OPTIMIZATION METHODS APPLIED TO ELECTRIC POWER SYSTEMS

JIASI KONG

Optimization Methods Applied to Electric Power Systems

By

Jiasi Kong, B.Eng

A thesis submitted to the School of Graduate Studies in partial fulfillment

of the requirements for the degree of Master of Engineering

Faculty of Engineering and Applied Science

Memorial University of Newfoundland

October 2009

St.John's Newfoundland and Labrador Canada

Abstract

The modern power system has been facing a tremendous challenge for utilities to maintain an economical, reliable and secure operation due to the increasing fuel cost, long distance transmission and intense market competition. This work investigates the application of optimization methods in power systems. Specifically, optimization methods to minimize total fuel cost and transmission loss for a specified load are considered. Multiobjective optimization focusing on the constraints related to the steady state operation including security constraints in power system is studied. Optimal Power Flow (OPF) including economic dispatch, security constrained optimal power flow and multiobjective optimization are the three key concepts of this thesis. Sequential quadratic programming is proposed and implemented as an optimization method for carrying out this research. Weighted sum method, a conventional multiobjective optimization method, is applied and implemented by Matlab Optimization Toolbox. A series of mutiobjective OPF case studies are presented in this research to show the performance and applications of the proposed optimization methods. The results from the case studies presented show that the tools are able to determine feasible, non-dominated optimal operation points that allow a system to operate economically and safely under a specified load demand.

Acknowledgements

I would like to thank and give my sincere gratitude to Dr. Benjamin Jeyasurya. His constant support, encouragement and guidance throughout this research experience are instrumental to my success at Memorial university of Newfoundland.

I would like to thank my parents, Wenjin and Fanrong for their encouragement and support throughout my time as a graduate student. Without them, this work would never have come in to existence.

Special thanks and appreciation are given to the Natural Sciences and Engineering Research Council of Canada and to Memorial University of Newfoundland for the financial support, which made this research possible. Thanks are also given to the Faculty of Engineering at Memorial University for providing the resources to carry out this research.

Contents

List of Figures

List of Tables

List of Abbreviations and Symbols

BFGS	Broyden, Fletcher, Goldfarb and Shannon Formula
EA	Evolutionary Algorithms
EMS	Energy Management System
H	Hessian Matrix
IP	Integer Programming
KT	Kuhn-Tucker
LP	Linear Programming
MVAR	Mega Volt Ampere Reactive
MOPSP	Multiobjectieve Partical Swarm Optimization
NLP	Nonlinear Programming
OPF	Optimal Power Flow
PSO	Particle Swarm Optimization
P	Active Power
p.u.	Per Unit
Q	Reactive Power
QP	Quadratic Programming
SCOPF	Security Constrained Optimal Power Flow
SQP	Sequential Quadratic Programming
WS	Weighted Sum mehod

Y	Bus Admittance Matrix
\$/hr	Dollar per Hour

Chapter 1

Introduction

Electric power plays an exceedingly important role in the life of the community and in the development of various sectors of economy [1,2]. The modern power system has been facing a tremendous challenge for the utilities to maintain an economical and reliable operation due to the increasing fuel cost and intense market competition. Furthermore, the growing demand for electrical energy coupled with the reduced cost of generation has led generating stations to be located hundreds of kilometers away from load centers. This in turn has caused the electric power system to face many challenges, such as transmission loss and the security. Thus, power utilities need efficient tools and aid to ensure that the electrical energy at desired demand can be produced in the best possible way in the most reliable, secure, and economic manner [3]. This is also the purpose of this research which is focused on optimizing the power system in economic and secure aspects.

One possible approach to improve the system operation is economic dispatch. By using the economic dispatch, the generators' power output can be varied within certain limits to support a particular load demand at the lowest possible fuel cost. Optimal Power Flow (OPF) is used to determine an optimal operating condition for power systems while considering the limitations of the equipments and other operating constraints [4,5]. Security Constrained Optimal Power Flow (SCOPF) determines a feasible, minimum cost operating point such that in the event of any possible contingency, the post contingency states will remain secure (within operating limits) [6]. Different objectives can be achieved using optimal power flow while ensures the security of the power system.

Many optimization methods have been developed to achieve the goals of the above studies. In recent years there has been interest in applying Multiobjective optimization for power system problems [2]. It can be considered as optimizing many objective functions subject to different constraints. For power system applications, these objective functions can be cost, transmission loss, voltage deviation etc.. Many of proposed methods for multiobjective optimization focus on the constraints related to the steady state operation. Security constraints (operation of the power system under credible contingencies) are not considered in detail [7].

1.1 Objectives of the Research

The focus of this research is to apply optimization methods for different power system problems. Specifically, optimization methods to minimize total fuel cost and transmission loss are considered. In addition, multiobjective optimization is also applied for power system planning and operation problems. Another important goal of the research is to include security constraints in the power system problems studies. The principal goals of this research are summarized as follows:

- To research the applications of the optimization techniques in the power system engineering field.
- To study the possible use of constrained optimization methods for power system economic dispatch and optimal power flow.
- To research and explore multiobjective optimization methodologies in power system operation.
- To include security constraints in power system optimization problems.

1.2 Organization of the Thesis

This thesis is organized as follows:

A discussion of optimization techniques is presented in chapter 2. The definition and classification of optimization problems are given. The optimal conditions and optimization algorithm are introduced. The classification, theories and features of general optimization techniques are discussed. The applications of optimization methods in power system are discussed.

Chapter 3 focuses on the economic dispatch for real power generation. It starts with discussing the relationship between fuel cost and the power generation. The typical economic dispatch problem is introduced. Following this, the economic dispatch of generation for minimization of the total operating cost neglecting and including transmission loss and generation limits are discussed. Next, the transmission loss formula is derived and the economic dispatch of generation based on the loss formula is obtained.

Case studies are discussed and illustrated.

Chapter 4 starts with the main principal of the Optimal Power Flow (OPF) and Security Constrained Optimal Power Flow (SCOPF). The algorithm for OPF and SCOPF are presented. The minimum cost and minimum transmission loss problems of OPF and SCOPF are considered as examples to discuss the application of the optimization techniques to solve OPF and SCOPF problems. Case studies are illustrated and discussed.

Chapter 5 starts with the comparison of single-objective optimization and multiobjective optimization, and then provides an overview of typical multiobjective optimization problems. The proposed optimization methods are introduced with an example. Case studies using weighted sum method are presented.

Chapter 6 presents the multiobjective optimization for power system including security constraints. Weighted sum method is proposed and applied to two power systems to solve an optimal power flow problem which is to minimize the total fuel cost and minimize the total transmission loss while ensuring that the power system is secure.

Chapter 7 highlights the key contributions of the research presented in this thesis along with suggestions for future work.

Chapter 2

Optimization Techniques

2.0 Introduction

Optimization is the process of determining the best result from a set of alternatives under certain given circumstances. It has been wildly applied to the engineering field to define economical reliable, secure, efficient systems as well as to devise plans and procedures to improve the operation of the existing systems. A number of optimization methods have been developed to solve various types of problems depending on their constraints and functions.

This chapter defines the concepts of engineering optimization problems and some associated concepts in section 2.1. Section 2.2 presents some algorithms to solve these problems. Nonlinear programming methods are presented in the later part of the research using an optimization package. Sequential quadratic programming method, a popular nonlinear direct method, is discussed in detail, and is applied to a simple numerical example. A brief application of the optimization techniques in power system is listed.

2.1 Statement of an Optimization Problem

The ultimate goal of engineering optimization is to minimize the effort or to maximize the benefit desired [8-10]. The effort required or the benefit desired in any practical situation can be expressed as an objective function. An optimization problem can be stated as follows:

The general single objective optimization problem can be formally defined as finding the vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

to minimize / maximize: $$f(X) \tag{2.1}$$

subject to: $$g_j(x) \le 0, j = 1, 2, \cdots, m \tag{2.2}$$

$$h_k(x) = 0, k = 1, 2, \cdots, p \tag{2.3}$$

$$x_i^{(L)} \le x_i \le x_i^{(U)}, i = 1, 2, \cdots, n \tag{2.4}$$

x is an n-dimensional vector called the design vector. In Equation 2.1, $f(X)$ is the objective function set. In Equationns. 2.2 and 2.3, $g_j(x)$ and $h_k(x)$, are inequality and equality constraints. $x_i^{(L)}$ and $x_i^{(U)}$ are the lower and upper bounds of variables, restricting each decision variable x_i to take a value within a lower $x_i^{(L)}$ and an upper $x_i^{(U)}$ bound [3,8]. The problem stated above is called a constrained optimization problem. Some optimization problems do not involve any constraints and can be stated as:

$$\text{Find} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

to minimize/maximize: $\qquad f(x) \qquad$ (2.5)

Such problems are named unconstrained optimization problems.

2.1.1 Objective Function

In a system design, there will be more than one feasible solution. The purpose of the optimization is to choose the best one of the alternatives. Thus a criterion is necessary to evaluate all the alternatives and decide the most suitable one. The criterion, with respect to which the design is optimized, when expressed as a function of design variables, is known as the criterion or merit or objective function [8,10]. The choice of the objective function depends on the nature of the problem.

2.1.2 Design Vector

Design vector is defined as a set of unknowns or variables which affect the value of the objective function. In general, certain quantities are usually fixed at the outset and these are called pre-assigned parameters. All the other quantities are treated as variables in the design process and are called design or decision variables [8]. For example, in the manufacturing problem, the variables could be the time spent on each activity or the efficiency of each machine.

2.1.3 Constraints

Constraints that represent limitations on the behavior or performance of the system are termed behavior or functional constraints [8]. They must be satisfied to produce an acceptable design. Equation 2.2 represents the inequality constraints and Equation 2.3 states the equality constraints. For example, in the power system, the maximum power generation of each generator is an inequality constraint. In a general optimization problem, the number of equality constraints j must be less than the number of the design variables n. If j equals n, then the problem can be solved by the equality constraints, which does not account as an optimization problem. If j is greater than n, then some of the constraints must be dependent on others and they are redundant. The variable bounds are considered as inequality constraints.

2.2 Classification of Optimization Problems

The optimization problems can be classified in several ways, such as based on the existence of constraints, the nature of the design variables, the physical structure of the problem, the nature of the equations involved and so on. Based on the nature of the expressions for the objective function and the constraints, optimization problems can be classified as Linear, Nonlinear and Quadratic Programming problems (LP, NLP and QP).

2.2.1 Linear Programming

If the objective functions and the constraints involved in the optimization problems are linear functions of the design variables, the mathematical programming problem is called a LP problem [8-10]. The standard form for a LP problem can be stated as:

Finding:
$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

to minimize/maximize:
$$f(X) = \sum_{i=1}^{n} c_i x_i \tag{2.6}$$

subject to:
$$\sum_{i=1}^{n} a_{ij} \times x_i = b_j, \quad j = 1,2,...m \tag{2.7}$$

$$x_i \geq 0, \quad i = 1,2,...,n \tag{2.8}$$

Where a_{ij}, b_j and c_i are coefficient of the corresponding functions.

2.2.2 Quadratic Programming

A quadratic programming problem is a nonlinear programming problem with a quadratic objective function and linear constraints. A QP problem can be formulated as [4,11]:

$$\text{Find} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

to minimize/maximize: $$f(X) = \frac{1}{2} x^T Q_x + c^T x \tag{2.9}$$

Subject to: $$Ax \leq b \tag{2.10}$$

$$x_i \geq 0, \quad i = 1, 2, \ldots, n \tag{2.11}$$

Where: c is a vector of objective coefficients

A is a matrix of constraint coefficients

b is a vector of right-hand sides of constraints

Q is a symmetric matrix

Comparing QP problem and LP problem, it is clear that the QP problem has a $\frac{1}{2} x^T Q_x$ term involved in the objective function, but the constraints functions are both linear.

2.2.3 Nonlinear Programming

If any of the functions among the objective functions and the constraints in equation 2.1, 2.2 and 2.3 is nonlinear, the problem is called a NLP problem. This is the most general programming problem and all other problems can be considered as the special cases of the NLP problems. Many optimization problems regarding electric power system are NLP problems, such as minimizing the generation cost. The associated algorithm is discussed in later section of the chapter.

2.3 Algorithms and Solutions

2.3.1 Local and Global Minimum

Global minimum is defined as for all $x \in S$ when $x^* \in S$ if $f(x^*) \le f(x)$, and then x^* is the global minimum of the function $f(x)$. A point is a local minimum if all other points in its neighborhood have a higher function value. The point $x^* = [x_1^*, x_2^*, ..., x_n^*]^T$ is a weak local minimum if there exist a $\delta > 0$ such that $f(x^*) \le f(x)$ for all x such that $\|x - x^*\| < \delta$. The point x^* is a strong local minimum if $f(x^*) \le f(x)$ for all x in a neighborhood of x^* [8-10]. The global minimum is also a local minimum if $f(x^*) \le f(x)$ for all $x \in R^n$. A local minimum may or may not be a global minimum but if a problem possesses a minimum then there is exactly one global minimum. The following figure illustrates the local and global minimum.

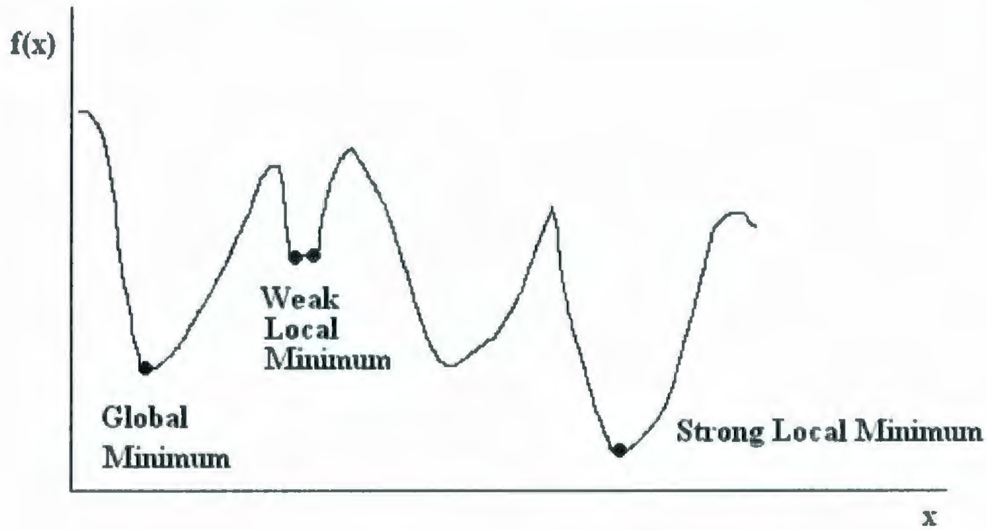

Fig. 2.1 Local Minimum and Global Minimum [8].

2.3.2 Multivariable Optimization with Constraints

This section is concerned with the solution of the problem stated in Equations 2.1, 2.2 and 2.3. The most common way to deal with inequality constraints is to transform it to equality constraints by adding nonnegative slack variables. The values of slack variables are unknown. Then the optimization problem becomes to

$$\text{Minimizie} \quad f(X) \tag{2.12}$$

$$\text{subject to} \quad G_j(X,Y) = g_j(x) + y_j^2 = 0, \quad j = 1,2,\ldots,m \tag{2.13}$$

where $Y = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{Bmatrix}$ is the vector of slack variables.

Now, this problem can be conveniently solved by linearly combining the objective, equality and inequality constraint functions into one function which is known as the Lagrange function *L*. The Lagrange function is formulated as:

$$L(X,Y,\lambda) = f(X) + \sum_{j=1}^{m} \lambda_j G_j(X,Y) + \sum_{k=1}^{p} \beta_k h_k(X) \tag{2.14}$$

where $\lambda = \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_m \end{Bmatrix}$ and $\beta = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{Bmatrix}$ are the vectors of Lagrange multipliers.

Based on the Lagrange function, the complete set of necessary conditions for optimizing a constrained minimization problem, known as Kuhn-Tucker (KT) conditions becomes [4,8,12]:

$$\nabla f + \sum_{j=1}^{m} \lambda_j \nabla g_j - \sum_{k=1}^{p} \beta_k \nabla h_k(X) \tag{2.15}$$

$$g_j(X) + y_j^2 = 0, \quad j = 1,2,\ldots,m \tag{2.16}$$

$$h_k(X) = 0, \quad k = 1,2,\ldots,p \tag{2.17}$$

$$\lambda_j g_j = 0, \quad j = 1,2,\ldots,m \tag{2.18}$$

$$\lambda_j \geq 0, \quad j = 1,2,\ldots,m \tag{2.19}$$

$$g_j \geq 0, \quad j = 1,2,\ldots,m \tag{2.20}$$

Equations 2.13 to 2.18 represent $(n+2m+p)$ equations with $(n+2m+p)$ unknowns, X, Y, λ, and β. The solutions of Equations 2.13 to 2.18 gives the optimum solution vector X^*, the Lagrange multiplier vector, λ^*, and the slack variable vector, Y^*. The Equations 2.14 and 2.15 ensure that the constraints Equations 2.2 and 2.3 are satisfied. The results under different assumptions (either λ^* or β^* is zero) are called KT points. The function value $f(z)$ should also be calculated to determine the value of the objective function.

2.4 Nonlinear Programming

Many methods are available to solve nonlinear constrained optimization problems. All the methods can be classified into two types: direct methods and indirect methods. In indirect methods, the constrained problem is solved as a sequence of unconstrained minimization problems by forming Lagrangian function. A penalty function maybe added to the Lagrangian function to force the constraints to be satisfied if one or more constraints are violated. Augmented Lagrangian multiplier method is an example of such methods [4,8].

In the direct methods, the constraints are handled in an explicit manner. Most direct methods build a sub-problem using linear approach or quadratic approach in order to approximate the objective function and constraint functions around the current points. The convergence direction is obtained by solving this sub optimization problem. The next point is reached by combining the direct and the step length generated by search methods [4]. There are many direct methods to solve the nonlinear problems, such as Heuristic search methods, generalized reduced gradient method and sequential linear programming method. However, Sequential Quadratic Programming (SQP) is one of the most popular direct methods of optimization.

2.4.1 Sequential Quadratic Programming

The Sequential Quadratic Programming (SQP) can be interpreted as Newton's method applied to the solution of the Kuhn-Tucker conditions. The SQR method has several attractions: the starting point can be infeasible, gradients of only active constraints are needed, equality constraints can be handled in addition to the inequalities, and the method can be proved to converge under certain assumptions [29]. In order to understand SQP method, two terms should be introduced first which are search direction d^k and step length α_k. The idea of solving an optimization problem is to start from an initial point to approach the solution iteratively, thus the design vector determined at each iteration can be expressed as the combination of d^k and α_k.

$$X_{k+1} = X_k + \alpha_k d^k \tag{2.21}$$

Linearly combining the objective function and constraints by Taylor series and adding a general quadratic term $\frac{1}{2} d^T [H] d$ in the linearized objective function, the quadratic programming subproblem is reformed as:

$$\text{Minimizing} \quad \nabla f(X)^T d + \frac{1}{2} d^T [H] d \tag{2.22}$$

$$\text{subject to:} \quad \beta_j g_j(X) + \nabla g_j(X)^T d \leq 0, \quad j = 1, 2, \cdots, m \tag{2.23}$$

$$\bar{\beta} h_k(X) + \nabla h_k(X)^T d = 0, \quad k = 1, 2, \cdots, p \tag{2.24}$$

$$\text{where } H = \nabla^2 f + \sum_{j=1}^{m} u_j \nabla^2 g_j + \sum_{k=1}^{p} u_k \nabla^2 h_k \tag{2.25}$$

Equation 2.22 is the new objective function. Equations 2.23 and 2.24 are constraint functions. H is a positive definite matrix that is taken initially as the identity matrix and is updated in subsequent iterations so as to converge to the Hessian matrix of the Lagrange function. β_j and $\bar{\beta}$ are constants used to ensure that the linearized constraints do not cut off the feasible space completely. Typical values of these constants are:

$$\bar{\beta} = 0.9;\ \beta_j = 0.9 \text{ if } g_j(X) \geq 0, \text{ otherwise, } \beta_j = 1. \qquad (2.26)$$

The subproblem of Equations 2.22-2.24 is a quadratic programming problem and can be solved using quadratic programming method. Once the search direction, d, is computed by solving Equations 2.22-2.24, the design vector can be updated using Equations 2.21. And then the Hessian matrix $[H]$ can be updated to improve the quadratic approximation in Equations 2.22-2.24. Broyden, Fletcher, Goldfarb, and Shannon (BFGS) formula is one of the methods applied to approximate the Hessian matrix [14]. The iterative process is terminated if there is no change in the objective function f for three consecutive iterations

The research focused in this thesis uses Matlab Optimization Toolbox [13], which uses sequential quadratic programming and linear search for constrained nonlinear problems.

2.4.2 Case Study of Sequential Quadratic Programming

A single objective constrained optimization problem [8] with two variables and three constraint functions is presented to illustrate the algorithm of the sequential quadratic programming method.

$$\text{Minimize } f(X) = 0.1x_1 + 0.05773x_2 \tag{2.27}$$

$$\text{subject to } g_1(X) = \frac{0.6}{x_1} + \frac{0.3464}{x_2} - 0.1 \le 0 \tag{2.28}$$

$$g_2(X) = 6 - x_1 \le 0 \tag{2.29}$$

$$g_3(X) = 7 - x_2 \le 0 \tag{2.30}$$

Setup the initial Points: $X_1 = (11.8765, \quad 7)$

f and $g_1, g_2,$ and g_3 are:

$$\begin{aligned} &f(X_1) = 1.5917; \\ &g_1(X_1) = 0; \\ &g_2(X_1) = -5.8765; \\ &g_3(X_1) = 0; \end{aligned}$$

The gradients of the objective and constraint function at X_1 are:

$$\nabla f = \begin{pmatrix} 0.1 \\ 0.05773 \end{pmatrix},$$

$$\nabla g_1(X_1) = \begin{pmatrix} \dfrac{-0.6}{x_1^2} \\ \dfrac{-0.3464}{x_2^2} \end{pmatrix}_{x_1} = \begin{pmatrix} -0.004254 \\ -0.007069 \end{pmatrix},$$

$$\nabla g_2(X_1) = \begin{pmatrix} -1 \\ 0 \end{pmatrix},$$

$$\nabla g_3(X_1) = \begin{pmatrix} 0 \\ -1 \end{pmatrix},$$

Hence the quadratic subproblem is:

$$\text{Minimize: } 0.1d_1 + 0.05773d_2 + 0.5d_1^2 + 0.5d_2^2 \tag{2.31}$$

$$\text{subject to: } g_1 = -0.004254d_1 - 0.007069d_2 \le 0 \tag{2.32}$$

$$g_2 = -5.8765 - d_1 \le 0 \tag{2.33}$$

$$g_3 = -d_2 \le 0 \tag{2.34}$$

solve this quadratic problem Equations 2.31-2.34 directly with the use of the Kuhn-Tucker conditions.

$$\text{Search Direction } \quad d = (-0.0476, \quad 0.0294)$$

Using linear search to calculate the step length:

$$\alpha = 1$$

Then the new design vector, X , can be determined as:

$$X_2 = X_1 + \alpha d = \begin{pmatrix} 11.8765 - 0.0476\alpha \\ 7 + 0.0294\alpha \end{pmatrix} = \begin{pmatrix} 10.9456 \\ 7.5699 \end{pmatrix}$$

The Hessian Matrix is the matrix of second partial derivatives and can be updated as:

$$H_2 = \begin{bmatrix} 0.2787 & 0.4395 \\ -0.4395 & 0.7399 \end{bmatrix}$$

The optimal solution can be achieved by keeping the iteration process by defining a new quadratic programming problem using Equations 2.22-2.26 and continuing the procedure until the stopping criteria is met. The following table lists the results at each iteration.

Table 2.1 Results at Each Iteration

Iteration Number	x	f(x)	Hessian	Search Direction
0 (initial guess)	11.8765, 7	1.59176		
1	11.8289, 7.0294	1.5887	0.2851, 0.442 0.442, 0.7320	-0.0476, 0.0294
2	10.9456, 7.5699	1.5316	0.2787, 0.4395 0.4395, 0.7399	-3.5591, 2.1772
3	10.3351, 8.171	1.5052	0.1132, 0.0757 0.0757, 0.1321	-0.5831, 0.5811
4	10.0309, 8.5868	1.4988	0.1193, 0.0729 0.0729, 0.0738	-0.3221, 0.4258
5	9.5392, 9.2441	1.4876	0.1190, 0.0746 0.0746, 0.0705	-0.5017, 0.6711
6	9.5004, 9.3987	1.4926	0.0717, 0.0123 0.0123, 0.0170	0.0293, 0.1433
7	9.4738, 9.4466	1.4927	0.0492, 0.0157 0.0157, 0.0210	0.0276, 0.0491
8	9.4639, 9.4642	1.4928	0.0489, 0.0158 0.0158, 0.0212	0.0115, 0.0203

The solution (9.4639, 9.4642) is found successfully after 8 iterations; the minimum of objective function value is 1.4928. Fig. 2.1 plots the calculated result at each iteration.

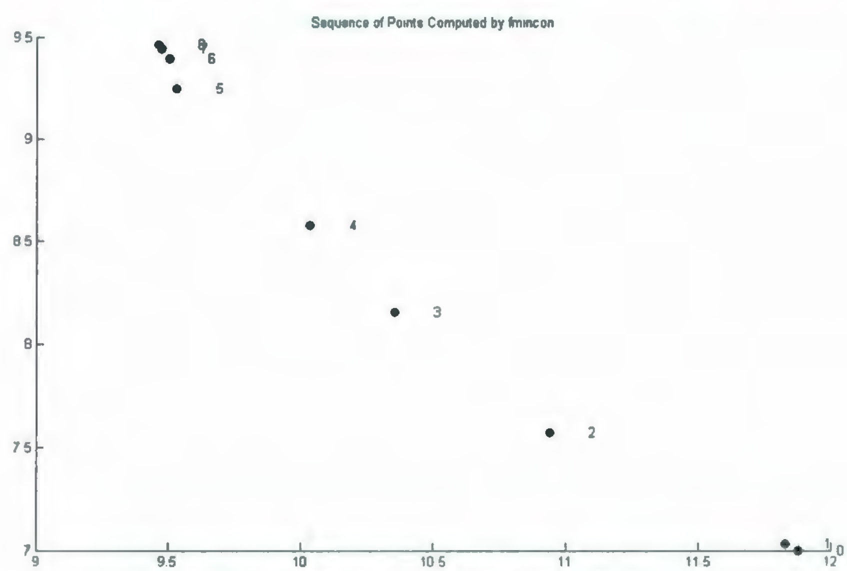

Fig. 2.2 Plot of x Values at Each Iteration.

2.5 Application of Optimization in Power System

Optimization problems in power systems are very challenging to solve because power systems are very large, geographically widely distributed and are influenced by many unexpected events. Therefore, it is necessary to employ the most efficient optimization methods to take full advantages in simplifying the formulation and implementation of the problem. This section briefly summaries the application of optimization techniques in the power system [14].

- Linear and quadratic programming methods are used to solve power systems problems with regards to optimal power flow, reactive power planning and active and reactive power dispatch [15,16].
- Nonlinear programming method has been applied to various areas of power system for optimal power flow, security constrained optimal power flow and hydrothermal scheduling [2,9,14].
- Integer and Mixed-Integer Programming method is employed to solve power system problems with regards to optimal reactive power planning, power system planning, unit commitment, and generation scheduling [14].
- Dynamic Programming method has been applied to various areas of power systems such as reactive power control, transmission planning and unit commitment [14].

The power system problems studied in this thesis can be formulated as a set of nonlinear equations. Hence, nonlinear programming method is more efficient to solve the problems.

2.6 Conclusion

Nonlinear programming method is the approach to start from an initial guess and to determine a 'descent direction' and step length in which objective function decreases in case of minimization problem. A large number of nonlinear programming methods are available, and are classified into direct and indirect methods. The sequential quadratic programming method is a typical direct method and perhaps one of the best optimization methods of optimization owing to its high speed of convergence. It is based on defining a quadratic subproblem at each iteration by linearizing the objective and constraint functions and specifying adjustable move limits.

This chapter has presented an overview of different optimization techniques along with the benefits of using it for solving difficult power system optimization problems. A simple numerical example is discussed to show the performance of sequential quadratic programming method. In the later part of this thesis, Matlab Optimization toolbox is used to solve the different power system operation problems.

Chapter 3

Economic Dispatch of Generation

3.0 Introduction

Electrical power generating stations are usually located hundreds of kilometers away from load centers and their fuel costs are different. Also, under normal operating conditions, the generation capacity is more than the total load demand and losses. Thus, there are many options for scheduling and planning generation [20]. This means that the real and reactive power provided by the generators can be adjusted within certain limits to meet the desired load demand with minimum fuel cost. This is also known as Optimal Power Flow (OPF). It can be achieved by minimizing the objective function, which the total fuel cost of the generating units, subject to the constraints that the sum of the powers generated must equal to the sum of the transmission loss and the power consumed by the loads. Economic dispatch is a special case of the OPF, which neglects the transmission line limits.

This chapter focuses on the economic dispatch of real power generation. It starts with introducing the concept of economic dispatch and typical economic dispatch problems. The economic dispatch of generation for minimization of the total operating cost neglecting and including transmission losses is obtained. Next, the transmission loss formula is derived and the economic dispatch of generation based on the loss formula is obtained. Case studies are presented to show the application of economic dispatch.

3.1 Operation Cost of a Conventional Power Plant

Fig. 3.1 shows the operating cost C of a fossil fuel generating unit versus real power output Pg. The fuel cost is a major portion of the available cost of operation [17,18]. The total cost of operation includes fuel, labor and maintenance costs. Only those costs that are a function of unit power output enter into the economic dispatch formulation, so only fuel cost is considered. From the shape of the fuel cost cure, it is clear that the fuel cost of all the generation units can be expressed as a quadratic function with positive coefficients and in terms of generation. It is expressed as:

$$C(P_g) = \alpha + \beta * P_g + \gamma * P_g^2 \tag{3.1}$$

where, α, β and γ are cost coefficients.

Thus, for an interconnected power system consisting of N units, the total fuel cost is:

$$C_t = \sum_{i=1}^{n_g} C_i = \sum_{i=1}^{n_g} (\alpha_i + \beta_i * P_i + \gamma_i * P_i^2) \tag{3.2}$$

where n_g is the number of generators.

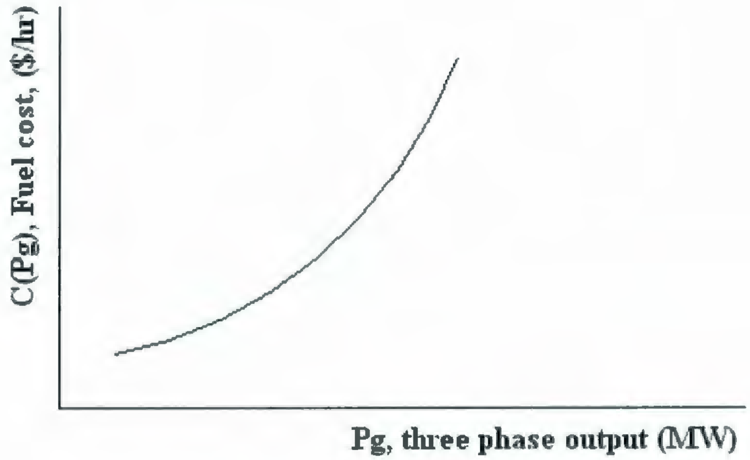

Fig. 3.1 Fuel Cost Versus Generation [17,20].

3.2 Economic Dispatch Problem

Figure 3.2 illustrates the configuration that will be studied in this section. This system consists of n generating units connected to only one bus serving a received electrical load P_D. The input to each unit, C_i, presents the generating cost of the unit. The output of each unit, P_{ng}, represents as the electrical power generated by the particular unit.

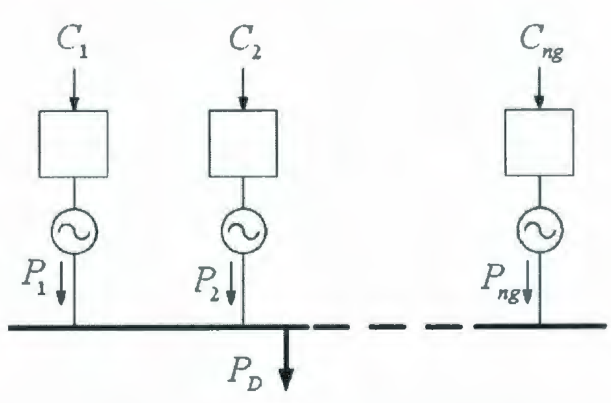

Fig. 3.2 Plants Connected to a Common Bus [19].

The objective of the economic dispatch is to determine the most efficient and low cost operation of a power system by dispatching the available electricity generation resources to supply the load on the system. The economic dispatch problem is to minimize the total fuel cost of generators in a power system, while satisfying the power balance and limits of generation units. It is formulated as:

$$\text{minimize} \quad C_t = \sum_{i=1}^{n_g} C_i = \sum_{i=1}^{n_g} (\alpha_i + \beta_i * P_i + \gamma_i * P_i^2) \tag{3.3}$$

$$\text{subject to}: \sum_{i=1}^{n_g} P_i = P_D + P_L \tag{3.4}$$

$$P_{i\min} \leq P_i \leq P_{i\max} \tag{3.5}$$

Where n_g is the total number of dispatchable generation units.

Ct is the total production cost, \$/hr.

C_i is the production cost of the ith unit, \$/ht.

α_i, β_i, and γ_i are cost coefficients of the *i*th unit

P_i is the real power generated by the *i*th unit, MW.

P_D is the total load demand, MW.

P_L is the transmission loss, MW.

$Pi\max$ is the upper permissible limit of real power generation, MW.

$Pi\min$ is the lower permissible limit of real power generation, MW.

Equation 3.3 expresses the objective function C_t, the total generation cost equals to the sum of the fuel cost of each generation unit. Equation 3.4 is the equality constraints. The total power generated by all the units equals to the sum of all the loads plus the total transmission loss. Equation 3.5 is inequality constraint and states the variable bounds. Each generating unit must not operate above its rating or below some minimum value. The design variables are the real power provided by each dispatchable generation unit. Transmission line limits are neglected for economic dispatch problems.

3.3 Algorithms of Economic Dispatch

Mathematically, the economic dispatch problem can be classified as a multivariable, or an nonlinear, or a constrained problem. Different software tools or algorithms have been proposed to solve economic dispatch problems, such as Lagrange multiplier, Lambda-Iteration method, Gradient methods, Newton method and Dynamic programming [1,2,15,17,18]. This section introduces the application of Lagrange multiplier method used to solve the economic dispatch problems. The concept of the Lagrange multiplier and the Kuhn-Tucker conditions are discussed in chapter 2.

In a power system, if the losses are ignored, at the minimum cost the following equations are satisfied [1]:

$$\lambda = \frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \cdots = \frac{dC_i}{dP_i} \qquad i = 1, \ldots, n_g \tag{3.6}$$

Equation 3.6 is the criterion for the solution to this problem: all units on the economic dispatch should operate at equal incremental operation cost, denoted by λ.

On the other hand, if the losses are considered, in order to minimize the total fuel cost, the following equations need to be satisfied:

$$\lambda = L_i \frac{dC_i}{dP_i} \qquad i = 1, \ldots, n_g \tag{3.7}$$

where $L_i = \dfrac{1}{1 - \dfrac{dP_L}{dP_i}}$.

The term $\frac{\partial P_L}{\partial P_i}$ is known as the incremental transmission loss, and L_i is called the penalty factor of plant *i*. Equation 3.7 shows that each unit that is not at a limit value operates such that its incremental operating cost $\frac{dC_i}{dP_i}$ multiplied by the penalty factor is the same.

An iterative solution can be obtained by the following steps:

1. Set iteration index *m*=1.
2. estimate *m*th value of λ.
3. Skip this step for all *m*>1. Determine initial unit outputs $P_i\,(i = 1, 2, ..., n)$. use $\frac{dC_i}{dP_i} = \lambda$ and read P_i from each incremental operating cost table. Transmission losses are neglected here.
4. Compute $\frac{\partial P_L}{\partial P_i}$ and $\frac{\partial C_i}{\partial P_i}, (i = 1, 2, ..., n)$
5. Determine updated values of unit output $P_i, (i = 1, 2, ..., n)$. Read P_i from each incremental operating cost table. If P_i exceeds a limit value, set P_i to the limit value.
6. Compare P_i determined in step 5 with the previous value $(i = 1, 2, ..., n)$. If the change in each unit output is less than a specified tolerance ε_1, go to step 6.

Otherwise return to step 4.

7. Compute P_L using B coefficients.

8. If $\left|(\sum_{i=1}^{n} P_i) - P_L - P_T\right|$ is less than a specified tolerance ε_1, stop the iteration. Otherwise, set increment m by 1 and return to step 2.

3.4 Economic Dispatch Neglecting Losses and no Generator Limits

The transmission losses can be neglected, if the transmission distance is very short and the load density is very high. Hence the power generated by all the units is equal to the sum of the loads. This is also the simplest economic dispatch problem. In this case, the system configuration and line impedances are not considered. Also, the power output of any generator is not restricted. Thus the problem is to find the real power generation for each unit such that the objective function (total production cost) as defined as Equation 3.1 is minimum, subject to the constraint give as:

$$\sum_{i=1}^{n_g} P_i = P_D \tag{3.8}$$

A criterion for the solution to this problem is Equation 3.14 that is: All units on economic dispatch should operate at equal incremental operation cost. The number of the equations is $n_g + 1$ and the number of the unknown variables is $n_g + 1$ as well, so identical solutions exist. A simple example of economic dispatch problem neglecting generation limits and line losses is shown below. It is the example cited from Glover's text book [1]. The fuel cost functions for three thermal plants in \$/hr are given by:

$$C_1 = 10P_1 + 0.016P_1^2 \quad \$/hr$$
$$C_2 = 8P_2 + 0.018P_2^2 \quad \$/hr$$
$$C_3 = 12P_3 + 0.018P_3^2 \quad \$/hr$$

The total load is 392 MW.

Using Equation 3.6, the minimum total production cost occurs when

$$\frac{dC_1}{dP_1} = 10 + 0.032P_1 = \lambda$$
$$\frac{dC_2}{dP_2} = 8 + 0.036P_2 = \lambda$$
$$\frac{dC_3}{dP_3} = 12 + 0.036P_3 = \lambda$$

$$P_1 + P_2 + P_3 = 392$$

Convert the above equations to matrix form:

$$\begin{bmatrix} 0.032 & 0 & 0 & -1 \\ 0 & 0.036 & 0 & -1 \\ 0 & 0 & 0.036 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \lambda \end{bmatrix} = \begin{bmatrix} -10 \\ -8 \\ -12 \\ 392 \end{bmatrix}$$

Then, the dispatched generation of each unit, the incremental operating cost and the total cost are:

$$P_1 = 141\,MW$$
$$P_2 = 181\,MW$$
$$P_3 = 70\,MW$$
$$\lambda = 14.52\,\$/Whr$$
$$C_T = 4694\,\$/hr$$

3.5 Economic Dispatch Including Losses and Generation Limits

As mentioned in previous sections, the transmission loss is one major factor that affects the optimal dispatch of generation. Furthermore, the power output of any unit should not exceed its rating nor should it be below that necessary for stable operation. The above two conditions are the constraints need to be considered in economic dispatch problems. This section discusses two case studies of economic dispatch problem with losses and generation limits. The following test cases were coded in Matlab R2007a [13]. The Matlab Optimization Tool box was used to determine the real power output of each dispatchable generating unit and the total production cost in order to solve the economic dispatch problems.

3.5.1 Economic Dispatch for a 5-bus Power System

The goal of the economic dispatch for the 5-bus, 3 generator power system [17] shown in Fig. 3.3 is to minimize total generator fuel costs while adhering to generator real power limits and the power flow equation. The limits, fuel cost coefficients and the system parameters are stated in Appendix A. The total load demanded by this system is 147MW and 93.1 MVAR. This system has 3 unknown variables which are 3 generator real power output variables. The reference bus is located at bus 1.

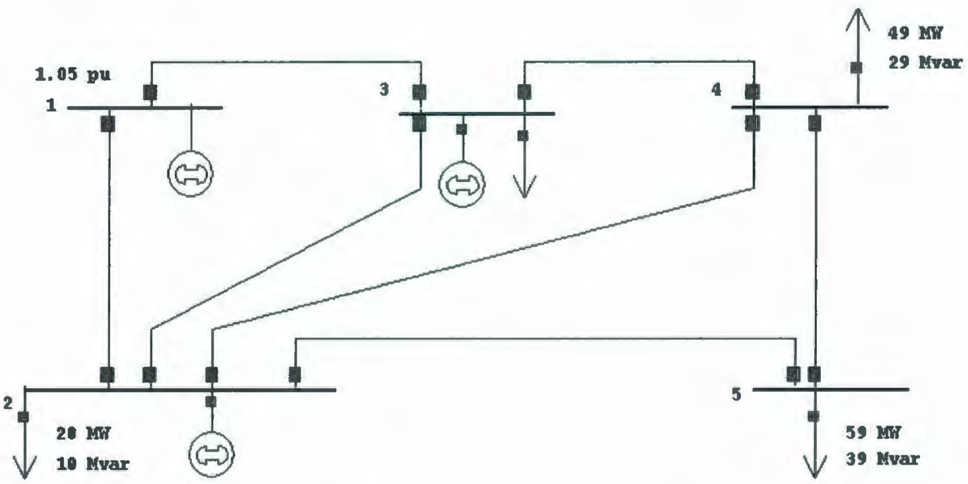

Fig. 3.3 One Line Diagram for the 5-Bus Power System [17].

Using the cost coefficients of the 3 generators in this 5-bus system, the generation cost function of each generator is:

$$C_T = 10P_1 + 0.016P_1^2 + 373.5 + 8P_2 + 0.018P_2^2 + 403.6 + 12P_3 + 0.018P_3^2 + 253.2 \quad \$/\text{hr}$$

where

P_1, real power generated by generator on bus 1;

P_2, real power generated by generator on bus 2;

P_3, real power generated by generator on bus 3.

B-coefficients are used to define the transmission loss in order to define the equality constraint and will be discussed later. Also the generation limits of each generator are considered as the inequality constraints. Table 3.1 shows the economic dispatch results. The total fuel cost is 2554.5$ with loss 3.2 MW. Using the economic dispatch, the system researches the minimum fuel cost. Also, the generation outputs are changed.

Table 3.1 Economic Dispatch Case study for 5-Bus System

	Number	Name	Base Case	Economic Dispatch
Generation(MW)	**1**	**Bus-1**	83	50.56
	2	**Bus-2**	40	96.97
	3	**Bus-3**	28	2.67
Total Generation (MW)			151	150.2
Total Load (MW)			147	147
Total Loss (MW)			4	3.2
Total Hourly Cost ($/hr)			2660	2554.5

3.5.2 Economic Dispatch for a 39-Bus Power System

The goal of the economic dispatch for the 39-bus, 10 generator power system [24] shown in Fig. 3.4 is to minimize total generator fuel costs while adhering to generator real power limits and the power flow equation. The limits, fuel cost coefficients and the system parameters are stated in Appendix B. The total load demanded by this system is 6150.1MW and 1408.9 MVAR. This system has 10 unknown variables which are 10 generator real power output variables. The reference bus is located at bus 31.

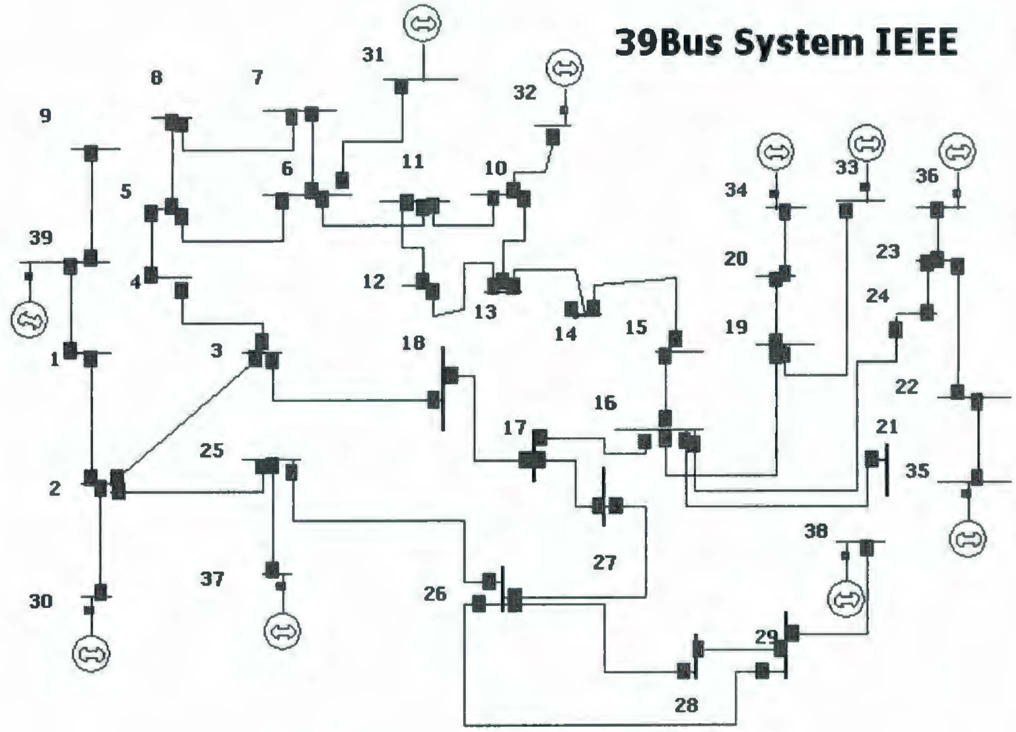

Fig. 3.4 One Line Diagram for 39-Bus Power System [24]

The objective function (total fuel cost) is formed by fuel cost coefficient. The transmission loss is defined using B-coefficient. The algorithm of B-coefficient is discussed in the later section of this chapter. Table 3.2 shows the economic dispatch results. The total fuel cost is 61838$/hr with loss 48.4 MW. In order to achieve the minimum fuel cost, the active power output of each generator is changed. The total fuel cost researches the minimum value.

Table 3.2 Economic Dispatch Case study for 39-Bus System

	Number	Name	Base Case	Economic Dispatch
Generation(MW)	**1**	**Bus-30**	250	236.19
	2	**Bus-31**	650	560.98
	3	**Bus-32**	580	638.32
	4	**Bus-33**	750	645.68
	5	**Bus-34**	550	517.67
	6	**Bus-35**	600	656.73
	7	**Bus-36**	658	568.46
	8	**Bus-37**	652	552.09
	9	**Bus-38**	900	869.42
	10	**Bus-39**	1000	952
Total Generation (MW)			6290	6198.5
Total Load (MW)			6150.1	6150.1
Total Loss (MW)			139.9	48.4
Total Hourly Cost ($/hr)			64182	61838

3.6 Derivation of Loss Formula

The system loss can be formed by loss coefficient or B-coefficient method, which is developed by Kron and adopted by Kirchmayer [17]. The system losses function can be expressed in terms of the generator's active power outputs. This section discusses the algorithm of B-coefficients, and two case studies are provided.

Assume the total system losses over all buses is:

$$P_L + Q_L = \sum_{i=1}^{n} V_i I_i^* = \sum_{i=1}^{n}\sum_{j=1}^{n} I_i Z_{ij} I_j^* \tag{3.9}$$

$$Z_{ij} = R_{ij} + jX_{ij} \tag{3.10}$$

where,

P_L and Q_L, are the real and reactive power loss of the system

Z_{ij}, is the bus impedance matrix,

I_i and I_j, are the injected bus currents.

R_{ij}, is the real part of the bus impedance matrix

Z_{ij} is the imaginary part of the bus impedance matrix

Since the bus impedance matrix is symmetrical, $Z_{ij} = Z_{ji}$, The real power loss equation can be rewritten as:

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} I_i R_{ij} I_j^* = I_{bus}{}^T R_{bus} I_{bus}{}^* \tag{3.11}$$

Assume that the individual bus currents of load buses vary as a constant complex fraction of the total load current:

$$I_{lk} = l_k I_D \tag{3.12}$$

where

l_k, is the complex fraction,

I_{lk}, is the individual bus currents,

$I_{D,}$ is the total load current.

Assume bus 1 to be the reference bus (slack bus), then voltage at bus 1 V_1 is:

$$V_1 = \sum_{i=1}^{n_g} Z_{1i} I_{gi} + \sum_{k=1}^{n_d} Z_{1k} I_{Lk} \tag{3.13}$$

where,

n_g*, is the number of generator buses*

n_d*, is the number of load buses*

Substituting Equation 3.27 into Equation3.28, then get:

$$V_1 = \sum_{i=1}^{n_g} Z_{1i} I_{gi} + I_D T \tag{3.14}$$

where

$$T = \sum_{k=1}^{n_d} l_k Z_{1k} \tag{3.15}$$

If I_0 is defined as the current flowing away from bus 1, with all other load current set to zero, then V_1 is:

$$V_1 = -Z_{11} I_0 \tag{3.16}$$

Substitute Equation 3.31 into Equation 3.29 and Equation 3.27, the load currents become:

$$I_{Lk} = \rho_k \sum_{i=1}^{n_g} Z_{1i} I_{gi} + \rho_k Z_{11} I_0 \tag{3.17}$$

where

$$\rho_k = -\frac{l_k}{T} \tag{3.18}$$

Reform the generator currents with the above relation in matrix form, then get:

$$\begin{bmatrix} I_{g1} \\ I_{g2} \\ \vdots \\ I_{L1} \\ I_{L2} \\ \vdots \\ I_{Ln_d} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ \rho_1 Z_{11} & \rho_1 Z_{12} & \cdots & \rho_1 Z_{1n_g} & \rho_1 Z_{11} \\ \rho_2 Z_{11} & \rho_2 Z_{11} & \cdots & \rho 2 Z_{1n_g} & \rho_2 Z_{11} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho_k Z_{11} & \rho_k Z_{12} & \cdots & \rho_k Z_{1n_g} & \rho_k Z_{11} \end{bmatrix} \begin{bmatrix} I_{g1} \\ I_{g1} \\ \vdots \\ I_{gnLg} \\ \\ \\ \\ I_0 \end{bmatrix} \tag{3.19}$$

In short form:

$$I_{bus} = CI_{new} \tag{3.20}$$

Substituting Eqn. 3.34 into Eqn. 3.26, then get:

$$P_L = I_{new}{}^T C^T R_{bus} C^* I_{bus}{}^* \tag{3.21}$$

Also the generator current at bus i can be formed as:

$$I_{gi} = \frac{P_{gi} - Q_{gi}}{V_i^*} = \psi_i P_{gi} \tag{3.22}$$

where
$$\psi_i = \frac{1 - j\dfrac{Q_{gi}}{P_{gi}}}{V_i^*} \tag{3.23}$$

Adding the current I_0 to the column vector current I_{gi} in Equation 3.37 results in:

$$\begin{bmatrix} I_{g1} \\ I_{g2} \\ \vdots \\ I_{gn_g} \\ I_0 \end{bmatrix} = \begin{bmatrix} \psi_1 & 0 & \cdots & 0 & 0 \\ 0 & \psi_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \psi_1 & 0 \\ 0 & 0 & \cdots & 0 & I_0 \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gn_g} \\ 1 \end{bmatrix} \tag{3.24}$$

Or in short form

$$I_{new} = \psi P_{G1} \tag{3.25}$$

where
$$P_{G1} = \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gn_g} \\ 1 \end{bmatrix} \tag{3.26}$$

Substituting Equation 3.40 into Equation 3.36, the loss equation becomes:

$$P_L = P_{G1}^T \psi^T C^T R_{bus} C^* \psi^* P_{G1}^* \tag{3.27}$$

The resultant matrix from Eqn. 3.42 is complex, so the real power loss is:

$$P_L = P_{G1}^T R[H] P_{G1}^* \tag{3.28}$$

where

$$H = \psi^T C^T R_{bus} C^* \psi^* \tag{3.29}$$

H is also known as Hermitian matrix, and the real part of H is found from:

$$R[H] = \frac{H + H^*}{2} \tag{3.30}$$

The above matrix is partitioned as follows:

$$R[H] = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n_g} & B_{01}/2 \\ B_{21} & B_{22} & \cdots & B_{2n_g} & B_{02}/2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ B_{n_g1} & B_{n_g2} & \cdots & B_{n_gn_g} & B_{0n_g}/2 \\ B_{01}/2 & B_{02}/2 & \cdots & B_{0n_g}/2 & B_{00} \end{bmatrix} \tag{3.31}$$

Then the power loss can be calculated by Eqn.3.43:

$$\begin{aligned} P_L = & \begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gn_g} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n_g} \\ B_{21} & B_{22} & \cdots & B_{2n_g} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n_g1} & B_{n_g2} & \cdots & B_{n_gn_g} \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gn_g} \end{bmatrix} \\ & + \begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gn_g} \end{bmatrix} \begin{bmatrix} B_{01}/2 \\ B_{02}/2 \\ \vdots \\ B_{0n_g}/2 \end{bmatrix} + B_{00} \end{aligned} \tag{3.32}$$

The actual computational procedure is slightly more complex than that indicated above. The calculation of B-coefficients is coded in Matlab using the above algorithm. Two case studies (5-Bus System and 39-Bus System) are provided to demonstrate the computation.

3.6.1 B-coefficients for 5-bus Power System

The 5-Bus power system adheres to previous case study in section 3.5.1. The purpose of this study is to determine the B-coefficient in terms of real power output of generators in order to define the power loss. The power data is given in Appendix A. A program is developed in Matlab to calculate the B-coefficients for different cases. Three generators involved in this power system, thus the dimension of B -matrix is 4x4. The power loss can be calculated using Eqn. 3.32 and B-matrix. Table 3.3 presents the B-coefficients.

Table 3.3 B-coefficients for 5-Bus Power System

B [3x3]	0.0245	0.0102	0.0030
	0.0103	0.0231	0.0016
	0.0030	0.0016	0.0185
B0	0.0004	0.0030	0.0015
B00	0.000191		

According to the calculated B-coefficient, the total power loss is

$$P_L = P_g^2 * (B/100) + P_g * B0 + B00 * 100 = 3.2\ MW$$

3.6.2 B-coefficients for 39-bus Power System

The 39-Bus power system adheres to previous case study in section 3.5.2. The purpose of this study is to determine the B-coefficient in terms of real power output of generators in order to define the power loss. The power data is given in Appendix B. The same MatLab program used to calculate the B-coefficients for this case. 10 generators involved in this power system, thus the dimension of B -matrix is 11x11. The power loss can be calculated using Equation 3.32 and B-matrix. Table 3.4 presents the B-coefficients.

Table 3.4 B-coefficient for 39-Bus Power System

	0.1313	0.1292	0.1264	0.1135	0.1184	0.1209	0.1170	0.0994	0.0412	0.1095
	0.1292	0.1290	0.1264	0.1150	0.1183	0.1200	0.1173	0.1035	0.0428	0.1158
	0.1264	0.1264	0.1238	0.1128	0.1159	0.1175	0.1150	0.1019	0.0420	0.1141
	0.1135	0.1150	0.1128	0.1042	0.1056	0.1063	0.1051	0.0965	0.0397	0.1096
B	0.1184	0.1183	0.1159	0.1056	0.1086	0.1100	0.1076	0.0953	0.0393	0.1067
(10^{-3})	0.1209	0.1200	0.1175	0.1063	0.1100	0.1119	0.1089	0.0946	0.0391	0.1051
	0.1170	0.1173	0.1150	0.1051	0.1076	0.1089	0.1068	0.0954	0.0394	0.1072
	0.0994	0.1035	0.1019	0.0965	0.0953	0.0946	0.0954	0.0936	0.0383	0.1089
	0.0412	0.0428	0.0420	0.0397	0.0393	0.0391	0.0394	0.0383	0.0157	0.0444
	0.1095	0.1158	0.1141	0.1096	0.1067	0.1051	0.1072	0.1089	0.0444	0.1281
B0	0.0012	0.0004	0.0003	-0.0003	0.0003	0.0007	0.0002	-0.0015	-0.0005	-0.0024
B00	0.0875									

According to the calculated B-coefficient and Eqn. 3.47, the total power loss is

$$P_L = P_g^{\,2} * (B/100) + P_g * B0 + B00 * 100 = 48.4\ MW$$

3.7 Conclusions

This chapter has provided an overview of key economic dispatch concepts along with the benefits of using it for solving optimal dispatch problems. The transmission losses, the operating efficiencies of generators and fuel cost are major factors influencing optimum dispatch of power generation. By using the economic dispatch, the generators' power output can be varied within certain limits to support a particular load demand at the lowest possible fuel cost. The economic dispatch is achieved by minimizing the objective function, which is the total fuel cost of the generating units, subject to the constraints that the sum of the powers generated must equal to the sum of the transmission loss and the power consumed by the loads. A Matlab program is developed to determine the B-coefficients in order to estimate the transmission loss. Two sample cases are studied to present the constrained optimization method for power system economic dispatch. Certain optimization tools have been investigated, which have powerful functions to solve the economic dispatch problems such as Optimization Toolbox in the Matlab and PowerWorld Simulator. Economic dispatch has one significant shortcoming. It ignores the limits imposed by the devices in the transmission system. With the worldwide trend toward deregulation of the electric utility industry, the transmission system is becoming a significant constraint. The solution to the problem of optimizing the generation while enforcing the transmission lines is to combine economic dispatch with the power flow. The result is known as the Optimal Power Flow (OPF), which is discussed in Chapter 4.

Chapter 4

Optimal Power Flow and Security Constrained Optimal Power Flow

4.0 Introduction

The idea of Optimal Power Flow (OPF) was defined in the early 1960s as an extension of the conventional economic dispatch to determine the optimal settings for control variables while respecting various constraints [20,25]. The OPF provides a useful support to the operator to overcome many difficulties in the real time control and operation planning of power systems [26-28]. Depending on the specific objectives and constraints, there are different OPF formulations. A secure power system is one with low probability of blackout or equipment damage [4]. Security Constrained Optimization Power Flow (SCOPF) is defined as an optimal power flow which takes into account of the outages of certain transmission lines or equipment.

The typical objectives of OPF and SCOPF problems are minimization of the total fuel cost, minimization of the transmission loss, maximization of the degree of security of a system, or a combination of some of them. OPF and SCOPF are considered as static, constrained, nonlinear, optimization problems. They have been widely used in most of today's Energy Management System (EMS) which monitors and controls the operation of power system.

The chapter starts with giving a fundamental understanding of power flow analysis. The formulations of OPF and SCOPF problems and algorithm and different tools for defining solutions are introduced. Optimal power flow problems for minimizing fuel cost and minimizing transmission loss are discussed with two case studies (7-Bus Power System from Power World Simulator [29] and 26-Bus Power System [17]). In the final part of this chapter, a summary of the OPF and SCOPF problems is given.

4.1 Review of Power Flow Study

A reliable power system operation under normal balanced three phase steady-state conditions requires four conditions, which are [1]:

- Generation supplies the demand (load) plus loss.
- Bus voltage magnitudes remain close to the rated value.
- Generators do not operate over the production limits for active power and reactive power.
- Transmission lines and transformers are not overloaded.

Power flow analysis is the tool used for investigating these requirements. Power flow study is used in planning, control, economic scheduling and operation of existing electric power system. It is required for many other analyses such as transient stability and contingency studies [25].

The power flow problem is the computation of bus voltage magnitudes and angles at each bus in a power system for specified demand. In addition, the real and reactive power flows for all equipments interconnecting the buses and equipment losses can be calculated. The power flow problem is formulated as a set of nonlinear algebraic equations suitable for computer solution. These equations are known as power flow equations, which are expressed in terms of bus voltage magnitudes, phase angles at each bus and bus admittance matrix.

Four variables are associated with each bus k : voltage magnitude P_k, phase angle δ_k, net real power P_k and reactive power Q_k supplied to the bus. At each bus, two of these variables are specified as input data, and the other two are unknowns to be computed by the power flow program. The power delivered to bus k as seen in Fig. 4.1 is separated into generator and load terms. These are:

$$P_k = P_{Gk} - P_{Lk} \tag{4.1}$$

$$Q_k = Q_{Gk} - Q_{Lk} \tag{4.2}$$

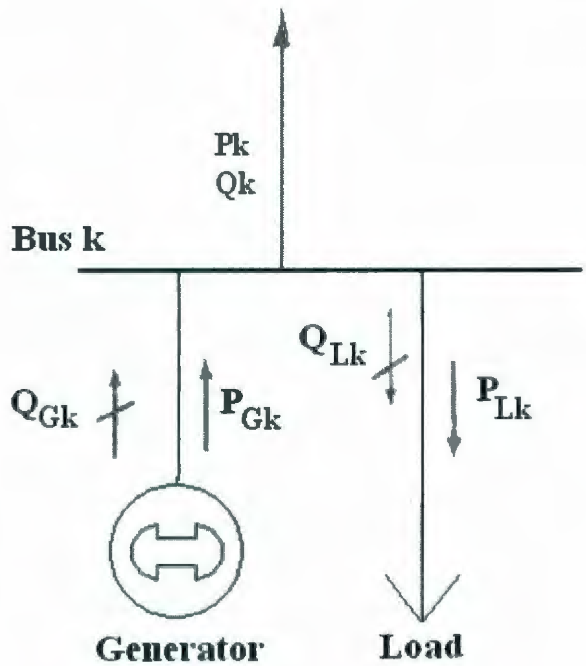

Fig. 4.1 General bus with Generation, Load, and Outgoing Lines [1].

Each bus k is categorized into one of the following three bus types:

1. Slack Bus- The bus voltage magnitude and phase angle are specified, typically 1.0 p.u. with phase angle 0°. This bus is selected to provide additional real and reactive power to supply transmission loss, the active power (P) and reactive power (Q) are unknowns and need to be calculated.
2. Voltage Controlled Bus- It is also called PV bus. Active power (P) and bus voltage magnitude are specified. They are usually the generating stations.
3. Load Bus- It is also known as PQ bus, and the load active power and reactive power are specified.

The power flow problem consists of a given transmission network where all lines are represented by a π- equivalent circuit. The power flow equations can be expressed in term of Y-bus (Bus admittance), bus voltage and phase angles as:

$$P_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn}), \quad k = 1, 2, \cdots, N \tag{4.3}$$

$$Q_k = V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn}), \quad k = 1, 2, \cdots, N \tag{4.4}$$

Where P_k and Q_k are the active and reactive power bus injections at bus *k*, V_k and V_n are bus voltage magnitudes, Y_{kn} is the magnitude of the element (*k,n*) of the power system's admittance matrix, θ_{kn} is the angle of the element (*k,n*) of the power system's admittance matrix, δ_k and δ_n are the bus voltage angles, and N is the number of system buses contained in the system

In matrix form:

$$P_k + jQ_k = V_k (YV)^*, \quad k = 1, 2, \cdots, N \tag{4.5}$$

Y is the bus admittance matrix. *V* is the bus voltage matrix.

The mathematical formulation of the power flow problem results in a system of algebraic nonlinear equations which can be solved by iterative techniques. Many techniques are available, such as Gauss-Seidel and Newton-Raphson methods. The information obtained from the power flow studies includes the magnitude and phase angle of voltages at each bus and the active and reactive power flow in each line.

4.2 Optimal Power Flow

The goal of OPF is to provide the electric utility with suggestions to optimize the current power system state with respect to various objectives under various constraints [30]. In the most general formulation the OPF is a single objective, large scale, non-convex optimization problem. It can be achieved by minimizing or maximizing the general objective functions while satisfying the constraints. The specified variables are real and reactive power at PQ buses, active powers and voltage magnitude at PV buses, and voltages and angles at slack buses. OPF problems are formulated as a set of nonlinear equations, which can be stated as [4,20,25]:

$$\text{Finding the vectors } x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

$$\begin{array}{l} \text{which minimizing or maximizing:} \\ f_1(x,u), f_2(x,u) \ldots f_n(x,u) \end{array} \tag{4.6}$$

$$\begin{array}{l} \text{Subject to:} \\ g(x,u) = 0, \end{array} \tag{4.7}$$

$$h(x,u) \leq 0, \tag{4.8}$$

$$(x,u)^{(L)} \leq (x,u) \leq (x,u)^{(U)} \tag{4.9}$$

It can be described as minimizing the general objective function $f_n(x,u)$ while satisfying the constraints $g(x,u)=0$ and $h(x,u) \leq 0$, where $g(x,u)$ represents nonlinear equality constraints (power flow equations) and $h(x,u)$ is nonlinear inequality constraints (transmission line limits) on the vectors x and u. Eqn. 4.9 defines the bounds of design vectors. The vector x contains the dependent variables including bus voltage magnitudes, phase angles and the reactive power output of generators designed for bus voltage control. The vector x also includes constant parameters, such as reference bus angles, noncontrolled generator MW, MVAR and outputs, noncontrolled load on fixed voltage, line parameters, and so on. The vector u consists of control variables involving [25,31]:

- Active and reactive power generation
- Phase shifter angles
- Net interchange
- Load MW and MVAR(load shedding)
- Direct current transmission line flows
- Control voltage settings
- Transformer tap setting
- Line switching

Typical goals of OPF problems are minimization of the total fuel cost, the minimization of the active power loss, minimization of reactive power planning cost and minimization of bus voltage deviation.

4.2.1 Objective Functions

The purpose of objective functions is to mathematically express the goals of an optimization process. Minimization fuel cost and the active power losses are two of the most common OPF goals, and are discussed in this thesis for OPF studies.

4.2.1.1 Total Fuel Cost

Fuel cost minimization is primarily a planning problem. The objective function based on generation operating cost can be expressed as:

$$f(x,u)=\sum_{i=1}^{n_g}(\alpha_i+\beta_i*P_{g_i}+\gamma_i*(P_{g_i})^2) \qquad (4.10)$$

where

$f(x,u)$ is the total fuel cost, dollar per hr (\$/hr),

P_{g_i} is the active power output of the *ith* unit, megawatts (MW),

α_i, β_i and γ_i are the cost coefficients of the *ith* generator.

n_g, is the number of generators.

4.2.1.2 Active Power Losses

Active power loss minimization is also known as loss minimization, which is a useful tool in conjunction with planning objective, providing optimal solutions for planning purposes [25]. It can be expressed in different ways, such as in terms of voltage and impedance, or active power outputs. Expressing the loss formula using voltage in polar forms turns out to be more complicated. The equation listed below defines the loss formulation in terms of active power outputs and loads, which is the difference between the total generation and the total load demands.

$$f(x,u) = \sum_{i=1}^{ng} P_{gen_i}(x,u) - \sum_{i=1}^{nl} P_{load_i} \tag{4.11}$$

where,

P_{gen_i}, is the active power output of the ith bus,

P_{load_i}, is the load demand of the ith bus,

ng, is the number of generators.

nl, is the number of loads.

4.2.2 Control Variables

Unknown variables can be classified to dependent variables and independent variables. The dependent variables (known as state variables) include bus voltage magnitude and phase angle. Independent variables are usually the control variables in an OPF problem which includes active power outputs of generation units, generator voltages, transformer tap ratios and values of switchable shunt capacitors and inductors.

4.2.3 Constraints

Constraints contained within the OPF problem are put in place in order to ensure that the solutions obtained by solving the OPF are feasible for practical power system operations. This section discusses typical operational constraints used in OPF formulation.

4.2.3.1 Equality Constraints

OPF equality constraints are represented by the power flow equations. These equations define the physical link between scheduled generation and load demand and cannot be violated as they define the state variable conditions for a given system operating point [15]. The following equations are the equality constraints:

$$P_{Gk} - P_{Lk} - P_k = 0 \tag{4.12}$$

$$Q_{Gk} - Q_{Lk} - Q_k = 0 \tag{4.13}$$

Where:

P_{Gk} and Q_{Gk} are the active and reactive power generation at bus *k*.

P_{Lk} and Q_{Lk} is the load at bus *k*.

P_k and Q_k are the net active and net reactive power injections at bus *k*.

4.2.3.2 Inequality Constraints

The inequality constraints define the tolerable limits on both state variables and equipment usage. Important limitations used in the OPF problem are as follows:

$$V_{k\min} \le V_k \le V_{k\max},, \quad k = 1, 2, \cdots m \tag{4.14}$$

$$P_{k\min} \le P_k \le P_{k\max}, \quad k = 1, 2, \cdots n_g \tag{4.15}$$

$$P_{kn\min} \le P_{kn} \le P_{kn\max}, \quad k = 1, 2, \cdots m \tag{4.16}$$

Where:

V_k is the bus voltage magnitude at bus *k*.

P_k is generation power at bus *k*.

P_{kn} is the power flow between bus *k* and *n*

m is the number of buses.

n_g is the number of generators.

Bus voltage magnitudes must be held between a certain ranges in order to ensure that equipment is operating under design specifications. Allowable bus voltage levels depend on the nominal voltages that are applied to the bus. As an example, a typical tolerable voltage range for a 138kv bus is within $\pm 5\%$ of this value while buses with voltages of 345kv and over should be within $\pm 10\%$ [16]. Eqn. 4.14 ensures that the bus voltage varies in an acceptable range. Generation limits on active power is a result of the generators' design characteristics. Eqn. 4.15 defines the range of active power outputs. All transmission lines have a limit for maximum MVA transfer to ensure the system is operated safely. Eqn. 4.16 define the transmission limit of each line to ensure that the power flow is not overloaded.

4.3 The Solution of Optimal Power Flow

Optimal power flow algorithms are designed to find an AC power flow solution which optimizes a performance function, such as fuel costs or network losses, while at the same time enforcing the loading limits imposed by the system equipment, such as voltage and transmission loading limits. For example, when fuel costs are minimized, an optimal schedule of generator active power outputs, transformer tap settings and controllable voltage settings are determined which produce the minimum operating costs while at the same avoiding any violation [33].

OPF problems can be mathematically formed as nonlinear constrained optimization problems. System size and the number of unknown variables significantly affect the difficulty of solving OPF. As the size of the system increasing, solving OPF problem is more difficulty. Almost every mathematical programming approach that can be applied to this problem has been researched and developed to computer codes. Mainly these are Gradient Method, Newton's method, Linear Programming (LP) method and Interior Point (IP) method.

Gradient method is computationally well suited for large systems, however it is slow in convergence and is difficult to solve the OPF problem with inequality constrains. Newton's method is powerful with fast convergence for OPF, but it may generate problems with inequality constraints as well. LP method is one of the fully developed methods now in common use. It easily handles the constraints, and effectively solves the nonlinear OPF problems by linearization.

Sequential quadratic programming method has been used to solve OPF problems in this research. Matlab Optimization Toolbox is used to implement the OPF algorithm. 'fmincon' is the command used to call and solve constrained nonlinear functions in the main program. Objective function and constraints equations are written in different 'm' files to be the function files.

4.4 Optimal Power Flow Minimizing Generation Cost

Minimizing generation cost is to reduce the total fuel cost, which is primarily an operational planning problem. The objective is to minimize the fuel cost. Based on previous research of OPF problems, the minimizing generation cost problem can be mathematically formed as:

Finding the vectors $P_g = \begin{bmatrix} P_{g_1} \\ P_{g_2} \\ \vdots \\ P_{g_n} \end{bmatrix}, \quad V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_m \end{bmatrix}$

which minimizing:

$$f(P_g, V, \theta) = \sum_{k=1}^{n_g} (\alpha_k + \beta_k * P_{g_k} + \gamma_k * (P_{g_k})^2) \tag{4.17}$$

Subject to:

$$P_{g_k} - P_{Lk} - P_k = 0 \tag{4.18}$$

$$Q_{g_k} - Q_{L_k} - Q_k = 0 \tag{4.19}$$

$$V_{k\min} \le V_k \le V_{k\max}, \quad k = 1,2,\cdots m \tag{4.20}$$

$$P_{g_k\min} \le P_{g_k} \le P_{g_k\max}, \quad k = 1,2,\cdots n_g \tag{4.21}$$

$$P_{kn\min} \le P_{kn} \le P_{kn\max}, \quad k = 1,2,\cdots m, \quad n = 1,2,\cdots m \tag{4.22}$$

where:

$f(P_g, V, \theta)$ is the total fuel cost, dollar per hr ($/hr).

α, β and γ are the cost coefficients of the generator

P_{g_k} and Q_{g_k} is generation power at bus *k*.

V_k is the bus voltage magnitude at bus *k*.

P_{L_k} and Q_{L_k} is the load at bus *k*.

P_k and Q_k are the net active and net reactive power injections at bus *k*.

P_{kn} is the power flow between bus *k* and *n*.

$P_{g_k\min}$ and $P_{g_k\max}$ are the minimum and maximum active power output of generation unit *k*.

$P_{kn\min}$ and $P_{kn\max}$ are the lower and upper bounds on the active power flow between bus *k* and *n*.

$V_{k\min}$ and $V_{k\max}$ are lower and upper bounds on the voltage magnitude at bus *k*.

n_g is the number of generator.

m and n is the number of buses.

Eqn. 4.17 establishes the objective function. Eqns. 4.18 and 4.19 defines the equality constraints (power flow equations). Eqns. 4.20-4.22 are inequality constraints, which clarify the upper and lower bounds on the voltage magnitude, active power outputs and power flow between buses. Active power outputs of dispatchable generation units are control variables need to be solved to achieve the optimal operation of the minimum fuel cost.

Two case studies are discussed in this section to illustrate the constrained optimization method for OPF. One is 7-bus power system, which is a sample case adopted from PowerWorld Simulator [29]. The other one is 26-bus power system, which a case from H. Saadat's book [17]. All the studies use the Optimization Toolbox available in Matlab [13]. For some aspects of the studies, PowerWorld Simulator is also used. The fuel costs of all the generating units are represented using cubic cost models.

4.4.1 Case Study of Minimizing Cost for 7-bus Power System

The goal of the OPF for the 7-bus system is to minimize generator fuel costs while adhering to power flow equations, specified branch flow (MVA), bus voltage magnitudes, slack generator active power limits. It contains 5 generators, 5 loads, 7 buses and 11 transmission lines, and bus 7 is the slack bus. The limits, fuel cost coefficients and the system parameters are found in Appendix C. Fig 4.2 shows the single line diagram of the 7-bus power system considered. The total loads are 760 MW and 130 MVAR. For the base case, the total fuel cost is 16939$/hr and the transmission loss is 7.9 MW.

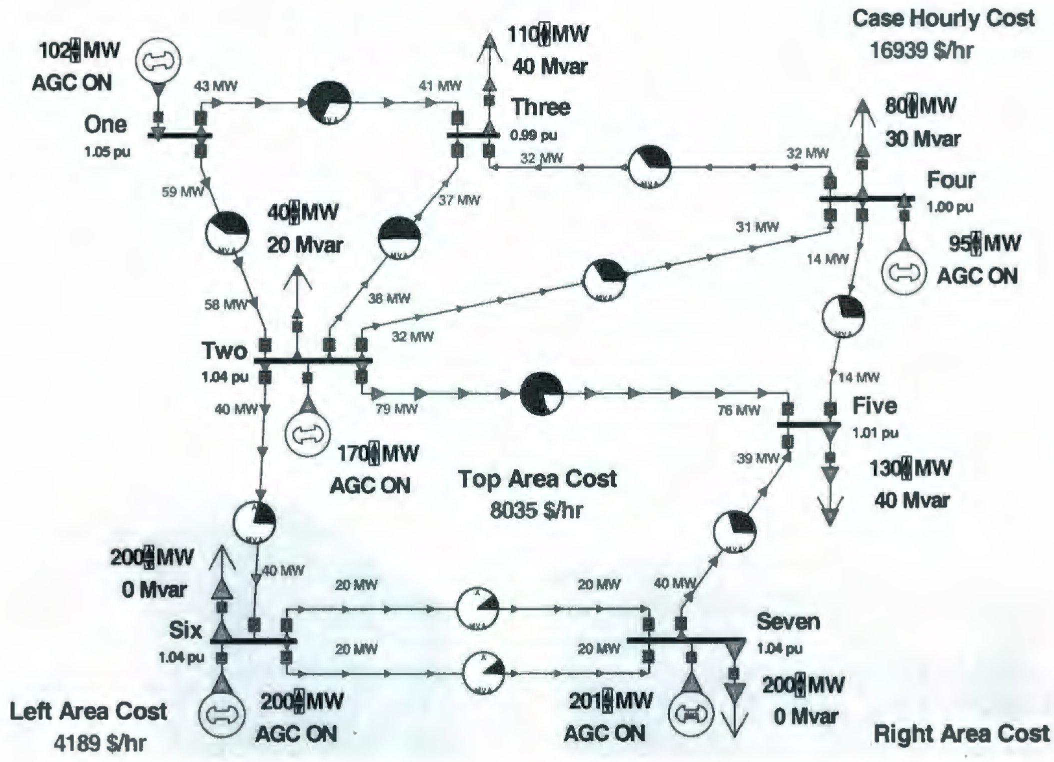

Fig. 4.2 One Line Diagram of the 7-Bus Power System [29]

This system has 12 unknown variables in total: 4 controllable variables and 8 dependent variables. Controllable variables include 4 generators active power output variables. Dependent variables are 2 load bus voltage magnitude variables and 6 phase angles. The formulation of OPF can be defined based on the Eqns. 4.17-4.22. Matlab Optimization Toolbox is applied to achieve the main goal (minimizing the generation cost).

Table 4.1 summarizes the results of OPF without considering security constraints. Power and loss are in MW, and Voltage is in per unit. Cost is in $/hr. The total hourly cost is 16375$/hr and the loss is 10.6 MW. The economic dispatch results are also calculated. Table 4.1 illustrates that the economic dispatch has the lowest cost and the base case has the highest cost. With the dispatch pattern obtained by economic dispatch, transmission line 2-5 and line 4-5 are violated 124% and 189% respectively. The cost of OPF is less than that of the base case, but it is more expensive than that of economic dispatch. With the operation pattern obtained from OPF, none of transmission lines is overloaded. The power flow on Line 2-5 and line 4-5 reaches the maximum limits. The bus voltage magnitudes maintain a reliable level (approximate 1 p.u.).

Table 4.1 OPF of 7-Bus Power System, Minimizing Generation Cost

MINIMIZING COST	P1(MW)	P2(MW)	P4(MW)	P6(MW)	P7(MW)	Cost($/hr)
Base Case	102	170	95	200	201	16939
Economic Dispatch	196	288	128	164	0	16226
OPF	126	230	71	291	52	16371

4.4.2 Case Study of Minimizing Cost for 26-bus Power System

The 26-bus power system is a test system from H. Saadat's book [17]. The goal of the OPF for the 26-bus system seen is to minimize generator fuel costs while satisfying all the power flow constraints. It includes 6 generators, 23 loads, 26 buses and 46 transmission lines, and bus 1 is the slack bus. The limits, fuel cost coefficients and the system parameters are found in Appendix D. Fig 4.3 shows the single line diagram of the 26-bus power system considered. The total loads are 947.3 MW and 484.5 MVAR. For the base case, the total fuel cost is 23946$/hr and the transmission loss is 9.4 MW.

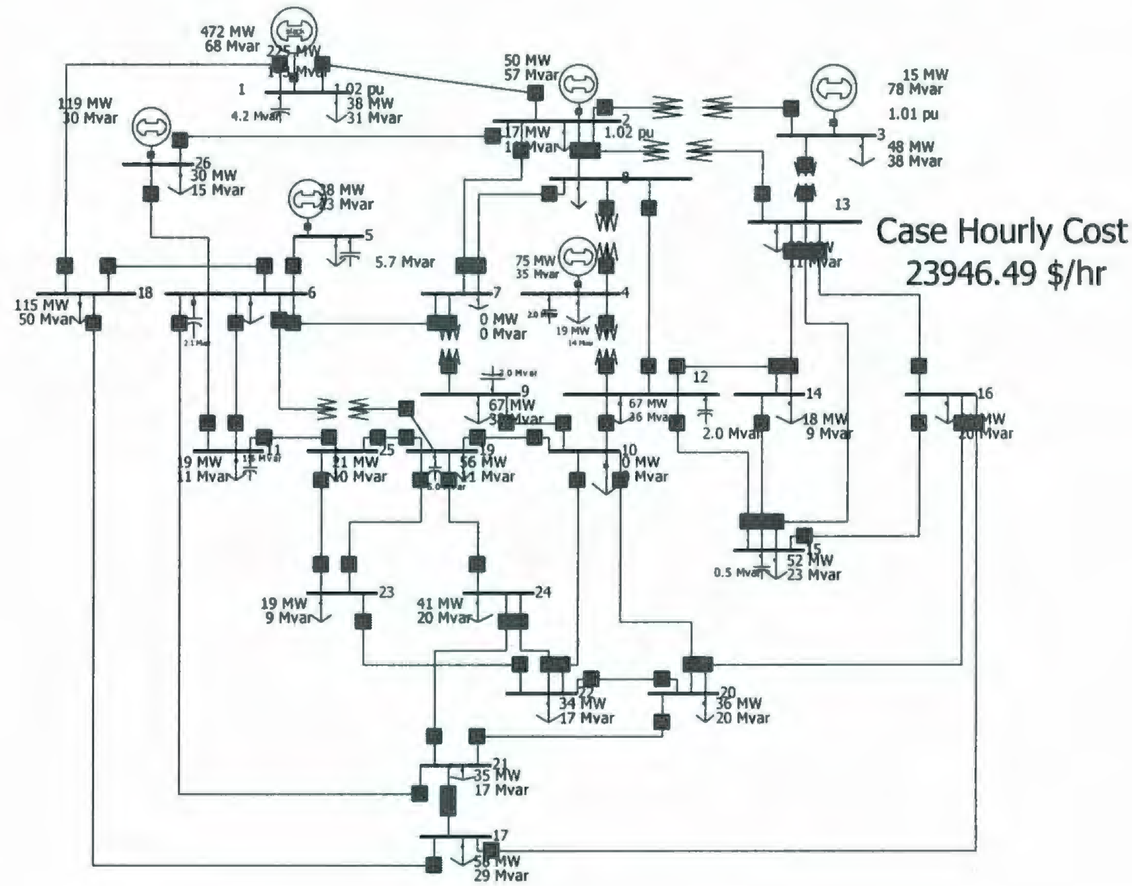

Fig. 4.3 One Line Diagram of 26-Bus Power System [17]

The formulation of OPF for 26-bus power system can be formed according to Eqns. 4.17-4.22. Power flow equations are considered as equality constraints. Upper and lower bounds of bus voltage magnitudes, active power output and power flow are used to setup the inequality constraints. This system has 40 unknown variables in total: 5 controllable variables and 35 dependent variables. Controllable variables include 5 generators active power output variables. Dependent variables consist of 20 load bus voltage magnitude variables and 15 phase angle variables. Matlab Optimization Toolbox is applied to achieve the main goal (minimizing the generation cost).

Table 4.1 summarizes the results of OPF including transmission line limits and neglecting security constraints. Power and Loss are in MW, and Voltage is in per unit. Cost is in \$/hr. The total hourly cost is 22646\$/hr and the loss is 7.3 MW. The economic dispatch results are also listed for comparison. Table 4.1 illustrates that the results obtained from OPF studies are very similar but not identical to the economic dispatch case. The economic dispatch achieves the lowest fuel cost, and the OPF is slightly higher than it, which is about \$2 more. This is because more constraints (power flow at each bus and transmission line limits) are involved in optimal power flow. The bus voltage magnitudes maintain a reliable level (approximate 1 p.u.).

Table 4.2 OPF of 26-Bus Power System, Minimizing Generation Cost

MINIMIZIN G COST	P1 (MW)	P2 (MW)	P3 (MW)	P4 (MW)	P5 (MW)	P26 (MW)	Cost (\$/hr)	Loss (MW)
Base Case	472	50	15	75	225	119	23946	9.4
Economic Dispatch	381	125	213	85	114	34	22644	7.4
OPF	378	120	209	80	118	50	22646	7.3

4.5 Optimal Power Flow Minimizing Loss

Minimizing power loss is to minimize transmission loss, which is another primary application of OPF. The expression for the overall transmission loss accumulated in a power system is defined in Eqn. 4.11. Based on previous research of OPF problems, the minimizing power loss problem can be mathematically formed as:

Finding the vectors $P_g = \begin{bmatrix} P_{g_1} \\ P_{g_2} \\ \vdots \\ P_{g_n} \end{bmatrix}$, $V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m \end{bmatrix}$, $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_m \end{bmatrix}$

which minimizing:

$$f(P_g, V, \theta) = \sum_{i=1}^{n_g} P_{g_i} - \sum_{i=1}^{n_l} P_{L_i} \tag{4.23}$$

Subject to:

$$P_{g_k} - P_{Lk} - P_k = 0 \tag{4.24}$$

$$Q_{g_k} - Q_{L_k} - Q_k = 0 \tag{4.25}$$

$$V_{k\min} \leq V_k \leq V_{k\max}, \quad k = 1, 2, \cdots m \tag{4.26}$$

$$P_{g_k \min} \leq P_{g_k} \leq P_{g_k \max}, \quad k = 1, 2, \cdots n_g \tag{4.27}$$

$$P_{kn\min} \leq P_{kn} \leq P_{kn\max}, \quad k = 1, 2, \cdots m, \quad n = 1, 2, \cdots m \tag{4.28}$$

Where:

$f(P_g, V, \theta)$ is the total power loss, MW.

P_{g_k} and Q_{g_k} is generation power at bus *k*.

V_k is the bus voltage magnitude at bus *k*.

P_{L_k} and Q_{L_k} is the load at bus *k*.

P_k and Q_k are the net active and net reactive power injections at bus *k*.

P_{kn} and Q_{kn} are the power flow between bus *k* and *n*.

$P_{g_k \min}$ and $P_{g_k \max}$ are the minimum and maximum active power output of generation unit *k*.

$P_{kn\min}$ and $P_{kn\max}$ are the lower and upper bounds on the active power flow between bus k and n.

$V_{k\min}$ and $V_{k\max}$ are lower and upper bounds on the voltage magnitude at bus k.

n_g is the number of generators.

n_l is the number of loads.

m and n is the number of buses.

Equation 4.23 establishes the objective function which is the expression of total power loss. The constraints for loss minimization are similar to those discussed earlier for cost minimization. Equations 4.24 and 4.25 define the equality constraints (power flow equations). Equations 4.26, 4.27 and 4.28 are inequality constraints, which clarify the upper and lower bounds on the voltage magnitude, active power outputs and power flow between buses. In the formulation for loss minimization, active power outputs of dispatchable generation units are used as control variables. The 7-bus power system [29] and H. Saadat 26-bus power system [17] are applied to illustrate the constrained optimization method for OPF. All the studies use the optimization tool box available in Matlab [13]. For some aspects of the studies, PowerWorld Simulator is also used.

4.5.1 OPF Minimizing Loss for 7-bus Power System

Case study 4.5.1 repeats the case study 4.4.1 power system, except that the objective is to minimize power loss. The base case of 7-bus power system is displayed in Fig. 4.2. The goal is to minimize generator fuel costs while adhering to power flow equations, specified branch flow (MVA), bus voltage magnitudes, slack generator active power limits. The limits, fuel cost coefficients and the system parameters are found in Appendix C. The total loads are 760 MW and 130 MVAR. For the base case, the transmission loss is 7.9 MW and the total fuel cost is 16939$/hr.

The formulation OPF regarding minimizing loss for 7-bus system is established using Eqns. 4.24-4.29. 12 unknown variables in total: 4 controllable variables and 8 dependent variables. Controllable variables include 4 generators active power output variables. Dependent variables are 2 load bus voltage magnitude variables and 6 phase angles. The formulation of OPF can be defined based on the Eqns. 4.23-4.28. Matlab Optimization Toolbox is applied to achieve the main goal (minimizing the generation cost).

Table 4.3 summarizes the results of OPF with consideration of transmission line limits. Power and loss are in MW; voltage is in per unit, and cost is in $/hr. The total transmission loss is reduced to 3.25 MW with hourly cost 17150$/hr. Table 4.3 illustrates that the operation pattern obtained from OPF with minimum loss has the lowest losses than that of the base case and the OPF (minimizing cost), but it is the most expensive one. The bus voltage magnitudes maintain a reliable level (approximate 1 p.u.).

Table 4.3 OPF of Minimizing Loss for 7-bus Power System

	P1 (MW)	P2 (MW)	P4 (MW)	P6 (MW)	P7 (MW)	Loss (MW)	Cost ($/hr)
Base Case	102	170	95	200	201	7.9	16939
OPF (Minimum Cost)	126	230	71	291	52	10.4	16371
OPF(Minimum Loss)	100	150	109	150	254	3.25	17150

4.5.2 OPF Minimizing Loss for 26-bus Power System

Case study 4.5.2 repeats the base case of 26-bus power system seen in Fig. 4.3. The goal is to minimize generator fuel costs while satisfying the power operation constraints, such as power flow equations, specified branch flow (MVA), bus voltage magnitudes and slack generator active power limits. The limits, fuel cost coefficients and the system parameters are found in Appendix D. The total loads are 947.3 MW and 484.5 MVAR. For the base case, the transmission loss is 9.4 MW and the total fuel cost is 23946$/hr.

The formulation of OPF with minimizing loss for 26-bus system is established using Eqns. 4.23-4.28. 40 unknown variables in total: 5 controllable variables which are the generator active power outputs, and 35 dependent variables which includes are 20 load bus voltage magnitude variables and 15 phase angles. Matlab Optimization Toolbox is applied to achieve the main goal (minimizing the transmission loss).

Table 4.4 summarizes the results of OPF of minimizing loss for 26-bus power system. The operation patterns of OPF of minimizing cost and the base case are listed in the table as well. The result obtained from OPF of minimizing loss is that the transmission loss is reduced to 6.4 MW with hourly cost 23177$/hr. Table 4.4 illustrates that the operation pattern obtained from OPF with minimum loss has the lowest losses than that of the base case and the OPF (minimizing cost), but it is the most expensive one. Thus, less transmission loss results in more generation cost.

Table 4.4 OPF of Minimizing Loss for 26-bus Power System

	P1 (MW)	P2 (MW)	P3 (MW)	P4 (MW)	P5 (MW)	P26 (MW)	Cost ($/hr)	Loss (MW)
Base Case	472	50	15	75	225	119	23946	9.4
OPF(Minimum Cost)	378	120	209	80	118	50	22646	7.3
OPF(Minimum Loss)	341	97	147	150	150	68	22876	6.26

4.6 Security Constrained Optimal Power Flow

Enforcing branch limits while optimization the generation will directly help to ensure that the power system is performing economically. However, the optimum operation conditions for a power system will often result in violation of system security. A secure power system is one where the power system continuous to operate even after some contingencies, such as generator and transmission line outages. Programs which can make control adjustments to the base or pre-contingency operation to prevent violations in the post contingency conditions are called Security Constrained Optimal Power Flow or SCOPF [15]. SCOPF is an optimal power flow taking into account outages of certain transmission lines or equipment. A SCOPF solution would be secure during all credible contingencies or can be made secure by corrective means depending on the level of security enforced in the optimization. A power system is typically classified into many security levels [34]. A security level 1 is a system where all load is supplied no operating limits are violated and no limit violations occur in the event of contingencies. Security level 2 is a system where all load is supplied no operating limits are violated and any violation caused by a contingency can be corrected by appropriate control action without loss of load [4]. Only security level 1 is considered.

The SCOPF can be achieved by adding security constrains into the OPF, which ensure the bounds on components during the contingency condition. For example, the following constraints might be incorporated:

$$V_{k,\min} \le V_k(\text{with line l outage}) \le V_{k,\max}, k = 1,2,\cdots,n \tag{4.29}$$

$$P_{ij,\min} \le P_{ij}(\text{with line l outage}) \le P_{ij,\max}, j = 1,2,\cdots,n, i = 1,2,\cdots,n. \tag{4.30}$$

Eqns. 4.29 and 4.30 implies that the SCOPF would prevent the post-contingency voltage on bus k and the post-contingency line flow on line ij from exceeding their limits for an outage line l. For a SCOPF analysis, constraints under both normal operation condition and outages situation are considered, and they can be a large set depends on the size of the network.

4.6.1 Objective Functions

The objective functions of SCOPF study are similar to the OPF. Minimizing the total fuel cost and minimizing the active power losses are two goals discussed in this thesis for SCOPF studies. The objective functions of these goals are expressed in Eqns. 4.10 and 4.11.

4.6.2 Control Variables

The SCOPF uses two kinds of variables: dependent variables and independent variables. The dependent variables (known as state variables) include bus voltage magnitude and phase angle. Independent variables (or the control variables) are the variables that may be set and controlled by the optimization algorithms. They include active power outputs of generation units, generator voltages, transformer tap ratios and values of switchable shunt capacitors and inductors. In addition to that, these variables exist both in pre-contingency case and post-contingency case. Considering an n-bus power system, if the number of control variables (dispatchable generator) is i and the number of state variables (bus voltage and phase angle) is j, then the total number of unknown variables for the system under normal operation condition is $i+j$. If one transmission line outage is considered, then a set of new state variables will be added. Hence the total number of state variables is $2j$, the number of control variable is still i, and the total number of unknown variable will be increased to $i+2j$. Therefore, if m transmission line outages are considered, the total number of unknown variable is $i+(m+1)\times j$.

4.6.3 Constraints

The purpose of constraints contained within the SCOPF problem is to ensure that the solutions obtained by solving the SCOPF are feasible for practical power system operations. Similar to the constraints of OPF, SCOPF has equality and inequality constraints as well. The main difference is that the SCOPF can also classified into intact system security constraints and single outage security constraints.

Intact system security constraint is also known as the N Security Constraints (SC). Satisfying of these constraints means that line flows, voltage magnitudes and reactive generated powers are kept within their allowed limits in the intact system. The other type is single outage security constraints, known as the N-1 security constraints. The previous variables are met in any state resulting from an outage of a single network element [34].

4.6.3.1 Equality Constraints

Each outage case is characterized by a new set of equation. Constraints for both normal condition and contingencies (outage) condition should be focused. Hence, constraints can be formed as [14]:

$$\sum_{i=1}^{nb} P_i(x,u) = 0, \sum_{i=1}^{nb} Q_i(x,u) = 0 \tag{4.31}$$

$$\sum_{i=1}^{nb} P'_i(x,u) = 0, \sum_{i=1}^{nb} Q'_i(x,u) = 0 \tag{4.32}$$

where:

x and u are state variables and control variables.

P_i and Q_i are the power flows at bus i in normal state.

P'_i and Q'_i are the power flows at bus i when transmission line n is blackout.

nb is the number of bus.

Eqn. 4.31 expresses the power flow equation at certain buses when the system is operated under normal condition. Eqn. 4.32 presents the power flow equations when the outages of certain transmission lines are happened.

4.6.3.2 Inequality Constraints

The security assessment should be executed without violation in normal condition and after contingencies. Tolerable limits both under normal operating conditions and post contingencies should be satisfied. Important limitations used in the OPF problem are as follows [4]:

$$V_{k\min} \le V_k \le V_{k\max},, \quad k = 1, 2, \cdots m \tag{4.33}$$

$$P_{gk\min} \le P_{gk} \le P_{gk\max}, \quad k = 1, 2, \cdots n_g \tag{4.34}$$

$$P_{kn\min} \le P_{kn} \le P_{kn\max}, \quad k = 1, 2, \cdots m \tag{4.35}$$

The following inequality constraints are required under contingency states:

$$V_{k\min} \le V'_k \le V_{k\max},, \quad k = 1, 2, \cdots m \tag{4.36}$$

$$P_{kn\min} \le P'_{kn} \le P_{kn\max}, \quad k = 1, 2, \cdots m \tag{4.37}$$

Where:

V_k is the bus voltage magnitude at bus k in normal conditions.

P_{gk} is generation power at bus k in normal conditions.

P_{kn} and Q_{kn} are the power flow between bus k and n in normal conditions.

V'_k is the bus voltage magnitude at bus k in contingency state.

P'_{kn} is the power flow between bus k and n in contingency state.

m is the number of buses.

n_g is the number of generators.

The number of contingency cases determines the size of the constraints of SCOPF problems. If there are n contingencies are considered, the number of SCOPF constraints is $(n+1)$ times the number of OPF constraints.

4.7 Solution of Security Constrained Optimal Power Flow

Security assessment can be classified as static security assessment and dynamic security assessment [7]. The study presented in this thesis considers only static security. The SCOPF starts by solving the system by OPF with N constraints to find an operating point, and then contingency analysis is run which identifies the potential contingency cases. If there is no constraint violation, then the solution of SCOPF is obtained by the OPF. If a security violation is caused by outages, the complete security constraints is added, and then the OPF and each of the contingency power flows is re-executed until the OPF has solved with all contingency constraints met. This new optimal operating point ensures that after any single line outage there are no voltage or branch limits violations. Many techniques are used to identify contingencies and the order of contingencies that simplifies the SCOPF problems. PowerWorld Simulator has the contingency analysis function which provides the detail contingency violation information. The potential worst contingency case can be easily recognized. Similar to the OPF studies, Matlab Optimization Toolbox is used to implement the SCOPF algorithm. Sequential Quadratic Programming is used by Matlab Optimization Toolbox to solve nonlinear constrained problems.

4.8 Security Constrained Optimal Power Flow Minimizing Generation Cost

The objective is to minimize the fuel cost. The SCOPF regarding minimizing generation cost problem can be mathematically formed as:

$$\text{Finding the vectors } u = \begin{bmatrix} P_{g_1} \\ P_{g_2} \\ \vdots \\ P_{g_n} \end{bmatrix}, \quad x = \begin{bmatrix} V_1 & V_1' \\ V_2 & V_2' \\ \vdots & \vdots \\ V_m & V_m' \\ \theta_1 & \theta_1' \\ \theta_2 & \theta_2' \\ \vdots & \vdots \\ \theta_m & \theta_m' \end{bmatrix}$$

to minimize:

$$f(x,u) = \sum_{k=1}^{ng} (\alpha_k + \beta_k * P_{g_k} + \gamma_k * (P_{g_k})^2) \tag{4.38}$$

subject to:

$$\sum_{i=1}^{m} P_i(x,u) = 0, \sum_{i=1}^{m} Q_i(x,u) = 0 \tag{4.39}$$

$$\sum_{i=1}^{m} P'_i(x,u) = 0, \sum_{i=1}^{m} Q'_i(x,u) = 0 \tag{4.40}$$

$$V_{k\min} \le V_k \le V_{k\max}, \quad k = 1,2,\cdots m \tag{4.41}$$

$$P_{g_k\min} \le P_{g_k} \le P_{g_k\max}, \quad k = 1,2,\cdots n_g \tag{4.42}$$

$$P_{kn\min} \le P_{kn} \le P_{kn\max}, \quad k = 1,2,\cdots m \tag{4.43}$$

$$V_{k\min} \le V'_k \le V_{k\max},, \quad k = 1,2,\cdots m \tag{4.44}$$

$$P_{kn\min} \le P'_{kn} \le P_{kn\max}, \quad k = 1,2,\cdots m, \quad n = 1,2,\cdots m \tag{4.45}$$

Where:

$f(x,u)$ is the total fuel cost, dollar per hr ($/hr).

α, β and γ are the cost coefficients of the generator

P_{g_k} is generation power at bus *k*.

V_k is the bus voltage magnitude at bus *k* in normal conditions.

P_{L_k} and Q_{L_k} is the load at bus *k* in normal conditions.

P_i and Q_i are the active and reactive power flow at bus *k* in normal condition.

P_{kn} is the power flow between bus *k* and *n* in normal conditions.

$P_{g_k\min}$ and $P_{g_k\max}$ are the minimum and maximum active power output of generation unit *k*.

$P_{kn\min}$ and $P_{kn\max}$ are the lower and upper bounds on the active power flow between bus *k* and *n*.

$V_{k\min}$ and $V_{k\max}$ are lower and upper bounds on the voltage magnitude at bus *k*.

$P_i^{'}$ and $Q_i^{'}$ are the active and reactive power flow at bus k in contingency state.

$V_k^{'}$ is the bus voltage magnitude at bus k in contingency state.

$P_{g_k}^{'}$ is generation power at bus k in contingency state.

$P_{kn}^{'}$ is the power flow between bus k and n in contingency state.

m is the number of buses.

n_g is the number of generators.

Active power outputs of generation units are control variables need to be solved to achieve the optimal operation of the minimum fuel cost. Vector x contains the total state variables for normal condition and contingencies case. $V_1', V_2', \cdots, V_m'$, and $\theta_1', \theta_2', \cdots, \theta_m'$ represent the state variables required for an outage of transmission line. Eqn. 4.38 establishes the objective function. Eqn. 4.39 is the equality constraints (power flow equations) under normal condition. Eqn. 4.40 defines the equality constraints (power flow equations) under contingency state (one line outage). Eqns. 4.41-4.43 are inequality constraints under normal condition, which clarify the upper and lower bounds on the voltage magnitude, active power outputs and power flow between buses. Eqns. 4.44 and 4.45 are inequality constraints when a contingency is considered, which ensure that the resulting voltages and flows would still be within limit. Considering the OPF analysis of a system under normal condition includes n buses, x unknown state variables, j equality constraints and p inequality constraints, if there are q contingency cases involved, and then the total number of unknown state will be $(q+1)x$, also the total number of equality and inequality constraints will be $(q+1)j$ and $(q+1)p$ respectively.

Two case studies are discussed in this section to illustrate the constrained optimization method for SCOPF. One is the 7-bus power system, and the base case is as same as that of OPF in section 4.4.1. The other one is 26-bus power system, and the base case is as same as that of OPF in section 4.4.2. All the studies use the optimization tool box available in Matlab [13]. For some aspects of the studies, PowerWorld Simulator is also used. The fuel costs of all the generating units are represented using cubic cost models. The SCOPF study focuses on single transmission line outage only. In the optimization problems, the constraints considering transmission line outage are included.

4.8.1 SCOPF Minimizing Cost for 7-bus Power System

Repeating the example in section 4.4.1 (7-bus system OPF), the goal of the SCOPF for the 7-bus system is to minimize generator fuel costs while adhering to power flow equations, specified branch flow (MVA), bus voltage magnitudes, slack generator active power limits. The limits, fuel cost coefficients and the system parameters are found in Appendix C. The single line diagram of the 7-bus power system can be seen in Fig 4.2. The total loads are 760 MW and 130 MVAR. For the base case, the total fuel cost is 16939$/hr and the transmission loss is 7.9 MW.

The formulation of SCOPF can be defined based on the Eqns. 4.38-4.45. Matlab Optimization Toolbox is applied to achieve the main goal (minimizing the generation cost). In order to ensure secure power operation, the outage of each transmission line need to be studied. For OPF analysis, the outages of transmission line 2-5 and 2-6 cause security violations. In other words, the system will be violated, if either line is out. Table 4.5 gives the final results of SCOPF. Power and Loss are in MW, and Voltage is in per unit. Cost is in $/hr. The table illustrates that in order to keep the system to operate securely and optimally during individual single line outage, the generation output of each generator and cost may change. The summary of cost data presented in Table 4.5 shows that the cost for SCOPF ($17020) is higher than the cost for OPF ($16371). However, for the generation dispatch of SCOPF, the system can operate without any violation. As seen from Table 4.5, the cost is increased in order to maintain a secure power operation system. The bus voltage magnitudes maintain a reliable level (approximate 1 p.u.). The results presented in Table 4.5 are obtained by Matlab and verified by PowerWorld Simulator.

Table 4.5 SCOPF of 7-Bus Power System, Minimizing Generation Cost

MINIMIZING COST	**P1 (MW)**	**P2 (MW)**	**P4 (MW)**	**P6 (MW)**	**P7 (MW)**	**Cost ($/hr)**	**Line Violation**
Base Case	102	170	95	200	201	16939	Line 4-5
OPF	126	230	71	291	52	16371	Line 2-5 and line 2-6
SCOPF	100	150	50	245	219	17020	No violation

4.8.2 SCOPF Minimizing Cost for 26-bus Power System

Repeating the base case in section 4.4.2 (26-bus system OPF [17]), the goal of the SCOPF for the 26-bus system is to minimize generator fuel costs while ensuring the security constraints. For the 26-bus system, the SCOPF study has focused on single transmission line outage only. The limits, fuel cost coefficients and the system parameters are found in Appendix D. The single line diagram of the 26-bus power system can be seen in Fig 4.3. The total loads are 947.3 MW and 484.5 MVAR. For the base case, the total fuel cost is 23946$/hr and the transmission loss is 9.4 MW.

The formulation of SCOPF can be defined using the Equations 4.38-4.45. The objective functions stay same for the OPF study. The constraints considering transmission line outage are included in the optimization problems. Considering the outage of transmission line 1-18 as an example, which is the potential worst-contingency case, the results are given in Table 4.6. Power is in MW, Voltage is in per unit, and Cost is in $/hr. The total hourly cost for SCOPF is 22846$/hr with the loss 6.8 MW. The table illustrates that the output generation of each unit and total fuel cost are changed during this outage to keep the system to operate securely and optimally. The solutions of OPF are also included in the Table 4.6. As seen in Table 4.6 the cost of SCOPF ($22846) in normal operation is higher than cost of normal OPF ($22646), but the system can operate without any violation during the outage of line 1-18. Hence, higher cost required when the security constraints are included. The results presented in Table 4.5 are obtained by Matlab and verified by PowerWorld Simulator.

Table 4.6 SCOPF of 26-Bus Power System, Minimizing Generation Cost

MINIMIZING COST	P1 (MW)	P2 (MW)	P3 (MW)	P4 (MW)	P5 (MW)	P26 (MW)	Cost ($/hr)	Line Violation
Base Case	472	50	15	75	225	119	23946	Line 1-2, 1-18, 4-12, 5-6 and 18-17
OPF	378	120	209	80	118	50	22646	line1-18 and 4-12
SCOPF	276	134	229	112	151	53	22846	Line 4-12

4.9 Security Constrained Optimal Power Flow Minimizing Loss

The objective is to minimize the active power loss during the transmission. Based on previous research of SCOPF problems, the SCOPF regarding minimizing active power loss can be mathematically formed as:

$$\text{Finding the vectors } u = \begin{bmatrix} P_{g_1} \\ P_{g_2} \\ \vdots \\ P_{g_n} \end{bmatrix}, \quad x = \begin{bmatrix} V_1 & V_1' \\ V_2 & V_2' \\ \vdots & \vdots \\ V_m & V_m' \\ \theta_1 & \theta_1' \\ \theta_2 & \theta_2' \\ \vdots & \vdots \\ \theta_m & \theta_m' \end{bmatrix}$$

To minimize:

$$f(x,u) = \sum_{k=1}^{n_g} P_{g_k}(x,u) - \sum_{k=1}^{n_l} P_{L_k} \tag{4.46}$$

Subject to:

$$\sum_{i=1}^{m} P_i(x,u) = 0, \sum_{i=1}^{m} Q_i(x,u) = 0 \tag{4.47}$$

$$\sum_{i=1}^{m} P'_i(x,u) = 0, \sum_{i=1}^{m} Q'_i(x,u) = 0 \tag{4.48}$$

$$V_{k\min} \le V_k \le V_{k\max}, \quad k = 1,2,\cdots m \tag{4.49}$$

$$P_{g_k \min} \le P_{g_k} \le P_{g_k \max}, \quad k = 1,2,\cdots n_g \tag{4.50}$$

$$P_{kn\min} \le P_{kn} \le P_{kn\max}, \quad k = 1,2,\cdots m \tag{4.51}$$

$$V_{k\min} \le V'_k \le V_{k\max},, \quad k = 1,2,\cdots m \tag{4.52}$$

$$P_{kn\min} \le P'_{kn} \le P_{kn\max}, \quad k = 1,2,\cdots m, \quad n = 1,2,\cdots m \tag{4.53}$$

where:

$f(x,u)$ is the total loss, MW.

P_{g_k} is generation power at bus *k*.

V_k is the bus voltage magnitude at bus *k* in normal conditions.

P_{L_k} and Q_{L_k} is the load at bus *k* in normal conditions.

P_i and Q_i are the active and reactive power flow at bus *k* in normal condition.

P_{kn} is the power flow between bus *k* and *n* in normal conditions.

$P_{g_k \min}$ and $P_{g_k \max}$ are the minimum and maximum active power output of generation unit *k*.

$P_{kn\min}$ and $P_{kn\max}$ are the lower and upper bounds on the active power flow between bus *k* and *n*.

$V_{k\min}$ and $V_{k\max}$ are lower and upper bounds on the voltage magnitude at bus *k*.

$P_i^{'}$ and $Q_i^{'}$ are the active and reactive power flow at bus k in contingency state.

$V_k^{'}$ is the bus voltage magnitude at bus k in contingency state.

$P_{g_k}^{'}$ is generation power at bus k in contingency state.

$P_{kn}^{'}$ is the power flow between bus k and n in contingency state.

n_g is the number of buses.

n_l is the number of buses.

m and n is the number of buses.

Vector u defines all the control variables (active power outputs of dispatchable generators) need to be solved to achieve the objective function. Vector x defines the state variables both in normal condition and contingency state. Equation 4.46 defines the objective function, which is total transmission loss. Equations 4.47, 4.49-4.50 define the equality constraints (power flow equations) and inequality constraints (operation limits of bus voltage magnitude, branch limits and generation limits) in normal condition. Equations 4.48, 4.52-4.53 express the equality and inequality constraints when a transmission line outage is considered. Those constraints prevent the post-contingency voltage on buses or the post-contingency flow on transmission lines from exceeding their limits for the outage of a transmission line. The constraints considering transmission line outages for minimizing transmission loss are similar to those for the SCOPF regarding minimizing total fuel cost. For example, if there are m contingency cases need to be considered, and then the number of the total constraints for SCOPF study is $(m+1)$ times the number of constraints for OPF study. The size of the state variables and security constraints depend on the number of contingency cases considered.

The 7-bus power system [17] and the 26-bus power system [4] are discussed in this section to illustrate the SCOPF study for minimizing the active power loss. The base cases of above two power systems are as same as those of OPF in the sections 4.4.1 and 4.4.2. All the studies use the optimization tool box available in Matlab [13]. For some aspects of the studies, PowerWorld Simulator is also used. The SCOPF study focuses on single transmission line outage only.

4.9.1 SCOPF Minimizing Loss for 7-bus Power System

The data of 7-bus power system considered in this study are given in Appendix C. The one line diagram can be seen in Fig 4.2. The total loads are 760 MW and 130 MVAR. For the base case, the transmission loss is 7.9 MW and the total fuel cost is 16939$/hr. The goal is to minimize the total active power loss while satisfying all the power flow equations, specified branch flow (MVA), bus voltage magnitudes, slack generator active power limits both in normal condition and contingency condition. In the optimization problem, the constraints considering transmission line outage are included. For OPF analysis, if the outage of transmission line 2-5 happens, the system will be violated. Hence in order to ensure the most secure operation manner, the outage of transmission line 2-5 is considered for SCOPF.

The formulation of OPF regarding minimizing loss for 7-bus system can be formed using Eqns. 4.46-4.53. Matlab Optimization Toolbox is applied to achieve the main goal (minimizing the transmission loss). The controllable variables include 4 generators active power output variables. State variables are load bus voltage magnitude and phase angles. The more contingency cases included, the more state variables are required. Table 4.7 illustrates the final results of SCOPF. Power and loss are in MW; voltage is in per unit, and cost is in $/hr. The table shows that the generation output of each generator and the total cost are changed in order to keep the system to operate securely and optimally during the contingency cases. For the results of SCOPF, the transmission loss (3.31 MW) and cost ($17243) obtained from the SCOPF are slightly larger than those for the OPF. However, the system can be operated without any violation. This implies that more cost is required to maintain a secure operation system, also the minimum loss from OPF may not be ensured in SCOPF. The bus voltage magnitudes maintain a reliable level (approximate 1 p.u.). The results presented in Table 4.7 are obtained by Matlab and verified by PowerWorld Simulator.

Table 4.7 SCOPF of Minimizing Loss for 7-Bus Power System

	P1 (MW)	P2 (MW)	P4 (MW)	P6 (MW)	P7 (MW)	Loss (MW)	Cost ($/hr)	**Line Violation**
Base Case	102	170	95	200	201	7.9	16939	Line 4-5, 160%
OPF (Loss)	100	150	109	150	254	3.25	17150	Line 2-5
SCOPF(Loss)	100	150	92	150	271	3.31	17243	No violation

4.9.2 SCOPF Minimizing Loss for 26-bus Power System

The goal of this study is to minimize generator fuel costs for 26-bus power system while satisfying the power operation constraints such as intact system SC and single outage security constraints. The one line diagram of 26-bus system can be seen in Fig. 4.3. The limits, fuel cost coefficients and the system parameters are found in Appendix D. The total loads are 947.3 MW and 484.5 MVAR. For the base case, the transmission loss is 9.4 MW and the total fuel cost is 23946$/hr. For the 26-bus system, the SCOPF study focuses on single transmission line outage only. The complete security constraints are included in the optimization problem.

The SCOPF for 26-bus power system can be formulated as a set of nonlinear optimization problem using Eqns. 4.46-4.53. The control variables are active power outputs of 5 dispatchable generators. The state variables are the bus voltage magnitudes and phase shift for both normal and contingency conditions. The constraints are used to prevent voltage magnitudes and branch flow under pre-contingency and post contingency operation from exceeding their operation limits.

Consider the outage for line 1-18 as an example; Table 4.8 gives the results of SCOPF. Power and loss are in MW; voltage is in per unit, and cost is in $/hr. For the results from SCOPF, the transmission loss is reduced to 6.33 MW, and the hourly cost is increased to 23076$/hr. The active power outputs of each generator are changed to maintain a secure manner during the outage of line 1-18. As seen in Tables 4.4 and 4.8, the transmission loss and cost obtained from the SCOPF are slightly higher than those for the OPF. However, the system is secured even if the outage of transmission line 1-18 is happened. This implies that when security constraints are included, the minimum transmission loss will be increased compares to those for the OPF study, and the corresponding fuel cost is increased as well. The results presented in Table 4.8 are obtained using Matlab and verified using PowerWorld Simulator.

Table 4.8 SCOPF of Minimizing Loss for 26-Bus Power System

	P1 (MW)	P2 (MW)	P3 (MW)	P4 (MW)	P5 (MW)	P26 (MW)	Loss (MW)	Cost ($/hr)	Line Violation
Base Case	472	50	15	75	225	119	9.4	23946	Line 1-2, 1-18, 4-12, 5-6 and 18-17
OPF	341	97	147	150	150	68	6.26	22876	Line1-18, 4-12
SCOPF	277	123	169	150	164	70	6.33	23076	Line 4-12

4.10 Conclusions

This chapter has discussed the fundamentals of the OPF and SCOPF and the solutions to solve it. With OPF, the power system can be scheduled to optimize a certain objective while satisfying a set of operational constraints imposed by equipment limitations. SCOPF is combined with OPF to redispatch generation, and adjust voltages to meet the contingency constraints. The transmission line outages are considered in this chapter. The solutions obtained from SCOPF ensure that the system is operated at an acceptable condition even if some contingencies occur. Both OPF and SCOPF can be mathematically formed as a set of nonlinear equations. Matlab Optimization Toolbox is used to implement programs and solve these optimization problems. Case studies are illustrated and discussed to present the application of OPF and SCOPF. Objective of minimizing fuel cost and minimizing active power loss are considered in these studies. As seen from the case studies, the fuel cost of a power system at OPF setting is higher than that of a power system at Economic Dispatch setting. This is because that the transmission line limits and other limits may be violated in Economic Dispatch. For minimizing active power loss case, the optimal operation pattern obtained from SCOPF is more expensive than that of OPF. This is because that of the need to ensure that the power system operates economically both in normal condition and during contingencies.

The studies presented in this chapter show that the nonlinear programming based optimization techniques can handle OPF and SCOPF problems efficiently. For larger power system, the number of contingency cases will increase. As a result, the number of security constraints and the number of variables in the optimization problem will increase significantly.

Chapter 5

Engineering Multiobjective Optimization

5.0 Introduction

Optimization is an act of finding one or more feasible solutions which correspond to extreme values of one or more objectives. The typical aims of these solutions are either to maximize possible benefit or minimize possible cost. Some real world problems can be defined by a single objective; the task of finding the best result is called single objective optimization. However, most problems involve more than one criterion. The problems that involve more than one objective functions are known as multi-objective optimization problems. The use of multi-objective optimization technologies allows the management of different objectives, and gives indications on the consequences of the decision with respect to all the objective functions considered [8,35-37].

The aim of this chapter is to discuss the fundamental of multiobjective optimization and the most commonly used Weighted Sum (WS) algorithm which can effectively solve multiobjective optimization problems. This chapter is organized as follows: Section 5.1 discusses the difference between single objective and multiobjective optimization. Section 5.2 presents an overview of the typical multiobjective optimization problems. Section 5.3 states the concept of Pareto optimal for minimization problems. A simple example is also illustrated. Section 5.4 presents the WS algorithms. Case studies using WS method are presented in section 5.5. Section 5.6 provides some concluding remarks.

5.1 Single and Multiple Objective Optimizations

Since most problems in the real life have many objectives that need to be satisfied, it is possible that these objectives conflict with each other. In general, it is easy to find a single optimal solution, when the objective functions have been considered independently. However, it is impossible to find a single solution that fits all the objectives. Hence, instead of giving a single optimal solution, a set of trade off solutions gives the values of the entire objective functions acceptable to the decision maker, which are also known as Pareto set. Each solution in the Pareto set has an important characteristic: the improvement in one of the objectives results in the worsening of at least one other objective [38]. The final solution that can be chosen from the set depends on the decision maker.

Car-buying making decision problem is a common example of multiobjective optimization problems with conflicting objective functions. This example is from the book by K. Deb [35,37]. Assuming that there is a need to buy a car, most people would like to select the one with the less cost and more comfort. Normally an inexpensive car is likely to be less comfortable as shown in Fig. 5.1. For people whose only target is comfort, the optimal choice is option 2 which has the highest comfort level. For people whose objective is cost, and then the optimal choice is option 1 which has the lowest cost. These solutions are illustrated in Fig. 5.1. Between these two extreme solutions, a number of trade-off options (option A, B and C) with different costs and comfort levels exist between option 1 and option 2. Customers are able to make a decision based on weighing two objectives. None of these solutions is the best with respect to both objectives, thus all options in this set are optimal solutions for this multiobjective optimization problem rather than a single optimal solution. This is also the fundamental difference between a single-objective and a multi-objective optimization.

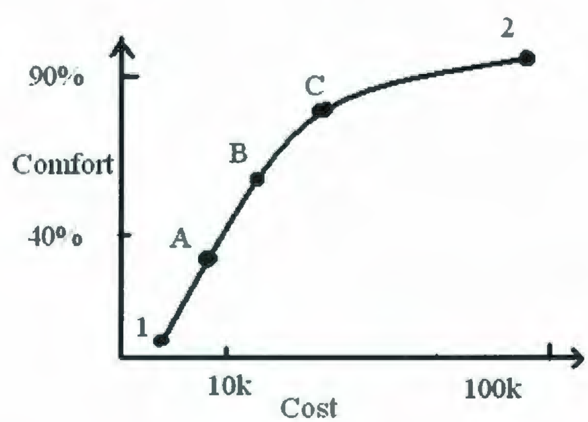

Fig. 5.1 Trade-off Solutions for Buying a Car – Cost vs Comfort [37].

5.2 Multiobjective Optimization

A typical multiobjective optimization problem is to find a vector of decision variables which satisfies the constraints and optimizes a vector function whose elements represent the objective functions. These functions form a mathematical description of performance criteria which are usually in conflict with each other. Hence, the term "optimize" means finding such a solution which would give the values of all the objective functions acceptable to the decision maker [32,39].

The general MOP can be formally defined as finding the vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

to minimize/maximize: $$f_1(X), f_2(X) \ldots f_n(X) \tag{5.1}$$

Subject to: $$\begin{aligned} &g_j(X) \leq 0, j = 1,2,\cdots,J \\ &h_k(X) = 0, k = 1,2,\cdots,K \\ &x_i^{(L)} \leq x_i \geq x_i^{(U)}, i = 1,2,\cdots,n \end{aligned} \tag{5.2}$$

In Equation 5.1, $f_n(X)$ is the objective function set. X is a vector of n decision variables: $x_1, x_2, \ldots, x_n$. In Equation 5.2, $g_j(X)$ and $h_k(X)$ are inequality and equality constraints. $x_i^{(L)}$ and $x_i^{(U)}$ are lower and upper bounds of variables, restricting each decision variable x_i to take a value within a lower $x_i^{(L)}$ and an upper $x_i^{(U)}$ bound.

5.3 Dominated and Non-dominated Solution

The concept of domination is applied to find the optimal solution set of MOPs. Comparing two solutions A and B, a solution A is said to dominate the other solution Y, if both conditions 1 and 2 are true [35,37]:

1. The solution A is no worse than Y in all objectives;
2. The solution A is strictly better than Y in at least one objective.

That is, A dominates B when it is as good as B regarding each objective, and there is at least one objective with respect to which A is better than B. If any of the above condition is violated, solution A does not dominate solution B. If A dominates B, it also means:

- B is dominated by A;
- A is non-dominated by B.

A sample optimization problem with two objective functions subject to minimization is illustrated in Fig. 5.2. Comparing solutions A and B, A is better than B in both of objective functions, 1 and 2. Hence, both above conditions are satisfied, and solution A dominates solution B. Comparing A and C, C is better than A in both of objectives, so C dominates A. Since the concept of domination is the method of comparing solutions applied to multi-objective, most multiobjective optimization algorithms use this concept to search for non-dominated solutions.

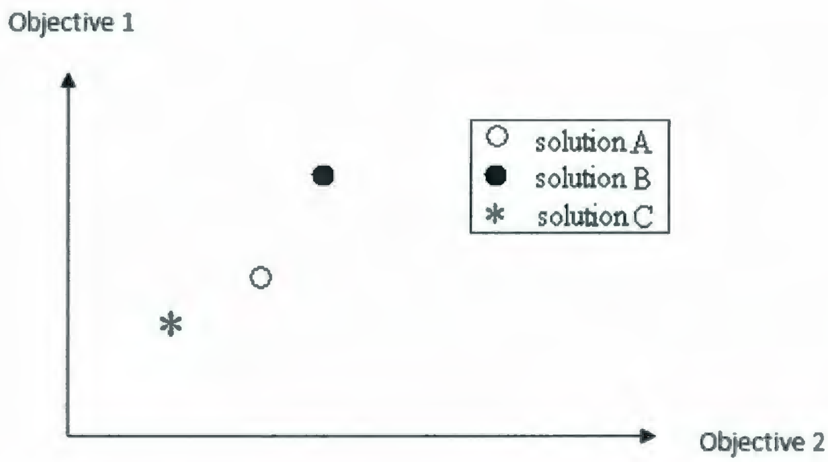

Fig. 5.2 Solution A Dominates Solution B

In the entire objective space, a non-dominated solution is defined as any feasible solution such that no other feasible solution is strictly better than it with respect to all objectives [32]. These non-dominated solutions are known as Pareto Optimal solutions. The plot of the objective functions whose non-dominated vectors are in the Pareto optimal set is called the Pareto front.

Continuing the above example, Fig. 5.3 illustrates the optimal solutions between two objective functions. The solid curve is the Pareto front which contains all the non-dominated solution for the multiobjective optimization problem. The Pareto front line shows that as the value of objective 1 increasing, the objective 2 is decreasing. However, none of the solution on the line fits these two objectives better than any other.

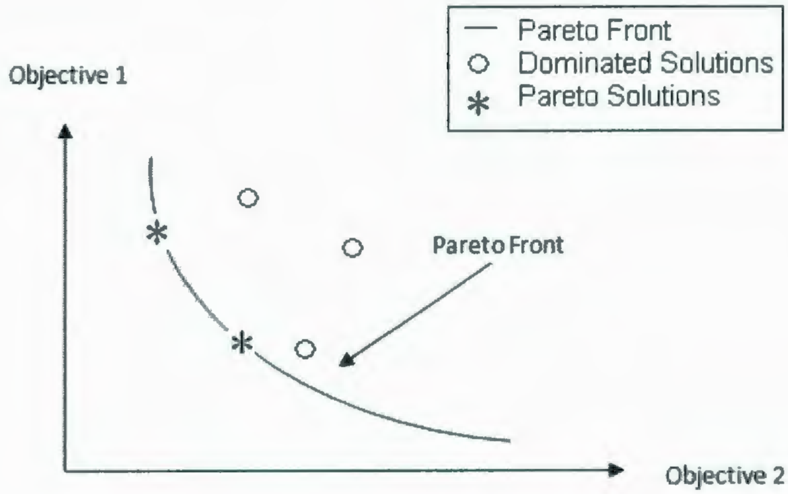

Fig. 5.3 Pareto Front of a Bi-objective Functions Optimization Problem

The following example of multi-objective optimization problem with two objective functions and one decision variable illustrates the optimal solutions and Pareto front. This example is adopted from Matlab Tutorial [40].

$$\begin{aligned} &\text{Minimize:} \quad F(x) = [objective1(x); \quad objective2(x)]; \\ &\text{where,} \quad objective1 \quad (x) = (x+2)^2 - 10, \\ &\qquad\qquad objective2 \quad (x) = (x-2)^2 + 20. \end{aligned} \tag{5.3}$$

The above two objective functions are quadratic equations. Fig. 5.4 plots the objective functions when the variable x varies between -10 and 10. The two objectives have their minima at x = -2 and x = +2 respectively. However, in a multi-objective problem, x = -2, x = 2, and any solution in the range -2 <= x <= 2 is equally optimal. There is no single solution to this multi-objective problem. Fig.5.5 presents the corresponding Pareto front, which contains the set of solutions. All solutions on the Pareto front are optimal.

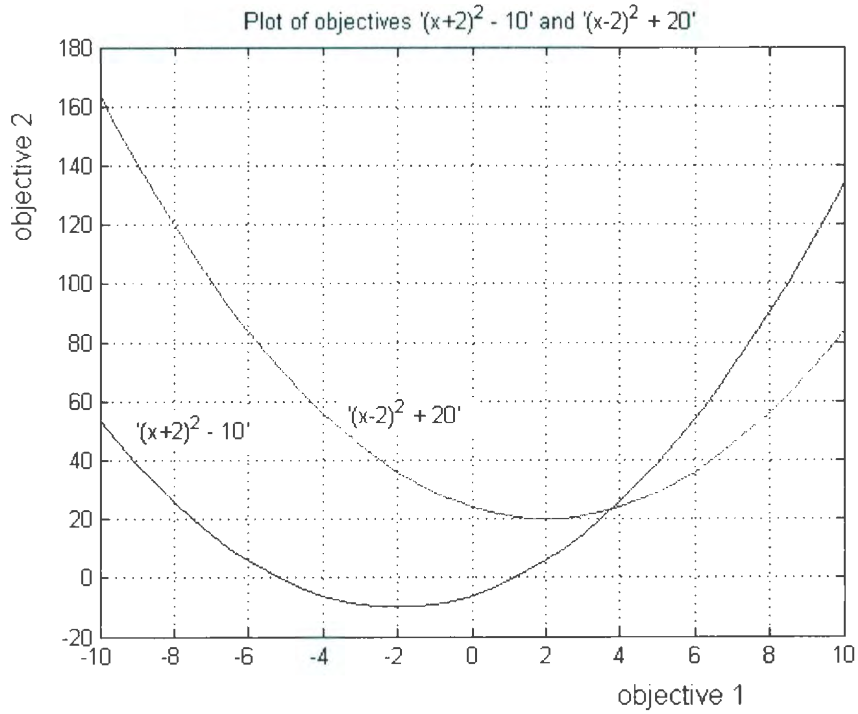

Fig. 5.4 Plots of Objective Functions

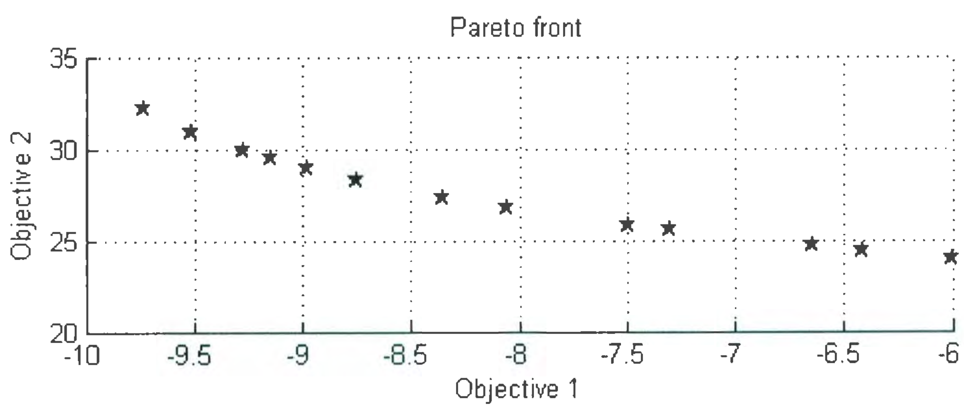

Fig. 5.5 Plots of Pareto Front

5.4 Weighted Sum Method

Along with the application of multiobjective optimization, significant amount of algorithms have been developed to solve optimization problems. Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and Evolutionary Algorithms (EA) are some of the techniques that have been proposed recently. Weighted Sum (WS) is the most common and the simplest classical approach used to determine the Pareto set. WS allows finding each point in the Pareto set by using traditional optimization techniques applied to a nonlinear constrained problem. It scalarizes a set of objectives into a single function by multiplying each objective with a weight factor. The weight of an objective function is usually chosen in proportion to the objective's relative importance in the problem [8,37,41]. The Pareto front can be obtained by changing the weights among the objective functions. ε-constaint method, weighted metric methods, value function methods, goal programming methods are some of the other available methods. For the multiobjective optimization studies, presented in this thesis, the weighted sum method is used [7]. The WS is stated as:

$$\begin{array}{lll} \textit{Minimizing}: & f(x)=\sum_{m=1}^{M} w_m * f_m(x), & m=1,2,\ldots M \\ \textit{Subject to}: & g_j(X)\leq 0, & j=1,2,\cdots,J \\ & h_k(X)=0, & k=1,2,\cdots,K \\ & x_i^{(L)}\leq x_i \geq x_i^{(U)}, & i=1,2,\cdots,n \end{array} \quad (5.4)$$

In Equation 5.4, w_m is the weight of the m-th objective function. Normally the sum of chosen weights is one ($\sum_{m=1}^{M} w_m = 1, \quad m=1,2,\ldots M$). Different weight values of an objective function result in multiple solutions of the decision variables. Therefore, decision makers can use their experience and any preference information of the Pareto set to choose the final operating point by varying the weights of objective functions.

5.5 Case Studies

Two case studies of multiobjective optimization problems are discussed in this section. The first case has objective functions 2 variables and no constraints. The second case is a MOP with 2 objective functions, 4 variables and 2constraints.

5.5.1 Case Study 1

The two-objective functions to be minimized are:

$$f_1 = x_1^4 - 10x_1^2 + x_1x_2 + x_2^4 - x_1^2x_2^2 \tag{5.5}$$

$$f_2 = x_2^4 - x_1^2x_2^2 + x_1^4 + x_1x_2 \tag{5.6}$$

The two functions are combined into a single objective function (unconstrained) based on the weighted sum method. The solution corresponding to different weights is determined using an unconstrained minimization method. Fig. 5.6 shows the Pareto front obtained by WS. The decay curve shown in the figure illustrates the non-dominated solutions of the Pareto front. From the figure, it is clear that the objectives conflict with each other. Hence, a single optimal solution cannot satisfy both objectives. The trade off region provides multiple solutions to meet the requirement of the objectives.

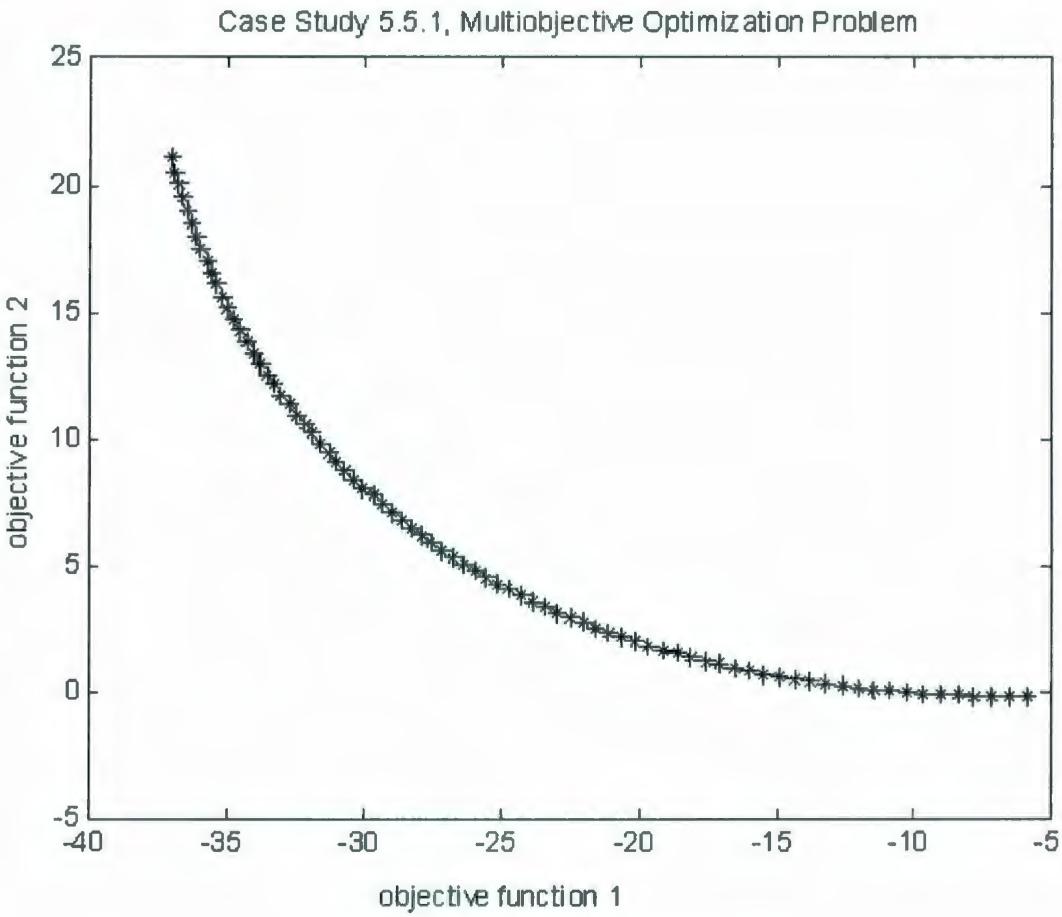

Fig. 5.6 Pareto Front of Case Study 1

5.5.2 Case Study 2

Case 2 is about a multiobjective optimization problem with minimizing two objectives, which is a simple power system example from the book by Bergen and vittal[18]. 2 objective functions, 2 equality constraints and 4 unknown variables are included. Function 1 f_1 presents the generation cost, and function 2 f_2 states the transmission loss. The problem is to minimize:

$$x_a = 8 - 4\cos(x_1) - 8\sin(x_1) - 4\cos(x_2) - 8\sin(x_2) \tag{5.7}$$

$$f_1 = 100(x_a + x_3 + x_4 - 5) \tag{5.8}$$

$$f_2 = 1450 + 800(x_a + x_3) + 750x_4 + 15x_a^{\ 2} + 5x_3^{\ 2} + 10x_4^{\ 2} \tag{5.9}$$

subject to

$$x_3 + 4\cos(x_1) - 8\sin(x_1) + 4\cos(x_1 - x_2) + 8\sin(x_1 - x_2) - 8 = 0 \tag{5.10}$$

$$x_4 + 4\cos(x_2) - 8\sin(x_2) + 4\cos(x_1 - x_2) - 8\sin(x_1 - x_2) - 13 = 0 \tag{5.11}$$

where x_1, x_2, x_3 and x_4 are unknown variables

By using WS, these objective functions can be linearly combined as a simple objective function. Constrained optimization method is applied to determine the solutions corresponding to different weights. Fig. 5.7 illustrates the non-dominated solutions of the Pareto front by WS. The trade off region provides multiple solutions to meet the requirement of the objectives. The figure shows that the objectives conflict with each other. The minimum fuel cost does not ensure the minimum transmission loss. More cost is required to reduce the transmission loss. The Pareto front contains all the optimal solutions, which allows the operator to select a solution by observing a wide range of options.

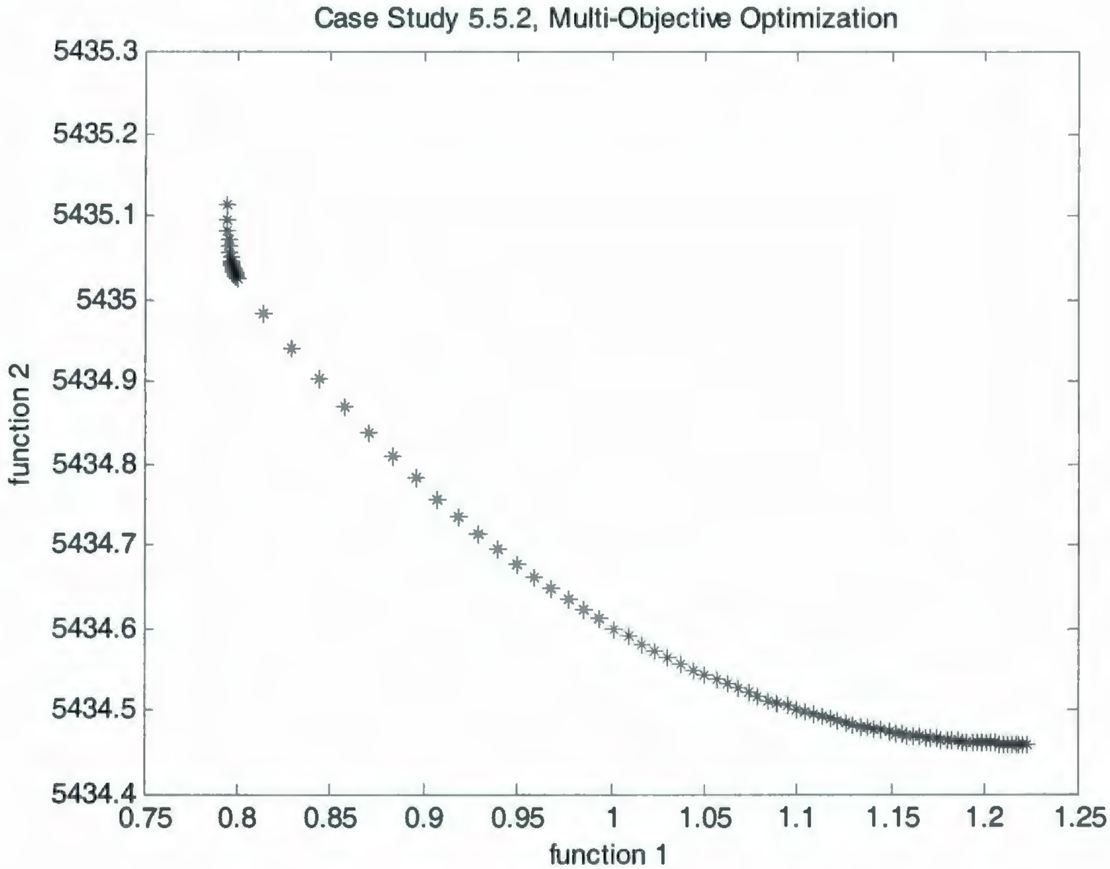

Fig. 5.7 Pareto Front of Case Study 5.5.2

5.6 Conclusions

This chapter has provided an overview of multiobjective optimization. Pareto Optimal is used to define a set of acceptable trade-off optimal solutions which is also known as non-inferior solutions. A number of classical multiobjective optimization algorithms are available to solve problems. Most algorithms convert the multiple objective functions into a single objective function by using some user defined procedures. Weighted sum method is one of the most simple and common approach that linearly combines all the objectives into one function by using weight factors. The weight factors are user defined which promotes more appropriate roles for the participants in the planning and decision making process. The Pareto front contains all the optimal solutions which are computed by varying the weight factors of objective functions. It allows the operator to select a solution by observing a wide range of options. Two simple numerical examples are presented to illustrate the performance of weighted sum method. In the next chapter (chapter 6), instead of implementing a program for weighted sum algorithm, Matlab Optimization Toolbox is employed to solve the multiobjective optimization problems in power system.

Chapter 6

Application of Multi-objective Optimization in Power Systems

6.0 Introduction

Along with the liberalization of modern power system, the multiple objective programming and planning represents a very useful generalization of more traditional single objective approaches to planning problems [42,43]. The consideration of multiple objectives brings three major advantages [38]. First, it allows the management of different objectives. Second, it provides more opportunities for operators to plan and make decision. Third, it gives indications on the consequences of the decision with respect to all the objective functions considered. Multiobjective optimization can be considered as optimizing many objective functions subject to different constraints. For power system applications, these objectives functions can be total fuel cost and total transmission losses [34,38,44].

Many tools are available to solve the multiobjective optimization problems. Chapter 5 introduced the fundamentals of multiobjective optimization and the solutions associated with it. However, many of the proposed methods for multiobjective optimization in power system focus on the constraints related to the steady state operation only. Security constraints (i.e. operation of the power system under credible contingencies) are not considered in detail [7]. The aim of this chapter is to evaluate the application of multiobjective optimization for power system including security constraints.

Chapter 4 discussed challenging power systems optimization problems known as the Optimal Power Flow (OPF) and Security Constrained Optimal Power Flow (SCOPF) problems. The outages of transmission lines are considered only. When security constraints are involved, the size of the optimization problem depends on the number of contingency cases considered. The OPF and SCOPF can be achieved by dispatching generation and adjust voltages to minimize the objective functions while satisfies the normal constraints and contingency constraints.

Two case studies are performed using two different power systems. Two objective functions are involved in the optimization problems. For both case studies, the multiobjective optimization is required to compute a feasible and non-dominated set of generation patterns that minimizes fuel cost and minimizes transmission loss while maintaining security constraints.

This chapter is organized as follows: section 6.1 gives an overview of common multiobjective optimization, and the formulation of the problem for power system application including security constraints is discussed. Case studies (7-bus power system [29] and 26-bus power system [17]) using simple power system models are presented in section 6.3 and 6.3. The goal of these two case studies is to minimize the total fuel cost and the transmission loss. Section 6.4 provides some concluding remarks.

6.1 Mutiobjective Optimization Problem Formulations

Two aspects of the optimal power flow problem considered here are to minimize the total fuel cost and minimize the total transmission loss for specified loading conditions. All the generating units are assumed to be thermal with the fuel cost expressed as a cubic function of the output of the generating units. The objective function to minimize the fuel cost of generation is formed as the sum of the fuel cost for all the available generating units:

$$f_1(x,u)=\sum_{k=1}^{n_g}(\alpha_k+\beta_k * P_{g_k}+\gamma_k *(P_{g_k})^2) \tag{6.1}$$

where:

P_{g_k} is the active power output at the kth generating unit.

n_g is the number of generators in the system.

α, β and γ are cost coefficients.

x and u are state and control variables.

The objective function to minimize the system transmission loss is defined as:

$$f_2(x,u)=\sum_{i=1}^{ng}P_{g_i}-\sum_{i=1}^{nl}P_{L_i} \tag{6.2}$$

where:

P_{L_i} is the load at bus *i*.

ng is the number of generators.

nl is the number of loads.

x presents the state variables which include the bus voltage magnitudes and phase angles. The size of x depends on the number of the contingency cases considered. u represents the control variables which are the active power outputs of dispachable generators. The constraints include equality constraints (power flow equations) and inequality constraints (operation limits of bus voltage, transmission line and generator).

The main contribution of the multiobjective optimization problem presented in this thesis is the inclusion of security constraints. The goal of the multiobjective problems is to ensure that the present operating condition meets the minimum cost and loss criteria as well as remain secure considering the possibility of any credible contingency [7]. For the studies presented here, the contingencies considered are the outage of transmission lines (one at a time). For each contingency, the equality and inequality constraints corresponding to that operating condition must be included in the problem formulation. This will increase the number of variables (x) in the optimization problem significantly. However, the solution will satisfy the desired operating criteria and ensure that the power system is secure [7].

The power system multiobjective optimization problem with security constraints included is formed as a set of nonlinear equations, which is [7,16,34]:

$$\text{Finding the vectors } u = \begin{bmatrix} P_{g_1} \\ P_{g_2} \\ \vdots \\ P_{g_n} \end{bmatrix}, \quad x = \begin{bmatrix} v_1 & v_1' \\ v_2 & v_2' \\ \vdots & \vdots \\ v_m & v_m' \\ \theta_1 & \theta_1' \\ \theta_2 & \theta_2' \\ \vdots & \vdots \\ \theta_m & \theta_m' \end{bmatrix}.$$

$$\text{to minimize:} \quad f_1(x,u) \text{ and } f_2(x,u) \tag{6.3}$$

$$\text{subject to} \quad \sum_{i=1}^{m} P_i(x,u) = 0, \sum_{i=1}^{m} Q_i(x,u) = 0 \tag{6.4}$$

$$\sum_{i=1}^{m} P_i^{'}(x,u) = 0, \sum_{i=1}^{m} Q_i^{'}(x,u) = 0 \tag{6.5}$$

$$V_{k\min} \le V_k \le V_{k\max}, \quad k = 1,2,\cdots m \tag{6.6}$$

$$P_{g_k \min} \le P_{g_k} \le P_{g_k \max}, \quad k = 1,2,\cdots ng \tag{6.7}$$

$$P_{kn\min} \le P_{kn} \le P_{kn\max}, \quad k = 1,2,\cdots m \tag{6.8}$$

$$V_{k\min} \le V_k^{'} \le V_{k\max}, \quad k = 1,2,\cdots m \tag{6.9}$$

$$P_{kn\min} \le P_{kn}^{'} \le P_{kn\max}, \quad k = 1,2,\cdots m, \quad n = 1,2,\cdots m \tag{6.10}$$

where:

$f_1(x,u)$ is the total fuel cost, \$/hr.

$f_2(x,u)$ is the total loss, MW.

P_{g_k} is generation power at bus k.

V_k is the bus voltage magnitude at bus k in normal conditions.

P_{L_k} and Q_{L_k} is the load at bus k in normal conditions.

P_i and Q_i are the active and reactive power flow at bus k in normal condition.

P_{kn} is the power flow between bus k and n in normal conditions.

$P_{g_k \min}$ and $P_{g_k \max}$ are the minimum and maximum active power output of generation unit k.

$P_{kn\min}$ and $P_{kn\max}$ are the lower and upper bounds on the active power flow between bus k and n.

$V_{k\min}$ and $V_{k\max}$ are lower and upper bounds on the voltage magnitude at bus k.

P'_i and Q'_i are the active and reactive power flow at bus k in contingency state.

V'_k is the bus voltage magnitude at bus k in contingency state.

P'_{kn} is the power flow between bus k and n in contingency state.

m is the number of buses.

ng is the number of generator buses.

The formulation of the multiobjective optimization mentioned above, is similar to the SCOPF study in chapter 4. The only difference is that two objectives are involved to compute the optimal solutions. Equation 6.3 represents a set of objective functions, which are minimization of the total fuel cost and minimization of the transmission loss. The vector x consists of dependent variables (bus voltage magnitudes and phase angles). The vector u consists of control variable (real power generation). $v_1', v_2', \cdots, v_m'$, and $\theta_1', \theta_2, \cdots, \theta_m'$ are unknown state variables that are required by the optimization problem when the contingency occurs. Equations 6.4 and 6.5 represent nonlinear equality constraints (power flow equations) under normal condition and contingency state. Equations 6.8-6.10 define the nonlinear inequality constraints of vector arguments x and u which are the voltage magnitude restrictions, generation restrictions and power flow limits. When the outages of transmission lines are considered, a new set of equality and inequality constraints are required based on the contingency cases, and these constraints are also known as security constraints. Hence, the power flow equations (Equation 6.5) at each bus for the outage of one line are formed and included in the problem. The post-contingency bus voltage magnitudes (Equation 6.9) and the post-contingency power flow (Equation 6.10) on each transmission line need to be limited for the outage of one line. The more contingency cases are considered the larger size of the security constraints is resulted for SCOPF. If the optimization problem only focuses on the multiobjective optimal power flow, then the security constraints (Equations 6.5, 6.9-6.10) are not required.

Many methods are available to solve the multiobjective optimization problem, such as Evalutionary Algorithm, Partical Swarm Optimization and Conventional methods. One of the most common, popular and simple classical approach is Weighted Sum (WS) which is also proposed for the multiobjective optimization case studies in this thesis. It determines each point in the Pareto set by using traditional optimization techniques applied to a nonlinear constrained problem. It linearly combines all the objective functions into an overall function by using scalar factors. These scalar factors are also known as weight factor which are chosen in proportion to the objectives relative importance in the problem. The Pareto front which contains optimal solutions can be obtained by varying the weight factors of objective functions. Thus, the two objective optimization problem can be combined into a single function using the WS approach. The final objective function therefore can be expressed as:

$$f(x,u) = w_1 * f_1(x,u) + w_2 * f_2(x,u) \tag{6.3}$$

$$w_1 + w_2 = 1 \tag{6.4}$$

where w_1 and w_2 are the two weights which represent the importance of the objective functions. $f_1(x,u)$ is the objective function for minimizing the cost and $f_2(x,u)$ is the objective function of minimizing the total transmission loss. The operator can decide which objective is more important based on the weight factor corresponding to a specific objective.

In order to perform the studies, weighted sum method, a classical optimization method, is coded in MatLab. Moreover, the PowerWorld Simulator is used to identify the potential contingency cases and verify the results under contingency state.

6.2 7-Bus Case Study

In this section, a multiobjective optimization problem is performed where the goal of optimization is to minimize both the total fuel cost and the active power transmission loss for specified loads. These objective functions were discussed in detail in chapter 4 and previous section (6.1). Security constraints are included to obtain optimal solutions for the outages of the transmission lines. The weighted sum approach is used to solve the optimization problem. The 7-bus power system used for this study is shown in Fig. 6.1[4]. This system has 5 generators, 7 buses, 11 transmission lines and bus 7 is the reference bus. The limits, fuel cost coefficients and the system parameters are listed in Appendix C. The fuel costs of all the generating units are represented using cubic cost models. The total loads are 760 MW and 130 MVAR. For the base case, the total fuel cost is 16939$/hr and the transmission loss is 7.9 MW. Also the 7-bus base case does not ensure either a security or an economy operation manner. However, all generator bus voltages are maintained at 1p.u.

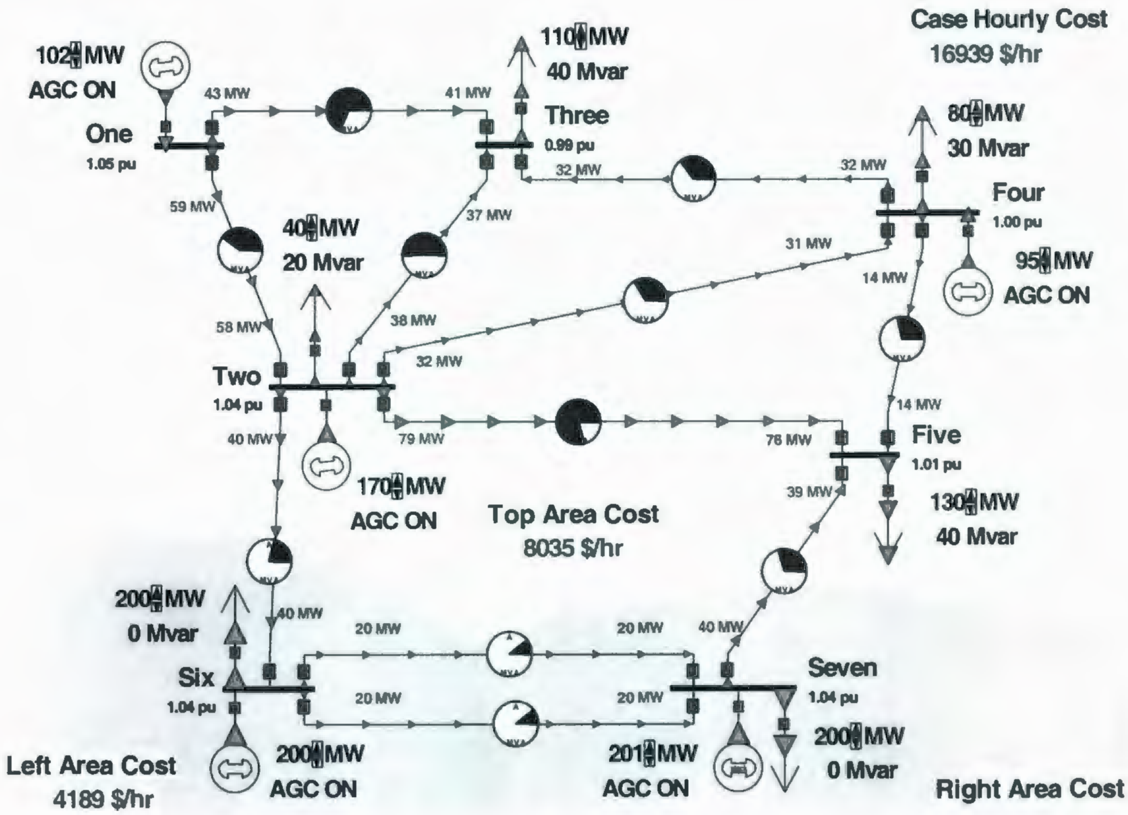

Fig. 6.1. Seven Bus Power System

6.2.1 OPF for7-Bus Power System

The goal of multiobjective optimal power flow is to minimize the total fuel cost and the transmission losses while satisfying all the constraints. Security constraints are not included. The multiobjective OPF problem can be expresses as a set of nonlinear functions using Equations 6.1, 6.2, 6.4, and 6.6-6.8. Equations 6.1 and 6.2 are two objective functions: the total fuel cost and the transmission losses. Equation 6.4 defines the equality constraints which are the power flow equations at each bus. Equations 6.6-6.8 are inequality constraints which set up the boundaries of bus voltage magnitude and branch flow. In total, there are 12 unknown variables, 8 equality constraints and 25 inequality constraints. Table 6.1 lists the number of each parameter used for this case study.

Table 6.1 Multionjective OPF parameter for 7-bus Power System [7]

<table>
<tr><td colspan="2">Control Variable</td><td>4</td></tr>
<tr><td colspan="2">State Variable</td><td>8</td></tr>
<tr><td colspan="2">Equality Constraints
(Power Flow Equations)</td><td>8</td></tr>
<tr><td rowspan="3">Inequality Constraints</td><td>Transmission line</td><td>11</td></tr>
<tr><td>Bus Voltage</td><td>4</td></tr>
<tr><td>Active Power Outputs</td><td>10</td></tr>
</table>

This study is performed using weighted sum method. Matlab is applied to implement the code to solve the optimization problem. Table 6.1, summarizes the results of multiobjective optimization without considering contingency constraints. Multiple optimal solutions can be obtained by varying the weight factors corresponding to the importance of objectives. The more weight factors are used, the more optimal solutions can be computed. 40 pairs of weight factors are used in this study. Table 6.2 lists the results of the OPF studies for 7-bus power system without the consideration of security constraints. When the fuel cost is the most important objective, the corresponding weight factor is 1 and the minimum cost will be computed. Similarly, if the system transmission losses is considered as the most important objective, the minimum loss can be achieved, but minimum fuel cost can not be ensured. More cost is required to reduce the transmission loss.

Table 6.2 Mutliobjective OPF of 7-bus Power System (Without Security Constraints)[7]

Weight Factor of Loss	Transmission Loss (MW)	Weight Factor of Cost	Generation Cost ($/hr)
0	10.050	1	16371
0.2	3.933	0.8	16804
0.4	3.408	0.6	17003
0.6	3.310	0.4	17095
0.8	3.292	0.2	17132
1	3.290	0	17150

Fig. 6.2 shows the optimal solutions set for the 7-bus power system provided by the multiobjective optimization without considering the security constraints. Any star point on the Pareto front is an optimal solution, but it does not guarantee a secure power operation when a single line outage occurs. Each of the solutions contained in the frontier is feasible as no operation constraint was found to be violated. The curve shows that the multiple solutions are well distributed near the area of minimum loss, but a large gap around the area of minimum cost. This can be improved by involving more weight factors. The Pareto front shows that minimization of the total fuel cost and transmission line losses are in direct conflict with each other. Clearly from Fig. 6.2, with more money spent on generation, lower line losses are achieved. The trade off region provides more solutions for the operator to choose an option.

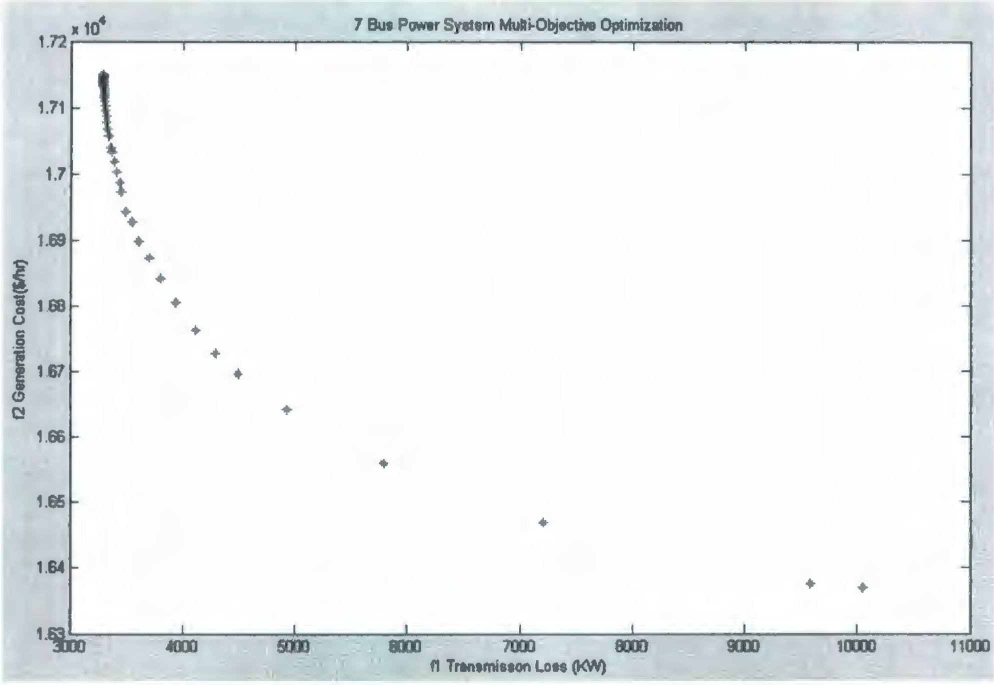

Fig. 6.2. Pareto Front for the 7-bus Power System (without security) [7]

6.2.2 7-Bus Power System for SCOPF

The goal of multiobjective optimal power flow is to minimize the total fuel cost and the transmission losses while satisfying all the constraints under normal condition and contingency cases. Thus, security constraints are included. The one line diagram of 7-bus power system is shown in Fig 6.1, and the system information is introduced in Appendix C. In order to ensure the most secure power operation, the outage of each transmission line needs to be studied. PowerWorld Simulator is used for contingency analysis which determines the violation of the system when single transmission line is out. The analysis results show that the outages of transmission lines (line 2-5 and 2-6) result in all the violations. Therefore, the contingencies with respect to the outage of only two transmission lines (line 2-5 and 2-6) are considered.

The multiobjective SCOPF problem can be expressed as a set of nonlinear functions using Eqns. 6.1, 6.2, 6.4, 6.5 and 6.6-6.10. Eqns. 6.1 and 6.2 are two objective functions: the total fuel cost and the transmission losses which are same as the OPF case. Eqn. 6.4 defines the equality constraints, which are the power flow equations at each bus when the system is operated under normal condition. Eqns. 6.6-6.8 are inequality constraints under normal condition which enhance the operation limits of bus voltage magnitude, active power output of each dispatchable generator and branch flow. When security constraints are considered, the problem is formulated in such a way that state variables, equality and inequality constraints corresponding to the outage of some transmission lines are included. $v_1', v_2', \cdots, v_m'$, and $\theta_1', \theta_2, \cdots, \theta_m'$ are unknown state variables that are required by the optimization problem when the contingency is happened. Eqns. 6.5 represents nonlinear equality constraints, which are power flow equations at each bus for the outages of transmission lines. Eqns. 6.9-6.10 define the nonlinear inequality constraints of vector arguments x and u which are the voltage magnitude restrictions, generation restrictions and branch power flow limits. The post-contingency bus voltage magnitudes (Eqn. 6.9) and the post-contingency power flow (Eqn. 6.10) on each transmission line need to be limited for the outage of one line. The more contingency cases are considered the larger sizes of the security constraints and state variables are resulted for SCOPF. Outages of 2 transmission lines (line 2-5 and 2-6) are considered, therefore, there are 28 variables, 24 equality constraints and 75 inequality constraints in total. Comparing this total amount to the OPF case, the total number of parameters for SCOPF is three times the total number for OPF. Table 6.3 lists the number of each parameter used for this case study.

Table 6.3 Multionjective SCOPF parameter for 7-bus Power System [7]

<table>
<tr><td colspan="2">Control Variable</td><td>4</td></tr>
<tr><td colspan="2">State Variable</td><td>24</td></tr>
<tr><td colspan="2">Equality Constraints
(Power Flow Equations)</td><td>24</td></tr>
<tr><td rowspan="3">Inequality Constraints</td><td>Transmission line</td><td>33</td></tr>
<tr><td>Bus Voltage</td><td>12</td></tr>
<tr><td>Active Power Outputs</td><td>30</td></tr>
</table>

This SCOPF study is computed using weighted sum method, which is implementated using Matlab Optimization Toolbox. Multiple optimal solutions can be obtained by varying the weight factors corresponding to the importance of objectives. The number of the weight factors determines the number of the optimal solutions can be obtained. 40 pairs of weight factors are applied to this study, so 40 pairs of optimal solutions (the fuel cost and the transmission losses) are computed. Table 6.4, summarizes the results of multiobjective optimization with the consideration of security constraints. Ultimately, the choice of the optimal solution is up to the planner. There may be many reasons for choosing one solution over another. For example, when the total fuel cost of generation is considered as the most important objective, the minimum cost should be chosen with the maximum corresponding weight factor (weight factor = 1). Similarly, if the system transmission losses is primary importance and the fuel cost is the secondary one, the minimum loss should be chosen, but minimum fuel cost can not be ensured. The more cost is required to reduce the transmission loss. However, if the fuel cost and the transmission losses are of the same importance, the best solution would be contained somewhere in the middle of the Pareto front as neither objective is minimized but a better balance between the two objectives is realized. As seen from Tables 6.2 and 6.4, the cost is higher when security constraints are included.

Table 6.4 Multiobjective OPF of 7-bus Power System (Secure) [7]

Weight Factor of Loss	Transmission Loss (MW)	Weight Factor of Cost	Generation Cost ($/hr)
0	4.41	1	17021
0.2	3.58	0.8	17150
0.4	3.38	0.6	17213
0.6	3.34	0.4	17243
0.8	3.34	0.2	17243
1	3.34	0	17243

Fig. 6.3 shows the Pareto front provided by the multiobjective optimization with considering the security constraints. Any point on the Pareto front is an optimal solution, and it guarantees a secure power system with respect to the outage of a single transmission line. Each of the solutions listed in the frontier is feasible as no operation constraint was found to be violated, even under contingency situation. The solution set is well distributed over the Pareto front, but there are a few air gaps over the frontier around the area of the minimum cost. These gaps can be improved by adding more weight factors. The Pareto front shows that the objectives of minimization of the total fuel cost and transmission line losses are conflict with each other. Clearly from Fig. 6.3, with more money spent on generation, lower line losses are achieved. The power system operator can select a particular solution from these multiple solutions.

As a result of these allocations, it is found that the maximum fuel cost scheme has a cost of 17243 $/hr, and the minimum cost scheme has a cost of 17021 $/hr. The maximum and minimum cost solutions have transmission line losses of 3.34 MW and 4.41 MW respectively. Thus, by allowing an additional spending of $222, it is possible to reduce the transmission losses by 24.2% and the system is secure considering the contingency cases.

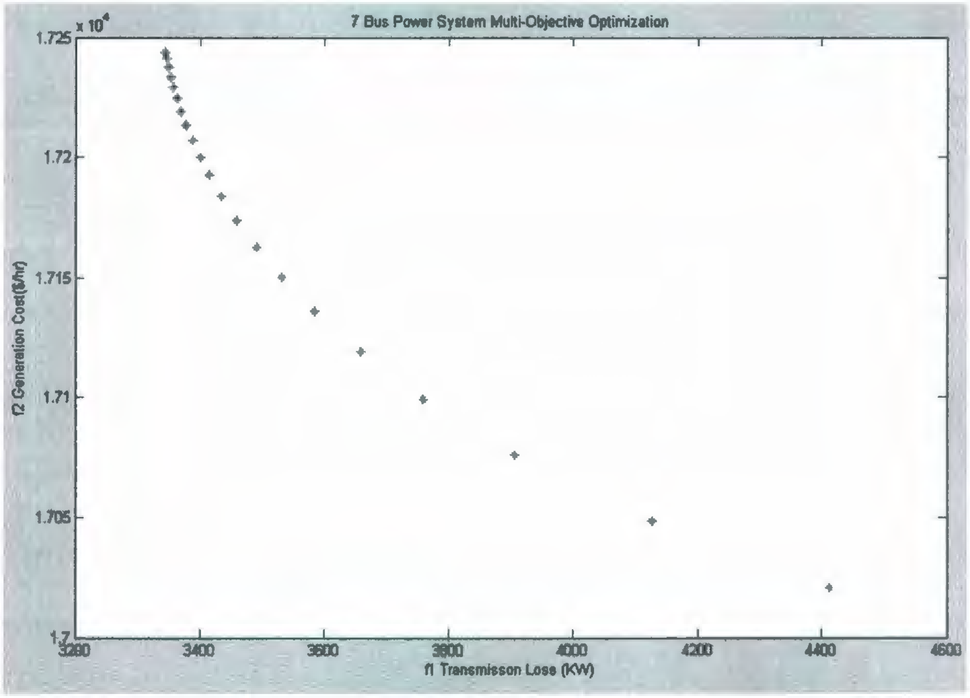

Fig. 6.3. Pareto Front for the 7-bus Power System (secure) [7]

For the OPF study, the maximum cost scheme is 17150 $/hr, and the minimum cost scheme has a cost of 16371 $/hr. The maximum and minimum cost solutions have transmission line losses of 3.29 MW and 10.05 MW respectively. Comparing the results of OPF and SCOPF, it shows that the minimum loss schemes of two studies are similar (0.05 MW difference), and the corresponding fuel costs are similar as well (only $100 difference). The most important feature of the SCOPF is that any solution of SCOPF ensures a secure operation manner when the outages of transmission lines occur. If the fuel cost is of primary concern, the minimum fuel cost obtained from the SCOPF study is significantly higher than that of the OPF study ($ 650 higher). However, the solution of OPF does not ensure the security of the system operation. If the security is of great concern, the solutions of SCOPF may be very attractive and should be preferred. Thus, higher cost is required in order to achieve a secure operation.

6.3 26-Bus Case Study

In this section, a multiobjective optimization case study is performed where the goal of optimization is to minimize the total fuel cost and the total active power transmission losses for specified loads. The cost objective is identical to the cost objective used for the 7-bus case study, but with different cost coefficients and control variables. The objective of total transmission loss is expressed in Eqn. 6.2. OPF and SOPF studies are both discussed in this section. For the SCOPF study, security constraints are required to obtain optimal solutions for the outages of the transmission lines. These two objectives can be formed as a nonlinear optimization problem which can be solved by weighted sum method. The 26-bus power system used for this study is shown in Fig. 6.4 [17]. The system consists of 6 generators, 46 transmission lines/ transformers, 26 loads, and bus 1 is the reference bus. The loads total 947.3 MW and 484.5 MVAR. All system parameters along with the initial load demand and generation schedule are available in Appendix D using an apparent power base of 100 MVA. For the base case, the total fuel cost is 23946

$/hr and the transmission loss is 9.4MW.

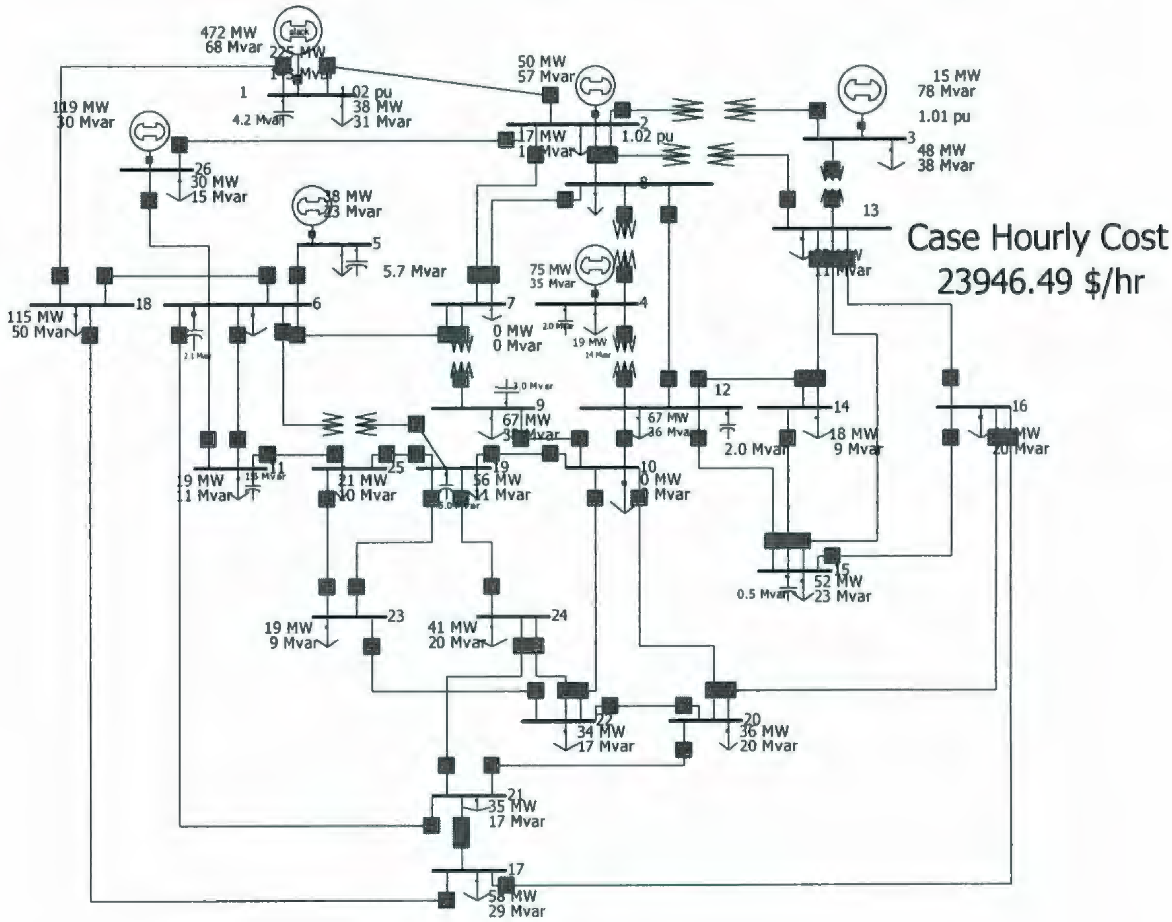

Fig. 6.4 26-bus Power System

6.3.1 OPF for 26-Bus Power System

The goal of multiobjective optimal power flow is to minimize the total fuel cost and the transmission losses while satisfying all the constraints. Security constraints are not included. The multiobjective OPF problem can be expresses as a set of nonlinear functions using Equations 6.1, 6.2, 6.4, and 6.6-6.8. Equations 6.1 and 6.2 are two objective functions: the total fuel cost and the transmission losses. Eqn. 6.4 defines the equality constraints which are the power flow equations at each bus. Eqns. 6.6-6.8 are inequality constraints which set up the boundaries of bus voltage magnitude, generator production and branch flow. The bus voltage magnitude should be restricted within 0.95p.u. and 1.05p.u., and the limits of generator production and transmission line are specified by the manufacture. In total, there are 40 unknown variables, 45 equality constraints and 98 inequality constraints. Table 6.5 lists the number of each parameter used for this case study.

Table 6.5 Multionjective OPF parameter for 26-bus Power System [7]

Control Variable		5
State Variable		35
Equality Constraints (Power Flow Equations)		45
Inequality Constraints	**Transmission line**	46
	Bus Voltage	40
	Active Power Outputs	12

The fuel costs of all the generating units are represented using cubic cost models. This study is computed using weighted sum method. The following table, Table 6.6, summarizes the results of multiobjective optimization without considering contingency constraints. Multiple optimal solutions can be obtained by varying the weight factors corresponding to the importance of objectives. The more weight factors are used, the more optimal solutions can be computed, and the more accurate Pareto front can be plotted. As seen from Tables 6.5 and 6.3, an increased parameter size is used over the case study in section 6.2, since 26-bus power system is large scale and has many more operational constraints. The larger parameter size used in this case study causes the longer processing time. 20 pairs of weight factors are used in this study. Table 6.6 summaries the results of the OPF studies for 26-bus power system without the consideration of security constraints. The multiple optimal solutions provide more options for the operator to make a selection. For example, if the fuel cost is the most important objective and the transmission loss is the secondary importance, the minimum cost solution should be chosen. Similarly, if the system transmission losses is more important than the fuel cost, the minimum loss solution should be considered, but minimum fuel cost can not be ensured. Higher cost is required to reduce the transmission loss. Table 6.6 shows that the maximum fuel cost scheme (22974 $/hr) and the minimum fuel cost scheme (22646 $/hr) solutions have transmission loss of 6.26 MW and 7.25 MW respectively. By additional spending of $328, it is possible to reduce the transmission losses by 13.7%.

Table 6.6 Multiobjective OPF of 26-bus Power System (Without Security Constraints)[7]

Weight Factor of Loss	**Transmission Loss (MW)**	**Weight Factor of Cost**	**Generation Cost ($/hr)**
0	7.25	1	22646
0.2	7.15	0.8	22648
0.4	7.01	0.6	22654
0.6	6.81	0.4	22676
0.8	6.47	0.2	22764
1	6.26	0	22974

Fig. 6.5 shows the optimal solutions set for the 26-bus power system provided by the multiobjective optimization without considering the security constraints. Any point on the Pareto front is an optimal solution, but it does not violate any operating constraints. Regardless of the slightly discrepancy between the last two solutions near the area of the minimum loss, each of the solutions contained in the frontier is non-dominated and well distributed. The Pareto front also shows that minimization of the total fuel cost and transmission line losses conflict with each other. Hence, with more money spent on generation, lower line losses are achieved. The trade off region provides more solutions for the operator to choose an option.

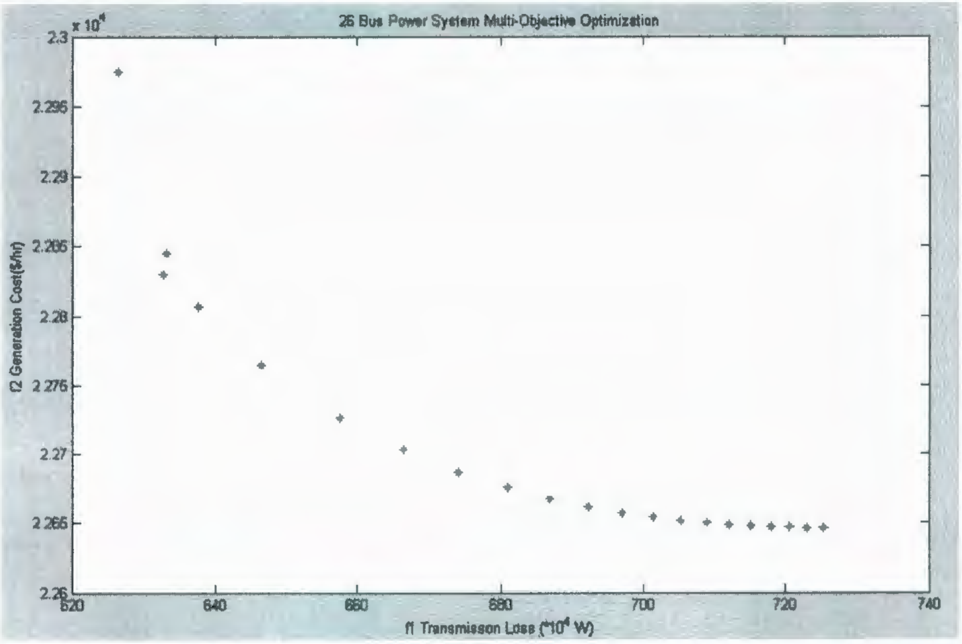

Fig. 6.5. Pareto Front for the 26-bus Power System (without secure) [7]

6.3.2 SCOPF for 26-Bus Power System

The goal of this study is to minimize the total fuel cost and the transmission losses while satisfying all the constraints under normal condition and contingency cases. Thus, security constraints are included. The one line diagram of 26-bus power system is shown in Fig 6.4, and the system information is introduced in Appendix D. To simplify the problem, contingency with respect to the outage of only one transmission line (line 1-18) is considered. Line 1-18 outage is the potential worst-contingency cases, which is verified using PowerWorld Simulator.

Using Eqns. 6.1, 6.2 and 6.4-6.10, The multiobjective SCOPF problem can be expresses as a set of nonlinear functions. Eqns. 6.1 and 6.2 are two objective functions: the total fuel cost and the transmission losses. Eqn. 6.4 defines the equality constraints for normal operation, which is the net power at each bus is zero. Eqns. 6.6-6.8 are inequality constraints under normal condition which enhance the operation limits of bus voltage magnitude, active power output of each dispatchable generator and branch flow. When security constraints are considered, the problem is formulated in such a way that state variables, equality and inequality constraints corresponding to the outage of some transmission lines are included. $v_1', v_2', \cdots, v_m'$, and $\theta_1', \theta_2, \cdots, \theta_m'$ are unknown state variables that are required by the optimization problem for the outage of the transmission line. Eqns. 6.5 represents nonlinear equality constraints, which are power flow equations at each bus for a contingency case. Eqns. 6.9-6.10 define the nonlinear inequality constraints of vector arguments x and u, which are the voltage magnitude restrictions, generation restrictions and branch power flow limits. The post-contingency bus voltage magnitudes and the post-contingency power flow on each transmission line can be restricted using Eqns. 6.9-6.10 for the outage of a line. The size of the state variables and security constraints depends on the number of the contingency case. Only the outage of transmission lines 1-18 is considered, therefore, there are 75 variables, 90 equality constraints and 196 inequality constraints in total. Table 6.7 lists the number of each parameter used for this case study. As seen from Tables 6.5 and 6.7, the total number of parameters for SCOPF is two times the total number for OPF.

Table 6.7 Multionjective SCOPF parameter for 26-bus Power System [7]

<table>
<tr><td colspan="2">Control Variable</td><td>5</td></tr>
<tr><td colspan="2">State Variable</td><td>70</td></tr>
<tr><td colspan="2">Equality Constraints
(Power Flow Equations)</td><td>90</td></tr>
<tr><td rowspan="3">Inequality Constraints</td><td>Transmission line</td><td>92</td></tr>
<tr><td>Bus Voltage</td><td>80</td></tr>
<tr><td>Active Power Outputs</td><td>24</td></tr>
</table>

This SCOPF study is computed using weighted sum method, which is implementated using Matlab Optimization Toolbox. Multiple optimal solutions can be obtained by varying the weight factors corresponding to the importance of objectives. The number of the weight factors determines the number of the optimal solutions can be obtained. Since the larger amount of SCOPF parameters results in longer processing time, 20 pairs of weight factors are used. Hence, 20 pairs of optimal solutions (the fuel cost and the transmission losses) are computed. Table 6.8 presents the results of SCOPF studies when the security constraints are considered. There are many reasons for choosing one solution over another. For example, when the total fuel cost of generation is considered as the most important objective, the minimum cost should be chosen. Similarly, if the system transmission losses is primary importance and the fuel cost is the secondary one, the minimum loss should be chosen, but minimum fuel cost can not be ensured. As seen from Tables 6.6 and 6.8, the cost is higher when security constraints are included. For example, the minimum fuel cost induces 23076 $/hr for SCOPF study while the minimum fuel cost of OPF induces 22974 $/hr. The SCOPF solution is higher than the OPF solution ($102 difference). However, the OPF solution does not ensure the system security when the contingency on line 1-18 occurs. The most important advantage of SCOPF over OPF is that any optimal solution obtained from SCOPF enhances the security of the 26-bus power system when the outage of line 1-18 is happened.

Table 6.8 Multiobjective OPF of 26-bus Power System (Secure) [7]

Weight Factor of Loss	Transmission Loss (MW)	Weight Factor of Cost	Generation Cost ($/hr)
0	6.83	1	22846
0.2	6.79	0.8	22846
0.4	6.75	0.6	22848
0.6	6.62	0.4	22862
0.8	6.39	0.2	22915
1	6.30	0	23076

Fig. 6.6 shows the Pareto front which contains the optimal solutions set for the 26 bus power system provided by the multiobjective optimization with security constraints. Each of the points listed in the frontier is an optimal solution, and it guarantees a secure

power system with respect to the outage of a single transmission line. The power system operator can select a particular solution from these multiple solutions. Regardless of the minor discrepancy near the area of the minimum loss, each of the solutions contained in the frontier is non-dominated and well distributed. The Pareto front shows that the objectives of minimization of the total fuel cost and transmission line losses are conflict with each other. With more money spent on generation, lower line losses are achieved.

It is apparent from the Table 6.4 and Fig. 6.3 that the maximum fuel cost scheme has a cost of 23076 $/hr, and the minimum cost scheme has a cost of 22846 $/hr. The maximum and minimum cost solutions have transmission line losses of 6.3 MW and 6.83 MW respectively. Thus, by allowing an additional spending of $ 230, it is possible to reduce the transmission losses by 7.6% and the system is secure when line 1-18 is out.

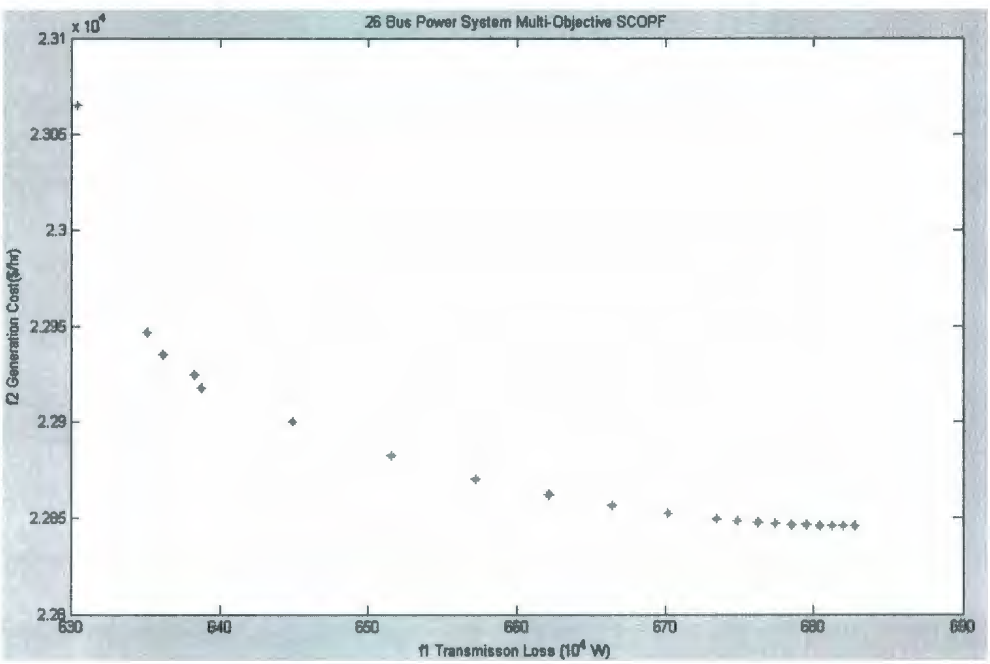

Fig. 6.6. Pareto Front for the 26-bus Power System (secure) [7]

Comparing the results of OPF and SCOPF, it shows that the minimum loss schemes of two studies are similar (0.04 MW difference), and the corresponding fuel costs are similar as well (only $102 difference). For minimum cost scheme, the SCOPF solution has a cost of $22846 which is $200 higher than the result of OPF. However, the SCOPF provides lower transmission loss which is 0.42 MW less than the loss from the OPF study. In addition, the SCOPF results ensure a secure operation even if the outage of line 1-18 is happened. Thus, with extra $200 spending, a more secure and less transmission loss operation can be achieved, and the solutions of SCOPF may be very attractive and should be preferred.

6.4 Conclusions

This chapter has presented multiobjective optimization for power system including security constraints. OPF and SCOPF case studies have been performed on a 7-bus power system and a 26-bus power system, and the objectives are the minimization the total fuel cost and the minimization of the transmission losses. For OPF and SCOPF studies, the weighted sum approach has successfully computed the feasible non-dominated solution set. The Pareto front contains all the optimal solutions, and they are well distributed over the frontier. The results also showed that the weighted sum method correctly identified the trade off region between fuel cost and transmission line losses. The power system operator can select a particular solution from these multiple solutions. One of the major advantages of the SCOPF is that any solution of SCOPF ensures a secure operation manner. Thus, the system will not be violated even if the outages of transmission lines occurs. The solutions of OPF may have less cost or less losses, but they cannot guarantee a secure operation under contingency cases. Hence, if the security is of great concern, the solutions of SCOPF may be very attractive and should be preferred. The Pareto front obtained from all case studies show that the objectives of minimization of the total fuel cost and transmission line losses are conflict with each other. With more money spent on generation, lower line losses and more security of the system can be achieved. Furthermore, for larger power systems, when security constraints are included, the number of variables in the optimization problem will increase significantly, and the processing time of the weight sum approach also increases.

Chapter 7

Conclusions and Future Work

In this thesis, optimization has been shown as one of the challenging problems in power system operation as its formulation is single objective or multiobjective, nonlinear, highly constrained and of large scale. Many optimization methods have been researched, and the methods proposed in this thesis have been very successful in achieving the goal of obtaining economic, reliable and secure operation scheme. Minimization of the operation cost (fuel cost) and minimization of transmission losses while ensuring a secure system is a primary problem for power system planning and operation. Combining multiobjective optimization methods with optimal power flow, the generators' power output can be varied within certain limits to support a specified load demand, and the above objective can be achieved. The multiobjective optimization is achieved by minimizing the objective functions subject to the constraints under normal condition and contingency state. Only the outages of transmission lines are considered as contingencies in the optimal power flow study. Weighted sum, a classical multiobjective optimization method, is implemented by MatLab Optimization Toolbox and is employed to solve all the multiobjective optimization problems.

Case Studies have been presented throughout the thesis to illustrate the performance of the constrained nonlinear optimization method (single objective and multiple objectives) for solving power system optimization problems. This thesis has considered three key problems for power systems: economy, transmission loss and security. The results of the work presented in the thesis show that the proposed approaches have the potential to benefit electric power utilities.

7.1 Summary of the Research and Contribution of the Thesis

The main contributions of this thesis can be summarized as follows:

1. Conventional optimization methods including their application to power system optimization problems is investigated. A case study is presented to show the algorithm of the proposed Sequential Quadratic Programming method.
2. Optimal power flow problems including economic dispatch and security constraints optimal power flow are studied for different objectives. B-coefficients are used to express the total transmission loss to solve the economic dispatch problem.
3. The concept of the Pareto front that highlighted important implications of multiobjective optimization are discussed. Two typical multiobjective optimization problems are presented to show the performance of the proposed Weighted Sum method.
4. The application of the multiobjective optimization to two power system case studies including security constraints is presented. The benefit of optimizing the

power system with security constraints is discussed.

5. A program is developed using Matlab software tool based on the weighted sum method for handling multiobjective optimization problems.
6. Two technical papers [3,7] related to the application of optimization methodologies to power system optimal power flow are published.

7.2 Recommendations for Future Work

Two areas are proposed for future research:

1. An investigation of alternate strategies for solving multiobjective optimization problems
2. A further research for reactive power planning and voltage profile improvement.

When considering security constraints, the problem is formulated in such a way that the equality and inequality constraints corresponding to the outage of some transmission lines are included. Since the number of variables has increased, the gradient-based algorithm takes a lot of time to converge. Furthermore, the proposed multiobjective optimization method, weighted sum method, cannot find certain Pareto-optimal solutions in the case of a nonconvex objective space. This leads one to investigate alternate strategies for multiobjective optimization. A recently proposed method called "multiobjective particle swarm optimization" (MOPSP) is a strong candidate to meet these challenges. Further research is required to explore this and investigate its suitability for multiobjective power system optimization problems including security constraints [7].

Under normal operating conditions, the system transmission loss is considered for reactive power dispatch and for improving the voltage profile. Moreover, this will result in a reduction in the cost of installing extra equipment for reactive power generation and voltage adjustment. A possible area of research is to investigate of multiobjective optimization problem for the reactive power planning and voltage profile improvement.

References

[1] J. D. Glover, M. S. Sarma, *Power System Analysis and Design*, Brooks/Cole, Pacific Grove, CA, 2002.

[2] D. P. Kothari, J. S. Dhillon, *Power System Optimization*, Prentice-Hall of India Private Limited, New Delhi, 2004.

[3] J. Kong, B. Jeyasurya, "Constrained Optimization Method for Power System Economic Dispatch", *18th Annual Newfoundland Electrical and Computer Engineering Conference,* November 2008, St.John's Newfoundland.

[4] C. Shu, *Application of Optimization Methods for Power System Economic Operation and Transfer Capability Evaluation*, M. Eng. Thesis, 2003, Memorial University of Newfoundland, St. John's Newfoundland.

[5] D. I. Sun, B. Ashley, A. Hughes, W. F. Tinney, "Optimal Power Flow by Newton Approach", *IEEE Transactions on Power Apparatus and Systems,* Vol. PAS-103, No.10, October 1984, pp. 2864-2880.

[6] W. Qiu, A. J. Flueck, F. Tu, " A New Parallel Algorithm for Security Constrained Optimal Powre Flow With a Nonlinear Interior Power Method", *Power Engineering Society General Meeting,* Vol. 1, June, 2005, pp. 447-453.

[7] J. Kong, B. Jeyasurya, "Multiobjective Power System Optimization Including Security Constratins", Accepted for presentation at *the 15th Annual Conference on Intelligent System Applications to Power Systems*, November 2009, Brazil.

[8] S. S. Rao, *Engineering Optimization Theory and Practice,* John Wiley & Sons, Toronto, ON, 1996.

[9] R. Baldick, *Applied Optimization Formulation and Algorithms for Engineering Systems*, Cambridge University Press, Cambridge, UK, 2006.

[10] A. D. Belegundu, T. R. Chandrupatla, *Optimization Concepts and Applications in Engineering*, Prentice-Hall, Upper Saddle River, New Jersey, 1999.

[11] S. G. Nash and A. Sofer, *Linear and Nonlinear Programming*, McGraw-Hill, New York, NY, 1996.

[12] G. V. Reklaitis, A. Racindran, K. M. Ragsdell, *Engineering Optimization Methods and Applications*, John Wiley & Sons, Toronto ON, 1983.

[13] MATLAB, version 7.4, Mathworks, Inc., USA, 2007.

[14]R. C. Bansal, "Optimization Methods for Electric Power System: An Overview", *International Journal of Energing Electric Power Systems*, Vol. 2, Issue 1, 2005.

[15]A. J. Wood, B. F. Wollenberg, *Power Generation Operation and Control*, John Wiley & Sons, Toronto, ON, 1984

[16]L. L. Lai, *Intelligent System Applications in Power Engineering: Evolutionary Programming and Neural Networks*. John Wiley & Sons, Chichester, England,1998.

[17]H. Saadat, *Power System Analysis*, McGraw-Hill, Toronto, ON, 1999.

[18]A. R. Bergen, V. Vittal, *Power System Analysis*, Prentice-hall, Upper Saddle River, New Jersey, 2000.

[19]S. Panta, S. Premrudeepreechacharn, S. Nuchpraynoon, C. Dechthummarong, S. Janjornmanit, S. Yachiangkam, "Optimal Economic Dispatch for Power Generation Using Artificial Neural Network", *IEEE Power Engineering Conference,* 2007, Dec. 2007, pp. 1343-1348.

[20] K. Bhattacharya, M. H. J. Bollen, J. E. Daalder, *Operation of Restructured Power Systems, Kluwer Academic Publishers*, Kluwer Academic Publishers Group, Norwell, Boston, US, 2001.

[21]C. L. Chen, N. M. Chen, "Direct Search Method for Solving Economic Dispatch Problem Considering Transmission Capacity Constraints", *IEEE Transactions on Power Systems*, Vol.16, No.4, November 2001, pp. 764-769.

[22]Y. Wallach, *Calculations and Porgrams for Power System Networks*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1986.

[23]A. B. M. Nasiruzzaman, M. G Rabbani, "Implementation of Genetic Algorithm and Fuzzy Logic in Economic Dispatch Problem", *International Conference on Electrical and Computer Engineering*, December, 2008, pp 360-365.

[24]IEEE 39 Bus Power System, [online] available: http://psdyn.ece.wisc.edu/IEEE_benchmarks/menu1.htm.

[25]J. A. Momoh, *Electric Power System Applications of Optimization*, Marcel Dekker, New York, NY, 2001.

[26]Y. Xia and K. W. Chan, "Dynamic constrained optimal power flow using semi-infinite programming," *IEEE Transaction on Power System*, vol. 21, no. 3, August 2006, pp. 1455–1457.

[27]Y. Zhang and Z. Ren, "Optimal reactive power dispatch considering costs of

adjusting the control devices," *IEEE Transaction on Power System*, vol. 20, no. 3, Aug. 2005, pp. 1349–1356.

[28] A. M. Azmy, "Optimal Power Flow to Manage Voltage Profiles in Interconnected Networks Using Expert Systems", *IEEE Transaction on Power System*, vol. 22, no. 4, Aug. 2007, pp. 1622–1626.

[29] Power World Simulator Version 12. Power World Corporation, USA.

[30] Y. H. Song, *Modern Optimisation Techniques in Power Systems*, Kluwer Academic Publishers, Norwell, MA, 1999.

[31] J. C. Das, *Power System Analysis Short Circuit Load Flow and Harmonics*, Marcel Dekker, New York, NY, 2002

[32] S. Small, *Multiple Objective Reactive Power Planning Using Genetic Algorithms*, M. Eng. Thesis, 2007, Memorial University of Newfoundland, St. John's Newfoundland.

[33] R. C. Nurchett, H. H. Happ, K. A. Wirgau, "Large Scale Optimal Power Flow", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-101, No. 10, October 1982, pp. 3722-3732.

[34] Z. Li, Z. Duan, Y. Li, "Comparison and Classification of Objective-Converting Methods in Multi-objective Reactive Power Optimization Using Contour Lines", IEEE Power and Energy Society General Meeting, July, 2008, pp 1-8.

[35] Y. Arslanoglu, *Genetic Algorithm for Personnel Assignment Problem with Multiple Objectives*, M. Eng., The Graduate School of Natural and Applied Sciences of Middle East Technical University, Turkey. [online] available: http://delta.cs.cin vestav. mx/~ccoello/EMOO/masterstheses.html

[36] C. A. Coello Coello, G. B. Lamont, D. A. Van Veldhuizen, *Evolutionary Algorithms for Solving Multi-Objective Problems*, Springer, USA, 2007.

[37] K. Deb, *Multi-Objective Optimization Using Evolutionary Algorithms*, John Wiley & Sons Ltd., Chichester, England, 2001.

[38] A. Berizzi, C. Bovo, M. Innorta, P. Marannino, "Multiobjective Optimization Techniques Applied to Modern Power Systems", *IEEE Power Engineering Society Winter Meeting*, Vol. 3, January 2001, pp. 1503-1508.

[39] A. Osyczka, *Multicriterion Optimization in Engineering with FORTRAN programs*. Ellis Horwood Ltd., 1984.

[40]The MathWorks, Inc. "Genetic Algorithm and Direct Search Toolbox", [online] available:http://www.mathworks.com/products/gads/demos.html?file=/products/demos/shipping/gads/gamultiobjfitness.html

[41]I. Y. Kim, O. L. de Weck, "Adaptive Weighted-Sum method for Bi-objective Optimization: Pareto Front Generation", *Structural and Multidisciplinary Optimization*, vol.31, no.2, 2006, pp.105-116.

[42]C. L. Wadhwa, N. K. Jain, "Multiple Objective Optimal Load Flow: a New Perspective", *IEEE Proceeding,* Vol. 137, No. 1, January 1990, pp 13-18.

[43]U. Nangia, N. K. Jain, V. L. Wadhwa, " Optimal Weight Assessment Based on a Range of Objectives in a Multiobjective Optimal Load Flow Study", *IEE Proc. –Gener. Transm. Distrib.*, Vol. 145, No.1, January, 1998, pp 65-69.

[44]M. A. Abido, "Multiobjective Evolutionary Algorithms for Electric Power Dispatch Problem", *IEEE Transactions on Evolutionary Computation*, Vol. 10, No.3 June, 2008, pp 315-329.

Appendix A: 5-Bus Power System Data

Appendix A contains the information about the 5-Bus Power system [20] discussed in the thesis. The one line diagram is shown in Fig. A.1. The line characteristics, generations, loads and generation fuel cost coefficients are presented in tables A.1, A.2, A.3 and A.4 respectively.

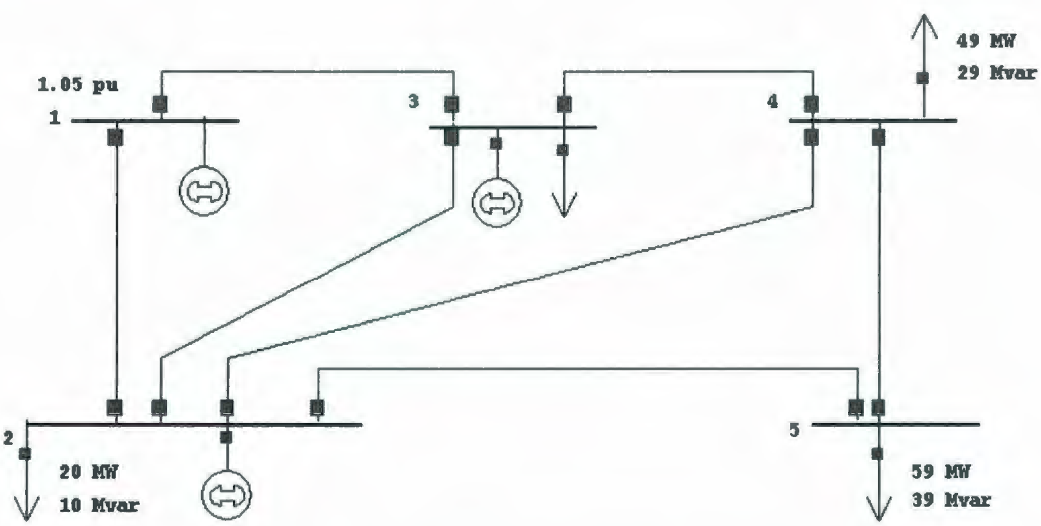

Fig. A.1 One Line Diagram of the 5-Bus Power System

Table A.1: Line Characteristics for 5-Bus Power System (100MVA base)

Line No.	From Bus	To Bus	Resistance (p.u)	Reactance (p.u)	Line Charging (p.u)	Line Limit (MVA)
1	1	2	0.02	0.06	0.06	150
2	1	3	0.08	0.24	0.05	150
3	2	3	0.06	0.18	0.04	120
4	2	4	0.06	0.18	0.04	100
5	2	5	0.04	0.12	0.03	200
6	3	4	0.01	0.03	0.02	222
7	4	5	0.08	0.24	0.05	60

Table A. 2 Generation Schedule and Generator Limits for 5-Bus Power System

Bus	Real Power Generation (MW)	Maximum Real Power Generation (MW)	Minimum Real Power Generator (MW)
1	58.79	400	-9900
2	120	500	-9900
3	60	1000	-9900

Table A. 3 Load Demand for 5-Bus Power System

Bus	Real Power Load (MW)	Reactive Power Load (MVAR)
2	19.6	9.8
3	19.6	4.7
4	49	29.4
5	58.8	39.2

Table A.4 Generator Fuel Cost Coefficients for 5-Bus Power System

Bus	α	β	γ
1	373.5	10	0.016
2	403.6	8	0.018
3	253.2	12	0.018

Appendix B: IEEE 39-Bus Power System Data

Appendix B gives the information about the IEEE 39-Bus Power system [25] discussed in the thesis. The one line diagram is shown in Fig. B.1. The line characteristics, generations, loads and generation fuel cost coefficients are presented in tables B.1, B.2, B.3 and B.4 respectively.

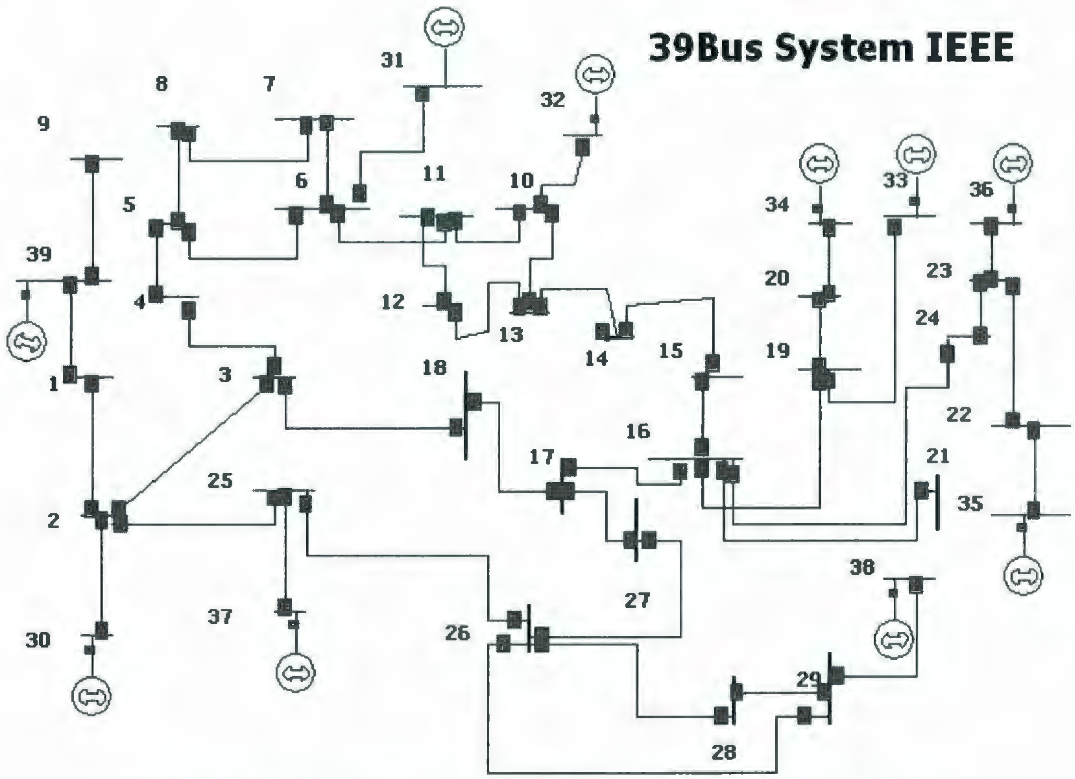

Fig. B.1 One Line Diagram of the IEEE 39-Bus Power System

Table B.1: Line Characteristics for the IEEE 39-Bus Power System (100MVA base)

Line No.	From Bus	To Bus	Resistance (p.u)	Reactance (p.u)	Line Charging (p.u)	Line Limit (MVA)
1	2	1	0.0035	0.0411	0.6987	600
2	39	1	0.001	0.025	0.75	1000
3	3	2	0.0013	0.0151	0.2572	500
4	25	2	0.007	0.0086	0.146	500
5	2	30	0	0.0181	0	500
6	4	3	0.0013	0.0213	0.2214	500
7	18	3	0.0011	0.0133	0.2138	500
8	5	4	0.0008	0.0128	0.1342	600
9	14	4	0.0008	0.0129	0.1382	500
10	6	5	0.0002	0.0026	0.0434	1200
11	8	5	0.0008	0.0112	0.1476	900
12	7	6	0.0006	0.0092	0.113	900
13	11	6	0.0007	0.0082	0.1389	480
14	6	31	0	0.025	0	2500
15	8	7	0.0004	0.0046	0.078	900
16	9	8	0.0023	0.0363	0.3804	900
17	39	9	0.001	0.025	1.2	900
18	11	10	0.0004	0.0043	0.0729	600
19	13	10	0.0004	0.0043	0.0729	600
20	10	32	0	0.02	0	2500
21	12	11	0.0016	0.0435	0	500
22	12	13	0.0016	0.0435	0	500
23	14	13	0.0009	0.0101	0.1723	600
24	15	14	0.0018	0.0217	0.366	600
25	16	15	0.0009	0.0094	0.171	600
26	17	16	0.0007	0.0089	0.1342	600
27	19	16	0.0016	0.0195	0.304	2500
28	21	16	0.0008	0.0135	0.2548	600
29	24	16	0.0003	0.0059	0.068	600
30	18	17	0.0007	0.0082	0.1319	600
31	27	17	0.0013	0.0173	0.3216	600
32	19	20	0.0007	0.0138	0	2500
33	19	33	0.0007	0.0142	0	2500
34	20	34	0.0009	0.018	0	2500
35	22	21	0.0008	0.014	0.2565	900
36	23	22	0.0006	0.0096	0.1846	600
37	22	35	0	0.0143	0	2500
38	24	23	0.0022	0.035	0.361	600
39	23	36	0.0005	0.0272	0	2500

40	26	25	0.0032	0.0323	0.513	600
41	25	37	0.0006	0.0232	0	0
42	27	26	0.0014	0.0147	0.2396	600
43	28	26	0.0043	0.0474	0.7802	600
44	29	26	0.0057	0.0625	1.029	600
45	29	28	0.0014	0.0151	0.249	600
46	29	38	0.0008	0.0156	0	2500

Table B. 2 Generation Schedule and Generator Limits for the IEEE 39-Bus Power System

Bus	Real Power Generation (MW)	Maximum Real Power Generation (MW)	Minimum Real Power Generator (MW)
30	340	350	50
31	658.8	650	50
32	735.4	800	50
33	540	750	50
34	600	650	50
35	670	750	50
36	550	750	50
37	600	750	50
38	8903	900	50
39	1052.2	1200	50

Table B. 3 Load Demand for the IEEE 39-Bus Power System

Bus	Real Power Load (MW)	Reactive Power Load (MVAR)
3	322	2.4
4	500	184
7	233.8	84
8	522	176
12	8.5	88
15	320	153
16	329	32.3
18	158	30
20	680	103
21	274	115
23	247.5	84.6
24	308.6	-92.2

25	224	47.2
26	139	17
27	281	75.5
28	206	27.6
29	283.5	26.9
31	9.2	4.6
39	1104	250

Table B.4 Generator Fuel Cost Coefficients for the IEEE 39-Bus Power System

Bus	α	β	γ
30	0	6.9	0.0193
31	0	3.7	0.0111
32	0	2.8	0.0104
33	0	4.7	0.0088
34	0	2.8	0.0128
35	0	3.7	0.0094
36	0	4.8	0.0099
37	0	3.6	0.0113
38	0	3.7	0.0071
39	0	3.9	0.0064

Appendix C: 7-Bus Power System Data

Appendix C contains the information about the 7-Bus Power system [29] discussed in the thesis. The one line diagram is shown in Fig. C.1. The line characteristics, generations, loads and generation fuel cost coefficients are presented in tables C.1, C.2, C.3 and C.4 respectively.

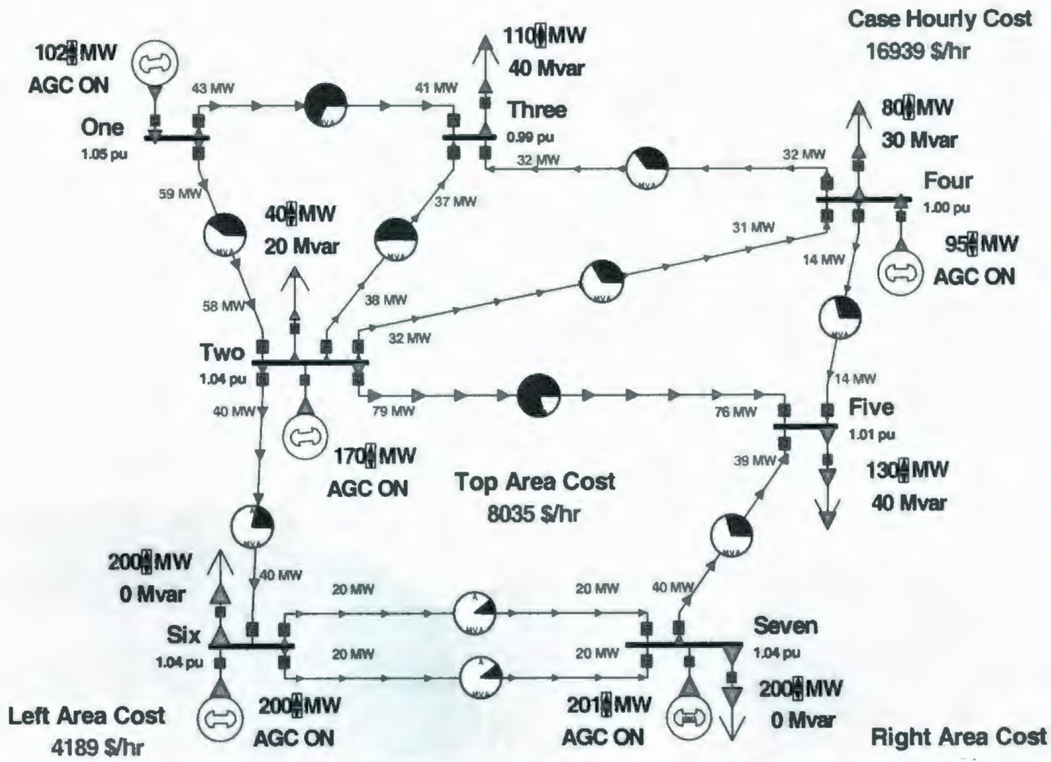

Fig. C.1 One Line Diagram of the 7-Bus Power System

Table C.1: Line Characteristics for 7-Bus Power System (100MVA base)

Line No.	From Bus	To Bus	Resistance (p.u)	Reactance (p.u)	Line Charging (p.u)	Line Limit (MVA)
1	1	2	0.01	0.06	0.06	150
2	1	3	0.04	0.24	0.05	165
3	2	3	0.03	0.18	0.04	80
4	2	4	0.03	0.18	0.04	100
5	2	5	0.02	0.12	0.03	130
6	2	6	0.01	0.06	0.05	200
7	3	4	0.005	0.03	0.02	100
8	4	5	0.04	0.24	0.05	30
9	7	5	0.01	0.06	0.04	200
10	6	7	0.04	0.24	0.05	200
11	6	7	0.04	0.24	0.05	200

Table C. 2 Generation Schedule and Generator Limits for 7-Bus Power System

Bus	Real Power Generation (MW)	Maximum Real Power Generation (MW)	Minimum Real Power Generator (MW)
1	102	400	100
2	170	500	150
4	95	200	50
6	200	500	150
7	201	600	0

Table C. 3 Load Demand for 7-Bus Power System

Bus	Real Power Load (MW)	Reactive Power Load (MVAR)
2	40	20
3	110	40
4	80	30
5	130	40
6	200	0
7	200	0

Table C.4 Generator Fuel Cost Coefficients for 7-Bus Power System

Bus	α	β	γ
1	373.5	7.62	0.002
2	403.61	7.52	0.0014
4	253.24	7.84	0.0013
6	388.93	7.57	0.0013
7	194.28	7.77	0.0019

Appendix D: 26-Bus Power System Data

Appendix D gives the information about the 26-bus Power system [17] discussed in the thesis. The one line diagram is shown in Fig. D.1. The line characteristics, generations, loads and generation fuel cost coefficients are presented in tables D.1, D.2, D.3 and D.4 respectively.

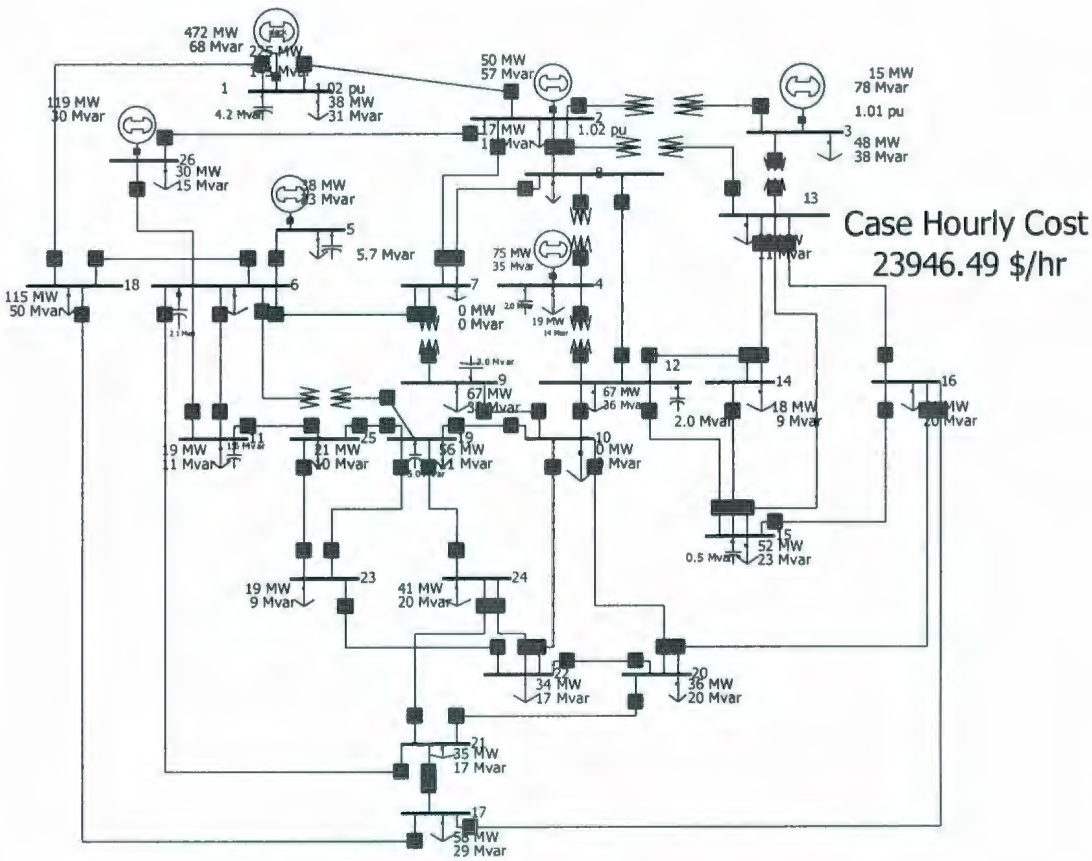

Fig. D.1 One Line Diagram of the 26-Bus Power System

Table D.1: Line Characteristics for the 26-Bus Power System (100MVA base)

Line No.	From Bus	To Bus	Resistance (p.u)	Reactance (p.u)	Line Charging (p.u)	Line Limit (MVA)
1	1	2	0.00055	0.0048	0.06	250
2	1	18	0.0013	0.0115	0.12	410
3	2	3	0.00146	0.0513	0.1	170
4	2	7	0.0103	0.0586	0.036	110
5	2	8	0.0074	0.0321	0.078	175
6	2	13	0.00357	0.0967	0.05	100
7	2	26	0.0323	0.1967	0	100
8	3	13	0.0007	0.00548	0.001	265
9	4	8	0.0008	0.024	0.0002	110
10	4	12	0.0016	0.0207	0.03	140
11	5	6	0.0069	0.03	0.198	300
12	6	7	0.00535	0.0306	0.0021	150
13	6	11	0.0097	0.057	0.0002	50
14	6	18	0.00374	0.0222	0.0024	175
15	6	19	0.0035	0.066	0.09	116
16	6	21	0.005	0.09	0.0452	100
17	8	7	0.0012	0.00693	0.0002	100
18	7	9	0.00095	0.0429	0.05	116
19	8	12	0.002	0.018	0.04	160
20	9	10	0.00104	0.0493	0.002	80
21	12	10	0.00247	0.0132	0.02	175
22	10	19	0.0547	0.236	0	50
23	10	20	0.0066	0.016	0.002	80
24	10	22	0.0069	0.0298	0.01	85
25	11	25	0.096	0.27	0.02	50
26	11	26	0.0165	0.097	0.008	200
27	12	14	0.0327	0.0802	0	100
28	12	15	0.018	0.0598	0	50
29	13	14	0.0046	0.0271	0.002	105
30	13	15	0.0116	0.061	0	100
31	13	16	0.01793	0.0888	0.002	75
32	14	15	0.0069	0.0382	0	100
33	15	16	0.0209	0.0512	0	100
34	16	17	0.099	0.06	0	100
35	16	20	0.0239	0.0585	0	100
36	18	17	0.0032	0.06	0.076	111.4
37	21	17	0.229	0.445	0	50
38	19	23	0.03	0.131	0	50

39	19	24	0.03	0.125	0.004	50
40	25	19	0.119	0.2249	0.008	50
41	20	21	0.0657	0.157	0	50
42	22	20	0.015	0.0366	0	100
43	21	24	0.0476	0.151	0	100
44	23	22	0.029	0.099	0	100
45	24	22	0.031	0.088	0	100
46	25	23	0.0987	0.1168	0	50

Table D. 2 Generation Schedule and Generator Limits for the 26-Bus Power System

Bus	Real Power Generation (MW)	Maximum Real Power Generation (MW)	Minimum Real Power Generator (MW)
1	472.44	500	100
2	50	200	50
3	15	300	80
4	75	150	50
5	225	200	50
26	119.23	120	50

Table D. 3 Load Demand for the 26-Bus Power System

Bus	Real Power Load (MW)	Reactive Power Load (MVAR)
1	38.25	30.75
2	16.5	11.25
3	48	37.5
4	18.75	14.25
5	37.5	22.5
6	57	21.75
7	0	0
8	0	0
9	66.75	37.5
10	0	0
11	18,75	11.25
12	66.75	36
13	23.25	11.25
14	18	9
15	52.5	23.25

16	41.25	20.25
17	58.5	28.5
18	114.75	50.25
19	56.25	11.25
20	36	20.25
21	34.5	17.25
22	33.75	16.5
23	18.75	9
24	40.5	20.25
25	21	9.75
26	30	15

Table D.4 Generator Fuel Cost Coefficients for the 26-Bus Power System

Bus	α	β	γ
1	240	7	0.007
2	200	10	0.0095
3	220	8.5	0.009
4	200	11	0.009
5	220	10.5	0.008
26	190	12	0.0075