CRITICAL REFLECTIVE THINKING IN EUCLIDEAN GEOMETRY: FOR GRADE NINE MATHEMATICS STUDENTS

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CRITICAL REFLECTIVE THINKING IN EUCLIDEAN GEOMETRY:

FOR GRADE NINE MATHEMATICS STUDENTS

by

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Abstract

The vision of the mathematics classroom that is presented in the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (1989) has inspired many of us to want to change the way in which we teach. Indeed, the NCTM has said that many of the mathematics classrooms need to change as transmission models of teaching no longer fit what we know about the teaching and learning process. Learning is no longer viewed as a passive exercise and many have come to believe that students must think, be mentally active, generate meaning, and construct their own understanding.

This however, is not a sudden realization as Dewey (1933) and Polya (1957), for example, have both recognized the power of reflection and the need for purposeful critical thinking. Polya (1987) noted that as teachers what we need to teach is purposeful thinking (Dossey, 1988). Polya (1987) also claims that such thinking can be identified with problem solving (Taback, 1992). John Dossey, the President of the NCTM, has stated that the knowledge, skills, attitudes and purposeful thinking discussed in Polya's publications of 1945, 1954 and 1962 have provided the foundation for their work as a Council (Dossey, 1988). Thus, this change called for by the NCTM is not so much of a new initiative in theory as it is in practice.

Likewise, this project is grounded in similar theory and signifies a shift away from the transmissive practices of the past. It signifies a shift towards a more transformative way of thinking and away from textbook learning, rote memorization and the mastery of traditional school subjects through traditional methodologies. This

project is not a research paper but is instead an alternate unit of Euclidean Geometry which requires an alternate teaching style. It is something that other teachers can lay hands on and use in their classroom as a means of promoting critical reflection and higher order thinking skills. While this unit may represent a transition in approach it does reflect the changes called for -- and the standards of -- the NCTM (1989) and the Intermediate Mathematics Curriculum Guide (1995). Thus, it is the general purpose of this unit to implement those changes in an environment of cooperative learning. The unit will introduce students to the dynamics of group structure, encourage students to reflect and employ a problem centred discovery approach to mathematics.

As Vermette (1994) states, "cooperative learning is a powerful and engaging strategy worthy of thoughtful implementation by all high school teachers" (p.38). Conard (1988) goes even further and suggests that cooperative learning methods are important not only for academic achievement but also for our survival as human beings in a complex world. Indeed, several cooperative learning models have been developed for use as teaching tools however, the model employed in this unit resembles what Sharan and Sharan (1990) describe as group investigation.

The unit is based on critical reflection, problem solving and communication and thus literature reviews are provided for each. There is also a literature review on the process of change as this unit does represent a change for both student and teacher. Chapter Three, titled Methodology, provides a brief description and analysis of the unit whereas Chapter Four is the complete unit itself. The project then concludes with reflections on my experiences with the unit and suggestions for prospective users.

Dedicated to ...

the loving memory of my grandfather

Harold Stanley Morgan

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First, I would like to express sincere gratitude to my advisor, Dr. David Reid, for his time effort and unfailing commitment to this project.

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Preface

As a mathematics teacher, I had come to realize that many mathematics students seemed to lack the skills of critical reflection and higher order thinking. My teaching experience coupled with my reading as a graduate student are the sources of my inspiration and initiative, and it is from there that this project and unit of Critical Reflective Thinking in Euclidean Geometry for grade nine students has come. This unit of geometry represents one means of meeting the challenge of promoting critical reflection in the classroom and as such is offered for other teachers to use and to reflect upon.

The project itself consists of five chapters. Chapter One provides the historical context, purpose, and significance of the unit itself as it relates to the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (1989) and the Intermediate Mathematics Curriculum Guide (1995). Chapter Two consists of literature reviews on critical reflection, problem solving and journal writing as these are the concepts which provide the foundation for the unit. Chapter Two also provides a literature review on the change process as this unit does represent a change for both the student and the teacher. Chapter Three, titled Methodology, provides a brief description and analysis of the unit whereas Chapter Four is the complete unit itself, with introduction, purpose, objectives and significance, provided in a form so that it may be easily reproduced and used by other educators in the classroom. The project then concludes with reflections on my experiences with the unit and suggestions for prospective users.

Chapter 1: Introduction

This project is offered as something that other teachers can utilize in their classroom as a means of promoting higher order thinking skills, critical thinking, reflection and greater mathematical understanding. This project is not a research paper, but is instead an alternate unit of mathematics which requires an alternate teaching style as called for by the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (1989). The unit is supported by literature reviews and followed up with my personal reflections. I analyze the unit and my experiences when implementing this unit with reference to current theory and research regarding reflection, critical thinking, problem solving and journal writing.

Wheatley (1992) professes that reflection plays a critically important role in mathematics learning and that encouraging reflection results in greater mathematics achievement. Powell and Ramnauth (1992) claim that learning does not occur from experiences alone and that learners must reflect on their experiences. Krulik and Rudnick (1994) have noted that there has been an increased emphasis placed on reasoning skills and that these higher order thinking skills can be achieved by expanding the "reflect" category which is the final step in the heuristic method.

Peixotto (1993) notes that "what's wrong with education is that too much of it is not education, it is instruction ... and that instruction considers the mind a vessel of certain capacity into which a collection of approved items should be fitted by experts" (p. 1). Krulik and Rudnick (1994) making note of the same problem suggest that teachers try reflecting in their classrooms. The concept of reflective thinking is a reaction to the

overly technical and simplistic view of teaching that dominated the 1980's and a recognition that teaching is a complex, situation specific and dilemma ridden endeavour (Sparks-Langer & Colton, 1991). It signifies a shift away from the transmissive position towards a new transformative way of thinking. This is a shift away from textbook learning, rote memorization and the mastery of traditional school subjects through traditional methodologies towards a more holistic approach where the curriculum and the student are seen to interpenetrate each other (Miller &Seller, 1990). Wheatley (1992) goes on to explain that "there are serious limitations in the explain-practice method of instruction and active learning and that even performing self generated mathematical operations does not have the power which results from reflecting on the activity ... reflective abstraction is central to the theory of constructivism" (p. 529).

Certainly, there is a need for improved teaching practices as many of us have experienced mathematics as a collection of rules, rituals and routines. Berenson and Carter (1995) state that learning has been equated with producing an exact copy of what is in the teachers's mind and that such learning fails to produce life long learners. There needs to be a change and a shift towards classrooms as mathematical communities where logic, reasoning and problem solving are emphasized. There needs to be a shift towards classrooms where teachers and students engage in and reflect on mathematics for themselves. Where critical thinking is the norm for both students and teachers alike. Polya (1987) has noted that as teachers what we need to teach is purposeful thinking and

that this would require a shift in and a broadening of our traditional goals (Dossey, 1988). Litecky (1992) making note of the same problem emphasizes the importance of and the relationship between, critical thinking, higher order thinking skills and increased student comprehension. Certainly, there can be no doubt that what is needed is a shift from the transmissive to a more transformative way of teaching. As Andrews and Huffman-Joley (1991) say, "there is a need for a holistic curriculum which highlights the students' own learning for we can no longer view students as empty vessels to be filled" (p. 82).

Clarke, Waywood and Stephens (1993) have made the observation that the NCTM (1989) has listed problem solving and communication as their first two standards. They go on to state that these standards will facilitate construction and sharing of mathematical meaning and promote student reflection. Scott, Williams and Hyslip (1992) have concluded that if mathematics is to be taught as communication and reasoning, standards two and three of the NCTM (1989), as opposed to rules and rote memorization, then students must be provided with experiences that will facilitate growth in their mathematical understanding. Friel (1993) summarizes the changes in the learning environment as proposed by the NCTM when she notes that there needs to be a shift in classroom environments towards: classrooms as mathematical communities, logic and mathematical evidence as verification, mathematical reasoning and problem solving.

As I reflected on my students, my classroom and my teaching I came to realize that many of my methods as a mathematics teacher were transmissive and that explain

practice and rote memorization were a real part of my repertoire as a teacher. I came to realize that I, my classroom and my students needed a change. We needed to move towards a more transformative ideal of teaching and learning. We needed to move away from text centred instruction and rote memorization towards a more active learning environment where students constructed their own mathematical meaning and where both the teacher and the student were critical and reflective thinkers.

This realization was indeed a painful admission, but was a conclusion that I came to when I began to ponder on the difficulties that many students were having in their transition from high school to post secondary institutions. This coupled with poor performances in mathematics competitions, which were problem oriented, and low test scores in critical thinking categories on CTBS led me to believe that it was indeed a time for change.

Real change is gradual and takes time thus, while I did change some teaching practices in the high school my major initiative was in grade nine. The Euclidean Geometry Unit that I designed as a result was the beginning of that initiative. The unit is based on critical reflection, problem solving and communication. I believe that it provides the student with those experiences needed to facilitate growth in mathematical understanding. It requires the student to reflect on their experiences in the classroom, think critically about their reasoning and construct their own mathematical understanding. The unit is to be carried out according to the theory of cooperative learning and would

also require full class discussion and journal writing. Indeed, I believe that this unit represents some of that change called for by the NCTM (1989). "We must acknowledge our own and others' excuse for maintaining the status quo" (Andrews & Huffman-Joley, 1991, p. 82). This project represents a move away from that status quo.

Historical Context

What is learning and how do people learn? These questions and their possible answers have been the source of much philosophical debate through the ages. Plato, Aristotle and John Locke pondered over these questions and yet we are still debating these issues today. Indeed, human learning has been puzzling psychologists and educators for many years. Previously associationism or behavioursim dominated the thinking of psychologists. Now, however, there is a more cognitive view of learning. Learning is no longer viewed as a passive exercise. According to Wittrock, for example, learning involves mental activity. This means that students must think, be mentally active, generate meaning, construct their own understanding and relate new ideas to each other and to prior knowledge (Seifert, 1995). Thus, the role of the teacher now is to help the students to do just that. In doing this however, the teacher will now need to: be aware of the importance of prior knowledge, understand the encoding process, teach and demonstrate learning strategies to students and help students become metacognitive as well as self instructive. Indeed, this way of thinking is revolutionary in comparison to the

associationism or behaviourist ideals that dominated much of the thinking in the twentieth century.

The cognitive view of learning suggests that learners need to be mentally active and that they need to organize, elaborate and build relationships between new ideas and prior knowledge. It further suggests that they need to regulate and control their thinking and engage in self instruction which will teach them task specific strategies as well as a general self control strategy (Seifert, 1995). The associationism view of learning basically was the explanation of the responses made to stimuli. If a person was given approval for a response then the association between the stimuli and response was strengthened. For example, if the teacher asked a question such as "On what continent is Canada located?", and the student replied "North America" then the teacher would acknowledge the correct answer and praise the child. The approval of the teacher would serve to strengthen the relationship between Canada and North America. On the other hand, if the child had given an incorrect answer and the teacher chastised the student, then the chastisement should eliminate any connection between Canada and the incorrect response.

In the period from 1930 to World War II laboratory research was dominant. The behaviourist theories of this period were referred to as S-R theories because they described learning as a link between a stimulus and a response (Gredler, 1992). Clark Hull's Theory, called the hypothetico deductive system, was based on the belief that

behaviour was focused on satisfying certain needs. That is, all our behaviours are controlled by certain bodily and social needs. Although Hull's theory was dominant during the 1940's, B.F. Skinner's principles gained popularity in the next decade. Skinner introduced ideas that were also refinements to the associationism theory. He suggested that behaviour, consisting of stimulus and response, was shaped by the environment and that learning was a gradual process (Seifert, 1995).

As time moved on, philosophical and empirical problems developed with the associationism view of learning and behaviour. This theory reduced humans to the level of animals or biological machines where their behaviour was simply a function of their surrounding environment. In this view of learning the learner's thoughts and beliefs were omitted. Self determination and free choice played no part in explaining behaviour. Yet, Bandura noted that a person's beliefs about him/herself was an important determinant of behaviour, in addition to environmental factors. Evidence fails to bear out the data and conclusions of this behaviourist theory (Seifert, 1995).

By the latter part of the twentieth century many other researchers began to consider the importance of the learner's thoughts and beliefs -- or their mental activity. Wittrock, for example, claimed that learning involved mental activity and that in order to learn students must think, be mentally active and generate meaning. He then goes on to say that:

> The brain is not a passive consumer of information ... the stored memories and information processing strategies of our cognitive system interact with the sensory information received from the environment, selectively attend to this information, relate it to memory and actively construct meaning for it.
>
> (as cited in Gredler, 1992, p. 171)

Thus, according to Wittrock learning occurs only when students generate meaning, construct their own understanding and relate new ideas to each other and to prior knowledge. This view of learning would have serious implications for the role of teacher and thus their perceptions of students.

Purpose and Objectives of the Unit

The attitudes which students hold toward mathematics have been shown to correlate with mathematics achievement and most teachers realize the importance of how students feel about the subject. Most teachers also realize that they should help students feel free to explore a variety of ideas in reaching solutions and verifying their own thinking. Confidence in mathematics is fostered by encouraging students to explore mathematical concepts, ask questions, discuss their ideas and make mistakes. Students who enjoy mathematics and work in a constructive environment feel comfortable and confident and in turn perform better and learn more. This discovery approach to learning

is also well suited to the small group structure. Students learn mathematics by doing mathematics. With proper (subtle) guidance by the teacher, students can discover relationships, plan procedures, formulate definitions, make conjectures, construct examples and counterexamples, and solve problems. By listening to students' ideas and by having them listen to one another, the teacher can establish an attitude of mutual respect (Intermediate Mathematics Curriculum Guide, 1995).

This unit on Euclidean geometry introduces students to the dynamics of group structures, encourages students to listen to each other and most importantly employs such a discovery approach to mathematics. Students explore a variety of ideas in a small group setting and construct their own meaning for the mathematical problems presented. Students are then asked to reflect on this new found knowledge and record their thoughts in a journal. It is imperative that the students be encouraged to reflect and record their thoughts. For thinking to grow, the important measure is not the number of questions done, but the quality of thought put into tackling the questions and into reviewing the effort (Mason, Burton & Stacey, 1996, p. 150).

While this unit may represent a transition in approach it does reflect the changes called for -- and the standards of -- the NCTM (1989) and the Intermediate Mathematics Curriculum Guide (1995). Thus, it is the general purpose of this unit to implement these changes and to foster critical thinking, reflection and higher order thinking skills in mathematics. The specific objectives or intended learning outcomes however, would be

the same as those found in the related curriculum guide for this unit does not represent such a departure from content as it does from technique, approach or teaching style. Thus, the specific learning outcomes or objectives for the unit would be as follows:

1. → Understand the concepts of point, ray, line, line segment and angle.
   → Identify / name correctly, a point, line, ray, line segment and angle.
2. → Identify / classify complementary and supplementary angles.
   → Understand and apply the equality of opposite angles.
   → Calculate the measures of unknown angles.
3. → Classify triangles by sides.
   → Classify triangles by angle measures.
   → Find the missing values of a triangle, given certain information.
   → Understand that the sum of the angles in a triangle is 180°.
   → Understand the relationship between exterior and interior angles .
4. → Identify parallel lines, alternate, corresponding and interior angles.
   → Use parallel line, transversal relationship to find angle measures.
5. → Identify polygons and regular polygons.
   → Classify polygons by the number of sides.
   → Calculate the measures of interior angles of a polygon.
6. → Describe the properties conditions for congruent triangles.
7. → Derive information about triangles from congruency relationships.

8. → Derive new information from angle properties. Including vertically opposite angles, corresponding angles, alternate angles, angle bisector, perpendicular bisector, etc.
9. → Recognize various properties of segments and angles constructed within a circle. These properties include: a) The relationship between central and inscribed angle. b) The relationship between two inscribed angles drawn on the same arc. c) The size of a inscribed angle subtended by a diameter. d) The relationship between opposite angles of an inscribed quadrilateral. e) The relationship between chord length and distance from the centre. f) The use of right bisectors to determine centres of circles. g) The relationship between a chord and a perpendicular that passes through the centre of a circle.
10. → Integrate the various properties of segments and angles within circles to find missing measures.
11. → Recognize the properties of similar figures.
12. → Apply the properties of similar figures.
13. → Solve problems using Pythagorean relationship.

(Intermediate Mathematics Curriculum Guide, 1995)

Significance of the Unit

When Dossey (1988) was writing about the work of Polya he noted that Polya (1987) claimed that as teachers we should be teaching purposeful thinking which would require a shift or move away from traditional goals and methods. Polya (1987) also claimed that teaching and learning should be driven by three basic principles. The principle of activity which would mean that mathematical understanding would be developed as a personal construct and developed through involvement. The principle of motivation which would mean that learning was the result of performing an action that calls for reflection and perseverance as opposed to repetitive drill and finally, the principle of consecutive phases with those phases being exploration, assimilation and construction. This unit of Euclidean Geometry teaches that purposeful thinking. Mathematical understanding is developed as a personal construct and developed through involvement. Students are given a problem, an action to perform, and are required to: explore, assimilate and construct their own mathematical meaning, understanding and relationships through personal reflection.

Furthermore, the unit is based on critical reflection, problem solving and communication. It provides the student with those experiences needed to facilitate growth in mathematical understanding. It requires the student to reflect on their experiences in the classroom, think critically about their reasoning and construct their own mathematical understanding. This is a shift away from textbook learning, rote

memorization and the mastery of traditional school subjects through traditional methodologies towards a more holistic approach.

While this unit may represent a transition in approach it does reflect the changes called for -- and the standards of -- the NCTM (1989) and the Intermediate Mathematics Curriculum Guide (1995) as listed and outlined below.

Summary of Changes in Instructional Practices in 9-12 Mathematics

- The active involvement of students in constructing and applying mathematical ideas.
- Problem solving as a means as well as a goal of instruction.
- Effective questioning techniques that promote student interaction.
- The use of a variety of instructional formats (small groups, individual explorations, peer instruction, whole class discussions, project work).
- The use of calculators and computers as tools for learning.
- Student communication of mathematical ideas orally and in writing.
- The establishment and application of the interrelatedness of mathematical topics.

- The systematic maintenance of student learnings and embedding review in the context of new topics and problem situations.
- The assessment of learning as an integral part of instruction.

(NCTM, 1989, p. 129)

Standard 1: Mathematics as Problem Solving

In grades 9-12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can --

- use, with increasing confidence, problem solving approaches to investigate and understand mathematical context;
- apply integrated mathematical problem solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems from situations within and outside mathematics;
- apply the process of mathematical modelling to real world problem situations.

(NCTM, 1989, p. 137)

Standard 2: Mathematics as Communication

In grades 9-12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can --

- reflect upon and clarify their thinking about mathematical ideas and relationships;
- formulate mathematical definitions and express generalizations discovered through investigations;
- express mathematical ideas orally and in writing;
- read written presentations of mathematics with understanding;
- ask clarifying and extending questions related to mathematics they have read or heard about;
- appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.

(NCTM, 1989, p. 140)

Standard 3: Mathematics as Reasoning

In grades 9-12, the mathematical curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can --

- make and test conjectures;
- formulate counterexamples;
- follow logical arguments;
- judge the validity of arguments;
- construct simple valid arguments; and so that, in addition, college intending students can --
- construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

(NCTM, 1989, p.143)

Standard 7: Geometry from a Synthetic Perspective

In grades 9-12, the mathematics curriculum should include the continued study of the geometry of two and three dimensions so that all students can --

- interpret and draw three dimensional objects;
- represent problem situations with geometric models and apply properties of figures;
- classify figures in terms of congruence and similarity and apply these relationships;

- deduce properties of, and relationships between, figures from given assumptions; and so that, in addition, college intending students can --
- develop an understanding of an axiomatic system through investigating and comparing various geometries.

(NCTM, 1989, p. 157)

Problem Solving

The ability to solve problems is the principle reason for studying mathematics. A true problem involves situations or events for which there is no immediate or obvious solution. If a student knows the solution to a problem automatically, it cannot be considered a true problem. Likewise, whether a given situation represents a true problem is determined by the previous experience of the student. That which is a problem for one student may be 'routine' for another. In problem solving it is sometimes the case that more than one solution is possible or, indeed, no answer is found. Some problems require considerable time to deliberate the possibilities. Teachers should take deliberation time into account when assigning problems to students.

At the intermediate level students should be exposed to a number of problem situations which extend the strategies that were developed in the elementary grades. Strategies previously developed are revisited at an increased level of sophistication and new strategies are introduced. The focus of instruction, therefore, should be on the strategies required to facilitate successful problem solving. There are two types of problems, nonroutine (process) problems and routine (application) problems. At the intermediate level specific time and attention should be given to nonroutine problems. Routine problem solving is derived from the application of concepts and skills while utilizing the strategies developed with nonroutine problems.

(Intermediate Mathematics Curriculum Guide, 1995, p. 14)

Communication

The communication process of speaking, reading, writing and listening are important to the teaching and learning of mathematics. Talking about mathematics provides students with opportunities to clarify their thinking and is critical in the development and understanding of mathematical concepts. Students working in small groups can learn to solve problems by discussing them. When students write about these

problems and their approach to problems they will develop a greater understanding of the mathematical concepts involved. This writing forces students to reflect and it is this reflection that reconfirms their understanding. Open ended problems provide these opportunities for writing and greater understanding of mathematics.

(Intermediate Mathematics Curriculum Guide, 1995, p. 187)

Critical Thinking

An inherent part of the learning process is developing a knowledge of oneself as a learner and the self regulation of one's own learning activity. A major goal of all education is that students become self regulated learners, thereby seeing themselves as problem solvers. Students should be conscious of how they solve problems and have the ability to monitor their own thinking.

(Intermediate Mathematics Curriculum Guide, 1995, p. 9)

Cooperative Small Group Learnings

Provision of varied organizational structures increases the likelihood of meeting the developmental needs of intermediate students and prepares them to deal with and respond to the increasing demand for

cooperation, involvement, and individual responsibility in an information age. Peer group relations are very important for intermediate students. In cooperative learning structures, students work together and share responsibility for each other's learning through sharing of goals, resources, and rewards. Students increase their understanding of others' perspectives and learn to accommodate themselves to the perspectives of others. This becomes especially apparent when students are involved in problem solving activities. They are exposed to the thinking of others and through that exposure recognize that there are often many strategies which can be applied to solve a given problem.

Cooperative learning structures help students develop more positive attitudes toward themselves and others, as well as the whole process of teaching, and learning. Low risk situations allow students to contribute according to their different capabilities. They feel accepted and supported by peers and confident in their ability to contribute to the group effort.

Cooperative learning has also been found to be a powerful tool for shaping socio-academic behaviour. Students help each other learn as ideas are communicated in terms they understand. Through discussion with peers, students are exposed to ideas which may extend and augment their

> own understanding. Higher level reasoning, divergent thinking, and problem solving skills will result as students clarify their own thinking.
>
> (Intermediate Mathematics Curriculum Guide, 1995, p. 174)

Summary

As I reflected on my students, my classroom and my teaching I came to realize that many of my methods as a mathematics teacher were transmissive and that explain practice and rote memorization were a real part of my repertoire. I came to realize that I, my classroom and my students needed a change. Indeed, many teachers may see the need for improved teaching practices as many of us have experienced mathematics as a collection of rules, rituals and routines. As Berenson and Carter (1995) note, traditionally learning has been equated with producing an exact copy of what was in the teachers's mind. Previously, associationism or behavioursim dominated the thinking of psychologists. Now, however, there is a more cognitive view of learning. Learning is no longer viewed as a passive exercise. Students must think, be mentally active, generate meaning, and construct their own understanding. The National Council of Teachers of Mathematics (1989) have also stated that many mathematics classrooms need to change as transmission models of teaching no longer fit what we know about learning.

This project is offered as a potential means for promoting higher order thinking skills. Indeed, this is an alternate unit of mathematics which requires an alternate

teaching style. It signifies a shift away from the transmission practices of the past towards a more transformative way of thinking. This is a shift away from textbook learning, rote memorization and the mastery of traditional school subjects through traditional methodologies.

The Euclidean Geometry Unit is based on critical thinking, problem solving and communication. It requires the student to reflect on their experiences in the classroom, think critically about their reasoning and construct their own mathematical understanding. It introduces students to the dynamics of group structures, encourages students to listen to each other and utilizes a discovery approach to mathematics as students explore a variety of ideas in a small group setting.

While this unit may represent a transition in approach it does reflect the changes called for -- and the standards of -- the NCTM (1989) and the Intermediate Mathematics Curriculum Guide (1995). Thus, it is the general purpose of this unit to implement these changes and to foster critical thinking, reflection and higher order thinking skills in mathematics. The specific objectives or intended learning outcomes however, would be the same as those found in the curriculum guide for this unit does not represent such a departure from content as it does from technique, or teaching style.

Chapter 2: Selected Review of the Literature

This unit of Euclidean geometry is based on critical reflection, problem solving and communication and thus literature reviews are provided for each. There is also a literature review on the process of change as this unit does represent a change for both student and teacher.

Critical Thinking and Reflection

According to the National Council of Teachers of Mathematics (1989) many mathematics classrooms need to change. However, many teachers will ask the question "how do we know if we need to change?". The answer to this question quite simply put is to become a reflective practitioner (Hart, Schultz, Najee-ullah & Nash, 1992).

When reading educational journals, articles or indeed anything related to the field of education one cannot help but notice the recurring theme of reflection. It is an idea that permeates through educational reading from educational programs, university mission statements, curriculum guides, school policies to government reform. Reflection has become one of the most popular issues in teacher education with vast amounts of literature proclaiming the success of reflective practitioners (Copeland, Birmingham, Cruz & Lewin, 1993). However, Sparks-Langer and Colton (1991) remind us that reflection is not a new idea or concept as Dewey (1933) made reference to it in his early works. Yet, it was Schon (1983) who made the idea of reflection popular when he began to write about reflective practice in education and other professions. The basic questions that need to be asked now in relation to reflection and education are: "What is

reflection?", "Why reflect?", "What does one reflect about and for what purpose?", "Is there really an opportunity for reflective thinking or reflective practice in the current school setting?" and "Is reflection limited to teacher practice or can reflection be extended to student practice?".

What is Reflection

Dewey (1933) originally defined reflection as the, "active, persistent, and careful consideration of any belief, or supposed form of knowledge, in the light of the grounds that support it" (Bullough Jr., 1989, p. 18). Ross (1989) defines reflection as a way of thinking about educational matters that involves the ability to make rational choices while assuming responsibility for those choices. Many authors define reflective thinking as a problem solving process. Shulman, for example, defines reflection as the process of reviewing, reconstructing, reenacting and critically analyzing one's own performance while grounding explanations in evidence (Sparks-Langer, Simmons, Pasch, Colton & Starko, 1990). Goodman defines reflection as the process of recognizing, responding, reframing, experimenting and examining (Ross, 1989). Korthagen and Wubbels (1991) define reflection as the mental process of structuring or restructuring, an experience, existing knowledge, problem or insight. In light of these definitions of reflection it certainly seems that reflection would be an important tool for developing critical thinking skills for both teacher and student alike.

Why Reflect

As for the question, "why reflect?" Hart, Schultz, Najee-ullah and Nash (1992) state that reflection is a means of self assessment that makes you aware of how you teach. Williams (1992) notes, "that when teachers see themselves as learners ... and spend some of their energy trying to understand their students and their student's perspectives, they become less attached to pedagogical techniques and move quickly to a responsive and reflective way of teaching" (p. 2). Regan (1993) concluded that good teachers think about what they are doing and why they are doing it, or in other words , engage in reflection about their practice. While Ross (1989) goes on to note that, "the reflective teacher engages in thoughtful reconsideration of all that happens in the classroom with an eye towards improvement" (p. 23).

> Reflectivity is not simply a matter of pausing when facing a problem to think it through, but is a part of the ongoing practice of professionals as they interrupt and respond to situations that are "indeterminate" in order to achieve their aims.
>
> (Bullough Jr., 1989, p. 20)

Different authors describe different characteristics or attributes of a reflective teacher. Copeland, Birmingham, Cruz and Lewin (1993), for example, put forth a list of what they call twelve critical attributes that would indicate a teacher's stance towards reflection. Korthagen and Wubbels (1991) offer a list of four critical attributes of

reflective student teachers as well as a list of eight characteristics which correlate to reflectivity. Regardless of the different lists promoted by the different authors most of them agree with Colton and Sparks-Langer (1993) when they say that reflective teachers are motivated to grow, have a sense of self efficacy and believe that they can make a difference in the lives of students. These teachers also, learn continuously, serve the best interests of their students and are in short -- humane and loving.

> A reflective/analytic teacher is one who makes teaching decisions on the basis of a conscious awareness and careful consideration of -- the assumptions on which the decisions are based -- and the technical, educational and ethical consequences of those decisions. Those decisions are made before during and after teaching actions. In order to make these decisions, the reflective/analytic teacher must have an extensive knowledge of the content to be taught, pedagogical and theoretical options, characteristics of individual students and the situational constraints in the classroom, school and society in which they work.
>
> (Regan 1993, p. 191)

As for reflection and the student, Wheatley (1992) professes that reflection plays a critically important role in mathematics learning and that encouraging reflection results in greater mathematics achievement. Powell and Ramnauth (1992) claim that learning does not occur from experiences alone and that learners must reflect on their experiences.

Krulik and Rudnick (1994) have noted that there has been an increased emphasis placed on reasoning skills and that these higher order thinking skills can be achieved by expanding the "reflect" category which is the final step in the heuristic method.

The concept of reflective thinking is a reaction to the overly technical and simplistic view of teaching that dominated the 1980's and a recognition that teaching is a complex, situation specific and dilemma ridden endeavour (Sparks-Langer & Colton, 1991). It signifies a shift away from the transmission position towards a new transformative way of thinking. This is a shift away from textbook learning, rote memorization and the mastery of traditional school subjects through traditional methodologies towards a more holistic approach where the curriculum and the student are seen to interpenetrate each other (Miller & Seller, 1990). As Andrews and Huffman-Joley (1991) say, "there is a need for a holistic curriculum which highlights the students own learning for we can no longer view students as empty vessels to be filled" (p. 82). Indeed, this seems to be the shift that the NCTM(1989) is calling for.

Rote learning, recitation and strict discipline are all characteristic of the transmission position, as is Joseph Lancaster's "monitor system". In Lancaster's monitor system, children were not expected to engage in independent thinking or analysis. William T. Harris is also characteristic of the transmission position with his belief that the textbook was the centre of the curriculum, as is Franklin Bobbitt with his mechanistic view of teaching and belief that the curriculum should consist of activities that can readily

be identified and measured. Henry Clifton Morrison was one of the strongest spokesmen for the transmission position. Morrison claimed that education was an essentially passive process and that reading was the basis of the school curriculum as it allowed access to the rest of the curriculum (Miller & Seller, 1990).

On the other end of the spectrum of thought about education is the transformation position. It is here that reflective thinking and self analysis is valued and encouraged for both students and teachers alike. Self actualization, self transcendence and social involvement are the principle goals of this orientation. Here, learning is focused on the integration of the physical, cognitive, affective and spiritual, while the curriculum centres around self inquiry and social change. This school of thought provided the foundations for movements in education such as the open education movement and the social change movement. The transformation position also boasts a humanistic orientation towards education with an emphasis on values, self concept and self evaluation. Gerald Weinstein and his self science education is characteristic of this orientation. Weinstein proposes a process for achieving self knowledge, known as the trumpet, which applies problem solving skills to self inquiry which greatly resembles the processes for reflective thinking (Miller & Seller, 1990).

However, this transformation, or shift represented by the increased popularity of reflective thinking does not come without its problems, cautions and limits. For example, "the criteria that have become attached to the label of reflective practice are so diverse

that important conceptual differences among different practices are masked by the use of common rhetoric" (Zeichner, 1990, p. 55). "It is important that authors who discuss reflection make it clear what exactly they mean by this term ... as a comparison of publications by such authors as Zeichner (1983), Schon (1987), Cruickshank (1981), Ross (1987), and Korthagen (1985) suggest that there are fundamental differences between their approaches" (Korthagen & Wubbels, 1991, p. 18).

Furthermore, one should be cautioned that although thoughtful teachers who reflect about their practice are more desirable than thoughtless teachers who are ruled by tradition authority and circumstance -- teachers actions are not necessarily better merely because they are more deliberate and intentional (Zeichner, 1990). "One might be reflective about a problem and yet embrace solutions that are ethically irresponsible thus, reflectivity necessarily needs to be grounded in a social ideal useful for judging the desirability of proposed solutions" (Bullough, Jr., 1989, p. 16). One should also be cautioned that there is a, "danger of being drawn beyond the knowledge base to the employment of practices that are founded only in assumptions, rhetoric, and belief in what should be" (Copeland, Birmingham, Cruz & Lewin, 1993 , p. 348).

Reflection is also limited by the fact that "reflection must have a substance basis. Reflection on hearsay or unconfirmed opinion, or thought without experience can result in a mere propagation of ignorance -- while on the other hand, uncritical acceptance is miseducative, since it cuts short possibilities for further education and growth and is anti

theoretical to the purpose of education itself" (Roth, 1989, p. 34). The guiding factor for reflective practice then should be that, "irrespective of the tradition or context in which an individual articulates his or her aims, it must be the case that the aims and their practical consequences be neither repressive or discriminatory" (Zeichner, 1990, p. 59). The goal of reflective practice is student improvement (Copeland, Birmingham, Cruz & Lewin, 1993, p. 353).

What Does One Reflect About and for What Purpose

It can be concluded that reflection is important however, what does one reflect about and for what purpose? Bullough, Jr. (1989) answers this question quite simply by saying that one should reflect about "professional problems". Hart, Schultz, Najee-ullah and Nash (1992) say that teachers should reflect on: the nature of mathematical problems, their communication, students' communication and the learning environment. Friel (1993) answers the question of purpose when she says that for teachers the reflective practice: is central to an analysis of teaching, is a crucial part of making professional decisions, encourages and develops alternate approaches, and promotes listening to -- and learning about students and how they construct mathematical ideas. A good reflective teacher is a reflective decision maker.

> He/she is a thoughtful person intrinsically motivated to analyze a situation, set goals, plan, and monitor actions, evaluate results, and reflect on their

> own professional thinking. They consider the immediate and long term social and ethical implications of their decisions. Technical proficiency is no longer enough; morals and democratic principles must also guide the reflective teacher's action.
>
> (Colton & Sparks-Langer, 1993, p.45)

Zeichner (1990) defines reflective teaching as -- a process of moral deliberation, where teachers confront the difficult questions about what counts as good reasons for educational actions by reflecting about their educational actions and about the institutional, social and political contexts in which these actions are carried out.

Korthagen and Wubbels (1991) note that the nature and purpose of reflection are linked to a person's fundamental views about good teaching and the role of the teacher. They go on to note that authors like Zeichner, Carr, and Kemmis, for example, believe that the objects of reflection are primarily the moral, ethical, political and instrumental issues embedded in teachers' everyday thinking. Authors like Cruickshank, however, take a more technical approach and believe that the object of reflection is the effectiveness of instructional strategies in attaining given ends. Authors like Ross, on the other hand, consider the object of reflection to be rationality and responsibility where teachers are seen as professionals -- accountable for the way they teach -- rather than someone who is merely teaching a prescribed curriculum.

There seems to be general agreement that reflection is a desirable professional behaviour, but somewhat lesser agreement on purpose and exactly what that behaviour entails (Copeland, Birmingham, Cruz & Lewin, 1993). However, most transformational writers refer Van Manen's (1977) hierarchy when writing about critical reflection. The first level in this hierarchy, known as the technical level, is concerned with the effective application of skills and teaching knowledge in the classroom setting. The second level, the practical level, involves reflection about the assumptions underlying specific classroom practices as well as the consequences of particular strategies. The third and final level of this hierarchy is what is known as the critical level. At this level, reflection centres around the moral, ethical and other types of normative criteria related either directly or indirectly to the classroom. Note, this is a hierarchy and assumes that one would progress through the levels of reflection and that one level is not mutually exclusive from its predecessor (Regan, 1993).

To find the answers of object and purpose as they relate to students and reflection we can turn to Friel (1993) as he summarizes the changes in the learning environment as proposed by the National Council of Teachers of Mathematics (1989). The NCTM has stated that there needs to be a shift in classroom environments towards: classrooms as mathematical communities, logic and mathematical evidence as verification, mathematical reasoning and problem solving. Thus, for students, the object of their

reflection will be their classroom experiences -- their learning -- for the purpose of developing higher order thinking skills -- critical thinking skills.

In general, Schon (1983) explains that there are three types of reflection: reflection-in-action, reflection-on-action, and reflection-for-action and that we undertake reflection, not so much to revisit the past, but to guide future action (Regan, 1993). For the student -- reflection will improve thinking skills, allow for meta-cognition, self assessment and construction of meaning. For the teacher, reflection will allow for self assessment and improved teaching practices.

Certainly, there is a need for improved teaching practices as many of us have experienced mathematics as a collection of rules, rituals and routines. Now, however, there needs to be a change and a shift towards classrooms as mathematical communities where logic, reasoning and problem solving are emphasized. There needs to be a shift towards classrooms where teachers and students engage in and reflect on mathematics for themselves. Where critical thinking is the norm for both students and teachers alike. There needs to be a shift from the transmissive to a more transformative position. As Andrews and Huffman-Joley (1991) have previously noted, "there is a need for a holistic curriculum" (p. 82). To this end, reflection will become an increasingly important tool for both the teacher and the student as teachers reflect on their teaching and as students reflect on their learning.

Is There Really an Opportunity for Reflective Teaching

Do teachers really have the opportunity for such critical reflective practice in the current school setting? Is there any opportunity for these transformative teachers to analyze situations, set goals, plan and monitor actions, evaluate results, and reflect on their own professional thinking? Is there any opportunity for teachers to consider the immediate and long term social and ethical implications of their decisions? Is there any opportunity for teachers to confront the difficult questions about what counts as good reasons for educational actions by reflecting about their educational actions and about the institutional, social and political contexts in which these actions are carried out or do teachers simply implement programs designed by others?

To answer some of these questions one should first consider the ever present curriculum guide and its possible affects on the school setting. True, the transformative teacher will consider the curriculum to be much more than this prescribed content but -- it is the "primary resource" and thus worth considering (Intermediate Mathematics Curriculum Guide, 1995).

In the preface of the Intermediate Mathematics Curriculum Guide (1995) the teacher is told that the "Guide" represents core content expected of intermediate students and "must" represent the basis upon which "decisions" regarding instructional "procedures", the utilization of "resources" and "assessment" techniques are made ... that the guide "reflects the nature and needs of the adolescent"... that every teacher should

have a copy of this guide ... and that this is the primary resource for mathematics instruction at the intermediate level" (Intermediate Mathematics Curriculum Guide, 1995, p. iii).

This "Guide" does more than restrict content. It restricts instructional procedures, resources and instructional techniques. It even suggests that they, "the government", have reflected upon and know the nature and needs of the students. A look through the Table of Contents is enough to tell how encompassing and restrictive this guide really is. For example, we see titles such as: The Nature of Thinking and Learning of Intermediate Mathematics, The Nature and Needs of the Adolescent Learner, The Development of Mathematical Thinking, The Prescribed Intermediate Mathematics Curriculum, The Pedagogical Aspects of Intermediate Mathematics Instruction and The Evaluation of the students Performance in Intermediate Mathematics (Intermediate Mathematics Curriculum Guide, 1995).

The Mathematics 1300 Curriculum Guide (1993) goes a little further and suggests that if your teaching styles do not match those outlined in the guide then you should change. In the preface of this guide it is stated that "this guide presents a vision of the mathematics curriculum and, as such, represents what is intended ... the objectives in this guide represent the core program for the targeted students and must represent the basis upon which teachers make decisions regarding instruction, use of resources, and evaluation. Such decisions may necessitate a reshaping of some of the current teaching

practices" (Mathematics 1300 Curriculum Guide, 1993, p. iv).

It may be argued that the only thing that is really restricted by these curriculum guides is the content. Some may argue that the guide is only information to be reflected upon. It may also be argued that this prescribed curriculum represents only a prescribed core and that the teacher is free to supplement and add topics as desired. It may further be argued that this prescribed curriculum represents quantity and that the teacher controls and decides upon quality.

Whatever the argument, it must be recognized that it is an argument and that while there may very well be opportunities for reflection it is certainly not encouraged by the system itself. If a teacher wants to be a critically reflective practitioner he certainly can find the opportunity in his day to day teaching. However, others may find themselves asking: "Why reflect about the nature and needs of students?", "Why reflect about underlying rationale?", "Why reflect about, teaching strategies or pedagogy or anything else for that matter?" -- it's all in the curriculum guide. There is a real possibility for teachers to become nothing more than implementers of programs prescribed and designed by others -- if they are not morally committed to teaching. Transformative teachers have this needed sense of purpose and moral commitment (Sergiovanni, 1995).

Does government reform encourage reflective thinking or practice? Miller and Seller (1990) have noted that history has shown that during difficult economic times the transmission position becomes more dominant. While the transformative position may be

more politically acceptable, and some of the language used in government reform may indicate a tendency towards that position, government reform is certainly in line with the transmissive position. In the government document Adjusting the Course, Part II, Improving the Conditions for Learning (Government of Newfoundland and Labrador, 1994) one can see the language of the transformative thinker. For example, "Our goal for education is to transform society", "increased tolerance and respect for others", "balance between top-down and bottom-up reform" and "the school is the primary unit for change" is language characteristic of the transformative position.

However, there is also strong evidence of the transmissive position. For example, there is much emphasis placed on economics, standards, accountability, core curriculum and curriculum implementation. The document discusses, for example:.a provincial foundation curriculum, primary core areas consisting of language, mathematics, science and technology, achievement standards grounded in provincial standards, common curriculum and common standards, a new model for curriculum implementation, and a professional development centre that will facilitate provincial curriculum implementation (Government of Newfoundland, 1994).

Rote learning, recitation and strict discipline are all characteristic of the transmission position. If we recall, the philosophies of William T. Harris and Franklin Bobbitt are also characteristic of the transmission position as is Joseph Lancaster's philosophy and expectation that students were not to engage in independent thinking or

analysis and Henry Clifton Morrison who claimed that education was an essentially passive process (Miller & Seller, 1990). Is this where the educational reform is leading us? Elmore (1992) states that the central concern for effective schools is the students' performance on standardized tests which are ultimately unable to measure real learning or the development of understanding. After reading the government document Adjusting the Course, Part II, Improving the Conditions for Learning (Government of Newfoundland and Labrador, 1994) with its emphasis on accountability and standardized test scores there is no doubt as to the underlying meaning of effective. Teachers who are transformative, critically reflective and who encourage their students to be the same are not likely to find support or encouragement here.

> Reflection involves both conscious understanding of and actions in school on solving our daily problems. These problems will not go away by themselves, after all. But it also requires critically reflective practices that alter the material and ideological conditions that cause the problems we are facing as educators in the first place. Clearly, the major impediment to the reflective approach has been recent attempts to reform schooling by ensuring that what goes on inside schools is directly responsive to the economic needs outside of schools.
>
> (Smyth, 1989, p. 3)

Levin (1992) concluded that while the individual is important in shaping what the organization does the organization is also important in shaping what the individual thinks and does. Currently, teachers are being pressured to be productive and effective but not necessarily innovative or critically reflective. Thus, with the focus on accountability and effectiveness -- teachers, anxious to keep their jobs in a time of economic constraint, will embrace the familiar -- teaching by telling -- and not necessarily the innovation of reflective practice for teacher and student. Smith (1996) states that "teachers need to believe that their teaching actions have significant causal impacts on their students learning ... develop a sense of efficacy ... and good telling ... clearly defines what the central acts of teaching are and what counts as evidence of student learning" (p. 393). Thus, the current system does little to encourage the change called for by the National Council of Teachers of Mathematics (1989) and in fact, makes it very easy and indeed comforting for teachers to maintain the status quo in the mathematics classroom. As Smith (1996) notes, teachers can feel pressure from many sources such as students, parents and even administrators and colleagues to compromise the reform principles and return to teaching by telling.

Thus, after a brief look at the curriculum guide, government reform, organizational structure and some of the pressures placed on teachers we can conclude that while there may very well be opportunities for reflection there seems to be little encouragement from within the system itself. However, "teachers need to have a critical

insight into their roles in school and examine critically the value of the knowledge they teach and the function of schooling generally in society ... teachers must do more than simply implement programs designed by others" (Doyle, Kennedy, Ludlow, Ross & Singh, 1994, p. 33).

"Transmission models of teaching no longer fit what we know about learning ... we must acknowledge our own and others' excuse for maintaining the status quo" (Andrews & Huffman-Joley, 1991, p. 82). "Rational analysis of one's own behaviour as a teacher is an important tool in one's professional development" (Korthagen, 1993, p. 321). "Teachers acquire some professional knowledge from packaged educational principles and skills but the bulk of their learning comes through continuous action and reflection on everyday problems" (Sparks-Langer & Colton, 1991, p. 40).

Indeed, although reflection may not be encouraged or supported by current educational reform or the organizational structure itself, teachers will need to embrace the ideal of critical thinking and the concept of reflective practice and divorce themselves from the comforts of the status quo. A brief glance at some of the goals of the teacher education programs will indicate just how important these ideas really are. "The faculty in the elementary program at the University of Florida have identified the development of critical thinking as the primary goal of their teacher preparation program" (Ross, 1989, p. 22). "The Collaboration for the Improvement of Teacher Education (CITE) is a pre-student teaching program that promotes students' reflective thinking about curriculum,

methods and sociopolitical issues through blocked classes and structured field experiences" (Sparks-Langer, Simmons, Pasch, Colton & Starko, 1990, p. 23). "The preparation of teachers as professional educators has been the focus of several related national reform efforts in recent years ... the goal of these reform efforts, in short, has been the development of teachers who will engage in reflective practice as an integral and continuous component of their teaching" (Regan, 1993, p. 189). "We believe that the vision of teacher as reflective practitioner will be a crucial link in efforts to revitalize the profession ... the theme of teacher as reflective professional should be pursued vigorously in programs of training" (Roth, 1989, p. 35). "In fact it is often difficult to get time to reflect. Yet, we are convinced that unless we can help interns make real time for critical reflection, there is little hope for them to be professional or transformative. We see little value in training educational technicians" (Doyle, Kennedy, Ludlow, Ross & Singh, 1994, p. ii).

Certainly, reflection seems to be very important. For the teacher, reflection will allow for self assessment and improved teaching practices. However, teachers must also remember that reflection is an important tool for the mathematics student and students in general as it improves thinking skills, allows for meta cognition, self assessment and construction of meaning. So, even though reflection may not be encouraged or supported by current educational reform or the organizational structure itself teachers will indeed need to embrace the ideal of critical thinking and the concept of reflective practice for not

only themselves but for their students as well. Krulik and Rudnick (1994) suggest that teachers try reflecting in their classrooms while Wheatley (1992) explains that "there are serious limitations in the explain practice method of instruction and active learning and that even performing self generated mathematical operations does not have the power which results from reflecting on the activity" (p. 529).

Reflection in the Classroom

How can this reflective practice be extended to the student and what does reflection look like in the classroom? To answer this final question we can look at some specific examples of things currently being done at various grade levels. Wilson (1991), for example, examines grade one students writing and solving their own word problems and notes that the process combines reading and critical thinking. Raphel (1993) presents two seasonally related investigations to fourth graders regarding the number of candles needed for the eight nights of Hanukkah and the number of presents given during the twelve days of Christmas to develop critical thinking skills. Ward and Conderman (1995) describe a functional mathematics activity to help secondary students with mild disabilities develop critical thinking, calculation and reasoning skills by evaluating the rent to own option for furnishing an apartment. Powell and Ramnauth (1992) propose using multiple entry logs in which students reflect on mathematics problems as a focal point for teacher student interactions.

Reflection and other activities to develop critical thinking skills are important to all students from kindergarten to college. When writing about critical thinking in business education Sormunen (1992) notes that developing critical thinking activities will enhance the learning process. When writing about assessment in mathematics Raymond (1994) goes on to conclude that:

> reflection assignments challenge students to reflect on both the cognitive and metacognitive aspects of how they think about mathematical tasks ... and presents the student with a formal means of self assessment of both content knowledge and personal growth.
>
> (Raymond, 1994, p. 16)

The following unit of Euclidean Geometry that I have designed would also be a means of extending this reflective practice to the teacher and student in the classroom.

Wheatley (1992) professes that encouraging reflection results in greater mathematics achievement. Regan (1993) concluded that good teachers think about what they are doing and why they are doing it, or in other words, engage in reflection about their practice. Through reflection we further our own understanding and generate knowledge to inform future practice (Killion & Todnem, 1991). If students were given the opportunity to reflect, their ability to learn and their understanding of learning would improve -- reflection is a critical aspect of learning and consequently necessary for better teaching (Fedele, 1996). In classrooms where teacher join students in shared inquiry ...

teachers view learning as transformational ... and agree that reflective inquiry fuels teaching and learning (Patterson, 1996).

So, even though reflection may not be encouraged or supported by current educational reform or the organizational structure itself teachers might want to embrace the ideal of critical thinking and the concept of reflective practice for not only themselves but for their students as well. Teachers can continue to teach by telling and perpetuate student learning by rote memorization or they can embrace real change as this reflective practice does indeed represent real change and not only for the student but the teacher as well. Teachers can reflect on their teaching and encourage student to reflect on their learning or they can maintain the status quo. This unit represents one possible way to embrace that change.

Problem Solving

The vision of the mathematics classroom that is presented in the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (1989) has inspired many of us to want to change the way in which we teach. "We want to pose challenging problems ..." (McLeod, 1993, p.761).

The teaching of problem solving in mathematics is becoming a prime concern for teachers. However, for these teachers to teach problem solving effectively they will need to be problem solvers themselves and will need to teach mathematical problem solving as

a process rather than a product. Research indicates that skilful problem solvers maintain an open and flexible mind and that such problem solvers are the product of discovery learning (Taback, 1992). The goal of this reform is to teach for understanding. One way to teach for understanding is to avoid instruction of isolated facts, and, instead focus on the big ideas. A theory of cognitive development that would allow you to do so would be constructivism (Greenes, 1995). Greenes (1995) goes on to say that, "by learning and using the process of investigation to explore the big ideas in various contexts, students gain a robust and more lasting understanding of the concepts ... understanding and discovery are facilitated by a curriculum that has a strong component of exploration, investigation and is problem based" (p. 85).

However, many teachers are aware that students do not understand key concepts or indeed have even documented student misconceptions about key ideas in mathematics (Perkins & Blythe, 1994). "Learners must spend the larger part of their time with activities that ask them to generalize, find new examples, carry out applications and work through other understanding performances ... most school activities are not performances that demonstrate understanding. Rather they build knowledge or routine skill" (Perkins & Blythe, 1994, p.6). Schoenfeld (1983) suggests that there are serious flaws in the show and tell method of teaching where students are asked to master procedures as it is possible for students to master the wrong procedures because mastery here is based on memorization as opposed to understanding. With this show and tell method of teaching

students are considered to be passive receptacles as opposed to active agents in the construction of their own knowledge.

Unlike the traditional perspectives that treat mathematics as a codified body of knowledge to be taught constructivists consider mathematics to be the activity of constructing relationships and patterns. Here the goal is learning and mathematics is effectively learned only by experimenting, questioning, reflecting, discovering, inventing and discovering. Favourable conditions for such learning exist when a student is faced with a task for which there is no known procedure available, or in short, a problematic situation. When students engage in such learning activities for the purpose of constructing relationships there is potential for significant results and when mathematics is taught from such a problem centred perspective there is potential for significant benefit to the students (Wheatley, 1991).

Simon (1995) writing from the constructivist perspective states that," a teacher's job is to propose a learning situation within which students seek a response to the milieu ... where students accept the problem as their problem ... where learning involves being able to use the ideas beyond the narrow context of the original problem situation" (p.120). Wheatley (1991) also writing from a constructivist perspective explains that what is a problem for one may not be for another and that it is the teacher's job to select tasks which are problematic and that: are accessible to everyone at the start, invite students to make decisions, encourage what if questions, encourage students to use their own

methods, promote discussion and communication, are replete with patterns, lead somewhere, have an element of surprise, are enjoyable and are extendable.

Wheatley (1991) presents problem centred learning as an alternative to the explain practice method which is so characteristic of traditional mathematics instruction. The core of problem centred learning is a set of problematic tasks that will guide students to construct effective ways of thinking about the subject. Here knowledge is not passively received but is rather actively built by students constructing their own mathematical meaning. From this constructivist perspective it is believed that knowledge is related to the action and experiences of the learner and when the learner reflects on those actions their knowledge is even more powerful. It is this reflective abstraction that is the basis of learning and the mechanism for constructing mathematical knowledge.

Taback (1992) when commenting on the work of Polya (1987) notes that as teachers we should be teaching students to think and that such thinking can be identified with problem solving. Barba (1990) offers many definitions of problem solving but, sums up her thoughts by saying, "whatever our definition, we must recognize that problem solving is a multifaceted construct that involves questions about the way(s) we think and the ways we learn" (p. 32). Quinlan (1989) comments that students need to be convinced that it is worthwhile spending time struggling over mathematics problems as learning can and should occur during the time of trying to solve a problem. He also goes on to say that the reflective strategy will mean that the student will: think about their

thinking, analyze the ways that they have been attacking the problem, make inferences and pursue further explorations. Furthermore, this reflective strategy will be an assurance against students wasting time. "When what we experience differs from the expected or intended, disequilibrium results and our adaptive (learning) process is triggered. Reflection on successful adaptive operations (reflective abstraction) leads to new or modified concepts (Simon, 1995, p.115).

"Problem solving per se ought to involve higher order thinking skills ... it involves facts, rules, skills and strategies called heuristics. Heuristics include: identifying necessary information, generalizing, restating the problem, looking for patterns and writing mathematical statements" (Barba, 1990, p. 34). Schoenfeld (1983) warns however, that a teaching theory based solely on heuristics is doomed to failure. If we train students on problems where the technique always works and then test them on more of the same then we are not really dealing with problem solving. He then goes on to say that a problem is only a problem if a student does not know how to solve it and if in fact that student wants to solve it. The value to working such a problem lies within the solution process itself and its reward is an increase in one's knowledge. Quinlan (1989) shares Schoenfeld's ideals and describes a novel setting which presents students with a problem solving situation. This problem solving situation was just that because the teacher had not drilled the students with similar questions and what was required was inferential thinking and careful analysis of a novel setting.

Barba (1990) does go on to say however, that students who are given the opportunity to solve problems become problem solvers and that these problem solvers not only solve problems, but analyze their thinking and reflect on their own problem solving strategies. Students learn by doing and by reflecting on what they have done. This reflective thinking is considered to be a method of problem solving. Polya (1957) also saw the value of reflection. Indeed, it is his four phase model for problem solving that is commonly used and cited in research today. Polya's plan for solving problems required you to: understand the problem, devise a plan, carry out the plan and look back. However, this last step of looking back is most often neglected when problem solving even though such reflection can consolidate knowledge, develop the ability to solve problems and lead to greater mathematical understanding (Taback, 1992).

Greenes (1995) when commenting on the work of Steffe(1990) describes three periods in the teaching learning process that bears some resemblance to Polya's four phase model. The first period is the period of construction where students pass through the stages of engagement, exploration, explanation, elaboration and evaluation. The next period is the period of retroactive thematization where both the teacher and students review, discuss and reflect on solutions. The final period is the period of assimilating generalization where further discussion, analysis and reflection leads to some generalizations. Indeed, there are many common concepts between the two models.

Yet, the question remains does constructivist teaching actually work. While theory may tell us so is it actually better then traditional methods when put to the test? Kamii and Lewis (1991) put it to the test in actual classrooms and compared their results to a control group who received traditional instruction. They came to the conclusion that when put to the test constructivist teaching methods were no better than traditional methods when they compared scores on achievement tests and in fact traditional instruction appeared to be more effective. However, when they analyzed the results a little more closely their data lead them to some familiar conclusions. For example, tests become ends in themselves and tests stress lower, rather than higher order thinking. In fact they state quite clearly that the, "data consistently demonstrates with statistical significance the superiority of a constructivist program if higher order thinking is evaluated. The reason for the discrepancy lies in the fact that achievement tests are created within the framework of an obsolete conception of school mathematics" (Kamii & Lewis, 199, p.7).

Wheatley (1991) concluded that much of the current school practice reflects a behaviouristic set of assumptions where associations are strengthened through repetition and learning is rule oriented. Such a transmission view of education no longer fits the needs of the students. Simon (1995) sums it up quite nicely as he quotes Richards (1991) in saying,

> It is necessary for the mathematics teacher to provide a structure and a set of plans that support the development of informed exploration and reflective inquiry without taking initiative or control away from the student. The teacher must design tasks and projects that stimulate students to ask questions, pose problems and set goals. Students will not become active learners by accident, but by design, through the use of the plans that we structure to guide exploration and inquiry.
>
> (Simon, 1995, p.118)

Journal Writing

> Communication is at the heart of classroom experiences which stimulate learning ... classroom environments that place particular communication demands on students can facilitate the construction and sharing of mathematical meaning and promote student reflection on the nature of the mathematical meanings they are required to communicate.
>
> (Clarke, Wayward & Stephens, 1993, p.235)

"Communicating about mathematics orally and in writing not only enriches the learning of mathematics, but also makes mathematics relevant and meaningful to students" (Scott, William & Hyslip, 1992, p.18). These authors go on to say: that journal writing is clearly an effective and motivating experience, that journals have great

potential for helping students, that these activities stimulate children to think beyond rote answers and make unique connections and associations. Scott, Williams and Hyslip (1992) were writing about journal writing in the second grade when they stated that, "in journal writing, students reflect on experiences and organize their thoughts in order to communicate clearly ... children move beyond the mere performance of techniques and toward a deeper understanding of mathematics" (p. 15). Truly, this is something that all math teachers would desire at all grade levels. Perkins and Blythe (1994) state that "to learn for understanding, students need criteria, feedback and opportunities for reflection from the beginning and throughout any sequence of instruction" (p. 7). These seem to be things that can be provided by the journal.

Burns (1995) also believes in the power of journal writing as she states, "writing in math class has two major benefits. It supports student's learning because, in order to get ideas on paper, children must organize, clarify and reflect on their thinking ... their writing is also a window into what they understand, how they approach ideas and what misconceptions they harbour" (p. 40). Burns (1995) also goes on to offer strategies to help implement journal writing in the classroom. She makes such suggestions as: talk with students about the purpose of their writing, use student writing in classroom instruction, have students discuss their ideas before writing and provide prompts. Stix (1994) further summarizes some of the advantages of journal wring as discussed by other authors. Here it is stated that journal writing: helps students to construct and make

concepts meaningful for themselves, can expose misconceptions that otherwise might go unnoticed , allows students to clarify issues for themselves and enables students to retain and remember information better.

Wayward (1992) claims that journals can be classified as either recount, summary or dialogue. Journals classified as recount simple record, reinforce facts but, do little to advance learning. Summary writing would be a progression upwards. Here important ideas are recognized and reordered. With dialogue writing however there is integration of ideas and analyzing of arguments. These ideas and classifications of journal are again reiterated by Clarke, Wayward and Stephens (1993). "It is in the dialogue mode that students begin to focus on the ideas ... students begin to relate the mathematics being taught to what they are learning and to their ability to connect new ideas with what they already know ... through their writing they show that they are actively constructing mathematics" (p. 248).

McIntosh (1991) states that, "writing to learn in the mathematics classroom is a means for teachers to help students learn and to assess whether their students are learning what they are trying to teach" (p. 423). Then, however, she goes to describe several different types of writing in the classroom as opposed to just journal writing. Indeed, the term journal wring has come to mean many different things for many different people and may even encompass all distinctions made by McIntosh.

One form of writing in the classroom is the log. Students write in learning logs to reflect on what they are learning and to learn while they are reflecting. Student are given either: open ended writing task such as the expectation that they write in their logs every day , guided writing tasks such as responding to questions based on newly presented concepts, or specific writing tasks which would include how-tos, definitions and troubleshooting. How-tos would require explanations about how to do something such as bisect an angle. Definitions would require students to write their own explanation of a concept and would therefore be more likely to be understood as opposed to merely memorized. Troubleshooting would require students to explain the errors that they made or misconceptions that they may have had (McIntosh, 1991). Other forms of writing described by McIntosh are journals which are less formal then logs, expository writing whose primary purpose is to explain and creative writing.

Miller (1992), like McIntosh, believes in the value of writing in mathematics and also makes a distinction between various types of writing in the classroom. Miller (1992) however, unlike McIntosh, does not make a distinction between journals and logs but states that, "journal writing is brainstorming with oneself in which thinking about learning mathematics is done in writing ... journal writing can benefit students cognitively by leading them to summarize and reflect on the mathematics they are learning" (p. 2). Expository writing which is the second form of writing identified by Miller (1992) asks students to explain in writing their thinking about a nonroutine problem. She goes on to

conclude that students doing expository writing have better problem solving skills then students following traditional methods. Transactional writing, the third form identified here, seems somewhat similar to what McIntosh (1991) calls creative writing.

The NCTM (1989) has recommended that "the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can reflect upon and clarify their thinking about mathematical ideas and relationships" (p. 140). Furthermore, they state that, "the very act of communicating clarifies thinking and forces students to engage in doing mathematics" (p. 214). Indeed, writing about mathematics whether it be in the form of logs, journals, expository or creative writing, "suggests a significant move away from the traditional chalk and talk method of teaching mathematic" (Grossman, Smith & Miller, 1993, p. 4).

The Change Process

"Personal attitudes sometimes inhibit the likelihood of experiencing the satisfaction which is to be found in searching for solutions to challenging problems or in simply exploring possibilities" (Quinlan, 1989, p. 20). "A great deal of anxiety is created when new techniques are used" (Berenson & Carter, 1995, p. 186). "Even when students are working on problems in small groups, those who are accustomed to traditional instruction are especially susceptible to feelings of frustration. This kind of emotional response to mathematical tasks is often related to the beliefs that students hold about

mathematics, about themselves and about teaching mathematics" (McLeod, 1993, p. 762). "One (might get) the impression that it should be relatively easy for the teacher who sows some heuristic oats in the classroom to reap a harvest of budding young problem solvers. That just is not true." (Schoenfeld, 1983, p.43).

Norwood and Carter (1994) when commenting on change and journal writing note that students are more resistant to change after the school year has started, therefore, the journal writing process should be introduced at the beginning of the year. Wayward (1992) came to the conclusion in his study that journal writing enhanced student learning but the process took years and a conscious teacher effort. Clarke, Wayward and Stephens (1993) when commenting on the journal writing process state that, "teachers can play a crucial role in helping students to assume control over their learning ... the key appears to be to encourage students to question themselves when they do not understand rather than be dependent upon their teacher to tell them whether they understand" (p. 248).

When Fullan (1991) talks about change or planning and implementing change in general he offers several statements for consideration. For example, he claims that: people need pressure to change, effective change takes time, and changing the culture of institutions is the real agenda as opposed to implementing single innovations. Fullan also states that there are three dimensions at stake in implementing any new program or policy with those being the possible use of new materials, new teaching approaches and an alteration in beliefs.

Schoenfeld (1987) notes that if you want to affect student's mathematical problem solving behaviours then you had better understand that behaviour and in doing so you would need to look at students' cognitive resources, problem solving strategies, executive or control behaviour and their belief system. For instance, many students believe that: formal proofs or justifications are not necessary, all mathematics problems can be solved in ten minutes or less, only geniuses are capable of discovering, creating and understanding mathematics, and that formal mathematics and proof have nothing to do with discovery or invention. Certainly, such beliefs would stand in the way of mathematical reform and change. However, Schoenfeld (1987) goes on to note that these beliefs are created mostly by environment and for the student that would mean the classroom coupled with the school climate and culture. For example, traditional mathematics has been based on mastery, memorization and exercises as opposed to real problematic situations. Tests have traditionally emphasized low order thinking or recall. Traditional mathematics has been a solitary activity and applied problems have been stripped of the complexities of the real world. Schoenfeld (1987) concludes by saying that, " the result is that students do not learn the delicate art of mathematizing ... the culture of schooling stands as an obstacle to school reform. Real curricular reform must in part involve a reform of school culture" (p.37).

McLeod (1993) continues the argument started by Schoenfeld (1987). McLeod (1993) states that, "students' experiences in traditional classrooms often lead them to

develop beliefs about mathematics that generate negative responses to problem solving" (p. 762). Traditionally students have been lead to believe that mathematics involves mainly following rules. Thus, if students are given non-routine problems with no rule to follow they may respond negatively. Traditionally students have been lead to believe that mathematics is dependent more on ability then effort and that only geniuses can be creative in mathematics. Again, students with such a belief may respond negatively, get frustrated or even quit when presented with a non-routine problem for which it is perceive unreasonable to expect any success. Traditionally students have been presented with exercises as opposed to problematic situations and as a result have the belief that all problems can be solved in two minutes or less. This belief may also cause students to respond negatively when presented with real problems that require real persistence. McLeod (1993) concludes that, "before students can become proficient problem solvers, they may have to change their beliefs" (p. 762). Schoenfeld (1983) adds that, "doing mathematics is an intensely personal and emotional enterprise. Instruction that fails to take this into account is necessarily dry and lifeless. It does justice to neither to the students nor to mathematics" (p.45).

McLeod (1993) goes on to say that students will need to know that frustration is normal, that problems take time, effort, and a variety of strategies and will need to experience the joy and satisfaction of solving problems. To create an environment and climate conducive to problem solving teachers can: present themselves as problem

solvers, let students know about their own struggles, make frequent use of cooperative groups, encourage discussion and make it clear that they value problem solving by assessing students' performances regularly. Wilson (1994) making note of this last point comments that, "nontraditional activities used reasoning, reflecting, and communicating more than routine procedures but, since the nontraditional activities were not graded, they were not valued by students" (p. 414). Polya (1987) has been quoted as saying that, "we must know what we believe in, what we support, and where we are and are not achieving our goals" (Dossey, 1988, p.292).

When Smith (1996) talks about change, or planning and implementing change, he notes that teachers may have feelings of uncertainty as changes in teaching means taking risks. Teachers may also feel pressure from students, parents, administrators or colleges to abandon the change and return to the traditional teaching by telling. "The transmission of knowledge is a social process that depends on the cooperation and goodwill of all concerned. Teachers who are insecure about their mathematics, who feel unprepared to deal with new bodies of knowledge, and who are unhappy at having new curricula crammed down their throats, are hardly the most effective communicators of that knowledge"(Schoenfeld, 1983, p.44).

Smith (1996) explains that, "the reform of school mathematics content, learning and teaching undermines the base for teachers' sense of efficacy that teaching by telling provides" (p. 388). Traditionally teaching by telling meant that mathematics was a fixed

set of facts and procedures which the teacher could master. It also meant that mathematics was taught by telling -- which of course provided the teacher with a relatively detailed model of what they were to do. Furthermore, it meant that students learned by listening and that all mathematical problems were known and found in textbooks. Teaching by telling simplified issues of planning, classroom management and established a warm, orderly, and respectful classroom. However, teaching by telling is no longer acceptable as mathematics is no longer seen as a fixed collection of facts and procedures nor can students learn as passive listeners. Knowledge is gained through exploration, analysis and proof. Students need to understand, reason and develop a sense of their own mathematical power .

> This active view of learning mathematics substantially changes what teachers must do to enable learning ... in reality no one can teach mathematics. Effective teachers are those who can stimulate (or facilitate) students to learn mathematics ... and students learn mathematics well only when they construct their own mathematical understanding.
>
> (Smith, 1996, p.394)

Smith (1996) also explains that with this shift towards understanding, explanation and problem solving, teachers are often at a loss to know what and how to teach and as a result are tempted to return to the telling mode. What is needed are new sources of efficacy and failure to find such will seriously limit the impact of reform. McLeod (1993)

adds that "we will encounter many difficulties as we move towards that ideal classroom of the future" (p.761). However, he further adds that :

> Students should learn to value mathematics, become confident about mathematics, and become problem solvers. If students hold onto their traditional beliefs about mathematics, their affective reactions to problem solving will make it hard for them to value mathematics or to feel confident about their mathematical ability. However, teachers can help students change their beliefs.
>
> (McLeod, 1993, p.763)

Summary

The vision of the mathematics classroom that is presented in the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (1989) has inspired many of us to want to change the way in which we teach. Others may ask the question "how do we know if we need to change?". The answer to this question seems to be found with reflection. As Regan (1993) notes good teachers think about what they are doing and why they are doing it, or in other words, engage in reflection about their practice. As we reflect many of us will be able to recognize those serious flaws in the show and tell method of teaching referred to by

Schoenfeld (1983). Indeed, traditional methodologies have emphasized lower order thinking and repetition as opposed to higher order thinking and understanding.

Both Taback (1992) and Quinlan (1989) comment on the relationships between thinking, understanding and problem solving and conclude that problem solving per se ought to involve higher order thinking skills and understanding. One method of teaching for such understanding through problem solving is constructivism. Unlike the traditional perspectives that treat mathematics as a codified body of knowledge to be taught constructivists believe that mathematics is effectively learned only by experimenting, questioning, reflecting, inventing and discovering.

Communication in and reflection on mathematics is also important as Scott, William and Hyslip (1992) state "communicating about mathematics orally and in writing not only enriches the learning of mathematics, but also makes mathematics relevant and meaningful to students ... in journal writing, students reflect on experiences and organize their thoughts in order to communicate clearly ... children move beyond the mere performance of techniques and toward a deeper understanding of mathematics" (p. 15). If students were given the opportunity to reflect, their ability to learn and their understanding of learning would improve -- reflection is a critical aspect of learning and consequently necessary for better teaching (Fedele, 1996). This thinking signifies a shift away from the transmission position towards a more transformative way of teaching and learning. Indeed, it signifies a significant change.

When Fullan (1991) talks about change or planning and implementing change he offers several statements for consideration. For example he claims that: people need pressure to change, effective change takes time, and changing the culture of institutions is the real agenda as opposed to implementing single innovations. Fullan also states that there are three dimensions at stake in implementing any new program or policy with those being the possible use of new materials, new teaching approaches, and an alteration in beliefs.

Although it may not be encouraged or supported by current educational reform or the organizational structure itself teachers might want to embrace the ideal of critical thinking and the concept of reflective practice for not only themselves but for their students as well. Teachers can continue to teach by telling and perpetuate student learning by rote memorization or they can embrace real change as this reflective practice does indeed represent real change for both the student and teacher. Teachers can reflect on their teaching and encourage student to reflect on their learning or they can maintain the status quo. This unit of Euclidean Geometry represents one possible way to embrace that change.

Chapter 3: Methodology

Setting and Population

The unit is to be implemented in the grade nine mathematics classroom according to the theory of cooperative learning. The teacher will stimulate learning or act as a facilitator between the known and the unknown for the students. The students will be seated in groups of four for the purpose of group discussion, exploration, analysis and reflection on the given activities. There will be time for full class discussion where students will present and share their ideas, their strategies and their thinking. There will also be time for individual journal writing and personal reflection but, the majority of the time in class will be spent in small groups as students strive to construct their own mathematical meaning.

Time Frame

The unit of Euclidean geometry is designed for approximately twenty -two, fifty minute classes, however, group discussion, class discussion or journal writing should not be sacrificed for the sake of time. Indeed, questions of time, progression and the apparent abandonment of all structure will have to be addressed by each individual teacher to see how they align with their individual philosophy of learning. I would suggest that the unit has a set lesson structure based upon suggested time periods of fifty minutes, emphasizing the word "suggested" as the underlying philosophy of individual and group exploration does not lend itself well to time constraints. The key to successful implementation will be flexibility. All students and groups should be given the

opportunity to explore, analyze and reflect on each activity before full class discussion. All groups should come together for a full class discussion after each activity where the students can share their ideas, their strategies and their thinking. Note, this is a discussion driven by students and only facilitated by the teacher. The teacher must not resort to telling at this stage for the purpose of hurrying things along. Finally, there should be time given for personal reflection and journal writing so that the students understand that this is an important process and one that is valued by the teacher. This is a significant departure from the traditional where the recitation of material took up most of the available class time and where the teacher felt that their primary responsibility was to cover subject matter in didactic lectures (Paul, 1992, p. 17).

Description and Analysis of Unit

This unit of Euclidean Geometry is based on critical reflection, problem solving and communication. I believe that it provides the student with those experiences needed to facilitate growth in mathematical understanding. The unit introduces students to the dynamics of group structures, encourages students to listen to each other and most importantly employs a problem centred discovery approach to mathematics. Students explore a variety of ideas in a small group setting and construct their own meaning for the mathematical problems presented. The students are required to reflect on their experiences in the classroom, think critically about their reasoning and construct their

own mathematical understanding. Furthermore, the unit is to be carried out according to the theory of cooperative learning and requires full class discussion and journal writing.

Confidence in mathematics is fostered by encouraging students to explore mathematical concepts, ask questions, discuss their ideas and make mistakes. Students who enjoy mathematics and work in a constructive environment feel comfortable and confident and in turn perform better and learn more. This discovery approach to learning is also well suited to the small group structure. Students learn mathematics by doing mathematics. With proper (subtle) guidance by the teacher, students can discover relationships, plan procedures, formulate definitions, make conjectures, construct examples and counterexamples, and solve problems. By listening to students' ideas and by having them listen to one another, the teacher can establish an attitude of mutual respect (Intermediate Mathematics Curriculum Guide, 1995).

However, teachers should be aware that there are many factors that impinge upon the teaching of mathematics and as a result the implementation of this unit. The curriculum guides, government reforms, principals and parents previously mentioned represent just some of those factors. Levin (1992) stated that the organization is important in shaping what the individual thinks and does. Here the organization is represented not only by principals but by such things as textbooks, resources, district supports, time constraints and workload as well. As a result many of these things may have a bearing upon the teaching of mathematics in general and more specifically the

successful implementation of this unit. As Smith (1996) notes, teachers can feel pressure from many sources to return to teaching by telling.

Paul (1992) makes several suggestions to educators for redesigning their instruction. For example he offers the following as some general tactical recommendations:

1. Assess the amount of content as well as the way that it is covered.
2. Analyze the logic of what is taught.
3. As often as possible use cooperative learning as a teaching tool.
4. Let the student's knowledge, perceptions, misconceptions and attitudes determine the starting point.
5. Speak less so that students can think more.
6 Do not be a mother robin.
7 Focus on fundamental and powerful concepts with high generalizability.
8. Present concepts, as far as possible, in the context of their use as functional tools for the solution of real problems and the analysis of significant issues.
9. Develop specific strategies for cultivating critical reading, writing, speaking and listening.
10. Think aloud in front of the students.

11. Regularly question the students.
12. Call frequently on students who do not have their hands up.
13. Use concrete examples wherever possible to illustrate abstract concepts and thinking.
14. Require regular writing for class.
15. Spell explicitly the intellectual standards of the system of grading.
16. In general, design all activities and assignments, including readings, so that students must think their way through them.
17. Keep the logic of the most basic concepts in the foreground.
18. Let the students know what they are in for.

(Paul, 1992, p. 19-20)

It is this general list that provides the foundation for this unit of Euclidean geometry. The unit begins with a pre-lesson activity to build group dynamics as the entire unit is based on a theory of cooperative learning. Wheatley (1991) states that, "students can profit greatly by working together ... when students work in small groups they are stimulated by challenges to their ideas and thus recognize the need to reorganize and reconceptualize. The very act of formulating an expression of their views promotes reflection which then leads to revision" (p. 18). It is at this time the students are told what they are in for. As Paul (1992) suggests discussion at this time centres around active and passive learning and the need to develop student's abilities to think.

From the unit outline one can see that the following or first three lessons are based on the discussion, pretest and review of what is believed to be prior knowledge. Again, Paul (1992) suggests that we should let the student's knowledge, perceptions and misconceptions and attitudes determine the starting point. Thus, the first three lessons are trying to determine a starting point for individual students and groups. "This approach avoids the error of teaching material students already know and the error of presenting ideas which are, at the same time, beyond the student's level of comprehension" (Wheatley, 1991, p.17).

Table 3.1: UNIT OUTLINE

			Pre-lesson activity (Group dynamics)
1	1 - 5		Discussion
2	Pretest		Test on outcomes 1-5 (see appendix)
3			Review 1-5 / Discuss pre-test
4	6	#1	Congruent Triangles
5		#2	Congruency Game
6	7		Worksheet (see Pasko (1995, pp. 140 - 151) for sample)
7	8	#3	VOAT / Angle Bisector
8		#4	Creating an Angle Bisector
9		#5	Angles and Parallel Lines
10		#6	Perpendicular Bisector
11	9	#7	Cyclic Angles
12			
13		#8	Relations between Chords and Centre
14	10		Worksheet (see Pasko (1995, pp. 140 - 151) for sample)
15	11	#9	Similar Triangles
16	12		Worksheet (see Pasko (1995, pp. 140 - 151) for sample)
17	13	#10	Pythagorean Theorem
18			Worksheet (see Pasko (1995, pp. 140 - 151) for sample)
19			Journal discussion
20			Group Test (see appendix)
21			Individual Test (see appendix)

The students are then presented with activities, ten in total, which outline general tasks and problems. The core of problem centred learning is a set of problematic tasks that will guide students to construct effective ways of thinking about the subject. Knowledge is not passively received, but is rather actively built by students constructing their own mathematical meaning. Thus, these tasks are accessible to everyone at the start, invite students to make decisions, encourage what if questions, encourage students to use their own methods, promote discussion and communication, are replete with patterns, lead somewhere, have an element of surprise, are enjoyable and extendable (Wheatley 1991).

Furthermore, these tasks focus not so much on isolated facts as they do on larger concepts or big ideas. Greenes (1995) states that, "by learning and using the process of investigation to explore the big ideas in various contexts, students gain a robust and more lasting understanding of the concepts ... understanding and discovery are facilitated by a curriculum that has a strong component of exploration, investigation and is problem based" (p. 85). Also, many of these tasks and their concluding journal prompts are in the form of open ended questions as Hancock (1995) notes, "many open ended questions usually offer students multiple approaches to the problem by placing little constraint on students' method of solution" (p. 496).

Here the goal is learning and mathematics is effectively learned only by experimenting, questioning, reflecting, discovering, inventing and discovering.

Favourable conditions for such learning exist when a student is faced with a task for which there is no known procedure available or in short a problematic situation. When students engage in such learning activities for the purpose of constructing relationships there is potential for significant results and when mathematics is taught from such a problem centred perspective there is potential for significant benefit to the students (Wheatley, 1991).

Dossey (1988) states that' "learning mathematics is not doing piles of homework and repetitive drill but rather performing an action that calls for reflection and perseverance" (p. 292). Yet, the unit does include optional worksheets which may be considered drill and practice. However, it is explained in the introduction of the unit itself that the unit represents a transition in approach to mathematical teaching and thus, sample worksheets may be used as the individual teacher sees the need or ignored completely if one wishes to take a more constructive approach to the unit. Never the less, it is imperative that the students be encouraged to reflect and record their thoughts, for thinking to grow, the important measure is not the number of questions done but the quality of thought put into tackling the questions and into reviewing the effort (Mason, Burton & Stacey, 1996, p. 150). Perkins and Blythe (1994) add that, "many performances are too routine to be understanding performances however, such routine performances have their importance too, but they do not build understanding" (p. 6). As such, it is up to the individual teacher to decide if the worksheets are routine, if they are

routine for all students, or if they have some importance to individual students -- and maybe those decisions can be given to the students. Either way the worksheets are optional and do not make up the main crux of the unit which is still problem centred.

Two other key components of this unit are discussion and journal writing. As students work in their small groups they share ideas, discuss strategies and debate possible solutions. "Perhaps the greatest benefit of the investigative component lies in the verbal interplay when individual participants take turns in presenting and defending their conjectures ... the opportunity to share one's thoughts in a supportive setting, without fear of being wrong, can only enhance the problem solving process" (Taback, 1992, p. 255). Then at the end of the day or after each group has come to some conclusion about each activity there is a full class discussion where groups share their ideas, their strategies and their thinking. "Class discussion, where students share their solutions, provides a forum for students to construct explanations of their reasoning. In the process of telling others how they thought about a problem, students elaborate and refine their thinking and deepen their understanding" (Wheatley, 1991, p.19).

"Two chief ways of actively promoting thinking are through speaking and writing" (Litecky, 1992, p. 86). "Writing allows students an opportunity to examine their thoughts" (Miller, 1992, p.8). "The ability to articulate and express concepts in written discourse is crucial in students' ability to learn with understanding, rather than resort to memorization or imitation of process" (Grossman, Smith & Miller, 1993, p.4). "Getting

students to articulate their own thinking at the point where they are coming to terms with a new idea, or meeting difficulty, is essential to helping many to move into the more reflective mode of writing, characterized as dialogue" (Clarke, Wayward & Stephens, 1993, p.248). Therefore, as the students work through the unit they are asked to keep a daily journal of what they do, try, and think. The journal is a place for the students to ultimately reflect on their own thinking and consolidate their own understanding. "Reflective thinking is considered to be a method of problem solving that holds that students learn by doing and by thinking about what they did" (Barba, 1990, p.35). "The habit of logical reflection on what one learns is a key to critical thinking ... critical thinking enables a person to achieve genuine knowledge rather than mere recall" (Paul, 1992, p. 16).

Finally, the unit is concluded with a full class discussion of insights made in individual journals, a group test and an individual test. Raymond (1994) notes that where students work cooperatively to solve problems, group problem solving tests are a natural means of evaluating students learning. The individual test then is a way to ensure that every one in the group shares that learning.

Greenes (1995) has stated that there are five processes in the investigative approach with those being: observation and formulation of questions, gathering of information, analysis of information, evaluation of conclusions and communication of results. Wheatley (1991) claims that problem centred learning has three components:

tasks, groups and sharing. This unit of Euclidean geometry encompasses all of these processes and components and represents a marked shift from the traditional methodologies.

Greenes (1995) when quoting the American Association for the Advancement of Science (1989) states that:

> The present curricula in science and mathematics are overstuffed and undernourished. They emphasize the learning of answers more than the exploration of questions, memory at the expense of critical thinking, bits and pieces of information instead of understanding in context, recitation over argument, reading in lieu of doing. They fail to encourage students to work together, to share ideas and information freely with each other (p. 90).

This unit emphasizes that exploration, critical thinking, understanding and doing.

Chapter 4: Euclidean Geometry Unit

Euclidean Geometry

Grade Nine

Mathematical Problem Solving Unit

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Chapter Four is the complete unit with introduction, purpose, objectives and significance, provided in a form so that it may be easily reproduced and used by other educators in the classroom. As a result, readers may notice some repetition of content from Chapter One. This repetition is deliberate so that Chapter Four can stand alone and separate from the rest of the project. I do hereby give permission to other educators to copy and use this unit in their teaching.

The attitudes which students hold toward mathematics have been shown to correlate with mathematics achievement and most teachers realize the importance of how students feel about the subject. Most teachers also realize that they should help students feel free to explore a variety of ideas in reaching solutions and verifying their own thinking. Confidence in mathematics is fostered by encouraging students to explore mathematical concepts, ask questions, discuss their ideas and make mistakes. Students who enjoy mathematics and work in a constructive environment feel comfortable and confident and in turn perform better and learn more. This discovery approach to learning is also well suited to the small group structure. Students learn mathematics by doing mathematics. With proper (subtle) guidance by the teacher, students can discover relationships, plan procedures, formulate definitions, make conjectures, construct examples and counterexamples, and solve problems. By listening to students' ideas and by having them listen to one another, the teacher can establish an attitude of mutual respect (Intermediate Mathematics Curriculum Guide, 1995).

This unit on Euclidean geometry introduces students to the dynamics of group structures, encourages students to listen to each other and most importantly employs a discovery approach to mathematics. Students explore a variety of ideas in a small group setting and construct their own meaning for the mathematical problems presented. Students are then asked to reflect on this new found knowledge and record their thoughts in a journal. This unit represents a transition in approach to mathematical teaching and

therefore the sample worksheets may be used as the individual teacher sees the need or ignored completely if one wishes to take a more constructive approach to the unit. However, it is imperative that the students be encouraged to reflect and record their thoughts, for thinking to grow, the important measure is not the number of questions done but the quality of thought put into tackling the questions and into reviewing the effort (Mason, Burton & Stacey, 1996, p. 150). It should also be noted that the first five outcomes are assumed to be prior knowledge and it is here that the unit starts. A discussion and pre-test will refresh ideas for students and ensure that the teacher has not assumed too much. It should also allow the student to experience a degree of success and thus enhance their confidence at the onset of the unit. While this unit may represent a transition in approach it does reflect the changes called for -- and the standards of -- the NCTM (1989) and the Intermediate Mathematics Curriculum Guide (1995).

Summary of Changes in Instructional Practices in 9-12 Mathematics

- The active involvement of students in constructing and applying mathematical ideas.
- Problem solving as a means as well as a goal of instruction.
- Effective questioning techniques that promote student interaction.
- The use of a variety of instructional formats (small groups, individual explorations, peer instruction, whole class discussions, project work).

- The use of calculators and computers as tools for learning.
- Student communication of mathematical ideas orally and in writing.
- The establishment and application of the interrelatedness of mathematical topics.
- The systematic maintenance of student learnings and embedding review in the context of new topics and problem situations.
- The assessment of learning as an integral part of instruction.

(NCTM, 1989, p. 129)

Standard 1: Mathematics as Problem Solving

In grades 9-12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can --

- use, with increasing confidence, problem solving approaches to investigate and understand mathematical context;
- apply integrated mathematical problem solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems from situations within and outside mathematics;

- apply the process of mathematical modelling to real world problems.

(NCTM, 1989, p. 137)

Standard 2: Mathematics as Communication

In grades 9-12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can --

- reflect upon and clarify their thinking about mathematical ideas and relationships;
- formulate mathematical definitions and express generalizations discovered through investigations;
- express mathematical ideas orally and in writing;
- read written presentations of mathematics with understanding;
- ask clarifying and extending questions related to mathematics they have read or heard about;
- appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.

(NCTM, 1989, p. 140)

Standard 3: Mathematics as Reasoning

In grades 9-12, the mathematical curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can --

- make and test conjectures;
- formulate counterexamples;
- follow logical arguments;
- judge the validity of arguments;
- construct simple valid arguments; and so that, in addition, college intending students can --
- construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

(NCTM, 1989, p.143)

Standard 7: Geometry from a Synthetic Perspective

In grades 9-12, the mathematics curriculum should include the continued study of the geometry of two and three dimensions so that all students can --

- interpret and draw three dimensional objects;
- represent problem situations with geometric models and apply properties of figures;
- classify figures in terms of congruence and similarity and apply these relationships;
- deduce properties of, and relationships between, figures from given assumptions; and so that, in addition, college intending students can --
- develop an understanding of an axiomatic system through investigating and comparing various geometries.

(NCTM, 1989, p. 157)

Problem Solving

The ability to solve problems is the principle reason for studying mathematics. A true problem involves situations or events for which there is no immediate or obvious solution. If a student knows the solution to a problem automatically, it cannot be considered a true problem. Likewise, whether a given situation represents a true problem is determined by the previous experience of the student. That which is a problem for one student may be 'routine' for another. In problem solving it is sometimes

the case that more than one solution is possible or, indeed, no answer is found. Some problems require considerable time to deliberate the possibilities. Teachers should take deliberation time into account when assigning problems to students.

At the intermediate level students should be exposed to a number of problem situations which extend the strategies that were developed in the elementary grades. Strategies previously developed are revisited at an increased level of sophistication and new strategies are introduced. The focus of instruction, therefore, should be on the strategies required to facilitate successful problem solving. There are two types of problems, nonroutine (process) problems and routine (application) problems. At the intermediate level specific time and attention should be given to nonroutine problems. Routine problem solving is derived from the application of concepts and skills while utilizing the strategies developed with nonroutine problems.

(Intermediate Mathematics Curriculum Guide, 1995, p. 14)

Communication

The communication process of speaking, reading, writing and listening are important to the teaching and learning of mathematics.

Talking about mathematics provides students with opportunities to clarify their thinking and is critical in the development and understanding of mathematical concepts. Students working in small groups can learn to solve problems by discussing them. When students write about these problems and their approach to problems they will develop a greater understanding of the mathematical concepts involved. This writing forces students to reflect and it is this reflection that reconfirms their understanding. Open ended problems provide these opportunities for writing and greater understanding of mathematics.

(Intermediate Mathematics Curriculum Guide, 1995, p. 187)

Critical Thinking

An inherent part of the learning process is developing a knowledge of oneself as a learner and the self regulation of one's own learning activity. A major goal of all education is that students become self regulated learners, thereby seeing themselves as problem solvers. Students should be conscious of how they solve problems and have the ability to monitor their own thinking.

(Intermediate Mathematics Curriculum Guide, 1995, p. 9)

Cooperative Small Group Learnings

Provision of varied organizational structures increases the likelihood of meeting the developmental needs of intermediate students and prepares them to deal with and respond to the increasing demand for cooperation, involvement, and individual responsibility in an information age. Peer group relations are very important for intermediate students. In cooperative learning structures, students work together and share responsibility for each other's learning through sharing of goals, resources, and rewards. Students increase their understanding of others' perspectives and learn to accommodate themselves to the perspectives of others. This becomes especially apparent when students are involved in problem solving activities. They are exposed to the thinking of others and through that exposure recognize that there are often many strategies which can be applied to solve a given problem.

Cooperative learning structures help students develop more positive attitudes toward themselves and others, as well as the whole process of teaching, and learning. Low risk situations allow students to contribute according to their different capabilities. They feel accepted and supported by peers and confident in their ability to contribute to the group effort.

Cooperative learning has also been found to be a powerful tool for shaping socio-academic behaviour. Students help each other learn as ideas are communicated in terms they understand. Through discussion with peers, students are exposed to ideas which may extend and augment their own understanding. Higher level reasoning, divergent thinking, and problem solving skills will result as students clarify their own thinking.

(Intermediate Mathematics Curriculum Guide, 1995, p. 174)

Table 4.1: UNIT OUTLINE

			Pre-lesson activity (Group dynamics)
1	1 - 5		Discussion
2	Pretest		Test on outcomes 1-5 (see appendix)
3			Review 1-5 / Discuss pre-test
4	6	#1	Congruent Triangles
5		#2	Congruency Game
6	7		Worksheet (see Pasko (1995, pp. 140 - 151) for sample)
7	8	#3	VOAT / Angle Bisector
8		#4	Creating an Angle Bisector
9		#5	Angles and Parallel Lines
10		#6	Perpendicular Bisector
11	9	#7	Cyclic Angles
12			
13		#8	Relations between Chords and Centre
14	10		Worksheet (see Pasko (1995, pp. 140 - 151) for sample)
15	11	#9	Similar Triangles
16	12		Worksheet (see Pasko (1995, pp. 140 - 151) for sample)
17	13	#10	Pythagorean Theorem
18			Worksheet (see Pasko (1995, pp. 140 - 151) for sample)
19			Journal discussion
20			Group Test (see appendix)
21			Individual Test (see appendix)

Outcomes

1. → Understand the concepts of point, ray, line, line segment and angle.
   → Identify / name correctly, a point, line, ray, line segment and angle.
2. → Identify / classify complementary and supplementary angles.
   → Understand and apply the equality of opposite angles.
   → Calculate the measures of unknown angles.
3. → Classify triangles by sides.
   → Classify triangles by angle measures.
   → Find the missing values of a triangle, given certain information.
   → Understand that the sum of the angles in a triangle is 180°.
   → Understand the relationship between exterior and interior angles .
4. → Identify parallel lines, alternate, corresponding and interior angles.
   → Use parallel line, transversal relationship to find angle measures.
5. → Identify polygons and regular polygons.
   → Classify polygons by the number of sides.
   → Calculate the measures of interior angles of a polygon.
6. → Describe the properties conditions for congruent triangles.
7. → Derive information about triangles from congruency relationships.

8. → Derive new information from angle properties. Including vertically opposite angles, corresponding angles, alternate angles, angle bisector, perpendicular bisector, etc.

9. → Recognize various properties of segments and angles constructed within a circle. These properties include: a) The relationship between central and inscribed angle. b) The relationship between two inscribed angles drawn on the same arc. c) The size of a inscribed angle subtended by a diameter. d) The relationship between opposite angles of an inscribed quadrilateral. e) The relationship between chord length and distance from the centre. f) The use of right bisectors to determine centres of circles. g) The relationship between a chord and a perpendicular that passes through the centre of a circle.

10. → Integrate the various properties of segments and angles within circles to find missing measures.

11. → Recognize the properties of similar figures.

12. → Apply the properties of similar figures

13. → Solve problems using Pythagorean relationship.

(Intermediate Mathematics Curriculum Guide, 1995)

Table 4.2: ACTIVITY / TEXT CORRELATION

1	Congruent Triangles	Math Power	464
2	Congruency Game	Math in Context	421
3	VOAT / Angle Bisector	Minds on Math	416
4	Creating an Angle Bisector	Math in Context	66
5	Angles and Parallel Lines	Minds on Math	424
6	Perpendicular Bisector	MathPower	483
7	Cyclic Angles	Minds on Math	444
8	Relations between Chords and Centre	Minds on Math	437
9	Similar Triangles	Math in Context	422
10	Pythagorean Theorem	Minds on Math	194

Group Development Activities

The following activities are optional and not necessarily a required element of the geometry unit. It is recommended that if students are not comfortable with working as a group that these activities, or similar ones, be conducted as an introduction to the unit. The purpose of such activities are to familiarize the student with the structure of a group, the role of a leader and the individual members. These are also good opportunities for

students to see how each member contributing their part combine to successfully obtain a goal.

Activities

The students are given approximately fifty minutes to complete two tasks. The first task involves large groups of eight to twelve students. The groups are given a square milk crate, and asked to stand all students at once, on top of the box for ten seconds. After several minutes of trying different positions and methods of standing, students will discover that the task is easily done by pairing with each pair standing on opposite sides of the box. All pairs of students are added until everyone is on the box, forming a structure that looks similar to an inverted cone.

For the second task the students are asked to sit as if they were on a chair, with their hands on their heads and feet flat on the floor, without using anything but one another. This can be done by standing in a circle and sitting on the knees of the person behind. The teacher can then conclude by discussing group dynamics in general and how each member plays an important role to form a group and complete a given task.

Activity 1: Congruent Triangles

*** The following activity is to be done in groups of four ***

Exploration 1

1. Follow these steps to construct ΔABC where AB = 7 cm, BC = 8 cm, and AC = 10 cm.
   (a) Draw line segments AB = 7 cm.
   (b) Set your compasses to a radius of 8 cm. Then, use B as a centre and draw an arc.
   (c) Set your compasses to a radius of 10 cm. Then, use A as a centre and draw an arc to intersect the first arc. Label the point of intersection C.
   (d) Draw BC and AC.
2. Is it possible to construct ΔDEF where DE = 7 cm, EF = 8 cm, and DF = 10 cm so that the size and shape of ΔDEF are different from the size and shape of ΔABC? Explain.

Exploration 2

1. Construct ΔGHI so that GH = 5 cm, and GI = 7 cm.
2. Is it possible to construct ΔJKL with JK = 5 cm, and JL = 7 cm so that the size and shape of ΔJKL are different from the size and shape of ΔGHI? Explain.

Exploration 3

1. Construct ΔMNO so that $\angle M = 35°$, $\angle N = 65°$, and $\angle O = 80°$.

2. Is it possible to construct ΔPQR with $\angle P = 35°$, $\angle Q = 65°$, and $\angle R = 80°$ so that the size and shape of ΔPQR are different from the size and shape of ΔMNO? Explain.

Exploration 4

1. Construct ΔRST so that $\angle R = 70°$, RS = 6 cm, and RT = 7 cm.

2. Is it possible to construct ΔXYZ with $\angle X = 70°$, XY = 6 cm, and XZ = 7 cm so that the size and shape of ΔXYZ are different from the size and shape of ΔRST? Explain.

Exploration 5

1. Construct ΔABC so that $\angle A = 40°$, AB = 7 cm, and $\angle B = 55°$.

2. Is it possible to construct ΔDEF with $\angle D = 40°$, DE = 7 cm, and $\angle E = 55°$ so that the size and shape of ΔDEF are different from the size and shape of ΔABC? Explain.

Exploration 6

1. Construct ΔPQR so that $\angle P = 35°$ and PQ = 7 cm.

2. Is it possible to construct ΔXYZ with $\angle X = 35°$ and $XY = 7$ cm, so that the size and shape of ΔXYZ are different from the size and shape of ΔPQR? Explain.

Problem

Which sets of 3 facts need to be given so that only 1 triangle can be constructed?

Journal Entry

Congruent triangles are triangle are triangles that have the same size and shape. What information do you need to determine if triangles are congruent?

Activity 2: Congruency Game

*** The following activity is to be done in groups of four ***

1. Construct a spinner showing four spaces: ASA, SAS, SSS, NOT CONGRUENT
2. Each person in the group is to then construct six pairs of congruent triangles and three pairs of non congruent triangles -- with the appropriate information showing on each triangle.
3. Decide who spins first. Spin the spinner. Find pairs of triangles that match what you spin.
4. You receive one point for each correct answer. You lose one point for each incorrect answer.
4. Once a triangle has been named, it cannot be named again. If you do, you lose one point.
5. The player with the most points when all the triangles have been eliminated is the winner.

Journal Entry

In your journal, describe how playing this game helped you in your understanding of congruent triangles.

Activity 3: VOAT / Angle Bisector

*** The following activity is to be done in groups of four ***

You will need a piece of waxed paper and a protractor

Exploration 1

Fold the paper along a slanted line. Unfold it to show a crease.

Exploration 2

Fold the paper along a line that intersects the first one. Unfold the paper to show two creases.

- Four angles are formed by the creases.
- Which of these angles are equal?
- Check your answers by measuring with a protractor or folding the paper. Then use your paper model to explain why the angles are equal. These pairs of equal angles are called opposite angles.

An angle bisector is a line which divides an angle into two equal angles. You can use your paper model to explore angle bisectors.

Exploration 3

Fold the paper so that one arm of one angle lies on top of the other arm. Make a crease and unfold the paper. This crease is the angle bisector.

Repeat with one of the other angles.

- What do you notice about the bisectors? Why do you think this happened? Compare your result with other students. Did everyone get the same result?
- In your classroom, find an example of intersecting lines. To your partner, describe the angles that are formed.

Journal Entry

Why are vertically opposite angles equal?

Activity 4: Creating an Angle Bisector

*** The following activity is to be done in groups of four ***

Exploration 1

(a) How are the following words alike? How are they different?

bicycle bilingual bicentennial bilateral biweekly

(b) What is the meaning of each word in (a)?

(c) Refer to the meanings in (b). What do you think the prefix "bi" means?

(d) What do you think is meant by the term "bisect"?

Exploration 2

(a) Describe how you can use tracing paper to bisect an angle.

(b) Giver your description to your partner. Have your partner try to bisect an angle using your description. How reasonable was your description?

Exploration 3

(a) Describe how you can use a compass and a straight edge to bisect an angle.

(b) Giver your description to your partner. Have your partner try to bisect an angle using your description. How reasonable was your description?

Journal Entry

What is the relationship between bisected angles and congruent triangles?

Activity 5: Angles and Parallel Lines

*** The following activity is to be done in groups of four ***

You can use paper triangles to investigate some properties of parallel lines. You'll need paper, scissors, a ruler, and a protractor.

Exploration 1

Make a scalene triangle.

Exploration 2

Draw a line connecting the midpoints of two sides and identify two angles on either side (but each on the same side) that open in the same direction.

- The base of the triangle and the line joining the midpoints are parallel lines. The side of the triangle is a transversal. The marked angles are called corresponding angles.
- When a transversal intersects two parallel lines, what do you think is true about the corresponding angles?
- Find another pair of corresponding angles. Mark these angles.

Exploration 3

Fold the triangle along the line joining the midpoints, so that the top vertex lies on the base. Draw a line along each edge of this folded portion.

Exploration 4

Mark the angle between the top fold and one of these lines. Now mark the angle that is between the opposite side of this line and the base of the triangle. Look at the marked angles. How do you think their measures compare? Check your prediction. Mark the angles on your triangle to show what you have discovered.

- The marked angles are called alternate angles.
- When a transversal intersects two parallel lines, what do you think is true about the alternate angles?
- Find another pair of alternate angles. Mark these angles.

Journal Entry

Will you get the same results concerning corresponding and alternate angles with a right, isosceles and an equilateral triangle? Are there any differences or special cases that you see?

Activity 6: Perpendicular Bisector

*** The following activity is to be done in groups of four. ***

You will need a piece of waxed paper, compass and a protractor

Exploration 1

Draw a line 10 cm long. With a ruler and protractor construct its perpendicular bisector.

Exploration 2

Draw a line 10 cm long on a piece of wax paper or tracing paper and then fold the paper so as to make a crease along its perpendicular bisector (note: do not use your protractor)

Exploration 3

Draw a line 10 cm long. With your compass draw two semi circles (with a radius larger then half your line) each with its centre at opposite ends of your segment. Join the points of intersection. What do you notice?

Journal Entry

Why are we able to construct perpendicular bisectors with just a compass. What mathematical principles are we using?

Activity 7: Cyclic Angles

*** The following activity is to be done in groups of four. ***

You will need several sheets of paper, some light cardboard, a piece of corrugated cardboard, some tacks, scissors, a ruler, a protractor, and a pair of compasses.

Exploration 1

1. Construct a circle. Draw any diameter AB.
2. Make a point C on the circle. Join AC and BC.
3. Measure $\angle C$.
4. Repeat exercises 2 and 3 for other positions of C on the circle. What do you notice?
5. Repeat exercises 1 to 4 for other circles. Are the results the same?

Exploration 2

1. Place a sheet of paper on the corrugated cardboard, and insert two tacks. Cut a piece of cardboard to form a right angle. Position the corner of the cardboard against the tacks. Move the cardboard, keeping it against the tacks. Observe how the position of the corner of the cardboard changes. Use a pencil to mark some of these positions. What figure do you think is formed by the corner of the cardboard?

2. How is this related to your observations in Exploration 1.

Exploration 3

1. Construct a circle. Draw any chord DE (not a diameter).
2. Mark a point F on the circle. Join DF and EF.
3. Measure $\angle F$.
4. Repeat exercises 2 and 3 for other positions of F on the circle. What do you notice?
5. Repeat exercises 1 to 4 for other circles. Are the results the same?

Exploration 4

1. Place a sheet of paper on the corrugated cardboard, and insert two tacks. Cut a piece of cardboard to form an acute angle. Position the corner of the cardboard against the tacks. Move the cardboard, keeping it against the tacks. Observe how the position of the corner of the cardboard changes. Use a pencil to mark some of these positions. What figure do you think is formed by the corner of the cardboard?
2. How is this related to your observations in Exploration 3.

Exploration 5

1. Construct a circle. Mark the centre O.
2. Draw any chord GH. Join GO and HO.
3. Mark a point J on the circle. Join GJ and HJ.
4. Measure ∠O and ∠J. What do you notice?
5. Repeat exercises 3 and 4 for other positions of J on the circle.
6. Repeat exercises 2 to 5 for other chords. Did you always get the same results?

Exploration 6

1. Construct a circle.
2. Mark any four points A, B, C, and D on the circle and join to form a quadrilateral.
3. Measure ∠A and ∠C. Measure ∠B and ∠D. What do you notice?
4. Repeat exercises 2 and 3 for other positions of the points on the circle. Did you always get the same results?

Journal Entry

Angles with their vertex on the circumference of a circle are called "inscribed angles". Angles with their vertex on the centre point of a circle are called "central angles". What do you know about inscribed angles and central angles in relation to each other and the chords that subtend them?

Activity 8: Relations Between Chords and Centre

*** The following activity is to be done in groups of four. ***

You will need paper, a ruler, a protractor, and a pair of compasses. A line segment joining any two points on a circle is called a chord.

Exploration 1

1. Construct a circle. Draw any chord AB. Construct the perpendicular bisector of the chord. Through what point does the perpendicular bisector appear to pass?
2. Repeat exercise 1 for other chords and other circles. Did you always get the same results?

Exploration 2

1. Construct a circle. Draw any chord CD. Locate the midpoint M of the chord. Join it to the centre O of the circles. Measure $\angle OMC$ and $\angle OMD$. What do you notice?
2. Repeat exercise 1 for other chords and other circles. Did you always get the same results?

Exploration 3

1. Construct a circle. Draw any chord EF. Construct a line through the centre O of the circle, that is perpendicular to EF. Let N be the point where the perpendicular meets EF. Measure NE and NF. What do you notice?
2. Repeat exercise 1 for other chords and other circles. Did you always get the same result?

Journal Entry

Will chords of equal length be the same distance from the centre, how can you prove your answer? How can we find the centre of a circle using perpendicular bisectors?

Activity 9: Similar Triangles

*** The following activity is to be done in groups of four ***

Exploration 1

1. Did you know that, for some movies, special effects experts create miniature sets and models in their studios? A miniature set is created before the similar life sized versions are constructed.

   (a) How are the miniature set and similar life sized set alike? How are they different.

   (b) Describe situations in which similar constructions could be helpful.

Exploration 2

2. Draw a 5cm square and a 10 cm square

   (a) What do you think is meant by the word "similar"? Do the squares look similar?

   (b) Use a dictionary to find a definition "similar". Based on the definition, are the squares similar?

Exploration 3

3. (a) Draw a triangle ABC

(b) Measure the sides of your triangle ABC.

(c) Multiply the lengths of each side by 3 and draw the corresponding triangle and label it XYZ.

(d) Do the triangles look similar?

(e) Measure the angles in each triangle. What do you notice?

(f) Repeat the process multiplying the lengths by 1/2. What do you notice ?

Exploration 4

4. (a) Draw two 30 - 60 - 90 triangles of different sizes.

(b) Label the triangles ABC and XYZ so that angle A matches angle X etc.

(c) Calculate the ratios AB/XY, BC/YZ, and CA/ZX. What do you notice?

(d) Repeat for other triangles. Are your results the same?

Journal Entry

Similar triangles have three equal angles and equal ratios for corresponding sides. Can we say that triangles are similar if we are only told that they have two equal angles?

Activity 10: The Pythagorean Theorem

*** The following activity is to be done in groups of four ***

Any triangle that has a right angle in it is called a right triangle. The side opposite the right angle is called the hypotenuse. The other two sides are sometimes called the legs. More than 4000 years ago the Babylonians knew a special property that is satisfied by a right triangle, and no other kind of triangle. It was first proven by the ancient Greeks, and is named after the Greek mathematician Pythagoras. To investigate the Pythagorean Theorem, follow these steps.

1. Copy several different squares onto 1-cm grid paper. Determine the area of each square and write the area on each.
2. Cut out the squares.
3. Take three of the squares and arrange them so that their edges form a triangle. Find three squares that form a right triangle when you do this (you made need to make and cut out more squares). Try to get as many different right triangles as you can. When you think you have a right triangle, how can you be certain that it is a right triangle?
4. For each right triangle you get, record the areas of the three squares in a table.

Journal Entry

What is the relationship between the areas of the squares on the legs of a right triangle and the area of the square on the hypotenuse?

Chapter 5: Personal Reflections

In this section, I will provide personal reflections, in the format of a step by step or daily journal, so that teachers planning to use this unit may see the progression that they may expect from their students as they work through this unit of Euclidean geometry. Note, these reflections are based on my own trials of this material and this approach. I will also offer suggestions and make note of some concerns, or potential problem areas that prospective teachers might want to be aware of as they implement this unit. These potential problem areas, or concerns, are not concerns with the unit itself but instead difficulties that students might have in making the transition from the traditional, explain practice method of teaching towards this new problem centred style of learning. For most students, this unit will be transitional and, as a result, prospective teachers can expect some transitional difficulties and some concerns in relation to the change process in general -- as noted in the literature review. However, these growing pains are just that and do not last long. They are only mentioned here as an issue of awareness and as a means of encouragement. In this section, the focus is more on selected specifics of the unit as opposed to generalizations and statements on problem solving, journal writing and critical thinking. This is done so that prospective users, or teachers, can follow and see some of the transitions made and the issues raised as they occur.

This unit on Euclidean geometry begins by introducing students to the dynamics of group structures. Pre-lesson activities are designed for the purpose of familiarizing students with the structure of a group, the role of a leader and the parts played by the individual members. This is, indeed, an excellent opportunity for students to see how

each member, contributing their part, combines to successfully obtain a goal. However, in a grade nine class where there has been limited group work of this nature, there may be a need for additional work on group dynamics. Group work is not uncommon to students. Indeed, students have probably had numerous occasions to work in groups throughout their school career. However, the group work that they would have experienced in the past will be somewhat different from what they will experience in this unit. I would suggest that, in the past, groups would often have been more social and individual members less dependent on one another. The work done may have easily been completed by individual students. Now, however, the students will need to share ideas, listen to each other, debate concepts, and reflect to find the best possible solution. Individual members will be much more dependent on one another and the success of the group will depend largely on the contributions of its individual members. This also implies that the skills and habits of listening, sharing and reflecting are very important. However, it is these very skills and habits that I have found lacking in grade nine students.

The first task in this pre-lesson activity requires groups of eight to twelve students to stand, all students at once, on top of a milk crate for ten seconds. For the second task, the students are asked to sit down as if they were on a chair, with their hands on their heads and their feet flat on the floor, without using anything but one another. I should note that, in the past, I have not discussed the roles of group members or assigned

roles within the selected groups before these activities. Instead, I have used these experiences, their success or apparent lack thereof to introduce and discuss some of the underlying ideas in the theory of cooperative group learning. At that time, we would discuss the roles of each member in the group and the necessity of listening, sharing, debating and reflecting. As for the pre-lesson activities themselves, a natural leader usually emerged and directed the group through different possibilities until a solution was found. However, this was not always the case. Occasionally, the dominant figure would not listen to the other students in the group. As a result, there was no reflection or sharing of ideas and no progression towards the one best solution. Quite often, one person took charge and attempted individual suggestions at random as each suggestion successively failed. There was no joint effort, no collaboration or sharing of ideas. The groups attempted or initiated the suggestions of individuals and not those of the collective group. Again, the difference is subtle but one that needs to be recognized.

The pre-lesson activities excited the students and seemed to provide all the necessary motivation to continue the unit. However, before continuing with the unit, there should be a large group discussion on group dynamics and the importance of sharing, listening and reflecting to arrive at a collaborative solution. This is such an important idea that the fifty minutes set aside in this unit may not be sufficient. A teacher may need to supplement, depending upon the needs of the class, with similar activities

until these skills are developed further before adding the additional task of a mathematical problem.

From there, the unit proceeds with a full class, as opposed to small group, discussion of the first five outcomes. A disruptive or uninterested student, at this point, will have more of a detrimental effect on a group of four than a larger group. Once again, however, time may become an issue. The unit has set aside one fifty minute period, however, in the past I have taken as many as two fifty minute periods to conclude a satisfactory discussion. By now, it should be becoming apparent that the key to this problem centred style of learning, and thus this unit, is flexibility. The unit should not be driven by the unit outline but instead by the abilities and needs of the students. What may take one class, one lesson may take another class, three lessons. What may take one group, twenty minutes may take another group, forty. I have definitely found a need for flexibility. The discussion itself, I initiated with a prompt, similar to a question/answer period, but this usually lasted for only a few minutes and soon discussion between the students erupted. I say erupted because students were generally excited when they realized that they could make a contribution and that they were discussing something that they understood. Before too long, discussion between the students did not need to be prompted with questions. They started to actually listen to each other and reflect on what was being said. I have not prohibited discussion nor even tried to pace it. I would let the

discussion run its course and only throw out the next objective for discussion when the class reached a conclusion.

The activities and explorations that make up the balance of this unit employ a discovery approach to mathematics. Students explore a variety of ideas in a small group setting and are expected to construct their own meaning for the mathematical problems presented. My initial experiences with the unit made it quite clear to me that if I was going to maintain the integrity and the underlying philosophy of the unit, then the individual groups would have to be given the necessary time to meet their individual needs. This also meant that the different groups would progress at different rates through the various activities. Indeed, questions of time, progression and the apparent abandonment of all structure will have to be addressed by each individual teacher to see how it aligns with their individual philosophy of learning. I would suggest that the unit has a set lesson structure based upon suggested time periods of fifty minutes. These are simply suggested time periods, as the underlying philosophy of individual and group exploration does not lend itself well to time constraints.

I have not imposed my thoughts upon the individual groups or resorted to teaching by telling and thus I have given the groups the time they needed to explore their ideas and construct their own meaning. Initially, this was a large step for me and my students who were used to being held together as a group. True, it does take a period of adjustment and the first few days may be quite challenging. After the initial introduction of this idea,

however, things soon settled down and even worked well. Thus, I would suggest that the activities be treated as such and that the students be given the opportunity to learn. I would suggest that the teacher not be bound by structured lessons or class periods and that individual groups be given the time to explore.

This then will raise the question as to the role of the teacher. Initially, I knew that I was not going to lecture, demonstrate or even model but, I was not sure of what I was going to do. I knew that I needed to guard against imposing my ideas upon the groups. So I threw it open and decided that I would define my role as we progressed through the unit. It soon became apparent that my first role was to be an encourager. The students were venturing into the unknown. They were used to being shown the way and given the why, and now they were being asked to find each. At first, I was bombarded with questions of "How do we do this?" or "Why is this so?" and this was a difficult pattern to break. Certainly, it was a habit that I had cultivated in them and on the other side it was also difficult for me to not break out into full explanations. So, I would move from group to group and sit in listening to discussions and encourage them to try. As time passed the students did not need as much encouragement to try a problem. They soon warmed up to the idea and even seemed to enjoy it. Instead they needed encouragement to try something else and thus on times I provided them with other considerations. Therefore, as a teacher, I circulated through the groups listening , encouraging students, providing positive reinforcement for gains made and offering guidance and direction when needed.

With activity one may come some other fundamental questions regarding the unit. Some questions may include: How does a teacher ensure that individual students or individual groups, for that matter, have understood the concepts presented in the given activity?; and, How does a teacher ensure that the groups have not come to false conclusions? Full class discussion is one of the underlying principles of this unit as is critical thinking, problem solving, journal writing and cooperative learning. After all groups have finished an activity, all groups meet to present, talk about and share their ideas and findings. This will ensure the teacher that the concepts are understood and will allow the student to learn from each other as they share and reflect upon the ideas presented. Note, this discussion is a student driven process and not directed by the teacher who would impose their thinking on the class.

The apparent lack of structure and the fundamental necessity of class discussion may also have consequences for the journal or at least raise questions about the purpose of the journal. Some examples may be: When is a journal kept?; Are entries made daily?; Are journal prompts discussed before writing?; If journal prompts are discussed then is the journal really a reflection of the individual students thoughts or those of the group?; Should journal entries be made before or after the large group discussion of each activity, or after journal entries are made?; and, Should students be encouraged to add to journals after group discussions? All of these questions can be answered after the teacher decides upon the purpose of the journal.

I have treated the journal as what McIntosh (1991) calls a log with open ended writing tasks, guided writing tasks and specific questions regarding how to and definition. I believe that a journal is an opportunity for individuals to reflect upon their thoughts and ideas. I do not think that a journal entry is simply an exercise where students provide answers to given problems and prompts. Therefore, I have encouraged students to keep a daily journal in which they reflect upon the discussions and ideas of the day and ask themselves questions of why. A journal will have daily entries and will reflect the thoughts of the individual students as they work in their groups through the given activities. Students were also encouraged to reflect upon the prompt given at the end of each activity and make connections with it, the activity and other questions that they raised themselves in their journal. These journals are ideally what Wayward (1992) classifies as dialogue journals. Finally, students were encouraged to share their reflections as found in their individual journal entries during the next class. It should be noted that these are high expectations for journals and may even be difficult habits to establish -- but habits worth establishing never-the-less.

With the first activity may also come questions about what to do with the student who is not contributing to the group and the student who is dominating the group. I had hoped that with encouragement and time, both of these problems would work themselves out and so, initially, I spent much of my time sitting in groups trying to get some students to consider the ideas of others while encouraging others to participate. This problem was

not as big as I expected since the general nature of the hands on activities provided most of the motivation needed. However, there were some students who were absolutely disinterested in the process at first and had to be encouraged constantly to participate. In other groups, there were situations created because of a great discrepancy in abilities which allowed one student to dominate, even intimidate, and become impatient with other members of the group. Here too, I had to sit in on group discussions and encourage proper protocol. At first, these were my primary tasks as facilitator and I spent much of my time trying to establish the working dynamics of the group. As a result, the unit may get off to a slow start.

By the time I have reached activity two, The Congruency Game, I have usually established a consciousness in all students so that they are at least aware of other group members and are willing to share ideas. However, there may still be students in the class who have been unwilling to contribute to the process but I have noticed a change in this attitude as the class continued with activity two. As the groups continued to play, the disinterested students were more inclined to participate.

By now, attention may be turning to the more finer aspects of the activities themselves. Teachers may now have the time to reflect, and may come to the realization, as I did, that their students are not problem solvers. If answers to students' problems are not immediately forth coming they turn to the teacher for an answer. As I have previously noted, this was a mind set created by years of regimented math and as such, a

hard mind set to overcome.

My initial experiences with this unit in the classroom left me wondering about how much change I was asking the students to make and how quickly I was expecting them to make it. By the time that we got to activity three I was wondering if the class was expected to make too big of a change too quickly. I continue to debate whether or not the first three activities could be more transitional. By this, I mean maybe the class could work as a large group on the first activity and the teacher could take a more active role. Then, with activity two, perhaps the class could be divided in half and likewise the active role of the teacher. Activity three would then bring a further down sizing in groups and less dependence again on the teacher. Again, this is a reflection of some of the flexibility needed as these decisions will have to be made by individual teachers reflecting on their individual classes. However, it is definitely something to consider with a class that is not used to this group work, problem solving and this discovery approach to mathematics.

By the end of the third activity, however, I noticed that students were more inclined to spend time working on their problems and realized that they were not going to be given the answers. I have also noticed that some groups were turning to some of the resource books in the class to find some of their answers. Some teachers may take exception to this and want the students to discover for themselves. Right now, however, I am of the opinion that if a student can research and find their own answer then they are problem solving as they would in the real world.

Teachers may also want to be aware of students who fail to make any connections between vertically opposite angles, congruent triangles and similar triangles. I have found that most students had a good grasp on vertically opposite angles and angle bisectors but were at a loss for ideas when it came to commenting on the journal entry of why vertically opposite angles were equal. We have worked as a large group to make connections to congruent triangles and many students seemed amazed at the suggestion of making triangles out of their intersecting lines. However, when they got the idea that they were not limited by the steps of the activity the ideas soon started to flow. I would suggest that the structure of the activity with its numbered steps and explorations may inhibit students or lead them to believe that their discoveries are to be made only from what is presented to them in the structure of the activity. Teachers may need to continue to emphasize that these activities are starting points and that the students are free to explore as they see fit and are not limited to the confines of the steps or structure of the activity. All students may not have, as of yet, broken the confines of their structured past and will need to be encouraged to make connections and extensions wherever possible.

Teachers will notice that students adjust quickly and maybe even quicker than they themselves. For example, after the initial period of adjustment, I assumed that activity four would be slightly more predictable than what I had initially experienced in the first three activities. However, students took advantage of their new found freedom and it soon became clear that not even I fully understood the ramifications of the problem

solving process that we were going through. The activity itself went well and all groups seemed on task and involved in the process. When we went to the large group discussion I found out just how involved they were. In exploration two, for example, students were asked to bisect an angle with tracing paper -- which I thought was a pretty routine process. I assumed that all would tell me that they drew an angle, folded it so that one arm was on top of the other and that the crease would be the angle bisector. When we finished we had twice as many ways to construct angle bisectors as groups. The enthusiasm with which they shared their ideas was unbelievable. It was as if we just discovered a whole new world -- and I guess we really did. We were finally seeing a move away from what I would have expected to be routine towards a more problematic extension of the idea. Students, now comfortable with the process, were asking more of those "what if ..." questions.

I have also noted that the journal prompt following activity four has been rather effective. Effective journal prompts that stimulate the student are difficult to create and are only going to be discovered by an ongoing process of experimentation and reflection by the teacher. For example, in activity five, students are told that the line joining the midpoints of the sides of a triangle is parallel to the base but they were never asked why. I realize that ultimately these are the types of questions that we would want students to ask themselves.

Teachers may also find that cultural connotations about language may create some difficulty. For example, activity five creates some trouble when the students are instructed to make a scalene triangle. For some students that meant to draw a scalene triangle while for other it meant physically make the triangle by drawing it and cutting it out. Those people that drew the triangle ran into all kinds of difficulties at step four when they are expected to draw a line along the edge of a fold. Even when I asked them to reread their initial instructions they do not pick up on the error. I would suggest that difficulties, such as the cultural interpretation of the word "make", are by their very nature unavoidable. If there is a positive note to this, however, it would be that students will become more critically reflective as they come to contemplate the problem, the essence of the problem and the words used to formulate it once they do find themselves stuck.

Some students may find themselves temporarily stuck again in exploration number two of activity seven. Many of the students have just discovered the relationship between cyclic angles subtended by diameters but run into considerable difficulty in the next exploration when they construct triangles as opposed to angles. Again, it comes back to the power of critical reflection which is what we are trying to develop. Some teachers may believe that the problems here are avoidable and indeed, they would be with the simple inclusion of a diagram. However, the admirable state of being stuck is a part of the problem solving process and not necessarily something to be avoided (Mason, Burton, & Stacey, 1996).

While many of my students were not natural problem solvers they are now well on their way to developing this skill. At first, many of the students would not persist and would look to me for answers while many others were getting lost in the instructions of the activities. Step three, in activity six, for example, I remember caused one group considerable difficulty. They drew semi circles everywhere except where they were supposed to. I watched and wondered but refused to tell. Certainly, they refused to ask. Finally, when they managed to put it together their expressions of accomplishment were obvious. Sometimes reading the instructions themselves are a problem while other times the assumptions we make, like those made about make a scalene triangle, are a problem. I believe this unit has opened my eyes along with the eyes of my students.

I would suggest that this unit presents an equal number of opportunities for teacher reflection as it does student reflection. As Doyle, Kennedy, Ludlow, Ross and Singh (1994) have stated, "teachers need to have a critical insight into their roles in school and examine critically the value of the knowledge they teach and the function of schooling generally in society ... teachers must do more than simply implement programs designed by others" (p. 33). I would invite teachers to reflect, experiment and question as they implement this unit. The structure is not regimented and in many places not sequential. It lends itself rather well to teacher experimentation. That has been and will continue to be the process for myself.

In conclusion, I would remind future users, or prospective teachers, to spend considerable time developing group skills at the beginning. Those pre-lesson activities are there for a reason and to serve a purpose. Teachers may want to emphasize the importance of working in groups not only in the classroom but elsewhere. Maybe they will want to make some connections to the real world to demonstrate the importance of working in groups and solving one's own problems. Teachers may also want to reflect on the necessity of making the first three activities more transitional depending upon the needs of their students. I would advise other teachers to guard against the temptations of imposing their ideas or time constraints upon the students as I would suggest that their primary concern be with the concepts of exploration, reflection and problem solving. It will also be important to recognize the different abilities of the different groups and the need for allowing them to progress at their own rate. I would strongly advise against creating competition between the groups or even comparing one group to another. Such activities on the teacher's part, I believe, will be detrimental to real learning -- or real problem solving, as confidence in mathematics is fostered by encouraging students to explore mathematical concepts, ask questions, discuss their ideas and make mistakes. Students who enjoy mathematics and work in a constructive environment feel comfortable and confident and, in turn, perform better and learn more.

This unit of Euclidean geometry introduces students to the dynamics of group structures, encourages students to listen to each other and most importantly employs a

discovery approach to mathematics. Students explore a variety of ideas in a small group setting and construct their own meaning for the mathematical problems presented. Students are then asked to reflect on this new found knowledge and record their thoughts in a journal. While this unit may represent a transition in approach it does reflect the changes called for -- and the standards of -- the NCTM (1989) and the Intermediate Mathematics Curriculum Guide (1995). Truly a worthwhile and enjoyable experience.

Conclusion

The vision of the mathematics classroom that is presented in the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (1989) has inspired many of us to want to change the way in which we teach. Indeed, the NCTM has said that much of the current mathematics instruction needs to change as transmission models of teaching no longer fit what we know about the teaching and learning process. Learning is no longer viewed as a passive exercise and many have come to believe that students must think, be mentally active, generate meaning, and construct their own understanding. Certainly, this thinking signifies a shift away from the transmission position towards a more transformative way of teaching and learning.

This project also signifies a shift away from the transmissive past. It signifies a shift towards a new transformative way of thinking and away from textbook learning, rote

memorization and the mastery of traditional school subjects through traditional methodologies. While this project is not a research paper it is something that other teachers can reflect on and utilize in their classroom as a means of promoting higher order thinking skills. Fedele (1996) has noted that if students are given the opportunity to reflect, their ability to learn and their understanding of learning would improve -- reflection is a critical aspect of learning and consequently necessary for better teaching.

Likewise, there has been an increased emphasis placed on problem solving and communication. Quinlan (1989) comments that students need to be convinced that it is worthwhile spending time struggling over mathematics problems as learning can and should occur during the time of trying to solve a problem. Scott, William and Hyslip (1992) add that "communicating about mathematics orally and in writing not only enriches the learning of mathematics, but also makes mathematics relevant and meaningful to students ... in journal writing, students reflect on experiences and organize their thoughts in order to communicate clearly ... children move beyond the mere performance of techniques and toward a deeper understanding of mathematics" (p. 15). This unit of Euclidean Geometry is based upon this critical reflection, problem solving and communication.

This unit provides the student with those experiences needed to facilitate growth in mathematical understanding. The unit introduces students to the dynamics of group structures, encourages students to listen to each other and most importantly employs a

problem centred discovery approach to mathematics. Students explore a variety of big ideas in a small group setting and construct their own meaning for the mathematical problems presented. The students are required to reflect on their experiences in the classroom, think critically about their reasoning and construct their own mathematical understanding. Furthermore, the unit is carried out according to the theory of cooperative learning and requires full class discussion and journal writing. However, it is also important to remember that this unit also signifies a potential change for teacher, student and institution and as a result may cause considerable anxiety. One should remember that real change takes time and sustained effort.

While this unit may represent a transition in approach it does reflect the changes called for -- and the standards of -- the NCTM (1989) and the Intermediate Mathematics Curriculum Guide (1995). The general purpose of this unit is to implement those changes and to foster critical reflection and higher order thinking skills in mathematics. For me it was and continues to be a truly worthwhile and enjoyable experience. I would invite others to reflect upon its possibilities for their students.

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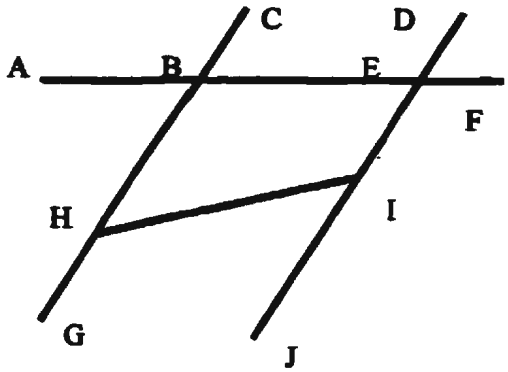
Appendix A: Pre-Test (sample)

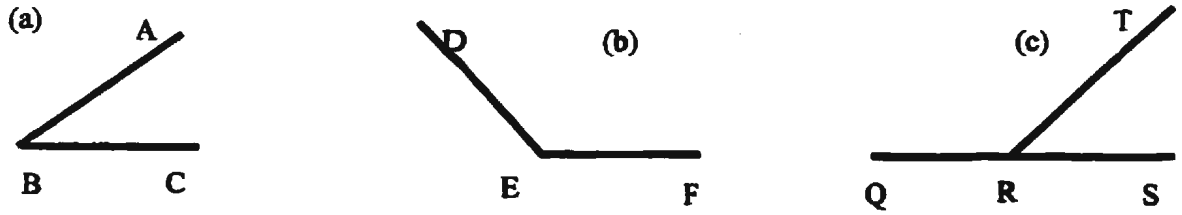
Grade 9 Euclidean Geometry Pre-Test Name_____



1. Using the above diagram name two :
   a) Points
   b) Lines
   c) Rays
   d) Line Segments
   e) Angles
   f) Corresponding Angles
   g) Parallel lines
   h) Alternate Angles

2. Using a protractor draw and classify the following angles.
   a) ∠ABC=90°
   b) ∠DEF=180°
   c) ∠GHI=35°
   d) ∠JKL=105°
   e) ∠TUV=215°

3. Measure and classify the following angles.

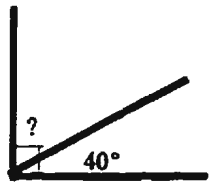


4. Draw and give the relevant measures of each type of triangle..
   (a) Equilateral
   (b) Equiangular
   (c) Scalene
   (d) Isosceles

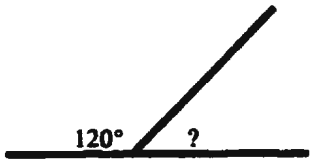
5. Calculate the sum of the interior angles of an Hexagon.

6. Find the measures of the unknown angles.

(a)



(b)



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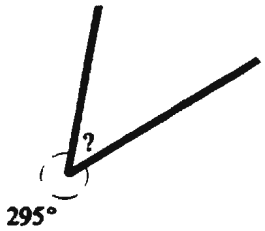

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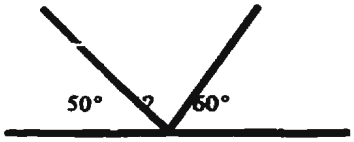

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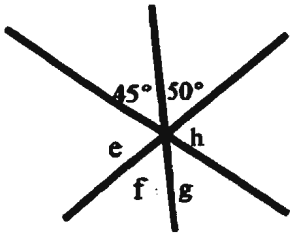

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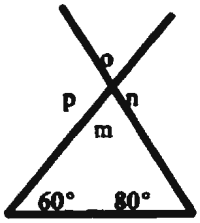

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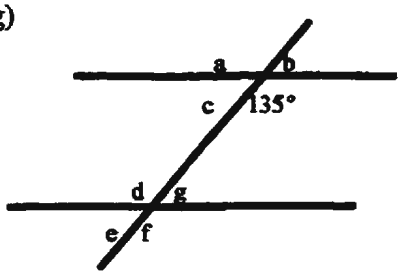

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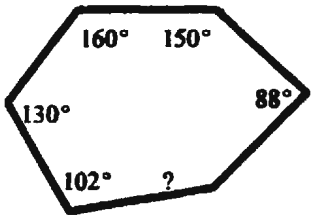

Appendix B: Group Test (sample)

GROUP TEST

EUCLIDEAN GEOMETRY NAME:_____

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1. How many pairs of congruent triangles are there on the flag of Newfoundland and Labrador?

2. Decide which of the following are always congruent, sometimes congruent, or never congruent. Illustrate your answer with diagrams.

   (a) 2 triangles with the same perimeter.

   (b) 2 rectangles with the same area.

   (c) 2 squares with the same perimeter.

   (d) 2 rectangles with the same perimeter.

3. A Pythagorean Triple is a set of 3 whole numbers that can be the lengths of the sides of a right triangle. The numbers 3, 4, and 5 make up the simplest Pythagorean Triple. With a classmate, list the other Pythagorean Triples in which no number is more than 50.

4. If the circumference of a circle is 60 cm and the central angle $\angle AOC$ is 60°, how long is arc AC?

5. The 5-m flagpole casts a 4-m shadow at the same time of day as a building casts a 30-m shadow. How tall is building?

6. Two quadrilaterals are similar if the corresponding angles are equal and the ratios of the lengths of the corresponding sides are equal.

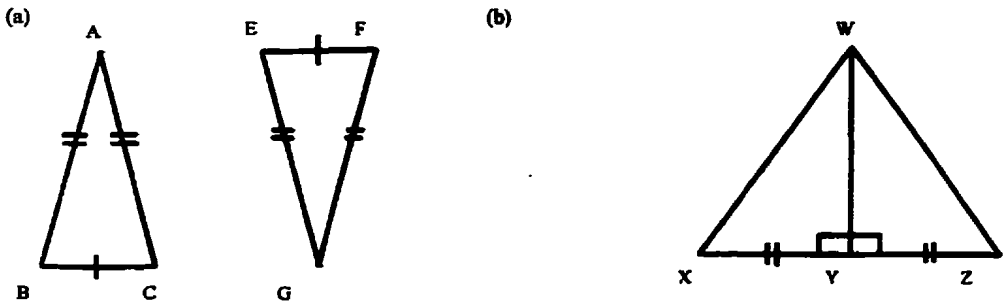
   (a) Are all squares similar? Explain.

   (b) Are all rectangles similar? Explain.

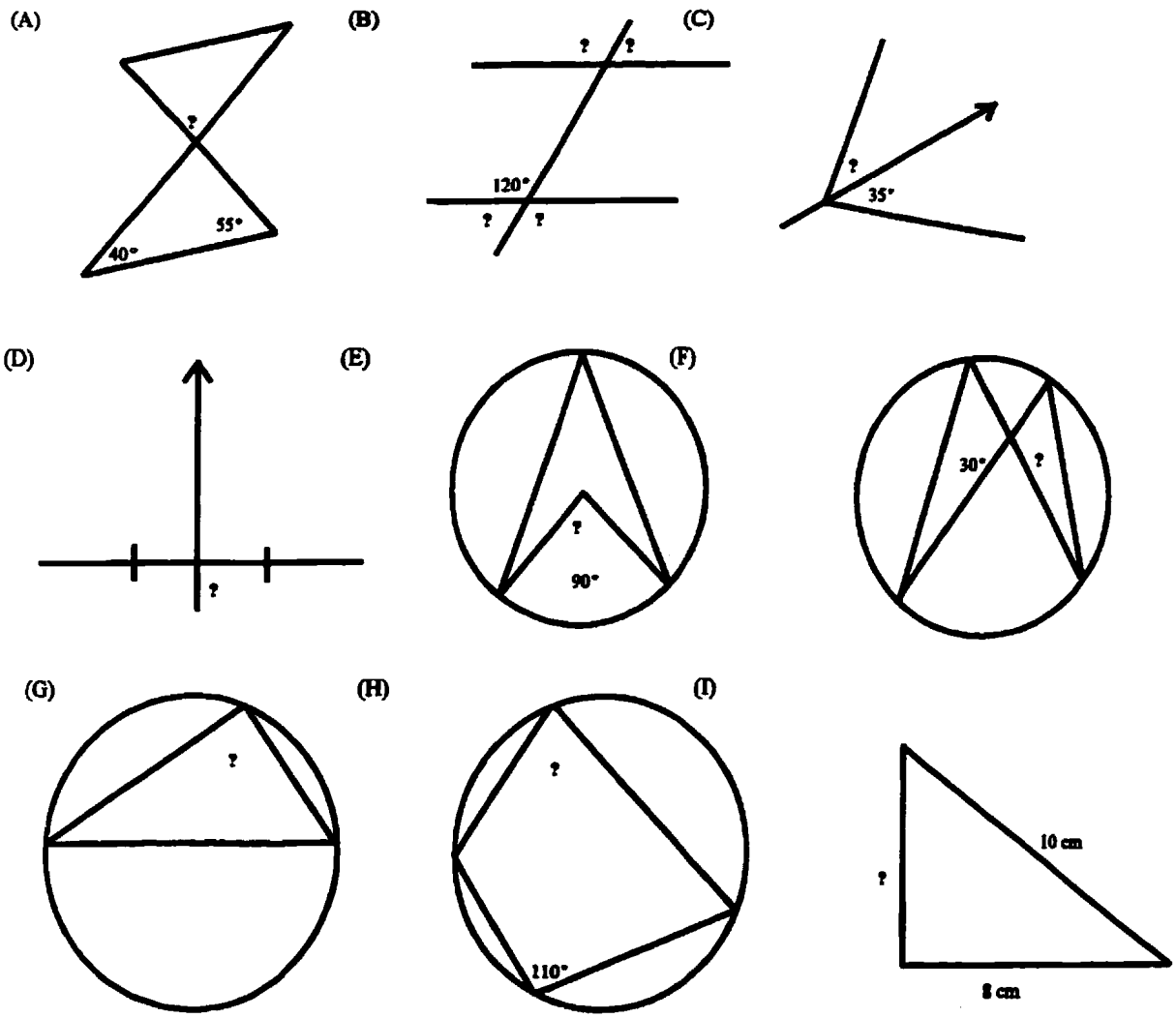
Appendix C: Individual Test (sample)

Individual Unit Test Euclidean Geometry Grade 9 Name_____

1. State why each pair of triangles are congruent and list all the corresponding equal parts.



2. Find the missing measures.



3. Construct a set of perpendicular lines using a compass.

4. Calculate the sum of the interior angles of polygons with the following number of sides.
   (a) five
   (b) thirty

5. Triangles ABC and DEF are similar. If AB is 12 cm, BC is 15 cm and DE is 8 cm what is the measure of side EF ?

6. The side of a tepee is 12 meters high. If the diameter is 13 meters, then how high should the center pole be ?