

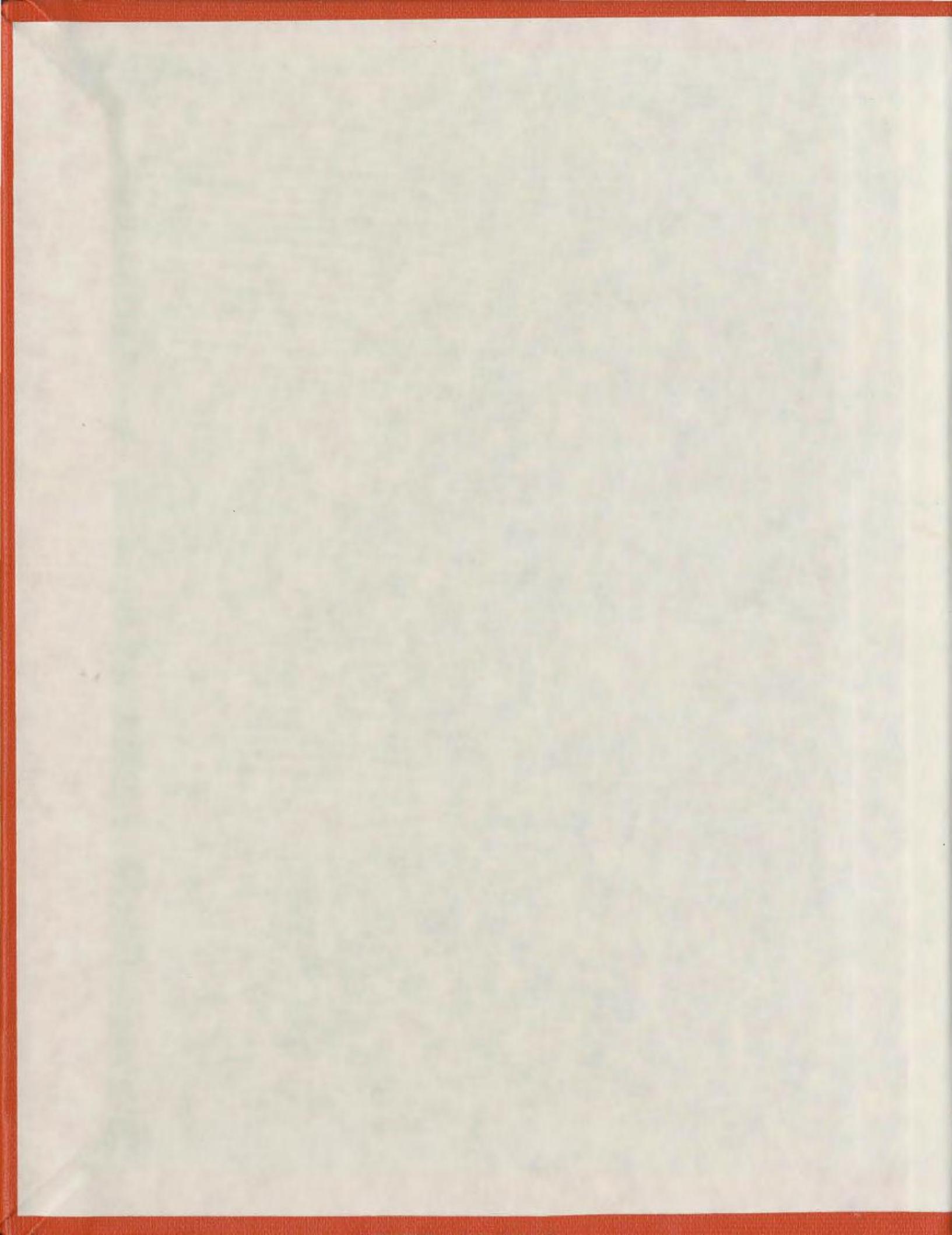
ECONOMIC OPERATION OF VARIABLE-HEAD  
HYDRO-THERMAL ELECTRIC POWER SYSTEMS  
USING NEWTON METHOD

CENTRE FOR NEWFOUNDLAND STUDIES

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Economic Operation of Variable-Head  
Hydro-Thermal Electric Power Systems  
Using Newton Method

by



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A Thesis submitted in partial fulfillment  
of the requirements for the degree of  
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TABLE OF CONTENTS

LIST OF TABLES .....	(iv)
LIST OF FIGURES .....	(v)
LIST OF SYMBOLS .....	(vii)
ACKNOWLEDGEMENTS .....	(ix)
ABSTRACT .....	(x)
1. INTRODUCTION	
1.1 BACKGROUND .....	1
1.2 SCOPE OF THE THESIS .....	2
2. A HISTORICAL REVIEW	
2.1 A REVIEW OF THE DEVELOPMENTS IN THE PROBLEM OF OPTIMAL VARIABLE-HEAD HYDRO-THERMAL DISPATCH .....	4
3. FORMULATION OF THE PROBLEM	
3.1 COORDINATION EQUATIONS FOR VARIABLE-HEAD HYDRO SYSTEMS .....	7
3.2 DISCRETE COORDINATION EQUATIONS .....	13
3.3 APPLICATION OF THE NEWTON-RAPHSON METHOD IN THE PROBLEM ..	15
3.4 GENERATION OF INITIAL ESTIMATES .....	20
4. PERFORMANCE EVALUATION	
4.1 INTRODUCTION .....	24
4.2 TEST SYSTEM ONE AND CHARACTERIZATION TESTS .....	24
4.2.1 Test System One Description .....	24
4.2.2 Computational Results .....	26

4.2.3 Characterization Tests Description .....	31
4.2.4 Characterization Test One .....	31
4.2.5 Characterization Test Two .....	36
4.2.6 Characterization Test Three .....	42
4.3 TEST SYSTEM TWO .....	42
4.3.1 Test System Two Description .....	42
4.3.2 Computational Results .....	46
4.4 TEST SYSTEM THREE .....	56a
4.4.1 Test System Three Description .....	56a
4.4.2 Computational Results .....	56a
5. COORDINATION EQUATIONS FOR ECONOMIC OPERATION OF POWER SYSTEMS WITH HYDRO PLANTS ON THE SAME STREAM	
5.1 INTRODUCTION .....	68
5.2 FORMULATION .....	68
5.3 DIRECT OPTIMALITY CONDITIONS .....	71
5.4 COORDINATION EQUATIONS .....	72
5.5 WATER-WORTH .....	74
6. CONCLUSIONS AND FUTURE WORK	
6.1 CONCLUSIONS .....	76
6.2 FUTURE WORK .....	76
7. BIBLIOGRAPHY AND REFERENCES .....	
APPENDIX A. FIRST-ORDER PARTIAL DERIVATIVES .....	83
APPENDIX B. THE COMPUTER PROGRAM .....	89

APPENDIX C. COMPUTER PRINTOUT	135
APPENDIX D. THE AUGMENTED OBJECTIVE FUNCTIONAL	165
APPENDIX E. REDUCING THE HYDRO OPTIMALITY CONDITIONS	170

LIST OF TABLES

4.2-1	OPTIMUM DISPATCH SCHEDULE. TABULATED DATA .....	29
4.2-2	INCREMENTAL COST OF POWER AND WATER-WORTH COEFFICIENTS. TABULATED DATA .....	32
4.2-3	HEAD VARIATIONS. TABULATED DATA .....	34
4.2-4	CHARACTERIZATION TEST ONE. TABULATED DATA .....	38
4.2-5	CHARACTERIZATION TEST TWO. TABULATED DATA .....	41
4.2-6	CHARACTERIZATION TEST THREE. TABULATED DATA .....	44
4.3-1	OPTIMUM DISPATCH SCHEDULE. TABULATED RESULTS .....	49
4.3-2	OPTIMUM DISPATCH SCHEDULE. TABULATED RESULTS .....	51
4.3-3	VARIATIONS IN $\lambda(t)$ AND $v(t)$ . TABULATED RESULTS .....	54
4.3-4	NET HEAD VARIATIONS. TABULATED RESULTS .....	56
4.4-1	OPTIMAL DISPATCH SCHEDULE. TABULATED RESULTS .....	59
4.4-2	VARIATIONS IN $\lambda(t)$ and $v(t)$ . TABULATED RESULTS .....	63
4.4-3	VARIATION IN $h(t)$ . TABULATED RESULTS .....	67

LIST OF FIGURES

FIGURE 3.1.	The Jacobian Matrix and Partition Details .....	17
FIGURE 3.2.	Structure and Partitioning of $\underline{f}(\underline{x}^m)$ and $\Delta \underline{x}^m$ .....	18
FIGURE 4.2-1.	Maximum Relative Error Versus Number of Iterations..	27
FIGURE 4.2-2.	Optimal Dispatch Schedule .....	28
FIGURE 4.2-3.	Variation in $\lambda(t)$ .....	30
FIGURE 4.2-4.	Variation in $v(t)$ .....	30
FIGURE 4.2-5.	Variation in $h(t)$ .....	33
FIGURE 4.2-6.	Variation in Head with Increased $B(t)$ .....	35
FIGURE 4.2-7.	Variation in $v(t)$ with Increased $B(t)$ .....	35
FIGURE 4.2-8.	Variation in Daily Fuel Costs with Increased $B(t)$ ..	37
FIGURE 4.2-9.	Variation in Head Variation with Increased $P_d(t)$ ..	39
FIGURE 4.2-10.	Variation in $\lambda(t)$ with Increased $P_d(t)$ .....	39
FIGURE 4.2-11.	Variation in Fuel Cost with Increased $P_d(t)$ .....	40
FIGURE 4.2-12.	Variation in Head Variation with Increased Inflow .....	43
FIGURE 4.2-13.	Variation in Daily Fuel Costs with Increased Inflow .....	43

FIGURE 4.3-1. Maximum Relative Error Versus Number of Iterations .....	47
FIGURE 4.3-2. Optimum Dispatch Schedule .....	48
FIGURE 4.3-3. Variation in $P_h(t)$ .....	50
FIGURE 4.3-4. Variation in $P_s(t)$ .....	50
FIGURE 4.3-5. Variation in $\lambda(t)$ .....	53
FIGURE 4.3-6. Variation in $v(t)$ .....	53
FIGURE 4.3-7. $h(t)$ Versus $t$ .....	55
FIGURE 4.4-1. Variation in Maximum Relative Error With Iterations .....	58
FIGURE 4.4-2. Optimal Dispatch Schedule .....	59
FIGURE 4.4-3. Variation in $P_h(t)$ .....	62
FIGURE 4.4-4. Variation in $P_s(t)$ .....	62
FIGURE 4.4-5. Variation in $\lambda(t)$ .....	63
FIGURE 4.4-6. Variation in $v(t)$ .....	63
FIGURE 4.4-7. Variation in $h(t)$ .....	67
FIGURE B-1. Algorithm Flowchart .....	90

LIST OF SYMBOLS

$F$	Fuel cost function.
$J$	Objective functional which is to be minimized.
$P_{s_i}(t)$	Thermal power output for the $i$ th plant at time $t$ .
$P_{h_i}(t)$	Hydro power output for the $i$ th plant at time $t$ .
$a_{s_i}, b_{s_i}, y_{s_i}$	Fuel cost model coefficients for the $i$ th plant.
$a_{h_i}, b_{h_i}, y_{h_i}$	Hydro plant model coefficients for the $i$ th plant.
$a_0, a_1, a_2$	Reservoir model coefficients for the $i$ th plant.
$h_i(t)$	The net head for the $i$ th plant at time $t$ .
$q_i(t)$	The biquadratic function defining the discharge of the $i$ th plant at time $t$ .
$i_i(t)$	The $i$ th reservoir's natural inflow at time $t$ .
$\phi_i(P_h)$	Hydro plant model equation for the $i$ th plant at time $t$ .
$\psi_i(h)$	The reservoir model equation for the $i$ th plant at time $t$ .
$P_i(t)$	The transmission losses for time $t$ .
$P_D(t)$	The system power demand at time $t$ .
$b_i$	The amount of available water for the $i$ th plant.

(viii)

$B_{ij}, K_L$

System's loss coefficients.

$v_i$

The water-worth coefficient of the  $i$ th plant.

$\lambda(t)$

The incremental cost of power for time  $t$ .

$K$

System constant.

$Q_i(t)$

Amount of water discharged from the  $i$ th plant at  
time  $t$ .

$a_i$

The surface area of the  $i$ th reservoir.

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ABSTRACT

In this thesis the optimum generation schedules for systems with variable-head hydro plants are developed. The scheduling problem is solved by use of the Newton-Raphson method where the coordination equations developed by Richard-Kron are employed. Several power systems containing different numbers of plants are considered.

The problem's formulation is profiled and details of the development of the coordination equations with variable-head hydro systems and the discretization of these equations for computer solution are outlined.

Also highlighted are the difficulties encountered with excess computer processing time due to large arrays and how these problems are resolved. Methods for generating the initial estimates of the variables are presented.

Results from three test systems are given and, in addition, evaluation tests for the algorithm's response to changes in the system's variables are detailed.

Coordination equations for systems with water transport delay problems resulting from being hydraulically coupled are presented.

In addition, complete documentation of the computer program is included.

CHAPTER I  
INTRODUCTION

1.1 BACKGROUND

In any power system one of the major goals is to attain optimum economic dispatch. This involves scheduling the generation at various generating stations to meet the system's power demand while keeping the power production costs to a minimum. Usually, this scheduling covers prescribed periods of time. In addition to operating economics the optimum operation of a power system is governed by other restrictions. The ability to fractionally reduce power production costs still has priority with electric utility companies. Also, knowledge of the optimum dispatch schedule allows for better planning and design of any future equipment additions to power systems. It is for these reasons that the problem of economic dispatch has been so extensively researched (refer Ch. II).

In problems concerning economic dispatch it is customary to consider the cost of operation only. Such consideration does not take into account the expenses of labour, capital, start-up and shut-down related to the duration of the down time of a specific unit. Hence, an accurate knowledge of the manner in which the total operating cost of each generating unit varies with the instantaneous output is essential.

The hydro-thermal optimization problem involves the planning of the usage of a limited resource over a prescribed period of time. The resource being the amount of water available for generation. In some systems the use of this water is governed by social factors including

irrigational and navigational commitments. Other systems, with hydro plants located on the same stream have special problems concerning water transport delay which affects the net head and discharge levels. Thus, the conditions which exist over the entire optimization interval must be taken into account when determining the optimum dispatch schedule. For instance, a system with a large reservoir may require an optimization period of a year. Still another system with a small to moderate storage capacity may find an optimization interval of a day or a week more useful.

### 1.2 SCOPE OF THE THESIS

In this thesis the optimum generation schedules for hydro-thermal systems with variable-head hydro plants are developed. The scheduling problem is solved by use of the Newton-Raphson method where the coordination equations (21,27) are employed. Several power systems containing different numbers of plants are considered.

The historical background on the problem is presented in Chapter II. In this presentation all previous work concerning optimum hydro-thermal scheduling for systems with variable-head hydro plants is detailed.

In Chapter III the problem's formulation is profiled. Sub-sections (3.1) and (3.2) detail the development of the coordination equations for variable-head hydro systems and the discretization of these equations for computer solution. In sub-section (3.3) it is shown how the Newton-Raphson method is applied to the problem. This section also highlights

the difficulties encountered with excess computer processing time due to large arrays and how these problems are resolved. Sub-section (3.4) outlines the methods for generating the initial values of the variables.

Chapter IV presents the results from three test systems. In addition to these tests, an evaluation of the program's performance is given. This evaluation tests for the algorithm's response to changes in the system's variables.

The coordination equations for systems having hydro plants with water transport delay problems resulting from being hydraulically coupled are given in Chapter V. This chapter looks at several different arrangements and the resulting coordination equations.

In Chapter VI, the major conclusions of the work are presented. Also outlined in this chapter are the areas in which further work may be conducted.

A full listing and description of the computer program is presented in Appendix B.

## CHAPTER II

### A HISTORICAL REVIEW

#### 2.1 A REVIEW OF THE DEVELOPMENTS IN THE PROBLEM OF OPTIMAL VARIABLE-

##### HEAD HYDRO-THERMAL DISPATCH

Much of the work done on optimal hydro-thermal dispatch in the past revolved around the assumption that for short range studies the effect of head variations could be neglected. However, Ricard [27] in 1940 relaxed this constant head principle. In his paper, he presented a set of coordination equations for systems with net head variations and negligible transmission losses.

Fourteen years later in 1954, Cypser [12] reported on a method he had developed which utilized variational calculus. As with Ricard, he neglected transmission losses and in addition concentrated his attention on the long-range scheduling problem. Cypser tested his method on a system containing one thermal and one hydro plant with varied success.

Then in 1958, Glimm and Kirchmayer [21] in a comprehensive paper, detailed the expansion of the basic coordination equations to include transmission losses. They tested their method on various model systems using the technique of numerical integration. One of the important results of this paper was the demonstration of equivalence of Ricard's, Kron's, and Cypser's equations. This the authors show using variational calculus techniques.

After this presentation by Glimm and Kirchmayer, the reports on optimizing the dispatching schedule for variable-head hydro-thermal

systems became more frequent and researchers began to try new approaches. Hence, only two years later in 1960 Arismunander [1] utilizing variational calculus and employing all the necessary and sufficient conditions for optimality, arrived at the required scheduling equations.

The following year Dandeno [15] reported on the computational experience gained by applying the coordination equations to an actual operating system. The computer algorithm he used proceeded to the solution by linearizing the non-linear equations, solving them by the Gauss-Seidel method for the Power values, and then adjusting the constraint multipliers accordingly. However, large amounts of computer time and heavy core requirements were the major drawbacks of the method.

In 1962 Drake et.al. [16], using basically the same computer algorithm as Dandeno, applied variational methods to functional systems. This method, although more successful than Dandeno's, did not resolve all of the problems surrounding the optimizing procedure.

The work continued and in 1964 Dahlin [13] presented his maximum principle approach to the problem. The basis of this approach was developed by Pontryagin. Later in 1966 Dahlin along with Shen [14] detailed the application of his method to several types of systems. The numerical analysis was performed on a test system consisting of one thermal and one hydro plant, and was mainly for the purpose of exploring the convergence behavior.

The next notable work came in 1971 when Bonaert and El-Abiad [8] reported on a method known as decomposition in which the hydro-thermal system was subdivided into hydro and thermal subsystems. In addition to decomposition, the technique also used perturbations to arrive at

the required result.

In 1972 another method was proposed by El-Hawary and Christensen [18]. This procedure utilized functional analytic minimum norm formulation. It was pointed out that this method eliminated the multipliers associated with the linear constraints in the control vector.

The decomposition method surfaced again in 1980 when Soares, Lyra, and Tavares [29] reported on a "coordinated" decomposition technique.

In this procedure the solution was obtained through a tri-level hierarchical calculation structure.

To conclude the review, the work presented herein utilizes the coordination equations and obtains the solution using the Newton-Raphson method.

### CHAPTER III:

#### FORMULATION OF THE PROBLEM

##### 3.1 COORDINATION EQUATIONS FOR VARIABLE-HEAD HYDRO SYSTEMS

The coordination equations for variable-head hydro systems are an extension of those used for fixed-head systems. For both types of systems the classical approach of variational calculus is utilized giving the optimality conditions in terms of Ricard's equations. To arrive at the optimal strategy for variable-head hydro systems, the equations for fixed-head systems are developed and then extended to the variable-head case.

To begin, it is assumed that the reservoir is large enough so that any variation in net head may be neglected. It is also assumed that the fuel cost for the thermal units is

$$F = \sum_{i=1}^{N_s} F_i(P_{s_i}) \quad (3.1)$$

where  $N_s$  is the number of thermal plants and  $F_i(P_{s_i})$  is defined as

$$F_i(P_{s_i}) = a_{s_i} + b_{s_i} P_{s_i}(t) + c_{s_i} P_{s_i}^2(t) \quad (i=1, \dots, N_s) \quad (3.2)$$

it is required to minimize the objective functional given by

$$J = \int_0^{T_f} F dt \quad (3.3)$$

while satisfying the active power balance equation given by

$$\sum_{i=1}^{N_s} P_{s_i}(t) + \sum_{i=1}^{N_h} P_{h_i}(t) - P_L(t) = P_D(t) \quad (3.4)$$

In the above equation  $P_{h_i}(t)$  is the output of the  $i$ th hydro unit,  $N_h$  is the number of hydro plants in the system,  $P_L(t)$  is the transmission loss and  $P_D(t)$  is the system's power demand.

The volume of water available,  $b_i$ , for generation is also taken into consideration through the requirement

$$\int_0^{T_f} q_i(t) dt = b_i \quad (i=1, \dots, N_h) \quad (3.5)$$

Here  $q_i(t)$  is the rate of water discharge at the  $i$ th plant. This is a biquadratic function of effective head and active power generation according to the model suggested by Glimm-Kirchmayer [21], given by

$$q_i(t) = K \psi_i(h) \phi_i(p) \quad (i=1, \dots, N_h) \quad (3.6)$$

where the dependence on net head is expressed as

$$\psi_i(h) = a_{0i} + a_{1i} h_i(t) + a_{2i} h_i(t)^2 \quad (i=1, \dots, N_h) \quad (3.7)$$

The dependence on active power generation is indicated by

$$\phi_i(p) = \alpha_{hi} + \beta_{hi} p_i(t) + \gamma_{hi} p_i(t)^2 \quad (i=1, \dots, N_h) \quad (3.8)$$

In the above

$K$  = constant of proportionality.

The parameters  $a_{0i}$ ,  $a_{1i}$ ,  $a_{2i}$ ,  $\alpha_{hi}$ ,  $\beta_{hi}$ , and  $\gamma_{hi}$  are assumed available.

The cost functional  $J$  is now augmented to include the volume of water constraint. This is accomplished by using constant multipliers,  $v_i$ , as follows

$$J = \int_0^{T_f} [F + \sum_{i=1}^{N_h} v_i q_i(t)] dt \quad (3.9)$$

The power balance equation is also included through the use of the multiplier function  $\lambda(t)$ . In this way, the constrained minimization problem of (3.1) is changed to an unconstrained problem of minimizing the following augmented cost functional

$$J = \int_0^{T_f} \left[ F + \sum_{i=1}^{N_h} v_i q_i(t) + \lambda(t) [P_D(t) - \sum_{i=1}^{N_s} P_{s_i}(t) - \sum_{i=1}^{N_h} P_{h_i}(t) + P_L(t)] \right] dt \quad (3.10)$$

Through variational calculus techniques, the optimality conditions for fixed-head systems are obtained as

$$\beta_{s_i} + 2\gamma_{s_i} P_{s_i}(t) + \lambda(t) [C_i + 2 \sum_{j=1}^{N_g} B_{ij} P_j(t)] = 0, \quad (i=1, N_s) \quad (3.11)$$

$$\nu_i [\beta_{h_i} + 2\gamma_{h_i} P_{h_i}(t)] + \lambda(t) [C_i + 2 \sum_{j=1}^{N_g} B_{ij} P_j(t)] = 0, \quad (i=1, N_h) \quad (3.12)$$

where

$$C_i = B_{i0} - 1, \quad (i=1, N_g) \quad (3.13)$$

Note that it is assumed here that  $Kg$  is set to unity for fixed head hydro plants.

When solved, these equations yield active powers, the incremental cost of power,  $\lambda_i(t)$ , and the base water worth,  $v_i$ . However, complete solution requires that the following constraint equations be adhered to:

$$\int_0^{T_f} q_i(t) dt = b_i \quad (i=1, N_h) \quad (3.14)$$

$$K_L + P_D(t) + \sum_{i=1}^{N_g} C P_i(t) \\ + \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P_i(t) B_{ij} P_j(t) = 0 \quad (3.15)$$

The assumption of negligible head variation is now relaxed and is replaced by the assumption that operation of the hydro units results in a change in the net head of the reservoirs. Thus, for hydro plants with variable-head characteristics, the generation of active power is regulated by the rate of discharge,  $q_i(t)$ , and also by the volume of water discharged,  $Q_i(t)$ , given by

$$Q_i(t) = \int_0^t q_i(t) dt \quad (i=1, N_h) \quad (3.16)$$

The cost functional is still constrained by the active power balance equation and the volume of water available, however, the decision variables are now extended to include  $P_s(t)$  and  $Q_i(t)$ .

Again, using variational calculus methods, it is found that the coordination equations for variable-head systems are equivalent to those of the fixed-head case with the exception that the water conversion coefficient,  $v_i$ , is now variable and is defined by

$$v_i(t) = v_{i_0} \exp \left[ \int_0^t \left[ \frac{1}{s} \partial q_i(t) / \partial h_i(t) \right] dt \right], \quad (i=1, N_h) \quad (3.17)$$

The integrand in the above is denoted by  $M(t)$  which is written as

$$M(t) = \frac{K}{s_i} \phi_i(P_h) [a_{1i} + 2a_{2i} h_i(t)], \quad (i=1, N_h) \quad (3.18)$$

By assuming that  $M(t)$  is constant over the interval  $[0, T_f]$ , equation 3.17 may be written as

$$v_i(t) = v_{i_0} \exp[M_i t]. \quad (3.19)$$

The coordination equations for the variable-head hydro system are given as

$$\beta_{s_i} + 2\gamma_{s_i} p_{s_i}(t) + \lambda(t) [c_i + 2 \sum_{j=1}^{N_g} B_{ij} p_j(t)] = 0, \quad (i=1, N_s) \quad (3.20)$$

$$\begin{aligned} & \{v_{i_0} \exp[M_i t]\} \left\{ 2 - \frac{2\alpha_{hi} + \beta_{hi} p_{hi}(t)}{\phi_i(P_h)} \right\} \left( \frac{q_i(t)}{p_{hi}(t)} \right) \\ & + \lambda(t) [c_i + 2 \sum_{j=1}^{N_g} B_{ij} p_j(t)] = 0, \quad (i=1, N_h) \end{aligned} \quad (3.21)$$

As before, complete solution requires the observance of the following constraint equations

$$\int_0^{T_f} [K \phi_i(h) \phi_i(P_h)] dt = b_i \quad (i=1, N_h) \quad (3.22)$$

$$K_{L_0} + P_D(t) + \sum_{i=1}^{N_g} C_i p_i(t) + \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P_i(t) B_{ij} p_j(t) = 0 \quad (3.23)$$

For variable-head hydro systems an additional constraint is imposed which accounts for the effect of head variation and has the form

$$h_i(t) = h_{0,i}(t) + \frac{1}{s_i} \int_{t_0}^t [i_i(t) - q_i(t)] dt \quad (i=1, N_h) \quad (3.24)$$

where  $i_i(t)$  is the natural inflow assuming a vertical sided reservoir.

Under the foregoing assumptions the optimality conditions for a variable-head hydro system are given by

$$\beta_{s,i} + 2\gamma_{s,i} p_{s,i}(t) + \lambda(t) [c_i + 2 \sum_{j=1}^{N_g} b_{i,j} p_j(t)] = 0, \quad (i=1, N_s) \quad (3.25)$$

$$\left( v_0 \exp[M_i(t)] \left( 2 - \frac{2\alpha_{h,i} + \beta_{h,i} p_{h,i}(t)}{\phi_i(p_h)} \right) \right) \left\{ \frac{q_i(t)}{p_{h,i}(t)} \right\} \\ + \lambda(t) [c_i + 2 \sum_{j=1}^{N_g} b_{i,j} p_j(t)] = 0, \quad (i=1, N_h) \quad (3.26)$$

$$\int_0^{T_f} q_i(t) dt = b_i \quad (i=1, N_h) \quad (3.27)$$

$$K_{L,D} + p_D(t) + \sum_{i=1}^{N_g} c_i p_i(t) + \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p_i(t) b_{i,j} p_j(t) = 0 \quad (3.28)$$

$$h_i(t) = h_{0,i}(t) + \frac{1}{s_i} \int_{t_0}^t [i_i(t) - q_i(t)] dt \quad (i=1, N_h) \quad (3.29)$$

As before

$$c_i = b_{i,i} - 1 \quad (i=1, N_g) \quad (3.30)$$

The dynamic equations 3.25 to 3.29 are non-linear and a discrete form is required for digital solution. This problem is treated in the next section.

### 3.2 DISCRETE COORDINATION EQUATIONS

To discretize equations (3.25) to (3.29) it is assumed that the optimization interval  $[0, T_f]$  is divided into  $N_T$  discrete intervals. The discrete time index is denoted by  $t_k$ . Equation (3.25) is denoted by  $f_{s_i}(t)$  and is straight-forward as far as the discretization process is concerned. Similarly equations (3.26) and (3.28) are denoted by  $f_{h_i}(t)$  and  $f_D(t)$ , respectively. The volume of water constraint given by equation (3.27) is replaced by a summation assuming that the discrete intervals to be of equal length,  $\Delta$ . The reservoir equation (3.29) is replaced by the equivalent form

$$h_i(t + \Delta) = h_i(t) + \frac{1}{s_i} \int_1^{t+\Delta} [i_i(t) - q_i(t)] dt, \quad (i=1, N_h) \quad (3.31)$$

Thus

$$f_{i_1}(t) = h_i(t) - h_i(t + \Delta) + \frac{1}{s_i} \int_1^{t+\Delta} [i_i(t) - q_i(t)] dt = 0 \quad (i=1, N_h) \quad (3.32)$$

or

$$f_{i_1}(t_k) = h_i(t_k) - h_i(t_k + 1) + \frac{\Delta}{s_i} [i_i(t_k) - q_i(t_k)] = 0 \quad (i=1, N_h) \quad (3.33)$$

The discretization process results in a set of nonlinear algebraic equations which must be solved and these equations are:

$$f_{s_i}(t_k) = \beta_{s_i} + 2\gamma_{s_i} p_{s_i}(t_k) + \lambda(t_k) [c_i + 2 \sum_{j=1}^{N_g} B_{i,j} p_j(t_k)] = 0, \quad (i=1, N_s) \quad (3.34)$$

$$f_{h_i}(t_k) = v_{0_i} \exp[Mt] \left\{ 2 - \frac{2\alpha_h + \beta_h p_{h_i}(t_k)}{\phi_i(p_h)} \right\} \left\{ \frac{q_i(t_k)}{p_{h_i}(t_k)} \right\} \\ + \lambda(t_k) [c_i + 2 \sum_{j=1}^{N_g} B_{i,j} p_j(t_k)] = 0, \quad (i=1, N_h) \quad (3.35)$$

$$f_D(t_k) = K_L + p_D(t_k) + \sum_{i=1}^{N_g} c_i p_i(t_k) \\ + \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p_i(t_k) B_{i,j} p_j(t_k) \quad (3.36)$$

$$f_{i_i}(t_k) = h_i(t_k) - h_i(t_k - 1) + \frac{\Delta}{s_i} [i_i(t_k) - q_i(t_k)] = 0, \quad (i=1, N_h) \quad (3.37)$$

$$f_{b_i}(t_k) = \sum_{k=0}^{N_T-1} [k_i \psi_i(h) \phi_i(p_h)] \Delta - b_i = 0, \quad (i=1, N_h) \quad (3.38)$$

Each of the above equations applies at the following discrete instants

$$f_{s_i}(t_k): t_k = 0, 1, \dots, N_T - 1$$

$$f_{h_i}(t_k): t_k = 0, 1, \dots, N_T - 1$$

$$f_D(t_k): t_k = 0, 1, \dots, N_T - 1$$

$$f_i(t_k): t_k = 0, 1, \dots, N_T - 2$$

The unknowns are  $v_{0_i}$  and the following

$$p_s(t_k) : t_k = 0, 1, \dots, N_T - 1$$

$$p_{h_i}(t_k) : t_k = 0, 1, \dots, N_T - 1$$

$$\lambda(t_k) : t_k = 0, 1, \dots, N_T - 1$$

$$h_i(t_k) : t_k = 1, \dots, N_T - 1$$

The interval over which  $h_i(t_k)$  is found, i.e.  $(1, N_T - 1)$  has its basis in the assumption that the reservoir levels  $h_i(0)$  are known. This is described more fully in section 3.4.

### 3.3 APPLICATION OF THE NEWTON-RAPHSON METHOD TO THE PROBLEM

The Newton-Raphson method requires solving on each iteration the following set of linear equations

$$f(x^m) + J\Delta x^m = 0 \quad (3.39)$$

In the above,  $f$  is the vector nonlinear function to be solved and  $J$  is the Jacobian matrix consisting of the first-order partial derivatives of the functions with respect to the unknown variables (Appendix A) which comprise the vector  $x$ . The index  $m$  denotes the iteration number.

The size of the Jacobian matrix depends on the number of units in the system and the number of time intervals in the period. For each hydro plant there are two variables per time instant, for each thermal plant only one variable per time instant. In addition, one variable  $v_{0_i}$  per hydro plant is encountered. Also included are the values of

the incremental cost of power,  $\lambda(t_k)$ , for each time interval.

Solving for  $\Delta x^m$  requires that the inverse of the Jacobian matrix,  $J^{-1}$ , be obtained. This is achieved by (1); ... or (2); by determining the inverse of the matrix as a whole or two; by utilizing the method of matrix partitioning.

The first method provides a direct path to  $J^{-1}$ . The Jacobian matrix is set up and its inverse is obtained directly through a commercially available inversion routine. This method performs well when considering small systems. However, with larger systems, 4 plants or more, the amount of time required to obtain  $J^{-1}$  is such that it is no longer feasible.

The main objective of the second method is to restrict the application of the inversion routine to a matrix of the smallest possible dimensions.

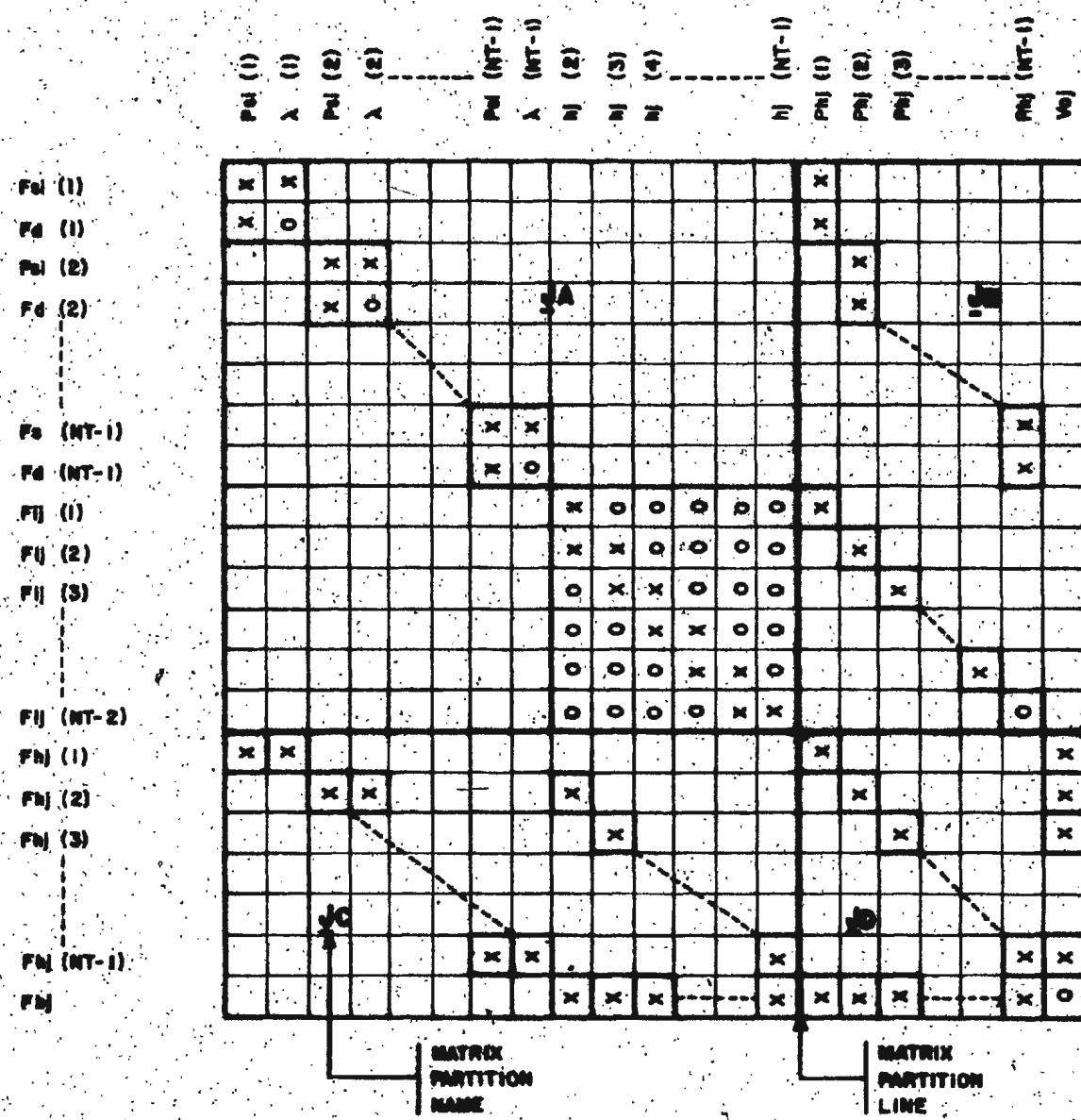
To accomplish this a method of matrix partitioning is employed. The Jacobian matrix,  $J$ , the nonlinear function vector,  $f(x^m)$ , and the unknown variables,  $\Delta x^m$ , are divided up or partitioned so that a more efficient procedure may be used.

To begin, the Jacobian matrix is structured as shown in Figure 3.1. This arrangement obtains the maximum benefit from the sparseness of the matrix. Figure 3.1 also details the partitioning of the matrix and identifies the submatrices.

Similarly, the vectors  $f(x^m)$  and  $\Delta x^m$  have partitions which are structured according to Figure 3.2.

Equation (3.38) is now rewritten in terms of these partitioned matrices

FIGURE 3.1  
THE JACOBIAN MATRIX  
AND  
PARTITION DETAILS



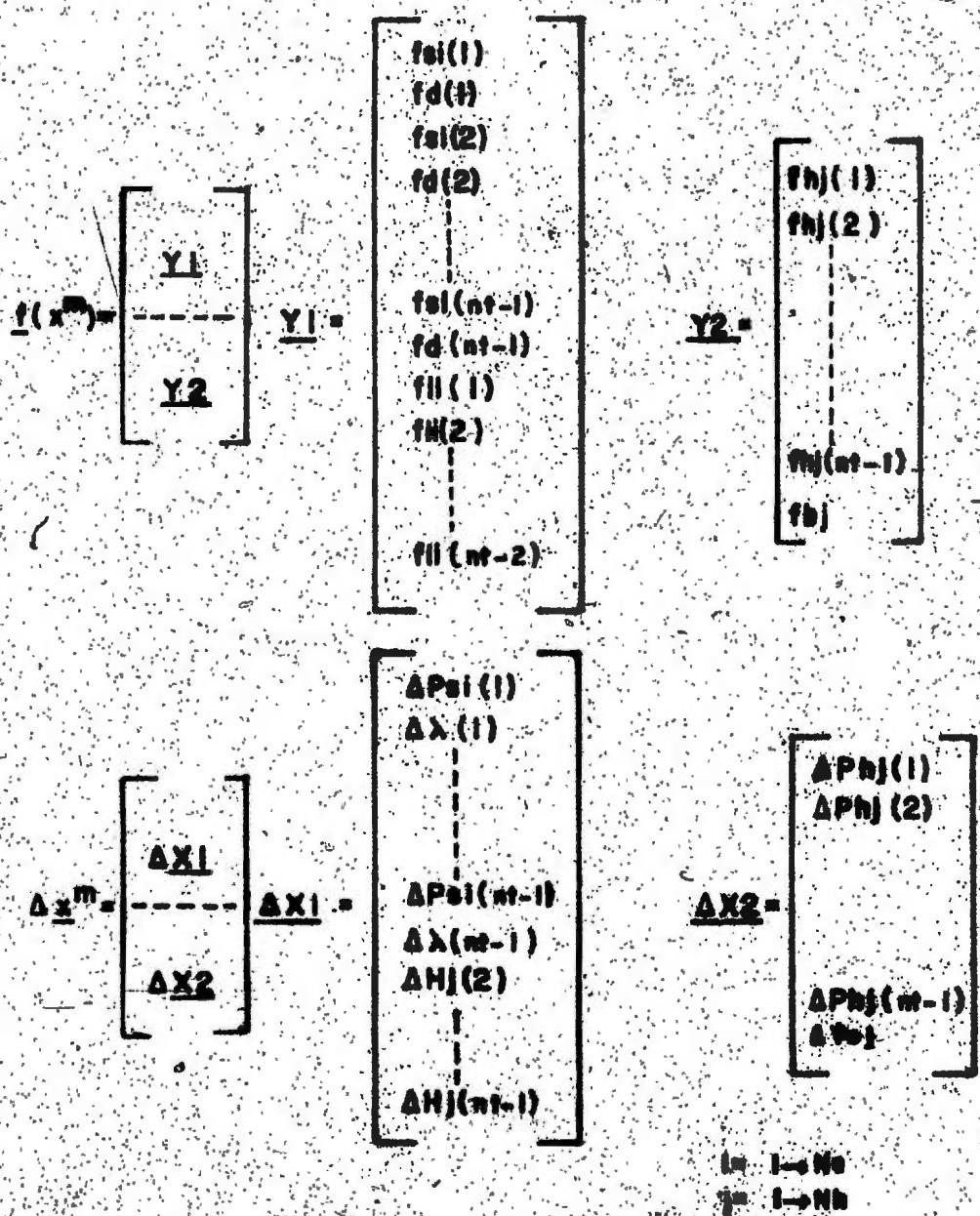
NOTE: i)  $x$  = PARTIAL DERIVATIVE OF THE FUNCTION (row element) WITH RESPECT TO THE VARIABLE (column element) SEE APPENDIX 'A'.

ii) ALL OTHER ELEMENTS ARE ZERO.

iii)  $i = 1 \rightarrow N_d$

$j = 1 \rightarrow N_d$

FIGURE 3.2  
STRUCTURE AND  
PARTITIONING OF  
 $f(x^m)$  AND  $\Delta x^m$



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} J_A & J_B \\ J_C & J_D \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \quad (3.40)$$

or in terms of  $F(x^m)$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \times \begin{bmatrix} J_A & J_B \\ J_C & J_D \end{bmatrix} = - \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (3.41)$$

Expanding equation (3.41) gives

$$J_A x_1 + J_B x_2 = -y_1 \quad (3.42)$$

$$J_C x_1 + J_D x_2 = -y_2 \quad (3.43)$$

Through algebraic manipulation the following relationships are

obtained

$$x_1 = -J_A^{-1} [y_1 - J_B x_2] \quad (3.44)$$

$$x_2 = -[J_D - J_C J_A^{-1} J_B]^{-1} [y_2 - J_C J_A^{-1} y_1] \quad (3.45)$$

Closer examination of the submatrix,  $J_A$ , reveals that it is block diagonal and, hence, its inverse may be found simply by inverting each subblock. This reduces the computational time substantially. The other inversion which is performed is on a matrix which has the same dimensions as the submatrix,  $J_D$ .

As an example of the amount of reduction which results, a system with three hydro plants and three thermal plants is examined. The resulting Jacobian matrix has the dimensions [240 x 240] for an interval of 24 time periods.

This is a very large matrix to invert by method one. However, solution by method two requires inverting the blocks of  $J_A$  with dimensions  $[4 \times 4]$  for  $I=1, N$  and  $[23 \times 23]$  for  $I=N+1$ . The inversion required for  $X_1$ , i.e.  $[J_D - J_C^{-1} J_B]^{-1}$ , has a dimension equal to  $J_D$  of  $[75 \times 75]$ . It is evident that method two is superior and indeed this is reflected in the fact that a successful solution was obtained for all three test systems as detailed in Chapter IV.

### 3.4. GENERATION OF INITIAL ESTIMATES

In problems such as this where the solution is obtained using iterative techniques, a good initial estimate of the variables is essential if the number of iterations and, hence, the computational time is to be minimized.

The methods by which these initial values may be obtained are numerous. Therefore, it is necessary to make certain assumptions and develop particular models to produce an initial variable estimation procedure which contributes to the overall efficiency of the program. The method described herein produces satisfactory initial estimates of the variables.

The first assumption made is that the level of the reservoir is monitored and would be available as input to the program in the form of  $h_i(t_o)$ . Thus, the number of initial values to be determined is reduced by one.

Knowing  $h_i(t_o)$ ,  $\psi_1(h)$  is calculated using equation (3.7). By further assuming that  $R_i(t_o)$  is constant over the time interval  $t_k$ , it is possible to write equation (3.27) as

$$T_f K \psi_1(h) [a_{h_i} + b_{h_i} p_{h_i}(t_o) + y_{h_i} p_{h_i}(t_o)^2] - b_i = 0, \quad (i=1-N_h) \quad (3.46)$$

Rearranging gives

$$a_{h_i} + [b_{h_i} p_{h_i}(t_o) + y_{h_i} p_{h_i}(t_o)^2] - \left[ \frac{b_i}{T_f K \psi_1(h)} \right] = 0 \quad (3.47)$$

$$\text{Letting } b_i = \frac{b_i}{T_f K \psi_1(h)} \quad (3.48)$$

gives

$$[y_{h_i}] p_{h_i}(t_o)^2 + [b_{h_i}] p_{h_i}(t_o) + [a_{h_i} - b_i] = 0 \quad (3.49)$$

$p_{h_i}(t_o)$  is now obtained by applying the quadratic formula and taking the positive root of the equation.

Once  $p_{h_i}(t_o)$  and  $p_{h_i}(t_o)$  are known,  $p_{s_i}(t_o)$  is quickly estimated from

$$p_{s_i}(t_o) = \left[ \frac{p_i(t_o)}{\frac{D(t_o)}{N_s}} \right] \left[ 1 - \frac{p_D(t_o)}{\frac{D(t_o)}{N_h}} \right] \sum_{j=1}^{N_h} p_{h_j}(t_o) \quad (3.50)$$

For simplicity it is assumed that the value for  $p_{s_i}(t_k)$  is equal for all plants,  $i=1-N_s$  over the interval  $[0, N_T - 1]$ , i.e.  $p_{s_1}(0) = p_{s_2}(0) = p_{s_{N_s}}(0)$ ,  $p_{s_1}(1) = p_{s_2}(1) = p_{s_{N_s}}(1)$ , etc.. The same is assumed for  $p_{h_i}(t_k)$  and  $h_i(t_k)$ . However, this does not imply that  $p_{s_1}(0) = p_{s_1}(1)$ , etc..

An initial value of  $\lambda(t_k)$  is now calculated from

$$\lambda(t_0) = \frac{s_i + 2 \sum_{j=1}^{N_g} s_i s_j p_i(t_0)}{c_s + 2 \sum_{j=1}^{N_g} b_j p_j(t_0)}, \quad (i=1 \rightarrow N_s) \quad (3.51)$$

and  $v_{0,i}$  is calculated directly from equation (3.35).

Once the values of the variables for all plants at time,  $t_0$ , are known, it remains only to determine values for the remainder of the study interval  $[1, N_T - 1]$ .

The first method developed to find these variables assumes a flat profile such that:

$$p_{h_i}(t_k) = p_{h_i}(t_0) \quad (3.52)$$

$$p_{s_j}(t_k) = p_{s_j}(t_0) \quad t_k = 1 \rightarrow N_T - 1 \quad (3.53)$$

$$\lambda(t_k) = \lambda(t_0) \quad j = 1 \rightarrow N_s \quad (3.54)$$

$$h_i(t_k) = h_i(t_0) \quad (3.55)$$

This method performs well, but on examination it is found that better results are obtained when an adjustment factor is used. Such a factor is based upon the ratio of the power demand at time instant,  $t_k'$ , to the initial power demand. In other terms

$$\text{fact}(t_k') = \frac{p_D(t_0)}{p_D(t_k')} \quad (3.56)$$

The variables at time instant,  $t_k'$ , are adjusted as follows

$$p_{h_i}(t_k) = p_{h_i}(t_0) \times \text{fact}(t_k) \quad (3.57)$$

$$p_{s_j}(t_k) = p_{s_j}(t_0) \times \text{fact}(t_k) \quad (3.58)$$

$$\lambda(t_k) = \lambda(t_0) \times \text{fact}(t_k) \quad (3.59)$$

The profile for the net head over the interval was assumed flat due to its characteristically slow variation with time.

CHAPTER IV  
PERFORMANCE EVALUATION

4.1 INTRODUCTION

In this chapter the results of the application of the computer algorithm to three hydro-thermal test systems are presented. The first system consists of one thermal unit and one variable-head hydro unit. Characterization tests are performed on this system to determine the algorithm's ability to adjust to variations in system parameters. The second test system examined contains two thermal plants and two variable-head hydro plants while the third system has two thermal units and five variable-head hydro units.

4.2 TEST SYSTEM ONE AND CHARACTERIZATION TESTS

4.2.1 Test System One Description

Test system one consists of one variable-head hydro plant and one thermal plant. Both of these supply power to a common grid over transmission lines with losses.

The models for the fuel cost, hydro plant performance and reservoir variation are represented by quadratic equations of the form

$$F(P_{s_1}) = a_{s_1} + b_{s_1} P_{s_1}(t) + c_{s_1} P_{s_1}(t)^2$$

$$\phi(P_{h_1}) = a_{h_1} + b_{h_1} P_{h_1}(t) + c_{h_1} P_{h_1}(t)^2$$

$$\psi(h_1) = a_{01} + a_{11} h_1(t) + a_{21} h_1(t)^2$$

where the quadratic coefficients are given as

$$a_{s1} = 1.0$$

$$a_{h1} = 1.0$$

$$B_{s1} = 2.7$$

$$B_{h1} = 0.1$$

$$Y_{s1} = 3.0 \times 10^{-3}$$

$$Y_{h1} = 1.0 \times 10^{-4}$$

$$a_{01} = 1.0$$

$$a_{11} = -0.2237$$

$$a_{21} = 1.0 \times 10^{-3}$$

The transmission loss coefficients are

$$B_{10} = 0$$

$$B_{20} = 0$$

$$B_{ss} = 0$$

$$B_{sh} = 0$$

$$B_{hs} = 0$$

$$B_{hh} = 1.43 \times 10^{-4}$$

$$K_{L0} = 0$$

The data for the reservoir is

$$\text{Area} = 10 \text{ mi}^2$$

$$\text{Available water} = 2.5 \times 10^9 \text{ cf}$$

Net head<sup>a</sup> (initial) = 205 ft.

Natural inflow =  $12 \times 10^3$  cfs

The test interval covers a 24 hour period and is subdivided into 24-1 hour discrete time instances.

#### 4.2.2 Computational Results

For test system one the program converged in seven iterations to an error criterion of  $1.0 \times 10^{-4}$  (see Figure 4.2-1) and required 33.25 seconds of cpu time for solution.

The optimal dispatch schedule is obtained and presented in Figure 4.2-2 and Table 4.2-1. It is observed that the hydro plant produced on an average 85% of the total power demand plus transmission losses.

This high percentage is in keeping with the guidelines of the program criteria to reduce thermal generation and, hence, fuel cost to a minimum.

The calculated daily fuel cost for the system under these conditions is \$9,844.70.

The variation in the incremental cost of power,  $\lambda(t)$ , is shown in Figure 4.2-3. The curve shows that  $\lambda(t)$  and  $P_i(t)$  vary with time in the same manner. That is, the curves have the same shape. This is to be expected since the incremental cost of power should increase or decrease in accordance with the variation in the power demand.

The water worth coefficient curve,  $w(t)$ , is detailed in Figure 4.2-4. The variation shown complies with earlier predictions that the optimization procedure conserves water at the start by keeping  $\lambda(t)$  high at first and then slowly decreasing it.

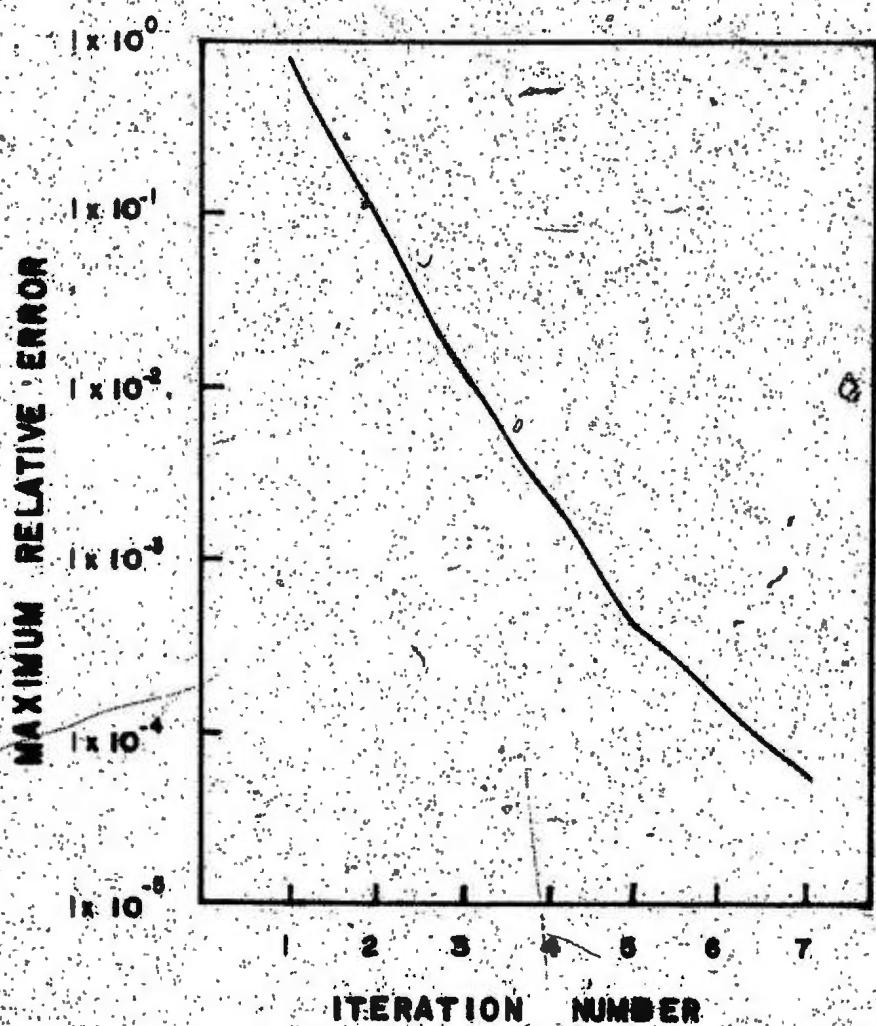


FIGURE 4.2-1. Maximum Relative Error Versus Number of Iterations.

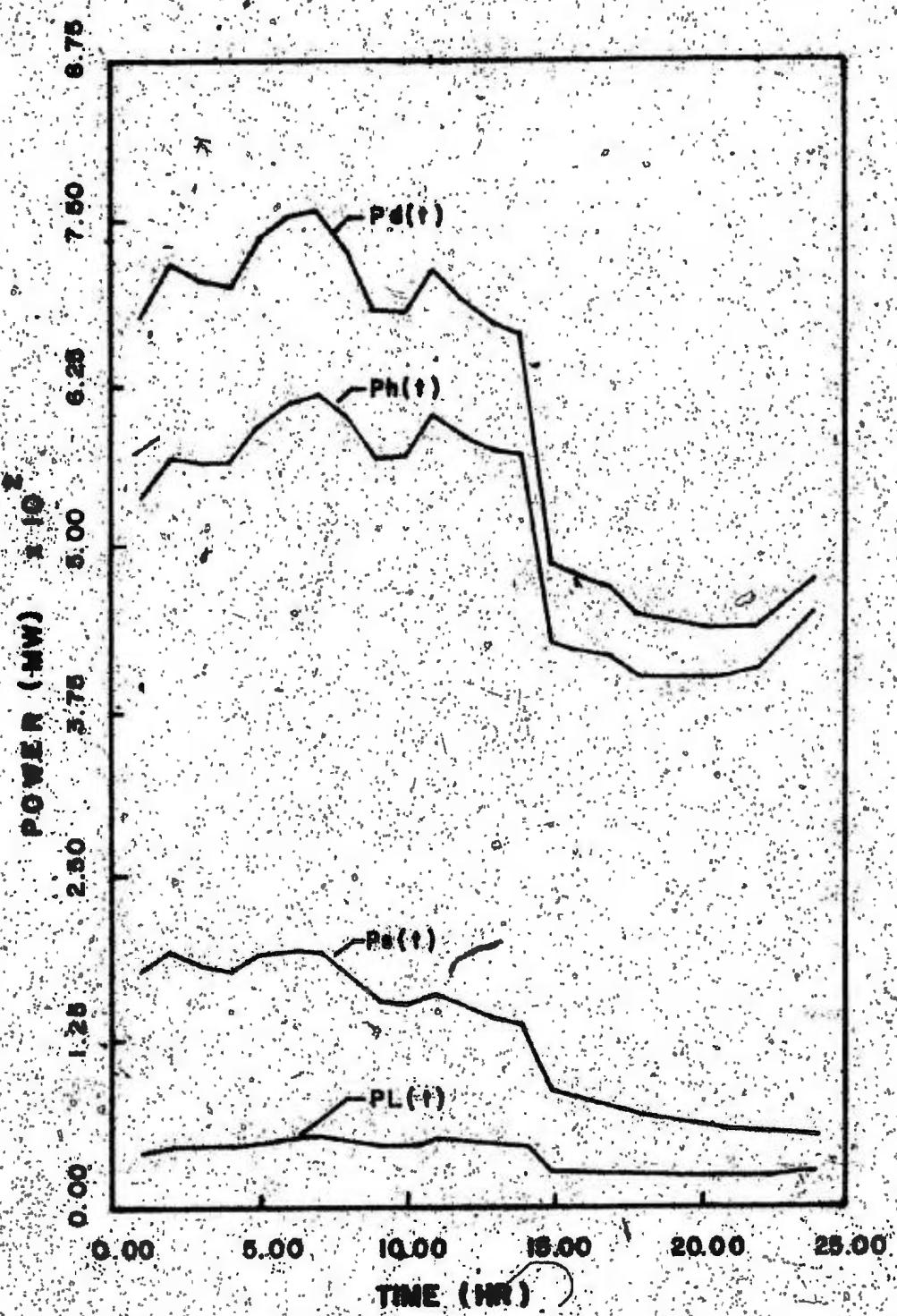


FIGURE 4.2-2. Optimal Dispatch Schedule.

TABLE 4.2-1  
OPTIMUM DISPATCH SCHEDULE. TABULATED DATA.

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	HYDRO PLANT NO. 1 MW	THERMAL PLANT NO. 1 MW
1	681	42.07	542.42	180.65
2	722	46.82	572.19	196.63
3	708	46.03	567.33	186.69
4	703	46.16	568.14	181.01
5	741	51.17	598.18	193.98
6	758	53.98	614.39	197.59
7	761	55.12	620.83	195.28
8	732	52.05	603.34	180.72
9	685	46.71	571.50	160.20
10	683	47.10	573.92	156.18
11	716	52.29	604.71	163.58
12	692	49.53	588.53	153.00
13	675	47.75	577.85	144.90
14	666	47.08	573.77	139.31
15	491	26.18	427.89	89.29
16	481	25.58	422.96	83.63
17	473	25.17	419.54	78.63
18	451	23.34	403.98	70.35
19	448	23.42	404.67	66.75
20	443	23.28	403.52	62.76
21	441	23.46	405.01	59.45
22	444	24.17	411.14	57.03
23	461	26.56	430.98	56.59
24	480	29.43	453.69	55.75

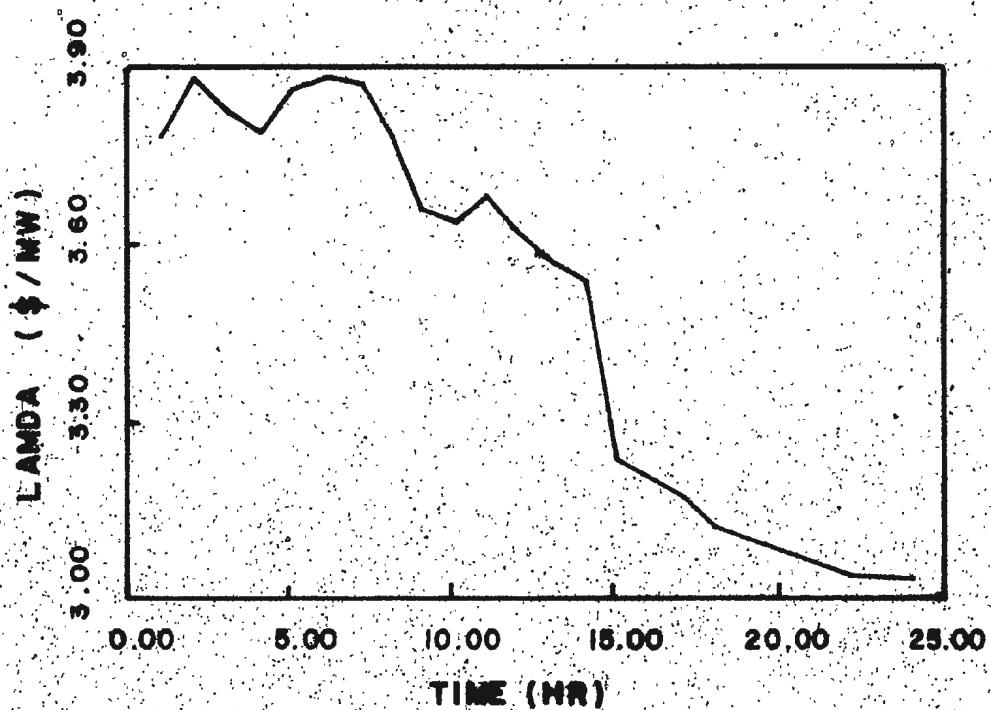


FIGURE 4.2-3. Variation in  $\lambda(t)$ .

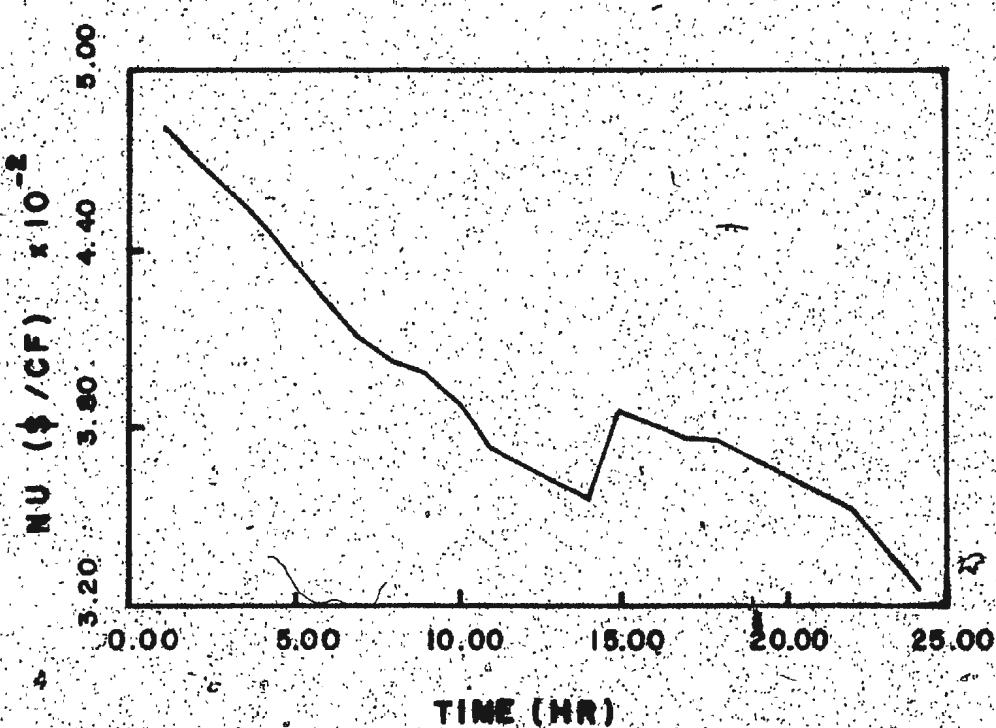


FIGURE 4.2-4. Variation in  $\nu(t)$ .

The values for  $\lambda(t)$  and  $v(t)$  are tabulated in Table 4.2-2. The variation in net head is presented in Figure 4.2-5 and Table 4.2-3. Since the inflow to the reservoir is constant, the value of the net head decreases as expected.

Actual computer print-out for this system is found in Appendix C.

#### 4.2.3 Characterization Tests Description

The characterization tests are carried out on the system described in 4.2.2 to evaluate the computer algorithm.

Three tests are performed on the system. In the first test the available water is varied. In the second test the power demand,  $P_d(t)$ , is changed while in the third test the natural inflow,  $i(t)$ , to the reservoir is altered. The tests and results are described herein.

#### 4.2.4 Characterization Test One

In this first test the power demand,  $P_d(t)$ , and the reservoir's natural inflow are held constant while varying the available water,  $B$ . Then, the effects of the changing  $B$  on the head variation, the water worth coefficient,  $v(t)$  and the daily fuel cost are examined.

Figure 4.2-6 shows how the head variation was affected by altering  $B$ . This head variation refers to the difference between the minimum and maximum head levels over the test interval. As expected, the head variation increased almost linearly with increases in  $B$ . The reason for this can be deduced from Figure 4.2-7 which shows that as  $B$  increases the water worth coefficient,  $v(t)$  and, hence, the cost of water decreases.

**TABLE 4.2-2**  
**INCREMENTAL COST OF POWER AND**  
**WATER-WORTH COEFFICIENTS. TABULATED DATA.**

TIME PERIOD HR	NU PLANT NO. 1 S/CF	LAMBDA \$/MW
1	0.04812	3.784
2	0.04698	3.880
3	0.04589	3.820
4	0.04481	3.786
5	0.04347	3.864
6	0.04219	3.886
7	0.04102	3.872
8	0.04025	3.784
9	0.03983	3.661
10	0.03888	3.637
11	0.03734	3.682
12	0.03681	3.618
13	0.03622	3.569
14	0.03551	3.536
15	0.03854	3.236
16	0.03809	3.202
17	0.03761	3.172
18	0.03751	3.122
19	0.03696	3.100
20	0.03647	3.077
21	0.03591	3.057
22	0.03520	3.042
23	0.03400	3.040
24	0.03267	3.034

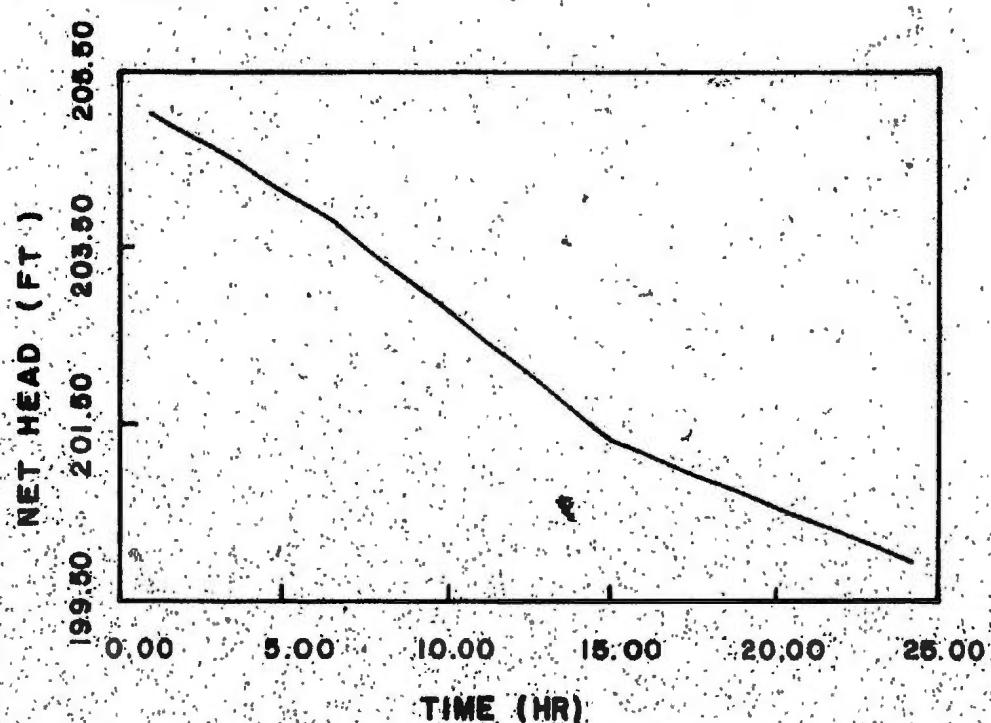


FIGURE 4.2-5. Variation in  $h(t)$ .

TABLE 4.2-3  
HEAD VARIATIONS.  
TABULATED DATA.

TIME PERIOD HR	NET HEAD PLANT NO. 1 FT
1	205.00
2	204.81
3	204.60
4	204.38
5	204.15
6	203.90
7	203.62
8	203.32
9	203.04
10	202.78
11	202.51
12	202.21
13	201.91
14	201.62
15	201.32
16	201.17
17	201.02
18	200.86
19	200.72
20	200.58
21	200.44
22	200.29
23	200.13
24	199.95

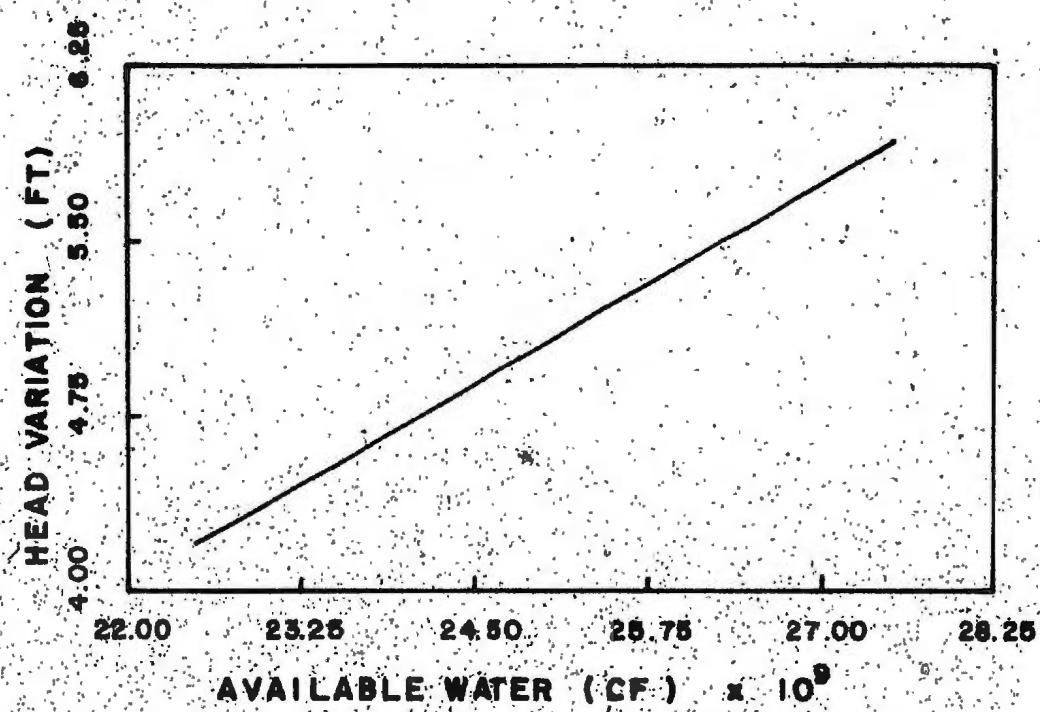


FIGURE 4.2-6. Variation in Head with Increased  $B(t)$ .

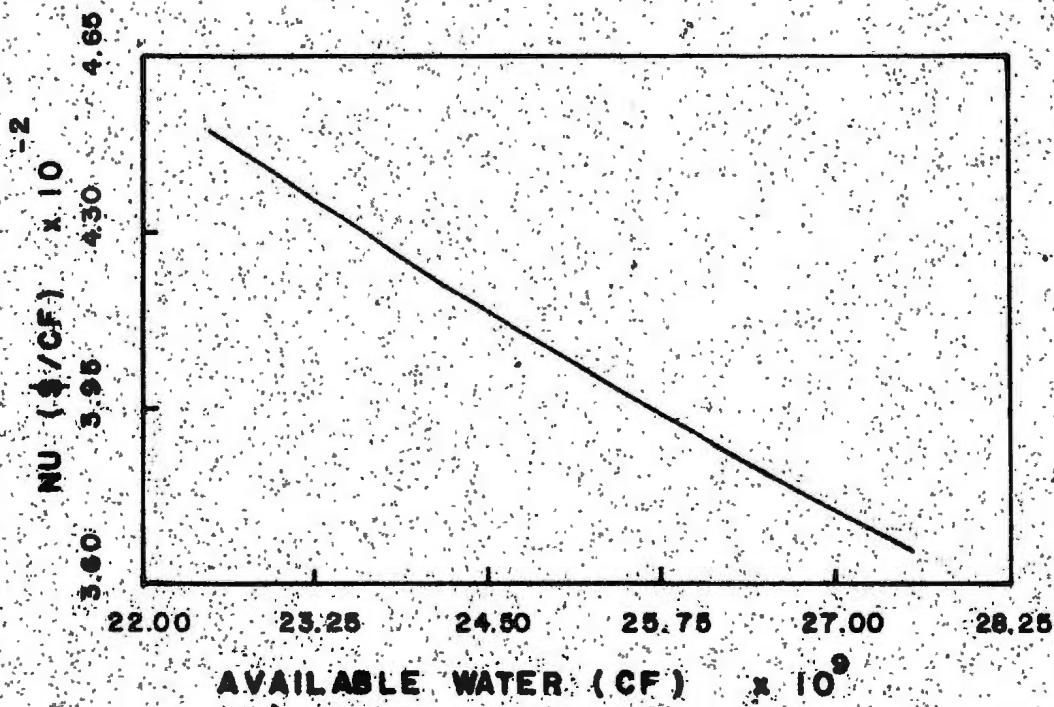


FIGURE 4.2-7. Variation in  $v(t)$  with increased  $B(t)$ .

This means that more hydro power will be generated and that the thermal output and, hence, daily fuel costs will be reduced. Such is the case as Figure 4.2-8 clearly indicates.

Data for the three curves are tabulated in Table 4.2-4.

#### 4.2.5 Characterization Test Two

In this second test the natural inflow and available water is held constant and the power demand is varied to determine the effects on the head variations, incremental cost of power,  $\lambda(t)$ , and the daily fuel cost.

The head variation remains fairly constant (Figure 4.2-9) as  $P_d(t)$  increases since the inflow and  $B$  are held constant which results in an increase in  $v(t)$ .

The incremental cost of power  $\lambda(t)$  increases as  $P_d(t)$  increases which is exactly as expected. This result is demonstrated by Figure 4.2-10.

Examination of Figure 4.2-11 shows that the daily fuel cost also increases with increases in  $P_d(t)$ . This is not unexpected since constant hydro plant characteristics result in no increase in hydro output. Therefore, the slack must be taken up by the thermal unit and, hence, an increase in the daily fuel cost.

The data for these curves is found in Table 4.2-5.

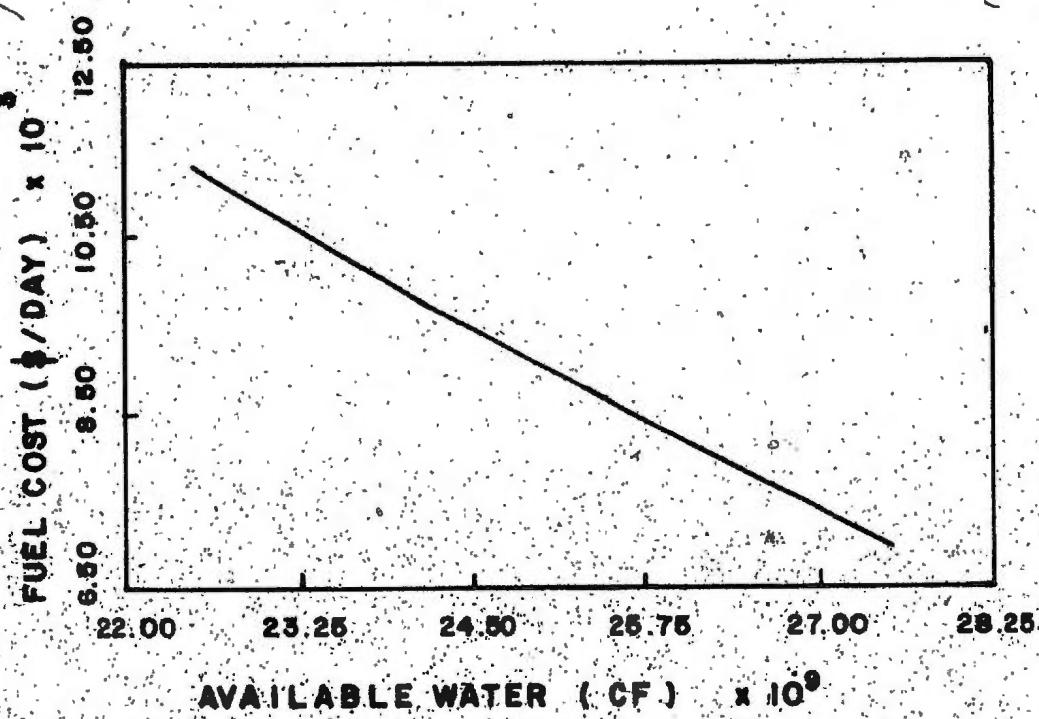


FIGURE 4.2-8. Variation in Daily Fuel Costs with Increased  $B(t)$ .

TABLE 4.2-4  
CHARACTERIZATION TEST.  
TABULATED RESULTS.

B cf x 10 <sup>9</sup>	v (av) \$/cf x 10 <sup>-2</sup>	HEAD VARIATION FT	FUEL COST \$/DAY
22.5	4.503	4.21	11325.96
23.0	4.407	4.38	10839.67
23.5	4.314	4.55	10365.86
24.0	4.224	4.72	9904.07
24.5	4.137	4.90	9453.86
25.0	4.052	5.07	9014.82
25.5	3.977	5.24	8586.54
26.0	3.891	5.41	8186.66
26.5	3.814	5.58	7760.80
27.0	3.739	5.76	7362.64
27.5	3.666	5.93	6973.84

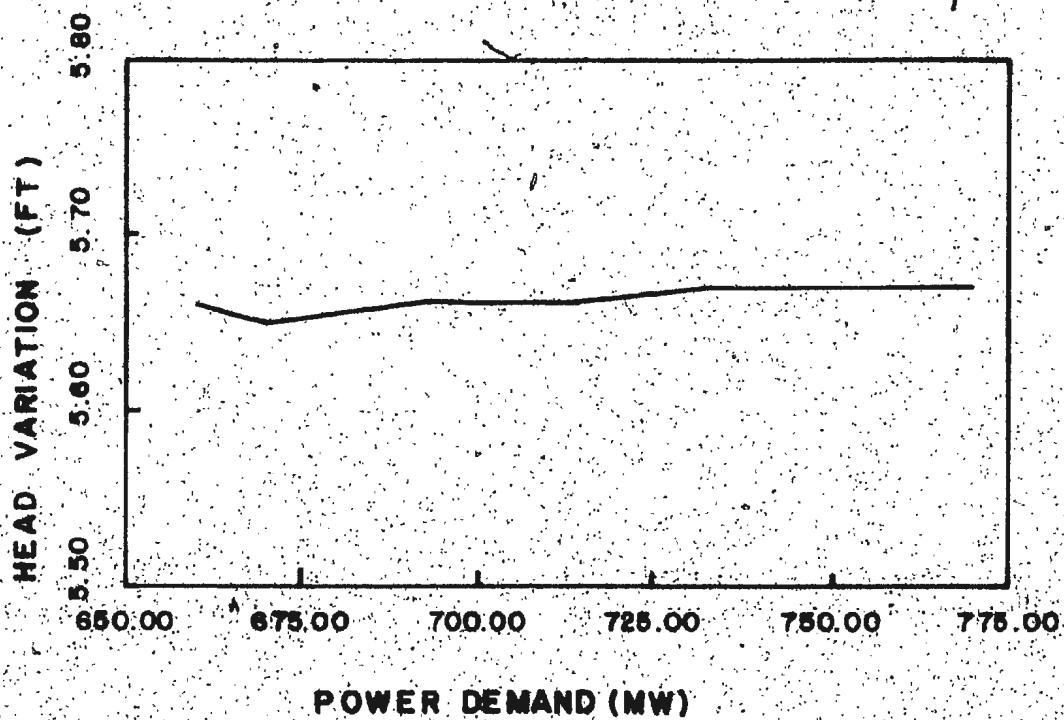


FIGURE 4.2-9. Variation in Head variation with Increased  $P_d(t)$ .

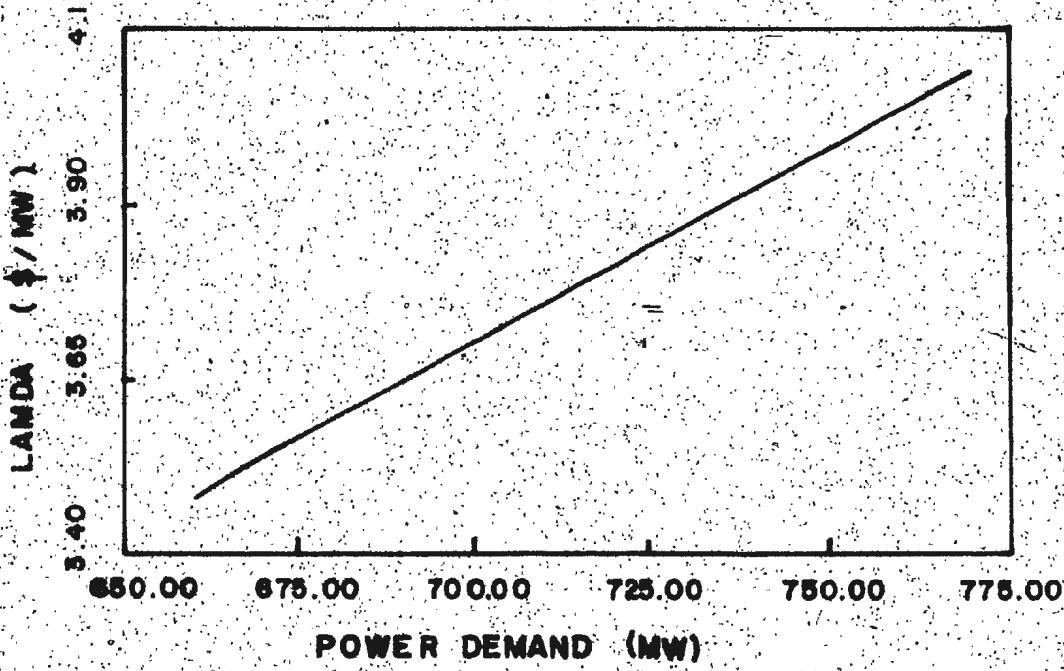


FIGURE 4.2-10. Variation in  $\lambda(t)$  with Increased  $P_d(t)$ .

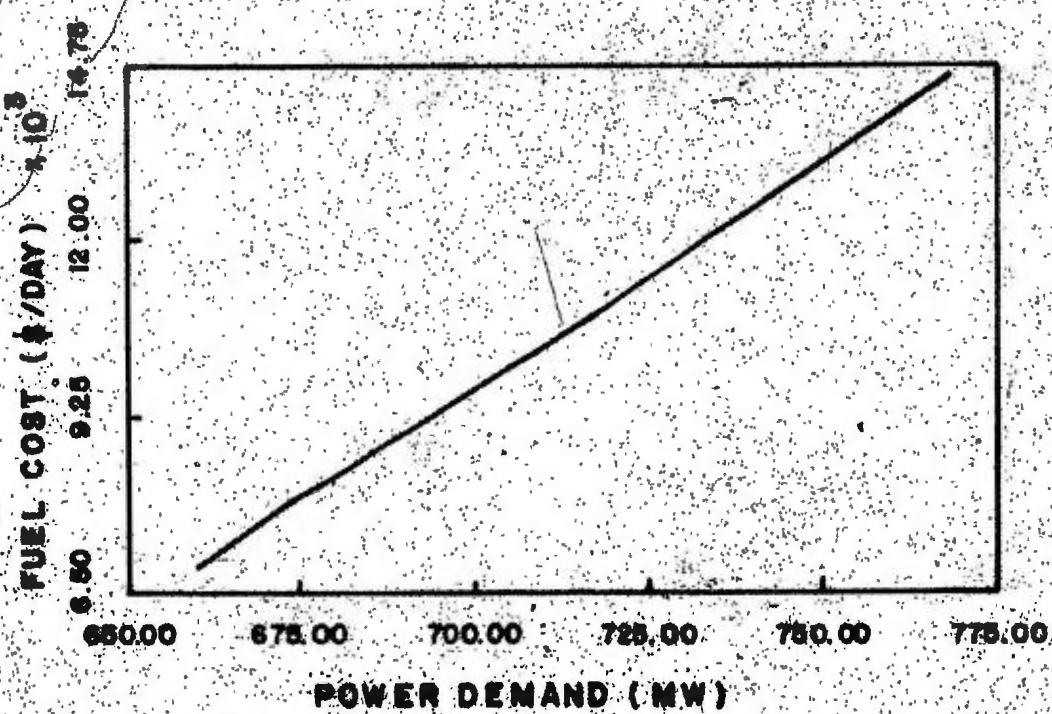


FIGURE 4.2-11. Variation in Fuel Cost with Increased  $P_d(t)$ .

TABLE 4.2-5  
CHARACTERIZATION TEST TWO.  
TABULATED RESULTS.

POWER DEMAND MW	HEAD VARIATION FT	$\lambda$ (AV) \$/MW	FUEL COST \$/DAY
660	5.66	3.483	6897.66
670	5.65	3.545	7685.68
690	5.66	3.654	9011.92
710	5.66	3.763	10377.32
730	5.67	3.872	11781.92
750	5.67	3.981	13225.74
770	5.67	3.091	14708.82

#### 4.2.6 Characterization Test Three

In this final test, the power demand and available water are held constant and the natural inflow to the reservoir is varied. The effects on the head variation and the daily fuel cost are presented in Figures 4.2-12 and 4.2-13, respectively.

Figure 4.2-12 shows that as the inflow is increased, the head variation decreases to such a point where the reservoir water must be spilled. This is not surprising and further, such increased head should result in a lowering of the daily cost which is indeed what happens as is shown in Figure 4.2-13.

Table 4.2-6 presents the data for the curves.

### 4.3 TEST SYSTEM TWO

#### 4.3.1 Test System Two Description

Test system two consists of two thermal and two variable-head hydro plants. All units are supplying power to a common grid over transmission lines with losses.

The quadratic models used to represent the fuel costs, hydro plants performances, and reservoir variations are of the same form as those for test system one. The quadratic coefficients are as follows

$$\alpha_{s_1} = 1.0$$

$$\alpha_{s_2} = 1.0$$

$$\beta_{s_1} = 2.7$$

$$\beta_{s_2} = 2.733$$

$$\gamma_{s_1} = 3.0 \times 10^{-3}$$

$$\gamma_{s_2} = 2.998 \times 10^{-3}$$

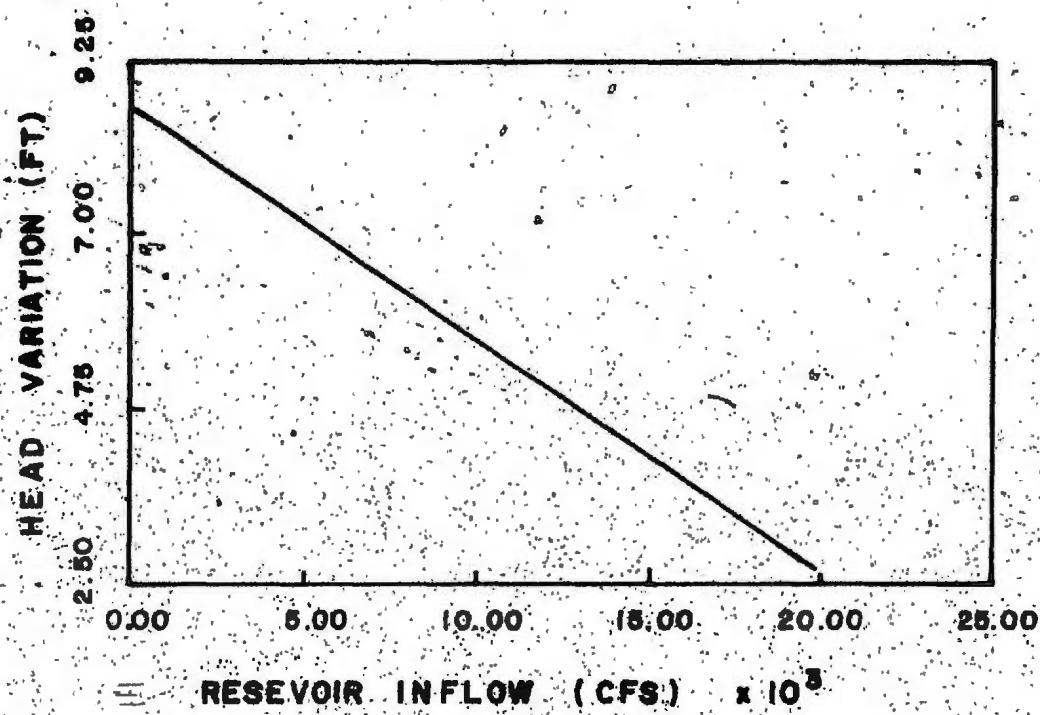


FIGURE 4.2-12. Variation in Head with Increased Inflow.

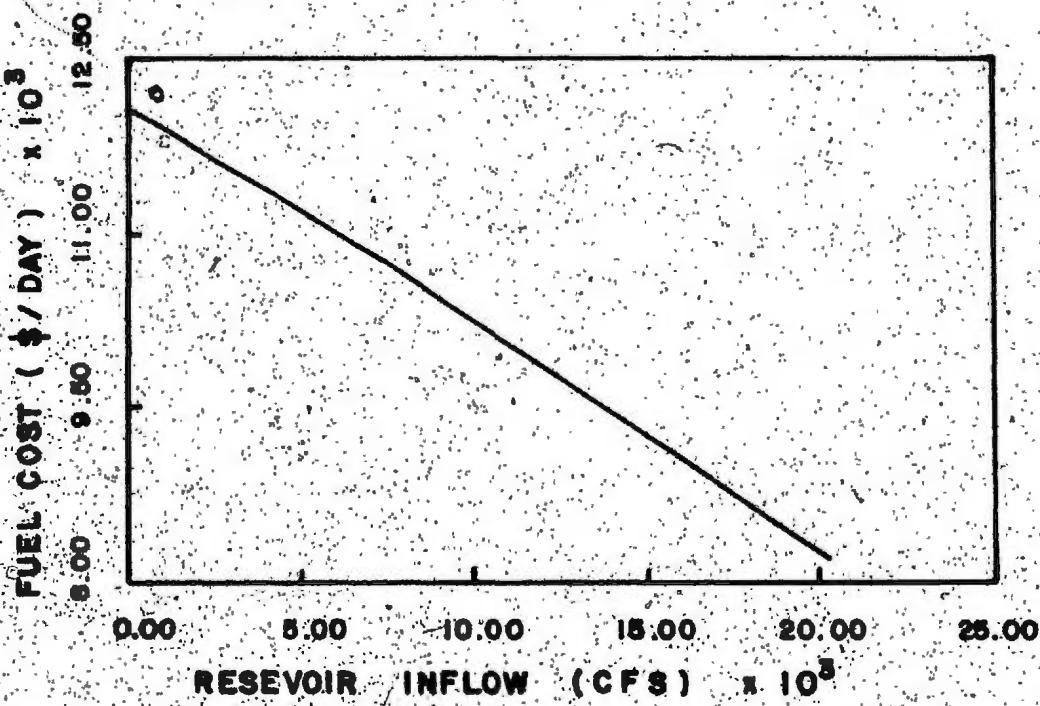


FIGURE 4.2-13. Variation in Daily Fuel Costs with Increased Inflow.

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TABLE 4.2-6  
CHARACTERIZATION TEST THREE.  
TABULATED DATA.

RESERVOIR INFLOW CFS	HEAD VARIATION FT	FUEL COST \$/DAY
0	8.63	12071.22
5	7.13	11182.63
10	5.64	10247.18
15	4.14	9262.04
20	2.65	8224.55

$$a_{h_1} = 1.0$$

$$a_{h_2} = 1.0$$

$$s_{h_1} = 0.1$$

$$s_{h_2} = 0.998 \times 10^{-1}$$

$$\gamma_{h_1} = 1.0 \times 10^{-4}$$

$$\gamma_{h_2} = 1.002 \times 10^{-4}$$

$$a_{0_1} = 1.0$$

$$a_{0_2} = 1.0$$

$$a_{1_1} = -0.2237$$

$$a_{1_2} = -0.2243$$

$$a_{2_1} = 1.0 \times 10^{-3}$$

$$a_{2_2} = 0.993 \times 10^{-3}$$

The transmission loss coefficient matrix is

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.43 \times 10^{-5} & 0 \\ 0 & 0 & 0 & 1.43 \times 10^{-5} \end{bmatrix}$$

Other loss coefficients are

$$B_{1_0} = 0$$

$$B_{3_0} = 0$$

$$B_{2_0} = 0$$

$$B_{4_0} = 0$$

$$K_{L_0} = 0$$

The data for reservoir one is

$$\text{Area} = 10 \text{ mi}^2$$

$$\text{Available water} = 2.5 \times 10^9 \text{ cfs}$$

$$\text{Net head (initial)} = 205 \text{ ft}$$

$$\text{Natural inflow} = 5.5 \times 10^3 \text{ cfs}$$

The data for reservoir two is

$$\text{Area} = 18 \text{ mi}^2$$

$$\text{Available water} = 2.25 \times 10^9 \text{ cfs}$$

$$\text{Net head (initial)} = 206 \text{ ft}$$

$$\text{Natural inflow} = 11 \times 10^3 \text{ cfs}$$

Again the test interval covered a 24 hour period which was subdivided into 24, one hour intervals.

#### 4.3.2 Computational Results

For test system two the program converged in 13 iterations to an error criterion of  $1 \times 10^{-4}$  (see Figure 4.3-1) and required 731 seconds of cpu time for solution.

The optimal dispatch schedule is obtained and presented in Figure 4.3-2 and Tables 4.3-1 and 4.3-2. Again, the thermal generation was minimized and the hydro generation accounted for an average of 80% of the total power requirements. The daily fuel costs for thermal plants one and two were found to be \$11,764.85 and \$11,413.87, respectively.

Figure 4.3-3 is an enlarged version of the  $P_h(t)$  portion of the composite graph. Note that as time progresses  $P_h(t)$  carries more of

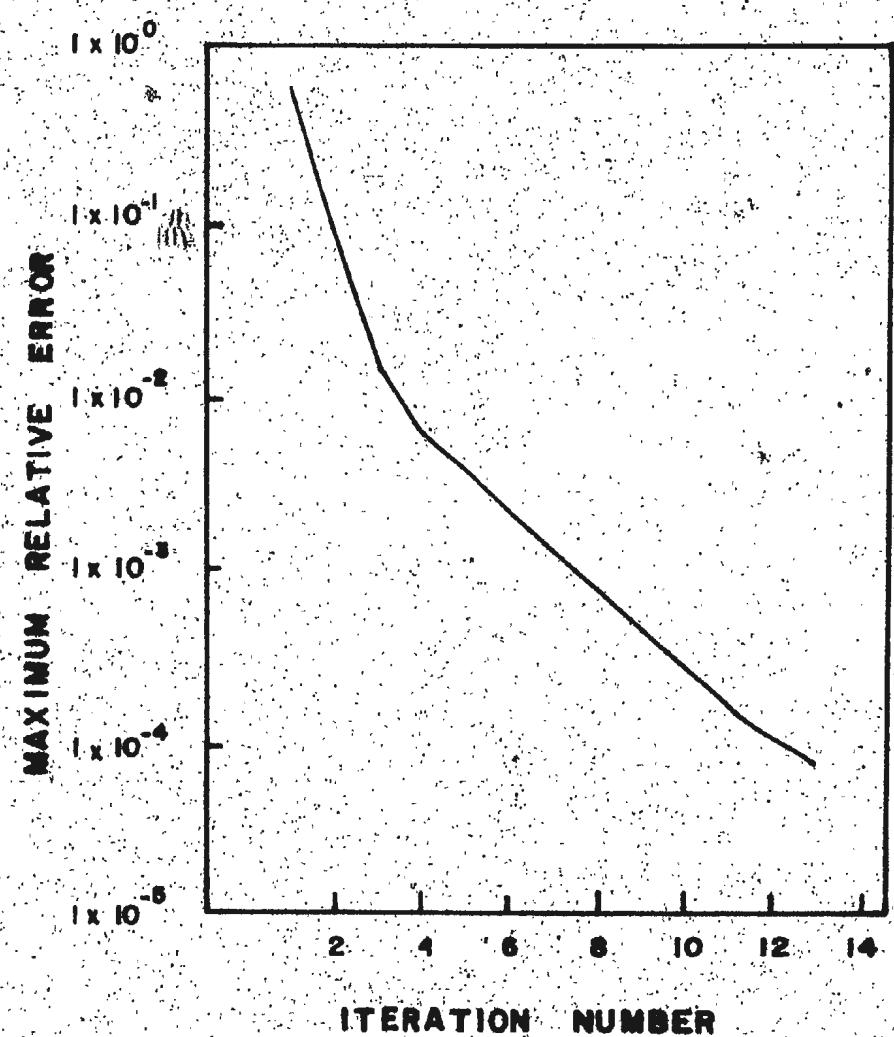


FIGURE 4.3-1. Maximum Relative Error Versus Number of Iterations.

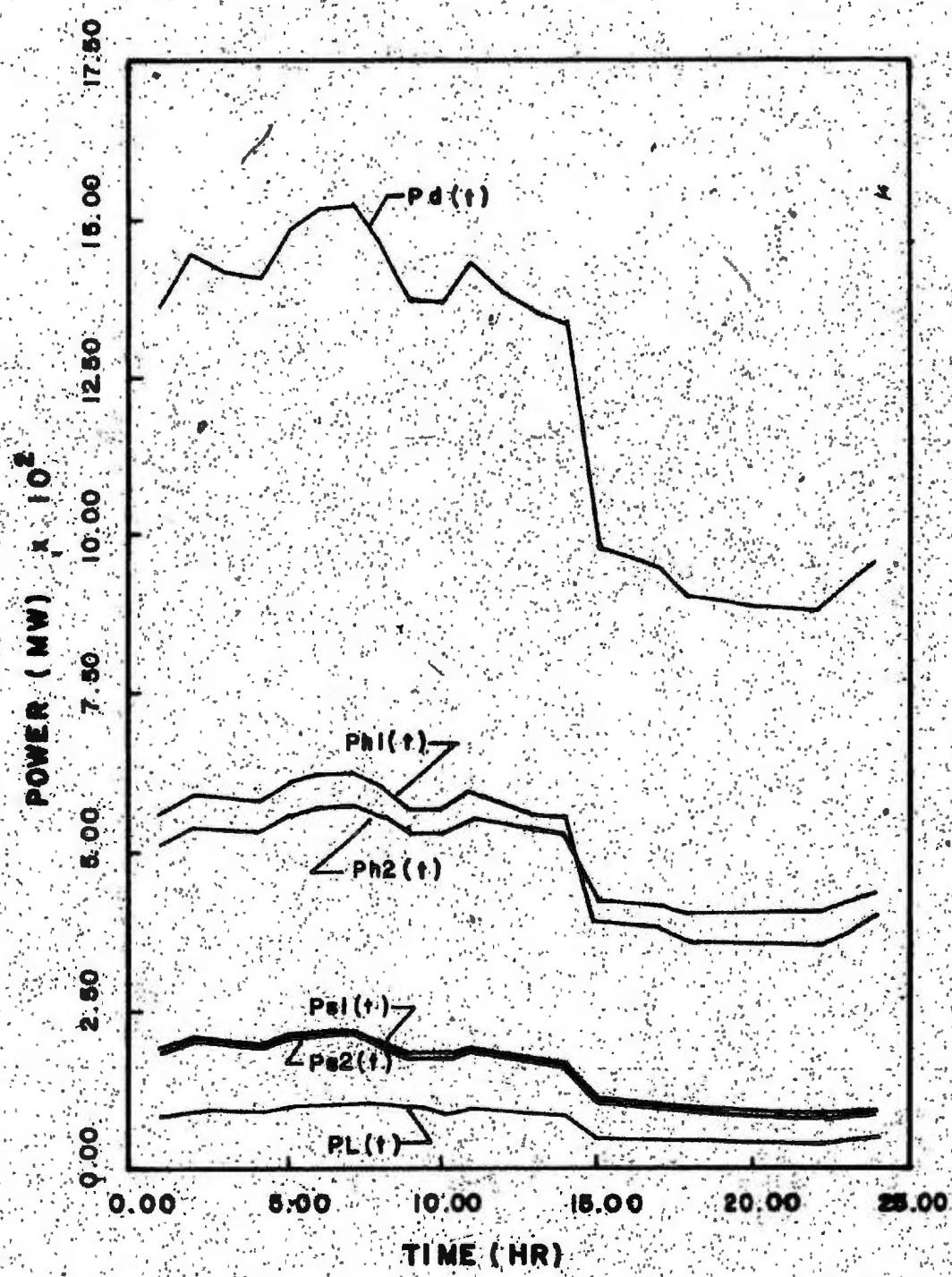


FIGURE 4.3-2. Optimum Dispatch Schedule.

TABLE 4.3-1  
OPTIMUM DISPATCH SCHEDULE.  
TABULATED RESULTS.

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW
1	1362	82.29
2	1444	90.86
3	1416	97.02
4	1406	88.26
5	1482	97.02
6	1516	101.54
7	1522	102.90
8	1464	96.60
9	1370	86.25
10	1366	86.33
11	1432	94.88
12	1384	89.38
13	1350	85.65
14	1332	83.89
15	982	47.43
16	962	46.03
17	946	44.97
18	902	41.53
19	896	41.34
20	886	40.81
21	882	40.78
22	888	41.63
23	922	45.11
24	970	50.31

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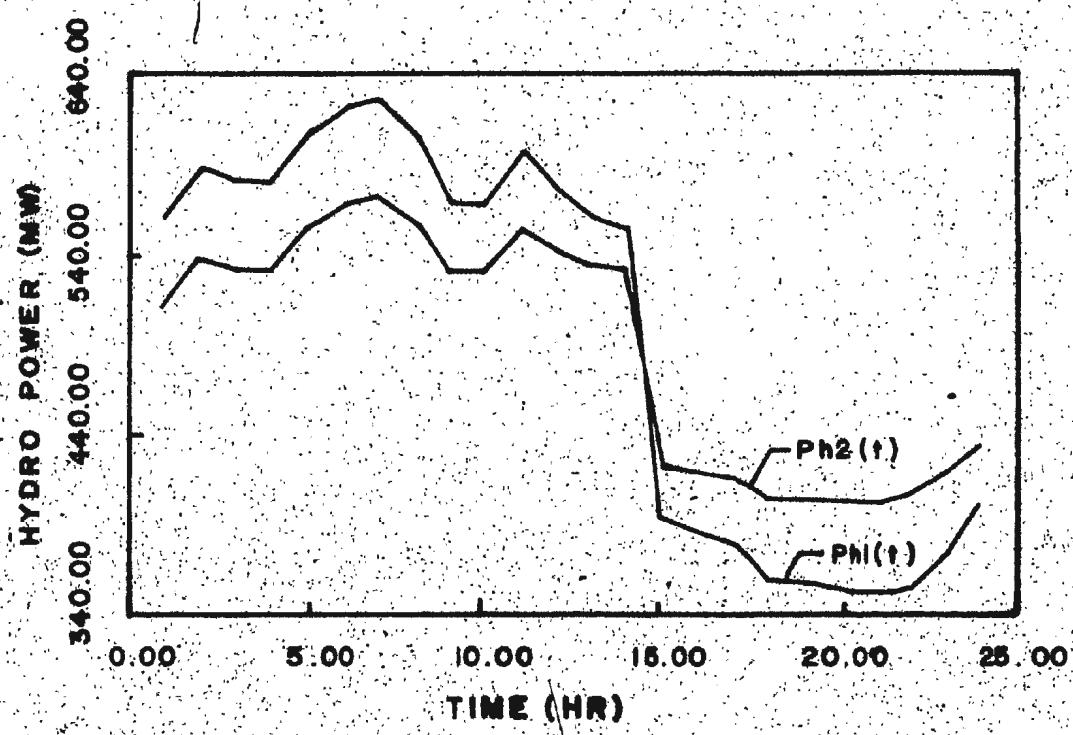


FIGURE 4.3-3. Variation in  $P_h(t)$ .

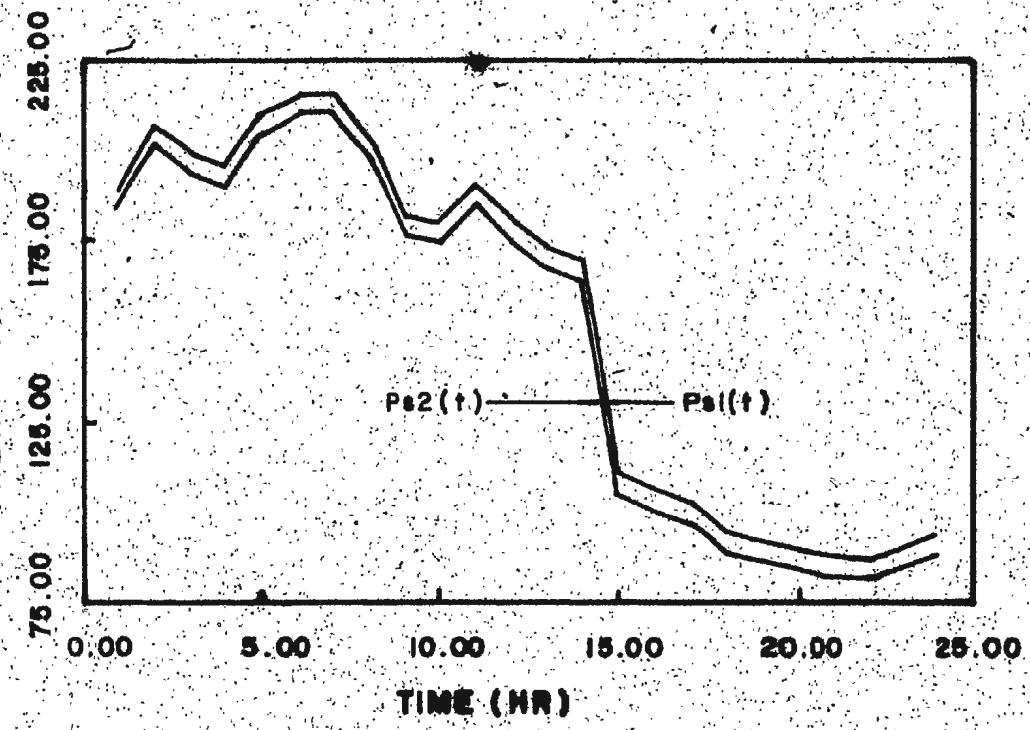


FIGURE 4.3-4. Variation in  $P_s(t)$ .

TABLE 4.3-2.  
OPTIMUM DISPATCH SCHEDULE.  
TABULATED RESULTS.

TIME PERIOD HR	HYDRO PLANT NO. 1 MW	HYDRO PLANT NO. 2 MW	THERMAL PLANT NO. 1 MW	THERMAL PLANT NO. 2 MW
1	560.26	511.42	188.99	183.61
2	588.05	538.14	207.02	201.65
3	580.83	531.71	198.75	193.38
4	579.28	530.72	194.82	189.45
5	607.54	556.23	210.31	204.95
6	621.60	568.92	216.19	210.83
7	625.48	573.02	215.88	210.52
8	604.22	557.17	202.29	196.92
9	567.74	529.95	181.97	176.59
10	567.31	530.94	179.73	174.35
11	597.17	553.98	190.55	185.18
12	576.13	541.41	180.62	175.23
13	560.92	533.24	173.44	168.05
14	552.81	530.14	169.17	163.78
15	392.55	421.39	110.46	105.03
16	384.64	417.06	105.88	100.45
17	378.25	414.05	102.06	96.62
18	358.94	401.98	94.02	88.58
19	356.84	402.22	91.86	86.42
20	352.70	401.21	89.17	83.73
21	351.30	402.18	87.37	81.93
22	354.86	406.40	86.91	81.46
23	373.60	419.35	89.80	84.35
24	402.14	436.03	93.79	88.35

the load than  $P_{h_1}(t)$ . This is as predicted, since reservoir two has a larger area and greater inflow. The higher starting value of  $P_{h_1}(t)$  can be attributed to the larger starting value for the available amount of water.

Figure 4.3-4 again is an expanded portion of the composite showing  $P_s(t)$ . As expected, the curves for  $P_{s_1}(t)$  and  $P_{s_2}(t)$  are very similar in shape as well as values.

Figures 4.3-5 and 4.3-6 present the variations in  $\lambda(t)$  and  $v(t)$ , respectively, with time. As for the two plant system,  $\lambda(t)$  varies with the power demand and  $v(t)$  decreases as the net head,  $h(t)$ , decreases. It is important to note that the decrease in  $v_2(t)$  is less steep than  $v_1(t)$ . The reason for this is evident from Figure 4.3-7 which shows the net head variations for the two reservoirs. The decrease in  $h_2(t)$  is less than that of  $h_1(t)$ , hence, the difference in  $v_2(t)$  and  $v_1(t)$ :

This lesser decrease in  $h_1(t)$  is due to the larger inflow and greater area of reservoir one. The tabulated data for these curves is given in Tables 4.3.3 and 4.3.4.

Actual computer print-out for this system is found in Appendix C.

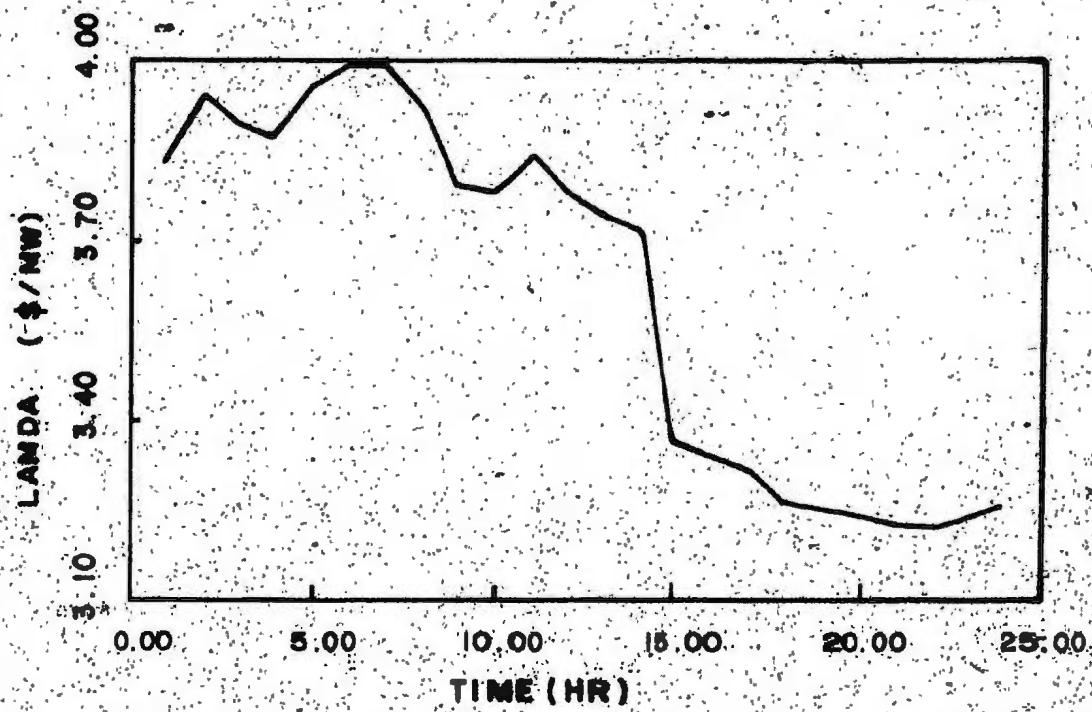


FIGURE 4.3-5. Variation in  $\lambda(t)$ .

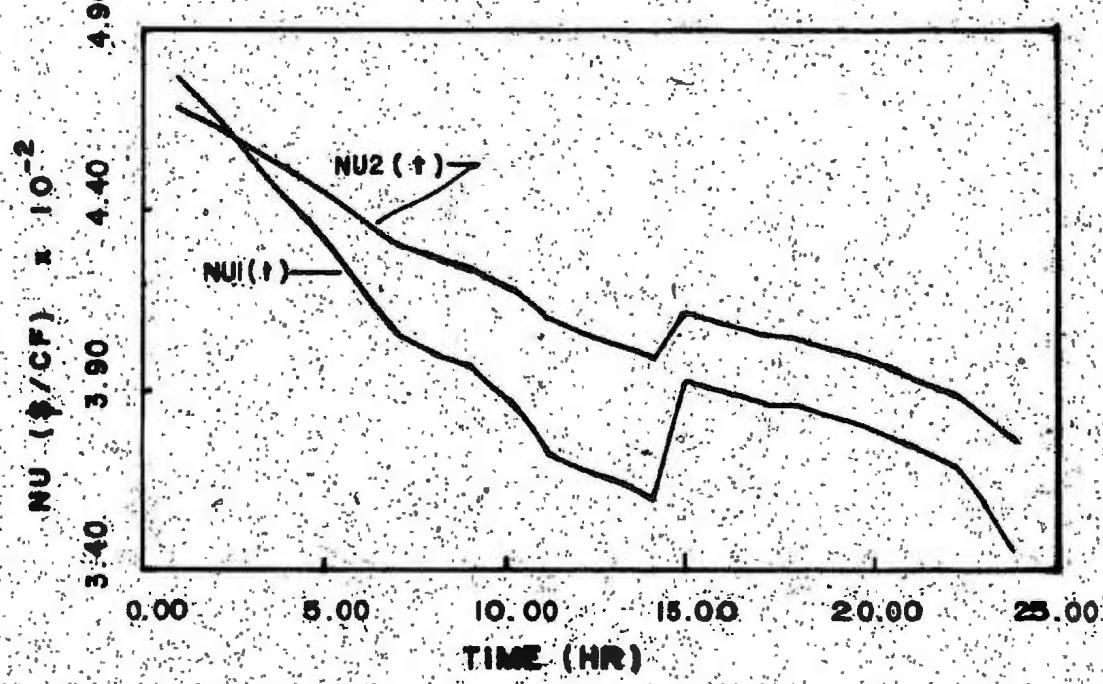


FIGURE 4.3-6. Variation in  $\nu(t)$ .

TABLE 4.3-3  
VARIATIONS IN  $\lambda(t)$  AND  $v(t)$ .

TABULATED RESULTS.

TIME PERIOD HR	$v(\pm)$ PLANT NO. 1 S/CF	$v(t)$ PLANT NO. 2 S/CF	$\lambda(t)$ \$/MM
1	0.04769	0.04684	3.834
2	0.04651	0.04624	3.942
3	0.04541	0.04566	3.893
4	0.04433	0.04509	3.869
5	0.04300	0.04438	3.962
6	0.04175	0.04370	3.997
7	0.04063	0.04307	3.995
8	0.03993	0.04263	3.914
9	0.03861	0.04236	3.792
10	0.03874	0.04183	3.778
11	0.03727	0.04101	3.843
12	0.03688	0.04066	3.784
13	0.03643	0.04027	3.741
14	0.03586	0.03983	3.715
15	0.03921	0.04116	3.363
16	0.03889	0.04086	3.335
17	0.03856	0.04055	3.312
18	0.03861	0.04042	3.264
19	0.03822	0.04007	3.251
20	0.03790	0.03976	3.235
21	0.03751	0.03940	3.224
22	0.03697	0.03897	3.221
23	0.03591	0.03834	3.239
24	0.03447	0.03760	3.263

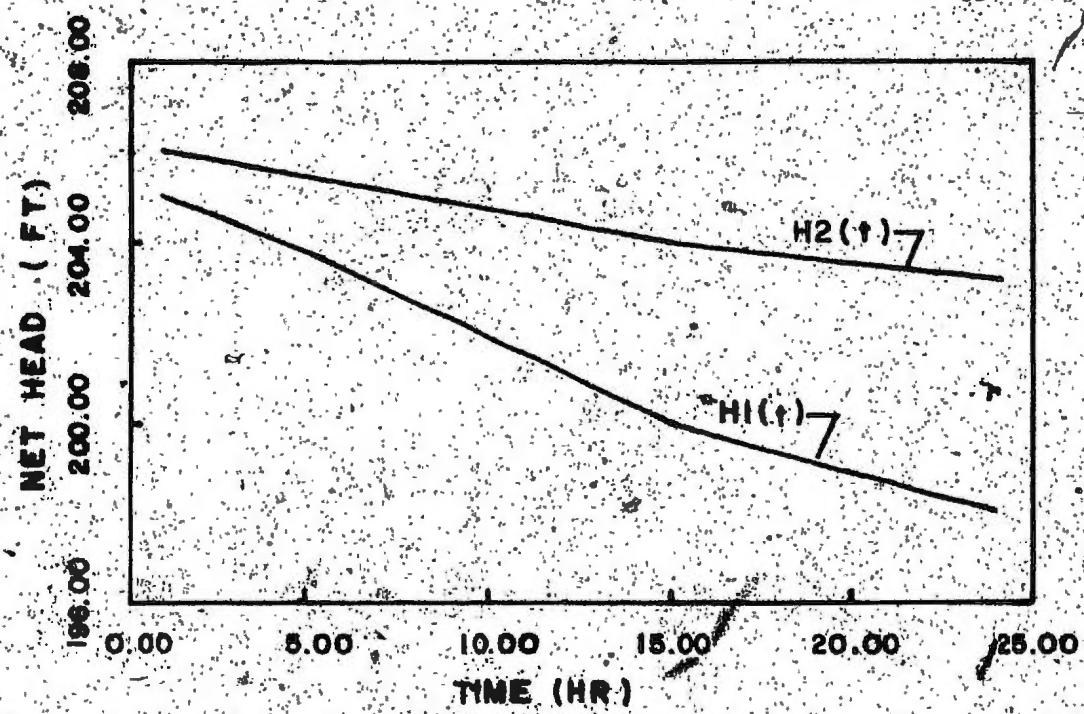


FIGURE 4.3-7.  $h(t)$  Versus  $t$ .

TABLE 4.3-4  
NET HEAD VARIATIONS.  
TABULATED RESULTS.

TIME PERIOD HR	NET HEAD PLANT NO. 1 FT	NET HEAD PLANT NO. 2 FT
1	205.00	206.00
2	204.71	205.88
3	204.40	205.75
4	204.08	205.62
5	203.75	205.48
6	203.39	205.34
7	203.01	205.18
8	202.61	205.01
9	202.22	204.86
10	201.86	204.72
11	201.50	204.57
12	201.09	204.41
13	200.70	204.26
14	200.31	204.11
15	199.93	203.96
16	199.70	203.86
17	199.48	203.77
18	199.26	203.68
19	199.05	203.60
20	198.85	203.51
21	198.65	203.43
22	198.44	203.34
23	198.23	203.25
24	198.00	203.15

#### 4.4 TEST SYSTEM THREE

##### 4.4.1 Test System Three Description

Since no data is available for larger systems, the basic hydro unit used for test one and the two thermal units of test two were scaled so that the algorithm could be tested on a system containing five hydro units and two thermal units.

The basic data for the system is found in the preceding two test descriptions and the system power demand is found in Appendix C.

##### 4.4.2 Computational Results

The program converged in seven iterations to an error criterion of  $1 \times 10^{-4}$  (see Figure 4.4-1) and required 35 min cpu time. This is not exceptional when one considers the size of the system.

The optimal dispatch schedule is obtained and is presented in Figure 4.4-2 and Table 4.4-1. As can be seen, the thermal generation was reduced to a minimum and again the hydro power output supplied approximately 80% of the total power requirements. The daily fuel costs for thermal plant one is \$16,693.21 and \$17,020.42 for thermal plant two.

Figures 4.4-3 and 4.4-4 are enlarged views of the hydro and thermal power output,  $P_h(t)$  and  $P_s(t)$ , respectively.

Figures 4.4-5 and 4.4-6 show how  $\lambda(t)$  and  $v(t)$  varies with time. As with the other two test systems, the shape and, hence, the functions of the curves act as earlier predicted. Tabulated data for these curves is found in Table 4.4-2.

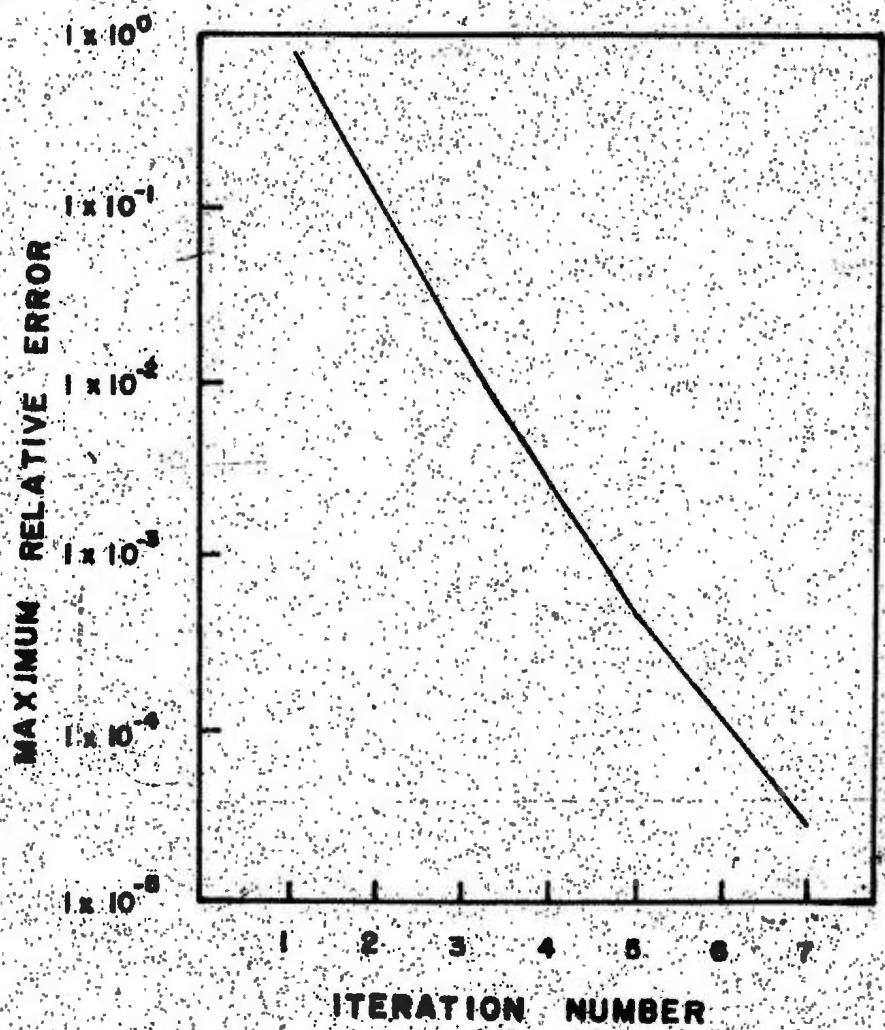


FIGURE 4.4-1. Variation in Maximum Relative Error with Iterations.

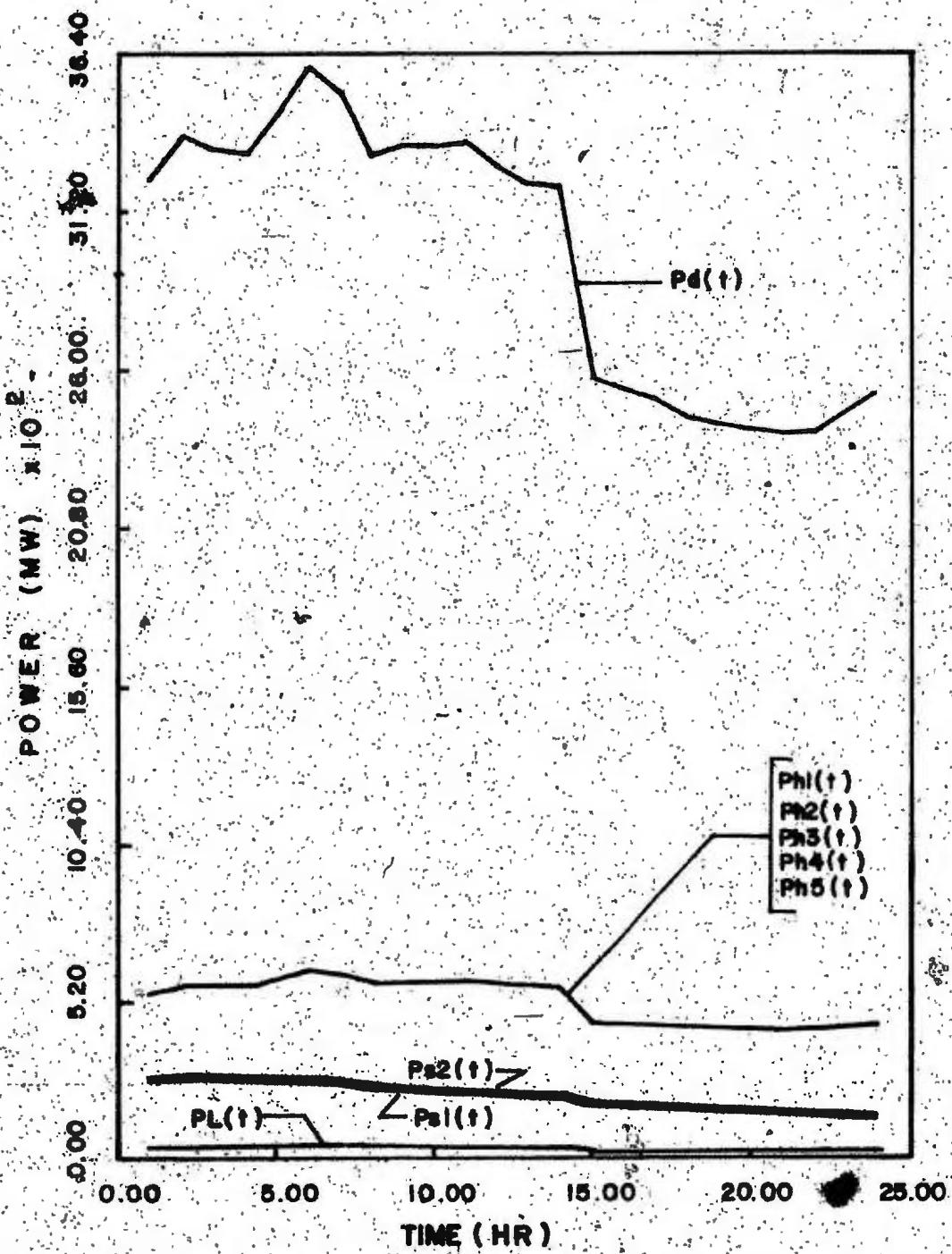


FIGURE 4.4-2. Optimal Dispatch Schedule.

TABLE 4.4-1. OPTIMAL DISPATCH SCHEDULE. TABULATED RESULTS.

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	THERMAL PLANT NO. 1 MW	THERMAL PLANT NO. 2 MW	HYDRO PLANT NO. 1 MW	HYDRO PLANT NO. 2 MW	HYDRO PLANT NO. 3 MW	HYDRO PLANT NO. 4 MW	HYDRO PLANT NO. 5 MW
1	3223	33.99	263.16	268.23	545.09	545.27	544.90	545.08	545.26
2	3367	37.35	271.21	276.29	571.35	571.54	571.15	571.32	571.49
3	3318	36.59	260.92	266.00	565.51	565.70	565.31	565.49	565.66
4	3300	36.49	253.66	258.74	564.79	564.99	564.60	564.77	564.95
5	3434	39.88	258.33	263.41	590.42	590.63	590.21	590.36	590.52
6	3593	44.11	263.60	268.68	620.98	621.22	620.74	620.88	621.02
7	3503	42.15	252.48	257.56	607.02	607.25	606.80	606.95	607.10
8	3302	37.51	235.63	240.72	572.60	572.79	572.41	572.59	572.78
9	3338	38.67	232.24	237.33	581.39	581.59	581.19	581.37	581.55
10	3331	38.78	226.79	231.88	582.19	582.39	582.00	582.18	582.36
11	3346	39.44	222.32	227.41	587.11	587.32	586.91	587.09	587.27
12	3262	37.56	214.80	219.89	572.92	573.10	572.74	572.94	573.15

TABLE 4.4-1. OPTIMAL DISPATCH SCHEDULE. TABULATED RESULTS. CONT'D.

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	THERMAL PLANT NO. 1 MW	THERMAL PLANT NO. 2 MW	HYDRO PLANT NO. 1 MW	HYDRO PLANT NO. 2 MW	HYDRO PLANT NO. 3 MW	HYDRO PLANT NO. 4 MW	HYDRO PLANT NO. 5 MW
13	3203	36.32	208.47	213.56	563.39	563.56	563.22	563.45	563.67
14	3191	36.26	203.57	208.67	562.93	563.10	562.77	563.00	563.23
15	2559	22.34	183.36	188.46	441.66	441.68	441.64	442.06	442.47
16	2524	21.84	177.85	182.96	436.76	436.78	436.75	437.16	437.58
17	2496	21.48	172.75	177.85	433.13	433.14	433.12	433.54	433.95
18	2437	20.53	167.17	172.28	423.36	423.36	423.36	423.79	424.21
19	2408	20.14	162.38	167.49	419.40	419.40	419.40	419.82	420.25
20	2391	19.99	157.92	163.03	417.75	417.76	417.75	418.17	418.60
21	2384	20.01	153.66	158.78	418.07	418.08	418.06	418.48	418.89
22	2393	20.35	149.50	154.62	421.62	421.64	421.60	421.99	422.38
23	2454	21.76	144.97	150.09	435.97	436.03	435.91	436.23	436.56
24	2520	23.34	139.88	145.00	451.60	451.71	451.49	451.71	451.94

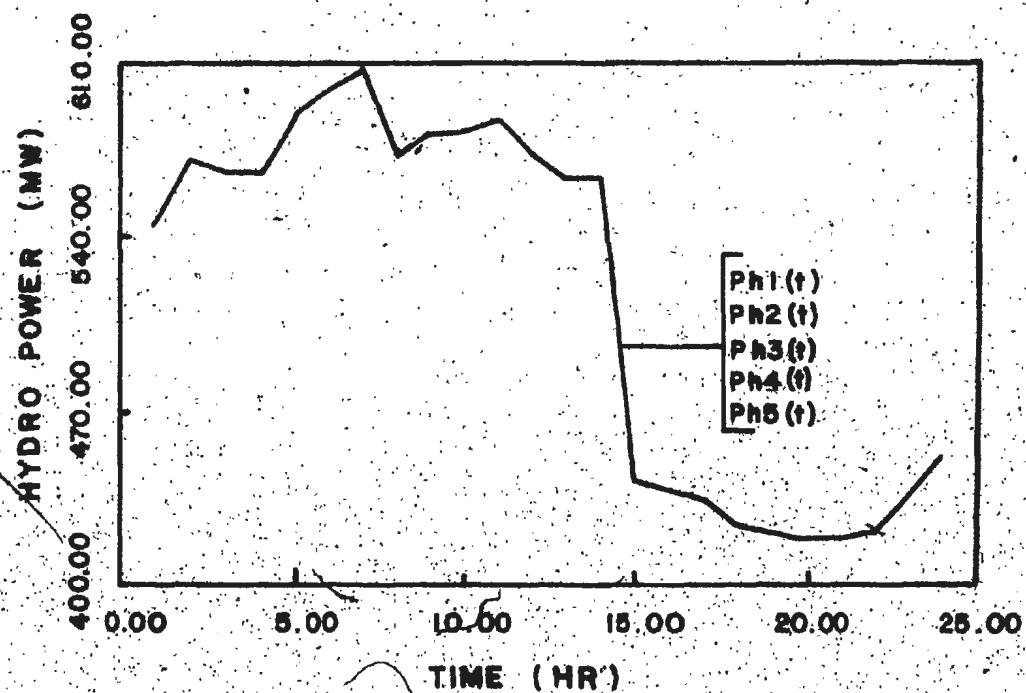


FIGURE 4.4-3. Variation in  $P_h(t)$ .

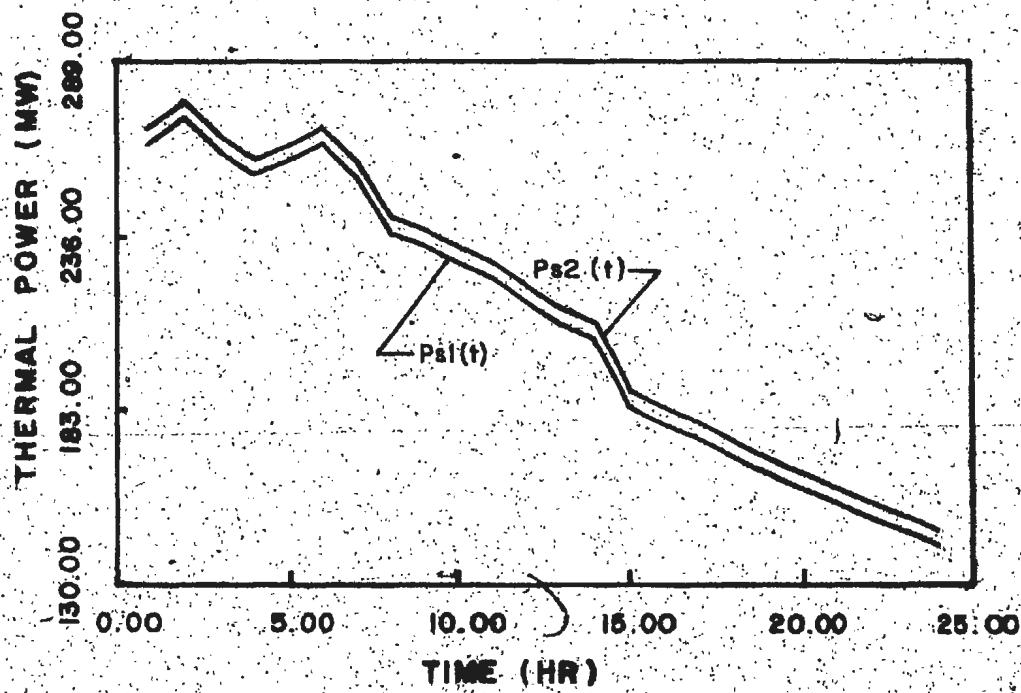


FIGURE 4.4-4. Variation in  $P_s(t)$ .

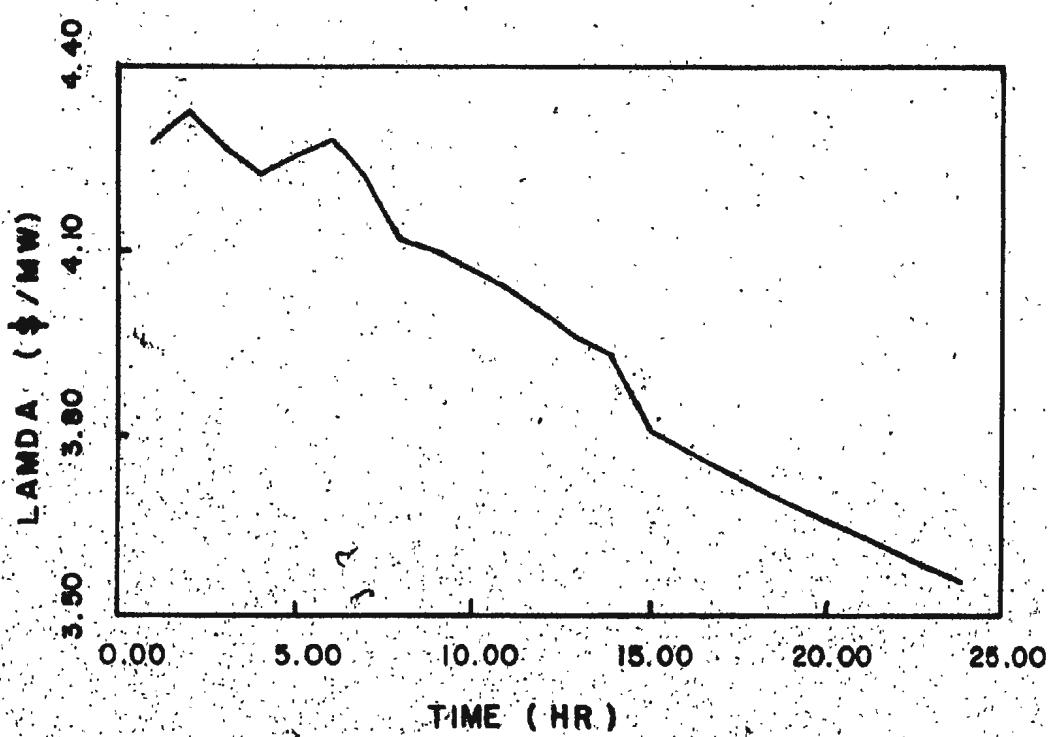


FIGURE 4.4-5. Variation in  $\lambda(t)$ .

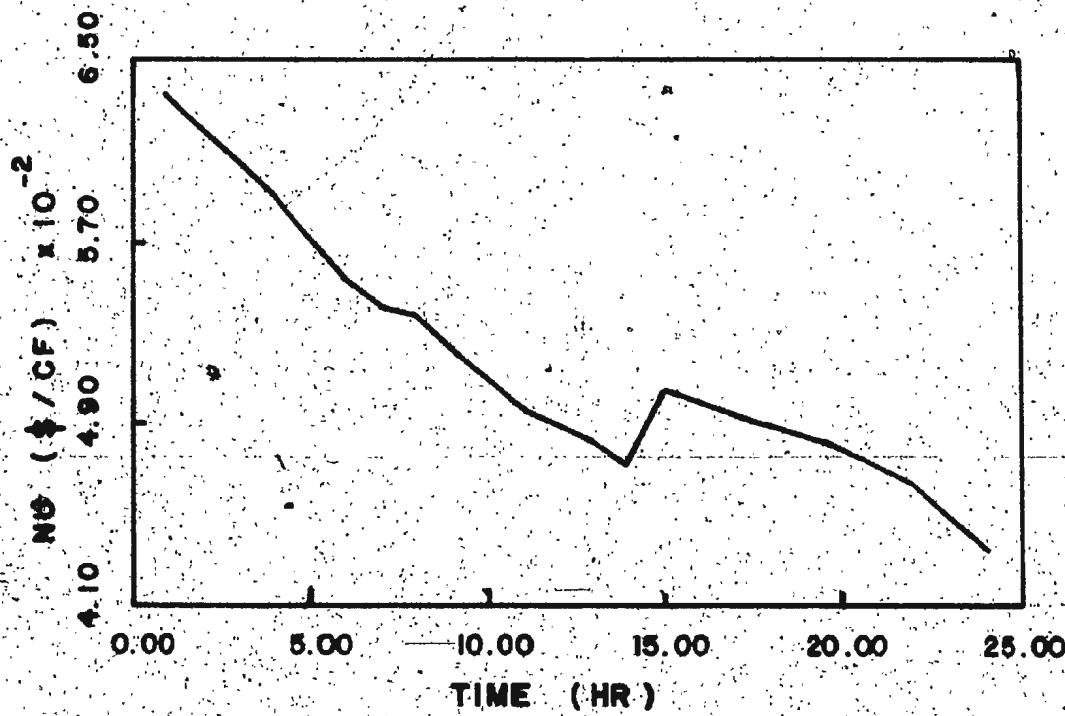


FIGURE 4.4-6. Variation in  $v(t)$ .

TABLE 4.4-2. VARIATIONS IN  $\lambda(t)$  AND  $v(t)$ .

TIME PERIOD HR	NU PLANT NO. 1 S/CF	NU PLANT NO. 2 S/CF	NU PLANT NO. 3 S/CF	NU PLANT NO. 4 S/CF	NU PLANT NO. 5 S/CF	LAMBDA \$/MW
1	0.06329	0.06331	0.06326	0.06328	0.06330	4.279
2	0.06169	0.06171	0.06167	0.06169	0.06171	4.327
3	0.06030	0.06032	0.06027	0.06029	0.06031	4.266
4	0.05891	0.05893	0.05889	0.05891	0.05892	4.222
5	0.05713	0.05715	0.05711	0.05713	0.05715	4.250
6	0.05515	0.05517	0.05514	0.05516	0.05517	4.282
7	0.05404	0.05406	0.05402	0.05404	0.05406	4.215
8	0.05353	0.05355	0.05351	0.05353	0.05355	4.114
9	0.05207	0.05209	0.05205	0.05207	0.05209	4.093
10	0.05084	0.05086	0.05082	0.05084	0.05086	4.061
11	0.04951	0.04953	0.04950	0.04952	0.04953	4.034
12	0.04883	0.04884	0.04881	0.04883	0.04884	3.989

TABLE 4.4-2. VARIATIONS IN  $\lambda(t)$  AND  $\nu(t)$ . CONT'D.

TIME PERIOD HR	NU PLANT NO. 1 S/CF	NU PLANT NO. 2 S/CF	NU PLANT NO. 3 S/CF	NU PLANT NO. 4 S/CF	NU PLANT NO. 5 S/CF	LAMBDA \$/MW
13	0.04807	0.04808	0.04805	0.04806	0.04908	3.951
14	0.04704	0.04706	0.04702	0.04704	0.04705	3.921
15	0.05052	0.05054	0.05050	0.05051	0.05052	3.800
16	0.04991	0.04993	0.04988	0.04989	0.04990	3.767
17	0.04927	0.04929	0.04925	0.04926	0.04926	3.736
18	0.04891	0.04894	0.04889	0.04890	0.04891	3.703
19	0.04835	0.04837	0.04833	0.04834	0.04835	3.674
20	0.04772	0.04774	0.04770	0.04770	0.04771	3.648
21	0.04701	0.04703	0.04699	0.04699	0.04700	3.622
22	0.04616	0.04617	0.04614	0.04614	0.04615	3.597
23	0.04478	0.04480	0.04477	0.04478	0.04478	3.570
24	0.04333	0.04335	0.04332	0.04333	0.04334	3.539

In Figure 4.4-7, the net head is presented. This curve also performs as expected. Table 4.4-3 gives the tabulated results. Actual computer printout is found in Appendix C.

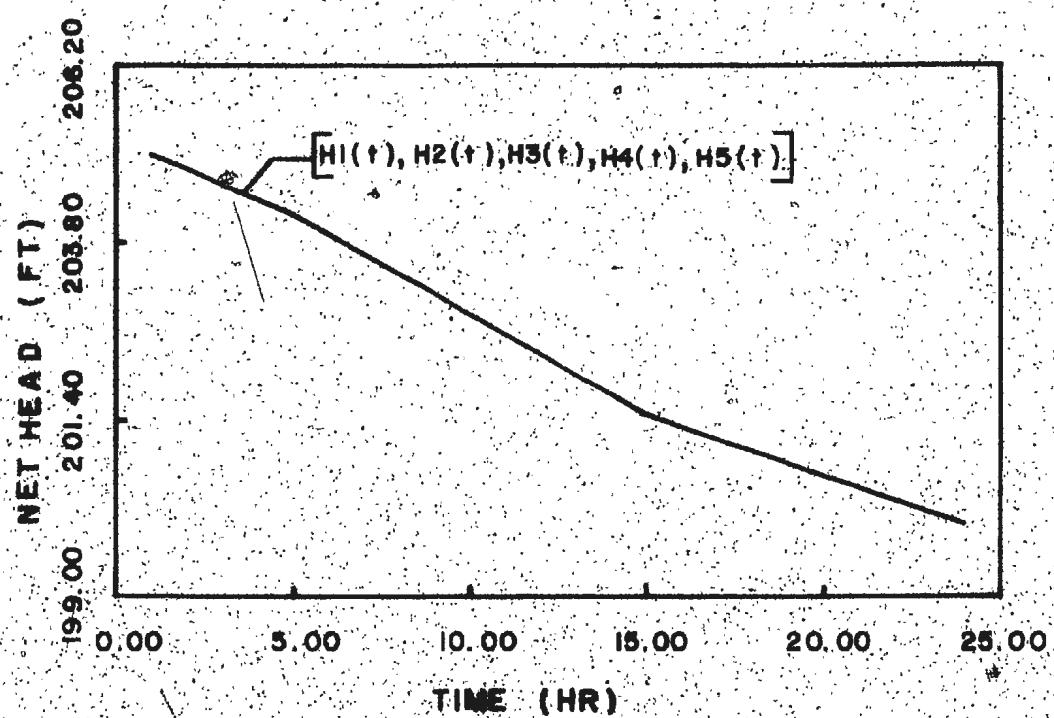


FIGURE 4.4-7. Variation in  $h(t)$ .

TABLE 4.4-3  
VARIATION IN  $h(t)$ . TABULATED RESULTS.

TIME PERIOD HR	NET HEAD PLANT NO. 1 FT	NET HEAD PLANT NO. 2 FT	NET HEAD PLANT NO. 3 FT	NET HEAD PLANT NO. 4 FT	NET HEAD PLANT NO. 5 FT
1	205.00	205.00	205.00	205.00	205.00
2	204.81	204.81	204.81	204.81	204.81
3	204.59	204.59	204.59	204.59	204.59
4	204.38	204.38	204.38	204.38	204.38
5	204.16	204.16	204.16	204.16	204.16
6	203.91	203.91	203.91	203.91	203.91
7	203.62	203.62	203.62	203.62	203.62
8	203.34	203.34	203.34	203.34	203.34
9	203.09	203.09	203.09	203.09	203.09
10	202.82	202.82	202.82	202.82	202.82
11	202.55	202.55	202.55	202.55	202.55
12	202.26	202.26	202.26	202.26	202.26
13	201.98	201.98	201.98	201.98	201.99
14	201.71	201.71	201.71	201.71	201.71
15	201.43	201.43	201.43	201.43	201.43
16	201.26	201.26	201.26	201.26	201.26
17	201.09	201.09	201.10	201.10	201.10
18	200.93	200.93	200.93	200.93	200.93
19	200.77	200.77	200.77	200.77	200.77
20	200.62	200.62	200.62	200.62	200.62
21	200.46	200.46	200.46	200.46	200.46
22	200.30	200.30	200.30	200.30	200.30
23	200.13	200.13	200.13	200.13	200.13
24	199.95	199.95	199.95	199.95	199.95

CHAPTER VCOORDINATION EQUATIONS FOR ECONOMIC OPERATIONOF POWER SYSTEMS WITH HYDRO PLANTSON THE SAME STREAM5.1 INTRODUCTION

A newly developed set of coordination equations for electric power systems with hydro plants on the same stream is presented. Time delay of flow between plants is taken into account. The resulting equations represent a natural extension of the pioneering Kron-Ricard equations from which the useful concepts of water-worth can be obtained easily.

This preserves the intuitive form of the coordination equations and provides insights into the physical meaning of the variables concerned.

5.2 FORMULATION

The system considered is assumed to contain one thermal plant whose active power generation is denoted by  $P_1$  and a hydro subsystem. The objective of the economic scheduling problem is to minimize the thermal cost represented by  $F$  over the optimization interval  $[0, T]$ .

$$J = \int_0^T F[P_1(t)] dt \quad (541)$$

The function  $F$  is assumed available. The optimization is to be carried out while satisfying the hydro subsystem constraints as well as the electric network constraints. For simplicity we assume that only one

constraint is used which requires an active power balance.

$$f_D(t) = \sum_{i=1}^N P_i(t) - P_L(t) - P_D(t) = 0. \quad (5.2)$$

The system demand,  $P_D$ , is assumed given for the optimization interval.

Power losses are denoted by  $P_L$ . The number of system plants is denoted by  $N$ .

The power system considered is assumed to contain three hydro plants on the same stream. This is the minimum number required for a general formulation. The plants are ordered as follows:

Plant 2 up-stream plant

Plant 3 intermediate plant

Plant 4 down-stream plant

It is assumed that the performance characteristic of each hydro plant is modeled by the Glimm-Kirchmayer model

$$\frac{f_i}{q_i}(t) = q_i - K_i \psi_i(h_i) \phi_i(P_i) = 0. \quad (5.3)$$

The rate of water discharge in  $m^3/sec$  is denoted by  $q_i$ , the effective head in meters is denoted by  $h_i$ , and  $P_i$  denotes the active power generation in MW. The functions  $\psi$  and  $\phi$  are given by

$$\psi(h) = a_0 + a_1 h + a_2 h^2$$

$$\phi(P) = b_0 + b_1 P + b_2 P^2$$

The model assumes the availability of the parameters  $K$ ,  $a_0$ ,  $a_1$ ,  $a_2$ ,  $b_0$ ,  $b_1$ , and  $b_2$  for each plant.

Each of the hydro plants is assumed to draw water from a vertical-sided reservoir whose surface area is  $s$  in  $m^2$ . The reservoir's natural water inflow is denoted  $i(t)$  in  $m^3/s$ . A forecast of  $i(t)$  is assumed available over the optimization interval  $[0, T]$ . The continuity equation for reservoir 2, at the up-stream plant is thus

$$\begin{aligned} f_{h_2}(t) = & s_2 h_2(t) - s_2 h_2(0) - \int_0^t i_2(z) dz \\ & + \int_0^t q_2(z) dz = 0 \end{aligned} \quad (5.4)$$

As the intermediate reservoir, the inflow is composed of a natural component  $i_3(t)$  as well as that due to the controlled discharge  $q_2$  of the up-stream plant delayed by  $\tau_{23}$  hours. As a result, the continuity equation for reservoir three is expressed as

$$\begin{aligned} f_{h_3}(t) = & s_3 h_3(t) - s_3 h_3(0) - \int_0^t i_3(z) dz \\ & + \int_0^t [q_3(z) - q_2(z-\tau_{23})] dz = 0 \end{aligned} \quad (5.5)$$

In a similar way, the continuity equation for reservoir four is given by

$$\begin{aligned} f_{h_4}(t) = & s_4 h_4(t) - s_4 h_4(0) - \int_0^t i_4(z) dz \\ & + \int_0^t [q_4(z) - q_3(z-\tau_{34})] dz = 0 \end{aligned} \quad (5.6)$$

The water draw-down constraints over the optimization interval for the system are given by

$$\int_{b_i}^t q_i(z) dz - v_i = 0 \quad (5.7)$$

The volumes  $v_2, v_3, v_4$  are assumed available and correspond to hydraulic requirements such as navigational, irrigation or other constraints.

### 5.3 DIRECT OPTIMALITY CONDITIONS

The optimal operation strategy is described by a set of optimality conditions obtained by forming an augmented objective functional as indicated in Appendix D. This takes the form

$$J_A = \int_0^T I_A(t) dt \quad (5.8)$$

Optimality is attained by setting the derivatives of  $I_A$  with respect to control variables to equal zero. We choose initially to have both discharge,  $q_i$ , and head,  $h$ , to be control variables in addition to the active power generations.

Differentiating with respect to the thermal active power generation we obtain

$$\frac{\partial I_A}{\partial P_1} = \frac{\partial F}{\partial P_1} + \lambda(t) \left[ \frac{\partial P_L}{\partial P_1} - 1 \right] = 0 \quad (5.9)$$

This is the familiar all-thermal, equal incremental cost expression.

Differentiating with respect to the hydro active power generation we get

$$\frac{\partial I_A}{\partial P_i} = \lambda(t) \left[ \frac{\partial P_L}{\partial P_i} - 1 \right] - m_i(t) \frac{\partial q_i}{\partial P_i} = 0 \quad (5.10)$$

Taking the derivatives of  $I_A$  with respect to  $h_i$ , we obtain

$$\frac{\partial I_A}{\partial h_i} = s_i n_i(t) - m_i(t) \frac{\partial q_i}{\partial h_i} = 0 \quad (5.11)$$

Taking the derivatives with respect to each of the  $q_i$ 's we get

$$\frac{\partial I_A}{\partial q_2} = v_{o_2} + m_2(t) + [n_2(T) - n_2(t)] - N_3(t) = 0 \quad (5.12)$$

$$\frac{\partial I_A}{\partial q_3} = v_{o_3} + m_3(t) + [n_3(T) - n_3(t)] - N_4(t) = 0 \quad (5.13)$$

$$\frac{\partial I_A}{\partial q_4} = v_{o_4} + m_4(t) + [n_4(T) - n_4(t)] - 0 = 0 \quad (5.14)$$

Equations (5.9) through (5.14) together with (5.2) through (5.7) constitute the optimality conditions necessary for solving the problem.

#### 5.4 COORDINATION EQUATIONS

The direct optimality conditions obtained above can be reduced to a form that represents a direct extension of the well-known Kron-Ricard's equations by simply eliminating the functions  $m_i(t)$  and  $n_i(t)$ . The algebraic details of the process are given in Appendix E. The resulting equations consist of the following.

For the thermal plant we have from Equation (5.9),

$$L_1 \frac{\partial P_1}{\partial P_1} = \lambda(t) \quad (5.15)$$

For the up-stream plant

$$L_2 v_2(t) \frac{\partial q_2}{\partial P_2} = \lambda(t) \quad (5.16)$$

where

$$v_2(t) = v_2 \phi_2(t, o) + \int_o^t \phi_2(t, \sigma) u_2(\sigma) d\sigma \quad (5.17)$$

For the intermediate plant

$$L_3 v_3(t) \frac{\partial q_3}{\partial P_3} = \lambda(t) \quad (5.18)$$

where

$$v_3(t) = v_3 \phi_3(t, o) + \int_o^t \phi_3(t, \sigma) u_3(\sigma) d\sigma \quad (5.19)$$

For the down-stream plant

$$L_4 v_4(t) \frac{\partial q_4}{\partial P_4} = \lambda(t) \quad (5.20)$$

where

$$v_4(t) = v_4 \phi_4(t, o) \quad (5.21)$$

In the above

$$\phi_i(t, t_o) = \exp \left[ \int_{t_o}^t M_i(z) dz \right] \quad (5.22)$$

$$M_i(z) = \frac{1}{s_i} \frac{\partial q_i}{\partial h_i} \quad (5.23)$$

$$\begin{aligned}
 u_i(\sigma) &= m_{i+1}(t + \tau_{i(i+1)}) \\
 0 \leq t &\leq T - \tau_{i(i+1)} \\
 &= 0 \quad T - \tau_{34} < t \leq T
 \end{aligned} \tag{5.24}$$

$$m_4(t) = -v_4 \exp \left\{ \int_0^t M_4(\sigma) d\sigma \right\} \tag{5.25}$$

$$m_3(t) = -v_3 \phi_3(E, 0) - \int_0^t \phi_3(t, \sigma) u_3(\sigma) d\sigma \tag{5.26}$$

$$m_2(t) = -v_2 \phi_2(t, 0) - \int_0^t \phi_3(t, \sigma) u_2(\sigma) d\sigma \tag{5.27}$$

As is common in conventional theory, the loss penalty factors are given by

$$L_i = \frac{1}{1 - \frac{\partial p}{\partial p_i}} \tag{5.28}$$

### 5.5 WATER-WORTH

The functions  $v_i(t)$  in Equations (5.16), (5.18), and (5.20) represent the water-worth functions for the hydro plants considered. In the classical theory for economic operation for fixed head hydro plants these functions are constant and represent the conversion factors necessary to convert the incremental discharge into an equivalent incremental cost. When hydro plants are of the variable head type, the water-worth is no longer a constant for each plant (for the given allowable volume of water available). Instead, the water-worth increases

over the optimization interval as long as discharge exceeds the natural inflow into the reservoir. This concept has been discovered by Kron and Ricard and illustrated vividly by Glimm and Kirchmayer for plants that are hydraulically isolated.

The coordination equations derived here for systems with plants on the same stream provide us with the basis for obtaining water-worth functions  $v_i(t)$  defined by Equations (5.17), (5.19), and (5.21).

Inspection of each of the equations reveals that the water-worth is made of two components. The first is the coupling-free term  $v_{i,i}(t,o)$  while the second component pertains to coupling and represents a penalty for discharge arriving at a plant further down-stream. Note that for the down-stream plant only the coupling-free term exists:

## CHAPTER VI

### CONCLUSIONS AND FUTURE WORK

#### 6.1 CONCLUSIONS

An algorithm based on the Newton-Raphson iterative technique is developed for application to the problem of optimal economic operation of variable-head hydro electric power systems. When this algorithm is applied to several test systems, the results show that it is successful in obtaining the optimal hydro-thermal dispatching schedule.

The computer evaluation of the behavior of the variables under varying system constraint conditions is consistent with the predetermined analytical observations. In other words, the variables react as predicted to changes in the system.

The amount of computer storage and computational time required for the successful solution of the problem is significantly reduced by exploiting the sparsity of the Jacobian matrix. As is pointed out in Section 6.2, even further savings are realizable through this process.

The coordination equations for hydraulically coupled variable-head hydro-thermal electric power systems are developed for several configurations of systems.

#### 6.2 FUTURE WORK

As in any type of research work there is always one more detail that could be accomplished before the topic is brought to a close. The following items constitute the details which form possible starting points

for any continuing work on this topic.

It is shown in the preceeding chapters how exploitation of the sparsity of the Jacobian matrix results in reduced core space and computer time. As mentioned, it is possible to further exploit this sparsity and obtain more savings. This additional work should be directed towards performing the matrix manipulations on an elemental basis. This will require further partitioning of the submatrices detailed in Chapter two and taking advantage of the sparsity of the block diagonal matrix  $J_A$ .

Another path would be to work towards improving the methods used to obtain the initial estimates of the variables. Such an improvement will increase the overall efficiency of the program and will also reduce the computational time required for solution.

As another point, the optimality equations for hydraulically-coupled hydro plants in hydro-thermal systems should be implemented. As well, the program could be expanded to include pumped storage units.

As a last item, an indepth sensitivity analysis of the program should be performed to determine, if any, the limitations of the program that are not obvious through the testing that has been performed.

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APPENDIX A

APPENDIX AFIRST-ORDER PARTIAL DERIVATIVES

This appendix lists the first order partial derivatives which form the elements of the Jacobian matrix.

The partial derivatives of  $f_{s_i}(t_k)$  are obtained on the basis of Equation (3.34) as

$$\frac{\partial f_{s_i}(t_k)}{\partial p_{s_i}(t_k)} = 2\gamma_{s_i} + 2 \sum_{j=1}^{N_s} B_{i,j} \lambda(t_k) \quad (i=1 \rightarrow N_s) \quad (A.1)$$

$$\frac{\partial f_{s_i}(t_k)}{\partial h_i(t_k)} = 2 \sum_{j=1}^{N_h} B_{i,j} \lambda(t_k) \quad (i=1 \rightarrow N_s) \quad (A.2)$$

$$\frac{\partial f_{s_i}(t_k)}{\partial \lambda(t_k)} = c_i + 2 \sum_{j=1}^{N_g} B_{i,j} p_j(t) \quad (i=1 \rightarrow N_s) \quad (A.3)$$

$$\frac{\partial f_{s_i}(t_k)}{\partial h_i(t_k)} = 0 \quad (A.4)$$

$$\frac{\partial f_{s_i}(t_k)}{\partial v_i} = 0 \quad (A.5)$$

The partial derivatives of  $f_{h_i}(t_k)$  are obtained on the basis of Equation (3.35) as

$$\frac{\partial f_{h_i}(t_k)}{\partial p_{s_i}(t_k)} = 2 \sum_{j=1}^{N_s} B_{i,j} \lambda(t_k) \quad (i=1 \rightarrow N_h) \quad (A.6)$$

$$\begin{aligned} \frac{\partial f_{h_i}(t_k)}{\partial p_{h_i}(t_k)} &= [K v_i(t_k) \psi_i(h) [2\gamma_{h_i} + \frac{K t_k}{s_i} (a_{1,i} + 2a_{2,i}(t_k)) x \\ &\quad (\beta_{h_i} + 2\gamma_{h_i} p_{h_i}(t_k)^2] + 2 \sum_{j=1}^{N_h^*} B_{i,j} \lambda(t_k)] \quad (i=1 \rightarrow N_h) \end{aligned} \quad (A.7)$$

$$\frac{\partial f_{h_i}(t_k)}{\partial \lambda(t_k)} = C_i + 2 \sum_{j=1}^{N_g} B_{i,j} p_j(t) \quad (i=1 \rightarrow N_h) \quad (A.8)$$

$$\begin{aligned} \frac{\partial f_{h_i}(t_k)}{\partial h_i(t_k)} &= K v_i(t_k) (\beta_{h_i} + 2\lambda_{h_i} p_{h_i}(t_k) x \\ &\quad [a_{1,i} + 2a_{2,i}(h_i(t_k) + \frac{K t_k}{s_i} (\phi_i(p_h) \psi_i(h))]) \quad (i=1 \rightarrow N_h) \end{aligned} \quad (A.9)$$

$$\frac{\partial f_{h_i}(t_k)}{\partial v_{o_i}} = [\frac{v_i(t_k)}{v_{o_i}}] [K \psi_i(h)] x (\beta_{h_i} + 2\gamma_{h_i} p_{h_i}(t_k)) \quad (i=1 \rightarrow N_h) \quad (A.10)$$

The partial derivatives of  $f_{D_k}(t_k)$  are obtained on the basis of Equation (3.36) as

$$\frac{\partial f_{D_k}(t_k)}{\partial p_{s_i}(t_k)} = \sum_{i=1}^{N_s} C_i + 2 \sum_{j=1}^{N_g} B_{i,j} p_j(t) \quad (i=1 \rightarrow N_s) \quad (A.11)$$

$$\frac{\partial f_D(t_k)}{\partial p_{h_i}(t_k)} = \sum_{i=1}^{N_h} c_i + 2 \sum_{j=1}^{N_g} B_{ij} p_j(t) , \quad (i=1 \rightarrow N_h) \quad (A.12)$$

$$\frac{\partial f_D(t_k)}{\partial \lambda(t_k)} = 0 \quad (A.13)$$

$$\frac{\partial f_D(t_k)}{\partial h_i(t_k)} = 0 \quad (A.14)$$

$$\frac{\partial f_D(t_k)}{\partial v_i} = 0 \quad (A.15)$$

The partial derivatives of  $f_{i_1}(t_k)$  are obtained on the basis of Equation (3.37) as

$$\frac{\partial f_{i_1}(t_k)}{\partial p_{s_i}(t_k)} = 0 \quad (A.16)$$

$$\frac{\partial f_{i_1}(t_k)}{\partial p_{h_i}(t_k)} = -\frac{A}{s_i} K \psi_i(h) (B_{h_i} + 2\gamma_{h_i} p_{h_i}(t_k)) , \quad (i=1 \rightarrow N_h) \quad (A.17)$$

$$\frac{\partial f_{i_1}(t_k)}{\partial \lambda(t_k)} = 0 \quad (A.18)$$

$$\frac{\partial f_{i_k}(t_k)}{\partial h_{i_k}} = 1 - \frac{\Delta}{s_i} K \dot{y}_i(p) \times (a_{1_i} + a_{2_i} h_{i_k}) \quad (i=1 \rightarrow N_h) \quad (A.19)$$

$$\frac{\partial f_{i_k}(t_k)}{\partial h_{i_k+1}} = -1 \quad (A.20)$$

$$\frac{\partial f_{i_k}(t_k)}{\partial v_i} = 0 \quad (A.21)$$

The partial derivatives of  $f_{b_i}$  are obtained on the basis of Equation (3.38) as

$$\frac{\partial f_{b_i}}{\partial p_{s_i}} = 0 \quad (A.22)$$

$$\frac{\partial f_{b_i}}{\partial p_{h_i}} = K \dot{y}_i(h) \times (B_{h_i} + 2\lambda_{h_i} p_i(t_k)) \quad (i=1 \rightarrow N_h) \quad (A.23)$$

$$\frac{\partial f_{b_i}}{\partial \lambda(t_k)} = 0 \quad (A.24)$$

$$\frac{\partial f_{b_i}}{\partial h_i} = K \dot{y}_i(p) \times (a_{1_i} + 2a_{2_i} h_i(t)) \quad (i=1 \rightarrow N_h) \quad (A.25)$$

$$\frac{\partial f_{b_i}}{\partial v_{\sigma_i}} = 0$$

(A.26)

APPENDIX B

APPENDIX B  
THE COMPUTER PROGRAM

B-1 INTRODUCTION

The computer program presented herein solves for the optimum hydro-thermal dispatch schedule. The algorithm, described in the text, revolves around the Newton-Raphson technique where the solution is obtained by iterative techniques.

The program logic is detailed in the flowchart presented in Figure B-1. The program itself is written in fortran language and has an approximate storage requirement of 1500 program lines.

The sections which follow detail the variables used in the calculations, a full listing of the computer program, a description of the matrix inversion routine, and some basic data on the computer.

B-2 PROGRAM VARIABLE DEFINITION LIST

The following list contains the definitions and dimensions of the terms used in the program. They are cataloged according to their function within the program.

Control Terms

N            number of discrete time intervals into which the study period is divided

NUMS        number of thermal plants in the system

NUMH        number of hydro plants in the system

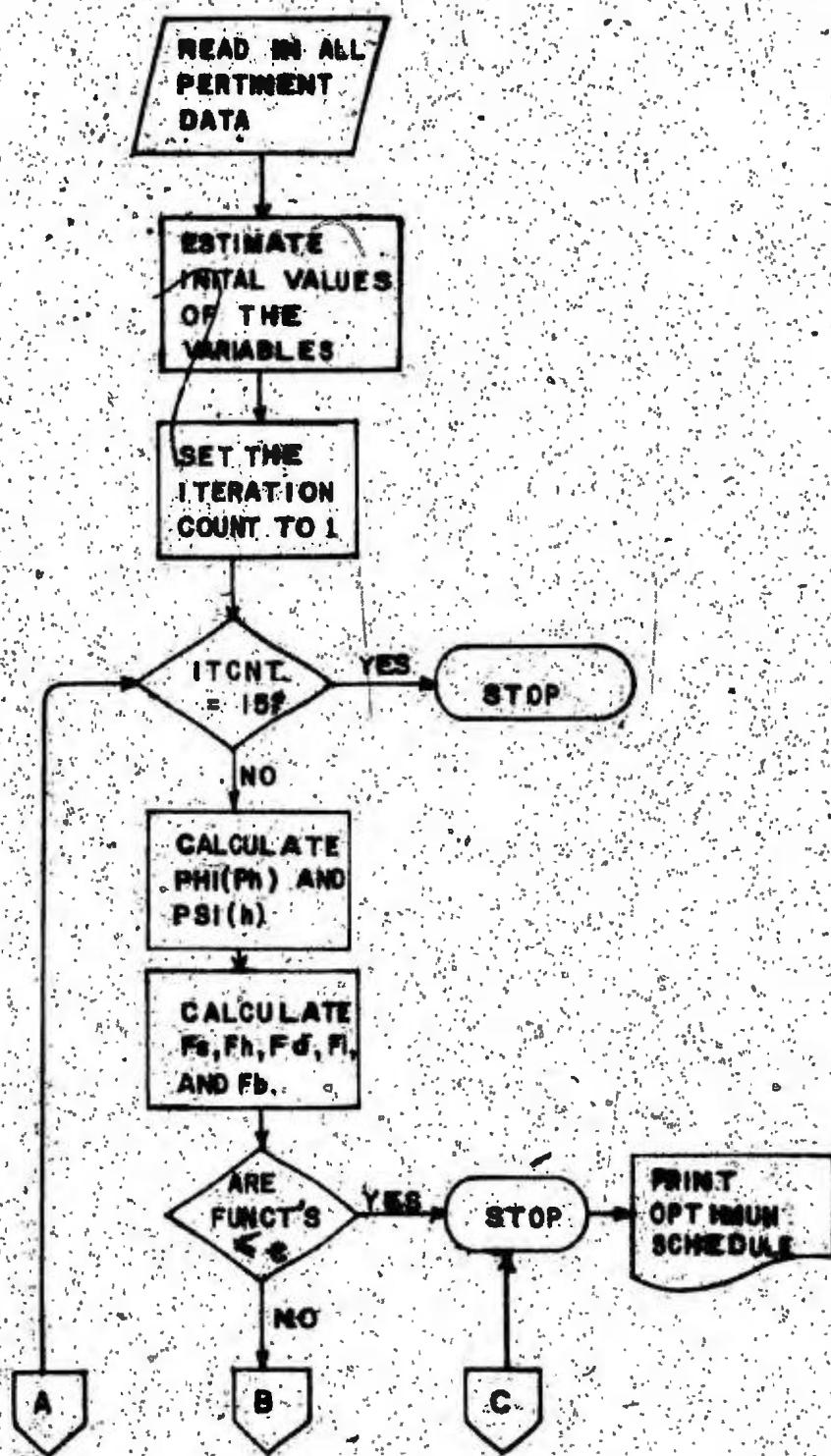


FIGURE B-1. Algorithm Flowchart.

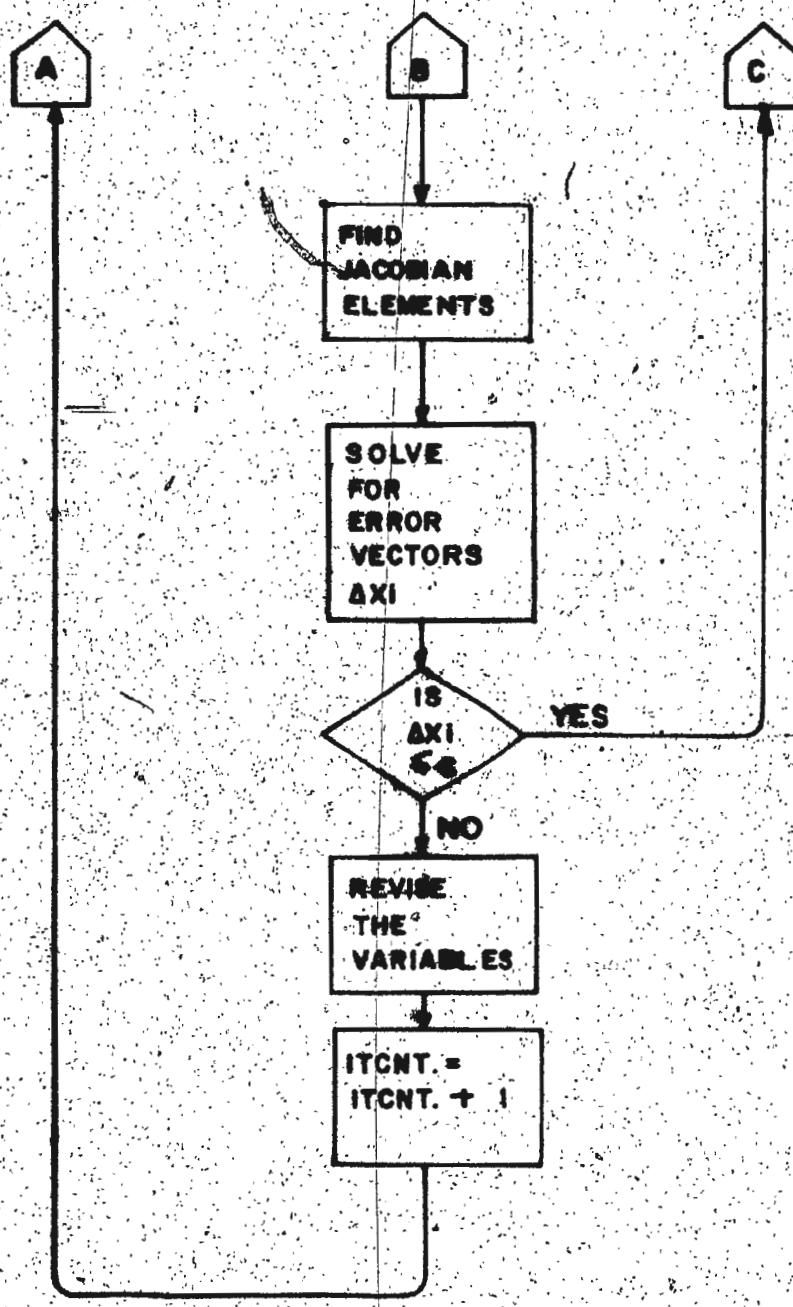


FIGURE B-1 (Cont'd). Algorithm Flowchart.

ITCNT : number of iterations completed by the program.

Variable Dimension Values

LAP	IR
LAPO	IC
LYO	IIR
LYT	IIC
NH	IRR
NHA	IIC
NS	NSA
MA	IA

Optimization Variables

Ps (NUMS,N) generated thermal power for plant NUMS at time period N.

Ph (NUMH,N) generated hydro power for plant NUMH at time period N.

h (NUMH,N) the reservoir level of plant NUMH at time N.

LAM(N) the incremental cost of power  $\lambda(t)$  for time instant N.

NEW (NUMH) the water worth coefficient  $v_o$  for the reservoir of plant NUMH.

Quadratic Model Coefficients

ALPHAH (NUMH)  $\alpha_h$  for plant NUMH.

BETAH (NUMH)  $\beta_h$  for plant NUMH.

GAMAH (NUMH)  $\gamma_h$  for plant NUMH.

BETAS (NUMS)  $\beta_s$  for plant NUMS.

GAMAS (NUMS)	$\lambda_s$ for plant NUMS.
AZ (NUMH)	$a_0$ for plant NUMH.
AO (NUMH)	$a_1$ for plant NUMH.
AT (NUMH)	$a_2$ for plant NUMH.

#### Transmission Loss Coefficients

BSS	
BSH	
BHS	These transmission losses are for interconnections
BHH	and other modes. S refers to thermal and H refers
CH	to hydro.
CS	
KLO	

#### System Constants and Data

PD(N)	Power demand at time N.
X(NUMH)	System's constant factor for plant NUMH.
DELT(I)	Length of time subintervals I.

#### Partial Derivatives (see Appendix A)

DFBH (NUMH,N)	$\frac{\partial [F_b]}{\partial P_h(t)}$
DFBD (NUMH,N)	$\frac{\partial [F_b]}{\partial h(t)}$
DFIH (NUMH,N)	$\frac{\partial [F_i]}{\partial P_h(t)}$
DFID (NUMH,N)	$\frac{\partial [F_i]}{\partial h(t)}$

DFIDP (NUMH, N)

$$\frac{\partial [F_1]}{\partial h(t+1)}$$

DFDS (NUMS, N)

$$\frac{\partial [F_d]}{\partial P_s(t)}$$

DFDH (NUMH, N)

$$\frac{\partial [F_d]}{\partial P_h(t)}$$

DFHS (NUMH, N)

$$\frac{\partial [F_h]}{\partial P_s(t)}$$

DFHH (NUMH, N)

$$\frac{\partial [F_h]}{\partial P_h(t)}$$

DFHL (NUMH, N)

$$\frac{\partial [F_h]}{\partial \lambda(t)}$$

DFHD (NUMH, N)

$$\frac{\partial [F_h]}{\partial P(t)}$$

DFHN (NUMH, N)

$$\frac{\partial [F_h]}{\partial v_o}$$

DFSS (NUMS, N)

$$\frac{\partial [F_s]}{\partial P_s(t)}$$

DFSH (NUMH, N)

$$\frac{\partial [F_s]}{\partial \lambda(t)}$$

Jacobian Submatrices

AP(LAP, LAP)

Submatrices of JA.

APO(LAPO, LAPO)

Inverse of matrix JA.

AINV(LYO, LY0)

BP(LY0, LYT)

Submatrices of J. See text for further reference.

CP(LYT, LY0)

DP(LYT, LYT)

Reservoir Data

FLO(NUMH,N) Natural inflow to the reservoir of plant NUMH.

EPCE(NUMH,N)  $\psi(t)$  - quadratic model of reservoir variation.

S(NUMH) Reservoir surface area for plant NUMH.

B(NUMH) Available amount of water from reservoir of plant NUMH.

System Output

LOSS(N) Transmission losses.

NU(t) Water worth coefficient  $v(t)$  determined as

$$v_o(t) \exp[Mt] (NEW(NUMH) * P(NUMH,N))$$

P(NUMH,N) Time variable portion of NU(t).

Q(NUMH,N) Hydro plant discharge.

FCST(NUMS) Thermal plant daily fuel cost.

Function Variables

PHI(NUMH,N)  $\phi(P_h)$  - quadratic model for the hydro plant performance.

FS(NUMS,N)

FH(NUMH,N)

FI(NUMH,N)

FD(N)

FB(NUMH)

YO(LYO)

YT(LYO)

XO(LYO)

XT(LYO)

System equations. See text for further discussion and explanation.

Vectors containing values of the functions for iteration i.

Vectors containing the error vectors generated by the program.

Variables, Vectors and Matrices used to Determine the MaximumRelative Error

F0 (LYO)	DECX1 (LYO)
FOO (LYO)	DELX2 (LYT)
FT (LYT)	TESTX
FTT (LYT)	

Vectors and Matrices used to Determine the Error Vectors

R1 (LYT, LYO)	R5 (LYT)
R2 (LYT, LYT)	R6 (LYO)
R3 (LYT, LYT)	R7 (LYO)
R4 (LYT)	

General Purpose (dummy) Variables

F1	F11	TF
F6	F12	CBA
F2	AC	CC
F7	SUM	FA
C	FF1	AA
Z	FF2	FB
X1	FF3	AB
X2	WW	FC
X3	WX	EE
F3	WY	FF
F8	WZ	P
F4	FP4	TEMP1
F9	FF6	ABC

F5            E            FAC  
F10          BP

B-3 THE COMPUTER PROGRAM LISTING

The following pages present the listing of the computer program in full. The dimensions shown are for the first test system of one thermal and one hydro plant for a 24-hour time period.



OF EPC(E,T). THIS QUADRATIC EQUATION MODELS THE RESEVIOR VARIATIONS.

SUBROUTINE T IS RESPONSIBLE FOR CALCULATING THE VALUES OF THE SYSTEM EQUATIONS THAT ARE TO BE MINIMIZED. ONCE THESE VALUES ARE DETERMINED, THE SUBROUTINE THEN POSITIONS THEM IN TWO ARRAYS Y0 AND YT. THESE AND THEIR STRUCTURES ARE FURTHER DESCRIBED IN THE TEXT.

SUBROUTINE JAC CALCULATES THE  
THE VALUES OF THE PARTIAL  
DERIVATIVES OF THE SYSTEM  
EQUATIONS AND POSITIONS THEM  
WITHIN THE SUBMATRICES AS  
DESCRIBED IN THE TEXT. IT  
ALSO DETERMINES THE INVERSE  
OF THE BLOCK DIAGONAL MATRIX  
NAMED ' JA '.

SUBROUTINE GUESS IS THE SUBROUTINE WHICH GENERATES THE INITIAL ESTIMATES THAT ARE USED IN THE SOLUTION OF THE OPTIMAL STRATEGY.

SUBROUTINES MINVRD AND SUBMXD ARE THE COMMERCIALY OBTAINED SUBROUTINES WHICH TOGETHER FIND THE INVERSE OF THE MATRICES.

SUBROUTINES X1MULT AND X2MULT ARE SUBROUTINES DEVELOPED TO HANDLE THE MATRIX MULTIPLICATION FOUND IN THE MAIN PROGRAM.

```

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION PS(1,24),PH(1,24),LAM(24),PD(24)
DIMENSION H(1,24),FLO(1,24),DELT(24)
DIMENSION PHI(1,24),EPCE(1,24),FB(1)
DIMENSION FS(1,24),FH(1,24),FI(1,24),FD(24)
DIMENSION IR(2),IC(2),IIR(23),IIC(23)
DIMENSION IRR(25),ICC(25),BHS(1,1)

```

```

DIMENSION LOSS(24),NUMBEH(1),NUMBES(1)
DIMENSION ALPHAH(1),BETAH(1),GAMAH(1)
DIMENSION AZ(1),AO(1),AT(1),CH(1)
DIMENSION BETAS(1),GAMAS(1),CS(1)
DIMENSION BSS(1,1),BHH(1,1),BSH(1,1)
DIMENSION NEW(1),S(1),K(1),FCST(1)
DIMENSION Q(1,24),P(1,24),NU(1,24)
DIMENSION B(1),DELX1(71),DELX2(25)
DIMENSION DFBH(1,24),DFBD(1,24),
1 DFTH(1,24),DFID(1,24),DFIDP(1,24),
1 DFDS(1,24),DFDH(1,24),DFHS(1,24),
1 DFHH(1,24),DFHL(1,24),DFHD(1,24),
4 DFHN(1,24),DFSS(1,24),DFSH(1,24),
5 DFSL(1,24)
DIMENSION AP(2,2),APO(23,23)
DIMENSION YO(71),YT(25)
DIMENSION AINV(71,71),BP(71,25)
DIMENSION CP(25,71),DP(25,25)
DIMENSION R1(25,71),R2(25,25)
DIMENSION R3(25,25),R4(25)
DIMENSION R5(25),R6(71),R7(71)
DIMENSION X0(71),XT(71)
DIMENSION FOO(71),FTT(25)
DIMENSION FO(71),FT(25)
REAL*8 NU,KLO,LAM,K,NEW,LOSS
OPEN(UNIT=10,NAME='STORE.DAT',ACCESS='SEQUENTIAL',
1 TYPE='OLD',DISPOSE='SAVE',FORM='FORMATTED')
OPEN(UNIT=6,NAME='OUT.DAT',TYPE='NEW',DISPOSE='SAVE'

```

C-----

C

C READ IN THE CONTROL TERMS

C-----

C

C N----NUMBER OF DISCRETE INTERVALS

C NUMH--NUMBER OF HYDRO PLANTS

C NUMS--NUMBER OF THERMAL PLANTS

C-----

C

123 READ(10,123)N,NUMH,NUMS

FORMAT(3I10)

C-----

C

C READ IN THE VARIABLES

C-----

C

C THESE VARIABLES ARE THE COEFFICIENTS  
FOR THE QUADRATIC MODELS OF: THE FUEL  
COSTS (BETAS,GAMAS),THE HYDRO PLANTSC PERFORMANCE CHARACTERISTICS (ALPHAH,  
C BETAH,GAMAH);AND THE RESEVOIR VARIATIONS

C AZ,AO,AT). ALSO INCLUDED ARE THE  
 C SYSTEM'S PROPORTIONALITY CONSTANT (K),  
 C THE RESEVOIR AREA (S) AND THE AVAILABLE  
 C AMOUNT OF WATER (B). THE TRANSMISSION  
 C LOSS COEFFICIENTS (BSS,BSH,BHS,BHH,CS,  
 C CH,KLO) ARE READ IN AS WELL.  
 C-----

CALL RONE(ALPHAH,BETAH,GAMAH,AZ,AO,AT,  
 1 BETAS,GAMAS,CS,CH,BSS,BSH,BHH,N,KLO,K,S,NUMH,  
 2 NUMS,B,BHS)

C-----  
 C WRITE OUT THE VARIABLES  
 C-----

```

      WRITE(6,1000)
1000  FORMAT(1H1,///)
      WRITE(6,1001)NUMS
1001  FORMAT(/,10X,'NUMS='I10)
      WRITE(6,1002)NUMH
1002  FORMAT(/,10X,'NUMH='I10)
      DO 6001 NH=1,NUMH
      WRITE(6,1003)ALPHAH(NH)
1003  FORMAT(/,10X,'ALPHAH=',E30.20)
      WRITE(6,1004)BETAH(NH)
1004  FORMAT(/,10X,'BETAH=',E30.20)
      WRITE(6,1005)GAMAH(NH)
1005  FORMAT(/,10X,'GAMAH=',E30.20)
      WRITE(6,1006)AZ(NH)
1006  FORMAT(/,10X,'AZ(A-ZERO)=',E30.20)
      WRITE(6,1007)AO(NH)
1007  FORMAT(/,10X,'AO(A-ONE)=',E30.20)
      WRITE(6,1008)AT(NH)
1008  FORMAT(/,10X,'AT(NH)=',E30.20)
      WRITE(6,1009)CH(NH)
1009  FORMAT(/,10X,'CH=',E30.20)
      WRITE(6,1010)BHH(NH,NH)
1010  FORMAT(/,10X,'BHH=',E30.20)
      WRITE(6,1021)B(NH)
1021  FORMAT(/,10X,'B=',E30.20)
      WRITE(6,1020)S(NH)
1020  FORMAT(/,10X,'S=',E30.20)
      6001 CONTINUE
      DO 6002 NS=1,NUMS
      WRITE(6,1011)BETAS(NS)
1011  FORMAT(/,10X,'BETAS=',E30.20)
      WRITE(6,1012)GAMAS(NS)
1012  FORMAT(/,10X,'GAMAS=',E30.20)
      WRITE(6,1013)CS(NS)
1013  FORMAT(/,10X,'CS=',E30.20)
```

C  
C ESTIMATE THE INITIAL VALUES.  
C-----  
C  
C THESE INITIAL VALUES ARE THE INITIAL  
C GUESSES USED IN THE ITERITIVE  
C PROCESS OF THE NEWTON-RAPHSON  
C METHOD.  
C  
C PS---THERMAL POWER  
C PH---HYDRO POWER  
C LAM---THE INCREMENTAL COST OF  
C POWER COEFFICIENT.  
C DELT---THE LENGTH OF THE DISCRETE  
C INTERVAL  
C PHI---THE QUADRATIC MODEL FOR  
C THE HYDRO PLANT PERFORMANCE  
C CHARACTERIC  
C EPCE--THE QUADRATIC MODEL FOR  
C RESEVOIR VARIATION.  
C Q----THE BIQUADRATIC MODEL FOR  
C THE RESEVOIR DISCHARGE.  
C-----  
C CALL GUESS(PS,PH,LAM,H,PD,DELT,AZ,AO,AT,  
1 B,S,K,N,ALPHAH,BETAH,GAMAH,KLO,CS,CH,BSS,  
2 BSH,BHH,BETAS,GAMAS,NEW,NUMS,NUMH,PHI,EPCE,  
3 Q,BHS)  
C-----  
C  
C BEGIN THE ITERATION COUNT  
C AND CHECK TO SEE THAT THE  
C THE NUMBER OF ITERATIONS  
C DOES NOT EXCEED THE PRESET  
C LIMIT.  
C-----  
C ITCNT=1  
902 CONTINUE  
IF(ITCNT.EQ.15)GOTO 4080  
C-----  
C  
C REVISE FO AND FT  
C-----  
C  
C THESE VECTORS ARE USED IN  
C THE CALCULATION OF THE

```

C      MAXIMUM RELATIVE ERROR
C
C-----
C      L=1
C      DO 9077 I=1,N
C      DO 9078 NS=1,NUMS
C      FO(L)=PS(NS,I)
9078  CONTINUE
C      FO(L)=LAM(I)
9077  L=L+1
C      DO 9079 I=2,N
C      DO 9079 NH=1,NUMH
C      FO(L)=H(NH,I)
9079  L=L+1
C
C      L=1
C      DO 9080 I=1,N
C      DO 9080 NH=1,NUMH
C      FT(L)=PH(NH,I)
9080  L=L+1
C      DO 9081 NH=1,NUMH
C      FT(L)=NEW(NH)
9081  L=L+1
C
C-----CALCULATE THE VALUES
C      OF PHI(T) AND EPCE(T)
C-----THESE ARE THE QUADRATIC MODELS
C      OF THE HYDRO PLANT'S PERFORMANCE
C      AND THE RESEVOIR VARIATION
C      RESPECTIVELY.
C-----CALL THREE(ALPHAH,BETAH,GAMAH,PH,PHI,N,NUMH)
C      CALL FOUR(AZ,AO,AT,H,EPCE,N,NUMH)
C
C-----SET UP THE VECTORS YO AND YT
C      AND DETERMINE IF THEY
C      ARE WITHIN THE ALLOWABLE
C      LIMIT.
C
C-----THESE VECTORS ARE DESCRIBED IN
C      IN THE TEXT. BRIEFLY, THEY ARE
C      THE FUNCTIONS WHICH ARE TO BE
C      MINIMIZED. ONCE THE VALUE OF
C      THE FUNCTIONS ARE CALCULATED
C      THEY ARE THEN COMPARED TO A

```

C MINIMUM CITERION VALUE WHICH  
C IS THE TEST FOR CONVERGENCE.  
C

C-----  
C CALL T(FS,FH,FD,EI,FB,PS,PH,LAM,H,EPCE,  
1 PHI,K,DELT,PD,KLO,YO,YT,BETAS,GAMAS,GAMAH,  
2 BETAH,CH,CS,BSS,BHS,BSH,BHH,B,S,FLO,N,NEW,  
3 AO,AT,NUMS,NUMH,P,Q,LYO,LYT)

C-----  
C DO 4000 I=1,LYO  
1 IF(DABS(YO(I)).GT.1.E-5) GOTO 4020  
2 IF(I.EQ.LYO) GOTO 4010  
4000 CONTINUE  
4010 DO 4011 I=1,LYT  
1 IF(DABS(YT(I)).GT.1.E-5) GOTO 4020  
2 IF(I.EQ.LYT) GOTO 4030  
4011 CONTINUE

C-----  
C DETERMINE THE ELEMENTS OF THE  
C JACOBIAN MATRIX AND SET UP THE  
C PARTITIONED MATRICIES.

C-----  
C 4020 CONTINUE  
C CALL JAC(ALPHAH,BETAH,GAMAH,GAMAS,EPCE,LAM,  
1 DELT,PHI,BSS,BHS,BSH,CS,CH,N,K,S,H,PH,  
2 PS,PD,KLO,AO,AT,NEW,NUMS,NUMH,P,Q,IR,IIC,IIR,IIC,  
3 NU,LYO,LYT,DFSS,DFSH,DFSL,DFHS,DFHH,DFHL,  
4 DFHD,DFHN,DFDS,DFDH,DFIH,DFID,DFBH,DFBD,  
5 DFIDP,LAP,LAPO,AINV,BP,CP,DP,AP,APO)

C-----  
C CALCULATE THE DIFFERENCE ELEMENT  
C VECTORS AND DETERMINE IF THEY  
C ARE WITHIN THE TOLERANCE LIMIT.

C-----  
C CALL X1MULT(CP,AINV,R1,LYT,LYO,LYO)

C-----  
C CALL X1MULT(R1,BP,R2,LYT,LYO,LYT)

C-----  
C DO 9000 I=1,LYT  
C DO 9000 J=1,LYT  
C R3(I,J)=DP(I,J)-R2(I,J)  
9000 CONTINUE

```

MA=LYT
IA=LYT
CALL MINVRD(R3,IA,MA,DET,IER,IRR,ICC)

C
C   CALL X2MULT(R1,Y0,R4,LYT,LY0)
C
C   DO 9001 I=1,LYT
C       R5(I)=YT(I)-R4(I)
9001   CONTINUE
C
C   CALL X2MULT(R3,R5,XT,LYT,LYT)
C
C   CALL X2MULT(BF,XT,R6,LY0,LYT)
C
C   DO 9002 I=1,LY0
C       R7(I)=Y0(I)-R6(I)
9002   CONTINUE
C
C   CALL X2MULT(AINV,R7,X0,LY0,LY0)
C
C-----
```

REVISE THE VARIABLES

```

C-----
```

---

```

L=1
DO 3000 I=1,N
DO 3001 NS=1,NUMS
PS(NS,I)=PS(NS,I)-X0(L)
3001   L=L+1
LAM(I)=LAM(I)-X0(L)
3000   L=L+1
C
C   DO 3002 I=2,N
DO 3002 NH=1,NUMH
H(NH,I)=H(NH,I)-X0(L)
3002   L=L+1
C
C   L=1
DO 3003 I=1,N
DO 3003 NH=1,NUMH
PH(NH,I)=PH(NH,I)-XT(L)
3003   L=L+1

```

```

C
C
      DO 3004 NH=1,NUMH
      NEW(NH)=NEW(NH)-XT(L)
      L=L+1
      CONTINUE
C-----
C
      SET UP THE NEW
      VECTORS FOO AND FTT
C-----
C
      THESE VECTORS ARE USED IN
      THE CALCULATION OF THE
      MAXIMUM RELATIVE ERROR
C-----
      L=1
      DO 9070 I=1,N
      DO 9071 NS=1,NUMS
      FOO(L)=PS(NS,I)
      9071 L=L+1
      FOO(L)=LAM(I)
      9070 L=L+1
      DO 9072 I=2,N
      DO 9072 NH=1,NUMH
      FOO(L)=H(NH,I)
      9072 L=L+1
      C
      C
      L=1
      DO 9073 I=1,N
      DO 9073 NH=1,NUMH
      FTT(L)=PH(NH,I)
      9073 L=L+1
      DO 9074 NH=1,NUMH
      FTT(L)=NEW(NH)
      9074 L=L+1
      C-----
      C
      CALCULATE THE RELATIVE
      ERROR AND DETERMINE IF
      IT IS WITHIN THE
      ALLOWABLE LIMITS.
C-----
      DO 9015 I=1,LYO
      DELX1(I)=(FO(I)-FOO(I))/FO(I)
      9015 CONTINUE
      DO 9016 I=1,LYT
      DELX2(I)=(FT(I)-FTT(I))/FT(I)
      9016 CONTINUE
      C
      C

```

```

TESTX=DABS(DELX1(1))
DO 5105 NN=2,LYO
IF(DABS(DELX1(NN)).GT.TESTX) TESTX=
1 DABS(DELX1(NN))
5105 CONTINUE
DO 5106 NN=1,LYT
IF(DABS(DELX2(NN)).GT.TESTX) TESTX=
1 DABS(DELX2(NN))
5106 CONTINUE
WRITE(6,5107)ITCNT,TESTX
5107 FORMAT(/,10X,'THE MAXIMUM RELATIVE ERROR FOR'
1 ,/,10X,'ITERATION NUMBER',I3,/,,10X,' IS ',
2 E20.10,/)

C
C
DO 9010 I=1,LYO
IF(DABS(DELX1(I)).GT.1.E-5) GOTO 4070
IF (I.EQ.LYO)GOTO 9011
9010 CONTINUE
9011 DO 9012 I=1,LYT
IF(DABS(DELX2(I)).GT.1.E-5) GOTO 4070
IF(I.EQ.LYT) GOTO 4030
9012 CONTINUE
C
C-----REVISE THE ITERATION COUNT
C
C-----4070 CONTINUE
ITCNT=ITCNT+1
GOTO 902
C-----CALCULATE THE THERMAL
C-----FUEL COSTS AND THE SYSTEM
C-----LOSSES.
C-----4030 CONTINUE
C
C
FCST(NS)=0.
DO 4435 I=1,N
DO 4435 NS=1,NUMS
FCST(NS)=FCST(NS)+(1+BETAS(NS)*PS(NS,I)+  

1 (GAMAS(NS)*PS(NS,I)**2.))
4435 CONTINUE
C
C
DO 4445 I=1,N

```



C  
C

```
      WRITE(6,5056)
5056  FORMAT(30X,'P A G E  1',////////////)
      WRITE(6,5051)
      WRITE(6,5057)
      FORMAT(25X,'TIME',1X,<NUMS>(3X,'THERMAL'))
5057  WRITE(6,5058)
      FORMAT(24X,'PERIOD',<NUMS>(4X,'PLANT'))
5058  WRITE(6,5059)(NUMBES(NS),NS=1,NUMS)
5059  FORMAT(30X,<NUMS>(5X,'NO:',I2))
      WRITE(6,5060)
      FORMAT(25X,'(HR)',1X,<NUMS>(5X,'MW',3X))
5060  WRITE(6,5051)
      DO 5061 I=1,N
5061  WRITE(6,5062)I,(PS(NS,I),NS=1,NUMS)
      FORMAT(26X,I2,2X,<NUMS>(3X,F7.2))
      CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)
C
      WRITE(6,5063)
5063  FORMAT(30X,'P A G E  2',////////////)
      WRITE(6,5051)
      WRITE(6,5064)
      FORMAT(25X,'TIME',1X,<NUMH>(4X,'HYDRO',1X))
5064  WRITE(6,5065)
      FORMAT(24X,'PERIOD',<NUMH>(4X,'PLANT'))
5065  WRITE(6,5066)(NUMBEH(NH),NH=1,NUMH)
5066  FORMAT(30X,<NUMH>(5X,'NO:',I2))
      WRITE(6,5067)
      FORMAT(25X,'(HR)',1X,<NUMH>(5X,'MW',3X))
5067  WRITE(6,5051)
      DO 5068 I=1,N
5068  WRITE(6,5069)I,(PH(NH,I),NH=1,NUMH)
      FORMAT(26X,I2,2X,<NUMH>(3X,F7.2))
      CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)
C
      WRITE(6,5070)
5070  FORMAT(30X,'P A G E  3',////////////)
      WRITE(6,5051)
      WRITE(6,5071)
      FORMAT(25X,'TIME',1X,<NUMH>(3X,'NET',2X))
5071  WRITE(6,5072)
      FORMAT(30X,<NUMH>(3X,'HEAD',1X))
5072  WRITE(6,5073)
      FORMAT(24X,'PERIOD',<NUMH>(3X,'PLANT'))
```

```
      WRITE(6,5074)(NUMBEH(NH),NH=1,NUMH)
5074    FORMAT(30X,<NUMH>(3X,'NO:',I2))
      WRITE(6,5075)
5075    FORMAT(26X,'HR',2X,<NUMH>(3X,'FT',3X))
      WRITE(6,5051)
      DO 5076 I=1,N
      WRITE(6,5077)I,(H(NH,I),NH=1,NUMH)
5077    FORMAT(26X,I2,2X,<NUMH>(2X,F6.2))
      CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)

C
      WRITE(6,5078)
5078    FORMAT(30X,'P A G E 4',/|||||||||||||)
      WRITE(6,5051)
      WRITE(6,5079)
5079    FORMAT(25X,'TIME',5X,'POWER',5X,'POWER',5X,'LAMBDA')
      WRITE(6,5080)
5080    FORMAT(24X,'PERIOD',4X,'DEMAND',4X,'LOSSES')
      WRITE(6,5081)
5081    FORMAT(26X,'HR',8X,'MW',8X,'MW',7X,'$/MW')
      WRITE(6,5051)
      DO 5082 I=1,N
      WRITE(6,5083)I,PD(I),LOSS(I),LAM(I)
5083    FORMAT(26X,I2,5X,F7.2,3X,F7.2,4X,F6.3)
      CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)

C
      WRITE(6,5084)
5084    FORMAT(30X,'P A G E 5',/|||||||||||||)
      WRITE(6,5051)
      WRITE(6,5085)
5085    FORMAT(25X,'TIME'1X,<NUMH>(4X,'NU',3X))
      WRITE(6,5086)
5086    FORMAT(24X,'PERIOD',<NUMH>(3X,'PLANT',1X))
      WRITE(6,5087)(NUMBEH(NH),NH=1,NUMH)
      FORMAT(30X,<NUMH>(3X,'NO:',I2,1X))
      WRITE(6,5088)
5088    FORMAT(26X,'HR',2X,<NUMH>(4X,'$/CF',1X))
      WRITE(6,5051)
      DO 5089 I=1,N
      WRITE(6,5090)I,(NU(NH,I),NH=1,NUMH)
5090    FORMAT(26X,I2,2X,<NUMH>(2X,F7.5))
      CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)

C
      WRITE(6,5092)
5092    FORMAT(30X,'P A G E 6',/|||||||||||||)
```

```

      WRITE(6,5051)
      WRITE(6,5093)
5093   FORMAT(25X,'TIME',1X,<NUMH>(3X,'RESEVIOR'))
      WRITE(6,5094)
5094   FORMAT(26X,'HR',2X,<NUMH>(4X,'INFLOW',1X))
      WRITE(6,5095)
5095   FORMAT(30X,<NUMH>(5X,'PLANT',1X))
      WRITE(6,5096)(NUMBEH(NH),NH=1,NUMH)
5096   FORMAT(30X,<NUMH>(5X,'NO:',I2,1X))
      WRITE(6,5097)
5097   FORMAT(26X,'HR',2X,<NUMH>(5X,'CFS',3X))
      WRITE(6,5051)
      DO 5098 I=1,N
      WRITE(6,5099)I,(FLO(NH,I),NH=1,NUMH)
5099   FORMAT(26X,I2,2X,<NUMH>(3X,F8.1))
5098   CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)

C
      WRITE(6,5100)
5100   FORMAT(30X,'P A G E 7',/////////////)
      WRITE(6,5051)
      WRITE(6,5101)
5101   FORMAT(24X,'THE VALUE OF THE COST FUNCTIONS FOR
1    ,/20X,'THE THERMAL GENERATING PLANTS IN ($/DAY)
2    'ARE',//)
      WRITE(6,5051)
      DO 5103 NS=1,NUMS
      WRITE(6,5102)NUMBES(NS),FCST(NS)
5102   FORMAT(/,24X,'PLANT NO:',I3,5X,'FUEL COST '
     F8.2)
5103   CONTINUE
      WRITE(6,5104)
5104   FORMAT(/)
      WRITE(6,5051)
      WRITE(6,5050)

C#####%
C#####%
C#####% END OF PRINT #####
C#####% SECTION #####
C#####%
C#####%
C#####% GOTO 9999
9999   .CONTINUE
      CLOSE(UNIT=10)
      CLOSE(UNIT=6)
      STOP
      END
C
C#####

```

```

C               S U B R O U T I N E
C               R O N E
C
C               SUBROUTINE RONE(ALPHAH,BETAH,GAMAH,AZ,AO,AT,
1             BETAS,GAMAS,CS,CH,BSS,BSH,BHH,N,KLO,K,S,NUMH,
2             NUMS,B,BHS)
C               IMPLICIT REAL*8 (A-H,O-Z)
C               DIMENSION K(NUMH),ALPHAH(NUMH)
C               DIMENSION BETAH(NUMH),GAMAH(NUMH)
C               DIMENSION AZ(NUMH),AO(NUMH),AT(NUMH)
C               DIMENSION BH(BH(NUMH,NUMH),S(NUMH),B(NUMH))
C               DIMENSION BETAS(NUMS),GAMAS(NUMS),BHS(NUMH,NUMS)
C               DIMENSION CS(NUMS),BSS(NUMS,NUMS),BSH(NUMS,NUMH)
C               DIMENSION CH(CH(NUMH)
C               REAL*8 KLO,K
C
C               DO 100 NH=1,NUMH
C               READ(10,111)ALPHAH(NH),BETAH(NH),GAMAH(NH)
100             CONTINUE
C
C               DO 200 NH=1,NUMH
C               READ(10,111)AZ(NH),AO(NH),AT(NH)
111             FORMAT(3E10.5)
200             CONTINUE
C
C               DO 201 NH=1,NUMH
C               READ(10,1)CH(NH),S(NH)
1             FORMAT(2E10.5)
201             CONTINUE
C
C               DO 202 NH=1,NUMH
C               READ(10,2)B(NH),K(NH)
2             FORMAT(2E10.5)
202             CONTINUE
C
C               DO 101 NS=1,NUMS
C               READ(10,3)BETAS(NS),GAMAS(NS),CS(NS)
3             FORMAT(3E10.5)
101             CONTINUE
C
C               READ(10,5)KLO
5             FORMAT(E10.5)
C
C               DO 102 NSA=1,NUMS
C               DO 102 NSB=1,NUMS
C               READ(10,6)BSS(NSA,NSB)
6             FORMAT(E10.5)
102             CONTINUE
C

```

```
DO 104 NS=1,NUMS
DO 104 NH=1,NUMH
READ(10,8)BSH(NS,NH)
FORMAT(E10.5)
CONTINUE
C
DO 105 NH=1,NUMH
DO 105 NS=1,NUMS
READ(10,9)BHS(NH,NS)
FORMAT(E10.5)
CONTINUE
C
DO 103 NHA=1,NUMH
DO 103 NHB=1,NUMH
READ(10,7)BHH(NHA,NHB)
FORMAT(E10.5)
CONTINUE
C
RETURN
END
C
C      S U B R O U T I N E
C      R T W O
C
SUBROUTINE RTWO(PD,FLO,DELT,N,H,NUMH)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DELT(N),PD(N)
DIMENSION FLO(NUMH,N),H(NUMH,N)
DO 100 I=1,N
DO 100 NH=1,NUMH
READ(10,1)FLO(NH,I)
1 FORMAT(E10.5)
100 CONTINUE
I=1
DO 101 NH=1,NUMH
READ(10,2)H(NH,I)
2 FORMAT(E10.5)
101 CONTINUE
DO 102 I=1,N
READ(10,3)PD(I),DELT(I)
3 FORMAT(2E10.5)
WRITE(6,4002)I,PD(I)
4002 FORMAT(5X,'I=',I3,' PD=',E20.10)
WRITE(6,4003)I,DELT(I)
4003 FORMAT(5X,'I=',I3,' DELT=',E20.10)
102 CONTINUE
RETURN
END
```

```

C
C***** SUBROUTINE
C***** THREE
C
C
SUBROUTINE THREE(ALPHAH,BETAH,GAMAH,PH,
1 PHI,N,NUMH,PD)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION ALPHAH(NUMH),BETAH(NUMH),GAMAH(NUMH)
DIMENSION PD(NUMH),PHI(NUMH,N)
DIMENSION PH(NUMH,N)
DO 1 I=1,N
DO 1 NH=1,NUMH
1 PHI(NH,I)=ALPHAH(NH)+BETAH(NH)*PH(NH,I)+  

1 GAMAH(NH)*PH(NH,I)**2.
1 CONTINUE
1 RETURN
END

C
C***** SUBROUTINE
C***** FOUR
C
C
SUBROUTINE FOUR(AZ,AO,AT,H,EPCE,N,NUMH)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION EPCE(NUMH,N),H(NUMH,N+1)
DIMENSION AZ(NUMH),AO(NUMH),AT(NUMH)
DO 1 I=1,N
DO 1 NH=1,NUMH
1 EPCE(NH,I)=AZ(NH)+AO(NH)*H(NH,I)+  

1 AT(NH)*H(NH,I)**2.
1 CONTINUE
1 RETURN
END

C
C***** SUBROUTINE
C***** "T"
C
C
SUBROUTINE T(FS,FH,FD,FI,FB,PS,PH,LAM,H,EPCE,
1 PHI,K,DELT,PD,KLO,YO,YT,BETAS,GAMAS,GAMAH,BETAH,  

2 CH,CS,BSS,BHS,BSH,BHH,B,S,FLO,N,NEW,AO,AT,NUMS,  

3 NUMH,P,Q,LYO,LYT)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION FS(NUMS,N),FH(NUMH,N),FD(N),S(NUMH)
DIMENSION FI(NUMH,N),PS(NUMS,N),PH(NUMH,N)
DIMENSION LAM(N),H(NUMH,N),PD(N),BSS(NUMS,NUMS)

```

```

DIMENSION PHI(NUMH,N), BSH(NUMS,NUMH), BHS(NUMH,NUMS)
DIMENSION EPCE(NUMH,N), Q(NUMH,N), BHH(NUMH,NUMH)
DIMENSION BETAS(NUMS)
DIMENSION AO(NUMH), AT(NUMH)
DIMENSION GAMAH(NUMH), BETAH(NUMH)
DIMENSION P(NUMH,N), GAMAS(NUMS)
DIMENSION FLO(NUMH,N), DELT(N), K(NUMH)
DIMENSION NEW(NUMH)
DIMENSION FB(NUMH), B(NUMH), CS(NUMS), CH(NUMH)
DIMENSION YO(LYO), YT(LYT)
REAL*8 KLO,K,LAM,NEW

C
C      CALCULATE FS(T)
C
DO 1 I=1,N
DO 1 NS=1,NUMS
F1=0.
DO 2 NH=1,NUMH
F1=F1+(2.*BSH(NS,NH)*PH(NH,I))
F6=0.
DO 15 NSA=1,NUMS
F6=F6+2.*BSS(NS,NSA)*PS(NSA,I)
FS(NS,I)=(BETAS(NS)+(2.*GAMAS(NS)*PS(NS,I))
1 LAM(I)*(CS(NS)+F6+F1)
CONTINUE

C
C      CALCULATE Q(T)
C
DO 3 I=1,N
DO 3 NH=1,NUMH
Q(NH,I)=K(NH)*EPCE(NH,I)*PHI(NH,I)
CONTINUE

C
C      CALCULATE FH(T)
C
DO 4 I=1,N
DO 4 NH=1,NUMH
F2=0.
DO 5 NS=1,NUMS
F2=F2+(2.*BHS(NH,NS)*PS(NS,I))
P(NH,I)=(K(NH)/S(NH))*0.00012913*
1 PHI(NH,I)*(AO(NH)+2.*AT(NH)*H(NH,I))
2 *DELT(I)*(I-1)
F7=0.
DO 16 NHA=1,NUMH
F7=F7+2.*BHH(NH,NHA)*PH(NHA,I)
C=CH(NH)+F2+F7
Z=2.-(((2.*GAMAH(NH))+(BETAH(NH)*PH(NH,I)))
1 PHI(NH,I))
FH(NH,I)=(NEW(NH)*DEXP(P(NH,I))*Z*

```

```

1  (Q(NH,I)/PH(NH,I)))+C*LAM(I)
4  CONTINUE
C
C  CALCULATE FI(T)
C
C  J=N-1
J=N-1
DO 6 I=1,J
DO 6 NH=1,NUMH
X1=H(NH,I)-H(NH,I+1)
X2=(DELT(I)/S(NH))*0.00012913
X3=FL0(NH,I)-Q(NH,I)
FI(NH,I)=X1+X2*X3
6  CONTINUE
C
C  CALCULATE FD(T)
C
C  DO 10 I=1,N
DO 10 I=1,N
F3=0.
DO 7 NS=1,NUMS
F3=F3+(CS(NS)*PS(NS,I))
F8=0.
JJJ=NUMS-1
DO 17 NSA=1,JJJ
DO 17 NSA=2,NUMS
F8=F8+BSS(NS,NSA)*(PS(NS,I)*PS(NSA,I))
F4=0.
DO 8 NH=1,NUMH
F4=F4+(CH(NH)*PH(NH,I))
F9=0.
JJ=NUMH-1
DO 18 NH=1,JJ
DO 18 NHA=2,NUMH
F9=F9+BHH(NH,NHA)*(PH(NH,I)*PH(NHA,I))
F5=0.
DO 9 NS=1,NUMS
DO 9 NH=1,NUMH
F5=F5+(BHS(NH,NS)*PS(NS,I)
1 *PH(NH,I))
F10=0.
DO 19 NH=1,NUMH
DO 19 NS=1,NUMS
F10=F10+(BSH(NS,NH)*PH(NH,I)*PS(NS,I))
19 C
F11=0.
DO 20 NH=1,NUMH
F11=F11+BHH(NH,NH)*PH(NH,I)**2
20 C
F12=0.
DO 21 NS=1,NUMS
F12=F12+BSS(NS,NS)*PS(NS,I)**2
21

```

```

C
10      FD(I)=KLO+PD(I)+F3+F4+F5+F8+F9+F11+F12+F10
C      CONTINUE
C
C      CALCULATE FB(T)
C
11      DO 11 NH=1,NUMH
        SUM=0.
        DO 12 I=1,N
          AC=Q(NH,I)*DELT(I)*3600.
          SUM=SUM+AC
12      CONTINUE
        FB(NH)=B(NH)-SUM
11      CONTINUE
C"#####
C      SET UP THE VECTOR YO
C
C"#####
C      L=1
        DO 1001 I=1,N
          DO 1000 NS=1,NUMS
            YO(L)=FS(NS,I)
1000      L=L+1
            YO(L)=FD(I)
1001      L=L+1
            J=N-1
            DO 1002 I=1,J
              DO 1003 NH=1,NUMH
                YO(L)=FI(NH,I)
1003      L=L+1
1002      CONTINUE
C"#####
C      SET UP THE VECTOR YT
C
C"#####
C      L=1
        DO 1004 I=1,N
          DO 1005 NH=1,NUMH
            YT(L)=FH(NH,I)
1005      L=L+1
1004      CONTINUE
          DO 1006 NH=1,NUMH
            YT(L)=FB(NH)
1006      L=L+1
C
C
C

```

```

C
C*****          S U B R O U T I N E
C*****          J A C
C
C
1   SUBROUTINE JAC(ALPHAH,BETAH,GAMAH,GAMAS,EPCE,LAM,
2   DELT,PHI,BSS,BHS,BSH,BHH,CS,CH,N,K,S,H,PH,
3   PS,PD,KLO,AO,AT,NEW,NUMS,NUMH,P,Q,IR,IC,IIR,IIG,
4   NU,LYO,LYT,DFSS,DFSH,DFSL,DFHS,DFHH,DFHL,
5   DFHD,DFHN,DFDS,DFDH,DFIH,DFID,DFBH,DFBD,
5.  DFIDP,LAP,LAPO,AINV,BP,CP,DP,AP,APO)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION ALPHAH(NUMH),BETAH(NUMH),GAMAH(NUMH)
DIMENSION GAMAS(NUMS),EPCE(NUMH,N),LAM(N)
DIMENSION DELT(N),PHI(NUMH,N),BSS(NUMS,NUMS)
DIMENSION BSH(NUMH,NUMS),BHH(NUMH,NUMH),CS(NUMS)
DIMENSION CH(NUMH),K(NUMH),S(NUMH),H(NUMH,N)
DIMENSION PH(NUMH,N),PS(NUMS,N),PD(N),AO(NUMH)
DIMENSION AT(NUMH),IR(LAP),IC(LAP)
DIMENSION IIR(LAPO),IIC(LAPO),BHS(NUMH,NUMS)
DIMENSION P(NUMH,N),Q(NUMH,N)
DIMENSION NU(NUMH,N),NEW(NUMH)
DIMENSION DFBH(NUMH,N),DFBD(NUMH,N),
1  DFIH(NUMH,N),DFID(NUMH,N),DFIDP(NUMH,N),
2  DFDS(NUMS,N),DFDH(NUMH,N),DFHS(NUMH,N),
3  DFHH(NUMH,N),DFHL(NUMH,N),DFHD(NUMH,N),
4  DFHN(NUMH,N),DFSS(NUMS,N),DFSH(NUMS,N),
5  DFSL(NUMS,N),AP(LAP,LAP),APO(LAPO,LAPO)
DIMENSION AINV(LYO,LYO),BP(LYO,LYT)
DIMENSION CP(LYT,LYO),DP(LYT,LYT)
REAL*8 KLO,K,LAM,NEW,NU

C
C*****          CALCULATE THE ELEMENTS OF THE
C*****          JACOBIAN MATRIX.
C
C
DO 100 I=1,N
C
DO 101 NS=1,NUMS
FF1=0.
DO 10 NSA=1,NUMS
FF1=FF1+BSS(NS,NSA)*LAM(I)
10 FFSS(NS,I)=2.*((GAMAS(NS)+FF1)
101 CONTINUE

```

```

C
IF(NUMS.EQ.NUMH)GOTO 90
IF(NUMS.LT.NUMH)GOTO 91
IF(NUMS.GT.NUMH)GOTO 92

C
90   DO 102 NS=1,NUMS
      F1=0.
      DO 103 NH=1,NUMH
      F1=F1+(2.*BSH(NS,NH)*LAM(I))
      DFSH(NS,I)=F1
      CONTINUE

C
102   DO 106 NH=1,NUMH
      DO 107 NS=1,NUMS
      F3=F3+(2.*BHS(NH,NS)*LAM(I))
      DFHS(NH,I)=F3
      CONTINUE

C
106   GOTO 93

C
91   DO 1020 NS=1,NUMS
      DO 1030 NH=1,NUMH
      F1=0.
      F1=F1+(2.*BSH(NS,NH)*LAM(I))
      DFSH(NS,I)=F1

C
1020  DO 1060 NH=1,NUMS
      F3=0.
      DO 1070 NS=1,NUMS
      F3=F3+(2.*BHS(NH,NS)*LAM(I))
      DFHS(NH,I)=F3

C
1060  GOTO 93

C
92   DO 1021 NS=1,NUMH
      F1=0.
      DO 1031 NH=1,NUMH
      F1=F1+(2.*BSH(NS,NH)*LAM(I))
      DFSH(NS,I)=F1

C
1021  DO 1061 NH=1,NUMH
      F3=0.
      DO 1071 NS=1,NUMH
      F3=F3+(2.*BHS(NH,NS)*LAM(I))
      DFHS(NH,I)=F3

C
1061  GOTO 93

C
93   DO 104 NS=1,NUMS

```

```

FF2=0.
DO 11 NSA=1,NUMS
FF2=FF2+2.*BSS(NS,NSA)*PS(NSA,I)
F2=0.
DO 105 NH=1,NUMH
F2=F2+(2.*BSH(NS,NH)*PH(NH,I))
DFSL(NS,I)=CS(NS)+FF2+F2
CONTINUE
C
DO 108 NH=1,NUMH
FF3=0.
DO 12 NHA=1,NUMH
FF3=FF3+2.*BHH(NH,NHA)
FF3=FF3*LAM(I)
NU(NH,I)=NEW(NH)*DEXP(P(NH,I))
WW=(BETAH(NH)+2.*GAMAH(NH)*PH(NH,I))
WX=(AO(NH)+2.*AT(NH)*H(NH,I))
WY=(K(NH)*DELT(I))/S(NH)
WZ=2.*GAMAH(NH)
DFHH(NH,I)=K(NH)*NU(NH,I)*EPCE(NH,I)
*(WZ+WY*(WW*2.)*0.00012913)+FF3
CONTINUE
C
DO 109 NH=1,NUMH
FF4=0.
DO 13 NHA=1,NUMH
FF4=FF4+2.*BHH(NH,NHA)*PH(NHA,I)
F4=0.
DO 110 NS=1,NUMS
F4=F4+(2.*BHS(NH,NS)*PS(NS,I))
DFHL(NH,I)=CH(NH)+F4+FF4
CONTINUE
C
DO 112 NH=1,NUMH
DFHN(NH,I)=(NU(NH,I)/NEW(NH))*(
1 (2.*GAMAH(NH)*PH(NH,I)+BETAH(NH))
2 *K(NH)*EPCE(NH,I))
CONTINUE
C
DO 113 NS=1,NUMS
FF5=0.
DO 14 NSA=1,NUMS
FF5=FF5+2.*BSS(NS,NSA)*PS(NSA,I)
F5=0.
DO 114 NH=1,NUMH
F5=F5+(BSH(NS,NH)*PH(NH,I))
DFDS(NS,I)=CS(NS)+FF5+F5
CONTINUE
C
DO 115 NH=1,NUMH

```



C THE INVERSE OF JA  
 C  
 C #####  
 LL=1  
 MM=1  
 DO 500 I=1,N  
 DO 499 II=1,LAP  
 DO 499 J=1,LAP  
 AP(II,J)=0.  
 499 CONTINUE  
 L=1  
 M=1  
 DO 501 NS=1,NUMS  
 AP(L,M)=DFSS(NS,I)  
 L=L+1  
 M=L  
 L=NUMS+1  
 M=1  
 DO 502 NS=1,NUMS  
 AP(L,M)=DFDS(NS,I)  
 502 M=M+1  
 M=NUMS+1  
 L=1  
 DO 503 NS=1,NUMS  
 AP(L,M)=DFSL(NS,I)  
 503 L=L+1  
 DO 517 III=1,LAP  
 DO 517 JJ=1,LAP  
 517 CONTINUE  
 C  
 C FIND THE INVERSE  
 C  
 MA=LAP  
 IA=LAP  
 CALL MINVRD(AP,IA,MA,DET,IER,IR,IC)  
 C POSITION THE BLOCK IN THE  
 C MATRIX AINV  
 C  
 MM=LL  
 DO 504 II=1,LAP  
 DO 505 J=1,LAP  
 AINV(LL,MM)=AP(II,J)  
 505 MM=MM+1  
 LL=LL+1  
 504 MM=MM-LAP  
 LL=LAP\*I+1  
 500 CONTINUE  
 C  
 C

C SET THE I' TH + 1 BLOCK

C  
C  
C  
DO 509 I=1,LAPO  
DO 509 J=1,LAPO  
APO(I,J)=0.  
CONTINUE

509

C  
C

L=1  
M=1  
J=N-1  
DO 510 I=1,J  
DO 511 NH=1,NUMH  
APO(L,M)=DFIDP(NH,I)  
L=L+1

511  
510

M=M+1  
CONTINUE  
IF(N.EQ.1)GOTO 516

M=1  
L=NUMH+1  
J=N-1  
DO 512 I=2,J  
DO 513 NH=1,NUMH  
APO(L,M)=DFID(NH,I)  
L=L+1

513  
512

M=M+1  
CONTINUE

C  
C  
C

INVERT THE BLOCK

MA=LAPO  
IA=LAPO  
CALL MINVRD(APO,IA,MA,DET,IER,IIR,IIC)

C  
C  
C

POSITION THE INVERSE  
WITHIN THE MATRIX  
AINV

C  
C  
C

516 LL=(LAP\*N)+1  
MM=LL  
DO 514 I=1,LAPO  
DO 515 J=1,LAPO  
AINV(LL,MM)=APO(I,J)  
MM=MM+1  
LL=LL+1  
MM=MM-LAPO

515  
514

```

C
C
C*****SET UP THE MATRIX." JB"
C
C*****DO 519 I=1,LYO
C      DO 519 J=1,LYT
C      BP(I,J)=0.
519   CONTINUE
C
C
C      IF(NUMS.LE.NUMH)GOTO520
C      IF( NUMS.GT.NUMH ) GOTO 525
C
C
520   M=1
      L=1
      DO 521 I=1,N
      DO 522 NS=1,NUMS,
      BP(L,M)=DFSH(NS,I)
      M=M+1
522   L=L+1
      L=((NUMS+1)*I)+1
521   M=(NUMH*I)+1
      GOTO 530
C
C
C
525   M=1
      L=1
      DO 526 I=1,N
      DO 527 NS=1,NUMH
      BP(L,M)=DFSH(NS,I)
      M=M+1
527   L=L+1
      M=(NUMH*I)+1
526   L=((NUMS+1)*I)+1
C
C
C
530   M=1
      L=NUMS+1
      DO 531 I=1,N
      DO 532 NH=1,NUMH
      BP(L,M)=DFDH(NH,I)
      M=M+1
532   L=L+NUMS+1
C

```



```
DO 551 I=1,N
DO 552 NH=1,NUMH
CP(L,M)=DFHL(NH,I)
552 L=L+1
551 M=M+NUMS+1
C
L=NUMH+1
M=(NUMS*N)+1+N
DO 553 I=2,N
DO 553 NH=1,NUMH
CP(L,M)=DFHD(NH,I)
L=L+1
553 M=M+1
C
C M=(NUMS*N)+1+N
DO 555 I=2,N
L=(NUMH*N)+1
DO 554 NH=1,NUMH
CP(L,M)=DFBD(NH,I)
L=L+1
554 M=M+1
555 CONTINUE
C"#####
C
C      SET UP THE MATRIX " JD "
C
C"#####
DO 556 I=1,LYT
DO 556 J=1,LYT
DP(I,J)=0
556 CONTINUE
L=1
M=1
DO 560 I=1,N
DO 561 NH=1,NUMH
DP(L,M)=DFHH(NH,I)
L=L+1
M=M+1
560 CONTINUE
C
C
L=1
DO 562 I=1,N
M=(NUMH*N)+1
DO 563 NH=1,NUMH
DP(L,M)=DFHN(NH,I)
L=L+1
M=M+1
563
```

562 CONTINUE

C

C

C

M=1

DO 564 I=1,N

L=(NUMH\*N)+1

DO 565 NH=1,NUMH

DP(L,M)=DFBH(NH,I)

L=L+1

565

M=M+1

564

CONTINUE

C

RETURN

END

C

C

C

C

C

SUBROUTINE

GUESS

C

C

1 SUBROUTINE GUESS(PS,PH,LAM,H,PD,DELT,AZ,AO,AT,B,  
 2 S,K,N,ALPHAH,BETAH,GAMAH,KLO,CS,CH,BSS,BSH,BHH,  
 2 BETAS,GAMAS,NEW,NUMS,NUMH,PHI,EPCE,Q,BHS)  
 IMPLICIT REAL\*8(A-H,O-Z)  
 DIMENSION PS(NUMS,N),PH(NUMH,N),LAM(N),H(NUMH,N)  
 DIMENSION PD(N),EPCE(NUMH,N),PHI(NUMH,N),Q(NUMH,N)  
 DIMENSION DELT(N),AZ(NUMH),AO(NUMH),AT(NUMH),B(NUMH)  
 DIMENSION S(NUMH),K(NUMH),ALPHAH(NUMH),BHS(NUMH,NUMS)  
 DIMENSION BETAH(NUMH),GAMAH(NUMH),CS(NUMS),CH(NUMH)  
 DIMENSION BSS(NUMS,NUMS),BSH(NUMS,NUMH),BETAS(NUMS)  
 DIMENSION GAMAS(NUMS),BHH(NUMH,NUMH),NEW(NUMH)  
 REAL\*8 LAM,NEW,K,KLO

C

TF=0.

DO 10 I=1,N

TF=TF+DELT(I)

10

CONTINUE

I=1

DO 11 NH=1,NUMH

EPCE(NH,I)=AZ(NH)+(AO(NH)\*H(NH,I))+

(AT(NH)\*H(NH,I)\*\*2.)

BP=B(NH)/(TF\*K(NH)\*EPCE(NH,I)\*3600.)

XX=DSQRT((BETAH(NH)\*\*2.)-(4.\*GAMAH(NH))

\*(ALPHAH(NH)-BP)))

PH(NH,I)=(-BETAH(NH)+XX)/(2.\*GAMAH(NH))

11 CONTINUE

CBA=0.

DO 777 NH=1,NUMH

```

777    CBA=CBA+PH(NH,I)
      DO 110 NS=1,NUMS
      CC=1.-(CBA/PD(I))
      PS(NS,I)=CC*(PD(I)/NUMS)
110    CONTINUE
      C
      C
      NS=1
      FA=0.
      DO 30 NSA=1,NUMS
      30    FA=FA+2.*BSS(NS,NSA)*PS(NSA,I)
      AA=BETAS(NS)+(2.*GAMAS(NS)*PS(NS,I))
      DO 13 NH=1,NUMH
      FB=0.
      DO 31 NSA=1,NUMH
      31    FB=FB+2.*BSH(NSA,NH)*PH(NH,I)
      AB=CS(NS)+FA+FB
      13    CONTINUE
      LAM(I)=(-AA/AB)
      C
      C
      DO 14 NH=1,NUMH
      PHI(NH,I)=(ALPHAH(NH)+(BETAH(NH)*PH(NH,I))
      1 )+(GAMAH(NH)*PH(NH,I)**2.)
      14    CONTINUE
      DO 15 NH=1,NUMH
      FC=0.
      DO 32 NHA=1,NUMH
      32    FC=FC+2.*BHH(NH,NHA)*PH(NH,I)
      Q(NH,I)=K(NH)*EPCE(NH,I)*PH(NH,I)
      EE=((2.*ALPHAH(NH))+(BETAH(NH)*
      1 PH(NH,I)))/PHI(NH,I)
      FF=Q(NH,I)/PH(NH,I)
      P=(PHI(NH,I)/S(NH))*AO(NH)+(2.*AT(NH)*
      1 H(NH,I)).*DELT(I)*0.00012913
      TEMP1=CH(NH)+FC
      ABC=0.0
      DO 16 NS=1,NUMS
      ABC=ABC+2.0*BHS(NH,NS)*PS(NS,I)
      CONTINUE
      ABC=ABC+TEMP1
      NEW(NH)=(-LAM(I)*ABC)/((2.-EE)*FF
      1 *DEXP(P))
      CONTINUE
      DO 20 NS=1,NUMS
      DO 20 I=2,N
      FAC=PD(I)/PD(1)
      PS(NS,I)=PS(NS,1)*FAC
      20    CONTINUE
      DO 22 NH=1,NUMH

```

```

DO 22 I=2,N
FAC=PD(I)/PD(1)
EPCE(NH,I)=EPCE(NH,1)*FAC
PH(NH,I)=PH(NH,1)*FAC
Q(NH,I)=Q(NH,1)*FAC
PH(NH,I)=PH(NH,1)*FAC
H(NH,I)=H(NH,I-1)
CONTINUE
DO 24 I=2,N
FAC=PD(I)/PD(1)
LAM(I)=LAM(I)*FAC
CONTINUE
RETURN
END
C
C***** SUBROUTINE ***** M I N V R D ***** C
C
SUBROUTINE MINVRD(A,IA,MA,DETA,IER,IR,IC)
REAL*8 A(IA,IA),DETA,PIV,PIV1,TEMP
DIMENSION IR(MA),IC(MA)
IER=0
DETA=1.
DO 1 I=1,MA
IR(I)=0
IC(I)=0
1 DO 123 IJKL=1,MA
CALL SUBMXD(A,IA,IA,MA,MA,IR,IC,I,J)
PIV=A(I,J)
DETA=PIV
IF(PIV.EQ.0.0D0) GOTO 17
IR(I)=J
IC(J)=(I)
PIV=1.D0/PIV
DO 5 K=1,MA
A(I,K)=A(I,K)*PIV
A(I,J)=PIV
DO 9 K=1,MA
IF(K.EQ.I) GOTO 9
PIV1=A(K,J)
DO 8 L=1,MA
A(K,L)=A(K,L)-PIV1*A(I,L)
A(K,J)=PIV1
CONTINUE
PIV1=A(I,J)
DO 11 K=1,MA
A(K,J)=-PIV1*A(K,J)
A(I,J)=PIV1
11

```

```

123  CONTINUE
12   DO 16 I=1,MA
      K=IC(I)
      M=IR(I)
      IF(K.EQ.I)GOTO 16
      DETA=-DETA
      DO 14 L=1,MA
      TEMP=A(K,L)
      A(K,L)=A(I,L)
      A(I,L)=TEMP
14   DO 15 L=1,MA
      TEMP=A(L,M)
      A(L,M)=A(L,I)
15   A(L,I)=TEMP
      IC(M)=K
      IR(K)=M
16   CONTINUE
      RETURN
17   IER=1
      RETURN
      END
C
C***** SUBROUTINE ***** SUBMXD *****
C
C
SUBROUTINE SUBMXD(A,IA,JA,MA,NA,IR,IC,I,J)
REAL*8 A(IA,JA),TEST,X,DABS
DIMENSION IR(MA),IC(NA)
I=0
J=0
TEST=0.D0
DO 5 K=1,MA
IF(IR(K).NE.0) GOTO 5
DO 4 L=1,NA
IF (IC(L).NE.0) GOTO 4
X=DABS(A(K,L))
IF(X.LT.TEST) GOTO 4
I=K
J=L
TEST=X
4  CONTINUE
5  CONTINUE
      RETURN
      END
C
C***** SUBROUTINE ***** X1MULT *****
C

```

```
C*****  
C  
C      SUBROUTINE X1MULT(AM,BM,RM,NA,NAMB,MB)  
C      IMPLICIT REAL*8(A-H,O-Z)  
C      DIMENSION AM(NA,NAMB),BM(NAMB,MB),RM(NA,MB)  
C      DO 30 I=1,NA  
C      DO 20 J=1,MB  
C          RM(I,J)=0.  
C          DO 10 K=1,NAMB  
C              RM(I,J)=RM(I,J)+AM(I,K)*BM(K,J)  
C 10      CONTINUE  
C 20      CONTINUE  
C 30      CONTINUE  
C      RETURN  
C      END  
C*****  
C*****  
C      S U B R O U T I N E  
C*****  
C      X 2 M U L T  
C*****  
C  
C      SUBROUTINE X2MULT(AM,BM,RM,NA,MB)  
C      IMPLIGIT REAL*8(A-H,O-Z)  
C      DIMENSION AM(NA,MB),BM(MB),RM(NA)  
C      DO 30 I=1,NA  
C          RM(I)=0.  
C          DO 20 K=1,MB  
C              RM(I)=RM(I)+AM(I,K)*BM(K)  
C 20      CONTINUE  
C 30      CONTINUE  
C      RETURN  
C      END  
C*****  
C*****  
C#####  
C#####  
C#####      E N D   O F   P R O G R A M  
C#####  
C#####  
C#####
```

#### B-4 MATRIX INVERSION ROUTINE

The subroutines MINVRD and SUBMND found in the listing are developed by the University of Waterloo of Waterloo and is available as part of a scientific package. This matrix inversion routine is available in either single (MINVRS) or double (MINVRD) precision.

The main purpose of the program is to compute the inverse of a matrix by the direct method.

The subroutine is called in the following way:

```
CALL MINVRD (A, IA, DET, IER, IR, IC)
```

where

A is dimensioned with absolute size IA by IA and the portion of matrix being used by the subprogram is represented by MA.

IR and IC are dimensioned to MA - they are work vectors.

IER is an error parameter usually set at 0 but equals 1 if A is singular.

The accuracy of the routine is dependent upon the size of the given matrix. The round-off error is minimized by searching for the largest pivotal element at each stage of the process. The greatest accuracy is 13 digits with MINVRD, and 4 for MINVRS.

The routine takes the matrix stored in A and computes the inverse of it by the direct method. It does this by searching for the largest pivotal element at each stage of the procedure. The result is stored in position A. The determinant  $|A|$  is also calculated and is stored in DET. The matrix A is destroyed.

The core requirements for MINVRD is 2688 Bytes for the object code.

#### B-5 COMPUTER SPECIFICATIONS

The computer used in this research belongs to Memorial University of Newfoundland. The equipment and software is produced by Digital Equipment of Canada. The hardware of the system is the VAX 11/780 computer and peripheral support equipment. The software is Version 3.0, VAX/VMS and in particular the language used for the program was VAX-11 Fortran (Fortran 77).

The computer has 4.0 Mb of main storage and 706 Mb of disk storage. In addition, the tape drive units are capable of operating on 1600 or 6250 bpi.

APPENDIX C

APPENDIX CCOMPUTER PRINTOUTC-1 INTRODUCTION

This appendix presents the actual form of the computer printout from the program.

It has been manually transferred to a word processing device to obtain an output of suitable quality. Otherwise, there are no alterations.

## ECONOMIC DISPATCH

---

SCHEDULE

---

TEST

SYSTEM

NO: 1

---

---

## COMPUTATIONAL RESULTS

THE SOLUTION WAS OBTAINED IN 7 ITERATIONS

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 1  
IS 0.9003693759E+00

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 2  
IS 0.9790824988E-01

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 3  
IS 0.1397912817E-01

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 4  
IS 0.2885521034E-02

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 5  
IS 0.6953570515E-03

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 6  
IS 0.1929799536E-03

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 7  
IS 0.5060176689E-04

TIME      THERMAL  
PERIOD    PLANT  
NO: 1  
(HR)      MW

1	180.65
2	196.63
3	186.69
4	181.01
5	193.96
6	197.59
7	195.28
8	180.72
9	160.20
10	156.18
11	163.58
12	153.00
13	144.90
14	139.31
15	89.29
16	83.63
17	78.63
18	70.35
19	66.75
20	62.76
21	59.45
22	57.03
23	56.59
24	55.75

TIME PERIOD (HR)	HYDRO PLANT NO: 1 MW
1	542.42
2	572.19
3	567.33
4	568.14
5	598.18
6	614.39
7	620.83
8	603.34
9	571.50
10	573.92
11	604.71
12	588.53
13	577.85
14	573.77
15	427.89
16	422.96
17	419.54
18	403.98
19	404.67
20	404.52
21	405.01
22	411.14
23	430.98
24	453.69

TIME PERIOD HR	NET HEAD PLANT NO: 1 FT
1	205.00
2	204.81
3	204.60
4	204.38
5	204.15
6	203.90
7	203.62
8	203.32
9	203.04
10	202.78
11	202.51
12	202.21
13	201.91
14	201.62
15	201.32
16	201.17
17	201.02
18	200.86
19	200.72
20	200.58
21	200.44
22	200.29
23	200.13
24	199.95

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	LAMBDA \$/MW
1	681.00	42.07	3.784
2	722.00	46.82	3.880
3	708.00	46.03	3.820
4	703.00	46.16	3.786
5	741.00	51.17	3.864
6	758.00	53.98	3.886
7	761.00	55.12	3.872
8	732.00	52.05	3.784
9	685.00	46.71	3.661
10	683.00	47.10	3.667
11	716.00	52.29	3.682
12	692.00	49.53	3.618
13	675.00	47.75	3.569
14	666.00	47.08	3.536
15	491.00	26.18	3.236
16	481.00	25.58	3.202
17	473.00	25.17	3.172
18	451.00	23.34	3.122
19	448.00	23.42	3.100
20	443.00	23.28	3.077
21	441.00	23.46	3.057
22	444.00	24.17	3.042
23	461.00	26.56	3.040
24	480.00	29.43	3.034

TIME      NU  
PERIOD    PLANT  
             NO: 1  
HR        \$/CF

1	0.04812
2	0.04698
3	0.04589
4	0.04481
5	0.04347
6	0.04219
7	0.04102
8	0.04025
9	0.03983
10	0.03888
11	0.03734
12	0.03681
13	0.03622
14	0.03551
15	0.03854
16	0.03809
17	0.03761
18	0.03751
19	0.03696
20	0.03647
21	0.03591
22	0.03520
23	0.03400
24	0.03267

TIME HR	RESEVIOR INFLOW PLANT NO: 1
HR	CFS
1	12000.0
2	12000.0
3	12000.0
4	12000.0
5	12000.0
6	12000.0
7	12000.0
8	12000.0
9	12000.0
10	12000.0
11	12000.0
12	12000.0
13	12000.0
14	12000.0
15	12000.0
16	12000.0
17	12000.0
18	12000.0
19	12000.0
20	12000.0
21	12000.0
22	12000.0
23	12000.0
24	12000.0

-----  
THE VALUE OF THE COST FUNCTIONS FOR  
THE THERMAL GENERATING PLANTS IN (\$/DAY) ARE

PLANT NO: 1 FUEL COST 9844.70

## ECONOMIC DISPATCH

-----  
SCHEDULE  
-----TEST  
SYSTEM-----  
N O : 2  
-----

### COMPUTATIONAL RESULTS

THE SOLUTION WAS OBTAINED IN 13 ITERATIONS

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 1  
IS 0.5500958074E+00

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 2  
IS 0.5925722180E-01

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 3  
IS 0.1329054893E-01

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 4  
IS 0.6463546968E-02

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 5  
IS 0.3478227069E-02

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 6  
IS 0.2019128443E-02

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 7  
IS 0.1204589334E-02

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 8  
IS 0.7322724724E-03

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 9  
IS 0.4512366086E-03

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 10  
IS 0.2809106443E-03

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 11  
IS 0.1762657394E-03

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 12  
IS 0.1113009484E-03

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 13  
IS 0.7063917420E-04

TIME PERIOD (HR)	THERMAL PLANT		THERMAL PLANT	
	NO: 1 MW	NO: 2 MW	NO: 1 MW	NO: 2 MW
1	188.99	183.61		
2	207.02	201.65		
3	198.75	193.38		
4	194.82	189.45		
5	210.31	204.95		
6	216.19	210.83		
7	215.88	210.52		
8	202.29	196.92		
9	181.97	176.59		
10	179.73	174.35		
11	190.55	185.18		
12	180.62	175.23		
13	173.44	168.05		
14	169.17	163.78		
15	110.46	105.03		
16	105.88	100.45		
17	102.06	96.62		
18	94.02	88.58		
19	91.86	86.42		
20	89.17	83.73		
21	87.37	81.93		
22	86.91	81.46		
23	89.80	84.35		
24	93.79	88.35		

TIME PERIOD (HR)	HYDRO PLANT NO: 1 MW	HYDRO PLANT NO: 2 MW
1	560.26	511.42
2	588.05	538.14
3	580.83	531.71
4	579.28	530.72
5	607.54	556.23
6	621.60	568.92
7	625.48	573.02
8	604.22	557.17
9	567.74	529.95
10	567.31	530.94
11	597.17	553.98
12	576.13	541.41
13	560.92	533.24
14	552.81	530.14
15	392.55	421.39
16	384.64	417.06
17	378.25	414.05
18	358.94	401.98
19	356.84	402.22
20	352.70	401.21
21	351.30	402.18
22	354.86	406.40
23	373.60	419.35
24	402.14	436.03

TIME PERIOD HR	NET HEAD PLANT NO: 1 FT	NET HEAD PLANT NO: 2 FT
1	205.00	206.00
2	204.71	205.88
3	204.40	205.75
4	204.08	205.62
5	203.75	205.48
6	203.39	205.34
7	203.01	205.18
8	202.61	205.01
9	202.22	204.86
10	201.86	204.72
11	201.50	204.57
12	201.09	204.41
13	200.70	204.26
14	200.31	204.11
15	199.93	203.96
16	199.70	203.86
17	199.48	203.77
18	199.26	203.68
19	199.05	203.60
20	198.85	203.51
21	198.65	203.43
22	198.44	203.34
23	198.23	203.25
24	198.00	203.15

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	LAMBDA \$/MW
1	1362.00	82.29	3.834
2	1444.00	90.86	3.942
3	1416.00	88.67	3.893
4	1406.00	88.26	3.869
5	1482.00	97.02	3.962
6	1516.00	101.54	3.997
7	1522.00	102.90	3.995
8	1462.00	96.60	3.914
9	1370.00	86.25	3.792
10	1366.00	86.53	3.778
11	1432.00	94.88	3.843
12	1384.00	89.38	3.784
13	1350.00	85.65	3.741
14	1332.00	83.89	3.715
15	982.00	47.43	3.363
16	962.00	46.03	3.335
17	946.00	44.97	3.312
18	902.00	41.53	3.264
19	896.00	41.34	3.251
20	886.00	40.81	3.235
21	882.00	40.78	3.224
22	881.00	41.63	3.221
23	922.00	45.11	3.239
24	960.00	50.31	3.263

TIME PERIOD	NU PLANT NO: 1 HR	NU PLANT NO: 2 \$/CF
----------------	----------------------------	-------------------------------

1	0.04769	0.04684
2	0.04651	0.04624
3	0.04541	0.04566
4	0.04433	0.04509
5	0.04300	0.04438
6	0.04175	0.04370
7	0.04063	0.04307
8	0.03993	0.04263
9	0.03961	0.04236
10	0.03874	0.04183
11	0.03727	0.04101
12	0.03688	0.04066
13	0.03643	0.04027
14	0.03586	0.03983
15	0.03921	0.04116
16	0.03889	0.04086
17	0.03856	0.04055
18	0.03861	0.04042
19	0.03822	0.04007
20	0.03970	0.03976
21	0.03751	0.03940
22	0.03697	0.03897
23	0.03591	0.03834
24	0.03447	0.03760

TIME HR	RESEVIOR		RESEVIOR	
	INFLOW PLANT NO: 1	CFS	INFLOW PLANT NO: 2	CFS
1	5500.0		11000.0	
2	5500.0		11000.0	
3	5500.0		11000.0	
4	5500.0		11000.0	
5	5500.0		11000.0	
6	5500.0		11000.0	
7	5500.0		11000.0	
8	5500.0		11000.0	
9	5500.0		11000.0	
10	5500.0		11000.0	
11	5500.0		11000.0	
12	5500.0		11000.0	
13	5500.0		11000.0	
14	5500.0		11000.0	
15	5500.0		11000.0	
16	5500.0		11000.0	
17	5500.0		11000.0	
18	5500.0		11000.0	
19	5500.0		11000.0	
20	5500.0		11000.0	
21	5500.0		11000.0	
22	5500.0		11000.0	
23	5500.0		11000.0	
24	5500.0		11000.0	

-----  
THE VALUE OF THE COST FUNCTIONS FOR  
THE THERMAL GENERATING PLANTS IN (\$/DAY) ARE  
-----

PLANT NO: 1 FUEL COST 11764.85

PLANT NO: 2 FUEL COST 11413.87

-----

## ECONOMIC DISPATCH

~~SCHEDULE~~

TEST

SYSTEM

NO: 3

COMPUTATIONAL RESULTS

---

THE SOLUTION WAS OBTAINED IN 7 ITERATIONS

---

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 1  
IS 0.8491699406E+00

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 2  
IS 0.1134254453E+00

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 3  
IS 0.1611677021E-01

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 4  
IS 0.3041146059E-02

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 5  
IS 0.6151115247E-03

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 6  
IS 0.1286717119E-03

THE MAXIMUM RELATIVE ERROR FOR  
ITERATION NUMBER 7  
IS 0.2744106656E-04

TIME PERIOD (HR)	THERMAL PLANT NO: 1 MW	THERMAL PLANT NO: 2 MW
1	263.76	268.84
2	271.83	276.90
3	261.53	266.61
4	254.26	259.34
5	258.93	264.01
6	264.21	269.29
7	253.08	258.16
8	236.22	241.30
9	232.83	237.91
10	227.37	232.46
11	222.90	227.99
12	215.37	220.47
13	209.04	214.13
14	204.13	209.23
15	183.91	189.02
16	178.40	183.51
17	173.29	178.40
18	167.71	172.82
19	162.92	168.03
20	158.45	163.57
21	154.19	159.31
22	150.03	155.14
23	145.49	150.61
24	140.40	145.51

TIME HR	HYDRO PLANT NO: 1 MW	HYDRO PLANT NO: 2 MW	HYDRO PLANT NO: 3 MW	HYDRO PLANT NO: 4 MW	HYDRO PLANT NO: 5 MW
1	544.87	544.87	544.87	544.87	544.87
2	571.12	571.12	571.12	571.12	571.12
3	565.29	565.29	565.29	565.29	565.29
4	564.57	564.57	564.57	564.57	564.57
5	590.19	590.19	590.19	590.19	590.19
6	620.72	620.72	620.72	620.72	620.72
7	606.78	606.78	606.78	606.78	606.78
8	572.39	572.39	572.39	572.39	572.39
9	581.18	581.18	581.18	581.18	581.18
10	581.99	581.99	581.99	581.99	581.99
11	586.90	586.90	586.90	586.90	586.90
12	572.74	572.74	572.74	572.74	572.74
13	563.22	563.22	563.22	563.22	563.22
14	562.77	562.77	562.77	562.77	562.77
15	441.64	441.64	441.64	441.64	441.64
16	436.74	436.74	436.74	436.74	436.74
17	433.11	433.11	433.11	433.11	433.11
18	423.25	423.25	423.25	423.25	423.25
19	419.39	419.39	419.39	419.39	419.39
20	417.75	417.75	417.75	417.75	417.75
21	418.06	418.06	418.06	418.06	418.06
22	421.60	418.06	418.06	418.06	418.06
23	435.90	435.90	435.90	435.90	435.90
24	451.47	451.47	451.47	451.47	451.47

TIME HR	NET HEAD PLANT NO: 1 FT	NET HEAD PLANT. NO: 2 FT	NET HEAD PLANT NO: 3 FT	NET HEAD PLANT NO: 4 FT	NET HEAD PLANT NO: 5 FT
1	205.00	205.00	205.00	205.00	205.00
2	204.81	204.81	204.81	204.81	204.81
3	204.59	204.59	204.59	204.59	204.59
4	204.38	204.38	204.38	204.38	204.38
5	204.16	204.16	204.16	204.16	204.16
6	203.91	203.91	203.91	203.91	203.91
7	203.62	203.62	203.62	203.62	203.62
8	203.34	203.34	203.34	203.34	203.34
9	203.09	203.09	203.09	203.09	203.09
10	202.82	202.82	202.82	202.82	202.82
11	202.55	202.55	202.55	202.55	202.55
12	202.26	202.26	202.26	202.26	202.26
13	201.98	201.98	201.98	201.98	201.98
14	201.71	201.71	201.71	201.71	201.71
15	201.43	201.43	201.43	201.43	201.43
16	201.26	201.26	201.26	201.26	201.26
17	201.10	201.10	201.10	201.10	201.10
18	200.93	200.93	200.93	200.93	200.93
19	200.77	200.77	200.77	200.77	200.77
20	200.62	200.62	200.62	200.62	200.62
21	200.46	200.46	200.46	200.46	200.46
22	200.30	200.30	200.30	200.30	200.30
23	200.13	200.13	200.13	200.13	200.13
24	199.95	199.95	199.95	199.95	199.95

TIME PERIOD	POWER DEMAND MW	POWER LOSSES MW	LAMBDA \$/MW
HR			
1	3223.00	38.96	4.283
2	3367.00	37.31	4.331
3	3318.00	36.56	4.269
4	3300.00	36.46	4.226
5	3434.00	39.85	4.254
6	3593.00	44.08	4.285
7	3503.00	42.12	4.218
8	3302.00	37.48	4.117
9	3338.00	38.64	4.097
10	3331.00	38.75	4.064
11	3346.00	39.41	4.037
12	3262.00	37.53	3.992
13	3203.00	36.29	3.954
14	3191.00	36.23	3.925
15	2559.00	22.32	3.803
16	2524.00	21.82	3.770
17	2496.00	21.46	3.740
18	2437.00	20.51	3.706
19	2408.00	20.13	3.678
20	2391.00	19.97	3.651
21	2384.00	20.00	3.625
22	2393.00	20.34	3.600
23	2454.00	21.74	3.573
24	2520.00	23.32	3.542

TIME HR	NU PLANT NO: 1	NU PLANT NO: 2	NU PLANT NO: 3	NU PLANT NO: 4	NU PLANT NO: 5
	\$/CF	\$/CF	\$/CF	\$/CF	\$/CF
1	0.06332	0.06332	0.06332	0.06332	0.06332
2	0.06173	0.06173	0.06173	0.06173	0.06173
3	0.06033	0.06033	0.06033	0.06033	0.06033
4	0.05894	0.05894	0.05894	0.05894	0.05894
5	0.05716	0.05716	0.05716	0.05716	0.05716
6	0.05519	0.05519	0.05519	0.05519	0.05519
7	0.05407	0.05407	0.05407	0.05407	0.05407
8	0.05356	0.05356	0.05356	0.05356	0.05356
9	0.05210	0.05210	0.05210	0.05210	0.05210
10	0.05087	0.05087	0.05087	0.05087	0.05087
11	0.04954	0.04954	0.04954	0.04954	0.04954
12	0.04886	0.04886	0.04886	0.04886	0.04886
13	0.04809	0.04809	0.04809	0.04809	0.04809
14	0.04706	0.04706	0.04706	0.04706	0.04706
15	0.05054	0.05054	0.05054	0.05054	0.05054
16	0.04992	0.04992	0.04992	0.04992	0.04992
17	0.04929	0.04929	0.04929	0.04929	0.04929
18	0.04893	0.04893	0.04893	0.04893	0.04893
19	0.04837	0.04837	0.04837	0.04837	0.04837
20	0.04773	0.04773	0.04773	0.04773	0.04773
21	0.04702	0.04702	0.04702	0.04702	0.04702
22	0.04617	0.04617	0.04617	0.04617	0.04617
23	0.04480	0.04480	0.04480	0.04480	0.04480
24	0.04335	0.04335	0.04335	0.04335	0.04335

TIME HR.	RESEVIOR	RESEVIOR	RESEVIOR	RESEVIOR	RESEVIOR
	INFLOW PLANT NO: 1 CFS	INFLOW PLANT NO: 2 CFS	INFLOW PLANT NO: 3 CFS	INFLOW PLANT NO: 4 CFS	INFLOW PLANT NO: 5 CFS
1	12000.0	12000.0	12000.0	12000.0	12000.0
2	12000.0	12000.0	12000.0	12000.0	12000.0
3	12000.0	12000.0	12000.0	12000.0	12000.0
4	12000.0	12000.0	12000.0	12000.0	12000.0
5	12000.0	12000.0	12000.0	12000.0	12000.0
6	12000.0	12000.0	12000.0	12000.0	12000.0
7	12000.0	12000.0	12000.0	12000.0	12000.0
8	12000.0	12000.0	12000.0	12000.0	12000.0
9	12000.0	12000.0	12000.0	12000.0	12000.0
10	12000.0	12000.0	12000.0	12000.0	12000.0
11	12000.0	12000.0	12000.0	12000.0	12000.0
12	12000.0	12000.0	12000.0	12000.0	12000.0
13	12000.0	12000.0	12000.0	12000.0	12000.0
14	12000.0	12000.0	12000.0	12000.0	12000.0
15	12000.0	12000.0	12000.0	12000.0	12000.0
16	12000.0	12000.0	12000.0	12000.0	12000.0
17	12000.0	12000.0	12000.0	12000.0	12000.0
18	12000.0	12000.0	12000.0	12000.0	12000.0
19	12000.0	12000.0	12000.0	12000.0	12000.0
20	12000.0	12000.0	12000.0	12000.0	12000.0
21	12000.0	12000.0	12000.0	12000.0	12000.0
22	12000.0	12000.0	12000.0	12000.0	12000.0
23	12000.0	12000.0	12000.0	12000.0	12000.0
24	12000.0	12000.0	12000.0	12000.0	12000.0

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THE VALUE OF THE COST FUNCTIONS FOR  
THE THERMAL GENERATING PLANTS IN (\$/DAY) ARE

PLANT NO: 1 FUEL COST 16747.22

PLANT NO: 2 FUEL COST 17074.42

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APPENDIX D.

## APPENDIX D

THE AUGMENTED OBJECTIVE FUNCTIONAL

The first step in the optimization process requires that each constraint  $f_k$  be paired with an appropriate multiplier function to form the produce  $F_k$ . The proper balance equation is paired with the multiplier function  $\lambda(t)$ , to form

$$F_D(t) = \lambda(t) f_D(t)$$

The discharge models  $f_{q_i}$  are paired with the multiplier functions  $m_i(t)$ , to form

$$F_{q_i}(t) = m_i(t) f_{q_i}(t) \quad (D.1)$$

Finally, the continuity equations  $f_{h_i}(t)$  are paired with the multiplier function  $n_i(t)$ , to form

$$F_{h_i}(t) = n_i(t) f_{h_i}(t) \quad (D.2)$$

The inclusion of the water draw-down constraint requires constant multipliers  $v_o$  to pair with  $j_{b_i}$  to form  $J_{b_i}$  according to

$$J_{b_i} = v_o j_{b_i} \quad (D.3)$$

This follows since  $j_{b_i}$  represent definite integrals and not time functions.

The next step is to form the individual contributions  $J_k$  to the

original objective by first defining:

$$J_D = \int_0^T F_D(t) dt \quad (D.4)$$

$$J_{q_i} = \int_0^T F_{q_i}(t) dt \quad (D.5)$$

$$J_{h_i} = \int_0^T f_{h_i}(t) dt \quad (D.6)$$

The augmented objective functional is thus given by

$$J_A = J + J_D + \sum_{i=2}^4 (J_{q_i} + J_{h_i} + J_{b_i}) \quad (D.7)$$

Let us note here that all components of  $J_A$  are explicit functions of the control variables, with the exception of  $J_{h_i}$  which needs some transformation.

Consider, for example, the function  $f_{h_2}(t)$  where the integral of the control variable  $q_2(t)$  appears. The part of  $J_{h_2}$  corresponding to this term is transformed as follows

$$\int_0^T q_2(z) dz = \int_0^T [n_2(T) - n_2(t)] q_2(t) dt \quad (D.8)$$

The corresponding terms in  $f_{h_3}$  and  $f_{h_4}$  are treated similarly. The last term in both  $f_{h_3}$  and  $f_{h_4}$  needs special treatment as well.

$$\begin{aligned}
 & \int_0^T \dot{n}_3(t) \int_0^t q_2(z-\tau_{23}) dz dt = \\
 & + \int_{-T_{23}}^0 [n_3(T) - n_3(t+\tau_{23})] q_2(t) dt \\
 & + \int_0^{T-t_{23}} [n_3(T) - n_3(t+\tau_{23})] q_2(t) dt. \quad (D.9)
 \end{aligned}$$

With the above mentioned transformations, we can thus write the relevant elements of the augmented functional  $J_{h_i}$  as:

$$J_{h_2} = \int_0^T [s_2 \dot{n}_2(t) h_2(t) + [n_2(T) - n_2(t)] q_2(t)] dt \quad (D.10)$$

$$\begin{aligned}
 J_{h_3} = & \int_0^T [s_3 \dot{n}_3(t) h_3(t) + [n_3(T) - n_3(t)] q_3(t) \\
 & - N_3(t) q_2(t)] dt \quad (D.11)
 \end{aligned}$$

$$\begin{aligned}
 J_{h_4} = & \int_0^T [s_4 \dot{n}_4(t) h_4(t) + [n_4(T) - n_4(t)] q_4(t) \\
 & - N_4(t) q_3(t)] dt. \quad (D.12)
 \end{aligned}$$

In the above we define

$$\begin{aligned}
 N_3(t) &= n_3(T) - n_3(t+\tau_{23}) \\
 0 &\leq t \leq T - \tau_{23} \\
 &= 0 \quad T - \tau_{23} \leq t \leq T \quad (D.13)
 \end{aligned}$$

$$\begin{aligned} N_4(t) &= n_4(T) - n_4(t+\tau_{34}) \\ &\quad 0 \leq t \leq T - \tau_{34} \\ &= 0 \quad T - \tau_{34} \leq t \leq T \end{aligned} \tag{D.14}$$

In writing  $J_h$  elements we drop terms that are independent of the control variables.

APPENDIX E

## APPENDIX E

### REDUCING THE HYDRO-OPTIMALITY CONDITIONS

Let us start with the down-stream plant optimality conditions (5.14) and (5.11), the derivatives of (5.14) with respect to time gives us

$$\dot{m}_4(t) = \dot{n}_4(t) \quad \dots \quad (E.1)$$

Define

$$M_4(t) = \frac{1}{s} \frac{\partial q_4}{\partial h_4} \quad \dots \quad (E.2)$$

Thus, Equations (5.11) and (E.1) combine to give the first order differential equation

$$\dot{m}_4(t) = m_4(t) M_4(t) \quad \dots \quad (E.3)$$

The solution of (E.3) is

$$m_4(t) = m_4(0) \exp \left[ \int_0^t M_4(t) dt \right] \quad \dots \quad (E.4)$$

From Equation (5.14), we have

$$m_4(0) = n_4(0) - n_4(T) - v_4 \quad \dots \quad (E.5)$$

To conform with the classical theory, we let

$$v_4 = -m_4(0) \quad \dots \quad (E.6)$$

As a result, the optimality equation (5.10) for plant 4 is

$$v_4 \exp \int_0^t M_4(t) dt \frac{\partial q_4}{\partial p_4} = \lambda(t) \left[ 1 - \frac{\partial p_L}{\partial p_4} \right] \quad (E.7)$$

This is exactly Kron-Ricards equation for uncoupled hydro plants.

Consider next the intermediate plant, number 3, Equation (5.13) upon differentiation gives

$$\dot{m}_3(t) = \dot{n}_3(t) = \dot{N}_4(t) \quad (E.8)$$

From (D.14), we get

$$\begin{aligned} \dot{m}_3(t) &= \dot{n}_3(t) - \dot{n}_4(t + \tau_{34}) \\ &\quad 0 \leq t \leq T - \tau_{34} \\ &= \dot{n}_3(t) \quad T - \tau_{34} \leq t \leq T \end{aligned} \quad (E.9)$$

Using (E.1), we further obtain

$$\begin{aligned} \dot{m}_3(t) &= \dot{n}_3(t) - \dot{m}_4(t + \tau_{34}) \\ &\quad 0 \leq t \leq T - \tau_{34} \\ &= \dot{n}_3(t) \quad T - \tau_{34} \leq t \leq T \end{aligned}$$

Equation (5.11) for plant 3, is written as

$$\dot{n}_3(t) = m_3(t) M_3(t) \quad (E.10)$$

where

$$M_3(t) = \frac{1}{s_3} \frac{\partial q_3}{\partial h_3} \quad (E.11)$$

Using Equation (E.9) in (E.10), we thus write

$$\dot{m}_3(t) = m_3(t) M_3(\pm) - u_3(t) \quad (E.12)$$

In Equation (E.14) we have

$$\begin{aligned} u_3(t) &= \dot{m}_4(t + \tau_{34}) & 0 \leq t \leq T - \tau_{34} \\ &= 0 & T - \tau_{34} \leq t \leq T \end{aligned}$$

On the basis of Equation (E.13), we have Equation (E.9) written as

$$\dot{m}_3(t) = \dot{n}_3(t) - u_3(t) \quad (E.14)$$

Equation (E.12) is an inhomogeneous linear equation whose solution is given by

$$m_3(t) = \phi_3(t, 0) m_3(0) - \int_0^t \phi_3(t, \sigma) u_3(\sigma) d\sigma \quad (E.15)$$

where

$$\phi_3(t, t_0) = \exp \left[ \int_{t_0}^t M_3(z) dz \right] \quad (E.16)$$

To conform with conventional theory let

$$v_3 = -m_3(0) \quad (E.17)$$

As a result, the optimality condition (5.10) for plant 3, is written as

$$(v_3 \phi_3(t, 0) + \int_0^t \phi_3(t, \sigma) u_3(\sigma) d\sigma) \frac{\partial q_3}{\partial p_3} = \lambda(t) [1 - \frac{\partial p_1}{\partial p_3}] \quad (E.18)$$

In a similar fashion we can arrive at the following relation for plant 2

$$\{v_{2,2}(t,\sigma) + \int_0^t \phi_2(t,\sigma) u_2(\sigma)^2 d\sigma\} \frac{\partial q_2}{\partial p_2} = \lambda(t) [1 - \frac{\partial p_L}{\partial p_2}] \quad (E.19)$$





