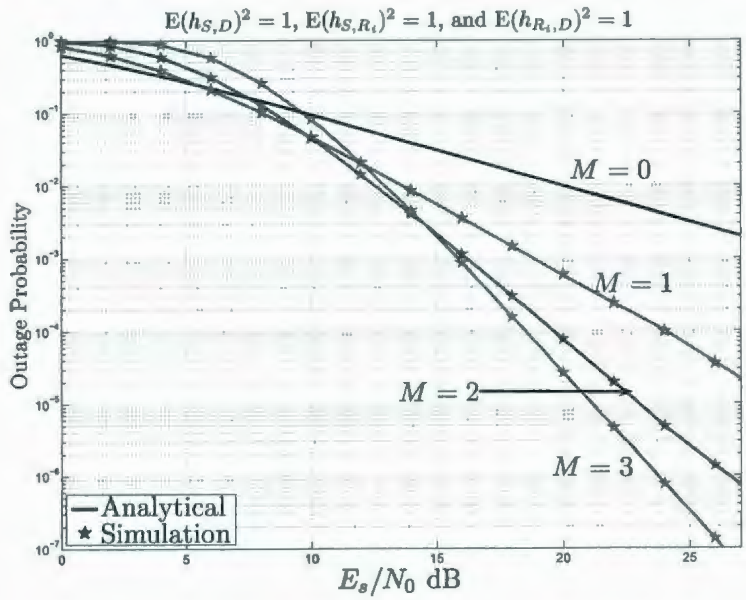

Figure 3.7: Outage probability of the differential adaptive decode-and-forward EGC with $M = 0, \cdots, 3$.

Chapter 4

Performance Analysis of Best-Relay Selection Cooperative Diversity Networks Over Rayleigh Fading Channels

In the previous two chapters, all the relay nodes relay the source signal to the destination using orthogonal channels (time slots, carriers or codes) to avoid cochannel interference. Hence, for a regular cooperative diversity network with M relays, we need $M + 1$ channels (one for the direct link and M for the M indirect links). This means that the number of required channels increases linearly with the number of relays. The resulting increased demand for channels constitutes the fundamental drawback of all cooperative diversity networks. As a consequence, if this problem is not addressed, cooperative diversity generally looses attractiveness for high-rate, high efficiency communications. In this chapter and following two chapters, we are going to address this problem and try to find some solutions.

In this chapter, we investigate the performance of the best-relay selection scheme where the "best" relay only participates in relaying. Therefore, two channels only are needed in this case (one for the direct link and the other one for the best indirect link) regardless of the total number of relays (M). The best relay is selected as the relay node that can achieve the highest signal-to-noise ratio (SNR) at the destination node. In this chapter a general mathematical probability model is developed and the performance of the best-relay selection for both schemes amplify-and-forward and adaptive decode-and-forward cooperative networks will be investigated. In particular, closed form expressions for the error probability, outage probability, average channel capacity and the moments of the destination SNR are derived over independent and non-identical Rayleigh fading channels. Results show that the best-relay selection not only reduces the number of required channels but also can maintain a full diversity order (which is achievable by the regular cooperative diversity system but with much more amount of resources).

4.1 General Background

The best-relay selection scheme for cooperative networks has been introduced in [40], and the authors showed that this scheme has the same diversity order as the regular cooperative diversity in terms of the outage probability. However, this important result was given using semi-analytical asymptotic analysis at high SNR (without deriving a closed-form expression for the outage probability). The best-relay selection scheme introduced in [40] requires no explicit communication among the relays, and assumes no prior knowledge of network geometry. It is based on instantaneous wireless channel measurements and reciprocity. This method does not need global channel state information at the relays or destination; hence, this method is suitable for most wireless communication systems. Briefly, based on the instantaneous channel conditions, each relay will feed a timer placed on it with a value that is inversely proportional to channel conditions, and the timer of the relay with the best channel conditions will expire first. All relays, while waiting for their timer to reduce to zero (i.e. to expire), are in listening mode. As soon as they hear another relay forwarding information (the best relay), they back off.

In [41], the authors presented an asymptotic analysis (at high SNR values) only of the symbol error probability of amplify-and-forward best-relay selection scheme, and compared it with the regular cooperative systems. The authors showed that best-relay selection scheme maintains full diversity order in terms of the symbol error probability. In [42], the authors analyzed the capacity outage probability of the best-relay selection scheme with decode-and-forward, and they showed that it outperforms distributed space-time codes for networks with more than three relaying nodes. This gain is due to the efficient use of power by the best-relay selection scheme networks. However, to the best of our knowledge, no one has derived closed form expressions for the symbol error probability and outage probability of the cooperative diversity network using the best-relay selection scheme at any SNR (not only high SNR values).

4.2 System Model

As shown in Figure 4.1, the system model is similar to that used in the previous two chapters except that in best-relay cooperative diversity network, only one relay (the best relay) would participate in retransmitting the source signal to the destination.

4.2.1 Amplify-and-Forward

Assuming that the relaying gain equals $\sqrt{1/\left(h^2_{S,R_i}E_s + N_0\right)}$, where E_s is the transmitted signal energy of the source, it was shown in chapter 2 that the end-to-end SNR of the indirect link $S \rightarrow, R_i \rightarrow, D$ can be written as

$$\gamma_{S,R_i,D} = \frac{\gamma_{S,R_i}\gamma_{R_i,D}}{\gamma_{S,R_i} + \gamma_{R_i,D} + 1} \tag{4.1}$$

where $\gamma_{S,R_i} = h^2_{S,R_i}E_s/N_0$ is the instantaneous SNR between S and R_i, $\gamma_{R_i,D} = h^2_{R_i,D}E_s/N_0$ is the instantaneous SNR between R_i and D. The best relay will be selected as the one

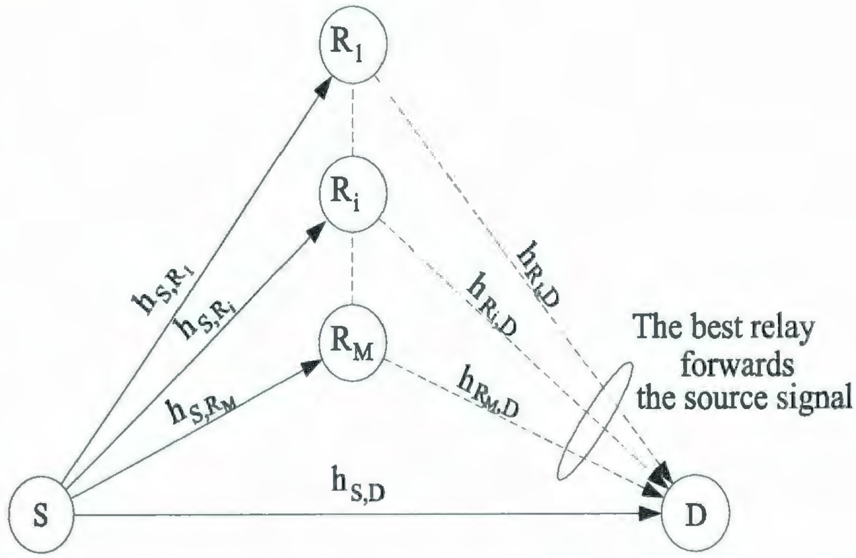

Figure 4.1: Illustration of a multi branch cooperative diversity network.

that achieves the highest end-to-end SNR of the indirect link. Then, assuming that MRC technique is employed at the destination node, the total SNR at the destination node can be written as

$$\gamma_{AF} = \gamma_{S,D} + \max_{i \in M} \left(\frac{\gamma_{S,R_i} \gamma_{R_i,D}}{\gamma_{S,R_i} + \gamma_{R_i,D} + 1} \right) \tag{4.2}$$

where $\gamma_{S,D} = h_{S,D}^2 E_s/N_0$ is the instantaneous SNR between S and D. In order to use the total SNR in the outage and error performance calculations, (4.1) should be expressed in a more mathematically tractable form. To achieve it, we proposed in chapter 2 a tight upper bound for $\gamma_{S,R_i,D}$, given by

$$\gamma_{S,R_i,D} \le \min\left(\gamma_{S,R_i}, \gamma_{R_i,D}\right) = \gamma_i \tag{4.3}$$

The PDF of γ_i can be expressed in terms of the average SNR $\bar{\gamma}_{S,R_i} = \mathrm{E}\left(h_{S,R_i}^2\right) E_s/N_0$ and $\bar{\gamma}_{R_i,D} = \mathrm{E}\left(h_{R_i,D}^2\right) E_s/N_0$ as, where $\bar{\gamma}_i = \bar{\gamma}_{S,R_i}\bar{\gamma}_{R_i,D}/\left(\bar{\gamma}_{S,R_i} + \bar{\gamma}_{R_i,D}\right)$. Using the value of γ_i

we can rewrite the total SNR in (4.2) as

$$\gamma_{AF} \leq \gamma_{S,D} + \gamma_b \tag{4.4}$$

where This approximation of the end-to-end SNR in (4.4) is analytically more tractable than the exact value in (4.2); and as a result, this facilitates the derivation of the SNR statistics (CDF, PDF, and MGF). This approximation is shown to be accurate enough, especially at medium and high SNR values as will be discussed in the results section.

4.2.2 Decode-and-Forward

In decode-and-forward scheme, assuming time division multiplexing for simplicity[1], in the first time slot the source broadcasts its signal to the destination node and the set of M-relay nodes as well. We define the decoding set (C) as the set of relays with the ability to fully decode the source message correctly. That is, the relay node is said to belong to the decoding set provided that the channel between the source and the relay node is sufficiently good to allow for successful decoding. In the second time slot, the best relay from the decoding set C decodes and forwards (retransmits) the source information to the destination. Then, the destination combines the signals of the direct link and the best indirect link using maximum ratio combining (MRC) technique.

$$\gamma_{DF} = \gamma_{S,D} + \max_{i \in C} (\gamma_{R_i,D}) \tag{4.5}$$

4.3 Error Performance Analysis

In this section, we derive closed-form expressions for the error probability of the amplify-and-forward and decode-and-forward relaying over independent non-identical Rayleigh fading channels.

[1]Frequency division multiplexing and code division multiplexing can also be used.

4.3.1 Amplify-and-Forward

Since we assume that the MRC technique is employed at the destination, the error probability is evaluated for coherent reception only. When multi-channel coherent reception is used, we can calculate the error probability by averaging the multi-channel conditional error probability $P(e|\gamma_{S,D},\gamma_b) = A \operatorname{erfc}\left(\sqrt{\beta(\gamma_{S,D}+\gamma_b)}\right)$, over the joint random variables representing the SNR values of the direct and indirect links $(\gamma_{S,D},\gamma_b)$. Since the random variables $(\gamma_{S,D},\gamma_b)$ are assumed to be independent, the joint PDF $f_{\gamma_{S,D},\gamma_b}(\gamma_{S,D},\gamma_b)$ can be given by $f_{\gamma_{S,D}}(\gamma_{S,D}) f_{\gamma_b}(\gamma_b)$. Therefore, error probability can be determined as follows

$$P(e) = \int_0^\infty \int_0^\infty P(e|\gamma_{S,D},\gamma_b) f_{\gamma_{S,D}}(\gamma_{S,D}) f_{\gamma_b}(\gamma_b)\, d\gamma_{S,D} d\gamma_b \tag{4.6}$$

Using the alternative definition of the erfc $(\bullet)$ function as [23, Chapter 4]

$$\operatorname{erfc}(x) = \frac{2}{\pi}\int_0^{\pi/2} \exp\left(-\frac{x^2}{\sin^2(\theta)}\right) d\theta \tag{4.7}$$

and by substituting (4.7) into (4.6), we obtain

$$P(e) = \frac{2}{\pi}\int_0^\infty \int_0^\infty \int_0^{\pi/2} \exp\left(\frac{-\beta\gamma_{S,D}}{\sin^2(\theta)}\right) \exp\left(\frac{-\beta\gamma_b}{\sin^2(\theta)}\right) f_{\gamma_{S,D}}(\gamma_{S,D}) f_{\gamma_b}(\gamma_b)\, d\theta d\gamma_{S,D} d\gamma_b \tag{4.8}$$

Since the order of integration can be interchanged [23], we obtain

$$P(e) = \frac{2}{\pi}\int_0^{\pi/2} M_{\gamma_{S,D}}\left(\frac{\beta\gamma_{S,D}}{\sin^2(\theta)}\right) M_{\gamma_b}\left(\frac{\beta\gamma_b}{\sin^2(\theta)}\right) d\theta \tag{4.9}$$

where $M_{\gamma_{S,D}}(s)$ and $M_{\gamma_b}(s)$ are the MGF of $\gamma_{S,D}$ and γ_b, respectively. In order to find $P(e)$ we need to find the PDF (and then the MGF) of $\gamma_{S,D}$ and γ_b. Since $h_{S,D}$ is a Rayleigh distributed random variable, the PDF of $\gamma_{S,D}$ has an exponential distribution with a mean $\bar{\gamma}_{S,D}$; hence, the MGF of $\gamma_{S,D}$ can be easily found as

$$M_{\gamma_{S,D}}(s) = \frac{1}{1+s\bar{\gamma}_{S,D}} \tag{4.10}$$

The PDF of γ_b, $f_{\gamma_b}(\gamma)$, can be found as follows. The CDF of γ_b can be written as $F_{\gamma_b}(\gamma) = \mathrm{P}(\gamma_b \leq \gamma)$, which can be obtained as

$$F_{\gamma_b}(\gamma) = \prod_{i=1}^{M} \left(1 - \exp\left(-\frac{\gamma}{\bar{\gamma}_i}\right)\right) \tag{4.11}$$

Then, the PDF can be found by taking the derivative of (4.11) with respect to γ, and after doing some manipulations, $f_{\gamma_b}(\gamma)$, can be written as

$$f_{\gamma_b}(\gamma) = \sum_{n=1}^{M} (-1)^{n+1} \sum_{k_1=1}^{M-n+1} \sum_{k_2=k_1+1}^{M-n+2} \cdots \sum_{k_n=k_{n-1}+1}^{M} \prod_{j=1}^{n} \exp\left(-\frac{\gamma}{\bar{\gamma}_{k_j}}\right) \sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}} \tag{4.12}$$

By using the PDF in (4.12), the MGF can be written as

$$M_{\gamma_b}(s) = \int_0^{\infty} e^{-s\gamma} \sum_{n=1}^{M} (-1)^{n+1} \sum_{k_1=1}^{M-n+1} \sum_{k_2=k_1+1}^{M-n+2} \cdots \sum_{k_n=k_{n-1}+1}^{M} \prod_{j=1}^{n} \exp\left(-\frac{\gamma}{\bar{\gamma}_{k_j}}\right) \sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}} d\gamma \tag{4.13}$$

and this integral can be evaluated in a closed form as

$$M_{\gamma_b}(s) = \sum_{n=1}^{M} (-1)^{n+1} \sum_{k_1=1}^{M-n+1} \sum_{k_2=k_1+1}^{M-n+2} \cdots \sum_{k_n=k_{n-1}+1}^{M} \frac{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}}{s + \sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}} \tag{4.14}$$

Substituting (4.14) and (4.10) in (4.9) and evaluating the integration with the help of [23, Chapter 5], $P(e)$ can be written in a closed form as

$$\begin{aligned} P(e) &= \sum_{n=1}^{M} (-1)^{n+1} \sum_{k_1=1}^{M-n+1} \sum_{k_2=k_1+1}^{M-n+2} \cdots \sum_{k_n=k_{n-1}+1}^{M} \\ &\times \left[1 - \frac{\frac{1}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}}}{\frac{1}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}} - \bar{\gamma}_{S,D}} \sqrt{\frac{\frac{\beta}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}}}{1 + \frac{\beta}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}}}} + \frac{\bar{\gamma}_{S,D}}{\frac{1}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}} - \bar{\gamma}_{S,D}} \sqrt{\frac{\beta\bar{\gamma}_{S,D}}{1 + \beta\bar{\gamma}_{S,D}}}\right] \end{aligned} \tag{4.15}$$

Asymptotic Analysis of the Error Probability

Although the expression for $P(e)$ in (4.15) enables numerical evaluation of the system performance and it is not computationally intensive, this expression does not offer insight into the effect of the different parameters (e.g. the number of relays M) that influence the system performance. In this subsection, we aim at expressing $P(e)$ in a simpler form in such a way we can see the effect of the different parameters as $\bar{\gamma}_{S,D}, \bar{\gamma}_{S,R_i}$ and $\bar{\gamma}_{R_i,D} \to \infty$.

The advantage of our accurate approximate solution obtained previously for the total SNR is that we have a closed form expression for the PDF. For this obtained PDF, the technique developed in [43] can be used to find asymptotic behavior of $P(e)$ at high SNR. If the approximate PDF of $\gamma_{S,D}$ and γ_b can be written as $f_{\gamma_{S,D}}(\gamma) = a_{S,D}\gamma^{t_{S,D}} + o(\gamma)$ and $f_{\gamma_b}(\gamma) = a_b\gamma^{t_b} + o(\gamma)$, respectively where $t_{S,D}$ and t_b are positive integers, $a_{S,D}$ and a_b are constants and $o(\gamma)$ is a high order polynomial function of γ. For $\gamma_{S,D}$ the value of $a_{S,D}$ is: $a_{S,D} = 1/\bar{\gamma}_{S,D}$ and $t_{S,D} = 0$ [43], while for γ_b the value of a_b and t_b can be found as follows. Using the series expansion, the CDF in (4.11) can be easily rewritten and approximated as

$$\begin{aligned} F_{\gamma_b}(\gamma) &= \prod_{i=1}^{M}\left(1-\left(1-\frac{\gamma}{\bar{\gamma}_i}+\frac{\gamma^2}{2\bar{\gamma}_i}-\frac{\gamma^3}{6\bar{\gamma}_i}+\cdots\right)\right) \\ &\approx \gamma^M\prod_{i=1}^{M}\frac{1}{\bar{\gamma}_i}+o(\gamma) \end{aligned} \tag{4.16}$$

From (4.16), the values of a_b and t_b are as follows

$$a_b = M\prod_{i=1}^{M}\frac{1}{\bar{\gamma}_i} \quad \text{and} \quad t_b = M-1 \tag{4.17}$$

Then, the approximate PDF of γ_{AF} can be written as [43]

$$f_{\gamma_{approx,AF}}(\gamma) = \frac{1}{\bar{\gamma}_{S,D}}\prod_{i=1}^{M}\frac{1}{\bar{\gamma}_i}\gamma^M \tag{4.18}$$

Note that asymptotic error probability is $P(e) \to A\int_0^{\infty}\operatorname{erfc}\left(\sqrt{B\gamma}\right) f_{approx,AF}(\gamma)\,d\gamma$, and after doing the integration, the asymptotic $P(e)$ can be written as

$$P(e) \approx D(M) \frac{1}{\bar{\gamma}_{S,D}} \prod_{i=1}^{M} \frac{1}{\bar{\gamma}_i} \gamma^M \tag{4.19}$$

where $D(M) = \frac{A\Gamma(M+3/2)}{\beta^{M+1}\sqrt{\pi}}$. In order to see the effect of increasing number of branches explicitly, we assume a special case where all the channels are identical $(\bar{\gamma}_1 = \bar{\gamma}_2 = \cdots = \bar{\gamma}_M = \bar{\gamma}_{S,D} = \bar{\gamma})$, then (4.19) can be written as $P(e) \to D(M) \left(\frac{1}{\bar{\gamma}}\right)^{M+1}$. It can clearly be seen that the diversity order is equal to $M+1$. This means that the diversity order increases linearly with the number of relays although we use one relay only.

4.3.2 Decode-and-Forward

In order to simplify the PDF calculation, we visualize the wireless cooperative network depicted in Fig. 4.1 as effectively having $M+1$ paths between the source and destination. Then, we define a random variable ξ_i representing the received instantaneous SNR at the destination on the i^{th} indirect link ($S \to R_i \to D$). Since the relay is assumed to forward the source information only if it is decoded correctly, we can write the PDF of ξ_i as

$$\begin{aligned} f_{\xi_i}(x) &= f_{\xi_i | R_i \text{ Decodes Incorrectly}}(x) \Pr(R_i \text{ Decodes Incorrectly}) \\ &+ f_{\xi_i | R_i \text{ Decodes Correctly}}(x) \Pr(R_i \text{ Decodes Correctly}) \end{aligned} \tag{4.20}$$

The PDF of the instantaneous SNR from S to R_i (γ_{S,R_i}) is $f_{\gamma_{S,R_i}}(x) = \frac{1}{\bar{\gamma}_{S,R_i}} \exp(-x/\bar{\gamma}_{S,R_i})$, where $\bar{\gamma}_{S,R_i} = \mathbf{E}\left(h^2_{S,R_i}\right) E_s/N_0$ is the average SNR between S and R_i and $\mathbf{E}(\bullet)$ is the statistical average operator. As in chapter 2, the PDF of ξ_i can be expressed as

$$f_{\xi_i}(x) = B_i \delta(x) + (1 - B_i) \frac{1}{\bar{\gamma}_{R_i,D}} \exp\left(-\frac{x}{\bar{\gamma}_{R_i,D}}\right) \tag{4.21}$$

where

$$B_i = \int_0^{\infty} \operatorname{erfc}\left(\sqrt{\beta \gamma_{S,R_i}}\right) f_{\gamma_{S,R_i}}\left(\gamma_{S,R_i}\right) d\gamma_{S,R_i} = \left[1 - \sqrt{\frac{\beta \bar{\gamma}_{S,R_i}}{\beta \bar{\gamma}_{S,R_i} + 1}}\right] \tag{4.22}$$

The unconditional PDF of the SNR at the destination given in (4.21) represents the ith cascaded link ($S \rightarrow R_i \rightarrow D$) and accounts for the possible incorrect detection of the source message as well as the fading on the ith relay to destination link. By using this PDF, the total SNR at the destination γ_{DF} can be rewritten as

$$\gamma_{DF} = \gamma_{S,D} + \max_{i \in M}\left(\xi_i\right) = \gamma_{S,D} + \chi \tag{4.23}$$

where $\chi = \max_{i \in M}\left(\xi_i\right)$. Note that the expressions of γ_{DF} in (4.23) and (4.5) are equivalent; however, the expression given in (4.23) is analytically more tractable than the expression given in (4.5). As a result, this facilitates the derivation of the total SNR statistics (CDF and PDF). Since $\gamma_{S,D}$ and χ are independent, then the PDF of their summation (γ_{DF}) is the convolution of the two PDFs. Since $h_{S,D}$ is Rayleigh fading, it is straight forward to show that the PDF of $\gamma_{S,D}$ is $f_{\gamma_{S,D}}(x) = \frac{1}{\bar{\gamma}_{S,D}} \exp\left(-x/\bar{\gamma}_{S,D}\right)$, where $\bar{\gamma}_{S,D} = \mathbf{E}\left(h_{S,D}^2\right) E_s/N_0$ is the average SNR between S and D. Therefore, we need to find the PDF of χ in order to find the PDF of γ_{DF}. The CDF of χ can be written as

$$F_{\chi}(x) = \Pr\left(\max_{i \in M} \xi_i \leq x\right) = \prod_{i=1}^{M} \Pr\left(\xi_i \leq x\right) \tag{4.24}$$

Then, the PDF of ξ_i can be found by taking the derivative of (4.24) with respect to x, and after some manipulations can be written as

$$\begin{aligned} f_{\chi}(x) = &\left(\prod_{i=1}^{M} B_i\right) \delta(x) + \sum_{k=1}^{M} (-1)^{k+1} \sum_{\lambda_1=1}^{M-k+1} \sum_{\lambda_2=\lambda_1+1}^{M-k+2} \cdots \sum_{\lambda_k=\lambda_{k-1}+1}^{M} \\ &\times \prod_{i=1}^{k} \left(1 - B_{\lambda_i}\right) \exp\left(-x/\bar{\gamma}_{R_{\lambda_i},D}\right) \sum_{i=1}^{k} \frac{1}{\bar{\gamma}_{R_{\lambda_i},D}} \end{aligned} \tag{4.25}$$

As a special case, where all the links are identical (i.e., $\bar{\gamma}_{S,R_i} = \bar{\gamma}_{R_i,D} = \bar{\gamma}$), the PDF of χ can be simplified as

$$f_{\chi}(x) = B^M \delta(x) + \sum_{k=1}^{M} (-1)^{k+1} \binom{M}{k} (1-B)^k \frac{k}{\bar{\gamma}} \exp(-xk/\bar{\gamma}) \tag{4.26}$$

where $B_1 = \cdots = B_M = B$.

By using the PDFs of $\gamma_{S,D}$ and χ and doing the convolution and some manipulations, the PDF of γ_{DF} can be written as

$$\begin{aligned} f_{\gamma_{DF}}(x) = {} & \left(\prod_{i=1}^{M} B_i\right) \frac{1}{\bar{\gamma}_{S,D}} \exp\left(-\frac{x}{\bar{\gamma}_{S,D}}\right) + \sum_{k=1}^{M} (-1)^{k+1} \sum_{\lambda_1=1}^{M-k+1} \sum_{\lambda_2=\lambda_1+1}^{M-k+2} \cdots \sum_{\lambda_k=\lambda_{k-1}+1}^{M} \\ & \times \frac{\prod_{i=1}^{k} (1-B_{\lambda_i})}{\bar{\gamma}_{S,D} - 1/\sum_{i=1}^{k} 1/\bar{\gamma}_{R_{\lambda_i},D}} \left[\exp(-x/\bar{\gamma}_{S,D}) - \prod_{i=1}^{k} \exp\left(-x/\bar{\gamma}_{R_{\lambda_i},D}\right)\right] \end{aligned} \tag{4.27}$$

Again as a special case, where all the links are identical, the PDF of γ_{DF} can be simplified to

$$f_{\gamma_{DF}}(x) = B^M \delta(x) + \sum_{k=1}^{M} (-1)^{k+1} \binom{M}{k} \frac{(1-B)^k}{\bar{\gamma}_{S,D} - \frac{\bar{\gamma}}{k}} \left[\exp\left(\frac{-x}{\bar{\gamma}_{S,D}}\right) - \exp\left(\frac{-xk}{\bar{\gamma}}\right)\right] \tag{4.28}$$

Hence, the error probability can be written as $P_b(e) = \int_0^{\infty} P_b(e \mid \gamma_{DF}) f_{\gamma_{DF}}(\gamma_{DF}) \, d\gamma_{DF}$, using (4.27) and doing the integration, the exact closed-form expression for the error probability of multi-branch adaptive decode-and-forward with the best-relay selection scheme cooperative diversity networks with M-relay nodes can be obtained as

$$P_b(e) = \left(\prod_{i=1}^{M} B_i\right)\left[1-\sqrt{\frac{\beta\bar{\gamma}_{S,D}}{\beta\bar{\gamma}_{S,D}+1}}\right] + \sum_{k=1}^{M}(-1)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M}\prod_{i=1}^{k}(1-B_{\lambda_i})$$
$$\times\left[1+\frac{1}{\bar{\gamma}_{S,D}-\frac{1}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}}}\left(\frac{1}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}}\sqrt{\frac{\beta}{\beta+\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}}}-\bar{\gamma}_{S,D}\sqrt{\frac{\beta\bar{\gamma}_{S,D}}{\beta\bar{\gamma}_{S,D}+1}}\right)\right] \quad (4.29)$$

Asymptotic Analysis of the Error Probability

By using the same method that has been used for the amplify-and-forward scheme, the approximate the PDF of γ_{DF} can be written as

$$f_{\gamma_{DF}}(\gamma) \approx a_{S,D}a_b\gamma^{M-1} + o(\gamma) \quad (4.30)$$

where the values of a_b and $a_{S,D}$ are as follows

$$a_b = M\prod_{i=1}^{M}\frac{1}{\bar{\gamma}_{R_i,D}} \quad \text{and} \quad a_{S,D} = \frac{1}{\bar{\gamma}_{S,D}} \quad (4.31)$$

Since the asymptotic error probability is $P(e) \to A\int_0^\infty \operatorname{erfc}\left(\sqrt{\beta\gamma}\right) f_{approx,DF}(\gamma)\, d\gamma$, and after doing the integration, the asymptotic $P(e)$ can be written as

$$P(e) \approx Z(M)\frac{1}{\bar{\gamma}_{S,D}}\prod_{i=1}^{M}\frac{1}{\bar{\gamma}_i}\gamma^M \quad (4.32)$$

where $Z(M) = \frac{A\Gamma(M+3/2)}{\beta^{M+1}\sqrt{\pi}}$. In order to see the effect of increasing number of branches explicitly, we assume a special case where all the channels are identical $(\bar{\gamma}_{R_1,D} = \bar{\gamma}_{R_2,D} = \cdots = \bar{\gamma}_{R_M,D} = \bar{\gamma}_{S,D} = \bar{\gamma})$, then (4.32) can be written as $P(e) \to Z(M)\left(\frac{1}{\bar{\gamma}}\right)^{M+1}$. It can clearly be seen that the diversity order is equal to $M+1$. This means that the diversity order increases linearly with the number of relays although we use one relay only. Also, it should be mentioned here that the best-relay adaptive decode-and-forward scheme outperforms the best-relay amplify-and-forward scheme. The reason of that is that $\bar{\gamma}_{R_i,D} \geq \bar{\gamma}_i$ since $\bar{\gamma}_i = \bar{\gamma}_{S,R_i}\bar{\gamma}_{R_i,D}/\left(\bar{\gamma}_{S,R_i} + \bar{\gamma}_{R_i,D}\right)$.

4.4 Outage Probability Analysis and Other Performance Criteria

In this section, we derive closed-form expressions for the outage probability of the amplify-and-forward and adaptive decode-and-forward relaying over independent non-identical Rayleigh fading channels.

4.4.1 Amplify-and-Forward

The CDF of the total end-to-end SNR using the best-relay selection cooperative diversity can be found as follows [23]

$$F_{\gamma_{AF}}(\gamma) = \Im^{-1}\left[\frac{M_{\gamma_{S,D}}(s)\, M_{\gamma_b}(s)}{s}\right]_{s=\gamma} \tag{4.33}$$

where $\Im^{-1}$ denotes the inverse Laplace transform. This inverse Laplace transform can be performed analytically[2], and the CDF of the total SNR can be expressed as

$$\begin{aligned} F_{\gamma_{AF}}(\gamma) = &\sum_{n=1}^{M}(-1)^{n+1}\sum_{k_1=1}^{M-n+1}\sum_{k_2=k_1+1}^{M-n+2}\cdots\sum_{k_n=k_{n-1}+1}^{M} \\ &\times\left(1-\frac{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}-\bar{\gamma}_{S,D}}\exp\left(\sum_{j=1}^{n}\frac{-\gamma}{\bar{\gamma}_{k_j}}\right)+\frac{\bar{\gamma}_{S,D}}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}-\bar{\gamma}_{S,D}}\exp\left(\frac{-\gamma}{\bar{\gamma}_{S,D}}\right)\right) \end{aligned} \tag{4.34}$$

The outage probability (P_{out}) is defined as the probability that the channel average mutual information (I_{AF}) falls below the required rate R. For the amplify-and-forward best-relay

[2] By doing the multiplication first and then using the partial fraction method

selection cooperative diversity networks, P_{out} can be written as

$$\begin{aligned} P_{out} &= \Pr\left(I_{AF} \leq R\right) \\ &= \Pr\left(\frac{1}{2}\log_2\left(1+\gamma_{S,D}+\gamma_b\right) \leq R\right) \\ &= \Pr\left(\gamma_{S,D}+\gamma_b \leq 2^{2R}-1\right) \end{aligned} \tag{4.35}$$

Hence, P_{out} is actually the CDF of γ_{AF} evaluated at $2^{2R}-1$; therefore, $P_{out} = F_{\gamma_{AF}}\left(2^{2R}-1\right)$.

In order to find the other statistics of the total SNR we have to find the PDF of γ_{AF}, which can be found directly by finding the derivative of the CDF, $F_{\gamma_{AF}}(\gamma)$, given in (4.34) with respect to γ yielding

$$\begin{aligned} f_{\gamma_{AF}}(\gamma) = & \sum_{n=1}^{M}(-1)^{n+1}\sum_{k_1=1}^{M-n+1}\sum_{k_2=k_1+1}^{M-n+2}\cdots\sum_{k_n=k_{n-1}+1}^{M} \\ & \times\left(\frac{1}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}-\bar{\gamma}_{S,D}}\exp\left(\sum_{j=1}^{n}\frac{-\gamma}{\bar{\gamma}_{k_j}}\right)-\frac{1}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}-\bar{\gamma}_{S,D}}\exp\left(\frac{-\gamma}{\bar{\gamma}_{S,D}}\right)\right) \end{aligned} \tag{4.36}$$

The lth moments of γ_{AF} can be found using (4.36) in a closed form as

$$\mu^l = \sum_{n=1}^{M}(-1)^{n+1}\sum_{k_1=1}^{M-n+1}\sum_{k_2=k_1+1}^{M-n+2}\cdots\sum_{k_n=k_{n-1}+1}^{M}\left(\frac{\bar{\gamma}_{S,D}^{l+1}}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}-\bar{\gamma}_{S,D}}-\frac{\left(\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right)^{l+1}}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}-\bar{\gamma}_{S,D}}\right) \tag{4.37}$$

where $\Gamma(\bullet)$ is the gamma function [22, eq. (8.310.1)]. By setting $l=1$ in (4.37), the average total SNR ($\bar{\gamma}_{AF}$) can be obtained. Furthermore, the first two moments of γ_{AF} can be used in order to evaluate the amount of fading at the destination. The the amount of fading is defined as the ratio of the variance to the square mean of γ_{AF} $\left(\mu_2/\bar{\gamma}_{AF}^2-1\right)$.

4.4.2 Decode-and-Forward

In order to find P_{out}, we have to find the CDF of γ_{DF}, which can be found directly by integrating its PDF given in (4.27), yielding

$$F_{\gamma_{DF}}(x) = \left(\prod_{i=1}^{M} B_i\right)\left(1-\exp\left(-x/\bar{\gamma}_{S,D}\right)\right) + \sum_{k=1}^{M}(-1)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M}\prod_{i=1}^{k}\left(1-B_{\lambda_i}\right)$$
$$\times\left[1+\frac{1}{\bar{\gamma}_{S,D}-1/\sum_{i=1}^{k}1/\bar{\gamma}_{R_{\lambda_i},D}}\left(\frac{\prod_{i=1}^{k}\exp\left(-x/\bar{\gamma}_{R_{\lambda_i},D}\right)}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}}-\bar{\gamma}_{S,D}\exp\left(-x/\bar{\gamma}_{S,D}\right)\right)\right] \quad (4.38)$$

Hence, P_{out} is actually the CDF of γ_{DF} evaluated at $2^{2R}-1$; therefore, $P_{out} = F_{\gamma_{AF}}\left(2^{2R}-1\right)$.

The l^{th} moments of the total SNR at the destination $\mu_l = \mathbf{E}\left(\gamma_{DF}^l\right)$ can be written as $\mu_l = \int_0^\infty \gamma_{DF}^l f_{\gamma_{DF}}(\gamma_{DF})\, d\gamma_{DF}$. After substituting the expression of $f_{\gamma_{DF}}(\gamma_{DF})$ and solving the integration and some necessary manipulations, the closed-form expression of the SNR moments can be expressed as

$$\mu_l = \Gamma(l+1)\left(\prod_{i=1}^{M} B_i\right)\bar{\gamma}_{S,D}^l + \Gamma(l+1)\sum_{k=1}^{M}(-1)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M}\prod_{i=1}^{k}\left(1-B_{\lambda_i}\right)$$
$$\times\left[\frac{1}{\bar{\gamma}_{S,D}-1/\sum_{i=1}^{k}1/\bar{\gamma}_{R_{\lambda_i},D}}\left(\left(\frac{1}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}}\right)^{l+1}-\bar{\gamma}_{S,D}^{l+1}\right)\right] \quad (4.39)$$

By setting $l = 1$ in (4.39), the average of the total SNR at the destination can be obtained in an exact closed form. For the special case where all the links are identical, the moments of γ_{DF} can be simplified as

$$\mu_l = \Gamma(l+1)B^M\bar{\gamma}_{S,D}^l + \Gamma(l+1)\sum_{k=1}^{M}(-1)^{k+1}\binom{M}{k}\frac{(1-B)^k}{\bar{\gamma}_{S,D}-\bar{\gamma}/k}\left((\bar{\gamma}/k)^{l+1}-\bar{\gamma}_{S,D}^{l+1}\right) \quad (4.40)$$

The channel capacity, in the Shannon's sense, is an important performance metric since it provides the maximum achievable transmission rate under which the errors are recover-able. The average channel capacity can be expressed as

$$\bar{C} = \frac{BW}{2} \int_0^{\infty} \log_2 (1 + \gamma_{DF}) f_{\gamma_{DF}} (\gamma_{DF}) d\gamma_{DF} \tag{4.41}$$

where BW is the transmitted signal bandwidth. By substituting (4.27) into (4.41), the average channel capacity can be obtained in closed-form as

$$\bar{C} = \frac{BW}{2\ln(2)} \left(\prod_{i=1}^{M} B_i \right) \bar{\gamma}_{S,D} e^{\frac{1}{\bar{\gamma}_{S,D}}} E_1 \left(\frac{1}{\bar{\gamma}_{S,D}} \right) + \frac{BW}{2\ln(2)} \sum_{k=1}^{M} (-1)^{k+1} \sum_{\lambda_1=1}^{M-k+1} \sum_{\lambda_2=\lambda_1+1}^{M-k+2} \cdots \sum_{\lambda_k=\lambda_{k-1}+1}^{M}$$
$$\times \left[\frac{\prod_{i=1}^{k} (1 - B_{\lambda_i})}{\bar{\gamma}_{S,D} - \frac{1}{\sum_{i=1}^{k} 1/\bar{\gamma}_{R_{\lambda_i},D}}} \left(\frac{E_1 \left(\sum_{i=1}^{k} \frac{1}{\bar{\gamma}_{R\lambda_i,D}} \right)}{\sum_{i=1}^{k} \frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}} \prod_{i=1}^{k} e^{\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}} - \bar{\gamma}_{S,D} e^{\frac{1}{\bar{\gamma}_{S,D}}} E_1 \left(\frac{1}{\bar{\gamma}_{S,D}} \right) \right) \right] \tag{4.42}$$

where $E_1(\bullet)$ is the exponential integral and it is given by $E_1(x) = \int_x^{\infty} \frac{\exp(-t)}{t} dt$ [22]. Finally, for the special case where all the links are identical, the CDF of γ_{DF} can be simplified as

$$\bar{C} = \frac{BW}{2\ln(2)} B^M \bar{\gamma}_{S,D} \exp(1/\bar{\gamma}_{S,D}) E_1 (1/\bar{\gamma}_{S,D}) + \frac{BW}{2\ln(2)} \sum_{k=1}^{M} (-1)^{k+1} \binom{M}{k} \frac{(1-B)^k}{\bar{\gamma}_{S,D} - \bar{\gamma}/k}$$
$$\times \left(\frac{E_1(k/\bar{\gamma})}{k/\bar{\gamma}} \exp(k/\bar{\gamma}) - \bar{\gamma}_{S,D} \exp(1/\bar{\gamma}_{S,D}) E_1 (1/\bar{\gamma}_{S,D}) \right) \tag{4.43}$$

4.5 Numerical Results and Analysis

In this section, we show numerical results of the analytical bit error rate (BER) and P_{out} for binary phase shift keying (BPSK) modulation. We plot the performance curves in terms of average BER and outage probability versus the SNR of the transmitted signal (E_s/N_0 dB) where E_s is the transmit energy signal. We also show the results of the computer simulations for verification.

4.5.1 Amplify-and-Forward

Figure 4.2 shows the BER performance of the best-relay selection scheme for different values of the number of relays (M). It can be noticed form Figure 4.2 that the derived lower bound is tight enough, especially at medium and high SNR values. For example, the exact BER (from simulation) for $M = 3$ at $E_s/N_0 = 15$ dB equals 7×10^{-6} while the analytical BER is 5×10^{-6}. This trend (the tightness of our bound) is valid at different values of M as shown in Figure 4.2. We can also notice that the BER decreases significantly with the increase in the number of relays (M) since the diversity gain and the virtual antenna array gain are monotonically increasing functions of M.

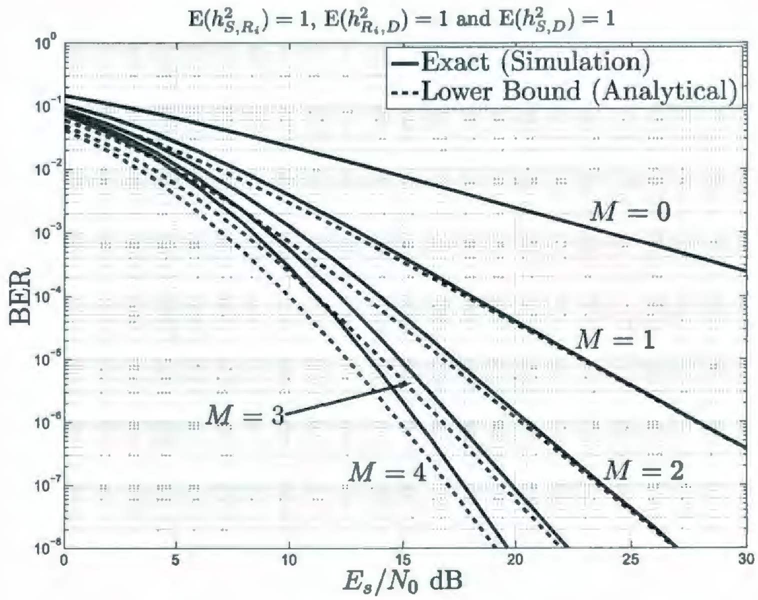

Figure 4.2: Error performance for the best-relay selection scheme over Rayleigh fading channels.

Figure 4.3 shows the P_{out} performance for $R = 1$ bit/sec/Hz. Again, it is obvious that the derived lower bound and the simulation results are in excellent agreement. It should be noted that for Figures 4.2 and 4.3 the tightness of the derived lower bounds (for BER and P_{out}) improves as E_s/N_0 increases; however, both bounds (for BER and P_{out}) slightly lose their tightness at low E_s/N_0 values, particularly when M increases.

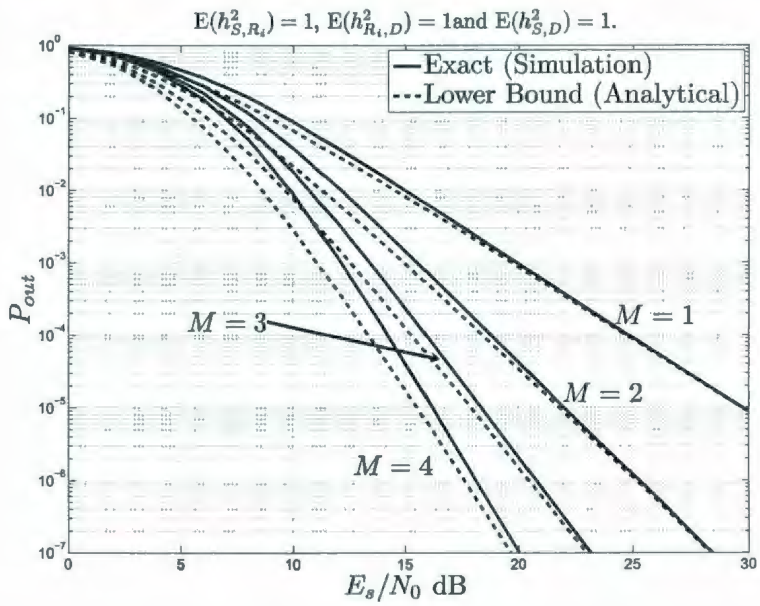

Figure 4.3: Outage performance for path the best-relay selection scheme over Rayleigh fading channels.

Figures 4.4 and 4.5 compare the performance of the best-relay selection scheme and the regular cooperative diversity in terms of the BER and P_{out} for different values of M. To make the comparisons fair, the transmitted power of the $M + 1$ transmitting nodes (the source node plus the M relays) in the regular cooperative system is set to $E_s = E_i = 1/(M + 1)$. For the best-relay selection scheme we have only two node (the source and the best relay), so $E_s = E_1 = 1/2$. From Figure 4.4, we can see an interesting result that the best-relay selection cooperative diversity scheme outperforms the regular cooperative diversity in terms of the BER. Also, we can see that as M increases this improvement also increases. This behavior is due to the efficient use of power by the best-relay selection scheme.

Figure 4.5 depicts the outage probability for $R = 1$ bit/sec/Hz. Figure 4.5 shows the dramatic improvement of the best-relay selection cooperative diversity over the regular one in terms of outage probability. In this figure, as M increases the outage probability of the regular cooperative diversity does not necessarily improve. Actually at low and medium SNR values, the outage probability increases. This is due to the fact that with regular cooperative diversity networks, when the number of relays increases, more channels are needed for relaying; hence, it becomes more difficult to achieve the required rate (R). This behavior is completely avoided in the best-relay selection scheme because we need only two orthogonal channels for transmissions regardless of the number of relays. Hence, increasing the number of relays in the best-relay selection scheme always improves the outage probability without any additional channel resources. This improvement does not depend on the value of E_s/N_0, unlike regular cooperative networks, where the value of E_s/N_0 determines whether increasing the number of relays will decrease the outage probability or not. For instance, increasing the number of relays (M) from 1 to 2, will reduce P_{out} regardless of the value of E_s/N_0, if the best-relay selection scheme is used. However, if regular cooperative diversity is employed, increasing M from 1 to 2 reduces P_{out} only if $E_s/N_0 > 23$ dB.

4.5.2 Decode-and-Forward

Figures 4.6 and 4.7 show the BER and outage probability of the best-relay selection adaptive decode-and-forward scheme at different values of the number of cooperating nodes ($M = 0, 1, 2, \text{and } 3$). Observe that the analytical results and the simulation results are in excellent agreement. From Figures 4.6 and 4.7, we can also notice that the number of cooperating relays (M) has a strong impact of the performance enhancement since the achieved diversity order is equal to $M + 1$.

Figures 4.8 and 4.9 show the performance comparison between the best-relay selection decode-and-forward scheme and the regular cooperative diversity where all relays participate in sending the source signal to the destination from chapter 2. From Figures 4.8 and 4.9, we can see an interesting result that the best-relay selection cooperative diversity scheme outperforms the regular cooperative diversity in terms of error probability. Also,

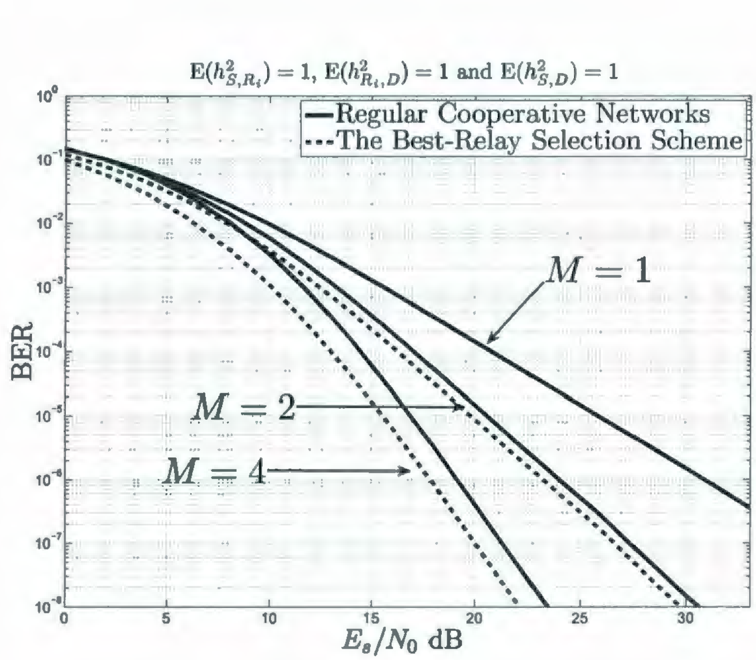

Figure 4.4: Comparison between the regular cooperative diversity and the best-relay selection scheme over Rayleigh fading channels (Note that for $M = 1$ the regular and best-relay selection scheme are the same).

we can see that as M increases this improvement also increases. This behavior is due to the efficient use of power in the best-relay selection scheme.

Figure 4.10 shows the substantial improvement of the best-relay selection cooperative diversity over the regular one in terms of average channel capacity. In particular, many interesting points can be drawn from Figure 4.10. First, the best-relay selection scheme outperforms the direct transmission in the low and medium SNR. Even more, the best-relay selection scheme can outperform the direct transmission at higher SNR if the total number of relays (M) can be increased. Second, the best-relay selection outperforms the regular cooperative diversity in all region of SNR. Furthermore, it is clearly seen that the gap between the best-relay selection scheme and regular cooperative diversity performance increases as M increases. Third, the average channel capacity of the best-relay selection scheme increases as M increases because the only dominant factor here is the SNR, which increases as M increases. On the other hand, the capacity of the regular cooperative diversity sys-

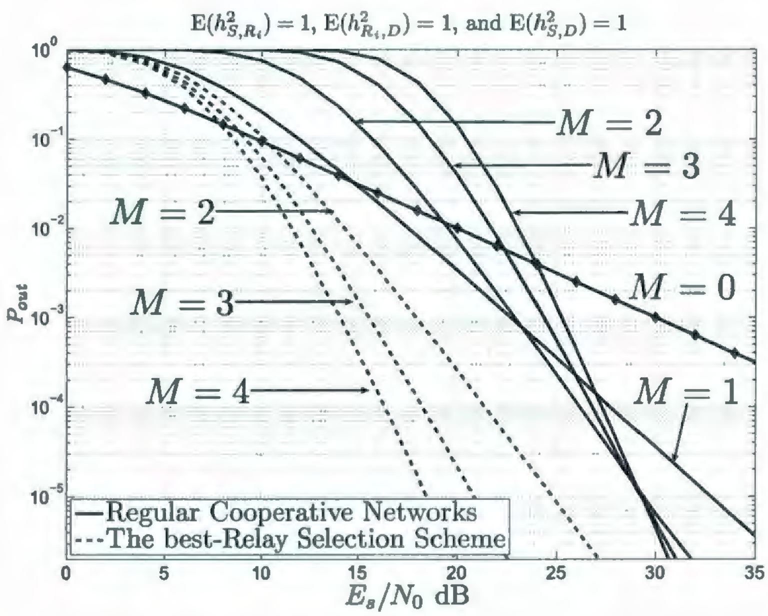

Figure 4.5: Comparison between the regular cooperative diversity and the best-relay selection scheme over Rayleigh fading channels (Note that for $M = 1$ the regular and best-relay selection scheme are the same).

tem decreases as M increases. The reason of this behavior is that, in regular cooperative diversity, there are two conflicting factors that affect the normalized average channel capacity, which is defined as $\frac{\bar{C}}{BW} = \frac{1}{M+1}\log_2(1+\gamma_{MRC})$. The first factor is the equivalent SNR improvement (γ_{MRC}). When M increases, the equivalent SNR at the MRC output increases, which improves the capacity. The second factor is the number of required orthogonal channels. When M increases, the number of required orthogonal channels ($= M+1$) will increase, which reduces the channel capacity. Results show that the second factor is more dominant, and as a result, the capacity of the regular cooperative diversity system always decreases with M.

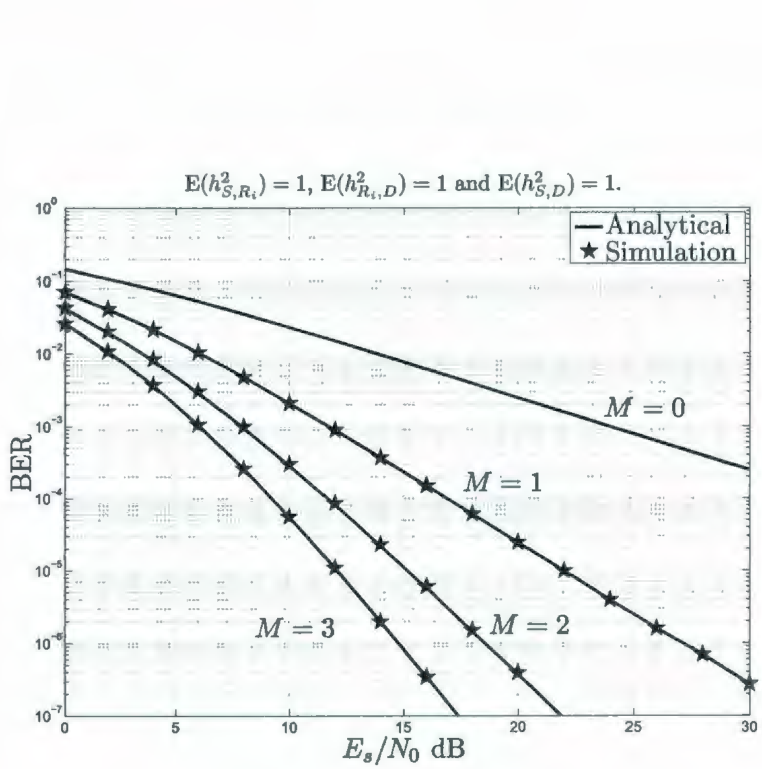

Figure 4.6: Error performance for the best- relay selection scheme over Rayleigh fading channels.

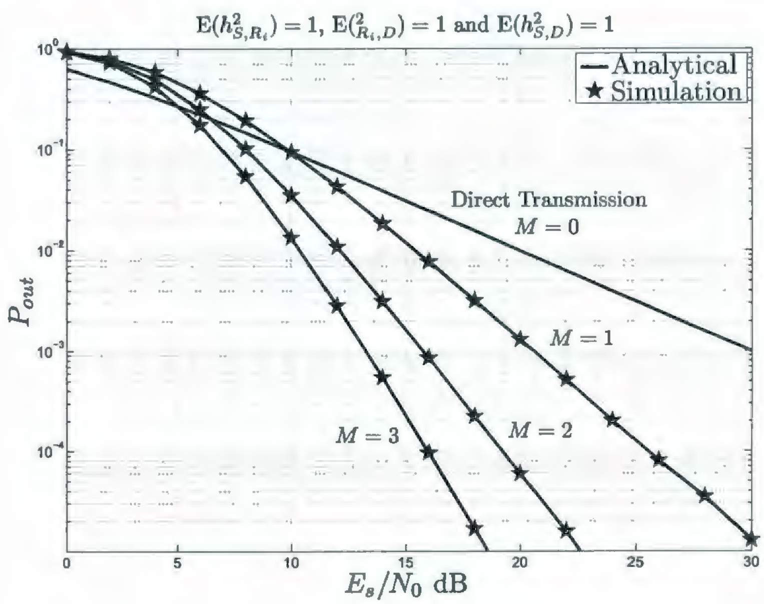

Figure 4.7: Outage performance for path the best- relay selection scheme over Rayleigh fading channels.

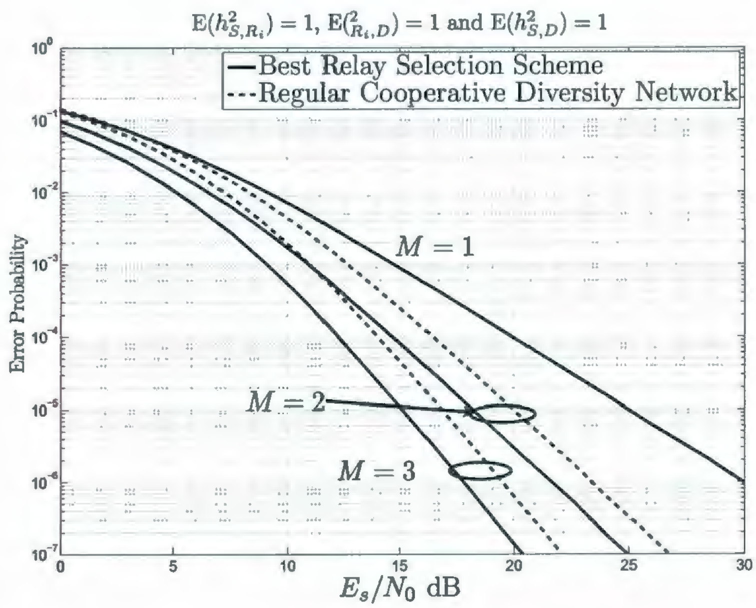

Figure 4.8: Comparison between the regular cooperative diversity with the best-relay selection cooperative diversity over Rayleigh fading channels. Note that for $M = 1$ regular and path-selection schemes have the same error performance.

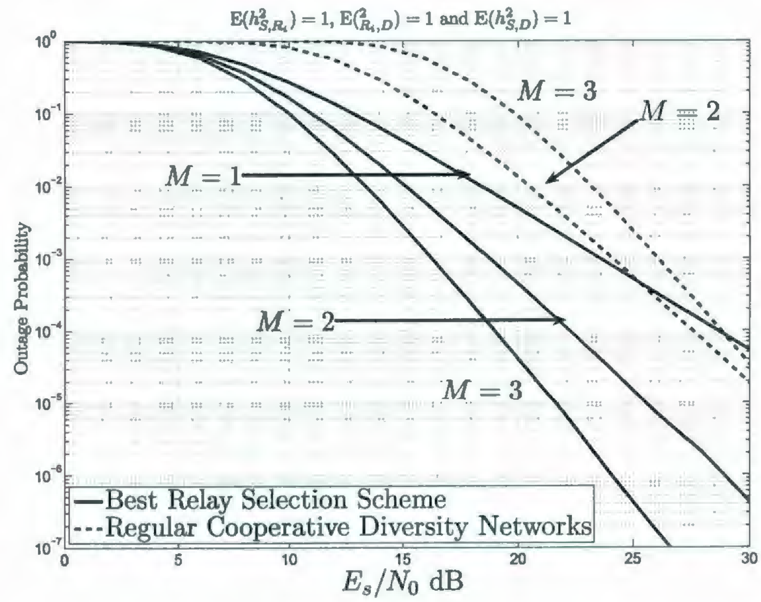

Figure 4.9: Outage probability comparison between the regular cooperative diversity with the best-relay selection cooperative diversity over Rayleigh fading channels. Note that for $M = 1$ regular and path-selection schemes have the same outage probability.

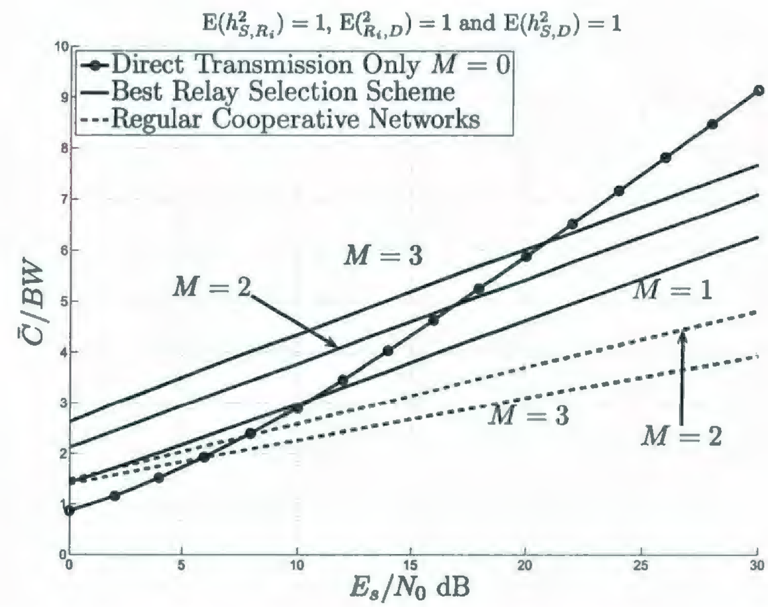

Figure 4.10: Outage probability comparison between the regular cooperative diversity with the best-relay selection cooperative diversity over Rayleigh fading channels. Note that for $M = 1$ regular and path-selection schemes have the same outage probability.

Chapter 5

Performance Analysis of Incremental Relaying Cooperative Networks Over Rayleigh Fading Channels

In this chapter the end-to-end performance of the incremental relaying cooperative diversity networks equipped with amplify-and-forward or decode-and-forward relays over independent non-identical Rayleigh fading channels will be investigated. In order to improve the spectral efficiency, incremental relaying cooperative diversity networks limit the relaying process to the necessary condition by exploiting limited feedback from the destination terminal, e.g., a single bit indicating the success or failure of the direct transmission. If the destination provides a negative acknowledgment via feedback the pre-determined relay retransmits instead of the source in an attempt to exploit spatial diversity by combining the signals that the destination receives from the source and the relay. Closed-form expressions for error probability and the outage probability are determined. Analysis proves that the incremental relaying cooperative relaying networks can achieve the maximum possible diversity and save the channel resources.

5.1 System Model

In Figure 5.1, the information source (S) and the destination (D) communicate over a channel with a slow and frequency-flat Rayleigh fading coefficient $h_{S,D}$. A relay terminal participates by providing the destination with a second copy of the original signal (when it is necessary) through a two hop-link with Rayleigh fading coefficients $h_{S,R}$ and $h_{R,D}$. We assume that all the additive white Gaussian noise (AWGN) terms in the three links ($S \rightarrow D$, $S \rightarrow R$ and $R \rightarrow D$) have equal variance N_0 and all the channels coefficients ($h_{S,D}, h_{S,R}, h_{R,D}$) are independent of each other. All terminals are equipped with a single antenna.

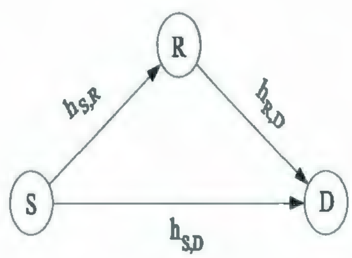

Figure 5.1: Illustration of a cooperative diversity network.

Assuming time division multiplexing for simplicity, in the first time slot, the source sends its signal. Both the relay and the destination receive faded noisy versions of this signal. Based on the quality of the received signal at the destination, the destination decides whether the relay should send another copy of the source signal or not. For sufficient signal quality, the relay will not send another copy of the source signal and the source will send a new message in the second time slot. For insufficient signal quality at the destination, the relay in the second time slot will forward the received source signal to the destination.

Mathematically speaking, the received signal from the source at the destination

$(y_{S,D}(t))$ and at the relay $(y_{S,R}(t))$ can be written as

$$\begin{aligned} y_{S,D}(t) &= h_{S,D}\sqrt{E_s}x(t) + n_1(t) \\ y_{S,R}(t) &= h_{S,R}\sqrt{E_s}x(t) + n_2(t) \end{aligned} \quad (5.1)$$

where E_s is the transmitted signal energy, $x(t)$ is a transmitted symbol signal with unit energy and $n_1(t)$ and $n_2(t)$ are the AWGN terms. In the second time slot, if it is necessary, the relay will process the received signal generates a signal $x_r(t)$ and transmits it to the destination. The received signal at the destination from the relay is given by

$$y_{R,D}(t) = h_{R,D}\sqrt{E_s}x_r(t) + n_3(t) \quad (5.2)$$

where $n_3(t)$ is the AWGN term of the $R \to D$ link.

We will focus on the case that the forwarding decision at the destination is made on the basis of the SNR forwarding threshold (γ_0), which defines the minimum SNR for which the destination can detect the signal successfully without the need of the relayed signal. While a large value of γ_0 lowers the probability of error, it reduces the bandwidth efficiency because the relay will forward the signal more often but this, of course, will increase the diversity benefit. Note that for direct transmission only, γ_0 is equal to 0 and for regular cooperative diversity networks in which the relay always resend the source signal to destination, γ_0 is equal to ∞.

5.2 Error Performance Analysis

In this section, we derive closed-form expressions for the error probability for the two relaying schemes, amplify-and-forward and decode-and-forward. The average unconditional error probability of the combined signal using the incremental relaying technique for both schemes, amplify-and-forward and decode-and-forward can be written as

$$P(e) = \Pr(\gamma_{S,D} \leq \gamma_0) \times P_{div}(e) + (1 - \Pr(\gamma_{S,D} \leq \gamma_0)) \times P_{direct}(e) \tag{5.3}$$

where $\gamma_{S,D}$ is the instantaneous SNR between S and D, $P_{div}(e)$ is the average probability that an error occurs in the combined diversity transmission from S and R to the D. The fading parameter $h_{S,D}$ follows the Rayleigh distribution; therefore, $\gamma_{S,D}$ follows the exponential distribution. Hence, it is straightforward to show that

$$\Pr(\gamma_{S,D} \leq \gamma_0) = 1 - \exp\left(-\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right) \tag{5.4}$$

where $\bar{\gamma}_{S,D} = \mathrm{E}\left(h_{S,D}^2\right) E_s/N_0$ is the average SNR between S and D and $P_{direct}(e)$ is the probability of error at the destination given that the destination decides that the relay should not forward the source signal, which case the destination needs to rely only on the direct signal from the source. For several Gray bit-mapped constellations employed in practical systems, the conditional error probability takes the form of $a \times \mathrm{erfc}\left(\sqrt{b\gamma_{S,D}}\right)$, with $\mathrm{erfc}(x)$ being the erfc-function defined as $\mathrm{erfc} = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp\left(-x^2\right) dx$ and (a, b) are constants depending on the type of modulation (e.g. BPSK: $a = 0.5$ and $b = 1$, QPSK: $a = 0.5$ and $b = 0.5$), then the corresponding average error probability ($P_{direct}(e)$) can be written as

$$P_{direct}(e) = \int_0^\infty P_{direct}(e|\gamma)\, f_{\gamma_{S,D}}(\gamma|\gamma_{S,D} > \gamma_0)\, d\gamma \tag{5.5}$$

where $P_{direct}(e|\gamma)$ is conditional error probability and $f_{\gamma_{S,D}}(\gamma|\gamma_{S,D} > \gamma_0)$ is the conditional PDF of $\gamma_{S,D}$ given that $\gamma_{S,D}$ is greater than the threshold value γ_0. The conditional PDF

$f_{\gamma_{S,D}}(\gamma|\gamma_{S,D} > \gamma_0)$ can be found to be as

$$f_{\gamma_{S,D}}(\gamma|\gamma_{S,D} > \gamma_0) = \begin{cases} 0, & \text{if } \gamma \leq \gamma_0; \\ \frac{\exp(\gamma_0/\bar{\gamma}_{S,D})}{\bar{\gamma}_{S,D}} \exp\left(\frac{-\gamma}{\bar{\gamma}_{S,D}}\right), & \text{if } \gamma > \gamma_0 \end{cases} \tag{5.6}$$

Substituting (5.6) into (5.5) and by solving the integration and doing some necessary manipulations, the average error probability can be written in a closed-from as

$$P_{direct}(e) = a \operatorname{erfc}\left(\sqrt{b\gamma_0}\right) - a \exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right) \sqrt{\frac{b\bar{\gamma}_{S,D}}{1 + b\bar{\gamma}_{S,D}}} \operatorname{erfc}\left(\sqrt{\gamma_0 \left(b + 1/\bar{\gamma}_{S,D}\right)}\right) \tag{5.7}$$

Note that for $\gamma_0 = 0$, $a = 0.5$ and $b = 1$, we obtain the well known probability of error for BPSK transmission over Rayleigh fading channel [39]. Furthermore, we should note that as γ_0 increases, the $P_{direct}(e)$ decreases, which plays an important role for high SNR region as we will see later. If the relay should resend another copy of the source signal, then the destination combines the signals it receives from source and relay using maximum ratio combining (MRC). The expression of $P_{div}(e)$ depends on the relaying scheme that will be used at the relay as discussed in the following subsections.

5.2.1 Amplify-and-Forward

In the amplify-and-forward scheme, the relayed signal ($x_r(t)$) is an amplified version of $y_{S,R}(t)$ and can be written as

$$x_r(t) = Gx(t) \tag{5.8}$$

where G is the gain factor. In case of the relay to maintain the unit energy transmitted signal, this gain can be written as

$$G = \sqrt{\frac{1}{E_s h_{S,R}^2 + N_0}} \tag{5.9}$$

In (5.9), we choose the amplifier gain to depend upon the fading coefficient $h_{S,R}$, which the relay can estimate with high accuracy. The choice of this gain aims to invert the fading effect of the first hop to limit the output energy from the relay to be E_s.

In order to calculate $P_{div}(e)$ we need to know the outpout SNR at the destination. By assuming that MRC technique is employed at the destination, the instantaneous output SNR is the sum of the instantaneous SNRs of the direct and the indirect (cascaded) links

$$\gamma_{AF} = \gamma_{S,R,D} + \gamma_{S,D} \tag{5.10}$$

where $\gamma_{S,R,D}$ is the equivalent instantaneous SNR for $S \to R \to D$. By using the same method adopted in chapter 2, the equivalent SNR can be written as

$$\gamma_{S,R,D} = \frac{\gamma_{S,R}\gamma_{R,D}}{\gamma_{S,R} + \gamma_{R,D} + 1} \tag{5.11}$$

The output SNR γ_{AF} can be approximated by its upper bound (γ_b) as follows

$$\gamma_{AF} \leq \gamma_b = \gamma_{S,D} \min\left(\gamma_{S,R}, \gamma_{R,D}\right) \tag{5.12}$$

where $\min(x, y)$ is the minimum value of x and y. The approximate SNR value (γ_b) is analytically more tractable than the exact value (γ_{AF}); and as a result, this facilitates the derivation of the SNR statistics (CDF and PDF).

Since $\gamma_{S,R}$ and $\gamma_{R,D}$ are exponentially distributed, the PDF of $\min\left(\gamma_{S,R}, \gamma_{R,D}\right)$ is also exponential with a mean $\bar{\gamma} = \frac{\bar{\gamma}_{S,R}\bar{\gamma}_{R,D}}{\bar{\gamma}_{S,R}+\bar{\gamma}_{R,D}+1}$. The unconditional error rate ($P_{div}(e)$) can be computed for the two cases $\bar{\gamma}_{S,R} \neq \bar{\gamma}_{R,D}$ and $\bar{\gamma}_{S,R} = \bar{\gamma}_{R,D}$ as follows.

By defining a new variable $Z = \gamma_{S,D} + \min\left(\gamma_{S,R}, \gamma_{R,D}\right)$; then the average error probability $P_{div}(e)$ can be written as

1. **Case of $\bar{\gamma}_{S,D} \neq \bar{\gamma}$:**

$$P_{div}(e) = a \int_0^{\infty} f_Z(z|\gamma_{S,D} \leq \gamma_0) \operatorname{erfc}\left(\sqrt{bz}\right) dz \tag{5.13}$$

where $f_Z(z|\gamma_{S,D} \leq \gamma_0)$ can be derived as

$$f_z(z|\gamma_{S,D} \leq \gamma_0) = \begin{cases} \frac{\exp(-z/\bar{\gamma}) - \exp(-z/\bar{\gamma}_{S,D})}{(\bar{\gamma}-\bar{\gamma}_{S,D})(1-\exp(-\gamma_0/\bar{\gamma}_{S,D}))}, & \text{if } z \leq \gamma_0; \\ \frac{1-\exp(-\gamma_0/(1/\bar{\gamma}_{S,D}-1/\bar{\gamma}))}{(\bar{\gamma}-\bar{\gamma}_{S,D})(1-\exp(-\gamma_0/\bar{\gamma}_{S,D}))} \exp(-z/\bar{\gamma}), & \text{if } z > \gamma_0 \end{cases} \tag{5.14}$$

Then, $P_{div}(e)$ reduces to

$$\begin{aligned} P_{div}(e) = & \frac{a}{(\bar{\gamma}-\bar{\gamma}_{S,D})\left(1-\exp\left(-\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)} \left\{ \bar{\gamma}_{S,D}\sqrt{\frac{b\bar{\gamma}_{S,D}}{1+b\bar{\gamma}_{S,D}}} \operatorname{erf}\left(\sqrt{\lambda}\right) - \bar{\gamma}\sqrt{\frac{b\bar{\gamma}}{1+b\bar{\gamma}}} \operatorname{erf}\left(\sqrt{\zeta}\right) \right. \\ & + \exp\left(\frac{-\gamma_0}{\frac{1}{\bar{\gamma}_{S,D}}+\frac{1}{\bar{\gamma}}}\right)\left(\bar{\gamma}_{S,D}\exp\left(\frac{\gamma_0}{\bar{\gamma}}\right) - \bar{\gamma}\exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right) \operatorname{erfc}\left(\sqrt{b\gamma_0}\right) - (\bar{\gamma}_{S,D}-\bar{\gamma}) \\ & \left. + \left(1-\exp\left(\frac{-\gamma_0}{\frac{1}{\bar{\gamma}_{S,D}}-\frac{1}{\bar{\gamma}}}\right)\right)\left[\bar{\gamma}\exp\left(\frac{-\gamma_0}{\bar{\gamma}}\right)\operatorname{erfc}\left(\sqrt{b\gamma_0}\right) - \bar{\gamma}\sqrt{\frac{b\bar{\gamma}}{1+b\bar{\gamma}}}\operatorname{erfc}\left(\sqrt{\zeta}\right)\right]\right\} \end{aligned} \tag{5.15}$$

where $\lambda = \frac{\gamma_0(1+b\bar{\gamma}_{S,D})}{\bar{\gamma}_{S,D}}$ and $\zeta = \frac{\gamma_0(1+b\bar{\gamma})}{\bar{\gamma}}$.

2. **Case of $\bar{\gamma}_{S,D} = \bar{\gamma}$:**

   In this case $f_Z(z|\gamma_{S,D} \leq \gamma_0)$ can be written as

$$f_Z(z|\gamma_{S,D} \leq \gamma_0) = \begin{cases} \frac{z\exp(-z/\bar{\gamma}_{S,D})}{\bar{\gamma}_{S,D}^2(1-\exp(-\gamma_0/\bar{\gamma}_{S,D}))}, & \text{if } z \leq \gamma_0; \\ \frac{\gamma_0}{\bar{\gamma}_{S,D}^2(1-\exp(-\gamma_0/\bar{\gamma}_{S,D}))} \exp(-z/\bar{\gamma}_{S,D}), & \text{if } z > \gamma_0 \end{cases} \tag{5.16}$$

Substituting (5.16) into (5.13) and doing the integral and after some simplifications

$P_{div}(e)$ can be written as

$$P_{div}(e) = \frac{a}{1-\exp\left(-\frac{\gamma_0}{\bar{\gamma}}\right)} \times \left\{ 1 + \frac{0.5}{1+b\bar{\gamma}}\sqrt{\frac{4b\gamma_0}{\pi}}\exp\left(\frac{-\gamma_0}{1+b\bar{\gamma}}\right) - \frac{\bar{\gamma}+\gamma_0}{\bar{\gamma}}\exp\left(\frac{-\gamma_0}{\bar{\gamma}}\right) \right.$$
$$\times \operatorname{erfc}\left(\sqrt{b\gamma_0}\right) - \sqrt{\frac{b\bar{\gamma}}{1+b\bar{\gamma}}}\operatorname{erf}\left(\sqrt{\lambda}\right) - \frac{0.5}{1+b\bar{\gamma}}\sqrt{\frac{b\bar{\gamma}}{1+b\bar{\gamma}}}\operatorname{erf}\left(\sqrt{\lambda}\right)$$
$$\left. + \frac{\gamma_0}{\bar{\gamma}}\exp\left(\frac{-\gamma_0}{\bar{\gamma}}\right)\operatorname{erfc}\left(\sqrt{b\gamma_0}\right) - \frac{\gamma_0}{\bar{\gamma}^2}\sqrt{\frac{b\bar{\gamma}}{1+b\bar{\gamma}}}\operatorname{erfc}\left(\sqrt{\lambda}\right) \right\} \quad (5.17)$$

By substituting $P_{div}(e)$, (5.7) and (5.4) into (5.3) we can have a closed-form expression for the error probability of the amplify-and-forward incremental relaying cooperative relaying protocol over Rayleigh flat fading channels.

5.2.2 Decode-and-Forward

In decode-and-forward scheme, the relayed signal ($x_r(t)$) is firstly detected and then re-transmitted to the destination. Thus, the resulting error probability $P_{div}(e)$ can be written as:

$$P_{div}(e) = P_{(S,R)}(e)\, P_x(e) + \left(1 - P_{(S,R)}(e)\right) P_{com}(e) \quad (5.18)$$

where $P_{(S,R))}(e)$ is the probability of error at the relay, $P_x(e)$ is the probability that an error happens at the destination given that the relay decoded unsuccessfully, and $P_{com}(e)$ is the probability that an error happens at the destination given that the relay decoded correctly. Therefore, two mutually exclusive error events may lead to a decision error at the destination. Firstly, there is the possibility of a decision error at the relay, which occurs with probability $P_{(S,R)}(e)$ and depends on the average SNR at the relay $\bar{\gamma}_{S,R} = \mathbf{E}\left(h_{S,R}^2\right) E_s/N_0$. The probability of error at the relay can be written as [23]

$$P_{(S,R)}(e) = a\left(1 - \sqrt{\frac{b\bar{\gamma}_{S,R}}{1+b\bar{\gamma}_{S,R}}}\right) \quad (5.19)$$

In case of a decision error at the relay, the relay retransmits an erroneous signal to the destination; thus leading to error propagation. The error probability due to error propagation (P_x) can be bounded with the worst case value, $P_x < 0.5$ [44].

Secondly, when the relay decoded correctly (with probability $(1 - P_{(S,R)}(e))$, it forwards a freshly correct signal to the destination. This is the desired case of useful spatial diversity, since the destination combines this signal with the version it has received from the source. The resulting error rate $P_{com}(e)$ depends on $\bar{\gamma}_{S,D} = \mathbf{E}\left(h_{S,D}^2\right) E_s/N_0$ and $\bar{\gamma}_{R,D} = \mathbf{E}\left(h_{R,D}^2\right) E_s/N_0$ for the channel from the relay to the destination. By defining a new random variable X; then the average error probability $P_{com}(e)$ can be written as

1. **Case of $\bar{\gamma}_{S,D} \neq \bar{\gamma}_{S,D}$:** By defining a new variable $X = \gamma_{S,D} + \gamma_{R,D}$, then the average error probability $P_{com}(e)$ can be written as

$$P_{com}(e) = a \int_0^{\infty} f_X (x|\gamma_{S,D} \leq \gamma_0) \operatorname{erfc}\left(\sqrt{bx}\right) dx \tag{5.20}$$

where $f_X (x|\gamma_{S,D} \leq \gamma_0)$ can be derived as

$$f_X (x|\gamma_{S,D} \leq \gamma_0) = \begin{cases} \frac{\exp(-x/\bar{\gamma}_{R,D}) - \exp(-x/\bar{\gamma}_{S,D})}{(\bar{\gamma}_{R,D} - \bar{\gamma}_{S,D})(1 - \exp(-\gamma_0/\bar{\gamma}_{S,D}))}, & \text{if } x \leq \gamma_0; \\ \frac{1 - \exp(-\gamma_0/(1/\bar{\gamma}_{S,D} - 1/\bar{\gamma}_{R,D}))}{(\bar{\gamma}_{R,D} - \bar{\gamma}_{S,D})(1 - \exp(-\gamma_0/\bar{\gamma}_{S,D}))} \exp\left(-x/\bar{\gamma}_{R,D}\right), & \text{if } x > \gamma_0 \end{cases} \tag{5.21}$$

Substituting (5.21) into (5.20) and doing the integral and after some simplifications

$P_{com}(e)$ can be obtained as

$$P_{com}(e) = \frac{a\left(1-\exp\left(-\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)^{-1}}{\left(\bar{\gamma}_{R,D}-\bar{\gamma}_{S,D}\right)}\left\{\bar{\gamma}_{S,D}\sqrt{\frac{b\bar{\gamma}_{S,D}}{1+b\bar{\gamma}_{S,D}}}\operatorname{erf}\left(\sqrt{\lambda}\right)-\bar{\gamma}_{R,D}\sqrt{\frac{b\bar{\gamma}_{R,D}}{1+b\bar{\gamma}_{R,D}}}\operatorname{erf}\left(\sqrt{\zeta}\right)\right.$$
$$+\exp\left(\frac{-\gamma_0}{\frac{1}{\bar{\gamma}_{S,D}}+\frac{1}{\bar{\gamma}_{R,D}}}\right)\left(\bar{\gamma}_{S,D}\exp\left(\frac{\gamma_0}{\bar{\gamma}_{R,D}}\right)-\bar{\gamma}_{R,D}\exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)\operatorname{erfc}\left(\sqrt{b\gamma_0}\right)$$
$$-\left(\bar{\gamma}_{S,D}-\bar{\gamma}_{R,D}\right)+\left(1-\exp\left(\frac{-\gamma_0}{\frac{1}{\bar{\gamma}_{S,D}}-\frac{1}{\bar{\gamma}_{R,D}}}\right)\right)$$
$$\left.\times\left[\bar{\gamma}_{R,D}\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{R,D}}\right)\operatorname{erfc}\left(\sqrt{b\gamma_0}\right)-\bar{\gamma}_{R,D}\sqrt{\frac{b\bar{\gamma}_{R,D}}{1+b\bar{\gamma}_{R,D}}}\operatorname{erfc}\left(\sqrt{\zeta}\right)\right]\right\} \tag{5.22}$$

where $\lambda = \frac{\gamma_0(1+b\bar{\gamma}_{S,D})}{\bar{\gamma}_{S,D}}$ and $\zeta = \frac{\gamma_0(1+b\bar{\gamma}_{R,D})}{\bar{\gamma}_{R,D}}$.

2. **Case of $\bar{\gamma}_{S,D} = \bar{\gamma}_{S,D}$:**

In this case $f_X\left(x|\gamma_{S,D}\leq\gamma_0\right)$ can be derived as

$$f_X\left(x|\gamma_{S,D}\leq\gamma_0\right) = \begin{cases} \frac{x\exp(-x/\bar{\gamma}_{S,D})}{\bar{\gamma}_{S,D}^2\left(1-\exp(-\gamma_0/\bar{\gamma}_{S,D})\right)}, & \text{if } x\leq\gamma_0; \\ \frac{\gamma_0}{\bar{\gamma}_{S,D}^2\left(1-\exp(-\gamma_0/\bar{\gamma}_{S,D})\right)}\exp\left(-x/\bar{\gamma}_{S,D}\right), & \text{if } x>\gamma_0 \end{cases} \tag{5.23}$$

Substituting (5.23) into (5.20) and doing the integral and after some simplifications Pcom(e) can be written as

$$P_{div}(e) = \frac{a}{1-\exp\left(-\frac{\gamma_0}{\bar{\gamma}_{R,D}}\right)}$$
$$\times\left\{1+\frac{0.5}{1+b\bar{\gamma}_{R,D}}\sqrt{\frac{4b\gamma_0}{\pi}}\exp\left(\frac{-\gamma_0}{1+b\bar{\gamma}_{R,D}}\right)-\frac{\bar{\gamma}_{R,D}+\gamma_0}{\bar{\gamma}_{R,D}}\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{R,D}}\right)\right.$$
$$\times\operatorname{erfc}\left(\sqrt{b\gamma_0}\right)-\sqrt{\frac{b\bar{\gamma}_{R,D}}{1+b\bar{\gamma}_{R,D}}}\operatorname{erf}\left(\sqrt{\lambda}\right)-\frac{0.5}{1+b\bar{\gamma}_{R,D}}\sqrt{\frac{b\bar{\gamma}_{R,D}}{1+b\bar{\gamma}_{R,D}}}\operatorname{erf}\left(\sqrt{\lambda}\right)$$
$$\left.+\frac{\gamma_0}{\bar{\gamma}_{R,D}}\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{R,D}}\right)\operatorname{erfc}\left(\sqrt{b\gamma_0}\right)-\frac{\gamma_0}{\bar{\gamma}_{R,D}^2}\sqrt{\frac{b\bar{\gamma}_{R,D}}{1+b\bar{\gamma}_{R,D}}}\operatorname{erfc}\left(\sqrt{\lambda}\right)\right\} \tag{5.24}$$

By substituting (5.19) and (5.22) or (5.24) into (5.18) and then substituting (5.4), (5.7) and (5.18) into (5.3) we can have a closed-form expression for the error probability of the decode-and-forward bandwidth efficient cooperative relaying protocol over Rayleigh flat fading channels.

5.3 Outage Probabilities Analysis

In this subsection, we derive closed-form expressions for the outage probability (P_{out}). In incremental relaying cooperative diversity networks, if the SNR of direct link at the destination is less than the threshold value γ_0, the destination will need assistance from the relay to send another copy of the source signal. In this case, the relay will send another copy of the signal but there is still a probability that the overall SNR at the destination is less than γ_0, and in this subsection we will determine this probability.

5.3.1 Amplify-and-Forward

The outage probability can be easily derived as

$$\begin{aligned} P_{out} &= \Pr(\gamma_{S,D} \leq \gamma_0) \Pr(\gamma_{S,R,D} + \gamma_{S,D} \leq \gamma_0 | \gamma_{S,D} \leq \gamma_0) \\ &= \Pr(\gamma_{S,R,D} + \gamma_{S,D} \leq \gamma_0) \\ &= \Pr(\gamma_{AF} \leq \gamma_0) \end{aligned} \tag{5.25}$$

By using the same approximation for γ_{AF} in (5.12) (i.e., using γ_b) and noting that γ_b follows the exponential distribution; then, P_{out} can be derived and written in a closed form as

$$P_{out} = \begin{cases} 1 + \frac{\bar{\gamma}}{\bar{\gamma}_{S,D} - \bar{\gamma}} \exp\left(\frac{-\gamma_0}{\bar{\gamma}}\right) - \frac{\bar{\gamma}_{S,D}}{\bar{\gamma}_{S,D} - \bar{\gamma}} \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right), & \text{if } \bar{\gamma}_{S,D} \neq \bar{\gamma}; \\ 1 - \frac{\gamma_0 + \bar{\gamma}_{S,D}}{\bar{\gamma}_{S,D}} \exp\left(-\gamma_0 / \bar{\gamma}_{S,D}\right), & \text{if } \bar{\gamma}_{S,D} = \bar{\gamma} \end{cases} \tag{5.26}$$

5.3.2 Decode-and-Forward

The outage probability can be easily derived as

$$\begin{aligned}
P_{out} =& \Pr(\gamma_{S,D} \leq \gamma_0) \Pr[\min(\gamma_{S,R}, \gamma_{S,D} + \gamma_{R,D} \leq \gamma_0) | \gamma_{S,D} \leq \gamma_0] \\
=& \Pr(\gamma_{S,D} \leq \gamma_0) [1 - \Pr(\gamma_{S,D} + \gamma_{R,D} > \gamma_0 | \gamma_{S,D} \leq \gamma_0) \Pr(\gamma_{S,R} > \gamma_0 | \gamma_{S,D} \leq \gamma_0)] \\
=& \Pr(\gamma_{S,D} \leq \gamma_0) \left[1 - \frac{\Pr(\gamma_{S,D} \leq \gamma_0) - \Pr(\gamma_{S,D} + \gamma_{R,D} \leq \gamma_0)}{\Pr(\gamma_{S,D} \leq \gamma_0)} \Pr(\gamma_{S,R} > \gamma_0)\right] \quad (5.27)
\end{aligned}$$

where $\Pr(\gamma_i \leq \gamma_0) = 1 - \exp(-\gamma_0/\bar{\gamma}_i)$ with $i \in \{h_{S,D}, h_{R,D}\}$. The first term in $\min(\bullet)$ function represents the outage probability at the relay and the second term represents the outage probability at the destination when MRC is used. Finally after simplification, the outage probability can be written as:

$$P_{out} = \Pr(\gamma_{S,R} \leq \gamma_0) \Pr(\gamma_{S,D} \leq \gamma_0) + \Pr(\gamma_{S,R} > \gamma_0) \Pr(\gamma_{S,R} + \gamma_{R,D} \leq \gamma_0) \quad (5.28)$$

The term $\Pr(\gamma_{S,R} + \gamma_{R,D} \leq \gamma_0)$ can be easily derived as:

$$\Pr(\gamma_{S,R} + \gamma_{R,D} \leq \gamma_0) = \begin{cases} 1 + \frac{\bar{\gamma}_{R,D} \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{R,D}}\right) - \bar{\gamma}_{S,D} \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)}{\bar{\gamma}_{S,D} - \bar{\gamma}_{R,D}}, & \text{if } \bar{\gamma}_{S,D} \neq \bar{\gamma}_{R,D}; \\ 1 - \frac{\gamma_0 + \bar{\gamma}_{S,D}}{\bar{\gamma}_{S,D}} \exp(-\gamma_0/\bar{\gamma}_{S,D}), & \text{if } \bar{\gamma}_{S,D} = \bar{\gamma}_{R,D} \end{cases} \quad (5.29)$$

Then P_{out} of decode-and-forward technique can be written as

$$P_{out} = \begin{cases} \left(1 - \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,R}}\right)\right)\left(1 - \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\right) + \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,R}}\right) & \\ \times \left[1 + \frac{\bar{\gamma}_{R,D} \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{R,D}}\right) - \bar{\gamma}_{S,D} \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)}{\bar{\gamma}_{S,D} - \bar{\gamma}_{R,D}}\right], & \text{if } \bar{\gamma}_{S,D} \neq \bar{\gamma}_{R,D}; \\ \left(1 - \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,R}}\right)\right)\left(1 - \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\right) + \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,R}}\right) & \\ \times \left[1 - \frac{\gamma_0 + \bar{\gamma}_{S,D}}{\bar{\gamma}_{S,D}} \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\right], & \text{if } \bar{\gamma}_{S,D} = \bar{\gamma}_{R,D} \end{cases} \quad (5.30)$$

5.4 Normalized Throughput Analysis

Throughput is the amount of data which is successfully transmitted per unit time, that is $R/\mathbf{E}(T)$ [45], where R bits/sec/Hz is the target rate, and $\mathbf{E}(T)$ is the average delay, i.e., the expected number of transmissions (original transmission plus retransmission) is given by

$$\mathbf{E}(T) = 1 \times \Pr(T = 1) + 2 \times \Pr(T = 2) \tag{5.31}$$

where 1 represents only one time slot needed and 2 represents two time slots needed. Now, consider the probability of each of the two cases. In case 1, the transmission takes only one time slot. The probability of successful transmission after the first time slot is given by $\Pr(T = 1) = p_1 = \exp(-\gamma_0/\bar{\gamma}_{S,D})$, and the probability of successful transmission after the two time slots is given by $\Pr(T = 2) = 1 - \exp(-\gamma_0/\bar{\gamma}_{S,D}) = 1 - p_1$. Finally, after simple manipulations, the throughput can be written as

$$\text{Throughput} = \frac{R}{2 - p_1} \tag{5.32}$$

From (5.32) we can see as p_1 decreases the throughput decreases. For regular cooperative diversity networks, $p_1 = 0$ and the throughput is $R/2$, while for direct transmission alone, $p_1 = 1$ and the throughput is R (the best value). It is clearly seen the throughput of the incremental cooperative diversity techniques lies between $R/2$ and R.

5.5 Numerical Results and Analysis

In this section, we show numerical results of the analytical bit error rate (BER) for binary phase shift keying (BPSK) modulation, and outage probability. We plot the performance curves of the average BER, and outage probability versus the SNR of the transmitted signal (E_s/N_0 dB). We also show the results of the computer simulations for verification. Asymmetric network geometry is examined where the relay is located across the straight line connecting the source and the destination. Direct path length $S \rightarrow D$ is normalized to be equal to 1. We also denote d as the distance between the source and the relay. In all presented results, the path-loss exponent α is assumed to be equal to 3.

Figures 5.2 and 5.3 show the bit error rate (BER) for different values of γ_0 for both schemes amplify-and-forward and decode-and-forward respectively, using the incremental relaying technique. Observe that the results obtained using the closed-form expressions derived in this paper and the simulation results are in excellent agreement. Figures 5.2 and 5.3 demonstrate that the cooperation significantly improves the BER performance in comparison with the direct transmission. This is expected because the cooperation benefits from the diversity gain as well as from the path-loss reduction. Also, Figure 5.2 shows that as γ_0 increases, the error performance improves because we will benefit more from the diversity. Figure 5.2 shows also that at high SNR, the error performance for incremental relaying scheme tends to be parallel with direct transmission, which means that the system achieves virtual antenna array gain but not diversity gain. This is because at high SNR, the destination will rarely need any retransmission from the relay. Finally, from Figures 5.2 and 5.3 we can see that in the amplify-and-forward scheme, increasing γ_0 will always increase the error performance until we reach the optimum one which is $\gamma_0 = \infty$ where the relay always resends a copy of the source signal (regular cooperative diversity networks). Actually an increasing γ_0 for decode-and-forward will not always improves the error performance. In particular we need to be careful in choosing γ_0 in decode-and-forward. As expected for $\gamma_0 = \infty$, the performance will not improve in all region of SNR due to the error propagation from the relay, since the relay will always resend the source signal and there is a probability to send an erroneous signal (the improvement can be seen in very high SNR because of a very small probability of error propagation. This behavior will be clearer when we discuss Figure 5.6).

Figure 5.4 shows the throughput of incremental relaying cooperative diversity network for $R = 1$ bit/s/Hz. Figure 5.4 shows when the SNR detection threshold (γ_0) increases the throughput decreases. Figure 5.4 also shows that incremental relaying cooperative diver-

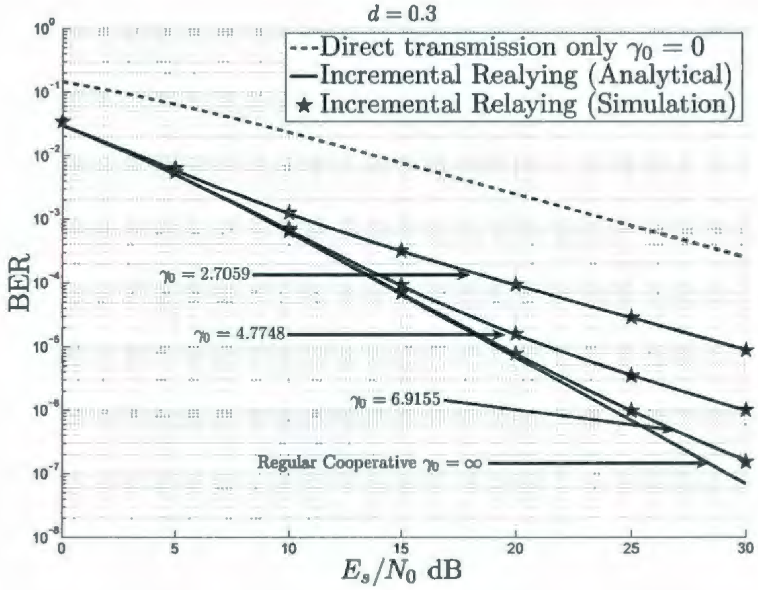

Figure 5.2: Error probability performance for amplify-and-forward incremental relaying for different values of γ_0. Note that direct transmission equivalent to $\gamma_0 = 0$ and regular cooperative diversity equivalent to $\gamma_0 = \infty$.

sity network gives high throughput (very close to direct system) at medium and high SNR while incremental relaying has much better error performance. For example, for incremental relaying at 15 dB, the throughput reaches 0.85 with $P(e) = 7 \times 10^{-5}$ for amplify-and-forward, $P(e) = 1 \times 10^{-4}$ for decode-and-forward with $\gamma_0 = 6.92$, while direct transmission has throughput of 1 and $P(e) = 10^{-2}$. Hence, we can conclude that incremental relaying has a significant improvement on error performance with a very high throughput (very close to the direct transmission throughput and higher than the regular cooperative diversity).

Figure 5.5 shows the outage probability performance defined in the previous section. From Figure 5.5, we can conclude that as γ_0 increases the outage probability increases. The reason is that as γ_0 increases the probability that the summation of the SNR of the combined signal is less than γ_0 will also increase. Furthermore, amplify-and-forward scheme slightly outperforms decode-and-forward scheme.

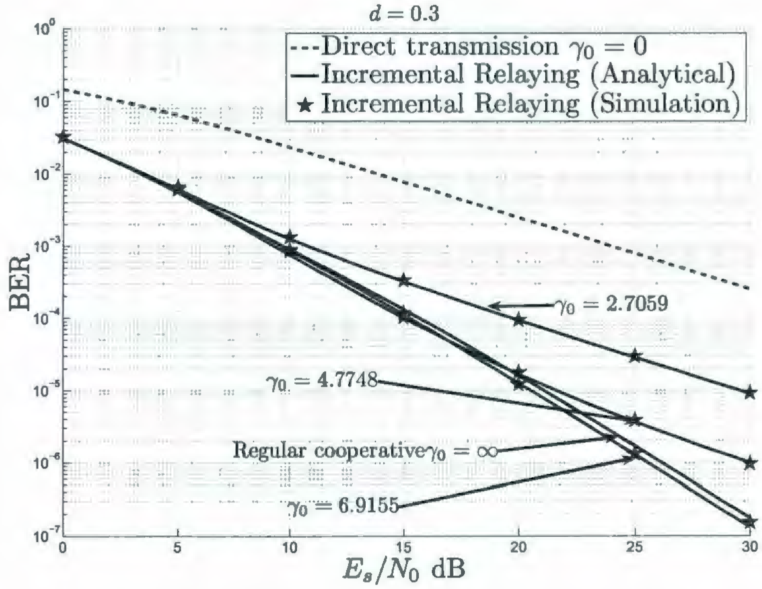

Figure 5.3: Error probability performance for decode-and-forward incremental relaying for different values of γ_0. Note that direct transmission equivalent to $\gamma_0 = 0$ and regular cooperative diversity equivalent to $\gamma_0 = \infty$.

Figure 5.6 shows the effect of the location of the relay for different values of γ_0. From this figure it can be seen that the best location for amplify-and-forward scheme is in the middle between the source and the relay and this location will not change even if we change the value of γ_0. Also, for amplify-and-forward increasing γ_0 will always improve the overall system performance. For decode-and-forward, we can see that the location of the relay plays an important role on the performance of the system. In decode-and-forward scheme the best place for the relay is to be nearer to the source. The reason of this trend is that when the relay is close to the source error propagation rarely happens due to the good channel between the source and the relay. Also, we can see that increasing γ_0 has a bad impact in some locations and this is due to the error propagation problem. Finally, we can notice that amplify-and-forward in some locations has dramatic superior performance over decode-and-forward.

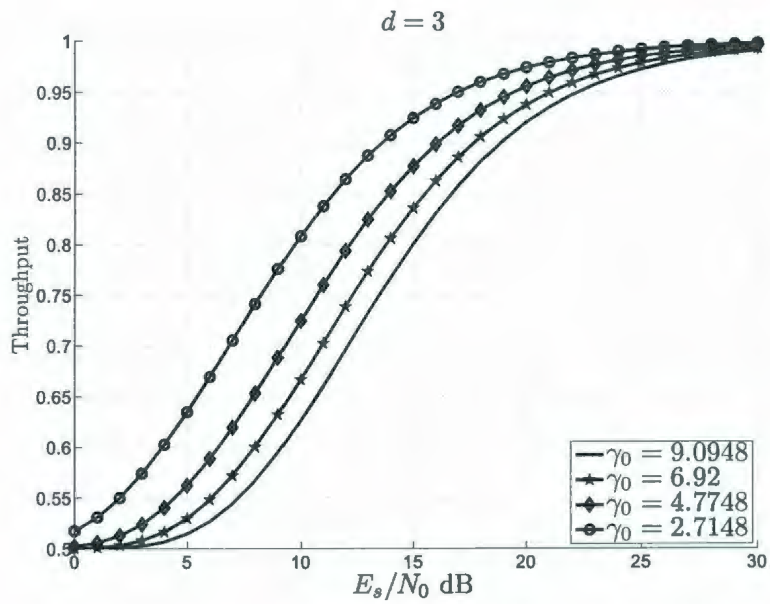

Figure 5.4: Throughput of incremental relaying system for different values of γ_0.

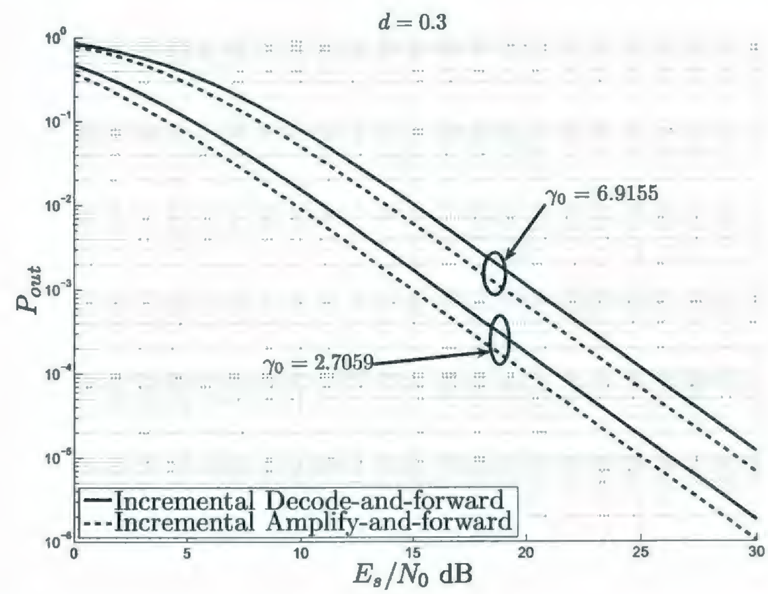

Figure 5.5: Outage probability performance for incremental relaying for different values of γ_0.

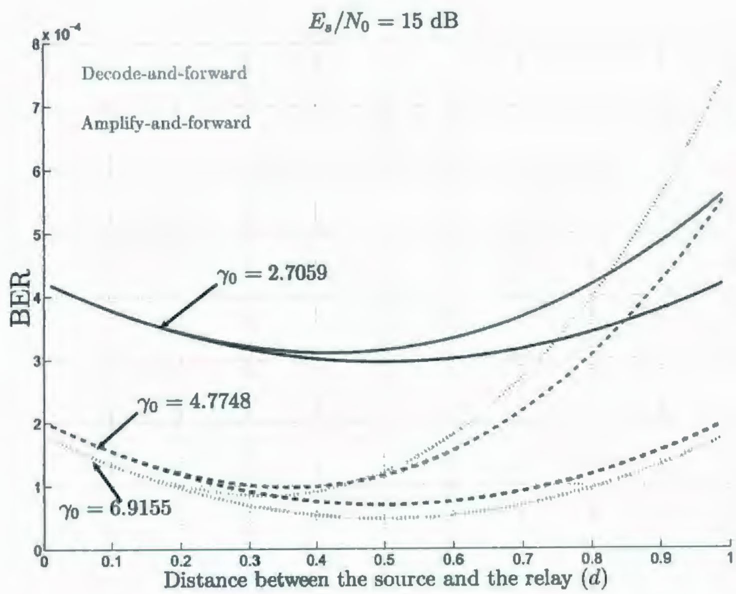

Figure 5.6: of the location on the error probability for incremental relaying technique for different values of γ_0.

Chapter 6

Performance Analysis of Incremental-Best-Relay Technique over Rayleigh Fading Channels

The previous chapter has studied the incremental relaying algorithm (with a single-relay) to overcome the inefficient use of the channel resource. In many cases, there is more than one cooperating relay in the network. Hence, we propose the incremental-best-relay scheme as a logical extension to the incremental relaying technique for the multiple relay case. In this scheme, we combine the incremental relaying technique studied in the previous chapter with the best-relay scheme discussed in chapter 4.

The incremental-best-relay cooperative relaying networks, like incremental relaying, exploits limited feedback from the destination terminal, e.g., a single bit indicating the success or failure of the direct transmission. If the destination provides a negative acknowledgment via feedback, in that case only the best relay among the M relays retransmits the source signal instead of the source itself in an attempt to exploit spatial diversity by combining the signals that the destination receives from source and the best relay. It can be seen that in this new algorithm we have two features. First, we restrict the relaying process (wasting of two time slots) to the necessary conditions. Second, to achieve high diversity order (increase the error performance) without sacrificing the spectral efficiency, only one (the best relay) should retransmit another copy of the source signal to the destination. Furthermore, this algorithm will not increase the hardware complexity compared to the incremental relying, since the same ACK/NACK can be used.

The end-to-end performance of the incremental-best-relay cooperative diversity networks equipped with the amplify-and-forward or adaptive decode-and-forward relays over independent non-identical Rayleigh fading channels will be investigated. In particular, closed-form expressions for error probability and the outage probability are determined. Analysis proves that the incremental-best-relay cooperative relaying can achieve the maximum possible diversity and enhance the spectral efficiency of the whole wireless cooperative diversity networks.

6.1 System Model

As shown in Figure 6.1 and similar to the system model used in the previous chapters, a source node (S) and a destination node (D) communicate over a channel with a flat Rayleigh fading coefficient ($h_{S,D}$). A number of potential relaying nodes $R_i, (i = 1, \cdots, M)$ are willings participate to relay the signal to provide the destination with another copy of the original signal. The channel coefficients between S and R_i (h_{S,R_i}) and between R_i and D ($h_{R_i,D}$) are also flat Rayleigh fading coefficients. In addition, $h_{S,D}$, h_{S,R_i} and $h_{R_i,D}$ are mutually-independent and non-identical. We also assume here that additive white Gaussian noise (AWGN) terms of all links have zero mean and equal variance (N_0).

All the M relays and the destination receive faded noisy versions of this broadcasted signal. Based on the quality of the received signal at the destination, the destination decides whether relaying is needed or not. For sufficient signal quality, all the relays do nothing and the source will send a new message in the second time slot. For insufficient signal quality at the destination, only the best relay in the second time slot will forward

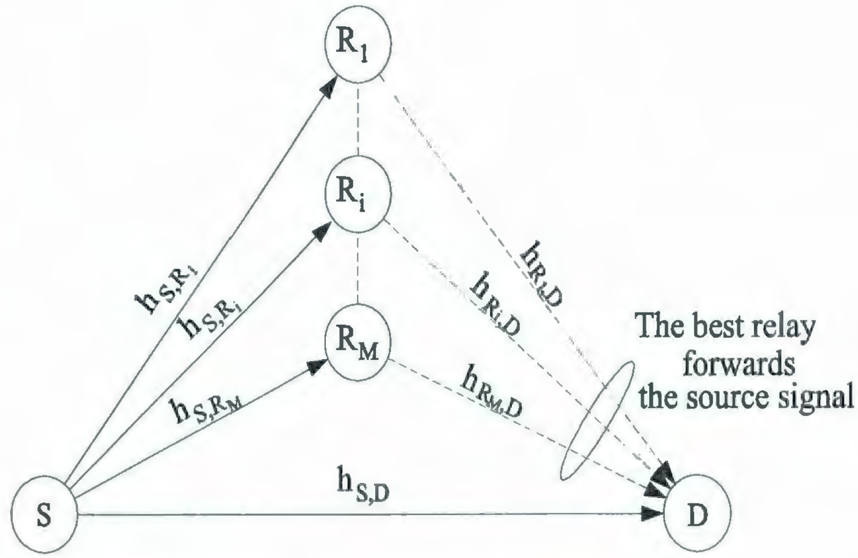

Figure 6.1: Illustration of a multi branch cooperative diversity network.

the received source signal to the destination. In the case of insufficient signal quality, the destination combines the two signals using MRC techniques.

We will assume that the forwarding decision at the destination is made on the basis of the SNR forwarding threshold (γ_0), which defines the minimum SNR for which the destination can detect the signal successfully without the need of the relayed signal. As in the previous chapter, a large value of γ_0 lowers the probability of error, which reduces the bandwidth efficiency because the best relay among M relays will forward the signal more often but this, of course, will increase the diversity benefit. Note that for direct transmission only, γ_0 is equal to 0 and for best-relay cooperative diversity networks in which the best-relay always forwards the source signal to destination, γ_0 is equal to ∞. Finally, the mathematical expressions for the received signal at the relays and the destination from the source and the received signal from the best relay at the destination will be the same as stated in chapters 4 and 5.

6.2 Error Performance Analysis

In this section, we derive closed-form expressions for the error probability for the incremental-best-relay cooperative diversity networks. Similar to the incremental relaying, the average unconditional error probability of the combined signal using incremental-best-relay technique for both schemes, amplify-and-forward or decode-and-forward can be written as

$$P(e) = \Pr(\gamma_{S,D} \leq \gamma_0) \times P_{div}(e) + (1 - \Pr(\gamma_{S,D} \leq \gamma_0)) \times P_{direct}(e) \tag{6.1}$$

where $\gamma_{S,D}$ is the instantaneous SNR between S and D, $P_{div}(e)$ is the average probability that an error occurs in the combined diversity transmission from S and R to the D. The fading parameter $h_{S,D}$ follows the Rayleigh distribution; therefore, $\gamma_{S,D}$ follows the exponential distribution. Hence, it is straightforward to show that

$$\Pr(\gamma_{S,D} \leq \gamma_0) = 1 - \exp\left(-\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right) \tag{6.2}$$

where $\bar{\gamma}_{S,D} = \mathrm{E}\left(h_{S,D}^2\right) E_s/N_0$ is the average SNR between S and D and $P_{direct}(e)$ is the probability of error at the destination given that the destination decides that the relay should not forward the source signal, which case the destination needs to rely only on the direct signal from the source. For several Gray bit-mapped constellations employed in practical systems, the conditional error probability takes the form of $a \times \mathrm{erfc}\left(\sqrt{b\gamma_{S,D}}\right)$, with $\mathrm{erfc}(x)$ being the erfc-function defined as $\mathrm{erfc} = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp\left(-x^2\right) dx$ and (a, b) are constants depending on the type of modulation (e.g. BPSK: $a = 0.5$ and $b = 1$, QPSK: $a = 0.5$ and $b = 0.5$), then the corresponding average error probability ($P_{direct}(e)$) can be written as

$$P_{direct}(e) = \int_0^{\infty} P_{direct}(e|\gamma)\, f_{\gamma_{S,D}}(\gamma|\gamma_{S,D} > \gamma_0)\, d\gamma$$
$$= a\,\mathrm{erfc}\left(\sqrt{b\gamma_0}\right) - a\exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\sqrt{\frac{b\bar{\gamma}_{S,D}}{1+b\bar{\gamma}_{S,D}}}\mathrm{erfc}\left(\sqrt{\gamma_0\left(b+1/\bar{\gamma}_{S,D}\right)}\right) \quad (6.3)$$

If the best relay among M relays should forward another copy of the source signal, then the destination combines the received signals from source and the best relay using MRC. The expression of $P_{div}(e)$ depends the relaying scheme that will be used at the relays.

6.2.1 Amplify-and-Forward

In the case of amplify-and-forward and when the relaying is needed, $P_{div}(e)$ can be calculated by knowing the equivalent SNR at the destination of the indirect link ($S \to R_i \to D$). By assuming that MRC technique is employed at the destination, the instantaneous output SNR is the sum of the instantaneous SNRs of the direct and the best indirect link (the indirect link that gives the highest SNR at the destination)

$$\gamma_{AF} = \gamma_{S,D} + \max_{i \in M}\left[\frac{\gamma_{S,R_i}\gamma_{R_i,D}}{\gamma_{S,R_i} + \gamma_{R_i,D} + 1}\right] \quad (6.4)$$

By using the same approximation adopted in chapter 2, the equivalent SNR can be written as

$$\gamma_{AF} \leq \gamma_{S,D} + \max_{i \in M}\left[\min\left(\gamma_{S,R_i}\gamma_{R_i,D}\right)\right] = \gamma_{S,D} + \gamma_b \quad (6.5)$$

The approximate SNR value (γ_b) is analytically more tractable than the exact value. The PDF of γ_b can be written as [see Chapter 4 for derivation]

$$f_{\gamma_b}(\gamma) = \sum_{n=1}^{M}(-1)^{n+1}\sum_{k_1=1}^{M-n+1}\sum_{k_2=k_1+1}^{M-n+2}\cdots\sum_{k_n=k_{n-1}+1}^{M}\prod_{j=1}^{n}\exp\left(-\frac{\gamma}{\bar{\gamma}_{k_j}}\right)\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}} \quad (6.6)$$

By defining a new variable $Z = \gamma_{S,D} + \gamma_b$, then the average error probability $P_{div}(e)$ can be written as

$$P_{div}(e) = a \int_0^{\infty} f_Z\left(z|\gamma_{S,D} \leq \gamma_0\right) \operatorname{erfc}\left(\sqrt{bz}\right) dz \tag{6.7}$$

where $f_Z\left(z|\gamma_{S,D} \leq \gamma_0\right)$ is the PDF of Z given that $\gamma_{S,D} \leq \gamma_0$. After doing some necessary manipulations, $f_Z\left(z|\gamma_{S,D} \leq \gamma_0\right)$ can be written as

$$\begin{aligned} f_z\left(z|\gamma_{S,D} \leq \gamma_0\right) = &\left(1 - \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\right) \\ \times &\begin{cases} \sum_{n=1}^{M} (-1)^{n+1} \sum_{k_1=1}^{M-n+1} \sum_{k_2=k_1+1}^{M-n+2} \cdots \sum_{k_n=k_{n-1}+1}^{M} \\ \times \left(\frac{1}{\frac{1}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}} - \bar{\gamma}_{S,D}} \exp\left(\sum_{j=1}^{n} \frac{-\gamma}{\bar{\gamma}_{k_j}}\right) - \frac{1}{\frac{1}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}} - \bar{\gamma}_{S,D}} \exp\left(\frac{-\gamma}{\bar{\gamma}_{S,D}}\right)\right), & \text{if } z \leq \gamma_0; \\ \sum_{n=1}^{M} (-1)^{n+1} \sum_{k_1=1}^{M-n+1} \sum_{k_2=k_1+1}^{M-n+2} \cdots \sum_{k_n=k_{n-1}+1}^{M} \\ \times \left(\frac{1 - \exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}} - \sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}\right)\right)}{\frac{1}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}} - \bar{\gamma}_{S,D}} \exp\left(-z \sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}\right)\right), & \text{if } z > \gamma_0 \end{cases} \end{aligned} \tag{6.8}$$

Then $P_{div}(e)$ reduces to

$$\begin{aligned} P_{div}(e) = &\sum_{n=1}^{M} (-1)^{n+1} \sum_{k_1=1}^{M-n+1} \sum_{k_2=k_1+1}^{M-n+2} \cdots \sum_{k_n=k_{n-1}+1}^{M} \\ &\times \frac{a}{\left(\omega - \bar{\gamma}_{S,D}\right)\left(1 - \exp\left(-\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)} \left\{ \bar{\gamma}_{S,D} \sqrt{\frac{b\bar{\gamma}_{S,D}}{1 + b\bar{\gamma}_{S,D}}} \operatorname{erf}\left(\sqrt{\lambda}\right) - \omega \sqrt{\frac{b\omega}{1 + b\omega}} \operatorname{erf}\left(\sqrt{\zeta}\right) \right. \\ &+ \exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}} + \frac{1}{\omega}\right)\right)\left(\bar{\gamma}_{S,D} \exp\left(\frac{\gamma_0}{\omega}\right) - \omega \exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right) \operatorname{erfc}\left(\sqrt{b\gamma_0}\right) - \left(\bar{\gamma}_{S,D} - \omega\right) \\ &\left. + \left(1 - \exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}} - \frac{1}{\omega}\right)\right)\right)\left[\omega \exp\left(\frac{-\gamma_0}{\omega}\right) \operatorname{erfc}\left(\sqrt{b\gamma_0}\right) - \omega \sqrt{\frac{b\omega}{1 + b\omega}} \operatorname{erfc}\left(\sqrt{\zeta}\right)\right] \right\} \end{aligned} \tag{6.9}$$

where $\omega = \left(\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}\right)^{-1}$, $\lambda = \frac{\gamma_0\left(1 + b\bar{\gamma}_{S,D}\right)}{\bar{\gamma}_{S,D}}$ and $\zeta = \frac{\gamma_0(1 + b\omega)}{\omega}$.

By substituting (6.9), (6.3) and (6.2) into (6.1) we can have a closed-form expression for the error probability of the incremental-best-relay amplify-and-forward cooperative relaying protocol over Rayleigh flat fading channels.

6.2.2 Decode-and-Forward

In decode-and-forward scheme, The source broadcasts its signal to the set of M-relay nodes and the destination node. We define the decoding set (C) as the set of relays with the ability to fully decode the source message correctly. Then, the best relay from the decoding set C decodes and forwards (retransmits) the source information to the destination (where the retransmission will happen only for insufficient signal quality between S and D). Then, the destination combines the direct and the best indirect link using MRC. Hence if the relaying is needed, the equivalent SNR at the destination can be written as

$$\gamma_{DF} = \gamma_{S,D} + \max_{i \in C} (\gamma_{R_i,D}) \tag{6.10}$$

As stated in chapter 4, this expression is difficult to handle. An easier expression which has been derived in chapter 4 can be written as

$$\gamma_{DF} = \gamma_{S,D} + \max_{i \in M} (\xi_i) = \gamma_{S,D} + \chi \tag{6.11}$$

where $\chi = \max_{i \in M} (\xi_i)$, and ξ_i is the equivalent SNR of the indirect link ($S \rightarrow R_i \rightarrow D$). As mentioned in chapter 4, the expressions of γ_{DF} in (6.10) and (6.11) are equal; however, the expression given in (6.11) is analytically more tractable than the expression given in (6.10). It is known that the PDF of $\gamma_{S,D}$ is $f_{\gamma_{S,D}} = (1/\bar{\gamma}_{S,D}) \exp(-\gamma/\bar{\gamma}_{S,D})$, where $\bar{\gamma}_{S,D}$ is the average SNR between S and D. Furthermore, the PDF of $\chi = \max_{i \in M} (xi_i)$ has been derived in chapter 4 as

$$f_X(x) = \left(\prod_{i=1}^{M} B_i\right)\delta(x) + \sum_{k=1}^{M}(-1)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M}$$
$$\times\prod_{i=1}^{k}(1-B_{\lambda_i})\exp\left(-x/\bar{\gamma}_{R_{\lambda_i},D}\right)\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}} \tag{6.12}$$

By defining a new variable $X = \gamma_{S,D} + \chi$, then the average error probability $P_{div}(e)$ can be written as

$$P_{div}(e) = a\int_0^{\infty} f_X(x|\gamma_{S,D} \le \gamma_0)\,\mathrm{erfc}\left(\sqrt{bx}\right)dx \tag{6.13}$$

where $f_X(x|\gamma_{S,D} \le \gamma_0)$ is the PDF of X given that $\gamma_{S,D} \le \gamma_0$. After doing some necessary manipulations, $f_X(x|\gamma_{S,D} \le \gamma_0)$ can be written as

$$f_X(x|\gamma_{S,D} \le \gamma_0) = \left(1-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)$$
$$\times\begin{cases} \left(\prod_{i=1}^{M} B_i\right)\frac{\exp\left(-\frac{x}{\bar{\gamma}_{S,D}}\right)}{\bar{\gamma}_{S,D}} + \sum_{k=1}^{M}(-1)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M} & \\ \times\frac{\prod_{i=1}^{k}(1-B_{\lambda_i})}{\bar{\gamma}_{S,D}-1/\sum_{i=1}^{k}1/\bar{\gamma}_{R_{\lambda_i},D}}\left[\exp\left(-x/\bar{\gamma}_{S,D}\right)-\prod_{i=1}^{k}\exp\left(-x/\bar{\gamma}_{R_{\lambda_i},D}\right)\right], & \text{if } x \le \gamma_0; \\ \sum_{k=1}^{M}(-1)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M} & \\ \times\frac{\prod_{i=1}^{k}(1-B_{\lambda_i})}{\bar{\gamma}_{S,D}-1/\sum_{i=1}^{k}1/\bar{\gamma}_{R_{\lambda_i},D}}\exp\left(-x\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right) & \\ \times\left[1-\exp\left(-\gamma_0\left(1/\bar{\gamma}_{S,D}-\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right)\right)\right], & \text{if } x > \gamma_0 \end{cases} \tag{6.14}$$

Substituting (6.14) into (6.13) and doing the integral and after some simplifications $P_{div}(e)$ can be obtained as

$$
\begin{aligned}
P_{div}(e) = & \frac{a\prod_{i=1}^{M} B_i}{1-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)}\left[1-\sqrt{\frac{b\bar{\gamma}_{S,D}}{1+b\bar{\gamma}_{S,D}}}\operatorname{erf}\left(\sqrt{\frac{\gamma_0\left(1+b\bar{\gamma}_{S,D}\right)}{\bar{\gamma}_{S,D}}}\right)-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\operatorname{erfc}\left(\sqrt{b\gamma_0}\right)\right] \\
& + \sum_{k=1}^{M}(-1)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M} \\
& \times \frac{a}{\left(\nu-\bar{\gamma}_{S,D}\right)\left(1-\exp\left(-\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)}\left\{\bar{\gamma}_{S,D}\sqrt{\frac{b\bar{\gamma}_{S,D}}{1+b\bar{\gamma}_{S,D}}}\operatorname{erf}\left(\sqrt{\lambda}\right)-\nu\sqrt{\frac{b\nu}{1+b\nu}}\operatorname{erf}\left(\sqrt{\zeta}\right)\right. \\
& + \exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}}+\frac{1}{\nu}\right)\right)\left(\bar{\gamma}_{S,D}\exp\left(\frac{\gamma_0}{\nu}\right)-\nu\exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)\operatorname{erfc}\left(\sqrt{b\gamma_0}\right)-\left(\bar{\gamma}_{S,D}-\nu\right) \\
& \left. + \left(1-\exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}}-\frac{1}{\nu}\right)\right)\right)\left[\nu\exp\left(\frac{-\gamma_0}{\nu}\right)\operatorname{erfc}\left(\sqrt{b\gamma_0}\right)-\nu\sqrt{\frac{b\nu}{1+b\nu}}\operatorname{erfc}\left(\sqrt{\zeta}\right)\right]\right\}
\end{aligned}
\tag{6.15}
$$

where $\nu = \left(\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right)^{-1}$, $\lambda = \frac{\gamma_0\left(1+b\bar{\gamma}_{S,D}\right)}{\bar{\gamma}_{S,D}}$ and $\zeta = \frac{\gamma_0(1+b\nu)}{\nu}$.

By substituting (6.2), (6.3) and (6.15) into (6.1) we can have a closed-form expression for the error probability of the incremental-best-relay adaptive decode-and-forward cooperative relaying protocol over Rayleigh flat fading channels.

Finally, although the derived error probabilities for both relaying schemes have multiple summations/multiplications, all these summations/multiplications are finite and easy to handle numerically.

6.3 Asymptotic Error Performance

Here, in this section we are going to simplify the error probability expressions derived in the previous section by focusing on the high SNR region. In the high SNR region we can assume that $\Pr(\gamma_{S,D} \leq \gamma_0)$ goes to zero and this simplifies (6.1) to

$$
\begin{aligned}
P(e) &= \underbrace{\Pr(\gamma_{S,D} \leq \gamma_0)}_{\to 0} P_{div} + \underbrace{(1 - \Pr(\gamma_{S,D} \leq \gamma_0))}_{\to 1} P_{direct} \\
&= a \int_{\gamma_0}^{\infty} \frac{\exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)}{\bar{\gamma}_{S,D}} \operatorname{erfc}\left(\sqrt{b\gamma}\right) d\gamma \\
&= a \operatorname{erfc}\left(\sqrt{b\gamma_0}\right) - a \exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right) \sqrt{\frac{b\bar{\gamma}_{S,D}}{1 + b\bar{\gamma}_{S,D}}} \operatorname{erfc}\left(\sqrt{\gamma_0 \left(b + 1/\bar{\gamma}_{S,D}\right)}\right) \quad (6.16)
\end{aligned}
$$

From this equation we can find two important things. First, at high SNR (SNR $>> \gamma_0$), the error performance for the incremental relaying with the best relay selection cooperative diversity network tends to be parallel with direct transmission, which means that the system achieves virtual antenna array gain but not diversity gain (significant improvement compared to the direct system alone). Second, as γ_0 increases we can find from the second line of (6.16) that the $P_{direct}(e)$ will also reduce since the region of the integration will decrease. This means, as we will see in the result sections, that for approximately SNR $< \gamma_0$ the incremental-best-relay diversity network will achieve a diversity order of $M + 1$ and for high SNR (SNR $>> \gamma_0$) the incremental-best-relay cooperative diversity network will achieve virtual array gain with diversity order of 1.

6.4 System Outage Probability Performance

In this subsection, we derive closed-form expressions for the SNR outage probability (P_{out}). In best-relay incremental relaying cooperative diversity networks, if the SNR of the direct link at the destination is less than the threshold value γ_0, the destination will need assistance from the relay to send another copy of the source signal. In this case, the best relay will send another copy of the signal but there is still a probability that the overall SNR at the destination is less than γ_0, and in this subsection we will determine this probability.

6.4.1 Amplify and Forward

The outage probability for amplify-and-forward can be derived as

$$\begin{aligned} P_{out} &= \Pr\left(\gamma_{S,D} + \max_{i\in M}\left[\min\left(\gamma_{S,R_i}, \gamma_{R_i,D}\right)\right] \leq \gamma_0 | \gamma_{S,D} \leq \gamma_0\right) \Pr\left(\gamma_{S,D} \leq \gamma_0\right) \\ &= \Pr\left(\gamma_{S,D} + \gamma_b \leq \gamma_0\right) \end{aligned} \tag{6.17}$$

Since the two links (direct and best indirect links) are independent, then the outage probability can be found by finding the convolution of the two PDFs to find the overall PDF and then finding the CDF. After doing some necessary manipulations, then P_{out} can be derived and written in a closed form as

$$\begin{aligned} P_{out} =& \sum_{n=1}^{M} (-1)^{n+1} \sum_{k_1=1}^{M-n+1} \sum_{k_2=k_1+1}^{M-n+2} \cdots \sum_{k_n=k_{n-1}+1}^{M} \\ & \times \left(1 - \frac{\frac{1}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}}}{\frac{1}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}} - \bar{\gamma}_{S,D}} \exp\left(\sum_{j=1}^{n} \frac{-\gamma_0}{\bar{\gamma}_{k_j}}\right) + \frac{\bar{\gamma}_{S,D}}{\frac{1}{\sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}}} - \bar{\gamma}_{S,D}} \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\right) \end{aligned} \tag{6.18}$$

6.4.2 Decode and Forward

In the case of decode-and-forward, the outage probability can be derived as follows

$$\begin{aligned} P_{out} &= \Pr\left(\gamma_{S,D} + \chi \leq \gamma_0 | \gamma_{S,D} \leq \gamma_0\right) \Pr\left(\gamma_{S,D} \leq \gamma_0\right) \\ &= \Pr\left(\gamma_{S,D} + \chi \leq \gamma_0\right) \end{aligned} \tag{6.19}$$

Since the two links (direct and best indirect links) are independent, then the outage probability can be found by finding the convolution of the two PDFs to find the overall PDF and then finding the CDF. After doing some necessary manipulations, then P_{out} can be derived and written in a closed form as

$$P_{out} = \left(\prod_{i=1}^{M} B_i\right)(1-\exp(-\gamma_0/\bar{\gamma}_{S,D})) + \sum_{k=1}^{M}(-1)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M}\prod_{i=1}^{k}(1-B_{\lambda_i})$$
$$\times\left[1+\frac{1}{\bar{\gamma}_{S,D}-1/\sum_{i=1}^{k}1/\bar{\gamma}_{R_{\lambda_i},D}}\left(\frac{\prod_{i=1}^{k}\exp\left(-\gamma_0/\bar{\gamma}_{R_{\lambda_i},D}\right)}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}}-\bar{\gamma}_{S,D}\exp(-\gamma_0/\bar{\gamma}_{S,D})\right)\right] \tag{6.20}$$

6.5 Normalized Throughput Analysis

Similar to chapter 5, the expected number of transmissions (original transmission plus retransmission) is given by

$$\mathbf{E}(T) = 1\times \Pr(T=1) + 2\times\Pr(T=2) \tag{6.21}$$

where $\Pr(T=1)$ represents the probability that only one time slot needed and $\Pr(T=1)$ represents the probability that two time slots needed. Now, consider the probability of each of the two cases. In case 1, the transmission takes only one time slot. The probability of successful transmission after the first time slot is given by $\Pr(T=1) = p_1 = \exp(-\gamma_0/\bar{\gamma}_{S,D})$, and the probability of successful transmission after the two time slots is given by $\Pr(T=2) = 1-\exp(-\gamma_0/\bar{\gamma}_{S,D}) = 1-p_1$. Finally, after simple manipulations, the throughput can be written as

$$\text{Throughput} = \frac{R}{2-p_1} \tag{6.22}$$

Note that the incremental relaying technique studied in the previous chapter and the incremental-best-relay cooperative diversity has the same throughput. This means that the main advantage of the incremental-best-relay diversity is to achieve the same spectral efficiency of the incremental relaying technique with much improvement in the error performance as we will see in the next section.

6.6 Numerical Results and Analysis

In this section, we show numerical results of the analytical BER for binary phase shift keying (BPSK) modulation, and outage probability. We plot the performance curves of the average BER, and outage probability versus the SNR of the transmitted signal (E_s/N_0 dB). We also show the results of the computer simulations for verification.

Figures 6.2 and 6.3 compare the error performance of the incremental-best-relay cooperative diversity with that of the regular cooperative diversity networks (all the relay participate), best-relay scheme and conventional direct system for different values of γ_0 and for $M = 3$. From these two figures we can conclude the following points:

First, as expected, the best-relay scheme outperforms all other cooperative diversity networks and can achieve $M + 1$ diversity order for the entire SNR region (regular cooperative also achieve $M + 1$ diversity order but with less virtual array gain). However, the incremental-best-relay cooperative diversity can achieve the same error performance of the best-relay scheme at low SNR while at high SNR (for approximately SNR $> \gamma_0$) the error performance will tend to have diversity order of 1 and virtual array gain[1]. From Figures 6.2 and 6.3 it is apparent that the virtual array gain depends on the value of γ_0: for high values of γ_0 we have higher virtual array gain and vice versa. It should be also noted that when $\gamma_0 = \infty$ we go back to the best relay scheme. Furthermore, for high values of γ_0 the region where the incremental-best-relay cooperative diversity outperforms the regular cooperative diversity networks also increases.

Second, it can be seen from Figures 6.2 and 6.3 that there is a critical SNR point where the error performance will no longer has $M + 1$ diversity order and this critical value actually depends on γ_0. Even more, it can be seen that this critical value starts earlier for adaptive decode-and-forward scheme compared to amplify-and-forward scheme. In addition, it can be seen that adaptive decode-and-forward scheme slightly outperforms

[1]If the error rate is plotted versus the SNR on a log-log scale the diversity order can be interpreted as the slope of the so-obtained curve whereas the virtual array gain corresponds to the horizontal position of the curve.

the amplify-and-forward scheme. It is worthy to mention that this improvement of the adaptive decode-and-forward comes with extra complexity at the relays compared with the amplify-and-forward scheme. Finally, it should be noted that for high SNR, amplify-and-forward and decode-and-forward have exactly the same error performance because they have the same asymptotic error performance which validates equation (6.16).

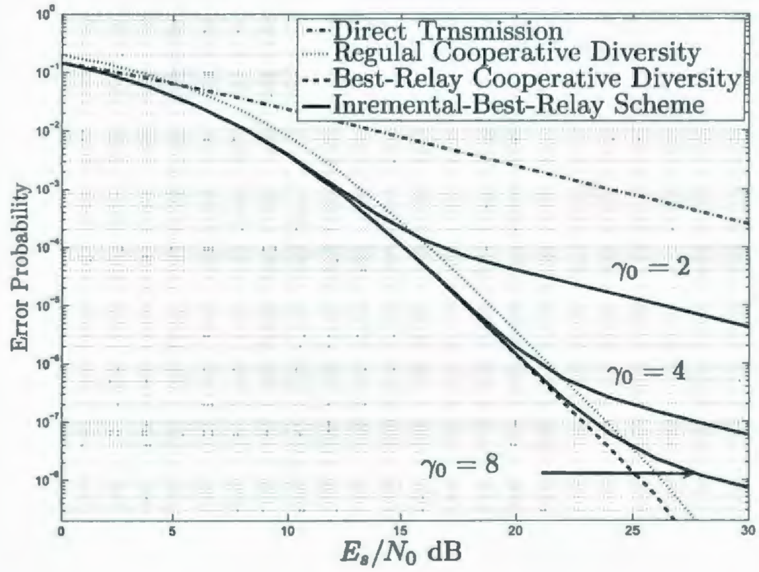

Figure 6.2: Error performance for the decode-and-forward incremental-best- relay selection scheme over Rayleigh fading channels.

It should be noted that the value of γ_0 is an important factor to determine the error performance of the whole system. Actually this value can be determined based on the quality of service and transmission delay that could be accepted (voice, video or data transmission). In Figure 6.4, we show how many time slots we need to complete the whole transmission of a message for different values of γ_0. We should note that for best-relay scheme we need two time slots regardless of the number of the relays, for regular cooperative diversity we need $M + 1$time slots (number of the relays will increase the transmission delay between the source and the destination) and one time slot for direct transmission (ideal

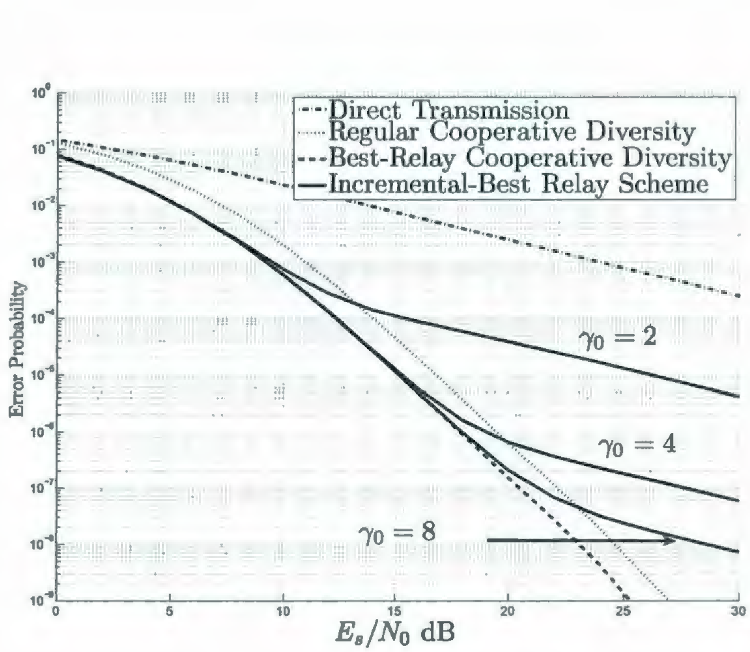

Figure 6.3: Error performance for the amplify-and-forward incremental-best- relay selection scheme over Rayleigh fading channels.

case). It is obvious that increasing γ_0 will increase the number of time slots needed to complete the transmission and vice versa. Therefore, the value of γ_0 depends on the quality of service to be afford. For example, for $\gamma_0 = 8$ at SNR = 15 dB, the average number of time slots needed to complete transmission of a message is 1.2 and the error probability of 10^{-4} while for direct transmission the error is 10^{-2}. Furthermore, for $\gamma_0 = 8$ at SNR = 20 dB, the average number of time slots needed to complete transmission of a message is almost 1 (almost the same as the direct link) with error probability of 10^{-6} while the direct link has error probability of 3×10^{-3}. This big improvement (more than 3 orders of magnitude) can be used in a hidden way to increase the data rate by going further to higher modulation level. It should be mentioned that the throughput of the incremental-best relay scheme is the same as the incremental scheme.

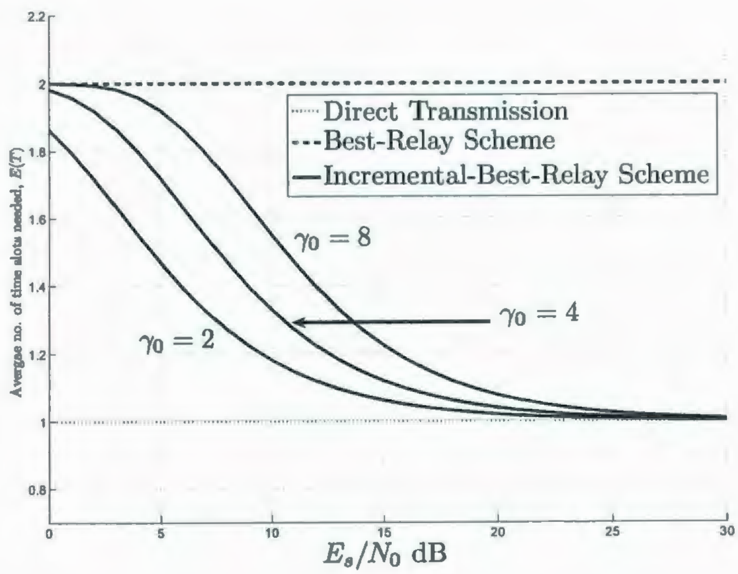

Figure 6.4: Error performance for the amplify-and-forward incremental-best- relay selection scheme over Rayleigh fading channels.

Chapter 7

Conclusion and Future Work

Cooperative diversity techniques have been proposed to achieve transmission diversity without the need of using multiple antenna elements by exploiting the broadcasting nature of the wireless channel. The main goal of cooperative diversity is to create a virtual MIMO system by forming a virtual antenna array, in which each element of the array represents a single node having a single antenna.

We have discussed various aspects of cooperative diversity networks over different fading channel models. The main conclusions of this thesis are as follows:

1. Tight performance bounds for dual-hop and multi-branches cooperative diversity networks over independent non-identical Nakagami-m fading channels have been investigated. Our numerical results show that the derived error rate and outage probability are a tight lower bound particularly at medium and high SNR.

2. Fixed and adaptive decode-and-forward cooperative diversity have been analyzed in terms of the error probability and the outage probability over independent non-identical flat Nakagami-m fading channels. A closed-form expression for the error probability was determined. From the theoretical results, we can say the following: First, the source-relay link has the most influence on the error performance and thus the best location of the relay is near to the source. Second, the proper choice of the

forwarding threshold depends on the value of the SNR.

3. With respect to the number of hops, the following was found

    - It is apparent that, in general, relay power is better used when it adds a cooperative branch than when it adds a hop in an existent branch. The reason is that adding more branches will increase the diversity order but adding more hops will increase the coding gain only. In cellular scenarios, the two-hop scheme leads to identical resource allocation for conventional and cooperative relaying.
    - Limiting the number of hops strongly simplifies combining, resource allocation, and scheduling.

4. We examined the performance of amplify-and-forward relaying scheme using a differential non-coherent modulation EGC cooperative wireless communication systems. We have obtained analytical closed-form expressions of the distribution for the instantaneous SNR, the outage probability and the error performance. We have shown that differential EGC technique for amplify-and-forward and adaptive decode-and-forward relay scheme offer remarkable diversity advantage (despite their simplicity) over direct transmission and conventional non-cooperative relaying over Nakagami-*m* fading channels.

5. An interesting point that can be extracted from Chapters 2 and 3 is that coherent MRC and differential EGC cooperative diversity networks outperform the traditional MISO system. This is due to that the cooperative network can benefit from the nonlinearity relationship between the power and the distance. This advantage leads to high average SNR will be received at the destination compared to the traditional MISO. If we place more than one antenna at the destination this behavior also will be hold and cooperative diversity in some applications can outperform traditional MIMO System.

6. In this thesis, important performance metrics of the best-relay cooperative diversity networks, operating over independent, but not necessarily identically distributed Rayleigh fading channels was addressed. Based on the derived PDF formula of the end-to-end SNR, novel closed-form expressions for the average error probability, outage probability, average channel capacity and average output SNR were obtained. Computer simulation results verified the accuracy and the correctness of the proposed analysis.

7. Selecting the best-relay cooperative diversity technique has a strong advantage in saving the channels. In selecting the best-relay technique increasing M will increase the diversity order without sacrificing the spectral efficiency.

8. Incremental cooperative diversity network is an efficient technique that can be used to save the channel resources and use these channel resources only when it is necessary. The closed-form error performance expressions of the incremental relaying scheme as well as outage probabilities are derived. Its performance analysis illustrates that this protocol dramatically outperforms the direct transmission system under any scenario of error performance threshold, relay position and transmit power without increasing implementation complexity.

9. The incremental-best-relay cooperative diversity goes one step further than incremental relaying technique by combining the best relay and incremental relaying schemes together. Our results show that this scheme can achieve diversity order of $M + 1$ in low SNR region and virtual array gain in the high SNR region. Also, the incremental-best-relay cooperative diversity has variable data rate and depends on the value of the threshold value used and the region of SNR. Furthermore, the incremental-best-relay cooperative diversity can achieve high error performance without a very small reduction in the data rate.

7.1 Future Work and Open Challenges

As a future work, the following subjects for further investigations can be suggested:

1. Studying the impact of the relay(s) location on the performance analysis and try to find the optimum location of the relay that will increase the performance of the system. This study is important especially for decode-and-forward technique since the decode-and-forward scheme is non-linear which means the mid-point is not the optimum one.

2. Analyzing the effect of the fading parameters on the performance of the best-relay, incremental relaying and incremental-best-relay schemes. This study is vital because, for example, in the cellular system the fading parameters of the mobile-base station, mobile-mobile channels are different. This means that we need to go to more general fading channels like Nakagami-m fading channels.

3. Studying cooperative diversity networks over fading channel that can be used to model the multi-path, shadowing and path-loss like generalized gamma fading channel.

4. Analyzing optimum power allocations. How we should distribute the power between the source and the relays to achieve the best performance? This also leads to increase the complexity of the system. This study should reveal if we distribute the power evenly will give us acceptable performance without increasing the system complexity.

5. Studying the cross layer issue as this is an important subject especially for the advanced relaying scheme introduced in this thesis.

6. In this thesis, little regard is given to the complexity of the receivers/transmitters required to implement the various protocols introduced. Future work should examine this issue more thoroughly to ensure that the improved results are achievable by

practical systems.

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Appendix A

Derivation of the equation (2.5)

We consider amplify and forward model where relays simply amplify the signal received from the source. Assuming that S and R_i transmit through orthogonal channels, the destination D receives $M+1$ independent copies of the signal $x(t)$, transmitted by the source

$$\begin{aligned} y_{S,D}(t) &= h_{S,D}\sqrt{E_s}x(t) + n_{S,D}(t) \\ y_{R_i,D}(t) &= h_{R_i,D}G_i\left(h_{S,R_i}\sqrt{E_s}x(t) + n_{S,R_i}(t)\right) = h_{S,R_i}G_ih_{R_i,D}\sqrt{E_s}x(t) + n_{eq}(t) \end{aligned} \tag{A.1}$$

where $n_{eq}(t) = h_{R_i,D}G_in_{S,R_i}(t) + n_{R_i,D}(t)$ and G_i is the amplification factor which will be discussed later. The receiver collects these copies with a maximum ratio combiner (MRC). We emphasize that the noise terms $n_{S,D}(t)$ and $n_{eq}(t)$ do not have identical power because $n_{eq}(t)$ includes a noise contribution at the intermediate stage; for this reason, the MRC should be preceded by a noise normalization step. With this combining rule, the resulting SNR of the decision variable is

$$\gamma_{equ,AF} = h_{S,D}^2\frac{E_s}{N_0} + \sum_{i=1}^{M}\frac{\left(G_ih_{R_i,D}h_{S,R_i}\right)^2E_s}{\left(1+\left(G_ih_{S,R_i}\right)^2\right)N_0} \tag{A.2}$$

Here, we choose the amplification factor G_i; to maintain constant instantaneous power output, equal to the original transmitted power

$$G_i^2 = \frac{1}{E_s h_{S,R_i}^2 + N_0} \tag{A.3}$$

Eq. (A.3) needs that the relay can estimate the channel fading h_{S,R_i}. Substituting (A.3) into (A.2) we obtain

$$\gamma_{equ,AF} = \gamma_{S,d} + \sum_{i=1}^{M} \frac{\gamma_{S,R_i}\gamma_{R_i,D}}{\gamma_{S,R_i} + \gamma_{R_i,D} + 1} \tag{A.4}$$

Appendix B

Performance Analysis of Multi-hops Multi-Branches Wireless Cooperative Diversity Networks over Non-identical Nakagami-m Fading Channels

We consider a cooperative system with $M+1$ diversity branches $B_0, B_1, \cdots, B_M$ as shown in Figure B.1, where by convention the diversity branch B_0 corresponds to the direct path with fading coefficient f. Each of the remaining M branches $B_1, \cdots, B_M$ is composed of N relays $R_1, \cdots, R_{N-1}$, $i \in \{1, \cdots, M\}$. The flat Nakagami-m fading channel coefficients between the relays $R_{i,j}$ and $R_{i,j+1}$ of branch B_i are denoted by $h_{i,j+1}$, with $h_{i,1}$ being the coefficient between the source and the first relay and $h_{i,N}$ being that between the last relay and the destination. In addition all the fading channels are mutually-independent and non-identical.

We define the average per-hop SNRs as $\bar{\gamma}_{i,j} = \mathbf{E}\left(h_{i,j}^2\right) E_s/N_0$, where E_s is the trans-

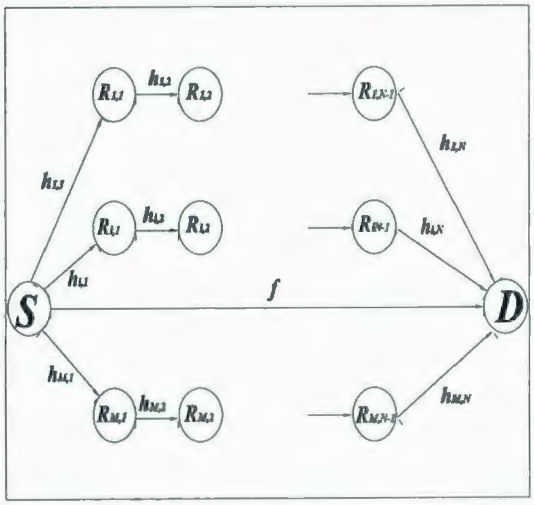

Figure B.1: Illustration of the N-hop M-branche communication system.

mitted signal energy and N_0 is the one sided power spectral density of the additive white Gaussian noise (AWGN) and we also define $f_{\gamma_{i,j}}(\gamma)$ to be the PDF of $\gamma_{i,j}$.

Assuming MRC at the destination node, the total SNR at the destination node can be written as

$$\gamma_{equ} = \gamma_f + \sum_{i=1}^{M} \left[\prod_{j=1}^{N} \left(1 + \frac{1}{\gamma_{i,j}} \right) - 1 \right]^{-1} \tag{B.1}$$

The total SNR for cooperative diversity networks can be approximated by its upper bound (γ_b) as follows

$$\gamma_{equ} \leq \gamma_b = \gamma_f + \sum_{i=1}^{M} \gamma_i \tag{B.2}$$

where $\gamma_i = \min(\gamma_{i,1}, \gamma_{i,2}, \cdots, \gamma_{i,N})$. The approximate SNR value in (B.2) is analytically more tractable than the exact value in (B.1); and as a result, this facilitates the derivation of the SNR statistics (CDF, PDF, and MGF).

Assuming the independency in all branches and hops, then the MGF of γ_b can be written as

$$M_{\gamma_b}(s) = M_{\gamma_f}(s) \prod_{i=1}^{M} M_{\gamma_i}(s) \tag{B.3}$$

where $M_{\gamma_f}(s)$ and $M_{\gamma_i}(s)$ are the MGF of γ_f and γ_i, respectively . Since the assumption that f is modeled by Nakagami-m distribution; it can be easily shown that

$$M_{\gamma_f}(s) = \left(1 + s\frac{\bar{\gamma}_f}{m_f}\right)^{-m_f} \tag{B.4}$$

where m_f is the Nakagami-m fading parameter of f and $\bar{\gamma}_f = \mathbf{E}\left(f^2\right) E_s/N_0$. In order to find $M_{\gamma_i}(s)$, we find the CDF of γ_i as follows

$$\begin{aligned} F_{\gamma_i}(\gamma) &= 1 - \prod_{j=1}^{N} \Pr\left(\gamma_{i,j} > \gamma\right) \\ &= 1 - \prod_{j=1}^{N} \frac{1}{\Gamma\left(m_{i,j}\right)} \Gamma\left(m_{i,j}, \frac{m_{i,j}}{\bar{\gamma}_{i,j}}\gamma\right) \end{aligned} \tag{B.5}$$

where $\Gamma(\bullet, \bullet)$ is the incomplete gamma function [22, eq.(8.350.2)], $\Gamma(\bullet)$ is the gamma function [22, eq. (8.310.1)], and $m_{i,j}$ are the Nakagami-m fading parameters of $h_{i,j}$.

Thus, the PDF can be found by taking the derivative of (B.5) with respect to γ, yielding

$$\begin{aligned} f_{\gamma_i}(\gamma) &= \sum_{j=1}^{N} \left[f_{\gamma_{i,j}}(\gamma) \prod_{n=1, n\neq j}^{N} \left(1 - F_{i,n}(\gamma)\right) \right] \\ &= \sum_{j=1}^{N} \left[\left(\frac{m_{i,j}}{\bar{\gamma}_{i,j}}\right)^{m_{i,j}} \frac{\gamma^{m_{i,j}-1}}{\Gamma\left(m_{i,j}\right)} \exp\left(-\frac{m_{i,j}}{\bar{\gamma}_{i,j}}\right) \prod_{n=1, n\neq j}^{N} \frac{1}{\Gamma\left(m_{i,n}\right)} \Gamma\left(m_{i,n}, \frac{m_{i,n}}{\bar{\gamma}_{i,n}}\gamma\right) \right] \end{aligned} \tag{B.6}$$

In order to get a closed form expression of the MGF of γ_i we assume that $m_{i,j}$ takes only integer values, then the approximate results for outage probability or error performance for non-integer values of $m_{i,j}$ can be obtained using interpolation of the final results obtained for integer values. Since the MGF of γ_i can be written as

$$M_{\gamma_i}(s) = \int_0^{\infty} f_{\gamma_i}(\gamma)\exp(-s\gamma)\,d\gamma \tag{B.7}$$

Then integral can be evaluated in a closed form as follows. By expressing the incomplete gamma function by its finite series as

$$\Gamma(k,x) = (k-1)!\exp(-x)\sum_{p=0}^{k-1}\frac{x^p}{p!} \tag{B.8}$$

for arbitrary integer value (k). Then, the PDF in (B.6) can be rewritten as

$$\begin{aligned} f_{\gamma_i}(\gamma) = &\exp\left(-\gamma\sum_{n=1}^{N}\frac{m_{i,n}}{\bar{\gamma}_{i,n}}\right)\sum_{j=1}^{N}\frac{\left(\frac{m_{i,j}}{\bar{\gamma}_{i,j}}\right)^{m_{i,j}}}{\Gamma(m_{i,j})}\sum_{k_1=0}^{m_{i,1}-1}\sum_{k_2=0}^{m_{i,2}-1}\cdots\sum_{k_{j-1}=0}^{m_{i,j-1}-1}\sum_{k_{j+1}=0}^{m_{i,j+1}-1}\cdots\sum_{k_N=0}^{m_{i,N}-1} \\ &\times(\gamma)^{m_{i,j}-1+\sum_{n=1, n\neq j}^{N}k_n}\left(\prod_{n=1,n\neq j}^{N}\frac{(m_{i,n}/\bar{\gamma}_{i,n})^{k_n}}{k_n!}\right) \end{aligned} \tag{B.9}$$

Finally, the MGF of γ_i after rearranging, performing all the manipulation required, can be written in a closed form as

$$\begin{aligned} M_{\gamma_i}(s) = &\sum_{j=1}^{N}\frac{\left(\frac{m_{i,j}}{\bar{\gamma}_{i,j}}\right)^{m_{i,j}}}{\Gamma(m_{i,j})}\sum_{k_1=0}^{m_{i,1}-1}\sum_{k_2=0}^{m_{i,2}-1}\cdots\sum_{k_{j-1}=0}^{m_{i,j-1}-1}\sum_{k_{j+1}=0}^{m_{i,j+1}-1}\cdots\sum_{k_N=0}^{m_{i,N}-1} \\ &\times\frac{\Gamma\left(m_{i,j}+\sum_{n=1,n\neq j}^{N}k_n\right)}{\left(s+\sum_{n=1}^{N}\frac{m_{i,n}}{\bar{\gamma}_{i,n}}\right)^{\sum_{n=1, n\neq j}^{N}k_n+m_{i,j}}}\left(\prod_{n=1,n\neq j}^{N}\frac{(m_{i,n}/\bar{\gamma}_{i,n})^{k_n}}{k_n!}\right) \end{aligned} \tag{B.10}$$

By substituting (B.10) and (B.4) into (B.3) we obtain a closed form expression of M_{γ_b}, which can be written as

$$
\begin{aligned}
M_{\gamma_b}(s) &= \left(1+s\frac{\bar{\gamma}_f}{m_f}\right)^{-m_f} \prod_{i=1}^{M}\left\{\sum_{j=1}^{N}\frac{\left(\frac{m_{i,j}}{\bar{\gamma}_{i,j}}\right)^{m_{i,j}}}{\Gamma(m_{i,j})}\sum_{k_1=0}^{m_{i,1}-1}\sum_{k_2=0}^{m_{i,2}-1}\cdots\sum_{k_{j-1}=0}^{m_{i,j-1}-1}\sum_{k_{j+1}=0}^{m_{i,j+1}-1}\cdots\sum_{k_N=0}^{m_{i,N}-1}\right. \\
&\quad\left.\times\frac{\Gamma\left(m_{i,j}+\sum_{n=1,n\neq j}^{N}k_n\right)}{\left(s+\sum_{n=1}^{N}\frac{m_{i,n}}{\bar{\gamma}_{i,n}}\right)^{\sum_{n=1, n\neq j}^{N}k_n+m_{i,j}}}\left(\prod_{n=1,n\neq j}^{N}\frac{(m_{i,n}/\bar{\gamma}_{i,n})^{k_n}}{k_n!}\right)\right\}
\end{aligned}
\tag{B.11}
$$

Using M_{γ_b}, the error rate can be evaluated for a wide variety of M-ary modulations (such as M-ary phase-shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM))[23].

Figure B.2 shows the BER performance at different values of the number of cooperating nodes (M), for $N = 3$ and for arbitrary values of fading parameters and average SNRs. It is clear that the BER bound is tight enough especially in medium and high SNR regime. From Figure B.2 we can also notice that the number of cooperating relays (M) has a strong impact of the performance enhancement and the achieved diversity order.

It should be noted from Figure B.2 that the tightness of the error performance improves as E_s/N_0 increases; however, the proposed lower bound slightly loses its tightness at low E_s/N_0 values particularly when M increases. This is due to the fact that the accuracy of total SNR approximation (in (B.2)) improves as E_s/N_0 increases.

In Figure B.3, an asymmetric network geometry is examined where the relays are distributed in equal distances on a straight line between S and D. Direct path length $S \rightarrow D$ is normalized to be 1. In all presented results, the path-loss exponent $\alpha = 3$ is under investigation. We can see that the simulation results well match the theoretical ones. This shows that the theoretical error performance expressions are almost exact. In addition, Figure B.3 demonstrates that the cooperation significantly improves the BER performance in comparison with direct transmission. These results are obvious because the cooperation benefits from diversity gain as well as from path-loss reduction.

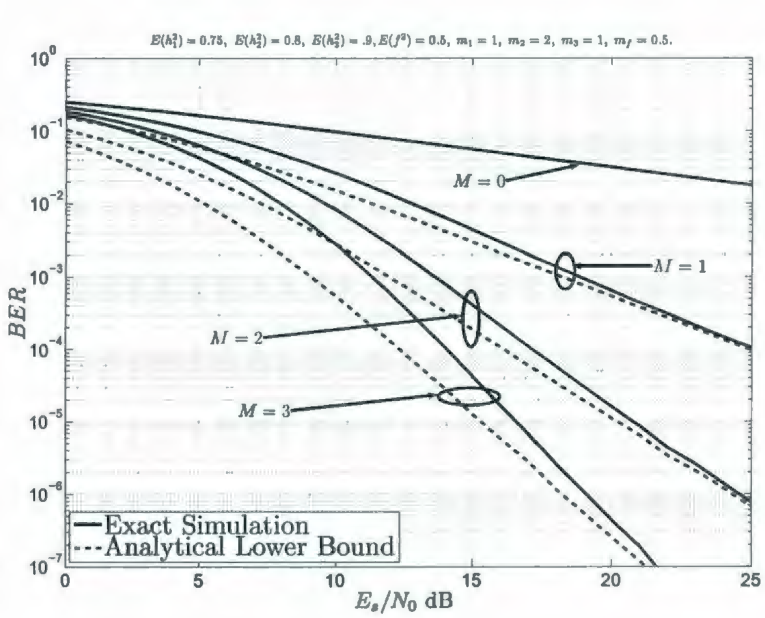

Figure B.2: BER performance for several number of cooperative paths ($M = 0, 1, 2$ and 3) and $N = 3$.

It is apparent from Figures B.2 and B.3; (as expected) that in general, relay power is better used when it adds a cooperative branch than when it adds a hop in an existent branch. The reason of that adding more branches will increase the diversity order but adding more hops will increase the virtual array gain only. It can be proven easily that similar results can be seen for outage probability performance.

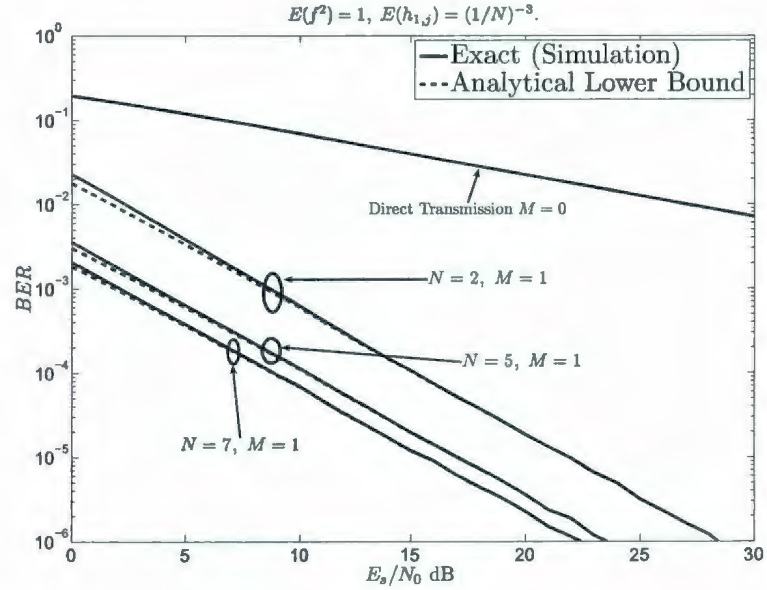

Figure B.3: BER performance for several number of multi hop ($N = 2, 5$ and 7) and $M = 1$(diversity order of 2)

Appendix C

Performance of the SNR Threshold Decode-and-Forward Cooperative Diversity Networks over Nakagami-m Fading Channels

Consider a cooperative communications in a dual-hop wireless network where information is transmitted from a source S to a destination D with the assistance of a relay R as shown in Figure C.1. All terminals are equipped with single-antenna transceivers and sharing the same frequency band. In addition, each terminal can not transmit and receive at the same time to mitigate the implementation complexity. Time division multiplexing (TDM) is used for channel access in this section.

In this appendix we will consider the adaptive decode-and-forward scheme which the forwarding decision at the relay is made on the basis of an SNR threshold. The forwarding threshold γ_0 defines the minimum instantaneous SNR for which the relay will decode and forward the signal received from the source. Note the following tradeoff: while a large γ_0 lowers the probability of a decoding error (thus minimizing the risk of error propaga-

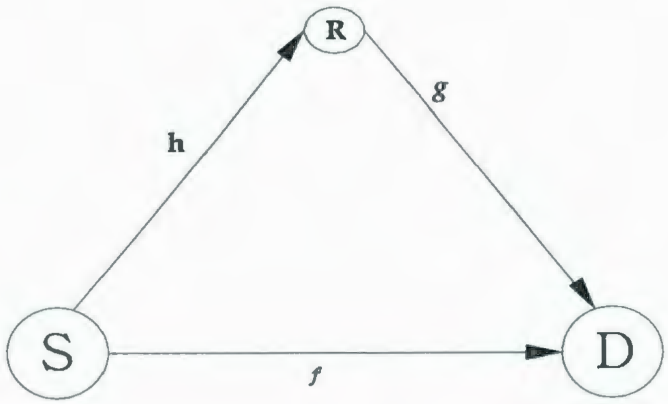

Figure C.1: Illustration of a multi branch cooperative diversity network.

tion induced by the forwarding relay), it decreases the number of cases in which the relay decodes and forwards, thereby reducing diversity benefits. For convenience of presentation and simplicity, we assume a BPSK modulation for all the transmissions. However, the derivations can be extended to MPSK modulation.

The general error probability of the decode-and-forward scheme can be written as

$$P_{e2e}(e) = P(dec)\, P_{div}(e) + (1 - P(dec))\, P_{non-coop}(e) \tag{C.1}$$

where, $P(dec)$ is the probability that the relay decodes and forwards, $P_{div}(e)$ is the probability that an error occurs in the combined diversity transmission from source and relay to the destination, and $P_{non-coop}(e)$ is the probability of error given that the relay decides not to decode. $P(dec)$ can be written as

$$P(dec) = \Pr(\gamma_h > \gamma_0) = \frac{1}{\Gamma(m_h)} \Gamma\left(m_h, \frac{m_h}{\bar{\gamma}_h} \gamma_0\right) \tag{C.2}$$

where, m_h is the Nakagami-m fading parameter between the source and relay, $\bar{\gamma}_h$ =

$\mathbf{E}\left(h^2\right) E_s/N_0$ is the average SNR from the source to the relay, $\Gamma\left(\bullet, \bullet\right)$ is the incomplete gamma function [22, eq.(8.350.2)], and $\Gamma\left(\bullet\right)$ is the gamma function [22, eq. (8.310.1)]. It should be noted that for fixed decode-and-forward scheme $P(dec) = 1$, since the relay always sends another copy to the destination.

If the relay decides to decode and forward, the destination combines the signals it receives from source and relay. Then the resulting error probability $P_{div}(e)$ can be written as:

$$P_{div}\left(e\right) = P_{(S,R)}\left(e\right) P_{prop}\left(e\right) + \left(1 - P_{(S,R)}\left(e\right)\right) P_{coop}\left(e\right) \tag{C.3}$$

where $P_{(S,R)}\left(e\right)$ is the probability of error at the relay, $P_{prop}\left(e\right)$ is the probability that an error happens in the cooperative transmission from the source and the relay to the destination given that the relay decoded unsuccessfully (propagation error), and $P_{coop}\left(e\right)$ is the probability that an error happens in the cooperative transmission from the source and the relay to the destination given that the relay decoded correctly.

The average bit error rate (BER) at the relay when BPSK modulation is used can be written as (The proof is at the end of this section):

$$\begin{aligned} P_{(S,R)}\left(e\right) &= Q\left(\sqrt{2\gamma_0}\right) - \frac{(m_h - 1)!}{2\sqrt{\pi}\Gamma\left(m_h, \frac{m_h}{\bar{\gamma}_h}\gamma_0\right)} \\ &\times \sum_{k=0}^{m_h-1} \frac{1}{k!}\left(\frac{m_h}{\bar{\gamma}_h}\right)^k \left(1 + \frac{m_h}{\bar{\gamma}_h}\right)^{-k-1/2} \Gamma\left(k + 1/2, \left(1 + \frac{m_h}{\bar{\gamma}_h}\right)\gamma_0\right) \end{aligned} \tag{C.4}$$

Note that for a special case $\gamma_0 = 0$, and $m_h = 1$ (Rayleigh channel), we obtain the well known probability of error for BPSK transmission over Rayleigh fading channel.

In case of a decision error, the relay transmits an erroneous signal to the destination, thus leading to error propagation. The error probability due to error propagation ($P_{prop}\left(e\right)$)

can be well approximated as (The proof is at the end of this section):

$$P_{prop}(e) = 1 - \left(\frac{m_g}{\bar{\gamma}_g}\right)^{m_g} \frac{1}{\Gamma(m_g)} \sum_{n=0}^{m_f-1} \frac{1}{n!} \left(\frac{m_f}{\bar{\gamma}_f}\right)^n \frac{\Gamma(m_g+n)}{(m_g/\bar{\gamma}_g + m_f/\bar{\gamma}_f)^{m_g+n}} \tag{C.5}$$

where m_g is the Nakagami-m fading parameter between the relay and destination, $\bar{\gamma}_g = \mathbf{E}(g^2) E_s/N_0$ is the average SNR from the relay to the destination and $\bar{\gamma}_f = \mathbf{E}(f^2) E_s/N_0$ is the average SNR from the source to the destination.

The $P_{coop}(e)$ denotes the probability of error at the destination after combining the signals received from the source and the relay, given that the relay has correct detection. This error can be written as (The proof is at the end of this section):

$$\begin{aligned} P_{coop}(e) &= \sum_{k=1}^{m_f} \Xi_1 \frac{(2k-1)!!}{2^k k!} \left(\frac{m_f}{\bar{\gamma}_f}\right)^{m_f} {}_2F_1\left(k, k+1/2, k+1; -\frac{m_f}{\bar{\gamma}_f}\right) \\ &+ \sum_{k=1}^{m_g} \Xi_2 \frac{(2k-1)!!}{2^k k!} \left(\frac{m_g}{\bar{\gamma}_g}\right)^{m_g} {}_2F_1\left(k, k+1/2, k+1; -\frac{m_g}{\bar{\gamma}_g}\right) \end{aligned} \tag{C.6}$$

where $(2k-1)!!$ is the double factorial notation denoting the product of only odd integers from 1 to $(2k-1)$, ${}_2F_1(\bullet,\bullet;\bullet;\bullet)$ is the Gauss' hypergeometric function defined in [22, eq. (9.100)], m_f is the Nakagami-m fading parameter between the source and destination, $\Xi_1 = (-1)^{m_g} \frac{(m_g/\bar{\gamma}_g)^{m_g}(m_f+m_g-k-1)!}{(m_f/\bar{\gamma}_f)^{k-m_f}(m_g-1)!(m_f-k)!}$ and $\Xi_2 = (-1)^{m_f} \frac{(m_f/\bar{\gamma}_f)^{m_f}(m_g+m_f-k-1)!}{(m_g/\bar{\gamma}_g)^{k-m_g}(m_f-1)!(m_g-k)!}$.

Finally, if the relay decides not to decode, then the destination needs to rely on the direct channel from the source to the destination. Then the corresponding error probability can be written as [23]

$$P_{non-coop}(e) = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}_f}{m_f+\bar{\gamma}_f}} \sum_{k=0}^{m_f-1} \binom{2k}{k} \left(\frac{1}{4} - \frac{\bar{\gamma}_f}{4(m_f+\bar{\gamma}_f)}\right)^k\right] \tag{C.7}$$

By substituting (C.4), (C.5) and (C.6) into (C.3) and then substituting (C.2), (C.3) and (C.7) into (C.1) we can have a closed-form expression for the end-to-end error probability of the SNR threshold adaptive decode-and-forward scheme over Nakagami-m flat fading channels.

The error performance analysis given above can be used for an arbitrary threshold γ_0. However, we can easily compute the optimum threshold that minimizes the error probability for the SNR threshold adaptive decode-and-forward cooperative diversity. This optimum threshold can be determined by setting the derivative of $P_{e2e}(e)$ with respect to γ_0 equal to zero. Since the derivative of $P_{e2e}(e)$ with respect to γ_0 can be written as

$$\frac{\partial P_{e2e}(e)}{\partial \gamma_0} = \frac{\partial \left(P\left(dec\right) P_{div}\left(e\right)\right)}{\partial \gamma_0} + \frac{\partial \left(\left(1 - P\left(dec\right)\right)\right)}{\partial \gamma_0} P_{non-coop}\left(e\right) \tag{C.8}$$

Then by setting (C.8) equal to zero and using Leibnitz differentiation rule [22], we can compute the optimum threshold (denoted as γ_{opt}) as

$$Q\left(\sqrt{2\gamma_{opt}}\right) = \frac{P_{non-coop}\left(e\right) - P_{coop}\left(e\right)}{P_{prop}\left(e\right) - P_{coop}\left(e\right)} \Rightarrow \gamma_{opt} = \frac{1}{2}\left[Q^{-1}\left(\frac{P_{non-coop}\left(e\right) - P_{coop}\left(e\right)}{P_{prop}\left(e\right) - P_{coop}\left(e\right)}\right)\right]^2 \tag{C.9}$$

where Q^{-1} is the inverse Q-function.

An asymmetric network is examined where the relay is located on a straight line between the source and destination. Direct path length $S \rightarrow D$ is normalized to be 1. In all presented results, the path-loss exponent α is equal to 3.

We verify the accuracy of the BER expressions in (C.1) using Monte-Carlo simulations. The results are depicted in Figure C.2 for $\gamma_0 = 0, 0.4106, 1.3530, 2.3874$ and γ_{opt}. We can see that the simulation results match the theoretical ones very well. This shows that the theoretical BER expressions are almost exact. In addition, Figure C.2 demonstrates that the cooperation significantly improves the BER performance in comparison with direct transmission. These results are obvious because the cooperation benefits from the diversity gain as well as from the path-loss reduction. Figure C.2 shows that adaptive decode-and-forward scheme outperforms the fixed decode-and-forward scheme regardless of the SNR value; hence we will focus on the adaptive decode-and-forward scheme in the following results.

Figure C.2 also shows the influence of γ_0 on the BER performance. It is evident that when γ_0 is large, the diversity order is reduced since the probability that the relay retransmits the source data is small. Also, the small threshold increases the percentage of incorrect detection at the relay and thus decaying the performance of the receiver. This point is clearly shown in Figure γ_0. It is obvious that the optimum BER curves lie below all the others. Moreover, based on (C.9), we find the list of the optimum thresholds γ_{opt} corresponding to each E_s/N_0 in Table C.1. Table C.1 shows that the values of γ_{opt} increases with respect to the increase of E_s/N_0.

Since the analysis agrees with the simulation, we will use only the theoretical formulas in the following remaining results.

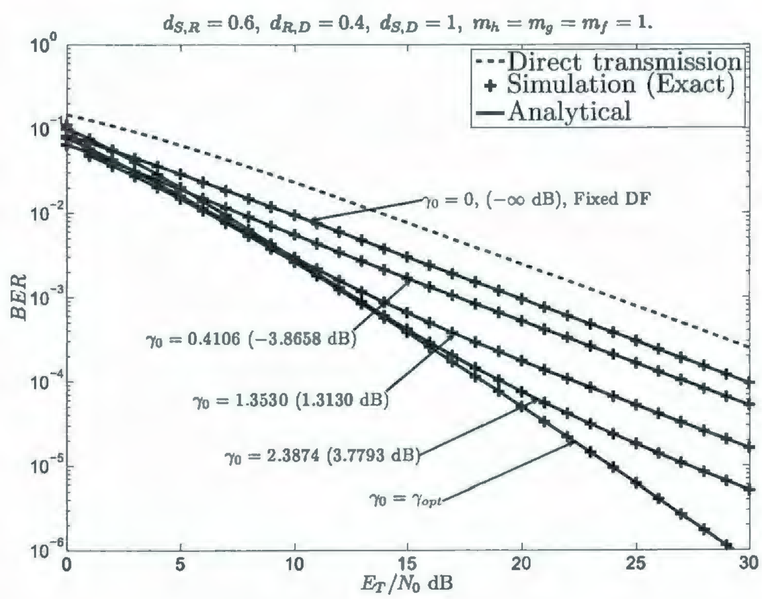

Figure C.2: BER of the adaptive decode-and-forward cooperative diversity and direct link for different values of γ_0.

Table C.1: The optimum threshold

E_T/N_0	5dB	10dB	15dB	20dB	25dB	30dB
γ_{opt}	1.084	1.888	2.823	3.839	4.882	4.946

Figure C.3 shows the effect of m_h on the error performance when m_g and m_f are constant. It can be shown from Figure C.3 that as mh increases the error performance improves until it approaches the ideal case which is two transmit diversity (the source and the relay send the same signal to the destination), especially for moderate to high SNR. This behavior occurs because as m_h increases $P_{S,R}$ significantly decreases; hence the relay will send a freshly corrected new signal to the destination with high probability getting the ideal case (two transmit diversity) at the destination. Note that for low SNR $P(dec)$ is not close to 1 and this is why for this region the error performance is not close to the ideal case (two transmit diversity). It should be noted that the two transmit ideal case is controlled by the values of m_g and m_f. Hence, obviously if m_g and m_f are improved with improving mh the end-to-end error will be improved.

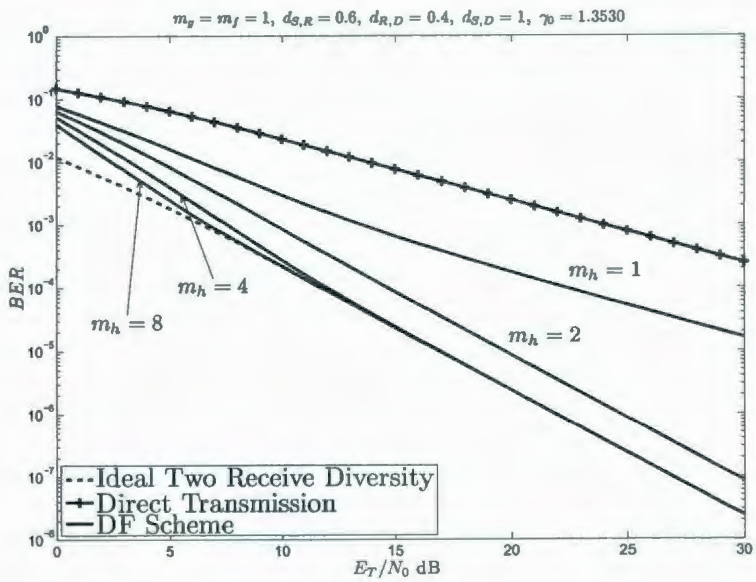

Figure C.3: BER of the adaptive decode-and-forward cooperative diversity for different values of fading parameter m_h.

The effect of m_g can be seen in Figure C.4. It can be noticed that m_g improves the BER until it approaches the value of m_h (which in this case equals 3). Then there is almost no improvement (the two lines for $m_g = 2$ and 3 are overlapped in Figure C.4). The reason of this trend is that when the relay-destination channel is worse than the source-relay channel, the relay-destination channel will have the most influence on the end-to-end error at the destination. As the relay-destination channel is getting better, the most influence on the error will transfer to the source-relay channel due to error propagation. Mathematically speaking, when m_g increases the second term of (C.3) will reduce significantly but the first term will not change (does not depend on mg), leaving the first term (error propagation term) is the dominant term of (C.3).

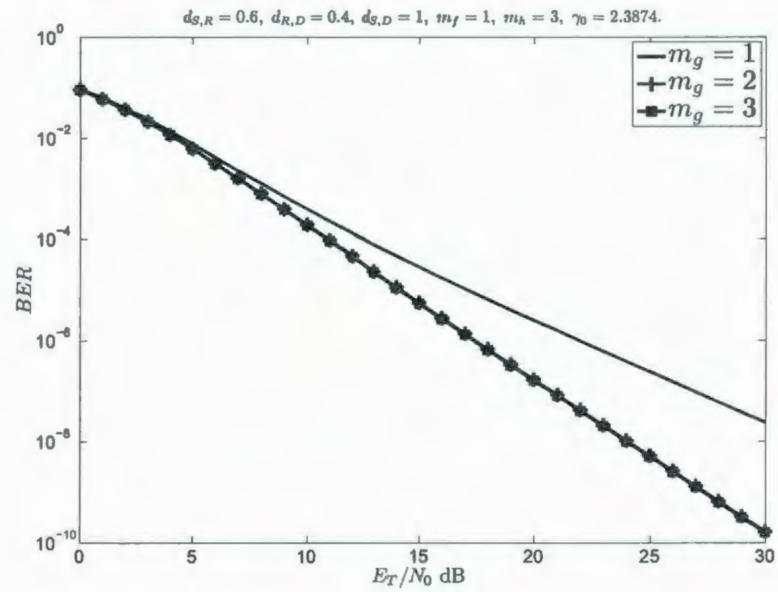

Figure C.4: BER of the adaptive decode-and-forward cooperative diversity for different values of fading parameter m_g.

C.1 Proof of equation (C.4)

The general average probability of error between the source and relay is given by

$$P_{(S,R)}(e) = \int_0^\infty P_{(S,R)}(e|\gamma)\, f_{\gamma_h}(\gamma|\gamma_h > \gamma_0)\, d\gamma \tag{C.10}$$

We consider a case of communication over a Nakagami-m fading channel where detection is performed only when the instantaneous SNR exceeds a threshold γ_0. The resulting PDF of the effective SNR after many algebraic simplifications can be written as:

$$f_{\gamma_h}(\gamma|\gamma_h > \gamma_0) = \begin{cases} 0, & \text{if } \gamma \leq \gamma_0; \\ \frac{(m_h/\bar{\gamma}_h)^{m_h}\gamma^{m_h-1}}{\Gamma(m_h, \gamma_0 m_h/\bar{\gamma}_h)} \exp\left(-\gamma m_h/\bar{\gamma}_h\right), & \text{if } \gamma > \gamma_0 \end{cases} \tag{C.11}$$

Now by introducing

$$u(\gamma) = \int f_{\gamma_h}(\gamma|\gamma_h > \gamma_0) = \frac{\Gamma\left(m_h, \gamma\frac{m_h}{\bar{\gamma}_h}\right)}{\Gamma\left(m_h, \gamma_0\frac{m_h}{\bar{\gamma}_h}\right)} \tag{C.12}$$

by partial integration of (C.13) we have

$$P_{(S,R)}(e) = u(\gamma)\, Q\left(\sqrt{2\gamma}\right)|_{\gamma=\gamma_0}^{\infty} - \int_{\gamma_0}^{\infty} u(\gamma)\frac{\partial Q\left(\sqrt{2\gamma}\right)}{\partial\gamma}d\gamma \tag{C.13}$$

Applying the definition of the Q-function and using Leibnitz' rule [22, eq. (0.410)], then after simplifications (C.13) can be written as

$$P_{(S,R)}(e) = Q\left(\sqrt{2\gamma_0}\right) - \frac{1}{2\sqrt{\pi}}\int_{\gamma_0}^{\infty}\frac{1}{\sqrt{\gamma}}u(\gamma)\exp(-\gamma)\,d\gamma \tag{C.14}$$

With the help of [22, eq. 8.352.2] and some algebraic simplifications, it can be easily shown that integral in (C.14) can be reduced to (C.4).

C.2 Proof of equation (C.5)

Since we assume BPSK modulation, without loss of generality, we assume that the source sends +1 symbol, the relay sends −1 symbol and an error occurs if the destination decides that −1 is send. For convenience of presentation, we utilize discrete time complex equivalent base-band models to express all signals. The received signals from the source and the relay are

$$\begin{aligned} y_{S,D} &= f\sqrt{E_s}x_s + n_1 \\ y_{R,D} &= g\sqrt{E_s}x_r + n_3 \end{aligned} \tag{C.15}$$

note in this case that $x_r = -x_s$, after MRC, we can use the following decision variable

$$\begin{aligned} y_{MRC} &= \frac{f^*\sqrt{E_s}}{N_0}y_{S,D} + \frac{g^*\sqrt{E_r}}{N_0}y_{R,D} \\ &= \left(\frac{f^2E_s}{N_0} - \frac{g^2E_s}{N_0}\right)x_s + \frac{f^*\sqrt{E_s}}{N_0}n_1 + \frac{g^*\sqrt{E_s}}{N_0}n_3 \\ &= (\gamma_f - \gamma_g)\,x_s + \tilde{n} \end{aligned} \tag{C.16}$$

where $\tilde{n} = \frac{f^*\sqrt{E_s}}{N_0}n_1 + \frac{g^*\sqrt{E_s}}{N_0}n_3$. The effective noise $\tilde{n}$ is also a Gaussian random variable with zero mean and variance equal to $(\gamma_f + \gamma_g)$. Since the destination assumes that both the source and relay send the same symbol, the optimal decision rule at the destination is:

$$\hat{x} = \begin{cases} 1, & \text{if } y_{MRC} > 0; \\ -1, & \text{if } y_{MRC} < 0 \end{cases} \tag{C.17}$$

The probability of error propagation given the SNRs of $R \to D$ and $S \to D$ channels is

$$\begin{aligned} P_{prop}(e|\gamma_f, \gamma_g) &= \Pr(y_{MRC} < 0|\gamma_f, \gamma_g) \\ &= \Pr(\tilde{n} > (\gamma_f - \gamma_g)\,|\gamma_f, \gamma_g) \\ &= Q\left(\frac{\gamma_f - \gamma_g}{\sqrt{(\gamma_f + \gamma_g)/2}}\right) \end{aligned} \tag{C.18}$$

Then, P_{prop}

$$\begin{aligned} P_{prop}(e) &= \int\!\!\int P_{prop}(e|\gamma_f, \gamma_g)\, f_{\gamma_f}(\gamma)\, f_{\gamma_g}(\gamma)\, d\gamma_f d\gamma_g \\ &= \int_0^\infty\!\!\int_0^\infty Q\left(\frac{\gamma_f - \gamma_g}{\sqrt{(\gamma_f + \gamma_g)/2}}\right) f_{\gamma_f}(\gamma)\, f_{\gamma_g}(\gamma)\, d\gamma_f d\gamma_g \end{aligned} \tag{C.19}$$

Due to the complexity of the exact expression given in (C.19), we use a high SNR approximation for $P_{prop}(e)$. If we assume $\gamma_f >> 1$, $\gamma_g >> 1$ and $\zeta = \gamma_g/\gamma_f$ then

$$\begin{aligned} Q\left(\frac{\gamma_f - \gamma_g}{\sqrt{(\gamma_f + \gamma_g)/2}}\right) &= Q\left(\sqrt{\sqrt{\gamma_f}\frac{1-\zeta}{\sqrt{(1+\zeta)/2}}}\right) \\ &= \begin{cases} 1, & \text{if } \zeta > 1; \\ -1, & \text{if } \zeta < 1 \end{cases} \end{aligned} \tag{C.20}$$

where we use

$$Q(x) \approx \begin{cases} 1, & \text{if } x >> 1; \\ 0, & \text{if } x << 1 \end{cases} \tag{C.21}$$

Substituting (C.21) in (C.19), we obtain

$$\begin{aligned} P_{prop}(e) &= \Pr(\zeta > 1) = \Pr(\gamma_f < \gamma_g) \\ &= \int_0^\infty \int_0^{\gamma_g} \left(\frac{m_f}{\bar{\gamma}_f}\right)^{m_f} \gamma_f^{m_f-1} \exp\left(-\gamma_f m_f/\bar{\gamma}_f\right) \left(\frac{m_g}{\bar{\gamma}_g}\right)^{m_g} \gamma_g^{m_g-1} \exp\left(-\gamma_g m_g/\bar{\gamma}_g\right) d\gamma_f d\gamma_g \end{aligned} \tag{C.22}$$

and this double integrals can be evaluated in a closed form as in (C.5)

C.3 Proof of equation (C.6)

It is known that the average error probability of MRC of two branches can be written as $P_{coop}(e|\gamma_f,\gamma_g) = Q\left(\sqrt{2(\gamma_f+\gamma_g)}\right)$ [23]. And also the average error probability can be written as $P_{coop}(e) = \int_0^\infty Q\left(\sqrt{2\gamma_{eq}}\right) f_{\gamma_e}(\gamma_{eq})\, d\gamma_{eq}$ where $f_{\gamma_e}(\gamma_{eq})$ is the PDF of $\gamma_{eq} = \gamma_f + \gamma_g$. By noting that γ_f and γ_g are independent random variables, then the PDF of γ_{eq} can be written as:

$$f_{\gamma_{eq}}(\gamma_{eq}) = \int_0^{\gamma_{eq}} f_{\gamma_f}(x) f_{\gamma_g}(\gamma_{eq} - x)\, dx \tag{C.23}$$

and using [22, eq. (3.383.1)], it can be expressed as

$$\begin{aligned} f_{\gamma_{eq}}(\gamma_{eq}) &= \frac{(1-m_f-m_g)_{m_f} (m_g/\bar{\gamma}_g - m_f/\bar{\gamma}_f)^{1-m_f-m_g}}{(\bar{\gamma}_g/m_g)^{m_g} (\bar{\gamma}_f/m_f)^{m_f} (m_f+m_g-1)(m_f-1)!} \\ &\times \exp(-\gamma_{eq} m_g/\bar{\gamma}_g) \left\{ \sum_{k=0}^{m_g-1} \frac{(1-m_g)_k [-\gamma_{eq}(m_f/\bar{\gamma}_f - m_g/\bar{\gamma}_g)]^k}{k!\,(2-m_f-m_g)_k} \right. \\ &- \left. \exp(-\gamma_{eq}(m_f/\bar{\gamma}_f - m_g/\bar{\gamma}_g)) \sum_{k=0}^{m_f-1} \frac{(1-m_f)_k [-\gamma_{eq}(m_g/\bar{\gamma}_g - m_f/\bar{\gamma}_f)]^k}{k!\,(2-m_g-m_f)_k} \right\} \end{aligned} \tag{C.24}$$

where $(n)_k \Gamma(n+k)/\Gamma(k)$ is the Pochammer symbol. By substituting (C.24) into (C.23), and this integral can be evaluated via [22, eq.(6.455.1)] by noting that $Q(\bullet)$ can be expressed as an incomplete Gamma function. Therefore, the $P_{coop}(e)$ can be derived in closed form as in (C.6).

Appendix D

Derivation of equation (3.14)

By following the same method in [36], we let $X = \gamma_{S,R_i}\gamma_{R_i,D}$ and $X = \bar{\gamma}_{S,R_i} + \gamma_{R_i,D} + 1$. The PDF of $\gamma_{S,R_i,D}$ is determined as follows

$$\begin{aligned} f_{\gamma_{S,R_i,D}}(\gamma) &= \int_{\bar{\gamma}_{S,R_i}+1}^{\infty} |y|\, f_{X,Y}(\gamma y, y) dy \\ &= \int_0^{\infty} |\bar{\gamma}_{S,R_i} + 1 + u|\, f_{X,Y}(\gamma(\bar{\gamma}_{S,R_i} + 1 + u), \bar{\gamma}_{S,R_i} + 1 + u) du \end{aligned} \tag{D.1}$$

where $f_{X,Y}(x,y) = f_{X|Y}(x|y) f_y(y)$ denotes the joint PDF of X and Y. the marginal PDF of Y is given by

$$f_Y(y) = \frac{(y - \bar{\gamma}_{S,R_i} - 1)^{m_{R_i,D}-1}}{\Gamma(m_{R_i,D})} \left(\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right)^{m_{R_i,D}} \exp\left(\frac{-m_{R_i,D}\,(y - \bar{\gamma}_{S,R_i} - 1)}{\bar{\gamma}_{R_i,D}}\right) \tag{D.2}$$

The conditional PDF $f_{X|Y}(x|y)$ is given by

$$f_{X|Y}(x|y) = \frac{1/\Gamma(m_{S,R_i})}{|y - \bar{\gamma}_{S,R_i} - 1|} \left(\frac{x}{y - \bar{\gamma}_{S,R_i} - 1}\right)^{m_{S,R_i}-1} \left(\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}\right)^{m_{S,R_i}} \exp\left(\frac{x m_{S,R_i}/\bar{\gamma}_{S,R_i}}{(y - \bar{\gamma}_{S,R_i} - 1)}\right) \tag{D.3}$$

Substituting (D.3) and (D.2) into (D.1), after some straightforward manipulations we obtain

$$f_{\gamma_{S,R_i,D}}(\gamma) = \frac{\gamma^{m_{S,R_i}-1}\left(\frac{m_{S,R_i}}{\bar{\gamma}_{S,R_i}}\right)^{m_{S,R_i}}}{\Gamma(m_{S,R_i})\Gamma(m_{R_i,D})}\left(\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}\right)^{m_{R_i,D}} \exp\left(\frac{-\gamma m_{S,R_i}}{\bar{\gamma}_{S,R_i}}\right)$$
$$\times \int_0^{\infty} \frac{(1+\bar{\gamma}_{S,R_i}+u)^{m_{S,R_i}}}{u^{1-m_{R_i,D}+m_{S,R_i}}} \exp\left(-\frac{m_{R_i,D}}{\bar{\gamma}_{R_i,D}}u - \frac{1}{u}\frac{\gamma m_{S,R_i}\left(1+\bar{\gamma}_{S,R_i}\right)}{\bar{\gamma}_{S,R_i}}\right) du \quad \text{(D.4)}$$

By assuming m_{S,R_i} is a positive integer, and using binomial theorem of the expression $(1+\bar{\gamma}_{S,R_i}+u)^{m_{S,R_i}}$, the equation in (D.4) with the help of [22, eq. 3.478.4] can be written as in (3.14).