THE EFFECT OF NUMBER SIZE AND ORDERING OF DATA ON THE DIFFICULTY OF SELECTED ARITHMETIC WORD PROBLEMS

MARY SHARON KIELEY

THE EFFECT OF NUMBER SIZE AND ORDERING OF DATA ON THE DIFFICULTY OF SELECTED ARITHMETIC WORD PROBLEMS

by

© Mary Sharon Kieley, B.A.(Ed.), B.A.

A Thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Education

Department of Curriculum and Instruction
Memorial University of Newfoundland

St. John's Newfoundland

April 1983

ABSTRACT

The purpose of this study was to investigate the effects of number size and ordering of data on the difficulty of arithmetic word problems. The study was conducted at level one of the senior high school system in Newfoundland with 220 students enrolled in the practical mathematics course at that level. The students were randomly selected and each was asked to complete one of six tests in a 30 minute time period. Each test consisted of 12 two-step word problems incorporating the operations of multiplication - addition; division - addition; multiplication - subtraction; division - subtraction, with three items per instrument testing each set of operations. The tests were designed as follows: (1) small numbers with ordered data; (2) large numbers with ordered data; (3) small numbers and large numbers with ordered data; (4) small numbers with unordered data; (5) large numbers with unordered data; (6) small numbers and large numbers with unordered data.

It was concluded that there was no significant interaction effect between number size and ordering of

data, but there was a significant effect for the number size variable and for the order variable. Whether the data was ordered or unordered, problems containing large numbers as data were the most difficult for students in the practical level of mathematics at level one of the senior high school; problems containing a combination of small numbers and large numbers were of intermediate difficulty, and problems containing small numbers were the least difficult. Also, whether the numbers were small, large, or a combination of small and large, problems containing unordered data were more difficult than problems containing ordered data.

Implications for teaching were discussed and recommendations for future research in arithmetic word problems were proposed.

ACKNOWLEDGEMENTS

The writer would like to thank all the students, teachers, and principals for their cooperation in this study.

Sincere appreciation is also extended to my supervisor, Dr. Lionel Mendoza, for his guidance, interest, and encouragement as well as to Dr. Dale Drost for his advice and interest.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

THE PROBLEM

<u>Rationale for the Study</u>

At its annual meeting in 1980 the National Council of Teachers of Mathematics (1980) announced its <u>Agenda for Action: Recommendations for School Mathematics of the 1980s</u>. One of the major recommendations was that: "Problem solving must be the focus of school mathematics in the 1980s." (p. 2) Among the suggestions for action were that the mathematics curriculum be organized around problem solving, and that researchers and funding agencies give priority in the 1980s to investigations into the nature of problem solving and to effective ways to develop problem solvers. Wong (1979), in a research report prepared for the Educational Research Institute of British Columbia, presented a similar recommendation for action:

> Teachers and teacher educators should stress the overriding importance of problem-solving in mathematics and they should attempt to teach their students various strategies to employ in attempting to solve problems in mathematics. (p. 23)

The PRISM Canada Project (1981), in its implications for action, reinforced the relevance to the Canadian scene of the recommendations of the National Council of Teachers of Mathematics. The researchers summarized their findings on problem solving by stating that:

> Problem solving must be a major organizing element in the mathematics curriculum. . . . Weaving problem solving experiences throughout the mathematics curriculum will develop methods of thinking and logical reasoning. (p. 61)

One particular aspect of problem solving, namely word problems, has received much attention in the problem solving literature. Any teacher who has taught mathematics at any level 'knows' that students experience difficulty in the solution of word problems. Such comments as the following have probably been heard in most classrooms: "I like algebra, except for word problems.", "Word problems have always been my downfall.", "I never did learn how to do word problems." Teachers, too, have their own feelings as to why students experience difficulty in this area. "They can't read.", "They can't think.", "They can't compute.", "They are careless.", "They can't decide what to do."

These statements may be true for some students but further research into the area should aid in the identification of specific reasons why children experience difficulty in the solution of word problems. West (1977) felt that it is necessary to diagnose the ailment before a cure can be prescribed.

Zweng (1979) and Knifong and Holton (1977) each concluded that the majority of students who solved problems incorrectly could read word problems orally and could retell the story in their own words; that is, they could comprehend the problem. From these studies, together with those carried out for the first mathematics assessment of the National Assessment of Educational Progress (NCTM, 1978) it was possible to conclude that deciding which operation(s) to perform, not lack of computational skill, was the major stumbling block to successful problem solving.

In 1979 the NAEP released the results of its second survey of Mathematics achievement (NCTM, 1980). It was concluded that, although computational skills remained satisfactory, the ability to solve problems had declined during the period from 1973 to 1978. This report offered no definitive answers as to why children cannot solve word problems, but did provide some insights regarding children's ability to solve word problems. The authors of the report indicated that children were quite successful in solving most problems that could be solved by a single arithmetic operation. However, almost every problem that could not be solved by a single routine application of a single arithmetic operation caused 'much' difficulty.

A key step to improving problem solving skills is to identify the reasons for this inability to solve such word problems. One answer seems to lie in an analysis of the problem and the factors contained in the structure of word problems that contribute to their difficulty.

These concerns regarding the solution of word problems become more acute when considering those students commonly labelled as 'slow students'. According to Smith (1968), 'retarded' learners are reasonably like other children in the areas of computation, but have greater difficulty in identifying and understanding which process should be used in problem solving. This being the case, a study designed to determine the specific causes of difficulty for such slower learners with regard to word problems seemed appropriate. The results of such a study could provide a basis for developing instructional strategies to minimize these difficulties.

Purpose of the Study

The problem investigated in the current study concerned the effect of number size and ordering of data on the difficulty of selected two-step arithmetic word problems. The population selected for the study consisted of students in the practical level of mathematics at level one of the senior high school. With respect to the problem, the following research questions were investigated:

1. Does the size of the numbers used in arithmetic word problems affect the difficulty of the problem?

2. Does the order in which the data are presented affect the difficulty of arithmetic word problems?

All problems used in the study involved two steps. The following null hypotheses were tested to provide answers to the research questions:

1. There is no significant difference in the difficulty of arithmetic word problems using small numbers, large numbers, and a combination of small numbers and large numbers.

2. There is no significant difference in difficulty between arithmetic word problems using ordered data and arithmetic word problems using unordered data.

3. There is no significant interaction effect between number size and ordering of data in arithmetic word problems.

Limitations

The limitations of the study arise out of the type of student used as subjects, the types of problems used in the tests, and the sampling procedure. First, the study was conducted with students in the practical level of mathematics at level one of senior high school and hence, the results may not be generalizable to students in the academic or advanced levels of mathematics or to students at levels two and three of the senior high school. Second, the problems dealt with specific sets of operations and thus, the results may not be generalizable to other types of word problems or to word problems using a different combination of operations. Third, the sample of 10 schools out of all high schools in Newfoundland was not representative of all geographic areas of the province, thus, the results may not be generalizable to other areas of Newfoundland or to areas outside of Newfoundland.

Definition of Terms

Certain terms or phrases used through the study are defined as follows:

Large Number: A number of four digits or more.

Small Number: A number of two digits or less.

Ordered Data: Data presented in the order in which they can be used to solve the word problem.

Unordered Data: Data presented in an order different from the order in which they can be used to solve the word problem.

<u>Arithmetic Word Problem</u>: A written description of a quantitative situation in which the necessary operation(s) for a solution must be provided by the pupil.

<u>Two-Step Problem</u>: A problem in which two different arithmetic operations are required in the solution.

<u>Practical Level</u>: Practical level mathematics courses are oriented towards students who have had minimal success with mathematics, and who expect to pursue careers in which practical mathematics is needed rather than abstract mathematics. (Division of Instruction, Department of Education, Government of Newfoundland and Labrador, 1982)

<u>Academic Level</u>: Academic level mathematics courses are designed to meet the needs of students who have average ability in mathematics. (Division of Instruction, Department of Education, Government of Newfoundland and Labrador, 1982)

<u>Advanced Level</u>: Advanced level mathematics courses are designed for students with considerable interest and ability in mathematics. (Division of Instruction, Department of Education, Government of Newfoundland and Labrador, 1982)

<u>Level One</u>: Level one is the first year of high school under the Newfoundland system.

<u>Level Two</u>: Level two is the second year of high school under the Newfoundland system.

<u>Level Three</u>: Level three is the third year of high school under the Newfoundland system.

CHAPTER II

REVIEW OF RELATED LITERATURE

The purpose of this chapter is to review the literature concerned with the solution of arithmetic word problems. The chapter is organized into five sections: (1) a general overview of problem solving is presented; (2) the general literature pertaining to the solution of arithmetic word problems is reviewed; (3) number size and its relationship to problem difficulty is discussed; (4) ordering of data and its relationship to problem difficulty is considered with respect to two-step arithmetic word problems; (5) additional research pertaining to two-step arithmetic word problems is presented.

Problem Solving

According to Polya (1962), for a problem to exist, the person seeking the solution must be required to "search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim." (p. 117) Willoughby (1967) summarized the thoughts of many mathematics educators by indicating the following facets of the definition of a problem: (1) an individual's unrealized goal which may be indicated by a vague uneasiness or a specific verbalized question; (2) some form of blocking that prevents the individual from reaching his goal; (3) consideration by the individual of ways of avoiding the blocking and his trial of certain procedures. Kantowski (1980) concluded that a problem is a situation for which the individual who confronts it has no algorithm that will guarantee a solution. The person's relevant knowledge must be put together in a new way to solve the problem. Thus, a key aspect of a problem is that of 'blocking' of the path to a successful solution. This blocking, in turn, leads to a trial of possible solutions. If an attempt leads to a removal of the block then the problem has been solved.

Solution of Arithmetic Word Problems

One particular area of problem solving that has received much consideration is that of verbal quantitative problems, or word problems. Kramer (1975) defined a verbal quantitative problem as:

> a described situation that involves a quantitative question for which the individual has no ready answer... Such a quantitative problem may be expressed as a number question, the

> answer to which cannot be given by an habitual response of the individual who faces the problem. (p. 379)

Although a definite pattern in the process of solving word problems has not been discovered, some steps have been identified in the literature. The number of steps reportedly involved in the solution of word problems varies, depending upon the authority in question. Since the early part of this century educators have expressed a variety of opinions on how word problems may best be solved. (Thorndike, 1921; Durell, 1928; Grossnickle, 1933; Foran, 1934; Brownell, 1942; Brewer, 1956; Polya, 1957, 1962)

However, as Kramer (1975) noted, the different theories on problem solving contained some common elements: (1) the problem is recognized and a goal is set; thus presuming the realization of a difficulty; (2) the individual starts the process of deliberation and tries to decide the process or processes he should use to reach the goal. He uses all available information to select that process which will help him overcome the difficulty; (3) a process is selected and the tentative solution is tested, with the result leading to acceptance or rejection of the process; if it is rejected, another solution or hypothesis must be formulated and tested. This process continues until a process which leads to a correct solution is found.

Thus, from an analysis of the literature it was concluded that one key to solving arithmetic word problems is analyzing and organizing the data and selecting a procedure. If, however, these processes are used and students still find difficulty in reaching a solution to a word problem, then other factors need to be considered.

Many studies have been undertaken in an effort to understand the process of problem solving, with much emphasis being placed on structural variables. Segalla (1973) stated:

> In structural variable studies we assume the existence of factors in the structure of arithmetic word problems that contribute to the difficulty level of the problem. (p. 1)

Days, Wheatley, and Kulm (1979) felt that certain task variables such as problem structure or manner of organization, problem context, problem length, magnitude of the numbers, and data placement might affect problem solving performance. They conducted a study with 58 eighth grade mathematics students in the midwestern United States and found that problem structure had a significant effect on the number of different problem-solving processes used. They concluded that structural variables should be given proper consideration when studying or teaching problem solving.

Begle (1979) reported findings from a survey of the empirical literature on critical variables in mathematics education. He concluded that, while format variables and structural variables had both been studied to some extent, and that there had been some significant effects discovered, not enough had been done to draw any broad conclusions. The research related to the factors of particular relevance to this study is now reviewed.

Number Size

There has been little research reported in the problem solving literature with respect to number size and its relationship to the level of difficulty of arithmetic word problems, per se. However, there seemed to be agreement that smaller numbers tend to cause less confusion than larger numbers and hence, students are more successful solving word problems containing smaller numbers. Hooten (1975) reported on the extensive Soviet research in the psychology of mathematical instruction. Because that research was conducted over a 25 year period, details of specific studies were not reported. However, the conclusion of relevance to this study was that temporarily replacing large numbers with small numbers in the problem was useful. Such substitutions, according to the reports, were a particular form of concretization since they helped to bring the problem closer to the pupil's experience.

In a study of 162 children in grades three, four, five and six, Zweng (1979) questioned whether a restatement of the problem with smaller numbers would help the children solve problems. She designed a hint for those students who were experiencing difficulty. Those unable to solve a problem were instructed to attempt a problem identical to the original problem, except the number size had been decreased. To determine the effectiveness of the 'smaller numbers' hint, the number of times the hint helped was divided by the number of times the hint was given and a percentage was thus computed. For students in grade three the hint was effective for 32% of the tries; for grade four 57% of the tries; for grade five 35% and for grade six, 60% of the tries. The overall effectiveness was 46% of the tries. With respect to ability level, the 'smaller numbers' hint was effective in 46% of the tries with students of high ability (87th to 99th percentile on Mathematics Problem Solving Subtest of Iowa Tests of Basic Skills); in 54% of the tries with students of average ability

(65th to 85th percentile) and in 37% of the tries with students of low ability (20th to 63rd percentile). She thus concluded that for all grade levels and all ability groups, decreasing the size of the numbers was very effective in helping students solve problems.

Ordering of Data

Burns and Yonally (1964) asked:

> If problems are stated with numerical data not given in the order in which they are needed to solve the problem, will pupils solve as many of them successfully as problems stated with numerical data given in the order in which they are used to solve the problem? (p. 267)

As a result of their research with 95 fifth grade students in eastern Kansas they concluded that students were less successful in obtaining the correct answer to arithmetic word problems when the data were presented in some order other than the order in which they should be used to solve the problem. Of the 95 students who were tested, 38 of them scored higher on the proper order type of problems, 29 of them scored higher on the mixed order type and 28 of them received the same score on each type. A statistical analysis of the entire group indicated that the difference was significant at the 5% level of significance. They further concluded that arithmetic reasoning ability was positively related to student success with problems which presented numerical data in mixed order.

Suydam and Weaver (1970) gave an overview of research studies which pertain to the verbal problem solving ability of elementary school children. In their summary of the findings of research with respect to the ordering of data they stated that there was some evidence to show that the order in which the processes are presented in multi-step problems may affect their difficulty and that significantly higher scores resulted when numerical data were presented in the order in which they would be needed to solve the problem.

Hooten (1975) suggested that it is very important to vary the arrangement of the numerical data in a word problem. He also suggested that children should be given problems to solve in which the data are not ordered. His suggestion was based upon the results of studies carried out with primary and elementary children in the Soviet Union over the previous 25 years. He hypothesized that these children were so conditioned by traditional ordered textbook problems that when presented with unordered problems they failed to analyze the problem, thus accounting for the low level of success with unordered problems.

From a study with 157 first-year algebra students Ingle (1975) reported that structural variables that proved to be significant in predicting word problem difficulty were more closely related to order of presentation of the data than to translation of a word problem into an equation, per se. She further concluded that the role of problem variables in a given word problem model was somewhat dependent upon the ability level of students who worked the problems.

Rosenthal and Resnick (1971, 1974) conducted two studies on the effect of problem form and sequence of information on problem difficulty for students at the elementary level. The earlier experiment was conducted with 29 subjects in grade three and the later experiment was conducted with 34 subjects, also in grade three, with all 63 subjects solving 32 word problems. They concluded that when data appeared in the reverse order to that which was needed for solution the problem posed maximum difficulty, suggesting that in such unordered problems, subjects' normal processing habits were disrupted.

Stover (1979) investigated the effect of instruction in changing the format of a mathematical word problem upon the problem solving ability of sixth grade students. More specifically, the study attempted to show that three common format or structural variables are in part causally related to the difficulty of the problem itself. The researchers concluded that the order of numerical presentation was one variable that contributed heavily to mathematical problem solving difficulty in sixth graders. They further suggested that students can be taught, through direct instruction, to successfully manipulate this variable themselves within an educationally efficient time frame of three days.

Schulman (1981) conducted studies with 105 fourth grade students in Massachusets. The researchers concluded that there was a significant difference in the ability of fourth grade students to solve the types of problems used in the study. One type of problem dealt with data organization and students experienced the greatest difficulty with such problems, with a mean score of 44% correct. The need to develop and assess instructional techniques and materials designed to improve the ability of students to recognize all problem conditions, and to interpret pertinent arithmetic language in problem statements was suggested by the researchers.

Two-Step Word Problems

Zweng (1979) studied both one-step and two-step word problems with 162 students at grades three, four, five and six. She found that for two-step problems achievement was considerably lower than for single-step problems. The analysis was conducted across grade level and across ability level. She found that with respect to grade level, by the sixth grade, most single step problems could be solved by children of all ability levels. The results for two-step problems were, however, less encouraging for the researcher, for success in solving two-step problems did not increase across grade levels. For the problems used in the study, overall, achievement remained almost constant. With respect to ability level, there was very little difference in achievement for single-step problems, but for two-step problems there was a high positive correlation between achievement and ability, with a marked increase in performance from low to average and from average to high ability students.

In a study with 105 fourth grade students Schulman (1981) investigated the ability of these students to solve one-step and two-step word problems. She concluded that students experienced the least difficulty solving one-step problems with a mean score of 82% correct, and hence experienced more difficulty with two-step word problems. She hypothesized that these results could be due to the greater amount of attention paid to one-step problems in the elementary school curriculum, and suggested the need to develop and assess instructional techniques and materials designed to improve the ability of students to solve other types of problems, particularly two-step problems.

In the report of the second NAEP (NCTM, 1980) it was suggested that the primary area of concern should not be simple one-step problems, but multi-step problems that require more than an application of a single arithmetic operation. From the assessment results, it was concluded that students have relatively little difficulty solving problems that only require them to choose the correct operation. Their difficulties with multi-step problems seemed to result from their interpretation that problem solving simply involves choosing the appropriate arithmetic operation and applying it to the numbers given in the problem.

Summary

From reviewing the literature on solving arithmetic word problems, evidence was found that number size and ordering of data do contribute to the level of difficulty of word problems. Problems with larger numbers tend to be more difficult than problems with smaller numbers and unordered problems tend to be more difficult than ordered problems. There is also some evidence that children do not perform as well on multi-step problems as they do on single-step problems.

CHAPTER III

METHODOLOGY

In this chapter, the methodology used in the investigation is described. The population, sample, and the pilot study and the related results are discussed. The instruments used in the main study are outlined and the procedures for the main study are presented. Finally, the methods used to analyze the data are discussed.

Population and Sample

The population consisted of students enrolled in the practical level of mathematics at level one of the senior high school in Newfoundland. The sample was selected from 10 schools and consisted of 220 students. The schools selected for the study were among those in which the investigator knew a teacher on a personal basis so as to ensure 100% participation of the schools contacted. These schools represented several areas of the Avalon Peninsula and western Newfoundland. The contact person in each school, in turn, contacted the teachers who were teaching the practical mathematics course at level one and requested their cooperation in the study.

Pilot Study

A pilot study was carried out for several reasons; first, to ensure that the time allotted for the completion of the test items was sufficient; second, to check the difficulty of the individual items and the complete tests; third, to ascertain the best method of indicating the solution to each word problem; and fourth, to determine any difficulties that might have existed with the written instruments.

Four intact classes, with a total of 84 students, were used for the first pilot study. Three pilot tests were developed, each test consisting of 12 items, for a total of 36 one-step and two-step word problems. The 12 one-step items on the pilot test included items using the operations of addition, subtraction, multiplication, and division crossed with small numbers, large numbers, and a combination of small numbers and large numbers. For the two-step problems, 24 representative problems were chosen from the possible combinations of number size, order, and appropriate sets of operations.

On one pilot test the students were permitted to use a calculator to compute the answer to the problem and were asked to indicate, on the paper, the equations which they used to solve the problem, as well as the numerical solution. On a second pilot test the students were also permitted to use a calculator to compute the answer to the problem but were to indicate on the paper only the final numerical solution. On the third pilot test a calculator was not used and the students were instructed to indicate only the equations they would use to solve the problem and not to calculate the final answer. An example of how the solution was to be written was printed on the instruction sheet for each pilot test. All three pilot tests were administered during the same class period for a given class, with approximately one-third of each class assigned to each pilot test. Students were given 30 minutes to complete each test.

As a result of the first pilot study several decisions were made. First, the time allotted for the completion of the test items was sufficient. Second, two items proved to be too difficult for the sample and consequently were removed from the main study. Third, the sets of operations to be used in the problems in the main study were chosen. These sets of operations were: multiplication - addition; division - addition; multiplication - subtraction; and division - subtraction. These sets of operations were chosen since they are the only possible combinations that do not yield a correct answer when used in an unordered manner. Fourth, the method of indicating the solution to each word problem was chosen. Students in the main study would be instructed to indicate, on the paper, the equations which they used to solve the problem, as well as the numerical solution. This allowed a judgment as to whether answers were wrong because of incorrect operations, incorrect symbolism, or computational errors. It also made it possible to determine if the correct operations had been used, but in the incorrect order. Fifth, the written instructions and the design of the tests posed no difficulties. Sixth, since 94% of the students were successful on the one-step word problems, they were dropped from the final tests. Seventh, students in the main study would be encouraged to use a calculator to compute the numerical solutions since the use of a calculator facilitated the solutions in the pilot study.

Once the initial pilot was complete and the items selected for the tests to be used in the main study, a second pilot was conducted in one class with 24 students. The instruments used were those designed for the main study and this pilot was carried out as a final check on those instruments. Since there were no apparent difficulties with the items, the instruments for the main study were approved.

Instruments

The final instruments consisted of six separate tests. Each of the tests consisted of 12 two-step word problems designed to cover the following sets of operations, with each test containing 3 items for a particular set of operations. The sets of operations were multiplication - addition; division - addition; multiplication - subtraction; and division - subtraction. The tests are described below, with the complete text of each test being included in Appendix A.

Test # 1 This test consisted of 12 problems using small numbers and ordered data.

Test # 2 This test consisted of 12 problems using large numbers and ordered data.

Test # 3 This test consisted of 12 problems using a combination of small numbers and large numbers and ordered data.

Test # 4 This test consisted of 12 problems using small numbers and unordered data.

Test # 5 This test consisted of 12 problems using large numbers and unordered data.

Test # 6 This test consisted of 12 problems using a combination of small numbers and large numbers and unordered data.

Procedure

Each of the six tests was administered to each of the classes participating in the study. In each class all tests were administered simultaneously so that all six tests were completed by different students, with approximately one-sixth of each class completing each test.

The teacher administering the tests was instructed to randomly assign the tests to the students in the class. It was suggested, however, that, if a student did not have a calculator he/she should be given either test #1 or test #4 since these tests did not require as much tedious computation as the remaining tests. This suggestion was made to help avoid frustration on the part of the student when faced with large numbers to compute by hand. It is recognized, however, that this may limit the generalizability of the results of the study.

The students were asked to complete each test in a 30 minute time period. They were instructed that they could use a calculator to assist them in solving the problems but they were to indicate all equations which they used to reach the solution, as well as the numerical solution. On all tests, space was provided after each problem for students to write the solution.

Analysis of Data

Since some students indicated from the equations that were written that they did know how to solve the problem, but were unable to obtain a correct numerical solution because of a computational error, the analysis was conducted with respect to two different sets of criteria.

Each of the six tests was graded for each subject completing that test on the basis of (1) correct equation and (2) correct numerical solution and one mark indicating the total number of solutions correct on both aspects was recorded for each student. A two-factor ANOVA was then used to test differences among means and a Scheffé test was used on the number size factor where appropriate.

Each of the six tests was then graded for each subject completing that test on the basis of (1) correct equation and (2) incorrect numerical solution as a result of computational errors. A second mark, indicating the number of correct equations, was thus recorded for each student. A second ANOVA was conducted and a second Scheffé test used where applicable.

Both sets of ANOVA and Scheffé tests were used to test the following null hypotheses:

Hypothesis 1: There is no significant difference in the difficulty of arithmetic word problems using small numbers, large numbers, and a combination of small numbers and large numbers.

<u>Hypothesis 2</u>: There is no significant difference in the difficulty between arithmetic word problems using ordered data and arithmetic word problems using unordered data.

<u>Hypothesis 3</u>: There is no significant interaction effect between number size and ordering of data in arithmetic word problems.

CHAPTER IV

RESULTS

In this chapter the results of the study are presented. First, the dependent variables are identified and defined. Then the research questions are presented and answered with respect to the results of the analysis of variance and the Scheffé test for each of the dependent variables. Finally, a summary of the results is presented.

Identification of dependent variables

In order to facilitate a discussion of the analysis of the results the dependent variables are identified and defined.

Each of the six tests used in the study was graded for each subject completing that particular test on the basis of (1) correct equation and (2) correct numerical solution, and one mark indicating the total number of solutions correct on both aspects was recorded for each student. This mark, which was referred to as MARK 1, indicated that the student had a solution completely correct.

Each of the six tests was then graded for each subject completing that test on the basis of (1) correct equation and (2) incorrect numerical solution as a result of computational errors. A second mark indicating the number of correct equations was recorded for each student. This mark was referred to as MARK 2.

Thus, a mark out of 12, indicating the number correct, of the 12 questions per test, with respect to both equation and numerical solution was the dependent variable for the first analysis. Then, a mark out of 12, indicating the number of correct equations only was used as the dependent variable for the second analysis.

Research Questions

The purpose of this study was to answer two questions:

Question 1: Does the size of the numbers used in arithmetic word problems affect the difficulty of the problems?

Question 2: Does the order in which the data are presented affect the difficulty of arithmetic word problems?

To answer these questions an analysis of variance was conducted on each of the two dependent variables. To facilitate the presentation of results, each research question is answered in light of the analysis of the data for each of MARK 1 and MARK 2.

Results for MARK 1

The means of each test for MARK 1, the means for each category of number size and the means for both the ordered and unordered categories are presented in table 1.

Table 1

Means for MARK 1

	Small	Small/Large	Large	Row Means
Ordered	10.02 n = 41	7.67 n = 36	7.00 n = 37	8.29
Unordered	8.08 n = 40	6.19 n = 32	4.74 n = 34	6.43
Column Means	9.06	6.97	5.92	

An analysis of variance with unequal n's and disproportional cell frequencies (Glass and Stanley, 1970) was conducted to determine if the differences between the means were significant for the number size variable and the order variable. The effect of an interaction between number size and ordering of data was also investigated. The results of the ANOVA are presented in table 2.

Table 2

Analysis of variance with MARK 1
as the dependent variable

Source of Variation	SS	df	MS	F
Order (A)	5.40	1	5.40	16.76*
Number Size (B)	10.50	2	5.25	16.29*
A x B	0.16	2	0.08	0.24
Within Cells	2511.24	214	0.32	

* $p < 0.01$

From the analysis of the data it was concluded that no significant interaction effect between number size and ordering of data was evident, thus the null hypothesis with respect to interaction was accepted for MARK 1. The relationship among the means of the data is depicted graphically in figure 1.

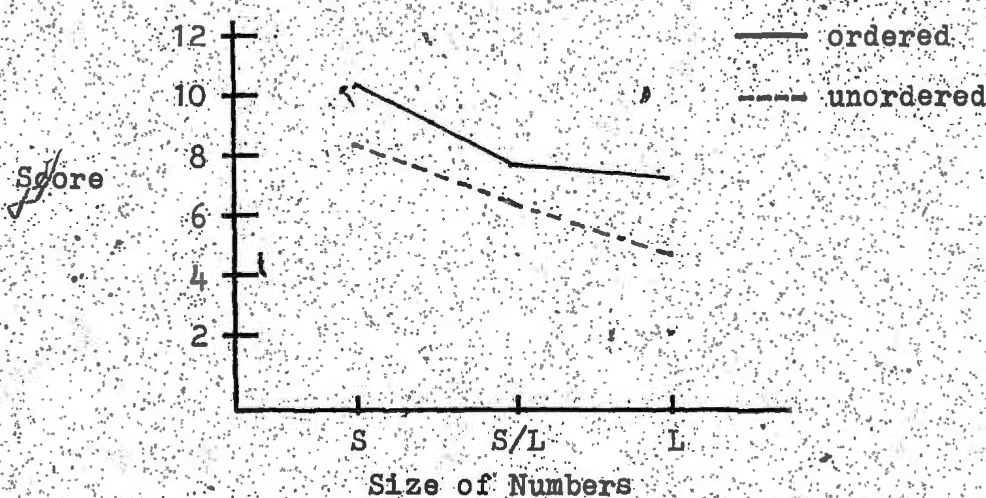

Fig. 1 Relationship among means of tests for MARK 1

Since there was no significant effect of an interaction between number size and ordering of data, the results dealing with the main effects of number size and of order were interpretable. Because of the significant effect indicated for the number size variable in the analysis of variance, a Scheffé test was performed on the means of the tests for each category of number size. The results of the Scheffé test indicated F - ratios of 34.08 for the small versus large contrast, 22.39 for the small versus small/large contrast, and -10.95 for the large versus small/large contrast. All ratios were significant at the 0.01 level of significance.

It was thus concluded that the size of the numbers used in arithmetic word problems did affect the level of difficulty of the problems. Whether the data was ordered or unordered, tests containing large numbers as data had significantly lower means than tests containing a combination of small numbers and large numbers as data, which, in turn, had significantly lower means than tests containing small numbers as data. Hence, the null hypothesis of no significant effect of number size on the difficulty of arithmetic word problems was rejected for MARK 1.

From the analysis of variance it was also concluded that a significant effect for the order variable existed. Whether the data were presented as small numbers, large numbers, or a combination of small numbers and large numbers, the tests which presented the data in an unordered manner had significantly lower means than the tests which presented the data in an ordered manner. Thus, the null hypothesis of no significant effect of ordering of data on the difficulty of arithmetic word problems was rejected for MARK 1.

Results for MARK 2

The means of each test for MARK 2, the means for each category of number size and for both the ordered and unordered categories are presented in table 3.

Table 3

Means for MARK 2

	Small	Small/Large	Small	Row Means
Ordered	10.39 n = 41	8.53 n = 36	8.41 n = 37	9.16
Unordered	8.58 n = 40	6.69 n = 32	5.41 n = 34	6.99
Column Means	9.49	7.67	6.97	

An analysis of variance with unequal n's and disproportional cell frequencies (Glass and Stanley, 1970) was conducted to determine if the differences between the means were significant for the number size variable and the order variable. The effect of an interaction between number size and ordering of data was also investigated. The results of the ANOVA are presented in table 4.

Table 4

Analysis of variance with MARK 2 as the dependent variable

Source of Variation	SS	df	MS	F
Order (A)	7.37	1	7.37	22.78*
Number Size (B)	7.10	2	3.55	10.96*
A x B	0.46	2	0.23	0.70
Within Cells	2518.53	214	0.32	

* $p < 0.01$

From the analysis of the data it was concluded that no significant interaction effect between number size and ordering of data was evident, thus the null hypothesis with respect to interaction was accepted for MARK 2. The relationship among the means of the tests is depicted graphically in figure 2.

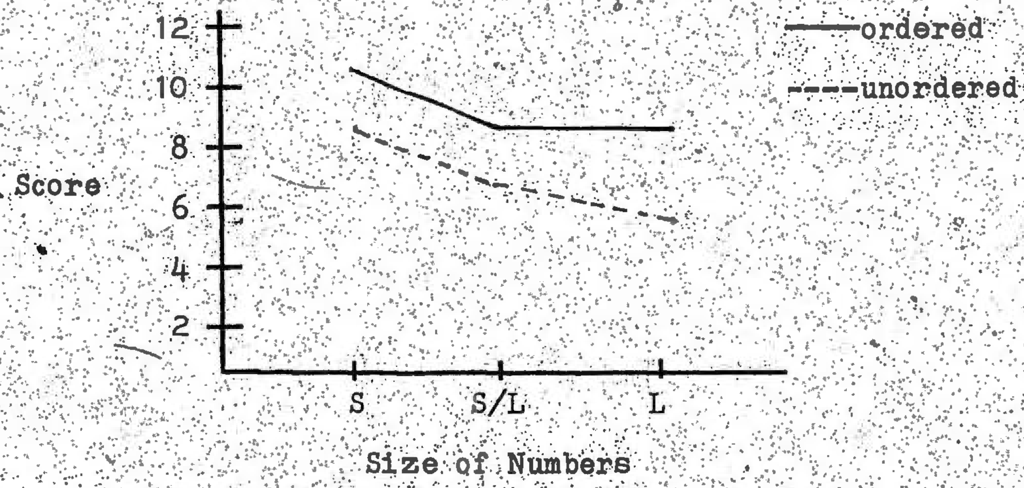

fig. 2 Relationship among means of tests for MARK 2

Since there was no significant effect of an interaction between number size and ordering of data, the results dealing with the main effects of number size and of order were interpretable. Because of the significant effect indicated for the number size variable in the analysis of variance, a Scheffé test was performed on the means of the tests for each category of number size. The results of the Scheffé test indicated F - ratios of 27.28 for the small versus large contrast, 19.59 for the small versus small/large contrast and -7.15 for the large versus small/large contrast. All ratios were significant at the 0.01 level of significance.

It was thus concluded that the size of the numbers used in arithmetic word problems did affect the level of difficulty of the problems. Whether the data was ordered or unordered, tests containing large numbers as data had significantly lower means than tests containing a combination of small numbers and large numbers as data, which, in turn, had significantly lower means than tests containing small numbers as data. Hence, the null hypothesis of no significant effect of number size on the difficulty of arithmetic word problems was rejected for MARK 2.

From the analysis of variance it was also concluded that a significant effect for the order variable existed. Whether the data were presented as small numbers, large numbers, or a combination of small numbers and large numbers, the tests which presented the data in an unordered manner had significantly lower means than the tests which presented the data in an ordered manner. Thus, the null hypothesis of no significant effect of ordering of data on the difficulty of arithmetic word problems was rejected for MARK 2.

<u>Summary</u>

In summary, the results for both dependent variables were similar. For both MARK 1 and MARK 2, the null hypothesis of no significant interaction effect between number size and ordering of data was accepted.

For both MARK 1 and MARK 2, the null hypothesis of no significant effect of number size on the difficulty of arithmetic word problems was rejected. Therefore, in answer to question 1: The size of the numbers used in arithmetic word problems does affect the difficulty of the problems.

For both MARK 1 and MARK 2, the null hypothesis of no significant effect of ordering of data on the difficulty of arithmetic word problems was rejected. Therefore, in answer to question 2: The order in which the data are presented does affect the difficulty of arithmetic word problems.

CHAPTER V

CONCLUSIONS, IMPLICATIONS, RECOMMENDATIONS

In this chapter an overview of the study as well as a statement of the conclusions resulting from an analysis of the data is presented. Then implications for teaching are discussed and recommendations for future research are presented.

Overview

This study was designed to test the effects of number size and ordering of data on the difficulty of arithmetic word problems. The study was conducted at level one of the senior high school system in Newfoundland with students enrolled in the practical mathematics course at that level. Each subject was asked to complete one of six tests which were designed as follows: (1) small numbers with ordered data; (2) large numbers with ordered data; (3) small numbers and large numbers with ordered data; (4) small numbers with unordered data; (5) large numbers with unordered data; (6) small numbers and large numbers with unordered data. The tests were then graded with respect to each of two criteria: (1) both equation and numerical solution correct and (2) equation correct but numerical solution incorrect. The first criteria of both equation and numerical solution correct resulted in the dependent variable identified as MARK 1 while the second criteria of correct equation only resulted in the dependent variable identified as MARK 2. The means of these tests were then subjected to an analysis of variance for each of the two sets of criteria and, where applicable, a Scheffé test was used to determine if the factors of number size and order did influence the difficulty of arithmetic word problems.

Conclusions

As a result of the analysis of the data on each of the two sets of criteria it was found that whether the dependent variable was MARK 1 or MARK 2 the results were the same. It was concluded that, with respect to number size, those tests which contained large numbers of four digits or more as data had the lowest means and, hence, could be concluded to be the most difficult. Those tests which contained small numbers of two digits or less as data had the highest means and hence could be concluded to be the least difficult. Those tests containing both small numbers and large numbers as data had means between the two previously mentioned and hence were of intermediate difficulty.

An indication of the difficulty of the problems in the tests with large numbers is the number of students who failed to attempt specific problems as is indicated in Appendix B. This failure on the part of students to attempt the problems could have been due to lack of knowledge as to how to produce a solution to the problem. If a student did not understand a problem or did not know how to produce a solution then that student may have skipped that problem completely and moved on to those which were perceived to be less difficult. It could also be due to non-availability of a calculator to aid in computation. It was indicated on some papers that the student did not use a calculator and hence could have been frustrated with such large numbers. Also, without the aid of a calculator, some students may have spent too much time on the computation of the earlier problems in the test and then, may not have had sufficient time to complete the entire paper.

Apart from those questions not attempted, the number and type of errors made by the subjects who completed the tests containing large numbers as data indicated an uncertainty on the part of the subjects as to how the problems were to be approached. The list of errors and the frequency of each error for each problem on each of the six tests is presented in Appendix C and gives an indication of how each problem was approached. Upon comparison of the errors made in solving problems containing small numbers as data and similar problems containing large numbers as data, it can be concluded that number size does influence the level of difficulty of arithmetic word problems. Furthermore large numbers caused more difficulty than small numbers, or a combination of small numbers and large numbers, for students at the level studied.

Thus, since no significant interaction effect was found, it was concluded that, whether the data was ordered or unordered, problems containing large numbers as data were the most difficult for students in the practical level of mathematics at level one of the senior high school; problems containing a combination of small numbers and large numbers were of intermediate difficulty, and problems containing small numbers were the least difficult.

With respect to ordering of data, it was determined that there was a significant difference between the means of those tests with ordered data and those tests with unordered data. Those tests on which data were presented in an ordered manner had higher means than those tests on which data were presented in an unordered manner, and hence, it was concluded that problems which present data in an unordered manner were more difficult than problems which present data in an ordered manner.

In many cases, as is evident from the list of errors in Appendix C, subjects failed to recognize that data was unordered and hence set up and solved equations based upon the order of presentation of that data. Also, there was some evidence that some subjects did recognize that they were dealing with an unordered problem but failed to place the data in an order that would lead to the solution of the problem. This failure on the part of students to solve the problem correctly could arise from lack of practice in recognition of an unordered problem. This lack of practice could be the result of insufficient instruction in our school system regarding that particular aspect of problem solving. It could also be due to insufficient knowledge of how to rearrange the data for a correct solution if an unordered problem is recognized. Some errors listed in Appendix C indicate that the student may have recognized that the problem was unordered but was unable to set up and solve a correct equation. In some cases, data were used which 'fit together', for example, two numbers which should have been added or subtracted may have been divided simply because one number was divisible by the other. These errors seem to indicate that students sometimes set up equations based upon the properties of the numbers themselves, rather than on their relationship to each other in the problem as a whole.

It was thus concluded that, whether the numbers were small, large, or a combination of small and large, problems containing unordered data were more difficult for students in the practical level of mathematics at level one of the senior high school, while problems containing data which were ordered were less difficult.

As a result of the analysis of the method by which students recorded their equations and the numerical solutions it was concluded that some students could produce correct numerical solutions while recording equations using incorrect symbolism. A list of such incorrect symbolism with correct numerical solutions, as well as the frequency of response, is included in Appendix D. It is evident that, even though the students knew how to complete the word problem and could produce the correct numerical solution, they did not know how to correctly code the equation, especially when division was involved.

One factor which may influence such symbolism could stem from initial activity with division when students wrote division equations as $a\overline{)b}$ rather than $b \div a$. They apparently did not recognize the non-commutativity of division and continued to write the divisor before the dividend in such equations. It was evident that the students knew how to perform the division correctly since not all equations could be completed from division facts alone, but rather, required long division either manually or with a calculator.

It is also possible to conclude that some students may have performed the operations on the numbers exactly as they were recorded in the equation and, upon finding an unreasonable quotient, decided to try the numbers in reverse order and thus attain a more reasonable, and in this case, correct, answer.

There was also evidence to indicate that, as a result of using a calculator, at least one student failed to produce correct solutions. A list of these errors is included in Appendix E. With the initial equation written correctly, the student produced the quotient that would result if the numbers were entered into the calculator in reverse order, and then proceeded to use that portion of the quotient that best fit in with the remaining number in the problem. This type of error resulted from the failure of the student to recognize the unreasonableness of the quotient and a possible over-confidence that a calculator always gives a 'right' answer.

Again, the student knew how to solve the problems as was in evidence from the correct order of operations, but, as a result of a computational error which resulted from incorrect entry of the data into a calculator, the numerical solution was incorrect.

From further inspection of the list of errors, it was concluded that word problems, per se, whether the numbers are small, large, or a combination of small and large, or whether the data are ordered or unordered, presented difficulty for some students. The errors did not occur solely in the problems with large numbers or solely in the problems with unordered data. There were errors recorded for each problem of each test and this may be a result of the type of student used in the study. Since the practical level of mathematics in Newfoundland schools is designed for those students who have experienced little success in mathematics in the past, even the 'easier' type of problems presented may have been too difficult for some students to solve. While these same students may have very little difficulty with computation in itself, they were often unable to decide on the appropriate computation in the context of a word problem.

Implications for Teaching

Since the current mathematics course in the practical level at level one of the senior high school in Newfoundland has a textbook which is problem oriented, a knowledge of those factors in a problem which make that problem more difficult for the student could aid the teacher in guiding the student to a successful solution of the problem. For some students, number size and ordering of data did influence the difficulty of arithmetic word problems. If a teacher at that level could recognize those aspects of number size and ordering of data that would make the problem more difficult, that teacher could give some instruction to the student on how to approach and to analyze such a problem.

Such instruction would, of necessity, require much caution on the part of the teacher in order to ensure that any translation of the problem did not alter the meaning of the problem or the original relationships among the numbers. Simply replacing large numbers with small numbers would not be useful unless original relationships such as relative size or divisibility and the original meaning of the situation were maintained. Only after these aspects are considered should the translation be presented as an aid to the solution of the original problem. Likewise, a rearrangement of the data might involve more than a simple rearrangement of the numbers. In such cases, it may be necessary to change the structure of the sentence(s) in order to make the new situation conform to the meaning of the original situation. Such a change in sentence structure might, for some students, be unsuitable, for they might not recognize it as a translation of the original problem. Even though a teacher may be aware of the factors of number size and ordering of data, instructions on translation of the problem into a simpler form may be difficult and may require much preparation.

In an attempt to create, in the student, an awareness that number size and ordering of data do affect the difficulty of arithmetic word problems a teacher might ensure that, when using word problems as examples in instruction, different kinds of word problems which incorporate the factors of number size and ordering of data are used. If a particular textbook uses only small numbers and/or ordered data in examples then the teacher could supplement these with problems containing large numbers or a combination of small numbers and large numbers and also problems with unordered data. Instruction using such examples could be followed by supplementary exercises, especially if the textbook exercises are not sufficient in this area.

To further increase awareness of the factors that make arithmetic word problems more difficult, a teacher might include such aspects as number size and ordering of data when planning lessons on how to approach word problems.

As a result of the types of errors that were made in symbolism and in computation there might be some argument for special instruction, at this level, in how equations are to be set up, with particular emphasis on the non-commutativity of division and subtraction. This area would appear to be a very suitable area of instruction for the use of the calculator as an instructional aid rather than just a tool to aid in computation. This type of instruction should ensure that the student gain a good working knowledge of order of entry of data with respect to the order of operations in the equation.

It would seem, then, that the major implication for teaching is 'awareness'. An awareness on the part of the teacher of the factors that influence the difficulty of arithmetic word problems should aid that teacher in providing learning experiences that would allow students to attain greater success in the solution of arithmetic word problems.

Recommendations for Future Research

Since the study was conducted with only students in the practical level of mathematics at level one of the senior high school, its results may not be generalizable to other levels of mathematics or to other grade levels in the school system. Further research could focus on the effect of number size and ordering of data on word problems used at the elementary and junior high school grades; in the academic and advanced levels of mathematics at level one and in the practical, academic and advanced levels of mathematics at levels two and three of the senior high school system. Also, since the study was conducted in 10 of the senior high schools in the province, future research might include a different cross-section of schools, or a larger number of schools, or schools outside the province of Newfoundland and Labrador.

The problems used in the tests were two-step word problems which dealt with specific sets of operations and thus the results may not be generalizable to other multi-step word problems or to word problems using a different combination of operations. Future research in this area might concentrate on other multi-step word problems or on word problems using sets of operations different from those used in this study in order to give a more comprehensive view of the role of number size and ordering of data.

Future research of a more general nature with respect to word problems could concentrate on the types of errors that students make in the solution of word problems. These errors could be in computation, either with or without a calculator, or could be in recording the equations and/or solutions to the word problems.

BIBLIOGRAPHY

BIBLIOGRAPHY

Begle, Edward G. Critical Variables in Mathematics Education: Findings from a Survey of the Empirical Literature. Washington, D.C.: Mathematical Association of America and the National Council of Teachers of Mathematics, 1979.

Brewer, Shirley S. "The Scientific Method of Problem Solving," The Arithmetic Teacher, III (April, 1956), 117 - 118.

Brownell, William A. "Problem Solving", The Psychology of Learning, Forty-first Yearbook of the National Society for the Study of Education, Part II, Chicago: University of Chicage Press, 1942, 415 - 443.

Burns, P.C. and J.L. Yonally. "Does the Order of Presentation of Numerical Data in Multisteps Arithmetic Problems Affect their Difficulty?" School Science and Mathematics, LXIV (April, 1964), 267 - 270.

Days, H.C., G.H. Wheatley and G. Kulm. "Problem Structure, Cognitive Level, and Problem-Solving Performance," Journal for Research in Mathematics Education, X, No. 2 (1979), 135 - 144.

Division of Instruction. Course Description, Advanced Mathematics 1201. Department of Education, Government of Newfoundland and Labrador, 1982.

Division of Instruction. Course Description, Consumer Mathematics 1202. Department of Education, Government of Newfoundland and Labrador, 1982.

Division of Instruction. Course Description, Academic Mathematics 1203. Department of Education, Government of Newfoundland and Labrador, 1982.

Durell, Fletcher. "Solving Problems in Arithmetic," School Science and Mathematics, XXVII (November, 1928), 925 - 935.

Foran, T.G. "Methods of Teaching Problem Solving in Arithmetic," Catholic Educational Review, XXXII (May, 1934), 269 - 282.

Glass, G.V. and J.C. Stanley. Statistical Methods in Education and Psychology. Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1970. Ch. 17.

Grossnickle, Foster E. "Cues in Division Problems Given in Nine Representative Textbooks in Arithmetic," Elementary School Journal, XXXII (February, 1933), 452 - 461.

Hooten, J.R. ed. Teaching Arithmetic in the Elementary School. Vol. XIV, Soviet Studies in the Psychology of Learning and Teaching Mathematics. Virginia: National Council of Teachers of Mathematics, 1975.

Ingle, Josephine A. "Prediction of Word Problem Difficulty on the Basis of Problem Characteristics". (Doctoral Dissertation, George Peabody College for Teachers, 1975) Dissertation Abstracts International, 1975, 36, 2157 A (University Microfilms No. 75-22273.)

Kantowski, Mary G. "Some Thoughts on Teaching for Problem Solving," Krulik, Stephen and Robert Reys, ed. Problem Solving in School Mathematics. 1980 Yearbook of the National Council of Teachers of Mathematics. Virginia: National Council of Teachers of Mathematics, 1980.

Knifong, J. Dan and Boyd D. Holtan. "A Search for Reading Difficulties among Erred Word Problems," Journal for Research in Mathematics Education, LXXXIII (1977), 227 - 30.

Kramer, Klaas. Teaching Elementary School Mathematics. Boston: Allyn and Bacon, Inc., 1975.

National Council of Teachers of Mathematics. An Agenda for Action: Recommendations for School Mathematics of the 1980s. Virginia: National Council of Teachers of Mathematics, Inc., 1980.

National Council of Teachers of Mathematics. Results from the First Mathematics Assessment of the National Assessment of Educational Progress. Virginia: National Council of Teachers of Mathematics, 1978.

National Council of Teachers of Mathematics. Results from the Second Mathematics Assessment of the National Assessment of Educational Progress. Virginia: National Council of Teachers of Mathematics, 1980.

Polya, George. How to Solve It. Garden City, New York: Doubleday, 1957.

Polya, George. Mathematical Discovery. New York: John Wiley and Sons, Inc., Vol. 1, 1962.

Rosenthal, Daniel J. and Lauren B. Resnick. "The Sequence of Information in Arithmetic Word Problems." Paper presented at the meeting of the American Educational Research Association, New York, N.Y. Feb. 4 - 7, 1971.

Rosenthal, Daniel J. and Lauren B. Resnick. "Children's Solution Processes in Arithmetic Word Problems," Journal of Educational Psychology, LXVI (December, 1974), 817 - 825.

Schulman, Linda A. "The Ability of Fourth Grade Students to Solve Five Types of Mathematical Problems." (Doctoral Dissertation, Boston University School of Education, 1981) Dissertation Abstracts International, 1982, 42, 3895 A - 3896 A (University Microfilms No. 82 - 03906.)

Segally, Angelo. "Using Structural Variables to Predict Word-Problem Solving Difficulty for Junior College Arithmetic Students". Paper presented at the annual meeting of the American Educational Research Association, New Orleans, Louisiana, February, 1973.

Smith, Robert M. Clinical Teaching: Methods of Instruction for the Retarded. New York: McGraw Hill Book Co., 1968.

Stover, Georgia, B. "Structural Variables Affecting Mathematical Word Problem Difficulty in Sixth Graders". (Doctoral Dissertation, University of San Francisco, 1979) Dissertation Abstracts International, 1982, 42, 5050 - A (University Microfilms No. 82 - 11361

Suydam, Marilyn, and J. Fred Weaver. Overview... Verbal Problem Solving, Set. B. Using Research: A Key to Elementary School Mathematics. Pennsylvania State University, University Park. Center for Cooperative Research with Schools, 1970.

Thorndike, Edward L. The New Methods in Arithmetic. Chicago: Rand, McNally and Co., 1921.

University of Alberta. PRISM Canada (Priorities in School Mathematics.) Edmonton: Faculty of Education, University of Alberta, 1981.

West, Tommie A. "R_x for Verbal Problems: A Diagnostic - Prescriptive Approach," The Arithmetic Teacher, XXV (November, 1977), 57 - 58

Willoughby, Stephen S. Contemporary Teaching of Secondary School Mathematics. New York: John Wiley and Sons, Inc., 1967.

Wong, Shirley M. The Secondary Consumer Mathematics Curriculum in British Columbia - Present and Proposed. Report # 79:23 of the Educational Research Institute of British Columbia. Vancouver, June 30, 1979.

Zweng, Marilyn J. "Children's Strategies of Solving Verbal Problems. Final Report." National Institute of Education (DHEW), Washington, D.C., August 1, 1979.

Zweng, Marilyn J. "The Problem of Solving Story Problems," The Arithmetic Teacher, XXVII (September, 1979), 2 - 3.

APPENDIX A

INSTRUMENTS

Test # 1

Instructions to students

In this test you may use a calculator to compute the answer to each of the word problems.

You must show all equations which you use to reach your solution.

Example: John had 12 apples and bought 10 more. If he gave away 15 apples, how many apples does he have?

(Answer) 12 + 10 = 22
22 - 15 = 7

1. Joe had 11 photo albums with 9 pictures in each album. He received 9 pictures from Tooton's today. How many pictures does he have now?

2. Mrs. Jones distributed 96 candies among 16 children. Later, she gave each child 4 more candies. How many candies did each child receive?

3. The student council has 23 people making Winter Carnival buttons and each person can make 8 buttons. If 11 of the buttons are broken, how many are not broken?

4. A teacher has 75 bookmarks to distribute among 3 students. If each student had to give away 15 of these bookmarks, how many bookmarks did each student keep?

5. Mr. Thomas distributed 60 posters among 5 children. Later, he gave each child 10 more posters. How many posters did each child receive?

6. Mom has 12 children making toys for a fair and each child can make 8 toys. If 4 of the toys were broken, how many were unbroken?

7. The principal had 84 books to distribute among 6 students. If each student had to sell 3 of these books, how many books could each student keep?

8. Fabricville had 15 rolls of velvet material with 6 m of velvet on each roll. The store received 18 m of velvet from the factory today. How many metres of velvet do they have now?

9. Susan bought 10 boxes of peanuts to sell for school lunches and each box contained 24 bags of peanuts. If 8 of the bags are broken, how many bags are unbroken?

10. The Arts and Culture Centre had 96 opera tickets to distribute among 12 tourist information officers in the province. If each officer had to sell 6 of these tickets, how many tickets could each officer keep?

11. Roncalli School had 19 classes with 31 students in each class. The principal accepted 8 students from Nova Scotia this month. How many students does the school have now?

12. Miss Smith shared 24 books among 8 children in the hospital. Later, she gave each child 4 more books. How many books did each child receive?

Test # 2

Instructions to Students

In this test you may use a calculator to compute the answer to each of the word problems.

You must show all equations which you use to reach your solution.

Example: John had 12 apples and bought 10 more. If he gave away 15 apples, how many apples does he have?

(Answer) 12 + 10 = 22

22 - 15 = 7

1. A newspaper office had 4576 photo albums with 1250 pictures in each album. They received 2850 pictures from their photographers today. How many pictures do they have now?

2. In a contest, $250 000 was distributed among 2500 people. If each person later received $1250 more, how much money did each person receive?

3. Workers in a glass factory packed 1298 boxes of ashtrays with 2400 ashtrays in each box. If 1081 of the ashtrays were broken, how many were unbroken?

4. Ultramar Canada Inc. had 6 734 000 glasses to distribute among 5180 service stations. If each service station gave away 1295 glasses in a competition, how many glasses did each service station keep?

5. A company distributed a bonus of $3 442 494 among 2838 employees. If each employee later received $1419 more, how much money did each employee receive?

6. K-Mark Department Stores across Canada bought 2400 cases of greeting cards and each case contained 1440 cards. If 1217 of the cards were damaged how many were not damaged?

7. The organizers of a TROOPER concert tour had 4 534 950 tickets to distribute among 3675 areas where the group would perform. If each area sold 1225 tickets, how many tickets were unsold in each area?

8. A warehouse had 5238 rolls of material with 1200 m of material on each roll. The warehouse received 3217 m of material from the factory today. How many metres of material do they have now?

9. An electronics firm ordered 1000 boxes of wire and each box contained 1800 m of wire. If 1329 m of wire was damaged in shipping, how many metres of wire were not damaged?

10. A publishing company distributed 3 175 102 magazines among 2506 drugstores across Canada. If each drugstore sold 1253 magazines, how many magazines were unsold in each drugstore?

11. A city library had 1500 shelves with 1200 books on each shelf. They received 1182 books from the publishers this month. How many books do they have now?

12. A publishing company distributed 1 539 180 encyclopedias among 3018 sales offices. If each office later received 1006 more, how many encyclopedias did each office receive?

Test # 3

Instructions to students

In this test you may use a calculator to compute the answer to each of the word problems.

You must show all equations which you use to reach your solution.

Example: John had 12 apples and bought 10 more. If he gave away 15 apples, how many apples does he have?

(Answer) 12 + 10 = 22

22 - 15 = 7

1. The newspaper office had 1000 photo albums with 95 pictures in each album. They received 58 pictures from their photographers today. How many pictures do they have now?

2. A school distributed 3600 tickets among 90 students. If each student later received 15 more tickets, how many tickets did each student have to sell?

3. Woolco Department Stores have 1893 stores across North America and each store has 93 employees. If 75 763 employees are men, how many employees are women?

4. Newfoundland Hydro distributed 6030 pins among 45 employees. If each employee sold 90 of these pins, how many pins did each employee return to the company?

5. A company distributed a bonus of $9630 among 18 employees. If each employee later received $1926 more, how much did each employee receive?

6. A factory had 1102 people making party hats and each person can make 36 hats. If 1802 of the hats were damaged, how many were not damaged?

7. A company had 9056 vacuum cleaners to distribute among 8 sales offices. If each sales office sold 1129 vacuum cleaners, how many vacuum cleaners were unsold in each office?

8. A warehouse had 2819 rolls of material with 72 m of material on each roll. The warehouse received 1895 m of material from the factory today. How many metres of material do they now have?

9. The Kinsman Club in a city has 2600 members and each member is asked to sell 5 tickets on a car. If 18 tickets were not sold, how many were sold?

10. VOCM had 2590 bumper stickers to distribute among 74 service stations across the province. If each service station gave away 32 bumper stickers, how many bumper stickers did each service station keep?

11. A store had 2350 boxes of bars with 8 bars in each box. They received 2400 bars from their suppliers today. How many bars do they now have?

12. A publishing company distributed 1092 books among 28 salesmen. If each salesman later received 7 more books, how many books did each salesman receive?

Test # 4

Instructions to Students

In this test you may use a calculator to compute the answer to each of the word problems.

You must show all equations which you use to reach your solution.

Example: John had 12 apples and bought 10 more. If he gave away 15 apples, how many apples does he have?

(Answer) 12 + 10 = 22
22 - 15 = 7

1. The school received 8 library books from the school board. Each of the 19 classes also collected 9 books. How many books will be in the library?

2. At a party, Mrs. Jones gave each child 2 balloons. Later, she distributed 50 more balloons among the 10 children at the party. How many balloons did each child receive at the party?

3. The class president paid $45 for a calculator on which he sold tickets. If he sold 98 tickets at $2 each, how much profit did he make?

4. Each student on a team had to sell 12 buttons. If Fred distributed 96 buttons among 6 students on the team, how many buttons could each student keep?

5. Mr. Thomas gave each student 12 notebooks. Later, he distributed 36 notebooks among the 4 students. How many notebooks did each student now have?

6. The coach paid $8 for a prize for the team's fund raising drive. If the team sold 15 tickets at $2 each, how much profit did they make?

7. Each member of the cheerleading team had to sell 4 bars. If the captain distributed 48 bars among the 8 girls, how many bars could each girl keep?

8. Marie received 50 stamps from a friend today. She already had 19 packages of stamps with 6 stamps in each package. How many stamps does she now have?

9. John lost 8 of his cassette tapes. If he had 12 cases with 24 tapes in each case, how many tapes does he have now?

10. Each player on a team had to sell 5 tickets on a car. If the team manager distributed 80 tickets among the 10 players, how many tickets could each player keep?

11. Household Finance hired 7 employees this month. They already had 28 offices across Canada with 12 employees in each office. How many employees do they have now?

12. Each student in a club sold 4 tickets on a stereo. Later, the teacher distributed 72 tickets among the 8 students in the club. How many tickets did each student sell on the stereo?

Test # 5

Instructions to Students

In this test you may use a calculator to compute the answer to each of the word problems.

<u>You must show all equations which you use to reach your solution.</u>

Example: John had 12 apples and bought 10 more. If he gave away 15 apples, how many apples does he have?

(Answer) 12 + 10 = 22

22 - 15 = 7

1. The newspaper office received 1230 stories from their writers this month. They already had 2983 files with 1250 stories in each file. How many stories do they have now?

2. Each winner in a lottery received $1250. Later, each of the 2500 winners shared in $150 000. How much money did each winner now have?

3. General Motors paid out $7923 in taxes last month. If they sold 1009 cars at $8500 each, how much profit did they make?

4. Each branch office of American Motors must contribute $4500 to the Heart Foundation. The company president shared $6 942 000 among the 1500 offices. How much money will each office have after they have contributed to the Heart Foundation?

5. Each employee of a company received a bonus of $1000. Later, the company distributed another bonus of $1 722 000 among the 3000 employees. How much did each employee now have?

6. An electronics company rejected 4763 damaged components last year. If they had 1230 employees and each employee made 2570 components, how many components were not rejected?

7. Each sales office for an encyclopaedia company must sell 2000 books. If the company distributed 2 686 000 books among the 1000 sales offices, how many books could each sales office keep?

8. A fish plant processed 2530 kg of fish today. They had already processed 5295 boxes with 1000 kg of fish in each box. How many kg of fish have they processed now?

9. A store manager found 2583 damaged rolls of film in todays shipment. If there were 1008 cases of film and each case contained 1000 rolls of film, how many rolls of film were not damaged?

10. Each salesman for a magazine company must sell 1800 magazines this year. If the company distributed 9 768 600 magazines among the 5400 salesmen, how many magazines could each salesman keep?

11. A department store received 1250 record albums for a special sale. They already had 1100 cases with 1440 albums in each case. How many record albums do they have now?

12. Each sales office for ABC Publishing Co. received 2048 cookbooks. Later, the company distributed 1 625 088 cookbooks among the 1024 offices. How many cookbooks did each office have now?

Test # 6

Instructions to Students

In this test you may use a calculator to compute the answer to each of the word problems.

You must show all equations which you use to reach your solution.

Example: John had 12 apples and bought 10 more. If he gave away 15 apples, how many apples does he have?

(Answer) 12 + 10 = 22

22 - 15 = 7

1. A computer club received 93 programs from their members this month. They already had 1750 tapes with 25 programs on each tape. How many programs do they have now?

2. Each student in a class received 10 tickets to sell. Later, the teacher distributed 1200 tickets among the 30 students. How many tickets did each student now have?

3. Canadian Tire stores across Canada have 51 630 employees who are men. If there are 1021 stores with 98 employees in each store, how many employees are women?

4. The employees of a company had to sell 80 pens each. If the company distributed 3360 pens among the 40 employees, how many pens could each employee keep?

5. Each stamp collector in a club received 1000 stamps when they joined the club. Later, the club president distributed 2000 stamps among the 50 members. How many stamps does each member now have?

6. Ultramar paid out $8475 in city taxes last year. If they sold 9576 barrels of oil at $40 per barrel, how much profit did they make?

7. Each Household Finance office must contribute $1200 to their social fund. The president shared a bonus of $8400 among the 6 offices. How much money will each office have after they have contributed to their social fund?

8. A movie theatre canteen received 5410 bars from their suppliers today. They already had 1092 boxes of bars with 12 bars in each box. How many bars do they now have?

9. The boys club needed $99 to pay for softball equipment. If there are 1003 members in the club and each member raised $2, how much money did they have after buying they softball equipment?

10. Each student on a summer employment program sold 30 copies of the Enchclopedia of Newfoundland and Labrador. If the publisher distributed 6000 copies among 60 students, how many copies did each student return?

11. A clothing factory received 3250 buttons today. They already had 2573 packages of buttons with 6 buttons in each package. How many buttons do they have now?

12. Mrs. Brown collected 9 postcards on a trip. Later, her travel club distributed 1404 postcards among its 54 members. How many postcards does she now have?

APPENDIX B

NUMBER OF STUDENTS WHO FAILED TO
ATTEMPT INDIVIDUAL PROBLEMS

Table 5

NUMBER OF STUDENTS WHO FAILED TO ATTEMPT INDIVIDUAL PROBLEMS

	Test #1	Test #2	Test #3	Test #4	Test #5	Test #6
Problem 1	-	-	-	2	-	1
Problem 2	-	1	-	2	-	-
Problem 3	-	-	2	-	1	9
Problem 4	-	2	1	2	6	1
Problem 5	-	1	1	-	4	1
Problem 6	-	1	1	-	6	1
Problem 7	-	1	1	1	4	5
Problem 8	-	4	5	1	3	4
Problem 9	-	6	1	2	4	4
Problem 10	1	6	1	1	7	6
Problem 11	1	7	2	1	11	8
Problem 12	1	10	2	1	11	8
	n = 41	n = 37	n = 36	n = 40	n = 34	n = 32

The number of students assigned a specific test is indicated at the bottom of table 5.

APPENDIX C

LIST OF ERRORS FOR EACH PROBLEM
OF EACH TEST AND FREQUENCY
OF EACH ERROR

In this Appendix the specific errors made for each problem of each test and the frequency of each error are presented.

The correct equations and correct numerical solution are included in parentheses for each problem.

SPECIFIC ERRORS MADE FOR EACH PROBLEM OF TEST # 1

PROBLEM 1 (11 x 9 = 99, 99 + 8 = 107)

ERROR	FREQUENCY
9 + 8 = 17	4
9 x 11 = 99, 99 + 8 = 917	1
11 + 9 = 20, 20 + 8 = 28	1
11 x 9 = 99, 99 + 9 = 108	1

PROBLEM 2 (96 ÷ 16 = 6, 6 + 4 = 10)

ERROR	FREQUENCY
96 ÷ 16 = 6.25, 6.25 + 4 = 10.25	1
96 ÷ 16 = 3, 16 x 4 = 84	1
96 x 16 = 1536, 1536 + 64 = 1600	1
96 + 4 = 100, 100 ÷ 16 = 6 R4	1
96 - 16 = 80, 80 - 4 = 76	1
96 ÷ 4 = 16, 24 + 4 = 28	1

PROBLEM 3 (23 x 8 = 184, 184 - 11 = 173)

ERROR	FREQUENCY
23 + 8 = 31, 31 - 11 = 20	2
23 x 8 = 184	1
23 x 8 = 184, 184 - 11 = 174	1
23 x 8 = 32	1
23 x 8 = 184, 184 - 184 = 0	1

PROBLEM 4 (75 ÷ 3 = 25, 25 - 15 = 10)

ERROR	FREQUENCY
75 ÷ 15 = 60, 30 ÷ 60 = 20	1
75 - 3 = 72, 72 - 15 = 57	1

PROBLEM 5 (60 ÷ 5 = 12, 12 + 10 = 22)

ERROR	FREQUENCY
60 - 5 = 55, 55 - 10 = 45	1
5 ÷ 60 = 12, 12 + 10 = 10	1

PROBLEM 6 (12 x 8 = 96, 96 - 4 = 92)

ERROR	FREQUENCY
12 + 8 = 20, 20 - 4 = 16	2
12 x 8 = 96, 12 x 4 = 48	1
12 x 8 = 96, 96 - 8 = 88	2
12 x 8 = 96, 96 - 4 = 94	1
12 ÷ 3 = 4, 4 - 4 = 0	1

PROBLEM 7 (84 ÷ 6 = 14, 14 − 3 = 11)

ERROR	FREQUENCY
84 ÷ 6 = 14, 84 ÷ 3 = 28	1
84 ÷ 6 = 18, 18 − 3 = 15	1
84 − 6 = 78, 78 − 3 = 75	1
84 + 6 = 90, 90 − 3 = 87	1
84 ÷ 6 = 14, 14 − 3 = 12	1
84 ÷ 6 = 14, 14 − 3 = 9	1
84 ÷ 6 = 14, 12 − 3 = 9	1

PROBLEM 8 (15 x 6 = 90, 90 + 18 = 108)

ERROR	FREQUENCY
15 x 6 = 90, 18 ÷ 6 = 3	1
18 − 6 = 12	1
15 x 6 = 90, 15 x 24 = 360	1
18 + 6 = 24	5
18 x 6 = 108	2
15 x 6 = 90, 18 x 6 = 108, 108 + 90 = 198	1
15 x 6 = 90, 90 − 18 = 72	4
15 + 6 = 21, 21 + 18 = 39	1
18 + 6 = 24, 24 x 15 = 360	1

PROBLEM 9 (10 x 24 = 240, 240 − 8 = 232)

ERROR	FREQUENCY
10 x 16 = 160, 24 − 8 = 16	1
10 x 24 = 244, 244 − 8 = 236	1
240 ÷ 10 = 24, 24 − 8 = 16	1
24 − 8 = 16	3
10 + 24 = 34, 34 − 8 = 26	1

PROBLEM 10 (96 ÷ 12 = 8, 8 − 6 = 2)

ERROR	FREQUENCY
96 x 12 = 1152, 6 ÷ 96 = 16	1
96 − 12 = 84, 84 − 6 = 78	1
96 ÷ 12 = 8, 8 − 4 = 4	1

PROBLEM 11 (19 x 31 = 589, 589 + 8 = 597)

ERROR	FREQUENCY
19 + 31 = 50, 50 + 8 = 58	1
19 x 31 = 499, 499 + 8 = 507	1
19 x 31 = 489, 489 − 8 = 481	1
19 x 31 = 589, 589 + 8 = 197	1
19 x 31 = 76, 76 + 8 = 14	1
19 x 31 = 589, 589 − 8 = 581	2

PROBLEM 12 (24 ÷ 8 = 3, 3 + 4 = 7)

ERROR	FREQUENCY
24 ÷ 8 = 3, 4 had 3 books, 4 had 4 books	1
24 ÷ 8 = 4, 4 + 4 = 8	1
24 - 8 = 16, 16 - 4 = 12	1
24 ÷ 8 = 3, 3 + 4 = 12	1
8 ÷ 24 = 192, 192 + 4 = 196	1
24 ÷ 8 = 3, 8 ÷ 4 = 12	1

SPECIFIC ERRORS MADE FOR EACH PROBLEM OF TEST # 2

PROBLEM 1 (4576 x 1250 = 5 720 000, 5 720 000 + 2850 = 5 722 850)

ERROR	FREQUENCY
1250 + 2850 = 4100	7
4576 + 1250 = 5826, 5826 - 2850 = 2976	2
4576 x 1250 = 595 000, 595 000 + 2850 = 597 850	1
4576 x 1250 = 5 720 000, 5 720 000 + 2850 = 7 722 850	1
4576 x 1250 = 1 144 000, 1 144 000 + 2850 = 1 146 850	1
4576 x 1250 = 5 720 000, 5 720 000 - 2850 = 5 717 150	1
4576 + 1250 + 2850 = 8676	1
4576 - 1250 = 3326, 3326 + 2850 = 6176	1
4576 x 1250 = 366 080, 366 080 + 2850 = 368 930	1
4576 x 1250 = 5 515 000, 5 515 000 + 2850 = 5 517 850	1

PROBLEM 2 (250 000 ÷ 2500 = 100, 1250 + 100 = 1350)

ERROR	FREQUENCY
250 000 ÷ 2500 = 200, 200 + 1250 = 1450	2
250 000 ÷ 2500 = 100	1
250 000 ÷ 2500 = 10, 10 + 1250 = 1260	4
250 000 ÷ 2500 = 0.01, 0.01 x 1250 = 12.50	1
2500 ÷ 250 000 = 100, 100 x 1250 = 12 500	1
250 000 ÷ 2500 = 100, 1250 ÷ 100 = 12.50	1
250 000 + 1250 = 251 250	1
250 000 ÷ 2500 = 1000, 1000 + 1250 = 2250	1

PROBLEM 3 (1298 x 2400 = 3 115 200, 3 115 200 - 1081 = 3 114 119)

ERROR	FREQUENCY
2400 - 1081 = 1319	4
2400 x 1298 = 3 115 200, 3 115 200 - 1081 = 3 114 120	1
1298 x 240 = 311 520, 311 520 - 1081 = 310 439	1
1298 x 2400 = 2 647 200, 2 647 200 - 1081 = 2 646 119	1
1998 x 2400 = 4 795 200, 4 795 200 - 1081 = 478 419	1
2400 x 1298 = 3 115 200, 3 115 200 - 1081 = 31 152 010	1
1298 + 2400 = 3698, 3698 - 1081 = 2617	1
1298 x 2400 = 3 115 200, 3698 - 1081 = 3 116 281	1
1198 x 2400 = 311 520, 311 520 - 1081 = 310 440	1
1298 x 2400 = 3 015 200, 3 015 200 - 1081 = 3 014 119	1
1081 x 2400 = 2 594 400	1
2400 - 1081 = 3319	1

PROBLEM 4 (6 734 000 ÷ 5180 = 1300, 1300 - 1295 = 5)

ERROR	FREQUENCY
6 734 000 ÷ 5180 = 131, 1295 - 131 = 1104	1
6 734 000 ÷ 5180 = 0.0 007 692, 7692 - 1295 = 6397	1
6 734 000 - 1295 = 632 705, 632 705 ÷ 5180 = 1299 R75	1
6 734 000 ÷ 5180 = 121, 1295 + 121 = 1416	1
6 734 000 ÷ 5180 = 1210, 1295 - 1210 = 85	1
5180 x 1295 = 6 708 100, 6 734 000 - 6 708 100 = 25 900	1
6 734 000 ÷ 5180 = 7692, 7692 - 1295 = 6397	1
6 734 000 ÷ 5180 = 121, 5180 - 1295 = 3985	1

PROBLEM 5 (3 442 494 ÷ 2838 = 1213, 1213 + 1419 = 2632)

ERROR	FREQUENCY
3 442 494 ÷ 2838 = 1213, 1213 + 1419 = 2732	1
3 442 494 ÷ 2838 = 1214.055, 1214.055 + 1419 = 1215.474	1
3 442 494 ÷ 2838 = 1213, 1213 x 1419 = 1 721 249	1
3 442 494 ÷ 2838 = 0.0008244, 8244 + 1419 = 9663	1
3 442 494 ÷ 2838 = 1213	1
3 442 494 ÷ 2838 = 121 R8514	1
3 442 494 ÷ 2838 = 1214.05, 1214.05 + 1419 = 2633.05	1
8244 + 1419 = 9663	1

PROBLEM 6 (2400 x 1440 = 3 456 000, 3 456 000 - 1217 = 3 454 783)

ERROR	FREQUENCY
2400 x 1440 = 432 000, 432 000 - 1217 = 430 783	1
1440 - 1217 = 223	2
2400 x 1440 = 3 456 000, 3 456 000 ÷ 1217 = ___	1
2400 x 1440 = 3 513 600, 3 513 600 ÷ 1217 = 2887, (2887 - 1217 = 1670	1
2400 ÷ 1440 = ___	1
2400 ÷ 1440 = 165	1
1440 - 1217 = 423	1
2440 x 1440 = 3 513 600, 3 513 600 - 1217 = 3 512 377	1
2400 x 1440 = 3 168 000, 3 168 000 - 1217 = 3 166 783	1
2400 + 1440 = 3840, 3840 - 1217 = 2623	1
2400 x 144 = 345 600, 345 600 - 1217 = 344 383	1
2400 + 1440 = 3840, 3840 = 1217 = 2663	1
2400 x 1440 = 219 600, 219 600 = 1217 = 218 383	1

PROBLEM 7 (4 534 950 ÷ 3675 = 1234, 1234 - 1225 = 9)

ERROR	FREQUENCY
4 534 950 ÷ 3675 = 1234, 1235 - 1225 = 10	1
3675 - 1225 = 2450	1
4 534 950 ÷ 3675 = 1234, 1242 ÷ 1225 = 1	1
4 534 950 ÷ 3675 = __	3
4 534 950 ÷ 3675 = 1234, 3675 x 1225 = 4 501 875, 4 534 950 - 4 501 875 = 33 075	1
4 534 950 ÷ 3675 = 1261.3, 1261.3 - 1225 = 11 388	1
4 534 950 ÷ 3675 = 0.0008103, 8103 - 1225 = 6878	1
3675 x 1225 = 4 501 875, 4 534 950 - 4 501 875 = 33 075	1
3675 x 1225 = 4 501 875, 4 501 875 -453 490 = 4 048 385	1
4 534 950 ÷ 3675 = 12 330, 12 330 - 1225 = 11 105	1
4 534 950 ÷ 3675 = 8103, 8103 - 1225 = 6878	1

PROBLEM 8 (5238 x 1200 = 6 285 600, 6 285 600 + 3217 = 6 288 817)

ERROR	FREQUENCY
5238 ÷ 1200 = 4.365, 3217 x 4.365 = 14 042.205	1
5238 ÷ 1200 = 4.365, 3217 + 4.365 = 3221.365	1
3217 + 1200 = 4417	1
5238 x 1200 = 1 099 980, 1 099 980 + 3217 = 1 103 197	1
5238 x 1200 = 6 285 600, 6 285 600 - 3217 = 6 282 383	1
5238 x 1200 = 6 285 600, 6 285 600 x 3217 = 2022.20775	1
5238 ÷ 1200 = 4.365, 4 365 + 3217 = 7582	1
5238 ÷ 1200 = 4, 4 x 3217 = 12 868	1
5238 x 1200 = 6 285 600, 6 285 600 ÷ 3217 = 18	1
1200 x 3217 = 3 860 400	1
5238 + 1200 = 6438, 6438 - 3217 = 8459	1
5328 x 1200 = 6 393 600, 6 393 600 + 3217 = 6 396 817	1
5238 x 1200 = 6 285 600, 6 285 600 + 3217 = 6 288 877	1
5238 + 3217 = 8455, 8455 - 1200 = 7255	1
5238 - 3217 = 2021	1

PROBLEM 9 (1000x 1800 = 1 800 000, 1 800 000 - 1329 =1 798 671

ERROR	FREQUENCY
1800 x 1000 = 1 800 000, 1 800 000 - 1329 = 1 796 871	1
1800 ÷ 1000 = 1.8, 1.8 - 1329 = 1327.2	1
1800 - 1329 = 471	1
1000 x 1800 = 1 800 000, 1 800 000 - 1329 = 1 798 617	1
1000 x 1800 = 1 800 000, 1 800 000 - 1329 = 179 871	1
1000 ÷ 1800 = 18, 1329 - 18 = 1211	1
1000 + 1800 = 2800, 2800 - 1329 = 1471	2
1000 x 1800 = 1 800 000, 1 800 000 - 1329 = 1 786 671	1
1800 - 1329 = 471, 1000 - 471 = 529	1
1800 - 1329 = 571	1

PROBLEM 10 (3 175 102 ÷ 2506 = 1267, 1267 - 1253 = 14)

ERROR	FREQUENCY
3 175 102 ÷ 2506 = 1267, 1267 - 1253 = 1214	1
3 175 102 ÷ 2506 = 12 402.742, 12 402.742 - 1253 = 11 149.742	1
2506 - 1253 = 1253	2
3 175 102 ÷ 2506 = ___	1
3 175 102 ÷ 2506 = 1267, 2506 x 1253 = 3 140 018, 3 175 102 - 3 140 018 = 35 084	1
3 175 102 ÷ 2506 = .0007892, 7892 - 1253 = 6639	1
3 175 102 ÷ 2506 = 1267, 1267 - 1253 = 1142	1
3 175 102 - 2506 = 3 172 596, 3 172 596 + 1253 = 3 173 849	1
2506 x 1553 = 4 871 818, 4 871 818 - 3775 = ___	1

PROBLEM 11 (1500 x 1200= 1 800 000, 1 800 000 + 1182 = 1 801 182)

ERROR	FREQUENCY
1500 x 1200 = 1 800 000, 1 800 000 + 1182 = 1 798 818	5
1200 + 1182 = 2382	2
1500 x 1200 = 180 000, 180 000 + 1182 = 181 182	1
1200 x 1182 = 1 418 400	1
1500 + 1200 = 2700, 2700 - 1182 = 1518	1
1500 x 1200 = ___, 1182 + ___	1

PROBLEM 12 (1 539 180 ÷ 3018 = 510, 510 +1006 = 1516)

ERROR	FREQUENCY
1 539 108 ÷ 3018 = 509.97614, 509.97614 + 1006 = 1515.9761	1
1 539 180 ÷ 3018 = 500, 500 + 1006 = 1506	1
3018 + 1006 = 4024	2
1 539 180 ÷ 3018 = .0019607, 19607 + 1006 = 20 613	1
1 539 180 - 3018 = 1 536 162, 1 536 162 + 1006 = 1 537 168	1
3081 + 1006 = 4087	1

SPECIFIC ERRORS MADE FOR EACH PROBLEM OF TEST # 3

PROBLEM 1 (1000 x 95 = 95 000, 95 000 + 58 = 95 058)

ERROR	FREQUENCY
1000 x 95 = 95 000, 95 000 - 58 = 94 942	4
95 + 58 = 153	7
1000 x 95 = 95 000, 95 000 + 58 = 59 058	1
1000 - 95 = 905, 905 - 58 = 847	1
1000 ÷ 95 = 10.526, 10.526 + 58 = 68.526	1
1000 + 95 = 1095, 1095 + 58 = 1153	2
1000 + 95 = 1095, 1095 - 58 = 1037	1
1000 - 95 = 1005, 1005 + 58 = 1063	1
95 - 58 = 37, 37 x 95 = 1515	1

PROBLEM 2 (3600 ÷ 90 = 40, 40 + 15 = 55)

ERROR	FREQUENCY
90 ÷ 3600 + 15 = 0.375	1
3600 ÷ 90 = 40, 40 x 15 = 600	2
3600 ÷ 90 = 4, 4 + 15 = 19	1
3600 + 15 = 3615	1
3600 ÷ 15 = 240, 240 + 15 = 555	1
3600 - 90 = 3510, 3510 + 15 = 3525	1
90 x 15 = 1350, 1350 + 3600 = 4950	1
90 + 15 = 105	1

PROBLEM 3 (1893 x 93 = 176 049, 176 049 - 75 763 = 100 286)

ERROR	FREQUENCY
1893 x 93 = 176 049, 176 049 ÷ 75 763 = 2.3	2
1893 ÷ 93 = 20, 20 ÷ 75 763 = 3788.15	1
1893 x 93 = 176 049, 176 049 - 75 763 = 100 283	1
1893 x 93 = 176 049, 176 049 - 75 763 = 101 286	1
1893 + 93 = 1986, 75 763 - 1986 = 73 777	1
75 763 ÷ 93 = 814.6 = 815	2
1893 x 93 = 175 949, 175 949 - 75 763 = 99 986	1
1893 x 93 = 174 749, 174 749 - 75 763 = 98 986	1
1893 x 93 = 176 049, 176 049 - 75 763 = 99 986	1
1893 x 93 = 176 046, 176 046 - 75 763 = 100 283	1
1893 x 93 = 173 049, 173 049 - 75 763 = 97 286	1

PROBLEM 4 (6030 ÷ 45 = 134, 134 − 90 = 44)

ERROR	FREQUENCY
6030 ÷ 45 = 134, 6030 − 4050 = 1980	1
134 x 90 = 12 060, 6030 ÷ 45 = 134	1
45 x 90 = 4050, 6030 − 4050 = 1980, 1980 ÷ 45 = 22	1
90 x 45 = 4050, 6030 − 4050 = 1980	2
45 ÷ 6030 = 124	1
6030 ÷ 45 = 134, 134 ÷ 9 = 1.48 = 1.50	1
6030 x 45 = 271 350, 45 x 90 = 4050, 271 350 − 4050 = 267 300	1
6030 ÷ 45 = 134, 134 − 90 = 40	1
6030 ÷ 40 = 125, 125 − 90 = 35	1

PROBLEM 5 (9630 ÷ 18 = 535, 535 + 1926 = 2461)

ERROR	FREQUENCY
9630 + 1926 = 11 556, 11 556 ÷ 18 = 642	3
9630 ÷ 18 = 535, 535 + 1926 = 2561	2
9630 ÷ 18 = 535	1
9630 ÷ 18 = 535, 535 + 1926 = 1070	1
9630 ÷ 18 = 5305, 5305 + 1926 = 7231	1
9630 ÷ 18 = 535, 1926 − 535 = 1391	2
9630 ÷ 18 = 536.11, 536.11 + 1926 = 2462.11	1
9630 ÷ 18 = 535, 535 x 1926 = 1 030 410	1
9630 x 18 = 173 340, 173 340 + 1926 = 175 266	1

PROBLEM 6 (1102 x 36 = 39 672, 39 672 − 1802 = 37 870)

ERROR	FREQUENCY
1102 x 36 = 39 672, 39 672 ÷ 1802 = 22	2
1102 x 36 = 39 672, 39 672 − 1802 = 700	1
1102 x 36 = 39 678, 39 678 − 1802 = 37 876	2
1102 + 36 = 1138, 1138 ÷ 1802 = 1.58	1

PROBLEM 7 (9056 ÷ 8 = 1132, 1132 − 1129 = 3)

ERROR	FREQUENCY
9056 ÷ 8 = 1132, 1132 ÷ 1129 = 1	2
9056 x 8 = 72 448, 72 448 − 1129 = 71 319	2
1129 x 8 = 9032, 9056 − 9032 = 23	1

PROBLEM 8 (2819 x 72 = 202 968, 202 968 + 1895 = 204 863)

ERROR	FREQUENCY
2819 ÷ 72 = 39.15, 39.15 + 1895 = 1934.15	1
2819 x 72 = 202 968, 202 968 ÷ 1895 = 107.10	1
1895 + 72 = 1967	3
2819 x 72 = 202 968	1
2819 ÷ 72 = 3915, 3915 + 1895 = 5810	1
2819 + 0 = 2819, 2819 + 1895 = 4714	1
2819 x 72 = 205 968, 205 968 -1895 = 204 073	1
1895 ÷ 72 = 26	1
2819 ÷ 72 = 39, 39 + 1895 = 1934	1
1895 + 72 = 1967, 2819 - 1967 = 852	1
2819 x 72 = 202 768, 202 768 + 1895 = 204 663	1
2819 + 72 = 2891, 2891 + 1895 = 4786	1
2819 x 72 = 212 968, 212 968 + 1895 = 214 863	1

PROBLEM 9 (2600 x 5 = 13 000, 13 000 - 18 = 12 982)

ERROR	FREQUENCY
2600 ÷ 18 = 144	1
2600 x 5 = 13 000, 13 000 - 18 = 1282	1
2600 ÷ 5 = 520, 520 - 18 = 502	7
2600 - 18 = 2582	1
2600 ÷ 5 = 520	2
2600 + 5 = 2605, 2605 ÷ 18 = 144.72	1
2600 x 5 = 13 000, 13 000 - 18 = 12 989	1
2600 ÷ 5 = 520, 520 ÷ 18 = 28.9 = 29	1
2600 x 5 = 13 000, 1300 - 18 = 1282	1

PROBLEM 10 (2590 ÷ 74 = 35, 35 - 32 = 3)

ERROR	FREQUENCY
74 x 32 = 2368, 2590 - 2368 = 222	3
2590 ÷ 74 = 39, 39 - 32 = 7	1
2590 ÷ 74 = 305, 305 - 32 = 273	1
2590 ÷ 74 = 35, 35 + 32 = 67	1
74 - 32 = 42	1

PROBLEM 11 (2350 x 8 = 18 800, 18 800 + 2400 = 21 200)

ERROR	FREQUENCY
2350 ÷ 8 = 293.75, 293.75 + 2400 = 2693.75	5
2350 x 8 = 18 800, 18 800 - 2400 = 16 400	2
2350 ÷ 8 = 293.75	1
2350 x 8 = 16 800, 16 800 + 2400 = 19 200	1
2350 ÷ 8 = 295, 295 + 2400 = 2695	1
2350 ÷ 8 = 293, 2350 + 2400 = 4750	1
2350 x 8 = 18 800, 18 800 + 2400 = 11 200	1
2350 x 8 = 19 200, 19 200 + 2350 = 21 550	1
2350 + 2400 = 4750, 4750 x 8 = 38 000	1
2400 + 8 = 2408	1

PROBLEM 12 (1092 ÷ 28 = 39, 39 + 7 = 46)

ERROR	FREQUENCY
10 920 ÷ 28 = 136, 136 + 7 = 143	1
1092 ÷ 28 = 39, 39 x 7 = 273	2
1092 ÷ 7 = 156, 156 + 7 = 233	1
1092 x 28 = 30 576, 30 576 + 7 = 30 583, 30 583 ÷ 28 = 1092	1
1092 ÷ 28 = 37, 37 + 7 = 44	2
1092 ÷ 28 = 38, 38 + 7 = 45	1
1092 ÷ 28 = 38, 38 x 7 = 266	1
28 + 7 = 35	1

SPECIFIC ERRORS MADE FOR EACH PROBLEM OF TEST # 4

PROBLEM 1 (19 x 9 = 171, 171 + 8 = 179)

ERROR	FREQUENCY
8 + 19 = 27, 27 + 9 = 36	1
19 x 9 = 108	1
19 x 9 = 171, 171 + 8 = 178	1
8 + 9 = 17, 19 x 17 = 323	2
19 + 8 + 9 = 36	2
19 x 9 = 172, 172 + 8 = 180	2
19 x 9 = 171, 171 + 9 = 180	1
8 + 9 = 17	2
8 + 19 = 27, 19 x 9 = 171	1
19 + 9 = 28, 28 - 8 = 20	1

PROBLEM 2 (50 ÷ 10 = 5, 5 + 2 = 7)

ERROR	FREQUENCY
2 x 10 = 20, 20 + 50 = 70	2
10 ÷ 50 = 5	1
50 x 2 = 100, 100 ÷ 10 = 10	1
2 x 10 = 20, 10 ÷ 50 = 5	1
2 + 50 = 52, 52 ÷ 10 = 5	1
50 ÷ 10 = 5	1
52 ÷ 10 = 5.2	1
50 + 10 = 60, 60 - 2 = 58	1
50 ÷ 10 = 5, 5 - 2 = 3	1
50 x 20 = 1000	1

PROBLEM 3 (98 x 2 = 196, 196 - 45 = 151)

ERROR	FREQUENCY
98 ÷ 2 = 49	2
2 ÷ 98 = 46	1
98 x 2 = 1.96	1
98 x 2 = 196	2
98 x 2 = 196, 196 ÷ 45 = 4	1
45 + 98 = 143, 143 ÷ 2 = 71	1
45 + 45 = 90, 98 - 90 = 8	1
98 x 2 = 19.60, 45	1
92 x 2 = 184, 184 - 45 = 139	1
98 x 2 = 96, 96 + 45 = 141	1
45 + 98 = 143, 143 = 2 - 141	1

PROBLEM 4 (96 ÷ 6 = 16, 16 - 12 = 4)

ERROR	FREQUENCY
96 ÷ 12 = 8	1
6 ÷ 96 = 16	1
96 ÷ 6 = 16	4
96 - 12 = 84, 6 ÷ 84 = 24	1
96 - 12 = 84	1
96 ÷ 6 = 14, 14 - 12 = 2	1
96 ÷ 6 = 15, 15 - 12 = 3	1
96 + 12 = 108, 108 ÷ 96 = 1.125	1

PROBLEM 5 (36 ÷ 4 = 9, 9 + 12 = 21)

ERROR	FREQUENCY
36 ÷ 4 = 9	2
36 ÷ 4 = 9, 9 + 12 = 18	1
4 ÷ 36 = 12	1
36 - 12 = 24, 24 - 4 = 20	1
36 + 12 = 48, 48 ÷ 4 = 12	2
36 ÷ 4 = 9, 9 + 12 = 31	1
12 ÷ 4 = 3, 36 ÷ 4 = 9	1
36 ÷ 4 = 9, 12 - 9 = 3	2
12 x 36 = 436, 436 ÷ 4 = 19	1
36 - 12 = 24, 24 + 4 = 28	1
48 x 36 = 288	1

PROBLEM 6 (15 x 2 = 30, 30 - 8 = 22)

ERROR	FREQUENCY
15 ÷ 2 = 7.5 = 8	1
15 x 2 = 30	5
15 x 2 = 40	1
15 ÷ 2 = 7, 7 + 8 = 15	1
15 ÷ 2 = 525 = 53	1
8 + 8 = 16, 30 - 16 = 14	1
15 x 2 = 30, 30 - 8 = 24	1
15 + 8 = 23, 23 - 2 = 21	1

PROBLEM 7 (48 ÷ 8 = 6, 6 - 4 = 2)

ERROR	FREQUENCY
48 ÷ 8 = 6	2
8 ÷ 48 = 6	1
52 - 8 = 43	1
48 ÷ 4 = 12	1
48 ÷ 8 = 6, 6 x 4 = 24	1
48 ÷ 8 = 6, 6 + 4 = 10	1
8 x 4 = 32, 48 - 32 = 16, 16 ÷ 4 = 4	1
12 ÷ 8 = 1.5	1

PROBLEM 8 (19 x 6 = 114, 114 + 50 = 164)

ERROR	FREQUENCY
19 x 6 = 114, 111 + 50 = 151	1
19 + 56 = 114, 114 + 50 = 164	1
19 x 6 = 123, 123 + 50 = 173	1
50 + 19 = 69, 69 + 19 = 88	1
19 x 6 = 96, 96 + 50 = 146	2
19 x 6 = 116, 116 + 50 = 166	1
50 ÷ 6 = 8, 8 + 9 = 27	1
50 ÷ 6 = 8 R2	1
50 + 19 = 69, 69 ÷ 6 = 11.5	1

PROBLEM 9 (12 x 24 = 288, 288 - 8 = 280)

ERROR	FREQUENCY
24 x 12 = 288, 288 - 8 = 36	1
12 x 24 = 286, 286 - 8 = 278	1
12 x 8 = 32, 32 - 24 = 8, 24 + 4 = 28	1
24 - 8 = 16	2
24 x 12 = 288, 288 ÷ 8 = 36	1
24 + 12 = 36, 36 - 8 = 28	1
12 x 24 = 288	1
24 x 12 = 368, 368 - 8 = 360	1
12 - 8 = 4, 24 - 1 = 23	1
24 x 12 = 298, 298 - 8 = 290	1

PROBLEM 10 (80 ÷ 10 = 8, 8 - 5 = 3)

ERROR	FREQUENCY
80 ÷ 10 = 8	1
80 - 10 = 8, 8 + 5 = 13	2
80 - 10 = 70, 70 - 5 = 65	1
5 x 10 = 50, 80 - 50 = 30	1
10 x 5 = 50, 50 + 80 = 130, 130 ÷ 10 = 13	1
80 ÷ 10 = 8, 8 x 5 = 40	1
80 + 5 = 85, 85 x 10 = 850	1
5 x 10 = 50, 80 - 50 = 30, 30 ÷ 5 = 6	2
5 x 10 = 50, 50 - 80 = 30	1
80 ÷ 5 = 16, 16 - 10 = 6	1

PROBLEM 11 (28 x 12 = 336, 336 + 7 = 343)

ERROR	FREQUENCY
28 + 12 = 40, 40 + 7 = 47	2
28 x 12 = 430, 430 + 7 = 437	1
28 x 12 = 326, 326 - 7 = 319	1
7 + 12 = 19	1
28 ÷ 7 = 4, 28 x 12 = 336	1
28 x 7 = 196, 196 + 7 = 203	1
7 + 28 = 35, 35 - 12 = 23	1
28 x 12 = 336, 12 x 7 = 84, 336 + 84 = 420	1
12 + 7 = 18	1
28 x 12 = 336, 336 + 7 = 342	1
12 + 7 = 24	1
28 ÷ 7 = 4, 4 + 12 = 16	1

PROBLEM 12 (72 ÷ 8 = 9, 9 + 4 = 13)

ERROR	FREQUENCY
72 ÷ 8 = 9	4
72 - 8 = 64, 64 - 4 = 60	1
72 x 8 = 576, 576 + 32 = 608	1
72 ÷ 8 = 9, 9 x 4 = 36	1
72 - 32 = 40	1
72 ÷ 8 = 9, 8 - 4 = 5	2
72 ÷ 8 = 9, 9 x 4 = 32	1
8 ÷ 72 = 8, 8 - 4 = 4	1
8 x 4 = 32, 72 ÷ 8 = 9, 9 + 32 = 41	2

SPECIFIC ERRORS MADE FOR EACH PROBLEM OF TEST # 5

PROBLEM 1 (2983 x 1250 = 3 728 750, 3 728 750 + 1230 = 3 729 980)

ERROR	FREQUENCY
2983 ÷ 1250 = 2.3864, 2.3864 x 1230 = 2935.272	1
1230 + 1250 = 2480	8
2983 x 1250 = 1 043 750, 1 043 750 + 1230 = 1 044 980	1
1230 + 2983 + 1250 = 5463	2
2983 + 1230 = 4213, 4213 - 1230 = 2983	1
2983 + 1250 = 4233	2
1250 + 1230 = 2480, 2983 - 2480 = 503	1
2983 x 1250 = 37 388 750, 37 388 750 + 1230 = 37 389 980	1
1230 + 2983 = 4223	1
1250 x 2983 = 3 728 650, 3 728 650 + 1230 = 3 729 880	1
2983 x 1250 = 3 528 750, 3 528 750 + 1230 = 3 529 980	1

PROBLEM 2 (150 000 ÷ 2500 = 60, 60 + 1250 = 1310)

ERROR	FREQUENCY
150 000 ÷ 2500 = 60, 12.5 x 60 = 75 000	1
150 000 ÷ 2500 = 60	7
150 000 ÷ 1250 = 120	1
2500 ÷ 150 000 = 60, 60 + 1250 = 1850	1
150 000 + 1250 = 151 250, 151 250 ÷ 2500 = 60.5, 60.5 + 1250 = 1310.5	1
1250 + 150 000 = 151 250, 151 250 ÷ 2500 = 60.5	2
1250 + 150 000 = 151 250, 15 125 ÷ 250 = 65	1
2500 ÷ 150 000 = 1 666 667, 1 666 667 + 1250 = 1 667 917	1
1250 x 60 = 75 000	1
150 000 ÷ 250 = 600	1
2500 x 1250 = 3 125 000, 3 125 000 ÷ 150 000 = 20	1
150 000 ÷ 2500 = 6000, 6000 + 1250 = 7250	1
150 000 + 1250 = 151 250	1
2843 ÷ 1250 = 15 000	1
2500 ÷ 150 000 = 0.0166666	1

PROBLEM 3 (1009 x 8500 = 8 576 500, 8 576 500 - 7923 = 8 568 577)

ERROR	FREQUENCY
8500 x 1009 = 8 576 500, 85 765 - 7923 = 77 842	1
8500 x 1009 = 8 576 500	2
8500 x 1009 = 857.65	1
1009 x 8500 = 8576.5, 8576.5 ÷ 7923 = 1082.48	1
8500 - 7923 = 577	1
8500 x 1009 = 8 565 490, 8 565 490 + 7293 = 8 568 587	1
8500 x 1009 = 1 576 500, 1 576 500 - 7923 = 1 568 568	1
8500 - 7923 = 677	1
1009 + 7933 = 849 727, 857 650 - 7293 = 849 727	1
8500 + 1009 = 9509, 7923 - 9509 = 1586	1

PROBLEM 4 (6 942 000 ÷ 1500 = 4628, 4628 − 4500 = 128)

ERROR	FREQUENCY
6 942 000 ÷ 1500 = 4628	2
6 942 000 ÷ 4500 = 1543	1
1500 x 4500 = 6 750 000, 6 942 000 − 6 750 000 = 192 000	2
1500 x 4500 = 6 750 000, 6 750 000 + 6 942 000 = 13 692 000	1
6 942 000 − 4500 = 6 937 500, 6 937 500 ÷ 1500 = 1541.6	1
6 942 000 − 4500 = 6 937 500, 6 937 500 ÷ 1500 = 46 274.5	1
6 942 000 − 4500 = 6 937 500, 6 937 500 ÷ 1500 = 4625	2
6 942 000 ÷ 1500 = 4628, 4628 x 4500 = 20 826 000	1
694 200 − 4500 = 689 700, 689 700 ÷ 1500 = 459.8	1
4500 − 6 942 000 = 6 937 500, 1500 ÷ 6 937 500 = 0.0021023	1
6 942 000 ÷ 1500 = 4638	1
6 942 000 ÷ 1500 = 4228	1
4500 x 1500 = 6 250 000	1
6 942 000 ÷ 1500 = 563 000, 552 000 ÷ 4500 = 558 700	1
6 942 000 ÷ 1500 = 4928, 4928 − 4500 = 428	1
6 942 000 ÷ 1500 = 4628, 4628 + 4500 = 9128	1

PROBLEM 5 (1 722 000 ÷ 3000 = 574, 574 + 1000 = 1574)

ERROR	FREQUENCY
1 722 000 ÷ 3000 = 574	4
3000 x 1000 = 300 000, 1 722 000 ÷ 3000 = 574	1
1 722 000 + 1000 = 1 723 000, 1 723 000 ÷ 3000 = 574.3	4
1 722 000 − 1000 = 1 721 000, 1 721 000 ÷ 3000 = 5614	1
1 722 000 ÷ 300 = 574, 574 x 1000 = 574 000	1
1 722 000 ÷ 3000 = 0.0017421	1
3000 ÷ 1 722 000, 17 421, 17 421 + 100 = 18 421	1
1 722 000 ÷ 3000 = 3740, 3740 + 1000 = 4740	1

PROBLEM 6 (1230 x 2570 = 3 161 100, 3 161 100 − 4763 = 3 156 337)

ERROR	FREQUENCY
2570 x 1230 = 3 161 100, 3 161 100 ÷ 4763 = 664	1
1230 x 2570 = 3 161 100, 3 161 100 − 4763 = 311 347	1
1230 x 2570 = 2 961 100, 2 961 100 − 4763 = 2 956 347	1
2570 + 1230 = 3800	1
1230 + 2570 = 6260, 6260 ÷ 4763 = 1.3142977	1
4763 − 2570 = 2133	1
4763 − 2570 = 2193, 2193 + 4763 = 6956	1
2570 − 1230 = 1340	2
4763 − 2570 = 2193	2
2570 x 1230 = 3 161 100, 3 161 100 − 4763 = 3 156 437	1
1230 x 2570 = 3 171 100, 3 171 100 − 4763 = 3 166 337	1
1230 + 2570 = 3800, 3800 − 4763 = 963	1
1230 x 2570 = 31 631 100, 31 631 100 ÷ 4763 = 1505	1

PROBLEM 7 (2 686 000 ÷ 1000 = 2686, 2686 - 2000 = 686)

ERROR	FREQUENCY
2 686 000 x 2000 = 5 372 000, 5 372 000 ÷ 1000 = 5372	1
2 686 000 ÷ 1000 = 2686	5
2000 x 1000 = 2 000 000, 2 000 000 ÷ 2 686 000 = 1.343	1
2 686 000 ÷ 1000 = 3723, 3723 - 2000 = 1723	1
2 686 000 ÷ 2000 = 1343	1
2 686 000 - 2000 = 2 684 000, 2 684 000 ÷ 1000 = 2684	2
2 686 000 ÷ 2000 = 1343, 1343 - 1000 = 343, 343 ÷ 1000 = 2	1
2 686 000 ÷ 1000 = 372	1
2000 - 2 686 000 = 2 684 000, 1000- 2 684 000 = 2 687 000	1
2 686 000 ÷ 1000 = 26 860, 26 860 - 2000 = 24 860	1
2 686 000 ÷ 1000 = 268.6, 2000 - 268.6 = 1731.4	1
2 686 000 ÷ 1000 = 26 860	1

PROBLEM 8 (5295 x 1000 = 5 295 000, 5 295 000 + 2530 = 5 297 530)

ERROR	FREQUENCY
5295 x 1000 = 5 295 000, 5 295 000 + 2530 = 5 297 350	1
5295 x 1000 = 5 295 000, 5 295 000 + 5295 = 5 300 295	1
5295 x 1000 = 5 295 000, 5 295 000 x 2530 = 133.20450	1
5295 x 1000 = 5 295 000, 5 295 000 - 2530 = 5 292 470	1
2530 + 5290 = 7820, 7820 ÷ 1000 = 7.8	1
2530 + 5295 = 7825	4
5295 + 1000 = 6295, 6295 + 2530 = 8825	2
5295 x 1000 = 5 295 000	2
5295 x 1000 = 5 295 000, 5 295 000 + 2530 = 5 297 530	1
2530 + 1000 = 3530	1
5295 + 1000 = 6295, 6295 + 2530 = 8825	1
5295 ÷ 1000 = 5.295	1
2530 + 5.295 = 2535.295	1
5295 x 1000 = 5 295 000, 5 295 000 + 2530 = 532 030	1

PROBLEM 9 (1008 x 1000 = 1 008 000, 1 008 000 - 2583 = 1 005 417)

ERROR	FREQUENCY
1008 x 1000 = 1008 000, 1 008 000 ÷ 2583 = 390.2439	2
2583 ÷ 1008 = 2.5625, 2.5626 x 1000 = 2562.5	1
1008 + 1000 = 2008	1
1008 - 2583 = -1575, - 1575 x 1000 = 1575	1
1008 x 1000 = 1 008 000, 1 008 000 - 2583 = 98 217	1
1008 - 1000 = 8	2
1008 x 1000 = 1008 000	1
2583 - 1000 = 1583, 1583 + 1008 = 2591	1
1008 x 1000 = 1008 000, 1008 000 ÷ 2583 = 3	1
2583 - 1008 = 1575	1
1008 ÷ 1000 = 1.008	1

PROBLEM 10 (9 768 600 ÷ 5400 = 1809, 1809 - 1800 = 9)

ERROR	FREQUENCY
9 768 600 ÷ 5400 = 1809	6
9 768 600 ÷ 5400 = 1800	1
1800 ÷ 9 768 600 = 5427	1
9 768 600 - 1800 = 9 766 800, 5400 ÷ 9 766 800 = 1808	1
5400 ÷ 1800 = 3	1
9 768 600 - 1800 = 9 766 800, 9 766 800 ÷ 5400 = 1808.6667	1
1800 x 5400 = 9 720 000, 9 768 600 - 9 720 000 = 48 600	1
1800 - 9 768 600 = 958 860, 5400 ÷ 958 860 = 36 316	1

PROBLEM 11 (1100 x 1440 = 1 584 000, 1 584 000 + 1250 = 1 585 250)

ERROR	FREQUENCY
1440 x 1250 = 1 800 000	1
1100 x 1440 = 1 584 000	3
1100 + 1440 = 2540, 2540 + 1250 = 3790	3
1100 x 1440 = 1 144 000, 1 144 000 + 1250 = 1 145 250	1
1250 + 1440 = 2690, 2690 x 1110 = 2 959 000	1
1440 x 1000 = 1 440 000, 1 440 000 + 1250 = 1 441 250	1

PROBLEM 12 (1 625 088 ÷ 1024 = 1587, 1587 + 2048 = 3635)

ERROR	FREQUENCY
1 625 088 ÷ 1024 = 1587	3
1 625 088 + 2048 = 1 627 136, 1 627 136 ÷ 1024 = 1589	2
1 625 088 ÷ 1024 = 1587, 2048 - 1587 = 461	2
1 625 088 ÷ 1024 = 5002	1
1 625 088 ÷ 1024 = 1587, 1587 + 2048 = 5222	1
1 625 088 ÷ 1024 = 6301	1
2048 x 1024 = 2 097 152, 2 097 152 - 1 625 088 = 472 064	1
1 625 088 + 2048 = 1 627 136	1

SPECIFIC ERRORS MADE FOR EACH PROBLEM OF TEST # 6

PROBLEM 1 (1750 x 25 = 43 750, 43 750 + 93 = 43 843)

ERROR	FREQUENCY
93 + 25 = 118	4
1750 ÷ 25 = 70, 70 + 93 = 163	4
1750 x 25 = 43 750, 93 x 25 = 2325, 43 750 + 2325 = 46 975	1
93 x 25 = 2325	1
1750 ÷ 25 = 875, 875 + 93 = 968	1
93 + 25 = 118, 1750 - 118 = 1632	1
93 + 1750 = 2843, 2843 ÷ 25 = 117 R18	1
1750 x 25 = 8750, 8750 + 93 = 8840	1

PROBLEM 2 (1200 ÷ 30 = 40, 40 + 10 = 50)

ERROR	FREQUENCY
1200 x 30 = 36 000, 3600 ÷ 10 = 360	1
1200 ÷ 30 = 40	9
1200 ÷ 30 = 40, 30 x 10 = 300, 300 + 40 = 340	1
1200 ÷ 30 = 40 = 1500	1
1200 ÷ 30 = 40, 40 x 30 = 1200	1

PROBLEM 3 (1021 x 98 = 100 058, 100 058 - 51 630 = 48 428)

ERROR	FREQUENCY
51 630 + 1021 = 52 651, 52 651 ÷ 98 = 537.26	1
1021 x 98 = 100 058, 51 630 ÷ 100 058 = 1.9379819	1
1021 x 98 = 100 058, 51 630 + 100 058 = 151 572	1
1021 x 98 = 100 058	1
1021 x 98 = 100 058, 100 058 - 51 630 = 38 428	1
1021 x 98 = 100 758, 100 758 ÷ 98 = 128 R14	1
51 630 - 1021 = 50 609, 1021 - 98 = 923, 60 609 - 923 = 49 686	1
110 058 - 51 630 = 58 428	1
1021 x 98 = 100 658, 100 658 - 51 630 = 48 028	1
51 630 x 1021 = 52 714 230, 52 714 230 - 98 = 52 714 132	1
1021 x 98 = 100 058, 100 058 - 51 630 = 48 420	1
1021 ÷ 98 = 10 or 11	1

PROBLEM 4 (3360 ÷ 40 = 84, 84 - 80 = 4)

ERROR	FREQUENCY
3360 ÷ 40 = 84	7
80 x 40 = 3200, 3360 - 3200 = 160	4
3360 ÷ 40 = 84, 84 + 80 = 164	1
3360 ÷ 40 = 83	1
3360 ÷ 40 = 42, 42 x 80 = 3360	1
3360 ÷ 40 = 84, 84 x 40 = 3360 = 0	1
3360 ÷ 40 = 840, 840 ÷ 80 = 8.5	1

PROBLEM 5 (200 ÷ 50 = 40, 40 + 1000 = 1040)

ERROR	FREQUENCY
2000 + 1000 = 3000, 3000 ÷ 50 = 60	1
2000 + 1000 = 3000, 3000 x 50 = 1500	1
2000 ÷ 50 = 40	4
2000 ÷ 50 = 40, 40 x 50 = 2000	1
2000 + 1000 = 3000	1
2000 + 1000 = 3000, 3000 x 50 = 150 000	1
2000 x 50 = 100 000, 100 000 + 1000 = 101 000	1

PROBLEM 6 (9576 x 40 = 383 040, 383 040 - 8475 = 374 565)

ERROR	FREQUENCY
8475 + 9576 = 18 051, 18 051 ÷ 40 = 451.28	1
9576 x 40 = 383 040, 383 040 ÷ 8475 = 45.19646	1
9576 x 40 = 383 040	2
9576 x 40 = 383 040, 383 040 - 8475 = 383 040	1
9576 ÷ 40 = 239 = 2.39	1
9576 ÷ 40 = 239.4, 239.4 x 40 = 9576 = 23.94	1
9576 x 40 = 383 040, 383 040 - 8475 = 274 565	1
383 040 - 8475 = 383 565	1
9576 ÷ 40 = 239.4, 8475 - 239.4 = 8235.60	1
9576 x 40 = 38 304, 38 304 - 8475 = 29 829	1
9576 x 40 = 383 040, 383 040 - 8476 = 374 564	1
9576 ÷ 40 = 239.16	1

PROBLEM 7 (8400 ÷ 6 = 1400, 1400 - 1200 = 200)

ERROR	FREQUENCY
1200 x 6 = 7200, 7200 + 8400 = 15 600	1
8400 ÷ 6 = 1400, 1400 + 1200 = 2600	5
8400 x 6 = 50 400, 50 400 - 1200 = 49 200	1
8400 ÷ 6 = 1400	1
8400 ÷ 6 = 1400, 6 ÷ 1200 = 200	1
1400 + 1200 = 2600	1
8400 x 6 = 50 400, 8400 - 1200 = 7200	1
1200 x 6 = 7200, 8400 - 7200 = 1200, 1200 ÷ 6 = 300	1
1200 x 6 = 7200, 8400 - 7200 = 1200	1

PROBLEM 8 (1092 x 12 = 13 104, 13 104 + 5410 = 18 514)

ERROR	FREQUENCY
1092 ÷ 12 = 91, 91 + 5410 = 5501	2
649.20 + 13 104 = 78 024	1
5410 + 1092 = 6502	1
1092 ÷ 12 = 91, 5410 - 91 = 5319	1
5410 + 1092 = 6502, 6502 x 12 = 78 024	2
1092 x 12 = 13 104, 13 104 + 5410 = 18 614	1
5410 + 1092 = 6502, 6502 ÷ 12 = 541.8 = 542	1
13 102 + 65 020 = 78 122	1
1092 x 2 = 2184, 2184 + 5410 = 7594	1

PROBLEM 9 (1003 x 2 = 2006, 2006 - 99 = 1907)

ERROR	FREQUENCY
1003 x 2 = 2006, 2006 - 1003 = 1003	1
10 030 ÷ 2 = 5015, 5015 - 99 = 4916	1
1003 x 2 = 2006	2
2006 - 99 = 78.94	1
1003 ÷ 2 = 501.5, 501.5 - 99 = 402.5	1
1003 ÷ 2 = 501 = 5.1	1
99 x 2 = 198, 1003 - 198 = 805	1

PROBLEM 10 (6000 ÷ 60 = 100, 100 - 30 = 70)

ERROR	FREQUENCY
6000 ÷ 60 = 100	3
60 x 30 = 1800, 6000 ÷ 1800 = 3.33	1
60 x 30 = 1800, 6000 - 1800 = 4200	2
6000 ÷ 60 = 1000, 1000 - 30 = 970	2
6000 ÷ 30 = 200, 6000 - 60 = 5940	1
6000 ÷ 30 = 200, 200 - 30 = 170	1

PROBLEM 11 (2573 x 6 = 15 438, 15 438 + 3250 = 18 688)

ERROR	FREQUENCY
2573 ÷ 6 = 428.5, 428.5 + 3250 = 36 785	1
19500 + 15 438 = 34 938	1
2573 ÷ 6 = 428	1
2573 x 6 = 15 438, 15 438 + 3250 = 18 588	1
2573 ÷ 6 = 428, 428 + 3250 = 3678	1
3250 + 2573 = 5823, 5823 ÷ 6 = 970.5	1
3250 + 2573 = 5823, 5823 x 6 = 34 938	2
2573 x 6 = 15 438, 15 438 + 3250 = 18 598	1
2573 x 6 = 15 438, 3250 x 6 = 19 800 = 34 938	1

PROBLEM 12 (1404 ÷ 54 = 26, 26 + 9 = 35)

ERROR	FREQUENCY
1404 + 9 = 1413, 1413 + 54 = 1467	1
1400 ÷ 54 = 27, 27 + 9 = 36	1
1404 ÷ 54 = 26, 26 - 9 = 17	1
1404 x 54 = 65 816	1
1404 ÷ 54 = 26	1

APPENDIX D

SUMMARY OF CASES OF INCORRECT
SYMBOLISM WITH CORRECT
NUMERICAL SOLUTION

Table 6

SUMMARY OF CASES OF INCORRECT SYMBOLISM WITH CORRECT NUMERICAL SOLUTION

SYMBOLISM	FREQUENCY OF RESPONSE
16 ÷ 96 = 6, 6 + 4 = 10	4
23 x 8 = 184, 184 ÷ 11 = 173	1
3 ÷ 75 = 25, 25 - 15 = 10	2
5 ÷ 60 = 12, 12 + 10 = 22	1
6 ÷ 84 = 14, 14 - 3 = 11	2
12 ÷ 96 = 8, 8 - 6 = 2	3
8 ÷ 24 = 3, 3 + 4 = 7	2
2500 ÷ 250 000 = 100, 100 + 1250 = 1350	2
5180 ÷ 6 734 000 = 1300, 1300 - 1295 = 5	1
2506 ÷ 3 175 102 = 1267, 1267 - 1253 = 14	1
8 ÷ 9056 =1132, 1132 - 1129 = 3	1
10 ÷ 50 = 5, 5 + 2 = 7	2
6 ÷ 96 = 16, 16 - 12 = 4	1
4 ÷ 36 = 9, 9 + 12 = 21	2
8 ÷ 48 = 6, 6 - 4 = 2	1
10 ÷ 80 = 8, 8 - 5 = 3	2
8 ÷ 72 = 9, 9 + 4 = 13	1

APPENDIX E

SUMMARY OF CASES OF CORRECT ORDER OF OPERATIONS BUT INCORRECT ENTRY OF DATA INTO CALCULATOR

Table 7

SUMMARY OF CASES OF CORRECT ORDER OF OPERATIONS

BUT INCORRECT ENTRY OF DATA INTO CALCULATOR

EQUATIONS AND SOLUTIONS
6 734 000 ÷ 5180 = 0.0007692; 7692 - 1295 = 6397
3 442 494 ÷ 2838 = 0.0008244; 8244 + 1419 = 9663
4 534 950 ÷ 3675 = 0.0008103; 8103 - 1225 = 6878
3 175 102 ÷ 2506 = 0.0007892; 7892 - 1253 = 6639
1 539 180 ÷ 3018 = 0.0019607; 19 607 + 1006 = 20 613