AN INVESTIGATION OF THE VAN HIELE LEVELS OF THINKING IN GEOMETRY AT THE BEGINNING OF THE NINTH GRADE

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by

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ABSTRACT

This study was motivated by the controversy which exists over the approach to teaching geometry in grade nine. Which approach should be used to teach geometry at this grade level: inductive or deductive? This has been an issue of concern in Newfoundland and Labrador during the last five years.

The main purpose of the study was to investigate the level of thinking of grade nine students in geometry at the beginning of the school year. A second important aspect of the study was related to the text materials used to teach the geometry strand in grade eight. Also, it was attempted to determine if the mental development of grade nine students in geometry in Newfoundland and Labrador differed from those of students in the United States.

The sample consisted of 1 004 grade nine students at the beginning of the school year in Newfoundland and Labrador. However, 75 students were eliminated from the sample because they were repeating grade nine (46) or using an alternative textbook series (29). Consequently, 929 students were included for data analysis.

The students were required to provide information relative to their grade last year, the textbook used to study geometry in grade eight, and placement in grade nine this school year: advanced, academic, or practical. This information was utilized in data analysis.

The students were administered a modified version of the van Hiele Geometry Test. This test included four levels of multiple-choice

questions: Recognition, Analysis, Ordering, and Deduction. There were five questions at each level for a total of 20 questions.

The students were classified according to the van Hiele theory of mental development in geometry. Each student was assigned a level: Recognition, Analysis, Ordering, or Deduction. It was possible to classify 88.7 percent of the sample using a criterion of 3 out of 5 items correct at each level. When a criterion of 4 out of 5 items correct at each level was applied, it was possible to classify 95 percent into a van Hiele level. In the case of 3 out of 5, the majority of students were at the recognition and analysis levels of the van Hiele theory. In the case of 4 out of 5, the majority of students were at the recognition level or below recognition level. The major finding of the investigation was that students at the beginning of grade nine are not prepared for deductive reasoning according to the van Hiele model.

The second major conclusion related to the textbooks used to teach geometry in grade eight and the level of thinking of students at the beginning of grade nine. It was found that the level of thinking of students in geometry at the beginning of grade nine was independent of the textbook used for geometry instruction in grade eight when a criterion of 3 out of 5 was applied. However, a significant difference was found when the 4 out of 5 criterion was used. The level of thinking in geometry was dependent on the text used for geometry instruction.

The third major conclusion related to homogeneous populations and levels of mental development in geometry. There was a significant difference in the level of mental development in geometry of grade nine students in Newfoundland and Labrador and those in the United States. The level of cognitive development in geometry was higher for students in the United States.

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CHAPTER I

STATEMENT OF THE PROBLEM

> To some minds this "urge" comes early; to others it comes late; to a few it comes not at all. Until it comes, however, the pupil can profit but little from the study of demonstrative geometry. (Reeve, 1930, p. 11)

Reeve (1930) identified two types of geometry: informal, which includes intuitive and experimental geometries, and demonstrative geometry which is a desire or "urge" to prove. Fifty years ago, mathematics educators were interested in developing a "prognostic test in demonstrative geometry" to determine who could profit by deductive thinking (p. 14).

When do students reach a stage in their mental development where they can reason deductively? Piaget indicated a transition between the stage of concrete operations and the formal operational stage. He stated that the child enters the stage of adult reasoning or formal operations near the age of eleven or twelve. (Adler, 1971, p. 214)

Deductive reasoning is considered to be a sophisticated behaviour. This ability is the final form of all mathematical reasoning. The geometry component in senior high school mathematics courses requires deductive thinking. Students are expected to be able to write a formal proof or synthesize. Usiskin (1982) stated: "Geometry proof is a high level task which would seem to make cognitive demands in the areas of spatial reasoning, abstract reasoning, and problem solving" (p. 88).

Proof-making is considered a complex terminal behaviour which is built on a number of prerequisite skills. These skills are hierarchial and require a considerable amount of instructional time. There is general agreement among mathematics educators (Hendrix, 1961; Allendoerfer, 1969; Kane, 1975; Hiatt, 1979; Hoffer, 1981; Brown, 1982) that inductive conjecturing is a prerequisite to formal proof writing or deductive reasoning.

There is some concensus among mathematics educators that geometry is a basic skill at the junior high school level (Sanders and Dennis, 1968; Peterson, 1973; Sherard, 1981). In grades seven and eight, the geometry strand is informal and inductive. However, there is some uncertainty at the ninth grade level regarding the approach to the teaching of geometric concepts and properties. What reasoning should be used in geometry with these students -- inductive or deductive? It seems that some students in this grade go through a transitory period from inductive to deductive reasoning. In the ninth grade, the geometry studied should reflect the student's mental development as it relates to geometric thinking. Despite their chronological age, the mental development of many ninth graders may not be sufficient to reason deductively. The degree of emphasis given to the teaching of informal geometry in earlier grades is a critical variable in terms of student success in grade nine.

In Newfoundland and Labrador, the geometry component of junior high school mathematics has been a focal point for much discussion since September 1981. There have been two main issues: (i) the approach used in ninth grade geometry - inductive and/or deductive; and (ii) the quality of informal geometry being taught before grade nine, especially grades seven and eight. An overview of the current junior high mathematics curriculum, as well as some background, is necessary.

The junior high mathematics curriculum currently has two distinct structures, one designed for grades seven and eight and another for grade nine. In grades seven and eight, there are core units at each grade level with enrichment and introductory topics. There are two units of geometry in the core for each grade. In grade nine, there are three courses designed for different levels of mathematical ability: advanced, academic, and practical. In the advanced and academic courses, geometry is 50 percent of the prescribed mathematics curriculum with the major difference being in the approach.

The advanced course was designed for students with above average ability in mathematics. The students are introduced to an abstract mathematical system early in the ninth grade. The treatment of geometry is rigorous and proof-oriented. Students are expected to prove triangles congruent by the end of grade nine. The textbook, Geometry (Moise and Downs, 1975), authorized for this course was written for tenth grade students in the United States who study geometry in a single year. At the end of the ninth grade, the attrition rate in a number of classes is above what might be considered normal.

The academic course was designed for students with average ability in mathematics. The course is studied by the majority of students in Newfoundland and Labrador and in many all-grade schools, it is the only course offered. The geometry portion of this course uses the inductive approach to establish geometric concepts and properties of geometric figures. The new text authorized for academic students is Math Is/ Geometry, Book 1 (Ebos et al, 1981).

In September 1979, two text series were authorized for grades seven and eight: School Mathematics and Math Is. When these texts were implemented, schools had the option to select one or the other depending on the ability of the students and the grouping policy of the school - homogeneous versus heterogeneous. At that time, it was suggested by pilot teachers that the Math Is series was more suited for students with average or above average ability in mathematics. Also, it was suggested that the School Mathematics series was more appropriate for average or below average ability in mathematics. There are some differences in the textbook series which have had an impact on their use in the classroom.

The School Mathematics series was written in the United States in the early 1970's. However, in 1974, these texts were Canadianized and metricated for use in Canadian schools. The series is a continuation of the mathematics program used in the elementary school with the same teaching strategies being recommended. The texts were teacher-oriented with a detailed teacher's edition. As well, the texts seemed more appropriate for heterogeneous grouping in mathematics or the all-grade schools since the texts contain graded exercises. Consequently, in 1979, the majority of schools adopted the School Mathematics series.

The Math Is series was written in Canada by indigenous authors in the mid-1970's. The series extends from grades seven to twelve and uses a 'problem solving' approach. The texts are student-oriented with very little direction for the teacher. The first edition of these books did not contain graded exercises. It was adopted in very few schools and mainly by those which had a homogeneous grouping policy in grades seven and eight.

It is important to note the location of the geometry in the two textbook series. In _School Mathematics_, the geometry modules are located at the end of the textbooks. In _Math Is_, the geometry chapters are integrated throughout the textbooks. If mathematics is taught using the textbook approach, there may not be enough instructional time to teach geometry in _School Mathematics_.

Since September 1981, with the implementation of _Math Is/Geometry, Book 1_, there has been much discussion on the suitability of the geometry content in _School Mathematics_ and _Math Is_ as a prerequisite to grade nine academic mathematics. Teachers have found the geometry in _Math Is_ to be more applicable as a prerequisite. Many schools have moved to implement the _Math Is_ texts in grades seven and eight. Currently, approximately 70 percent of students are using _Math Is_ in this province. This series is being used in both homogeneous and heterogeneous groups as well as all-grade schools.

In summary, the geometry portion of the academic mathematics course at the grade nine level was modified to accommodate teachers' concerns that the previous course was too proof-oriented. This shift from deductive reasoning to the inductive approach at this grade level has been controversial. There are some teachers who feel that deductive reasoning should remain in grade nine despite their students having limited success. The introduction of a new geometry course in grade nine has raised fundamental questions relating to the student's mental development in geometry and the quality of geometry taught in the previous grade using different textbooks.

I. PURPOSE OF THE STUDY

Geometry is part of the core requirements in the junior high mathematics program in Newfoundland and Labrador. Students should be provided with experiences which are appropriate to their level of thinking. In this study answers to three questions relating to students' mental development in the ninth grade are sought.

Question 1: Are students at the beginning of the ninth grade prepared for deductive reasoning?

Question 2: Are there any differences between the van Hiele levels of mental development in geometry of grade nine students who were taught using the textbooks: Math Is/2 and School Mathematics 2?

Question 3: Are there any differences between the van Hiele levels of mental development in geometry of grade nine students in Newfoundland and Labrador and those of students in the United States?

II. LIMITATIONS OF THE STUDY

This study was conducted in the province of Newfoundland and Labrador in the fall of 1983. The following delimitations were imposed:

1. Students sampled were enrolled in grade nine for the first time in September 1983. Students repeating grade nine were eliminated from the sample.

2. Students sampled were taught geometry in grade eight from <u>School Mathematics</u> 2 or <u>Math Is/2</u>. Students taught using other textbooks were eliminated from the sample.

3. The study did not attempt to determine the amount of instructional time spent on geometry in grade eight.

4. The researcher is the Mathematics Consultant with the Department of Education.

III. SIGNIFICANCE OF THE STUDY

Over the next two years, the junior high school curriculum in Newfoundland and Labrador will be reorganized. There are a number of implications for mathematics, especially at the grade nine level. Hence, this research should have an impact on the design of the mathematics program in junior high school and, in particular, the geometry component. The level of thinking in geometry should be an important consideration in curriculum development.

Freudenthal (1973) examined the role of geometry in the mathematics curriculum. He stated that: "Geometry is not only deductivity" (p. 402). He suggested that some students will never build deductive systems, but they must still learn mathematics. He maintained that some are pushed to a higher level in the learning process too early and aided by algorithms (p. 416).

Hoffer (1981) suggested that geometry includes more than proof. He identified five skills in geometry - visual, verbal, drawing, logical, and applied. Further, he maintained that informal activities and investigations in each of these skill areas would be beneficial before writing out a proof. He suggested that formal proof be postponed until the students are prepared to work with a deductive system.

> ... by beginning formal proofs too early in a geometry course, we may not account for those students who have not yet reached a sufficiently high level of mental development to enable them to function adequately at the formal level. (Hoffer, 1981, p. 14)

In September 1983, the Department of Education, Division of Instruction (Curriculum), appointed a Junior High Mathematics Curriculum Committee. One of the terms of reference is to develop a teaching guide for junior high school mathematics. How much geometry should be included at each grade level? What teaching approaches should be used at each grade level?

The authorization of textbooks is also of considerable significance. The most appropriate text materials should be used in developing geometric concepts and principles. The geometry component in the various texts should match the student's level of thinking. Should textbooks authorized for grade nine students include material on deductive reasoning?

In summary, the junior high mathematics is being revised and restructured to consist of grades seven to nine, a new junior high mathematics curriculum guide is being developed and new textbooks will be authorized for September 1984. Hence, the major significance of this study is using the results in curriculum development at the junior high school level.

IV. THE EXPERIMENTAL SETTING

The following is an overview of the experimental design. A more detailed account is reported in Chapter III.

The population from which the sample was drawn consisted of all grade nine students in Newfoundland and Labrador. Twenty schools were randomly selected giving a sample of 1004 students. However, there were 46 students repeating grade nine who were eliminated from the sample. Also, there were 29 students who studied mathematics in grade eight using an alternate textbook. Consequently, only 929 students were suitable for data analysis.

The students were administered a modified version of the van Hiele Geometry Test in September 1983 (Appendix B). The test contained 20 multiple-choice items based on writings of the van Hieles. The items required different levels of mental development in geometry: Recognition, Analysis, Ordering, and Deduction.

Data were collected with respect to the students' status at the time of testing. They were asked to indicate their grade last year, the textbook used to study geometry in grade eight, and their placement in grade nine mathematics courses. These data are summarized in tables in Chapter IV.

V. OUTLINE OF REPORT

A review of related research is presented in Chapter II. In Chapter III, the design of the study, the instrumentation, testing procedures, and methods used to analyze the data are discussed. The results of the data analyses, interpretations and conclusions are contained in Chapter IV. In the final chapter, a summary of the study, a discussion of the results and implications, and suggestions for further research are provided.

CHAPTER II

REVIEW OF RELATED RESEARCH

This chapter contains a review of related literature on the van Hiele theory of mental development in geometry. A short historical overview of the theory is presented. The theory and its properties are described in detail. A summary of three major research projects in the United States on the theory is presented. Finally, implications of the theory are discussed.

I. HISTORICAL OVERVIEW OF THE VAN HIELE THEORY

The van Hiele theory was developed in the late 1950's by two high school teachers, Dina van Hiele-Geldof and her husband Pierre Marie van Hiele in the Netherlands. P. van Hiele (1957) formulated the scheme and psychological principles, while D. van Hiele-Geldof (1957) focused on the didactics experiments to raise a student's thought level. (Hoffer, 1982, p. 4)

Freudenthal (1973) described the theory in some detail, especially the work of D. van Hiele-Geldof. Hence, it was brought to the attention of mathematics educators in Western Europe. However, the theory received little attention in North America, in particular the United States, until the mid-1970's. Wirszup (1974) formally introduced the van Hiele theory to American audiences. He described breakthroughs in the teaching of geometry in Russia and the van Hiele theory. Hoffer (1982) maintained that it was Wirszup's presentation that attracted the attention of American educators to the van Hieles' work (p. 11). Therefore, the theory has been the focus of research in geometry during the last decade in the United States.

II. OVERVIEW OF THE VAN HIELE THEORY

The van Hiele theory deals with cognitive development in geometry. The theory provides a rationale for describing why many students have difficulty with geometry, as well as identifying some solutions in relationship to curriculum development and classroom instruction. Both Usiskin (1982) and Hoffer (1982) identified three main components of the van Hiele model. The components consist of (i) existence of levels; (ii) properties of the levels; and, (iii) phases of learning.

Existence of Levels

The van Hieles identified five levels of mental development in geometry. Wirszup (1976) gave a detailed description of the levels which were used in the Russian research. Also, he pointed out the various descriptions of behaviour given by Freudenthal (1973), as well as the descriptions given by P. M. van Hiele in 1959. Hoffer (1981) described the levels after visiting with P. M. van Hiele in the Netherlands. Usiskin (1982), Geddes (1982), and Burger (1982) gave descriptions of these behaviours. A number of source documents of the van Hieles' writings were examined to find quotes that described behaviours of students at a given level. Geddes (1982) referred to them as "descriptors of van Hiele levels" (pp. 7-10). Burger (1982) identified certain reasoning phenomena which he referred to as "level indicators" (pp. 23-25). Wirszup (1976) described the levels in detail, while Hoffer (1981) gave the descriptions names for each of the levels.

Level I: Recognition (Hoffer, 1981, p. 13)

> This initial level is characterized by the perception of geometric figures in their totality as entities. Figures are judged according to their appearance. The pupils do not see the parts of the figure, nor do they perceive the relationships among components of the figure and among the figures themselves. They cannot even compare figures with common properties with one another. The children who reason at this level distinguish figures by their shape as a whole. They recognize, for example, a rectangle, a square, and other figures. They conceive of the rectangle, however, as completely different from the square. When a six-year-old is shown what a rhombus, a rectangle, a square, and a parallelogram are, he is capable of reproducing these figures without error on a "geoboard of Gattegno", even in difficult arrangements. The child can memorize the names of these figures relatively quickly, recognizing the figures by their shapes alone, but he does not recognize the square as a rhombus, or the rhombus as a parallelogram. To him, these figures are still completely distinct. (Wirszup, 1976, p. 77)

Level II: Analysis (Hoffer, 1981, p. 14)

> The pupil who has reached the second level begins to discern the components of the figures; he also establishes relationships among these components and relationships between individual figures. At this level, he is therefore able to make an analysis of the figures perceived. This takes place in the process (and with the help) of observations, measurements, drawings, and model-making. The properties of the figures are established

experimentally; they are described, but not yet formally defined. These properties which the pupil has established serve as a means of recognizing figures. At this stage, the figures act as the bearers of their properties, and the student recognizes the figures by their properties. That a figure is a rectangle means that it has four right angles, that the diagonals are equal, and that the opposite sides are equal. However, these properties are still not connected with one another. For example, the pupil notices that in both the rectangle and the parallelogram of general type the opposite sides are equal to one another, but he does not yet conclude that a rectangle is a parallelogram. (Wirszup, 1976, pp. 77-78)

Level III: Ordering (Hoffer, 1981, p. 14)

Students who have reached this level of geometric development establish relations among the properties of a figure and among the figures themselves. At this level there occurs a logical ordering of the properties of a figure and of classes of figures. The pupil is now able to discern the possibility of one property following from another, and the role of definition is clarified. The logical connections among figures and properties of figures are established by definitions. However, at this level the student still does not grasp the meaning of deduction as a whole. The order of logical conclusion is established with the help of the textbook or the teacher. The child himself does not yet understand how it could be possible to modify this order, nor does he see the possibility of constructing the theory proceeding from different premises. He does not yet understand the role of axioms, and cannot yet see the logical connection of statements. At this level deductive methods appear in conjunction with experimentation, thus permitting other properties to be obtained by reasoning from some experimentally obtained properties. At the third level, a square is already viewed as a rectangle and as a parallelogram. (Wirzsup, 1976, p. 78)

LEVEL IV: Deduction (Hoffer, 1981, p. 14)

> At the fourth level, the students grasp the significance of deduction as a means of constructing and developing all geometric theory. The transition to this level is assisted by the pupils' understanding of the role and the essence of axioms, definitions, and theorems; of the logical structure of a proof; and of, the analysis of the logical relationships between concepts and statements.
>
> The students can now see the various possibilities for developing a theory proceeding from various premises. For example, the pupil can now examine the whole system of properties and features of the of the parallelogram by using the textbook definition of a parallelogram: A parallelogram is a quadrilateral in which the opposite sides are parallel. But he can also construct another system based, say, on the following definition: A parallelogram is a quadrilateral, two opposite sides of which are equal and parallel. (Wirszup, 1976, p. 78)

Level V: Rigor (Hoffer, 1981, p. 14)

> This level of intellectual development in geometry corresponds to the modern (Hilbertian) standard of rigor. At this level, one attains an abstraction from the concrete nature of objects and from the concrete meaning of the relations connecting these objects. A person at this level develops a theory without making any concrete interpretation. Here geometry acquires a general character and broader applications. For example, several objects, phenomena or conditions serve as "points", and any set of "points" serves as a "figure", and so on. (Wirszup, 1976, p. 79)

Properties of the Levels

The van Hieles identified properties of the levels. Usiskin (1982) and Hoffer (1982) described these properties and assigned names.

Property 1: (Fixed Sequence) A person cannot be at van Hiele level n without having gone through level n-1.

Property 2: (Adjacency) At each level of thought what was intrinsic in the preceding level becomes extrinsic in the current level.

Property 3: (Distinction) Each level has its own linguistic symbols and its own network of relationships connecting those symbols.

Property 4: (Separation) Two persons who reason at different levels cannot understand each other.

Property 5: (Attainment) The learning process which leads to complete understanding at the next higher level has five phases, approximately but not strictly sequential, entitled: inquiry, directed orientation, explanation, free orientation, integration. (Usiskin, 1982, pp. 4-6)

Phases of Learning

This aspect of the theory is considered by the van Hieles to be of special significance. They believed that cognitive development in geometry is directly related to quality and quantity of instruction. Usiskin (1982) stated: "Van Hiele (1959) is more optimistic than Piaget, believing that cognitive development in geometry can be accelerated by instruction." (p. 5) Also, Geddes (1982) observed: "Progress from one level to the next, assert the van Hieles, is more dependent upon instruction than on age or biological maturation, and types of instructional experiences can affect progress (or lack of it)." (p. 6)

The van Hiele model is based on instructional time rather than biological growth of the child. The writings of the van Hieles indicate a considerable amount of instructional time is necessary to move from one level to the next. Usiskin (1982) reported the results of Dina van

Hiele-Geldof's didactics experiments. He stated:

> Dina van Hiele (1957) reports having been able to lead students from level 1 (Recognition) to 3 (Ordering) in 70 lessons, 20 lessons to go from level 1 (Recognition) to level 2 (Analysis) and 50 more lessons to go from level 2 (Analysis) to level 3 (Ordering). (Usiskin, 1982, p. 39)

Hoffer (1982) observed that the van Hieles proposed "a prescription for organizing instruction" (p. 2). He described in some detail the process of movement from one level to the next (pp. 5-6).

> Phase 1. Inquiry. The teacher engages the students in conversations about the objects of the study to be pursued. The teacher learns how the students interpret the words and gives the students some understanding of what topic is to be studied. Questions are raised and observations made that use the vocabulary and objects of the topic and set the stage for further study.
>
> Phase 2. Directed orientation. The teacher carefully sequences activities for student exploration by which students begin to realize what direction the study is taking, and they become familiar with the characteristic structures. Many of the activities in this phase are one-step tasks which elicit specific responses.
>
> Phase 3. Expliciting. The students with minimal prompting by the teacher and building on previous experiences refine their use of the vocabulary and express their opinions about the inherent structures of the study. During the phase, the students begin to form the system of relations of the study.
>
> Phase 4. Free Orientation. The students now encounter multi-step tasks or different ways. They gain experience in finding their own way or resolving the tasks. By orienting themselves, many of the relations between the objects of the study become explicit to the students.
>
> Phase 5. Integration. The students now review the methods at their disposal and form an overview. The objects and relations are unified and internalized into a new domain of thought. The teacher aids this process by providing global surveys of what the students already know being careful not to present new or discordant ideas.

The van Hieles maintained that the student must go through these phases of learning before attaining the next level of thought. Hoffer (1982) stated: "At the close of the fifth phase the new level of thought is attained" (p. 6).

Other Aspects of the van Hiele Model

Usiskin (1982) and Hoffer (1982) identified other characteristics of the van Hiele theory. Usiskin (1982) referred to elegance, comprehensiveness, and wide applicability (p. 6). Hoffer (1982) proposed that the van Hiele theory could be applied to topics other than geometry. For example, he organized logic, geometric transformations, and real numbers into levels of thought (pp. 30-32). He also stated: "In the Netherlands the levels have been used to structure courses in chemistry and economics" (p. 30). Further, he maintained that the model provides us with a "blueprint" to interpret each topic that we want students to learn (p. 36). Mayberry (1983) suggested that the levels were hierarchical and discrete with respect to different topics (p. 68).

III. RESEARCH ON THE VAN HIELE THEORY

Coxford (1978) provided direction for three research projects to better understand the van Hiele model in geometry and cognitive structures. Since 1979, there have been three major studies conducted in the United States. Hoffer (1982) maintained that all three research projects contributed to the needs identified by Coxford. Also, he identified each study as they relate to Coxford's suggestions. (pp. 18-19)

(i) The gathering of data to compare cognitive structures and developmental stages - The Chicago Project

(ii) An analysis of the effects of instruction on cognitive structures - The Brooklyn Project

(iii) Longitudinal case studies - The Oregon Project

The Chicago Project: Cognitive Development and Achievement in Secondary School Geometry

The Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project was the most comprehensive of the three projects. It began in 1979 as a three-year study conducted by Usiskin and funded by the National Institute of Education (NIE). It was completed in June 1982 at the University of Chicago.

> The fundamental purpose of this project is to test the ability of the van Hiele theory to describe and predict the performance of students in secondary school geometry. (Usiskin, 1982, p. 8)

The CDASSG project was a classical experiment involving a sample of 2 699 students in 99 classes in 13 schools in 5 states. All of the students were enrolled in a one-year geometry course in the tenth grade.

The project used four tests which were administered in September 1980 and May 1981. One test which is of special interest is the van Hiele

Geometry Test constructed by the staff of the CDASSG project. This is a multiple-choice test dealing directly with the van Hiele levels. This test was designed from quotes of the van Hieles themselves regarding student behaviours. It was used to determine the student's van Hiele level at the beginning of geometry in the tenth grade. The test was given to students in the fall and again in the spring in order to determine changes in van Hiele levels after a year's study of geometry.

Usiskin (1982), the CDASSG project's principal investigator, arrived at a number of important conclusions regarding the van Hiele theory and geometry. The most significant ones are identified below.

The theory can be used to classify students into van Hiele levels of mental development in geometry. Usiskin (1982) concluded: "Over two-thirds and perhaps as many as nine-tenths of students respond to test items in ways which make it easy to assign them a van Hiele level." (p. 80).

The CDASSG project revealed a great variability in the amount of change in van Hiele levels from fall to spring after eight months instruction in geometry. Usiskin (1982) stated: "... About a third of the students stay at the same level or go down (!), about a third go up one level, and about a third go up two or more levels". (p. 81).

Usiskin (1982) found evidence to support the claims of Wirszup (1976) and Hoffer (1981). They claimed that the majority of high school students who have difficulty in high school geometry are at the first level of development in geometry. They maintained that the course that the students were studying demanded the <u>fourth</u> level of thought. They further stated that their prerequisite experiences in elementary and junior high school may be insufficient background to enable them to write proofs. Usiskin (1982) stated:

> Taken together, Conclusions 7 and 9 support the claims of Izaak Wirszup and Alan Hoffer that many if not most students in the United States enter geometry at van Hiele levels that are too low to insure success and that the geometry course, as presently taught, does not improve their understanding (as measured by van Hiele levels) enough to get that success. (Usiskin, 1982, p. 84)

Usiskin (1982) indicated that many students are not learning even the simplest geometry notions in junior high school. Some students entering the high school geometry course could not identify simple figures such as triangles, squares, rectangles, and parallelograms. Furthermore, he concluded that many students leave high school with very little knowledge of geometry.

The CDASSG project provided guidance for future research. Usiskin (1982) confirmed that the use of the van Hiele theory can explain why many students have trouble learning and performing in the geometry classroom. Furthermore, he indicated that half of the students who enroll in deductive geometry experience very little or no success with proof. (p. 89)

> This study confirms the need for systematic geometry instruction before high school if we desire greater geometry knowledge and proof-writing success among our students. (Usiskin, 1982, p. 89)

The Brooklyn Project: Geometric Thinking Among Adolescents in Inner City Schools

This project began in November 1979 and was completed in January 1982. It was conducted by Geddes and sponsored by the National Science Foundation (NSF). The main part of the study involved a clinical investigation on a one-to-one basis with 40 inner city adolescents for eight 45-minute sessions using four instructional modules developed on the van Hiele model. The modules were patterned after the experiments in Dina van Hiele's thesis.

> The general purpose of this phase is to determine whether the van Hiele model provides a reasonable structure for describing and understanding geometry learning as it takes place in the context of formal schooling. (Geddes, 1982, p. 2)

The instructional modules were intended to facilitate movement through the van Hiele levels. Hoffer (1982) stated: "The instructional modules that were developed by the Brooklyn project did contribute to student movement through the lower levels on certain topics". (p. 26)

The Brooklyn project evaluated several school textbook series for their geometric content and their relationship to the van Hiele model. This evaluation provided some insight into the quality of geometry which students are learning in grades 1 to 8. Geddes (1982) found some "gaps" existed in van Hiele terms. She found level one - recognition experiences to be sufficient, but a lack of extensive level two - analysis experiences. (p. 22)

> There are also frequent gaps in level in individual text pages, where the exposition is at a higher level than the exercises required of the student. Tests are usually at the lowest level. (Geddes, 1982, p. 23)

The Oregon Project: Assessing Children's Development in Geometry

This project began in September 1979 and concluded in February 1982. It was conducted by Burger and also sposnored by the National Science Foundation (NSF). The study used clinical interviews involving students in grades 1 to 12 in three states. The tasks and scripts were administered to over 70 students in two 45-minute sessions.

> The study described here is an investigation of children's reasoning processes in geometry and of the usefulness of the van Hiele levels in describing their reasoning. (Burger, 1982, p. 1)

The project staff developed two sequences of tasks and companion scripts. One set of tasks related to triangles and the other to quadrilaterals. The triangle activities were: drawing triangles, identifying and defining triangles, and sorting triangles. The quadrilateral activities included: drawing quadrilaterals, identifying and defining quadrilaterals, sorting quadrilaterals, identification of a "mystery figure" from its properties (What's my shape?), and establishing the logical equivalence of several geometrical definitions. (Burger, 1982, p. 1 and C1)

Burger (1982) identified "level indicators" from the interviews. In other words, certain reasoning phenomena were observed that could be interpreted as indicators of a particular van Hiele level of reasoning (p. 23).

Hoffer (1982) described the Oregon project in some detail since he was a staff member. He stated: "The van Hiele model provides us with a peephole through which we can use our mathematical eye to view children's interaction with mathematics" (p. 19). As well, he made some observations relating to all three projects. He discussed the language used by the students in geometry, in particular, the middle grades through junior high school. For example, many students have difficulty identifying and naming triangles. He examined the students' perceptions of geometric figures in relation to orientation and textbook position. It seems that the textbook position plays an important role on the students' perceptions of triangles and rectangles. He observed the students' ability to reason in geometry. He stated: "Students are for the most part unable to contrast definitions, postulates, and theorems" (p. 24). He concluded that there are many instances of disharmony in the teaching and learning of mathematics. There are levels of communication that differ between children, teachers, and textbooks.

IV. IMPLICATIONS OF THE VAN HIELE THEORY

The van Hiele model provides direction for curriculum development and classroom instruction in geometry at all grade levels from Kindergarten to senior high school. It is a comprehensive theory which can be applied to the whole of teaching and learning of geometry.

Curriculum Development

The van Hiele levels provide a plan for organization of geometric content at the various grade levels. The fixed sequence property suggests that students must go through the sequence of levels in a specific way. Therefore, it appears necessary for students to recognize geometric figures; analyze the properties of geometric figures; and, logically order geometric figures and their relationships before beginning deductions. Consequently, activities which require the first three levels should be included in the mathematics curriculum before senior high school. For example, primary mathematics might contain recognition activities using concrete materials; elementary mathematics might contain informal geometric activities which require analysis of figures; and, ordering activities might be the main focus of junior high school geometry. As a result, students might be better prepared for proof-writing or deductive reasoning in senior high school mathematics.

It seems that each topic must be examined separately in relation to the van Hiele levels. A student may be at one level in studying triangles and a different level for quadrilaterals. Therefore, it is essential to examine each topic in geometry when planning the curriculum.

The adjacency and distinction properties provide some criteria for textbook selection for different grade levels. The adjacency property indicates the activities should be organized in a contiguous manner. Activities which are implicit at one level become explicit at the next level. The distinction property provides an indication of the level of difficulty of the material in relationship to van Hiele levels. This type of activity would imply that the individual should be familiar with the van Hiele theory in order to select appropriate geometric materials.

Classroom Instruction

The <u>separation</u> property of the van Hiele theory has implications for classroom instruction in geometry. Levels of communication in the geometry classroom are an important consideration. If the student is operating at one level and the teacher at a higher level, there will be a lack of understanding. The student cannot understand the language of the teacher.

The <u>attainment</u> property provides the teacher with an approach to instruction in geometry. The phases of learning indicate how the teacher should operate to lead students from one level to the next. These phases have implications for instructional time in geometry at all grade levels.

SUMMARY

The van Hiele theory was developed in the late 1950's by two high school teachers in the Netherlands. The theory was applied to curriculum changes in geometry in both the Netherlands and the Soviet Union.

The theory relates cognitive development and thinking in geometry. There are three main components of the theory: Existence of levels, properties of the levels, and phases of learning. The theory is based on the quality and quantity of instruction in geometry.

The theory has the potential to explain why many students have trouble with geometry and, in particular, deductive reasoning. Students must receive instruction in three prior levels of thinking before deduction is understood. A considerable amount of instructional time is required to move from one level to the next level in the sequence.

The van Hiele theory has received considerable attention in the United States since Izaak Wirszup introduced the theory to mathematics educators in 1974. Burger (1982), Geddes (1982), Hoffer (1982), and Usiskin (1982) have conducted extensive research in the United States on this theory. In 1979, after visiting P. M. van Hiele, Hoffer wrote a secondary school geometry text, Geometry, A Model of the Universe, which incorporated the van Hiele theory.

The van Hiele model can be used to examine cognitive development in geometry of students in Newfoundland and Labrador. To date, research on the theory in North America has been limited to the United States with the exception of a study by Taaffe (1983) in this province. The theory has wide applicability and can be used to examine levels of thinking in geometry of ninth grade students. The results of the study can be used to determine appropriate geometric experiences for students at this grade level.

CHAPTER III

THE EXPERIMENTAL DESIGN OF THE STUDY

The main purposes of this study were to investigate the level of thinking in geometry of students at the beginning of grade nine, to determine if the student's level of mental development in geometry is influenced by the geometry content taught in previous years, specifically grade eight, and to compare the levels of mental development of grade nine students in Newfoundland and Labrador with those of students in the United States. In this chapter, the experimental design of the study, a description of the population and sampling procedures, instrumentation and procedures, and the purpose of the pilot study are outlined. The questions which the study attempted to answer and the methods used to analyze the data are also stated.

I. DESIGN OF THE STUDY

POPULATION

The population for this study consisted of approximately 12 000 students in Newfoundland and Labrador enrolled in grade nine in the 1983-84 school year. They were enrolled in three different mathematics courses: Advanced, Academic, and Practical. In grade eight, these students studied mathematics using the authorized texts: Math Is/2 (Ebos et al, 1975) or

School Mathematics 2 (Fleenor et al, 1974). There are two core units of informal geometry prescribed. There is also a geometry strand from Kindergarten to grade eight.

SAMPLING PROCEDURES

A sample of 1 004 students was randomly selected from 20 schools in 20 school districts. However, only students who studied grade eight the previous year and studied geometry from Math Is/2 or School Mathematics 2 were used for data analysis. Consequently, there were 46 students repeating grade nine which were eliminated from the sample. There were 29 students who studied mathematics in grade eight using an alternate textbook, and these students were also eliminated from the data analysis.

The sample contained students from both rural and urban communities in Newfoundland and Labrador. As well, it contained students from different administrative organizations: 10 central high schools (7-12); 3 junior high schools (7-9); and 7 all-grade schools (K-12).

The sample was stratified into two categories, text used in grade eight for geometry and placement in grade nine mathematics. The students were required to check three questions at the top of the answer sheet. They are as follows:

1. Which grade did you study last year?

   _____ (a) Grade 8

   _____ (b) Grade 9

2. Which text did you have for geometry last year?

_____ (a) Math Is/2

_____ (b) School Mathematics 2

_____ (c) Math Is/Geometry, Book 1

_____ (d) Geometry - Moise and Downs

_____ (e) Mathematics for Daily Use

_____ (f) Other texts

3. Which mathematics class are you enrolled this year in grade 9?

_____ (a) Advanced

_____ (b) Academic

_____ (c) Practical

II. INSTRUMENTATION

In this section the major instrument used to answer the questions stated in Chapter I is described.

THE VAN HIELE GEOMETRY TEST

The original van Hiele Geometry Test developed by CDASSG staff consisted of 25 multiple-choice items with 5 items at each level: Recognition, Analysis, Ordering, Deduction, and Rigor. The questions for each level were based on quotes of the van Hieles themselves regarding student behaviours to be expected at each level. A total of nine works were examined by the CDASSG staff. A list of the behaviours indicative of each van Hiele level as identified in the CDASSG project are presented in Appendix A. (Usiskin, 1982, pp. 9-12)

The van Hiele Geometry Test was designed to be administered in a 35-minute time limit. It was piloted and used extensively in the CDASSG project (1980-82). It was constructed for use with seventh to twelfth grade students, but 56 percent of the sample in the CDASSG project was in the tenth grade. Ages of the students ranged from 11 to 20 with 96 percent of the students between the ages of 14 and 17. (Usiskin, 1982, p. 16)

The major instrument used in the current study was a modified version of the van Hiele Geometry Test developed by the CDASSG project in 1980. The modified van Hiele Geometry Test consisted of the first 20 items on the original test; that is, the items dealing with the first four levels: Recognition, Analysis, Ordering, and Deduction. The last five items on the original test were excluded since the existence and/or testability of level 5 (Rigor) had been questioned. (Usiskin, 1982, p. 79) A copy of the modified van Hiele Geometry Test is contained in Appendix B with appropriate instructions and answer sheet.

III. PILOT STUDY

A pilot study of the first 20 items of the van Hiele Geometry Test, involving 40 students in grade eight, was conducted in March 1983. The purposes of this pilot study were to examine the following:

1. the necessity of including items 16-20 on Deduction.

2. the length of the test. It was important for administration of the test to determine if 30 minutes was a suitable time period for completion of the test.

On the basis of this pilot study, it was decided that the items at the deduction level would be included and the time limit for the test would be 30 minutes.

IV. TEST ADMINISTRATION

The van Hiele Geometry Test and answer sheets were sent to Mathematics/Science Co-ordinators and Assistant Superintendents responsible for Mathematics on August 31, 1983. The following instructions were given to the school district personnel:

1. They were asked to administer the test before the end of September, 1983.

2. The test was to be administered to <u>all</u> grade 9 students (Advanced, Academic, and Practical) in the schools identified in the random sample.

3. The students were given a <u>school number</u> to place at the top of the answer sheet.

4. The students were to check three questions at the top of the answer sheet regarding their grade level last year, textbook used for geometry last year, and placement in mathematics this year before beginning the test.

5. The time allowed for the test was to be exactly 30 minutes.

6. The answer sheets were to be returned immediately after testing was completed.

7. The tests could be retained at the District Office for future use in other schools.

V. ANALYSIS

This study was concerned with three questions with respect to students' level of mental development in geometry at the beginning of the ninth grade. These questions, along with the corresponding statistical analysis used to test the hypotheses, or describe the data collected, are given below.

Question 1

Are students at the beginning of the ninth grade prepared for deductive reasoning?

This question was answered by administering a modified version of the van Hiele Geometry Test to 1 004 grade nine students in early September. The students were classified into a van Hiele level using two criteria: 3 out of 5 (60%) items and 4 out of 5 (80%) items correct at each of the levels.

Usiskin (1982) discussed the probability of Type I and Type II errors and the choice of criterion. He maintained that the 4 of 5 criterion avoids about 5 percent of cases in which Type I error may be expected to manifest itself. As well, he stated that the 3 of 5 criterion avoids about 7 percent of cases in which Type II error may be expected to appear (pp. 23-24).

Tables were constructed to show the numbers and percentages of students at various van Hiele levels using each of the 3 out of 5 and 4 out of 5 criterion. Also, a cross tabulation matrix was constructed to determine the number of students that met both criteria.

Question 2

Are there any differences between the van Hiele levels of mental development in geometry of grade nine students who were taught using the textbooks: **Math Is/2** and **School Mathematics 2**?

Null Hypothesis: The van Hiele levels of thinking in geometry of grade nine students and the text used for geometry instruction in grade eight are independent.

In order to answer the second question, the null hypothesis was tested using the chi-square test of independence of van Hiele level and text used for geometry instruction in grade eight. Tables were constructed for both 3 out of 5 and 4 out of 5 criteria and the 0.05 level of significance was applied.

Question 3

Are there any differences between the van Hiele levels of mental development in geometry of grade nine students in Newfoundland and Labrador and those of students in the United States?

Null Hypothesis: There is no significant difference in the van Hiele levels of mental development in geometry of grade nine students in Newfoundland and Labrador and those of students in the United States.

To answer the third question, the null hypothesis was tested using the chi-square test for homogeneity of van Hiele levels of students in Newfoundland and Labrador and students in the United States. The samples were considered separate and distinct and randomly selected from homogeneous populations.

Contingency tables were constructed for both 3 out of 5 and 4 out of 5 criteria using the van Hiele levels of students in Newfoundland and Labrador and those of students in the United States. The fall results of the CDASSG project were used for the latter group of students. The level of significance selected was 0.05 in both instances.

CHAPTER IV

THE RESULTS OF THE INVESTIGATION

In this chapter the results of the testing are presented. An analysis of the data is utilized to answer the questions which provided the impetus for this study. The van Hiele levels of students in geometry at the beginning of grade nine are examined. The effects of text materials used for geometry instruction in grade eight is analyzed in relation to the students' van Hiele levels. Finally, the van Hiele levels of students in Newfoundland and Labrador are compared with those of students in the United States.

The population in this study consisted of all grade nine students in the province of Newfoundland and Labrador for the school year, 1983-84. Data were collected relative to placement of students in grade nine and the textbook used for geometry instruction in grade eight. A breakdown of the sample with respect to textbooks used to study geometry in grade eight and placement in mathematics classes in grade nine is given in Table I.

The mathematics program in grade nine is designed for distinct levels of difficulty: advanced, academic, and practical. The advanced students consisted of 13.8 percent of the sample and studied geometry using the authorized text: Geometry (Moise and Downs, 1975). The academic students comprised 74.7 percent of the sample and studied geometry using the authorized text: Math Is/Geometry, Book 1 (Ebos et al, 1981). Finally, the practical students consisted of 11.5 percent of the sample and

TABLE I

BREAKDOWN OF SAMPLE BY TEXTBOOK USED FOR GEOMETRY INSTRUCTION IN GRADE EIGHT AND PLACEMENT IN THE NINTH GRADE.

Grade VIII Text / Grade IX Placement	Math Is/2		School Mathematics 2		Totals	
	#	%	#	%	#	%
Advanced Text: Geometry - Moise & Downs	128	13.8	0	0	128	13.8
Academic Text: Math Is/ Geometry, Book 1	488	52.5	206	22.2	694	74.7
Practical Text: Mathematics for Daily Use	48	5.2	59	6.3	107	11.5
Totals	664	71.5	265	28.5	929	100.0

studied some informal geometry from a practical viewpoint using the authorized text: Mathematics for Daily Use (Hayden et al, 1981).

The breakdown in the percentage of students who used Math Is/2 and School Mathematics 2 in grade eight is also presented in Table I. Seventy-one and a half percent of the students studied Math Is/2 and 28.5 percent of the students studied geometry from School Mathematics 2. Students enrolled in advanced mathematics in grade nine did not study School Mathematics 2 in grade eight.

In summary, the majority of grade nine students (88.5 percent) are enrolled in advanced or academic mathematics. In these courses, 50 percent of the instructional time is allocated to the teaching of geometry.

Question 1

Are students at the beginning of the ninth grade prepared for deductive reasoning?

The answer to this question was obtained by administering a modified version of the van Hiele Geometry Test to 1 004 grade nine students in early September. However, 75 students were eliminated from the sample because they were repeaters (46 students) or they studied geometry using an alternate textbook in grade eight (29 students). Therefore, the test results of 929 students were suitable for data analysis.

The students were classified into a van Hiele level of thinking in geometry: Recognition, Analysis, Ordering, or Deduction. A parallel analysis of data using two criteria: 3 out of 5 (60%) items and 4 out of 5 (80%) items correct was applied at each of the levels. These criteria are referred to as the weaker (3 out of 5) and the stricter (4 out of 5) criterion respectively. (Usiskin, 1982, p. 23)

In order to assign a van Hiele level to a student, it was necessary to reach the criterion at all levels from 1 to n and at no other levels greater than n so as to be classified at level n. Students who did not meet the criterion at the recognition level (level 1) were considered as being below recognition (level 0).

There were a number of students classified as "nofit", meaning that each of these students satisfied the indicated criterion at some level n but not at all levels below n. The theory requires that the students must move through the levels in a fixed sequence: A person cannot be at level n without having gone through level n-1.

The numbers and percentages of students at each of the van Hiele levels and the textbook used in grade eight for geometry instruction are presented in Table II. Also, the number of students who were below the recognition level, and those who did not fit the theory are included. The criterion used in Table II was 3 out of 5 (60%) items correct at each level. It was possible to classify 88.7 percent of the students into a van Hiele level or below recognition level. Only 9 students or less than 1 percent are at the <u>deduction</u> level at the beginning of grade nine. However, using this weaker criterion, approximately 7 percent were at the <u>ordering</u> level or <u>deduction</u> levels at the beginning of the school year in Newfoundland and Labrador.

TABLE II

NUMBERS AND PERCENTAGES OF STUDENTS AT EACH VAN HIELE LEVEL USING A 3 OUT OF 5 CRITERION

Level Grade IX \ Text Grade VIII	Math Is/2		School Mathematics 2		Totals	
	#	%	#	%	#	%
Below Recognition	44	6.6	22	8.3	66	7.1
Recognition	314	47.3	130	49.1	444	47.8
Analysis	190	28.6	62	23.4	252	27.1
Ordering	39	5.9	14	5.3	53	5.7
Deduction	9	1.4	0	0	9	0.97
Nofit	68	10.3	37	14.0	105	11.3
Totals	664	71.5	265	28.5	929	100.0

The numbers and percentages of students at each of the van Hiele levels and the textbook used in grade eight for geometry instruction are reported in Table III. The criterion used in this instance was 4 out of 5 (80%) items correct at each level. It was possible to classify 95 percent of the students into a van Hiele level or below recognition using this stricter criterion. At the beginning of grade nine there were no students at the <u>deduction</u> level; an indication that they were not ready for deductive reasoning. In fact, there were only 9 students, or less than 1 percent, at the <u>ordering</u> level at the beginning of the school year.

A crosstabulation matrix of student van Hiele levels under the 3 out of 5 and the 4 out of 5 criterion is given in Table IV. Those students whose van Hiele levels are the same under the two criteria are identified on the main diagonal of the matrix. Only 43.9 percent of students (408 of 929) were assigned the same van Hiele level under the two criteria. There were no students at the deduction level that met both criteria on the modified van Hiele Geometry Test.

On the basis of the above results, students at the beginning of grade nine in Newfoundland and Labrador are not ready for deductive reasoning in geometry. The majority are at the <u>recognition</u> level and/or <u>analysis</u> level if a 3 out of 5 criterion is applied. Using a 4 out of 5 criterion, the majority of students are at the <u>recognition</u> level and/or below recognition. Approximately 27 percent can meet both criteria at the <u>recognition</u> level. A discussion of these results is found in Chapter V.

TABLE III

NUMBERS AND PERCENTAGES OF STUDENTS AT EACH VAN HIELE LEVEL USING A 4 OUT OF 5 CRITERION

Level Grade IX \ Text Grade VIII	Math Is/2		School Mathematics 2		Totals	
	#	%	#	%	#	%
Below Recognition	229	34.5	110	41.5	339	36.5
Recognition	310	46.7	114	43.0	424	45.6
Analysis	86	13.0	25	9.4	111	11.9
Ordering	9	1.4	0	0	9	0.97
Deduction	0	0	0	0	0	0
Nofit	30	4.5	16	6.0	46	5.0
Totals	664	71.5	265	28.5	929	100.0

TABLE IV

CROSSTABULATION OF STUDENTS FITTING VAN HIELE LEVELS WITH 3 OUT OF 5 CRITERION AND THE 4 OUT OF 5 CRITERION

3 out of 5 \ 4 out of 5	Below Recognition	Recognition	Analysis	Ordering	Deduction	No Fit	Totals
Below Recognition	66 (7.1)	0	0	0	0	0	66
Recognition	189	255 (27.4)	0	0	0	0	444
Analysis	38	116	81 (8.7)	0	0	17	252
Ordering	3	18	20	6 (0.6)	0	6	53
Deduction	0	1	3	3	0 (0.0)	2	9
No Fit	43	34	7	0	0	21 (2.3)	105
Totals	339	424	111	9	0	46	929

Question 2

Are there any differences between the van Hiele levels of mental development in geometry of grade nine students who were taught using the textbooks: **Math Is/2** and **School Mathematics 2**?

Null Hypothesis: The van Hiele levels of thinking in geometry of grade nine students and the text used for geometry instruction in grade eight are independent.

The null hypothesis was tested using the chi-square test of independence of van Hiele levels and textbook used for geometry instruction in grade eight. Tests were conducted for both 3 out of 5 and 4 out of 5 criteria using a two-way contingency table. The level of statistical significance selected for this test was the 0.05 level. The number of degrees of freedom was 5. Therefore, a chi-square value equal to or greater than 11.07 was necessary for rejection of the null hypothesis. However, it must be noted that the expected frequency should be equal to or greater than 5 in at least 80 percent of the cells and the number of degrees of freedom must be greater than one. (Runyon and Haber, 1971, p. 253)

Table V is a contingency table for the 3 out of 5 criterion to test independence. The expected frequency was greater than 5 in over 90 percent of the cells. Therefore, the chi-square value was found to be 8.84 which resulted in acceptance of the null hypothesis.

Table VI is a contingency table for the 4 out of 5 criterion to test independence. Since the frequencies in the cells for the deduction level wre both zero, this row was deleted. The number of degrees of

TABLE V

CONTINGENCY TABLE FOR 3 OUT OF 5 CRITERION

TO TEST INDEPENDENCE

Level Grade IX \ Text Grade VIII	Math Is/2	School Mathematics 2	Totals
Below Recognition	44 (6.6)	22 (8.3)	66
Recognition	314 (47.3)	130 (49.1)	444
Analysis	190 (28.6)	62 (23.4)	252
Ordering	39 (5.9)	[illegible] (5.3)	53
Deduction	9 (1.4)	0 (0.0)	9
Nofit	68 (10.3)	37 (14.0)	105
Totals	664	265	929

TABLE VI

CONTINGENCY TABLE FOR 4 OUT OF 5 CRITERION

TO TEST INDEPENDENCE

Level Grade IX \ Text Grade VIII	Math Is/2	School Mathematics 2	Totals
Below Recognition	229 (34.5)	110 (41.5)	339
Recognition	310 (46.7)	114 (43.0)	424
Analysis	86 (13.0)	25 (9.4)	111
Ordering	9 (1.4)	0 (0.0)	9
Deduction	0 (0.0)	0 (0.0)	0
Nofit	30 (4.5)	16 (6.0)	46
Totals	664	265	929

freedom was reduced to 4. Therefore, a chi-square value equal to or greater than 9.49 was required for rejection of the null hypothesis. The expected frequency was greater than 5 in 90 percent of the cells.

The chi-square value was found to be 9.65 which resulted in rejection of the null hypothesis: The van Hiele levels of thinking in geometry of grade nine students and the text used for geometry instruction in grade eight are independent.

In summary, the level of thinking of students was independent of the textbook used for geometry instruction in grade eight when classified into a van Hiele level using the 3 out of 5 criterion. However, the level of thinking of students was dependent on the textbook used for geometry instruction in grade eight when the 4 out of 5 criterion was applied. The percentages of students at recognition, analysis, and ordering levels were higher for those who were taught geometry in grade eight using Math Is/2. As well, the percentage of students below the recognition level was smaller. Some possible reasons for this phenomenon are discussed in Chapter V.

Question 3

Are there any differences between the van Hiele levels of mental development in geometry of grade nine students in Newfoundland and Labrador and those of students in the United States?

Null Hypothesis: There is no significant difference in the van Hiele levels of mental development in geometry of grade nine students in Newfoundland and Labrador and those of students in the United States.

The null hypothesis was tested using the chi-square test for homogeneity of van Hiele levels and distinct samples of students in Newfoundland and Labrador and students in the United States. Contingency tables were constructed for both 3 out of 5 and 4 out of 5 criteria using the fall van Hiele levels of students in the CDASSG project in the United States.

The level of significance selected was 0.05 in both instances. The number of degrees of freedom was 4. Therefore, the critical chi-square value was 11.07 or greater for rejection of the null hypothesis.

Table VII is a contingency table for the 3 out of 5 criterion to test homogeneity. The chi-square value was found to be 133.19 which resulted in rejection of the null hypothesis.

Table VIII is a contingency table for the 4 out of 5 criterion to test homogeneity. The chi-square value was found to be 40.88 which also resulted in rejection of the null hypothesis.

In summary, there was a significant difference in the van Hiele levels of mental development in geometry of grade nine students in Newfoundland and Labrador and those of students in the United States. The percentages of students at the ordering and deduction levels were higher for students in the United States as presented in the CDASSG project using the 3 out of 5 criterion. However, the percentages of students at the analysis and ordering levels were higher for students in the United States when the 4 out of 5 criterion was applied. Some possible reasons for this inference are discussed in Chapter 5.

TABLE VII

CONTINGENCY TABLE FOR 3 OUT OF 5 CRITERION

TO TEST HOMOGENEITY

Level \ Sample	Newfoundland and Labrador	United States	Totals
Below Recognition	66 (7.1)	158 (6.7)	224
Recognition	444 (47.8)	900 (38.1)	1 344
Analysis	252 (27.1)	596 (25.2)	848
Ordering	53 (5.7)	270 (11.4)	323
Deduction	9 (1.0)	80 (3.4)	89
Nofit	105 (11.3)	357 (15.1)	462
Totals	929	2 361	3 290

TABLE VIII

CONTINGENCY TABLE FOR 4 OUT OF 5 CRITERION
TO TEST HOMOGENEITY

Level \ Sample	Newfoundland and Labrador	United States	Totals
Below Recognition	339 (36.5)	726 (30.7)	1 065
Recognition	424 (45.6)	1,008 (42.7)	1 432
Analysis	111 (11.9)	338 (14.3)	449
Ordering	9 (1.0)	93 (3.9)	102
Deduction	0 (0.0)	5 (0.2)	5
Nofit	46 (5.0)	191 (8.1)	237
Totals	929	2 361	3 290

SUMMARY

The data collected in the study relative to the three major questions given in Chapter I have been presented in this chapter.

It was found that the students at the beginning of grade nine were not at the van Hiele deduction level of thinking in geometry. The student's level of thinking in geometry was found to be independent of the text used for geometry instruction in grade eight when the students were classified into a van Hiele level using 3 out of 5 criterion. However, when a 4 out of 5 criterion was used to classify students, the van Hiele level was dependent on the textbook used for grade eight geometry instruction. In particular, students who studied Math Is/2 had slightly higher van Hiele levels. Finally, the van Hiele levels of students in Newfoundland and Labrador at the beginning of grade nine were significantly different than those of students in the United States as determined by the CDASSG project in 1982. The van Hiele levels of students at the beginning of grade ten in the United States were higher than those of students at the beginning of grade nine in Newfoundland and Labrador.

A discussion of the findings as well as implications and recommendations is presented in the final chapter.

CHAPTER V

SUMMARY, DISCUSSION AND IMPLICATIONS, AND RECOMMENDATIONS

I. SUMMARY OF INVESTIGATION

In this study an attempt was made to determine the van Hiele level of thinking in geometry at the beginning of ninth grade. The relationship between the text materials used to study geometry in grade eight and the student's level of thinking in geometry at the beginning of the ninth grade was investigated. Also, the levels of mental development in geometry of students in Newfoundland and Labrador were compared with those of students in the United States.

In order to gather the necessary data a sample of grade nine students was randomly selected and a modified version of the van Hiele Geometry Test was administered in September 1983.

SAMPLE

The sample of 1 004 grade nine students was randomly drawn from the grade nine population in Newfoundland and Labrador. However, only students who studied geometry in grade eight using the prescribed texts: Math Is/2 or School Mathematics 2 were used for data analysis. Also, those students repeating grade nine were eliminated from the study. Consequently, only 929 students were suitable for the study.

It was assumed that these students were representative of grade nine mathematics students in this province. They were enrolled in three different mathematics programs in grade nine depending on their ability to do mathematics. The students came from 20 different schools with three different administrative arrangements: all-grade, junior high, and central high school.

INSTRUMENTATION

A modified version of the van Hiele Geometry Test was used in the present study. (Appendix B) This test was originally constructed by the staff of the CDASSG project based on the writings of the van Hieles'. (Appendix A) This test was used with 2 700 students in the United States. The original test contained 25 multiple-choice items on different levels of mental development in geometry: Recognition, Analysis, Ordering, Deduction, and Rigor. Only the first four levels were used with grade nine students in Newfoundland and Labrador as a result of piloting.

The van Hiele Geometry Test was written by students in the sample in September before geometry was studied in grade nine. Each student was classified into a van Hiele level of thinking in geometry using two criteria: 3 out of 5 and 4 out of 5 items correct at each level. Contingency tables were used to determine if the level of thinking in geometry is independent of the textbook used for geometry instruction in grade eight. Also, contingency tables were constructed to determine if the van Hiele levels of thinking in geometry of students in Newfoundland and Labrador and those of students in the United States are significantly different.

CONCLUSIONS

There were three major conclusions reached based on the van Hiele testing at the beginning of grade nine.

1. Students at the beginning of grade nine were not at the van Hiele deduction level of thinking in geometry.

2. The level of thinking of students was independent of the textbook used for geometry instruction in grade eight when classified into a van Hiele level using the 3 out of 5 criterion. However, the level of thinking of students was dependent on the textbook used for geometry instruction in grade eight when the 4 out of 5 criterion was applied.

3. There was a significant difference in the van Hiele levels of students in Newfoundland and Labrador and those of students in the United States. Students in this province were at lower van Hiele levels at the beginning of grade nine than those of students in the United States at the beginning of grade ten.

II. DISCUSSION AND IMPLICATION OF THE FINDINGS

The results of this study were presented in detail in the previous chapter as they relate to the three major questions. In this chapter, the findings are discussed.

The first major conclusion indicated that grade nine students are not ready for a study of deductive reasoning at the beginning of the school year. The major of students were at the recognition (47.8 percent) and analysis (27.1 percent) levels of the van Hiele theory when a 3 out of 5 criterion was used to classify the students into a level. In order to reach the deduction level, the students would have to move through the ordering level. However, only a small number of students (5.7 percent) were at the ordering level at the beginning of the school year. These students with the aid of instruction may reach the deduction level before the end of grade nine.

The results are very discouraging when a 4 out of 5 criterion was used to classify the students. In this instance, the majority of students were at the recognition (45.6 percent) or below recognition (36.5 percent) levels. Even more disturbing was the large number of students below the recognition level in geometry in this province. It seems that these students were entering grade nine with very little knowledge of geometry. They have difficulty with recognition of shapes which is an objective of kindergarten.

From a crosstabulation of results, it was determined that the majority of students were at the recognition level at the beginning of the ninth grade. One possible reason for this could be a lack of instructional time allocated to the teaching of geometry from kindergarten to grade eight. Roberts (1979) conducted a study of time spent on teaching geometry in the elementary school in Newfoundland and Labrador. He reported that the mean time spent on geometry to be 2.53 weeks per year, thus making this reason plausible (p. 63). This study did not investigate the amount of instructional time spent on geometry in previous grades. This is an issue which must be given some attention in future by mathematics educators in Newfoundland and Labrador.

The results of this study have implications for geometry instruction in junior high school. The instruction should fit the cognitive level of the student. It would appear that for the majority of students instruction at the analysis and ordering levels would be most appropriate. Therefore, deductive reasoning should be a vertical enrichment topic for some students toward the end of grade nine rather than a major component of a prescribed course such as advanced mathematics.

The second major conclusion related to the kinds of geometry experiences encountered prior to grade nine. As a result of parallel analysis of results using two criteria, the conclusion is dichotomous. Hence, it is necessary to discuss both aspects of this conclusion.

The student's level of thinking in geometry was found to be independent of the textbook using a criterion of 3 out of 5 items correct at each van Hiele level. Although the percentage of students at the analysis level (28.6) was greater for Math Is/2, it must be remembered that in some instances these students are average or above average in mathematics ability. The data in Table I indicated that all students enrolled in advanced mathematics in grade nine studied Math Is/2 in grade eight.

When the guessing factor was reduced using a 4 out of 5 criterion, there was a marginal significant difference between the student's van Hiele level in geometry at the beginning of grade nine and textbooks used for geometry instruction in grade eight. The students who were taught geometry in grade eight from Math Is/2 tended to have higher van Hiele levels. The results in Table III indicated that approximately 42 percent of students who were taught geometry from School Mathematics 2 were below the recognition level at the beginning of the ninth grade. At the same time, approximately 35 percent of students who were taught geometry from Math Is/2 were below the recognition level.

There is one possible explanation for this lack of knowledge of geometric terminology. The teachers in grade eight may have taught mathematics using the page-by-page approach. In this instance, the geometry in School Mathematics 2 is at the end of the text whereas some geometry in Math Is/2 is located in the middle of the text. Teachers may not have had enough instructional time to teach geometry.

Furthermore, there is a belief among some teachers of mathematics that geometry begins in grade nine. Geometry is not considered a basic skill from kindergarten to grade eight. It has been considered as the domain of senior high school (Grades 9-12). This belief has permeated the primary and elementary schools, in particular, where the majority of teachers have very little training in teaching mathematics and especially geometry.

The third major conclusion related to the van Hiele levels of students in Newfoundland and Labrador and those of students in the United States. The van Hiele levels of students in the United States were significantly higher than those of students at the beginning of grade nine in Newfoundland and Labrador.

In making such a comparison, there are some important considerations. It must be remembered that 56 percent of the sample in the CDASSG project entered the tenth grade in the fall. Usiskin (1982) also stated:

"In the United States, secondary school geometry is usually studied in a single year, normally in the tenth grade". (p. 1) This is an important factor since the students were likely a year older in most instances.

Usiskin (1982) found that when 3 out of 5 was used in the fall and level 5 (Rigor) was excluded, 85 percent of the students could be classified. He also found 3 percent of the students at the <u>deduction</u> level. (p. 98) This is higher than the results for Newfoundland and Labrador, but the students may have had some geometry instruction in grade nine as well as previous grades.

Usiskin (1982) found that: "The tougher 4 out of 5 criterion minimizes the chance of a student being at a level of guessing" (p. 79). Also, he found in the fall that 92 percent of the students could be classified in a van Hiele level if level 5 (Rigor) was excluded and the 4 out of 5 criterion was applied. In this instance, he found 31 percent below recognition and 43 percent at recognition.

III. RECOMMENDATIONS FOR FURTHER RESEARCH

Student thinking in geometry at the beginning of grade nine in Newfoundland and Labrador was analyzed in this study. It has been suggested that there may be a number of factors contributing to the extremely low levels of student thinking in geometry at the beginning of grade nine. Instructional time spent on geometry, instructional strategies and/or text materials utilized for geometry instruction must be taken into consideration.

The following recommendations for further research are suggested:

1. That a similar study be conducted at the end of grade nine to indicate if students are prepared for deductive reasoning in the senior high mathematics courses.

2. That a year long study be conducted with the same sample at a specific grade level in order to determine the amount of instructional time required to move from one level of thinking to the next in geometry.

3. That a study be conducted at the end of the sixth grade using actual shapes: square, rectangle, triangle, parallelogram, circle, etc. This research would help to determine the appropriate geometry for junior high school.

4. That a study be conducted with teachers of mathematics at the primary, elementary, and junior high schools to determine their level of thinking in geometry.

BIBLIOGRAPHY

Adler, I. "Piaget on the Learning of Mathematics". In Aichele, D. B. and Reys, R. E. (Eds.), Readings in Secondary School Mathematics. Boston, Prindle, Weber & Schmidt, 1971, 210-221.

Allendoerfer, C. B. "The Dilemma in Geometry" The Mathematics Teacher. March 1969, 165-169.

Brown, R. G. "Making Geometry A Personal and Inventive Experience". The Mathematics Teacher, September 1982, 442-446.

Burger, W. F. "Using the van Hiele Model to Describe Reasoning Processes in Geometry". Paper presented at American Educational Research Association, March 1982.

Coxford, A. "Research Directions in Geometry". In R. Lesh & D. Mierkiewicz (Eds.), Recent Research Concerning the Development of Spatial and and Geometric Concepts. Columbus, Ohio: ERIC/SMEAC, 1978.

Ebos, F. et al. Math Is/Geometry, Book 1. Toronto: Nelson/Canada Limited, 1981.

________. Math Is/2. Toronto: Nelson/Canada Limited, 1975.

Fleenor, C. R. et al. School Mathematics 2. Don Mills: Addison-Wesley Canada Limited, 1974.

Freudenthal, H. Mathematics As An Educational Task. Dardrecht, Holland: D. Reidel, 1973.

Geddes, D. "An Investigation of the van Hiele Model of Thinking in Geometry Among Adolescents". Project report presented at the National Council of Teachers of Mathematics Annual Meeting, Toronto, Canada, April 1982.

Hayden, M. P. et al. Mathematics For Daily Use. Toronto: Doubleday Canada Limited, 1980.

Hendrix, G. "Learning by Discovery". The Mathematics Teacher, May 1961, 290-297.

________. "The Psychological Appeal of Deductive Proof". The Mathematics Teacher, November 1961, 515-520.

Hiatt, A. A. "Basic Skills: What Are They?" The Mathematics Teacher, February 1979, 141-144.

Hoffer, A. Geometry, A Model of the Universe. Menlo Park: Addison-Wesley Publishing Company, 1979.

_________. "Geometry Is More Than Proof". The Mathematics Teacher, January 1981, 7-18.

_________. "Van Hiele Based Research". Eugene, OR: Department of Mathematics, University of Oregon, 1982.

Kane, R. B. "On the Proof-Making Task". The Mathematics Teacher, February 1975, 89-94.

Labinowicz, E. "Piaget's Stages - Levels of Children's Thinking". The Piaget Primer. Menlo Park: Addison-Wesley Publishing Company, 1980, 60-95.

Mayberry, J. "The van Hiele Levels of Geometry Thought in Undergraduate Preservice Teachers". Journal for Research in Mathematics Education, 1983. Vol. 14, No. 1, 58-69.

Moise, E. and Downs, F. Geometry. Menlo Park: Addison-Wesley Publishing Company, 1975.

Orleans, J. B. "A Unit of Demonstrative Geometry for the Ninth Year". The Teaching of Geometry. W. D. Reeve (Ed.). 5th yearbook of National Council of Teachers of Mathematics, Washington, National Council of Teachers of Mathematics, 1930.

Peterson, J. C. "Informal Geometry in Grades 7-14". Geometry in the Mathematics Curriculum. K. B. Henderson (Ed.). 36th yearbook of National Council of Teachers of Mathematics, Washington, National Council of Teachers of Mathematics, 1973.

Reeve, W. D. "The Teaching of Geometry". The Teaching of Geometry. W. D. Reeve (Ed.). 5th yearbook of National Council of Teachers of Mathematics, Washington, National Council of Teachers of Mathematics, 1930.

Roberts, B. A. "A Study of the Time Spent on Teaching Geometry in the Elementary School and Its Relationship to Teacher Attitudes Toward Geometry". Unpublished Masters thesis, Memorial University of Newfoundland, 1979.

Runyon, R. P. and Haber, A. "Statistical Inference with Categorical Variables". Fundamentals of Behavioural Statistics. Menlo Park: Addison-Wesley Co., 2nd. Edition, 1971, 242-256.

Sanders, W. J. and Dennis, J. R. "Congruence Geometry for Junior High School". The Mathematics Teacher, April 1968, 354-369.

Seidlin, J. "A Unit of Demonstrative Geometry for the Ninth Year". The Teaching of Geometry. W. D. Reeve (Ed.). 5th yearbook of National Council of Teachers of Mathematics, Washington, National Council of Teachers of Mathematics, 1930.

Sherard, W. H. "Why Is Geometry A Basic Skill?" The Mathematics Teacher, January 1981, 19-21.

Smith, S. A. "What Does A Proof Really Prove?" The Mathematics Teacher. May 1968, 483-484.

Taaffe, G. "An Investigation into the van Hiele Levels and Proof-Writing Achievement of Level I Geometry Students in Newfoundland". Unpublished Masters thesis, Memorial University of Newfoundland, 1983.

Usiskin, Z. "Van Hiele Levels and Achievement in Secondary School Geometry". Report of CDASSG Project. Chicago, The University of Chicago, 1982.

Wirszup, I. "Breakthroughs in the Psychology of Learning and Teaching Geometry". In J. L. Martin (Ed.), Space and Geometry: Papers From A Research Workshop. Columbus, Ohio: ERIC/SMEAC, 1976.

APPENDIX A:

BEHAVIOURS AT EACH VAN HIELE LEVEL

identified by

the CDASSG project

APPENDIX A

BEHAVIOURS AT EACH VAN HIELE LEVEL

In 1979-80, all of the van Hiele writings available to the CDASSG project personnel were examined for quotes that described behaviours of students at a given level. A total of nine works were examined, four originally written in English, five translated into English from Dutch, German, or French. The following is a list of behaviours, sorted by level. (Usiskin, 1982, pp. 9-12)

<u>Level 1 (their base level, level 0)</u>

(P.M., 1958-59)

1. "Figures are judged according to their appearance."
2. "A child recognizes a rectangle by its form, shape.
3. "... and the rectangle seems different to him from a square."
4. "When one has shown to a child of six, a six year old child, what a rhombus is, what a rectangle is, what a square is, what a parallelogram is, he is able to produce those figures without error on a geoboard of Gattegno, even in difficult situations."
5. "a child does not recognize a parallelogram in a rhombus."
6. "the rhombus is not a parallelogram. The rhombus appears ... as something quite different."

(P.M., 1968)

7. "when one says that one calls a quadrilateral whose four sides are equal a rhombus this statement will not be enough to convince the beginning student [from which I deduce that this is his level 0] that the parallelograms which he calls squares are part of the set of rhombuses."

(P.M., 1979)

8. (on a question involving recognition of a tilted square as a square) "basic level, because you can see it!"

Level 2 (their first level)

(P.M., 1957)

1. "He is able to associate the name 'isosceles triangle' with a specific traingle, knowing that two of its sides are equal, and draw the subsequent conclusion that the two corresponding angles are equal."

(Dina, 1957; P.M. and Dina, 1958)

2. "... a pupil who knows the properties of the rhombus and can name them, will also have a basic understanding of the isosceles triangle = semirhombus."

3. "The figures are the supports (lit. 'supports' in French) of their properties."

4. "That a figure is a rectangle signifies that it has four right angles, it is a rectangle, even if the figure is not traced very carefully."

5. "The figures are identified by properties. (E.g.) If one is told that the figure traced on the blackboard possesses four right angles, it is a rectangle, even if the figure is not traced very carefully."

6. "The properties are not yet organized in such a way that a square is identified as being a rectangle."

(P.M., 1959)

7. "The child learns to see the rhombus as an equilateral quadrangle with identical opposed angles and interperpendicular diagonals that bisect both at each other and the angles."

8. (a middleground between this and the next level) "Once the child gets to the stage where it knows the rhombus and recognizes the isosceles triangles for a semi-rhombus, it will also be able to determine offhand a certain number of properties of the equilateral triangle."

9. "Once it has been decided that a structure is an 'isosceles triangle' the child will also know that a certain number of governing properties must be present, without having to memorize them in this special case."

(P.M., 1976)

10. "The inverse of a function still belongs to the first thought level."

11. "Resemblance, rules of probability, powers, equations, functions, revelations, sets - with these you can go from zero to the first thought level."

Level 3 (their second level)

(Dina, 1957)

1. "Pupils ... can understand what is meant by 'proof' in geometry. They have arrived at the second level of thinking."

(P.M., 1957)

2. "He can manipulate the interrelatedness of the characteristics of geometric patterns."

3. "e.g., if on the strength of general congruence theorems, he is able to deduce the equality of angles or linear segments of specific figures."

(P.M., 1958-59)

4. "The properties are ordered [lit. 'ordonnet']. They are deduced from each other: one property precedes or follows another property."

5. "The intrinsic significance of deduction is not understood by the student."

6. "The square is recognized as being a rectangle because at this level definitions of figures come into play."

(P.M., 1959)

7. "the child ... [will] recognize the rhombus by mens of certain of its properties ... because, e.g., it is a quadrangle whose diagonals bisect each other perpendicularly."

8. "It [the child] is not capable of studying geometry in the strictest sense of the word."

9. "The child knows how to reason in accordance with a deductive logical system ... this is not however, identical with reasoning on the strength of formal logic."

(P.M., 1976)

10. "the question about whether the inverse of a function is a function belongs to the second thought level."

11. "The understanding of implication, equivalence, negation of an implication belongs to the second thought level."

(P.M., 1978)

12. "they are able to understand more advanced thought structure such as: 'the parallelism of the lines implies (according to the signal character) the presence of a saw, and therefore (according to their symbolic character) equality of the alternate-interior angles'."

13. "I [the student] can learn a definition by heart. No level. I can understand that definitions may be necessary: second level."

14. "... you know what is meant by it [the use of 'some' and 'all'] second level."

Level 4 (their third level)

(P.M., 1957)

1. "He will reach the third level of thinking when he starts manipulating the intrinsic characteristics of relations. For example: if he can distinguish between a proposition and its reverse" [sic. meaning our converse]

(Dina, 1957)

2. "We can start studying a deductive system of propositions i.e., the way in which the interdependency of relations is effected. Definitions and propositions now come within the pupils' intellectual horizon."

3. "Parallelism of the lines implies equality of the corresponding angles and vice versa."

(P.M. and Dina, 1958)

4. "The pupil will be able, e.g. to distinguish between a proposition and its converse."

5. "it (is) ... possible to develop an axiomatic system of geometry".

(P.M., 1958-59)

6. "The mind is occupied with the significance of deduction, of the converse of a theorem, of an axiom, of the conditions necessary and sufficient."

(P.M., 1968)

7. "... one could tell him (the student) that in a proof it is really a question of knowing whether these theses are true or not, or rather of the relationship between the truth of these theses and of some others. Without their understanding such relationships we cannot explain to the student that one has to have recourse to axioms." [I induced the level from the first part of this statement; he never identifies the level.]

Level 5 (their fourth level)

(Dina, 1957)

1. "A comparative study of the various deductive systems within the field of geometrical relations is ... reserved for those, who have reached the fourth level ...".

(P.M. and Dina, 1958)

2. "finally at the fourth level (hardly attainable in secondary teaching) logical thinking itself can become a subject matter."

3. "The axiomatics themselves belong to the fourth level."

(P.M., 1958-59)

4. "one doesn't ask such questions as: what are points, lines, surfaces, etc.? ... Figures are defined only by symbols connected by relationships. To find the specific meaning of the symbols, one must turn to lower levels where the specific meaning of these symbols can be seen.

APPENDIX B

1. Van Hiele Geometry Test
   (Directions)

2. Van Hiele Geometry Test

3. Van Hiele Geometry Test
   (Answer Sheet)

4. Van Hiele Geometry Test
   (Answers)

VAN HIELE GEOMETRY TEST*

Directions

Do not open this test booklet until you are told to do so.

This test contains 20 questions. It is not expected that you know everything on this test.

When you told to begin:

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on your answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on this test booklet.
4. If you want to change an answer, completely erase the first answer.
5. You will have 30 minutes for this test.

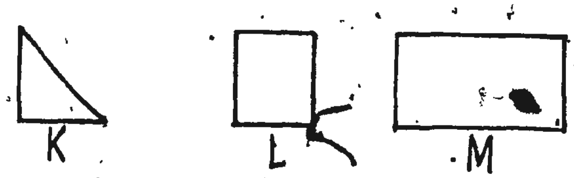
Wait until your teacher says that you may begin.

*This test is based on the work of P. M. van Hiele.

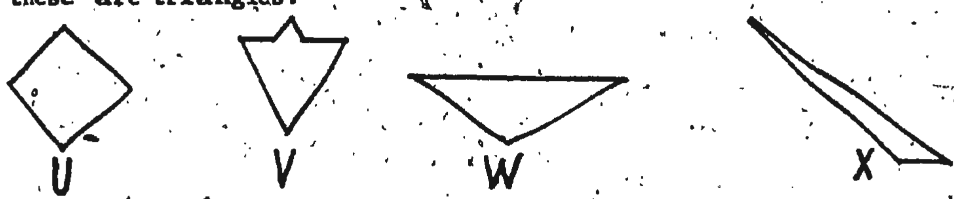
VAN HIELE GEOMETRY TEST

1. Which of these are squares?

(A) K only
(B) L only
(C) M only
(D) L and M only
(E) All are squares.

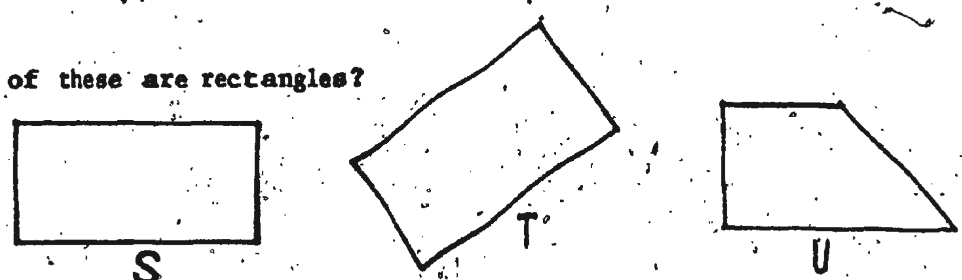


2. Which of these are triangles?



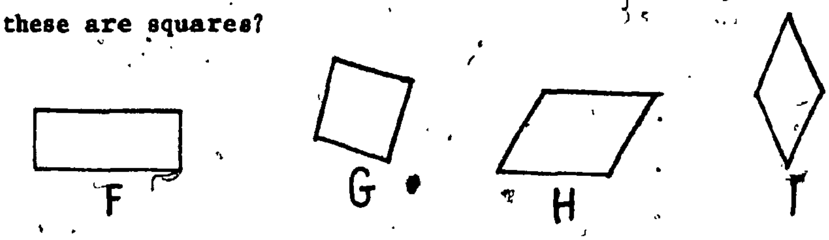
(A) None of these are triangles.
(B) V only
(C) W only
(D) W and X only
(E) V and W only

3. Which of these are rectangles?



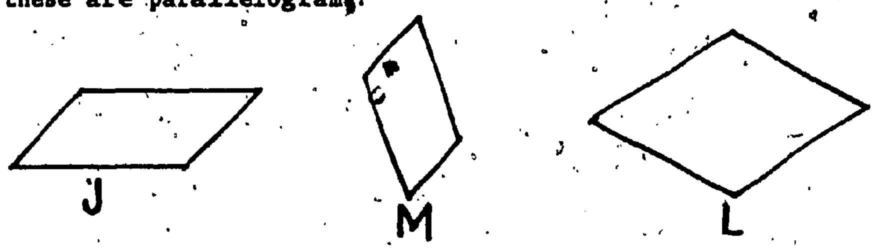
(A) S only
(B) T only
(C) S and T only
(D) S and U only
(E) All are rectangles.

4. Which of these are squares?



(A) None of these are squares.
(B) G only
(C) F and G only
(D) G and I only
(E) All are squares.

5. Which of these are parallelograms?

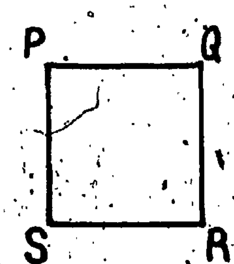


(A) J only
(B) L only
(C) J and M only
(D) None of these are parallelograms.
(E) All are parallelograms.

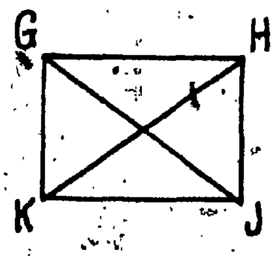
6. PQRS is a square.

Which relationship is true in all squares?

(A) $\overline{PR}$ and $\overline{RS}$ have the same length.
(B) $\overline{QS}$ and $\overline{PR}$ are perpendicular.
(C) $\overline{PS}$ and $\overline{QR}$ are perpendicular.
(D) $\overline{PS}$ and $\overline{QS}$ have the same length.
(E) Angle Q is larger than angle R.



7. In the rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the <u>diagonals</u>.



Which of (A)-(D) is <u>not</u> true in <u>every</u> rectangle?

(A) There are four right angles.

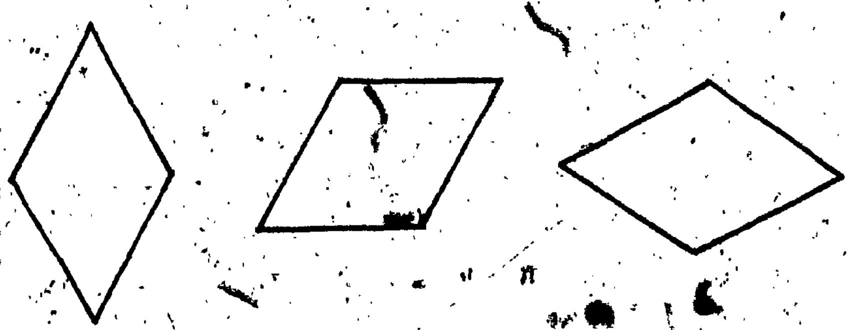
(B) There are four sides.

(C) The diagonals have the same length.

(D) The opposite sides have the same length.

(E) All of (A)-(D) are true in every rectangle.

8. A <u>rhombus</u> is a 4-sided figure with all sides of the same length. Here are three examples.



Which of (A)-(D) is <u>not</u> true in <u>every</u> rhombus?

(A) The two diagonals have the same length.

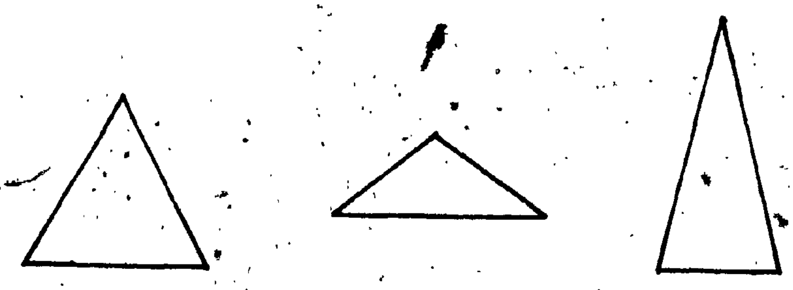
(B) Each diagonal bisects two angles of the rhombus.

(C) The two diagonals are perpendicular.

(D) The opposite angles have the same measure.

(E) All of (A)-(D) are true in every rhombus.

9. An isosceles triangles is a triangle with two sides of equal length. Here are three examples.



Which of (A)-(D) is true in every isosceles triangle?

(A) The three [illegible] must have the same length.

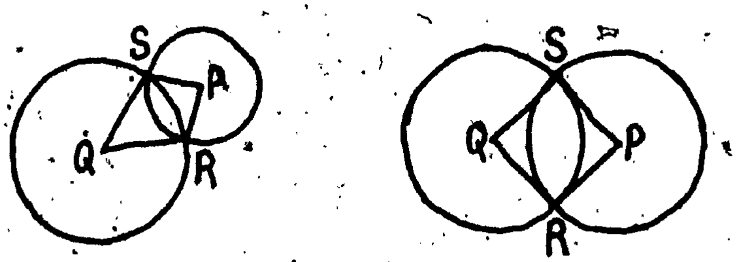
(B) One side must have twice the length of another side.

(C) There must be at least two angles with the same measure.

(D) The three angles must have the same measure.

(E) None of (A)-(D) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)-(D) is <u>not</u> always true?

(A) PRQS will have two pairs of sides of equal length.

(B) PRQS will ahve at least two angles of equal measure.

(C) The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.

(D) Angles P and Q will have the same measure.

(E) All of (A)-(D) are true.

11. Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which is correct?

(A) If 1 is true, then 2 is true.

(B) If 1 is false, than 2 is true.

(C) 1 and 2 cannot both be true.

(D) 1 and 2 cannot both be false.

(E) None of (A)-(D) is correct.

12. Here are two statements.

Statement S: Δ ABC has three sides of the same length.

Statement T: In Δ ABC, $\angle$B and $\angle$C have the same measure.

Which is correct?

(A) Statements S and T cannot both be true.

(B) If S is true, then T is true.

(C) If T is true, then S is true.

(D) If S is false, than T is false.

(E) None of (A)-(D) is correct.

13. Which of these can be called rectangles?

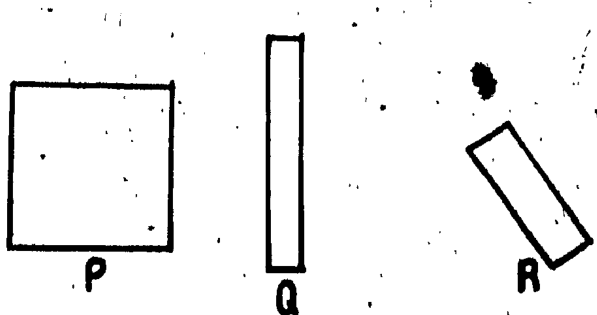
(A) All can.

(B) Q only

(C) R only

(D) P and Q only

(E) Q and R only



14. Which is true?

(A) All properties of rectangles are properties of all squares.

(B) All properties of squares are properties of all rectangles.

(C) All properties of rectangles are properties of all parallelograms.

(D) All properties of squares are properties of all parallelograms.

(E) None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?

(A) opposite sides equal

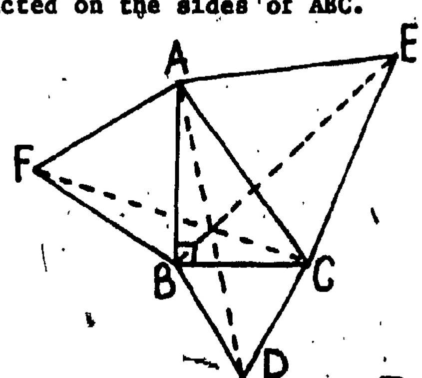
(B) diagonals equal

(C) opposite sides parallel

(D) opposite angles equal

(E) none of (A)-(D)

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(B) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(C) In any right triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(D) In any triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(E) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.

Property S: It is a square.

Property R: It is a rectangle.

Which is true?

(A) D implies S which implies R.

(B) D implies R which implies S.

(C) S implies R which implies D.

(D) R implies D which implies S.

(E) R implies S which implies D.

18. Here are two statements.

I. If a figure is a rectangle, its diagonals bisect each other.

II. If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

(A) To prove I is true, it is enough to prove that II is true.

(B) To prove II is true, it is enough to prove that I is true.

(C) To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.

(D) To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.

(E) None of (A)-(D) is correct.

19. In geometry:

(A) Every term can be defined and every true statement can be proved true.

(B) Every term can be defined but it is necessary to assume that certain statements are true.

(C) Some terms must be left undefined but every true statement can be proved true.

(D) Some terms must be left undefined and it is necessary to have some statements which are assumed true.

(E) None of (A)-(D) is correct.

20. Examine these three sentences.

(1) Two lines perpendicular to the same line are parallel.

(2) A line that is perpendicular to one of two parallel lines is perpendicular to the other.

(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

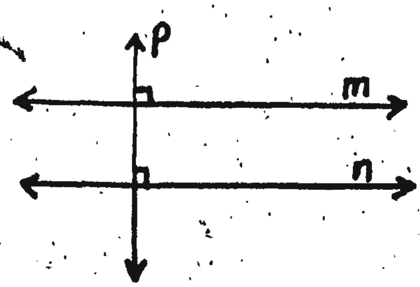
(A) (1) only

(B) (2) only

(C) (3) only

(D) Either (1) or (2)

(E) Either (2) or (3)



Name of School ____________________

VAN HIELE GEOMETRY TEST

ANSWER SHEET

Cross out the correct answer.

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E
16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E

Answers

1.	B	
2.	D	
3.	C	RECOGNITION
4.	B	
5.	E	
6.	B	
7.	E	
8.	A	ANALYSIS
9.	C	
10.	D	
11.	C	
12.	B	
13.	A	ORDERING
14.	A	
15.	B	
16.	C	
17.	C	
18.	D	DEDUCTION
19.	D	
20.	A	