GUYED TOWER RESPONSE TO IRREGULAR WAVES CONSIDERING LINEAR AND NON LINEAR MOORING CABLES

MOMEN ABDEL-MAGID AHMED WISHAHY

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by

© Momen Abdel-Magid Wishahy, B. Eng.

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To my Parents

ABSTRACT

The study presents the dynamic analysis procedure for predicting the guyed tower response to irregular waves. Hydrodynamic interaction is taken into account by the added water mass concept and the fundamental frequencies are determined using i) a lumped parameter two dimensional beam model and ii) three dimensional truss finite element model. Effect of the mooring guylines is simulated using one dimensional boundary elements. The example structure analyzed is the Exxon test guyed tower erected in 89.3 m deep water in the Gulf of Mexico. The measured wave height-time history reported by Exxon is used to determine the wave forces. Irregular wave forces are computed using linearized Morison's equation. The nonlinearity of the mooring system is considered using NONSAP finite element program. The tower response in terms of offset-time history to wave forces is determined for both linear and nonlinear cable behaviour. Numerical results obtained in this investigation are within the maximum values accepted by current design criteria. The computed frequencies and the responses agree reasonably well with the available measured values.

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CHAPTER I

INTRODUCTION

1.1 Introduction

With increasing demand for oil, oil exploration has moved from land to the shallow water (up to 100 m depth). Over the past thirty years fixed jacket-type platforms were considered to be the most economical offshore structure for drilling and production. The fundamental period of the fixed platforms (2 sec) is much smaller than the peak period of the sea states (8 sec and more). The dynamic amplification factor (DAF) being very close to unity, the structural response will be equal to that of the static case.

As the exploration and drilling continues into the deeper water (200 to 600 m) the structural frequency decreases and the DAF becomes larger than unity and moves closer to the resonance peak resulting in large displacements and stresses.

The concept of the offshore compliant structures is that the lowest natural structure sway frequency is much smaller than the wave frequency resulting in the DAF less than unity. Consequently, the principal structure inertial forces generally oppose and reduce the applied wave forces as well as the structure displacement. The compliant structure concept provides feasible and economical solution in deep waters. Several compliant structures are fabricated such as Tension Leg Platform, Semi-submersible Platform, Buoyant Tower, Guyed Tower, etc.

1.2 Objectives

In the present study, the guyed tower response to irregular waves will be determined. The hydrodynamic interaction will be taken into account considering the drag and inertia effects and the added water mass concept. Fundamental frequencies will be determined using: i) a lumped parameter two dimensional beam model; and ii) a three dimensional truss finite element model. The nonlinear effect of the mooring system restoring force will be taken into consideration and the theoretical results compared with the available measured values.

CHAPTER II

LITERATURE REVIEW

Theoretical studies have been made on the guyed tower by researchers and limited field data published by Exxon Production Research Co. are based on the large scale model installed in the Gulf of Mexico during October, 1975.

Finn [1] described the basic concept of the Exxon guyed tower and illustrated the design procedures to ensure structural safety under severe environmental conditions. Finn and Young [2] discussed a large scale model-test guyed tower installed in a water depth of 89.3 m in the Gulf of Mexico along with structural design, installation procedures, dynamic response, guying system behaviour, spud-can (foundation) behaviour and tower construction.

Finn, Wardell and Loftin [3] described in detail the test guyed tower and the prototype. They discussed the main features of the structural design, fabrication and installation of a guyed tower designed for 350 m water depth in the North Sea. Several guyed tower design alternatives and the sensitivity of the structural response to the principal design variables were also discussed. Mangiavacchi, Abott and Hanna [4] discussed the design criteria, fabrication, installation and system response of a guyed tower supported by piles.

Mes [5,6,7,8] discussed the concept, static analysis of preliminary guy design, approximate calculation procedures and design for large deflections and other nonlinear effects. Basu and Dutta [9] discussed methods of dynamic analysis under the action of waves and currents. The equations of tower motion were solved in the time-domain using the Newmark method incorporating the nonlinearities associated with the time-varying displacement, velocity and depth of immersion of the tower. The Mode Summation Method and the Mode Acceleration Method were used to obtain numerical solution for Exxon test tower subjected to regular waves. Basu and Dutta [9] also presented results of dynamic analysis under wave loading. The tower was modelled as a line member and the effect of the guy system simulated by a nonlinear elastic translational spring at the guy attachment point. The relationship between the horizontal force and displacement for the guylines was determined by an initial value finite segment technique. Triantafyllou, Kardomateas and Bliek [10] discussed the cable statics and the concept of the effective tension and obtained analytic solutions including the elasticity effects. The effect of elasticity and catenary shape were studied including the cable dynamics. Analytic solutions were developed using perturbation techniques. Hanna, Mangiavacchi and Suhendra [11] discussed the nonlinear dynamic analysis, structural idealization, hydrodynamic forces, wind forces, forces due to P-delta effect and nonlinear mooring and soil effects. Murray [12] tested a guyed tower model for motion analysis in a wave tank under regular waves, measured the deck displacements and guy line tensions using rotary and ring transducers respectively and compared physical model test results with the analytical values.

CHAPTER III

EXXON GUYED TOWER

3.1 Introduction

Exxon Production Research Co. proposed Exxon's Guyed Tower (Fig. 1) as one of the compliant offshore structures. The compliant tower has a period of 25-40 seconds, depending on the water depth and the horizontal force-displacement relationship of the mooring system; the structure will jump over the highest-energy band of the wave spectrum and absorb safely the energy of waves up to 30 m height.

The guyed tower concept is feasible in water depth range from 180 to 700 meters. Over 700 m water depth, although the concept is still applicable, the structure suitability will be governed by economical consideration since the total cost increases exponentially with depth. On the other hand, for water depth less than 180 m, the structure rigidity increases causing an increase in the structure frequency implying that the DAF (Fig. 2) approaches the resonance peak. Also in shallow water the bending stresses in the conductors become excessive due to the large displacement of the tower.

3.2 General Description

3.2.1 Deck

The deck is designed to support all the needed equipment for drilling and production. A two-or-three-level integrated deck with 7500 tonnes deck payload and production rates in excess of 100,000 barrels per day is used to avoid large space between the deck legs. The conductors run from the deck down through guides on the tower frame passing through sleeves provided in the tower base or in the spud can. Because the tower tilts a maximum of two degrees during severe storms, the conductors would be designed to withstand safely the bending stresses at the mud line. Studies have indicated that conventional conductor designs are adequate as long as the tilt of the tower does not exceed 2 degrees. Crude oil could be transported through a pipeline or by offshore terminals.

3.2.2 Tower

The tower frame is relatively a slender structure and the members of the tower are made of steel pipes with 4 to 16 main legs extending from foundation at mud line to the deck bottom. The main legs are connected together by a system of horizontal and diagonal members designed to resist the shear stresses.

The tower is held upright by several guylines providing lateral support. Since the mooring lines are attached to the structure near the center of the applied wave forces, they absorb most of the environmental forces. As a result, large overturning moments are not resisted by the tower's slender frame with consequent reduction in the total amount of steel required for fabricating the structure. Due to buoyancy the gravity loads supported by the foundation are reduced.

3.2.3 Guying System

The tower is supported laterally by several guylines - up to 24 lines - arranged symmetrically and radially around the tower. The maximum tension in any individual guyline is limited to 50% of the breaking strength of the wire rope. The guyline should last for 20 years with a very little loss in the strength due to corrosion. If, however, two adjacent lines fail for any reason, the structure is in no danger of collapse since the guying system is designed to be highly redundant. Proper tension is maintained by clamps on the deck operated by a hydraulic jacking system.

The guylines run vertically downward from the deck to fairleads (Fig. 1) located under the water surface, at center of the wave loads. The position of the fairleads provides two main advantages:

i) Absorbing most of the environmental loads.

ii) Provide clear passage and prevent possible interference with shipping operations due to its position under the keel of the passing vessels.

The cable travels down the fairleads at 60° from the vertical to a clump weight resting on the sea floor. Beyond the clump weight, the guyline runs to either an anchor pile or a conventional drag anchor such as Boss anchor.

The clump weight guying system has several advantages. Using the clumps, a shorter guyline can be used and a steeper angle than with conventional catenary lines can be obtained. The clump weights are designed to lift off the bottom, during the passage of large storm waves, softening the mooring system to permit the tower to deflect more with waves. Thus the tension in the cables increases slightly and remains at a value less than the maximum allowable guyline tension. This gives the guying system a greater energy-absorbing capacity at large offsets.

The mooring system makes use of chains, chain fairleads and chain stoppers for guy-to-tower connection. The main advantages of this system are:

i) Chain has a low strength-to-weight ratio providing small departure angle of the guy with the vertical where it approaches the fairlead. Therefore the high tension load transferred to the deck is eliminated.

ii) Eliminates strand wear in a fairlead.

iii) Allows for positive stopping against a chain link.

iv) The ease of tuning the guy by lengthening or shortening.

v) Eliminates abrasion around the sheave.

vi) Less heat sensitive than steel wire rope and easily inspected.

The composite guy system - steel wire combined with link chain components - is the most suitable system to meet the specified requirements of the wire ropes, the corrosion protection requirements, deployment and handling, kinking and behaviour at overload.

3.2.4 Foundation

The basic function of a guyed tower foundation is to provide vertical, lateral and torsional restraint; moreover the rotational restraint must be relatively weak to minimize the tower bending stress. Two types of foundation are proposed:

i) Pile foundation

ii) Spud-can foundation.

The pile foundation is a critical element of the overall design. It must allow the necessary compliancy to the tower within the allowable pile stresses. This is particularly critical when the tower is subjected to high waves and undergoes large lateral deflections. Selection of the number of piles, their location and spacing are based on the load carrying capacity and the desired structural compliancy. When the piles are too close to provide pivot support, the potential problems derived from group behaviour arise and generally they are incapable of providing sufficient torsional restraint. Depending on the soil properties the pile may need to be driven to a very large depth to ensure sufficient axial load capacity. Also the piles may require special design in the vicinity of the mud line to withstand the high axial and bending stresses.

Spud-can foundation (Fig. 1), which is basically a truss-reinforced stiffened shell, offers a solution to most of these problems. It provides a buoyancy and pre-load chamber during the tower installation. The foundation would be forced to penetrate into the sea floor by placing ballast material, such as drilling mud or iron tailings, into the spud-can. The penetration would continue slowly during the deck setting operation. Once a desirable penetration had been reached, the ballast material would be removed. The spud-can foundation essentially provides a crude pivot because the rotational restraint offered by the soil is generally very small relative to the overturning moment.

Torsion should be considered in design since torsional response is influenced by an eccentric placement of conductors and deck mass. The tower frame is sufficiently stiff in torsion since the diagonal and horizontal members with minimum slenderness ratio, are designed to resist shear stresses and hydrostatic collapse. The torsional frequency is dominated by the torsional soil stiffness at the base. The spud-can structure develops high torsional stiffness, since a large volume of soil must deform for the spud-can to displace torsionally. Both spud-can and pile foundation, with their relative merits and demerits, can be used for the guyed tower and selection of the appropriate foundation type is based on site specification.

3.3 Current Design Criteria

The following criteria have been suggested by Exxon Production Research Co. [3]:

- The maximum tension in any cable should not exceed 50% of the cable breaking strength.
- The structure withstands safely the environmental loads even when a specified number of mooring lines becomes ineffective.
- The tower tilt should not exceed 2 degrees.
- The effect of corrosion on cable strength is negligible for the life time of 20 years.
- The tower frame and spud-can foundation are fabricated using the standard procedures for small tubular space frame.

CHAPTER IV

TEST GUYED TOWER

4.1 Introduction

A one-fifth scale test guyed tower (Fig. 3) was installed in the Gulf of Mexico during October 1975 in 89.3 m of water in order to model a 30 m North Sea design wave with 6 m winter storm waves that occurs in the Gulf of Mexico [2].

4.2 Objectives

The guyed tower test was designed to obtain data and information for:

i) Comparison of analytical predictions with measured response, guying system behaviour, and spud can behaviour in an actual offshore environment.

ii) Demonstrating the practicality of proposed installation procedures including towing of the structure to location, upending and setting on bottom by adding preload ballast to spud can as well as deploying the guylines, clump weights, anchors and tensioning the guylines to ensure proper setting of the anchors.

iii) Gaining practical operation and maintenance experience in actual sea environment.

4.3 Fabrication

The test-guyed tower truss-work and spud-can were constructed in one piece using standard fabrication procedures for small tubular space frame. Two 3 m diameter, 20 m long auxiliary buoyancy tanks were attached to the upper end of the structure to provide the required tower floatation during upending. The spud-can was constructed of metal plate and was tested for water tightness.

4.4 Deck

The deck weight is 670 kN. The deck and the tower were equipped with instrumentation to measure simultaneously wave height, current magnitude and direction, orthogonal deck accelerations, orthogonal tower inclinations, tension in the guylines and bending strains in the conductors.

4.5 Tower

The 6x6 m test structure (Fig. 3) had four 40.64 cm (16 in) diameter legs with bracing members 21.91 cm (8 5/8 in) diameter. It has ten 21.91 cm dummy well conductors, nine of which are driven 18 m through sleeves in the spud-can into the sea bottom. The tenth conductor was terminated in the spud-can and used to place and remove the ballast mud. The tower weight and buoyancy per meter run are 22 kN, 16.1 kN respectively.

4.6 Guying System

The test tower has a dual guying system. During winter the tower is guyed by eight 19 mm (0.75 in) guylines. Each guyline is secured at the deck by a cable tensioning unit which allows line adjustments using hydraulic control and passes vertically from the deck under sheave fairleads located 5.33 m below mean water level. From the fairleads, each line extends down at 60° angle from the vertical to a 6 tonnes (submerged weight) clump weight on the sea floor. Beyond the clump weight the line is secured by a 0.59 tonnes Boss anchor. This guying system models the twenty 8.89 cm (3.5 in) bridge strands of the prototype. However during the hurricane season waves exceed the design wave for the model, thus necessitating to tension four additional 6.39 cm (2.75 in) guylines to provide additional support during hurricanes. The hurricane lines are secured in tensioners on the deck and pass vertically under friction fairleads (Fig. 3) with their centers located 5.33 m below the design mean water level. From the friction fairleads, the lines pass down to 75 tonnes (submerged weight) clump weights and out to tandem 12 and 6 tonnes Boss anchors. The hurricane lines were left slack during the winter months while data were taken.

4.7 Foundation

The spud-can penetration into the soil was 7.8 m after the removal of the drilling mud. Since the mud was removed the spud-can remained essentially at the same penetration.

CHAPTER V

PROBLEM FORMULATION

5.1 Introduction

The response of the test-guyed tower is predicted using two types of finite element models. The dynamic behaviour of the guylines, along with foundation, damping, and added mass, is essentially nonlinear. However, both models assume linearized foundation and mooring line springs, constant damping and added mass. The nonlinear behaviour of the mooring cables is assumed only for the truss element model.

5.2 Modelling

The guyed test tower was modelled by:

i) 2-dimensional beam element with linear mooring system;

ii) 3-dimensional truss element with both linear and nonlinear mooring system.

5.2.1 2-Dimensional Beam Element

The tower is represented by an equivalent 2-dimensional lumped mass beam element model (Fig. 4). The masses are lumped at 19 nodal points. The beam elements have constant flexural rigidity (EI) and each nodal point has two degrees of freedom-lateral translation and rotational displacements. The axial deformation and the rotational inertia of the nodes being relatively small are neglected. The foundation is modelled as a hinged support. The mooring system is simulated by a single horizontal linear weightless equivalent spring.

5.2.2 3-Dimensional Truss Element

The tower is discretized by a 3-dimensional truss element model (Fig. 5) with 80 nodes and 346 members Each nodal point has 3 degrees of freedom - vertical, horizontal and lateral translations. The guylines are idealized as four link members with a 60° angle of departure from the vertical. The pretensioning in the guylines is represented by initial strain in the link members. The model base consists of four hinged supports 0.3 m apart to simulate the foundation torsional resistance and free tower rotation on the base.

5.2.3 Added Mass

In both the models masses are lumped at the nodal points taking into account the added mass effect for the immersed portion of the tower. Because the tower members and conductors are made of pipes, the inertia coefficient C_I is taken as 2. The added mass is computed by multiplying the mass of the displaced water by (C_I-1). The deck mass is lumped at the topmost node (nodes along with the contribution from the tower itself. The guying system mass and its added contribution being relatively small are neglected.

CHAPTER VI

GUYING SYSTEM ANALYSIS

6.1 Introduction

The selection of the cable system is of primary importance in the guyed tower structural system since the lateral support under the environmental loadings is mainly provided by the mooring lines. The general requirements of the mooring system to guarantee satisfactory tower response are:

i) The sway period of the tower should be larger than the period of the design wave to achieve the desired dynamic amplification factor.

ii) The tower deflections should be acceptable under environmental loads.

iii) Sufficiently redundant mooring system to ensure acceptable platform behaviour with a specified number of mooring lines missing.

The test-guyed tower guying system is analyzed considering the geometrical nonlinearity for the static condition. Significant forces in the study of mooring lines immersed in the sea are gravity forces, wave forces, current forces, etc. However only the gravity forces (self weight) are considered in the following analysis.

6.2 Assumptions

The following assumptions are made in the static analysis:

i) The tower deflection is equal to the cable system deflection.

ii) The cables support only tension.

iii) Frictionless horizontal seabed.

iv) Only self weight is considered, neglecting the current and wave forces on the cables and clump weights.

v) The cable has no bending stiffness.

vi) Tower motion only in the Y-Z plane is considered.

vii) The pile anchoring is represented by a hinged support.

6.3 Single Cable Behaviour

In order to determine the behaviour of a cable to applied loads, the following parameters should be specified:

i) vertical and horizontal distances between cable supports;

ii) angle of inclination of the guyline;

iii) length of the cable;

iv) unit weight (submerged) of the cable;

v) weight of clump weight per unit length and its total weight;

vi) pretensioning force in the cable.

The cable analysis is complex due to nonlinear structure response. The following factors cause nonlinearity in the cable behaviour:

i) Change in configuration (sag), which is nonlinear and depends on the geometry and stress in the cable (geometric nonlinearity).

ii) Load-deflection behaviour of the cables and their zero stiffness in compression (material nonlinearity).

6.4 Single Cable Modelling

The finite element model shown in Fig. 6 represents one of the eight cables supporting the test tower. The cable is discretised into fourteen segments, while the nodes are connected together with linear springs of stiffness

$$K_i = \frac{A_{st}\ E_s}{L_i} , \qquad (6.1)$$

where:

K_i = stiffness of line segment;

A_{st} = metallic cable cross-sectional area;

E_s = Young's modulus;

L_i = segment length.

The clump weight area is discretised into four equal segments. The five nodes where the clump weight is lumped are connected together with stiffer linear springs simulating the chain clump. The seabed is simulated by nonlinear springs having very high compressive stiffness and insignificant tensile stiffness to allow the clump weight to displace only above the seabed. Each node has two degrees of freedom, horizontal and vertical directions. The weight of each segment is concentrated at the nodal points at both ends.

6.5 Single Cable Analysis

The tension-horizontal displacement relationship of the guyline is obtained using the nonlinear Structural Analysis Program (NONSAP) [13]. The geometrically nonlinear finite element procedure including large displacement effects with small strains are assumed in the calculation of element forces. An iterative technique is applied by assuming initial configuration of the cable system and correcting the configuration using successive solutions. The equation for determining the initial configuration for the unstretched cable (Fig. 7) assuming constant horizontal tension has been modified to include the initial slope angle and is given by:

$$c_1 = \tan\theta_o + \sqrt{1 + \tan^2\theta_o} \; ; \tag{6.2}$$

$$z = \frac{T_h}{2w}\left[\left(c_1 e^{\frac{wy}{T_h}} - \frac{1}{c_1} e^{\frac{-wy}{T_h}}\right) - \left(c_1 + \frac{1}{c_1}\right)\right] \; ; \tag{6.3}$$

$$s = \frac{T_h}{2w}\left(C_1 e^{\frac{wy}{T_h}} - \frac{1}{c_1} e^{\frac{-wy}{T_h}}\right) - \frac{T_h \tan\theta_o}{w} \; ; \tag{6.4}$$

$$T = \frac{T_h}{2}\left(\frac{2wz}{T_h} + c_1 + \frac{1}{c_1}\right) \; ; \tag{6.5}$$

$$\theta = \cos^{-1}(T_h/T) \; . \tag{6.6}$$

where

θ_o = cable inclination angle with the horizontal axis at the origin ($y = z = 0$);

y, z = horizontal and vertical coordinate of a point on the cable curve, respectively;

T_h = constant horizontal tension component along the cable;

T = tension at any point (y, z);

w = weight of cable per unit length;

s = length of the cable measured from the origin;

θ = cable inclination angle with the horizontal axis at point (y, z).

The total stiffness of the mooring system is calculated using

$$K_I = \sum_{i=1}^{m} K_i \cos \psi_i \quad , \qquad (6.7)$$

where

K_I = total mooring system stiffness;

K_i = instantaneous stiffness of each mooring line in its radial direction;

ψ_i = the angle between each mooring line and the direction of deflection;

m = number of mooring lines.

The total horizontal restoring force of the mooring system vs the tower offset is plotted in Fig. 9. It is observed that the mooring system restoring force is a nonlinear function of the tower offset. However, in the linear analysis stage the relation is assumed to be a straight line with a constant horizontal stiffness of 98.1 kN/m.

CHAPTER VII

ANALYSIS

7.1 Introduction

The finite element package programmes NONSAP [13] and SAP-IV [14] are used to calculate the test-guyed tower response to irregular wave forces. For 100% and 75% mooring stiffnesses, the following cases are considered in the study:

i) free vibration for both beam and truss element models;

ii) linear tower response for both beam and truss element models;

iii) nonlinear response of truss element model considering nonlinearity of the mooring system with full capacity.

7.2 Equation of Motion and Mode of Vibration

The equation of motion of a multidegree-of-freedom system is given by,

$$[M]\{\ddot{v}(t)\} + [C]\{\dot{v}(t)\} + [K]\{v(t)\} = \{F(t)\}, \qquad (7.1)$$

where:

[M] = diagonal lumped mass matrix;

[C] = structural damping matrix;

[K] = stiffness matrix;

{F(t)} = time dependent load vector;

and

$\{\ddot{v}(t)\}$, $\{\dot{v}(t)\}$ and $\{v(t)\}$ = nodal time dependent acceleration, velocity and displacement vectors, respectively.

In a dynamic analysis, the time step affects the solution accuracy. In order to select an appropriate time step, it was necessary to solve for the fundamental frequencies of the system. The equation of motion for a free undamped system can be expressed as

$$[M]\{\ddot{v}(t)\} + [K]\{v(t)\} = \{0\}. \tag{7.2}$$

Assuming that the free-vibration motion is harmonic,

$$\{v(t)\} = \{\phi\}\, e^{i\omega t}, \tag{7.3}$$

where:

$\{\phi\}$ = shape of the system which does not change with time, only varies in the amplitude;

ω = angular natural frequency of vibration of structural system in any one of its natural modes.

Substituting equation (7.3) into equation (7.2) and solving,

$$[[K] - \omega^2 [M]]\{\phi\} = \{0\}. \tag{7.4}$$

Applying Cramer's rule, equation (7.4) is reduced to,

$$\{\phi\} = \frac{\{0\}}{\det\ [[K] - \omega^2[M]]}. \tag{7.5}$$

where det $[[K] - \omega^2 [M]]$ represents determinant of the matrix of the system. A nontrivial solution is possible only when the determinant in the denominator is equal to zero

$$\det [[K] - \omega^2[M]] = \{0\} . \tag{7.6}$$

Expanding the frequency equation (7.6) gives a polynominal of the Nth degree which yields the eigenvalues (frequencies) ω^2 for the N degrees of freedom system.

The mode shapes are computed by substituting these eigenvalues into equation (7.4). Every eigenvalue results in a distinct eigenvector (mode shape). The lowest 15 mode of vibrations and the associated natural frequencies are calculated for the beam element model using SAP-IV (Table 1). For the truss element model the lowest 15 modes of vibration and the corresponding frequencies are computed using NONSAP (Table 1).

7.3 Evaluation of Damping Matrix

In solving the equation of motion, (Eqn. 7.1), Rayleigh's approach is assumed to determine the damping matrix [C]. The damping effect is considered to be proportional to the mass and stiffness matrices as

$$[C] = a_0 [M] + a_1[K] , \tag{7.7}$$

where a_0, a_1 are mass and stiffness proportionality factors, respectively.

Applying the orthogonality condition the resulting damping can be expressed by modal damping ratios. The damping ratio for the Nth mode is given by

$$\xi_n = a_0/2\omega_n + a_1\omega_n/2 \quad , \tag{7.8}$$

where:

ω_n = the modal frequency;

ξ_n = damping ratio.

The factors a_0 and a_1 can be evaluated by specifying damping ratios for the lowest two frequencies and solving the resulting two simultaneous equations (7.8). The guyed tower can be considered to be heavily damped when the response is large and lightly damped for small motions [10]. S.Y. Hanna [11] uses a value of ξ = 0.03 - 0.05 for a lightly damped system, whereas a value of 0.02 has been reported in Ref. 15. In the present investigation the viscous damping is taken as 2% of the critical damping, i.e. ξ = 0.02.

7.4 Wave Loads

The wave record (Fig. 10) corresponding to a storm as reported by Exxon [2] is used to calculate the irregular wave loads on the tower using the computer program IRWAF [15, 16] based on linearized Morison's equation and the linear (Airy) wave theory. The wave forces exerted on an incremental vertical circular piling length are determined using Morison's equation given by

$$dF = \frac{1}{2} C_D \rho \, DU|U| \, dS + C_I \, \rho \frac{\pi D^2}{4} \dot{U} \, dS \, , \tag{7.9}$$

where:

D = pile diameter;

ρ = water density;

U = horizontal component of water particle velocity;

$\dot{U}$ = horizontal component of water particle acceleration;

dS = incremental length of the pile;

C_D = drag coefficient;

C_I = mass or inertia coefficient.

The drag force component in Morison's equation is proportional to the square of the water velocity regardless of the sign. This nonlinear effect is approximated by method of equivalent linearization [16]. The regular wave profile is given by:

$$\eta = a \cos(\omega t) , \tag{7.10}$$

where:

a = wave amplitude;

ω = wave frequency.

The total force exerted on a vertical cylindrical pile with ends at depths z_1, z_2 (Fig. 11) is given by

$$F_T = C_1 \cos(\omega t) - C_2 \sin(\omega t) , \tag{7.11}$$

where:

$$C_1 = \frac{\gamma C_D D\, a^2}{2\pi \sinh(2kd)} [\sinh(2kd - 2kz_1) - \sinh(2kd - 2kz_2) + 2k(z_2 - z_1)];$$

$$C_2 = \frac{\gamma C_I \pi D^2 a}{4 \cosh (kd)} [\sinh (kd - kz_1) - \sinh (kd - kz_2)] ;$$

γ = specific weight of water;

a = wave amplitude;

d = water depth;

t = time;

L = wave length;

k = $2\pi/L$;

F_T = total force.

The irregular wave profile (t) is digitized with N discrete points at time interval Δt and decomposed into N/2+1 cosine waves and N/2 sine waves using Fast Fourier Transform of Cooley and Tukey. The force coefficients C_1 and C_2 are evaluated for each frequency of the decomposed profile. The total irregular wave force on an element whose ends are at depths z_1 and z_2 is given by:

$$F_T(t) = \sum_{s=0}^{N/2} [C_{1s} \cos(\omega_s t) - C_{2s} \sin(\omega_s t)] + \sum_{s=0}^{N/2} [C_{1s} \sin(\omega_s t) + C_{2s} \cos(\omega_s t)] , \qquad (7.12)$$

where

$$\omega_s = \frac{2\pi s}{N\Delta t} , \qquad s = 0, 1, \ldots N/2.$$

The wave record (Fig. 10) digitized into 305 points has a time interval $\Delta t = 0.5$ sec. Equivalent vertical members are used to represent the horizontal and the inclined diagonal members of the guyed tower.

7.5 Forced Response

The response of the guyed tower to irregular wave forces is computed by solving the equations of motion, (Eqn. 7.1), in the time domain using a step-by-step direct integration procedure. In the linear dynamic analysis, the diagonal lumped mass matrix [M] is obtained by adding the structural lumped mass and the hydrodynamic added mass matrices. The structural damping matrix [C] is proportional to the mass and stiffness matrices (Eqn. 7.7). The linear stiffness matrix [K] is triangularized and then used in the step-by-step solution. For each time step, displacements, velocities and accelerations are computed using the effective load vector.

In the nonlinear dynamic analysis, the linear and nonlinear elements are defined and the stiffness matrix [K] is determined by summing the structural linear stiffness matrix $[K_l]$ of the linear elements and nonlinear stiffness matrix $[K_n]$, which includes the nonlinear effect of the mooring system. The mass and damping matrices are computed as in the linear dynamic analysis. The Rayleigh damping coefficient a_1 (Eqn. 7.7) is applied to the linear stiffness matrix. A nonlinear effective load vector and the corresponding displacements, velocities and accelerations are determined at each time step. The nonlinear stiffness matrix $[K_n]$ and corresponding nonlinear effective load vector are updated at the specified time steps. The forcing function is based on the wave record digitized at 0.5 sec time interval. However, in a step-by-step integration accuracy of the solution depends on the time step interval Δt which could be chosen such that

$$\Delta t < 0.1 \, T \, , \qquad (7.13)$$

where T is the period of the highest mode to be included in the response. A larger time step (i.e., $\Delta t > 0.1$ T) will filter the participation of the higher modes from the predicted response. The time step Δt was chosen to be 0.05 sec to include the effect up to the third mode. In the nonlinear dynamic analysis, the nonlinear stiffness matrix was updated every five time-steps, i.e. every 0.25 sec, to obtain a reasonable accuracy without expending an excessive computing time.

CHAPTER VIII

RESULTS AND DISCUSSIONS

8.1 Frequencies and Mode Shapes

The lowest 15 modes of vibration and the associated natural frequencies are calculated for the beam element model using SAP-IV [13] and presented in Table 1. The first (sway) mode has a period of 10.76 sec. When the stiffness of the guying system (spring) is reduced by 25%, to simulate slack in the mooring lines, the period increased to 12.48 sec. For the truss element model the lowest 15 modes of vibration and the corresponding frequencies are obtained using NONSAP [13]. The first mode shape corresponds to sway with a period of 10.96 sec. The stiffness reduction by 25% in one of the four link members simulating the mooring system, caused an increase in the period corresponding to the lowest frequency (11.68 sec) (Table 1). The predicted highest sway period of 11 sec compares closely with the measured value of 11.9 sec [2]. The first (sway) mode shape is nearly linear (Figs. 4,5) and does not contain significant flexural bending. Since the tower frame is considerably rigid compared to the guyline restoring force, the first mode depends mainly on the mooring system stiffness. The second mode, or first flexural mode, has the characteristics of a simply supported beam with an overhang. Since the sway deflections are considerably large compared to the flexural deflections, the flexural mode is nearly independent of the flexibility of the guyline support, and is only affected by the tower stiffness. This explains the differences in the higher modes between the beam and truss element models - the truss model has varying stiffness within any panel while the beam element has constant stiffness along the tower.

8.2 Effect of Mooring System on Response

Fig. 8 shows the cable tension - fairlead offset relationship. Although the measured tower offset is within 70 cm, the analysis was extended to include larger offset for studying effect of the clump weight. It is noticeable that the clump weight lifts off the sea bottom when the fairlead deflects 1.8 m, softening the mooring system. The total horizontal restoring force - tower offset relationship (Fig. 9) is shown for both nonlinear and linear analyses. The linear horizontal stiffness of the mooring system is assumed as 98.1 kN/m, while its variation is considered nonlinear with tower offset in the nonlinear analysis. The power spectral density of the wave amplitude shown in Fig. 12 suggests that the dominant wave frequency is 0.125 Hz (8 sec period). Fig. 13 shows the power spectral density for the wave forces at the first node below the mean water level with a predominant frequency of the forcing function around 0.125 Hz. Figs. 14 and 15 show the power spectral density for the deck displacement based on the beam and truss element models, to the wave loads with full capacity of the mooring system in the linear case. It is observed that there are two response peaks, the first at the structural resonance frequency of 0.1 Hz and the second at the frequency of 0.125 Hz corresponding to the wave load spectrum. Figs. 16 and 17 show the power spectral density for the deck response with 25% reduction in cable stiffness. The spectra can be seen to have two closely spaced similar peaks. However, the peak response has been reduced sharply due to the shift in the natural frequency of the system from that of the applied dynamic forces, with a consequent reduction in the dynamic amplification factor. The computed responses in Figs. 18 and 19 show a maximum deck displacement of 65 cm and 56.5 cm for beam and truss element models, respectively. The response values compare well with a measured maximum of 69 cm. The maximum displacement occurs with a lag, shortly after the maximum wave force is applied. When the mooring system is softened by 25% of its capacity, the response in Figs. 20 and 21 decreased sharply to only 37.5 cm for the beam element model. As for the truss element model the response decreased to 41 cm. The reduction in the case of truss element model is less than that of the beam element model because of the reduced rigidity in the direction of the failing cable only.

The maximum deck acceleration and velocity were computed as 0.31 m/sec and 0.4 m/sec, respectively for 100% mooring capacity while the corresponding values decreased to 0.285 m/sec and 0.31 m/sec, respectively when the mooring system operated at 75% of its capacity. The maximum effective horizontal guy forces is found to be 5 tonnes with the maximum base shear value of 0.43 tonnes for 100% mooring capacity. It can be seen from Fig. 22 that much of the wave load is carried by the guylines and only a small portion is transferred to the foundation. With 25% reduction in guyline stiffness, the maximum horizontal restoring force is computed to be 2.2 tonnes. It is observed from Fig. 22 that the high moments are resisted by the top portion of the tower only and especially at the attachment points of guylines to tower.

In the nonlinear analysis of the truss element model, the computed maximum deck displacement (Fig. 23) is 51 cm representing 74% of the field measured value, while the computed offset in the linear analysis case is 81% of the measured value. In the present analysis the effect of wind and current loads are neglected, which, if taken into account, would improve the predicted response value. The maximum deck acceleration and velocity are 0.236 m/sec and 0.28 m/sec, respectively, while the maximum horizontal restoring force is 4.32 tonnes.

CHAPTER IX

CONCLUSIONS

A major portion of the horizontal wave loading is carried by the guylines and only a small part of the shear is transmitted to the tower base. The maximum moments on the structure occur at the attachment points of the guylines to the tower. However, the stresses in this region may not be critical because of the relatively small hydrostatic pressure and the compressive stresses. The tower is expected to have a very favourable fatigue resistance due to the low flexural response to cyclic loading. Difficulties may arise in the case of the stiff sea floor, as the foundation will not behave as an ideal pivot. The space truss model of the guyed tower with three degrees of freedom for each node, in contrast to a space frame tower with six degrees of freedom, can be used to predict the response reasonably well as it depends mainly on the mooring system stiffness and not on the tower rigidity. NONSAP [13] can be used to simulate the cable stiffness with a reasonable accuracy. The mooring system exhibits very little nonlinearity until clump weight lifts off the sea bottom and can be considered linear in the response computation with a reasonable accuracy. At large tower displacements, the clump weight lifts off the bottom, which softens the mooring system and introduces significant nonlinearity affecting the response. The tower is not operationally weather sensitive and remains stable during a design storm with a 25% reduction in mooring stiffness due to a cable failure. The analytical procedures presented in the study can be used for predicting the guyed tower response to irregular waves.

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TABLES AND FIGURES

Table 1-a

FREQUENCIES AND TYPE OF VIBRATIONS
FOR BEAM ELEMENT COMPUTER MODEL

	100% Mooring Stiffness		75% Mooring Stiffness		Type of Vibration
	Frequency Hz	Period Sec	Frequency Hz	Period Sec	
1	.093	10.760	.0818	12.470	Sway
2	.498	2.010	.4975	2.010	Flexure
3	1.670	.599	1.666	.600	"
4	3.630	.276	3.612	.277	"
5	6.430	.156	6.391	.156	"
6	6.830	.146	6.799	.147	"
7	10.050	.099	10.00	.100	"
8	14.380	.069	14.340	.069	"
9	19.260	.052	19.250	.052	"
10	20.070	.0498	20.050	.050	"
11	24.82	.0403	24.820	.0403	"
12	31.23	.0320	31.160	.0320	"
13	33.29	.0300	33.27	.0300	"
14	38.12	.0260	37.93	.0264	"
15	44.54	.0225	44.11	.0227	"

Table 1-b

FREQUENCIES AND TYPE OF VIBRATIONS
FOR TRUSS ELEMENT COMPUTER MODEL

	100% Mooring Stiffness		75% Mooring Stiffness		Type of Vibration
	Frequency Hz	Period Sec	Frequency Hz	Period Sec	
1	.091	10.96	.0856	11.68	Sway
2	.091	10.96	.091	10.96	Sway
3	.243	4.12	.243	4.12	Torsion
4	1.064	.94	1.064	.94	Flexure
5	1.266	.79	1.266	.79	Flexure
6	2.594	.38	2.594	.38	Torsion
7	2.911	.34	2.911	.34	Flexure
8	3.598	.28	3.598	.28	Flexure
9	5.145	.19	5.145	.19	Torsion
10	5.968	.17	5.968	.17	Flexure
11	6.37	.16	6.37	.16	Flexure
12	6.48	.16	6.48	.16	Flexure
13	7.51	.13	7.51	.13	Torsion
14	9.307	.11	9.307	.11	Flexure
15	9.524	.11	9.524	.11	Flexure

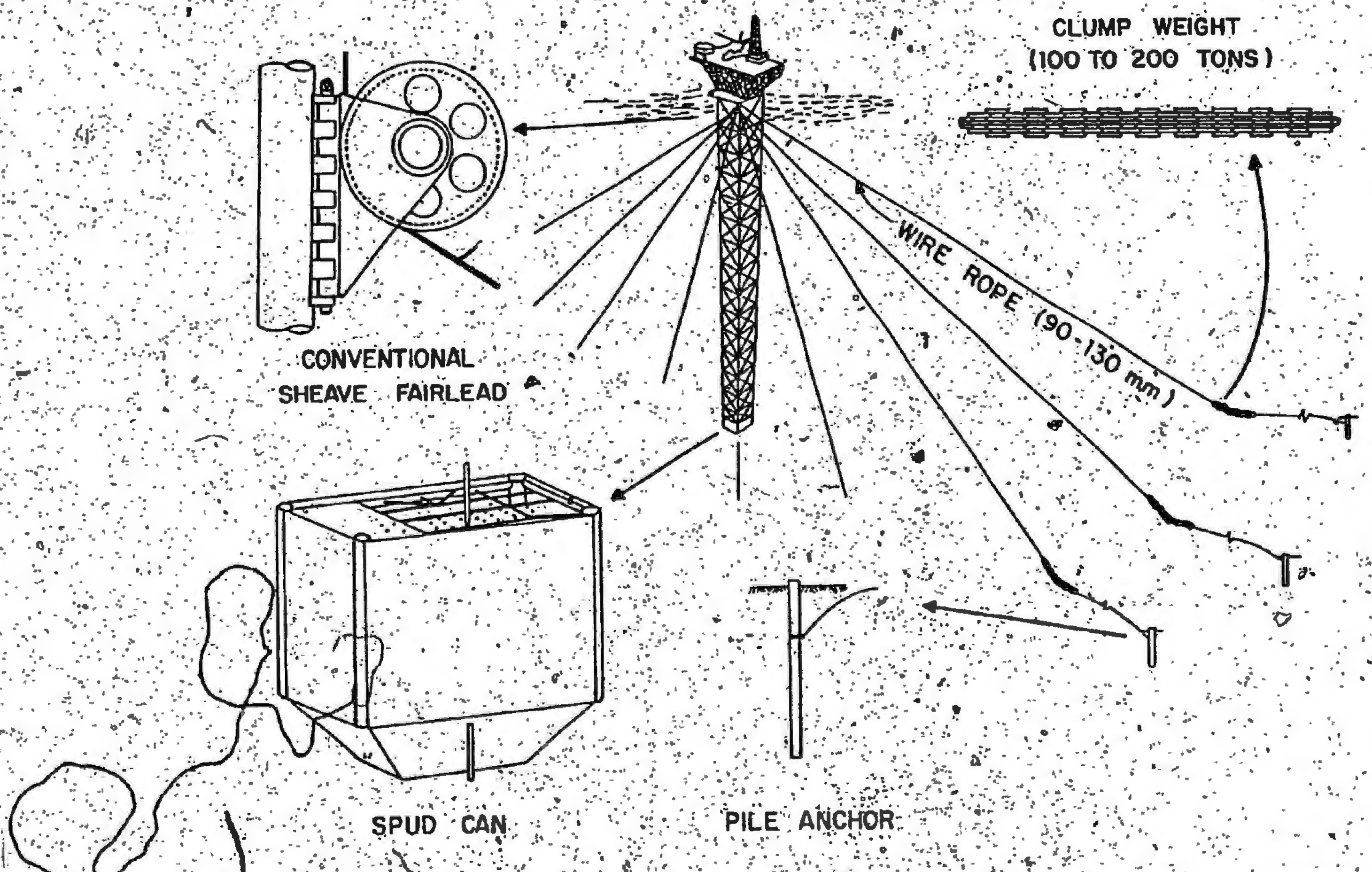

Fig. 1. Exxon's guyed tower bottom-founded and supported by anchored guylines attached near the water surface. [Ref. 3]

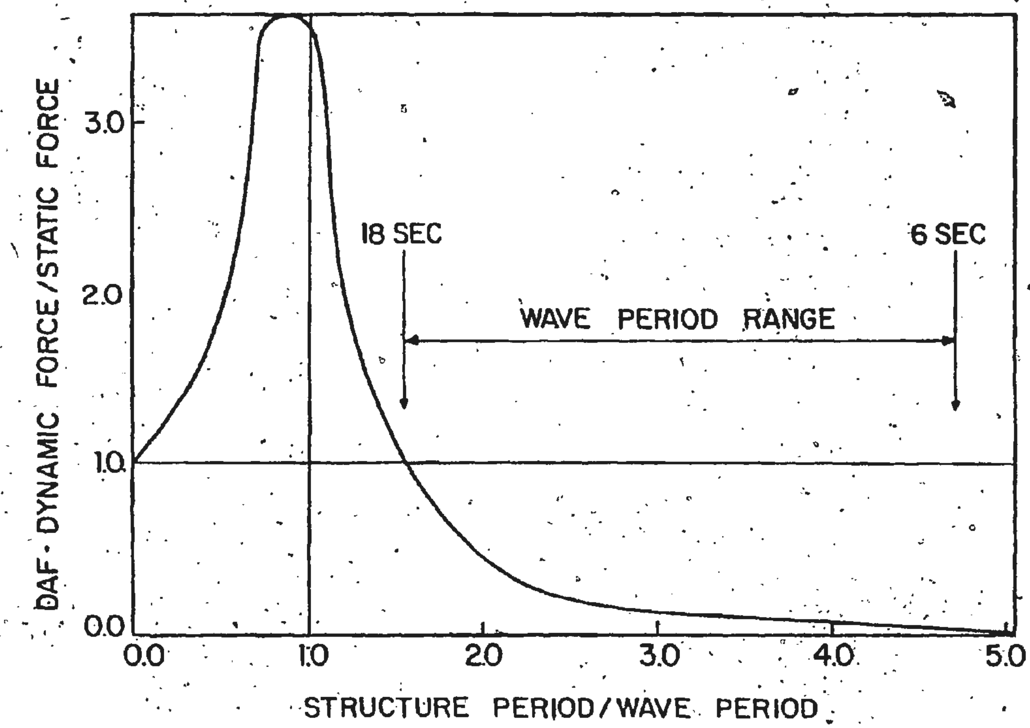

Fig. 2. The dynamic amplification factor vs. period ratio for simple sine waves (the frequency response function.) [Ref. 1]

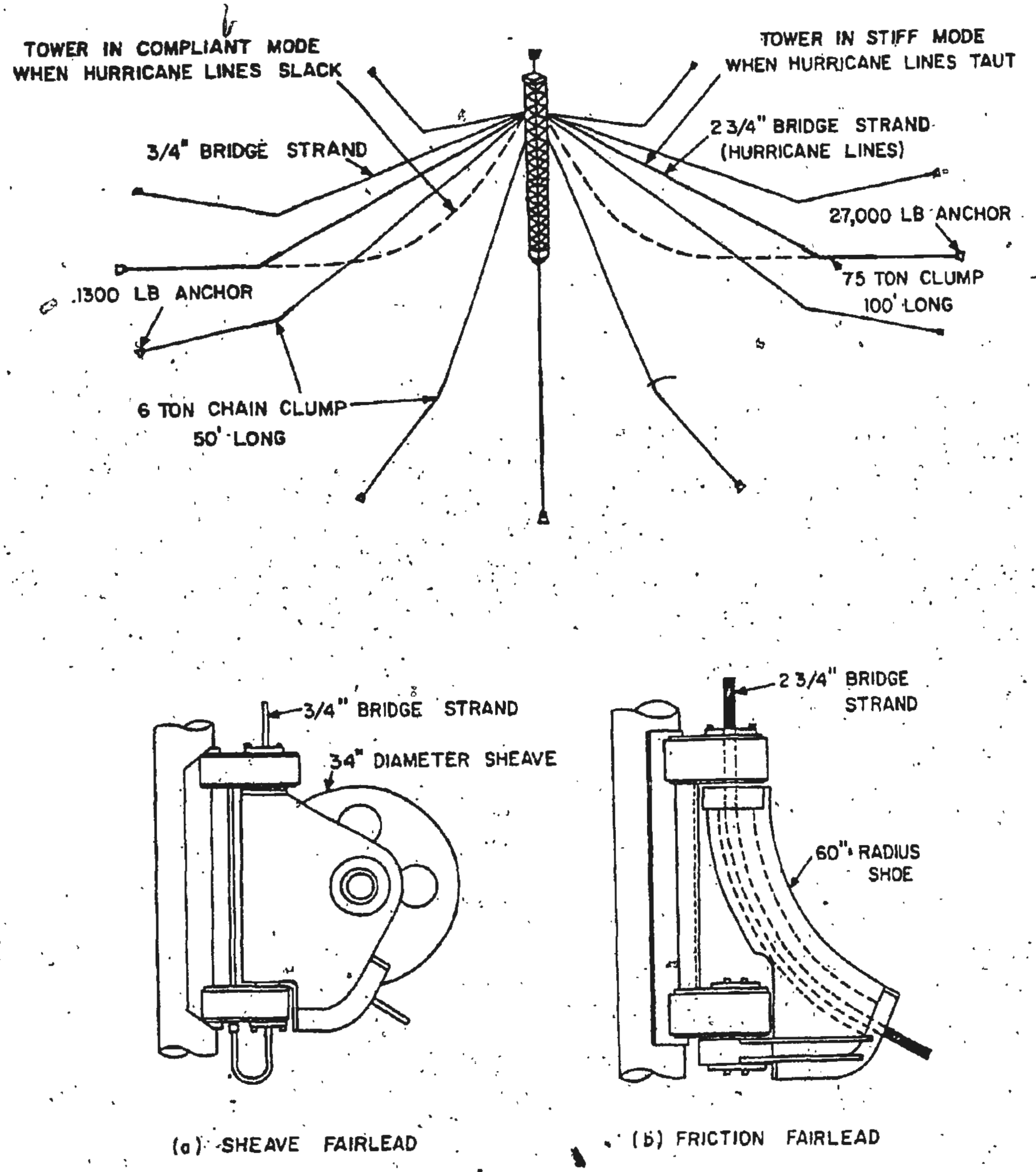

Fig. 3. Guying system for test tower. [Ref. 2]

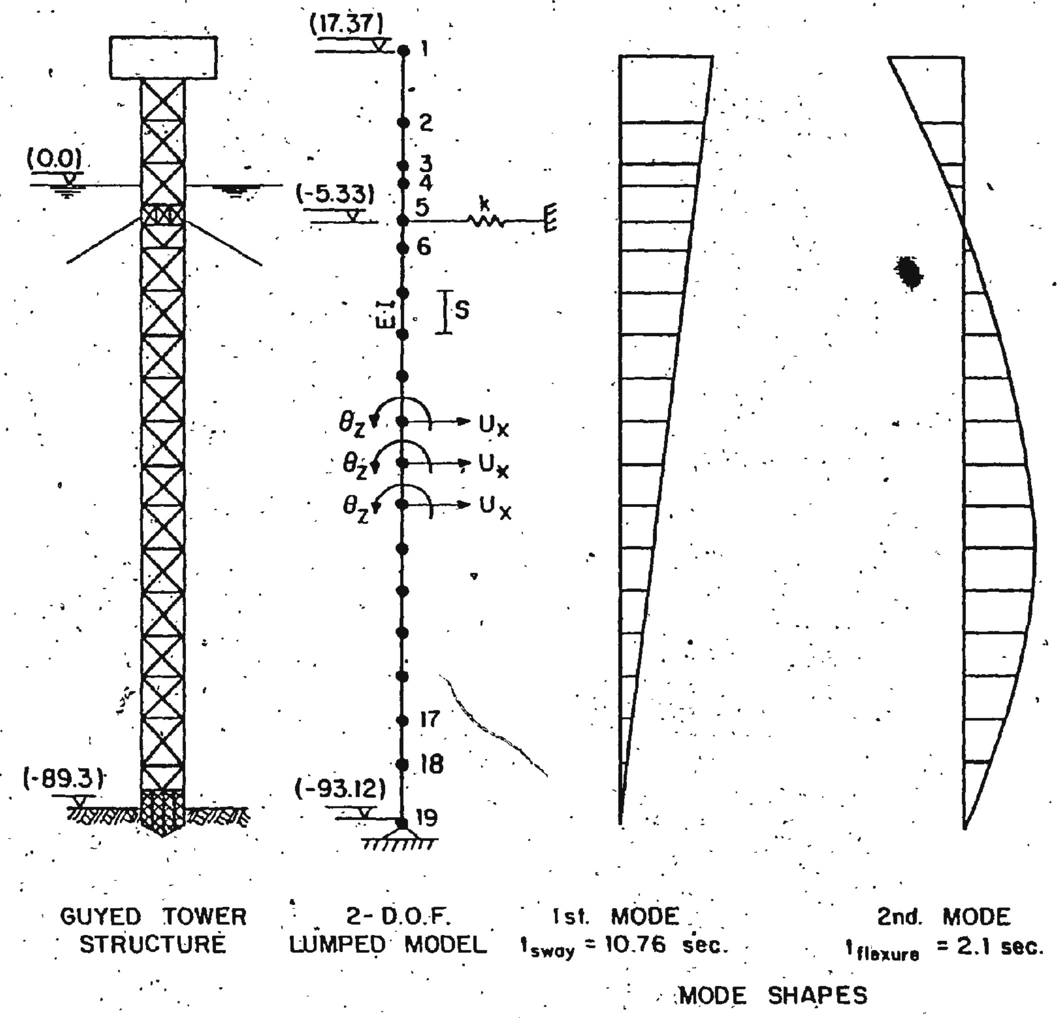

Fig. 4. 2-D Beam element model.

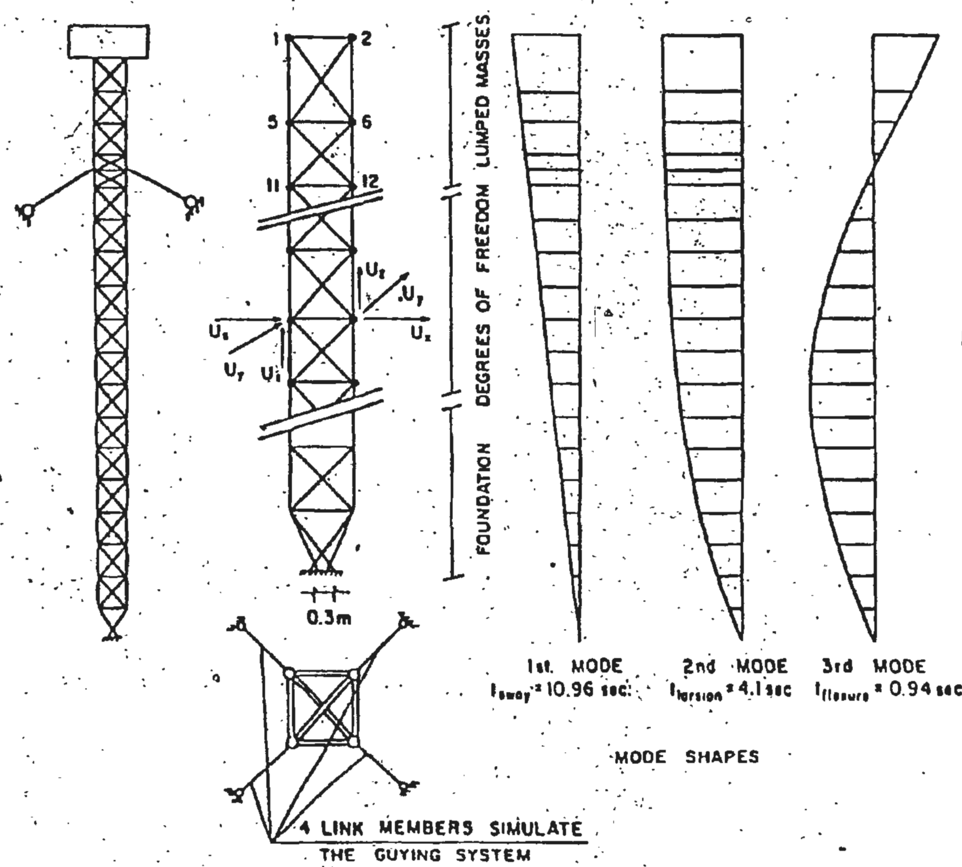

Fig. 5. 3-D truss element model.

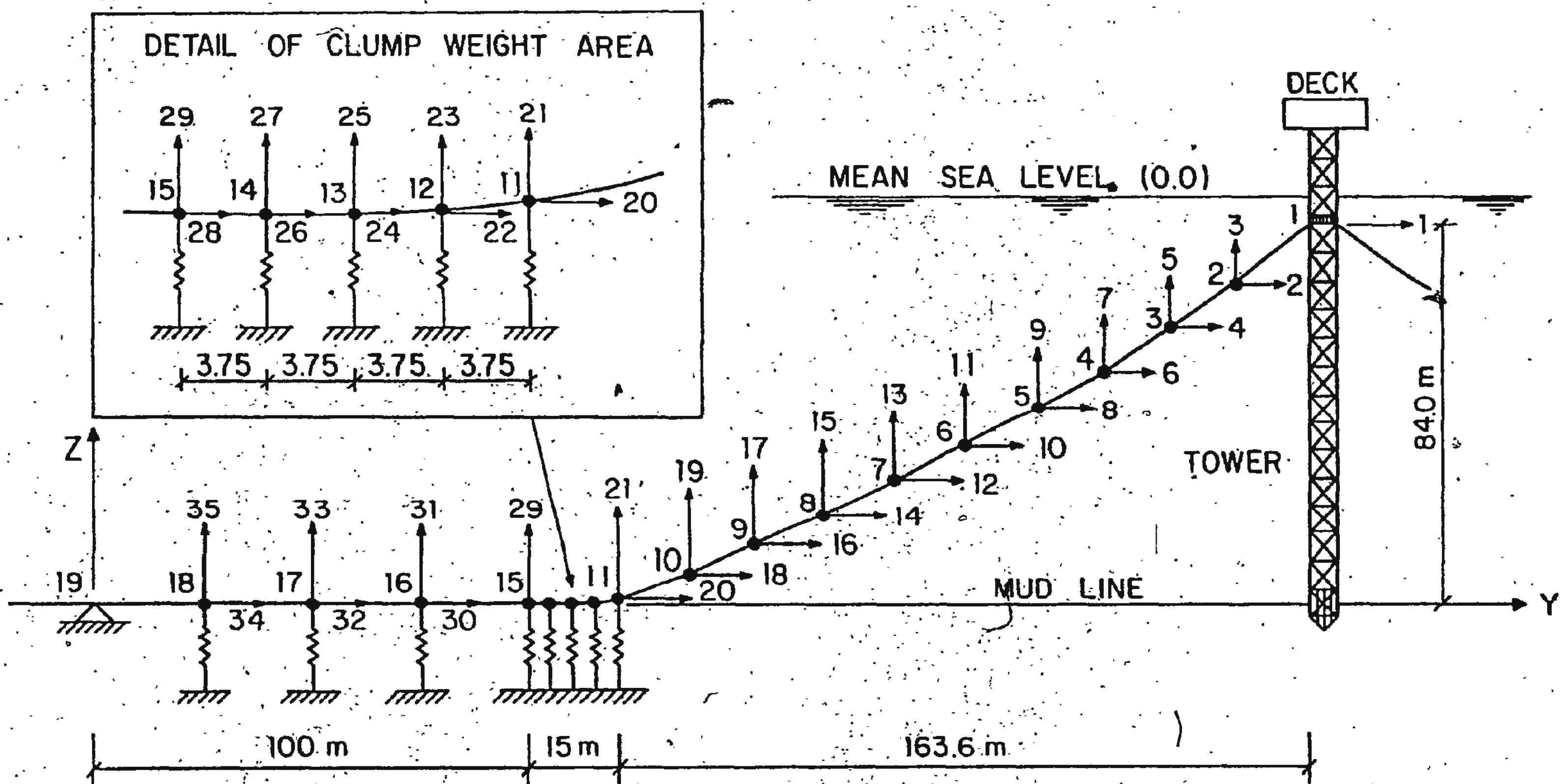

Fig. 6. Finite element model of Test Guyed Tower mooring system..

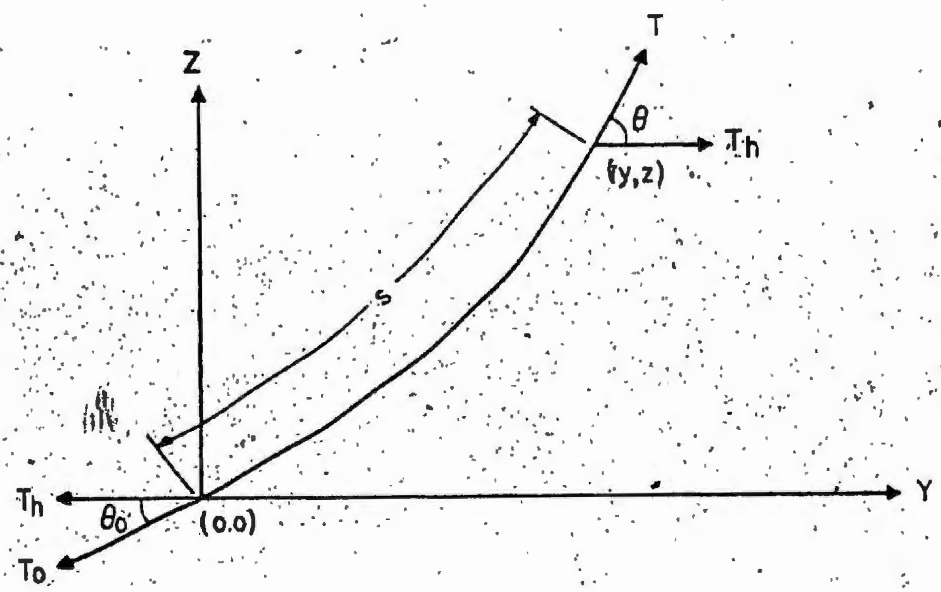

Fig. 7. Unstretched cable configuration.
(T_h = constant)

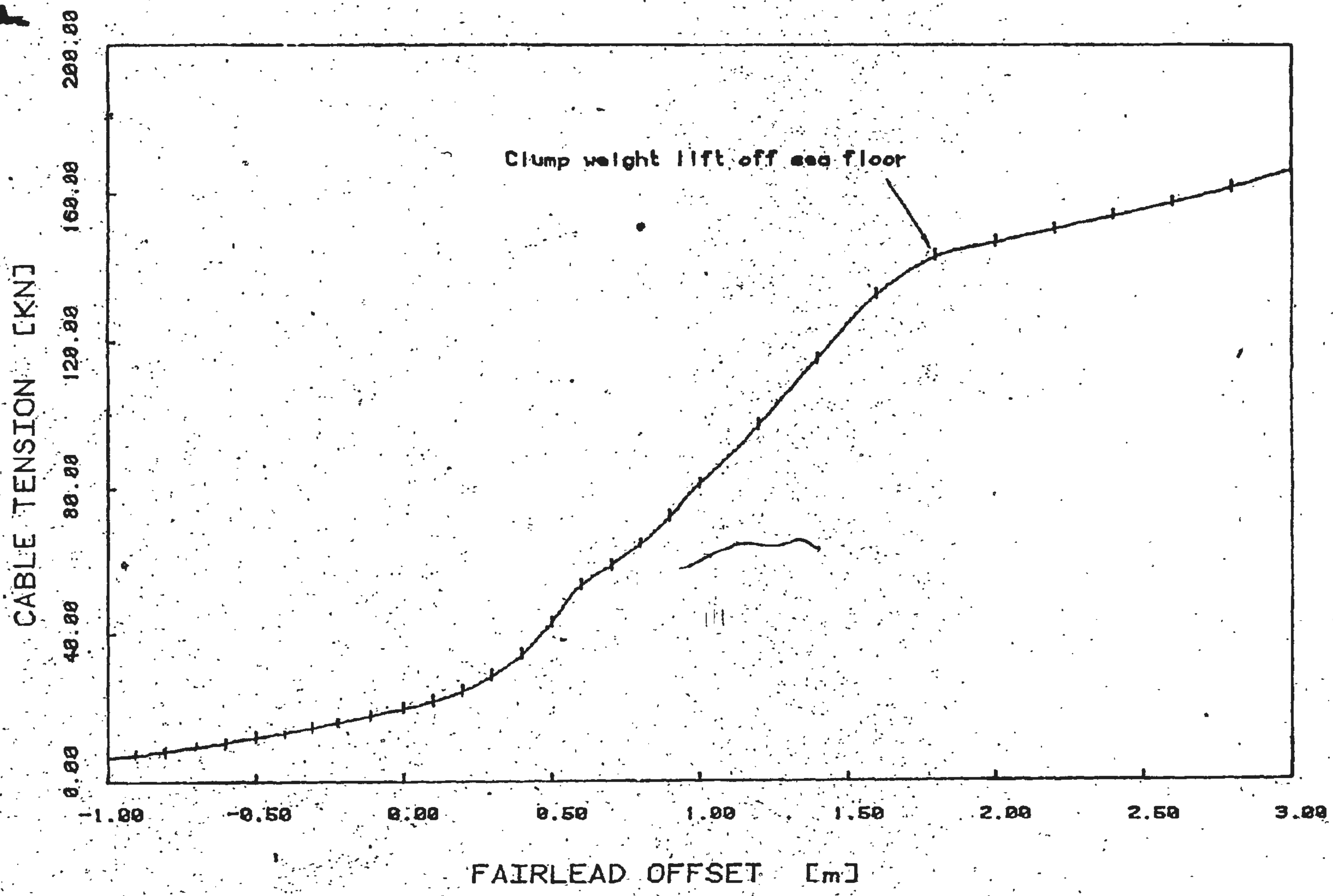
Clump weight lift off sea floor
CABLE TENSION [KN]
0.00
40.00
80.00
120.00
160.00
200.00
-1.00
-0.50
0.00
0.50
1.00
1.50
2.00
2.50
3.00
FAIRLEAD OFFSET [m]

Fig. 8

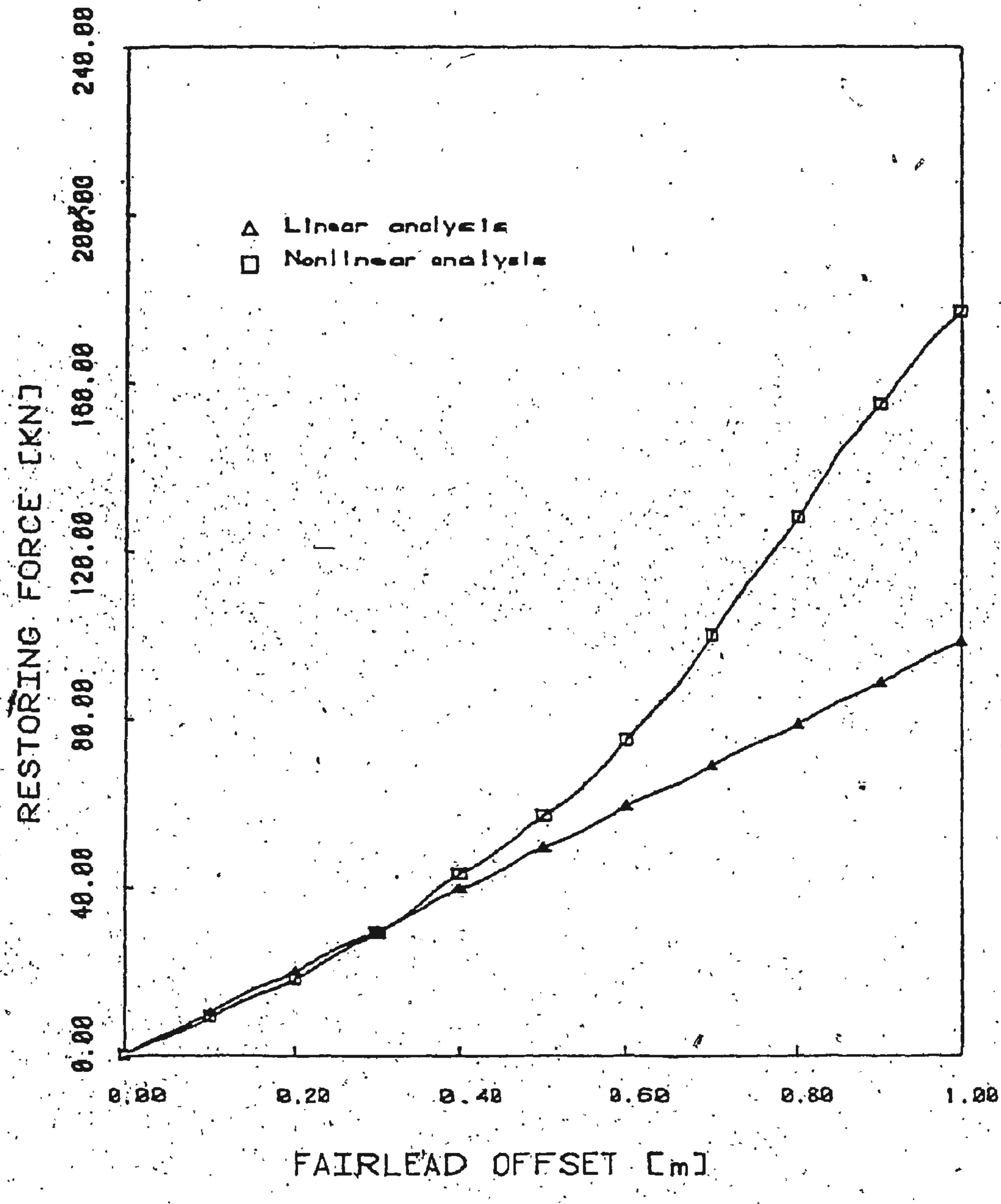

Fig. 9. TOTAL HORIZONTAL RESTORING FORCE

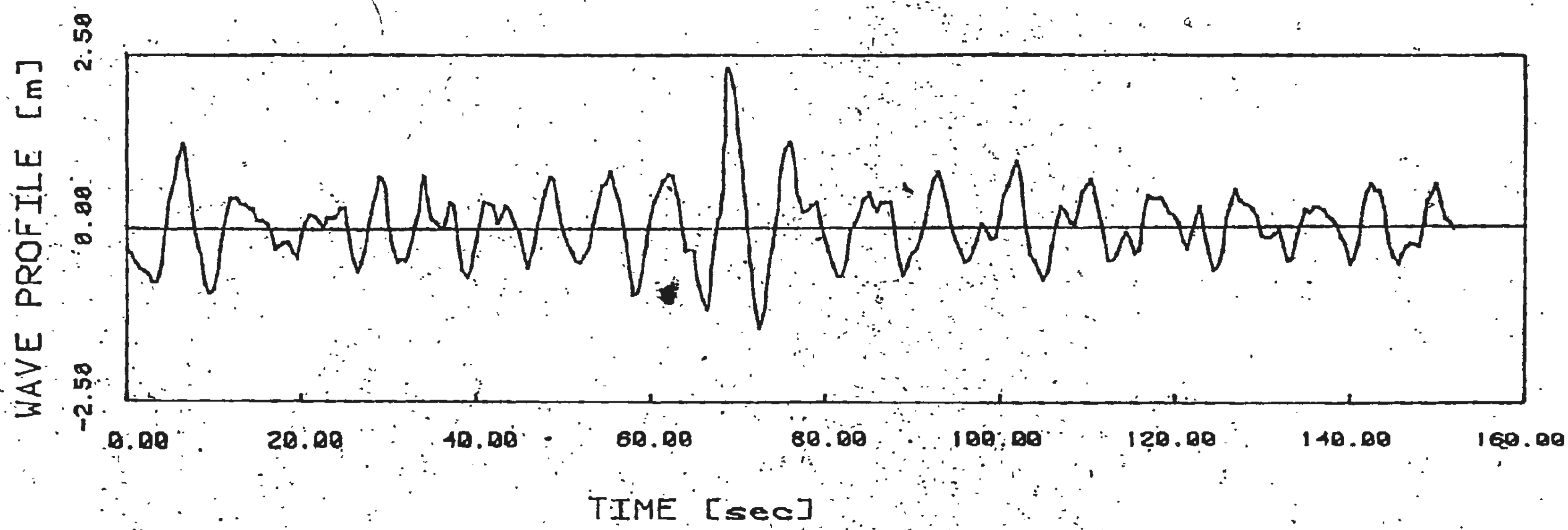

Fig.10 DIGITIZED WAVE PROFILE

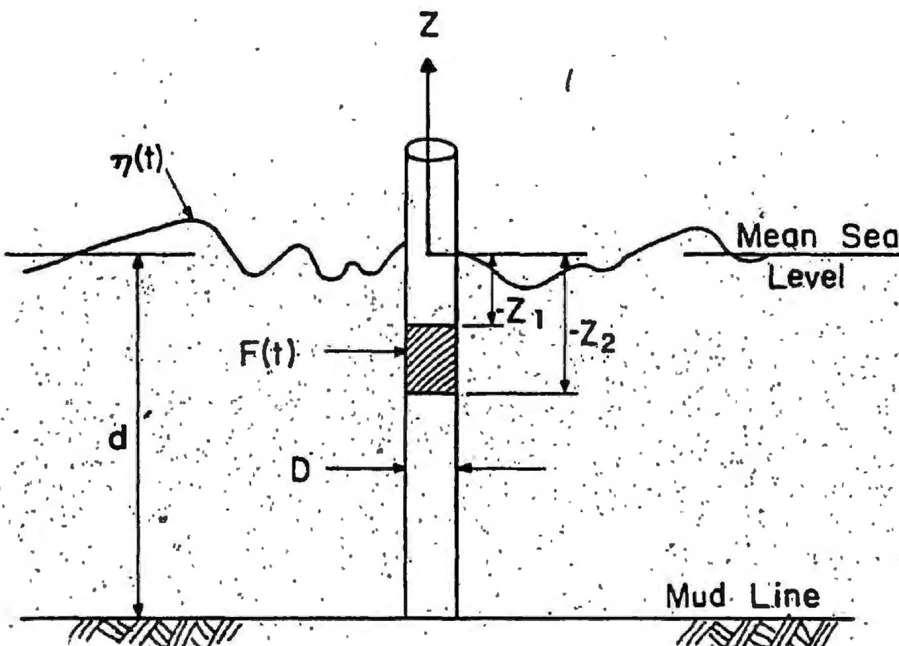

Fig. 1.1. Conventions and symbols of wave forces. [Ref. 14]

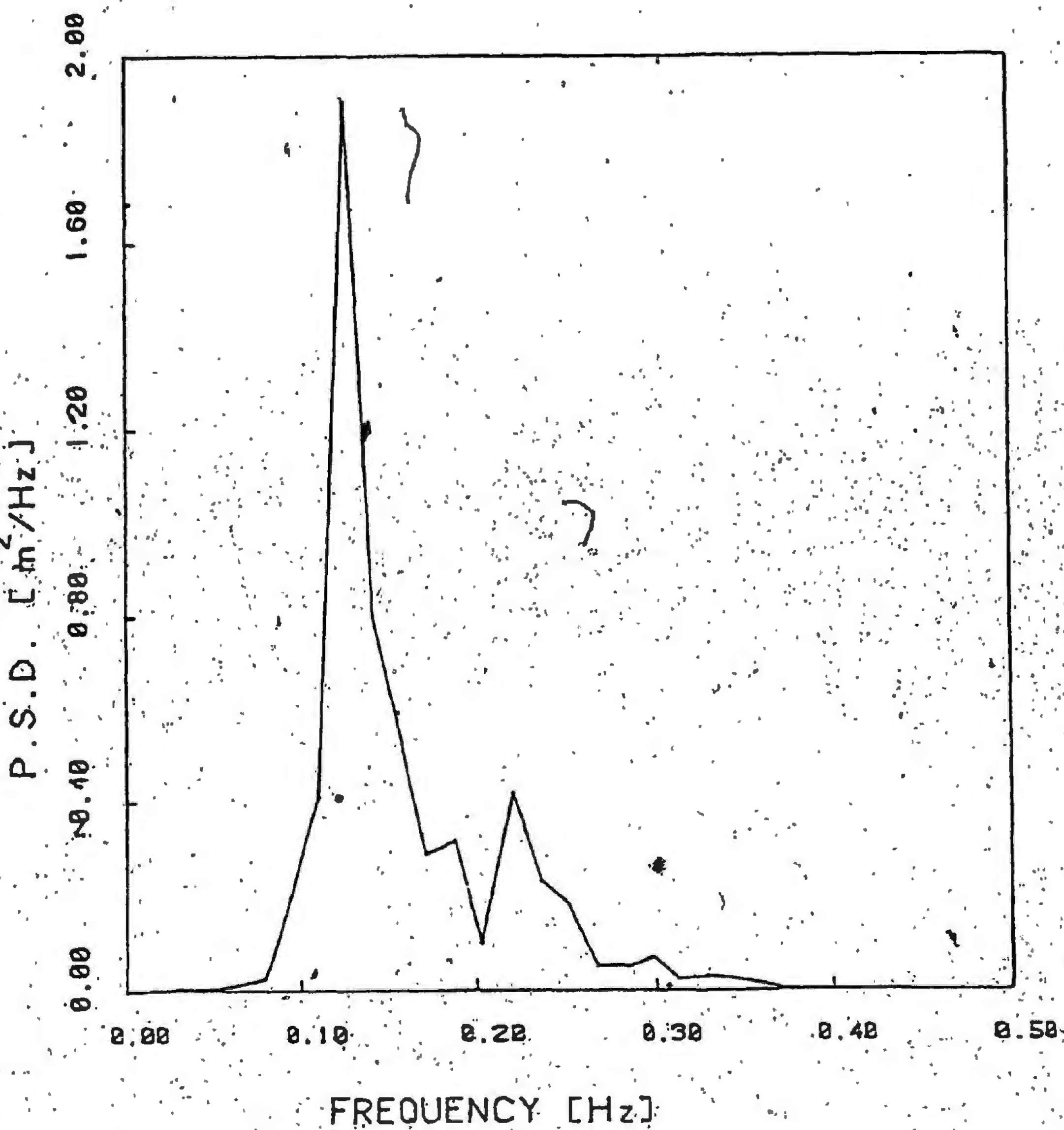

Fig. 12 POWER SPECTRAL DENSITY FOR WAVE PROFILE

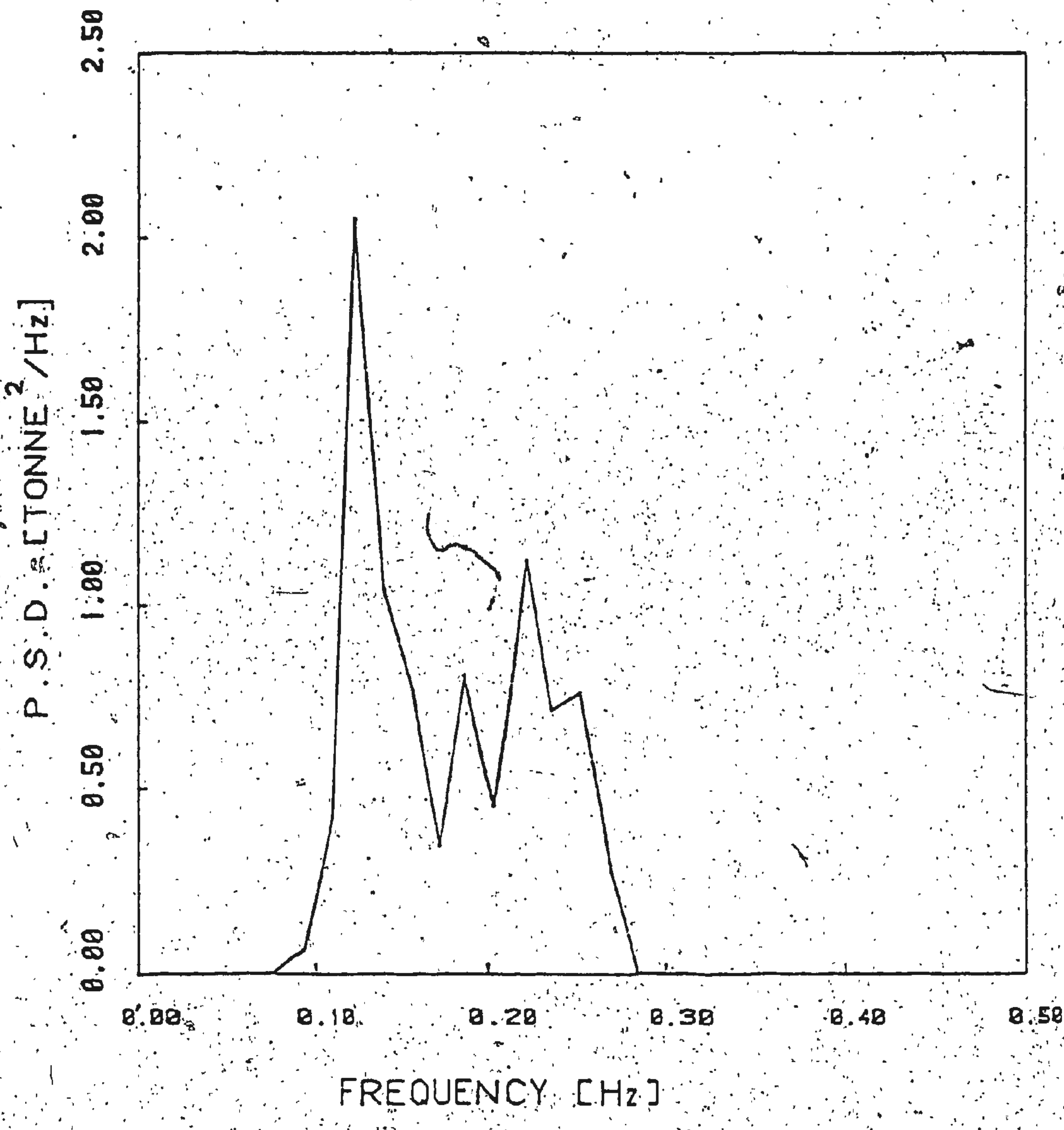

Fig.13. POWER SPECTRAL DENSITY FOR WAVE LOADS AT FIRST NODE BELOW WATER SURFACE

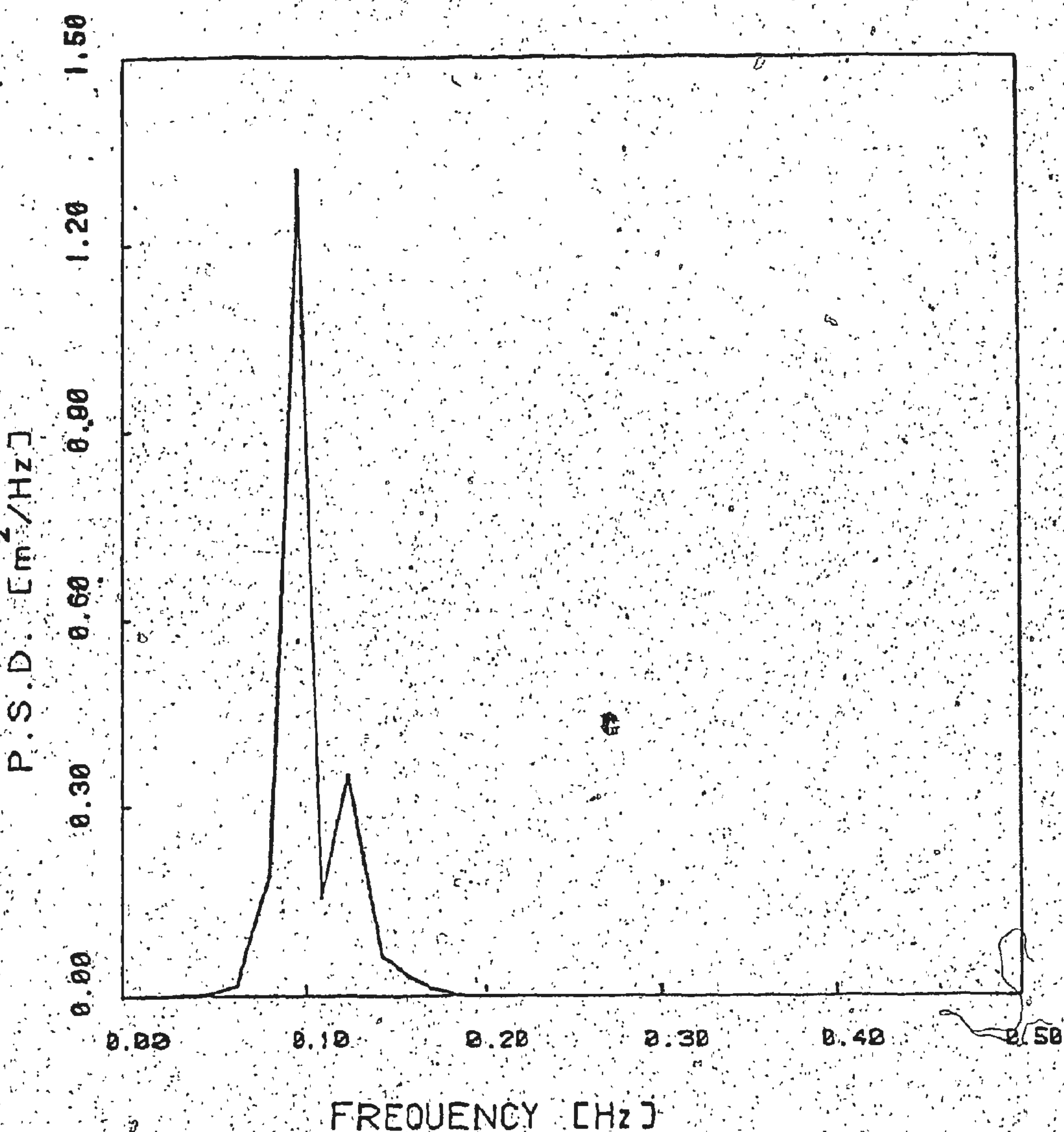

Fig.14 POWER SPECTRAL DENSITY FOR TOWER RESPONSE USING BEAM ELEMENT MODEL [100% mooring capacity]

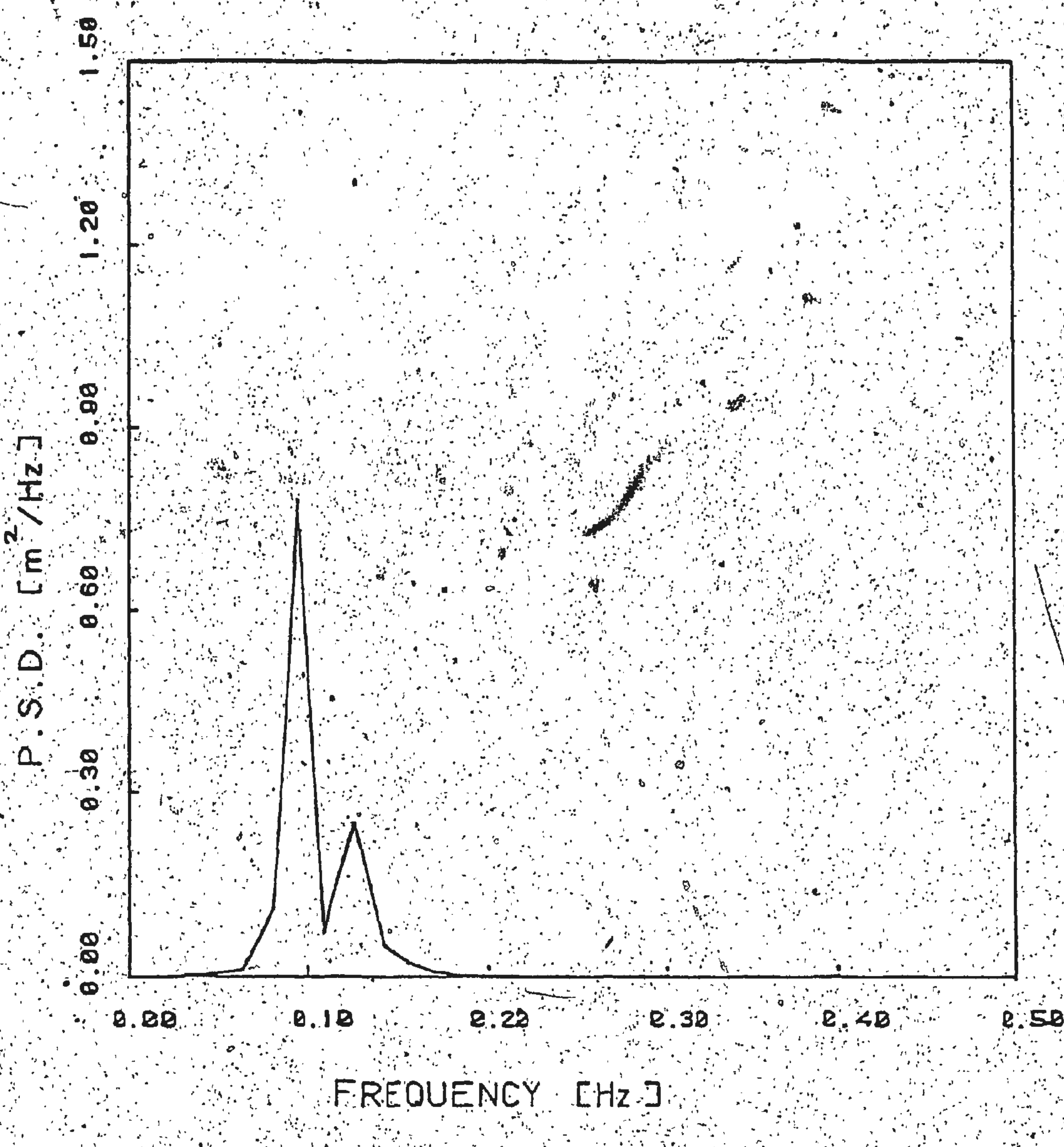

Fig.15 POWER SPECTRAL DENSITY FOR TOWER RESPONSE USING TRUSS ELEMENT MODEL [100% mooring capacity]

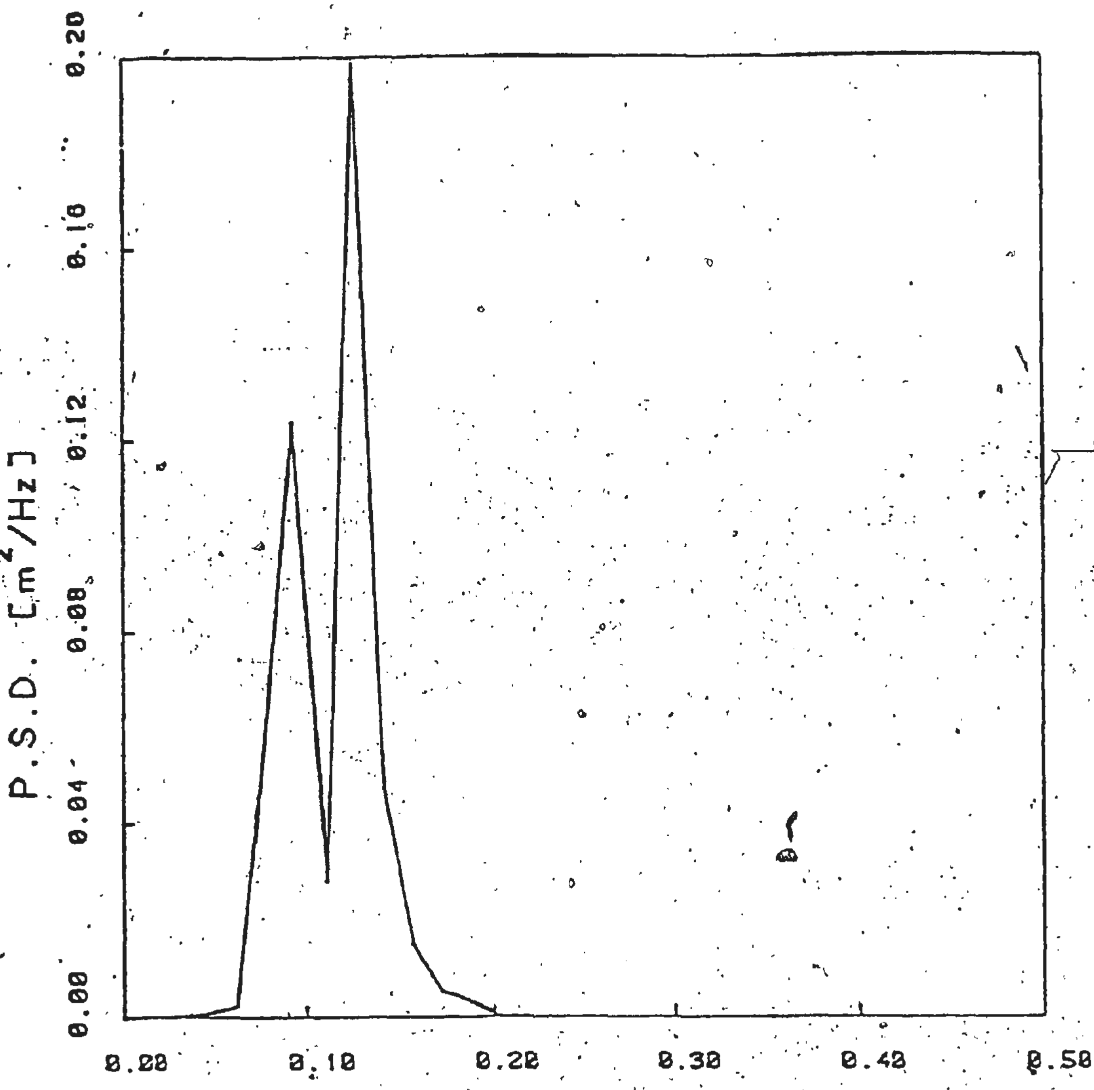

Fig.16 POWER SPECTRAL DENSITY FOR TOWER RESPONSE USING BEAM ELEMENT MODEL [75% mooring capacity]

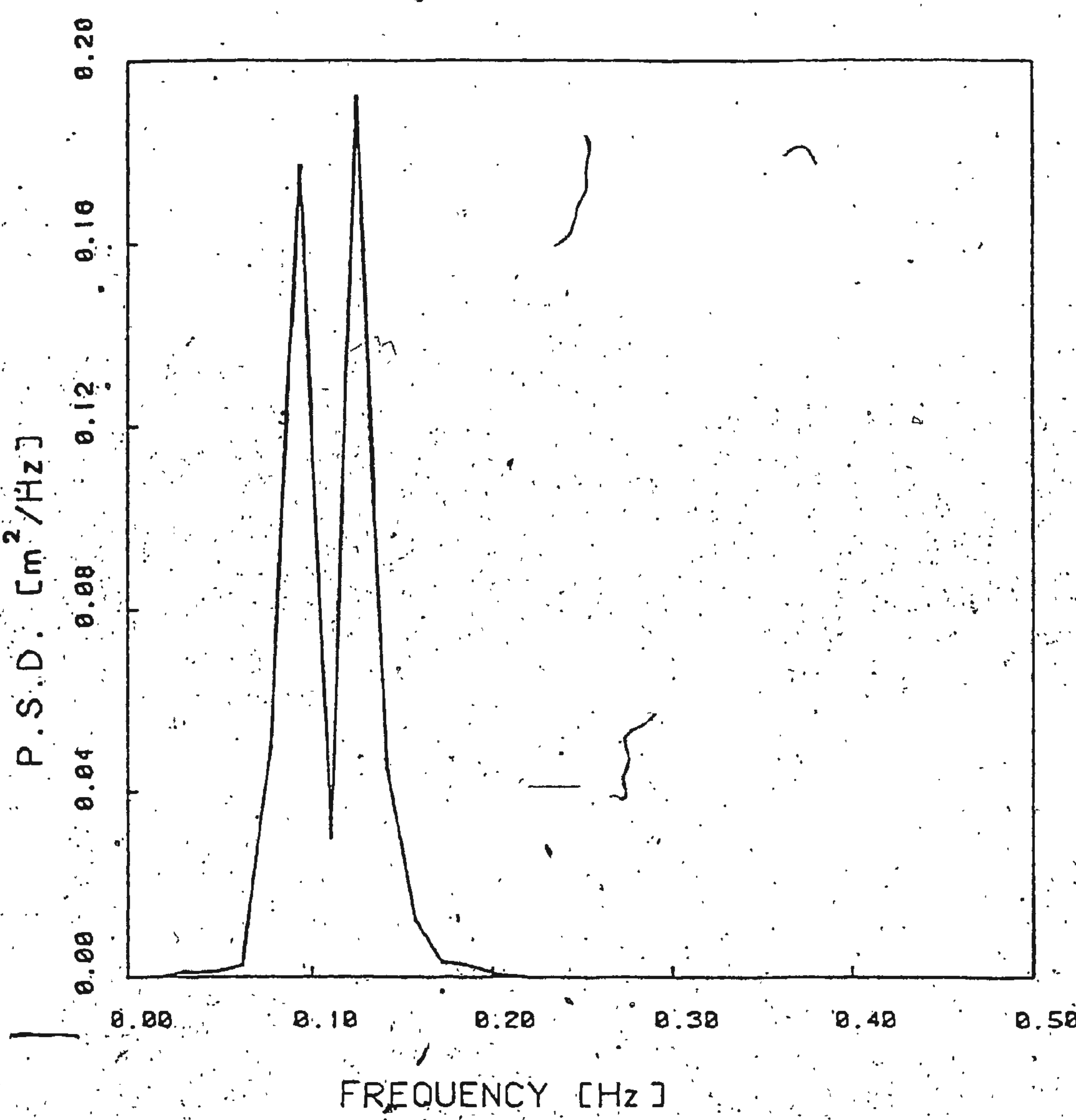

Fig. 17 POWER SPECTRAL DENSITY FOR TOWER RESPONSE USING TRUSS ELEMENT MODEL [75% mooring capacity]

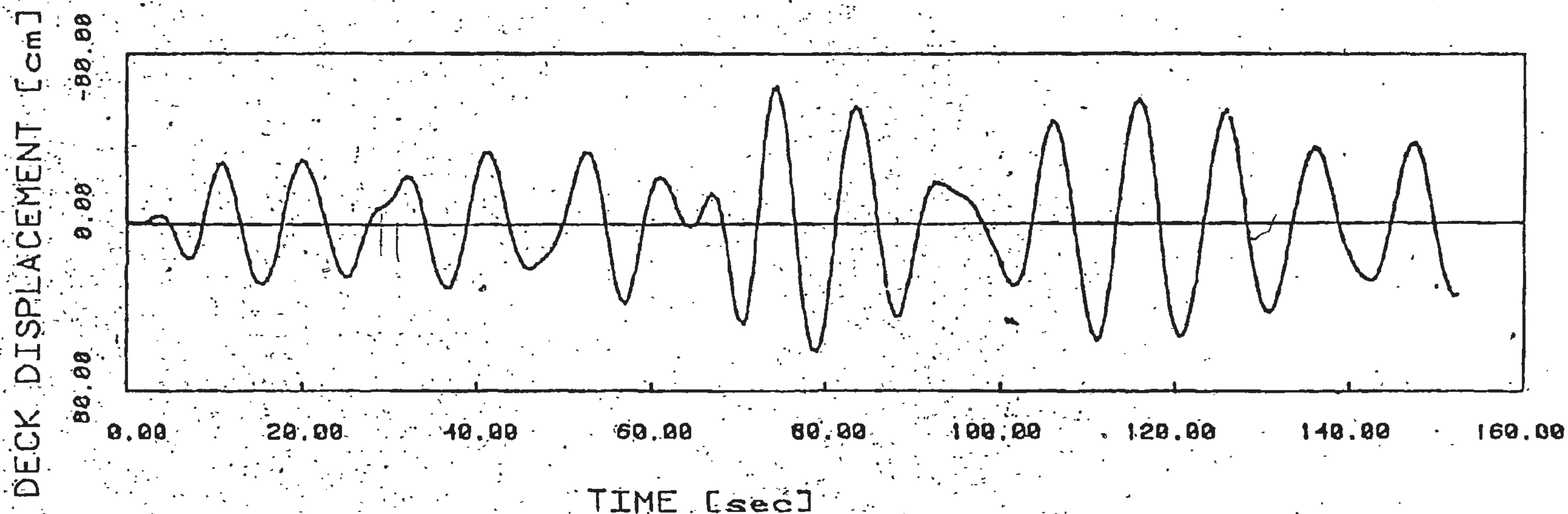

Fig. 18 PREDICTED TOWER RESPONSE USING BEAM ELEMENT MODEL

[100% mooring capacity, linear analysis]

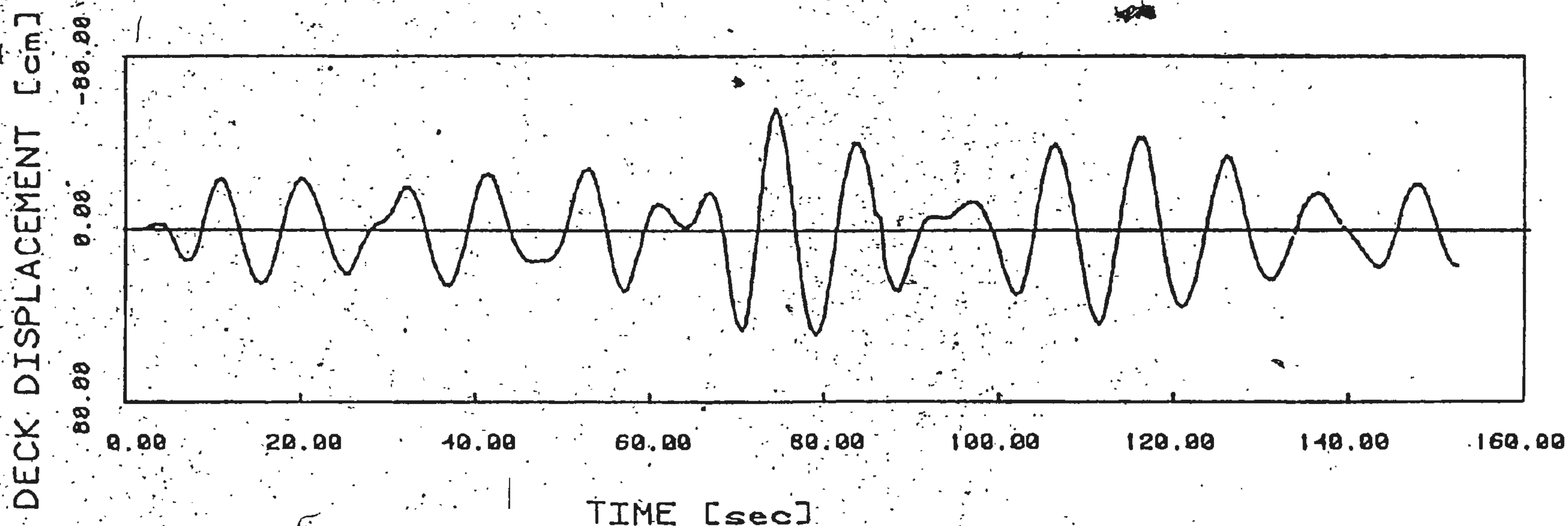

Fig 19 PREDICTED TOWER RESPONSE USING TRUSS ELEMENT MODEL

[100% mooring capacity, linear analysis]

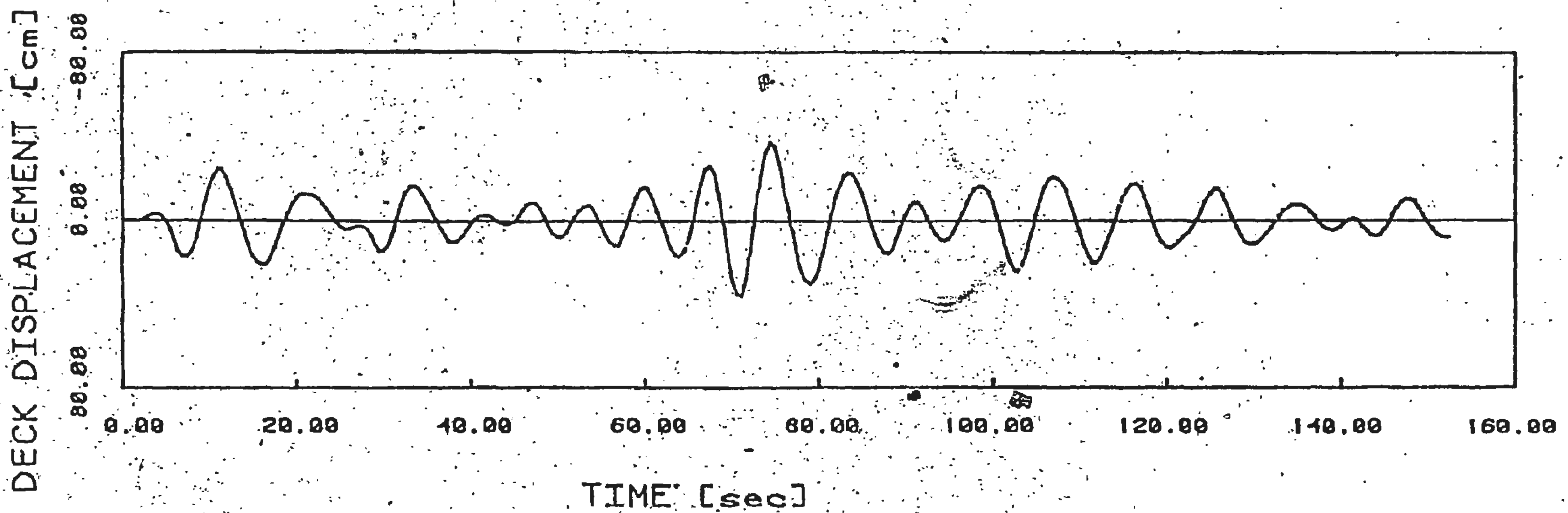

Fig 20 PREDICTED TOWER RESPONSE USING BEAM ELEMENT MODEL

[75% mooring capacity, linear analysis]

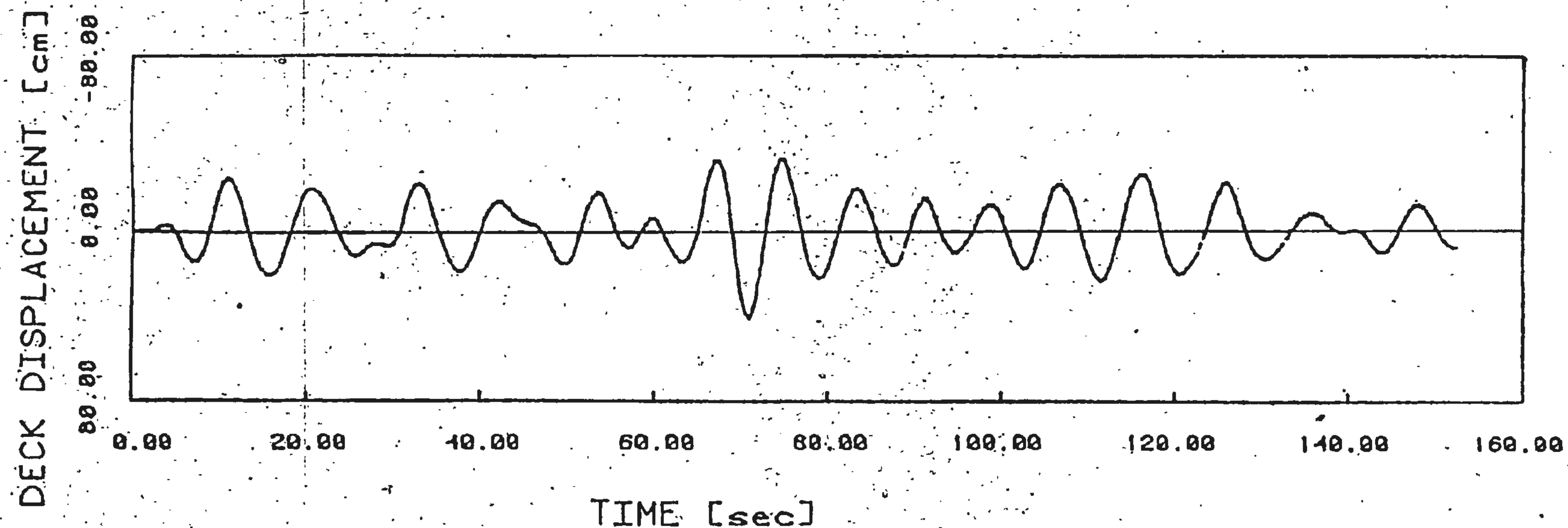

Fig.21 PREDICTED TOWER RESPONSE USING TRUSS ELEMENT MODEL

[75% mooring capacity, linear analysis]

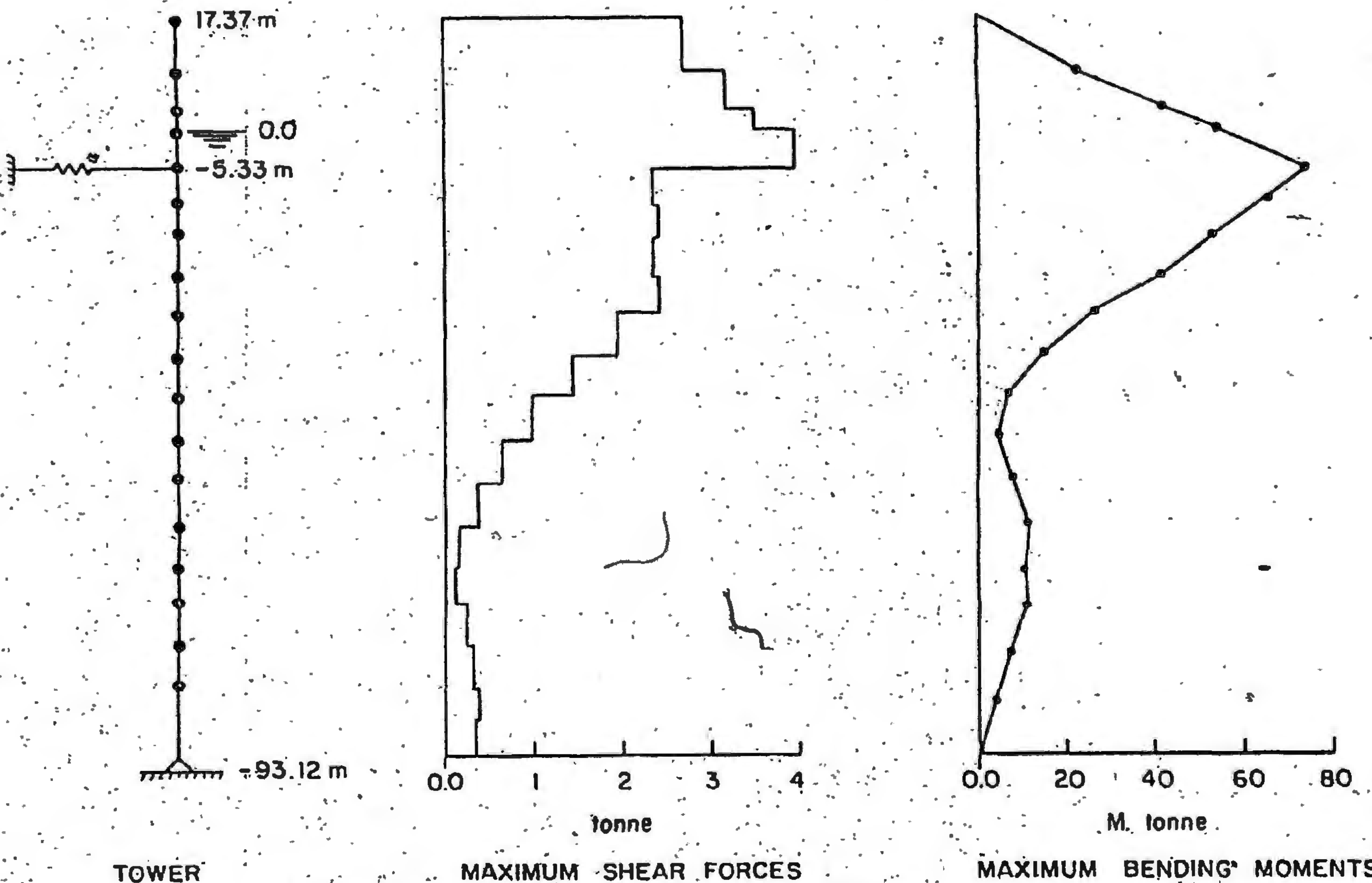

Fig. 22 Maximum shear and bending moment.

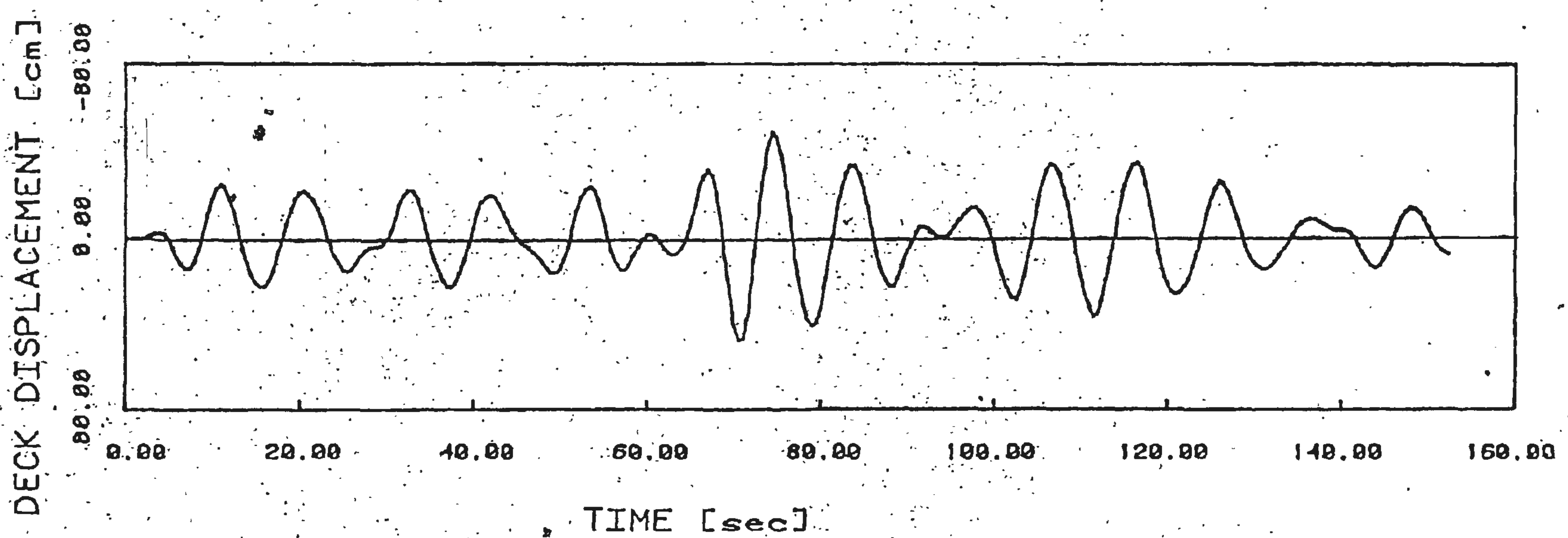

Fig.23 PREDICTED TOWER RESPONSE USING TRUSS ELEMENT MODEL

[100% mooring capacity, nonlinear analysis]