CIRCLES IN A DYNAMIC SOFTWARE ENVIRONMENT

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Circles in a Dynamic Software Environment

by

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Abstract

This project consists of two parts: a review of the relevant literature on the new pedagogical issues facing educators today as it applies to the mathematics classroom and a *circles unit* designed to address these issues. The literature review focuses on the pedagogical issues of constructivism, collaboration and reflective inquiry, the role of the teacher, and the nature of proof. A brief explanation of dynamic geometry software and its capabilities in the classroom is included since this unit is developed around the dynamic geometry software, *The Geometer's Sketchpad*, that allows for individual exploration and discovery. This software has caused much excitement as teachers begin to explore its capabilities in their classrooms. The unit is based on the standards put forth by the National Council of Teachers of Mathematics to change the focus of math courses towards the process of discovery rather than the facts discovered. This *circles unit* is intended to be used in classrooms as enrichment or as a supplement to a unit on circles in courses such as Mathematics 2200 in Newfoundland and Labrador. A brief report on both teacher and students' reactions to this unit is also included.

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Introduction

The *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics (NCTM), 1989a) calls for new goals or expectations of school mathematics. It focuses attention away from the classic form of teacher delivery to a more constructivist approach where the teacher acts as a facilitator of learning. It "portrays mathematics as activity and process, not simply as a body of content to be mastered" (NCTM, 1989a, p.vi). As a result, the *Standards* call for

- increased activity in student constructions and applications of mathematical ideas.
- increased use of problem solving as a *means* of learning.
- increased variety of instruction - cooperative group, individual exploration, projects, and whole-class instruction.
- increased use of technology (calculators and computers) as tools for learning.

(NCTM, 1989a, p.vi)

This report describes the theoretical background to and development of materials for a unit on *circles* as outlined in the Academic Level II Mathematics

Course for Newfoundland High Schools (i.e., students approximately 15 years old). However, it could easily be adapted for Junior High or other High School students for enrichment.

Literature Review

Pedagogical Issues

Constructivism

One of the most demanding challenges in education today is how to make learning relevant. To accomplish this challenge, a teacher/facilitator must understand the learning process so he/she can assign learning tasks that allow the students to develop more sophisticated learning and thinking skills. One model of learning that has come forth recently in educational reform is the *constructivist model*. Simon (1995) contends that constructivism derives from the philosophical position that we as human beings have no access to an objective reality, that is, a reality independent from our way of knowing it. Rather, we construct our knowledge of our world from our perceptions and experiences, which are themselves mediated through our previous knowledge. Learning, contends Wheatley (1991), is a process by which human beings adapt to their experiential world. From a constructivist perspective, we have no way of knowing whether a concept matches an objective reality. Our concern, explains Wheatley, is whether it works (fits in with our experiential world). To clarify, a concept works or is visible to the extent that it does what we need it to do: to make sense of our perceptions or data, to make an accurate prediction, to solve a problem, or to accomplish a certain goal. When what we experience is different from the expected or intended, disequilibrium results and our adaptive (learning) process is triggered. Reflection on successful adaptive operations leads to new or modified concepts (Simon, 1995).

It is necessary for the teacher/ facilitator to provide a structure and a set of plans that support the development of informed exploration and reflective inquiry based on the student's own experience of the materials, not some predetermined truths or interpretations of them. Throughout this *circles unit*, students will construct and manipulate figures to discover conjectures. They will then be encouraged to test these conjectures and explain why they believe that the conjectures are true.

Collaboration

The concept of collaborative learning, the grouping and pairing of students for the purpose of achieving an academic goal has been widely researched and advocated throughout the professional literature. The term *collaborative learning* refers to an instruction method in which students at various performance levels work together in small groups towards a common goal. The students are responsible for one another's learning as well as their own. Thus the success of one student helps other students to be successful. Legere (1991) and other proponents of collaborative learning claim that the active exchange of ideas within small groups not only increases interest among the participants but also promotes critical thinking. According to Silver (1994), there is persuasive evidence that collaborative teams achieve higher levels of thought and retain information longer than students who work quietly as individuals. The shared learning gives students an opportunity to engage in discussion, take responsibility for their own learning and thus become critical thinkers.

Group diversity in terms of knowledge and experience contribute positively to the learning process. Critical-thinking skills develop best in an atmosphere of dialogue, interchange, and problem solving not merely listening to lectures. Legere (1991) and other educators believe that collaboration is not merely an adjunct to lecturing but a vehicle for the learning process itself. Bruner (1987) contends that collaborative learning methods improve problem-solving strategies because the students are confronted with different interpretations of the given situation. Students need to recognize that the application of mathematics is a flexible, creative process with a variety of approaches. Nichols (1996) claims that the peer support system makes it possible for the learner to internalize both external knowledge and critical thinking skills and to convert them into tools for intellectual functioning.

In this unit, students will work in groups. The setting will be informal to facilitate discussion and interaction. This group interaction will help the students to learn from each other's scholarship, skills and experiences while developing their own ideas and conjectures. The students will have to go beyond mere statements of opinion by giving reasons for their judgments and conjectures and reflecting upon the criteria employed in making these conjectures. McLeod (1993) states that discussing the problems in small groups helps the students see that everybody struggles with a problem at some time and that communicating your ideas to another student can help both of you understand the problem better. These heterogeneous groups according to Johnson & Johnson (1985) offer a wealth of background knowledge and perspectives to different real-world problems.

Reflective Inquiry

The National Council of Teachers of Mathematics in its *Curriculum and Evaluation Standards for School Mathematics* (1989b) advocates mathematics teaching "through activities that encourage students to explore mathematics, to gather evidence and make conjectures and to reason and communicate mathematically as they discuss and write ideas that use the language of mathematics" (p. vii.). In this unit the students must communicate their thought processes. This communication should reveal not only successful attempts but also attempts that lead to blind alleys. This form of metacognition, thinking about one's own thinking (even if done in a group), will help the students clarify their thought processes and reflect on their ideas and reasoning skills (Hatfield and Bitter, 1991). This approach is far more valuable to the students than merely supplying the answer. From the teacher point of view, according to Hatfield and Bitter, this method of looking back permits a look inside the students' heads to examine thought patterns and processes in which they were engaged.

During the past decade, many articles have been written on using the heuristic method in the mathematics classroom to improve the problem-solving skills of students. Krulik and Rudnick (1994) contended that Polya's plan for problem solving- read, plan, solve and look back-, has proven to be an effective pedagogical way to improve students' performance. However, Polya's fourth step, looking back, is perhaps the most neglected phase of problem-solving. Even fairly good students when they have obtained the solution of a problem, shut their books and look for something else. Doing so, according to Polya (1980), they miss an important and instructive phase of the work. By looking back at the completed solution, by reconsidering and reexamining the result and path that led to it, they could consolidate their knowledge and develop their ability to solve problems. Research indicates that skilful problem solvers use a format strategy more often than poor problem solvers who seem to rely more on a random trial and error strategy.

From a constructivist perspective, knowledge is a learner's activity. Wheatley (1992) says that our knowledge resembles a fabric- a network of information, images, relationships, error, hypotheses, inconsistencies, gaps, feelings, anticipations, inferences, hunches, rules, generalizations and so forth. When learners reflects on their actions, knowledge is constructed. Reflective abstraction, according to Wheatley (1992), is the mechanism of constructing knowledge. In mathematics learning, reflection is characterized by distancing oneself from the action of doing mathematics. Thiessen (1995) found that it is one thing to solve a problem and it is quite another to take one's own action as an object of reflection. In the process of reflection, schemes of schemes are constructed, a second-order construction. Persons who reflect have greater control over their thinking and can decide which of several paths to take. It is not enough for students to complete tasks. They must be encouraged to reflect on their activity.

In this unit, students will be asked "What do you notice?", "Do your observations still hold true?", and "Can you explain why this conjecture is true?" These kinds of questions acknowledge the students' mathematics. As students present alternative interpretations to their group, they are made aware that others did not see it the same way as they had. This leads to reflection on their own interpretation. Does their interpretation still make sense to them? Is there a conflict? If so, how should it be changed? This shows reflection and possibly modification of cognitive structures to account for conflict which they now realize. Here the students are free to construct their own mathematics rather than try to determine what they were suppose to do. It would be a different matter had the students been told to look at their drawing and see if they got it "right". This language can imply that there is just one way of thinking about this figure and the students' task is to see it that way. By asking "What do you notice?" and "Why do you think this is true?" students are encouraged to give meaning to their experiences in ways that make sense to them (Parker, 1991). Miller (1991) alleges that as a variety of interpretations are presented, students should realize that diversity and creativity have a place in mathematics.

The Nature of Proof

The NCTM (NCTM, 1989b) states that students should be encouraged to refine their thinking, gradually leading them to understand the limitations of visual and empirical justification so that they discover and begin to use some form of formal proof. Fawcett (1938) in *The Nature of Proof*, suggests that the first step in any classroom is to arouse the student's interest in order for him/her to think about the subject in his/her own way. According to Fawcett, when this is done, the spirit of discovery is encouraged and preserved. A good teacher knows that " to have searched and found, leaves a pupil a different person from what he would be if he merely understands and accepts the results of others' search and formulation" (p.23). Fawcett presents general principles and methods to aid in the students' development of this sense of discovery:

1 No formal textbook will be used as students will create their own

texts as discoveries are made.

2. Students are given the opportunities to make discoveries for themselves without first knowing what has to be proved.
3. Students make their own generalizations about their discoveries.
4. Major emphasis is not on the statement proved but rather on the method.

This circle unit is designed around the above principles. Students will create their own list of conjectures as they discover and test their findings. They will make these discoveries themselves with the aid of a partner and then take part in a larger discussion group at the end of each activity. Most attention will be placed on the actual method and explanation of discovery rather than the written formal statements.

The Role of the Teacher

The role of the teacher in the classroom has shifted from the primary role of information giver to that of facilitator, guide and learner. Teaching based on a "constructivist" view of learning must be guided by knowledge of the conceptual advances that students need to make for various mathematical topics and of the processes by which they make these advances. Our instructional goals are cognitive rather than behavioural and seek to mould students' own personal mathematical ideas. From this perspective, the essential pedagogical task is not to instill "correct ways of doing" but rather to guide children's constructive activities until they eventually find viable techniques. Such guidance must necessarily start from points that are accessible to the children. In order to establish these starting points, we must first gain insight into the children's conceptual structure and methods, no matter how wayward or ineffective they might seem (Simon, 1995). It is the intention of this unit that teachers will have the opportunity to see first hand the cognitive strategies of their students in the reflection process as they guide them along the path of learning. This would give a teacher an important way of assessing the students' learning.

Dynamic Geometry Software

Dynamic Geometry is described by Schattschneider and King (1997) as "active, exploratory geometry carried out with interactive computer software" (preface). Since its creation, dynamic geometry has received favorable reports from the classroom by students and teachers (e.g., DeVilliers, 1995; Clements and Battista, 1994; and Battista, 1995). With the implementation of this interactive software, the focus of teaching geometry in the classroom has shifted from pencil drawn sketches to more accurate computer assisted drawings. What is exciting about dynamic software, according to Schattschneider and King (1997) is its very nature: the ability to grab or stretch any objects (arbitrary segments or points for example), not dependent on any other objects. As each object is moved, all other objects in the drawing automatically self-adjust, thereby preserving all dependent relationships and constraints. But what is dynamic geometry software good for?

1. Accuracy of construction. All constructions and measurements are completely accurate. As figures are manipulated, all measurements adjust accurately.

2. Visualization. Dynamic geometry can help the students *see* what is true. Students can construct, revise and manipulate figures to create a better understanding of their own concepts. As Clements and Battista (1994) assert, by allowing students to investigate continuous variation directly, dynamic geometry environments can be used to help students build mental constructs that are useful for analytic thinking. Kaput (1992) contends that students acquire rich and varied kinds of mental representations interacting with dynamic media in a short period

of time.

3. Exploration and Discovery. Dynamic geometry allows students to test their own mathematical ideas and conjectures in a visual manner. This will engage the students more fully in their own learning, according to Schattschneider and King (1997). Students using this dynamic software often make surprising discoveries that were not planned. This leads to a greater understanding of their own mathematical ideas and often empowers them to go forward and test challenge themselves (De Villiers, 1997).

4. Proof. The National Council of Mathematics (NCTM, 1989b) proposes that meaningful justification of ideas be a major goal of the geometry curriculum. Students should be required to explain and justify their ideas. Many researchers (De Villiers, 1997; Galindo, 1998; and Clements and Battista, 1994) contend that dynamic geometry software encourages students towards the need for proofs. When students make their own conjectures from their own constructions and investigations, they realize that it is not enough to say it is true because of the measurements. Rather they see a need to create a proof explaining their conjectures. Futhermore, says Galindo (1998), when some of their findings are challenged by their classmates, students realize that a more formal proof is needed to justify their ideas.

This unit is created using the dynamic software *Geometer's Sketchpad.*

Conclusion

Simon (1995) states that if we are to produce truly "literate" students in mathematics, the teacher/facilitator must design tasks and projects that stimulate students to ask questions, pose problems, and set goals. Students will not become active learners by accident, but by design, through the use of the plans structured to guide exploration and inquiry. This unit is structured around this goal.

Problem Addressed by the Project

The teaching population in Newfoundland is getting older and as a result many of our teaching methods need reevaluation and modification. This *circles unit* was designed with this need in mind. It is hoped that this unit will be valuable to seasoned teachers in the field by demonstrating how, using technology, to incorporate these new pedagogies, discussed in the literature review, in the mathematics classroom.

At the present time, there are few resources available to mathematics teachers to aid in the implementation of the new Atlantic Provinces Education Foundation (APEF) curriculum, which is strongly influenced by the NCTM standards. Dynamic geometry, particularly *Geometer's Sketchpad*, will play a large role in this new curriculum by enhancing mathematics learning/teaching. Dynamic geometry provides a means for students to construct and manipulate shapes thus providing an exciting environment in which students could make conjectures and then test them out (NCTM, 1989b). One dynamic geometry software package that has been piloted by our Department of Education is *Geometer's Sketchpad*. However, unless teachers have previous experience using this dynamic geometry, creating activities with this software can be a time consuming, difficult task. This *circles unit* is designed to address this problem. It is hoped that teachers with limited previous dynamic geometry experience will use this unit in their classrooms, either as enrichment or in place of particular topics concerning the circle.

Rationale

Learning to write proofs has long been an objective of the high school mathematics courses. However, research has shown that students have difficulties with the concept of proof. One argument put forth explaining this difficulty is that students fail to see anything meaningful in the written proof. Frequently, teachers are asked "Why do we have to do these proofs?" and "When will I ever use this again?" Schoenfeld (1987) contends that after completing these proof-related activities, students do not seem to understand geometry concepts any better nor do they seem to be able to relate what they have learned to other situations. Because of this failing of traditional methods the National Council of Teachers of Mathematics (1989b) proposes that meaningful justification of ideas must be an important goal for geometry instruction.

Dynamic geometry, such as the Geometer's Sketchpad can be used effectively towards meeting this goal. This software allows students to create simple geometric figures, explore relationships in these figures, make conjectures about these properties and test those conjectures. Galindo (1998) concludes that "these activities have promise for moving students towards meaningful justification for their ideas in a way that traditional axiomatic approaches to proof never did" (p.77).

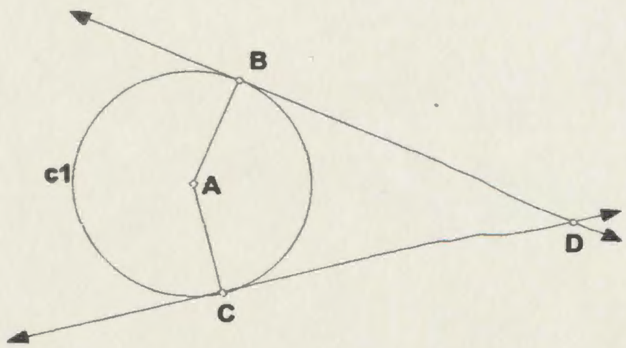
In this unit, students will study certain properties of the circle. Students will first construct diagrams and then manipulate particular parts of their construction to form conjectures about the mathematical relationships they have discovered. (Because students may have very limited experience with this dynamic geometry software, instructions are included within each construction.) They will then be encouraged to test their conjectures by using the tools available within *Geometer's Sketchpad*: built in measurement that adjusts as the figure changes and calculation tools. Because students will be working in groups, it is hoped that they will be encouraged to build validating arguments for their findings that will survive the scrutiny of others. It is the intention of this unit that as students realize that they have to validate their arguments, they will see the need to formalize their argument in some form of a proof to convince others that their conjectures are true.

Physical Description of the Unit

The unit consists of four topics of the circle:

1. Discovering Chord Properties
2. Discovering Tangent Properties
3. Discovering Circle and Angle Properties
4. Discovering Secant, Tangent and Chord Properties.

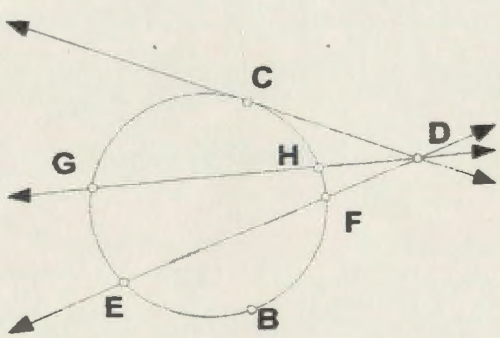
Each of the four topics will be divided into separate investigations related to each of the main topic. For example, congruent chords will be an investigation in the Discovering Chord Properties Topic. Each investigation will consist of four parts - construction, investigation, conjectures and/or justification and finally group or class discussion. At the end of some sections there will be further investigations or activities. These activities will encourage students to apply what they have learned to find other conjectures. Finally, at the back of the unit there will be projects designed around the unique aspects of circles and their properties. The unit will be designed for students working in pairs through the first three parts of each investigation and then meeting with a larger group for the group discussions. This could be handled differently depending upon the physical makeup of the class. It will also be assumed that students will have access to a computer lab. The design of the entire unit was influenced by the new pedagogies of constructivism, collaboration, and reflective inquiry.

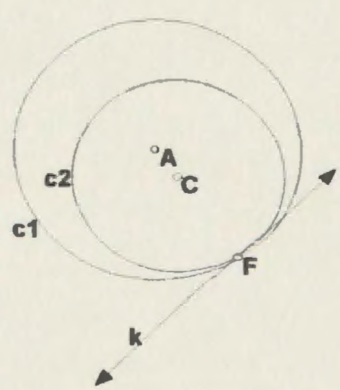


Circles

using

Geometer's Sketchpad





Note to teacher

This *circles unit* is designed for students and teachers with no previous knowledge of Geometer's Sketchpad. Each section includes instructions to complete all constructions. The unit is ready to be handed out to students. Permission is granted to individual teachers to photo copy and use any or all parts of this unit in their classes.

Grouping: This unit is designed for group work. Groups of two are recommended but that would depend on the availability of computers and the size of classes. It would also work with larger groups. It is further suggested, where possible, to pair students with little computer knowledge with more computer literate students as computer familiarity may aid in the use of this software.

Time frame: Each section is designed to take approximately two class periods of 40 minutes. However, all sections can be adapted for longer or shorter periods depending on individual classes.

Contents

Assignment

Investigation 4.2: Tangent Secant Theorem

Investigation 4.3: Tangent Chord Property

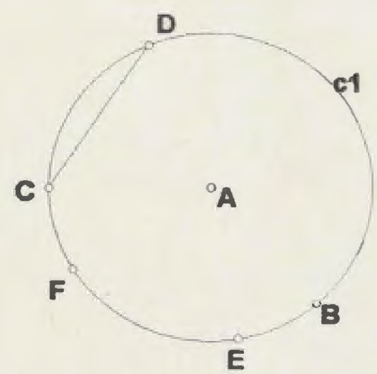
Project: Constructing a Sketchpad Kaleidoscope

Discovering Chord Properties

Investigation 1.1 Congruent Chords

Construction

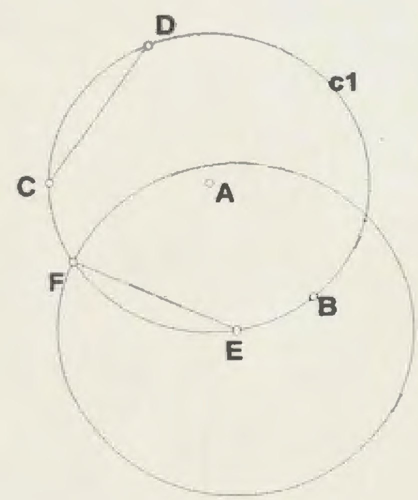
Step 1: Construct circle AB. (Label center point A and the point on the circumference B.)



Step 2 : Construct points C and D on the circle.(Click on the circle and then select the *construct menu* and *point on the circle*.) Label each point.

Step 3 : Construct chord CD. (Hold the shift key down and click on point C and point D. Then, select the *construct menu* and *segment.*)

Step 4: Construct point E on the circle, then construct a circle with center E and a radius equal to the length of chord CD. (Hold the Shift key down and click on point E and the chord CD. Then select the *construct menu* and click on *circle by center and radius.*

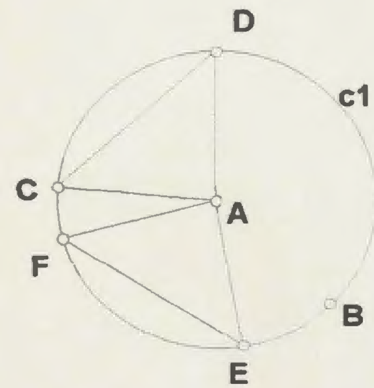


Step 5 Construct a point F at one of the intersections of the two circles. (Hold the Shift key down and click the two circles. Then select the *construct menu* and click *point at intersection.)* Label the point F. *Construct* chord EF. (Step 3)

Step 6: Hide the second circle.(Click on the circle, select the *display menu* and *hide circle.*)

Step 7: Construct chords AC, AD, AE and AF. (Step 2)

Step 8: Save your construction as "Cir 1a".



Investigation

1. Measure chords CD and EF . (To measure, click on the chord, select the *measure menu* and click *length.*) What do you notice?

2. Measure the central angles ∠CAD and∠ EAF. (To measure, hold the shift key down and click on point C, then vertex point A and then point D.Select the *measure menu* and click *angle.*) What do you notice?

3. Move point C or D to confirm that this relationship holds true for all congruent chords of any length.

4. Measure Arcs CD and EF. (Hold the Shift key down and click on the circle and the two points defining the arc. Then select the *measure menu* and click both *arc length and arc angle*) What do you notice?

5. Measure the distances from chord CD and chord EF to center point A. (Hold the Shift key down and click on the chord and the center point. Then select *measure menu* and *distance.*)What do you notice? Move point C or D to confirm your findings.

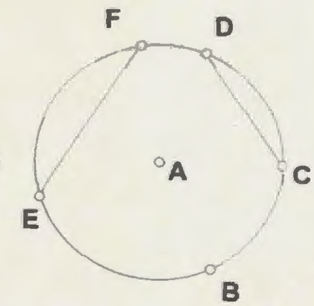
Conjectures: Write your conjectures below.

Explain why your conjecture(s) holds true. Write you explanation down and discuss it with your group(s). What did you discover?

Discovering Chord Properties

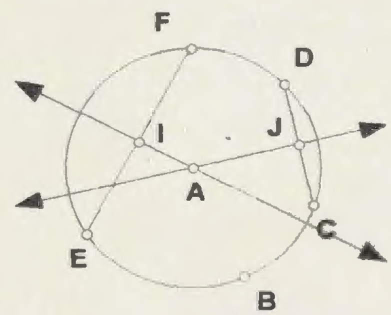
Investigation 1.2 Chords and Perpendicular Lines

Construction



Step 1: Construct circle AB.

Step 2: Construct two non congruent, nonparallel chords and CD and EF. (Steps 2 and 3, construction 1.1)



Step 3: Construct a perpendicular line from the center A to both chords CD and EF (Hold the Shift key down and click both the center point A and the chord. Then select the *construct menu* and *perpendicular line*.)

Step 4: Construct a point at the intersection of each chord and perpendicular line. (Hold the Shift key down and click both the chord and the perpendicular line. Then select the *construct menu* and *point at intersection.*)

Step 5. Save your construction as "cir 1b".

Investigation

1. Measure segments FI and EI. What do you notice?

2. Measure segments DJ and CJ. What do you notice?

3. Move points F, E, D and C around the circle. What do you notice? Do your observations

hold true for chords of any length?.

4. Make a conjecture below on your observations:

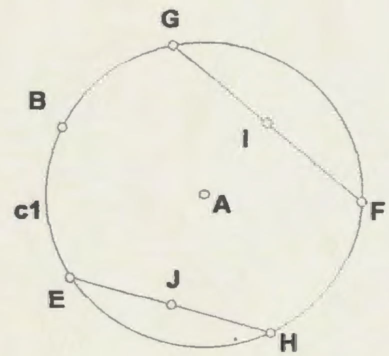
5. Review your conjectures. Can you explain why your conjecture(s) holds true. Write your explanation down.

6. Discuss your findings with your group(s).

Discovering Chord Properties

Investigation 1.3 Chords, Midpoints and Perpendicular Lines

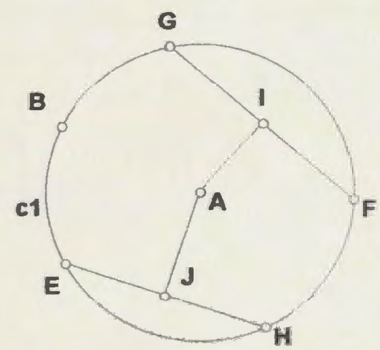
Construction



Step 1: Construct circle AB.

Step 2: Construct two nonparallel chords that are not diameters. (Steps 2 and 3, construction 1.1)

Step 3: Construct the midpoint of both chords. (Click on the chord and then select the *construct menu* and *point at midpoint*.) Label points I and J.



Step 4: Construct a segment from the midpoint of each chord and the center of the circle. (Hold the Shift key down and click the midpoint and the chord. Then, select the *construct menu* and *segment*.)

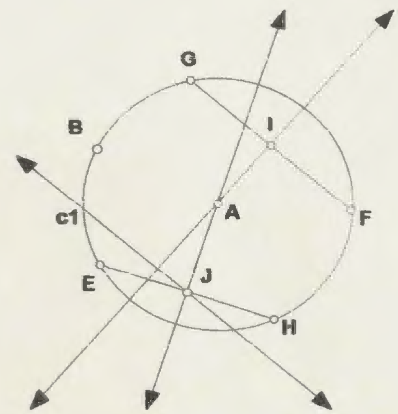
Investigation

1. What is special about these segments ? Measure $\angle GIA$ and $\angle HJA$.(Investigation 1.1, #2) What do you notice?

2. Move around the endpoints of each chord. Measure the angles again. Does your observation still hold true?

Further Construction

Step 5: Construct perpendicular bisectors of each chord. (Hold the Shift key down, and click the chord and midpoint. Then select the *construct menu* and *perpendicular line*.)



Investigation

1 What is special about the point of intersection?

2. Move the endpoints of each chord around the circle. What is happening?

3. Write your observations down in the form of conjectures.

Further Construction

Step 6: Retrieve file "cir 1a" and construct the perpendicular bisector of each congruent chord. (Construct the midpoint and then a perpendicular line from that point.)

Investigation:

1. What do you notice about the intersection of these two perpendicular bisectors?

2. Applying your observations, how could you find the center of any circle? Test your theory.

3. Write your observations down in the form of conjectures.

Can you explain why each of your conjectures hold true? Write your explanation down and discuss it with your group(s). Do you all agree?

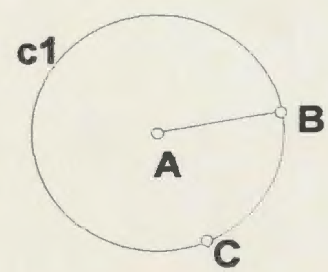
Discovering Tangent Properties

Investigation 2.1 Secants and Tangents

A line that intersects a circle in exactly one point is called a ***tangent*** and the point of intersection is called the ***point of tangency***.

A line that intersects a circle at two points is called a ***secant***.

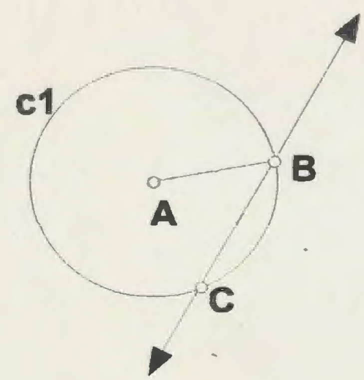
Construction



Step 1: Construct circle AB.

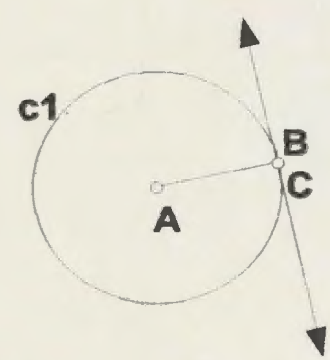
Step 2: Construct radius AB. (Constructed the same way as a chord.)

Step 3: Construct point C on circle AB.



Step 4: Construct a line (Secant BC) through points B and C. (In the tool menu, on the side, select line. Hold down the Shift key and click points B and C. Then select the *construct menu* and *line*.)

Investigation



1. Measure $\angle ABC$. (Investigation 1.1, #2)
2. Drag point C around the circle towards point B. What happens to $\angle ABC$ as point C gets closer to point B?
3. Drag point C so that it is right on top of point B. What is the measure of $\angle ABC$?

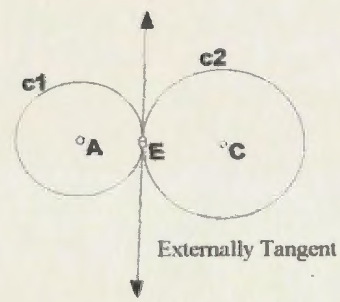
4. When points B and C coincide, your line intersect the circle at only one point. Is it still a secant? Why or why not?

5. What is the relationship between a tangent and a radius to the point of tangency?

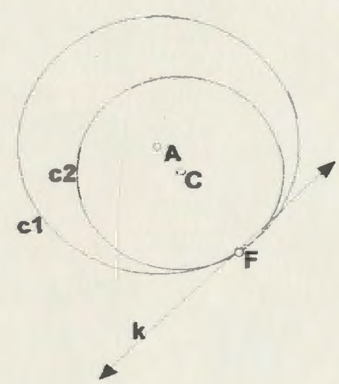
6. How can you use this relationship to devise a method for constructing a tangent to a circle

Conjecture: Write your conjectures and method of construction down.

Further Investigation



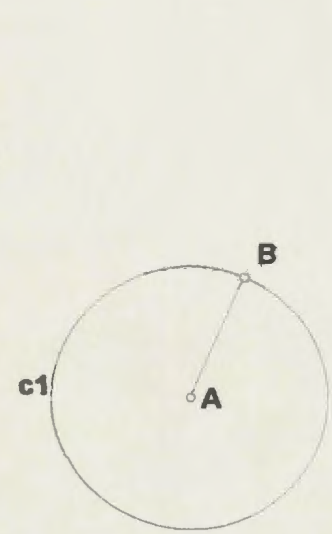
See if you can devise a method or methods for constructing externally or internally tangent circles (circles that intersect in a single point) Discuss your method with others in your group. Write your constructions down.



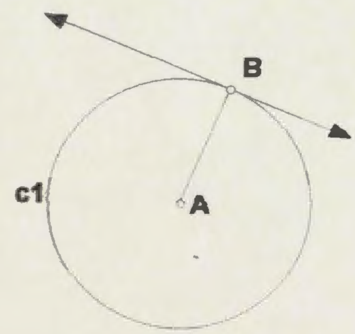
Discovering Tangent Properties

Investigation 2.2 Tangent Segments

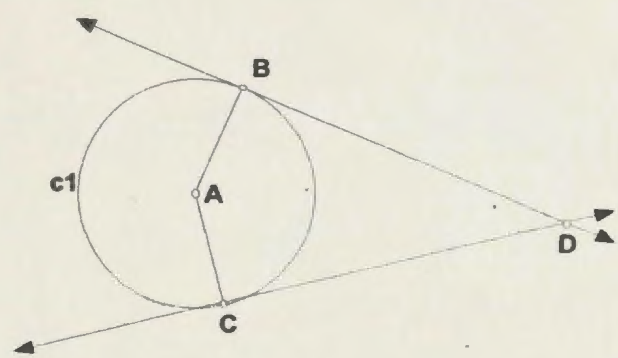
Construction



Step 1: Construct circle AB and chord AB (radius AB).



Step 2: Construct a line perpendicular to chord AB through B. (Hold Shift Key down and click on chord AB and point B. Then select the *construct menu* and *perpendicular line.*) This line is tangent to the circle.



Step 3: Construct a second chord AC and a tangent through C.(Same construction as Step 2 above.)

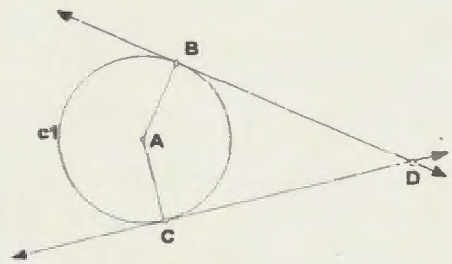
Step 4: Construct a point of intersection of the two tangents. Label the point of intersection D.(Hold the Shift key down and click on the two tangents. Then select the *construct menu* and *point of intersection.)*

Investigation

1. Measure the lengths of BD and CD.
2. Move points C, B, or A to see if this relationship holds for all segments tangent to a circle from a point outside the circle.
3. Write your conjecture(s) down.
4. Can you explain why your conjecture holds true? Discuss your findings with your group(s).

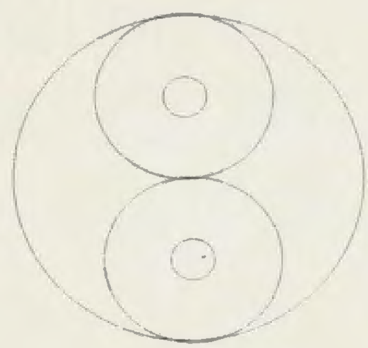
Further Investigation: Group Activity.

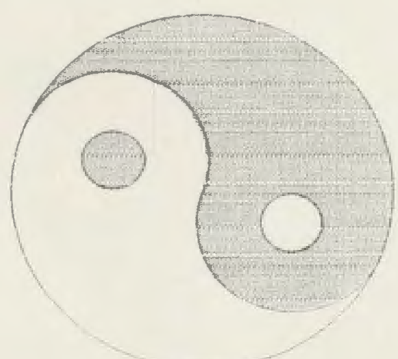
1. Investigate and state a conjecture about the quadrilateral formed by two tangent segments to a circle and the two radii to the points of tangency. Explain why you think your conjecture is true. Write your steps down.



2. State a conjecture for planes in space, tangent to a sphere. Make a sketch. Test your conjecture with physical objects and explain why you think your conjecture is true.

3. In Chinese philosophy, all things are divided into two natural principles, yin and yang. Yin represents the earth , characterized by darkness, cold, or wetness. Yang represents the heavens, characterized by light, heat, or dryness. The two principles combine to produce the harmony of nature. The symbol for yin and yang is shown at the right. Construct your own yin and yang symbol. Start with one large circle. Then construct two circles with half the diameter that are internally tangent to the large circle and externally tangent to each other. Finally, construct small circles that are concentric to the two inside circles. Devise a method to colour your symbol.

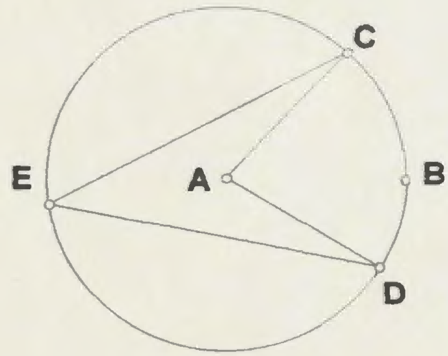




Discovering Circle and Angle Properties

Investigation 3.1 Inscribed Angles, Central Angles and Arcs

Construction



Step 1: Construct circle AB

Step 2: Construct points C and D on the circle. (Step 2, Construction 1.1)

Step 3: Construct radii AC and AD to create a central angle ∠CAD (Step 3, construction 1.1).

Step 4: Construct point E on the circle.

Step 5: Construct chords EC and ED. To create an inscribed angle ∠ CED.

Step 6. Save as " Invest 31"

Investigation

1. Measure ∠CAD, ∠CED, and arc angle CD.(Steps 2 and 4, Investigation 1.1)
2. Move point C or D. Do you see a relationship between the measure of the central angle ∠CAD , the inscribed angle ∠CED and the arc angle CD?
3. Move point E. What is happening? Why? Can you make any conjectures?

Conjecture Write your conjectures below.

Can you explain why your conjectures are true? Write your explanation down and discuss your results with your group(s).

Further Investigation

1. What if the measure of $\angle CAD$ was more than 180°? Move point C or D so that its measure is greater than 180°. What is happening to the measure of the arc CD? Why?

2. When an arc is greater than a semi-circle, it is called a ***major arc***. Can you make a conjecture about the measure of the Major Arc CD ? Measure Major Arc CD. (Hold the Shift key down and click points C and D, point E which is on the arc and the circle. Select the *measure menu* and *arc length.*) Was your conjecture correct? Why?

3. Experiment with inscribed angles on major arcs. Does your previous conjecture still hold true? Why or why not?

Conjectures: Write your findings down in the form of conjectures.

Can you explain why these conjectures are true? Write your explanation down and discuss your results with your group(s).

Discovering Circle and Angle Properties

Investigation 3.2 Inscribed Angles Intercepting the Same Arc.

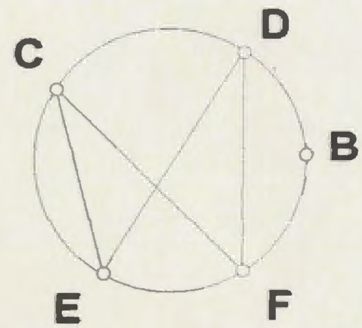
Construction

Step 1: Construct circle AB.

Step 2: Construct four points C, D, E, and F on the circle.

Step 3: Construct chords DF and DE.

Step 4: Construct chords CF and CE



Investigation

1. Measure ∠ECF and ∠ EDF. What did you notice?

2. Move point C around the circle. What is happening?

3.

Conjecture. Write a conjecture on your findings.

Can you explain why these conjectures are true? Write your explanation down and discuss your results with your group(s).

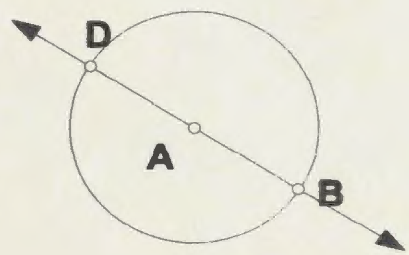
Discovering Circle and Angle Properties

Investigation 3.3 Inscribed Angles on the Diameter

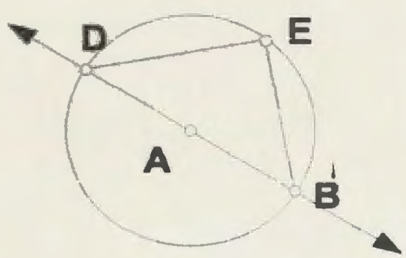
Construction

Step 1: Construct circle AB

Step 2: Construct line AB. (First select a line in the tool menu. Next, hold the Shift key down and click on points A and B. Select the *construct menu* and *line.*)



Step 3: Construct point D at the intersection of the circle and the line. (Hold the Shift key down and click both the circle and the line. Select the *construct menu* and *point at intersection.*)



Step 4: Construct point E on the circle.

Step 5: Construct Chords DE and BE.

Investigation

1. Measure ∠DEB.

2. Move point E around the circle. What have you discovered?

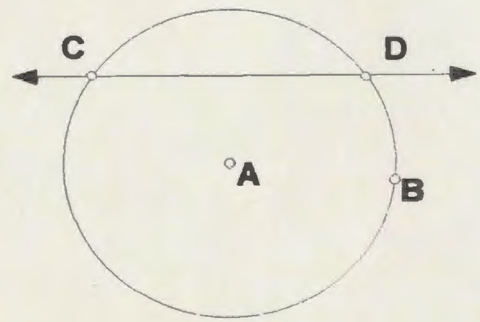
Conjecture. Write your findings in the form of a conjecture.

Can you explain why these conjectures are true? Write your explanation down and discuss your results with your group(s).

Discovering Circle and Angle Properties

Investigation 3.4 Parallel Lines and Intercepted Arcs.

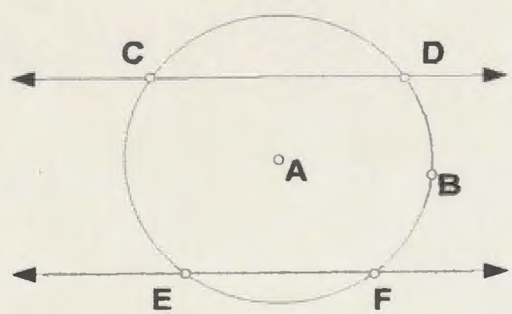
Construction



Step 1: Construct circle AB.

Step 2: Construct points C and D on the circle. (Step 2, Construction 1.1)

Step 3: Construct line CD. (Hold the Shift key down, and click on points C and D and select *line* from the *tool bar*. The select the *construction menu* and *line.*)



Step 4: Construct point E on the circle.

Step 5: Construct a line through point E parallel to line CD. (Hold the Shift key down and click on point E and the line CD. Then select *line* from the tool bar and select the *construct menu* and *parallel line*.)

Step 6: Construct point F as the point of intersection between this line and the circle. (Hold the Shift key down and click on both the line and the circle. Select the construction menu and point at intersection.)

Investigation

1. Measure arcs CE and DF. Move points C, D, E and B. What can you says about those arcs?

Discovering Circle and Angle Properties

Investigation 3.5 Cyclic Quadrilaterals

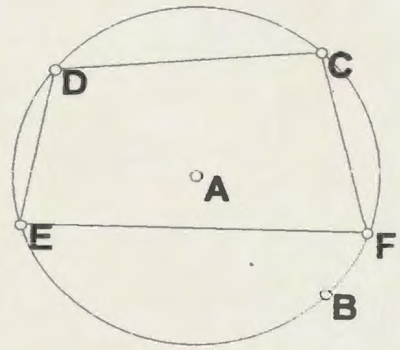
A quadrilateral whose vertices are all on the same circle is called a ***cyclic quadrilateral***.

Construction

Step 1: Construct circle AB.

Step 2: Construct four points C, D, E, and F on the circle.

Step 3: Join CD, DE, FE, and FC. (See diagram)(Step 3, Construction 1.1)

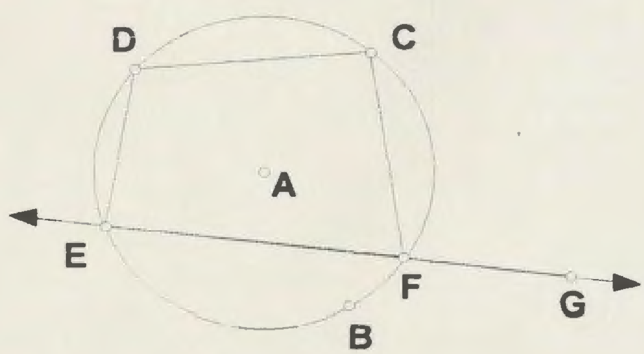


Investigation

1. Measure all four inscribed angles. What do you notice?
2. Move point D around the circle. What is happening? Do your observations still hold true?
3. Move point C around the circle? What is happening? Do your observations still hold true?
4. Experiment with other cyclic quadrilaterals.

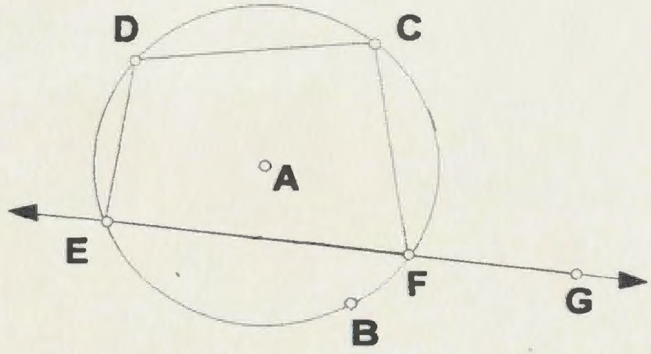
Further Construction

1. Construct a line through points E and F. (Investigation 2.1, Step 4) Construct a point on the right side of the line (see diagram) and label it G.



Further Investigation

1. Measure ∠CFG and ∠ CDE. What do you notice? Move around points C, D and F. Are your observations holding true?



2. Test your observations by constructing other secants and measuring external angles and corresponding interior angles. Do your observations still hold true?

Review investigations 3.4 and 3.5.

Conjectures: Write your findings in the form of conjectures

Can you explain why these conjectures are true? Write your explanation down and discuss your results with your group(s).

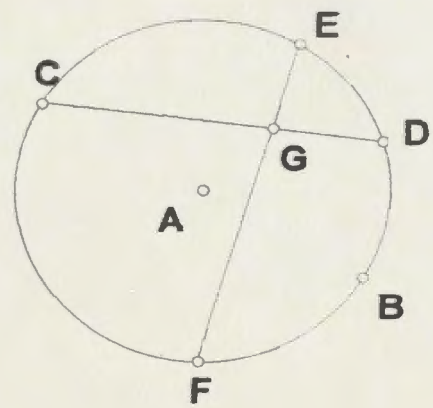
Assignment

1. Take each of your conjectures and write its converse. Are these conjectures true? Why or why not? Write your reasonings down and discuss each with your group(s).

Discovering Secant, Tangent and Chord Properties

Investigation 4.1 Chord Theorem

Construction



Step 1: Construct circle AB.

Step 2: Construct Chord CD on circle AB. (Construction 1.1, Steps 2 and 3)

Step 3: Construct Chord EF on circle AB so that it intersects chord CD.

Step 4: Construct point G as the intersection of these two chords. (Investigation 1.2, Step 4)

Investigation

1. Measure CG and EG. Measure GF and GD.
2. Now select the display menu and select preferences. Change the measure of distance to thousandths. Measure the lengths of CG, EG, GF and GD.
3. Perform the following calculations: $CG \cdot GD$ and $EG \cdot GF$. What do you notice?
4. Move point C or D. Does your observation still hold true?
5. Write a conjecture about your observations.

Conjecture:

Explain why you believe that this conjecture is true. Write your reasoning down and discuss with your group(s).

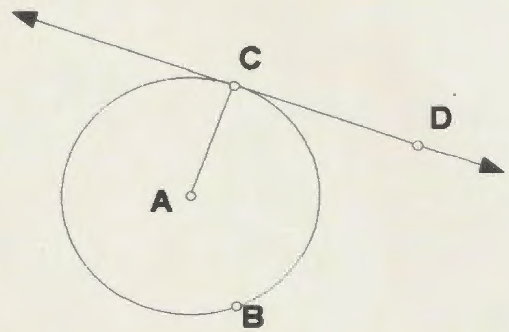
Assignment

1. Use *Geometer's Sketchpad* to show that $CG \cdot GD = EG \cdot GF$ (Hint: Use similar triangles and angles subtended on the same arc.) Write your steps down and discuss with your group(s).

Discovering Secant, Tangent and Chord Properties

Investigation 4.2 Tangent Secant Theorem

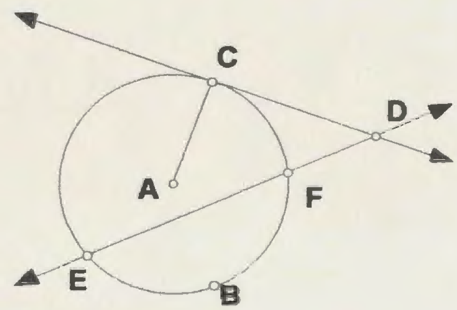
Construction



Step 1: Construct circle AB.

Step 2: Construct a tangent to the circle and label the point of tangency C. (Investigation 2.2, Step 2.)

Step 3: Construct an external point on the tangent and label it D. (Click on the line and then select the *construct menu* and *point on object*. Move the point slightly to the right on the line. See diagram)



Step 4: Construct a point E on the bottom left hand side of the circle.

Step 5: Construct a line through points D and E. (Investigation 2.1, Step 4)

Step 6: Construct the other point of intersection of that line. And label it F. Note: You have constructed a secant EF.

Investigation

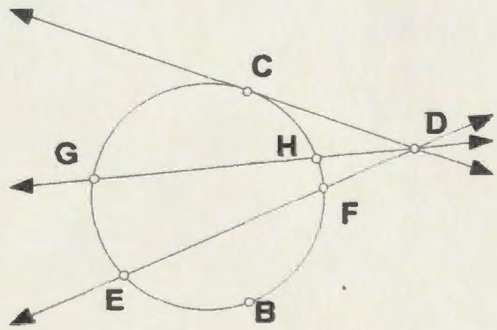
1. Measure segment CD

2. Measure segments ED and FD. Multiply these two measurements. How does the result compare to the measure in #1? (Hint CD^2)

3. Move point C or D around the circle. Does your observation still hold true?

4. Write a conjecture about your observations.

Further Investigation



1. Construct another secant through the circle at point D. (Steps 4, 5 and 6 above) Label the secant GH. See diagram.

2. Measure DG and DH. Calculate DE · DF and DG · DH. What have you discovered?

3. Move point F or H. Does your observation still hold true?

4. Write a conjecture about your observations.

Conjecture

Can you explain why your conjectures are true? Write your explanations down and discuss with your group(s).

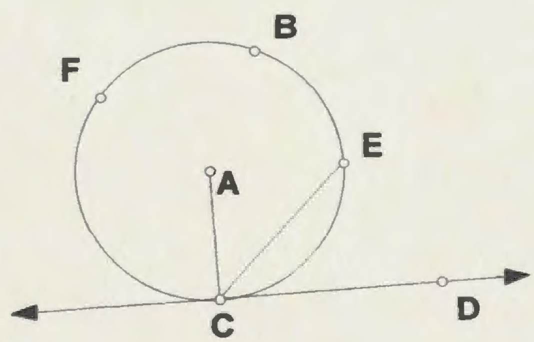
Discovering Secant, Tangent and Chord Properties

Investigation 4.3 Tangent Chord Property

Construction

Step 1: Construct circle AB.

Step 2: Construct a tangent to circle AB at a point C. (Investigation 2.2, Steps 1 and 2.) Label the tangent CD.



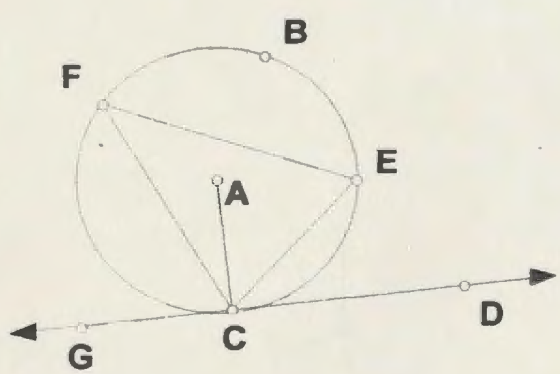
Step 3: Construct a point E on the right side of the circle (see diagram).

Step 4: Construct chord BE. (Investigation 1.1, Step 3)

Step 5: Construct a point F on the left hand side of the circle (see diagram).

Step 6: Construct chords FC and FE.

Investigation



1. Measure ∠ ECD and ∠EFC. What do you notice?
2. Move point E. Does your observations still hold true when ∠ECD is a right angle? An obtuse angle?
3. Construct a point G to the left of point C on the tangent.
4. Measure ∠FCG and ∠FEC. Do these measurements support your observations?

5. State your observation in the form of a conjecture.

Conjecture

Can you explain why these conjectures are true? Write your explanation down and discuss your results with your group(s)

Review all your conjectures and explanations and comprise a list. Compare your list with other groups. Do you all have the same conjectures? Do you all agree?

Project: Constructing a Sketchpad Kaleidoscope

Note: Each step must be completed before proceeding to the next step.

Step 1: Open a new sketch and construct a many-sided polygon. (Select the *file menu* and *new sketch*. Then use the *segment (line) tool* to construct a polygon with many sides . Make it long and slender.) See *figure 1* on the right.

Figure 1

Step 2: Construct several polygon interiors with your polygon. Shade them different colors.

a. Make sure all objects are deselected by selecting the *arrow tool* and clicking in any blank space.

b. Hold the shift key down and select three or four points in clockwise or counterclockwise order.

c. Go to the *construct menu* and select *polygon interior*. Then select the *display menu* and select a shade and/or color for your polygon interior.

d. Deselect objects by clicking in any blank space. Repeat steps b, c, and d until you polygon has several different polygon interiors with different colors or shades. See *figure 2* on the right.

Figure 2

Step 3: Mark the bottom vertex point (point A in figure 2) as the center. Hide the points and rotate the polygon by an angle of 60°.

a. Deselect objects by clicking on any blank space.

b. Select the bottom vertex point and then select the *transform menu* and *mark center.*

c. Click on the *point tool* and select the *edit menu* and *select all points*. Then select the *display menu* and chose *hide points.*

d. Click on the arrow tool and drag until a box appears around your figure and select. Then select the *transform menu* and *rotate.* (Note make sure that the figure is the only object inside the box.) Then type 60° and select OK. (You may pick a different factor of 360 if you wish)

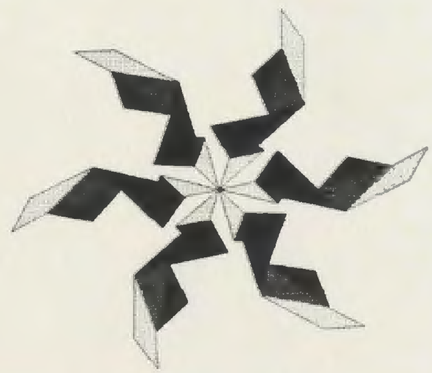


Figure 3

e. Continue this rotation (steps d and e) until you have completed your kaleidoscope.

Step 4: Construct circles with their centers at the center of your kaleidoscope.

a. Select the *display menu* and *show all hidden.*

b. You should see the points on the original arm reappear.

c. Deselect objects by clicking in any blank space.

d. Select the *circle tool* and then click on the center point of your kaleidoscope. Drag a circle with a radius a little larger than the outside edge of your kaleidoscope.

e. Using the *circle tool* again, click on the center point and drag a circle approximately one half the radius of the other circle.

f. Repeat this for a circle with a radius approximately one-third the radius of your kaleidoscope. See figure 4.

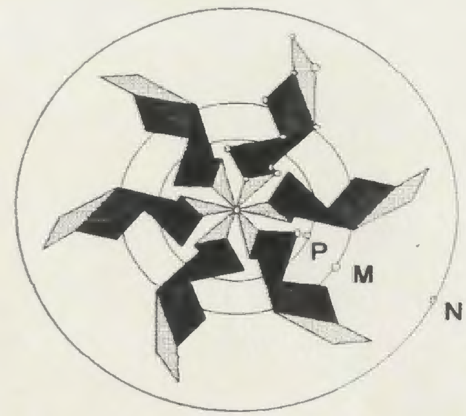


Figure 4

*Note: Make sure you release your mouse point in a blank space between two arms of your kaleidoscope. Look at points P, M, and N in figure 4. You do not want the outside control points of your circles to be constructed on any part of your kaleidoscope.

Step 6: Animate points of your kaleidoscope on the three circles.

a. Select the *selection arrow tool* and deselect objects by clicking in any blank space.

b. Hold the Shift Key down. Select one point on the original polygon near the outside circle and select the outside circle. Do not click on one of the control points of the circles- Points P, M or N.

c. While you continue to hold the Shift Key down, select a point near the middle circle and then select the middle circle. Select a point near the smallest circle and select the smallest circle.

d. Select the edit menu, select Action button and drag to the right and choose animation. Click on Animate on the Animate dialog box.

e. When the button appears, double click on it to start the animation.. Watch your kaleidoscope turn!

f. Deselect objects by clicking in any blank space. To hide all points, click on the *points tool*. Go to the *display menu* and choose *hide points*. Click on the *circle tool* and select *all the circle*s and *hide circle*. To hide labels P, M, and N, select each label while holding down the Shift Key and then select the *display menu* and *hide captions*.

(Key Curriculum Press, 1998, 182-185)

Investigation

1. Describe the effect that the animated circles have on the kaleidoscope? Does the size of each circle have any effect? Explain.

2. What would happen if you only had two animated circles? What would happen if you had more than three? Test your theories out.

3. Create your own kaleidoscope and get your partner to recreate it. What assumptions or theories did you use in its creation.

Student Reaction

Note: This unit was presented to my class of 18 students in five class periods at the end of this school year. The Technology Education Teacher graciously gave up his lab for one week. Because of this time constraint, only the first topic was completed.

The overall reaction of the students was positive. Most students were very excited about going to the lab for Math class. The first day was spent introducing the software to the students. The second day, the students started the first topic, 'Discovering Chord Properties'. Two of the students were absent from the first class, so they were not familiar with the software. However because they were working in groups of two, each of their partners explained her steps as she went through the exercises. Students had little difficulty working through Investigation 1.1 and showed enthusiasm as they were manipulating their diagrams. All groups discovered the same conjectures:

1. If two chords in a circle are congruent, then they determine two central angles that are congruent.
2. If two chords in a circle are congruent, then their arcs are congruent.

3. Congruent chords are equidistance from the centre of the circle.

The remaining three days were spent on either Investigation 1.2: Chords and Perpendicular Lines or Investigation 1.3: Chords, Midpoints and Perpendicular Lines.

Observations

I noticed immediately that the students were completely involved in the investigations. There were discussions going on all around me about their diagrams. A few times I had to intercede because the conversations became too animated. Another surprise was that one particular student, who seemed bored all year, now suddenly was going around helping others and discussing ideas intelligently. Obviously he enjoyed being in the lab working through math investigations rather than being in the classroom. A few other students showed more interest than ever before and were completely involved, even in the discussions at the end of each class. Two of the female students were not overly excited about working through the investigations but these were the students who missed two of the five classes. As I reflected at the end of each class, I realized that I had a much more eager and vocal class. They could not wait to get to the lab to do their constructions and bragged to other students about the program that they were using. One student told another Math teacher that it was awesome!

Towards the end of the week, I had Advanced Math students coming to me and asking me to take them down to the lab to do this topic. (Both academic and advanced students were doing this topic at the same time.) Obviously I was doing something right in the students' minds because I had many enthusiastic students for the first time.

The students were not formally tested on this topic because of lack of time. However on the final exam, all but two students (the absentees) receive full marks on the questions concerning this topic. I discussed this fact with the student who showed little interest until now and he said, "Those questions were easy . I didn't have to learn them 'cause I already knew 'em!" He only knew them because of the activities in the lab! There are many factors that could have influenced this successful outcome: variation of delivery, collaboration, method of reflection, ease of the topic etc. The bottom line was that all students knew these conjectures through their own constructions, manipulations and discussions.

As I was looking through the students' notebooks I was surprised at the efforts of some students explaining "why?" Many had produced good logical arguments for their conjectures. One group, for example, argued that since all radii of the one circle are equal and given that the chords are congruent, then two congruent triangles are formed using the SSS Postulate. If that is the case, then the central angles must be equal because they are corresponding angles. One students added that further manipulation of one or more points would not alter their findings. I asked them about these justifications and they said that they felt the need to explain why because they knew from manipulating their constructions that their conjectures were true. They felt challenged (Galindo, 1998). Many of the groups used this argument of congruent triangles which was very surprising since the topic of congruency had not been discussed in detail since the previous year. Others used written arguments describing what they saw during the manipulations of their constructions. One student, for example, when explaining that congruent chords cut at congruent arcs, reasoned that when he placed one chord over the other, the two arcs were exactly the same measure. However, when these students became part of the larger group discussion at the end of each class, they opted to write the written formal proof. However, when confronted with the justifications of others they realized that the formal written proof was a better justification. This decision was left entirely up to each individual student. The above findings support the literature on collaboration and reflective inquiry (e.g., Leger, 1991; Silver, 1994; Bruner, 1987; Wheatley, 1992; and Parker, 1991).

Teacher Reaction

This unit was presented to a class of Education Graduate Students during Summer Session. All were Math teachers but from different school levels. The overall reaction to the unit was positive. One teacher commented that he was really 'turned on' to *Geometer's Sketchpad* now that he had an opportunity to work through a few exercises. He also commented that he did not even know that this type of software existed and he could not wait to implement it in his classes this fall. He believed that his students would really enjoy this discovery method using technology and he would enjoy it as well. The unit was also seen by another teacher as providing a different means of presenting a topic rather than the chalkboard. According to this teacher, it provided a means of letting the students discover for themselves. A few teachers commented on how the questions were asked, e.g., "What do you notice?". It was agreed that this kind of question would acknowledge the students mathematics. By asking "What do you notice?" students are encouraged to give meaning to their experiences in a way that makes sense to them. One teacher remarked that children modify their strategies by listening to others explain their justifications. According to this teacher, students are forced to reflect on their own solutions to see if modifications are necessary. This was supported by my class. Also, it was suggested that this method of asking questions could improve problem-solving skills and other forms of reasoning as students reflect on their own justification. These comments support the literature on collaboration and reflective inquiry (e.g., Bruner, 1987; Simon, 1995; and Hatfield and Bitter, 1991).

A final comment was that as teachers experimented with this unit, it opened up ideas to use Geometer's Sketchpad in other ways. Overall teachers were impressed with the software and its capabilities and the unit. Most agreed that they would try to implement it in their courses next fall. That was the intention behind the design of this unit.

Author's note

Many issues have come up during the design of this unit (e.g., dynamic software verses other methods of delivery; slow students verses average or above average students and group work verses individual work) that I have not answered or addressed. Hopefully, this project will serve as an incentive for others to develop their own projects to research student learning in geometry, in particular using dynamic software.

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