DEDUCTIVE REASONING IN EUCLIDEAN GEOMETRY: AN INTERMEDIATE LEVEL UNIT

JOANNE HOGAN

DEDUCTIVE REASONING IN EUCLIDEAN GEOMETRY:

AN INTERMEDIATE LEVEL UNIT

by

Joanne Hogan

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Abstract

Proof has traditionally been the touchstone of mathematics. It is at the heart of mathematics as students explore, make conjectures, and try to convince themselves and others about the truth or falsity of such conjectures. Reasoning is a necessary component if proving is seen as explaining deductively. By its nature, proof should promote understanding and as such can be a valuable part of the curriculum. Yet students and teachers often find the study of proof difficult, and a debate within mathematics education is currently underway about the extent to which proof should play a role in mathematics. A reexamination of the role and nature of proof in the curriculum is needed.

This project is designed with the purpose of creating occasions for deductive reasoning while following the provincial curriculum objectives as outlined for intermediate mathematics students in Newfoundland and Labrador. It builds upon the two documents produced by the National Council of Teachers of Mathematics (NCTM): the *Curriculum and Evaluation Standards for School Mathematics* (1989) and the *Professional Standards for Teaching Mathematics* (1991).

The project consists of a unit plan for Euclidean Geometry at the intermediate level. It stresses a method of teaching deductive reasoning that highlights student involvement and teacher facilitation. A case has been made for establishing a classroom atmosphere that encourages students to explore and investigate geometry problems, to ask questions, to engage in divergent thinking, and to use logical reasoning to develop convincing arguments (NCTM, 1992). This project capitalizes on the opportunities such topics provide to bring deductive reasoning into the intermediate mathematics class. The aim is to offer teachers a resource that supports the instruction of deductive reasoning in their own classrooms.

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Introduction

Ask students what they like most and what they dislike most about geometry. To what they like, there is a wide range of mildly stated answers. To what they dislike, there is only one strong answer: *proof*. By itself, like-dislike should not be used to determine the curriculum. But deductive proof is one of the characteristics of mathematics responsible for the central role mathematics plays in Western thought (Chazan, 1993). Thus a hated and feared idea is conceivably the most important.

I currently teach mathematics in an intermediate school setting. Through extensive reading and examination of researchers' ideas on the subject, as well as reflection on my own ideas and practices as an educator, I have decided that deductive reasoning deserves a place in the intermediate mathematics curriculum. However, in my own classroom, and I suspect in many other mathematics classes, students are asked to accept too much on blind faith without discovery or proof. This has led me to develop a Euclidean Geometry unit that offers opportunities for deductive reasoning in the intermediate mathematics class. The approach taken moves away from a textbook orientation and toward the reasoning processes of students. The sequence of activities that comprise the unit is based on the belief that deductive reasoning is of paramount importance at this grade level. It contributes to students' attitudes towards mathematics and proof at the senior high level. If students are not expected to prove things when they are first introduced, and instead merely accept them as fact, why would they see a need to prove the same result the following year? In grade 8, students learn to identify and use angle relationships associated with parallel lines via inductive reasoning. This intuitive, inductive knowledge sometimes interferes with students' understanding of the need to do this same work deductively – a curriculum objective in the senior high program. They may fail to recognize the crucial role that proof plays in convincing us of its validity.

The intermediate grades have to bridge the gap between the elementary grades and senior high. All too often it seems students become inhibited when they reach the intermediate level. Gradually, the interest in exploration to which they had grown accustomed in the primary and elementary grades disintegrates. They settle into a routine where they learn to accept things too readily. As educators, we must teach students to think and communicate effectively. Implementing and developing proofs can be one effective strategy in accomplishing these goals.

I do not want to use an approach so rigid that it conveys an impression that the style of the response – for example, the two-column proof format – is more important than its mathematical quality. The main function of proof in the classroom is the promotion of understanding. Hersh (1993) believes that

> … the role of proof in the classroom is different from its role in research. In research its role is to convince. In the classroom, convincing is no problem. Students are all too easily convinced. What a proof should do for the student is provide insight into *why* something is true (p.396).

Students need to be encouraged to explain their reasoning in their own words. As Hanna (1995) indicates, a proof actually becomes legitimate only when it leads to real mathematical understanding. The most important challenge to mathematics educators in the context of proof is to enhance its role in the classroom by finding more effective ways of using it as a vehicle to promote mathematical understanding. I have to present opportunities for students to refine their own thoughts and language by sharing ideas with their peers and with me. I can impart to my students a greater understanding of proof and a particular mathematical topic by concentrating my attention on meaning, rather than on formal derivation. Such is the purpose of this project.

Literature Review

What is Deductive Reasoning?

In 1933 Christofferson (as cited in Fawcett, 1938) summarized the ideal geometry classroom culture as one that would help "develop an attitude of mind which always tends to analyse situations, to understand their interrelationships, to question hasty conclusions, to express clearly, precisely and accurately non-geometric as well as geometric ideas" (p.28). Years later, reform documents such as the *Professional Standards for Teaching Mathematics* (NCTM, 1991) echo this by stressing that teachers should consider mathematics as "... a process involving problem solving, reasoning, and communication" (p.95). In line with this, the *Curriculum and Evaluation Standards for School Mathematics* [further referred to as *the Standards*] (NCTM, 1989) identify five goals for students:

- That they learn to value mathematics;
- That they become confident in their ability to do mathematics;
- That they become mathematical problem solvers;
- That they learn to communicate mathematically; and
- That they learn to reason mathematically.

The subject of mathematical proof is not a simple one. The world of mathematics as viewed by the professional mathematicians is a far different world than that seen by a grade eight student. Yet the idea of proof may be the quintessential theme of mathematics. Proof was traditionally included in the school curriculum to provide a way of learning geometrical facts and following a reasoned argument. For many students it became synonymous with a set of facts and processes to be learned and reproduced. If proof is to be a feature of our curriculum, it should provide students with the opportunity to prove things for themselves.

The appropriate inclusion of proof in the school curriculum raises further issues. The concept of proof does not imply any one particular activity or process. However, there are criteria to which activities and processes must conform if we are to accept them as proofs. Educators must decide what we want students to know about proof, what kinds of proof are acceptable, and the degree of rigor to be expected of students.

Fawcett (1938) understood that the purpose of studying proofs was to "...cultivate critical and reflective thought" (p.1) but the mathematics education community, then and now, struggles with the instructional methods to achieve this goal. To overcome these difficulties it becomes necessary to examine what proof and deductive reasoning involve.

Of proof, Schoenfeld (1988) says it is "... a coherent chain of argumentation in which one or more conclusions are deduced, in accord with certain well-specified rules of deduction from two sets of givens..." (p.157). The "givens" he refers to are either hypotheses or a set of accepted facts. The *Standards* (NCTM, 1989) define deductive reasoning as "the method by which the validity of a mathematical assertion is finally established" (p.143). Similarly, understanding proof has been defined as understanding that "a formal proof of a mathematical statement confers on it the attribute of a priori, universal validity" (Fischbein and Kedem, 1982, p.128).

Reid (1996a) questions the role of deductive reasoning or proving in mathematics as the verification of conjectures. He suggests that proving is used primarily as a way to explore and explain mathematical ideas. Lampert (1990) argues that what is important is developing and defending strategies, making hypotheses, and rising to the challenges of articulating and defending the assumptions that led to the formation of these hypotheses. Hoyles (1997) suggests that proofs done in school should aim to provide insight as to why a statement is true and throw light upon the mathematical structures under study rather than seek only to verify correctness. Hadas and Hershkowitz (1998) say deductive reasoning serves two purposes: convincing and explaining. Similarly, de Villiers (as cited in Hadas and Hershkowitz, 1998) argues that convincing oneself and others of the truth of a conjecture should take precedence over the process of proving. The important role of proof is explaining.

Schoenfeld (1994) summarizes three roles of proof: first, it should provide air-tight mathematical arguments leading to certainty; second, proof can be seen as a way of communicating with others the ideas resulting from clear thinking, and need not be conceived as a formal ritual; third, proving is a way of thinking, exploring, and coming to understand. Schoenfeld believes students have so little appreciation for proof, and so little apparent aptitude for it due in large part to the way they encounter proof in school. There tends to be an overriding classroom emphasis on form over content. In addition to this, proof rarely has any personal meaning or explanatory power for students. "Students believe that proof-writing is a ritual to be engaged in, rather than a productive endeavor" (Schoenfeld, 1994, p.75). Hanna (1989) concludes that current mathematical practices and philosophies do not lean towards extreme formalism in proof.

Proof is different for different people at any one time. What is a proof for a five-year-old is unlikely to be a proof for a grade 8 student, and similarly, what constitutes proof in an intermediate setting will be different than what constitutes proof in a senior high mathematics class. Depending on the position taken, the main role of proof will either be verification or explanation. Both views can offer support for teaching proof.

When Can Students Learn to Deal With Proof?

The NCTM suggests that reasoning is to have a role in all of mathematics from the earliest grades on up. If students are to become proficient with proof, then reasoning and proof-related tasks must be features of the entire K-12 curriculum. There is evidence that reasonable progress can be made when students are provided with well-planned activities relevant to their current level of thinking (Mayberry, 1983). As such, a continued development of proof throughout the whole curriculum is likely to produce some visible progress in this area.

In a study involving students in grades 4 through 7, an analysis is done of children's reasoning in solving hands-on and written problems in which they must process a number of clues to determine a solution is complete (English, 1996). The informal deductive problems are chosen to introduce students to processes of logic and present them with "... novel problem situations in which they must develop their own reasoning strategies, in contrast to applying taught rules or algorithms" (p.329). The results were somewhat surprising. On most problems, students in the younger grades displayed superior reasoning. It was found that grade 6 and 7 students showed little consideration of alternate models while solving problems and rarely applied checking, monitoring, or validation processes. In an effort to turn things around in the future, English (1996) makes the suggestion that it is important to actively encourage children's continued development of mathematical reasoning. However there was no examination or explanation of why there was not an increase in sophistication of reasoning across the grades.

Maher and Martino (1996) studied a young girl's development of the idea of mathematical proof. Their monitoring of this development spans grades 1 through 5 inclusive. The significant progress demonstrated by the girl offers insights into the process by which children may learn to prepare proofs, written in a setting that encourages the development of their ideas. The researchers attribute her success to various factors: she was given many opportunities to build multiple representations of the problems she encountered; she persevered in trying to make sense of situations; she was interested in reexamining and reconstructing earlier ideas through discussing her thinking with other classmates. She responded to teacher/researcher challenges to explain and justify her thinking. This research indicates that students at this level can deal effectively with proof. Focus will not be on formal development but rather on communicating and justifying one's ideas. David and Lopes (1998) cite an example of a grade 5 teacher carrying his students into making an informal deduction. The question was *What is the sum of the angles in a quadrilateral?* After receiving an answer of 360°, a proof is requested. One student says "... *it suffices to divide the quadrilateral into two triangles and that each triangle has 180°*" (p.237). Although the purpose here was to identify teaching methods that could be contributing to students' flexible thinking, evidence of deductive reasoning was seen in the grade 5 class.

Schoenfeld (1994) also believes proof can be embedded into the curriculum at all levels. He refers to the classroom discourse witnessed in a grade three classroom. Of this, he says "... it convinced me that it is possible to have mathematics classes be communities in which mathematical sense-making takes place" (p.76). In this setting, ideas were put forth as conjectures, discussed, and defined or rejected based on sound mathematical thinking. If students develop meaningful arguments of their own in an attempt to understand something, they are more likely to accept the proofs and see their importance. Proofs would be seen as a natural part of students' mathematics if they grew up in a mathematical culture where discourse, thinking things through, and convincing were important parts of doing mathematics (Schoenfeld, 1994). Burns (1985) shares this view:

> Children's classroom experiences need to lead them to make predictions, formulate generalizations, justify their thinking, consider how ideas can be expanded or shifted, look for alternative approaches and search for those insights that, rather than converging toward an answer, open up new areas to investigate (p.17).

The research discussed above strongly suggests students in the primary and elementary grades can deal with proof. The focus at this level should be placed on informal communication of deductive reasoning and students should be encouraged to develop their own mathematical arguments. Introduction to deductive reasoning at an early age could be beneficial to its study at the intermediate level, thus potentially smoothing the transition from informal to formal proof.

Perks and Prestage (1995) pose a question that is well worth our attention: *Is it possible that we leave working on proofs too late?* Proof can be used and developed to various levels of complexity. There is evidence, even in lessons based on typical textbook problems, of students at the intermediate level proposing, comparing, and justifying solutions (Rodd, 1997; Sawada, 1997).

Deductive Reasoning at the Intermediate Level

In *The Mathematical Experience* Davis and Hersh (1981) comment on the resistance to proof on the part of students. They suggest such resistance surfaces as the level of difficulty increases because students lack the experience and the strategies needed. "They don't know how to fiddle around" (p.283). Arsac, Balacheff, and Mante (1992) conducted a study involving 13- and 14-year-olds where the focus was on the learning of mathematical proof. The data gathered showed that although students had discussions about proposed solutions, initially they did not prove those solutions. However, when prompted by the teacher they were able to provide a mathematical proof. This research highlights two points: first, students often seem to be lacking the motivation to prove; second, opportunities need to be provided for our students to acquire the necessary experience so as to smoothen the transition from informal to formal proof.

Ask secondary students what a proof is in mathematics. The most common response might be "*Oh, that's what we do in geometry where there is a vertical line forming two columns, at the top of which are the headings 'statements' and 'reasons'.*" They might also recall knowing a proof was finished when the last statement in the left column was the same as the statement following the words at the top: *to prove*. From past experience, these are characteristics my own students have associated with proof. In my intermediate classes, the proving we were doing involved congruence. When first introduced, students were completely overwhelmed with the process. As we progressed, many of them had mastered this process of writing two-column proofs without any true understanding of what they were expressing. This became evident on several occasions. Some students would draw statements and reasons' charts for every question they were asked. When they realized this wasn't necessary they constantly sought reinforcement that they only used such a form when the word *prove* was in the question. Proof quickly became limited to the two-column format. Another problem arose with the commonly used reason: ***Corresponding parts of congruent triangles are congruent***. It came to be associated with the last reason provided for every proof. Again, when they realized the incorrectness of this they sought a rule that applied to all situations. These are examples of cases where learning to give back formal symbols from memory is not producing an ability to recognize and communicate a logical flow of ideas. Leonard (1997) suggests that we should work to instill in students the idea that "... proving something is the process of beginning with an assumption or a given and proceeding logically to a conclusion thus convincing by offering arguments" (p.204). He believes this could do much for the learning of mathematics.

An important component of deductive reasoning is feeling the need for a proof (Reid and Dobbin, 1998; Hadas and Hershkowitz, 1998). In an experiment conducted by Furinghetti and Paola (1997), a class of 14-year-old students performed well when proving a statement on natural numbers. In this case, the researchers are of the opinion that the need to prove is coming from outside influences like the textbook, the teacher, or classmates. This is consistent with Reid and Dobbin's (1998) belief that students usually feel the need to prove to ensure good marks. Interestingly, Furinghetti and Paola (1997) suggest that the wording of the text influences the way students work. They say the directions *"Prove..."* instead of *"Is...?"* push students towards argumentation rather than conjecturing. However, more work would have to be done on this aspect before drawing definite conclusions. Despite the fact that proving is often done to fill external needs, it is possible to construct situations in classrooms where students will have a need for proof, either to verify or to explain. Hadas and Hershkowitz (1998) suggest that surprising findings and not being able to check all cases inductively lead to a need for a deductive proof. This has important implications for planning activities.

Sekiguchi (1996) suggests that current instructional practices pertaining to mathematical proofs have serious defects, and need to be changed. In an eighth grade mathematics class in Japan, an attempt was made to frame the learning of proof around three guiding principles:

1. Proof is considered explanation.
2. Statements must be backed by reasons.
3. Writing style is important.

The teacher planned his instruction in anticipation of students' difficulties with proof. He used techniques such as a box-filling format proof, where a partial proof is given and students have to fill in missing statements or reasons.

Some positive results were seen. As students gradually learned ways to present a proof, they had to be able to specify the contextual information of a proof, not explicitly written: where, how, and why particular assumptions, definitions, and theorems were used. However problems with proving were encountered and the degree of student participation in constructing and discussing proofs was not always high. Sekiguchi (1996) believes it would be difficult to understand and appreciate proof without the use of writing. However, this is an area where students experienced difficulty. This difficulty with writing proofs seems to have distracted students from true deductive reasoning (Sekiguchi, 1996). Although current teaching of proof was criticized at the start, instruction did not sway too far from traditional methods in this study. The focus remained on the form of a proof. *Fill-in-the-blanks* proofs were used quite frequently. While such a technique could be beneficial if used in moderation, extensive use draws attention to the narrowness of one particular type of proof. Overemphasis on the formality of proof may cause misconceptions and lack of appreciation for proof.

Some success in proving was reported here. Students were capable of presenting proofs, but difficulty with writing did cause problems. The proofs tended to be of the traditional two-column format. If the intention is to promote deductive reasoning based on true understanding on the part of the students, instruction must reflect this intent.

In a study conducted with eighth grade classes (Mariotti, Bussi, Boero, Ferri, and Garuti, 1997), it was found that although students did not produce formal proofs, they did produce and verify conjectures. Throughout the process they used deductive reasoning which, in the opinions of the researchers, shared some crucial aspects with the construction of a mathematical proof . This particular experiment concerned the study of the parallelism of shadows of two non-parallel sticks. It has to be noted, as the researchers acknowledged, that the success of this study cannot be automatically transferred to other fields of experience, namely traditional geometry theorems.

However, findings here do support early introduction to theorems in suitable fields of experience.

Boero, Garuti, and Sibilla (1995) found similar results. Seventh grade students were able to either produce a proof or at least take steps in that direction. It was shown to be feasible for students at this level to be involved in proving statements of theorems in appropriate educational contexts, dependent on the role of the teacher.

Students at the intermediate level are capable of reasoning deductively, but the ability is often not acted upon in the classroom. Mathematical reasoning at this level is contingent upon many factors. The teacher plays a large role in determining whether these factors exist. One thing teachers need is a reservoir of models and examples of how to design mathematics instruction so that it fosters students' engagement. The challenge for intermediate teachers is to present mathematics as an exciting discipline that is relevant and accessible to all students.

Turner, Rossman Styers, and Daggs (1997) spent time experimenting with approaches that would interest students in challenging mathematics while supporting them in constructing meaning. When developing lessons, they took into account things that would most likely involve young adolescents in mathematics. These included the use of principles of challenging work, student autonomy, and student collaboration. An analysis of the data collected indicates that statistically significant changes occurred in students' attitudes. After spending a year in this program, students rated the classroom as a place where they wanted to be and where the emphasis was on learning and thinking. They also expressed a preference for challenging work. These results show that it is possible to design instruction that is both mathematically rich and engaging for students in the intermediate grades.

Waring, Orton, and Roper (1998) investigated whether students could be led to develop their capabilities in proving within an enhanced environment. Their work involved 14 and 15-year-olds and was piloted with classes of 12- and 13-year-olds. The researchers considered whether appropriate curriculum activities involving pattern might lead to improved skills in proving. Results show that it was appropriate to use pattern as a motivator for the study of proof. Perhaps more importantly, the awareness of the need for proof had been raised. Students were likely to want to try to provide a proof and, even if they could not complete one on their own, they were receptive to the teacher then leading them to a conclusive proof.

Turner, Rossman Styers, and Daggs (1997) as well as Waring, Orton, and Roper (1998) found that classroom conditions can contribute to improvement in students' deductive reasoning. In particular, the types of activities students reason about proved to play a significant role.

A review of the literature shows that students in the intermediate grades are capable of reasoning deductively. However, it is unlikely that many will spontaneously exhibit this ability in a mathematical context. Therefore, if we as educators value proof, we must be aware of the vital role we play. We must offer students opportunities for engaging in activities that encourage them to want to look for proofs and to justify and explain their findings. Specifically, we must focus on reasoning that involves making a conjecture about where to begin and following up the conjecture with a deductive proof.

Based on this, I have produced a resource unit that will aid teachers in their attempts to create a classroom environment where deductive reasoning prevails.

Overview of Project

Description

In this the "Information Society" there exists much more information than any individual or group could ever know in a lifetime. A great demand exists for learners who can access and use information to problem solve. While certain content will remain essential in itself, increasingly content will be important as a vehicle to facilitate processes such as problem-solving, learning-to-learn skills and metacognitive skills (Department of Education and Training, 1995). Logical thinking involves consistent and rational reasoning. The unit of study is Euclidean Geometry. The *Intermediate Mathematics Curriculum Guide* (Department of Education and Training, 1995) states the following objectives to be achieved by the end of grade 8:

1. Determine, through construction, the conditions sufficient to produce a unique triangle.
2. Identify properties of and minimum conditions for congruent triangles.
3. Identify and use the angle relationships associated with parallel lines.
4. Analyse quadrilaterals to determine properties and interrelationships.
5. Determine missing angle measures by applying angle relationships.

Through varied experiences, this unit provides students with opportunities to "... apply logical reasoning to develop the ability to construct valid arguments and to evaluate the arguments of others" (Department of Education and Training, 1995, p.9). During the intermediate grades, students are learning to recognize and apply deductive reasoning. This capability begins to formalize as students progress from grades 7 to 9.

Typically students are told that the interior angles in any triangle have a sum of 180 degrees. If there is any active involvement from students, it probably extends to having them measure the angles in several triangles and draw a conclusion. The intermediate geometry curriculum affords many opportunities for students to explore their environment and to learn and enjoy many new aspects and applications of mathematics in their world. This unit provides opportunities for students to be active participants in a mathematics class where importance is placed on deductive reasoning.

For the duration of this unit, the suggested environment is one that is highlighted by a cooperative learning atmosphere. A sequence of activities is presented to the class; students work in small groups to discuss the problem in an attempt to arrive at a solution agreeable to all members. This is followed up with group presentations engaging the entire class in discussion. Instruction will cover the curriculum outcomes outlined previously, while also encouraging and expecting convincing arguments for all conclusions reached. A more detailed description of the classroom setup and the teacher's role in this process follows.

<u>The Classroom Climate</u>

> Mathematics, as we teach it, is too often like walking on a path that is carefully laid out through the woods; it never comes up against any cliffs or thickets; it is all nice and easy.
>
> Albers and Alexanderson, 1985, p.231

Commonly, mathematics is associated with certainty. Lampert (1990) summarizes it nicely. Doing mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical truth is determined when the answer is ratified by the teacher. Research has indicated that mathematical reasoning is actually discouraged in many classrooms as a result of the traditional view that something is true because the teacher or book says so (Schoenfeld, 1985). Beliefs about how to do mathematics are acquired through years of watching, listening, and practising. As such, it is important for educators to keep in mind that students develop their sense of what it means to *do mathematics* from their actual experience with it. Henningsen and Stein (1997) remind us that students' primary opportunities to experience mathematics as a discipline come in their participation in classroom activities. They conducted a study at the middle school level to look at the mathematical disposition of students. One of the activities which characterizes this disposition is thinking and reasoning in flexible ways: conjecturing, generalizing, justifying, and communicating one's mathematical ideas. Findings suggest that high level activity is possible at the intermediate level. However, several factors contributed to students engaging at high levels. These included factors related to the appropriateness of the task for the students and to supportive actions by teachers, such as scaffolding and consistently pressing students to provide meaningful explanations or make meaningful connections. A unit such as this one, with an emphasis on deductive reasoning, requires a shift in attitude and practice, both on the part of the teacher and student.

According to Vygotsky's theory (as cited in Leikin and Zaslavsky, 1997) children learn well when they are involved in group activities. Leikin and Zaslavsky (1997) have found that in a cooperative, small-group setting students' activeness increases. There is a shift toward students' on-task verbal interactions; various opportunities for students to receive help arise; students exhibit positive attitudes toward cooperative learning. The findings of the study also show that small-group cooperative learning facilitates a higher level of learning activities. It is the intent here to create such an environment, where the focus will be on deductive reasoning.

The evolving nature of our society suggests that there is a continual move away from an assembly-line way of work and a move toward collaborative teamwork. Business and industry increasingly demand that people be prepared to work in a cooperative, problem solving environment. In response to this, the Department of Education is calling for a mathematics curriculum which has problem solving at its core and which facilitates the development of collaborative skills and responsibility for one's own learning within such a context.

The classroom setup for the present Euclidean Geometry unit takes each of these points into consideration. Students are no longer told step-by-step what to do. They work in pairs to explore, discuss, and explain their findings. The teacher becomes a facilitator.

The Teacher's Role

Proof is often perceived as being rigid and formal. It is commonly left out of any work in mathematics until students reach high school, often leading to difficulty when they finally confront the idea of proof. This outcome is not inevitable. Intermediate school students are capable of grasping the basic logic of proof and should be given the opportunity to encounter it. The curriculum should be flexible enough for students to continue working on a problem or pursue a new idea. To a certain extent, the mandated provincial curriculum places constraints on the delivery of the mathematics course. However we cannot ignore the influences of our own daily decisions concerning curriculum organization and sequencing. Although every grade 8 mathematics teacher in Newfoundland and Labrador possesses a copy of the *Intermediate Mathematics Curriculum Guide*, there are many variations in how that curriculum is delivered and experienced. Teachers make decisions and choices, which can have important consequences on students' learning.

The design of this project assumes certain classroom conditions for students' development of mathematical ideas. The role of the teacher shifts from conveyor of information to one of moderator and observer of children's thinking. As teachers monitor the thinking of their students, they are better able to pose timely questions encouraging students to build deeper mathematical understanding and assess the progress of their reasoning. Since group work is predominant throughout this unit of work, there is a suggested shift in the teacher's role from telling to guiding. Research has shown that teachers' interference can play a significant role in the development of students' thinking (Coles, 1993; McGuinness, 1993; Tanner and Jones, 1995; David and Lopes, 1998). How does one know when to ask questions of students? What types of questioning will they most benefit from? Taking into consideration that it is the teacher who develops and then delivers lessons, it is also the teacher who plays a significant part in whether there is opportunity for deductive reasoning to occur. The teacher must design tasks and projects that stimulate students to ask questions, pose problems, and set goals. However, as Simon (1995) points out, there are two faces to planning, the extremes of which will not result in optimal conditions. If the teacher leads students to a particular response, no real learning takes place. However, if the teacher has no plan, learning will probably not occur either. The challenge I am trying to meet with the use of this resource package is to integrate the teacher's goals and direction for learning with students' mathematical thinking and learning.

Students will not become active learners by accident. Simply putting them in groups will not ensure cooperative learning takes place. Student participation is essential. Maher and Martino (1996) present certain conditions that promote student conversation in classrooms. Students may be encouraged to share and discuss their findings when opportunities are provided for them to work in a variety of social settings. They also recommend teachers refrain from telling students what to do, as well as allow teaching to guided by student thinking. These conditions suggest a shift in the teacher's role from telling to guiding.

Henningsen and Stein (1997) conducted a study of mathematics classes in middle schools. The purpose was to investigate the classroom factors that either hinder or support students' engagement in high-level mathematical thinking and reasoning. Their findings have important implications for any teacher who expects students to be actively engaged in doing mathematics. The teacher is responsible for selecting appropriate tasks, allotting appropriate amounts of time for the tasks, keeping the focus on understanding rather than on the correctness or completeness of the answer, and managing the class. Each of these roles must be carried out when delivering the current unit, as deductive reasoning can certainly be classified as high level mathematical thinking. Perhaps the most challenging role of the teacher here is to proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demands of the task. At the point in the class where whole group discussion takes over, the teacher has to assume a more active role. The responsibility for the search of the problem's solution and for the decision about its validity are left entirely to the students during a certain period of time. Later on, the teacher concludes and summarizes the activity. As Schoenfeld points out "... if we believe that learning mathematics is empowering and that there is a mathematical way of thinking that has value and power, then our classroom practices must reflect those beliefs" (1994, pp.60-61).

Students have to be given the message that what is important is careful reasoning and building arguments that can be scrutinized and revised. A degree of formalization may be necessary, but emphasis must be placed on the clarity of ideas (Hanna, 1989). I am advocating an approach to proving that rejects rigorous presentation of a traditional proof in favor of understanding and explanation. This is important at the intermediate level. As Reid (1996b) indicates, in teaching proof, we must help students formulate their own deductive reasoning before introducing formal proofs. However, reading, interpreting, and presenting formal proofs will eventually be expected. The challenge is to move students to a more formalized level of proving without sacrificing true deductive reasoning.

The unit of study for this project views mathematical proof more as a matter of individual construction than the result of the transmission of information. Here mathematics is viewed as a dynamic, growing, and changing discipline; students are viewed as active learners and teachers as facilitators of learning.

<u>Note to the Teacher</u>

At every level of schooling, and for all students, reform documents recommend that mathematics students should make conjectures, abstract mathematical properties, explain their reasoning, validate their assertions, and discuss and question their own thinking and the thinking of others. The mathematics teacher must provide opportunities for informed exploration and reflective inquiry without taking initiative or control away from the student (Simon, 1995). This resource package will help teachers create a classroom culture in which such student activity can occur.

The geometry activities that follow are intended to provide intermediate mathematics teachers with easily accessible materials, making the inclusion of deductive reasoning in the curriculum a little less challenging. These activities are the teacher's versions. Student activity sheets follow in the appendix. Material in Roman font is for your instruction; material in Italics are suggested instructions to give to students.

As a teacher comes to know his/her own strengths and those of students, variations in teaching methodologies result. As such, teacher instructions are general and the primary focus is on the student activities. Although not specifically outlined in the activities that follow, homework complements class work and is a regular part of this unit. Textbooks for grade 8 mathematics can be used frequently in this area to provide reinforcement for the objectives covered in the unit.

Permission is given to teachers to reproduce class sets of the student activity sheets for use in their own classrooms. The activity descriptions can be reproduced with the provisions that they remain as a set and acknowledgement of authorship is given.

Euclidean Geometry: An Activity Based Unit

ACTIVITY 1
Congruent Triangles

TERMINOLOGY:
A. *congruent*
B. *contained angle*
C. *contained side*

Activity 1-(a)
MATCH THE TRIANGLES

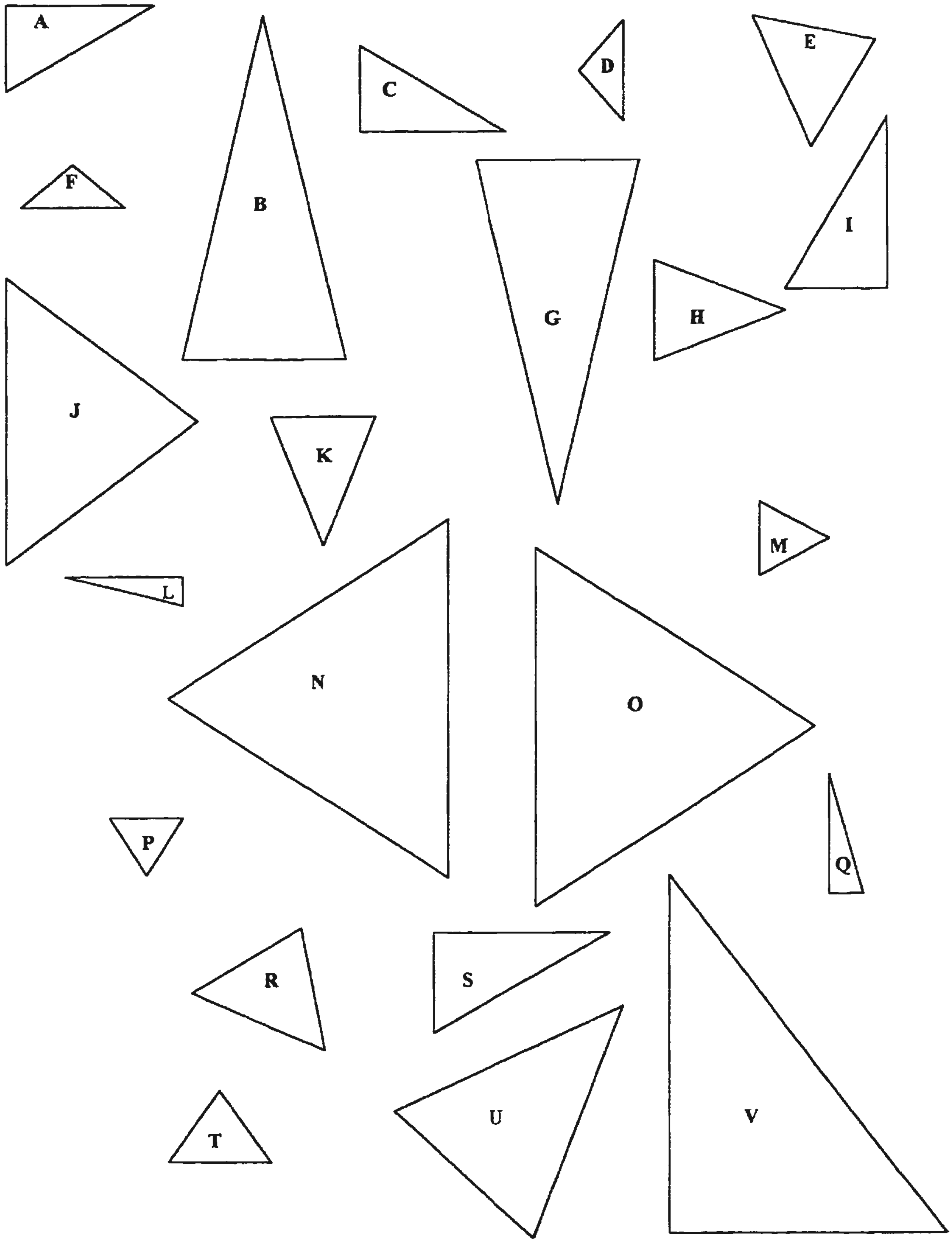
Play this game with a partner.

1. Toss a coin to decide which player starts the game. Players then take turns selecting two triangles that are the same size and shape. Rulers and protractors cannot be used to measure the triangles. If the selection is correct, the player receives two points. If not, the other player receives one point.

2. Using tracing paper, verify whether the two triangles in each pair are the same shape and size. Cross out the matching pairs.

3. After all possible triangles have been matched, add up the scores. The player with the highest score is the winner.

(Diagrams follow on the next page)


A
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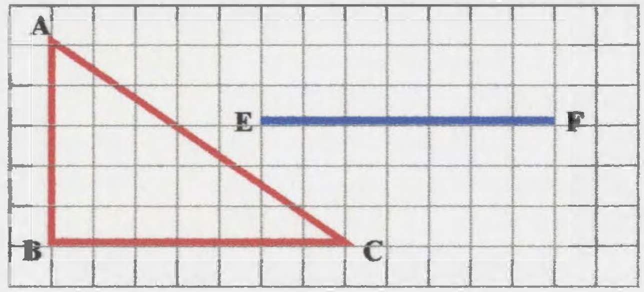
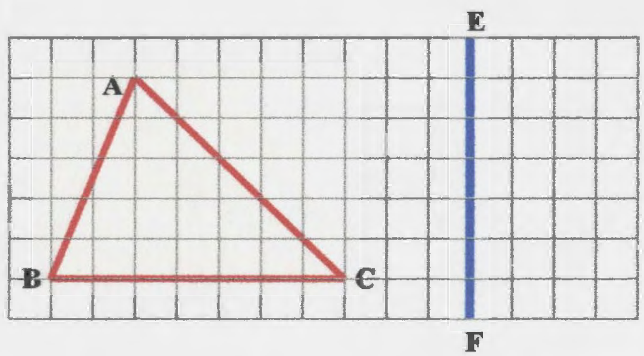
Activity 1-(b)

For each example below, copy the given triangle and line segment onto your grid paper. Locate a point D to make triangle DEF the same size as triangle ABC. Is there more than one place that D could be located?





Activity 1-(c)

SSS, SAS, ASA

Revisit the definition of **congruent**.

If two triangles have the same shape and size, then the three angles and the three sides in one triangle must be equal to the three corresponding angles and the three corresponding sides in the other triangle.

Part I

A. Follow these steps to construct ∆ABC where AB = 7 cm, BC = 8 cm, and AC = 10 cm.

- *Draw line segment AB = 7 cm.*
- *Set your compass to a radius of 8 cm. Then use B as a center and draw an arc.*
- *Set your compass to a radius of 10 cm. Then use A as a center and draw an arc to intersect the first arc. Label the point of intersection C.*
- *Draw BC and AC.*

B. Is it possible to construct ∆DEF where DE = 7 cm, EF = 8 cm, and DF = 10 cm so that the size and shape of ∆DEF are different from the size and shape of ∆ABC? Explain.

Part II

A. Construct ∆GHI so that GH = 5 cm and GI = 7 cm.

B. Is it possible to construct ∆JKL with JK = 5 cm and JL = 7 cm so that the size and shape of ∆JKL are different from the size and shape of ∆GHI? Explain.

Part III

A. Construct $\triangle MNO$ so that $\angle M = 35°$, $\angle N = 65°$, and $\angle O = 80°$.

B. Is it possible to construct $\triangle PQR$ with $\angle P = 35°$, $\angle Q = 65°$, and $\angle R = 80°$ so that the size and shape of $\triangle PQR$ are different from the size and shape of $\triangle MNO$? Explain.

Part IV

A. Construct $\triangle RST$ so that $\angle R = 70°$, RS = 6 cm, and RT = 7 cm.

B. Is it possible to construct $\triangle XYZ$ with $\angle X = 70°$, XY = 6 cm, and XZ = 7 cm so that the size and shape of $\triangle XYZ$ are different from the size and shape of $\triangle RST$? Explain.

Part V

A. Construct ΔTUV so that $\angle T = 50°$, TU = 4 cm, and UV = 6 cm.

B. Is it possible to construct ΔWXY with $\angle W = 50°$, WX = 4 cm, and XY = 6 cm so that the size and shape of ΔWXY are different from the size and shape of ΔTUV? Explain.

Part VI

A. Construct $\triangle ABC$ so that $\angle A = 40°$, AB = 7 cm, and $\angle B = 55°$.

B. Is it possible to construct $\triangle DEF$ with $\angle D = 40°$, DE = 7 cm, and $\angle E = 55°$ so that the size and shape of $\triangle DEF$ are different from the size and shape of $\triangle ABC$? Explain.

Part VII

A. Construct $\triangle PQR$ so that $\angle P = 35^\circ$ and $PQ = 7$ cm.

B. Is it possible to construct $\triangle XYZ$ with $\angle X = 35^\circ$, and $XY = 7$ cm, so that the size and shape of $\triangle XYZ$ are different from the size and shape of $\triangle PQR$? Explain.

Conclusion

Use your results from the first 7 activities to decide which 3 conditions need to be given so that only one triangle can be constructed.

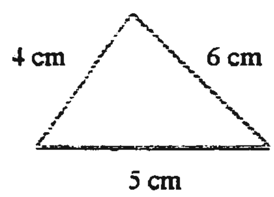
Once all groups come to a conclusion, these results should be discussed with the class.

In your notebook, summarize your findings.

Note: Teachers may wish to introduce the notation SSS, SAS, and ASA here.

Activity 1-(d)

To change the shape of this triangle, you have to change the length of at least one of the sides. ***Why?***



Activity 1-(e)

Decide which of the following are always congruent, sometimes congruent, or never congruent. ***Explain your answers.*** *(You may use diagrams to illustrate your responses).*

A. 2 triangles with the same perimeter.

B. 2 rectangles with the same area.

C. 2 squares with the same perimeter.

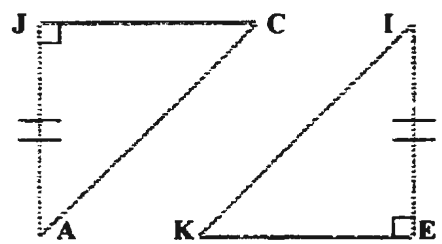
D. 2 rectangles with the same perimeter.

ACTIVITY 2
Proving Triangles Congruent

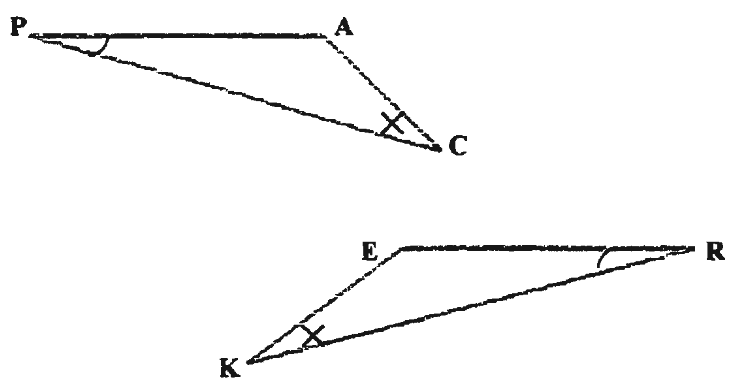
Activity 2-(a)

For each of the following pairs of triangles, two equal corresponding parts are marked. What is the fewest number of additional corresponding parts that must be equal to ensure that the two triangles are congruent? ***Explain.***

(A)



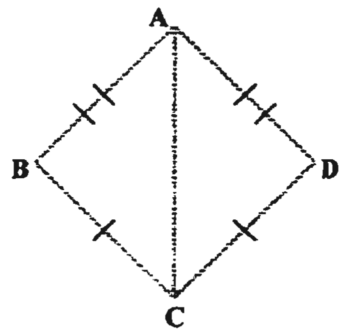
(B)



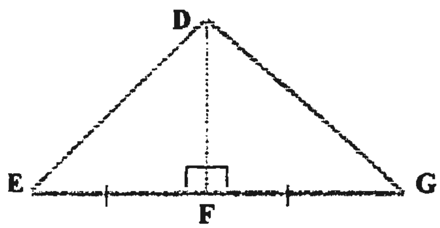
Activity 2-(b)

Identify the pairs of congruent triangles in each diagram. Justify your responses.

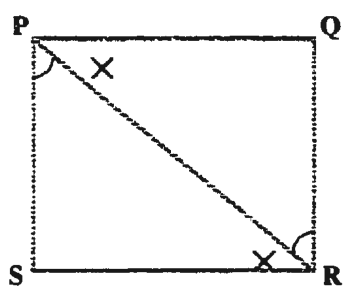
(A)



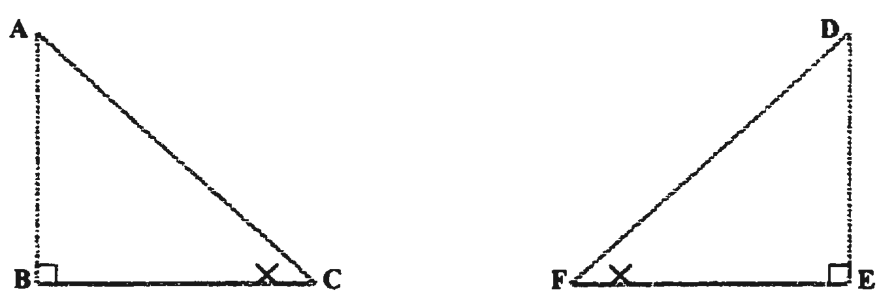
(B)



(C)



(D)



ACTIVITY 3
Parallel Lines

TERMINOLOGY:

D. *parallel lines*
E. *transversal*
F. *alternate angles*
G. *corresponding angles*
H. *vertically opposite angles*
I. *co-interior angles*

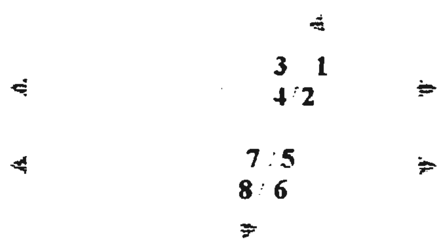
Activity 3-(a)

A. *Describe the most common use of parallel lines that you encounter.*

B. *Work with your partner to list other uses of parallel lines.*

Activity 3-(b)

Display the following diagram.



There are four types of angles associated with parallel lines cut by a transversal:

Co-interior angles
Alternate angles
Corresponding angles
Vertically Opposite angles

Identify the types associated with each of the following pairs of angles.

$\angle 2$ and $\angle 5$
$\angle 4$ and $\angle 5$
$\angle 3$ and $\angle 7$
$\angle 1$ and $\angle 4$

Based on the location of these angles, name other pairs of the same type.

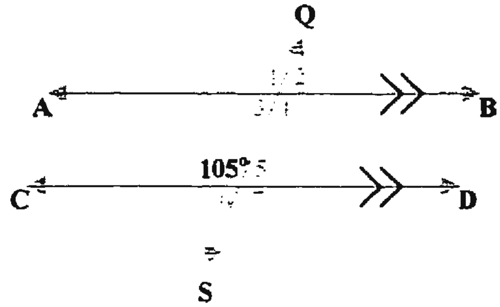
Measure the eight angles in the diagram above formed by the parallel lines intersected by a transversal.

How many pairs of each type of angles did you find?
Was the relationship between the particular pairs of angles consistent for all such pairs?
Make a ***conjecture*** *about each type of angle.*

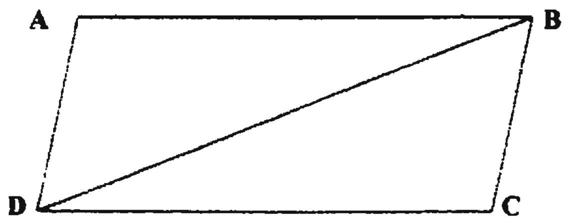
If you know that vertically opposite angles are ALWAYS congruent, and that corresponding angles are ALWAYS congruent, could you explain why the other kinds of angles are related in the ways they are?

Activity 3-(c)
Missing Measures

Find all the missing angle measures. Justify your answers.



Activity 3-(d)



If AB is parallel to CD and AD is parallel to CB, is ΔABD congruent to ΔCDB? ***Explain.***

Activity 3-(e)

Write a problem that involves angles and parallel lines.
Exchange the problem with your partner. Solve the problems and discuss the answers.

ACTIVITY 4
Triangle Angle Sum

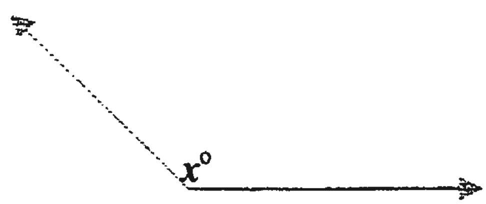
TERMINOLOGY:

J. *angle*
K. *triangle*
L. *acute triangle*
M. *right triangle*
N. *obtuse triangle*
O. *scalene triangle*
P. *isosceles triangle*
Q. *equilateral triangle*

Activity 4-(a)

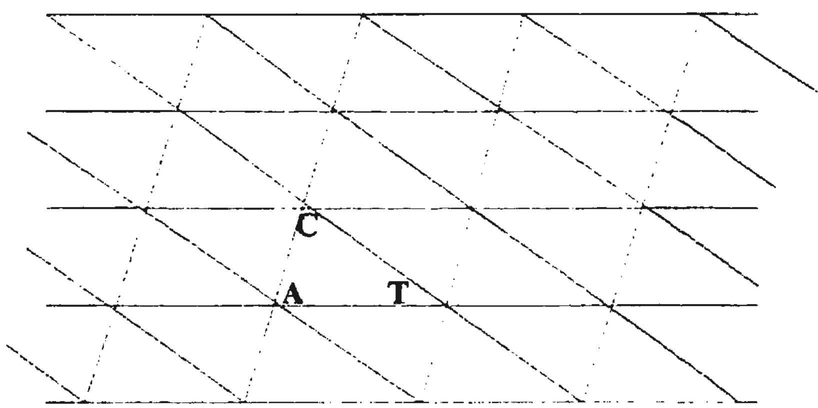
Students' knowledge of angle measurement is important, especially their use of benchmark angles such as 90°, 180°, and 45°, to estimate the size of given angles.

*Estimate the measurement of angle **x** in degrees.*
Explain or show how you got your answer.



Activity 4-(b)

The grid shown below consists of sets of parallel lines.



1. Outline ΔCAT on this grid. Choose three different colors. With the first color, color in ∠C and all the angles that are congruent to ∠C. With the second color, color in ∠A and all the angles that are congruent to ∠A. With the third color, color in ∠T and all angles that are congruent to ∠T.

2. What do you observe about the angles around C? Around A? Around T?

   Find straight angles in the colored diagram. What colors are the angles that make up each straight angle?

3. What can you say about the sum of the measures of ∠C, ∠A, and ∠T?

   Could you say the same things about the three angles on other triangles?

   Could you say the same thing about all triangles?
   Justify your conclusion.

***** Activity 4-(b) is adapted from the NCTM Addenda Series/Grades 5-8: *Geometry in the Middle Grades***

Activity 4-(c)

Is it possible for a triangle to have two right angles?
Explain.

Is it possible for a triangle to have more than one obtuse angle?
Explain.

Consider triangles A through I, classified according to their angles and sides. Some types of triangles named in the table exist, while others do not. Draw the types of triangles that exist and label them with the appropriate letters.

	SCALENE	ISOSCELES	EQUILATERAL
ACUTE	A	B	C
RIGHT	D	E	F
OBTUSE	G	H	I

If a triangle does not exist, explain why.

ACTIVITY 5
Quadrilaterals

TERMINOLOGY: R. *quadrilateral*
S. *diagonal*

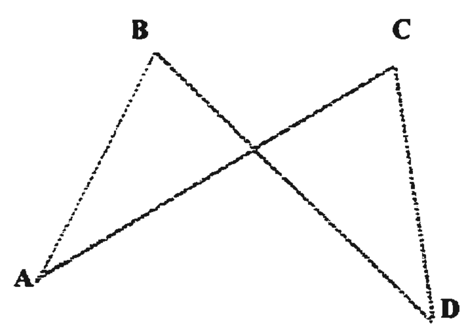
Activity 5-(a)
DEFINING QUADRILATERALS

A student was asked to define a quadrilateral.

"A quadrilateral is what you get if you take 4 points A, B, C, D in a plane and join them with straight lines."

Do you agree or disagree with this definition? ***Why?***

Once students have given their opinions on the above definition, display the following diagram.



Is this a quadrilateral? ***Explain*** *why or why not.*

Formulate your own definition of a quadrilateral.

A class discussion should follow leading to a definition agreed upon by the group.

Activity 5-(b)
Sum of the Measures of the Angles of Quadrilaterals

The following represents one student's effort to write a proof of the quadrilateral angle sum.

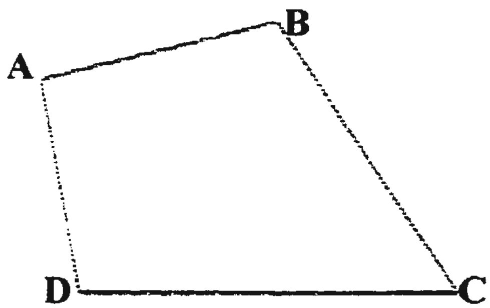
Explain *what is right or wrong about it.*

> **Every quadrilateral can be made into two triangles by joining a diagonal. Each triangle has 180° and 2 x 180° = 360°. Therefore, the sum of the angles of a quadrilateral must be 360°.**

Write your own proof to improve upon this one.

Activity 5-(c)
Missing Measures

What is the measure of the unknown angle?



$\angle A = 91^\circ$

$\angle B = 113^\circ$

$\angle C = ?$

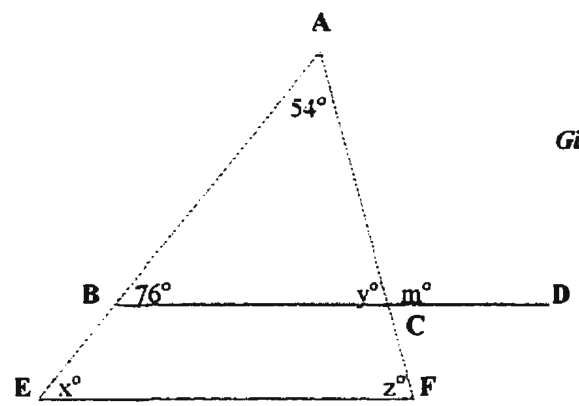
$\angle D = 104^\circ$

***** Activity 5 is an adaptation of an activity from an article entitled "Mathematics as Reasoning" in the May 1995 issue of *Mathematics Teacher*.**

ACTIVITY 6
"Piecing it all Together"

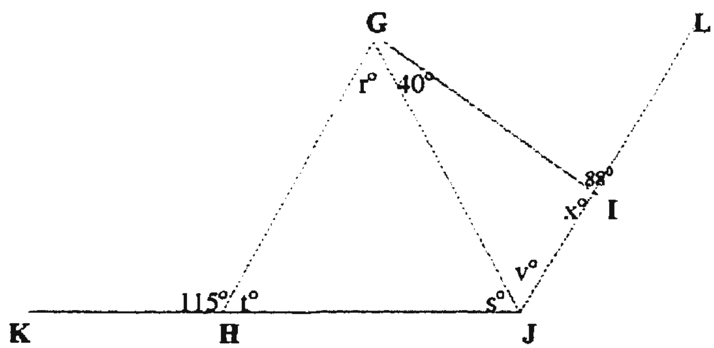
Find all the unknown angle measures, as indicated.
***Explain** how you found each of the missing measures.*
Note: Diagrams are not drawn to scale.

(A)



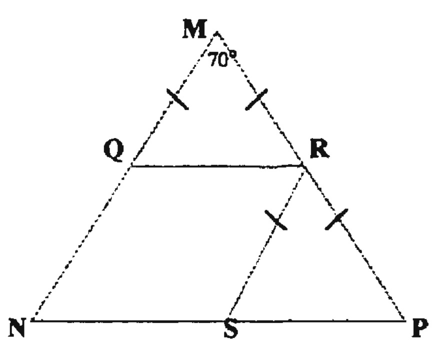
Given: **BD || EF**

(B)



Given:
∠HGI is a right angle

(C)



Given:
QR || NS

NQ || SR

Hint: *You may find it helpful to first list all the angles in this diagram.*

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APPENDIX

Student Activity Sheets

Permission is given to reproduce the following activity sheets for use in the intermediate mathematics class.

Student Activity Sheet #1

EUCLIDEAN GEOMETRY

Terminology

Congruent
Contained Angle
Contained Side
Parallel Lines
Transversal
Alternate Angles
Corresponding Angles
Vertically Opposite Angles

Co-Interior Angles
Angle
Triangle
Acute Triangle
Right Triangle
Obtuse Triangle
Scalene Triangle
Isosceles Triangle
Equilateral Triangle

Quadrilateral
Diagonal

Additional Definitions:

Activity 1-(a)
MATCH THE TRIANGLES

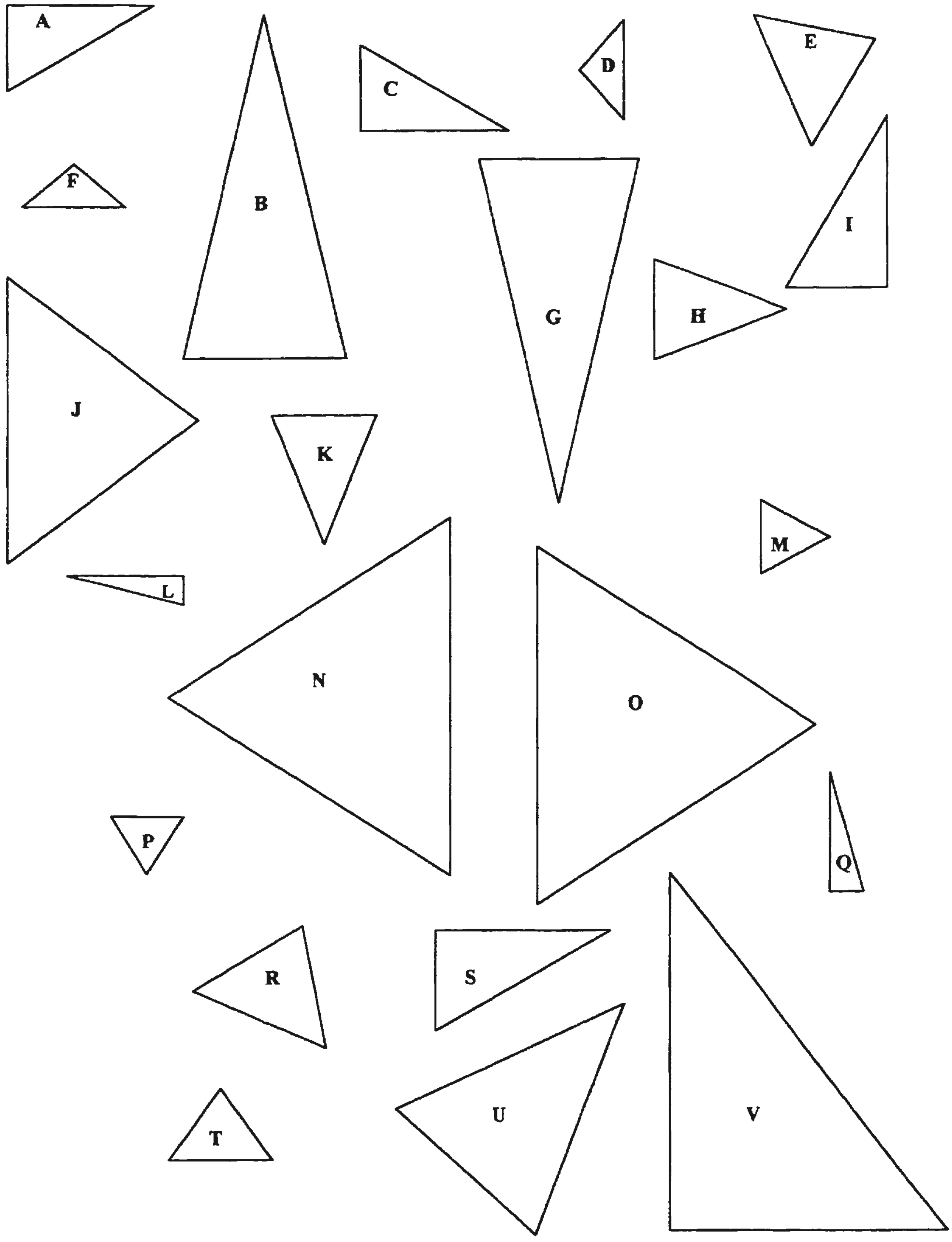
Play this game with a partner.

1. Toss a coin to decide which player starts the game. Players then take turns selecting two triangles that are the same size and shape. Rulers and protractors cannot be used to measure the triangles. If the selection is correct, the player receives two points. If not, the other player receives one point.

2. Using tracing paper, verify whether the two triangles in each pair are the same shape and size. Cross out the matching pairs.

3. After all possible triangles have been matched, add up the scores. The player with the highest score is the winner.

(Diagrams follow on the next page)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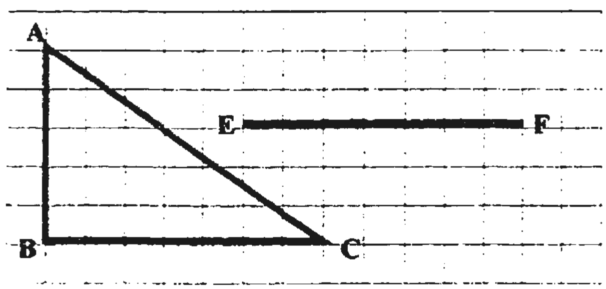
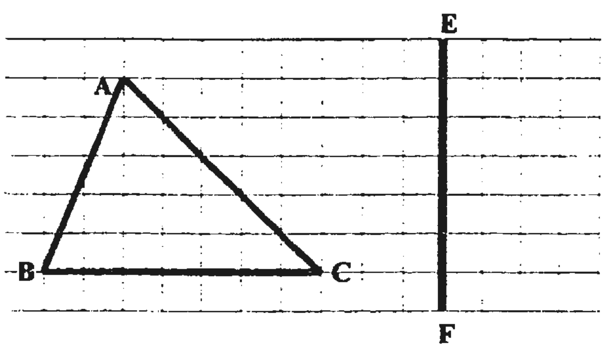
Student Activity Sheet #3

Activity 1-(b)

For each example below, copy the given triangle and line segment onto your grid paper. Locate a point D to make triangle DEF the same size as triangle ABC. Is there more than one place that D could be located?





Activity 1-(c)

SSS, SAS, ASA

Part I

A. Follow these steps to construct $\triangle ABC$ where AB = 7 cm, BC = 8 cm, and AC = 10 cm.

- *Draw line segment AB = 7 cm.*
- *Set your compass to a radius of 8 cm. Then use B as a center and draw an arc.*
- *Set your compass to a radius of 10 cm. Then use A as a center and draw an arc to intersect the first arc. Label the point of intersection C.*
- *Draw BC and AC.*

B. Is it possible to construct $\triangle DEF$ where DE = 7 cm, EF = 8 cm, and DF = 10 cm so that the size and shape of $\triangle DEF$ are different from the size and shape of $\triangle ABC$? Explain.

Part II

A. Construct $\triangle GHI$ so that GH = 5 cm and GI = 7 cm.

B. Is it possible to construct $\triangle JKL$ with JK = 5 cm and JL = 7 cm so that the size and shape of $\triangle JKL$ are different from the size and shape of $\triangle GHI$? Explain.

Part III

A. Construct $\triangle MNO$ so that $\angle M = 35^\circ$, $\angle N = 65^\circ$, and $\angle O = 80^\circ$.

B. Is it possible to construct $\triangle PQR$ with $\angle P = 35^\circ$, $\angle Q = 65^\circ$, and $\angle R = 80^\circ$ so that the size and shape of $\triangle PQR$ are different from the size and shape of $\triangle MNO$? Explain.

Part IV

A. Construct $\triangle RST$ so that $\angle R = 70^\circ$, $RS = 6$ cm, and $RT = 7$ cm.

B. Is it possible to construct $\triangle XYZ$ with $\angle X = 70^\circ$, $XY = 6$ cm, and $XZ = 7$ cm so that the size and shape of $\triangle XYZ$ are different from the size and shape of $\triangle RST$? Explain.

Part V

A. Construct ΔTUV so that $\angle T = 50^\circ$, $TU = 4$ cm, and $UV = 6$ cm.

B. Is it possible to construct ΔWXY with $\angle W = 50^\circ$, $WX = 4$ cm, and $XY = 6$ cm so that the size and shape of ΔWXY are different from the size and shape of ΔTUV? Explain.

Part VI

A. Construct $\triangle ABC$ so that $\angle A = 40^\circ$, $AB = 7$ cm, and $\angle B = 55^\circ$.

B. Is it possible to construct $\triangle DEF$ with $\angle D = 40^\circ$, $DE = 7$ cm, and $\angle E = 55^\circ$ so that the size and shape of $\triangle DEF$ are different from the size and shape of $\triangle ABC$? Explain.

Part VII

A. Construct $\triangle PQR$ so that $\angle P = 35^\circ$ and $PQ = 7$ cm.

B. Is it possible to construct $\triangle XYZ$ with $\angle X = 35^\circ$, and $XY = 7$ cm, so that the size and shape of $\triangle XYZ$ are different from the size and shape of $\triangle PQR$? Explain.

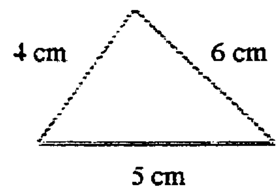
Conclusion

Use your results from the first 7 activities to decide which 3 conditions need to be given so that only one triangle can be constructed.

In your notebook, summarize your findings.

Activity 1-(d)

To change the shape of this triangle, you have to change the length of at least one of the sides. ***Why?***



Student Activity Sheet #6

Activity 1-(e)

Decide which of the following are always congruent, sometimes congruent, or never congruent. ***Explain your answers.*** *(You may use diagrams to illustrate your responses).*

A. 2 triangles with the same perimeter.

B. 2 rectangles with the same area.

C. 2 squares with the same perimeter.

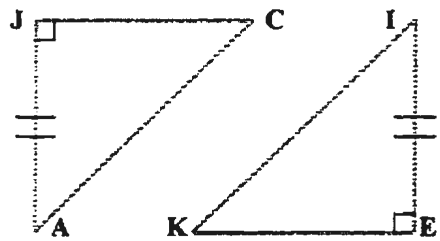
D. 2 rectangles with the same perimeter.

Student Activity Sheet #7

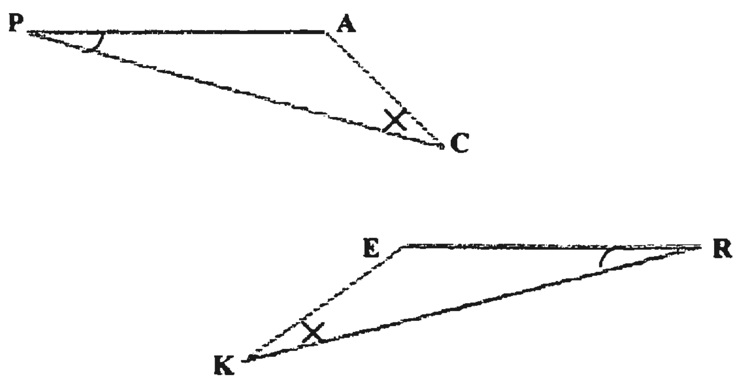
Activity 2-(a)

For each of the following pairs of triangles, two equal corresponding parts are marked. What is the fewest number of additional corresponding parts that must be equal to ensure that the two triangles are congruent? ***Explain.***

(A)



(B)

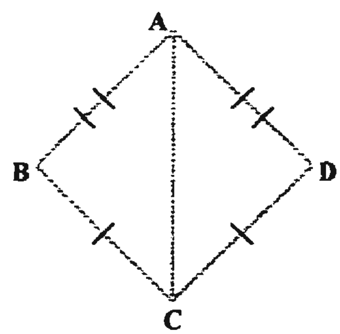


Student Activity Sheet #8

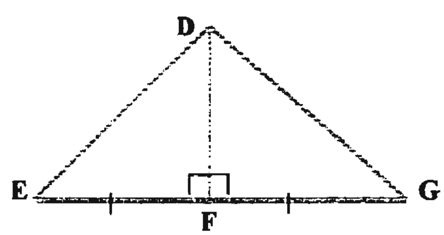
Activity 2-(b)

Identify pairs of congruent triangles in each diagram. Justify your responses.

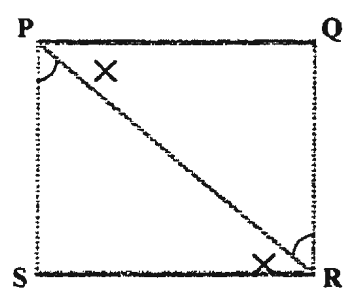
(A)



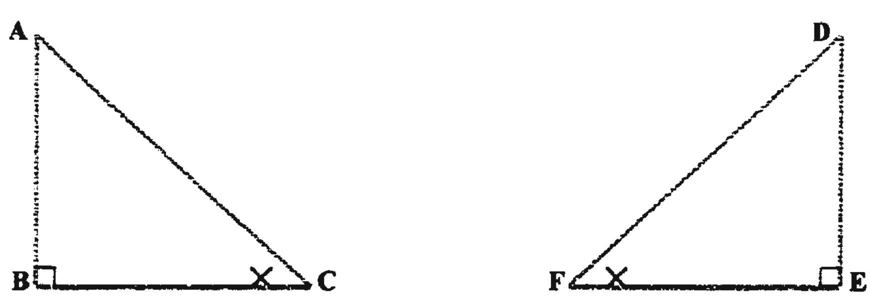
(B)



(C)



(D)



Student Activity Sheet #9

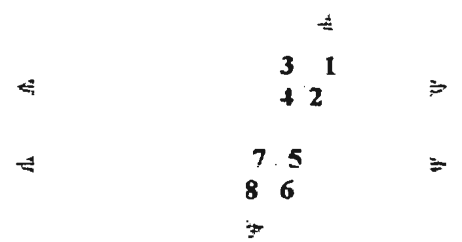
Activity 3-(a)

A. *Describe the most common use of parallel lines that you encounter.*

B. *Work with your partner to list other uses of parallel lines.*

Activity 3-(b)

Given the following diagram:



there are four types of angles associated with parallel lines cut by a transversal.

- **Co-interior angles**
- **Alternate angles**
- **Corresponding angles**
- **Vertically Opposite angles**

Identify the types associated with each of the following pairs of angles.

$\angle 2$ and $\angle 5$
$\angle 4$ and $\angle 5$
$\angle 3$ and $\angle 7$
$\angle 1$ and $\angle 4$

Based on the location of these angles, name other pairs of the same type.

Measure the eight angles in the diagram above formed by the parallel lines intersected by a transversal.

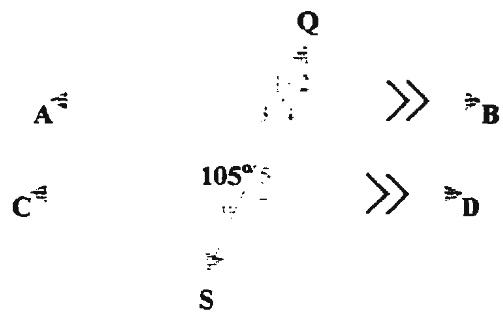
How many pairs of each type of angles did you find?
Was the relationship between the particular pairs of angles consistent for all such pairs?
Make a ***conjecture*** *about each type of angle.*

If you know that vertically opposite angles are ALWAYS congruent, and that corresponding angles are ALWAYS congruent, could you explain why the other kinds of angles are related in the ways they are?

Student Activity Sheet #11

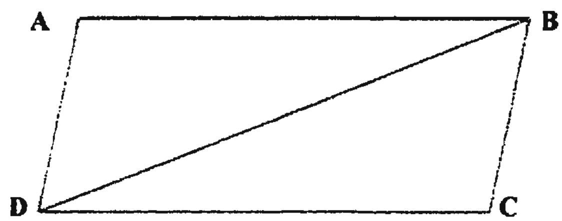
Activity 3-(c)
Missing Measures

Find all the missing angle measures. Justify your answers.



Student Activity Sheet #12

Activity 3-(d)



If AB is parallel to CD and AD is parallel to CB, is ΔABD congruent to ΔCDB? ***Explain.***

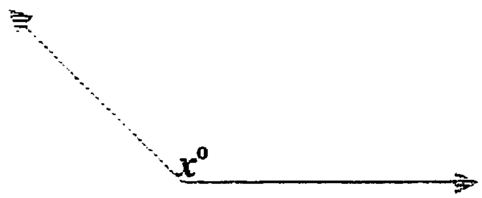
Student Activity Sheet #13

Activity 3-(e)

Write a problem that involves angles and parallel lines.
Exchange the problem with your partner. Solve problems and discuss the answers.

Activity 4-(a)

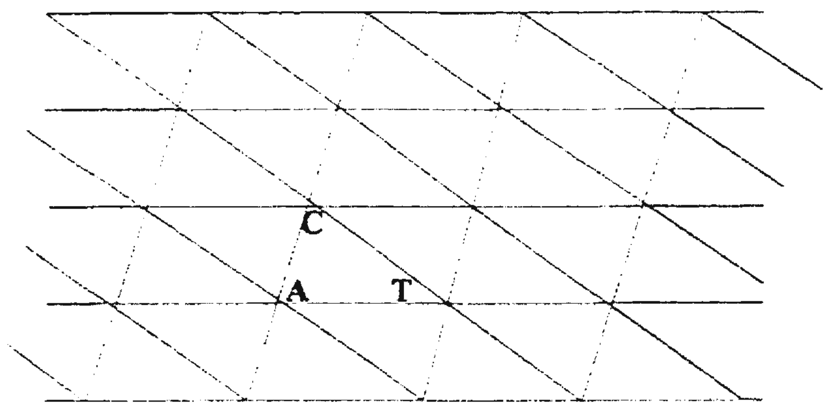
*Estimate the measurement of angle **x** in degrees.*
Explain or show how you got your answer.



Student Activity Sheet #15

Activity 4-(b)

The grid shown below consists of sets of parallel lines.



1. Outline ΔCAT on this grid. Choose three different colors. With the first color, color in ∠C and all the angles that are congruent to ∠C. With the second color, color in ∠A and all angles that are congruent to ∠A. With the third color, color in ∠T and all angles that are congruent to ∠T.

2. What do you observe about the angles around C? Around A? Around T?

   Find straight angles in the colored diagram. What colors are the angles that make up each straight angle?

3. What can you say about the sum of the measures of ∠C, ∠A, and ∠T?

   Could you say the same things about the three angles on other triangles?

   Could you say the same thing about all triangles?
   Justify your conclusion.

***** Activity 4-(b) is adapted from the NCTM Addenda Series/Grades 5-8: *Geometry in the Middle Grades***

Student Activity Sheet 7-16

Activity 4-(c)

Is it possible for a triangle to have two right angles?
Explain.

Is it possible for a triangle to have more than one obtuse angle?
Explain.

Consider triangles A through I, classified according to their angles and sides. Some types of triangles named in the table exist, while others do not. Draw the types of triangles that exist and label them with the appropriate letters.

	SCALENE	ISOSCELES	EQUILATERAL
ACUTE	**A**	**B**	**C**
RIGHT	**D**	**E**	**F**
OBTUSE	**G**	**H**	**I**

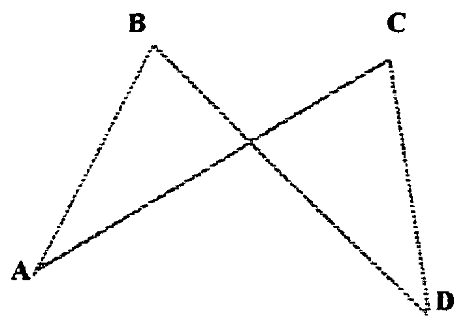
If a triangle does not exist, explain why.

Activity 5-(a)
DEFINING QUADRILATERALS

A student was asked to define a quadrilateral.

"A quadrilateral is what you get if you take 4 points A, B, C, D in a plane and join them with straight lines."

Do you agree or disagree with this definition? ***Why?***



Is this a quadrilateral? **Explain** *why or why not.*

Formulate your own definition of a quadrilateral.

Student Activity Sheet #18

Activity 5-(b)
Sum of the Measures of the Angles of Quadrilaterals

The following represents one student's effort to write a proof of the quadrilateral angle sum.
Explain *what is right or wrong about it.*

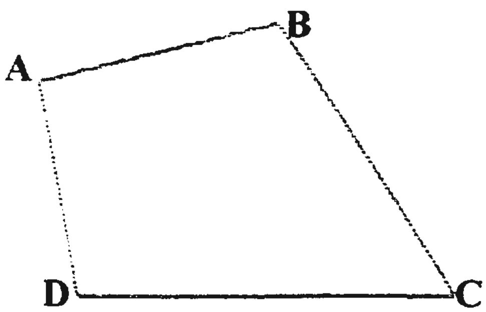
"Every quadrilateral can be made into two triangles by joining a diagonal. Each triangle has 180º and 2 x 180º = 360º. Therefore, the sum of the angles of a quadrilateral must be 360º."

Write your own proof to improve upon this one.

Student Activity Sheet #19

Activity 5-(c)
Missing Measures

What is the measure of the unknown angle?



∠A = 91°

∠B = 113°

∠C = ?

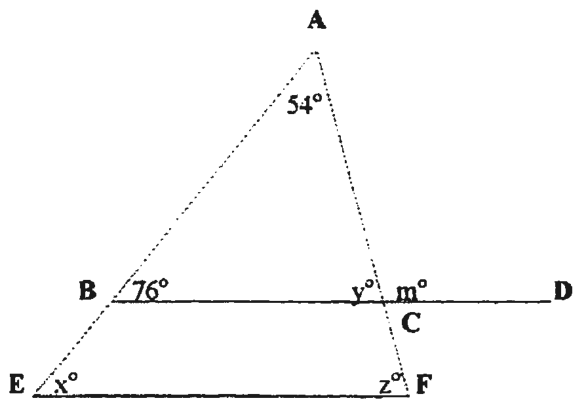
∠D = 104°

***** Activity 5 is an adaptation of an activity from the May 1995 issue of *Mathematics Teacher*.**

ACTIVITY 6
"Piecing it all Together"

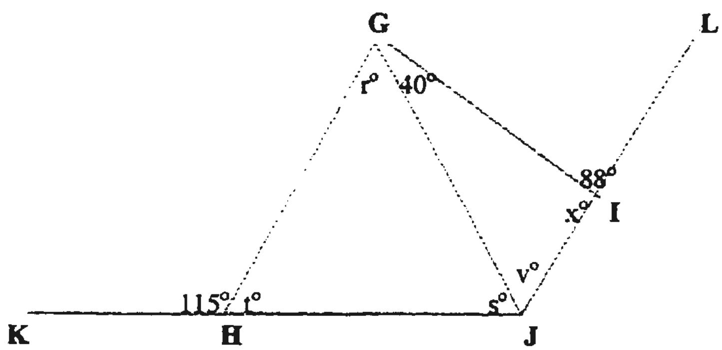
Find all the unknown angle measures, as indicated.
***Explain** how you found each of the missing measures.*
Note: Diagrams are not drawn to scale.

(A)



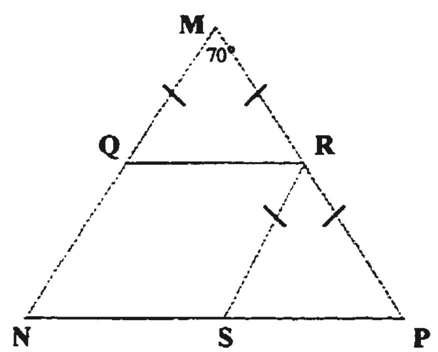
Given:
BD || EF

(B)



Given:
∠HGI is a right angle

(C)



Given:
QR || NS

NQ || SR

***Hint:** You may find it helpful to first list all the angles in this diagram.*