THE EFFECTS OF SEX, LEVEL OF ABILITY, AND DIFFERENTIAL AMOUNTS OF PRACTICE ON IMMEDIATE AND DELAYED ACHIEVEMENT IN MATHEMATICS

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by

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A Thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Education

Department of Curriculum and Instruction
Memorial University of Newfoundland
August 1978

St. John's Newfoundland

ABSTRACT

The purpose of this study was to examine the relationship of drill and practice to mathematics achievement in skill and concept acquisition in grade five. To do this the experimenter examined the following major questions. Do any of the variables of sex, level of ability, and amount of practice result in significantly different achievement on an immediate posttest of concepts and skills or on a delayed posttest? In addition, are there any significant interactions among any of these variables on the immediate or delayed posttests?

To investigate these questions a unit on addition of fractions for grade five was developed, implemented, and evaluated. The study was conducted using 140 grade five students in five classes from both urban and rural Newfoundland communities. Students in each class were randomly divided into three groups. Each group was randomly assigned to one of three treatment conditions: five practice exercises, ten practice exercises, or fifteen practice exercises. Classes were held once a day for a total of fifteen class sessions.

To determine the students' achievement on the unit, two tests were administered. The first, the immediate posttest, was given at the end of the fifteen class sessions

following a review session. The second, the delayed post-test, was given one month later to test retention of the material covered in the class sessions. Both of these tests were constructed by the experimenter and were designed to test whether the behavioral objectives of the unit had been achieved. In an attempt to eliminate inadequacies, both the addition of fractions unit and the tests were piloted in a grade six classroom prior to conducting the study.

The data were collected and analyzed using a three factor analysis of variance procedure. Treatment differences were significant at the .05 level of significance on the immediate posttest, but not on the delayed posttest. To determine where the significant differences lay specifically, a Scheffé test was performed on the data. The results of the test indicated that students receiving fifteen practice exercises achieved significantly higher than those receiving five or ten. Achievement of students receiving five and ten practice exercises was not significantly different.

There were significant sex differences in achievement on both the immediate and delayed posttests with females scoring significantly higher than males. In addition, high ability students scored significantly higher than lower ability students on both tests.

There was no significant "sex by treatment" interaction on either the immediate or delayed posttests. In addition, there was no significant "ability by treatment" interaction on either of the tests. There was, however, a significant "sex by ability" interaction on both the immediate and delayed posttests. High ability males and females obtained approximately equivalent results, whereas lower ability females scored considerably higher than lower ability males on both tests. As indicated by the "sex by ability" interaction, the obtained sex difference in achievement occurred mainly in the lower ability groups.

Following the Scheffé procedure finding, students receiving the five and ten practice exercises were collapsed into one group and the data were reanalyzed. The results of this further analysis were similar to those of the initial analysis. Notable exceptions were that the treatment differences were significant at the .05 level on both the immediate and delayed posttests. Students receiving fifteen practice exercises achieved significantly higher than students in the combined five and ten practice groups on both tests. Also, the "ability by treatment" interaction on the immediate posttest was significant. High ability students achieved approximately the same results whether they received five, ten, or fifteen practice exercises. Lower ability students receiving five and ten exercises achieved approximately the same results, however, those receiving fifteen practice exercises achieved considerably higher results.

On the basis of these findings it was concluded that for grade five students practice does have an effect on mathematics achievement. However, the ability of the student must be considered in assigning that practice. The researcher recommended that further research be conducted at other grade levels and in other areas of the curriculum.

ACKNOWLEDGEMENTS

The writer wishes to express his grateful appreciation to the people who provided assistance and cooperation in conducting and reporting the study.

Gratitude is expressed to Dr. Dale Drost and Dr. Glyn Wooldridge for their direction, helpful criticism, and advice during the development and completion of this thesis.

Thanks are expressed to Mary Kennedy whose use of English grammar is something else; to my parents who put up with my eating and sleeping habits; and to Beth who provided moral support and patient understanding.

Sincere thanks are also expressed to the principals, and especially the teachers and students for their cooperation.

Finally, I dedicate this thesis to Len and Norma who learned to cope with graduate work before their time.

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CHAPTER I

INTRODUCTION

Purpose of the Study

The purpose of this study was to examine the relationship of drill and practice to mathematics achievement in skill and concept acquisition. Research on the effects of drill and practice in the past indicates that no consensus has been reached on the benefits of drill and practice. Researchers are divided on the issue of whether or not it is beneficial in improving achievement. Because of the lack of consensus this study was designed to examine this relationship in Newfoundland schools and to attempt to answer some of the questions related to the issue. If practice is beneficial, how much practice is beneficial, and to which students is it of benefit?

Definition of the Problem

The problem under investigation in this study was: Do different amounts of practice result in different levels of achievement in mathematics? In addition, is there any relationship between students' ability level and the amount of practice necessary to master a concept? Is there any relationship between sex of pupil and the amount of practice necessary to master a concept? Specifically, if the amount and type of instruction is held relatively constant, would different groups, each given a different amount of drill and practice, attain different levels of mathematical achievement on immediate posttesting? Furthermore, does the amount of practice have any effect on later retention as measured by a delayed posttest? In this study an attempt was made to determine the effects, if any, which these factors have on immediate achievement and delayed retention.

Background to the Problem

Two of the major schools of psychology have disagreed about the effects of drill on retention of learning. According to Brandt (1973), the Skinnerian school contends that drill is not only needed, but that intensive drill has a positive influence on learning and therefore improved retention will result. The Gestaltist school, he says, recognizes the importance of drill, but contends that because of the unique nature of the individual any prescribed program of intensive drill is questionable in its applicability.

Gay (1972) reported that, "In the search for ways to improve retention, one variable that has fairly consistently been found to have an effect is practice" (p. 466). Underwood (1964), a learning psychologist, has also reported evidence to this effect. Unfortunately, as Gay (1972) states, these findings have not been critically applied to the classroom. For example, in mathematics the number of practice examples assigned to students learning a new concept or skill has typically been uniform for all students and determined by the teacher or textbook. Generally, inadequate attention has been given to individual differences and the possibility that different students may require different numbers of exercises. In fact, if one ignores the students' ability level in assigning practice, any given amount of practice is probably uneconomical overlearning for some students and inefficient underlearning for others.

Most teachers, especially mathematics teachers, belong to either the Skinnerian or Gestaltist school of thought, as evidenced by the type of mathematics programs of instruction existing in our present schools. Traditionally, teachers tended to be Skinnerians. They tended to place great emphasis on practice and drill, hailing it as the solution to the understanding and retention problems of students. Modernist teachers, those teachers who are supporters of the recent individualized mathematics program movements, tend to be Gestaltists. This is evidenced by their procedure of assigning practice on the basis of the individual's ability and need. However, there also exist those teachers at the opposite extreme from the Skinnerians who believe that practice and drill add nothing to the mathematical understanding of the students, and therefore practice and drill are unnecessary and of no benefit in promoting retention of concepts and skills.

Gay (1972), in a study of practice assignment methods with grade eight pupils, concluded that traditional methods of mathematics instruction whereby all students receive the same amount of practice were not conducive to promotion of retention. However, Brandt (1973) found that practice amounts had no significant effect on arithmetic achievement. Brandt varied the amount of drill that different groups of students received and found that there were no significant differences in achievement. This seems to indicate that the results of studies to date have not been conclusive and more research is needed to resolve the issue. The results of Brandt's study call into question the benefits of using drill and practice at all. This is especially true when one considers the fact that one of his treatment groups received no practice exercises.

Principles and Uses of Drill

Sueltz (1953) lists several principles which have a basis in experiment and tested teaching practices. He states that these principles are generally recognized as sound and applicable to many situations in the teaching of mathematics. Two of these principles are:

> 1. Drill should be done with correct processes lest a child practice errors which need to be remedied later.
>
> 2. It seems that pupils of lower ability require more drill than more able students. (pp. 196-197)

Sueltz goes on to say that there is now general agreement that drill and practice, or recurring experience, is useful:

1. To gain proficiency in handling a mathematical process or procedure after it has been studied or its usefulness established.

2. To enhance or enlarge the understanding of a concept whose basic principle or idea has been established. (p. 197)

Ausubel (1968) reiterates these uses of practice or recurring experience, and adds that:

> Practice increases the dissociability strength of newly-learned meanings for a given trial and thereby facilitates their retention; it enhances the learner's responsiveness to subsequent presentations of the same material; it enables the learner to profit from inter-trial forgetting; and it facilitates the learning and retention of related new learning tasks. (p. 274)

In this sense both Sueltz (1953) and Ausubel (1968) have suggested that practice and drill are only useful after the concepts and skills have been developed instructionally by the teacher, and the student has a basic knowledge of the ideas to be practiced. Sueltz ends his commentary by saying that drill is both important and necessary in learning mathematics. However, because of the many variables which a teacher can only partially control it is not possible to prescribe precisely when, where, and how much practice is necessary.

Present Status of Mathematics Achievement

In a recent issue of Time magazine (November, 1977), a review of recent innovations in the schools and the present state of education gives an inside view of some classrooms in the United States. In a statement on declining performance reflected in student achievement, the Time writer stated, "After more than a decade of vaunted innovations, all signs indicate that today's students are more poorly prepared in basic skills than were their predecessors" (p. 58).

The National Advisory Committee on Mathematical Education (NACOME), in an overview and analysis of mathematics in our schools, stated that, "Recent trends in achievement vary according to grade level, geographical region, and instructional emphasis" (p. 103). Furthermore, as measured on the Scholastic Aptitude Test (SAT), the broadest measure of nationwide educational achievement, college entrance scores have been falling slowly but steadily since 1962. Average mathematics scores have fallen from over 500 to 472. Perhaps more significantly, the NACOME report states, is the fact that the percentage of scores above 600 in mathematics declined from 20.2 to 16.4 (p. 107). This result indicates that the expectations that gifted students would at least benefit from the infusion of money, technology, and new material were not realized.

The NACOME writers state further that the results of the California testing program showed a decline in mathematics achievement during the years 1969-1973. For example, in grade six the median score on the Comprehensive Test of Basic Skills mathematics test dropped from 47 in 1969 to 38 in 1971, and then remained constant for two years (p. 104). Related to this is the five year National Longitudinal

Study of Mathematical Abilities (NLSMA, 1961). The NACOME report states that, "The study showed that there is a clear trend for modern textbooks to be associated with poorer performance on computation scales, however, they are associated with widely varied patterns of performance on the other scales" (p. 111). Some of these scales are comprehension, application, and analysis.

The beginning of this decline in achievement scores on national standardized tests in the United States and Canada coincided almost exactly with the beginning of modern mathematics programs which the NACOME writers define as "the multitude of mathematics education concerns and developments of the period 1955-1975" (p. 137). Probably the major feature of modern mathematics programs of interest to the present study is their emphasis on understanding and a de-emphasis on drill and practice. This is in direct opposition to Sueltz's (1953) and Ausubel's (1968) statements on the usefulness of practice and drill. An argument could be made that drill does not lead to increased understanding. This may be true if drill and practice is being assigned only for the sake of drill; however, drill that is assigned for the expressed purpose of furthering understanding through use of concepts and skills already taught can be made to play a vital role in a child's intellectual development. In addition, drill and practice on these concepts and skills can be designed to have practical application in the child's real world experience.

Significance of the Study

The author is aware that very little coordinated effort has been made to examine the ways in which students learn. Educational agencies have made individual and sporadic attempts to examine this issue. For example, studies such as those by Shipp and Deer (1960) and Zahn (1966) have, to a degree, established that higher achievement results if over half of class time is devoted to instruction of a developmental nature. However, they have not established whether it is at all beneficial to provide practice and drill after this instruction has been completed.

While it is recognized that such small studies have limited generalizability, larger studies are frequently not possible because of a lack of accessible funding. In the meantime, studies such as the present investigation will add needed evidence on the procedure of assigning practice in the classroom as part of an overall instructional strategy.

The NACOME report recommended that there be continuing attempts to find a sound empirical basis for the recommendation of particular patterns, methods, and materials of instruction. It further recommended that, "Once goals are clearly established concerning desired computational ability, research is needed to identify the techniques, and balance of rationalization and practice that are optimal for attainment of these goals" (p. 144).

While this is a worthy undertaking, it is beyond the scope of this study. However, this study finds its significance in that it is part of a continuing attempt to identify the relative benefits of different practice and drill amounts to increasing mathematical achievement.

As was previously stated, most mathematics teachers tend to belong to either the Skinnerian or Gestaltist school of thought on the usefulness of drill and practice. This study was an attempt to partially answer some of the questions surrounding the issue of drill and practice use in mathematics. It is recognized that such a small study, with its limited scope and sample size, will not solve the issue definitely. It attempted, however, to fulfill the recommendations of the NACOME report. Given that improving mathematics achievement is an established worthwhile goal, this study was an attempt to examine the methods of assigning practice for their relative usefulness in attaining this worthwhile goal.

Definition of Terms

For the purpose of this study, the following definitions are stipulated:

Practice is defined as recurring experiences, and refers to those aspects of learning that possess elements of similarity or sameness which repeat or recur. In this study, the word practice will be used within the framework that some authors such as Sueltz (1953) refer to simply as "drill."

Amount of practice refers to the number of practice exercises that a student is assigned on a concept after instruction has taken place. Students were assigned to one of three treatment groups given three different amounts of practice exercises. Treatment A was given five practice exercises; Treatment B--ten practice exercises; and Treatment C--fifteen practice exercises.

Ability refers to the average of achievement grades in composite language arts and mathematics at the end of grade four.

High ability students refers to those grade five students who obtained an average of greater than 75 in composite language arts and mathematics scores at the end of grade four.

Lower ability students refers to a composite of medium and low ability students. These students were those who obtained an average of less than 75 in composite language arts and mathematics scores at the end of grade four.

Immediate posttest refers to a test which measured concepts and skills taught during the designated instructional period, and administered on the day immediately following completion and review of the instructional unit.

Delayed posttest refers to a test which measured concepts and skills taught during the designated instructional period, and administered one month after completion of the instructional unit.

Delimitations

The results of this study are delimited in their generalizability to the area under investigation--mathematics--in particular, the addition of fractions. Because research has indicated that the amount of practice necessary to completely master a concept varies in different subject areas, and in the use of material of lesser or greater difficulty, it is unlikely that these results could be generalized to other areas of the curriculum without further research.

The generalizability of the results of this study are further limited to the grade level of the students involved in the study--grade five. The findings of research in the area indicate that the effects of practice vary with the grade level of students, as well as with content material. However, it may be possible to generalize to students in one grade above and below the grade five level students being considered here, since they have basically the same psychological and physiological characteristics.

Organization of the Thesis

The statement of the problem, the background of the problem, the significance of the study, the definition of terms used, and a description of the delimitations of the study have been presented in this chapter.

A review of the literature follows in Chapter II with an emphasis on the effects of practice and drill on learning and retention of mathematics concepts and skills. The research has generally taken the form of a comparison of some drill and practice procedures with other modes of instruction. Research findings related to sex differences in achievement will be presented. Research on homework of the drill and practice type will be reported to establish that it is a potential source of influence, and therefore for the purposes of this study must be eliminated. A summary of the review of the literature, and suggestions from that research will conclude Chapter II.

Chapter III contains the design of the study, including: the selection of the sample and a brief description of the population, the selection and preparation of the instructional materials, the choice and preparation of instruments employed, the statement of the hypotheses, the procedures used in the investigation, further limitations of the results, the statistical tests used in the analysis of results and significance levels accepted.

A report of the findings will be presented in Chapter IV. A discussion of the reported findings is presented in Chapter V. The study is summarized and conclusions drawn from the study are presented. Recommendations evolving from the study conclude the thesis.

CHAPTER II

REVIEW OF LITERATURE

The following review of related literature and commentary makes one become immediately aware that there is no consensus of opinion as to the usefulness or lack of usefulness of practice and drill in mathematics learning. This review is a presentation of findings of studies by researchers who have attempted to establish the benefits of drill as an aid in improving mathematics achievement.

Rousseau (1972), in a study of different methodologies for teaching arithmetic at the grade four level, concluded that there were no significant differences in retention of division algorithms. However, for extension of the algorithms to cases of slightly different or greater difficulty the rote algorithm, or drill and practice procedure, is superior to the other modes of presentation such as distributive, quotitive and partitive. By "cases of slightly greater difficulty" Rousseau means the application of knowledge and skills to problem solving situations.

Wright (1970) did a study involving a comparison of a modern mathematics program which emphasized understanding and a traditional mathematics program which emphasized drill and practice. He concluded that there were no differences between groups when tested on a traditional mathematics test of achievement requiring recall of knowledge of materials learned. However, students in the modern mathematics program scored significantly higher on modern mathematics tests of understanding of concepts. This is in almost direct opposition to Rousseau's (1972) finding. If one examines Bloom's (1956) taxonomy of cognitive skills and abilities, one finds that the levels are arranged in ascending order of complexity: knowledge, comprehension, application, analysis, synthesis and evaluation. According to the taxonomy the comprehension (understanding) stage is at a more complex level than straight recall of knowledge. In Rousseau's study the drill and practice (rote) algorithm was superior when extending learning to cases of greater understanding; however, Wright reported that the modern program was superior when extending learning to cases of greater understanding.

Riedesel (1970), in a review of research contributions to elementary school mathematics, reported that researchers have found that rote rule or drill, and meaning methods produce about the same results when immediate computational ability is used as a criterion; however, the meaning method is superior to drill and practice if long term retention is used as the criterion.

Davies (1972), using a computer-assisted-instruction program (grades 2-6) in drill and practice in elementary school mathematics, found that students on the computer drill-and-practice program did significantly better than those not using the computer. Students preferred the media to other types of instruction such as texts because the system gave individual drill and practice, and improved competency. This study has a direct bearing on the purpose of the present study. From Davies' study it appears that the method of assigning practice from a textbook, whereby every student receives the same treatment, is not a desirable practice at least not for all students. From the results of the study and the comments by students, it appears that students perceive a need for practice, but this practice must be assigned on an individualized needs basis.

In a study of method of feedback using practice worksheets in grade five, Morrell (1970) found that all groups using the practice worksheets obtained significant achievement gains. Under these treatment conditions all students received the same amount of practice. Relating the results of this study to Davies' (1972) study it might be hypothesized that even more significant gains might have been made if this practice had been assigned on the basis of individual need.

Shipp and Deer (1960) and Zahn (1966) reveal that in elementary school mathematics maximum achievement is obtained when over half the time is devoted to developmental activities. Utilizing four treatment groups A, B, C, and D with amount of time spent on developmental activities being 67 per cent, 56 per cent, 44 per cent, and 33 per cent, respectively, both researchers reported a trend toward higher achievement when the per cent of class time spent on developmental activities was increased. In both studies the A and B groups scored significantly higher than group C and D on tests of achievement. These studies are evidence against the belief that more practice makes for better understanding. The results of these studies do not imply that practice is not beneficial. However, analysis of the results does imply that the greatest benefits of practice are achieved only after the concept has been well developed through instruction.

Preston (1974) distinguished between two phases of acquisition of permanent knowledge--a learning phase and a remembering phase. The learning phase is the time required to master the material; whereas the remembering phase involves the ability to store, retain, and retrieve this knowledge after a given interval. Preston studied the effects of devices such as extra practice on retention. He found that in tests of retention there were no significant differences between practice and no-practice groups after three and five weeks. The instructional procedures varied for different treatment groups only in the final stage--one group received practice and the control group received no practice. The results lend partial support to Riedesel's (1970) findings in that the instructional or developmental activities appear to be the most important promoter of retention. The results imply that practice adds very little to the improvement of retention.

Cohen (1970) found that a conventional textbook and chalkboard approach using drill and practice produced significantly higher gains in achievement than a laboratory approach on a teaching unit on fractional concepts and computation. The laboratory approach which utilized a variety of manipulative and multisensory materials along with a student centered teaching approach required much more time than the conventional approach. No significant differences were found on tests of understanding of concepts, but the conventional approach group scored significantly higher on computational ability tests.

Puglisi (1970) conducted an experiment with grade six classes to determine if the effects of programmed and drill type supplemented self-instruction on mathematics achievement differ. Three supplemental self-instruction groups were used: (i) Teacher-directed using programmed materials; (ii) Pupil-directed using programmed materials, and (iii) Teacher-directed using drill materials. Finding no significant differences, Puglisi concluded that in terms of their effect on mathematics achievement each procedure warrants equal consideration. This result indicates that if students need supplementary work after completing regular classroom instruction, a drill and practice type supplement is as beneficial as further instruction using programmed material in improving achievement.

Hirschi (1971) carried out a study with grade six students concerned with improving the arithmetic computational skills of intermediate grade students. Suppes' and Jerman's Individualized Mathematics Drill and Practice Program was selected for use in the study. This program provides for students of differing abilities, allows for some self-pacing, and is geared to the student's level. Hirschi compared this program with regular classroom instruction whereby all students of differing abilities receive the same amount of practice. Hirschi concluded that the drill and practice program was significantly greater than the regular classroom instruction in terms of achievement gains in computational skills. This result indicates that computational skills of sixth graders can be improved by using an individualized drill program in the regular classroom. This result lends further support to a statement made earlier in Chapter I that any practice or drill program must take into consideration the nature of the individual's need for practice, and must be assigned in proportion to that need. The results of the Hirschi study also lend support to Davies' (1972) findings. While Davies employed a computerized program in his study, in both cases the major feature of the programs was the assignment of drill and practice on the basis of the needs of individual students.

Gay (1972) adds to this by stating that students instructed by the fixed method of assigning practice exercises were inferior to students in the retention index group in terms of achievement. Under the fixed method of assigning practice everybody received the same amount, whereas students in the retention index group were assigned practice on an individual needs basis. The fixed method was also inferior to the choice group where students decided how many practice exercises were needed. This superiority was demonstrated on measures of acquisition, immediate retention, and delayed retention. This result seems to indicate that the high levels of forgetting that are often reported, such as by Tyler (1934), may be a result of the inadequate strategies used for providing or assigning practice to students after a topic has been taught. Tyler reported that if students are examined on a subject a month or a year after they have written the original exam, their scores show an alarming drop.

Ausubel (1968) supported this suggestion when he concluded that if adequate attention were paid to such considerations as optimal review, i.e., in proportion to needs, students might retain over a lifetime most of the important ideas they learn in school.

Schubert (1972) states that, "Most teachers feel that in a general way the learning that takes place during a given period of time is proportional to the amount of practice or repetition a child engages in" (p. 80). Consistent with this principle, he says, is the belief that the pupil should practice if he wishes to learn. This statement lends support to the statement made earlier that most teachers belong to either the Skinnerian or Gestaltist school of thought on practice procedures. However, it also indicates that most classroom teachers assign practice as a general aid instead of assigning practice on the basis of student needs. The questions that arise from this statement are: How much practice is beneficial? Do different amounts of practice affect achievement? Does ability of the students determine how much practice is needed? This, as has already been stated, is the purpose of the present study.

Brandt (1973) studied the effects of drill and practice in the classroom. He assigned students to three different drill groups--those receiving no drill exercises; ten drill exercises; and twenty-five drill exercises. Using a pretest-posttest design he obtained results which indicated no significant differences in mathematics achievement between any of the drill groups. This study is somewhat similar to the present study in actual design; however, the study by Brandt failed to account for other variables that could have potentially influenced the findings of his research. These potential influences are: (1) sex of the student; (2) amount of practice performed outside the classroom, i.e., homework; (3) ability levels of students; and (4) amount and type of classroom presentation of instructional materials. In addition, the study by Brandt failed to examine the long term effects of such practice differences, as Campbell and Stanley (1963) suggest should be done.

Sex and Ability

The following review of research on the sex and ability variables as potential factors in assigning practice will give some indication as to the status of these variables.

Carruth (1970), in a study of grades four, five and six, found that the sex of a pupil had no significant effect on current achievement after different practice treatments. However, he found that previous levels of mathematics achievement, intelligence, and reading ability were the most important variables having an effect. These variables accounted for 65 per cent of the variance in achievement of students in the drill and practice groups. In addition, Hirschi (1971), in a study of the effects of a practice and drill program, discussed earlier, reported that there were no significant differences in arithmetic computational skills between boys and girls. This result adds support to the Carruth (1970) study. Furthermore, Hirschi reported that students of higher ability are more likely to achieve a higher score in arithmetic computational tests.

Also supporting these findings of no significant sex differences is a study by Grant (1971). He found that mean differences from compulsory practice homework treatments between boys and girls on tests of computation and concepts were not significant. Grant also reported that there were no significant differences between low and high IQ groups. This result is contrary to what Carruth (1970) and Hirschi (1971) reported.

However, Layton (1932) and Renay (1938), in research on sex differences, indicated that superiority of either sex varies with the nature of the material. In addition, the results of the previously described study by Gay (1972) suggest that the superiority of instructional methods is dependent on sex. Gay reported that the index method of assigning practice was better for females, but males did better if given a choice of amount of practice. This result was similar on both immediate and delayed posttests.

Adding further support to the findings of significant differences between sexes is a study by Engle and Lerch (1971) who stated, "It seems reasonable to reject the hypothesis of no differences between boys and girls in ability to respond correctly to computational type exercises" (p. 333). From the analysis of the results of the study it was concluded that girls were better than boys in responding to both conceptual and computational types of exercises.

Morrell (1970) found significant sex differences in achievement on practice worksheets. However, the interactions that resulted were further complicated by previous achievement differences. An analysis of the interaction of sex and previous achievement indicated that high achieving boys scored significantly higher than high achieving girls on practice worksheets; while low and medium achieving girls did better than medium and low achieving boys. These results generally coincided with the findings of Unkel (1966) and Parsley (1964).

The research cited here emphasizes the disparity of opinions and conclusions that exist in the research community on the effects which the sex and abilities of students have on achievement using practice and drill programs. However, the general consensus seems to be that, at least at the elementary school level, the sex and ability of the student does affect achievement. The direction of these sex differences in achievement may vary with the nature of the material being learned, and the level of prior achievement. In any case, prior research evidence has been substantive enough to suggest that when studying the effects of practice assignments to students, sex and ability are variables to consider as having a potential effect on achievement.

Amount and Type of Instruction

As previously mentioned, Shipp and Deer (1960) and Zahn (1966) revealed that in elementary school mathematics maximum achievement is obtained when over half the time is devoted to developmental teaching instead of student practice. These studies indicate that the amount of class time devoted to instruction does affect achievement. These conclusions indicate that for the present study, in order to determine the effects which different amounts of practice have on achievement, the amount of actual classroom instruction must be held relatively constant in all classrooms.

Literally thousands of studies have been carried out in an attempt to determine the best type of classroom presentation or instructional strategy to use. These studies have compared and contrasted numerous modes of presentation such as the discovery approach, expository approach, laboratory, activity, individualization, grouping, programmed instruction, problem solving, sequencing, computer-assisted instruction, etc. The question that arises is: Which is the best mode of presentation?

Some of the larger studies which have been carried out in recent years have attempted to answer this question. Results are quite contradictory and vary from one study to the other. Brophy and Evertson (1976) reported that many kinds of teaching may work in many subjects at a certain grade level, but they may only work for certain students. Rosenshine (1974) reported that after examining grade level and subject area variations of teaching behavior and student achievement, none of the results were particular to any combination of grade level or subject area. In addition, MacDonald (1976) concluded that no teaching practices correlated with pupil achievement in both arithmetic and reading at the second and fifth grade levels.

There appears to be no one best answer. After examining these and other numerous studies, the answer to the above question can probably best be determined by examining pupils' abilities, pupil needs, the nature of the program content, environmental setting, and physical plant facilities. The only real agreement among researchers is that different modes of presentation have different effects on achievement for different students. This indicates that for the present study, in order to determine the effects which different amounts of practice have on achievement, the type of classroom instruction must be held relatively constant in all classrooms.

Homework

As has already been stated, another factor to be considered as influencing the results of practice treatments in the classroom is the amount of homework the student does, or the amount of extra practice the student gets outside the practice treatment of the experiment.

Homework, as referred to here, is of the reinforcement or practice type, i.e., extra practice that is assigned to the student to be completed at home. As the following research review demonstrates, the effects of practice homework on achievement are not at all firmly established.

Otto (1950) stated that, "Homework is not significantly related to achievement as measured by teachers' marks and standardized tests" (p. 380). Opposed to this is a finding by Vincent (1937) who, in a study of homework versus no homework in grades five and six, found that results slightly favored the homework group. However, Mulry (1961) stated that, "There is little conclusive evidence available concerning the positive or negative effects of home study, either the regular assigned homework, or the voluntary assignments" (p. 49).

Cooke and Brown (1935) and Teahan (1935) found no significant differences between groups receiving homework and those receiving no homework. However, Crawford and Carmichael (1937), in a three year study involving grades five to eight, found differences favoring the homework group. In a follow-up study these same researchers found that those same students in the no homework group received lower achievement scores in high school. Again, the results are contradictory; however, a long term study such as that carried out by Crawford and Carmichael (1937) adds considerable strength to the argument for assigning homework.

Goldstein (1960), in a summary of these earlier studies, concluded that "regularly assigned homework favors higher academic achievement . . . however, it is . . . more important at some grade levels than at others, in some subjects than in others, and for some pupils than for others" (p. 221).

These statements give some idea as to the lack of consensus of opinion as to the relative benefits of homework versus no homework. To give a clearer picture of the status of homework a look at further research might be helpful. More recent research on the issue indicates no change in the status of homework.

Koch (1965), in testing the effects of homework in grade six arithmetic, found significant differences favoring the homework group on tests of arithmetic concepts. He concluded that homework of the reinforcing type can increase arithmetic achievement; however, with the available data it was impossible to say that homework will increase achievement in arithmetic as a general statement of effect.

Gray and Allison (1971) reported no significant differences on tests of computation and understanding between homework and no homework groups. However, Maertens and Johnston (1972) reported, in a study of the effects of drill and practice homework on achievement in fifth and sixth grade arithmetic, that in every case the means of the homework group were higher than the no homework group on computational and problem solving tests.

Because of the contradictory findings and the possibility that homework does have an effect (whether negative or positive), it was deemed desirable to eliminate this variable as a potential source of influence. By taking this and other safeguards the researcher attempted to eliminate sources of influence other than the experimental treatment effects. The researcher realized that it is nearly impossible to have a completely controlled environment in a regular classroom; nevertheless, it was in the interest of reliability of results that such obvious influences be eliminated or reduced.

Summary of Literature Review

The research studies reviewed in this chapter on the effects of drill and practice contain many conflicting results. The one agreement among all the research findings is that practice is not detrimental to achievement, i.e., is not an inhibiting factor. Most studies reviewed in this chapter have agreed that practice and drill are effective in promoting computational achievement. On the issue surrounding understanding of concepts researchers have been less than unanimous in their findings. Research findings indicate that maximal conceptual understanding is achieved through classroom instruction followed by practice. In order for practice to achieve its greatest effect it appears that practice and drill must be assigned on the basis of the individual's need for that practice. In addition, research findings have suggested that practice and drill interact with such variables as sex and level of previous achievement. However, researchers' conclusions regarding the direction of these interactions are not in complete accord.

As has already been stated, the purpose of this study was to examine the effects of amount of practice on immediate achievement and long term retention. Given the contradictory findings in the area of the relatedness of practice and achievement, an attempt was made to resolve the direction of this relationship and provide evidence either for or against the use of practice exercises as a means of improving mathematical achievement.

To delve further into this relationship and the question of how much practice to assign, other variables that previous researchers have suggested may influence the effects were investigated. These variables included amount of practice, level of prior achievement of the students, sex, and the interaction effects of these variables on immediate and delayed mathematical achievement.

CHAPTER III

PROCEDURE AND STATISTICAL DESIGN

Introduction

The design of this study and the procedures employed in conducting the research are presented and elaborated upon in this chapter. The following headings will be used: (1) sample; (2) experimental design; (3) procedure; (4) instrumentation; (5) limitations, assumptions and controls; (6) hypotheses; and (7) statistical tests and significance levels.

Sample

The sample for this study consisted of two grade five classes from two schools in the St. John's urban area, two grade five classes from a school in the vicinity of St. John's (urban/rural), and one grade five class from a school in a rural Newfoundland community. This sample, therefore, contained students from different cultural and socio-economic backgrounds. There were a total of 140 students in the sample.

For the purpose of this study and given the limited resources, it was decided that the maximum number of students that could be handled was 150. These schools were chosen for the following reasons: (1) It was possible to maintain close contact with these schools; (2) these schools agreed to participate in the study; and (3) the grade five teachers were enthusiastic about participating in the study.

These schools were not selected for any particular reason other than those listed above. The researcher had had no previous contact with any of these schools. It is therefore assumed that these students are representative of grade five students in Newfoundland.

Subjects were classified as above average or below average in ability according to grade four classroom performance in mathematics and language arts. Each class was randomly subdivided into thirds. A third of each class was randomly assigned to one of three treatment conditions: five practice exercises, ten practice exercises, or fifteen practice exercises.

<u>Experimental Design</u>

The design for this study was a modification of Design 6 as given by Campbell and Stanley (1963). Design 6 is a Posttest-Only Control Group Design:

R	X	O
R		O

This design, as described by Campbell and Stanley, controls for threats to internal validity such as history, maturation, testing, instrumentation, regression, selection, mortality, and the interaction effects of these factors.

The modified Campbell and Stanley (1963) Design 6 was used according to this format:

R	X_0	O_1	O_2
R	X_1	O_1	O_2
R	X_2	O_1	O_2

where R represents the random assignment of students to the treatment conditions, Xs represent the treatment conditions, and O_1 and O_2 represent the immediate and delayed posttests, respectively.

This design controls for pretesting effects by eliminating the pretest for all groups, assuming that the selection procedure has equated the groups. Campbell and Stanley warn us against pinning all our experimental evaluation of teaching methods on immediate posttests and recommend that posttesting should be carried out at various intervals after instruction. This should be included as part of the design. It was for this reason that the delayed posttest was included. This gives the special advantage of determining the effects of the treatments over time. In fact, Campbell and Stanley state that, "Multiple O's should be an orthodox requirement in any study of teaching methods" (p. 33).

To further test the effects of other variables selected for examination such as sex of the student and initial ability, this design was further modified to examine the relationships between this variable and the treatment conditions.

In summary, a 2 x 3 x 2 factorial design was employed using the variables treatment, sex, and ability. This design is summarized in Figure 1. The relationship to Campbell and Stanley's Design 6 is also represented.

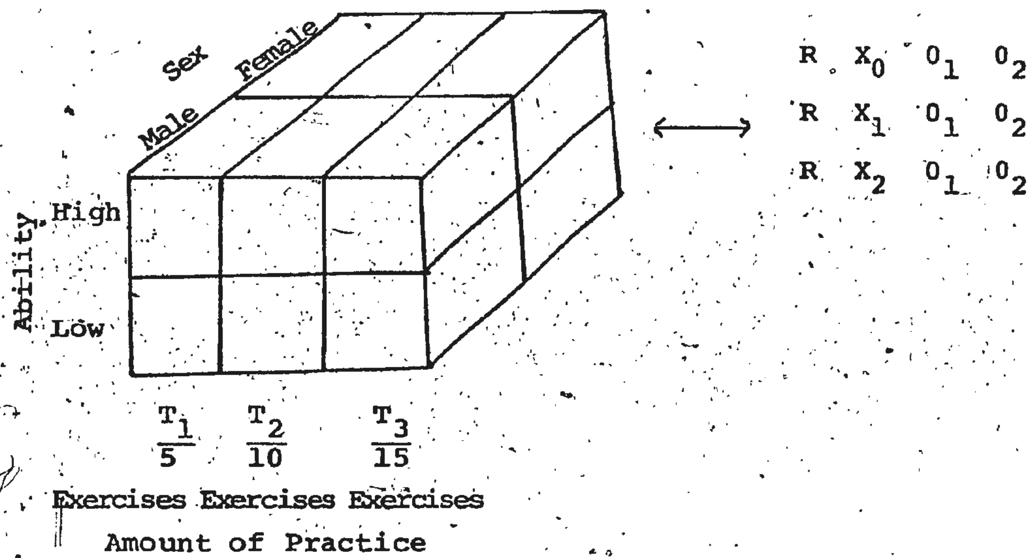

Figure 1. Factorial Design employed in the study and its relationship to Campbell and Stanley's Design 6.

Procedure

An equal number of students in each class was randomly assigned to each of the three treatment conditions: Treatment A--five practice exercises; Treatment B--ten practice exercises; and Treatment C--fifteen practice exercises. Following assignment to the treatment conditions, records of the previous years language arts and mathematics scores were obtained to determine whether students would be in the high or low ability group in the subsequent analysis of data.

All subjects were taught a unit on addition of fractions. This topic was chosen to concur with the grade five curriculum program topic being taught at that time in the schools. The unit was designed by the researcher in consultation with content and learning experts in the field of mathematics education. It was designed to take students from the basic concept of a fraction through to addition of fractions with unlike denominators--including opportunities for application to everyday events. Examples of the objectives of the unit were: (1) to develop a strong conception of fraction units; and (2) to develop skills in manipulating and comparing fractions in terms of their relative size ordering. Fractions are presented in the unit as an extension of the whole number system and the operation of addition is introduced as being the same operation used with whole numbers. Activities are included which involve the child in both manipulation of physical models and teacher-child communication. The actual instructional objectives of the unit are included in Appendix A together with the unit.

This instructional unit was designed for use by the instructor and contains a sequential step by step development of fractional concepts. However, it does offer some suggestions for alternative development to suit different students. For example, in developing the concept of equivalent fractions, activities are included at the concrete, semi-abstract, and abstract level of instruction. Throughout the unit the teacher has the option of using an expository or activity approach, or some combination of these approaches.

This unit was piloted in a grade six classroom for a four week period. This was done to detect any development flaws and presentation problems before implementation in the experimental study. The minor changes recommended by the teacher were made in consultation with learner and content specialists in the area of mathematics education.

Before this study began, all teachers were given the instructional unit in order to familiarize them with its content. Teachers were given directions on how to proceed with instruction (see instructional unit in Appendix A). Teachers were instructed to follow the package presentation as closely as possible, and to note any deviations. There were no significant deviations as teachers noted that the unit was fairly comprehensive in scope and well sequenced.

Teachers were directed to ask at the end of instructional time in each class period, "Are there any questions?" Most questions were asked and answered at this time. Teachers were instructed to attempt to minimize their helping the students (in terms of further instruction) after the practice exercises were distributed. This procedure was followed as closely as possible, although some teachers did report helping students who were having a difficult time with the practice exercises. Students who were assigned to the five and ten practice exercise treatments and completed the exercises before the end of the period were assigned reading or allowed to work at other courses until the other students finished.

Class sessions were run for 40-50 minutes each day. Subjects were taught by the regular classroom teacher for 50-60 per cent of the time, followed by the appropriate number of exercises for the practice treatment to which the student was assigned. These completed exercises were collected by the teacher, corrected, and returned to the students either at the end of the session or at the beginning of the next class session. Five class sessions were held per week--one each day. These sessions continued for three weeks for a total of fifteen sessions. There were two exceptions to this time schedule since other school activities conflicted with mathematics class time. This prolonged the treatment time in these cases. However, all possible attempts were made to keep the instruction time and duration approximately equal for all groups. It must be remembered that this unit was run as part of the regular school curriculum and in a school situation it is not entirely possible to have a completely controlled situation.

After completion of the instructional unit, all subjects in all conditions were given a one class review session. On the day immediately following the review session, all subjects were given a posttest on all the concepts and skills taught over the three week period (see Appendix B). These results were collected by the classroom teacher and corrected. The researcher checked the grading of these tests.

To further test the effects of the different practice treatments on retention over time, another posttest was given one month after termination of instruction (see Appendix C). During this time the teacher continued with his (her) regular instruction of other topics to be covered in the grade five mathematics curriculum.

All results of both immediate posttest and delayed posttest were tabulated and statistically analyzed using the appropriate statistical tests to be discussed later in this chapter.

Instruments

Ability:--The researcher did not devise an instrument to measure ability levels of students as scores indicating ability were available on all students. To measure ability, year-end achievement scores in both language arts and mathematics at the end of grade four were used. These scores were available from all schools participating in the study. These scores for each student were arrived at by taking the average of scores in the many component parts of each program. For example, in language arts scores were available on vocabulary, oral reading, comprehension, phonics, and spelling. To obtain an overall language arts score for a student the mean was taken of scores on these component parts.

These scores were accepted as indicative of ability levels of students because according to Fransecky and Debes

(1972) achievement in both language arts and mathematics is based on the students' ability to encode and decode a digital communications system. Visual literacy theorists such as Fransecky and Debes define a digital code as one whose components or signs in no way reflect on the visual image of the object described. Digital codes facility is the ability to work with abstract symbols such as words and numbers, and this they feel is precisely what is involved in language arts and mathematics study particularly in the primary and early elementary grades.

Aiken (1971) has reported a strong, positive relationship between intelligence and mathematical achievement, and between mathematical ability and mathematics achievement. Muscio (1962) has also reported evidence to this effect. In addition, Muscio reported that mathematical ability was found to be related to certain reading factors. When intelligence was partialled out the relationship between mathematical ability and mathematical achievement was still a significant relationship. Armstrong (1975) concluded that, based on the available evidence, mathematical ability and intelligence are strongly and positively related to mathematical achievement.

Given this strong positive relationship and a statement by Armstrong (1975) that performance implies or reflects the intelligence and mathematical ability of the student, it was decided to accept achievement scores at the end of grade four as indicative of the ability levels of the students.

<u>Immediate and Delayed Posttests</u>:--These tests were designed by the researcher to test the stated behavioral objectives of the instructional unit on addition of fractions. To establish the validity of the tests, four learner and content specialists in the field of mathematics education were consulted. These people examined all test items for such things as relevance to behavioral objectives and difficulty level. Furthermore, these people examined each test item to determine if the wording of the question was suitable for grade five students. To further establish their validity, these tests were carefully scrutinized by an instructional development expert at Memorial University of Newfoundland, who was involved in providing guidance in the development and testing of instructional materials.

These people recommended that minor changes were needed and aided in making these changes. In addition, these instruments were tested in a pilot study in the fall of 1977. The teacher who piloted the program made recommendations for minor changes in the wording of a few test items. Some of these recommended changes were made on the further advice of instructional development experts at Memorial University of Newfoundland.

The split-half method was used to test the reliability of the tests. Two subtests were formed, one consisting of the odd numbered items and the other consisting of the even numbered items. Scores were obtained on the two halves and these were correlated. The half-test reliability coefficient for the immediate posttest was 0.84. Using the Spearman-Brown formula the reliability coefficient for the whole test was 0.91. The half-test reliability coefficient for the delayed posttest was 0.89. Using the Spearman-Brown formula the reliability coefficient for the whole test was 0.94.

To further test the reliability of the immediate and delayed posttests, a Pearson Product-Moment Correlation Coefficient was calculated for students' scores on the two tests. The resulting correlation coefficient for the two tests was 0.83.

Each test consisted of nineteen test items designed to cover all stated behavioral objectives. Some examples of test items follow (see Appendices B and C for complete tests).

I. Explain the difference between a factor and a multiple. Use an example if necessary.

II. Tell whether these pairs of fractions are equivalent. Show how you get your answer.

(a) $3/5$ and $2/3$ (b) $5/8$ and $10/16$

Students were permitted to write on paper other than the test paper that was distributed to them. Tests were collected at the end of the class period, corrected by the classroom teacher and checked by the researcher. For most of the test items it was fairly easy to grade in terms of a correct or incorrect response. Where explanations were required the teacher and researcher judged as to whether the student's response was correct.

Assumptions, Limitations, and Controls for Extraneous Variables

In Chapter I a deliberate decision was made to delimit the results of this study to grade five level students and to the subject area of mathematics, in particular to the addition of fractions. In addition, there are other factors which could potentially limit the results.

The results of this study were limited by the degree to which the teacher variable was controlled. Teacher variables, as researchers such as Palardy (1969) and Brophy and Good (1970) agree, do affect student achievement. This study attempted to control, as much as possible, the teacher variable by familiarizing teachers with the instructional unit on addition of fractions and by having all teachers follow basically the same procedure. This procedure eliminated, or at least reduced, the variable effects of teacher performance. To further control for the teacher variables, each classroom contained subjects from each of the practice treatment groups.

The results were also limited in their generalizability by the degree to which this sample is representative of the population. Since selection had to be done by classroom, and was limited to schools agreeing to participate in the experiment, the researcher attempted to make the sample as representative as possible by approaching schools from urban, urban/rural, and rural areas. Furthermore, in an attempt to equate groups the researcher assigned students randomly to the different treatment conditions.

The results are also limited in their generalizability by the sample size. The sample size (140) was only a small portion of the total population of grade five students; however, by the selection process explained above and by random assignment to treatments it was felt that the sample was representative of the population and that treatment groups were relatively equal. As was previously mentioned, none of the schools that were approached refused to participate in the study.

As was explained in Chapter II, researchers are divided over the issue of homework effects on achievement. However, most researchers agree that it does have an effect. For this reason, and others previously explained, an original unit on addition of fractions was designed. Using an original unit for instruction controlled to some extent the possibility of the student doing extra practice work at home. Students had no textbook or notebook to take home, and teachers did not assign any homework during the study. Due to the randomization process, it was assumed that the amount of extra work undertaken by the student, if any, was equal over all treatment conditions.

To ensure that the Hawthorne effect was controlled in all groups, all students were told that they were participating in an experiment.

All mathematics instruction occurred at the regularly scheduled time every day for all classes. Should time of day have an effect on a student's performance, this procedure would have eliminated this variable as a potential source of influence. Instruction occurred early in the morning or early in the afternoon. At these times students are generally more alert and less fatigued. This procedure attempted to eliminate fatigue factors which could potentially affect performance differentially if the instruction had been given on the last class period of the day. Also, this procedure eliminated differential fatigue factors among students due to different instruction in other subject areas.

Finally, it is assumed that the controls for the extraneous variables discussed here, which could have potentially influenced the conditions and results, adequately minimized the effects of these variables.

Hypotheses

On the basis of the contradictory findings in previous research on the relationship between the variables of sex, level of ability, amount of practice, and mathematical achievement, the following null hypotheses were tested in this study:

1. There are no significant differences between groups having different amounts of practice exercises on an immediate posttest of mathematical concepts and skills.

2. There are no significant differences between groups having different amounts of practice exercises on a delayed posttest of mathematical concepts and skills.
3. There are no significant sex differences on an immediate posttest of mathematical concepts and skills.
4. There are no significant sex differences on a delayed posttest of mathematical concepts and skills.
5. There are no significant ability differences on an immediate posttest of mathematical concepts and skills.
6. There are no significant ability differences on a delayed posttest of mathematical concepts and skills.
7. There is no significant interaction between the sex of the student and amount of practice on an immediate posttest of mathematical concepts and skills.
8. There is no significant interaction between the sex of the student and amount of practice on a delayed posttest of mathematical concepts and skills.
9. There is no significant interaction between the level of ability of students and the amount of practice on an immediate posttest of mathematical concepts and skills.
10. There is no significant interaction between the level of ability of students and the amount of practice on a delayed posttest of mathematical concepts and skills.
11. There is no significant interaction between the sex

and level of ability of the students on an immediate posttest of mathematical concepts and skills.

12. There is no significant interaction between the sex and level of ability of the students on a delayed posttest of mathematical concepts and skills.

13. There is no significant interaction between sex, level of ability, and amount of practice on an immediate posttest of mathematical concepts and skills.

14. There is no significant interaction between sex, level of ability, and amount of practice on a delayed posttest of mathematical concepts and skills.

Statistical Model

The statistical model used to tabulate and analyze the data is outlined in Figure 2.

	Male				Female				Total
	Treatment A	Treatment B	Treatment C	Sub Total	Treatment A	Treatment B	Treatment C	Sub Total	
High	X_{111}	X_{121}	X_{131}	$X_{1.1.}$	X_{112}	X_{122}	X_{132}	$X_{1.2.}$	$X_{1...}$
Low	X_{211}	X_{221}	X_{231}	$X_{2.1.}$	X_{212}	X_{222}	X_{232}	$X_{2.2.}$	$X_{2...}$
Total	$X_{.11.}$	$X_{.21.}$	$X_{.31.}$	$X_{..1.}$	$X_{.12.}$	$X_{.22.}$	$X_{.32.}$	$X_{..2.}$	$X_{....}$

Figure 2. Statistical model used in the study to organize and analyze data.

It is assumed that the scores X_{ijkn} can be thought of in terms of the following linear model:

$$X_{ijkn} = \mu + \alpha_i + \beta_j + \gamma_k + \gamma\beta_{ij} + \alpha\gamma_{ik} + \beta\gamma_{jk} + \alpha\beta\gamma_{ijk} + e_{ijkn}$$

where, X_{ijkn} is the n^{th} score in the i^{th} row, the j^{th} column and the k^{th} layer,

μ is the population mean of all observations,

α_i is the main effect of ability i,

β_j is the main effect of treatment j,

γ_k is the main effect of sex of the student,

$\alpha\beta_{ij}$, $\alpha\gamma_{ik}$, and $\beta\gamma_{jk}$ are the first order interactions of the main effects,

$\alpha\beta\gamma_{ijk}$ represents the second order interaction of the main effects,

e_{ijkn} is the error, or "residual" component that accounts for variation of observation within the ijk^{th} cell.

Given that these are the hypothetical main and interaction effects underlying the data, it is assumed for example that:

$$X_{1121} = \mu + \alpha_1 + \beta_1 + \gamma_2 + \alpha\beta_{11} + \alpha\gamma_{12} + \beta\gamma_{12} + \alpha\beta\gamma_{112} + e_{1121}$$

On the basis of this statistical model, the hypotheses which have been stated substantively, could also be stated in the statistical terminology of the model. For example:

Hypothesis 1:--There are no significant differences between groups having different amounts of practice exercises on an immediate posttest of mathematical concepts and skills.

$$H_o: \mu_{.1..} = \mu_{.2..} = \mu_{.3..}$$

Hypothesis 7:--There is no significant interaction between the sex of the student and amount of practice on an immediate posttest of mathematical concepts and skills.

$$H_o: \mu_{.jk.} - \mu_{.j..} - \mu_{..k.} + \mu = 0$$

for all the μ_{ijk}'s.

All other hypotheses could be stated in this statistical form.

Statistical Tests and Significance Levels

Results of the immediate posttest were tabulated separately using the tabulation procedure outlined in Figure 2. A separate table was used to tabulate the delayed posttest results.

The results of the immediate posttest were subjected to a 2 x 3 x 2 analysis of variance: three-way classification for a fixed model ($n>1$). The results of the delayed posttest were also subjected to the same analysis.

All stated hypotheses were tested at the 0.05 significance level.

Where the treatment effect was significant, a "studentized range" test as given by Scheffé (1953) was used to determine where the significant differences lay specifically. Due to the fact that ~~the~~ Scheffé procedure is more rigorous than multiple t-tests, it was decided to employ a less rigorous significance level (.10) as suggested by Scheffé (1959).

All possible comparisons were formulated and tested so as to fully utilize all information in the data tables.

Chapter IV contains a complete analysis of the results using the statistical tests and significance levels outlined in this chapter. Results will be discussed in Chapter V after a careful analysis has been completed and all hypotheses tested.

CHAPTER IV

ANALYSIS OF RESULTS

Introduction

In this chapter the data, collected according to the procedures outlined in Chapter III, are examined in terms of the stated hypotheses. This chapter also includes some a posteriori data analysis which were not anticipated prior to carrying out the study.

To test the stated hypotheses a three-factor analysis of variance was conducted on the results of the immediate and delayed posttest. Because of losses due to mortality and incomplete data collection, the total number of subjects included in the analysis was 130.

The analysis of variance of the immediate posttest and delayed posttest results are summarized in Tables 1 and 2.

It was necessary to test the hypotheses involving the interactions of the variables sex, level of ability, and amount of practice prior to examining and interpreting the main effects of each of these variables. This order of reporting was adopted because these interactions had a decided influence on the separate outcomes of each of these variables.

TABLE 1

ANALYSIS OF VARIANCE OF IMMEDIATE POSTTEST RESULTS

Main Effects	SS	DF	MS	F	Significance Level
Treatment	178	2	89	4.51	.013
Sex	90	1	90	4.55	.035
Ability	1827	1	1827	92.5	< .001
2-Way Interactions					
Sex by Treatment	71.4	2	35.7	1.81	.169
Ability by Treatment	103.2	2	51.6	2.61	.078
Sex by Ability	110	1	110	5.58	.020
3-Way Interaction					
Sex by Ability by Treatment	5.2	2	2.6	.130	.878
Error	2330	118	19.75		
Total	4715	129	36.6		

TABLE 2

ANALYSIS OF VARIANCE OF DELAYED POSTTEST RESULTS

Main Effects	SS	DF	MS	F	Significance Level
Treatment	160	2	80	2.24	.111
Sex	158	1	158	4.42	.038
Ability	2447	1	2447	68.2	< .001
2-Way Interactions					
Sex by Treatment	139	2	69.5	1.94	.148
Ability by Treatment	88	2	44	1.23	.297
Sex by Ability	144	1	144	4.01	.048
3-Way Interaction					
Sex by Ability by Treatment	30	2	15	.411	.664
Error	4234	118	35.9		
Total	7400	129	57.4		

3-Way Interactions

Hypotheses 13 and 14, which stated that there is no significant interaction between the variables sex, level of ability, and amount of practice on the immediate and delayed posttests were both accepted at the .05 level of significance. The results of the analysis indicated that there was no significant 3-way interaction on either the immediate or delayed posttests. The F-ratios are included in summary Tables 1 and 2. A detailed breakdown of cell sizes and means is presented in Tables 3 and 4.

TABLE 3

BREAKDOWN OF MEAN ACHIEVEMENT SCORES ON THE IMMEDIATE POSTTEST BY SEX, LEVEL OF ABILITY, AND TREATMENT GROUP

	Male				Female				
	Treatment A	Treatment B	Treatment C	Sub Total	Treatment A	Treatment B	Treatment C	Sub Total	Total
High	30.4 n=10	32 n=8	30.9 n=8	31.0	31.6 n=12	29.4 n=12	31.8 n=15	31.0	31.0
Low	19.5 n=15	20.8 n=13	24.6 n=12	21.4	24.9 n=8	22.7 n=9	28.1 n=8	25.1	22.9
Total	23.9	25.1	27.1	25.2	28.9	26.5	30.5	28.7	26.9

TABLE 4

BREAKDOWN OF MEAN ACHIEVEMENT SCORES ON THE DELAYED POST-TEST BY SEX, LEVEL OF ABILITY, AND TREATMENT GROUP

	Male				Female				
	Treatment A	Treatment B	Treatment C	Sub Total	Treatment A	Treatment B	Treatment C	Sub Total	Total
High	27.6 n=10	32.5 n=8	31.0 n=8	30.2	31.4 n=12	28.9 n=12	30.9 n=15	30.5	30.3
Low	17.7 n=15	17.9 n=13	22.3 n=12	19.1	24.0 n=8	21.3 n=9	25.8 n=8	23.6	20.9
Total	21.7	23.5	25.8	23.5	28.4	25.6	29.1	27.8	25.6

2-Way Interactions

Hypotheses 11 and 12 which stated that there is no significant interaction between the variables sex and level of ability on the immediate and delayed posttests were both rejected at the .05 level of significance. The results of the analysis indicated that there was a significant interaction on the immediate posttest ($p < .05$) and on the delayed posttest ($p < .05$). These data are summarized in Tables 1 and 2. A graphic representation of these interactions is presented in Figure 3 and Figure 4.

In examining these figures it was apparent that high ability males scored as high on both the immediate and delayed posttests as did high ability females. On the immediate

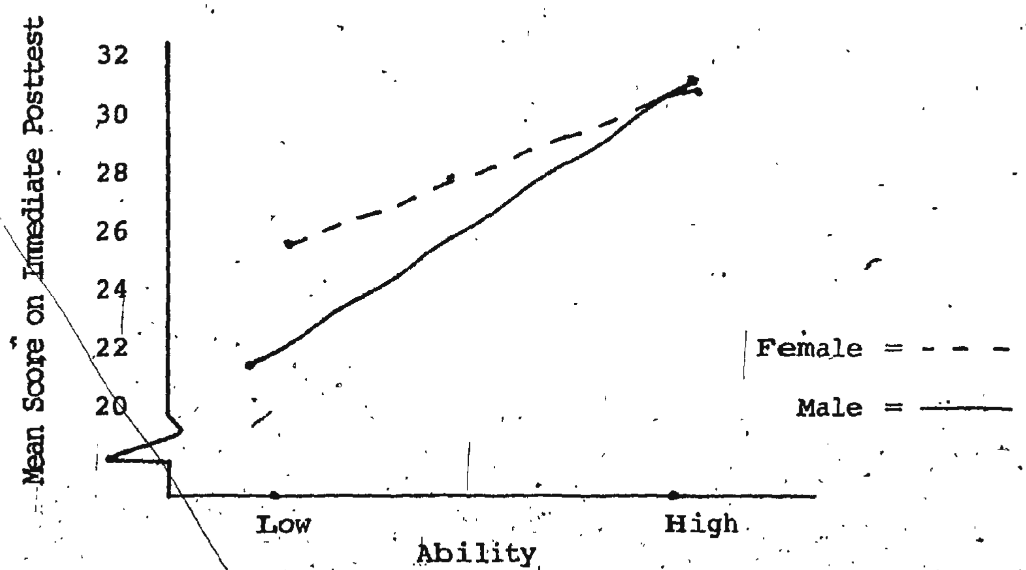

Figure 3. "Sex by Ability" Interaction on the Immediate Posttest.

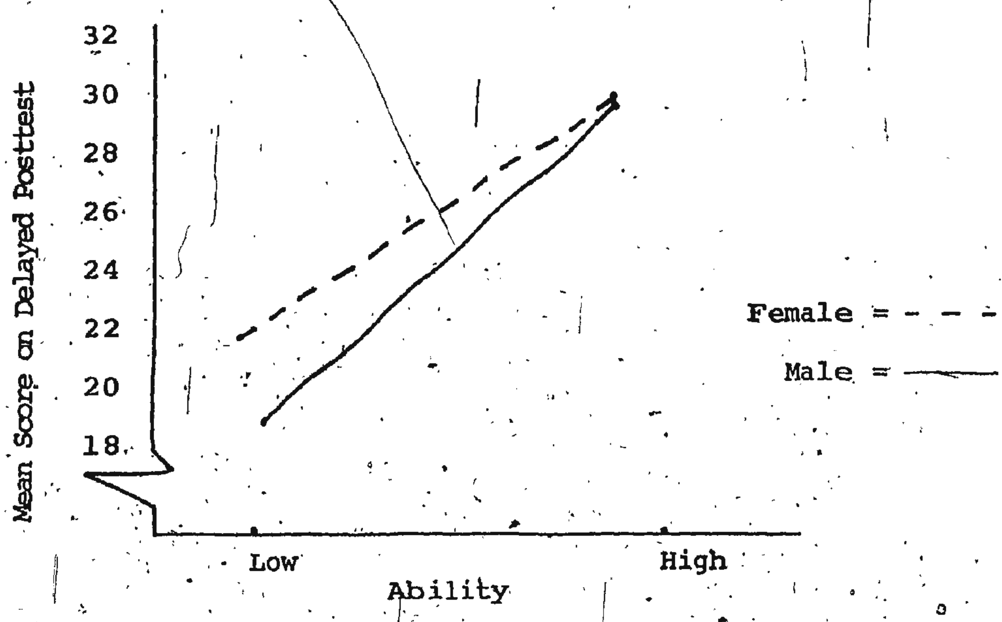

Figure 4. "Sex by Ability" Interaction on the Delayed Posttest.

posttest the mean score for both high ability males and females was 31.0. On the delayed posttest high ability males achieved a mean score of 30.2; high ability females achieved a mean score of 30.5.

Low ability females, however, scored higher on both the immediate and delayed posttests than did low ability males. On the immediate posttest low ability females achieved a mean score of 25.1, whereas low ability males achieved a mean score of 21.4. On the delayed posttest low ability females obtained a mean score of 23.6, whereas low ability males obtained a mean score of 19.1.

Hypotheses 7 and 8 which stated that there is no significant "sex by treatment" interaction on either the immediate or delayed posttests were both accepted at the .05 level of significance. The analysis indicated that there was no significant "sex by treatment" interaction on either the immediate or delayed posttests. The F-ratios obtained were 1.81 and 1.94, respectively. Actual significance levels are included in summary Tables 1 and 2. This result indicated that achievement on the tests under any of the treatment conditions was not dependent on the sex of the person.

Hypotheses 9 and 10 which stated that there is no significant interaction between level of ability and treatment conditions on either the immediate posttest or delayed posttest were both accepted at the .05 level of significance. The results of the analysis indicated that there was no "ability by treatment" interaction which was significant at the .05 level. The F-ratios obtained on the immediate and delayed posttests were 2.61 and 1.23, respectively. However, the "ability by treatment" interaction on the immediate posttest was close enough to significance (p = 0.78) as to make it an educationally significant interaction. This result is graphically represented in Figure 5.

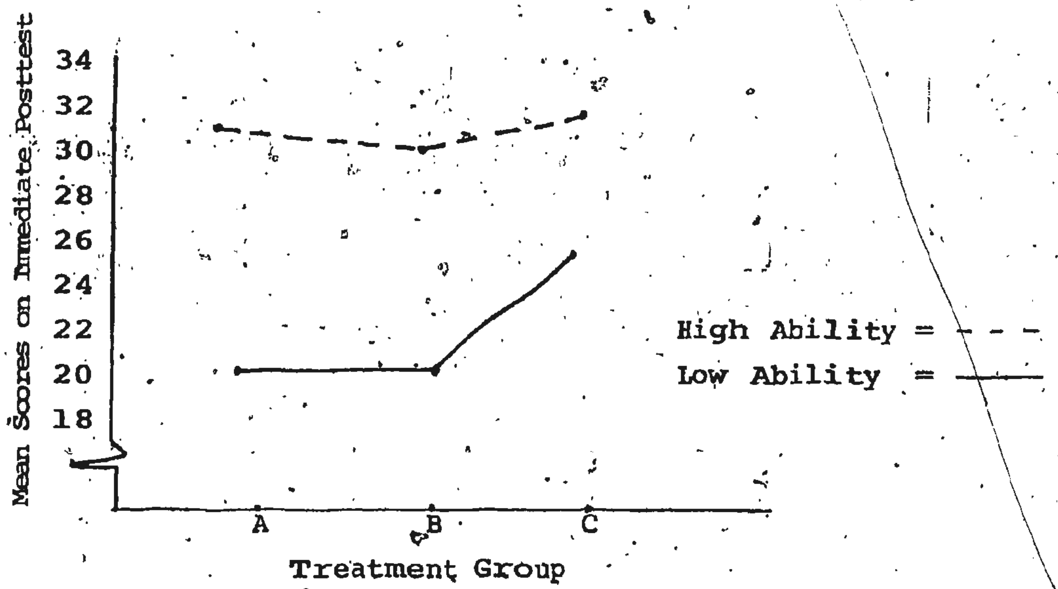

Figure 5. "Ability by Treatment" Interaction on the Immediate Posttest.

The pictorial diagram of this interaction indicated that for high ability students the amount of practice exercises whether five, ten, or fifteen did not affect the results significantly. High ability students receiving treatment A, i.e., five practice exercises, obtained a mean score of 31.0; those receiving treatment B, i.e., ten practice exercises, obtained a mean score of 30.5; and those receiving treatment C, i.e., fifteen practice exercises, obtained a mean score of 31.5. The overall mean difference in achievement between high ability students receiving treatments A, B, and C was only 1.0.

An examination of the performance of lower ability students revealed that they performed as well under treatment A, as they did under treatment B. Mean scores for lower ability students receiving treatment A was 21.4, and for lower ability students receiving treatment B was 21.6--a mean difference of 0.2. However, lower ability students receiving treatment C performed considerably better, in terms of achievement on the immediate posttest, than those lower ability students receiving treatments A and B. Lower ability students receiving treatment C obtained a mean score of 26 on the immediate posttest. The mean difference between lower ability students receiving treatments A, B, and C was 4.7.

Interpreting this result, it appeared that high ability students obtained approximately equivalent achievement scores on the immediate posttest whether they received five, ten, or fifteen practice exercises. However, lower ability students receiving fifteen practice exercises achieved significantly higher ($p = 0.078$) than they did with five or ten practice exercises. This result implied that when assigning practice exercises in mathematics to grade five level students of lower ability, it is advantageous--in terms of achievement--to assign at least fifteen practice exercises.

Main Effects

Hypotheses 5 and 6, which stated that there is no difference in achievement between low ability and high ability students on either the immediate or delayed posttests were both rejected at the .05 level of significance. The F-ratios are included in summary Tables 1 and 2. On both tests higher ability students scored significantly higher than lower ability students.

High ability students obtained mean results of 31.0 and 30.3, respectively, on the immediate and delayed posttest. Low ability students obtained mean results of 22.9 and 20.9, respectively, on the immediate and delayed posttests. This result is not surprising as it was partially expected that high ability students would achieve superior results.

Hypotheses 3 and 4, which stated that there are no significant sex differences in achievement on the immediate and delayed posttests were both rejected at the .05 level of significance. The analysis indicated that females scored significantly higher on both tests. On the immediate posttest females obtained a mean score of 28.7; males obtained a mean score of 25.2. On the delayed posttest females obtained a mean result of 27.8; males obtained a mean score of 23.5.

At this point it should be noted that this difference between males and females, as indicated by the interaction between sex and ability, occurred mainly in the lower ability group. According to the analysis of variance result and Figures 1 and 2, there were no significant sex differences in the higher ability group, but females scored significantly higher in the lower ability group.

Hypothesis 1, which stated that there is no significant difference in achievement between groups receiving different amounts of practice exercises on the immediate posttest, was rejected at the .05 level of significance. The analysis results indicated significant differences between practice treatments. Students receiving five practice exercises obtained a mean score of 26.1 on the immediate posttest; those receiving ten practice exercises obtained a mean score of 25.8; and those receiving fifteen practice exercises obtained a mean score of 28.9.

To determine where the significant differences between the treatments lay, a Scheffé test was carried out using .10 as the significance level.

The results of the analysis indicated that students in treatment C achieved significantly higher results than students in treatment A ($p < .10$), and treatment B ($p < .10$). However, achievement differences between students receiving treatments A and B were not significantly different.

When these results were examined in the light of the two-way interaction between ability and treatment conditions depicted in Figure 3, it was apparent that the significant differences between the treatments occurred in the low ability groups. High ability students obtained approximately equivalent results in all three treatment conditions. Low ability students achieved approximately equivalent results in treatments A and B, but obtained significantly higher results in treatment C.

Hypothesis 2, which stated that there is no significant difference in achievement between students receiving different amounts of practice exercises on the delayed posttest, was accepted at the .05 level of significance. The analysis indicated no significant differences in achievement on the delayed posttest ($p > .05$) between groups receiving different amounts of practice. The F-ratio obtained from the analysis of treatment differences on the delayed posttest was 2.24. Students receiving five practice exercises obtained a mean score of 24.7 on the delayed posttest; those receiving ten practice exercises obtained a mean score of 24.6; and those receiving fifteen practice exercises obtained a mean score of 27.6.

The analysis indicated that the significant differences obtained between treatment conditions on the immediate posttest had, to some degree, dissipated by the time the delayed posttest was administered a month later.

Further Analysis

The Scheffé test of immediate posttest results demonstrated that students receiving treatment C achieved significantly higher results on the immediate posttest than students receiving treatment A and treatment B. Achievement differences between students receiving treatments A and B were not significant. On the basis of this finding it was decided to collapse treatment groups A and B into one group and reanalyze the results using the three factor analysis of variance.

Tables 5 and 6 summarize the results of the analysis of variance of the immediate posttest and delayed posttest after treatment groups A and B had been collapsed.

TABLE 5

ANALYSIS OF VARIANCE OF IMMEDIATE POSTTEST SCORES AFTER COLLAPSING TREATMENT GROUPS A AND B

Main Effects	SS	DF	MS	F	Significance Level
Treatment	176	1	176	8.98	.003
Sex	89	1	89	4.51	.036
Ability	1830	1	1830	92.9	< .001
2-Way Interactions					
Sex by Treatment	3.7	1	3.7	.189	.665
Ability by Treatment	104	1	104	5.29	.023
Sex by Ability	107	1	107	5.42	.022
3-Way Interaction					
Sex by Ability by Treatment	4.5	1	4.5	.228	.634
Error	2402	122	19.7		

TABLE 6

ANALYSIS OF VARIANCE OF DELAYED POSTTEST SCORES AFTER COLLAPSING TREATMENT GROUPS A AND B

Main Effects	SS	DF	MS	F	Significance Level
Treatment	160	1	160	4.43	.037
Sex	158	1	158	4.38	.039
Ability	2449	1	2449	67.8	<.001
2-Way Interactions					
Sex by Treatment	5.8	1	5.8	.161	.689
Ability by Treatment	56	1	56	1.543	.217
Sex by Ability	132	1	132	3.65	.058
3-Way Interaction					
Sex by Ability by Treatment	1.6	1	1.6	.044	.835
Error	4409	122	36.1		

The results of this a posteriori analysis again indicated that there was no three-way interaction between the variables: sex, level of ability, and amount of practice on either the immediate posttest or the delayed posttest.

Two-way interactions under this analysis remained basically unchanged. There was no significant "sex by treatment" interaction on either the immediate or delayed posttests. The "sex by ability" interaction, under this analysis, was again significant with low ability females achieving significantly higher results on the immediate and delayed posttests than low ability males. High ability males and females scored at levels that were approximately equivalent.

The results of the analysis for the "ability by treatment" interaction were different from the results of the initial analysis. Previously, the "ability by treatment" interaction on the immediate posttest was significant at the .078 level. However, having collapsed treatment groups A and B into one group, the analysis indicated that the "ability by treatment" interaction was significant at the .05 level. This "ability by treatment" interaction is graphically depicted in Figure 6.

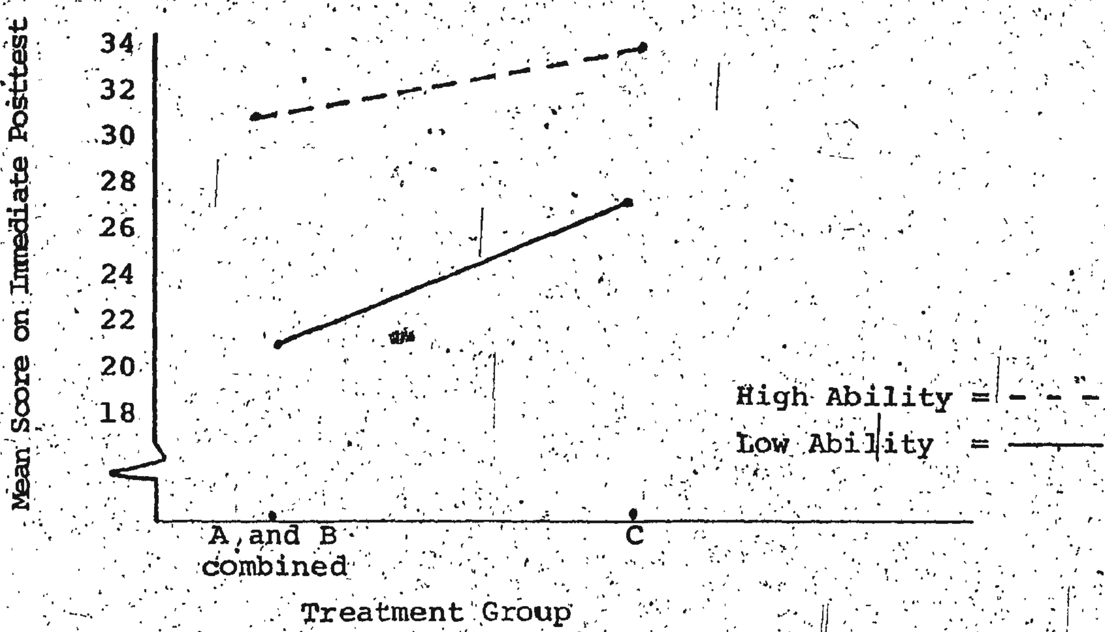

Figure 6. "Ability by Treatment" Interaction on the Immediate Posttest after Treatment Groups A and B have been collapsed into one Group.

Figure 6 indicates that high ability students performed as well under the combined treatment groups A and B as they did in treatment C on the immediate posttest. High ability students receiving treatment C obtained a mean score of 31.5, while their counterparts in the combined treatment groups A and B obtained a mean score of 30.8. Low ability students receiving treatment C, however, achieved significantly higher results than their counterparts in combined treatment groups A and B. Low ability students receiving treatment C obtained a mean score of 26.0, whereas low ability students in combined treatment groups A and B obtained a mean score of 21.5.

The result again indicated that high ability students achieved as well with a small amount of practice as they did with a larger amount of practice. For lower ability students, it appeared that they need more practice to achieve higher results. The results also demonstrated that for lower ability students the amount of practice necessary to achieve higher results was more than ten practice exercises.

The analysis indicated no significant "ability by treatment" interaction on the delayed posttest.

The results of this á posteriori analysis indicated that the main effects of sex, level of ability, and amount of practice were all significant at the .05 level on both the immediate and delayed posttests. Females scored significantly higher than males and higher ability students scored significantly higher than lower ability students on both the immediate and delayed posttests. The obtained F-ratios and significance levels are included in Tables 5 and 6.

The analysis results for treatment differences varied slightly from the first analysis. As indicated in Table 5 there were significant differences between the treatments on the immediate posttest. Students in treatment group C achieved significantly higher results ($p < .05$) than the combined treatment groups A and B on the immediate posttest. The mean score for treatment C students was 28.9, whereas students in combined groups A and B obtained a mean score of 25.9.

By combining treatment groups A and B, however, the treatment differences were also significant on the delayed posttest (refer to Table 6 for a summary of data). Analysis of delayed posttest results indicated that treatment group C achieved significantly higher results on the delayed posttest than the combined groups A and B. Students receiving treatment C obtained a mean score of 27.6 on the delayed posttest, whereas students in the combined groups A and B obtained a mean score of 24.6. Therefore, on the basis of this a posteriori analysis, Hypothesis 2 was also rejected at the .05 level of significance.

Summary of Significant Findings

In summary, the major findings of the study were: (1) females scored significantly higher than males on both the immediate and delayed posttests; (2) high ability students did better than low ability students--high ability males and females were approximately equivalent, whereas low ability females performed better than low ability males. This accounted for most of the discrepancy between the sexes; (3) treatment group C students, i.e., those who received fifteen practice exercises performed significantly better on the posttests than students in group A, who received five practice exercises, and group B, who received ten practice exercises. Treatment groups A and B were not significantly different in terms of achievement on posttests; (4) high ability students in either groups A, B, or C scored approximately equivalent results on the immediate posttest, this was also true on the delayed posttest; (5) low ability students scored approximately the same in groups A and B, but those in group C scored significantly better on the posttests than those in groups A and B.

These results are discussed further in the concluding chapter of this report.

CHAPTER V

SUMMARY, DISCUSSION, CONCLUSIONS, AND RECOMMENDATIONS

In this chapter a summary of the study, a discussion of the results in light of the major questions stated, the conclusions drawn from the results of the study, their implications for education, and some recommendations for future research are presented.

Summary

The purpose of this study was to examine the relationship of drill and practice to mathematics achievement in skill and concept acquisition in grade five mathematics. To do this the experimenter examined these major questions: Do any of the variables of sex, level of ability, or amount of practice result in significantly different achievement on either an immediate or delayed posttest of concepts and skills? Moreover, are there any interactions between any of these variables? To investigate these questions a unit on addition of fractions was developed, implemented, and evaluated.

This study was conducted using 140 grade five students in five classes from both urban and rural Newfoundland communities. Students in each class were randomly assigned to three equal groups. Each group was randomly assigned to one of three treatment conditions. Treatment A students received five practice exercises per class session; Treatment B students received ten practice exercises; and Treatment C students received fifteen practice exercises. Classes were held once a day for a total of fifteen class sessions.

In order to determine student achievement on the unit, two tests were administered. The first, the immediate posttest, was given at the end of the fifteen lessons following a review session. The second, the delayed posttest, was given one month later to measure retention of the material covered in the class sessions. Both of these tests were constructed by the experimenter and were designed to test whether the behavioral objectives of the unit had been achieved. In an attempt to eliminate inadequacies, both the addition of fractions unit and the tests were piloted in a grade six classroom prior to conducting the study.

The data were collected and analyzed using a three factor analysis of variance procedure. The results were reported in Chapter IV.

Discussion of Results

The analysis indicated that there were significant differences in achievement between treatment groups on the immediate posttest. The Scheffé procedure was used and it was found that there were significant differences between treatment groups A and C, and treatment groups B and C. There were, however, no significant differences between groups A and B. In other words, for students who received five or ten practice exercises on a concept, there was no significant difference on immediate achievement. However, when the amount of practice was increased to fifteen practice exercises, there was a significant gain in achievement.

A possible explanation for this result is the psychological phenomenon "overlearning." According to Kolesnik (1970) overlearning is the repetition of a task continued beyond the point of initial mastery or bare comprehension. Its purpose is to produce a high degree of retention. Relating this to the results of the study, it may be argued that a leveling effect occurred. Up to ten practice exercises there was no notable difference in achievement, but once the number of practice exercises exceeded ten, an "overlearning" phenomenon was exhibited. This resulted in increased retention which was demonstrated by higher achievement on the immediate posttest.

It is interesting that this effect was only demonstrated with students of lower ability. This is precisely what Sueltz (1953) stated in his principles of drill and practice. For students of higher ability the amount of practice had no significant effect. That is, high ability students achieved as well with five practice exercises as they did with ten or fifteen practice exercises. This result has great implications for assignment of practice to elementary school students in mathematics classes. If, as demonstrated in the present study, ability is the major determinant of the effectiveness of assigned practice, teachers should be more selective in how much practice they assign to different students. Only in this way can we, as Ausubel (1968) states, have efficient and optimum use of class time.

The initial analysis indicated that the significant differences obtained between the practice treatments on the immediate posttest had dissipated somewhat by the time the delayed posttest was administered a month later. This finding, of itself, is not surprising, but when studied in light of the "overlearning" theory is noteworthy.

Apparently, the amount of practice did affect immediate achievement results, but when a delay of one month was introduced and students were retested the differential effects of different amounts of practice had lessened. If an "overlearning" effect was actually the phenomenon being demonstrated, then the fifteen-practice-exercise group should have shown a higher degree of retention even a month later. However, this was not the case as all groups showed an approximately equivalent drop in retention from immediate to delayed posttest.

Because the achievement of students in treatment groups A and B was not significantly different, it was decided to combine these two groups into one and reanalyze the results. This analysis demonstrated significant differences in delayed posttest achievement between treatment group C and the combined groups A and B. This result added supporting evidence that the supposed "overlearning" effect was indeed a valid explanation, and that the effect continued over the one month period. On the delayed posttest treatment group C, which received fifteen practice exercises per concept, demonstrated a higher degree of retention, or achieved significantly higher results than treatment groups A and B.

The analysis indicated that high ability students achieved significantly higher results on both the immediate and delayed posttests than did lower ability students. Given Aiken's (1971) statement that research has consistently reported a high positive relationship between ability and mathematical achievement, this result is not surprising. In addition, Kolesnik (1970) reported that previous academic achievement measures are strong predictors of subsequent academic performance. Given these relationships and the fact that the ability measure used in this study was the academic achievement of pupils in the previous year, one would expect higher ability students to obtain significantly higher results.

The results of the analysis indicated that females achieved significantly higher results than males on both the immediate and delayed posttests. However, there was also a significant "sex by ability" interaction on both tests. Under this interaction effect the achievement means of high ability males and females were approximately the same on each of the immediate and delayed posttests, whereas lower ability females scored considerably higher than lower ability males on both posttests. It is apparent that the obtained sex difference was largely concentrated in the lower ability group.

According to Kolesnik (1970) a majority of studies at the elementary school level have shown that girls, on the average, achieve higher results than boys. This difference may be accounted for in a number of ways, including: neurological, environmental, age of school entry, and sex role definition. Neurologically, the difference might be related to the fact that girls of elementary school age are one to two years more mature than their male counterparts with respect to physical, mental, and social factors. According to Kolesnik, studies have generally found that girls excel boys in verbal abilities and in most aspects of linguistic development. Suggestions are that girls are more verbal than boys because of early development and closer interactions with their mother. When they go to school their oral communication competencies are better and as a result girls learn to read several months before boys. Kolesnik further reports that educators generally agree that there is a high degree of relationship between verbal precosity and mathematical ability. If one is handicapped in reading, then it is also generally true that he is handicapped in mathematics. These relationships place boys at a disadvantage which still remains at the grade five level. Kolesnik reports that even in arithmetic at the elementary level, girls usually surpass boys in the mechanics of computation, outperforming them in both speed and accuracy.

However, in examining the results of the present study, how can we account for the fact that the sex difference occurred mainly in the lower ability groups? It has been argued that boys, far more frequently than girls, do not use the scholastic abilities they possess. According to Waetjen (1962) and Maccoby (1966), boys outnumber girls as underachievers by two or three to one. Because of our society's definition of the male sex role, by the time a male child enters school there is already a tendency toward underachievement. Shaw and MaCuen (1960) noted that for male achievers and underachievers, there was a statistically significant difference in their grade point average as early as grade three. This difference increased as they progressed from grades three to ten. For girls, however, significant differences between achievers and underachievers did not occur until the ninth grade. In comparison with the high achievers, Shaw and MaCuen noted that the problem with male underachievers becomes steadily more serious as they progress through school. Given the way we have defined our sex roles, the psychological atmosphere of the elementary school is more favorable to girls in terms of achievement potential. According to Kolesnik, if a young male is verbally proficient, he may be perceived by his peer group as having more in common with girls than with other boys his own age.

These are possible explanations for the finding of sex differences in achievement only in the lower ability groups. This finding is similar to other findings by other researchers such as Morrell (1970), Parsley (1964), and Unkel (1966). This result is particularly apparent in Parsley's findings regarding lower ability boys and girls. These researchers, and others, generally report that for the reasons previously discussed, lower ability males are generally disproportionately disadvantaged in terms of success at school.

Conclusions

The conclusions in this study are limited to grade five level students and to the subject area of mathematics. As was previously stated, it may be possible to generalize to students in grades four or six. However, due to the nature of the findings, the researcher is cautious about doing so.

In summary, the data analysis led to the following conclusions:

1. There was a difference in achievement between treatment groups, with students in treatment group C achieving significantly higher results on the immediate posttest. This result was also found on the delayed posttest. Thus, fifteen practice exercises were more effective than five or ten practice exercises in promoting higher achievement in mathematics at the grade five level.
2. The obtained treatment differences in achievement were concentrated mainly in the lower ability groups.

Achievement for high ability students was approximately equivalent for all treatment groups.

3. There was a difference in achievement between the sexes, with females scoring significantly higher than males on the immediate posttest.
4. Females also achieved significantly higher results than males on the delayed posttest.
5. The obtained sex differences in achievement were concentrated in the lower ability groups. Achievement differences between sexes in the high ability group were not significant.
6. The major factor which determined the amount of practice which was most effective and appropriate was the ability level of students.

Recommendations

As a result of the study, the following recommendations are made for further research and applications for teachers:

1. A similar study should be conducted with a larger sample and over a longer period of time to further investigate the effects of practice on achievement.
2. To determine whether these findings are generalizable to other topics in the mathematics curriculum, similar studies should be conducted with other mathematics topics at this and other grade levels.
3. To determine whether these findings are generalizable to other subject areas, studies should be conducted

in other areas of the curriculum, and at other grade levels.

4. Future studies in examining the effects of practice on students of a particular grade level should place greater emphasis on, or give greater consideration to, the age differences of students within that grade level.

5. Teachers should reexamine their policy of assigning mathematics practice exercises in the classroom. The findings of this study indicate that the amount of practice does affect mathematics achievement, at least at the grade five level. Given the interaction effects of ability and treatments, it is suggested that this policy of reexamination if carried out at regular intervals could potentially eliminate inefficiencies in practice assignments. By assigning practice on the basis of the student's ability, teachers may actually guarantee more efficient use of class time and higher mathematics achievement.

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APPENDIX A

UNIT ON ADDITION OF FRACTIONS

INSTRUCTIONAL OBJECTIVES

1. Given a fraction of the form $^a/b$, the student will be able to identify the parts of the fraction, and define what each part means.
2. Given a fraction of the form $^a/b$, the student will demonstrate his understanding of the concept of a fraction by drawing a region using shaded/unshaded areas to represent the amount.
3. Given a shaded/unshaded region, the student will be able to identify the fractional amount of the region represented by the shaded area.
4. Given a fraction of the form $^a/b$, the student will be able to list three other fractions which are equivalent to the given fraction.
5. Using fractional strips, the student will demonstrate his understanding of equivalent fractions by constructing and explaining (orally) a physical example.
6. Given two fractions of the form $^a/b$ and $^c/d$, the student will be able to determine whether or not they are equivalent.
7. Given three fractions of the form $^a/b$ with different denominators, the student will construct a number line and place the fractions in their correct order.
8. Given some whole number, the student will list all the factors of that number.

9. Given two whole numbers, the student will be able to determine the greatest common factor of the two numbers.
10. The student will demonstrate his understanding of the concept of a factor, by devising his own example to explain the notion.
11. Given a fraction $^a/_b$ not in lowest terms, the student will be able to reduce that fraction to lowest terms by removing the G.C.F.
12. Given two fractions of the form $^a/_b$ and $^c/_d$ (with the same denominator), the student will be able to find the sum by jumping on the number line.
13. Given a mixed number, the student will be able to break the number into whole and fractional parts.
14. Given a mixed number, the student will demonstrate his understanding of the concept of mixed numbers by constructing a figure to represent the number using shaded/unshaded regions.
15. Given a mixed number, the student will be able to change the number into fraction form of $^a/_b$.
16. The student will explain, by using an example, how to determine whether a fraction is greater than or less than one.
17. Given two mixed numbers with the fractional parts having the same denominator, the student will be able to find the sum by adding the whole parts, adding the fractional parts, and finding the combined sum of whole and fractional parts.

18. Given two whole numbers, the student will list some of the multiples of each number, and find the lowest common multiple (L.C.M.) of both numbers.
19. The student will demonstrate that he understands the difference between a multiple and a factor by giving an example.
20. Given two fractions $^a/_b$ and $^c/_d$ (with different denominators), the student will determine a common denominator for the two fractions.
21. Given two fractions of the form $^a/_b$ and $^c/_d$ (with different denominators), the student will be able to find the sum of the fractions by finding a common denominator, writing each given fraction as an equivalent fraction using the common denominator, and adding the fractions together. Reduce answers to lowest term.
22. Given two mixed numbers with the fractional parts having different denominators, the student will add the whole parts, then add the fractional parts by first finding a common denominator, and finally add the whole and fractional parts together.
23. Given some practical problems involving addition of fractions, the student will demonstrate his understanding of the addition of fractions concept by applying his knowledge of fractions to solving these problems.

TEACHER'S GUIDEBOOK

Concept of Fractions

Draw figure A and figure B on the blackboard.

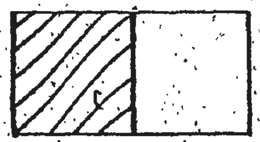


FIGURE A

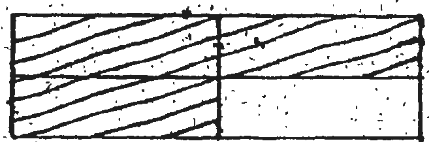


FIGURE B

Now shade in one of the parts of figure A (as shown). Ask the students what number we can use to represent the shaded area in figure A. The students, who are already familiar with fractions should respond "½". Explain to the pupils that the number means that one part out of two parts is shaded.

The 2 is called the denominator.

The 1 is called the numerator.

Involve the students by asking them what the denominator and numerator tell us. Students should arrive at the conclusion that the denominator tells how many equal portions the figure is divided into. The numerator tells how many of the parts are shaded. This idea may involve a small amount of explanation and discussion.

Show the students another figure--figure B. Shade in 3 of the 4 parts (as shown). Now ask the students how we could represent the shaded area in figure B. Go through the process of asking how many parts it is divided into and how many of these parts are shaded.

Answer = $^3/_4$.

The 4 is called the ______________.

The 3 is called the ______________.

Inquire of the students what the number 2/4 means. Represent this by a figure on the blackboard.

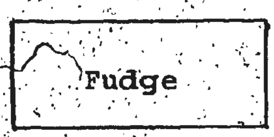
Try to make the students think ahead by asking them how we would represent the area of figure B if all the parts were shaded.

Answer = 4/4 (4 shaded parts out of 4 parts).

We could also say that we have the whole region shaded, or 1 whole region. In this way, and by using other examples, try to get students to make the discovery that 4/4 and 1 represent the same idea and therefore are equal. Use as many examples as necessary. For example: 3/3, 5/5, 6/6. Students should arrive at the conclusion that if a fraction equals 1, then the numerator and denominator are equal.

Give the students a practical problem to work with. Afterwards explain the problem on the blackboard.

<u>Situation</u>: Jane made some fudge for her friend Bill. She poured the fudge into a pan in the shape of a rectangle.



Then she cut the fudge into 10 pieces of equal size. While she had the fudge in the fridge to cool, Bill ate 3 of the pieces.

1) What fraction describes how much Bill ate?

2) What is the denominator of the fraction?

3) What is the numerator?

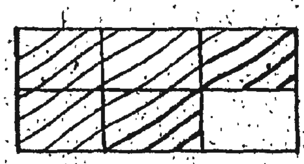
4) What does the denominator tell you?

5) What does the numerator tell you?

Assign the exercises.

Concept of Fractions

1. Give the fractional amount represented by the shaded area:

 = ____________

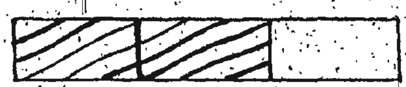
2. In the fraction $4/7$:

   (a) The 7 is called the ____________

   (b) The 4 is called the ____________

3. Draw a shaded region to represent the fraction $7/8$.

4. What fraction of the long strip has been shaded?



5. 

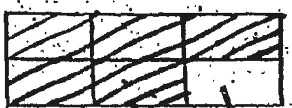
   (a) How many of the triangles are shaded?

   (b) How many triangles are there in all?

   (c) What fraction tells how many of the triangles are shaded?

Concept of Fractions

1. Give the fractional amount represented by the shaded area:

 = ____________

2. In the fraction $^4/_7$:

   (a) The 7 is called the ____________

   (b) The 4 is called the ____________

3. (a) The denominator of a fraction tells us ____________
   ____

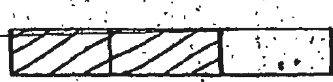
   (b) The numerator of a fraction tells us ____________
   ____

4. Draw a shaded region to represent the fraction $^3/_5$.

5. Draw a shaded region to represent the fraction $^7/_8$.

What fraction of each long strip has been shaded?

6. 

7. 

8. 

9. 

10. 

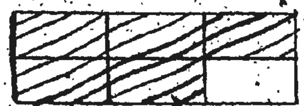
   (a) How many of the triangles are shaded?

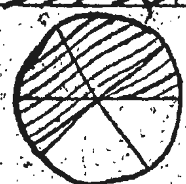
   (b) How many triangles are there in all?

   (c) What fraction tells how many of the triangles are shaded?

Concept of Fractions

Give the fractional amount represented by the shaded area:

1.  = ____________

2.  = ____________

3. In the fraction $4/7$:

(a) The 7 is called the ____________

(b) The 4 is called the ____________

4. (a) The denominator of a fraction tells us ____________

____

(b) The numerator of a fraction tells us ____________

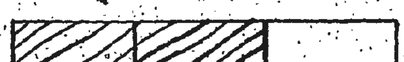
____

5. Draw a shaded region to represent the fraction $3/5$.

6. Draw a shaded region to represent the fraction $7/8$.

What fraction of each long strip has been shaded?

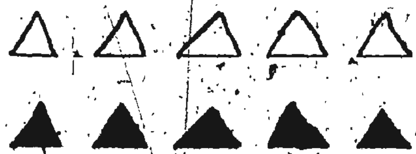
7. 

8. 

9. 

10. 

11. 

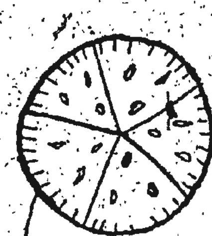
12. 

(a) How many of the triangles are shaded?

(b) How many triangles are there in all?

(c) What fraction tells how many of the triangles are shaded?

Mother baked an apple pie. Then she cut the pie into 5 equal pieces. For dessert Bob ate 3 pieces of the pie.



13. What fraction represents the amount of the pie that Bob ate? Ans.= ____

14. Each piece of pie can be represented by the fraction ____________.

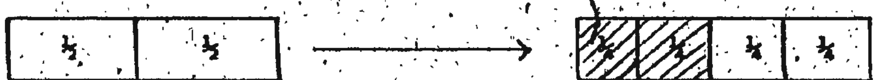
15. There are ____________ $^1/_5$'s in 1 whole pie.

Equivalent Fractions

Purpose: To teach students the idea that the fractional part of a whole may be named by more than one fraction.

Distribute to each student a cardboard strip approximately 20 cm long. Have them divide it into two equal parts and colour one of the parts. (To divide the strip into two equal parts, they can fold the strip in half before they colour).

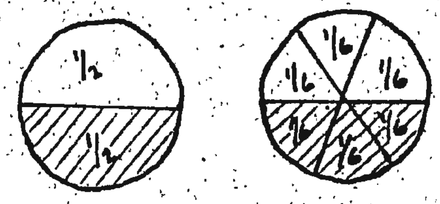
Now after the students have folded the paper, demonstrate that they can fold it in half again.



$^2/_4$ of the paper is now coloured. The child should make the discovery that $^1/_2$ and $^2/_4$ represent the shaded area. Therefore $^1/_2 = {}^2/_4$.

Perform the same exercise with other shaded areas.

For example: $^1/_2 = {}^3/_6$.



The students should recall that fractions which name the same amount are called equivalent fractions. If students insist on calling them equal fractions do not attempt to correct them as they are expressing the correct notion

Comparative Fraction Strips

1 whole															
1/2	1/2														
1/3	1/3	1/3													
1/4	1/4	1/4	1/4												
1/5	1/5	1/5	1/5	1/5											
1/6	1/6	1/6	1/6	1/6	1/6										
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8								
1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10						
1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12				
1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16

Distribute to each child the sheet with comparative fractions strips on it. Have the students cut out the strips and ask them to compare the fractional parts.

The children by comparing these strips should come to realize that:

$$1/2 = 2/4 = 3/6 = 4/8 = 5/10 = 6/12, \text{ etc.}$$

$$1/3 = 2/6 = 4/12 = 3/9, \text{ etc.}$$

$$1/4 = 2/8 = 3/12 = 4/16, \text{ etc.}$$

As students are doing these comparisons walk around and help or guide the students by giving helpful hints on comparison. Do not tell them the answers. Remember they have been exposed to this topic before.

Have students continue to expand their own fractions chart using the ideas they discovered in the previous exercise.

Take one of the examples (e.g., 1/2) and show the students how to generate equivalent fractions.

Ask the students what happens when we multiply a number by 1. Most students know the answer, but it is a good idea to demonstrate this principle. Multiply any number by 1 and we get back the same number we started with. Remind students that we can write:

$$1 = 2/2 = 3/3 = 4/4 = 5/5, \text{ etc.}$$

Using this principle we can generate equivalent fractions

$1/2 = 1/2 \cdot 1$ $\qquad$ $1/2 = 1/2 \times 2/2 = 2/4$

$1/2 = 1/2 \cdot 1$ $\qquad$ $1/2 = 1/2 \times 3/3 = 3/6$

$1/2 = 1/2 \cdot 1$ $\qquad$ $1/2 = 1/4 \times 4/4 = 4/8$

$$1/2 = 2/4 = 3/6 = 4/8 \text{ (etc.)}$$

Have students refer to the comparative fractions strips to see that the results are correct.

Do another example and ask students to suggest the method.

$2/3 = 2/3 \cdot 1 = 2/3 \cdot 2/2 = 4/6$

$2/3 = 2/3 \cdot 1 = 2/3 \cdot 3/3 = 6/9$

$2/3 = 4/6 = 6/9$ (etc.)

Place a couple of exercises on the blackboard for students to try.

a) $1/4$ b) $2/5$

Afterwards have the students generate or make their own equivalent fractions chart by using the multiplication by 1 idea, or if necessary by using comparative fraction strips.

After students have completed these exercises pick out a couple of pairs of equivalent fractions and ask the students to look at these fractions, tell if they are equivalent, and try to discover a way to tell if they are equivalent or not.

Eg.: $1/3$ and $6/18$

$2/3$ and $8/12$

Some students may answer that $1/3 = 1/3 \times 6/6 = 6/18$. This is correct, however there is a shorter way to tell if two fractions are equivalent.

$1/3 = 6/18$ If we cross-multiply $1/3 \; 6/18$ $(3 \times 6 = 1 \times 18)$ we get the same result. Now, ask students to check the second example to see if the rule works.

Place a couple of examples on the blackboard and ask students to decide whether or not they are equivalent.

Eg.: a) $^{5}/_{2} = {}^{10}/_{4}$ b) $^{3}/_{4} = {}^{9}/_{12}$ c) $^{3}/_{1} = {}^{12}/_{4}$

If students are able to master this idea assign seatwork exercises.

Equivalent Fractions

Decide which pairs of fractions are equivalent.

1. $^{1}/_{4}$ and $^{2}/_{6}$

2. $^{2}/_{8}$ and $^{1}/_{4}$

In the sentence below you are to replace each by a whole number. Choose a whole number that makes the sentence true.

3. $^{3}/_{8}$ is equivalent to $\;/_{24}$

4. List two equivalent fractions for $^{2}/_{5}$.

5. Draw two shaded regions showing that $^{1}/_{3}$ and $^{3}/_{9}$ are equivalent.

Equivalent Fractions

Decide which pairs of fractions are equivalent.

1. $^1/_4$ and $^2/_6$
2. $^2/_8$ and $^1/_4$
3. $^1/_2$ and $^2/_6$

In the sentences below you are to replace each by a whole number. For each sentence choose a whole number that makes the sentence true.

4. $^1/_2$ is equivalent to $\square/8$.
5. $^{12}/_8$ is equivalent to $^2/_\square$.
6. $^{13}/_{13}$ is equivalent to $\square/1$.
7. $^3/_8$ is equivalent to $\square/24$.
8. List three equivalent fractions for $^1/_4$.
9. List two equivalent fractions for $^2/_5$.
10. Draw two shaded regions showing that $^1/_3$ and $^3/_9$ are equivalent.

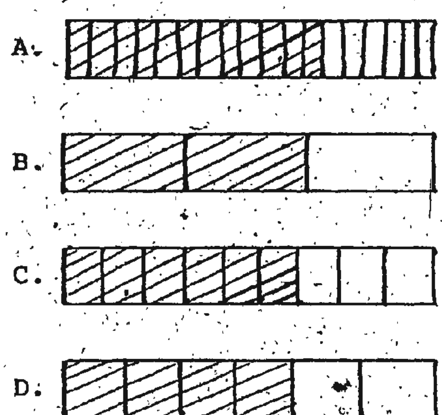
Equivalent Fractions

Decide which pairs of fractions are equivalent.

1. $^1/_4$ and $^2/_6$
2. $^2/_8$ and $^1/_4$
3. $^3/_{12}$ and $^2/_6$
4. $^1/_2$ and $^2/_6$

In the sentences below you are to replace each $\square$ by a whole number. For each sentence choose a whole number that makes the sentence true.

5. $^1/_2$ is equivalent to $\square/8$.
6. $^{12}/_8$ is equivalent to $^2/_\square$.
7. $^{13}/_{13} = \square/1$.
8. $^3/_8 = \square/24$.
9. What four equivalent fractions are shown in the diagram? (Look at the shaded parts).



10. List three equivalent fractions for $^1/_4$.
11. List two equivalent fractions for $^2/_5$.

12. Jane invited 6 people to a party. She made a cake and sliced it into 6 pieces of the same size so that each person might have $1/6$ of the cake. Then Jane decided that each person should have 2 small pieces rather than one large piece. Draw a picture to show how the cake looked after Jane made the second set of cuts.

13. Draw a picture to show how the cake would look if Jane sliced it so that each person would get 4 small pieces of the same size.

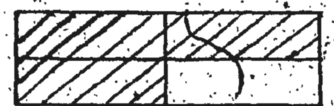
14. Give two fractions different from $1/6$ that can be used to represent each person's share of the cake.

15. Draw two shaded regions showing that $1/3$ and $3/9$ are equivalent.

Ordering of Fractions

Up until now, most of the fractions we have talked about are fractions whose numerators are less than the denominator. These fractions all show amounts less than one whole unit.

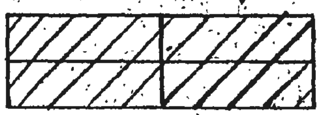
Eg.



Shaded area = $^3/_4$

We have also looked briefly at fractions whose numerator equals the denominator.

Eg.



Shaded area = $^4/_4$

1 whole shaded region

So when we talk about 1 whole region we represent it by a fraction whose numerator is equal to the denominator.

What about numbers greater than one? How can these be represented by fractions? Look at these three examples. Place them on the blackboard and ask students to try to represent the shaded area by a fraction.

Figure A

Figure B

Figure C

In Figure A, there are two regions shown. These regions are divided into 4 parts. How much of each region does each part represent?

Answer: $^1/_4$

How many $^1/_4$'s are shaded? Answer: 5

The numerator tells how many parts are shaded = <u>5</u>; the denominator tells how many parts the region are divided into = <u>4</u>.

Fractional number = $^5/_4$.

Explain to students that more than one whole region is shaded in Figure A. If one whole region were shaded, we could represent it by $^4/_4$.

Proceed with Figures B and C, and ask the same type of questions. Try to draw out the students' thinking. Afterwards, ask the students if they have found any way to tell if a fraction is less than, equal to, or greater than 1.

As students respond, write the discoveries on the blackboard and give examples.

If the numerator is less than the denominator, the fraction is less than 1.

Eg.: $^3/_4$, $^7/_8$, $^3/_5$

If the numerator is greater than the denominator, the fraction is equal to 1.

Eg.: $^3/_3$, $^5/_5$, $^7/_7$

If the numerator is greater than the denominator, then the fraction is greater than 1.

Eg.: $^7/_5$, $^6/_3$, $^9/_6$

Ordering and Comparing and Comparing Fractions

Procedure

Suppose we had two fractions and we wanted to tell which was the largest and which is the smallest. How could we decide?

For example, $^{3}/_{5}$ and $^{4}/_{5}$. Which is larger?

$^{3}/_{5}$ $^{4}/_{5}$

$^{4}/_{5}$ is larger than $^{3}/_{5}$

Now suppose we were given a set of fractions ($^{1}/_{6}$, $^{3}/_{6}$, $^{2}/_{6}$, $^{5}/_{6}$, $^{9}/_{6}$, $^{6}/_{6}$) and asked to put them in their proper order. How could we do it? Remember that all the denominators are the same. Ask the students to represent each of these amounts by figures if they don't really know.

The correct order from smallest to largest is ($^{1}/_{6}$, $^{2}/_{6}$, $^{3}/_{6}$, $^{5}/_{6}$, $^{6}/_{6}$, $^{9}/_{6}$).

Ask the students:

1. How many of these fractions are less than 1?
2. Name a fraction that is equal to $^{1}/_{2}$.
3. Name a fraction that is equal to 1.
4. Name a fraction that is greater than 1.

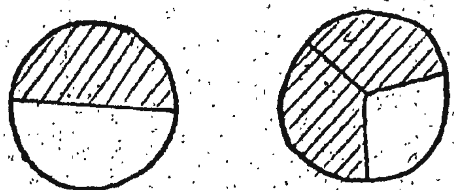
Now suppose we had two fractions like $^{1}/_{2}$ and $^{2}/_{3}$. Which fraction is greater: $^{1}/_{2}$ or $^{2}/_{3}$?

How can we decide?

Call to the students' attention the fact that the denominators of the fractions are not the same, and it is not as easy as the first example.

Is there any way we can decide which fraction is greater?

Students may suggest drawing regions and shading in the amounts. Try this.



From the figure, students can see that $^2/_3$ is greater than $^1/_2$. However, try to explain to students that to draw regions to decide which fractions are larger all the time is a very long process.

Ask students if there is another way we can decide which fraction is larger.
Since students have been introduced to this topic before, mention to them that we can use equivalent fractions.
Allow students time to think about this.
Remember when we did equivalent fractions we said that:

$$^1/_2 = {^2/_4} = {^3/_6} = {^4/_8} = {^5/_{10}} = {^6/_{12}}$$

Also, we wrote $^1/_2 = {^4/_6} = {^6/_9} = {^8/_{12}} = {^{10}/_{15}}$.
(Using equivalent fractions chart or multiplication by 1).

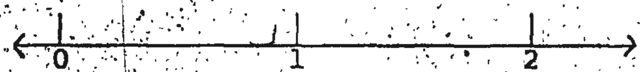
Ask students to compare the two lists to see if any one of the equivalent fractions for $^1/_2$ and $^2/_3$ have the same denominator. Students will readily see that $^1/_2 = {^3/_6}$ and $^2/_3 = {^4/_6}$.

Now we can decide which fraction is greater. Asking if $^2/_3$ is greater than or less than $^1/_2$ is the same as asking if $^4/_6$ is greater than or less than $^3/_6$.

$$2/3 = 4/6 \text{ is greater than } 1/2 = 3/6$$

Repeat this exercise with other examples like $3/4$ and $5/6$ and $3/10$, etc.

Now suppose we have a number line like this and we want to put a list of numbers in their correct position ($1/2$, $2/3$, $5/4$):



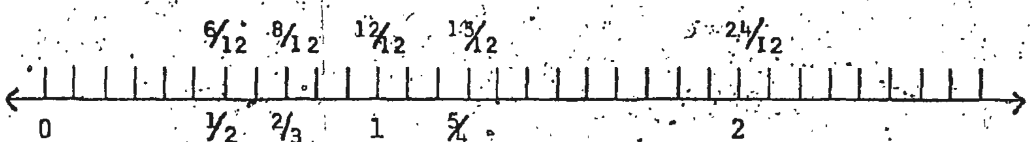
First of all, we have to decide which is the largest and which is the smallest. Before we can decide this, we must change the fractions to equivalent fractions with the same denominator. Ask the students to refer to the equivalent fractions charts they have constructed to find these fractions:

$$1/2 = 6/12; \quad 2/3 = 8/12; \quad 5/5 = 15/12$$

Which is the smallest? Answer = $\underline{1/2}$

Which is the largest? Answer = $\underline{5/4}$

To graph these fractions, we must divide each unit into 12 parts and then place the fractions in their correct order.



Try another example with the students doing the exercise themselves.

$$3/2, \; 1/4, \; 7/8, \; 9/8$$

Assign the practice exercizes.

Ordering Fractions

1. Use equivalent fractions to compare the two fractions. Decide whether the first fraction is less than, equal to, or greater than the second fraction: $^{2}/_{3}$ -- $^{3}/_{5}$.

2. Are $^{1}/_{8}$, $^{3}/_{8}$, $^{5}/_{8}$, $^{7}/_{8}$, $^{9}/_{8}$, listed in order from the greatest to the smallest?

3. Arrange these fractions in order from the smallest to the largest: $^{3}/_{5}$, $^{1}/_{5}$, $^{7}/_{5}$, $^{2}/_{5}$, $^{8}/_{5}$, $^{5}/_{5}$.

4. Mary wanted to buy $^{3}/_{4}$ of a metre of material. The clerk told her that there was $^{5}/_{8}$ of a metre in stock. Was this enough material for Mary?

5. How can we decide if a fraction is greater than 1?

Ordering Fractions

Use equivalent fractions to compare the two fractions in each exercise. Decide whether the first fraction is less than, equal to, or greater than the second fraction.

1. $^2/_3$ -- $^3/_5$

2. $^5/_6$ -- $^7/_9$

3. Are $^1/_8$, $^3/_8$, $^5/_8$, $^7/_8$, $^9/_8$, listed in order from the greatest to the smallest?

4. Graph the fractions in exercise 4 on a number line.

5. Arrange these fractions in order from the smallest to the largest: $^3/_2$, $^1/_4$, $^3/_4$, $^7/_8$.

6. Mary wanted to buy $^3/_4$ of a metre of material. The clerk told her that there was $^5/_8$ of a metre in stock. Was this enough material for Mary?

   Tell whether these fractions are greater than, less than, or equal to 1.

7. $^3/_4$

8. $^6/_5$

9. $^7/_7$

10. How can we decide if a fraction is greater than 1?

Ordering Fractions

Use equivalent fractions to compare the two fractions in each exercise. Decide whether the first fraction is less than, equal to, or greater than the second fraction.

1. $2/3$ -- $3/5$
2. $4/7$ -- $3/5$
3. $5/6$ -- $7/9$
4. Are $1/8$, $3/8$, $5/8$, $7/8$, $9/8$, listed in order from the greatest to the smallest?
5. Graph the fractions in exercise 4 on a number line.

Arrange the fractions in order from the smallest to the largest.

6. $3/5$, $1/5$, $7/5$, $2/5$, $8/5$, $5/5$
7. $3/2$, $1/4$, $3/4$, $7/8$
8. $1/3$, $1/2$, $5/6$, $1/6$
9. Mary wanted to buy $3/4$ of a metre of material. The clerk told her that there was $5/8$ of a metre in stock. Was this enough material for Mary?
10. Joe was baking bread for his mother. The recipe said to put in $3/4$ of a teaspoon of salt. Joe put in $6/8$ of a teaspoon. Was this too little, too much, or just enough salt?

Tell whether these fractions are greater than, less than, or equal to 1.

11. $3/4$

12. $6/5$

13. $7/7$

14. How can we decide if a fraction is greater than 1?

15. Graph on a number line these fractions:

$1/6$, $1/3$, $1/2$

Lowest Terms Fractions

Ask students to recall what a factor is. A <u>factor</u> is any number that will divide into another number. For example: What are the factors of 8? Remind students that when we ask for the factors of 8, we are asking, "What are the numbers that divide into 8?"

Students will readily tell you that the numbers that divide into 8 are 1, 2, 4, and 8. Therefore the factors of 8 = (1, 2, 4, 8). Write these on the blackboard.

Now ask the students what the factors of 12 are. Factors of 12 = (1, 2, 3, 4, 6, 12). Write these on the blackboard.

Now that students have some notion of what factors are again, ask them if 8 and 12 have any factors in common. In other words, are there any numbers that divide into 8 that also divide into 12.

By referring to the lists that they have given you, students should tell you that the numbers that divide into 8, and also into 12 are (1, 2, and 4). Thus the common factors of 8 and 12 are (1, 2, 4).

Now what is the greatest common factor of 8 and 12? If students do not respond to this question, ask them what is the largest number that will divide into both 8 and 12. This can be demonstrated from the list of common factors (1,2,4). The largest of the common factors is <u>4</u>.

<u>Definition</u>: The largest number that divides into two given numbers is called the <u>Greatest Common Factor</u> of these numbers.

Greatest Common Factor = G.C.F.

Repeat the same procedure a few times using other examples.

Examples: (a) 10 and 30 (b) 16 and 24

Allow students time to try an example themselves. This is an excellent review of multiplication facts and forces students to practice them if they are not strong in this area.

Briefly recall equivalent fractions again. Take an example like $^2/_3 = {}^4/_6 = {}^6/_9 = {}^8/_{12} = {}^{10}/_{15}$, etc. These fractions were built up by multiplying by 1 in the form of $^2/_2$, $^3/_3$, or $^4/_4$, etc.

$$^2/_3 \times {}^2/_2 = {}^4/_6 \qquad {}^2/_3 \times {}^3/_3 = {}^6/_9$$

Now, if we take a fraction like $^6/_9$, can we write it in a simpler way? Is there an equivalent fraction for $^6/_9$ in which the numerator and denominator are smaller?

Students will see that there are two fractions equivalent to $^6/_9$ which have a smaller numerator and denominator. These are $^4/_6$ and $^2/_3$. The smallest equivalent fraction for $^6/_9$ is $^2/_3$.

Remind students that $^6/_9 = \frac{2 \times 3}{3 \times 3}$. Ask them to notice that 3 divides into both the numerator and the denominator. In other words 6 and 9 have a common factor of 3.

Show them that if we divide a number by 1 we always get back the same number. Eg. $7 \div 1 = 7$. So since 6 and 9 have a common factor of 3, suppose we were to divide $^6/_9$ by 1.

$$6/9 \div 1 \quad (1 \text{ can be written as } 3/3)$$

so $6/9 \div 3/3 = \frac{6 \div 3}{9 \div 3} = 2/3$. $\quad 6/9 = 2/3$

Referring to the comparative fractions strips demonstrate to the students or let them see for themselves that this is in fact true.

Try another example like $12/16$. Can it be written as an equivalent fraction that has a smaller numerator and denominator? Are there any numbers that divide into 12 and also into 16? If some of the students suggest 2, then use this procedure.

We can remove this common factor of 2.

$$12/16 = \frac{12 \div 2}{16 \div 2} = 6/8$$

Now ask if $6/8$ can be written as an equivalent fraction with a smaller numerator and denominator.

Eg., Are there any numbers that divide into 6 and also into 8.

Student response = <u>2</u>.

Now we can remove a 2, $6/8 = \frac{6 \div 2}{8 \div 2} = 3/4$

So $12/16 = 6/8 = 3/4$.

Can we find a simpler equivalent fraction for $12/16$? Are there any numbers that divide into 3 and also into 4?

Response = <u>1</u>.

So $3/4 = \frac{3 \div 1}{4 \div 1} = 3/4$.

The simplest equivalent fraction for $12/16$ is $3/4$.

<u>Definition</u>: A fraction is said to be in simplest form, or in lowest terms if the largest number that divides into the

numerator and denominator is 1. Repeat this example and show students that finding the G.C.F. of the numerator and denominator and removing this factor reduces the fraction to lowest terms.

Factors of 12 = (1, 2, 3, 4, 6, 12) $^{12}/_{16} = \frac{12 \div 4}{16 \div 4} = {}^{3}/_{4}$
Factors of 16 = (1, 2, 4, 8, 16)

Common Factors = (1, 2, 4)
G.C.F. = (4)

Take another example $^{15}/_{18}$ and repeat the above procedure.

Is it in simplest form? Why or why not?

What is the G.C.F. of 15 and 18?

Divide out the G.C.F.

$^{15}/_{18} = \frac{15 \div 3}{18 \div 3} = {}^{5}/_{6}.$

Now ask if $5/_{6}$ is the lowest terms fraction for $15/_{18}$. Why?

(Refer to definition).

Assign exercises.

Lowest Terms Fractions

1. List all the factors of each number in order from the smallest to the largest: 36

2. List all the common factors of this pair of numbers: 12 and 24

3. Reduce this fraction to simplest form. Hint: Find the G.C.F. first and divide it into the numerator and denominator. $^{6}/_{8}$ = ________.

4. Reduce to lowest terms: $^{25}/_{10}$.

5. Reduce to lowest terms: $^{7}/_{13}$.

Lowest Terms Fractions

List all the factors of each number in order from the smallest to the largest.

1. 36

2. 28

List all the common factors of each pair of numbers.

3. 24 and 27

4. 12 and 24

5. Find the greatest common factor (G.C.F.) of: 12 and 24

6. What are the common factors of the numerator and denominator of $^{10}/_{15}$?

7. How can we decide if a fraction is in simplest form? (You can use an example).

8. What is a factor? (You can use an example).

9. Reduce this fraction to simplest form. Hint: Find the G.C.F. first and divide it into the numerator and denominator. $^{6}/_{8}$ = __________.

10. Reduce to lowest terms: $^{25}/_{10}$.

Lowest Terms Fractions

List all the factors of each number in order from the smallest to the largest.

1. (a) 24 2. 36

   (b) 30 3. 28

List all the common factors of each pair of numbers.

4. 24 and 27

5. 12 and 24

6. Find the greatest common factor (G.C.F.) of: 12 and 24

7. Find the greatest common factor (G.C.F.) of: 24 and 27

8. In the list of equivalent fractions $1/2 = 2/4 = 3/6 = 4/8$, etc.: Is there one fraction you would consider the simplest fraction? Why?

9. What are the common factors of the numerator and denominator of $10/15$?

10. How can we decide if a fraction is in simplest form? (You can use an example).

11. What is a factor? (You can use an example).

12. Reduce this fraction to simplest form. Hint: Find the G.C.F. first and divide it into the numerator and denominator. $6/8 =$ __________.

13. Reduce to lowest terms: $^{25}/_{10}$

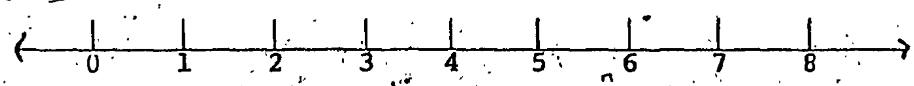
14. Reduce to lowest terms: $^{7}/_{13}$.

15. Reduce to lowest terms: $^{50}/_{25}$.

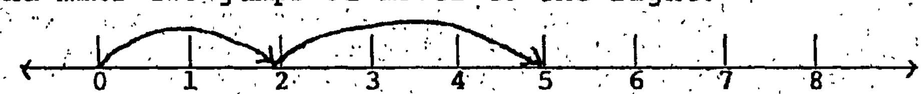
Adding Fractions with the Same Denominator

To help students see how to find the sum of two rational numbers, first show them how we can add whole numbers on a number line.

Draw a whole number line on the blackboard.



Suppose we want to add 2 + 3 on the number line. We start at 0 and make two jumps or moves to the right.



How many units long is the first move?

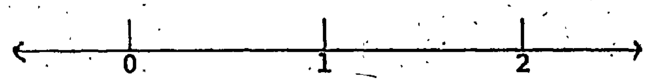
How many units long is the second move?

The two moves take you from point 0 to what point?

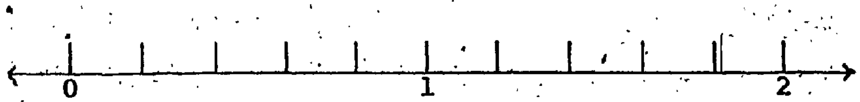
What point on the number line corresponds to 2 + 3?

Now ask the students how we could add two fractions like $^2/_5 + ^1/_5$.

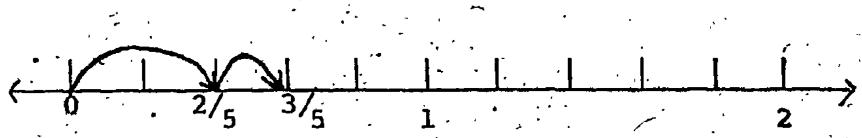
If a number line is constructed make sure the units are large enough.



Since we are adding fifths each unit must be divided into 5 equal parts.

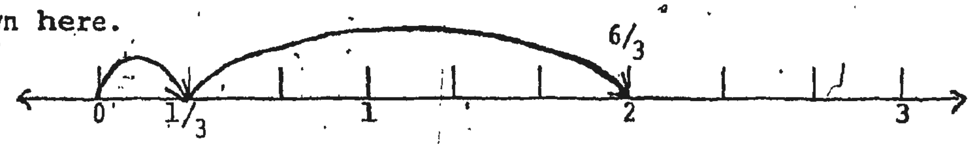


Now start at zero and jump $^2/_5$, then jump $^1/_5$.



What point do you arrive at on the number line? Do you see that $^2/_5 + ^1/_5 = ^3/_5$?

Draw another number line on the blackboard like the one shown here.



Ask the students how many parts each unit is divided into. Now on the number line represent by jumping that $^1/_3 + ^5/_3 = ^6/_3$

$$^6/_3 = \frac{6 \div 3}{3 \div 3} = ^2/_1 = 2$$

Now assign a problem for the students.

Problem: On a number line find the sum of $^5/_4$ and $^6/_4$. First divide each unit into 4 equal parts, then show a move of $^5/_4$ and a move of $^6/_4$ units.

$$^5/_4 + ^6/_4 = \_\_\_\_\_$$

Demonstrate the example on the blackboard, and if students have mastered the idea, assign a few more practical exercises.

Exercises: Add these fractions on separate number lines.

(a) $^6/_3 + ^5/_3 =$ ____ (b) $^0/_4 + ^3/_4 =$ ____ (c) $^1/_2 + ^4/_2 =$ ____

By now students should have arrived at a method for adding fractions without using a number line. Ask if anyone knows a rule for adding fractions. Most students will say that you must add the numerators and the denominator stays the same.

Take one of the previous examples already done on a number line.

eg. $^{2}/_{5} + ^{1}/_{5} = ^{3}/_{5}$ when we add on a number line.

Suppose we use our new rule of adding the numerator, and keeping the denominator the same. Show this on the blackboard

$$^{2}/_{5} + ^{1}/_{5} = \frac{2+1}{5} = ^{3}/_{5}$$

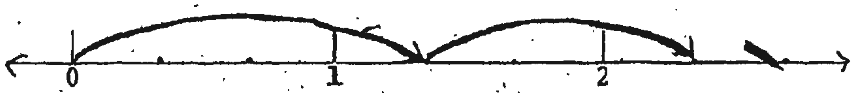
Since we get the same result our rule must be true. Check out a few more examples using both number lines and the rule to see if we always get the same result. Assign the practice exercises.

Adding Fractions with the Same Denominator

Find the sum.

1. $\frac{2}{8} + \frac{3}{8}$ 2. $\frac{5}{6} + \frac{2}{6}$ 3. $\frac{3}{9} + \frac{6}{9}$

4. Write the equation suggested by the number line:



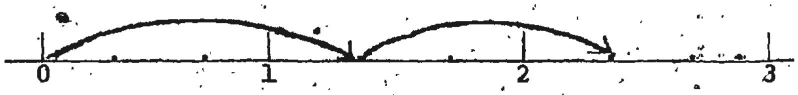
5. Bill painted $\frac{1}{5}$ of a fence. Jim painted $\frac{2}{5}$ of the fence. How much did they paint altogether?

Adding Fractions with the Same Denominator

Find the sum.

1. $2/8 + 3/8$
2. $9/4 + 2/4$
3. $5/6 + 2/6$
4. $3/9 + 6/9$
5. $6/7 + 3/7$
6. $5/3 + 0/3$

Write the equation suggested by each number line.

7. 

8. 

9. Bill painted $1/5$ of a fence. Jim painted $2/5$ of the fence. How much did they paint altogether?

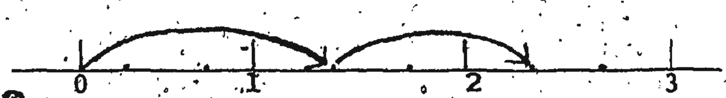
10. Ann bought $3/10$ of a lb. of grapes. Mary bought $4/10$ of a lb. of grapes. How much did they buy altogether?

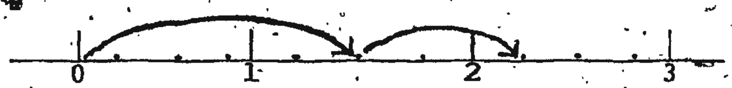
Adding Fractions with the Same Denominator

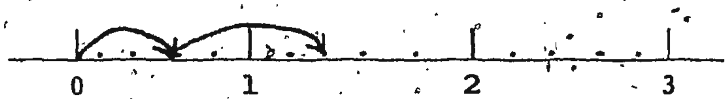
Find the sum.

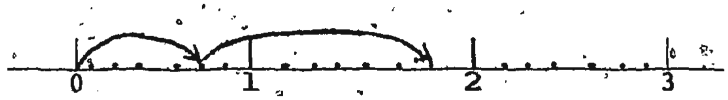
1. $2/8 + 3/8$
2. $9/4 + 2/4$
3. $5/6 + 2/6$
4. $4/5 + 3/5$
5. $3/9 + 6/9$
6. $7/8 + 2/8$
7. $6/7 + 3/7$
8. $5/3 + 0/3$

Write the equation suggested by each number line.

9. 

10. 

11. 

12. 

13. Bill painted $1/5$ of a fence. Jim painted $2/5$ of the fence. How much did they paint altogether?

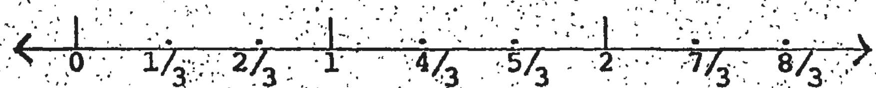
14. Ann bought $3/10$ of a lb. of grapes, Mary bought $4/10$ of a lb. of grapes. How much did they buy altogether?

15. Andy rowed a boat for $1/4$ of a mile. Bill then rowed the boat for $1/4$ of a mile. How far did they row altogether?

Concept of Mixed Numbers

We have already looked at fractions that are greater than 1, such as $5/_3$ and $7/_4$ etc. We noticed that for fractions greater than one whole unit, the numerator is greater than the denominator.

Let's look at some of these numbers again. Draw a number line on the blackboard like the one shown.



Ask "What number corresponds to point A?" Most students will answer $8/_3$. This answer is correct, but also – – $8/_3 = 2 + 2/_3$. Remind students that 2 can be written as $6/_3$. Therefore,

$8/_3 = 6/_3 + 2/_3$.

(using addition).

Therefore, $8/_3 = 2 + 2/_3$.

We write $2 + 2/_3$ as $2\ 2/_3$. It reads "Two and two thirds."

$$\begin{aligned} 2\ 2/_3 = 2 + 2/_3 &= 1 + 1 + 2/_3 \\ &= 3/_3 + 3/_3 + 2/_3 \\ &= 6/_3 + 2/_3 \\ &= 8/_3 \end{aligned}$$

Fractions in the form of $2\ 2/_3$ are called <u>mixed numbers</u>. They contain a whole number + a fraction.

Take another example like $3\ 3/_4$.

Show that $3\ 3/_4 = 3 + 3/_4$ on a number line. If we wanted to write $3\ 3/_4$ as a number in fraction form $a/_b$ would the numerator be larger than the denominator? Why?

Answer = Number greater than one.

$$3\ 3/_4 = 3 + 3/_4 = (1 + 1 + 1) + 3/_4$$
$$= 4/_4 + 4/_4 + 4/_4 + 3/_4$$
$$= 12/_4 + 3/_4 = 15/_4$$

so $3\ 3/_4 = 15/_4$.

Check back to the number line to see if in fact this is true.

Again explain that:

(a) if a fraction is less than one the numerator is less than the denominator;

(b) if a fraction is equal to one the numerator is equal to the denominator;

(c) if a fraction is greater than one the numerator is greater than the denominator.

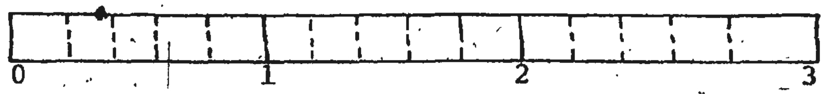
Try a couple of extra examples with students like $2\ 1/_4$ and $3\ 3/_5$, then assign a couple for students to work at by drawing their own number lines.

For examples (a) $17/_5$ (b) $4\ 1/_2$ (c) $1\ 4/_5$

In this way students will grasp the idea of changing from a mixed number to fractional form and vice versa.

If students have difficulty with this exercise, coloured strips of construction paper can be cut out and divided into units.

Example



This will allow students concrete experience in dealing with mixed numbers. Drawing strips and colouring appropriate amounts will achieve the same purpose.

When correcting the exercises work examples out both ways; from fractions to mixed numbers and from mixed numbers to fractions.

Example: $8/_7 = 7/_7 + 1/_7 \quad (7/_7 = 1)$ $\qquad 1\ 1/_7 = 1 + 1/_7$

$= 1 + 1/_7 \qquad\qquad\qquad\qquad = 7/_7 + 1/_7$

$= 1\ 1/_7 \qquad\qquad\qquad\qquad\quad = 8/_7$

This in a simple way is a basic introduction to finding a common denominator. Now ask students if there is a shorter way we could change a fraction like $2\ 1/_4$ to a fraction of the form $a/_b$ without breaking the mixed number apart and drawing a number line.

Demonsrtate to students that if we take $2\ 1/_4$, we can change to $9/_4$ by multiplying 2×4 and adding 1/over the denominator 4.

Explain to them that the process is the same as the one demonstrated above only this time it is in a shorter form.

$2\ 1/_4 = 2 + 1/_4 = (1 + 1) + 1/_4$ (Each 1 is $4/_4$ or four quarters).

Since we have 2 whole units, we have 2×4 quarters, or 8 quarters plus we also have another $1/_4$.

$$2\ 1/_4 = \frac{(2 \times 4) + 1}{4} = \frac{8 + 1}{4} = 9/_4$$

Take another example like $3\ 2/_5$. Demonstrate by using the number line that $3\ 2/_5 = 17/_5$

Also show that $3\ 2/_5 = 3 + 2/_5 = (1 + 1 + 1) + 2/_5$

$$= 5/_5 = 5/_5 = 5/_5 = 2/_5$$

$$= 15/_5 + 2/_5$$

$$= 17/_5$$

Proceed to show also that

$$3\ 2/_5 = \frac{(3 \times 5) + 2}{5} = \frac{15 + 2}{5} = 17/_5$$

In the same way fractions larger than 1 can be changed to mixed numbers by a shorter method.

Take an example like $15/_4$.

Show on a number line that $15/_4 = 12/_4 + 3/_4$

$$= 3\ 3/_4.$$

$$\frac{12 \div 4}{4 \div 4} = 3/_1 = 3$$

Now ask students if they can see a short method of changing fractions greater than 1 to mixed numbers.

$$15/_4 = 15 \div 4 = 3 \text{ and } 3 \text{ remainder.}$$

Most students remember this process.

Ask them the question about the remainder. What is the remainder? Is it 3 apples or candy or quarters or thirds?

By breaking $15/_4$ up into $4/_4 + 4/_4 + 4/_4 + 3/_4$ show students that $15/_4$ contains 3 whole units + $3/_4$ of another unit.

$$15/4 = 4\overline{)15} \quad \frac{12}{3} \; 3/4 \qquad 3 + 3/4 = 3\ 3/4$$

Do other examples like $11/3$, $13/15$, and $9/7$. Allow time for students to work along also.

Assign exercises.

Concept of Mixed Numbers

1. Express each fraction as a mixed number.

   (a) $8/_7$ (b) $14/_9$

2. Express each fraction as a mixed number.

   (a) $11/_5$ (b) $13/_4$

3. Change this mixed number to a fraction $a/_b$.

   $2\ 1/_5$ = ______.

4. Give the missing numerator:

   (a) $12/_7 = 7/_7 + \ /_7$ (b) $9/_6 = 6/_6 + \ /_6$.

5. Give the missing numerator:

   (a) $9/_8 = 1 + \ /_8$ (b) $13/_5 = 2 + \ /_5$.

Concept of Mixed Numbers

1. Express each fraction as a mixed number.

   (a) $8/_7$ (b) $14/_9$

2. Express each fraction as a mixed number.

   (a) $11/_5$ (b) $13/_4$

3. Change this mixed number to a fraction $a/_b$.

   $4\ 1/_3$ = ________

4. Change this mixed number to a fraction $a/_b$.

   $8\ 1/_4$ = ________

5. Change this mixed number to a fraction $a/_b$.

   $2\ 1/_5$ = ________

6. Give the missing numerator:

   (a) $5/_3 = 3/_3 + \ /_3$ (b) $12/_{10} = 10/_{10} + \ /_{10}$

7. Give the missing numerator:

   $8/_6 = 1 + \ /_6$ (b) $5/_5 = 1 + \ /_5$

8. Give the missing numerator:

   (a) $9/_8\ 1 + \ /_8$ (b) $13/_5 = 2 + \ /_5$

9. Which is larger, $2\ 1/_3$ or $2\ 1/_2$?

10. Bill ate $3/_7$ of a pie, Jim ate $6/_7$ of a pie. How much pie did they eat all together?

Concept of Mixed Numbers

1. Express each fraction as a mixed number.

   (a) $8/_7$ (b) $14/_9$

2. Express each fraction as a mixed number.

   (a) $19/_3$ (b) $23/_6$

3. Express each fraction as a mixed number.

   (a) $11/_5$ (b) $13/_4$

4. Change this mixed number to a fraction $a/_b$.

   $4\ 1/_3$ = ________

5. Change this mixed number to a fraction $a/_b$.

   $8\ 1/_4$ = ________

6. Change this mixed number to a fraction $a/_b$.

   $2\ 1/_5$ = ________

7. Give the missing numeral:

   (a) $5/_3 = 3/_3 + \ /_3$ (b) $12/_{10} = 10/_{10} + \ /_{10}$

8. Give the missing numeral:

   (a) $12/_7 = 7/_7 + \ /_7$ (b) $9/_6 = 6/_6 + \ /_6$

9. Give the missing numerator:

   (a) $8/_6 = 1 + \ /_6$ (b) $5/_5 = 1 + \ /_5$

10. Give the missing numerator:

    (a) $9/_8 = 1 + \ /_8$ (b) $13/_5 = 2 + \ /_5$

11. Which is larger, 2 $1/_3$ or 2 $1/_2$?

12. If cheese comes in $1/_2$ kg containers, how much would 9 containers weigh?

13. Bill ate $3/_7$ of a pie, Jim ate $6/_7$ of a pie. How much pie die they eat all together?

14. The distance between Jim's house and Joe's house is $1/_3$ km. Jim walks this distance 10 times a day. How far does he walk?

15. Which is smaller 1 $1/_2$ or 1 $3/_4$?

Adding Mixed Numbers with the Same Denominator

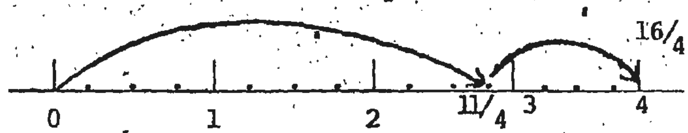
Refer back to adding fractions with the same denominator.

Example: $2/_3 + 5/_3 = \frac{2+5}{3} = 7/_3 = 2\ 1/_3$.

Now, how can we add mixed numbers whose fractional parts have the same denominator?

Suppose we are asked to add $2\ 3/_4 + 1\ 1/_4$. Can you think of a way to add these mixed numbers?

To present the idea of adding mixed numbers refer to a number line or a fractional strip.



$2\ 3/_4 = 11/_4$ $\qquad$ $1\ 1/_4 = 5/_4$

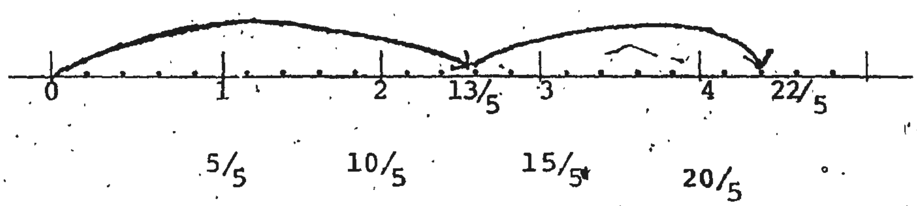
$2\ 3/_4 + 1\ 1/_4 = 11/_4 + 5/_4 = 16/_4 = 4/_1 = 4\ (\frac{16 \div 4}{4 \div 4}) = 4/_1 = 4$

$2\ 3/_4 + 1\ 1/_4 = 4$

Try another example (use either strips or a number line).

$2\ 3/_5 + 1\ 4/_5$ $\qquad$ $2\ 3/_5 = 13/_5$ $\qquad$ $1\ 4/_5 = 9/_5$

$2\ 3/_5 + 1\ 4/_5 = 13/_5 + 9/_5 = 22/_5 = 4\ 2/_5$



yet. This will develop later on.

Try presenting a few examples using both methods.

$$\begin{array}{r} 2\ 3/4 \\ +\ 1\ 1/4 \\ \hline 3 + 4/4 \\ \hline 3 + 1 = 4 \end{array}$$

$$\boxed{\begin{array}{ll} 2 & 3/4 \\ +1/3 & +\ 1/4 \\ \hline & +\ 4/4 \end{array}} \quad \boxed{4/4 \ = 1}$$

Present as many examples as necessary.

$$\begin{array}{r} 2\ 3/5 \\ +1\ 4/5 \\ \hline 3 + 7/5 \\ \hline 3 + 1 + 2/5 \\ \hline = 4\ 2/5 \end{array}$$

$$\boxed{\begin{array}{ll} 2 & 3/5 \\ +1/3 & +\ 4/5 \\ \hline & 7/5 \end{array}} \quad \boxed{7/5 = 1\ 2/5}$$

Assign exercises.

Do as many examples using strips, number lines, or the method already shown until students have mastered the idea. Let them work on some examples, before you actually demonstrate the solution. This will get them involved in the process and create a challenge. As students are doing the exercises walk around and help by explaining to those who are having difficulty. At this time students are ready to proceed with another faster approach--a computational approach.

$$2\ 3/_4 + 1\ 1/_4 = 2 + 3/_4 + 1 + 1/_4$$

Now add the whole units, then add the fractional units. Ask students, "how many whole units do we have?"

Answer = $(2 + 1) = 3$

Also we have $3/_4 + 1/_4$ fractional parts of a unit.

$$3/_4 + 1/_4 = 4/_4$$

So we have 3 whole units + $4/_4$ of another unit

$4/_4 = 1$

So we have $3 + 1 = 4$

$$\begin{aligned} 2\ 3/_4 + 1\ 1/_4 &= 2 + 3/_4 + 1 + 1/_4 \\ &= (2 + 1) + (3/_4 + 1/_4) \\ &= 3 + 4/_4 \\ &= 3 + 1 \\ &= 4 \end{aligned}$$

Some students may work better on a vertical form, so try to present both a horizontal and a vertical view. Do not try to force the student into one mode of thinking just

Adding Mixed Numbers with Same Denominator

Add these mixed numbers. Reduce to lowest terms if possible.

1. $3\ 2/_3 + 5\ 2/_3$ 2. $2\ 1/_7 + 5\ 4/_7$ 3. $4\ 4/_9 + 3\ 5/_9$

Add, using a number line. Write your answer as a mixed number.

4. $2\ 1/_4 + 3\ 1/_4$

5. Mary's little finger is $4\ 1/_2$ cm long; Ann's little finger is $4\ 1/_2$ cm long. How long are their little fingers together?

Adding Mixed Numbers with Same Denominator

Add these mixed numbers. Reduce to lowest terms if possible.

1. $3\ 2/_3 + 5\ 2/_3$
2. $2\ 1/_4 + 6$
3. $2\ 3/_5 + 4\ 3/_5$
4. $4\ 4/_9 + 3\ 5/_9$
5. $4\ 1/_2 + 2$

Add these mixed numbers using a number line. Write your answers as mixed numbers.

6. $2\ 1/_4 + 3\ 1/_4$
7. $1\ 1/_3 + 2\ 2/_3$
8. $1\ 1/_5 + 2\ 3/_5$
9. $3\ 1/_2 + 2\ 1/_2$
10. Mary's little finger is $4\ 1/_2$ cm long. Ann's little finger is $4\ 1/_2$ cm long. How long are their little fingers together?

Adding Mixed Numbers with Same Denominator

Add these mixed numbers. Reduce to lowest terms if possible.

1. $3\frac{2}{3} + 5\frac{2}{3}$
2. $2\frac{1}{4} + 6$
3. $2\frac{3}{5} + 4\frac{3}{5}$
4. $2\frac{1}{7} + 5\frac{4}{7}$
5. $4\frac{4}{9} + 3\frac{5}{9}$
6. $4\frac{1}{2} + 2$

Add these mixed numbers using a number line. Write your answers as mixed numbers.

7. $2\frac{1}{4} + 3\frac{1}{4}$
8. $1\frac{1}{3} + 2\frac{2}{3}$
9. $1\frac{1}{5} + 2\frac{3}{5}$
10. $2\frac{1}{5} + 1$
11. $3\frac{1}{2} + 2\frac{1}{2}$
12. $2\frac{3}{4} + 2\frac{1}{4} + 1\frac{1}{4}$
13. Bill caught a trout that weighed $1\frac{1}{4}$ kilograms; Joe caught a trout that weighed $2\frac{3}{4}$ kilograms. How much did the trout weigh altogether?
14. Mary's little finger is $4\frac{1}{2}$ cm long; Ann's little finger is $4\frac{1}{2}$ cm long. How long are the little fingers together?
15. The distance from Dog Cove to Bread Cove is $1\frac{2}{5}$ km; the distance from Bread Cove to Long Cove is $1\frac{4}{5}$ km. How far is from Dog Cove to Long Cove?

Adding Fractions with Unlike Denominators

Suppose we want to find the sum of $1/_6 + 1/_3$. Recall from adding fractions with the same denominator that $1/_6 + 1/_6 = 2/_6$ and $1/_3 + 1/_3 = 2/_3$.

Now if we add $1/_6 + 1/_3$ what do you think the answer will be? Have the students notice that in this case we have two different denominators. Using paper strips with the units different in terms of number of parts demonstrate a couple of examples.

$1/_6 + 1/_3 = 1/_6 + 2/_6 = 3/_6$

1/3	1/3	1/3

$1/_3 = 2/_6$

1/6	1/6	1/6	1/6	1/6	1/6

A few of the fast paced students may immediately see the principle involved.

Another example: $3/_4 + 1/_2 = 3/_4 + 2/_4 = 5/_4$

$1/_2 = 2/_4$

$1/_2$	$1/_2$

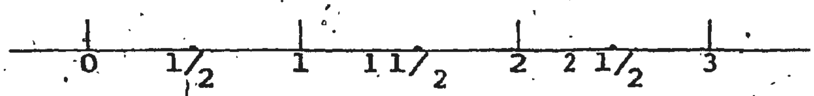
$1/_4$	$1/_4$	$1/_4$	$1/_4$

Start by explaining to the students that when adding fractions which have different denominators we must change these fractions so that they can be written with the same denominators or a <u>common denominator</u>.

This can be demonstrated to students by taking an example like $1/_2 + 1/_5$ and drawing a number line on the blackboard.

We now find a problem with dividing the units into fractional parts. Do we divide them into $1/2$'s or $1/5$'s?

Suppose we try $1/2$'s:



Now add $1/2 + 1/5$. Starting at zero and jumping $1/2$ a unit is easy, but the second jump is $1/5$. How far do we jump? Students should see that in order to add two fractions they must have a common denominator.

Recall the comparing fractions lesson. How did we find a common denominator?

Do another example $1/3 + 1/4$. Students should see by looking at a number line that we cannot add these fractions in this form.

Ask students how they decided which fraction was larger $1/4$ or $1/3$. In order to decide, these fractions had to be written with the same denominator. This required the use of an equivalent fractions list.

<u>Example</u>: $1/3 = 2/6 = 3/9 = 4/12 = 5/15 = 6/18 = 7/21$

$$1/4 = 2/8 = 3/12 = 4/16 = 5/20 = 6/24 = 7/28$$

To find equivalent fractions for $1/4$ and $1/3$ which have the same denominator we look through the list and find that

$$1/3 = 4/12 \text{ and } 1/4 = 3/12$$

Therefore $1/3$ is larger than $1/4$.

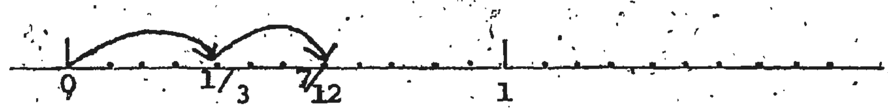
The same idea can now be applied to adding two fractions with unlike denominators

$1/_3 + 1/_4$

Remember when we added these fractions on the number line we found that we couldn't add because the denominators were unequal.

However, if we write $1/_3 = 4/_{12}$ and $1/_4 = 3/_{12}$ we can now write $1/_3 + 1/_4 = 4/_{12} + 3/_{12} = 7/_{12}$.

Demonstrate this on the number line. Notice that the unit must be divided into 12 parts.



$1/_3 + 1/_4 = 7/_{12}$.

One way to add fractions with unlike denominators on a number line is to find a common denominator using equivalent fractions.

Do a few other examples on the blackboard for the students. Explain the process very thoroughly. Allow students to work through the examples themselves. This will get students involved in the guided discovery process. Students can use the equivalent fractions chart they used previously.

Some examples that could be used:

(a) $1/_2 + 1/_5$ (b) $1/_3 + 1/_6$ (c) $2/_3 + 1/_5$

Be very careful with example (b). Have students note that for $1/_3 + 1/_6$ the common denominator is 6. This is because

3 divides into 6. This example will be explained more fully later.

When students are adept at adding fractions using the equivalent fractions chart, try to wean them away from it. If students become dependent on the chart then the explanation device has been overused. Be careful to see that this does not happen.

One way to reduce dependence on the equivalent fractions chart is to introduce the idea of finding a common denominator by using <u>multiples</u>.

Try the example $1/_3 + 1/_4$.

Ask students if anyone remembers what multiples are.

Explain by listing the multiples of 3.

Multiples of 3: 0, 3, 6, 9, 12, 15, 18, 21, 24, 27, . . .

Now ask, "What are the multiples of 4?"

Multiples of 4: 0, 4, 8, 12, 16, 20, 24, 28, 32, . . .

Students should see that the <u>multiples</u> of 3 and 4 are all the numbers that 3 and 4 divide into.

This idea is not to be confused with the concept of a factor. The factors of 6 are not the same as the multiples of 6. Now in order to add $1/_3 + 1/_4$ we must find a denominator that is a multiple of both 3 and 4. That is, both 3 and 4 divide into this denominator.

By looking at the table the child can see that it shows two multiples that are the same for both 3 and 4.

Multiples of 3 and 4: 12 and 24.

These numbers are called common multiples of 3 and 4.

Explain to the students that when we are finding a common denominator for two fractions, we find the lowest or smallest common multiple.

In the example $1/_3 + 1/_4$ the smallest number that 3 and 4 will divide into (other than zero) is 12.

Now when adding $1/_3$ we change our fractions to an equivalent fraction with

$$\underline{+1/_4}$$

the common denominator of 12.

$$1/_3 = /_{12} \qquad 1/_4 = /_{12}$$

$$1/_3 \times 4/_4 = 4/_{12} \qquad 1/_4 \times 3/_3 = 3/_{12}$$

$$1/_3 = 4/_{12}$$

$$\underline{+1/_4} \quad \underline{+3/_{12}}$$

$$7/_{12}$$

This process is probably the best way for students to convert to the common denominator. Also, a lot of students understand the process better in a vertical formation.

Demonstrate these other examples also.

(a) $1/_2 + 1/_5$ (b) $1/_3 + 1/_6$ (c) $2/_3 + 1/_5$

Allow students time to grasp the ideas and try some themselves. Place particular emphasis on example (b); explain to students that 6 is a multiple of 3.

Demonstrate some examples in a horizontal formation, but do not force students to do exercises in a particular way.

Let them become familiar with the ideas, the horizontal approach will develop later when students may better understand it.

Examples:

$$\frac{1}{3} + \frac{1}{4}$$

$$= \frac{4 + 3}{12}$$

$$= \frac{7}{12}$$

Follow the arrows.

3 divides into 12 (four times).

4 times 1 is 4.

4 divides into 12 (three times).

3 times 1 is 3.

Ask students to construct a multiples table for the numbers from 2 to 12. This is a good exercise to develop the concept of multiples and will mean more to the students if they construct it themselves.

Now present another pair of fractions to add $2/5 + 3/4$. Ask students to refer to multiples table to find the common denominator. The child looks in the 5's row and in the 4's row. He finds that the L.C.M. is 20. Now he must write $2/5$ and $3/4$ as equivalent fractions with a denominator of 20.

$$\begin{array}{lcl} \ 2/5 & \times\ 4/4 & 8/20 \\ +\ 3/4 & \times\ 5/5 & 15/20 \\ \hline & & 23/20 = 1\ 3/20 \end{array}$$

The vertical form also has the added benefit of writing the equivalent fractions side by side as in the equivalent fractions tables.

If a child finds a common multiple that is not an L.C.M. do not discourage him or tell him he is wrong. Rather, when he finds his answer make sure he reduces it to a lowest terms fraction.

Assign exercises.

Addition of Fractions with Unlike Denominators

1. List the first 4 multiples of:

   (a) 6 (b) 7

2. Find the lowest common multiple (L.C.M.) of:

   (a) 5 and 6 (b) 3 and 7

Copy each exercise and give the missing numerator. Find the sum.

3. $\frac{1}{2} = \frac{\ }{6}$

   $+\frac{1}{3} = +\frac{\ }{6}$

4. $\frac{5}{6} + \frac{1}{3}$

   $= \frac{\ }{6} + \frac{\ }{6} =$

5. Find the sum. Give the answer in lowest terms.

   $\frac{7}{8}$

   $+\frac{1}{2}$

Addition of Fractions with Unlike Denominators

1. List the first 4 multiples of:

   (a) 6 (b) 7

2. List the first 5 multiples of:

   (a) 4 (b) 5

3. Find the L.C.M. of: (4 and 16)

4. Find the L.C.M. of: (9 and 12)

Copy each exercise and give the missing numerator. Find the sum.

5. $\frac{1}{2} = \frac{\ }{6}$
   $+\ \frac{1}{3} + \frac{\ }{6}$

6. $\frac{1}{4} = \frac{\ }{8}$
   $+\ \frac{3}{8} + \frac{\ }{8}$

7. $\frac{5}{6} + \frac{1}{3} = \frac{\ }{6} + \frac{\ }{6} =$

Find the sums. Give answers in lowest terms.

8. $\frac{1}{3}$
   $+\ \frac{1}{6}$

9. $\frac{5}{6} + \frac{1}{9}$

10. Jim studied history for $\frac{1}{3}$ of an hour, then he studied religion for $\frac{1}{4}$ of an hour. How much time did he spend studying? Was this more than, less than or equal to $\frac{1}{2}$ hour?

Addition of Fractions with Unlike Denominators

1. List the first 4 multiples of:

   (a) 6 (b) 7

2. List the first 5 multiples of:

   (a) 4 (b) 5

3. Find the lowest common multiple (L.C.M.) of:

   (a) 5 and 6 (b) 3 and 7

4. Find the L.C.M. of: (4 and 16)

5. Find the L.C.M. of: (9 and 12)

Copy each exercise and give the missing numerator. Find the sum.

6. $\frac{1}{2} = \frac{\ }{6}$, $+ \frac{1}{3} + \frac{\ }{6}$

7. $\frac{1}{4} = \frac{\ }{8}$, $+ \frac{3}{8} + \frac{\ }{8}$

8. $\frac{7}{12} = \frac{\ }{12}$, $+ \frac{1}{3} + \frac{\ }{12}$

9. $\frac{5}{6} + \frac{1}{3}$

   $= \frac{\ }{6} + \frac{\ }{6} =$

10. $\frac{1}{8} + \frac{5}{12}$

    $= \frac{\ }{24} + \frac{\ }{24}$

Find the sums. Give answers in lowest terms.

11. $\frac{1}{3} + \frac{1}{6}$

12. $\frac{5}{6} + \frac{1}{9} =$

13. $\frac{2}{3} + \frac{2}{5}$

14. $\frac{7}{8} + \frac{1}{2}$

15. Jim studied history for $\frac{1}{3}$ of an hour, then he studied religion for $\frac{1}{4}$ of an hour. How much time did he spend studying? Was this more than, less than or equal to $\frac{1}{2}$ hour?

The Addition of Mixed Numbers with Unlike Denominators

Proceed with the teaching of addition of mixed numbers with unlike denominators using the method for adding mixed numbers with like denominators. Combine with this, the method for adding fractions with unlike denominators.

Demonstrate a couple of examples on the blackboard.

$3\,{}^1/_4 + 2\,{}^2/_3$ Find the common denominator for fractional parts.

Multiples of 4: 0, 4, 8, 12, 16

Multiples of 3: 0, 3, 6, 9, 12, 15

Lowest common multiple = 12

$$\begin{aligned} 3\,{}^1/_4 &= 3\,{}^3/_{12} \\ +\,2\,{}^2/_3 &= 2\,{}^8/_{12} \\ &= 5\,{}^{11}/_{12} \end{aligned} \qquad {}^1/_4 \begin{smallmatrix}\times 3\\ \times 3\end{smallmatrix} = {}^3/_{12} \qquad {}^2/_3 \begin{smallmatrix}\times 4\\ \times 4\end{smallmatrix} = {}^8/_{12}$$

(add whole parts, then add the fractional parts)

Take another example and demonstrate

$$\begin{aligned} 2\,{}^3/_4 &= 2\,{}^{15}/_{20} \\ +\,4\,{}^3/_5 &= 4\,{}^{12}/_{20} \\ \hline & \;\;6\,{}^{27}/_{20} \end{aligned}$$

L.C.M. of 4 and 5 is 20

$${}^3/_4 \begin{smallmatrix}\times 5\\ \times 5\end{smallmatrix} = {}^{15}/_{20} \qquad {}^3/_5 \begin{smallmatrix}\times 4\\ \times 4\end{smallmatrix} = {}^{12}/_{20}$$

$${}^{27}/_{20} = 1\,{}^7/_{20}$$

$$\begin{aligned} 6\,{}^{27}/_{20} &= 6 + {}^{27}/_{20} \\ &= 6 + 1\,{}^7/_{20} \\ &= 7\,{}^7/_{20} \end{aligned}$$

Assign a couple of exercises that students can work at, and explain as you do the examples on the blackboard. Assign the practice exercises.

Addition of Mixed Numbers with Unlike Denominators

1. What common denominator would you use to add?

$$\begin{array}{r} 3\ 1/3 \\ +\ 2\ 1/5 \\ \hline \end{array}$$

2. Fill in the missing numerator and add:

$$\begin{array}{r} 2\ 1/5 \\ +\ 1\ 1/4 \\ \hline \end{array} = \begin{array}{r} 2/20 \\ +\ 1/20 \\ \hline \end{array}$$

3. Find the sum:

$$\begin{array}{r} 2\ 1/4 \\ +\ 3\ 1/2 \\ \hline \end{array}$$

4. Find the sum:

$1\ 1/3 + 5\ 1/6$

5. Find the sum:

$$\begin{array}{r} 2\ 1/4 \\ +\ 2\ 1/6 \\ \hline \end{array}$$

Addition of Mixed Numbers with Unlike Denominators

1. What common denominator would you use to add?

$$\begin{array}{r} 3\,\frac{1}{3} \\ +\ 2\,\frac{1}{5} \\ \hline \end{array}$$

2. What common denominator would you use to add?

$$\begin{array}{r} 2\,\frac{1}{4} \\ +\ 3\,\frac{1}{8} \\ \hline \end{array}$$

3. Fill in the missing numerator and add:

$$\begin{array}{r} 3\,\frac{1}{4} \\ +\ 2\,\frac{1}{2} \\ \hline \end{array}$$

4. Fill in the missing numerator and add:

$$\begin{array}{rcr} 2\,\frac{1}{5} & = & \frac{2}{20} \\ +\ 1\,\frac{1}{4} & + & \frac{1}{20} \\ \hline + & & \end{array}$$

5. Find the sum:

$$\begin{array}{r} 2\,\frac{1}{4} \\ +\ 3\,\frac{1}{2} \\ \hline \end{array}$$

6. Find the sum:

$$\begin{array}{r} 3\,\frac{2}{5} \\ +\ 5\,\frac{4}{7} \\ \hline \end{array}$$

7. Find the sum:

$1\,\frac{1}{3} + 5\,\frac{1}{6}$

8. Find the sum:

$$\begin{array}{r} 2\,\frac{1}{4} \\ +\ 2\,\frac{1}{6} \\ \hline \end{array}$$

9. Find the sum:

$1\,\frac{5}{5} + 2\,\frac{2}{3}$

10. Jim ate $4\,\frac{1}{2}$ apples, and Bob ate $3\,\frac{3}{4}$ apples. How many apples did they eat in all?

Addition of Mixed Numbers with Unlike Denominators

1. What common denominator would you use to add?

$$\begin{array}{r} 3\ 1/3 \\ +\ 2\ 1/5 \\ \hline \end{array}$$

2. What common denominator would you use to add?

$$\begin{array}{r} 2\ 1/4 \\ +\ 3\ 1/8 \\ \hline \end{array}$$

3. Fill in the missing numerator and add:

$$\begin{array}{rcr} 3\ 1/4 & = & 3\ \ \ /4 \\ +\ 2\ 1/2 & & +\ 2\ \ \ /4 \\ \hline \end{array}$$

4. Fill in the missing numerator and add:

$$\begin{array}{rcr} 2\ 1/5 & = & 2\ \ \ /20 \\ +\ 1\ 1/4 & & +\ 1\ \ \ /20 \end{array}$$

5. Find the sum:

$$\begin{array}{r} 2\ 1/4 \\ +\ 3\ 1/2 \\ \hline \end{array}$$

6. Find the sum:

$$\begin{array}{r} 3\ 2/5 \\ +\ 5\ 4/7 \\ \hline \end{array}$$

7. Find the sum:

$1\ 1/3 + 5\ 1/6$

8. Find the sum:

$$\begin{array}{r} 2\ 1/4 \\ +\ 2\ 1/6 \\ \hline \end{array}$$

9. Find the sum:

$$\begin{array}{r} 3\ 1/3 \\ +\ 6\ 4/9 \\ \hline \end{array}$$

10. Find the sum:

$1\ 5/7 + 2\ 2/3$

11. Find the sum:

$2\ 1/_2 + 4\ 2/_3$

12. Find the sum:

$3\ 2/_5 + 2\ 1/_3$

13. Jim ate $4\ 1/_2$ apples. Bob ate $3\ 3/_4$ apples. How many apples did they eat in all?

14. For a walk-a-thon, Ann walked $5\ 2/_5$ km, Ralph walked $4\ 1/_6$ km. How far did they walk altogether?

15. Ann, Mike, and Joe decided to save their money to buy Christmas presents. Ann saved $2\ 1/_4$ dollars, Mike saved 4 dollars, and Joe saved $3\ 3/_{10}$ dollars. How many dollars did they save altogether?

Concluding Comments

Review the complete unit with the students, giving individual help where needed. Encourage students to make up their own examples for review and practice. It may help if they study in groups.

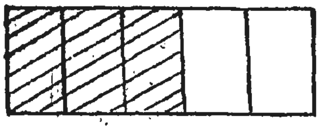
The exam can be written in a class period. Students may use their own paper when writing the exam.

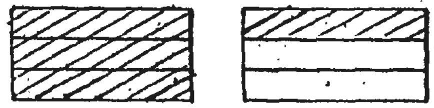
APPENDIX B

IMMEDIATE POSTTEST

Immediate Posttest

1. Give the fractional amount of the region represented by the shaded area.

(a) 

(b) 

2. In the fraction $3/5$

(a) The 5 is called the ________________.

(b) The 3 is called the ________________.

(c) The denominator tells us ________________________.

(d) The numerator tells us __________________________.

3. Draw a shaded region to represent these fractions:

(a) $3/4$ (b) $2/7$

4. List three equivalent fractions for each of the following:

(a) $2/3$ (b) $1/6$

5. Tell whether these pairs of fractions are equivalent. Show how you get your answer.

(a) $3/5$ and $2/3$ (b) $5/8$ and $10/16$

6. Find and list all the factors of:

(a) 16 (b) 28

7. Find the G.C.F. of:

(a) 24 and 32 (b) 10 and 40

8. Reduce each of these fractions to lowest terms:

(a) $6/9$ (b) $15/20$ (c) $16/24$

9. Using a number line, add each of the following. Check your answer:

(a) $3/_5 + 4/_5$ (b) $5/_6 + 2/_6$

10. Tell whether greater than, less than, or equal to by placing the correct word in the blanks.

(a) $2/_3$ ______ $3/_4$ (b) $5/_6$ ______ $4/_5$ (c) $4/_5$ ______ $16/_{20}$

11. Construct a shaded region to represent the mixed number: $1\ 2/_5$

12. Change these mixed numbers to fraction form $a/_b$

(a) $1\ 1/_4$ (b) $2\ 2/_3$

13. Find the sum:

(a) $2\ 1/_7 + 3\ 4/_7$ (b) $4\ 3/_5 + 2\ 4/_5$

14. List 5 multiples of 7.

15. Find the lowest common multiple of:

(a) 8 and 6 (b) 3 and 15

16. Explain the difference between a factor and multiple. Use an example if necessary.

17. Find the sum: Reduce answers to lowest terms:

(a) $5/_6 + 1/_3$ (b) $1/_5 + 3/_4$ (c) $1\ 1/_2 + 2\ 1/_6$

(d) $\begin{array}{r} 3\ 1/_2 \\ +\ 4\ 1/_5 \\ \hline \end{array}$

18. (a) Mary ate $2/_9$ of an apple pie. Jane ate $3/_9$ of the pie. How much did they eat altogether?

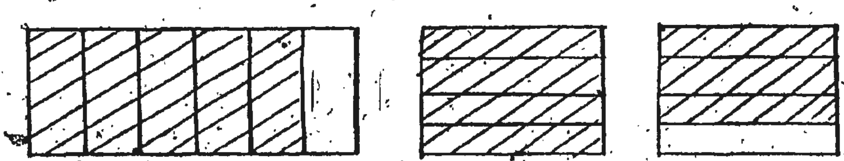
19. (b) Jim walked 2 $1/_3$ km. Mike walked 3 $1/_2$ km. How far did they walk altogether?

APPENDIX C

DELAYED POSTTEST

Delayed Posttest

1. Give the fractional amount of the region represented by the shaded area.



2. Draw a shaded region to represent these fractions:

   (a) $2/3$ (b) $4/5$

3. In the fraction $4/3$:

   (a) The 3 is called the ______________.

   (b) The 4 is called the ______________.

   (c) The denominator tells us ______________________.

   (d) The numerator tells us ______________________.

4. Tell whether these pairs of fractions are equivalent. Show how you get your answer.

   (a) $4/7$ and $8/14$ (b) $3/4$ and $2/3$

5. List three equivalent fractions for each of the following:

   (a) $3/5$ (b) $1/3$

6. List all the factors of:

   (a) 12 (b) 28

7. Find the greatest common factor (G.C.F.) of:

   (a) 16 and 24 (b) 10 and 30

8. Reduce each of these fractions to lowest terms:

   (a) $5/10$ (b) $12/16$ (c) $12/18$

9. Using a number line, add each of the following. Check your answer.

(a) $2/_7 + 3/_7$ (b) $5/_6 + 3/_6$

10. Construct a shaded region to represent the mixed number --1 $1/_4$.

11. Tell whether greater than, less than, or equal to by placing the correct word in the blank.

(a) $3/_4$ ____ $9/_{12}$ (b) $3/_5$ ____ $4/_7$ (c) $1/_5$ ____ $1/_3$

12. Change these mixed numbers to fractions of the form $a/_b$.

(a) 1 $2/_5$ (b) 2 $3/_4$

13. Find the sum:

(a) 2 $1/_7$ + 3 $3/_7$ (b) 4 $3/_5$ + 2 $2/_5$

14. List 5 multiples of 4.

15. Find the lowest common multiple (L.C.M.) of:

(a) 4 and 5 (b) 3 and 9

16. Explain the difference between a factor and a multiple. Use an example, if necessary.

17. Find the sum. Reduce answers to lowest terms (if possible).

(a) $5/_6 + 1/_3$ (b) $1/_4 + 2/_7$ (c) 1 $1/_3$ + 2 $1/_6$

(d) 2 $1/_2$
+ 5 $3/_8$

18. Mary walked $2/_7$ of a kilometer. Jane walked $4/_7$ of a kilometer. How far did they walk altogether?

19. In an apple eating contest, Ann ate 2 $1/_4$ apples and Joan ate 3 $1/_2$ apples. How many apples did they eat altogether?