A Study of Bias in the Naive Estimator in Longitudinal Linear Mixed-effects Models with Measurement Error and Misclassification in Covariates

by

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Abstract

This research presents a generalized least square approach to estimate the parameters in a longitudinal linear mixed-effects model. In this model, we consider measurement error and misclassification in the covariates. Moreover, a classical measurement error for continuous covariates, and misclassification for discrete covariates up to three categories, is considered. Through simulation studies, we observe the impact of each parameter of the model on the bias of the naive estimation, when the other parameters stay unchanged.

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Dedication

I dedicate my dissertation work to my family who love me unconditionally.

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Chapter 1

Introduction

In longitudinal studies, responses are determined by multiple factors (covariates) collected over time. (e.g. Laird and Ware (1982), Pinheiro (2005), and Verbeke (2009)). These models are widely applied in different areas of research such as biostatistics, epidemiology, and statistical genetics (Henderson, Kempthorne, Searle, and von Krosigk (1959), McLean, Sanders, and Stroup (1991), Page and Magnus. (2012), Abarin, Wu, Warrington, Pennell, and Briollais (2012), and many more). In this research, we consider a longitudinal linear mixed-effects model. The model contains a repeatedly measured response y_{it}, continuous predictors X_{it} subject to measurement error, and classified predictor G_i subject to misclassification. We consider the model error term ϵ_{it} with an autoregressive model with lag one (AR(1)) to generate the correlation between the time points. Autoregressive models with lag one are widely applied in science (e.g. Zhang, Zhang, Young, and Li (2014), Dakos, Carpenter, Brock, Ellison, Guttal, Ives, ... & Scheffer (2012)). We also consider a time independent random effect γ_i for a specific individual. As both X and G are random variables, in the first chapter, we calculated the marginal moments of the response in order to obtain a closed-form for the parameter estimates using Generalized Least Square (GLS) estimation (Nelder and Wedderburn (1972)).

There are many studies using longitudinal data that assume that all covariates are measured accurately (e.g. Parsons, et al (2001), Yarkiner, Hunter, O'Neil, and De Lusignan (2013)). In practice, however, there are occurrences when some variables in the model of interest cannot be observed exactly, usually due to instrument or sampling error. It is well-known that measurement error (ME) and/or misclassification have negative impacts on the parameter estimation (Fuller (1986), Carroll and Stefanski (2006), Abarin, Li, Wang, and Briollais (2012), Abarin and Wang (2012)). Naive estimators that ignore the errors in covariates are typically inconsistent. However, the direction and the magnitude of the bias can be quite complex as the naive estimates are functions of the unknown model parameters. Depending on its magnitude and nature, in some cases the bias can be ignored (Wang et al (1998), Buzas, Stefanski, and Tosteson. (2014)), reduced (Eisenhower, Mathiowetz, & Morganstein, (1991), Cheng, Branscum, & Stamey (2010)), or corrected (Batistatou, and McNamee, (2012), Wang, et al (1999), Spiegelman, McDermott, and Rosner (1997)).

In our model, we consider ME with a random error satisfying a classical ME model. The marginal moments of the response in terms of observed covariates are calculated. It is, however, extremely difficult to assess the bias in the naive estimator as a function of the response covariance matrix. In this research, it is assumed that the covariance matrix of the response is known. However, actually, this matrix is itself a function of the unknown model parameters. We therefore evaluate the bias caused by the ME through simulation studies in Chapter 2. Simulation studies help us to observe the change in the bias of the naive estimator when we change one model parameter while keeping the others unchanged.

We also consider misclassification in the categorical predictor, where in Chapter 3, the true covariate G is subject to error. The binary case with two categories of "success" and "failure" is very common in application, such as in gender classification or smoking status. Sensitivity and specificity play a very important role in misclassification studies. They are statistical measures of performance of a binary classification, also known in statistics as classification function. Sensitivity, which is also called the true positive rate, or the recall rate in some fields of research, measures the proportion of actual positives which are correctly identified, such as the percentage of sick people who are correctly identified as having the condition. Specificity, which is sometimes called the true negative rate, measures the proportion of negatives which are correctly identified, such as the percentage of healthy people who are correctly identified as not having the condition. These two measures are closely related to the concepts of type I and type II errors. A perfect predictor would be described as 100 percent sensitive, which is predicting all people from the sick group as sick, and 100 percent specific, which is not predicting anyone from the healthy group as sick; however, theoretically any predictor will possess a minimum error bound known as the Bayes error rate (Fawcelt, Tom (2006)). Since in practice classification with more than two categories are often applied, we extend our assessment to the case with three categories. Comparing to the binary case, in this case there are four more misclassification cases. We assessed the bias in the naive estimates by modifying the misclassification rates in the trinomial case.

In Chapter 4, where we combine ME and misclassification, we calculate the marginal moments of the response in terms of the observed covariates. Through extensive simulation studies, we assess the impact of ME on the bias in the naive estimator in the presence of misclassification, and vice versa.

This dissertation is organized as follows. Chapter 1 describes the standard linear longitudinal mixed-effects model and the assumptions required for the model. Chapter 2 presents the model with ME only. Simulation studies are conducted to evaluate the bias in the naive estimators. Chapter 3 presents the model with misclassification only, which includes two separate parts; the binary case and the case with three categories. Simulation studies in this chapter are also conducted for these two different cases. Finally, we consider the model with both ME and misclassification in Chapter 4, where through simulation studies, we assess the performance of the naive estimators, comparing with the other chapters. The last chapter draws a conclusion about the bias in the naive estimators.

Chapter 2

Longitudinal Linear Mixed-effects Model

In this chapter, we consider the following longitudinal linear mixed-effects model

$$y_{it} = X_{it}'\beta + \gamma_i + G_i\alpha + \epsilon_{it}, \quad i = 1, \cdots, k, \quad t = 1, \cdots, T, \tag{2.1}$$

where $y_{it} \in I\!R$ is the response at time point t for the ith individual, $\gamma \in I\!R$ is the individual random effect with mean zero and variance σ_γ^2, and $\alpha \in I\!R$ is the coefficient of the classification variable. Moreover, $X_{it} \in I\!R^p$ is the random continuous predictor with coefficient β, independent of $G_i \in I\!R$, which is the random classified predictor.

In model (2.1), ϵ_{it} is an error term that follows an AR(1), such that $\epsilon_{it} = \rho\epsilon_{i,t-1} + a_{it}$ and $|\rho| < 1$. In this autoregressive model, a_{it} is a random error term with mean zero and variance σ_a^2, independent of γ_i. The model error term ϵ_{it} can, therefore, be expressed as $\epsilon_{it} = \sum_{t=0}^{\infty} \rho^t a_{it}$. It is then straightforward to conclude that

$$E(\epsilon_{it}|X_{it}, G_i) = 0, \quad var(\epsilon_{it}|X_{it}, G_i) = \frac{\sigma_a^2}{1-\rho^2}, \;\; cov(\epsilon_{it}, \epsilon_{iu}|X_{it}, G_i) = \frac{\sigma_a^2 \rho^{|t-u|}}{1-\rho^2}.$$

Now, using the above model specifications, we write the marginal mean, variance, and covariance of the response as follows.

$$E(y_{it}|X_{it,}G_i) = E_\gamma(E(y_{it}|X_{it}, G_i, \gamma_i)) = X_{it}'\beta + G_i\alpha \tag{2.2}$$

$$\begin{aligned}
& var(y_{it}|X_{it}, G_i) \\
= \;& var_\gamma(E(y_{it}|X_{it}, G_i, \gamma_i)) + E_\gamma(var(y_{it}|X_{it}, G_i, \gamma_i)) \\
= \;& var_\gamma(X_{it}'\beta + G_i\alpha + \gamma_i|X_{it}, G_i) + E_\gamma(\frac{\sigma_a^2}{1-\rho^2}|X_{it}, G_i) \\
= \;& \sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}.
\end{aligned} \tag{2.3}$$

When $t \neq u$,

$$\begin{aligned}
& cov(y_{it}, y_{iu}|X_{it}, G_i) \\
= \;& cov_\gamma(E(y_{it}|X_{it}, G_i, \gamma_i), E(y_{iu}|X_{iu}, G_i, \gamma_i)) + E_\gamma(cov(y_{it}, y_{iu}|X_{it}, X_{iu}, G_i, \gamma_i)) \\
= \;& cov_\gamma((X_{it}'\beta + G_i\alpha + \gamma_i), (X_{iu}'\beta + G_i\alpha + \gamma_i)|X_{it}, X_{iu}, G_i) + E_\gamma(cov(y_{it}, y_{iu}|X_{it}, X_{iu}, G_i, \gamma_i)) \\
= \;& var(\gamma_i) + E_\gamma(\frac{\sigma_a^2\rho^{|t-u|}}{1-\rho^2}) \\
= \;& \sigma_\gamma^2 + \frac{\sigma_a^2\rho^{|t-u|}}{1-\rho^2}
\end{aligned} \tag{2.4}$$

The Generalized Least Square (GLS) estimate of $\theta = (\beta', \alpha)'$ has a closed-form as follows (Sutradhar (2011)).

$$\hat{\theta} = \begin{pmatrix} \hat{\beta} \\ \\ \hat{\alpha} \end{pmatrix} = \left[\sum_{i=1}^{n} \begin{pmatrix} X_i \\ \\ \mathbf{1}_T G_i \end{pmatrix} \Sigma_i^{-1} (X_i : \mathbf{1}_T' G_i) \right]^{-1} \sum_{i=1}^{n} \begin{pmatrix} X_i \\ \\ \mathbf{1}_T G_i \end{pmatrix} \Sigma_i^{-1} y_i, \tag{2.5}$$

where $X_i = (X_{i1}, \ldots, X_{ip})'$, $X_{ip} = (X_{i1p}, \cdots, X_{iTp})'$, $\mathbf{1}_T$ is a T dimensional column vector of ones, y_i is $(y_{i1}, \cdots, y_{iT})'$, and Σ_i is the covariate matrix of y_i which satisfies:

1. $var(y_{it}|X_{it}, G_i) = \sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}$,

2. for $t \neq u$, $cov(y_{it}, y_{iu}|X_{it}, G_i) = \sigma_\gamma^2 + \frac{\sigma_a^2 \rho^{|t-u|}}{1-\rho^2}$.

The covariace matrix of the estimated parameters, condition on X and G, as follows.

$$var(\hat{\theta}) = \left[\sum_{i=1}^{n} \begin{pmatrix} X_i \\ \\ \mathbf{1}_T G_i \end{pmatrix} \Sigma_i^{-1} (X_i' : \mathbf{1}_T G_i) \right]^{-1}$$

Chapter 3

Longitudinal Linear Mixed-effects Model with Measurement Error

We now consider model (2.1) with measurement error (ME). In this model, the true predictor X_{it} is not observed; Instead W_{it} is observed with a random error term satisfying a classical measurement error model

$$W_{it} = X_{it} + U_{it}, \tag{3.1}$$

where U_{it} is the p-dimensional measurement error, independent of X_{it}, with mean vector zero and variances $\sigma^2_{u_1}, \cdots, \sigma^2_{u_p}$. We assume that measurement errors for any two covariates are independent, irrespective of their occurrence times.

In this model with ME, we can write the marginal moments of the response, only in terms of the observed covariate W. Therefore, by model assumptions and the law of iterative expectation (McClave, Sincich (2013)), we have

$$\begin{aligned}
& E(y_{it}|W_{it}, G_i) \\
= \quad & E_{X|W}(E(y_{it}|X_{it}, W_{it}, G_i)|W_{it}, G_i)) \\
= \quad & E_{X|W}(E(y_{it}|X_{it}, G_i)|W_{it}, G_i)) && (3.2) \\
= \quad & E_{X|W}(X'_{it}\beta + G_i\alpha|W_{it}, G_i) && (3.3) \\
= \quad & E(X'_{it}|W_{it})\beta + G_i\alpha && (3.4)
\end{aligned}$$

where equation (3.2) is true, since W_{it} is assumed to be surrogate, meaning that it can not provide any more information about the distribution of the response, given the information provided by X. Moreover, equation (3.3) follows from equation (2.2).

Similarly, we can calculate the marginal variance and covariance of the response as follows.

$$\begin{aligned}
& var(y_{it}|W_{it}, G_i) \\
= \quad & var_{X|W}(E(y_{it}|X_{it}, W_{it}, G_i)|W_{it}, G_i) + E_{X|W}(var(y_{it}|X_{it}, W_{it}, G_i)|W_{it}, G_i) \\
= \quad & var_{X|W}(E(y_{it}|X_{it}, G_i)|W_{it}, G_i) + E_{X|W}(var(y_{it}|X_{it}, G_i)|W_{it}, G_i) \\
= \quad & var_{X|W}(X'_{it}\beta + G_i\alpha|W_{it}, G_i) + E_{X|W}(\sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}|W_{it}, G_i) && (3.5) \\
= \quad & \beta' var(X_{it}|W_{it})\beta + \sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}
\end{aligned}$$

Note that equation (3.5) is concluded from (2.2) and (2.3). Furthermore, when $t \neq u$,

$$\begin{aligned}
& cov(y_{it}, y_{iu}|W_{it}, W_{iu}, G_i) \\
= \;& cov_{X|W}(E((y_{it}|X_{it}, W_{it}, G_i)|W_{it}, G_i), E((y_{iu}|X_{iu}, W_{iu}, G_i)|W_{iu}, G_i)) \\
+ \;& E_{X|W}(cov((y_{it}|X_{it}, W_{it}, G_i)|W_{it}, G_i), ((y_{iu}|X_{iu}, W_{iu}, G_i)|W_{iu}, G_i)) \\
= \;& cov_{X|W}(E((y_{it}|X_{it}, G_i)|W_{it}, G_i), E((y_{iu}|X_{iu}, G_i)|W_{iu}, G_i)) \\
+ \;& E_{X|W}(cov((y_{it}|X_{it}, G_i)|W_{it}, G_i), ((y_{iu}|X_{iu}, G_i)|W_{iu}, G_i)) \qquad (3.6) \\
= \;& cov((X'_{it}\beta + G_i\alpha|W_{it}, G_i), (X'_{iu}\beta + G_i\alpha|W_{iu}, G_i)) \\
+ \;& E_{X|W}(\sigma_\gamma^2 + \frac{\sigma_a^2\rho^{|t-u|}}{1-\rho^2}|W_{it}, W_{iu}, G_i)) \qquad (3.7) \\
= \;& \beta' cov(X'_{it}, X'_{iu}|W_{it}, W_{iu})\beta + \sigma_\gamma^2 + \frac{\sigma_a^2\rho^{|t-u|}}{1-\rho^2},
\end{aligned}$$

where equation (3.6) is true, since W_{it} is assumed to be surrogate, and equation (3.7) comes from equations (2.2) and (2.4).

The *naive* GLS estimate of the model parameters based on the observed W rather than X, is expressed as follows.

$$\hat{\theta}_n = \begin{pmatrix} \hat{\beta}_n \\ \hat{\alpha}_n \end{pmatrix} = \left[\sum_{i=1}^{n} \begin{pmatrix} W'_i \\ \mathbf{1}_T G_i \end{pmatrix} \Sigma_i^{*^{-1}} (W'_i : \mathbf{1}_T G_i) \right]^{-1} [\sum_{i=1}^{n} \begin{pmatrix} W'_i \\ \mathbf{1}_T G_i \end{pmatrix} \Sigma_i^{*^{-1}} y_i], \qquad (3.8)$$

where Σ_i^* is the matrix of variance and covariance of y_i based on W_i, which satisfying:

1. $var(y_{it}|W_{it}, G_i) = \beta' var(X_{it}|W_{it})\beta + \sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}$,

2. for $t \neq u$, $cov(y_{it}, y_{iu}|W_{it}, W_{iu}, G_i) = \beta' cov(X'_{it}, X'_{iu}|W_{it}, W_{iu})\beta + \sigma_\gamma^2 + \frac{\sigma_a^2 \rho^{|t-u|}}{1-\rho^2}$.

The covariance matrix of $\hat{\theta}_n$ conditioned on W_i and G_i, can be expressed as

$$Cov(\hat{\theta}_n) = \left[\sum_{i=1}^{n} \begin{pmatrix} W'_i \\ \\ G_i \end{pmatrix} \Sigma_i^{*^{-1}} (W'_i : \mathbf{1}_T G_i) \right]^{-1}.$$

3.1 Simulation Studies

It is well-known that ME and/or misclassification have negative impacts on the estimating parameters. (e.g. Fuller (1986), Carroll and Stefanski (2006)). Naive estimators are typically inconsistent. However, the direction and the magnitude of the bias can be quite complex. In the last chapter, we provided the closed-form of the GLS estimator of the parameters. It should be noted that for the GLS estimator, it was assumed that the covariance matrix of the response is known. Actually, this assumption is unrealistic, as the covariance matrix is a function of unknown model parameters. As a result, the bias in the naive estimators change according to any changes in the model parameters. In this section, using simulation studies, we examine the direction and magnitude of the bias in the naive estimators.

For each simulation scenario, we present the set ups and the results, separately.

Here, we first present the common setups for all scenarios of this chapter. For $T = 4$ time points, we generated $p = 2$ independent continuous time-invariant predictors from a uniform distribution $U(0, 1)$. The random effect γ was generated from a normal distribution with mean zero. Except in the scenario that σ^2_γ changes, it was set to be one. The categorical time-invariant G was generated from a binary distribution with probability of success $\pi = 0.4$. The regression model parameters were set to be $\alpha = 0.2$ and $\beta = (1, 0.5)'$, except the cases that they changed. The model error term, ϵ, follows a first order auto-regressive model, such that $\epsilon_{it} = \rho_1\epsilon_{i,t-1} + a_{it}$ and $|\rho_1| < 1$. We generated a_{it} from a normal distribution with mean zero. Except when they change, we set ρ_1 and σ^2_a to be 0.8 and 1, respectively.

For the classic measurement error models, each U_{1t} and U_{2t} follow a first order auto-regressive model with standard normal error, and autocorrelation lag parameters ρ_2 and ρ_3, respectively. ρ_2 and ρ_3 were both set for 0.8, unless they change.

We selected 500 as the sample size for all the scenarios, except when it changes. For each of the sample sizes, 1000 Monte Carlo replicates were simulated and the Monte-Carlo mean estimates and standard errors of the estimators were computed. All computations were done in R version 3.0.1.

For the following scenarios, we modify one model parameter at a time, while keeping other parameters unchanged. Table 3.1 shows the selected range, as well as the steps for every parameter.

Table 3.1: The range and increment steps for each model parameters

Parameter	Range	Step
ρ_1	$(-1, 1)$	0.1
ρ_2	$(-1, 1)$	0.1
ρ_3	$(-1, 1)$	0.1
σ_a^2	$(0, 2)$	0.2
σ_γ^2	(0,2)	0.2
$\sigma_{u_1}^2$	(0,2)	0.2
$\sigma_{u_2}^2$	(0,2)	0.2
α	(-3,3)	0.5
β_1	(-3,3)	0.5
β_2	(-3,3)	0.5
π	(0,1)	0.1
n	(100,1000)	100

3.1.1 Scenario 1: Bias analysis for different values of ρ_1

In this scenario, we intend to observe the behaviour of the bias in the naive estimates of $\theta = (\beta', \alpha)'$ when ρ_1, the autocorrelation lag parameter for the model error term, changes. As ρ_1 changes from -1 to 1, the bias in the naive estimator of β_1 looks like a convex function with minimum value at $\rho_1 = 0.6$. Its value is zero at $\rho_1 = 0.2$. The bias in the naive estimator of β_2 is relatively unchanged. The bias in the naive estimator of α looks like a concave function with its maximum point at approximately $\rho_1 = 0.6$. It, however, does not reach zero. It is interesting to observe that the variabilities of the naive estimators of β decrease as ρ_1 changes from -1 to 1, while the variability of the estimator of α has the opposite behaviour. Figure 3.1 and Table 3.2, present these behaviours.

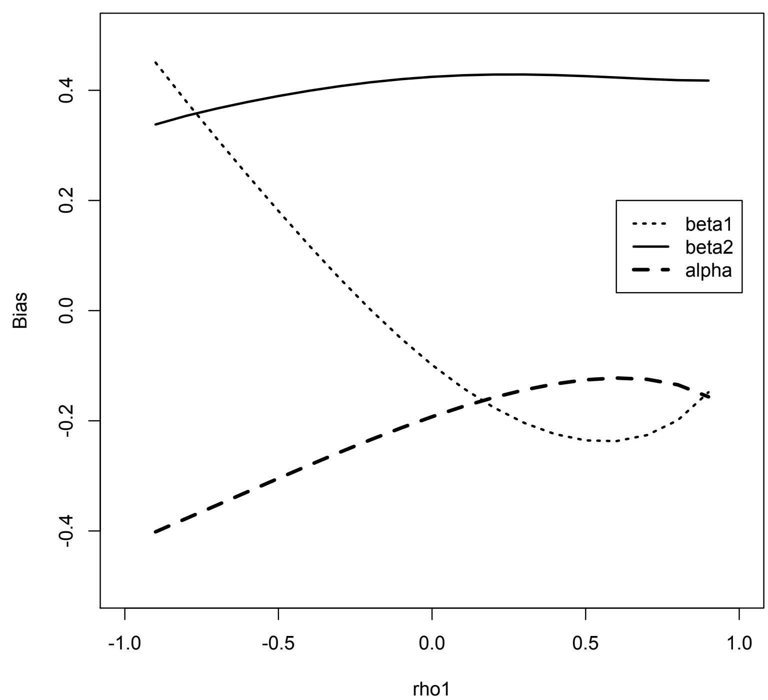

Figure 3.1: Bias of the naive estimators for different values of ρ_1

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\rho_1 = -0.9$	-0.4016	0.0451	0.4505	0.0035	0.3380	0.0039
$\rho_1 = -0.8$	-0.3774	0.0447	0.3798	0.0034	0.3537	0.0038
$\rho_1 = -0.7$	-0.3531	0.0443	0.3119	0.0032	0.3670	0.0037
$\rho_1 = -0.6$	-0.3288	0.0440	0.2456	0.0031	0.3789	0.0036
$\rho_1 = -0.5$	-0.3045	0.0438	0.1809	0.0029	0.3896	0.0035
$\rho_1 = -0.4$	-0.2805	0.0436	0.1182	0.0028	0.3992	0.0034
$\rho_1 = -0.3$	-0.2571	0.0436	0.0582	0.0026	0.4076	0.0033
$\rho_1 = -0.2$	-0.2344	0.0437	0.0015	0.0025	0.4146	0.0032
$\rho_1 = -0.1$	-0.2128	0.0439	-0.0509	0.0024	0.4203	0.0031
$\rho_1 = 0$	-0.1926	0.0442	-0.0984	0.0023	0.4245	0.0030
$\rho_1 = 0.1$	-0.1742	0.0448	-0.1402	0.0022	0.4273	0.0029
$\rho_1 = 0.2$	-0.1579	0.0456	-0.1756	0.0021	0.4287	0.0028
$\rho_1 = 0.3$	-0.1440	0.0470	-0.2038	0.0020	0.4287	0.0027
$\rho_1 = 0.4$	-0.1330	0.0493	-0.2241	0.0020	0.4277	0.0026
$\rho_1 = 0.5$	-0.1255	0.0530	-0.2356	0.0019	0.4257	0.0025
$\rho_1 = 0.6$	-0.1223	0.0591	-0.2369	0.0019	0.4232	0.0025
$\rho_1 = 0.7$	-0.1247	0.0692	-0.2259	0.0018	0.4206	0.0024
$\rho_1 = 0.8$	-0.1346	0.0873	-0.1990	0.0017	0.4184	0.0022
$\rho_1 = 0.9$	-0.1565	0.1276	-0.1481	0.0013	0.4177	0.0018

Table 3.2: Bias and standard error of the naive estimators for the different values of ρ_1

3.1.2 Scenario 2: Bias analysis for different values of ρ_2 and ρ_3

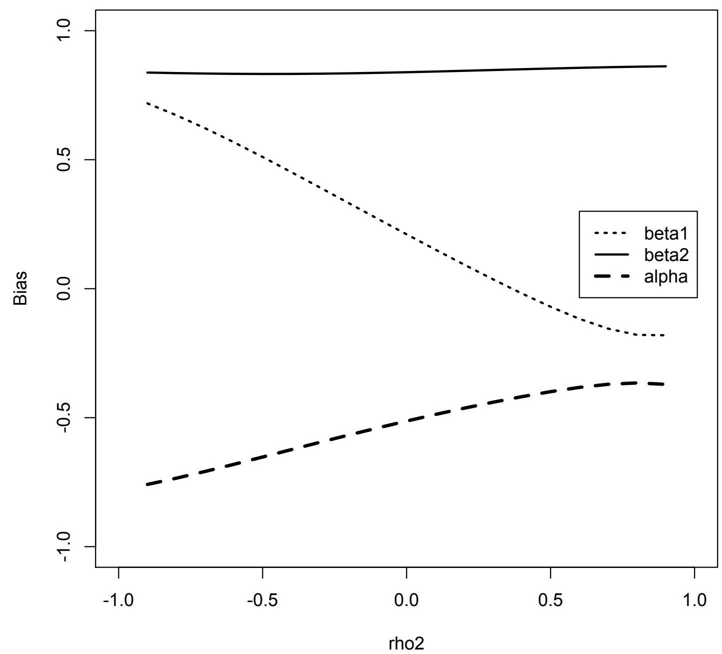

Figure 3.2: Bias of the naive estimators for different values of ρ_2

Figure 3.2 and Table 3.3, show the downward bias of the naive estimator of β_1 to approximately $\rho_2 = 0.5$, as the correlation parameter of the first measurement error increases from -1 to 1. The bias in the naive estimator of β_2 seems to stay unchanged. It is interesting to find that the correlation parameter of the first measurement error has no effect on the estimation of the coefficient parameter of the second continuous

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\rho_2 = -0.9$	-0.7588	0.1252	0.7183	0.0006	0.8377	0.0032
$\rho_2 = -0.8$	-0.7347	0.1236	0.6726	0.0007	0.8356	0.0032
$\rho_2 = -0.7$	-0.7086	0.1219	0.6221	0.0008	0.8340	0.0031
$\rho_2 = -0.6$	-0.6810	0.1201	0.5678	0.0009	0.8328	0.0030
$\rho_2 = -0.5$	-0.6525	0.1182	0.5107	0.0010	0.8323	0.0030
$\rho_2 = -0.4$	-0.6238	0.1164	0.4516	0.0012	0.8325	0.0029
$\rho_2 = -0.3$	-0.5952	0.1145	0.3916	0.0013	0.8334	0.0028
$\rho_2 = -0.2$	-0.5671	0.1127	0.3312	0.0014	0.8348	0.0027
$\rho_2 = -0.1$	-0.5397	0.1109	0.2708	0.0014	0.8367	0.0027
$\rho_2 = 0$	-0.5132	0.1090	0.2108	0.0015	0.8391	0.0026
$\rho_2 = 0.1$	-0.4876	0.1070	0.1515	0.0016	0.8418	0.0025
$\rho_2 = 0.2$	-0.4633	0.1047	0.0932	0.0017	0.8446	0.0025
$\rho_2 = 0.3$	-0.4402	0.1020	0.0364	0.0017	0.8476	0.0024
$\rho_2 = 0.4$	-0.4187	0.0988	-0.0184	0.0018	0.8506	0.0023
$\rho_2 = 0.5$	-0.3992	0.0950	-0.0702	0.0018	0.8535	0.0022
$\rho_2 = 0.6$	-0.3827	0.0905	-0.1169	0.0018	0.8561	0.0021
$\rho_2 = 0.7$	-0.3706	0.0852	-0.1551	0.0018	0.8585	0.0020
$\rho_2 = 0.8$	-0.3654	0.0794	-0.1791	0.0018	0.8605	0.0018
$\rho_2 = 0.9$	-0.3708	0.0735	-0.1806	0.0017	0.8620	0.0017

Table 3.3: Bias and standard error of the naive estimators for the different values of ρ_2

predictor. However, it affects the bias in the naive estimator of the categorical variable. As we can see, the naive estimator of α has a downward bias to a minimum at around $\rho_2 = 0.8$. More, interestingly, the variabilities of the estimators of α and β_2

decline, while the bias of the estimate of β_1 increases, as ρ_2 modifies over its range.

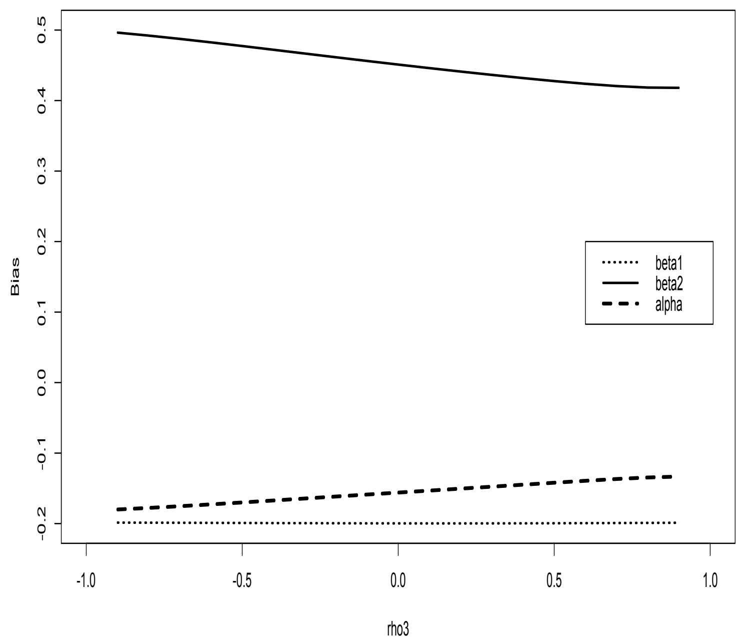

Figure 3.3: Bias of the naive estimators for different values of ρ_3

Figure 3.3 and Table 3.4 show a slight decrease in the bias of both naive estimators of β_2 and α, as the correlation parameter of the second measurement error changes from -1 to 1. The bias in the naive estimator of β_1 seems to stay unchanged. Since ρ_3 is the correlation parameter for the second measurement error, it has no effect on the estimation of the coefficient parameter of the first continuous predictor. Similar to the case of ρ_2, the variabilities of the estimators of α and β_2 decrease, with the increase in ρ_3, while the variability of the estimate of β_1 increases.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\rho_3 = -0.9$	-0.1800	0.0863	-0.1986	0.0017	0.4964	0.0004
$\rho_3 = -0.8$	-0.1778	0.0863	-0.1987	0.0017	0.4921	0.0005
$\rho_3 = -0.7$	-0.1753	0.0864	-0.1988	0.0017	0.4875	0.0006
$\rho_3 = -0.6$	-0.1727	0.0864	-0.1990	0.0017	0.4825	0.0007
$\rho_3 = -0.5$	-0.1700	0.0865	-0.1991	0.0017	0.4773	0.0008
$\rho_3 = -0.4$	-0.1672	0.0865	-0.1993	0.0017	0.4721	0.0009
$\rho_3 = -0.3$	-0.1644	0.0866	-0.1994	0.0017	0.4667	0.0011
$\rho_3 = -0.2$	-0.1616	0.0866	-0.1995	0.0017	0.4614	0.0012
$\rho_3 = -0.1$	-0.1588	0.0867	-0.1996	0.0017	0.4562	0.0013
$\rho_3 = 0$	-0.1560	0.0868	-0.1997	0.0017	0.4510	0.0014
$\rho_3 = 0.1$	-0.1532	0.0868	-0.1997	0.0017	0.4460	0.0015
$\rho_3 = 0.2$	-0.1504	0.0869	-0.1997	0.0017	0.4411	0.0016
$\rho_3 = 0.3$	-0.1476	0.0869	-0.1997	0.0017	0.4364	0.0017
$\rho_3 = 0.4$	-0.1448	0.0870	-0.1996	0.0017	0.4319	0.0018
$\rho_3 = 0.5$	-0.1420	0.0871	-0.1995	0.0017	0.4276	0.0020
$\rho_3 = 0.6$	-0.1393	0.0871	-0.1994	0.0017	0.4238	0.0021
$\rho_3 = 0.7$	-0.1367	0.0872	-0.1992	0.0017	0.4206	0.0022
$\rho_3 = 0.8$	-0.1346	0.0873	-0.1990	0.0017	0.4184	0.0022
$\rho_3 = 0.9$	-0.1333	0.0873	-0.1988	0.0016	0.4180	0.0023

Table 3.4: Bias and standard error of the naive estimators for the different values of ρ_3

3.1.3 Scenario 3: Bias analysis for different values of σ_a^2

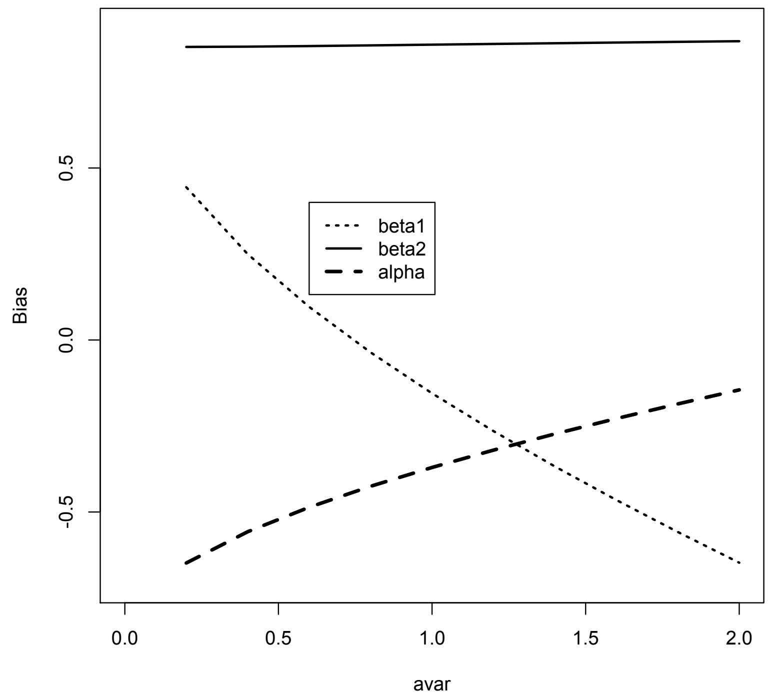

Figure 3.4: Bias of the naive estimators for different values of the variability of model error term

From Figure 3.4 and Table 3.5 we can see that when the variance of a_{it} increases from 0.2 to 2 with 0.2 increments, the bias of the naive estimator of the coefficient of the second continuous predictor, stays unchanged. The bias of the naive estimators of β_1 and α change in opposite directions, as σ_a^2 increases. The variabilities of the three estimators increase with the increase in σ_a^2.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma_a^2 = 0.2$	-0.6483	0.0514	0.4441	0.0008	0.8519	0.0008
$\sigma_a^2 = 0.4$	-0.5574	0.0582	0.2495	0.0012	0.8527	0.0012
$\sigma_a^2 = 0.6$	-0.4863	0.0665	0.0963	0.0014	0.8544	0.0015
$\sigma_a^2 = 0.8$	-0.4253	0.0756	-0.0360	0.0016	0.8564	0.0017
$\sigma_a^2 = 1.0$	-0.3706	0.0852	-0.1551	0.0018	0.8585	0.0020
$\sigma_a^2 = 1.2$	-0.3202	0.0951	-0.2649	0.0020	0.8606	0.0022
$\sigma_a^2 = 1.4$	-0.2730	0.1053	-0.3677	0.0021	0.8627	0.0025
$\sigma_a^2 = 1.6$	-0.2284	0.1157	-0.4651	0.0023	0.8647	0.0028
$\sigma_a^2 = 1.8$	-0.1859	0.1263	-0.5581	0.0025	0.8667	0.0031
$\sigma_a^2 = 2.0$	-0.1450	0.1370	-0.6474	0.0027	0.8686	0.0034

Table 3.5: Bias and standard error of the naive estimators for the different values of the model error term

3.1.4 Scenario 4: Bias analysis for different values of σ_γ^2

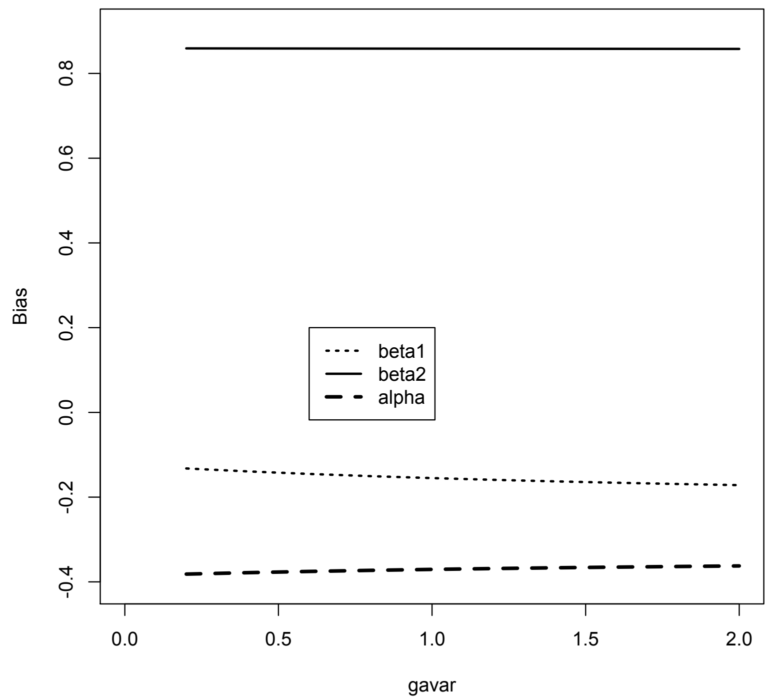

Figure 3.5: Bias of the naive estimators for different values of σ_γ^2

In this scenario, we observe the behaviour of the bias in the naive estimates of $\theta = (\beta', \alpha)'$ when σ_γ^2, the variability of the random effect, increases. As it can be seen in Figure 3.5 and Table 3.6, the increase in the variance of γ has a small effect on the bias of the naive estimators of all the coefficient parameters. However, it is not the case with their variabilities. The variabilities of the estimators of β_1 and β_2 show a

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma_\gamma^2 = 0.2$	-0.3816	0.0430	-0.1323	0.0016	0.8592	0.0015
$\sigma_\gamma^2 = 0.4$	-0.3783	0.0534	-0.1393	0.0016	0.8590	0.0016
$\sigma_\gamma^2 = 0.6$	-0.3754	0.0639	-0.1452	0.0017	0.8588	0.0018
$\sigma_\gamma^2 = 0.8$	-0.3728	0.0745	-0.1505	0.0018	0.8586	0.0019
$\sigma_\gamma^2 = 1.0$	-0.3706	0.0852	-0.1551	0.0018	0.8585	0.0020
$\sigma_\gamma^2 = 1.2$	-0.3686	0.0959	-0.1591	0.0019	0.8584	0.0021
$\sigma_\gamma^2 = 1.4$	-0.3668	0.1066	-0.1628	0.0019	0.8582	0.0022
$\sigma_\gamma^2 = 1.6$	-0.3652	0.1174	-0.1661	0.0019	0.8581	0.0022
$\sigma_\gamma^2 = 1.8$	-0.3637	0.1281	-0.1690	0.0020	0.8580	0.0023
$\sigma_\gamma^2 = 2.0$	-0.3624	0.1389	-0.1717	0.0020	0.8579	0.0024

Table 3.6: Bias and standard error of the naive estimators for the different values of σ_γ^2

slight increase, while the variance of the estimator of the coefficient of the categorical variable increases with the increase in σ_γ^2.

3.1.5 Scenario 5: Bias analysis for different values of $\sigma^2_{u_1}$

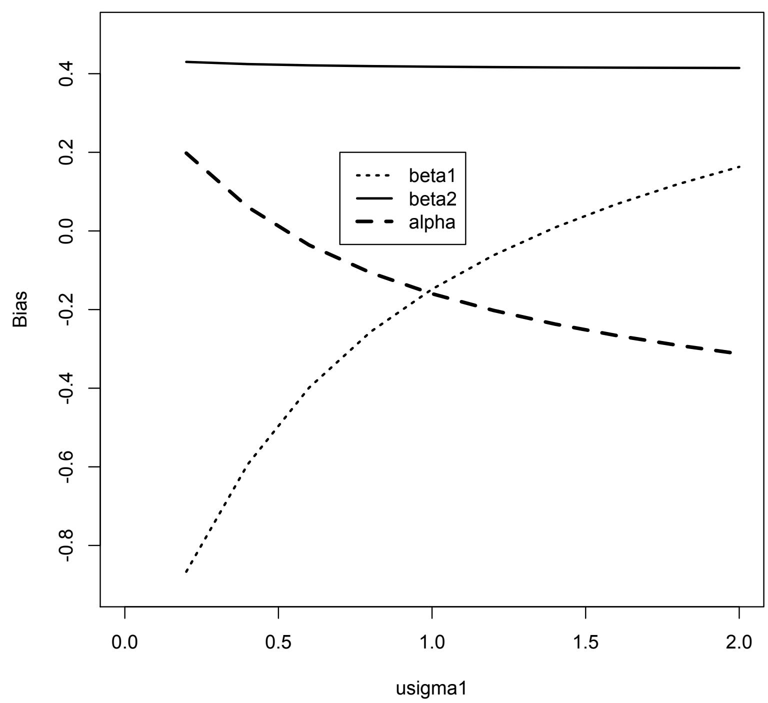

Figure 3.6: Bias of the naive estimators for different values of $\sigma^2_{u_1}$

As the measurement error on X_1 increases, we expect to observe that the bias of the naive estimator of β_1 increases. Surprisingly, the bias has a downward shape to its possible minimum value (zero), before it moves upward. With smaller magnitude, the bias of the naive estimator of α shows similar behaviour. The difference in the two, however, is the direction of the biases. (Figure 3.6) As we may expect, the change in

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma^2_{u_1} = 0.2$	0.1979	0.1130	-0.8668	0.0061	0.4300	0.0028
$\sigma^2_{u_1} = 0.4$	0.0610	0.1027	-0.5938	0.0035	0.4244	0.0026
$\sigma^2_{u_1} = 0.6$	-0.0360	0.0955	-0.3985	0.0024	0.4212	0.0024
$\sigma^2_{u_1} = 0.8$	-0.1062	0.0898	-0.2566	0.0018	0.4192	0.0023
$\sigma^2_{u_1} = 1.0$	-0.1597	0.0850	-0.1481	0.0015	0.4178	0.0022
$\sigma^2_{u_1} = 1.2$	-0.2021	0.0808	-0.0619	0.0013	0.4168	0.0021
$\sigma^2_{u_1} = 1.4$	-0.2369	0.0771	0.0087	0.0011	0.4160	0.0020
$\sigma^2_{u_1} = 1.6$	-0.2660	0.0738	0.0681	0.0010	0.4154	0.0020
$\sigma^2_{u_1} = 1.8$	-0.2909	0.0707	0.1188	0.0010	0.4149	0.0019
$\sigma^2_{u_1} = 2.0$	-0.3125	0.0680	0.1630	0.0009	0.4145	0.0019

Table 3.7: Bias and standard error of the naive estimators for the different values of $\sigma^2_{u_1}$

the ME on the first continuous covariate has no effect on the coefficient of the second continuous predictor, β_2. From Table 3.7, we observe that the increase in ME on X_1 decreases the variabilities in the naive estimators of the three parameters.

3.1.6 Scenario 6: Bias analysis for different values of $\sigma^2_{u_2}$

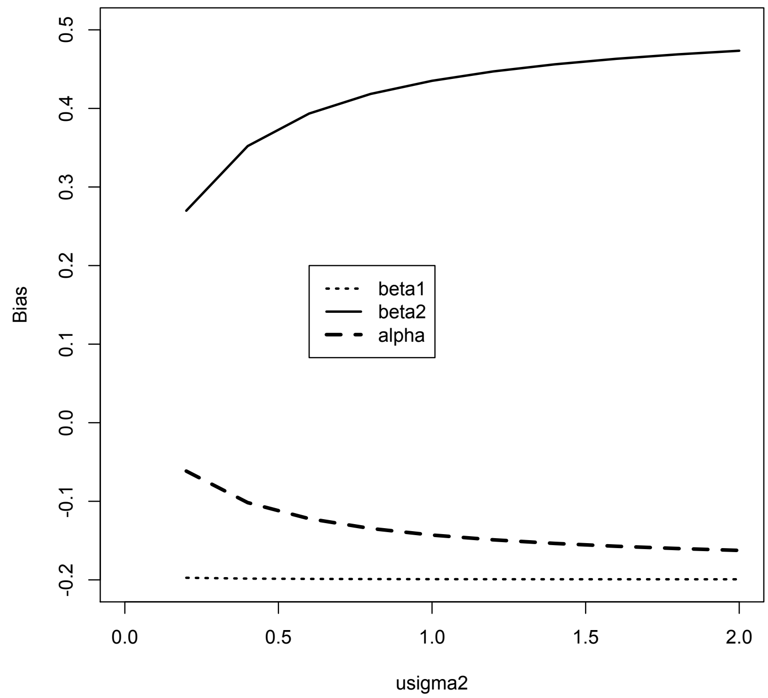

Figure 3.7: Bias of the naive estimators for different values of $\sigma^2_{u_2}$

As we expected, the increase in ME of X_2 has no impact on the estimate of the coefficient of X_1, β_1. As we see in Figure 3.7 and Table 3.8, the bias in the naive estimator of β_1 stays unchanged, as $\sigma^2_{u_2}$ increases from 0 to 2. However, the bias in the naive estimators of β_2 has a sharp increase first, and then continues to increase, slowly. Similarly, when the variance increases from 0 to 0.5, the bias in the naive

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma^2_{u_2} = 0.2$	-0.0616	0.0888	-0.1974	0.0015	0.2698	0.0054
$\sigma^2_{u_2} = 0.4$	-0.1017	0.0880	-0.1984	0.0016	0.3521	0.0037
$\sigma^2_{u_2} = 0.6$	-0.1222	0.0875	-0.1988	0.0016	0.3935	0.0028
$\sigma^2_{u_2} = 0.8$	-0.1346	0.0873	-0.1990	0.0017	0.4184	0.0022
$\sigma^2_{u_2} = 1.0$	-0.1430	0.0871	-0.1991	0.0017	0.4351	0.0019
$\sigma^2_{u_2} = 1.2$	-0.1490	0.0870	-0.1992	0.0017	0.4471	0.0016
$\sigma^2_{u_2} = 1.4$	-0.1536	0.0869	-0.1992	0.0017	0.4561	0.0014
$\sigma^2_{u_2} = 1.6$	-0.1572	0.0868	-0.1992	0.0017	0.4632	0.0013
$\sigma^2_{u_2} = 1.8$	-0.1601	0.0867	-0.1993	0.0017	0.4688	0.0012
$\sigma^2_{u_2} = 2.0$	-0.1625	0.0867	-0.1993	0.0017	0.4734	0.0011

Table 3.8: Bias and standard error of the naive estimators for the different values of $\sigma^2_{u_2}$

estimators of α has faster rise and then slow increase later. The variabilities in the estimators of α and β_1 seem to stay unchanged. However, the increase in ME of X_2 decreases the variability of the naive estimator of its coefficient, making it an even worse estimator.

3.1.7 Scenario 7: Bias analysis for different values of α

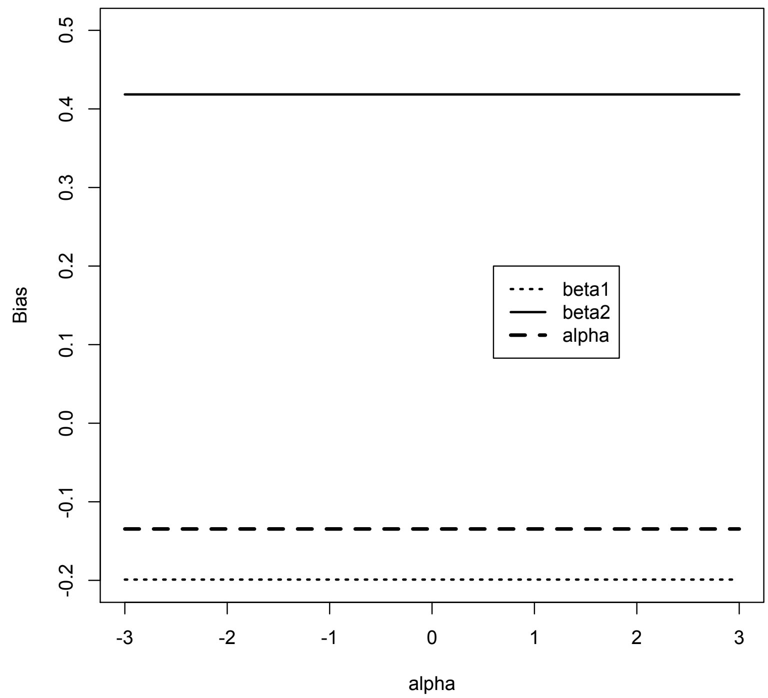

Figure 3.8: Bias of the naive estimators for different values of α

From Figure 3.8 and Table 3.9, it is clear that the bias in the naive estimators of all the coefficients stay unchanged as α increases from -3 to 3. Since this model has only measurement errors, and G has no correlation with the covariates with measurement error, the change of α does not affect the bias of the naive estimators of the coefficients. The same happens to their variabilities.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\alpha = -3.0$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = -2.5$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = -2.0$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = -1.5$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = -1.0$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = -0.5$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = 0$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = 0.5$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = 1.0$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = 1.5$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = 2.0$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = 2.5$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022
$\alpha = 3.0$	-0.1346	0.0873	-0.199	0.0017	0.4184	0.0022

Table 3.9: Bias and standard error of the naive estimators for the different values of α

3.1.8 Scenario 8: Bias analysis for different values of β_1

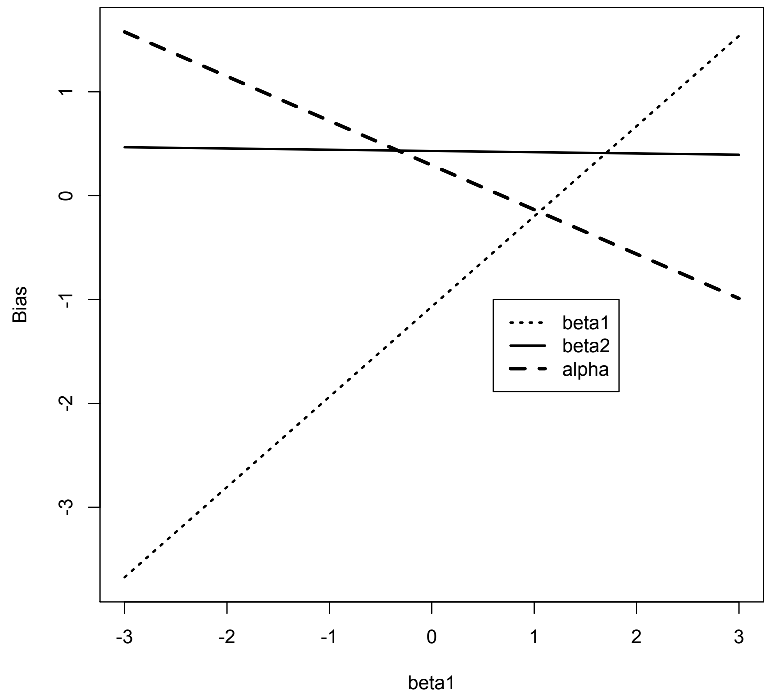

Figure 3.9: Bias of the naive estimators for different values of β_1

Figure 3.9 and Table 3.10 show that the biases of the naive estimators of β_1 and α have dramatic changes as β_1 increases from -3 to 3. The bias of the naive estimator of β_1 increases from negative values to zero, as β_1 changes from negative to around one, then it becomes positive as β_1 increses to 3. The bias of the naive estimator of α, however, decreases from positive values to negative values, as β_1 moves in the opposite direction. The bias in the naive estimator of β_2 seems to remain unchanged. Similar

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\beta_1 = -3.0$	1.5764	0.1083	-3.6746	0.0025	0.4663	0.0048
$\beta_1 = -2.5$	1.3625	0.1049	-3.2401	0.0022	0.4603	0.0043
$\beta_1 = -2.0$	1.1487	0.1018	-2.8057	0.0019	0.4544	0.0038
$\beta_1 = -1.5$	0.9348	0.0988	-2.3712	0.0016	0.4484	0.0034
$\beta_1 = -1.0$	0.7209	0.0960	-1.9368	0.0014	0.4424	0.0030
$\beta_1 = -0.5$	0.5070	0.0935	-1.5023	0.0013	0.4364	0.0027
$\beta_1 = 0$	0.2932	0.0911	-1.0679	0.0013	0.4304	0.0024
$\beta_1 = 0.5$	0.0793	0.0891	-0.6335	0.0014	0.4244	0.0023
$\beta_1 = 1.0$	-0.1346	0.0873	-0.1990	0.0017	0.4184	0.0022
$\beta_1 = 1.5$	-0.3485	0.0858	0.2354	0.0019	0.4125	0.0023
$\beta_1 = 2.0$	-0.5623	0.0846	0.6699	0.0022	0.4065	0.0025
$\beta_1 = 2.5$	-0.7762	0.0837	1.1043	0.0026	0.4005	0.0028
$\beta_1 = 3.0$	-0.9901	0.0832	1.5388	0.0029	0.3945	0.0032

Table 3.10: Bias and standard error of the naive estimators for the different values of β_1

to the last scenario, changes in the coefficient parameters of one continuous predictor has no effects on the estimate of the coefficient of the other continuous predictor. It is interesting to observe the variabilities in the three estimators. The variability in the estimator of α decreases with the increase in β_1, making it more sensitive to the coefficient's sign. For the estimators of the two βs, however, variabilities are concave function of β_1, with minimum value near $\beta_1 = 0$, for $Var(\beta_1)$ and $\beta_1 = 1$ for $Var(\beta_2)$.

3.1.9 Scenario 9: Bias analysis for different values of β_2

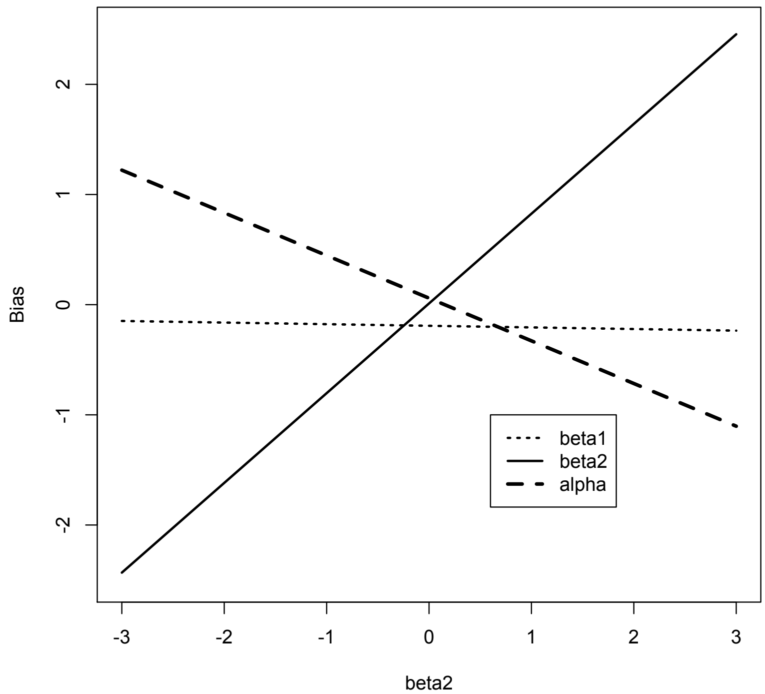

Figure 3.10: Bias of the naive estimators for different values of β_2

From Figure 3.10 and Table 3.11 we can see that the impact of ME is quite dramatic on the bias of the naive estimators of β_2 and α, as β_2 increases from -3 to 3. More specifically, the naive estimator of β_2 underestimates β_2 when it is negative, and overestimates it when it becomes positive. On the contrary, the bias of the naive estimator of α decreases from a positive value to a negative value. This is similar to the behaviour of the bias in the naive estimator of α when we increase β_1 from -3 to

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\beta_2 = -3.0$	1.2213	0.0951	-0.1478	0.0020	-2.4325	0.0035
$\beta_2 = -2.5$	1.0276	0.0930	-0.1551	0.0017	-2.0252	0.0032
$\beta_2 = -2.0$	0.8339	0.0911	-0.1624	0.0015	-1.6180	0.0029
$\beta_2 = -1.5$	0.6402	0.0896	-0.1697	0.0013	-1.2107	0.0026
$\beta_2 = -1.0$	0.4465	0.0885	-0.1770	0.0012	-0.8034	0.0024
$\beta_2 = -0.5$	0.2528	0.0877	-0.1844	0.0013	-0.3961	0.0023
$\beta_2 = 0$	0.0591	0.0873	-0.1917	0.0014	0.0112	0.0022
$\beta_2 = 0.5$	-0.1346	0.0873	-0.1990	0.0017	0.4184	0.0022
$\beta_2 = 1.0$	-0.3283	0.0877	-0.2063	0.0019	0.8257	0.0023
$\beta_2 = 1.5$	-0.5220	0.0884	-0.2136	0.0022	1.2330	0.0025
$\beta_2 = 2.0$	-0.7157	0.0896	-0.2210	0.0025	1.6403	0.0027
$\beta_2 = 2.5$	-0.9094	0.0911	-0.2283	0.0029	2.0476	0.0030
$\beta_2 = 3.0$	-1.1031	0.0929	-0.2356	0.0032	2.4548	0.0033

Table 3.11: Bias and standard error of the naive estimators for the different values of β_2

3. The bias of the naive estimator of β_1 seems to remain unchanged. Similarly, it is implied that the change of the coefficient parameter of one continuous predictor has no effect on the coefficient parameter of the other continuous predictor. Similar to the last scenario, the variability of the estimator of α seems to be sensitive to the sign of β_2, decreases as β_2 changes from negative values to positive ones. For the estimators of the two βs, variabilities have roughly concave shapes, with minimum value around $\beta_2 = 0$.

3.1.10 Scenario 10: Bias analysis for different values of π

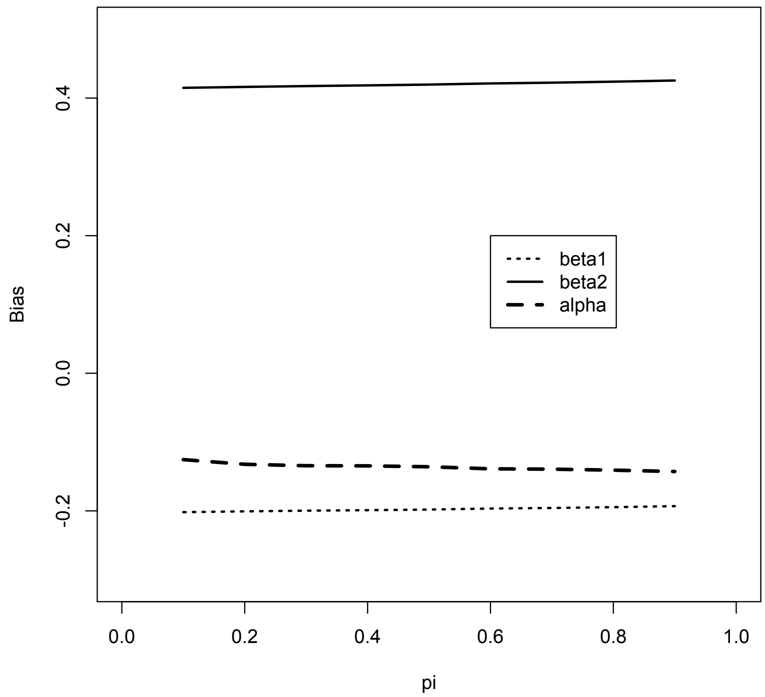

Figure 3.11: Bias of the naive estimators for different values of π

As it can be seen in Figure 3.11 and Table 3.12, the probability of classified predictor G seems to have no effect on the bias of the naive estimators for all the coefficient parameters. The variabilities, however, change as π changes from 0.1 to 0.9. Table 3.12 shows the impact of changes of π on the estimator of α. More specifically, the decrease in the variance of $\hat{\alpha}$ makes it a very poor estimator. Interestingly, the variabilities in both estimators of β_1 and β_2 have convex shapes, with maximum values

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\pi = 0.1$	-0.1255	0.2123	-0.2019	0.0010	0.4149	0.0014
$\pi = 0.2$	-0.1323	0.1424	-0.2007	0.0013	0.4162	0.0018
$\pi = 0.3$	-0.1344	0.1120	-0.1997	0.0016	0.4174	0.0021
$\pi = 0.4$	-0.1346	0.0873	-0.1990	0.0017	0.4184	0.0022
$\pi = 0.5$	-0.1359	0.0713	-0.1981	0.0017	0.4196	0.0023
$\pi = 0.6$	-0.1389	0.0586	-0.1966	0.0016	0.4213	0.0022
$\pi = 0.7$	-0.1394	0.0487	-0.1958	0.0016	0.4224	0.0022
$\pi = 0.8$	-0.1408	0.0363	-0.1946	0.0014	0.4238	0.0019
$\pi = 0.9$	-0.1428	0.0242	-0.1932	0.0010	0.4255	0.0014

Table 3.12: Bias and standard error of the naive estimators for the different values of π

around $\pi = 0.5$.

3.1.11 Scenario 11: Bias analysis for different values of n

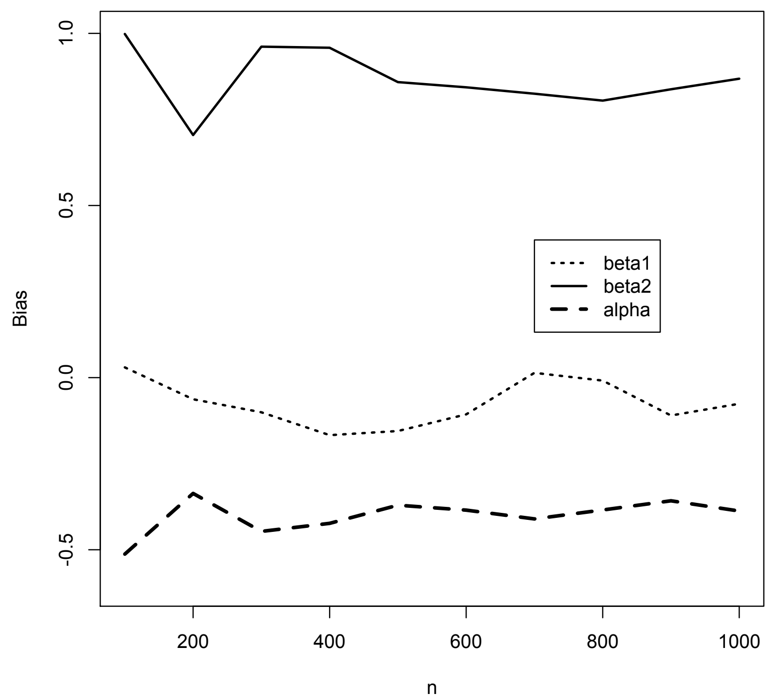

Figure 3.12: Bias of the naive estimators for different values of n

As it was expected, the bias in the naive estimators for all the coefficient parameters do not improve with the increase in the sample size. (Figure 3.12) The variabilities of the estimators, however, decline with the larger sample size. (Table 3.13) Therefore, with more observations, the naive estimators become very poor estimators!

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$n = 100$	0.2559	0.1786	0.01691	0.0038	0.5533	0.0033
$n = 300$	0.2083	0.1105	0.1389	0.0022	0.5075	0.0022
$n = 500$	0.1345	0.0872	0.1990	0.0016	0.4184	0.0022
$n = 700$	0.1583	0.0742	0.0316	0.0014	0.3757	0.0016
$n = 900$	0.1061	0.0646	0.1543	0.0013	0.3870	0.0014

Table 3.13: Bias and standard error of the naive estimators for the different values of n

Chapter 4

Longitudinal Linear Mixed-effects Model with Misclassification

Now, we consider model (2.1) with misclassification, only. In this model, the true predictor G_i is not observed. Instead, G_i^* is observed with random misclassification. We first consider both G and G^* to be binary variables with values 0 and 1. The conditional probability of G^* given G is displayed below.

$$\theta_{ij} = P(G^* = i|G = j); \;\; i = 0, 1; \; j = 0, 1$$

where,

$$\sum_{i=0}^{1} \theta_{ij} = 1, j = 0, 1.$$

In literature, θ_{11}, or the probability of the correct classification of the success, is called *sensitivity*, and θ_{00}, or the probability of correctly classifying the failure, is called *specificity*. In this model with misclassification, we can write the marginal moments of the response in terms of the observed covariate G^*. Therefore, by model assumptions and the law of iterative expectation (McClave & Sincich (2013)), we have

$$\begin{aligned}
& E(y_{it}|X_{it}, G_i^*) \\
= \;& E_{G|G^*}(E(y_{it}|X_{it}, G_i^*, G_i))|X_{it}, G_i^*) \\
= \;& E_{G|G^*}(E(y_{it}|X_{it}, G_i))|X_{it}, G_i^*) \qquad (4.1) \\
= \;& E_{G|G^*}(X_{it}'\beta + G_i\alpha|X_{it}, G_i^*) \\
= \;& X_{it}'\beta + E(G_i|G_i^*)\alpha \qquad (4.2) \\
= \;& X_{it}'\beta + P(G_i = 1|G_i^*)\alpha. \qquad (4.3)
\end{aligned}$$

where equation (4.1) is true, since G_i^* is assumed to be surrogate. We can express $P(G_i = 1|G_i^*)$ based on the model parameters. In pratice, $P(G)$, or the probability of success for the true variable, is usually known. We define $P(G = 1)$ to be π. Hence, by The Bayes' Law,

$$\begin{aligned}
& P(G_i = 1|G_i^* = 0) \\
= \;& \frac{P(G_i^* = 0|G_i = 1)P(G_i = 1)}{P(G_i^* = 0|G_i = 0)P(G_i = 0) + P(G_i^* = 0|G_i = 1)P(G_i = 1)} \\
= \;& \frac{\theta_{01}\pi}{\theta_{00}(1-\pi) + \theta_{01}\pi}. \qquad (4.4)
\end{aligned}$$

Similarly,

$$\begin{aligned}
& P(G_i = 1|G_i^* = 1) \\
= \;& \frac{P(G_i^* = 1|G_i = 1)P(G_i = 1)}{P(G_i^* = 1|G_i = 0)P(G_i = 0) + P(G_i^* = 1|G_i = 1)P(G_i = 1)} \\
= \;& \frac{\theta_{11}\pi}{\theta_{10}(1-\pi) + \theta_{11}\pi} \qquad (4.5)
\end{aligned}$$

Next, we consider both G and G^* with three categories, with status 0,1, and 2. The conditional probabilities are then as follows,

$$\theta_{ij} = P(G^* = i|G = j); \quad i = 0, 1, 2; \; j = 0, 1, 2$$

where,

$$\sum_{i=0}^{2} \theta_{ij} = 1, \quad j = 0, 1, 2.$$

Similar to the binary case, we express $P(G_i = 1|G_i^*)$ and $P(G_i = 2|G_i^*)$, based on model parameters. Defining $P(G_i = 1) = \pi_1$ and $P(G_i = 2) = \pi_2$, we have

$$\begin{aligned} P(G_i^* = 0) &= \sum_{j=0}^{2} P(G_i^* = 0|G_i = j)P(G_i = j) = \theta_{00}(1-\pi_1-\pi_2) + \theta_{01}\pi_1 + \theta_{02}\pi_2, \\ P(G_i^* = 1) &= \sum_{j=0}^{2} P(G_i^* = 1|G_i = j)P(G_i = j) = \theta_{10}(1-\pi_1-\pi_2) + \theta_{11}\pi_1 + \theta_{12}\pi_2, \\ P(G_i^* = 2) &= \sum_{j=0}^{2} P(G_i^* = 2|G_i = j)P(G_i = j) = \theta_{20}(1-\pi_1-\pi_2) + \theta_{21}\pi_1 + \theta_{22}\pi_2. \end{aligned}$$

Therefore, we can calculate the conditional probabilities of G given G^* as follows.

$$\begin{aligned} & P(G_i = 1|G_i^* = 0) \\ = \; & \frac{P(G_i^* = 0|G_i = 1)P(G_i = 1)}{P(G_i^* = 0)} \\ = \; & \frac{\theta_{01}\pi_1}{\theta_{00}(1-\pi_1-\pi_2) + \theta_{01}\pi_1 + \theta_{02}\pi_2}, \end{aligned} \tag{4.6}$$

$$P(G_i = 2|G_i^* = 0) = \frac{\theta_{02}\pi_2}{\theta_{00}(1-\pi_1-\pi_2) + \theta_{01}\pi_1 + \theta_{02}\pi_2}, \tag{4.7}$$

$$P(G_i = 1|G_i^* = 1) = \frac{\theta_{11}\pi_1}{\theta_{10}(1-\pi_1-\pi_2) + \theta_{11}\pi_1 + \theta_{12}\pi_2}, \tag{4.8}$$

$$P(G_i = 2|G_i^* = 1) = \frac{\theta_{12}\pi_2}{\theta_{10}(1-\pi_1-\pi_2) + \theta_{11}\pi_1 + \theta_{12}\pi_2}, \tag{4.9}$$

$$P(G_i = 1|G_i^* = 2) = \frac{\theta_{21}\pi_1}{\theta_{20}(1-\pi_1-\pi_2) + \theta_{21}\pi_1 + \theta_{22}\pi_2}, \tag{4.10}$$

$$P(G_i = 2|G_i^* = 2) = \frac{\theta_{22}\pi_2}{\theta_{20}(1-\pi_1-\pi_2) + \theta_{21}\pi_1 + \theta_{22}\pi_2}. \tag{4.11}$$

Similarly, by the model assumptions and the law of iterative variance, we can calculate the marginal variance and covariance of response as follows.

$$
\begin{aligned}
& var(y_{it}|X_{it}, G_i^*) \\
= \;& var_{G|G^*}(E((y_{it}|X_{it}, G_i^*, G_i)|X_{it}, G_i^*)) + E_{G|G^*}(var((y_{it}|X_{it}, G_i^*, G_i)|X_{it}, G_i^*)) \\
= \;& var_{G|G^*}(E((y_{it}|X_{it}, G_i)|X_{it}, G_i^*)) + E_{G|G^*}(var((y_{it}|X_{it}, G_i)|X_{it}, G_i^*)) \qquad (4.12) \\
= \;& var_{G|G^*}(X_{it}'\beta + G_i\alpha|X_{it}, G_i^*) + E_{G|G^*}(\sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}|X_{it}, G_i^*) \qquad (4.13) \\
= \;& var(G_i|G_i^*)\alpha^2 + \sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}. \qquad (4.14)
\end{aligned}
$$

When $t \neq u$,

$$
\begin{aligned}
& cov(y_{it}, y_{iu}|X_{it}, G_i^*) \\
= \;& cov_{G|G^*}(E((y_{it}|X_{it}, G_i, G_i^*)|X_{it}, G_i^*), E((y_{iu}|X_{iu}, G_i, G_i^*)|X_{iu}, G_i^*)) \\
+ \;& E_{G|G^*}(cov(((y_{it}|X_{it}, G_i, G_i^*)|X_{it}, G_i^*), ((y_{iu}|X_{iu}, G_i, G_i^*)|X_{iu}, G_i^*))) \\
= \;& cov_{G|G^*}(E((y_{it}|X_{it}, G_i)|X_{it}, G_i^*), E((y_{iu}|X_{iu}, G_i)|X_{iu}, G_i^*)) \\
+ \;& E_{G|G^*}(cov(((y_{it}|X_{it}, G_i)|X_{it}, G_i^*), ((y_{iu}|X_{iu}, G_i)|X_{iu}, G_i^*))) \qquad (4.15) \\
= \;& cov_{G|G^*}((X_{it}'\beta + \alpha G_i|X_{it}, G_i^*), (X_{iu}'\beta + \alpha G_i|X_{iu}, G_i^*)) \\
+ \;& E_{G|G^*}(\sigma_\gamma^2 + \frac{\sigma_a^2\rho^{|t-u|}}{1-\rho^2}|X_{it}, X_{iu}, G_i^*) \qquad (4.16) \\
= \;& var(G_i|G_i^*)\alpha^2 + \sigma_\gamma^2 + \frac{\sigma_a^2\rho^{|t-u|}}{1-\rho^2} \qquad (4.17)
\end{aligned}
$$

Equations (4.12) and (4.15) are true, since G^* is a surrogate (Hogg & Craig (2004)). Moreover, equations (4.13) and (4.16) result from equations (2.2), (2.3), and (2.4).

Now, we further calculate $var(G_i|G_i^*)$ as required for both $var(y_{it}|X_i, G_i^*)$ and $cov(y_{it}, y_{iu}|X_i, G_i^*)$. Similar to the marginal mean, we first consider the binary case.

$$var(G_i|G_i^* = 0) = E(G_i^2|G_i^* = 0) - (E(G_i|G_i^* = 0))^2$$

$$= 1^2 * P(G_i = 1|G_i^* = 0) - (1 * P(G_i = 1|G_i^* = 0))^2$$

$$= \frac{\theta_{01}\pi}{\theta_{00}(1-\pi) + \theta_{01}\pi} - (\frac{\theta_{01}\pi}{\theta_{00}(1-\pi) + \theta_{01}\pi})^2 \tag{4.18}$$

$$var(G_i|G_i^* = 1) = E(G_i^2|G_i^* = 1) - (E(G_i|G_i^* = 1))^2$$

$$= 1^2 * P(G_i = 1|G_i^* = 1) - (1 * P(G_i = 1|G_i^* = 1)^2$$

$$= \frac{\theta_{11}\pi}{\theta_{10}(1-\pi) + \theta_{11}\pi} - (\frac{\theta_{11}\pi}{\theta_{10}(1-\pi) + \theta_{11}\pi})^2 \tag{4.19}$$

where equations (4.18) and (4.19) come from (4.4) and (4.5).

Moving to three categories, we have,

$$\begin{aligned}
& var(G_i|G_i^* = 0) \\
= \; & E(G_i^2|G_i^* = 0) - (E(G_i|G_i^* = 0))^2 \\
= \; & 1^2 * P(G_i = 1|G_i^* = 0) + 2^2 * P(G_i = 2|G_i^* = 0) \\
- \; & (1 * P(G_i = 1|G_i^* = 0) + 2 * P(G_i = 2|G_i^* = 0))^2 \\
= \; & \frac{\theta_{01}\pi_1}{\theta_{00}(1-\pi_1-\pi_2) + \theta_{01}\pi_1 + \theta_{02}\pi_2} + 4\frac{\theta_{02}\pi_2}{\theta_{00}(1-\pi_1-\pi_2) + \theta_{01}\pi_1 + \theta_{02}\pi_2} \\
- \; & (\frac{\theta_{01}\pi_1}{\theta_{00}(1-\pi_1-\pi_2) + \theta_{01}\pi_1 + \theta_{02}\pi_2} + 2\frac{\theta_{02}\pi_2}{\theta_{00}(1-\pi_1-\pi_2) + \theta_{01}\pi_1 + \theta_{02}\pi_2})^2
\end{aligned}$$

where, in the last equations, the two conditional probabilities come from (4.6) and (4.7). Similarly, based on the conditional probabilities calculated in (4.8)–(4.11), we have

$$\begin{aligned}
var(G_i|G_i^* = 1) &= \frac{\theta_{11}\pi_1}{\theta_{10}(1-\pi_1-\pi_2)+\theta_{11}\pi_1+\theta_{12}\pi_2} + 4\frac{\theta_{12}\pi_2}{\theta_{10}(1-\pi_1-\pi_2)+\theta_{11}\pi_1+\theta_{12}\pi_2} \\
- & (\frac{\theta_{11}\pi_1}{\theta_{10}(1-\pi_1-\pi_2)+\theta_{11}\pi_1+\theta_{12}\pi_2} + 2\frac{\theta_{12}\pi_2}{\theta_{10}(1-\pi_1-\pi_2)+\theta_{11}\pi_1+\theta_{12}\pi_2})^2, \\
var(G_i|G_i^* = 2) &= \frac{\theta_{21}\pi_1}{\theta_{20}(1-\pi_1-\pi_2)+\theta_{21}\pi_1+\theta_{22}\pi_2} + 4\frac{\theta_{22}\pi_2}{\theta_{20}(1-\pi_1-\pi_2)+\theta_{21}\pi_1+\theta_{22}\pi_2} \\
- & (\frac{\theta_{21}\pi_1}{\theta_{20}(1-\pi_1-\pi_2)+\theta_{21}\pi_1+\theta_{22}\pi_2} + 2\frac{\theta_{22}\pi_2}{\theta_{20}(1-\pi_1-\pi_2)+\theta_{21}\pi_1+\theta_{22}\pi_2})^2.
\end{aligned}$$

The *naive* GLS estimator of the model coefficient parameters based on the observed G^*, (rather than G), is expressed as follows.

$$\hat{\theta}_n = \begin{pmatrix} \hat{\beta}_n \\ \hat{\alpha}_n \end{pmatrix} = [\sum_{i=1}^{n} \begin{pmatrix} X_i' \\ \mathbf{1}_T G_i^* \end{pmatrix} {\Omega_i^*}^{-1} (X_i' : \mathbf{1}_T G_i^*)]^{-1} [\sum_{i=1}^{n} \begin{pmatrix} X_i' \\ \mathbf{1}_T G_i^* \end{pmatrix} {\Omega_i^*}^{-1} y_i],$$

where Ω_i^* is the matrix of variance and covariance of y_i based on G_i^*, with the following elements.

1. $var(y_{it}|X_{it}, G_i^*) = \sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}$,

2. for $t \neq u$, $cov(y_{it}, y_{iu}|X_{it}, X_{iu}, G_i^*) = \sigma_\gamma^2 + \frac{\sigma_a^2 \rho^{|t-u|}}{1-\rho^2}$.

Similar to the model with ME only, the covariance matrix of $\hat{\theta}_n$ can be expressed as

$$Cov(\hat{\theta}_n) = \left[\sum_{i=1}^{n} \begin{pmatrix} X_i' \\ \mathbf{1}_T G_i^* \end{pmatrix} \Omega_i^{*^{-1}} (X_i' : \mathbf{1}_T G_i^*) \right]^{-1} .$$

4.1 Simulation Studies

Misclassification can affect both bias and variabilities of the parameter estimates. (Fuller (1996), Carrol and Stefanski (2006), Buonaccorsi (2010)). It is, however, very difficult to assess the bias in the naive estimator as a function of the *true* response covariance matrix. In this research, it is assumed that the covariance matrix of the response is known. However, in reality, this matrix is itself a function of the unknown model parameters. We therefore evaluate the bias caused by the misclassification through simulation studies.

1. **Categorical variable with two categories**

   We first consider the case where both G and G^* are binary. For $T = 4$ time points, we generated $p = 2$ independent continuous time-invariant predictors from a uniform distribution $U(0, 1)$. The random effect γ was generated from a normal distribution with mean zero. Except in the scenario that σ_γ^2 changes, it was set to one. The categorial time-invariant G was generated from a binary distribution with probability of success $\pi = 0.4$, except in the scenario where it changed. The regression model parameters were set to be $\alpha = 1$ and $\beta = (1, 0.5)'$. The model error term, ϵ, follows a first order auto-regressive model, such that $\epsilon_{it} = \rho_1 \epsilon_{i,t-1} + a_{it}$ and $|\rho_1| < 1$. We generated a_{it} from a normal distribution with mean zero. Except when they changed, we set ρ_1 and σ_a^2 to be 0.8 and 1, respectively.

For the misclassification models, the observed categorial time-invariant G^* was generated based on G. Specificity (θ_{00}) and sensitivity (θ_{11}) were set to be 0.9 and 0.8, respectively, unless changed in the relative scenarios.

Similar to the case with ME only, we selected 500 as the sample size for all the scenarios, except the last scenario when it changes. For each of the sample sizes, 1000 Monte Carlo replicates were simulated and the Monte-Carlo mean estimates and standard errors of the estimators were computed.

For the following scenarios, we modify one model parameter at a time, while keeping other parameters unchanged. Table 4.1 shows the selected range as well as the steps for every parameter.

Table 4.1: The range and step of the model parameters

Parameter	Range	Step
ρ_1	$(-1, 1)$	0.1
σ_a^2	$(0, 2)$	0.2
σ_γ^2	(0,2)	0.2
α	(-3,3)	0.5
β_1	(-3,3)	0.5
β_2	(-3,3)	0.5
θ_{00}	(0,1)	0.1
θ_{11}	(0,1)	0.1
π	(0,1)	0.1
n	(100,1000)	100

4.1.1 Scenario 1: Bias analysis for different values of ρ_1

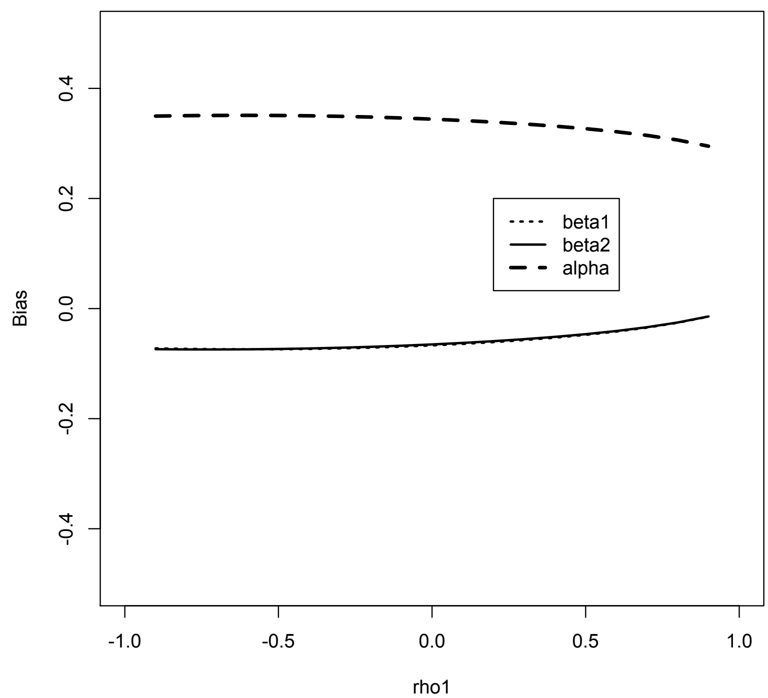

Figure 4.1: Bias of the naive estimators for different values of ρ_1

As ρ_1, the autocorrelation lag parameter for the model error term, changes from -1 to 1, the absolute values of the bias of the three naive estimators slightly decrease. The variabilities of the naive estimators of β decrease as ρ_1 changes from -1 to 1, while the variability of the estimator of α has the opposite behaviour. (Figure 4.1 and Table 4.2)

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\rho_1 = -0.9$	0.3497	0.0751	-0.0723	0.0227	-0.0738	0.0236
$\rho_1 = -0.8$	0.3505	0.0767	-0.0732	0.0233	-0.0742	0.0240
$\rho_1 = -0.7$	0.3509	0.0781	-0.0738	0.0238	-0.0743	0.0243
$\rho_1 = -0.6$	0.3510	0.0795	-0.0741	0.0242	-0.0740	0.0245
$\rho_1 = -0.5$	0.3508	0.0808	-0.0739	0.0245	-0.0734	0.0245
$\rho_1 = -0.4$	0.3502	0.0821	-0.0734	0.0247	-0.0725	0.0245
$\rho_1 = -0.3$	0.3493	0.0835	-0.0724	0.0248	-0.0712	0.0244
$\rho_1 = -0.2$	0.3480	0.0850	-0.0710	0.0248	-0.0695	0.0242
$\rho_1 = -0.1$	0.3463	0.0866	-0.0692	0.0246	-0.0675	0.0239
$\rho_1 = 0$	0.3442	0.0883	-0.0670	0.0244	-0.0652	0.0235
$\rho_1 = 0.1$	0.3417	0.0903	-0.0642	0.0241	-0.0624	0.0231
$\rho_1 = 0.2$	0.3388	0.0927	-0.0610	0.0236	-0.0592	0.0226
$\rho_1 = 0.3$	0.3354	0.0955	-0.0572	0.0230	-0.0556	0.0220
$\rho_1 = 0.4$	0.3314	0.0989	-0.0529	0.0223	-0.0514	0.0212
$\rho_1 = 0.5$	0.3268	0.1033	-0.0478	0.0214	-0.0465	0.0203
$\rho_1 = 0.6$	0.3213	0.1091	-0.0418	0.0201	-0.0408	0.0191
$\rho_1 = 0.7$	0.3146	0.1175	-0.0346	0.0184	-0.0339	0.0175
$\rho_1 = 0.8$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.0150
$\rho_1 = 0.9$	0.2950	0.1623	-0.0146	0.0116	-0.0143	0.0110

Table 4.2: Bias and Standard Error for the different values of ρ_1

4.1.2 Scenario 2: Bias analysis for different values of σ_a^2

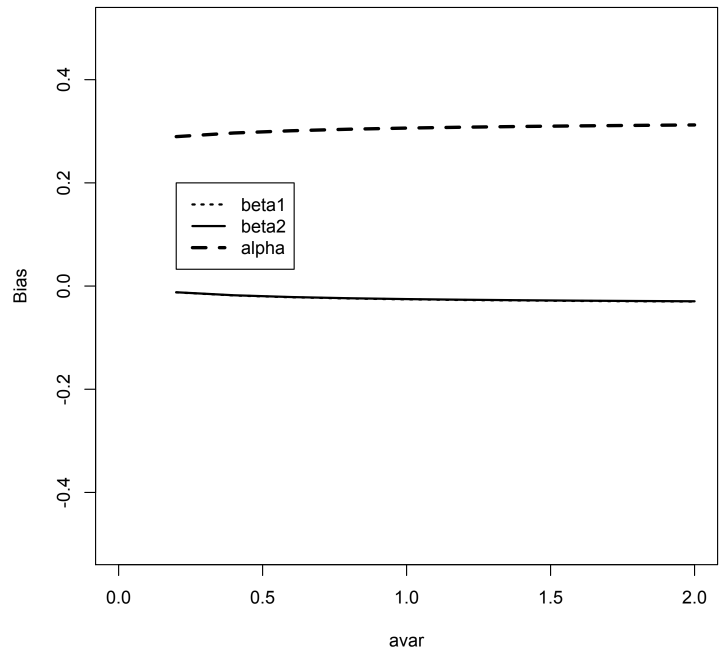

Figure 4.2: Bias of the naive estimators for different values of the variability of model error term

From Figure 4.2, we can see that the bias of the naive estimators of all the coefficient parameters seem to stay unchanged. However, from Table 4.3, we see that the variabilities of all the estimators increase slightly. This means that the change of the variability of model error term creates more conservative estimators for α, β_1 and β_2.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma_a^2 = 0.2$	0.2896	0.0693	-0.0123	0.0043	-0.0120	0.0041
$\sigma_a^2 = 0.4$	0.2967	0.0854	-0.0182	0.0077	-0.0178	0.0073
$\sigma_a^2 = 0.6$	0.3010	0.1010	-0.0218	0.0106	-0.0213	0.0100
$\sigma_a^2 = 0.8$	0.3040	0.1163	-0.0241	0.0133	-0.0236	0.0126
$\sigma_a^2 = 1.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.0150
$\sigma_a^2 = 1.2$	0.3079	0.1463	-0.0271	0.0183	-0.0265	0.0174
$\sigma_a^2 = 1.4$	0.3093	0.1610	-0.0281	0.0208	-0.0275	0.0197
$\sigma_a^2 = 1.6$	0.3105	0.1758	-0.0289	0.0232	-0.0283	0.0219
$\sigma_a^2 = 1.8$	0.3114	0.1904	-0.0296	0.0256	-0.0290	0.0242
$\sigma_a^2 = 2.0$	0.3123	0.2050	-0.0301	0.0279	-0.0295	0.0264

Table 4.3: Bias and Standard Error for the different values of the variability of model error term

4.1.3 Scenario 3: Bias analysis for different values of σ_γ^2

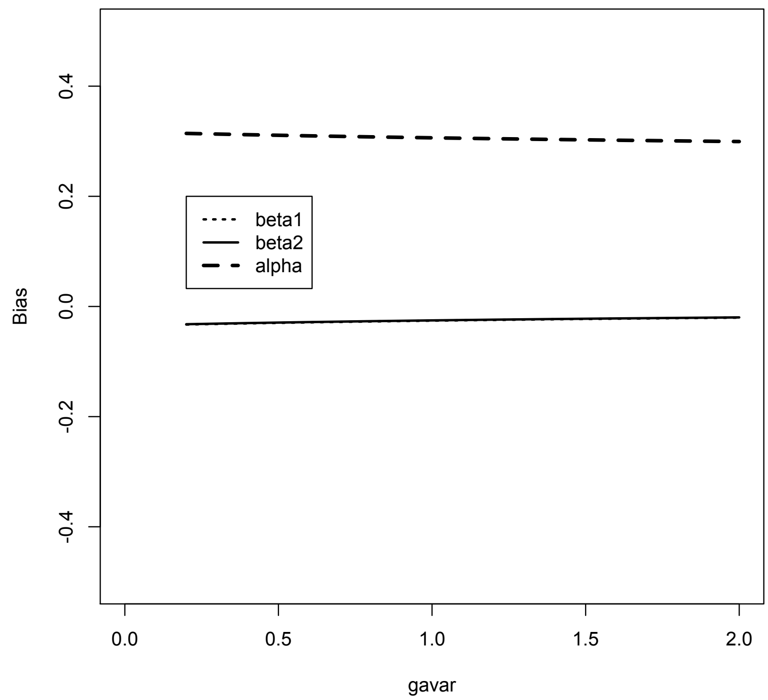

Figure 4.3: Bias of the naive estimators for different values of the variability in the random effect

As presented in Figure 4.3, interestingly, the bias of the naive estimators of all the coefficient parameters seem to stay unchanged when σ_γ^2 changes from 0 to 2. However, from Table 4.4, we see that the variabilities of all the three estimators slightly decrease. We can conclude that increase in the variance of γ, in the presence of misclassification, provides a very poor estimator for α with large

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma_\gamma^2 = 0.2$	0.3142	0.0938	-0.0328	0.0140	-0.0322	0.0133
$\sigma_\gamma^2 = 0.4$	0.3119	0.1030	-0.0307	0.0145	-0.0301	0.0138
$\sigma_\gamma^2 = 0.6$	0.3098	0.1123	-0.0289	0.0150	-0.0283	0.0142
$\sigma_\gamma^2 = 0.8$	0.3079	0.1218	-0.0273	0.0154	-0.0267	0.0146
$\sigma_\gamma^2 = 1.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.0150
$\sigma_\gamma^2 = 1.2$	0.3046	0.1410	-0.0245	0.0162	-0.0240	0.0154
$\sigma_\gamma^2 = 1.4$	0.3032	0.1506	-0.0233	0.0166	-0.0228	0.0157
$\sigma_\gamma^2 = 1.6$	0.3019	0.1603	-0.0222	0.0169	-0.0217	0.0160
$\sigma_\gamma^2 = 1.8$	0.3006	0.1700	-0.0212	0.0172	-0.0207	0.0163
$\sigma_\gamma^2 = 2.0$	0.2994	0.1798	-0.0203	0.0175	-0.0199	0.0166

Table 4.4: Bias and Standard Error for the different values of the variability in the random effect

bias and small variability. However, it has almost no impact on estimating β_1 and β_2.

4.1.4 Scenario 4: Bias analysis for different values of α

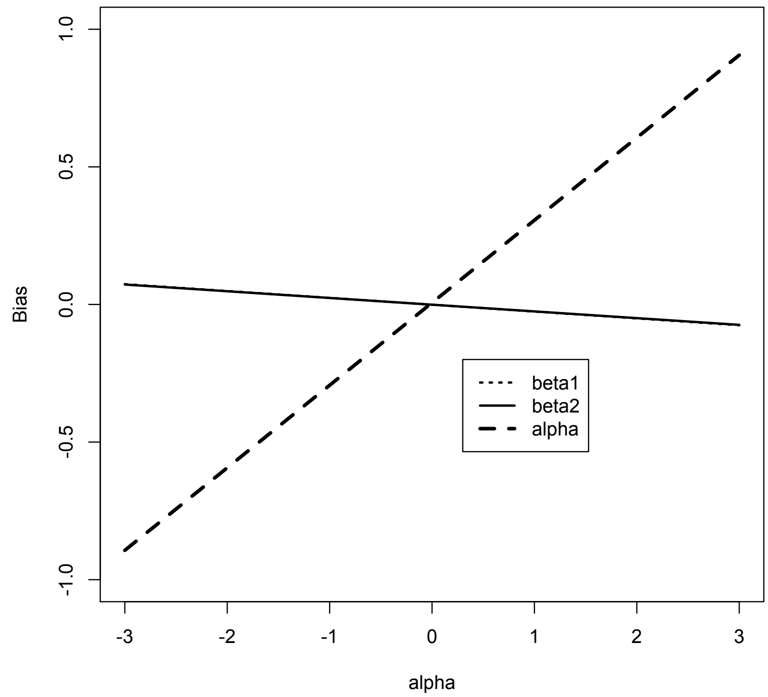

Figure 4.4: Bias of the naive estimators for different values of α

From Figure 4.4 and Table 4.5, we are not surprised to see that the bias in the naive estimators of all the continuous coefficient parameters stay unchanged as α increases from -3 to 3. Since the covariates are generated independently, the change in α has no effects on β_1 and β_2. However, the bias in the naive estimator of α increases sharply, from -1 to 1. Clearly, when $\alpha = 0$, the bias in the naive estimator of α is almost zero. The changes in α seems to have no

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\alpha = -3.0$	-0.8934	0.1381	0.0737	0.0173	0.0725	0.0161
$\alpha = -2.5$	-0.7434	0.1355	0.0613	0.0168	0.0603	0.0157
$\alpha = -2.0$	-0.5935	0.1334	0.0488	0.0164	0.0480	0.0153
$\alpha = -1.5$	-0.4435	0.1318	0.0364	0.0161	0.0358	0.0150
$\alpha = -1.0$	-0.2936	0.1306	0.0240	0.0158	0.0236	0.0148
$\alpha = -0.5$	-0.1436	0.1300	0.0115	0.0157	0.0114	0.0147
$\alpha = 0$	0.0063	0.1299	-0.0009	0.0156	-0.0008	0.0147
$\alpha = 0.5$	0.1563	0.1304	-0.0134	0.0157	-0.0131	0.0148
$\alpha = 1.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.0150
$\alpha = 1.5$	0.4561	0.1328	-0.0382	0.0161	-0.0375	0.0153
$\alpha = 2.0$	0.6061	0.1348	-0.0507	0.0165	-0.0497	0.0157
$\alpha = 2.5$	0.7560	0.1373	-0.0631	0.0169	-0.0619	0.0162
$\alpha = 3.0$	0.9060	0.1402	-0.0755	0.0174	-0.0741	0.0168

Table 4.5: Bias and Standard Error for the different values of α

effect on the variability of the naive estimators.

4.1.5 Scenario 5: Bias analysis for different values of β_1 and β_2

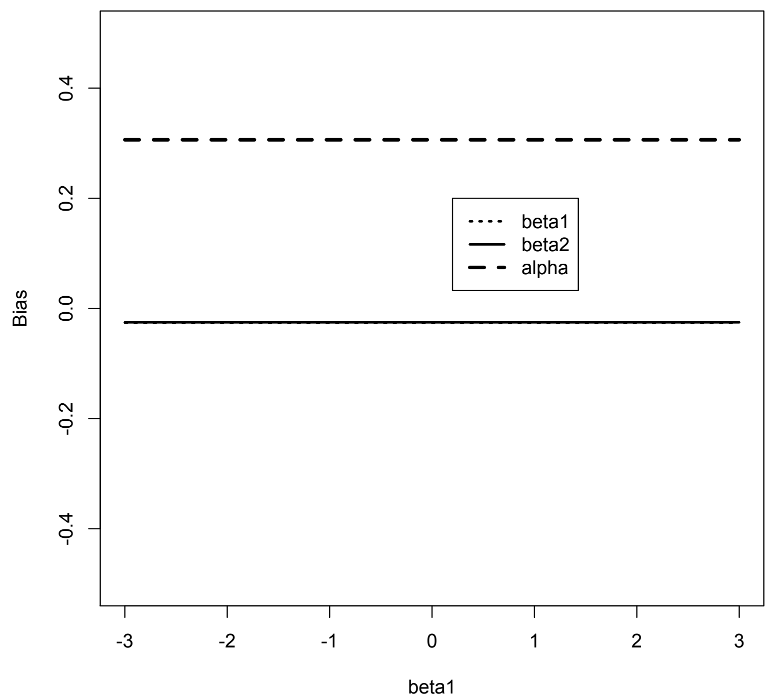

Figure 4.5: Bias of the naive estimators for different values of β_1

Figures 4.5 and 4.6, and Tables 4.7 and 4.6 show that the bias in the naive estimators of β_1, β_2 and α stay unchanged as β_1 and β_2 increase from -3 to 3. Since the model has misclassification only, and all the variables are independent, the change of the coefficient parameter of continuous predictors has no impact

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\beta_1 = -3.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = -2.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = -2.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = -1.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = -1.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = -0.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = 0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = 0.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = 1.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = 1.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = 2.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = 2.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_1 = 3.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015

Table 4.6: Bias and Standard Error for the different values of β_1

on the bias of the naive estimators. The variabilities also remain unchanged.

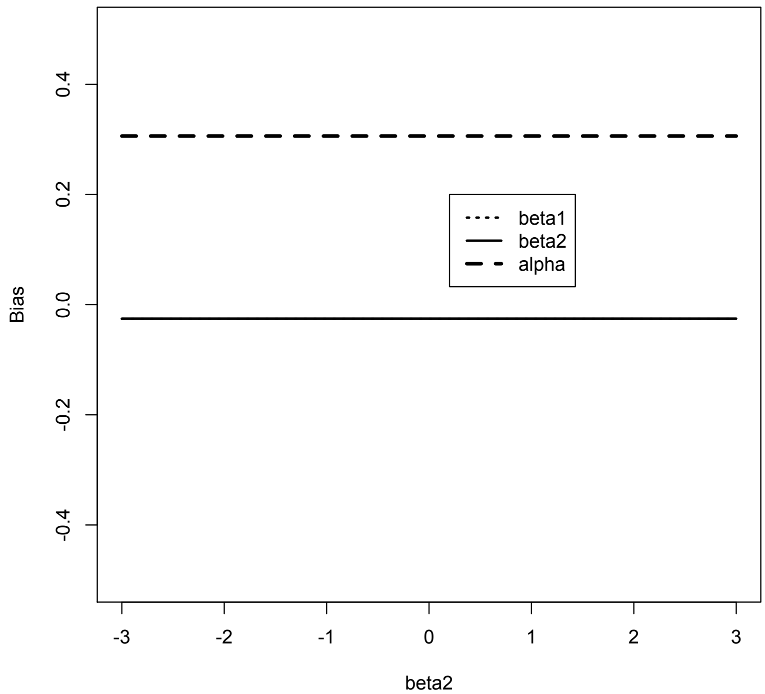

Figure 4.6: Bias of the naive estimators for different values of β_2

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\beta_2 = -3.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = -2.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = -2.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = -1.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = -1.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = -0.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = 0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = 0.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = 1.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = 1.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = 2.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = 2.5$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015
$\beta_2 = 3.0$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.015

Table 4.7: Bias and Standard Error for the different values of β_2

4.1.6 Scenario 6: Bias analysis for different values of θ_{00}

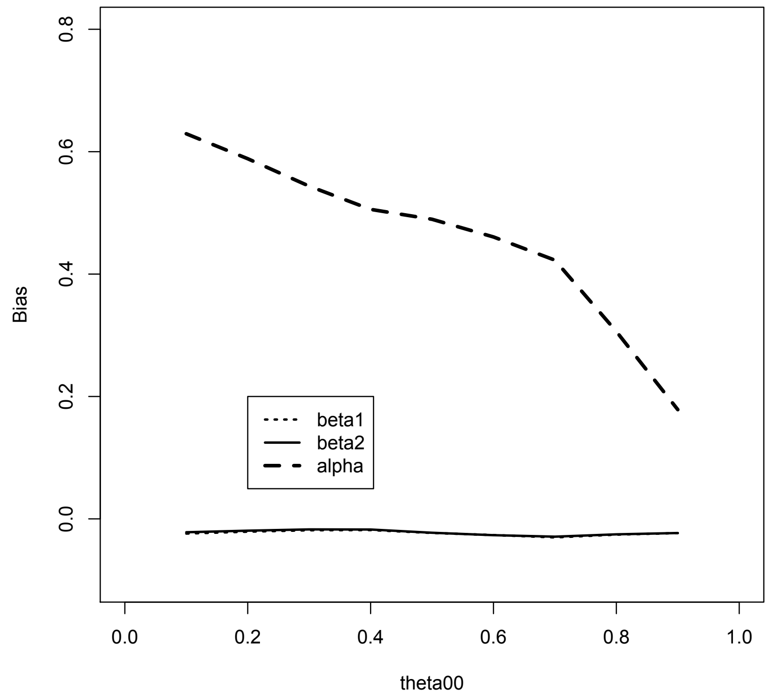

Figure 4.7: Bias of the naive estimators for different values of specificity

From Figure 4.7 and Table 4.8, it is obvious to find that the bias in the naive estimators of β_1 and β_2 stay almost unchanged, as the specificity increases from 0 to 1. However, the bias in the naive estimator of α decreases with the increase in specificity. More specifically, when θ_{00} changes from 0.7 to 0.9, the decrease in the bias speeds up. Hence, the higher the probability of correct classification of the failure, the smaller the bias in the naive estimator of α. Since the probability

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{00} = 0.1$	0.6293	0.0465	-0.0241	0.0107	-0.0218	0.0102
$\theta_{00} = 0.2$	0.5885	0.0566	-0.0210	0.0122	-0.0193	0.0117
$\theta_{00} = 0.3$	0.5437	0.0660	-0.0186	0.0133	-0.0172	0.0127
$\theta_{00} = 0.4$	0.5057	0.0744	-0.0183	0.0138	-0.0174	0.0131
$\theta_{00} = 0.5$	0.4895	0.0876	-0.0230	0.0148	-0.0227	0.0139
$\theta_{00} = 0.6$	0.4607	0.1010	-0.0264	0.0155	-0.0266	0.0145
$\theta_{00} = 0.7$	0.4227	0.1185	-0.0303	0.0162	-0.0291	0.0153
$\theta_{00} = 0.8$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.0150
$\theta_{00} = 0.9$	0.1785	0.1440	-0.0232	0.0151	-0.0231	0.0141

Table 4.8: Bias and Standard Error for the different values of specificity

of success in the population is set to be 0.4 in this scenario, increase in specificity increases the correct classification of the majority of the population. It should also be mentioned that the increase in the specificity enlarges the variability of the naive estimator of α, making it overall a more conservative estimator.

4.1.7 Scenario 7: Bias analysis for different values of θ_{11}

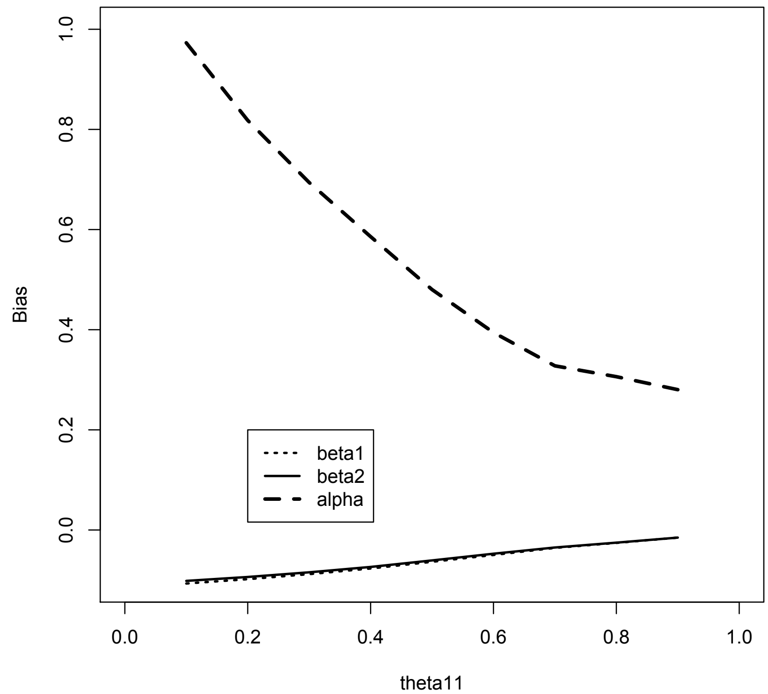

Figure 4.8: Bias of the naive estimators for different values of sensitivity

Figure 4.8 and Table 4.9 show that as sensitivity increases from zero to 1, the bias in the naive estimators of β_1 and β_2 has a small improvement from negative values to almost zero, making them better estimators. In addition, the bias of the naive estimator of α decreases from a positive value to smaller values, while its variability decreases. Therefore, increasing the sensitivity improves the overall estimation of all the coefficient parameters.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{11} = 0.1$	0.6293	0.0465	-0.0241	0.0107	-0.0218	0.0102
$\theta_{11} = 0.2$	0.5885	0.0566	-0.0210	0.0122	-0.0193	0.0117
$\theta_{11} = 0.3$	0.5437	0.0660	-0.0186	0.0133	-0.0172	0.0127
$\theta_{11} = 0.4$	0.5057	0.0744	-0.0183	0.0138	-0.0174	0.0131
$\theta_{11} = 0.5$	0.4895	0.0876	-0.0230	0.0148	-0.0227	0.0139
$\theta_{11} = 0.6$	0.4607	0.1010	-0.0264	0.0155	-0.0266	0.0145
$\theta_{11} = 0.7$	0.4227	0.1185	-0.0303	0.0162	-0.0291	0.0153
$\theta_{11} = 0.8$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.0150
$\theta_{11} = 0.9$	0.1785	0.1440	-0.0232	0.0151	-0.0231	0.0141

Table 4.9: Bias and Standard Error for the different values of sensitivity

4.1.8 Scenario 8: Bias analysis for different values of π

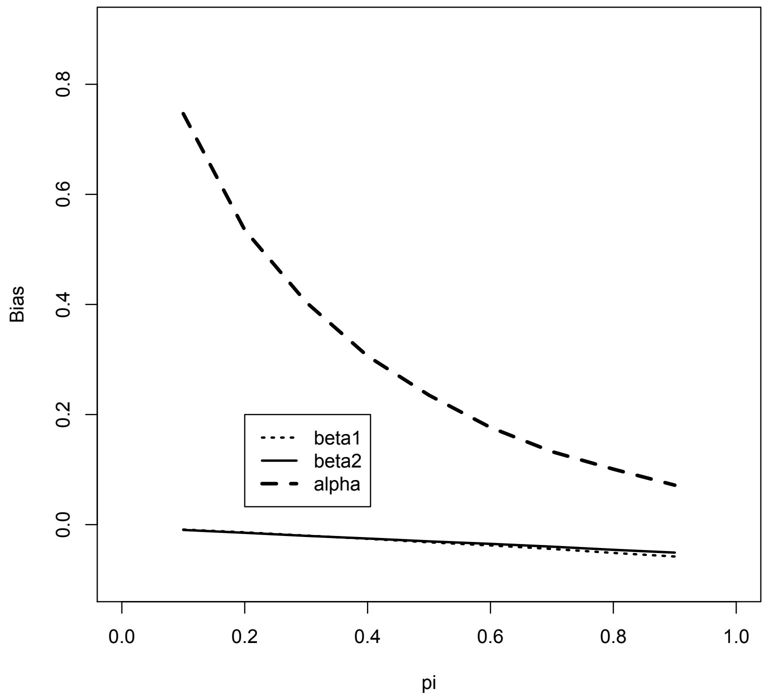

Figure 4.9: Bias of the naive estimators for different values of π

From Figure 4.9 and Table 4.10, we find that the bias in the naive estimators for the coefficient parameters of continuous predictors seems to remain almost unchanged as the probability of success in the classified predictor G increases from 0.1 to 0.9. However, it has impact on the bias in the naive estimator of the coefficient of the classified predictor. The bias in the naive estimator of α decreases from a positive value to almost zero. It means increasing the value

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\pi = 0.1$	0.7466	0.1305	-0.0090	0.0093	-0.0097	0.0084
$\pi = 0.2$	0.5355	0.1411	-0.0140	0.0125	-0.0149	0.0114
$\pi = 0.3$	0.4041	0.1402	-0.0200	0.0147	-0.0204	0.0137
$\pi = 0.4$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.0150
$\pi = 0.5$	0.2351	0.1201	-0.0321	0.0164	-0.0304	0.0159
$\pi = 0.6$	0.1765	0.1068	-0.0378	0.0164	-0.0350	0.0158
$\pi = 0.7$	0.1326	0.0921	-0.0442	0.0158	-0.0400	0.0153
$\pi = 0.8$	0.1008	0.0733	-0.0514	0.0138	-0.0458	0.0135
$\pi = 0.9$	0.0717	0.0526	-0.0580	0.0107	-0.0508	0.0105

Table 4.10: Bias and Standard Error for the different values of π

of π helps to reduce the bias in the naive estimator of α. This is because both sensitivity and specificity for this scenario are relatively high. It should also be mentioned that the reduction in the variability of $\hat{\alpha}$, makes an overall more precise estimator for α.

4.1.9 Scenario 9: Bias analysis for different values of n

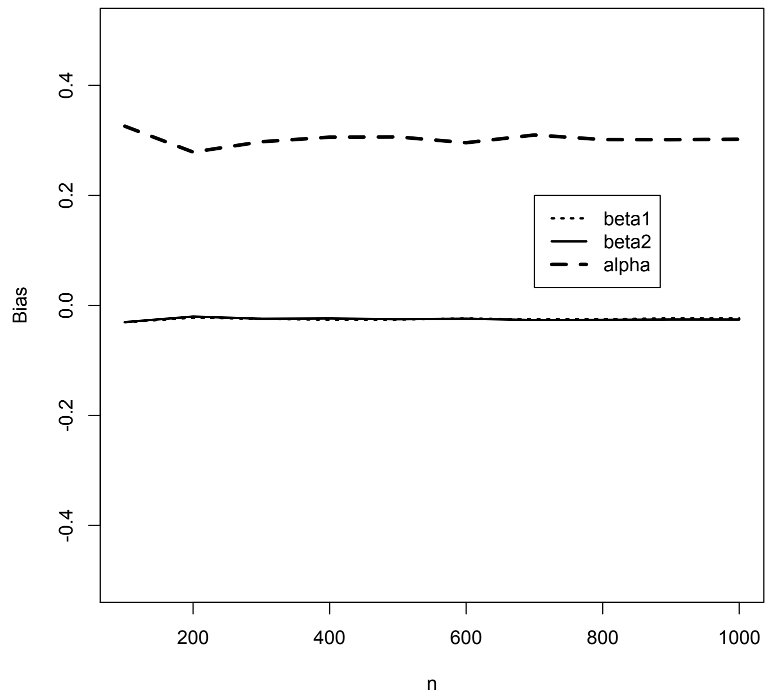

Figure 4.10: Bias of the Naive Estimators for different values of n

As Figure 4.10 and Table 4.11 present, the bias in the naive estimators of all the coefficient parameters seem to stay unchanged. More interestingly, it reduces the variabilities, making even worse estimators. Hence, increasing the sample size does not improve any of the naive estimators of the coefficient parameters.

2. **Categorical variable with three categories**

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$n = 100$	0.3257	0.2857	-0.0308	0.0383	-0.0304	0.0367
$n = 200$	0.2786	0.2030	-0.0224	0.0250	-0.0204	0.0235
$n = 300$	0.2974	0.1729	-0.0246	0.0207	-0.0244	0.0195
$n = 400$	0.3058	0.1451	-0.0261	0.0172	-0.0238	0.0162
$n = 500$	0.3062	0.1314	-0.0258	0.0158	-0.0253	0.0150
$n = 600$	0.2956	0.1198	-0.0239	0.0142	-0.0242	0.0140
$n = 700$	0.3097	0.1149	-0.0256	0.0134	-0.0268	0.0138
$n = 800$	0.3014	0.1043	-0.0252	0.0126	-0.0264	0.0132
$n = 900$	0.3012	0.0978	-0.0237	0.0112	-0.0258	0.0118
$n = 1000$	0.3019	0.0949	-0.0238	0.0106	-0.0258	0.0117

Table 4.11: Bias and Standard Error for the different values of n

Next, we consider three categories for the classified predictor. The common set ups for T, p, the continuous covariates, the regression parameters, the model error term and its lag correlation, and the random effects are the same as the binary case.

The categorical time-invariant G was generated from a trinomial distribution with probability $P(G = 1) = \pi_1 = 0.4$ and $P(G = 2) = \pi_2 = 0.4$, except in the scenarios that they changed. For the misclassification models, the categorical time-invariant G^* was generated from G, with conditional probabilities $\theta_{00} = 0.8$, $\theta_{01} = 0.1$, $\theta_{02} = 0.3$, and $\theta_{10} = 0.1$. Moreover, θ_{11} was set to 0.1, and θ_{22} to 0.4, except they changed in their scenarios, respectively. The other three conditional probabilities can be calculated by equation $\sum_{i=0}^{2} \theta_{ij} = 1$, for every $j = 0, 1, 2$.

Here again, we selected 500 samples for all scenarios. For each sample size, 1000 Monte Carlo replicates were simulated and the Monte-Carlo mean estimates and standard errors of the estimators were computed. For the following scenarios, we modify one model parameter at a time, while keeping other parameters unchanged. Table 4.12 shows the selected range, as well as the steps for every parameter.

Table 4.12: The range and step of the model parameters

Parameter	Range	Step
ρ_1	$(-1, 1)$	0.1
σ_a^2	$(0, 2)$	0.2
σ_γ^2	(0,2)	0.2
α	(-3,3)	0.5
β_1	(-3,3)	0.5
β_2	(-3,3)	0.5
θ_{00}	(0,1)	0.1
θ_{01}	(0,1)	0.1
θ_{02}	(0,0.6]	0.1
θ_{10}	(0,0.2]	0.01
θ_{11}	(0,1)	0.1
θ_{22}	(0,0.7]	0.1
π_1	(0,0.6]	0.1
π_2	(0,0.6]	0.1
n	(100,1000)	100

4.1.10 Scenario 1: Bias analysis for different values of ρ_1

As ρ_1, the autocorrelation lag parameter for the model error term, changes from -1 to 1, the absolute values of the bias of the three naive estimators slightly decreases. The variabilities of the naive estimators of β decrease as ρ_1 changes from -1 to 1, while the variability of the estimator of α has the opposite behaviour. (Figure 4.11 and Table 4.13)

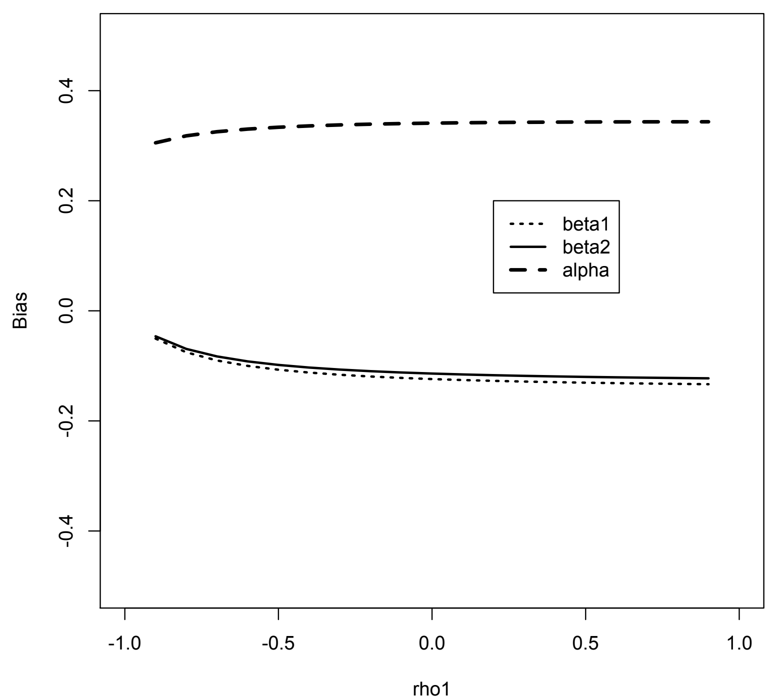

Figure 4.11: Bias of the naive estimators for different values of ρ_1

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\rho_1 = -0.9$	0.3053	0.0251	-0.0504	0.0050	-0.0463	0.0048
$\rho_1 = -0.8$	0.3179	0.0302	-0.0754	0.0084	-0.0692	0.0081
$\rho_1 = -0.7$	0.3254	0.0353	-0.0902	0.0113	-0.0829	0.0109
$\rho_1 = -0.6$	0.3302	0.0404	-0.1000	0.0139	-0.0919	0.0134
$\rho_1 = -0.5$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\rho_1 = -0.4$	0.3360	0.0506	-0.1122	0.0186	-0.1031	0.0180
$\rho_1 = -0.3$	0.3378	0.0557	-0.1162	0.0209	-0.1068	0.0202
$\rho_1 = -0.2$	0.3391	0.0607	-0.1193	0.0231	-0.1097	0.0224
$\rho_1 = -0.1$	0.3402	0.0658	-0.1219	0.0253	-0.1120	0.0246
$\rho_1 = 0$	0.3410	0.0708	-0.1240	0.0275	-0.1140	0.0267
$\rho_1 = 0.1$	0.3417	0.0758	-0.1258	0.0297	-0.1156	0.0288
$\rho_1 = 0.2$	0.3422	0.0808	-0.1272	0.0318	-0.1170	0.0309
$\rho_1 = 0.3$	0.3426	0.0858	-0.1285	0.0340	-0.1182	0.0330
$\rho_1 = 0.4$	0.3429	0.0908	-0.1296	0.0361	-0.1192	0.0351
$\rho_1 = 0.5$	0.3431	0.0957	-0.1306	0.0383	-0.1201	0.0372
$\rho_1 = 0.6$	0.3433	0.1007	-0.1314	0.0404	-0.1208	0.0393
$\rho_1 = 0.7$	0.3434	0.1057	-0.1321	0.0426	-0.1215	0.0414
$\rho_1 = 0.8$	0.3435	0.1107	-0.1328	0.0447	-0.1221	0.0435
$\rho_1 = 0.9$	0.3435	0.1156	-0.1333	0.0468	-0.1226	0.0456

Table 4.13: Bias and standard error of the naive estimators for the different values of ρ_1

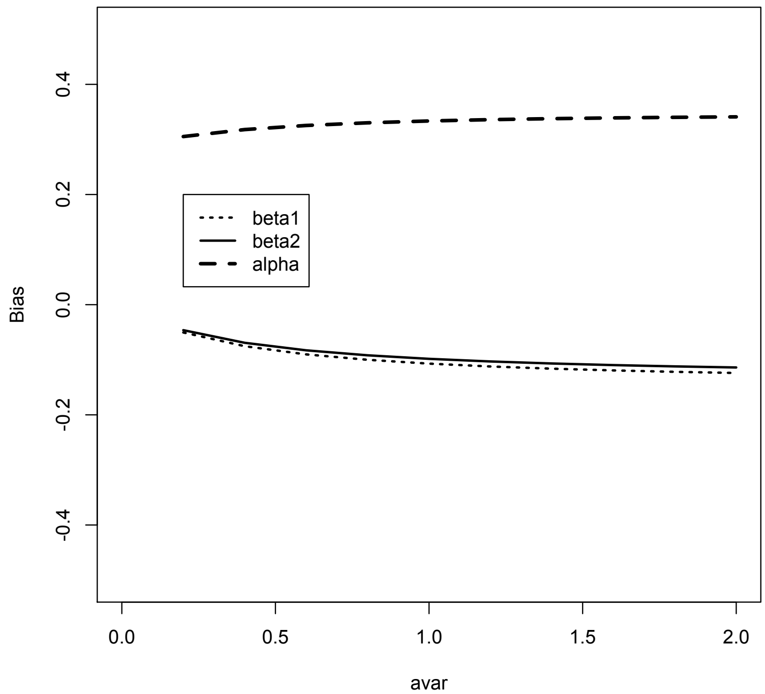

Figure 4.12: Bias of the naive estimators for different values of the variability of model error term

4.1.11 Scenario 2: Bias analysis for different values of σ_a^2

In this scenario, we observe a similar trend to the last scenario. More specifically, when the variance of a_{it} changes from 0 to 2, Figure 4.12 and Table 4.14 show that the bias in the naive estimators of β_1 and β_2 decrease slightly. However, the bias in the naive estimator of α increases slightly. It also seems to enlarge the variabilities of the naive estimators of β_1, β_2, and α.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma_a^2 = 0.2$	0.3053	0.0251	-0.0504	0.0050	-0.0463	0.0048
$\sigma_a^2 = 0.4$	0.3179	0.0302	-0.0754	0.0084	-0.0692	0.0081
$\sigma_a^2 = 0.6$	0.3254	0.0353	-0.0902	0.0113	-0.0829	0.0109
$\sigma_a^2 = 0.8$	0.3302	0.0404	-0.1000	0.0139	-0.0919	0.0134
$\sigma_a^2 = 1.0$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\sigma_a^2 = 1.2$	0.3360	0.0506	-0.1122	0.0186	-0.1031	0.0180
$\sigma_a^2 = 1.4$	0.3378	0.0557	-0.1162	0.0209	-0.1068	0.0202
$\sigma_a^2 = 1.6$	0.3391	0.0607	-0.1193	0.0231	-0.1097	0.0224
$\sigma_a^2 = 1.8$	0.3402	0.0658	-0.1219	0.0253	-0.1120	0.0246
$\sigma_a^2 = 2.0$	0.3410	0.0708	-0.1240	0.0275	-0.1140	0.0267

Table 4.14: Bias and standard error of the naive estimators for the different values of the variability of model error term

4.1.12 Scenario 3: Bias analysis for different values of σ_γ^2

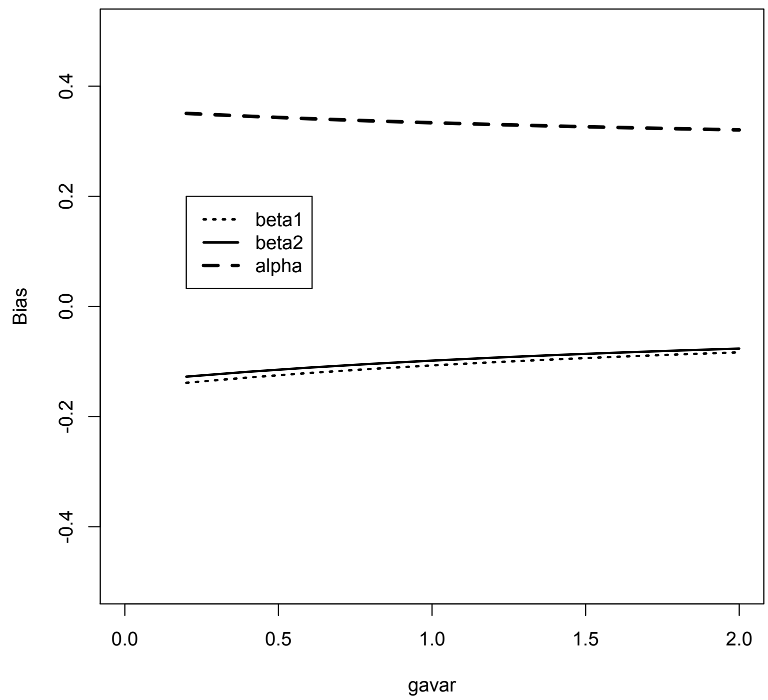

Figure 4.13: Bias of the Naive Estimators for different value of the variability in the random effect

Figure 4.13 and Table 4.15 show that the bias in all the naive estimators decrease, as we increase the variance of γ from 0 to 2. More specifically, the estimators of β_1 and β_2 with downward bias, and the naive estimator of α with an upward bias, decline with the increase in σ_γ^2. We can also observe that the change in the variance of γ slightly increases the variabilities of the three estimators.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma_\gamma^2 = 0.2$	0.3506	0.0340	-0.1386	0.0160	-0.1274	0.0154
$\sigma_\gamma^2 = 0.4$	0.3454	0.0368	-0.1291	0.0160	-0.1186	0.0154
$\sigma_\gamma^2 = 0.6$	0.3410	0.0397	-0.1208	0.0161	-0.1110	0.0155
$\sigma_\gamma^2 = 0.8$	0.3370	0.0426	-0.1135	0.0162	-0.1043	0.0156
$\sigma_\gamma^2 = 1.0$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\sigma_\gamma^2 = 1.2$	0.3304	0.0485	-0.1012	0.0164	-0.0930	0.0159
$\sigma_\gamma^2 = 1.4$	0.3276	0.0515	-0.0961	0.0165	-0.0882	0.0160
$\sigma_\gamma^2 = 1.6$	0.3250	0.0545	-0.0914	0.0167	-0.0839	0.0161
$\sigma_\gamma^2 = 1.8$	0.3227	0.0575	-0.0871	0.0168	-0.0800	0.0163
$\sigma_\gamma^2 = 2.0$	0.3206	0.0605	-0.0833	0.0169	-0.0764	0.0164

Table 4.15: Bias and standard error of the naive estimators for the different values of the variability in the random effect

4.1.13 Scenario 4: Bias analysis for different values of α

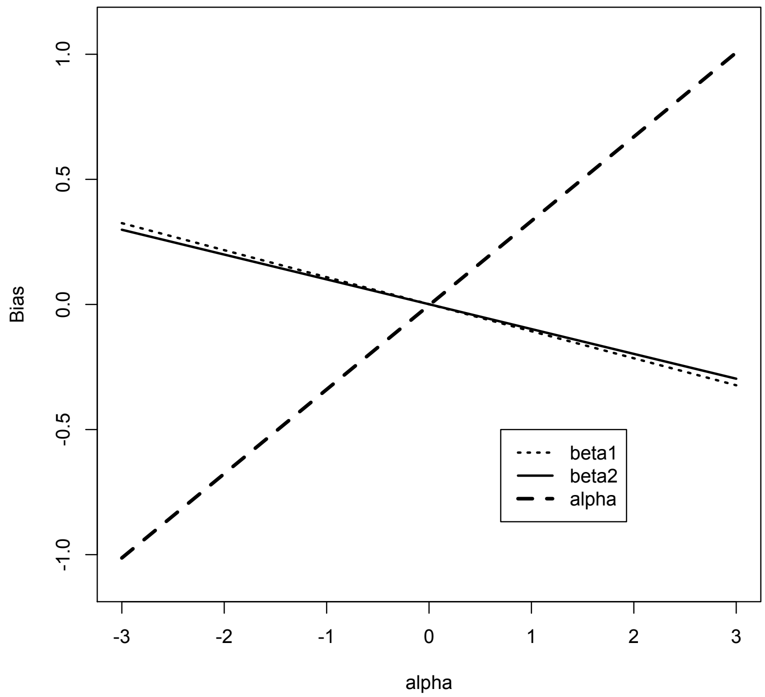

Figure 4.14: Bias of the naive estimators for different values of α

Figure 4.14 and Table 4.16 present dramatic change in the bias of the naive estimators. It is clear to see that the bias in the naive estimators of β_1 and β_2 decrease, significantly, from positive values to negative values, as α increases from -3 to 3. However, the bias in the naive estimate of α increases significantly, from -1 to 1. We can see that when $\alpha = 0$, the bias in the naive estimators of β_1, β_2, and α approach zero. The change in α, however, does not seem to affect the variabilities of the three

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\alpha = -3.0$	-1.0132	0.0547	0.3251	0.0248	0.2987	0.0226
$\alpha = -2.5$	-0.8449	0.0516	0.2711	0.0221	0.2491	0.0202
$\alpha = -2.0$	-0.6765	0.0489	0.2171	0.0196	0.1994	0.0181
$\alpha = -1.5$	-0.5082	0.0467	0.1631	0.0175	0.1498	0.0163
$\alpha = -1.0$	-0.3398	0.0451	0.1090	0.0158	0.1002	0.0149
$\alpha = -0.5$	-0.1715	0.0442	0.0550	0.0148	0.0506	0.0141
$\alpha = 0$	-0.0031	0.0440	0.0010	0.0145	0.0009	0.0140
$\alpha = 0.5$	0.1652	0.0444	-0.0530	0.0150	-0.0487	0.0146
$\alpha = 1.0$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\alpha = 1.5$	0.5019	0.0473	-0.1610	0.0181	-0.1479	0.0174
$\alpha = 2.0$	0.6702	0.0496	-0.2150	0.0204	-0.1976	0.0194
$\alpha = 2.5$	0.8386	0.0524	-0.2691	0.0229	-0.2472	0.0217
$\alpha = 3.0$	1.0069	0.0557	-0.3231	0.0257	-0.2968	0.0242

Table 4.16: Bias and standard error of the naive estimators for the different values of α

estimators.

4.1.14 Scenario 5: Bias analysis for different values of β_1 and β_2

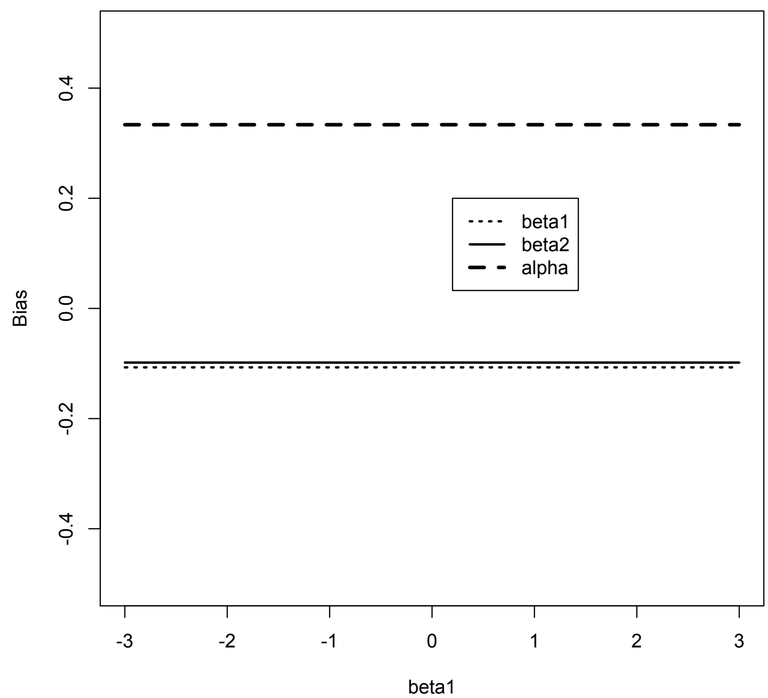

Figure 4.15: Bias of the naive estimators for different values of β_1

Similar to the binary case, change in the coefficient parameter of the continuous predictor has no impact on the bias in the naive estimators. (Figures 4.15 and 4.15, and Tables 4.17 and 4.18)

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\beta_1 = -3.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = -2.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = -2.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = -1.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = -1.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = -0.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = 0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = 0.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = 1.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = 1.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = 2.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = 2.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_1 = 3.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157

Table 4.17: Bias and standard error of the naive estimators for the different values of β_1

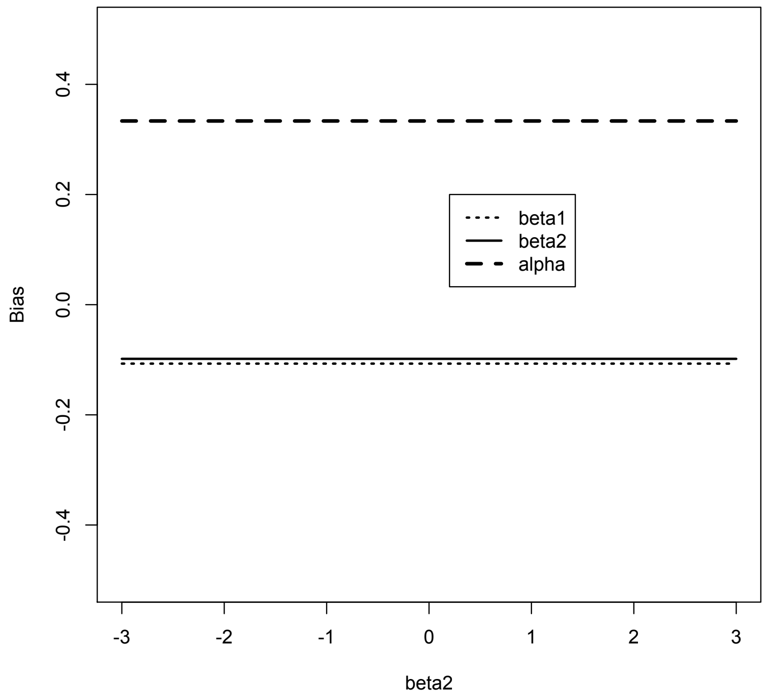

Figure 4.16: Bias of the naive estimators for different values of β_2

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\beta_2 = -3.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = -2.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = -2.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = -1.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = -1.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = -0.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = 0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = 0.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = 1.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = 1.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = 2.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = 2.5$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157
$\beta_2 = 3.0$	0.3335	0.0455	-0.107	0.0163	-0.0983	0.0157

Table 4.18: Bias and standard error of the naive estimators for the different values of β_2

4.1.15 Scenario 6: Bias analysis for different values of θ_{00}

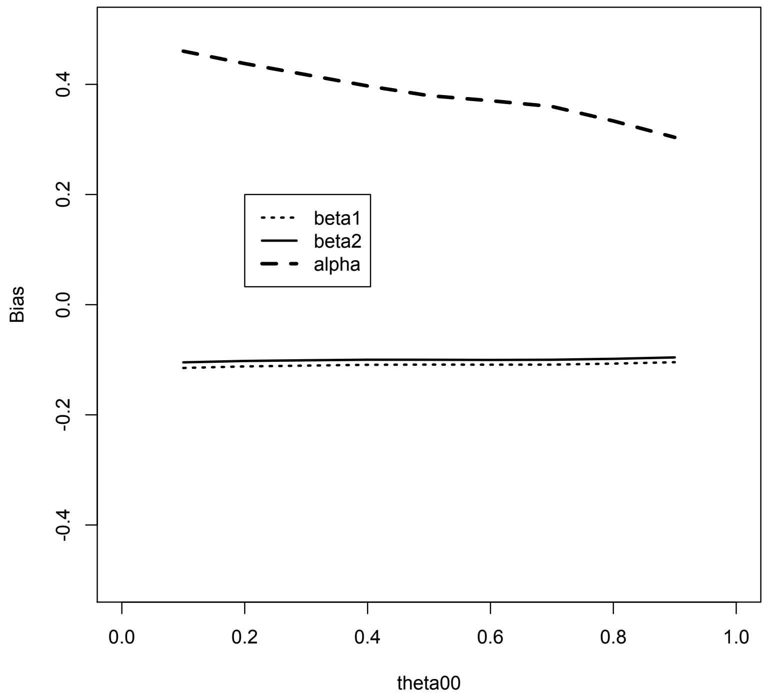

Figure 4.17: Bias of the Naive Estimators for different values of θ_{00}

In this scenario, as Figure 4.17 and Table 4.19 show, we observe that the bias in the naive estimators of β_1 and β_2 seem to stay unchanged, as θ_{00} increases from 0 to 1. However, the bias in the naive estimator of α decreases with the increase in θ_{00}. Since θ_{00} is the parameter associated with the classified predictor, its change seems to only affect the estimation of the coefficient parameter of the classified predictor. It does not, however, affect the variabilities.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{00} = 0.1$	0.4603	0.0333	-0.1149	0.0141	-0.1047	0.0135
$\theta_{00} = 0.2$	0.4378	0.0363	-0.1121	0.0150	-0.1022	0.0143
$\theta_{00} = 0.3$	0.4175	0.0388	-0.1106	0.0157	-0.1010	0.0151
$\theta_{00} = 0.4$	0.3971	0.0411	-0.1091	0.0161	-0.0999	0.0154
$\theta_{00} = 0.5$	0.3795	0.0430	-0.1088	0.0161	-0.1000	0.0155
$\theta_{00} = 0.6$	0.3703	0.0446	-0.1089	0.0163	-0.1003	0.0156
$\theta_{00} = 0.7$	0.3598	0.0447	-0.1088	0.0164	-0.1000	0.0158
$\theta_{00} = 0.8$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\theta_{00} = 0.9$	0.3037	0.0466	-0.1045	0.0162	-0.0958	0.0157

Table 4.19: Bias and standard error of the naive estimators for the different values of θ_{00}

4.1.16 Scenario 7: Bias analysis for different values of θ_{01}

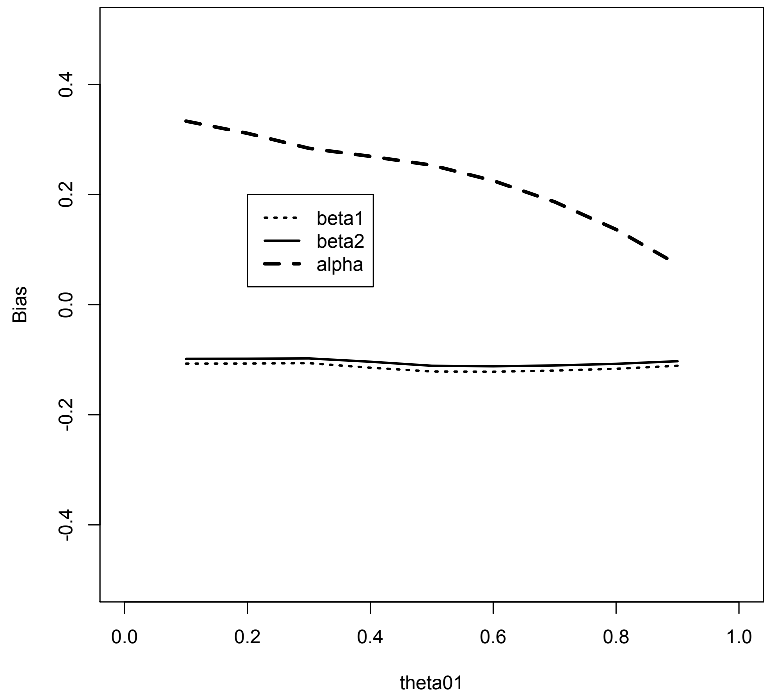

Figure 4.18: Bias of the naive estimators for different values of θ_{01}

Figure 4.18 and Table 4.20 show that the bias in the naive estimators of β_1 and β_2 stay unchanged as θ_{01} increases from 0 to 1. Nevertheless, the bias in the naive estimator of α decreases with the increase in θ_{01}. This may sound surprising, as θ_{01} is a probability of misclassifying $G = 1$ when $G^* = 0$. However, in our setup, as θ_{01} increases, θ_{21} decreases. For example, when $\theta_{01} = 0.9$, (since $\theta_{11} = 0.1$), θ_{21} would be only 0.1. In our setup, $P(G = 2) = 0.4$ and $P(G = 0) = 0.2$. This means that for

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{01} = 0.1$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\theta_{01} = 0.2$	0.3114	0.0468	-0.1069	0.0161	-0.0981	0.0156
$\theta_{01} = 0.3$	0.2842	0.0473	-0.1062	0.0157	-0.0976	0.0151
$\theta_{01} = 0.4$	0.2696	0.0497	-0.1145	0.0158	-0.1036	0.0152
$\theta_{01} = 0.5$	0.2535	0.0505	-0.1214	0.0150	-0.1109	0.0146
$\theta_{01} = 0.6$	0.2253	0.0524	-0.1217	0.0149	-0.1119	0.0145
$\theta_{01} = 0.7$	0.1869	0.0561	-0.1197	0.0153	-0.1104	0.0149
$\theta_{01} = 0.8$	0.1367	0.0594	-0.1164	0.0155	-0.1074	0.0151
$\theta_{01} = 0.9$	0.0731	0.0649	-0.1109	0.0162	-0.1027	0.0157

Table 4.20: Bias and standard error of the naive estimators for the different values of θ_{01}

the majority of the population, the misclassification rate is low. That is why we have lower bias in this case. This parameter slightly increases the variability in the naive estimate of α. However, it does not seem to affect the variabilities in the estimates of the β parameters.

4.1.17 Scenario 8: Bias analysis for different values of θ_{02}

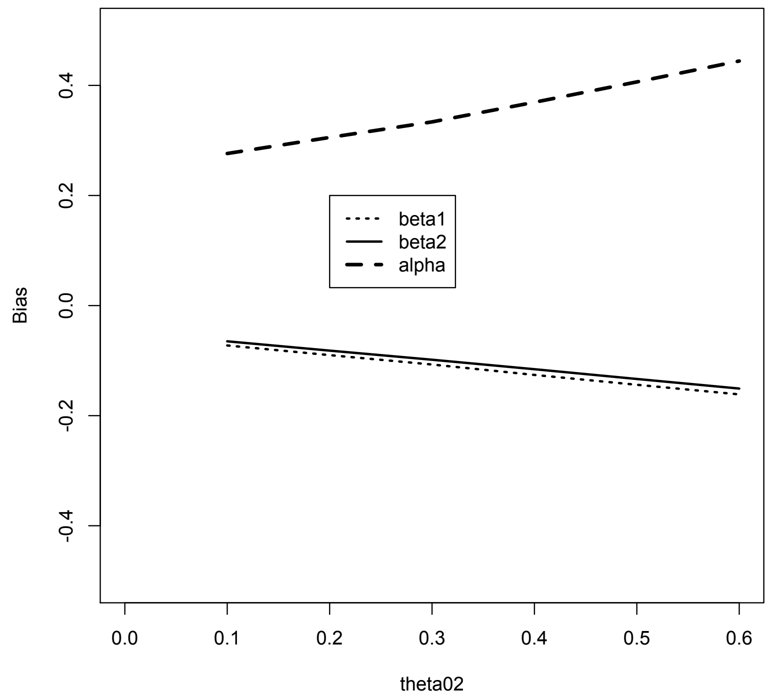

Figure 4.19: Bias of the naive estimators for different values of θ_{02}

It is interesting to find that when we change θ_{02} from 0.1 to 0.6, the absolute value of the bias in the naive estimators of β_1 and β_2 increase. Meanwhile, (as it may be expected), the bias in the naive estimator of α increases with the increase in θ_{02}. Figure 4.19 and Table 4.21 present these results. It does not, however, seem to affect the variabilities.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{02} = 0.1$	0.2763	0.0430	-0.0725	0.0154	-0.0649	0.0149
$\theta_{02} = 0.2$	0.3055	0.0443	-0.0900	0.0159	-0.0819	0.0154
$\theta_{02} = 0.3$	0.3337	0.0456	-0.1071	0.0163	-0.0984	0.0158
$\theta_{02} = 0.4$	0.3694	0.0467	-0.1260	0.0167	-0.1156	0.0161
$\theta_{02} = 0.5$	0.4063	0.0480	-0.1439	0.0172	-0.1334	0.0164
$\theta_{02} = 0.6$	0.4443	0.0495	-0.1614	0.0176	-0.1508	0.0168

Table 4.21: Bias and standard error of the naive estimators for the different values of θ_{02}

4.1.18 Scenario 9: Bias analysis for different values of θ_{10}

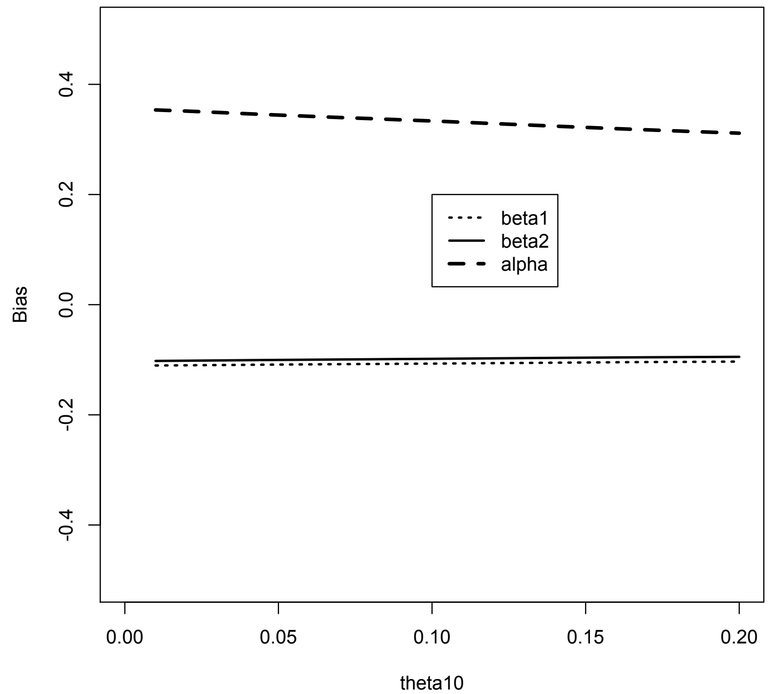

Figure 4.20: Bias of the Naive Estimators for different values of θ_{10}

Because of the setups for probability of misclassifications given $G = 0$, θ_{10} can only change from 0 to 0.2. The change in the bias of the parameters may not be clear, as the range of the values are small. We could still observe some of the patterns from Figure 4.20 and Table 4.22. The bias in the naive estimators of β_1 and β_2 seem to stay unchanged, as θ_{10} increases from 0 to 0.2. However, the bias in the naive estimator of α decreases with the increase in θ_{10}. This could be explained the same

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{10} = 0.01$	0.3536	0.0453	-0.1104	0.0163	-0.1021	0.0158
$\theta_{10} = 0.02$	0.3515	0.0454	-0.1101	0.0163	-0.1017	0.0158
$\theta_{10} = 0.03$	0.3491	0.0454	-0.1096	0.0163	-0.1012	0.0158
$\theta_{10} = 0.04$	0.3466	0.0454	-0.1092	0.0163	-0.1007	0.0158
$\theta_{10} = 0.05$	0.3442	0.0455	-0.1087	0.0163	-0.1003	0.0158
$\theta_{10} = 0.06$	0.3421	0.0455	-0.1084	0.0163	-0.0999	0.0158
$\theta_{10} = 0.07$	0.3398	0.0456	-0.1080	0.0163	-0.0995	0.0158
$\theta_{10} = 0.08$	0.3378	0.0457	-0.1077	0.0163	-0.0991	0.0158
$\theta_{10} = 0.09$	0.3356	0.0456	-0.1073	0.0163	-0.0987	0.0158
$\theta_{10} = 0.10$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\theta_{10} = 0.11$	0.3312	0.0455	-0.1066	0.0163	-0.0979	0.0157
$\theta_{10} = 0.12$	0.3288	0.0457	-0.1062	0.0163	-0.0974	0.0157
$\theta_{10} = 0.13$	0.3265	0.0457	-0.1058	0.0163	-0.0970	0.0157
$\theta_{10} = 0.14$	0.3241	0.0457	-0.1053	0.0163	-0.0966	0.0157
$\theta_{10} = 0.15$	0.3219	0.0457	-0.1049	0.0163	-0.0962	0.0157
$\theta_{10} = 0.16$	0.3196	0.0458	-0.1045	0.0163	-0.0958	0.0157
$\theta_{10} = 0.17$	0.3174	0.0459	-0.1042	0.0163	-0.0955	0.0157
$\theta_{10} = 0.18$	0.3154	0.0460	-0.1039	0.0163	-0.0952	0.0157
$\theta_{10} = 0.19$	0.3133	0.0460	-0.1036	0.0163	-0.0950	0.0157
$\theta_{10} = 0.20$	0.3114	0.0461	-0.1033	0.0163	-0.0947	0.0157

Table 4.22: Bias and standard error of the naive estimators for the different values of θ_{10}

way as we did in Scenario 7 for θ_{01}.

4.1.19 Scenario 10: Bias analysis for different values of θ_{11}

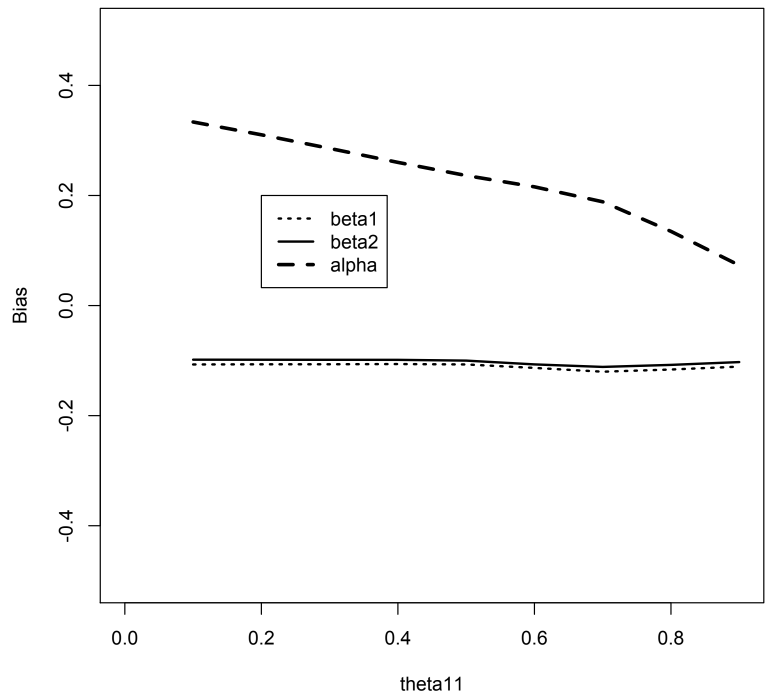

Figure 4.21: Bias of the naive estimators for different values of θ_{11}

In this scenario, we observe θ_{11} from 0.1 to 0.9. From Figure 4.21 and Table 4.23, with the increase in θ_{11}, the bias in the naive estimators of β_1 and β_2 seem to remain unchanged. The bias in the naive estimator of α, however, decreases. This could be expected, as it is the probability of correct classification of category one. This parameter, as well as θ_{10}, does not affect the variabilities of the three estimators.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{11} = 0.1$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\theta_{11} = 0.2$	0.3103	0.0482	-0.1067	0.0166	-0.0984	0.0160
$\theta_{11} = 0.3$	0.2856	0.0503	-0.1066	0.0167	-0.0985	0.0161
$\theta_{11} = 0.4$	0.2602	0.0530	-0.1062	0.0169	-0.0986	0.0164
$\theta_{11} = 0.5$	0.2363	0.0553	-0.1070	0.0169	-0.1000	0.0164
$\theta_{11} = 0.6$	0.2157	0.0582	-0.1133	0.0174	-0.1069	0.0167
$\theta_{11} = 0.7$	0.1886	0.0609	-0.1202	0.0166	-0.1115	0.0162
$\theta_{11} = 0.8$	0.1348	0.0617	-0.1162	0.0161	-0.1078	0.0157
$\theta_{11} = 0.9$	0.0731	0.0649	-0.1109	0.0162	-0.1027	0.0157

Table 4.23: Bias and standard error of the naive estimators for the different values of θ_{11}

4.1.20 Scenario 11: Bias analysis for different values of θ_{22}

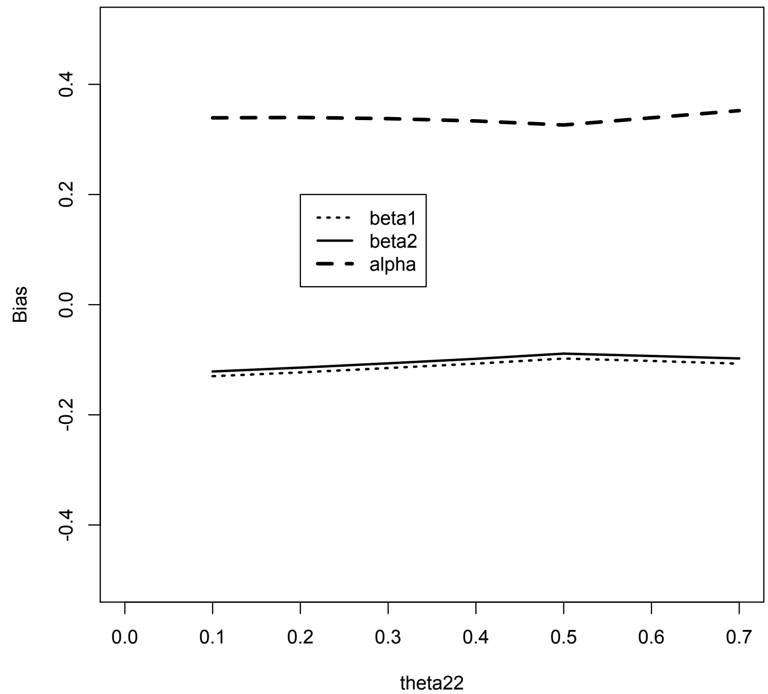

Figure 4.22: Bias of the naive estimators for different values of θ_{22}

It is surprising (as it is the probability of correct classification of $G = 2$), to find that the absolute value of the bias in the naive estimators of the three parameters remains (more or less) unchanged as θ_{22} increases from 0 to 0.7. (Figure 4.22 and Table 4.24) This may be explained by the fact that even with the maximum value of θ_{22}, we still have 30% misclassification in $G = 0$, as in this case $\theta_{02} = 0.3$ and $\theta_{12} = 0$. In a different setup, it could improve the naive estimate of α (similar to θ_{11}).

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{22} = 0.1$	0.3392	0.0532	-0.1299	0.0173	-0.1213	0.0166
$\theta_{22} = 0.2$	0.3399	0.0505	-0.1229	0.0170	-0.1143	0.0164
$\theta_{22} = 0.3$	0.3378	0.0480	-0.1151	0.0166	-0.1065	0.0161
$\theta_{22} = 0.4$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\theta_{22} = 0.5$	0.3262	0.0442	-0.0978	0.0160	-0.0889	0.0155
$\theta_{22} = 0.6$	0.3393	0.0450	-0.1021	0.0179	-0.0932	0.0170
$\theta_{22} = 0.7$	0.3523	0.0447	-0.1071	0.0178	-0.0976	0.0171

Table 4.24: Bias and standard error of the naive estimators for the different values of θ_{22}

4.1.21 Scenario 12: Bias analysis for different values of π_1

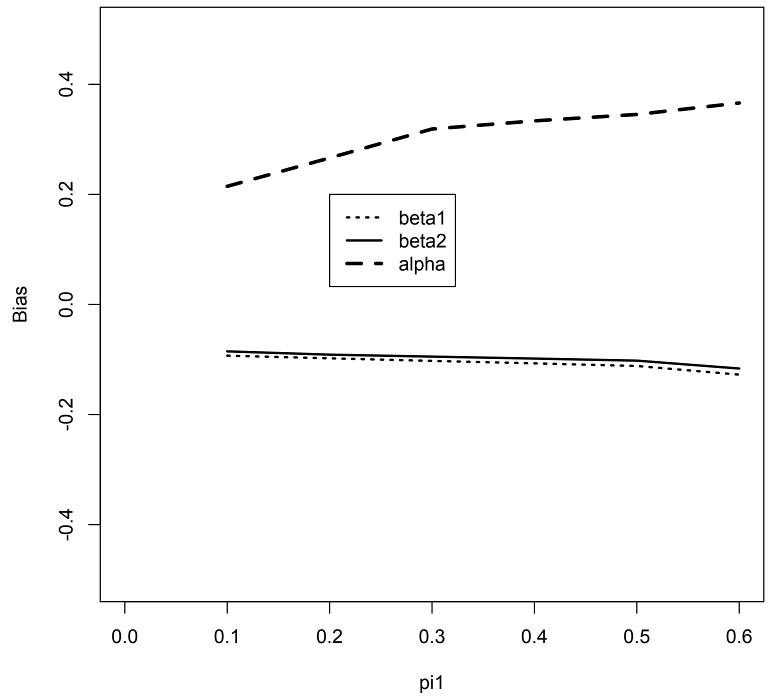

Figure 4.23: Bias of the naive estimators for different values of π_1

From Figure 4.23 and Table 4.25, we find that the bias in the naive estimators for the coefficient parameters of continuous predictors seems to stay unchanged, as the probability of classified predictor $G = 1$ increases from 0.1 to 0.6. The change in the probability of classified predictor $G = 1$ does not affect the bias in the naive estimators of the coefficient parameters of continuous predictors. However, the probability of success of classified predictor $G = 1$ has impact on the bias in the naive estimator of

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\pi_1 = 0.1$	0.2146	0.0819	-0.0931	0.0164	-0.0853	0.0161
$\pi_1 = 0.2$	0.2662	0.0720	-0.0979	0.0174	-0.0912	0.0168
$\pi_1 = 0.3$	0.3188	0.0569	-0.1025	0.0176	-0.0948	0.0169
$\pi_1 = 0.4$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\pi_1 = 0.5$	0.3452	0.0363	-0.1118	0.0148	-0.1021	0.0142
$\pi_1 = 0.6$	0.3660	0.0304	-0.1275	0.0156	-0.1164	0.0148

Table 4.25: Bias and standard error of the naive estimators for the different values of π_1

the coefficient parameter of the classified predictor. The bias in the naive estimator of α enlarges and the variability declines, as π_1 increases. This makes the naive estimate of α a very poor estimator for the parameter.

4.1.22 Scenario 13: Bias analysis for different values of π_2

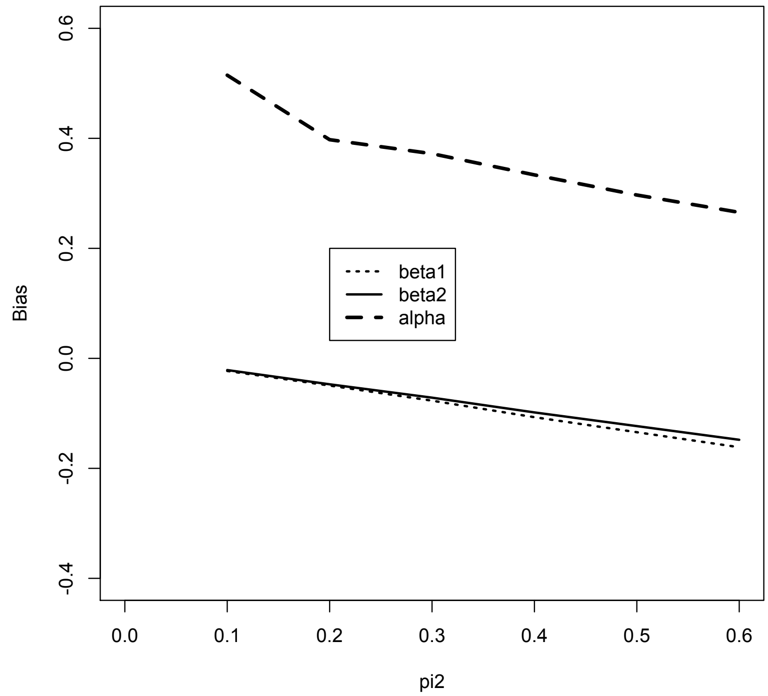

Figure 4.24: Bias of the naive estimators for different values of π_2

We are surprised to find that the probability of classified predictor $G = 2$ not only increases the bias in the naive estimate of α, but also affects the bias in the naive estimators of the coefficient parameters of continuous predictors in the same way, as it increases from 0.1 to 0.6. Similar to the case for π_1, it also decreases the variability of the naive estimate of α. (Figure 4.24 and Table 4.26)

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\pi_2 = 0.1$	0.5148	0.1001	-0.0229	0.0124	-0.0213	0.0120
$\pi_2 = 0.2$	0.3976	0.0821	-0.0494	0.0157	-0.0472	0.0150
$\pi_2 = 0.3$	0.3721	0.0614	-0.0769	0.0176	-0.0715	0.0168
$\pi_2 = 0.4$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$\pi_2 = 0.5$	0.2970	0.0389	-0.1343	0.0150	-0.1232	0.0142
$\pi_2 = 0.6$	0.2654	0.0301	-0.1616	0.0126	-0.1481	0.0118

Table 4.26: Bias and standard error of the naive estimators for the different values of π_2

4.1.23 Scenario 14: Bias analysis for different values of n

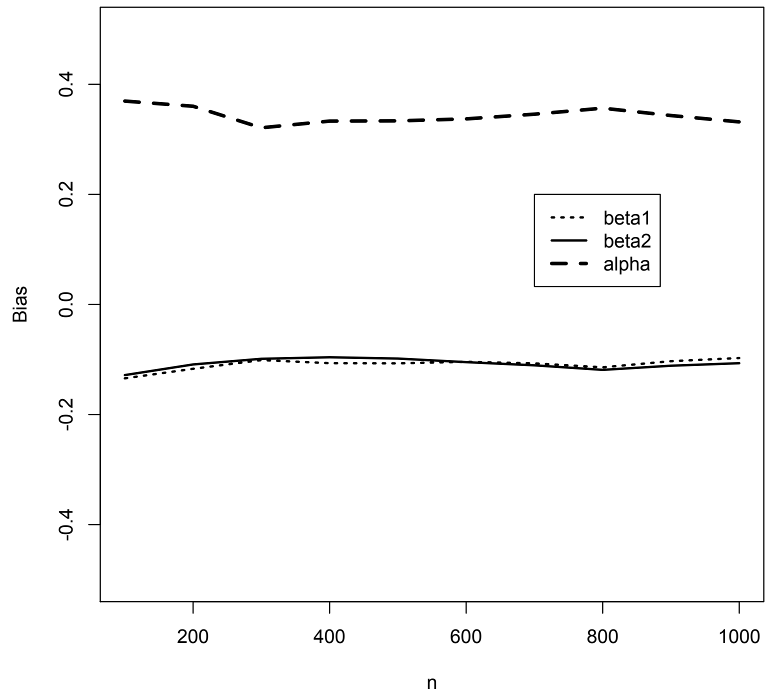

Figure 4.25: Bias of the naive estimators for different values of n

Last but not the least, we observe the behaviour of the naive estimates of the parameters for the case that the sample size changes from 100 to 1000. From Figure 4.25 and Table 4.27, as we could expect, the bias in the naive estimators of all the coefficient parameters remain (more or less) unchanged. Therefore, unfortunately, the increase in sample size does not help with the naive estimation of the coefficient parameters. More interestingly, it decreases the variabilities, which makes very poor

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$n = 100$	0.3695	0.1073	-0.1342	0.0400	-0.1283	0.0398
$n = 200$	0.3601	0.0715	-0.1169	0.0260	-0.1090	0.0235
$n = 300$	0.3210	0.0609	-0.1011	0.0208	-0.0987	0.0204
$n = 400$	0.3332	0.0515	-0.1067	0.0174	-0.0959	0.0168
$n = 500$	0.3335	0.0455	-0.1070	0.0163	-0.0983	0.0157
$n = 600$	0.3371	0.0427	-0.1042	0.0144	-0.1048	0.0146
$n = 700$	0.3457	0.0406	-0.1070	0.0138	-0.1106	0.0143
$n = 800$	0.3566	0.0372	-0.1142	0.0132	-0.1188	0.0140
$n = 900$	0.3432	0.0347	-0.1031	0.0119	-0.1113	0.0127
$n = 1000$	0.3317	0.0331	-0.0973	0.0111	-0.1067	0.0120

Table 4.27: Bias and standard error of the naive estimators for the different values of n

estimates for the parameters.

Chapter 5

Longitudinal Linear Mixed-effects Model with Measurement Error and Misclassification

In this chapter, we consider the most challenging case, where model (2.1) has both measurement error (ME) and misclassification. In this model, the true covariates, X_{it} and G_i are not observed, and instead, W_{it} and G_i^* are observed with error, respectively. By the model assumptions and the law of iterative expectation, we can write the marginal moments of the response as follows.

$$
\begin{aligned}
& E(y_{it}|W_{it}, G_i^*) \\
= \; & E_{X|W}(E(y_{it}|X_{it}, W_{it}, G_i^*)|W_{it}, G_i^*) \\
= \; & E_{G|G^*}(E_{X|W}(E(y_{it}|X_{it}, W_{it}, G_i, G_i^*)|X_{it}, W_{it}, G_i^*)|W_{it}, G_i^*) \\
= \; & E_{G|G^*}(E_{X|W}(E(y_{it}|X_{it}, G_i)|X_{it}, W_{it}, G_i^*)|W_{it}, G_i^*) \qquad (5.1) \\
= \; & E_{G|G^*}(E_{X|W}(X_{it}'\beta + \alpha G_i|X_{it}, W_{it}, G_i^*)|W_{it}, G_i^*) \\
= \; & E_{G|G^*}((E_{X|W}(X_{it}'\beta|W_{it}) + \alpha G_i)|W_{it}, G_i^*) \\
= \; & E(X_{it}'|W_{it})\beta + \alpha E(G_i|G_i^*)
\end{aligned}
$$

Equation (5.1) comes from the assumption that both G_i^* and W_i are surrogates. We continue with similar techniques to calculate the marginal variance and covariance of the response.

$$
\begin{aligned}
& var(y_{it}|W_{it}, G_i^*) \\
= \quad & var_{X|W}(E(y_{it}|X_{it}, W_{it}, G_i^*)|W_{it}, G_i^*) + E_{X|W}(var(y_{it}|X_{it}, W_{it}, G_i^*)|W_{it}, G_i^*) \\
= \quad & var_{X|W}(E(y_{it}|X_{it}, G_i^*)|W_{it}, G_i^*) + E_{X|W}(var(y_{it}|X_{it}, G_i^*)|W_{it}, G_i^*) \qquad (5.2) \\
= \quad & var_{X|W}((X_{it}'\beta + \alpha E(G_i|G_i^*))|W_{it}, G_i^*) \\
+ \quad & E_{X|W}(var(G_i|G_i^*)\alpha^2 + \sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}|W_{it}, G_i^*) \qquad (5.3) \\
= \quad & \beta' var(X_{it}'|W_{it})\beta + \alpha^2 var(G_i|G_i^*) + \sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}
\end{aligned}
$$

Equation (5.2) is true, since W_i is assumed to be surrogate. Moreover, equation (5.3) is true because of equations (4.2) and (4.14).

When $t \neq u$, we have

$$
\begin{aligned}
& cov(y_{it}, y_{iu}|W_{it}, G_i^*) \\
= \;& cov_{X|W}(E((y_{it}|X_{it}, W_{it}, G_i^*)|W_{it}, G_i^*), E((y_{iu}|X_{iu}, W_{iu}, G_i^*)|W_{iu}, G_i^*) \\
+ \;& E_{X|W}(cov(((y_{it}|X_{it}, W_{it}, G_i^*)|W_{it}, G_i^*), ((y_{iu}|X_{iu}, W_{iu}, G_i^*)|W_{iu}, G_i^*))) \quad (5.4) \\
= \;& cov_{X|W}(E((y_{it}|X_{it}, G_i^*)|W_{it}, G_i^*), E((y_{iu}|X_{iu}, G_i^*)|W_{iu}, G_i^*) \\
+ \;& E_{X|W}(cov(((y_{it}|X_{it}, G_i^*)|W_{it}, G_i^*), ((y_{iu}|X_{iu}, G_i^*)|W_{iu}, G_i^*))) \quad (5.5) \\
= \;& cov_{X|W}((X_{it}'\beta + \alpha E(G_i|G_i^*)|W_{it}, G_i^*), (X_{iu}'\beta + \alpha E(G_i|G_i^*)|W_{iu}, G_i^*)) \\
+ \;& E_{X|W}(\alpha^2 var(G_i|G_i^*) + \sigma_\gamma^2 + \frac{\sigma_a^2\rho^{|t-u|}}{1-\rho^2}|W_{it}, W_{iu}, G_i^*) \quad (5.6) \\
= \;& \beta' cov((X_{it}', X_{iu}')|W_{it}, W_{iu})\beta + var(G_i|G_i^*)\alpha^2 + \sigma_\gamma^2 + \frac{\sigma_a^2\rho^{|t-u|}}{1-\rho^2}
\end{aligned}
$$

Equation (5.5) is true, since W_i is assumed to be surrogate, and equation (5.6) is true because of equations (4.2) and (4.17).

The *naive* GLS estimate of the model parameters based on the observed W and G^* rather than X and G, is expressed as follows.

$$
\hat{\theta}_n = \begin{pmatrix} \hat{\beta}_n \\ \hat{\alpha}_n \end{pmatrix} = \left[\sum_{i=1}^{n} \begin{pmatrix} W_i' \\ \mathbf{1}_T G_i^* \end{pmatrix} \Phi_i^{*^{-1}} (W_i' : \mathbf{1}_T G_i^*) \right]^{-1} [\sum_{i=1}^{n} \begin{pmatrix} W_i' \\ \mathbf{1}_T G_i^* \end{pmatrix} \Phi_i^{*^{-1}} y_i], \quad (5.7)
$$

where Φ_i^* is the matrix of variance and covariance of y_i based on W_i and G^*, which satisfying:

1. $var(y_{it}|W_{it}, G_i^*) = \beta' var(X_{it}'|W_{it})\beta + \alpha^2 var(G_i|G_i^*) + \sigma_\gamma^2 + \frac{\sigma_a^2}{1-\rho^2}$,

2. for $t \neq u$, $cov(y_{it}, y_{iu}|W_{it}, W_{iu}, G_i^*) = \beta' cov((X_{it}', X_{iu}')|W_{it}, W_{iu})\beta + var(G_i|G_i^*)\alpha^2 + \sigma_\gamma^2 + \frac{\sigma_a^2 \rho^{|t-u|}}{1-\rho^2}$.

The covariance matrix of $\hat{\theta}_n$ conditioned on W_i and G_i^*, can be expressed as

$$Cov(\hat{\theta}_n) = \left[\sum_{i=1}^n \begin{pmatrix} W_i' \\ G_i^* \end{pmatrix} \Phi_i^{*^{-1}} (W_i' : \mathbf{1}_T G_i^*) \right]^{-1}.$$

5.1 Simulation Studies

In the last two chapters, we observed that either measurement error or misclassification could affect the bias and variabilities of the parameter estimates. In this chapter, we consider both measurement errors and misclassification in the model.

We now present the common set ups for all the scenarios. For $T = 4$ time points, we generated $p = 2$ independent continuous time-invariant predictors from a uniform distribution $U(0, 1)$. The random effect γ was generated from a normal distribution with mean zero. Except in the scenario that σ_γ^2 changes, it was set to be one. The categorical time-invariant G was generated from a trinomial distribution with probabilities $P(G = 1) = \pi_1 = P(G = 2) = \pi_2 = 0.4$, except in the scenarios they changed. The regression model parameters were set to be $\alpha = 0.7$ and $\beta = (1, 0.5)'$. The model error term, ϵ, follows a first order auto-regressive model, such that $\epsilon_{it} = \rho_1 \epsilon_{i,t-1} + a_{it}$ and $|\rho_1| < 1$. We generated a_{it} from a normal distribution with mean zero. Except when they changed, we set ρ_1 and σ_a^2 to be 0.8 and 1, respectively.

For the classical measurement error, each U_{1t} and U_{2t} follow a first order auto-regressive model with standard normal error, and autocorrelation lag parameters ρ_2 and ρ_3, respectively. ρ_2 and ρ_3 were both set for 0.8, unless they changed.

For the misclassification, the categorical time-invariant G^* was generated from G. Similar to the last chapters, 500 was selected as the sample size for all the scenarios, except when it changed. For each of the sample sizes, 1000 Monte Carlo replicates were simulated and the Monte-Carlo mean estimates and standard errors of the estimators were computed.

For the following scenarios, we change one model parameter at a time and keep the others constant. Table 5.1 shows the selected range as well as the steps for every parameter.

Table 5.1: The range and step of the model parameters

Parameter	Range	Step
ρ_1	$(-1, 1)$	0.1
ρ_2	$(-1, 1)$	0.1
ρ_3	$(-1, 1)$	0.1
σ_a^2	$(0, 2)$	0.2
σ_γ^2	(0,2)	0.2
$\sigma_{u_1}^2$	(0,2)	0.2
$\sigma_{u_2}^2$	(0,2)	0.2
α	(-3,3)	0.5
β_1	(-3,3)	0.5
β_2	(-3,3)	0.5
θ_{00}	(0,1)	0.1
θ_{01}	(0,1)	0.1
θ_{02}	(0,0.6]	0.1
θ_{10}	(0,0.2]	0.01
θ_{11}	(0,1)	0.1
θ_{22}	(0,0.7]	0.1
π_1	(0,0.6]	0.1
π_2	(0,0.6]	0.1
n	(100,1000)	100

5.1.1 Scenario 1: Bias analysis for different values of ρ_1

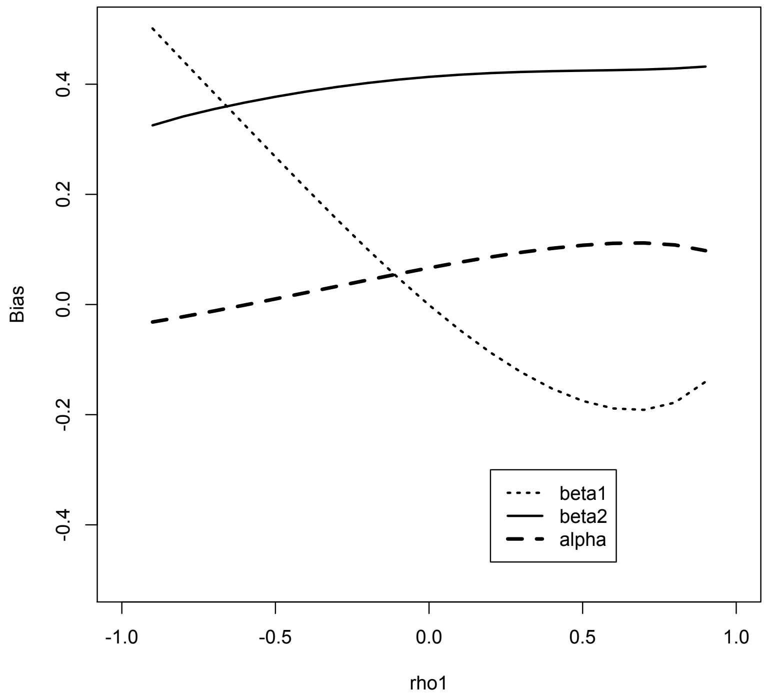

Figure 5.1: Bias of the naive estimators for different values of ρ_1

As ρ_1 increases from -1 to 1, Figure 5.1 and Table 5.2 display that the bias in the naive estimator of β_1 declines from positive values to negative values, and then increases to roughly $\rho_1 = 0.7$. However, the bias in the naive estimator of β_2 increases from positive values to even larger value, while the bias in the naive estimator of α increases slowly from negative to small positive values. The variability of the naive estimator of α increases with the change in ρ_1, while the variabilities of the other two

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\rho_1 = -0.9$	-0.0317	0.0158	0.5012	0.0076	0.3253	0.0083
$\rho_1 = -0.8$	-0.0221	0.0156	0.4422	0.0076	0.3415	0.0083
$\rho_1 = -0.7$	-0.0118	0.0154	0.3840	0.0076	0.3548	0.0084
$\rho_1 = -0.6$	-0.0010	0.0152	0.3259	0.0075	0.3666	0.0084
$\rho_1 = -0.5$	0.0102	0.0150	0.2681	0.0075	0.3772	0.0083
$\rho_1 = -0.4$	0.0216	0.0149	0.2108	0.0074	0.3866	0.0082
$\rho_1 = -0.3$	0.0330	0.0148	0.1547	0.0072	0.3950	0.0081
$\rho_1 = -0.2$	0.0444	0.0147	0.1002	0.0071	0.4022	0.0080
$\rho_1 = -0.1$	0.0556	0.0147	0.0481	0.0069	0.4083	0.0078
$\rho_1 = 0$	0.0664	0.0147	-0.0009	0.0066	0.4133	0.0075
$\rho_1 = 0.1$	0.0767	0.0148	-0.0462	0.0064	0.4172	0.0072
$\rho_1 = 0.2$	0.0862	0.0149	-0.0872	0.0061	0.4202	0.0069
$\rho_1 = 0.3$	0.0947	0.0151	-0.1229	0.0057	0.4223	0.0064
$\rho_1 = 0.4$	0.1020	0.0156	-0.1525	0.0053	0.4237	0.0059
$\rho_1 = 0.5$	0.1076	0.0163	-0.1750	0.0049	0.4246	0.0054
$\rho_1 = 0.6$	0.1111	0.0176	-0.1887	0.0043	0.4255	0.0047
$\rho_1 = 0.7$	0.1117	0.0199	-0.1912	0.0037	0.4266	0.0040
$\rho_1 = 0.8$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\rho_1 = 0.9$	0.0978	0.0351	-0.1402	0.0019	0.4320	0.0020

Table 5.2: Bias and standard error for the different values of ρ_1

estimators decrease.

5.1.2 Scenario 2: Bias analysis for different values of ρ_2

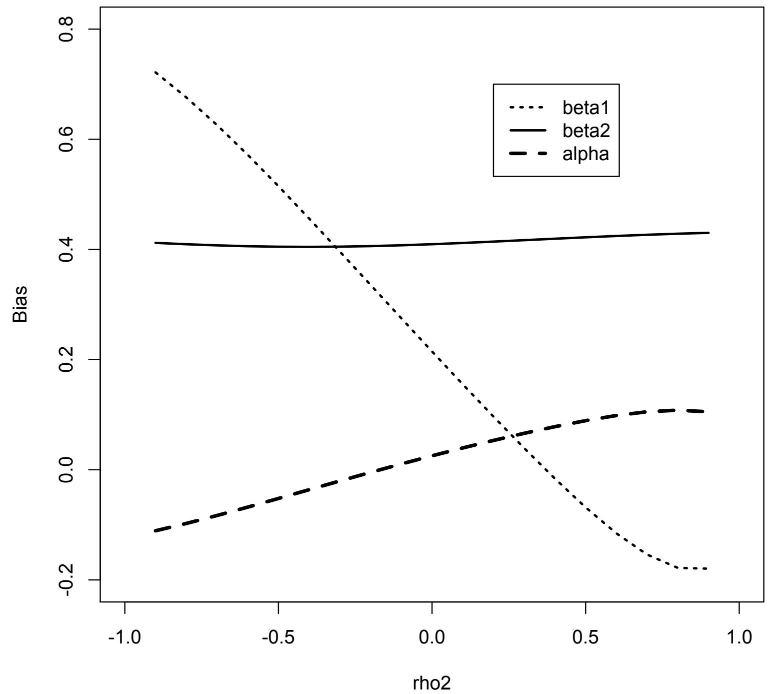

Figure 5.2: Bias of the naive estimators for different values of ρ_2

From Figure 5.2 and Table 5.3, we see that the bias of the naive estimator of β_1 decreases from positive values to negative values as the value of ρ_2 increases from -1 to 1. On the contrary, the bias in the naive estimator of α increases from negative values to positive values. In addition, the bias in the naive estimator of β_2 seems to remain unchanged, as changing the correlation parameter of one covariate with measurement error has no effect on the estimation of the coefficient parameter of

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\rho_2 = -0.9$	-0.1108	0.0379	0.7215	0.0006	0.4119	0.0041
$\rho_2 = -0.8$	-0.0975	0.0375	0.6760	0.0008	0.4095	0.0040
$\rho_2 = -0.7$	-0.0831	0.0370	0.6259	0.0009	0.4074	0.0040
$\rho_2 = -0.6$	-0.0679	0.0365	0.5718	0.0010	0.4059	0.0039
$\rho_2 = -0.5$	-0.0521	0.0360	0.5149	0.0012	0.4049	0.0039
$\rho_2 = -0.4$	-0.0362	0.0354	0.4560	0.0013	0.4047	0.0038
$\rho_2 = -0.3$	-0.0203	0.0349	0.3960	0.0014	0.4051	0.0038
$\rho_2 = -0.2$	-0.0047	0.0343	0.3356	0.0016	0.4060	0.0037
$\rho_2 = -0.1$	0.0105	0.0337	0.2750	0.0017	0.4075	0.0037
$\rho_2 = 0$	0.0253	0.0332	0.2148	0.0018	0.4095	0.0036
$\rho_2 = 0.1$	0.0396	0.0325	0.1552	0.0019	0.4117	0.0036
$\rho_2 = 0.2$	0.0532	0.0318	0.0965	0.0020	0.4142	0.0035
$\rho_2 = 0.3$	0.0662	0.0310	0.0392	0.0022	0.4168	0.0034
$\rho_2 = 0.4$	0.0782	0.0300	-0.0161	0.0023	0.4195	0.0034
$\rho_2 = 0.5$	0.0892	0.0289	-0.0683	0.0024	0.4220	0.0033
$\rho_2 = 0.6$	0.0985	0.0275	-0.1154	0.0026	0.4244	0.0032
$\rho_2 = 0.7$	0.1053	0.0260	-0.1539	0.0027	0.4266	0.0032
$\rho_2 = 0.8$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\rho_2 = 0.9$	0.1052	0.0228	-0.1796	0.0031	0.4301	0.0030

Table 5.3: Bias and standard error for the different values of ρ_2

the other continuous predictor with ME. Interestingly, modifying ρ_2 increases the variability of the naive estimator of β_1, while it decreases the variability of the two others.

5.1.3 Scenario 3: Bias analysis for different values of ρ_3

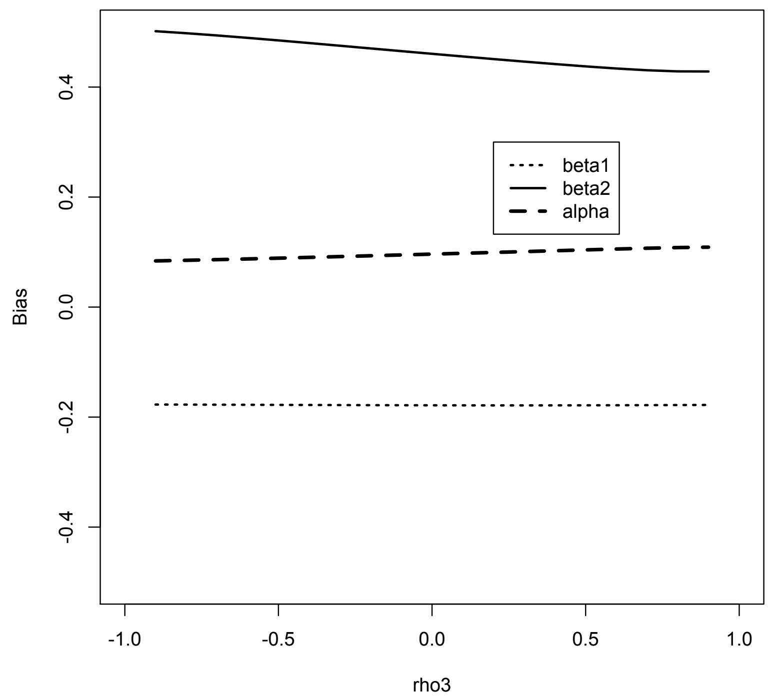

Figure 5.3: Bias of the naive estimators for different values of ρ_3

Figure 5.3 and Table 5.4 show that changing the correlation parameter of the second measurement has no effect on the estimation of the coefficient of the first continuous predictor. Therefore, the bias in the naive estimator of β_1 remains unchanged. However, the bias in the naive estimator of β_2 decreases slightly as the correlation parameter increases from -1 to 1. In addition, the positive bias in the naive estimator of α increases slightly. This change seems to have no impact on the variabilities of β_1

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\rho_3 = -0.9$	0.0840	0.0241	-0.1772	0.0029	0.5016	0.0004
$\rho_3 = -0.8$	0.0850	0.0242	-0.1773	0.0029	0.4979	0.0005
$\rho_3 = -0.7$	0.0863	0.0242	-0.1775	0.0029	0.4939	0.0006
$\rho_3 = -0.6$	0.0876	0.0242	-0.1776	0.0029	0.4895	0.0007
$\rho_3 = -0.5$	0.0889	0.0242	-0.1778	0.0029	0.4849	0.0008
$\rho_3 = -0.4$	0.0904	0.0242	-0.1780	0.0029	0.4801	0.0009
$\rho_3 = -0.3$	0.0918	0.0242	-0.1782	0.0029	0.4752	0.0011
$\rho_3 = -0.2$	0.0933	0.0242	-0.1783	0.0029	0.4703	0.0012
$\rho_3 = -0.1$	0.0948	0.0242	-0.1785	0.0029	0.4654	0.0013
$\rho_3 = 0$	0.0963	0.0243	-0.1786	0.0029	0.4605	0.0014
$\rho_3 = 0.1$	0.0978	0.0243	-0.1787	0.0029	0.4557	0.0016
$\rho_3 = 0.2$	0.0993	0.0243	-0.1787	0.0029	0.4509	0.0017
$\rho_3 = 0.3$	0.1009	0.0243	-0.1787	0.0029	0.4463	0.0019
$\rho_3 = 0.4$	0.1024	0.0243	-0.1787	0.0029	0.4419	0.0021
$\rho_3 = 0.5$	0.1040	0.0243	-0.1786	0.0029	0.4376	0.0023
$\rho_3 = 0.6$	0.1055	0.0243	-0.1785	0.0029	0.4338	0.0025
$\rho_3 = 0.7$	0.1070	0.0244	-0.1783	0.0029	0.4306	0.0028
$\rho_3 = 0.8$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\rho_3 = 0.9$	0.1089	0.0244	-0.1779	0.0029	0.4284	0.0034

Table 5.4: Bias and standard error for the different values of ρ_3

and α. However, it increases the variability of β_2.

5.1.4 Scenario 4: Bias analysis for different values of σ_a^2

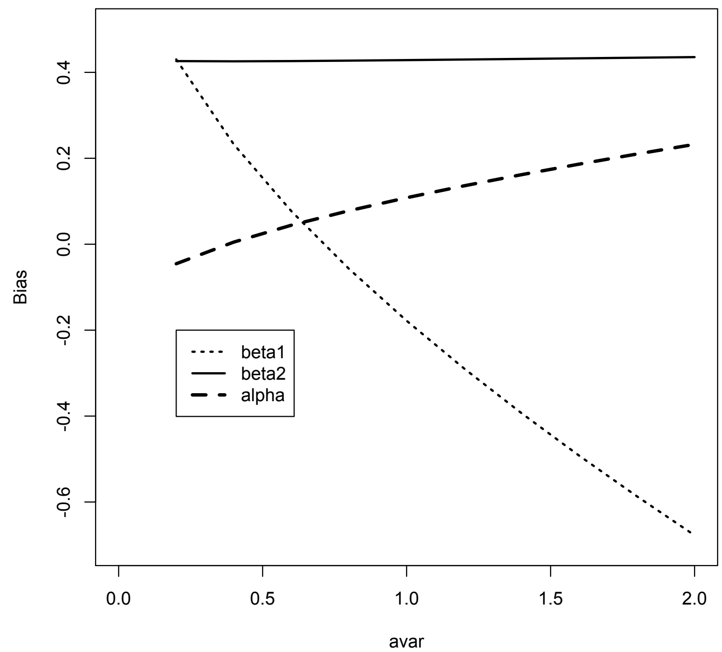

Figure 5.4: Bias of the naive estimators for different values of the variability of model error term

By changing the variance of a_{it} from 0 to 2, we found that the bias in the naive estimators of β_2 seems to stay unchanged. The bias in the naive estimator of β_1 decreases sharply, from positive values to negative ones, and the bias in the naive estimator of α increases from around 0, as the the variance of a_{it} increases from 0 to 2. Figure 5.4 and Table 5.5 present these results.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma_a^2 = 0.2$	-0.0455	0.0167	0.4302	0.0014	0.4263	0.0015
$\sigma_a^2 = 0.4$	0.0050	0.0181	0.2324	0.0021	0.4257	0.0021
$\sigma_a^2 = 0.6$	0.0443	0.0199	0.0768	0.0024	0.4263	0.0026
$\sigma_a^2 = 0.8$	0.0780	0.0220	-0.0574	0.0027	0.4273	0.0029
$\sigma_a^2 = 1.0$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\sigma_a^2 = 1.2$	0.1360	0.0269	-0.2894	0.0031	0.4299	0.0033
$\sigma_a^2 = 1.4$	0.1620	0.0294	-0.3937	0.0032	0.4313	0.0035
$\sigma_a^2 = 1.6$	0.1865	0.0321	-0.4925	0.0033	0.4327	0.0037
$\sigma_a^2 = 1.8$	0.2099	0.0349	-0.5868	0.0034	0.4341	0.0039
$\sigma_a^2 = 2.0$	0.2324	0.0377	-0.6774	0.0036	0.4355	0.0041

Table 5.5: Bias and standard error for the different values of the variability of model error term

5.1.5 Scenario 5: Bias analysis for different values of σ_γ^2

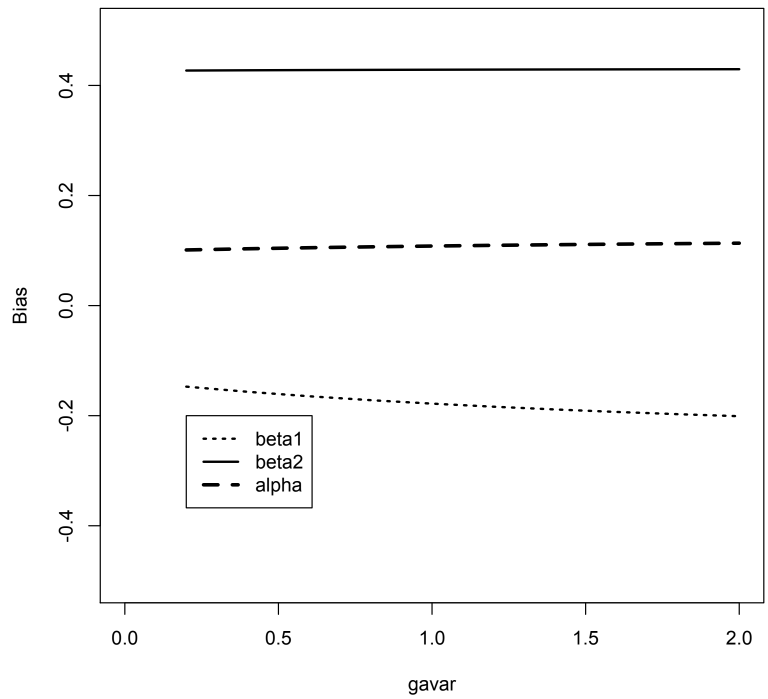

Figure 5.5: Bias of the naive estimators for different value of the variability in the random effect

From Figure 5.5 and Table 5.6, it is interesting to find that the bias of the naive estimators of the three parameters stay almost unchanged when we increase the variance of γ_i from 0 to 2. It, however, increases the variabilities of the naive estimators of α, while it decreases the variance of the naive estimates of the two βs.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma_\gamma^2 = 0.2$	0.1011	0.0141	-0.1473	0.0034	0.4271	0.0036
$\sigma_\gamma^2 = 0.4$	0.1033	0.0164	-0.1566	0.0032	0.4275	0.0034
$\sigma_\gamma^2 = 0.6$	0.1051	0.0190	-0.1647	0.0031	0.4279	0.0033
$\sigma_\gamma^2 = 0.8$	0.1068	0.0216	-0.1718	0.0030	0.4283	0.0032
$\sigma_\gamma^2 = 1.0$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\sigma_\gamma^2 = 1.2$	0.1095	0.0272	-0.1837	0.0028	0.4288	0.0030
$\sigma_\gamma^2 = 1.4$	0.1106	0.0300	-0.1887	0.0028	0.4290	0.0030
$\sigma_\gamma^2 = 1.6$	0.1116	0.0329	-0.1932	0.0027	0.4292	0.0029
$\sigma_\gamma^2 = 1.8$	0.1126	0.0358	-0.1973	0.0027	0.4293	0.0029
$\sigma_\gamma^2 = 2.0$	0.1134	0.0388	-0.2010	0.0026	0.4295	0.0029

Table 5.6: Bias and Standard Error for the different values of the variability in the random effect

5.1.6 Scenario 6: Bias analysis for different values of $\sigma^2_{u_1}$

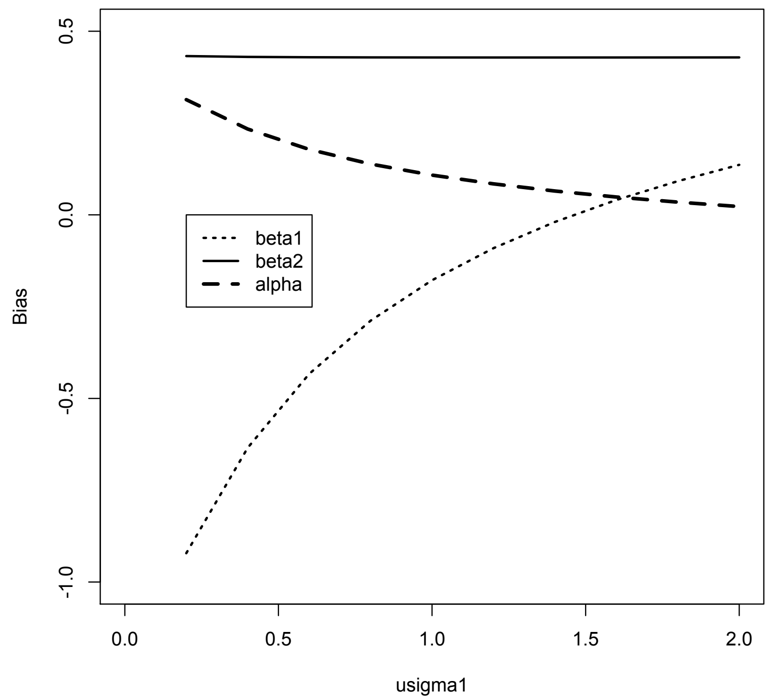

Figure 5.6: Bias of the naive estimators for different value of $\sigma^2_{u_1}$

From Figure 5.6 and Table 5.7, it is clear to see that the bias in the naive estimator of β_1 increases as the variance of its own measurement error increases from 0 to 2. However, the bias in the naive estimator of α decreases to almost 0. In addition, the bias and variability of the naive estimates of β_2 stay unchanged. However, it decreases the variabilities of the estimates of β_1 and α, making even poorer estimators for these

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma^2_{u_1} = 0.2$	0.3137	0.0334	-0.9215	0.0065	0.4324	0.0032
$\sigma^2_{u_1} = 0.4$	0.2337	0.0300	-0.6344	0.0044	0.4301	0.0031
$\sigma^2_{u_1} = 0.6$	0.1782	0.0277	-0.4333	0.0036	0.4292	0.0031
$\sigma^2_{u_1} = 0.8$	0.1384	0.0259	-0.2883	0.0032	0.4287	0.0031
$\sigma^2_{u_1} = 1.0$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\sigma^2_{u_1} = 1.2$	0.0843	0.0231	-0.0908	0.0027	0.4285	0.0031
$\sigma^2_{u_1} = 1.4$	0.0648	0.0220	-0.0194	0.0026	0.4285	0.0031
$\sigma^2_{u_1} = 1.6$	0.0485	0.0210	0.0405	0.0025	0.4285	0.0031
$\sigma^2_{u_1} = 1.8$	0.0346	0.0201	0.0917	0.0024	0.4286	0.0031
$\sigma^2_{u_1} = 2.0$	0.0225	0.0193	0.1362	0.0023	0.4287	0.0031

Table 5.7: Bias and standard error for the different values of $\sigma^2_{u_1}$

two parameters.

5.1.7 Scenario 7: Bias analysis for different values of $\sigma^2_{u_2}$

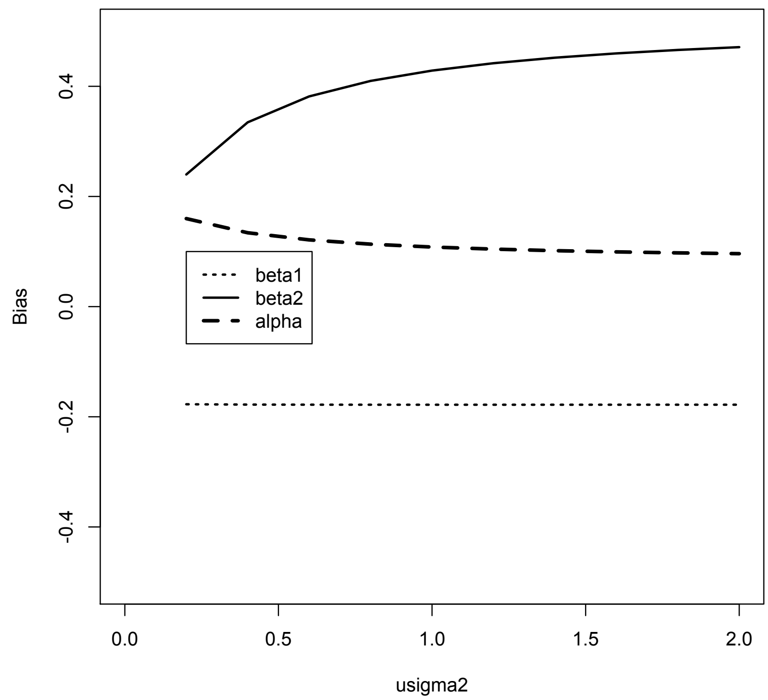

Figure 5.7: Bias of the naive estimators for different value of $\sigma^2_{u_2}$

Surprisingly, from Figure 5.7 and Table 5.8, the bias of the naive estimates of β_1 and α stay relatively unchanged as the variance of measurement error of the other continuous covariate increases from 0 to 2. However, as it was expected, the bias in the naive estimator of β_2 increases, with the increase of its own measurement error. Except for the decrease in the variance of $\hat{\beta}_2$, this change has no impact on the

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\sigma^2_{u_2} = 0.2$	0.1598	0.0249	-0.1773	0.0028	0.2399	0.0056
$\sigma^2_{u_2} = 0.4$	0.1341	0.0246	-0.1778	0.0028	0.3347	0.0043
$\sigma^2_{u_2} = 0.6$	0.1212	0.0245	-0.1780	0.0029	0.3817	0.0037
$\sigma^2_{u_2} = 0.8$	0.1134	0.0244	-0.1781	0.0029	0.4098	0.0033
$\sigma^2_{u_2} = 1.0$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\sigma^2_{u_2} = 1.2$	0.1044	0.0243	-0.1781	0.0029	0.4419	0.0029
$\sigma^2_{u_2} = 1.4$	0.1016	0.0243	-0.1781	0.0029	0.4520	0.0028
$\sigma^2_{u_2} = 1.6$	0.0994	0.0243	-0.1781	0.0029	0.4598	0.0027
$\sigma^2_{u_2} = 1.8$	0.0976	0.0243	-0.1781	0.0029	0.4660	0.0026
$\sigma^2_{u_2} = 2.0$	0.0961	0.0243	-0.1780	0.0029	0.4711	0.0025

Table 5.8: Bias and standard error for the different values of $\sigma^2_{u_2}$

variabilities of the other two estimators.

5.1.8 Scenario 8: Bias analysis for different values of α

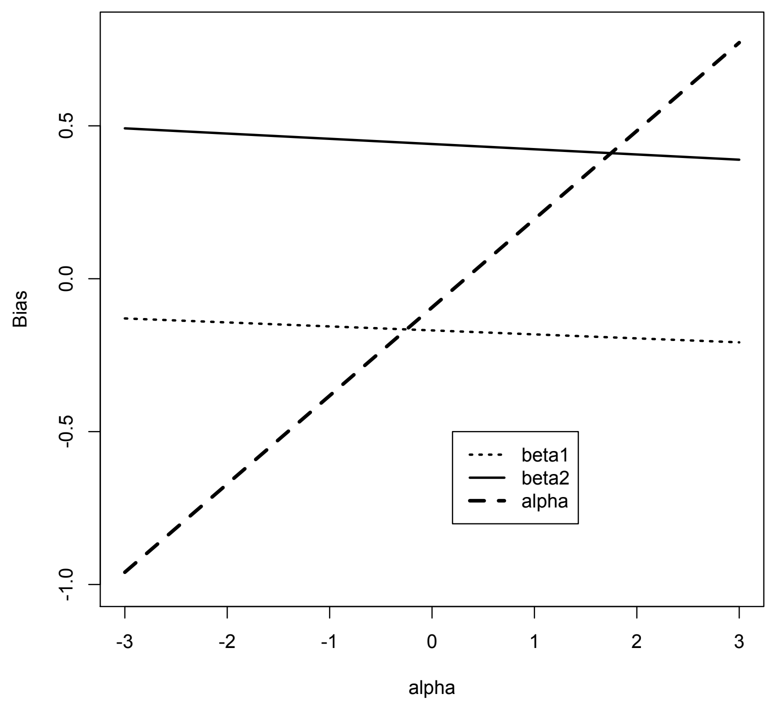

Figure 5.8: Bias of the naive estimators for different values of α

The results of changing α from -3 to 3 are summarized in Figure 5.8 and Table 5.9. Since the model has misclassification as well, the change of α effects the estimation of α. The bias in the naive estimate of α increases sharply from -1 to 1. We can see that when $\alpha = 0$, the bias in the naive estimator of α is approximately zero. There is a slight incline in the bias of the estimators of β_1 and β_2. Interestingly, the three variabilities have U shapes. The three variances decline slowly to around $\alpha = 0$,

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\alpha = -3.0$	-0.9599	0.0454	-0.1299	0.0094	0.4918	0.0110
$\alpha = -2.5$	-0.8156	0.0397	-0.1364	0.0078	0.4832	0.0092
$\alpha = -2.0$	-0.6712	0.0345	-0.1429	0.0062	0.4747	0.0074
$\alpha = -1.5$	-0.5269	0.0297	-0.1494	0.0046	0.4661	0.0056
$\alpha = -1.0$	-0.3826	0.0259	-0.1560	0.0031	0.4576	0.0038
$\alpha = -0.5$	-0.2382	0.0233	-0.1625	0.0017	0.4491	0.0023
$\alpha = 0$	-0.0939	0.0224	-0.1690	0.0013	0.4405	0.0015
$\alpha = 0.5$	0.0505	0.0235	-0.1755	0.0023	0.4320	0.0025
$\alpha = 1.0$	0.1948	0.0262	-0.1820	0.0038	0.4234	0.0041
$\alpha = 1.5$	0.3391	0.0302	-0.1885	0.0054	0.4149	0.0058
$\alpha = 2.0$	0.4835	0.0349	-0.1950	0.0069	0.4063	0.0076
$\alpha = 2.5$	0.6278	0.0402	-0.2016	0.0085	0.3978	0.0094
$\alpha = 3.0$	0.7722	0.0459	-0.2081	0.0102	0.3892	0.0112

Table 5.9: Bias and standard error for the different values of α

before they increase again.

5.1.9 Scenario 9: Bias analysis for different values of β_1

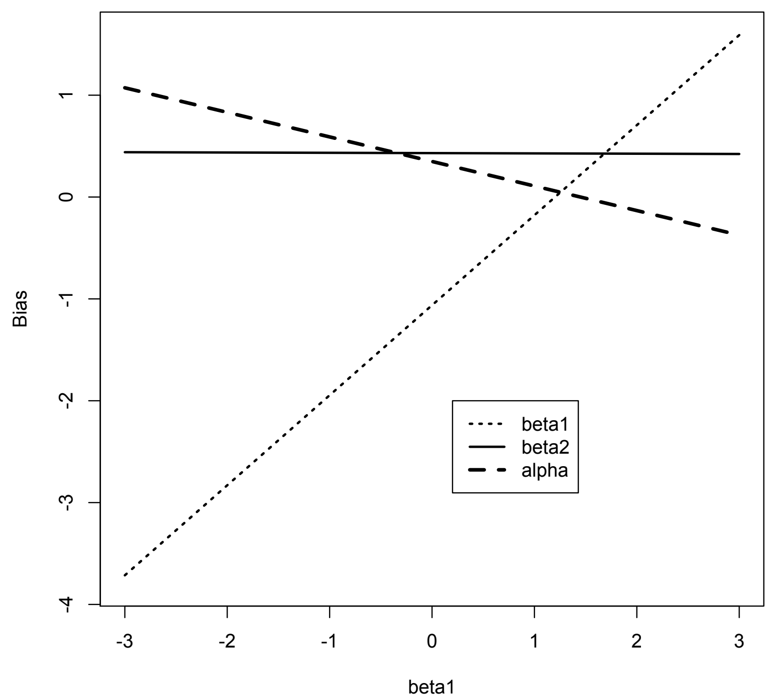

Figure 5.9: Bias of the naive estimators for different values of β_1

In this scenario, we observe the behaviours of the three naive estimates as β_1 increases from -3 to 3. Figure 5.9 and Table 5.10 show that the bias in the naive estimator of β_2 stays unchanged, as β_1 increases from -3 to 3. We could expect that when we change β_2, the bias in the naive estimator of β_1 remains unchanged. In addition, the bias in the naive estimator of β_1 increases sharply, from negative values to positive values. In the contrary, the bias in the naive estimator of α decreases from

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\beta_1 = -3.0$	1.0725	0.0302	-3.7138	0.0025	0.4401	0.0035
$\beta_1 = -2.5$	0.9520	0.0292	-3.2719	0.0023	0.4387	0.0033
$\beta_1 = -2.0$	0.8314	0.0283	-2.8299	0.0022	0.4372	0.0031
$\beta_1 = -1.5$	0.7109	0.0275	-2.3879	0.0022	0.4358	0.0030
$\beta_1 = -1.0$	0.5904	0.0267	-1.9460	0.0022	0.4343	0.0029
$\beta_1 = -0.5$	0.4698	0.0260	-1.5040	0.0023	0.4329	0.0028
$\beta_1 = 0$	0.3493	0.0254	-1.0620	0.0025	0.4314	0.0029
$\beta_1 = 0.5$	0.2287	0.0248	-0.6201	0.0027	0.4300	0.0029
$\beta_1 = 1.0$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\beta_1 = 1.5$	-0.0123	0.0240	0.2639	0.0032	0.4271	0.0033
$\beta_1 = 2.0$	-0.1329	0.0238	0.7058	0.0034	0.4256	0.0035
$\beta_1 = 2.5$	-0.2534	0.0236	1.1478	0.0037	0.4242	0.0038
$\beta_1 = 3.0$	-0.3740	0.0236	1.5898	0.0040	0.4227	0.0041

Table 5.10: Bias and standard error for the different values of β_1

positive values to negative values. Modifying β_1 increases the variabilities of the $\hat{\beta}$s, and has almost no impact on the variability of the naive estimate of α.

5.1.10 Scenario 10: Bias analysis for different values of β_2

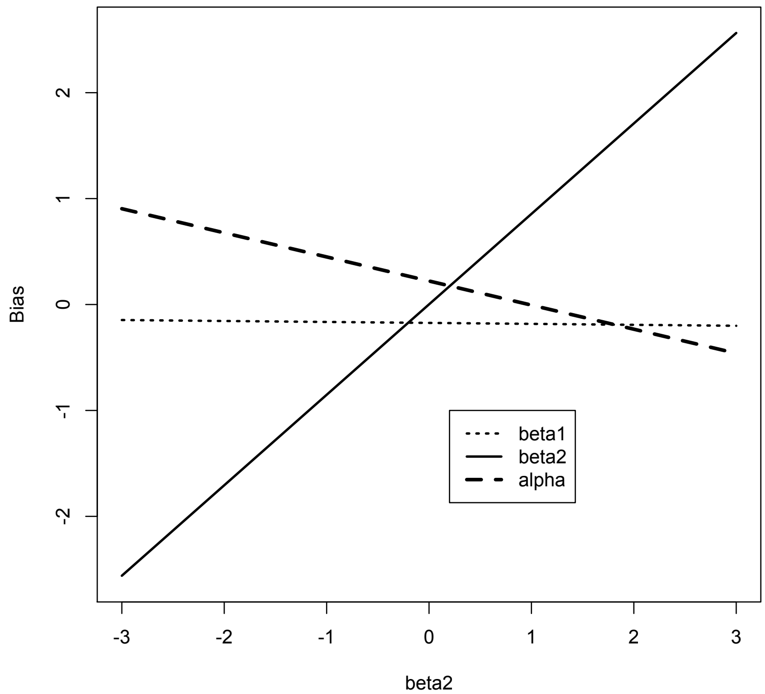

Figure 5.10: Bias of the naive estimators for different values of β_2

It was expected that the bias of the naive estimator of β_1 would remain unchanged as β_2 increased from -3 to 3. (Figure 5.10 and Table 5.11) Nevertheless, the bias of the naive estimator of β_2 increases sharply, from negative to positive values. On the contrary, the bias in the naive estimator of α decreases slightly from positive values to negative values. Similar to the last scenario, the variability of the estimate of α remains unchanged, while the variances of the other two estimates increase slightly.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\beta_2 = -3.0$	0.9043	0.0260	-0.1464	0.0023	-2.5604	0.0030
$\beta_2 = -2.5$	0.7906	0.0254	-0.1509	0.0022	-2.1334	0.0029
$\beta_2 = -2.0$	0.6769	0.0249	-0.1555	0.0022	-1.7064	0.0028
$\beta_2 = -1.5$	0.5631	0.0246	-0.1600	0.0022	-1.2794	0.0028
$\beta_2 = -1.0$	0.4494	0.0243	-0.1645	0.0023	-0.8525	0.0028
$\beta_2 = -0.5$	0.3357	0.0242	-0.1691	0.0025	-0.4255	0.0029
$\beta_2 = 0$	0.2219	0.0242	-0.1736	0.0027	0.0015	0.0030
$\beta_2 = 0.5$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\beta_2 = 1.0$	-0.0055	0.0246	-0.1826	0.0032	0.8555	0.0033
$\beta_2 = 1.5$	-0.1193	0.0250	-0.1872	0.0035	1.2825	0.0035
$\beta_2 = 2.0$	-0.2330	0.0255	-0.1917	0.0038	1.7095	0.0037
$\beta_2 = 2.5$	-0.3467	0.0261	-0.1962	0.0041	2.1365	0.0039
$\beta_2 = 3.0$	-0.4605	0.0268	-0.2007	0.0044	2.5635	0.0042

Table 5.11: Bias and standard error for the different values of β_2

5.1.11 Scenario 11: Bias analysis for different values of θ_{00}

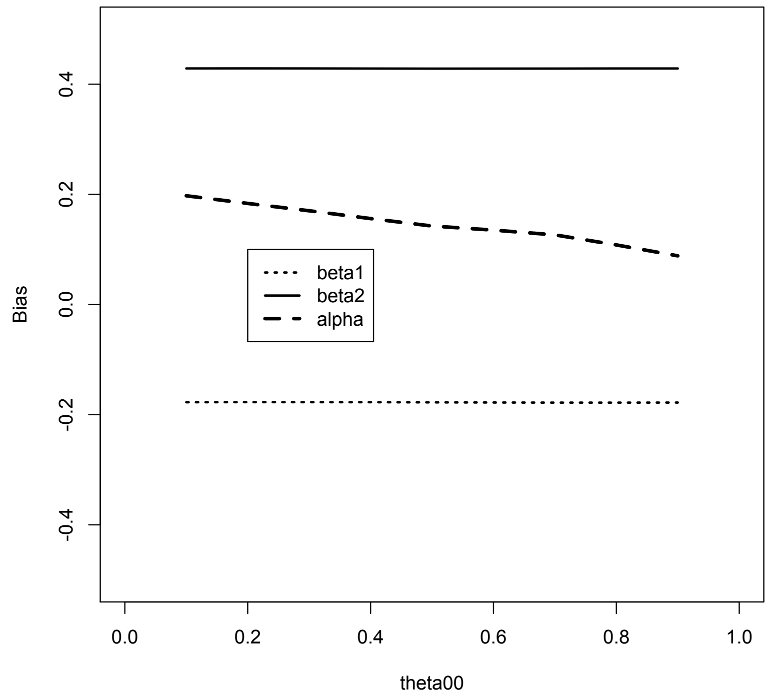

Figure 5.11: Bias of the naive estimators for different values of θ_{00}

We are surprised to find that the bias in the naive estimators of β_1 and β_2 remains unchanged when we change θ_{00} from 0 to 1. However, Figure 5.11 and Table 5.12 show that the bias in the naive estimator of α decreases with the increase in θ_{00}. Since the change in θ_{00} affects the classified predictor, the bias and variability in the naive estimators of coefficient parameters of continuous predictors do not change. On the other hand, the variability of the naive estimate of α increases as the probability of

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{00} = 0.1$	0.1974	0.0192	-0.1774	0.0032	0.4286	0.0032
$\theta_{00} = 0.2$	0.1835	0.0205	-0.1773	0.0031	0.4287	0.0032
$\theta_{00} = 0.3$	0.1702	0.0216	-0.1773	0.0032	0.4286	0.0032
$\theta_{00} = 0.4$	0.1560	0.0227	-0.1774	0.0032	0.4284	0.0032
$\theta_{00} = 0.5$	0.1425	0.0236	-0.1777	0.0031	0.4283	0.0032
$\theta_{00} = 0.6$	0.1351	0.0244	-0.1779	0.0030	0.4283	0.0032
$\theta_{00} = 0.7$	0.1264	0.0240	-0.1781	0.0030	0.4284	0.0032
$\theta_{00} = 0.8$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\theta_{00} = 0.9$	0.0884	0.0251	-0.1780	0.0028	0.4285	0.0030

Table 5.12: Bias and standard error for the different values of θ_{00}

the correct classification increases, making it a more conservative estimator.

5.1.12 Scenario 12: Bias analysis for different values of θ_{01}

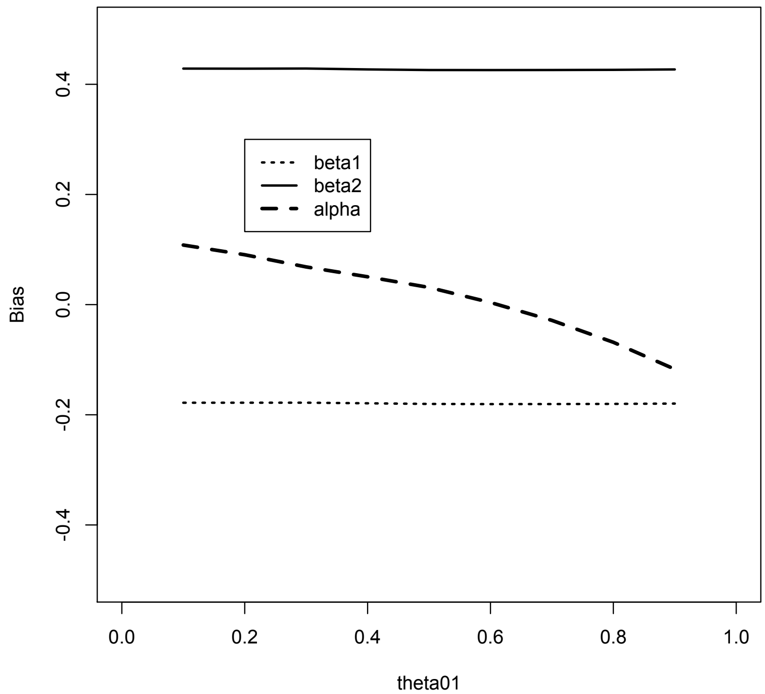

Figure 5.12: Bias of the naive estimators for different values of θ_{01}

Figure 5.12 and Table 5.13 show that the bias in the naive estimators of β_1 and β_2 seem to stay unchanged as θ_{01} increases. However, the absolute bias in the naive estimator of α increases with the increase in θ_{01}. As we change θ_{01} from 0.1 to 0.9, there is a slight decline in the variances of the naive estimates of β_1 and β_2. The variability in $\hat{\alpha}$ seems to increase.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{01} = 0.1$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\theta_{01} = 0.2$	0.0906	0.0249	-0.1781	0.0029	0.4284	0.0031
$\theta_{01} = 0.3$	0.0685	0.0253	-0.1780	0.0028	0.4287	0.0030
$\theta_{01} = 0.4$	0.0506	0.0264	-0.1791	0.0028	0.4270	0.0030
$\theta_{01} = 0.5$	0.0312	0.0266	-0.1803	0.0026	0.4259	0.0028
$\theta_{01} = 0.6$	0.0039	0.0274	-0.1806	0.0025	0.4259	0.0028
$\theta_{01} = 0.7$	-0.0286	0.0290	-0.1805	0.0026	0.4260	0.0028
$\theta_{01} = 0.8$	-0.0684	0.0307	-0.1802	0.0026	0.4263	0.0028
$\theta_{01} = 0.9$	-0.1172	0.0332	-0.1796	0.0026	0.4270	0.0028

Table 5.13: Bias and standard error for the different values of θ_{01}

5.1.13 Scenario 13: Bias analysis for different values of θ_{02}

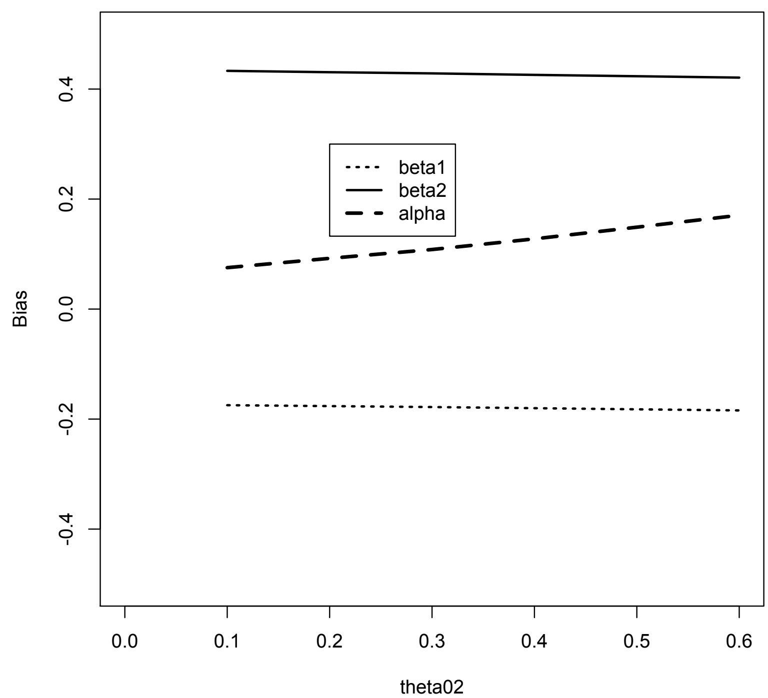

Figure 5.13: Bias of the naive estimators for different values of θ_{02}

In this scenario, θ_{02} is changed from 0.1 to 0.6. Figure 5.13 and Table 5.14 show that the bias in the naive estimators of β_1 and β_2 remains unchanged as θ_{02} increases from 0 to 0.6. However, the bias in the naive estimator of α increases with the increase in θ_{02}. Hence, decreasing the probability of misclassification, θ_{02}, can improve the naive estimator of α. Interestingly, unlike the case for θ_{00} and θ_{01}, the change in θ_{00} has almost no impact on the variability of the naive estimate of α. It, however,

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{02} = 0.1$	0.0753	0.0231	-0.1747	0.0026	0.4331	0.0028
$\theta_{02} = 0.2$	0.0922	0.0238	-0.1764	0.0027	0.4307	0.0030
$\theta_{02} = 0.3$	0.1083	0.0244	-0.1781	0.0029	0.4285	0.0031
$\theta_{02} = 0.4$	0.1276	0.0250	-0.1802	0.0030	0.4258	0.0032
$\theta_{02} = 0.5$	0.1489	0.0258	-0.1822	0.0031	0.4234	0.0033
$\theta_{02} = 0.6$	0.1706	0.0267	-0.1842	0.0032	0.4208	0.0034

Table 5.14: Bias and standard error for the different values of θ_{02}

slightly increases the variances of the estimates of β_1 and β_2.

5.1.14 Scenario 14: Bias analysis for different values of θ_{10}

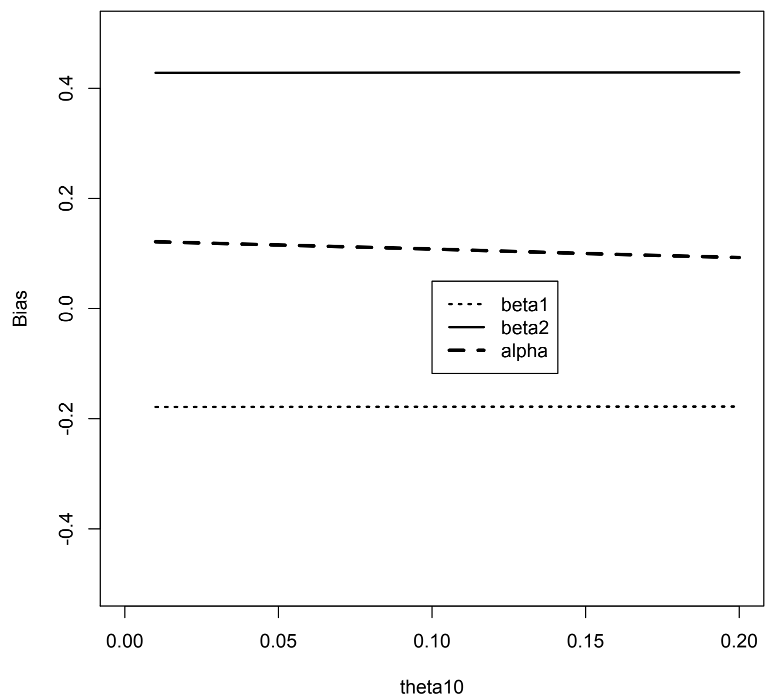

Figure 5.14: Bias of the naive estimators for different values of θ_{10}

It is clear from Figure 5.14 and Table 5.15 that the bias in the naive estimators of β_1 and β_2 remains unchanged as θ_{10} increases from 0 to 0.2. The bias in the naive estimator of α decreases slightly with the increase in θ_{10}. Increasing the value of θ_{10} does not seem to have any impact on the variabilities of the three estimates.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{10} = 0.01$	0.1214	0.0242	-0.1785	0.0030	0.4281	0.0032
$\theta_{10} = 0.02$	0.1199	0.0242	-0.1784	0.0029	0.4282	0.0032
$\theta_{10} = 0.03$	0.1185	0.0242	-0.1784	0.0029	0.4282	0.0031
$\theta_{10} = 0.04$	0.1170	0.0243	-0.1783	0.0029	0.4283	0.0031
$\theta_{10} = 0.05$	0.1155	0.0243	-0.1783	0.0029	0.4283	0.0031
$\theta_{10} = 0.06$	0.1141	0.0244	-0.1782	0.0029	0.4283	0.0031
$\theta_{10} = 0.07$	0.1126	0.0244	-0.1782	0.0029	0.4284	0.0031
$\theta_{10} = 0.08$	0.1112	0.0244	-0.1782	0.0029	0.4285	0.0031
$\theta_{10} = 0.09$	0.1097	0.0244	-0.1781	0.0029	0.4285	0.0031
$\theta_{10} = 0.10$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\theta_{10} = 0.11$	0.1065	0.0244	-0.1781	0.0029	0.4286	0.0031
$\theta_{10} = 0.12$	0.1049	0.0245	-0.1780	0.0029	0.4286	0.0031
$\theta_{10} = 0.13$	0.1033	0.0245	-0.1780	0.0029	0.4287	0.0031
$\theta_{10} = 0.14$	0.1017	0.0245	-0.1779	0.0029	0.4287	0.0031
$\theta_{10} = 0.15$	0.1001	0.0245	-0.1779	0.0029	0.4288	0.0031
$\theta_{10} = 0.16$	0.0985	0.0245	-0.1779	0.0029	0.4288	0.0031
$\theta_{10} = 0.17$	0.0970	0.0246	-0.1778	0.0029	0.4288	0.0030
$\theta_{10} = 0.18$	0.0956	0.0246	-0.1778	0.0029	0.4288	0.0030
$\theta_{10} = 0.19$	0.0942	0.0246	-0.1778	0.0029	0.4289	0.0030
$\theta_{10} = 0.20$	0.0928	0.0247	-0.1778	0.0029	0.4289	0.0030

Table 5.15: Bias and standard error for the different values of θ_{10}

5.1.15 Scenario 15: Bias analysis for different values of θ_{11}

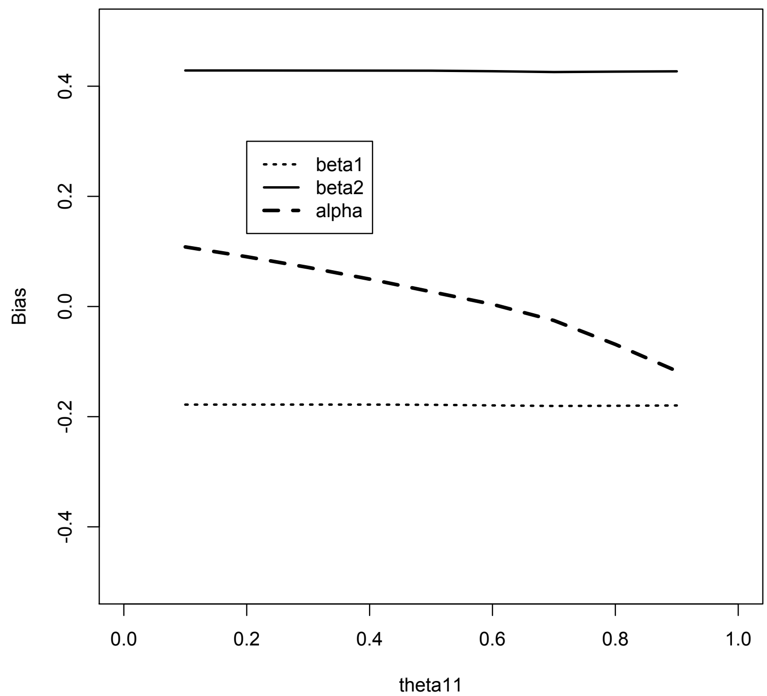

Figure 5.15: Bias of the naive estimators for different values of θ_{11}

In this scenario, we observe the behaviour of the naive estimates as θ_{11} increases from 0.1 to 0.9. From Figure 5.15 and Table 5.16, we can see that the bias in the naive estimators of β_1 and β_2 remains unchanged. On the other hand, the bias in the naive estimator of α decreases from a positive value to almost zero near $\theta_{11} = 0.5$, before it continues to change on the negative side. It also increases the variability of $\hat{\alpha}$, but decreases the variances of the other two estimates.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{11} = 0.1$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\theta_{11} = 0.2$	0.0904	0.0257	-0.1781	0.0029	0.4286	0.0031
$\theta_{11} = 0.3$	0.0710	0.0267	-0.1780	0.0029	0.4284	0.0031
$\theta_{11} = 0.4$	0.0498	0.0280	-0.1781	0.0029	0.4283	0.0031
$\theta_{11} = 0.5$	0.0269	0.0290	-0.1784	0.0029	0.4282	0.0031
$\theta_{11} = 0.6$	0.0040	0.0300	-0.1795	0.0029	0.4272	0.0031
$\theta_{11} = 0.7$	-0.0253	0.0315	-0.1805	0.0028	0.4257	0.0029
$\theta_{11} = 0.8$	-0.0687	0.0319	-0.1802	0.0026	0.4265	0.0029
$\theta_{11} = 0.9$	-0.1172	0.0332	-0.1796	0.0026	0.4270	0.0028

Table 5.16: Bias and standard error for the different values of θ_{11}

5.1.16 Scenario 16: Bias analysis for different values of θ_{22}

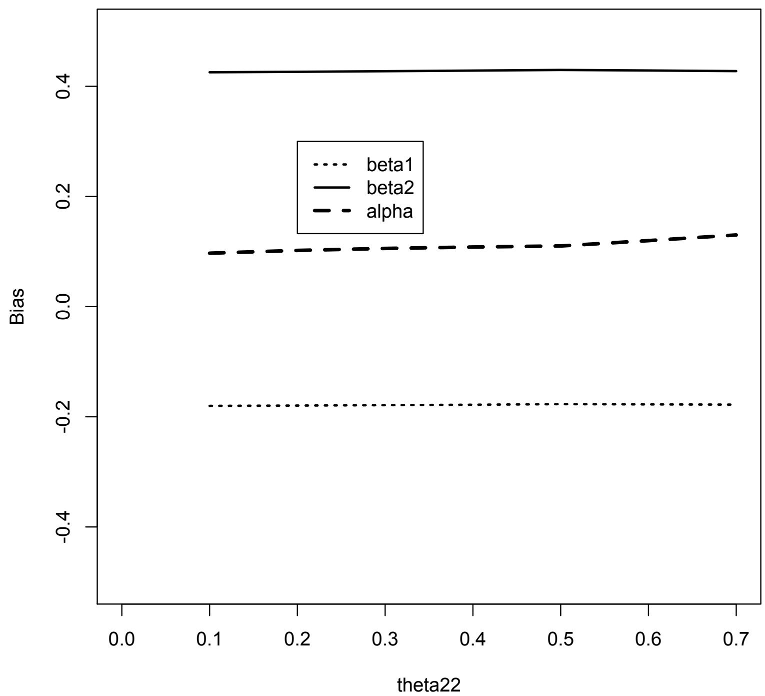

Figure 5.16: Bias of the naive estimators for different values of θ_{22}

With changing θ_{22} from 0.1 to 0.7, it is not surprising to find that the bias in the naive estimators of β_1 and β_2 does not change. However, the bias in the naive estimator of α increases slightly. This is the case only in our setup, as increasing the probability of correct classification generally improves the naive estimate of α. Figure 5.16 and Table 5.17 display these results. It does not seem to affect any variabilities.

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\theta_{22} = 0.1$	0.0970	0.0291	-0.1802	0.0033	0.4255	0.0033
$\theta_{22} = 0.2$	0.1020	0.0272	-0.1796	0.0031	0.4263	0.0033
$\theta_{22} = 0.3$	0.1057	0.0258	-0.1789	0.0030	0.4274	0.0032
$\theta_{22} = 0.4$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\theta_{22} = 0.5$	0.1102	0.0234	-0.1770	0.0028	0.4297	0.0030
$\theta_{22} = 0.6$	0.1198	0.0236	-0.1775	0.0029	0.4287	0.0032
$\theta_{22} = 0.7$	0.1301	0.0233	-0.1779	0.0030	0.4276	0.0032

Table 5.17: Bias and standard error for the different values of θ_{22}

5.1.17 Scenario 17: Bias analysis for different values of π_1

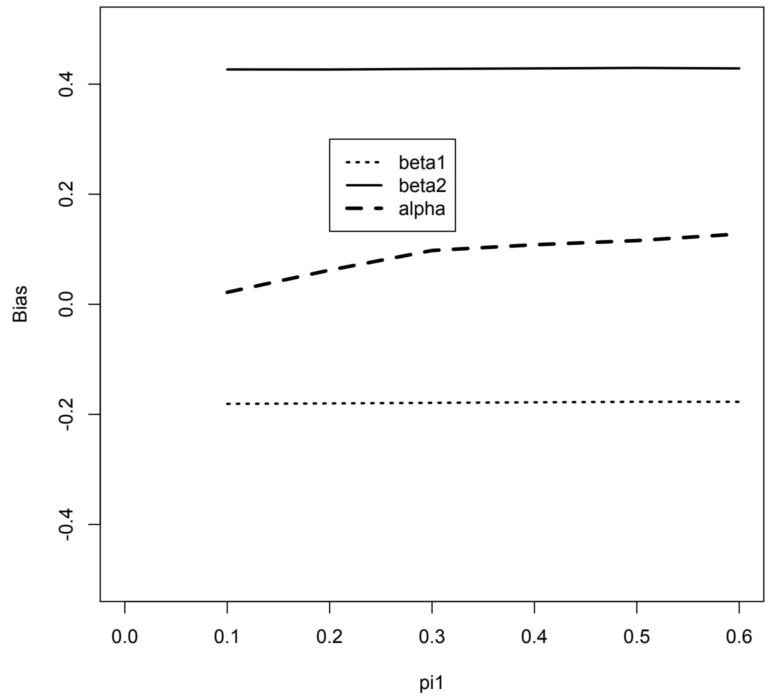

Figure 5.17: Bias of the naive estimators for different values of π_1

From Figure 5.17 and Table 5.18, we find that the bias in the naive estimators for the coefficient parameters of continuous predictors seems to stay unchanged as the probability of $G = 1$ increases from 0.1 to 0.6. The change in the probability of classified predictor $G = 1$ does not affect the bias in the naive estimators of the coefficient parameters of continuous predictors. However, this probability has impact on the bias in the naive estimator of the coefficient parameter of the classified

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\pi_1 = 0.1$	0.0218	0.0461	-0.1809	0.0028	0.4267	0.0031
$\pi_1 = 0.2$	0.0621	0.0400	-0.1801	0.0029	0.4265	0.0032
$\pi_1 = 0.3$	0.0977	0.0305	-0.1790	0.0030	0.4277	0.0032
$\pi_1 = 0.4$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\pi_1 = 0.5$	0.1157	0.0200	-0.1771	0.0029	0.4295	0.0030
$\pi_1 = 0.6$	0.1278	0.0164	-0.1772	0.0031	0.4286	0.0031

Table 5.18: Bias and standard error for the different values of π_1

predictor. The bias in the naive estimator of α increases with the increase in π_1. It also decreases the variability of the same estimator, leaving it to be an even poorer estimate. It has, however, little to no impact on the variabilities of the other two estimates.

5.1.18 Scenario 18: Bias analysis for different values of π_2

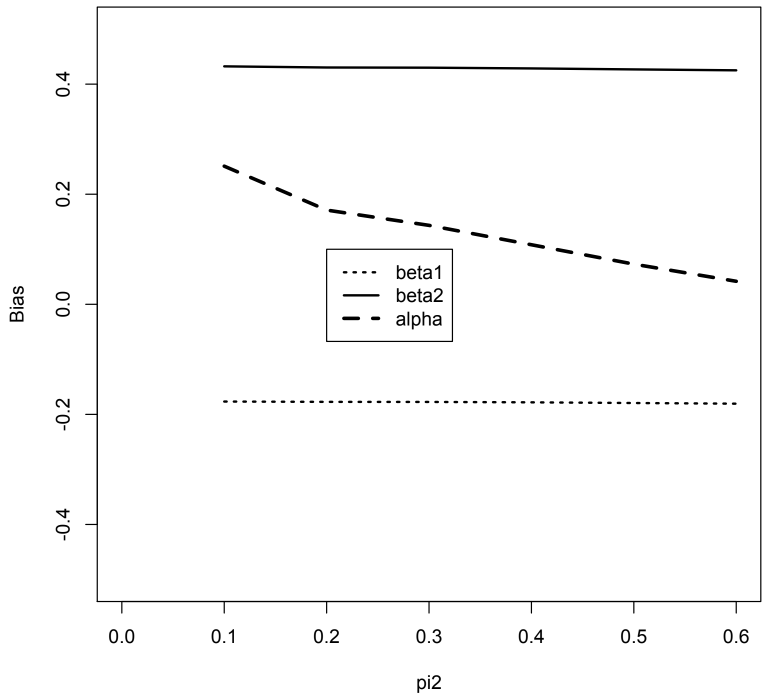

Figure 5.18: Bias of the naive estimators for different values of π_2

Figure 5.18 and Table 5.19 show that the bias in the naive estimators of the coefficient parameters of continuous predictors stays unchanged as the probability of $G = 2$ increase from 0.1 to 0.6. The change in the probability of classified predictor $G = 2$ does not affect the bias in the naive estimators of the coefficient parameters of continuous predictors. However, this probability decreases the bias in the naive estimate of α. It is also observed that the increase in π_2 increases the variabilities in

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$\pi_2 = 0.1$	0.2510	0.0591	-0.1767	0.0017	0.4323	0.0020
$\pi_2 = 0.2$	0.1712	0.0462	-0.1773	0.0023	0.4302	0.0025
$\pi_2 = 0.3$	0.1434	0.0334	-0.1774	0.0028	0.4299	0.0030
$\pi_2 = 0.4$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$\pi_2 = 0.5$	0.0730	0.0217	-0.1794	0.0030	0.4268	0.0031
$\pi_2 = 0.6$	0.0418	0.0174	-0.1806	0.0030	0.4251	0.0029

Table 5.19: Bias and standard error for the different values of π_2

the estimates of β_1 and β_2, but decreases the variance of $\hat{\alpha}$.

5.1.19 Scenario 19: Bias analysis for different values of n

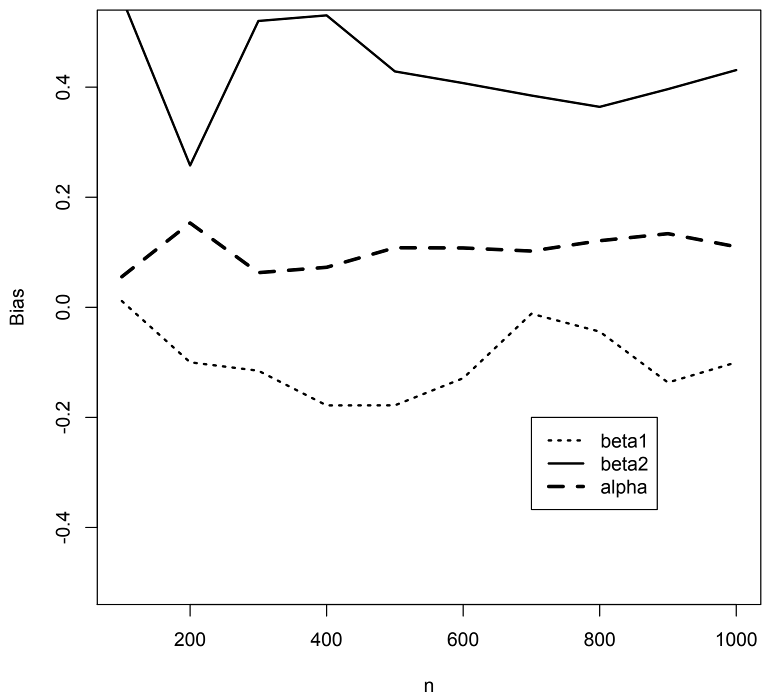

Figure 5.19: Bias of the naive estimators for different values of n

Last but not the least, we observe behaviour of the naive estimates, as sample size changes from 100 to 1000. From Figure 5.19 and Table 5.20, it is interesting to see that the bias in the naive estimators of all the coefficient parameters fluctuates randomly. However, increasing the sample size does not generally improve the naive estimators of the coefficient parameters. More interestingly, as the sample size increases, the variabilities of all the estimates decline, leaving the naive estimates to perform very

	$\hat{\alpha}$		$\hat{\beta}_1$		$\hat{\beta}_2$	
	Bias	SE	Bias	SE	Bias	SE
$n = 100$	0.0555	0.0522	0.0114	0.0078	0.5623	0.0069
$n = 200$	0.1532	0.0359	-0.0999	0.0053	0.2577	0.0040
$n = 300$	0.0629	0.0321	-0.1152	0.0041	0.5202	0.0034
$n = 400$	0.0727	0.0279	-0.1782	0.0034	0.5304	0.0033
$n = 500$	0.1082	0.0244	-0.1781	0.0029	0.4285	0.0031
$n = 600$	0.1078	0.0227	-0.1288	0.0028	0.4073	0.0026
$n = 700$	0.1022	0.0215	-0.0117	0.0026	0.3847	0.0023
$n = 800$	0.1209	0.0201	-0.0442	0.0026	0.3641	0.0023
$n = 900$	0.1338	0.0188	-0.1367	0.0024	0.3964	0.0022
$n = 1000$	0.1100	0.0178	-0.1001	0.0023	0.4310	0.0020

Table 5.20: Bias and standard error for the different values of n

poorly.

Chapter 6

Discussion

We use the Generalized Least Square method to estimate the parameters in a longitudinal linear mixed-effects model with measurement error and misclassification. It is well-known that ME and/or misclassification have negative impacts on the estimation of parameters. It is, however, very challenging to assess the bias through the closed-form naive estimates. As a result, we observe the bias as a function of all the model parameters. We should also mention in here that although the primary focus of this research is to evaluate the "bias" of the naive estimates, as a by-product, however, in each scenario we look at the variability of the estimates as well. This allows us to asses the overall performance of the naive estimates, especially for statistical inference. In here, we review and summarize the results of the simulation studies.

In the scenario where we change ρ_1, the correlation parameter for the model error term, the bias in the model with ME only has a behaviour similar to the case with both ME and misclassification. This means that misclassification has no impact on the naive estimates of this parameter. We also observe that the bias in all the parameters in the model with misclassification, whether two or three categories, behaves similarly when we change ρ_1. On the opposite side, the behavior of the bias in the model with misclassification are very similar, for both the binary case and three categories.

When the model has only ME, we observe that the bias in all the parameters is affected by ρ_2 and ρ_3, which are the correlation parameters for the measurement error term. Comparing the model with ME only versus the model with ME and misclassification, we again found very similar patterns.

In the case that we change the variance of a_{it}, we find that the biases of all the naive estimators in the models with ME (both with and without misclassification) have similar patterns. We found that the biases of all the naive estimates in the binary case are very similar to the case with three categories. These results could be expected, as the continuous and discrete variables were generated independently.

From our simulation studies, it seems that changing the variance of γ, which is the random effect term, has little to no impact on the bias of the naive estimates. In most cases, the biases in the naive estimators of α and β_2 stay unchanged. The bias in the naive estimator of β_1, however, decreases as the variance of γ increases from 0 to 2.

When α changes, it is clear to see that the biases in the naive estimators stay unchanged in the model with no misclassification. However, in the model with misclassification, the biases have similar patterns with or without ME. More specifically, the biases of the naive estimators of β_1 and β_2 decrease, while the bias in the naive estimator of α increases.

When β_1 changes from -3 to 3, in the model with misclassification only, it has no impact on the bias of the estimate of β_2. However, the bias of the naive estimator of β_1 increases, and the one for α decreases with the changes in β_1. These patterns are similar in the models with ME. It is expected that the bias in the naive estimator of β_2 stays almost unchanged, however, the impact on the estimate of α is surprising. We observe similar behaviours in the biases of β_1 and α, when β_2 modifies.

In the models with ME, we observe the effect of variabilities of the two measurement errors on the bias of the naive estimators. In both models with ME (with and without misclassification), when the variance of one measurement error increases, the bias of the naive estimator of the coefficient parameter of the other variable with ME does not change. However, the bias of the naive estimator of α decreases. As the two covariates with ME are independent, we could expect to see no impact on β parameters when the variability in the ME on one of the covariates increases. This result is consistent with literature (Carroll and Stefanski (2006)). However, the impact on the bias of $\hat{\alpha}$ is surprising, as G is also generated independently from the two continuous variables.

When misclassification is considered in the model, in the binary case, we find that both sensitivity and specificity have little to no impact on the bias of the naive estimates of β_1 and β_2. However, the increase of these conditional probabilities helps to reduce the bias in the naive estimator of α. When we consider the classified predictor with three categories, we observe the impact of six conditional probabilities, θ_{00}, θ_{01}, θ_{02}, θ_{10}, θ_{11}, and θ_{22} on the bias. Similar to the binary case, these probabilities have almost no impact on the bias of the naive estimates of β_1 and β_2. However, aside from θ_{02}, increases in these conditional probabilities reduces the bias in the naive estimator of α. In literature (e.g. Buonaccorsi. (2010)), increasing the probability of correct classification generally improves the naive estimate. However, improvement of the naive estimate of α when the four misclassification rates θ_{00}, θ_{01}, θ_{02}, and θ_{10} increase may sound surprising. This is as a result of our setting. Since increasing any of these probabilities decreases another misclassification with higher impact on the population, our results make sense.

In the model with ME only, probability of success for the classified predictor seems to have no impact on the naive estimates. However, in both models with misclassification, when π increases from 0 to 1, the biases in the naive estimators of β_1 and β_2 decrease slightly, while the bias in the naive estimator of α decreases quite significantly. Again, these results make sense for our set up with relatively high sensitivity and specificity. The bias would have different behaviour if we had low rates of correct classifications. In addition, when we consider the classified predictor with three categories (again in the model with no ME), as π_1 or π_2 increase from 0 to 1, the biases in the naive estimator of β_1 and β_2 decrease. However, the bias in the naive estimator of α increases as π_1 increases, and it decreases as π_2 increases. Interestingly, when the model has both ME and misclassification, the change in π_1 and π_2 has no effect on the bias in the naive estimators of β_1 and β_2. The bias in the naive estimator of α, however, has the same pattern as the case without ME.

Finally, we consider the effect of the sample size on the bias. When we increase the sample size from 100 to 1000, the performance of the naive estimators becomes poorer. More specifically, increasing the sample size does not reduce the bias, but reduces the variability. Therefore, the overall performance is worse! These results are also consistent with some literature in ME and misclassification. (Carroll and Stefanski (2006) and Buonaccorsi. (2010))

Chapter 7

Computer programs

7.1 R codes for Study of Bias in the Naive Estimator in Longitudinal Linear Mixed-effects Models

7.1.1 Model with Measurement Error and Misclassification

```
K<-1000 # Simulation ittiration number

T<-4

p<-2

n<-500 # Sample Size
```

```
r<-19

rho1<-0.8

rho2<-0.8

rho3<-0.8

avar<- 1

gavar<- 1

npar<-p+1

beta1<-1

beta2<-0.5

alpha<- 0.7

usigma1<-1

usigma2<-1

theta00<-0.8

theta01<-0.1

theta02<-0.3

theta10<-0.1

theta11<-0.1

theta22<-0.4

pi1<-0.4
```

```
pi2<-0.4

sigma<-as.matrix(array(0,dim=c(T,T)))

bias<-array(0,dim=c(npar,r))

se<-array(0,dim=c(npar,r))

for (m in 1:r)

 {

  rho1<-(m-10)*0.1

#  rho2<-(m-10)*0.1

#    rho3<-(m-10)*0.1

#   avar<-0.2*m

#     gavar<-0.2*m

# usigma1<-0.2*m

# usigma2<-0.2*m

#  alpha<-(m-7)*0.5

#   beta1<-(m-7)*0.5

#   beta2<-(m-7)*0.5

# theta00<-m*0.1

# theta01<-m*0.1

# theta02<-m*0.1
```

```
# theta10<-m*0.01

# theta11<-m*0.1

#theta22<-m*0.1

# pi1<-m*0.1

# pi2<-m*0.1

   beta<-c(beta1,beta2)

   theta<-c(beta1,beta2,alpha)

   theta12<-1-theta02-theta22

   theta20<-1-theta00-theta10

   theta21<-1-theta01-theta11

#   n<-m*100


for (i in 1:T)

for (l in 1:T){

sigma[i,l]<-gavar+avar*rho1^(abs(i-l))/(1-rho1^2)

}

Isig<-solve(sigma)


ThetaQL<-array(0,dim=c(npar,K))
```

```
ThetaQLn<-array(0,dim=c(npar,K))



X<-array(0,dim=c(p,T,n,K))

  set.seed(X)

 X[1,1:T,,]<-runif(array(dim=c(T,n)))

 X[2,1:T,,]<-runif(array(dim=c(T,n)))

# print(X)

 U<-array(0,dim=c(p,T,n,K))

 b<-array(0,dim=c(p,T,n,K))

   set.seed(b)

 b[1,,,]<-sqrt(usigma1)*matrix(rnorm(T*n),T,n,K) # Add set.seed

 b[2,,,]<-sqrt(usigma2)*matrix(rnorm(T*n),T,n,K)

 U[1,1,,]<-b[1,1,,]*sqrt(usigma1)

 U[2,1,,]<-b[2,1,,]*sqrt(usigma2)

 U[1,2,,]<-rho2*U[1,1,,]+b[1,2,,]

 U[1,3,,]<-rho2*U[1,2,,]+b[1,3,,]

 U[1,4,,]<-rho2*U[1,3,,]+b[1,4,,]

 U[2,2,,]<-rho3*U[2,1,,]+b[2,2,,]

 U[2,3,,]<-rho3*U[2,2,,]+b[2,3,,]
```

```
 U[2,4,,]<-rho3*U[2,3,,]+b[2,4,,]

#print(U)

 W<-X+U

# print(W)




GG<-array(0,dim=c(n,K))

GG[,]<-runif(array(dim=c(n,K)))

G <-array(0,dim=c(T,n,K))

set.seed(G)

for (j in 1:K)

{

  for(i in 1:n)

  {

    if(GG[i,j]>=0 &&GG[i,j]<(1-pi1-pi2))G[1,i,j]<-0

    if(GG[i,j]>=(1-pi1-pi2) &&GG[i,j]<.6)G[1,i,j]<-1

    if(GG[i,j]>=(1-pi2) &&GG[i,j]<=1)G[1,i,j]<-2

  }

}
```

```
G[2,,]<-G[1,,]

G[3,,]<-G[1,,]

G[4,,]<-G[1,,]

Gstar<- array(0,dim=c(T,n,K))

Gstar<-G


n0<-array(0,dim=c(K))

n1<-array(0,dim=c(K))

n2<-array(0,dim=c(K))

for (s in 1:K)

{

for (i in 1:n)

{

if(G[1,i,s]==0) {n0[s]<-n0[s]+1}

if(G[1,i,s]==1) {n1[s]<-n1[s]+1}

  if(G[1,i,s]==2) {n2[s]<-n2[s]+1}

}

k<-0
```

```
n00<-1

while(k<theta10*n0[s]&&n00<n)

{



if(G[1,n00,s]==0)

{

Gstar[1,n00,s]<-1

k<-k+1

}

n00<-n00+1

}

while(k<theta10*n0[s]+theta20*n0[s]&&n00<n)

{



  if(G[1,n00,s]==0)

  {

    Gstar[1,n00,s]<-2

    k<-k+1

  }
```

```
  n00<-n00+1

}

k<-0

n11<-1

while(k<theta01*n1[s]&&n11<n)

{



if(G[1,n11,s]==1) {Gstar[1,n1,s]<-0

k<-k+1}

n11<-n11+1



}

while(k<theta01*n1[s]+theta21*n1[s]&&n11<n)

{



  if(G[1,n11,s]==1) {Gstar[1,n11,s]<-2

                     k<-k+1}

  n11<-n11+1
```

```
}

k<-0

n22<-1

while(k<theta02*n2[s]&&n22<n)

{



  if(G[1,n22,s]==2)

  {

    Gstar[1,n22,s]<-0

    k<-k+1

  }

  n22<-n22+1

}

while(k<theta02*n2[s]+theta12*n2[s]&&n22<n)

{



  if(G[1,n22,s]==2)

  {

    Gstar[1,n22,s]<-1
```

```
    k<-k+1

  }

  n22<-n22+1

}

}

Gstar[2,,]<-Gstar[1,,]

Gstar[3,,]<-Gstar[1,,]

Gstar[4,,]<-Gstar[1,,]

eps<-array(0,dim=c(T,n,K))

 a<-array(0,dim=c(T,n,K))

 set.seed(a)

 a[,,]<-sqrt(avar)*matrix(rnorm(T*n),T,n,K) #set.seed

 eps[1,,]<-a[1,,]*sqrt(avar/(1-rho1^2))

 eps[2,,]<-rho1*eps[1,,]+a[2,,]
```

```
 eps[3,,]<-rho1*eps[2,,]+a[3,,]

 eps[4,,]<-rho1*eps[3,,]+a[4,,]

 #print(eps)

 ga<-array(0,dim=c(1,n,K))

 set.seed(ga)

 ga[,,]<-matrix(rnorm(n,0,gavar),n,K)


for (j in 1:K){

Y<-array(0,dim=c(T,n))

dermu<-array(0,dim=c(npar,T,n))

dermun<-array(0,dim=c(npar,T,n))

Left<-array(0,dim=c(npar,npar))

Right<-array(0,dim=c(npar,1))

Leftn<-array(0,dim=c(npar,npar))

Rightn<-array(0,dim=c(npar,1))


for (i in 1:n){

Y[,i]<-t(as.matrix(X[,,i,j]))%*%as.vector(beta)+ga[1,i,j]+eps[,i,j]+G[,i,j]*alpha

dermu<-rbind(X[,,i,j],t(G[,i,j]))
```

```
Left<-Left+dermu%*%Isig%*%cbind(t(X[,,i,j]),G[,i,j])

Right<-Right+dermu%*%Isig%*%as.vector(Y[,i])


dermun<-rbind(W[,,i,j],t(Gstar[,i,j]))

Leftn<-Leftn+dermun%*%Isig%*%cbind(t(W[,,i,j]),Gstar[,i,j])

Rightn<-Rightn+dermun%*%Isig%*%as.vector(Y[,i])

}


ThetaQL[,j]<- solve(Left,Right)

ThetaQLn[,j]<- solve(Leftn,Rightn)

}

se[1,m]<-sqrt(var(ThetaQLn[1,]))

se[2,m]<-sqrt(var(ThetaQLn[2,]))

se[3,m]<-sqrt(var(ThetaQLn[3,]))

bias[1,m]<-mean(ThetaQL[1,])-mean(ThetaQLn[1,])

bias[2,m]<-mean(ThetaQL[2,])-mean(ThetaQLn[2,])

bias[3,m]<-mean(ThetaQL[3,])-mean(ThetaQLn[3,])

}
```

```
bias1<-t(bias)

se1<-t(se)

result<-cbind(bias1[,3], se1[,3],bias1[,1],se1[,1],bias1[,2],se1[,2])

round(result,4)


 seq<-seq(-0.9,0.9,0.1)

 #seq<-seq(0.2,2,0.2)

# seq<-seq(-3,3,0.5)

# seq<-seq(0.1,0.6,0.1)

# seq<-seq(100,1000,100)


 Rbn1<-bias[1,]

 Rbn2<-bias[2,]

 Rbn3<-bias[3,]

 ptilda<-seq

 #opar <- par(mfrow=c(3,2))

 plot(ptilda,Rbn1,lwd=2, lty=3, type="l", ylim=c(-0.5,0.5),col=2,

 col.axis="darkred",col.main="darkred",col.sub="darkred",

 col.lab="darkred", xlim=c(-1,1), xlab="rho1", ylab="Bias",
```

```
main="Bias of the Naive Estimator for different value of rho1")

lines(ptilda,Rbn2,lwd=2, lty=1, type="l",col=3)

lines(ptilda,Rbn3,lwd=3, lty=2, col=7,type="l")

legend(0.2, 0.2, c("beta1","beta2","alpha"), lwd = c(2,2,3),

col=c(2,3,7), lty = c(3,1,2))
```

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