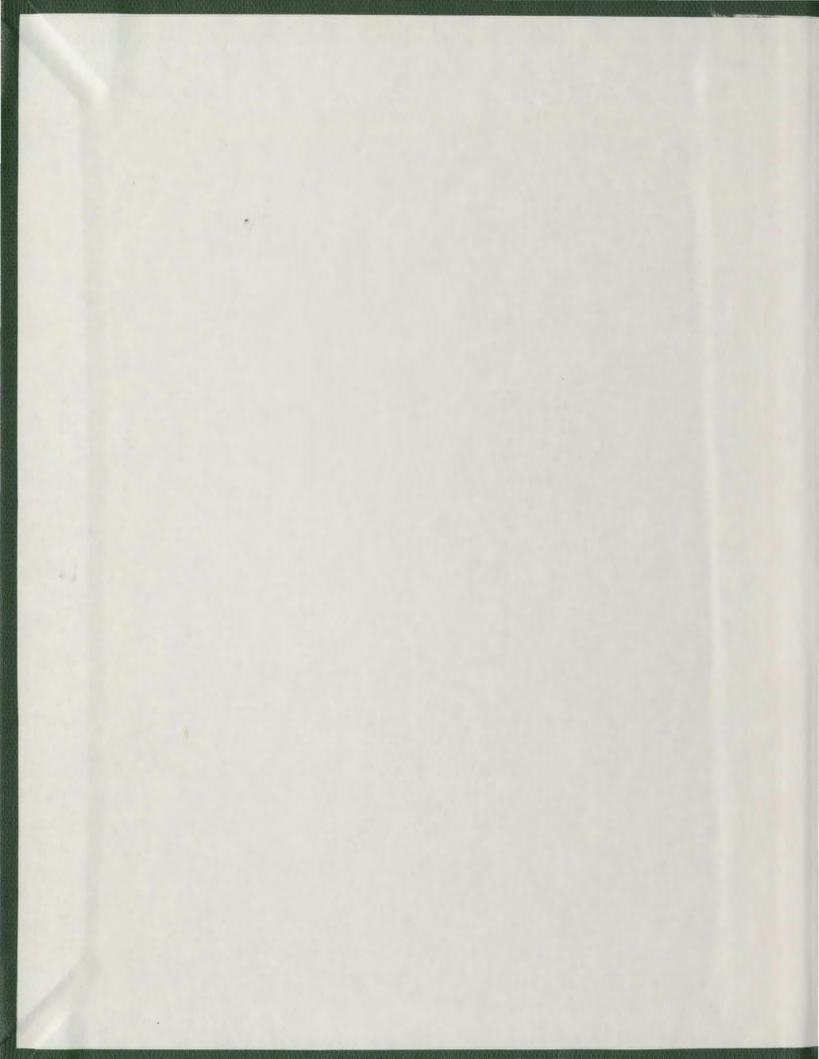
THE EFFECTS OF CONCRETE MATERIALS ON ATTITUDE AND ATTAINMENT OF OBJECTIVES OF STUDENTS TAUGHT A UNIT ON MEASUREMENT IN MATHEMATICS

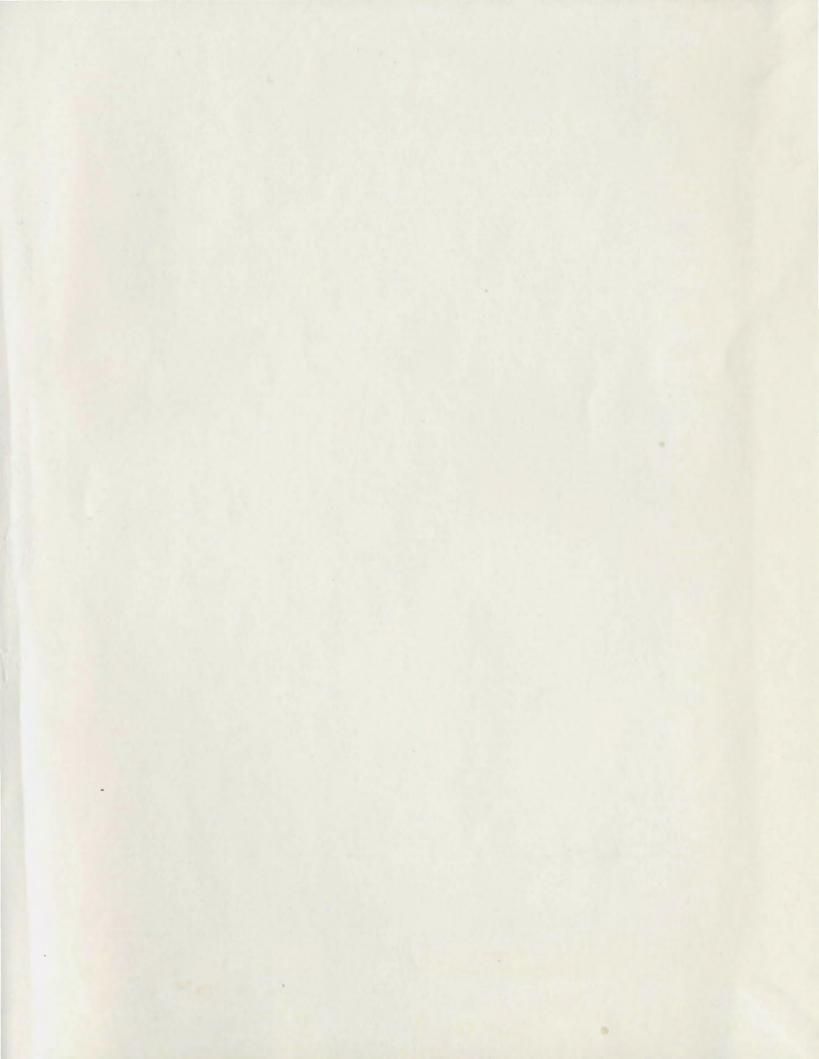
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JEREMIAH JOSEPH JESSO





THE EFFECTS, OF CONCRETE MATERIALS ON ATTITUDE AND

ATTAINMENT OF OBJECTIVES OF STUDENTS TAUGHT

A UNIT ON MEASUREMENT IN MATHEMATICS

An Internship Report

presented to

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ABSTRACT

The purposes of this study were to develop and evaluate activities to supplement a unit on 'measurement' in mathematics from the text-book School Mathematics I. The effects of the unit and the supplementary activities on student achievement and attitude toward mathematics were investigated. To do this, four questions were considered:

- 1. Can students attain competence with the mathematical concepts of 'measurement' in mathematics taught by an activity oriented approach?
- What are the effects of the activity approach to instruction on student attitude toward mathematics?
- 3. What are the attitudes of teachers toward the activity oriented approach?
- 4. What are the problems encountered in teaching 'measurement' by the activity oriented approach?

The study involved teaching an eleven-topic unit on 'measure-ment' to 30 students in two grade seven classes in a high school in western Newfoundland.—The unit was taken from the textbook School

Mathematics I and each topic was supplemented by activities developed by the investigator or selected from various mathematics education sources. The activities involved the use of concrete materials which the students manipulated.

To answer question (1), two achievement tests constructed by the investigator were administered to the students. Question (2) was Attitudes toward Mathematics' as a pretest and as a posttest. Question (3) was answered by administering to the teachers involved in the study, a ten-point questionnaire developed by the investigator. To answer question (4), the teachers were asked to keep a record of the problems encountered with the unit and the activities.

Analysis of the achievement test results indicated that the students achieved the 80% criterion on only one objective. The 80% criterion was also reached on two comprehension items and four computation items. Less than 80% was achieved on all of the application items. A dependent t-test for means was performed on the pretest-posttest attitude scores. A t-value of 3.51 indicated a significant positive change in student attitude toward mathematics at the .01 significance level.

Responses to the teacher questionnaire indicated that the teachers enjoyed teaching the material in the unit. They felt that the supplementary activities were particularly useful in getting the students involved in doing mathematics. They noted that one, problem encountered was the students' lack of experience in working with concrete materials.

The results of the study were inconclusive since no control group was used. Nor was there control for such extraneous variables as student background and differences in teacher and student abilities.

It was recommended that further studies be conducted in which these variables are controlled.

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CHAPTER, I

THE PROBLEM

In the past, areas of mathematics such as 'measurement' were often taught in a meaningless way through the lecture approach. The students listened as the teacher dictated formulae without giving the students an adequate understanding of the concepts involved. Today, mathematics educators are under greater pressure to encourage their students to become more actively involved in the learning of mathematics. Many of today's mathematics courses are designed to involve students in activities that deal with the studying of patterns, collecting and examining data, and making generalizations. Therefore, educators must seek out and implement new teaching approaches that will help the student become an active learner in mathematics.

Kieren and Vance (1968) claimed that the mathematics laboratory concept is one of the most interesting classroom strategies arising from modern cognitive learning theory. They pointed out that "the new mathematics laboratory not only includes different kinds of concrete manipulative materials, but it is far different in its purposes and methods from the old 'activity curriculum' which had its roots in social utility" (p. 33).

Learning theorists such as Bruner, Dienes, and Piaget have also emphasized the learning of mathematics through the use of concrete materials. Bruner (1966) pointed out that the child develops concepts

through three stages of representation: the enactive, the iconic, and the symbolic stage. Bruner said that in order to teach an idea or concept effectively, the teacher/first should provide the child with the opportunity to manipulate concrete materials. Kieren and Vance (1968) interpreted Bruner as saying that the next step in the learning process is to encourage the child to form images of the idea in the constructed forms. They noted that the final step is to develop a notational system "which describes the construction and yet is free of the manipulation and the image" (p. 34). The value of this approach is that the learner understands the abstract idea and has a stock of concrete images which embody the abstraction. Active learning also allows the child to go through the proper sequence of concept learning from the constructive-active to the analytic or symbolic levels. Therefore, the child is motivated by his own activity to continue to learn. concepts which allows for generalizations and creative behavior by the child.

Dienes (1960) also emphasized that the child can best learn mathematical ideas and concepts by first giving him the freedom to manipulate and experiment with concrete materials. He claimed that the system of teacher-centered class teaching should be replaced by "individual learning or learning in small groups from concrete material and written instructions with the teacher acting as a guide and counselor" (p. 29).

Kieren (1969) stated that "Davis (1967) observed that learning from physical materials added reality to the learning situation and provided an alternative to authoritarian teaching" (p. 513). Vance

(1970) noted that concrete materials serve several important functions:

- 1. They create interest and provide motivation.
- 2. They provide a real world setting for the problem to be investigated or concept to be developed.
- 3. They provide a physical means by which the learner can begin to solve the problem or explore the concept.
- 4. They provide the learner with a way of verifying his hypotheses and checking his calculations independently of, a teacher or textbook. (p. 16)

Learning theorists and educators agree that if a student becomes actively involved in the learning process, he is more highly motivated than a student who is a passive learner. One way that students can become actively involved in learning mathematics is through the manipulation of concrete materials. In the present study, the effects of concrete materials on achievement in mathematics and attitudes toward mathematics were examined.

Statement of the Problem

It was emphasized above that students can become actively involved in learning mathematics through the use of concrete materials. It was also noted that the use of concrete materials can help motivate the students to learn mathematics. Therefore, the problem under investigation in this study was to determine the effects of the use of concrete materials on achievement and attitudes toward mathematics. The use of concrete materials was investigated at the grade seven level where the students were taught a unit of work on 'measurement' in mathematics.

Purposes of the Study

The purposes of this study were to develop and evaluate activities to supplement the unit of work on 'measurement' in mathematics from the textbook School Mathematics I (Fleenor et al, 1974). The textbook unit and the supplementary activities involved finding perimeter, area, and volume of geometric figures. The lessons were taught with the aid of concrete materials such as the geoboard, paper, scissors, graph paper, dot paper, and rectangular solids which the students manipulated.

The effects of the unit and the supplementary activities on student achievement in mathematics and attitudes toward mathematics were investigated. The attitudes of the teachers toward the material in the unit were also investigated. To accomplish this, four questions were considered:

- 1. Can students attain competence with the mathematical concepts of 'measurement' in mathematics when taught by an activity oriented approach?
- 2. What effects do the activity oriented approach to learning the concepts of 'measurement' in mathematics have on student attitudes toward mathematics?
- 3. What are the attitudes of teachers toward the activity oriented approach?
- 4. What are the problems involved in teaching 'measurement' by the activity oriented approach?

Rationale for the Study

Many educators believe that how a topic in mathematics is taught is more important than the content of the topic itself. This view is supported by Dunn (1976) who noted that "what constitutes the subject matter of study is unimportant in itself, but that the vital thing is the activity which it stimulates" (p. 109). One approach to making mathematics stimulating to students is through the use of concrete materials.

Educators and learning theorists have made claims for the use of concrete materials in teaching and learning mathematics. Kieren (1969) stated that Biggs "claimed superiority for a multimodel environment for mathematics learning over a unimodel environment, which uses only one type of manipulative materials such as Cuisenaire rods" (p. 514). The use of a wide range of concrete materials in teaching and learning mathematics is also supported by Dienes (1967). He suggested that a mathematical concept can best be developed through the use of multiple concrete and game-like embodiments. Dienes (1967) also claimed that the use of varied concrete materials in mathematics learning can help the child learn to discover patterns and relationships among mathematical concepts.

Educators and researchers have emphasized the need and urgency for research into the use of concrete materials at all grade levels.

Kuhfitting (1974) pointed out that "the relative merits of teaching by the use of concrete training materials, as opposed to teaching abstractly, has received increased attention from educational researchers" (p. 104).

The urgency for research into the use of concrete materials has also

been emphasized by Friedman and Kieren. Friedman (1978) noted that "we should increase our efforts to determine those situations in which the strategy (manipulative materials strategy) is most promising" (p. 80). Kieren (1971) suggested that we have a long way to go in research before we can answer such questions as "For whom, for which topics, and with what materials are manipulative and play-like activities valuable?" (p. 232).

In the present study, the effectiveness of concrete materials were examined at the grade seven level with the concepts of measurement in mathematics.

Definition of Terms

Attitudes toward mathematics. The score obtained by the student on the 'Scale of Attitudes toward Mathematics' developed by Aiken (1979).

Mathematics achievement. The subject's score on the achievement. tests designed to determine the extent to which the objectives of the unit have been achieved.

Activity materials. Concrete materials such as geoboards, rubber bands, paper, scissors, dot paper, graph paper, and rectangular solids that the student can manipulate.

Activity oriented instruction. An instructional approach to teaching that uses activity oriented materials where the student is given the opportunity to work with the materials.

Delimitations of the Study

This study was designed with the following limitations:

- 1. The sample for this study consisted of two intact classes of 30 grade seven students selected from one high school in western

 Newfoundland. Neither the students nor the classes were randomly chosen.
- 2. The study was conducted over a six-week period.
- 3. The study dealt only with one unit of mathematics-- measurement'-- at the grade seven level.
- 4. The instruments used in the study to test for mathematics achievement were constructed by the investigator and were not standardized tests. However, they were designed to test the behavorial objectives of the unit (Appendix A).

Outline of the Study

In Chapter I, the purposes of the study and the rationale for the study were discussed. The delimitations of the study were also noted. Chapter II contains a review of the related literature. The procedures that were followed in conducting the study and the methods that were used to collect and analyze the data are presented in Chapter III. The results obtained from the analysis of the data are presented in Chapter IV. In Chapter V, the conclusions that were drawn from the study are summarized and implications and recommendations for further research into the problem are discussed.

CHAPTER II

REVIEW OF RELATED LITERATURE

Research studies relating specifically to activity oriented instruction at the grade seven level are scarce and the results of the studies that have been done in this area are contradictory. In this chapter, the results of two groups of studies that deal directly with activity learning or the use of concrete materials are discussed. The first group of studies is concerned with comparisons between activity oriented instruction and other methods of instruction. The second group of studies deals with the effects of the mathematics laboratory, an activity approach to instruction, on achievement in mathematics and attitude toward mathematics.

Studies Involving Activity Oriented Approaches

Kline (1976) argued that the new mathematics movement had little or no impact in many schools simply because much of the new material was still presented in the traditional expository manner. In this light, the effects of activity oriented instruction on achievement in mathematics were examined. Since this study was conducted with seventh grade students, the results of studies conducted with students at or near the seventh grade were examined first.

Suydam (1978) reported on research conducted in grades K-8 on activity-based teaching approaches, including studies done on the use

of manipulative materials. She reported that lessons using manipulative materials have a probability of producing greater mathematical achievement than do non-manipulative lessons. The use of both manipulative materials and pictorial representations is highly effective while symbolic treatments alone are less effective. Suydam stated that "the use of materials appears to be effective with children at all achievement levels, ability levels, and socioeconomic levels" (p. 155). Programs using the activity oriented approach and the use of mathematics laboratories" can be expected to result in achievement at least as high as when activities are not emphasized" (p. 155).

Kuhfitting (1974) investigated the effectiveness of guided discovery (learning with some help from the teacher) and concrete materials on mathematics learning. This method of instruction was compared to an abstract method of learning without the use of concrete materials. Forty grade seven students who were at least one standard deviation above or below the mean on two mathematics achievement tests were randomly assigned to one of four groups with five high and five low ability subjects in each group. The methods of teaching were labelled as concrete and abstract on the learning aids dimension, and maximal and intermediate guidance on the discovery dimension. The learning task involved converting American to old English currency and vice versa. The learning aids were models of coins. On the abstract dimension, only verbal references were made tofthe coin models. Significant differences were found in favor of concrete materials and intermediate goldance for the low ability groups but not for the high/ability Also, the intermediate guidance with concrete aids treatment

resulted in significantly greater transfer than the abstract treatment on the retention test. No significant differences were found between the posttest and retention test scores for the maximal guidance subjects on transfer.

Johnson (1971) studied the effects of three treatments on the achievement of objectives relating to perimeter, area, and volume by fourth, fifth, and sixth grade students. The maximum treatment used a semi-programmed text constructed by the experimenter with two sets of concrete physical models and instruments for each subject. The moderate treatment used the same semi-programmed text but without the models. The minimum treatment used the same text except that all drawings and illustrations were removed and verbal descriptions substituted. The author reported evidence "that a high degree of concreteness yields significantly higher means on achievement of objectives in the topics taught upon immediate measures as well as higher retention on subsequent measures."

Another study which examined the effects of concrete materials at the grade six level was done by Carmody (1971). He investigated the assumption that the use of concrete and semi-concrete materials can contribute significantly to the learning of mathematics at the elementary school level. Three sixth grade classes were randomly assigned to three experimental approaches—the symbolic approach, the semi-concrete approach, and the concrete approach. The material taught included topics on number bases, properties of even and odd numbers, and divisibility tests based on the decimal representation of numbers. Significant differences were found between the symbolic group and the semi-

concrete group in favor of the semi-concrete group on numeration and two transfer tests. A significant difference was also found in favor of the concrete aids group over the symbolic group on one transfer test. No significant differences were found between the concrete and the semi-concrete groups.

Purser (1973) conducted a study to determine if certain manipulative activities using measuring instruments were associated with student gains in achievement and retention scores in mathematics at the seventh-grade level. The experimental group received learning packages that utilized manipulative activities while the control group received learning packages that utilized paper-pencil type problems. It was concluded that students of all ability levels in the experimental treatment achieved significantly higher scores on the posttest and retention test than students of all ability levels in the control treatment.

The results of the four studies already examined indicate that the use of concrete materials in the teaching of mathematics can be effective depending on the amount of guidance given to the student. The effectiveness of concrete materials upon achievement in mathematics is also influenced by the ability level of the student. Since Suydam (1978) indicated that the use of concrete materials appears to be effective with children at all achievement levels, other studies conducted with students at grade levels other than the seventh grade were examined. These studies compared the results of activity oriented instruction to expository instruction and gave some indication of the grade levels at which activity materials are most effective.

Davidson (1973) measured the impact of concrete materials when used in conjunction with the textbook, on mathematical concept understanding by third and fourth grade children. Children in the experimental group were introduced to all concepts through the use of concrete materials followed by use of the adopted textbook. The control group did not use concrete materials, but they did use the adopted text and drill materials. Among grade three children, the experimental average—low IQ group had significantly greater conservation responses than the control group. At the grade four level, the high IQ experimental group had significantly greater conservation responses on the 'Conservation of Length' test than did the control group.

Results supporting Davidson's conclusions were found by Toney s (1968) who compared the achievement in understanding of basic mathematics between students who individually manipulated the instructional materials and those who saw only a teacher demonstration of the same materials. Grade four students were randomly assigned to two groups. The same lesson was taught to each group by the investigator. The presentation. of the lesson differed only in the manner in which the instructional materials were utilized. The experimental group was given the materials to manipulate individually while the control group saw only a teacher presentation of the materials. Toney reported that there were no statistically significant differences on the test for understanding of basic mathematical principles and the test for general mathematical achievement. However, the group using individually manipulated materials made greater gains in mathematics proficiency scores than the group seeing only a teacher demonstration of the material on both measuring

instruments (test for understanding and test for achievement).

Other studies indicated that student manipulation of concrete materials is not always more effective than teacher demonstrations of these materials on achievement in mathematics. Bisio (1971) compared the effectiveness of three methods of teaching addition and subtraction of like fractions to fifth grade students. In treatment A, no manipulative materials were used. In treatment B, the teacher used the manipulative materials as a demonstration for the students. In treatment C, both teachers and students used the manipulative materials. Bisio concluded that children taught addition and subtraction of like fractions with manipulative materials were at least equal on measures of a test involving addition and subtraction of like fractions to students taught without the use of manipulative materials. No significant differences were found between groups on the individual interview and posttest scores. No indications of unfavorable results from the use of the manipulative materials were found.

More positive results with the use of concrete materials were found by Bring (1972) who investigated the effects of concrete activities on achievement of objectives in metric and non-metric geometry with fifth and sixth grade students. The students were divided into two groups characterized by the amount and type of concrete activities they were exposed to during the experiment. Two posttests were given one week apart. Bring reported that students in the concrete activities classes achieved higher means than students in the deprived classes (classes which did not use concrete activities) but the difference was significant only on posttest II.

Positive results were also found with concrete materials at the high school level. Sobel (1954) investigated the relationships between the learning of certain algebraic topics and their methods of presentation with ninth-grade algebra students. The methods were (a) an abstract, verbalized, deductive method with concepts defined and presented by the teacher, and (b) a concrete, non-verbalized, inductive procedure with students guided, through experiences involving applications, to discover and verbalize concepts. A significant difference was found in favor of the concrete-discovery approach on mathematics achievement involving concepts and skills.

Monier (1977) investigated the effects of an activity approach to teaching geometry in certain high schools of Afghanistan. The activity approach involved a learning process using solution keys and practical activities to supplement lecture and textbook presentations. This activity approach was compared to a traditional method of teaching which consisted of lecture, textbook, and recitation based on memorization only. The results indicated that in comparison to the traditional approach, the activity approach significantly helped students (1) improve their performance in overall understanding of geometry, (2) achieve higher levels in creative thinking, (3) develop greater ability to explain geometric problems, (4) develop the ability to recall geometric concepts, and (5) develop greater ability in setting up complete proofs for geometric theorems.

Wilkinson (1971) examined the effectiveness of using supplementary materials in the teaching of eighth grade mathematics. The supplementary materials were mathematical objects, filmstrips, and films.

Wilkinson found that students who were taught mathematics with the use of supplementary materials did not show a significant gain in attitude over those who were taught by the traditional method. Using supplementary materials to teach understanding and concepts of mathematics to heterogeneously grouped students produced a significant gain over those who were grouped heterogeneously and taught by the traditional approach.

Two other studies were examined for the effects of concrete materials on learning of mathematical concepts and no significant differences between the activity approach and other approaches were found.

Johnson (1971) conducted a study to identify the effectiveness of using activity oriented lessons in seventh grade mathematics. Treatment A involved exclusive use of the textbook as the only mode of instruction. Treatment B was the activity treatment and involved exclusive use of instructional modes other than the textbook. Treatment C was the enriched treatment in which the textbook was supplemented by enrichment activities from treatment B. Johnson concluded that activity oriented lessons in seventh grade mathematics did not result in improved achievement over exclusively textbook-based or activity-enriched instruction. No differences were detected in achievement between activity enriched and textbook-based instruction. Low and middle ability students were aided in the learning of some concepts in seventh grade mathematics by the use of activity oriented lessons.

A study was carried out by Trueblood (1968) with fourth grade students to determine whether the students would achieve and retain more by (1) manipulating visual-tactual aids or (2) by observing and telling

the teacher how to manipulate such devices. The lessons involved exponential notation and non-decimal bases. The pupils taught by the second approach scored higher on the immediate posttest than the pupils taught by the first approach. However, the difference was only marginally significant (p = .10). Also, the pupils taught by the second approach did not retain significantly more than the pupils taught by the first approach. Both approaches resulted in a high degree of retention.

Although some negative results have been found with the activity oriented method of instruction, the majority of studies that have been examined in this section have found results in favor of the activity oriented approach. It appears that activity oriented instruction is effective with certain groups of students and with certain variations of the activity approach. From the studies examined in this section, it appears that the use of concrete materials can be effective in helping students achieve in mathematics. This study examined the effects of concrete materials upon attitude and achievement of seventh grade students in mathematics.

Studies Utilizing the Mathematics Laboratory Approach

The next group of studies that were examined dealt with the effects of the mathematics laboratory approach upon achievement in mathematics and attitude toward mathematics. Since instruction through the use of the mathematics laboratory is based on concrete materials, results of studies done in this area were relevant to this study.

Vance and Kieren (1971) pointed out that "laboratory activities are designed to lead to the development of a concept or the discovery

of a relationship" (p. 586). The concrete materials serve to create interest and motivation and to provide a real world setting for the solving of problems or the investigation of concepts. The students make use of physical objects or manipulative devices that will help them experiment and collect data relating to problems and concepts.

The laboratory method of teaching geometry was compared with a more conventional approach by Wilkinson (1971). The study was done with sixth grade classes. The laboratory units contained work sheets and manipulative materials. The control group was taught by teacher and textbook. Wilkinson reported that students taught by the laboratory method did as well as the conventionally instructed students on the geometry achievement posttest. However, the laboratory approach did not significantly affect pupils attitudes toward mathematics, but the method appeared more effective with students of low and middle intelligence.

The results of the study by Wilkinson indicated that the laboratory approach to the reaching of mathematics appears to be more effective with students of average or below-average mathematical ability. The results of other studies examined indicated that the mathematics laboratory was not more effective than other approaches to teaching mathematics.

Kujawa (1976) investigated whether a supplementary mathematics laboratory made a significant difference in fourth, fifth, and sixth grade students' mathematical achievement and attitude toward mathematics. Students in the experimental group attended a mathematics laboratory for 40 minutes a day, three days a week, for a period of 15 weeks. The

mathematics laboratory was supplementary to each student's regular mathematics class. Motivational and remedial exercises were provided for the students. The activities utilized were games, programmed materials, diagnostic instruments and puzzles. The researcher did all the teaching and grading. The laboratory was divided into three sections: (1) structured activities, (2) semi-structured activities, and (3) free choice activities. Kujawa concluded that no conclusive evidence was found to support the supplementary mathematics laboratory as an instructional approach.

Ropes (1973) studied changes in the attitude and performance of elementary school students after they had used manipulative materials and related activity sheets in a mathematics laboratory. An experimental mathematics laboratory was established in the second and sixth grades of a New York City elementary school. The experimental groups were instructed in a mathematics laboratory which contained a variety of manipulative materials and activity sheets related to each material. During each laboratory period, students worked in small groups with the activity sheets, and self-direction was emphasized. The control groups had no laboratory experiences and worked in regular mathematics classes. The authors found that students exposed to laboratory experiences showed no significant change in overall attitude toward mathematics when compared with students not having laboratory experiences. However, analysis of the results indicated "that mathematics laboratory students developed a greater awareness of the enjoyment to be derived from arithmetic and an increased liking for that subject." Mathematics Laboratory students did not score higher on a standardized test than the control groups.

Similar results were found by Smith (1974) who investigated the extent to which mathematics laboratory experiences helped middle school students to gain in mathematics achievement and to develop more positive attitudes toward mathematics. Sixth, seventh, and eighth grade students were chosen for this study. Laboratory activities for the experimental group were designed to correlate with the objectives and lessons of the regular mathematics classes. The control group was given conventional mathematics instruction with the same objectives in the regular class-room setting. Both groups were taught by the same teacher and laboratory instruction was compared with non-laboratory instruction. It was found that laboratory instruction in mathematics did not significantly affect the attitudes toward mathematics of middle school students and no significant difference in achievement scores between the two groups was found at any level.

Vance (cited in Vance and Kieren, 1971) studied the effects of a laboratory program on mathematics achievement. Six classes of seventh and eighth graders rotated on a once-a-day basis through ten activity lessons based on the use of concrete materials. Tests of achievement, retention, and transfer indicated that the students did learn new mathematical ideas through the laboratory approach, even though they learned slightly less than the students taught in the classroom situation. However, student reaction was more favorable to the laboratory setting than to the class setting.

Results of the studies examined indicated that the mathematics laboratory can help students learn mathematical ideas. Vance and Kieren (1971) pointed out that the mathematics laboratory promotes better atti-

tudes toward mathematics. Mathematics laboratories appear effective with particular groups of students such as low ability students. The laboratory approach can be a viable alternative as an activity oriented approach to the teacher-lecture or expository approach to teaching mathematics.

Summary

The results of some of the studies examined contained a considerable amount of evidence to support activity oriented instruction. The results of other studies indicated that other approaches to mathematics instruction were as effective or more effective than the use of the activity approach. However, the use of activity or concrete materials, whether in an ordinary classroom setting or in a formal mathematics laboratory setting seemed to be effective with students at all grade levels examined.

There were some indications in the studies by Kuhfitting(1974) and Johnson (1971) that students of below-average mathematical ability benefit more from the activity approach to learning mathematics than do students of higher mathematical ability.

The results found in the review of the literature indicate that more research is needed to determine the effectiveness of the activity approach on achievement in mathematics and attitude toward mathematics at different grade and ability levels. The purpose of the present study was to investigate the effectiveness of the use of concrete materials upon achievement and attitude toward mathematics with seventh grade students taught a unit on 'measurement' in mathematics.

CHAPTER III

DESIGN AND PROCEDURE

In this chapter, a description of the sample used during the investigation is presented. The instructional unit and supplementary activities are discussed. The development and administration of the instruments that were used during the study to collect the data are described. Results of the pilot study which was done to determine any problems with the activities are reported. The four questions asked in Chapter I and the methods of analysis for each question are discussed.

Description of the Sample

The subjects for this study were all 30 grade seven students at a high school in western Newfoundland. Neither the students nor teachers were randomly selected since there were only 30 grade seven students in the school. The teachers and students were already assigned to classes at the beginning of the school year. The majority of students were from families of low socioeconomic status. Many of their families were receiving government assistance because of a lack of employment in the area.

The two grade seven classes were described by their teachers as below average in general school achievement with respect to provincial norms. However, students in both classes ranged from low to high in mathematical ability.

Instructional Materials

The subjects in this study were taught the unit on measurement, Module 2 from the textbook School Mathematics I: The unit consisted of eleven topics which were found on pages C-36 to C-56 of School Mathematics I. Eleven behavorial objectives were written, each of which corresponded to one topic. These objectives are listed in Appendix A.

Each topic in the unit was supplemented by a supplementary activity. These supplementary activities were numbered from 1 to 11 to
correspond to the eleven topics in the unit. These activities are found
in Appendix B.

The 'Lost Area Puzzle' for activity 6 was adapted from Beardsley (1973). Activity 7 was adapted from Cohen (1970) and activity 10 was developed by Kulm (1975). The remaining supplementary activities were developed by the investigator.

Each activity contained an objective, a list of the concrete materials to be used, and the procedure for performing the activity.

Any student activity sheets that were used were included with the activity.

The activities involved the use of concrete materials such as the geoboard, paper, scissors, cardboard squares, and boxes. In order to meet the objective of each activity, the students used the concrete materials in instances such as measuring objects around the classroom or finding area and volume of given objects or figures. In several of the activities, work sheets were provided, on which the students could record measurements or answer questions.

Instruments

This section contains a description of the instruments that were used during the study to collect the data.

Student Achievement Tests

The two achievement tests, which are included in Appendix C, were developed by the investigator. Test I was developed to determine student achievement on objectives 1 to 5. Test II was used to determine student achievement on objectives 6 to 11. Each objective was tested at two or more cognitive levels of behavior using the classification system developed by Wilson (1971). According to Wilson, computation items are designed to require recall of basic facts and knowledge of terminology or the manipulation of problem elements according to rules the students have learned. For comprehension items, the emphasis is upon demonstrating understanding of concepts and their relationships, not upon using concepts to produce a solution. Application items require the student to use concepts in a specific context and in a way he has presumably practiced,

Table I shows the classification of each item on each test as either computation, comprehension, or application.

Student Attitude Scale

The 'Scale of Attitudes toward Mathematics' developed by Aiken (1979) was used to determine changes in student attitudes over the period during which the unit was taught. A copy of the attitude scale is found in Appendix D.

Table I
Classification of Test Items

		Co	gnitive Levels	
Objective 🔩	Test.	Cômputation	Comprehension	Application
1	İ	1 (a)	1(b), 1(c)	
2	Cr. ex.	2(a)	2(b)	2(c)
3		3(a), 3(c)		3(b)
4		4(a), 4(c)	4 (b)	
5		5(a)		5(c)
6	II	1(a), 1(b)		1(c)
7		2(a)		2(b)
8		3(a)	3(b)	
9			4(a)	4(b)
10		5(a)	5(b)	
V11:		6(a)	6(b)	6(c)

Aiken (1979) administered the 'Scale of Attitudes toward Mathematics' to a sample of 50 boys and 50 girls, aged 11-15 in each of grades 6, 7, and 8 of randomly selected middle schools in Tehran. The reliability coefficient for the scale ranged from .81 to .91.

The scale consisted of 24 items which were designed to assess student attitudes toward mathematics. The responses for each item on the scale ranged from strongly disagree (SD) to strongly agree (SA).

Twelve of the items on the scale were positively stated while the other twelve items were negatively stated.

The highest possible score that could be obtained on the scale was 96 which indicated a most positive attitude toward mathematics. The lowest possible score which indicated a most negative attitude toward mathematics was zero.

Teacher Questionnaire

To determine the attitudes of the teachers toward the instructional materials and the supplementary activities, a ten-item question-naire was developed by the investigator. This questionnaire is found in Appendix E. It consisted of ten questions which were designed to elicit teacher reactions to the usefulness of the supplementary activities as well as to the activity approach to teaching mathematics.

Procedure

The unit on measurement, including the supplementary activities, consisted of eleven topics which were taught over a six-week period.

The unit was taught to two grade seven classes by their regular mathematics teachers. The students were taught eight periods of mathematics

per six day cycle. Four of the mathematics periods in each cycle were 35 minutes long while the other four periods were 40 minutes long. Two mathematics periods were spent on each supplementary activity except for activities 5 and 10 which took three periods to complete. Two periods were spent on testing. The remaining periods were spent on the textbook material.

manual of the textbook School Mathematics I, as well as the concrete materials needed for each activity, were given to the students. The objective and procedures for each supplementary activity were explained to the students by the teachers. While the students worked on the activity, the teachers answered any questions asked by the students. The teachers also gave individual help where necessary. The supplementary activities were used to assist in developing some concepts and reinforcing the learning of others.

At the end of each topic, the teachers assigned the appropriate practice exercises and homework assignments.

The achievement tests and student scale were administered to the students by the regular mathematics teachers. The student attitude scale was given to the students as a pretest three days prior to the start of the unit and as a posttest three days after the unit was completed. The first achievement test was given to the students after five topics had been completed. The second achievement test was given at the completion of the unit.

During the teaching of the unit, the teachers kept checks on student progress and indications of changes in attitudes toward mathe-

matics. This was done by having the teachers make comments on class progress and attitudes toward the instructional materials.

The investigator kept in daily contact with the teachers and held regular conferences with them. During these conferences, the investigator and the teachers discussed any problems encountered with the unit or the supplementary activities. Student progress and changes in attitudes were also discussed at these conferences. During the teaching of the unit, the investigator provided the teachers with whatever help and advice was needed.

At the completion of the unit, both teachers were asked to complete the teacher questionnaire.

Pilot Study Report

Two months prior to the implementation of the study, a pilot study was done with ten regular grade eight students. The purpose of this pilot study was to evaluate the supplementary activities. This evaluation served to determine any problem that might be encountered with the supplementary activities and any revisions that might have to be made.

The supplementary activities were incorporated in the unit on 'measurement' in mathematics at the grade eight level by the regular mathematics teacher. After completing the unit, the teacher was asked to evaluate the supplementary activities by completing the sections of the 'teacher questionnaire' which dealt with the supplementary activities.

The teacher noted that the activities supplemented the material in the unit very well. He indicated that the supplementary activities

were less abstract than some of the material in the regular textbook.

They served as practical experience in reinforcing the concepts that

were being taught.

The teacher emphasized that the students became much more involved in learning mathematics when they were working on activities. Through the activities, they discovered and learned concepts. He stated during an informal conference with the investigator that the students looked forward to the activities with enthusiasm. They showed much more interest in learning the material in the unit on the days they knew they would be involved in the activities. Both teacher and students showed a great deal of interest in the activities. The teacher emphasized that the use of the supplementary activities made the teaching and learning of the unit on 'measurement' both enjoyable and meaningful.

The teacher pointed out that he had a very positive reaction to the activities. They enabled the students to start with the simple concepts and progress to more difficult ones. The activities gave the students the opportunity for enjoyment while learning mathematics. He noted that with the activities, there was not as much pressure on the students to arrive at the correct answer. The activities also enabled the students to work at their own pace.

The teacher stated that he would not omit any of the supplementary activities and could not suggest any that might be added. He indicated that one problem he encountered with the activities was that the students were not used to working with concrete materials. The students needed much supervision when they were using the materials. The teacher had to give much individual help to the students because they were

unfamiliar with working with activities of this type.

On the basis of the teacher's evaluation, no supplementary activities were omitted and none was added. None of the supplementary activities was altered. However, the two teachers involved in the main study were told about the problems encountered in the pilot study. They were asked to remain alert to student problems and to give individual help while the students were working with the activities.

Teacher Training Sessions

In order to familiarize the teachers with the purposes and procedures of the study, the investigator had several meetings with the teachers prior to the study. During these sessions, the following were discussed:

- 1. the purposes of the study,
- 2. the objectives of the unit,
- 3. the nature of the supplementary activities and where they should be used in each topic (Appendix F contains an outline of where the activities should be used in the unit).
- 4. the methods of teaching the unit;
- 5, the problems encountered with the activities in the pilot study,
- 6. the nature and purposes of the achievement tests and student attitude scale.

Prior to the study, the teachers were supplied with the necessary materials. Daily contact with the teachers was maintained over the six-week period during which the unit was being taught. During these daily meetings, problems with the unit and the activities were discussed. Any questions which the teachers had concerning the supplementary activities were also clarified.

Analysis

To answer question (1): Can students attain competence with the mathematical concepts of 'measurement' in mathematics taught by an activity oriented approach?, two achievement tests were administered to each of the 30 students. These achievement tests were designed by the investigator to evaluate the objectives of the unit.

Bloom (1968) stated that perhaps over 90 percent of students can master what we have to teach them. However, since the students in this study were described by their teachers as below average in general school achievement, an expected performance of 90% on each objective was considered too high. Therefore, it was decided by the investigator and teachers involved in the study that an expected performance of 80% for the students on each objective would be more realistic.

The sub-items for each objective were classified as computation, comprehension, or application. The average score for each sub-item was calculated to determine whether the 80% performance level was achieved for the sub-items.

A percent score was also calculated for the set of computation sub-items, the comprehension sub-items, and the application sub-items in order to determine whether the 80% performance level was achieved for each class of sub-items.

To answer question (2): What effects do the activity oriented approach to learning the concepts of 'measurement' in mathematics have on student attitudes toward mathematics?, a 'Scale of Attitudes toward

Mathematics developed by Aiken (1979) was administered to each of the 30 students. This scale was given as a pretest and as a posttest.

In order to analyze the student attitude scale data, the following null nypothesis was considered: There is no significant difference between the pretest attitude scale scores and the posttest attitude scale scores. This null hypothesis was tested using a dependent t-test.

To answer question (3): What are the attitudes of teachers toward the activity oriented approach?, the teacher questionnaire was completed by the two teachers after the unit of work on 'measurement' had been completed.

The responses made by the teachers to each item on the questionnaire were examined. The reactions of the teachers to the activity
approach and to the usefulness of the supplementary activities were
noted. These responses are outlined in the analysis of the results in
Chapter IV.

To answer question (4): What are the problems involved in teaching 'measurement' by the activity oriented approach?, the teachers were asked to make comments on student performance and attitude changes toward mathematics during the teaching of the unit. The teachers were also asked to note any problems they encountered with the unit or the supplementary activities. Question (8) of the teacher attitude questionnaire asked the teachers to discuss any problems encountered during the teaching of the unit.

These problems are stated in the analysis of the results in Chapter IV.

CHAPTER IV

ANALYSIS OF DATA

In this chapter, the data collected during the investigation is reported. The results of the analysis of the data relating to each of the four questions asked in Chapter I are stated.

Student Achievement

To answer question (1): Can students attain competence with the mathematical concepts of 'measurement' in mathematics taught by an activity oriented approach?, an analysis of the results of the two achievement tests was performed. In Table 2, the data for Test I is given while the data for Test II is reported in Table 3.

In each of Tables 2 and 3, the score assigned to each item on each test is given. The items on each test were classified as computation, comprehension, or application. The average score for each item is reported in the tables. These scores are also shown as percentages. The tables also contain the number of students who achieved 80% or more on each item.

Question (1) was answered by comparing the performance of the students on each objective to an expected performance of 80%. Question 5 on Test I corresponding to objective 5 was the only question on which the students met the 80% criterion. The observed performance on the computation item of this question was 93.8% while the observed performance for the application item was 70.3%.

Table 2

Objective	Item	Classification	Válue Assigned to Each Item	l Average Score	Percent Score on Each Item	# of Students Reaching Criterion	Percent Score on each Objective	# of Students Reaching Criterion
1.	1.a	Computation	4 7	2.83	. 68.3	17	75.7	17.
	b	Comprehension	10	8.20	82.0	23		
	c	Comprehension	6	4.20	70:0	21		
2	2.a	Computation	4	2.83	69.2	15	65:5	9
	Ъ	Comprehension	10	8.20	82.0	22		
	c	Application	6-	2.13	35.5			
3	3.a	Computation	6 -	5.87	96.7	29	70.2	12
	. "Ъ	Application	8-	3.63	45.4	,10 ; 🥳 ,		
	(1 c .)	Computation	6	4.60	76.7	20		
4	4.a	Computation	6	1.53	25.5	7	63.5	6.
	Ъ	Comprehension	8	5.37	67.1	17		
	c	Computation	6	5.80	96.7	29		
5	5.a	Computation	8	7.50	93.8	27	79.7	19
	b	Application	12	8.43	70.3	18		

Table 3
Data for Test II

Obje	ctive	Item	Classification			Percent Score on Each Item	# of Students Reaching Criterion	Percent Score on Each Objective	# of Students Reaching Criterion
	6	i.a.	Computation	6	4.10	68.3	20	65.9.	8
		ъ.	Computation	6	5.00	83.3	25		
		c ·	Application	. 8	4.07	50.9	15		
			en de la companya de		5 00				
		2.a		8.	5.20	65.0	15	55.3	
		Ь	Application		3.10	44.3	13		
	8	3.a	Computation	7	3.40	48.6		56.9	9
		Ъ	Comprehension	8	5.13	64.1	16		
	9	4.a	Comprehension	6.5	4.07	66.7	.20	68.0	11
		Ъ	Application	9	6.20	68.9	20		
1	0	5.a	Computation	6	2.97	49.5	15	49.8	1
		Ъ	Comprehension	9	4.50	50.0	15		
. 1	1	.6.a∷	Computation	.4	3.87	-96.8	29	65.0	13
	a.	ь	Comprehension	7	3.33	47.6			
		C	Application	9	5.80	64.4	18		

The 80% criterion was also achieved on the sub-items of some of the other questions. The comprehension items for questions 1 and 2 on Test I had observed performances greater than 80%. The students also achieved the expected performance of 80% on computation items 3(a), 4(c), 5(a), and 11(a). The students achieved less than 80% on all of the application items.

Of the 30 students in the sample, the number reaching criterion on the individual computation items ranged from 7 to 29. For 7 of the 13 computation items, over two-thirds of the students reached criterion on the individual items.

The number of students reaching criterion for each of the comprehension items ranged from 5 to 23. The range for the application items was 7 to 20. Two-thirds of the students reached criterion on four of the individual comprehension items and on one application item. Over half of the students reached criterion on 7 of the 8 individual comprehension items and on 4 of the 7 individual application items. In order to compare student achievement on the three classifications of items, Table 4 contains the percent scores for the computation items, the comprehension items, and the application items for each of the two achievement tests. The total percent score for each class of item is also reported.

The percent scores shown in Table 4 for all three classifications of items were all less than 80% for both achievement tests. However, the students achieved the highest total percent score on the computation items and the lowest total percent score on the application items.

Table 4
Percent Scores According to Classification of Items

	Classification of Items
Test Computation	Comprehension Application
76.8	76.4 54.6
II 66.3	56.6 58.1
Total 71.8	67.1 56.6
	The state of the s

Student Attitudes

To answer question (2): What reffects do the activity oriented approach to learning the concepts of 'measurement' in mathematics have on student attitudes toward mathematics?, an analysis of the results of the student attitude scale was performed.

The difference scores for the pretent and posttest raw scores, were computed. A dependent t-test for means was then performed on the set of difference scores. In Table 5, the mean and standard deviation for each of the pretest and posttest scores are reported.

Table 5

Data for Student Attitude Scale

2 to 10 to 1	** , ** 1 4 1			1 10
Stranger Commence		+	and the second	t-value
	$\mathbf{n}_{r,j}$, $\mathbf{n}_{r,j}$	^ ^ (, _, , , , , , , , , , , , , , , , , ,	5	r-Aarne.
		the first of the fact	and the second of	
Pretest	30 5	8.97	3.38	11/201 - 6
				3.51*
Posttest	30	0.70	5.53	
		1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		

^{*}P < 0.01

The dependent t-value of 3.51 indicated a significant positive change in student attitude toward mathematics over the six-week period during which the unit on 'measurement' was taught. The computed t-value was significant at the .01 level of significance. Therefore, the null hypothesis that 'there is no significant difference between the pretest attitude scale scores and the posttest attitude scale scores' was rejected.

Teacher Questionnaire

To answer question (3): What are the attitudes of teachers toward the activity oriented approach?, an analysis of the teacher questionnaire which was administered to the two teachers was performed.

In response to question (1) on the questionnaire, both teachers indicated that they enjoyed teaching the material in the unit. One teacher indicated that she enjoyed teaching the material because she liked geometry. The other teacher noted that he enjoyed teaching the material because the students enjoyed learning it.

In response to question (2), both teachers indicated that the supplementary activities were very useful in both the development and reinforcement of the concepts being taught. One teacher noted that the main value of the supplementary activities was in creating interest in teaching and learning the unit. The other teacher noted that concrete development of the concepts was very instrumental in keeping the students interested and involved in learning mathematics.

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In response to question (3), one teacher indicated that he would not omit any of the supplementary activities because they were all rele-

vant and purposeful. The other teacher pointed out that the activities which involved 'cutting and gluing' took too long.' She noted that the material in some of the activities could be covered by using solid objects such as cubes.

Both teachers did not indicate that they would add any additional supplementary activities to the unit. One teacher indicated that the material in the unit was suitable to the average mathematical ability student. However, she noted that some of the material required above average reading ability because the instructions required a great deal of reading. The other teacher felt that the material in the unit was suitable for grade seven students of all ability levels. However, he noted that the amount of time needed to teach the unit varied with the ability level. The high achieving students learned the concepts much more quickly and they were more able to make applications than the slower students.

In response to question (6), both teachers had positive attitudes toward the activity approach. One teacher noted that the activity approach held student interest better than the lecture approach and involved every student. The other teacher pointed out that the activity approach made the lessons much more interesting.

In questions (7) and (8), both teachers noted that at the beginning of the unit, the students were unsure of themselves because they lacked experience in working with concrete materials. Considerable time had to be spent making sure that all students knew what was expected of them. However, once the students knew what to do; they enjoyed working with the materials.

In response to question (9), one teacher stated that the activity approach to learning mathematics could be especially helpful with the slower students. The other teacher stated that the activity approach could be helpful in getting students to understand fractions and decimals because students divorce fractions and decimals from their actual meaning.

Both teachers responded positively to question (10). They noted that the material in the unit motivated students to learn the concepts. They stated that the material in the unit could also help individuals who have problems grasping the concepts.

In summary, both teachers enjoyed teaching the material in the unit. In their opinion, the supplementary activities motivated the students to learn mathematics and reinforced concepts already learned. The material in the unit and the activities could be adapted for use with other grade levels and with slower students. The supplementary activities were particularly useful in getting the students involved in learning mathematics. Both teachers would recommend the material in the unit to other teachers.

Problems with the Activity Approach

To answer question (4): What are the problems involved in teaching 'measurement' by the activity oriented approach?, the problems noted by the teachers during the teaching of the unit were analyzed.

Comments made on question (8) of the teacher questionnaire were also analyzed.

One problem noted by both teachers was that the students lacked experience in working with concrete materials. Therefore, a great deal time had to be spent making sure that the students knew how to work with the concrete materials.

Another problem noted by the teachers was that the students were not accustomed to working in groups. The students needed to be reminded about the use of the materials and the need for care with the materials. Both teachers noted that building student confidence in using the materials was very important.

The teachers also emphasized that with the supplementary activities, much individual help had to be given. Some of the students had
trouble understanding how to do the activities. Therefore, considerable
time was spent in explaining to the students how the activities were to
be done.

CHAPTER V

SUMMARY, CONCLUSIONS AND DISCUSSION, IMPLICATIONS
AND RECOMMENDATIONS

Summary

The purposes of this study were to develop and evaluate activities to supplement the unit on 'measurement' in mathematics at the grade seven level. The study also investigated the effects of the unit on 'measurement' and the supplementary activities on student attitudes and achievement in mathematics. To do this, four questions were considered:

- 1. Can students attain competence with the mathematical concepts of 'measurement' in mathematics taught through an activity approach?
- 2. What effects do the activity approach to learning the concepts of measurement have on student attitudes toward mathematics?
- 3. What are the attitudes of teachers toward the activity approach to teaching mathematics?
- 4. What are some problems encountered in teaching measurement by the activity approach?

The study was conducted over a six-week period with two classes of 30 regular grade seven students. The unit on measurement was taken from the grade seven textbook School Mathematics I. Some of the supplementary activities were developed by the investigator while other supplementary activities were selected from various mathematics educa-

To answer question (1), two achievement tests were designed by the investigator. These tests were used to determine whether the behavorial objectives of the unit were achieved. Question (2) was answered by administering Aiken's (1979) 'Scale of Attitudes toward Mathematics' as a pretest and as a posttest. Question (3) was answered by having the two teachers involved in the study complete the teacher questionmaire. To answer question (4), the teachers were asked to note any problems they encountered with the unit and with the activities.

Conclusions and Discussion

The analysis of the student achievement tests indicated that the students achieved the expected performance of 80% on one objective only. This objective dealt with finding areas of regions by counting or estimating the number of square units in the regions. The students achieved 93.8% on the computation part of this objective and 70.3% on the application part.

Observed performances greater than 80% were found on the comprehension items for two of the objectives. Performances greater than 80% were also observed on some of the computation items for four of the objectives. When the items on the two achievement tests were classified as computation, comprehension, or application, the mean scores were less than 80% for each classification.

The teachers were asked to give reasons why the students did not achieve the 80% expected performance on all of the objectives.

Both teachers noted that a number of the students had reading problems. They indicated that students with reading problems did poorly on the comprehension and application test items which required more reading

than the computation items. The students achieved less than 80% on all of the application items. The teachers stated that, in their opinion, the students with poor reading ability had trouble understanding what was required in the application problems.

It appeared that a number of students did poorly on the achievement tests because they did not understand some of the key concepts in the unit, or because they could not distinguish between concepts. Some of the students misinterpreted 'width' for 'length' on one of the test items, and as a result, got the wrong answer. On another test item, many students had trouble distinguishing between the concepts of 'perimeter' and 'area.' Some students did not fully understand such concepts as 'volume' and 'surface area.'

The teachers emphasized that problems which involved more than mere application of simple formulae were poorly done. Some of the application problems were not attempted by many students. On one test item, no diagram was included with the problem. None of the students drew diagrams to help them solve the problem. Consequently, many students did poorly on this item. On another test item, students were unable to find the length of a rectangle when the area and width were given.

In summary, poor reading ability and lack of knowledge of key concepts on the part of many students are suggested as the main reasons why the 80% expected performance level was not achieved for all objectives.

To determine any changes in student attitudes toward mathematics over the six-week period, the 'Scale of Attitudes toward Mathematics' was given as a pretest and as a posttest. A dependent t-test indicated a significant positive change in student attitudes toward mathematics.

The teachers were asked to suggest reasons for negative or positive changes in student attitudes toward the unit and the activities over the six-week period. They noted that the students showed a positive attitude toward the supplementary activities and became involved in using the concrete materials. Some students were particularly interested in cutting out geometric figures and constructing boxes. The students also enjoyed measuring objects around the classrooms. In the teachers' opinions, the students appeared to be motivated to do mathematics from their involvement with the activities.

At the beginning of the unit, the students were not familiar with working with concrete materials. Some of the students were not sure of how the activities were to be done. However, after they knew how to work with the concrete materials, they showed more positive attitudes toward the activity approach.

An analysis of the attitude questionnaire which was administered to the two teachers involved in the study indicated that the teachers enjoyed teaching the unit on 'measurement' through the activity approach. They noted that, even though the 80% criterion was not reached for all objectives, the supplementary activities were helpful in developing some concepts and reinforcing others. Both teachers indicated that they would recommend the material in the unit to other teachers because of its motivational value. They also suggested that the unit and the activities could be adapted to different grade and ability levels.

Both teachers were very positive in their comments toward the activities and the unit. They felt that the activity approach to teaching mathematics enabled the students to become involved in learning

mathematics. As active learners, the students derived more enjoyment from the learning of mathematics.

Both teachers commented that, with the supplementary activities, emphasis was put on the development of concepts rather than on trying to arrive at the correct answer. The activities enabled the students to work at their own pace.

The teachers were asked to note the problems which they encountered with the activity approach. One problem that received emphasis was the unfamiliarity of the students in working with concrete materials. Because the students were not sure of how to go about doing the activities, the instructions needed to be explained carefully. Much individual help had to be given while the students were involved in the activities. Because the students were not accustomed to working in groups, the teachers had to maintain more class supervision during this unit than during other units.

However, the teachers felt that if students were exposed to more activities involving concrete materials, problems such as those encountered in the unit would not be as evident.

In summary, the results of the study were inconclusive and further investigation in this area is needed. Since no control group was used in this study, the results obtained may not be attributed with certainty to the activity approach used to teach the unit. Extraneous variables such as student background, student ability, and differences in teacher ability may have influenced the results obtained from the study. Future studies of this type should control for such variables as those mentioned above.

Implications and Recommendations

This study has several implications for teaching and learning mathematics through the activity approach. One implication is that the activity approach to teaching areas of mathematics such as 'measurement' can help students develop more positive attitudes toward mathematics. The activity approach can be used to help arouse interest and motivate learning. Through this approach, students become involved in doing mathematics.

The analysis of the teacher questionnaire revealed that the teachers had a positive attitude toward the unit and the supplementary activities. They noted that the supplementary activities were particularly useful in getting students interested and involved in learning mathematics. However, they felt that the students in this study were not used to working with concrete materials. Therefore, the six-week period of time over which the study was carried out may not have been sufficient to enable the students to become competent with the activity approach. It is recommended that in future studies of this type, students be given ample time to adapt to the activity approach.

There was some indication in the analysis of the teacher questionnaire that the supplementary activities could be adapted to meet the needs of students of different grade and ability levels. It is recommended for future studies in this area that the effects of supplementary activities on achievement in and attitude toward mathematics be examined at different grade levels with students of high, medium, and low mathematics ability levels.



- Aiken, G.J. Attitudes toward mathematics and science in Iranian middle schools. School Science and Mathematics, 1979, 79, 229-234.
- Beardsley, L.M. Activities and ideas for teaching math. New York: Parker Publishing Co., Inc., 1973.
- Bisio, R.M. Effects of manipulative materials on understanding operations with fractions in grade V. <u>Dissertation Abstracts International</u>, 1971, 32, 833A.
- Bloom, B.S. Learning for mastery. <u>U.C.L.A. Evaluation Comment</u>, 1968, <u>1</u>, 49-71.
- Bring, C.R. Effects of varying concrete activities on the achievementof objectives in metric and non-metric geometry by students of grades five and six. <u>Dissertation Abstracts International</u>, 1972, 32, 3775A.
- Bruner, J. Toward a theory of instruction. Cambridge: Harvard University Press, 1966.
- Carmody, L.M. A theoretical and experimental investigation into the role of concrete and semi-concrete materials in the teaching of elementary school mathematics. <u>Dissertation Abstracts International</u>, 1971, 31, 3407A.
- Cohen, D. The area of a triangle. In D.B. Harrison (Ed.), Active Learning in Mathematics: A Set of Resource Materials for Teachers. Calgary: Albertá Teachers Association, 1970.
- Davidson, J.E. The impact of selected concrete materials on the understanding of certain mathematical concepts by grade 3 and grade 4 students. <u>Dissertation Abstracts International</u>, 1973, <u>33</u>, 6232A.
- Dienes, Z.P. <u>Building up mathematics</u>. London: Hutchinson Educational Ltd., 1960.
- Dienes, Z.P. Some basic processes involved in mathematics learning. In J. Scandura (Ed.), Research in Mathematics Education. Washington, D.C.: NCTM, 1967.
- Dunn, J.A. Discovery, creativity, and school mathematics: A review of research. Educational Review, 1976, 28, 102-117.
- Fleenor, C.R., Eicholz, R.E., and O'Daffer, P. School Mathematics I.
 Don Mills: Additon-Wesley (Canada), Ltd., 1974.
- Friedman, M. The manipulative materials strategy: The latest pied piper? Journal for Research in Mathematics Education, 1978, 9, 78-80.

- Johnson, R.E. The effects of activity oriented lessons on the achievement and attitudes of seventh grade students in mathematics. <u>Dissertation Abstracts International</u>, 1971, 32, 305A.
- Johnson, R.L. Effects of varying concrete activities on achievement of objectives in perimeter, area, and volume by students of grades four, five, and six. Dissertation Abstracts International, 1971, 31, 4624A.
- Kieren, T.E. Activity learning. Review of Educational Research, 1969, 39, 509-522.
- Kieren, T.E. and Vance, J.H. The theory of active learning: Its application in a mathematics workshop. Manitoba Journal of Education, 1968, 4, 33-40.
- Kline, M. NACOME: Implications for curriculum design. <u>Mathematics</u>
 <u>Teacher</u>, 1976, 69, 449-454.
- Kubfitting, P.K.E. The relative effectiveness of concrete aids in discovery learning. School Science and Mathematics, 1974, 74, 104-108.
- Kujawa, E. The effects of a supplementary mathematics laboratory on the mathematical achievement and attitude toward mathematics of students in the intermediate grades, four, five, and six. Dissertation Abstracts International, 1976, 37, 3378A.
- Kulm, B. Volume and surface area. Mathematics Teacher, 1975, 68, 583-586.
- Monier, M.L. Some effects of an activity approach to teaching geometry in the high schools of Afghanistan. Dissertation Abstracts International, 1977, 38, 2630A-2631A.
- Purser, J.D. The relation of manipulative activities, achievement and retention, in a seventh-grade mathematics class: An exploratory study. <u>Dissertation Abstracts International</u>, 1973, 34, 3255A-3256A.
- Ropes, G.H. The effects of a mathematics laboratory on elementary school students. <u>Dissertation Abstracts International</u>, 1973, 33, 4250A.
- Smith, E.D.B. The effects of laboratory instruction upon achievement in and attitude toward mathematics of middle school students.

 <u>Dissertation Abstracts International</u>, 1974, 34, 3715A.
- Sobel, M.A. A comparison of two methods of teaching certain topics in ninth grade algebra. <u>Dissertation Abstracts International</u>, 1954, 14, 1647.

- Suydam, M.N. Activity-based learning in elementary school mathematics:

 Recommendations from research. Resources in Education, 1978, 13,

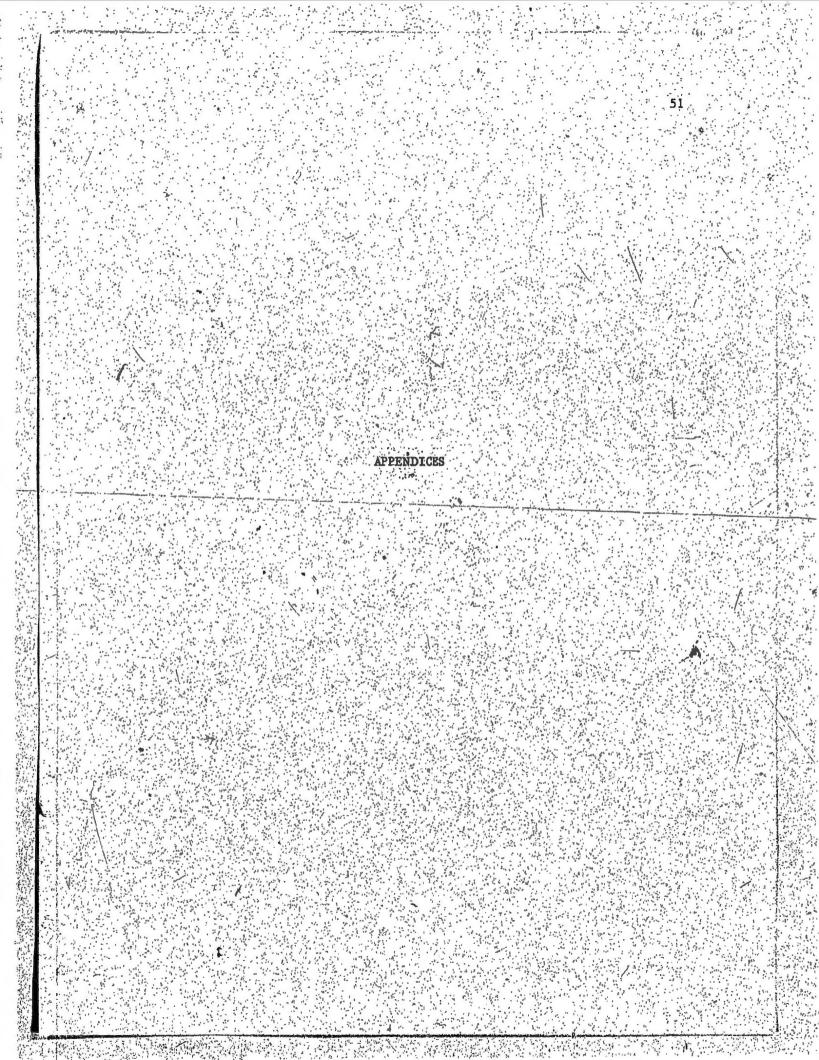
 154-155. (ERIC Document Reproduction Service No. ED 144 840).
- Toney, J.A.S. The effectiveness of individual manipulation of instructional materials as compared to a teacher demonstration in developing understanding in mathematics. <u>Dissertation Abstracts International</u>, 1968, 29, 1831A-1832A.
- Trueblood, C.R. A comparison of two techniques for using visual-tactual devices to teach exponents and non-decimal bases in elementary school mathematics. <u>Dissertation Abstracts International</u>, 1968, 29, 190A.
- Vance, J.H. Establishing a mathematics laboratory program. In D.B.

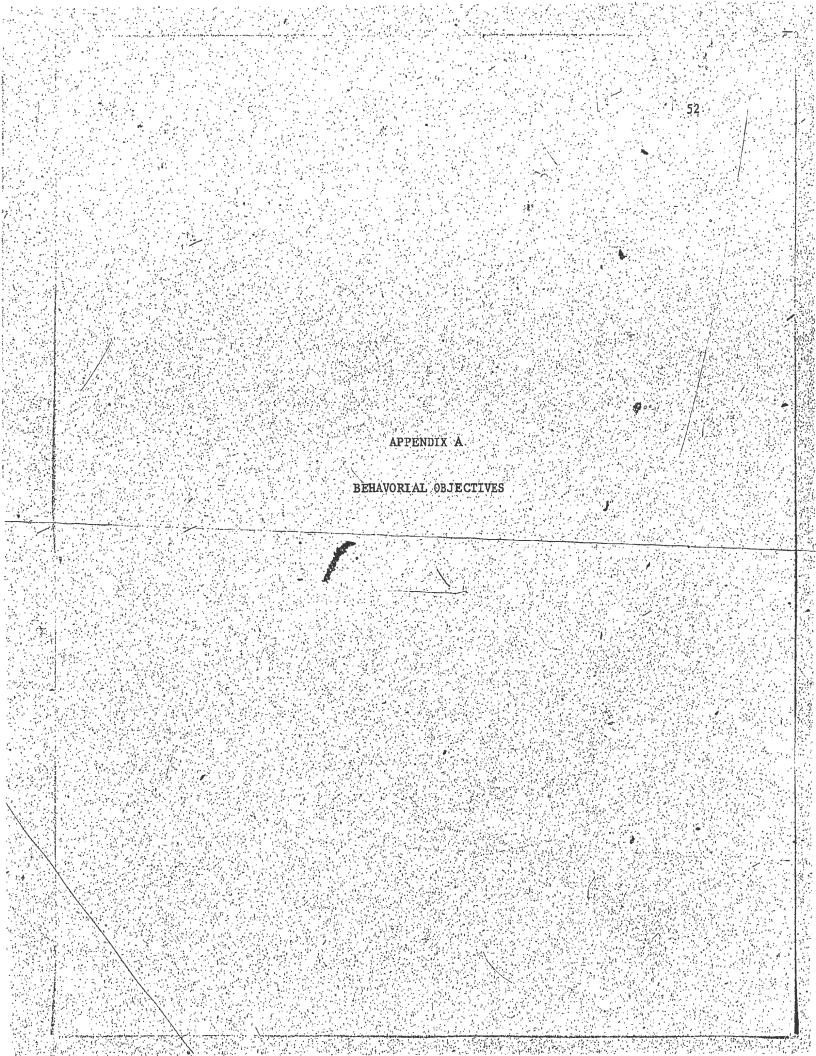
 Harrison (Ed.), Active Learning in Mathematics—A Set of Resource

 Materials for Teachers. Calgary: Alberta Teachers Association,

 1970.
- Vance, J.H. and Kieren, T.E. Laboratory settings in mathematics: What does research say to the teacher. The Arithmetic Teacher, 1971, 18, 585-589.
- Wilkinson, G.G. The effect of supplementary materials upon academic achievement in and attitude toward mathematics among eighth grade students. Dissertation Abstracts International, 1971, 32, 1994A.
- Wilkinson, J.D. A laboratory method to teach geometry in selected sixth grade mathematics classes. <u>Dissertation Abstracts International</u>, 1971, 31, 4637A.
- Wilson, J.W. Evaluation of learning in secondary school mathematics.

 In B.S. Bloom, J.T. Hastings, and G.F. Madaus (Eds.), Handbook on Formative and Summative Evaluation of Student Learning. New York: McGraw-Hill, Inc., 1971.





BEHAVORIAL OBJECTIVES

- 1. The student can select an appropriate unit and use it to find the measure of an object.
- 2. The student can measure and record lengths of segments using metric units.
- 3. (a) The student can find lengths of polygonal paths and perimeters of polygons.
 - (b) The student can use the concept of perimeter to solve examples of real-life problems.
- 4. The student can state the greatest possible error for a given measure of length.
- 5. The student can find the area of a region by counting or estimating square units.
- 6. (a) The student can find the areas of rectangles by using area formulas.
 - (b) The student can use the formula for finding the area of a rectangle to solve examples of real-life problems.
 - (c) The student can write a formula for finding areas of parallelograms.
 - (d) The student can use the formula to find the areas of parallelograms.
- 7. (a) The student can use the formula $A = \frac{1}{2} \cdot b \cdot h$ to find the area of any triangle.
 - (b) The student can use the formula for finding the area of a triangle to solve examples of real-life problems.
- 8. (a) The student can verify the Pythagorean Theorem for particular cases.
 - (b) The student can use the Pythagorean Theorem to find the length of a side of a given right triangle.
- 9. (a) The student can find the volume of a rectangular solid by using the formula V = 1 · w · h.
 - (b) The student can use the concept of volume to solve examples of real-life problems.

10. Given a rectangular solid, the student can find the surface area of the solid.

11. The student can express the capacity of appropriate containers in litres or in subunits or multiples of litres.

APPENDIX B

SUPPLEMENTARY ACTIVITIES

Activity 1

Objective: The student can select an appropriate unit and use it to

Materials: Cardboard, scissors, masking tapa.

Procedure: Have students construct their own ruler from heavy cardboard by using a unit of their choice. A piece of masking
tape could be placed along the edge of the cardboard. The
student can then mark off the unit on the masking tape
along the cardboard strip. The students could then use
this ruler to measure the lengths of various objects as
indicated by the teacher.

Activity 2

Objective: The student can measure and record lengths of segments using metric units.

Materials: Metre sticks, centimetre rulers, activity sheet.

Procedure: Have students estimate the length of each object listed on the activity sheet. Then have them measure the length of each object and record their results on the activity sheet.

Have them determine how close their actual measurements were to their estimations.

Activity Sheet

In the table below, you will find a list of items that you can measure in your classroom. Decide on a suitable unit of measurement such as the centimetre, decimetre, or metre. First, estimate the length of the object, and record your result. Then measure the length of the object and record your result. Then tell how close your estimation was to the actual measurement.

	The second secon		Actual	How much were
	Object	Estimate	Measurement	you over or under
.0.	Length of chalkboard			
2.	Width of your desk top			
3.	Length of a sheet of o			
15.	paper			
4.	Width of your exercise book			
5.	Width of your hand			
6.	Width of the classroom			
7.	Height of the door			
0	Length of a stick of			
	chalk			
9	Thickness of your math			
	book	7		
10	Height of your desk	9		
1 1			the same of the same of	

Activity 3

Objective: The student can find lengths of polygonal paths and peri-

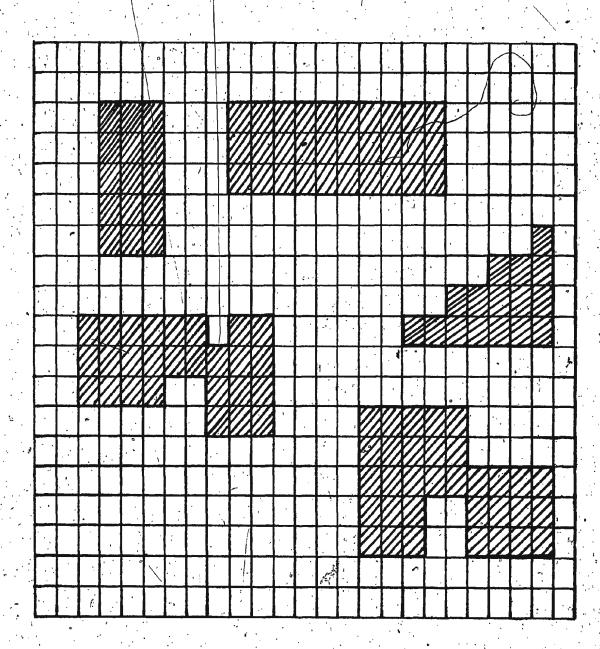
Materials: Activity sheet, unit squares.

Procedure: In activity (a), have students find the perimeter of each
geometric figure shown on the activity sheet. In activity
(b), have students arrange the nine unit squares to form

- (a) a region with the smallest possible perimeter,
- (b) a region with the greatest possible perimeter.

Activity Sheet

(a) On the grid below, a number of geometric figures are shown. Find the perimeter of each of these geometric figures.



Activity Sheet

(b) Take nine unit squares and, without overlapping any squares, arrange them to form as many regions as you can. Record your results in a table like the one shown below:

# of sides of region	perimeter of region		

- (a) Which region has the smallest perimeter?
- (b) Which region has the largest perimeter?.

Objective: The student can state the greatest possible error for a given measure of length.

Materials: Metre sticks.

Procedure:

Have students work in groups of three to determine the perimeter of the classroom floor in metres using metre sticks. Have each group of students record their measure of the perimeter in a chart drawn on the chalkboard. When all groups have completed their measurements:

- (a) Compare the results in the different groups.
- (b) Discuss ways in which the measurements might be in error.
- (c) Find the greatest possible error for each measure-
- (d) Discuss ways of finding the best estimate of the actual perimeter such as averaging all of the measurements.

Objective: The student can find the area of a region by counting or estimating square units.

Materials: Activity sheet, graph paper.

Procedure: In activity (a), have students find the area of each
figure shown on the activity sheet by counting unit
squares. In activity (b), have students draw on dot paper,
geometric figures with given areas. In activity (c), have
students trace their hands on graph paper and determine
its area. In activity (d), have students find the area of
a semi-circle by counting unit squares.

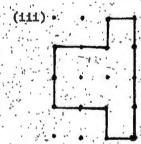
(a) For this activity, our unit square will be the one shown below.

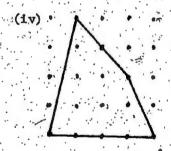


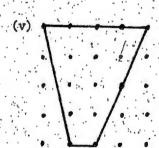
Find the area of each geometric figure shown below by counting unit squares.

(1)

(11)







- (b) On your dot paper, draw geometric figures having the following areas:
 - (1) 12 square units.
 - (11) 7 square units.
 - (iii) 10 square units.
 - (iv) 15 square units.

- (c) Trace your hand on the graph paper provided. Then find its area in square units.
- (d) On graph paper, find the area of a semi-circle by counting the square units.

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Objective: The student can write a formula for finding the areas of parallelograms.

Materials: Scissors, activity sheet.

Procedure: Have the students perform the activities indicated on the activity sheets. Then have them answer the questions related to the activities.

A. Using your paper and scissors, cut out a parallelogram in the shape of the one shown below.

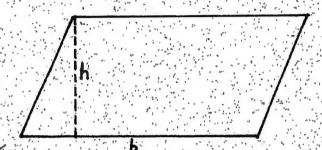


Figure I

Then taking the parallelogram, fold the two triangles as shown below. Then take your scissors and cut off one of the triangles along the fold.

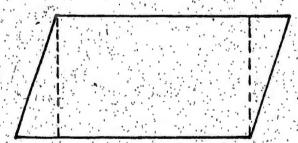


Figure II.

Using the triangle you have cut off, form the rectangle shown below.

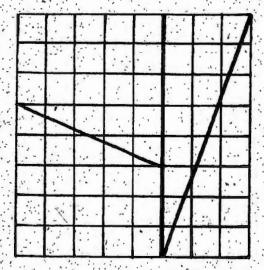


Figure III

- Questions:, 1. Are Figures I and III the same size? Why or why not?
 - 2. If the length of Figure III is 12 cm and the width of Figure III is 8 cm, what is its area?
 - 3. Are Figure I and III the same height? Why or why not?
 - 4. What is the area of Figure I?
 - 5. If b is the length of the base and h is the height of the parallelogram in Figure I, write a formula for finding the area of the parallelogram,

B. Lost Area Puzzle

The 8 x 8 grid has 64 square units. Mark and cut as shown below.



Using the four places, that you have cut, form a 5 by 13 rectangular figure. Why does your new figure have an extra square unit?

Objective: To determine methods of finding the area of triangular regions which can be formed on a geoboard.

Materials: Geoboard and rubber bands.

Procedure:

Let the shaded area represent one square unit on the geoboard.

How many square units are within this figure?

What is the area of this shape? ____ square units

How many square units are within this triangle?

What is the area of the following triangles?"



Did you find an easy way to figure out the areas of triangles? Explain how you did this.



Make this triangle on the geoboard.

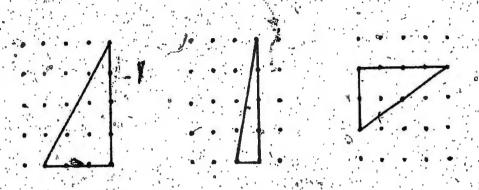


With another elastic band make a rectangle.

The area of the rectangle is square units.

The area of the triangle is square units.

Use this method to find the areas of the triangles below:

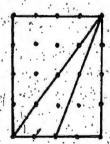




The last triangle can be thought of as being made up of two triangles.



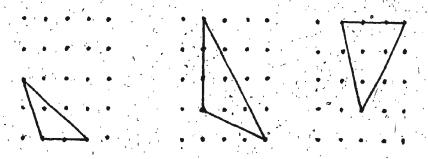
How would you find the area of this triangle?



One way would be to complete the rectangles as shown and subtract two triangular areas.

Area = 16 sq. units - 8 sq. units - 4 sq. units = 4 sq. units.

Find the area of these triangles:.



Given a triangle, choose a side you can determine the length of and call this side the base of the triangle. The height of the triangle is the length of the other side of the rectangle enclosing the triangle.

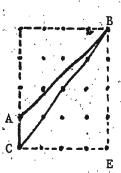
Example 1

In triangle ABC, if BC is the base,
CD is the height = 4 units.

Example 2

In triangle ABC, if Asis the base, CE is the height = 4 units.





For each of the triangles below, find the lengths of the base and the height. Also find the area.

height area

height ____

area

base height

If you know the length of the base and the height of a triangle, how can you find its area?

Objective: The student can verify the Pythagorean Theorem for particular cases.

Materials: Strips of wood 12 cm long, 16 cm long, and 20 cm long, wood glue.

Procedure: Have students work in groups to construct a carpenter's square. Have students bring with them to school strips of wood 12 cm long, 16 cm long, and 20 cm long. After the students have glued their strips together, students can check whether or not their square is a right-angled square by checking whether or not the corners of the classroom, chalkboard, etc. are square. If any squares are not right-angled, a discussion could then follow as to why this happened.

Objective: The student can find the volume of a rectangular solid by using the formula V = 1 · w · h.

Materials: Two boxes.

Procedure: The teacher presents to the class two boxes which have the same volume but appear to hold different amounts. The teacher asks the class which box holds more. The students can then measure the boxes and find the volume of each. A discussion on why the boxes appear to hold different amounts but have the same volume would follow.

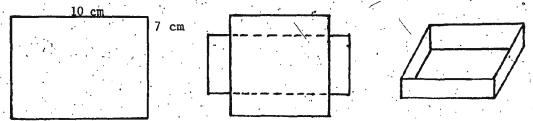
Objectives: Given a pectangular solid, the student can:

- (1) find the volume of the rectangular solid by using the formula $V = 1 \cdot w \cdot h$.
- (2) find the surface area of the solid.

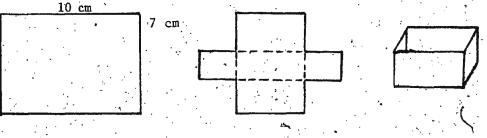
Materials: Activity sheets 1-3, scissors, tape, centimetre ruler.

Procedure: Provide the students with the materials listed above and copies of sheets 1-3. Students may work together in groups of three and each student construct one of the boxes. In Part A; each box on sheet 3 is made from a rectangle 10 cm by 7 cm. In Part B, students have to find the surface area of each box.

A cardboard box manufacturer has a flat piece of cardboard measuring 10 cm. by 7 cm. The manufacturer is given an order to form boxes with open tops. This is done by cutting out the corners.



If he cuts larger corners, the box will be taller but the base smaller.



Part A

- 1. The manufacturer wishes to find the box with the greatest volume.

 Follow the directions on sheet 3 to form boxes with open tops.
- 2. Estimate which box has the greatest volume.
- 3. Measure each box and fill in the first three columns.

	Length(1)	Width(w)	Height(h)	.Volume(v)			
Box 1	· cm	cm	em	cm ³			
Box 2	' cm	cm	_ cm	cm ³			
Box 3'	-cm	cm	cm	cm ³			

The volume of the box is the product of the length, the width, and the height. $V = 1 \cdot w \cdot h$.

- 4. Compute the volume of each box and complete the last column.
- 5. Which box has the greatest volume?

Part B

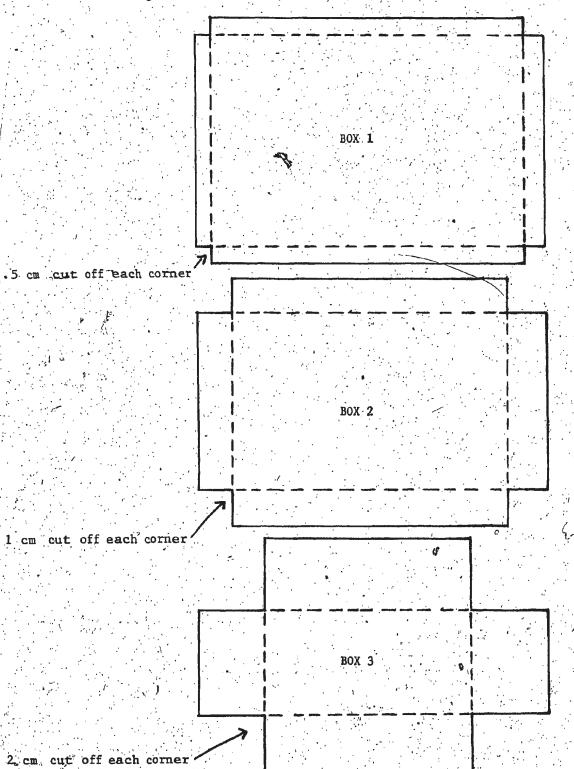
The cardboard box manufacturer wants to find the box with the least surface area. The surface area of a box is the sum of the area of its faces.

Find the surface area of the three boxes.

	Box 1	Box 2	Box 3
Area of side (1.h)		•	
Area of side (1.h)			
Area of end (w·h)	, ,		
Area of end (w·h)	e e e e e e e e e e e e e e e e e e e		
Area of bottom (1.w)		1	
Total surface area			

Which box has the least surface area?

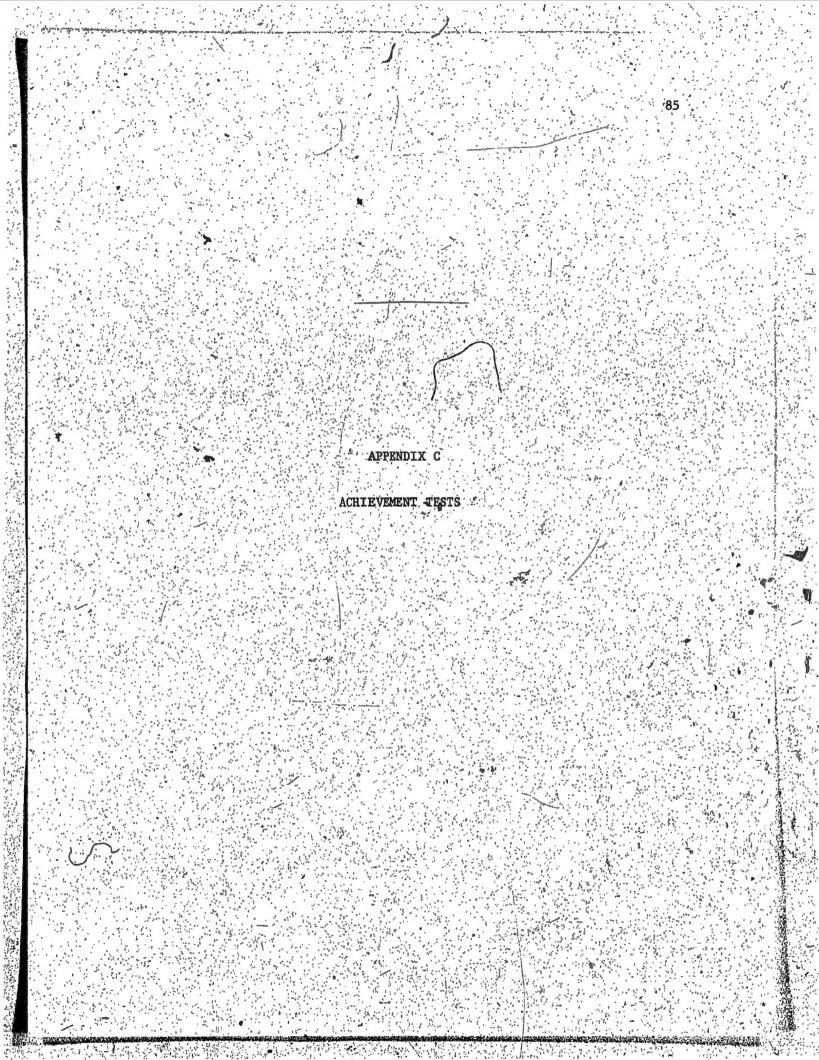
Cut out the three figures shown below.



Objective: The student can express the capacity of appropriate containers in litres or in subunits or multiples of litres.

Materials: Box, carton, or container.

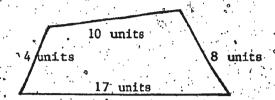
Procedure: Have students bring a box, carton, or some other container to school. Have the students find the capacity of the containers in cubic centimetres and also in millilitres or litres. Have students make a display of the containers with labels stating the capacity of each.



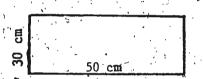
Test 1

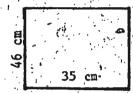
- 1. (a) Using an appropriate unit, give the width of your test paper.
 - (b) Would you be finding length, area, or volume when measuring each of the following?
 - (1) the amount of water needed to fill a container.
 - (ii) distance from the school to your home.
 - (iii) size of the classroom floor.
 - (iv) height of the school.
 - (v) the amount of ice cream in a container.
 - (c) What would be an appropriate unit for measuring the width of your pencil?
- 2. (a) Find the length of the segment shown below to the nearest whole centimetre? What is the length of the segment in millimetres?
 - (b) Which unit of length, the metre, kilometre, centimetre, or millimetre would you use to measure each of the following?
 - (1) the length of the school.
 - (ii) the distance from Stephenville to Corner Brook.
 - (iii) the width of your hand.
 - (iv) the thickness of a quarter.
 - (v) your height.
 - (c) An engineer planned to build a road 227 cm wide. He has a measuring tape which is expressed in metres but not centimetres. Help him by expressing 227 cm in metres.

3. (a) Find the perimeter of the following geometric figure:



- (b) The distance around a rectangular field is 100 metres. The length of the field is 34 metres. How many metres wide is the field?
- (c) Two rectangles are shown below:

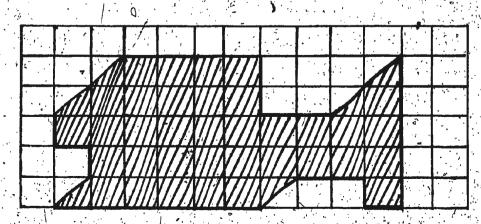




Which rectangle has the greatest perimeter? What is the difference between the two perimeters?

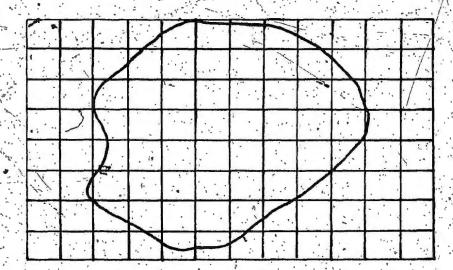
- (a) If you were to measure an object and find its length to be
 20 centimetres, what would be the largest and smallest measures
 possible for its actual length?
 - (b) We are told that the distance between two cities in Canada is about 1700 kilometres. Could the actual distance be as much as 1785 kilometres? Why or why not?
 - (c) Find the relative error for a measurement of 25 cm.
- 5. (a) In the diagram below, unit squares are indicated by the grid.

 Find the area of the figure shown.



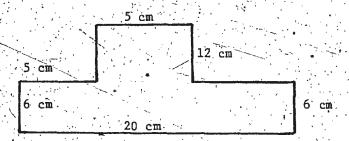
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(b) A farmer has drawn a diagram of a section of his farmland. In his diagram which is shown below, one centimetre represents one metre. Estimate the area of the farmland shown in the diagram.

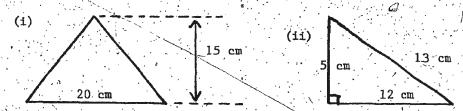


Test 2

1. (a) Find the area of the following geometric figure.



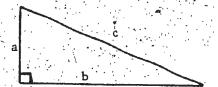
- (b) The base of a parallelogram is 32 cm long, and the height of the parallelogram is 7 cm. Calculate the area of the parallelogram.
- (c) A construction company plans to construct an office building which is to have a rectangular floor area of 1200 square metres. The building is to be 30 metres wide. How long must the building be?
- 2. (a) Find the area of each triangle shown below:



(b) A building sign is to be in the shape of the triangle shown below. Its area is to be 6 square metres. Its base is to be 4 metres long. How high should the sign be?

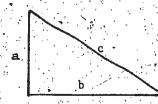


3. (a) In the right triangle shown below, use the Pythagorean Theorem to complete each part.



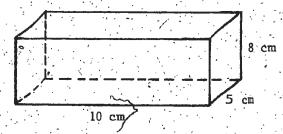
If $c^2 = 169$ and $a^2 = 25$, then $b^2 = and b = an$

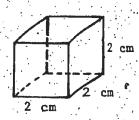
(b) Consider the triangle shown below:



If a = 3, b = 5, and c = 7, is the triangle a right triangle? Why or why not?

- (a) How many unit cubes would you need to make a rectangular box.
 6 units long, 4 units wide, and 3 units high?
- (b) A container in the shape of a rectangular box and a cube are shown below:



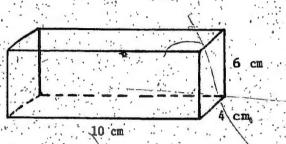


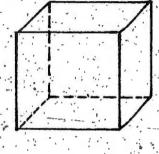
Suppose the container is filled with water and the cube is dipped into the container. The cube is filled with water and then removed from the container. What volume of water remains in the container?

is the surface area of the box?

(b) Two rectangular solids having different shapes are shown below.

One solid has dimensions shown and the other solid has no dimensions shown. Find dimensions for the second solid so that the two solids have the same volume but different surface areas.

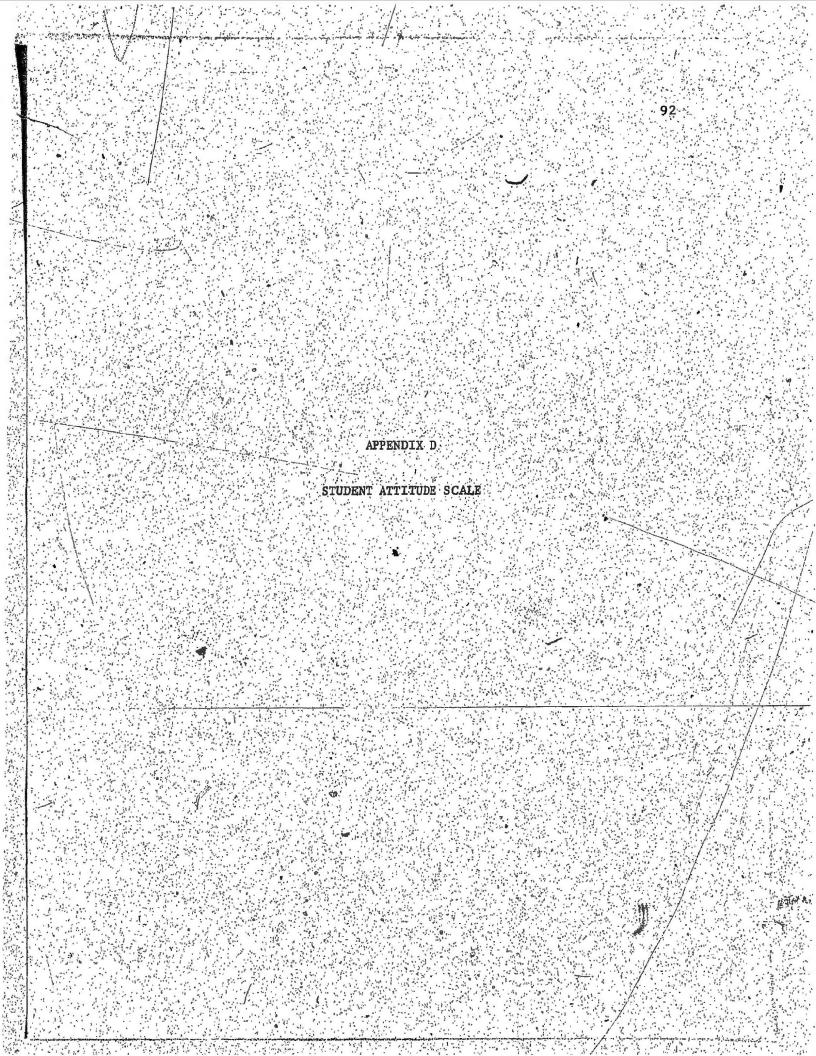




6. (a) One litre of water has a volume of

. cubic centimetres.

- (b) An ice cream container is in the shape of a cube whose edges are 20 cm long. How many millilitres of ice cream does the container hold? How many litres does the container hold?
- (c) A druggist has two medicine containers. One container has a volume of 1200 cubic centimetres when full and the other container has a capacity of 1500 millilitres. Which container has the greater capacity? By how much?



SCALE OF ATTITUDES TOWARD MATHEMATICS.

Directions: Write your name in the upper right-hand corner. Then draw a circle around the letter(s) indicating how strongly you agree or disagree with each statement: SD (Strongly Disagree), D (Disagree), U (Undecided), A (Agree), SA (Strongly Agree).

1.	Mathematics is not a very interesting SD subject.	D	U	A	SA
i			1.		1 2
2.	I want to develop my mathematical skills SD	D.	TT .	A	SA
- 2 1/4 *	and study this subject more.	D	1. 2	A.	, OA
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3.	Mathematics is a very worthwhile and SD.	D	Ü	Α.	SA
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	uncomfortable.			4. 1	N 47.7
5.	I have usually enjoyed studying mathe-			1 - 3"	
	matics in school.	D	. U	Α	SA
7	mattes in school,	*			
6.	I don't want to take any more mathe-				
-: ;	matics than I have to.	D	Ω.: -	A	SA
4 4				250	
7,:	Other subjects are more important to		- 1	3	
	people than mathematics.	ט	U	. A	SA
			12	1.1	, , ,
8.	I am very calm when studying mathe-	n	71	/	SA
	matics.	7.3			JA
			1, 11,		
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9.	I have seldom liked studying mathe-	, a	U	A	SA
9.	I have seldom liked studying mathematics.	D	ΰ	À	SA
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10.	matics. I am interested in acquiring further sp knowledge of mathematics. Methematics helps to develop the mind	D D	U U	A	SA
10.	I am interested in acquiring further knowledge of mathematics. Mathematics helps to develop the mind and teaches a person to think. Mathematics makes me feel uneasy and	D D	U U	A	SA SA
10.	matics. I am interested in acquiring further knowledge of mathematics. Mathematics helps to develop the mind and teaches a person to think. Mathematics makes me feel uneasy and contact the state of	D D	U U	A	SA
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10: 11. 12.	I am interested in acquiring further knowledge of mathematics. Mathematics helps to develop the mind and teaches a person to think. Mathematics makes me feel uneasy and confused. Mathematics is enjoyable and stimu— SD	D D	U	A	SA SA

	Mathematics is not especially important in everyday life.	SD	D ₁	U	A	SA
16.	Trying to understand mathematics doesn't make me anxious.	SD	. D	Ù	A	SA
17.	Mathematics is dull and boring.	SD	, D	U	A	SA:
18.	I plan to take as much mathematics as I can during my education.	SD	D	Ü	Ą	SA
19.	Mathematics has contributed greatly to the advancement of civilization.	,SD	D	, U	, A	SA
20.	Mathematics is one of my most dreaded subjects.	-SD	D	Ü	A	SA
21.	I like trying to solve new problems in mathematics.	SD	D	U	A	SA
22.	I am not motivated to work very hard on mathematics lessons.	\$D	D	U ,	Ar	SA
23.	Mathematics is not one of the most important subjects for people to study.	SD	Ď.	U	A	SA
24.	I don't get upset when trying to do mathematics lessons.	SD	D	U,	A	SA

Directions for Scoring

The Mathematics Attitude Scale can be scored on four subscale variables. Enjoyment in Mathematics (items 1, 5, 9, 13, 17, and 21); Motivation in Mathematics (items 2, 6, 10, 14, 18, and 22); Importance of Mathematics (items 3, 7, 11, 15, 19, and 23); and Fear (absence) of Mathematics (items 4, 8, 12, 16, 20, and 24). Items 1, 4, 6, 7, 9, 12, 14, 15, 17, 20, 22, and 23 are scored according to the following key: SD = 4, D = 3, U = 2, A = 1, SA = 0. Items 2, 3, 5, 8, 10, 11, 13, 16, 18, 19, 21, and 24 are scored according to the following key: SD = 0, D = 1, U = 2, A = 3, SA = 4. The four subscale scores can also be combined to yield a total score.

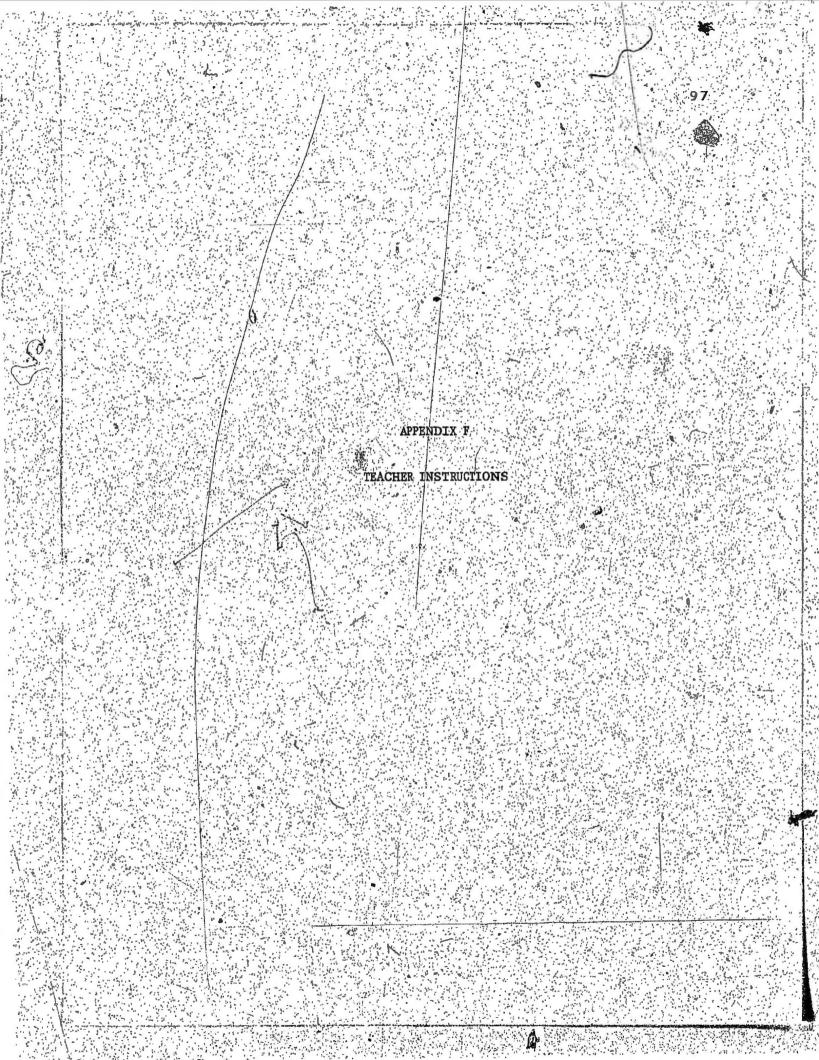
APPENDIX E TEACHER QUESTIONNAIRE

TEACHER QUESTIONNAIRE

- 1. Did you enjoy teaching the material in this unit? Why or why not?
- 2. Comment on the usefulness of the supplementary activities in developing or reinforcing the concepts taught.
- 3. Would you omit any of the supplementary activities? Which ones?
 Why?
- 4. Are there any supplementary activities you would like to add to the unit? Explain.
- 5. For which grade seven ability level do you think the material in this unit is suitable—the below average student, the average student, or the above average student? Explain your answer.
- 6. Do you think that the activity approach to teaching measurement is more effective, than other approaches such as the lecture approach?

 Why or why not?
- 7. What were some of the students! reactions to the use of the concrete materials?
- 8. What were some problems you encountered in teaching this unit through the activity approach?
- 9. Would you consider using the activity approach with other areas of mathematics at the grade seven level and at other grade levels?

 Why or why not?
- 10. Would you recommend the material in this unit to other teachers?
 Why or why not?



TEACHER INSTRUCTIONS

For the purposes of this study, eleven lessons on 'measurement' have been selected from the unit on 'Geometry and Measurement' from the text book School Mathematics I. These eleven lessons are included in the pages from C-36 to C-56. You have been given eleven behavorial objectives, one objective corresponding to each lesson.

Each lesson is divided into five sections:

- (a) Preparation
- (b) Investigation
- (c) Discussion
- (d) Utilization
- (e) Extension

These five sections for each lesson are described in the teacher manual of the textbook School Mathematics I. The teacher is urged to read carefully these five sections for each lesson.

Use of Supplementary Activities

Lesson #1: 'What is the Measurement Process?'

In Lesson #1, the supplementary activity is to be used in the 'Extension' section of the lesson. This activity will reinforce the basic concepts of measurement taught in this lesson. This activity will also provide the students with practical experience in choosing a unit to construct a ruler and using the ruler to measure the lengths of objects.

Lesson #2: Units of Length

The supplementary activity for lesson #2 is to be used in the 'Utilization' section of the lesson after the students have completed the practice exercises. This activity will provide the students with additional practice in estimating and measuring the lengths of objects and recording results.

Lesson #3: Polygonal Paths and Perimeter

For this lesson, supplementary activity (a) is to be used after the 'Discussion' section in the textbook is completed. This activity will reinforce the concept of perimeter and is to be completed before the students start the practice exercises.

Supplementary activities (b) and (c) for this lesson are to be used as enrichment activities in the Extension' section of this lesson.

Lesson #4: Greatest Possible Error

The supplementary activity for this lesson is to be used in the Extension section of the lesson. This activity will reinforce the

concepts taught in this lesson. This activity will also provide the students with practical experience in determining the greatest possible error for a measurement.

Lesson #5: 'Units of Area'

Supplementary activity (a) 4s to be used in the Discussion section of this lesson. This activity develops the concept of area.

Supplementary activities (b), (c) and (d) are to be used in the, 'Extension' section of the lesson and provides the students with further practice in estimating and calculating area by counting unit squares.

Lesson #6: 'Finding Areas of Rectangles and Parallelograms'

Supplementary activity A is to be used in the 'Discussion' section of this lesson. This activity helps students see the relationship between the area of a parallelogram and the area of a rectangle. This activity also helps the students develop the formula for finding the areas of parallelograms.

Supplementary activity B is a recreational activity and is to be used in the 'Extension' section of the lesson.

Lesson #7: 'Finding Areas of Triangles'

The supplementary activity for this lesson replaces the 'Investigation' section of this lesson. This activity shows the student how the area of a triangle is related to the area of a rectangle. This activity also develops the concept of area of a triangle and helps the students develop a formula for finding the area of a triangle.

Lesson #8: 'The Pythagorean Theorem'

For this lesson; the supplementary activity reinforces the concepts learned in this lesson. This activity also provides the students with a practical application of the Pythagorean Theorem. This activity is to be used in the 'Extension' section of this lesson.

Lesson #9: 'Finding Volume

The activity for this lesson is a discussion activity and supplements the 'Discussion' section of this lesson. This activity further develops the concept of volume. This activity also gives the students the opportunity to see that two boxes can have different shapes, and still have the same volume.

Lesson #10: 'Volume and Surface Area'

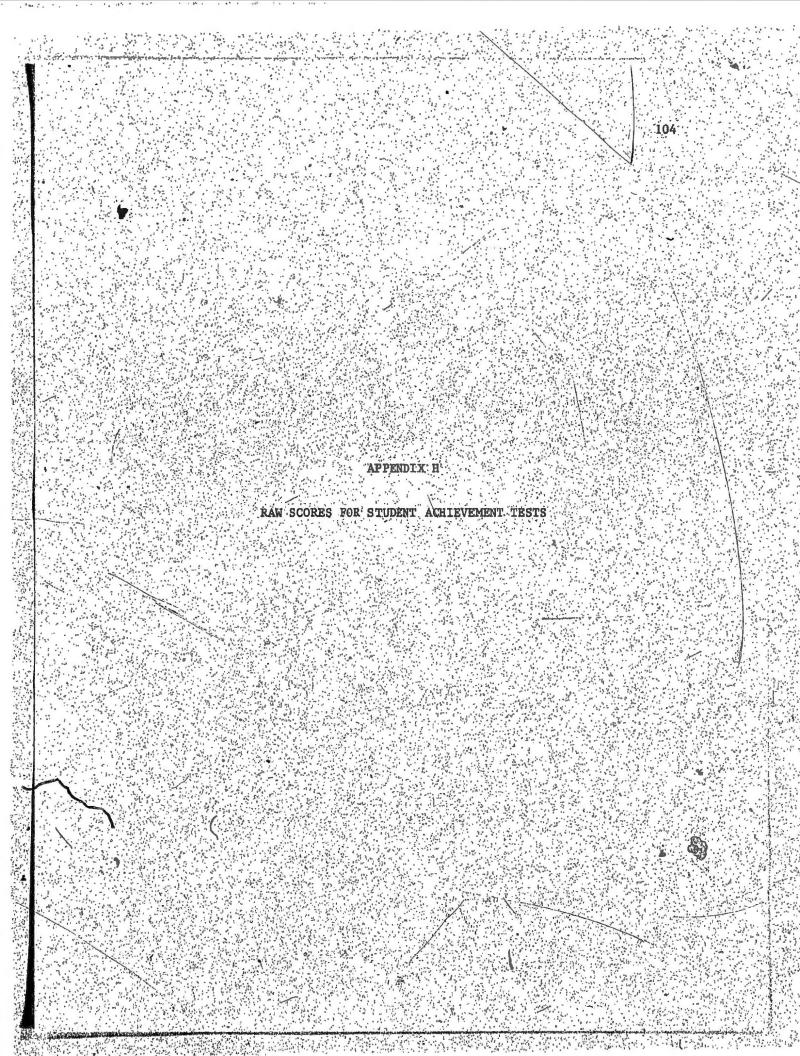
The supplementary activity for this lesson is to be used in the 'Extension' section of this lesson. It presents the students with a real-life problem and provides them with practical expenience in comparing volumes and surface areas of boxes having different shapes.

Lesson #11: 'Metric Measures of Capacity'

The activity for this lesson is an enrichment activity and is to be used in the 'Extension' section of this lesson. It provides the students with further practice in finding capacities of containers in metric units.' This activity also gives the students the opportunity to compare the capacity of containers having different sizes and shapes.

RAW SCORES FOR STUDENT ATTITUDE SCALE

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Raw Scores for Student Achievement Test I

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Raw Scores for Student Achievement Test II

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