AN INVESTIGATION IN AC/DC LOAD FLOW ANALYSIS USING NEWTON'S METHOD AND EXTENTIONS

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by

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LIST OF SYMBOLS

a	Transformer tap ratio
a_i	Tap ratio of the inverter's transformer
a_r	Tap ratio of the rectifier's transformer
B_{pq}	Reactive portion of the line admittance between bus p and bus q
e_{ba}	Commutation voltage when phases b and a are conducting
e_p	Active portion of the complex bus voltage of bus p
Δe_p	Voltage correction for the active portion of the voltage at bus p
$\Delta e'_p$	Updated voltage correction for the active portion as calculated using the second order correction factors
f_p	Reactive portion of the complex bus voltage of bus p
Δf	Error vector as obtained using Taylor expansion
Δf_p	Voltage correction for the reactive portion at bus p
$\Delta f'_p$	Updated voltage correction for the reactive portion at bus p
G_{pq}	Active portion of the line admittance between bus p and bus q
$\underline{H}_i$	Hessian matrix of second order derivatives
i	Subscript "i" denotes the inverter side of HVDC link
I_D	DC current over the HVDC link

I_D^{sp}	Specified DC current over the HVDC link
I_p	Sum of currents flowing into bus p
I_p^*	Conjugate of I_p
$\underline{J}$	Jacobian matrix of first order derivatives
$K1, K_2$	Constants used in the rectifier and inverter voltage drop equations
M.Q.S.O.N.R.	Modified quasi-second order Newton-Raphson
N.R.	Newton-Raphson
n	Total number of buses in system
P_{D_i}	DC power at inverter
$P_{D_i}^{sp}$	Specified DC power at inverter
ΔP_{D_i}	DC power mismatch at inverter
$\Delta P'_{D_i}$	Updated DC power mismatch at inverter as calculated using the second order correction factors
P_{D_r}	DC power at rectifier
ΔP_{D_r}	DC power mismatch at rectifier
$\Delta P'_{D_r}$	Updated DC power mismatch at rectifier
P_p	Total active power entering bus p
P_{PSCHED}	Scheduled active power at bus p
ΔP_p	Active power mismatch at bus p
$\underline{\Delta P'}$	Updated active power bus mismatch matrix
Q_{D_i}	Reactive power required by inverter
Q_{D_r}	Reactive power required by rectifier
Q_p	Total calculated reactive power at bus p

$X_1, X_2, X_3, X_4,$ X_5, X_6, X_7, X_8	These are eight (8) additional terms (first order derivatives) added to the Jacobian matrix, J, due to the HVDC link
$X'_1, X'_2, X'_3, X'_4,$ X'_5, X'_6, X'_7, X'_8	Updated additional terms in the Jacobian matrix, J, as calculated using the second order correction factors
X_i	Second order reactive power correction term due to inverter
X_{c_i}	Commutating reactance of the inverter's AC source
X_{c_r}	Commutating reactance of the rectifier's AC source
X_r	Second order reactive power correction term due to rectifier
$\underline{\Delta x}$	Increment vector as obtained using Taylor expansion
$\underline{\Delta x}^T$	Transpose of increment vector
Y_{pq}	Sum of all admittances connected between bus p and bus q
Z_i	Second order DC power correction factor for inverter
Z_r	Second order DC power correction factor for rectifier
α	The value of alpha determines the portion of the second order correction factor which is used to update the bus mismatches and the portion used to update the diagonal elements of the AC Jacobian submatrices and also the additional Jacobian elements due to the HVDC link

α_r	Delay angle of rectifier
$\Delta\alpha_r$	Rectifier's delay angle mismatch
$\Delta\alpha_r'$	Updated value of rectifier's delay angle mismatch
β_i	Ignition advance angle of inverter
δ_r	Extinction angle of rectifier
γ_i, δ_i	Extinction angle of inverter
$\Delta\gamma_i$	Inverter's extinction angle mismatch
$\Delta\gamma_i'$	Updated value of inverter's extinction angle mismatch
μ_i	Overlap angle of inverter
μ_i	Overlap angle of rectifier
$\cos\psi_i$	Power factor of inverter
$\cos\psi_r$	Power factor of rectifier
ε	Specified tolerance for convergence

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ABSTRACT

In this thesis AC/DC load flow problem is formulated on the basis of the conditions established by realistic systems. The non-linear AC/DC load flow equations, thus formulated, are solved first by using the Newton-Raphson technique. The method is then extended to using the alpha-modified quasi second order Newton-Raphson (alpha-M.Q.S.O.N.R.) iterative method by including the second order terms of the Taylor series.

The AC/DC load flow equations are developed in rectangular form. Only one equation per converter station is necessary. For one HVDC link, two rows and two columns (with only eight elements) are added to the AC Jacobian matrix of the Newton-Raphson procedure. Suitable algorithms are proposed to obtain the solution.

The proposed algorithms are extensively tested on four different test systems. With the Newton-Raphson method, computations for all test systems coverged in three iterations. The effect of DC link resistance, and initial guesses for voltages and coverter angles on the performance of the Newton-Raphson technique is also investigated. The performance of the alpha-modified quasi second order Newton-Raphson method is analyzed for a range of alpha values. The convergence rate with the alpha-modified quasi second order Newton-Raphson method varies with the value of alpha chosen. In three of the four test systems, the covergence performance of the alpha-modified quasi second order Newton-Raphson method for certain alpha values is better than that of the Newton-Raphson method.

CHAPTER 1

INTRODUCTION

1.1 Background

Solution of the load flow problem is necessary for effective operation of electric power systems; load flow simulations are utilized for day-to-day analysis for network contingency evaluation and for planning network expansion.

In modern power systems the majority of the generating stations are in close proximity to the load centres. Due to the exploitation of these local generating sites and increased pressure from concerned environmental groups, it is becoming increasingly desirable to locate such generating stations far from the population centres. Such expansion into the remote areas results in the transmission of large amounts of power over very long distances. For reasons of economy and system reliability this is often accomplished by High Voltage Direct Current (HVDC) transmission. In order to accommodate DC links the conventional AC load flow solution must be appropriately modified. This modification is due to the dissimilarity in the behaviour of the HVDC and the HVAC transmission networks. Thus it is increasingly important to develop algorithms which will merge the two subsystems to allow for integrated system analysis.

1.2 Scope of the Thesis

In this thesis, an existing AC load flow program, as given by Wellon [29], is extended to account for the presence of HVDC links. A major portion of the work centres around the application of the first order Newton-Raphson method for solving the load flow problem; moreover, the application of the alpha-modified quasi-second order Newton-Raphson (alpha-M.Q.S.O.N.R.) method for AC/DC applications is also investigated. In order to be compatible with the original program, the rectangular coordinate system is adopted. Chapter 2 is the core of the thesis and provides the necessary load flow equations for AC systems having HVDC links. Using these equations, the Newton-Raphson technique is then tested for solving the integrated AC/DC load flow problem.

Application of the alpha-modified quasi-second order Newton-Raphson method for solving the AC/DC load flow problem is investigated in Chapter 3. The Jacobian and the second order terms necessary for the procedure are detailed. Special considerations for the efficient calculation of the additional terms required by the method are also discussed in this chapter.

Chapter 4 is devoted to highlighting the results of extensive testing of the proposed algorithms. First, details of four well known test systems are given. The performance of the first order Newton-Raphson method in solving the four test systems is given next. Included also are the results of sensitivity analyses related to the effect of the initial conditions and the DC link resistance on the convergence of the method. The chapter is concluded with results pertaining to the application of the alpha-modified quasi-second order Newton-Raphson method. Chapter 5 contains concluding remarks and suggestions for future work.

1.3 A Historical Review of AC/DC Load Flow

One of the first papers reporting on accommodation of HVDC links in a standard AC load flow program is that by Barker and Carré [6] in 1966. In their work a successive overrelaxation iterative method has been used to solve the AC/DC load flow equations.

Subsequently, in 1969, Sato and Arrillaga [24] reported on work in which they investigated the effect of HVDC links on the accuracy and convergence rates of the standard AC programs. The method presented involved the simulation of the DC link whose outcome was then utilized in an existing AC load flow program. The algorithms were tested on the AEP 14 Bus Power System. This test system was later on used by many other authors as a standard test system for AC/DC applications.

The work by Braunagel, Kraft and Whysong [7] in 1976 centered around including the equations for DC converters and transmission lines directly in a Newton AC power flow. The Jacobian of the AC system was modified to account for the DC equations. The DC equations were arranged such that the reactive power consumed by the converter was assumed constant. With tolerances of less than 5 MW and 5 Mvars mismatch per bus, it is reported that the addition of the DC equations in the AC Newton power flow did not increase the number of iterations needed for convergence of the AC system alone. The method used by Braunagel, Kraft and Whysong was Newton-Raphson in polar form.

In 1977, Reeve, Fahmy and Stott [21] presented a Newton-based algorithm for solving the load flow problem for multiterminal HVDC systems. The methodology adopted in their work as to alternate between the AC and DC system load flows.

An integrated approach to the AC/DC load flow problem was reported by Arrillaga and Bodger [2] in 1977. The technique presented relied on solving the AC part by means of the fast-decoupled method. It should be noted that the formulation presented in that paper accounted for the transformer reactance twice. A clarification of this point can be found in Arrillaga, Harker and Turner [5] in 1980.

The application of fast decoupled methods to integrated AC-DC systems was given by El-Marsafawy and Mathur [14] in 1979. The DC system was formulated in such a way that any multiterminal system of any configuration could be easily accommodated. The algorithms were tested on a number of test systems. The DC link in each test sytem is delivering a constant MW at the inverter end. It is worth noting that for each of the systems tested four iterations were required to achieve convergence.

In 1981, Ong and Hamzei-nejad [19] presented a method for solving AC/DC load flow problems with a variety of converter controls and operating conditions of the DC system. Their algorithms have the capability to handle discrete tap-step and tap limits of the converter transformer to set overcurrent limit at the DC terminal. The iterations are performed on the DC voltage equations using a digital current reference balancer to update the DC currents. The DC and AC load flows are performed sequentially. The proposed DC algorithms are flexible and can be combined with any existing AC load flow program.

CHAPTER 2

PROBLEM FORMULATION

2.1 Introduction

In its simplest form the AC/DC load flow problem can be represented as a DC link interfacing two functional AC networks. As shown in Figure 2.1, this link has several components with characteristics that contribute additional complexities to the overall load flow problem.

The formulation of equations of an integrated AC/DC system is done in two parts. In section 2.2 equations for these AC nodes (buses) which are not directly connected to the DC link are formulated. In section 2.3 some background pertaining to the operation of converters (rectifier and inverter) is given and the necessary equations at the rectifier and inverter end of the DC link are developed. A brief review of the Newton-Raphson method is given in section 2.4. This is followed by section 2.5 where equations using the Newton-Raphson technique are formulated and the method for solving the integrated AC/DC load flow problem is outlined.

2.2 Equations for AC Buses

2.2.1 Types of Buses and System Modelling

Normally there are three different types of nodes or buses identified in an electric power system. These are: the load bus, the generator or voltage-controlled bus, the slack or swing bus. A bus in an AC system is said to be completely defined if the following four parameters are specified at that bus: total active power (P) entering or leaving the bus, total reactive power (Q) entering or leaving the bus, magnitude of the bus voltage ($|V|$), and angle of the bus voltage (δ). For each bus two of these variables are known and two are unknown. Thus,

for a load bus: P and Q are specified

$|V|$ and δ are unknown

for a generator or voltage-controlled bus:

P and $|V|$ are specified

Q and δ are unknown

At a generator bus the voltage magnitude is controlled by supplying reactive power (Q) from the generator.

for the swing or slack bus:

$|V|$ and δ are specified

P and Q are unknowns

The swing bus is essentially a generator bus and it supplies the difference between the specified real and the calculated reactive power into the system at the other generator buses and the total system output plus losses.

Since the purpose of this thesis is to modify an existing Newton-Raphson AC load flow program so as to accommodate HVDC links, the modelling and programming of the AC system is not described in great detail. The basic AC load flow program considered in this study is developed in [29]. Transmission lines are represented by their Pi-equivalents. If a transformer with off-nominal turns ratio is connected to a line as shown in Figure 2.2, an equivalent Pi-model with parameters as shown in Figure 2.3 is obtained in the program. This equivalent Pi-model is then used in the formulation of the static load flow equations.

2.2.2 Load Flow Equations for AC Buses

To begin with, consider the following well established AC load flow equations. Nodal analysis technique is followed in the formulation of these equations.

At a load bus, p

$$P_p = \sum_{q=1}^{n} \{e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq})\} \quad (2.1)$$

$$Q_p = \sum_{q=1}^{n} \{f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq})\} \quad (2.2)$$

At a voltage controlled bus, c,

$$P_c = \sum_{q=1}^{n} \{e_c (e_q G_{cq} + f_q B_{cq}) + f_c (f_q G_{cq} - e_q B_{cq})\} \quad (2.3)$$

$$|V_c|^2 = e_c^2 + f_c^2 \quad (2.4)$$

where

$V_p \triangleq$ voltage at bus p

$= e_p + j f_p$ (in rectangular coordinates)

$V_q \triangleq$ voltage at bus q

$= e_q + j f_q$

$V_c \triangleq$ voltage at bus c

$= e_c + j f_c$

G_{pq}, $B_{pq} \triangleq$ active and reactive components of the series admittance (y_{pq}) of the line connecting bus p and bus q.

The active and reactive powers at the swing line are determined at the end of the load flow solution since it supplies the system transmission (line and transformer) losses and the difference between the system load and the total generation at the voltage controlled buses. Hence no iterative solution is required for the unknown parameters (P and Q) of the swing bus.

2.3 Equations for AC Buses Interfacing DC Link

2.3.1 Basic Definitions and Assumptions

With reference to Figure 2.1 the DC link is between bus r and bus i of the AC system. The subscripts r and i used here refer to rectifier and inverter, respectively. At bus r (the rectifier bus) the AC power is converted to DC through rectification and is then wheeled over DC transmission lines or cables. At bus i (the inverter bus) DC power is converted back to AC through inversion to be fed into the AC system. The DC transmission scheme could be monopolar, bipolar, or homopolar depending on the economics and the degree of reliability desired.

HVDC converter (rectifier or inverter) circuits are formed by connecting valve groups or bridges in various ways. In modern HVDC systems, two or more bridges in series on the DC side are usually needed for achieving as high a direct voltage as required for economical transmission. The circuit arrangement which is most commonly used in converters is the Three-Phase Two-Way (graetz) Circuit commonly referred to as the Three-Phase Bridge Converter Circuit. This is due to:

(a) low peak inverse voltage (PIV),

(b) low transformer volt-ampere rating for both the primary and secondary sides,

(c) low valve volt-ampere rating, and

(d) the simplest transformer connection.

This circuit arrangement is shown in Figure 2.4. The valves are represented by diode symbols. The direct voltage across the bridge converter is controlled by delaying the firing of the valves. The delay angle, also referred to as firing or ignition angle in HVDC literature, is denoted by α_r and corresponds to a time delay of α_r/ω seconds. In Figure 2.4 the numbers associated with these valves indicate their firing order.

The relationship between the direct voltage (V_D) and the firing angle (α_r), as given in [13], is

$$V_D = \frac{3\sqrt{3}}{\pi} E_m \cos \alpha_r$$

$$= V_{DO} \cos \alpha_r \tag{2.5}$$

where E_m is the maximum value of the AC phase voltage and V_{DO} is the average direct voltage with

$$\alpha_r = 0$$

α_r cannot be greater than one hundred and eighty (180) degrees. For

$$\alpha_r = 90°$$

the direct voltage is zero and for

$$\alpha_r > 90°$$

the direct voltage is negative. Since the current in the valves cannot flow in the reverse direction, this situation ($V_D < 0$) denotes a reversal of power flow and hence inverter operation.

The fundamental component of the AC line current is in phase with the AC source voltage for $\alpha_r = 0$. The converter in this case does not consume any reactive power. With ignition delay current and voltage are no longer in phase and the converter draws reactive power from the AC system.

During the sequential firing of the valves, the current transfer from one valve to another in the same row does not take place simultaneously due to the presence of the leakage inductance in the AC source transformer windings. Thus there is a finite time during which two phases and three valves conduct at the same time: in two of the valves the current diminishes and in the third valve the current increases. This period of commutation is called the angle of overlap and is denoted by μ_r. Thus the conduction begins at

$$\omega t = \alpha_r$$

and ends at

$$\omega t = \delta_r$$

where δ_r is known as the extinction angle and is defined by the following relationship,

$$\delta_r = \alpha_r + \mu_r \qquad (2.6)$$

The symbols α_r and δ_r for delay angle and extinction angle, respectively are used exclusively for rentifier operation where they have values between zero and ninety degrees. These angles hold values ranging between ninety to one hundred and eighty (180) degrees for inverter operation. However, in this case, it is a more common practice to define the ignition advance angle β_i and the extinction angle γ_i by their advance from the instant ($\omega t = 180°$ for ignition of value 3 and extinction of value 1, see Figures 2.4 and 2.5) when the communication voltage (e_{ba}) is zero and decreasing.

The following fundamental assumptions are made in the formulation of the equations representing the AC/DC converter:

(i) The AC source supplies a balanced steady-state sinusoidal voltage and current of constant frequency.

(ii) The voltages and currents of higher harmonics generated by the converter are filtered out and do not appear on the AC side.

(iii) The converter transformers are resistance-free.

(iv) The direct voltage and current are constant and ripple-free.

(v) The converter valves are ideal so that the forward resistance is zero and the inverse resistance is infinite. Also, one assumes that these valves have no arc voltage drop.

(vi) The converter valves ignite at equal intervals of the AC cycle.

2.3.2 AC/DC Converter Model

The basic converter model used in this thesis is shown in Figure 2.6. The area surrounded by the dotted line is the representation of the converter. The converter transformer is an ideal On-Load-Tap-Changing (OLTC) transformer and its leakage reactance is included in the commutating reactance of the converter's (rectifier's or inverter's) AC source.

Consider the single-line representation of a DC line as illustrated in Figure 2.7. The two AC buses between which the DC link exists are bus r and bus i denoting the rectifier end and the inverter end, respectively. The respective AC voltage and current levels at these two buses are

$$|V_r| \angle \theta_r \ , \ |I_r| \angle \phi_r$$

and

$$|V_i| \angle \theta_i \ , \ |I_i| \angle \phi_i$$

The On-Load-Tap-Changing transformers have tap ratios a_r and a_i with taps on the DC side in both cases.

2.3.3 Basic Converter Equations

The well known basic converter equations that have been developed in the literature [13, 18] are given below. These equations are valid only for a single-bridge converter and are in per-unit form. The per-unit system chosen is such that the same base MVA and base voltage are used on the AC and DC sides. Thus

$$(VA_{base})_{AC} = (P_{base})_{AC} = (Q_{base})_{AC} = (P_{base})_{DC} = P_{base} \tag{2.7}$$

$$(V_{base})_{AC} = (V_{base})_{DC} = V_{base} \tag{2.8}$$

$$(I_{base})_{AC} = \frac{P_{base}}{\sqrt{3}\, V_{base}} \tag{2.9}$$

$$(I_{base})_{DC} = \frac{P_{base}}{V_{base}} \tag{2.10}$$

From equations (2.9) and (2.10),

$$(I_{base})_{DC} = \sqrt{3}\,(I_{base})_{AC} \tag{2.11}$$

The direct voltage in terms of the rectifier delay angle, the commutating reactance, and the voltage on the AC side of the rectifier transformer is written as

$$V_{D_r} = K_1\, a_r\, |V_r| \cos \alpha_r - K_2\, I_D\, X_{c_r} \tag{2.12}$$

where

$$K_1 = \frac{3\sqrt{2}}{\pi} \tag{2.13}$$

$$K_2 = \frac{3}{\pi}$$

V_{D_r} is the direct voltage at the rectifier.

α_r is the delay angle of the rectifier.

$|V_r|$ is the magnitude of the AC voltage at bus r.

a_r is the tap ratio of the rectifier transformer.

I_D is the direct current in the link.

X_{c_r} is the commutating reactance of the rectifier's AC source.

The direct voltage at the rectifier may also be written as a function of the rectifier power factor and the rectifier AC bus voltage,

$$V_{D_r} = K_1 \, a_r \, |V_r| \cos \psi_r \tag{2.15}$$

where $\cos \psi_r$ is the power factor of the rectifier.

The direct current in terms of the rectifier parameters and the rectifier AC bus voltage is given by the following relationship:

$$I_D = \frac{a_r \, |V_r|}{\sqrt{2} \, X_{c_r}} [\cos \alpha_r - \cos (\alpha_r + \mu_r)] \tag{2.16}$$

where μ_r is the overlap angle of the rectifier.

The reactive power requirement of a rectifier is dependent predominantly on two factors: the delay angle and the AC bus voltage according to:

$$Q_{D_r} = \frac{3}{2\pi} \frac{a_r^2 \, |V_r|^2}{X_{c_r}} \sin \mu_r \, [1 - \cos (2\alpha_r + \mu_r)] \tag{2.17}$$

Because the inverter model is similar to that of the rectifier, equations similar to (2.12), (2.15), (2.16), and (2.17) apply for the inverter operation as well.

$$V_{D_i} = K_1 \, a_i \, |V_i| \cos \gamma_i - K_2 \, I_D \, X_{c_i} \tag{2.18}$$

$$V_{D_i} = K_1 \, a_i \, |V_i| \cos \psi_i \tag{2.19}$$

$$I_D = \frac{a_i \, |V_i|}{\sqrt{2} \, X_{c_i}} [\cos \gamma_i - \cos (\gamma_i + \mu_i)] \tag{2.20}$$

$$Q_{D_i} = \frac{3}{2\pi} \frac{a_i^2 |V_i|^2}{X_{c_i}} \sin \mu_i [1 - \cos (2\gamma_i + \mu_i)] \quad (2.21)$$

where

V_{D_i} is the direct voltage at the inverter.

γ_i is the extinction angle of the inverter.

$|V_i|$ is the magnitude of the AC voltage at bus i.

a_i is the tap ratio of the inverter transformer.

I_D is the direct current in the link.

X_{c_i} is the commutating reactance of the inverter's AC source.

$\cos \psi_i$ is the power factor of the inverter.

The DC voltage drop across the link is given by:

$$V_{D_r} - V_{D_i} = R_D I_D \quad (2.22)$$

where R_D is the resistance of the DC line.

2.3.4 Load Flow Equations for the Interfacing Buses

For the rectifier AC bus r the following active and reactive balance equations are written:

$$P_r = \sum_{q=1}^{n} \{e_r (e_q G_{rq} + f_q B_{rq}) + f_r (f_q G_{rq} - e_q B_{rq})\} + P_{D_r} \quad (2.23)$$

$$Q_r = \sum_{q=1}^{n} \{f_r (e_q G_{rq} + f_q B_{rq}) - e_r (f_q G_{rq} - e_q B_{rq})\} + Q_{D_r} \quad (2.24)$$

where P_{D_r} is the DC power over the link on the rectifier side, and Q_{D_r} is the reactive power absorbed by the rectifier as given by equation (2.17). P_{D_r} is given by multiplying equation (2.12) by I_D,

$$P_{D_r} = K_1 a_r |V_r| I_D \cos \alpha_r - K_2 I_D^2 X_{c_r} \quad (2.25)$$

Thus, the active power (MW) over the rectifier end of the DC link and the reactive absorption (Mvar) of the rectifier are treated as a load on the rectifier AC bus.

Equations (2.26) and (2.27) provide the active and reactive power balance for the inverter AC bus i.

$$P_i = \sum_{q=1}^{n} \{e_i (e_q G_{iq} + f_q B_{iq}) + f_i (f_q G_{iq} - e_q B_{iq})\} - P_{D_i} \qquad (2.26)$$

$$Q_i = \sum_{q=1}^{n} \{f_i (e_q G_{iq} + f_q B_{iq}) - e_i (f_q G_{iq} - e_q B_{iq})\} + Q_{D_i} \qquad (2.27)$$

where P_{D_i} is the DC power over the link on the inverter side, and Q_{D_i} is the reactive power absorbed by the inverter as given by equation (2.21). As in the rectifier case, P_{D_i} is given by multiplying equation (2.18) by I_d. Hence

$$P_{D_i} = K_1 a_i |V_i| I_D \cos \gamma_i - K_2 I_D^2 X_{c_i} \qquad (2.28)$$

Thus, the representation of the DC link on the inverter side is that of a load which draws negative active power and positive reactive power from the inverter AC bus.

2.4 Review of the Newton-Raphson Method

The Taylor expansion for a multi-variable function f is written as

$$f(x_1 + \Delta x_1, x_2 + \Delta x_2, \ldots)$$

$$= f(x_1, x_2, \ldots) + \frac{1}{1!} \sum_{k=1}^{n} (\Delta x_k) \frac{\partial f}{\partial x_k}$$

$$+ \frac{1}{2!} \sum_{k=1}^{n} \sum_{\ell=1}^{n} (\Delta x_k)(\Delta x_\ell) \frac{\partial^2 f}{\partial x_k \partial x_\ell}$$

$$+ \frac{1}{3!} \sum_{k=1}^{n} \sum_{\ell=1}^{n} \sum_{m=1}^{n} (\Delta x_k)(\Delta x_\ell)(\Delta x_m) \frac{\partial^3 f}{\partial x_k \partial x_\ell \partial x_m} + \ldots \qquad (2.29)$$

As an approximation, the Newton-Raphson Method assumes convergence after the first two terms of the above series. Thus we have

$$f(x_1+\Delta x_1,\ x_2+\Delta x_2,\ \ldots) - f(x_1,\ x_2,\ \ldots) = \sum_{k=1}^{n} (\Delta x_k)\ \frac{\partial f}{\partial x_k} \qquad (2.30)$$

or

$$\Delta f = \sum_{k=1}^{n} (\Delta x_k)\ \frac{\partial f}{\partial x_k} \qquad (2.31)$$

If there are a number of multivariable functions (f_1, f_2, f_3, etc.), equation (2.31) can conveniently be written as

$$\underline{\Delta f} = \underline{J}\ \underline{\Delta x}$$

or

$$\underline{\Delta x} = \underline{J}^{-1}\ \underline{\Delta f} \qquad (2.32)$$

Here $\underline{\Delta x}$ = increment vector

$\underline{J}$ = matrix of first order partial derivative coefficients referred to as the Jacobian matrix

$\underline{\Delta f}$ = error vector

During each iteration equation (2.32) is solved for the increment matrix $\underline{\Delta x}$. Iterations are performed until the solution is reached.

2.5 Formulation of Equations Using Newton-Raphson Method

2.5.1 Formulation for the AC Load Buses

The active and reactive powers entering or leaving a bus in a power system are functions of system bus voltages. Thus, at any bus p,

$$P_p = P_p\ (V_1,\ V_2,\ \ldots) \qquad (2.33)$$

$$Q_p = Q_p\ (V_1,\ V_2,\ \ldots) \qquad (2.34)$$

Applying equation (2.30) to the above equations gives

$$P_p(V_1 + \Delta V_1, V_2 + \Delta V_2, \ldots) - P_p(V_1, V_2, \ldots) = \sum_{k=1}^{n} (\Delta V_k) \frac{\partial P_p}{\partial V_k} \quad (2.35)$$

$$Q_p(V_1 + \Delta V_1, V_2 + \Delta V_2, \ldots) - Q_p(V_1, V_2, \ldots) = \sum_{k=1}^{n} (\Delta V_k) \frac{\partial Q_p}{\partial V_k} \quad (2.36)$$

The left hand sides of equations (2.35) and (2.36) represent the errors or mismatches at bus p in the active power and the reactive power, respectively. Let ΔP_p be the active power mismatch and ΔQ_p be the reactive power mismatch at bus p, and let, in rectangular coordinates,

$$V_k = e_k + j\, f_k \quad (2.37)$$

Then equations (2.35) and (2.36) take the form

$$\Delta P_p = \sum_{k=1}^{n} \left[\frac{\partial P_p}{\partial e_k} \Delta e_k \right] + \sum_{k=1}^{n} \left[\frac{\partial P_p}{\partial f_k} \Delta f_k \right] \quad (2.38)$$

$$\Delta Q_p = \sum_{k=1}^{n} \left[\frac{\partial Q_p}{\partial e_k} \Delta e_k \right] + \sum_{k=1}^{n} \left[\frac{\partial Q_p}{\partial f_k} \Delta f_k \right] \quad (2.39)$$

Equations (2.38) and (2.39) are linear in nature and they express the changes in active and reactive powers in terms of the components of the system bus voltages. For a system of n buses where the nth bus is the swing bus, the set of equations to be solved are written in matrix form as

$$\begin{bmatrix} \Delta P_1 \\ \cdots \\ \Delta P_{n-1} \\ \hline \Delta Q_1 \\ \cdots \\ \Delta Q_{n-1} \end{bmatrix} = \left[\begin{array}{cc|cc} \frac{\partial P_1}{\partial e_1} & \frac{\partial P_1}{\partial e_{n-1}} & \frac{\partial P_1}{\partial f_1} & \frac{\partial P_1}{\partial f_{n-1}} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial P_{n-1}}{\partial e_1} & \frac{\partial P_{n-1}}{\partial e_{n-1}} & \frac{\partial P_{n-1}}{\partial f_1} & \frac{\partial P_{n-1}}{\partial f_{n-1}} \\ \hline \frac{\partial Q_1}{\partial e_1} & \frac{\partial Q_1}{\partial e_{n-1}} & \frac{\partial Q_1}{\partial f_1} & \frac{\partial Q_1}{\partial f_{n-1}} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial Q_{n-1}}{\partial e_1} & \frac{\partial Q_{n-1}}{\partial e_{n-1}} & \frac{\partial Q_{n-1}}{\partial f_1} & \frac{\partial Q_{n-1}}{\partial f_{n-1}} \end{array}\right] \begin{bmatrix} \Delta e_1 \\ \cdots \\ \Delta e_{n-1} \\ \hline \Delta f_1 \\ \cdots \\ \Delta f_{n-1} \end{bmatrix} \quad (2.40)$$

In the above equation the square coefficient matrix whose elements are first order partial derivatives is referred to as the Jacobian matrix. Equation (2.40) is also written as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} \tag{2.41}$$

The elements of J_1, J_2, J_3 and J_4 are derived in Appendix A.

2.5.2. Formulation for the A.C. Voltage-Controlled Buses

The following equations determine a voltage-controlled bus, c,

$$P_c = \sum_{q=1}^{n} \{e_c (e_q G_{cq} + f_q B_{cq}) + f_c (f_q G_{cq} - e_q B_{cq})\} \tag{2.3}$$

$$|V_c|^2 = e_c^2 + f_c^2 \tag{2.4}$$

The active power mismatch (ΔP) relationship as given by equation (2.38), developed in the previous section, would also apply to this case. The voltage mismatch equation is developed using the same procedure and is given as follows:

$$\Delta|V_c|^2 = \sum_{k=1}^{n} [\frac{\partial|V_c|^2}{\partial e_k} \Delta e_k] + \sum_{k=1}^{n} [\frac{\partial|V_c|^2}{\partial f_k} \Delta f_k] \tag{2.42}$$

Hence for a system of n buses where bus n-1 is the voltage-controlled (generator) bus and bus n is the swing bus, the following matrix equation is written:

$$\begin{bmatrix} \Delta P_1 \\ \cdots \\ \Delta P_{n-1} \\ \hline \Delta Q_1 \\ \cdots \\ \Delta Q_{n-2} \\ \hline \Delta|V_{n-1}|^2 \end{bmatrix} = \left[\begin{array}{ccc|ccc} \partial P_1/\partial e_1 & & \partial P_1/\partial e_{n-1} & \partial P_1/\partial f_1 & \cdots & \partial P_1/\partial f_{n-1} \\ \cdots\cdots & & \cdots\cdots & \cdots\cdots & & \cdots\cdots \\ \partial P_{n-1}/\partial e_1 & \cdots & \partial P_{n-1}/\partial e_{n-1} & \partial P_{n-1}/\partial f_1 & & \partial P_{n-1}/\partial f_{n-1} \\ \hline \partial Q_1/\partial e_1 & \cdots & \partial Q_1/\partial e_{n-1} & \partial Q_1/\partial f_1 & \cdots & \partial Q_1/\partial f_{n-1} \\ \cdots\cdots & & \cdots\cdots & \cdots\cdots & & \cdots\cdots \\ \partial Q_{n-2}/\partial e_1 & & \partial Q_{n-2}/\partial e_{n-1} & \partial Q_{n-2}/\partial f_1 & \cdots & \partial Q_{n-2}/\partial f_{n-1} \\ \hline \partial|V_{n-1}|^2/\partial e_1 & & \partial|V_{n-1}|^2/\partial e_{n-1} & \partial|V_{n-1}|^2/\partial f_1 & & \partial|V_{n-1}|^2/\partial f_{n-1} \end{array}\right] \begin{bmatrix} \Delta e_1 \\ \cdots \\ \Delta e_{n-1} \\ \hline \Delta f_1 \\ \cdots \\ \\ \Delta f_{n-1} \end{bmatrix} \qquad (2.43)$$

In condensed form, equation (2.43) is also written as

$$\begin{bmatrix} P \\ \hline Q \\ \hline |V|^2 \end{bmatrix} = \left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \\ \hline J_5 & J_6 \end{array}\right] \begin{bmatrix} e \\ \hline f \end{bmatrix} \qquad (2.44)$$

In the above equation J_1, J_2, J_3, and J_4 are the same as in (2.41). The elements of J_5 and J_6 are also derived in Appendix A.

2.5.3 Formulation for the Interfacing Buses

Since the HVDC link is represented in this thesis as a load on both the rectifier and inverter AC buses, the treatment of these two buses for the Newton-Raphson applications is the same as in the foregoing for the load buses. The equations for the converters and for the interfacing AC buses can be arranged to maintain

(i) constant converter power (P_{D_r} or P_{D_i}), or

(ii) constant DC current (I_D) over the link, or

(iii) constant DC voltage at either end of the link, or

(iv) constant reactive power absorbed either in the rectifier or in the inverter, or

(v) a combination of the above.

In the following formulation power delivered to the AC system at the inverter (P_{D_i}) is maintained while holding fixed the direct current (I_D) over the HVDC link, i.e.

$$P_{D_i} = P_{D_i}^{sp} \tag{2.45}$$

$$I_D = I_D^{sp} \tag{2.46}$$

The subscript 'sp' will be used to signify that a quantity is set at a specified value. The condition set by equations (2.45) and (2.46) is the preferred mode of operation in most electric power utilities with HVDC link. The DC power at the inverter is given by the following simple relationship,

$$P_{D_i}^{sp} = V_{D_i} I_D^{sp} \tag{2.47}$$

which readily gives V_{D_i}, the inverter DC voltage. Thus

$$V_{D_i} = \frac{P_{D_i}^{sp}}{I_D^{sp}} \tag{2.48}$$

In light of the above, equation (2.22) which gives the voltage drop across the link is written as

$$V_{D_r} - \frac{P_{D_i}^{sp}}{I_D^{sp}} = R_D I_D^{sp}$$

or $$V_{D_r} = \frac{P^{sp}_{D_i}}{I^{sp}_D} + R_D\, I^{sp}_D \tag{2.49}$$

I_D and V_{D_r} being known the DC power at the rectifier is

$$P_{D_r} = V_{D_r}\, I^{sp}_D \tag{2.50}$$

Thus the following parameters pertaining to the DC link are now known.

Originally specified: P_{D_i} , I_D

Calculated: V_{D_i} , V_{D_r} , P_{D_r}

It is obvious from the above equations that, once P_{D_i} and I_D are specified, no iterative procedure is required for computing V_{D_i}, V_{D_r}, and P_{D_r}. These variables are computed as per equations (2.48), (2.49), and (2.50) before the iterations begin.

The control of the converters is realized

(A) by varying the delay angle (α_r) of the rectifier

(B) by varying the extinction angle (γ_i) of the inverter, and

(C) by adjusting the taps of the converter on-load-tap-changing transformers (a_r, a_i).

Given α_r, $|V_r|$, γ_i, and $|V_i|$, the transformer tap ratios a_r and a_i are calculated using equations (2.12) and (2.18), respectively.

$$a_r = \frac{V_{D_r} + K_2\, I^{sp}_D\, X_{c_r}}{K_1\, |V_r|\, \cos \alpha_r} \tag{2.51}$$

$$a_i = \frac{V_{D_i} + K_2\, I^{sp}_D\, X_{c_i}}{K_1\, |V_i|\, \cos \gamma_i} \tag{2.52}$$

The active and reactive power balance equations for the rectifier bus, r, and for the inverter bus, i, as a function of system variables are:

$$P_r = P_r\,(V_1, V_2, \ldots) + P_{D_r}\ \text{(calculated)} \tag{2.53}$$

$$Q_r = Q_r\,(V_1, V_2, \ldots) + Q_{D_r}\,(V_r, \alpha_r) \tag{2.54}$$

$$P_i = P_i\,(V_1, V_2, \ldots) - P^{sp}_{D_i} \tag{2.55}$$

$$Q_i = Q_i\,(V_1, V_2, \ldots) + Q_{D_i}\,(V_i, \gamma_i) \tag{2.56}$$

Applying equation (2.30) to the above equations results in

$$P_r(V_1+\Delta V_1, V_2+\Delta V_2, \ldots) - P_r(V_1, V_2, \ldots) = \sum_{k=1}^{n} (\Delta V_k)\,\frac{\partial P_r}{\partial V_k} \tag{2.57}$$

$$[Q_r(V_1+\Delta V_1, V_2+\Delta V_2, \ldots) + Q_{D_r}(V_r+\Delta V_r, \alpha_r+\Delta\alpha_r)]$$

$$- [Q_r(V_1, V_2, \ldots) + Q_{D_r}(V_r, \alpha_r)$$

$$= \{\sum_{k=1}^{n} (\Delta V_k)\,\frac{\partial Q_r}{\partial V_k}\} + (\Delta V_r)\,\frac{\partial Q_{D_r}}{\partial V_r} + (\Delta\alpha_r)\,\frac{\partial Q_{D_r}}{\partial \alpha_r} \tag{2.58}$$

$$P_i(V_1+\Delta V_1, V_2+\Delta V_2, \ldots) - P_i(V_1, V_2, \ldots) = \sum_{k=1}^{n} (\Delta V_k)\,\frac{\partial P_i}{\partial V_k} \tag{2.59}$$

$$[Q_i(V_1+\Delta V_1, V_2+\Delta V_2, \ldots) + Q_{D_i}(V_i+ V_i, \gamma_i+\Delta\gamma_i)]$$

$$- [Q_i(V_1, V_2, \ldots) + Q_{D_i}(V_i, \gamma_i)]$$

$$= \{\sum_{k=1}^{n} (\Delta V_k)\,\frac{\partial Q_i}{\partial V_k}\} + (\Delta V_i)\,\frac{\partial Q_{D_i}}{\partial V_i} + (\Delta\gamma_i)\,\frac{\partial Q_{D_i}}{\partial \gamma_i} \tag{2.60}$$

The left-hand sides of the above equations give the bus mismatches for active and reactive powers. In rectangular coordinates (see (2.37)) these equations may be written as

$$\Delta P_r = \sum_{k=1}^{n} [\frac{\partial P_r}{\partial e_k} \Delta e_k] + \sum_{k=1}^{n} [\frac{\partial P_r}{\partial f_k} \Delta f_k] \quad (2.61)$$

$$\Delta Q_r = \sum_{k=1}^{n} [\frac{\partial Q_r}{\partial e_k} \Delta e_k] + \sum_{k=1}^{n} [\frac{\partial Q_r}{\partial f_k} \Delta f_k]$$

$$+ \frac{\partial Q_{D_r}}{\partial e_r} \Delta e_r + \frac{\partial Q_{D_r}}{\partial f_r} \Delta f_r + \frac{\partial Q_{D_r}}{\partial \alpha_r} \Delta \alpha_r \quad (2.62)$$

$$\Delta P_i = \sum_{k=1}^{n} [\frac{\partial P_i}{\partial e_k} \Delta e_k] + \sum_{k=1}^{n} [\frac{\partial P_i}{\partial f_k} \Delta f_k] \quad (2.63)$$

$$\Delta Q_i = \sum_{k=1}^{n} [\frac{\partial Q_i}{\partial e_k} \Delta e_k] + \sum_{k=1}^{n} [\frac{\partial Q_i}{\partial f_k} \Delta f_k]$$

$$+ \frac{\partial Q_{D_i}}{\partial e_i} \Delta e_i + \frac{\partial Q_{D_i}}{\partial f_i} \Delta f_i + \frac{\partial Q_{D_i}}{\partial \gamma_i} \Delta \gamma_i \quad (2.64)$$

where

ΔP_r is the total active power mismatch at the rectifier bus.

ΔQ_r is the total reactive power mismatch at the rectifier bus.

ΔP_i is the total active power mismatch at the inverter bus.

ΔQ_i is the total reactive power mismatch at the inverter bus.

Two equations, one per converter, are utilized to account for the performance of the rectifier and inverter in the Newton-Raphson method. These are equations (2.25) and (2.28) of Section 2.3.4. Now that I_D is specified, we have

$$P_{D_r} = K_1\, a_r\, |V_r|\, (I_D^{sp}) \cos\alpha_r - K_2\, (I_D^{sp})^2\, X_{c_r} \qquad (2.65)$$

and

$$P_{D_i} = K_1\, a_i\, |V_i|\, (I_D^{sp}) \cos\gamma_i - K_2\, (I_D^{sp})^2\, X_{c_i} \qquad (2.66)$$

Recalling from equations (2.51), and (2.52) of this section that a_r is known once $|V_r|$ and α_r are known, and a_i is known once $|V_i|$ and γ_i are known, hence the only unknowns in the above equations are: $|V_r|$, α_r in (2.65), and $|V_i|$, γ_i in (2.66). Thus, we have

$$P_{D_r} = P_{D_r}(e_r, f_r, \alpha_r) \qquad (2.67)$$

$$P_{D_i} = P_{D_i}(e_i, f_i, \gamma_i) \qquad (2.68)$$

Using Taylor expansion as a basis and executing the procedure outlined previously in this section, the following equations are derived.

$$\Delta P_{D_r} = \frac{\partial P_{D_r}}{\partial e_r}\Delta e_r + \frac{\partial P_{D_r}}{\partial f_r}\Delta f_r + \frac{\partial P_{D_r}}{\partial \alpha_r}\Delta\alpha_r \qquad (2.69)$$

$$\Delta P_{D_i} = \frac{\partial P_{D_i}}{\partial e_i}\Delta e_i + \frac{\partial P_{D_i}}{\partial f_i}\Delta f_i + \frac{\partial P_{D_i}}{\partial \gamma_i}\Delta\gamma_i \qquad (2.70)$$

This completes the formulation using Newton-Raphson method for a system having HVDC link. Consider a power system of n buses with the following description:

(A) There are m voltage-controlled buses, one of which (the nth bus) is the swing bus.

(B) The HVDC link is between the bus r (rectifier) and bus i (inverter) of the system.

The matrix equation (2.71) for this system is shown on the following page. In condensed form, equation (2.71) is written as shown in equation (2.72).

$$
\begin{bmatrix}
\Delta P_1 \\ \cdots \\ \Delta P_r \\ \Delta P_i \\ \cdots \\ \Delta P_{n-1} \\ \Delta Q_1 \\ \cdots \\ \Delta Q_r \\ \Delta Q_i \\ \cdots \\ \Delta Q_{n-m} \\ \Delta|V_{n-m+1}|^2 \\ \cdots \\ \Delta|V_{n-1}|^2 \\ \Delta P_{D_r} \\ \Delta P_{D_i}
\end{bmatrix}
=
\left[\begin{array}{cccc|cccc|cc}
\frac{\partial P_1}{\partial e_1}\cdots & & & \cdots\frac{\partial P_1}{\partial e_{n-1}} & \frac{\partial P_1}{\partial f_1}\cdots & & & \cdots\frac{\partial P_1}{\partial f_{n-1}} & & \\
 & \frac{\partial P_r}{\partial e_r} & & & & \frac{\partial P_r}{\partial f_r} & & & & \\
 & & \frac{\partial P_i}{\partial e_i} & & & & \frac{\partial P_i}{\partial f_i} & & & \\
\frac{\partial P_{n-1}}{\partial e_1}\cdots & & & \cdots\frac{\partial P_{n-1}}{\partial e_{n-1}} & \frac{\partial P_{n-1}}{\partial f_1} & & & \cdots\frac{\partial P_{n-1}}{\partial f_{n-1}} & & \\
\hline
\frac{\partial Q_1}{\partial e_1}\cdots & & & \cdots\frac{\partial Q_1}{\partial e_{n-1}} & \frac{\partial Q_1}{\partial f_1}\cdots & & & \cdots\frac{\partial Q_1}{\partial f_{n-1}} & & \\
 & \frac{\partial Q_r}{\partial e_r}+\frac{\partial Q_{D_r}}{\partial e_r} & & & & \frac{\partial Q_r}{\partial f_r}+\frac{\partial Q_{D_r}}{\partial f_r} & & & \frac{\partial Q_{D_r}}{\partial \alpha_r} & \\
 & & \frac{\partial Q_i}{\partial e_i}+\frac{\partial Q_{D_i}}{\partial e_i} & & & & \frac{\partial Q_i}{\partial f_i}+\frac{\partial Q_{D_i}}{\partial f_i} & & & \frac{\partial Q_{D_i}}{\partial \gamma_i} \\
\frac{\partial Q_{n-m}}{\partial e_1}\cdots & & & \cdots\frac{\partial Q_{n-m}}{\partial e_{n-1}} & \frac{\partial Q_{n-m}}{\partial f_1}\cdots & & & \cdots\frac{\partial Q_{n-m}}{\partial f_{n-1}} & & \\
\hline
\frac{\partial|V_{n-m+1}|^2}{\partial e_1}\cdots & & & \cdots\frac{\partial|V_{n-m+1}|^2}{\partial e_{n-1}} & \frac{\partial|V_{n-m+1}|^2}{\partial f_1}\cdots & & & \cdots\frac{\partial|V_{n-m+1}|^2}{\partial f_{n-1}} & & \\
\cdots & & & \cdots & \cdots & & & \cdots & & \\
\frac{\partial|V_{n-1}|^2}{\partial e_1}\cdots & & & \cdots\frac{\partial|V_{n-1}|^2}{\partial e_{n-1}} & \frac{\partial|V_{n-1}|^2}{\partial f_1}\cdots & & & \cdots\frac{\partial|V_{n-1}|^2}{\partial f_{n-1}} & & \\
\hline
 & \frac{\partial P_{D_r}}{\partial e_r} & & & & \frac{\partial P_{D_r}}{\partial f_r} & & & \frac{\partial P_{D_r}}{\partial \alpha_r} & \\
 & & \frac{\partial P_{D_i}}{\partial e_i} & & & & \frac{\partial P_{D_i}}{\partial f_i} & & & \frac{\partial Q_{D_i}}{\partial \gamma_i}
\end{array}\right]
\begin{bmatrix}
\Delta e_1 \\ \cdots \\ \Delta e_r \\ \Delta e_i \\ \cdots \\ \Delta e_{n-1} \\ \Delta f_1 \\ \cdots \\ \Delta f_r \\ \Delta f_i \\ \cdots \\ \Delta f_{n-1} \\ \Delta \alpha_r \\ \Delta \gamma_i
\end{bmatrix}
\tag{2.71}
$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta |V|^2 \\ \Delta P_{D_r} \\ \Delta P_{D_i} \end{bmatrix} = \begin{bmatrix} J_1 & J_2 & & \\ \mathcal{J}_3 & \mathcal{J}_4 & X_5 & X_7 \\ J_5 & J_6 & & \\ X_1 & X_3 & X_6 & \\ X_2 & X_4 & & X_8 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \\ \Delta \alpha_r \\ \Delta \gamma_i \end{bmatrix} \qquad (2.72)$$

It is noted that the portion of equation (2.72) surrounded by thick lines corresponds to AC equation (2.44) with modified J_3 and J_4. There are eight additional terms added to the Jacobian matrix and two additional terms to the error vector and the increment vector.

Comparing equations (2.71) and (2.72), we have

$$X_1 = \frac{\partial P_{D_r}}{\partial e_r} \qquad (2.73)$$

$$X_2 = \frac{\partial P_{D_i}}{\partial e_i} \qquad (2.74)$$

$$X_3 = \frac{\partial P_{D_r}}{\partial f_r} \qquad (2.75)$$

$$X_4 = \frac{\partial P_{D_i}}{\partial f_i} \qquad (2.76)$$

$$X_5 = \frac{\partial Q_{D_r}}{\partial \alpha_r} \qquad (2.77)$$

$$X_6 = \frac{\partial P_{D_r}}{\partial \alpha_r} \qquad (2.78)$$

$$X_7 = \frac{\partial Q_{D_i}}{\partial \gamma_i} \qquad (2.79)$$

$$X_8 = \frac{\partial P_{D_i}}{\partial \gamma_i} \tag{2.80}$$

$$\mathcal{J}_3(r,r) = \frac{\partial Q_r}{\partial e_r} + \frac{\partial Q_{D_r}}{\partial e_r} \tag{2.81}$$

$$\mathcal{J}_3(i,i) = \frac{\partial Q_i}{\partial e_i} + \frac{\partial Q_{D_i}}{\partial e_i} \tag{2.82}$$

$$\mathcal{J}_4(r,r) = \frac{\partial Q_r}{\partial f_r} + \frac{\partial Q_{D_r}}{\partial f_r} \tag{2.83}$$

$$\mathcal{J}_4(i,i) = \frac{\partial Q_i}{\partial f_i} + \frac{\partial Q_{D_i}}{\partial f_i} \tag{2.84}$$

The new elements of the Jacobian matrix as well as those which modify J_3 and J_4 are derived in Appendix B.

2.5.4 Outline of the Method

The iterative computations of the load flow equations utilizing the Newton-Raphson method proceed in the following steps:

Step 1 - Assemble the bus admittance matrix [Y]

Step 2 - Calculate V_{D_i}, V_{D_r}, and P_{D_r} using equations (2.48), (2.49) and (2.50), respectively.

Step 3 - Initialize

(A) real and imaginary components of the bus voltages, e_p and f_p

(B) reactive powers (Q_c) at the voltage controlled buses

(C) delay angle (α_r) of the rectifier, and

(D) extinction angle (γ_i) of the inverter.

Step 4 - Calculate

P_p using equation (2.1)

Q_p using equation (2.2)

$|V_c|^2$ using equation (2.4)

a_r using equation (2.51)

a_i using equation (2.52)

Q_{D_r} using equation (2.17)

Q_{D_i} using equation (2.21)

P_{D_r} using equation (2.25) and

P_{D_i} using equation (2.28).

Step 5 - Calculate errors using the following equations:

$$\Delta P_p = P_{pSCHED} - P_p \tag{2.85}$$

$$\Delta P_r = P_{\gamma SCHED} - P_r - P_{D_r} \text{ (known from Step 2)} \tag{2.86}$$

$$\Delta P_i = P_{iSCHED} - P_i + P_{D_i}^{sp} \tag{2.87}$$

$$\Delta Q_p = Q_{pSCHED} - Q_p \tag{2.88}$$

$$\Delta Q_r = Q_{rSCHED} - Q_r - Q_{D_r} \text{ (calculated in Step 4)} \tag{2.89}$$

$$\Delta Q_i = Q_{iSCHED} - Q_i - Q_{D_i} \text{ (calculated in Step 4)} \tag{2.90}$$

$$\Delta|V_c|^2 = |V_{cSCHED}|^2 - |V_c|^2 \tag{2.91}$$

$$\Delta P_{D_r} = P_{D_r} \text{ (known from Step 2)} - P_{D_r} \text{ (calculated in Step 4)} \tag{2.92}$$

$$\Delta P_{D_i} = P_{D_i}^{sp} - P_{D_i} \text{ (calculated in Step 4)} \tag{2.93}$$

Step 6 - Test for convergence. If ΔP_p, ΔP_r, ΔP_i, ΔQ_p, ΔQ_r, ΔQ_i, $\Delta|V_c|^2$, ΔP_{D_r}, and ΔP_{D_i} are all less than the specified tolerance (ε), solution is attained. If not, continue.

Step 7 - Calculate the elements of the Jacobian matrix.

Step 8 - Solve the matrix equation (2.72) for Δe, Δf, $\Delta \alpha_r$ and $\Delta \gamma_i$. Thus, compute

$$\begin{bmatrix} \Delta e \\ \hdashline \Delta f \\ \hdashline \Delta \alpha_r \\ \hdashline \Delta \gamma_i \end{bmatrix} = \begin{bmatrix} J_1 & J_2 & & \\ J_3 & J_4 & X_5 & X_7 \\ J_5 & J_6 & & \\ X_1 & X_3 & X_6 & \\ X_2 & X_4 & & X_8 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \hdashline \Delta Q \\ \hdashline \Delta |V|^2 \\ \hdashline \Delta P_{D_r} \\ \hdashline \Delta P_{D_i} \end{bmatrix} \tag{2.94}$$

Step 9 - Update the bus voltages (e and f) and the converter angles (α_r and γ_i).

$$e^{(j+1)} = e^{(j)} + \Delta e^{(j)} \tag{2.95}$$

$$f^{(j+1)} = f^{(j)} + \Delta f^{(j)} \tag{2.96}$$

$$\alpha_r^{(j+1)} = \alpha_r^{(j)} + \Delta \alpha_r^{(j)} \tag{2.97}$$

$$\gamma_i^{(j+1)} = \gamma_i^{(j)} + \Delta \gamma_i^{(j)} \tag{2.98}$$

Step 10 - Return to Step 4 for the next iteration.

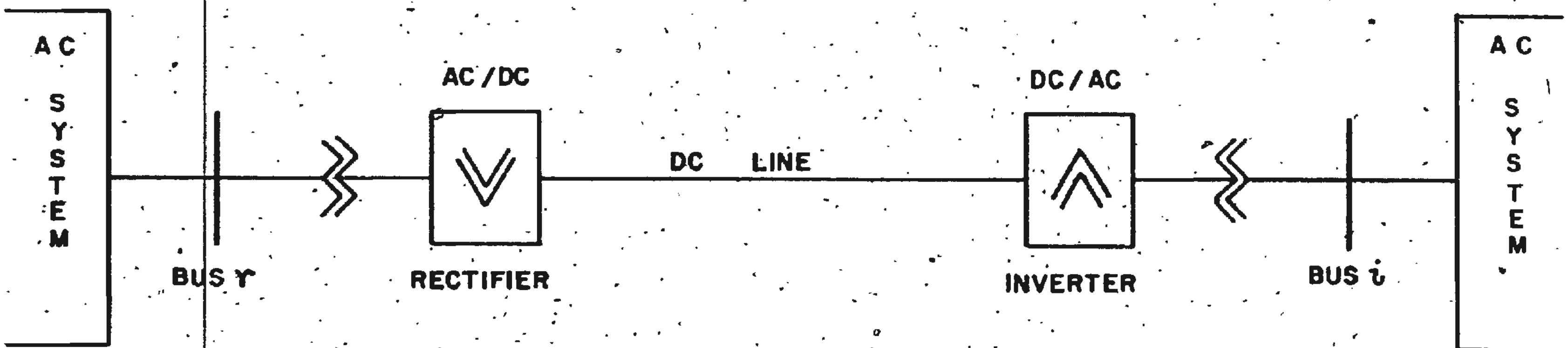


FIGURE 2.1 Representation of a HVDC Link in an AC System

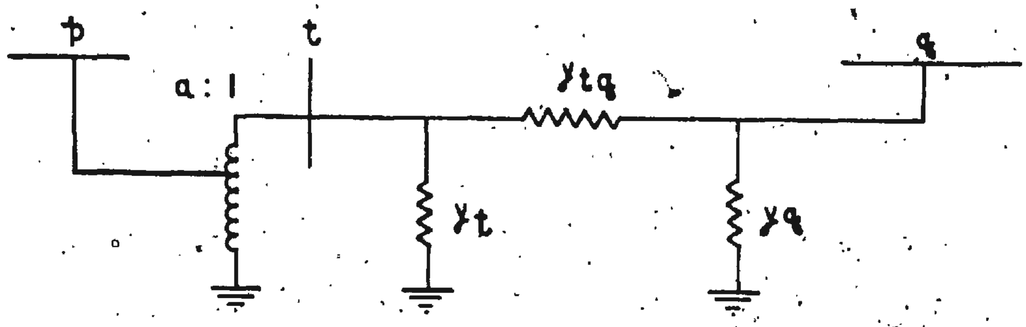


FIGURE 2.2 Transformer with Off-Nominal Turns Ratio and a Transmission Line

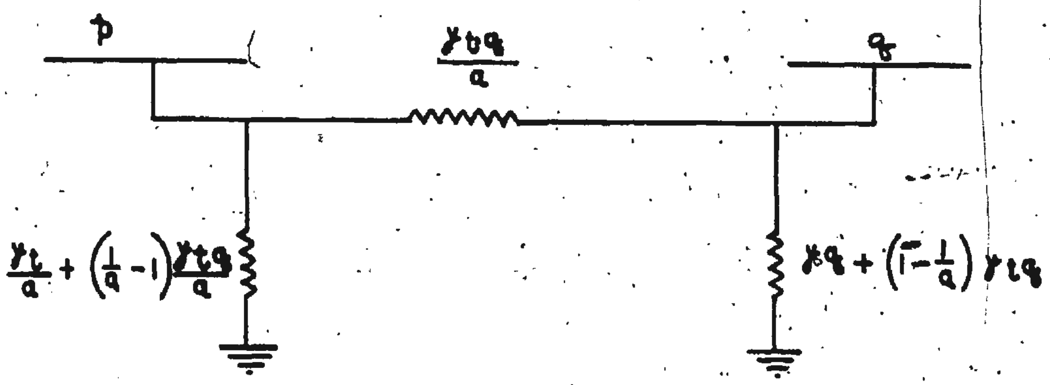


FIGURE 2.3 Equivalent Pi-model of the Line with the Transformer of Figure 2-2 in Terms of Admittance and Off-Nominal Turns Ratio

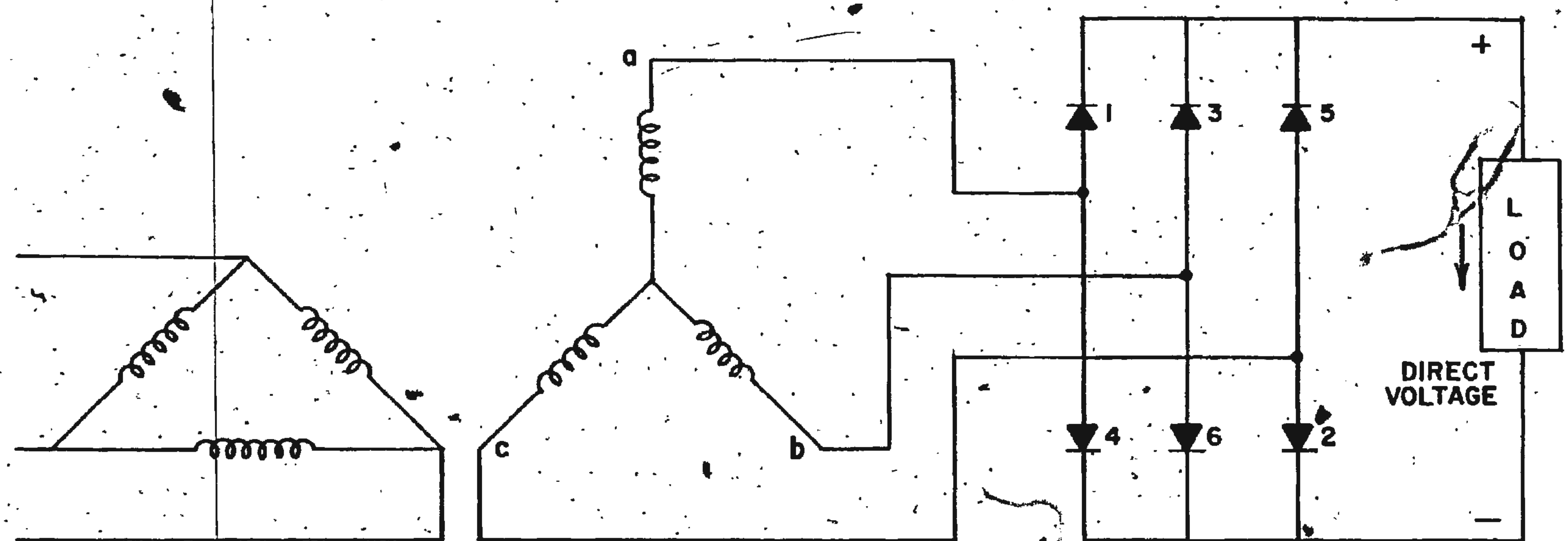


FIGURE 2.4 Three-Phase Bridge Converter Circuit

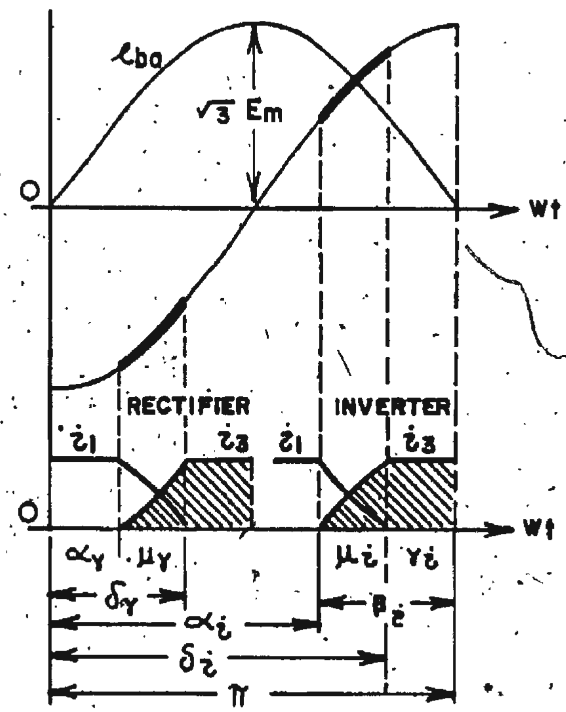


FIGURE 2.5 : Definition of Angles for Converter Operation

i_1 = current in valve 1

i_3 = current in valve 3

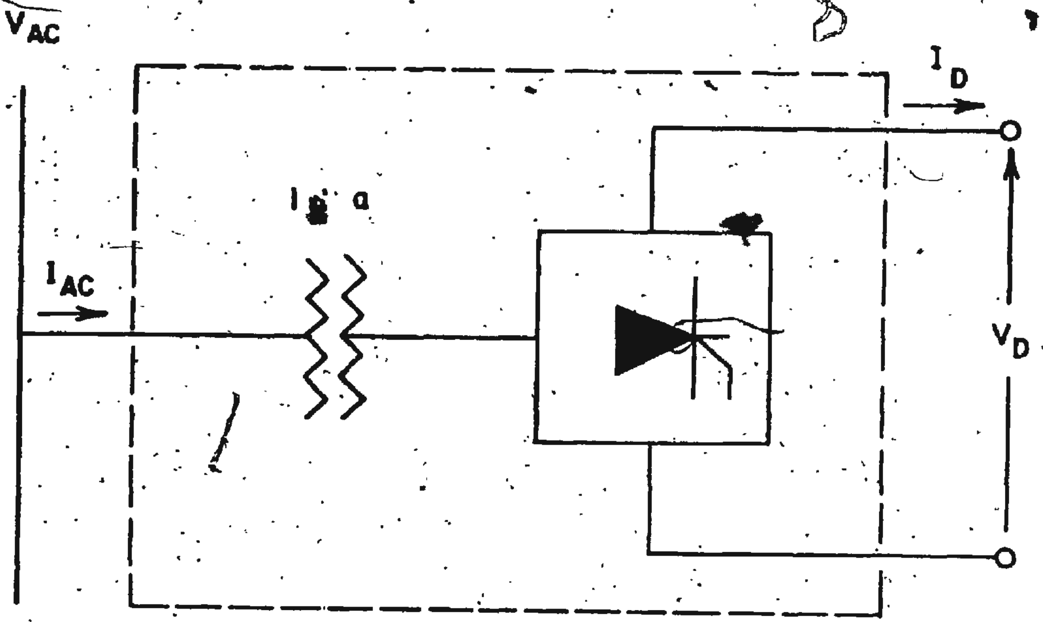


FIGURE 2.6 Bridge Converter Model (Single Phase Representation)

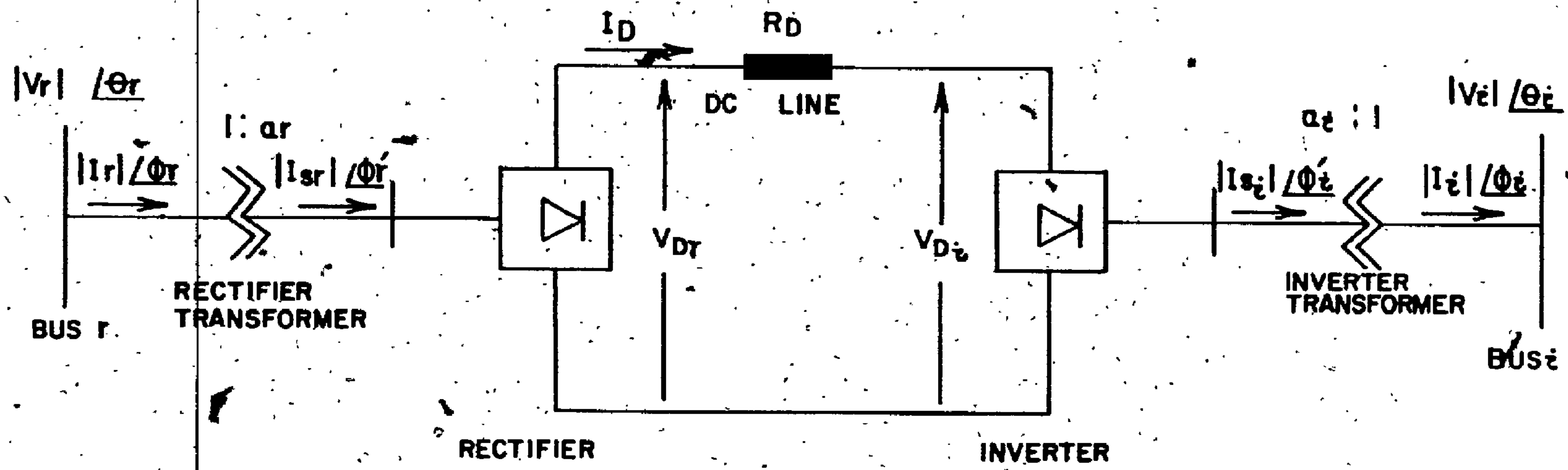


FIGURE 2.7 Single Line Representation of a DC Link

CHAPTER 3

APPLICATION OF ALPHA-MODIFIED QUASI-SECOND ORDER NEWTON-RAPHSON METHOD

3.1 Introduction

The theory of the alpha-modified quadi-second order Newton-Raphson method for solving AC load flow problems is fully developed in [29]. This theory is extended in this chapter to the analysis of integrated AC/DC systems.

Section 3.2 of this chapter explains the underlying concept of the alpha-modified quasi-second order Newton-Raphson method. The formulation of the AC/DC load flow equations using this method is dealt with in Section 3.3.1 and, in the last section, a step-by-step outline of the method is discussed.

3.2 Basic Concept of Alpha-Modified Quasi-Second Order Newton-Raphson Method

The second order Newton-Raphson method, an extension of the Newton-Raphson method, is obtained by utilizing the first and second order terms of the Taylor expansion. Thus

$$f_m(x_1+\Delta x_1,\ x_2+\Delta x_2,\ \ldots) = f_m(x_1,\ x_2,\ \ldots) + \sum_{k=1}^{n} \frac{\partial f_m}{\partial x_k} (\Delta x_k) + \frac{1}{2} \sum_{k=1}^{n} \sum_{\ell=1}^{n} \frac{\partial^2 f_m}{\partial x_k\ \partial x_\ell} (\Delta x_k)(\Delta x_\ell) \qquad (3.1)$$

For a number of multivariable functions (f_1, f_2, etc.), equation (3.1) can be written as

$$\underline{\Delta f}_m = \underline{J}_m \underline{\Delta x} + \frac{1}{2} \underline{\Delta x}^T \underline{H}_m \underline{\Delta x} \tag{3.2}$$

where

Δf_m = error in mth component

$\underline{J}_m$ = mth row of the Jacobian matrix of the first order elements

$\underline{H}_m$ = Hessian matrix whose elements are the second order partial derivatives of f_m

$\underline{\Delta x}$ = increment vector

The underlying concept of an alpha-modified second order Newton-Raphson method can be exhibited by rewriting equation (3.2) as follows:

$$\Delta f_m - \alpha \left(\frac{1}{2} \underline{\Delta x}^T \underline{H}_m \underline{\Delta x}\right) = \{\underline{J}_m + (1-\alpha) \left(\frac{1}{2} \underline{\Delta x}^T \underline{H}_m\right)\} \underline{\Delta x} \tag{3.3}$$

Thus the increment vector Δx is written in vector form as

$$\underline{\Delta x} = \{\underline{J} + (1-\alpha) \left(\frac{1}{2} \underline{\Delta x}^T \underline{H}\right)\}^{-1} \{\Delta f - \alpha \left(\frac{1}{2} \underline{\Delta x}^T \underline{H} \underline{\Delta x}\right)\} \tag{3.4}$$

Comparing equation (3.4) with equation (2.32), it is noted that in the alpha-modifed second order technique, the Jacobian matrix, J, as well as the error vector, Δf, are modified. In equation (3.4), for $\alpha = 1.0$ only the error vector (Δf) is modified. In the literature, this particular case is referred to as the quasi-second order Newton-Raphson (Q.S.O.N.R.) method. A number of possible methods of implementing the second order correction factors to the original Newton-Raphson technique based on equation (3.2) are described in [29]. On such method is shown above and is considered further in this thesis for AC/DC applications.

3.3 Application of Alpha-Modified Quasi-Second Order Newton-Raphson (α-M.Q.S.O.N.R.) Method

3.3.1. Formulation

Equation (3.3) of the previous section is the basis for developing the load flow equations to be used in the α-M.Q.S.O.N.R. method. For compatibility the same notations as used in [29] are followed. First, we apply equation (3.3) to the AC buses (excluding those two which interface the HVDC link). Thus,

$$\Delta P_p - \alpha R_p = \sum_{k=1}^{n-1} \left(\frac{\partial P_p}{\partial e_k} \Delta e_k \right) + \sum_{k=1}^{n-1} \left(\frac{\partial P_p}{\partial f_k} \Delta f_k \right) + (1-\alpha)\, R_p \qquad (3.5)$$

$$\Delta Q_p - \alpha T_p = \sum_{k=1}^{n-1} \left(\frac{\partial Q_p}{\partial e_k} \Delta e_k \right) + \sum_{k=1}^{n-1} \left(\frac{\partial Q_p}{\partial f_k} \Delta f_k \right) + (1-\alpha)\, T_p \qquad (3.6)$$

$$\Delta |V_c|^2 - \alpha U_c = \sum_{k=1}^{n-1} \left(\frac{\partial |V_c|^2}{\partial e_k} \Delta e_k \right) + \sum_{k=1}^{n-1} \left(\frac{\partial |V_c|^2}{\partial f_k} \Delta f_k \right) + (1-\alpha)\, U_c \qquad (3.7)$$

In the above equations R_p, T_p and U_c are the second order active power, reactive power, and voltage squared correction factors, respectively, and are given as

$$R_p = \Delta e_p\, (CR1)_p + \Delta f_p\, (CR2)_p \qquad (3.8)$$

$$T_p = \Delta f_p\, (CR1)_p - \Delta e_p\, (CR2)_p \qquad (3.9)$$

$$U_c = \Delta e_c\, (\Delta e_c) + \Delta f_c\, (\Delta f_c) \qquad (3.10)$$

where

$$(CR1)_p = \sum_{k=1}^{n-1} (B_{pk}\, \Delta f_k + G_{pk}\, \Delta e_k) \qquad (3.11)$$

$$(CR2)_p = \sum_{k=1}^{n-1} (G_{pk}\, \Delta f_k - B_{pk}\, \Delta e_k) \qquad (3.12)$$

Substituting equations (3.8), (3.9), and (3.10) into equations (3.5), (3.6), and (3.7), we get

$$\Delta P_p - \alpha[\Delta e_p (CR1)_p + \Delta f_p (CR2)_p]$$

$$= \sum_{\substack{k=1 \\ k\neq p}}^{n-1} \left(\frac{\partial P_p}{\partial e_k} \Delta e_k \right) + \sum_{\substack{k=1 \\ k\neq p}}^{n-1} \left(\frac{\partial P_p}{\partial f_i} \Delta f_i \right)$$

$$+ \frac{\partial P_p}{\partial e_p} \Delta e_p + \frac{\partial P_p}{\partial f_p} \Delta f_p + (1-\alpha) [\Delta e_p (CR1)_p + \Delta f_p (CR2)_p]$$

$$= \sum_{\substack{k=1 \\ k\neq p}}^{n-1} \left(\frac{\partial P_p}{\partial e_k} \Delta e_k \right) + \left[\frac{\partial P_p}{\partial e_p} + (1-\alpha) (CR1)_p \right] \Delta e_p$$

$$+ \sum_{\substack{k=1 \\ k\neq p}}^{n-1} \left(\frac{\partial P_p}{\partial f_k} \Delta f_k \right) + \left[\frac{\partial P_p}{\partial f_p} + (1-\alpha) (CR2)_p \right] \Delta f_p \qquad (3.13)$$

$$\Delta Q_p - \alpha[\Delta f_p (CR1)_p - \Delta e_p (CR2)_p]$$

$$= \sum_{\substack{k=1 \\ k\neq p}}^{n-1} \left(\frac{\partial Q_p}{\partial e_k} \Delta e_k \right) + \left[\frac{\partial Q_p}{\partial e_p} - (1-\alpha) (CR2)_p \right] \Delta e_p$$

$$+ \sum_{\substack{k=1 \\ k\neq p}}^{n-1} \left(\frac{\partial Q_p}{\partial f_k} \Delta f_k \right) + \left[\frac{\partial Q_p}{\partial f_p} + (1-\alpha) (CR1)_p \right] \Delta f_p \qquad (3.14)$$

$$\Delta |V_c|^2 - \alpha[\Delta e_c (\Delta e_c) + \Delta f_c (\Delta f_c)]$$

$$= \sum_{\substack{k=1 \\ k\neq c}}^{n-1} \left(\frac{\partial |V_c|^2}{\partial e_k} \Delta e_k \right) + \left[\frac{\partial |V_c|^2}{\partial e_c} + (1-\alpha)(\Delta e_c) \right] \Delta e_c$$

$$+ \sum_{\substack{k=1 \\ k\neq c}}^{n-1} \left(\frac{\partial |V_c|^2}{\Delta f_k} \Delta f_k \right) + \left[\frac{\partial |V_c|^2}{\partial f_c} + (1-\alpha) (\Delta f_c) \right] \Delta f_c \qquad (3.15)$$

At the rectifier AC bus, r, the following two equations are written:

$$\Delta P_r - \alpha R_r = \sum_{k=1}^{n-1} \left(\frac{\partial P_r}{\partial e_k} \Delta e_k \right) + \sum_{k=1}^{n-1} \left(\frac{\partial P_r}{\partial f_k} \Delta f_k \right) + (1-\alpha)\ R_r \tag{3.16}$$

$$\Delta Q_r - \alpha(T_r+X_r) = \sum_{k=1}^{n-1} \left(\frac{\partial Q_r}{\partial e_k} \Delta e_k \right) + \sum_{k=1}^{n-1} \left(\frac{\partial Q_r}{\partial f_k} \Delta f_k \right)$$

$$+ \frac{\partial Q_{D_r}}{\partial e_r} \Delta e_r + \frac{\partial Q_{D_r}}{\partial f_r} \Delta f_r + \frac{\partial Q_{D_r}}{\partial \alpha_r} \Delta \alpha_r$$

$$+ (1-\alpha)\ (T_r+X_r) \tag{3.17}$$

In the above equations R_r and (T_r+X_r) are the second order active and reactive power correction factors, respectively. The factor X_r is due to the reactive power consumption of the rectifier. These factors are defined as follows:

$$R_r = \Delta e_r\ (CR1)_r + \Delta f_r\ (CR2)_r \tag{3.18}$$

$$T_r = \Delta f_r\ (CR1)_r - \Delta e_r\ (CR2)_r \tag{3.19}$$

$$X_r = \Delta e_r\ (XDUM9) + \Delta f_r\ (XDUM11) + \Delta \alpha_r\ (XDUM5) \tag{3.20}$$

where

$$(CR1)_r = \sum_{k=1}^{n-1} (B_{rk}\ \Delta f_k + G_{rk}\ \Delta e_k) \tag{3.21}$$

$$(CR2)_r = \sum_{k=1}^{n-1} (G_{rk}\ \Delta f_k - B_{rk}\ \Delta e_k) \tag{3.22}$$

$$XDUM9 = \Delta f_r\ (XDC34) + \frac{1}{2} \Delta e_r\ (XDC31) \tag{3.23}$$

$$XDUM11 = \Delta \alpha_r\ (XDC36) + \frac{1}{2} \Delta f_r\ (XDC32) \tag{3.24}$$

$$XDUM5 = \Delta e_r\ (XDC35) + \frac{1}{2} \Delta \alpha_r\ (XDC33) \tag{3.25}$$

The factors used in equations (3.23), (3.24), and (3.25) are defined by the following equations:

$$XDC31 = \frac{\partial^2 Q_{D_r}}{\partial e_r^2} \tag{3.26}$$

$$XDC32 = \frac{\partial^2 Q_{D_r}}{\partial f_r^2} \tag{3.27}$$

$$XDC33 = \frac{\partial^2 Q_{D_r}}{\partial \alpha_r^2} \tag{3.28}$$

$$XDC34 = \frac{\partial^2 Q_{D_r}}{\partial e_r \, \partial f_r} \tag{3.29}$$

$$XDC35 = \frac{\partial^2 Q_{D_r}}{\partial e_r \, \partial \alpha_r} \tag{3.30}$$

$$XDC36 = \frac{\partial^2 Q_{D_r}}{\partial f_r \, \partial \alpha_r} \tag{3.31}$$

The second order partial differential coefficients as expressed by equations (3.26) through (3.31) are derived in Appendix B. Equation (3.18) when substituted into equation (3.16) gives:

$$\begin{aligned} &\Delta P_r - \alpha[\Delta e_r (CR1)_r + \Delta f_r (CR2)_r] \\ &= \sum_{\substack{k=1 \\ k\neq r}}^{n-1} \left(\frac{\partial P_r}{\partial e_k} \Delta e_k \right) + \left[\frac{\partial P_r}{\partial e_r} + (1-\alpha) (CR1)_r \right] \Delta e_r \\ &+ \sum_{\substack{k=1 \\ k\neq r}}^{n-1} \left(\frac{\partial P_r}{\partial f_k} \Delta f_k \right) + \left[\frac{\partial P_r}{\partial f_r} + (1-\alpha) (CR2)_r \right] \Delta f_r \end{aligned} \tag{3.32}$$

Substituting equations (3.19) and (3.20) into equation (3.17), we get

$$\Delta Q_r - \alpha[\Delta f_r\ (CR1)_r - \Delta e_r\ (CR2)_r]$$

$$- \alpha[\Delta e_r\ (XDUM9) + \Delta f_r\ (XDUM11) + \Delta\alpha_r\ (XDUM5)]$$

$$= \sum_{\substack{k=1\\k\neq r}}^{n-1} \left(\frac{\partial Q_r}{\partial e_k}\Delta e_k\right) + \sum_{\substack{k=1\\k\neq r}}^{n-1} \left(\frac{\partial Q_r}{\partial f_k}\Delta f_k\right)$$

$$+ \frac{\partial Q_r}{\partial e_r}\Delta e_r + \frac{\partial Q_r}{\partial f_r}\Delta f_r + \frac{\partial Q_{D_r}}{\partial e_r}\Delta e_r + \frac{\partial Q_{D_r}}{\partial f_r}\Delta f_r$$

$$+ \frac{\partial Q_{D_r}}{\partial \alpha_r}\Delta\alpha_r + (1-\alpha)\ [\Delta f_r\ (CR1)_r - \Delta e_r\ (CR2)_r]$$

$$+ (1-\alpha)\ [\Delta e_r\ (XDUM9) + \Delta f_r\ (XDUM11) + \Delta\alpha_r\ (XDUM5)]$$

$$= \sum_{\substack{k=1\\k\neq r}}^{n-1} \left(\frac{\partial Q_r}{\partial e_k}\Delta e_k\right) + \left[\frac{\partial Q_r}{\partial e_r} + \frac{\partial Q_{D_r}}{\partial e_r} - (1-\alpha)\ (CR2)_r + (1-\alpha)\ (XDUM9)\right]\Delta e_r$$

$$+ \sum_{\substack{k=1\\k\neq r}}^{n-1} \left(\frac{\partial Q_r}{\partial f_k}\Delta f_k\right) + \left[\frac{\partial Q_r}{\partial f_r} + \frac{\partial Q_{D_r}}{\partial f_r} + (1-\alpha)\ (CR1)_r + (1-\alpha)\ (XDUM11)\right]\Delta f_r$$

$$+ \left[\frac{\partial Q_{D_r}}{\partial \alpha_r} + (1-\alpha)\ (XDUM5)\right]\Delta\alpha_r \qquad (3.33)$$

Now, at the inverter AC bus (i) the following equations, which are similar to those at the rectifier AC bus, are written for the α-M.Q.S.O.N.R. method. These are:

$$\Delta P_i - \alpha R_i = \sum_{k=1}^{n-1} \left(\frac{\partial P_i}{\partial e_k}\Delta e_k\right) + \sum_{k=1}^{n-1} \left(\frac{\partial P_i}{\partial f_k}\Delta f_k\right) + (1-\alpha)\ R_i \qquad (3.34)$$

$$\Delta Q_i - \alpha(T_i+X_i) = \sum_{k=1}^{n-1} \left(\frac{\partial Q_i}{\partial e_k} \Delta e_k \right) + \sum_{k=1}^{n-1} \left(\frac{\partial Q_i}{\partial f_k} \Delta f_k \right)$$

$$+ \frac{\partial Q_{D_i}}{\partial e_i} \Delta e_i + \frac{\partial Q_{D_i}}{\partial f_i} \Delta f_i + \frac{\partial Q_{D_i}}{\partial \gamma_i} \Delta \gamma_i + (1-\alpha)(T_i+X_i) \quad (3.35)$$

R_i and (T_i+X_i) are respectively the second order active and reactive power correction factors for the inverter AC bus. The following equations define these factors:

$$R_i = \Delta e_i \, (CR1)_i + \Delta f_i \, (CR2)_i \quad (3.36)$$

$$T_i = \Delta f_i \, (CR1)_i - \Delta e_i \, (CR2)_i \quad (3.37)$$

$$X_i = \Delta e_i \, (XDUM10) + \Delta f_i \, (XDUM12) + \Delta \gamma_i \, (XDUM7) \quad (3.38)$$

where

$$(CR1)_i = \sum_{k=1}^{n-1} (B_{ik} \, \Delta f_k + G_{ik} \, \Delta e_k) \quad (3.39)$$

$$(CR2)_i = \sum_{k=1}^{n-1} (G_{ik} \, \Delta f_k - B_{ik} \, \Delta e_k) \quad (3.40)$$

$$XDUM10 = \Delta f_i \, (XDC40) + \frac{1}{2} \Delta e_i \, (XDC37) \quad (3.41)$$

$$XDUM12 = \Delta \gamma_i \, (XDC42) + \frac{1}{2} \Delta f_i \, (XDC38) \quad (3.42)$$

$$XDUM7 = \Delta e_i \, (XDC41) + \frac{1}{2} \Delta \gamma_i \, (XDC39) \quad (3.43)$$

The factors used in equations (3.41), (3.42) and (3.43) are defined below.

$$XDC37 = \frac{\partial^2 Q_{D_i}}{\partial e_i^2} \quad (3.44)$$

$$XDC38 = \frac{\partial^2 Q_{D_i}}{\partial f_i^2} \quad (3.45)$$

$$XDC39 = \frac{\partial^2 Q_{D_i}}{\partial \gamma_i^2} \tag{3.46}$$

$$XDC40 = \frac{\partial^2 Q_{D_i}}{\partial e_i \, \partial f_i} \tag{3.47}$$

$$XDC41 = \frac{\partial^2 Q_{D_i}}{\partial e_i \, \partial \gamma_i} \tag{3.48}$$

$$XDC42 = \frac{\partial^2 Q_{D_i}}{\partial f_i \, \partial \gamma_i} \tag{3.49}$$

The derivations of the second order partial differential coefficients as shown in equations (3.44) through (3.49) are contained in Appendix B. Adapting the same procedure as used in the foregoing for the rectifier AC bus, the following equations for the inverter AC bus result.

$$\Delta P_i - \alpha[\Delta e_i \ (CR1)_i + \Delta f_i \ (CR2)_i]$$
$$= \sum_{\substack{k=1 \\ k \neq i}}^{n-1} \left(\frac{\partial P_i}{\partial e_k} \Delta e_k \right) + \left[\frac{\partial P_i}{\partial e_i} + (1-\alpha) \ (CR1)_i \right] \Delta e_i$$
$$+ \sum_{\substack{k=1 \\ k \neq i}}^{n-1} \left(\frac{\partial P_i}{\partial f_k} \Delta f_k \right) + \left[\frac{\partial P_i}{\partial f_i} + (1-\alpha) \ (CR2)_i \right] \Delta f_i \tag{3.50}$$

$$\Delta Q_i - \alpha[\Delta f_i \ (CR1)_i - \Delta e_i \ (CR2)_i]$$
$$- \alpha[\Delta e_i \ (XDUM10) + \Delta f_i \ (XDUM12) + \Delta \gamma_i \ (XDUM7)]$$
$$= \sum_{\substack{k=1 \\ k \neq i}}^{n-1} \left(\frac{\partial Q_i}{\partial e_k} \Delta e_k \right) + \left[\frac{\partial Q_i}{\partial e_i} + \frac{\partial Q_{D_i}}{\partial e_i} + (1-\alpha) \ (CR2)_i + (1-\alpha) \ (XDUM10) \ \Delta e_i \right]$$

$$+ \sum_{\substack{k=1 \\ k \neq i}}^{n-1} \left(\frac{\partial Q_i}{\partial f_k} \Delta f_k \right) + \left[\frac{\partial Q_i}{\partial f_i} + \frac{\partial Q_{D_i}}{\partial f_i} + (1-\alpha)(CR1)_i + (1-\alpha)(XDUM12) \right] \Delta f_i$$

$$+ \left[\frac{\partial Q_{D_i}}{\partial \gamma_i} + (1-\alpha)\ (XDUM7) \right] \Delta \gamma_i \tag{3.51}$$

As described in Chapter 2, the last two equations used in the Newton-Raphson technique for the AC/DC load flow are the converter performance equations (2.65) and (2.66). These equations after the application of equation (3.3) take the following form:

$$\Delta P_{D_r} - \alpha Z_r = \frac{\partial P_{D_r}}{\partial e_r} \Delta e_r + \frac{\partial P_{D_r}}{\partial f_r} \Delta f_r + \frac{\partial P_{D_r}}{\partial \alpha_r} \Delta \alpha_r + (1-\alpha)\ Z_r \tag{3.52}$$

$$\Delta P_{D_i} - \alpha Z_i = \frac{\partial P_{D_i}}{\partial e_i} \Delta e_i + \frac{\partial P_{D_i}}{\partial f_i} \Delta f_i + \frac{\partial P_{D_i}}{\partial \gamma_i} \Delta \gamma_i + (1-\alpha)\ Z_i \tag{3.53}$$

In equations (3.52) and (3.53) Z_r and Z_i are the second order DC power correction factors on the DC side of rectifier and inverter, respectively. These correction factors are defined by the equations given below:

$$Z_r = \Delta e_r\ (XDUM1) + \Delta f_r\ (XDUM3) + \Delta \alpha_r\ (XDUM6) \tag{3.54}$$

$$Z_i = \Delta e_i\ (XDUM2) + \Delta f_i\ (XDUM4) + \Delta \gamma_i\ (XDUM8) \tag{3.55}$$

where

$$XDUM1 = \Delta f_r\ (XDC1) + \frac{1}{2} \Delta e_r\ (XDC4) \tag{3.56}$$

$$XDUM3 = \Delta \alpha_r\ (XDC3) + \frac{1}{2} \Delta f_r\ (XDC5) \tag{3.57}$$

$$XDUM6 = \Delta e_r\ (XDC2) + \frac{1}{2} \Delta \alpha_r\ (XDC6) \tag{3.58}$$

$$XDUM2 = \Delta f_i\ (XDC7) + \frac{1}{2} \Delta e_i\ (XDC10) \tag{3.59}$$

$$XDUM4 = \Delta\gamma_i \; (XDC9) + \frac{1}{2} \Delta f_i \; (XDC11) \tag{3.60}$$

$$XDUM8 = \Delta e_i \; (XDC8) + \frac{1}{2} \Delta\gamma_i \; (XDC12) \tag{3.61}$$

The factors used in equations (3.56) through (3.61) are the second order partial differential coefficients which are expressed by the following equations and are derived in Appendix B.

$$XDC1 = \frac{\partial^2 P_{D_r}}{\partial e_r \; \partial f_r} \tag{3.62}$$

$$XDC2 = \frac{\partial^2 P_{D_r}}{\partial e_r \; \partial \alpha_r} \tag{3.63}$$

$$XDC3 = \frac{\partial^2 P_{D_r}}{\partial f_r \; \partial \alpha_r} \tag{3.64}$$

$$XDC4 = \frac{\partial^2 P_{D_r}}{\partial e_r^2} \tag{3.65}$$

$$XDC5 = \frac{\partial^2 P_{D_r}}{\partial f_r^2} \tag{3.66}$$

$$XDC6 = \frac{\partial^2 P_{D_r}}{\partial \alpha_r^2} \tag{3.67}$$

$$XDC7 = \frac{\partial^2 P_{D_i}}{\partial e_i \; \partial f_i} \tag{3.68}$$

$$XDC8 = \frac{\partial^2 P_{D_i}}{\partial e_i \; \partial \gamma_i} \tag{3.69}$$

$$XDC9 = \frac{\partial^2 P_{D_i}}{\partial f_i \, \partial \gamma_i} \tag{3.70}$$

$$XDC10 = \frac{\partial^2 P_{D_i}}{\partial e_i^2} \tag{3.71}$$

$$XDC11 = \frac{\partial^2 P_{D_l}}{\partial f_i^2} \tag{3.72}$$

$$XDC12 = \frac{\partial^2 P_{D_i}}{\partial \gamma_i^2} \tag{3.73}$$

Substituting (3.54) into (3.55) and (3.55) into (3.53), the following equations result.

$$\Delta P_{D_r} - \alpha[\Delta e_r \; (XDUM1) + \Delta f_r \; (XDUM3) + \Delta \alpha_r \; (XDUM6)]$$
$$= [\frac{\partial P_{D_r}}{\partial e_r} + (1-\alpha)(XDUM1)] \; \Delta e_r + [\frac{\partial P_{D_r}}{\partial f_r} + (1-\alpha) \; (XDUM3)] \; \Delta f_r$$
$$+ [\frac{\partial P_{D_r}}{\partial \alpha_r} + (1-\alpha) \; (XDUM6)] \; \Delta \alpha_r \tag{3.74}$$

$$\Delta P_{D_i} - \alpha[\Delta e_i \; (XDUM2) + \Delta f_i \; (XDUM4) + \Delta \gamma_i \; (XDUM8)]$$
$$+ [\frac{\partial P_{D_i}}{\partial e_i} + (1-\alpha) \; (XDUM2)] \; \Delta e_i + [\frac{\partial P_{D_i}}{\partial f_i} + (1-\alpha) \; (XDUM4)] \; \Delta f_i$$
$$+ [\frac{\partial P_{D_i}}{\partial \gamma_i} + (1-\alpha) \; (XDUM8)] \; \Delta \gamma_i \tag{3.75}$$

Equations (3.13), (3.14), (3.15), (3.32), (3.33), (3.50), (3.51), (3.74) and (3.75) are used for AC/DC load flow analysis in α-M.Q.S.O.N.R. technique and are written in compact matrix form as follows:

$$\begin{bmatrix} \Delta P' \\ \Delta Q' \\ (\Delta|V|^2)' \\ \Delta P'_{D_r} \\ \Delta P'_{D_i} \end{bmatrix} = \begin{bmatrix} J'_1 & J'_2 & & \\ \tilde{J}'_3 & \tilde{J}'_4 & X'_5 & X'_7 \\ J'_5 & J'_6 & & \\ X'_1 & X'_3 & X'_6 & \\ X'_2 & X'_4 & & X'_8 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \\ \Delta \alpha_r \\ \Delta \gamma_i \end{bmatrix} \qquad (3.76)$$

It is noted that the off-diagonal terms of the Jacobian matrices J'_1, J'_2, $\tilde{J}'_3$, $\tilde{J}'_4$, J'_5 and J'_6 are the same as those for the Newton-Raphson method but the diagonal terms are modified by the second order correction factors. The eight additional terms X'_1, X'_2, ..., X'_8 due to the HVDC link have the second order correction factors added to them. The power error vector is also modified to account for the second order terms of the Taylor series. During each iteration, equation (3.76) is solved for the voltage increments at buses and the angle increments at converters. Just how the α-M.Q.S.O.N.R. technique is applied in conjunction with the Newton-Raphson method is discussed in the next section.

3.3.2 Outline of the Method

The step-by-step procedure for applying the α-M.Q.S.O.N.R. technique to solve the AC/DC load flow equations is as follows:

Step 1 - Assemble the bus admittance matrix [Y].

Step 2 - Calculate V_{D_i}, V_{D_r}, and P_{D_r} using equations (2.48), (2.49) and (2.50), respectively.

Step 3 - Initialize

(A) real and imaginary components of the bus voltages, e_p and f_p,

(B) reactive powers (Q_c) at the voltage controlled buses,

(C) delay angle (α_r) of the rectifier, and

(D) extinction angle (γ_i) of the inverter.

Step 4 - Calculate

P_p using equation (2.1)

Q_p using equation (2.2)

$|V_c|^2$ using equation (2.4)

a_r using equation (2.51)

a_i using equation (2.52)

Q_{D_r} using equation (2.17)

Q_{D_i} using equation (2.21)

P_{D_r} using equation (2.25) and

P_{D_i} using equation (2.28).

Step 5 - Calculate errors using the following equations:

$$\Delta P_p = P_{pSCHED} - P_p \qquad (2.85)$$

$$\Delta P_r = P_{rSCHED} - P_r - P_{D_r} \text{ (known from Step 2)} \qquad (2.86)$$

$$\Delta P_i = P_{iSCHED} - P_i + P_{D_i}^{sp} \qquad (2.87)$$

$$Q_p = Q_{pSCHED} - Q_p \qquad (2.88)$$

$$\Delta Q_r = Q_{rSCHED} - Q_r - Q_{D_r} \text{ (calculated in Step 4)} \qquad (2.89)$$

$$\Delta Q_i = Q_{iSCHED} - Q_i - Q_{D_i} \text{ (calculated in Step 4)} \qquad (2.90)$$

$$\Delta |V_c|^2 = |V_{cSCHED}|^2 - |V_c|^2 \qquad (2.91)$$

$$\Delta P_{D_r} = P_{D_r} \text{ (known from Step 2)} - P_{D_r} \text{ (calculated in Step 4)} \qquad (2.92)$$

$$\Delta P_{D_i} = P_{D_i}^{sp} - P_{D_i} \text{ (calculated in Step 4)} \qquad (2.93)$$

Step 6 – Test for convergence. If ΔP_p, ΔP_r, ΔP_i, ΔQ_p, ΔQ_r, ΔQ_i, $\Delta|V_c|^2$, ΔP_{D_r}, and ΔP_{D_i} are all less than the specified tolerance (ε), solution is attained. If not, continue.

Step 7 – Calculate the elements of the Jacobian matrix.

Step 8 – Solve equation (2.72) for Δe, Δf, $\Delta\alpha_r$, and $\Delta\gamma_i$. Thus, compute

$$\begin{bmatrix} \Delta e \\ \hline \Delta f \\ \hline \Delta\alpha_r \\ \hline \Delta\gamma_i \end{bmatrix} = \left[\begin{array}{c|c|c|c} J_1 & J_2 & & \\ \hline \tilde{J}_3 & \tilde{J}_4 & X_5 & X_7 \\ \hline J_5 & J_6 & & \\ \hline X_1 & X_3 & X_6 & \\ \hline X_2 & X_4 & & X_8 \end{array}\right]^{-1} \begin{bmatrix} \Delta P \\ \hline \Delta Q \\ \hline \Delta|V|^2 \\ \hline \Delta P_{D_r} \\ \hline \Delta P_{D_i} \end{bmatrix} \qquad (2.94)$$

Step 9 – With the values of Δe, Δf, $\Delta\alpha_r$, and $\Delta\gamma_i$ from Step 8, calculate the second order correction factors as follows:

R_p using equation (3.8)

R_r using equation (3.18)

R_i using equation (3.36)

T_p using equation (3.9)

T_r using equation (3.19)

T_i using equation (3.37)

U_c using equation (3.10)

X_r using equation (3.20)

X_i using equation (3.38)

Z_r using equation (3.54)

Z_i using equation (3.55)

Step 10 - Test for convergence with the (first order) Newton-Raphson method. If the convergence criterion is met with the first order voltage corrections, the procedure would stop and there would be no need to go for the second order voltage corrections. One way to do this would have been to calculate the bus power and voltage mismatches right after Step 8 and then test for convergence. Realizing that this check would be carried out during every iteration and thus it would involve a lot of computation time (probably much more than performing the second half of the last iteration), a much less time consuming approach is utilized. In this method, the second order correction factors are calculated based on the first order voltage corrections, as done in Step 9. If all the factors are lower than the tolerance (ε), the updated voltage corrections would be the same as non-updated values and hence the sign of convergence. In this case the program is said to converge in half of the last iteration. Otherwise, go to the next step.

Step 11 - Update the power and voltage magnitude mismatches by subtracting the second order correction factors of Step 9 from the original mismatches of Step 5. Thus,

$$\Delta P'_p = \Delta P_p - \alpha R_p \tag{3.77}$$

$$\Delta P'_r = \Delta P_r - \alpha R_r \tag{3.78}$$

$$\Delta P'_i = \Delta P_i - \alpha R_i \tag{3.79}$$

$$\Delta Q'_p = \Delta Q_p - \alpha T_p \tag{3.80}$$

$$\Delta Q'_r = \Delta Q_r - \alpha (T_r + X_r) \tag{3.81}$$

$$\Delta Q'_i = \Delta Q_i - \alpha (T_i + X_i) \tag{3.82}$$

$$(\Delta|V_c|^2)' = \Delta|V_c|^2 - \alpha U_c \tag{3.83}$$

$$\Delta P'_{D_r} = \Delta P_{D_r} - \alpha Z_r \tag{3.84}$$

$$\Delta P'_{D_i} = \Delta P_{D_i} - \alpha Z_i \tag{3.85}$$

Step 12 - Calculate the modified Jacobian elements of J_1', J_2', $\tilde{J}_3'$, $\tilde{J}_4'$, J_5' and J_6' as well as the modified additional terms X_1', X_2', ... X_8' due to the HVDC link.

$$J_1'(p,k) = \frac{\partial P_p}{\partial e_k}, \quad k \neq p \tag{3.86}$$

$$= \frac{\partial P_p}{\partial e_i} + (1-\alpha)\,(CR1)_k, \quad k = p \tag{3.87}$$

$$J_2'(p,k) = \frac{\partial P_p}{\partial f_k}, \quad k \neq p \tag{3.88}$$

$$= \frac{\partial P_p}{\partial f_k} + (1-\alpha)\,(CR2)_k, \quad k = p \tag{3.89}$$

$$\tilde{J}_3'(p,k) = \frac{\partial Q_p}{\partial e_k}, \quad k \neq p, r, i \tag{3.90}$$

$$= \frac{\partial Q_p}{\partial e_k} - (1-\alpha)\,(CR2)_k, \quad k = p \neq r,i \tag{3.91}$$

$$\tilde{J}_3'(r,r) = \frac{\partial Q_r}{\partial e_r} + \frac{\partial Q_{D_r}}{\partial e_r} - (1-\alpha)(CR2)_r + (1-\alpha)(XDUM9) \tag{3.92}$$

$$\tilde{J}_3'(i,i) = \frac{\partial Q_i}{\partial e_i} + \frac{\partial Q_{D_i}}{\partial e_i} - (1-\alpha)(CR2)_i + (1-\alpha)(XDUM10) \tag{3.93}$$

$$\tilde{J}_4'(p,k) = \frac{\partial Q_p}{\partial f_k}, \quad k \neq p,r,i \tag{3.94}$$

$$= \frac{\partial Q_p}{\partial f_k} + (1-\alpha)(CR1)_k, \quad k = p \neq r,i \tag{3.95}$$

$$\tilde{J}_4' \ (r,r) = \frac{\partial Q_r}{\partial f_r} + \frac{\partial Q_{D_r}}{\partial f_r} + (1-\alpha) \ (CR1)_r + (1-\alpha)(XDUM11) \quad (3.96)$$

$$\tilde{J}_4' \ (i,i) = \frac{\partial Q_i}{\partial f_i} + \frac{\partial Q_{D_r}}{\partial f_i} + (1-\alpha)(CR1)_i + (1-\alpha)(XDUM12) \quad (3.97)$$

$$J_5' (c,k) = \frac{\Delta|V_c|^2}{\partial e_k} = 0 \quad , k \neq c \quad (3.98)$$

$$= \frac{\Delta|V_c|^2}{\partial e_k} + (1-\alpha) \ (\Delta e_c) \ , k = c \quad (3.99)$$

$$\tilde{J}_6' \ (c,k) = \frac{\Delta|V_c|^2}{\partial f_k} = 0 \quad , k \neq c \quad (3.100)$$

$$= \frac{\Delta|V_c|^2}{\partial f_k} + (1-\alpha) \ (\Delta f_c) \ , k = c \quad (3.101)$$

$$X_1' = \frac{\partial P_{D_r}}{\partial e_r} + (1-\alpha) \ (XDUM1) \quad (3.102)$$

$$X_2' = \frac{\partial P_{D_i}}{\partial e_i} + (1-\alpha) \ (XDUM2) \quad (3.103)$$

$$X_3' = \frac{\partial P_{D_r}}{\partial f_r} + (1-\alpha) \ (XDUM3) \quad (3.104)$$

$$X_4' = \frac{\partial P_{D_i}}{\partial f_i} + (1-\alpha) \ (XDUM4) \quad (3.105)$$

$$X_5' = \frac{\partial Q_{D_r}}{\partial \alpha_r} + (1-\alpha) \ (XDUM5) \quad (3.106)$$

$$X_6' = \frac{\partial P_{D_r}}{\partial \alpha_r} + (1-\alpha) \ (XDUM6) \quad (3.107)$$

$$X_7' = \frac{\partial Q_{D_i}}{\partial \gamma_i} + (1-\alpha) \ (XDUM7) \quad (3.108)$$

$$X_8' = \frac{\partial P_{D_i}}{\partial \gamma_i} + (1-\alpha) \ (XDUM8) \quad (3.109)$$

Step 13 - With the mismatch values calculated in Step 11 and the modified Jacobian matrix elements obtained in Step 12, solve the following matrix equation for the voltage and angle corrections.

$$\begin{bmatrix} \Delta e' \\ \Delta f' \\ \Delta \alpha_r' \\ \Delta \gamma_i' \end{bmatrix} = \begin{bmatrix} J_1' & J_2' & & \\ \tilde{J}_3' & \tilde{J}_4' & X_5' & X_7' \\ J_5' & J_6' & & \\ X_1' & X_3' & X_6' & \\ X_2' & X_4' & & X_8' \end{bmatrix}^{-1} \begin{bmatrix} \Delta P' \\ \Delta Q' \\ (\Delta |V|^2)' \\ \Delta P_{D_r}' \\ \Delta P_{D_i}' \end{bmatrix} \quad (3.110)$$

Step 14 - With the voltage and angle corrections obtained in Step 13, update the bus voltages and converter angles.

$$e_k^{(j+1)} = e_k^{(j)} + \Delta e_k'^{(j)} \quad (3.111)$$

$$f_k^{(j+1)} = f_k^{(j)} + \Delta f_k'^{(j)} \quad (3.112)$$

$$\alpha_r^{(j+1)} = \alpha_r^{(j)} + \Delta \alpha_r'^{(j)} \quad (3.113)$$

$$\gamma_i^{(j+1)} = \gamma_i^{(j)} + \Delta \gamma_i'^{(j)} \quad (3.114)$$

Step 15 - Return to Step 4 for the next iteration.

Steps 1 to 10 constitute one-half iteration of the alpha-M.Q.S.O.N.R. technique.

CHAPTER 4

COMPUTATIONAL RESULTS

4.1 Introduction

The purpose of this chapter is to summarize results of extensive computational testing of the proposed algorithms indicated in Chapters 2 and 3.

Due to the unavailability of standard AC/DC test systems, the methods proposed in this thesis are tried on existing AC test systems modified to include a DC link. In each AC test system, an AC line is replaced by a DC line which delivers the same power at the inverter end as the active power transported by the replaced AC line. After comparing resistances of DC lines and AC lines, it is found that on the same base MVA and kV the p.u. resistance of a DC line's conductor is approximately one-sixth of that of the corresponding AC case of equal length. This approach is utilized in arriving at a resistance for the DC line in all test systems. Typical values for all other parameters are assumed. The data for all test systems are contained in Appendix C.

4.2 Test Systems

The details of four test systems available in the literature with the modifications necessary for the HVDC link inclusion are given here.

4.2.1 Test System A

This test system has five buses and is given in [26]. It consists of

(i) three load buses

(ii) two voltage-controlled buses, one of which is the swing bus

(iii) seven lines

A two terminal DC link is established by replacing the AC line 2-3 as shown in Appendix C, Figure C-1. The data for this test system are given in Tables C-1, C-2, C-3, and C-4 of Appendix C.

4.2.2 Test System B

This test system is given in [10] and has eight buses. The particulars of this system are as follows:

(i) four load buses

(ii) four voltage-controlled buses, one of which acts as the swing bus

(iii) fourteen lines

The AC line 2-3 is replaced by a two terminal DC link as illustrated in Figure C-2 of Appendix C. The system data are given in Appendix C, Tables C-5, C-6, C-7, and C-8.

4.2.3 Test System C

This is American Electric Power Corporation's fourteen bus test system [17] and consists of:

(i) nine load buses

(ii) five voltage-controlled buses, one of which is the swing bus

(iii) twenty lines

(iv) three transformers with off-nominal turns ratios

(v) one static capacitor

A two terminal DC link is set up by replacing the AC line 4-5 as illustrated in Figure C-3 of Appendix C. The data for this test system are given in Appendix C, Tables C-9, C-10, C-11, C-12, and C-13.

4.2.4 Test System D

This is a fifty-seven bus system and is one of the standard IEEE test systems [17]. It consists of:

(i) fifty load buses

(ii) seven voltage-controlled buses, one of which acts as the swing bus

(iii) eighty lines

(iv) seventeen transformers with off-nominal turns ratios

(v) three static capacitors

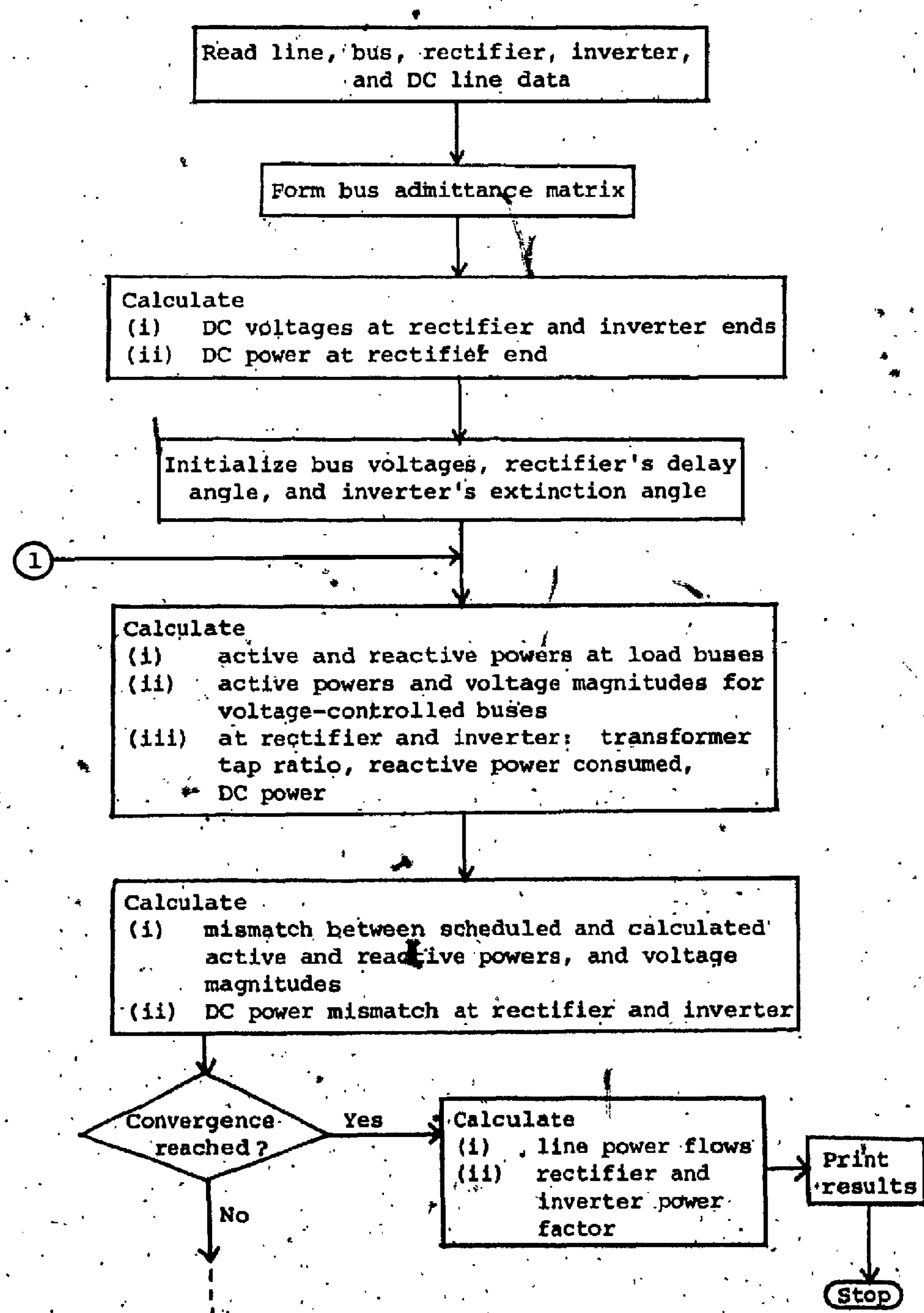
The AC line 47-48 is replaced by a two terminal DC link as shown in Figure C-4 of Appendix C. The system data are given in Appendix C, Tables C-14 through C-20.

4.3 Computer Program

The AC program used in this thesis is the one developed in [29]. This program utilized the FORTRAN IV language and was run on an IBM 370/158 model II with SVS operating system owned by Newfoundland and Labrador Computer Services (NLCS). In order to be compatible, the DC extension has also been written in the FORTRAN IV language and the integrated AC/DC program has been run on the same system.

Sparsity subroutines as developed by the Atomic Energy Research Establishment (U.K.) in Harwell, England [9] are utilized in the program. Appendix D contains the specification sheets of these subroutines. The listings are given in Appendix F.

Figure 4.1 provides a flow chart which basically demonstrates the procedure involved in Newton-Raphson and alpha-modified quasi-second order Newton-Raphson techniques. It is obvious that the alpha-modified


Read line, bus, rectifier, inverter, and DC line data
Form bus admittance matrix
Calculate
(i) DC voltages at rectifier and inverter ends
(ii) DC power at rectifier end
Initialize bus voltages, rectifier's delay angle, and inverter's extinction angle
1
Calculate
(i) active and reactive powers at load buses
(ii) active powers and voltage magnitudes for voltage-controlled buses
(iii) at rectifier and inverter: transformer tap ratio, reactive power consumed, DC power
Calculate
(i) mismatch between scheduled and calculated active and reactive powers, and voltage magnitudes
(ii) DC power mismatch at rectifier and inverter
Convergence reached ?
Yes
No
Calculate
(i) line power flows
(ii) rectifier and inverter power factor
Print results
Stop

- continued

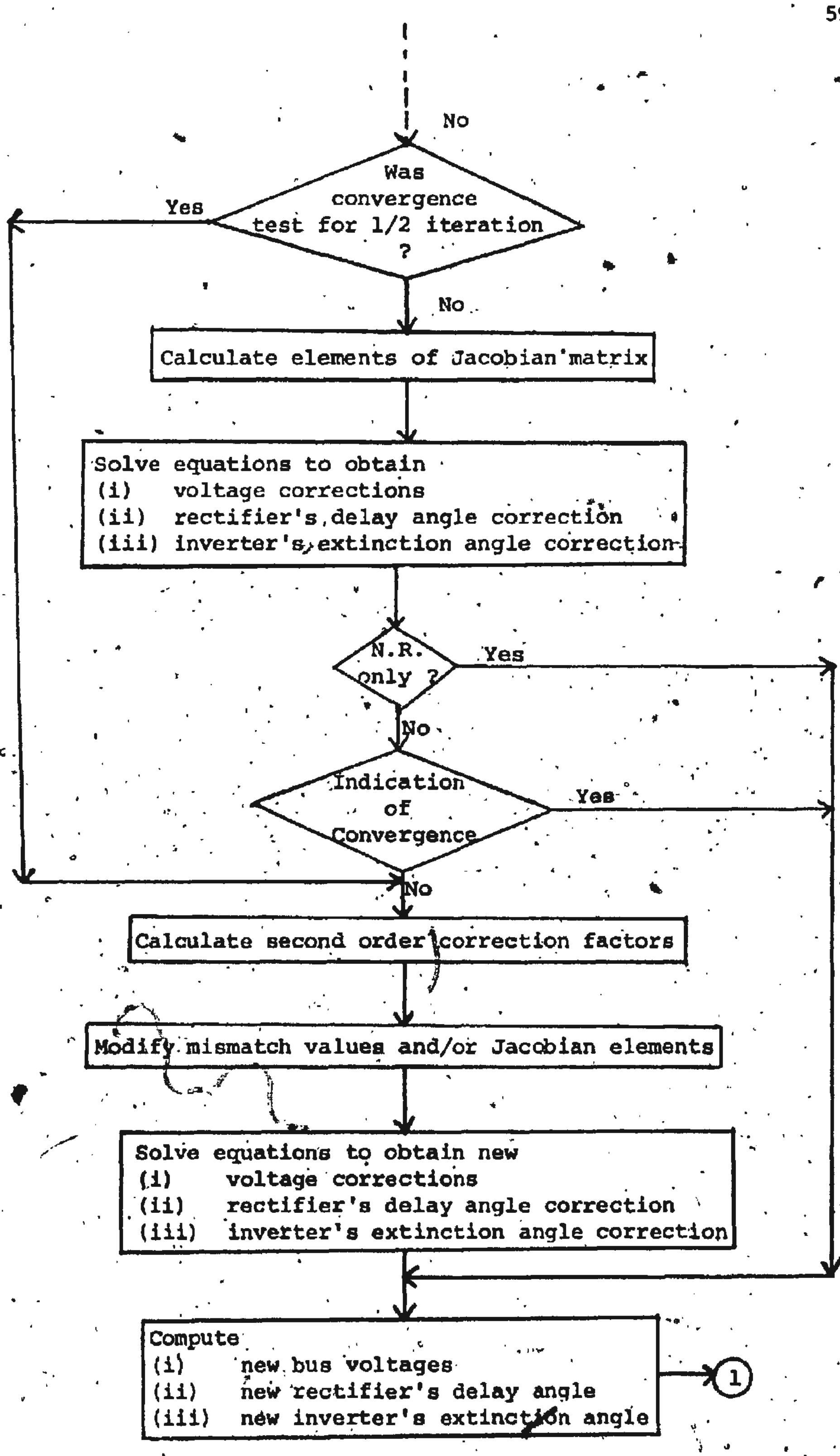


Figure 4.1 Flow Chart for Newton-Raphson and Alpha-Modified Quasi Second Order Newton-Raphson Methods

quasi-second order Newton-Raphson program can be obtained by applying only minor modifications to the Newton-Raphson program. Appendix F contains the complete listing of the program. The computer variable names and their dimensions are given in Appendix E.

4.4 Computational Experience with the Newton-Rapshon Method

4.4.1 General

In this section, the results of extensive testing of the proposed first order algorithms will be discussed. The systems tested are those described in Section 4.2

First with a flat start (i.e. initial guesses for voltages = 1.0 +j0.0 p.u.) and a tolerance of 0.0001 p.u. for all AC and DC variables, each test system required three iterations to converge. The initial guesses for the rectifier delay angle and inverter extinction angle are 8° and 15°, respectively. The range for delay angle in these load flows is from 7° to 18° and that for extinction angle is from 12° to 20°. Figures 4.2, 4.5, 4.8, and 4.11 show the convergence characteristics with flat start for test systems A, B, C, and D, respectively. The actual solutions are contained in Appendix G.

4.4.2 Effect of Initial Conditions on the Newton-Raphson Method

As mentioned earlier, the program requires initial guesses for system voltage levels and converter (rectifier and inverter) angles before iterations begin. A number of load flow cases were run with different combinations of initial guesses for voltage levels and converter angles. It is found that the initial voltage guesses have a significant impact on the convergence pattern of the program. Two different voltage levels ($1.0\ \angle 0.0^\circ$ and $1.0\ \angle 30^\circ$) as initial guesses are tried and it is observed that, in all cases, flat start is by far the best.

It has been stated in Section 4.4.1 that with a flat start (i.e. 1.0 + j0.0) on voltage levels and with delay and extinction angles initially set at 8° and 15°, respectively, all test systems converged in three iterations for a tolerance of 0.0001 p.u. When the starting voltage level was changed to 1.0 $\underline{/30°}$ (i.e. 0.866 + j0.5) with all other conditions remaining the same as before, the test systems A (5-Bus), B (8-Bus), and C (14-Bus) took one more iteration to converge whereas the test system D (57-Bus) took three more iterations to converge. Figures 4.3, 4.6, 4.9, and 4.12 when compared with Figures 4.2, 4.5, 4.8 and 4.11, respectively, present the difference in convergence patterns of non-flat start and flat start computations.

Changing the starting delay and extinction angles to 7° and 12° (from 8° and 15°, respectively) and with flat start did not change the number of iterations (three) for convergence for test systems A, B, and C while it took four iterations (from three) for test system D. The largest mismatches in test systems A, B, C, and D are plotted against the number of iterations in Figures 4.4, 4.7, 4.10, and 4.13, respectively. This shows that for systems with only a few buses, the initial guess for delay and extinction angles is not that critical.

4.4.3 <u>Effect of DC Link Resistance on the Newton-Raphson Method</u>

Due to the unavailability of a suitable resistance for the DC link, each test system was examined with the following DC resistance values:

(i) One-sixth of the resistance of replaced AC line in p.u. The reason for utilizing this approach is explained in Section 4.1.

(ii) The same resistance as considered by [14]. This value is 0.00334 p.u.

(iii) Twice the resistance considered by [14]. Hence 0.00668 p.u. Thus the performance of the proposed method is tested for a wide range of DC link resistance values. It is interesting to note that, for all these DC resistances, each test system converged in three iterations for a tolerance of 0.0001 p.u. This shows that the convergence of the proposed method is not altered by a change in the DC link resistance.

4.5 Computational Experience with the Alpha-Modified Quasi-Second Order Newton-Raphson Method

The computational results of the application of alpha-modified quasi second order Newton-Raphson (α-M.Q.S.O.N.R.) method to the solution of AC/DC load flow problem are discussed in this section. From the theory presented in Chapter 3, it can be concluded that even though any value of alpha can be used, its selection would be crucial to the convergence of the method. In order to set up the alpha-modified quasi second order load flow equations a portion of the second order correction factors is subtracted from the bus mismatches of the first order and a portion is added to the Jacobian elements of the first order. These portions are regulated by the magnitude of alpha selected. Load flow computations for each test system were performed with alpha values ranging from 0.0 to 2.3 in increments of 0.1. The tolerance considered in all cases was 0.0001 p.u.

The number of iterations required for convergence for test systems A, B, C, and D are shown in Figures 4.14, 4.15, 4.16, and 4.17, respectively. It is noted from Figure 4.14 that the test system A converged in a minimum of 2.5 iterations for a number of alpha values. The largest mismatches for one such alpha value is plotted in Figure 4.18.

The test system B converged for all the alpha values considered. As shown in Figure 4.15, the number of iterations to convergence varied from 2 to 5. It is also observed that higher alpha values resulted in more iterations to meet the same tolerance. The convergence characteristics of this test system for alpha equals 0.0 is shown in Figure 4.19.

The test system C converged only for two alpha values as shown in Figure 4.16. The minimum number of iterations required for convergence was 3.5. Figure 4.20 gives the variation in the largest mismatches during iterations for the test system C.

The test system D converged for all alpha values except for when alpha was greater than 1.0. It is worth noting in Figure 4.17 that for those alpha values for which the convergence was reached, the same number of iterations (2) was required. Figure 4.21 illustrates the convergence pattern of the test system D.

The maximum mismatches with α-M.Q.S.O.N.R. method for each of the test systems are contained in Appendix H.

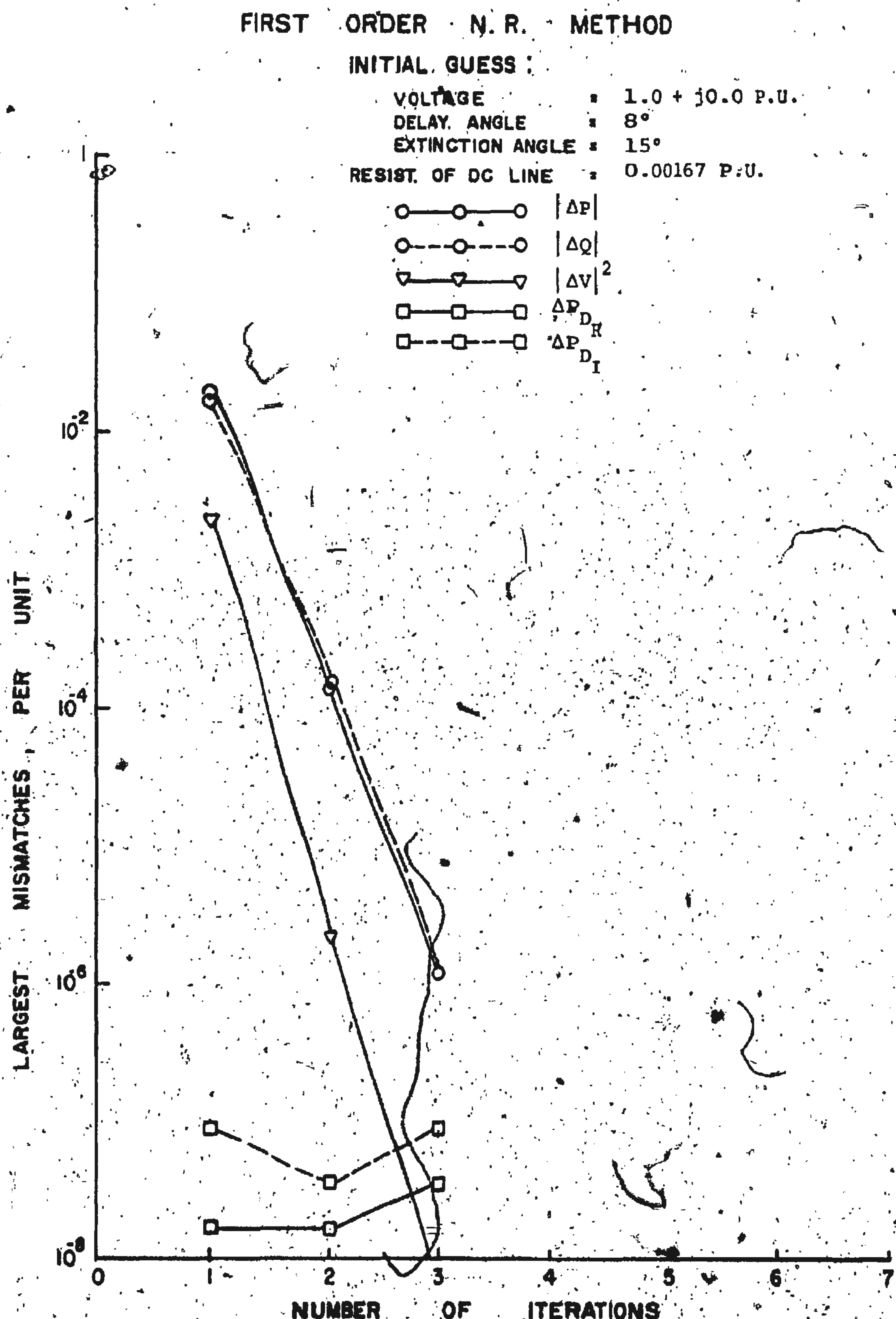


FIGURE 4 : 2 Convergence Characteristics of Test System A (N.R. Method)

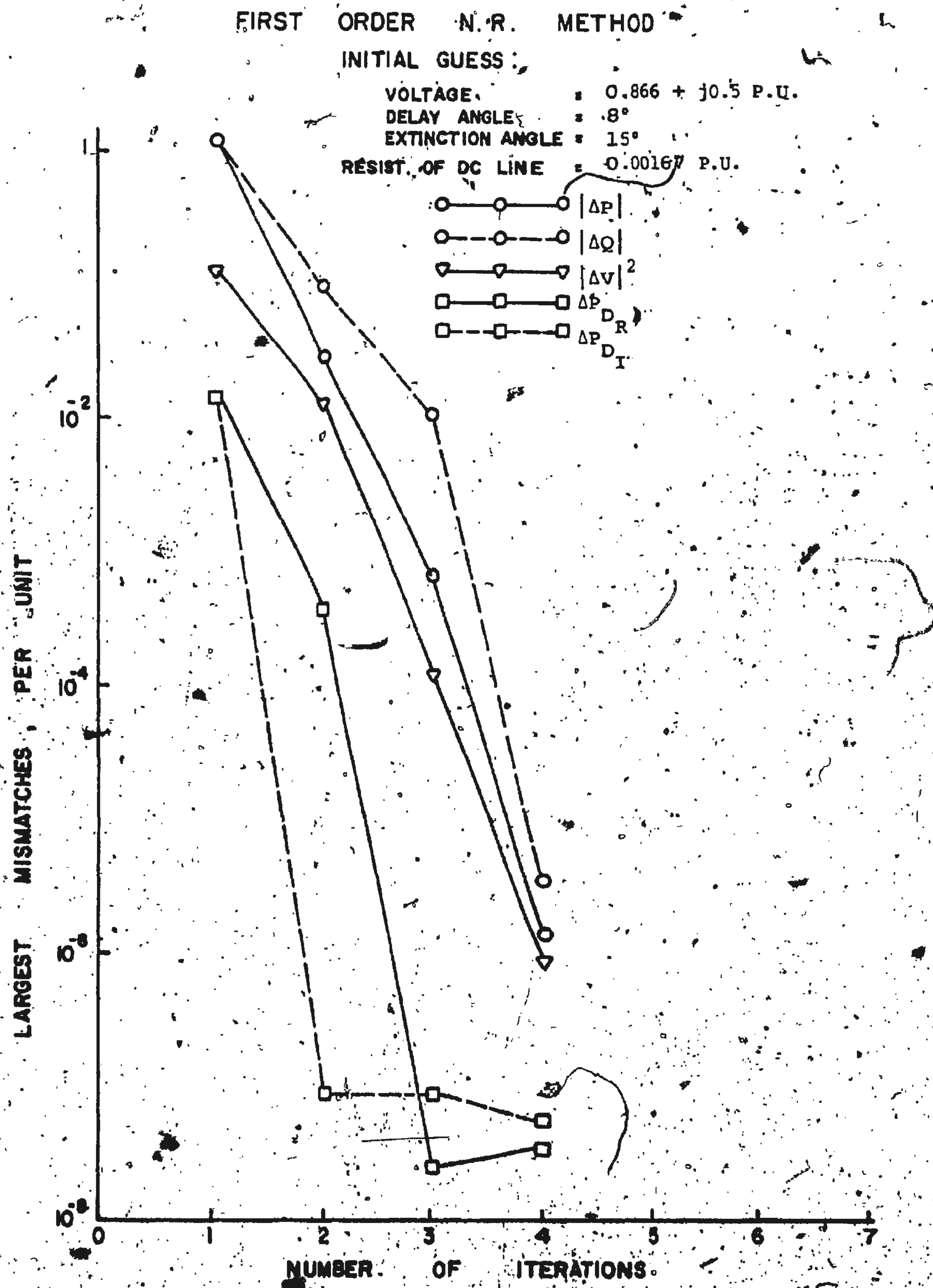


FIGURE 4.3 Convergence Characteristics of Test System A (N.R. Method)

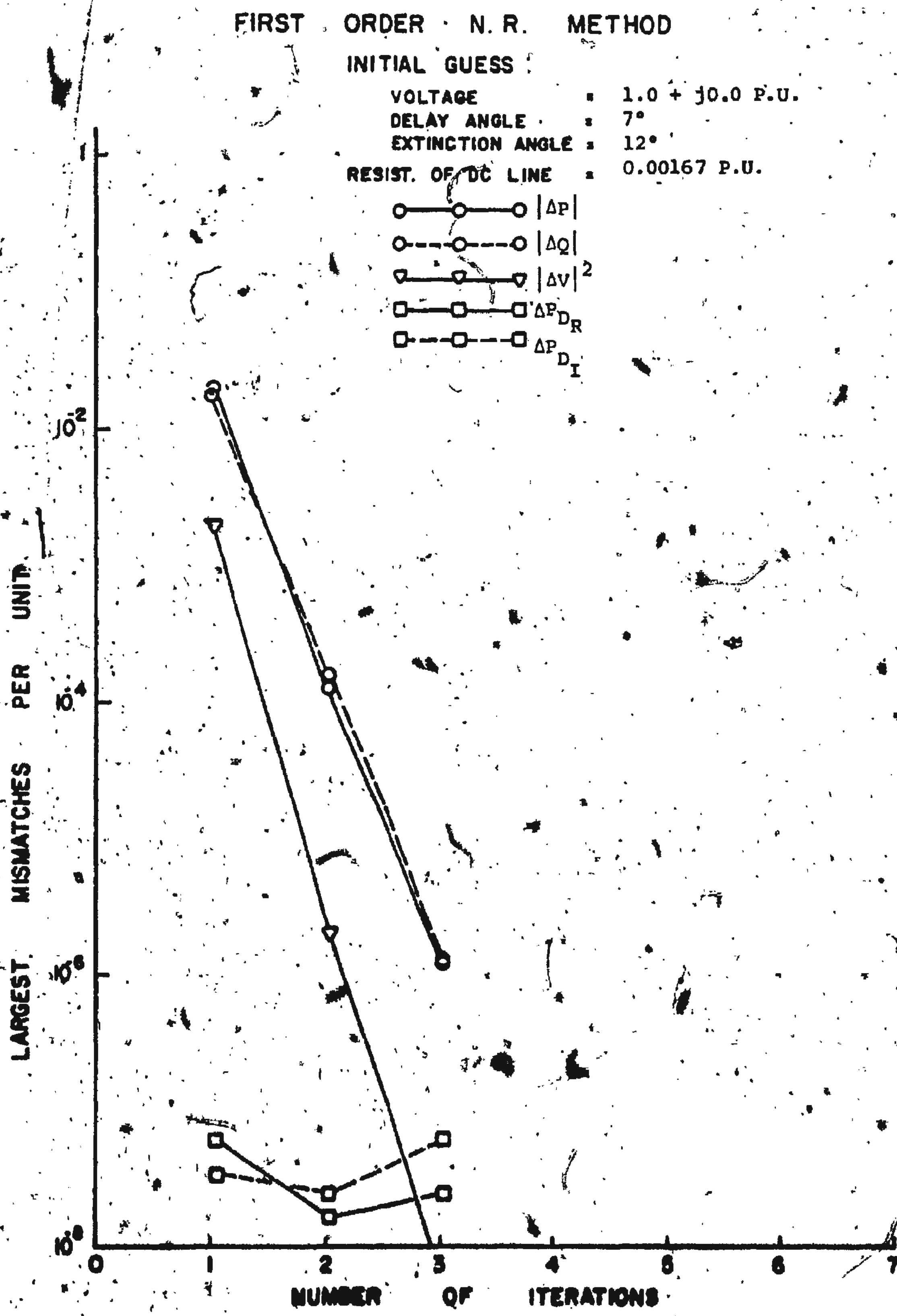


FIGURE 4.4 Convergence Characteristics of Test System A (N.R. Method)

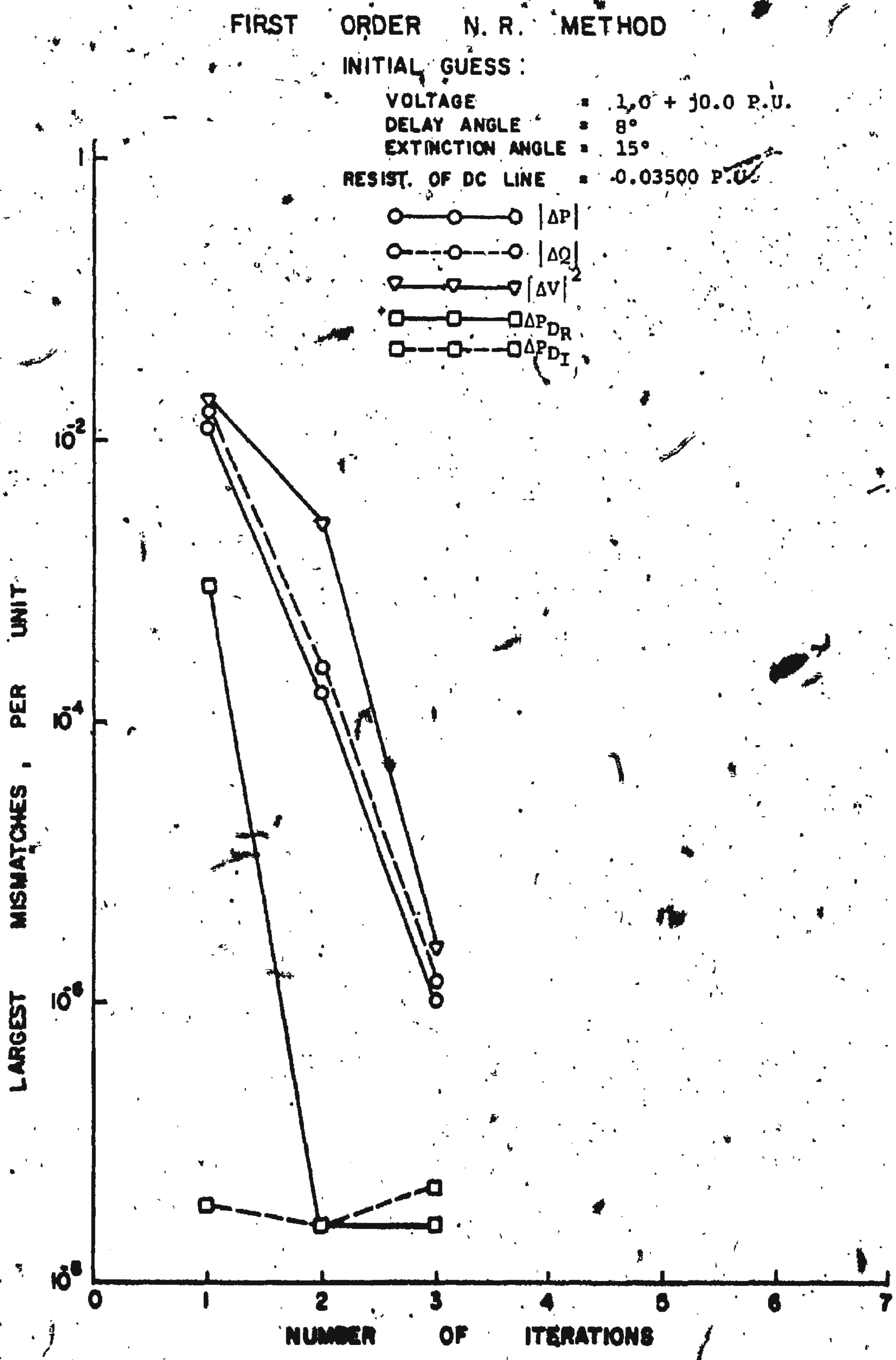


FIGURE A.5 Convergence Characteristics of Test System B (N.R. Method)

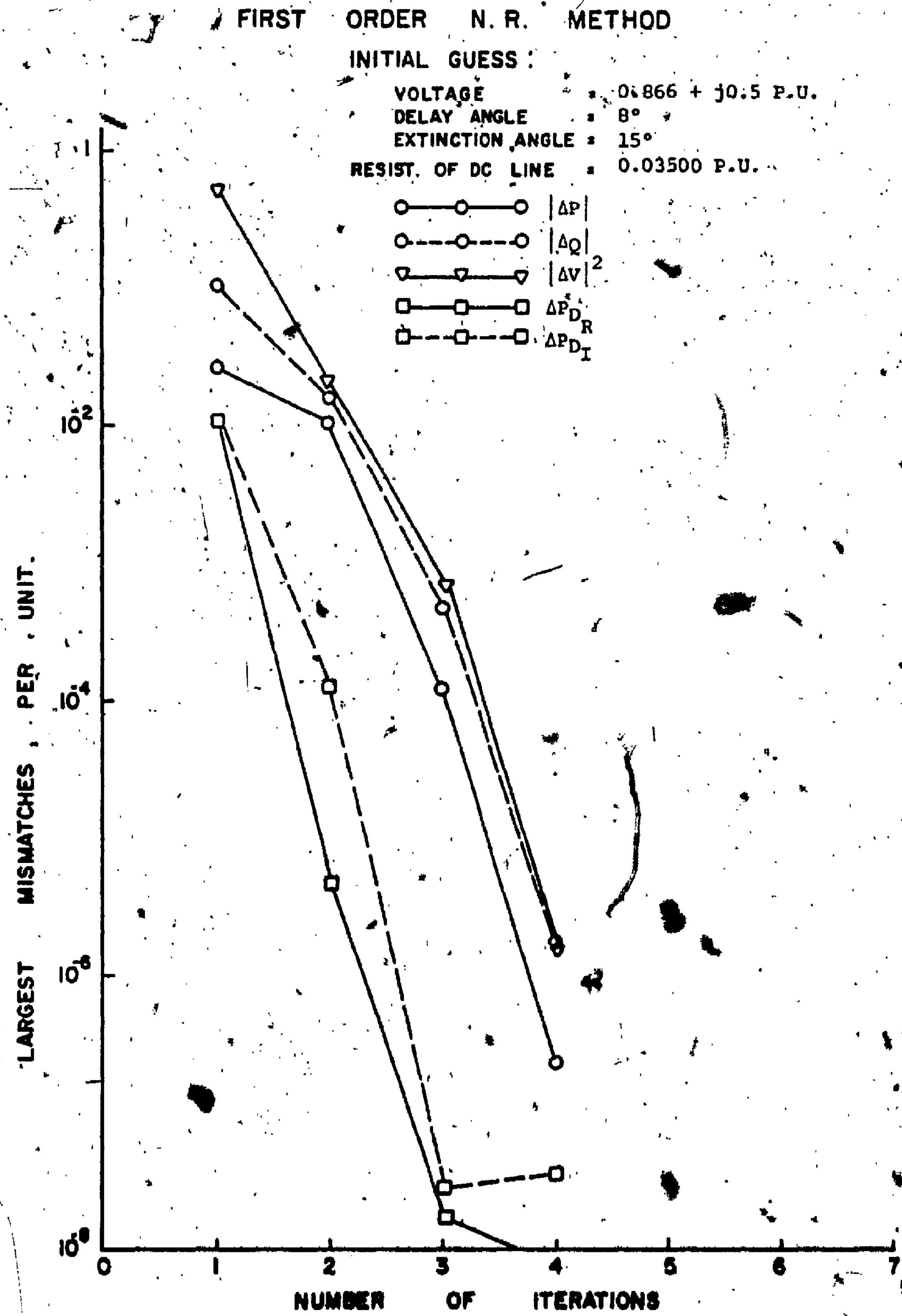


FIGURE 4.6 Convergence Characteristics of Test System B (N.R. Method)

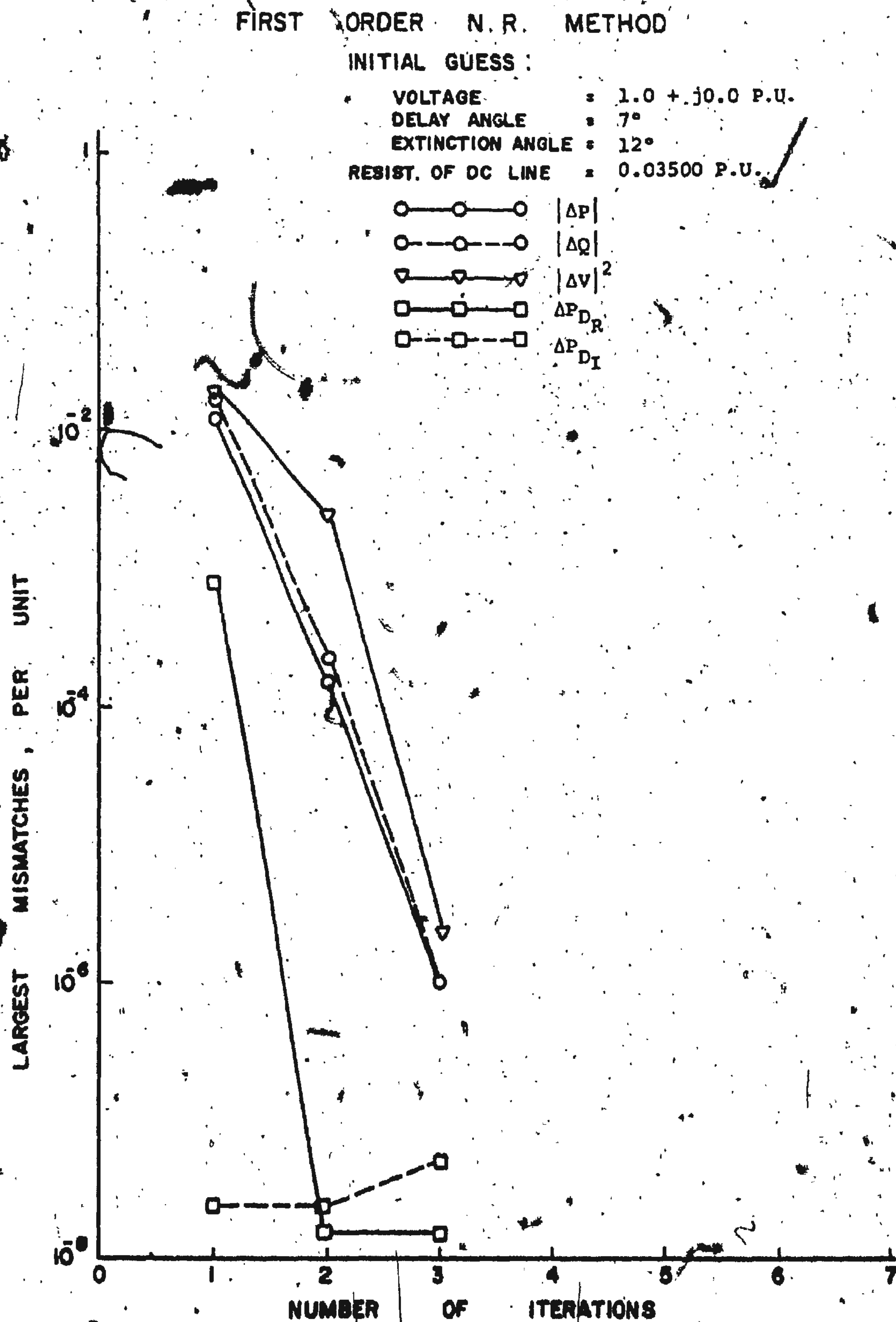


FIGURE 4:7 Convergence Characteristics of Test System B (N.R. Method)

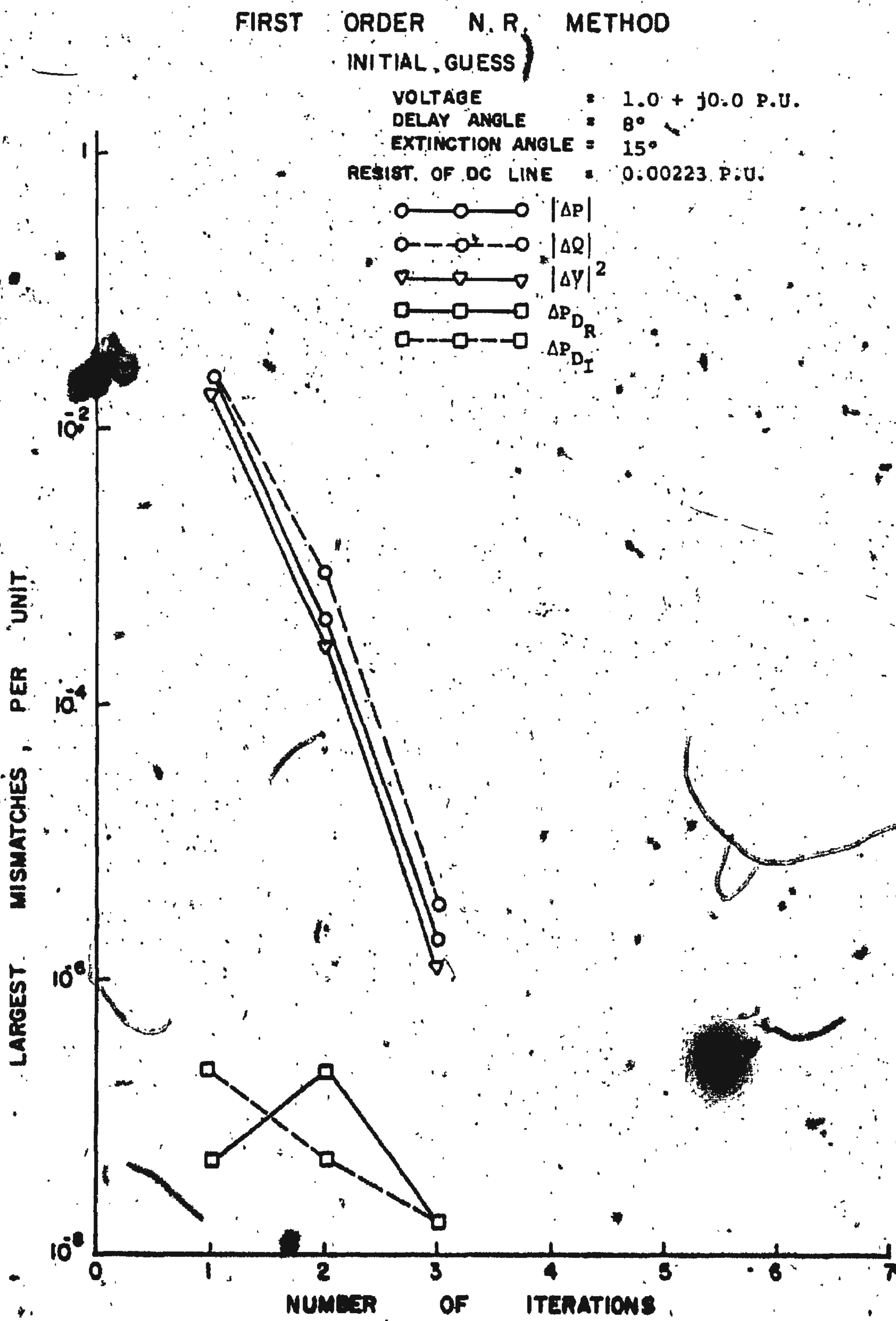


FIGURE 4.8 Convergence Characteristics of Test System C (N.R. Method)

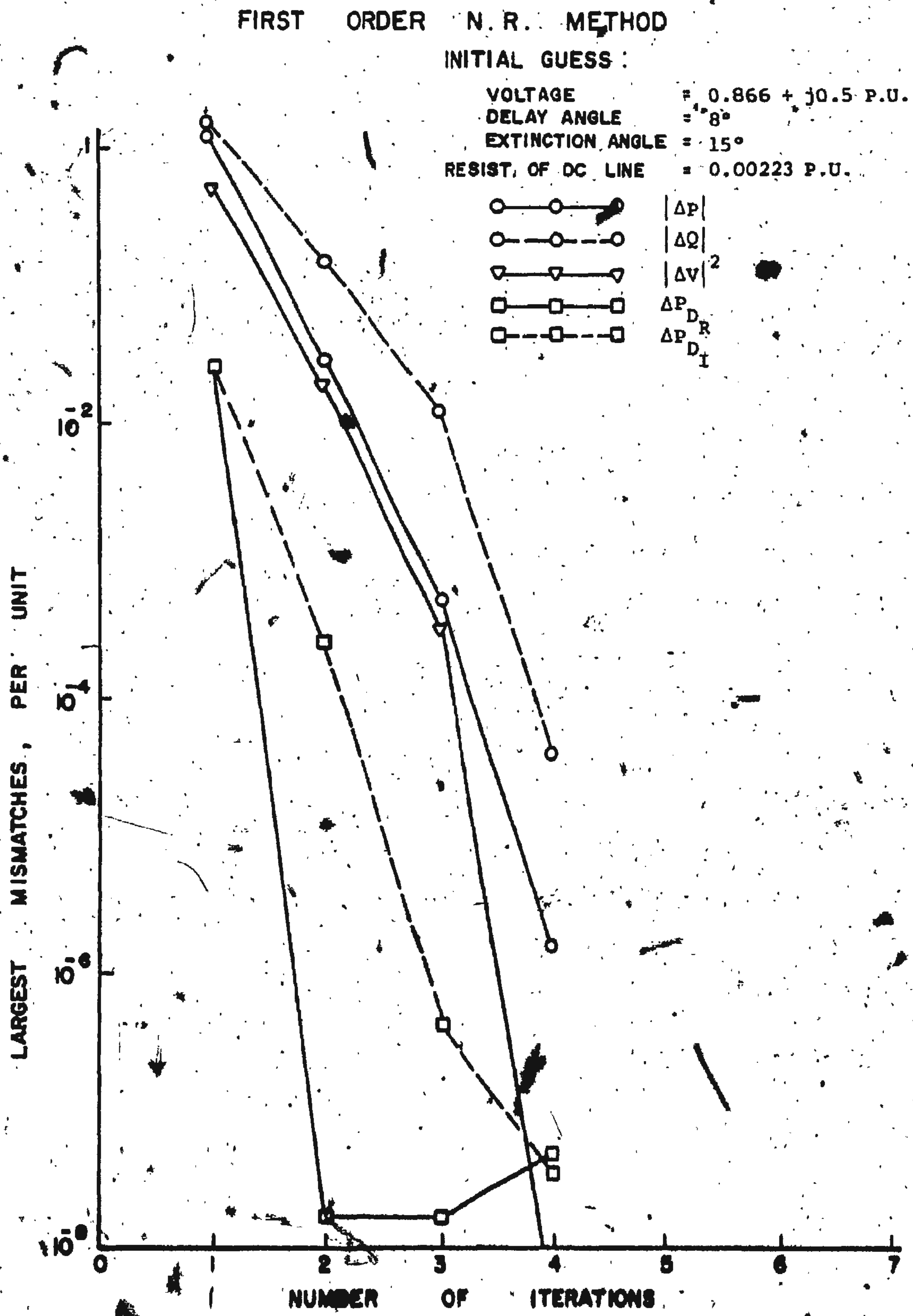


FIGURE 4.9 Convergence Characteristics of Test System C (N.R. Method)

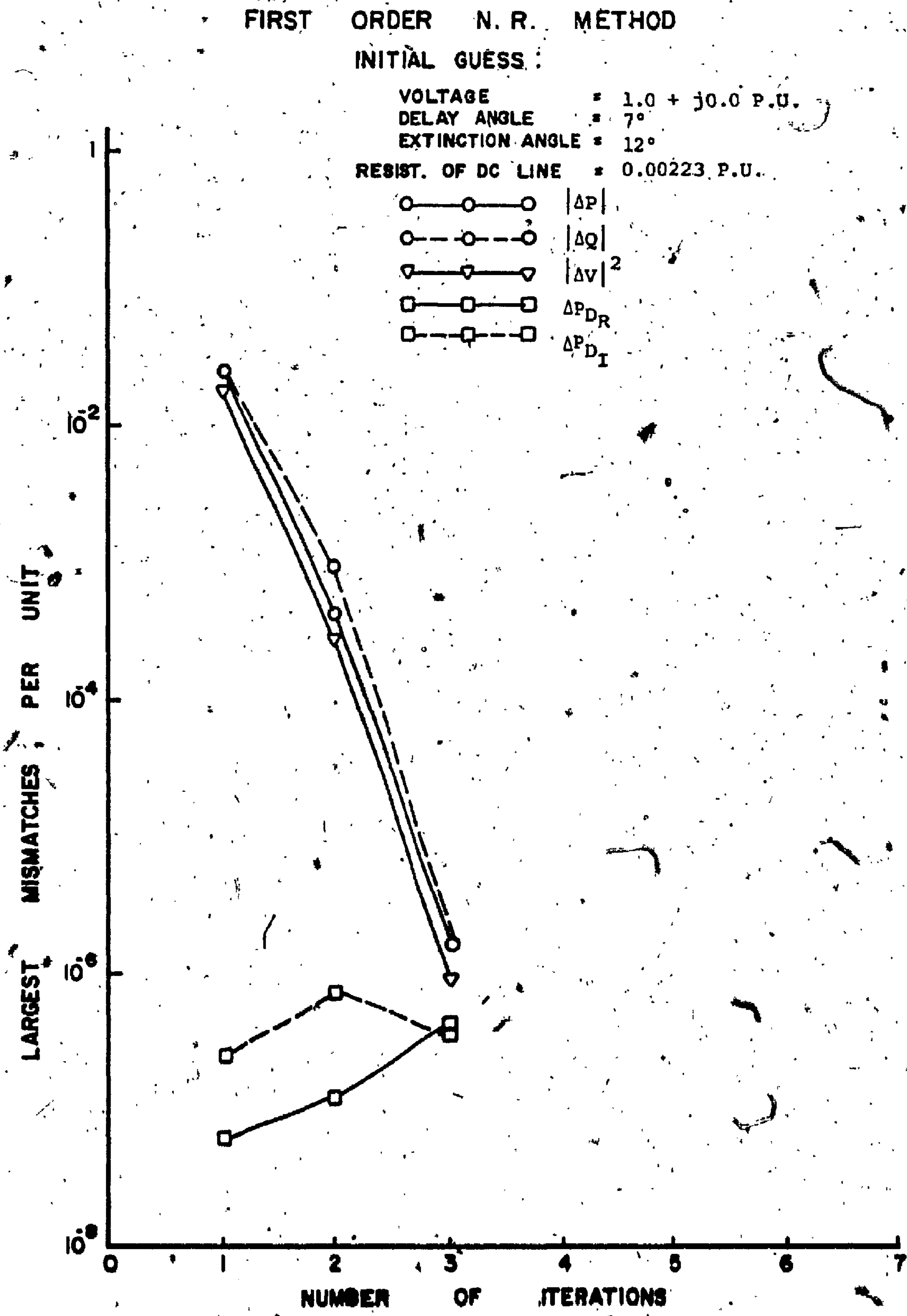


FIGURE 4.10 Convergence Characteristics of Test System C (N.R. Method)

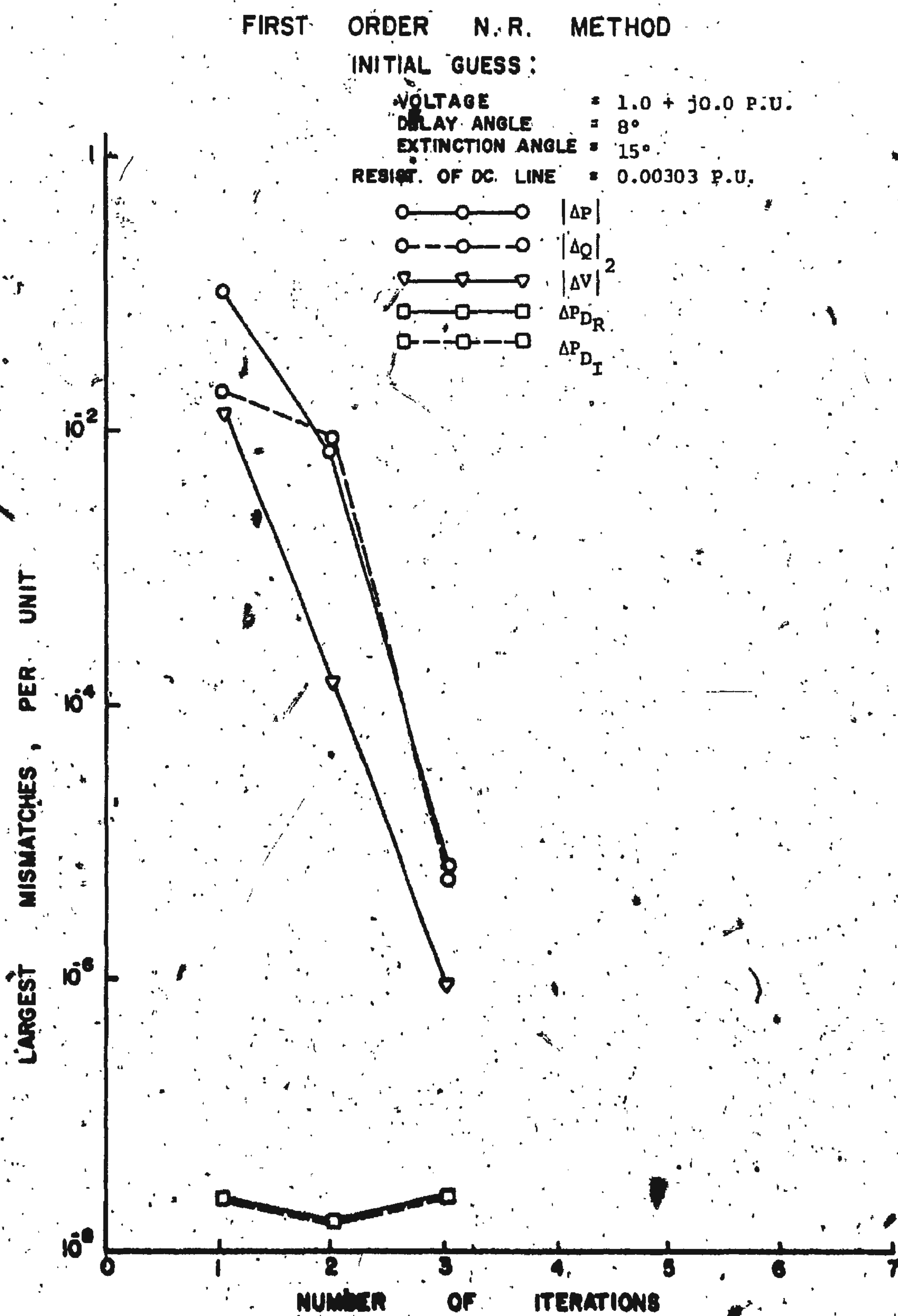


FIGURE 4.11 Convergence Characteristics of Test System D (N.R. Method)

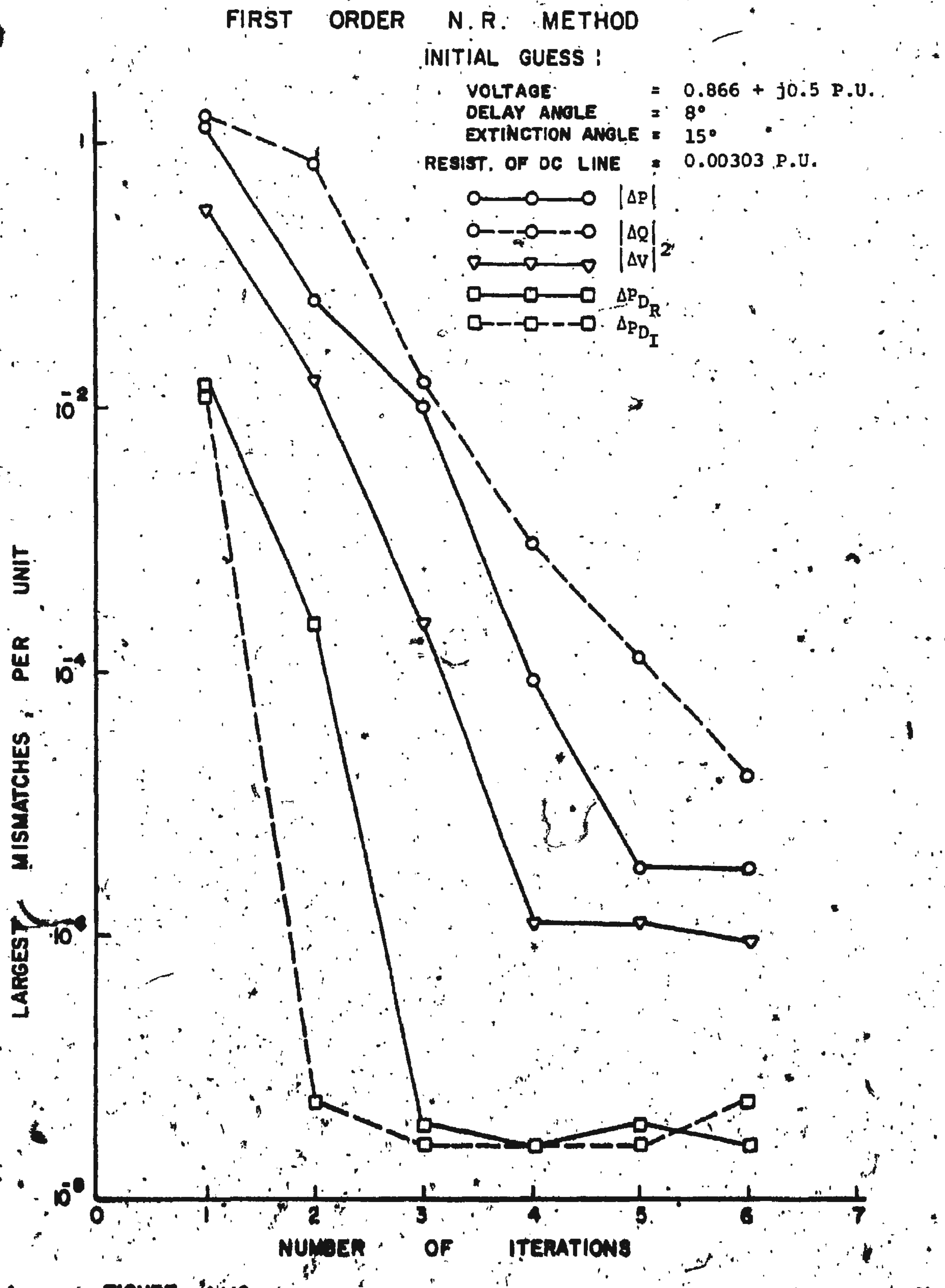


FIGURE 4.12 Convergence Characteristics of Test System D (N.R. Method)

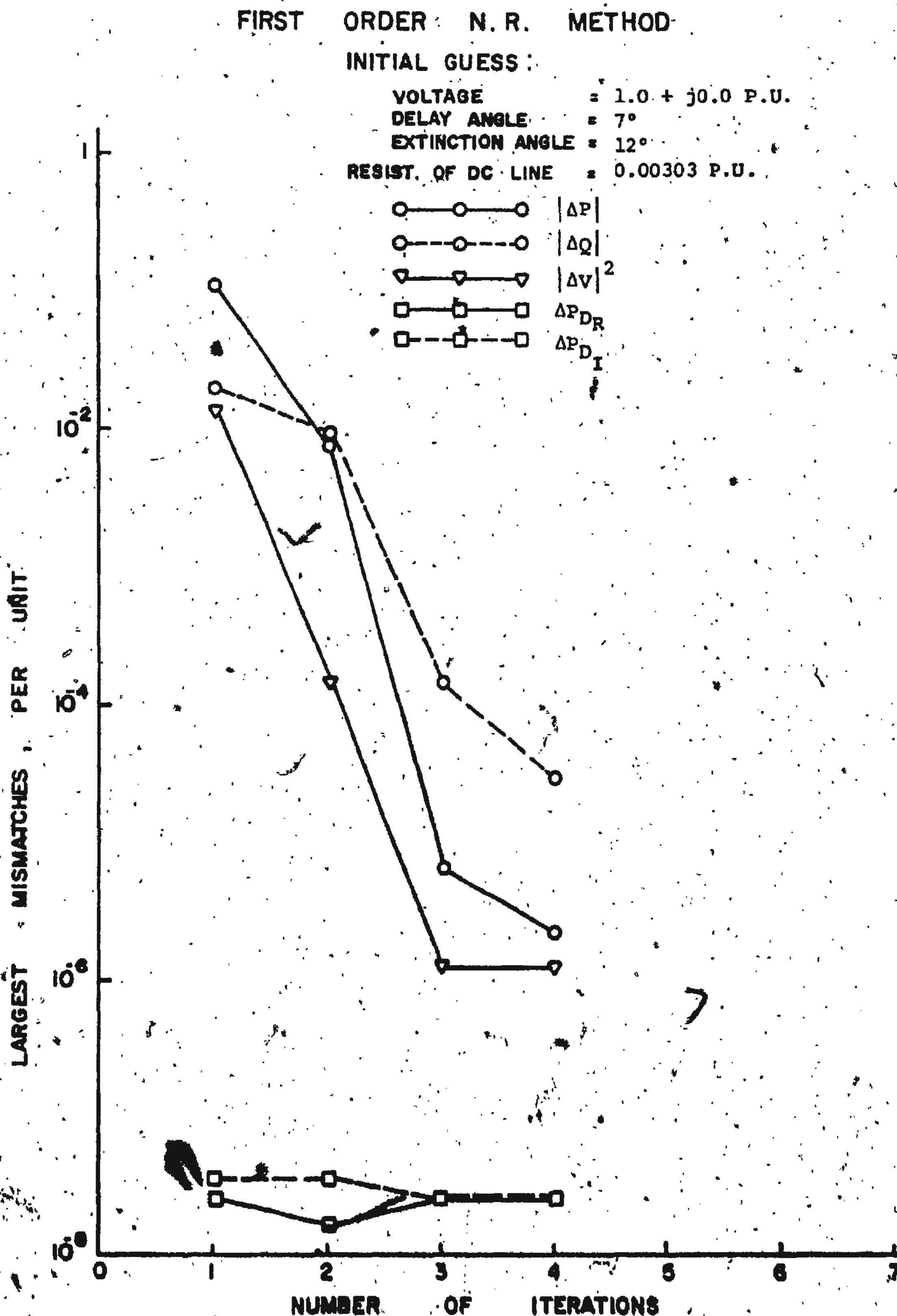


FIGURE 4:13 Convergence Characteristics of Test System D (N.R. Method)

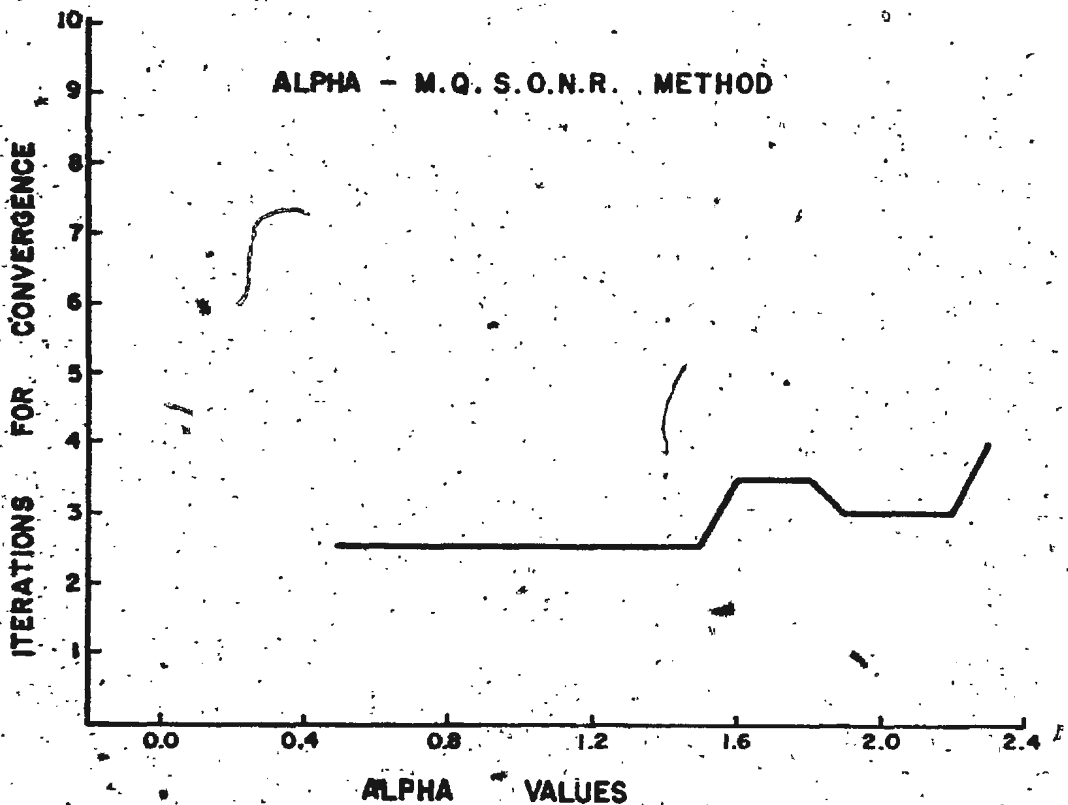


FIGURE 4.14 Iterations to Convergence for Test System A (Alpha-M.Q.S.R.N.R. Method)
(Tolerance = 0.0001 P.U.)

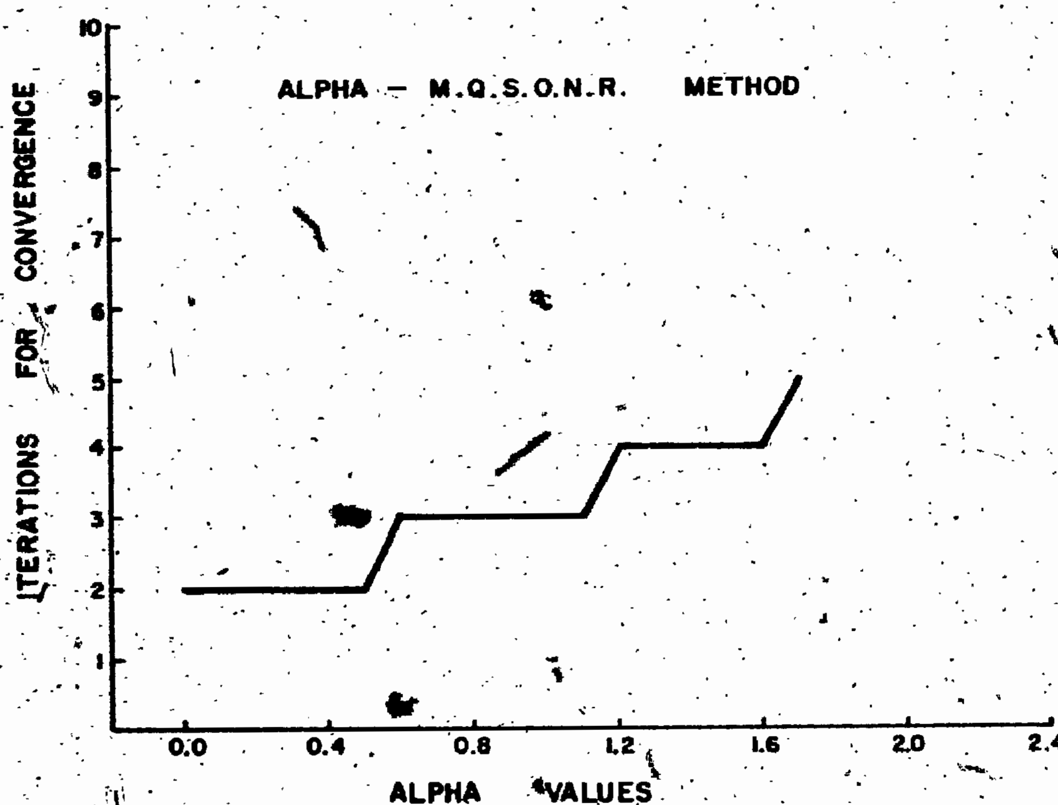


FIGURE 4 : 15 Iterations to Convergence for Test System B (Alpha-M.Q.S.O.N.R. Method) (Tolerance = 0.0001 P.U.)

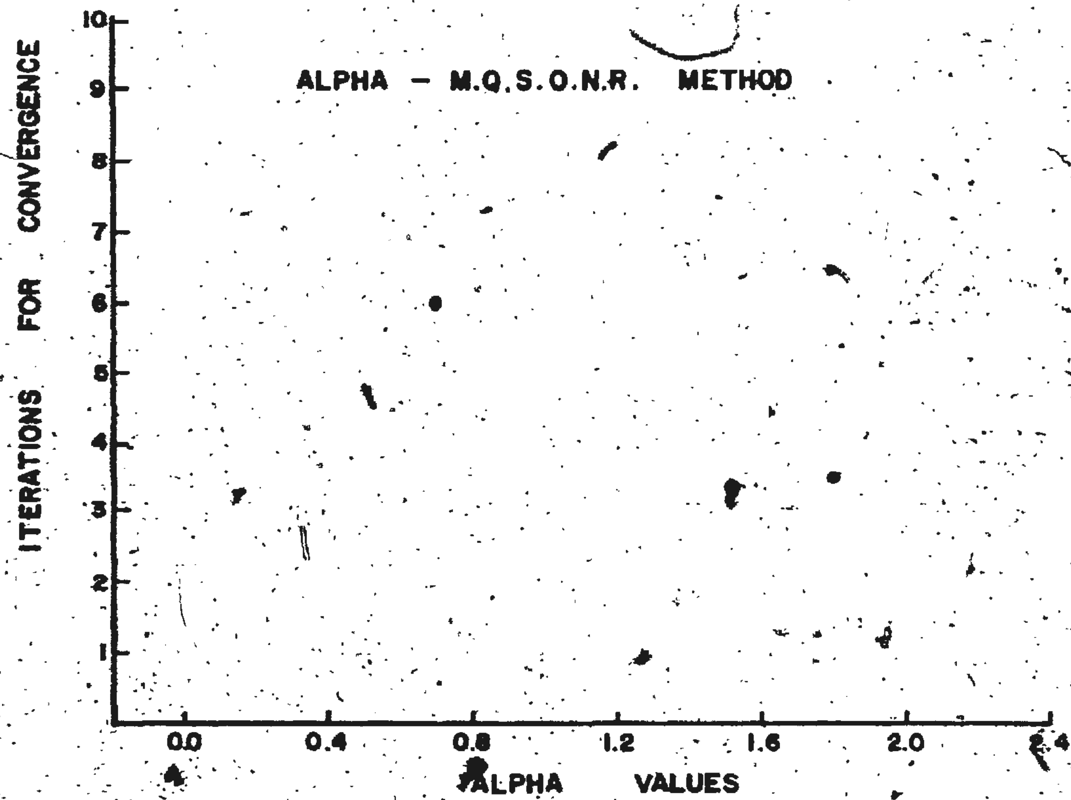


FIGURE 4.16 Iterations to Convergence for Test System C (Alpha-M.Q.S.O.N.R. Method)
(Tolerance = 0.0001 P.U.)

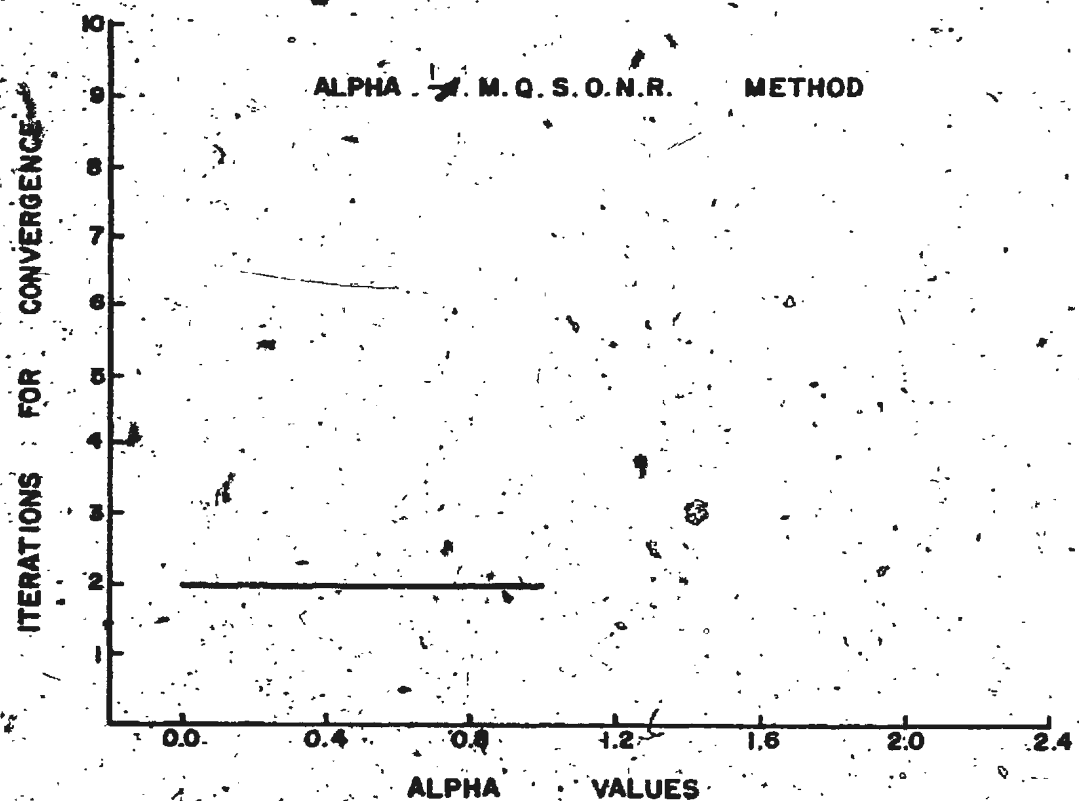


FIGURE 4.17 Iterations to Convergence for Test System D (Alpha-M.Q.S.O.N.R. Method)
(Tolerance = 0.0001 P.U.)

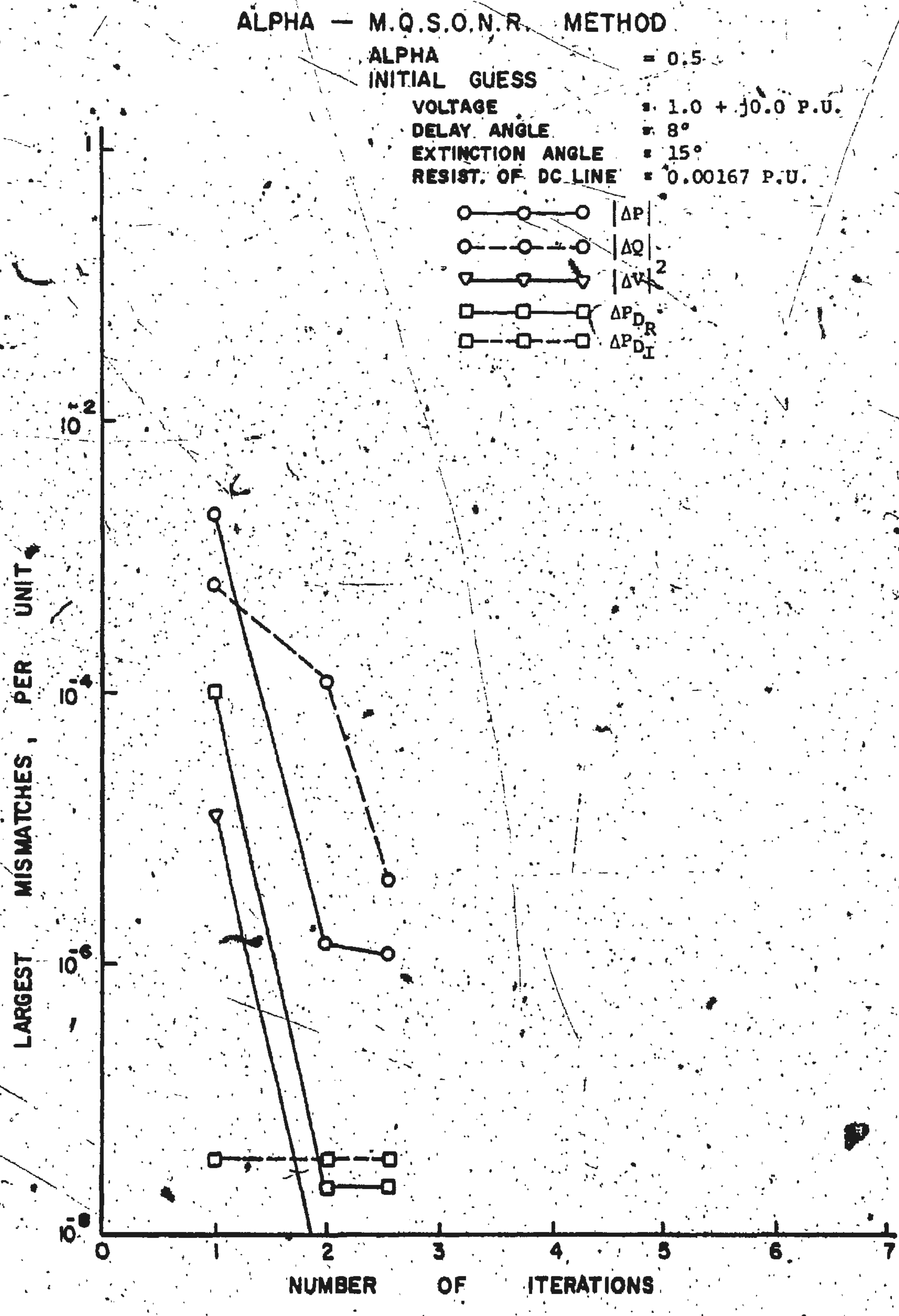


FIGURE 4.18 Convergence Characteristics of Test System A (Alpha-M.Q.S.O.N.R. Method)

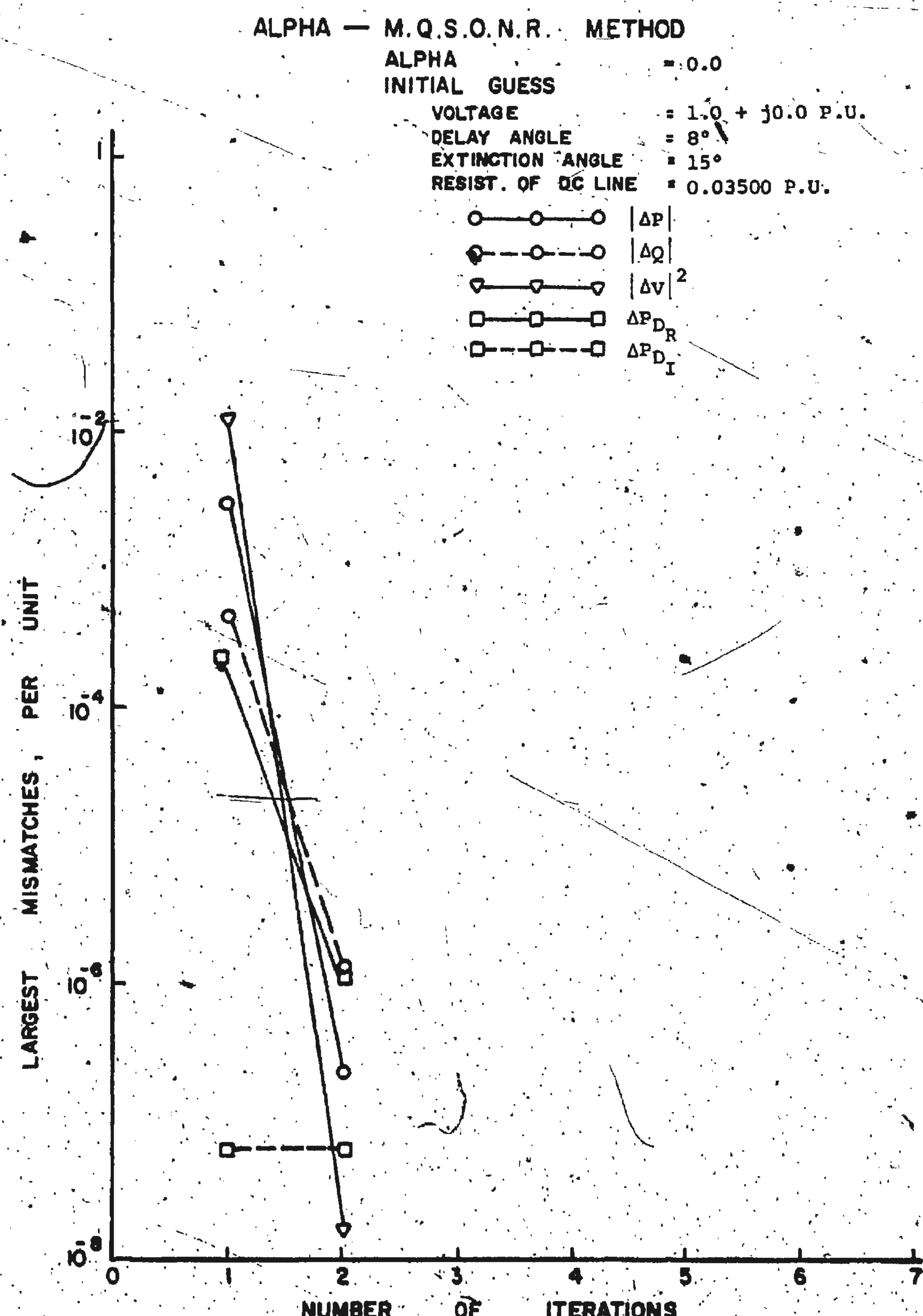


FIGURE 4.19 Convergence Characteristics of Test System B (Alpha-M.Q.S.O.N.R. Method)

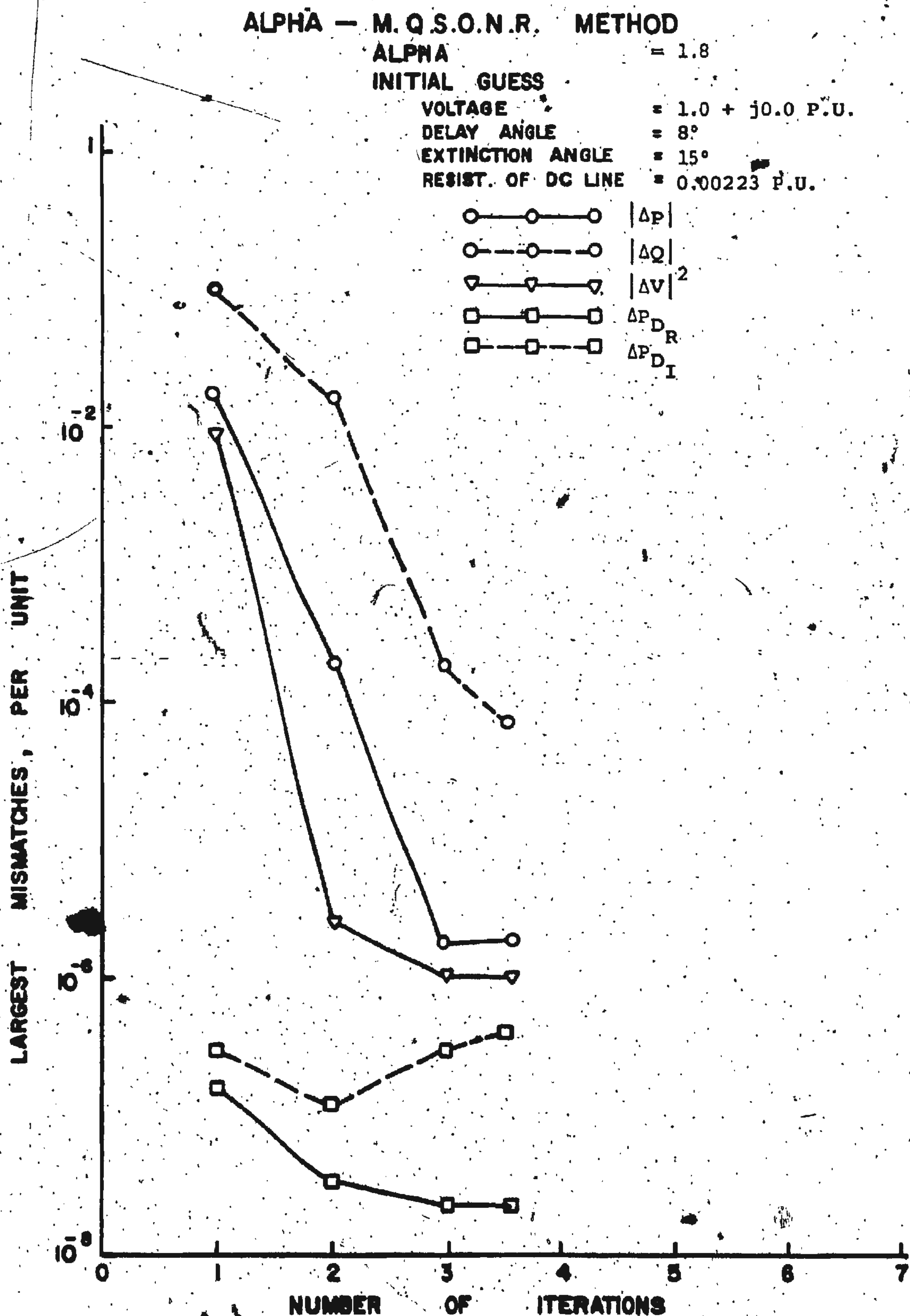


FIGURE 4:20 Convergence Characteristics of Test System C (Alpha-M.Q.S.O.N.R. Method)

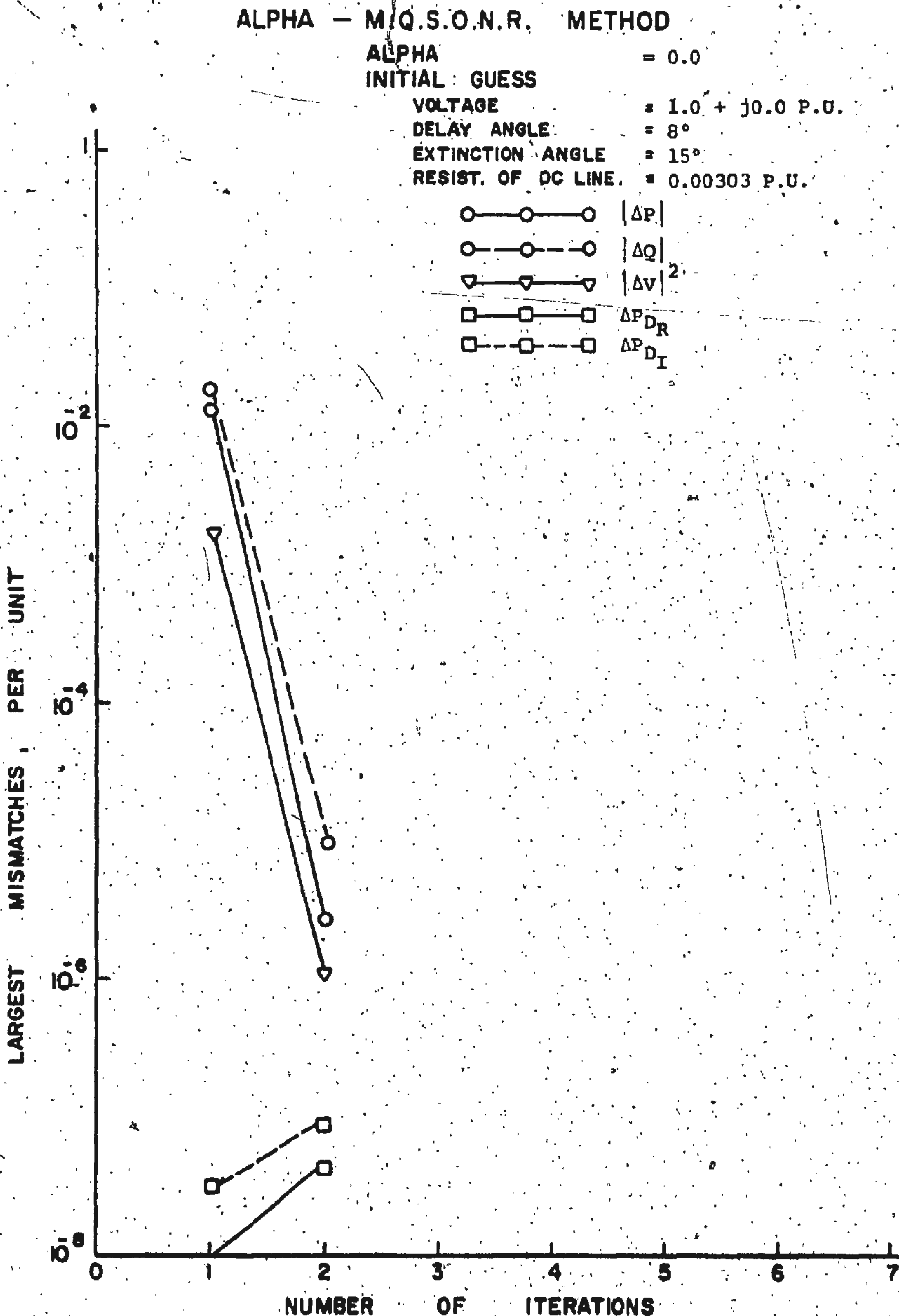


FIGURE 4.21 Convergence Characteristics of Test System D (Alpha-M.Q.S.O.N.R. Method)

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

An alternative formulation of the DC system performance modelling was developed. The formulation was such that only two equations per link were necessary. This was achieved by considering all the necessary constraints imposed by realistic operation requirements. The AC/DC load flow problem was then formulated in the rectangular form to take advantage of the quadratic nature of the resulting equations.

Computational experience with the application of Newton-Raphson method as well as alpha-modified quasi-second order Newton-Rapshon method is reported in the thesis. Using Newton-Raphson method, convergence was attained in three iterations to a tolerance of 10^{-4} with a flat start. The performance of the alpha-modified quasi-second order Newton-Raphson method depends on the values of the alpha chosen. The number of iterations required for convergence varies from a best of 2 iterations to 3.5 iterations with non-convergence in some cases.

5.2 Suggestions for Future Work

A number of areas for future research are suggested:

(1) The question of the optimum value of α has to be addressed. This area has many exciting prospects not only for load flow solution but also in other fields.

(2) Incorporation of realistic load models in the formulation should be investigated as the dependence of active and reactive power in the load on voltage values can be crucial to the acceptability of the results.

(3) The problem of contingency evaluation in AC/DC systems should be considered with the results of this investigation forming the basis for resolution.

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APPENDIX A

JACOBIAN ELEMENTS

(FIRST ORDER)

A.1 Jacobian Elements of J_1, J_2, J_3, J_4, J_5, and J_6

These elements are the first order partial derivatives of the load flow equations (2.1),(2.2) and (2.4).

Elements of J_1 and J_2: are the first order partial derivatives of the active bus powers.

Off-diagonal elements of J_1:

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}, \quad q \neq p \tag{A-1}$$

Diagonal elements of J_1:

$$\frac{\partial P_p}{\partial e_p} = 2 e_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq}) \tag{A-2}$$

Off-diagonal elements of J_2:

$$\frac{\partial P_p}{\partial f_q} = e_p B_{pq} + f_p G_{pq}, \quad q \neq p \tag{A-3}$$

Diagonal elements of J_2:

$$\frac{\partial P_p}{\partial f_p} = 2 f_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_p B_{pq}) \tag{A-4}$$

Elements of J_3 and J_4: are the first order partial derivatives of the reactive bus powers.

Off-diagonal elements of J_3:

$$\frac{\partial Q_p}{\partial e_q} = e_p B_{pq} + f_q G_{pq}, \quad q \neq p \tag{A-5}$$

Diagonal elements of J_3:

$$\frac{\partial Q_p}{\partial e_p} = 2\, e_p\, B_{pp} - \sum_{\substack{q=1\\ q\neq p}}^{n} (f_q\, G_{pq} - e_q\, B_{pq}) \tag{A-6}$$

Off-diagonal elements of J_4:

$$\frac{\partial Q_p}{\partial f_q} = -\, e_p\, G_{pq} + f_p\, B_{pq}\,, \quad q \neq p \tag{A-7}$$

Diagonal elements of J_4:

$$\frac{\partial Q_p}{\partial f_p} = 2\, f_p\, B_{pp} + \sum_{\substack{q=1\\ q\neq p}}^{n} (e_q\, G_{pq} + f_q\, B_{pq}) \tag{A-8}$$

Elements of J_5 and J_6: are the first order partial derivatives of the square of the bus voltage magnitudes.

$$\frac{\partial |V_p|^2}{\partial e_q} = 0 \qquad q = p, \text{ elements of } J_5 \tag{A-9}$$

$$\frac{\partial |V_p|^2}{\partial e_p} = 2e_p \qquad \text{elements of } J_5 \tag{A-10}$$

$$\frac{\partial |V_p|^2}{\partial f_q} = 0 \qquad q = p, \text{ elements of } J_6 \tag{A-11}$$

$$\frac{\partial |V_p|^2}{\partial f_p} = 2f_p \qquad \text{elements of } J_6 \tag{A-12}$$

A.2 Jacobian Elements that Modify J_3 and J_4 Due to HVDC Link

These elements are the first order partial derivatives of the reactive powers consumed by the converters as given by equations (2.17) and (2.21).

The following first order partial derivatives are added to the elements of J_3.

$$\frac{\partial Q_{D_r}}{\partial e_p} = 0 \qquad p \neq r \tag{A-13}$$

$$\frac{\partial Q_{D_r}}{\partial e_r} = \frac{3}{2\pi} \frac{a_r^2\ (2e_r)}{X_{c_r}} \sin \mu_r\ [1 - \cos\ (2\alpha_r + \mu_r)]$$

$$+ \frac{3}{2\pi} \frac{a_r^2 |V_r|^2}{X_{c_r}} [\sin \mu_r\ \sin\ (2\alpha_r + \mu_r) \frac{\partial \mu_r}{\partial e_r}$$

$$+ \{1 - \cos\ (2\alpha_r + \mu_r)\} \cos \mu_r \frac{\partial \mu_r}{\partial e_r}$$

$$= \frac{Q_{D_r}}{|V_r|^2} (2e_r) + \frac{Q_{D_r} \sin\ (2\alpha_r + \mu_r)}{1 - \cos\ (2\alpha_r + \mu_r)} \frac{\partial \mu_r}{\partial e_r} + \frac{Q_{D_r}}{\sin \mu_r} \cos \mu_r \frac{\partial \mu_r}{\partial e_r}$$

$$= Q_{D_r} \left[\frac{2e_r}{|V_r|^2} + \frac{\partial \mu_r}{\partial e_r} \left\{ \frac{1}{\tan \mu_r} + \frac{\sin\ (2\alpha_r + \mu_r)}{1 - \cos\ (2\alpha_r + \mu_r)} \right\} \right] \tag{A-14}$$

$\frac{\partial \mu_r}{\partial e_r}$ is derived in A.4.

$$\frac{\partial Q_{D_i}}{e_p} = 0 \qquad p \neq i \tag{A-15}$$

$$\frac{\partial Q_{D_i}}{\partial e_i} = \frac{3}{2\pi} \frac{a_i^2\ (2e_i)}{X_{c_i}} \sin \mu_i\ [1 - \cos\ (2\gamma_i + \mu_i)]$$

$$+ \frac{3}{2\pi} \frac{a_i^2 |V_i|^2}{X_{c_i}} [\sin \mu_i\ \sin\ (2\gamma_i + \mu_i) \frac{\partial \mu_i}{\partial e_i}$$

$$+ \{1 - \cos\ (2\gamma_i + \mu_i)\} \cos \mu_i \frac{\partial \mu_i}{\partial e_i}]$$

$$= \frac{Q_{D_i}}{|V_i|^2} (2e_i) + \frac{Q_{D_i} \sin\ (2\gamma_i + \mu_i)}{1 - \cos\ (2\gamma_i + \mu_i)} \frac{\partial \mu_i}{\partial e_i} + \frac{Q_{D_i}}{\sin \mu_i} \cos \mu_i \frac{\partial \mu_i}{\partial e_i}$$

$$= Q_{D_i} \left[\frac{2e_i}{|V_i|^2} + \frac{\partial \mu_i}{\partial e_i} \left\{ \frac{1}{\tan \mu_i} + \frac{\sin\ (2\gamma_i + \mu_i)}{1 - \cos\ (2\gamma_i + \mu_i)} \right\} \right] \tag{A-16}$$

$\frac{\partial \mu_i}{\partial e_i}$ is derived in A.4.

The first order partial derivatives that are added to the elements of J_4 are given below.

$$\frac{\partial Q_{D_r}}{\partial f_p} = 0 \qquad p \neq r \tag{A-17}$$

$$\frac{\partial Q_{D_r}}{\partial f_r} = Q_{D_r} \left[\frac{2f_r}{|V_r|^2} + \frac{\partial \mu_r}{\partial f_r} \left\{ \frac{1}{\tan\mu_r} + \frac{\sin (2\alpha_r+\mu_r)}{1 - \cos (2\alpha_r+\mu_r)} \right\}\right] \tag{A-18}$$

$$\frac{\partial Q_{D_i}}{\partial f_p} = 0 \qquad p \neq i \tag{A-19}$$

$$\frac{\partial Q_{D_i}}{\partial f_i} = Q_{D_i} \left[\frac{2f_i}{|V_i|^2} + \frac{\partial \mu_i}{\partial f_i} \left\{ \frac{1}{\tan \mu_i} + \frac{\sin (2\gamma_i+\mu_i)}{1 - \cos (2\gamma_i+\mu_i)} \right\}\right] \tag{A-20}$$

$\frac{\partial \mu_r}{\partial f_r}$ and $\frac{\partial \mu_i}{\partial f_i}$ are derived in A.4.

A.3 Jacobian Elements Due to HVDC Link

The eight additional terms (X_1, X_2, ..., X_8) in the Jacobian matrix as mentioned in Section 2.5.3 are

$$X_1 = \frac{\partial P_{D_r}}{\partial e_r} = \frac{\partial}{\partial e_r} \text{[Equation (2.25)]}$$

$$= \frac{K_1 \, a_r \, e_r \, I_D}{|V_r|} \cos \alpha_r \tag{A-21}$$

$$X_2 = \frac{\partial P_{D_i}}{\partial e_i} = \frac{\partial}{\partial e_i} \text{[Equation (2.28)]}$$

$$= \frac{K_1 \, a_i \, e_i \, I_D}{|V_i|} \cos \gamma_i \tag{A-22}$$

$$X_3 = \frac{\partial P_{D_r}}{\partial f_r} = \frac{\partial}{\partial f_r} \text{ [Equation (2.25)]}$$

$$= \frac{K_1 a_r f_r I_D}{|V_r|} \cos \alpha_r \tag{A-23}$$

$$X_4 = \frac{\partial P_{D_i}}{\partial f_i} = \frac{\partial}{\partial f_i} \text{ [Equation (2.28)]}$$

$$= \frac{K_1 a_i f_i I_D}{|V_i|} \cos \gamma_i \tag{A-24}$$

$$X_5 = \frac{\partial Q_{D_r}}{\partial \alpha_r} = \frac{\partial}{\partial \alpha_r} \text{ [Equation (2.17)]}$$

$$= \frac{3}{2\pi} \frac{a_r^2 |V_r|^2}{X_{c_r}} [\sin \mu_r \sin (2\alpha_r + \mu_r) (2 + \frac{\partial \mu_r}{\partial \alpha_r})$$

$$+ \{1 - \cos (2\alpha_r + \mu_r)\} \cos \mu_r \frac{\partial \mu_r}{\partial \alpha_r}]$$

$$= \frac{Q_{D_r}}{\sin \mu_r [1 - \cos (2\alpha_r + \mu_r)]} [2 \sin \mu_r \sin (2\alpha_r + \mu_r)$$

$$+ \sin \mu_r \sin (2\alpha_r + \mu_r) \frac{\partial \mu_r}{\partial \alpha_r}$$

$$+ \{1 - \cos (2\alpha_r + \mu_r) \cos \mu_r \frac{\partial \mu_r}{\partial \alpha_r}]$$

$$= Q_{D_r} [\frac{2 \sin (2\alpha_r + \mu_r)}{1 - \cos (2\alpha_r + \mu_r)} + \frac{\partial \mu_r}{\partial \alpha_r} \{ \frac{1}{\tan \mu_r} + \frac{\sin (2\alpha_r + \mu_r)}{1 - \cos (2\alpha_r + \mu_r)} \}] \tag{A-25}$$

$\frac{\partial \mu_r}{\partial \alpha_r}$ is derived in A.4.

$$X_6 = \frac{\partial P_{D_r}}{\partial \alpha_r} = \frac{\partial}{\partial \alpha_r} \text{ [Equation (2.25)]}$$

$$= (-1.0)\, K_1\, a_r\, |V_r|\, I_D \sin \alpha_r \tag{A-26}$$

$$X_7 = \frac{\partial Q_{D_i}}{\partial \gamma_i} = \frac{\partial}{\partial \gamma_i} \text{[Equation (2.21)]}$$

$$= \frac{3}{2\pi} \frac{a_i^2 |V_i|^2}{X_{c_i}} [\sin \mu_i \sin (2\gamma_i+\mu_i) (2 + \frac{\partial \mu_i}{\partial \gamma_i})$$

$$+ \{1 - \cos (2\gamma_i+\mu_i)\} \cos \mu_i \frac{\partial \mu_i}{\partial \gamma_i}]$$

$$= \frac{Q_{D_i}}{\sin \mu_i [1 - \cos (2\gamma_i+\mu_i)]} [2 \sin \mu_i \sin (2\gamma_i+\mu_i)$$

$$+ \sin \mu_i \sin (2\gamma_i+\mu_i) \frac{\partial \mu_i}{\partial \gamma_i}$$

$$+ \{1 - \cos (2\gamma_i+\mu_i)\} \cos \mu_i \frac{\partial \mu_i}{\partial \gamma_i}]$$

$$= Q_{D_i} [\frac{2 \sin (2\gamma_i+\mu_i)}{1 - \cos (2\gamma_i+\mu_i)} + \frac{\partial \mu_i}{\partial \gamma_i} \{ \frac{1}{\tan \mu_i} + \frac{\sin (2\gamma_i+\mu_i)}{1 - \cos (2\gamma_i+\mu_i)} \}] \quad \text{(A-27)}$$

$\frac{\partial \mu_i}{\partial \gamma_i}$ is derived in A.4.

$$X_8 = \frac{\partial P_{D_i}}{\partial \gamma_i} = \frac{\partial}{\partial \gamma_i} \text{[Equation (2.28)]}$$

$$= (-1.0) K_1 a_i |V_i| I_D \sin \gamma_i \quad \text{(A-28)}$$

A.4 Derivations for Some Additional Partial Derivatives

To Derive $\partial \mu_r/\partial e_r$, $\partial \mu_r/\partial f_r$, and $\partial \mu_r/\partial \alpha_r$

Equation (2.16) may also be written as

$$\cos (\alpha_r+\mu_r) = \cos \alpha_r - \frac{\sqrt{2}\, X_{c_r} I_D}{a_r |V_r|} \quad \text{(A-29)}$$

The partial derivatives of this equation are taken with respect to e_r, e_f, and α_r realizing that

$$|V_r| = (e_r^2 + f_r^2)^{1/2} \tag{A-30}$$

Thus,

$$\frac{\partial\mu_r}{\partial e_r} = \left[\frac{\cos(\alpha_r+\mu_r) - \cos\alpha_r}{\sin(\alpha_r+\mu_r)} \right] \frac{e_r}{|V_r|^2} \tag{A-31}$$

$$\frac{\partial\mu_r}{\partial f_r} = \left[\frac{\cos(\alpha_r+\mu_r) - \cos\alpha_r}{\sin(\alpha_r+\mu_r)} \right] \frac{f_r}{|V_r|^2} \tag{A-32}$$

$$\frac{\partial\mu_r}{\partial\alpha_r} = \left[\frac{\sin\alpha_r}{\sin(\alpha_r+\mu_r)} \right] - 1.0 \tag{A-33}$$

To Derive $\partial\mu_i/\partial e_i$, $\partial\mu_i/\partial f_i$, and $\partial\mu_i/\partial\gamma_i$

In this case, equation (2.20) is rewritten as

$$\cos(\gamma_i+\mu_i) = \cos\gamma_i - \frac{\sqrt{2}\, X_{c_i} I_D}{a_i\, |V_i|} \tag{A-34}$$

and its partial derivatives are taken with respect to e_i, f_i, and γ_i realizing that

$$|V_i| = (e_i^2 + f_i^2)^{1/2} \tag{A-35}$$

Thus,

$$\frac{\partial\mu_i}{\partial e_i} = \left[\frac{\cos(\gamma_i+\mu_i) - \cos\gamma_i}{\sin(\gamma_i+\mu_i)} \right] \frac{e_i}{|V_i|^2} \tag{A-36}$$

$$\frac{\partial\mu_i}{\partial f_i} = \left[\frac{\cos(\gamma_i+\mu_i) - \cos\gamma_i}{\sin(\gamma_i+\mu_i)} \right] \frac{f_i}{|V_i|^2} \tag{A-37}$$

$$\frac{\partial\mu_i}{\partial\gamma_i} = \left[\frac{\sin\gamma_i}{\sin(\gamma_i+\mu_i)} \right] - 1.0 \tag{A-38}$$

APPENDIX B

JACOBIAN ELEMENTS

(SECOND ORDER)

B.1. Second Order Partial Derivatives of Q_{D_r}

From equation (2.17), we can write that

$$Q_{D_r} = Q_{D_r}(e_f, f_r, \alpha_r) \quad . \tag{B-1}$$

Taylor expansion of equation (B-1) gives

$$Q_{D_r}(e_r + \Delta e_r, f_r + \Delta f_r, \alpha_r + \Delta\alpha_r) - Q_{D_r}(e_r, f_r, \alpha_r)$$

$$= [\frac{\partial Q_{D_r}}{\partial e_r}\Delta e_r + \frac{\partial Q_{D_r}}{\partial f_r}\Delta f_r + \frac{\partial Q_{D_r}}{\partial \alpha_r}\Delta \alpha r]$$

$$+ [\Delta e_r \Delta f_r \frac{\partial^2 Q_{D_r}}{\partial e_r \partial f_r} + \Delta e_r \Delta\alpha_r \frac{\partial^2 Q_{D_r}}{\partial e_r \partial \alpha_r} + \Delta f_r \Delta\alpha_r \frac{\partial^2 Q_{D_r}}{\partial f_r \partial \alpha_r}]$$

$$+ \frac{1}{2}[\Delta e_r^2 \frac{\partial^2 Q_{D_r}}{\partial e_r^2} + \Delta f_r^2 \frac{\partial^2 Q_{D_r}}{\partial f_r^2} + \Delta\alpha_r^2 \frac{\partial^2 Q_{D_r}}{\partial \alpha_r^2}]$$

$$= [\frac{\partial Q_{D_r}}{\partial e_r}\Delta e_r + \frac{\partial Q_{D_r}}{\partial f_r}\Delta f_r + \frac{\partial Q_{D_r}}{\partial \alpha_r}\Delta \alpha_r]$$

$$+ [\Delta e_r \Delta f_r (XDC34) + \Delta e_r \Delta\alpha_r (XDC35) + \Delta f_r \Delta\alpha_r (XDC36)]$$

$$+ \frac{1}{2}[\Delta e_r^2 (XDC31) + \Delta f_r^2 (XDC32) + \Delta\alpha_r^2 (XDC33)] \quad . \tag{B-2}$$

The derivations for the first order differential coefficients of equation (B-2) are carried out in Appendix A while those for the second order are performed here.

Derivation for XDC31

Rewriting equation (A-14), we have

$$\frac{\partial Q_{D_r}}{\partial e_r} = Q_{D_r} \left[XER1 + \frac{\partial \mu_r}{\partial e_r} (FACTR2) \right] \tag{B-3}$$

where

$$XER1 = \frac{2e_r}{|v_r|^2} = 2e_r \; (e_r^2 + f_r^2)^{-1} \tag{B-4}$$

$$FACTR2 = \frac{1}{\tan\mu_r} + \frac{\sin(2\alpha_r + \mu_r)}{1 - \cos(2\alpha_r + \mu_r)} \tag{B-5}$$

Then, using equation (B-3)

$$XDC31 = \frac{\partial^2 Q_{D_r}}{\partial e_r^2}$$

$$= Q_{D_r} \left[\frac{\partial (XER1)}{\partial e_r} + \frac{\partial \mu_r}{\partial e_r} \frac{\partial (FACTR2)}{\partial e_r} + \frac{\partial^2 \mu_r}{\partial e_r^2} (FACTR2) \right]$$

$$+ \frac{\partial Q_{D_r}}{\partial e_r} \left[XER1 + \frac{\partial \mu_r}{\partial e_r} (FACTR2) \right] \tag{B-6}$$

The partial derivatives occuring in equation (B-6) are derived below.

Using equation (B-4)

$$\frac{\partial (XER1)}{\partial e_r} = (2.0) \left[\frac{-2e_r^2}{|v_r|^4} + \frac{1}{|v_r|^2} \right] \tag{B-7}$$

Using equation (B-5)

$$\frac{\partial(\text{FACTR2})}{\partial e_r} = \frac{\partial}{\partial \mu_r} [\cot \mu r + \frac{-\sin(2\alpha_r + \mu_r)}{1 - \cos(2\alpha_r + \mu_r)}] \frac{\partial \mu_r}{\partial e_r}$$

$$= - [\frac{1}{\sin^2 \mu_r} + \frac{1}{1 - \cos(2\alpha_r + \mu_r)}] \frac{\partial \mu_r}{\partial e_r} \quad \text{(B-8)}$$

Equation (A-31) may be written as

$$\frac{\partial \mu_r}{\partial e_r} = (\text{FACTR1})\ e_r\ (e_r^2 + f_r^2)^{-1} \quad \text{(B-9)}$$

where

$$\text{FACTR1} = \frac{\cos(\alpha_r + \mu_r) - \cos \alpha_r}{\sin(\alpha_r + \mu_r)} \quad \text{(B-10)}$$

Using equation (B-9)

$$\frac{\partial^2 \mu_r}{\partial e_r^2} = (\text{FACTR1})\ [\frac{-2e_r^2}{|v_r|^4} + \frac{1}{|v_r|^2}] + \frac{\partial(\text{FACTR1})}{\partial e_r} \frac{e_r}{|v_r|^2} \quad \text{(B-11)}$$

Using equation (B-10)

$$\frac{\partial(\text{FACTR1})}{\partial e_r} = [\frac{\cos(\alpha_r + \mu_r) \cos \alpha_r - 1}{\sin^2(\alpha_r + \mu_r)}] \frac{\partial \mu_r}{\partial e_r} \quad \text{(B-12)}$$

$\frac{\partial Q_{D_r}}{\partial e_r}$ is given by equation (B-3).

<u>Derivation for XDC32</u>

Rewriting equation (A-18), we get

$$\frac{\partial Q_{D_r}}{\partial f_r} = Q_{D_r}\ [\text{XFR1} + \frac{\partial \mu_r}{\partial f_r}\ (\text{FACTR2})] \quad \text{(B-13)}$$

where

$$XFR1 = \frac{2f_r}{|V_r|^2} = 2\, f_r\, (e_r^2 + f_r^2)^{-1} \qquad \text{(B-14)}$$

and FACTR2 is defined by equation (B-5).

Then, from equation (B-13)

$$XDC32 = \frac{\partial^2 Q_{D_r}}{\partial f_r^2}$$

$$= Q_{D_r} \left[\frac{\partial}{\partial f_r} (XFR1) + \frac{\partial \mu_r}{\partial f_r} \frac{\partial}{\partial f_r} (FACTR2) + \frac{\partial^2 \mu_r}{\partial f_r^2} (FACTR2)\right]$$

$$+ \frac{\partial Q_{D_r}}{\partial f_r} \left[XFR1 + \frac{\partial \mu_r}{\partial f_r} (FACTR2)\right] . \qquad \text{(B-15)}$$

The partial derivatives on the right-hand-side of equation (B-15) are derived below.

From equation (B-14)

$$\frac{\partial (XFR1)}{\partial f_r} = (2.0) \left[\frac{-2f_r^2}{|V_r|^4} + \frac{1}{|V_r|^2}\right] . \qquad \text{(B-16)}$$

Using equation (B-5)

$$\frac{\partial (FACTR2)}{\partial f_r} = - \left[\frac{1}{\sin^2 \mu_r} + \frac{1}{1 - \cos(2\alpha_r + \mu_r)}\right] \frac{\partial \mu_r}{\partial f_r} . \qquad \text{(B-17)}$$

Equation (A-32) may be written as

$$\frac{\partial \mu_r}{\partial f_r} = (FACTR1)\; f_r\; (e_r^2 + f_r^2)^{-1} \qquad \text{(B-18)}$$

where FACTR1 is already defined in equation (B-10). Then, from equation (B-18)

$$\frac{\partial^2 \mu_r}{\partial f_r^2} = (\mathrm{FACTR1}) \left[\frac{-2f_r^2}{|v_r|^4} + \frac{1}{|v_r|^2}\right] + \frac{\partial(\mathrm{FACTR1})}{\partial f_r} \frac{f_r}{|v_r|^2} \qquad \text{(B-19)}$$

Using equation (B-10)

$$\frac{\partial(\mathrm{FACTR1})}{\partial f_r} = \left[\frac{\cos(\alpha_r + \mu_r)\cos\alpha_r - 1}{\sin^2(\alpha_r + \mu_r)}\right] \frac{\partial \mu_r}{\partial f_r} \qquad \text{(B-20)}$$

$\frac{\partial Q_{D_r}}{\partial f_r}$ is given by equation (B-13).

Derivation for XDC33

We rewrite equation (A-25) as follows:

$$\frac{\partial Q_{D_r}}{\partial \alpha_r} = Q_{D_r} \left[\mathrm{XALR1} + \frac{\partial \mu_r}{\partial \alpha_r} (\mathrm{FACTR2})\right] \qquad \text{(B-21)}$$

where

$$\mathrm{XALR1} = \frac{2 \sin(2\alpha_r + \mu_r)}{1 - \cos(2\alpha_r + \mu_r)} \qquad \text{(B-22)}$$

and FACTR2 is already defined by equation (B-5). Then, from equation (B-21)

$$\mathrm{XDC33} = \frac{\partial^2 Q_{D_r}}{\partial \alpha_r^2}$$

$$= Q_{D_r} \left[\frac{\partial(\mathrm{XALR1})}{\partial \alpha_r} + \frac{\partial \mu_r}{\partial \mu_r} \frac{\partial(\mathrm{FACTR2})}{\partial \alpha_r} + \frac{\partial^2 \mu_r}{\partial \alpha_r^2} (\mathrm{FACTR2})\right]$$

$$+ \frac{\partial Q_{D_r}}{\partial \alpha_r} \left[\mathrm{XALR1} + \frac{\partial \mu_r}{\partial \alpha_r} (\mathrm{FACTR2})\right] \qquad \text{(B-23)}$$

Given below are the derivations for the partial derivatives occuring on the right-hand-side of equation (B-23). Using equation (B-22)

$$\frac{\partial(\text{XALR1})}{\partial\alpha_r} = \frac{(-2.0)\ [2 + \partial\mu_r/\partial\mu_r]}{1 - \cos\ (2\alpha_r + \mu_r)} \quad \text{(B-24)}$$

From equation (B-5)

$$\frac{\partial(\text{FACTR2})}{\partial\alpha_r} = \frac{(-1.0)}{\sin^2\mu_r}\ \frac{\partial\mu_r}{\partial\alpha_r} + \frac{(-1.0)\,[2 + \partial\mu_r/\partial\alpha_r]}{1 - \cos(2\alpha_r + \mu_r)} \quad \text{(B-25)}$$

From equation (A-33)

$$\frac{\partial^2\mu_r}{\partial\alpha_r^2} = \frac{\sin\mu_r - \sin\alpha_r\ \cos(\alpha_r + \mu_r)\,(\partial\mu_r/\partial\alpha_r)}{\sin^2(\alpha_r + \mu_r)} \quad \text{(B-26)}$$

$\frac{\partial Q_{D_r}}{\partial\alpha_r}$ and $\frac{\partial\mu_r}{\partial\alpha_r}$ are given by equations (B-21) and (A-33), respectively.

<u>Derivation for XDC34</u>

Taking the partial derivative of equation (B-13) with respect to e_r, we have

$$\text{XDC34} = \frac{\partial^2 Q_{D_r}}{\partial e_r\ \partial f_r}$$

$$= Q_{D_r}\ \left[\frac{\partial(\text{XFR1})}{\partial e_r} + \frac{\partial\mu_r}{\partial f_r}\ \frac{\partial(\text{FACTR2})}{\partial e_r} + \frac{\partial^2\mu_r}{\partial e_r\ \partial f_r}\ (\text{FACTR2})\right]$$

$$+ \frac{\partial Q_{D_r}}{\partial e_r}\ [\text{XFR1} + \frac{\partial\mu_r}{\partial f_r}\ (\text{FACTR2})] \quad \text{(B-27)}$$

The partial derivatives occuring in equation (B-27) are derived in the following equations.

Using equation (B-14)

$$\frac{\partial(XFR1)}{\partial e_r} = \frac{-4\ e_r\ f_r}{(e_r^2 + f_r^2)^2} = \frac{-4\ e_r\ f_r}{|v_r|^4} \quad . \tag{B-28}$$

Using equation (B-18)

$$\frac{\partial^2 \mu_r}{\partial e_r\ \partial f_r} = (-2.0)\ (FACTR1)\ \frac{e_r\ f_r}{|v_r|^4} + \frac{\partial(FACTR1)}{\partial e_r}\ \frac{f_r}{|v_r|^2} \quad . \tag{B-29}$$

$\frac{\partial Q_{D_r}}{\partial e_r}$, $\frac{\partial(FACTR2)}{\partial e_r}$, and $\frac{\partial(FACTR1)}{\partial e_r}$ are given by equations (B-3), (B-8), and (B-12), respectively.

<u>Derivation for XDC35</u>

Taking the partial derivative of equation (B-21) with respect to e_r, we get

$$XDC35 = \frac{\partial^2 Q_{D_r}}{\partial e_r\ \partial \alpha_r}$$

$$= Q_{D_r}\left[\frac{\partial(XALR1)}{\partial e_r} + \frac{\partial \mu_r}{\partial \alpha_r}\frac{\partial(FACTR2)}{\partial e_r} + \frac{\partial^2 \mu_r}{\partial e_r\ \partial \alpha_r}(FACTR2)\right]$$

$$+ \frac{\partial Q_{D_r}}{\partial e_r}\left[XALR1 + \frac{\partial \mu_r}{\partial \alpha_r}(FACTR2)\right] \quad . \tag{B-30}$$

The partial derivatives which occur in equation (B-30) are derived below.

From equation (B-22)

$$\frac{\partial(XALR1)}{\partial e_r} = \left[\frac{-\ 2.0}{1 - \cos(2\alpha_r + \mu_r)}\right]\frac{\partial \mu_r}{\partial e_r} \quad . \tag{B-31}$$

$\frac{\partial Q_{D_r}}{\partial e_r}$, and $\frac{\partial \mu_r}{\partial \alpha_r}$ are given by equations (B-3), and (A-33), respectively.

Using equation (A-33)

$$\frac{\partial^2 \mu_r}{\partial e_r\, \partial \alpha_r} = (-1.0)\ \frac{\sin \alpha_r \cos(\alpha_r + \mu_r)}{\sin^2(\alpha_r + \mu_r)}\ \frac{\partial \mu_r}{\partial e_r} \quad . \tag{B-32}$$

$\frac{\partial (FACTR2)}{\partial e_r}$, and $\frac{\partial \mu_r}{\partial e_r}$ are given by equations (B-8), and (B-9), respectively.

Derivation for XDC36

Taking the partial derivative of equation (B-21) with respect to f_r, we obtain

$$XDC36 = \frac{\partial^2 Q_{D_r}}{\partial f_r\, \partial \alpha_r}$$

$$= Q_{D_r}\ [\frac{\partial (XALR1)}{\partial f_r} + \frac{\partial \mu_r}{\partial \alpha_r}\ \frac{\partial (FACTR2)}{\partial f_r} + \frac{\partial^2 \mu_r}{\partial f_r\, \partial \alpha_r}\ (FACTR2)]$$

$$+ \frac{\partial Q_{D_r}}{\partial f_r}\ [XALR1 + \frac{\partial \mu_r}{\partial \alpha_r}\ (FACTR2)] \quad . \tag{B-33}$$

The partial derivatives of equation (B-33) are derived in the following equations.

From equation (B-22)

$$\frac{\partial (XALR1)}{\partial f_r} = [\frac{-2.0}{1 - \cos(2\alpha_r + \mu_r)}]\ \frac{\partial \mu_r}{\partial f_r} \quad . \tag{B-34}$$

$\frac{\partial Q_{D_r}}{\partial f_r}$, and $\frac{\partial \mu_r}{\partial \alpha_r}$ are given by equations (B-13), and (A-33), respectively.

Using equation (A-33)

$$\frac{\partial^2 \mu_r}{\partial f_r \ \partial \alpha_r} = \frac{(1.0) \ \sin \alpha_r \ \cos(\alpha_r + \mu_r)}{\sin^2(\alpha_r + \mu_r)} \ \frac{\partial \mu_r}{\partial f_r} \qquad \text{(B-35)}$$

$\frac{\partial (FACTR2)}{\partial f_r}$, and $\frac{\partial \mu_r}{\partial f_r}$ are given by equations (B-17), and (B-18), respectively.

B.2 Second Order Partial Derivatives of Q_{D_i}

We can write from equation (2.21) that

$$Q_{D_i} = Q_{D_i} \ (e_i, \ f_i, \ \gamma_i) \qquad \text{(B-36)}$$

Expanding this equation using the Taylor series gives

$$Q_{D_i} \ (e_i + \Delta e_i, \ f_i + \Delta f_i, \ \gamma_i + \Delta\gamma_i) - Q_{D_i} \ (e_i, \ f_i, \ \gamma_i)$$

$$= [\frac{\partial Q_{D_i}}{\partial e_i} \Delta e_i + \frac{\partial Q_{D_i}}{\partial f_i} \Delta f_i + \frac{\partial Q_{D_i}}{\partial \gamma_i} \Delta\gamma_i]$$

$$+ [\Delta e_i \ \Delta f_i \ \frac{\partial^2 Q_{D_i}}{\partial e_i \ \partial f_i} + \Delta e_i \ \Delta\gamma_i \ \frac{\partial^2 Q_{D_i}}{\partial e_i \ \partial \gamma_i} + \Delta f_i \ \Delta\gamma_i \ \frac{\partial^2 Q_{D_i}}{\partial f_i \ \partial \gamma_i}]$$

$$+ \frac{1}{2} [\Delta e_i^2 \ \frac{\partial^2 Q_{D_i}}{\partial e_i^2} + \Delta f_i^2 \ \frac{\partial^2 Q_{D_i}}{\partial f_i^2} + \Delta\gamma_i^2 \ \frac{\partial^2 Q_{D_i}}{\partial \gamma_i^2}]$$

$$= [\frac{\partial Q_{D_i}}{\partial e_i} \Delta e_i + \frac{\partial Q_{D_i}}{\partial f_i} \Delta f_i + \frac{\partial Q_{D_i}}{\partial \gamma_i} \Delta\gamma_i]$$

$$+ [\Delta e_i \ \Delta f_i \ (XDC40) + \Delta e_i \ \Delta\gamma_i \ (XDC41) + \Delta f_i \ \Delta\gamma_i \ (XDC42)]$$

$$+ \frac{1}{2} [\Delta e_i^2 \ (XDC37) + \Delta f_i^2 \ (XDC38) + \Delta\gamma_i^2 \ (XDC39)] \qquad \text{(B-37)}$$

The first order partial differential coefficients of equation (B-37) are derived in Appendix A. The derivation for the second order coefficients are presented in this section.

Derivation for XDC37

Equation (A-16) may be written as

$$\frac{\partial Q_{D_i}}{\partial e_i} = Q_{D_i} \; [XEI1 + \frac{\partial \mu_i}{\partial e_i} \; (FACTI2)] \qquad (B-38)$$

where

$$XEI1 = \frac{2e_i}{|V_i|^2} = 2e_i \; (e_i^2 + f_i^2)^{-1} \qquad (B-39)$$

$$FACTI2 = \frac{1}{\tan \mu_i} + \frac{\sin(2\gamma_i' + \mu_i)}{1 - \cos(2\gamma_i + \mu_i)} \qquad (B-40)$$

Then, using equation (B-38)

$$XDC37 = \frac{\partial^2 Q_{D_i}}{\partial e_i^2}$$

$$= Q_{D_i} \; [\frac{\partial (XEI1)}{\partial e_i} + \frac{\partial \mu_r}{\partial e_i} \; \frac{\partial (FACTI2)}{\partial e_i} + \frac{\partial^2 \mu_i}{\partial e_i^2} \; (FACTI2)]$$

$$+ \frac{\partial Q_{D_i}}{\partial e_i} \; [XEI1 + \frac{\partial \mu_i}{\partial e_i} \; (FACTI2)] \qquad (B-41)$$

The first order elements of equation (B-41) are derived below.

Using equation (B-39)

$$\frac{\partial (XEI1)}{\partial e_i} = (2.0) \; [\frac{-2e_i^2}{|V_i|^4} + \frac{1}{|V_i|^2}] \qquad (B-42)$$

Using equation (B-40)

$$\frac{\partial(\text{FACTI2})}{\partial e_i} = -\left[\frac{1}{\sin^2\mu_i} + \frac{1}{1 - \cos(2\gamma_i + \mu_i)}\right]\frac{\partial\mu_i}{\partial e_i} \qquad \text{(B-43)}$$

Rewriting equation (A-36) as

$$\frac{\partial\mu_i}{\partial e_i} = (\text{FACTI1})\, e_i\, (e_i^2 + f_i^2)^{-1} \qquad \text{(B-44)}$$

where

$$\text{FACTI1} = \frac{\cos(\gamma_i + \mu_i) - \cos\gamma_i}{\sin(\gamma_i + \mu_i)} \qquad \text{(B-45)}$$

we obtain

$$\frac{\partial^2\mu_i}{\partial e_i^2} = (\text{FACTI1})\left[\frac{-2e_i^2}{|V_i|^4} + \frac{1}{|V_i|^2}\right] + \frac{\partial(\text{FACTI1})}{\partial e_i}\,\frac{e_i}{|V_i|^2} \qquad \text{(B-46)}$$

Using equation (B-45)

$$\frac{\partial(\text{FACTI1})}{\partial e_i} = \left[\frac{\cos(\gamma_i + \mu_i)\cos\gamma_i - 1}{\sin^2(\gamma_i + \mu_i)}\right]\frac{\partial\mu_i}{\partial e_i} \qquad \text{(B-47)}$$

$\frac{\partial Q_{D_i}}{\partial e_i}$ is given by equation (B-38).

<u>Derivation for XDC38</u>

Equation (A-20) may be written as

$$\frac{\partial Q_{D_i}}{\partial f_i} = Q_{D_i}\left[\text{XFI1} + \frac{\partial\mu_i}{\partial f_i}(\text{FACTI2})\right] \qquad \text{(B-48)}$$

where

$$XFI1 = \frac{2f_i}{|v_i|^2} = 2f_i \ (e_i^2 + f_i^2)^{-1} \qquad \text{(B-49)}$$

and FACTI2 is defined by equation (B-40).

Using equation (B-48), we have

$$XDC38 = \frac{\partial^2 Q_{D_i}}{\partial f_i^2}$$

$$= Q_{D_i} \left[\frac{\partial(XFI1)}{\partial f_i} + \frac{\partial \mu_i}{\partial f_i} \frac{\partial(FACTI2)}{\partial f_i} + \frac{\partial^2 \mu_i}{\partial f_i^2} (FACTI2)\right]$$

$$+ \frac{\partial Q_{D_i}}{\partial f_i} \left[XFI1 + \frac{\partial \mu_i}{\partial f_i} (FACTI2)\right] \quad . \qquad \text{(B-50)}$$

The first order partial derivatives of equation (B-50) are given below.

From equation (B-49)

$$\frac{\partial(XFI1)}{\partial f_i} = (2.0) \left[\frac{-2f_i^2}{|v_i|^4} + \frac{1}{|v_i|^2}\right] \quad . \qquad \text{(B-51)}$$

From equation (B-40)

$$\frac{\partial(FACTI2)}{\partial f_i} = - \left[\frac{1}{\sin^2 \mu_i} + \frac{1}{1 - \cos(2\gamma_i + \mu_i)}\right] \frac{\partial \mu_i}{\partial f_i} \quad . \qquad \text{(B-52)}$$

Rewriting equation (A-37) as

$$\frac{\partial \mu_i}{\partial f_i} = (FACTI1) \ f_i \ (e_i^2 + f_i^2)^{-1} \qquad \text{(B-53)}$$

where FACTI1 is given by equation (B-45), we obtain

$$\frac{\partial^2 \mu_i}{\partial f_i^2} = (FACTI1) \left[\frac{-2f_i^2}{|v_i|^4} + \frac{1}{|v_i|^2}\right] + \frac{\partial(FACTI1)}{\partial f_i} \ \frac{f_i}{|v_i|^2} \quad . \qquad \text{(B-54)}$$

Using equation (B-45)

$$\frac{\partial(\text{FACTI1})}{\partial f_i} = \left[\frac{\cos(\gamma_i + \mu_i)\cos\gamma_i - 1}{\sin^2(\gamma_i + \mu_i)}\right]\frac{\partial \mu_i}{\partial f_i} \quad \text{(B-55)}$$

$\frac{\partial Q_{D_i}}{\partial f_i}$ is given by equation (B-48).

Derivation for XDC39

Equation (A-27) is written as

$$\frac{\partial Q_{D_i}}{\partial \gamma_i} = Q_{D_i}\left[\text{XGAI1} + \frac{\partial \mu_i}{\partial \gamma_i}(\text{FACTI2})\right] \quad \text{(B-56)}$$

where

$$\text{XGAI1} = \frac{2\sin(2\gamma_i + \mu_i)}{1 - \cos(2\gamma_i + \mu_i)} \quad \text{(B-57)}$$

and FACTI2 is defined by equation (B-40).

Using equation (B-56)

$$\text{XDC39} = \frac{\partial^2 Q_{D_i}}{\partial \gamma_i^2}$$

$$= Q_{D_i}\left[\frac{\partial(\text{XGAI1})}{\partial \gamma_i} + \frac{\partial \mu_i}{\partial \gamma_i}\frac{\partial(\text{FACTI2})}{\partial \gamma_i} + \frac{\partial^2 \mu_i}{\partial \gamma_i^2}(\text{FACTI2})\right]$$

$$+ \frac{\partial Q_{D_i}}{\partial \gamma_i}\left[\text{XGAI1} + \frac{\partial \mu_i}{\partial \gamma_i}(\text{FACTI2})\right] \quad \text{(B-58)}$$

The first order partial derivatives occuring in equation (B-58) are derived in the following equations.

Using equation (B-57)

$$\frac{\partial(\text{XGAI1})}{\partial\gamma_i} = \frac{(-2.0)\ [2 + \partial\mu_i/\partial\gamma_i]}{1 - \cos(2\gamma_i + \mu_i)} \quad . \tag{B-59}$$

From equation (B-40)

$$\frac{\partial(\text{FACTI2})}{\partial\gamma_i} = \frac{(-1.0)}{\sin^2\mu_i}\ \frac{\partial\mu_i}{\partial\gamma_i} + \frac{(-1.0)\ [2 + \partial\mu_i/\partial\gamma_i]}{1 - \cos(2\ \gamma_i + \mu_i)} \quad . \tag{B-60}$$

Taking the partial derivative of equation (A-38) with respect to γ_i, we obtain

$$\frac{\partial^2\mu_i}{\partial\gamma_i^2} = \frac{\sin\mu_i - \sin\gamma_i\cos(\gamma_i + \mu_i)(\partial\mu_i/\partial\gamma_i)}{\sin^2(\gamma_i + \mu_i)} \quad . \tag{B-61}$$

$\frac{\partial Q_{D_i}}{\partial\gamma_i}$ as used in equation (B-58) is given by equation (B-56).

Derivation for XDC40

Taking the partial derivative of equation (B-48) with respect to e_i, we get

$$\text{XDC40} = \frac{\partial^2 Q_{D_i}}{\partial e_i\ \partial f_i}$$

$$= Q_{D_i}\ [\frac{\partial(\text{XFI1})}{\partial e_i} + \frac{\partial\mu_i}{\partial f_i}\ \frac{\partial(\text{FACTI2})}{\partial e_i} + \frac{\partial^2\mu_i}{\partial e_i\ \partial f_i}\ (\text{FACTI2})]$$

$$+ \frac{\partial Q_{D_i}}{\partial e_i}\ [\text{XFI1} + \frac{\partial\mu_i}{\partial f_i}\ (\text{FACTI2})] \quad . \tag{B-62}$$

The partial derivatives occuring in equation (B-62) are defined in the following equations.

Using equation (B-49)

$$\frac{\partial(XFI1)}{\partial e_i} = \frac{-4e_i f_i}{|V_i|^4} \quad \text{(B-63)}$$

Using equation (B-53)

$$\frac{\partial^2 \mu_i}{\partial e_i\ \partial f_i} = (-2.0)(FACTI1)\ \frac{e_i f_i}{|V_i|^4} + \frac{\partial(FACTI1)}{\partial e_i}\ \frac{f_i}{|V_i|^2} \quad \text{(B-64)}$$

$\frac{\partial Q_{D_i}}{\partial e_i}$, $\frac{\partial(FACTI2)}{\partial e_i}$, and $\frac{\partial(FACTI1)}{\partial e_i}$ are given by equations (B-38), (B-43), and (B-47), respectively.

Derivation for XDC41

If we take the partial derivative of equation (B-56) with respect to e_i, we obtain

$$XDC41 = \frac{\partial^2 Q_{D_i}}{\partial e_i\ \partial \gamma_i}$$

$$= Q_{D_i}\ [\frac{\partial(XGAI1)}{\partial e_i} + \frac{\partial \mu_i}{\partial \gamma_i}\ \frac{\partial(FACTI2)}{\partial e_i} + \frac{\partial^2 \mu_i}{\partial e_i\ \partial \gamma_i}\ (FACTI2)]$$

$$+ \frac{\partial Q_{D_i}}{\partial e_i}\ [XGAI1 + \frac{\partial \mu_i}{\partial \gamma_i}\ (FACTI2)] \quad \text{(B-65)}$$

The partial derivatives of equation (B-65) are defined as follows:

$$\frac{\partial(XGAI1)}{\partial e_i} = [\frac{-2.0}{1 - \cos(2\gamma_i + \mu_i)}]\ \frac{\partial \mu_i}{\partial e_i} \quad \text{(B-66)}$$

$\frac{\partial Q_{D_i}}{\partial e_i}$, and $\frac{\partial \mu_i}{\partial \gamma_i}$ are given by equations (B-38), and (A-38), respectively.

From equation (A-33)

$$\frac{\partial^2 \mu_i}{\partial e_i \, \partial \gamma_i} = \frac{(-1.0) \sin \gamma_i \cos(\gamma_i + \mu_i)}{\sin^2(\gamma_i + \mu_i)} \; \frac{\partial \mu_i}{\partial e_i} \quad \text{(B-67)}$$

$\frac{\partial(\text{FACTI2})}{\partial e_i}$, and $\frac{\partial \mu_i}{\partial e_i}$ are expressed by equations (B-43), and (B-44), respectively.

Derivation for XDC42

If we take the partial derivative of equation (B-56) with respect to f_i, we get

$$\text{XDC42} = \frac{\partial^2 Q_{D_i}}{\partial f_i \, \partial \gamma_i}$$

$$= Q_{D_i} \left[\frac{\partial(\text{XGAI1})}{\partial f_i} + \frac{\partial \mu_i}{\partial \gamma_i} \; \frac{\partial(\text{FACTI2})}{\partial f_i} + \frac{\partial^2 \mu_i}{\partial f_i \, \partial \gamma_i} (\text{FACTI2})\right]$$

$$+ \frac{\partial Q_{D_i}}{\partial f_i} \left[\text{XGAI1} + \frac{\partial \mu_i}{\partial \gamma_i} (\text{FACTI2})\right] \quad \text{(B-68)}$$

The partial derivatives occuring in equation (B-68) are defined as follows:

$$\frac{\partial(\text{XGAI1})}{\partial f_i} = \left[\frac{-2.0}{1 - \cos(2\gamma_i + \mu_i)}\right] \frac{\partial \mu_i}{\partial f_i} \quad \text{(B-69)}$$

$\frac{\partial Q_{D_i}}{\partial f_i}$, and $\frac{\partial \mu_i}{\partial \gamma_i}$ are given by equations (B-48), and (A-38), respectively.

Using equation (A-33)

$$\frac{\partial^2 \mu_i}{\partial f_i \, \partial \gamma_i} = \frac{(-1.0) \sin \gamma_i \cos(\gamma_i + \mu_i)}{\sin^2(\gamma_i + \mu_i)} \; \frac{\partial \mu_i}{\partial f_i} \quad \text{(B-70)}$$

$\frac{\partial(\text{FACTI2})}{\partial f_i}$ and $\frac{\partial \mu_i}{\partial f_i}$ are given by equations (B-52) and (B-53), respectively.

B.3 Second Order Partial Derivatives of P_{D_r}

Recalling equation (2.67)

$$P_{D_r} = P_{D_r}(e_r, f_r, \alpha_r)$$

and applying the Taylor expansion to it, we get

$$P_{D_r}(e_r + \Delta e_r, f_r + \Delta f_r, \alpha_r + \Delta\alpha_r) - P_{D_r}(e_r, f_r, \alpha_r)$$

$$= [\frac{\partial P_{D_r}}{\partial e_r}\Delta e_r + \frac{\partial P_{D_r}}{\partial f_r}\Delta f_r + \frac{\partial P_{D_r}}{\partial \alpha_r}\Delta\alpha_r]$$

$$+ [\Delta e_r \Delta f_r \frac{\partial^2 P_{D_r}}{\partial e_r \partial f_r} + \Delta e_r \Delta\alpha_r \frac{\partial^2 P_{D_r}}{\partial e_r \partial \alpha_r} + \Delta f_r \Delta\alpha_r \frac{\partial^2 P_{D_r}}{\partial f_r \partial \alpha_r}$$

$$+ \frac{1}{2}[\Delta e_r^2 \frac{\partial^2 P_{D_r}}{\partial e_r^2} + \Delta f_r^2 \frac{\partial^2 P_{D_r}}{\partial f_r^2} + \Delta\alpha_r^2 \frac{\partial^2 P_{D_r}}{\partial \alpha_r^2}]$$

$$= [\frac{\partial P_{D_r}}{\partial e_r}\Delta e_r + \frac{\partial P_{D_r}}{\partial f_r}\Delta f_r + \frac{\partial P_{D_r}}{\partial \alpha_r}\Delta\alpha_r]$$

$$+ [\Delta e_r \Delta f_r (\text{XDC1}) + \Delta e_r \Delta\alpha_r (\text{XDC2}) + \Delta f_r \Delta\alpha_r (\text{XDC3})$$

$$+ \frac{1}{2}[\Delta e_r^2 (\text{XDC4}) + \Delta f_r^2 (\text{XDC5}) + \Delta\alpha_r^2 (\text{XDC6})] \qquad \text{(B-71)}$$

The first order partial derivatives of equation (B-71) have been derived in Appendix A while the second order terms are derived in the following equations.

Derivation for XDC1

Using equation (A-23)

$$XDC1 = \frac{\partial^2 P_{D_r}}{\partial e_r \, \partial f_r}$$

$$= \frac{(-1.0)\ K_1\ a_r\ f_r\ I_D\ \cos\alpha_r}{|V_r|^3} \qquad \text{(B-72)}$$

Derivation for XDC2

Using equation (A-26)

$$XDC2 = \frac{\partial^2 P_{D_r}}{\partial e_r \, \partial \alpha_r}$$

$$= \frac{(-1.0)\ K_1 a_r\ e_r\ I_D\ \sin\alpha_r}{|V_r|} \qquad \text{(B-73)}$$

Derivation for XDC3

Using equation (A-26)

$$XDC3 = \frac{\partial^2 P_{D_r}}{\partial f_r \, \partial \alpha_r}$$

$$= \frac{(-1.0)\ K_1\ a_r\ f_r\ I_D\ \sin\alpha_r}{|V_r|} \qquad \text{(B-74)}$$

Derivation for XDC4

Using equation (A-21)

$$XDC4 = \frac{\partial^2 P_{D_r}}{\partial e_r^2}$$

$$= (K_1 \, a_r \, I_D \cos\alpha_r) \left[\frac{-e_r^2}{|V_r|^3} + \frac{1}{|V_r|}\right] \qquad \text{(B-75)}$$

<u>Derivation for XDC5</u>

Using equation (A-23)

$$XDC5 = \frac{\partial^2 P_{D_r}}{\partial f_r^2}$$

$$= (K_1 \, a_r \, I_D \cos\alpha_r) \left[\frac{-f_r^2}{|V_r|^3} + \frac{1}{|V_r|}\right] \qquad \text{(B-76)}$$

<u>Derivation for XDC6</u>

Using equation (A-27)

$$XDC6 = \frac{\partial^2 P_{D_r}}{\partial \alpha_r^2}$$

$$= (-1.0) \, K_1 \, a_r \, |V_r| \, I_D \cos\alpha_r \qquad \text{(B-77)}$$

<u>B.4 Second Order Partial Derivatives of P_{D_i}</u>

Recalling equation (2.68)

$$P_{D_i} = P_{D_i}(e_i, f_i, \gamma_i)$$

and expanding it using the Taylor series, we obtain

$$P_{D_i}(e_i + \Delta e_i, f_i + \Delta f_i, \gamma_i + \Delta\gamma_i) - P_{D_i}(e_i, f_i, \gamma_i)$$

$$= [\frac{\partial P_{D_i}}{\partial e_i} \Delta e_i + \frac{\partial P_{D_i}}{\partial f_i} \Delta f_i + \frac{\partial P_{D_i}}{\partial \gamma_i} \Delta\gamma_i]$$

$$+ [\Delta e_i \; \Delta f_i \frac{\partial^2 P_{D_i}}{\partial e_i \; \partial f_i} + \Delta e_i \; \Delta\gamma_i \frac{\partial^2 P_{D_i}}{\partial e_i \; \partial \gamma_i} + \Delta f_i \; \Delta\gamma_i \frac{\partial^2 P_{D_i}}{\partial f_i \; \partial \gamma_i}]$$

$$+ \frac{1}{2} [\Delta e_i^2 \frac{\partial^2 P_{D_i}}{\partial e_i^2} + \Delta f_i^2 \frac{\partial^2 P_{D_i}}{\partial f_i^2} + \Delta\gamma_i^2 \frac{\partial^2 P_{D_i}}{\partial \gamma_i^2}]$$

$$= [\frac{\partial P_{D_i}}{\partial e_i} \Delta e_i + \frac{\partial P_{D_i}}{\partial f_i} \Delta f_i + \frac{\partial P_{D_i}}{\partial \gamma_i} \Delta\gamma_i]$$

$$+ [\Delta e_i \; \Delta f_i \; (XDC7) + \Delta e_i \; \Delta\gamma_i \; (XDC8) + \Delta f_i \; \Delta\gamma_i \; (XDC9)]$$

$$+ \frac{1}{2} [\Delta e_i^2 \; (XDC10) + \Delta f_i^2 \; (XDC11) + \Delta\gamma_i^2 \; (XDC12)] \qquad (B\text{-}78)$$

The first order partial derivatives of equation (B-78) have been derived in Appendix A. The derivations for the second order terms of this equation are performed in the following equations.

Derivation for XDC7

Using equation (A-24)

$$XDC7 = \frac{\partial^2 P_{D_i}}{\partial e_i \; \partial f_i}$$

$$= \frac{(-1.0) \; K_1 \; a_i \; e_i \; f_i \; I_D \; \cos\gamma_i}{|V_i|^3} \qquad (B\text{-}79)$$

Derivation for XDC8

Using equation (A-28)

$$XDC8 = \frac{\partial^2 P_{D_i}}{\partial e_i \, \partial \gamma_i}$$

$$= \frac{(-1.0)\ K_1\ a_i\ e_i\ I_D\ \sin\gamma_i}{|V_i|} \quad \text{(B-80)}$$

Derivation for XDC9

Using equation (A-28)

$$XDC9 = \frac{\partial^2 P_{D_i}}{\partial f_i \, \partial \gamma_i}$$

$$= \frac{(-1.0)\ K_1\ a_i\ f_i\ I_D\ \sin\gamma_i}{|V_i|} \quad \text{(B-81)}$$

Derivation for XDC10

Using equation (A-22)

$$XDC10 = \frac{\partial^2 P_{D_i}}{\partial e_i^2}$$

$$= (K_1\ a_i\ I_D\ \cos\gamma_i)\ \left[\frac{-e_i^2}{|V_i|^3} + \frac{1}{|V_i|}\right] \quad \text{(B-82)}$$

Derivation for XDC11

Using equation (A-24)

$$XDC11 = \frac{\partial^2 P_{D_i}}{\partial f_i^2}$$

$$= (K_1 \; a_i \; I_D \; \cos\gamma_i) \; [\frac{-f_i^2}{|V_i|^3} + \frac{1}{|V_i|}] \quad . \qquad \text{(B-83)}$$

Derivation for XDC12

Using equation (A-28)

$$XDC12 = \frac{\partial^2 P_{D_i}}{\partial \gamma_i^2}$$

$$= (-1.0) \; K_1 \; a_i \; |V_i| \; I_D \; \cos\gamma_i \quad . \qquad \text{(B-84)}$$

APPENDIX C

TEST SYSTEMS DATA

(1) Test Systems A

(2) Test Systems B

(3) Test Systems C

(4) Test Systems D

Table C-1: Line Data for Test System A

Line Designation	Resistance p.u.*	Reactance p.u.*	Line Charging p.u.*
3-1	0.08	0.24	0.025
2-3	0.01	0.03	0.010
4-1	0.04	0.12	0.015
4-3	0.06	0.18	0.020
4-2	0.06	0.18	0.020
5-2	0.08	0.24	0.025
5-4	0.02	0.06	0.030

* These values are in p.u. on a 100 MVA base. The line charging is one-half of the total charging of the line.

Table C-2: Bus Data for Test System A

Bus	Generation		Load		Voltage	
Number	MW	Mvar	MW	Mvar	Mag p.u.	Angle deg.
1	0.0	0.0	60.0	10.0	Unspec.	Unspec.
2	0.0	0.0	45.0	15.0	Unspec.	Unspec.
3	0.0	0.0	40.0	5.0	Unspec.	Unspec.
4	40.0	30.0	20.0	10.0	1.047	Unspec.
5	Unspec.	(Swing)	0.0	0.0	1.06	0.0

Table C-3: Generator Reactive Capability Data for Test System A

Bus Number	Underexcited Mvar Capability	Overexcited Mvar Capability
4	-10	50

Table C-4: Specified DC Link Parameters for Test System A

Rectifier bus number	=	2
Inverter bus number	=	3

For rectifier:

Commutating reactance (p.u.)	=	0.10
Delay angle - initial guess (deg.)	=	8
Delay angle - minimum (deg.)	=	7
Delay angle - maximum (deg.)	=	18
Transformer tap ratio - minimum	=	0.95
Transformer tap ratio - maximum	=	1.15

For inverter:

Commutating reactance (p.u.)	=	0.10
Extinction angle - initial guess (deg.	=	15
Extinction angle - minimum (deg.)	=	12
Extinction angle - maximum (deg.)	=	18
Transformer tap ratio - minimum	=	0.95
Transformer tap ratio - maximum	=	1.15

Power to be maintained over DC link at the inverter end (MW)[1]	=	18.88
Resistance[2] of DC line (p.u.)	=	0.00167
DC current (p.u.)	=	0.15

(1) This power is equal to the active power carried by the replaced AC line between bus 2 and bus 3.

(2) Adjusted DC resistance = $(\frac{1}{6})$ x resistance of the replaced AC line between bus 2 and bus 3.

Table C-5: Line Data for Test System B

Line Designation	Resistance p.u.*	Reactance p.u.*	Line Charging p.u.*
8-7	1.00	7.00	0.0005
8-3	0.20	1.00	0.0
8-4	0.30	3.00	0.0
8-5	0.80	6.50	0.0003
5-4	0.35	2.00	0.0
5-6	0.75	6.30	0.0006
1-6	0.10	1.50	0.0
2-6	0.25	2.30	0.0
7-6	1.10	8.10	0.0008
7-2	0.32	3.00	0.0
2-3	0.21	1.00	0.0
3-4	0.20	1.30	0.0
4-1	0.16	2.10	0.0
2-1	0.21	3.11	0.0

* The values are in p.u. on a 100 MVA base. The line charging is the total line charging of the line.

Table C-6: Bus Data for Test System B

Bus Number	Generation MW	Generation Mvar	Load MW	Load Mvar	Voltage Mag. p.u.	Voltage Angle deg.
1	0.0	0.0	0.0	10.0	Unspec.	Unspec.
2	25.0	0.0	0.0	0.0	Unspec.	Unspec.
3	0.0	0.0	22.0	13.0	Unspec.	Unspec.
4	25.0	20.0	0.0	0.0	Unspec.	Unspec.
5	0.0	Unspec.	20.0	0.0	1.0	Unspec.
6	15.0	Unspec.	0.0	0.0	1.0	Unspec.
7	0.0	Unspec.	23.3	0.0	1.0	Unspec.
8	Unspec.	(SWING)	0.0	0.0	1.0	0.0

Table C-7: Generator Reactive Capability Data for Test System B

Bus Number	Underexcited Mvar Capability	Overexcited Mvar Capability
5	-10.0	10.0
6	-10.0	10.0
7	-10.0	10.0

Table C-8: Specified DC Link Parameters for the Test System B

Rectifier bus number	=	2
Inverter bus number	=	3
For rectifier:		
Commutating reactance (p.u.)	=	0.10
Delay angle - initial guess (deg.)	=	8
Delay angle - minimum (deg.)	=	7
Delay angle - maximum (deg.)	=	18
Transformer tap ratio-minimum	=	0.95
Transformer tap ratio-maximum	=	1.15
Filter susceptance (p.u.)	=	0.031
For inverter:		
Commutating reactance (p.u.)	=	0.10
Extinction angle-initial guess (deg.)	=	15
Extinction angle - minimum (deg.)	=	12
Extinction angle - maximum (deg.)	=	20
Transformer tap ratio-minimum	=	0.95
Transformer tap ratio-maximum	=	1.15
Filter susceptance (p.u.)	=	0.041
Power to be maintained over DC link at the inverter end (MW)[1]	=	12.75
Resistance[2] of the DC line (p.u.)	=	0.035
~~DC current (p.u.)~~	~~=~~	~~0.11~~

(1) This power is made equal to the active power carried by the replaced AC line between bus 2 and bus 3.

(2) Adjusted DC resistance $= (\frac{1}{6})$ x resistance of the replaced AC line between bus 2 and bus 3.

Table C-9: Line Data for Test System C

Line Designation	Resistance p.u.*	Reactance p.u.*	Line Charging p.u.*
14-13	0.0194	0.0592	0.0528
14-4	0.0540	0.2230	0.0492
13-12	0.0470	0.1980	0.0438
13-5	0.0581	0.1763	0.0374
13-4	0.0569	0.1739	0.0340
12-5	0.0670	0.1710	0.0346
4-5	0.0133	0.0421	0.0128
5-7	0.0	0.2091	0.0
5-9	0.0	0.5562	0.0
4-11	0.0	0.2520	0.0
11-8	0.0950	0.1989	0.0
11-3	0.1229	0.2558	0.0
11-2	0.0661	0.1303	0.0
7-10	0.0	0.1761	0.0
7-9	0.0	0.1100	0.0
9-6	0.0318	0.0845	0.0
9-1	0.1271	0.2704	0.0
6-8	0.0820	0.1921	0.0
3-2	0.2209	0.1999	0.0
2-1	0.1709	0.3480	0.0

* These values are in p.u. on a 100 MVA base. The line charging is the total line charging of the line.

Table C-10: Bus Data for Test System C

Bus Number	Generator MW	Generator Mvar	Load MW	Load Mvar	Voltage Mag. p.u.	Voltage Angle deg.
1	0.0	0.0	14.9	5.0	Unspec.	Unspec.
2	0.0	0.0	13.5	5.8	Unspec.	Unspec.
3	0.0	0.0	6.1	1.6	Unspec.	Unspec.
4	0.0	0.0	7.6	1.6	Unspec.	Unspec.
5	0.0	0.0	47.8	3.9	Unspec.	Unspec.
6	0.0	0.0	9.0	5.8	Unspec.	Unspec.
7	0.0	0.0	0.0	0.0	Unspec.	Unspec.
8	0.0	0.0	3.5	1.8	Unspec.	Unspec.
9	0.0	0.0	29.5	16.6	Unspec.	Unspec.
10	0.0	17.4	0.0	0.0	1.09	Unspec.
11	0.0	12.24	11.2	7.5	1.07	Unspec.
12	0.0	23.4	94.2	19.0	1.01	Unspec.
13	40.0	4.0	21.7	12.7	1.045	Unspec.
14	Unspec.	(SWING)	0.0	0.0	1.06	0.0

Table C-11: Transformer Data for Test System C

Transformer Designation	Tap Setting
5-7	0.978
5-9	0.969
4-11	0.932

Table C-12: Static Capacitor Data for Test System C

Bus Number	Susceptance p.u.*
9	0.19

* Susceptance in p.u. on a 100 MVA base.

Table C-13: Specified DC Link Parameters for Test System C

Rectifier bus number = 4
Inverter bus number = 5

For rectifier:

Commutating reactance (p.u.) = 0.10
Delay angle - initial guess (deg.) = 8
Delay angle - minimum (deg.) = 7
Delay angle - maximum (deg.) = 18
Transformer tap ratio - minimum = 0.95
Transformer tap ratio - maximum = 1.15
Filter susceptance (p.u.) = 0.4902

For inverter:

Commutating reactance (p.u.) = 0.10
Extinction angle - initial guess (deg.) = 15
Extinction angle - minimum (deg.) = 12
Extinction angle - maximum (deg. = 18
Transformer tap ratio - minimum = 0.95
Transformer tap ratio - maximum = 1.15
Filter susceptance (p.u.) = 0.6301

Power to be maintained over DC link at the inverter end (MW)[1]	=	58.55
Resistance[2] of DC line (p.u.)	=	0.00223
DC current (p.u.)	=	0.456

(1) This power is equal to the active power carried by the replaced AC line between bus 4 and bus 5.

(2) Adjusted DC resistance = $(\frac{1}{6})$ x resistance of the replaced AC line between bus 4 and bus 5.

Table C-14: Line Data for Test System D

Line Designation	Resistance p.u.*	Reactance p.u.*	Line Charging p.u.*
57-2	0.0083	0.0280	0.0645
2-3	0.0298	0.0850	0.0409
3-4	0.0112	0.0366	0.0190
4-5	0.0625	0.1320	0.0129
4-6	0.0430	0.1480	0.0174
6-7	0.0200	0.1020	0.0138
6-8	0.0339	0.1730	0.0235
8-9	0.0099	0.0505	0.0274
9-10	0.0369	0.1679	0.0220
9-11	0.0258	0.0848	0.0109
9-12	0.0648	0.2950	0.0386
9-13	0.0481	0.1580	0.0203
13-14	0.0132	0.0434	0.0055
13-15	0.0269	0.0869	0.0115
57-15	0.0178	0.0910	0.0494
57-16	0.0454	0.2060	0.0273
57-17	0.0238	0.1080	0.0143
3-15	0.0162	0.0530	0.0272
4-18	0	0.555	0
4-18	0	0.43	0
5-6	0.0302	0.0611	0.0062
7-8	0.0139	0.0712	0.0097
10-12	0.0277	0.1262	0.0164
11-13	0.0223	0.0732	0.0094
12-13	0.0178	0.0580	0.0302
12-16	0.0180	0.0813	0.0108
12-17	0.0397	0.1790	0.0238
14-15	0.0171	0.0547	0.0074
18-19	0.4610	0.6850	0
19-20	0.2830	0.4340	0
20-21	0	0.7767	0
21-22	0.0736	0.1170	0
22-23	0.0099	0.0152	0
23-24	0.1660	0[illegible]560	0.0042
24-25	0	1.182	0
24-25	0	1.23	0
24-26	0	0.0473	0
26-27	0.1650	0.2540	0
27-28	0.0618	0.0954	0
28-29	0.0418	0.0587	0
7-29	0	0.0648	0
~~25-30~~	~~0.1350~~	~~0.2020~~	~~0~~
30-31	0.3260	0.4970	0
31-32	0.5070	0.7550	0
32-33	0.0392	0.0360	0
32-34	0	0.9530	0

Table C-14 Cont'd

Line Designation	Resistance p.u.*	Reactance p.u.*	Line Charging p.u.*
34-35	0.0520	0.0780	0.0016
35-36	0.0430	0.0537	0.0008
36-37	0.0290	0.0366	0
37-38	0.0651	0.1009	0.0010
37-39	0.0239	0.0379	0
36-40	0.0300	0.0466	0
22-38	0.0192	0.0295	0.0010
11-41	0	0.7490	0
41-42	0.2070	0.3520	0
41-43	0	0.4120	0
38-44	0.0289	0.0585	0.0010
15-45	0	0.1042	0
14-46	0	0.0735	0
46-47	0.0230	0.0680	0.0016
47-47	0.0182	0.0233	0
48-49	0.0834	0.1290	0.0024
49-50	0.0801	0.1280	0
50-51	0.1386	0.2200	0
10-51	0	0.0712	0
13-49	0	0.1910	0
29-52	0.1442	0.1870	0
52-53	0.0762	0.0984	0
53-54	0.1878	0.2320	0
54-55	0.1732	0.2265	0
11-43	0	0.1530	0
44-45	0.0624	0.1242	0.0020
40-56	0	1.1950	0
56-41	0.5530	0.5490	0
56-42	0.2125	0.3540	0
39-1	0	1.3550	0
1-56	0.1740	0.2600	0
38-49	0.1150	0.1770	0.0030
38-48	0.0312	0.0482	0
9-55	0	0.1205	0

* Impedance and line charging susceptance in p.u. on a 100000kVA base

Line charging: one-half of total charging of line.

Table C-15: Bus Data for Test System D

Bus Number	Generation MW	Generation Mvar	Load MW	Load Mvar	Voltage Mag. p.u.	Voltage Angle deg.
57**	Unspec.	(SWING)	55.0	17.0	1.04	0.0
2	0	0	3.0	88.0	1.01	Unspec.
3	40	0	41.0	21.0	0.985	Unspec.
4	0	0	0	0	Unspec.	Unspec.
5	0	0	13.0	4.0	Unspec.	Unspec.
6	0	0	75.0	2.0	0.98	Unspec.
7	0	0	0	0	Unspec.	Unspec.
8	450	0	150.0	22.0	1.005	Unspec.
9	0	0	121.0	26.0	0.98	Unspec.
10	0	0	5.0	2.0	Unspec.	Unspec.
11	0	0	0	0	Unspec.	Unspec.
12	310	0	377.0	24.0	1.015	Unspec.
13	0	0	18.0	2.3	Unspec.	Unspec.
14	0	0	10.5	5.3	Unspec.	Unspec.
15	0	0	22.0	5.0	Unspec.	Unspec.
16	0	0	43.0	3.0	Unspec.	Unspec.
17	0	0	42.0	8.0	Unspec.	Unspec.
18	0	0	27.2	9.8	Unspec.	Unspec.
19	0	0	3.3	0.6	Unspec.	Unspec.
20	0	0	2.3	1.0	Unspec.	Unspec.
21	0	0	0	0	Unspec.	Unspec.
22	0	0	0	0	Unspec.	Unspec.
23	0	0	6.3	2.1	Unspec.	Unspec.
24	0	0	0	0	Unspec.	Unspec.
25	0	0	6.3	3.2	Unspec.	Unspec.
26	0	0	0	0	Unspec.	Unspec.
27	0	0	9.3	0.5	Unspec.	Unspec.
28	0	0	4.6	2.3	Unspec.	Unspec.
29	0	0	17.0	2.6	Unspec.	Unspec.
30	0	0	3.6	1.8	Unspec.	Unspec.
31	0	0	5.8	2.9	Unspec.	Unspec.
32	0	0	1.6	0.8	Unspec.	Unspec.
33	0	0	3.8	1.9	Unspec.	Unspec.
34	0	0	0	0	Unspec.	Unspec.
35	0	0	6.0	3.0	Unspec.	Unspec.
36	0	0	0	0	Unspec.	Unspec.
37	0	0	0	0	Unspec.	Unspec.
38	0	0	14.0	7.0	Unspec.	Unspec.
39	0	0	0	0	Unspec.	Unspec.
40	0	0	0	0	Unspec.	Unspec.
41	0	0	6.3	3.0	Unspec.	Unspec.
42	0	0	7.1	4.4	Unspec.	Unspec.
43	0	0	2.0	1.0	Unspec.	Unspec.
44	0	0	12.0	1.8	Unspec.	Unspec.
45	0	0	0	0	Unspec.	Unspec.
46	0	0	0	0	Unspec.	Unspec.
47	0	0	29.7	11.6	Unspec.	Unspec.

Table C-15: Continued

Bus Number*	Generation MW	Generation Mvar	Load MW	Load Mvar	Voltage Mag. p.u.	Voltage Angle deg.
48	0	0	0	0	Unspec.	Unspec.
49	0	0	18.0	8.5	Unspec.	Unspec.
50	0	0	21.0	10.5	Unspec.	Unspec.
51	0	0	18.0	5.3	Unspec.	Unspec.
52	0	0	4.9	2.2	Unspec.	Unspec.
53	0	0	20.0	10.0	Unspec.	Unspec.
54	0	0	4.1	1.4	Unspec.	Unspec.
55	0	0	6.8	3.4	Unspec.	Unspec.
56	0	0	7.6	2.2	Unspec.	Unspec.
1	0	0	6.7	2.0	Unspec.	Unspec.

* This is prior to the bus renumbering necessary to enable the regulated buses to have the higher numbers.

Table C-16: Generator Reactive Capability Data for Test System D

Bus Number	Underexcited Mvar Capability	Overexcited Mvar Capability
2	-17	50
3	-10	60
6	-8	25
8	-140	200
9	-3	9
12	-50	155

Table C-17: Transformer Data for Test System D

Transformer designation	Tap Setting*
4-18	0.97
4-18	0.978
7-29	0.967
9-55	0.94
10-51	0.93
11-41	0.955
11-43	0.958
13-49	0.895
14-46	0.9
15-45	0.955
21-20	1.043
24-25	1.000
24-25	1.000
24-26	1.043
34-32	0.975
39-1	0.98
40-56	0.958

*Off-nominal turns ratio, as determined by the actual transformer-tap positions and the voltage basis. In the case of nominal turns ratio, this would equal 1.

Table C-18: Static Capacitor Data for Test System D

Bus Number	Susceptance* p.u.
18	0.1
25	0.059
53	0.063

* Susceptance in p.u. on a 100 MVA base

Table C-19: Bus Numbers that are Changed to Allow the Regulated Buses to have the Largest Numbers

Bus Number		Bus Number
2	becomes	51
3	becomes	52
6	becomes	53
8	becomes	54
9	becomes	55
12	becomes	56
51	becomes	2
52	becomes	3
53	becomes	6
54	becomes	8
55	becomes	9
56	becomes	12

Table C-20: Specified DC Link Parameters for Test System D

Rectifier bus number	=	47
Inverter bus number	=	48
For rectifier:		
Commutating reactance (p.u.)	=	0.10
Delay angle - initial guess (deg.)	=	8
Delay angle - minimum (deg.)	=	7
Delay angle - Maximum (deg.)	=	18
Transformer tap ratio - minimum	=	0.85
Transformer tap ratio - maximum	=	1.15
For inverter:		
Commutating reactance (p.u.)	=	0.10
Extinction angle - initial guess (deg.)	=	15
Extinction angle - minimum (deg.)	=	12
Extinction angle - maximum (deg.)	=	20
Transformer tap ratio - minimum	=	0.85
Transformer tap ratio - maximum	=	1.15
Power to be maintained over DC link at the inverter end (MW)[1]	=	17.59
Resistance[2] of DC line (p.u.)	=	0.00303
DC current (p.u.)	=	0.15

(1) This power is equal to the active power carried by the replaced AC line between bus 47 and bus 48.

(2) Adjusted DC resistance = $(\frac{1}{6})$ x resistance of the replaced AC line between bus 47 and bus 48.

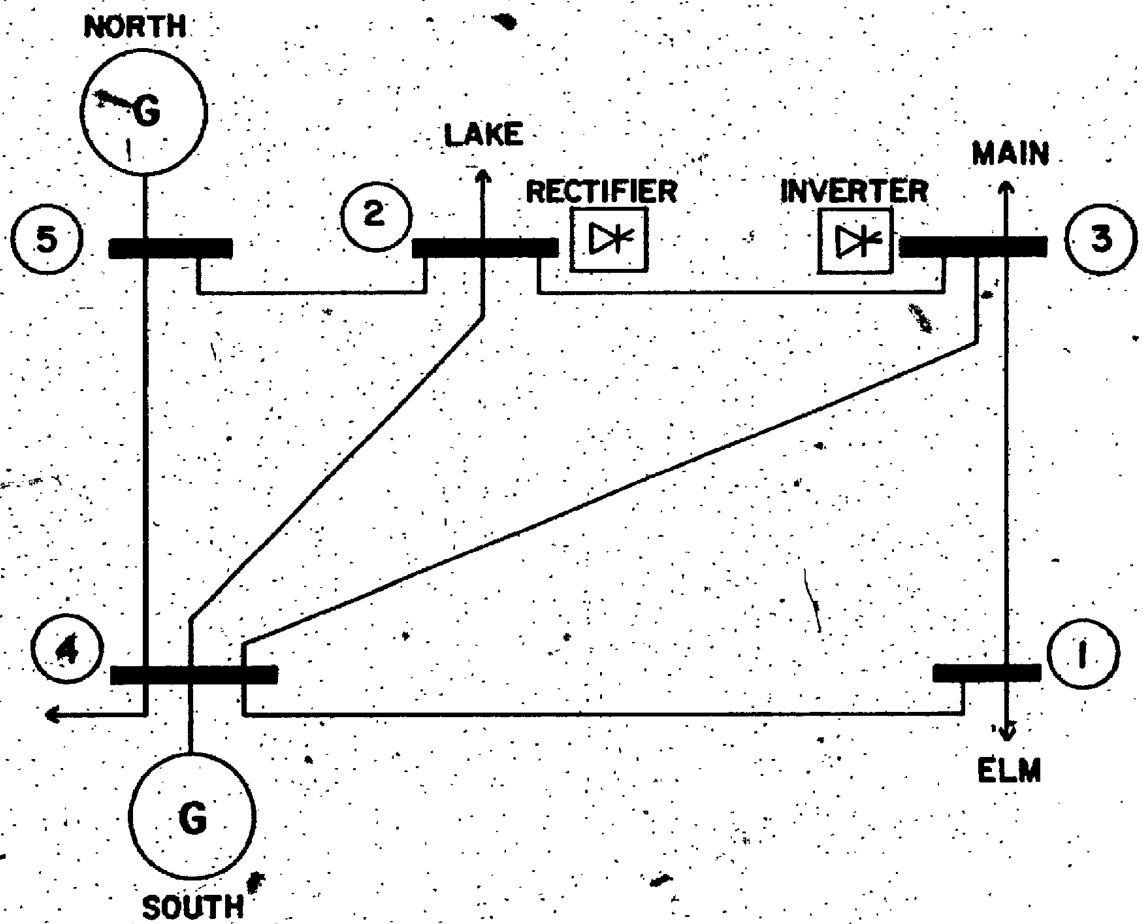

FIGURE C - I Test System A
[5-Bus Power System of Stagg and El-Abiad]

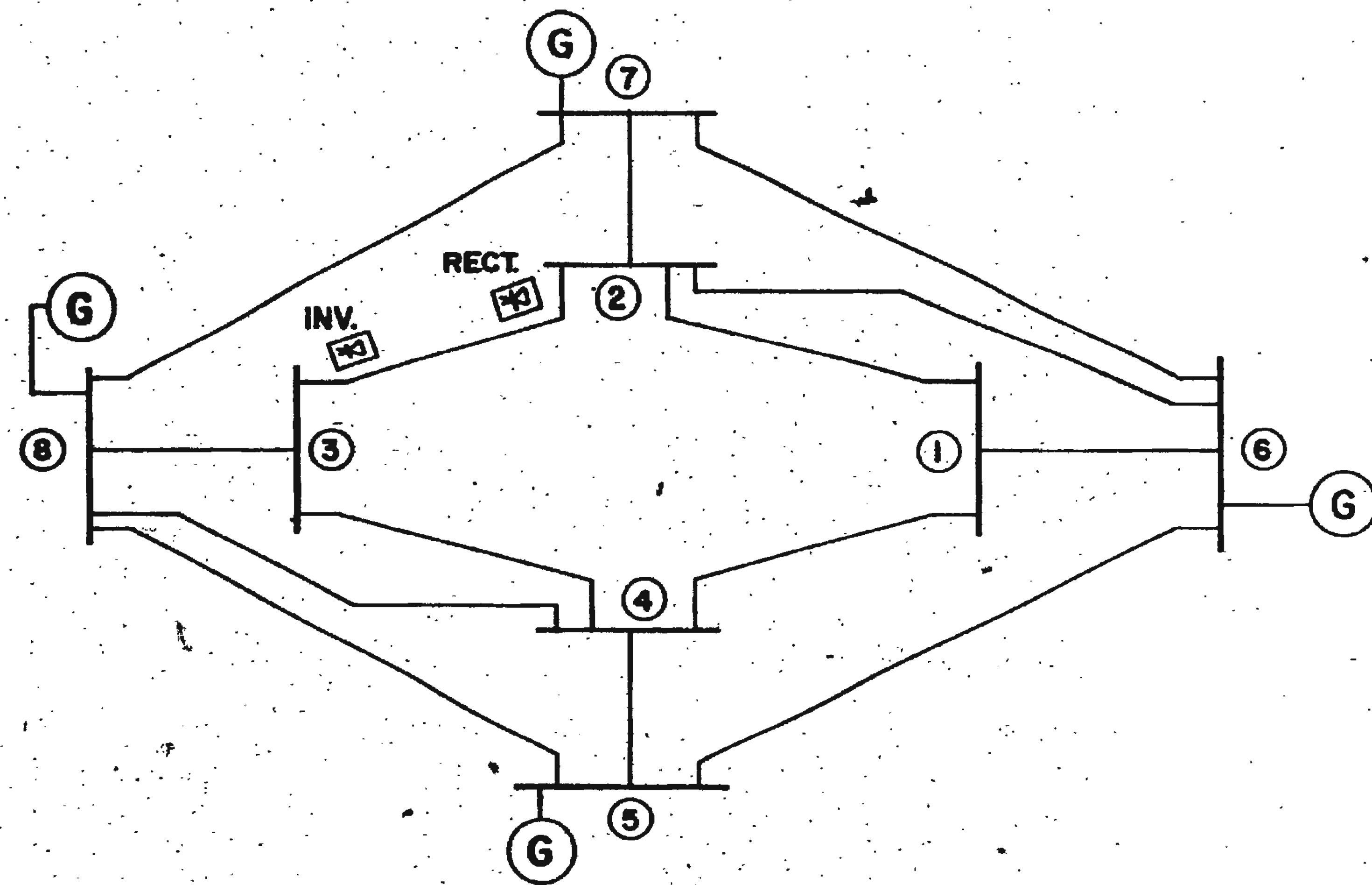

FIGURE C - 2 Test System B
[8-Bus Power System of Elgerd]

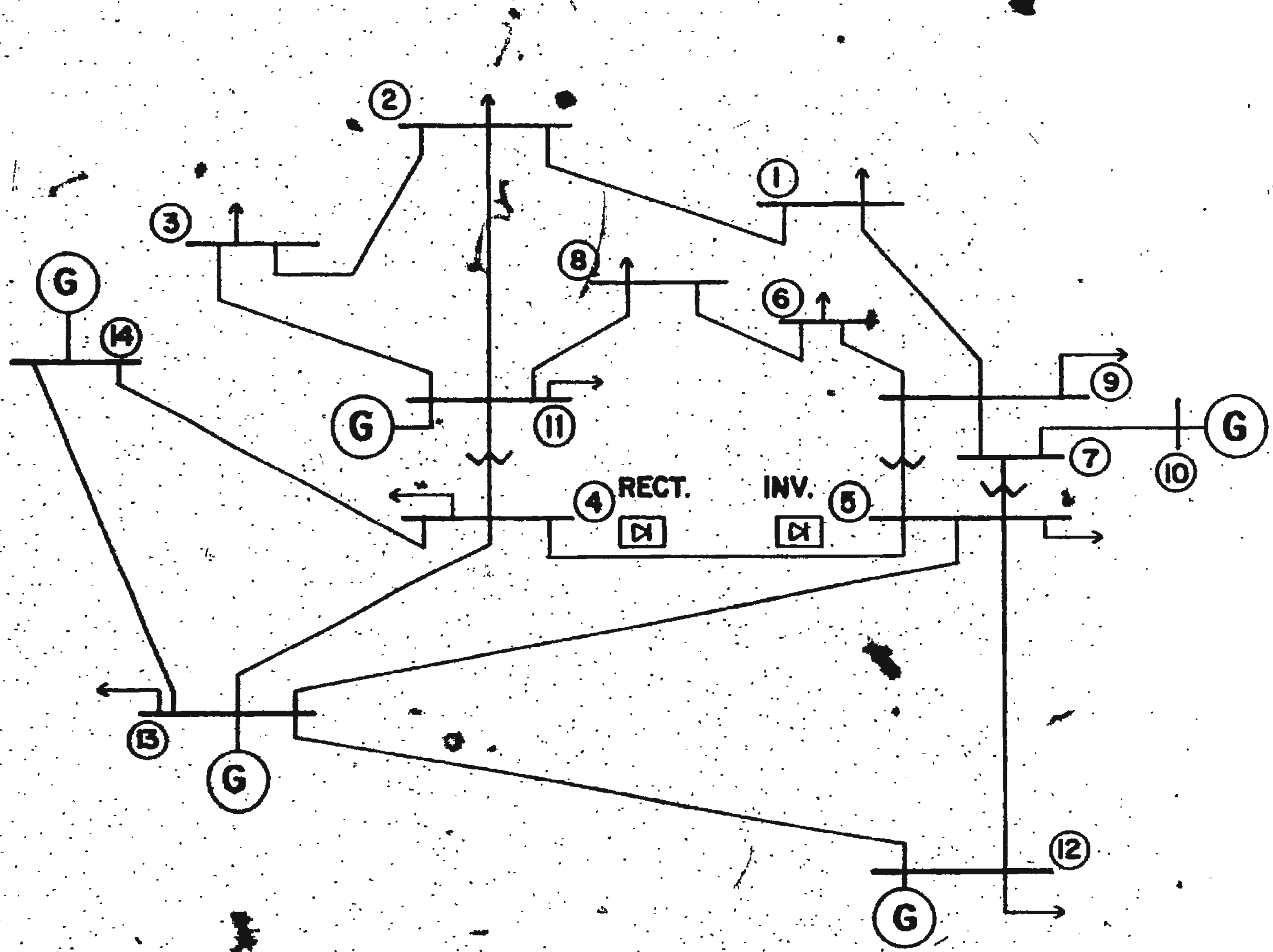

FIGURE G - 3 Test System C
[AEP 14-Bus Power System]

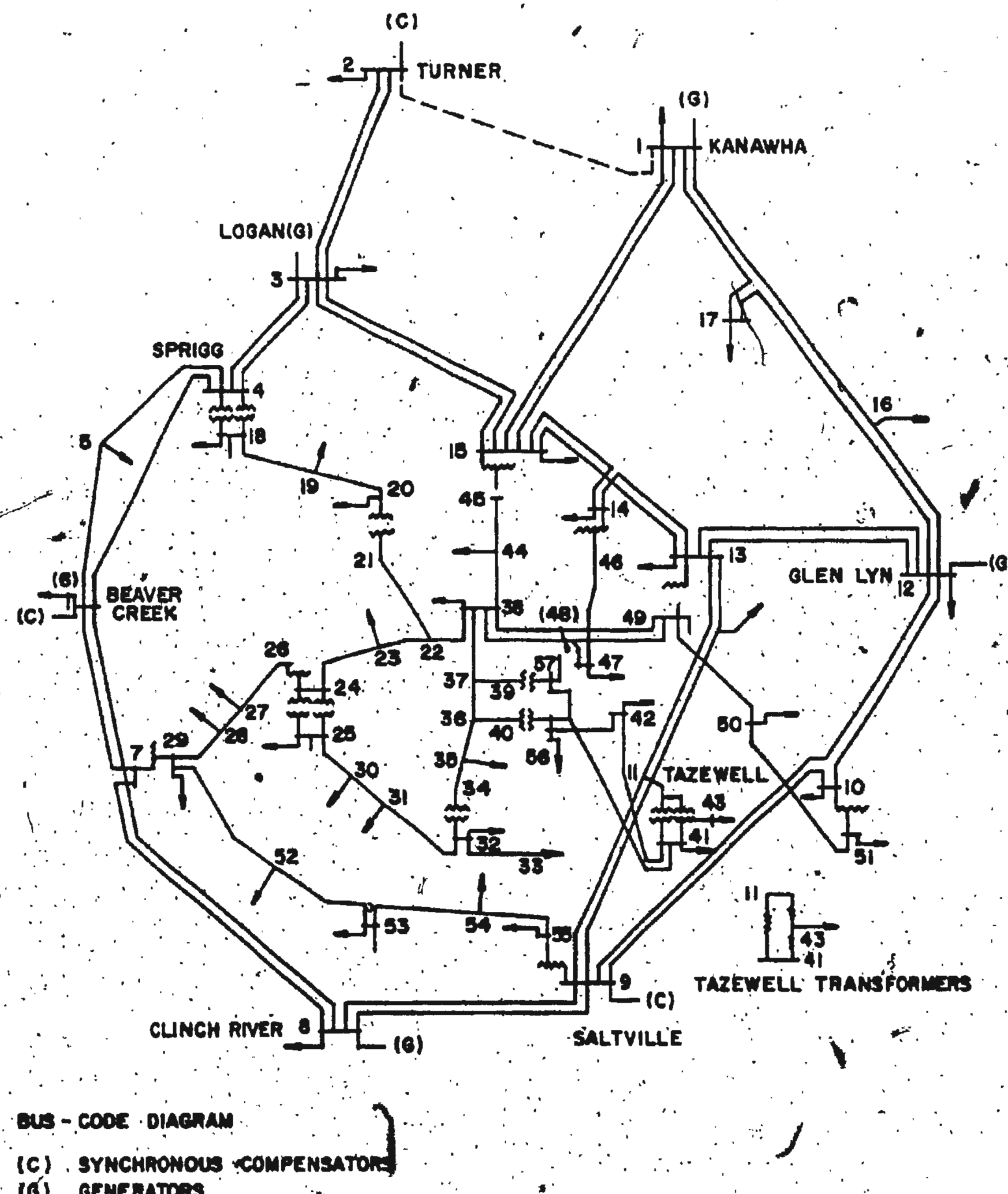

FIGURE C-4 : Test System D
[IEEE 57-Bus Power System]

APPENDIX D

SUBROUTINE SPECIFICATION SHEETS

In this appendix a description of the sparsity directed programs for solving linear sets of equations is given. These were developed at the U.K. Atomic Energy Establishments' Harwell Laboratories.

1. Purpose

To calculate equilibration factors for the rows and columns of a sparse nxn matrix A, which, if applied before Gaussian elimination with pivoting, will make the choice of pivots more likely to lead to low growth of round-off errors. The scaling factors are integral powers 16**R_i (for row i) and 16**C_j (for column j) of 16 (but this base can easily be changed to suit a computer which uses a different radix for floating-point operations). Thus pivots should be chosen as if the matrix elements had been—

$$b_{ij} = a_{ij} * 16 ** (R_i + C_j) \qquad (1)$$

The matrix A is stored in the condensed form used by MA18A. Here the non-zero elements are stored linearly by columns, with row numbers in a parallel INTEGER*2 array and with pointers to the start and finish of each column in a smaller INTEGER*2 array.

2. Calling Sequence and Argument List

```
CALL MC12A(A,IRN,IP,N,NP,ISC,WS,ISING)
```

where

A — is a REAL array which contains the non-zero elements of the matrix to be scaled, with all elements of a column consecutive.

IRN — is an INTEGER*2 array in which IRN(K) contains i if A(K) holds a_{ij}.

IP — is an INTEGER*2 array of dimension (NP,2), where NP $\geq$ N+1. IP(m,2) contains j, the column number of the column which is stored mth in sequence, and IP (j,1) contains k, where A(k) is the first element of column j. IP(IP(m+1,2),1) thus points to the first element of A beyond those for column j. IP(N+1,2) contains N+1, and IP(N+1,1) points to the first unused element of array A.

N — is the order of A.

NP — is the first dimension of IP.

ISC — is an INTEGER*2 array of dimensions (NP,2) in which integer scaling powers are returned, R_i in ISC(j,1) and C_j in ISC(j,2) (see §3 below).

WS — is a workspace array holding 4*N REAL*4 numbers.

ISING — (INTEGER*4) is set on return to 0 normally, but if row I or column j of A is found to consist only of zero elements, ISING is set to I or -J respectively. The scaling factors returned for non-zero rows and columns are correct. For several zero rows or columns, only the last one detected is returned in ISING.

Note: The ouptut and workspace reuqirements for identical for single and double-precision versions, only array A being declared REAL*8 in the latter.

3. Method

The variables p_i and c_j are chosen to minimize the function

$$\phi = \sum_{i,j} (f_{ij}-p_i-c_j)^2 \qquad (2)$$

where

$$f_{ij} = \log |a_{ij}|/\log 16 \qquad (3)$$

and the summation is over pairs i,j for which $a_{ij} \neq 0$. This is done to sufficient accuracy in only a few matrix-by vector multiplications. Then R_i and C_j are obtained by rounding $-p_i$ and $-c_j$ to integers. See Curtis and Reid (1971) for further information.

Use of this method gives far better results on sparse matrices than scaling to equilibrate row and column norms, and MC12A/AD is called by MA18A/AD before factorising a matrix.

Reference

Curtis, A. R. and Reid, J. K. "On the automatic scaling of matrices for Gaussian elimination", AERE note TP.444 (1971).

1. Purpose

This subroutine solves a general sparse NxN system of linear equations

$$\sum_{j=1}^{N} a_{ij} x_j = b_i, \ i = 1, 2, \ldots, N$$

(i.e. find $x = A^{-1}b$) or related problems.

There are four entries:

(a) MA18A decomposes A into triangular factors using a pivotal strategy designed to compromise between maintaining sparsity and controlling loss of accuracy through roundoff.

(b) MA18B uses the factors produced by MA18A (or MA18C) to find $A^{-1}b$, $(A^T)^{-1}b$, Ab or A^Tb.

(c) MA18C factorises a new matrix A of the same pattern, using the pivotal sequence determined by an earlier entry to MA18A.

(d) MA18D loads the elements of a new matrix A into a storage array in the sequence required by MA18C, calling a user-supplied subroutine to obtain each column of A and using indexing information stored by MA18A.

It is envisaged that MA18C may be called many times for one call of MA18A, so it is much faster. Also it is expected that MA18B may be called with many different vectors for the same matrix A.

2. Argument Lists

```
CALL MA18A (A,IRN,IP,N,NP,G,U,IA)

CALL MA18B (A,IRN,IP,N,NP,W,B,MTYPE)

CALL MA18C (A,IRN,IP,N,NP,G)

CALL MA18D (A,IRN,IP,N,NP,W,NAME)
```

A is a REAL*4 (or REAL*8 for the D versions) array of dimension IA holding the non-zero elements of the matrix A on entry to MA18A or MA18C and the elements of the triangular factors on exit. Elements are stored by columns. For entry to MA18A, they must be in natural row order within each column and the columns must be in natural order; that is a_{ij} precedes $a_{k\ell}$

if $j < \ell$ or if $j = \ell$ and $i < k$. Thus a typical order might be $a_{11}, a_{31}, a_{12}, a_{23}, a_{53}, a_{54}, a_{45}, a_{55}$. Before entry to MA18C, the elements of A should be set by calling MA18D. A is altered by MA18A, by MA18C and by MA18D.

IRN is an INTEGER*2 of dimension IA*2, whose first IA elements are used to hold row numbers and whose remaining elements are used for workspace by MA18A only. If a_{ij} is held in A(K) then IRN(K) must contain i; for the above example IRN would contain 1,3,5,1,4,2,5,5,4,5, IRN is altered by MA18A.

IP is an INTEGER*2 array of dimensions (NP,13) where NP $\geq$ 1. Before entry to MA18A the values of IP(J,1), J=1, ..., N should be set to the subscript in array A of the first element of column j of the matrix and IP(N+1,1) should be set to the subscript of the first unused location in A; thus in the above example IP would contain 1,4,6,8,9,11. The contents of IP are altered by MA18A. ((IP (I,J), I=1,N+1), J=1,5) should be left undisturbed between a MA18A entry and a subsequent entry to MA18B/C/D, or (for J=3,4) to MA18A if the previous scaling factors are to be used (see §4). The rest of IP is available as workspace. An equivalence should be used to ensure that IP starts on a 4-byte boundary. MA18C uses the whole of IP as workspace if it obtains new scaling factors (see §4).

N (INTEGER*4) is the order of the matrix A.

NP (INTEGER*4) is the first dimension of the array IP and should be at least N+1.

G (REAL*4 or REAL*8 for the D version) is an output parameter used to indicate the possible growth of errors during the elimination. Normally MA18A and MA18C scale the rows and columns of the matrix (see §4) so that the comparisons used in choosing each pivot will be reasonable. The maximum difference between the floating-point exponent of any element at any stage of the elimination and the floating-point exponent of the initial largest element in its column is evaluated; G is set to the computer rounding error times 16 to the power of this integer. It is thus an estimate of the relative perturbation on the elements of A. It is set to -1 in the event of an error, such as singularity of the matrix or lack of space, preventing successful execution.

U (REAL*4 or REAL*8 for the D version) is a number set by the user in the range $0 < U \leq 1$ to control the choice of pivots: if $U > 1$ it is reset to 1 and if $U \leq 0$ it is reset to the relative floating point accuracy. When searching a row/column for a pivot any element less than U times the largest element in the row/column is excluded. Thus decreasing U biases the algorithm towards maintaining sparsity at the expense of G and vice-versa. The value 0.25 has been found satisfactory in test examples.

IA (INTEGER*4) indicates the size of arrays A and IRN. The number of elements in the decomposed form of A is limited to lA which may not exceed (32767-N) because of the use of INTEGER*2 indices.

W is a REAL*4 (or REAL*8 for the D version) working array of dimension at least N. W(1) may be equivalenced with an element on IP beyond IP(N+1,5) to save space.

B is a REAL*4 (or REAL*8 for the D version) array of dimension N used to hold b on entry and $A^{-1}b$, $(A^T)^{-1}b$, Ab or A^Tb on exit.

MTYPE is an INTEGER*4 variable controlling the action of MA18B. It should have the value 1,2,3, or 4 according to whether $A^{-1}b$, $(A^T)^{-1}b$, Ab or A^Tb is required. If MA18B is called erroneously, it sets MTYPE=0 before return.

NAME is the name of a user supplied subroutine called by MA18D. It must be declared EXTERNAL by the calling program. It is called by

CALL NAME (N,W,J)

and should return all the non-zero elements a_{IJ} of column J of the next matrix in W(I), without altering other elements of W. MA18D will call NAME N times (with J values <u>not</u> in sequence), and load the new matrix into array A.

3. <u>Use of the Entries</u>

Between a call of MA18A and a subsequent call of MA18B, MA18C or MA18D, the contents of (IRN(I), I=1, IA), ((IP(I,J), I=1,N+1), J=1,5) N,NP should not be altered. IP(N+1,2) is set to N+1 on successful completion of MA18A, or to zero if an error has been detected. IP(N+1,3) is set to zero normally but to 1 if MA18C detects a zero pivot.

By examining IP(N+1,2) and for MA18B, IP(N=L,3) subsequent entry points can conclude that MA18A has not been previously entered, or that it or MA18C diagnosed an error. This causes an error diagnostic from the new entry. G is set to -1 by MA18A or MA18C if an error is found, so that the calling program can test for success by examining the sign of G on return. MA18B detects two types of error: those due to invalid entry as described above, which it signals to the calling program by setting MTYPE=0; or if it finds MTYPE out of range, when it leaves it unaltered.

Execution of MA18C is much faster than MA18A, but it is important to check the value of G on successful return from MA18C, in case the old pivotal sequence is poor, for the new matrix, from the roundoff point of view. If G is too large to be acceptable as a relative perturbation on A, the arrays A, IRN, IP should be reset and MA18A re-entered. If there is ample space in arrays A and IRN, it may be worthwhile to increase the value of U. The number of locations used in A or IRN is (IP(N+1,1)-1). G should in any case be monitored to detect error returns.

4. Subroutines Called and Common Block

MA18A (an in some circumstances MA18C) calls subroutine MC12A to obtain row and column scaling factors for the matrix. The application of this scaling is controlled by a parameter in a named common block, which also contains the output stream number for diagnostic messages. The common statement is

COMMON/MA18E/JP,JSCALE

and the default values are JP-6, JSCALE=1.

By including this statement in his program, the user can, if he wishes, charge the stream number JP or the scaling parameter JSCALE.

NB: This should be done by Fortran instructions, not by BLOCK DATA. The significance of JP is obvious; that of JSCALE is as follows:

JSCALE	Scaling action
< 0	Scaling factors determined during an earlier call to MA18A (or MA18C) are applied to the current matrix.
= 0	No scaling is done (i.e. all scaling factors are set to 1.0).
= 1	MC12A is called by MA18A to obtain scaling factors, but the action with MA18C is as for JSCALE < 0.
> 1	MC12A is called both by MA18A and by MA18C.

If JSCALE > 1, MA18C will change all 13 columns of array IP, otherwise it will use only the first 5 columns.

The time overhead for calling MC12A is significant on MA18C, but not usually on MA18A. Moreover, use of scaling factors with MA18C affects only the value of G, not the pivotal sequence as with MA18A.

5. Error Diagnostics

A number of error conditions are diagnosed which prevent successful completion. Most are detected by MA18A, since the other entry points are used only after this one has either succeeded with one set of matrix elements in the prescribed sparsity pattern, or has recorded its failure for them. The following messages may be printed on stream JP by MA18:

(i) ERROR RETURN FROM MA18A BECAUSE THE ELEMENT HELD IN A(k) IS OUT OF ORDER
This message covers sequence errors in the indexing information supplied in IRN, IP; k gives the location at which the error was detected.

(ii) ERROR RETURN FROM MA18A BECAUSE THE MATRIX IS SINGULAR. COLUMN (or ROW) j IS DEPENDENT ON THE REST.
This message covers two cases, (a) where the indexing information specifies no non-zero elements in column (or row) j, (b) where after elimination on the column or row all the elements eligible as pivots are zero.

(iii) ERROR RETURN FROM MA18A BECAUSE IA IS TOO SMALL. SPACE RAN OUT WHEN ELIMINATING ON PIVOT i.
This message appears when there is insufficient room to store a new non-zero element generated in elimination operations using the ith pivot. Thus if i << N much more space will probably be needed, but if i is nearly equal to N just a little more may suffice.

(iv) ERROR RETURN FROM MA18B BECAUSE MTYPE = m WHICH IS OUT OF RANGE.
This message needs no comment.

(v) ERROR RETURN FROM MA18B (or MA18C or MA18D) BECAUSE PREVIOUS ENTRY TO MA18A (or MA18C) GAVE ERROR RETURN.
This message is given with MA18C in the second position only if MA18B occurs in the first position. In that case, the error detected by MA18C was a zero pivot, which may not occur on subsequent re-entry to MA18C with a further new matrix.

(vi) ERROR RETURN FROM MA18B (or MA18C or MA18D) BECAUSE NO PREVIOUS ENTRY TO MA18A.
This message is given if IP(N+1,2) is found to have a value which is neither N+1 (after a successful exit from MA18A) nor 0 (after an error return from MA18A).

(vii) ERROR RETURN FROM MA18C BECAUSE ZERO PIVOT (i j).
This message signifies that a_{ij} was found to be zero when it was due to be used as a pivot by MA18C.

(viii) ERROR RETURN FROM MA18C BECAUSE MA12A HAS GIVEN ERROR RETURN WITH IS = j.
This message signifies that all the elements of the new matrix in row i (if i > 0) or in column (-i) (if i < 0) have been found to be zero.

6. Method

The subroutine is described in detail in AERE Report R.6844, which should be consulted for certain details if it is planned to transfer the sub-routine to a computer other than the System/360.

```
C       SUBROUTINE VSRTPM (A,LA,IR)                                     VSRT0010
C                                                                       VSRT0020
C-VSRTPM --------S/D-----LIBRARY L---------------------------------------VSRT0030
C-VSORTP                                                                VSRT0040
C                                                                       VSRT0050
C   FUNCTION         VSRTPM - SORT ARRAYS BY ABSOLUTE VALUE -           VSRT0060
C                                 PERMUTATIONS RETURNED                 VSRT0070
C                    VSORTP - SORT ARRAYS BY ALGRBRAIC VALUE -          VSRT0080
C                                 PERMUTATIONS RETURNED                 VSRT0090
C   USAGE                   - CALL VSRTPM (A.LA.IR)                     VSRT0100
C                           - CALL VSORTP (A.LA.IR)                     VSRT0110
C   PARAMETERS       A      - ON INPUT, CONTAINS THE ARRAY TO BE        VSRT0120
C                             SORTED ON OUTPUT, A CONTAINS THE          VSRT0130
C                             SORTED ARRAY                              VSRT0140
C                    LA     - INPUT VARIABLE CONTAINING THE NUMBER      VSRT0150
C                             OF ELEMENTS IN THE ARRAY TO BE SORTED     VSRT0160
C                    IR(LA) - ON INPUT, IR CONTAINS THE INTEGER         VSRT0170
C                             VALUES 1,2,....LA. SEE PROGRAMMING        VSRT0180
C                             NOTES.                                    VSRT0190
C                           - ON OUTPUT, IR CONTAINS A RECORD OF THE    VSRT0200
C                             PERMUTATIONS MADE ON THE VECTOR A         VSRT0210
C   PRECISION               - SINGLE/DOUBLE                             VSRT0220
C   LANGUAGE                - FORTRAN                                   VSRT0230
C                                                                       VSRT0240
C-----------------------------------------------------------------------VSRT0250
```

CALL VSRTPM (A,LA,IR)

CALL VSORTP (A,LA,IR)

Purpose

VSRTPM sorts any LA consecutive elements of a vector into ascending sequence by absolute value, keeping a record in IR of the permutations to the vector A. That is, the elements of IR are moved in the same manner as are the elements in A as A is being sorted.

VSORTP sorts any LA consecutive elements of a vector into ascending sequence by algebraic value, keeping a record in IR of the permutations to the vector A. That is, the elements of IR are moved in the same manner as are the elements in A as A is being sorted.

Algorithm

VSRTPM/VSORTP uses the algorithm declared in IMSL routine VSORTM/VSORTA.

Programming Notes

1. IR and A must have dimension at least LA.

2. The vector IR must be initialized before entering VSRTPM/VSORTP. Ordinarily, IR(1) = 1, IR(2) = 2, ..., IR(LA) = LA. For wider applicability, any integer that is to be associated with A(I) for I = 1, 2, ..., LA may be entered into IR(I).

3. If entry VSRTPM is used, A is replaced by the sorted absolute values of its elements, on output.

Example

CALL VSORTP (A,LA,IR)

Input:	Output:
A = (10.,9.,8.,7.,6.,5.,4.,3.,2.,1.)	A = (1.,2.,3.,4.,5.,6.,7.,8.,9.,10.)
LA = 10	IR =(10, 9, 8, 7, 6, 5, 4, 3, 2, 1)
IR = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)	

APPENDIX E

DEFINITIONS OF COMPUTER VARIABLE NAMES AND THEIR DIMENSIONS

E.1 Definitions of Computer Variable Names

Definitions of the following ten variables are given in Appendix D.

A, (IA+IEXTRA), G, IP, IRN, MTYPE, N, NP, U, and W

NAME	DESCRIPTION
AA	Inputed flag which tells if total line shunt admittances to ground is inputed (AA=0) or only one-half of total value is inputed (AA=1).
A1	A vector which holds the row numbers of the non-zero elements of the upper (or lower) triangle of the bus admittance matrix.
A2	A vector which tells us how many non-zero upper triangle bus admittance matrix elements that have been at the commencement of a given column.
ALPHA	The value determines the portion of the second order correction factor which is used to update the bus mismatches and the portion used to update the diagonal elements of the AC Jacobian submatrices and also the additional Jacobian elements due to the HVDC link.
ALPHAR	Delay angle of the rectifier in degrees (input the starting value).
ALPINI	Initial value of the rectifier's delay angle in degrees.
ALPMAX	Upper limit of the rectifier's delay angle in degrees.
ALPMIN	Lower limit of the rectifier's delay angle in degrees.
BB	Inputed flag which indicates whether the transformer inputed information is the tap setting (BB=1) or the turns ratio (BB=0).

BETAI	Ignition advance angle of the inverter.
CC	Inputed flag which determines whether a Newton-Raphson load flow is to be solved (CC=0) or one using the alpha M.Q.S.O.N.R. method (CC=1).
COSALP	Cosine of the rectifier's delay angle.
COSGAM	Cosine of the inverter's extinction angle.
D	Diagonal elements of the bus admittance matrix.
D1, D2, De, D4	Vectors which contain the locations of the diagonal elements of the Jacobian submatrices within the vector of the non-zero elements of the Jacobian, JACOB.
DELK	Contains the iteration number for which the corresponding maximum mismatch values are stored.
DELMAX	The specified tolerance required in order for convergence to occur.
DELMXP	Maximum active power mismatch.
DELMXQ	Maximum reactive power mismatch.
DELMXV	Maximum voltage squared mismatch.
DELP	Variable which stores the value which DELMXP has at the end of an iteration.
DELPDI	DC power mismatch at the inverter.
DELPDR	DC power mismatch at the rectifier.
DELQ	Variable which stores the value which DELMXQ contains at the end of an iteration.
DELS	Temporary storage for updated bus mismatches, needed when there is an unsuccessful convergence check at the one-half iteration point.

DELTAP	Active power bus mismatch.
DELTAQ	Reactive power bus mismatch.
DELTAS	Vector which contains all the bus mismatches.
DELTAV	Voltage squared bus mismatch.
DELV	Stores the value which DELMXV contains after an iteration.
DSERY, DSHTY, DSHTYD, DTMR	Temporary storage for line data when the line numbers are being rearranged.
E	Real component of bus voltage.
EB	Bus number at the receiving end of a line.
EB1	Temporary storage for EB's when line numbers are being rearranged.
ERROR1	Voltage and converter angle corrections after the first half of an iteration.
F	Imaginary component of the bus voltage.
GAMINI	Initial value of the inverter's extinction angle in degrees.
GAMMAI	Extinction angle of the inverter in degrees (input the starting value).
GAMMAX	Upper limit of the inverter's extinction angle in degrees.
GAMMIN	Lower limit of the inverter's extinction angle in degrees.
IA	The number of non-zero elements in the Jacobian matrix.
ID	DC current in the HVDC link.
IDC1	Gives the location of (i) the DC power mismatch at the rectifier in the vector ERROR1 and (ii) the rectifier's delay angle mismatch in the vector DELTAS.

IDC2	Gives the location of (i) the DC power mismatch at the inverter in the vector ERROR1, and (ii) the inverter's delay angle mismatch in the vector DELTAS.
IEXTRA	Workspace required for the sparsity subroutine, MA18A.
JACA1	Vector which contains the row numbers of all the non-zero elements of the Jacobian matrix.
JACA2	Vector which tells how many non-zero Jacobian matrix elements there are up to the commencement of a given column.
JACOB	Non-zero elements of the Jacobian matrix.
K	Iteration counter.
K0	The number of voltage regulated buses, not including the slack bus.
K1	The number of load buses.
K2	The number of buses, not including the slack bus.
KDEL	Records the number of times that a convergence check is performed.
KDUM1, KDUM2, KDUM3, KDUM4, KDUM5, KDUM6, KDUM7, KDUM8	Contain the locations of X_1, X_2, X_3, X_4, X_5, X_6, X_7, and X_8, respectively (see equation 2.72) in the Jacobian matrix.
KKK	If one-half an iteration is successful in converging, the value is set to 5. Otherwise its value is 0.
KKSAVE	Indicator as to whether the load flow solution ends in a half iteration.
KMAX	Maximum number of iterations allowed.
KNECT	Records the number of lines connected to the slack bus.

KSAVE	Number of iterations to convergence for a particular alpha value.
KTOT	The number of non-zero elements in the bus admittance matrix, excluding the row and column corresponding to the swing bus number (NB).
LINE	Contains line numbers.
LINE1	Temporary storage for line numbers during line renumbering sequence.
MAXY	Total number of non-zero elements in the upper (or lower) triangle of the bus admittance matrix.
MB	The number of voltage controlled buses.
MUINV	Overlap angle of the inverter.
MURECT	Overlap angle of the rectifier.
NB	The total number of buses.
NEWA1	Vector containing the row numbers of all the non-zero elements of the bus admittance matrix.
NEWA2	Vector containing running totals, column by column, of the number of non-zero elements in the bus admittance matrix.
NEWY	Vector containing all the non-zero elements of the bus admittance matrix.
NINV	The inverter bus number.
NL	The total number of lines.
NRECT	The rectifier bus number.
P	Calculated active bus power.
PDI	DC power at the inverter (input initially the DC power to be maintained at the inverter end).

PDISP	Specified DC power at the inverter.
PDR	DC power at the rectifier.
PDRSP	Calculated DC power at the rectifier.
PFI	Power factor of the inverter.
PFR	Power factor of the rectifier.
PSCHED	Scheduled active bus power.
Q	Calculated reactive bus power.
QDI	Calculated reactive power required by the inverter.
QDR	Calculated reactive power required by the rectifier.
QSCHED	Scheduled reactive bus power.
R	Second order correction factor for active power.
RDC	Resistance of the DC link.
RECORD	Records the number of lines connected to the sending buses.
S	Complex conjugate bus power.
SAVEP	Vector which stores the maximum active power mismatch for every iteration.
SAVEPI	Vector which stores the DC power mismatch at inverter for every iteration.
SAVEPR	Vector which stores the DC power mismatch at rectifier for every iteration.
SAVEQ	Vector which stores the maximum reactive power mismatch for every iteration.
SAVEV	Vector which stores the maximum voltage squared mismatch for every iteration.
SB	Bus number at the sending end of a line.

SBCAB	Static capacitor or shunt reactor connected to the sending bus of a line.
SERY	Line series admittance.
SERZ	Line series impedance.
SHTY	Line charging susceptance.
SHTYB	Line charging represented by admittance shunted to ground at the receiving end of a line.
SINALP	Sine of the rectifier's delay angle.
SINGAM	Sine of the inverter's extinction angle.
SPDI	Second order DC power correction term at the inverter bus.
SPDR	Second order DC power correction term at the rectifier bus.
SUM1, SUM2	Summation portions of the first order mismatch correction factors.
SUM3, SUM4	Summation portions of the second order mismatch correction factors.
T	Second order correction factors for reactive power.
TAPI	Tap ratio of the inverter's transformer.
TAPR	Tap ratio of the rectifier's transformer.
TFMR	Inputed off-norminal tap setting of a transformer.
TMAXI	Maximum tap ratio of the inverter's transformer.
TMAXR	Maximum tap ratio of the rectifier's transformer.
TMINI	Minimum tap ratio of the inverter's transformer.
TMINR	Minimum tap ratio of the rectifier's transformer.
U2	Second order correction factors for the square of the bus voltage.
V	Complex bus voltage.

VDI	DC voltage at the inverter end of the link.
VDR	DC voltage at the rectifier end of the link.
VINV, VMAGI	Magnitude of the voltage on the AC side of the inverter's transformer.
VRECT, VMAGR	Magnitude of the voltage on the AC side of the rectifier's transformer.
VSCHED	Scheduled bus voltage magnitude.
VTINV	Magnitude of the voltage on the DC side of the inverter's transformer.
VTRECT	Magnitude of the voltage on the DC side of the rectifier's transformer.
VV	Complex conjugate of the bus voltage.
XCI	Commutating reactance of the inverter's AC source.
XCR	Commutating reactance of the rectifier's AC source.
XI	Second order reactive power correction term due to inverter.
XR	Second order reactive power correction term due to rectifier.
Y	Non-diagonal elements of the upper (or lower) triangle of the bus admittance matrix.

E.2 Mathematical Notations

E.2.1 First Order Differential Coefficients

DMUDER = $\partial\mu_r/\partial e_r$, DMUDFR = $\partial\mu_r/\partial f_r$

DMUDAL = $\partial\mu_r/\partial\alpha_r$, DMUDGA = $\partial\mu_i/\partial\gamma_i$

DMUDEI = $\partial\mu_i/\partial e_i$, DMUDFI = $\partial\mu_i/\partial f_i$

DQRDER = $\partial Q_{D_r}/\partial e_r$, DQRDFR = $\partial Q_{D_r}/\partial f_r$

DQRDAL = $\partial Q_{D_r}/\partial\alpha_r$, DQIDGA = $\partial Q_{D_i}/\partial\gamma_i$

DQIDEI = $\partial Q_{D_i}/\partial e_i$, DQIDFI = $\partial Q_{D_i}/\partial f_i$

DPDDER = $\partial P_{D_r}/\partial e_r$, DPDDEI = $\partial P_{D_i}/\partial e_i$

DPDDFR = $\partial P_{D_r}/\partial f_r$, DPDDFI = $\partial P_{D_i}/\partial f_i$

DPDDAL = $\partial P_{D_r}/\partial \alpha_r$, DPDDGA = $\partial P_{D_i}/\partial \gamma_i$

The expressions for these first order differential equations are given in Appendix B.

E.2.2 The variables XDUM1, XDUM2, XDUM3, XDUM4, XDUM5, XDUM6, XDUM7, XDUM8, XDUM9, XDUM10, XDUM11, and XDUM12 are used in the calculation of the second order correction factors and are defined in Chapter 3.

E.2.3 The following variable names represent the second order differential coefficients and are defined in Chapter 3.

XDC1 , XDC2 , XDC3 , XDC4 ,

XDC5 , XDC6 , XDC7 , XDC8 , XDC9 ,

XDC10 , XDC11 , XDC12 , XDC31 , XDC32 ,

XDC33 , XDC34 , XDC35 , XDC36 , XDC37 ,

XDC38 , XDC39 , XDC40 , XDC41 , XDC42 .

E.2.4 The variable names given below are used to simplify the computing algorithm of the program and are described in Appendix B.

XER1, XFR1, XEI1, XFI1, XALR1,

XGAI1, FACTR1, FACTR2, FACTI1, FACTI2

E.3 Dummy Variables

ANGL, ANGR, FACTEN, FACTMO,

FACTNO, FACTRE

FACT4, FACT5, FACT10, FACT11, FACT21, FACT22,

FACT23, FACT24, FACT25, FACT26, FACT27,

FACT28, FACT29, FACT30, FACT31, FACT32,

FACT33, FACT34, FACT35, FACT36, FACT37,

FACT38, FACT39, FACT40, FACT41, FACT42,

FACT43, FACT44, FACT45, FACT46, FACT47,

FACT48, FACT49, FACT50, FACT51,

FACT52, FACT53, FACT54, FACT55,

FACT56, FACT57, FACT58, FACT59,

FACT60, FACT61, FACT62

XDC13, XDC14, XDC15, XDC16, XDC17,

XDC18, XDC19, XDC20, XDC21, XDC22,

XDC23, XDC24, XDC25, XDC26, XDC27,

XDC28, XDC29, XDC30, XDC45

XSAM1, XSAM2, XSAM3, XSAM4

Dimensions of Variables

NB $\triangleq$ number of buses

NL $\triangleq$ number of AC lines

MB $\triangleq$ number of voltage controlled buses, including the slack bus.

VARIABLE	DIMENSION
A	IEXTRA + dimension of JACOB
A1	NL
A2	NB
CR1	NB-1
CR2	NB-1
D	NB
D1, D2, D3, D4	NB-1
DELK	2*(KMAX)
DELP	2*(KMAX)
DELPI	2*(KMAX)
DELPR	2*(KMAX)
DELQ	2*(KMAX)
DELS	2*(NB-1)+2
DELTAS	2*(NB-1)+2
DELV	2*(KMAX)
DSERY	NL
DSHTY	NL
DSHTYB	NL
DTFMR	NL
E	NB
EB	NL

EB1	NL
ERROR1	2*(NB-1)+2
F	NB
IBUS	NB
IR	[(2*NB-1)+2, 13]
IRN	2*(IEXTRA + dimension of JACOB)
JACA1	8* (number of AC lines not connected to the swing bus) +4*(NB-1)+8
JACA2	(2*NB-1)+2
JACOB	same as for JACA1
LINE	NL
LINE1	NL
NEWA1	2*NL+NB
NEWA2	NB+1
NEWY	2*NL+NB
P	NB
PSCHED	NB
Q	NB
QSCHED	NB
RECORD	NB
SB	NL
SBCAP	NL
SERY	NL
SHTY	NL
SHTYB	NL
SUM1	NB-1

SUM2	NB-1
SUM3	NB-1
SUM4	NB-1
TFMR	NL
V	NB
VSCHED	MB-1
W	2*(NB-1)+2
Y	NL

For running a series of load flows with different values, we also have:

KSAVE	number of alpha values +1
KKSAVE	number of alpha values +1
SAVEP	(number of alpha values)*(KMAX)
SAVEPI	(number of alpha values)*(KMAX)
SAVEPR	(number of alpha values)*(KMAX)
SAVEQ	(number of alpha values)*(KMAX)
SAVEV	(number of alpha values)*(KMAX)

APPENDIX F

PROGRAM LISTING

```
C     This program was originally written (April 1979) by O. K. Wellon for A. C.
C     applications and later on modified (October 1983) to include HVDC links
C     by B. N. Singh

C
      COMPLEX Y,V,S,SS,SP,IBUS,SUMS,VV,D,NEWY
      COMPLEX SHTY,DSHTY,ZSER,SERY,DSERY,SHTYB,DSHTYB,S[illegible]E,SERZ
      INTEGER DUMMY,DUM1,DUM2,RECORD,EB1,LINE,LINE1
      INTEGER SB,EB,A1,A2,D1,D2,D3,D4,AA,BB,CC
      REAL*8 ALPHA
      REAL ID,MURECT,MUINV,KAY1,KAY2
      REAL JACOB,MAGN
      INTEGER*2 JACA1,JACA2,IP(115,13),IRN(3664)
      DIMENSION D1(56),D2(56),D3(56),D4(56),CR1(56),CR2(56)
      DIMENSION E(57),F(57),V(57),P(57),Q(57),D(57),A2(57),RECORD(57)
      DIMENSION PSCHED(57),QSCHED(57),IBUS(57)
      DIMENSION SHTY(79),SHTYB(79),SERY(79),DSHTY(79),DSHTYB(79)
      DIMENSION DSERY(79),LINE(79),LINE1(79),EB1(79),Y(79),A1(79)
      DIMENSION TFMR(79),DTFMR(79),SBCAP(79)
      DIMENSION SB(79),EB(79),W(114),DELTAS(114),ERROR1(114),A(1832)
      DIMENSION NEWY(215),NEWA1(215),NEWA2(58),VSCHED(6)
      DIMENSION JACOB(832),JACA1(832),JACA2(115)
      DIMENSION DELK(20),DELP(20),DELQ(20),DELV(20)
      DIMENSION DELS(114),DELPR(20),DELPI(20),SAVEPR(110),SAVEPI(110)
      DIMENSION SAVEP(110),SAVEQ(110),SAVEV(110),KSAVE(12),KKSAVE(12)
      COMMON /MEMORY/ JACOB,JACA1,JACA2
      EXTERNAL RELOAD
    1 WRITE(6,2)
2     FORMAT(1H1,/////,29X,'     57- BUS SYSTEM ')
C
      IEXTRA=1000
C     READ NUMBER OF BUSSES, NUMBER OF LINES, NUMBER OF VOLTAGE CONTROL
C     BUSSES INCLUDING SLACK BUS, AND FLAGS
C
      READ(5,74)ALPHA
74    FORMAT(F10.3)
      READ(5,100)AA,BB,CC
      READ(5,100)NB,NL,MB
      WRITE(6,76)ALPHA,AA,BB,CC,NB,NL,MB
76    FORMAT(' ',//,29X,'ALPHA    AA    BB    CC    NB    NL    MB',/,F3
     12.1[illegible],3I5,3I7)
C
C        PRINTOUT OF CHANGES MADE IN BUS NUMBERS
C
      DO 210 J=1,NB
      RECORD(J)=0
      D(J)=0.0
210   CONTINUE
C
C     READ LINE NUMBER, STARTING BUS, END BUS, LENGTH, SHUNT ADMITTANCE
```

```
C     IN P.U. PER UNIT LENGTH,SERIES IMPEDANCE IN P.U. PER UNIT LENGTH
C
      WRITE(6,102)
102   FORMAT(///,50X,10(1H*),' L I N E  D A T A ',10(1H*),/,/,27X,'LINE'
     1,2X,'SB',3X,'EB',2X,'TFMR RATIO',2X,'SHUNT ADMITTANCE',6X,'SERIES
     2IMPEDANCE',6X,'SB STATIC CAP.',/)
      DO 108 I=1,NL
      READ(5,100)J,SB(I),EB(I),SERZ,SHTY(I),TFMR(I),SBCAP(I)
100   FORMAT(3I5,5F10.4,F1.0,F9.4)
      IF(AA.EQ.0.)GO TO 75
      SHTY(I)=2*SHTY(I)
75    WRITE(6,104)I,SB(I),EB(I),TFMR(I),SHTY(I),SERZ,SBCAP(I)
104   FORMAT(' ',24X,3I5,F9.3,4X,2F9.4,4X,2F9.4,7X,F3.1,F8.4)
      IF(TFMR(I).EQ.0.)GO TO 106
      IF(PP.EQ.0.)GO TO 80
      B=TFMR(I)
      GO TO 85
80    B=1.+TFMR(I)/100.
85    SERY(I)=1./(B*SERZ)
      SHTYB(I)=(1.0/SERZ)*(1.0-(1.0/B))+SHTY(I)/2.
      SHTY(I)=SHTY(I)/B**2-SERY(I)+SERY(I)/B
      SHTY(I)=2.*SHTY(I)
      GO TO 108
  106 SERY(I)=1./SERZ
      SHTYB(I)=SHTY(I)/2.
108   CONTINUE
C     ----------------------------------------------------------------
C
C     SPARSE STORAGE OF ADMITTANCE MATRIX
C     ( INCLUDES REARRANGING LINE NUMBERS IF NECESSARY )
C
C
C
C        ASSEMBLE DIAGONAL ELEMENTS
C
      DO 119 I=1,NL
      Y(I)=0.0
      L=SB(I)
      M=EB(I)
      D(L)=D(L)+SERY(I)+SHTY(I)/2.+SBCAP(I)
      D(M)=D(M)+SERY(I)+SHTYB(I)
119   CONTINUE
      KNECT=0
      DO 220 I=1,NL
C
C        THE SENDING AND END BUS DESIGNATIONS ARE REVERSED IF THE S.B.
```

```
C        NUMBER IS LARGER THAN THE E.B. NUMBER
C
      IF(SB(I).LT.EB(I))GO TO 30
      IF(TFMR(I).NE.0.0)TFMR(I)=1./TFMR(I)
      DUMMY=SB(I)
      SB(I)=EB(I)
      EB(I)=DUMMY
30    CONTINUE
C
C        RECORD THE NUMBER OF LINES CONNECTED TO EACH SENDING BUS
C
      RECORD(SB(I))=RECORD(SB(I))+1
      IF(EB(I).EQ.NB)KNECT=KNECT+1
      LINE(I)=I
C
  220 CONTINUE
      KNECT=KNECT+RECORD(NB)
C
C        REARRANGE THE LINES IN AN ORDER ACCEPTABLE FOR THE FORMATION
C        OF THE SPARSE BUS ADMITTANCE VECTOR
C
      CALL VSRTP(SB,NL,LINE)
      DO 31 I=1,NL
31    EB1(I)=EB(LINE(I))
      DUM1=0
      DUM2=0
      DO 35 I=1,NB
      IF(RECORD(I).EQ.0)GO TO 35
      DUMMY=RECORD(I)
      DO 32 J=1,DUMMY
      DUM1=DUM1+1
      EB1(J)=EB1(DUM1)
32    LINE1(J)=LINE(DUM1)
      CALL VSRTP(EB1,DUMMY,LINE1)
      DO 34 K=1,DUMMY
      DUM2=DUM2+1
      EB(DUM2)=EB1(K)
34    LINE(DUM2)=LINE1(K)
35    CONTINUE
C
C        REARRANGE THE LINE DATA TO BE COMPATIBLE WITH THE
C        NEW LINE NUMBERS
C
      DO 36 I=1,NL
      DTFMR(I)=TFMR(I)
      DSHTY(I)=SHTY(I)
```

```
      DSHTYB(I)=SHTYB(I)
36    DSERY(I)=SERY(I)
      DO 37 I=1,NL
      TFMR(I)=DTFMR(LINE(I))
      SHTY(I)=DSHTY(LINE(I))
      SHTYP(I)=DSHTYB(LINE(I))
37    SERY(I)=DSERY(LINE(I))
C
      WRITE(6,38)
38    FORMAT(///,40X,10(1H*),' R E O R D E R E D   L I N E   D A T A ',1
     10(1H*),//,18X,'NEW',5X,'OLD',/,18X,'LINE',4X,'LINE',5X,'SB',3X,'EB
     2',2X,'TFMR RATIO',2X,'SHUNT ADMITTANCE',6X,'SERIES IMPEDENCE',6X,'
     3SB STATIC CAP.',/)
      DO 40 I=1,NL
      SERZ=1./SERY(I)
      WRITE(6,39)I,LINE(I),SB(I),EB(I),TFMR(I),SHTY(I),SERZ,SBCAP(LINE(I
     1))
39    FORMAT(' ',12X,3I8,2X,I3,F9.3,3X,2F9.4,4X,2F9.4,7X,F3.1,F8.4)
40    CONTINUE
C
      J1=1
      LL=0
      MM=0
      A2(1)=1
      DO 128 I=1,NL
      L=SB(I)
      M=EB(I)
      IF(LL.EQ.0) GO TO 122
      IF(L.EQ.LL)GO TO 124
      IF(L.EQ.LL+1)GO TO 122
      DUM1=L-2
      DO 120 II=LL,DUM1
120   A2(II+1)=J1
122   A2(L)=J1
      GO TO 126
124   IF(M.EQ.MM) J1=J1-1
126   A1(J1)=M
      Y(J1)=Y(J1)-SERY(I)
      J1=J1+1
      LL=L
      MM=M
128   CONTINUE
      MAXY=J1-1
      IF(L.EQ.NB-1) GO TO 134
      DUM1=NB-1
      DO 132 I=L,DUM1
```

```
132   A2(I+1)=J1
134   CONTINUE
      A2(NB)=J1
      NRL=MAXY
      WRITE(6,1665)MAXY,A2(NB)
1665  FORMAT(' ',///,5X,'MAXY=',I4,5X,'A2(NB)=',I4,///)
      WRITE(6,1331)(I,D(I),I=1,NB)
1331  FORMAT(' ',40(3(4X,'* D(',I3,')=',2F12.4),//))
      KB=0
      K2=NB-1
      DO 1334 I=1,K2
      KA=A2(I+1)-A2(I)
      IF(KA.EQ.0)GO TO 1334
      DO 1333 J=1,KA
      KB=KB+1
      IF(A2(I).NE.KB)GO TO 1332
      WRITE(6,1335)I,A2(I)
1335  FORMAT(' ',/,65X,'A2(',I3,')=',I4)
1332  WRITE(6,1330)KB,Y(KB),KB,A1(KB)
1330  FORMAT(' ',5X,'Y(',I3,')=',2F12.4,4X,'* A1(',I3,')=',I4,//)
1333  CONTINUE
1334  CONTINUE
C
C   RESTORAGF OF SPARSE ACMITTANCE ELEMENTS IN A MANNER MORE EASILY
C        USED WHEN THE JACOBIAN IS FORMED.
C
      NEWY(1)=D(1)
      NEWA1(1)=1
      NEWA2(1)=1
      IA=A2(2)-A2(1)
      DO 136 I1=1,IA
      KA=I1+1
      NEWY(KA)=Y(I1)
136   NEWA1(KA)=A1(I1)
      NEWA2(2)=KA+1
      DO 152 I=2,NB
      DUM1=I-1
      DO 144 I3=1,DUM1
      IF(A2(I3).EQ.A2(I3+1)) GO TO 144
      KB=A2(I3+1)-1
      DUM2=A2(I3)
      DO 142 I2=DUM2,KB
      IF(A1(I2).NE.I) GO TO 138
      KA=KA+1
      NEWY(KA)=Y(I2)
      NEWA1(KA)=I3
```

```
      GO TO 144
138   IF(A1(I2).GT.I) GO TO 144
142   CONTINUE
144   CONTINUE
      KA=KA+1
      NEWY(KA)=D(I)
      NEWA1(KA)=I
      IF(I.EQ.NB)GO TO 148
      IF(A2(I).EQ.A2(I+1)) GO TO 148
      KC=A2(I+1)-1
      DUM2=A2(I)
      DO 146 I4=DUM2,KC
      KA=KA+1
      NEWY(KA)=Y(I4)
146   NEWA1(KA)=A1(I4)
148   NEWA2(I+1)=KA+1
152   CONTINUE
      MDUM=2*NRL+NB
      WRITE(6,1442)(I,NEWY(I),NEWA1(I),I=1,MDUM)
1442  FORMAT(' ',3X,'I=',I4,3X,'NEWY=',2F14.4,8X,'NEWA1=',I3,/)
      NNB=NB+1
      WRITE(6,1443)(I,NEWA2(I),I=1,NNB)
1443  FORMAT(' ',40(6(//,2X,'* NEWA2(',I3,')=',I4)))
C
C     ------------------------------------------------------------------
C
C     READ IN SPECIFIED BUS DATA: REAL POWER, REACTIVE POWER, REFERENCE
C     VOLTAGE V(SB), VOLTAGE CONTROL BUS MAGNITUDES AND REACTIVE
C     POWER LIMITS
C
44    K0=NB-1
      K1=NB-MB
      K2=NB-1
      READ (5,150) (PSCHED(I),I=1,K2)
      READ (5,150) (QSCHED(I),I=1,K1)
      READ (5,150) E(NB), F(NB), (VSCHED(I),I=1,K0)
  150 FORMAT(8F10.3)
      WRITE(6,160)(PSCHED(I),I=1,K2)
160   FORMAT(1H1,//,' P(SCHEDULED)(I) =',11F10.3,//,50(18X,11F10.3,//))
      WRITE(6,161)(QSCHED(I),I=1,K1)
161   FORMAT(1H0,//,' Q(SCHEDULED)(I) =',11F10.3,//,50(18X,11F10.3,//))
      WRITE(6,162)E(NB),F(NB),(VSCHED(I),I=1,K0)
162   FORMAT(//,10X,' V(SB) =',2F10.3,///6X,' VSCHED(I) =',11F10.3,//,50
     1(18X,11F10.3,//))
C-----------------------------------------------------------------------
C     READ   D C   I N P U T    I N F O R A T I O N
```

```
C-----------------------------------------------------------------------
C       READ SPECIFIED PARAMETERS FOR RECTIFIER
        READ(5,22)NRECT,XCR,ALPHAR,ALPMIN,ALPMAX,TMINR,TMAXR
C       READ SPECIFIED PARAMETERS FOR INVERTER
        READ(5,22)NINV,XCI,GAMMAI,GAMMIN,GAMMAX,TMINI,TMAXI
C       READ DC LINE RESISTANCE,DC CURRENT SETTING,CONSTANT POWER TO BE
C       MAINTAINED ON THE INVERTER END
        READ(5,173)RDC,ID,PDI
 173    FORMAT(3F15.5)
        PI=3.1415927
        KAY1=3.*SQRT(2.0)/PI
        KAY2=3.0/PI
        ROOT2=SQRT(2.0)
        WRITE(6,28)
        WRITE(6,51)NRECT,NINV
  51    FORMAT(////,31X,'S P E C I F I E D   P A R A M E T E R S   F O R
       1    D C   L I N K',/,31X,67(1H-),/,31X,'RECTIFIER BUS NUMBER
       2    =',I4,/,31X,'INVERTER BUS NUMBER                =',I4)
        WRITE(6,41)XCR,ALPHAR,ALPMIN,ALPMAX,TMINR,TMAXR
  41    FORMAT(///,31X,'FOR RECTIFIER',/,31X,13(1H-),/,31X,'COMM. REACTANC
       1E  (P.U.)          =',F10.5,/,31X,'DELAY ANGLE-INI. GUESS (DEG.)=',F10
       2.5,/,31X,'DELAY ANGLE-MIN. (DEG.)          =',F10.5,/,31X,'DELAY ANGLE
       3-MAX. (DEG.)          =',F10.5,/,31X,'TRANSFORMER TAP RATIO-MIN.   =',
       4F10.5,/,31X,'TRANSFORMER TAP RATIO-MAX.   =',F10.5)
        WRITE(6,52)XCI,GAMMAI,GAMMIN,GAMMAX,TMINI,TMAXI,PDI
  52    FORMAT(///,31X,'FOR INVERTER',/,31X,12(1H-),/,31X,'CCMM. REACTANCE
       1  (P.U.)          =',F10.5,/,31X,'EXT. ANGLE-INI. GUESS (DEG.) =',F10.
       25,/,31X,'EXTINCTION ANGLE-MIN. (DEG.) =',F10.5,/,31X,'EXTINCTION A
       3NGLE-MAX. (DEG.) =',F10.5,/,31X,'TRANSFORMER TAP RATIO-MIN.   =',F
       410.5,/,31X,'TRANSFCRMFR TAP RATIO-MAX.   =',F10.5,///,31X,'POWER T
       5O BE MAINTAINED OVER',/,31X,'DC LINK @ INVERTER END(P.U.)=',F10.5)
        WRITE(6,53)RDC,ID
  53    FORMAT(///,31X,'RESIST. OF DC LINE  (P.U.)   =',F10.5,///,31X,'DC
       1CURRENT   (P.U.)              =',F10.5)
  22    FORMAT(I5,6F10.5)
        ALPHAR=(PI/180.0)*ALPHAR
        GAMMAI=(PI/180.0)*GAMMAI
        ALPMIN=(PI/180.0)*ALPMIN
        ALPMAX=(PI/180.0)*ALPMAX
        GAMMIN=(PI/180.0)*GAMMIN
        GAMMAX=(PI/180.0)*GAMMAX
        READ(5,105)KMAX,DELMAX
 105    FORMAT(I5,F10.7)
        ALPINI=ALPHAR
        GAMINI=GAMMAI
        WRITE(6,28)
```

```
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C       INITIALIZE UNKNOWN VOLTAGES AND REACTIVE POWERS
C
C       CALCULATE DC PARAMETERS AND INITIALIZE OTHERS RELATED TO DC LINK
C-----------------------------------------------------------------------
        VDI=PDI/ID
        VDR=VDI+RDC*ID
        PDISP=PDI
        PDRSF=VDR*ID
        IJ=0
        IJK=0
        DO 1019 I=1,25
        KSAVE(I)=0
1019    CONTINUE
101     CONTINUE
        ALPHAR=ALPINI
        GAMMAI=GAMINI
        DO 250 I=1,K2
        E(I)=1.0
        F(I)=0.0
250     CONTINUE
        K5=K1+1
        DO 251 I=K5,K2
251     Q(I)=0.0
C-----------------------------------------------------------------------
C
        R=0.0
        T=0.0
        L=0
        K=0
        KKK=0
        KDEI=0
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
900     CONTINUE
        DO 255 I=1,K2
        CR1(I)=0.0
        CR2(I)=0.0
255     CONTINUE
901     CONTINUE
C
C       CALCULATE THE REAL AND REACTIVE POWERS
C
        DELMXP=0.0
        DELMXQ=0.0
        DELMXV=0.0
```

```
      IB=0
      DELPDR=0.0
      DELPDI=0.0
C-----------------------------------------------------------------------
C     CALCULATE   R E A C T I V E   P O W E R S   AT CONVERTERS
C-----------------------------------------------------------------------
      COSALP=COS(ALPHAR)
      SINALP=SIN(ALPHAR)
      COSGAM=COS(GAMMAI)
      SINGAM=SIN(GAMMAI)
      VRECT=SQRT((E(NRECT)**2)+(F(NRECT)**2))
      TAPR=(VDR+KAY2*ID*XCR)/(KAY1*VRECT*COSALP)
      IF(TAPR.GE.TMINR.AND.TAPR.LE.TMAXR)GO TO 42
      IF(TAPR.GT.TMAXR)TAPR=TMAXR
      IF(TAPR.LT.TMINR)TAPR=TMINR
   42 VTRECT=TAPR*VRECT
      MURECT=ARCOS(COSALP-(ROOT2*XCR*ID)/VTRECT)-ALPHAR
      WRITE(6,9001)MURECT,VTRECT,VRECT,TAPR
      ANGR=2.*ALPHAR+MURECT
      FACTRE=(SIN(MURECT))*(1.0-COS(ANGR))
      VINV=SQRT((E(NINV)**2)+(F(NINV)**2))
      QDR=(3.*(VTRECT**2)*FACTRE)/(2.*PI*XCR)
      TAPI=(VDI+KAY2*ID*XCI)/(KAY1*COSGAM*VINV)
      IF(TAPI.GE.TMINI.AND.TAPI.LE.TMAXI)GO TO 43
      IF(TAPI.GT.TMAXI)TAPI=TMAXI
      IF(TAPI.LT.TMINI)TAPI=TMINI
   43 VTINV=TAPI*VINV
      MUINV=ARCOS(COSGAM-(ROOT2*XCI*ID)/VTINV)-GAMMAI
      BETAI=GAMMAI+MUINV
      ANGI=2.*GAMMAI+MUINV
      WRITE(6,9001)MUINV,VTINV,VINV,TAPI
      WRITE(6,9001)GAMMAI,BETAI
      FACTIN=(SIN(MUINV))*(1.0-COS(ANGI))
      QDI=(3.*(VTINV**2)*FACTIN)/(2.*PI*XCI)
      PDR=KAY1*VTRECT*ID*COSALP-KAY2*(ID**2)*XCR
      PDI=KAY1*VTINV*ID*COSGAM-KAY2*(ID**2)*XCI
      WRITE(6,9001)PDR,KAY1,ID,VTRECT,COSALP,KAY2,XCR
      WRITE(6,9001)PDI,KAY1,ID,VTINV,COSGAM,KAY2,XCI
 9001 FORMAT(7(F9.5,5X))
      FACTR1=(COS(ALPHAR+MURECT)-COSALP)/SIN(ALPHAR+MURECT)
      DMUDER=FACTR1*E(NRECT)/(VRECT**2)
      DMUDFR=FACTR1*F(NRECT)/(VRECT**2)
      DMUDAL=(SIN(ALPHAR)/SIN(ALPHAR+MURECT))-1.0
      FACTI1=(COS(GAMMAI+MUINV)-COSGAM)/SIN(GAMMAI+MUINV)
      DMUDEI=FACTI1*E(NINV)/(VINV**2)
      DMUDFI=FACTI1*F(NINV)/(VINV**2)
```

```
      DMUDGA=(SIN(GAMMAI)/SIN(GAMMAI+MUINV))-1.0
      FACTR2=(1.0/TAN(MURECT))+(SIN(ANGR)/(1.0-COS(ANGR)))
      DQRDER=QDR*((2.*E(NRECT)/(VRECT**2))+DMUDER*FACTR2)
      DQRDFR=QDR*((2.*F(NRECT)/(VRECT**2))+DMUDFR*FACTR2)
      DQRDAL=QDR*((2.*SIN(ANGR)/(1.-COS(ANGR)))+DMUDAL*FACTR2)
      FACTI2=(1.0/TAN(MUINV))+(SIN(ANGI)/(1.0-COS(ANGI)))
      DQIDEI=QDI*((2.*E(NINV)/(VINV**2))+DMUDEI*FACTI2)
      DQIDFI=QDI*((2.*F(NINV)/(VINV**2))+DMUDFI*FACTI2)
      DQIDGA=QDI*((2.*SIN(ANGI)/(1.-COS(ANGI)))+DMUDGA*FACTI2)
      DO 261 I=1,K2
      IA=NEWA2(I+1)-NEWA2(I)
      IF(IA.EQ.0) GO TO 261
      SUM1=0.0
      SUM2=0.0
      DO 262 J=1,IA
      IB=IB+1
      IC=NEWA1(IB)
      SUM1=SUM1+E(IC)*REAL(NEWY(IB))-F(IC)*AIMAG(NEWY(IB))
      SUM2=SUM2+F(IC)*REAL(NEWY(IB))+E(IC)*AIMAG(NEWY(IB))
  262 CONTINUE
      P(I)=E(I)*SUM1+F(I)*SUM2
      Q(I)=F(I)*SUM1-E(I)*SUM2
C
C     CALCULATE MAXIMUM REAL AND REACTIVE POWER ERRORS
C
      DELTAP=PSCHED(I)-P(I)
      IF(I.EQ.NINV)DELTAP=DELTAP+PDISP
      IF(I.EQ.NRECT)DELTAP=DELTAP-PDRSP
      DELTAS(I)=DELTAP
      IF(ABS(DELTAP).GT.DELMXP)DELMXP=ABS(DELTAP)
      IF(I.GT.K1)GO TO 261
      DELTAQ=QSCHED(I)-Q(I)
      IF(I.EQ.NINV)DELTAQ=DELTAQ-QDI
      IF(I.EQ.NRECT)DELTAQ=DELTAQ-QDR
      DELTAS(I+K2)=DELTAQ
      IF(ABS(DELTAQ).GT.DELMXQ)DELMXQ=ABS(DELTAQ)
  261 CONTINUE
C
      WRITE(6,351)
  351 FORMAT(1HO,15X,5(1H*),' POWER ERROR1 MATRIX ',5(1H*),//,15X,'DELTA
     1 P',10X,'DELTA Q',5X,'DELTA V-SQUARED',/)
      DO 353 I=1,K1
      WRITE(6,352)I,DELTAS(I),DELTAS(I+K2)
  352 FORMAT(' ',I5,2F17.7,/)
  353 CONTINUE
C
```

```
C         IF THERE ARE VOLTAGE CONTROLLED BUSSES
C
      IF(KQ.EQ.0)GO TO 273
      DO 272 I=K5,K2
      V2=E(I)**2+F(I)**2
      DELTAV=VSCHED(I-K1)**2-V2
      DELTAS(I+K2)=DELTAV
      IF(ABS(DELTAV).GT.DELMXV)DELMXV=ABS(DELTAV)
      WRITE(6,271)I,DELTAS(I),DELTAS(I+K2)
271   FORMAT(' ',I5,F17.7,17X,F17.7)
272   CONTINUE
      IDC1=2*NB-1
      IDC2=2*NB
      DELPDR=PDRSP-PDR
      DELPDI=PDISP-PDI
      DELTAS(IDC1)=DELPDR
      DELTAS(IDC2)=DELPDI
      WRITE(6,23)DELPDR,DELPDI
   23 FORMAT('0',10X,'DELPDR=',F15.8,20X,'DELPDI=',F15.8)
273   CONTINUE
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C     TEST FOR CONVERGENCE OF SYSTEM
C
      KDEL=KDEL+1
      DELK(KDEL)=FLOAT(K)+FLOAT(KKK)/10.
      DELP(KDEL)=DELMXP
      DELQ(KDEL)=DELMXQ
      DELV(KDEL)=DELMXV
      DELPR(KDEL)=DELPDR
      DELPI(KDEL)=DELPDI
      WRITE(6,303) DELMXP,DELMXQ,DELMXV
303   FORMAT(' ',///,5X,'DELTAP MAX. =',F15.8,/,5X,'DELTAQ MAX. =',F15.8
     1,/,5X,'DELTAV MAX. =',F15.8,/)
      IF(DELMXP.LE.DELMAX.AND.DELMXQ.LE.DELMAX.AND.DELMXV.LE.DELMAX.AND.
     1DELPDR.LE.DELMAX.AND.DELPDI.LE.DELMAX)GO TO 950
      IF(KKK.NE.5) GO TO 222
      KDEL=KDEL-1
      KKK=0
      K=K+1
      GO TO 1066
222   CONTINUE
      IF(K.GE.KMAX)GO TO 1000
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C     CALCULATE BUS CURRENTS
```

```
C      CALCULATE ELEMENTS OF THE JACOBIAN MATRIX
C
       K3=K2*2
       K=K+1
       WRITE(6,279)K,KKK
279    FORMAT(1H1,/,4X,'ITERATION NO. ',I2,'.',I1,//,9X,'CONJUGATE',/,9X,
      1'OF  POWER',24X,'CURRENT')
       DO 320 I=1,K2
       S=CMPLX(P(I),-Q(I))
       VV=CMPLX(E(I),-F(I))
       IBUS(I)=S/VV
       WRITE(6,281)S,IBUS(I)
 320   CONTINUE
281    FORMAT(' ',/,2F12.5,7X,2F12.5)
C
       KB=0
       KTOT=NEWA2(NB+1)-(2*(NEWA2(NB+1)-NEWA2(NB))-1)-1
       WRITE(6,5001)KTOT
5001   FORMAT('0',/////,10X,'KTOT=',I5,//)
       DO 186 I=1,K2
       JACA2(I)=KB+1
       KA=NEWA2(I+1)-NEWA2(I)
       KD=0
       IF(KA.EQ.NB) KA=KA-1
180    CONTINUE
C
C         FORMULATE J1 AND J4
C
       DUMMY=NEWA2(I)
       IF(NEWA1(DUMMY+KA-1).EQ.NB) KA=KA-1
       DUM1=DUMMY+KA-1
       DO 182 I1=DUMMY,DUM1
       KB=KB+1
       KG=2*KTOT+KB+KA+2
       JACOB(KB)=E(NEWA1(I1))*REAL(NEWY(I1))+F(NEWA1(I1))*AIMAG(NEWY(I1))
       JACOB(KG)=-JACOB(KB)
       IF(NEWA1(I1).GT.M1) JACOB(KG)=0.0
       JACA1(KB)=NEWA1(I1)
       JACA1(KG)=NEWA1(I1)+K2
       IF(NEWA1(I1).NE.I) GO TO 182
       JACOB(KB)=JACOB(KB)+REAL(IBUS(NEWA1(I1)))
       JACOB(KG)=JACOB(KG)+REAL(IBUS(NEWA1(I1)))
       IF(NEWA1(I1).GT.K1) JACOB(KG)=2.*F(NEWA1(I1))
       D1(I)=KB
       D4(I)=KG
       IF(I.EQ.NRECT.AND.JACA1(KG).EQ.(K2+NRECT))GO TO 65
```

```
      IF(I.EQ.NINV.AND.JACA1(KG).EQ.(K2+NINV))GO TO 66
      GO TO 182
   65 JACOB(KG)=JACOB(KG)+DQRDFR
      GO TC 182
   66 JACOB(KG)=JACOB(KG)+DQIDFI
  182 CONTINUE
C
C        FORMULATE J2 AND J3
C
      DUMMY=NEWA2(I)
      DO 184 I1=DUMMY,DUM1
      KB=KB+1
      KH=2*KTOT+KB-KA+2
      JACOB(KH)=-E(NEWA1(I1))*AIMAG(NEWY(I1))+F(NEWA1(I1))*REAL(NEWY(I1)
     1)
      JACOB(KB)=JACOB(KH)
      IF(NEWA1(I1).GT.K1) JACOB(KB)=0.0
      JACA1(KH)=NEWA1(I1)
      JACA1(KB)=NEWA1(I1)+K2
      IF(NEWA1(I1).NE.I) GO TO 184
      JACOB(KB)=JACOB(KB)-AIMAG(IBUS(NEWA1(I1)))
      IF(NEWA1(I1).GT.K1) JACOB(KB)=2.*E(NEWA1(I1))
      JACOB(KH)=JACOB(KH)+AIMAG(IBUS(NEWA1(I1)))
      D2(I)=KH
      D3(I)=KB
      IF(I.EQ.NRECT.AND.JACA1(KB).EQ.(K2+NRECT))GO TO 67
      IF(I.EQ.NINV.AND.JACA1(KB).EQ.(K2+NINV))GO TO 68
      GO TC 184
   67 JACOB(KB)=JACOB(KB)+DQRDER
      GO TO 184
   68 JACOB(KB)=JACOB(KB)+DQIDEI
  184 CONTINUE
      IF(I.EQ.NRECT)GO TC 170
      IF(I.EQ.NINV)GO TO 171
      GO TC 172
  170 KDUM1=KB+1
      KB=KB+1
      KDUM3=KG+1
      KG=KG+1
      GO TC 172
  171 KDUM2=KB+1
      KB=KB+1
      KDUM4=KG+1
      KG=KG+1
  172 JACA2(I+K2+1)=KG+1
  186 CONTINUE
```

```
      JACA2(NB)=2*KTOT+3
C-------------------------------------------------------------------------
C     CALCULATION OF JACCBIAN ELEMENTS DUE TO DC LINK START FROM HERE
C-------------------------------------------------------------------
      VMAGR=SQRT(E(NRECT)**2+F(NRECT)**2)
      VMAGI=SQRT(E(NINV)**2+F(NINV)**2)
      FACTMO=KAY1*TAPR*ID*COSALP/VMAGR
      DPDDER=FACTMO*E(NRECT)
      JACOB(KDUM1)=DPDDER
      JACA1(KDUM1)=IDC1
      FACTNO=KAY1*TAPI*ID*COSGAM/VMAGI
      DPDDEI=FACTNO*E(NINV)
      JACOB(KDUM2)=DPDDEI
      JACA1(KDUM2)=IDC2
      DPDDFR=FACTMO*F(NRECT)
      JACOB(KDUM3)=DPDDFR
      JACA1(KDUM3)=IDC1
      DPDDFI=FACTNO*F(NINV)
      JACOB(KDUM4)=DPDDFI
      JACA1(KDUM4)=IDC2
      KDUM5=4*KTOT+5
      JACOB(KDUM5)=DQRDAL
      JACA1(KDUM5)=K2+NRECT
      JACA2(IDC1)=KDUM5
      KDUM6=KDUM5+1
      DPDDAL=(-1.0)*KAY1*TAPR*VMAGR*ID*SIN(ALPHAR)
      JACOB(KDUM6)=DPDDAL
      JACA1(KDUM6)=IDC1
      KDUM7=KDUM6+1
      JACOB(KDUM7)=DQIDGA
      JACA1(KDUM7)=K2+NINV
      JACA2(IDC2)=KDUM7
      KDUM8=KDUM7+1
      DPDDGA=(-1.0)*KAY1*TAPI*VMAGI*ID*SIN(GAMMAI)
      JACOB(KDUM8)=DPDDGA
      JACA1(KDUM8)=IDC2
      JACA2(2*K2+3)=4*KTOT+9
C      PRINT  JACOBIAN  VECTOR
C
      WRITE(6,403)
403   FORMAT(1H1,//,5X,' THE JACOBIAN SPARSITY VECTOR AND ROW LOCATION V
     1ECTOR, JACA1',//,2(4X,'I',5X,'JACOB(I)',5X,'JACA1(I)'),/)
      NJACOB=8*(NRL-KNECT)+(NB-1)*4+8
      N2=NJACOB/2
      DO 404 I=1,N2
      N3=I+N2
```

```
      WRITE(6,405)I,JACOB(I),JACA1(I),N3,JACOB(N3),JACA1(N3)
404   CONTINUE
405   FORMAT(' ',/,1X,I4,F14.6,I10,'*',I6,F14.6,I10)
      MNB=2*NB-1+2
      WRITE(6,1444)(I,JACA2(I),I=1,MNB)
1444  FORMAT(' ',//,6(3X,'  JACA2(',I3,')=',I4))
      WRITE(6,1448)
1448  FORMAT(' ',////,4X,'I',4X,'D1(I)',3X,'D2(I)',3X,'D3(I)',3X,'D4(I)'
     1,/)
      DO 1447 I=1,K2
      WRITE(6,1446)I,D1(I),D2(I),D3(I),D4(I)
1447  CONTINUE
1446  FORMAT(' ',/,2X,I3,4I8)
C- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      IA=JACA2(2*K2+3)-1
      WRITE(6,1664)NJACOB,IA
1664  FORMAT(' ',///,5X,'NJACOB=',I5,5X,'IA=',I5,///)
      IF((K+KKK/5).LE.2) GO TO 1770
C
C        USE MA18D TO LOAD THE JACOBIAN NONZERO ELEMENTS INTO THEIR
C        PROPER PLACES IN THE ORIGINAL 'A' MATRIX. THEN MA18C FACTORS
C        THIS MATRIX.
C
      DO 1549 J=1,IA
1549  A(J)=JACOB(J)
      CALL MA18D(A,IRN,IP,N,NP,W,RELOAD)
      CALL MA18C(A,IRN,IP,N,NP,G)
      WRITE(6,1554)G
      GO TO 1559
1770  CONTINUE
C
      N=2*(NB-1)+2
      NP=N+1
      DO 1551 I=1,IA
      A(I)=JACOB(I)
      IRN(I)=JACA1(I)
1551  CONTINUE
      DO 1552 I=1,NP
      IP(I,1)=JACA2(I)
1552  CONTINUE
C
C        SET PARAMETER, U, CALL MA18A TO FACTOR JACOB AND CHECK FOR ERROR.
C
      U=0.25
      CALL MA18A(A,IRN,IP,N,NP,G,U,IA+IEXTRA)
      WRITE(6,1554)G
```

```
1554  FORMAT('OVALUE OF G( RELATIVE PERTURBATION OF A ):',/,41('*'),//,E
     115.7,///)
      IF((G.EQ.-1.0).OR.(IP(N+1,2).EQ.0)) GO TO 1553
C
C        SOLVE FOR THE VOLTAGE CORRECTIONS (FIRST ORDER)
C
1559  MTYPE=1
      CALL MA18B(A,IRN,IP,N,NP,W,DELTAS,MTYPE)
      IF(MTYPE.EQ.0) GO TO 1556
      GO TO 1558
C
1553  WRITE(6,1555)
1555  FORMAT('-INVALID RETURN FROM MA18A')
 1556  WRITE(6,1557)
1557  FORMAT('-INVALID RETURN FROM MA18B')
C
1558  CONTINUE
C
      WRITE(6,366)
366   FORMAT(1H0,///,5(1H*),' VOLTAGE ERROR1 MATRIX ',5(1H*),//,15X,'DEL
     1TA E',10X,'DELTA F',/)
      DO 367 I=1,K2
      ERROR1(I)=DELTAS(I)
      ERROR1(I+K2)=DELTAS(I+K2)
367   WRITE(6,352) I,ERROR1(I),ERROR1(I+K2)
      ERROR1(IDC1)=DELTAS(IDC1)
      ERROR1(IDC2)=DELTAS(IDC2)
      WRITE(6,47)ERROR1(IDC1)
      WRITE(6,48)ERROR1(IDC2)
  47  FORMAT('0',10X,'DELTA ALPHAR=',F17.7)
  48  FORMAT('0',10X,'DELTA GAMMAI=',F17.7)
      IF(CC.EQ.0.)GO TO 411
C
C------------------------------------------------------------------------
C     SECOND ORDER CORRECTION FOR DC RELATED ELEMENTS STARTS FROM HERE
C------------------------------------------------------------------------
      XDC1=(-1.0)*KAY1*TAPR*E(NRECT)*F(NRECT)*ID*COSALP/(VMAGR**3)
      XDC2=(-1.0)*KAY1*TAPR*E(NRECT)*ID*SINALP/VMAGR
      XDC3=(-1.0)*KAY1*TAPR*F(NRECT)*ID*SINALP/VMAGR
      FACT4=((-1.0)*(E(NRECT)**2)/(VMAGR**3))+(1.0/VMAGR)
      XDC4=KAY1*TAPR*ID*COSALP*FACT4
      FACT5=((-1.0)*(F(NRECT)**2)/(VMAGR**3))+(1.0/VMAGR)
      XDC5=KAY1*TAPR*ID*COSALP*FACT5
      XDC6=(-1.0)*KAY1*TAPR*VMAGR*ID*COSALP
      XDUM1=ERROR1(K2+NRECT)*XDC1+0.5*ERROR1(NRECT)*XDC4
      XDUM3=ERROR1(IDC1)*XDC3+0.5*ERROR1(K2+NRECT)*XDC5
```

```
      XDUM6=ERROR1(NRECT)*XDC2+0.5*ERROR1(IDC1)*XDC6
      XDC7=(-1.0)*KAY1*TAPI*E(NINV)*F(NINV)*ID*COSGAM/(VMAGI**3)
      XDC8=(-1.0)*KAY1*TAPI*E(NINV)*ID*SINGAM/VMAGI
      XDC9=(-1.0)*KAY1*TAPI*F(NINV)*ID*SINGAM/VMAGI
      FACT10=((-1.0)*(E(NINV)**2)/(VMAGI**3))+(1.0/VMAGI)
      XDC10=KAY1*TAPI*ID*COSGAM*FACT10
      FACT11=((-1.0)*(F(NINV)**2)/(VMAGI**3))+(1.0/VMAGI)
      XDC11=KAY1*TAPI*ID*COSGAM*FACT11
      XDC12=(-1.0)*KAY1*TAPI*VMAGI*ID*COSGAM
      XDUM2=ERROR1(K2+NINV)*XDC7+0.5*ERROR1(NINV)*XDC10
      XDUM4=ERROR1(IDC2)*XDC9+0.5*ERROR1(K2+NINV)*XDC11
      XDUM8=ERROR1(NINV)*XDC8+0.5*ERROR1(IDC2)*XDC12
      JACOB(KDUM1)=JACOB(KDUM1)+(1.-ALPHA)*XDUM1
      JACOB(KDUM2)=JACOB(KDUM2)+(1.-ALPHA)*XDUM2
      JACOB(KDUM3)=JACOB(KDUM3)+(1.-ALPHA)*XDUM3
      JACOB(KDUM4)=JACOB(KDUM4)+(1.-ALPHA)*XDUM4
      JACOB(KDUM6)=JACOB(KDUM6)+(1.-ALPHA)*XDUM6
      JACOB(KDUM8)=JACOB(KDUM8)+(1.-ALPHA)*XDUM8
      SPDR=ERROR1(NRECT)*XDUM1+ERROR1(K2+NRECT)*XDUM3+ERROR1(IDC1)*XDUM6
      SPDI=ERROR1(NINV)*XDUM2+ERROR1(K2+NINV)*XDUM4+ERROR1(IDC2)*XDUM8
      DELPDR=DELPDR-SPDR*ALPHA
      DELTAS(IDC1)=DELPDR
      DELPDI=DELPDI-SPDI*ALPHA
      DELTAS(IDC2)=DELPDI
C     SECOND ORDER DERIVATIVE OF QDR W.R.T. ALPHAR
      XALR1=2.0*SIN(ANGR)/(1.0-COS(ANGR))
      XDC13=(-2.0)*(2.0+DMUDAL)/(1.0-COS(ANGR))
      XDC14=((1.0/SIN(MURECT))**2)*DMUDAL+((2.0+DMUDAL)/(1.0-COS(ANGR)))
      XDC14=(-1.0)*XDC14
      XDC15=(SIN(MURECT)-SINALP*COS(ALPHAR+MURECT)*DMUDAL)/((SIN(ALPHAR+
     1MURECT))**2)
      FACT21=XDC13+DMUDAL*XDC14+XDC15*FACTR2
      FACT22=XALR1+DMUDAL*FACTR2
      XDC33=QDR*FACT21+DGRDAL*FACT22
C     SECOND ORDER DERIVATIVE OF QDI W.R.T. GAMMAI
      XGAI1=2.0*SIN(ANGI)/(1.0-COS(ANGI))
      XDC16=(-2.0)*(2.0+DMUDGA)/(1.0-COS(ANGI))
      XDC17=((1.0/SIN(MUINV))**2)*DMUDGA+((2.0+DMUDGA)/(1.0-COS(ANGI)))
      XDC17=(-1.0)*XDC17
      XDC18=(SIN(MUINV)-SINGAM*COS(GAMMAI+MUINV)*DMUDGA)/((SIN(GAMMAI+MU
     1INV))**2)
      FACT23=XDC16+DMUDGA*XDC17+XDC18*FACTI2
      FACT24=XGAI1+DMUDGA*FACTI2
      XDC39=QDI*FACT23+DGIDGA*FACT24
C     SECOND ORDER DERIVATIVE OF QDR W.R.T. E(NRECT)
      XER1=2.0*E(NRECT)/(VMAGR**2)
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```
      FACT25=(-2.0*(E(NRECT)**2)/(VMAGR**4))+(1.0/(VMAGR**2))
      XDC19=2.0*FACT25
      FACT26=(1.0/((SIN(MURECT))**2))+(1.0/(1.0-COS(ANGR)))
      XDC20=(-1.0)*FACT26*DMUDER
      FACT61=(COS(ALPHAR+MURECT)*COSALP-1.0)/((SIN(ALPHAR+MURECT))**2)
      XDC43=FACT61*DMUDER
      XDC21=FACTR1*FACT25+(XDC43*E(NRECT)/(VMAGR**2))
      FACT27=XDC19+DMUDER*XDC20+XDC21*FACTR2
      FACT28=XER1+DMUDER*FACTR2
      XDC31=QDR*FACT27+DQRDER*FACT28
C     SECOND ORDER DERIVATIVE OF QDR W.R.T. F(NRECT)
      XFR1=2.0*F(NRECT)/(VMAGR**2)
      FACT29=(-2.0*(F(NRECT)**2)/(VMAGR**4))+(1.0/(VMAGR**2))
      XDC22=2.0*FACT29
      XDC23=(-1.0)*FACT26*DMUDFR
      XDC44=FACT61*DMUDFR
      XDC24=FACTR1*FACT29+(XDC44*F(NRECT)/(VMAGR**2))
      FACT30=XDC22+DMUDFR*XDC23+XDC24*FACTR2
      FACT31=XFR1+DMUDFR*FACTR2
      XDC32=QDR*FACT30+DQRDFR*FACT31
C     SECOND ORDER DERIVATIVE OF QDI W.R.T. E(NINV)
      XEI1=2.0*E(NINV)/(VMAGI**2)
      FACT32=(-2.0*(E(NINV)**2)/(VMAGI**4))+(1.0/(VMAGI**2))
      XDC25=2.0*FACT32
      FACT33=(1.0/((SIN(MUINV))**2))+(1.0/(1.0-COS(ANGI)))
      XDC26=(-1.0)*FACT33*DMUDEI
      FACT62=(COS(GAMMAI+MUINV)*COSGAM-1.0)/((SIN(GAMMAI+MUINV))**2)
      XDC45=FACT62*DMUDEI
      XDC27=FACTI1*FACT32+(XDC45*E(NINV)/(VMAGI**2))
      FACT34=XDC25+DMUDEI*XDC26+XDC27*FACTI2
      FACT35=XEI1+DMUDEI*FACTI2
      XDC37=QDI*FACT34+DQIDEI*FACT35
C     SECOND ORDER DERIVATIVE OF QDI W.R.T. F(NINV)
      XFI1=2.0*F(NINV)/(VMAGI**2)
      FACT36=(-2.0*(F(NINV)**2)/(VMAGI**4))+(1.0/(VMAGI**2))
      XDC28=2.0*FACT36
      XDC29=(-1.0)*FACT33*DMUDFI
      XDC46=FACT62*DMUDFI
      XDC30=FACTI1*FACT36+(XDC46*F(NINV)/(VMAGI**2))
      FACT37=XDC28+DMUDFI*XDC29+XDC30*FACTI2
      FACT38=XFI1+DMUDFI*FACTI2
      XDC38=QDI*FACT37+DQIDFI*FACT38
C     SECOND ORDER DERIVATIVE OF QDR W.R.T. E(NRECT),F(NRECT)
      FACT39=(-4.0)*E(NRECT)*F(NRECT)/(VMAGR**4)
      FACT40=DMUDFR*XDC20
      XSAM1=(-2.0)*FACTR1*E(NRECT)*F(NRECT)/(VMAGR**4)
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      XSAM2=XDC43*F(NRECT)/(VMAGR**2)
      XDC47=XSAM1+XSAM2
      FACT41=XDC47*FACTR2
      FACT42=XFR1+DMUDFR*FACTR2
      XDC34=QDR*(FACT39+FACT40+FACT41)+DQRDER*FACT42
C     SECOND ORDER DERIVATIVE OF QDR W.R.T. E(NRECT),ALPHAR
      FACT43=(-2.0)*DMUDER/(1.0-COS(ANGR))
      FACT44=DMUDAL*XDC20
      XDC48=(-1.0)*SINALF*COS(ALPHAR+MURECT)*DMUDER/((SIN(ALPHAR+MURECT)
     1)**2)
      FACT45=XDC48*FACTR2
      FACT46=XALR1+DMUDAL*FACTR2
      XDC35=QDR*(FACT43+FACT44+FACT45)+DQRDER*FACT46
C     SECOND ORDER DERIVATIVE OF QDR W.R.T. F(NRECT),ALPHAR
      FACT47=(-2.0)*DMUDFR/(1.0-COS(ANGR))
      FACT48=DMUDAL*XDC23
      XDC49=(-1.0)*SINALF*COS(ALPHAR+MURECT)*DMUDFR/((SIN(ALPHAR+MURECT)
     1)**2)
      FACT49=XDC49*FACTR2
      XDC36=QDR*(FACT47+FACT48+FACT49)+DQRDFR*FACT46
C
      XDUM9=ERROR1(K2+NRECT)*XDC34+0.5*ERROR1(NRECT)*XDC31
      XDUM11=ERROR1(IDC1)*XDC36+0.5*ERROR1(K2+NRECT)*XDC32
      XDUM5=ERROR1(NRECT)*XDC35+0.5*ERROR1(IDC1)*XDC33
C     SECOND ORDER DERIVATIVE OF QDI W.R.T. E(NINV),F(NINV)
      FACT50=(-4.0)*E(NINV)*F(NINV)/(VMAGI**4)
      FACT51=DMUDFI*XDC26
      XSAM3=(-2.0)*FACTI1*E(NINV)*F(NINV)/(VMAGI**4)
      XSAM4=XDC45*F(NINV)/(VMAGI**2)
      XDC50=XSAM3+XSAM4
      FACT52=XDC50*FACTI2
      FACT53=XFI1+DMUDFI*FACTI2
      XDC40=QDI*(FACT50+FACT51+FACT52)+DQIDEI*FACT53
C     SECOND ORDER DERIVATIVE OF QDI W.R.T. E(NINV),GAMMAI
      FACT54=(-2.0)*DMUDEI/(1.0-COS(ANGI))
      FACT55=DMUDGA*XDC26
      XDC51=(-1.0)*SINGAM*COS(GAMMAI+MUINV)*DMUDEI/((SIN(GAMMAI+MUINV))*
     1*2)
      FACT56=XDC51*FACTI2
      FACT57=XGAI1+DMUDGA*FACTI2
      XDC41=QDI*(FACT54+FACT55+FACT56)+DQIDEI*FACT57
C     SECOND ORDER DERIVATIVE OF QDI W.R.T. F(NINV),GAMMAI
      FACT58=(-2.0)*DMUDFI/(1.0-COS(ANGI))
      FACT59=DMUDGA*XDC29
      XDC52=(-1.0)*SINGAM*COS(GAMMAI+MUINV)*DMUDFI/((SIN(GAMMAI+MUINV))*
     1*2)
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      FACT60=XDC52*FACII2
      XDC42=QDI*(FACT58+FACT59+FACT60)+DQIDFI*FACT57
      XDUM10=ERROR1(K2+NINV)*XDC40+0.5*ERROR1(NINV)*XDC37
      XDUM12=ERROR1(IDC2)*XDC42+0.5*ERROR1(K2+NINV)*XDC38
      XDUM7=ERROR1(NINV)*XDC41+0.5*ERROR1(IDC2)*XDC39
C     ----------------------------------------------------------------
C     CALCULATE SECOND ORDER CORRECTION DUE TO RECTIFIER AND INVERTER
C
      XR=ERROR1(NRECT)*XDUM9+ERROR1(K2+NRECT)*XDUM11+ERROR1(IDC1)*XDUM5
      XI=ERROR1(NINV)*XDUM10+ERROR1(K2+NINV)*XDUM12+ERROR1(IDC2)*XDUM7
C     ----------------------------------------------------------------
C
C     CALCULATE SECOND ORDER CORRECTION FOR REAL AND REACTIVE POWERS
71    CONTINUE
      WRITE(6,407)
407   FORMAT(1H0,///,'  NO.',15X,'SUM3',17X,'SUM4',15X,'R',18X,'I',15X,'
     1R*ALPHA',12X,'T*ALPHA',7X,'L')
      L=0
      IB=0
      DO 1662 I=1,K2
      IA=NEWA2(I+1)-NEWA2(I)
      IF(IA.EQ.0)GO TO 1662
      SUM3=0.0
      SUM4=0.0
      DO 1661 J=1,IA
      IB=IB+1
      IC=NEWA1(IB)
      IF(IC.EQ.NB) GO TO 1661
      SUM3=SUM3-AIMAG(NEWY(IB))*ERROR1(IC+K2)+REAL(NEWY(IB))*ERROR1(IC)
      SUM4=SUM4+REAL(NEWY(IB))*ERROR1(IC+K2)+AIMAG(NEWY(IB))*ERROR1(IC)
1661  CONTINUE
      CR1(I)=SUM3
      CR2(I)=SUM4
      R=SUM3*ERROR1(I)+SUM4*ERROR1(I+K2)
      T=-SUM4*ERROR1(I)+SUM3*ERROR1(I+K2)
      IF(ABS(R).LT.DELMAX.AND.ABS(T).LT.DELMAX)L=L+1
      RO=R
      TO=T
      R=R*ALPHA
      T=T*ALPHA
      XR=XR*ALPHA
      XI=XI*ALPHA
      WRITE(6,408)I,SUM3,SUM4,RO,TO,R,T,L
408   FORMAT(1H0,/,3X,I3,F20.8,F21.8,2F19.8,2F19.8,I6)
  603 DELTAP=PSCHED(I)-P(I)-R
      IF(I.EQ.NINV)DELTAP=DELTAP+PDISP
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      IF(I.EQ.NRECT)DELTAP=DELTAP-PDRSP
  607 DELTAS(I)=DELTAP
      DELS(I)=DELTAP
      IF(I.GT.K1) GO TO 1662
  604 DELTAQ=QSCHED(I)-Q(I)-T
      IF(I.EQ.NINV)DELTAQ=DELTAQ-QDI-XI
      IF(I.EQ.NRECT)DELTAQ=DELTAQ-QDR-XR
      DELTAS(I+K2)=DELTAQ
      DELS(I+K2)=DELTAQ
 1662 CONTINUE
      DELS(IDC1)=DELPDR
      DELS(IDC2)=DELPDI
      IF(L.EQ.K2.AND.ABS(DELPDR).LT.DELMAX.AND.ABS(DELPDI).LT.DELMAX)GO
     1TO 412
 1066 CONTINUE
      IF(L.NE.K2) GO TO 1067
      DO 333 I=1,K2
      DELTAS(I)=DELS(I)
      DELTAS(I+K2)=DELS(I+K2)
  333 CONTINUE
      DELTAS(IDC1)=DELS(IDC1)
      DELTAS(IDC2)=DELS(IDC2)
 1067 CONTINUE
      IF(ALPHA.EQ.0.AND.K0.EQ.0)GO TO 433
C
      WRITE(6,368)
  368 FORMAT(1H1,15X,5(1H*),' POWER ERROR2 MATRIX ',5(1H*),//,15X,'DELTA
     1 P',10X,'DELTA Q',5X,'DELTA V-SQUARED',/)
      DO 369 I=1,K1
  369 WRITE(6,352) I,DELTAS(I),DELTAS(I+K2)
  441 IF(K0.EQ.0)GO TO 444
C
C      IF THERE ARE VOLTAGE CONTROLLED BUSSES
C
      DO 442 I=K5,K2
      U2=ERROR1(I)**2+ERROR1(I+K2)**2
      U2=U2*ALPHA
      DELTAV=VSCHED(I-K1)**2-E(I)**2-F(I)**2-U2
      DELTAS(I+K2)=DELTAV
      WRITE(6,271)I,DELTAS(I),DELTAS(I+K2)
  442 CONTINUE
  444 CONTINUE
      WRITE(6,23)DELDC1,DELDC2
C
      IF(ALPHA.EQ.1.)GO TO 414
  433 DO 33 I=1,K2
```

```
      JACOB(D1(I))=JACOB(D1(I))+(1.-ALPHA)*CR1(I)
      JACOB(D4(I))=JACOB(D4(I))+(1.-ALPHA)*CR1(I)
      JACOB(D2(I))=JACOB(D2(I))+(1.-ALPHA)*CR2(I)
      JACOB(D3(I))=JACOB(D3(I))-(1.-ALPHA)*CR2(I)
33    CONTINUE
C
C        IF THERE ARE VOLTAGE CONTROLLED BUSES
C
      IF(NB.EQ.1)GO TO 1706
      DO 1705 I=K5,K2
      JACOB(D3(I))=JACOB(D3(I))+(1.-ALPHA)*ERROR1(I)+(1.-ALPHA)*CR2(I)
      JACOB(D4(I))=JACOB(D4(I))+(1.-ALPHA)*ERROR1(I+K2)-(1.-ALPHA)*CR1(I
     1)
1705  CONTINUE
1706  CONTINUE
C     ----------------------------------------------------------------
C     SOME ADDITIONAL CORRECTIONS DUE TO DC ELEMENTS
C     ----------------------------------------------------------------
      JACOB(D3(NRECT))=JACOB(D3(NRECT))+(1.-ALPHA)*XDUM9
      JACOB(D3(NINV))=JACOB(D3(NINV))+(1.-ALPHA)*XDUM10
      JACOB(D4(NRECT))=JACOB(D4(NRECT))+(1.-ALPHA)*XDUM11
      JACOB(D4(NINV))=JACOB(D4(NINV))+(1.-ALPHA)*XDUM12
      JACOB(KDUM5)=JACOB(KDUM5)+(1.-ALPHA)*XDUM5
      JACOB(KDUM7)=JACOB(KDUM7)+(1.-ALPHA)*XDUM7
C
C
      IA=JACA2(2*K2+3)-1
      IF(K.EQ.1) GO TO 1703
      DO 1649 I=1,IA
1649  A(I)=JACOB(I)
      CALL MA18D(A,IRN,IP,N,NP,W,RELOAD)
      CALL MA18C(A,IRN,IP,N,NP,G)
      WRITE(6,1554)G
      GO TO 414
C
1703  CONTINUE
      N=2*(NB-1)+2
      NP=N+1
      DO 1700 I=1,IA
      A(I)=JACOB(I)
      IRN(I)=JACA1(I)
1700  CONTINUE
      DO 1701 I=1,NP
      IP(I,1)=JACA2(I)
1701  CONTINUE
C
```

```
C         SET PARAMETER, U, CALL MA18A TO FACTOR JACOB AND CHECK FOR ERROR
C
          U=0.25
          CALL MA18A(A,IRN,IP,N,NP,G,U,IA+IEXTRA)
          WRITE(6,1554)G
          IF((G.EQ.-1.0).OR.(IP(N+1,2).EQ.0)) GO TO 1553
C
C            SOLVE FOR THE SECOND ORDER VOLTAGE CORRECTIONS
C
C
414       MTYPE=1
          CALL MA18B(A,IRN,IP,N,NP,W,DELTAS,MTYPE)
          IF(MTYPE.EQ.0) GO TO 1556
C
          WRITE(6,371)
 371      FORMAT(1H0,///,5(1H*),' VOLTAGE ERROR2 MATRIX ',5(1H*),//,15X,'DEL
         1TA E',10X,'DELTA F',/)
          DO 372 I=1,K2
372       WRITE(6,352) I,DELTAS(I),DELTAS(I+K2)
          WRITE(6,47)DELTAS(IDC1)
          WRITE(6,48)DELTAS(IDC2)
C
C
C         DETERMINE THE NEW REAL AND IMAGINARY BUS VOLTAGES
C
411       DO 420 I=1,K2
          E(I)=E(I)+DELTAS(I)
420       F(I)=F(I)+DELTAS(I+K2)
          ALPHAR=ALPHAR+DELTAS(IDC1)
          IF(ALPHAR.LE.ALPMAX.AND.ALPHAR.GE.ALPMIN)GO TO 174
          IF(ALPHAR.LT.ALPMIN)ALPHAR=ALPMIN
 174      GAMMAI=GAMMAI+DELTAS(IDC2)
          IF(ALPHAR.GT.ALPMAX)ALPHAR=ALPMAX
          IF(GAMMAI.GT.GAMMAX)GAMMAI=GAMMAX
          IF(GAMMAI.LE.GAMMAX.AND.GAMMAI.GE.GAMMIN)GO TO 900
          IF(GAMMAI.LT.GAMMIN)GAMMAI=GAMMIN
          GO TO 900
C
C
C       IF DOING A HALF ITERATION
C
412       DO 413 I=1,K2
          E(I)=E(I)+ERROR1(I)
413       F(I)=F(I)+ERROR1(I+K2)
          ALPHAR=ALPHAR+DELTAS(IDC1)
          IF(ALPHAR.LE.ALPMAX.AND.ALPHAR.GE.ALPMIN)GO TO 175
          IF(ALPHAR.LT.ALPMIN)ALPHAR=ALPMIN
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      IF(ALPHAR.GT.ALPMAX)ALPHAR=ALPMAX
  175 GAMMAI=GAMMAI+DELTAS(IDC2)
      IF(GAMMAI.LE.GAMMAX.AND.GAMMAI.GE.GAMMIN)GO TO 176
      IF(GAMMAI.LT.GAMMIN)GAMMAI=GAMMIN
      IF(GAMMAI.GT.GAMMAX)GAMMAI=GAMMAX
  176 IF(KKK.EQ.0)K=K-1
      KKK=5-KKK
      GO TO 901
C
C     CONVERGENCE OBTAINED - CALCULATE SLACK BUS POWER
C
  950 SUM5=CMPLX(0.0,0.0)
      DUMMY=NEWA2(NB+1)-1
      DUM1=NEWA2(NB)
      DO 509 I=1,NB
  509 V(I)=CMPLX(E(I),F(I))
      DO 510 I=DUM1,DUMMY
      DUM2=NEWA1(I)
      SUM5=SUM5+NEWY(I)*V(DUM2)
  510 CONTINUE
      P(NB)=REAL(SUM5*CONJG(V(NB)))
      Q(NB)=-AIMAG(SUM5*CONJG(V(NB)))
C
C     WRITE OUT BUS DATA
C
      IF(CC.EQ.1.)GO TO 503
   62 FORMAT(//)
   84 FORMAT(//////////)
      WRITE(6,502)K
  502 FORMAT(////////,26X,5(1H*),'  FIRST ORDER NEWTON-RAPHSON ITERATIVE
     1 TECHNIQUE CONVERGED IN',I2,' ITERATIONS ',5(1H*),//////,31X,'BUS'
     2,7X,'VOLTAGE',8X,'MAGNITUDE',2X,'PHASE(DEGS)',2X,'REAL POWER',2X,'
     3REACTIVE POWER',/)
      GO TO 505
  503 WRITE(6,511)K,KKK
  511 FORMAT(////////,26X,5(1H*),' SECOND ORDER NEWTON-RAPHSON ITERATIVE
     1 TECHNIQUE CONVERGED IN',I2,'.',I1,' ITERATIONS ',//////,31X,'BUS'
     2,7X,'VOLTAGE',8X,'MAGNITUDE',2X,'PHASE(DEGS)',2X,'REAL POWER',2X,'
     3REACTIVE POWER',/)
  505 DO 512 I=1,NB
      PHASE=ATAN2(F(I),E(I))*57.29578
      MAGN=CABS(V(I))
   86 FORMAT(////,31X,'BUS',7X,'VOLTAGE',8X,'MAGNITUDE',2X,'PHASE(DEGS)'
     1,2X,'REAL POWER',2X,'REACTIVE POWER',/)
  512 WRITE(6,513)I,V(I),MAGN,PHASE,P(I),Q(I)
  513 FORMAT(' ',26X,I7,2X,2F8.4,4X,F7.4,4X,F9.5,6X,F8.4,4X,F8.4,/)
```

```
C
C     CALCULATE AND WRITE OUT LINE FLOWS
C
      WRITE(6,514)
  514 FORMAT(///////,45X,10(1H*),' LINE FLOWS ',10(1H*),//,38X,'LINE',4X
     1,'SB',3X,'EB',5X,'REAL POWER',3X,'REACTIVE POWER',/)
      DO 515 I=1,NL
      L=SB(I)
      M=EB(I)
      SS=CONJG(V(L))*(V(L)-V(M))*SERY(I)+CONJG(V(L))*V(L)*(SHTY(I)/2.0)
      SR=CONJG(V(M))*(V(M)-V(L))*SERY(I)+CONJG(V(M))*V(M)*(SHTY(I)/2.0)
      SS=CONJG(SS)
      SR=CONJG(SR)
   83 WRITE(6,516)I,L,M,SS
      WRITE(6,57)M,L,SR
      WRITE(6,58)
  515 CONTINUE
   57 FORMAT(' ',42X,2I5,2F13.4)
   58 FORMAT(/)
  516 FORMAT(' ',37X,3I5,2F13.4)
C----------------------------------------------------------------------
C     CALCULATE AND WRITE OUT DC PARAMETERS
C----------------------------------------------------------------------
      PFI=VDI/(KAY1*TAPI*CABS(V(NINV)))
      PFR=VDR/(KAY1*TAPR*CABS(V(NRECTI))
      DEGF=180.0/PI
      ALPHAR=DEGF*ALPHAR
      BETAI=DEGF*BETAI
      GAMMAI=DEGF*GAMMAI
      MURECT=DEGF*MURECT
      MUINV=DEGF*MUINV
   61 FORMAT(///)
      WRITE(6,54)
   54 FORMAT(///,31X,37(1H=),/,31X,'D C   L I N K   P A R A M E T E R
     1S',/,31X,37(1H=))
      WRITE(6,55)NRECT,ALPHAR,VDR,XCR,TAPR,PFR,QDR,VRECT,MURECT,PDR
   55 FORMAT(//,31X,'FOR RECTIFIER',/,31X,13(1H-),/,31X,'BUS NUMBER
     1                    =',I4,/,31X,'DELAY ANGLE (DEG.)
     2=',F10.3,/,31X,'DC VOLTAGE (P.U.)                 =',F10.5,/,31X,'
     3COMM. REACTANCE (P.U.,SPECIFIED)  =',F10.5,/,31X,'TRANSFORMER TAP
     4RATIO                 =',F10.5,/,31X,'POWER FACTOR
     5  =',F10.5,/,31X,'REACTIVE POWER CONSUMED (P.U.)     =',F10.5,/,31X
     6,'AC BUS VOLTAGE (P.U.)               =',F10.5,/,31X,'OVERLAP ANGLE
     7(DEG.)                =',F10.3,/,31X,'DC POWER (P.U.)
     8   =',F10.5)
      WRITE(6,56)NINV,GAMMAI,PDI,XCI,VDI,TAPI,PFI,QDI,VINV,MUINV,BETAI
```

```
 56   FORMAT(//,31X,'FOR INVERTER',/,31X,12(1H-),/,31X,'BUS NUMBER
     1                =',I4,/,31X,'EXTINCTION ANGLE (DEG.)                =
     2',F10.3,/,31X,'DC POWER (P.U.,SPECIFIED)              =',F10.5,/,31X,'C
     3OMM. REACTANCE (P.U.,SPECIFIED)  =',F10.5,/,31X,'DC VOLTAGE (P.U.)
     4                =',F10.5,/,31X,'TRANSFORMER TAP RATIO
     5 =',F10.5,/,31X,'POWER FACTER                          =',F10.5,/,31X,
     6'REACTIVE POWER CONSUMED (P.U.)     =',F10.5,/,31X,'AC BUS VOLTAGE
     7(P.U.)              =',F10.5,/,31X,'OVERLAP ANGLE (DEG.)
     8    =',F10.3,/,31X,'IGNITION ADVANCE ANGLE (DEG.)        =',F10.3)
      WRITE(6,59)RDC,IC
 59   FORMAT(///,31X,'RESIST. OF DC LINE (P.U.,SPEC.)     =',F10.5,///,31X
     1,'DC CURRENT (P.U.,SPECIFIED)              =',F10.5)
      WRITE(6,28)
 28   FORMAT(1H1)
      GO TC 1005
1000  WRITE(6,1001)KMAX,CELMAX
1001  FORMAT(/,/,/,/,3X,'THE MAXIMUN ALLOWED NUMBER OF ITERATIONS HAS BE
     1EN PERFORMED ','(',I2,')','WITHOUT ACHIEVING CONVERGENCE TO THE DE
     2SIREC ACCURACY ','(',F8.5,')','.')
1005  CONTINUE
      WRITE(6,84)
3023  WRITE(6,1004)DELMAX
1004  FORMAT(//,15X,'THE REQUIRED ACCURACY WAS',F8.5)
      WRITE(6,58)
      WRITE(6,1011)
1011  FORMAT(//,15X,'THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WE
     1RE:',//,13X,'ITERATION',3X,'DELTA P',7X,'DELTA Q',7X,'DELTA V**2',
     27X,'CELTA PDR',7X,'DELTA PDI',/,13X,9(1H-),3X,7(1H-),7X,7(1H-),7X,
     310(1H-),7X,9(1H-),7X,9(1H-),/)
      DELPR(1)=ABS(DELPR(1))
      DELPI(1)=ABS(DELPI(1))
      WRITE(6,1012)DELK(1),DELP(1),DELQ(1),DELV(1),DELPR(1),DELPI(1)
1012  FORMAT(15X,F4.1,1X,F14.8,2F14.8,3X,F14.8,3X,F14.8,/)
C
C         SAVING MAXIMUM MISMATCH VALUES OF EACH ITERATION
C                   WITH THE PRESENT ALPHA VALUE
C
      DO 1013 I=2,KDEL
      IJ=IJ+1
      SAVFP(IJ)=DELP(I)
      SAVEQ(IJ)=DELQ(I)
      SAVEV(IJ)=DELV(I)
      SAVEFR(IJ)=DELPR(I)
      SAVEFI(IJ)=DELPI(I)
      DELPR(I)=ABS(DELPR(I))
      DELPI(I)=ABS(DELPI(I))
```

```
      WRITE(6,1012)DELK(I),DELP(I),DELQ(I),DELV(I),DELPR(I),DELPI(I)
1013  CONTINUE
      IJK=IJK+1
      KSAVE(IJK)=KDEL-1
      KKSAVE(IJK)=KKK
      IF(CC.NE.1) GO TO 1070
      WRITE(6,1009) ALPHA
1009  FORMAT(/,10X,'THE VALUE OF ALPHA WAS',F6.2)
      ALPHA=ALPHA+0.1
      IF(ALPHA.GT.2.3)GO TO 1006
      GO TO 101
1006  CONTINUE
1070  STOP
      END
      SUBROUTINE RELOAD(N,W,J)
      DIMENSION W(114)
      REAL JACOB(832)
      INTEGER*2 JACA1(832),JACA2(115)
      COMMON /MEMORY/ JACOB,JACA1,JACA2
      II1=JACA2(J)
      II2=JACA2(J+1)-1
      DO 1772 I=II1,II2
1772  W(JACA1(I))=JACOB(I)
      RETURN
      END
      SUBROUTINE ISRANK (IX,IO,N)
C
C        THIS SUBROUTINE WILL SORT THE INTEGER*2 ARRAY IX INTO
C        ASCENDING ORDER SUCH THAT  IX(IO(I)) < IX(IO(I+1)) FOR
C        I = 1,N
C        THE CODE IS A NEAR COPY OF THE ISRANK ROUTINE OF WATFIV
C        AND IS DESIGNED TO TAKE PLACE OF THE HARWELL ROUTINE
C        KB10AS IN THE SPARSITY ROUTINE MA18AD.
C
C                                                     G. SOMERTON
C
C
C
      IMPLICIT INTEGER*2 (I-M)
      DIMENSION IX(N),IO(N)
      IONE = 1
      ITWO = 2
      IS1 = IONE
      IF1 = N
C
C        THE FOLLOWING CODE IS ENTIRELY THE WATFIV ROUTINE ISRANK
C        EXCEPT EVERYTHING IS IN INTEGER*2 ARITHMETIC.
```

```
C
      IO(IS1) = IS1
      IS2 = IS1+IONE
      DO 40  I=IS2,IF1
      IS = IX(I)
      M = I-IONE
      DO 10 J=IS1,M
      IOJ = IO(J)
      IF(IS.GT.IX(IOJ)) GO TO 20
   10 CONTINUE
      IO(I)=I
      GO TO 40
   20 IM = J+1
      IZ=I+IM
      DO 30 IB=IM,I
      IA=IZ-IB
   30 IO(IA)=IO(IA-IONE)
      IO(J)=I
   40 CONTINUE
C
C        THE FOLLOWING SECTION OF CODE CHANGES THE IO POINTERS FROM
C        DESCENDING TO ASCENDING ORDER.
C
      N2 = N/ITWO
      DO 50 I=1,N2
      II = I-IONE
      ITEMP = IO(I)
      IO(I) = IO(IF1-II)
   50 IO(IF1-II) = ITEMP
      RETURN
      END
      SUBROUTINE MA18A (A,IND,IW,N,NP,G,U,IA)
      REAL*4 EPS/1E-6/,ZERO/0.0,ONE/1.0/
      REAL*8 ROWCOL(2)/8H    ROW ,8H COLUMN /
C     EPS IS THE RELATIVE ACCURACY OF FLOATING-POINT COMPUTATION.
      LOGICAL*1 LFAC,MFAC(4)
      INTEGER*2 IND(IA,4),IW(NP,13),IFAC(2)
       EQUIVALENCE(IFAC(1),FAC,LFAC,DAK),(JFAC,MFAC(1))
      REAL*8 ROWCOL(2)/8H    ROW ,8H COLUMN /,ZERO/0D0/,ONE/1D0/
      DIMENSION A(IA),IK(2),IC(2),JC(2),JP(2)
      COMMON/MA1 8ED/LP,JSCALE
C     MATRIX ELEMENTS ARE HELD IN A(K),K=1,2,...,KA.
C      ON ENTRY IND(K,1) HOLDS THE ROW NUMBER OF THE ELEMENT HELD IN
C A(K).  IN THE MAIN BODY OF THE SUBROUTINE IND(K,1),IND(K,2) HOLD THE
C ADDRESS CF THE NEXT ELEMENT IN THE ROW/COLUMN IF THERE IS ONE AND
C FOR THE LAST ELEMENT IN THE ROW/COLUMN HOLD (IA+ THE ROW/COLUMN
```

```
C NUMBER). THESE NUMBERS ARE NEGATED IF THEY POINT TO ELEMENTS THAT
C HAVE BEEN IN A PIVOTAL COLUMN/ROW. FINALLY IND(K,1) IS RESET TO THE
C ROW NUMBER OF THE ELEMENT HELD IN A(K).
C      ON ENTRY AND ON EXIT IW(I,1) CONTAINS THE ADDRESS OF THE FIRST
C ELEMENT OF COLUMN I AND IW(N+1,1) CONTAINS THE ADDRESS OF THE FIRST
C UNUSED ELEMENT IN A. ON EXIT IW(I,2) HOLDS THE COLUMN NUMBERS IN
C PIVOTAL ORDER. AFTER A SUCCESSFULL ENTRY IW(N+1,2)=N+1 AND IW(N+1,3)=0;
C AFTER AN UNSUCCESSFUL ENTRY IW(N+1,2)=0. IN THE MAIN BODY OF THE
C SUBROUTINE IW(I,1),IW(I,2) HOLD THE ADDRESS OF THE FIRST ELEMENT OF
C THE I TH ROW/COLUMN AND ARE NEGATED IF THE FIRST ELEMENT HAS BEEN
C IN A PIVOTAL COLUMN/ROW.
      KA=IW(N+1,1)-1
C      IW(I,3),IW(I,4) HOLD THE LOGS TO BASE 16 OF THE ROW/COLUMN
C SCALING FACTORS USED.
C      IW(I,5),IW(I,6) HOLD THE POSITION IN THE ORDERING BY NUMBER OF
C NON-ZEROS OF THE LAST ROW/COLUMN TO HAVE LESS THAN I NON-ZERO
C ELEMENTS OR ZERO IF NONE HAVE LESS THAN I NON-ZERO ELEMENTS.
C ON EXIT IW(I,5) HOLDS THE POSOTION OF THE I TH ROW IN THE PIVOTAL
C ORDERING.
C      IW(I,7),IW(I,8) HOLD THE NUMBER OF NON-ZEROS IN THE I TH ROW/
C COLUMN.
C      IW(I,9),IW(I,10) HOLD THE POSITION OF THE I TH ROW/COLUMN
C IN THE ORDERING BY NUMBER OF NON-ZEROS.
C      IW(I,11),IW(I,12) HOLD ROW/COLUMN NUMBERS IN PIVOTAL ORDER FOR
C I<IP AND IN ORDER OF INCREASING NUMBERS OF NON-ZEROS OTHERWISE.
C      IW(I,13) HOLDS THE EXPONENT OF THE MAXIMAL ELEMENT IN THE I TH
C COLUMN OF THE SCALED VERSION OF THE ORIGINAL MATRIX.
      U=DMIN1(ONE,DMAX1(U,EPS*ONE))
      N1=N+1
C     FIND SCALING FACTORS.
      DO 1 I=1,N1
1     IW(I,2)=I
      J1=3
      IF(JSCALE)6,8,2
2      J=2*(KA/2)+3
      CALL MC12AD(A,IND,IW,N,NP,IW(1,3),IW(1,5),I)
3     L=1
      IR=IABS(I)
      IF(I.LT.0)L=2
      IF(I.NE.0) GO TO 560
6     J1=5
8     DO 10 I=1,N1
      IW(I,2)=IW(I,1)
      IW(I,11)=I
      IW(I,12)=I
      IW(I,13)=0
```

```
      DO 10 J=J1,9
10    IW(I,J)=0
C     SCALE THE MATRIX, SET ROW AND COLUMN LINKS, FIND FIRST ELEMENTS
C OF THE ROWS, COUNT THE NUMBER OF NON-ZEROS IN THE ROWS AND COLUMNS
C AND FIND EXPONENTS OF MAXIMAL COLUMN ELEMENTS.
      IG=0
      DO 30 J=1,N
      FAC=ONE
C     TEMPORARILY WE USE IW(I,9) TO HOLD THE ADDRESS OF THE LAST NON-
C ZERO ENCOUNTERED IN THE I TH ROW.
      K1=IW(J,2)
      K2=IW(J+1,2)-1
      IW(J,8)=K2-K1+1
      IL=0
      AMAX=ZERO
      DO 20 K=K1,K2
      I=IND(K,1)
      IF(I.LE.IL) GO TO 520
      IL=I
C     ON THE IBM 360 THE FOLLOWING TWO INSTRUCTIONS ARE EQUIVALENT TO
C     FAC=16.**( IW(I,3)+IW(J,4) )
      JFAC=65+IW(I,3)+IW(J,4)
      LFAC=MFAC(4)
      A(K)=A(K)*FAC
      AMAX=DMAX1(AMAX,DABS(A(K)))
      IND(K,1)=I+IA
      IND(K,2)=K+1
      KL=IW(I,9)
      IF(KL.LE.0)IW(I,1)=K
      IF(KL.GT.0)IND(KL,1)=K
      IW(I,9)=K
20    IW(I,7)=IW(I,7)+1
      IND(K2,2)=IA+J
      DAK=AMAX
C      ON THE IBM 360 THE FOLLOWING INSTRUCTION SETS JFAC TO THE
C     SMALLEST INTEGER GREATER THAN ALOG16(DAK)+64
      MFAC(4)=LFAC
30    IW(J,13)=JFAC
C
C     SET UP THOSE VECTORS IN IW ASSOCIATED WITH ORDERING BY NUMBERS
C OF NON-ZEROS.
      DO 130 L=1,2
      CALL ISRANK (IW(1,L+6),IW(1,L+10),N)
C     CHECK FOR NULL ROW OR COLUMN.
      IR=IW(1,L+10)
      IF(IW(IR,L+6).LE.0)GO TO 560
```

```
      DO 110 I=1,N
      J=IW(I,L+10)
      IW(J,L+8)=I
      NZ=IW(J,L+6)
110   IF(NZ.NE.N)IW(NZ+1,L+4)=I
      J=0
      DO 130 I=1,N
      IF(IW(I,4+L).EQ.0)IW(I,L+4)=J
130   J=IW(I,L+4)
C
C     NOW PERFORM THE MAIN ELIMINATION.
      DO 298 IP=1,N
C     FIND THE PIVOT. WE DO THIS BY SEARCHING A ROW/COLUMN. THE
C NEXT ROW/COLUMN TO BE USED IS JC(L)=IW(IK(L),L+10).
133   IK(1)=IP
      IK(2)=IP
C     JCOST IS THE COST OF THE CHEAPEST PIVOT SO FAR FOUND.
      JCOST=N*N
135   DO 140 L=1,2
      JC(L)=IW(IK(L),L+10)
140   IC(L)=IW(JC(L),L+6)
C     ICOST IS THE MINIMAL POSSIBLE COST OF A PIVOT NOT SO FAR FOUND.
      ICOST=(IC(1)-1)*(IC(2)-1)
      IF(JCOST.LE.ICOST)GO TO 160
      L=1
      IF(IC(1).GT.IC(2))L=2
      IR=JC(L)

C     FIND THE MAXIMAL ELEMENT IN ROW/COLUMN UNDER CONSIDERATION.
      AMAX=ZERO
      K=IW(IR,L)
      GO TO 143
142   K=IND(-K,L)
143   IF(K.LT.0)GO TO 142
      KK=K
      GO TO 145
144   AMAX=DMAX1(AMAX,DABS(A(KK)))
      KK=IND(KK,L)
145   IF(KK.LE.IA) GO TO 144
      IF(AMAX.EQ.ZERO) GO TO 560
      AU=AMAX*U
      L3=3-L
      IK(L)=IK(L)+1
C     NOW CONSIDER THE ELEMENTS IN THE ROW/COLUMN IN TURN.
146   IF(DABS(A(K)).LT.AU) GO TO 150
      KK=K
```

```
147   KK=IND(KK,L3)
      IF(KK.LE.IA) GO TO 147
      KCOST=(IC(L)-1)*(IW(KK-IA,9-L)-1)
      IF(KCOST.GE.JCOST) GO TO 150
      JCOST=KCOST
      KP=K
      JP(L)=IR
      JP(L3)=KK-IA
      IF(JCOST.LE.ICOST) GO TO 160
150   K=IND(K,L)
      IF(IA-K)135,146,146
C
C     REARRANGE THE LINKS SO THAT THE PIVOTAL ROW AND COLUMN ARE IN
C CORRECT PIVOTAL SEQUENCE.
160   DO 188 L=1,2
C     "MOVE" THE PIVOTAL COLUMN FIRST AND THEN THE PIVOTAL ROW.
      IK(3-L)=JP(3-L)
      K=IABS(IW(IK(3-L),3+L)+0)
C     K POINTS TO AN ELEMENT IN THE PIVOTAL COLUMN/ROW.
C     KM POINTS TO ITS PREDESSOR IN ITS ROW/COLUMN.
C     KL POINTS TO THE LAST ELEMENT THAT HAS BEEN PIVOTAL IN ITS
C ROW/COLUMN.
165   KO=K
170   KO=IABS(IND(KO,L)+0)
      IF(KO.LE.IA)GO TO 170
      IK(L)=KO-IA
      KL=KO
      KO=IW(KO-IA,L)
      IF(IW(IK(L),L+6).LE.0)GO TO 174
C     ON THE REMOVAL OF THE ELEMENT A(IK(1),IK(2)) THE FOLLOWING
C INSTRUCTIONS ARE USED TO UPDATE THE NUMBERS OF ELEMENTS IN THE
C CORRESPONDING ROW AND COLUMN AND MAKE CONSEQUENT CHANGES TO THE
C ORDERING BY NUMBER OF NON-ZEROS.
      DO 171 LM=1,2
      IR=IK(LM)
      NZ=IW(IR,LM+6)-1
      IW(IR,LM+6)=NZ
      JPOS=IW(NZ+1,LM+4)+1
      IPOS=IW(IR,LM+8)
      IF(IPOS.EQ.JPOS)GO TO 171
      JR=IW(JPOS,LM+10)
      JJ=IW(IPOS,LM+10)
      IW(IPOS,LM+10)=IW(JPOS,LM+10)
      IW(JPOS,LM+10)=JJ
      JJ=IW(IR,LM+8)
      IW(IR,LM+8)=IW(JR,LM+8)
```

```
      IW(JM,LM+8)=JJ
171   IW(N2+1,LM+4)=JPOS
      GO TO 174
172   KL=-KO
      KO=IND(KL,L)
174   IF(KO.LT.0)GO TO 172
      KM=KL
      GO TO 178
176   KM=KO
      KO=IND(KO,L)
178   IF(KO.NE.K) GO TO 176
      IF(KL.EQ.KM) GO TO 182
      IND(KM,L)=IND(K,L)
      IF(KL.LE.IA)GO TO 183
      IND(K,L)=IW(IK(L),L)
180   IW(IK(L),L)=-K
      GO TO 186
182   IF(KM-IA)184,184,180
183   IND(K,L)=IND(KL,L)
184   IND(KL,L)=-K
186   K=IABS(IND(K,3-L)+0)
      IF(K.LE.IA) GO TO 165
188   CONTINUE
C
C     OVERWRITE THE ELEMENTS OF THE PIVOTAL COLUMN BY MULTIPLIERS
C AND PERFORM THE ELIMINATION.
      K=IND(KP,2)
      GO TO 295
190   M=K
193   M=IND(M,1)
      IF(M.LE.IA)GO TO 193
      L=M-IA
      A(K)=A(K)/A(KP)
      KI=KP
      KL=K
      GO TO 280
195   M=KI
200   M=IND(M,2)
      IF(M.LE.IA)GO TO 200
      JL=M-IA
      IF(JI-JL)210,275,270
C
C     CREATE A NEW NON-ZERO IN POSITION (L,JI).
210   KA=KA+1
      IF(KA.GT.IA)GO TO 580
      A(KA)=ZERO
```

```
      IK(1)=L
      IK(2)=JI
      IND(KA,1)=IND(KLAST,1)
      IND(KLAST,1)=KA
C     THE CRDER OF THE COLUMN LINKS DOES NOT MATTER SO WE PUT THE
C NEW ELEMENT AS THE SECCND IN ITS COLUMN.
      IND(KA,2)=IND(KI,2)
      IND(KI,2)=KA
C     ON THE ADDITION OF THE ELEMENT A(IK(1),IK(2)) THE FOLLOWING
C INSTRUCTIONS ARE USED TO UPDATE IW.
      DO 250 LM=1,2
      IR=IK(LM)
      NZ=IW(IR,LM+6)
      IW(IR,LM+6)=NZ+1
      JPOS=IW(NZ+1,LM+4)
      JR=IW(JPOS,LM+10)
      IF(IR.EQ.JR)GO TO 250
      IPOS=IW(IR,LM+8)
      JJ=IW(IPOS,LM+10)
      IW(IPOS,LM+10)=IW(JPOS,LM+10)
      IW(JPOS,LM+10)=JJ
      JJ=IW(IR,LM+8)
      IW(IR,LM+8)=IW(JR,LM+8)
      IW(JR,LM+8)=JJ
250   IW(NZ+1,LM+4)=JPOS-1
      KL=KA
      GO TO 275
265   M=KI
267   M=IND(M,2)
      IF(M.LE.IA)GO TO 267
      JI=M-IA
270   KLAST=KL
      KL=IND(KL,1)
      IF(IA-KL)210,195,195
275   A(KL)=A(KL)-A(K)*A(KI)
      DAK=DABS(A(KL))
C     ON THE IBM 360 THE FOLLOWING INSTRUCTION SETS JFAC TO THE
C FLOATING-POINT EXPONENT OF DAK.
      MFAC(4)=LFAC
      IG=MAX0(IG,JFAC-IW(JI,13))
280   KI=IND(KI,1)
      IF(KI.LE.IA)GO TO 265
290   K=IND(K,2)
295   IF(K.LE.IA)GO TO 190
298   CONTINUE
C
```

```
C       SCAN BY ROWS REPLACING ROW LINKS BY ROW NUMBERS.
        DO 310 I=1,N
        K=IABS(IW(I,1)+0)
300     KK=IND(K,1)
        IND(K,1)=I
        K=IABS(KK)
        IF(K.LE.IA)GO TO 300
310     CONTINUE
C
C
C       SCAN BY COLUMNS REPLACING COLUMN LINKS BY ORDERING NUMBERS.
        J=1
        DO 330 I=1,N
        K=IABS(IW(IW(I,12),2)+0)
        IW(IW(I,12),1)=J
320     KK=IND(K,2)
        IND(K,2)=J
        J=J+1
        K=IABS(KK)
        IF(K.LE.IA)GO TO 320
330     CONTINUE
        IW(N+1,1)=J
C
C       REORDER.
        KA=J-1
        DO 360 I=1,KA
        IF(I.EQ.IND(I,2))GO TO 360
        A1=A(I)
        I1=IND(I,1)
        J = I
350     K=IND(J,2)
        IND(J,2)=J
        A2=A(K)
        I2=IND(K,1)
        A(K)=A1
        IND(K,1)=I1
        A1=A2
        I1=I2
        J=K
        IF(K.NE.I)GO TO 350
360     CONTINUE
C
C       SET REMAINING VECTORS IN PREPARATION FOR FACTOR AND OPERATE.
        DO 370 I=1,N1
        IW(I,2)=IW(I,12)
370     IW(I,5)=IW(I,9)
```

```
C
C      RESTORE THE MATRIX TO ITS UNEQUILIBRATED STATE.
       FAC=ONE
       DO 400 J=1,N
       JCOL=IW(J,2)
       K1=IW(JCOL,1)
       K2=IW(IW(J+1,2),1)-1
       IO=IW(JCOL,4)
       DO 400 K=K1,K2
       JRO=IND(K,1)
C      ON THE IBM 360 THE FOLLOWING TWO INSTRUCTIONS ARE EQUIVALENT TO
C      FAC=16.0**(-IO-IW(JRO,3))
       JFAC=65-IO-IW(JRO,3)
       LFAC=MFAC(4)
       A(K)=A(K)*FAC
400    IF(IW(JRO,5).EQ.0)IO=-IW(JRO,3)
       IW(N+1,3)=0
       G=EPS*16D0**IG
       RETURN
C
C      THE FOLLOWING INSTRUCTIONS IMPLEMENT THE FAILURE EXITS.
500    WRITE(LP,510)
510    FORMAT('+ERROR RETURN FROM MA18AD BECAUSE')
       IW(N1,2)=0
       G=-ONE
       RETURN
520    WRITE(LP,530)K
       GO TO 500
530    FORMAT(//34X,'THE ELEMENT HELD IN A(',I5,') IS OUT OF ORDER')
560    WRITE(LP,570)ROWCOL(L),IR
570    FORMAT(//34X
      1,'THE MATRIX IS SINGULAR.',A8,I4,' IS DEPENDENT IN THE REST')
       GO TO 500
580    WRITE(LP,590)IP
590    FORMAT(//34X,'IA IS TOO SMALL. SPACE RAN OUT WHEN ELIMINATING'
      1,' ON PIVOT',I5)
       GO TO 500
       END
       BLOCK DATA
       COMMON /MA18ED/ JP,JSCALE
       DATA JP/6/
       DATA JSCALE/1/
       END
       SUBROUTINE MA18B (A,IRN,IP,N,NP,AWS,AVECT,MTYPE)
C
```

```
C
C      THIS ENTRY OPERATES ON THE VECTOR AVECT WITH THE FOLLOWING
C      MATRICES ACCORDING TO THE VALUE OF MTYPE
C
C         MTYPE = 1      A**(-1)
C         MTYPE = 2      A**(-T)
C         MTYPE = 3      A
C         MTYPE = 4      A**T
C
C      JP IS STREAM NUMBER FOR DIAGNOSTICS
C      JSCALE CONTROLS EQUILIBRATION
       COMMON /MA18ED/ JP,JSCALE
C
C       ARND IS ROUNDING ERROR ESTIMATE
       DATA ARND/1.D-15/
       REAL*8 ZERO /0D0/
C       ANAME CONTAINS NAMES FOR USE IN DIANOSTICS
       REAL*8 ANAME(5)/' MA18AD',' MA18BD ',' MA18CD ',' MA18DD ',
     X' MC12AD '/
       LOGICAL LOG
C       ON S/360 NEXT TWO STATEMENTS PERMIT ACCES TO EXPONENT OF REAL
C       EQUIV BETWEEN AU AND JU IS ALSO USED IN ALL IMPLEMENTATIONS
       EQUIVALENCE (AU,JU,LU),(JUU,LUU(1))
       LOGICAL*1 LU,LUU(4)
C       DIMENSIONS OF ARGUMENT ARRAYS COMMON TO ALL ENTRIES
       INTEGER*2 IRN(1),IP(NP,5)
       DIMENSION A(1)
C       DIMENSIONS SPECIAL TO THIS ENTRY
       DIMENSION AWS(1)
       DIMENSION AVECT(1)
       KERR=1
       MT=MTYPE
       MTYPE=0
       KENTRY=2
C       TEST FOR INVALID ENTRY
       IF(IP(N+1,2).EQ.0) GO TO 2205
       IF(IP(N+1,2).NE.N+1)GO TO 2207
       KERR=3
       IF(IP(N+1,3).NE.0) GO TO 2205
       LOG=.FALSE.
       MTYPE=MT
C       JUMP ACCORDING TO REQUIRED OPERATION
       IF(MTYPE.LE.0.OR.MTYPE.GT.4) GO TO 2204
       GO TO (101,102,103,104),MTYPE
C       SET UP FOR MTYPE =1,4, PERMUTING AVECT TO AWS
104    LOG=.TRUE.
```

```
101    DO 110 J=1,N
110    AWS(IP(J,5))=AVECT(J)
       AU=ZERO
C       FIRST DIVIDE BY L OR MULTIPLY BY L**T
       DO 140 J=1,N
       J1=IP(IP(J+1,2),1)
141    J1=J1-1
       JSEQ=IP(IRN(J1),5)
       IF(JSEQ.EQ.J) GO TO 111
       IF(LOG)GO TO 1412
       AWS(JSEQ)=AWS(JSEQ)-AWS(J)*A(J1)
       GO TO 141
1412   AWS(J)=AWS(J)+AWS(JSEQ)*A(J1)
       GO TO 141
111    IF(.NOT.LOG)JU=J1
C       SAVE PIVOT POSITION FOR MTYPE=1
C       SET ZERO FOR MTYPE=4
 140   AVECT(IP(J,2))=AU
C      NOW DIVIDE BY U OR MULTIPLY BY U**T
       J=N
143    JCOL=IP(J,2)
       J1=IP(JCOL,1)
       IF(LOG)GO TO 144
C       RECOVER PIVOT POSITION IN JU
       AU=AVECT(JCOL)
       AVECT(JCOL)=AWS(J)/A(JU)
144    JSEQ=IP(IRN(J1),5)
       IF(LOG)GO TO 1442
       IF(JSEQ.EQ.J)GO TO 119
       AWS(JSEQ)=AWS(JSEQ)-AVECT(JCOL)*A(J1)
       GO TO 1443
1442   AVECT(JCOL)=AVECT(JCOL)+AWS(JSEQ)*A(J1)
       IF(JSEQ.EQ.J)GO TO 119
1443   J1=J1+1
       GO TO 144
119    J=J-1
       IF(J.GT.0) GO TO 143
       GO TO 3004
C       ENTRY FOR MTYPE=2,3
103    LOG=.TRUE.
C       FIRST DIVIDE BY U**T OR MULTIPLY BY U
102    DO 120 J=1,N
       JCOL=IP(J,2)
       J1=IP(JCOL,1)
121    JSEQ=IP(IRN(J1),5)
       IF(JSEQ.EQ.J) GO TO 122
```

```
      IF(LCG) GO TO 1212
      AVECT(JCOL)=AVECT(JCOL)-AWS(JSEQ)*A(J1)
      GO TC 1213
1212  AWS(JSEQ)=AWS(JSEQ)+A(J1)*AVECT(JCOL)
1213  J1=J1+1
      GO TC 121
122   IF(LOG) GO TO 132
      AWS(JSEQ)=AVECT(JCCL)/A(J1)
      GO TC 120
132   AWS(JSEQ)=AVECT(JCCL)*A(J1)
120   CONTINUE
C      NOW DIVIDE BY L**T OR MULTIPLY BY L
      J=N
123   J1=IP(IP(J+1,2),1)
124   J1=J1-1
      JRO=IRN(J1)
      IF(IP(JRO,5).EQ.J) GO TO 125
      IF(LOG) GO TO 1242
      AWS(J)=AWS(J)-AVECT(JRO)*A(J1)
      GO TC 124
1242  AVECT(JRO)=AVECT(JRO)+AWS(J)*A(J1)
      GO TC 124
125   AVECT(JRO)=AWS(J)
      J=J-1
      IF(J.GT.0) GO TO 123
      GO TC 3004
C
      ENTRY MA18CD(A,IRN,IP,N,NP,   AGRO)
C      ON ENTRY, THE ARRAYS ARE AS SET UP BY MA18AD, EXCEPT THAT A
C     CONTAINS THE ELEMENTS OF A NEW MATRIX IN UNFACTORISED FORM.
C     ON EXIT, THE CONTENTS OF A HAVE BEEN REPLACED BY THE L/U DECOMP-
C     OSITION OF THIS NEW MATRIX, EXACTLY AS ON EXIT FROM MA18AD.
C      THE ARGUMENT AGRC IS SET TO THE GROWTH ESTIMATE AS FOR MA18AD.
C      NEW SCALING FACTORS ARE CALCULATED AND STORED IF JSCAL.GT.1, CR
C     IF JSCALE=0 WHEN THEY WILL ALL BE UNITY.      OTHERWISE, THE OLD
C     SCALING FACTORS ARE USED.     THE PIVOTAL SEQUENCE SET BY MA18AD
C     IS ALWAYS USED.
      IP(N+1,3)=0
      KERR=1
      AGRO=-1D0
      KENTRY=3
C      TEST VALIDITY OF ENTRY
      IF(IP(N+1,2).EQ.0)GO TO 2205
      IF(IP(N+1,2).NE.N+1)GO TO 2207
C      GET SCALING FACTORS IF INDICATED
      IF(JSCALE.LE.1)GC TO 991
```

```
C      SAVE COLUMN 5 IN COLUMN 13
      DO 898 I=1,N
898   IP(I,13)=IP(I,5)
C     MC12AD USES COLUMNS 5 TO 12 AS WORKSPACE
      CALL MC12AD(A,IRN,IP,N,NP,IP(1,3),IP(1,5),IS)
C      RESTORE COLUMN 5
      DO 897 I=1,N
897   IP(I,5)=IP(I,13)
      IF(IS.NE.0)GO TO 2208
991   IF(JSCALE.NE.0) GO TO 892
C      SET ZERO SCALING POWERS IF JSCALE=0
      DO 895 L=1,2
      DO 895 K=1,N
895   IP(K,L+2)=0
892   JUU=0
      IAG=0
C      OPERATE ON COLUMNS IN SEQUENCE
      DO 56 J=1,N
      JST=IP(IP(J,2),1)
      JND=IP(IP(J+1,2),1)-1
C      FIND APPROX LOG OF MAX ELEMENT IN COLUMN
      IA0=0
      DO 57 J1=JST,JND
      AU=DABS(A(J1))
C      ON SYSTEM/360, NEXT STATEMENT SETS JUU SO THAT
C      16**(JUU-64).GT.AU.GE.16**(JUU-65)
      LUU(4)=LU
57    IA0=MAX0(IA0,JUU+IP(IRN(J1),3))
C      SCAN THROUGH COLUMN, ELIMINATING WITH ELEMENTS OF U AND
C     BRANCHING WHEN PIVOT FOUND
      ASSIGN 581 TO JPIV
      DO 58 J1=JST,JND
      GO TO JPIV,(581,582)
581   KSEQ=IP(IRN(J1),5)
C      BRANCH ON PIVOT
      IF(KSEQ.EQ.J) GO TO 59
C      ELEMENT OF U - GET OLD PIVOTAL COLUMN
      KST=IP(IP(KSEQ,2),1)
      KND=IP(IP(KSEQ+1,2),1)-1
      AMULT=A(J1)
      L1=J1+1
C      SCAN DOWN COLUMNS KEEPING IN STEP
      DO 60 K1=KST,KND
      KRO=IRN(K1)
C      SKIP IRRELEVANT ELEMENTS
      IF(IP(KRO,5).LE.KSEQ)GO TO 60
```

```
62       IF(KRO.EQ.IRN(L1))GO TO 63
      L1 = L1+1
      GO TO 62
C       ELIMINATION STEP
63    A(L1)=A(L1)-AMULT*A(K1)
      AU=DABS(A(L1))
C       ON S/360,NEXT STATEMENT SETS JUU AS ABOVE
      LUU(4)=LU
C       UPDATE LOG OF GROWTH ESTIMATE
      IAG=MAX0(IAG,JUU+IP(IRN(L1),3)-IAO)
      L1=L1+1
60    CONTINUE
      GO TO 58
C       TEST FOR ZERO PIVOT
59    IF(A(J1).EQ.ZERO)GO TO 1011
      AMULT=A(J1)
      ASSIGN 582 TO JPIV
      GO TO 58
C       DIVIDE ELEMENTS OF L BY PIVOT
582   A(J1)=A(J1)/AMULT
58    CONTINUE
56    CONTINUE
C       SET GROWTH ESTIMATE
      ARGO=ARND*16D0**IAG
      GO TO 3004
      ENTRY MA18DD(A,IRN,IP,N,NP,AWS,NAME)
      KERR=1
      KENTRY=4
C       TEST VALIDITY OF ENTRY
      IF(IP(N+1,2).EQ.0)GO TO 2205
      IF(IP(N+1,2).NE.N+1)GO TO 2207
C       INITIALLY CLEAR COLUMN

2402  DO 301 J=1,N
      AWS(J)=ZERO
301   CONTINUE
C       COLUMNS IN PIVOTAL ORDER
      DO 303 J=1,N
      JCOL=IP(J,2)
C       LOAD COLUMN
      CALL NAME(N,AWS,JCOL)
C       COPY TO REQUIRED PLACE AND RESET TO ZERO
      J1=IP(JCOL,1)
      J2=IP(IP(J+1,2),1)-1
      DO 302 L1=J1,J2
      JRO=IRN(L1)
```

```
      A(L1)=AWS(JRO)
302   AWS(JRO)=ZERO
303   CONTINUE
      GO TO 3004
C       DIAGNOSTIC PRINTING
2204  WRITE(JP,2304)ANAME(2),MTYPE
2304  FORMAT('0ERROR RETURN FROM',A8,'BECAUSE MTYPE =',I5,' WHICH IS OUT
     X OF RANGE')
      GO TO 3004
2205  WRITE(JP,2305)ANAME(KENTRY),ANAME(KERR)
2305  FORMAT('0ERROR RETURN FROM',A8,'BECAUSE PREVIOUS ENTRY TO',A8,
     X'GAVE ERROR RETURN')
      GO TO 3004
2207  WRITE(JP,2307) ANAME(KENTRY),ANAME(1)
2307  FORMAT('0ERROR RETURN FROM',A8,'BECAUSE NO PREVIOUS ENTRY TO',A8)
      GO TO 3004
1011  WRITE(JP,2309)ANAME(3),IRN(J1),IP(J,2)
2309  FORMAT('0ERROR RETURN FROM',A8,'BECAUSE ZERO PIVOT (',2I4,')')
      IP(N+1,3)=1
      GO TO 3004
2208  WRITE(JP,2308) ANAME(3),ANAME(5),IS
2308  FORMAT('0ERROR RETURN FROM',A8,'BECAUSE',A8,'HAS GIVEN ERROR ',
     X'RETURN WITH IS=',I4)
3004  RETURN
      END
      SUBROUTINE MC12A (A,IND,IP,N,NP,DIAG,RES,IS)
      INTEGER*2 IND(1),IP(NP,2),DIAG(NP,2)
      REAL*4 RES(N,4)
C       DIAG IS USED TO RETURN INTEGER SCALING POWERS, AND TO HOLD
C     COUNTS OF NON-ZEROS IN ROWS AND COLUMNS DURING EXECUTION.
C     IT IS SET TO 0 ON SUCCESSFUL COMPLETION, TO I IF ROW I HAS ONLY
C        ZERO ELEMENTS, TO -I IF COLUMN I HAS ONLY ZERO ELEMENTS
C      RES IS A WORKSPACE ARRAY. COLUMNSI AND 2 HOLD NON-ZERO HALF
C          OF RESIDUAL VECTOR FOR TWO CONSECUTIVE ITERATIONS.   COLUMN
C          3 HOLDS COLUMN SCALING POWERS, AND COLUMN 4 HOLDS THEIR
C          CHANGES OVER A DOUBLE ITERATION.
C
      DATA SMIN/.01/
C     SMIN IS USED IN A CONVERGENCE TEST ON (RESIDUAL NORM1**2
      INTEGER*2 JU(2)
      LOGICAL*1 IU,IW(3)
      EQUIVALENCE (UU,IW(1)),(U,IU,JU(1))
      UU=100.
      IS=0
C     INITIALISE FOR ACCUMULATION OF SUMS AND PRODUCTS
      DO 2 L=1,2
```

```
      DO 2 I=1,N
      RES(I,L)=0.
      RES(I,L+2)=0.
    2 DIAG(I,L)=0
      DO 3 J=1,N
      I2=IP(J,2)
      K1=IP(I2,1)
      K2=IP(IP(J+1,2),1)-1
      IF(K1.GT.K2) GO TO 3
      DO 3 K=K1,K2
      I1=IND(K)
      U=DABS(A(K))
      IF(U.EQ.0)GO TO 3
C     ON THE IBM 360 THE FOLLOWING TWO INSTRUCTIONS FIND THE SMALLEST
C     INTEGER GREATER THAN ALOG16(U).
      IW(2)=IU
      U=UU-64.
C     COUNT NON-ZEROS IN ROW AND COLUMN
      DIAG(I1,1)=DIAG(I1,1)+1
      DIAG(I2,2)=DIAG(I2,2)+1
      RES(I1,1)=RES(I1,1)+U
52    RES(I2,3)=RES(I2,3)+U
    3 CONTINUE
C     COMPUTE RHS VECTORS TESTING FOR ZERO ROW OR COLUMN
      J=0
      JU(1)=17920
      DO 8 I=1,N
      J=J+DIAG(I,1)
      DO 9 L=1,2
      IF(DIAG(I,L).GT.0 ) GO TO 13
      DIAG(I,L)=1
      IS=I*(3-2*L)
   13 CONTINUE
C     ON IBM 360 NEXT INSTRUCTION SETS 0 TO VALUE OF POSITIVE INTEGER
      JU(2)=DIAG(I,L)
    9 RES(I,2*L-1)=RES(I,2*L-1)/U
8     CONTINUE
      SM=SMIN*J
C     SWEEP TO COMPUTE INITIAL RESIDUAL VECTOR
      DO 10 J1=1,N
      J=IP(J1,2)
      K1=IP(J,1)
      K2=IP(IP(J1+1,2),1)-1
      IF(K1.GT.K2)GO TO 10
      DO 10 K=K1,K2
       IF(A(K).EQ.0D0) GO TO 10
```

```
      I=IND(K)
C      ON IBM 360 NEXT INSTRUCTION SETS U TO VALUE OF PSITIVE INTEGER
      JU(2)=DIAG(I,1)
      RES(I,1)=RES(I,1)-RES(J,3)/U
   10 CONTINUE
C     INITIALISE ITERATION
      E=0.
      E1=0.
      Q=1.
      S=0.
      DO 11 I=1,N
C      ON IBM 360 NEXT INSTRUCTION SETS U TO VALUE OF POSITIVE INTEGER
      JU(2)=DIAG(I,1)
   11 S=S+U*RES(I,1)**2
      L=2
      IF(S.LE.SM) GO TO 101
C     ITERATION STEP
   20 EM=E*E1
C    SWEEP THROUGH MATRIX TO UPDATE RESIDUAL VECTOR
      DO 22 J3=1,N
      J1=IP(J3,2)
      K1=IP(J1,1)
      K2=IP(IP(J3+1,2),1)-1
      IF(K1.GT.K2) GO TO 22
      DO 22 K=K1,K2
      IF(A(K).EQ.0D0) GO TO 22
      J2=(2-L)*(J1-IND(K))
      I=J1-J2
      J=IND(K)+J2
      RES(I,L)=RES(I,L)+RES(J,3-L)
   22 CONTINUE
      S1=S
      S=0.
      DO 23 I=1,N
      V=-RES(I,L)/Q
C      ON IBM 360 NEXT INSTRUCTION SETS U TO VALUE OF POSITIVE INTEGER
      JU(2)=DIAG(I,L)
      RES(I,L)=V/U
   23 S=S+V*RES(I,L)
      E1=E
      E=Q*S/S1
      Q1=Q
      Q=1.-E
      M=3-I
      IF(S.GT.SM) GO TO 27
      E=M-1
```

```
      M=1
      Q=1.
   27 IF(L.EQ.2) GO TO 25
      QM=Q*Q1
      DO 26 I=1,N
      RES (I,4)=(EM*RES(I,4)+RES(I,2))/QM
   26 RES(I,3)=RES(I,3)+RES(I,4)
   25 L=M
      DO 24 I=1,N
C       ON IBM 360 NEXT INSTRUCTION SETS U TO VALUE OF POSITIVE INTEGER
      JU(2)=DIAG(I,L)
   24 RES(I,L)=RES(I,L)*U*E
      IF(S.GT.SM)GO TO 20
C       SWEEP THROUGH MATRIX TO GET ROW SCALING POWERS
  101 DO 103 J1=1,N
      J=IP(J1,2)
      K1=IP(J,1)
      K2=IP(IP(J1+1,2),1)-1
      IF(K1.GT.K2)GO TC 103
      DO 103 K=K1,K2
      U=DABS(A(K))
      IF(U.EQ.0.)GO TO 103
C       ON IBM 360 NEXT TWO INSTRUCTIONS FIND THE SMALLEST INTEGER
C     LESS THAN ALOG16(U)
      IW(2)=IU
      U=UU-64.
      I=IND(K)
      RES(I,1)=RES(I,1)+RES(J,3)-U
  103 CONTINUE
C       CONVERT POWERS TO INTEGERS
      JU(1)=17920
      DO 104 I=1,N
C     ON IBM 360 NEXT INSTRUCTION SETS 0 TO VALUE OF PASITIVE INTEGER
      JU(2)=DIAG(I,1)
      V=RES(I,1)/U
      DIAG(I,1)=V+SIGN(0.5,V)
  104 DIAG(I,2)=-(RES(I,3)+SIGN(0.5,RES(I,3)))
      RETURN
      END
```

APPENDIX G

SOLUTIONS OF TEST SYSTEMS

Table G-1: Solution of Test System A, Bus Information

***** FIRST ORDER NEWTON-RAPHSON ITERATIVE TECHNIQUE CONVERGED IN 3 ITERATIONS *****

BUS	VOLTAGE		MAGNITUDE	PHASE(DEGS)	REAL POWER	REACTIVE POWER
1	1.0103	-0.1086	1.0161	-6.13351	-0.6000	-0.1000
2	1.0095	-0.0855	1.0131	-4.84061	-0.6388	-0.2019
3	1.0149	-0.0935	1.0192	-5.26183	-0.2112	-0.1149
4	1.0457	-0.0512	1.0470	-2.80484	0.2000	0.2873
5	1.0600	0.0	1.0600	0.0	1.2969	-0.0194

Table G-2: Solution of Test System A, Line Flows

*********** LINE FLOWS ***********

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	3	-0.0628	-0.0173
	3	1	0.0631	-0.0336
2	1	4	-0.5372	-0.0827
	4	1	0.5485	0.0848
3	2	4	-0.2445	-0.1259
	4	2	0.2486	0.0958
4	2	5	-0.3944	-0.0760
	5	2	0.4067	0.0592
5	3	4	-0.2743	-0.0813
	4	3	0.2789	0.0523
6	4	5	-0.8760	0.0544
	5	4	0.8902	-0.0786

Table G-3: Solution of Test System A, DC Link Information

D C   L I N K   P A R A M E T E R S

FOR RECTIFIER

BUS NUMBER	=	2
DELAY ANGLE (DEG.)	=	12.990
DC VOLTAGE (P.U.)	=	1.25892
COMM. REACTANCE (P.U.,SPECIFIED)	=	0.10000
TRANSFORMER TAP RATIO	=	0.95501
POWER FACTOR	=	0.96345
REACTIVE POWER CONSUMED (P.U.)	=	0.05188
AC BUS VOLTAGE (P.U.)	=	1.01315
OVERLAP ANGLE (DEG.)	=	4.743
DC POWER (P.U.)	=	0.18884

FOR INVERTER

BUS NUMBER	=	3
EXTINCTION ANGLE (DEG.)	=	17.060
DC POWER (P.U.,SPECIFIED)	=	0.18880
COMM. REACTANCE (P.U.,SPECIFIED)	=	0.10000
DC VOLTAGE (P.U.)	=	1.25867
TRANSFORMER TAP RATIO	=	0.96744
POWER FACTER	=	0.94524
REACTIVE POWER CONSUMED (P.U.)	=	0.06485
AC BUS VOLTAGE (P.U.)	=	1.01920
OVERLAP ANGLE (DEG.)	=	3.795
IGNITION ADVANCE ANGLE (DEG.)	=	20.855

RESIST. OF DC LINE (P.U.,SPEC.) = 0.00167

DC CURRENT (P.U.,SPECIFIED) = 0.15000

Table G-4: Maximum P.U. Mismatches for Test System A (N.R. Method)

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE:

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.08557677	0.06474459	0.00470161	0.00000006	0.00000030
2.0	0.00036383	0.00050789	0.00000954	0.00000006	0.00000018
3.0	0.00000250	0.00000244	0.0	0.00000018	0.00000030

Table G-5: Solution of Test System B, Bus Information

***** FIRST ORDER NEWTON-RAPHSON ITERATIVE TECHNIQUE CONVERGED IN 3 ITERATIONS *****

BUS	VOLTAGE		MAGNITUDE	PHASE(DEGS)	REAL POWER	REACTIVE POWER
1	0.9235	0.1469	0.9351	9.03843	-0.0000	-0.1000
2	0.9388	0.1412	0.9494	8.55126	0.1221	-0.0399
3	0.9367	0.0078	0.9367	0.47791	-0.0925	-0.1655
4	1.0543	0.1158	1.0607	6.27039	0.2500	0.2000
5	0.9901	-0.1401	1.0000	-8.05148	-0.2000	0.0268
6	0.9826	0.1856	1.0000	10.69535	0.1500	0.0711
7	0.9511	-0.3090	1.0000	-17.99625	-0.2330	0.0998
8	1.0000	0.0	1.0000	0.0	0.0303	0.0474

Table G-6: Solution of Test System B, Line Flows

*********** LINE FLOWS ***********

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	0.0021	-0.0044
	2	1	-0.0021	0.0045
2	1	4	0.0185	-0.0568
	4	1	-0.0178	0.0653
3	1	6	-0.0206	-0.0388
	6	1	0.0208	0.0421
4	2	6	-0.0175	-0.0187
	6	2	0.0177	0.0204
5	2	7	0.1417	0.0022
	7	2	-0.1345	0.0646
6	3	4	-0.0886	-0.0711
	4	3	0.0917	0.0902
7	3	8	-0.0039	-0.0585
	8	3	0.0047	0.0624
8	4	5	0.1356	0.0249
	5	4	-0.1296	0.0088
9	4	8	0.0406	0.0195
	8	4	-0.0400	-0.0141

Table G-6: continued

********** LINE FLOWS **********

LINE	SB	EB	REAL POWER	REACTIVE POWER
10	5	6	-0.0493	0.0140
	6	5	0.0513	0.0020
11	5	8	-0.0210	0.0040
	8	5	0.0214	-0.0013
12	6	7	0.0602	0.0066
	7	6	-0.0562	0.0224
13	7	8	-0.0423	0.0128
	8	7	0.0442	0.0004

Table G-7: Solution of Test System B, DC Link Information

D C   L I N K   P A R A M E T E R S

FOR RECTIFIER		
BUS NUMBER	=	2
DELAY ANGLE (DEG.)	=	15.672
DC VOLTAGE (P.U.)	=	1.16294
COMM. REACTANCE (P.U.,SPECIFIED)	=	0.10000
TRANSFORMER TAP RATIO	=	0.95059
POWER FACTOR	=	0.95421
REACTIVE POWER CONSUMED (P.U.)	=	0.03992
AC BUS VOLTAGE (P.U.)	=	0.94937
OVERLAP ANGLE (DEG.)	=	3.316
DC POWER (P.U.)	=	0.12792
FOR INVERTER		
BUS NUMBER	=	3
EXTINCTION ANGLE (DEG.)	=	13.713
DC POWER (P.U.,SPECIFIED)	=	0.12750
COMM. REACTANCE (P.U.,SPECIFIED)	=	0.10000
DC VOLTAGE (P.U.)	=	1.15909
TRANSFORMER TAP RATIO	=	0.95169
POWER FACTER	=	0.96277
REACTIVE POWER CONSUMED (P.U.)	=	0.03554
AC BUS VOLTAGE (P.U.)	=	0.93672
OVERLAP ANGLE (DEG.)	=	3.724
IGNITION ADVANCE ANGLE (DEG.)	=	17.437
RESIST. OF DC LINE (P.U.,SPEC.)	=	0.03500
DC CURRENT (P.U.,SPECIFIED)	=	0.11000

Table G-8: Maximum P.U. Mismatches for Test System B (N.R. Method)

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE:

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.01821926	0.04911101	0.06433010	0.00310290	0.00000018
2.0	0.00051880	0.00117624	0.00504494	0.00000012	0.00000012
3.0	0.00000107	0.00000250 *	0.00001049	0.00000012	0.00000024

Table G-9: Solution of Test System C, Bus Information

***** FIRST ORDER NEWTON-RAPHSON ITERATIVE TECHNIQUE CONVERGED IN 3 ITERATIONS *****

BUS	VOLTAGE		MAGNITUDE	PHASE(DEGS)	REAL POWER	REACTIVE POWER
1	1.0039	-0.2949	1.0463	-16.37068	-0.1490	-0.0500
2	1.0157	-0.2769	1.0528	-15.24618	-0.1350	-0.0580
3	1.0200	-0.2754	1.0565	-15.11060	-0.0610	-0.0160
4	1.0186	-0.1569	1.0307	-8.75826	-0.6620	-0.2348
5	1.0380	-0.2048	1.0580	-11.16141	0.1075	-0.1782
6	1.0259	-0.2857	1.0649	-15.56071	-0.0900	-0.0580
7	1.0475	-0.2610	1.0795	-13.99205	0.0000	0.0000
8	1.0275	-0.2761	1.0639	-15.04146	-0.0350	-0.0180
9	1.0338	-0.2866	1.0728	-15.49317	-0.2950	-0.1660
10	1.0577	-0.2635	1.0900	-13.99205	0.0000	0.0649
11	1.0372	-0.2630	1.0700	-14.23006	-0.1120	-0.0641
12	0.9854	-0.2217	1.0100	-12.67938	-0.9420	-0.1915
13	1.0410	-0.0912	1.0450	-5.00834	0.1830	0.0107
14	1.0600	0.0	1.0600	0.0	2.3253	-0.2185

Table G-10: Solution of Test System C, Line Flows

*********** LINE FLOWS ***********

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	2	-0.0576	0.0093
	2	1	0.0581	-0.0082
2	1	9	-0.0914	-0.0593
	9	1	0.0928	0.0622
3	2	3	-0.0157	-0.0023
	3	2	0.0157	0.0023
4	2	11	-0.1774	-0.0476
	11	2	0.1795	0.0515
5	3	11	-0.0767	-0.0183
	11	3	0.0774	0.0197
6	4	11	0.4477	0.1787
	11	4	-0.4477	0.5563
7	4	13	-0.3871	0.0368
	13	4	0.3953	-0.0484
8	4	14	-0.7226	0.0703
	14	4	0.7496	-0.0125
9	5	7	0.2758	0.0185
	7	5	-0.2758	0.2487

Table G-10: continued

********** LINE FLOWS **********

LINE	SB	EB	REAL POWER	REACTIVE POWER
10	5	9	0.1591	0.0433
	9	5	-0.1591	0.1039
11	5	12	0.2451	0.1836
	12	5	-0.2390	-0.2052
12	5	13	-0.5724	0.2817
	13	5	0.5942	-0.2570
13	6	8	-0.0432	0.0240
	8	6	0.0434	-0.0236
14	6	9	-0.0468	-0.0820
	9	6	0.0471	0.0827
15	7	9	0.2758	0.0692
	9	7	-0.2758	-0.0616
16	7	10	-0.0000	-0.0643
	10	7	0.0000	0.0649
17	8	11	-0.0784	0.0056
	11	8	0.0789	-0.0045
18	12	13	-0.7030	0.0137
	13	12	0.7258	0.0362

Table G-10: continued

********** LINE FLOWS **********

LINE	SB	EB	REAL POWER	REACTIVE POWER
19	13	14	-1.5323	0.2800
	14	13	1.5757	-0.2060

Table G-11: Solution of Test System C, DC Link Information

```
*****************************************
D C   L I N K   P A R A M E T E R S
*****************************************

FOR RECTIFIER
*************
BUS NUMBER                                  =   4
DELAY ANGLE (DEG.)                          =   15.295
DC VOLTAGE (P.U.)                           =   1.28501
COMM. REACTANCE (P.U.,SPECIFIED)            =   0.10000
TRANSFORMER TAP RATIO                       =   0.98956
POWER FACTOR                                =   0.93296
REACTIVE POWER CONSUMED (P.U.)              =   0.21884
AC BUS VOLTAGE (P.U.)                       =   1.03065
OVERLAP ANGLE (DEG.)                        =   10.369
DC POWER (P.U.)                             =   0.58596

FOR INVERTER
************
BUS NUMBER                                  =   5
EXTINCTION ANGLE (DEG.)                     =   15.125
DC POWER (P.U.,SPECIFIED)                   =   0.58550
COMM. REACTANCE (P.U.,SPECIFIED)            =   0.10000
DC VOLTAGE (P.U.)                           =   1.28399
TRANSFORMER TAP RATIO                       =   0.96249
POWER FACTER                                =   0.93369
REACTIVE POWER CONSUMED (P.U.)              =   0.21716
AC BUS VOLTAGE (P.U.)                       =   1.05797
OVERLAP ANGLE (DEG.)                        =   10.449
IGNITION ADVANCE ANGLE (DEG.)               =   25.574


RESIST. OF DC LINE (P.U.,SPEC.)             =   0.00223

DC CURRENT (P.U.,SPECIFIED)                 =   0.45600
```

Table G-12: Maximum P.U. Mismatches for Test System C (N.R. Method)

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE:

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.09778798	0.10946256	0.07344055	0.00000024	0.00000048
2.0	0.00206715	0.00301510	0.00136280	0.00000048	0.00000024
3.0	0.00000811	0.00001800	0.00000191	0.00000006	0.00000006

Table G-13: Solution of Test System D, Bus Information

***** FIRST ORDER NEWTON-RAPHSON ITERATIVE TECHNIQUE CONVERGED IN 3 ITERATIONS *****

BUS	VOLTAGE		MAGNITUDE	PHASE(DEGS)	REAL POWER	REACTIVE POWER
1	0.9158	-0.2706	0.9550	-16.46347	-0.0670	-0.0200
2	1.0233	-0.2287	1.0485	-12.59649	-0.1800	-0.0530
3	0.9590	-0.1958	0.9788	-11.54189	-0.0490	-0.0220
4	0.9725	-0.1254	0.9806	-7.34636	-0.0000	0.0000
5	0.9655	-0.1454	0.9764	-8.56621	-0.1300	-0.0400
6	0.9473	-0.2065	0.9696	-12.29646	-0.2000	-0.1000
7	0.9747	-0.1305	0.9834	-7.62774	0.0000	0.0000
8	0.9747	-0.2027	0.9956	-11.74840	-0.0410	-0.0140
9	1.0122	-0.1937	1.0306	-10.83461	-0.0680	-0.0340
10	0.9646	-0.1959	0.9843	-11.47906	-0.0500	-0.0200
11	0.9577	-0.1721	0.9731	-10.18980	0.0	0.0
12	0.9228	-0.2642	0.9599	-15.97521	-0.0760	-0.0220
13	0.9639	-0.1663	0.9781	-9.78741	-0.1800	-0.0230
14	0.9586	-0.1582	0.9715	-9.37236	-0.1050	-0.0530

Table G-13: continued

BUS	VOLTAGE		MAGNITUDE	PHASE(DEGS)	REAL POWER	REACTIVE POWER
15	0.9796	-0.1234	0.9873	-7.17907	-0.2200	-0.0500
16	1.0012	-0.1562	1.0134	-8.86882	-0.4300	-0.0300
17	1.0129	-0.0958	1.0174	-5.40103	-0.4200	-0.0800
18	0.9778	-0.2035	0.9988	-11.75439	-0.2720	-0.0980
19	0.9384	-0.2185	0.9635	-13.10507	-0.0330	-0.0060
20	0.9288	-0.2183	0.9541	-13.22450	-0.0230	-0.0100
21	0.9687	-0.2192	0.9932	-12.75250	0.0000	0.0000
22	0.9695	-0.2180	0.9937	-12.67133	-0.0000	-0.0000
23	0.9681	-0.2190	0.9925	-12.74437	-0.0630	-0.0210
24	0.9611	-0.2255	0.9872	-13.20391	0.0000	-0.0000
25	0.9195	-0.3036	0.9683	-18.26903	-0.0630	-0.0320
26	0.9243	-0.2113	0.9481	-12.87463	-0.0000	0.0000
27	0.9567	-0.1946	0.9763	-11.49916	-0.0930	-0.0050
28	0.9768	-0.1810	0.9935	-10.49987	-0.0460	-0.0230
29	0.9935	-0.1719	1.0083	-9.81505	-0.1700	-0.0260
30	0.8970	-0.3061	0.9477	-18.84061	-0.0360	-0.0180
31	0.8666	-0.3077	0.9196	-19.54933	-0.0580	-0.0290
32	0.8832	-0.2986	0.9323	-18.67874	-0.0160	-0.0080

Table G-13: continued

BUS	VOLTAGE		MAGNITUDE	PHASE(DEGS)	REAL POWER	REACTIVE POWER
33	0.8808	-0.2985	0.9300	-18.71994	-0.0380	-0.0190
34	0.9156	-0.2284	0.9437	-14.00735	-0.0000	-0.0000
35	0.9234	-0.2261	0.9507	-13.76110	-0.0600	-0.0300
36	0.9339	-0.2239	0.9604	-13.48381	-0.0000	0.0000
37	0.9433	-0.2226	0.9692	-13.27905	-0.0000	0.0000
38	0.9726	-0.2158	0.9962	-12.50962	-0.1400	-0.0700
39	0.9413	-0.2231	0.9673	-13.33143	0.0	0.0
40	0.9311	-0.2239	0.9576	-13.52021	-0.0000	-0.0000
41	0.9617	-0.2417	0.9916	-14.10970	-0.0630	-0.0300
42	0.9249	-0.2567	0.9599	-15.51075	-0.0710	-0.0440
43	0.9879	-0.1984	1.0076	-11.35830	-0.0200	-0.0100
44	0.9826	-0.2035	1.0035	-11.70159	-0.1200	-0.0180
45	1.0162	-0.1658	1.0296	-9.26670	0.0	0.0
46	1.0470	-0.2058	1.0670	-11.12099	0.0000	0.0000
47	1.0208	-0.2279	1.0459	-12.58366	-0.4730	-0.1616
48	0.9831	-0.2124	1.0057	-12.19261	0.1759	-0.0517
49	0.9974	-0.2277	1.0231	-12.85806	-0.1800	-0.0850
50	0.9859	-0.2348	1.0134	-13.39819	-0.2100	-0.1050

Table G-13: continued

BUS	VOLTAGE		MAGNITUDE	PHASE(DEGS)	REAL POWER	REACTIVE POWER
51	1.0098	-0.0210	1.0100	-1.18960	-0.0300	-0.8876
52	0.9796	-0.1029	0.9850	-5.99416	-0.0100	-0.2012
53	0.9687	-0.1482	0.9800	-8.69897	-0.7500	-0.0013
54	1.0019	-0.0789	1.0050	-4.50498	3.0000	0.4119
55	0.9663	-0.1636	0.9800	-9.60841	-1.2100	-0.2081
56	0.9981	-0.1847	1.0150	-10.48501	-0.6700	1.0768
57	1.0400	0.0	1.0400	0.0	4.2376	1.1260

Table G-14: Solution of Test System D, Line Flows

********** LINE FLOWS **********

LINE	SB	EB	REAL POWER	REACTIVE POWER
1	1	12	-0.0290	0.0016
	12	1	0.0291	-0.0013
2	1	39	-0.0380	0.0062
	39	1	0.0380	0.0244
3	2	10	-0.3039	2.2693
	10	2	0.3039	0.1497
4	2	50	0.1239	0.0895
	50	2	-0.1210	-0.0848
5	3	6	0.1242	-0.0036
	6	3	-0.1230	0.0052
6	3	29	-0.1732	-0.0184
	29	3	0.1778	0.0243
7	4	5	0.1387	-0.0456
	5	4	-0.1374	0.0237
8	4	18	0.1398	0.0275
	18	4	-0.1398	0.0964
9	4	18	0.1790	0.0159
	18	4	-0.1790	0.1034

Table G-14: continued

********** LINE FLOWS **********

LINE	SB	EB	REAL POWER	REACTIVE POWER
10	4	52	-0.6004	0.0547
	52	4	0.6047	-0.0775
11	4	53	0.1429	-0.0525
	53	4	-0.1419	0.0224
12	5	53	0.0074	-0.0636
	53	5	-0.0073	0.0520
13	6	8	-0.0770	-0.0460
	8	6	0.0786	0.0480
14	7	29	0.6040	0.1479
	29	7	-0.6040	0.9654
15	7	53	0.1766	-0.0134
	53	7	-0.1760	-0.0100
16	7	54	-0.7806	-0.1344
	54	7	0.7896	0.1613
17	8	9	-0.1196	-0.0620
	9	8	0.1228	0.0661
18	9	55	-0.1908	1.0610
	55	9	0.1908	0.1054

Table G-14: continued

********** LINE FLOWS **********

LINE	SB	EB	REAL POWER	REACTIVE POWER
19	10	55	-0.1730	0.0449
	55	10	0.1743	-0.0814
20	10	56	-0.1809	-0.2146
	56	10	0.1830	0.1912
21	11	13	-0.1021	-0.0444
	13	11	0.1024	0.0275
22	11	41	0.0922	0.0403
	41	11	-0.0922	0.0936
23	11	43	0.1364	0.0552
	43	11	-0.1364	0.5426
24	11	55	-0.1265	-0.0510
	55	11	0.1270	0.0318
25	12	40	-0.0344	0.0379
	40	12	0.0344	0.0340
26	12	41	-0.0553	0.0012
	41	12	0.0571	0.0007
27	12	42	-0.0155	0.0094
	42	12	0.0156	-0.0092

Table G-14: continued

********** LINE FLOWS **********

LINE	SB	EB	REAL POWER	REACTIVE POWER
28	13	14	-0.1039	0.1747
	14	13	0.1045	-0.1832
29	13	15	-0.4877	0.0473
	15	13	0.4945	-0.0476
30	13	49	0.3136	0.4076
	49	13	-0.3136	0.9959
31	13	55	-0.0206	-0.0249
	55	13	0.0207	-0.0139
32	13	56	0.0162	-0.6550
	56	13	-0.0089	0.6188
33	14	15	-0.6877	-0.0599
	15	14	0.6963	0.0733
34	14	46	0.4782	0.1901
	46	14	-0.4782	3.4601
35	15	45	0.3721	0.0489
	45	15	-0.3721	0.9463
36	15	52	-0.3337	0.1231
	52	15	0.3360	-0.1687

Table G-14: continued

*********** LINE FLOWS ***********

LINE	SB	EB	REAL POWER	REACTIVE POWER
37	15	57	-1.4492	-0.2476
	57	15	1.4882	0.3457
38	16	56	0.3369	-0.1011
	56	16	-0.3348	0.0885
39	16	57	-0.7669	0.0711
	57	16	0.7933	-0.0087
40	17	56	0.4950	-0.0978
	56	17	-0.4854	0.0920
41	17	57	-0.9150	0.0178
	57	17	0.9343	0.0394
42	18	19	0.0468	0.0204
	19	18	-0.0456	-0.0186
43	19	20	0.0126	0.0126
	20	19	-0.0125	-0.0124
44	20	21	-0.0105	0.0024
	21	20	0.0105	0.1092
45	21	22	-0.0105	0.0023
	22	21	0.0105	-0.0023

Table G-14: continued

*********** LINE FLOWS ***********

LINE	SB	EB	REAL POWER	REACTIVE POWER
46	22	23	0.0942	0.0176
	23	22	-0.0941	-0.0175
47	22	38	-0.1047	-0.0153
	38	22	0.1049	0.0156
48	23	24	-0.0311	-0.0035
	24	23	-0.0310	-0.0045
49	24	25	0.0686	0.0182
	25	24	-0.0686	-0.0118
50	24	25	0.0714	0.0189
	25	24	-0.0714	-0.0123
51	24	26	-0.1090	-0.0326
	26	24	0.1090	-1.5015
52	25	30	0.0770	0.0474
	30	25	-0.0758	-0.0457
53	26	27	-0.1091	-0.0332
	27	26	0.1114	0.0369
54	27	28	-0.2044	-0.0419
	28	27	0.2073	0.0463

Table G-14: continued

********** LINE FLOWS **********

LINE	SB	EB	REAL POWER	REACTIVE POWER
55	28	29	-0.2533	-0.0692
	29	28	0.2562	0.0733
56	30	31	0.0398	0.0277
	31	30	-0.0390	-0.0264
57	31	32	-0.0190	-0.0026
	32	31	0.0192	0.0029
58	32	33	0.0381	0.0191
	33	32	-0.0380	-0.0190
59	32	34	-0.0733	-0.0300
	34	32	0.0733	-0.0088
60	34	35	-0.0733	-0.0372
	35	34	0.0737	0.0350
61	35	36	-0.1337	-0.0649
	36	35	0.1347	0.0648
62	36	37	-0.1692	-0.0988
	37	36	0.1704	0.1003
63	36	40	0.0345	0.0341
	40	36	-0.0344	-0.0340

Table G-14: continued

*********** LINE FLOWS ***********

LINE	SB	EB	REAL POWER	REACTIVE POWER
64	37	38	-0.2084	-0.1248
	38	37	0.2125	0.1292
65	37	39	0.0381	0.0245
	39	37	-0.0380	-0.0244
66	38	44	-0.2422	-0.0035
	44	38	0.2439	0.0049
67	38	48	-0.1708	-0.0861
	48	38	0.1719	0.0879
68	38	49	-0.0444	-0.1252
	49	38	0.0463	0.1221
69	41	42	0.0885	0.0381
	42	41	-0.0866	-0.0348
70	41	43	-0.1164	-0.0358
	43	41	0.1164	0.0420
71	44	45	-0.3639	-0.0229
	45	44	0.3721	0.0351
72	46	47	0.4782	0.1735
	47	46	-0.4730	-0.1615

Table G-14: continued

********** LINE FLOWS **********

LINE	SB	EB	REAL POWER	REACTIVE POWER
73	48	49	0.0040	-0.1395
	49	48	-0.0024	0.1370
74	49	50	0.0897	0.0212
	50	49	-0.0890	-0.0202
75	51	52	0.9786	-0.0466
	52	51	-0.9506	0.0450
76	51	57	-1.0086	-0.8409
	57	51	1.0218	0.7497
77	53	54	-0.4248	-0.0657
	54	53	0.4312	0.0522
78	54	55	1.7792	0.1984
	55	54	-1.7477	-0.0916
79	55	56	0.0249	-0.1584
	56	55	-0.0239	0.0863

Table G-15: Solution of Test System D, DC Link Information

D C   L I N K   P A R A M E T E R S

FOR RECTIFIER

BUS NUMBER	=	47
DELAY ANGLE (DEG.)	=	11.814
DC VOLTAGE (P.U.)	=	1.17312
COMM. REACTANCE (P.U.,SPECIFIED)	=	0.10000
TRANSFORMER TAP RATIO	=	0.85889
POWER FACTOR	=	0.96701
REACTIVE POWER CONSUMED (P.U.)	=	0.04555
AC BUS VOLTAGE (P.U.)	=	1.04589
OVERLAP ANGLE (DEG.)	=	5.401
DC POWER (P.U.)	=	0.17597

FOR INVERTER

BUS NUMBER	=	48
EXTINCTION ANGLE (DEG.)	=	13.979
DC POWER (P.U.,SPECIFIED)	=	0.17590
COMM. REACTANCE (P.U.,SPECIFIED)	=	0.10000
DC VOLTAGE (P.U.)	=	1.17267
TRANSFORMER TAP RATIO	=	0.90060
POWER FACTER	=	0.95867
REACTIVE POWER CONSUMED (P.U.)	=	0.05164
AC BUS VOLTAGE (P.U.)	=	1.00574
OVERLAP ANGLE (DEG.)	=	4.765
IGNITION ADVANCE ANGLE (DEG.)	=	18.744

RESIST. OF DC LINE (P.U.,SPEC.)	=	0.00303
DC CURRENT (P.U.,SPECIFIED)	=	0.15000

Table G-16: Maximum P.U. Mismatches for Test System D (N.R. Method)

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE:

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.33344090	0.08863091	0.03046989	0.00000012	0.00000012
2.0	0.00870963	0.00942615	0.00038815	0.00000006	0.00000006
3.0	0.00002658	0.00002318	0.00000095	0.00000012	0.00000012

APPENDIX H

MAXIMUM MISMATCHES DURING LOAD FLOWS WITH ALPHA-M.Q.S.O.N.R. METHOD

Table H-1: Maximum P.U. Mismatches for Test System A (Alpha-M.Q.S.O.N.R. Method)

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00702041	0.00256771	0.00004196	0.00000018	0.00000018
2.0	0.00702441	0.00198662	0.00000095	0.00000024	0.00000018
3.0	0.00702137	0.00195575	0.00000095	0.00000018	0.00000030
4.0	0.00702107	0.00197732	0.00000095	0.00000030	0.00000024
5.0	0.00702143	0.00195152	0.00000095	0.00000018	0.00000018
6.0	0.00702566	0.00198519	0.00000095	0.00000006	0.00000030
7.0	0.00702214	0.00194860	0.00000095	0.00000006	0.00000024
8.0	0.00702316	0.00197870	0.00000095	0.00000018	0.00000036
9.0	0.00702351	0.00195324	0.00000095	0.00000012	0.00000012
10.0	0.00702375	0.00197738	0.00000095	0.00000012	0.00000024

Table H-1: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.1)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00648183	0.00245893	0.00004101	0.0	0.00000030
2.0	0.00648654	0.00203925	0.00000095	0.00000024	0.00000018
3.0	0.00648582	0.00201654	0.00000191	0.00000024	0.00000042
4.0	0.00648654	0.00204176	0.00000095	0.00000012	0.00000042
5.0	0.00648570	0.00201845	0.00000191	0.00000006	0.00000024
6.0	0.00648981	0.00204885	0.00000095	0.00000024	0.00000024
7.0	0.00648928	0.00202650	0.00000191	0.00000024	0.00000006
8.0	0.00649124	0.00205135	0.00000095	0.00000006	0.00000042
9.0	0.00648850	0.00202245	0.00000191	0.00000012	0.00000024
10.0	0.00648946	0.00204927	0.00000095	0.00000012	0.00000024

Table H-1: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.2)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00595140	0.00235558	0.00004101	0.00000006	0.00000030
2.0	0.00595134	0.00211537	0.00000095	0.00000018	0.00000018
3.0	0.00594759	0.00208938	0.00000095	0.00000018	0.00000024
4.0	0.00594538	0.00210232	0.00000095	0.00000024	0.00000012
5.0	0.00594127	0.00207275	0.00000095	0.00000018	0.00000018
6.0	0.00594687	0.00209904	0.[illegible]00095	0.00000006	0.00000024
7.0	0.00594544	0.00207567	0.00000095	0.00000012	0.00000018
8.0	0.00595140	0.00210589	0.00000095	0.0	0.00000024
9.0	0.00594658	0.00207442	0.00000095	0.00000024	0.00000036
10.0	0.00595063	0.00210071	0.00000095	0.00000006	0.00000024

Table H-1: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.3)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00542206	0.00226057	0.00003910	0.00000763	0.00000018
2.0	0.00542450	0.00220960	0.00000095	0.00003517	0.00000030
3.0	0.00541979	0.00218630	0.00000191	0.00000024	0.00000024
4.0	0.00541759	0.00219804	0.00000095	0.00000024	0.00000018
5.0	0.00541717	0.00218576	0.00000191	0.00000024	0.00000024
6.0	0.00541723	0.00220364	0.00000095	0.00000018	0.00000030
7.0	0.00540984	0.00217777	0.00000191	0.00000012	0.00000030
8.0	0.00540876	0.00219578	0.00000095	0.00000018	0.00000024
9.0	0.00540882	0.00218159	0.00000191	0.00000012	0.00000030
10.0	0.00540942	0.00219911	0.00000095	0.00000018	0.00000024

Table H-1: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.4)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00489527	0.00228971	0.00003719	0.00006533	0.00000018
2.0	0.00489706	0.00230116	0.00000191	0.00003207	0.00000030
3.0	0.00489467	0.00227833	0.00000191	0.00000018	0.00000030
4.0	0.00489557	0.00228530	0.00000095	0.00000018	0.00000018
5.0	0.00489181	0.00226384	0.00000191	0.00000024	0.00000024
6.0	0.00488967	0.00227338	0.00000095	0.00000012	0.00000012
7.0	0.00488836	0.00226146	0.00000191	0.00000024	0.00000024
8.0	0.00488722	0.00226963	0.00000095	0.00000012	0.00000012
9.0	0.00488538	0.00225198	0.00000191	0.00000030	0.00000018
10.0	0.00488698	0.00226533	0.00000095	0.00000012	0.00000024

Table H-1: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.5)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00437266	0.00240874	0.00003433	0.00012630	0.00000018
2.0	0.00000381	0.00024211	0.0	0.00000012	0.00000018
2.5	0.00000304	0.00002056	0.0	0.00000012	0.00000018

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.6)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00384957	0.00252891	0.00003052	0.00019062	0.00000018
2.0	0.00000316	0.00023353	0.0	0.00000006	0.00000030
2.5	0.00000405	0.00001216	0.0	0.00000006	0.00000030

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.7)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00333148	0.00267571	0.00002670	0.00025880	0.00000030
2.0	0.00000310	0.00022501	0.0	0.00000024	0.00000030
2.5	0.00000405	0.00001204	0.0	0.00000006	0.00000030

Table H-1: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.8)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00281358	0.00282979	0.00002289	0.00033146	0.00000030
2.0	0.00000203	0.00022668	0.0	0.00000036	0.00000030
2.5	0.00000411	0.00001198	0.0	0.00000018	0.00000030

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.9)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00229561	0.00299567	0.00001907	0.00040901	0.00000030
2.0	0.00000197	0.00022763	0.0	0.00000072	0.00000030
2.5	0.00000405	0.00001198	0.0	0.00000060	0.00000030

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00177783	0.00317931	0.00001431	0.00049192	0.00000018
2.0	0.00000221	0.00023228	0.0	0.00000113	0.00000030
2.5	0.00000501	0.00001955	0.0	0.00000095	0.00000018

Table H-1: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.1)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00177550	0.00336492	0.00000858	0.00058067	0.00000030
2.0	0.00000238	0.00023353	0.0	0.00000203	0.00000030
2.5	0.00000501	0.00001531	0.0	0.00000197	0.00000030

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.2)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00184667	0.00358313	0.00000286	0.00067621	0.00000030
2.0	0.00000286	0.00025147	0.0	0.00000340	0.00000030
2.5	0.00000703	0.00002088	0.0	0.00000340	0.00000018

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.3)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00189543	0.00402525	0.00000381	0.00077879	0.00000018
2.0	0.00000304	0.00026488	0.0	0.00000608	0.00000018
2.5	0.00000405	0.00003213	0.0	0.00000614	0.00000018

Table H-1: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.4)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00195366	0.00462903	0.00000954	0.00088960	0.00000030
2.0	0.00000310	0.00027233	0.0	0.00000995	0.00000018
2.5	0.00000405	0.00004942	0.0	0.00001037	0.00000018

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.5)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00198150	0.00528201	0.00001812	0.00100958	0.00000018
2.0	0.00000238	0.00029892	0.0	0.00001627	0.00000018
2.5	0.00000232	0.00007735	0.0	0.00001723	0.00000018

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.6)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00201499	0.00599670	0.00002575	0.00113952	0.00000030
2.0	0.00000358	0.00032032	0.0	0.00002736	0.00000018
3.0	0.00000423	0.00027877	0.0	0.00000125	0.00000030
3.5	0.00000226	0.00001770	0.0	0.00000101	0.00000030

Table H-1: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.7)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00203049	0.00676636	0.00003433	0.00128096	0.00000018
2.0	0.00000292	0.00035542	0.0	0.00004506	0.00000018
3.0	0.00000679	0.00029713	0.0	0.00000244	0.00000030
3.5	0.00000250	0.00001800	0.0	0.00000179	0.00000030

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.8)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00245714	0.00760177	0.00004387	0.00143498	0.00000018
2.0	0.00000644	0.00038999	0.00000191	0.00007540	0.00000030
3.0	0.00001234	0.00033975	0.00000191	0.00000387	0.00000018
3.5	0.00000924	0.00002360	0.00000191	0.00000304	0.00000030

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.9)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00300944	0.00850146	0.00005341	0.00160289	0.00000030
2.0	0.00000197	0.00063057	0.0	0.00012749	0.00000030
3.0	0.00000280	0.00003034	0.0	0.00000012	0.00000030

Table H-1: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00356913	0.00946325	0.00006294	0.00178653	0.00000030
2.0	0.00000358	0.00104783	0.0	0.00022095	0.00000030
3.0	0.00000566	0.00003499	0.0	0.00000006	0.00000030

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.1)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00413769	0.01049379	0.00007343	0.00198740	0.00000006
2.0	0.00000465	0.00177577	0.0	0.00039685	0.00000030
3.0	0.00000179	0.00002933	0.0	0.00000072	0.00000030

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.2)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00471818	0.01158909	0.00008392	0.00220728	0.00000030
2.0	0.00000912	0.00308238	0.00000191	0.00075197	0.00000030
3.0	0.00000936	0.00009513	0.00000191	0.00000757	0.00000018

Table H-1: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.3)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.60000002	0.08237749	0.09620857	0.00000012	0.00000036
1.0	0.00530779	0.01275620	0.00009632	0.00244772	0.00000018
2.0	0.00000197	0.00565263	0.0	0.00154018	0.00000018
3.0	0.00001091	0.00081618	0.00000191	0.00012147	0.00000030
4.0	0.00000942	0.00000632	0.00000191	0.00000006	0.00000030

Table H-2: Maximum P.U. Mismatches for Test System B (Alpha-M.Q.S.O.N.R. Method)

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00561035	0.00218794	0.02051163	0.00135380	0.00000024
2.0	0.00000048	0.00000405	0.00000006	0.00000256	0.00000012

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.1)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00482166	0.00233528	0.02107757	0.00172520	0.00000018
2.0	0.00000036	0.00000639	0.0	0.00000560	0.00000024

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.2)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00404727	0.00277346	0.02153778	0.00211340	0.00000018
2.0	0.00000072	0.00001058	0.00000012	0.00001186	0.00000030

Table H-2: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.3)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00328654	0.00381130	0.02204323	0.00252092	0.00000024
2.0	0.00000119	0.00001782	0.0	0.00002420	0.00000024

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.4)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00258428	0.00489634	0.02254963	0.00294977	0.00000018
2.0	0.00000155	0.00003281	0.00000036	0.00004900	0.00000024

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.5)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00300229	0.00603759	0.02305984	0.00340164	0.00000012
2.0	0.00000274	0.00006859	0.00000042	0.00009894	0.00000018

Table H-2: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.6)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00368756	0.00723100	0.02357864	0.00387722	0.00000024
2.0	0.00000530	0.00015951	0.00000197	0.00020200	0.00000012
3.0	0.00000018	0.00000268	0.00000012	0.0	0.00000018

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.7)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00451708	0.00848991	0.02411079	0.00437640	0.00000018
2.0	0.00000983	0.00040350	0.00000632	0.00042307	0.00000012
3.0	0.00000042	0.00000045	0.00000012	0.00000006	0.00000018

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.8)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00530893	0.00980812	0.02466011	0.00489253	0.00000012
2.0	0.00001782	0.00108883	0.00001574	0.00092500	0.00000018
3.0	0.00000024	0.00000072	0.00000006	0.00000137	0.00000018

Table H-2: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.9)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00606245	0.01119739	0.02523041	0.00531924	0.00000012
2.0	0.00002956	0.00252332	0.00003052	0.00178075	0.00000012
3.0	0.00000024	0.00001136	0.00000012	0.00001466	0.00000018

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00677508	0.01265967	0.02582169	0.00545681	0.00000012
2.0	0.00003219	0.00310471	0.00003302	0.00199050	0.00000024
3.0	0.00000031	0.00002515	0.00000024	0.00002480	0.00000006

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.1)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00744128	0.01419073	0.02643490	0.00562340	0.00000024
2.0	0.00003558	0.00378667	0.00003511	0.00222534	0.00000018
3.0	0.00000024	0.00005358	0.00000024	0.00084256	0.00000030

Table H-2: Continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.2)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00805306	0.01579624	0.02705956	0.00583303	0.00000024
2.0	0.00004231	0.00457013	0.00003630	0.00248820	0.00000012
3.0	0.00000054	0.00011300	0.00000042	0.00007433	0.00000012
4.0	0.00000066	0.00009203	0.00000012	0.00000006	0.00000012

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.3)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00859743	0.01747662	0.02768040	0.00610483	0.00000018
2.0	0.00005060	0.00545619	0.00003690	0.00278229	0.00000012
3.0	0.00000083	0.00023315	0.00000060	0.00013310	0.00000018
4.0	0.00000042	0.00000185	0.00000012	0.00000006	0.00000030

Table H-2: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.4)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00905496	0.01922315	0.02826118	0.00646704	0.00000018
2.0	0.00005943	0.00644260	0.00003678	0.00311077	0.00000018
3.0	0.00000125	0.00048120	0.00000089	0.00024539	0.00000012
4.0	0.00000024	0.00000167	0.00000012	0.00000006	0.00000012

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.5)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00939649	0.02101803	0.02873898	0.00695986	0.00000018
2.0	0.00007021	0.00751965	0.00003529	0.00347859	0.00000012
3.0	0.00000197	0.00100448	0.00000155	0.00047153	0.00000018
4.0	0.00000024	0.00000262	0.00000006	0.00000030	0.00000018

Table H-2: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.6)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.25000000	0.19999993	0.0	0.010672[illegible]7	0.00766033
1.0	0.00957662	0.02282661	0.02899647	0.00764567	0.00000018
2.0	0.00008351	0.00867692	0.00003231	0.00389284	0.00000018
3.0	0.00000376	0.00216530	0.00000256	0.00096041	0.00000018
4.0	0.00000012	0.00001407	0.00000012	0.00000376	0.00000024

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.7)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00952536	0.02458280	0.02882195	0.00862330	0.00000018
2.0	0.00010180	0.00985831	0.00002623	0.00436950	0.00000018
3.0	0.00000751	0.00502843	0.00000[illegible]07	0.00214326	0.00000018
4.0	0.00000030	0.00020825	0.00000030	0.00007308	0.00000024
5.0	0.00000030	0.00002898	0.00000018	0.00000006	0.00000018

Table H-2: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.8)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00913024	0.02615201	0.02781677	0.01005846	0.00000024
2.0	0.00015992	0.01082632	0.00003088	0.00489396	0.00000024
3.0	0.00001723	0.01167801	0.00001145	0.00476152	0.00000018
4.0	0.00001580	0.01166289	0.00001150	0.00476396	0.00000018
5.0	0.00001580	0.01166303	0.00001150	0.00476396	0.00000024
6.0	0.00001591	0.01166302	0.00001162	0.00476396	0.00000024
7.0	0.00001597	0.01166296	0.00001162	0.00476396	0.00000018
8.0	0.00001580	0.01166304	0.00001168	0.00476396	0.00000018
9.0	0.00001597	0.01166298	0.00001162	0.00476396	0.00000006
10.0	0.00001585	0.01166307	0.00001156	0.00476396	0.00000018

Table H-2: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.9)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00820410	0.03890704	0.02521420	0.01225007	0.00000024
2.0	0.00028265	0.00884502	0.00003403	0.00521225	0.00000018
3.0	0.00002402	0.01078709	0.00001460	0.00490493	0.00000018
4.0	0.00001997	0.01074473	0.00001442	0.00491208	0.00000018
5.0	0.00002015	0.01074568	0.00001460	0.00491184	0.00000012
6.0	0.00002021	0.01074557	0.00001460	0.00491196	0.00000024
7.0	0.00002015	0.01074565	0.00001454	0.00491184	0.00000012
8.0	0.00002015	0.01074558	0.00001460	0.00491184	0.00000018
9.0	0.00002009	0.01074569	0.00001460	0.00491196	0.00000012
10.0	0.00002027	0.01074526	0.00001466	0.00491196	0.00000030

Table H-2: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00773680	0.06514025	0.01941895	0.01579273	0.00000018
2.0	0.00059861	0.00445702	0.00010204	0.00591350	0.00000012
3.0	0.00003898	0.00964656	0.00002164	0.00508791	0.00000012
4.0	0.00002581	0.00952566	0.00001794	0.00510812	0.00000012
5.0	0.00002646	0.00952902	0.00001889	0.00510746	0.00000018
6.0	0.00002640	0.00952881	0.00001895	0.00510758	0.00000024
7.0	0.00002629	0.00952892	0.00001884	0.00510746	0.00000018
8.0	0.00002635	0.00952886	0.00001884	0.00510758	0.00000018
9.0	0.00002640	0.00952880	0.00001884	0.00510758	0.00000018
10.0	0.00002646	0.00952867	0.00001884	0.00510758	0.00000018

Table H-2: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.1)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.00861746	0.11396098	0.00932157	0.02204579	0.00000012
2.0	0.00180346	0.01022971	0.00037670	0.00821370	0.00000018
3.0	0.00010085	0.00801135	0.00003451	0.00534827	0.00000012
4.0	0.00003439	0.00791069	0.00002223	0.00536692	0.00000018
5.0	0.00003564	0.00791380	0.00002515	0.00536633	0.00000030
6.0	0.00003564	0.00791352	0.00002509	0.00536644	0.00000012
7.0	0.00003576	0.00791321	0.00002509	0.00536644	0.00000012
8.0	0.00003564	0.00791343	0.00002509	0.00536644	0.00000018
9.0	0.00003558	0.00791341	0.00002509	0.00536650	0.00000018
10.0	0.00003576	0.00791329	0.00002515	0.00536650	0.00000018

Table H-2: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.2)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.01866311	0.22460043	0.02224112	0.03490019	0.00003278
2.0	0.01997977	0.37875879	0.02186871	0.05139595	0.00008869
3.0	0.08304948	0.11858171	0.01711231	0.00457090	0.00000024
4.0	0.00518620	0.00565808	0.00934792	0.00787449	0.00000018
5.0	0.00073329	0.00586497	0.00027525	0.00568122	0.0
6.0	0.00004834	0.00577885	0.00003207	0.00570256	0.00000024
7.0	0.00004953	0.00578740	0.00003415	0.00570107	0.00000024
8.0	0.00004965	0.00578671	0.00003415	0.00570118	0.00000024
9.0	0.00004953	0.00578665	0.00003415	0.00570118	0.00000018
10.0	0.00004953	0.00578697	0.00003415	0.00570118	0.00000012

Table H-2: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.3)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.25000000	0.19999993	0.0	0.01067227	0.00766033
1.0	0.07212287	0.61353475	0.11526108	0.07142597	0.00012648
2.0	0.37753928	0.10478967	0.18859577	0.01510763	0.00044626
3.0	2.63076878	5.45419788	4.64019775	0.35071045	0.00944036
4.0	1.51791573	1.89185715	2.45224762	0.17998922	0.00673085
5.0	0.17361474	0.71484983	0.22169590	0.08491468	0.00000018
6.0	0.11704749	0.00956633	0.01256859	0.00000012	0.00000018
7.0	0.00219887	0.01696402	0.00458956	0.00000006	0.00000024
8.0	0.00000352	0.00326288	0.00000024	0.0	0.00000018
9.0	0.00000286	0.00101335	0.00000012	0.00000012	0.00000018
10.0	0.00000280	0.00066318	0.00000012	0.00000006	0.00000024

Table H-3: Maximum P.U. Mismatches for Test System C (Alpha-M.Q.S.O.N.R. Method)

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.02893180	0.04135370	0.00337219	0.00000024	0.00000048
2.0	0.00000739	0.00419271	0.00000286	0.00000036	0.00000048
3.0	0.00001085	0.00340092	0.00000191	0.00000060	0.00000048
4.0	0.00000751	0.00307024	0.00000286	0.00000018	0.00000012
5.0	0.00001180	0.00276554	0.00000286	0.00000048	0.00000018
6.0	0.00001234	0.00249487	0.00000286	0.00000012	0.00000012
7.0	0.00001460	0.00224215	0.00000191	0.00000012	0.0
8.0	0.00001997	0.00204813	0.00000191	0.00000024	0.00000024
9.0	0.00002450	0.00186127	0.00000191	0.00000024	0.00000030
10.0	0.00002360	0.00169033	0.00000191	0.00000024	0.00000024

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.1)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.02677882	0.03969610	0.00321198	0.00000054	0.00000048
2.0	0.00000989	0.00399172	0.00000286	0.00000012	0.00000048
3.0	0.00001067	0.00322217	0.00000191	0.00000006	0.00000048
4.0	0.00001711	0.00292665	0.00000191	0.00000036	0.00000048
5.0	0.00000882	0.00264841	0.00000095	0.00000018	0.00000018
6.0	0.00002086	0.00237685	0.00000286	0.00000012	0.00000036
7.0	0.00001776	0.00213575	0.00000095	0.00000018	0.00000042
8.0	0.00002676	0.00191391	0.00000191	0.00000006	0.00000012
9.0	0.00002235	0.00170749	0.00000191	0.0	0.00000018
10.0	0.00003183	0.00153369	0.00000286	0.00000012	0.00000060

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.2)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.02463937	0.03797364	0.00301456	0.00000018	0.00000048
2.0	0.00000781	0.00382233	0.00000286	0.00000030	0.00000048
3.0	0.00001979	0.00310385	0.00000191	0.00000012	0.00000048
4.0	0.00001776	0.00281703	0.00000095	0.00000006	0.00000054
5.0	0.00002015	0.00254488	0.00000286	0.00000048	0.00000024
6.0	0.00001591	0.00229955	0.00000191	0.00000036	0.00000012
7.0	0.00002009	0.00208324	0.00000191	0.00000030	0.00000024
8.0	0.00001800	0.00186634	0.00000191	0.00000036	0.00000048
9.0	0.00003082	0.00166065	0.00000191	0.00000042	0.00000060
10.0	0.00002944	0.00149435	0.00000191	0.00000048	0.00000066

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.3)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.02250141	0.03615886	0.00278473	0.00000048	0.00000048
2.0	0.00001061	0.00369167	0.00000286	0.00000018	0.00000048
3.0	0.00000644	0.00296676	0.0000019[illegible]	0.00000072	0.00000048
4.0	0.00001389	0.00267595	0.00000191	0.00000024	0.00000054
5.0	0.00001264	0.00241548	0.00000191	0.00000066	0.00000030
6.0	0.00002098	0.00218767	0.00000191	0.00000024	0.00000054
7.0	0.00002384	0.00197852	0.00000095	0.00000066	0.00000006
8.0	0.00003731	0.00177610	0.00000191	0.00000006	0.00000030
9.0	0.00003767	0.0015832[illegible]	0.00000095	0.00000018	0.00000054
10.0	0.00002879	0.00143784	0.00000191	0.00000042	0.00000024

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.4)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.02036399	0.03433406	0.00251865	0.00000048	0.00000048
2.0	0.00000638	0.00356734	0.00000191	0.00000018	0.00000048
3.0	0.00001180	0.00287098	0.00000191	0.00000006	0.00000048
4.0	0.00001073	0.00259674	0.00000286	0.00000036	0.00000006
5.0	0.00001556	0.00234354	0.00000191	-0.00000030	0.00000072
6.0	0.00001484	0.00211638	0.00000286	0.00000018	0.00000095
7.0	0.00001603	0.00192279	0.00000191	0.00000036	0.00000018
8.0	0.00001030	0.00173277	0.00000286	0.00000012	0.00000054
9.0	0.00001436	0.00155407	0.00000191	0.00000024	0.00000036
10.0	0.00002158	0.00141627	0.00000286	0.00000066	0.00000060

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.5)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.01872796	0.03261226	0.00221348	0.00000006	0.00000048
2.0	0.00000703	0.00345677	0.00000286	0.00000006	0.00000048
3.0	0.00001794	0.00280386	0.00000286	0.0	0.00000089
4.0	0.00001568	0.00252080	0.00000191	0.00000030	0.00000030
5.0	0.00001580	0.00228512	0.00000286	0.00000024	0.00000083
6.0	0.00002635	0.00207835	0.00000191	0.00000030	0.00000018
7.0	0.00003636	0.00187147	0.00000286	0.00000024	0.00000012
8.0	0.00003588	0.00168788	0.00000191	0.0	0.00000018
9.0	0.00004607	0.00152147	0.00000286	0.0	0.0
10.0	0.00005215	0.00138372	0.00000191	0.0	0.00000036

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.6)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.01812935	0.03115958	0.00187206	0.00000048	0.00000089
2.0	0.00000751	0.00340968	0.00000191	0.00000024	0.00000048
3.0	0.00001836	0.00275773	0.00000191	0.00000018	0.00000048
4.0	0.00002158	0.00247961	0.00000286	0.00000018	0.00000012
5.0	0.00002867	0.00221920	0.00000191	0.00000036	0.00000072
6.0	0.00002784	0.00201291	0.00000095	0.00000018	0.00000048
7.0	0.00003558	0.00182676	0.00000191	0.00000036	0.00000024
8.0	0.00004119	0.00166547	0.00000286	0.0	0.00000006
9.0	0.00004786	0.00150698	0.00000191	0.00000018	0.00000054
10.0	0.00004923	0.00137532	0.00000191	0.00000030	0.00000083

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.7)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.01801169	0.03027296	0.00148773	0.00000030	0.00000048
2.0	0.01796657	0.02308077	0.00004101	0.00000018	0.00000048
3.0	0.01799482	0.01989794	0.00004292	0.00000024	0.00000054
4.0	0.01797467	0.01703763	0.00004292	0.00000042	0.00000054
5.0	0.01799244	0.01445287	0.00004387	0.00000006	0.00000018
6.0	0.00000924	0.00000733	0.00000191	0.00000066	0.00000024

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.8)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.01791656	0.03028744	0.00105858	0.00000018	0.00000048
2.0	0.01788825	0.02297139	0.00002766	0.00000036	0.00000048
3.0	0.01789689	0.01975375	0.00002861	0.00000012	0.00000012
4.0	0.01788926	0.01698661	0.00002766	0.0	0.00000030
5.0	0.01789463	0.01442802	0.00003052	0.00000036	0.00000012
6.0	0.01789093	0.01209742	0.00003052	0.00000030	0.00000048
7.0	0.01790339	0.01001608	0.00003052	0.00000066	0.00000030
8.0	0.01790947	0.00808233	0.00003147	0.00000036	0.00000030
9.0	0.01791835	0.00639558	0.00003338	0.00000036	0.00000083
10.0	0.01792026	0.00479490	0.00003147	0.00000024	0.00000012

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.9)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.01788372	0.03176457	0.00067806	0.00000006	0.00000048
2.0	0.01786822	0.02406347	0.00001717	0.00000030	0.00000048
3.0	0.01786041	0.02078515	0.00001717	0.00000018	0.00000024
4.0	0.01787174	0.01791668	0.00001717	0.00000006	0.00000054
5.0	0.01787043	0.01526147	0.00001812	0.00000018	0.00000024
6.0	0.01787060	0.01288259	0.00001621	0.00000006	0.00000054
7.0	0.01786786	0.01072222	0.00001907	0.00000006	0.00000060
8.0	0.01787430	0.00877589	0.00001812	0.00000006	0.00000036
9.0	0.01787102	0.00704157	0.00002003	0.00000006	0.00000072
10.0	0.01787621	0.00543642	0.00001907	0.00000048	0.00000030

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.01793659	0.03527588	0.00106621	0.00000054	0.00000048
2.0	0.01793927	0.02673703	0.00000954	0.00000024	0.00000048
3.0	0.01791835	0.02317947	0.00000858	0.00000036	0.00000066
4.0	0.01793110	0.02013075	0.00000763	0.00000036	0.00000024
5.0	0.01792294	0.01724333	0.00000858	0.00000018	0.00000036
6.0	0.01792669	0.01471210	0.00000763	0.00000018	0.00000107
7.0	0.01791805	0.01237631	0.00000858	0.00000018	0.00000006
8.0	0.01792258	0.01029760	0.00000858	0.0	0.00000060
9.0	0.01790506	0.00840831	0.00000858	0.00000018	0.00000066
10.0	0.01791793	0.00668114	0.00000954	0.0	0.00000060

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.1)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.01811308	0.04164743	0.00151825	0.0	0.00000048
2.0	0.00000745	0.00533444	0.00000286	0.00000012	0.00000048
3.0	0.00001597	0.00431532	0.00000286	0.00000012	0.00000089
4.0	0.00002503	0.00391644	0.00000191	0.00000006	0.00000054
5.0	0.00002348	0.00354117	0.00000191	0.00000018	0.00000072
6.0	0.00002229	0.00319427	0.00000095	0.00000006	0.00000030
7.0	0.00001979	0.00287229	0.00000191	0.00000006	0.00000083
8.0	0.00002253	0.00260520	0.00000191	0.0	0.00000042
9.0	0.00001890	0.00234330	0.00000191	0.00000012	0.00000060
10.0	0.00001872	0.00211817	0.00000191	0.00000018	0.00000030

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.2)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PCR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.01843798	0.05177557	0.00217724	0.0	0.00000048
2.0	0.00000572	0.00665629	0.00000191	0.00000036	0.00000048
3.0	0.00001520	0.00538522	0.00000095	0.00000024	0.00000048
4.0	0.00002486	0.00484818	0.00000191	0.00000018	0.00000107
5.0	0.00002760	0.00438231	0.00000095	0.00000018	0.0
6.0	0.00002927	0.00396395	0.00000191	0.0	0.00000060
7.0	0.00003308	0.00360399	0.00000191	0.00000018	0.00000048
8.0	0.00003356	0.00326455	0.00000191	0.00000048	0.00000054
9.0	0.00003004	0.00294298	0.00000191	0.00000012	0.00000072
10.0	0.00003386	0.00264025	0.00000095	0.00000030	0.0

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.3)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.01895654	0.06680143	0.00290012	0.0	0.00000048
2.0	0.00000751	0.00854921	0.00000191	0.0	0.00000048
3.0	0.00001228	0.00690955	0.00000095	0.0	0.00000048
4.0	0.00001448	0.00623715	0.00000095	0.00000012	0.00000072
5.0	0.00001687	0.00563395	0.00000095	0.00000006	0.00000042
6.0	0.00002563	0.00509250	0.00000095	0.00000030	0.00000030
7.0	0.00001878	0.00458652	0.00000191	0.00000030	0.00000018
8.0	0.00001580	0.00413483	0.00000191	0.00000012	0.00000024
9.0	0.00001242	0.00374228	0.00000191	0.0	0.00000012
10.0	0.00002164	0.00339764	0.00000191	0.00000018	0.00000066

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.4)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.01969516	0.08798259	0.00369358	0.0	0.00000048
2.0	0.00001031	0.01124716	0.00000191	0.00000012	0.00000048
3.0	0.00001585	0.00912374	0.00000191	0.00000036	0.00000048
4.0	0.00001782	0.00825065	0.00000095	0.00000018	0.00000072
5.0	0.00001454	0.00745362	0.00000191	0.00000006	0.0
6.0	0.00001150	0.00672680	0.00000191	0.00000024	0.00000048
7.0	0.00001401	0.00607377	0.00000095	0.0	0.00000077
8.0	0.00002950	0.00549060	0.00000286	0.00000018	0.00000083
9.0	0.00002885	0.00497085	0.00000191	0.00000018	0.00000042
10.0	0.00004315	0.00448465	0.00000191	0.00000042	0.00000024

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.5)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.02068359	0.12358415	0.00455379	0.0	0.00000030
2.0	0.00001115	0.00937444	0.00000095	0.00000030	0.00000048
3.0	0.00001502	0.00758946	0.00000191	0.0	0.00000089
4.0	0.00001544	0.00685400	0.00000095	0.00000018	0.00000048
5.0	0.00001138	0.00619352	0.00000191	0.00000018	0.00000060
6.0	0.00002313	0.00558573	0.00000191	0.0	0.00000012
7.0	0.00001734	0.00504911	0.00000095	0.00000024	0.00000060
8.0	0.00003189	0.00456309	0.00000191	0.00000012	0.00000030
9.0	0.00003129	0.00413609	0.00000095	0.00000012	0.00000006
10.0	0.00004536	0.00375038	0.00000191	0.00000006	0.00000060

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.6)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.02527082	0.17458928	0.00566292	0.0	0.00000036
2.0	0.00004572	0.00534517	0.00000095	0.00000006	0.00000048
3.0	0.00004905	0.00432616	0.00000286	0.00000018	0.00000048
4.0	0.00005227	0.00392044	0.00000191	0.00000006	0.00000048
5.0	0.00004703	0.00352639	0.00000286	0.00000006	0.00000072
6.0	0.00004858	0.00317550	0.00000095	0.00000024	0.00000048
7.0	0.00005203	0.00287694	0.00000095	0.00000012	0.00000095
8.0	0.00005394	0.00260478	0.00000095	0.00000018	0.00000012
9.0	0.00005633	0.00237852	0.00000191	0.00000030	0.0
10.0	0.00005817	0.00215584	0.00000286	0.00000054	0.00000066

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.7)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.03722054	0.23961186	0.00722313	0.0	0.00000048
2.0	0.00017840	0.01700336	0.00000477	0.00000018	0.00000077
3.0	0.00018752	0.01531398	0.00000191	0.00000018	0.00000089
4.0	0.00017554	0.01379740	0.00000191	0.00000006	0.00000042
5.0	0.00018370	0.01245183	0.00000191	0.00000012	0.00000006
6.0	0.00017852	0.01123410	0.00000191	0.0	0.00000018
7.0	0.00018066	0.01012218	0.00000191	0.00000024	0.00000012
8.0	0.00017095	0.00910556	0.00000286	0.00000042	0.00000024
9.0	0.00017238	0.00820392	0.00000191	0.00000006	0.00000024
10.0	0.00016832	0.00738877	0.00000191	0.0	0.00000030

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.8)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.05208731	0.31858557	0.00900173	0.00000042	0.00000054
2.0	0.00082368	0.06064147	0.00001431	0.00000018	0.00000036
3.0	0.00000829	0.00081366	0.00000191	0.00000012	0.00000054
3.5	0.00000954	0.00008440	0.00000191	0.00000012	0.00000066

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.9)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.07026058	0.39370632	0.01095200	0.0	0.00000054
2.0	0.00217837	0.12750298	0.00003242	0.0	0.00000066
3.0	0.00003076	0.00604516	0.00000286	0.0	0.00000036
4.0	0.00002772	0.00539166	0.00000191	0.00000042	0.0
5.0	0.00003099	0.00483137	0.00000095	0.00000006	0.00000036
6.0	0.00002193	0.00429630	0.00000191	0.0	0.00000042
7.0	0.00002092	0.00384361	0.00000191	0.0	0.00000018
8.0	0.00001675	0.00342238	0.00000191	0.00000012	0.00000006
9.0	0.00001335	0.00305325	0.00000286	0.00000006	0.00000054
10.0	0.00001556	0.00272232	0.00000191	0.00000018	0.00000107

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.09208047	0.48450649	0.01285172	0.00176668	0.00000006
2.0	0.00534827	0.24246895	0.00007057	0.0	0.00000006
3.0	0.00036407	0.04198092	0.00000572	0.00000048	0.00000072
4.0	0.00043952	0.03690463	0.00000191	0.00000042	0.00000060
5.0	0.00037903	0.03325325	0.00000191	0.00000012	0.00000048
6.0	0.00042468	0.02932107	0.00000095	0.00000018	0.00000054
7.0	0.00038546	0.02636230	0.00000286	0.00000012	0.00000054
8.0	0.00040656	0.02328813	0.00000191	0.00000024	0.00000018
9.0	0.00038487	0.02091277	0.00000095	0.00000024	0.00000006
10.0	0.00040364	0.01851934	0.00000191	0.0	0.00000018

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.1)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.11786211	0.61209488	0.01368332	0.01939762	0.00000006
2.0	0.01150107	0.40892398	0.00011444	0.0	0.00000054
3.0	0.00228941	0.17131460	0.00002289	0.00000042	0.00000018
4.0	0.00016028	0.02352756	0.00000381	0.0	0.00000036
5.0	0.00018132	0.02073789	0.00000191	0.0	0.00000036
6.0	0.00016397	0.01865619	0.00000191	0.00000006	0.00000060
7.0	0.00018108	0.01649505	0.00000095	0.00000018	0.00000018
8.0	0.00016719	0.01480222	0.00000191	0.00000024	0.00000018
9.0	0.00017095	0.01309276	0.00000191	0.00000012	0.00000072
10.0	0.00016850	0.01175171	0.00000191	0.00000012	0.00000066

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.2)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.44171846	3.78103256	0.03112030	0.12432271	0.00000054
2.0	0.04016417	0.60680610	0.00095463	0.00576061	0.00000054
3.0	0.00830591	0.43120950	0.00004864	0.00000042	0.00000006
4.0	0.00340825	0.23092401	0.00003433	0.0	0.00000054
5.0	0.00055331	0.06151515	0.00000763	0.0	0.00000083
6.0	0.00001234	0.00286239	0.00000191	0.00000042	0.00000018
7.0	0.00001454	0.00253016	0.00000095	0.0	0.00000060
8.0	0.00001848	0.00225967	0.00000095	0.00000054	0.00000018
9.0	0.00002289	0.00199592	0.00000191	0.00000012	0.00000060
10.0	0.00002152	0.00179303	0.00000191	0.00000012	0.00000030

Table H-3: continued

THE REQUIRED ACCURACY WAS 0.00010

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 2.3)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	0.94199997	0.31972605	0.18809986	0.00000006	0.00000036
1.0	0.21691346	3.17039680	0.07389927	0.13817358	0.00000095
2.0	90.3940582	311.237549	12.1884909	2.54718304	0.45037246
3.0	147.400650	326.155029	47.7430267	1.89806175	0.14040726
4.0	575.789062	1364.42358	67.0831604	2.30134773	4.76142120
5.0	1027.87061	2734.74927	185.531326	1.82776070	7.15369129
6.0	205.070526	583.325684	34.3058319	0.18470383	3.05787945
7.0	44.8822327	124.550934	5.35421658	0.00955981	1.06998539
8.0	193.816528	561.489258	7.04989338	3.18460560	0.07454765
9.0	7388.43359	26245.7461	103.798416	24.1378174	2.58178902
10.0	1376.79785	4877.47266	29.3840942	10.1113777	1.05452728

Table H-4: Maximum P.U. Mismatches for 57 Bus System (Alpha-M.Q.S.Q.N.R. Method)

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.03799820	0.06913525	0.00403047	0.0	0.00000018
2.0	0.00001485	0.00003039	0.00000191	0.00000024	0.00000030

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.1)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.03660774	0.05968871	0.00404888	0.00000006	0.00000024
2.0	0.00001299	0.00003449	0.00000095	0.00000012	0.00000024

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.2)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.03758234	0.05069891	0.00404549	0.00000018	0.00000018
2.0	0.00001955	0.00003089	0.00000191	0.0	0.00000012

Table H-4: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.3)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.03917456	0.04211516	0.00402737	0.00000012	0.00000030
2.0	0.00001746	0.00003157	0.00000191	0.00000018	0.00000006

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.4)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.04058498	0.03394242	0.00398445	0.00003046	0.00000024
2.0	0.00001542	0.00002209	0.00000286	0.0	0.00000024

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.5)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.04183650	0.02845360	0.00391674	0.00104880	0.00000024
2.0	0.00001941	0.00003303	0.00000191	0.00001037	0.00000018

Table H-4: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.6)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.04289687	0.02363753	0.00382328	0.00137490	0.00000024
2.0	0.00001878	0.00005892	0.00000191	0.00002521	0.00000006

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.7)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.04378515	0.01890737	0.00370693	0.00155127	0.00000030
2.0	0.00001472	0.00006727	0.00000191	0.00002724	0.00000012

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.8)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA PDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.04448688	0.01520540	0.00356770	0.00176066	0.00000018
2.0	0.00001149	0.00007413	0.00000191	0.00002939	0.00000024

Table H-4: continued

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 0.9)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.04496545	0.02529636	0.00340176	0.00201297	0.00000024
2.0	0.00002028	0.00008034	0.00000191	0.00003183	0.00000024

THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WERE: (ALPHA = 1.0)

ITERATION	DELTA P	DELTA Q	DELTA V**2	DELTA PDR	DELTA FDI
0.0	3.00000000	1.71975040	0.03960001	0.00000006	0.00000030
1.0	0.04678822	0.01643609	0.00322342	0.00155938	0.00000018
2.0	0.00001085	0.00005789	0.00000191	0.00003451	0.00000030