ESTIMATION OF WAVE-INDUCED SHIP HULL BENDING MOMENT FROM SHIP MOTION MEASUREMENTS
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Estimation of Wave-Induced Ship Hull Bending Moment from Ship Motion Measurements

By

© Jinsong Xu, M. Eng.

A thesis submitted to the School of Graduate Studies in partial fulfillment of the requirement for the degree of Ph.D.

Faculty of Engineering and Applied Science
Memorial University of Newfoundland
November 2000

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Abstract

A time-domain technique is developed for the real-time estimation of wave-induced vertical bending moment from the coupled heave and pitch motion measurements. The estimated values can be compared with those obtained from the strain gauge readings to ensure the validity of Hull Response Monitoring Systems (HRMS).

A general time-domain relationship between the vertical bending moment and the coupled heave and pitch motions was formulated through a Fourier transform of the frequency-domain mathematical model. The instantaneous bending moment value was approximated by a function of the heaving and pitching displacements, velocities, and accelerations. A neural network technique was developed to identify this unknown function through a learning process.

The application of the proposed technique to the experimental data demonstrated the validity of the methodology. The simulation results suggested that this technique could be used in conjunction with a time-domain simulation package to provide a numerical estimation model.
Acknowledgements

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<td>ABS</td>
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</tr>
<tr>
<td>DNV</td>
<td>Det Norske Veritas (Norway)</td>
</tr>
<tr>
<td>HRMS</td>
<td>Hull Response Monitoring System</td>
</tr>
<tr>
<td>IMO</td>
<td>International Maritime Organization</td>
</tr>
<tr>
<td>ISSC</td>
<td>The International Ship and Offshore Structure Congress</td>
</tr>
<tr>
<td>ITTC</td>
<td>The International Towing Tank Conference</td>
</tr>
<tr>
<td>Lloyd’s</td>
<td>Lloyd’s Register of Shipping (UK)</td>
</tr>
<tr>
<td>MLP</td>
<td>The Multilayer Perceptron</td>
</tr>
<tr>
<td>MUN</td>
<td>Memorial University of Newfoundland</td>
</tr>
<tr>
<td>NSRDC</td>
<td>Naval Ship Research and Development Center (USA)</td>
</tr>
<tr>
<td>ONR</td>
<td>Office of Naval Research (USA)</td>
</tr>
<tr>
<td>RAO</td>
<td>Response Amplitude Operator</td>
</tr>
<tr>
<td>RINA</td>
<td>The Royal Institution of Naval Architects (UK)</td>
</tr>
<tr>
<td>SNAME</td>
<td>The Society of Naval Architects and Marine Engineers (USA)</td>
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1. Introduction

1.1 Objective
The growth in the size and complexity of ship structures has provided an impetus for the increased use of Hull Response Monitoring Systems (HRMS). These systems are intended to enhance ship operational safety by monitoring and displaying ship response information. Since ship structural design is mainly based on the magnitude of the allowable hull girder bending moment, it would be preferable for the HRMS systems to display the instantaneous bending moment against allowable values in real time.

In most HRMS systems, bending moment values are obtained from strain gauge measurements. These transformations are usually based on the theoretical strain-moment relationship and need to be validated by some other estimation methods. Significant research has been conducted in an attempt to estimate hull girder bending moments from ship motion data. These estimated values can be compared with those transformed from the strain gauge readings to prevent any significant errors.

The main objective of this work is to develop a time-domain technique for estimating the wave-induced vertical bending moment from the coupled heave and pitch motions.

1.2 Research Outline
Ship response analysis could be carried out in three different domains, namely, the time domain, the frequency domain, and the probability domain. Numerous transformation techniques allow the researchers to move from one domain to another.
For real-time estimation in this work, a time-domain relationship between the wave-induced bending moment and vertical ship motions has to be established. It is convenient to determine a mathematical model in the frequency domain first, and then transform the model into the time domain. The frequency-domain model for the rigid-body ship motions and the wave-induced bending moment can be formulated using a strip theory. Through Fourier transform, the wave-induced bending moment is approximated by a function of the heaving and pitching displacements, velocities, and accelerations in the time domain. This function can be identified using artificial neural networks.

The important feature of neural networks is their ability to approximate arbitrary functions through on-line and off-line learning processes. A multilayer perceptron (MLP) network is employed in this work, and the backpropagation learning algorithm is used. The training data must be maximally representative and informative. For the present system, it is proved that the auto- and cross-correlation functions of the bending moment and ship motions are suitable training data. The correlation functions can be estimated from the ship response time histories.

The proposed technique is applied to experimental data as well as to simulated data. Model tests of 'R-class Icebreaker' ship model are carried out in the wave tank at Memorial University of Newfoundland. The model consists of two segments joined at the midship section by an aluminum bar. Four strain gauges are mounted on the joint bar to measure the midship bending moment. After the training process, the neural network produces the accurate estimates of the instantaneous bending moment. The simulation
results suggest that this technique can be used in conjunction with a time-domain simulation package to provide a numerical estimation model.

This work is based on a mathematical model for the rigid ship hull and a linear relationship between the wave-induced bending moment and the ship motions in the vertical plane. The high-frequency vibratory loads due to whipping and springing are not considered in the current work. It is promising to extend the present technique to include the high-frequency loads in the final estimation.

1.3 Organization

An extensive review of previously published work and available techniques is presented in Chapter 2. A full description of the ship model and the experimental program follows in Chapter 3. The mathematical formulation as well as the experimental analysis in the frequency domain is described in Chapter 4. The time-domain estimation technique is developed in Chapter 5. A numerical simulation procedure and the results are given in Chapter 6. Finally, the conclusions and recommendations are presented in Chapter 7.

More details of the numerical computations, the computer programs, and the experimental results are presented in Appendix I to Appendix III.
2. Literature Review

2.1 Hull Response Monitoring System

2.1.1 HRMS Overview
Ship response to the sea environment is a main concern for ship designers and operators. With the growth in ship size and structural complexity, it has been difficult for mariners to monitor various ship responses through their physical senses. Hull Response Monitoring Systems (HRMS) have been developed to measure and display the real-time ship motions and hull structural responses. The recorded data is also valuable for more sophisticated onshore analysis to develop and verify ship design criteria. HRMS development is spurred by regulatory bodies, classification societies, academic organizations, and ship owner/operators. IMO is developing a set of HRMS rules for bulk carriers over 20,000 DWT. ABS, Lloyd’s, and DNV have offered guidelines for HRMS systems. Some technical issues are also addressed in recent ISSC proceedings (Moan and Berge, 1997). A survey result from Slaughter et al. (1997) shows that there are more than 200 HRMS installations around the world. Representative installations include four crude oil tankers for BP Oil Company (Witmer and Lewis, 1995), and two other crude oil tankers for ARCO Marine (Lacey and Chen, 1995).

Ashcroft (1996) and Slaughter et al. (1997) present an overview of Hull Response Monitoring Systems. There are four major components in a basic HRMS system: a microprocessor, a visual display, data storage equipment, and a suite of sensors. Long baseline strain gauges are typically used to measure primary hull girder strain on the main deck. Accelerometers are used to measure ship motions. Some environmental sensors are
also available for installation. The microprocessor analyzes the raw data and interprets the signals. Immediate comparison can be made between measured and allowable values. Suitable information is displayed in real time. The storage of monitored data allows more sophisticated post voyage analysis. The basic HRMS system may be expanded to incorporate a wide variety of additional sensors. Some more extensive systems include automatic downloading of weather forecast information via satellite and advanced voyage-planning capability based on weather information. It is expected that HRMS systems will become a standard feature in future ships.

2.1.2 Characteristics of Major Ship Responses
The major environmental factor for a floating vessel is ocean waves. The wave elevation in a realistic seaway is always irregular and never repeats itself. Ship responses that may require monitoring are mainly ship motions, hull girder stresses, and wave loads. They are also random processes (Slaughter et al., 1997).

The six-degree-of-freedom ship motions are three translational (surge, sway, and heave) and three rotational (roll, pitch, and yaw). Roll, pitch, and heave are generally of most concern for ship motion analysis. Roll motion reduces reserve transverse stability and causes crew discomfort. The coupled heaving and pitching generate the vertical cargo acceleration and increase relative bottom velocity with respect to the waves. A detailed description of ship motions is given in Lewis (1989).

Full-scale stress data from Little and Lewis (1971), Vulovich et al. (1989) and other research programs have clarified the characteristics of hull girder stresses under actual service conditions. These characteristics are summarized by Mansour (1990). A typical
stress time history is the combination of three independent components: static stress, low-frequency wave-induced stress, and high-frequency vibratory stress. The static stress results mainly from the stillwater loads caused by the loading condition in stillwater and the thermal loads due to temperature difference in water and air. The wave-induced stress is associated with the loads resulting from the motions of the ship as a rigid body. The frequency range is relatively low and close in magnitude to the wave encounter frequency for a wave having a length nearly equal to the ship length. The vibratory stress is associated with loads resulting from the two-node mode vibration of the hull girder. The frequency range is relatively high and close to the two-node mode natural frequency of the hull girder. The vibration can be continuous "springing" excited by the high-frequency wave components. It also can be the transient "whipping" caused by bottom or bow flare slamming. The wave-induced stress and the vibratory stress do not always occur simultaneously. Quite often only the wave-induced component appears in a stress record. Due to the random nature of the ocean waves, both wave-induced stress and vibratory stress are random processes.

Wave loads exerted on a ship hull girder include bending moments and shear forces (Bhattacharyya, 1978). Similar to the hull girder stresses, the total bending moment consists of a static component, a wave-induced component, and a vibratory component (due to whipping and springing). From full-scale measurements and model tests results, the whipping bending moment may be as large as the wave-induced bending moment, while the springing bending moment is usually of low magnitude. Only in very long flexible ships such as Great Lakes bulk carriers, the springing loads are found to be
important. Jenson and Dogliani (1996) shows that a design wave bending moment probably can be derived without considering springing for normal merchant ship types. In most cases, only static and wave-induced bending moments are superimposed on each other. At an arbitrary longitudinal section, total wave-induced bending moments can be resolved into three components: vertical bending moment, horizontal bending moment, and torsional moment. Vertical bending moment dominates the structural design. Horizontal bending moment is generally much less important because of its smaller magnitude and the greater hull strength. Torsional moment is usually insignificant. Therefore, the wave-induced vertical bending moment represents a major component of hull girder bending moment for a ship at sea.

2.1.3 Monitoring Hull Girder Bending Moments
With the drastic increase in ship size since 1960’s, overloading of hull structures has been highlighted in many marine losses. Since ship structural design is based mainly on the magnitude of the allowable bending moment, it would be preferable for the HRMS system to display the instantaneous bending moment against allowable values in real time (Witmer and Lewis, 1995). The recorded data is also valuable for the structural reliability analysis, which requires more specific information about wave loads (Mansour, 1990).

In most full-scale measurements, the bending moment values are obtained from the strain gauge readings. Strains experienced by a ship’s deck structure normally comprise three components, namely, primary hull strain due to hull girder bending moments, secondary grillage strains and tertiary strains in local plate panels. Long baseline strain gauges are typically designed to measure primary hull strain by measuring the displacement of a
metal rod of about two meters in length. The rod is rigidly fixed at one end and free to move at the other. The coefficient of thermal expansion of the rod and deck material should be matched to minimize errors caused by thermal expansion. Experience has shown that long baseline strain gauges are more durable in exposed locations and less susceptible to local strain effects (Ashcroft, 1996). As long as the ship structure remains elastic, hull girder bending moment varies linearly with the primary hull strain. Ideally the transformation from strain readings to bending moment values should be based on a full-scale calibration. That is, a known bending moment is applied on the ship in calm water while the corresponding change in strain gauge reading is recorded (Little and Lewis, 1971). The “known” bending moments are usually applied in either of two ways: special ballast changes or normal cargo discharges. Full-scale calibration frequently takes several days and the strain gauges are responsive to thermal effects that may cause a gross change in hull stress. It is thus required to conduct full-scale calibrations on overcast, windless nights after the hull structure has reached a steady-state temperature. Even if the calibration were “thermal effects free”, the final measurements may still be affected by the diurnal thermal stress changes, particularly in the more tropic regions. Such thermal contamination has been fully discussed by Shi et al. (1996). As an alternative to full-scale calibrations, some HRMS systems obtain the linear strain-moment relationship from the structural analysis results. Witmer and Lewis (1995) used simple beam theory to obtain the bending moment as the product of the measured bending stress and the calculated section modulus. The use of finite element analysis should provide more accurate results. As indicated by Ashcroft (1996), an HRMS system should include a means for checking the system results to ensure its validity. Significant
research has been conducted in an attempt to estimate hull girder bending moments from wave information or ship motion measurements.

Theoretically it is easier to estimate the hull girder bending moment from wave information rather than from ship motion measurements. But shipboard environmental sensors are much less accurate than ship motion sensors. Some researchers are even back-calculating sea states as a function of ship motion parameters (Hua and Palmquist, 1995). It is expected that values of hull girder bending moments estimated from ship motion measurements should be more accurate than those estimated from wave information. Slaughter et al. (1997) quoted a technique proposed by Lovdahl, Lacey, and Chen in their presentation “Advances in Computer Based Onboard Voyage Planning” to the 1995 SNAME Joint California Sections Meeting. Their approach is based on the work of Kaplan (1995), but the accuracy of this approach is not within ABS guidelines for real-time value display. In Kaplan (1995), both wave-induced loads and slam-induced vibratory loads are incorporated into time-domain simulations. The rigid-body ship motion characteristics are calculated in the frequency domain using a linear strip theory. Although time histories of ship motions and wave loads are generated, the general time-domain relationships between the wave loads and ship motions are not established. Without a time-domain mathematical model, it is not convenient to estimate the instantaneous bending moments from ship motion measurements in real time.
2.2 Theories for Ship Response Analysis

2.2.1 Frequency-domain Theories
Extensive research has been conducted to study ship response in regular waves. The strip theory for heave and pitch motions in head waves (Korvin-Kroukovsky and Jacobs, 1957) was the first motion theory suitable for numerical computations with adequate accuracy. This theory was later extended by Jacobs (1958) to include wave-induced bending moments and shear forces for a ship in regular head seas. The more rigorous approach of Salvesen et al. (1970) has been widely accepted for computing the motions and loads of a rigid-body ship advancing at constant mean forward speed with arbitrary heading in regular sinusoidal waves.

Strip theory is used to determine the coefficients and exciting forces in the linear equations of ship motion. It is assumed that the vessel is a slender body moving in high-frequency waves. Consequently, the fluid flow velocities in the transverse direction are much greater than in the longitudinal direction, and the flow field around any cross section of the ship may be approximated by the assumed two-dimensional flow in that plane. If the ship body is divided into many narrow strips, the total effect on the ship is the integration of the effects on all individual strips. The essence of strip theory is thus to reduce a three-dimensional hydrodynamic problem to a series of two-dimensional problems. The most popular methods for solving the two-dimensional hydrodynamic problems are the boundary integral method (Frank, 1967) and the Lewis-form method (Lewis, 1929). Frank's method allows a more accurate description of the hull cross section, but requires more computational effort compared with the Lewis-form approach.
Strip theory methods have been extended to study the springing loads in flexible ship hulls. Troesch (1984) carried out both experiments and computations for springing response on a ship model joined amidships. Jensen and Dogliani (1996) performed springing load calculations within the framework of a nonlinear, quadratic strip theory. The complete solution to the springing problem requires consideration of the hull structural properties as well as the hydrodynamic excitation.

For ship motions in low-frequency waves, the ordinary slender-body theory has to be used instead of the strip theory. Newman and Sclavounos (1980) developed a unified theory to cover the whole frequency range. At low frequencies, the unified theory approaches the ordinary slender-body theory and yields terms that involve longitudinal interference between sections. For high frequencies, this longitudinal interference disappears and the results are identical to the strip theory.

The advent of large, high-speed computers has allowed the application of three-dimensional calculation methods, which take into account the 3-D effects and improve the accuracy of both local and global force predictions. The Rankine source methods and 3-D panel methods are being developed to replace the strip theory (Tan et al., 1996).

The equations for regular-wave responses involve frequency-dependent coefficients. For a given frequency they are all constants and the system has a solution. Essentially we are solving the problems in the frequency domain, and the regular-wave equations constitute a frequency-domain mathematical model (Ogilvie, 1964).
2.2.2 Linear-Random Theory

The theory for the response of a linear system to random excitation was developed in the realm of electrical engineering (Rice, 1944). It is essentially a combination of linear system theory and random process theory. St. Denis and Pierson (1953) first introduced this linear-random theory to the marine field for the study of irregular ship motions. Later application for the wave load evaluation helped in the development of the structural reliability analysis for marine structures (Mansour, 1990).

Lewis (1989) gives a description of the linear-random theory for marine applications. There are two crucial assumptions underlying the theory. First, the short-term ocean waves are stationary, zero-mean, Gaussian random processes. Secondly, ship responses are linear transformations of the wave elevation or slope. Based on these assumptions the probability structure and the statistical parameters of the wave elevation and the ship response are constant in the short term. Linear-random theory is intended for predicting the stationary statistics of the ship response. In a stationary, zero-mean, Gaussian process the only necessary statistical parameter is the variance. It is uniquely determined by the power spectrum. Because of the linearity assumption, the response spectrum is related to the wave spectrum through a Response Amplitude Operator (RAO). The Response Amplitude Operator is conventionally defined as the modulus of the frequency response function, and physically interpreted as the ratio of the response amplitude to the amplitude of a harmonic wave. The RAOs represent the frequency-domain relationship between ship responses and wave elevations. They can be obtained from either theoretical calculations or regular-wave experiments. Once the wave spectrum and RAOs are available, the ship response spectra and statistics can be easily determined.
The essence of linear-random theory is transforming the frequency-domain results into a probability domain. The statistical parameters as well as extreme values are suitable for use in reliability analyses and performance assessments.

2.2.3 Time-domain Theories
Although the application of linear strip theory in the frequency and probability domains has achieved remarkable success, there are a number of ship response problems that cannot be satisfactorily analyzed using such techniques. Bottom slamming, bow flare impact, and the resulting whipping are obvious examples. They are commonly dealt with in the time domain using estimated motion history and impact forces (Oliver, 1990). A comprehensive review of the theories and techniques for impact loading is given in Daidola and Mishkevich (1995). In order to analyze the combined responses, even linear rigid-body results have to be expressed in the form of a time history.

Kaplan (1995) describes a quasi-linear analysis procedure to determine both the rigid-body and hydroelastic vibratory responses due to slamming impacts. The rigid-body ship motions and wave loads are calculated in the frequency domain using a linear strip theory. These outputs in frequency response form are then expressed in time history form in order to be combined with the slam-induced responses that are determined in the time domain. The transformation is performed using a time-domain simulation method. A wave spectrum is considered as a sum of sinusoidal components, and the response time history is obtained as the linear superposition of responses to these individual wave components. The general time-domain equations are not formulated in this procedure.
In Oliver (1990), the simulation of ship motions and the hull girder loads is based on the time-domain equations. The assumed equation form is similar to that of the frequency domain, but the coefficients are calculated at each time step. The frequencies at which the coefficients are evaluated can be selected to be either peak frequencies of response spectra or the average frequencies during the previous two cycles of response. Here the time-domain equations are actually approximated using the frequency-domain models.

For the ship response in ocean waves, it is usually convenient to formulate a mathematical model in the frequency domain first, and then transform the results into the time domain (Hutchison, 1990). The frequency-domain equations are generally differential equations with frequency-dependent coefficients. Tick (1959) and Ogilvie (1964) have shown that these equations can be transformed to the time domain as convolution integral equations. Under certain conditions, they may be approximated by the constant-coefficient differential equations.
2.3 Experiments for Ship Response Analysis

2.3.1 Full-scale Measurements and Ship Model Tests

Full-scale measurement is the most direct approach to study ship response in actual seaways. More and more design criteria are based on the results of full-scale measurements. Typical programs are presented in Little and Lewis (1971) and Vulovich et al. (1989). This approach, however, has some disadvantages and difficulties. First of all, full-scale measurements are very expensive and difficult to implement. Secondly, it is difficult to measure the sea state, in particular the directional wave spectrum. Thirdly, it is impossible to fully control the experimental environment. Finally, full-scale measurements can not be used as a design tool for predicting the behavior of a proposed new design (Hutchison, 1990).

In contrast to full-scale measurements, ship model tests are controlled experiments. The desired sea environment can be achieved and maintained during the test. This approach is a useful design tool and relatively inexpensive. One major problem is the errors caused by scale effect due to viscous and surface tension forces. In some experiments, the physical dimensions of the test facility make it difficult to obtain long duration test runs for a model with forward speed (Hutchison, 1990).

During model tests, the motion measurements are relatively straightforward. But the experimental study of the hull girder bending moment requires a specially designed ship model. Lewis (1954) used a wooden model to determine the wave-induced bending moment. The model was segmented amidships and the bending moment was determined by measuring the relative deflection between the two segments. This model is essentially
rigid and does not include any hydroelastic effects. De Does (1960) measured bending moments at three locations along the model length. The models were made of fiberglass with wire resistance strain gauges imbedded in the walls. Wachnik and Schwartz (1963) segmented a ship model at seven stations and interconnected the model parts using an aluminum flexure beam. These models are essentially hydroelastic models and the hull vibratory responses are included in the measurements.

Recently, Hermanski (1993) measured the wave-induced bending moment on a rigid ship model. McTaggart (1997) used a hydroelastic model with a continuous flexible backbone to determine both the wave-induced and whipping bending moments.

2.3.2 Experimental Data Analysis and System Identification
Ship response data are recorded during ship model tests and full-scale measurements. Data analysis and system identification techniques have been employed to determine the response characteristics and the coefficients of the assumed mathematical model. Hutchison (1990) mentioned some applications in determining resistance and maneuvering characteristics for a full-scale ship. Application to the rolling model tests showed some good results.

A typical procedure for conducting system identification is described in Ljung (1999). The most fundamental step is the careful design of the experiments. The recorded response data must be representative and informative. After data acquisition and preparation, the form of the mathematical model has to be determined. This is the most difficult step in the procedure. In some cases, a model with several unknown physical parameters may be constructed using basic physical laws and other well-established
relationships. This is a "gray box" model. In other cases some standard mathematical structures, such as a linear time-invariant model and artificial neural networks, are assumed without reference to the physical background. The parameters of these models have no physical interpretation. This is a "black box" model. The final step in the procedure is to identify the unknown parameters using the experimental data through the application of estimation methods. The quality of the model is assessed from the performance when the model is used to reproduce the experimental data.

The analysis of the experimental data could be carried out in three different domains, namely, the time domain, the frequency domain, and the probability domain. Numerous transformation and analysis techniques allow the researchers to move from one domain to another (Hutchison, 1990). Typical data processing techniques include data filtering, mean and variance analysis, probability density analysis, auto-correlation analysis, cross-correlation analysis, auto-spectral density analysis, cross-spectral density analysis, impulse response analysis, and frequency response analysis. The details of data analysis techniques are fully presented in Bendat and Piersol (1986), and Ljung (1999).
2.4 Real-time Techniques for Ship Response Analysis

2.4.1 Application of Real-time Techniques
Real-time techniques have been developed rapidly in the past decades. Their application in the marine field has helped to improve the operational safety and efficiency. Real-time estimation is widely used in HRMS systems for monitoring instantaneous ship response. Real-time forecasting is developed for predicting motions or other responses over an interval of several seconds. Tan et al. (1993) gives a brief review of these techniques. The early attempts for real-time prediction of ship responses were made using Kalman filtering techniques. The mathematical model of the ship response is provided using seakeeping theory and based on the knowledge of the encountered wave spectrum and response transfer functions. The main difficulty in this approach is the estimation of the wave spectrum. Later development involved the application of on-line system identification techniques. Chung et al. (1990) developed an algorithm in which the wave excitation information is extracted from the ship motion data. The estimated wave excitation is extrapolated for the forecasting of the ship motions. Broome and Pittaras (1990) used the ARMAX models (Auto Regressive Moving Average with eXogenous input) to identify the ship motion equations in real time. No previous knowledge of the ship response is required. The mathematical model is formed on-line and conveniently updated whenever it is necessary.

Recent development is closely associated with the real-time control of ship response and machinery. The application details and technical issues are presented in Wilson (1997).
The focus of the real-time techniques is the construction of a mathematical model in the time domain.

2.4.2 Artificial Neural Networks
Artificial neural networks are a group of cellular computational structures that can be implemented in both hardware and software forms. One of their capabilities is to approximate arbitrary functions through an on-line or off-line learning process. This provides a means to identify and model complex systems without a priori knowledge of the physical mechanisms. In the past decade, they have become a very popular choice as a universal "black box" model for nonlinear systems (Ljung, 1999). A full discussion about network structures, functional capabilities, learning algorithms, and generalization performance is presented in Hush and Horne (1993).

Artificial neural networks are constructed from individual neurons, which are the basic computing units in the structure. Static networks are characterized by neuron equations that are memoryless, thus the output is a function only of the current inputs. Dynamic networks, on the other hand, are systems with memory. Their neuron equations are typically described by differential or difference equations. In the multilayer perceptron (MLP) network, which is the most widely used static network, individual neurons are arranged in successive layers with the sigmoid nonlinearity as neuron equation. Quite often each layer is fully connected to the adjacent layers and information is passed only forward from the input layer through the hidden layer(s) to the output layer. Linear neurons are commonly used in the output layer to make learning easier. The connecting weights between the layers are the trainable network parameters that fully determine the
relationship between the inputs and the outputs. During the supervised learning process, the neural network is presented with a set of input-output points and trained to implement a mapping that matches the sample points as closely as possible. The most popular learning method for the MLP is the backpropagation algorithm, which uses a gradient search technique to find the optimum values for the connecting weights. It is an iterative process of computing the gradient and adjusting the weight values until a minimum error is located. This learning process finally selects a "black box" model suitable for the underlying system.

The most important issue for the network implementation is the generalization performance, which is a measure of how well the network performs on the actual data outside the training set. A particular method, cross-validation, has been developed to monitor generalization performance during the learning process. It works by splitting the sample data into two sets: a training set that is used to train the network, and a test set that is used to measure the generalization performance of the network. During the learning process the performance of the network on the training data will continue to improve, but its performance on the test data will only improve to a point, beyond which it will start to degrade. It is at this point where the network starts to memorize the data instead of to learn the function. This is the point at which the learning process should be stopped. Cross-validation not only avoids any premature termination, but also improves the generalization performance of the network.

In the marine field, the MLP networks have been used for ship motion identification by Haddara & Hinchey (1995), and Haddara & Xu (1999). More applications are associated
with ship control systems. Hardier (1997) developed a maneuverability model using recurrent neural networks. Zhang et al. (1997) described an on-line trained neutral network controller (NNC) for the automatic ship control. The focus of these developments is to take advantage of the learning ability of the neural networks.
2.5 Review Summary

An extensive review of the literature indicates that Hull Response Monitoring System (HRMS) is a developing technology to measure and display the real-time ship response information. The major ship responses that may require monitoring are ship motions, hull girder stresses, and wave loads. The results of full-scale measurements and model tests have shown that the wave-induced vertical bending moment is the significant wave load component. Since ship structural design is mainly based on the magnitude of allowable bending moment, it would be preferable for the HRMS system to display the instantaneous bending moment against allowable values in real time. In most HRMS systems, the bending moment values are determined from the strain gauge readings based on linear strain-moment relationship. The transformation results may be affected by the calibration process and the computational inaccuracies. They need to be validated by some other estimation results. Significant research has been conducted in an attempt to estimate hull girder bending moments from ship motion readings.

Ship response analysis could be carried out in three different domains, namely, the time domain, the frequency domain, and the probability domain. Numerous theories and transformation techniques allow the researchers to move from one domain to another. Strip theory, unified theory, and 3-D theories have been developed to solve the regular-wave problems and formulate the frequency-domain equations. Linear-random theory transforms the frequency-domain results into probability domain. The resulting statistical parameters and extreme values are suitable for reliability analysis and performance assessment. The time-domain results are also transformed from the frequency domain.
Numerical simulations and Fourier transforms are most common techniques. In addition to theoretical analysis, full-scale measurements and ship model tests provide the experimental methods for ship response analysis. System identification techniques have been combined with the experimental data to determine the unknown coefficients and terms in ship response models.

More and more real-time techniques have been developed for estimation, forecasting, and control of ship responses. The fundamental element of the real-time techniques is the construction of a mathematical model in the time domain. Artificial neural networks have become a popular choice as a universal "black box" model of nonlinear systems. The peculiar advantage lies in their capability to approximate arbitrary functions through an on-line or off-line learning process. This provides a good means to identify and model the complex systems without a priori knowledge of physical mechanisms.
3. Model Tests

3.1 Experimental Set-up

3.1.1 General Arrangement
The objective of the present work is to identify the time-domain relationship between wave-induced vertical bending moment and coupled heave and pitch motions. Time histories of bending moment and ship motions have to be measured during model tests. The experimental technique involving a segmented model, as seen in Lewis (1954) and Hermanski (1993), is used in this work.

Model tests were carried out in MUN's wave tank using a 1:40 'R-class Icebreaker' ship model. The testing facility consists of a wave tank, an instrumented towing carriage, and a fully equipped control room. The wave tank has inside dimensions of 58.27m in length, 4.57m in wide, and 3.04m in depth. At one end of the wave tank is a hydraulically operated, piston type wave generator. At the other end is a parabolic beach for absorbing and dissipating the incident wave energy. Waves are created in the tank by the translatory motion of a waveboard, which is electronically controlled from the control room. The available wave frequencies range from 0.3Hz to 1.2Hz. The towing carriage over the wave tank is equipped with a dynamometer, which is able to measure the vertical motion up to 0.4m and rotations within a ±30°.

During the model tests, an 'R-class Icebreaker' ship model was positioned along the centerline of the wave tank, and attached to the towing carriage through the dynamometer. The model was free to move only in the vertical plane, and the forward
speed was provided by the towing carriage. In every test run, five parameters were measured and recorded in the form of time history. They are:

- Model Speed (m/sec)
- Wave Elevation (cm)
- Heave Displacement (cm)
- Angular Pitch Displacement (degree)
- Wave-induced Midship Bending Moment (Nm)

In order to measure the wave-induced bending moment amidships, the ship model was segmented at the midship section. The two segments were joined together by an aluminum bar, and the hull gap was sealed with medium-resistive latex medical band. The general arrangement is illustrated in Figure 1.

![Figure 1: Illustration of General Arrangement](image-url)
3.1.2 Model Description

The model used for the tests is a 1:40 scale 'R-class Icebreaker'. The hydrostatic particulars of the ship model are presented in Table 1, and the body plan is shown in Figure 2. The model hull was made of glass reinforced plastic and segmented at the midship section. A gap of about 5mm was left between the segments to allow the small deflection of the joint bar. This gap was sealed with medium-resistant latex medical band, which was attached to the hull surface using duct tape. When the ship model was out of the tank, three auxiliary bars were added to prevent the overload of the midship section. They were removed before each calibration and test run.

![Figure 1: Body Plan of 'R-class Icebreaker' Ship Model](image)

*Figure 2: Body Plan of 'R-class Icebreaker' Ship Model*
Table 1: Hydrostatic Particulars for 'R-class Icebreaker' Ship Model

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars (LPP), m</td>
<td>2.1985</td>
</tr>
<tr>
<td>Length of waterline (LWL), m</td>
<td>2.3250</td>
</tr>
<tr>
<td>Waterline beam at midships, m</td>
<td>0.4840</td>
</tr>
<tr>
<td>Waterline beam at maximum section, m</td>
<td>0.4840</td>
</tr>
<tr>
<td>Maximum waterline beam, m</td>
<td>0.4845</td>
</tr>
<tr>
<td>Draft at midships, m</td>
<td>0.1735</td>
</tr>
<tr>
<td>Draft at maximum section, m</td>
<td>0.1745</td>
</tr>
<tr>
<td>Draft at aft perpendicular, m</td>
<td>0.1790</td>
</tr>
<tr>
<td>Draft at forward perpendicular, m</td>
<td>0.1675</td>
</tr>
<tr>
<td>Equivalent level keel draft, m</td>
<td>0.1735</td>
</tr>
<tr>
<td>Maximum section forward at midships, m</td>
<td>-0.1850</td>
</tr>
<tr>
<td>Area of maximum section, m²</td>
<td>0.0773</td>
</tr>
<tr>
<td>Center of buoyancy forward of midships (LCB), m</td>
<td>-0.0080</td>
</tr>
<tr>
<td>Center of buoyancy above keel, m</td>
<td>0.0970</td>
</tr>
<tr>
<td>Wetted surface area, m²</td>
<td>1.3347</td>
</tr>
<tr>
<td>Volume of displacement, m³</td>
<td>0.1190</td>
</tr>
<tr>
<td>Center of floatation forward of midships (LCF), m</td>
<td>-0.0175</td>
</tr>
<tr>
<td>Center of floatation above keel, m</td>
<td>0.1735</td>
</tr>
<tr>
<td>Area of waterline plane, m²</td>
<td>0.899</td>
</tr>
<tr>
<td>Transverse metacentric radius (BM), m</td>
<td>0.122</td>
</tr>
<tr>
<td>Longitudinal metacentric radius (BML), m</td>
<td>2.4</td>
</tr>
<tr>
<td>Center of area of profile plane forward of midships, m</td>
<td>-0.0195</td>
</tr>
<tr>
<td>Center of area of profile plane above keel, m</td>
<td>0.0895</td>
</tr>
<tr>
<td>Area of profile plane, m²</td>
<td>0.3580</td>
</tr>
</tbody>
</table>

The platform structure, the aluminum bar, and the dynamometer formed a considerable proportion of the ballast on the model. Other lead ballast weights were distributed along
the model length and placed as low as possible to assure maximum transverse stability. The individual weight locations are listed in Table 2, and the model longitudinal weight distribution is plotted in Figure 3.

Figure 3: Weight Distribution of 'R-class Icebreaker' Model
Table 2: Weight Locations in ‘R-class Icebreaker’ Model

<table>
<thead>
<tr>
<th>Component</th>
<th>Mass (kg)</th>
<th>Aft-edge (cm) (from midship)</th>
<th>Fore-edge (cm) (from midship)</th>
<th>Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballast-1</td>
<td>12.005</td>
<td>42.5</td>
<td>70.5</td>
<td>28</td>
</tr>
<tr>
<td>Ballast-2</td>
<td>8.381</td>
<td>42.5</td>
<td>64.5</td>
<td>22</td>
</tr>
<tr>
<td>Ballast-3</td>
<td>1.964</td>
<td>42.5</td>
<td>51.5</td>
<td>9</td>
</tr>
<tr>
<td>Ballast-4</td>
<td>4.056</td>
<td>32.5</td>
<td>42.5</td>
<td>10</td>
</tr>
<tr>
<td>Ballast-5</td>
<td>4.075</td>
<td>32.5</td>
<td>42.5</td>
<td>10</td>
</tr>
<tr>
<td>Ballast-6</td>
<td>4.110</td>
<td>-14</td>
<td>-3</td>
<td>11</td>
</tr>
<tr>
<td>Ballast-7</td>
<td>4.285</td>
<td>-14</td>
<td>-3</td>
<td>11</td>
</tr>
<tr>
<td>Ballast-8</td>
<td>1.998</td>
<td>-36.5</td>
<td>-31.5</td>
<td>5</td>
</tr>
<tr>
<td>Ballast-9</td>
<td>1.996</td>
<td>-36.5</td>
<td>-31.5</td>
<td>5</td>
</tr>
<tr>
<td>Ballast-10</td>
<td>8.607</td>
<td>-58.5</td>
<td>-36.5</td>
<td>22</td>
</tr>
<tr>
<td>Ballast-11</td>
<td>12.005</td>
<td>-63.5</td>
<td>-36.5</td>
<td>27</td>
</tr>
<tr>
<td>Ballast-12</td>
<td>2.096</td>
<td>-83.5</td>
<td>-74.5</td>
<td>9</td>
</tr>
<tr>
<td>Dynamometer</td>
<td>7.3</td>
<td>-8</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Aft-platform</td>
<td>11.33</td>
<td>-27.5</td>
<td>-7.5</td>
<td>20</td>
</tr>
<tr>
<td>Center-platform</td>
<td>1.643</td>
<td>-7.5</td>
<td>7.5</td>
<td>15</td>
</tr>
<tr>
<td>Fore-platform</td>
<td>8.667</td>
<td>7.5</td>
<td>27.5</td>
<td>20</td>
</tr>
<tr>
<td>Joint bar</td>
<td>2.348</td>
<td>-22.5</td>
<td>22.5</td>
<td>45</td>
</tr>
<tr>
<td>Ship Hull</td>
<td>24.5</td>
<td>-110</td>
<td>110</td>
<td>220</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>121.366</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.1.3 Instrumentation

Five parameters were recorded during model tests. The wave elevation was measured using a capacitance type wave probe, which was installed on the carriage at a fixed location about 0.5m away from the model at the midship section. The forward speed was
measured by a velocity meter on the towing carriage. Heave displacement and angular pitch displacement were measured using the dynamometer located on the vertical line through the center of gravity. The wave-induced bending moment was determined from the strain of the aluminum bar, which was equipped with four strain gauges at the center. During model tests, a computer on the towing carriage was used to record the experimental data at a frequency of 50Hz.

The aluminum bar actually functioned as a bending moment transducer. It was machined from an aluminum 6061 T6 stock. The elastic part is 150mm long and extends 75mm fore-and-aft from the midship section. Its cross-sectional shape is 20mm×20mm square. The strain values were measured using four metal-foil strain gauges mounted along the longitudinal center of the bar. The four matched gauges were connected to form a full Wheatstone bridge, which compensated for temperature effects and other environmental noise. A schematic diagram is shown in Figure 4. This design is based on the work of Wachnik and Schwartz (1963).

![Schematic Diagram of the Aluminum Bar](image-url)
To install both the aluminum bar and the dynamometer across the midship section, a platform structure with three levels was mounted on the two segments. The upper-most level is the dynamometer platform, which has a cantilever form to avoid any disturbance to the moment transducer. The second level consists of two separate platforms mounted each on a different segment. The aluminum bar was rigidly bolted to these platforms at the two ends, leaving the middle part to deflect elastically. On the lowest level, the installation space was left for three auxiliary bars that were added after the experiments. These bars provided a protection for the model hull against breakage when the model was out of the tank. The whole platform structure was bolted to large wooden plates that were securely fastened to the ship model bottom with wood screws and epoxy bonding material. Figure 5 shows the ship model out of the wave tank with the auxiliary bars added in the lowest level. Figure 6 shows the ship model in the wave tank with the dynamometer attached to the cantilever platform.

Figure 5: Ship Model out of the Wave Tank
Figure 6: Ship Model in the Wave Tank
3.2 Experimental Preparation

3.2.1 Bending Moment Calibration
Prior to the model tests, one major concern is the calibration of the bending moment transducer. Since the bending moment is linearly proportional to the strain of the aluminum bar, the calibration was performed by applying a series of known loads to the bar. The calculated bending moments were combined with the strain gauge readings to determine the slope and offset constants using a least-square linear fit. In this experiment, two types of calibration were carried out for validating each other. They are the bench calibration and the in-situ calibration.

For the bench calibration the aluminum bar was removed from the model and set up as a cantilever beam. Several known weights were placed at the free end, and the corresponding bending moments at the strain gauge location were calculated as the product of loads and distance. These calculated values represent dynamic bending moments rather than total bending moments. The sagging moments were recorded as positive values, and the hogging moments were negative. The applied weights, calculated bending moments, and the strain gauge readings are listed in Table 3.

<table>
<thead>
<tr>
<th>Load W (kg)</th>
<th>Distance L (m)</th>
<th>Dynamic BM M (Nm)</th>
<th>Digital Reading X</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.215</td>
<td>0</td>
<td>31420</td>
</tr>
<tr>
<td>1</td>
<td>0.215</td>
<td>-2.107</td>
<td>31051</td>
</tr>
<tr>
<td>3</td>
<td>0.215</td>
<td>-6.321</td>
<td>30300</td>
</tr>
<tr>
<td>4</td>
<td>0.215</td>
<td>-8.428</td>
<td>29928</td>
</tr>
<tr>
<td>5</td>
<td>0.215</td>
<td>-10.535</td>
<td>29553</td>
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</tbody>
</table>
The in-situ calibration took place in the wave tank with the aluminum bar and the strain gauges mounted on the fully outfitted model. Some known weights were placed initially at the midship section, and the bending moment was recorded as zero. The weights were then separated into two equal sets, and moved the same distance in opposite directions. The calibration process is illustrated in Figure 7. The changes of the midship bending moment due to the weight movements were calculated as the product of the moved weight and the distance to the midship section. The changes in the sag direction were recorded as positive values, and those in the hog direction were negative. The final results of the in-situ calibration are listed in Table 4.

Figure 7: In-situ Calibration Illustration
Table 4: In-situ Calibration Results for Dynamic Bending Moment

<table>
<thead>
<tr>
<th>Moved Weight W (kg)</th>
<th>Distance L (m)</th>
<th>Dynamic BM M (Nm)</th>
<th>Digital Reading X</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>36534</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>-9.8</td>
<td>35007</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>-15.68</td>
<td>33991</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>-19.6</td>
<td>33326</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>-23.52</td>
<td>32649</td>
</tr>
<tr>
<td>4</td>
<td>0.7</td>
<td>-27.44</td>
<td>31910</td>
</tr>
<tr>
<td>4</td>
<td>0.8</td>
<td>-31.36</td>
<td>31254</td>
</tr>
<tr>
<td>4</td>
<td>0.9</td>
<td>-35.28</td>
<td>30534</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>-39.2</td>
<td>29827</td>
</tr>
</tbody>
</table>

Both bench calibration data and in-situ calibration data are plotted together in Figure 8. There is a perfect linear relationship between the midship dynamic bending moment and the strain gauge readings. The straight line passing through the bench calibration points is parallel to the line through the in-situ calibration points. After performing the least-squares linear fit, we got the bench calibration equation

\[ M = 0.005637X - 177.13 \]  \( (1) \)

and the in-situ calibration equation

\[ M = 0.005800X - 212.8 \]  \( (2) \)

The slope values in two equations are very close to each other. The difference in offset values is caused by the zero adjustment of the instrument. In bench calibration the initial value of the digital reading was set to 31420, while for in-situ calibration it was 36534.
These offset values, however, have no effect on the final results since wave-induced bending moments are always taken as the relative change with respect to the stillwater measurements. Therefore, the bench and in-situ calibrations are in excellent agreement.

![Calibration Results for Bending Moment](image)

Figure 8: Calibration Results for Bending Moment

Before the model tests, other measuring instruments were also calibrated. The positive direction for the heave displacement and the wave elevation was set vertically upward. For the angular pitch displacement, the positive direction was set bow upward.

### 3.2.2 Free Response Tests

Prior to the model tests in the waves, the free response tests were carried out to determine the natural frequencies of the ship responses. The model was excited and released in calm
water, and the declining responses were recorded. The coupled heave and pitch responses are shown in Figure 9, and the midship bending moment response is shown in Figure 10.

From the free response results, the natural frequency of the coupled heave and pitch motions is about 1.0Hz. For the midship bending moment, there are two components present in its free response data. While the low-frequency component corresponds to the coupled heave and pitch motions, the high-frequency component is associated with two-node mode ship hull vibration. The two-node mode natural frequency is about 6.5Hz.

![Heave Free Response](image)

![Pitch Free Response](image)

*Figure 9: Coupled Heave and Pitch Free Responses*
Figure 10: Midship Bending Moment Free Response
3.3 Scope of the Experiments

3.3.1 Stationary Tests in Head Waves
Stationary tests were carried out in regular head waves, JONSWAP random head waves, and broad-band random head waves.

Regular head waves with different frequencies were generated in the wave tank. In each test run, 120sec long experimental data were recorded in five channels. The initial part of the time history was obtained in calm water, which was used as the zero value for data analysis. The data files of regular wave tests are listed in Table 5.

Table 5: Stationary Tests in Regular Head Waves

<table>
<thead>
<tr>
<th>Wave Frequency (Hz)</th>
<th>Wave Height (cm)</th>
<th>Data Files</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>6</td>
<td>W3h6.dat</td>
</tr>
<tr>
<td>0.4</td>
<td>6</td>
<td>W4h6.dat</td>
</tr>
<tr>
<td>0.5</td>
<td>6</td>
<td>W5h6.dat</td>
</tr>
<tr>
<td>0.6</td>
<td>6</td>
<td>W6h6.dat</td>
</tr>
<tr>
<td>0.7</td>
<td>6</td>
<td>W7h6.dat</td>
</tr>
<tr>
<td>0.8</td>
<td>6</td>
<td>W8h6.dat</td>
</tr>
<tr>
<td>0.9</td>
<td>6</td>
<td>W9h6.dat</td>
</tr>
<tr>
<td>1.0</td>
<td>6</td>
<td>W10h6.dat</td>
</tr>
<tr>
<td>1.1</td>
<td>6</td>
<td>W11h6.dat</td>
</tr>
</tbody>
</table>

Six groups of JONSWAP head waves were generated in the wave tank. For the test runs within the same group, the wave time histories were generated from the same spectrum. In each test run, 500sec long experimental data were recorded in five channels. The initial
part of the time history was obtained in calm water, which was used as the zero value for data analysis. The data files of JONSWAP wave tests are listed in Table 6.

**Table 6: Stationary Tests in JONSWAP Head Waves**

<table>
<thead>
<tr>
<th>Peak Frequency $F_m$ (Hz)</th>
<th>Significant Wave Height $H_s$ (cm)</th>
<th>Data Files</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>5.0</td>
<td>J7h5c.dat</td>
</tr>
<tr>
<td>0.7</td>
<td>7.5</td>
<td>J7h75a.dat, J7h75b.dat, J7h75c.dat</td>
</tr>
<tr>
<td>0.5</td>
<td>5.0</td>
<td>J5h5a.dat, J5h5b.dat, J5h5c.dat</td>
</tr>
<tr>
<td>0.5</td>
<td>7.5</td>
<td>J5h75a.dat, J5h75b.dat, J5h75c.dat</td>
</tr>
<tr>
<td>0.6</td>
<td>5.0</td>
<td>J6h5a.dat, J6h5b.dat, J6h5c.dat</td>
</tr>
<tr>
<td>0.6</td>
<td>7.5</td>
<td>J6h75a.dat, J6h75b.dat, J6h75c.dat</td>
</tr>
</tbody>
</table>

Two groups of broad-band random wave tests were carried out in the wave tank. For the test runs within the same group, the wave time histories were generated from the same spectrum. In each test run, 500sec long experimental data were recorded in five channels. The initial part of the time history was obtained in calm water, which was used as the zero value for data analysis. The data files for broad-band random wave tests are listed in Table 7.
Table 7: Stationary Tests in Broad-band Random Head Waves

<table>
<thead>
<tr>
<th>Frequency Range</th>
<th>Significant Wave Height</th>
<th>Data Files</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (Hz)</td>
<td>H_s (cm)</td>
<td></td>
</tr>
<tr>
<td>0.3 ~ 1.2</td>
<td>7.5</td>
<td>B75a.dat, B75b.dat</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B75c.dat</td>
</tr>
<tr>
<td>0.3 ~ 1.2</td>
<td>5.0</td>
<td>B5a.dat, B5b.dat</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B5c.dat</td>
</tr>
</tbody>
</table>

3.3.2 Stationary Tests in Following Waves

Stationary tests were carried out in regular following waves, JONSWAP random following waves, and broad-band random following waves.

Regular following waves with different frequencies were generated in the wave tank. In each test run, 120sec long experimental data were recorded in five channels. The initial part of the time history was obtained in calm water, which was used as the zero value for data analysis. The data files for regular wave tests are listed in Table 8.

Six groups of JONSWAP waves were generated in the wave tank. For the test runs within the same group, the wave time histories were generated from the same spectrum. In each test run, 500sec long experimental data were recorded in five channels. The initial part of the time history was obtained in calm water, which was used as the zero value for data analysis. The data files for JONSWAP wave tests are listed in Table 9.
### Table 8: Stationary Tests in Regular Following Waves

<table>
<thead>
<tr>
<th>Wave Frequency (Hz)</th>
<th>Wave Height (cm)</th>
<th>Data Files</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>6</td>
<td>fw3h6.dat</td>
</tr>
<tr>
<td>0.4</td>
<td>6</td>
<td>fw4h6.dat</td>
</tr>
<tr>
<td>0.5</td>
<td>6</td>
<td>fw5h6.dat</td>
</tr>
<tr>
<td>0.6</td>
<td>6</td>
<td>fw6h6.dat</td>
</tr>
<tr>
<td>0.7</td>
<td>6</td>
<td>fw7h6.dat</td>
</tr>
<tr>
<td>0.8</td>
<td>6</td>
<td>fw8h6.dat</td>
</tr>
<tr>
<td>0.9</td>
<td>6</td>
<td>fw9h6.dat</td>
</tr>
<tr>
<td>1.0</td>
<td>6</td>
<td>fw10h6.dat</td>
</tr>
<tr>
<td>1.1</td>
<td>6</td>
<td>fw11h6.dat</td>
</tr>
</tbody>
</table>

### Table 9: Stationary Tests in JONSWAP Following Waves

<table>
<thead>
<tr>
<th>Peak Frequency Fm (Hz)</th>
<th>Significant Wave Height Hs (cm)</th>
<th>Test Data Files</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5.0</td>
<td>Fj5h5a.dat, Fj5h5b.dat</td>
</tr>
<tr>
<td>0.5</td>
<td>7.5</td>
<td>Fj5h75a.dat, Fj5h75b.dat</td>
</tr>
<tr>
<td>0.6</td>
<td>5.0</td>
<td>Fj6h5a.dat, Fj6h5b.dat</td>
</tr>
<tr>
<td>0.6</td>
<td>7.5</td>
<td>Fj6h75a.dat, Fj6h75b.dat</td>
</tr>
<tr>
<td>0.7</td>
<td>5.0</td>
<td>Fj7h5a.dat, Fj7h5b.dat</td>
</tr>
<tr>
<td>0.7</td>
<td>7.5</td>
<td>Fj7h75a.dat, Fj7h75b.dat</td>
</tr>
</tbody>
</table>

Two groups of broad-band random wave tests were carried out in the wave tank. For the test runs within the same group, the wave time histories were generated from the same spectrum. In each test run, 500sec long experimental data were recorded in five channels. The initial part of the time history was obtained in calm water, which was used as the
zero value for data analysis. The data files for broad-band random wave tests are listed in Table 10.

### Table 10: Stationary Tests in Broad-band Random Following Waves

<table>
<thead>
<tr>
<th>Frequency Range</th>
<th>Significant Wave Height</th>
<th>Test Data Files</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (Hz)</td>
<td>Hs (cm)</td>
<td></td>
</tr>
<tr>
<td>0.3 ~ 1.2</td>
<td>5.0</td>
<td>Fb5a.dat, Fb5b.dat</td>
</tr>
<tr>
<td>0.3 ~ 1.2</td>
<td>7.5</td>
<td>Fb75a.dat, Fb75b.dat</td>
</tr>
</tbody>
</table>

3.3.3 Towing Tests in Head Waves
Towing tests were carried out in calm water, regular head waves, and JONSWAP random head waves with a forward speed 0.5m/sec.

A towing test in calm water was carried out prior to each group of test runs. The purpose of these tests is to measure the ship responses due to the model wave system in calm water. In each test run, 90sec long experimental data were recorded in five channels. The initial part of the time history was obtained in stationary state, which was used as the zero value for data analysis.

Regular head waves with different frequencies were generated in the wave tank. In each test run, 90sec long experimental data were recorded in five channels. The initial part of the time history was obtained in calm water stationary state, which was used as the zero value for data analysis. Prior to each group of test runs, a towing test in calm water was carried out. The data files for the towing tests in regular wave and the corresponding calm water tests are listed in Table 11.
Table I: Towing Tests in Regular Head Waves

<table>
<thead>
<tr>
<th>Wave Frequency (Hz)</th>
<th>Wave Height (cm)</th>
<th>Calm Water Test</th>
<th>Data Files</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>6</td>
<td>C05b.dat</td>
<td>S5w3h6.dat</td>
</tr>
<tr>
<td>0.4</td>
<td>6</td>
<td>C05b.dat</td>
<td>S5w4h6.dat</td>
</tr>
<tr>
<td>0.5</td>
<td>6</td>
<td>C05b.dat</td>
<td>S5w5h6.dat</td>
</tr>
<tr>
<td>0.6</td>
<td>6</td>
<td>C05b.dat</td>
<td>S5w6h6.dat</td>
</tr>
<tr>
<td>0.7</td>
<td>6</td>
<td>C05b.dat</td>
<td>S5w7h6.dat</td>
</tr>
<tr>
<td>0.7</td>
<td>5</td>
<td>C05c.dat</td>
<td>S5w7h5.dat</td>
</tr>
<tr>
<td>0.8</td>
<td>5</td>
<td>C05c.dat</td>
<td>S5w8h5.dat</td>
</tr>
<tr>
<td>0.9</td>
<td>5</td>
<td>C05c.dat</td>
<td>S5w9h5.dat</td>
</tr>
<tr>
<td>1.0</td>
<td>5</td>
<td>C05c.dat</td>
<td>S5w10h5.dat</td>
</tr>
<tr>
<td>1.1</td>
<td>5</td>
<td>C05c.dat</td>
<td>S5w11h5.dat</td>
</tr>
</tbody>
</table>

JONSWAP random waves were generated in the wave tank. Due to the limited length of the wave tank, a single test run did not provide a data record long enough for random data analysis. Therefore, a JONSWAP wave time history of 450sec long was divided into seven segments, which were generated sequentially in the wave tank. A separate towing test was performed for each wave segment. The time histories from the seven segment runs were finally joined together to form an experimental data record of 350sec length. In each segment run, 120sec long experimental data were recorded in five channels. The initial part of the time history was obtained in calm water for the stationary model. This was used as the zero value for data analysis. Prior to each segment run, a towing test in
calm water was also carried out. The data files for towing tests in JONSWAP wave segments and the corresponding calm water tests are listed in Table 12.

Table 12: Towing Tests in JONSWAP Head Waves

<table>
<thead>
<tr>
<th>JONSWAP Wave</th>
<th>Calm Water Tests</th>
<th>Test Segments Data</th>
</tr>
</thead>
</table>
| $F_m = 0.7\text{Hz}$  
$H_s = 7.5\text{cm}$ | C05c.dat         | S5j7r1.dat, S5j7r2.dat               |
|               |                  | S5j7r3.dat, S5j7r4.dat              |
|               | C05d.dat         | S5j7r5.dat, S5j7r6.dat              |
|               |                  | S5j7r7.dat                          |
4. Frequency-domain Analysis

4.1 Mathematical Model

For a ship advancing at constant mean forward speed with arbitrary heading in regular waves, the six-degree-of-freedom motion responses and the wave-induced bending moment could be computed theoretically. Under the assumption of linear response and ship port/starboard symmetry, the vertical-plane motions are uncoupled from the horizontal-plane motions. Both the wave excitation forces and the resultant oscillatory motions are assumed to be linear and harmonic at the frequency of wave encounter $\omega_c$,

$$\omega_c = \omega_0 - \frac{\omega_0^2 U}{g} \cos \mu$$  \hspace{1cm} (3)$$

where $\omega_0$ is the wave circular frequency, $U$ is the ship forward speed, and $\mu$ is the wave direction angle. The equations of coupled vertical motions and wave-induced bending moment can be formulated using a strip theory. It is customary to use two sets of coordinate systems as shown in Figure 11: an inertia system $(x_0, y_0, z_0)$ and a body system $(x, y, z)$. The origin of the body system is attached to the center of gravity of the ship, with its $x$-axis pointing forward, $y$-axis pointing to port and $z$-axis pointing upward. The inertial system is fixed in space with its coordinate axes parallel to that of the body system before the ship motion starts.
In this work we use the equations given in Lewis (1989), which are based on the work of Salvesen et al. (1970). The equations of motion for a rigid-body ship are expressed as

\[
(M + A_{33}) \ddot{z} + B_{33} \dot{z} + C_{33} z + A_{35} \ddot{\theta} + B_{35} \dot{\theta} + C_{35} \theta = F_3 e^{i\omega t} \\
A_{23} \ddot{z} + B_{23} \dot{z} + C_{23} z + (I + A_{23}) \ddot{\theta} + B_{23} \dot{\theta} + C_{23} \theta = F_5 e^{i\omega t}
\]

where \(z\) is the heave displacement of the center of gravity, \(\theta\) is the angular pitch displacement, \(M\) is the mass of the ship, \(I\) is the mass moment of inertia; \(F_3\) and \(F_5\) are complex amplitudes of the exciting force and moment, respectively. The force and moment are given by the real part of \(F_j e^{i\omega t}\) \((j = 3, 5)\). The positive direction for \(z\) is vertically upward, and for \(\theta\) is bow downward. The added mass and moment \(A_{jk}\), damping coefficients \(B_{jk}\), and restoring coefficients \(C_{jk}\) are expressed in terms of the sectional coefficients as follows:
\[ A_{33} = \int a_{33} d\xi \quad B_{33} = \int b_{33} d\xi \]  
\[ A_{35} = -\int \xi a_{33} d\xi - \frac{U}{\omega_e^2} B_{33} \quad B_{35} = -\int \xi b_{33} d\xi + UA_{33} \]  
\[ A_{33} = -\int \xi a_{33} d\xi + \frac{U}{\omega_e^2} B_{33} \quad B_{33} = -\int \xi b_{33} d\xi - UA_{33} \]  
\[ A_{35} = \int \xi^2 a_{33} d\xi + \frac{U^2}{\omega_e^2} A_{33} \quad B_{35} = \int \xi^2 b_{33} d\xi + \frac{U^2}{\omega_e^2} B_{33} \]  
\[ C_{33} = \int c_{33} d\xi = \int \rho g B(\xi) d\xi \]  
\[ C_{35} = C_{33} = -\int \xi c_{33} d\xi = -\int \rho g B(\xi) d\xi \]  
\[ C_{55} = \int \xi^2 c_{33} d\xi = \int \rho g B(\xi) d\xi \]

For head sea conditions (\( \mu = 180^\circ \)), the wave excitation forces \( F_3 \) and \( F_5 \) can be simplified as

\[ F_3 = \xi a \int_L e^{ik\xi} e^{-kT(\xi)} [c_{33} - \omega_0(\omega_e a_{33} - ib_{33})] d\xi \]  
\[ F_5 = \xi a \int_L e^{i4\xi} e^{-kT(\xi)} \left\{ \xi [c_{33} - \omega_0(\omega_e a_{33} - ib_{33})] - \frac{U}{i\omega_e} \omega_0(\omega_e a_{33} - ib_{33}) \right\} d\xi \]

In equations (5) and (6), \( a_{33} \) is the sectional heave added mass, \( b_{33} \) is the sectional heave damping, \( c_{33} \) is the sectional restoring force \( \rho g B \), \( B \) is the sectional waterline beam, \( T \) is the mean sectional draft \( S/B \), and \( \xi_a \) is the incident wave amplitude. The sectional coefficients \( a_{33} \) and \( b_{33} \) are obtained by solving the two-dimensional hydrodynamic problems. All coefficients are frequency-dependent.

For a rigid ship hull, the wave-induced vertical bending moment at an arbitrary longitudinal location \( x_0 \) can be expressed in terms of the resulting motions. If the sag direction is taken positive, the expression is
\[ y(x_0) = A_y \ddot{z} + B_y \dot{z} + C_y z + D_y \dot{\theta} + E_y \theta + H_y \theta + y_x e^{i \omega t} \]  

(7)

where

\[ A_y = - \int_{x_0}^{\text{bow}} (m_n + a_{g3})(\xi - x_0) d\xi + \int_{x_0}^{\text{bow}} \frac{U}{\omega_e^2} b_{g3} d\xi \]  

(8)

\[ B_y = - \int_{x_0}^{\text{bow}} b_{g3}(\xi - x_0) d\xi - \int_{x_0}^{\text{bow}} U a_{g3} d\xi \]

\[ C_y = - \int_{x_0}^{\text{bow}} c_{g3}(\xi - x_0) d\xi \]

\[ D_y = \int_{x_0}^{\text{bow}} (m_n + a_{g3})\xi(\xi - x_0) d\xi + \int_{x_0}^{\text{bow}} \frac{U^2}{\omega_e^2} a_{g3} d\xi - \int_{x_0}^{\text{bow}} \frac{U}{\omega_e^2} b_{g3} x_0 d\xi \]

\[ E_y = \int_{x_0}^{\text{bow}} b_{g3}(\xi - x_0) d\xi + \int_{x_0}^{\text{bow}} \frac{U^2}{\omega_e^2} b_{g3} d\xi + \int_{x_0}^{\text{bow}} U a_{g3} x_0 d\xi \]

\[ H_y = \int_{x_0}^{\text{bow}} c_{g3}\xi(\xi - x_0) d\xi \]

The bending moment component \( y_x e^{i \omega t} \) results from the wave excitation forces, and the complex amplitude \( y_x \) can be simplified for head sea conditions as

\[ y_x = \zeta_n \int_{L} e^{ik\xi} e^{-ikT(\xi)} \left\{ (\xi - x_0)[c_{g3} - \omega_0 (\omega_e a_{g3} - ib_{g3})] - \frac{U}{i\omega_e} \omega_0 (\omega_e a_{g3} - ib_{g3}) \right\} d\xi . \]  

(9)

In linear theory, the steady-state harmonic responses \( z, \theta \), and \( y(x_0) \) have the same encounter frequency \( \omega_e \), but different phase shifts. Consequently, they can be expressed in the complex form as

\[ z = Z_n e^{i \omega t} \]

(10)

\[ \theta = \Theta_n e^{i \omega t} \]

\[ y(x_0) = Y_n (x_0) e^{i \omega t} \]
The complex amplitudes $z_a$, $\theta_a$, and $y_a(x_0)$ are functions of both the magnitude and phase of the responses. Substituting expression (10) into equations (4) and (7), and eliminating the common term $e^{i\omega t}$, the resulting equations are

\[
\begin{align*}
[-\omega_e^2(M + A_{33}) + C_{33} + i\omega_e B_{33}]z_a + (\omega_e^2 A_{35} + C_{35} + i\omega_e B_{35})\theta_a &= F_3 \\
[-\omega_e^2(I + A_{35}) + C_{35} + i\omega_e B_{35}]\theta_a + (\omega_e^2 A_{33} + C_{33} + i\omega_e B_{35})z_a &= F_5
\end{align*}
\]

and

\[
y_a(x_0) = (-A_y\omega_e^2 + C_y + i\omega_e B_y)z_a + (-D_y\omega_e^2 + H_y + i\omega_e E_y)\theta_a + y_5
\]

Equations (11) and (12) constitute the frequency-domain model for vertical ship motions and bending moment. The solution to equation (11) is easily obtained as

\[
\begin{align*}
z_a &= \frac{F_5S - F_3Q}{PS - QR} \\
\theta_a &= \frac{F_5P - F_3R}{PS - QR}
\end{align*}
\]

where

\[
\begin{align*}
P &= -\omega_e^2(M + A_{33}) + C_{33} + i\omega_e B_{33} \\
Q &= -\omega_e^2 A_{35} + C_{35} + i\omega_e B_{35} \\
R &= -\omega_e^2 A_{33} + C_{33} + i\omega_e B_{33} \\
S &= -\omega_e^2(I + A_{35}) + C_{35} + i\omega_e B_{35}
\end{align*}
\]

Substitution of (13) into (12) gives the solution for the wave-induced bending moment amplitude $y_a(x_0)$. The ratios between the complex response amplitudes and the wave amplitude are frequency response functions as shown below,
\[ H_z(\omega_r) = \frac{\varepsilon_a}{\xi_n} \]
\[ H_\theta(\omega_r) = \frac{\theta_a}{\xi_n} \]
\[ H_y(\omega_r) = \frac{y_a(x_0)}{\xi_n} \]  

(15)

The modulus of the frequency response function is a Response Amplitude Operator (RAO). These frequency-domain solutions could be obtained either from theoretical computations of two-dimensional hydrodynamic problems, or from experimental model tests in a wave tank.
4.2 Regular Waves Data Analysis

4.2.1 Frequency Analysis Method
Following Ljung (1999), if the input signal $u(t)$ to a linear system has the form

$$u(t) = u_0 \cos(\omega t + \alpha) \quad (16)$$

then the output signal $y(t)$ will be given by

$$y(t) = u_0 |H(\omega)| \cos[\omega t + \alpha + \arg H(\omega)] + v(t) + \text{transient} \quad (17)$$

where $v(t)$ is noise. The transient response eventually decays and only the steady state response remains. The amplitude and the phase of $H(\omega)$ are related to the amplitudes and phase angles of $u(t)$ and $y(t)$. This provides a means to estimate the frequency response function $H(\omega)$ from a series of sinusoidal-wave tests with different frequencies.

For each sinusoidal-wave test, the amplitudes and phase angles of $u(t)$ and $y(t)$ can be determined first from the time histories of the input $u(t)$ and the output $y(t)$, respectively. The existence of noise $v(t)$ in the signal makes it difficult to determine the required parameters accurately. Since the steady state part of $y(t)$ is harmonic, it is possible to average out the noise using the correlation method described by Ljung (1999). From the amplitudes and phase angles of $u(t)$ and $y(t)$, the frequency response function $H(\omega)$ can be calculated. The following algorithm was used in this work

$$|H(\omega)| = \frac{y_0(\omega)}{u_0(\omega)} \quad (18)$$

$$\arg H(\omega) = \arg y(\omega) - \arg u(\omega)$$
where

\[ y_0(\omega) = 2\sqrt{I_c^2(y) + I_s^2(y)} \]  
\[ \arg y(\omega) = \arctan \frac{I_s(y)}{I_c(y)} + k\pi, \quad (k = 0,1) \]

\[ I_c(y) = \frac{1}{N} \sum_{n=1}^{N} y[n] \cos\left(\frac{\omega}{f_s} n\right) \]

\[ I_s(y) = \frac{1}{N} \sum_{n=1}^{N} y[n] \sin\left(\frac{\omega}{f_s} n\right) \]

and

\[ u_0(\omega) = 2\sqrt{I_c^2(u) + I_s^2(u)} \]  
\[ \arg u(\omega) = \arctan \frac{I_s(u)}{I_c(u)} + k\pi, \quad (k = 0,1) \]

\[ I_c(u) = \frac{1}{N} \sum_{n=1}^{N} u[n] \cos\left(\frac{\omega}{f_s} n\right) \]

\[ I_s(u) = \frac{1}{N} \sum_{n=1}^{N} u[n] \sin\left(\frac{\omega}{f_s} n\right) \]

Here \( \omega \) is the circular encounter frequency and \( f_s \) is the sampling frequency taken as 50Hz. The effect of the noise \( v(t) \) diminishes to zero as the number of data points, \( N \) approaches infinity.

### 4.2.2 Results of Regular Wave Tests

From the frequency analysis of the regular wave tests data, we could determine the amplitudes and phase angles of the wave elevation, the heave displacement, the angular pitch displacement, and the wave-induced midship bending moment. Based on these values, the frequency response functions can be estimated. The modulus (RAO) was
calculated as the ratio of the response amplitude to the wave amplitude, and the phase angle of $H(\omega)$ is the phase difference between the response and the wave elevation.

The results of stationary tests in head waves are presented in Table 13 and Table 14. The results of stationary tests in following waves are presented in Table 15 and Table 16.

For each towing test in regular waves, a towing test in calm water was always performed previously. The mean values of the calm water test data were subtracted from the regular wave test data to determine the wave-induced responses. Since the wave probe was fixed to the towing carriage, the identified frequency was the wave encounter frequency. The results of towing tests in head waves are presented in Table 17 and Table 18.
### Table 13: Results of Stationary Tests in Regular Head Waves

<table>
<thead>
<tr>
<th>Wave</th>
<th>BM Amp. (cm)</th>
<th>BM Pha. (deg)</th>
<th>Heave Amp. (cm)</th>
<th>Heave Pha. (deg)</th>
<th>Pitch Amp. (deg)</th>
<th>Pitch Pha. (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W3h6</td>
<td>0.294</td>
<td>3.40</td>
<td>3.27</td>
<td>221</td>
<td>3.29</td>
<td>6.0</td>
</tr>
<tr>
<td>W4h6</td>
<td>0.392</td>
<td>3.31</td>
<td>3.83</td>
<td>205</td>
<td>3.05</td>
<td>5.8</td>
</tr>
<tr>
<td>W5h6</td>
<td>0.490</td>
<td>3.56</td>
<td>7.39</td>
<td>203</td>
<td>3.02</td>
<td>9.0</td>
</tr>
<tr>
<td>W6h6</td>
<td>0.588</td>
<td>2.82</td>
<td>8.89</td>
<td>205</td>
<td>1.94</td>
<td>8.5</td>
</tr>
<tr>
<td>W7h6</td>
<td>0.685</td>
<td>2.70</td>
<td>10.48</td>
<td>196</td>
<td>1.35</td>
<td>16.6</td>
</tr>
<tr>
<td>W8h6</td>
<td>0.784</td>
<td>3.28</td>
<td>13.75</td>
<td>188</td>
<td>0.88</td>
<td>23.8</td>
</tr>
<tr>
<td>W9h6</td>
<td>0.882</td>
<td>3.30</td>
<td>10.82</td>
<td>194</td>
<td>0.52</td>
<td>83.3</td>
</tr>
<tr>
<td>W10h6</td>
<td>0.980</td>
<td>4.13</td>
<td>6.14</td>
<td>222</td>
<td>2.22</td>
<td>181.3</td>
</tr>
<tr>
<td>W11h6</td>
<td>1.077</td>
<td>3.32</td>
<td>6.63</td>
<td>230</td>
<td>0.54</td>
<td>150.1</td>
</tr>
</tbody>
</table>

### Table 14: Frequency Response Functions of Stationary Tests in Regular Head Waves

| Wave | $|H_i|$ (Nm/cm) | $\text{ang}(H_i)$ (deg) | $|H_d|$ (cm/cm) | $\text{ang}(H_d)$ (deg) | $|H_0|$ (deg/cm) | $\text{ang}(H_0)$ (deg) |
|------|--------------|--------------------------|----------------|--------------------------|-----------------|--------------------------|
| W3h6 | 0.294        | 0.962                    | 221            | 0.968                    | 6.0             | 0.268                    | 73.5 |
| W4h6 | 0.392        | 1.157                    | 205            | 0.921                    | 5.8             | 0.226                    | 87.5 |
| W5h6 | 0.490        | 2.076                    | 203            | 0.848                    | 9.0             | 0.360                    | 81.1 |
| W6h6 | 0.588        | 3.152                    | 205            | 0.688                    | 8.5             | 0.730                    | 92.1 |
| W7h6 | 0.685        | 3.881                    | 196            | 0.500                    | 16.6            | 0.878                    | 101.7 |
| W8h6 | 0.784        | 4.192                    | 188            | 0.268                    | 23.8            | 0.720                    | 106.2 |
| W9h6 | 0.882        | 3.279                    | 194            | 0.158                    | 83.3            | 0.551                    | 104.1 |
| W10h6| 0.980        | 1.487                    | 222            | 0.538                    | 181.3           | 0.320                    | 132.2 |
| W11h6| 1.077        | 1.997                    | 230            | 0.163                    | 150.1           | 0.238                    | 166.3 |
### Table 15: Results of Stationary Tests in Regular Following Waves

<table>
<thead>
<tr>
<th>F (Hz)</th>
<th>Wave Amp. (cm)</th>
<th>BM Amp. (Nm)</th>
<th>BM Pha. (deg)</th>
<th>Heave Amp. (cm)</th>
<th>Heave Pha. (deg)</th>
<th>Pitch Amp. (deg)</th>
<th>Pitch Pha. (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>fw3h6</td>
<td>0.295</td>
<td>3.49</td>
<td>2.91</td>
<td>109</td>
<td>3.21</td>
<td>1.96</td>
<td>0.89</td>
</tr>
<tr>
<td>fw4h6</td>
<td>0.393</td>
<td>3.03</td>
<td>3.63</td>
<td>149</td>
<td>2.68</td>
<td>2.51</td>
<td>0.62</td>
</tr>
<tr>
<td>fw5h6</td>
<td>0.491</td>
<td>3.98</td>
<td>7.09</td>
<td>159</td>
<td>3.23</td>
<td>3.30</td>
<td>1.38</td>
</tr>
<tr>
<td>fw6h6</td>
<td>0.590</td>
<td>3.04</td>
<td>8.14</td>
<td>163</td>
<td>1.79</td>
<td>-2.01</td>
<td>1.89</td>
</tr>
<tr>
<td>fw7h6</td>
<td>0.689</td>
<td>3.09</td>
<td>11.29</td>
<td>177</td>
<td>1.41</td>
<td>2.07</td>
<td>2.35</td>
</tr>
<tr>
<td>fw8h6</td>
<td>0.787</td>
<td>3.40</td>
<td>13.69</td>
<td>190</td>
<td>0.72</td>
<td>-0.41</td>
<td>2.30</td>
</tr>
<tr>
<td>fw9h6</td>
<td>0.886</td>
<td>3.70</td>
<td>17.68</td>
<td>218</td>
<td>0.65</td>
<td>72.7</td>
<td>2.13</td>
</tr>
<tr>
<td>fw10h6</td>
<td>0.983</td>
<td>2.49</td>
<td>16.74</td>
<td>241</td>
<td>3.14</td>
<td>124.6</td>
<td>1.43</td>
</tr>
<tr>
<td>fw11h6</td>
<td>1.081</td>
<td>2.94</td>
<td>10.57</td>
<td>272</td>
<td>0.51</td>
<td>125.1</td>
<td>0.72</td>
</tr>
</tbody>
</table>

### Table 16: Frequency Response Functions of Stationary Tests in Regular Following Waves

| F (Hz) | |H_x| (Nm/cm) | ang(H_y) (deg) | |H_d| (cm/cm) | ang(H_d) (deg) | |H_0| (deg/cm) | ang(H_0) (deg) |
|--------|----------------|----------------|---------------|----------------|----------------|----------------|---------------|----------------|----------------|
| 0.295  | 0.834          | 109             | 0.920         | 1.96            | 0.254          | 255.5          |
| 0.393  | 1.198          | 149             | 0.884         | 2.51            | 0.203          | 279.3          |
| 0.491  | 1.781          | 159             | 0.812         | 3.30            | 0.347          | 271.6          |
| 0.590  | 2.678          | 163             | 0.589         | -2.01           | 0.620          | 271.2          |
| 0.689  | 3.652          | 177             | 0.456         | 2.07            | 0.760          | 277.7          |
| 0.787  | 4.027          | 190             | 0.212         | -0.41           | 0.676          | 285.4          |
| 0.886  | 4.778          | 218             | 0.176         | 72.7            | 0.576          | 287.1          |
| 0.983  | 6.723          | 241             | 1.261         | 124.6           | 0.576          | 284.2          |
| 1.081  | 3.595          | 272             | 0.172         | 125.1           | 0.246          | 337.4          |
Table 17: Results of Towing Tests in Regular Head Waves

<table>
<thead>
<tr>
<th>En. F (Hz)</th>
<th>Wave Amp. (cm)</th>
<th>BM Amp. (Nm)</th>
<th>BM Pha. (deg)</th>
<th>Heave Amp. (cm)</th>
<th>Heave Pha. (deg)</th>
<th>Pitch Amp. (deg)</th>
<th>Pitch Pha. (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S5w3h6 0.333</td>
<td>2.82</td>
<td>2.30</td>
<td>232.0</td>
<td>2.90</td>
<td>6.12</td>
<td>0.81</td>
<td>92.3</td>
</tr>
<tr>
<td>S5w4h6 0.450</td>
<td>2.23</td>
<td>3.12</td>
<td>218.9</td>
<td>2.24</td>
<td>7.65</td>
<td>0.96</td>
<td>94.4</td>
</tr>
<tr>
<td>S5w5h6 0.573</td>
<td>2.61</td>
<td>5.84</td>
<td>207.8</td>
<td>2.53</td>
<td>10.56</td>
<td>1.64</td>
<td>98.7</td>
</tr>
<tr>
<td>S5w6h6 0.702</td>
<td>3.02</td>
<td>9.71</td>
<td>190.0</td>
<td>2.41</td>
<td>7.56</td>
<td>2.22</td>
<td>99.2</td>
</tr>
<tr>
<td>S5w7h6 0.840</td>
<td>2.95</td>
<td>12.39</td>
<td>167.6</td>
<td>1.99</td>
<td>8.43</td>
<td>3.26</td>
<td>103.0</td>
</tr>
<tr>
<td>S5w7h5 0.841</td>
<td>2.57</td>
<td>11.73</td>
<td>163.1</td>
<td>1.68</td>
<td>3.04</td>
<td>2.77</td>
<td>99.7</td>
</tr>
<tr>
<td>S5w8h5 0.987</td>
<td>1.37</td>
<td>13.09</td>
<td>79.7</td>
<td>1.31</td>
<td>306.9</td>
<td>3.20</td>
<td>24.3</td>
</tr>
<tr>
<td>S5w9h5 1.135</td>
<td>2.90</td>
<td>4.86</td>
<td>-4.6</td>
<td>0.34</td>
<td>356.3</td>
<td>1.98</td>
<td>23.6</td>
</tr>
<tr>
<td>S5w10h5 1.292</td>
<td>3.23</td>
<td>7.65</td>
<td>-108.1</td>
<td>0.43</td>
<td>76.95</td>
<td>0.26</td>
<td>64.7</td>
</tr>
<tr>
<td>S5w11h5 1.454</td>
<td>3.12</td>
<td>4.65</td>
<td>-113.2</td>
<td>0.25</td>
<td>108.3</td>
<td>0.19</td>
<td>159.2</td>
</tr>
</tbody>
</table>

Table 18: Frequency Response Functions of Towing Tests in Regular Head Waves

| En. F (Hz) | |H| (Nm/cm) | ang(H_y) (deg) | |H| (cm/cm) | ang(H_x) (deg) | |H| (deg/cm) | ang(H_0) (deg) |
|------------|----------|-----------|----------------|----------|----------------|----------|-------------|
| 0.333 | 0.82 | 232.0 | 1.03 | 6.12 | 0.29 | 92.3 |
| 0.450 | 1.40 | 218.9 | 1.00 | 7.65 | 0.43 | 94.4 |
| 0.573 | 2.24 | 207.8 | 0.97 | 10.56 | 0.63 | 98.7 |
| 0.702 | 3.22 | 190.0 | 0.80 | 7.56 | 0.74 | 99.2 |
| 0.840 | 4.20 | 167.6 | 0.67 | 8.43 | 1.11 | 103.0 |
| 0.841 | 4.56 | 163.1 | 0.65 | 3.04 | 1.08 | 99.7 |
| 0.987 | 9.55 | 79.7 | 0.96 | 306.9 | 2.34 | 24.3 |
| 1.135 | 1.68 | -4.6 | 0.12 | 356.3 | 0.68 | 23.6 |
| 1.292 | 2.37 | -108.1 | 0.13 | 76.95 | 0.08 | 64.7 |
| 1.454 | 1.49 | -113.2 | 0.08 | 108.3 | 0.06 | 159.2 |

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4.3 Random Waves Data Analysis

4.3.1 Spectral Analysis Method

The Response Amplitude Operators (RAOs) can be estimated from the random test data using a spectral analysis method (Ljung, 1999). An algorithm in continuous form is given in Thomson (1981) as

\[ X(\omega) = \lim_{T \to \infty} \int_0^T x(t)e^{-i\omega t} dt \]

\[ Y(\omega) = \lim_{T \to \infty} \int_0^T y(t)e^{-i\omega t} dt \]

\[ S_{xx}(\omega) = \lim_{T \to \infty} \frac{1}{T} X(\omega)X^*(\omega) \]

\[ S_{yy}(\omega) = \lim_{T \to \infty} \frac{1}{T} Y(\omega)Y^*(\omega) \]

\[ |H(\omega)| = \frac{S_{yy}(\omega)}{S_{xx}(\omega)} \]

The discrete form used in this work is as follows,

\[ X(k) = \sum_{n=1}^{N} x[n]e^{-2\pi k \frac{n}{N}} \]

\[ Y(k) = \sum_{n=1}^{N} y[n]e^{-2\pi k \frac{n}{N}} \]

\[ S_{xx}(k) = \frac{1}{Nf_s} X(k)X^*(k) \]

\[ S_{yy}(k) = \frac{1}{Nf_s} Y(k)Y^*(k) \]

\[ |H(k)| = \frac{S_{yy}(k)}{\sqrt{S_{xx}(k)}} \]

\[ f(k) = f_s \frac{k}{N}, (0 \leq k \leq N-1) \]
From linear-random theory, the above RAO values should have a good agreement with the frequency response functions estimated from the regular wave tests.

4.3.2 Results of Random Wave Tests

For each stationary test in random waves, 500sec long experimental data were recorded in five channels. Since the initial 50sec segment consisted of the calm water data and the transient response data, only the latter 450sec long experimental data could be used for spectral analysis. For the towing test in random waves, 350sec long time histories were formed by joining seven segments. The estimated frequency was the encounter frequency.

For stationary tests in head waves, representative results of the JONSWAP random wave tests and the broad-band random wave tests are plotted in Figure 12 and Figure 13, respectively. The circle points in the figures are RAO values obtained from the stationary tests in regular head waves. Other test results are presented in Appendix III.

For stationary tests in following waves, representative results of the JONSWAP random wave tests and the broad-band random wave tests are plotted in Figure 14 and Figure 15, respectively. The circle points in the figures are RAO values obtained from the stationary tests in regular following waves. Other test results are presented in Appendix III.

For towing tests in head waves, the results of the JONSWAP random wave tests are plotted in Figure 16. The circle points in the figure are RAO values obtained from the towing tests in regular head waves.
Figure 12: Spectral Analysis Results of J5h75a.dat
Figure 13: Spectral Analysis Results of B75a.dat
Figure 14: Spectral Analysis Results of FjSh75a.dat
Figure 15: Spectral Analysis Results of Fb75b.dat
Figure 16: Spectral Analysis Results of JONSWAP Wave Towing Test
From Figure 12 to Figure 16, there is an excellent agreement between the RAOs obtained from both regular wave tests and random wave tests. According to linear-random theory, this agreement validates the assumption of linearity that is adopted in the current work. In addition, the accuracy of the experimental measurements is also validated.
4.4 Numerical Computations

In order to validate the experimental results theoretically, the frequency response functions of heave, pitch, and midship bending moment for head sea stationary conditions were computed using a strip theory. The two-dimensional sectional coefficients and excitation forces were determined by Lewis-form method. The computation details are described in Appendix I, and the results of the frequency response functions are presented in Table 19. The comparisons between the calculated results and the experimental results obtained from the regular wave tests are shown in Figure 17 to Figure 19. It should be noted that the positive direction for angular pitch displacement is bow downward in the computations, but bow upward in the experiments. This causes a 180° difference in pitch phase angles, as seen in Figure 19.

Table 19: Frequency Response Functions from Numerical Computations

| F (Hz) | |H_y| (Nm/cm) | ang(H_y) (deg) | |H_z| (cm/cm) | ang(H_z) (deg) | |H_a| (deg/cm) | ang(H_a) (deg) |
|-------|---|---|---|---|---|---|---|---|---|
| 0.392 | 0.6829 | 181.75 | 0.921 | -1.18 | 0.389 | 267.68 |
| 0.490 | 1.345 | 185.64 | 0.848 | -2.54 | 0.537 | 266.17 |
| 0.588 | 2.130 | 188.70 | 0.749 | -4.89 | 0.662 | 263.81 |
| 0.685 | 3.208 | 191.35 | 0.542 | -8.16 | 0.761 | 260.61 |
| 0.784 | 3.939 | 196.74 | 0.258 | -8.05 | 0.721 | 256.64 |
| 0.882 | 4.017 | 205.55 | 0.119 | 111.62 | 0.527 | 249.03 |
| 0.980 | 3.566 | 221.71 | 0.332 | 110.95 | 0.167 | 241.30 |
| 1.077 | 2.695 | 232.95 | 0.271 | 85.75 | 0.224 | 41.04 |
Figure 17: Frequency Response Functions for Midship Bending Moment

Figure 18: Frequency Response Functions for Heave
The computed frequency response functions of heave and pitch motions agree well with the experimental results for the frequency range from 0.4Hz to 0.9Hz. There is a discrepancy for frequencies 1.0Hz and 1.1Hz. This is the frequency range where resonance occurs. The assumption of linearity may not be valid in this range.

For the frequency response functions of wave-induced bending moment, the agreement between theoretical and experimental values is not as good as in the case of motions. This is expected since errors in the motion computations produce larger errors in the bending moment computations.

Due to time and resource constraints, we used a strip theory based on Lewis-form method. It has limitations in computational accuracy. A more sophisticated theory will yield better results.
4.5 Frequency-domain Results

In the previous sections, the obtained frequency response functions or RAOs expressed the frequency-domain relationship between ship responses and wave elevation. Following linear-random theory, they can be used to estimate the statistical parameters of the ship responses from the wave spectrum.

In this work, the main objective is the relationship between wave-induced bending moment and ship motions. The input signal is the heave or pitch motion instead of the wave elevation. Therefore, the RAOs should be calculated as the ratios between the bending moment amplitudes and the motion amplitudes. These amplitude values are available from both the experimental results and the computational results presented in the previous sections.

Based on the regular wave results given in Table 13, Table 15, and Table 17, the RAOs of the heave-moment system are calculated and plotted in Figure 20. The RAOs of the pitch-moment system are calculated and plotted in Figure 21. The results are dependent on the forward speed and the heading angle.

Given the power spectra of the ship motions, the statistical parameters of the wave-induced bending moment can be estimated from the above RAOs.
Figure 20: RAOs for Heave - Bending Moment System

Figure 21: RAOs for Pitch - Bending Moment System
Although the frequency-domain relationships shown in Figure 20 and Figure 21 are suitable for the statistical estimation of the bending moment parameters, they are not sufficient for real-time estimation of the instantaneous values. A time-domain relationship is needed for this purpose.
5. Time-domain Analysis

5.1 Mathematical Formulation

In this work, the objective is to estimate the instantaneous bending moment from ship motion measurements. This real-time estimation procedure requires a time-domain relationship between the wave-induced bending moment and ship motions. It can be obtained by transforming the frequency-domain model into the time domain.

The frequency-domain model of ship vertical motions and wave-induced bending moments is expressed by equations (11) and (12). For steady-state solutions, both the motions and the bending moments are sinusoidal with a frequency \( \omega \) equal to the encounter frequency. Therefore the equations can be written in the complex form as

\[
A(i\omega)X_z(\omega) + B(i\omega)X_\theta(\omega) = F_1(i\omega)U(\omega) \tag{23}
\]
\[
C(i\omega)X_z(\omega) + D(i\omega)X_\theta(\omega) = F_2(i\omega)U(\omega)
\]

and

\[
Y(\omega) = \alpha(i\omega)X_z(\omega) + \beta(i\omega)X_\theta(\omega) + L(i\omega)U(\omega) \tag{24}
\]

where \( Y(\omega), U(\omega), X_z(\omega) \) and \( X_\theta(\omega) \) are complex amplitudes of the bending moment \( y(t) \), wave elevation \( \zeta(t) \), heave displacement \( z(t) \) and pitch angle \( \theta(t) \), respectively. Other complex coefficients in equation (23) and (24) can be expressed in terms of the coefficients in equations (11) and (12).
\[ A(i\omega) = [C_{33} - \omega^2(M + A_{33})] + i(\omega B_{33}) \]  \hspace{1cm} (25)
\[ B(i\omega) = (C_{35} - \omega^2 A_{35}) + i(\omega B_{35}) \]
\[ C(i\omega) = (C_{53} - \omega^2 A_{53}) + i(\omega B_{53}) \]
\[ D(i\omega) = [C_{55} - \omega^2 (I + A_{55})] + i(\omega B_{55}) \]

\[ F_3(i\omega) = \frac{F_3}{U(\omega)} \]
\[ F_5(i\omega) = \frac{F_5}{U(\omega)} \]

and

\[ \alpha(i\omega) = (C_y - \omega^2 A_y) + i(\omega B_y) \]  \hspace{1cm} (26)
\[ \beta(i\omega) = (H_y - \omega^2 D_y) + i(\omega E_y) \]
\[ L(i\omega) = \frac{y_5}{U(\omega)} \]

The motion responses \( X_z(\omega) \) and \( X_\theta(\omega) \) can be obtained from equation (23) in terms of the wave complex amplitude \( U(\omega) \) as

\[
\begin{bmatrix}
X_z(\omega) \\
X_\theta(\omega)
\end{bmatrix}
= \begin{bmatrix} A(i\omega) & B(i\omega) \\ C(i\omega) & D(i\omega) \end{bmatrix}^{-1}
\begin{bmatrix}
F_3(i\omega) \\
F_5(i\omega)
\end{bmatrix}
U(\omega) \hspace{1cm} (27)
\]

Equation (27) can be rewritten in the following form

\[ U(\omega) = \frac{A(i\omega)D(i\omega) - B(i\omega)C(i\omega)}{D(i\omega)F_3(i\omega) - B(i\omega)F_5(i\omega)} X_z(\omega) \]  \hspace{1cm} (28)

or

\[ U(\omega) = \frac{A(i\omega)D(i\omega) - B(i\omega)C(i\omega)}{A(i\omega)F_3(i\omega) - C(i\omega)F_5(i\omega)} X_\theta(\omega) \]  \hspace{1cm} (29)
Substitution of equation (28) into equation (24) results in the following equation

\[ Y(\omega) = a(i\omega)X_z(\omega) + b(i\omega)X_\theta(\omega) \]  \hspace{1cm} (30)

where

\[ a(i\omega) = \alpha(i\omega) + \frac{A(i\omega)D(i\omega) - B(i\omega)C(i\omega)}{D(i\omega)F_3(i\omega) - B(i\omega)F_5(i\omega)}L(i\omega) \]  \hspace{1cm} (31)

\[ b(i\omega) = \beta(i\omega) \]

Equations (30) and (31) express the frequency-domain relationship between the bending moment and ship motions. This relationship is independent of the wave excitation. The complex coefficients \( a(i\omega) \) and \( b(i\omega) \) are frequency-dependent for any combination of ship speed and heading angle. To get estimates for the instantaneous bending moment, we have to transform equation (30) to the time domain.

Tick (1959) has shown that frequency-dependent equations actually represent systems that are described in the time domain by equations involving convolution integrals. Under certain conditions they may be approximated by constant-coefficient differential equations. Following the approach of Tick (1959), we can first apply the inverse Fourier transform on both sides of equation (30) and obtain a convolution integral equation in the time domain as

\[ y(t) = \int_0^\infty h_a(\tau)z(t-\tau)d\tau + \int_0^\infty h_b(\tau)\theta(t-\tau)d\tau \]  \hspace{1cm} (32)

where
Since this is a real-valued system, the complex coefficients in equation (30) must have Hermitian symmetry as

\[ a(-i\omega) = a^*(i\omega) \quad \text{(34)} \]
\[ b(-i\omega) = b^*(i\omega) \]

If \( a(\omega) \) and \( b(\omega) \) can be approximated by polynomials, their real parts must be even polynomials and the imaginary parts must be odd polynomials to satisfy the above Hermitian symmetry. The simplest approximation uses the second order polynomials as

\[ a(i\omega) = (A_1\omega^2 + A_2) + i(A_3\omega) \quad \text{(35)} \]
\[ b(i\omega) = (B_1\omega^2 + B_2) + i(B_3\omega) \]

\(-\infty < \omega < +\infty\)

where \( A_1, B_i \) are real-valued constants. Then,

\[ h_a(\tau) = -A_1\delta(\tau) + A_2\delta(\tau) + A_3\delta(\tau) \quad \text{(36)} \]
\[ h_b(\tau) = -B_1\delta(\tau) + B_2\delta(\tau) + B_3\delta(\tau) \]

where \( \delta(\tau) \) is the Dirac delta-function. Substituting (36) into equation (32) and performing the integration, we get a 2nd order constant-coefficient differential equation

\[ y(t) = -A_1\ddot{z}(t) + A_2\dot{z}(t) + A_3z(t) - B_1\dot{\theta}(t) + B_2\theta(t) + B_3\theta(t) \quad \text{(37)} \]

\(-\infty < \omega < +\infty\)
In the above derivation, the approximation given in equation (35) is assumed to be valid for the full frequency range \((-\infty, +\infty)\). Since the ship motions and the wave-induced bending moment are both narrow-band processes, we need only consider the approximation in a narrow-band frequency range. The approximation thus becomes

\[
a(i\omega) = (C_1 \omega^2 + C_3) + i(C_2 \omega) \\
b(i\omega) = (D_1 \omega^2 + D_3) + i(D_2 \omega)
\]

\[
\omega_1 < |\omega| < \omega_2
\]

Substitution of (38) into equation (30) results in

\[
Y(\omega) = [(C_1 \omega^2 + C_3) + i(C_2 \omega)]X_z(\omega) + [(D_1 \omega^2 + D_3) + i(D_2 \omega)]X_\theta(\omega)
\]

\[
\omega_1 < |\omega| < \omega_2
\]

Taking the inverse Fourier transform of both sides, we get

\[
y(t) = \frac{1}{2\pi} \int_\omega \omega^2 X_z(\omega)e^{i\omega t} d\omega + \frac{1}{2\pi} \int_\omega i\omega X_z(\omega)e^{i\omega t} d\omega + \frac{1}{2\pi} \int_\omega X_z(\omega)e^{i\omega t} d\omega
\]

\[
+ \frac{1}{2\pi} \int_\omega \omega^2 X_\theta(\omega)e^{i\omega t} d\omega + \frac{1}{2\pi} \int_\omega i\omega X_\theta(\omega)e^{i\omega t} d\omega + \frac{1}{2\pi} \int_\omega X_\theta(\omega)e^{i\omega t} d\omega
\]

\[
\omega_1 < |\omega| < \omega_2
\]

Using the following identities:

\[
\int_\omega \omega^2 X(\omega)e^{i\omega t} d\omega = -\frac{d^2}{dt^2} \left[ \int_\omega X(\omega)e^{i\omega t} d\omega \right]
\]

\[
\int_\omega i\omega X(\omega)e^{i\omega t} d\omega = \frac{d}{dt} \left[ \int_\omega X(\omega)e^{i\omega t} d\omega \right]
\]

equation (40) reduces to a second order constant-coefficient differential equation in the following form:
\begin{equation}
    y(t) = -C_1 \dot{z}(t) + C_2 \dot{z}(t) + C_3 z(t) - D_1 \dot{\theta}(t) + D_2 \dot{\theta}(t) + D_3 \theta(t)
    \label{42}
\end{equation}
\omega_1 < |\omega| < \omega_2

This is a time-domain approximation for a narrow-band process. For more complicated cases, the coefficients \( a(i\omega) \) and \( b(i\omega) \) may be approximated by higher-order polynomials, and the convolution integral equation (32) reduces to a constant-coefficient differential equation of a higher order.
5.2 Neural Networks Model

5.2.1 Structure and Algorithm
There are six unknown coefficients involved in the time-domain model given by equation (42). It is not easy to determine the individual values of these coefficients. Instead, we can express this parametric model in the following form:

\[ y(t) = F_y[\tilde{z}(t), \dot{z}(t), z(t), \dot{\bar{z}}(t), \dot{\bar{z}}(t), \theta(t)] \]  

A neural network model is employed in this work to approximate the function \( F_y \) in equation (43). This is achieved through a learning process using a set of known input-output data.

Artificial neural networks constitute a set of cellular computational structures that can be implemented in both hardware and software forms. The operation of neural networks can be viewed as that of nonlinear systems. Static networks are used to map the static input-output relationship, while recurrent networks are used to simulate dynamic systems. In the system given by equation (43), the output \( y(t) \) is a function only of the current six inputs. A typical static network, single-hidden-layer MLP network, is suitable to model this system. The network structure is illustrated in Figure 22.

This MLP network has six input neurons and one output neuron. A single bias neuron is added to each of the input and hidden layers. Data passing through the neurons in the hidden layer undergo a nonlinear transformation using a sigmoid function. Linear neurons are used in both the input and the output layers. All three layers are fully
connected as shown in Figure 22, and the weight values between the layers can be adjusted to map any input-output relationship.

If the first layer weights are denoted as $\alpha_{ij}$, the output of the $j^{th}$ hidden neuron is given as

$$u_{ij} = \left[1 + \exp\left(-\sum_{i=0}^{6} \alpha_{ij}x_i\right)\right]^{-1}$$

(44)
where the 0th component of the input vector, \( x_0 = 1 \), is the bias input, and \( \alpha_{ij} \) are the bias weights. Similarly, the second layer weights are denoted as \( \beta_j \), and the neural network output is given as

\[
u_o = \sum_{j=1}^{N_h} u_{nj} \beta_j
\]

(45)

where \( N_h \) is the number of the neurons in the hidden layer. It has to be chosen carefully to model the system properly.

The important feature of neural networks is their ability to approximate arbitrary functions through on-line and off-line learning processes. During the learning process, the neural network is presented with a set of input-output points and trained to implement a mapping that matches the sample points as closely as possible. The most popular learning method for the MLP is the backpropagation algorithm. It uses a gradient search technique to find the optimum weight values that minimize a criterion function. In this work the criterion function is the SUM-of-Squared-Error function defined as follows,

\[
J = \sum_{p=1}^{N} J_p
\]

(46)

\[
J_p = \frac{1}{2} [u_o(p) - y(p)]^2
\]

where \( u_o(p) \) is the network output for the \( p \)th training points, \( y(p) \) is the corresponding actual value, \( J_p \) is the squared error of the \( p \)th training points, and \( J \) is the total squared error for all training points. The weights of the network are adjusted iteratively according to the following algorithm,
where $\mu$ is a positive constant called the learning rate. To implement this algorithm we must have an expression for the partial derivative of $J_p$ with respect to each weight in the network. This can be derived from the output layer backwards (Hush and Horne, 1993), and the final results are given as

\begin{align}
\frac{\partial J_p}{\partial \beta_j} &= \frac{\partial J_p}{\partial u_o(p)} \frac{\partial u_o(p)}{\partial \beta_j} = [u_o(p) - y(p)]u_{hi} \\
\frac{\partial J_p}{\partial \alpha_i} &= \frac{\partial J_p}{\partial u_o(p)} \frac{\partial u_o(p)}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial \alpha_i} = [u_o(p) - y(p)]\beta_j u_{hi} (1 - u_{hi}) x_i
\end{align}

The results of equation (48) are substituted into equation (47) to implement the gradient search. The weights are typically initialized to small random values. The process of computing the gradient and adjusting the weights is repeated until a minimum value of $J$ is found,

\begin{align}
\frac{\partial J}{\partial \alpha_i} &= \sum_{p=1}^{N} \frac{\partial J_p}{\partial \alpha_i} = 0 \\
\frac{\partial J}{\partial \beta_j} &= \sum_{p=1}^{N} \frac{\partial J_p}{\partial \beta_j} = 0
\end{align}

A cross-validation technique can be used to monitor the generalization performance during the learning process.
5.2.2 Training Data Selection

Generalization performance is a measure of how well the network performs on the actual data outside the training set. It represents the quality of the training results. From Hush and Horne (1993), generalization performance is heavily influenced by the selection of the training data samples.

In this work, the following criteria are used for the training data selection:

1. Representative — The training data must satisfy the mathematical model of the underlying system.

2. Informative — The training data should include information covering the whole frequency range of the frequency-dependent system.

3. Efficient — The ratio of the information amount to training data number should be high enough to reduce the computational time.

4. Noiseless — The training data should have minimum noise effects.

Conventionally, the time histories from the experiments are used as the training data. They are naturally representative of the underlying system. In this work there are two groups of test data available, the regular-wave test data and the random-wave test data. Since the system given by equation (43) is a frequency-dependent system, the regular-wave time histories with the discrete frequency values are not informative on the whole frequency range. Only the random time histories may have enough information and can possibly be used as training data. However, in this case a long time history is needed to retrieve the complete information. This requires a large amount of computational time. In
addition, the random data obtained from model experiments or full-scale tests are usually contaminated by noise. They are not efficient or noiseless as the training data.

As an alternative to the raw test data, the statistical data is another source for the training data selection. Previous experience from similar studies (Xu and Haddara, 2001) shows that one set of the auto- and cross-correlation functions provides good training results. These correlation functions are Fourier transforms of the auto- and cross-spectral of the ship motions and the hull bending moment. They are informative on the whole frequency range. Since all correlation functions are in decay form, the complete information is carried on a limited number of data points. This provides an efficient training scheme. In addition, the averaging process involved in the calculation of the correlation functions greatly reduces the effect of noise on the data. Therefore, the correlation functions meet the most requirements of the training data. The only concern is that the selected correlation functions must satisfy the system model expressed in equation (43). One set of the training data is derived as follows.

Multiplying both sides of equation (30) by the complex conjugate $X_z^*(\omega)$, one gets

$$X_z^*(\omega)Y(\omega) = a(i\omega)X_z^*(\omega)X_z(\omega) + b(i\omega)X_z^*(\omega)X_\phi(\omega)$$

Thus the auto-spectral density $S_{zz}(\omega)$, cross-spectral density $S_{zb}(\omega)$ and $S_{zy}(\omega)$ are related as

$$S_{zy}(\omega) = a(i\omega)S_{zz}(\omega) + b(i\omega)S_{zb}(\omega)$$

Taking the inverse Fourier transform of equation (51) gives the time-domain expression
\[ R_{\gamma}(t) = \int_0^t h_a(\tau)R_{\gamma}(t-\tau)d\tau + \int_0^t h_b(\tau)R_{\gamma\theta}(t-\tau)d\tau \] (52)

where \( R_{zz}(\tau) \) is the auto-correlation function of the heave displacement \( z(t) \), \( R_{\gamma\gamma}(\tau) \) is the cross-correlation function between the heave displacement \( z(t) \) and the bending moment \( y(t) \), and \( R_{z\theta}(\tau) \) is the cross-correlation function between the heave displacement \( z(t) \) and the angular pitch displacement \( \theta(t) \). They can be estimated from the random time histories in the following form:

\[
R_{\gamma}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T z(t)y(t+\tau)dt
\] (53)

\[
R_{\gamma\gamma}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T z(t)z(t+\tau)dt
\]

\[
R_{z\theta}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T z(t)\theta(t+\tau)dt
\]

Equation (52) is identical to equation (32). Therefore, an expression similar to equation (43) can be used to relate the auto- and cross-correlation functions of the motions and the bending moment. This relationship is given as

\[
R_{\gamma\gamma}(\tau) = F_s[\ddot{R}_{\gamma\gamma}(\tau), \dot{R}_{\gamma\gamma}(\tau), \dot{R}_{\gamma\gamma}(\tau), \ddot{R}_{z\theta}(\tau), \dot{R}_{z\theta}(\tau), \dot{R}_{z\theta}(\tau)]
\] (54)

Equation (54) is equivalent to the system model given in equation (43). Therefore, we can conclude that samples from the correlation functions can be used for the training of the neural network.

**5.2.3 Training Procedure**

In this work, a commercial software package "NeuroShell 2" is used to create and train the single-hidden-layer MLP network. There are six input variables and one output
variable. In the training process, the input variables are correlation functions $R_z, R_{z\theta}$ and their derivatives, while the output variable is $R_{z\theta}$. The auto- and cross-correlation functions were estimated from the random time histories using equation (53). Their derivatives were obtained using the following algorithm:

$$
\hat{R}(t) = \frac{R(t + \Delta t) - R(t - \Delta t)}{2(\Delta t)}
$$

$$
\hat{\hat{R}}(t) = \frac{R(t + \Delta t) - 2R(t) + R(t - \Delta t)}{(\Delta t)^2}
$$

(55)

Since "NeuroShell 2" requires all variables to be in the range [0,1] or [-1,1], we need to know the real value range of each input variable and scale the actual input data to the required range. In order to achieve the uniform value ranges for different test data, all correlation functions and their derivatives were divided by value $R_z(0)$. In this work 10sec long correlation functions were calculated and normalized, and 480 sample points were selected for network training.

After the training points were imported into "NeuralShell 2" using the File Import module, the inputs/output variables and their real value ranges were specified in the Define Inputs And Outputs module. Generally, the real value ranges of the input and the output variables have to be specified close to those of the training data. Otherwise the network may lose its ability to spot small differences between the points. In this work, we specified the real value ranges of the variables to be 20% wider than those of the training data. This allows for some flexibility in future predictions.
The network architecture and the parameters were specified in the Design module. The single-hidden-layer MLP network was chosen in this work. There were six input neurons and one bias neuron in the input layer, and only one neuron in the output layer. The number of hidden neurons was set by default to 24 plus one bias neuron. In the MLP networks, the number of the hidden neurons determines how well a system can be modeled. If we use too many, the network will tend to memorize the training points instead of learning the relationship between the input and the output. This network will not generalize well. If we use too few, the network may not have enough "power" to learn the system well. Specifying the right number of the hidden neurons is a matter of trial and error, and the default number was calculated here with the following formula:

\[ N_{\text{neurons}} = \frac{1}{2}(N_{\text{input}} + N_{\text{output}}) + \sqrt{N_{\text{points}}} \]  \hspace{1cm} (56)

where \( N_{\text{input}} \) is 6, \( N_{\text{output}} \) is 1, and \( N_{\text{points}} \) is 480.

After determining the neuron number, we have to specify the scaling functions for the input layer and the activation functions for the hidden and output layers, respectively.

When the input variables are imported into the neural network, they must be scaled within the numerical range \([0,1]\) or \([-1,1]\) for the efficient operation of the network. In this work, the linear scaling functions were used for the input neurons

\[ f_1(x) = 1 - \frac{2(X_{\text{max}} - x)}{X_{\text{max}} - X_{\text{min}}} \]  \hspace{1cm} (57)

where \( x \) denotes the actual input to the neuron, \( f_1(x) \) is the output of the neuron, and \( X_{\text{max}} \) and \( X_{\text{min}} \) are maximum and minimum values of the corresponding variable, respectively.
The numerical range of \( f_1(x) \) is \([-1,1]\). These outputs from the input layer neurons were weighted and then passed to the hidden layer, and the neurons in the hidden layer produced their outputs based upon the sum of the weighted values passed to them. The activation function between the hidden neurons input and output was specified as a sigmoid function

\[
    f_h(y) = \frac{1}{1+e^{-y}}
\] (58)

The outputs \( f_h(y) \) were weighted and passed to the output layer. Since the network output is a continuous variable, the activation function for the output neuron was specified as a linear function

\[
    f_o(z) = z
\] (59)

The output \( f_o(z) \) is the final output of the neural network. The learning rate and momentum for all links were set to 0.1 and 0.1, respectively. The initial weight values were randomly set within the range \([-0.3, +0.3]\).

After the network architecture was decided, several parameters for the training process had to be specified in the Training Criteria module. The most important is the set-up of the NET-PERFECT function, which limits the over-training of the network and prevents the pure "memorization" of the training points. With the NET-PERFECT feature, the network is only trained to build the model that smoothly interpolates between close training points, thus the network could generalize well in later predictions. In the operation, a test set was created by randomly extracting 57 points from the 480 sample...
points. The network was trained using the training set of 423 points, but at specified intervals it was tested by computing the average error on the 57 test points. The test interval was set as 200 training events. We also specified that the network be saved on the best test set, thus NET-PERFECT saved the network weight values every time the average error of the test set reached a new minimum value. The training process was stopped after the minimum test error did not change for 40,000 training events.

After the network training was completed, other sets of data could be processed through the trained neural network in the *Apply To File* module. The network outputs could be compared with the actual values for validation. The statistical indicator $R^2$ was calculated as

$$R^2 = 1 - \frac{\sum_n (y_n - \hat{y}_n)^2}{\sum_n (y_n - \mu_y)^2}$$  \hspace{1cm} (60)

where $y_n$ is the actual value, $\hat{y}_n$ is the network output, and $\mu_y$ is the mean of the actual values. A perfect fit would result in an $R^2$ value of 1, a very good fit near 1, and a poor fit near 0. After the neural network has been validated, it can be used for prediction or real-time estimation.
5.3 Time-domain Results

The proposed technique was applied to the ‘R-class Icebreaker’ model test data. All test data was filtered using a low-pass filter with a cut-off frequency of 2.5Hz. The auto- and cross-correlation functions of each random test run were computed first. For every combination of forward speed and heading angle, one set of correlation functions was used for the network training. Other sets of correlation functions were imported into the trained network for validation. The validated network was then used for the real-time estimation of the midship bending moment in regular and random waves. The estimated values were compared with the actual values obtained from the experiments.

5.3.1 Stationary Tests in Head Waves

The correlation functions of the random test J6h75a were used for the network training. The training data sample consisted of 480 training points, which were selected from 10sec long correlation functions. The final $R^2$ value is 0.999. The training data and the results are plotted in Figure 23. The network weight values are presented in Table 20.

The correlation functions obtained from other test runs were used to validate the neural network model. The $R^2$ values are presented in Table 21. All correlation functions satisfied the trained neural network very well.
Figure 23: Training Results for Stationary Test in Head Waves
### Table 20: Network Weight Values for Stationary Tests in Head Waves

<table>
<thead>
<tr>
<th>Hidden</th>
<th>bias</th>
<th>Input 1</th>
<th>Input 2</th>
<th>Input 3</th>
<th>Input 4</th>
<th>Input 5</th>
<th>Input 6</th>
<th>Output</th>
</tr>
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<tr>
<td></td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>1</td>
<td>2</td>
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<td>bias</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.20</td>
</tr>
<tr>
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<td>-0.16</td>
<td>0.28</td>
<td>0.00</td>
<td>-0.11</td>
<td>0.51</td>
<td>-0.15</td>
<td>0.58</td>
</tr>
<tr>
<td>2</td>
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<td>0.25</td>
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<td>0.18</td>
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<td>0.03</td>
<td>-0.15</td>
<td>-0.05</td>
<td>-0.19</td>
</tr>
<tr>
<td>5</td>
<td>0.11</td>
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<td>0.31</td>
<td>-0.05</td>
<td>-0.20</td>
<td>0.03</td>
<td>0.15</td>
<td>-0.01</td>
</tr>
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<td>0.08</td>
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<td>-0.06</td>
<td>0.06</td>
<td>-0.22</td>
</tr>
<tr>
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<td>0.03</td>
<td>0.02</td>
<td>0.12</td>
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<td>0.56</td>
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<td>0.82</td>
</tr>
<tr>
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<td>-0.14</td>
<td>0.15</td>
<td>0.23</td>
<td>0.15</td>
<td>0.17</td>
</tr>
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<td>-0.21</td>
<td>0.15</td>
<td>0.03</td>
<td>-0.02</td>
<td>0.23</td>
<td>0.09</td>
</tr>
<tr>
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<td>0.24</td>
<td>-0.11</td>
<td>-0.18</td>
<td>0.01</td>
<td>-0.22</td>
<td>0.13</td>
<td>-0.05</td>
<td>0.21</td>
</tr>
<tr>
<td>11</td>
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<td>0.00</td>
<td>0.28</td>
<td>0.06</td>
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<tr>
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<td>0.05</td>
<td>-0.23</td>
<td>-0.01</td>
<td>-0.10</td>
<td>-0.52</td>
<td>0.22</td>
<td>-0.40</td>
</tr>
<tr>
<td>13</td>
<td>-0.15</td>
<td>0.13</td>
<td>-0.06</td>
<td>-0.04</td>
<td>0.43</td>
<td>-0.35</td>
<td>-0.24</td>
<td>-0.50</td>
</tr>
<tr>
<td>14</td>
<td>0.08</td>
<td>0.14</td>
<td>-0.22</td>
<td>-0.16</td>
<td>0.25</td>
<td>-0.28</td>
<td>0.35</td>
<td>-0.52</td>
</tr>
<tr>
<td>15</td>
<td>-0.23</td>
<td>-0.12</td>
<td>0.17</td>
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<td>0.05</td>
<td>-0.48</td>
<td>0.04</td>
<td>-0.65</td>
</tr>
<tr>
<td>16</td>
<td>-0.24</td>
<td>-0.17</td>
<td>-0.31</td>
<td>-0.10</td>
<td>0.19</td>
<td>-0.08</td>
<td>-0.26</td>
<td>-0.02</td>
</tr>
<tr>
<td>17</td>
<td>-0.05</td>
<td>0.39</td>
<td>-0.17</td>
<td>-0.03</td>
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<td>0.74</td>
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<td>0.79</td>
</tr>
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<td>18</td>
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<td>0.28</td>
<td>0.14</td>
<td>0.06</td>
<td>0.17</td>
<td>-0.19</td>
<td>0.07</td>
</tr>
<tr>
<td>19</td>
<td>-0.15</td>
<td>0.24</td>
<td>-0.11</td>
<td>-0.04</td>
<td>-0.09</td>
<td>-0.26</td>
<td>0.25</td>
<td>-0.33</td>
</tr>
<tr>
<td>20</td>
<td>-0.08</td>
<td>0.27</td>
<td>-0.13</td>
<td>0.33</td>
<td>-0.24</td>
<td>0.07</td>
<td>0.04</td>
<td>0.20</td>
</tr>
<tr>
<td>21</td>
<td>0.30</td>
<td>-0.04</td>
<td>-0.03</td>
<td>0.09</td>
<td>0.24</td>
<td>-0.38</td>
<td>0.25</td>
<td>-0.56</td>
</tr>
<tr>
<td>22</td>
<td>-0.25</td>
<td>-0.23</td>
<td>0.18</td>
<td>-0.22</td>
<td>0.21</td>
<td>-0.48</td>
<td>-0.18</td>
<td>-0.61</td>
</tr>
<tr>
<td>23</td>
<td>-0.17</td>
<td>0.02</td>
<td>0.23</td>
<td>-0.19</td>
<td>-0.24</td>
<td>0.31</td>
<td>0.16</td>
<td>0.22</td>
</tr>
<tr>
<td>24</td>
<td>-0.26</td>
<td>-0.16</td>
<td>0.11</td>
<td>0.17</td>
<td>-0.09</td>
<td>0.17</td>
<td>-0.26</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Table 21: Validation Results for Stationary Tests in Head Waves (Correlation Functions)

<table>
<thead>
<tr>
<th>Test Data</th>
<th>Test Conditions</th>
<th>Peak Frequency</th>
<th>Significant Wave Height</th>
<th>Random Seed</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training (J6h75a)</td>
<td>JONSWAP Head Waves</td>
<td>0.6 Hz</td>
<td>0.75cm</td>
<td>1</td>
<td>0.999</td>
</tr>
<tr>
<td>B5a</td>
<td>Broad-band Head Waves</td>
<td>0.3 ~ 1.2 Hz</td>
<td>5.0 cm</td>
<td>1</td>
<td>0.969</td>
</tr>
<tr>
<td>B5b</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>0.960</td>
</tr>
<tr>
<td>B5c</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>0.971</td>
</tr>
<tr>
<td>B75a</td>
<td>Broad-band Head Waves</td>
<td>0.3 ~ 1.2 Hz</td>
<td>7.5 cm</td>
<td>1</td>
<td>0.983</td>
</tr>
<tr>
<td>B75b</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>0.969</td>
</tr>
<tr>
<td>B75c</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>0.980</td>
</tr>
<tr>
<td>J5h5a</td>
<td>JONSWAP Head Waves</td>
<td>0.5 Hz</td>
<td>5.0 cm</td>
<td>1</td>
<td>0.963</td>
</tr>
<tr>
<td>J5h5b</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>0.982</td>
</tr>
<tr>
<td>J5h5c</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>0.984</td>
</tr>
<tr>
<td>J5h75a</td>
<td>JONSWAP Head Waves</td>
<td>0.5 Hz</td>
<td>7.5 cm</td>
<td>1</td>
<td>0.968</td>
</tr>
<tr>
<td>J5h75b</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>0.976</td>
</tr>
<tr>
<td>J5h75c</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>0.983</td>
</tr>
<tr>
<td>J6h5a</td>
<td>JONSWAP Head Waves</td>
<td>0.6 Hz</td>
<td>5.0 cm</td>
<td>1</td>
<td>0.996</td>
</tr>
<tr>
<td>J6h5b</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>0.998</td>
</tr>
<tr>
<td>J6h5c</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>0.996</td>
</tr>
<tr>
<td>J6h75a</td>
<td>JONSWAP Head Waves</td>
<td>0.6 Hz</td>
<td>7.5 cm</td>
<td>1</td>
<td>0.999</td>
</tr>
<tr>
<td>J6h75b</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>0.996</td>
</tr>
<tr>
<td>J6h75c</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>0.997</td>
</tr>
<tr>
<td>J7h5c</td>
<td>JONSWAP Head Waves</td>
<td>0.7 Hz</td>
<td>5.0 cm</td>
<td>3</td>
<td>0.992</td>
</tr>
<tr>
<td>J7h75a</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>0.992</td>
</tr>
<tr>
<td>J7h75b</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>0.995</td>
</tr>
<tr>
<td>J7h75c</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>0.996</td>
</tr>
</tbody>
</table>

For each regular wave test, 48 equally spaced points were selected from 10-second long time histories and imported into the neural network for validation. The results are shown in Table 22 and Figure 24. Excellent fits are obtained for the frequency range 0.4Hz to 0.9Hz. The agreement is not so good for frequencies in the range of 1.0Hz and 1.1Hz. This is expected since resonance occurs within this range.
Table 22: Validation Results for Stationary Tests in Regular Head Waves

<table>
<thead>
<tr>
<th>Test Data</th>
<th>Test Conditions</th>
<th>Frequency</th>
<th>Wave Height</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W4h6</td>
<td>Stationary Tests in Regular Head Waves</td>
<td>0.4 Hz</td>
<td>6.0 cm</td>
<td>0.813</td>
</tr>
<tr>
<td>W5h6</td>
<td></td>
<td>0.5 Hz</td>
<td></td>
<td>0.829</td>
</tr>
<tr>
<td>W6h6</td>
<td></td>
<td>0.6 Hz</td>
<td></td>
<td>0.899</td>
</tr>
<tr>
<td>W7h6</td>
<td></td>
<td>0.7 Hz</td>
<td></td>
<td>0.975</td>
</tr>
<tr>
<td>W8h6</td>
<td></td>
<td>0.8 Hz</td>
<td></td>
<td>0.937</td>
</tr>
<tr>
<td>W9h6</td>
<td></td>
<td>0.9 Hz</td>
<td></td>
<td>0.931</td>
</tr>
<tr>
<td>W10h6</td>
<td></td>
<td>1.0 Hz</td>
<td></td>
<td>0.715</td>
</tr>
<tr>
<td>W11h6</td>
<td></td>
<td>1.1 Hz</td>
<td></td>
<td>0.718</td>
</tr>
</tbody>
</table>

This trained neural network could be used for the real-time estimation of instantaneous bending moment on the frequency range [0.4Hz, 1.1Hz]. A typical example for 10sec
long estimates of J6h75a random test is shown in Figure 25. There are 480 points estimated, and $R^2$ value is 0.943.

![Figure 25: Estimation Example for Random Test J6h75a](image)

5.3.2 Stationary Tests in Following Waves
The correlation functions of the random test Fj6h75a were used for the network training. The training data sample consisted of 480 training points, which were selected from 10sec long correlation functions. The final $R^2$ value is 0.998. The training data and results are plotted in Figure 26. The neural network weight values are presented in Table 23.
Figure 26: Training Results for Stationary Tests in Following Waves
The correlation functions of other test runs were imported into the neural network for the validation. The $R^2$ values are presented in Table 24. The agreement is excellent for all sets of correlation functions with one exception. The broad-band following wave test $Fb5b$ produced a poor fit. This is expected since the analysis is based on the narrow-band process assumption.
### Table 24: Validation Results for Stationary Tests in Following Waves (Correlation Functions)

<table>
<thead>
<tr>
<th>Test Data</th>
<th>Test Conditions</th>
<th>Peak Frequency</th>
<th>Significant Wave Height</th>
<th>Random Seed</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training (Fj6h75a)</td>
<td>JONSWAP Following Waves</td>
<td>0.6 Hz</td>
<td>0.75 cm</td>
<td>1</td>
<td>0.998</td>
</tr>
<tr>
<td>Fb5a</td>
<td>Broad-band Following Waves</td>
<td>0.3 ~ 1.2 Hz</td>
<td>5.0 cm</td>
<td>1</td>
<td>0.903</td>
</tr>
<tr>
<td>Fb5b</td>
<td>Following Waves</td>
<td>0.3 ~ 1.2 Hz</td>
<td>7.5 cm</td>
<td>1</td>
<td>0.861</td>
</tr>
<tr>
<td>Fb75a</td>
<td>JONSWAP Following Waves</td>
<td>0.5 Hz</td>
<td>5.0 cm</td>
<td>1</td>
<td>0.981</td>
</tr>
<tr>
<td>Fb75b</td>
<td>Following Waves</td>
<td>0.5 Hz</td>
<td>7.5 cm</td>
<td>2</td>
<td>0.957</td>
</tr>
<tr>
<td>Fj5h5a</td>
<td>JONSWAP Following Waves</td>
<td>0.5 Hz</td>
<td>7.5 cm</td>
<td>1</td>
<td>0.947</td>
</tr>
<tr>
<td>Fj5h75b</td>
<td>Following Waves</td>
<td>0.5 Hz</td>
<td>7.5 cm</td>
<td>2</td>
<td>0.957</td>
</tr>
<tr>
<td>Fj6h5a</td>
<td>JONSWAP Following Waves</td>
<td>0.6 Hz</td>
<td>5.0 cm</td>
<td>1</td>
<td>0.995</td>
</tr>
<tr>
<td>Fj6h5b</td>
<td>Following Waves</td>
<td>0.6 Hz</td>
<td>7.5 cm</td>
<td>2</td>
<td>0.988</td>
</tr>
<tr>
<td>Fj6h75a</td>
<td>JONSWAP Following Waves</td>
<td>0.6 Hz</td>
<td>7.5 cm</td>
<td>1</td>
<td>0.998</td>
</tr>
<tr>
<td>Fj6h75b</td>
<td>Following Waves</td>
<td>0.6 Hz</td>
<td>7.5 cm</td>
<td>2</td>
<td>0.999</td>
</tr>
<tr>
<td>Fj7h5a</td>
<td>JONSWAP Following Waves</td>
<td>0.7 Hz</td>
<td>5.0 cm</td>
<td>1</td>
<td>0.971</td>
</tr>
<tr>
<td>Fj7h5b</td>
<td>Following Waves</td>
<td>0.7 Hz</td>
<td>7.5 cm</td>
<td>2</td>
<td>0.980</td>
</tr>
<tr>
<td>Fj7h75a</td>
<td>JONSWAP Following Waves</td>
<td>0.7 Hz</td>
<td>7.5 cm</td>
<td>1</td>
<td>0.986</td>
</tr>
</tbody>
</table>

For every regular wave test, 48 equally spaced points were selected from a 10-second long time history and imported into the neural network for validation. The results are shown in Table 25 and Figure 27. The neural network model is only valid on a narrow-band frequency range centered around the peak frequency of 0.6 Hz. This agrees with the narrow-band process assumption.
Table 25: Validation Results for Stationary Tests in Regular Following Waves

<table>
<thead>
<tr>
<th>Test Data</th>
<th>Test Conditions</th>
<th>Frequency</th>
<th>Wave Height</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fw3h6</td>
<td>Stationary Tests</td>
<td>0.3 Hz</td>
<td>6.0 cm</td>
<td>0.000</td>
</tr>
<tr>
<td>Fw4h6</td>
<td>in Regular</td>
<td>0.4 Hz</td>
<td></td>
<td>0.650</td>
</tr>
<tr>
<td>Fw5h6</td>
<td>Following Waves</td>
<td>0.5 Hz</td>
<td></td>
<td>0.852</td>
</tr>
<tr>
<td>Fw6h6</td>
<td></td>
<td>0.6 Hz</td>
<td></td>
<td>0.982</td>
</tr>
<tr>
<td>Fw7h6</td>
<td></td>
<td>0.7 Hz</td>
<td></td>
<td>0.998</td>
</tr>
<tr>
<td>Fw8h6</td>
<td></td>
<td>0.8 Hz</td>
<td></td>
<td>0.874</td>
</tr>
<tr>
<td>Fw9h6</td>
<td></td>
<td>0.9 Hz</td>
<td></td>
<td>0.749</td>
</tr>
<tr>
<td>Fw10h6</td>
<td></td>
<td>1.0 Hz</td>
<td></td>
<td>0.947</td>
</tr>
<tr>
<td>Fw11h6</td>
<td></td>
<td>1.1 Hz</td>
<td></td>
<td>0.689</td>
</tr>
</tbody>
</table>

Figure 27: Validation Results for Stationary Tests in Regular Following Waves
The trained network could be used for the real-time estimation of instantaneous bending moment in narrow-band waves. A typical example for 10-second long estimates of Fj6h75a random test is shown in Figure 28. There are 480 points estimated, and $R^2$ value is 0.959.

![Figure 28: Estimation Example for Random Test Fj6h75a](image)

5.3.3 Towing Tests in Head Waves
Due to the limited length of the tank, the time history from a single test run was not long enough for data analysis. Thus the data from seven test runs were joined together to form a single time history, and 10sec long correlation functions were computed for the network training. The training sample consisted of 480 points, and the final $R^2$ value is 0.995. The
training data and the results are plotted in Figure 29. The neural network weight values are presented in Table 26.

The training data shown in Figure 29 do not have good decay form. This is caused by discontinuity between different data segments. The accuracy of the neural network model may also be affected.

For each regular wave test, 48 equally spaced points were selected from a 10sec long time history and imported into the neural network for validation. The results are shown in Table 27 and Figure 30. The neural network model is valid only on a very narrow frequency range. This agrees with the narrow-band assumption adopted in the mathematical formulation. Another reason is the inaccuracies of the neural network model caused by discontinuity in joined time histories.

This network was also used for the real-time estimation of instantaneous bending moment in narrow-band waves. A typical example of 10sec long estimates of the JONSWAP random test is shown in Figure 31. There are 480 points estimated, and $R^2$ value is 0.819. The estimation result is not too bad even though the training data are not very good.
Figure 29: Training Results for Towing Tests in Head Waves
Table 26: Network Weight Values for Towing Tests in Head Waves

<table>
<thead>
<tr>
<th>Hidden</th>
<th>bias</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>bias</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>-0.23</td>
</tr>
<tr>
<td>1</td>
<td>0.28</td>
<td>-0.20</td>
<td>0.32</td>
<td>0.44</td>
<td>0.04</td>
<td>-0.02</td>
<td>-0.23</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>0.52</td>
<td>-0.16</td>
<td>-0.18</td>
<td>-0.40</td>
<td>-0.35</td>
<td>0.17</td>
<td>-0.26</td>
<td>-0.48</td>
</tr>
<tr>
<td>3</td>
<td>0.33</td>
<td>0.29</td>
<td>0.29</td>
<td>0.61</td>
<td>0.26</td>
<td>-0.54</td>
<td>0.49</td>
<td>0.88</td>
</tr>
<tr>
<td>4</td>
<td>0.71</td>
<td>-0.03</td>
<td>0.19</td>
<td>-0.14</td>
<td>-0.06</td>
<td>0.18</td>
<td>-0.11</td>
<td>-0.19</td>
</tr>
<tr>
<td>5</td>
<td>0.60</td>
<td>-0.25</td>
<td>0.27</td>
<td>-0.11</td>
<td>-0.31</td>
<td>0.45</td>
<td>0.06</td>
<td>-0.41</td>
</tr>
<tr>
<td>6</td>
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<td>0.30</td>
<td>0.21</td>
<td>0.26</td>
<td>0.17</td>
<td>0.11</td>
<td>0.10</td>
</tr>
<tr>
<td>7</td>
<td>0.42</td>
<td>-0.62</td>
<td>-0.84</td>
<td>2.00</td>
<td>-0.92</td>
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<td>0.67</td>
<td>1.10</td>
</tr>
<tr>
<td>8</td>
<td>0.79</td>
<td>-0.02</td>
<td>-0.07</td>
<td>-0.12</td>
<td>0.22</td>
<td>0.08</td>
<td>0.11</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>0.72</td>
<td>0.10</td>
<td>-0.13</td>
<td>0.31</td>
<td>0.11</td>
<td>-0.42</td>
<td>0.38</td>
<td>0.52</td>
</tr>
<tr>
<td>10</td>
<td>0.68</td>
<td>0.03</td>
<td>-0.10</td>
<td>0.06</td>
<td>-0.15</td>
<td>-0.19</td>
<td>-0.05</td>
<td>0.09</td>
</tr>
<tr>
<td>11</td>
<td>0.31</td>
<td>-1.09</td>
<td>0.58</td>
<td>2.06</td>
<td>0.69</td>
<td>1.78</td>
<td>-0.85</td>
<td>0.92</td>
</tr>
<tr>
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Table 27: Validation Results for Towing Tests (0.5m/sec)

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<tr>
<th>Test Data</th>
<th>Test Conditions</th>
<th>Encounter Fre.</th>
<th>Wave Height</th>
<th>R²</th>
</tr>
</thead>
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<tr>
<td>Training (JONSWAP)</td>
<td>Towing Tests (0.5m/s) in</td>
<td>Peak Fre.</td>
<td>Significant Height</td>
<td></td>
</tr>
<tr>
<td>S5w3h6</td>
<td>Regular Head Waves</td>
<td>0.333 Hz</td>
<td>6.0 cm</td>
<td>0.000</td>
</tr>
<tr>
<td>S5w4h6</td>
<td></td>
<td>0.450 Hz</td>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>S5w5h6</td>
<td></td>
<td>0.573 Hz</td>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>S5w6h6</td>
<td></td>
<td>0.702 Hz</td>
<td></td>
<td>0.520</td>
</tr>
<tr>
<td>S5w7h5</td>
<td></td>
<td>0.841 Hz</td>
<td>5.0 cm</td>
<td>0.988</td>
</tr>
<tr>
<td>S5w8h5</td>
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<td>0.987 Hz</td>
<td></td>
<td>0.724</td>
</tr>
<tr>
<td>S5w9h5</td>
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<td>0.135 Hz</td>
<td></td>
<td>0.000</td>
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<tr>
<td>S5w10h5</td>
<td></td>
<td>1.292 Hz</td>
<td></td>
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<td>S5w11h5</td>
<td></td>
<td>1.454 Hz</td>
<td></td>
<td>0.508</td>
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</table>

Figure 30: Validation Results for Regular Wave Towing Tests (0.5m/sec)
Figure 31: Estimation Example for JONSWAP Wave Towing Test (0.5m/sec)
6. Time-domain Simulations

6.1 Simulation Procedure

The single-hidden-layer MLP network is used in the present technique to learn the time-domain relationship between wave-induced bending moment and ship motions. In the previous sections, the auto- and cross-correlation functions were estimated from the experimental data for the training of neural networks. These correlation functions can also be estimated from the simulated time histories.

A time-domain simulation procedure based on frequency-domain solutions is used in this section. The frequency response functions are available from both theoretical computations and wave tank tests. The JONSWAP random waves were simulated using the following expression

\[ S(f) = \frac{5H_s^2 f_m^4}{16 f^5 \gamma^3} \exp(-\frac{5 f_m^4}{4 f^4}) \exp\left(-\frac{(f-f_m)^2}{2\sigma^2 f_m^2}\right) \]  

(61)

where the peak enhancement factor \( \gamma \) is 3.3, the shape parameter \( \sigma \) is 0.07 \( (f<f_m) \) or 0.09 \( (f>f_m) \), the significant height \( H_s \) is 7.5 cm, and the peak frequency \( f_m \) is 0.6 Hz. This is the wave spectrum used in the previous experiments. In the simulation process, the spectrum was divided into 51 equally spaced segments on a frequency range 0.4 Hz to 0.9 Hz. In each segment, the spectrum value was assumed to be constant over a frequency interval of 0.01 Hz. This spectrum value was then used to generate one regular wave component. The elevation of each wave component is expressed as
\[ \zeta_t(t) = \sqrt{2S(f, \Delta f)} \cos(2\pi f t + \varphi) \]
\[ \Delta f = 0.01hz \]

where the phase angle \( \varphi \) is a random variable with a uniform distribution.

Using the frequency response functions defined in equation (15), the harmonic heave, pitch and bending moment components were generated as

\[ z(t) = |H_z(\omega_r)\zeta_m| \cos(2\pi f t + \varphi_t + \text{ang}(H_z)) \]
\[ \phi(t) = |H_\phi(\omega_r)\zeta_m| \cos(2\pi f t + \varphi_t + \text{ang}(H_\phi)) \]
\[ y(t) = |H_y(\omega_r)\zeta_m| \cos(2\pi f t + \varphi_t + \text{ang}(H_y)) \]

The frequency response functions were computed and tested only at discrete frequency points 0.4Hz, 0.5Hz, 0.6Hz, 0.7Hz, 0.8Hz, 0.9Hz, 1.0Hz, and 1.1Hz. The values between these points were obtained using a linear interpolation method. The 450sec long random time histories of heave, pitch and bending moment were simulated by the superposition of 51 regular components,

\[ z(t) = \sum_{i=1}^{51} z_i(t) \]
\[ \phi(t) = \sum_{i=1}^{51} \phi_i(t) \]
\[ y(t) = \sum_{i=1}^{51} y_i(t) \]

The correlation functions were computed from the above simulated time histories using equation (53).
6.2 Simulation Results

For the stationary model in head waves, the frequency response functions were available from both strip theory computations and regular wave tests. The first simulation was based on the computed frequency response functions given in Table 19. The simulated correlation functions were compared with those estimated from the model test of J6h75a. This comparison is shown in Figure 32. There is a discrepancy in the heave-moment cross-correlation function. The second simulation was based on the experimental frequency response functions given in Table 14. The comparisons between the simulated correlation functions and the results of model test J6h75a are shown in Figure 33. There is an excellent agreement between the two sets of correlation functions.

Obviously, the discrepancy of the first simulation results shown in Figure 32 is caused by the inaccuracy of the computed frequency response functions. The second simulation results shown in Figure 33 have demonstrated the validity of this simulation method.

For the stationary model in following waves, the simulation was based on the experimental frequency response functions given in Table 16. The simulated correlation functions were compared with those estimated from the model test Fj6h75a. The comparison results are plotted in Figure 34. There is an excellent agreement between two sets of correlation functions.

For the towed model in head waves, the simulation was based on the experimental frequency response functions given in Table 18. The simulated correlation functions were compared with those estimated from the joined test data. The comparison results are plotted in Figure 35. There is a quite big discrepancy shown in the plot.
In Figure 35, both the experimental and the simulated correlation functions are not accurate enough. The experimental time history was formed by joining seven segments. Discontinuity between different segments must have some effects on the accuracy of the estimated correlation functions. In the simulation, the time history was generated using the frequency response functions. Since only values at 10 discrete frequency points were available, the values at other points were obtained by linear interpolations. For the towed model in head waves, the values of the frequency response functions have several big jumps between the 10 discrete points shown in Table 18. These jumps make it very difficult to obtain the accurate values from the linear interpolations. More values of the frequency response functions are needed, particularly within the range of jumps.

Generally, it is possible to obtain the correlation functions from the time-domain simulations. If the available frequency response functions are accurate and sufficient enough, the simulated correlation functions can be used for the training of neural networks. The calibrated neural network will provide a numerical estimation model.
Figure 32: Simulation Results for Stationary Model in Head Waves (Computed Frequency Response Functions)
Figure 33: Simulation Results for Stationary Model in Head Waves (Experimental Frequency Response Functions)
Figure 34: Simulation Results for Stationary Model in Following Waves
Figure 35: Simulation Results for Towed Model in Head Waves (0.5 m/s)
7. Conclusions and Recommendations

In this work, a time-domain technique was developed for estimating the wave-induced vertical bending moment from coupled heave and pitch motions. The estimated values can be compared with those obtained from the strain measurements to ensure the validity of Hull Response Monitoring Systems (HRMS).

The frequency-domain analysis showed that the relationship between vertical bending moment and coupled heave and pitch motions can be expressed by their frequency response functions or Response Amplitude operators (RAOs). They could be obtained from both theoretical computations and ship model experiments. This frequency-domain relationship is suitable for the estimation of the statistical parameters of the bending moment, but not sufficient for the real-time estimation of the instantaneous values.

A time-domain relationship between the bending moment and the ship motions was formulated in this work through a Fourier transform of the frequency-domain model. It was shown that the wave-induced vertical bending moment can be approximated by a function of the heaving and pitching displacements, velocities, and accelerations in the time domain.

An efficient neural network technique was developed in this work to identify the time-domain model through a learning process. It was proved herein that one can use a set of auto- and cross-correlation functions of the bending moment and the ship motions to train the neural network. This should reduce the computational time significantly and improve
the accuracy of the training results. The calibrated neural network model is expected to produce accurate estimates of the instantaneous bending moment.

The experimental data of an “R-class Icebreaker” ship model were used to validate the methodology. For the stationary model in both head waves and following waves, the estimates from the neural networks had an excellent agreement with the bending moment values measured from the experiments. For the towing test in head waves, a single time history was formed by joining seven short segments. The accuracy of the estimated correlation functions was affected by discontinuity in joined time history. Even with this defect, the bending moment estimates from the neural network had a good agreement with the experimental values. A more sophisticated test facility should produce long enough time histories and better training data.

From the verification results shown in Figure 24, Figure 27, and Figure 30, the proposed technique has variable effective range for different test conditions. In the head sea stationary tests, the technique is effective across the range of the tested frequencies with a slight degeneration around the motion resonance frequency. In the following sea stationary tests, it is more effective in the narrow band around the spectral peak frequency with sharp declines in effectiveness outside this region. In the head sea towing tests, the verification results are not very good due to the inaccuracies of the training data. The above variation of the effective range is caused by the approximation given in equation (35) and (38). For the head sea stationary condition, this approximation is valid for a broad-band frequency range. For the following sea stationary condition, it is only valid in a narrow band around the spectral peak frequency. To get an accurate
estimation model effective over the whole frequency range, we have to develop a technique based on the general time-domain model given in equation (32). It is a dynamic model involving convolution integrals, and the recurrent neural networks are suitable tools.

Due to the limitations of the MUN test facility, the experimental data for other directional seas and multidirectional seas are not available in this work. More validations are needed in further research.

After the technique was validated by the experimental data, a simulation method was also investigated to produce the simulated correlation functions as the training data. This time-domain simulation was based on the frequency-domain solutions. The comparisons between the simulation results and the experimental results showed that the accuracy of the simulated correlation functions was greatly determined by the accuracy of the frequency-domain solutions. It is suggested that the proposed technique can be used in conjunction with a time-domain simulation package to provide a numerical estimation model.

The current work is based on a mathematical model for a rigid ship hull and a linear relationship between the wave-induced bending moment and the vertical ship motions. This poses certain limitations on the range of applicability. Since neural networks can easily model nonlinear systems, the present technique can be extended to include the nonlinear effects in the estimation. In this case the time-domain mathematical model needs to be reformulated.
In addition to rigid-body wave loads, both the springing and whipping bending moments are also related to ship motions. The whipping bending moment may be as large as the wave-induced bending moment, while the springing loads are found to be important for very long flexible ships. It is possible to estimate these vibratory loads from ship motion information. In this case, the mathematical formulation of the time-domain model involves consideration of the hull structural properties as well as the hydrodynamic forces.
References


Appendix I  Strip Theory Computations

In this appendix, the coupled heave and pitch motions and the wave-induced midship bending moment of the rigid-body "R-class Icebreaker" ship model are solved in the frequency domain using a strip theory. The ship model is in head regular waves with zero forward speed.

From Lewis (1989) and Salvesen et al. (1970), the solution of the vertical ship motions in regular waves can be expressed by the complex amplitudes of the heave displacement $z_a$ (positive upward) and the angular pitch displacement $\theta_a$ (positive bow downward) as

$$z_a = \frac{F_3 S - F_5 Q}{PS - QR} \tag{65}$$

$$\theta_a = \frac{F_5 P - F_3 R}{PS - QR}$$

where

$$P = -\omega_e^2 (M + A_{33}) + C_{33} + i\omega_e B_{33} \tag{66}$$

$$Q = -\omega_e^2 A_{33} + C_{33} + i\omega_e B_{33}$$

$$R = -\omega_e^2 A_{33} + C_{33} + i\omega_e B_{33}$$

$$S = -\omega_e^2 (I + A_{33}) + C_{33} + i\omega_e B_{33}$$

For zero forward speed and head sea conditions, the hydrodynamic coefficients in the above equations can be expressed in terms of the sectional coefficients in the following form:
\[ A_{33} = \int a_{33} d\xi \quad B_{33} = \int b_{33} d\xi \]
\[ A_{35} = -\int \xi a_{33} d\xi \quad B_{35} = -\int \xi b_{33} d\xi \]
\[ A_{33} = -\int \xi a_{33} d\xi \quad B_{33} = -\int \xi b_{33} d\xi \]
\[ A_{35} = \int \xi^2 a_{33} d\xi \quad B_{35} = \int \xi^2 b_{33} d\xi \]

\[ C_{33} = \int c_{33} d\xi = \int \rho g B(\xi) d\xi \]
\[ C_{35} = C_{33} = -\int \xi c_{33} d\xi = -\int \xi \rho g B(\xi) d\xi \]
\[ C_{35} = \int \xi^2 c_{33} d\xi = \int \xi^2 \rho g B(\xi) d\xi \]

and the complex amplitudes of the wave excitation forces, \( F_3 \) and \( F_5 \), are simplified as

\[ F_3 = \gamma_a \int e^{i\xi} e^{-iky} \left[ c_{33} - \omega_0 (\omega_x a_{33} - ib_{33}) \right] d\xi \]
\[ F_5 = \gamma_a \int e^{i\xi} e^{-iky} \left[ c_{33} - \omega_0 (\omega_x a_{33} - ib_{33}) \right] d\xi \]

For a rigid ship hull, the complex amplitude of the wave-induced midship bending moment (positive in sag direction) can be solved in terms of the resulting motions,

\[ y_a(x_0) = (-A_x \omega_x^2 + C_y + i \omega_e B_x)z_n + (-D_y \omega_y^2 + H_y + i \omega_e E_y) \theta_n + y_\xi \]

where

\[ y_\xi = \gamma_a \int e^{i\xi} e^{-iky} \left[ (\xi - x_0) \left( c_{33} - \omega_0 (\omega_x a_{33} - ib_{33}) \right) \right] d\xi \]
\[ A_y = -\int_{x_0}^{\text{bow}} (m_n + a_{33})(\xi - x_0) d\xi \tag{71} \]
\[ B_y = -\int_{x_0}^{\text{bow}} b_{33} (\xi - x_0) d\xi \]
\[ C_y = -\int_{x_0}^{\text{bow}} c_{33} (\xi - x_0) d\xi \]
\[ D_y = \int_{x_0}^{\text{bow}} (m_n + a_{33})\xi (\xi - x_0) d\xi \]
\[ E_y = \int_{x_0}^{\text{bow}} b_{33}\xi (\xi - x_0) d\xi \]
\[ H_y = \int_{x_0}^{\text{bow}} c_{33}\xi (\xi - x_0) d\xi \]

The two-dimensional sectional coefficients \(a_{33}\) and \(b_{33}\) can be determined using Lewis-form method. From Bhattacharyya (1978), the added mass coefficient \(a_{33}\) is calculated as

\[ a_{33} = C \frac{\rho g B (\xi)^2}{8} \tag{72} \]

where the coefficient \(C\) for Lewis-form sections is plotted in Figure 36 as a function of the draft/beam ratio (\(B/T\)), the area coefficient of the section (\(S/\pi T^2\)), as well as a function of the circular frequency of oscillation. Similarly, the sectional damping coefficient \(b_{33}\) is given by

\[ b_{33} = \frac{\rho g^2 \bar{A}^2}{\omega_e^3} \tag{73} \]

where the coefficient \(\bar{A}\) for Lewis-form sections is plotted in Figure 37. It is assumed that the hydrodynamic coefficients for sections other than those of the mathematical Lewis form will not differ appreciably as long as the beam \(B\), draft \(T\), and the section area \(S\) are equal in both cases. The offsets of the full-scale “R-class Icebreaker” ship are presented in Table 28.
Figure 36: Coefficients $C$ for Lewis-form Sections in Heaving Motion (Bhattacharyya, 1978)
Figure 37: Coefficient $\bar{A}$ for Lewis-form Section in Heaving Motion (Bhattacharyya, 1978)
**Table 28: Offsets Table of the Full-scale “R-class Icebreaker” Ship**

<table>
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<tr>
<th>Z (m)</th>
<th>Station 20</th>
<th>Station 19</th>
<th>Station 18</th>
<th>Station 17</th>
<th>Station 16</th>
<th>Station 15</th>
<th>Station 14</th>
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<td>X=0.000m</td>
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The sectional geometric data of the full-scale "R-class Icebreaker" ship are calculated from the offset values and listed in Table 29. The strip theory computations for a 1:40 scale ship model in head wave conditions are performed using the scaled geometric data. The computational details at the frequency point 0.685 Hz are presented in Table 30.

Table 29: Sectional Data of the Full-scale "R-class Icebreaker" Ship

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<th>Area (m)</th>
<th>From LCB (m)</th>
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Table 30: Strip Theory Computations for 1:40 “R-class Icebreaker” Model

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\[ F(f,B_n) = \frac{\omega^2 B_n}{2g} \]

C is from Figure 36; A is from Figure 37.
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### Formulas

\[ a_{33} = \frac{C \rho b_n^2}{8} \]

\[ b_{33} = \frac{\rho g^2 A}{\omega^2} \]

\[ c_n = \rho g B_n \]

\[ m_n \text{ is weight distribution;} \]

\[ M_1 \text{ and } M_2 \text{ are first and second moments, respectively.} \]
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\[
T'(x) = \frac{S_n}{B_n}; \quad K = \frac{\omega_0^2}{g}; \quad f_3(R) = e^{-KT'(x)} \cos(Kx) c_n; \quad f_3(l) = e^{-KT'(x)} \sin(Kx) c_n;
\]

\[
h_3(R) = e^{-KT'(x)}[-\cos(Kx) \omega_x \omega_y a_{33} - \sin(Kx) \omega_y b_{33}]; \quad h_3(l) = e^{-KT'(x)}[-\sin(Kx) \omega_x \omega_y a_{33} + \cos(Kx) \omega_y b_{33}]
\]

\[
F_3(R) F_3(l) = 2424.899 942.785
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\[ P(R) = C_{33} - \omega_e^2 (m + A_{33}) \quad Q(R) = C_{35} - \omega_e^2 A_{35} \quad R(R) = C_{53} - \omega_e^2 A_{53} \quad S(R) = C_{55} - \omega_e^2 (I_{yy} + A_{55}) \]

\[ P(I) = \omega_e B_{33} \quad Q(I) = \omega_e B_{35} \quad R(I) = \omega_e B_{53} \quad S(I) = \omega_e B_{55} \]

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\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
Station & xc-X0 (m) & mn+a33 & Product & f(U) & An & Simpson & b33(xc-X0) & U*a33 & Bn & c33(xc-X0) & Cn \\
\hline
1 & 0.944 & 32.222 & 30.432 & 0 & -30.432 & 4 & 116.509 & 0 & -116.509 & 2128.79 & -2128.79 \\
2 & 0.839 & 48.626 & 40.773 & 0 & -40.773 & 2 & 186.775 & 0 & -186.775 & 2695.27 & -2695.274 \\
3 & 0.733 & 59.225 & 43.387 & 0 & -43.387 & 4 & 220.633 & 0 & -220.633 & 2853.75 & -2853.746 \\
4 & 0.627 & 154.155 & 96.597 & 0 & -96.597 & 2 & 253.947 & 0 & -253.947 & 2702.01 & -2702.007 \\
5 & 0.521 & 214.481 & 111.680 & 0 & -111.680 & 4 & 218.359 & 0 & -218.359 & 2360.07 & -2360.073 \\
6 & 0.415 & 183.181 & 75.974 & 0 & -75.974 & 2 & 140.352 & 0 & -140.352 & 1924.56 & -1924.546 \\
7 & 0.309 & 126.444 & 39.049 & 0 & -39.049 & 4 & 100.600 & 0 & -100.600 & 1445.15 & -1445.147 \\
8 & 0.203 & 137.049 & 27.804 & 0 & -27.804 & 2 & 61.101 & 0 & -61.101 & 949.35 & -949.3536 \\
9 & 0.097 & 125.916 & 12.208 & 0 & -12.208 & 4 & 29.199 & 0 & -29.199 & 453.68 & -453.6775 \\
\hline
X0(mid) & 0 & 124.789 & 0.000 & 0 & 0.000 & 1 & 0.000 & 0 & 0.000 & 0 & 0.000 \\
\hline
\end{tabular}
\caption{Station xc-X0 (m) mn+a33 Product f(U) An Simpson b33(xc-X0) U*a33 Bn c33(xc-X0) Cn}
\end{table}

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
Station & xc(m) & xc(mn+a33) & Product & f(U) & Dn & xc*b33 & Product & f(U) & En & xc*c33 & Hn \\
\hline
1 & 0.944 & 30.432 & 28.741 & 0 & 28.741 & 116.509 & 110.037 & 0 & 110.037 & 2128.79 & 2010.54 \\
2 & 0.839 & 40.773 & 34.188 & 0 & 34.188 & 186.775 & 156.611 & 0 & 156.611 & 2695.27 & 2259.99 \\
3 & 0.733 & 43.387 & 31.784 & 0 & 31.784 & 220.633 & 161.631 & 0 & 161.631 & 2853.75 & 2090.58 \\
4 & 0.627 & 96.597 & 60.530 & 0 & 60.530 & 253.947 & 159.129 & 0 & 159.129 & 2702.01 & 1693.15 \\
5 & 0.521 & 111.680 & 58.152 & 0 & 58.152 & 218.359 & 113.699 & 0 & 113.699 & 2360.07 & 1228.89 \\
6 & 0.415 & 75.974 & 31.510 & 0 & 31.510 & 140.352 & 58.211 & 0 & 58.211 & 1924.56 & 798.21 \\
7 & 0.309 & 39.049 & 12.059 & 0 & 12.059 & 100.600 & 31.068 & 0 & 31.068 & 1445.15 & 446.30 \\
8 & 0.203 & 27.804 & 5.641 & 0 & 5.641 & 61.101 & 12.396 & 0 & 12.396 & 949.35 & 192.60 \\
9 & 0.097 & 12.208 & 1.184 & 0 & 1.184 & 29.199 & 2.831 & 0 & 2.831 & 453.68 & 43.98 \\
\hline
X0(mid) & 0 & 0.000 & 0.000 & 0 & 0.000 & 0.000 & 0.000 & 0 & 0.000 & 0.000 & 0.000 \\
\hline
\end{tabular}
\caption{Station xc(m) xc(mn+a33) Product f(U) Dn xc*b33 Product f(U) En xc*c33 Hn}
\end{table}

\begin{align*}
A_n &= -(m_n + a_{33})(x_r - x_0); & B_n &= -b_{33}(x_r - x_0); & C_n &= -c_n(x_r - x_0); \\
D_n &= (m_n + a_{33})x_r(x_r - x_0); & E_n &= b_{33}x_r(x_r - x_0); & H_n &= c_nx_r(x_r - x_0).
\end{align*}

134
<table>
<thead>
<tr>
<th>Station</th>
<th>$f_3+h_3(R)$</th>
<th>Product</th>
<th>$f(U)$</th>
<th>Addition</th>
<th>$f_3+h_3(I)$</th>
<th>Product</th>
<th>$f(U)$</th>
<th>Addition</th>
</tr>
</thead>
<tbody>
<tr>
<td>FP</td>
<td>-564.024</td>
<td>-592.437</td>
<td>0</td>
<td>-592.437</td>
<td>784.012</td>
<td>823.507</td>
<td>0</td>
<td>823.51</td>
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<tr>
<td>1</td>
<td>-833.223</td>
<td>-786.937</td>
<td>0</td>
<td>-786.937</td>
<td>1558.090</td>
<td>1471.538</td>
<td>0</td>
<td>1471.54</td>
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<td>2</td>
<td>-833.551</td>
<td>-698.932</td>
<td>0</td>
<td>-698.932</td>
<td>2106.267</td>
<td>1766.122</td>
<td>0</td>
<td>1766.12</td>
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<td>3</td>
<td>-571.789</td>
<td>-418.879</td>
<td>0</td>
<td>-418.879</td>
<td>2551.844</td>
<td>1869.417</td>
<td>0</td>
<td>1869.42</td>
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<td>-191.199</td>
<td>0</td>
<td>-191.199</td>
<td>2835.099</td>
<td>1776.544</td>
<td>0</td>
<td>1776.54</td>
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<tr>
<td>5</td>
<td>227.477</td>
<td>118.447</td>
<td>0</td>
<td>118.447</td>
<td>2818.076</td>
<td>1467.372</td>
<td>0</td>
<td>1467.37</td>
</tr>
<tr>
<td>6</td>
<td>947.244</td>
<td>392.869</td>
<td>0</td>
<td>392.869</td>
<td>2485.117</td>
<td>1030.702</td>
<td>0</td>
<td>1030.70</td>
</tr>
<tr>
<td>7</td>
<td>1441.845</td>
<td>445.278</td>
<td>0</td>
<td>445.278</td>
<td>2199.875</td>
<td>679.376</td>
<td>0</td>
<td>679.38</td>
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<tr>
<td>8</td>
<td>1868.684</td>
<td>379.109</td>
<td>0</td>
<td>379.109</td>
<td>1785.436</td>
<td>362.220</td>
<td>0</td>
<td>362.22</td>
</tr>
<tr>
<td>9</td>
<td>2180.387</td>
<td>211.388</td>
<td>0</td>
<td>211.388</td>
<td>1374.321</td>
<td>133.24</td>
<td>0</td>
<td>133.24</td>
</tr>
</tbody>
</table>

| X0(mid) | 2409.561 | 0.000 | 0.000 | 0.000 | 0.000 | 909.999 | 0.000 | 0.000 | 0.000 |

$M_w(R) = \int_{x_0}^{b_{ow}} [f_3(R) + h_3(R)](x_r - x_0)dx_r$

$M_w(I) = \int_{x_0}^{b_{ow}} [f_3(I) + h_3(I)](x_r - x_0)dx_r$
The above computations are performed at each frequency value of the regular wave tests, and the resulting coefficients and excitation forces are presented in Table 31.

<table>
<thead>
<tr>
<th>f (hz)</th>
<th>P (R)</th>
<th>P (I)</th>
<th>Q (R)</th>
<th>Q (I)</th>
<th>R (R)</th>
<th>R (I)</th>
<th>S (R)</th>
<th>S (I)</th>
<th>F3 (R)</th>
<th>F3 (I)</th>
<th>F5 (R)</th>
<th>F5 (I)</th>
<th>Y =</th>
<th>B_y</th>
<th>C_y</th>
<th>D_y</th>
<th>E_y</th>
<th>H_y</th>
<th>Mw (R)</th>
<th>Mw (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.392</td>
<td>6674.607</td>
<td>5887.607</td>
<td>4978.069</td>
<td>4028.452</td>
<td>2845.297</td>
<td>1462.324</td>
<td>-127.046</td>
<td>1 -1964</td>
<td>162.694</td>
<td>154.762</td>
<td>149.977</td>
<td>134.844</td>
<td>122.506</td>
<td>-144.561</td>
<td>-1931.720</td>
<td>-1215.143</td>
<td>1236.769</td>
<td>975.096</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.490</td>
<td>5887.607</td>
<td>4978.069</td>
<td>4028.452</td>
<td>2845.297</td>
<td>1462.324</td>
<td>-127.046</td>
<td>1 -1964</td>
<td>162.694</td>
<td>154.762</td>
<td>149.977</td>
<td>134.844</td>
<td>122.506</td>
<td>-144.561</td>
<td>-1931.720</td>
<td>-1215.143</td>
<td>1236.769</td>
<td>975.096</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.588</td>
<td>4978.069</td>
<td>4028.452</td>
<td>2845.297</td>
<td>1462.324</td>
<td>-127.046</td>
<td>1 -1964</td>
<td>162.694</td>
<td>154.762</td>
<td>149.977</td>
<td>134.844</td>
<td>122.506</td>
<td>-144.561</td>
<td>-1931.720</td>
<td>-1215.143</td>
<td>1236.769</td>
<td>975.096</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.685</td>
<td>4028.452</td>
<td>2845.297</td>
<td>1462.324</td>
<td>-127.046</td>
<td>1 -1964</td>
<td>162.694</td>
<td>154.762</td>
<td>149.977</td>
<td>134.844</td>
<td>122.506</td>
<td>-144.561</td>
<td>-1931.720</td>
<td>-1215.143</td>
<td>1236.769</td>
<td>975.096</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0.784</td>
<td>2845.297</td>
<td>1462.324</td>
<td>-127.046</td>
<td>1 -1964</td>
<td>162.694</td>
<td>154.762</td>
<td>149.977</td>
<td>134.844</td>
<td>122.506</td>
<td>-144.561</td>
<td>-1931.720</td>
<td>-1215.143</td>
<td>1236.769</td>
<td>975.096</td>
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<tr>
<td>0.882</td>
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<td>-127.046</td>
<td>1 -1964</td>
<td>162.694</td>
<td>154.762</td>
<td>149.977</td>
<td>134.844</td>
<td>122.506</td>
<td>-144.561</td>
<td>-1931.720</td>
<td>-1215.143</td>
<td>1236.769</td>
<td>975.096</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the above calculations, we can get the following complex coefficients:

\[ P = P(R) + iP(I) \]
\[ Q = Q(R) + iQ(I) \]
\[ R = R(R) + iR(I) \]
\[ S = S(R) + iS(I) \]
\[ F_3 = F_3(R) + iF_3(I) \]
\[ F_5 = F_5(R) + iF_5(I) \]
\[ y_z = M_w(R) + iM_w(I) \]

Substitution of P, Q, R, S, F_3, and F_5 into equation (65) results in the complex amplitudes of the heave displacement \( z_a \) and the angular pitch displacement \( \theta_a \). Subsequent substitution of \( z_a \), \( \theta_a \), A_y, B_y, C_y, D_y, E_y, H_y, and \( y_z \) into equation (69) gives
the complex amplitude of the wave-induced midship bending moment. The modulus and phase shift could be determined from the complex amplitude. The final results are listed in Table 19. The MATLAB M-file for the complex solutions is presented below.

```
solution.m

% M-file for Solution Of Regular Wave Motion
P=motion(1)+i*motion(2);
Q=motion(3)+i*motion(4);
R=motion(5)+i*motion(6);
S=motion(7)+i*motion(8);
F3=motion(9)+i*motion(10);
F5=motion(11)+i*motion(12);
x3=(F3*S-F5*Q)/(P*S-Q*R);
x5=(F5*P-F3*R)/(P*S-Q*R);
% Computation of Midship Bending Moment
A=motion(13);
B=motion(14);
C=motion(15);
D=motion(16);
E=motion(17);
H=motion(18);
Mw=motion(19)+i*motion(20);
omega=motion(21);
BM=(C-A*omega^2+i*B*omega)*x3+(H-D*omega^2+i*E*omega)*x5+Mw;
% Magnitude and Phase
mag3=abs(x3);
mag5=abs(x5);
magBM=abs(BM);
if real(x3)>=0
    phase3=atan(imag(x3)/real(x3))*180/pi;
else
    phase3=(pi+atan(imag(x3)/real(x3)))*180/pi;
end
if real(x5)>=0
    phase5=atan(imag(x5)/real(x5))*180/pi;
else
    phase5=(pi+atan(imag(x5)/real(x5)))*180/pi;
end
if real(BM)>0
    phaseBM=atan(imag(BM)/real(BM))*180/pi;
else
    phaseBM=(pi+atan(imag(BM)/real(BM)))*180/pi;
end
result=[mag3,phase3;mag5,phase5;magBM,phaseBM]
```
Appendix II Data Analysis Programs

The computer programs used for the data analysis and numerical simulations are summarized in Table 32. All programs in this work are written as MATLAB M-files.

Table 32: Summary of the Computer Programs

<table>
<thead>
<tr>
<th>Stationary Tests in Head Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>hregu.m</strong></td>
</tr>
<tr>
<td><strong>hspec.m</strong></td>
</tr>
<tr>
<td><strong>hneural.m</strong></td>
</tr>
<tr>
<td><strong>htest1.m</strong></td>
</tr>
<tr>
<td><strong>htest3.m</strong></td>
</tr>
<tr>
<td><strong>hsimu.m</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stationary Tests in Following Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>fregu.m</strong></td>
</tr>
<tr>
<td><strong>fspec.m</strong></td>
</tr>
<tr>
<td><strong>fneural.m</strong></td>
</tr>
<tr>
<td><strong>ftest1.m</strong></td>
</tr>
<tr>
<td><strong>ftest3.m</strong></td>
</tr>
<tr>
<td><strong>fsimu.m</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Towing Tests in Head Waves (0.5m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>s5regu.m</strong></td>
</tr>
<tr>
<td><strong>s5sample.m</strong></td>
</tr>
<tr>
<td><strong>s5spec.m</strong></td>
</tr>
<tr>
<td><strong>s5neural.m</strong></td>
</tr>
<tr>
<td><strong>s5test1.m</strong></td>
</tr>
<tr>
<td><strong>s5test3.m</strong></td>
</tr>
<tr>
<td><strong>s5simu.m</strong></td>
</tr>
</tbody>
</table>
% Frequency Analysis of Regular Test (Head Wave Stationary Tests)

heave = 0.00140*data(:,1) + 87.5966;
pitch = 0.00232*data(:,2) + 70.5666;
speed = 0.00015*data(:,3) - 5.0490;
wave = 0.00050*data(:,4) - 17.3866;
bend = 0.00580*data(:,5) - 212.5687;

heave = heave(3001:6000) - mean(heave(1:100));
pitch = pitch(3001:6000) - mean(pitch(1:100));
wave = wave(3001:6000) - mean(wave(1:100));
bend = bend(3001:6000) - mean(bend(1:100));

f = t == [0:2999] / 50;

Icw = wave .* cos(2*pi*f*t) / 3000;
Isw = wave .* sin(2*pi*f*t) / 3000;
Ich = heave .* cos(2*pi*f*t) / 3000;
Ish = heave .* sin(2*pi*f*t) / 3000;
Icp = pitch .* cos(2*pi*f*t) / 3000;
Isp = pitch .* sin(2*pi*f*t) / 3000;
Icb = bend .* cos(2*pi*f*t) / 3000;
Isb = bend .* sin(2*pi*f*t) / 3000;

W = 2*sqrt(Icw^2 + Isw^2);
H = 2*sqrt(Ich^2 + Ish^2);
P = 2*sqrt(Icp^2 + Isp^2);
B = 2*sqrt(Icb^2 + Isb^2);

if Icw > 0
    phasew = (-atan(Isw/Icw)) * 180/pi;
else
    phasew = (pi - atan(Isw/Icw)) * 180/pi;
end

if Ich > 0
    phaseh = (-atan(Ish/Ich)) * 180/pi;
else
    phaseh = (pi - atan(Ish/Ich)) * 180/pi;
end

if Icp > 0
    phasep = (-atan(Isp/Icp)) * 180/pi;
else
    phasep = (pi - atan(Isp/Icp)) * 180/pi;
end

if Icb > 0
    phaseb = (-atan(Isb/Icb)) * 180/pi;
else
    phaseb = (pi - atan(Isb/Icb)) * 180/pi;
end

result = [f, W, H, phaseh - phasew, P, phasep - phasew, B, phaseb - phasew]
% Spectral Analysis of Random Test (Head Wave Stationary Tests)
heave = -0.00140*data(:,1) + 87.5966;
pitch = -0.00232*data(:,2) + 70.5666;
speed = 0.00015*data(:,3) - 5.0490;
wave = 0.00050*data(:,4) - 17.3866;
bend = 0.00580*data(:,5) - 212.5687;
heave = heave(2501:25000) - mean(heave(1:100));
pitch = pitch(2501:25000) - mean(pitch(1:100));
wave = wave(2501:25000) - mean(wave(1:100));
bend = bend(2501:25000) - mean(bend(1:100));

% Spectrum Estimation
fwave = fft(wave);
swave = fwave.*conj(fwave)/1125000;
heave = fft(heave);
sheave = fheave.*conj(fheave)/1125000;
pitch = fft(pitch);
spitch = fpitch.*conj(fpitch)/1125000;
bend = fft(bend);
sbend = fbend.*conj(fbend)/1125000;

f = [0:11249]/450;
raoh = sqrt(sheave./swave);
raop = sqrt(spitch./swave);
raob = sqrt(sbend./swave);

% Regular Wave Test Results
F = [0.294, 0.392, 0.490, 0.588, 0.685, 0.784, 0.882, 0.980, 1.077];
AW = [3.40, 3.31, 3.56, 2.82, 2.70, 3.28, 3.30, 4.13, 3.32];
AB = [3.27, 3.83, 7.39, 8.89, 10.48, 13.75, 10.82, 6.14, 6.63];
AH = [3.29, 3.05, 3.02, 1.94, 1.35, 0.88, 0.52, 2.22, 0.54];
AP = [0.91, 0.75, 1.28, 2.06, 2.37, 2.36, 1.82, 1.32, 0.79];
RAOH = AH./AW; RAOP = AP./AW; RAOB = AB./AW;

% Plotting
subplot(2,1,1)
plot(f(100:600), swave(100:600), 'r')
title ('Wave Spectrum'); ylabel ('S(f)')

subplot(2,1,2)
plot(f(100:600), raob(100:600), '-r', F, RAOB, 'ob')
legend ('Random Test', 'Regular Test')
title ('BM RAOs'); ylabel ('RAO')

figure
subplot(2,1,1)
plot(f(100:600), raoh(100:600), '-r', F, RAOH, 'ob')
title ('Heave RAOs'); ylabel ('RAO')

subplot(2,1,2)
plot(f(100:600), raop(100:600), '-r', F, RAOP, 'ob')
title ('Pitch RAOs')
ylabel ('RAO'); xlabel ('f(Hz)')
% Estimation of Correlation Functions for Training (Head Wave Stationary Tests)
heave=-0.00140*data(:,1)+87.5966;
pitch=-0.00232*data(:,2)+70.5666;
speed=0.00015*data(:,3)-5.0490;
wave=0.00050*data(:,4)-17.3866;
bend=0.00580*data(:,5)-212.5687;
heave=heave(2501:25000)-mean(heave(1:100));
pitch=pitch(2501:25000)-mean(pitch(1:100));
wave=wave(2501:25000)-mean(wave(1:100));
bend=bend(2501:25000)-mean(bend(1:100));

% Estimation of Auto- and Cross-correlation Functions
for J=1:500
    crosshh(J)=heave(1+(J-1):22500)*heave(1:22500-(J-1))/(22500-(J-1));
    crosshp(J)=pitch(1+(J-1):22500)*heave(1:22500-(J-1))/(22500-(J-1));
    crosshb(J)=bend(1+(J-1):22500)*heave(1:22500-(J-1))/(22500-(J-1));
end
MX=max(crosshh);
crosshh=crosshh/MX;
crosshp=crosshp/MX;
crosshb=crosshb/MX;

% Calculation of Velocity & Acceleration
for K=1:480
    I=K+10;
    dhh(K)=crosshh(I);
    vhh(K)=(crosshh(I+10)-crosshh(I-10))/(2*0.2);
    ahh(K)=(crosshh(I+10)-2*crosshh(I)+crosshh(I-10))/(0.2)^2;
    dhp(K)=crosshp(I);
    vhp(K)=(crosshp(I+10)-crosshp(I-10))/(2*0.2);
    ahp(K)=(crosshp(I+10)-2*crosshp(I)+crosshp(I-10))/(0.2)^2;
    dhb(K)=crosshb(I);
end
t=[1:480]/50;
result=[dhh' vhh' ahh' dhp' vhp' ahp' dhb'];
% An M-file for Regular Test Verification (Head Wave Stationary Tests)
heave=-0.00140*data(:,1)+87.5966;
pitch=-0.00232*data(:,2)+70.5666;
speed=0.00015*data(:,3)-5.0490;
wave=0.00050*data(:,4)-17.3866;
bend=0.00580*data(:,5)-212.5687;
heave=heave(3001:6001)-rnean(heave(1:100));
pitch=pitch(3001:6001)-rnean(pitch(1:100));
wave=wave(3001:6001)-rnean(wave(1:100));
bend=bend(3001:6001)-rnean(bend(1:100));

% Resampling Data
for J=1:50
crosshh(J)=heave(1+(J-1)*10:3001)'*heave(1:3001-(J-1)*10)/(3001-(J-1)*10);
crosshp(J)=pitch(1+(J-1)*10:3001)'*heave(1:3001-(J-1)*10)/(3001-(J-1)*10);
crosshb(J)=bend(1+(J-1)*10:3001)'*heave(1:3001-(J-1)*10)/(3001-(J-1)*10);
end
MX=max(crosshh);
crosshh=crosshh/MX;
crosshp=crosshp/MX;
crosshb=crosshb/MX;

% Calculation of Velocity & Acceleration
for K=1:48
  I=K+1;
  dhh(K)=crosshh(I);
  vhh(K)=(crosshh(I+1)-crosshh(I-1))/(2*0.2);
  ahh(K)=(crosshh(I+1)-2*crosshh(I)+crosshh(I-1))/(0.2)^2;
  dhp(K)=crosshp(I);
  vhp(K)=(crosshp(I+1)-crosshp(I-1))/(2*0.2);
  ahp(K)=(crosshp(I+1)-2*crosshp(I)+crosshp(I-1))/(0.2)^2;
  dhb(K)=crosshb(I);
  vhb(K)=(crosshb(I+1)-crosshb(I-1))/(2*0.2);
  ahb(K)=(crosshb(I+1)-2*crosshb(I)+crosshb(I-1))/(0.2)^2;
end
t=[1:48]/5;
result=[dhh' vhh' ahh' dhp' vhp' ahp' dhb'];
An M-file for Random Test Verification (Head Wave Stationary Tests)

heave = -0.00140 * data(:, 1) + 87.5966;
pitch = -0.00232 * data(:, 2) + 70.5666;
speed = 0.00015 * data(:, 3) - 5.0490;
wave = 0.00050 * data(:, 4) - 17.3866;
bend = 0.00580 * data(:, 5) - 212.5687;
heave = heave(2501:25000) - mean(heave(1:100));
pitch = pitch(2501:25000) - mean(pitch(1:100));
wave = wave(2501:25000) - mean(wave(1:100));
bend = bend(2501:25000) - mean(bend(1:100));

% Resampling
N
crosshh = heave(N+1:N+500);
crosshp = pitch(N+1:N+500);
crosshb = bend(N+1:N+500);
MX = max(crosshh);
crosshh = crosshh / MX;
crosshp = crosshp / MX;
crosshb = crosshb / MX;

% Calculation of Velocity & Acceleration
for K = 1:480
    I = K+10;
    dhh(K) = crosshh(I);
    vhh(K) = (crosshh(I+10) - crosshh(I-10)) / (2 * 0.2);
    ahh(K) = (crosshh(I+10) - 2 * crosshh(I) + crosshh(I-10)) / (0.2)^2;
    dhp(K) = crosshp(I);
    vhp(K) = (crosshp(I+10) - crosshp(I-10)) / (2 * 0.2);
    aph(K) = (crosshp(I+10) - 2 * crosshp(I) + crosshp(I-10)) / (0.2)^2;
    dbh(K) = crosshb(I);
end

t = (1:480) / 50;

% Frequency-domain Filter Implementation
fbend = fft(bend);
nfbend = zeros(size(fbend));
nfbend(1:1000) = fbend(1:1000);
nfbend(21502:22500) = fbend(21502:22500);
nbend = ifft(nfbend);
nbend = real(nbend);

fheave = fft(heave);
nfheave = zeros(size(fheave));
nfheave(1:1000) = fheave(1:1000);
nfheave(21502:22500) = fheave(21502:22500);
nheave = ifft(nfheave);
nheave = real(nheave);

fpitch = fft(pitch);
nfpitch = zeros(size(fpitch));
nfpitch(1:1000) = fpitch(1:1000);
nfpitch(21502:22500) = fpitch(21502:22500);
npitch = ifft(nfpitch);
npitch = real(npitch);

% Calculation for Filtered Values
for J=1:500
    ncrosshh(J)=nheave(1+N+(J-1));
    ncrosshp(J)=npitch(1+N+(J-1));
    ncrosshb(J)=nbend(1+N+(J-1));
end
ncrosshh=ncrosshh/MX;
ncrosshp=ncrosshp/MX;
ncrosshb=ncrosshb/MX;

% Calculation of Velocity & Acceleration
for K=1:480
    I=K+10;
    ndhh(K)=ncrosshh(I);
    nvhh(K)=(ncrosshh(I+10)-ncrosshh(I-10))/(2*0.2);
    nahh(K)=(ncrosshh(I+10)-2*ncrosshh(I)+ncrosshh(I-10))/(0.2)^2;
    ndhp(K)=ncrosshp(I);
    nvhp(K)=(ncrosshp(I+10)-ncrosshp(I-10))/(2*0.2);
    nahp(K)=(ncrosshp(I+10)-2*ncrosshp(I)+ncrosshp(I-10))/(0.2)^2;
    ndhb(K)=ncrosshb(I);
end
result=[ndhh' nvhh' nahh' ndhp' nvhp' nahp' ndhb'];
%Simulation of Ship Motions and Bending Moment (Head Wave Stationary Condition)

%JONSWAP Wave Spectrum
f1=0.4:0.01:0.6;
s1=0.21*7.5^2*0.6^4./f1.^5.*exp(-1.25*0.6^4./f1.^4).*3.3.^exp(-f1-0.6).^2/2/(0.07*0.6).^2);
f2=0.61:0.01:0.9;
s2=0.21*7.5^2*0.6^4./f2.^5.*exp(-1.25*0.6^4./f2.^4).*3.3.^exp(-f2-0.6).^2/2/(0.09*0.6).^2);
f=[f1,f2]';
s=[s1,s2]';

%Wave Simulation
phasew=rand(size(f))*2*pi;
ampliw=sqrt(2*s*0.01);

%Computed Frequency Transfer Functions
%f=[0.4:0.1:0.9]';
%Magh=[0.921,0.848,0.749,0.542,0.255,0.119]';
%Angleh=[-1.18,-2.54,-4.89,-8.16,-11.62]';
%Magp=[0.389,0.537,0.662,0.761,0.721,0.527]';
%Anglep=[267.7,266.2,263.8,260.6,256.6,249.0]';
%Magb=[0.683,1.345,2.130,3.208,3.939,4.017]';
%Angleb=[181.8,185.6,188.7,191.4,196.7,205.6]';

%Experimental Frequency Response Functions
F=[0.4:0.1:0.9]';
Magh=[0.921,0.848,0.688,0.5,0.268,0.158]';
Angleh=[5.8,9.85,16.6,23.8,83.3]';
Magp=[0.226,0.36,0.731,0.878,0.72,0.552]';
Anglep=[267.5,261.1,272.1,281.7,286.2,284.1]';
Magb=[1.1571,2.0758,3.1525,3.8815,4.1921,3.2788]';
Angleb=[205,203,205,196,188,194]';

%Heave Simulation
raoh=interp1(F,Magh,f);
phaseh=interp1(F,Angleh,f)*pi/180;
t=(0.02:0.02:450)';
heave=zeros(size(t));
for k=1:size(f)
    heave=heave+ampliw(k)*raoh(k)*cos(2*pi*f(k)*t+phasew(k)+phaseh(k));
end

%Pitch Simulation
raop=interp1(F,Mapg,f);
phasep=interp1(F,Anglep,f)*pi/180;
pitch=zeros(size(t));
for k=1:size(f)
    pitch=pitch+ampliw(k)*raop(k)*cos(2*pi*f(k)*t+phasew(k)+phasep(k));
end

%Bending Moment Simulation
raob=interp1(F,Mapb,f);
phaseb=interp1(F,Angleb,f)*pi/180;
bend=zeros(size(t));
for k=1:size(f)
    bend=bend+ampliw(k)*raob(k)*cos(2*pi*f(k)*t+phasew(k)+phaseb(k));
end

% Estimation of Correlation Functions
for J=1:500
  crosshh(J)=heave(1+(J-1):22500)'*heave(1:22500-(J-1))/(22500-(J-1));
  crosshp(J)=pitch(1+(J-1):22500)'*heave(1:22500-(J-1))/(22500-(J-1));
  crosshb(J)=bend(1+(J-1):22500)'*heave(1:22500-(J-1))/(22500-(J-1));
end

MX=max(crosshh);
crosshh=crosshh/MX;
crosshp=crosshp/MX;
crosshb=crosshb/MX;

% Calculation of Velocity & Acceleration
for K=1:480
  I=K+10;
  dhh(K)=crosshh(I);
  vhh(K)=(crosshh(I+10)-crosshh(I-10))/(2*0.2);
  ahh(K)=(crosshh(I+10)-2*crosshh(I)+crosshh(I-10))/(0.2)^2;
  dhp(K)=crosshp(I);
  vhp(K)=(crosshp(I+10)-crosshp(I-10))/(2*0.2);
  ahp(K)=(crosshp(I+10)-2*crosshp(I)+crosshp(I-10))/(0.2)^2;
  dhb(K)=crosshb(I);
end

T=(1:480)/50;
result=[dhh' vhh' ahh' dhp' vhp' ahp' dhb'];
```
% Frequency Analysis of Regular Test (Following Wave Stationary Test)
heave=-0.0013967*data(:,1)+88.866;
pitch=-(-0.00090696*data(:,2)+30.743);
wave=0.00103865*data(:,4)-34.1906;
bend=0.00705158*data(:,3)-249.699;
heave=heave(3001:6000)-mean(heave(1:100));
pitch=pitch(3001:6000)-mean(pitch(1:100));
wave=wave(3001:6000)-mean(wave(1:100));
bend=bend(3001:6000)-mean(bend(1:100));
t=[0:2999]'/50;
f
Icw=wave*cos(2*pi*f*t)/3000;
Isw=wave*sin(2*pi*f*t)/3000;
Ich=heave*cos(2*pi*f*t)/3000;
Ish=heave*sin(2*pi*f*t)/3000;
Icp=pitch*cos(2*pi*f*t)/3000;
Isp=pitch*sin(2*pi*f*t)/3000;
Icb=bend*cos(2*pi*f*t)/3000;
Isb=bend*sin(2*pi*f*t)/3000;
W=2*sqrt(Icw^2+Isw^2);
H=2*sqrt(Ich^2+Ish^2);
P=2*sqrt(Icp^2+Isp^2);
B=2*sqrt(Icb^2+Isb^2);
if Icw>=0
    phasew=(-atan(Isw/Icw))*180/pi;
else
    phasew=(pi-atan(Isw/Icw))*180/pi;
end
if Ich>=0
    phaseh=(-atan(Ish/Ich))*180/pi;
else
    phaseh=(pi-atan(Ish/Ich))*180/pi;
end
if Icp>=0
    phasep=(-atan(Isp/Icp))*180/pi;
else
    phasep=(pi-atan(Isp/Icp))*180/pi;
end
if Icb>=0
    phaseb=(-atan(Isb/Icb))*180/pi;
else
    phaseb=(pi-atan(Isb/Icb))*180/pi;
end
result=[f,W,H,phaseh-phasew,P,phasep-phasew,B,phaseb-phasew]
```
% Spectral Analysis of Random Test (Following Wave Stationary Tests)
heave=0.0013967*data(:,1)+88.866;
pitch=-(-0.00090696*data(:,2)+30.743);
wave=0.00103865*data(:,4)-34.1906;
bend=0.00705158*data(:,3)-249.699;
heave=heave(2501:25000)-mean(heave(1:100));
pitch=pitch(2501:25000)-mean(pitch(1:100));
wave=wave(2501:25000)-mean(wave(1:100));
bend=bend(2501:25000)-mean(bend(1:100));

% Spectrum Estimation
fwave=fft(wave);
swave=fwave.*conj(fwave)/1125000;
fh=0:22499)/450;
swave(1:11250)*2;
swave=swave(1:11250)*2;
sheave=sheave(1:11250)*2;
spitch=spitch(1:11250)*2;
raoh=sqrt(sheave./swave);
raop=sqrt(spitch./swave);
raob=sqrt(sbend./swave);

% Regular Wave Test Results
F=[0.285,0.393,0.491,0.590,0.689,0.787,0.886,0.983,1.081];
RAOB=[0.834,1.198,1.781,2.678,3.652,4.027,4.778,6.723,3.595];
RAOH=[0.920,0.884,0.589,0.456,0.212,0.176,1.261,0.172];
RAO=[0.254,0.203,0.347,0.620,0.760,0.676,0.576,0.576,0.246];

% Plotting
figure
subplot(2,1,1)
plot(f(100:600),raoh(100:600),'-r',F,RAOH,'ob')
title ('Heave RAOs'); ylabel ('RAO')
figure
subplot(2,1,1)
plot(f(100:600),raop(100:600),'-r',F,RAOP,'ob')
title ('Pitch RAOs')

figure
subplot(2,1,1)
plot(f(100:600),raoh(100:600),'-r',F,RAOH,'ob')
title ('Heave RAOs'); ylabel ('RAO')
figure
subplot(2,1,1)
plot(f(100:600),raop(100:600),'-r',F,RAOP,'ob')
title ('Pitch RAOs')
ylabel ('RAO'); xlabel ('f(Hz)')
% Estimation of Correlation Functions for Training (Following Wave
Stationary Test)
heave=-0.0013967*data(:,1)+88.866;
pitch=-(-0.00090696*data(:,2)+30.743);
wave=0.00103865*data(:,4)-34.1906;
bend=0.00705158*data(:,3)-249.699;
heave=heave(2501:25000)-mean(heave(1:100));
pitch=pitch(2501:25000)-mean(pitch(1:100));
wave=wave(2501:25000)-mean(wave(1:100));
bend=bend(2501:25000)-mean(bend(1:100));

% Estimation of Correlation Functions
for J=1:500
  crosshh(J)=heave(1+(J-1):22500)'*heave(1:22500-(J-1));
crosshp(J)=pitch(1+(J-1):22500)'*heave(1:22500-(J-1));
crosshb(J)=bend(1+(J-1):22500)'*heave(1:22500-(J-1));
crosshh(J)=crosshh(J)/{22500-(J-1));
crosshp(J)=crosshp(J)/(22500-(J-1));
crosshb(J)=crosshb(J)/(22500-(J-1));
end
MX=max(crosshh);
crosshh=crosshh/MX;
crosshp=crosshp/MX;
crosshb=crosshb/MX;

% Calculation of Velocity & Acceleration
for K=1:480
  I=K+10;
  dhh(K)=crosshh(I);
  vhh(K)=(-crosshh(I+10)-crosshh(I-10))/(2*0.2);
  ahh(K)=(crosshh(I+10)-2*crosshh(I)+crosshh(I-10))/(0.2)^2;
  dhp(K)=crosshp(I);
  vhp(K)=-crosshp(I+10)-crosshp(I-10)/(2*0.2);
  ahp(K)=(crosshp(I+10)-2*crosshp(I)+crosshp(I-10))/(0.2)^2;
  dhb(K)=crosshb(I);
end

t=[1:480]/50;
result=[dhh' vhh' ahh' dhp' vhp' ahp' dhb'];
An M-file for Regular Test Verification (Head Wave Stationary Tests)

heave = -0.00140 * data(:,1) + 87.5966;
pitch = -0.00232 * data(:,2) + 70.5666;
speed = 0.00015 * data(:,3) - 5.0490;
wave = 0.00050 * data(:,4) - 17.3866;
bend = 0.00050 * data(:,5) - 212.5687;

heave = heave(3001:6001) - mean(heave(1:100));
pitch = pitch(3001:6001) - mean(pitch(1:100));
wave = wave(3001:6001) - mean(wave(1:100));
bend = bend(3001:6001) - mean(bend(1:100));

Resampling Data
for J = 1:50
    crosshh(J) = heave(1 + (J-1)*10:3001)' * heave(1:3001-(J-1)*10) / (3001-(J-1)*10);
    crosshp(J) = pitch(1 + (J-1)*10:3001)' * heave(1:3001-(J-1)*10) / (3001-(J-1)*10);
    crosshb(J) = bend(1 + (J-1)*10:3001)' * heave(1:3001-(J-1)*10) / (3001-(J-1)*10);
end

MX = max(crosshh);
crosshh = crosshh / MX;
crosshp = crosshp / MX;
crosshb = crosshb / MX;

Calculation of Velocity & Acceleration
for K = 1:48
    I = K+1;
    dhh(K) = crosshh(I);
    vhh(K) = (crosshh(I+1) - crosshh(I-1)) / (2 * 0.2);
    ahh(K) = (crosshh(I+1) - 2 * crosshh(I) + crosshh(I-1)) / (0.2)^2;
    dhp(K) = crosshp(I);
    vhp(K) = (crosshp(I+1) - crosshp(I-1)) / (2 * 0.2);
    ahp(K) = (crosshp(I+1) - 2 * crosshp(I) + crosshp(I-1)) / (0.2)^2;
    dhb(K) = crosshb(I);
    vhb(K) = (crosshb(I+1) - crosshb(I-1)) / (2 * 0.2);
    ahb(K) = (crosshb(I+1) - 2 * crosshb(I) + crosshb(I-1)) / (0.2)^2;
end

t = (1:48) / 5;
result = [dhh' vhh' ahh' dhp' vhp' ahp' dhb'];

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% An M-file for Random Data Verification (Following Wave Stationary Test)
heave=-0.0013967*data(:,1)+88.866;
pitch=-(-0.00090696*data(:,2)+30.743);
wave=0.00103865*data(:,4)-34.1906;
bend=0.00705158*data(:,3)-249.699;
heave=heave(2501:25000)-mean(heave(1:100));
pitch=pitch(2501:25000)-mean(pitch(1:100));
wave=wave(2501:25000)-mean(wave(1:100));
bend=bend(2501:25000)-mean(bend(1:100));

% Resampling
N
crosshh=heave(N+1:N+500);
crosshp=pitch(N+1:N+500);
crosshb=bend(N+1:N+500);
MX=max(crosshh);
crosshh=crosshh/MX;
crosshp=crosshp/MX;
crosshb=crosshb/MX;

% Calculation of Velocity & Acceleration for K=1:480
for K=1:480
    I=K+10;
    dhh(K)=crosshh(I);
    vhh(K)=(crosshh(I+10)-crosshh(I-10))/(2*0.2);
    ahh(K)=(crosshh(I+10)-2*crosshh(I)+crosshh(I-10))/(0.2)^2;
    dhp(K)=crosshp(I);
    vhp(K)=(crosshp(I+10)-crosshp(I-10))/(2*0.2);
    ahp(K)=(crosshp(I+10)-2*crosshp(I)+crosshp(I-10))/(0.2)^2;
    dhb(K)=crosshb(I);
end

t=(1:480)/50;

% Frequency-domain Filter Implementation
fbend=fft(bend);
nfbend=zeros(size(fbend));
nfbend(1:1000)=fbend(1:1000);
nfbend(21502:22500)=fbend(21502:22500);
nbend=ifft(nfbend);
bend=real(nbend);

fheave=fft(heave);
nfheave=zeros(size(fheave));
nfheave(1:1000)=fheave(1:1000);
nfheave(21502:22500)=fheave(21502:22500);
nheave=ifft(nfheave);
nheave=real(nheave);

fpitch=fft(pitch);
nfpitch=zeros(size(fpitch));
nfpitch(1:1000)=fpitch(1:1000);
nfpitch(21502:22500)=fpitch(21502:22500);
npitch=ifft(nfpitch);
npitch=real(npitch);

% Calculation for Filtered Values
for J=1:500
    ncrosshh(J)=nheave(1+N+(J-1));
    ncrosshp(J)=npitch(1+N+(J-1));
    ncrosshb(J)=nbend(1+N+(J-1));
end
ncrosshh=ncrosshh/MX;
ncrosshp=ncrosshp/MX;
ncrosshb=ncrosshb/MX;

% Calculation of Filtered Velocity & Acceleration
for K=1:480
    I=K+10;
    ndhh(K)=ncrosshh(I);
    nvhh(K)=(ncrosshh(I+10)-ncrosshh(I-10))/(2*0.2);
    nahh(K)=(ncrosshh(I+10)-2*ncrosshh(I)+ncrosshh(I-10))/(0.2)^2;
    ndhp(K)=ncrosshp(I);
    nvhp(K)=(ncrosshp(I+10)-ncrosshp(I-10))/(2*0.2);
    nahp(K)=(ncrosshp(I+10)-2*ncrosshp(I)+ncrosshp(I-10))/(0.2)^2;
    ndhb(K)=ncrosshb(I);
end
result=[ndhh' nvhh' nahh' ndhp' nvhp' nahp' ndhb'];
%Simulation of Ship Motions and Bending Moment (Following Wave Stationary Test)
%JONSWAP Wave Spectrum
f1=0.4:0.01:0.6;
s1=0.21*7.5^2*0.6^4./f1.^5.*exp(-1.25*0.6^4./f1.^4).*3.3.*exp(-(f1-0.6).^2/2/(0.07*0.6)^2);
f2=0.61:0.01:0.9;
s2=0.21*7.5^2*0.6^4./f2.^5.*exp(-1.25*0.6^4./f2.^4).*3.3.*exp(-(f2-0.6).^2/2/(0.09*0.6)^2);
f=[f1,f2]';
s=[s1,s2]';
%Wave Simulation
phasew=rand(size(f))*2*pi;
amplitw=sqrt(2*s*0.01);

%Tested Frequency Transfer Functions
F=[0.4:0.1:0.9]';
Magh=[0.884,0.812,0.589,0.456,0.212,0.176]';
Angleh=[2.51,3.3,-2.01,2.066,-0.412,72.74]';
Magr=[0.203,0.347,0.62,0.76,0.676,0.576]';
Anglep=[279.3,271.6,271.2,277.7,285.4,287.1]';
Magb=[1.198,1.781,2.678,3.652,4.027,4.778]';
Angleb=[149.42,159,163.2,176.5,190.3,218.4]';

%Heave Simulation
raoh=interp1(F,Magh,f);
phaseh=interp1(F,Angleh,f)*pi/180;
t=[0.02:0.02:450]';
heave=zeros(size(t));
for k=1:size(f)
    heave=heave+amplitw(k)*raoh(k)*cos(2*pi*f(k)*t+phasew(k)+phaseh(k));
end

%Pitch Simulation
raop=interp1(F,Magp,f);
phasep=interp1(F,Anglep,f)*pi/180;
pitch=zeros(size(t));
for k=1:size(f)
    pitch=pitch+amplitw(k)*raop(k)*cos(2*pi*f(k)*t+phasew(k)+phasep(k));
end

%Bending Moment Simulation
raob=interp1(F,Magb,f);
phaseb=interp1(F,Angleb,f)*pi/180;
bend=zeros(size(t));
for k=1:size(f)
    bend=bend+amplitw(k)*raob(k)*cos(2*pi*f(k)*t+phasew(k)+phaseb(k));
end

% Estimation of Correlation Functions
for J=1:500
    crosshh(J)=heave(1+(J-1):22500)'*heave(1:22500-(J-1))/(22500-(J-1));
end
crosshh=crosshh/MX;
crosshp=crosshp/MX;
crosshb=crosshb/MX;
%
Calculation of Velocity & Acceleration
for K=1:480
    I=K+10;
    dhh(K)=crosshh(I);
    vhh(K)=(crosshh(I+10)-crosshh(I-10))/(2*0.2);
    ahh(K)=(crosshh(I+10)-2*crosshh(I)+crosshh(I-10))/(0.2)^2;
    dhp(K)=crosshp(I);
    vhp(K)=(crosshp(I+10)-crosshp(I-10))/(2*0.2);
    ahp(K)=(crosshp(I+10)-2*crosshp(I)+crosshp(I-10))/(0.2)^2;
    dhb(K)=crosshb(I);
end
T=[1:480]/50;
result=[dhh' vhh' ahh' dhp' vhp' ahp' dhb'];
% Frequency Analysis of Regular Wave Towing Test (0.5 m/sec)
cheave = -0.00140 * calm(:,1) + 87.5966;
cpitch = -0.00232 * calm(:,2) + 70.5666;
cspeed = 0.00015 * calm(:,3) - 5.0490;
cwave = 0.00050 * calm(:,4) - 17.3866;
cbend = 0.00580 * calm(:,5) - 212.5687;
cheave = cheave - mean(cheave(1:100));
cpitch = cpitch - mean(cpitch(1:100));
cspeed = cspeed - mean(cspeed(1:100));
cwave = cwave - mean(cwave(1:100));
cbend = cbend - mean(cbend(1:100));

sheave = -0.00140 * data(:,1) + 87.5966;
spitch = -0.00232 * data(:,2) + 70.5666;
sspeed = 0.00015 * data(:,3) - 5.0490;
swave = 0.00050 * data(:,4) - 17.3866;
sbend = 0.00580 * data(:,5) - 212.5687;
sheave = sheave - mean(sheave(1:100));
spitch = spitch - mean(spitch(1:100));
sspeed = sspeed - mean(sspeed(1:100));
swave = swave - mean(swave(1:100));
sbend = sbend - mean(sbend(1:100));

[mxi,I] = max(cspeed);
cheave = cheave(I+150:I+2649);
cpitch = cpitch(I+150:I+2649);
cspeed = cspeed(I+150:I+2649);
cwave = cwave(I+150:I+2649);
cbend = cbend(I+150:I+2649);

[mxj,J] = max(sspeed);
sheave = sheave(J+150:J+2649);
spitch = spitch(J+150:J+2649);
sspeed = sspeed(J+150:J+2649);
swave = swave(J+150:J+2649);
sbend = sbend(J+150:J+2649);
t = [0:2499]'/50;

% Find the Wave-induced Response
heave = sheave - mean(cheave);
pitch = spitch - mean(cpitch);
wave = swave - mean(cwave);
bend = sbend - mean(cbend);

speed = sspeed;

% Correlation Analysis for Amplitudes and Phase
Icw = wave' * cos(2*pi*f*t)/2500;
Isw = wave' * sin(2*pi*f*t)/2500;
Ich = heave' * cos(2*pi*f*t)/2500;
Ish = heave' * sin(2*pi*f*t)/2500;
Icp = pitch' * cos(2*pi*f*t)/2500;
Isp = pitch' * sin(2*pi*f*t)/2500;
Icb = bend' * cos(2*pi*f*t)/2500;
Isb = bend' * sin(2*pi*f*t)/2500;
\[ Aw = 2 \cdot \sqrt{I_{cw}^2 + I_{sw}^2}; \]
\[ Ah = 2 \cdot \sqrt{I_{ch}^2 + I_{sh}^2}; \]
\[ Ap = 2 \cdot \sqrt{I_{cp}^2 + I_{sp}^2}; \]
\[ Ab = 2 \cdot \sqrt{I_{cb}^2 + I_{sb}^2}; \]

if \( I_{cw} \geq 0 \)
\[ \text{phasew} = (-\text{atan}(I_{sw}/I_{cw})) \cdot \frac{180}{\pi}; \]
else
\[ \text{phasew} = (\pi - \text{atan}(I_{sw}/I_{cw})) \cdot \frac{180}{\pi}; \]
end

if \( I_{ch} \geq 0 \)
\[ \text{phaseh} = (-\text{atan}(I_{sh}/I_{ch})) \cdot \frac{180}{\pi}; \]
else
\[ \text{phaseh} = (\pi - \text{atan}(I_{sh}/I_{ch})) \cdot \frac{180}{\pi}; \]
end

if \( I_{cp} \geq 0 \)
\[ \text{phasep} = (-\text{atan}(I_{sp}/I_{cp})) \cdot \frac{180}{\pi}; \]
else
\[ \text{phasep} = (\pi - \text{atan}(I_{sp}/I_{cp})) \cdot \frac{180}{\pi}; \]
end

if \( I_{cb} \geq 0 \)
\[ \text{phaseb} = (-\text{atan}(I_{sb}/I_{cb})) \cdot \frac{180}{\pi}; \]
else
\[ \text{phaseb} = (\pi - \text{atan}(I_{sb}/I_{cb})) \cdot \frac{180}{\pi}; \]
end

result = \([f, Aw, Ah, phaseh-phasew, Ap, phasep-phasew, Ab, phaseb-phasew]\)
% An M-file for Data Preparation of Towing Tests (0.5m/sec)
cheave=-0.00140*calm(:,1)+87.5966;
cpitch=-0.00232*calm(:,2)+70.5666;
cspeed=0.00015*calm(:,3)-5.0490;
cwave=0.00050*calm(:,4)-17.3866;
cbend=0.00580*calm(:,5)-212.5687;
cheave=cheave-mean(cheave(1:100));
cpitch=cpitch-mean(cpitch(1:100));
cspeed=cspeed-mean(cspeed(1:100));
cwave=cwave-mean(cwave(1:100));
cbend=cbend-mean(cbend(1:100));

sheave=-0.00140*data(:,1)+87.5966;
spitch=-0.00232*data(:,2)+70.5666;
sspeed=0.00015*data(:,3)-5.0490;
swave=0.00050*data(:,4)-17.3866;
sbend=0.00580*data(:,5)-212.5687;
sheave=sheave-mean(sheave(1:100));
spitch=spitch-mean(spitch(1:100));
sspeed=ssspeed-mean(ssspeed(1:100));
swave=swave-mean(swave(1:100));
sbend=sbend-mean(sbend(1:100));

Find the Speed Segment for Analysis
[mxi, I]=max(cspeed);cheave=cheave(I+150:I+2649);
cpitch=cpitch(I+150:I+2649);
cspeed=cspeed(I+150:I+2649);
cwave=cwave(I+150:I+2649);
cbend=cbend(I+150:I+2649);

[mxj, J]=max(ssspeed);sheave=sheave(J+150:J+2649);
spitch=spitch(J+150:J+2649);
ssspeed=ssspeed(J+150:J+2649);
swave=swave(J+150:J+2649);
sbend=sbend(J+150:J+2649);

% Find the Wave-induced Response
heave=sheave-mean(cheave);
pitch=spitch-mean(cpitch);
wave=swave-mean(cwave);
bend=sbend-mean(cbend);
speed=ssspeed;
sample=[wave heave pitch bend];
% Spectral Analysis of Random Test (Head Wave Towing Test 0.5m/sec)
series=[ ];
calm=c05c;
data=s5j7r1;
s5sample
series=[series;sample];
data=s5j7r2;
s5sample
series=[series;sample];
data=s5j7r3;
s5sample
series=[series;sample];
data=s5j7r4;
s5sample
series=[series;sample];
calm=c05d;
data=s5j7r5;
s5sample
series=[series;sample];
data=s5j7r6;
s5sample
series=[series;sample];
data=s5j7r7;
s5sample
series=[series;sample];

% Read Data from Joined Time Series
wave=series(:,1);
heave=series(:,2);
pitch=series(:,3);
bend=series(:,4);

%Spectrum Estimation from Joined Time Series
f wave=fft(wave);
specw=f wave.* conj(f wave)/ (17500*50);
f heave=fft(heave);
spech=f heave.* conj(f heave)/ (17500*50);
f pitch=fft(pitch);
specp=f pitch.* conj(f pitch)/ (17500*50);
f bend=fft(bend);
specb=f bend.* conj(f bend)/ (17500*50);
specw=2*specw(1:8750);
spech=2*spech(1:8750);
specp=2*specp(1:8750);
specb=2*specb(1:8750);
f=[0:8749]’/350;
raoh=sqrt(spech./specw);
raop=sqrt(specp./specw);
raob=sqrt(specb./specw);

% Regular Wave Test Results
F=[0.333,0.450,0.573,0.702,0.840,0.841,0.987,1.135,1.292,1.454]’;
AW=[2.82,2.23,2.61,3.02,2.95,2.57,1.37,2.90,3.23,3.12]’;
AH=[2.90,2.24,2.53,2.41,1.99,1.68,1.31,0.34,0.43,0.25]’;
AP=[0.81,0.96,1.64,2.22,3.26,2.77,3.20,1.98,0.26,0.19];
AB=[2.30,3.12,5.84,9.71,12.39,11.73,13.09,4.86,7.65,4.65];
RAOH=AH./AW;
RAOP=AP./AW;
RAOB=AB./AW;
%Plotting
figure
subplot(2,1,1)
plot(f(100:600),specw(100:600),'r')
title('Wave Spectrum')
ylabel('S(f)')
subplot(2,1,2)
plot(f(100:600),raob(100:600),'-r',F,RAOB,'ob')
legend('Random Test','Regular Test')
title('BM RAOs')
ylabel('RAO')
figure
subplot(2,1,1)
plot(f(100:600),raoh(100:600),'-r',F,RAOH,'ob')
title('Heave RAOs')
ylabel('RAO')
subplot(2,1,2)
plot(f(100:600),raop(100:600),'-r',F,RAOP,'ob')
title('Pitch RAOs')
ylabel('RAO')
xlabel('f(Hz)')
sSneural.m

% Estimation of Correlation Functions for Training (Towing Test 0.5m/s)
series=[];
calm=c05c; data=s5j7r1;
s5sample
series=[series;sample];
data=s5j7r2;
s5sample
series=[series;sample];
data=s5j7r3;
s5sample
series=[series;sample];
data=s5j7r4;
s5sample
series=[series;sample];
calm=c05d; data=s5j7r5;
s5sample
series=[series;sample];
data=s5j7r6;
s5sample
series=[series;sample];
data=s5j7r7;
s5sample
series=[series;sample];
% Read Data from Joined Time Series
wave=series(:,1);
heave=series(:,2);
pitch=series(:,3);
bend=series(:,4);
% Estimation of Correlation Functions
for J=1:500
crosshh(J)=heave(1+(J-1):17500)'*heave(1:17500-(J - 1))/(17500-(J-1));
crosshp(J)=pitch(1+(J-1):17500)'*heave(1:17500-(J - 1))/(17500-(J-1));
crosshb(J)=bend(1+(J-1):17500)'*heave(1:17500-(J - 1))/(17500-(J-1));
end
MX=max({crosshh});
crosshh=crosshh/MX;
crosshp=crosshp/MX;
crosshb=crosshb/MX;
% Calculation of Velocity & Acceleration
for K=1:480
I=K+10;
dhh(K)=crosshh(I);
vhh(K)=(crosshh(I+10)-crosshh(I-10))/(2*0.2);
ahh(K)=(crosshh(I+10)-2*crosshh(I)+crosshh(I-10))/(0.2)^2;
dhp(K)=crosshp(I);
vhp(K)=(crosshp(I+10)-crosshp(I-10))/(2*0.2);
ahp(K)=(crosshp(I+10)-2*crosshp(I)+crosshp(I-10))/(0.2)^2;
dhb(K)=crosshb(I);
end
t=[1:480]/50;
result=[dhh' vhh' ahh' dhp' vhp' ahp' dhb'];
s5test1.m

% An M-file of Regular Test Verification (Towing Tests 0.5m/sec)
s5sample
% Resampling Data
for J=1:50
    crosshh(J)=heave(1+(J-1)*10:2500)'*heave(1:2500-(J-1)*10)/(2500-(J-1)*10);
    crosshp(J)=pitch(1+(J-1)*10:2500)'*heave(1:2500-(J-1)*10)/(2500-(J-1)*10);
    crosshb(J)=bend(1+(J-1)*10:2500)'*heave(1:2500-(J-1)*10)/(2500-(J-1)*10);
end
MX=max(crosshh);
crosshh=crosshh/MX;
crosshp=crosshp/MX;
crosshb=crosshb/MX;
% Calculation of Velocity & Acceleration
for K=1:48
    I=K+1;
    dhh(K)=crosshh(I);
    vhh(K)=(crosshh(I+1)-crosshh(I-1))/(2*0.2);
    ahh(K)=(crosshh(I+1)-2*crosshh(I)+crosshh(I-1))/(0.2)^2;
    dp(K)=crosshp(I);
    vhp(K)=(crosshp(I+1)-crosshp(I-1))/(2*0.2);
    aph(K)=(crosshp(I+1)-2*crosshp(I)+crosshp(I-1))/(0.2)^2;
    db(K)=crosshb(I);
    vhb(K)=(crosshb(I+1)-crosshb(I-1))/(2*0.2);
    abh(K)=(crosshb(I+1)-2*crosshb(I)+crosshb(I-1))/(0.2)^2;
end
t=[1:48]/5;
result=[dhh' vhh' ahh' dp' vhp' aph' db'];
% An M-file for Random Test Verification (Towing Test 0.5m/sec)

series=[];
calm=c05c;
data=s5j7r1;
s5sample
series=[series;sample];
data=s5j7r2;
s5sample
series=[series;sample];
data=s5j7r3;
s5sample
series=[series;sample];
data=s5j7r4;
s5sample
series=[series;sample];
calm=c05d;
data=s5j7r5;
s5sample
series=[series;sample];
data=s5j7r6;
s5sample
series=[series;sample];
data=s5j7r7;
s5sample
series=[series;sample];

% Read Data from Joined Time Series
wave=series(:,1);
heave=series(:,2);
pitch=series(:,3);
bend=series(:,4);

% Resampling
N


crosshh=heave(N+1:N+500);
crosshp=pitch(N+1:N+500);
crosshb=bend(N+1:N+500);
MX=max(crosshh);
crosshh=crosshh/MX;
crosshp=crosshp/MX;
crosshb=crosshb/MX;

% Calculation of Velocity & Acceleration
for K=1:480
    I=K+10;
    dhh(K)=crosshh(I);
vhh(K)=(crosshh(I+10)-crosshh(I-10))/(2*0.2);
    ahh(K)=(crosshh(I+10)-2*crosshh(I)+crosshh(I-10))/(0.2)^2;
    dhp(K)=crosshp(I);
vhp(K)=(crosshp(I+10)-crosshp(I-10))/(2*0.2);
    ahp(K)=(crosshp(I+10)-2*crosshp(I)+crosshp(I-10))/(0.2)^2;
    dhb(K)=crosshb(I);
end

t=[1:480]/50;
% Frequency-domain Filter Implementation

fbend=fft(bend);

nfend=zeros(size(fbend));
nfend(1:1000)=fbend(1:1000);
nfend(16502:17500)=fbend(16502:17500);
nbend=ifft(nfend);
nbend=real(nbend);

fheave=fft(heave);

nfheave=zeros(size(fheave));
nfheave(1:1000)=fheave(1:1000);
nfheave(16502:17500)=fheave(16502:17500);
nheave=ifft(nfheave);
nheave=real(nheave);

fpitch=fft(pitch);

nf.pitch=zeros(size(fpitch));
nf.pitch(1:1000)=fpitch(1:1000);
nf.pitch(16502:17500)=fpitch(16502:17500);
npitch=ifft(nfpitch);
npitch=real(npitch);

% Calculation for Filtered Values
for J=1:500
   ncrosshh(J)=nheave(I+N+(J-1));
   ncrosshp(J)=npitch(I+N+(J-1));
   ncrosshb(J)=nbend(I+N+(J-1));
end
ncrosshh=ncrosshh/MX;
ncrosshp=ncrosshp/MX;
ncrosshb=ncrosshb/MX;

% Calculation of Filtered Velocity & Acceleration
for K=1:480
   I=K+10;
   ndhh(K)=ncrosshh(I);
   nvhh(K)=(ncrosshh(I+10)-ncrosshh(I-10))/(2*0.2);
   nahh(K)=(ncrosshh(I+10)-2*ncrosshh(I)+ncrosshh(I-10))/(0.2)^2;
   ndhp(K)=ncrosshp(I);
   nvhp(K)=(ncrosshp(I+10)-ncrosshp(I-10))/(2*0.2);
   nahp(K)=(ncrosshp(I+10)-2*ncrosshp(I)+ncrosshp(I-10))/(0.2)^2;
   ndhb(K)=ncrosshb(I);
end
result=[ndhh' nvhh' nahh' ndhp' nvhp' nahp' ndhb'];
%Simulation of Ship Motions and Bending Moment (Towing Test 0.5 m/sec)
%JONSWAP Wave Spectrum
f1=0.5:0.01:0.7;
s1=0.21*7.5^2*0.7^4./f1.^5.*exp(-1.25*0.7^4./f1.^4).*3.3.*exp(-f1./0.7).^2/2/(0.07*0.7)^2);
f2=0.71:0.01:1.0;
s2=0.21*7.5^2*0.7^4./f2.^5.*exp(-1.25*0.7^4./f2.^4).*3.3.*exp(-f2./0.7).^2/2/(0.09*0.7)^2);
f0=[f1,f2];
f=f0+2*pi*f0.^2/9.8*0.5;
s=[s1,s2];
%Wave Simulation
phasew=rand(size(f))*2*pi;
ampliw=sqrt(2*s*0.01);

%Experimental Frequency Transfer Functions
F=[0.573,0.702,0.84,0.987,1.135,1.292,1.454]';
Magh=[0.97,0.8,0.67,0.96,0.12,0.13,0.08]';
Angleh=[10.56,7.56,8.43,-53.1,-3.7,76.95,108.3]';
Magp=[0.63,0.74,1.11,2.34,0.68,0.08,0.06]';
Anglep=[98.7,99.2,103.2,24.3,23.6,64.7,159.2]';
Magb=[2.24,3.22,4.2,9.55,1.68,2.37,1.49]';
Angleb=[207.8,190.1,67.6,79.7,-4.6,-108.1,-113.2]';

%Heave Simulation
raoh=interpl(F,Magh,f);
phaseh=interp1(F,Angleh,f*pi/180;
t=[0.02:0.02:450];
heave=zeros(size(t));
for k=1:size(f)
heave=heave+ampliw(k)*raoh(k)*cos(2*pi*f(k)*t+phasew(k)+phaseh(k));
end

%Pitch Simulation
raop=interpl(F,Magp,f);
phasep=interp1(F,Anglep,f*pi/180;
pitch=zeros(size(t));
for k=1:size(f)
pitch=pitch+ampliw(k)*raop(k)*cos(2*pi*f(k)*t+phasew(k)+phasep(k));
end

%Bending Moment Simulation
raob=interpl(F,Magb,f);
phaseb=interp1(F,Angleb,f*pi/180;
bend=zeros(size(t));
for k=1:size(f)
bend=bend+ampliw(k)*raob(k)*cos(2*pi*f(k)*t+phasew(k)+phaseb(k));
end

% Estimation of Correlation Functions
for J=1:500
  crosshh(J)=heave(1+(J-1):22500)'*heave(1:22500-(J-1))/(22500-(J-1));
  crosshp(J)=pitch(1+(J-1):22500)'*heave(1:22500-(J-1))/(22500-(J-1));
  crosshb(J)=bend(1+(J-1):22500)'*heave(1:22500-(J-1))/(22500-(J-1));
end
MX=max(crosshh);
% Calculation of Velocity & Acceleration
for K=1:480
    I=K+10;
    dhh(K)=crosshh(I);
    vhh(K)=(crosshh(I+10)-crosshh(I-10))/(2*0.2);
    ahh(K)=(crosshh(I+10)-2*crosshh(I)+crosshh(I-10))/(0.2)^2;
    dhp(K)=crosshp(I);
    vhp(K)=(crosshp(I+10)-crosshp(I-10))/(2*0.2);
    ahp(K)=(crosshp(I+10)-2*crosshp(I)+crosshp(I-10))/(0.2)^2;
    dhb(K)=crosshb(I);
end
T=[1:480]/50;
result=[dhh' vhh' ahh' dhp' vhp' ahp' dhb'];
Appendix III  Data Analysis Results

The RAOs results obtained from both the regular wave tests and the random wave tests are presented in this Appendix.

In the following figures, the circle points represent the RAOs values from the regular wave tests.
Figure 38: Spectral Analysis Results of JSkSa
Figure 39: Spectral Analysis Results of J5h75b.dat
Figure 40: Spectral Analysis Results of J6h5a.dat
Figure 41: Spectral Analysis Results of J6h75a.dat
Figure 42: Spectral Analysis Results of J7h5c.dat
Figure 43: Spectral Analysis Results of J7h75a.dat
Figure 44: Spectral Analysis Results of B5a.dat
Figure 45: Spectral Analysis Results of B75b.dat
Figure 46: Spectral Analysis Results of Fj5h5a.dat
Wave Spectrum

BM RAOs

Heave RAOs

Pitch RAOs

Figure 47: Spectral Analysis Results of Fj5h75h.dat
Figure 48: Spectral Analysis Results of Fj6h5a.dat
Figure 49: Spectral Analysis Results of Fj6h75a.dat
Figure 50: Spectral Analysis Results of Fj7h5a.dat
Figure 51: Spectral Analysis Results of FJ7h75a.dat
Figure 52: Spectral Analysis Results of Fb5a.dat
Figure 53: Spectral Analysis Results of Fb75a.dat