DYNAMIC ANALYSIS AND OPTIMAL DESIGN OF LATHE SPINDLES USING FINITE ELEMENT METHOD

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by

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ABSTRACT

This thesis presents the free vibration response, transient response and optimal design of a lathe under transient conditions. The optimal design is based on minimizing the maximum dynamic displacement response of the system.

The differential equation of motion characterizing the behaviour of a lathe spindle-workpiece system is formulated based on the Euler-Bernoulli's equation. The matrix differential equation of motion is obtained using finite element technique. The boundary condition at the live center is taken as hinged connection. In order to economize on CPU time and memory, the sytem matrices are condensed by using the dynamic condensation technique.

The free vibration response of the system is studied by varying parameters such as the bearing spacing, the bearing stiffness etc. The effects of these variations on the static deflection, the natural frequencies and the rate of decay of free oscillations of the system are analyzed. The condensation of the system matrices is done by selecting the appropriate number of masters by comparing the natural frequencies of the condensed and uncondensed systems. The free vibration response of the system is studied by assuming the initial velocity vector to be equal to zero.

A method comprising of the finite element technique and modal analysis is used for studying the system behaviour under transient cutting conditions. The response of the system due to an impulse and exponentially decaying pulse excitations is obtained. The effect of the variations of system design variables on the maximum dynamic displacement response is presented. Based on these variations the optimal values of the variables are obtained.

An optimal design of the lathe spindle-workpiece system using a nonlinear programming technique with bearing spacing, bearing stiffness and location of an external damper is obtained. The minimization of the maximum dynamic displacement response of the system is chosen as the objective for this optimization scheme. The optimal values of the design variables obtained by single parameter optimization are then compared with those obtained by multi-parameter optimization.

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NOMENCLATURE

$\cdot$	differentiation with respect to time
$\{\ \}$	vector
$[\ ]$	matrix
a_{ii}	diagonal element of [A]
b_{ii}	diagonal element of [B]
$c(x)$	damping coefficient
d	degrees of freedom condensed
$f_e(x,t)$	externally applied force
$g_j(X)$	constraint equation
$h_i(t-\tau)$	impulse response function
j	$\sqrt{-1}$
$k(x)$	stiffness coefficient
ℓ	length of an element
$m(x)$	mass per unit length
n	number of degrees of freedom
r	master degrees of freedom
$r_{i,j}$	random number
t	time variable
x_i	local position coordinate
$y_i(t)$	joint displacement
$[c]$	damping matrix of an element
$[k]$	stiffness matrix of an element
$[m]$	inertia matrix of an element
$\{y(t)\}$	local displacement vector
$\{\mathcal{Y}\}$	displacement vector in global coordinates
D_w	diameter of the workpiece

E	modulus of elasticity
$I(x)$	diametral moment of inertia
K_d	dynamic stiffness coefficient
K_f	front bearing stiffness
K_r	rear bearing location
L	length of the lathe spindle-workpiece system
L_b	bearing spacing
L_d	location of the external damper from the free end
$N_i(x_i)$	shape functions
$P(t)$	potential energy of the beam element
P^*	potential energy of the system
$R(t)$	kinetic energy of the element
R^*	kinetic energy of the system
T^*	time taken by the oscillations to reach 66.66 % of the initial displacement
W_m	maximum dynamic displacement
X	design vector
X_h	design vector corresponding to the largest value of objective function
X_r	design vector found by reflection
$[A^*]$	diagonal matrix
$[B^*]$	diagonal matrix
$[C]$	global damping matrix
$[C_c]$	global damping matrix of the condensed system
$[C_{rr}]$ $[C_{rd}]$ $[C_{dd}]$	sub matrices of $[C]$
$[I]$	identity matrix
$[K]$	global stiffness matrix

$[K_{rr}]$ $[K_{rd}]$ $[K_{dd}]$	sub matrices of [K]
$[K_c]$	global stiffness matrix of the condensed system
[L]	matrix of direction cosines
[M]	global inertia matrix
$[M_{rr}]$ $[M_{rd}]$ $[M_{dd}]$	sub matrices of [M]
$[M_c]$	inertia matrix of the condensed system
[O]	null matrix
$\{\bar{Y}\}$	global displacement vector
$\{Z(t)\}$	principal coordinate vector
β_1	real part of α
ω_1	imaginary part of α
α	a complex number
τ	decay constant of a single degree of freedom system
τ_m	decay constant of the lathe spindle system
θ	a real number
γ_i	eigen values
$\delta(t)$	dirac delta function
$\overline{\delta W}$	virtual work
$\{\psi\}$	a modal vector
$[\Phi]$	transformation matrix
$[\psi]$	modal matrix of the actual sytem
$[\psi_r]$	modal matrix of the condensed sytem
$[\eta]$	transpose of $[\psi_r]$

CHAPTER 1

INTRODUCTION

1.1 Machine Tools

Machine tools produce working parts for all machines and equipment used in all major divisions of industry. They are used to cut metals and to produce metal parts of all sizes and shapes to very close tolerances. The machining of metals generally involves cutting, shaping, or forming. Machine tools cut, shape, or form metal parts - (i) through the cutting and removal of metal chips by the use of edge tools, (ii) through shearing and (iii) through the controlled corrosive action of either chemicals, electricity or sound. In the third, more recently developed method, a combination of electrical and chemical action is used in machining ultra - hard materials.

Machine tools operate on either a reciprocating or a rotary type principle i.e, either the tool or the workpiece reciprocates or rotates. For example, the lathe utilizes a single point tool which removes metal in the form of chips as it travels longitudinally along a workpiece which is revolving. The relative motion between the tool and the workpiece is the essential principle in machining metals to produce workpieces having the desired shape, size and surface finish. In the next section, a brief description about the turning operation is presented.

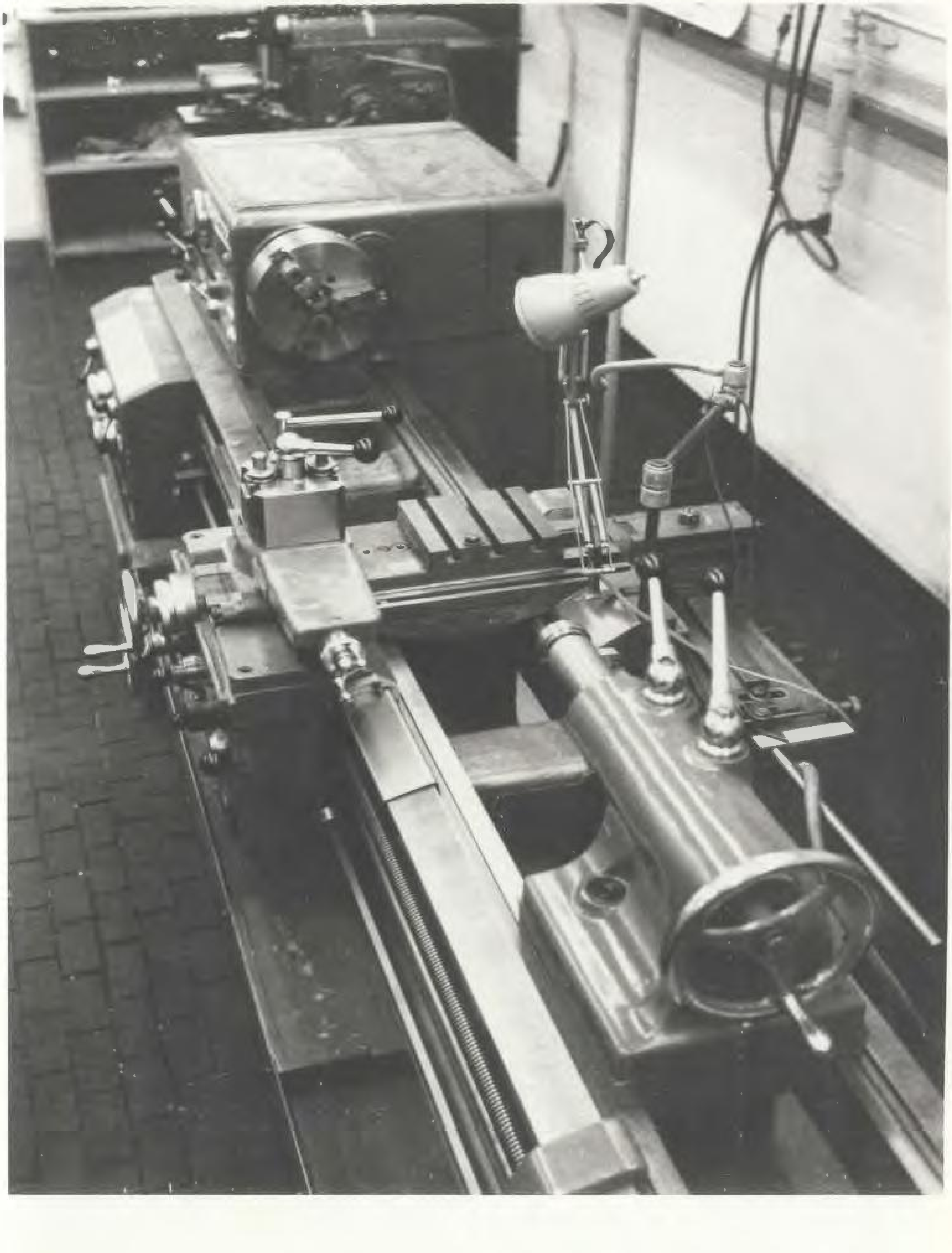
1.2 The Turning Operation

In a lathe, the metal is removed in the form of chips by a cutting tool moving along the workpiece as it revolves. A typical lathe is shown in Fig.1.1. The power generated in the motor is transmitted to the spindle through belts and gears. The spindle-workpiece system is shown in Fig.1.2. The spindle is supported by two bearings and the mass and the stiffness along the spindle are nonuniformly distributed along its axis. The chuck is normally considered to be an integral part of the spindle and therefore is to be included in any mathematical model. Cutting tools are made of high speed steel or carbide tip brazed or clamped to a solid piece. The tool is fed from right to left (refer to Fig.1.2) and the machining of a workpiece is carried out in two stages. The first stage usually called as roughing operation, consists of removing large amounts of material. In the second stage, known as finishing operation, very small amounts of material are removed to achieve dimensional accuracy and acceptable surface finish.

The performance of roller bearings is determined by the clearance. Normally, the bearings are preloaded to make the clearance as small as possible. The selection of bearing stiffness is done based on turning operation, since a lathe is essentially used for turning. There are small amounts of damping in the bearings in the form of frictional damping.

In a machine tool system both the internal and external dissipative forces exist. Internal damping, arising due to dissipation within the material, is in the form of hysteretic damping. The bearing location and the workpiece tool interface are key sources of external damping. These two damping effects can be combined without any significant loss of accuracy.

The cutting forces in the three orthogonal directions (refer to Fig.1.2) have very large fluctuations in their magnitude. The main cutting force is in the Z direction and it



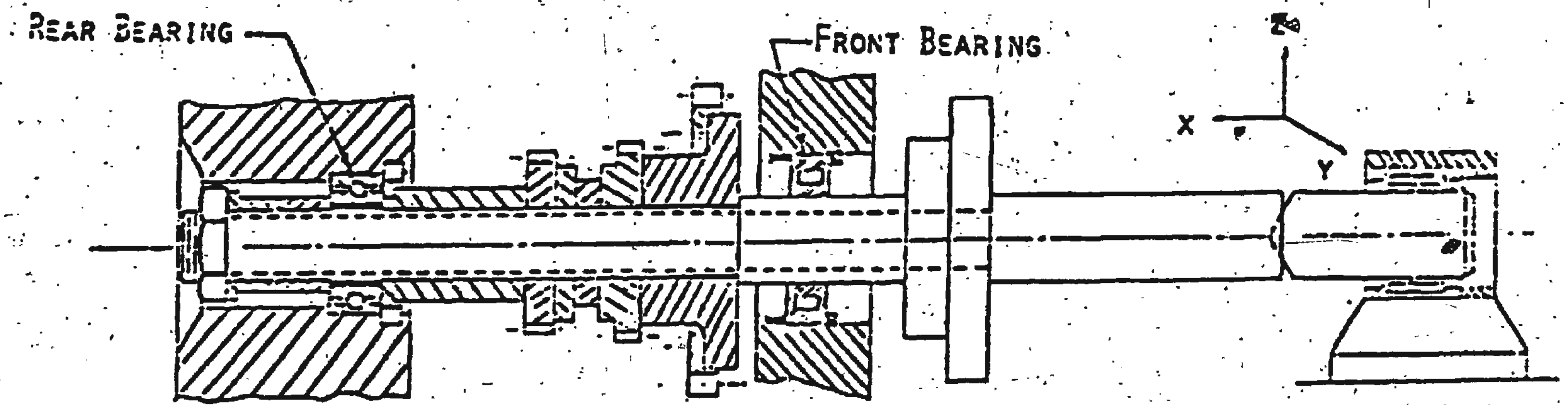


Fig. 1.2 Lathe Spindle-Workpiece System

has been found [1] to be the main force which controls the surface finish of the workpiece. These cutting forces cause vibrations in the spindle-workpiece system. This is discussed in the following section.

1.3 Vibrations in Machine Tools

The machining of metals and other materials is accompanied by relative vibration between the workpiece and the tool. These vibrations are due to: (i) inhomogeneities in the workpiece material; (ii) interrupted cutting; (iii) disturbances in the workpiece or tool drives; (iv) vibration transmitted through the foundation, and (v) vibration generated by the cutting process. The tolerable level of relative vibration between tool and workpiece in the roughing process is guided by tool life. In finishing operations, surface finish and machining accuracy determine the tolerable level. The control of these vibrations by a proper design of machine tools is thus essential for a longer tool life and good surface finish.

1.3.1 Vibrations Due to Inhomogeneities in the Workpiece

Hard spots in the material being worked, impart small shocks to the tool and the workpiece as a result of which vibrations are set up. These transient disturbances build up to vibrations of large amplitudes as a result of dynamic instability. During discontinuous chip removal, the segemntation of chip elements results in a fluctuation of the cutting thrust. If the frequencies of these fluctuations coincide with one of the natural frequencies of the structure, forced vibrations of appreciable amplitude occur. The breaking away of a built-up edge from the tool face also imparts impulses to the cutting tool which result in vibrations.

1.3.2 Vibrations Due to Interrupted Cutting

Impulses of appreciable magnitude may be imparted to the tool, when machining interrupted surfaces. The resulting vibration usually is not severe in case of single pointed tools but leads to serious effects when using tools of multiple cutting edges i.e, in the milling process.

1.3.3 Disturbances in the Workpiece and Tool Drive

There is a distinction between vibrations arising in drives that impart rotational motions and in drives that impart rectilinear motion. Vibrations resulting from rotating unbalanced masses, gear and belt drives, fall under the first group. The second group comprises free vibration resulting from shocks caused by reciprocating unbalanced masses and a type self-induced vibration frequently encountered with slide drives.

1.3.4 Vibrations Transmitted from other Machines

Vibration generated in machines is transmitted through the foundation to other machines and may set up forced vibration in these. The isolation of vibration transmitted through the floor is achieved by the use of vibration isolators.

1.3.5 Vibrations Generated by the Cutting Process

The cutting of metals is frequently accompanied by a violent vibration of workpiece and cutting tool known as machine tool chatter. Chatter is a self-induced vibration. It is highly detrimental to tool life and surface finish. In the elimination of chatter, cutting conditions are altered first. If this fails, an increase of stiffness between the tool and the workpiece and damping of disturbances impinging on the tool or improved tool design is resorted to.

1.4 Various Considerations in Machine-Tool Design

A machine tool is made up of several machine components. A good design of a machine tool requires that all of its components be designed with utmost care and precision. Thus a detailed static and dynamic analysis of each of the components is to be carried out to ensure a proper a design of the machine tool. The present investigation, which is, a study of the dynamic behavior of lathe spindles, is a step in that direction.

The important considerations in the design of machine tools are as follows:

(i) The stability of the structure,

(ii) The nature of the cutting forces,

(iii) The type of the damping,

(iv) The dynamic response ofthe workpiece under cutting force excitation, and

(v) The selection of optimal design parameters.

A literature survey on all the topics mentioned above, are presented in the following sections.

1.4.1 The Stability of the Machine Tool Structure

The machine tool structures are usually made of cast iron or mild steel. Analog and digital computers are often used in the design of machine tool frames which require complex mathematical analysis. Several researchers [2-4] analyzed the machine tool response by representing the structure by a number of lumped masses which were joined together by elastic and weightless beams. The response of each mode was computed separately and then they were added vectorially in the complex plane. A study of a machine tool drive and structure was carried out on an analog computer by Cuppan and Bollinger [5]. A system of differential equations was used to represent the machine tool structure and was solved for various system parameters. The theoretical results were then experimentally verified.

1.4.2 The Cutting Forces

Researchers in the field of manufacturing have been attempting to understand the nature of the cutting forces during a machining operation for more than a century. The machines of the cutting process was studied by Merchant [6,7]. Depending on the orientation of the cutting edge to the relative motion of the tool and the workpiece, the cutting process was divided into two parts; an orthogonal cutting and an oblique cutting. The theory and equations of the cutting process were developed, by which with little approximation, any orthogonal cutting process could be analyzed. Lee [8] studied the machining as a plastic flow of an ideal plastic material and thereby obtained the stress and the strain distribution in the system. An excellent treatment of the mechanics of the cutting process is given in [9].

1.4.3 The Damping of the Machine Tools

Several authors [10-12] have studied the effect of vibration on a machine tool and its effects on the quality of the workpiece produced. The dynamic stability is one of the factors affecting the quality of a machine tool. The dynamic stability determines the maximum material removal rate that can be attained without chatter. This is further related to the dynamic stiffness of the machine tool, which in turn depends on the static stiffness and on the damping of the machine tool structure.

The damping in a machine tool can be due to (a) structural damping, and (b) frictional damping. Frictional damping is the main source of energy dissipation in machine tools. This damping, in machine tools, causes energy dissipation at guideways, joints etc.

An excellent discussion on damping in machine tools can be found in [13-16].

1.4.4 The Flexural Vibration of Rotating Shafts.

The response of the machine tools can be assessed by making use of the information available from the area of stability of rotating shafts. The stability of symmetrical and unsymmetrical shafts was carried out in [17,18]. It has been reported that in the absence of internal damping, instability of transverse motion occurs only at certain critical speeds of rotation in case of symmetrical shafts, whereas, in case of unsymmetrical shafts, for a whole range of speeds. Kimball [16] suggested that the internal friction might be caused by elastic hysteresis which acts as a damping forces rotating with the shaft. However, Robertson [19] favoured a law by which damping forces were a function of change of strain. Neither of these theories have been found to be exact. The rotating viscous medium theory has been utilized in the present investigation. The coupling of the transverse and the torsional modes of vibration was studied by Bagci [20].

1.4.5 The Dynamic Response of Machine Tools

The study of the dynamic response under actual operating conditions enables the designer to estimate the performance of a machine tool. Since it is impractical to perform an analysis that includes the spindle, the drive system and the structure, a practical alternate solution would be to consider each system individually and to carry out the dynamic response study. In order to improve the performance of the machine tool, the response, at various points on the structure, has to be known. The literature survey on the dynamic response of machine tool spindles is presented in this section.

Sankar and Osman [21] studied the spindle-dynamics using two-degree-of-freedom system under stochastic excitation. The two equations were solved independently by assuming that there was no cross coupling. Rakhit [1] analyzed a lathe spindle-workpiece system by representing the system by two degrees of freedom. In this work, the actual cutting force was measured and its power spectral density was experimentally obtained by using a frequency analyzer. The mathematical model did not include the live center connection and the workpiece was held only at the chuck.

Various researchers have used analytical [22-24] and graphical [25] methods for analyzing the lathe spindle behavior under the static and the dynamic conditions. The multi-degree-of-freedom steady state analysis of a lathe spindle was carried out by Bollinger [22]. The spindle was represented by a seven degrees of freedom and the harmonic force was assumed to be acting on the chuck. In this analysis the boundary condition at the workpiece-live center interface was not included in the mathematical model. It was concluded that the approach, which uses the finite difference technique, to formulate the model of the spindle, and the solution of the equations using the analog computers, is an efficient and useful method for analyzing the variables involved in the optimal design of the spindle. It was also concluded that the external damper should be located at the free end of the spindle. In [24,26], the natural frequencies of the system were computed analytically and experimentally. Based on a root sum squared criterion, the boundary condition at the workpiece-live center interface was classified as hinged connection. The variations of the first five natural frequencies and the corresponding mode shapes, as a function of the bearing stiffness, were also studied. The application of the fairly recent technique, such as the finite element method to study the dynamic response of machine tool spindles, can be found in [26]. In this work, the dynamic response based optimal design of a lathe spindle under experimentally measured random cutting forces was carried out. The optimal design was based on minimizing the maximum mean square displacement response of the workpiece under the action of random cutting forces. The modal analysis in conjunction with the finite element techinque was used to calculate the mean square dispacement of the workpiece.

The dynamic characteristics such as dynamic response and the phase angle as a function of rotational speed of the spindle were experimentally studies by Morse [27] and Allemang [28].

1.4.6 The Acceptance Tests of Machine Tools

It has been recognized by researchers in the area of machine tools that static tests [29] only indicate whether the machine is properly manufactures. That is, the static acceptance tests do not give a complete description of the behaviour of machines under actual machining conditions. Tobias [30] introduces, for the first time, a method for testing the dynamic aspects of machine tools. Several dynamic acceptance tests [31-33] were later proposed. In [32,33] dynamic acceptance tests for machine tools were proposed based on a coefficient of dynamic stiffness. The mathematical model consisted of either a single degree or two degree-of-freedom nonlinenar system and the process was assumed to be stationary with a gaussian distribution. It was concluded that the proposed acceptance test determines the dynamic resistance available in a machine tool under actual cutting conditions.

1.4.7 Optimal Design

Design of stochastically excited lathe spindles based on optimal selection of parameters, can be found in [34]. In this work, a direct search optimization was used to select optimal bearing stiffness and bearing spacing.

In order to establish the quality of machine tool, there are several characteristics that are worth looking into. Some of these desirable characteristics are: (a) the damped free oscillations of the system subjected to initial displacement and (b) the transient response of the system under a shock or an impulse excitation.

Based on a review of the available literature, it can be inferred that studies such as the transient or the free vibration characteristics of a machine tool as a measure of its performance have not been done earlier. This is the objective of the present investigation. A brief description of the objectives is presented in the next section.

1.5 Objectives

In this thesis a method for obtaining the flexural response of the lathe spindle-workpiece system subjected to (a) a set of initial displacement excitation and (b) the transient excitation is developed. The system is modeled as a multi-degree-of freedom damped system using the finite element analysis. The system design parameters such as the bearing spacing, the bearing stiffness etc., have been varied with a view to optimize the desired characteristics such as the rate of decay of the damped free oscillations or the maximum dynamic displacement response of the system. With this in view, the following are the objectives:

1) The study of the static deflection characteristics of the system, using the finite element method.

2) The analysis of the free vibration response of the system due to a set of initial displacements.

3) The dynamic condensation of system matrices to achieve computational efficiency.

4) The development of a methodology for studying the transient response of the lathe spindle-workpiece system modelled as a multi-degree-of-freedom damped system.

5) A parametric study of design variables such as the bearing stiffness, the bearing spacing, and the damping on the maximum dynamic displacement response of the system.

6) The optimal selection of the system parameters to minimize the maximum dynamic displacement response of the system, employing a nonlinear programming technique.

In chapter 2, based on the finite element technique, the lathe spindle-workpiece system is formulated as a multi-degree-of-freedom damped system. The mathematical model consists of bearings represented by a linear spring and a viscous damper. Further, the boundary condition at the live center interface is taken as hinged connection. The dynamic condensation technique is discussed and the equations necessary for condensing the system matrices are developed.

In chapter 3, the free vibration behavior of the system is discussed. The criteria for the selection of the master-degrees-of-freedom for the condensation of system matrices, is presented. The static deflection characteristics of the system and the time required for the damped free oscillations to reach 66.66% of the initial displacements are obtained. The variation of the static deflection and decay rate of free oscillations as a function of design variables are studied.

In chapter 4, the method of obtaining the transient response of the lathe spindle-workpiece system is developed. The transient response of the system due to a unit impulse and exponentially decaying pulse are obtained. The parametric study on the maximum dynamic displacement response of the system is carried out.

Chapter 5 describes the optimum selection of parameters and the optimization technique used for the system under consideration. The system is optimized using a direct search method known as the complex method.

Conclusions and recommendations for future work are presented in chapter 6.

CHAPTER 2

THE MATHEMATICAL MODEL OF THE LATHE SPINDLE-WORKPIECE SYSTEM

2.1 Introduction

It is known that the surface finish of a workpiece is dependent upon its dynamic response due to the cutting forces. The higher the response, the rougher is the finish. The flexural response can be controlled by an optimum selection of parameters such as bearing spacing, bearing stiffness etc. In order to achieve the optimal conditions, a mathematical model of the lathe spindle-workpiece system is necessary. In this chapter a comprehensive mathematical model of the lathe spindle-workpiece system is developed. In the development of any mathematical model, there are several assumptions made. A detailed discussion of these assumptions is presented in the next section.

2.2 Assumptions and Justifications

Following are the several assumptions made in the formulation of the mathematical model :

1) Since there is no relative translation between the chuck and the workpiece, the workpiece is taken as an integral part of the spindle.

2) The effects of shear deformation and rotary inertia have been assumed to be negligible

3) The change in the diameter of the workpiece is assumed to be negligible since the depth of cut in the finishing operation is very small compared to the dimensions of the workpiece.

4) The spindle-workpiece system is taken to be statically and dynamically balanced and to be operating at a constant speed.

5) The dominant mode of vibration during the turning operations is considered to be the flexural vibration of the spindle-workpiece system. The torsional oscillations of the spindle-workpiece system due to the combined flexural torsional action of the metal cutting forces are assumed to be insignificant and therefore neglected in the mathematical model.

6) The bearings are assumed to be rigidly mounted in the housing due to the presence of an interference fit between the bearing and the housing.

7) The bearings are represented by a viscous damper and a linear spring in the model, since the inner races of the bearing are considered to exhibit some viscous damping and stiffness properties.

2.3 The Mathematical Model

The equation of motion of the spindle-workpiece system shown in Fig. 1.2 can be formulated by considering it as a bar in flexure. The Euler-Bernoulli's equation of motion, governing the flexural vibration of the spindle-workpiece system, can be written as [35]

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\,\frac{\partial^2 y}{\partial x^2}(x,t)\right] = -m(x)\,\frac{\partial^2 y}{\partial t^2}(x,t) - k(x)\,y(x,t) - c(x)\,\frac{\partial y}{\partial t}(x,t) + f_e(x,t) \tag{2.1}$$

E =	modulus of elasticity,
I(x) =	diametral moment of inertia,
m(x) =	mass per unit length,
y(x,t) =	deflection along the system,
k(x) =	stiffness coefficient,
c(x) =	damping coefficient,
$f_e(x,t)$ =	externally applied force, and,
t =	time.

The effects of the rotary inertia and shear deformation are neglected in Eq.(2.1) as indicated earlier. The terms on the right hand side of Eq.(2.1) correspond to the inertia force, the elastic force, the damping force and the externally applied force respectively. The boundary conditions to be satisfied in finding a solution of the Eq.(2.1), are:

(i) At the free end of the spindle the shear force and the bending moment are zero.

(ii) The boundary condition at the workpiece-live center connection can be assumed either as hinged or clamped. If assumed as hinged, the bending moment and the deflection, must be zero at the connecting point; or if the above mentioned connection is assumed as clamped, then the slope of the deflection curve and the deflection must be zero. It was found [24] that the hinged end-condition is a better representation of the actual behavior of the system. Therefore, in the present model, the workpiece-live center connection has been represented by a hinged connection.

The system is divided into twelve lumped elements as shown in Fig. 2.1. The fourth order partial differential equation, Eq.(2.1) can be solved to investigate the dynamic

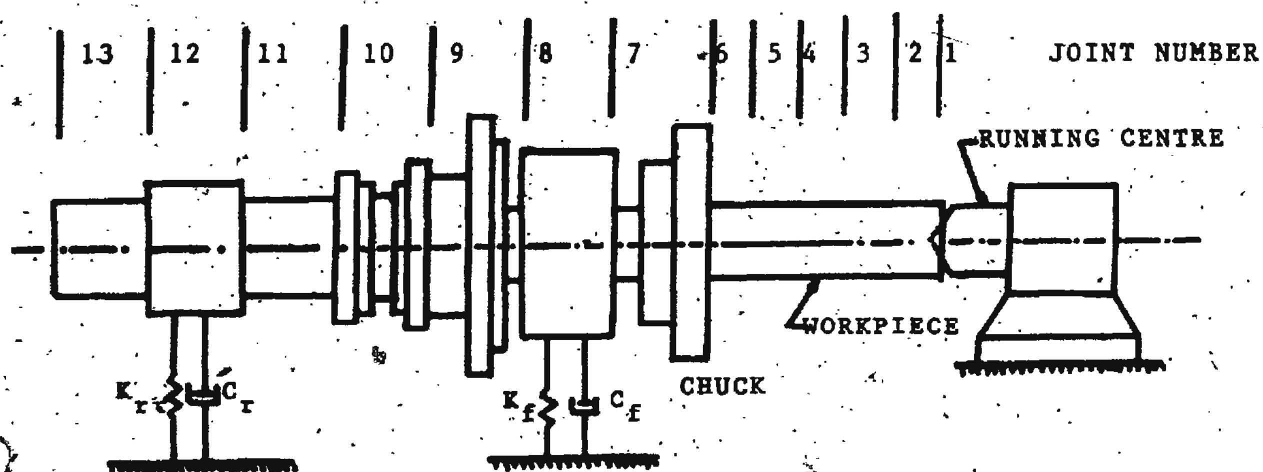


Fig. 2.1 Schematic Model of a Lathe Spindle-Workpiece System

behavior of the lathe spindle-workpiece system with the following boundary conditions;

At the live center, the boundary condition would be hinged connection which can be mathematically written as

$$y(x,t)|_{x=0} = 0 \text{ and } EI(x)\frac{\partial^2 y}{\partial x^2}(x,t)|_{x=0} = 0. \quad (2.2)$$

The boundary condition for the free end of the spindle can be mathematically represented as

$$EI(x)\frac{\partial^2 y}{\partial x^2}(x,t)|_{x=L} = 0,$$

$$\text{and } \frac{\partial}{\partial x}\left[EI(x)\frac{\partial^2 y}{\partial x^2}(x,t)\right]_{x=L} = 0. \quad (2.3)$$

Where L is the total length of the spindle-workpiece system. The variable x is measured from the live center towards the free end of the spindle.

2.4 The Method of Solution

Eq.(2.1) together with the associated boundary conditions as represented by Eqs.(2.2) and (2.3), can be solved by finite difference method as used by Bollinger [22], or it can be solved by finite element method [26]. In [26], the advantages of using the finite element method over the finite difference method are discussed. Therefore, in the present study, the finite element technique has been chosen for studying the dynamic behavior of the spindle-workpiece system.

2.4.1 The Finite Element Method

The idea behind the finite element method is to provide a formulation which can exploit digital computer automation for the analysis of complex systems. The method regards a complex structure as a finite assemblage of discrete elements, where every such element is a continuous structural member. The displacement at any point of the continuous element is expressed in terms of a finite number of displacements at the boundaries of the element. By requiring that the displacements be compatible and the internal forces in balance at certain points shared by several elements, where the points are known as joints, the entire structure is compelled to act as one entity.

A beam element in bending is shown in Fig.2.2. The beam element will be assumed to be a straight bar of uniform cross-section capable of resisting the axial forces and bending moments. Thus the element is subjected to shear forces $f_1(t)$, $f_3(t)$ and bending moments $f_2(t)$, $f_4(t)$ undergo both translational and rotational displacements. The displacement at any point in the beam is expressed as [36]

$$y(x_1,t) = \sum_{i=1}^{4} N_i(x_1)\, y_i(t) \tag{2.4}$$

Where, $N_i(x_1)$ is the shape function, x_1 is the local position coordinate and $y_i(t)$ is the joint displacement. It can be inferred from the Fig. 2.2 that

$$y(0,t) = y_1(t)\ ;\ \frac{\partial y}{\partial x_1}(x_1,t)\Big|_{x_1=0} = y_2(t),$$

$$y(l,t) = y_3(t)\ ,\ \frac{\partial y}{\partial x}(x_1,t)\Big|_{x_1=l} = y_4(t). \tag{2.5}$$

The solution of the differential equation governing the static bending of a uniform bar, with inertia term set to zero, gives the general expression for the shape functions. The set of shape functions is then found by applying the boundary conditions given by Eq.(2.5). The generalized displacement at any point on the beam is expressed as [36]

$$y(x_1,t) = (1 - \frac{3x_1^2}{l^2} + \frac{2x_1^3}{l^3})\, y_1(t) + (\frac{x_1}{l} - \frac{2x_1^2}{l^2} + \frac{x_1^3}{l^3})\, l\, y_2(t)$$

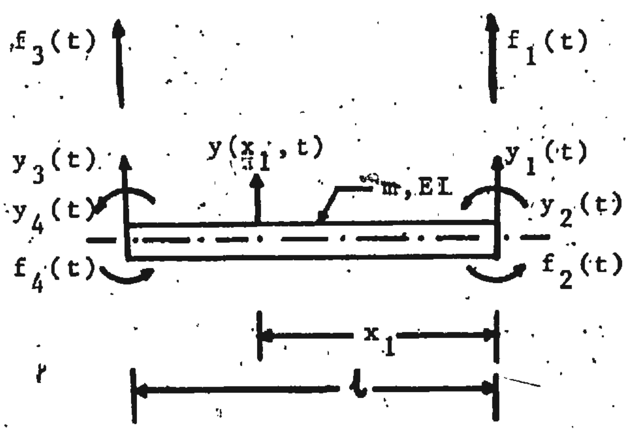


Fig. 2.2 Finite Element Model of One of the Elements of the System

$$+ \left(\frac{3x_1^2}{l^2} - \frac{2x_1^3}{l^3}\right) y_3(t) - \left(\frac{x_1^2}{l^2} + \frac{x_1^3}{l^3}\right) l \, y_4'(t) \tag{2.6}$$

The kinetic and potential energies of the beam element are given by

$$R(t) = \frac{1}{2} \int_0^l m \left[\frac{\partial y}{\partial t}(x_1, t) \right]^2 dx_1 \tag{2.7}$$

and

$$P(t) = \frac{1}{2} \int_0^l EI \left[\frac{\partial^2 y}{\partial x_1^2}(x_1, t) \right]^2 dx_1. \tag{2.7a}$$

The equation of motion for the element can be written by substituting Eqs.(2.7) and (2.7a) in Lagrange's equation of motion. After a few mathematical steps (shown in the Appendix A) the system of differential equations of motion for the element can be written as

$$[m]\{\ddot{y}(t)\} + [k]\{y(t)\} = \{f(t)\}. \tag{2.8}$$

In Eq.(2.8) [m], [k] are elemental inertia and stiffness matrices and {y(t)}, {f(t)} are local displacement and force vectors. The stiffness matrices for elements 7 and 11 (refer to Fig. 2.1), are different due to the presence of the bearings. The stiffness matrix for these elements is augmented by a matrix, which is given as

$$\begin{bmatrix} \frac{K_b}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{K_b}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{2.9}$$

where, K_b is the bearing stiffness at the element in consideration. In addition, a damping matrix will be present at these elements and is given by

$$[c] = \begin{bmatrix} \frac{C_b}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{C_b}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (2.10)$$

Where C_b is the damping present at the element in question.

The equations of motion for a beam element given by Eq.(2.8) are derived in local coordinates $\{y(t)\}$. The differential equations of motion for the beam element are to be written in terms of a set of global coordinates to obtain the equations of motion for the complete structure, Choosing a global reference system, the displacement components along the local coordinate system can be resolved into components along the global reference system by the transformation

$$\{y\} = [L]\{\bar{y}\} \qquad (2.11)$$

Where $[L]$ is the matrix of direction cosines and $\{\bar{y}\}$ is the displacement vector in global coordinates. The kinetic and the potential energies can be written in the form of the triple matrix products as

$$R = \frac{1}{2}\{\dot{y}\}^T [m] \{\dot{y}\}, \qquad (2.12)$$

and

$$P = \frac{1}{2}\{y\}^T [k] \{y\}; \qquad (2.12a)$$

Whereas the virtual work has the expression

$$\overline{\delta W} = \{\delta y\}^T \{f\}. \qquad (2.12b)$$

By introducing Eq.(2.11) into Eqs.(2.12), (2.12a) and (2.12b), the equation of motion of the beam element in the global coordinates can be written as

$$[\bar{m}]\{\ddot{y}(t)\}+[\bar{k}]\{\bar{y}(t)\}=\{\bar{f}(t)\} \tag{2.13}$$

Where

$$[\bar{m}]=[L]^T[m][L] \tag{2.14}$$

$$[\bar{k}]=[L]^T[k][L] \tag{2.14a}$$

$$\{\bar{f}(t)\}=[L]^T\{f(t)\} \tag{2.14b}$$

Assuming that the structure consists of n joint displacements and p elements, the relation between the element displacement vector and global displacement vector can be written as

$$\{\bar{y}\}_s=[H]_s\{\bar{Y}\} \quad s=1,\ldots,p \tag{2.15}$$

Where $[H]_s$ is a rectangular transformation matrix and s denotes the number of the elements. The matrix $[H]_s$ has as many rows as $\{\bar{y}\}_s$ and as many columns as $\{\bar{Y}\}$ has rows. The elements of every row of $[H]_s$ are all zero, with the exception of one element in each row which is equal to unity. The position of the unit element in every row is such that Eq.(2.15) represents an identity. It should be noted that a given joint displacement can occur in several of the vectors. By adding the contributions of all the elements the kinetic energy for the complete system can be written as

$$R^*=\frac{1}{2}\sum_{s=1}^{p}\{\dot{\bar{y}}\}_s^T[\bar{m}]_s\{\dot{\bar{y}}\}_s \tag{2.16}$$

Differentiation of Eq.(2.15) gives

$$\{\dot{\bar{y}}\}_s=[H]_s\{\dot{\bar{Y}}\} \quad s=1,\ldots,p \tag{2.17}$$

Combining Eqs.(2.17) and (2.13) one obtains

$$R^*=\frac{1}{2}\sum_{s=1}^{p}\{\dot{\bar{Y}}\}^T[H]_s^T[\bar{m}]_s[H]_s\{\dot{\bar{Y}}\}$$

$$= \frac{1}{2}\{\dot{Y}\}^T [M] \{\dot{Y}\} \tag{2.18}$$

Where

$$[M] = \sum_{s=1}^{p} [H]_s^T [\bar{m}]_s [H]_s \tag{2.19}$$

represents the inertia matrix of the system. Following the same procedure the potential energy can be written as

$$P^* = \frac{1}{2}\sum_{s=1}^{p} \{\bar{Y}\}^T [H]_s^T [\bar{k}]_s [H]_s \{\bar{Y}\}$$

$$= \frac{1}{2}\{\bar{Y}\}^T [K] \{\bar{Y}\} \tag{2.20}$$

Where,

$$[K] = \sum_{s=1}^{p} [H]_s^T [\bar{k}]_s [H]_s \tag{2.21}$$

is the stiffness matrix of the complete system. Similarly [C], the damping matrix of the complete system can be represented in the form

$$[C] = \sum_{s=1}^{p} [H]_s^T [c]_s [H]_s \tag{2.22}$$

Where $[c]_s$ is the damping matrix associated with the s^{th} element in the global coordinates. The virtual work must be the same, whether it is expressed in terms of virtual displacements and forces corresponding to the assemblage of elements or in terms of those corresponding to the complete system. Hence

$$\overline{\delta W} = \sum_{s=1}^{p} \{\delta Y\}^T [H]_s^T \{f\}_s$$

$$= \{\delta Y\}^T \{F\} \tag{2.23}$$

where {F} is the vector of the joint non-conservative forces for the complete system. Substituting Eqs.(2.18), (2.20), (2.22) and (2.23) into Lagrange's equation of motion, the matrix differential equation of the complete spindle-workpiece system can be derived in the form

$$[M]\{\ddot{Y}(t)\}+[C]\{\dot{Y}(t)\}+[K_t]\{Y(t)\}=\{F(t)\} \qquad (2.24)$$

In the present analysis, the matrices [M], [C] and [K] have a size of (26X26) obtained after assembling the elemental matrices.

2.4.2 The Boundary Conditions

Before solving Eq.(2.24), boundary conditions are to be applied to remove the rigid body mode. Since the live center connection is taken as hinged connection, the first component of the displacement vector {Y} would vanish. As a consequence of this, the size of matrices in Eq.(2.24) would be (25 × 25).

2.5 The Dynamic Condensation

The size of matrices in Eq.(2.24) is such that the dynamic response calculations require large CPU time and storage. When the system has too many degrees of freedom for economical treatment, these degrees of freedom can be reduced by invoking a technique known as dynamic condensation [37].

The degrees of freedom to be retained are known as the masters and the degrees of freedom to be discarded are known as the slaves. The selection of master degrees of freedom, r and slave degrees of freedom, d is carried out by scanning the diagonal terms of matrices [M] and [K]. The degree of freedom i, for which the ratio is largest is selected as the first slave. The process is continued, until the required number of slaves are chosen. This ensures an accurate representation of lower vibration modes in the reduced system.

The dynamical equation of motion is rearranged in the form [38]

$$\left[\begin{array}{c:c}[M_{rr}] & [M_{rd}] \\ \hdashline [M_{rd}]^T & [M_{dd}]\end{array}\right]\left\{\begin{array}{c}\ddot{Y}_r(t) \\ \hdashline \ddot{Y}_d(t)\end{array}\right\} + \left[\begin{array}{c:c}[C_{rr}] & [C_{rd}] \\ \hdashline [C_{rd}]^T & [C_{dd}]\end{array}\right]\left\{\begin{array}{c}\dot{Y}_r(t) \\ \hdashline \dot{Y}_d(t)\end{array}\right\}$$

$$+\left[\begin{array}{c:c}[K_{rr}] & [K_{rd}] \\ \hdashline [K_{rd}]^T & [K_{dd}]\end{array}\right]\left\{\begin{array}{c}Y_r(t) \\ \hdashline Y_d(t)\end{array}\right\} = \{ F(t) \} \tag{2.25}$$

The transformation matrix for reduction, is obtained based on the principle assumption: that for the lower frequencies, the inertia forces on slave degrees of freedom are far less important than elastic forces transmitted by the master degrees of freedom. The relation between the displacements of the actual and reduced system can be written as [38]

$$Y(t) = \left\{\begin{array}{c}Y_r(t) \\ Y_d(t)\end{array}\right\} = [\Phi]\{Y_r(t)\}$$

$$= \left[\begin{array}{c}[I] \\ \hdashline -[K_{dd}]^{-1}[K_{rd}]^T\end{array}\right]\{Y_r(t)\} \tag{2.26}$$

The dynamical equation for the condensed system can be written as

$$[M_c]\{\ddot{Y}_r(t)\} + [C_c]\{\dot{Y}_r(t)\}$$

$$+[K_c]\{Y_r(t)\} = \{F_c(t)\}, \tag{2.27}$$

Where

$$[M_c] = [\Phi]^T [M][\Phi], \tag{2.27a}$$

$$[C_c] = [\Phi]^T [C][\Phi], \tag{2.27b}$$

$$[K_c] = [\Phi]^T [K][\Phi], \tag{2.27c}$$

and

$$\{ F_c(t) \} = [\Phi]^T \{ F(t) \} \qquad (2.27d)$$

2.6 Conclusions

In this chapter, an appropriate mathematical model of the lathe spindle-workpiece system has been obtained. The matrix differential equation of motion is formulated by making use of the finite element technique. In order to achieve computational efficiency, the system matrices have been condensed by using the dynamic condensation technique.

CHAPTER 3

THE FREE VIBRATION ANALYSIS OF THE LATHE SPINDLE-WORKPIECE SYSTEM

3.1 Introduction

A better performance of a machine tool under actual machining conditions can be achieved by a design, based on both the static and dynamic analyses of the spindle-workpiece system. A design based on the static analysis is not sufficient because in actual machining operations, the spindle is subjected to dynamic cutting forces [29]. Various researchers have used analytical [22-24] and graphical methods [25] for analyzing the spindle behaviour under the static and dynamic conditions. The analytical methods used are: the lumped parameter method, the finite difference method, and the finite element method. The free vibration study of the lathe spindle using finite element method, was carried out in [26]. In this work the natural frequencies were obtained in two ways: analytically and experimentally. These values were compared and variation of the first five natural frequencies and corresponding mode shapes, as a function of the bearing stiffness were studied.

One of the several desirable characteristics of a lathe spindle-workpiece system is that its deflection due to the cutting forces should be as small as possible. It implies that the vibratory response of the system should be reduced. This requires high static or dynamic stiffness as the case may be. Another feature a machine tool spindle should have, is that, if it is given an initial displacement by applying a static force and allowed to restore to its original configuration by withdrawing this force, it must do so very quickly. This phenomenon is called the rate of decay of the free oscillations. There are several parameters which should be considered in the design of the lathe spindles such as the bearing spacing, the bearing stiffness, the workpiece diameter and the damping. A suitable choice of these parameters can lead to a design that would have the above mentioned desirable characteristics.

In the last chapter, the dynamic equation of motion of the system for the present investigation was derived using the finite element technique. Since the system matrices involved are large, a dynamic condensation technique was used to reduce the size of these matrices. In this way, the dynamic behaviour of the spindle could be analyzed using these reduced matrices.

In this chapter, the free vibration response of the system is studied by varying parameters such as the bearing spacing, the bearing stiffness etc. The effects of these variations on the static deflection, the natural frequencies, and the rate of decay of free oscillations are analyzed. To carry out these analyses, the system matrices are reduced first. This reduction process is done by selecting the appropriate number of masters by comparing the natural frequencies of the condensed and uncondensed systems. After this step, the free vibration response of the system is studied by applying a static force on the workpiece which provides the initial displacement vector. The initial velocity vector is assumed to be zero.

3.2 The Free Vibration Response of the System

The differential equation of motion of a lathe spindle subjected to initial displacements can be obtained from Eq.(2.24) by setting the right hand side vector equal to zero.

This can be written as

$$[M]\{\ddot{Y}(t)\}+[C]\{\dot{Y}(t)\}+[K]\{Y(t)\}=\{0\} \tag{3.1}$$

The free vibration response of the system is obtained by solving the above equation with the given initial conditions. In Eq.(3.1) [M], [C] and [K] are the inertia, the damping and the stiffness matrices of size (nxn). These matrices are real symmetrical matrices.

3.2.1 The Eigenvalues and the Eigenvectors of the System

The n second order differential equations in Eq.(3.1) can be uncoupled by rewriting them as 2n first order differential equations [39]. The reduced equations are formed by introducing a 2nx1 state vector, defined as

$$\{W(t)\}=\begin{Bmatrix}\dot{Y}(t)\\ Y(t)\end{Bmatrix} \tag{3.2}$$

Eq.(3.1), with the help of Eq.(3.2), can be rewritten as

$$[A]\{\dot{W}(t)\}+[B]\{W(t)\}=0 \tag{3.3}$$

Where,

$$[A]=\begin{bmatrix}[0] & [M]\\ [M] & [C]\end{bmatrix} \tag{3.4}$$

and

$$[B]=\begin{bmatrix}-[M] & [0]\\ [C] & [K]\end{bmatrix} \tag{3.4}$$

[A] and [B] are real symmetrical matrices of order 2n. Premultiplying Eq.(3.3) by $[A]^{-1}$, one can write

$$\{\dot{W}(t)\}-[H]\{W(t)\}=\{0\} \tag{3.5}$$

where $[H] = -[A]^{-1}[B]$. The eigenvalues of the system are obtained by assuming a solution of the form

$$\{W(t)\} = \{\Psi\} e^{\alpha t} \tag{3.6}$$

Where α is a complex number and $\{\Psi\}$ is a 2nx1 modal vector with complex elements. Substituting Eq.(3.6) in Eq.(3.5) yields

$$\alpha [I] \{\Psi\} - [H] \{\Psi\} = \{0\} \tag{3.7}$$

The characteristic equation of the system can be written as

$$|\alpha I - H| = 0 \tag{3.8}$$

Eq.(3.8) can be solved to obtain the 2n eigenvalues, which are necessarily complex conjugates. A modal vector $\{\Psi\}$ is found by substituting an eigenvalue α in Eq.(3.7). The modal matrix $[\Psi]$ can be obtained as a linear combination of the eigenvectors and is of the order 2n. This can be expressed as

$$[\Psi] = [\{\Psi\}_1 \{\Psi\}_2 \ldots\ldots \{\Psi\}_{2n}] \tag{3.9}$$

3.2.2 Free Vibration - Modal Analysis

The reduced equations in Eq.(3.3) can be uncoupled by means of modal matrix $[\Psi]$. Introducing a new state vector $\{Z(t)\}$ as defined by the transformation

$$\{W(t)\} = [\Psi] \{Z(t)\} \tag{3.10}$$

into Eq.(3.3) and premultiplying the resulting equation by $[\Psi]^T$, one obtains

$$[A^*] \{\dot{Z}(t)\} + [B^*] \{Z(t)\} = \{0\} \tag{3.11}$$

where,

$$[A^*] = [\Psi]^T [A][\Psi] ,$$

and

$$[B^*] = [\Psi]^T [B][\Psi] . \tag{3.12}$$

The matrices $[A^*]$ and $[B^*]$ are diagonal, hence Eq.(3.11) represents a set of 2n independent first order differential equations. The solution of Eq.(3.11) can be written as

$$Z_i(t) = Z_{io}\, e^{\alpha_i t} \qquad i=1,.....,2n \tag{3.13}$$

Where Z_{io} is the initial condition for the i^{th} coordinate. The initial conditions for each of these coordinates can be obtained using Eq.(3.10) and can be written as

$$\{Z(o)\} = [\Psi]^{-1} \{W(o)\} = [\Psi]^{-1} \left\{\begin{array}{c} \dot{Y}(o) \\ \hline Y(o) \end{array}\right\} \tag{3.14}$$

The eigenvalues α_i in Eq.(3.13) are in general complex and can be expressed as

$$\alpha_i = -\varsigma_i + j\, \omega_i \tag{3.15}$$

Thus, the solution in Eq.(3.13) can be written as

$$Z_i(t) = Z_{io}\, e^{-\varsigma_i t} \{\cos(\omega_i t) + \sin(\omega_i t)\} \tag{3.16}$$

For stable systems, ς_i is always positive. The displacement vector $\{Y(t)\}$ is obtained by using the transformation given in Eq.(3.10), which can be written as

$$\left\{\begin{array}{c} \dot{Y}(t) \\ \hline Y(t) \end{array}\right\} = \{W(t)\} = [\Psi]\{Z(t)\} . \tag{3.17}$$

3.3 The Dynamic Reduction of the System Matrices

In the static and dynamic analyses of systems having large degrees of freedom, the matrices involved are large. At the same time the displacement vector must be evaluated for different instants of time. Thus the number of computations become excessive. This requires large computer memory storage. In the case of spindle workpiece system it is known [1] that the quality of the workpiece is directly related to its dynamic response. Further, the joints on the workpiece have higher displacements as compared to joints on the spindle. Therefore the criteria for the spindle design should be to minimize the workpiece response.

The system matrices [M], [C] and [K] can be reduced by the dynamic reduction technique, described in chapter 2. The reduced matrices $[M_c]$, $[C_c]$ and $[K_c]$ can then be substituted in Eq.(3.1) and following the procedure outlined in section 3.2, one can obtain the expression for the dynamic displacement vector $\{Y_r(t)\}$ of master degrees of freedom as

$$\{ Y_r(t) \} = [\Psi_r] \{ Z_r(t) \} . \tag{3.18}$$

3.3.1 The Selection of the Number of Master Degrees of Freedom

A 12 hp Demoor type lathe, model no. 821A shown in Fig.1.1 is chosen for this investigation. The workpiece is assumed to be made of AISI 1020 steel. The variations in the mass and the stiffness along the spindle workpiece-system are shown in Figs.3.1 and 3.2 respectively. The values of the various parameters are given in Table 3.1.

The reduction of system matrices is done in such a way that the lower vibration modes are accurately represented in the condensed system. This is carried out by choosing the degrees of freedom having large mass to stiffness ratio as master degrees of freedom. In order to choose the number of master degrees of freedom for an effective reduction, the damped natural frequencies were computed using Eq.(3.8), for different number of master degrees of freedom. The results are shown in Table 3.2. In this table the

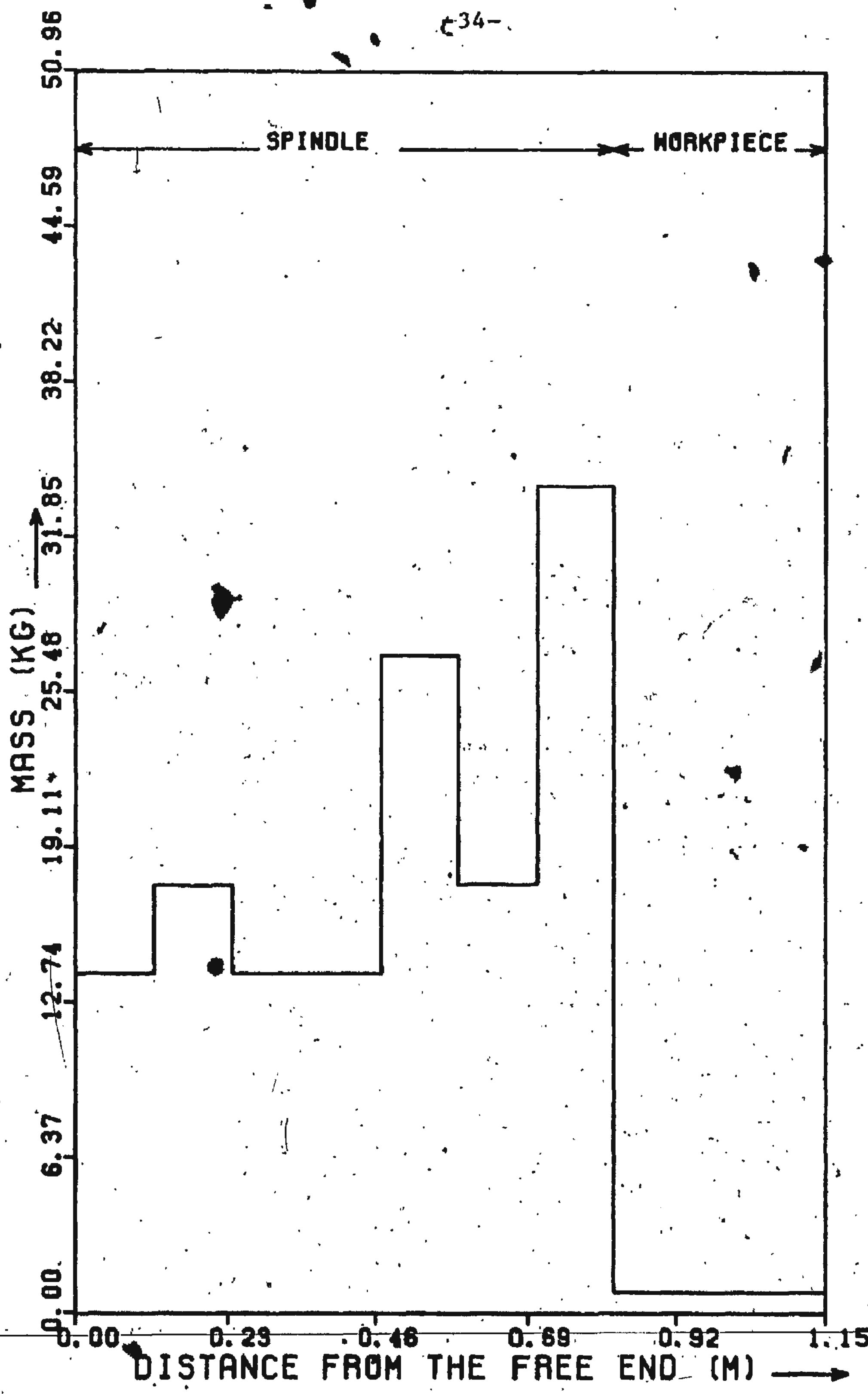


Fig. 3.1 The Variation of Mass Along the System

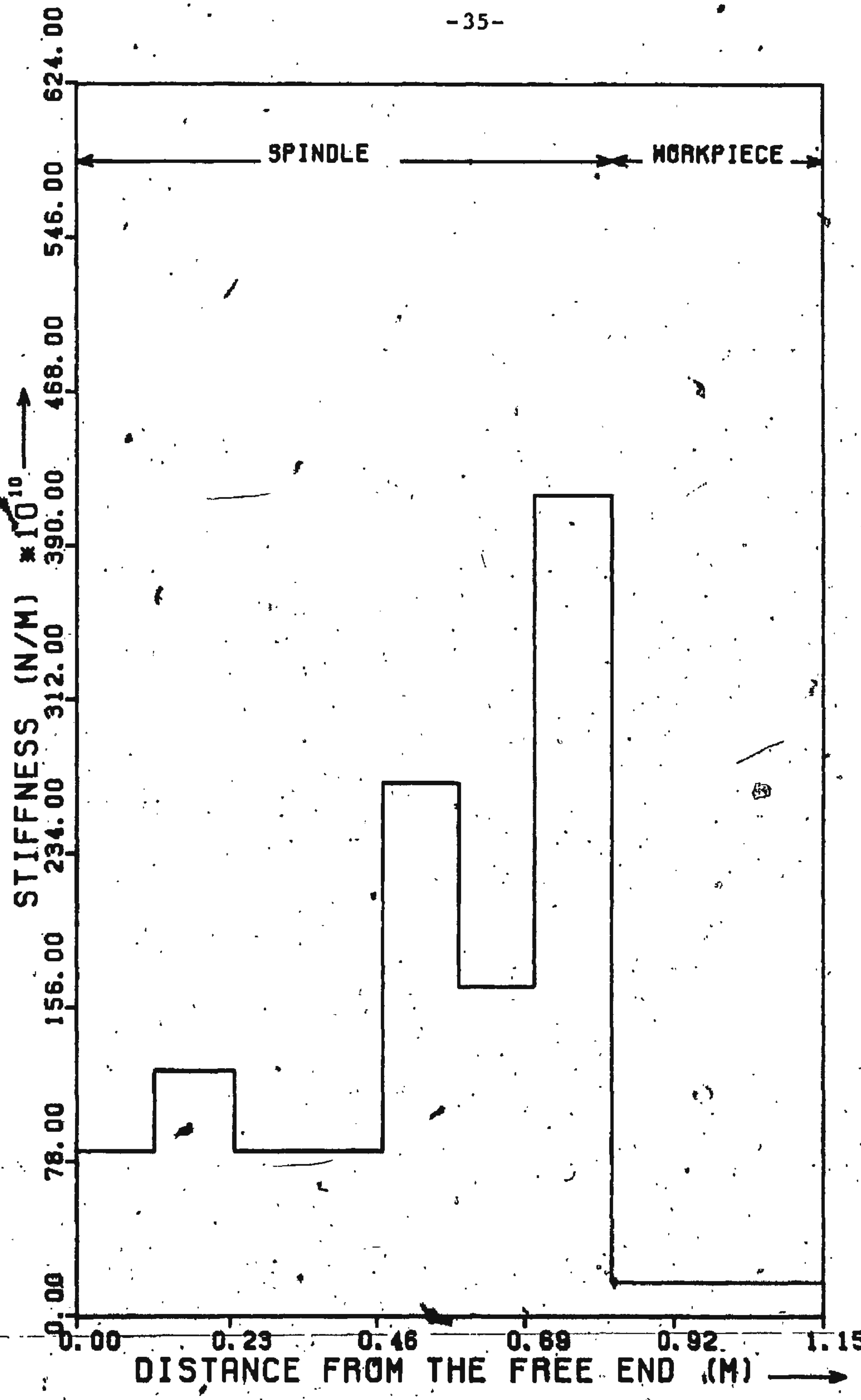


Fig. 3.2 The Variation of Stiffness Along the System

Table 3.1

Parameter Values of the Lathe Spindle-Workpiece System [26]

Parameters	Values of the Parameters
The Modulus of Elasticity	$206.456 \times 10^{9} \frac{N}{m}$
The Mass of the Chuck	34.012 Kg.
The Diameter of the Chuck	0.254 m
The Stiffness of the Front Bearing (K_F)	$2.2703939 \times 10^{9} \frac{N}{m}$
The Stiffness of the Rear Bearing (K_R)	$7.1172232 \times 10^{8} \frac{N}{m}$
The Damping at the Front Bearing Location (C_F)	$28.632 \times 10^{3} \frac{N-sec}{m}$
The Damping at the Rear Bearing Location (C_R)	$22.328 \times 10^{3} \frac{N-sec}{m}$
The Length of the Workpiece	0.3302 m
The Length of the Spindle	0.822325 m
The Diameter of Elements 1 - 5	0.051 m
The Diameter of Element 6	0.254 m
The Diameter of Element 7	0.200 m
The Diameter of Element 8	0.228 m
The Diameter of Elements 9,10 and 12	0.170 m
The Diameter of Element 11	0.188 m

Table 3.2

The Damped Natural Frequencies of the Condensed System

The Number of Master Degrees of Freedom	The First Natural Frequency (Hertz)		The Second Natural Frequency (Hertz)		The Third Natural Frequency (Hertz)		The Fourth Natural Frequency (Hertz)		The Fifth Natural Frequency (Hertz)	
	Frequency	Deviation	Frequency	Deviation	Frequency	Deviation	Frequency	Deviation	Frequency	Deviation
5	585.1821	68.4933	796.302	121.2565	1610.706	413.444	5384.271	3832.3230	12931.960	9214.427
6	520.6648	3.9760	780.2669	105.2214	1227.347	29.785	1697.180	145.2320	5416.960	1699.427
7	517.7612	1.0724	776.4613	101.4158	1218.882	21.320	1696.710	145.2336	529[illegible].472	1576.939
8	517.6447	0.9559	772.0693	97.4158	1218.360	20.798	1692.293	140.3450	4685.229	967.696
9	517.6258	0.9370	724.8812	49.8357	1214.135	16.573	1658.304	106.3560	3971.084	253.551
10	517.2473	0.5585	689.8925	14.8470	1210.175	12.613	1592.012	40.064	3959.630	242.097
11	516.7045	0.01570	675.8264	0.7809	1199.261	1.699	1555.252	3.304	3780.806	63.273
12	516.6294	−0.0594	674.7415	−0.3040	1197.647	−0.085	1552.000	0.052	3722.216	4.683
13	516.6220	−0.0668	674.7330	−0.3125	1197.644	−0.085	1551.999	0.151	3722.183	4.650
14	516.6306	−0.0582	674.7502	−0.2953	1197.644	−0.085	1551.988	0.040	3722.180	4.647
15	516.6199	−0.0689	674.7430	−0.3025	1197.648	−0.086	1551.992	0.044	3722.156	4.623
25	516.6888	0.0000	675.0455	0.0000	1197.562	0.0000	1551.948	0.000	3717.533	0.000

number of masters were varied between five to fifteen. It also shows the natural frequencies corresponding to twenty five degrees of freedom i.e., without any reduction. It is evident from this table that lesser are the number of masters, greater is the deviation of a particular natural frequency of the condensed system. As the number of master degrees of freedom are increased the natural frequencies of the condensed system approach the natural frequencies of the uncondensed system. An important feature of this table is that the deviations approach from the positive side, for all the natural frequencies. However, the deviations tend to be slightly negative as the damped natural frequencies of the condensed system approach the corresponding values of the uncondensed system, due to errors in the computations of the eigenvalues. Secondly, higher the natural frequency greater are the master degrees of freedom required to attain a certain deviation. For example, if there are nine master degrees of freedom, then the deviation for the first natural frequency is 0.937; whereas, for the fifth natural frequency, it is 253.551. Therefore, to include the effects of first five modes accurately, twelve degrees of freedom should be retained in the condensed system. This is because the deviation in the fifth natural frequency is within a reasonable amount.

3.4 The Static Deflection of the Spindle-Workpiece System

The static analysis can be carried out by equating the inertia and damping forces in Eq.(2.27) to zero, and each of the externally applied forces to a constant value, which then reduces to

$$[K_c]\{Y_r\} = \{F_c\}, \tag{3.19}$$

Where $[K_c]$ and $\{F_c\}$ are given by Eqs.(2.27c) and (2.27d). $\{Y_r\}$ is the vector containing the time independent deflections of master degrees of freedom. Eq.(3.19) can also be expressed as

$$\{Y_r\} = [K_e]^{-1}\{F_e\} \tag{3.20}$$

The deflections of master degrees of freedom can also be obtained by solving the original system. A procedure similar to the one used for the condensed system leads to the equation

$$\{Y\} = [K]^{-1}\{F\} \tag{3.21}$$

Where, {Y} is the vector containing the deflections of all the degrees of freedom of the lathe spindle system. In order to establish the dynamic condensation technique to the static case also, the deflections corresponding to the master degrees of freedom are obtained using Eq.(3.20) and compared with those obtained from Eq.(3.21). The results are presented in Table 3.3. As can be seen from this table, the deflections of master degrees are identical to the third decimal place.

3.5 The Decay Rate of the Damped Free Oscillations

The damped free oscillations of a single degree of freedom system is shown in Fig.3.3, where the bounding curve is obtained by passing an exponentially decaying curve through the maxima of each cycle. The equation for such a curve is given by [35]

$$U(t) = V_o\, e^{-rt} \tag{3.22}$$

Higher the value of r, faster is the decay rate. Using Eq.(3.22), the time T^*, required for the deflection U, to reach V^*, a certain percentage of the initial deflection, V_o, can be obtained. Different single-degree-of-freedom systems, subjected to the same initial excitation, will have a different value of r in Eq.(3.22). Therefore, the time interval during which the deflection, U, decays to a value V^*, will be different for each of these systems.

Table 3.3

The static deflection of the workpiece

Stiffness of the bearing = $2.2703939 \times 10^9 \frac{N}{m}$

Damping at the Front Bearing = $28631.831 \frac{N-sec}{m}$

Stiffness at the Rear Bearing = $7.1172232 \times 10^8 \frac{N}{m}$

Damping at the Rear Bearing = $22327.966 \frac{N-sec}{m}$

Diameter of the Workpiece = 0.0508 m

Bearing Spacing = 0.4699 m

Length of the Workpiece = 0.3302 m

$F1 = 0.3937 \times 10^7$

The Degree of freedom	The Static Deflection × F1 (m)	
	Twenty five degrees of freedom system (Eq. 13)	Twelve degrees of freedom system (Eq. 12)
3	8.9698769	8.9698762
5	14.208430	14.208425
7	12.916407	12.016380
9	6.9548609	6.9548063
11	2.8451432	2.8450790

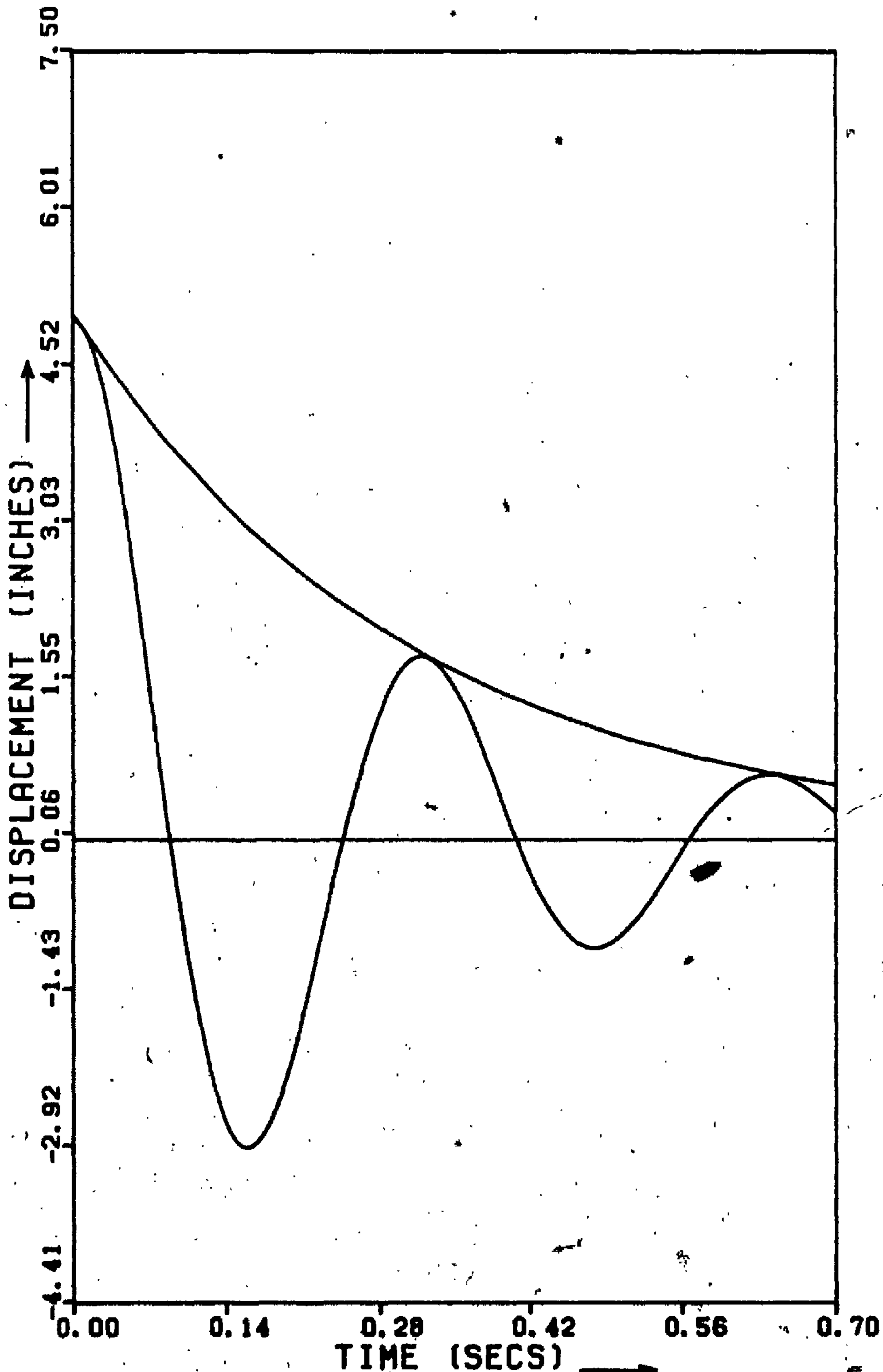


Fig. 3.3 The Damped Free Vibrations of Single-Degree of Freedom System

Thus the best system would be the one for which the value T *, is the minimum.

The multi-degree-of-freedom systems have similar curves but the response is due to the summation of all the modes. The amplitude in this case gets modulated because of the presence of several modes. In this case, the bounding curve can be obtained by fitting an exponentially decaying curve through the maxima of the oscillations by making use of the least square analysis. Once the equation for such a curve is obtained, the time T *, required for the oscillations to reach certain percentage of the initial displacement, can be obtained using an equation similar to Eq.(3.22), which can be written as

$$U_m = V_m e^{-\xi_m [illegible]} \quad (3.23)$$

3.6 The Effect of the System Parameters on the Damped Natural Frequencies

The free Vibration behaviour of the lathe spindle system is due to the interaction between the elastic forces, the inertia forces and the flexural rigidity of the system. The effect of the variation of the bearing spacing (L_b), the diameter of the workpiece (D_w) and the front bearing stiffness (K_f), on the damped natural frequencies is described below.

Fig.3.4 shows the effect of the variation of the bearing spacing on the damped natural frequencies. The bearing spacing is varied by fixing the front bearing and moving the rear bearing. The first natural frequency is most sensitive to changes in bearing spacing whereas the fifth natural frequency shows the least variation.

The effect of the variation of the workpiece diameter (D_w) on the damped natural frequencies is shown in Fig.3.5. Higher the diameter of the workpiece stiffer is the system. Therefore, higher should be the natural frequencies. The effect is very clearly

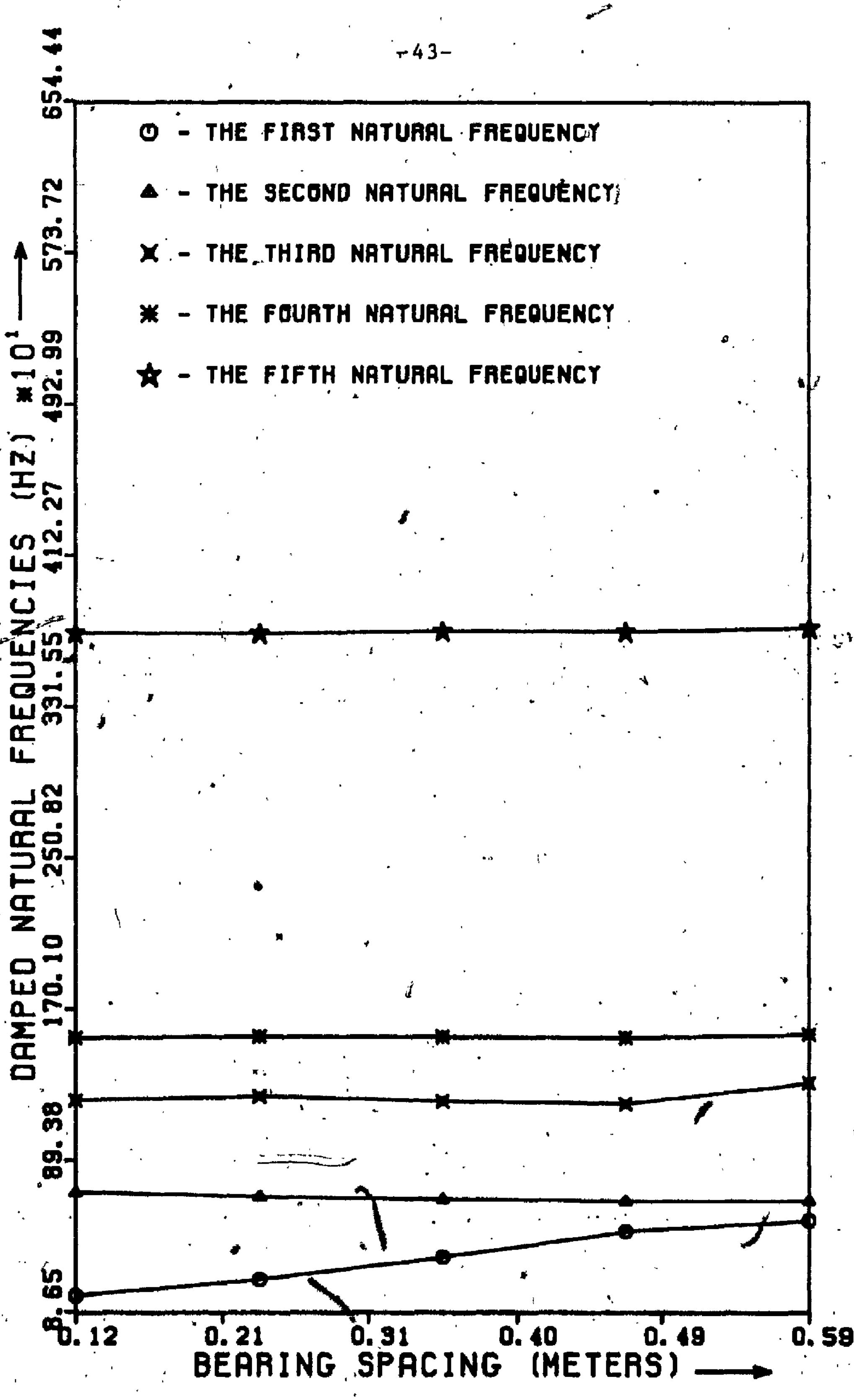


Fig. 3.4 Effect of the Bearing Spacing(L_b) on the Damped Natural Frequencies

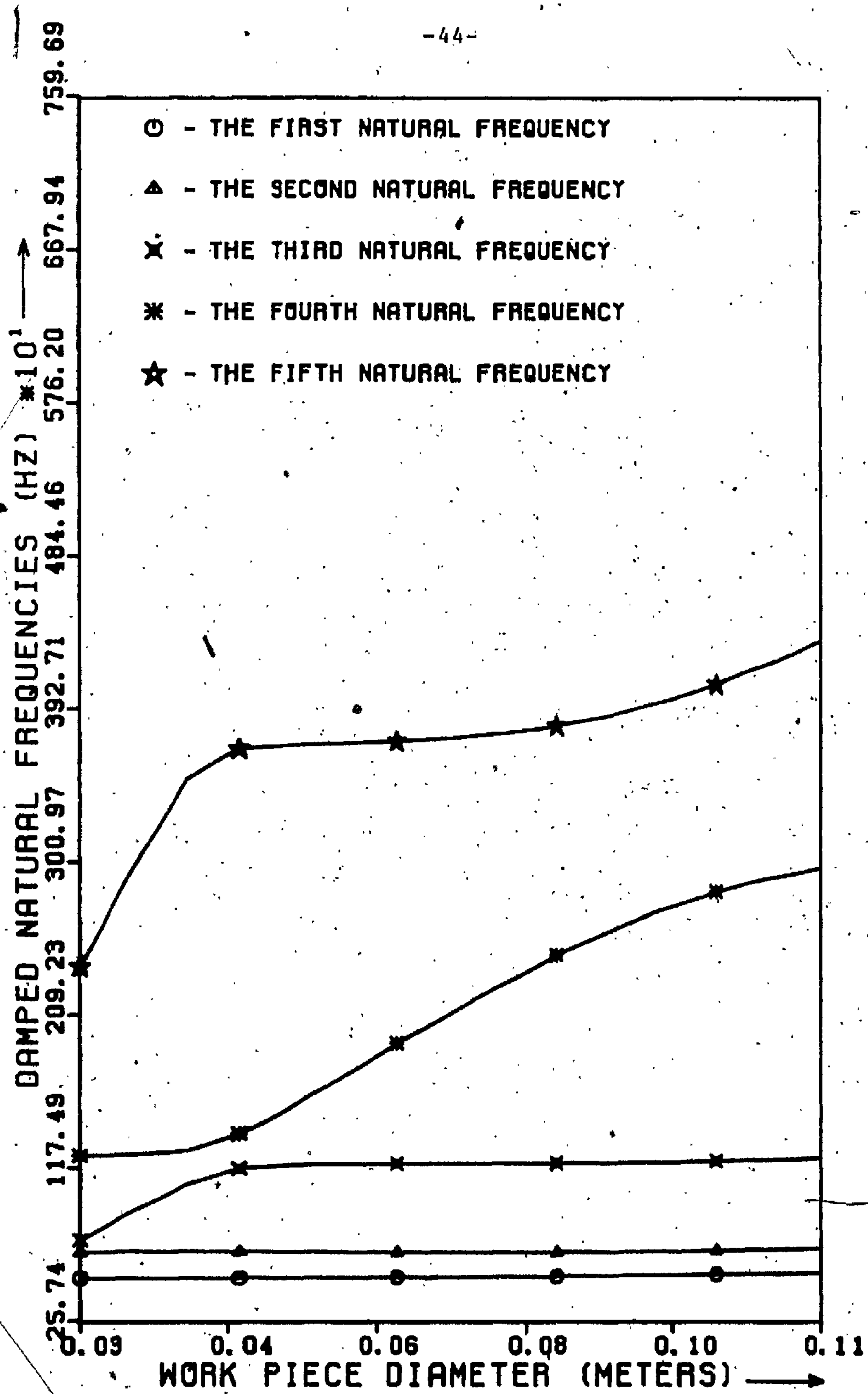


Fig. 3.5 Effect of the Workpiece Diameter(D_W) on the Damped Natural Frequencies

shown in this figure. This figure also shows that the larger diameter workpiece tends to affect the higher modes more than the lower modes.

The effect of the variation of the front bearing stiffness (K_f) is shown in Fig.3.6. The stiffness of the rear bearing is kept constant and the front bearing stiffness is varied from K_r to $5K_r$. The second natural frequency increases with the front bearing stiffness. The rest of the natural frequencies are not sensitive to changes in the values of K_f.

3.7 The Effect of the System Parameters on the Static Deflection of the System

The static deflection characteristics of the spindle-workpiece system were computed using Eq.(3.20). In this analysis, the minimum of the maximum deflection along the spindle-workpiece was chosen as the desired characteristic of the system. Several parameters such as the bearing spacing, the workpiece diameter and the front bearing stiffness were varied and their effect on the static deflection of the system was computed.

The effect of the bearing spacing, L_b, on the static deflection is shown in Fig.3.7. In this figure, the deflection at a given joint is maximum when the rear bearing is located at the middle of the element number eight (refer to Fig.2.1), which means that the bearing spacing is minimum. The minimum joint deflection is obtained if the rear bearing is located at the element number twelve. The effect of the diameter of the workpiece on the static deflection is shown in Fig.3.8. In this case, the deflections at any joint decrease as the diameter of the workpiece increases. This can be explained by the fact that the system stiffness increases as the diameter increases resulting in decreased deflections. Fig.3.9 shows the effect of the front bearing stiffness (K_f), on the system displacement. The maximum deflection at any given K_f, takes place at the middle of the workpiece. As K_f is increased the deflections decrease; but the rate of decrease in the deflection becomes less as K_f reaches a very high value.

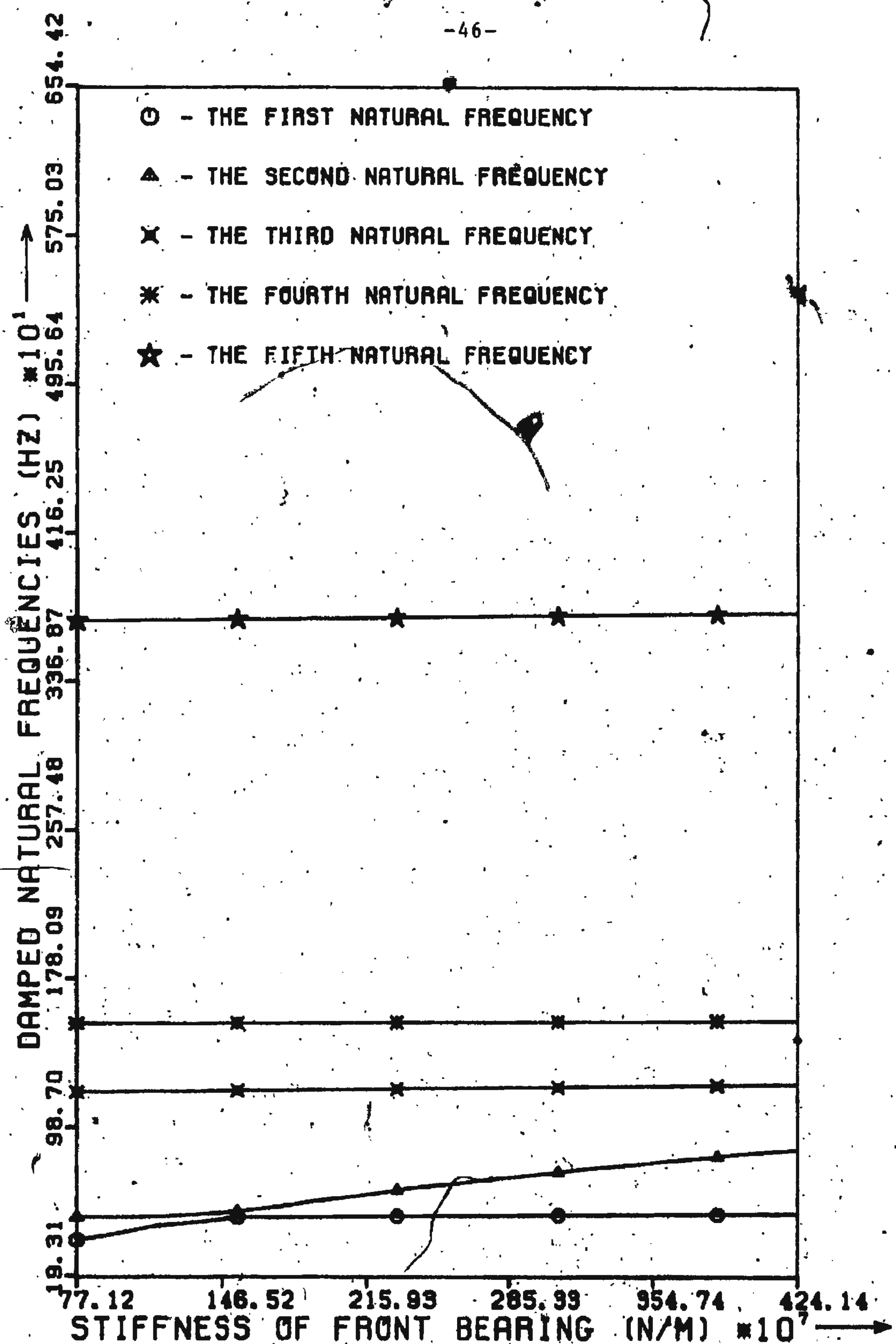


Fig. 3.6 Effect of the Front Bearing Stiffness(K_f) on the Damped Natural Frequencies

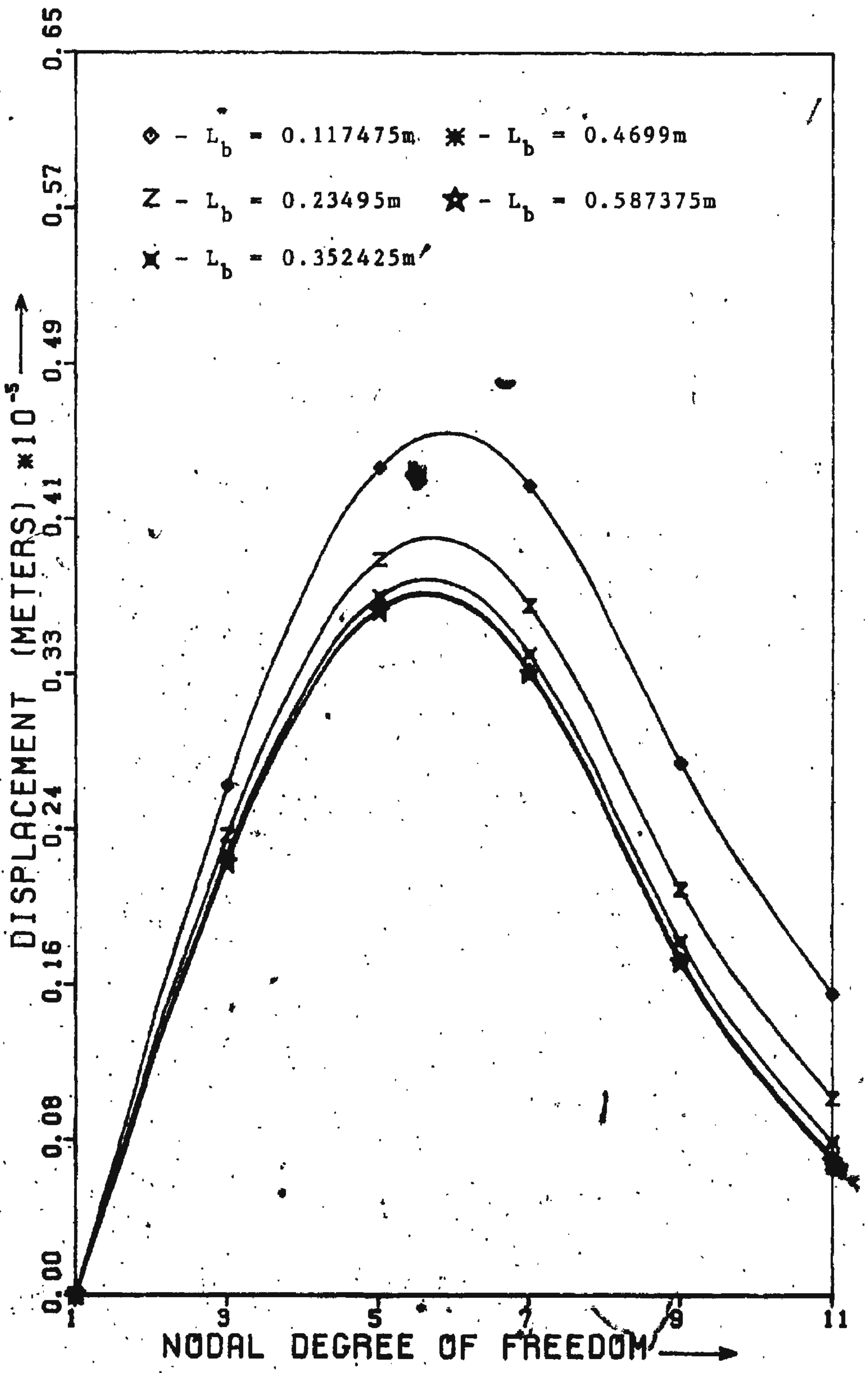


Fig. 3.7 Effect of the Bearing Spacing (L_b) on the System Deflection

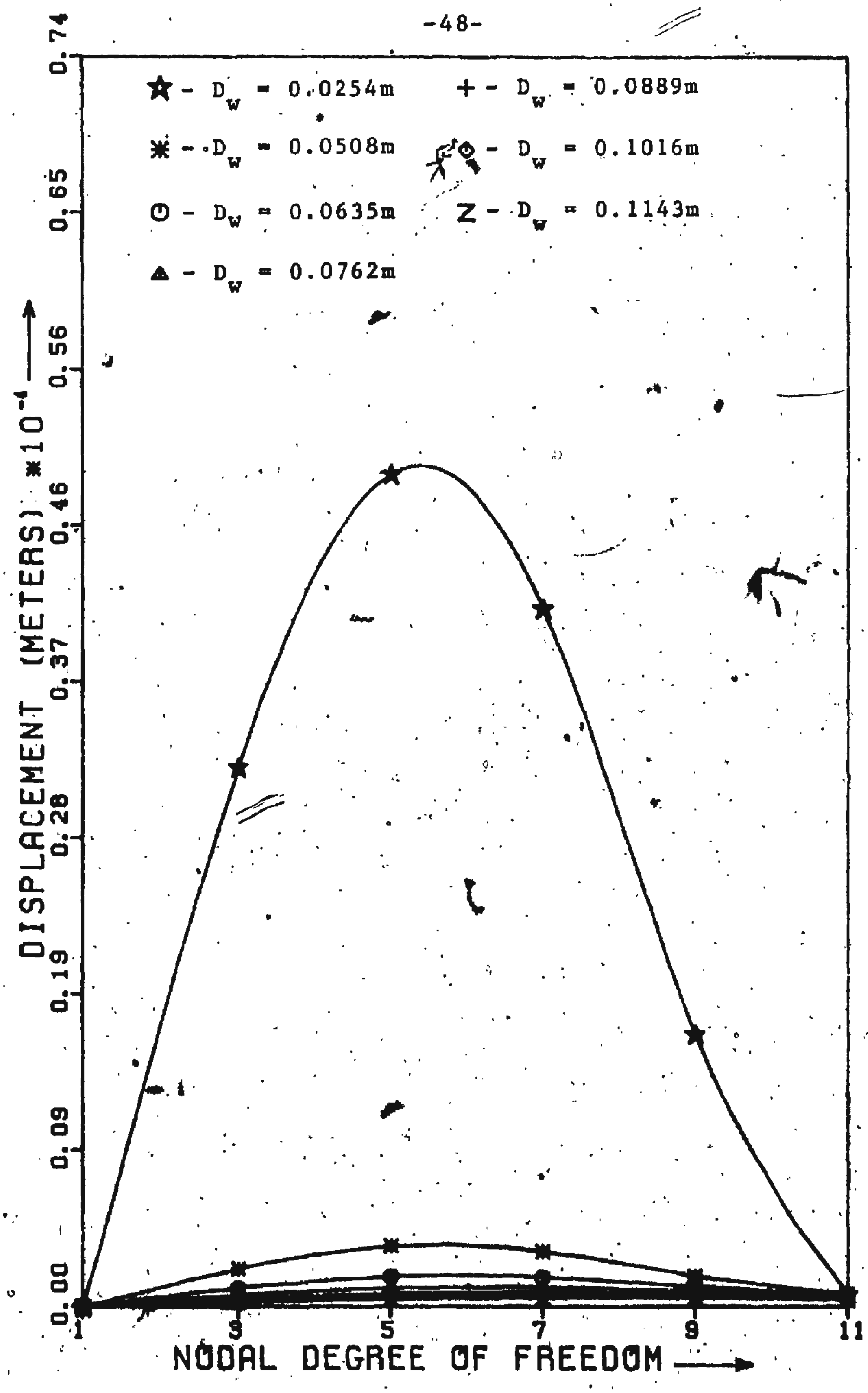


Fig. 3.8 Effect of the Workpiece Diameter(D_w) on the System Deflection

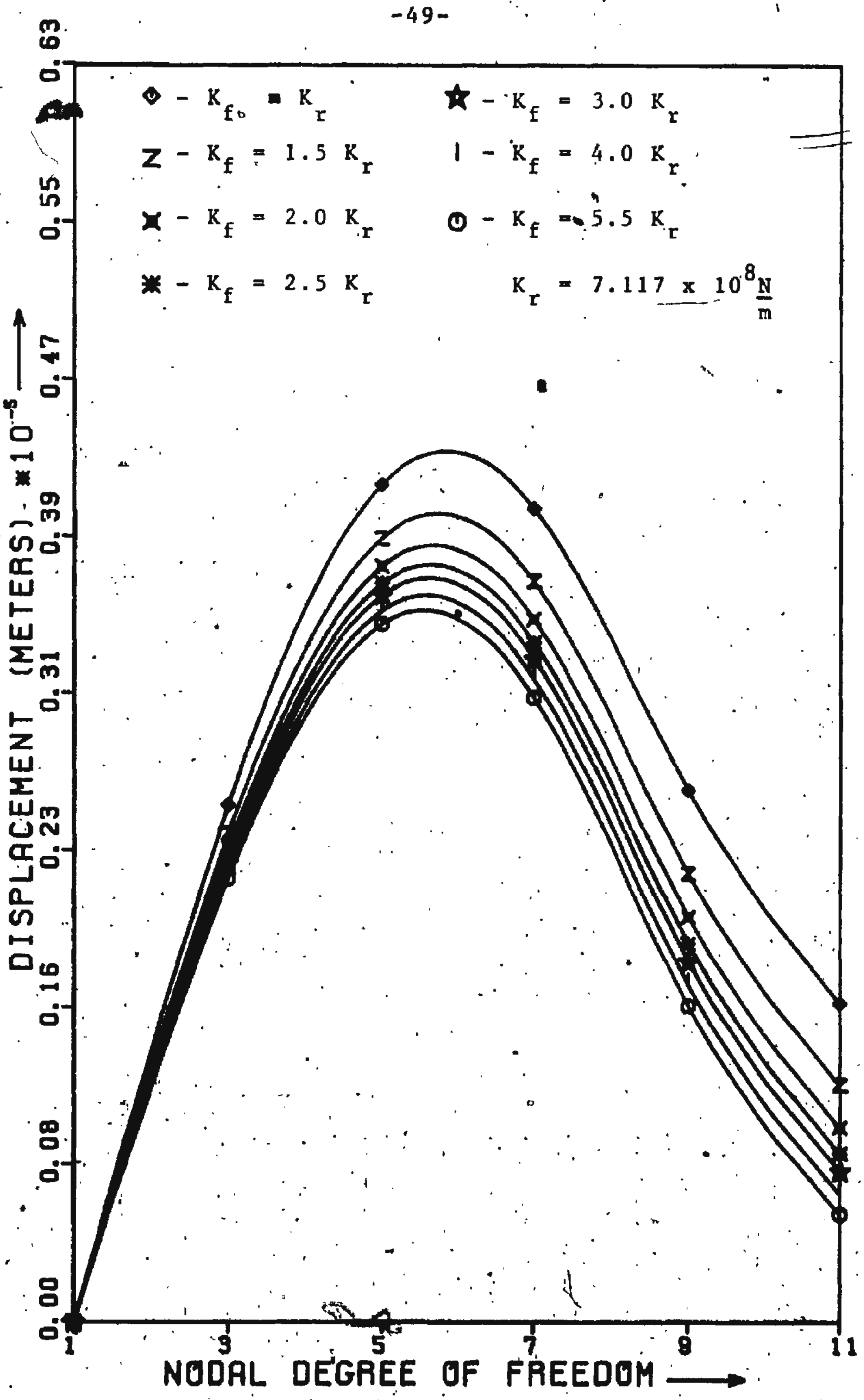


Fig. 3.9 Effect of the Front Bearing Stiffness(K_f) on the System Deflection

3.8 The Effect of System Parameters on the Decay Rate of Damped Free oscillations

The decay rate of the free oscillations were studied by applying a static force at the middle of the workpiece and calculating the deflections at the joints using the stiffness matrix only. The static force was then withdrawn and the time history of the free oscillations were computed using Eq.(3.20). The CPU time taken for obtaining the response using Eq.(3.20) was about one-third the time taken for obtaining the response using Eq.(3.21). A typical displacement time plot of a joint on the workpiece is shown in Fig.3.10. It shows the displacement history at the middle of the workpiece. As can be seen in this figure, the displacement peaks decay continuously as time increases. An exponentially decaying curve based on least square analysis, was fitted through the maxima of the amplitude modulated curve. This decay curve defines the upper bound of the free oscillations and can be very useful in the design of the lathe spindle-workpiece system.

Tables 3.4, 3.5, 3.6 and 3.7 show the variation of T^* which is the time corresponding to 66.66% of the initial displacement, of the exponentially decaying curve of the multi-degree-of-freedom system. Table 3.4 shows the variation of the T^* as a function of the rear bearing location. The bearing in the existing design, is located at the middle of the eleventh element. As this bearing is moved towards the front bearing, T changes quite significantly. It is minimum, if the rear bearing is located at the twelfth element; whereas, it is maximum, if it is located at the ninth element. Thus the desirable location for the rear bearing is at the twelfth element.

The effect of the variation of the workpiece diameter on T^* is shown Table 3.5. It is evident from this table that different diameter workpieces would have varying decay rates. There are several minima and maxima. The global maxima corresponds to the

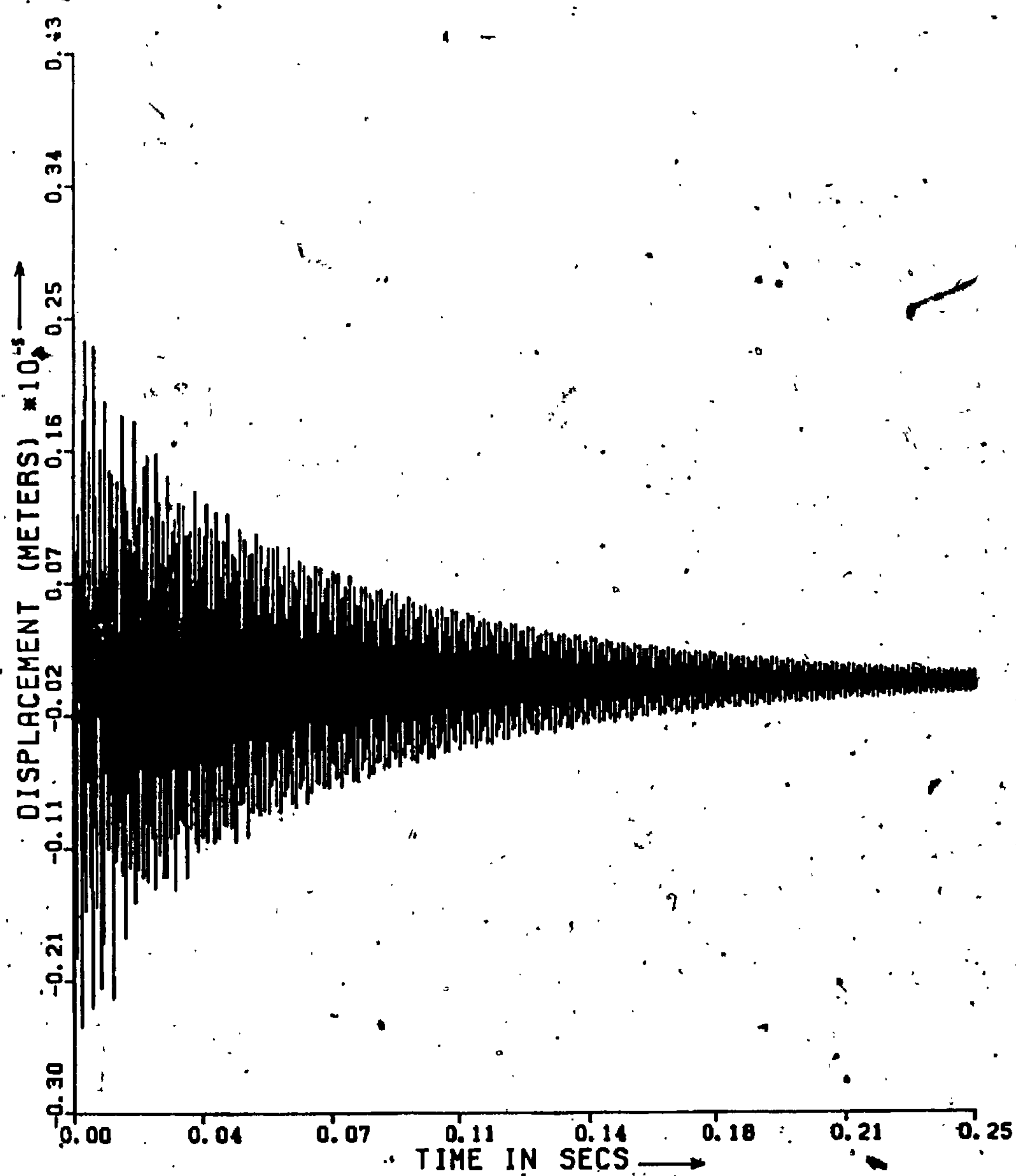


Fig. 3.10 Displacement-Time History at the Middle of the Workpiece

Table 3.4

The Effect of the Rear Bearing Location on T *

$K_f = 2.2703939 \times 10^9 \frac{N}{m}$ $C_f = 28.632 \times 10^3 \frac{N\text{-}sec}{m}$

$C_f = 22.328 \times 10^3 \frac{N\text{-}sec}{m}$ $\frac{K_f}{K_r} = 3.19$

Rear bearing location at joint number	T * × 10³ (secs)
12	1.9871
11	[illegible]80
10	5.5317
9	12.3929
8	10.6488

Table 3.5

The Effect of the Workpiece Diameter on T *

$K_f = 2.2703939 \times 10^{6} \frac{N}{m}$ $\qquad$ $C_f = 28.632 \times 10^{3} \frac{N\text{-}sec}{m}$

$C_f = 22.328 \times 10^{3} \frac{N\text{-}sec}{m}$ $\qquad$ $\frac{K_f}{K_r} = 3.19$

$F2 = 0.3937008 \times 10^{2}$

Workpiece Diameter × F2 (m)	T * × 10³ (secs)
1.00	9.4723
1.25	26.4974
1.50	13.7604
1.75	7.9632
2.00	6.9580
2.25	8.3750
2.50	4.2651
2.75	5.4031
3.00	3.3332
3.25	2.5969
3.50	2.5487
3.75	0.7676
4.00	1.2578
4.25	3.7970
4.50	3.3497

Table 3.6

The Effect of the Front Bearing Stiffness on T*

$K_f = 2.2703939 \times 10^9 \, \frac{N}{m}$ $\quad C_f = 28.632 \times 10^3 \, \frac{N\text{-}sec}{m}$

$C_f = 22.328 \times 10^3 \, \frac{N\text{-}sec}{m}$ $\quad \frac{K_f}{K_r} = 3.19$

Front Bearing Stiffness $\times K_r(\frac{N}{m})$	$T^* \times 10^3$ (secs)
1.0	6.55350
1.5	8.85780
2.0	8.09590
2.5	10.2533
3.0	8.23770
3.5	10.5750
4.0	6.71120
4.5	7.08360
5.0	7.01540
5.5	10.1874

Table 3.7

The Effect of the Location of the External Damper on T*

$K_f = 2.2703939 \times 10^9 \frac{N}{m}$ $\quad C_j = 28.632 \times 10^3 \frac{N\text{-}sec}{m}$

$C_f = 22.328 \times 10^3 \frac{N\text{-}sec}{m}$ $\quad \frac{K_f}{K_r} = 3.19$

External Damper Location at Element Number	$T^* \times 10^3$ (secs)
12	1.8772477
11	4.3256156
10	3.0192322
9	2.2037794
8	3.1337801
7	3.5782992
6	2.0722842

workpiece diameter of 0.032m.

The effect of the variation of the front bearing stiffness on the decay rate is shown in Table 3.6. There are several maxima and minima in this case also. The global maxima corresponds to a front bearing stiffness equal to 3.5 times the rear bearing stiffness. The global minima occurs for a front bearing stiffness equal to the rear bearing stiffness i.e., it requires a very soft front bearing. The front bearing stiffness is usually much more than the rear bearing stiffness.

External dampers can be used to increase the decay rates of the free oscillations. A given amount of damping would have different effect on the decay rate depending on the location of the damper. Table 3.7 shows the variation of the T^* corresponding to different damper locations. This table shows that the best result can be obtained by locating the damper at the twelfth element. If the damper is located at the sixth element, which is the location of the chuck, the decay rate would be quite fast.

3.9 Conclusions

In this chapter a method for the static and the dynamic analyses of lathe spindle-workpiece system is presented. The finite element technique is used to formulate the mathematical model of the lathe spindle system. The resulting matrices are then condensed, and the computations are carried out using the condensed matrices. The analyses leading to the design of the lathe spindle is based on the variations of parameters such as bearing spacing, diameter of the workpiece, bearing stiffness and damping. The first objective is to minimize the static deflection at the maximum deflection location. The second objective is to minimize T^*, the time taken by the damped free oscillations to reach 66.66% of the initial displacement. Based on this investigation the following conclusions can be drawn.

1) The damped natural frequencies increase with the increase in the bearing spacing and the diameter of the workpiece.

2) T * is minimum for a maximum bearing spacing.

3) T * is different for different diameters of the workpiece. There is no specific trend in this case.

4) T * is very low if the front bearing stiffness is low.

5) The free end of the spindle is the best location for applying external damper for the minimization of the time, T *.

6) The static deflection along the workpiece is minimum for a maximum bearing spacing.

7) The static deflection at any joint decreases as the diameter of the workpiece is increased.

8) The maximum static deflection, for any given value of K_f, takes place at the middle of the workpiece. As K_f is increased, the deflections decrease.

9) The dynamic condensation can also be applied in static analysis, without any loss of accuracy.

CHAPTER 4

THE TRANSIENT ANALYSIS OF THE LATHE SPINDLE-WORKPIECE SYSTEM

4.1 Introduction

In actual machining operations a lathe spindle-workpiece system is subjected to both the transient as well as the steady state excitations [40]. Therefore a lathe should be designed in such a way that it performs satisfactorily under both these excitations. Several researchers [30-33] have investigated the dynamic behaviour of machine tools and suggested that the dynamic stiffness coefficient, K_d, which is inversely related to the dynamic displacement response be considered as the most important parameter, as a measure of the performance of such systems. The mathematical model used in [32,33] consisted of either a single degree or two degrees of freedom nonlinear system and the cutting process was assumed to be stationary with a Gaussian distribution. The lathe is a complex structure, which requires sufficiently large degrees of freedom to analyze its behaviour. These researchers modeled the machine tool as a single or two degree of freedom system. In case of a single degree of freedom sytem an exact and simple relationship between K_d and the system parameters can be written [33]. However for two or multi degree of freedom systems [34], these expressions get involved. In such situations the dynamic performance of the machine tool can be assesed by defining K_d at a given degree of freedom as

$$(K_d)_i = \frac{1}{W_i},$$

Where, W_i represents the dynamic displacement response at i^{th} degree of freedom.

In this chapter a method comprising of the finite element method and the modal analysis is used for studying the system behaviour under an impulse and exponentially decaying pulse excitations. The effect of the variations of system design variables on the maximum dynamic displacement response, which is inversely related to K_d, has been studied. The maximum dynamic displacement response or the maximum response is defined as the global maximum of the response values along the system under a particular transient conditions. The computations are done on reduced matrices to economize on the computer memory storage and CPU time.

4.2 The Transient Response of the Lathe Spindle-Workpiece System

The differential equation of motion of a system subjected to externally applied forces is given by Eq.(2.27). This can be expressed as

$$[M]\{\ddot{Y}(t)\} + [C]\{\dot{Y}(t)\} + [K]\{Y(t)\} = \{F(t)\} \tag{2.27}$$

Introducing a 2nx1 state vector, $\{W(t)\}$, as defined by Eq.(3.2), the above equation can be rewritten as

$$[A]\{\dot{W}(t)\} + [B]\{W(t)\} = \{E(t)\} \tag{4.1}$$

Where the matrices [A], [B] are given by Eq.(3.4) and

$$\{E(t)\} = \begin{Bmatrix} 0 \\ F(t) \end{Bmatrix} \tag{4.2}$$

The modal matrix $[\Psi]$ is obtained by solving the homogeneous part of Eq.(4.2) as described in chapter 3. The set of equations given by Eq.(4.1) can be uncoupled by means of the modal matrix $[\Psi]$. Introducing a new state vector, $\{Z(t)\}$, as defined by Eq.(3.10) into Eq.(4.1) and premultiplying Eq.(4.1) by $[\Psi]^T$, one obtains

$$[A^*]\{\dot{Z}(t)\} + [B^*]\{Z(t)\} = \{N(t)\} \tag{4.3}$$

Where $[A^*]$, $[B^*]$ are diagonal matrices and can be obtained by Eq.(3.12). The vector $\{N(t)\}$ can be obtained by using the following relationship:

$$\{N(t)\} = [\Psi]^T \{E(t)\}. \tag{4.4}$$

Eq.(4.3) represents a set of 2n first order differential equations and can be expressed as

$$a_{ii}\, \dot{Z}_i(t) + b_{ii}\, Z_i(t) = N_i(t) \qquad i = 1, \ldots\ldots, 2n. \tag{4.5}$$

The particular solution, with zero initial conditions of the above equation, can be obtained by the convolution integral

$$Z_i(t) = \int_0^t h_i(t-\tau)\, N_i(\tau)\, d\tau \tag{4.6}$$

Where $h_i(t-\tau)$ is the impulse response function.

4.3 The Impulse Response

The system response due to a unit impulse input with zero initial conditions is called its impulse response. A unit impulse $\delta(t)$, is mathematically defined by the following relations

$$\delta(t) = 0 \qquad \text{for } t \neq 0$$

and

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \tag{4.7}$$

The impulse occurs at $t=0$. Any function having the properties defined as above is called the Dirac delta function. In pulse testing of real systems, an excitation can be considered as an impulse if its duration is very short compared with the natural period of the system. The equation of motion along the i^{th} coordinate, with an excitation $N_i(t) = \delta(t)$ can be expressed as

$$a_{ii}\,\dot{Z}_i(t) + b_{ii}\,Z_i(t) = \delta(t) \tag{4.8}$$

This can be also expressed as

$$\dot{Z}_i(t) - \gamma_i\,Z_i(t) = \frac{\delta(t)}{a_{ii}} \tag{4.9}$$

Where $\gamma_i = -\frac{b_{ii}}{a_{ii}}$. Assuming that the system is at rest before the unit impulse $\delta(t)$ is applied; this can be expressed as

$$Z_i(0^-) = 0 \tag{4.10}$$

Since $\delta(t)$ is applied at $t=0$, it is overwith at $t \geq 0^+$. Thus, (1) the system becomes unforced for $t \geq 0^+$, and (2) the energy input due to $\delta(t)$ becomes the initial conditions at $t = 0^+$. The initial conditions at $t = 0^+$, can be found by integrating Eq.(4.8) once for $0^- \leq t \leq 0^+$. Thus,

$$a_{ii}\int_{0^-}^{0^+}\frac{dZ_i}{dt}\,dt + b_{ii}\int_{0^-}^{0^+} Z_i\,dt = \int_{0^-}^{0^+}\delta(t)\,dt \tag{4.11}$$

From Eq.(4.7), the first integration of $\delta(t)$ gives a unit value. Hence the right side of the equation is equal to unity. Since $Z_i(t)$ does not become infinite, its integration over the infinitesimal interval is also zero. Thus,

$$a_{ii}\,[Z_i(0^+) - Z_i(0^-)] = 1 \tag{4.12}$$

Using Eq.(4.10) this reduces to

$$[Z_i(0^+)] = \frac{1}{a_{ii}} \tag{4.13}$$

Which represents the initial condition at $t = 0^+$, due to a unit impulse. The homogeneous equation equivalent to Eq.(4.8) is

$$a_{ii}\dot{Z}_i(t) + b_{ii}Z_i(t) = 0, \tag{4.14}$$

With the initial condition given by Eq.(4.13). However, the solution of Eq.(4.14) can be obtained as [41]

$$Z_i(t) = Z_i(0^+)\,e^{\eta_i t} = \frac{e^{\eta_i t}}{a_{ii}} \tag{4.15}$$

The impulse response function $h(t-\tau)$, due to a unit impulse applied at $t = \tau$, can be written as

$$h(t-\tau) = \frac{e^{\eta_i(t-\tau)}}{a_{ii}} \quad \text{for } t > \tau$$

$$h(t-\tau) = 0 \quad \text{for} \quad t < \tau \tag{4.16}$$

4.4 The Response of the Lathe Spindle System Due to a Unit Impulse

The dynamic response of the system subjected to a unit impulse applied at various locations can be obtained by writing the excitation vector $\{F(t)\}$ in the form [42]

$$\{F(t)\} = \begin{Bmatrix} 0 \\ \vdots \\ \delta_r(t) \\ \vdots \\ 0 \end{Bmatrix} \tag{4.17}$$

Where the impulse is applied along the r^{th} coordinate. The expression for the generalized force vector using Eq.(4.4), can be written as

$$\{N(t)\} = [\Psi]^T \left\{ \begin{array}{c} 0 \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ \delta_r(t) \\ \vdots \\ 0 \end{array} \right\}$$

$$= [\eta] \left\{ \begin{array}{c} 0 \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ \delta_r(t) \\ \vdots \\ 0 \end{array} \right\} \tag{4.18}$$

The component of the generalized force vector along the i^{th} coordinate, can be written as

$$N_i(t) = \eta_{i,n+r}\ \delta_r(t) \tag{4.19}$$

The dynamic equations of motion for a system subjected to an impulse along r^{th} coordinate, using Eq.(4.15) can be written as

$$a_{ii}\ \dot{Z}_i(t) + b_{ii}\ Z_i(t) = \eta_{i,n+r}\ \delta_r(t) \qquad i=1,\ldots,2n \tag{4.20}$$

This is a first order equation with the forcing term being an impulse equal to $\eta_{i,n+r}\ \delta_r(t)$. Hence, using Eq.(4.15) the response can be obtained as

$$Z_i(t) = \frac{\eta_{i,n+r}\ e^{\lambda_i t}}{a_{ii}} \qquad i=1,\ldots\ldots,2n \tag{4.21}$$

The response $\{W(t)\}$ can be obtained using Eqs.(3.2) and (3.10).

4.4.1 The Response of the Lathe Spindle System Due to an Exponentially Decaying Pulse

If the system under consideration is subjected to an exponentially decaying pulse instead of an impulse, an equation for the response of the system in principal coordinates can be written as

$$a_{ii}\,\dot{Z}_i(t) + b_{ii}\,Z_i(t) = \eta_{i,n+r}\,P_o\,e^{-\beta t} \qquad i = 1,\ldots,2n \tag{4.22}$$

Where,

$$P = P_o\,e^{-\beta t} \tag{4.23}$$

is the equation of the pulse. Assuming zero initial conditions, the solution of Eq.(4.22), using the convolution integral given by Eq.(4.6), can be written as

$$Z_i(t) = \int_0^t h_i(t-\tau)\,\eta_{i,n+r}\,P_o\,e^{-\beta\tau}\,d\tau \tag{4.24}$$

Substituting Eq.(4.16) into Eq.(4.24) and after doing the necessary integration one can write

$$Z_i(t) = \frac{\eta_{i,n+r}\,P_o\,e^{\gamma_i t}}{(\beta+\gamma_i)a_{ii}}\left[1 - e^{(\beta+\gamma_i)t}\right] \tag{4.25}$$

Again the dynamic response vector $\{w(t)\}$, can be obtained by making use of Eqs.(3.2) and (3.10).

4.5 The Effect of the Location of the Impulse and the Exponential Pulse on the Workpiece Response

The system matrices were reduced and the dynamic response of the entire system was obtained as described earlier, for various locations of unit impulse and exponentially decaying pulse. The maximum dynamic displacement response, W_m, at various joints for different locations of the pulse, is shown in Fig.4.1 and 4.2. It should be noted that the maximum displacements at the joints did not take place at the same instant of time due to the damping in the system. However, the maximum displacements at the joints were calculated to identify the sections where the coefficient of dynamic stiffness, K_d, would be the least. The maximum displacements at joints give a picture of the distribution of K_d along the spindle-workpiece system. As shown in Figs.4.1 and 4.2, the maximum displacement has the the highest value at the middle of the workpiece, for all the locations of the force. In addition to this, the maximum displacement at this location has a global maximum value, if the transient forces are also located at this position. In other words, if the transient forces are located at the joint number 2, the maximum dynamic response at this location is the highest of all the values obtained for transient forces located at other joints. It implies that the dynamic stiffness coefficient as defined in [32,33] at this location, is minimum. Therefore, in the design of the spindle, the parameters should be selected in such a way that the maximum displacement response (W_m), at the joint number 2, due to the force at this joint, is minimum.

4.5.1 The Effect of the Bearing Spacing on W_m

The effect of the variation of the bearing spacing on W_m, is shown in Figs.4.3 and 4.4. In this study, the front bearing was fixed and the location of the rear bearing was varied. As can be seen in Fig.4.3 the values of W_m go through a minima and maxima as the bearing spacing is changed. It is obvious from this figure that the bearing spacing corresponding to the point A (refer to Fig.4.3), would yield the best result. In Fig.4.4, the value of W_m decreases rapidly with the increase in spacing at first and then it has a constant value more or less. One can infer from both of these figures that the bearing spacing corresponding to the point A, shown in Fig.4.3 would be the optimum spacing.

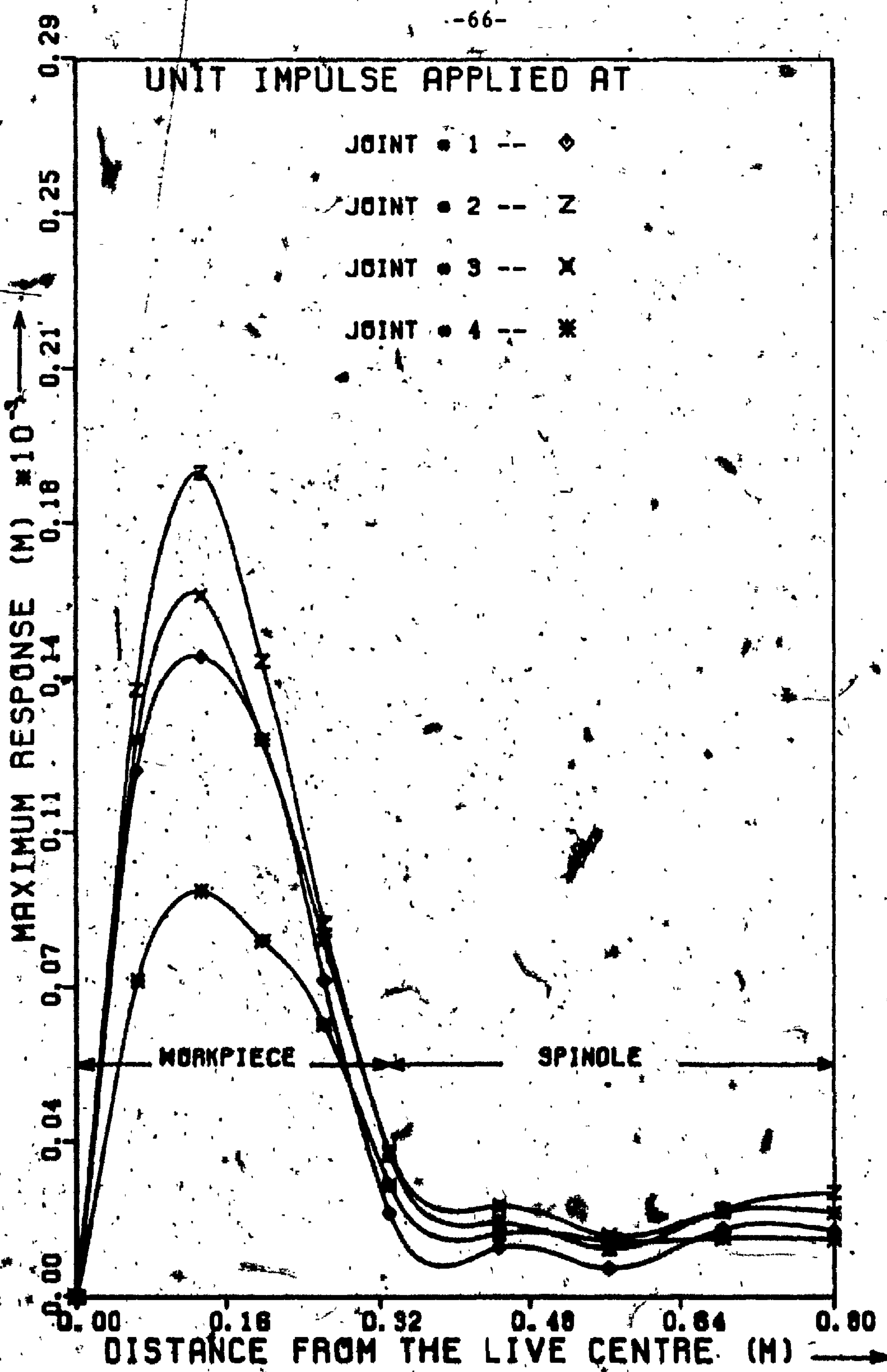


Fig. 4.1 Effect of the Location of the Unit Impulse on Maximum Displacement W_m

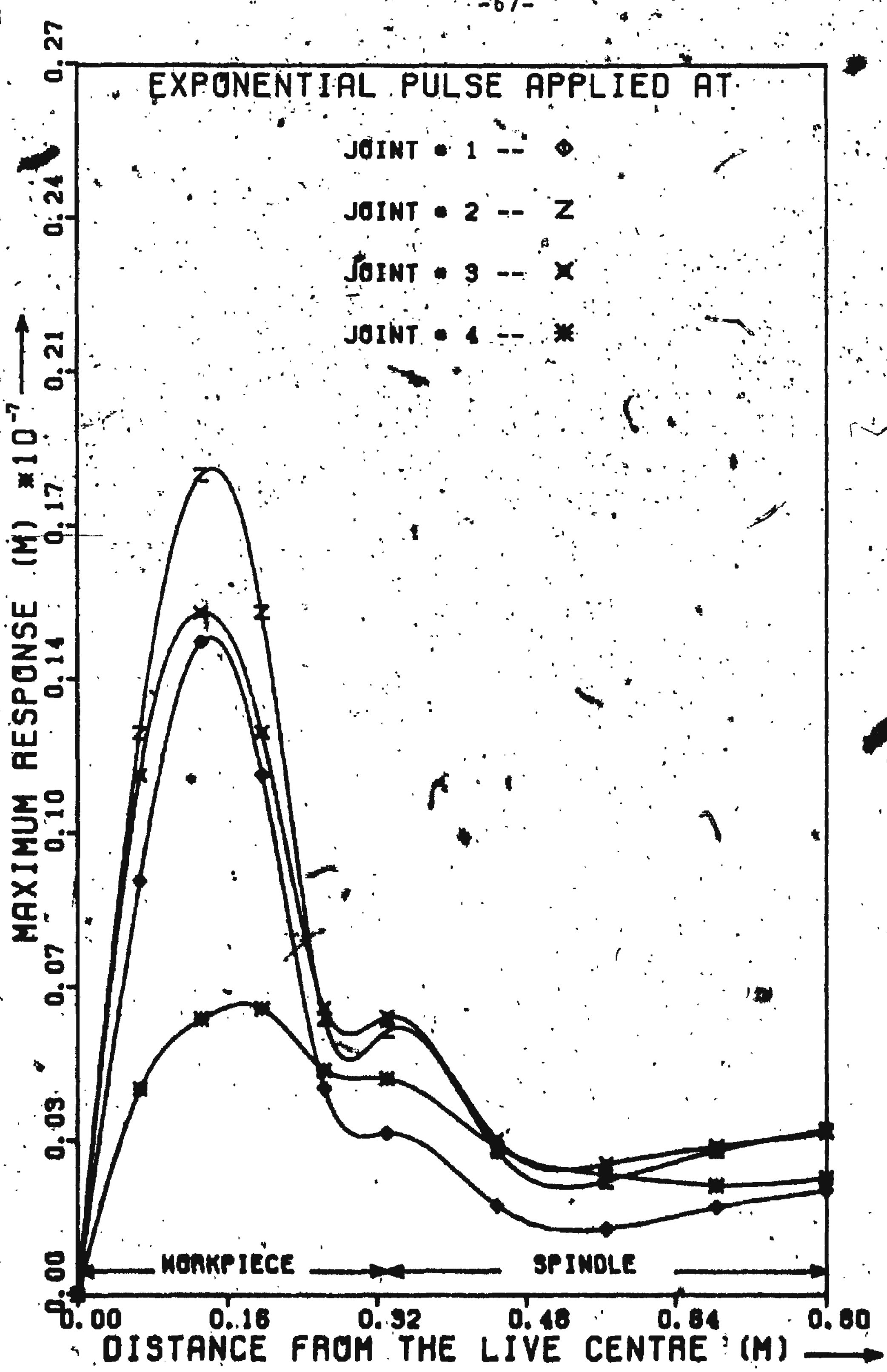


Fig. 4.2 Effect of the Location of the Exponential Pulse on Maximum Displacement W_m

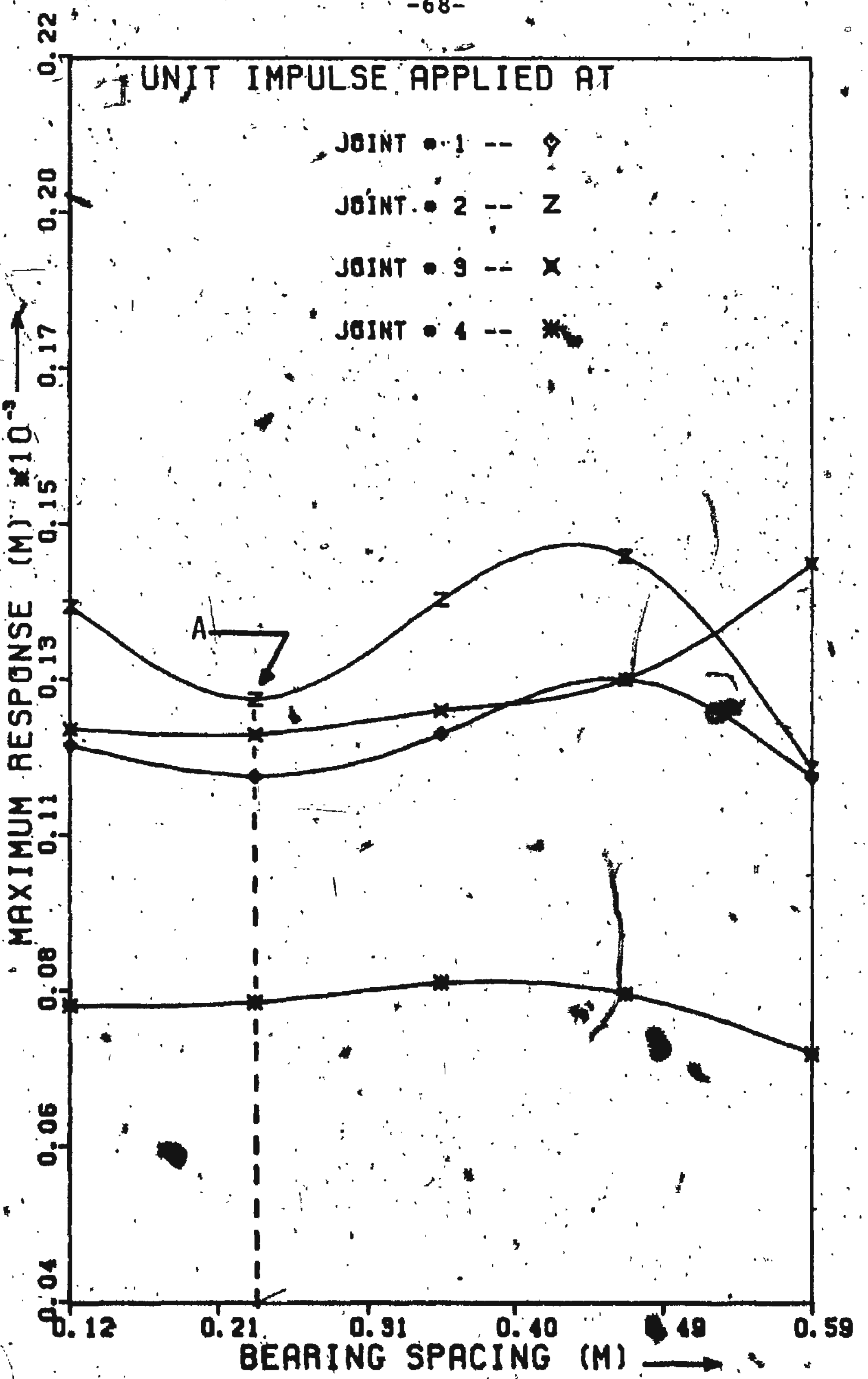


Fig. 4.3 Effect of the Bearing Spacing on Maximum Displacement W_m Due to a Unit Impulse

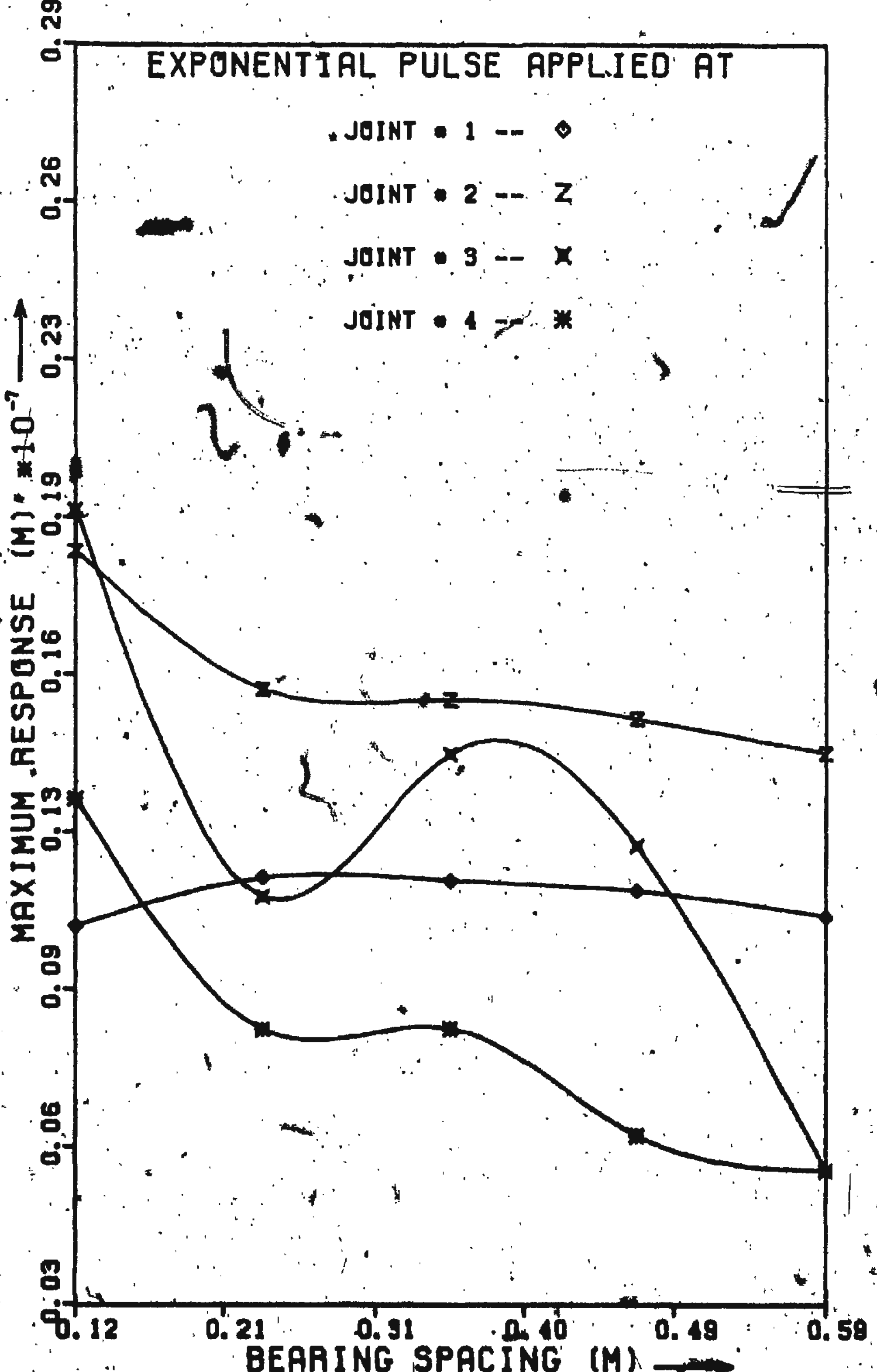


Fig. 4.4 Effect of the Bearing Spacing on Maximum Displacement W_m Due to an Exponential Pulse

4.5.2 The Effect of the Variation of the Front Bearing Stiffness on W_m

The change in W_m with the variation in the front bearing stiffness due to a unit impulse and the exponentially decaying pulse are shown in Figs. 4.5 and 4.6 respectively. In these figures, the value of W_m goes through several maxima and minima due to the stiffness variations. As can be seen in Fig. 4.5, the span of the stiffness variation has been divided into three regions. This subdivision of the span is based on the maximum value of W_m for a given location of the excitation. For example, in region I, the design should be based on the location of the excitation at joint number 2. This is because, one would like to minimize the maximum dynamic displacement W_m, for all the possible locations of the impulse. In region II, on the other hand, W_m has the highest value if the impulse is located at the joint number 1. In region III, again W_m has the highest value if the impulse is located at joint 2. The optimum stiffness for this case would be a stiffness value corresponding to the point B. The global optima in Fig.4.6 would be a stiffness value corresponding to the point C.

4.5.3 The Effect of the Location of an External Damper

The external dampers are used to reduce the vibration amplitudes of systems. Before these dampers are applied to the system, their location where maximum benefits could be realized, must be known. The location of the damper was varied along the spindle with this point in mind. The results obtained for the impulse and exponential pulse are shown in Figs.4.7 and 4.8 respectively. From these figures, it is clear that W_m undergoes different maxima and minima due to the variation in the location, in the case of impulse excitation; whereas the response does not change very much in the case of exponential pulse excitation. Secondly, the best location for this application would be at the chuck in both cases.

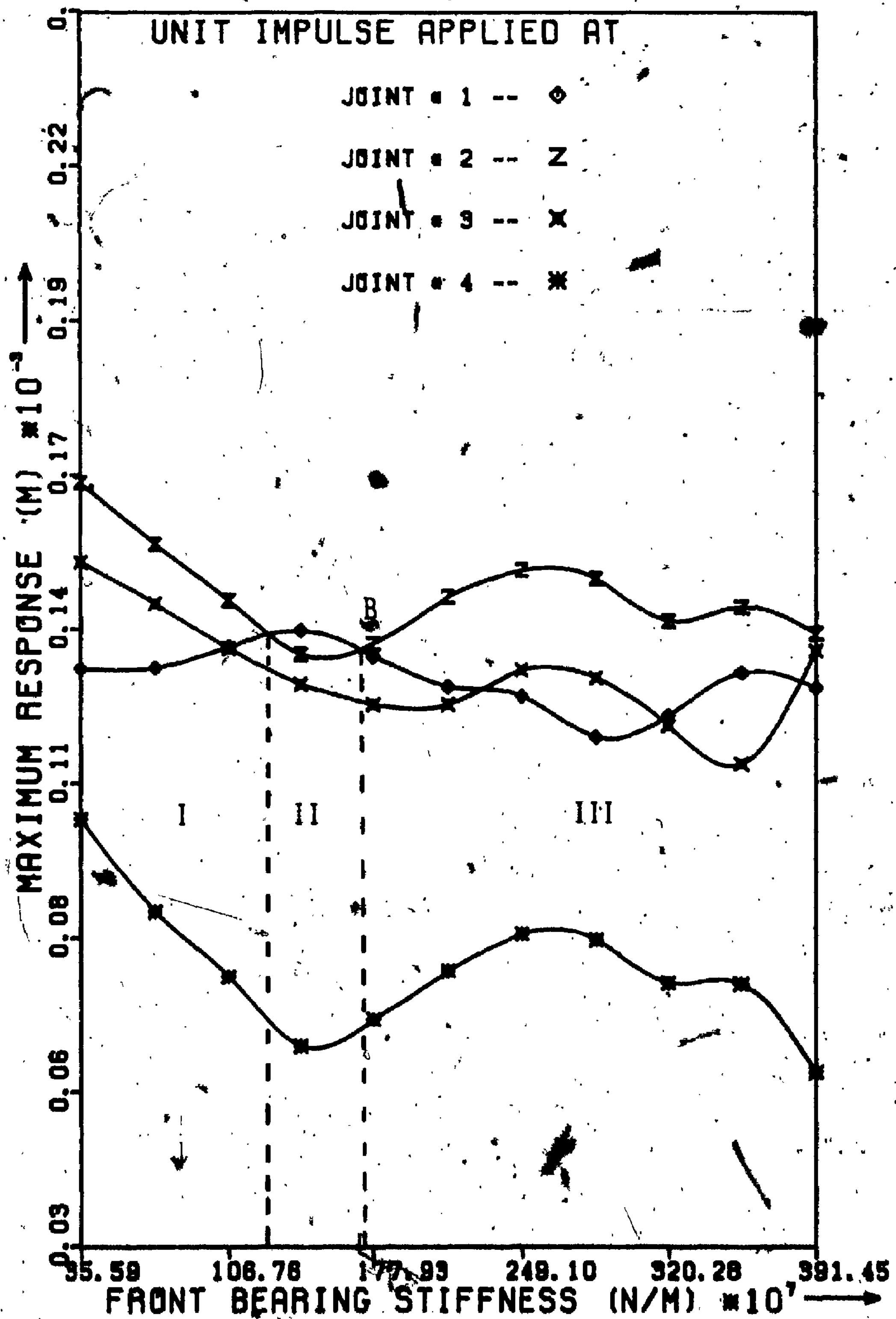


Fig. 4.6 Effect of the Front Bearing Stiffness on Maximum Displacement W_m Due to a Unit Impulse

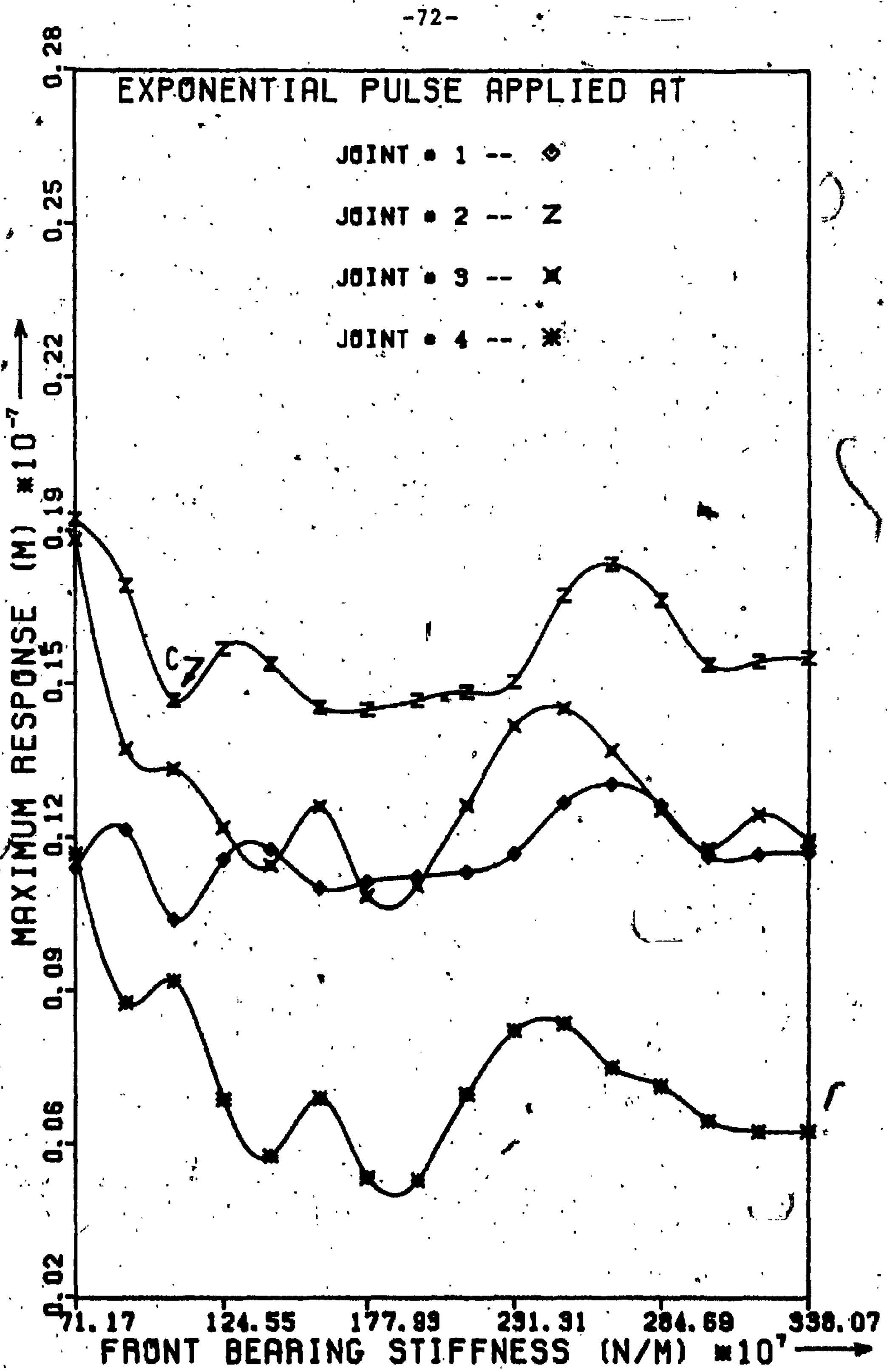


Fig. 4.6 Effect of the Front Bearing Stiffness on Maximum Displacement W_m Due to an Exponential Impulse

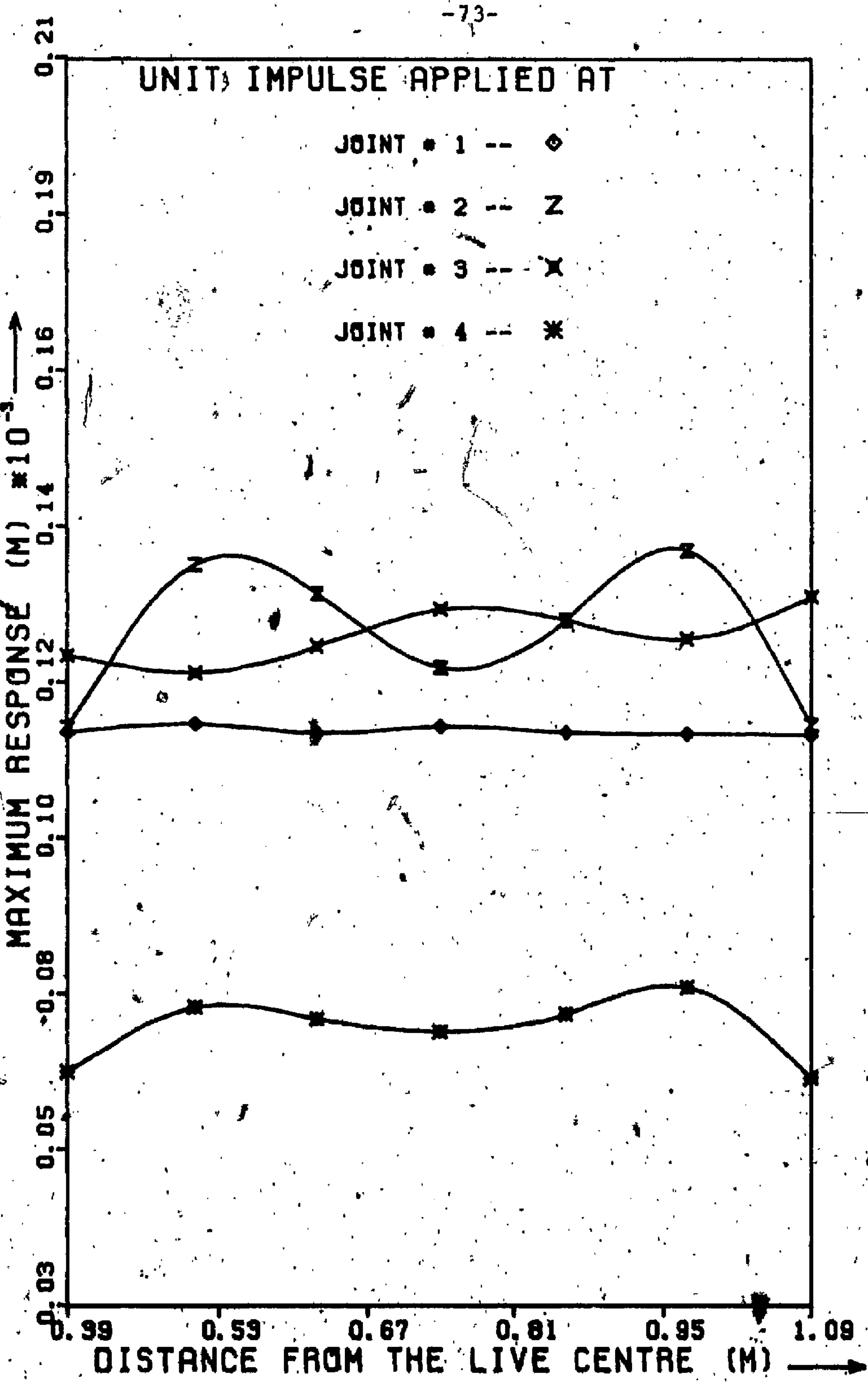


Fig. 4.7 Effect of the Location of an External Damper on Maximum Displacement W_m Due to a Unit Impulse

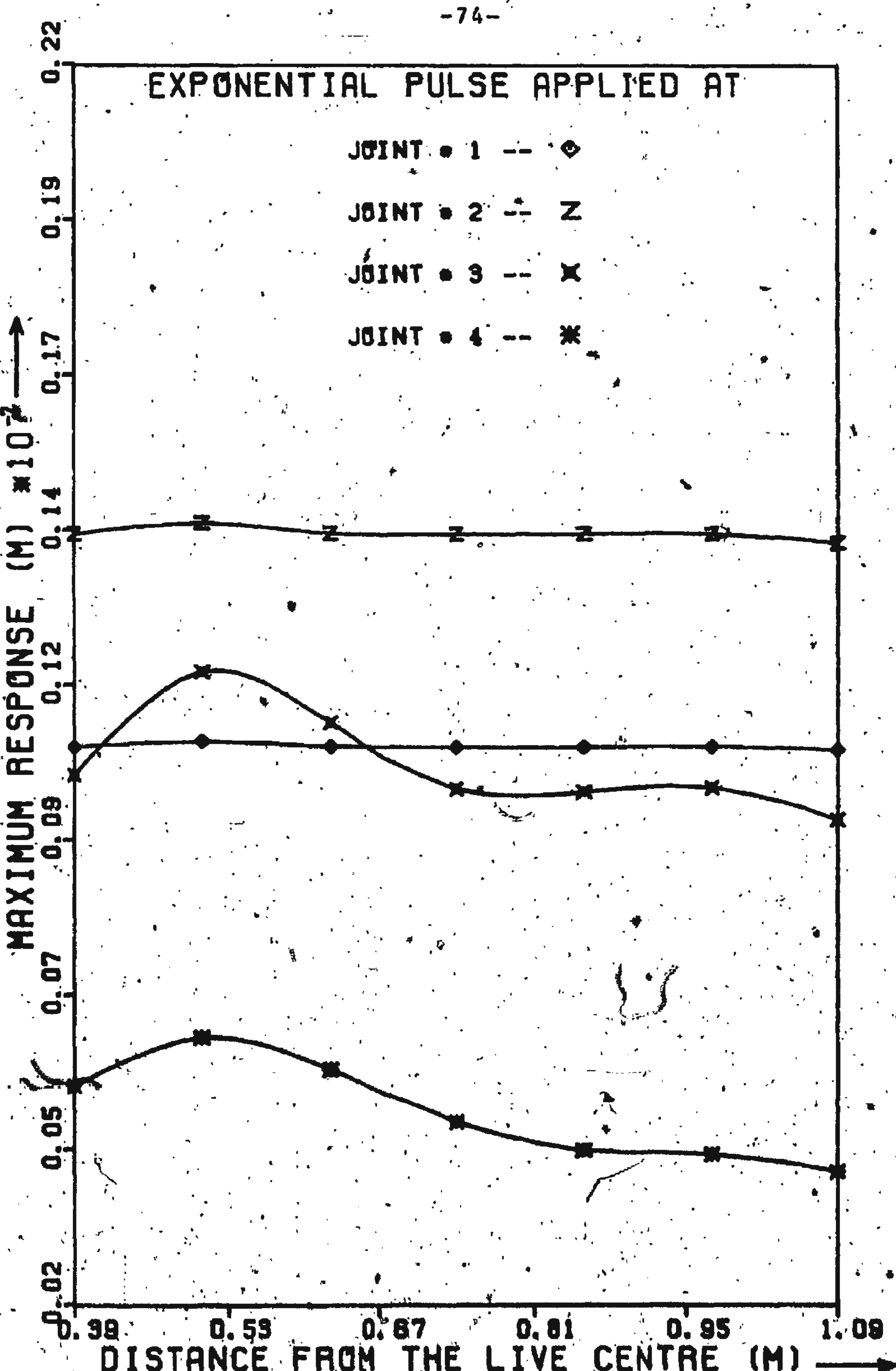


Fig. 4.8 Effect of the Location of an External Damper on Maximum Displacement W_m Due to an Exponential Pulse

4.5.4 The Effect of the Workpiece Diameter on W_m

Different diameters of workpiece are machined on a given lathe. Therefore, any spindle design should involve the study of the dynamic response due to the variations of the workpiece diameter. In the present analysis, the diameter of the workpiece was varied between 0.0254 m to 0.1016 m and its effect on W_m was studied. The results obtained are shown in Tables 4.1 and 4.2. It is evident from these tables, that the maximum value occurs, for all workpiece diameters, at joint 2, irrespective of the location of the force.

4.6 Conclusions

In this chapter the method of obtaining the transient response of the lathe spindle-workpiece system is presented. The method is a combination of the finite element technique, the modal analysis and the impulse response technique. The calculations are done on reduced matrices to save CPU time and to economize on the storage. The design of the system was based on the minimization of the maximum displacement response. Two types of transient excitations are considered. These are: the impulse excitation and the exponentially decaying pulse. The effects of the variations in the bearing spacing, the front bearing stiffness, the location of the external damper and the workpiece diameter, on maximum dynamic displacement response (W_m) were studied. Based on these studies the following conclusions can be drawn:

(1) The maximum dynamic displacement response, W_m, is maximum for the tool location at joint number 2.

(2) There exists an optimum bearing spacing for which W_m is minimum.

(3) There exists an optimum front bearing stiffness for which W_m is minimum.

(4) The maximum dynamic displacement response, W_m, decreases as the workpiece diameter increases.

Table 4.1

The Effect of the Workpiece Diameter on Maximum Displacement W_m Due to a Unit Impulse

$F3 = 39.370079 \times 10^4$

Diameter of the Workpiece, m	Maximum Displacement W_m Due to the Unit Impulse at Joint Number ×F3, m			
	1	2	3	4
0.0254	425.08848000	565.0735300	557.9979000	230.2254000
0.0381	107.37678300	155.0178900	125.5708500	64.7248560
0.0508	50.67078400	57.7865870	50.6352070	32.3718950
0.0635	29.23339400	32.35951500	30.9422660	19.8822490
0.0762	14.85818400	17.9692460	16.0917590	9.6898648
0.0889	11.39999400	11.3650470	11.3960000	7.1388460
0.1016	4.49369430	8.6506149	8.3978334	7.0762169

Table 4.2

The Effect of the Workpiece Diameter on Maximum Displacement W_m Due to an Exponential Pulse

$F4 = 39.370079 \times 10^4$

Diameter of the Workpiece m	Maximum Displacement w_m due to the Exponential pulse at Joint Number ×F4, m			
	1	2	3	4
0.0254	773.2139200	1011.198800	771.323630	287.719040
0.0381	126.8184600	188.5928200	176.133300	76.675201
0.0508	45.5005390	59.7388580	49.220603	25.093601
0.0635	24.6717690	36.2385750	31.764978	21.815707
0.0762	9.3592504	14.2240980	12.861356	10.337247
0.0889	7.6022069	13.5667820	13.216300	10.720977
0.1016	4.2751306	7.3224811	7.214740	7.0192300

CHAPTER 5

THE OPTIMAL DESIGN OF THE LATHE SPINDLE-WORKPIECE SYSTEM UNDER TRANSIENT CONDITIONS

5.1 Introduction

In chapter 4, the effects of the tool location, the bearing spacing, the front bearing stiffness, the diameter of the workpiece and the location of an external damper on the maximum dynamic displacement response, W_m, were presented. It was concluded that, for the tool location at joint number 2, the values of W_m attained a global maximum. The values of W_m, due to transient force modeled as an impulse, are higher than for a similar force modeled as an exponentially decaying pulse. Based on this analysis it was concluded that there exist optimum values of the bearing spacing, front bearing stiffness and location of the external damper for a minimum value of the maximum dynamic displacement response. In this chapter a method for finding the optimal parameter values is described. Actual optimization of the lathe spindle-workpiece system is carried out by making use of a nonlinear programming technique known as the ' complex method '.

5.2 Formulation of the Problem

The first task in optimization of any mechanical system is to select the design variables for the optimization. Based on the analysis presented in Chapter 4, three design parameters namely, the bearing spacing, the front bearing stiffness, and the location of an external damper, are chosen for the optimization. The values of the dynamic displacement response were found to be the highest for a unit impulse located at joint 2. Hence, the objective would be to minimize the maximum dynamic displacement response of the workpiece for the tool located at joint 2. The differential equation of motion of the system should be solved for several sets of design parameters before the optimum solution can be found. This requires large computer time. Therefore the computations should be done on reduced matrices to overcome this difficulty. The dynamic condensation technique described in chapter 2 was used to reduce the system matrices.

5.3 The Statement of the Optimization Problem

The optimization of the lathe spindle-workpiece system can be mathematically stated as follows:

$$\text{minimize } \max\{(W_m)_i\}^{*} \quad i = 1,\ldots,5 \tag{5.1}$$

the maximum dynamic displacement response of the workpiece

Subject to :

$$\begin{aligned} K_f^{(l)} &\leq K_f \leq K_f^{(u)} \\ L_b^{(l)} &\leq L_b \leq L_b^{(u)} \\ L_d^{(l)} &\leq L_d \leq L_d^{(u)} \end{aligned} \tag{5.2}$$

Where, l and u represent the lower and upper bound values of the design variables.

5.4 The Complex Method

The optimization of the present system was carried out using the complex method [42,43], which is normally used to solve constrained minimization problems of the type:

Minimize $f(\mathbf{X})$,

Where, $\mathbf{X}$ is a vector of variables $x_1, x_2, \ldots, x_q$ to be optimized, subject to

$$g_j(\mathbf{X}) \leq 0, j = 1, \ldots, m \tag{5.3}$$

$$x_i^{(l)} \leq x_i \leq x_i^{(u)}, i = 1, \ldots, q \tag{5.4}$$

The basic idea in the complex method is the formation of a sequence of geometric figures each having $p \geq q+1$, vertices in a q - dimensional space. The principle is to compare the values of the objective function at the q+1 vertices of the complex and move this complex generally towards the optimum point during the iterative process. The general direction of a search is taken in a direction away from the worst point. It is chosen so that the movement passes through the center of gravity of the remaining points.

5.4.1 The Iterative Procedure

1) The first step is to find $p \geq q+1$ points, satisfying all the m constraints. Alternatively, one can start with only one feasible point $\mathbf{X}_1$ and the remaining p-1 points can be found by generating random numbers and using the relation

$$x_{i,j} = x_i^{(l)} + r_{i,j}\,(x_i^{(u)} - x_i^{(l)})$$

$$i = 1, 2, \ldots, q \text{ and } j = 2, 3, \ldots, p \tag{5.5}$$

Where x_{ij} is the i^{th} component of the point $\mathbf{X}_j$, and $r_{i,j}$ is a random number lying in the interval (0,1). It is to be noted that the points $\mathbf{X}_2, \mathbf{X}_3, \ldots, \mathbf{X}_p$ are generated according to Eq.(5.5) and satisfy the side constraints, Eq.(5.4), but they may not satisfy the constraints given by Eq.(5.3).

As soon as a new point X_j is generated (j=2,...,p), it is found whether it satisfies all the constraints. If any of the constraints in Eq.(5.3) are violated, the trial point X_j is moved half way towards the centroid of the remaining, already accepted points. The centroid X_0 of already accepted points is given by

$$X_0 = \frac{1}{j-1} \sum_{l=1}^{j-1} X_l \qquad (5.6)$$

If the trial point X_j violates some of the constraints the process of moving halfway in towards the centroid X_0 is continued until a feasible point X_j is found. By proceeding in this fashion the required feasible points $X_2, X_3, \ldots, X_k$ are found.

2) The objective function is evaluated at each of the p points. If the vertex X_h corresponds to the largest function value, the process of reflection is used to find a new point X_r as

$$X_r = (1+\theta) X_0 - \theta X_h \qquad (5.7)$$

Where $\theta \geq 1$ (to start with) and X_0 is the centroid of all the vertices except X_h.

$$X_0 = \frac{1}{p-1} \sum_{l=1, l \neq h}^{p} X_l \qquad (5.8)$$

3) The point X_r is to be tested for feasibility. If the point X_r is feasible, and $f(X_r) < f(X_h)$ the point X_h is replaced by X_k, and the process of reflection is repeated with the new set of trial points by going to step 2. If $f(X_r) \geq f(X_h)$, X_r is found by reducing θ in Eq.(5.7) by a factor of 2 and is tested for the satisfaction of $f(X_r) < f(X_h)$. The procedure for finding a new point X_r with a reduced value of θ is repeated until the value of θ becomes smaller than a prescribed small quantity.

If an improved point X_r, with $f(X_r) < f(X_h)$ cannot be obtained, the point X_r is discarded and the whole procedure of reflection is restarted by using the point X_g instead of X_h. X_g is the point corresponding to the second highest function value.

4) If the reflected point X_r, at any stage violates any of the constraints it is moved half way in towards the centroid until it becomes feasible, i.e.,

$$(X_r)_{new} = \frac{1}{2}(X_o + X_r) \tag{5.9}$$

5) Whenever the complex gets modified by a replacement of X_h by X_r, it is tested for convergence. Convergence is assumed whenever the following condition is satisfied.

$$\left\{ \frac{1}{p} \sum_{j=1}^{p} [f(\underline{X}) - f(X_j)]^2 \right\}^{\frac{1}{2}} \leq \epsilon_2 \tag{5.10}$$

Where $\underline{X}$ is the centroid of all the p vertices of the current complex, and $\epsilon_2 > 0$ is a specified small number.

5.5 Results and Discussion of the Optimization

The constrained optimization results of three parameter optimization scheme are shown in Table 5.1. The results show that the existing values of the bearing stiffness and the bearing spacing are higher than the optimal values. It is to be noted that the values obtained by single parameter variations are very much different from those obtained by multi-parameter optimization.

Table 5.2 shows the improvement attained by minimizing the objective function values by multi-parameter optimization. In each of the single parameter optimizations the optimal objective function values are higher than the optimal values obtained by multi-parameter optimization. In other words, the optimal design using the complex method is an improvement over all the three single parameter optimal designs.

5.6 Conclusions

In this chapter a method for the optimal design of the lathe spindle-workpiece system is presented. The method is a nonlinear programming technique, called the complex method. The lathe spindle system was optimized by making use of this technique. Based on this study the following conclusions are drawn:

(1) The multi-parameter optimization by the constrained nonlinear programming technique leads to a design vector, which is quite different from the one obtained by single parameter optimization.

(2) There could be a significant decrease in maximum dynamic displacement response W_m, if the optimal design vector is used instead of the existing design vector. In other words, a modified design of the lathe spindle-workpiece system instead of the existing design, will improve the dynamic performance characteristics of the lathe.

Table 5.1

Optimal Values of the Three Design Variables

	Parameters		
	K_f $\frac{N}{m} \times 10^5$	L_b m	L_d
Lower Limit	3.55867107	0.00508	0.00508
Upper Limit	320.27496	0.587375	0.6985
Values in the Existing Design	22.703939	0.4699	—
Optimal Values Obtained by Single Parameter Variation	10.2070388	0.219964	0.6461125
Optimal Values by The Complex Method	21.746078	0.4086214	0.3649866

Table 5.2

Comparision of Optimal Values of the Objective Function

Type of Optimization	Design Variable(s) Optimized	Optimal Values of the Objective Function	Percent Deviation from Multi-Parameter Optimization
Single Parameter	Bearing Spacing L_b m	0.00013	6.54
Single Parameter	Front Bearing Stiffness (K_f)	0.00135	10.64
Single Parameter	Location of the External Damper	0.000125	2.44
Multi-Parameter	L_b, K_f & L_d	0.00012208	—

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 A Brief Discussion

The work presented in this thesis is a computer aided analytical investigation and design of lathe spindles. The methodology presented in this work, can be utilized in the design of a lathe spindle-workpiece system in order to achieve the required performance characteristics of the machine under actual cutting conditions. The design of a lathe spindle is influenced by several parameters such as the bearing spacing, the bearing stiffness, the location of an external damper etc. These parameters should be selected in such a way that the maximum dynamic displacement response is a minimum.

The finite element technique has been used to formulate the mathematical model of the lathe spindle-workpiece system. Since the system matrices involved in the model are large, they are condensed by making use of a technique known as computational efficiency.

The static deflection characteristics and the free vibration behaviour of the system subjected to initial displacement were analyzed using modal analysis. The actual computations were done on condensed matrices. The selection of the number of master degrees of freedom for condensation, was determined by an accurate representation of the first five natural frequencies. The effects of various parameters such as the bearing spacing, the bearing stiffness, on the static deflection, and on the free vibration response of the system, were analyzed. The dynamic response of the system was obtained by impulse response technique; and the transient behaviour of the spindle was analyzed by varying the above mentioned system parameters. Finally, the optimal design vector of the lathe spindle-workpiece system was obtained by nonlinear programming technique. The design variables for this purpose were: (a) the bearing spacing, (b) the front bearing stiffness, and (c) the location of the external damper.

6.2 Conclusions

Based on the present investigation the following conclusions can be drawn:

(1) The finite element technique and the modal analysis have been utilized in developing a mathematical model of the lathe spindle-workpiece system.

(2) The dynamic matrix condensation technique can be used to condense the system matrices to achieve computational efficiency, without incurring any significant loss of accuracy.

(3) The damped natural frequencies increase with the increase in the bearing spacing and the diameter of the workpiece.

(4) The time, T^*, is minimum corresponding to the maximum bearing spacing.

(5) Different diameters of the workpiece result in different values of T^*. There is no specific trend in this case.

(6) The front bearing stiffness should be very low for the time, T^*, to be a minimum.

(7) The free end of the spindle is the best location for applying external damper for minimizing T^*.

(8) The maximum dynamic displacement response, W_m, is maximum for the tool location at the joint number 2.

(9) There exists an optimum bearing spacing for which W_m is minimum.

(10) There exists an optimum front bearing stiffness for which W_m is minimum.

(11) The maximum dynamic displacement response, W_m, decreases as the diameter of the workpiece increases.

(12) The multi-parameter optimization by the constrained nonlinear programming technique leads to a design vector, which is quite different from the one obtained from single parameter optimization.

(13) There could be a significant decrease in the maximum dynamic displacement response, W_m, if the optimal design vector is used instead of the existing design vector.

(14) The proposed method can be used in the design process to achieve better performances of lathes under actual machining conditions.

6.3 Limitations of the Investigation

The present investigation is subject to the following limitations:

(1) The effects of rotary inertia and shear deformation have been assumed to be negligible.

(2) The spindle-workpiece system has been assumed to exhibit a linear behaviour.

(3) The bearing stiffness and the damping have been assumed to be independent of fre-

quency.

(4) The bearings are assumed to be exhibiting linear load versus deflection characteristic.

6.4 Recommendations for Future Work

The investigation presented is concerned with the study of the flexural response of the lathe spindle-workpiece system under actual machining conditions. This work can be extended to include the following:

(1) Nonlinear stiffness characteristics of the bearings supporting the spindle system.

(2) Damping of the bearing as a function of the frequency, and the structural damping of each of the elements.

(3) Refinements of the mathematical model to include the effects of shear deformation and rotary inertia. The elemental stiffness and inertia matrices respectively are given in Appendix C.

(4) An approach similar to this thesis can be extended to study the spindle-dynamics of other machine tools such as the hobbing, milling, shaping etc.

(5) The element matrices can be based on the axisymmetric solid element having asymmetric forces.

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APPENDIX A

MASS AND STIFFNESS MATRICES OF AN ELEMENT

The kinetic and potential energies of the beam element can be obtained by substituting Eq.(2.6) into Eqs.(2.7) and (2.7a). The kinetic energy can be written as [36]

$$R(t) = \frac{1}{2}\int_0^l m\left[\frac{\partial y}{\partial t}(x_1,t)\right]^2 dx_1$$

$$= \frac{1}{2}\frac{m\,l}{420}\,[\,156\dot{y}_1^2(t) + 4l^2\dot{y}_2^2(t) + 156\,\dot{y}_3^2(t)$$

$$+ 4\,l^2\dot{y}_4^2(t) + 2\times 22\,l\,\dot{y}_1(t)\dot{y}_2(t) + 2\times 54\,\dot{y}_1(t)\dot{y}_3(t)$$

$$- 2\times 13\,l\,\dot{y}_1(t)\dot{y}_4(t) + 2\times 13\,l\,\dot{y}_2(t)\dot{y}_3(t)$$

$$- 2\times 3\,l^2\dot{y}_2(t)\dot{y}_4(t) - 2\times 22\,l\,\dot{y}_3(t)\dot{y}_4(t)\,] \tag{A.1}$$

Similarly the potential energy can be written as

$$P(t) = \frac{1}{2}\int_0^l EI\left[\frac{\partial^2 y}{\partial x_1^2}(x_1,t)\right]^2 dx_1$$

$$= \frac{1}{2}\frac{EI}{l^3}\,[\,12\,y_1^2(t) + 4\,l^2 y_2^2(t) + 12\,y_3^2(t) + 4\,l^2 y_4^2(t)$$

$$+ 2\times 6l\,y_1(t)\,y_2(t) - 2\times 12\,y_1(t)\,y_3(t) + 2\times 6\,l\,y_1(t)\,y_4(t) - 2\times 6\,y_2(t)\,y_3(t)$$

$$+ 2\times 2l^2 y_2(t) y_4(t) - 2\times 6\, l\, y_3(t) y_4(t)] \qquad (A.2)$$

The virtual work can be written as [36]

$$\overline{\delta W} = \sum_{j=1}^{4} f_j(t)\, \delta u_j(t) \qquad (A.3)$$

Where

$$f_1(t) = \int_0^l f(x_1,t)\left(1 - \frac{3x_1^2}{l} + \frac{2x_1^3}{l^3}\right) dx_1 + f_1^*(t) \qquad (A.3a)$$

$$f_2(t) = \int_0^l f(x_1,t)\left(\frac{x_1}{l} - 2\frac{x_1^2}{l^2} + \frac{x_1^3}{l^3}\right) l\, dx_1 + f_2^*(t) \qquad (A.3b)$$

$$f_3(t) = \int_0^l f(x_1,t)\left(\frac{3x_1^2}{l} - \frac{2x_1^3}{l^3}\right) dx' + f_3^*(t) \qquad (A.3c)$$

$$f_4(t) = -\int_0^l f(x_1,t)\left(\frac{x_1^2}{l^2} - \frac{x_1^3}{l^3}\right) l\, dx + f_4^*(t) \qquad (A.3d)$$

are the joint forces, in which $f(x_1,t)$ is the distributed nonconservative force and $f_j^*(t)$ $(j=1,2,3,4)$ are the forces exerted by the adjacent elements on the element considered.

The Lagrange's equation of motion for the system can be expressed as

$$\frac{d}{dt}\left[\frac{\partial L^*(t)}{\partial \dot{y}_j}\right] - \frac{\partial L^*}{\partial y_j} = Q_j \qquad j=1,2,3,4 \qquad (A.4)$$

Where, $L^*(t) = R(t) - P(t)$.

Combining Eqs. (A.1), (A.2), (A.3) and (A.4) one can write the matrix differential equation of motion for the element as

$$[m]\{\ddot{y}(t)\} + [k]\{y(t)\} = \{f(t)\}$$

Where

$$[\,m\,] = \frac{m\,l}{420}\begin{bmatrix} 156 & 22l & 54 & 13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

and

$$[\,k\,] = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & -12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

APPENDIX B

DESCRIPTION AND LISTING OF THE COMPUTER PROGRAMS

The required computations were done on a Vax 11/780 digital computer, using a package of programs developed in **FORTRAN** language by the author. These programs make use of several subroutines available with the **IMSL** package. Each of the programs are self-sufficient and can be run on any system facilitated with a **FORTRAN** compiler. However, input data files and the **IMSL** subroutines mentioned at the begining of each of the listing, are essential for all these programs.

The first program **FVIBA** computes the static deflections, response at the middle of the workpiece and the damped natural frequencies of the system. Input data consists of values of system parameters and the static force applied. The elemental matrices are computed and global matrices are then obtained by assembling. This program uses the uncondensed matrices.

The second program **FVIBC** is a variable dimensioned program. This program can be run for different number of master degrees of freedom. The system matrices are condensed first, depending upon the number of master degrees of freedom chosen. The set of initial displacements is found out and the system response is computed for different instants of time. By least square analysis, a curve is fitted through the maxima of the response and its equation is found out.

TVIBI and **TVIBE** are the programs used for obtaining the response of the lathe spindle-workpiece system subjected to an impulse and exponentially decaying pulse excitations respectively. These programs are also variable dimensioned and can be run for different choices of master degrees of freedom. The computations are carried out using the condensed matrices.

Program **OPTM** is developed for the optimal design of the lathe spindle-workpiece system. The maximum displacement response of the workpiece subjected to a unit impulse which is the objective function, is computed by the subroutine **UREAL** . In this subroutine, the three design variables as described in chapter 5, are stored in the variable X. The distance of the rear bearing and external damper from the free-end, are stored in X(1) and X(2) respectively. X(3) contains the ratio of the front bearing stiffness to the rear bearing stiffness. Subroutine **CMPLX**, based on the complex method finds the optimal design vector.

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C***********************************
C
C LISTING OF THE PROGRAM ++ FVIBA ++
C FREE VIBRATION RESPONSE OF A MACHINE TOOL SPINDLE SPINDLE
C IMSL SUBROUTINES USED : LEQ1S LEQT1C EIGCC AND LINV2F
C
C***********************************
      DIMENSION SL1(12),SD1(12),Y1(12),GMM(25,25),GKK(25,25)
      DIMENSION GCC(25,25),GM(26,26),GK(26,26),GC(26,26),YOO(25)
      REAL KF,KR,ZO(25)
      COMPLEX W(50),SB(50),Z(50,50),ZI(50,50)
C************************************
      OPEN(UNIT=6,FILE='YZ.DAT',TYPE='OLD')
C************************************
      READ(6,*) NUE,NODES,NR
      READ(6,*) SE,KF,KR
      READ(6,*) D1,D2
      READ(6,*) WL,WD
      READ(6,*) (SD1(L),SL1(L),L=6,NUE)
      READ(6,*) F1,F2
      READ(6,*) D12
      DO 12 I=1,5
      SD1(I)=WD
      SL1(I)=WL
   12 CONTINUE
      CALL EKEM(SD1,SL1,SE,KF,KR,D1,D2,D12,NUE,NODES,NR,GK,GM,GC)
C************************************
      OPEN(UNIT=7,FILE='X30.DAT',TYPE='NEW')
C************************************
      DO 15 I=1,25
      DO 15 J=1,25
   15 GKK(I,J)=GK(I+1,J+1)
      CALL DEFL(GKK,F1,F2,YOO)
C************************************
      OPEN(UNIT=12,FILE='ZX1.DAT',TYPE='NEW')
C************************************
      WRITE(12,110)  WD
  110 FORMAT( 12X,'WORK PIECE DIAMETER IN INCHES ',F10.3,/)
      WRITE(12,901)
  901 FORMAT( 12X,'STIFFNESS VALUES OF FRONT & REAR BEARINGS ',/)
      WRITE(12,*) KF,KR
      WRITE(12,113) RR
  113 FORMAT( 12X,'RATIO OF FRONT BRG STIFF  TO REAR BRG STIFF ',F10.6,/)
      WRITE(12,112)  NR
  112 FORMAT( 12X,'REAR BEARING LOCATION AT ',I3,' ELEMENT',/)
      WRITE(12,120)    F1,F2
  120 FORMAT( 12X,'INITIAL DISPLACEMENTS IN INCHES DUE TO FORCES',
     1 F10.2,2X,F10.2,/)
      WRITE(12,*) (YOO(I),I=1,25)
      CALL XXX(GK,GM,GC,GKK,GMM,GCC,W,Z,ZI)
      WRITE(12,160)
  160 FORMAT( 12X,'DIAGNOL VALUES OF MASS MATRIX',/)
```

```
      WRITE(12,*) (GMM(I,I),I=1,25)
      WRITE(12,140)
 140  FORMAT(12X,'DIAGNOL VALUES OF  STIFFNESS MATRIX',/)
      WRITE(12,*) (GKK(I,I),I=1,25)
      WRITE(12,130)
 130  FORMAT(12X,'DIAGNOL VALUES OF  DAMPING MATRIX ',/)
      WRITE(12,*) (GCC(I,I),I=1,25)
      t2=0.25/1000.0
      DO 20 I=1,1000
      T1=FLOAT(I)*T2
      CALL RES(Z,ZI,YOO,W,T1,ZO,DWORK)
      WRITE(7,*) DWORK,T1
 20   CONTINUE
      WRITE(12,2002)
 2002 FORMAT(12X,'EIGEN VALUES',/)
      DO 2001  I=1,50
      W(I)=AIMAG(W(I))/(44.0/7.0)
      WRITE(12,*) W(I)
 2001 CONTINUE
 1000 STOP
      END
C*************************************
C SUBROUTINE FOR FINDING EIGENVALUES AND MODAL MATRIX
C*************************************
      SUBROUTINE XXX(GK,GM,GC,GKK,GMM,GCC,W,Z,ZI)
      DIMENSION GK(26,26),GM(26,26),GC(26,26),GKK(25,25),GMM(25,25)
      DIMENSION GCC(25,25),GMI(25,25),GMK(25,25),GMC(25,25),UN(25,25)
      INTEGER N,IA,IZ,M,IB,IER,IJOB,IDGT
      REAL WKAREA(700),WK(5100),WA(50)
      COMPLEX W(50),Z(50,50),ZN,DC(50,50),ZT(50,50)
      COMPLEX AC(50,50),BC(50,50),ACZ(50,50),BCZ(50,50),ZTAC(50,50)
      COMPLEX ZTBC(50,50),SB(50),Z1(50,50),ZI(50,50)
      DO 25 I=1,25
      DO 25 J=1,25
      GMM(I,J)=GM(I+1,J+1)
      GKK(I,J)=GK(I+1,J+1)
      GCC(I,J)=GC(I+1,J+1)
 25   CONTINUE
      DO 26 I=1,25
      DO 26 J=1,25
      AC(I,J)=CMPLX(0.0,0.0)
      AC(I,J+25)=CMPLX(GMM(I,J),0.0)
      AC(I+25,J)=CMPLX(GMM(I,J),0.0)
      AC(I+25,J+25)=CMPLX(GCC(I,J),0.0)
 26   CONTINUE
      DO 27 I=1,25
      DO 27 J=1,25
      BC(I,J)=CMPLX(-1.0*GMM(I,J),0.0)
      BC(I,J+25)=CMPLX(0.0,0.0)
      BC(I+25,J)=CMPLX(0.0,0.0)
      BC(I+25,J+25)=CMPLX(GKK(I,J),0.0)
 27   CONTINUE
```

```
      IA=25
      N=25
      IDGT=4
      CALL LINV2F(GMM,N,IA,GMI,IDGT,WKAREA,IER)
      CALL MATMP3(GMI,GKK,GMK,25,25,25)
      CALL MATMP3(GMI,GCC,GMC,25,25,25)
      DO 45 I=1,25
      DO 45 J=1,25
      GMK(I,J)=-1.0*GMK(I,J)
      GMC(I,J)=-1.0*GMC(I,J)
45    CONTINUE
      DO 54 I=1,25
      DO 54 J=1,25
      UN(I,J)=0.0
54    CONTINUE
      DO 55 I=1,25
      UN(I,I)=UN(I,I)+1.0
55    CONTINUE
      DO 60 I=1,25
      DO 60 J=1,25
      DC(I,J)=CMPLX(GMC(I,J),0.0)
      DC(I,J+25)=CMPLX(GMK(I,J),0.0)
      DC(I+25,J)=CMPLX(UN(I,J),0.0)
      DC(I+25,J+25)=CMPLX(0.0,0.0)
60    CONTINUE
      N=50
      IA=50
      IZ=50
      IJOB=2
      CALL EIGCC(DC,N,IA,IJOB,W,Z,IZ,WK,IER)
      DO 62 I=1,50
      DO 62 J=1,50
      ZT(I,J)=Z(J,I)
62    CONTINUE
      CALL MATM1(ZT,AC,ZTAC,50)
      CALL MATM1(ZT,BC,ZTBC,50)
      CALL MATM1(ZTAC,Z,ACZ,50)
      CALL MATM1(ZTBC,Z,BCZ,50)
      DO 64 I=1,50
      SB(I)=-1.0*(BCZ(I,I)/ACZ(I,I))
64    CONTINUE
      DO 80 I=1,50
      DO 80 J=1,50
      Z1(I,J)=Z(I,J)
80    CONTINUE
      DO 85 I=1,50
      DO 85 J=1,50
      ZI(I,J)=CMPLX(0.0,0.0)
85    CONTINUE
      DO 90 J=1,50
      ZI(J,J)=ZI(J,J)+CMPLX(1.0,0.0)
90    CONTINUE
```

```
      IA=50
      IB=50
      N=50
      M=50
      IJOB=O
      CALL LEQT1C(Z1,N,IA,ZI,M,IB,IJOB,WA,IER)
      RETURN
      END
C****************************************
C SUBROUTINE FOR FINDING THE RESPONSE AT THE MIDDLE OF THE WORKPIECE.
C****************************************
      SUBROUTINE RES(Z,ZI,YOO,W,T1,ZO,DWORK)
      COMPLEX Z(50,50),ZI(50,50),W(50),YO(50,1),V1(50,1)
      COMPLEX ZIO(50,1),ZRES(50,1),ZOR(50,1)
      REAL YOO(25),ZO(25)
      DO 105 I=1,50
      YO(I,1)=CMPLX(0.0,0.0)
 105  CONTINUE
      DO 110 I=1,25
      YO(I+25,1)=YO(I+25,1)+CMPLX(YOO(I),0.0)
 110  CONTINUE
      CALL RESP(W,T1,V1)
      CALL MAT(ZI,YO,50,ZIO)
      CALL MAS(ZIO,V1,ZRES,50)
      CALL MAT(Z,ZRES,50,ZOR)
      DO 120 I=1,25
      ZO(I)=REAL(ZOR(I+25,1))
 120  CONTINUE
      U1=ZO(4)
      U2=0.0
      U3=ZO(6)
      U4=0.0
      DWORK=(0.5*U1+(2.6/8.0)*U2+0.5*U3-(2.6/8.0)*U4)
      RETURN
      END
C*********************************
C SUBROUTINE FOR GLOBAL MASS & STIFFNESS MATRICES
C*********************************
      SUBROUTINE  EKEM(SD1,SL1,SE,KF,KR,D1,D2,D12,NUE,NODES,NR,GK,GM,GC)
      DIMENSION GM(26,26),GK(26,26),GC(26,26)
      DIMENSION SD1(12),SL1(12),EK(4,4),EM(4,4),EC(4,4)
      DIMENSION A(4,4),AK(4,4),AM(4,4),AC(4,4)
      DIMENSION SK(26,26),SM(26,26),SC(26,26)
      REAL KF,KR
      DO 20 I=1,26
      DO 20 J=1,26
      GM(I,J)=0.0
      GC(I,J)=0.0
 20   GK(I,J)=0.0
      KM1=1
      KM2=2
      DO 105 K=1,NUE
```

```
      SL=SL1(K)
      SD=SD1(K)
      JU1=KM1
      JU2=KM2
      CALL MATFOR(SD,SL,SE,K,EK,EM,EC)
      IF(K.EQ.7) GO TO 145
      IF(K.EQ.NR) GO TO 146
      IF(K.EQ.12) GO TO 149
      GO TO 148
145   EK(1,1)=EK(1,1)+0.5*KF
      EK(3,3)=EK(3,3)+0.5*KF
      EC(1,1)=EC(1,1)+0.5*D1
      EC(3,3)=EC(3,3)+0.5*D1
      GO TO 148
146   EK(1,1)=EK(1,1)+0.5*KR
      EK(3,3)=EK(3,3)+0.5*KR
      EC(1,1)=EC(1,1)+0.5*D2
      EC(3,3)=EC(3,3)+0.5*D2
      GO TO 148
149   EC(1,1)=EC(1,1)+0.5*D12
      EC(3,3)=EC(3,3)+0.5*D12
148   CALL TRNSF(A)
      CALL TRNSF1(EK,A,AK)
      CALL TRNSF1(EM,A,AM)
      CALL TRNSF1(EC,A,AC)
      DO 14 IP=1,26
      DO 14 KP=1,26
      SK(IP,KP)=0.0
      SH(IP,KP)=0.0
      SC(IP,KP)=0.0
 14   CONTINUE
      CALL ASSEM(AK,JU1,JU2,NODES,SK)
      CALL ASSEM(AM,JU1,JU2,NODES,SH)
      CALL ASSEM(AC,JU1,JU2,NODES,SC)
      DO 150 I1=1,26
      DO 150 J1=1,26
      GK(I1,J1)=GK(I1,J1)+SK(I1,J1)
      GM(I1,J1)=GM(I1,J1)+SH(I1,J1)
      GC(I1,J1)=GC(I1,J1)+SC(I1,J1)
150   CONTINUE
      KM1=KM1+2
      KM2=KM2+2
105   CONTINUE
      RETURN
      END
C*********************************
C SUBROUTINE FOR ELEMENTAL STIFFNESS & MASS MATRIX
C*********************************
      SUBROUTINE MATFOR(SD,SL,SE,K,EK,EM,EC)
      DIMENSION EC(4,4),EK(4,4),EM(4,4)
      RHO=0.2832958/386.0
      DO 4 NI=1,4
```

```
      DO 4 JI=1,4
      EC(NI,JI)=0.0
      EK(NI,JI)=0.0
    4 EM(NI,JI)=0.0
      EM(1,1)=156.0
      EM(1,2)=22.0*SL
      EM(1,3)=54.0
      EM(1,4)=-13.0*SL
      EM(2,1)=22.0*SL
      EM(2,2)=4.0*SL**2
      EM(2,3)=13.0*SL
      EM(2,4)=-3.0*SL**2
      EM(3,1)=54.0
      EM(3,2)=13.*SL
      EM(3,3)=156.0
      EM(3,4)=-22.0*SL
      EM(4,1)=-13.0*SL
      EM(4,2)=-3.0*SL**2
      EM(4,3)=-22.0*SL
      EM(4,4)=4.0*SL**2
      EK(1,1)=12.0
      EK(1,2)=6.0*SL
      EK(1,3)=-12.0
      EK(1,4)=6.0*SL
      EK(2,1)=6.0*SL
      EK(2,2)=4.0*SL**2
      EK(2,3)=-6.0*SL
      EK(2,4)=2.0*SL**2
      EK(3,1)=-12.0
      EK(3,2)=-6.0*SL
      EK(3,3)=12.0
      EK(3,4)=-6.0*SL
      EK(4,1)=6.0*SL
      EK(4,2)=2.0*SL**2
      EK(4,3)=-6.0*SL
      EK(4,4)=4.0*SL**2
      IF(K.LE.5) BD=0.0
      IF(K.GT.5) BD=2.5
      SI=(22.0/(7.0*64.0))*(SD**4-BD**4)
      SM=(22.0/28.0)*(SD**2-BD**2)*RHO
      FM=SM*SL/420.0
      FK=SE*SI/(SL**3)
      DO 5 I=1,4
      DO 5 J=1,4
      EM(I,J)=FM*EM(I,J)
      EK(I,J)=FK*EK(I,J)
    5 CONTINUE
      RETURN
      END
C**************************
C  FINDS THE TRANSPOSE OF A MATRIX
C**************************
```

```
      SUBROUTINE TRANSP(A,MX,MY,AT)
      DIMENSION A(4,4),AT(4,4)
      DO 1 I=1,MX
      DO 1 J=1,MY
    1 AT(J,I)=A(I,J)
      RETURN
      END
C****************************
C  SUBROUTINE FOR FINDING INITIAL DEFLECTIONS GIVEN BRG. DEFL.
C****************************
      SUBROUTINE DEFL(AC,F1,F2,YO)
      DIMENSION AC(25,25),A1(325),B(25),DET(25)
      INTEGER N,IB,IJOB,ICHNG(650),IER
      REAL  YO(25)
      DO 2 I=1,25
      B(I)=0.0
 2    CONTINUE
      B(4)=B(4)+F1
      B(6)=B(6)+F2
      JA=0
      I6=1
      DO 10 K=1,25
      JA=JA+1
      DO 10 KL=1,JA
      A1(I6)=AC(K,KL)
 10   I6=I6+1
      M=1
      N=25
      IB=25
      IJOB=0
      CALL LEQ1S(A1,N,B,M,IB,IJOB,ICHNG,DET,IER)
      DO 20 I=1,25
      YO(I)=B(I)
 20   CONTINUE
      RETURN
      END
C*************************
C SUBROUTINE FOR RESPONSE OF THE SPINDLE
C*************************
      SUBROUTINE RESP(W,T1,V)
      COMPLEX W(50),V(50,1)
      DO 5 I=1,50
      AR=REAL(W(I))*T1
      AI=AIMAG(W(I))*T1
      AR1=EXP(AR)
      XC=AR1*COS(AI)
      YC=AR1*SIN(AI)
      V(I,1)=CMPLX(XC,YC)
 5    CONTINUE
      RETURN
      END
C************************
```

```
C SUBROUTINE FOR COMPLEX MATRIX MULTIPLICATION
C*********************
      SUBROUTINE MAT(AC,BC,N,CC)
      COMPLEX AC(50,50),BC(50,1),CC(50,1)
      DO 40 I=1,N
      CC(I,1)=CMPLX(0.0,0.0)
      DO 40 J=1,N
   40 CC(I,1)=CC(I,1)+AC(I,J)*BC(J,1)
      RETURN
      END
C*************************
C SUBROUTINE FOR TRIPLE MATRIX PRODUCT
C**********************
      SUBROUTINE MATMP1(A1,B1,C1,L1,M1,N1)
      DIMENSION A1(4,4),B1(4,4),C1(4,4)
      DO 2 I=1,L1
      DO 2 J=1,N1
      C1(I,J)=0.0
      DO 2 K=1,M1
    2 C1(I,J)=C1(I,J)+A1(I,K)*B1(K,J)
      RETURN
      END
C*********************
C SUBROUTINE FOR ASSEMBLING
C********************
      SUBROUTINE ASSEM(AZ,JU1,JU2,NODES,SUM)
      DIMENSION AZ(4,4),SUM(26,26)
      DO 8 I=1,26
      DO 8 J=1,26
      SUM(I,J)=0.0
    8 CONTINUE
      JV1=JU1+NODES
      JV2=JU2+NODES
      DO 10 KI=1,4
      IF(KI.EQ.1) KO=JU1
      IF(KI.EQ.2) KO=JU2
      IF(KI.EQ.3) KO=JV1
      IF(KI.EQ.4) KO=JV2
      SUM(KO,JU1)=SUM(KO,JU1)+AZ(KI,1)
      SUM(KO,JU2)=SUM(KO,JU2)+AZ(KI,2)
      SUM(KO,JV1)=SUM(KO,JV1)+AZ(KI,3)
      SUM(KO,JV2)=SUM(KO,JV2)+AZ(KI,4)
   10 CONTINUE
      RETURN
      END
C************************
C SUBROUTINE FOR TRANSFORMATION MATRIX
C************************
      SUBROUTINE TRNSF(A)
      DIMENSION A(4,4)
      DO 12 I=1,4
      DO 12 J=1,4
```

```
      A(I,J)=0.0
   12 CONTINUE
      A(1,1)=A(1,1)+1.0
      A(2,2)=A(2,2)+1.0
      A(3,3)=A(3,3)+1.0
      A(4,4)=A(4,4)+1.0
      RETURN
      END
C************************
C SUBROUTINE FOR MATRIX PRODUCT
C************************
      SUBROUTINE TRNSP1(RR,A,RK)
      DIMENSION A(4,4),RR(4,4),AT(4,4),RTK(4,4),RK(4,4)
      DO 10 IP=1,4
      DO 10 JP=1,4
      RTK(IP,JP)=0.0
      RK(IP,JP)=0.0
   10 CONTINUE
      CALL TRANSP(A,4,4,AT)
      CALL MATMP1(AT,RR,RTK,4,4,4)
      CALL MATMP1(RTK,A,RK,4,4,4)
      RETURN
      END
C****************************
C SUBROUTINE FOR TRIPLE MATRIX PRODUCT
C*****************************
      SUBROUTINE MATMP3(AQ,BQ,CQ,I1,I2,I3)
      DIMENSION AQ(25,25),BQ(25,25),CQ(25,25)
      DO 9 I=1,I1
      DO 9 J=1,I2
      CQ(I,J)=0.0
      DO 9 K=1,I3
    9 CQ(I,J)=CQ(I,J)+AQ(I,K)*BQ(K,J)
      RETURN
      END
C********************************
C SUBROUTINE FOR MATRIX PRODUCT
C********************************
      SUBROUTINE MATM1(AA,BB,CC,K1)
      COMPLEX AA(50,50),BB(50,50),CC(50,50)
      DO 2 I=1,K1
      DO 2 J=1,K1
      CC(I,J)=CMPLX(0.0,0.0)
      DO 2 K=1,K1
      CC(I,J)=CC(I,J)+AA(I,K)*BB(K,J)
    2 CONTINUE
      RETURN
      END
C****************************
C SUBROUTINE FOR VECTOR PRODUCT
C**********************************
      SUBROUTINE MAS(AA,BB,CC,K1)
```

```
      COMPLEX AA(50,1),BB(50,1),CC(50,1)
      DO 1 I=1,50
      CC(I,1)=AA(I,1)*BB(I,1)
1     CONTINUE
      RETURN
      END
```

```
C****************************************
C
C LISTING OF THE PROGRAM ++ FVIBC ++
C PROGRAM FOR FREE-VIBRATION RESPONSE OF A LATHE SPINDLE
C - WORKPIECE SYSTEM
C THE SYSTEM MATRICES ARE CONDENSED FIRST
C AND THE RESPONSE DUE TO INITIAL DISPLACEMENTS
C IS THEN FOUND OUT
C IMSL SUBROUTINES : RLFOR LINV2F LEQIS LEQT1C & EIGCC
C
C****************************************
      DIMENSION SL1(12),SD1(12),Y1(12),GMM(25,25),GKK(25,25)
      DIMENSION GCC(25,25),GM(26,26),GK(26,26),GC(26,26)
      REAL KF,KR,GMR(25,25),GKR(25,25),GCR(25,25)
      INTEGER LN(25),LD(25)
      REAL F1(50,50),F2(50,50),F3(50,50),F4(50,50)
      REAL F5(50,50),F6(50,50),FF1(50,50)
      REAL FF2(50,50),FF3(50,50),FF4(50,50)
      REAL GMN(50,50),GKN(50,50),GCN(50,50),TR(50,50),TT(50,50)
      COMPLEX W(50),Z(50,50),FF5(50,50),HZ,HZ1
      REAL SA(50,50),SB(50,50)
      COMPLEX ZT(50,50),DM(50),DK(50),A4(50,50)
      COMPLEX A5(50,50),SR(50,50),V1(50,50),V2(50,50)
      COMPLEX G(50),AS(50,50),AC(50,50),PR(50,50),Z1(50,50),Z1T(50,50)
      REAL SF(50,50),PF(50,50),SR1(25,25),SR2(25,25)
      COMPLEX PR1(50,50),PR2(50,50),PR3(50,50),PR4(50,50)
      COMPLEX SQ1(50,50),SQ2(50,50),SQ3(50,50),SQ4(50,50)
      COMPLEX V3(50,50),V4(50,50),V5(50,50)
      COMPLEX ZI(50,50),ZS(50,50)
      COMPLEX ZO(25),YA1(50,1),YA2(50,1),YA3(50,1),
      COMPLEX YA4(50,1),YA5(50,1)
      REAL FR(25,1),FRD(25,1),YOO(25),BJ(25)
C*****************************
      OPEN(UNIT=6,FILE='BA.DAT',TYPE='OLD')
      READ(6,*) NSD,N1
C*****************************
      N5=N1-NSD
      N55=N5*2
      N2=N1*2
      CALL SPRTN(N1,N2,N5,N55,NSD,SL1,SD1,
     $ GMM,GKK,GCC,GMR,GKR,GCR,LN,
     $ LD,F1,F2,F3,F4,F5,F6,FF1,FF2,FF3,FF4,GMN,GKN,GCN,
     $ TR,TT,W,Z,FF5,DM,SA,SB,ZT,
     $ DK,A4,A5,SR,G,AS,AC,PR,Z1,Z1T,SF,PF,V1,V2,V3,V4,V5,
     $ SR1,SR2,PR1,PR2,PR3,PR4,SQ1,SQ2,SQ3,SQ4,FR,FRD,YOO,ZI,ZS,
     $ ZO,YA1,YA2,YA3,YA4,YA5,BJ)
      STOP
      END
      SUBROUTINE SPRTN(N1,N2,N5,N55,NSD,SL1,SD1,
     $ GMM,GKK,GCC,GMR,GKR,GCR,LN,
     $ LD,F1,F2,F3,F4,F5,F6,FF1,FF2,FF3,FF4,GMN,GKN,GCN,
     $ TR,TT,W,Z,FF5,DM,SA,SB,ZT,
```

```
     $ DK,A4,A5,SR,G,AS,AC,PR,Z1,Z1T,SF,PF,V1,V2,V3,V4,V5,
     $ SR1,SR2,PR1,PR2,PR3,PR4,SQ1,SQ2,SQ3,SQ4,FR,FRD,YOO,ZI,ZS,
     $ ZO,YA1,YA2,YA3,YA4,YA5,BJ)
      DIMENSION SL1(12),SD1(12)
      REAL SR1(N5,N5),SR2(N5,N5)
      COMPLEX PR1(N5,N5),PR2(N5,N5),PR3(N5,N5),PR4(N5,N5)
      COMPLEX SQ1(N1,N1),SQ2(N1,N1),SQ3(N1,N1),SQ4(N1,N1)
      REAL GM(26,26),GK(26,26),GC(26,26)
      REAL GMM(25,25),GKK(25,25),GCC(25,25)
      REAL KF,KR,GMR(N5,N5),GKR(N5,N5),GCR(N5,N5)
      INTEGER LN(N1),LD(N1)
      REAL F1(N5,N5),F2(N5,N5),F3(NSD,NSD)
      REAL F4(NSD,N5),F5(NSD,N5),F6(NSD,NSD)
      REAL FF1(N5,N5),FF2(N5,N5),FF3(N5,N5)
      REAL FF4(N5,N5),GMN(N1,N1),GKN(N1,N1)
      REAL GCN(N1,N1),TR(N1,N5),TT(N5,N1)
      COMPLEX W(N55),Z(N55,N55),ZT(N55,N55),DM(N55),HZ,HZ1
      REAL  SA(N55,N55),SB(N55,N55)
      REAL SF(N5,N5),PF(N55,N55)
      COMPLEX AS(N55,N55),AC(N55,N55)
      COMPLEX PR(N55,N55),Z1(N5,N55),Z1T(N55,N5)
      COMPLEX DK(N55),A4(N5,N5),A5(N5,N5)
      COMPLEX SR(N5,N5),G(N55),FF5(N55,N55)
      COMPLEX V1(N55,N55),V2(N55,N55),V3(N55,N55),V4(N5,N55)
      COMPLEX V5(N5,N5),ZI(N55,N55),ZS(N55,N55),ZO(N5),YA4(N55,1)
      COMPLEX YA1(N55,1),YA2(N55,1),YA3(N55,1),YA5(N55,1)
      REAL FR(N1,1),FRD(N5,1),YOO(N5),BJ(N5)
C******************************
      OPEN(UNIT=4,FILE='PAS.DAT',TYPE='OLD')
      READ(4,*) NUE,NODES,NR,ND
      READ(4,*) SE,KF,KR
      READ(4,*) D1,D2,D12
      READ(4,*) WL,WD
      READ(4,*) (SD1(L),SL1(L),L=6,NUE)
      READ(4,*) RR
C     KF=RR*KR
      DO 606 I=1,5
      SL1(I)=WL
      SD1(I)=WD
  606 CONTINUE
      CALL EKEM(SD1,SL1,SE,KF,KR,D1,D2,D12,NUE,NODES,NR,ND,GK,GM,GC)
      DO 25 I=1,N1
      DO 25 J=1,N1
      GKK(I,J)=GK(I+1,J+1)
      GMM(I,J)=GM(I+1,J+1)
      GCC(I,J)=GC(I+1,J+1)
   25 CONTINUE
      CALL MATLOK(N1,N2,N5,N55,NSD,NR,ND,KF,KR,GMM,GKK,GCC,
     $ GMR,GKR,GCR,LN,LD,F1,F2,F3,F4,F5,F6,FF1,
     $ FF2,FF3,FF4,GMN,GKN,GCN,TR,TT,W,Z,FF5,
     $ SA,SB,ZT,DM,DK,A4,A5,SR,G,
     $ AS,AC,PR,Z1,Z1T,SF,PF,V1,V2,V3,V4,V5,
```

```
     $ SR1,SR2,PR1,PR2,PR3,PR4,SQ1,SQ2,SQ3,SQ4,FR,FRD,YOO,ZI,ZS,
     $ ZO,YA1,YA2,YA3,YA4,YA5,BJ)
      RETURN
      END
      SUBROUTINE MATLOK(N1,N2,N5,N55,NSD,NR,ND,KF,KR,GMM,GKK,GCC,
     $ GMR,GKR,GCR,LN,LD,F1,F2,F3,F4,F5,F6,FF1,
     $ FF2,FF3,FF4,GMN,GKN,GCN,TR,TT,W,Z,FF5
     $ SA,SB,ZT,DM,DK,A4,A5,SR,G,
     $ AS,AC,PR,Z1,Z1T,SF,PF,V1,V2,V3,V4,V5,
     $ SR1,SR2,PR1,PR2,PR3,PR4,SQ1,SQ2,SQ3,SQ4,FR,FRD,YOO,ZI,ZS,
     $ ZO,YA1,YA2,YA3,YA4,YA5,BJ)
      REAL GMM(25,25),GKK(25,25),GCC(25,25)
      REAL GMR(N5,N5),GKR(N5,N5),GCR(N5,N5)
      INTEGER LN(N1),LD(N1)
      REAL F1(N5,N5),F2(N5,N5),F3(NSD,NSD)
      REAL F4(NSD,N5),F5(NSD,N5),F6(NSD,NSD)
      REAL FF1(N5,N5),FF2(N5,N5),FF3(N5,N5),FF4(N5,N5),KF,KR
      REAL GMN(N1,N1),GKN(N1,N1),GCN(N1,N1),TR(N1,N5),TT(N5,N1)
      COMPLEX  W(N55),Z(N55,N55),ZT(N55,N55),DM(N55),HZ,HZ1,SW(25,25)
      REAL SA(N55,N55),SB(N55,N55),SF(N5,N5),PF(N55,N55),SFF(25,25)
      COMPLEX AS(N55,N55),AC(N55,N55),PR(N55,N55),Z1(N5,N55)
      COMPLEX Z1T(N55,N5),FF5(N55,N55)
      COMPLEX  DK(N55),A4(N5,N5),A5(N5,N5),SR(N5,N5),G(N55)
      COMPLEX V1(N55,N55),V2(N55,N55),V3(N55,N55)
      COMPLEX V4(N5,N55),V5(N5,N5)
      REAL SF3(25,25),SF2(25,25),SR1(N5,N5),SR2(N5,N5)
      COMPLEX PR1(N5,N5),PR2(N5,N5),PR3(N5,N5),PR4(N5,N5)
      COMPLEX SQ1(N1,N1),SQ2(N1,N1),SQ3(N1,N1),SQ4(N1,N1)
      REAL FR(N1,1),FRD(N5,1),YOO(N5),BJ(N5)
      COMPLEX ZI(N55,N55),ZS(N55,N55),ZO(N5)
      COMPLEX YA1(N55,1),YA2(N55,1),YA3(N55,1)
      COMPLEX YA4(N55,1),YA5(N55,1)
      REAL XN1(1001),XN2(1001),TX1(1001)
      REAL TX2(1001),XN3(1001),TX3(1001)
      REAL ABC(1000,4),ACD(1000,6),PO(4,12)
      REAL XN11(1000),TX11(1000)
C********************************
      OPEN(UNIT=2,FILE='FORCE.DAT',TYPE='OLD')
      READ(2,*) FU1,FU2
      DO 2002 I=1,N1
      FR(I,1)=0.0
 2002 CONTINUE
      FR(2,1)=FU1
      FR(4,1)=FU2
      OPEN(UNIT=5,FILE='RED.DAT',TYPE='NEW')
      WRITE(5,3012)
 3012 FORMAT(12X,'FRONT & REAR BEARING STIFFNESS VALUES ',/)
      WRITE(5,*) KF,KR
      WRITE(5,3013) NR
 3013 FORMAT(12X,'LOCATION OF REAR BEARING AT ',I4,' ELEMENT ',/)
      WRITE(5,3017) ND
 3017 FORMAT(12X,'LOCATION OF THE EXTERNAL DAMPER AT ',I4,'ELEMENT',/)
```

```
      WRITE(5,3001)
3001  FORMAT(12X,'STIFFNESS MATRIX',/)
      WRITE(5,*) (GKK(I,I),I=1,25)
      WRITE(5,3002)
3002  FORMAT(12X,'MASS MATRIX ',/)
      WRITE(5,*) (GMM(I,I),I=1,25)
      CALL APX11(N1,GKK,GMM,LD,LN)
      WRITE(5,91) N5
 91   FORMAT(12X,'MASTERS = ',I4,//)
      WRITE(5,92)
 92   FORMAT(12X,'MASTER DEGREES OF FREEDOM',/)
      DO 94 IO=NSD+1,N1
      WRITE(5,*) LD(IO)
 94   CONTINUE
      WRITE(5,96) NSD
 96   FORMAT(12X,'SLAVES ',I4,/)
      WRITE(5,55)
55    FORMAT(12X,'*** DIAGNOL RATIOS *****',/)
      WRITE(5,*) (LD(I),I=1,N1)
      WRITE(5,*) (LN(I),I=1,N1)
      CALL APX222(N1,GMM,LN,NSD,GMN)
      WRITE(5,3004)
3004  FORMAT(12X,'ARRANGED MASS MATRIX ',/)
      WRITE(5,*) (GMN(I,I),I=1,N1)
      CALL APX222(N1,GKK,LN,NSD,GKN)
      WRITE(5,3005)
3005  FORMAT(12X,'ARRANGED STIFFNESS MATRIX',/)
      WRITE(5,*) (GKN(I,I),I=1,N1)
      CALL APX222(N1,GCC,LN,NSD,GCN)
      WRITE(5,3006)
3006  FORMAT(12X,'ARRANGED DAMPING MATRIX ',/)
      WRITE(5,*) (GCN(I,I),I=1,N1)
      CALL APX33(N1,NSD,N5,GKN,TR,TT,F1,F2,F3,F4,F5,F6)
      CALL APX44(N1,N5,NSD,TT,GMN,TR,GMR,F6)
      CALL APX44(N1,N5,NSD,TT,GKN,TR,GKR,F6)
      CALL APX44(N1,N5,NSD,TT,GCN,TR,GCR,F6)
      CALL APX55(N1,N5,NSD,TT,FR,FRD)
      DO 1212 I=1,N5
      FRD(I,1)=0.0
1212  CONTINUE
      FRD(2,1)=75.0
      FRD(3,1)=75.0
      CALL DEFL(N5,GKR,FRD,YOO,BJ)
      WARK=(YOO(2)+YOO(3))*0.5
      WRITE(5,3010)
3010  FORMAT(12X,'INITIAL DEFLECTIONS OF THE MASTERS',/)
      WRITE(5,*) (YOO(I),I=1,N5)
      WRITE(5,855) WARK
855   FORMAT(12X,'DEFL AT THE CENTER OF THE WORK PIECE = ',E20.10,/)
      WRITE(5,3007)
3007  FORMAT(12X,'REDUCED MASS MATRIX',/)
      WRITE(5,*) ((GMR(I,J),J=1,N5),I=1,N5)
```

```
      WRITE(5,3008)
3008  FORMAT(12X,'REDUCED STIFFNESS MATRIX',/)
      WRITE(5,*) ((GKR(I,J),J=1,N5),I=1,N5)
      WRITE(5,3009)
3009  FORMAT(12X,'REDUCED DAMPING MATRIX ',/)
      WRITE(5,*) ((GCR(I,J),J=1,5),I=1,N5)
      CALL XXX(N5,N55,GKR,GMR,GCR,W,Z,ZI,ZS,FF1,FF2,FF3,
     1 FF4,FF5,SA,SB)
      WRITE(5,501)
 501  FORMAT(12X,'EIGEN VALUES ',/)
      WRITE(5,*) (W(I),I=1,N55)
      T2=(0.25)/1000.0
      DO 22 I=1,1000
      TX11(I)=FLOAT(I)*T2
      TQ=TX11(I)
      CALL RES(N5,N55,Z,ZI,YOO,W,TQ,ZO,DWORK,YA1,YA2,YA3,YA4,YA5)
      XW11(I)=DWORK
C     WRITE(60,*) XW1(I),TX1(I)
22    CONTINUE
      DO 907 I=2,1001
      XW1(I)=XW11(I-1)
      TX1(I)=TX11(I-1)
907   CONTINUE
      XW1(1)=WARK
      TX1(1)=0.0
      DO 905 I=1,1000
      WRITE(60,*) XW1(I),TX1(I)
905   CONTINUE
      NCT=1
      DO 6501 I=1,1001
      IF(XW1(I).GT.0.0) GO TO 6502
      GO TO 6501
6502  XW2(NCT)=XW1(I)
      TX2(NCT)=TX1(I)
      NCT=NCT+1
6501  CONTINUE
      J1=1
      DO 6503 I=1,NCT
      J1=J1+1
      DO 6503 J=J1,NCT
      AA=XW2(I)
      AB=XW2(J)
      T11=TX2(I)
      T22=TX2(J)
      IF(AA.LT.AB) GO TO 6504
      GO TO 6503
6504  XW2(I)=AB
      XW2(J)=AA
      TX2(I)=T22
      TX2(J)=T11
6503  CONTINUE
      KX=1
```

```
      T33=TX2(1)
      J=1
      DO 6505 I=J,NCT
      T44=TX2(I)
      IF(T44.GE.T33) GO TO 6606
      GO TO 6505
 6606 XN3(KX)=XN2(I)
      TX3(KX)=TX2(I)
      KX=KX+1
      T33=TX2(I)
 6505 CONTINUE
C     DO 988 I=1,1000
C     WRITE(82,*) XN3(I),TX3(I)
C988  CONTINUE
      TX3(1)=0.0
      KX=KX-1
      CALL KRYPTON(XN3,TX3,6,ABC,ACD,PO)
      DO 3333 I=1,6
      XD=PO(1,2)*TX3(I)+PO(2,2)
      WRITE(9,*) XD,XN3(I)
 3333 CONTINUE
      WRITE(9,*) (PO(I,2),I=1,4)
      WRITE(5,211)
 211  FORMAT(12X,'EIGEN VALUES IN HERTZ ',/)
      DO 801 I=1,N55
      W(I)=AIMAG(W(I))/(44.0/7.0)
      WRITE(5,*) W(I)
 801  CONTINUE
 600  STOP
      END
C*******************************
C SUBROUTINE FOR EIGENVALUES, MODAL MATRIX
C*******************************
      SUBROUTINE XXX(NX,NY,GKK,GMM,GCC,W,Z,ZI,Z1,GMI,GMK,GMC,
     1 UN,DC,AC,BC)
      DIMENSION GKK(NX,NX),GMM(NX,NX),GCC(NX,NX)
      DIMENSION GMI(NX,NX),GMK(NX,NX),GMC(NX,NX),UN(NX,NX)
      INTEGER N,IA,IZ,M,IB,IER,IJOB,IDGT
      REAL WKAREA(700),WK(5100),WA(50)
      COMPLEX W(NY),Z(NY,NY),ZN,DC(NY,NY),ZI(NY,NY),Z1(NY,NY)
      REAL  AC(NY,NY),BC(NY,NY)
      DO 25 I=1,NX
      DO 25 J=1,NX
      AC(I,J)=0.0
      AC(I,J+NX)=GMM(I,J)
      AC(I+NX,J)=GMM(I,J)
      AC(I+NX,J+NX)=GCC(I,J)
  25  CONTINUE
      DO 30 I=1,NX
      DO 30 J=1,NX
      BC(I,J)=-1.0*GMM(I,J)
      BC(I,J+NX)=0.0
```

```
      BC(I+NX,J)=0.0
      BC(I+NX,J+NX)=GKK(I,J)
   30 CONTINUE
      IA=NX
      N=NX
      IDGT=4
      CALL LINV2F(GMM,N,IA,GMI,IDGT,WKAREA,IER)
      CALL MATMP3(NX,NX,NX,GMI,GKK,GMK)
      CALL MATMP3(NX,NX,NX,GMI,GCC,GMC)
      DO 45 I=1,NX
      DO 45 J=1,NX
      GMK(I,J)=-1.0*GMK(I,J)
      GMC(I,J)=-1.0*GMC(I,J)
   45 CONTINUE
      DO 54 I=1,NX
      DO 54 J=1,NX
      UN(I,J)=0.0
   54 CONTINUE
      DO 55 I=1,NX
      UN(I,I)=UN(I,I)+1.0
   55 CONTINUE
      DO 60 I=1,NX
      DO 60 J=1,NX
      DC(I,J)=CMPLX(GMC(I,J),0.0)
      DC(I,J+NX)=CMPLX(GMK(I,J),0.0)
      DC(I+NX,J)=CMPLX(UN(I,J),0.0)
      DC(I+NX,J+NX)=CMPLX(0.0,0.0)
   60 CONTINUE
      N=NY
      IA=NY
      IZ=NY
      IJOB=2
      CALL EIGCC(DC,N,IA,IJOB,W,Z,IZ,WK,IER)
      DO 101 I=1,NY
      DO 101 J=1,NY
      Z1(I,J)=Z(I,J)
  101 CONTINUE
      DO 102 I=1,NY
      DO 102 J=1,NY
      ZI(I,J)=CMPLX(0.0,0.0)
  102 CONTINUE
      DO 104 I=1,NY
      ZI(I,I)=CMPLX(1.0,0.0)
  104 CONTINUE
      IA=NY
      IB=NY
      N=NY
      M=NY
      IJOB=0
      CALL LEQT1C(Z1,N,IA,ZI,M,IB,IJOB,WA,IER)
      RETURN
      END
```

```
C*************************
C SUBROUTINE FOR GLOBAL MASS & STIFFNESS MATRICES
C*************************
      SUBROUTINE EKEM(SD1,SL1,SE,KF,KR,D1,D2,
     1 D12,NUE,NODES,NR,ND,GK,GM,GC)
      DIMENSION GM(26,26),GK(26,26),GC(26,26)
      DIMENSION SD1(12),SL1(12),EK(4,4),EM(4,4),EC(4,4)
      DIMENSION A(4,4),AK(4,4),AM(4,4),AC(4,4)
      DIMENSION SK(26,26),SH(26,26),SC(26,26)
      REAL KF,KR
      DO 20 I=1,26
      DO 20 J=1,26
      GM(I,J)=0.0
      GC(I,J)=0.0
   20 GK(I,J)=0.0
      KM1=1
      KM2=2
      DO 105 K=1,NUE
      SL=SL1(K)
      SD=SD1(K)
      JU1=KM1
      JU2=KM2
      CALL MATFOR(SD,SL,SE,K,EK,EM,EC)
      IF(K.EQ.7) GO TO 145
      IF(K.EQ.NR) GO TO 146
      GO TO 148
  145 EK(1,1)=EK(1,1)+0.5*KF
      EK(3,3)=EK(3,3)+0.5*KF
      EC(1,1)=EC(1,1)+0.5*D1
      EC(3,3)=EC(3,3)+0.5*D1
      GO TO 148
  146 EK(1,1)=EK(1,1)+0.5*KR
      EK(3,3)=EK(3,3)+0.5*KR
      EC(1,1)=EC(1,1)+0.5*D2
      EC(3,3)=EC(3,3)+0.5*D2
      GO TO 148
  148 IF(K.EQ.ND) GO TO 149
      GO TO 158
  149 EC(1,1)=EC(1,1)+0.5*D12
      EC(3,3)=EC(3,3)+0.5*D12
  158 CALL TRNSF(A)
      CALL TRNSF1(EK,A,AK)
      CALL TRNSF1(EM,A,AM)
      CALL TRNSF1(EC,A,AC)
      DO 14 IP=1,26
      DO 14 KP=1,26
      SK(IP,KP)=0.0
      SH(IP,KP)=0.0
      SC(IP,KP)=0.0
   14 CONTINUE
      CALL ASSEM(AK,JU1,JU2,NODES,SK)
      CALL ASSEM(AM,JU1,JU2,NODES,SH)
```

```
      CALL ASSEM(AC,JU1,JU2,NODES,SC)
      DO 150 I1=1,26
      DO 150 J1=1,26
      GK(I1,J1)=GK(I1,J1)+SK(I1,J1)
      GM(I1,J1)=GM(I1,J1)+SM(I1,J1)
      GC(I1,J1)=GC(I1,J1)+SC(I1,J1)
150   CONTINUE
      KM1=KM1+2
      KM2=KM2+2
105   CONTINUE
      RETURN
      END
C******************************
C SUBROUTINE FOR FINDING INITIAL DEFLECTIONS
C******************************
      SUBROUTINE DEFL(N5,AC,F1,YO,B)
      REAL AC(N5,N5),A1(325),B(N5),DET(25),F1(N5)
      INTEGER N,IB,IJOB,ICHNG(650),IER
      REAL YO(N5)
      DO 2 I=1,N5
      B(I)=F1(I)
  2   CONTINUE
      JA=0
      I6=1
      DO 10 K=1,N5
      JA=JA+1
      DO 10 KL=1,JA
      A1(I6)=AC(K,KL)
 10   I6=I6+1
      M=1
      N=N5
      IB=N5
      IJOB=0
      CALL LEQ1S(A1,N,B,M,IB,IJOB,ICHNG,DET,IER)
      DO 20 I=1,N5
      YO(I)=B(I)
 20   CONTINUE
      RETURN
      END
C*************************************
C SUBROUTINE FOR FINDING THE RESPONSE
C*************************************
      SUBROUTINE RES(N5,N55,Z,ZI,YOO,W,T1,
     $ ZO,DWORK,YO,V1,ZIO,ZRES,ZOR)
      COMPLEX Z(N55,N55),ZI(N55,N55),W(N55),YO(N55,1),V1(N55,1)
      COMPLEX ZIO(N55,1),ZRES(N55,1),ZOR(N55,1)
      REAL YOO(N5),ZO(N5)
      DO 105 I=1,N55
      YO(I,1)=CMPLX(0.0,0.0)
105   CONTINUE
      DO 110 I=1,N5
      YO(I+N5,1)=YO(I+N5,1)+CMPLX(YOO(I),0.0)
```

```
110     CONTINUE
        CALL RESP(N55,W,T1,V1)
        CALL MAR1(ZI,YO,N55,ZIO)
        CALL MAR2(ZIO,V1,ZRES,N55)
        CALL MAR1(Z,ZRES,N55,ZOR)
        DO 120 I=1,N5
        ZO(I)=REAL(ZOR(I+N5,1))
120     CONTINUE
        U1=ZO(2)
        U2=ZO(3)
        DWORK=0.5*(U1+U2)
        RETURN
        END
C**********************
C  SUBROUTINE FOR ELEMENTAL STIFFNESS & MASS  MATRIX
C**********************
        SUBROUTINE MATFOR(SD,SL,SE,K,EK,EM,EC)
        DIMENSION EC(4,4),EK(4,4),EM(4,4)
        RHO=0.2832958/386.0
        DO 4 NI=1,4
        DO 4 JI=1,4
        EC(NI,JI)=0.0
        EK(NI,JI)=0.0
4       EM(NI,JI)=0.0
        EM(1,1)=156.0
        EM(1,2)=22.0*SL
        EM(1,3)=54.0
        EM(1,4)=-13.0*SL
        EM(2,1)=22.0*SL
        EM(2,2)=4.0*SL**2
        EM(2,3)=13.0*SL
        EM(2,4)=-3.0*SL**2
        EM(3,1)=54.0
        EM(3,2)=13.*SL
        EM(3,3)=156.0
        EM(3,4)=-22.0*SL
        EM(4,1)=-13.0*SL
        EM(4,2)=-3.0*SL**2
        EM(4,3)=-22.0*SL
        EM(4,4)=4.0*SL**2
        EK(1,1)=12.0
        EK(1,2)=6.0*SL
        EK(1,3)=-12.0
        EK(1,4)=6.0*SL
        EK(2,1)=6.0*SL
        EK(2,2)=4.0*SL**2
        EK(2,3)=-6.0*SL
        EK(2,4)=2.0*SL**2
        EK(3,1)=-12.0
        EK(3,2)=-6.0*SL
        EK(3,3)=12.0
        EK(3,4)=-6.0*SL
```

```
      EK(4,1)=6.0*SL
      EK(4,2)=2.0*SL**2
      EK(4,3)=-6.0*SL
      EK(4,4)=4.0*SL**2
      IF(K.LE.5) BD=0.0
      IF(K.GT.5) BD=2.5
      SI=(22.0/(7.0*64.0))*(SD**4-BD**4)
      SM=(22.0/28.0)*(SD**2-BD**2)*RHO
      FM=SM*SL/420.0
      FK=SE*SI/(SL**3)
      DO 5 I=1,4
      DO 5 J=1,4
      EM(I,J)=FM*EM(I,J)
      EK(I,J)=FK*EK(I,J)
    5 CONTINUE
      RETURN
      END
C*********************
C  FINDS THE TRANSPOSE OF A MATRIX
C*******************
      SUBROUTINE TRANSP(A,MX,MY,AT)
      DIMENSION A(4,4),AT(4,4)
      DO 1 I=1,MX
      DO 1 J=1,MY
    1 AT(J,I)=A(I,J)
      RETURN
      END
C***********************************
C  FOR TRIPLE MATRIX PRODUCT
C***********************************
      SUBROUTINE MATMP1(A1,B1,C1,L1,M1,N1)
      DIMENSION A1(4,4),B1(4,4),C1(4,4)
      DO 2 I=1,L1
      DO 2 J=1,N1
      C1(I,J)=0.0
      DO 2 K=1,M1
    2 C1(I,J)=C1(I,J)+A1(I,K)*B1(K,J)
      RETURN
      END
C********************************
C   SUBROUTINE FOR ASSEMBLING
C********************************
      SUBROUTINE ASSEM(AZ,JU1,JU2,NODES,SUM)
      DIMENSION AZ(4,4),SUM(26,26)
      DO 8 I=1,26
      DO 8 J=1,26
      SUM(I,J)=0.0
    8 CONTINUE
      JV1=JU1+NODES
      JV2=JU2+NODES
      DO 10 KI=1,4
      IF(KI.EQ.1) KO=JU1
```

```
      IF(KI.EQ.2) KO=JU2
      IF(KI.EQ.3) KO=JV1
      IF(KI.EQ.4) KO=JV2
      SUM(KO,JU1)=SUM(KO,JU1)+AZ(KI,1)
      SUM(KO,JU2)=SUM(KO,JU2)+AZ(KI,2)
      SUM(KO,JV1)=SUM(KO,JV1)+AZ(KI,3)
      SUM(KO,JV2)=SUM(KO,JV2)+AZ(KI,4)
 10   CONTINUE
      RETURN
      END
C*******************************
C SUBROUTINE FOR TRANSFORMATION MATRIX
C*******************************
      SUBROUTINE TRNSF(A)
      DIMENSION A(4,4)
      DO 12 I=1,4
      DO 12 J=1,4
      A(I,J)=0.0
 12   CONTINUE
      A(1,1)=A(1,1)+1.0
      A(2,2)=A(2,2)+1.0
      A(3,3)=A(3,3)+1.0
      A(4,4)=A(4,4)+1.0
      RETURN
      END
C***********************
C  SUBROUTINE FOR MATRIX PRODUCT
C***********************
      SUBROUTINE TRNSF1(RR,A,RK)
      DIMENSION A(4,4),RR(4,4),AT(4,4),RTK(4,4),RK(4,4)
      DO 10 IF=1,4
      DO 10 JF=1,4
      RTK(IF,JF)=0.0
      RK(IF,JF)=0.0
 10   CONTINUE
      CALL TRANSP(A,4,4,AT)
      CALL MATMP1(AT,RR,RTK,4,4,4)
      CALL MATMP1(RTK,A,RK,4,4,4)
      RETURN
      END
C********************************
      SUBROUTINE MATMP3(I1,I2,I3,AQ,BQ,CQ)
      DIMENSION AQ(I1,I2),BQ(I2,I3),CQ(I1,I3)
      DO 9 I=1,I1
      DO 9 J=1,I2
      CQ(I,J)=0.0
      DO 9 K=1,I3
 9    CQ(I,J)=CQ(I,J)+AQ(I,K)*BQ(K,J)
      RETURN
      END
C********************************
C SUBROUTINE FOR CONDENSATION
```

```
C*********************************
      SUBROUTINE APX11(N1,ASM,ASK,LD,LN)
      DIMENSION ASM(N1,N1),ASK(N1,N1),R(25),RN(25)
      INTEGER LD(N1),LN(N1)
      DO 5 I=1,N1
      R(I)=ASM(I,I)/ASK(I,I)
 5    CONTINUE
      DO 8 I=1,N1
      RN(I)=R(I)
      LD(I)=I
 8    CONTINUE
      AA=0.0
      J1=1
      DO 15 I=1,N1
      J1=J1+1
      DO 15 J=J1,N1
      AA=RN(I)
      AB=RN(J)
      IM=LD(I)
      IN=LD(J)
      IF(AA.LT.AB) GO TO 20
      GO TO 15
 20   RN(I)=AB
      RN(J)=AA
      LD(I)=IN
      LD(J)=IM
 15   CONTINUE
      DO 60 I=1,N1
      LN(I)=LD(I)
 60   CONTINUE
      J1=1
      DO 70 I=1,N1
      IC=LN(I)
      J1=J1+1
      DO 70 J=J1,N1
      ID=LN(J)
      IF(IC.LT.ID) GO TO 80
      GO TO 70
 80   LN(J)=LN(J)-1
 70   CONTINUE
      RETURN
      END
C*********************************
C SUBROUTINE FOR CONDENSATION
C*********************************
      SUBROUTINE APX222(N1,A1,LN,NSD,A9)
      DIMENSION A1(N1,N1),A5(25,25),A9(N1,N1)
      INTEGER LN(N1)
      DO 5 K=1,NSD
      L1=LN(K)
      CALL APX22(N1,A1,L1,A5)
      DO 10 I=1,N1
```

```
      DO 10 J=1,N1
      A1(I,J)=A5(I,J)
10    CONTINUE
5     CONTINUE
      DO 77 I=1,N1
      DO 77 J=1,N1
      A9(I,J)=A1(I,J)
77    CONTINUE
      RETURN
      END
C*********************************
C SUBROUTINE FOR CONDENSATION
C*********************************
      SUBROUTINE APX22(N1,A1,LD,A3)
      DIMENSION A1(N1,N1),A3(N1,N1),A2(25,25)
      DO 5 I=1,N1
      DO 5 J=1,N1
      A2(I,J)=A1(I,J)
5     CONTINUE
      N2=N1-1
      DO 10 JJ=LD,N2
      JK=JJ+1
      DO 10 I=1,N1
      A2(I,JJ)=A1(I,JK)
10    CONTINUE
      DO 20 I=1,N1
      A2(I,N1)=A1(I,LD)
20    CONTINUE
      DO 6 I=1,N1
      DO 6 J=1,N1
      A3(I,J)=A2(I,J)
6     CONTINUE
      DO 30 JJ=LD,N2
      JK=JJ+1
      DO 30 I=1,N1
      A3(JJ,I)=A2(JK,I)
30    CONTINUE
      DO 40 I=1,N1
      A3(N1,I)=A2(LD,I)
40    CONTINUE
      RETURN
      END
C*****************************
C SUBROUTINE FOR CONDENSATION
C*****************************
      SUBROUTINE APX33(N1,NSD,N5,AK,T,TT,UN,AKM,AKS,AKT,AX,ASI)
      DIMENSION AK(N1,N1),T(N1,N5),TT(N5,N1),UN(N5,N5)
      REAL WKAREA(700),AKM(N5,N5),AKS(NSD,NSD),AKT(NSD,N5)
      REAL AX(NSD,N5),ASI(NSD,NSD)
      MD=N1-NSD
      DO 2 I=1,MD
      DO 2 J=1,MD
```

```
      AKM(I,J)=AK(I,J)
2     CONTINUE
      DO 4 I=1,NSD
      DO 4 J=1,NSD
      AKS(I,J)=AK(I+MD,J+MD)
4     CONTINUE
      DO 6 I=1,NSD
      DO 6 J=1,MD
      AKT(I,J)=AK(I+MD,J)
6     CONTINUE
      N=NSD
      IA=N
      IDGT=4
      CALL LINV2F(AKS,N,IA,ASI,IDGT,WKAREA,IER)
      DO 12 I=1,NSD
      DO 12 J=1,MD
      AX(I,J)=0.0
      DO 12 K=1,NSD
      AX(I,J)=AX(I,J)+ASI(I,K)*AKT(K,J)
12    CONTINUE
      DO 8 I=1,N1
      DO 8 J=1,N1
      UN(I,J)=0.0
8     CONTINUE
      DO 10 I=1,MD
      UN(I,I)=UN(I,I)+1.0
10    CONTINUE
      DO 22 I=1,MD
      DO 22 J=1,MD
      T(I,J)=UN(I,J)
22    CONTINUE
      DO 14 I=1,NSD
      DO 14 J=1,MD
      T(I+MD,J)=-1.0*AX(I,J)
14    CONTINUE
      DO 16 I=1,N1
      DO 16 J=1,MD
      TT(J,I)=T(I,J)
16    CONTINUE
      RETURN
      END
C********************************
C SUBROUTINE FOR CONDENDSATION
C********************************
      SUBROUTINE APX44(N1,N5,NSD,A1,A2,A3,A4,A12)
      DIMENSION A1(N5,N1),A2(N1,N1),A3(N1,N5),A12(N5,N1)
      REAL A4(N5,N5)
      MD=N1-NSD
      DO 10 I=1,MD
      DO 10 J=1,N1
      A12(I,J)=0.0
      DO 10 K=1,N1
```

```
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
 10   CONTINUE
      DO 20 I=1,MD
      DO 20 J=1,MD
      A4(I,J)=0.0
      DO 20 K=1,N1
      A4(I,J)=A4(I,J)+A12(I,K)*A3(K,J)
 20   CONTINUE
      RETURN
      END
C*****************************
C SUBROUTINE FOR CONDENSATION
C*****************************
      SUBROUTINE APX55(N1,N5,NSD,A1,A2,A3)
      REAL A1(N5,N1),A2(N1,1),A3(N5,1)
      DO 10 I=1,N5
      A3(I,1)=0.0
      DO 10 J=1,N1
      A3(I,1)=A3(I,1)+A1(I,J)*A2(J,1)
 10   CONTINUE
      RETURN
      END
C**************************
      SUBROUTINE CMAT1(A1,A2,A3,K1,A4,A12)
      COMPLEX A1(K1,K1),A3(K1,K1),A4(K1),A12(K1,K1)
      REAL A2(K1,K1)
      DO 1 I=1,K1
      DO 1 J=1,K1
      A12(I,J)=CMPLX(0.0,0.0)
      DO 1 K=1,K1
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
 1    CONTINUE
      DO 3 I=1,K1
      A4(I)=CMPLX(0.0,0.0)
      DO 3 K=1,K1
      A4(I)=A4(I)+A12(I,K)*A3(K,I)
 3    CONTINUE
      RETURN
      END
C********************************
      SUBROUTINE XAT(A1,B1,C1,K1,A4,AB)
      COMPLEX A1(K1,K1),B1(K1),C1(K1,K1),AB(K1,K1),A4(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      AB(I,J)=A1(I,J)*B1(J)
 2    CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      A4(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      A4(I,J)=A4(I,J)+AB(I,K)*C1(K,J)
 4    CONTINUE
```

```
      RETURN
      END
C***********************************
      SUBROUTINE XAT1(A1,A2,A3,K1,A4,A12)
      COMPLEX A1(K1,K1),A3(K1,K1),A12(K1,K1),A4(K1,K1)
      REAL A2(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      A12(I,J)=CMPLX(0.0,0.0)
      DO 2 K=1,K1
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
    2 CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      A4(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      A4(I,J)=A4(I,J)+A12(I,K)*A3(K,J)
    4 CONTINUE
      RETURN
      END
C*********************************
      SUBROUTINE SAT3(A1,B,C1,MR,NC,AC,AB)
      COMPLEX A1(MR,NC),C1(NC,MR),B(NC),AB(MR,NC),AC(MR,MR)
      DO 2 I=1,MR
      DO 2 J=1,NC
      AB(I,J)=A1(I,J)*B(J)
    2 CONTINUE
      DO 4 I=1,MR
      DO 4 J=1,MR
      AC(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,NC
      AC(I,J)=AC(I,J)+AB(I,K)*C1(K,J)
    4 CONTINUE
      RETURN
      END
C*********************************
      SUBROUTINE SAT4(A1,B1,C1,K1,D1,AB)
      COMPLEX A1(K1,K1),AB(K1,K1),C1(K1,K1),D1(K1,K1)
      REAL B1(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      AB(I,J)=CMPLX(0.0,0.0)
      DO 2 K=1,K1
      AB(I,J)=AB(I,J)+A1(I,K)*B1(K,J)
    2 CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      D1(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      D1(I,J)=D1(I,J)+AB(I,K)*C1(K,J)
    4 CONTINUE
      RETURN
```

```
      END
C******************************
      SUBROUTINE RESP(N55,W,T1,V)
      COMPLEX W(N55),V(N55,1)
      DO 5 I=1,N55
      AR=REAL(W(I))*T1
      AI=AIMAG(W(I))*T1
      AR1=EXP(AR)
      XC=AR1*COS(AI)
      YC=AR1*SIN(AI)
      V(I,1)=CMPLX(XC,YC)
 5    CONTINUE
      RETURN
      END
C******************************
      SUBROUTINE MAR2(AA,BB,CC,K1)
      COMPLEX AA(K1,1),BB(K1,1),CC(K1,1)
      DO 1 I=1,K1
      CC(I,1)=AA(I,1)*BB(I,1)
 1    CONTINUE
      RETURN
      END
C******************************
      SUBROUTINE MAR1(AC,BC,N,CC)
      COMPLEX AC(N,N),BC(N,1),CC(N,1)
      DO 40 I=1,N
      CC(I,1)=CMPLX(0.0,0.0)
      DO 40 J=1,N
      CC(I,1)=CC(I,1)+AC(I,J)*BC(J,1)
 40   CONTINUE
      RETURN
      END
C*****************************
C SUBROUTINE FOR LEAST SQUARE CURVE FITTING
C*****************************
      SUBROUTINE KRYPTON(XW,TW,NCT,XYW,PRED,B)
      INTEGER IX,N,MDP(3),IB,IP,IER
      REAL XW(1001),TW(1001)
      REAL XYW(NCT,4),RSQ,ALBP(2),ANOVA(13),B(4,12)
      REAL PRED(NCT,6)
      DOUBLE PRECISION WK(9044)
      OPEN(UNIT=11,FILE='FOR011.DAT',TYPE='OLD')
      READ(11,*) WW
C     WRITE(7,*) (XW(I),I=1,3)
C     DO 670 I=1,3
C     XW(I)=XW(I)+(PP/100.0)*XW(I)
C     WRITE(7,*) XW(I)
C 670 CONTINUE
      DO 600 I=1,NCT
      XW(I)=ALOG(XW(I))
 600  CONTINUE
      DO 101 I=1,NCT
```

```
      XYW(I,1)=TW(I)
      XYW(I,2)=XW(I)
      XYW(I,3)=1.0
101   CONTINUE
      XYW(1,3)=WW
      XYW(2,3)=WW
      XYW(3,3)=WW
      IX=NCT
      ALBP(1)=0.05
      N=NCT
      RSQ=100.0
      MDP(1)=1
      MDP(3)=0
      IB=4
      IP=NCT
      CALL RLFOR(XYW,IX,N,RSQ,MDP,ALBP,ANOVA,B,IB,
     1 PRED,IP,WK,IER)
      WRITE(7,*) MDP(2)
      WRITE(7,*) (B(I,2),I=1,4)
      RETURN
      END
```

```
C***********************************
C
C LISTING OF THE PROGRAM ++ TVIBI ++
C PROGRAM FOR IMPULSE RESPONSE OF A  LATHE SPINDLE
C - WORKPIECE SYSTEM
C IMSL SUBROUTINES : EIGCC & LINV2F
C
C*************************************
      DIMENSION SL1(12),SD1(12),Y1(12),GMM(25,25),GKK(25,25)
      DIMENSION GCC(25,25),GM(26,26),GK(26,26),GC(26,26)
      REAL KF,KR,GMR(25,25),GKR(25,25),GCR(25,25)
      INTEGER LN(25),LD(25)
      REAL F1(50,50),F2(50,50),F3(50,50),F4(50,50)
      REAL F5(50,50),F6(50,50),FF1(50,50),FF2(50,50)
      REAL FF3(50,50),FF4(50,50)
      REAL GMN(50,50),GKN(50,50),GCN(50,50),TR(50,50),TT(50,50)
      COMPLEX W(50),Z(50,50),FF5(50,50)
      REAL SA(50,50),SB(50,50),ERD(50,1)
      COMPLEX ZT(50,50),EE1(50,1),SS1(50,50),SS2(50,50)
      COMPLEX V1(50,50),V2(50,50)
      COMPLEX V3(50,50),V4(50,50),V5(50,50)
      COMPLEX ZS(50,50)
      COMPLEX ZO(25),YA1(50,1),YA2(50,1),YA3(50,1)
      COMPLEX YA4(50,1),YA5(50,1)
      REAL FR(25,1),FRD(25,1),YOO(25)
C****************************
      OPEN(UNIT=6,FILE='BA.DAT',TYPE='OLD')
      READ(6,*) NSD,N1
      N5=N1-NSD
      N55=N5*2
      N2=N1*2
      CALL MATLOK(N1,N2,N5,N55,NSD,SL1,SD1,
     $ GMM,GKK,GCC,GMR,GKR,GCR,LN,
     $ LD,F1,F2,F3,F4,F5,F6,FF1,FF2,FF3,FF4,GMN,GKN,GCN,
     $ TR,TT,W,Z,FF5,SA,SB,ZT,
     $ V1,V2,V3,V4,V5,
     $ FR,FRD,YOO,ZI,ZS,
     $ ZO,YA1,YA2,YA3,YA4,YA5,ERD,EE1,SS1,SS2)
      STOP
      END
      SUBROUTINE MATLOK(N1,N2,N5,N55,NSD,SL1,SD1,
     $ GMM,GKK,GCC,GMR,GKR,GCR,LN,
     $ LD,F1,F2,F3,F4,F5,F6,FF1,FF2,FF3,FF4,GMN,GKN,GCN,
     $ TR,TT,W,Z,FF5,SA,SB,ZT,
     $ V1,V2,V3,V4,V5,
     $ FR,FRD,YOO,ZI,ZS,
     $ ZO,YA1,YA2,YA3,YA4,YA5,ERD,EE1,SS1,SS2)
      REAL GMM(25,25),GKK(25,25),GCC(25,25),GMR(N5,N5)
      REAL GKR(N5,N5),GCR(N5,N5)
      INTEGER LN(N1),LD(N1)
      REAL GK(26,26),GM(26,26),GC(26,26)
      REAL F1(N5,N5),F2(N5,N5),F3(NSD,NSD),F4(NSD,N5)
```

```
      REAL F5(NSD,N5),F6(NSD,NSD)
      REAL FF1(N5,N5),FF2(N5,N5),FF3(N5,N5),FF4(N5,N5),KF,KR
      REAL GMN(N1,N1),GKN(N1,N1),GCN(N1,N1),TR(N1,N5),TT(N5,N1)
      COMPLEX  W(N55),Z(N55,N55),ZT(N55,N55)
      REAL SS1(N55,N55),SS2(N55,N55)
      REAL SA(N55,N55),SB(N55,N55),SL1(12),SD1(12)
      COMPLEX FF5(N55,N55)
      COMPLEX V1(N55,N55),V2(N55,N55),V3(N55,N55)
      COMPLEX V4(N5,N55),V5(N5,N5)
      REAL FR(N1,1),FRD(N5,1),YOO(N5),ERD(N55,1)
      COMPLEX ZI(N55,N55),ZS(N55,N55),ZO(N5),EE1(50,1)
      COMPLEX YA1(N55,1),YA2(N55,1),YA3(N55,1)
      COMPLEX YA4(N55,1),YA5(N55,1)
      REAL RIP1(1000),RIP2(1000),RIP3(1000)
      REAL RIP4(1000),RIP5(1000)
      REAL TIM(1000)
C********************************
      OPEN(UNIT=4,FILE='TRO4.DAT',TYPE='OLD')
      READ(4,*) NUE,NODES,NR,ND
      READ(4,*) SE,KF,KR
      READ(4,*) D1,D2,D12
      READ(4,*) WL,WD
      READ(4,*) (SD1(L),SL1(L),L=6,NUE)
      READ(4,*) RR
C     KF=RR*KR
      OPEN(UNIT=2,FILE='FORCE.DAT',TYPE='OLD')
      READ(2,*) NIC
      DO 606 I=1,5
      SL1(I)=WL
      SD1(I)=WD
 606  CONTINUE
      CALL EKEM(SD1,SL1,SE,KF,KR,D1,D2,D12,NUE,NODES,NR,ND,GK,GM,GC)
      DO 25 I=1,25
      DO 25 J=1,25
      GKK(I,J)=GK(I+1,J+1)
      GMM(I,J)=GM(I+1,J+1)
      GCC(I,J)=GC(I+1,J+1)
  25  CONTINUE
      DO 2002 I=1,N1
      FR(I,1)=0.0
 2002 CONTINUE
      FR(NIC,1)=1.0
C********************************
      OPEN(UNIT=5,FILE='RED.DAT',TYPE='NEW')
      WRITE(5,3012)
 3012 FORMAT(12X,'FRONT & REAR BEARING STIFFNESS VALUES ',/)
      WRITE(5,*) KF,KR
      WRITE(5,3013)  NR
 3013 FORMAT(12X,'LOCATION OF REAR BEARING AT ',I4,' ELEMENT ',/)
      WRITE(5,3009) ND
 3009 FORMAT(12X,'LOCATION OF THE EXTERNAL DAMPER AT ',I4,'ELEMENT',/)
      WRITE(5,3080)  WD
```

```
3080    FORMAT(12X,' DIAMETER OF THE WORK PIECE ',F10.5,/)
        WRITE(5,3001)
3001    FORMAT(12X,'STIFFNESS MATRIX',/)
        WRITE(5,*) (GKK(I,I),I=1,25)
        WRITE(5,3002)
3002    FORMAT(12X,'MASS MATRIX ',/)
        WRITE(5,*) (GMM(I,I),I=1,25)
        CALL APX11(N1,GKK,GMM,LD,LN)
        WRITE(5,91) N5
 91     FORMAT(12X,'MASTERS = ',I4,//)
        WRITE(5,92)
 92     FORMAT(12X,'MASTER DEGREES OF FREEDOM',/)
        DO 94 IO=NSD+1,N1
        WRITE(5,*) LD(IO)
 94     CONTINUE
        WRITE(5,96) NSD
 96     FORMAT(12X,'SLAVES ',I4,/)
        WRITE(5,55)
55      FORMAT(12X,'*** DIAGNOL RATIOS *****',/)
        WRITE(5,*) (LD(I),I=1,N1)
        WRITE(5,*) (LN(I),I=1,N1)
        CALL APX222(N1,GMM,LN,NSD,GMN)
        WRITE(5,3004)
3004    FORMAT(12X,'ARRANGED MASS MATRIX ',/)
        WRITE(5,*) (GMN(I,I),I=1,N1)
        CALL APX222(N1,GKK,LN,NSD,GKN)
        WRITE(5,3005)
3005    FORMAT(12X,'ARRANGED STIFFNESS MATRIX',/)
        WRITE(5,*) (GKN(I,I),I=1,N1)
        CALL APX222(N1,GCC,LN,NSD,GCN)
        WRITE(5,3006)
3006    FORMAT(12X,'ARRANGED DAMPING MATRIX ',/)
        WRITE(5,*) (GCN(I,I),I=1,N1)
        CALL APX33(N1,NSD,N5,GKN,TR,TT,F1,F2,F3,F4,F5,F6)
        CALL APX44(N1,N5,NSD,TT,GMN,TR,GMR,F6)
        CALL APX44(N1,N5,NSD,TT,GKN,TR,GKR,F6)
        CALL APX44(N1,N5,NSD,TT,GCN,TR,GCR,F6)
        CALL APX55(N1,N5,NSD,TT,FR,FRD)
        WRITE(5,3007)
3007    FORMAT(12X,'REDUCED MASS MATRIX',/)
        WRITE(5,*) ((GMR(I,J),J=1,N5),I=1,N5)
        WRITE(5,3008)
3008    FORMAT(12X,'REDUCED STIFFNESS MATRIX',/)
        WRITE(5,*) ((GKR(I,J),J=1,N5),I=1,N5)
        WRITE(5,3090)
3090    FORMAT(12X,'REDUCED DAMPING MATRIX ',/)
        WRITE(5,*) ((GCR(I,J),J=1,N5),I=1,N5)
        CALL XXX(N5,N55,GKR,GMR,GCR,FRD,W,Z,FF1,FF2,FF3,
     1  FF4,FF5,SA,SB,ERD)
        CALL TMAX(N5,N55,SA,SB,ERD,Z,SS1,SS2,EE1,V1,V2,V3,V4,YA4)
        TST=0.4/1000.0
        DO 3026 I=1,1000
```

```
      T1=FLOAT(I)*TST
      CALL RES(N5,N55,W,T1,SS1,EE1,Z,YA1,YA2,YA3,YOO)
      RIP1(I)=YOO(1)
      RIP2(I)=YOO(2)
      RIP3(I)=YOO(3)
      RIP4(I)=YOO(4)
      RIP5(I)=YOO(5)
      TIM(I)=T1
3026  CONTINUE
      WRITE(15,1501) NIC
1501  FORMAT(12X,'UNIT IMPULSE APPLIED AT ',I4,
     1' TH DEGREE OF FREEDOM',/)
      WRITE(15,1502) WD
1502  FORMAT(12X,'WORKPIECE DIAMETER =  ',F7.4,/)
      WRITE(15,1503) NR
1503  FORMAT(12X,'REAR BEARING LOCATION AT    ',I5,' ELEMENT',/)
      WRITE(15,1505) ND
1505  FORMAT(12X,'EXTERNAL DAMPER AT  ',I4,  'D.O.F',/)
      CALL MAXM(RIP1,TIM,RMAX,TCOR)
      WRITE(15,3028)
3028  FORMAT(12X,'MAX RESPONSES AT 2,4,6,8,10, DEGREES OF FREEDOM ',/)
      WRITE(15,*) RMAX,TCOR
      CALL MAXM(RIP2,TIM,RMAX,TCOR)
      WRITE(15,*) RMAX,TCOR
      CALL MAXM(RIP3,TIM,RMAX,TCOR)
      WRITE(15,*) RMAX,TCOR
      CALL MAXM(RIP4,TIM,RMAX,TCOR)
      WRITE(15,*) RMAX,TCOR
      CALL MAXM(RIP5,TIM,RMAX,TCOR)
      WRITE(15,*) RMAX,TCOR
C     DO 3040 I=1,1000
C     WRITE(60,*) RIP1(I),TIM(I)
C 3040 CONTINUE
      STOP
      END
C******************************
      SUBROUTINE XXX(NX,NY,GKK,GMM,GCC,FRD,W,Z,GMI,GMK,GMC,
     1 UN,DC,AC,BC,ERD)
      DIMENSION GKK(NX,NX),GMM(NX,NX),GCC(NX,NX),FRD(NY,1),ERD(NY,1)
      DIMENSION GMI(NX,NX),GMK(NX,NX),GMC(NX,NX),UN(NX,NX)
      INTEGER N,IA,IZ,M,IB,IER,IJOB,IDGT
      REAL WKAREA(700),WK(5100),WA(50)
      COMPLEX W(NY),Z(NY,NY),ZN,DC(NY,NY)
      REAL  AC(NY,NY),BC(NY,NY)
      DO 25 I=1,NX
      DO 25 J=1,NX
      AC(I,J)=0.0
      AC(I,J+NX)=GMM(I,J)
      AC(I+NX,J)=GMM(I,J)
      AC(I+NX,J+NX)=GCC(I,J)
25    CONTINUE
      DO 30 I=1,NX
```

```
      DO 30 J=1,NX
      BC(I,J)=-1.0*GMM(I,J)
      BC(I,J+NX)=0.0
      BC(I+NX,J)=0.0
      BC(I+NX,J+NX)=GKK(I,J)
30    CONTINUE
      DO 36 I=1,NX
      ERD(I,1)=0.0
      ERD(I+NX,1)=FRD(I,1)
36    CONTINUE
      IA=NX
      N=NX
      IDGT=4
      CALL LINV2F(GMM,N,IA,GMI,IDGT,WKAREA,IER)
      CALL MATMP3(NX,NX,NX,GMI,GKK,GMK)
      CALL MATMP3(NX,NX,NX,GMI,GCC,GMC)
      DO 45 I=1,NX
      DO 45 J=1,NX
      GMK(I,J)=-1.0*GMK(I,J)
      GMC(I,J)=-1.0*GMC(I,J)
45    CONTINUE
      DO 54 I=1,NX
      DO 54 J=1,NX
      UN(I,J)=0.0
54    CONTINUE
      DO 55 I=1,NX
      UN(I,I)=UN(I,I)+1.0
55    CONTINUE
      DO 60 I=1,NX
      DO 60 J=1,NX
      DC(I,J)=CMPLX(GMC(I,J),0.0)
      DC(I,J+NX)=CMPLX(GMK(I,J),0.0)
      DC(I+NX,J)=CMPLX(UN(I,J),0.0)
      DC(I+NX,J+NX)=CMPLX(0.0,0.0)
60    CONTINUE
      N=NY
      IA=NY
      IZ=NY
      IJOB=2
      CALL EIGCC(DC,N,IA,IJOB,W,Z,IZ,WK,IER)
      RETURN
      END
C******************************
      SUBROUTINE TMAX(NX,NY,SA,SB,ERD,Z,SS1,SS2,E2,S1,S2,ZT,V4,E1)
      REAL SA(NY,NY),SB(NY,NY),ERD(NY,1)
      COMPLEX S1(NY,NY),S2(NY,NY),ZT(NY,NY),Z(NY,NY),E1(NY,1)
      COMPLEX SS1(NY,NY),SS2(NY,NY),E2(NY,1)
      DO 10 I=1,NY
      DO 10 J=1,NY
      S1(I,J)=CMPLX(SA(I,J),0.0)
      S2(I,J)=CMPLX(SB(I,J),0.0)
10    CONTINUE.
```

```
      DO 20 I=1,NY
      E1(I,1)=CMPLX(ERD(I,1),0.0)
 20   CONTINUE
      DO 30 I=1,NY
      DO 30 J=1,NY
      ZT(I,J)=Z(J,I)
 30   CONTINUE
      CALL SPAX1(NY,ZT,S1,Z,SS1,V4)
      CALL SPAX1(NY,ZT,S2,Z,SS2,V4)
      CALL SPAX2(NY,ZT,E1,E2)
      RETURN
      END
C***********************
      SUBROUTINE SPAX1(K1,A,B,C,D,AB)
      COMPLEX A(K1,K1),B(K1,K1),C(K1,K1),AB(K1,K1),D(K1,K1)
      DO 10 I=1,K1
      DO 10 J=1,K1
      AB(I,J)=CMPLX(0.0,0.0)
      DO 10 K=1,K1
      AB(I,J)=AB(I,J)+A(I,K)*B(K,J)
 10   CONTINUE
      DO 20 I=1,K1
      DO 20 J=1,K1
      D(I,J)=CMPLX(0.0,0.0)
      DO 20 K=1,K1
      D(I,J)=D(I,J)+AB(I,K)*C(K,J)
 20   CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE SPAX2(K1,A,B,C)
      COMPLEX A(K1,K1),B(K1,1),C(K1,1)
      DO 10 I=1,K1
      C(I,1)=CMPLX(0.0,0.0)
      DO 10 J=1,K1
      C(I,1)=C(I,1)+A(I,J)*B(J,1)
 10   CONTINUE
      RETURN
      END
C***********************
      SUBROUTINE RES(NX,NY,W,T1,SS1,E1,Z,V1,ZPR,YT,YO)
      COMPLEX W(NY),Z(NY,NY),SS1(NY,NY),E1(NY,1)
      REAL YO(NX)
      COMPLEX V1(NY,1),ZPR(NY,1),YT(NY,1)
      CALL RESP(NY,W,T1,V1)
      DO 10 I=1,NY
      ZPR(I,1)=(V1(I,1)*E1(I,1))/SS1(I,I)
 10   CONTINUE
      CALL SPAX2(NY,Z,ZPR,YT)
      DO 20 I=1,NX
      YO(I)=REAL(YT(I+NX,1))
 20   CONTINUE
```

```
      RETURN
      END
C*****************
      SUBROUTINE MAXW(A,B,AMAX,BCOR)
      REAL A(1000),B(1000)
      AMAX=10.0E-24
      DO 90 I=1,1000
      A(I)=ABS(A(I))
 90   CONTINUE
      DO 50 K=1,1000
      IF(A(K).GT.AMAX) GO TO 60
      GO TO 50
 60   AMAX=A(K)
      BCOR=B(K)
 50   CONTINUE
      RETURN
      END
C******************************
C  SUBROUTINE FOR GLOBAL MASS & STIFFNESS MATRICES
      SUBROUTINE EKEM(SD1,SL1,SE,KF,KR,D1,D2,
     1 D12,NUE,NODES,NR,ND,GK,GM,GC)
      DIMENSION GM(26,26),GK(26,26),GC(26,26)
      DIMENSION SD1(12),SL1(12),EK(4,4),EM(4,4),EC(4,4)
      DIMENSION A(4,4),AK(4,4),AM(4,4),AC(4,4)
      DIMENSION SK(26,26),SM(26,26),SC(26,26)
      REAL KF,KR
      DO 20 I=1,26
      DO 20 J=1,26
      GM(I,J)=0.0
      GC(I,J)=0.0
 20   GK(I,J)=0.0
      KM1=1
      KM2=2
      DO 105 K=1,NUE
      SL=SL1(K)
      SD=SD1(K)
      JU1=KM1
      JU2=KM2
      CALL MATFOR(SD,SL,SE,K,EK,EM,EC)
      IF(K.EQ.7) GO TO 145
      IF(K.EQ.NR) GO TO 146
      GO TO 148
 145  EK(1,1)=EK(1,1)+0.5*KF
      EK(3,3)=EK(3,3)+0.5*KF
      EC(1,1)=EC(1,1)+0.5*D1
      EC(3,3)=EC(3,3)+0.5*D1
      GO TO 148
 146  EK(1,1)=EK(1,1)+0.5*KR
      EK(3,3)=EK(3,3)+0.5*KR
      EC(1,1)=EC(1,1)+0.5*D2
      EC(3,3)=EC(3,3)+0.5*D2
      GO TO 148
```

```
148    IF(K.EQ.ND) GO TO 149
       GO TO 158
149    EC(1,1)=EC(1,1)+0.5*D12
       EC(3,3)=EC(3,3)+0.5*D12
158    CALL TRNSF(A)
       CALL TRNSF1(EK,A,AK)
       CALL TRNSF1(EM,A,AM)
       CALL TRNSF1(EC,A,AC)
       DO 14 IP=1,26
       DO 14 KP=1,26
       SK(IP,KP)=0.0
       SH(IP,KP)=0.0
       SC(IP,KP)=0.0
 14    CONTINUE
       CALL ASSEM(AK,JU1,JU2,NODES,SK)
       CALL ASSEM(AM,JU1,JU2,NODES,SH)
       CALL ASSEM(AC,JU1,JU2,NODES,SC)
       DO 150 I1=1,26
       DO 150 J1=1,26
       GK(I1,J1)=GK(I1,J1)+SK(I1,J1)
       GM(I1,J1)=GM(I1,J1)+SH(I1,J1)
       GC(I1,J1)=GC(I1,J1)+SC(I1,J1)
150    CONTINUE
       KM1=KM1+2
       KM2=KM2+2
105    CONTINUE
       RETURN
       END
C****************************
C   SUBROUTINE FOR ELEMENTAL STIFFNESS & MASS MATRIX*
C****************************
       SUBROUTINE MATFOR(SD,SL,SE,K,EK,EM,EC)
       DIMENSION EC(4,4),EK(4,4),EM(4,4)
       RHO=0.2832958/386.0
       DO 4 NI=1,4
       DO 4 JI=1,4
       EC(NI,JI)=0.0
       EK(NI,JI)=0.0
  4    EM(NI,JI)=0.0
       EM(1,1)=156.0
       EM(1,2)=22.0*SL
       EM(1,3)=54.0
       EM(1,4)=-13.0*SL
       EM(2,1)=22.0*SL
       EM(2,2)=4.0*SL**2
       EM(2,3)=13.0*SL
       EM(2,4)=-3.0*SL**2
       EM(3,1)=54.0
       EM(3,2)=13.*SL
       EM(3,3)=156.0
       EM(3,4)=-22.0*SL
       EM(4,1)=-13.0*SL
```

```
      EM(4,2)=-3.0*SL**2
      EM(4,3)=-22.0*SL
      EM(4,4)=4.0*SL**2
      EK(1,1)=12.0
      EK(1,2)=6.0*SL
      EK(1,3)=-12.0
      EK(1,4)=6.0*SL
      EK(2,1)=6.0*SL
      EK(2,2)=4.0*SL**2
      EK(2,3)=-6.0*SL
      EK(2,4)=2.0*SL**2
      EK(3,1)=-12.0
      EK(3,2)=-6.0*SL
      EK(3,3)=12.0
      EK(3,4)=-6.0*SL
      EK(4,1)=6.0*SL
      EK(4,2)=2.0*SL**2
      EK(4,3)=-6.0*SL
      EK(4,4)=4.0*SL**2
      IF(K.LE.5) BD=0.0
      IF(K.GT.5) BD=2.5
      SI=(22.0/(7.0*64.0))*(SD**4-BD**4)
      SM=(22.0/28.0)*(SD**2-BD**2)*RHO
      FM=SM*SL/420.0
      FK=SE*SI/(SL**3)
      DO 5 I=1,4
      DO 5 J=1,4
      EM(I,J)=FM*EM(I,J)
      EK(I,J)=FK*EK(I,J)
    5 CONTINUE
      RETURN
      END
C********************
      SUBROUTINE TRANSP(A,MX,MY,AT)
      DIMENSION A(4,4),AT(4,4)
      DO 1 I=1,MX
      DO 1 J=1,MY
    1 AT(J,I)=A(I,J)
      RETURN
      END
C******************
      SUBROUTINE MATMP1(A1,B1,C1,L1,M1,N1)
      DIMENSION A1(4,4),B1(4,4),C1(4,4)
      DO 2 I=1,L1
      DO 2 J=1,N1
      C1(I,J)=0.0
      DO 2 K=1,M1
    2 C1(I,J)=C1(I,J)+A1(I,K)*B1(K,J)
      RETURN
      END
C******************
C SUBROUTINE FOR ASSEMBLING
```

```
C*******************
      SUBROUTINE ASSEM(AZ,JU1,JU2,NODES,SUM)
      DIMENSION AZ(4,4),SUM(26,26)
      DO 8 I=1,26
      DO 8 J=1,26
      SUM(I,J)=0.0
 8    CONTINUE
      JV1=JU1+NODES
      JV2=JU2+NODES
      DO 10 KI=1,4
      IF(KI.EQ.1) KO=JU1
      IF(KI.EQ.2) KO=JU2
      IF(KI.EQ.3) KO=JV1
      IF(KI.EQ.4) KO=JV2
      SUM(KO,JU1)=SUM(KO,JU1)+AZ(KI,1)
      SUM(KO,JU2)=SUM(KO,JU2)+AZ(KI,2)
      SUM(KO,JV1)=SUM(KO,JV1)+AZ(KI,3)
      SUM(KO,JV2)=SUM(KO,JV2)+AZ(KI,4)
10    CONTINUE
      RETURN
      END
C**********************
C  SUBROUTINE FOR TRANSFORMATION MATRIX
C**********************
      SUBROUTINE TRNSF(A)
      DIMENSION A(4,4)
      DO 12 I=1,4
      DO 12 J=1,4
      A(I,J)=0.0
 12   CONTINUE
      A(1,1)=A(1,1)+1.0
      A(2,2)=A(2,2)+1.0
      A(3,3)=A(3,3)+1.0
      A(4,4)=A(4,4)+1.0
      RETURN
      END
C*************************
      SUBROUTINE TRNSF1(RR,A,RK)
      DIMENSION A(4,4),RR(4,4),AT(4,4),RTK(4,4),RK(4,4)
      DO 10 IF=1,4
      DO 10 JF=1,4
      RTK(IF,JF)=0.0
      RK(IF,JF)=0.0
 10   CONTINUE
      CALL TRANSP(A,4,4,AT)
      CALL MATMP1(AT,RR,RTK,4,4,4)
      CALL MATMP1(RTK,A,RK,4,4,4)
      RETURN
      END
C**********************
      SUBROUTINE MATMP3(I1,I2,I3,AQ,BQ,CQ)
      DIMENSION AQ(I1,I2),BQ(I2,I3),CQ(I1,I3)
```

```
      DO 9 I=1,I1
      DO 9 J=1,I2
      CQ(I,J)=0.0
      DO 9 K=1,I3
9     CQ(I,J)=CQ(I,J)+AQ(I,K)*BQ(K,J)
      RETURN
      END
C**********************
      SUBROUTINE APX11(N1,ASM,ASK,LD,LN)
      DIMENSION ASM(N1,N1),ASK(N1,N1),R(25),RN(25)
      INTEGER LD(N1),LN(N1)
      DO 5 I=1,N1
      R(I)=ASM(I,I)/ASK(I,I)
5     CONTINUE
      DO 8 I=1,N1
      RN(I)=R(I)
      LD(I)=I
8     CONTINUE
      AA=0.0
      J1=1
      DO 15 I=1,N1
      J1=J1+1
      DO 15 J=J1,N1
      AA=RN(I)
      AB=RN(J)
      IM=LD(I)
      IN=LD(J)
      IF(AA.LT.AB) GO TO 20
      GO TO 15
20    RN(I)=AB
      RN(J)=AA
      LD(I)=IN
      LD(J)=IM
15    CONTINUE
      DO 60 I=1,N1
      LN(I)=LD(I)
60    CONTINUE
      J1=1
      DO 70 I=1,N1
      IC=LN(I)
      J1=J1+1
      DO 70 J=J1,N1
      ID=LN(J)
      IF(IC.LT.ID) GO TO 80
      GO TO 70
80    LN(J)=LN(J)-1
70    CONTINUE
      RETURN
      END
C**********************
      SUBROUTINE APX222(N1,A1,LN,NSD,A9)
      DIMENSION A1(N1,N1),A5(25,25),A9(N1,N1)
```

```
      INTEGER LN(N1)
      DO 5 K=1,NSD
      L1=LN(K)
      CALL APX22(N1,A1,L1,A5)
      DO 10 I=1,N1
      DO 10 J=1,N1
      A1(I,J)=A5(I,J)
10    CONTINUE
5     CONTINUE
      DO 77 I=1,N1
      DO 77 J=1,N1
      A9(I,J)=A1(I,J)
77    CONTINUE
      RETURN
      END
C**************************
      SUBROUTINE APX22(N1,A1,LD,A3)
      DIMENSION A1(N1,N1),A3(N1,N1),A2(25,25)
      DO 5 I=1,N1
      DO 5 J=1,N1
      A2(I,J)=A1(I,J)
5     CONTINUE
      N2=N1-1
      DO 10 JJ=LD,N2
      JK=JJ+1
      DO 10 I=1,N1
      A2(I,JJ)=A1(I,JK)
10    CONTINUE
      DO 20 I=1,N1
      A2(I,N1)=A1(I,LD)
20    CONTINUE
      DO 6 I=1,N1
      DO 6 J=1,N1
      A3(I,J)=A2(I,J)
6     CONTINUE
      DO 30 JJ=LD,N2
      JK=JJ+1
      DO 30 I=1,N1
      A3(JJ,I)=A2(JK,I)
30    CONTINUE
      DO 40 I=1,N1
      A3(N1,I)=A2(LD,I)
40    CONTINUE
      RETURN
      END
C**********************************
      SUBROUTINE APX33(N1,NSD,N5,AK,T,TT,UN,AKM,AKS,AKT,AX,ASI)
      DIMENSION AK(N1,N1),T(N1,N5),TT(N5,N1),UN(N5,N5)
      REAL NKAREA(700),AKM(N5,N5),AKS(NSD,NSD),AKT(NSD,N5)
      REAL AX(NSD,N5),ASI(NSD,NSD)
      ND=N1-NSD
      DO 2 I=1,ND
```

```
      DO 2 J=1,MD
      AKM(I,J)=AK(I,J)
2     CONTINUE
      DO 4 I=1,NSD
      DO 4 J=1,NSD
      AKS(I,J)=AK(I+MD,J+MD)
4     CONTINUE
      DO 6 I=1,NSD
      DO 6 J=1,MD
      AKT(I,J)=AK(I+MD,J)
6     CONTINUE
      N=NSD
      IA=N
      IDGT=4
      CALL LINV2F(AKS,N,IA,ASI,IDGT,WKAREA,IER)
      DO 12 I=1,NSD
      DO 12 J=1,MD
      AX(I,J)=0.0
      DO 12 K=1,NSD
      AX(I,J)=AX(I,J)+ASI(I,K)*AKT(K,J)
12    CONTINUE
      DO 8 I=1,N1
      DO 8 J=1,N1
      UN(I,J)=0.0
8     CONTINUE
      DO 10 I=1,MD
      UN(I,I)=UN(I,I)+1.0
10    CONTINUE
      DO 22 I=1,MD
      DO 22 J=1,MD
      T(I,J)=UN(I,J)
22    CONTINUE
      DO 14 I=1,NSD
      DO 14 J=1,MD
      T(I+MD,J)=-1.0*AX(I,J)
14    CONTINUE
      DO 16 I=1,N1
      DO 16 J=1,MD
      TT(J,I)=T(I,J)
16    CONTINUE
      RETURN
      END
C*********************************
      SUBROUTINE APX44(N1,N5,NSD,A1,A2,A3,A4,A12)
      DIMENSION A1(N5,N1),A2(N1,N1),A3(N1,N5),A12(N5,N1)
      REAL A4(N5,N5)
      MD=N1-NSD
      DO 10 I=1,MD
      DO 10 J=1,N1
      A12(I,J)=0.0
      DO 10 K=1,N1
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
```

```
 10     CONTINUE
        DO 20 I=1,MD
        DO 20 J=1,MD
        A4(I,J)=0.0
        DO 20 K=1,N1
        A4(I,J)=A4(I,J)+A12(I,K)*A3(K,J)
 20     CONTINUE
        RETURN
        END
C************************
        SUBROUTINE APX55(N1,N5,NSD,A1,A2,A3)
        REAL A1(N5,N1),A2(N1,1),A3(N5,1)
        DO 10 I=1,N5
        A3(I,1)=0.0
        DO 10 J=1,N1
        A3(I,1)=A3(I,1)+A1(I,J)*A2(J,1)
 10     CONTINUE
        RETURN
        END
C*************************
        SUBROUTINE CMAT1(A1,A2,A3,K1,A4,A12)
        COMPLEX A1(K1,K1),A3(K1,K1),A4(K1),A12(K1,K1)
        REAL A2(K1,K1)
        DO 1 I=1,K1
        DO 1 J=1,K1
        A12(I,J)=CMPLX(0.0,0.0)
        DO 1 K=1,K1
        A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
  1     CONTINUE
        DO 3 I=1,K1
        A4(I)=CMPLX(0.0,0.0)
        DO 3 K=1,K1
        A4(I)=A4(I)+A12(I,K)*A3(K,I)
  3     CONTINUE
        RETURN
        END
C************************
        SUBROUTINE XAT(A1,B1,C1,K1,A4,AB)
        COMPLEX A1(K1,K1),B1(K1),C1(K1,K1),AB(K1,K1),A4(K1,K1)
        DO 2 I=1,K1
        DO 2 J=1,K1
        AB(I,J)=A1(I,J)*B1(J)
  2     CONTINUE
        DO 4 I=1,K1
        DO 4 J=1,K1
        A4(I,J)=CMPLX(0.0,0.0)
        DO 4 K=1,K1
        A4(I,J)=A4(I,J)+AB(I,K)*C1(K,J)
  4     CONTINUE
        RETURN
        END
C***********************
```

```
      SUBROUTINE XAT1(A1,A2,A3,K1,A4,A12)
      COMPLEX A1(K1,K1),A3(K1,K1),A12(K1,K1),A4(K1,K1)
      REAL A2(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      A12(I,J)=CMPLX(0.0,0.0)
      DO 2 K=1,K1
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
2     CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      A4(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      A4(I,J)=A4(I,J)+A12(I,K)*A3(K,J)
4     CONTINUE
      RETURN
      END
C*****************************
      SUBROUTINE SAT3(A1,B,C1,MR,NC,AC,AB)
      COMPLEX A1(MR,NC),C1(NC,MR),B(NC),AB(MR,NC),AC(MR,MR)
      DO 2 I=1,MR
      DO 2 J=1,NC
      AB(I,J)=A1(I,J)*B(J)
2     CONTINUE
      DO 4 I=1,MR
      DO 4 J=1,MR
      AC(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,NC
      AC(I,J)=AC(I,J)+AB(I,K)*C1(K,J)
4     CONTINUE
      RETURN
      END
C***************************
      SUBROUTINE SAT4(A1,B1,C1,K1,D1,AB)
      COMPLEX A1(K1,K1),AB(K1,K1),C1(K1,K1),D1(K1,K1)
      REAL B1(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      AB(I,J)=CMPLX(0.0,0.0)
      DO 2 K=1,K1
      AB(I,J)=AB(I,J)+A1(I,K)*B1(K,J)
2     CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      D1(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      D1(I,J)=D1(I,J)+AB(I,K)*C1(K,J)
4     CONTINUE
      RETURN
      END
C************************
      SUBROUTINE RESP(N55,W,T1,V)
```

```
      COMPLEX W(N55),V(N55,1)
      DO 5 I=1,N55
      AR=REAL(W(I))*T1
      AI=AIMAG(W(I))*T1
      AR1=EXP(AR)
      XC=AR1*COS(AI)
      YC=AR1*SIN(AI)
      V(I,1)=CMPLX(XC,YC)
 5    CONTINUE
      RETURN
      END
C***********************
      SUBROUTINE MAR2(AA,BB,CC,K1)
      COMPLEX AA(K1,1),BB(K1,1),CC(K1,1)
      DO 1 I=1,K1
      CC(I,1)=AA(I,1)*BB(I,1)
 1    CONTINUE
      RETURN
      END
C*******************
      SUBROUTINE MAR1(AC,BC,N,CC)
      COMPLEX AC(N,N),BC(N,1),CC(N,1)
      DO 40 I=1,N
      CC(I,1)=CMPLX(0.0,0.0)
      DO 40 J=1,N
      CC(I,1)=CC(I,1)+AC(I,J)*BC(J,1)
 40   CONTINUE
      RETURN
      END
```

```
C*************************
C
C LISTING OF THE PROGRAM ++ TVIBE ++
C PROGRAM FOR FINDING THE RESPONSE OF A LATHE SPINDLE
C - WORKPIECE SYSTE DUE TO AN EXPONENETIALLY  DECAYING PULSE
C IMSL SUBROUTINES : LINV2F & EIGCC
C
C*************************
      DIMENSION SL1(12),SD1(12),Y1(12),GMM(25,25),GKK(25,25)
      DIMENSION GCC(25,25),GM(26,26),GK(26,26),GC(26,26)
      REAL KF,KR,GMR(25,25),GKR(25,25),GCR(25,25)
      INTEGER LN(25),LD(25)
      REAL F1(50,50),F2(50,50),F3(50,50),F4(50,50)
      REAL F5(50,50),F6(50,50),FF1(50,50)
      REAL FF2(50,50),FF3(50,50),FF4(50,50)
      REAL GMN(50,50),GKN(50,50),GCN(50,50)
      REAL TR(50,50),TT(50,50)
      COMPLEX W(50),Z(50,50),FF5(50,50)
      REAL SA(50,50),SB(50,50),ERD(50,1)
      COMPLEX ZT(50,50),EE1(50,1),SS1(50,50),SS2(50,50)
      COMPLEX V1(50,50),V2(50,50),YA6(50,1)
      COMPLEX V3(50,50),V4(50,50),V5(50,50)
      COMPLEX ZS(50,50)
      COMPLEX ZO(25),YA1(50,1),YA2(50,1),YA3(50,1)
      COMPLEX YA4(50,1),YA5(50,1)
      REAL FR([illegible],1),FRD(25,1),YOO(25)
C*********************
      OPEN(UNIT=6,FILE='BAA.DAT',TYPE='OLD')
      READ(6,*) NSD,N1
      N5=N1-NSD
      N55=N5*2
      N2=N1*2
      CALL MATLOK(N1,N2,N5,N55,NSD,SL1,SD1,
     $ GMM,GKK,GCC,GMR,GKR,GCR,LN,
     $ LD,F1,F2,F3,F4,F5,F6,FF1,FF2,FF3,FF4,GMN,GKN,GCN,
     $ TR,TT,W,Z,FF5,SA,SB,ZT,
     $ V1,V2,V3,V4,V5,
     $ FR,FRD,YOO,ZI,ZS,
     $ ZO,YA1,YA2,YA3,YA4,YA5,YA6,ERD,EE1,SS1,SS2)
      STOP
      END
      SUBROUTINE MATLOK(N1,N2,N5,N55,NSD,SL1,SD1
     $ GMM,GKK,GCC,GMR,GKR,GCR,LN,
     $ LD,F1,F2,F3,F4,F5,F6,FF1,FF2,FF3,FF4,GMN,GKN,GCN,
     $ TR,TT,W,Z,FF5,SA,SB,ZT,
     $ V1,V2,V3,V4,V5,
     $ FR,FRD,YOO,ZI,ZS,
     $ ZO,YA1,YA2,YA3,YA4,YA5,YA6,ERD,EE1,SS1,SS2)
      REAL GMM(25,25),GKK(25,25),GCC(25,25),GMR(N5,N5)
      REAL GKR(N5,N5),GCR(N5,N5)
      INTEGER LN(N1),LD(N1)
      REAL GK(26,26),GM(26,26),GC(26,26)
```

```
      REAL F1(N5,N5),F2(N5,N5),F3(NSD,NSD),F4(NSD,N5)
      REAL F5(NSD,N5),F6(NSD,NSD)
      REAL FF1(N5,N5),FF2(N5,N5),FF3(N5,N5),FF4(N5,N5),KF,KR
      REAL GMN(N1,N1),GKN(N1,N1),GCN(N1,N1),TR(N1,N5),TT(N5,N1)
      COMPLEX  W(N55),Z(N55,N55),ZT(N55,N55),SS1(N55,N55),SS2(N55,N55)
      REAL SA(N55,N55),SB(N55,N55)
      COMPLEX  FF5(N55,N55),YA6(N55,1)
      COMPLEX V1(N55,N55),V2(N55,N55),V3(N55,N55),V4(N5,N55),V5(N5,N5)
      REAL FR(N1,1),FRD(N5,1),YOO(N5),ERD(N55,1)
      COMPLEX ZI(N55,N55),ZS(N55,N55),ZO(N5),EE1(50,1)
      COMPLEX YA1(N55,1),YA2(N55,1),YA3(N55,1),YA4(N55,1),YA5(N55,1)
      REAL RIP1(1000),RIP2(1000),RIP3(1000),RIP4(1000),RIP5(1000)
      REAL TIM(1000),SL1(12),SD1(12)
C*******************************
      OPEN(UNIT=2,FILE='FORCE.DAT',TYPE='OLD')
      READ(2,*) NIC,DEC
C*******************************
      OPEN(UNIT=4,FILE='TRO5.DAT',TYPE='OLD')
      READ(4,*) NUE,NODES,NR,ND
      READ(4,*)  SE,KF,KR
      READ(4,*) D1,D2,D12
      READ(4,*) WL,WD
      READ(4,*) (SD1(L),SL1(L),L=6,NUE)
      READ(4,*) RR
      DO 606 I=1,5
      SD1(I)=WD
      SL1(I)=WL
606   CONTINUE
      CALL EKEM(SD1,SL1,SE,KF,KR,D1,D2,D12,NUE,NODES,NR,ND,GK,GM,GC)
      DO 25 I=1,25
      DO 25 J=1,25
      GKK(I,J)=GK(I+1,J+1)
      GMM(I,J)=GM(I+1,J+1)
      GCC(I,J)=GC(I+1,J+1)
 25   CONTINUE
      DO 2002 I=1,N1
      FR(I,1)=0.0
2002  CONTINUE
      FR(NIC,1)=1.0
C*******************************
      OPEN(UNIT=5,FILE='RED.DAT',TYPE='NEW')
      OPEN(UNIT=35,FILE='BR.DAT',TYPE='NEW')
      WRITE(35,350) NIC
350   FORMAT(12X,'PULSE APPLIED AT ', I4 ,'ELEMENT',/)
      WRITE(35,3012)
3012  FORMAT(12X,'FRONT & REAR BEARING STIFFNESS VALUES ',/)
      WRITE(35,*) KF,KR
      WRITE(35,3082)  WD
3082  FORMAT(12X,'DIAMETER OF THE WORKPIECE ',F8.5,/)
      WRITE(35,3013)  NR
3013  FORMAT(12X,'LOCATION OF REAR BEARING AT ',I4,' ELEMENT ',/)
      WRITE(35,3080)  ND
```

```
 3080 FORMAT(12X,'LOCATION OF THE EXTERNAL DAMPER AT',I5,'ELEMENT',/)
      WRITE(5,3001)
 3001 FORMAT(12X,'STIFFNESS MATRIX',/)
      WRITE(5,*) (GKK(I,I),I=1,25)
      WRITE(5,3002)
 3002 FORMAT(12X,'MASS MATRIX ',/)
      WRITE(5,*) (GMM(I,I),I=1,25)
      CALL APX11(N1,GKK,GMM,LD,LN)
      WRITE(5,91) N5
   91 FORMAT(12X,'MASTERS = ',I4,//)
      WRITE(5,92)
   92 FORMAT(12X,'MASTER DEGREES OF FREEDOM',/)
      DO 94 IO=NSD+1,N1
      WRITE(5,*) LD(IO)
   94 CONTINUE
      WRITE(5,96) NSD
   96 FORMAT(12X,'SLAVES ',I4,/)
      WRITE(5,55)
   55 FORMAT(12X,'*** DIAGNOL RATIOS',/)
      WRITE(5,*) (LD(I),I=1,N1)
      WRITE(5,*) (LN(I),I=1,N1)
      CALL APX222(N1,GMM,LN,NSD,GMN)
      WRITE(5,3004)
 3004 FORMAT(12X,'ARRANGED MASS MATRIX ',/)
      WRITE(5,*) (GMN(I,I),I=1,N1)
      CALL APX222(N1,GKK,LN,NSD,GKN)
      WRITE(5,3005)
 3005 FORMAT(12X,'ARRANGED STIFFNESS MATRIX',/)
      WRITE(5,*) (GKN(I,I),I=1,N1)
      CALL APX222(N1,GCC,LN,NSD,GCN)
      WRITE(5,3006)
 3006 FORMAT(12X,'ARRANGED DAMPING MATRIX ',/)
      WRITE(5,*) (GCN(I,I),I=1,N1)
      CALL APX33(N1,NSD,N5,GKN,TR,TT,F1,F2,F3,F4,F5,F6)
      CALL APX44(N1,N5,NSD,TT,GMN,TR,GMR,F6)
      CALL APX44(N1,N5,NSD,TT,GKN,TR,GKR,F6)
      CALL APX44(N1,N5,NSD,TT,GCN,TR,GCR,F6)
      CALL APX55(N1,N5,NSD,TT,FR,FRD)
      WRITE(5,3007)
 3007 FORMAT(12X,'REDUCED MASS MATRIX',/)
      WRITE(5,*) ((GMR(I,J),J=1,N5),I=1,N5)
      WRITE(5,3008)
 3008 FORMAT(12X,'REDUCED STIFFNESS MATRIX',/)
      WRITE(5,*) ((GKR(I,J),J=1,N5),I=1,N5)
      WRITE(5,3009)
 3009 FORMAT(12X,'REDUCED DAMPING MATRIX ',/)
      WRITE(5,*) ((GCR(I,J),J=1,N5),I=1,N5)
      CALL XXX(N5,N55,GKR,GMR,GCR,FRD,W,Z,FF1,FF2,FF3,
     1 FF4,FF5,SA,SB,ERD)
      CALL TWAX(N5,N55,SA,SB,ERD,Z,SS1,SS2,EE1,V1,V2,V3,V4,YA4)
      TST=0.4/1000.0
      T1=0.0
```

```
      DO 3026 I=1,1000
      T1=FLOAT(I)*TST
      CALL RES(N5,N55,W,T1,SS1,EE1,Z,DEC,YA1,YA6,YA2,YA3,YOO)
      RIP1(I)=YOO(1)
      RIP2(I)=YOO(2)
      RIP3(I)=YOO(3)
      RIP4(I)=YOO(4)
      RIP5(I)=YOO(5)
      TIM(I)=T1
 3026 CONTINUE
      CALL MAXW(RIP1,TIM,RMAX,TCOR)
      WRITE(35,3028)
 3028 FORMAT(12X,'MAX RESPONSES AT 2,4,6,8,10, DEGREES OF FREEDOM ',/)
      WRITE(35,*) RMAX,TCOR
      CALL MAXW(RIP2,TIM,RMAX,TCOR)
      WRITE(35,*) RMAX,TCOR
      CALL MAXW(RIP3,TIM,RMAX,TCOR)
      WRITE(35,*) RMAX,TCOR
      CALL MAXW(RIP4,TIM,RMAX,TCOR)
      WRITE(35,*) RMAX,TCOR
      CALL MAXW(RIP5,TIM,RMAX,TCOR)
      WRITE(35,*) RMAX,TCOR
C     DO 208 I=1,1000
C     WRITE(70,*) RIP1(I),TIM(I)
C 208 CONTINUE
      STOP
      END
C*********************************
      SUBROUTINE XXX(NX,NY,GKK,GMM,GCC,FRD,W,Z,GMI,GMK,GMC,
     1 UN,DC,AC,BC,ERD)
      DIMENSION GKK(NX,NX),GMM(NX,NX),GCC(NX,NX),FRD(NY,1),ERD(NY,1)
      DIMENSION GMI(NX,NX),GMK(NX,NX),GMC(NX,NX),UN(NX,NX)
      INTEGER N,IA,IZ,M,IB,IER,IJOB,IDGT
      REAL WKAREA(700),WK(5100),WA(50)
      COMPLEX W(NY),Z(NY,NY),ZN,DC(NY,NY)
      REAL  AC(NY,NY),BC(NY,NY)
      DO 25 I=1,NX
      DO 25 J=1,NX
      AC(I,J)=0.0
      AC(I,J+NX)=GMM(I,J)
      AC(I+NX,J)=GMM(I,J)
      AC(I+NX,J+NX)=GCC(I,J)
   25 CONTINUE
      DO 30 I=1,NX
      DO 30 J=1,NX
      BC(I,J)=-1.0*GMM(I,J)
      BC(I,J+NX)=0.0
      BC(I+NX,J)=0.0
      BC(I+NX,J+NX)=GKK(I,J)
   30 CONTINUE
      DO 36 I=1,NX
      ERD(I,1)=0.0
```

```
      ERD(I+NX,1)=FRD(I,1)
 36   CONTINUE
      IA=NX
      N=NX
      IDGT=4
      CALL LINV2F(GMM,N,IA,GMI,IDGT,WKAREA,IER)
      CALL MATMP3(NX,NX,NX,GMI,GKK,GMK)
      CALL MATMP3(NX,NX,NX,GMI,GCC,GMC)
      DO 45 I=1,NX
      DO 45 J=1,NX
      GMK(I,J)=-1.0*GMK(I,J)
      GMC(I,J)=-1.0*GMC(I,J)
 45   CONTINUE
      DO 54 I=1,NX
      DO 54 J=1,NX
      UN(I,J)=0.0
 54   CONTINUE
      DO 55 I=1,NX
      UN(I,I)=UN(I,I)+1.0
 55   CONTINUE
      DO 60 I=1,NX
      DO 60 J=1,NX
      DC(I,J)=CMPLX(GMC(I,J),0.0)
      DC(I,J+NX)=CMPLX(GMK(I,J),0.0)
      DC(I+NX,J)=CMPLX(UN(I,J),0.0)
      DC(I+NX,J+NX)=CMPLX(0.0,0.0)
 60   CONTINUE
      N=NY
      IA=NY
      IZ=NY
      IJOB=2
      CALL EIGCC(DC,N,IA,IJOB,W,Z,IZ,WK,IER)
      RETURN
      END
C********************************
      SUBROUTINE TWAX(NX,NY,SA,SB,ERD,Z,SS1,SS2,E2,S1,S2,ZT,V4,E1)
      REAL SA(NY,NY),SB(NY,NY),ERD(NY,1)
      COMPLEX S1(NY,NY),S2(NY,NY),ZT(NY,NY),Z(NY,NY),E1(NY,1)
      COMPLEX SS1(NY,NY),SS2(NY,NY),E2(NY,1)
      DO 10 I=1,NY
      DO 10 J=1,NY
      S1(I,J)=CMPLX(SA(I,J),0.0)
      S2(I,J)=CMPLX(SB(I,J),0.0)
 10   CONTINUE
      DO 20 I=1,NY
      E1(I,1)=CMPLX(ERD(I,1),0.0)
 20   CONTINUE
      DO 30 I=1,NY
      DO 30 J=1,NY
      ZT(I,J)=Z(J,I)
 30   CONTINUE
      CALL SPAX1(NY,ZT,S1,Z,SS1,V4)
```

```
      CALL SPAX1(NY,ZT,S2,Z,SS2,V4)
      CALL SPAX2(NY,ZT,E1,E2)
      RETURN
      END
C*************************
      SUBROUTINE SPAX1(K1,A,B,C,D,AB)
      COMPLEX A(K1,K1),B(K1,K1),C(K1,K1),AB(K1,K1),D(K1,K1)
      DO 10 I=1,K1
      DO 10 J=1,K1
      AB(I,J)=CMPLX(0.0,0.0)
      DO 10 K=1,K1
      AB(I,J)=AB(I,J)+A(I,K)*B(K,J)
 10   CONTINUE
      DO 20 I=1,K1
      DO 20 J=1,K1
      D(I,J)=CMPLX(0.0,0.0)
      DO 20 K=1,K1
      D(I,J)=D(I,J)+AB(I,K)*C(K,J)
 20   CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE SPAX2(K1,A,B,C)
      COMPLEX A(K1,K1),B(K1,1),C(K1,1)
      DO 10 I=1,K1
      C(I,1)=CMPLX(0.0,0.0)
      DO 10 J=1,K1
      C(I,1)=C(I,1)+A(I,J)*B(J,1)
 10   CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE RES(NX,NY,W,T1,SS1,E1,Z,DEC,V1,V11,ZPR,YT,YO)
      COMPLEX W(NY),Z(NY,NY),SS1(NY,NY),E1(NY,1),V11(NY,1)
      REAL YO(NX)
      COMPLEX V1(NY,1),ZPR(NY,1),YT(NY,1),A1,A2
      CALL RESP1(NY,W,T1,V1)
      CALL RESP2(NY,W,DEC,T1,V11)
      DO 10 I=1,NY
      A1=(CMPLX(DEC,0.0)+W(I))
      A2=A1*SS1(I,I)
      ZPR(I,1)=(E1(I,1)*V1(I,1)*(CMPLX(1.0,0.0)-V11(I,1)))/A2
 10   CONTINUE
      CALL SPAX2(NY,Z,ZPR,YT)
      DO 20 I=1,NX
      YO(I)=REAL(YT(I+NX,1))
 20   CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE MAXM(A,B,AMAX,BCOR)
      REAL A(1000),B(1000)
```

```
      AMAX=10.0E-25
      DO 90 I=1,1000
      A(I)=ABS(A(I))
   90 CONTINUE
      DO 50 K=1,1000
      IF(A(K).GT.AMAX) GO TO 60
      GO TO 50
   60 AMAX=A(K)
      BCOR=B(K)
   50 CONTINUE
      RETURN
      END
C********************************
C SUBROUTINE FOR GLOBAL MASS & STIFFNESS MATRICES
C********************************
      SUBROUTINE EKEM(SD1,SL1,SE,KF,KR,D1,D2,
     1 D12,NUE,NODES,NR,ND,GK,GM,GC)
      DIMENSION GM(26,26),GK(26,26),GC(26,26)
      DIMENSION SD1(12),SL1(12),EK(4,4),EM(4,4),EC(4,4)
      DIMENSION A(4,4),AK(4,4),AM(4,4),AC(4,4)
      DIMENSION SK(26,26),SM(26,26),SC(26,26)
      REAL KF,KR
      DO 20 I=1,26
      DO 20 J=1,26
      GM(I,J)=0.0
      GC(I,J)=0.0
   20 GK(I,J)=0.0
      KM1=1
      KM2=2
      DO 105 K=1,NUE
      SL=SL1(K)
      SD=SD1(K)
      JU1=KM1
      JU2=KM2
      CALL MATFOR(SD,SL,SE,K,EK,EM,EC)
      IF(K.EQ.7) GO TO 145
      IF(K.EQ.NR) GO TO 146
      GO TO 148
  145 EK(1,1)=EK(1,1)+0.5*KF
      EK(3,3)=EK(3,3)+0.5*KF
      EC(1,1)=EC(1,1)+0.5*D1
      EC(3,3)=EC(3,3)+0.5*D1
      GO TO 148
  146 EK(1,1)=EK(1,1)+0.5*KR
      EK(3,3)=EK(3,3)+0.5*KR
      EC(1,1)=EC(1,1)+0.5*D2
      EC(3,3)=EC(3,3)+0.5*D2
      GO TO 148
  148 IF(K.EQ.ND) GO TO 149
      GO TO 158
  149 EC(1,1)=EC(1,1)+0.5*D12
      EC(3,3)=EC(3,3)+0.5*D12
```

```
158     CALL TRNSF(A)
        CALL TRNSF1(EK,A,AK)
        CALL TRNSF1(EM,A,AM)
        CALL TRNSF1(EC,A,AC)
        DO 14 IP=1,26
        DO 14 KP=1,26
        SK(IP,KP)=0.0
        SH(IP,KP)=0.0
        SC(IP,KP)=0.0
 14     CONTINUE
        CALL ASSEM(AK,JU1,JU2,NODES,SK)
        CALL ASSEM(AM,JU1,JU2,NODES,SH)
        CALL ASSEM(AC,JU1,JU2,NODES,SC)
        DO 150 I1=1,26
        DO 150 J1=1,26
        GK(I1,J1)=GK(I1,J1)+SK(I1,J1)
        GM(I1,J1)=GM(I1,J1)+SH(I1,J1)
        GC(I1,J1)=GC(I1,J1)+SC(I1,J1)
150     CONTINUE
        KM1=KM1+2
        KM2=KM2+2
105     CONTINUE
        RETURN
        END
C*****************************
C  SUBROUTINE FOR ELEMENTAL STIFFNESS & MASS MATRIX
C*****************************
        SUBROUTINE MATFOR(SD,SL,SE,K,EK,EM,EC)
        DIMENSION EC(4,4),EK(4,4),EM(4,4)
        RHO=0.2832958/386.0
        DO 4 NI=1,4
        DO 4 JI=1,4
        EC(NI,JI)=0.0
        EK(NI,JI)=0.0
 4      EM(NI,JI)=0.0
        EM(1,1)=156.0
        EM(1,2)=22.0*SL
        EM(1,3)=54.0
        EM(1,4)=-13.0*SL
        EM(2,1)=22.0*SL
        EM(2,2)=4.0*SL**2
        EM(2,3)=13.0*SL
        EM(2,4)=-3.0*SL**2
        EM(3,1)=54.0
        EM(3,2)=13.*SL
        EM(3,3)=156.0
        EM(3,4)=-22.0*SL
        EM(4,1)=-13.0*SL
        EM(4,2)=-3.0*SL**2
        EM(4,3)=-22.0*SL
        EM(4,4)=4.0*SL**2
        EK(1,1)=12.0
```

```
      EK(1,2)=6.0*SL
      EK(1,3)=-12.0
      EK(1,4)=6.0*SL
      EK(2,1)=6.0*SL
      EK(2,2)=4.0*SL**2
      EK(2,3)=-6.0*SL
      EK(2,4)=2.0*SL**2
      EK(3,1)=-12.0
      EK(3,2)=-6.0*SL
      EK(3,3)=12.0
      EK(3,4)=-6.0*SL
      EK(4,1)=6.0*SL
      EK(4,2)=2.0*SL**2
      EK(4,3)=-6.0*SL
      EK(4,4)=4.0*SL**2
      IF(K.LE.5) BD=0.0
      IF(K.GT.5) BD=2.5
      SI=(22.0/(7.0*64.0))*(SD**4-BD**4)
      SM=(22.0/28.0)*(SD**2-BD**2)*RHO
      FM=SM*SL/420.0
      FK=SE*SI/(SL**3)
      DO 5 I=1,4
      DO 5 J=1,4
      EM(I,J)=FM*EM(I,J)
      EK(I,J)=FK*EK(I,J)
    5 CONTINUE
      RETURN
      END
C********************
      SUBROUTINE TRANSP(A,MX,MY,AT)
      DIMENSION A(4,4),AT(4,4)
      DO 1 I=1,MX
      DO 1 J=1,MY
    1 AT(J,I)=A(I,J)
      RETURN
      END
C********************
      SUBROUTINE MATMP1(A1,B1,C1,L1,M1,N1)
      DIMENSION A1(4,4),B1(4,4),C1(4,4)
      DO 2 I=1,L1
      DO 2 J=1,N1
      C1(I,J)=0.0
      DO 2 K=1,M1
    2 C1(I,J)=C1(I,J)+A1(I,K)*B1(K,J)
      RETURN
      END
C**********************
      SUBROUTINE ASSEM(AZ,JU1,JU2,NODES,SUM)
      DIMENSION AZ(4,4),SUM(26,26)
      DO 8 I=1,26
      DO 8 J=1,26
      SUM(I,J)=0.0
```

```
  8     CONTINUE
        JV1=JU1+NODES
        JV2=JU2+NODES
        DO 10 KI=1,4
        IF(KI.EQ.1) KO=JU1
        IF(KI.EQ.2) KO=JU2
        IF(KI.EQ.3) KO=JV1
        IF(KI.EQ.4) KO=JV2
        SUM(KO,JU1)=SUM(KO,JU1)+AZ(KI,1)
        SUM(KO,JU2)=SUM(KO,JU2)+AZ(KI,2)
        SUM(KO,JV1)=SUM(KO,JV1)+AZ(KI,3)
        SUM(KO,JV2)=SUM(KO,JV2)+AZ(KI,4)
 10     CONTINUE
        RETURN
        END
C****************************
        SUBROUTINE TRNSF(A)
        DIMENSION A(4,4)
        DO 12 I=1,4
        DO 12 J=1,4
        A(I,J)=0.0
 12     CONTINUE
        A(1,1)=A(1,1)+1.0
        A(2,2)=A(2,2)+1.0
        A(3,3)=A(3,3)+1.0
        A(4,4)=A(4,4)+1.0
        RETURN
        END
C***************************
        SUBROUTINE TRNSF1(RR,A,RK)
        DIMENSION A(4,4),RR(4,4),AT(4,4),RTK(4,4),RK(4,4)
        DO 10 IF=1,4
        DO 10 JF=1,4
        RTK(IF,JF)=0.0
        RK(IF,JF)=0.0
 10     CONTINUE
        CALL TRANSP(A,4,4,AT)
        CALL MATMP1(AT,RR,RTK,4,4,4)
        CALL MATMP1(RTK,A,RK,4,4,4)
        RETURN
        END
C**************************
        SUBROUTINE MATMP3(I1,I2,I3,AQ,BQ,CQ)
        DIMENSION AQ(I1,I2),BQ(I2,I3),CQ(I1,I3)
        DO 9 I=1,I1
        DO 9 J=1,I2
        CQ(I,J)=0.0
        DO 9 K=1,I3
  9     CQ(I,J)=CQ(I,J)+AQ(I,K)*BQ(K,J)
        RETURN
        END
C*************************
```

```
      SUBROUTINE APX11(N1,ASM,ASK,LD,LN)
      DIMENSION ASM(N1,N1),ASK(N1,N1),R(25),RN(25)
      INTEGER LD(N1),LN(N1)
      DO 5 I=1,N1
      R(I)=ASM(I,I)/ASK(I,I)
5     CONTINUE
      DO 8 I=1,N1
      RN(I)=R(I)
      LD(I)=I
8     CONTINUE
      AA=0.0
      J1=1
      DO 15 I=1,N1
      J1=J1+1
      DO 15 J=J1,N1
      AA=RN(I)
      AB=RN(J)
      IM=LD(I)
      IN=LD(J)
      IF(AA.LT.AB) GO TO 20
      GO TO 15
20    RN(I)=AB
      RN(J)=AA
      LD(I)=IN
      LD(J)=IM
15    CONTINUE
      DO 60 I=1,N1
      LN(I)=LD(I)
60    CONTINUE
      J1=1
      DO 70 I=1,N1
      IC=LN(I)
      J1=J1+1
      DO 70 J=J1,N1
      ID=LN(J)
      IF(IC.LT.ID) GO TO 80
      GO TO 70
80    LN(J)=LN(J)-1
70    CONTINUE
      RETURN
      END
C************************
      SUBROUTINE APX222(N1,A1,LN,NSD,A9)
      DIMENSION A1(N1,N1),A5(25,25),A9(N1,N1)
      INTEGER LN(N1)
      DO 5 K=1,NSD
      L1=LN(K)
      CALL APX22(N1,A1,L1,A5)
      DO 10 I=1,N1
      DO 10 J=1,N1
      A1(I,J)=A5(I,J)
10    CONTINUE
```

```
5     CONTINUE
      DO 77 I=1,N1
      DO 77 J=1,N1
      A9(I,J)=A1(I,J)
77    CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE APX22(N1,A1,LD,A3)
      DIMENSION A1(N1,N1),A3(N1,N1),A2(25,25)
      DO 5 I=1,N1
      DO 5 J=1,N1
      A2(I,J)=A1(I,J)
5     CONTINUE
      N2=N1-1
      DO 10 JJ=LD,N2
      JK=JJ+1
      DO 10 I=1,N1
      A2(I,JJ)=A1(I,JK)
10    CONTINUE
      DO 20 I=1,N1
      A2(I,N1)=A1(I,LD)
20    CONTINUE
      DO 6 I=1,N1
      DO 6 J=1,N1
      A3(I,J)=A2(I,J)
6     CONTINUE
      DO 30 JJ=LD,N2
      JK=JJ+1
      DO 30 I=1,N1
      A3(JJ,I)=A2(JK,I)
30    CONTINUE
      DO 40 I=1,N1
      A3(N1,I)=A2(LD,I)
40    CONTINUE
      RETURN
      END
C*******************************
      SUBROUTINE APX33(N1,NSD,N5,AK,T,TT,UN,AKM,AKS,AKT,AX,ASI)
      DIMENSION AK(N1,N1),T(N1,N5),TT(N5,N1),UN(N5,N5)
      REAL WKAREA(700),AKM(N5,N5),AKS(NSD,NSD),AKT(NSD,N5)
      REAL AX(NSD,N5),ASI(NSD,NSD)
      MD=N1-NSD
      DO 2 I=1,MD
      DO 2 J=1,MD
      AKM(I,J)=AK(I,J)
2     CONTINUE
      DO 4 I=1,NSD
      DO 4 J=1,NSD
      AKS(I,J)=AK(I+MD,J+MD)
4     CONTINUE
      DO 6 I=1,NSD
```

```
      DO 6 J=1,MD
      AKT(I,J)=AK(I+MD,J)
6     CONTINUE
      N=NSD
      IA=N
      IDGT=4
      CALL LINV2F(AKS,N,IA,ASI,IDGT,WKAREA,IER)
      DO 12 I=1,NSD
      DO 12 J=1,MD
      AX(I,J)=0.0
      DO 12 K=1,NSD
      AX(I,J)=AX(I,J)+ASI(I,K)*AKT(K,J)
12    CONTINUE
      DO 8 I=1,N1
      DO 8 J=1,N1
      UN(I,J)=0.0
8     CONTINUE
      DO 10 I=1,MD
      UN(I,I)=UN(I,I)+1.0
10    CONTINUE
      DO 22 I=1,MD
      DO 22 J=1,MD
      T(I,J)=UN(I,J)
22    CONTINUE
      DO 14 I=1,NSD
      DO 14 J=1,MD
      T(I+MD,J)=-1.0*AX(I,J)
14    CONTINUE
      DO 16 I=1,N1
      DO 16 J=1,MD
      TT(J,I)=T(I,J)
16    CONTINUE
      RETURN
      END
C**********************************
      SUBROUTINE APX44(N1,N5,NSD,A1,A2,A3,A4,A12)
      DIMENSION A1(N5,N1),A2(N1,N1),A3(N1,N5),A12(N5,N1)
      REAL A4(N5,N5)
      MD=N1-NSD
      DO 10 I=1,MD
      DO 10 J=1,N1
      A12(I,J)=0.0
      DO 10 K=1,N1
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
10    CONTINUE
      DO 20 I=1,MD
      DO 20 J=1,MD
      A4(I,J)=0.0
      DO 20 K=1,N1
      A4(I,J)=A4(I,J)+A12(I,K)*A3(K,J)
20    CONTINUE
      RETURN
```

```
      END
C***************************
      SUBROUTINE APX55(N1,N5,NSD,A1,A2,A3)
      REAL A1(N5,N1),A2(N1,1),A3(N5,1)
      DO 10 I=1,N5
      A3(I,1)=0.0
      DO 10 J=1,N1
      A3(I,1)=A3(I,1)+A1(I,J)*A2(J,1)
 10   CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE CMAT1(A1,A2,A3,K1,A4,A12)
      COMPLEX A1(K1,K1),A3(K1,K1),A4(K1),A12(K1,K1)
      REAL A2(K1,K1)
      DO 1 I=1,K1
      DO 1 J=1,K1
      A12(I,J)=CMPLX(0.0,0.0)
      DO 1 K=1,K1
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
  1   CONTINUE
      DO 3 I=1,K1
      A4(I)=CMPLX(0.0,0.0)
      DO 3 K=1,K1
      A4(I)=A4(I)+A12(I,K)*A3(K,I)
  3   CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE XAT(A1,B1,C1,K1,A4,AB)
      COMPLEX A1(K1,K1),B1(K1),C1(K1,K1),AB(K1,K1),A4(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      AB(I,J)=A1(I,J)*B1(J)
  2   CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      A4(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      A4(I,J)=A4(I,J)+AB(I,K)*C1(K,J)
  4   CONTINUE
      RETURN
      END
C***************************
      SUBROUTINE XAT1(A1,A2,A3,K1,A4,A12)
      COMPLEX A1(K1,K1),A3(K1,K1),A12(K1,K1),A4(K1,K1)
      REAL A2(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      A12(I,J)=CMPLX(0.0,0.0)
      DO 2 K=1,K1
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
```

```
2     CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      A4(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      A4(I,J)=A4(I,J)+A12(I,K)*A3(K,J)
4     CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE SAT3(A1,B,C1,MR,NC,AC,AB)
      COMPLEX A1(MR,NC),C1(NC,MR),B(NC),AB(MR,NC),AC(MR,MR)
      DO 2 I=1,MR
      DO 2 J=1,NC
      AB(I,J)=A1(I,J)*B(J)
2     CONTINUE
      DO 4 I=1,MR
      DO 4 J=1,MR
      AC(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,NC
      AC(I,J)=AC(I,J)+AB(I,K)*C1(K,J)
4     CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE SAT4(A1,B1,C1,K1,D1,AB)
      COMPLEX A1(K1,K1),AB(K1,K1),C1(K1,K1),D1(K1,K1)
      REAL B1(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      AB(I,J)=CMPLX(0.0,0.0)
      DO 2 K=1,K1
      AB(I,J)=AB(I,J)+A1(I,K)*B1(K,J)
2     CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      D1(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      D1(I,J)=D1(I,J)+AB(I,K)*C1(K,J)
4     CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE RESP1(N55,W,T1,V)
      COMPLEX W(N55),V(N55,1)
      DO 5 I=1,N55
      AR=REAL(W(I))*T1
      AI=AIMAG(W(I))*T1
      AR1=EXP(AR)
      XC=AR1*COS(AI)
      YC=AR1*SIN(AI)
      V(I,1)=CMPLX(XC,YC)
```

```
 5      CONTINUE
        RETURN
        END
C***********************
        SUBROUTINE RESP2(N55,W,DEC,T1,V2)
        COMPLEX W(N55),V2(N55,1),W1
        UN=CMPLX(-1.0,0.0)
        DO 10 I=1,NY
        W1=UN*(W(I)+CMPLX(DEC,0.0))
        AR=REAL(W1)*T1
        AI=AIMAG(W1)*T1
        XC=EXP(AR)
        X1=XC*COS(AI)
        X2=XC*SIN(AI)
        V2(I,1)=CMPLX(X1,X2)
 10     CONTINUE
        RETURN
        END
C*************************
        SUBROUTINE MAR2(AA,BB,CC,K1)
        COMPLEX AA(K1,1),BB(K1,1),CC(K1,1)
        DO 1 I=1,K1
        CC(I,1)=AA(I,1)*BB(I,1)
 1      CONTINUE
        RETURN
        END
C*************************
        SUBROUTINE MAR1(AC,BC,N,CC)
        COMPLEX AC(N,N),BC(N,1),CC(N,1)
        DO 40 I=1,N
        CC(I,1)=CMPLX(0.0,0.0)
        DO 40 J=1,N
        CC(I,1)=CC(I,1)+AC(I,J)*BC(J,1)
 40     CONTINUE
        RETURN
        END
```

```
C******************************
C
C LISTING OF THE PROGRAM ++ OPTM ++
C PROGRAM FOR OPTIMAL DESIGN OF A LATHE SPINLDE
C - WORKPIECE SYSTEM
C  DESIGN VARIABLES ARE : BEARING SPACING ,LOCATION OF
C AN EXTERNAL DAMPER AND STIFFNESS OF THE FRONT BEARING
C IMSL SUBROUTINES : EINV2F & EIGCC
C
C******************************
      LOGICAL GOOD, OPTVRT, RES
      DIMENSION XSTRT(3),B(3),A(3),X(3,4),XC(4),XS(4),XO(4)
      DIMENSION  OLD(3,4),FUN(4),GOOD(4),OPTVRT(4)
      DATA ISEED/1717/
      OPEN(UNIT=10,FILE='VER3.DAT',TYPE='NEW')
      OPEN(UNIT=6,FILE='VER33.DAT',TYPE='NEW')
      OPEN(UNIT=5,FILE='VERO.DAT',TYPE='OLD')
      REWIND 10
      READ(5,*) N,NP,K,M,MM,MAXM,IPRINT,IDATA,EPSI,DELTA
      READ(5,*) (A(I),I=1,N)
      READ(5,*) (B(I),I=1,N)
      READ(5,*) (XSTRT(I),I=1,N)
      READ(5,*) IX
      RES=.TRUE.
      IF (IX.EQ.0) RES=.FALSE.
      IF (.NOT. RES) GOTO 780
      READ(5,*) ((X(I,J),I=1,NP),J=1,K)
780   CALL COMPLX(NP,B,A,N,K,M,MM,MAXM,XSTRT,IPRINT,IDATA,
     $EPSI,DELTA,ISEED,X,U,FUN,XC,XO,XS,CEQS,OPTVRT,GOOD,OLD,
     $ RES)
      STOP
      END
C******************************
      SUBROUTINE COMPLX(NP,B,A,N,K,M,MM,MAXM,XSTRT,IPRINT,IDATA,
     $EPSI,DELTA,ISEED,X,U,FUN,XC,XO,XS,CEQS,OPTVRTX,GOOD,OLD,
     $RES)
      DIMENSION X(NP,K),XC(K),XS(K),XD(K),CEQS(1),A(N),B(MM),
     $OLD(NP,K),FUN(K),XSTRT(NP)
      LOGICAL BRIEF,IMPCSTR,OPTVRTX(K),GOOD(K),RES
C
C
C     PROGRAM TO OPTIMIZE USING COMPLEX METHOD.
C     THIS IMPLEMENTATION OF THE COMPLEX OPTIMIZATION IS A
C     MINIMIZATION PROBLEM
C
C
C     MULTI-PARAMETER OPTIMIZATION PROGRAM TO OPTIMIZE A MAXIMUM OF
C     FOUR PARAMETERS.
C     COMMON BLOCK REDEFINITION
C      /OLD/     AREA IN MEMORY WHERE THE  FUNCTION, AND
C     THEIR RESPECTIVE "OLD"(OR PREVIOUS)VALUES OF
C     THE VERTICES ARE RETAINED. IN THE EVENT OF A
```

```
C     TIME CONSUMING FUNCTION EVALUATION, THIS ENSURES
C     THAT THE SAME VERTICES ARE NOT REEVALUATED.
C     FUN(K) = VALUE RETURNED FROM FUNCTION EVALUATION
C     OLD(NP,K) = PREVIOUS  VERTEX VALUE
C
C      /LOWLIM/ AREA IN MEMORY WHERE THE LOWBOUNDS ARE RETAINED.
C      A(N)     = ARRAY OF LOWER EXPLICIT CONSTRAINTS.
C
C      /UPLIM/  AREA IN MEMORY WHERE THE UPPER BOUNDS ARE RETAINED.
C      B(N)     = ARRAY OF UPPER BOUNDS ON EXPLICIT CONSTRAINTS.
C      B(N+1)..B(MM) = ARRAY OF BOUNDS ON IMPLICIT CONSTRAINTS.
C
C      EXPLAINATION OF VARIABLES.
C
C
C  N  = NUMBER OF EXPLICIT CONSTRAINTS.
C  M  = NUMBER OF IMPLICIT CONSTRAINTS.
C  NP = NUMBER OF PARAMETERS.
C  K  = NUMBER OF VERTICES IN THE COMPLEX BOX
C  MM = NUMBER OF EXPLICIT PLUS IMPLICIT CONSTRAINTS(M+N)
C       EPSI      = REAL VARIABLE=>CONVERGENCE FACTOR
C       FCENT     = REAL VARIABLE=>FUNCTION VALUE ASSOCIATED WITH
C       CENTROID
C       FMAX      = REAL VARIABLE=> CONTAINS THE VALUE OF THE LARGEST
C       FUNCTION VALUE OF THE PRESENT ARRAY FUN
C       JMAX      = INTEGER VARIABLE=> POINTS TO LOCATION IN
C       ARRAY " X "THAT CONTAINS THE LARGEST ELEMENT
C       JMIN      = INTEGER VARIABLE=> POINTS TO SMALLEST ELEMENT
C       IN ARRAY " X ".
C       MM        = INTEGER VARIABLE=> SUM OF IMPLICIT AND EXPLICIT
C       CONSTRAINTS.
C       OPTVRTX(K)= LOGICAL ARRAY=> USED TO KEEP TRACK OF WHICH
C       VERTICES HAVE BEEN OPTIMIZED.
C       PMAX      = THE MAXIMUM FUNCTION VALUE AT THIS POINT.
C       EVALUATED AT THIS POINT IN THE PROGRAM
C       X(NP,K)   = REAL ARRAY=> LOCATIONS WHERE THE VALUES OF
C      THE VERTICES ARE RETAINED.
C       CEQS(M)= REAL ARRAY=> RESULTS OF FUNCTION EVALUATIONS.
C       WHICH MUST SATISFY THE CONSTRAINING EQUATIONS
C       XS(K)     = REAL ARRAY=> USED AS A TEMPORARY STORAGE
C       LOCATION FOR THE COLUMN IN THE "X" ARRAY
C       WHICH CONTAINS THE LARGEST ELEMENT.
C       XC(NP)    = REAL ARRAY=> USED TO STORE THE CENTROID.
C       FVAL      = REAL VARIABLE => RETAINS RESULT OF ONE FUNCTION
C       EVALUATION RETURNED BY FUNCTION SUBROUTINE;
C       DELTA     .= DISTANCE THAT A "BAD" VERTEX IS MOVED WITH IN
C      THE LEGAL BOUNDARIES AFTER AN EXPLICIT CONSTRAINT
C      HAS FAILED.
C      IMPCSTR  = LOGICAL VARIABLE INDICATING THAT THERE ARE
C      IMPLICIT CONSTRAINTS (IF TRUE).
C
C
```

```
C
C         RES = A LOGICAL VARIABLE WHICH INDICATES WHETHER THE
C         VALUES FOR ARRAY X HAVE BEEN PASSED.
C
C
C        INITIALIZATION OF THE ARRAYS
C
      NI = 5
      NO = 6
      DO 5 J = 1,K
      XS(J) = 0.0
      XC(J) = 0.0
      XO(J) = 0.0
      OPTVRTX(J) = .FALSE.
      GOOD(J) = .FALSE.
      DO 5 I = 1,NP
      OLD(I,J) = 0.0
      IF (.NOT. RES) X(I,J)=0.0
    5 CONTINUE

      DO 10 I = 1,NP
      X(I,1) = XSTRT(I)
   10 CONTINUE

      IF (IDATA .NE. 1)GOTO 20

      WRITE (NO,440)
      WRITE (10,440)
      WRITE (NO,450) N,M,NP
      WRITE (10,450) N,M,NP
      WRITE (NO,460)
      WRITE (10,460)
      WRITE (NO,490) (X(I,1),I=1,NP)
      WRITE (10,490) (X(I,1),I=1,NP)
      WRITE (NO,470)
      WRITE (10,470)
      WRITE (NO,480) (A(I),I=1,N)
      WRITE (10,480) (A(I),I=1,N)
      WRITE (NO,500)
      WRITE (10,500)
      WRITE (NO,510) (B(I),I=1,N)
      WRITE (10,510) (B(I),I=1,N)
      IF (M.EQ.0)GOTO 15
      J = N + 1
      WRITE(NO,515) (B(I),I=J,MM)
      WRITE(10,515) (B(I),I=J,MM)
  515 FORMAT(//25X,'UPPER BOUND ON IMPLICIT CONSTRAINTS',
     + 10(25X,G15.8/))
   15 WRITE (NO,520) EPSI
      WRITE (10,520) EPSI

C     INITIALIZATION OF CONSTANTS AND CREATION OF THE COMPLEX BOX
```

```
 20 IMPCSTR = .TRUE.
    IF (M .EQ. 0)IMPCSTR = .FALSE.
    BRIEF = .FALSE.
    IF (IPRINT .NE. 0)GOTO 30
    IPRINT = 00001717
    BRIEF = .TRUE.
 30 CONTINUE
    NUMFUN = 0
    ITER = 0
    STIME = SECNDS(T)
    IF (RES) GOTO 777
    DO 50 J=2,K
    DO 40 I=1,NP
    II = MOD(I,N)
    IF (II .EQ. 0)II = N
 40 X(I,J) = A(II)+RAN(ISEED)*(B(II)-A(II))
 50 CONTINUE
777 CONTINUE

    IF (BRIEF) GOTO 70
 60 WRITE (NO,540)
    WRITE (10,540)
    WRITE(10,*) K,NP
    WRITE(NO,*) K,NP
    WRITE (NO,560) ((X(I,J),I=1,NP),J=1,K)
    WRITE (10,560) ((X(I,J),I=1,NP),J=1,K)
 70 CONTINUE

100 CONTINUE

C     CHECK THAT THE STARTING VALUES AND THEIR RESPECTIVE
C     FUNCTION VALUES MEET BOTH THE EXPLICIT AND IMPLICIT CONSTRAINTS


101 CALL EXPLICT (X,NP,K,MM,A,B,N,DELTA,&102,&102)
102 IF (IMPCSTR) CALL IMPLICT (X,B,MM,NP,K,M,N,CEQS,XO,GOOD,
   1NO,&103,&101)
103 CONTINUE
    IF (BRIEF) GOTO 120
110 WRITE (NO,550)
    WRITE (10,550)
    WRITE(10,*) K,NP
    WRITE(NO,*) K,NP
    WRITE (NO,560) ((X(I,J),I=1,NP),J=1,K)
    WRITE (10,560) ((X(I,J),I=1,NP),J=1,K)

C     EVALUATION OF THE COMPLETED AND CHECKED COMPLEX BOX

120 FMAX = 10E25
130 DO 150 J=1,K
    JP = J
```

```
      CALL OBJFUN (X,FVAL,JT,NP,K,FUN,OLD,NUMFUN,XO)
  150 FUN(J) = FVAL
      ITER = ITER + 1
      BRIEF = .TRUE.
      IF ((MOD(ITER,IPRINT)) .EQ. 0)BRIEF = .FALSE.
      IF (ITER .GT. MAXM)GOTO 415

      IF (BRIEF) GOTO 170
      WRITE(NO,695) ITER
      WRITE(10,695) ITER
      WRITE (NO,580)
      WRITE (10,580)
      WRITE(10,*) K
      WRITE(NO,*)  K
      WRITE (NO,590) (FUN(J),J=1,K)
      WRITE (10,590) (FUN(J),J=1,K)

C     FINDING THE MAXIMUM FUNCTION VALUE

  170 FMAX = -10E25
      JMIN = 1
      DO 190 J=1,K
      IF ((FMAX).GT.(FUN(J)).OR.(FUN(J)).GE.FMAX) GOTO 180
      FMAX = FUN(J)
      JMAX = J
  180 CONTINUE
      IF ((FUN(JMIN)).LT.(FUN(J))) GOTO 190
      JMIN = J
  190 CONTINUE
  200 CONTINUE

C     SAVE THE ROW WHICH CONTAINS THE LARGEST FUNCTION VALUE.

      FMAX = FMAX
      DO 210 I=1,NP
      XS(I) = X(I,JMAX)
  210 CONTINUE

C     REJECT THIS MAXIMUM VERTEX AND FIND THE CENTROID.

      DO 230 I=1,NP
      SUM = 0.
      DO 220 J=1,K
      IF (J.EQ.JMAX) GOTO 220
      SUM = SUM+X(I,J)
  220 CONTINUE
      XC(I) = SUM/(K-1)
  230 CONTINUE

      IF (BRIEF) GOTO 250
  240 WRITE (NO,600) FUN(JMAX),JMAX
      WRITE (10,600) FUN(JMAX),JMAX
```

```
      WRITE (NO,610)
      WRITE (10,610)
      WRITE(NO,*) NP
      WRITE(10,*)  NP
      WRITE (NO,570) (XC(I),I=1,NP)
      WRITE (10,570) (XC(I),I=1,NP)
      WRITE (NO,540)
      WRITE (10,540)
      WRITE(10,*) K,NP
      WRITE(NO,*) K,NP
      WRITE (NO,560) ((X(I,J),I=1,NP),J=1,K)
      WRITE (10,560) ((X(I,J),I=1,NP),J=1,K)
C     CHECK IF THE CENTROID FITS THE EXPLICIT CONSTRAINTS

  250 CALL EXPLICT(XC,NP,1,MM,A,B,N,DELTA,&251,&270)
  251 IF (IMPCSTR) CALL IMPLICT(XC,B,MM,NP,1,M,N,CEQS,XO,GOOD,
     1NO,&300,&270)
      GOTO 300

C     AT THE POINT AN ELEMENT HAS BEEN FOUND WHICH DOES NOT
C     CONFORM TO THE CONSTRAINING EQUATIONS.

  270 DO 290 J=1,K
      IF (J.EQ.JMIN) GOTO 290
      DO 280 I=1,NP
  280 X(I,J) = X(I,JMIN)+RAN(ISEED)*(XC(I)-X(I,JMIN))
  290 CONTINUE
      PMAX = 10E25

C     NOW TO LOOP UP AND CHECK THE NEWLY GENERATED VERTICES.


      GOTO 100

  300 ALPHA = 1.3

C     EXECUTION RESUMES HERE IF ALL CONSTRAINTS ARE MET

      DO 310 I=1,NP
      X(I,JMAX) = 2.3*XC(I)-1.3*X(I,JMAX)
  310 CONTINUE
      IF (.NOT.BRIEF) WRITE (NO,620) NP
      IF(.NOT.BRIEF) WRITE(NO,720)(X(I,JMAX),I=1,NP)
      IF (.NOT.BRIEF) WRITE (10,620) NP
      IF(.NOT.BRIEF) WRITE(10,720) (X(I,JMAX),I=1,NP)

C     NOW EVALUATE THE FUNCTION AT THE NEW VERTEX AND CHECK THE
C     RESULTANT VALUE.
C      IF THE RESULTANT VALUE IS GREATER THAN THE OLD FUNCTION VALUE
C      THE VERTEX WHICH HAS THE NEXT HIGHEST FUNCTION VALUE IS
```

```
C      WORKED ON.

  314 CALL EXPLICT(X,NP,K,MM,A,B,N,DELTA,&315,&315)
  315 IF (IMPCSTR) CALL IMPLICT(X,B,MM,NP,K,M,N,CEQS,XO,GOOD,
     1NO,&316,&314)
  316 CONTINUE
      CALL OBJFUN (X,FVAL,JMAX,NP,K,FUN,OLD,NUMFUN,XO)
      FUNN = FVAL
      IF ((FUNN).LE.(FUN(JMAX))) GOTO 400

C     THE FOLLOWING CODE KEEPS TRACK OF WHICH VERTICES HAVE
C     ALREADY BEEN OPTIMIZED

      ITOGGLE = 0
      DO 320 J=1,K,1
      IF (OPTVRTX(J)) ITOGGLE = ITOGGLE+1
  320 CONTINUE
      IF (ITOGGLE.NE.K)GOTO 330
      WRITE(NO,680)
      WRITE(10,680)
  680 FORMAT(///10X,'*** ERROR IN COMPLEX OPTIMIZATION'
     1 /10X,'ALL VERTICES ARE UNSATISFIED')
      GOTO 415

C     MOVE VERTEX OF THE MAXIMUM FUNCTION VALUE TOWARDS THE CENTROID
C     UNTIL A "BETTER" FUNCTION VALUE IS FOUND. IF A BETTER
C     FUNCTION VALUE IS NOT FOUND, TRY THE NEXT LARGEST FUNCTION VALUE

  330 ALPHA = ALPHA/2.0
      DO 340 I=1,NP
  340 X(I,JMAX) = (1.0+ALPHA)*XC(I)-ALPHA*X(I,JMAX)
  344 CALL EXPLICT(X,NP,K,MM,A,B,N,DELTA,&345,&345)
  345 IF (IMPCSTR) CALL IMPLICT(X,B,MM,NP,K,M,N,CEQS,XO,GOOD,
     1NO,&346,&344)
  346 CONTINUE
      CALL OBJFUN (X,FVAL,JMAX,NP,K,FUN,OLD,NUMFUN,XO)
      FUNN = FVAL
      IF ((FUNN).LT.(FUN(JMAX))) GOTO 390
      IF (ALPHA.GT.0.00001) GOTO 330

C     RESTORE ORIGINAL STATUS OF THE VERTEX WHICH HELD THE
C     MAXIMUM FUNCTION VALUE.

48    OPTVRTX(JMAX) = .TRUE.
      DO 350 I=1,NP
  350 X(I,JMAX) = XS(I)
      FMAX = -10.0E+25

C     FIND THE NEXT LOWEST MAXIMUM FUNCTION

      DO 380 I = 1,K
      IF ((I .EQ. JMAX) .OR. (OPTVRTX(I)))GOTO 380
```

```
      IF ((FUN(I)) .LE. (FMAX))GOTO 380
      NEWMAX = I
      FMAX = FUN(NEWMAX)
  380 CONTINUE
      JMAX = NEWMAX
      GOTO 200
  390 CONTINUE
  400 FUN(JMAX) = FUNN

      IF (.NOT. BRIEF) WRITE (NO,630) NP
      IF(.NOT.BRIEF) WRITE(NO,730)(X(I,JMAX),I=1,NP)
      IF (.NOT. BRIEF) WRITE (10,630) NP
      IF(.NOT.BRIEF)  WRITE(NO,730)(X(I,JMAX),I=1,NP)
      ADD = 0.0

C     HERE THE SUM OF THE SQUARES OF THE DIFFERENCES BETWEEN THE
C     RETURNED ELEMENT AND THE CENTROID OF ACCEPTABLE VERTICES,
C     IS BEING CALCULATED

      DO 408 I = 1,NP
      SUM = 0.0
      DO 403 J = 1,K
      SUM = SUM+X(I,J)
  403 CONTINUE
      XC(I) = SUM/K
  408 CONTINUE

      CALL OBJFUN (X,FCENT,0,NP,K,FUN,OLD,NUMFUN,XC)
      DO 410 I=1,K
  410 ADD = ADD+(FCENT-FUN(I))**2.0
      AINDEX = ADD/K
      AINDEX = SQRT(AINDEX)

C     STOPPING CONDITION

      IF (AINDEX.LE.EPSI) GOTO 420
      FMAX = 10.E25
      GOTO 130

  415 WRITE(NO,690)ITER,JMIN,FUN(JMIN),K,(NP,(X(I,J),I=1,NP),J
     1    = 1,K)
      WRITE(10,690)ITER,JMIN,FUN(JMIN),K,(NP,(X(I,J),I=1,NP),J
     1    = 1,K)
  690 FORMAT(//5X,'PROGRAM FAILED TO FIND.OPTIMAL SOLUTION IN ',
     1  I6,' STEPS'/5X,'BEST FUNCTION VALUE WAS AT VERTEX ',I3,
     2  ' = ',G15.8,//20X,'FINAL COMPLEX '/5X,'=',G15.8)
      RETURN

  420 U = FUN(JMIN)
      ETIME = SECNDS(T) - STIME
      WRITE (NO,640) JMIN,FUN(JMIN)
      WRITE(NO,580)
```

```
      WRITE(10,580)
      WRITE(10,*) K
      WRITE(NO,*) K
      WRITE(NO,590) (FUN(J),J=1,K)
      WRITE(10,590) (FUN(J),J=1,K)
      WRITE(NO,640) JMIN,FUN(JMIN)
      WRITE(10,640) JMIN,FUN(JMIN)
      WRITE(10,650) NP
      WRITE(NO,650) NP
      WRITE (NO,750) (X(I,JMIN),I=1,NP)
      WRITE (10,750) (X(I,JMIN),I=1,NP)
      WRITE (NO,660)
      WRITE (10,660)
      WRITE(10,*) K,NP
      WRITE(NO,*) K,NP
      WRITE (NO,560) ((X(I,J),I=1,NP),J=1,K)
      WRITE (10,560) ((X(I,J),I=1,NP),J=1,K)
      WRITE (NO,670)NUMFUN,ETIME,ITER
      WRITE (10,670)NUMFUN,ETIME,ITER
      DO 700 I = 1,NP
  700 FUN(I) = X(I,JMIN)
      RETURN
  440 FORMAT (1H1////25X,'MULTIPLE PARAMETER OPTIMIZATION',////)
  450 FORMAT (///,25X,'*N=*',I2,25X,'*M=*',I2,25X,'*NP=*',I2)
  460 FORMAT (///,25X,'*INITIAL PARAMETERS*')
  470 FORMAT (//25X,'*LOWER BOUND*',//)
  480 FORMAT (25X,G15.8/)
  490 FORMAT (25X,G15.8/)
  500 FORMAT (///,25X,'*UPPER BOUND*',//)
  510 FORMAT (25X,G15.8,/)
  520 FORMAT (//,25X,'*EPSI=*',5X,G25.10,//)
  540 FORMAT (///,20X,'*COMPLETED COMPLEX*',///)
  550 FORMAT (///,20X,' COMPLETED AND CHECKED COMPLEX')
  560 FORMAT (10X,G15.8)
  570 FORMAT (10X,G15.8)
  580 FORMAT (/,25X,'VALUES OF FUNCTION*',/)
  590 FORMAT (10X,G15.8/)
  600 FORMAT (/,25X,'*MAXIMUM FUNCTION VALUE=*',G15.8/,25X,'*VERTEX NUMBER
     +AT MAXIMUM=*',I2)
  610 FORMAT (/,'*  CENTROID*')
  620 FORMAT (/,25X,'*NEW VERTEX=*',I2,/)
  720 FORMAT( 15X,G15.8,/)
  630 FORMAT (/,25X,'*ACCEPTABLE VERTEX*',I2,/)
  730 FORMAT( 15X,G15.8,/)
  640 FORMAT (////,'*  VECTOR *',I4,'*  OPTIMAL  VALUE= *',G15.8,///)
  650 FORMAT (///,25X,'*OPTIMAL VECTOR*',I2)
  750 FORMAT(//,10X,G15.8,//)
  660 FORMAT (///,'*      FINAL COMPLEX *',/////)
  670 FORMAT (///5X,'NUMBER OF FUNCTION EVALUATIONS = ',I8,
     1        /5X,'TOTAL COMPUTATION TIME',F10.4,' SECONDS',
     2 /5X,'TOTAL NUMBER OF ITERATIONS = ',I8)
  695 FORMAT(10X,'ITERATION NUMBER ',I10)
```

```
      END
C********************************
      SUBROUTINE EXPLICT (X,NP,K,MM,A,B,N,DELTA,*,*)

C     THIS ROUTINE CHECKS TO INSURE THAT THE EXPLICIT CONSTRAINTS
C     ARE SATISFIED.
C     IF THEY ARE SATISFIED THE PROGRAM RETURNS THROUGH "SUCCESS"
C     IF NOT, CONTROL IS RETURNED THROUGH FAILED.

      DIMENSION X(NP,K),A(N),B(MM)

      DO 40 J = 1,K
      DO 20 I = 1,N
      IF (X(I,J) .GE. A(I))GOTO 20
      IF (K .LE. 1)RETURN FAILURE
      X(I,J) = A(I) + DELTA
   20 CONTINUE
      DO 30 I = 1,N
      IF (X(I,J) .LE. B(I))GOTO 30
      IF (K .LE. 1)RETURN FAILURE
      X(I,J) = B(I) - DELTA
   30 CONTINUE
   40 CONTINUE
      RETURN SUCCESS
      END
C******************************
      SUBROUTINE IMPLICT (X,B,MM,NP,K,M,N,CEQS,XC,GOOD,NO,*,*)

C     THIS ROUTINE CHECKS TO INSURE THAT THE IMPLICIT CONSTRAINTS
C     ARE SATISFIED.
C     THE CALCULATION OF THE EQUATION(S) THAT DEFINE THE IMPLICIT
C     CONSTRAINTS IS FIRST PERFORMED BY CALLING THE USER DEFINED
C     ROUTINE "IMPEQS".   UPON COMPLETION THE RESULTS ARE CHECKED
C     IF THE IMPLICIT CONSTRAINTS ARE MET, CONTROL IS RETURNED
C     THROUGH SUCCESS, OTHERWISE CONTROL IS RETURNED THROUGH FAILED.


      DIMENSION X(NP,K),CEQS(M),XC(NP),B(MM)
      LOGICAL GOOD(K)


      DO 20 J = 1,K
      GOOD(J) = .TRUE.
      DO 5 I = 1,NP
    5 XC(I) = X(I,J)
      CALL CONST(XC,NP,CEQS)
      DO 10 I = 1,M
      IF (CEQS(I) .GT. B(N+I))GOOD(J) = .FALSE.
   10 CONTINUE
   20 CONTINUE

      DO 30 I = 1,NP
```

```
      XC(I) = 0.0
   30 CONTINUE
      IS = 0
      DO 50 J = 1,K
      IF (.NOT. GOOD(J))GOTO 50
      IS = IS + 1
      DO 40 I = 1,NP
      XC(I) = XC(I) + X(I,J)
   40 CONTINUE
   50 CONTINUE

      IF (K .GT. 1)GOTO 60
      IF (IS .EQ. K)RETURN SUCCESS
      RETURN FAILURE
   60 CONTINUE
      IF (IS.NE.0)GOTO 65
      WRITE(NO,100)
  100 FORMAT(///10X,'*** ERROR IN COMPLEX OPTIMIZATION'/
     1 10X,'ALL IMPLICIT CONSTRAINTS FAILED')
       STOP
   65 IF (IS .EQ. K)RETURN SUCCESS

      DO 70 I = 1,NP
      XC(I) = XC(I)/IS
   70 CONTINUE

      DO 90 J = 1,K
      IF (GOOD(J))GOTO 90
      DO 80 I = 1,NP
      X(I,J) = (X(I,J) + XC(I))/2.0
   80 CONTINUE
   90 CONTINUE
      RETURN FAILURE
      END
C********************************
      SUBROUTINE OBJFUN (X,FVAL,JT,NP,K,FUN,OLD,NUMFUN,XX)
C      THIS ROUTINE IS USED AS AN INTERFACE BETWEEN THE USERS
C      FUNCTION AND THE OPTIMIZATION PROGRAM.  THE PURPOSE HERE IS
C      TO CHECK IF THE FUNCTION HAS ALREADY BEEN EVALUATED FOR
C      THE VERTICIES PASSED.
C
      DIMENSION X(NP,K),FUN(K),OLD(NP,K),XX(NP)

      IF (JT .EQ. 0)GOTO 40
      DO 20 J = 1,K
      IKOUNT = 0
      DO 10 I = 1,NP
      IF (ABS(X(I,JT)) .LT. 1.0E-09)GOTO 10
      DIF = ABS(X(I,JT) - OLD(I,J))/(ABS(X(I,JT)))
      IF (DIF .LE. 1.0E-09)IKOUNT = IKOUNT + 1
   10 CONTINUE
      IF (IKOUNT .NE. NP)GOTO 20
```

```
      FVAL = FUN(J)
      RETURN
   20 CONTINUE
      DO 30 I = 1,NP
      OLD(I,JT) = X(I,JT)
      XX(I) = X(I,JT)
   30 CONTINUE
   40 CONTINUE
      CALL UREAL(XX,FVAL)
      NUMFUN = NUMFUN + 1
      RETURN
      END
C******************************
        SUBROUTINE CONST(X,NCONS,PHI)
        DIMENSION X(1),PHI(1)
        PHI(1)=X(1)-0.2
        PHI(2)=-X(1)+23.125
        PHI(3)=X(2)-0.2
        PHI(4)=-X(2)+27.75
        PHI(5)=X(3)-0.5
        PHI(6)=-X(3)+4.5
        RETURN
        END
C******************************
        SUBROUTINE EQUAL(X,PSI,NEQUS)
        DIMENSION X(1),PSI(1)
        RETURN
        END
C*********************************
C PROGRAM FOR IMPULSE RESPONSE OF A LATHE SPINDLE - WORKPIECE SYSTEM
C   THE FINITE ELEMENT MODELLED SPINDLE SYSTEM IS CONDENSED FIRST
C    THE RESPONSE DUE TO A UNIT IMPULSE IS THEN OBTAINED
C*********************************
        SUBROUTINE UREAL(X,U)
        REAL X(3)
        DIMENSION SL1(12),SD1(12),Y1(12),GMM(25,25),GKK(25,25)
        DIMENSION GCC(25,25),GM(26,26),GK(26,26),GC(26,26)
        REAL KF,KR,GMR(25,25),GKR(25,25),GCR(25,25)
        INTEGER LN(25),LD(25)
        REAL F1(50,50),F2(50,50),F3(50,50),F4(50,50)
        REAL F5(50,50),F6(50,50),FF1(50,50)
        REAL FF2(50,50),FF3(50,50),FF4(50,50)
        REAL GMN(50,50),GKN(50,50),GCN(50,50),TR(50,50),TT(50,50)
        COMPLEX WQC(50),Z(50,50),FF5(50,50)
        REAL SA(50,50),SB(50,50),ERD(50,1)
        COMPLEX ZT(50,50),EE1(50,1),SS1(50,50),SS2(50,50)
        COMPLEX V1(50,50),V2(50,50)
        COMPLEX V3(50,50),V4(50,50),V5(50,50)
        COMPLEX Z5(50,50)
        COMPLEX EO(25),YA1(50,1),YA2(50,1),YA3(50,1),YA4(50,1),YA5(50,1)
        REAL FR(25,1),FRD(25,1),YOO(25)
```

```
      NSD=13
      N1=25
      N5=N1-NSD
      N55=N5*2
      N2=N1*2
      DRB=X(1)
      DRD=X(2)
      RR=X(3)
      CALL MATLOK(N1,N2,N5,N55,NSD,SL1,SD1,GMM,GKK,
     $ GCC,GMR,GKR,GCR,LN,
     $ LD,F1,F2,F3,F4,F5,F6,FF1,FF2,FF3,FF4,GMN,GKN,GCN,
     $ TR,TT,WQC,Z,FF5,SA,SB,ZT,
     $ V1,V2,V3,V4,V5,
     $ FR,FRD,YOO,ZI,ZS,
     $ ZO,YA1,YA2,YA3,YA4,YA5,ERD,EE1,SS1,SS2,DRB,DRD,RR,U)
      RETURN
      END
      SUBROUTINE MATLOK(N1,N2,N5,N55,NSD,SL1,SD1,GMM,GKK,
     $ GCC,GMR,GKR,GCR,LN,
     $ LD,F1,F2,F3,F4,F5,F6,FF1,FF2,FF3,FF4,GMN,GKN,GCN,
     $ TR,TT,WQC,Z,FF5,SA,SB,ZT,
     $ V1,V2,V3,V4,V5,
     $ FR,FRD,YOO,ZI,ZS,
     $ ZO,YA1,YA2,YA3,YA4,YA5,ERD,EE1,SS1,SS2,DRB,DRD,RR,U)
      REAL GMM(25,25),GKK(25,25),GCC(25,25),GMR(N5,N5)
      REAL GKR(N5,N5),GCR(N5,N5)
      INTEGER LN(N1),LD(N1)
      REAL GK(26,26),GM(26,26),GC(26,26)
      REAL F1(N5,N5),F2(N5,N5),F3(NSD,NSD),F4(NSD,N5)
      REAL F5(NSD,N5),F6(NSD,NSD)
      REAL FF1(N5,N5),FF2(N5,N5),FF3(N5,N5),FF4(N5,N5),KF,KR
      REAL GMN(N1,N1),GKN(N1,N1),GCN(N1,N1),TR(N1,N5),TT(N5,N1)
      COMPLEX  WQC(N55),Z(N55,N55),ZT(N55,N55)
      COMPLEX SS1(N55,N55),SS2(N55,N55)
      REAL SA(N55,N55),SB(N55,N55),SL1(12),SD1(12)
      COMPLEX  FF5(N55,N55)
      COMPLEX V1(N55,N55),V2(N55,N55),V3(N55,N55)
      COMPLEX V4(N5,N55),V5(N5,N5)
      REAL FR(N1,1),FRD(N5,1),YOO(N5),ERD(N55,1)
      COMPLEX ZI(N55,N55),ZS(N55,N55),ZO(N5),EE1(50,1)
      COMPLEX YA1(N55,1),YA2(N55,1),YA3(N55,1),YA4(N55,1),YA5(N55,1)
      REAL RIP1(1000),RIP2(1000),RIP3(1000)
      REAL RIP4(1000),RIP5(1000)
      REAL TIM(1000),WPR(5)
      NUE=12
      NODES=2
      SE=30.0E+6
      KF=12.964286E+6
      KR=4.0640395E+6
      D1=127.496
      D2=163.492
      D12=0.0
```

```
      WL=2.6
      WD=2.0
      SD1(6)=10.0
      SD1(7)=7.9
      SD1(8)=8.99
      SD1(9)=6.68
      SD1(10)=6.68
      SD1(11)=7.4
      SD1(12)=6.68
      DO 902 I=6,12
      SL1(I)=4.625
902   CONTINUE
      KP=RR*KR
      DO 606 I=1,5
      SL1(I)=WL
      SD1(I)=WD
606   CONTINUE
      CALL EKEM(SD1,SL1,SE,KP,KR,D1,D2,D12,NUE,NODES,DRB,DRD,GK,GM,GC)
      DO 25 I=1,25
      DO 25 J=1,25
      GKK(I,J)=GK(I+1,J+1)
      GMM(I,J)=GM(I+1,J+1)
      GCC(I,J)=GC(I+1,J+1)
25    CONTINUE
      DO 2002 I=1,N1
      FR(I,1)=0.0
2002  CONTINUE
      FR(3,1)=1.0
      OPEN(UNIT=5,FILE='RED.DAT',TYPE='NEW')
      WRITE(5,3012)
3012  FORMAT(12X,'FRONT & REAR BEARING STIFFNESS VALUES ',/)
      WRITE(5,*) KP,KR
      WRITE(5,3013)  NR
3013  FORMAT(12X,'LOCATION OF REAR BEARING AT ',I4,' ELEMENT ',/)
      WRITE(5,3009) ND
3009  FORMAT(12X,'LOCATION OF THE EXTERNAL DAMPER AT ',I4,'ELEMENT',/)
      WRITE(5,3080)  WD
3080  FORMAT(12X,' DIAMETER OF THE WORK PIECE ',F10.5,/)
      WRITE(5,3001)
3001  FORMAT(12X,'STIFFNESS MATRIX',/)
      WRITE(5,*) (GKK(I,I),I=1,25)
      WRITE(5,3002)
3002  FORMAT(12X,'MASS MATRIX ',/)
      WRITE(5,*) (GMM(I,I),I=1,25)
      CALL APX11(N1,GKK,GMM,LD,LM)
      WRITE(5,91) MS
91    FORMAT(12X,'MASTERS = ',I4,//)
      WRITE(5,92)
92    FORMAT(12X,'MASTER DEGREES OF FREEDOM',/)
      DO 94 IO=NSD+1,N1
      WRITE(5,*) LD(IO)
94    CONTINUE
```

```
      WRITE(5,96) NSD
96    FORMAT(12X,'SLAVES ',I4,/)
      WRITE(5,55)
55    FORMAT(12X,'*** DIAGNOL RATIOS *****',/)
      WRITE(5,*) (LD(I),I=1,N1)
      WRITE(5,*) (LN(I),I=1,N1)
      CALL APX222(N1,GMM,LN,NSD,GMN)
      WRITE(5,3004)
3004  FORMAT(12X,'ARRANGED MASS MATRIX ',/)
      WRITE(5,*) (GMN(I,I),I=1,N1)
      CALL APX222(N1,GKK,LN,NSD,GKN)
      WRITE(5,3005)
3005  FORMAT(12X,'ARRANGED STIFFNESS MATRIX',/)
      WRITE(5,*) (GKN(I,I),I=1,N1)
      CALL APX222(N1,GCC,LN,NSD,GCN)
      WRITE(5,3006)
3006  FORMAT(12X,'ARRANGED DAMPING MATRIX ',/)
      WRITE(5,*) (GCN(I,I),I=1,N1)
      CALL APX33(N1,NSD,N5,GKN,TR,TT,F1,F2,F3,F4,F5,F6)
      CALL APX44(N1,N5,NSD,TT,GMN,TR,GMR,F6)
      CALL APX44(N1,N5,NSD,TT,GKN,TR,GKR,F6)
      CALL APX44(N1,N5,NSD,TT,GCN,TR,GCR,F6)
      CALL APX55(N1,N5,NSD,TT,FR,FRD)
      WRITE(5,3007)
3007  FORMAT(12X,'REDUCED MASS MATRIX',/)
      WRITE(5,*) ((GMR(I,J),J=1,N5),I=1,N5)
      WRITE(5,3008)
3008  FORMAT(12X,'REDUCED STIFFNESS MATRIX',/)
      WRITE(5,*) ((GKR(I,J),J=1,N5),I=1,N5)
      WRITE(5,3090)
3090  FORMAT(12X,'REDUCED DAMPING MATRIX ',/)
      WRITE(5,*) ((GCR(I,J),J=1,N5),I=1,N5)
      CALL XXX(N5,N55,GKR,GMR,GCR,FRD,WQC,Z,FF1,FF2,FF3,
     1 FF4,FF5,SA,SB,ERD)
      CALL TWAX(N5,N55,SA,SB,ERD,Z,SS1,SS2,EE1,V1,V2,V3,V4,YA4)
      TST=0.4/1000.0
      DO 3026 I=1,1000
      T1=FLOAT(I)*TST
      CALL RES(N5,N55,WQC,T1,SS1,EE1,Z,YA1,YA2,YA3,YOO)
      RIP1(I)=YOO(1)
      RIP2(I)=YOO(2)
      RIP3(I)=YOO(3)
      RIP4(I)=YOO(4)
      RIP5(I)=YOO(5)
      TIM(I)=T1
3026  CONTINUE
      CALL MAXWQC(RIP1,TIM,RMM,TCOR)
      WPR(1)=RMM
      CALL MAXWQC(RIP2,TIM,RMM,TCOR)
      WPR(2)=RMM
      CALL MAXWQC(RIP3,TIM,RMM,TCOR)
      WPR(3)=RMM
```

```
      CALL MAXWQC(RIP4,TIM,RMM,TCOR)
      WPR(4)=RMM
      CALL MAXWQC(RIP5,TIM,RMM,TCOR)
      WPR(5)=RMM
      CALL MAXW1(WPR,RMM)
      U=RMM
      RETURN
      END
C*******************
      SUBROUTINE MAXW1(A,AM)
      REAL A(5)
      AM=10.0E-25
      DO 10 I=1,5
      IF(A(I).GT.AM) AM=A(I)
10    CONTINUE
      RETURN
      END
C********************
      SUBROUTINE XXX(NX,NY,GKK,GMM,GCC,FRD,WQC,Z,GMI,GMK,GMC,
     1 UN,DC,AC,BC,ERD)
      DIMENSION GKK(NX,NX),GMM(NX,NX),GCC(NX,NX),FRD(NY,1),ERD(NY,1)
      DIMENSION GMI(NX,NX),GMK(NX,NX),GMC(NX,NX),UN(NX,NX)
      INTEGER N,IA,IZ,M,IB,IER,IJOB,IDGT
      REAL WKAREA(700),WK(5100),WA(50)
      COMPLEX WQC(NY),Z(NY,NY),ZN,DC(NY,NY)
      REAL  AC(NY,NY),BC(NY,NY)
      DO 25 I=1,NX
      DO 25 J=1,NX
      AC(I,J)=0.0
      AC(I,J+NX)=GMM(I,J)
      AC(I+NX,J)=GMM(I,J)
      AC(I+NX,J+NX)=GCC(I,J)
25    CONTINUE
      DO 30 I=1,NX
      DO 30 J=1,NX
      BC(I,J)=-1.0*GMM(I,J)
      BC(I,J+NX)=0.0
      BC(I+NX,J)=0.0
      BC(I+NX,J+NX)=GKK(I,J)
30    CONTINUE
      DO 36 I=1,NX
      ERD(I,1)=0.0
      ERD(I+NX,1)=FRD(I,1)
36    CONTINUE
      IA=NX
      N=NX
      IDGT=4
      CALL LINV2F(GMM,N,IA,GMI,IDGT,WKAREA,IER)
      CALL MATMP3(NX,NX,NX,GMI,GKK,GMK)
      CALL MATMP3(NX,NX,NX,GMI,GCC,GMC)
      DO 45 I=1,NX
      DO 45 J=1,NX
```

```
      GMK(I,J)=-1.0*GMK(I,J)
      GMC(I,J)=-1.0*GMC(I,J)
   45 CONTINUE
      DO 54 I=1,NX
      DO 54 J=1,NX
      UN(I,J)=0.0
   54 CONTINUE
      DO 55 I=1,NX
      UN(I,I)=UN(I,I)+1.0
   55 CONTINUE
      DO 60 I=1,NX
      DO 60 J=1,NX
      DC(I,J)=CMPLX(GMC(I,J),0.0)
      DC(I,J+NX)=CMPLX(GMK(I,J),0.0)
      DC(I+NX,J)=CMPLX(UN(I,J),0.0)
      DC(I+NX,J+NX)=CMPLX(0.0,0.0)
   60 CONTINUE
      N=NY
      IA=NY
      IZ=NY
      IJOB=2
      CALL EIGCC(DC,N,IA,IJOB,WQC,Z,IZ,WK,IER)
      RETURN
      END
C*******************************
      SUBROUTINE TWAX(NX,NY,SA,SB,ERD,Z,SS1,SS2,E2,S1,S2,ZT,V4,E1)
      REAL SA(NY,NY),SB(NY,NY),ERD(NY,1)
      COMPLEX S1(NY,NY),S2(NY,NY),ZT(NY,NY),Z(NY,NY),E1(NY,1)
      COMPLEX SS1(NY,NY),SS2(NY,NY),E2(NY,1)
      DO 10 I=1,NY
      DO 10 J=1,NY
      S1(I,J)=CMPLX(SA(I,J),0.0)
      S2(I,J)=CMPLX(SB(I,J),0.0)
   10 CONTINUE
      DO 20 I=1,NY
      E1(I,1)=CMPLX(ERD(I,1),0.0)
   20 CONTINUE
      DO 30 I=1,NY
      DO 30 J=1,NY
      ZT(I,J)=Z(J,I)
   30 CONTINUE
      CALL SPAX1(NY,ZT,S1,Z,SS1,V4)
      CALL SPAX1(NY,ZT,S2,Z,SS2,V4)
      CALL SPAX2(NY,ZT,E1,E2)
      RETURN
      END
C*******************************
      SUBROUTINE SPAX1(K1,A,B,C,D,AB)
      COMPLEX A(K1,K1),B(K1,K1),C(K1,K1),AB(K1,K1),D(K1,K1)
      DO 10 I=1,K1
      DO 10 J=1,K1
      AB(I,J)=CMPLX(0.0,0.0)
```

```
      DO 10 K=1,K1
      AB(I,J)=AB(I,J)+A(I,K)*B(K,J)
10    CONTINUE
      DO 20 I=1,K1
      DO 20 J=1,K1
      D(I,J)=CMPLX(0.0,0.0)
      DO 20 K=1,K1
      D(I,J)=D(I,J)+AB(I,K)*C(K,J)
20    CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE SPAX2(K1,A,B,C)
      COMPLEX A(K1,K1),B(K1,1),C(K1,1)
      DO 10 I=1,K1
      C(I,1)=CMPLX(0.0,0.0)
      DO 10 J=1,K1
      C(I,1)=C(I,1)+A(I,J)*B(J,1)
10    CONTINUE
      RETURN
      END
C***********************
      SUBROUTINE RES(NX,NY,WQC,T1,SS1,E1,Z,V1,ZPR,YT,YO)
      COMPLEX WQC(NY),Z(NY,NY),SS1(NY,NY),E1(NY,1)
      REAL YO(NX)
      COMPLEX V1(NY,1),ZPR(NY,1),YT(NY,1)
      CALL RESP(NY,WQC,T1,V1)
      DO 10 I=1,NY
      ZPR(I,1)=(V1(I,1)*E1(I,1))/SS1(I,I)
10    CONTINUE
      CALL SPAX2(NY,Z,ZPR,YT)
      DO 20 I=1,NX
      YO(I)=REAL(YT(I+NX,1))
20    CONTINUE
      RETURN
      END
C******************************
      SUBROUTINE MAXWQC(A,B,AMAX,BCOR)
      REAL A(1000),B(1000)
      AMAX=10.0E-24
      DO 90 I=1,1000
      A(I)=ABS(A(I))
90    CONTINUE
      DO 50 K=1,1000
      IF(A(K).GT.AMAX) GO TO 60
      GO TO 50
60    AMAX=A(K)
      BCOR=B(K)
50    CONTINUE
      RETURN
      END
```

```
C********************
C  SUBROUTINE FOR ELEMENTAL STIFFNESS & MASS MATRIX
C********************
      SUBROUTINE MATFOR(SD,SL,SE,K,EK,EM,EC)
      DIMENSION EC(4,4),EK(4,4),EM(4,4)
      RHO=0.2832958/386.0
      DO 4 NI=1,4
      DO 4 JI=1,4
      EC(NI,JI)=0.0
      EK(NI,JI)=0.0
    4 EM(NI,JI)=0.0
      EM(1,1)=156.0
      EM(1,2)=22.0*SL
      EM(1,3)=54.0
      EM(1,4)=-13.0*SL
      EM(2,1)=22.0*SL
      EM(2,2)=4.0*SL**2
      EM(2,3)=13.0*SL
      EM(2,4)=-3.0*SL**2
      EM(3,1)=54.0
      EM(3,2)=13.*SL
      EM(3,3)=156.0
      EM(3,4)=-22.0*SL
      EM(4,1)=-13.0*SL
      EM(4,2)=-3.0*SL**2
      EM(4,3)=-22.0*SL
      EM(4,4)=4.0*SL**2
      EK(1,1)=12.0
      EK(1,2)=6.0*SL
      EK(1,3)=-12.0
      EK(1,4)=6.0*SL
      EK(2,1)=6.0*SL
      EK(2,2)=4.0*SL**2
      EK(2,3)=-6.0*SL
      EK(2,4)=2.0*SL**2
      EK(3,1)=-12.0
      EK(3,2)=-6.0*SL
      EK(3,3)=12.0
      EK(3,4)=-6.0*SL
      EK(4,1)=6.0*SL
      EK(4,2)=2.0*SL**2
      EK(4,3)=-6.0*SL
      EK(4,4)=4.0*SL**2
      IF(K.LE.5) BD=0.0
      IF(K.GT.5) BD=2.5
      SI=(22.0/(7.0*64.0))*(SD**4-BD**4)
      SM=(22.0/28.0)*(SD**2-BD**2)*RHO
      FM=SM*SL/420.0
      FK=SE*SI/(SL**3)
      DO 5 I=1,4
      DO 5 J=1,4
      EM(I,J)=FM*EM(I,J)
```

```
      EK(I,J)=FK*EK(I,J)
    5 CONTINUE
      RETURN
      END
C*********************
C  FINDS THE TRANSPOSE OF A MATRIX A(MX,MY)
C**********************
      SUBROUTINE TRANSP(A,MX,MY,AT)
      DIMENSION A(4,4),AT(4,4)
      DO 1 I=1,MX
      DO 1 J=1,MY
    1 AT(J,I)=A(I,J)
      RETURN
      END
C**********************
      SUBROUTINE MATMP1(A1,B1,C1,L1,M1,N1)
      DIMENSION A1(4,4),B1(4,4),C1(4,4)
      DO 2 I=1,L1
      DO 2 J=1,N1
      C1(I,J)=0.0
      DO 2 K=1,M1
    2 C1(I,J)=C1(I,J)+A1(I,K)*B1(K,J)
      RETURN
      END
C************************
      SUBROUTINE ASSEM(AZ,JU1,JU2,NODES,SUM)
      DIMENSION AZ(4,4),SUM(26,26)
      DO 8 I=1,26
      DO 8 J=1,26
      SUM(I,J)=0.0
    8 CONTINUE
      JV1=JU1+NODES
      JV2=JU2+NODES
      DO 10 KI=1,4
      IF(KI.EQ.1) KO=JU1
      IF(KI.EQ.2) KO=JU2
      IF(KI.EQ.3) KO=JV1
      IF(KI.EQ.4) KO=JV2
      SUM(KO,JU1)=SUM(KO,JU1)+AZ(KI,1)
      SUM(KO,JU2)=SUM(KO,JU2)+AZ(KI,2)
      SUM(KO,JV1)=SUM(KO,JV1)+AZ(KI,3)
      SUM(KO,JV2)=SUM(KO,JV2)+AZ(KI,4)
   10 CONTINUE
      RETURN
      END
C**************************
      SUBROUTINE TRNSF(A)
      DIMENSION A(4,4)
      DO 12 I=1,4
      DO 12 J=1,4
      A(I,J)=0.0
   12 CONTINUE
```

```
      A(1,1)=A(1,1)+1.0
      A(2,2)=A(2,2)+1.0
      A(3,3)=A(3,3)+1.0
      A(4,4)=A(4,4)+1.0
      RETURN
      END
C*****************************
      SUBROUTINE TRNSF1(RR,A,RK)
      DIMENSION A(4,4),RR(4,4),AT(4,4),RTK(4,4),RK(4,4)
      DO 10 IF=1,4
      DO 10 JF=1,4
      RTK(IF,JF)=0.0
      RK(IF,JF)=0.0
   10 CONTINUE
      CALL TRANSP(A,4,4,AT)
      CALL MATMP1(AT,RR,RTK,4,4,4)
      CALL MATMP1(RTK,A,RK,4,4,4)
      RETURN
      END
C***************************
      SUBROUTINE MATMP3(I1,I2,I3,AQ,BQ,CQ)
      DIMENSION AQ(I1,I2),BQ(I2,I3),CQ(I1,I3)
      DO 9 I=1,I1
      DO 9 J=1,I2
      CQ(I,J)=0.0
      DO 9 K=1,I3
    9 CQ(I,J)=CQ(I,J)+AQ(I,K)*BQ(K,J)
      RETURN
      END
C***********************
      SUBROUTINE APX11(N1,ASM,ASK,LD,LN)
      DIMENSION ASM(N1,N1),ASK(N1,N1),R(25),RN(25)
      INTEGER LD(N1),LN(N1)
      DO 5 I=1,N1
      R(I)=ASM(I,I)/ASK(I,I)
    5 CONTINUE
      DO 8 I=1,N1
      RN(I)=R(I)
      LD(I)=I
    8 CONTINUE
      AA=0.0
      J1=1
      DO 15 I=1,N1
      J1=J1+1
      DO 15 J=J1,N1
      AA=RN(I)
      AB=RN(J)
      IM=LD(I)
      IN=LD(J)
      IF(AA.LT.AB) GO TO 20
      GO TO 15
   20 RN(I)=AB
```

```
      RN(J)=AA
      LD(I)=IN
      LD(J)=IM
15    CONTINUE
      DO 60 I=1,N1
      LN(I)=LD(I)
60    CONTINUE
      J1=1
      DO 70 I=1,N1
      IC=LN(I)
      J1=J1+1
      DO 70 J=J1,N1
      ID=LN(J)
      IF(IC.LT.ID) GO TO 80
      GO TO 70
80    LN(J)=LN(J)-1
70    CONTINUE
      RETURN
      END
C******************************
      SUBROUTINE APX222(N1,A1,LN,NSD,A9)
      REAL AX(25,25)
      DIMENSION A1(N1,N1),A5(25,25),A9(N1,N1)
      INTEGER LN(N1)
      DO 55 I=1,N1
      DO 55 J=1,N1
      AX(I,J)=A1(I,J)
55    CONTINUE
      DO 5 K=1,NSD
      L1=LN(K)
      CALL APX22(N1,AX,L1,A5)
      DO 10 I=1,N1
      DO 10 J=1,N1
      AX(I,J)=A5(I,J)
10    CONTINUE
5     CONTINUE
      DO 77 I=1,N1
      DO 77 J=1,N1
      A9(I,J)=AX(I,J)
77    CONTINUE
      RETURN
      END
C***********************
      SUBROUTINE APX22(N1,A1,LD,A3)
      DIMENSION A1(N1,N1),A3(N1,N1),A2(25,25)
      DO 5 I=1,N1
      DO 5 J=1,N1
      A2(I,J)=A1(I,J)
5     CONTINUE
      N2=N1-1
      DO 10 JJ=LD,N2
      JK=JJ+1
```

```
      DO 10 I=1,N1
      A2(I,JJ)=A1(I,JK)
10    CONTINUE
      DO 20 I=1,N1
      A2(I,N1)=A1(I,LD)
20    CONTINUE
      DO 6 I=1,N1
      DO 6 J=1,N1
      A3(I,J)=A2(I,J)
6     CONTINUE
      DO 30 JJ=LD,N2
      JK=JJ+1
      DO 30 I=1,N1
      A3(JJ,I)=A2(JK,I)
30    CONTINUE
      DO 40 I=1,N1
      A3(N1,I)=A2(LD,I)
40    CONTINUE
      RETURN
      END
C********************************
      SUBROUTINE APX33(N1,NSD,N5,AK,T,TT,UN,AKM,AKS,AKT,AX,ASI)
      DIMENSION AK(N1,N1),T(N1,N5),TT(N5,N1),UN(N5,N5)
      REAL WKAREA(700),AKM(N5,N5),AKS(NSD,NSD),AKT(NSD,N5)
      REAL AX(NSD,N5),ASI(NSD,NSD)
      MD=N1-NSD
      DO 2 I=1,MD
      DO 2 J=1,MD
      AKM(I,J)=AK(I,J)
2     CONTINUE
      DO 4 I=1,NSD
      DO 4 J=1,NSD
      AKS(I,J)=AK(I+MD,J+MD)
4     CONTINUE
      DO 6 I=1,NSD
      DO 6 J=1,MD
      AKT(I,J)=AK(I+MD,J)
6     CONTINUE
      N=NSD
      IA=N
      IDGT=4
      CALL LINV2F(AKS,N,IA,ASI,IDGT,WKAREA,IER)
      DO 12 I=1,NSD
      DO 12 J=1,MD
      AX(I,J)=0.0
      DO 12 K=1,NSD
      AX(I,J)=AX(I,J)+ASI(I,K)*AKT(K,J)
12    CONTINUE
      DO 8 I=1,N1
      DO 8 J=1,N1
      UN(I,J)=0.0
8     CONTINUE
```

```
      DO 10 I=1,MD
      UN(I,I)=UN(I,I)+1.0
10    CONTINUE
      DO 22 I=1,MD
      DO 22 J=1,MD
      T(I,J)=UN(I,J)
22    CONTINUE
      DO 14 I=1,NSD
      DO 14 J=1,MD
      T(I+MD,J)=-1.0*AX(I,J)
14    CONTINUE
      DO 16 I=1,N1
      DO 16 J=1,MD
      TT(J,I)=T(I,J)
16    CONTINUE
      RETURN
      END
C***************************
      SUBROUTINE APX44(N1,N5,NSD,A1,A2,A3,A4,A12)
      DIMENSION A1(N5,N1),A2(N1,N1),A3(N1,N5),A12(N5,N1)
      REAL A4(N5,N5)
      MD=N1-NSD
      DO 10 I=1,MD
      DO 10 J=1,N1
      A12(I,J)=0.0
      DO 10 K=1,N1
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
10    CONTINUE
      DO 20 I=1,MD
      DO 20 J=1,MD
      A4(I,J)=0.0
      DO 20 K=1,N1
      A4(I,J)=A4(I,J)+A12(I,K)*A3(K,J)
20    CONTINUE
      RETURN
      END
C************************
      SUBROUTINE APX55(N1,N5,NSD,A1,A2,A3)
      REAL A1(N5,N1),A2(N1,1),A3(N5,1)
      DO 10 I=1,N5
      A3(I,1)=0.0
      DO 10 J=1,N1
      A3(I,1)=A3(I,1)+A1(I,J)*A2(J,1)
10    CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE CMAT1(A1,A2,A3,K1,A4,A12)
      COMPLEX A1(K1,K1),A3(K1,K1),A4(K1),A12(K1,K1)
      REAL A2(K1,K1)
      DO 1 I=1,K1
      DO 1 J=1,K1
```

```
      A12(I,J)=CMPLX(0.0,0.0)
      DO 1 K=1,K1
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
    1 CONTINUE
      DO 3 I=1,K1
      A4(I)=CMPLX(0.0,0.0)
      DO 3 K=1,K1
      A4(I)=A4(I)+A12(I,K)*A3(K,I)
    3 CONTINUE
      RETURN
      END
C************************
      SUBROUTINE XAT(A1,B1,C1,K1,A4,AB)
      COMPLEX A1(K1,K1),B1(K1),C1(K1,K1),AB(K1,K1),A4(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      AB(I,J)=A1(I,J)*B1(J)
    2 CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      A4(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      A4(I,J)=A4(I,J)+AB(I,K)*C1(K,J)
    4 CONTINUE
      RETURN
      END
C*********************
      SUBROUTINE XAT1(A1,A2,A3,K1,A4,A12)
      COMPLEX A1(K1,K1),A3(K1,K1),A12(K1,K1),A4(K1,K1)
      REAL A2(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      A12(I,J)=CMPLX(0.0,0.0)
      DO 2 K=1,K1
      A12(I,J)=A12(I,J)+A1(I,K)*A2(K,J)
    2 CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      A4(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      A4(I,J)=A4(I,J)+A12(I,K)*A3(K,J)
    4 CONTINUE
      RETURN
      END
C**********************
      SUBROUTINE SAT3(A1,B,C1,MR,NC,AC,AB)
      COMPLEX A1(MR,NC),C1(NC,MR),B(NC),AB(MR,NC),AC(MR,MR)
      DO 2 I=1,MR
      DO 2 J=1,NC
      AB(I,J)=A1(I,J)*B(J)
    2 CONTINUE
      DO 4 I=1,MR
```

```
      DO 4 J=1,MR
      AC(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,NC
      AC(I,J)=AC(I,J)+AB(I,K)*C1(K,J)
4     CONTINUE
      RETURN
      END
C************************
      SUBROUTINE SAT4(A1,B1,C1,K1,D1,AB)
      COMPLEX A1(K1,K1),AB(K1,K1),C1(K1,K1),D1(K1,K1)
      REAL B1(K1,K1)
      DO 2 I=1,K1
      DO 2 J=1,K1
      AB(I,J)=CMPLX(0.0,0.0)
      DO 2 K=1,K1
      AB(I,J)=AB(I,J)+A1(I,K)*B1(K,J)
2     CONTINUE
      DO 4 I=1,K1
      DO 4 J=1,K1
      D1(I,J)=CMPLX(0.0,0.0)
      DO 4 K=1,K1
      D1(I,J)=D1(I,J)+AB(I,K)*C1(K,J)
4     CONTINUE
      RETURN
      END
C*************************
      SUBROUTINE RESP(N55,WQC,T1,V)
      COMPLEX WQC(N55),V(N55,1)
      DO 5 I=1,N55
      AR=REAL(WQC(I))*T1
      AI=AIMAG(WQC(I))*T1
      AR1=EXP(AR)
      XC=AR1*COS(AI)
      YC=AR1*SIN(AI)
      V(I,1)=CMPLX(XC,YC)
5     CONTINUE
      RETURN
      END
C***************************
      SUBROUTINE MAR2(AA,BB,CC,K1)
      COMPLEX AA(K1,1),BB(K1,1),CC(K1,1)
      DO 1 I=1,K1
      CC(I,1)=AA(I,1)*BB(I,1)
1     CONTINUE
      RETURN
      END
C*********************
      SUBROUTINE MAR1(AC,BC,N,CC)
      COMPLEX AC(N,N),BC(N,1),CC(N,1)
      DO 40 I=1,N
      CC(I,1)=CMPLX(0.0,0.0)
      DO 40 J=1,N
```

```
      CC(I,1)=CC(I,1)+AC(I,J)*BC(J,1)
   40 CONTINUE
      RETURN
      END
C**********************
      SUBROUTINE EREM(SD1,SL1,SE,KF,KR,D1,D2,
     1 D12,NUE,NODES,DRB,DRD,GK,GM,GC)
      DIMENSION GM(26,26),GK(26,26),GC(26,26)
      DIMENSION SD1(12),SL1(12),EK(4,4),EM(4,4),EC(4,4)
      DIMENSION A(4,4),AK(4,4),AM(4,4),AC(4,4)
      DIMENSION SK(26,26),SH(26,26),SC(26,26)
      REAL KF,KR,KR1,KR2
      INTEGER NP(2)
      DO 20 I=1,26
      DO 20 J=1,26
      GM(I,J)=0.0
      GC(I,J)=0.0
   20 GK(I,J)=0.0
      KM1=1
      KM2=2
      CALL LOCAT(KR,D2,D12,DRB,DRD,KR1,KR2,DR1,DR2,ED1,ED2,NP)
      NR=NP(1)
      ND=NP(2)
      DO 105 K=1,NUE
      SL=SL1(K)
      SD=SD1(K)
      JU1=KM1
      JU2=KM2
      CALL MATFOR(SD,SL,SE,K,EK,EM,EC)
      IF(K.EQ.7) GO TO 145
      IF(K.EQ.NR) GO TO 146
      GO TO 148
  145 EK(1,1)=EK(1,1)+0.5*KF
      EK(3,3)=EK(3,3)+0.5*KF
      EC(1,1)=EC(1,1)+0.5*D1
      EC(3,3)=EC(3,3)+0.5*D1
      GO TO 148
  146 EK(1,1)=EK(1,1)+KR1
      EK(3,3)=EK(3,3)+KR2
      EC(1,1)=EC(1,1)+DR1
      EC(3,3)=EC(3,3)+DR2
      GO TO 148
  148 IF(K.EQ.ND) GO TO 149
      GO TO 158
  149 EC(1,1)=EC(1,1)+0.5*ED1
      EC(3,3)=EC(3,3)+0.5*ED2
  158 CALL TRNSF(A)
      CALL TRNSF1(EK,A,AK)
      CALL TRNSF1(EM,A,AM)
      CALL TRNSF1(EC,A,AC)
      DO 14 IP=1,26
      DO 14 KP=1,26
```

```
      SK(IP,KP)=0.0
      SM(IP,KP)=0.0
      SC(IP,KP)=0.0
   14 CONTINUE
      CALL ASSEM(AK,JU1,JU2,NODES,SK)
      CALL ASSEM(AM,JU1,JU2,NODES,SM)
      CALL ASSEM(AC,JU1,JU2,NODES,SC)
      DO 150 I1=1,26
      DO 150 J1=1,26
      GK(I1,J1)=GK(I1,J1)+SK(I1,J1)
      GM(I1,J1)=GM(I1,J1)+SM(I1,J1)
      GC(I1,J1)=GC(I1,J1)+SC(I1,J1)
  150 CONTINUE
      KM1=KM1+2
      KM2=KM2+2
  105 CONTINUE
      RETURN
      END
C*******************************
      SUBROUTINE LOCAT(KR,D2,D12,DRB,DRD,KR1,KR2,DR1,DR2,ED1,ED2,NP)
      REAL KR1,KR2,KR,D(2)
      INTEGER NP(2)
      DO 100 I=1,2
      IF(I.EQ.1) DD=DRB
      IF(I.EQ.2) DD=DRD
      IF(0.0.LT.DD.AND.DD.LE.4.625) GO TO 10
      IF(4.625.LT.DD.AND.DD.LE.9.25) GO TO 20
      IF(9.25.LT.DD.AND.DD.LE.13.875) GO TO 30
      IF(13.875.LT.DD.AND.DD.LE.18.5) GO TO 40
      IF(18.5.LT.DD.AND.DD.LE.23.125) GO TO 50
      IF(23.125.LT.DD.AND.DD.LE.27.75) GO TO 60
   10 LOC=12
      GO TO 120
   20 LOC=11
      D(I)=DD-4.625
      GO TO 120
   30 LOC=10
      D(I)=DD-9.25
      GO TO 120
   40 LOC=9
      D(I)=DD-13.875
      GO TO 120
   50 LOC=8
      D(I)=DD-18.5
      GO TO 120
   60 LOC=7
      D(I)=DD-23.125
  120 NP(I)=LOC
  100 CONTINUE
      KR1=KR*(4.625-D(1))/4.625
      KR2=KR*D(1)/4.625
      DR1=D2*(4.625-D(1))/4.625
```

```
DR2=D2*D(1)/4.625
ED1=D12*(4.625-D(2))/4.625
ED2=D12*D(2)/4.625
RETURN
END
```

APPENDIX C

MASS AND STIFFNESS MATRICES OF AN ELEMENT WITH INERTIA AND SHEAR DEFORMATION EFFECTS INCLUDED

The effects of rotary inertia and shear deformation can be included in the elemental matrices. The expressions for the stiffness matrix [k] and inertia matrix [m] are as follows [45]:

$$[k] = \begin{bmatrix} \frac{12EI}{l^3(1+\phi_s)} & \frac{6EI}{l^2(1+\phi_s)} & -\frac{12EI}{l^3(1+\phi_s)} & \frac{6EI}{l^2(1+\phi_s)} \\ \frac{6EI}{l^2(1+\phi_s)} & \frac{(4+\phi_s)EI}{l(1+\phi_s)} & \frac{6EI}{l^2(1+\phi_s)} & \frac{(2-\phi_s)EI}{l(1+\phi_s)} \\ \frac{12EI}{l^3(1+\phi_s)} & \frac{6EI}{l^2(1+\phi_s)} & \frac{12EI}{l^3(1+\phi_s)} & \frac{6EI}{l^2(1+\phi_s)} \\ \frac{6EI}{l^2(1+\phi_s)} & \frac{(2-\phi_s)EI}{l(1+\phi_s)} & \frac{6EI}{l^2(1+\phi_s)} & \frac{(4+\phi_s)EI}{l(1+\phi_s)} \end{bmatrix}$$

$$[m] = \frac{\rho A l}{(1+\phi_s^2)} \begin{bmatrix} C1 & C2 & C4 & -C5 \\ C2 & C3 & C5 & C6 \\ C4 & C5 & C1 & -C2 \\ -C5 & C6 & -C2 & C3 \end{bmatrix}$$

$$+ \frac{\rho A l}{(1+\phi_s)^2}\left(\frac{r}{l}\right)^2 \begin{bmatrix} C7 & C8 & -C7 & C8 \\ C8 & C9 & -C8 & C10 \\ -C7 & -C8 & C7 & -C8 \\ C8 & C10 & -C8 & C11 \end{bmatrix}$$

Where

$$C1 = \frac{13}{35} + \frac{7}{10}\phi_s + \frac{1}{3}\phi_s^2$$

$$C2 = \left(\frac{11}{210} + \frac{11}{120}\phi_s + \frac{1}{24}\phi_s^2\right)l^2$$

$$C3 = \left(\frac{1}{105} + \frac{1}{60}\phi_s + \frac{1}{120}\phi_s^2\right)l^2$$

$$C4 = \left(\frac{9}{70} + \frac{3}{10}\phi_s + \frac{1}{6}\phi_s^2\right)l^2$$

$$C5 = \left(\frac{13}{420} + \frac{3}{40}\phi_s + \frac{1}{24}\phi_s\right)l^2$$

$$C6 = -\left(\frac{1}{140}+\frac{1}{60}\phi_s+\frac{1}{120}\phi_s+\frac{1}{120}\phi_s^2\right)l^2$$

$$C7 = \frac{6}{5}$$

$$C8 = \left(\frac{1}{10}-\frac{1}{2}\phi_s\right)l$$

$$C9 = \left(\frac{2}{15}+\frac{1}{6}\phi_s+\frac{1}{3}\phi_s^2\right)l^2$$

$$C10 = \left(\frac{-1}{30}-\frac{1}{6}\phi_s+\frac{1}{6}\phi_s^2\right)l^2$$

$$C11 = \left(\frac{2}{15}+\frac{1}{6}\phi_s+\frac{1}{3}\phi_s^2\right)l^2$$

Where $\phi_s = \frac{12EI}{GA\,l^2}$ is the shear-deformation parameter, G is the shear modulus and r is the radius of gyration.