# THE EFFECTS OF MEDIATION ON THE ATTITUDES AND ACHIEVEMENT OF MATHEMATICS STUDENTS IN STUDENT-DIRECTED AND TEACHER-DIRECTED LEARNING SITUATIONS



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# THE EFFECTS OF MEDIATION ON THE ATTITUDES AND ACHIEVEMENT OF MATHEMATICS STUDENTS IN STUDENT-DIRECTED AND TEACHER-DIRECTED LEARNING SITUATIONS.

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## ABSTRACT

This study was motivated by the increased use of multi-media and individualized instruction in the classroom. The present study was designed to investigate any effect of mediated (non-print) and non-mediated (print) instruction on mathematics attitudes, mathematics achievement, and the attitude-achievement relationship. The investigation was carried out using both student-directed and teacher-directed learning situations.

One hundred eighteen grade seven mathematics students from an elementary school were randomly assigned to four treatment groups; student-directed mediated, student-directed, teacher-directed mediated and teacher-directed. The students were given a mathematics attitude and mathematics achievement pretest. At the end of the study the students were given a mathematics attitude and mathematics achievement posttest. From this, a sample of 100, 25 from each group, was randomly selected for analysis purposes. The pretest scores were used as covariates for analysis.

A comparison of the mean attitude and mean achievement scores for the four treatment groups showed no significant difference in attitudes or achievement for any of the treatments.

Significant correlations between mathematics

attitudes and achievement were found on the pretest scores for the teacher-directed group, overall teacher-directed strategy, overall mediated strategy and overall non-mediated strategy.

Using the posttest scores of attitudes and achievement, significant correlations were found for the student-directed mediated group, student-directed group, teacher-directed group, overall student-directed and teacher-directed strategies, and overall mediated and non-mediated strategies.

An interpretation of these findings for the groups must consider the limitation of the short duration of the study.

The study concluded with several implications for education and some suggestions for further research.

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# CHAPTER I

# INTRODUCTION AND STATEMENT OF THE PROBLEM

## INTRODUCTION

The increased use of technology in education and the trend towards task analysis in program organization have placed increased emphasis on the search for the optimum mode of instruction for relating content. This, together with the knowledge explosion, has made it necessary for curriculum planners to attempt to determine the most effective and efficient mode of presentation of content for maximum student learning.

Molnar (1969), in reviewing the impact of educational technology on schools found that educational technology can improve learning effectiveness, positively change attitudes, and provide new learning experience economically and without restriction of time and frequency of use.

Gagné, in discussing media, learning theory and instruction, states that:

> Instruction needs to be arranged so that it will bring about the kind of change in a student which is called learning, and this requires learning theory. In attempting to bring about such a change the act of instruction is a matter of stimulating the student in certain ways - and here one has a choice of media to work with. Putting together ideas from these two domains of knowledge can yield some techniques and procedures of instruction which should make the process of learning an optimally effective one. (1970, p. 51)

The focus then, must be on the design of conditions for learning and the individual's interaction and progress under the different learning conditions in order to determine the optimum mode of instruction for effective student learning.

## I. PURPOSE OF THE STUDY

The purpose of this study was to determine the effects of four different treatments of a Structured Integers Package on the attitudes and achievement of grade seven mathematics students.

Explicitly, the study examined the following questions:

(1) Will mediation affect the attitudes and achievement of students in student-directed and teacher-directed learning situations in mathematics?

(2) Will student-directed learning have any different effect on students' attitudes and achievement than teacher-directed learning in mathematics?

(3) Will there be any relationship between students' attitudes and achievement in student-directed and teacher-directed learning situations?

Although some research has been done regarding student and teacher direction of learning, very little emphasis has been placed on the attitudes of students and the relationship between attitudes and achievement under these conditions. It was hoped that this investigation would provide specific information regarding the effect of these learning situations on attdtudes and achievement and the relationship between them using mediation to provide the learning situations investigated.

## II. MAJOR HYPOTHESES

(1) There is no significant difference between the mean scores of student achievement in mathematics in student-directed mediated learning; student-directed learning; teacher-directed mediated learning; and teacher-directed learning.

(2) There is no significant difference between the mean scores of student attitudes towards mathematics in student-directed mediated learning; student-directed learning; teacher-directed mediated learning; and teacher-directed learning.

(3) There is no significant relationship between attitudes and achievement in student-directed mediated learning; student-directed learning; teacher-directed mediated learning; and teacher-directed learning.

## III. DEFINITIONS

Mathematics Achievement. The subject's score on the achievement test designed for the study.

<u>Mathematics Attitude</u>. The subject's score on the Connelly Taxonomized Attitude Scale Objective II.

<u>Mediation</u>. The use of slides and/or audio tapes to present materials.

<u>Student-Directed</u>. The control of the subject in the rate of completion of the materials without interference from the teacher.

<u>Teacher-Directed</u>. The control of the teacher in the rate of presentation and completion of the materials.

## IV. LIMITATIONS

In interpreting the data of this study the following limitations should be considered:

(1) The short duration of the study may be a factor in interpretation of attitudes data since any change may be a reaction to a different than usual treatment and thus be shortlived.

(2) The novelty of the treatment to students may introduce a reactive effect.

(3) The presence of teachers and their awareness of the treatments may introduce some bias and limit the generalizability of the study. However, this was controlled as much as possible by defining the expected behavior of the teachers in each treatment group.

## V. SIGNIFICANCE

It was indicated in the introduction that increased use of educational technology and the knowledge explosion has made it necessary for curriculum planners to attempt to determine more effective and efficient modes of presentation of content for maximum student learning.

Briggs (1967) points out that the philosophy of "education for all" has developed a need for more individualized instruction and a more flexible curricula to meet the needs of all kinds of students. He points out that a fixed curriculum and lock-step methods of teaching are incompatible with achieving the multiple objectives of (a) education for all, (b) quality education, and (c) education for a rapidly changing society and new frontiers of knowledge.

Gagné (1970) emphasizes that learning is an individual matter and instructional processes must be developed and adopted to meet the individual's requirements.

Dienes (1960, 1963, 1966, 1967, 1970) in commenting on the basic processes involved in Mathematics learning points out that exploring and manipulating should be part of Mathematics lessons. Many different situations must be provided to enable children to make abstractions.

Biggs and MacLean (1969) in attempting to incorporate Jean Piaget's observations about active learning point out the following: Firstly, a child must be allowed to do things over and over again and thus reassure himself that what he has learned is true; and secondly, this practice should be enjoyable. The authors further emphasize that teachers should not give or teach a lesson, but provide a focus for learning.

These ideas may map the future pattern of education and their effects should be thoroughly studied in the classroom and in the context of their effects on student learning.

The significance of this study may be summarized as follows:

(1) Student-directed learning may be a major part of the educational setting in the near future. Its effects on student achievement, attitudes, and the attitude-achievement relationship should be studied.

(2) A comparison between student-directed learning and teacher-directed learning should be made to determine their effectiveness in attaining instructional goals.

(3) The influence of multi-media presentation of instructional content to provide for student-directed learning or as a supplement to teacher-directed learning should be investigated in the context of student attitudes, achievement, and the attitude-achievement relationship in order to determine any effect.

## VI. THE EXPERIMENTAL SETTING

The following is an overview of the experimental design. A more detailed account is reported in Chapter III.

The population from which the sample was drawn consisted of all grade seven students in an elementary school of the Burin Peninsula Integrated School Board. The students were randomly assigned to each of four treatment groups of which 25 students were randomly selected from each group for analysis purposes giving a sample of 100 students.

The students were then pretested for attitudes and achievement. These pretest scores were used as covariates for analysis.

The student-directed mediated group recieved a School Mathematics Study Group printed package introducing the Set of Integers and developing the unit of Integers to subtraction. Along with this package, they were given a series of slides and tapes directly related to the package and explaining the concepts of the package. These students worked individually or in groups to complete the work.

A teacher was present to answer direct questions on the topic.

The student-directed group recieved the same printed package as the above group. However, in place of tapes and slides, this group was given copies of what appears on each of the slides and a written script of what was presented in the tapes. These students worked in groups or individually to complete the work.

A teacher was present to answer direct questions on the topic.

The teacher-directed mediated group received the same printed package as the other groups. However, the slides were given to the teacher to be used in presentation to the class. The script of the tapes was given to the teacher and used as the foundation for the oral presentation of the topic to the students. In this way, the teacher was in control of the rate of movement of the students through the program.

The teacher-directed group received the same printed package as the other group. However, the teacher was given a copy of what appears in each slide and a written script of the tapes. The teacher was not allowed to give these to the group but used this as the foundation of the oral instruction to the group.

This experiment encompassed 18 forty-five minute mathematics classes.

The experiment was organized such that two teachers and two groups were involved at the same time period. In the next time period the other two groups were involved with the same two teachers. One teacher was involved with the student-directed treatments and the other with the teacher-directed treatments.

Each of the groups was given comparable numbers of each time period throughout the study.

At the end of this experiment all the students were given a posttest on attitudes and achievement which were analyzed using an analysis of covariance.

## VII. OUTLINE OF REPORT

A review of relevant literature is presented in Chapter II. Chapter III contains a detailed account of experimental design, testing procedures, and the research procedures used to test the hypotheses. The results of the data analyses is contained in Chapter IV. The final chapter, Chapter V, includes a summary and discussion of the findings, and contains some implications for education and further research.

## CHAPTER II

## REVIEW OF RELATED RESEARCH

The purpose of this chapter is to briefly discuss relevant studies in student-directed and teacher-directed instruction. In addition, investigations into studies dealing with the relationship between mediation and student-directed and teacher-directed learning are summarized and discussed.

Studies which report experimental work on student-directed and teacher-directed learning through mediation have been mainly comparative and thus both topics are treated together in this review.

### I. STUDENT-DIRECTED AND TEACHER-DIRECTED LEARNING

Studies which are related to the present investigation in at least one of its aspects are reported in this section. Most studies deal with student-directed learning as individualized instruction and with teacher-directed learning as traditional instruction.

Crosby (1960) reports that the first published record of individualized instruction appeared in 1894. In his article describing the "Pueblo Plan", W.P. Search, its originator, gave his impressions of individualization; no attempt was made to conduct an educational experiment. The work is mentioned here because of its historical significance in the investigation of student-directed learning.

In 1912, at the Experimental Elementary School of the San Francisco State Normal School, Burk and Ward developed definite techniques of individualized instruction. Burk's plan involved the elimination of class recitations, with each student working at his own rate in each subject using self-instructional pamphlets which were created for this purpose. Careful individual records were kept for each student. A study of the records showed that 100 out of 130 pupils had saved time by working on the individual plan. Over 75 per cent of the students completed more than a grade's work in a year.

Carleton Washburne, Superintendent of Schools in Winnetka, Illinois, who had been a teacher under Burk at San Francisco State Normal School, introduced Burk's plan into Winnetka schools with adaptations. The curriculum was divided into two parts: skills and knowledge to be mastered by all students, and opportunities for the development of self-expression. In the first, time varied for each student but all reached mastery. In the second, the student's work was directed by his interests and abilities. Half the morning and half the afternoon were devoted to group and creative activities. The other half of the school day was organized so that a student would do a unit of work, check his own answers and proceed to the next unit. After standardized tests had been administered, the students were compared by grades, achievement groups, and by mental age in the various subject areas. The Winnetka children did better work on these tests in every subject than students from comparable schools.

In 1922 a form of individualized instruction known as the Dalton Plan was introduced in a Dalton, Massachusetts public high school. This plan emphasized freedom, community living and student responsibility. Under the Dalton Plan, freedom was referred to as opportunity for uninterrupted pursuit of areas of interest; community living referred to replacing grade rooms and teachers with subject laboratories and specialists; and student responsibility referred to student budgeting of time in terms of ability and need to complete 'contract' jobs. The contracts were teacher constructed units of work.

W.H. Thompson (1933) experimentally tested the Dalton Plan at the Ohio State University Demonstration School. Thompson compared this school with Columbus public schools using batteries of standardized tests in every subject area at the beginning and end of an eight-week summer session. Mental age in reading, composition, spelling and arithmetic were measured. The same procedure was carried out the following summer. The Dalton group were superior on initial tests but comparison of gains showed no favorable effect for either group.

Mayer-Oakes (1936) studied the effect of the Dalton Plan on discipline problems, achievement on state tests and students' maximum use of their time. Five participating classes were set up and contracts prepared. The students worked alone or in groups. Standardized tests were given in the twelfth and twenty-eighth week. In addition, the scores on the state examination were also analyzed. Mayer-Oakes concluded that the Dalton Plan improved student's learning in the small high school since the standardized tests showed that Dalton Plan students were above the standard median scores in all cases except Algebra and English. Also, he found a 25 per cent gain in state test pass marks with 31 per cent more passes than the state average.

Shultz and Ohlsen (1949) tested the effectiveness of individualized instruction in providing remedial help to algebra students. Each experimenter taught an experimental and traditional control group. There were slight gains in the experimental groups, however, the gains were not significant. However, the experimenters pointed out that the experimental group students appeared to have a more satisfying experience and enjoyed the added responsibility and there were some favorable changes in mathematics attitudes. These observations were made from teacher-requested student reactions.

Bernstein (1955) used individualized instruction in setting up a small group clinic to overcome arithmetic deficiencies with ninth grade students. He found that individualized instruction enabled students to accomplish twice as much work in less time than those students taught in large remedial groups.

Crosby, Fremont and Mitzel (1960) in a study of the growth of achievement and attitudes in individualized instruction carried out probably one of the first replicable studies. The experiment involved 36 ninth grade algebra classes selected randomly and randomly assigned as experimental or traditional treatment groups. The dependent variables were achievement in algebra measured by 117-item examination and attitude measured by a 50 item Attitude Toward Mathematics Scale. The independent variable was a composite T-Score based on four sub-tests of the Differential Aptitude Tests and was used as a covariate item in analysis.

Cell size was set at 18 for equalization in analysis. The analysis showed no significant difference at the five per cent level. However, the experimenters argue that difficulties occurred in the classroom utilization of the experiment due to rigid classroom management thus biasing the experimental outcomes. Thus, they expressed the need for replication of this study and for other similar studies.

A study by Gold (1965) of the effects of self-directed learning on gifted elementary school children utilized a sample of 39 above average student in grades four, five and six. Reading, social studies and science were used to study achievement, study skills, divergent thinking but personal and social adjustment appeared to be favorably affected when self-direction was used.

Rainey (1965) studied the effects of directed and non-directed laboratory work in high school chemistry. The study used CBA as the non-directed approach and CHEM Study materials as the directed approach. The directed approach group were given lab exercises with specific and detailed instruction while the non-directed group were given the exercises without directions for their solutions. He found that the non-directed group were better able to recall specifics about each lab experiment. Learning of principles and approaches showed no significant difference between the two approaches.

Liedtke (1970) in a study of Mathematics Learning and Pupil Characteristics, examined the relationship between mathematics and pupil characteristics. He investigated intellectual ability, reading ability, reflectiveness, impulsiveness, socio-economic status, ability to make personal adjustment, ability to make social adjustment and sex. He carried out his investigation in a student-directed setting. The study was carried out over a period of four weeks with 51 student-directed subjects, 53 partially teacher-directed subjects, and 37 teacher-directed subjects. An Initial Learning Test was given at the end of the study and four weeks later the same test was given to obtain a measure of retention.

The results showed that: 1. for the student-directed setting none of the pupil characteristics considered could predict learning outcomes; 2. for students in the partially teacher-directed setting there existed significant relationships between initial learning and retention and the following factors: intelligence, personal adjustment, social adjustment and reading ability. Reflexive students had significantly higher mean scores than the impulsive students on the Retention test. There was no difference between means for boys and girls; 3. the learning outcomes for the teacher-directed subjects could be predicted on the basis of intellectual ability only. There existed no other significant relationships.

Kline (1971) studied the effectiveness of learning by self-directed and teacher-directed students in an Earth Science Curriculum Project open-ended lab block on soil. He divided 97 junior high school students into two control groups (teacher-directed) and two experimental groups (student-directed). Using IQ as the covariate, his analysis of the pre- and post-test lab scores showed no significant difference in cognitive understanding or achievement. A significant difference was found on reading difficulty, however, its seriousness was challenged since the student-directed group achieved as well as the teacher-directed group on the posttest.

Summary. Studies on student-directed and teacher-directed learning have mainly been observational and thus not replicable. The studies which parallel the present study to some extent, have failed to produce adequate experimental evidence to support or refute the purpose of this study. Much more experimental evidence is needed in this area and this present study may help this cause.

## II. MEDIATION IN STUDENT-DIRECTED AND TEACHER-DIRECTED LEARNING

The use of media to present instructional content has been widely studied for comparative effectiveness. The mediation used in the present study utilized slides and thus only research involving the use of slides will be reviewed.

Many of the methods of instruction have been based on a learning theory framework and in order to develop meaningful instruction for individuals and groups these theories must be explored. Many theories stress the need for divergent stimuli to enhance learning and this present study incorporates the use of media to provide this.

Piaget yields some important observations and research relevant to Mathematics. Piaget (1951, 1952, 1964) contains many suggestions as to how children develop and how they learn mathematical concepts. He points out that an individual's failure to grasp most basic concepts stems from affective, emotional blocking or inadequate preparation rather than lack of any special aptitude (Piaget, 1951). This leads to a very important need to provide the learner with concrete experience and practical action.

Dienes (1966) emphasizes that many different situations must be provided in order to enable children to make abstractions. He states that "the child must have the actual situations" (1966, p.32). He further emphasizes that if the proper stimulus situations are presented, children can engage in highly sophisticated logical thinking.

Dienes ideas seen to support the hypothesis by

Bruner (1960) that any subject can be taught to any child at any level in some honest form. Bruner points out the importanceof translating mathematics to the child's way of thinking. That is, the ideas should be presented in a concrete form at an early age and become more abstract as the child's thinking processes mature. Proper stimulus situations are of great importance in doing this effectively.

Gagné (1970), in relating his hierarchical learning theory with media, points out that instruction involves gaining and controlling attention, stimulating recall, guiding learning, providing feedback, arranging for remembering and assessing outcomes. It is these functions that are performed by various media of instruction and to a considerable degree by the learner himself. He considers that media may be required to achieve the kind of instruction that is most effective for learning.

Piaget, Bruner, Dienes and Gagné all express the need to make instruction meaningful and stimulating to learners. Learning is a developmental process and as a child explores his environment he becomes aware of the properties of and relations among ideas. As he becomes older these relations become more refined and result in highly structured concepts and eventually abstractions. To accomplish this, the child must be given concrete experiences and various stimulus situations. Instruction must encompass and integrate these ideas. The use of mediated approaches for instruction has possibilities for doing this.

Allen (1960), in a review of research of audio-visual communication devices, concluded that research to that time comparing slides with sound motion-picture instruction found in general that slides were as effective as motion-pictures in teaching factual information. His review summarized early studies by James (1924), McClusky (1924), Brown (1928), Goodman (1942), Jackson (1948), Anderson et al. (1951), Abramson (1952), Slattery (1953) and Lasser (1954). These studies covered a wide range of subject matter areas including spelling, nursing, mechanics, economics, social studies, health education and safety education. Also, grade levels from elementary school to university were involved.

Allen suggested that slides have been found effective because of special learning conditions for which they are especially suited such as individual pacing and student participation.

Campeau (1971) points out that no suitable studies were found in which the instructional effectiveness of slides alone was assessed. However, two studies using slides as part of multiple media instruction were found and are reviewed here because of their probable significance to this study.

Both studies were conducted by Edwards, Williams and Roderick (1968). The first study compared the performance of college students learning first-semester business machines skills in an audio-visual-tutorial laboratory with the performance of a control group taught in the traditional manner. Assignment to either experimental or control group was by lottery. The control class was taught by the same teacher who prepared the materials for the experimental groups, and both groups took identical final exams at the end of the term. During the term, experimental students attended an open lab at any time convenient to them and received their instruction in business machines skills through programmed materials presented by continuous-loop sound films, and slides and tapes, these media were housed in individual carrels. As measured by the final performance test, the experimental group learned significantly more than the control group. A difference of even greater magnitude, also statistically significant, was noted when the scores of those students who had no previous exposure to business machines were analyzed separately.

The second study was similar to the first but in a beginning typing course. Students in the comparison group were taught to type by a teacher who used traditional methods in presenting the complete course in beginning typing. Students in the experimental group covered the same material in the audio-visual-tutorial lab situation, during which they were exposed to films, slides and tapes, tapes alone and printed instructions and tests. Students sat in individual learning carrel units especially designed to house the audio-visual devices. These devices were used for particular types of learning tasks. For example, continuous-loop film cartridges with synchronized narration were used for units in which demonstrations were necessary. Format and rules for typing manuscripts, cards and letters were prepared on slides accompanied by sound tapes. Tape recordings provided skill-building drills. Timed production and straight-copy drills were prepared on tapes which were used by students at practice stations with portable cassette tape recorders, rather than in carrels. Results on an end-of-term performance exam consisting of three-minute timed writings indicated that students without prior typing training who learned beginning typing in the experimental situation significantly outperformed the traditionally trained group.

Summary. Research on mediation for-instructional presentation has been mainly comparative to other media. As a method of instruction, Edwards, Williams and Roderick

(1968) showed that it significantly outperformed traditional teaching. This present study is necessary to provide more information on the use of mediation as an alternative to traditional instruction in mathematics.

## III. ATTITUDES TOWARDS MATHEMATICS

Attitudes towards mathematics have been a great concern of many educators. The emphasis was to determine any relationship of attitudes to student learning. As Mayer (1966) notes, it is not necessary for students to like an activity or subject in order to use it, but most educators agree that anything done to influence a student should leave him with a favorable feeling.

A problem of the definition of the term "attitude" becomes evident when interpreting research concerning the measurement of attitudes towards mathematics. Aiken (1972) states that "terms such as attitude, value and appreciation refer to the affective objective of instruction" (p. 229). He further points out that the best definition can be found in an examination of the instrument used in a study. The most common instruments appear to be the Thurstone and Likert Scales and the Semantic Differential.

Knaupp (1973) observed that the lack of clarity in defining attitudes becomes a crucial problem in examining research done on student attitudes in mathematics. He notes the resulting problems as being the questionable validity of the measuring instrument, and the ineffective controls on extraneous variables in most classroom experiments.

Poffenberger and Norton (1956) suggested that the formation of attitudes towards mathematics is a complex one and caused by one experience building on another. This consideration must be foremost in any conclusions drawn from the literature.

Stright (1956) in a study of 1,023 students in grades 4, 5, and 6 reported that 80% of the students really liked mathematics. However, he questioned whether the self-reported responses reflected "true" feelings or "expected" feelings.

Fehon (1958), in her study of primary children, reported that children are already forming opinions about mathematics by grade three.

Dutton and Blum (1968) constructed and used a Likert-type scale to study the attitudes of 346 elementary school children who had at least one year of new math. They found that about 30% of the pupils had favorable attitudes towards the new math, 53% were neutral, and 17% disliked the subject a great deal. The study also confirmed findings of other studies that younger children have more positive attitudes than do older children when using new math. The children were found to have ambivalent feelings towards mathematics.

Neale and Proshek (1967) found that a variety of school-related attitudes became less positive as grades in school increased.

Ryan (cited in Neale, 1969) measured the attitudes of ninth-grade students at the beginning and end of the school year. He found a significant decline in mathematics attitudes.

Antonnen (cited in Neale, 1969) measured the attitudes of the same children over a six year period. He found that the mean attitude scores had declined a full standard deviation in that period.

Begle (1973), in a summary of two longitudinal studies conducted by SMSG, stated that:

> "Student attitudes towards mathematics seem to be favorable at the beginning of the fourth grade and improve slightly during the remainder of elementary school. However, at the beginning of junior high school, student attitudes towards mathematics begin a slow but steady drop that continues to the end of high school (pp. 212-13).

Investigations by Osborn (1965) and Woodall (1966) noted that these attitude changes were not affected by the nature of the curriculum to which the students had been exposed.

Lyda and Morse (1963), in a study of fourth grade students using the Dutton Arithmetic Attitude Scale, synthesized from their data that when meaningful methods of teaching arithmetic are used, changes in attitudes toward arithmetic take place. Negative attitudes become positive and the intensity of positive attitudes becomes enhanced.

Summary. Research on attitudes in mathematics show that attitudes are formed early and stablize as the child matures. Attitudes are generally favorable at an early age but begin to show a slow decline at the beginning of junior high and steadily get worse through to the end of high school. There is little research to show that this is curriculum based or can be improved by divergent methods of instruction. Further research is greatly needed in this aspect of mathematics attitudes.

## IV. THE ATTITUDE-ACHIEVEMENT RELATIONSHIP

A review of research on attitudes towards mathematics reveals a steady decline of attitudes after junior high school. Many educators view student attitudes as a determinant of mathematics achievement and thus, the effect of the steady decline in attitudes in relation to achievement makes necessary an investigation of research findings.

Ryan (cited in Neale, 1969), using grade nine students, and Husen (cited in Neale, 1969), using grade eight and grade twelve students, confirm this non-significant relationship. All concluded that there was no consistent relationship between attitudes and mathematics achievement.

Investigations of the relationship with groups within a grade have shown a few exceptions. Cristanello (cited in Neale, 1972) reported that there was a higher correlation between ability and mathematics achievement for the middle-attitude group than for the low or high-attitude group.

Shepps and Shepps (1971), using a sample of 26 sixth-grade students, reported a significant relationship between attitudes and achievement for girls.

Aiken (1969) and Fennema (1974) both support the significant relationship for girls. However, they note that at the university level the opposite is true.

Neale (1969), in reviewing research on the relationship of attitudes and achievement, argues that until new attitudes measures or new data show the contrary, it must be accepted that attitudes play a limited role in school achievement. Begle (1973) draws the same conclusion from the data collected by SMSG. He concluded that student attitudes towards mathematics was not a good predictor of achievement. The best predictor was previous mathematics achievement.

Aiken (1969) concludes that there is a need for longitudinal and "one-shot" studies to provide further information and to investigate the effect of variables such as teacher, parent, special abilities and curriculum on the relationship between attitudes and achievement.

Summary. Research on the relationship between attitudes and achievement in mathematics have shown very little evidence of any predictive relationship between the two.

More research in this area is needed with consideration of interacting variables such as parents, teachers, special abilities and the curriculum.

## V. SUMMARY

Studies by Crosby et al. (1960), Gold (1965), Rainey (1965) and Kline (1971) have shown little experimental evidence for utilization of student-directed over teacher-directed learning. However, studies by Edwards et al. (1968) have shown that a multi-media approach to teaching adults outperforms the traditional teacher-directed approach.

Research on student attitudes in mathematics have shown that a steady decline occurs at the junior high school level and this continues through high school. However, there is little evidence to show any effect of this decline on student achievement. As Neale (1969) and Begle (1973) point out, student attitudes cannot be considered a predictor of mathematics achievement. Much more experimental evidence is needed in this area giving consideration to a variety of possible interacting variables.

The purpose of the present study was to investigate the effect on attitudes and achievement and the attitude-achievement relationship in student-directed and teacher-directed instruction with and without non-print media in order to determine experimental evidence concerning mathematics attitudes, achievement and their relationship in varying instructional situations.

## CHAPTER III

## THE EXPERIMENTAL DESIGN

This chapter contains an explanation of the design that was employed and a description of the materials that were used in the study. A description of the test instruments that were used and the testing procedures employed are also included.

### I. SAMPLE AND PROCEDURES

The population consisted of the grade seven enrolment of an elementary school in the Burin Peninsula Integrated School Board System. This comprised a total of 118 students. The students were made available following a request to the Burin Peninsula Integrated School Board.

The total population was randomly assigned to one of four groups by assigning a number to each subject and using a table of random numbers (Glass & Stanley, 1970). Each of the four groups were then assigned to a method of instruction to be analyzed in this study. The students were then given an attitudes and achievement pretest. Table I shows a breakdown of the sample with respect to Method of Instruction and sex of students.

One group, consisting of 13 boys and 16 girls, was

TABLE I

GROUP, TREATMENT, AND SEX OF STUDENTS

| GROUP | SEX Boys | Girls | TREATMENT |
|---|---|---|---|
| Student Directed Mediated | 12 | 13 | Slides, audio tapes, workbook |
| Student Directed | 11 | 14 | Text handbook, workbook |
| Student Directed Mediated | 10 | 15 | Teacher instruction, slides, workbook |
| Teacher Directed | 15 | 10 | Teacher instruction, workbook |

assigned to the Student-Directed Mediated (SDM) method of instruction. Ten sets of slides and accompanying audio tapes were provided to the group along with individual workbooks. The students were advised to work alone or in groups and to use the slides and tapes to complete the workbook. The teacher present was given answer sheets which were provided to the students at the completion of each lesson. As each student completed the workbook the posttests for attitudes and achievement was administered.

A second group, consisting of 12 boys and 18 girls, was assigned to the Student-Directed (SD) method of instruction. Individual handbooks were given to each student containing the written text of what appeared in the slide-tape presentation used with the first group. The individual workbook was also give to each student. The students were advised to work alone or in groups and to read the handbook in order to obtain the necessary instruction to complete the workbook. The teacher present was provided with answer sheets which were provided to the students at the completion of each lesson. As each student completed the workbook the posttests for attitudes and achievement were administered.

The third group, consisting of 12 boys and 18 girls, was assigned to the Teacher-Directed Mediated (TDM) method of instruction. A teacher was provided with a set of slides and a written text to accompany the slides. The teacher integrated the slides with the lectures used in instructing the students in order that they be able to complete the individual workbooks provided. The teacher controlled the rate of movement of completion of the workbook and was responsible for providing instruction and correction. At the end of the program (18 forty-five minute periods) the students were administered the attitudes and achievement posttest.

The fourth group, consisting of 17 boys and 12 girls, was assigned to the Teacher-Directed (TD) method of instruction. The students were each given the individual workbook. The same teacher as in group three was provided with the workbook and a written text to aid in preparing instruction. The teacher was responsible for providing instruction and correction to the students in completion of the workbook. At the end of the program (18 forty-five minute periods) the students were administered the attitudes and achievement posttests. This group was set up to parallel the traditional teaching situation.

Using the table of random numbers (Glass & Stanley, 1970) and the previously assigned student numbers a sample of twenty-five students from each group was selected for analyses purposes.

## II. INSTRUMENTATION

In this section each of the instruments used to test the hypotheses stated in Chapter I is described.

### Mathematics Achievement Test

In this study a pretest and posttest to determine achievement was used. These tests were parallel forms designed to test attainment of objectives for which the methods of instruction were designed. These objectives were to be met by the methods of instruction and completion of the student workbook based on The Set of Integers (School Mathematics Study Group, 1970).

The pretest and posttest were designed around the pretest and Chapter test provided with the text, The Set of Integers, (SMSG, 1970) and covered questions ranging from introducing the Set of Integers through to number line subtraction with integers. Since both pretest and posttest were parallel forms designed to test the same objectives, the posttest only was administered to a sample of 25 grade eight students in February, 1976. A careful analysis of the test items provided necessary information for some revision in order that the objectives of the program would be tested. The conservative estimate of the reliability as given by the Kuder Richardson Formula 21 was 0.783. The revised forms of the pretest and posttest appears in Appendix A.

The Mathematics Achievement Tests consisted of nine items with several sub-items for each question. The items included questions about (a) integer values, (b) integer values using the number line, (c) integer opposites using the number line, (d) integer opposites as a function and graphing of the function, (e) solving integer addition and subtraction problems using the number line, and (f) Zero, the special integer. The inclusion of these items was consistent with the objectives and with the student handbook, The Set of Integers (SMSG, 1970, Revised). Table II shows the item numbers corresponding with the concept developed to meet each objective.

The achievement pretest was administered to all students after they had been randomly assigned to groups. The achievement posttest was administered to the students of the Student-Directed groups on an individual basis when the student completed the integer program. The Teacher-Directed groups were administered the posttest at the end of the integer program, determined by the teacher, which approximated 18 forty-five minute periods of class time. No time limit was set on completion of the test. Each student was given ample time to complete his responses to the questions.

TABLE II

CLASSIFICATION BY OBJECTIVE TASKS OF ITEMS ON THE MATHEMATICS ACHIEVEMENT TESTS

| OBJECTIVE TASK | ITEM NUMBERS | TOTAL VALUE OF ITEMS |
|---|---|---|
| Integer Value (P - N, P - P, N - N, N - 0) | 1. a,b,c,d | 4 |
| Integer Value Using Number Line | 2. a,b,c,d,e,f,g | 7 |
| Integer Opposites Using Number Line | 3. a,b,c,d | 4 |
| Integer Opposites as Function | 4. a | 5 |
| Graphing Integer Opposites | 4. b | 2 |
| Integer Addition & Subtraction Using the Number Line (P - P, P - N, N - P, N - N) | 5. a,b,c,d,e,f,g | 14 |
| Concept of Addition & Subtraction of Integers and Zero | 6. 7, 8, 9 | 4 |

* P = Positive, O = Zero, N = Negative

** Values were assigned by the tasks involved in answering questions

1 task answer = 1
2 task answer = 2

Attitude Scale

The Connelly Taxonomized Attitude Scale (Objective II) dealing with general student attitudes towards Mathematics was used as a pretest and posttest in this study. This attitude scale consisted of sixteen items of which six were Negatively stated and ten were Positively stated. The students were asked to check one of strongly agree, agree, no opinion, disagree or strongly disagree, depending on their reaction to the statements presented. The students were presented with a copy of the statements and a scoring sheet containing the choices. The values of 5 - strongly agree, to 1 - strongly disagree, were assigned to the positively stated items. The scale was reversed for the negatively stated items. Thus, the maximum negative attitude score was 16. A total score of 48 would indicate no overall opinion.

The items on the scale were classified into the four affective taxonomic levels of receiving, responding, valuing and organization. Table III gives a more complete breakdown of the item taxononomy levels and positive or negative statement of the item.

The attitude scale was administered as a pretest to all students after they were randomly assigned to groups. The attitude scale was administered as a post-test to all students directly before they were to complete

TABLE III

CLASSIFICATION OF ATTITUDE ITEMS BY TAXONOMIC LEVEL

| Taxonomic Level | Item Numbers | Total Number of Items |
|---|---|---|
| Receiving | *4, 6, 10, 14 | 4 |
| Responding | 3, 5, 9, 13 | 4 |
| Valuing | 2, 8, 12, 16 | 4 |
| Organization | 1, 7, 11, 15 | 4 |

* Underlined items are negatively stated

the achievement posttest.

The alpha reliability coefficient for the Connelly Taxonomized Attitude Scale (Objective II) is 0.87. The attitude scale appears in Appendix B.

## III. MATERIALS AND INSTRUCTION PROCEDURES

The major purposes of this study were to examine the role of media and amount of direction in instruction and its effect on students' attitudes and achievement.

An instructional kit of slides and accompanying audio tapes as well as a written student handbook was produced in order that the individual student or groups of students be able to receive instruction without teacher involvement. For one group space was provided in a classroom and the resource centre of the school for private and group reviewing of the slides and tapes.

The instructional handbook which covered the same material as the slides and tapes was given to the second group of students for their personal use. Directives were provided advising students when to complete exercises in the student workbook provided to all students.

The same teacher was present with both student-directed groups but worked as a resource person and not as a fountain of facts. The student controlled the pace of instruction and was tested when he considered he had met the objectives of the program about Integers.

Two other groups of students were provided with teacher directed instruction. Both groups were provided with the student workbook. The teacher was provided with a set of slides and a written text used by the student-directed groups. However, the teacher was instructed to and used the text to prepare an introduction to the topics to be completed which would parallel that received by the other group. One group received the teacher instruction complemented by the prepared slides while the other group received only the teacher introduction of topics (conventional teaching). The teacher assigned the work to be completed and the checking of answers was a group activity. The teacher controlled the pace of instruction and the students were tested as a group at the end of the instructional period.

Material was prepared for approximately eighteen, forty-five minute class periods or twelve days of Mathematics classes. Appendix C contains the materials provided for this study.

## IV. PILOT STUDY

A pilot study of the achievement test, involving 25 students who were considered to have attained the objectives of the instruction, was conducted in February, 1976. The purposes of this pilot study were to examine the following:

1. The wording of the question to ensure that students understood the tasks required.

2. The length of the test. It was important for administration of the tests to determine a suitable time period required for completion of the test.

3. To determine if the questions suitably tested the attainment of the objectives they were designed to test.

On the basis of this pilot study, the final modifications in the test and test procedures were made.

## V. ANALYSIS

An item analysis was carried out on the mathematics achievement test. The internal consistency of the test was assessed by using the conservative Kuder-Richardson Formula 21.

The effects on students' attitudes and achievement of student-directed mediated, student-directed, teacher-directed mediated, and teacher-directed methods of instruction were analyzed using analysis of covariance with the pretest used as covariate. The program ANCV 10 documented by the Division of Educational Research (1969) of the University of Alberta was used for this analyses utilizing the IBM 360 computer facilities of Memorial University.

# CHAPTER IV

## RESULTS OF THE INVESTIGATION

This chapter reports the results of testing the hypotheses. The analyses reported for the four treatments groups involved in the investigation were carried out using the program ANCV 10 developed by the Division of Educational Research at the University of Alberta. The Pearson produce moment correlation formula was used to analyse any relationship between mathematics attitudes and achievement. The IBM 360 computer of Memorial University was utilized for the analyses.

### I. ACHIEVEMENT DATA ANALYSES

The mathematics achievement pretest was administered to the total population and after completion of the treatments the mathematics achievement test was administered as a posttest. The mathematics pretest and posttest were parallel forms designed to measure attainment of the same objectives. From the population, a sample of 25 students from each of the four treatments was randomly selected for analyses purposes. The pretest was used as a covariate in the analyses in order to isolate any interference of previous learning from final achievement. Thus, a residual achievement scores was determined with a possible range from 0 to 40.

Table IV lists the pretest, posttest, and adjusted achievement mean scores of the four treatment groups.

An analysis of variance and an adjusted analysis of variance of the achievement scores for the treatment groups were carried out. Table V shows the analysis of variance of the posttest achievement scores and the probability level. A probability level of .05 or better was considered significant. The F-ratio (1.33) showed no significant difference between the posttest scores at the levels considered.

TABLE V

Analysis of Variance of Posttest Achievement Scores for Treatment Groups

| Source | df | Mean Square | F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 3 | 55.54 | 1.33 | 0.27 |
| Within | 96 | 41.88 | | |

The adjusted analyses of variance, using the pretest as the covariate to give residual achievement scores, was carried out. The adjusted F-ratio (0.74) showed no significant difference between the treatment groups for achievement. Table VI shows the Adjusted Analysis of

TABLE IV

Pretest, Posttest, and Adjusted Achievement Mean Scores for each Treatment Group

| Treatment | Mean Scores Pretest | Posttest | Adjusted |
|---|---|---|---|
| Student-directed Mediated | 15.80 | 34.88 | 32.53 |
| Student-directed | 11.16 | 32.00 | 31.97 |
| Teacher-directed Mediated | 6.12 | 31.72 | 34.21 |
| Teacher-directed | 11.32 | 33.72 | 33.60 |

variance of the achievement data and the probability level.

TABLE VI

Adjusted Analysis of Variance of Achievement Scores

| Source | df | Mean Square | Adjusted F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 3 | 23.23 | 0.74 | 0.53 |
| Within | 95 | 31.36 | | |

An analysis of the mathematics achievement scores of the student-directed and teacher-directed instructional groups was carried out to determine if overall instructional control would provide any difference in achievement.

Table VII shows the pretest, posttest and adjusted achievement mean scores for the student-directed and teacher-directed groups.

An analysis of variance of the posttest scores derived a F-ratio of 0.30 which was not significant. Table VIII shows the analysis of variance for the student-directed and teacher-directed posttest achievement scores.

TABLE VII

Combined Pretest, Posttest, and Adjusted Achievement Mean Scores for Student-Directed and Teacher-Directed Instructional Groups

| Treatment | Mean Scores Pretest | Posttest | Adjusted |
|---|---|---|---|
| Student-directed | 13.48 | 33.44 | 32.25 |
| Teacher-directed | 8.72 | 32.72 | 33.91 |

TABLE VIII

Analysis of Variance of Student-Directed and and Teacher-Directed Posttest Achievement Scores

| Source | df | Mean Square | F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 1 | 12.94 | 0.30 | 0.58 |
| Within | 98 | 42.60 | | |

An adjusted analysis of variance for the student-directed and teacher-directed achievement scores yielded a F-ratio of 1.99 which was not significant. Table IX shows the adjusted analysis of variance for student-directed and teacher-directed achievement scores.

TABLE IX

Adjusted Analysis of Variance for Student-Directed and Teacher-Directed Achievement Scores

| Source | df | Mean Square | Adjusted F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 1 | 61.27 | 1.99 | 0.16 |
| Within | 97 | | | |

An analysis of the mathematics achievement scores of the mediated and non-mediated instructional groups was carried out in order to determine if mediation would prvide any difference in achievement. Table X shows the

TABLE X

Combined Pretest, Posttest, and Adjusted Achievement

Mean Scores for the Mediated and Non-Mediated Instructional Groups

| Treatment | Mean Scores Pretest | Posttest | Adjusted |
|---|---|---|---|
| Mediated | 10.96 | 33.30 | 33.36 |
| Non-Mediated | 11.24 | 32.86 | 32.80 |

pretest, posttest and adjusted achievement mean scores for the mediated and non-mediated groups.

An analysis of variance of the achievement posttest scores of the mediated and non-mediated groups yielded a F-ratio of 0.11 which was not significant. Table XI shows the analysis of variance of the achievement posttest scores for the mediated and non-mediated groups.

TABLE XI

Analysis of Variance of the Achievement Posttest Scores for the Mediated and Non-Mediated Groups

| Source | df | Mean Square | F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 1 | 4.81 | 0.11 | 0.74 |
| Within | 98 | 42.68 | | |

An adjusted analysis of variance for the achievement scores of the mediated and non-mediated group yielded a F-ratio of 0.26 which was not significant at the accepted level. Table XII shows the adjusted analysis of variance of the achievement scores for the mediated and non-mediated groups.

TABLE XII

Adjusted Analysis of Variance of the Achievement Scores for the Mediated and Non-Mediated Groups

| Source | df | Mean Square | Adjusted F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 1 | 8.08 | 0.26 | 0.61 |
| Within | 97 | | | |

<u>Hypothesis One</u>

There is no significant difference between the mean scores of student achievement in mathematics in student-directed mediated learning; student-directed learning; teacher-directed mediated learning; and teacher-directed learning.

<u>Results</u>. The adjusted analyses of variance using the pretest as a covariate to produce a residual achievement score yielded a F-ratio of 0.74 in a comparison of the four treatments. In comparing the student-directed and teacher-directed groups an adjusted F-ratio of 1.99 was derived. A comparison of mediated and non-mediated groups on achievement yielded an adjusted F-ratio of 0.11. The F-ratio proved not significant at the 0.05 level or better.

<u>Conclusion</u>. On the basis of these results,

Hypothesis One was accepted. There was no significant difference between the mean scores of mathematics achievement for any of the treatment groups studied.

The results showed that the achievement of the groups was not significantly different regardless of the assigned treatment.

## II. ATTITUDE DATA ANALYSES

The Connelly Taxonomized Attitude Scale (Objective II) was administered to the total population as a pretest and after completion of the treatments as a posttest. A sample of 25 from each treatment group was randomly selected for analyses purposes. The pretest was used as a covariate in the analysis in order to produce a residual attitude score.

The possible range of the attitude scale was from 16 to 80. Table XIII lists the pretest, posttest and adjusted mean scores of attitudes for the four treatment groups.

An analysis of variance of the posttest attitude scores for the treatment groups was carried out. The porbability level was set at .05 or better for significance. The analysis of variance yielded a F-ratio of 0.54 which showed no significant difference between the mean posttest

TABLE XIII

Pretest, Posttest, and Adjusted Mean Attitude Scores for the Treatment Groups

| Treatment | Mean Score Pretest | Posttest | Adjusted |
|---|---|---|---|
| Student-directed Mediated | 61.04 | 62.12 | 60.89 |
| Student-directed | 57.80 | 59.88 | 60.28 |
| Teacher-directed Mediated | 57.88 | 62.76 | 63.12 |
| Teacher-directed | 57.68 | 60.28 | 60.74 |

scores of attitudes for the groups. Table XIV shows the analysis of variance for the posttest attitude scores and the probability level of the F-ratio.

TABLE XIV

Analysis of Variance of Posttest Attitude Scores for Treatment Groups

| Source | df | Mean Square | F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 3 | 48.77 | 0.54 | 0.66 |
| Within | 96 | 91.05 | | |

An adjusted analysis of variance using the pretest as a covariate yielded a F-ratio of 0.61 which showed no significant difference in the adjusted attitude scores for the treatment groups. Table XV shows the adjusted analysis of variance for the attitude data and the probability level of the F-ratio for the treatment groups.

TABLE XV

Adjusted Analysis of Variance of Attitude Scores for Treatment Groups

| Source | df | Mean Square | Adjusted F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 3 | 40.20 | 0.61 | 0.61 |
| Within | 95 | 66.05 | | |

An analysis of the mathematics attitude scores of the student-directed and teacher-directed instructional groups was carried out in order to determine if overall instructional control would provide any difference in attitudes. Table XVI shows the pretest, posttest, and adjusted attitude mean scores for the student-directed and teacher-directed groups.

An analysis of variance of the attitude posttest scores for the student-directed and teacher-directed groups yielded a F-ratio of 0.08 which was not significant. Table XVII shows the analysis of variance for the student-directed and teacher-directed attitude scores.

TABLE XVII

Analysis of Variance of Student-Directed and Teacher-Directed Posttest Attitude Scores

| Source | df | Mean Square | F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 1 | 6.81 | 0.08 | 0.79 |
| Within | 98 | 90.62 | | |

An adjusted analysis of variance for the student-directed and teacher-directed attitude scores yielded a F-ratio of 0.69 which was not significant. Table XVIII shows the adjusted analysis of variance of the attitude

TABLE XVI

Pretest, Posttest and Adjusted Attitude

Mean Scores for Student-Directed and Teacher-Directed Groups

| Treatment | Mean Scores Pretest | Posttest | Adjusted |
|---|---|---|---|
| Student-directed | 59.42 | 61.00 | 60.58 |
| Teacher-directed | 57.78 | 61.52 | 61.94 |

scores for the student-directed and teacher-directed groups.

TABLE XVIII

Adjusted Analysis of Variance of Attitude Scores for Student-Directed and Teacher-Directed Groups

| Source | df | Mean Square | Adjusted F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 1 | 45.34 | 0.69 | 0.41 |
| Within | 97 | 65.46 | | |

An analysis of the mathematics attitude scores of the mediated and non-mediated instructional groups was carried out in order to determine if mediation would provide any difference in attitudes. Table XIX shows the pretest, posttest, and adjusted attitude mean scores for the mediated and non-mediated groups.

An analysis of variance of the attitude posttest scores of the mediated and non-mediated groups yielded a F-ratio of 1.56 which was not significant at the accepted level. Table XX shows the analysis of variance of the attitude posttest scores for the mediated and non-mediated groups.

TABLE XIX

Combined Pretest, Posttest, and Adjusted Attitude Mean Scores for the Mediated and Non-Mediated Groups

| Treatment | Mean Scores Pretest | Mean Scores Posttest | Adjusted |
|---|---|---|---|
| Mediated | 62.44 | 62.44 | 62.04 |
| Non-Mediated | 57.74 | 60.08 | 60.51 |

TABLE XX

Analysis of Variance of the Attitude Posttest Scores for the Mediated and Non-Mediated Groups

| Source | df | Mean Square | F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 1 | 139.25 | 1.56 | 0.22 |
| Within | 98 | 89.27 | | |

An adjusted analysis of variance for the attitude scores of the mediated and non-mediated groups yielded a F-ratio of 0.86 which was not significant. Table XXI shows the adjusted analysis of variance of the attitude scores for the mediated and non-mediated groups.

TABLE XXI

Adjusted Analysis of Variance of the Attitude Scores for the Mediated and Non-Mediated Groups

| Source | df | Mean Square | Adjusted F-ratio | Probability Level |
|---|---|---|---|---|
| Group | 1 | 56.45 | 0.86 | 0.35 |
| Within | 97 | 65.35 | | |

<u>Hypothesis Two</u>

There is no significant difference between the mean scores of student attitudes in mathematics in student-directed mediated learning; student-directed learning; teacher-directed mediated learning; and teacher-directed learning.

Results. The adjusted analysis of variance using the pretest as a covariate to produce a residual attitude score yielded a F-ratio of 0.61 is a comparison of the four treatments. A comparison of the student-directed and teacher-directed groups yielded an adjusted F-ratio of 0.69. A comparison of the mediated and non-mediated groups on attitudes yielded an adjusted F-ratio of 0.86. The F-ratio proved not significant at the .05 level or better.

Conclusion. On the basis of these results, Hypothesis Two was accepted. There was no significant difference between the mean scores of mathemematics attitudes for any of the treatment groups.

The results showed that the attitude scores of all groups were not significantly different regardless of the assigned treatment.

## III. ATTITUDE-ACHIEVEMENT RELATIONSHIP DATA ANALYSIS

The mathematics achievement data derived from the pretest and posttest and the mathematics attitude data derived from the pretest and posttest administration of the Connelly Taxonomized Attitude Scale (Objective II) were correlated using the Pearson Product Moment Correlation Formula in order to determine any relationship between the attitudes and achievement of students in the treatment groups.

The pretest attitudes and achievement data was correlated in order to determine if any significant relationship existed prior to the treatment. Table XXII contains the correlations obtained on the pretest data for the four treatment groups.

TABLE XXII

Correlations Between Mathematics Attitudes and Achievement of Treatment Groups on Pretest

| Treatment | Pearson Correlation | Significance |
|---|---|---|
| Student-directed Mediated | 0.250 | NS |
| Student-directed | 0.097 | NS |
| Teacher-directed Mediated | 0.395 | NS |
| Teacher-directed | 0.559 | S* |

*Significantly related at the .01 level

To determine if any significant relationship existed between the combined student-directed and between the combined teacher-directed groups on the pretest, the attitude and achievement scores for these groups were correlated. Table XXIII contains the correlations obtained.

TABLE XXIII

Correlations Between Mathematics Attitudes and Achievement for the Combined Student-Directed and Teacher-Directed Groups on the Pretest

| Treatment | Pearson Correlation | Significance |
|---|---|---|
| Student-directed | 0.211 | NS |
| Teacher-directed | 0.425 | S* |

*Significantly related at the .01 level

To determine if any significant relationship existed between the combined mediated and between the combined non-mediated groups on the pretest, the attitude and achievement scores for these groups were correlated. Table XXIV contains the correlations obtained on the pretest data for the combined mediated and the combined non-mediated groups.

TABLE XXIV

Correlations Between Mathematics Attitudes and Achievement for the Combined Mediated and Non-Mediated Groups on the Pretest

| Treatment | Pearson Correlation | Significance |
|---|---|---|
| Mediated | 0.336 | S* |
| Non-Mediated | 0.288 | S* |

*Significantly related at the .05 level

To determine if any significant relationship between attitudes and achievement existed at the end of the instructional treatments, the posttest attitude and achievement scores were correlated. Table XXV contains the correlations obtained on the posttest data for the four treatment groups.

TABLE XXV

Correlations Between Mathematics Attitudes and Achievement of Treatment Groups on Posttest

| Treatment | Pearson Correlation | Signifinance |
|---|---|---|
| Student-directed Mediated | .552 | S** |
| Student-directed | .416 | S* |
| Teacher-directed Mediated | .385 | NS |
| Teacher-directed | .504 | S* |

*Significantly related at .05 level
**Significantly related at .01 level

To determine if any significant relationship existed between the combined student-directed and combined teacher-directed groups on the posttest, the attitudes and achievement scores were correlated.

Table XXVI contains the correlations obtained on the posttest data for the combined student-directed and combined teacher-directed groups.

TABLE XXVI

Correlations Between Mathematics Attitudes and Achievement for the Combined Student-Directed and Teacher-Directed Groups on the Posttest

| Treatment | Pearson Correlation | Significance |
|---|---|---|
| Student-directed | 0.490 | S* |
| Teacher-directed | 0.411 | S* |

*Significantly related at the .01 level

To determine if any significant relationship existed between the combined mediated and the combined non-mediated groups on the posttest, the attitude and achievement scores were correlated.

Table XXVII contains the correlations obtained on the posttest data for the combined mediated and combined non-mediated groups.

TABLE XXVII

Correlations Between Mathematics Attitudes and Achievement for the Combined Mediated and Non-Mediated Groups on the Posttest

| Treatment | Pearson Correlation | Significance |
|---|---|---|
| Mediated | 0.425 | S* |
| Non-Mediated | 0.461 | S* |

*Significantly related at the .01 level

Hypothesis Three

There is no significant relationship between mathematics attitudes and achievement in student-directed mediated learning; student-directed learning; teacher-directed mediated learning; and teacher-directed learning.

Results. The Pearson product moment correlation between mathematics attitudes and achievement was determined for the four instructional treatments and the combined treatment groups.

A significant relationship between the mathematics attitudes and achievement using the pretest scores was found for the teacher-directed group ($p \leq .01$); the combined teacher-directed group ($p \leq .01$); the combined mediated treatment groups ($p \leq .05$); and the combined non-mediated treatment groups ($p \leq .05$).

A significant relationship between the mathematics attitudes and achievement, using the posttest scores, was found for the student-directed mediated groups ($p \leq .01$); the student-directed group ($p \leq .05$); the teacher-directed group ($p \leq .05$); the combined student-directed treatment groups ($p \leq .01$); the combined teacher-directed treatment groups ($p \leq .01$); the combined mediated treatment groups ($p \leq .01$); and the combined non-mediated treatment groups ($p \leq .01$).

<u>Conclusion</u>. On the basis of the results, Hypothesis Three was accepted for the pretest achievement and attitude scores of the student-directed mediated group, the student-directed group, the teacher-directed mediated group and the combined student-directed groups; and on the posttest attitude and achievement scores for the teacher-directed mediated group. An analysis showed no significant relationship between the mathematics attitude and achievement scores for these groups.

Hypothesis Three was rejected for the attitude and achievement pretest scores of the teacher-directed group, the combined teacher-directed instruction groups, the combined mediated instruction groups, and the combined non-mediated instruction groups; and for the posttest scores of the student-directed mediated group, the student-directed group, the teacher-directed group, the combined student-directed instruction groups, the combined teacher-directed instruction groups, the combined mediated instruction groups, and the combined non-mediated instruction groups. An analysis of the scores for these groups showed a significant relationship between mathematics attitudes and achievement.

# IV. SUMMARY

This chapter contains the results and an analysis of the testing of the three hypotheses which were associated with the major purposes of the study as outlined in Chapter I.

The purpose of the study was to examine the effect on the mathematics attitudes, achievement, and attitude-achievement relationship of mediation in both student-directed and teacher-directed learning situations. This was studied by utilizing the four instructional treatments of student-directed with and without mediation, and teacher-directed with and without mediation.

It was found that none of the treatments produced significantly different achievement in, or attitude towards, mathematics. However, the results indicated that mathematics attitudes and achievement were significantly related for the teacher-directed group, combined teacher-directed instruction groups, combined mediated instruction groups and the combined non-mediated instruction groups when analyzing the pretest scores.

A significant relationship between mathematics attitudes and achievement using the posttest scores was found for the student-directed mediated group, the student-directed group, the teacher-directed group, the combined student-directed instruction groups, the combined teacher-directed groups, the combined mediated instruction groups and the combined non-mediated instruction groups. The attitude and achievement scores showed no significant relationship on the pretest or posttest for the teacher-directed mediated group.

Implications arising from these findings will be discussed in the next chapter.

# CHAPTER V

# SUMMARY, DISCUSSION AND IMPLICATIONS AND RECOMMENDATIONS

## I. SUMMARY OF THE INVESTIGATION

The present study was designed to investigate any effect of mediated (non-print) and non-mediated (print) instruction on mathematics attitudes, mathematics achievement, and the attitude-achievement relationship. The investigation was carried out using both student-directed and teacher-directed learning situations. Attention was focused on the achievement and attitudes of the students in the four instructional strategy groups and on an overall mediation and overall directedness effect.

In order to gather the necessary data a sample of grade seven students was randomly selected and several instruments were employed.

### Sample

The sample of 100 grade seven students was utilized for analyses purposes. The sample was drawn from the complete grade seven population of an elementary school within the Burin Peninsula Integrated School Board system. The population of 118 students had been randomly assigned to four treatment groups for the study and the sample of 100 students, 25 from each treatment group, were randomly selected for analyses purposes. It was assumed that these students were representative of grade seven mathematics students.

Instruments

A mathematics achievement pretest and posttest (Appendix A) was constructed for use in the present study. These tests were parallel forms consisting of nine items covering (a) integer values, (b) integer values using the number line, (c) integer opposites using the number line, (d) integer opposites as a function and graphing the function, (e) solving integer addition and subtraction using the number line and (f) Zero, the special integer. The pretest was administered to the population as a group prior to the treatment and at the end of the treatment the posttest was administered. The KR-21 conservative estimate of reliability was 0.783.

The Connelly Taxonomized Attitude Scale (Objective II) was administered as a pretest and posttest to the total population. This instrument measured general attitudes towards mathematics. The alpha reliability coefficient was given as 0.87.

The pretests were used as covariates in the analyses. Most of the analysis of the data was done by using a computer program supplied by the Division of Educational Research Services, Faculty of Education, at the University of Alberta. The Pearson correlation was used to analyze the relationship between attitudes and achievement.

## Conclusions

A summary of the findings will be presented on the basis of testing the hypotheses.

It was found that no significant difference existed between the mathematics achievement scores of students in student-directed mediated learning, student-directed learning, teacher-directed mediated learning or teacher-directed learning. An analysis of the overall student-directed and teacher-directed strategies and overall mediated and non-mediated strategies produced no significant difference in achievement.

No significant difference was found to exist between mathematics attitudes of students in student-directed mediated learning, student-directed learning, teacher-directed mediated learning and teacher-directed learning. An overall analysis of the student-directed and teacher-directed strategies and the mediated and non-mediated strategies produced no significant difference in attitudes.

On the basis of these findings, Hypothesis Two was accepted.

A significant relationship between attitudes and achievement in mathematics was found on the pretest scores for the teacher-directed group, overall teacher-directed strategy, overall mediated strategy and overall non-mediated strategy. Using the posttest scores, a significant relationship between mathematics attitudes and achievement was found for the student-directed mediated, student-directed, and teacher-directed groups, overall student-directed and teacher-directed strategies, and overall mediated and non-mediated strategies.

On the basis of these findings, Hypothesis Three was rejected for these groups and strategies.

## II. DISCUSSION AND IMPLICATIONS OF THE FINDINGS

The findings indicated that mathematics attitude and achievement scores were not significantly different for any of the instructional treatments studied.

It is evident from the findings that no instructional technique was any more successful than another. The utilization of a purely student-directed strategy does not clearly provide any optimum mode of instruction. Nor, will the use of print and non-print media to provide or supplement instruction meet this goal. As the literature indicates, teacher-directed instruction will be as successful in attaining instructional objectives as the other instructional strategies. However, the integration of the other techniques with teacher instruction may provide added stimulus situations to promote the attainment of an instructional objective without any significant interference with student achievement.

The importance of this finding must be considered in the light of classroom decision making. The matching of curriculum content with instructional strategy is a very important aspect in the total learning process.

The attitude scores for all groups were not significantly different. This result must be interpreted in light of the limitation of the short time duration of the study. The literature shows that attitudes are formed at about the grade four level and stablize as the child matures. The lack of a significant difference between the attitude scores of the groups may be attributed to this delimiting factor or to the students equally liking the treatments to which they were assigned.

The finding of a significant relationship between mathematics attitude and achievement scores for the teacher-directed group, overall teacher-directed strategy, and overall mediated and non-mediated strategies on the pretest scores; and for these same groups as well as the student-directed mediated and student-directed groups, and overall student-directed strategy on the posttest scores must be interpreted in light of the literature on the attitude-achievement relationship.

A significant correlational relationship between mathematics attitudes and mathematics achievement for the stated groups and strategies in this study must be noted. However, Jackson (cited in Neale, 1970) notes that this may be explained in two ways:

> (1) by the fact that students who achieve well receive more rewards than those who achieve poorly and as a consequence develop more favorable attitudes, or
>
> (2) by the fact that favorable attitudes in some way cause students to learn more. (p. 232).

Neale, (1969) and Aiken (1970) hypothesize that rather than good attitudes causing students to learn, it might be that learning causes favorable attitudes.

Any interpretation of these findings of this investigation must consider these limiting factors.

## III. RECOMMENDATIONS FOR FURTHER RESEARCH

This study analyzed mathematics attitudes and achievement scores for a sample of grade seven students randomly assigned to four treatment groups and results were completed on the basis of the groupings. A study of any effects of the treatments on high, medium and low achievers in mathematics would provide a more detailed basis for using these instructional procedures in mathematics instruction.

The time duration of this study was a limitation when studying attitudes towards mathematics. A second recommendation is that further materials be developed in order to greatly extend the time duration to provide a better basis for interpretating data on mathematics attitudes.

A third recommendation is that the relationship between attitudes and achievement in mathematics be further studied in varied instructional procedures to determine if any relationship exists using varied instructional procedures.

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# APPENDIX A

## MATHEMATICS ACHIEVEMENT PRETEST AND POSTTEST

## Pre-Test Exercises

These exercises are like the problems you will have on the chapter test. If you don't know how to do them, read the section again. If you still don't understand, ask your teacher.

1. Fill the blanks with $<$ or $>$ to show which integer of each pair is smaller.

(a) $0$ ____ $^-9$ (c) $^-5$ ____ $^-7$

(b) $^-2$ ____ $1$ (d) $12$ ____ $^-12$

2. On the number line below several points are named with letters. Imagine that an integer goes with each of these points. Make a list starting with the letter that goes with the smallest integer and ending with the letter that goes with the largest integer.


____ (smallest)

____

____

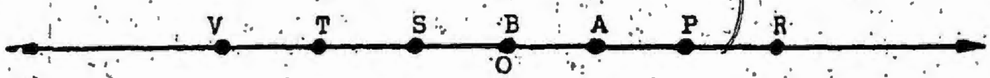
____

____

____

____ (largest)

3. On this number line letters are used to stand for positive and negative integers. Zero is shown on the line. In the blank beside each letter below write the letter that stands for its opposite.




(a) A __________

(b) V __________

(c) P __________

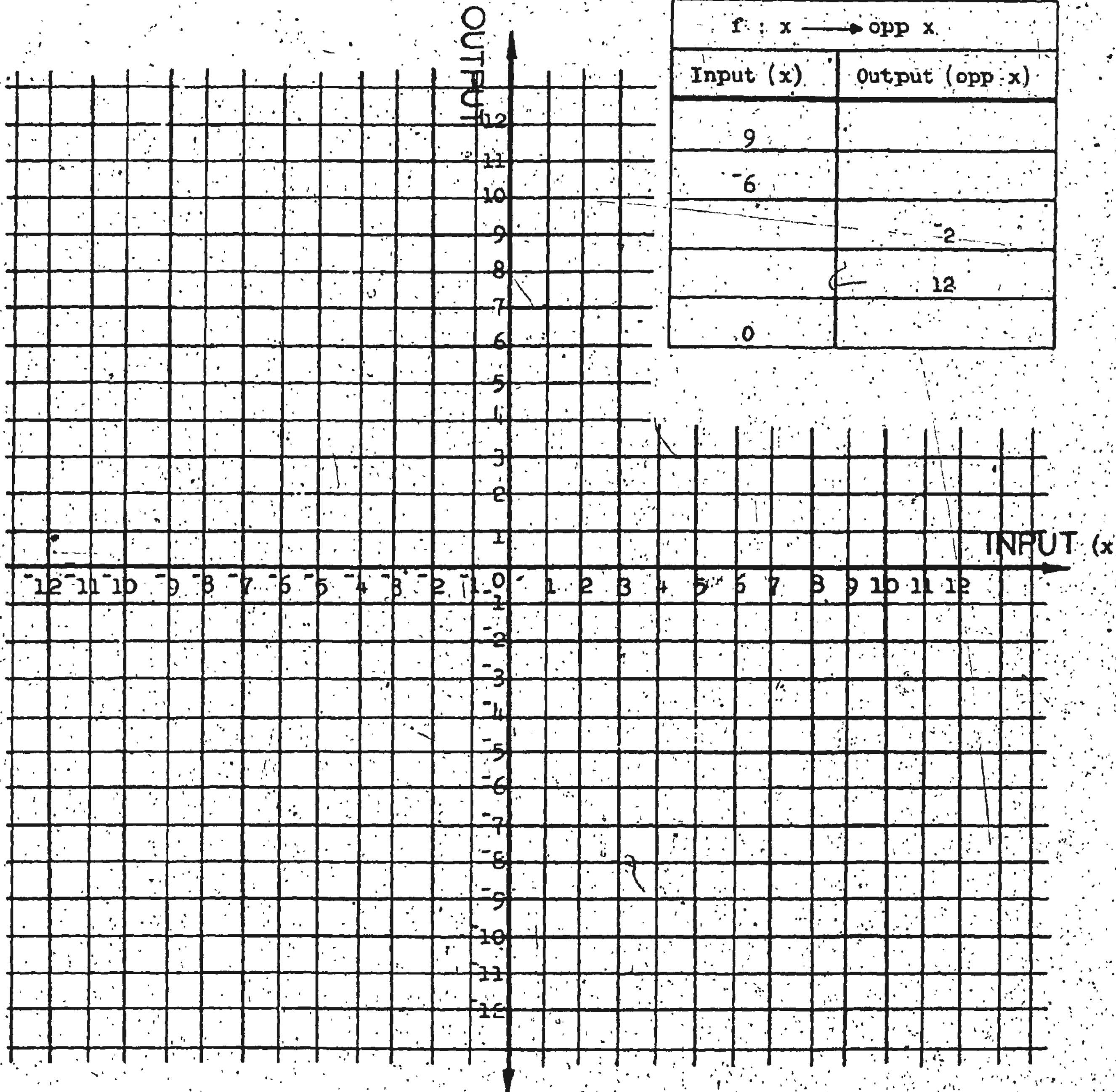
(d) B __________

4.

(a) Finish the table below for the function $f: x \longrightarrow \text{opp } x$.

(b) Use your table to graph the function on the coordinate plane below.

| $f: x \longrightarrow \text{opp } x$ | |
|---|---|
| Input (x) | Output (opp x) |
| 9 | |
| $^{-}6$ | |
| | $^{-}2$ |
| | 12 |
| 0 | |



5. Draw arrows on the following number lines that show these addition and subtraction problems. Write the answer to the problem in the blank. (Use the line below the problem to rewrite it if you need to!)

(a) 3 + 5 = ______

______________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(b) 3 + opp 5 = ______

______________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(c) 4 - 7 = ______

______________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(d) opp 2 + opp 3 = ______

______________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(e) opp 5 - opp 6 = ______

______________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(f) opp 7 + 10 = ______

______________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(g) 2 − opp 4 = ________

________________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

______ 6. If two integers are added together and their sum is zero, then the integers are:

(a) both greater than zero
(b) both negative
(c) opposites of each other
(d) different names of the same integer
(e) none of these

______ 7. If a positive integer is added to a negative integer the answer is always:

(a) positive
(b) zero
(c) negative
(d) larger than either of the integers
(e) none of the above

______ 8. If a negative integer is added to a negative integer the answer is always:

(a) positive
(b) zero
(c) negative
(d) larger than either of the integers
(e) none of these

______ 9. Two integers are added together and their sum is the same as one of them. The other integer has to be:

(a) positive
(b) zero
(c) negative
(d) opposite
(e) none of these

TEST

1. Fill the blanks with < or > to show which integer of each pair is smaller.

(a) 5 _____ 8 (c) $^-4$ _____ 0

(b) $^-3$ _____ $^-7$ (d) 2 _____ $^-15$

2. On the number line below several points are named with letters. Imagine that an integer goes with each of these points. Make a list starting with the letter that goes with the smallest integer and ending with the letter that goes with the largest integer.


______ (smallest)

______

______

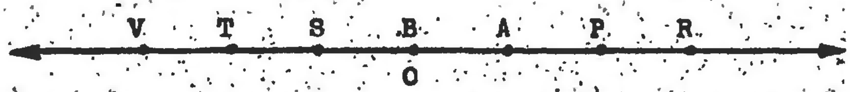
______

______

______

______ (largest)

3. On this number line letters are used to stand for positive and negative integers. Zero is shown on the line. In the blank beside each letter below write the letter that stands for its opposite.




(a) A __________

(b) V __________

(c) P __________

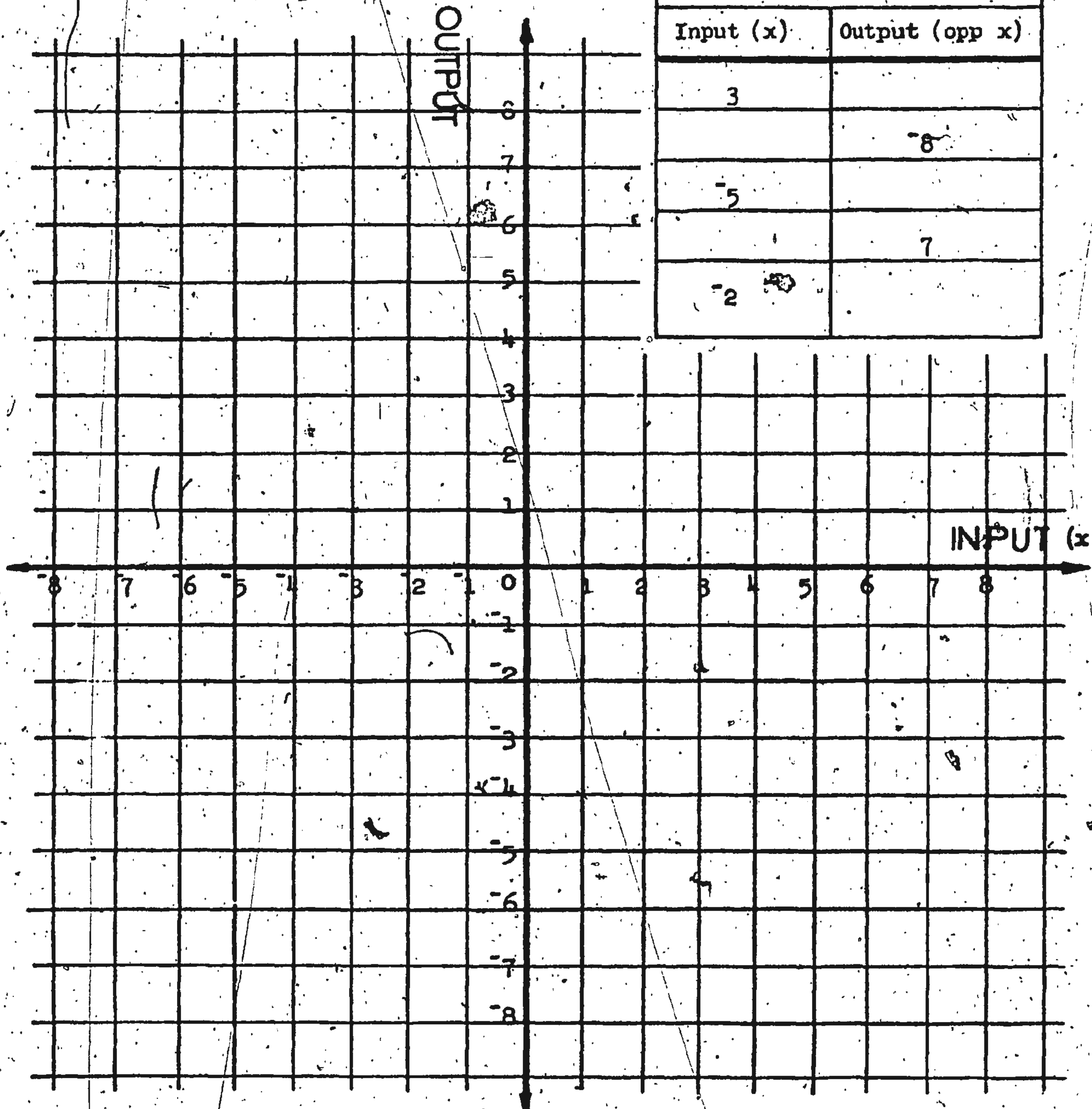
(d) B __________

4.

(a) Finish the table below for the function $f: x \longrightarrow \text{opp } x$.

(b) Use your table to graph the function on the coordinate plane below.

| $f : x \longrightarrow$ opp x | |
|---|---|
| Input (x) | Output (opp x) |
| 3 | |
| | ⁻8 |
| ⁻5 | |
| | 7 |
| ⁻2 | |



5. Draw arrows on the following number lines that show these addition and subtraction problems. Write the answer to the problem in the blank.

(a) $3 + 5 =$ ________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(b) $3 + \text{opp } 5 =$ ________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(c) $4 - 7 =$ ________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(d) $\text{opp } 3 - \text{opp } 8 =$ ________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(e) $6 - \text{opp } 2 =$ ________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(f) $\text{opp } 4 + 9 =$ ________

$^{-}10 \quad ^{-}9 \quad ^{-}8 \quad ^{-}7 \quad ^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$

(g) opp 1 + opp 3 = ______________

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8 9 10

______ 6. If two integers are added together and their sum is zero then the integers are:

(a) both greater than zero
(b) both negative
(c) opposites of each other
(d) different names of the same integer
(e) none of these

______ 7. If a positive integer is added to a negative integer the answer is always:

(a) positive
(b) zero
(c) negative
(d) larger than either of the integers
(e) none of the above

______ 8. If a negative integer is added to a negative integer the answer is always:

(a) positive
(b) zero
(c) negative
(d) larger than either of the integers
(e) none of these

______ 9. Two integers are added together and their sum is the same as one of them. The other integer has to be:

(a) positive
(b) zero
(c) negative
(d) opposite
(e) none of these

# APPENDIX B

## CONNELLY TAXONOMIZED ATTITUDE SCALE

## (OBJECTIVE II)

Directions: On your Answer Sheet, please grid your name. This is not a test and will not be used in any way to produce a grade for you. Your responses will be kept confidential. The items on this instrument are statements about mathematics. For each item select a response which best describes your impression on the statement. The response choices are:

A – Strongly agree
B – Agree
C – No Opinion
D – Disagree
E – Strongly disagree

1. I have nothing but contempt for mathematics.

2. I regard mathematics as a lasting tribute to man's ignorance.

3. I feel under a great strain in a mathematics class.

4. Mathematics makes me feel as though I'm lost in a jungle.

5. Mathematics makes me feel uncomfortable.

6. Mathematics is mainly pencil pushing.

7. The very existence of humanity depends on mathematics.

8. Mathematics may be compared to a great tree, ever putting forth new branches.

9. Mathematics is a subject which I have enjoyed studying in school.

10. My general attitude toward mathematics is favourable.

11. I feel mathematics is the greatest means for increasing the world's knowledge.

12. Mathematics is stimulating to me.

13. Working with various mathematical topics is fun.

14. I see nothing wrong with learning a variety of mathematical topics.

15. I feel mathematics helps make other subjects easier to understand.

16. Mathematics fascinates me.

# ANSWER SHEET

NAME ______________________

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1. | ____ | ____ | ____ | ____ | ____ |
| 2. | ____ | ____ | ____ | ____ | ____ |
| 3. | ____ | ____ | ____ | ____ | ____ |
| 4. | ____ | ____ | ____ | ____ | ____ |
| 5. | ____ | ____ | ____ | ____ | ____ |
| 6. | ____ | ____ | ____ | ____ | ____ |
| 7. | ____ | ____ | ____ | ____ | ____ |
| 8. | ____ | ____ | ____ | ____ | ____ |
| 9. | ____ | ____ | ____ | ____ | ____ |
| 10. | ____ | ____ | ____ | ____ | ____ |
| 11. | ____ | ____ | ____ | ____ | ____ |
| 12. | ____ | ____ | ____ | ____ | ____ |
| 13. | ____ | ____ | ____ | ____ | ____ |
| 14. | ____ | ____ | ____ | ____ | ____ |
| 15. | ____ | ____ | ____ | ____ | ____ |
| 16. | ____ | ____ | ____ | ____ | ____ |

A - Strongly Agree
B - Agree
C - No Opinion
D - Disagree
E - Strongly Disagree

# A P P E N D I X C

## WORKBOOK MATERIALS

The Set of Integers

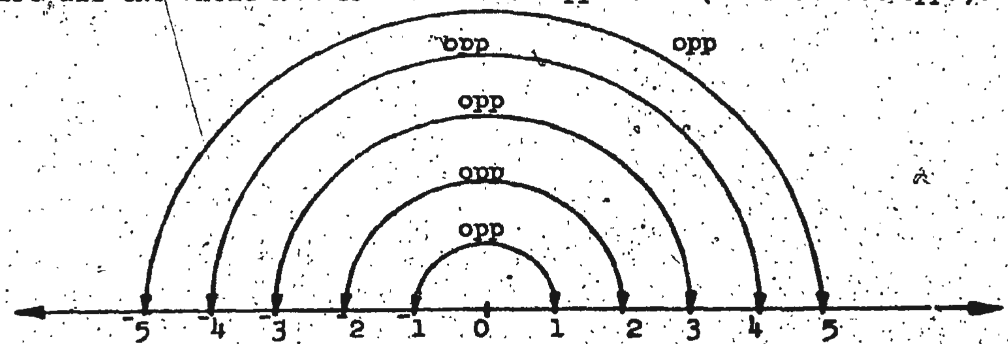
Workbook Materials

Produced by School Mathematics Study Group,
Stanford, California, 1970.

Adopted For This Study By
Frank Marsh
Memorial University of Nfld.
1976

LESSON 1

You have seen these numbers on the number line: ... $^{-}5$, $^{-}4$, $^{-}3$, $^{-}2$, $^{-}1$, 0, 1, 2, 3, 4, 5, ... . Since the number line goes on forever in both directions, these numbers keep going on and on forever. These are the integers (IN-tuh-jurs). There is no first one or last one. Maybe you have learned to call 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, and so on, "whole numbers". The integers are all the whole numbers and their opposites (abbreviated opp.).




The counting numbers 1, 2, 3, 4, 5, and so on, are called <u>positive</u> integers. The opposites of the counting numbers $^{-}1$, $^{-}2$, $^{-}3$, $^{-}4$, $^{-}5$, and so on, are called <u>negative</u> integers. 0 is neither positive nor negative. On the number line the point zero is called the <u>origin</u>.

## Class Discussion Exercises

When you start at 5 on the number line and go to the right, is the next number larger or smaller than 5 ? ________ All of the numbers to the right of 5 are ______________ than 5.
(larger or smaller)

When you start at 5 and go to the left, are the numbers larger or smaller than 5 ? ____________

No matter where you start on the number line, if you go to the ________ the numbers get larger, and if you go to the left, the numbers get ________.

The symbol used to mean smaller (less than) is this: $<$ . "3 is less (smaller) than 4" is written: $3 < 4$. Notice that the symbol $<$ "points to" the smaller number. $^{-}2 < {}^{-}1$ .

The symbol used to mean greater (larger) than is: $>$ . We write "6 is greater than 5" in this way: $6 > 5$. Notice again that the symbol $>$ still "points to" the smaller number.

## Exercises

1. Is there a largest positive integer? ______

2. Is there a smallest negative integer? ______

3. What is the smallest positive integer? ______

4. $^{-}1$ is the ______ (largest or smallest) negative integer.

5. (a) On the number line, $^{-}38$ is on the ______ (left or right) of $^{-}37$.

   (b) Is $^{-}38$ larger or smaller than $^{-}37$ ? ______

6. Which is larger, 2 or $^{-}5$ ? ______

7. 

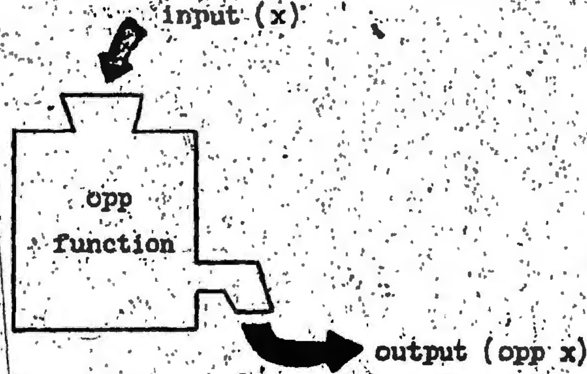

   On this number line, which is larger, the integer that goes with point A or the integer that goes with point B ? ______

8. Zero is larger than any negative integer. ______ (True or False)

9. Fill the blanks with either < or > to show which number of each pair is smaller. Remember that the symbol "points to" the smaller number.

   (a) 4 ______ 10
   (b) 1 ______ 6
   (c) 5 ______ opp 1
   (d) 2 ______ 0
   (e) opp 5 ______ opp 6
   (f) 7 ______ opp 3
   (g) opp 17 ______ opp 14
   (h) opp 4 ______ opp 2
   (i) opp 1 ______ 1
   (j) opp 11 ______ opp 15
   (k) opp 3 ______ 0
   (l) opp 5569 ______ 1

Opp Function




We will put values of x into this machine. The machine will output the opposite of the value of that input.

Say the value of x right now happens to be 4. We input 4 and the machine outputs opp 4 . Now let's assign x the value 3. What will the machine output? ______________ Suppose we assign opp 2 to x. The machine will output the opposite of this value which is 2. This function can be written this way:

$$f : x \longrightarrow \text{opp } x \,.$$

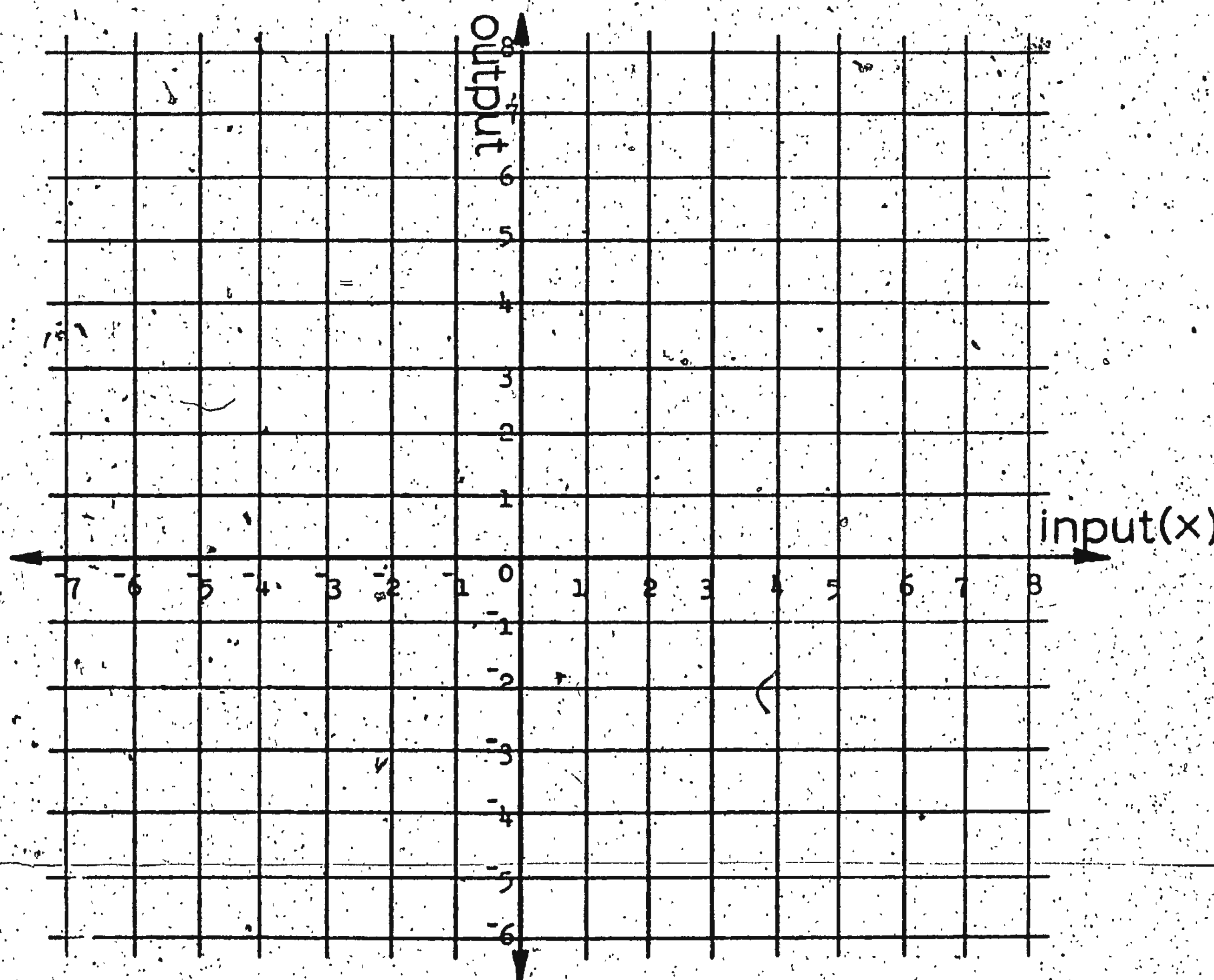
## Exercises

1. Finish the table of inputs and outputs for this function.

| Input (x) | Output (opp x) |
|---|---|
| 4 | opp 4 |
| 3 | opp 3 |
| opp 3 | 3 |
| opp 2 | |
| 1 | |
| 5 | |

Poor Copy

2. Graph the opposite function ($f: x \rightarrow \text{opp } x$) on this coordinate plane. Remember that you only need to plot two points to determine the line you draw. Plot a third point to check your work.




3. From your graph, what output do you get if you input zero?

____________________

4. From Question 3, would you say that zero is its own opposite?

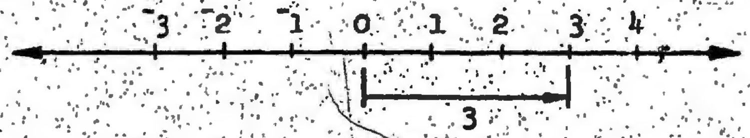
____________________

POOR COPY

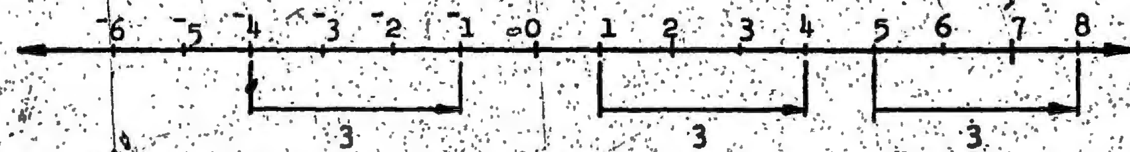
LESSON 2

Arrows

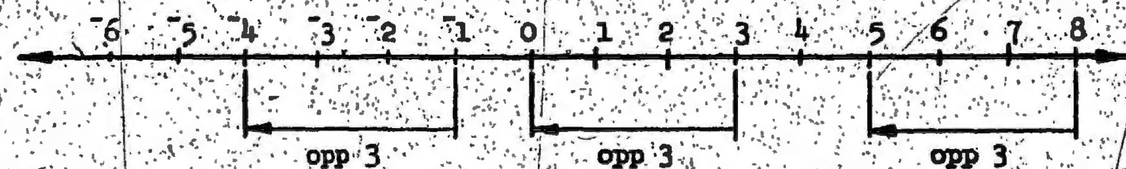
You have seen that numbers and their opposites are the same distance from zero on the number line, but they are in opposite directions from zero. Arrows are sometimes used to show the direction from zero to a number. The integer 3 can be shown as an arrow 3 units long and pointing to the right.




It doesn't matter where the arrow is on the number line. As long as it's 3 units long and pointing to the right, it still means 3.

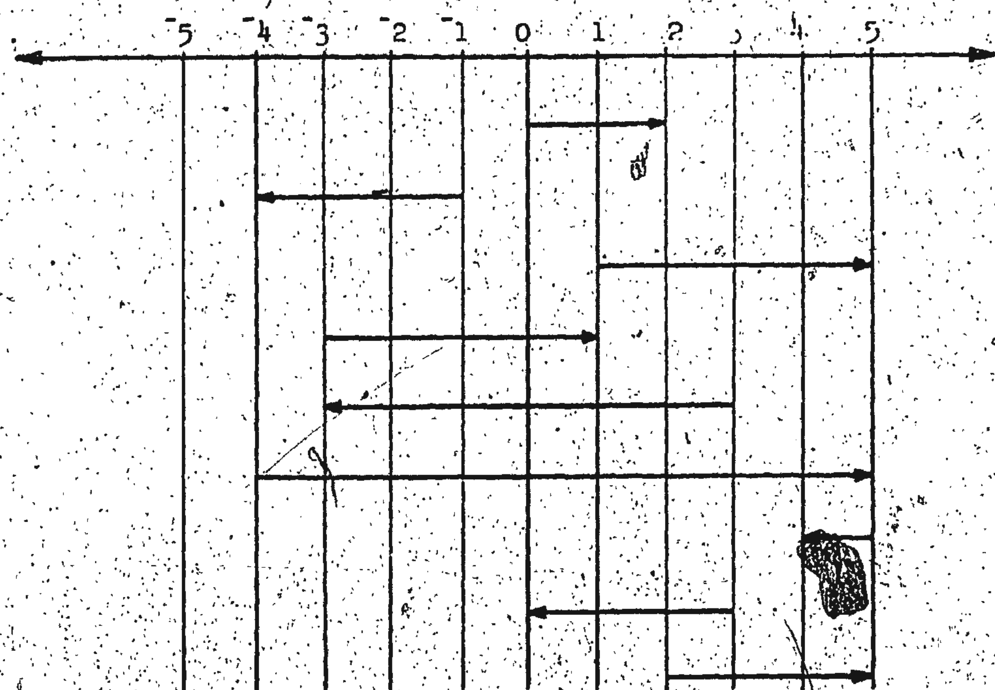



The opposite of 3 can be shown by an arrow 3 units long and pointing to the left.



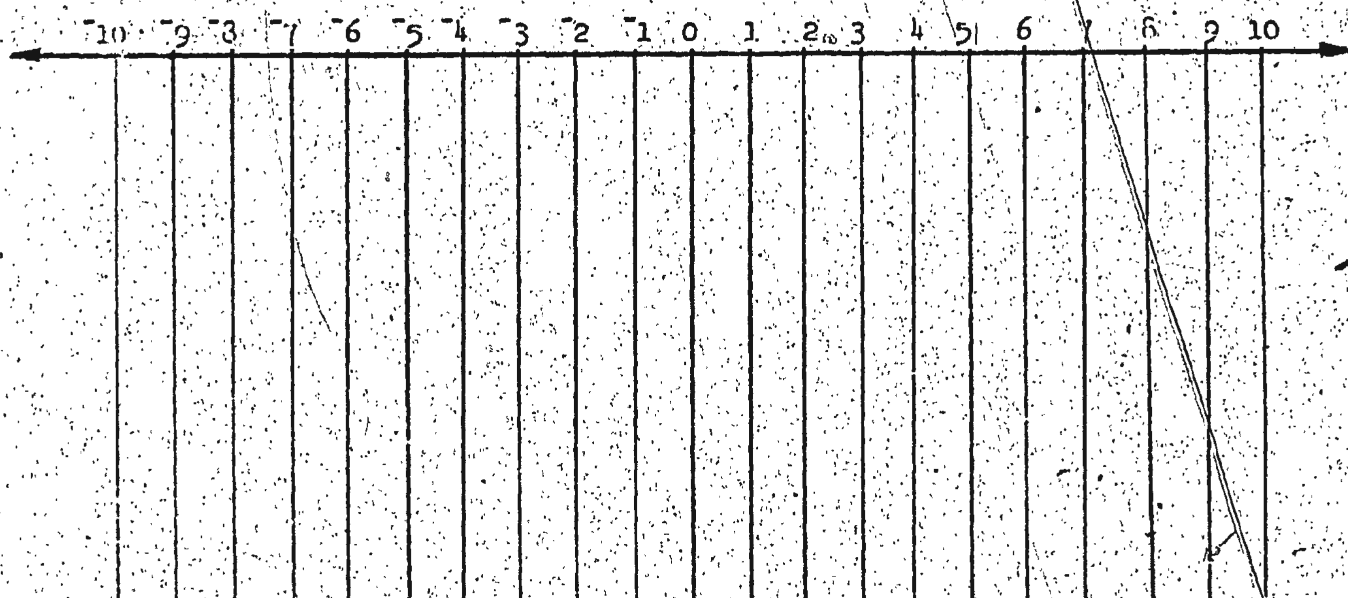
## Exercises

1. Below each arrow write the integer that the arrow represents.




2. Below the number line draw an arrow to represent each of these integers. Write the integer underneath the arrow.

5, 8, opp 3, opp 7, 10, opp 10



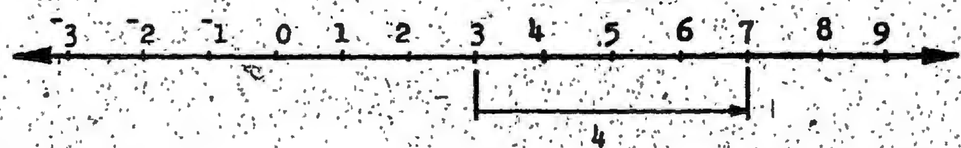
Addition of Integers

We can use our arrows to see how to add a negative integer to a positive integer. Let's start out with a simpler problem like $5 + 3 = \_\_\_$. On the number line we draw an arrow that starts at 5 and is 3 units long.

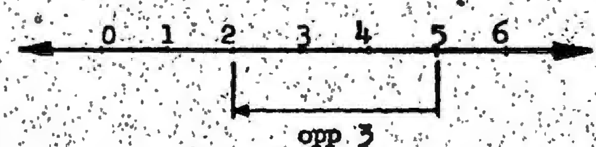

You see that the arrow "points" to 8 which is the correct answer to the problem $5 + 3 = \_\_\_$.

Here is another example.

$3 + 4 = \_\_\_$




Suppose we want to add 5 and opp 3. Again, we start at 5, but this time our arrow points to the left.

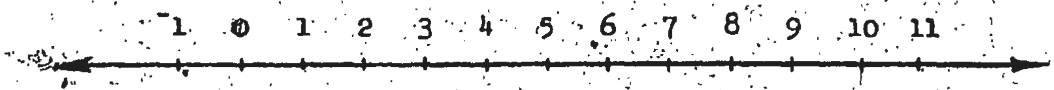



The arrow points to 2 so 5 + opp 3 = 2.

We got the same answer we would get if we subtracted 3 from 5!
$5 - 3 = 2$.

Class Discussion Exercises

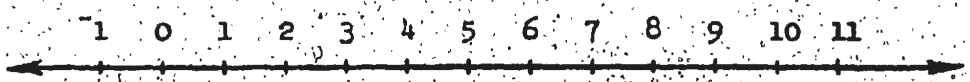
1. To solve the problem 6 + opp 5 = ? , draw an arrow on the number line to find the sum. Your arrow starts at 6 and is ____ units long. Because the arrow represents opp 5, it points to the ________.
(left or right)




6 + opp 5 = ________

and 6 - 5 = ________

2. 7 + opp 4 = ? . Use the number line. The arrow starts at ______. It is ______ units long and points to the ________.
(left or right)




7 + opp 4 = ________

and 7 - 4 = ________

3. 3 + opp 3 = ? . Use the number line. The arrow starts at ______. It is ______ units long and points to the ________.
(left or right)


3 + opp 3 = ________

and 3 - 3 = ________

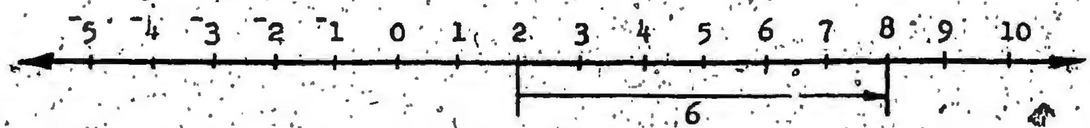
Every time we subtract, we get the same number as when we add the opposite of the subtrahend (the number we are subtracting).

POOR COPY

## Exercises

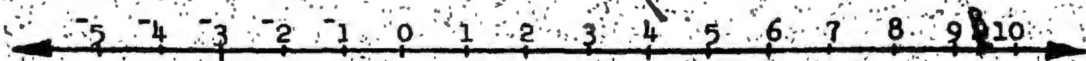
Work these addition problems using arrows on the number line. Draw the arrows first and then write your answer in the blank. The first one is done for you. Remember to start your arrow at the first number. Be sure it points in the correct direction.

1. $2 + 6 =$ 8

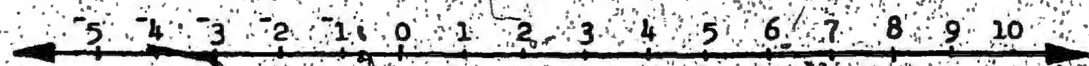



2. $4 + 3 =$ 7


3. $5 + 4 =$ ____

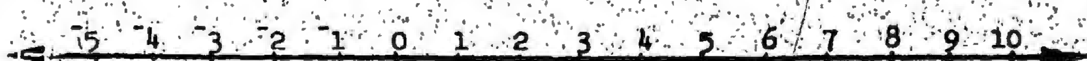



4. opp $3 + 4 =$ ____




5. $7 +$ opp $5 =$ ____


6. $2 +$ opp $6 =$ ____




7. $6 +$ opp $6 =$ ____

8. 0 + opp 7 = ______

$^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8

9. 8 + 5 = ______

$^-1$ 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

10. 12 + opp 15 = ______

$^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8 9 10 11 12

11. opp 6 + opp 4 = ______

$^-11$ $^-10$ $^-9$ $^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5

12. opp 3 + opp 2 = ______

$^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8 9 10 11

LESSON 3

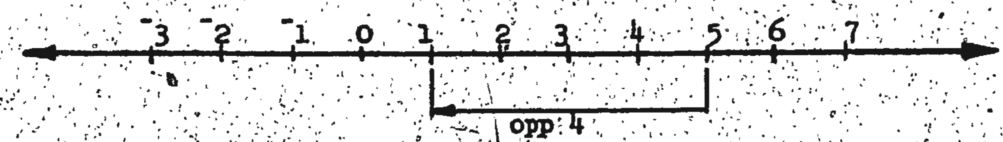
## Subtraction

We said before that when we subtract we really add the opposite of the subtrahend. The first thing we do with a subtraction problem is rewrite the problem as an addition problem and change the subtrahend to its opposite.

Here is an example:

$$5 - 4 = \underline{\ ?\ }$$

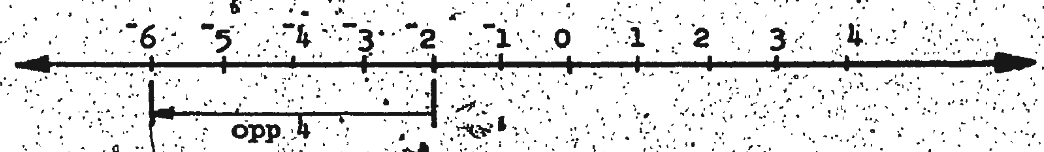
First, rewrite the problem like this: 5 + opp 4 = __?__ We can work this problem on the number line.




The answer we get from the number line is 1, so 5 - 4 = 1.

You can see why this method is important if you think about a problem like this: opp 2 subtract 4 = __?__

First, rewrite the problem as an addition problem: opp 2 + opp 4 = __?__ Then add on the number line.

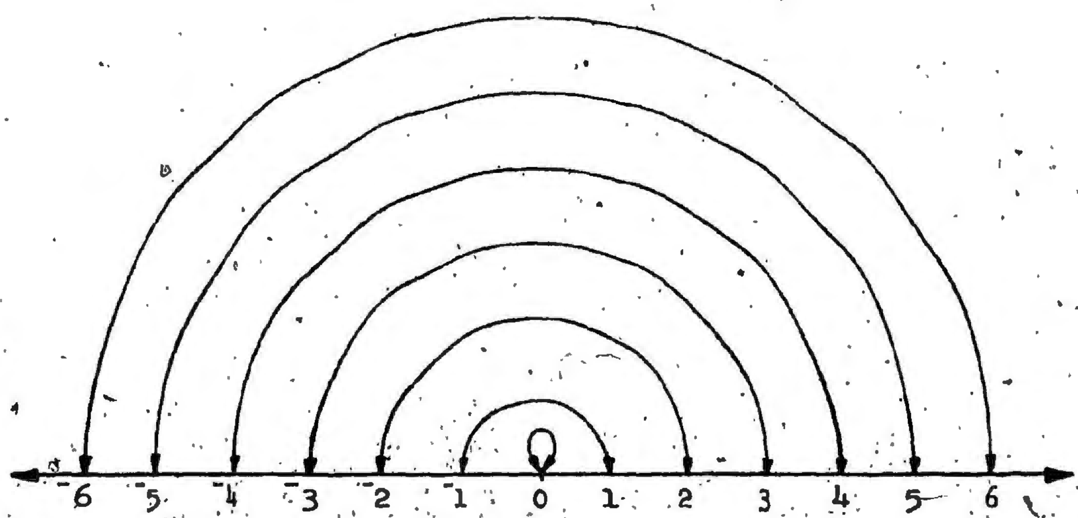



The answer is opp 6. opp 2 - 4 = opp 6. This may not be the answer you expected, but it is correct!

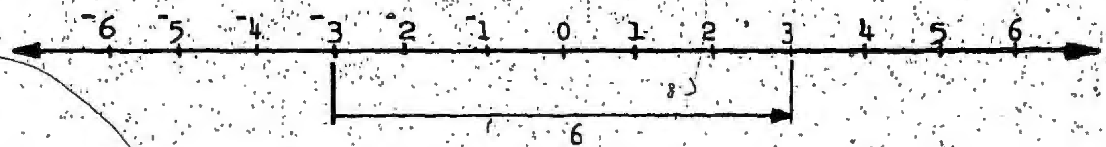
Let's try another example that's even a little harder.

Example: opp 3 subtract opp 6 = __?__

First, we need to find the opposite of opp 6.




You see that the opp of opp 6 is just 6 so we rewrite the problem this way: opp 3 + 6. On the number line it looks like this.




opp 3 + 6 = 3

so opp 3 − opp 6 = 3.

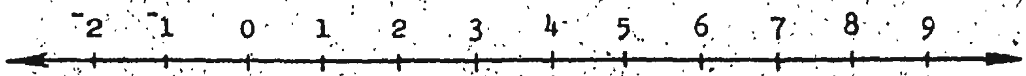
## Class Discussion Exercises

1. When you want to find the answer to the subtraction problem 5 - opp 2 = ____?____, you change the problem to an addition problem. You just copy the first number, 5 . What number are you subtracting from 5 ? ________ That is the subtrahend. What is the opposite of opp 2 ? ________ That is the number you add to 5 .

    Subtraction problem: 5 - opp 2 = ________

    Addition problem: 5 + 2 = ________

    The answer to both problems is the same. Work the addition problem on the answer line and write the answer to both problems.




2. Subtraction problem: 10 - 4 = ________

    Which number do you copy? ________

    What is the subtrahend? ________

    What is the opposite of the subtrahend? ________ That is the number you add to 10 .

    Subtraction problem: 10 - 4 = ________

    Addition problem: 10 + opp 4 = ________

    Work the addition problem on the number line and write the answer to both problems.


3. Subtraction problem: opp 2 - opp 5 = ________

    Which number do you copy? ________

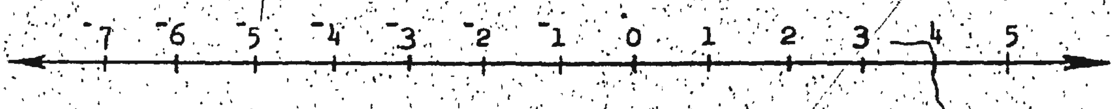
    What is the subtrahend? ________

    What is the opposite of the subtrahend? ________

    Subtraction problem: opp 2 - opp 5 = ________

    Addition problem: opp 2 + 5 = ________

    Work the addition problem on the number line and write the answer to both problems.

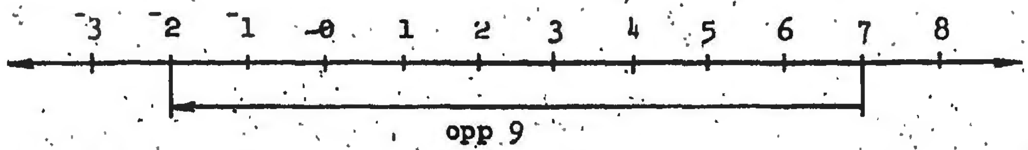


## Exercises

Rewrite each subtraction problem as an addition problem. Work the addition problem on the number line. Write your answer in the blank. The first one is done for you.

1. (subtraction problem) 7 - 9 = ?

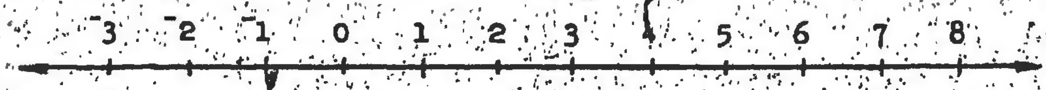
   (addition problem) 7 + opp 9 = ?




(answer) 7 - 9 = opp 2

2. (subtraction problem) 5 - 8 = ?

   (addition problem) __________




(answer) 5 - 8 = ________

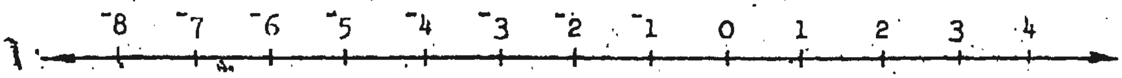
3. (subtraction problem) 3 - opp 3 = ?

   (addition problem) __________


(answer) 3 - opp 3 = ________

4. (subtraction problem) opp 8 - opp 12 = ?

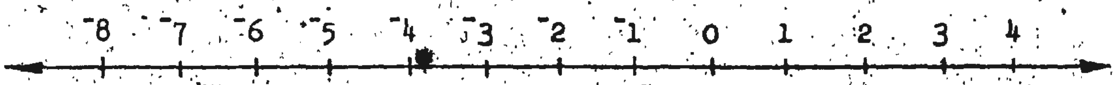
(addition problem) ______________________




(answer) opp 8 - opp 12 = ________

5. (subtraction problem) opp 5 - opp 3 = ?

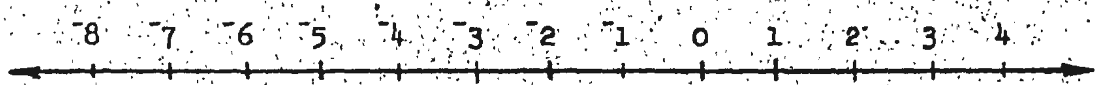
(addition problem) ______________________




(answer) opp 5 - opp 3 = ________

6. (subtraction problem) opp 3 - 4 = ?

(addition problem) ______________________




(answer) opp 3 - 4 = ________

7. (subtraction problem) opp 9 - opp 15 = ?

(addition problem) ______________________

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7

(answer) opp 9 - opp 15 = ______

8. (subtraction problem) opp 6 - opp 6 = ?

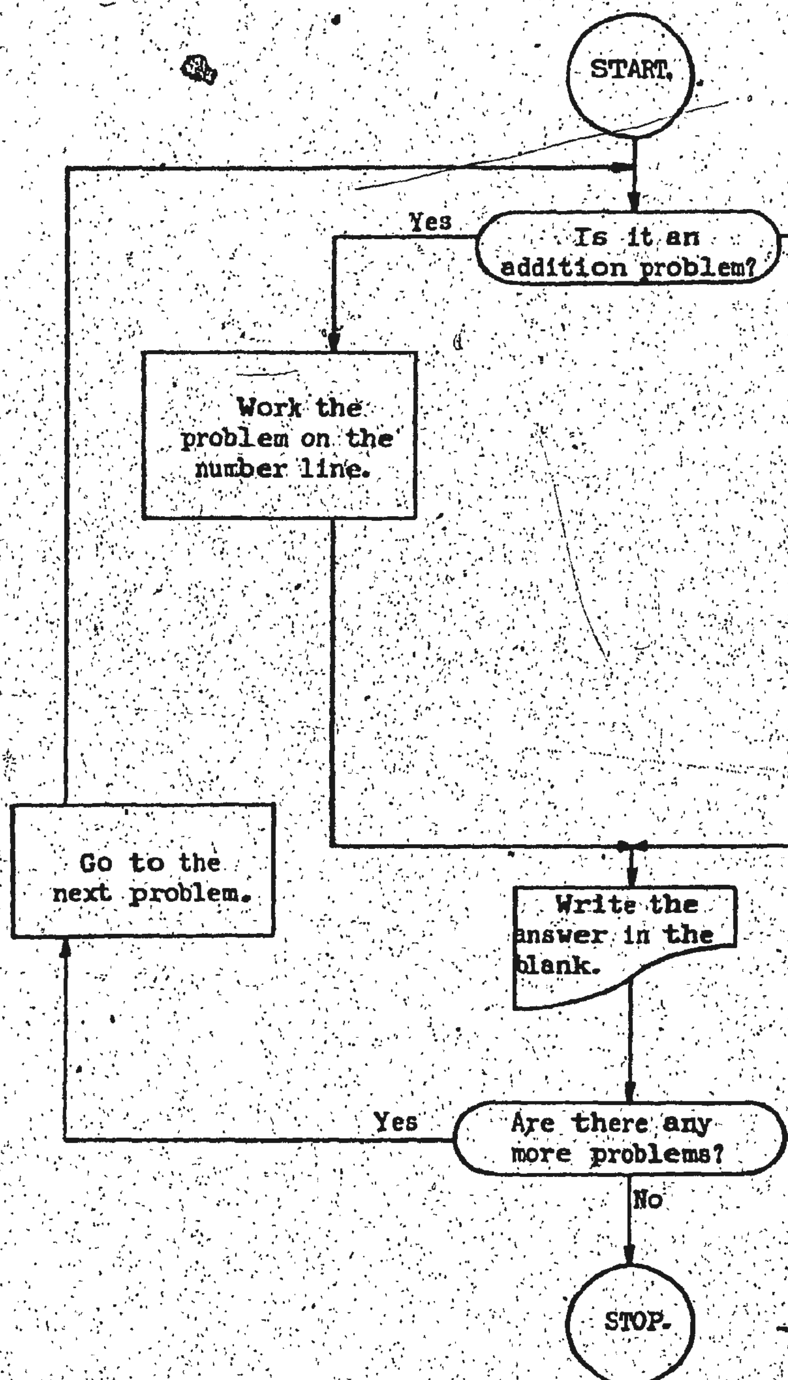
(addition problem) ______________________

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7

(answer) opp 6 - opp 6 = ______

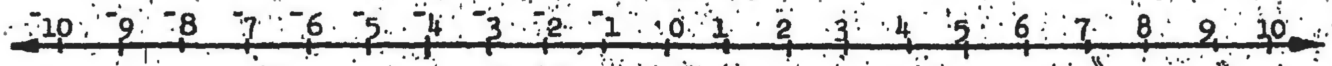
More Practice with Addition and Subtraction

The instructions for this exercise set are given in this flow chart.



## Exercises

1. 2 + 7 = ?

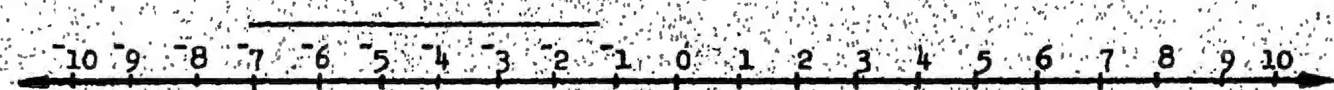



(answer) 2 + 7 = ______

2. opp 3 + opp 1 = ?


(answer) opp 3 + opp 1 = ______

3. opp 10 - opp 3 = ?

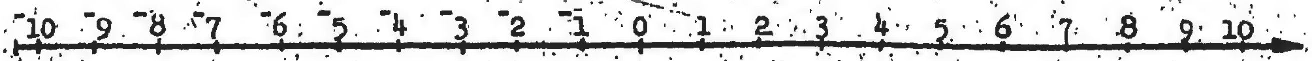



(answer) opp 10 - opp 3 = ______

4. 4 - 6 = ?


(answer) 4 - 6 = ______

5. 9 + opp 5 = ?




(answer) 9 + opp 5 = ______

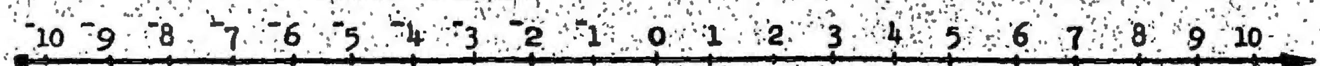
6. 10 + opp 12 = ?


(answer) 10 + opp 12 = ______

7. 8 - opp 2 = ?


(answer) 8 - opp 2 = ______

8. opp 8 - 2 = ?




(answer) opp 8 - 2 = ______

9. opp 8 - opp 2 = ?

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) opp 8 - opp 2 = ______

10. opp 8 + 11 = ?

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) opp 8 + 11 = ______

11. 8 + opp 8 = ?

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) 8 + opp 8 = ______

12. opp 7 - 2 = ?

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) opp 7 - 2 = ______

13. 6 - opp 3 = ?

$^-10$ $^-9$ $^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) 6 - opp 3 = ________

14. 7 - 5 = ?

$^-10$ $^-9$ $^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) 7 - 5 = ________

15. 5 - opp 5 = ?

$^-10$ $^-9$ $^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) 5 - opp 5 = ________

16. opp 2 + 6 = ?

$^-10$ $^-9$ $^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) opp 2 + 6 = ________

17. opp 9 + 7 = ?

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) opp 9 + 7 = ________

18. opp 2 - opp 9 = ?

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) opp 2 - opp 9 = ________

19. 1 - opp 9 = ?

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) 1 - opp 9 = ________

20. opp 1 + 1 = ?

$^{-}10$ $^{-}9$ $^{-}8$ $^{-}7$ $^{-}6$ $^{-}5$ $^{-}4$ $^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3 4 5 6 7 8 9 10

(answer) opp 1 + 1 = ________

# LESSON 4

Adding Integers That Are Both Positive or Both Negative

Drawing the number line each time we want to work an addition problem takes a lot of time and it is just plain hard work. It will be easier if you can begin to "see" the number line in your mind.

Pretend you have a "mental blackboard" with a number line on it. See if you can "look at" your mental blackboard to help you decide whether the answer to this problem is to the left or right of zero.

opp 16 + opp 19 = ___?___

Even if you have trouble adding the numbers it is clear that the answer will be negative.

You have probably noticed that on the number line instead of writing opp 1, opp 2, opp 3, and so on, we write $^-1$, $^-2$, $^-3$, and so on. We use the raised dash for the negative numbers because it is shorter than writing "opp" all the time. When you see $^-13$, think opp 13.

Exercises

1. In the following problems use your "mental blackboard" and write "-" if the answer is negative and "+" if the answer is positive. See if you can do these without drawing the number line. The first one is done for you.

| | |
|---|---|
| (a) $^-13 + {}^-5$ = ___-___ | (g) $28 + 29$ = ______ |
| (b) $^-17 + {}^-18$ = ______ | (h) $^-9 + {}^-19$ = ______ |
| (c) $^-27 + {}^-2$ = ______ | (i) $^-23 + {}^-29$ = ______ |
| (d) $29 + 14$ = ______ | (j) $23 + 29$ = ______ |
| (e) $13 + 21$ = ______ | (k) $^-15 + {}^-24$ = ______ |
| (f) $^-17 + {}^-17$ = ______ | (l) $^-30 + {}^-14$ = ______ |

2. When you add two negative numbers is the answer always negative? ______

______

3. When you add two positive numbers is the answer always positive? ______

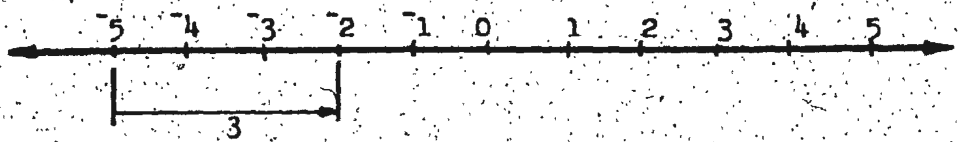
______

4. Use your addition tables to find the answer to each of the problems in Exercise 1. Write your answer in the blank beside the problem.

## Addition of a Positive and a Negative Integer

When you add a positive and a negative integer, first decide which integer is farther from zero. If the positive integer is farther from zero, then the answer is positive. Likewise, if the negative integer is farther from zero the answer is negative.

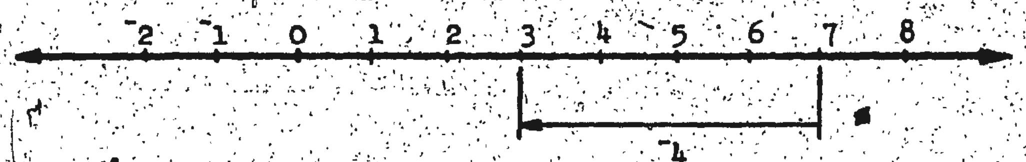
Example 1. $^-5 + 3 = ?$




The answer is $^-2$.

Which is farther from zero, $^-5$ or $3$ ? ____________
Do you see that the answer is also negative? ____________

Example 2. $^+7 + {}^-4 = ?$




Is the integer farther from zero positive or negative? ____________
We see from the number line that the answer is $^+3$. Is the sign of the answer the same as the sign of the number farther from zero? ____________

## Exercises

1. Finish this table.

| Problem | Number farthest from 0 | Sign of number farthest from 0 | Sign of the answer |
|---|---|---|---|
| (a) $^{-}7 + 8 = 1$ | 8 | + | + |
| (b) $^{-}4 + 2 = {}^{-}2$ | $^{-}4$ | - | - |
| (c) $^{-}18 + 13 = {}^{-}5$ | | | |
| (d) $29 + {}^{-}12 = 17$ | | | |
| (e) $15 + {}^{-}30 = {}^{-}15$ | | | |
| (f) $^{-}23 + 17 = {}^{-}6$ | | | |
| (g) $^{-}17 + 23 = 6$ | | | |
| (h) $8 + {}^{-}19 = {}^{-}11$ | | | |
| (i) $^{-}27 + 8 = {}^{-}19$ | | | |
| (j) $27 + {}^{-}8 = 19$ | | | |

2. In the blank write the sign (+ or -) of the number farthest from zero.

(a) $14 + {}^{-}5 =$ ______ (d) $19 + {}^{-}30 =$ ______

(b) $^{-}13 + 17 =$ ______ (e) $^{-}5 + 12 =$ ______

(c) $^{-}27 + 23 =$ ______ (f) $18 + {}^{-}13 =$ ______

3. Find the answer to each of the problems in Exercise 2 and write your answers in the blanks.

Integer Addition and Subtraction Exercises

Write the answer in the blank beside each problem. Try to do these without drawing number lines. Below each subtraction problem remember to rewrite it as an addition problem and then find the answer.

1. $7 + 8 =$ ______
2. $^{-}7 + {}^{-}8 =$ ______
3. $7 + {}^{-}8 =$ ______
4. $^{-}7 + 8 =$ ______
5. $9 - 11 =$ ______

 ____________________

6. $5 - {}^{-}4 =$ ______

 ____________________

7. $^{-}12 + {}^{-}17 =$ ______
8. $^{-}12 - {}^{-}17 =$ ______

 ____________________

9. $^{-}14 + 23 =$ ______
10. $^{-}27 - {}^{-}29 =$ ______

 ____________________

11. $^{-}19 - {}^{-}30 =$ ______

 ____________________

12. $^{-}29 + {}^{-}27 =$ ______
13. $18 + {}^{-}28 =$ ______
14. $17 + {}^{-}23 =$ ______
15. $24 + 23 =$ ______
16. $21 - 27 =$ ______

 ____________________

17. $15 - 10 =$ ______

 ____________________

18. $7 + {}^{-}7 =$ ______
19. $^{-}14 + 14 =$ ______
20. $^{-}23 + 23 =$ ______

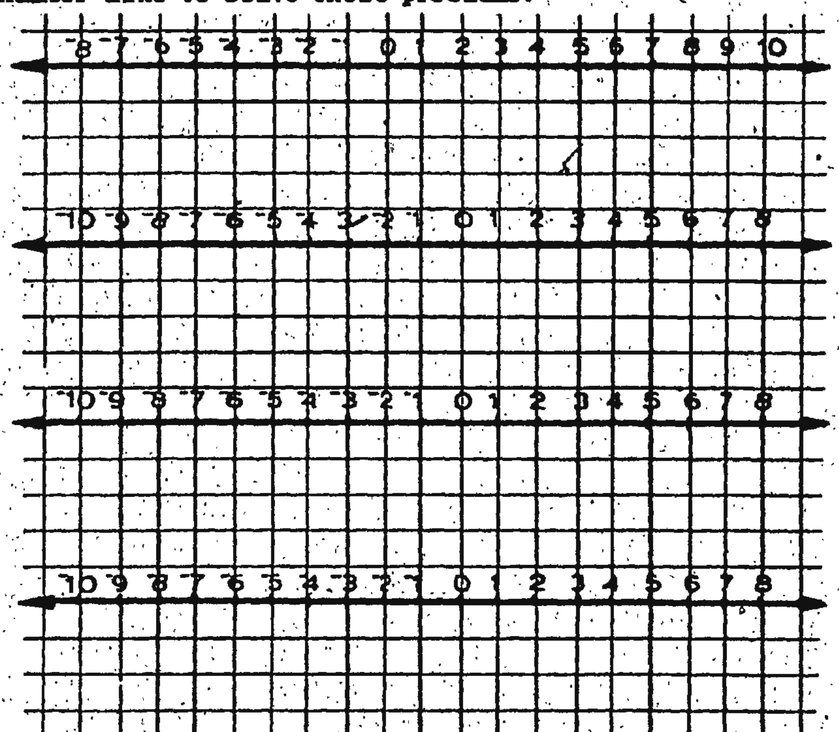
## Zero: A Special Integer

You have known for a long time that "nothing happens" when you add zero to a number. That is, when you add zero you get what you started with. In fact, we don't have an arrow to stand for zero on the number line. Mathematicians call zero the "identity number for addition".

Look at the last three problems on page 5-9. Can we add two integers and get zero for an answer? __________ Notice that the integers we added to get zero were the opposites of each other.

### Exercises

1. Use arrows on the number line to solve these problems.




(a) $^-5 + 5 =$ ____

(b) $8 + {}^-8 =$ ____

(c) $7 + {}^-7 =$ ____

(d) $^-10 + 10 =$ ____

2. When we add two integers that are opposites of each other, the answer is always ________________.

3. Without adding, write zero in the blank if the sum of the two integers is zero.

(a) 7 and $^{-}4$ ________

(b) $^{-}18$ and 18 ________

(c) $^{-}423$ and 423 ________

(d) 8 and $^{-}8$ ________

(e) 75 and $^{-}75$ ________

(f) $^{-}23$ and $^{-}25$ ________

(g) $^{-}27$ and 0 ________

(h) $^{-}1$ and 1 ________

(i) 1,296 and $^{-}1,296$ ________

(j) 30 and $^{-}300$ ________

**BRAIN BOOSTERS!**

(k) $(2 \times 10)$ and $^{-}20$ ________

(l) $(7 + 6)$ and $^{-}13$ ________

(m) $10^3$ and $^{-}1000$ ________

(n) $4^2$ and $^{-}16$ ________

(o) $^{-}120$ and 1200 ________

(p) $(3 \times 10)$ and $^{-}300$ ________

# A P P E N D I X D

## STUDENT-DIRECTED HANDBOOK

# THE SET OF INTEGERS

A Booklet for Student-Directed Learning

The Set of Integers

A Booklet for Student-Directed Learning

Developed By

Frank Marsh

Memorial University of Nfld.

1976

# INTRODUCTION

This presentation is designed to introduce the student to the Set of Integers of the Number System.

It deals with the basic ideas of Integers and develops the operations of addition and subtraction using the Number Line.

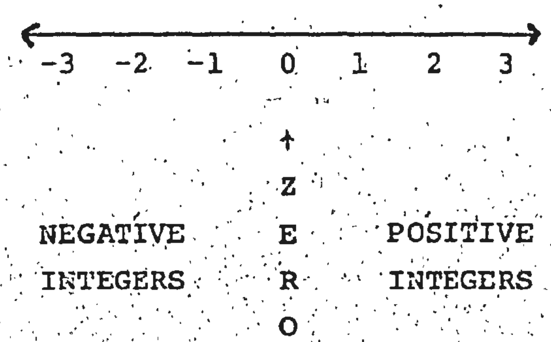
It was designed as a guide to be used along with a set of materials or text containing further explanations and exercises.

The main purpose of this booklet is to aid the students or a group of students in directing their own learning of this topic.

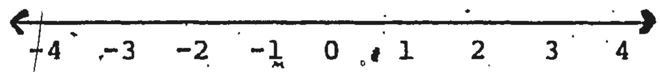
It is my hope that this booklet helps in the promotion of student self-directed learning in mathematics.

Frank Marsh

The Set of Integers




The Set of Integers is made up of the positive integers, zero, and the negative integers.




Numbers to the Right of Zero are Positive Integers.

Numbers to the Left of Zero are Negative Integers.

Zero is <u>neither</u> a positive <u>nor</u> a negative integer.

It is the MIDPOINT of the number line.

Is 5 a positive or negative Integer? Why?
Is -3 a positive or negative Integer? Why?
Is 0 a positive or negative Integer? Why?

If you can visualize the Number Line you will remember that numbers to the right of zero are positive, so <u>5</u> is a Positive Integer.

Numbers to the left of zero are negative, so <u>-3</u> is a Negative Integer.

0 is the midpoint of the number line, so 0 is neither positive or negative.

Numbers to the Right are Greater
Numbers to the Left are Smaller

$\longleftarrow$ -5 -4 -3 -2 -1 0 1 2 3 4 5 $\longrightarrow$

5 is Greater than 4
-3 is Less than -1

Numbers to the right are greater, for example, 5 is Greater than 4.

So then, numbers to the left are smaller, thus -3 is smaller than -1.

The symbol used to mean

Less than is <

Greater than is >

EG.: 5 is smaller than 6

& $5 < 6$

$-1$ is greater than $-3$

$-1 > -3$

Notice: The symbol < & > "Points To" the smaller number.

To symbolize greater than we use an arrow head pointing right.

To symbolize less than we use an arrow head pointing left.

To write 5 is less than 6 we write 5 arrow head pointing left 6. The arrow head points at 5.

To write -1 is greater than -3 we write -1 arrow head pointing right -3. The arrow head points at -3.

Notice: The arrow head always points at the smaller number.