ECONOMIC OPERATION OF VARIABLE-HEAD HYDRO-THERMAL ELECTRIC POWER SYSTEMS USING NEWTON METHOD

KENNETH PATRICK WALSH

Economic Operation of Variable-Head Hydro-Thermal Electric Power Systems Using Newton Method

by

© KENNETH P. WALSH, B.Eng.,

A Thesis submitted in partial fulfillment

of the requirements for the degree of

Master of Engineering

Department of Engineering

Memorial University of Newfoundland

November 1982

St. John's Newfoundland

TABLE OF CONTENTS

LIST OF TABLES .... (iv)

LIST OF FIGURES .... (v)

LIST OF SYMBOLS .... (vii)

ACKNOWLEDGEMENTS .... (ix)

ABSTRACT .... (x)

1. INTRODUCTION

1.1 BACKGROUND 1

1.2 SCOPE OF THE THESIS 2

2. A HISTORICAL REVIEW

2.1 A REVIEW OF THE DEVELOPMENTS IN THE PROBLEM OF OPTIMAL VARIABLE-HEAD HYDRO-THERMAL DISPATCH 4

3. FORMULATION OF THE PROBLEM

3.1 COORDINATION EQUATIONS FOR VARIABLE-HEAD HYDRO SYSTEMS 7

3.2 DISCRETE COORDINATION EQUATIONS 13

3.3 APPLICATION OF THE NEWTON-RAPHSON METHOD IN THE PROBLEM .. 15

3.4 GENERATION OF INITIAL ESTIMATES 20

4. PERFORMANCE EVALUATION

4.1 INTRODUCTION 24

4.2 TEST SYSTEM ONE AND CHARACTERIZATION TESTS 24

4.2.1 Test System One Description 24

4.2.2 Computational Results 26

4.2.3 Characterization Tests Description 31

4.2.4 Characterization Test One 31

4.2.5 Characterization Test Two 36

4.2.6 Characterization Test Three 42

4.3 TEST SYSTEM TWO 42

4.3.1 Test System Two Description 42

4.3.2 Computational Results 46

4.4 TEST SYSTEM THREE 56a

4.4.1 Test System Three Description 56a

4.4.2 Computational Results 56a

5. COORDINATION EQUATIONS FOR ECONOMIC OPERATION OF POWER SYSTEMS WITH HYDRO PLANTS ON THE SAME STREAM

5.1 INTRODUCTION 68

5.2 FORMULATION 68

5.3 DIRECT OPTIMALITY CONDITIONS 71

5.4 COORDINATION EQUATIONS 72

5.5 WATER-WORTH 74

6. CONCLUSIONS AND FUTURE WORK

6.1 CONCLUSIONS 76

6.2 FUTURE WORK 76

7. BIBLIOGRAPHY AND REFERENCES 78

APPENDIX A. FIRST-ORDER PARTIAL DERIVATIVES 83

APPENDIX B. THE COMPUTER PROGRAM 89

APPENDIX C. COMPUTER PRINTOUT 135

APPENDIX D. THE AUGMENTED OBJECTIVE FUNCTIONAL 165

APPENDIX E. REDUCING THE HYDRO OPTIMALITY CONDITIONS 170

LIST OF TABLES

4.2-1 OPTIMUM DISPATCH SCHEDULE. TABULATED DATA 29

4.2-2 INCREMENTAL COST OF POWER AND WATER-WORTH COEFFICIENTS. TABULATED DATA 32

4.2-3 HEAD VARIATIONS. TABULATED DATA 34

4.2-4 CHARACTERIZATION TEST ONE. TABULATED DATA 38

4.2-5 CHARACTERIZATION TEST TWO. TABULATED DATA 41

4.2-6 CHARACTERIZATION TEST THREE. TABULATED DATA 44

4.3-1 OPTIMUM DISPATCH SCHEDULE. TABULATED RESULTS 49

4.3-2 OPTIMUM DISPATCH SCHEDULE. TABULATED RESULTS 51

4.3-3 VARIATIONS IN $\lambda(t)$ AND $\nu(t)$. TABULATED RESULTS 54

4.3-4 NET HEAD VARIATIONS. TABULATED RESULTS 56

4.4-1 OPTIMAL DISPATCH SCHEDULE. TABULATED RESULTS 59

4.4-2 VARIATIONS IN $\lambda(t)$ and $\nu(t)$. TABULATED RESULTS 63

4.4-3 VARIATION IN $h(t)$. TABULATED RESULTS 67

LIST OF FIGURES

FIGURE 3.1 The Jacobian Matrix and Partition Details 17

FIGURE 3.2 Structure and Partitioning of $\underline{f}(x^m)$ and $\Delta \underline{x}^m$ 18

FIGURE 4.2-1. Maximum Relative Error Versus Number of Iterations.. 27

FIGURE 4.2-2. Optimal Dispatch Schedule 28

FIGURE 4.2-3. Variation in $\lambda(t)$ 30

FIGURE 4.2-4. Variation in $\nu(t)$ 30

FIGURE 4.2-5. Variation in h(t) 33

FIGURE 4.2-6. Variation in Head with Increased B(t) 35

FIGURE 4.2-7. Variation in $\nu(t)$ with Increased B(t) 35

FIGURE 4.2-8. Variation in Daily Fuel Costs with Increased B(t) .. 37

FIGURE 4.2-9. Variation in Head Variation with Increased $P_d(t)$ 39

FIGURE 4.2-10. Variation in $\lambda(t)$ with Increased $P_d(t)$ 39

FIGURE 4.2-11. Variation in Fuel Cost with Increased $P_d(t)$ 40

FIGURE 4.2-12. Variation in Head Variation with Increased Inflow 43

FIGURE 4.2-13. Variation in Daily Fuel Costs with Increased Inflow 43

FIGURE 4.3-1. Maximum Relative Error Versus Number of Iterations 47

FIGURE 4.3-2. Optimum Dispatch Schedule 48

FIGURE 4.3-3. Variation in $P_h(t)$ 50

FIGURE 4.3-4. Variation in $P_s(t)$ 50

FIGURE 4.3-5. Variation in $\lambda(t)$ 53

FIGURE 4.3-6. Variation in $\nu(t)$ 53

FIGURE 4.3-7. h(t) Versus t 55

FIGURE 4.4-1. Variation in Maximum Relative Error With Iterations 58

FIGURE 4.4-2. Optimal Dispatch Schedule 59

FIGURE 4.4-3. Variation in $P_h(t)$ 62

FIGURE 4.4-4. Variation in $P_s(t)$ 62

FIGURE 4.4-5. Variation in $\lambda(t)$ 63

FIGURE 4.4-6. Variation in $\nu(t)$ 63

FIGURE 4.4-7. Variation in h(t) 67

FIGURE B-1. Algorithm Flowchart 90

LIST OF SYMBOLS

F	Fuel cost function.
J	Objective functional which is to be minimized.
$P_{s_i}(t)$	Thermal power output for the ith plant at time t.
$P_{h_i}(t)$	Hydro power output for the ith plant at time t.
$\alpha_{s_i}, \beta_{s_i}, \gamma_{s_i}$	Fuel cost model coefficients for the ith plant.
$\alpha_{h_i}, \beta_{h_i}, \gamma_{h_i}$	Hydro plant model coefficients for the ith plant.
$a_{o_i}, a_{1_i}, a_{2_i}$	Reservoir model coefficients for the ith plant.
$h_i(t)$	The net head for the ith plant at time t.
$q_i(t)$	The biquadratic function defining the discharge of the ith plant at time t.
$i_i(t)$	The ith reservoir's natural inflow at time t.
$\phi_i(P_h)$	Hydro plant model equation for the ith plant at time t.
$\psi_i(h)$	The reservoir model equation for the ith plant at time t.
$P_i(t)$	The transmission losses for time t.
$P_D(t)$	The system power demand at time t.
b_i	The amount of available water for the ith plant.

B_{ij}, K_{L_o}	System's loss coefficients.
ν_i	The water-worth coefficient of the ith plant.
$\lambda(t)$	The incremental cost of power for time t.
K	System constant.
$Q_i(t)$	Amount of water discharged from the ith plant at time t.
δ_i	The surface area of the ith reservoir.

ACKNOWLEDGEMENTS

I would like at this time to express my thanks and appreciation to those people who helped make this thesis possible. Firstly, I would like to thank my supervisor, Dr. M.E. El-Hawary, for his guidance, help, and encouragement, and also Ms. Wanda Roy whose typing was so fast and accurate. Next, I want to acknowledge the help I obtained from the staff of both the Newfoundland and Memorial University computing services and especially Mr. Bernard Keogh who had so much time and patience. I also wish to thank Memorial University for their financial help and my fellow graduate students for their moral support. Last and foremost, I want to thank my wife whose understanding and support made it all possible.

ABSTRACT

In this thesis the optimum generation schedules for systems with variable-head hydro plants are developed. The scheduling problem is solved by use of the Newton-Raphson method where the coordination equations developed by Richard-Kron are employed. Several power systems containing different numbers of plants are considered.

The problem's formulation is profiled and details of the development of the coordination equations with variable-head hydro systems and the discretization of these equations for computer solution are outlined.

Also highlighted are the difficulties encountered with excess computer processing time due to large arrays and how these problems are resolved. Methods for generating the initial estimates of the variables are presented.

Results from three test systems are given and, in addition, evaluation tests for the algorithm's response to changes in the system's variables are detailed.

Coordination equations for systems with water transport delay problems resulting from being hydraulically coupled are presented.

In addition, complete documentation of the computer program is included.

CHAPTER I

INTRODUCTION

1.1 BACKGROUND

In any power system one of the major goals is to attain optimum economic dispatch. This involves scheduling the generation at various generating stations to meet the system's power demand while keeping the power production costs to a minimum. Usually, this scheduling covers prescribed periods of time. In addition to operating economics the optimum operation of a power system is governed by other restrictions. The ability to fractionally reduce power production costs still has priority with electric utility companies. Also, knowledge of the optimum dispatch schedule allows for better planning and design of any future equipment additions to power systems. It is for these reasons that the problem of economic dispatch has been so extensively researched (refer Ch. II).

In problems concerning economic dispatch it is customary to consider the cost of operation only. Such consideration does not take into account the expenses of labour, capital, start-up and shut-down related to the duration of the down time of a specific unit. Hence, an accurate knowledge of the manner in which the total operating cost of each generating unit varies with the instantaneous output is essential.

The hydro-thermal optimization problem involves the planning of the usage of a limited resource over a prescribed period of time. The resource being the amount of water available for generation. In some systems the use of this water is governed by social factors including irrigational and navigational committments. Other systems, with hydro plants located on the same stream have special problems concerning water transport delay which affects the net head and discharge levels. Thus, the conditions which exist over the entire optimization interval must be taken into account when determining the optimum dispatch schedule. For instance, a system with a large reservoir may require an optimization period of a year. Still another system with a small to moderate storage capacity may find an optimization interval of a day or a week more useful.

1.2 SCOPE OF THE THESIS

In this thesis the optimum generation schedules for hydro-thermal systems with variable-head hydro plants are developed. The scheduling problem is solved by use of the Newton-Raphson method where the coordination equations (21,27) are employed. Several power systems containing different numbers of plants are considered.

The historical background on the problem is presented in Chapter II. In this presentation all previous work concerning optimum hydro-thermal scheduling for systems with variable-head hydro plants is detailed.

In Chapter III the problem's formulation is profiled. Sub-sections (3.1) and (3.2) detail the development of the coordination equations for variable-head hydro systems and the discretization of these equations for computer solution. In sub-section (3.3) it is shown how the Newton-Raphson method is applied to the problem. This section also highlights the difficulties encountered with excess computer processing time due to large arrays and how these problems are resolved. Sub-section (3.4) outlines the methods for generating the initial values of the variables.

Chapter IV presents the results from three test systems. In addition to these tests, an evaluation of the program's performance is given. This evaluation tests for the algorithm's response to changes in the system's variables.

The coordination equations for systems having hydro plants with water transport delay problems resulting from being hydraulically coupled are given in Chapter V. This chapter looks at several different arrangements and the resulting coordination equations.

In Chapter VI, the major conclusions of the work are presented. Also outlined in this chapter are the areas in which further work may be conducted.

A full listing and description of the computer program is presented in Appendix B.

CHAPTER II

A HISTORICAL REVIEW

2.1 A REVIEW OF THE DEVELOPMENTS IN THE PROBLEM OF OPTIMAL VARIABLE-HEAD HYDRO-THERMAL DISPATCH

Much of the work done on optimal hydro-thermal dispatch in the past revolved around the assumption that for short range studies the effect of head variations could be neglected. However, Ricard [27] in 1940 relaxed this constant head principle. In his paper, he presented a set of coordination equations for systems with net head variations and negligible transmission losses.

Fourteen years later in 1954, Cypser [12] reported on a method he had developed which utilized variational calculus. As with Ricard, he neglected transmission losses and in addition concentrated his attention on the long-range scheduling problem. Cypser tested his method on a system containing one thermal and one hydro plant with varied success.

Then in 1958, Glimn and Kirchmayer [21] in a comprehensive paper, detailed the expansion of the basic coordination equations to include transmission losses. They tested their method on various model systems using the technique of numerical integration. One of the important results of this paper was the demonstration of equivalence of Ricard's, Kron's, and Cypser's equations. This the authors show using variational calculus techniques.

After this presentation by Glimn and Kirchmayer, the reports on optimizing the dispatching schedule for variable-head hydro-thermal systems became more frequent and researchers began to try new approaches. Hence, only two years later in 1960 Arismunander [1] utilizing variational calculus and employing all the necessary and sufficient conditions for optimality, arrived at the required scheduling equations.

The following year Dandeno [15] reported on the computational experience gained by applying the coordination equations to an actual operating system. The computer algorithm he used proceeded to the solution by linearizing the non-linear equations, solving them by the Gauss-Seidel method for the Power values, and then adjusting the constraint mutlipliers accordingly. However, large amounts of computer time and heavy core requirements were the major drawbacks of the method.

In 1962 Drake et.al. [16], using basically the same computer algorithm as Dandeno, applied variational methods to functional systems. This method, although more successful than Dandeno's, did not resolve all of the problems surrounding the optimizing procedure.

The work continued and in 1964 Dahlin [13] presented his maximum principle approach to the problem. The basis of this approach was developed by Pontryagin. Later in 1966 Dahlin along with Shen [14] detailed the application of his method to several types of systems. The numerical analysis was performed on a test system consisting of one thermal and one hydro plant and was mainly for the purpose of exploring the convergence behavior.

The next noteable work came in 1971 when Bonaert and El-Abiad [8] reported on a method known as decomposition in which the hydro-thermal system was subdivided into hydro and thermal subsystems. In addition to decomposition, the technique also used perturbations to arrive at the required result.

In 1972 another method was proposed by El-Hawary and Christensen [18]. This procedure utilized functional analytic minimum norm formulation. It was pointed out that this method eliminated the multipliers associated with the linear constraints in the control vector.

The decomposition method surfaced again in 1980 when Soares, Lyra, and Tavares [29] reported on a "coordinated" decomposition technique. In this procedure the solution was obtained through a tri-level hierarchical calculation structure.

To conclude the review, the work presented herein utilizes the coordination equations and obtains the solution using the Newton-Raphson method.

CHAPTER III

FORMULATION OF THE PROBLEM

3.1 COORDINATION EQUATIONS FOR VARIABLE-HEAD HYDRO SYSTEMS

The coordination equations for variable-head hydro systems are an extension of those used for fixed-head systems. For both types of systems the classical approach of variational calculus is utilized giving the optimality conditions in terms of Ricard's equations. To arrive at the optimal strategy for variable-head hydro systems, the equations for fixed-head systems are developed and then extended to the variable-head case.

To begin, it is assumed that the reservoir is large enough so that any variation in net head may be neglected. It is also assumed that the fuel cost for the thermal units is

$$F = \sum_{i=1}^{N_s} F_i(P_{s_i}) \tag{3.1}$$

where N_s is the number of thermal plants and $F_i(P_{s_i})$ is defined as

$$F_i(P_{s_i}) = \alpha_{s_i} + \beta_{s_i} P_{s_i}(t) + \gamma_{s_i} P_{s_i}^2(t) \, . \qquad (i=1, \ldots N_s) \tag{3.2}$$

It is required to minimize the objective functional given by

$$J = \int_0^{T_f} F dt \tag{3.3}$$

while satisfying the active power balance equation given by

$$\sum_{i=1}^{N_s} P_{s_i}(t) + \sum_{i=1}^{N_h} P_{h_i}(t) - P_L(t) = P_D(t) \quad . \qquad (3.4)$$

In the above equation $P_{h_i}(t)$ is the output of the ith hydro unit, N_h is the number of hydro plants in the system, $P_L(t)$ is the transmission loss and $P_D(t)$ is the system's power demand.

The volume of water available, b_i, for generation is also taken into consideration through the requirement

$$\int_0^{T_f} q_i(t)\,dt = b_i \quad . \qquad (i=1, \ldots N_h) \quad (3.5)$$

Here $q_i(t)$ is the rate of water discharge at the ith plant. This is a biquadratic function of effective head and active power generation according to the model suggested by Glimn-Kirchmayer [21], given by

$$q_i(t) = K\,\psi_i(h)\;\phi_i(P_h) \quad , \qquad (i=1, \ldots N_h) \quad (3.6)$$

where the dependence on net head is expressed as

$$\psi_i(h_i) = a_{0_i} + a_{1_i} h_i(t) + a_{2_i} h_i(t)^2 \quad . \qquad (i=1, \ldots N_h) \quad (3.7)$$

The dependence on active power generation is indicated by

$$\phi_i(P_h) = \alpha_{h_i} + \beta_{h_i}(t) + \gamma_{h_i} P_{h_i}(t)^2 \quad . \qquad (i=1, \ldots N_h) \quad (3.8)$$

In the above

K = constant of proportionality .

The parameters a_{0_i}, a_{1_i}, a_{2_i}, α_{h_i}, β_{h_i}, and γ_{h_i} are assumed available.

The cost functional J is now augmented to include the volume of water constraint. This is accomplished by using constant multipliers, ν_i, as follows

$$\tilde{J} = \int_0^{T_f} [F + \sum_{i=1}^{N_h} \nu_i q_i(t)] \, dt \quad . \tag{3.9}$$

The power balance equation is also included through the use of the multiplier function $\lambda(t)$. In this way, the constrained minimization problem of (3.1) is changed to an unconstrained problem of minimizing the following augmented cost functional

$$\tilde{J} = \int_0^{T_f} \{F + \sum_{i=1}^{N_h} \nu_i q_i(t) + \lambda(t) \, [P_D(t) - \sum_{i=1}^{N_s} P_{s_i}(t) - \sum_{i=1}^{N_h} P_{h_i}(t) + P_L(t)]\} \, dt \quad . \tag{3.10}$$

Through variational calculus techniques, the optimality conditions for fixed-head systems are obtained as

$$\beta_{s_i} + 2\gamma_{s_i} P_{s_i}(t) + \lambda(t) \, [C_i + 2 \sum_{j=1}^{N_g} B_{i_j} P_j(t)] = 0 \, , \quad (i=1,N_s) \tag{3.11}$$

$$\nu_i \, [\beta_{h_i} + 2\gamma_{h_i} P_{h_i}(t)] + \lambda(t) \, [C_i + 2 \sum_{j=1}^{N_g} B_{i_j} P_j(t)] = 0, \quad (i=1,N_h) \tag{3.12}$$

where

$$C_i = B_{i_o} - 1 \quad , \qquad (i=1,N_g) \quad . \tag{3.13}$$

Note that it is assumed here that $K\phi$ is set to unity for fixed head hydro plants.

When solved, these equations yield active powers, the incremental cost of power, $\lambda(t)$, and the base water worth, ν_i. However, complete solution requires that the following constraint equations be adhered to.

$$\int_0^{T_f} q_i(t)\, dt = b_i \,, \qquad (i=1,N_h) \qquad (3.14)$$

$$K_{L_o} + P_D(t) + \sum_{i=1}^{N_g} C_i P_i(t)$$

$$+ \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P_i(t)\, B_{ij}\, P_j(t) = 0 \,. \qquad (3.15)$$

The assumption of negligible head variation is now relaxed and is replaced by the assumption that operation of the hydro units results in a change in the net head of the reservoirs. Thus, for hydro plants with variable-head characteristics, the generation of active power is regulated by the rate of discharge, $q_i(t)$, and also by the volume of water discharged, $Q_i(t)$, given by

$$Q_i(t) = \int_0^{t} q_i(t)\, dt \,, \qquad (i=1,N_h) \,. \qquad (3.16)$$

The cost functional is still constrained by the active power balance equation and the volume of water available, however, the decision variables are now extended to include $P_{s_i}(t)$ and $Q_i(t)$.

Again, using variational calculus methods, it is found that the coordination equations for variable-head systems are equivalent to those of the fixed-head case with the exception that the water conversion coefficient, ν_i, is now variable and is defined by

$$\nu_i(t) = \nu_{i_o} \exp\left(\int_0^t \left[\frac{1}{s} \partial q_i(t)/\partial h_i(t)\right] dt \right), \qquad (i=1,N_h) . \qquad (3.17)$$

The integrand in the above is denoted by $M(t)$ which is written as

$$M(t) = \frac{K}{s_i} \phi_i(P_h) \left[a_{1_i} + 2a_{2_i} h_i(t)\right], \qquad (i=1,N_h) . \qquad (3.18)$$

By assuming that $M(t)$ is constant over the interval $[0,T_f]$, equation 3.17 may be written as

$$\nu_i(t) = \gamma_{0_i} \exp[M_i t] . \qquad (3.19)$$

The coordination equations for the variable-head hydro system are given as

$$\beta_{s_i} + 2\gamma_{s_i} P_{s_i}(t) + \lambda(t) \left[C_i + 2 \sum_{i=1}^{N_g} B_{i_j} P_j(t)\right] = 0, \qquad (i=1,N_s) \qquad (3.20)$$

$$\{\nu_{0_i} \exp[M_i t]\}\{2 - \frac{2\alpha_{h_i} + \beta_{h_i} P_{h_i}(t)}{\phi_i(P_h)}\} \{ \frac{q_i(t)}{P_{h_i}(t)} \}$$

$$+ \lambda(t) \left[C_i + 2 \sum_{j=1}^{N_g} B_{i_j} P_j(t)\right] = 0, \qquad (i=1,N_h) \qquad (3.21)$$

As before, complete solution requires the observance of the following constraint equations

$$\int_0^{T_f} [K \psi_i(h) \phi_i(P_h)] dt = b_i, \qquad (i=1,N_h) \qquad (3.22)$$

$$K_{L_o} + P_D(t) + \sum_{i=1}^{N_g} C_i P_i(t) + \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P_i(t) B_{i_j} P_j(t) = 0 . \qquad (3.23)$$

For variable-head hydro systems an additional constraint is imposed which accounts for the effect of head variation and has the form

$$h_i(t) = h_{0_i}(t) + \frac{1}{s_i}\int_0^t [i_i(t) - q_i(t)]\,dt \quad , \qquad (i=1,N_h) \qquad (3.24)$$

where $i_i(t)$ is the natural inflow assuming a vertical sided reservoir.

Under the foregoing assumptions the optimality conditions for a variable-head hydro system are given by

$$\beta_{s_i} + 2\gamma_{s_i} P_{s_i}(t) + \lambda(t)\,[C_i + 2\sum_{j=1}^{N_g} B_{i_j} P_j(t)] = 0 \ , \qquad (i=1,N_s) \qquad (3.25)$$

$$\{\nu_{0_i} \exp[M_i t]\}\{2 - \frac{2\alpha_{h_i} + \beta_{h_i} P_{h_i}(t)}{\phi_i(P_h)}\}\{\frac{q_i(t)}{P_{h_i}(t)}\}$$

$$+ \lambda(t)\,[C_i + 2\sum_{j=1}^{N_g} B_{i_j} P_j(t)] = 0 \ , \qquad (i=1,N_h) \qquad (3.26)$$

$$\int_0^{T_f} q_i(t)\,dt = b_i \ ; \qquad (i=1,N_h) \qquad (3.27)$$

$$K_{L_o} + P_D(t) + \sum_{i=1}^{N_g} C_i P_i(t) + \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} P_i(t)\, B_{i_j} P_j(t) = 0 \qquad (3.28)$$

$$h_i(t) = h_{0_i}(t) + \frac{1}{s_i}\int_0^t [i_i(t) - q_i(t)]\,dt \ ; \qquad (i=1,N_h) \ . \qquad (3.29)$$

As before

$$C_i = B_{i_o} - 1 \ , \qquad (i=1,N_g) \ . \qquad (3.30)$$

The dynamic equations 3.25 to 3.29 are non-linear and a discrete form is required for digital solution. This problem is treated in the next section.

3.2 DISCRETE COORDINATION EQUATIONS

To discretize equations (3.25) to (3.29) it is assumed that the optimization interval $[0,T_f]$ is divided into N_T discrete intervals. The discrete time index is denoted by t_k. Equation (3.25) is denoted by $f_{s_i}(t)$ and is straight-forward as far as the discretization process is concerned. Similarily equations (3.26) and (3.28) are denoted by $f_{h_i}(t)$ and $f_D(t)$, respectively. The volume of water constraint given by equation (3.27) is replaced by a summation assuming that the discrete intervals to be of equal length, Δ. The reservoir equation (3.29) is replaced by the equivalent form

$$h_i(t+\Delta) = h_i(t) + \frac{1}{s_i}\int_t^{t+\Delta} [i_i(t) - q_i(t)]\,dt \quad , \quad (i=1,N_h) \qquad (3.31)$$

Thus

$$f_{i_i}(t) = h_i(t) - h_i(t+\Delta) + \frac{1}{s_i}\int_t^{t+\Delta} [i_i(t) - q_i(t)]\,dt = 0 \qquad (i=1,N_h) \qquad (3.32)$$

or

$$f_{i_i}(t_k) = h_i(t_k) - h_i(t_k+1) + \frac{\Delta}{s_i}[i_i(t_k) - q_i(t_k)] = 0 \qquad (i=1,N_h). \qquad (3.33)$$

The discretization process results in a set of nonlinear algebraic equations which must be solved and these equations are:

$$f_{s_i}(t_k) = \beta_{s_i} + 2\gamma_{s_i} P_{s_i}(t_k) + \lambda(t_k)\left[C_i + 2\sum_{j=1}^{N_g} B_{ij} P_j(t_k)\right] = 0 \, , \qquad (i=1,N_s) \qquad (3.34)$$

$$f_{h_i}(t_k) = \{\nu_{0_i} \exp[Mt]\}\{2 - \frac{2\alpha_{h_i} + \beta_{h_i} P_{h_i}(t_k)}{\phi_i(P_h)}\}\{\frac{q_i(t_k)}{P_{h_i}(t_k)}\}$$

$$+ \lambda(t_k)\left[C_i + 2\sum_{j=1}^{N_g} B_{ij} P_j(t_k)\right] = 0 \, , \qquad (i=1,N_h) \qquad (3.35)$$

$$f_D(t_k) = K_{L_0} + P_D(t_k) + \sum_{i=1}^{N_g} C_i P_i(t_k)$$

$$+ \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} P_i(t_k) B_{ij} P_j(t_k) \qquad (3.36)$$

$$f_{i_i}(t_k) = h_i(t_k) - h_i(t_k - 1) + \frac{\Delta}{s_i}\left[i_i(t_k) - q_i(t_k)\right] = 0 \, , \qquad (i=1,N_h) \qquad (3.37)$$

$$f_{b_i}(t_k) = \sum_{k=0}^{N_T-1} \left[k_i \psi_i(h) \phi_i(P_h)\right]\Delta - b_i = 0 \, , \qquad (i=1,N_h) \qquad (3.38)$$

Each of the above equations applies at the following discrete instants

$$f_{s_i}(t_k): \quad t_k = 0, 1, \ldots, N_T-1$$

$$f_{h_i}(t_k): \quad t_k = 0, 1, \ldots, N_T-1$$

$$f_D(t_k): \quad t_k = 0, 1, \ldots, N_T-1$$

$$f_i(t_k): \quad t_k = 0, 1, \ldots, N_T-2 \, .$$

The unknowns are ν_{0_i} and the following

$$P_{s_i}(t_k): \quad t_k = 0, 1, \ldots, N_T-1$$

$$P_{h_i}(t_k): \quad t_k = 0, 1, \ldots, N_T-1$$

$$\lambda(t_k): \quad t_k = 0, 1, \ldots, N_T-1$$

$$h_i(t_k): \quad t_k = 1, \ldots, N_T-1 \quad .$$

The interval over which $h_i(t_k)$ is found, i.e. $(1, N_T-1)$ has its basis in the assumption that the reservoir levels $h_i(0)$ are known. This is described more fully in section 3.4.

3.3 APPLICATION OF THE NEWTON-RAPHSON METHOD TO THE PROBLEM

The Newton-Raphson method requires solving on each iteration the following set of linear equations

$$f(x^m) + J\Delta x^m = 0 \quad . \tag{3.39}$$

In the above, f is the vector nonlinear function to be solved and J is the Jacobian matrix consisting of the first-order partial derivatives of the functions with respect to the unknown variables (Appendix A) which comprise the vector x. The index m denotes the iteration number.

The size of the Jacobian matrix depends on the number of units in the system and the number of time intervals in the period. For each hydro plant there are two variables per time instant, for each thermal plant only one variable per time instant. In addition, one variable ν_{0_i} per hydro plant is encountered. Also included are the values of the incremental cost of power, $\lambda(t_k)$, for each time interval.

Solving for Δx^m requires that the inverse of the Jacobian matrix, J^{-1}, be obtained. This is achieved by (1); ... or (2); by determining the inverse of the matrix as a whole or two; by utilizing the method of matrix partitioning.

The first method provides a direct path to J^{-1}. The Jacobian matrix is set up and its inverse is obtained directly through a commercially available inversion routine. This method performs well when considering small systems. However, with larger systems, 4 plants or more, the amount of time required to obtain J^{-1} is such that it is no longer feasible.

The main objective of the second method is to restrict the application of the inversion routine to a matrix of the smallest possible dimensions.

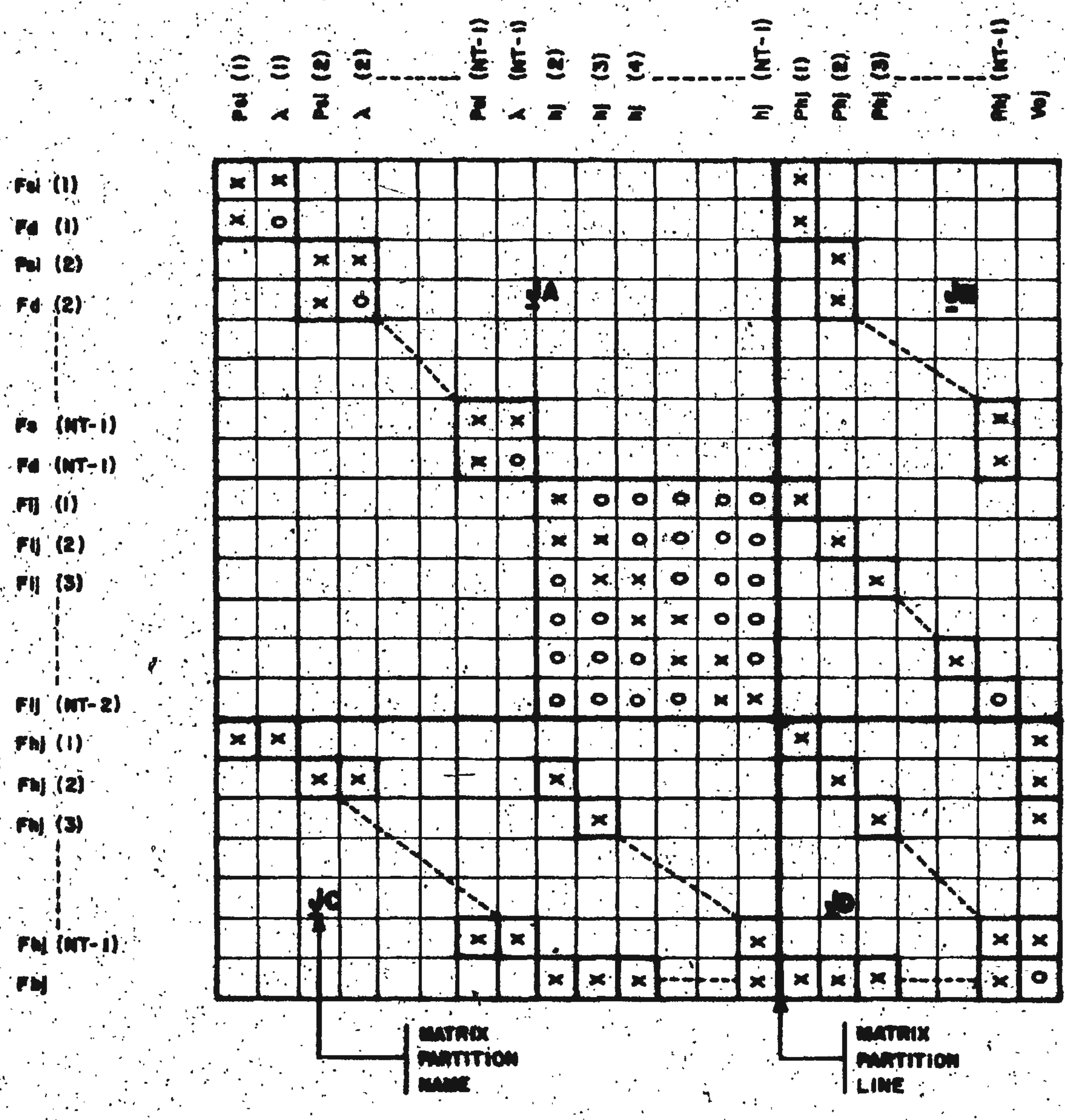
To accomplish this a method of matrix partitioning is employed. The Jacobian matrix, J, the nonlinear function vector, $f(x^m)$, and the unknown variables, Δx^m, are divided up or partitioned so that a more efficient procedure may be used.

To begin, the Jacobian matrix is structured as shown in Figure 3.1. This arrangement obtains the maximum benefit from the sparseness of the matrix. Figure 3.1 also details the partitioning of the matrix and identifies the submatrices.

Similarily, the vectors $f(x^m)$ and Δx^m have partitions which are structured according to Figure 3.2.

Equation (3.38) is now rewritten in terms of these partitioned matrices

FIGURE 3,1
THE JABOBIAN MATRIX AND PARTITION DETAILS



NOTE: i) X = PARTIAL DERIVATIVE OF THE FUNCTION (row element) WITH RESPECT TO THE VARIABLE (column element) SEE APPENDIX 'A'.

ii) ALL OTHER ELEMENTS ARE ZERO.

iii) i = 1 ⟶ Ns
j = 1 ⟶ Nh

FIGURE 3.2

STRUCTURE AND PARTITIONING OF $\underline{f}(x^m)$ AND $\Delta \underline{x}^m$

$$\underline{f}(x^m) = \begin{bmatrix} \underline{Y1} \\ \hdashline \underline{Y2} \end{bmatrix} \quad \underline{Y1} = \begin{bmatrix} fsi(1) \\ fd(1) \\ fsi(2) \\ fd(2) \\ \vdots \\ fsi(nt-1) \\ fd(nt-1) \\ fli(1) \\ fli(2) \\ \vdots \\ fli(nt-2) \end{bmatrix} \quad \underline{Y2} = \begin{bmatrix} fhj(1) \\ fhj(2) \\ \vdots \\ fhj(nt-1) \\ fbj \end{bmatrix}$$

$$\Delta \underline{x}^m = \begin{bmatrix} \Delta \underline{X1} \\ \hdashline \Delta \underline{X2} \end{bmatrix} \quad \Delta \underline{X1} = \begin{bmatrix} \Delta Psi(1) \\ \Delta \lambda(1) \\ \vdots \\ \Delta Psi(nt-1) \\ \Delta \lambda(nt-1) \\ \Delta Hj(2) \\ \vdots \\ \Delta Hj(nt-1) \end{bmatrix} \quad \Delta \underline{X2} = \begin{bmatrix} \Delta Phj(1) \\ \Delta Phj(2) \\ \vdots \\ \Delta Phj(nt-1) \\ \Delta Pbj \end{bmatrix}$$

$i = 1 \rightarrow Ns$

$j = 1 \rightarrow Nh$

$$\begin{bmatrix} \underline{Y}_1 \\ \underline{Y}_2 \end{bmatrix} + \begin{bmatrix} \underline{J}_A & \underline{J}_B \\ \underline{J}_C & \underline{J}_D \end{bmatrix} \times \begin{bmatrix} \underline{X}_1 \\ \underline{X}_2 \end{bmatrix} = 0 \qquad (3.40)$$

or in terms of $F(x^m)$

$$\begin{bmatrix} \underline{X}_1 \\ \underline{X}_2 \end{bmatrix} \times \begin{bmatrix} \underline{J}_A & \underline{J}_B \\ \underline{J}_C & \underline{J}_D \end{bmatrix} = - \begin{bmatrix} \underline{Y}_1 \\ \underline{Y}_2 \end{bmatrix} . \qquad (3.41)$$

Expanding equation (3.41) gives

$$\underline{J}_A \ \underline{X}_1 + \underline{J}_B \ \underline{X}_2 = - \underline{Y}_1 \qquad (3.42)$$

$$\underline{J}_C \ \underline{X}_1 + \underline{J}_D \ \underline{X}_2 = - \underline{Y}_2 \qquad (3.43)$$

Through algebraic manipulation the following relationships are obtained

$$\underline{X}_1 = - \underline{J}_A^{-1} [\underline{Y}_1 - \underline{J}_B \underline{X}_2] \qquad (3.44)$$

$$\underline{X}_2 = - [\underline{J}_D - \underline{J}_C \underline{J}_A^{-1} \underline{J}_B]^{-1} [\underline{Y}_2 - \underline{J}_C \underline{J}_A^{-1} \underline{Y}_1] . \qquad (3.45)$$

Closer examination of the submatrix, $\underline{J}_A$, reveals that it is block diagonal and, hence, its inverse may be found simply by inverting each subblock. This reduces the computational time substantially. The other inversion which is performed is on a matrix which has the same dimensions as the submatrix, $\underline{J}_D$.

As an example of the amount of reduction which results, a system with three hydro plants and three thermal plants is examined. The resulting Jacabian matrix has the dimensions [240 x 240] for an interval of 24 time periods.

This is a very large matrix to invert by method one. However, solution by method two requires inverting the blocks of J_A with dimensions [4 x 4] for I=1,N And [23 x 23] for I=N+1. The inversion required for $\underline{X}_1$, i.e. $[\underline{J}_D - \underline{J}_C \underline{J}_A^{-1} J_B]^{-1}$, has a dimension equal to $\underline{J}_D$ of [75 x 75]. It is evident that method two is superior and indeed this is reflected in the fact that a successful solution was obtained for all three test systems as detailed in Chapter IV.

3.4 GENERATION OF INITIAL ESTIMATES

In problems such as this where the solution is obtained using iterative techniques, a good initial estimate of the variables is essential if the number of iterations and, hence, the computational time is to be minimized.

The methods by which these initial values may be obtained are numerous. Therefore, it is necessary to make certain assumptions and develop particular models to produce an initial variable estimation procedure which contributes to the overall efficiency of the program. The method described herein produces satisfactory initial estimates of the variables.

The first assumption made is that the level of the reservoir is monitored and would be available as input to the program in the form of $h_i(t_o)$. Thus, the number of initial values to be determined is reduced by one.

Knowing $h_i(t_o)$, $\psi_1(h)$ is calculated using equation (3.7). By further assuming that $P\ (t_o)$ is constant over the time interval t_k, it is possible to write equation (3.27) as

$$T_f K\psi_i(h)\,[\alpha_{h_i} + \beta_{h_i} P_{h_i}(t_o) + \gamma_{h_i} P_{h_i}(t_o)^2] - b_i = 0\,, \quad (i=1\text{-}N_h)\,. \tag{3.46}$$

Rearranging gives

$$\alpha_{h_i} + [\beta_{h_i} P_{h_i}(t_o) + \gamma_{h_i} P_{h_i}(t_o)^2] - [\frac{b_i}{T_f K\psi_i(h)}] = 0\,. \tag{3.47}$$

Letting
$$b_{p_i} = \frac{b_i}{T_f K\psi_i(h)} \tag{3.48}$$

gives

$$[\gamma_{h_i}]\,P_{h_i}(t_o)^2 + [\beta_{h_i}]\,P_{h_i}(t_o) + [\alpha_{h_i} - b_{p_i}] = 0\,. \tag{3.49}$$

$P_{h_i}(t_o)$ is now obtained by applying the quadratic formula and taking the positive root of the equation.

Once $h_i(t_o)$ and $P_{h_i}(t_o)$ are known, $P_{s_i}(t_o)$ is quickly estimated from

$$P_{s_i}(t_o) = [\frac{P_D(t_o)}{N_s}][1 - \frac{P_D(t_o)}{\sum_{j=1}^{N_h} P_{h_j}(t_o)}]\,. \tag{3.50}$$

For simplicity it is assumed that the value for $P_{s_i}(t_k)$ is equal for all plants, $i=1\to N_s$ over the interval $[0,N_T-1]$, i.e. $P_{s_1}(0) = P_{s_2}(0) = P_{s_{N_s}}(0)$; $P_{s_1}(1) = P_{s_2}(1) = P_{s_{N_s}}(1)$; etc.. The same is assumed for $P_{h_i}(t_k)$ and $h_i(t_k)$. However, this does not imply that $P_{s_1}(0) = P_{s_1}(1)$, etc..

An initial value of $\lambda(t_k)$ is now calculated from

$$\lambda(t_o) = \frac{-\beta_{s_i} + 2\gamma_{s_i} P_{s_i}(t_o)}{C_{s_i} + 2 \sum_{j=1}^{N_g} B_{i_j} P_j(t_o)}, \qquad (i=1 \to N_s) \tag{3.51}$$

and ν_{0_i} is calculated directly from equation (3.35).

Once the values of the variables for all plants at time, t_o, are known, it remains only to determine values for the remainder of the study interval $[1, N_T-1]$.

The first method developed to find these variables assumes a flat profile such that

$$P_{h_i}(t_k) = P_{h_i}(t_o) \tag{3.52}$$

$$P_{s_j}(t_k) = P_{s_j}(t_o) \tag{3.53}$$

$$\lambda(t_k) = \lambda(t_o) \tag{3.54}$$

$$h_i(t_k) = h_i(t_o) \tag{3.55}$$

$$t_k = 1 \to N_T - 1, \quad i = 1 \to N_h, \quad j = 1 \to N_s$$

This method performs well, but on examination it is found that better results are obtained when an adjustment factor is used. Such a factor is based upon the ratio of the power demand at time instant, t_k, to the initial power demand. In other terms

$$\text{fact}(t_k) = \frac{P_D(t_o)}{P_D(t_k)} \tag{3.56}$$

The variables at time instant, t_k, are adjusted as follows

$$P_{h_i}(t_k) = P_{h_i}(t_o) \times \text{fact}(t_k) \qquad (3.57)$$

$$P_{s_j}(t_k) = P_{s_j}(t_o) \times \text{fact}(t_k) \qquad (3.58)$$

$$\lambda(t_k) = \lambda(t_o) \times \text{fact}(t_k) \qquad (3.59)$$

$$t_k = 1 \rightarrow N_T - 1, \quad j = 1 \rightarrow N_s, \quad i = 1 \rightarrow N_h$$

The profile for the net head over the interval was assumed flat due to its characteristically slow variation with time.

CHAPTER IV

PERFORMANCE EVALUATION

4.1 INTRODUCTION

In this chapter the results of the application of the computer algorithm to three hydro-thermal test systems are presented. The first system consists of one thermal unit and one variable-head hydro unit. Characterization tests are performed on this system to determine the algorithm's ability to adjust to variations in system parameters. The second test system examined contains two thermal plants and two variable-head hydro plants while the third system has two thermal units and five variable-head hydro units.

4.2 TEST SYSTEM ONE AND CHARACTERIZATION TESTS

4.2.1 Test System One Description

Test system one consists of one variable-head hydro plant and one thermal plant. Both of these supply power to a common grid over transmission lines with losses.

The models for the fuel cost, hydro plant performance and reservoir variation are represented by quadratic equations of the form

$$F(P_{s_1}) = \alpha_{s_1} + \beta_{s_1} P_{s_1}(t) + \gamma_{s_1} P_{s_1}(t)^2$$

$$\phi(P_{h_1}) = \alpha_{h_1} + \beta_{h_1} P_{h_1}(t) + \gamma_{h_1} P_{h_1}(t)^2$$

$$\psi(h_1) = a_{0_1} + a_{1_1} h_1(t) + a_{2_1} h_1(t)^2$$

where the quadratic coefficients are given as

$\alpha_{s_1} = 1.0$ $\alpha_{h_1} = 1.0$

$\beta_{s_1} = 2.7$ $\beta_{h_1} = 0.1$

$\gamma_{s_1} = 3.0 \times 10^{-3}$ $\gamma_{h_1} = 1.0 \times 10^{-4}$

$a_{0_1} = 1.0$

$a_{1_1} = -0.2237$

$a_{2_1} = 1.0 \times 10^{-3}$

The transmission loss coefficients are

$B_{1_0} = 0$ $B_{2_0} = 0$

$B_{s_s} = 0$ $B_{s_H} = 0$

$B_{H_s} = 0$ $B_{H_H} = 1.43 \times 10^{-4}$

$K_{L_0} = 0$

The data for the reservoir is

Area = 10 mi^2

Available water = 2.5×10^9 cf

Net head (initial) = 205 ft.

Natural inflow = 12×10^3 cfs

The test interval covers a 24 hour period and is subdivided into 24-1 hour discrete time instances.

4.2.2 Computational Results

For test system one the program converged in seven iterations to an error criterion of 1.0×10^{-4} (see Figure 4.2-1) and required 33.25 seconds of cpu time for solution.

The optimal dispatch schedule is obtained and presented in Figure 4.2-2 and Table 4.2-1. It is observed that the hydro plant produced on an average 85% of the total power demand plus transmission losses. This high percentage is in keeping with the guidelines of the program criteria to reduce thermal generation and, hence, fuel cost to a minimum. The calculated daily fuel cost for the system under these conditions is $9,844.70.

The variation in the incremental cost of power, $\lambda(t)$, is shown in Figure 4.2-3. The curve shows that $\lambda(t)$ and $P_d(t)$ vary with time in the same manner. That is, the curves have the same shape. This is to be expected since the incremental cost of power should increase or decrease in accordance with the variation in the power demand.

The water worth coefficient curve, $\nu(t)$, is detailed in Figure 4.2-4. The variation shown complies with earlier predictions that the optimization procedure conserves water at the start by keeping $\lambda(t)$ high at first and then slowly decreasing it.

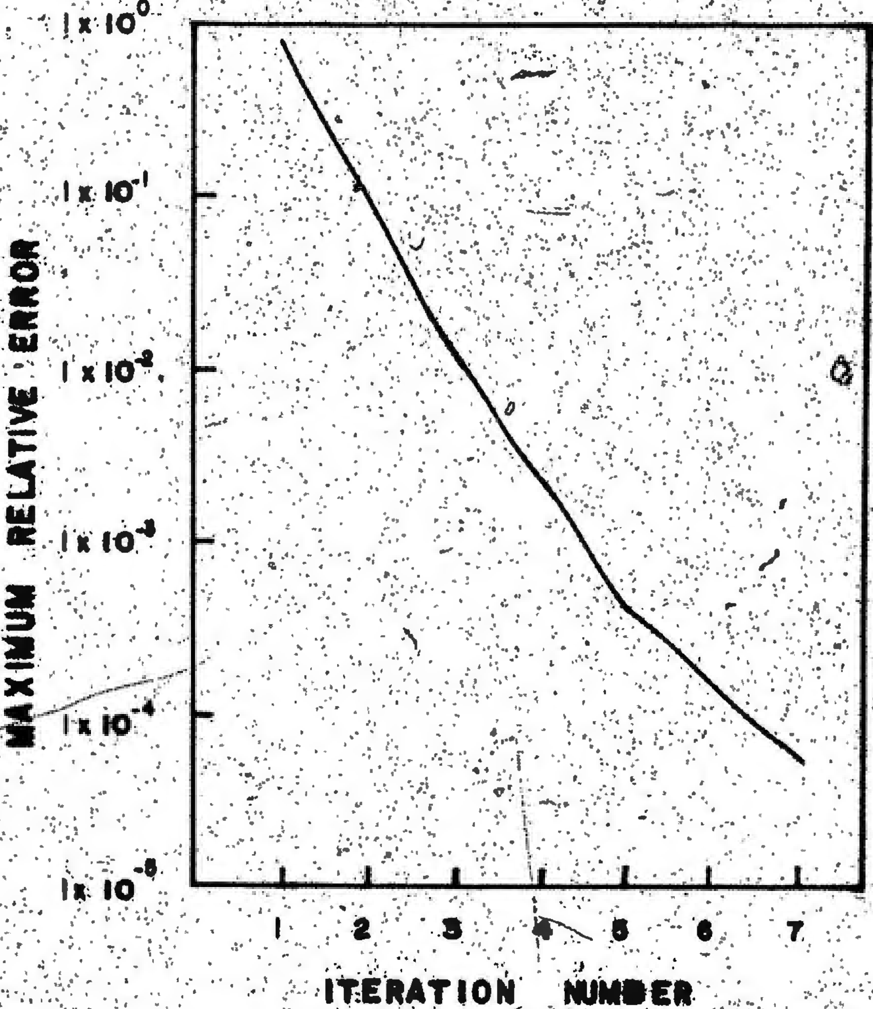


FIGURE 4.2-1. Maximum Relative Error Versus Number of Iterations.

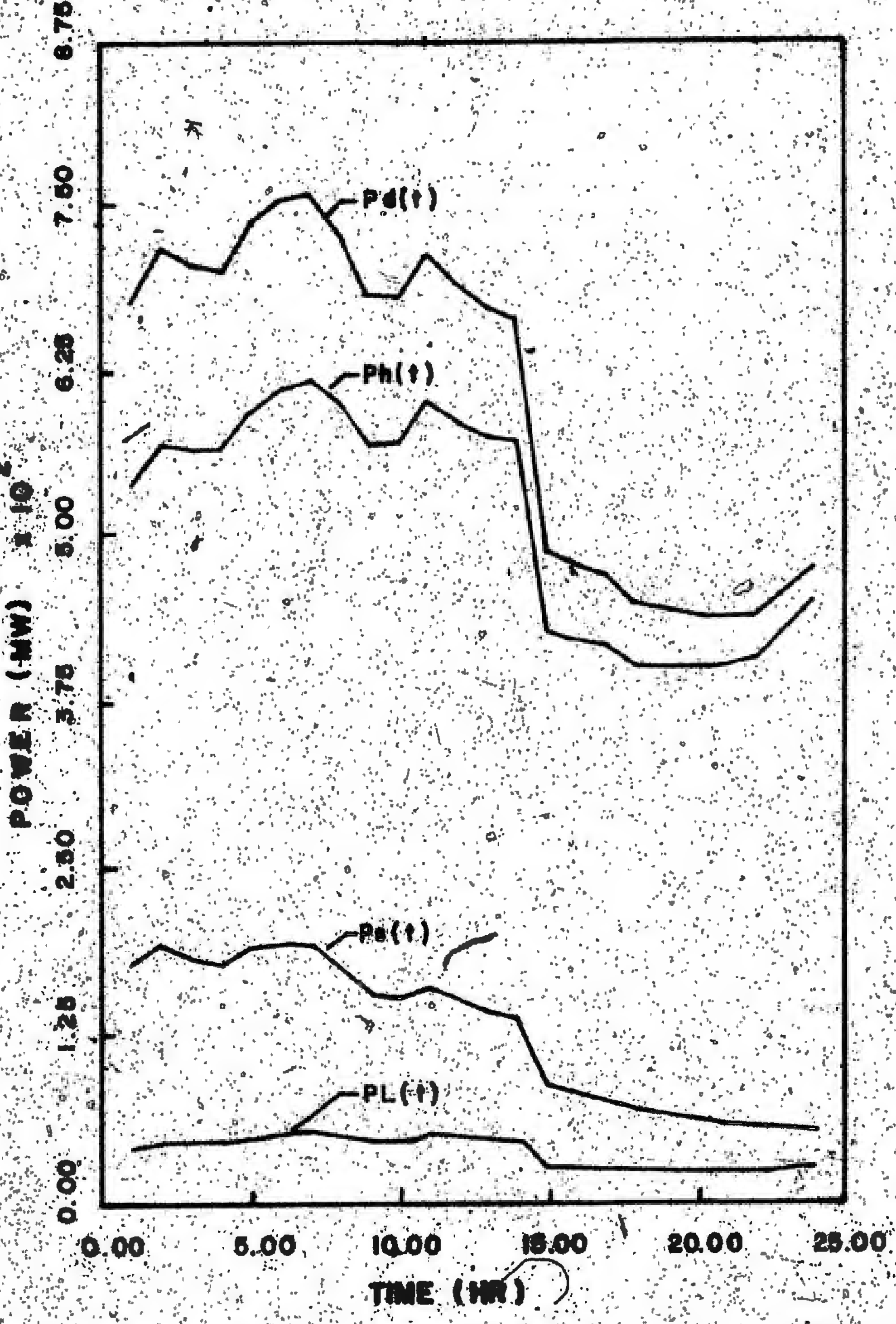


FIGURE 4.2-2. Optimal Dispatch Schedule.

TABLE 4.2-1

OPTIMUM DISPATCH SCHEDULE. TABULATED DATA.

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	HYDRO PLANT NO. 1 MW	THERMAL PLANT NO. 1 MW
1	681	42.07	542.42	180.65
2	722	46.82	572.19	196.63
3	708	46.03	567.33	186.69
4	703	46.16	568.14	181.01
5	741	51.17	598.18	193.98
6	758	53.98	614.39	197.59
7	761	55.12	620.83	195.28
8	732	52.05	603.34	180.72
9	685	46.71	571.50	160.20
10	683	47.10	573.92	156.18
11	716	52.29	604.71	163.58
12	692	49.53	588.53	153.00
13	675	47.75	577.85	144.90
14	666	47.08	573.77	139.31
15	491	26.18	427.89	89.29
16	481	25.58	422.96	83.63
17	473	25.17	419.54	78.63
18	451	23.34	403.98	70.35
19	448	23.42	404.67	66.75
20	443	23.28	403.52	62.76
21	441	23.46	405.01	59.45
22	444	24.17	411.14	57.03
23	461	26.56	430.98	56.59
24	480	29.43	453.69	55.75

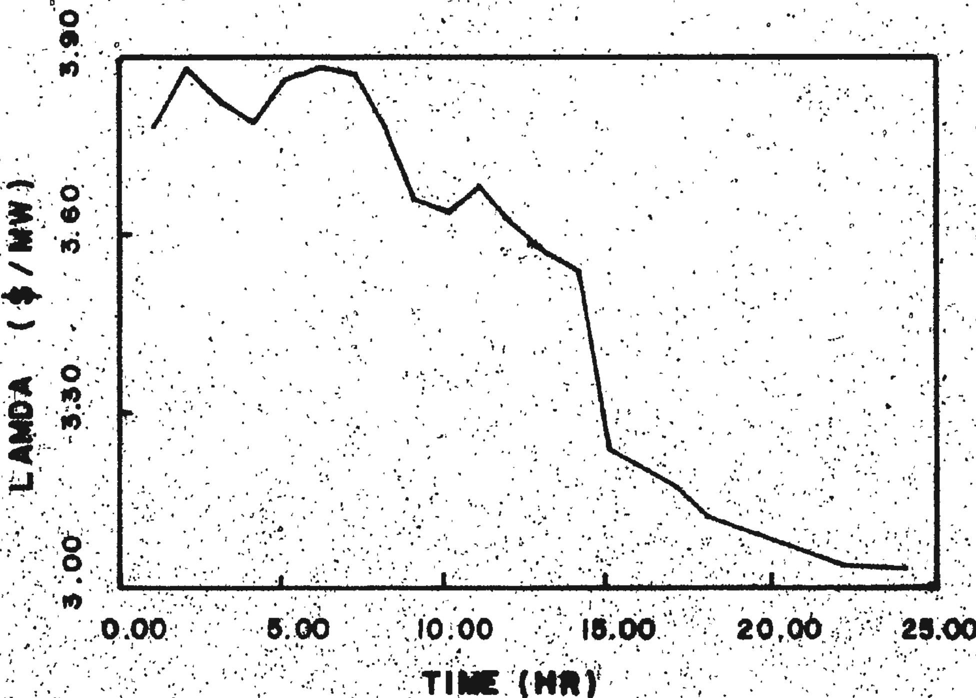


FIGURE 4.2-3. Variation in λ(t).

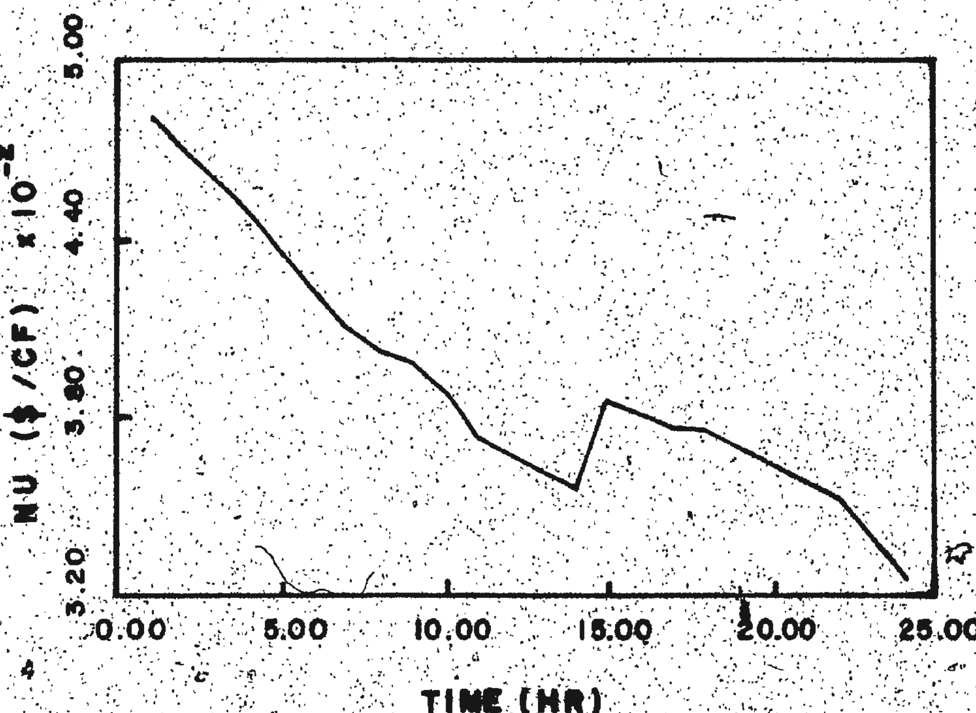


FIGURE 4.2-4. Variation in ν(t).

The values for $\lambda(t)$ and $\nu(t)$ are tabulated in Table 4.2-2. The variation in net head is presented in Figure 4.2-5 and Table 4.2-3. Since the inflow to the reservoir is constant, the value of the net head decreases as expected.

Actual computer print-out for this system is found in Appendix C.

4.2.3 Characterization Tests Description

The characterization tests are carried out on the system described in 4.2.2 to evaluate the computer algorithm.

Three tests are performed on the system. In the first test the available water is varied. In the second test the power demand, $P_d(t)$, is changed while in the third test the natural inflow, $i(t)$, to the reservoir is altered. The tests and results are described herein.

4.2.4 Characterization Test One

In this first test the power demand, $P_d(t)$, and the reservoir's natural inflow are held constant while varying the available water, B. Then, the effects of the changing B on the head variation, the water worth coefficient, $\nu(t)$ and the daily fuel cost are examined.

Figure 4.2-6 shows how the head variation was affected by altering B. This head variation refers to the difference between the minimum and maximum head levels over the test interval. As expected, the head variation increased almost linearly with increases in B. The reason for this can be deduced from Figure 4.2-7 which shows that as B increases the water worth coefficient, $\nu(t)$ and, hence, the cost of water decreases.

TABLE 4.2-2

INCREMENTAL COST OF POWER AND

WATER-WORTH COEFFICIENTS. TABULATED DATA.

TIME PERIOD HR	NU PLANT NO. 1 $/CF	LAMBDA $/MW
1	0.04812	3.784
2	0.04698	3.880
3	0.04589	3.820
4	0.04481	3.786
5	0.04347	3.864
6	0.04219	3.886
7	0.04102	3.872
8	0.04025	3.784
9	0.03983	3.661
10	0.03888	3.637
11	0.03734	3.682
12	0.03681	3.618
13	0.03622	3.569
14	0.03551	3.536
15	0.03854	3.236
16	0.03809	3.202
17	0.03761	3.172
18	0.03751	3.122
19	0.03696	3.100
20	0.03647	3.077
21	0.03591	3.057
22	0.03520	3.042
23	0.03400	3.040
24	0.03267	3.034

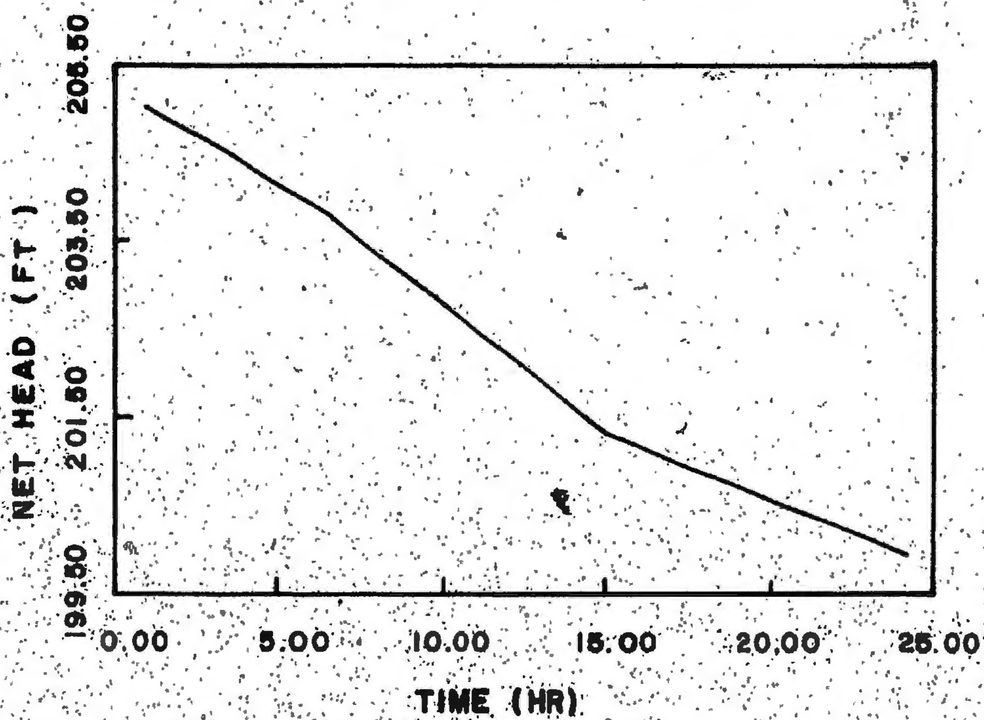


FIGURE 4.2-5. Variation in h(t).

TABLE 4.2-3

HEAD VARIATIONS.

TABULATED DATA.

TIME PERIOD HR	NET HEAD PLANT NO. 1 FT
1	205.00
2	204.81
3	204.60
4	204.38
5	204.15
6	203.90
7	203.62
8	203.32
9	203.04
10	202.78
11	202.51
12	202.21
13	201.91
14	201.62
15	201.32
16	201.17
17	201.02
18	200.86
19	200.72
20	200.58
21	200.44
22	200.29
23	200.13
24	199.95

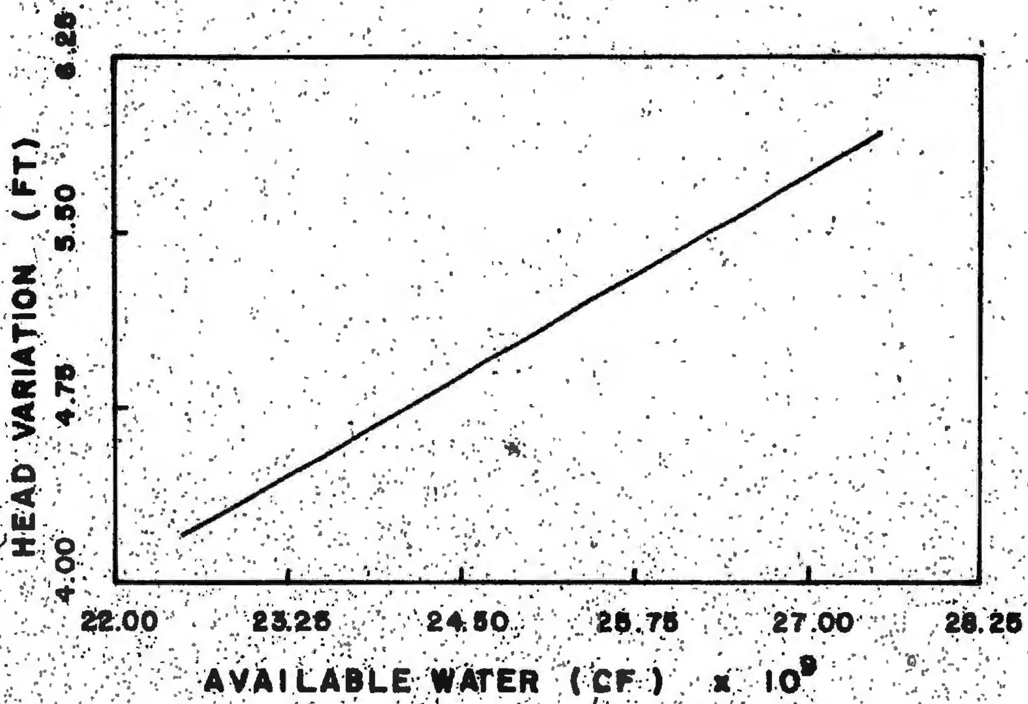


FIGURE 4.2-6. Variation in Head with Increased B(t).

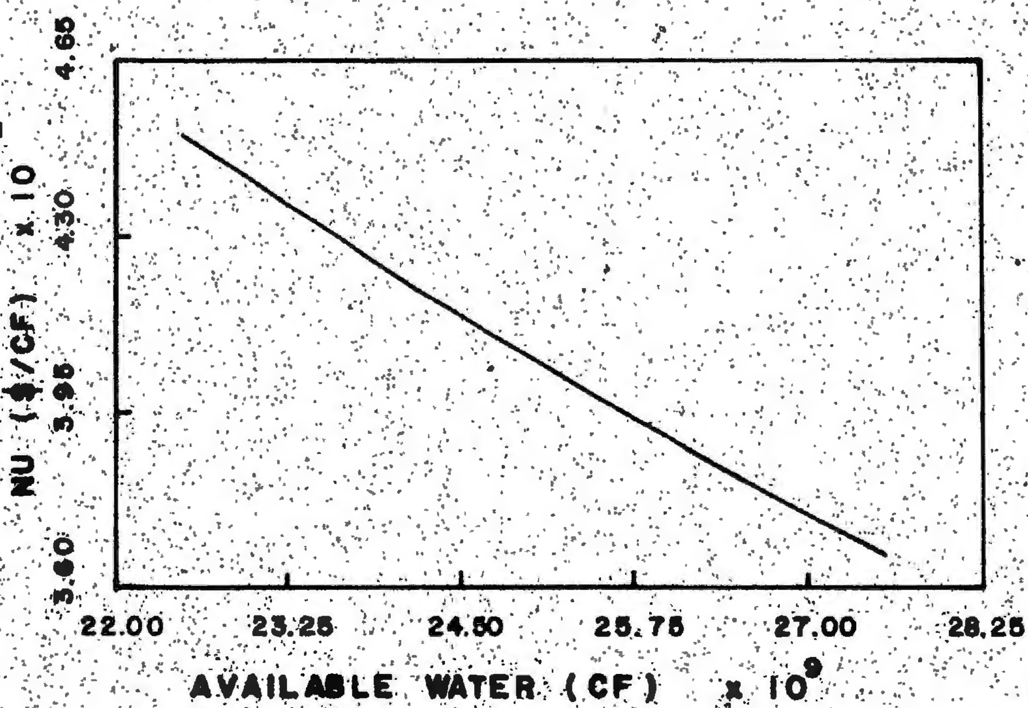


FIGURE 4.2-7. Variation in $\nu(t)$ with increased B(t).

This means that more hydro power will be generated and that the thermal output and, hence, daily fuel costs will be reduced. Such is the case as Figure 4.2-8 clearly indicates.

Data for the three curves are tabulated in Table 4.2-4.

4.2.5 Characterization Test Two

In this second test the natural inflow and available water is held constant and the power demand is varied to determine the effects on the head variations, incremental cost of power, $\lambda(t)$, and the daily fuel cost.

The head variation remains fairly constant (Figure 4.2-9) as $P_d(t)$ increases since the inflow and B are held constant which results in an increase in $\nu(t)$.

The incremental cost of power $\lambda(t)$ increases as $P_d(t)$ increases which is exactly as expected. This result is demonstrated by Figure 4.2-10.

Examination of Figure 4.2-11 shows that the daily fuel cost also increases with increases in $P_d(t)$. This is not unexpected since constant hydro plant characteristics result in no increase in hydro output. Therefore, the slack must be taken up by the thermal unit and, hence, an increase in the daily fuel cost.

The data for these curves is found in Table 4.2-5.

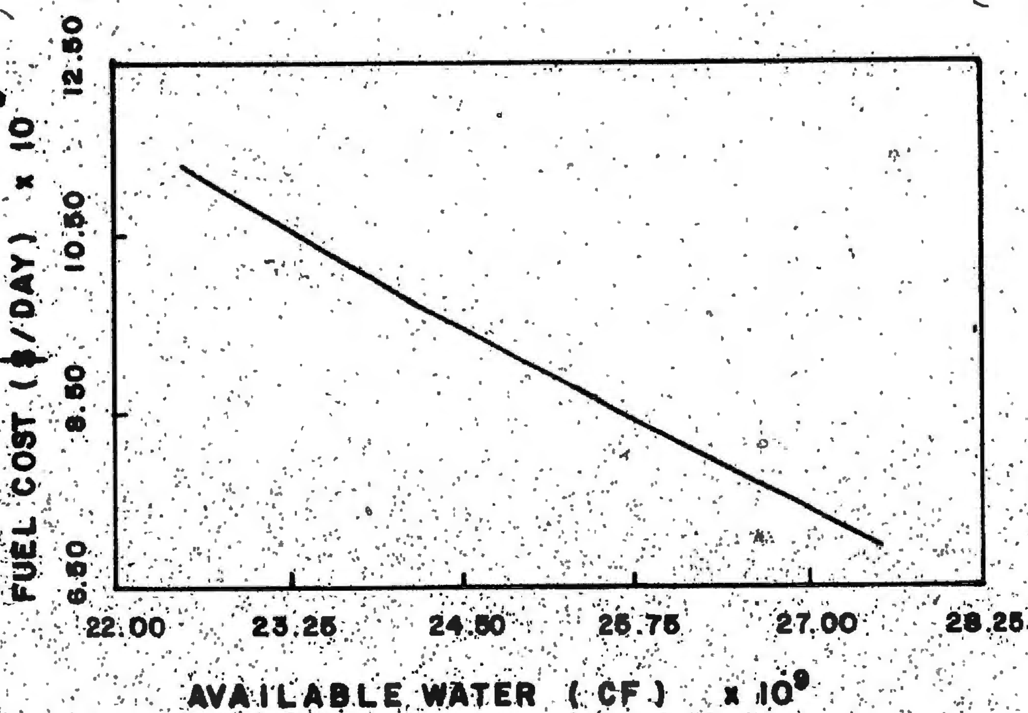


FIGURE 4.2-8. Variation in Daily Fuel Costs with Increased B(t).

TABLE 4.2-4

CHARACTERIZATION TEST.

TABULATED RESULTS.

B cf x 10^9	ν (av) \$/cf x 10^{-2}	HEAD VARIATION FT	FUEL COST \$/DAY
22.5	4.503	4.21	11325.96
23.0	4.407	4.38	10839.67
23.5	4.314	4.55	10365.86
24.0	4.224	4.72	9904.07
24.5	4.137	4.90	9453.86
25.0	4.052	5.07	9014.82
25.5	3.977	5.24	8586.54
26.0	3.891	5.41	8186.66
26.5	3.814	5.58	7760.80
27.0	3.739	5.76	7362.64
27.5	3.666	5.93	6973.84

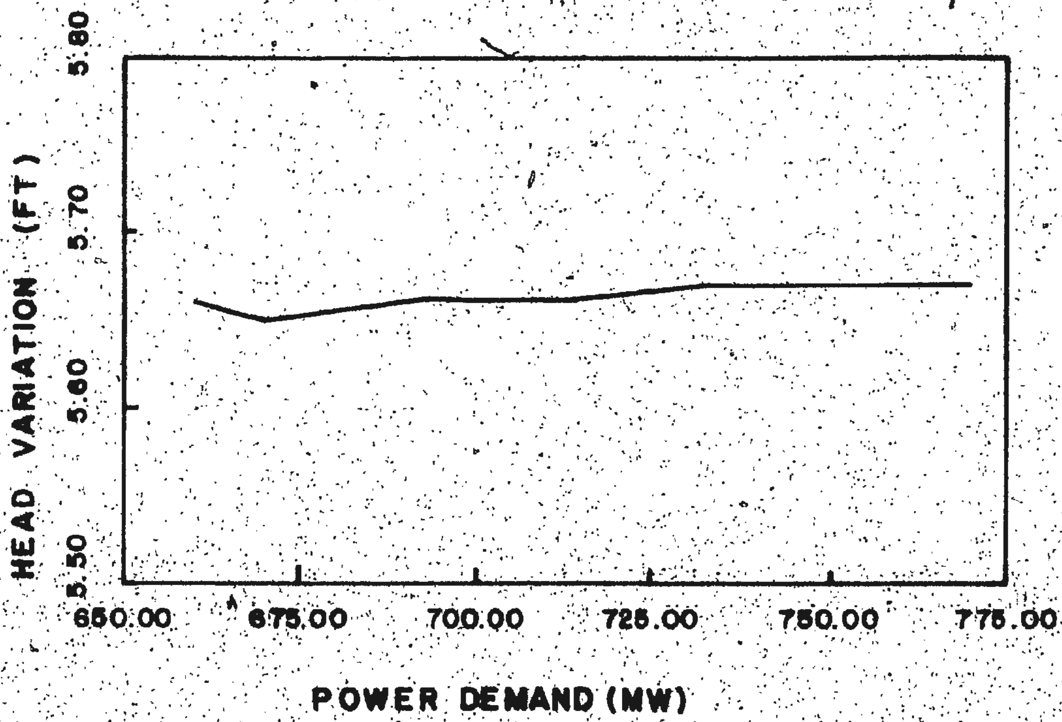


FIGURE 4.2-9. Variation in Head Variation with Increased $P_d(t)$.

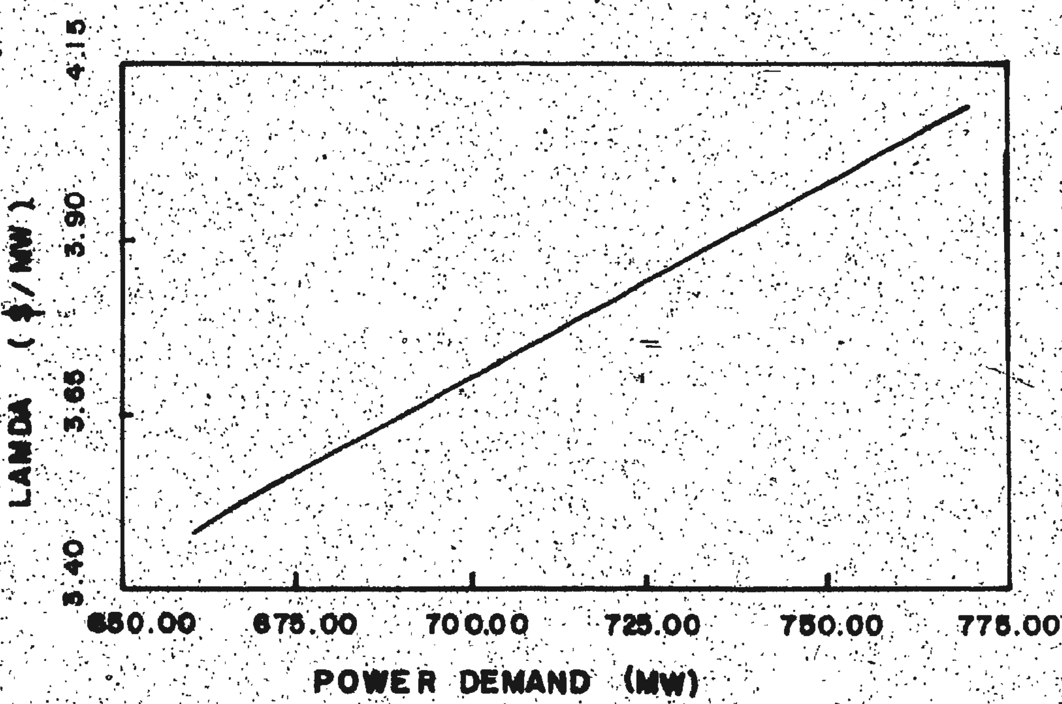


FIGURE 4.2-10. Variation in $\lambda(t)$ with Increased $P_d(t)$.

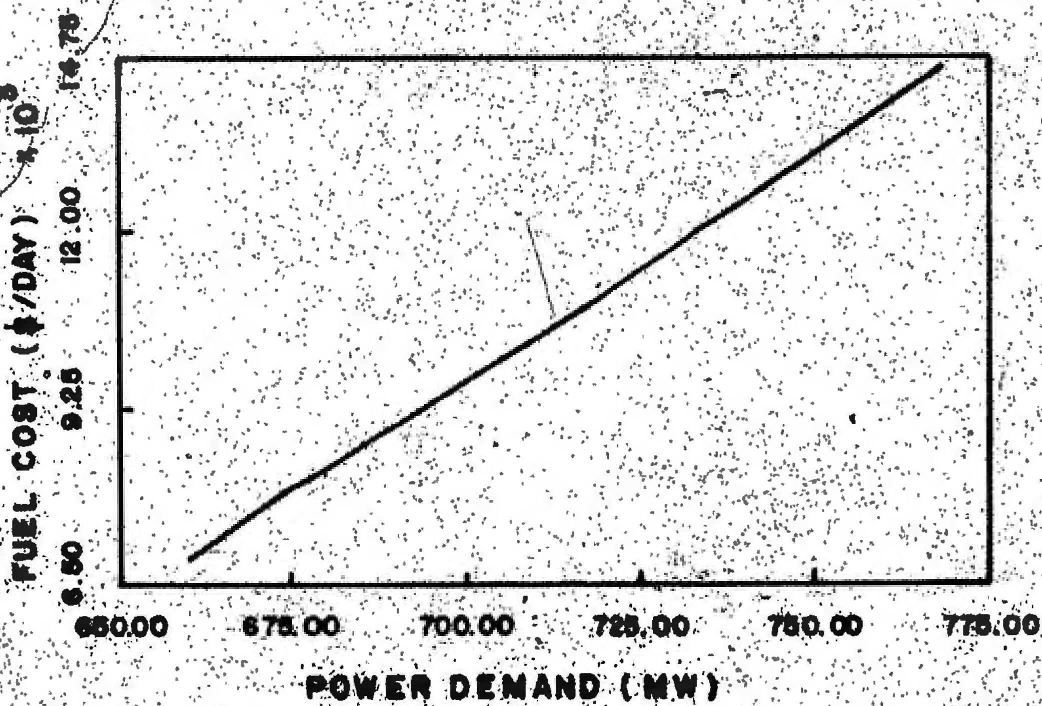


FIGURE 4.2-11. Variation in Fuel Cost with Increased $P_d(t)$.

TABLE 4.2-5

CHARACTERIZATION TEST TWO.

TABULATED RESULTS.

POWER DEMAND MW	HEAD VARIATION FT	λ (av) $/MW	FUEL COST $/DAY
660	5.66	3.483	6897.66
670	5.65	3.545	7685.68
690	5.66	3.654	9011.92
710	5.66	3.763	10377.32
730	5.67	3.872	11781.92
750	5.67	3.981	13225.74
770	5.67	3.091	14708.82

4.2.6 Characterization Test Three

In this final test, the power demand and available water are held constant and the natural inflow to the reservoir is varied. The effects on the head variation and the daily fuel cost are presented in Figures 4.2-12 and 4.2-13, respectively.

Figure 4.2-12 shows that as the inflow is increased, the head variation decreases to such a point where the reservoir water must be spilled. This is not surprising and further, such increased head should result in a lowering of the daily cost which is indeed what happens as is shown in Figure 4.2-13.

Table 4.2-6 presents the data for the curves.

4.3 TEST SYSTEM TWO

4.3.1 Test System Two Description

Test system two consists of two thermal and two variable-head hydro plants. All units are supplying power to a common grid over transmission lines with losses.

The quadratic models used to represent the fuel costs, hydro plants performances, and reservoir variations are of the same form as those for test system one. The quadratic coefficients are as follows

$$\alpha_{s_1} = 1.0 \qquad \alpha_{s_2} = 1.0$$

$$\beta_{s_1} = 2.7 \qquad \beta_{s_2} = 2.733$$

$$\gamma_{s_1} = 3.0 \times 10^{-3} \qquad \gamma_{s_2} = 2.998 \times 10^{-3}$$

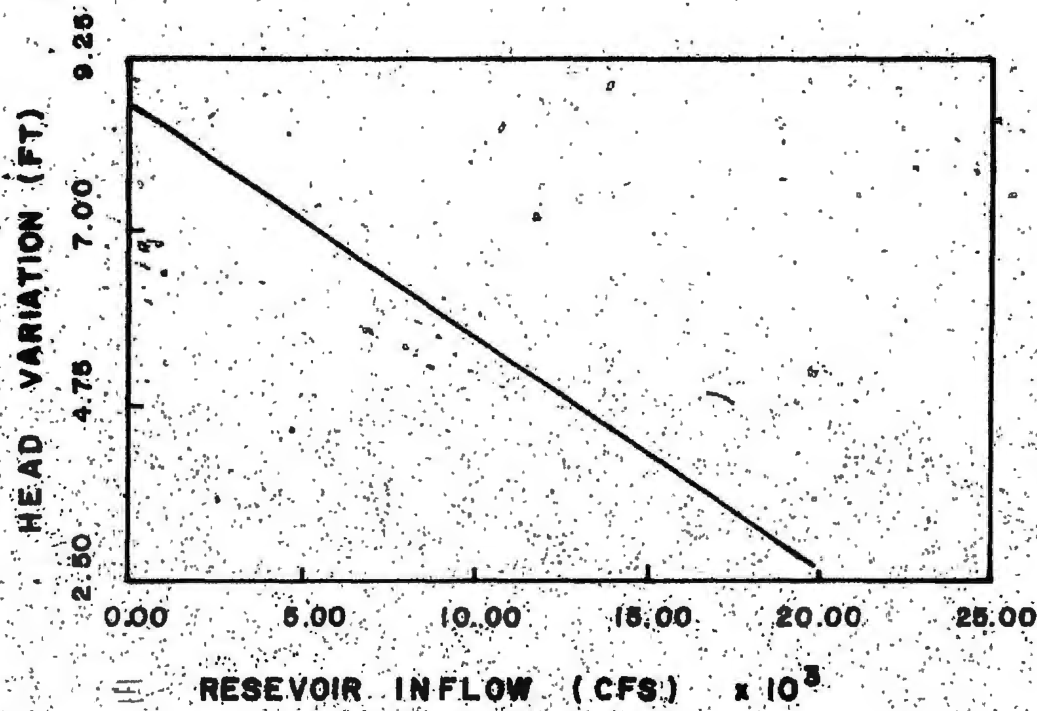


FIGURE 4.2-12. Variation in Head with Increased Inflow.

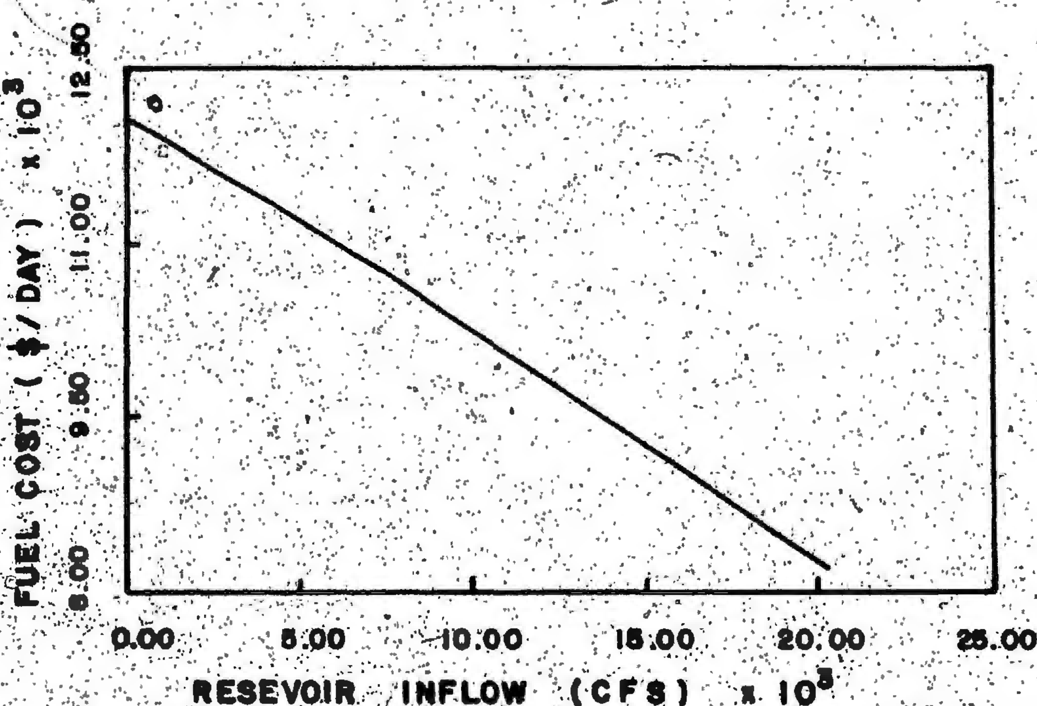


FIGURE 4.2-13. Variation in Daily Fuel Costs with Increased Inflow.

TABLE 4.2-6

CHARACTERIZATION TEST THREE.

TABULATED DATA.

RESERVOIR INFLOW CFS	HEAD VARIATION FT	FUEL COST $/DAY
0	8.63	12071.22
5	7.13	11182.63
10	5.64	10247.18
15	4.14	9262.04
20	2.65	8224.55

$\alpha_{h_1} = 1.0$ $\alpha_{h_2} = 1.0$

$\beta_{h_1} = 0.1$ $\beta_{h_2} = 0.998 \times 10^{-1}$

$\gamma_{h_1} = 1.0 \times 10^{-4}$ $\gamma_{h_2} = 1.002 \times 10^{-4}$

$a_{0_1} = 1.0$ $a_{0_2} = 1.0$

$a_{1_1} = -0.2237$ $a_{1_2} = -0.2243$

$a_{2_1} = 1.0 \times 10^{-3}$ $a_{2_2} = 0.993 \times 10^{-3}$

The transmission loss coefficient matrix is

$$B_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.43 \times 10^{-5} & 0 \\ 0 & 0 & 0 & 1.43 \times 10^{-5} \end{bmatrix}$$

Other loss coefficients are

$B_{1_0} = 0$ $B_{3_0} = 0$

$B_{2_0} = 0$ $B_{4_0} = 0$

$K_{L_0} = 0$

The data for reservoir one is

Area = 10 mi^2

Available water = 2.5×10^9 cf

Net head (initial) = 205 ft

Natural inflow = 5.5×10^3 cfs .

The data for reservoir two is

Area = 18 mi^2

Available water = 2.25×10^9 cf

Net head (initial) = 206 ft

Natural inflow = 11×10^3 cfs .

Again the test interval covered a 24 hour period which was subdivided into 24, one hour intervals.

4.3.2 Computational Results

For test system two the program converged in 13 iterations to an error criterion of 1×10^{-4} (see Figure 4.3-1) and required 731 seconds of cpu time for solution.

The optimal dispatch schedule is obtained and presented in Figure 4.3-2 and Tables 4.3-1 and 4.3-2. Again, the thermal generation was minimized and the hydro generation accounted for an average of 80% of the total power requirements. The daily fuel costs for thermal plants one and two were found to be \$11,764.85 and \$11,413.87, respectively.

Figure 4.3-3 is an enlarged version of the $P_h(t)$ portion of the composite graph. Note that as time progresses $P_{h_2}(t)$ carries more of

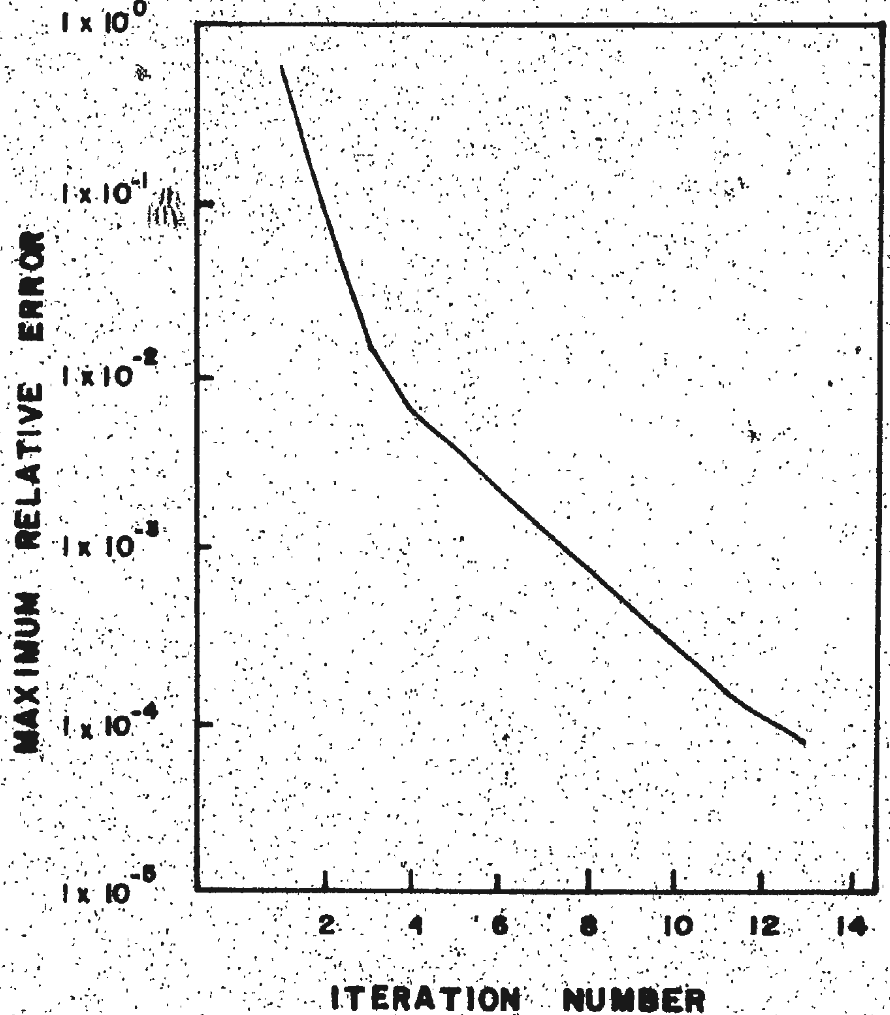


FIGURE 4.3-1. Maximum Relative Error Versus Number of Iterations.

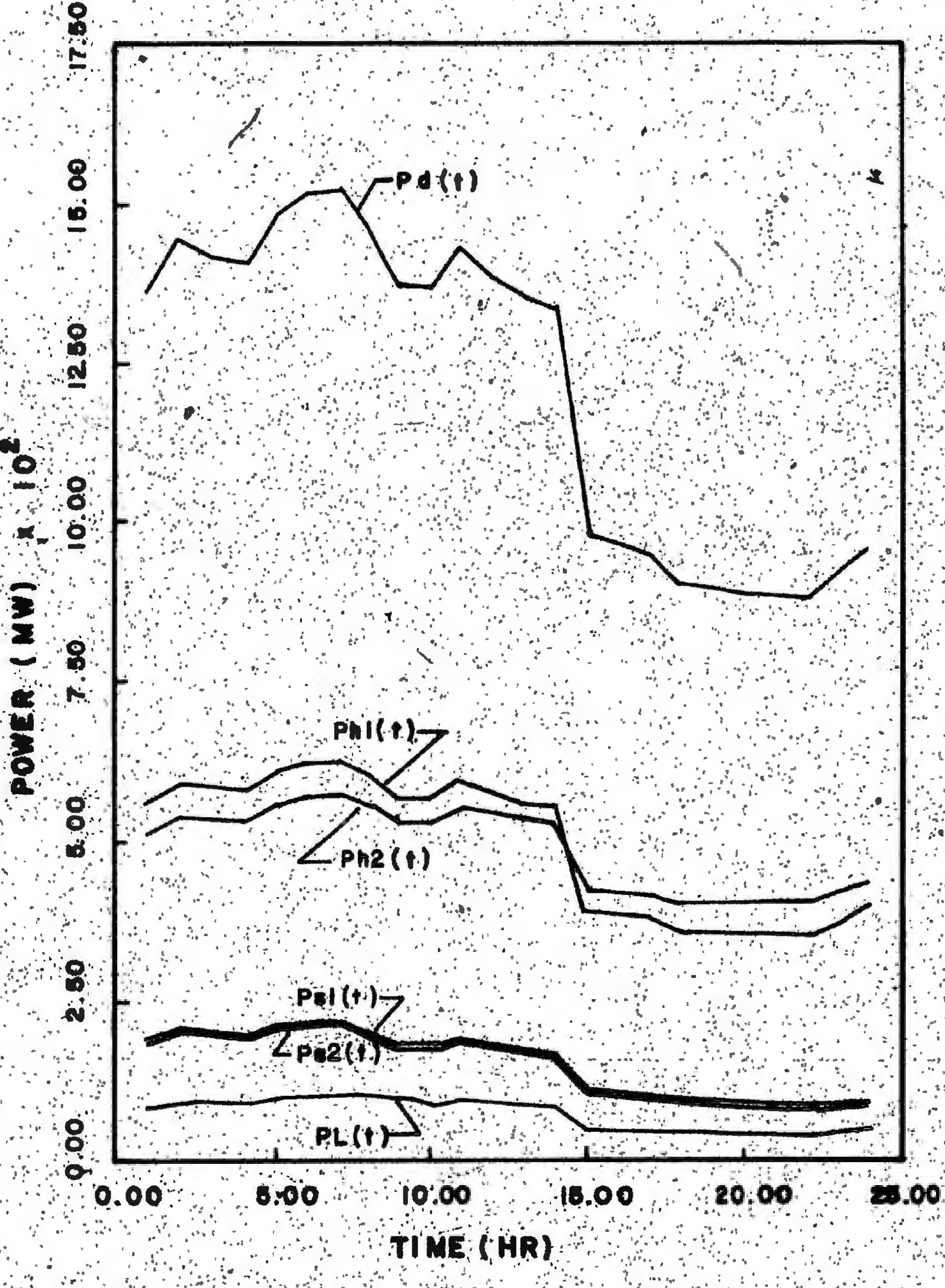


FIGURE 4.3-2. Optimum Dispatch Schedule.

TABLE 4.3-1

OPTIMUM DISPATCH SCHEDULE.

TABULATED RESULTS.

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW
1	1362	82.29
2	1444	90.86
3	1416	97.02
4	1406	88.26
5	1482	97.02
6	1516	101.54
7	1522	102.90
8	1464	96.60
9	1370	86.25
10	1366	86.33
11	1432	94.88
12	1384	89.38
13	1350	85.65
14	1332	83.89
15	982	47.43
16	962	46.03
17	946	44.97
18	902	41.53
19	896	41.34
20	886	40.81
21	882	40.78
22	888	41.63
23	922	45.11
24	970	50.31

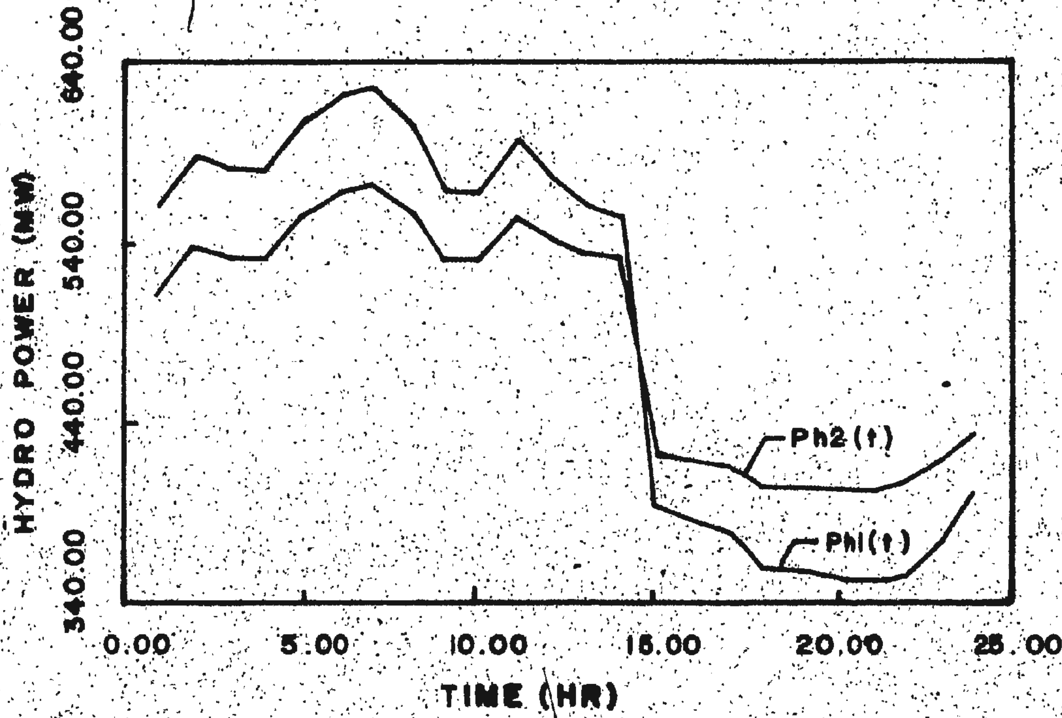


FIGURE 4.3-3. Variation in $P_h(t)$.

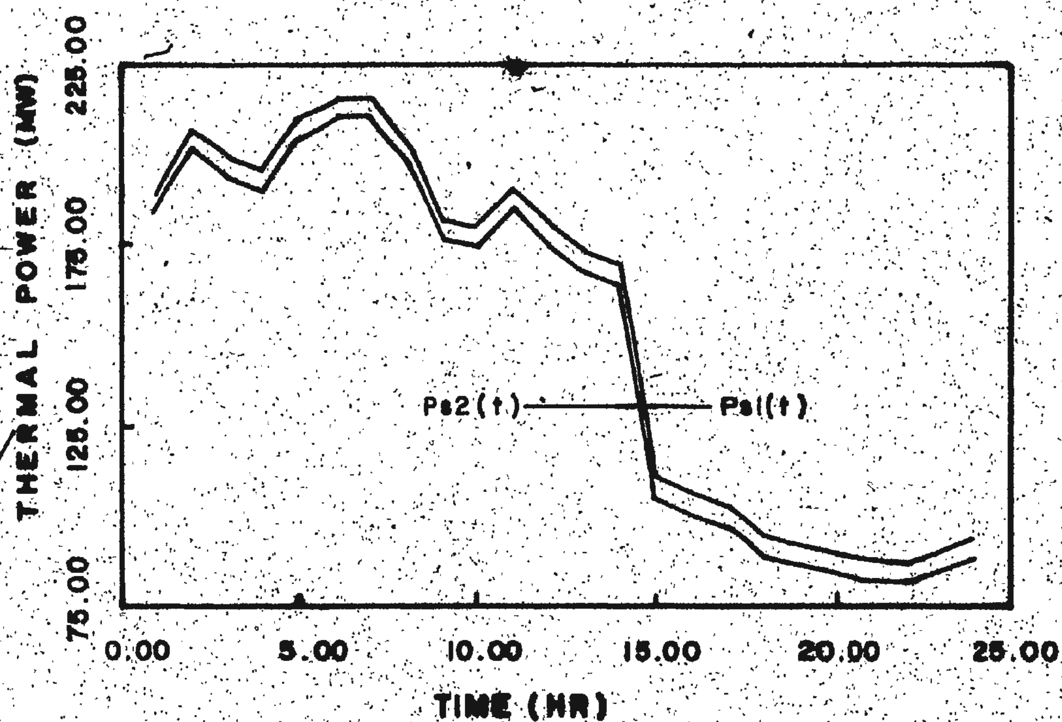


FIGURE 4.3-4. Variation in $P_s(t)$.

TABLE 4.3-2.

OPTIMUM DISPATCH SCHEDULE.

TABULATED RESULTS.

TIME PERIOD HR	HYDRO PLANT NO. 1 MW	HYDRO PLANT NO. 2 MW	THERMAL PLANT NO. 1 MW	THERMAL PLANT NO. 2 MW
1	560.26	511.42	188.99	183.61
2	588.05	538.14	207.02	201.65
3	580.83	531.71	198.75	193.38
4	579.28	530.72	194.82	189.45
5	607.54	556.23	210.31	204.95
6	621.60	568.92	216.19	210.83
7	625.48	573.02	215.88	210.52
8	604.22	557.17	202.29	196.92
9	567.74	529.95	181.97	176.59
10	567.31	530.94	179.73	174.35
11	597.17	553.98	190.55	185.18
12	576.13	541.41	180.62	175.23
13	560.92	533.24	173.44	168.05
14	552.81	530.14	169.17	163.78
15	392.55	421.39	110.46	105.03
16	384.64	417.06	105.88	100.45
17	378.25	414.05	102.06	96.62
18	358.94	401.98	94.02	88.58
19	356.84	402.22	91.86	86.42
20	352.70	401.21	89.17	83.73
21	351.30	402.18	87.37	81.93
22	354.86	406.40	86.91	81.46
23	373.60	419.35	89.80	84.35
24	402.14	436.03	93.79	88.35

the load than $P_{h_1}(t)$. This is as predicted, since reservoir two has a larger area and greater inflow. The higher starting value of $P_{h_1}(t)$ can be attributed to the larger starting value for the available amount of water.

Figure 4.3-4 again is an expanded portion of the composite showing $P_s(t)$. As expected, the curves for $P_{s_1}(t)$ and $P_{s_2}(t)$ are very similar in shape as well as values.

Figures 4.3-5 and 4.3-6 present the variations in $\lambda(t)$ and $\nu(t)$, respectively, with time. As for the two plant system, $\lambda(t)$ varies with the power demand and $\nu(t)$ decreases as the net head, $h(t)$, decreases. It is important to note that the decrease in $\nu_2(t)$ is less steep than $\nu_1(t)$. The reason for this is evident from Figure 4.3-7 which shows the net head variations for the two reservoirs. The decrease in $h_2(t)$ is less than that of $h_1(t)$, hence, the difference in $\nu_2(t)$ and $\nu_1(t)$.

This lesser decrease in $h_1(t)$ is due to the larger inflow and greater area of reservoir one. The tabulated data for these curves is given in Tables 4.3.3 and 4.3.4.

Actual computer print-out for this system is found in Appendix C.

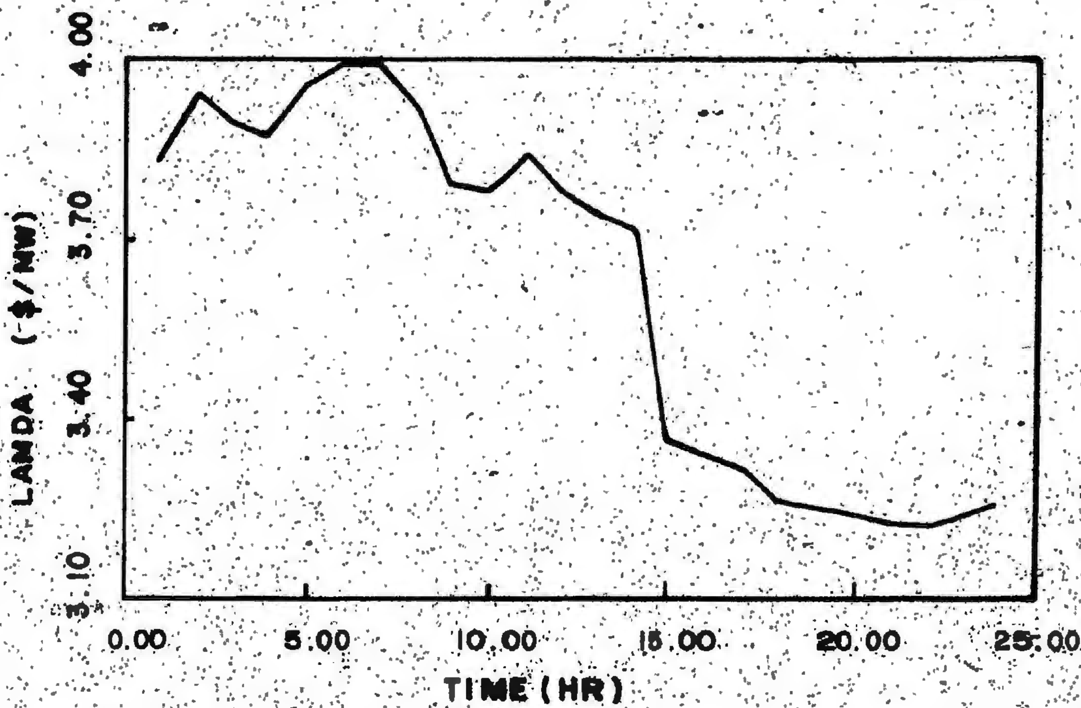


FIGURE 4.3-5. Variation in λ(t).

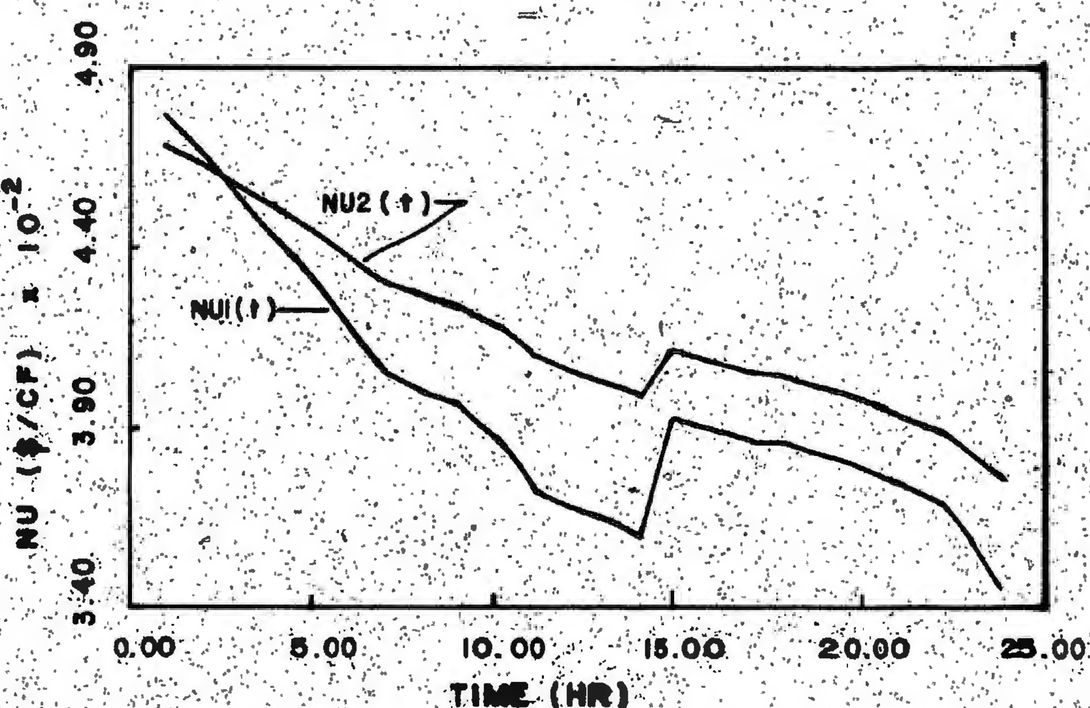


FIGURE 4.3-6. Variation in ν(t).

TABLE 4.3-3

VARIATIONS IN λ(t) AND ν(t).

TABULATED RESULTS.

TIME PERIOD HR	ν(t) PLANT NO. 1 $/CF	ν(t) PLANT NO. 2 $/CF	λ(t) $/MW
1	0.04769	0.04684	3.834
2	0.04651	0.04624	3.942
3	0.04541	0.04566	3.893
4	0.04433	0.04509	3.869
5	0.04300	0.04438	3.962
6	0.04175	0.04370	3.997
7	0.04063	0.04307	3.995
8	0.03993	0.04263	3.914
9	0.03[illegible]61	0.04236	3.792
10	0.03874	0.04183	3.778
11	0.03727	0.04101	3.843
12	0.03688	0.04066	3.784
13	0.03643	0.04027	3.741
14	0.03586	0.03983	3.715
15	0.03921	0.04116	3.363
16	0.03889	0.04086	3.335
17	0.03856	0.04055	3.312
18	0.03861	0.04042	3.264
19	0.03822	0.04007	3.251
20	0.03790	0.03976	3.235
21	0.03751	0.03940	3.224
22	0.03697	0.03897	3.221
23	0.03591	0.03834	3.239
24	0.03447	0.03760	3.263

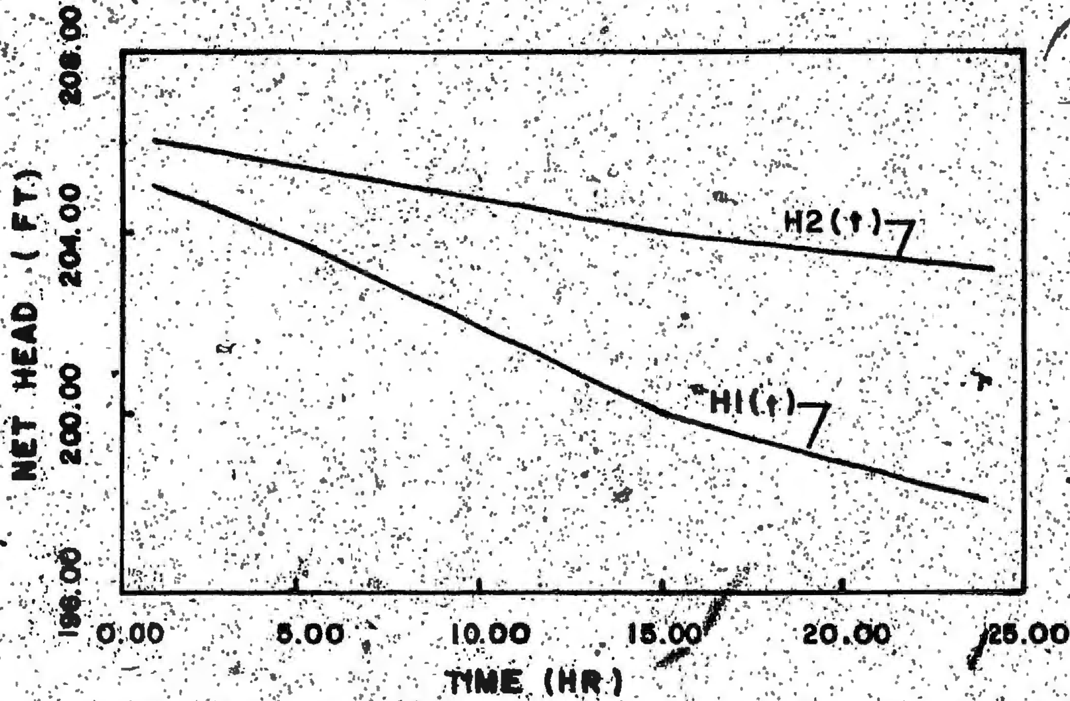


FIGURE 4.3-7. h(t) Versus t.

TABLE 4.3-4

NET HEAD VARIATIONS.

TABULATED RESULTS.

TIME PERIOD HR	NET HEAD PLANT NO. 1 FT	NET HEAD PLANT NO. 2 FT
1	205.00	206.00
2	204.71	205.88
3	204.40	205.75
4	204.08	205.62
5	203.75	205.48
6	203.39	205.34
7	203.01	205.18
8	202.61	205.01
9	202.22	204.86
10	201.86	204.72
11	201.50	204.57
12	201.09	204.41
13	200.70	204.26
14	200.31	204.11
15	199.93	203.96
16	199.70	203.86
17	199.48	203.77
18	199.26	203.68
19	199.05	203.60
20	198.85	203.51
21	198.65	203.43
22	198.44	203.34
23	198.23	203.25
24	198.00	203.15

4.4 TEST SYSTEM THREE

4.4.1 Test System Three Description

Since no data is available for larger systems, the basic hydro unit used for test one and the two thermal units of test two were scaled so that the algorithm could be tested on a system containing five hydro units and two thermal units.

The basic data for the system is found in the preceeding two test descriptions and the system power demand is found in Appendix C.

4.4.2 Computational Results

The program converged in seven iterations to an error criterion of 1×10^{-4} (see Figure 4.4-1) and required 35 min cpu time. This is not exceptional when one considersthe size of the system.

The optimal dispatch schedule is obtained and is presented in Figure 4.4-2 and Table 4.4-1. As can be seen, the thermal generation was reduced to a minimum and again the hydro power output supplied approximately 80% of the total power requirements. The daily fuel costs for thermal plant one is $16,693.21 and $17,020.42 for thermal plant two.

Figures 4.4-3 and 4.4-4 are enlarged views of the hydro and thermal power output, $P_h(t)$ and $P_s(t)$, respectively.

Figures 4.4-5 and 4.4-6 show how $\lambda(t)$ and $\nu(t)$ varies with time. As with the other two test systems, the shape and, hence, the functions of the curves act as earlier predicted. Tabulated data for these curves is found in Table 4.4-2.

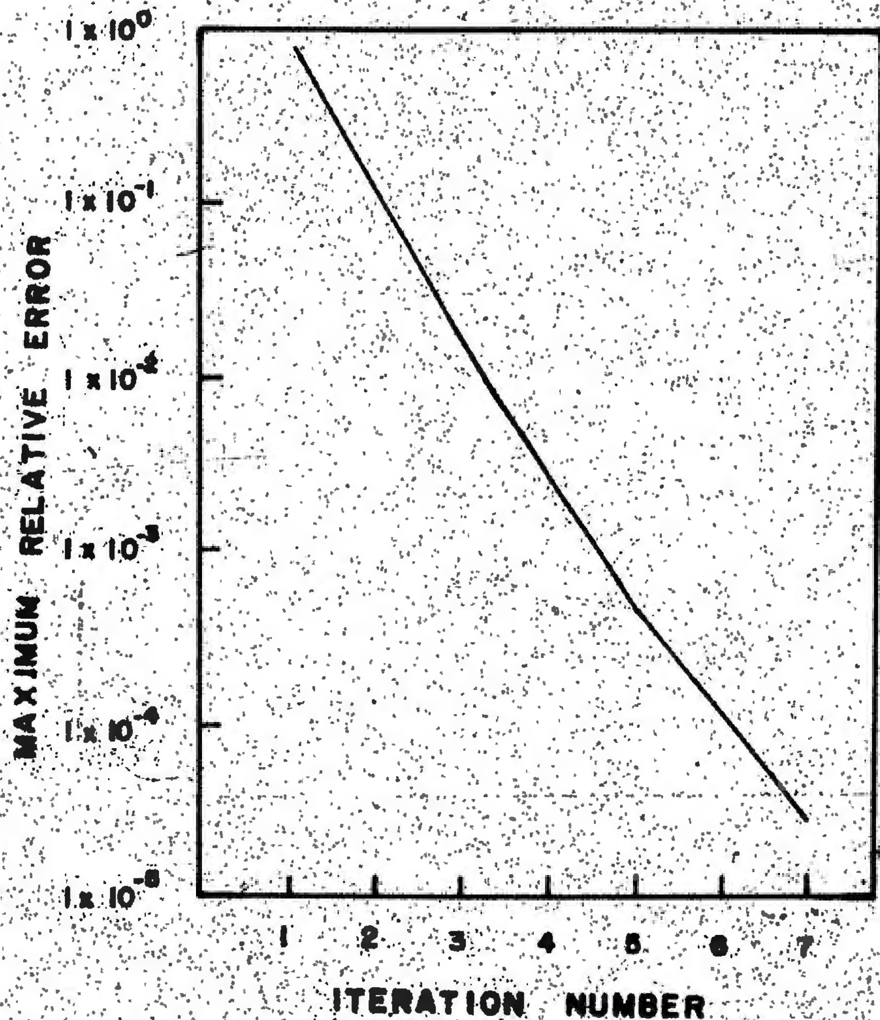


FIGURE 4.4-1. Variation in Maximum Relative Error with Iterations.

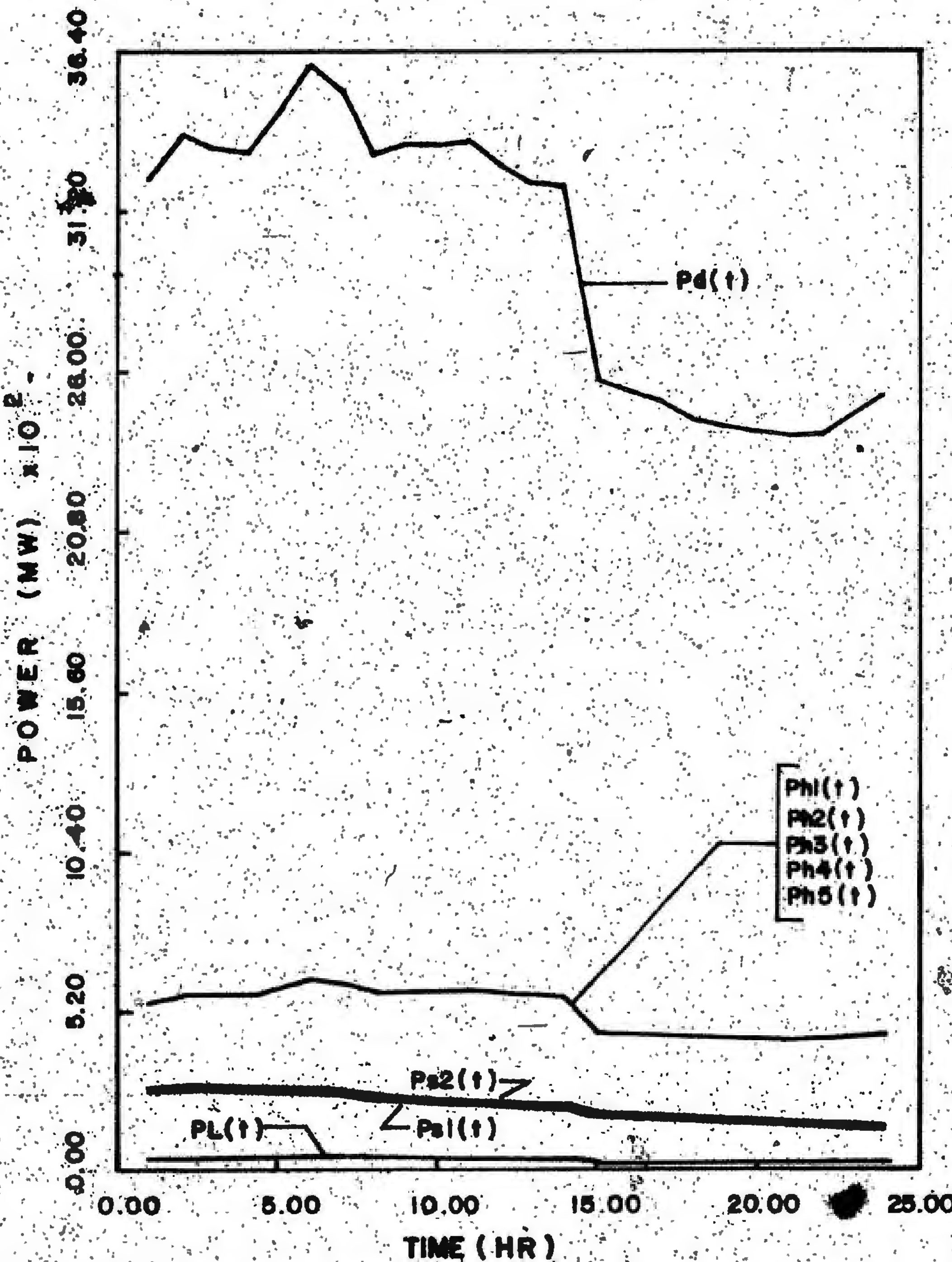


FIGURE 4.4-2. Optimal Dispatch Schedule.

TABLE 4.4-1. OPTIMAL DISPATCH SCHEDULE. TABULATED RESULTS.

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	THERMAL PLANT NO. 1 MW	THERMAL PLANT NO. 2 MW	HYDRO PLANT NO. 1 MW	HYDRO PLANT NO. 2 MW	HYDRO PLANT NO. 3 MW	HYDRO PLANT NO. 4 MW	HYDRO PLANT NO. 5 MW
1	3223	33.99	263.16	268.23	545.09	545.27	544.90	545.08	545.26
2	3367	37.35	271.21	276.29	571.35	571.54	571.15	571.32	571.49
3	3318	36.59	260.92	266.00	565.51	565.70	565.31	565.49	565.66
4	3300	36.49	253.66	258.74	564.79	564.99	564.60	564.77	564.95
5	3434	39.88	258.33	263.41	590.42	590.63	590.21	590.36	590.52
6	3593	44.11	263.60	268.68	620.98	621.22	620.74	620.88	621.02
7	3503	42.15	252.48	257.56	607.02	607.25	606.80	606.95	607.10
8	3302	37.51	235.63	240.72	572.60	572.79	572.41	572.59	572.78
9	3338	38.67	232.24	237.33	581.39	581.59	581.19	581.37	581.55
10	3331	38.78	226.79	231.88	582.19	582.39	582.00	582.18	582.36
11	3346	39.44	222.32	227.41	587.11	587.32	586.91	587.09	587.27
12	3262	37.56	214.80	219.89	572.92	573.10	572.74	572.94	573.15

TABLE 4.4-1. OPTIMAL DISPATCH SCHEDULE. TABULATED RESULTS. CONT'D.

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	THERMAL PLANT NO. 1 MW	THERMAL PLANT NO. 2 MW	HYDRO PLANT NO. 1 MW	HYDRO PLANT NO. 2 MW	HYDRO PLANT NO. 3 MW	HYDRO PLANT NO. 4 MW	HYDRO PLANT NO. 5 MW
13	3203	36.32	208.47	213.56	563.39	563.56	563.22	563.45	563.67
14	3191	36.26	203.57	208.67	562.93	563.10	562.77	563.00	563.23
15	2559	22.34	183.36	188.46	441.66	441.68	441.64	442.06	442.47
16	2524	21.84	177.85	182.96	436.76	436.78	436.75	437.16	437.58
17	2496	21.48	172.75	177.85	433.13	433.14	433.12	433.54	433.95
18	2437	20.53	167.17	172.28	423.36	423.36	423.36	423.79	424.21
19	2408	20.14	162.38	167.49	419.40	419.40	419.40	419.82	420.25
20	2391	19.99	157.92	163.03	417.75	417.76	417.75	418.17	418.60
21	2384	20.01	153.66	158.78	418.07	418.08	418.06	418.48	418.89
22	2393	20.35	149.50	154.62	421.62	421.64	421.60	421.99	422.38
23	2454	21.76	144.97	150.09	435.97	436.03	435.91	436.23	436.56
24	2520	23.34	139.88	145.00	451.60	451.71	451.49	451.71	451.94

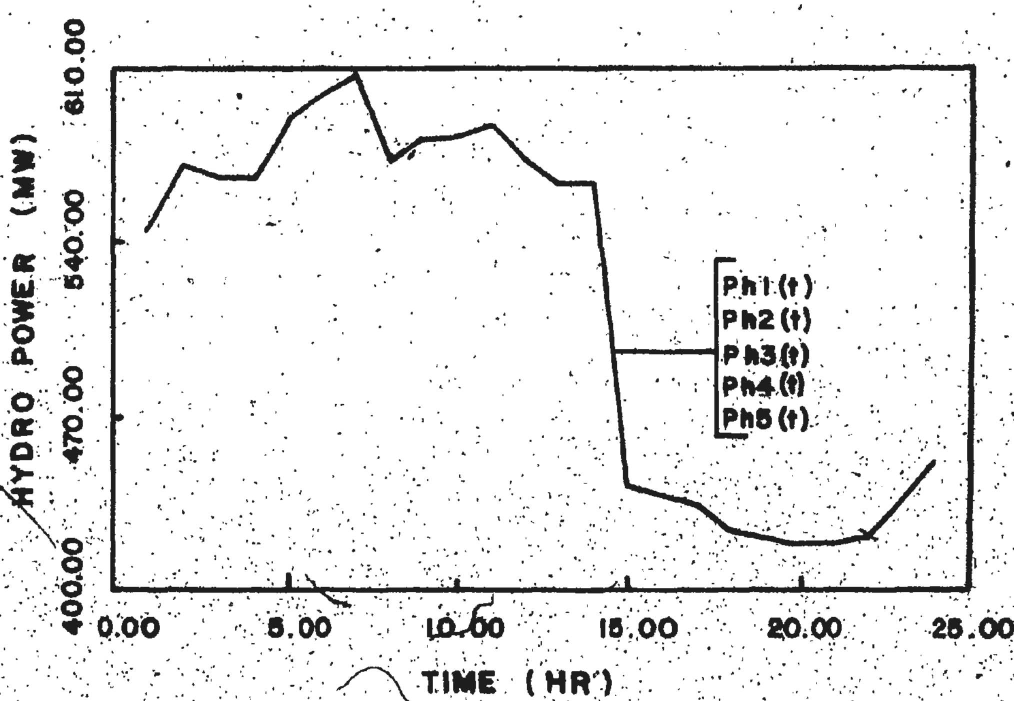


FIGURE 4.4-3. Variation in $P_h(t)$.

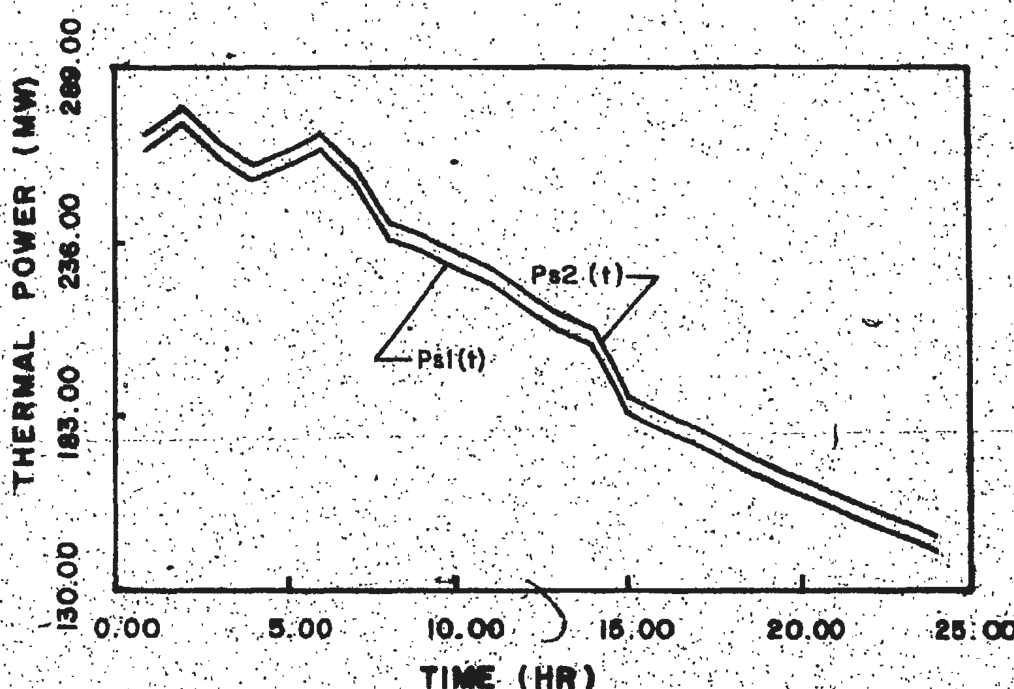


FIGURE 4.4-4. Variation in $P_s(t)$.

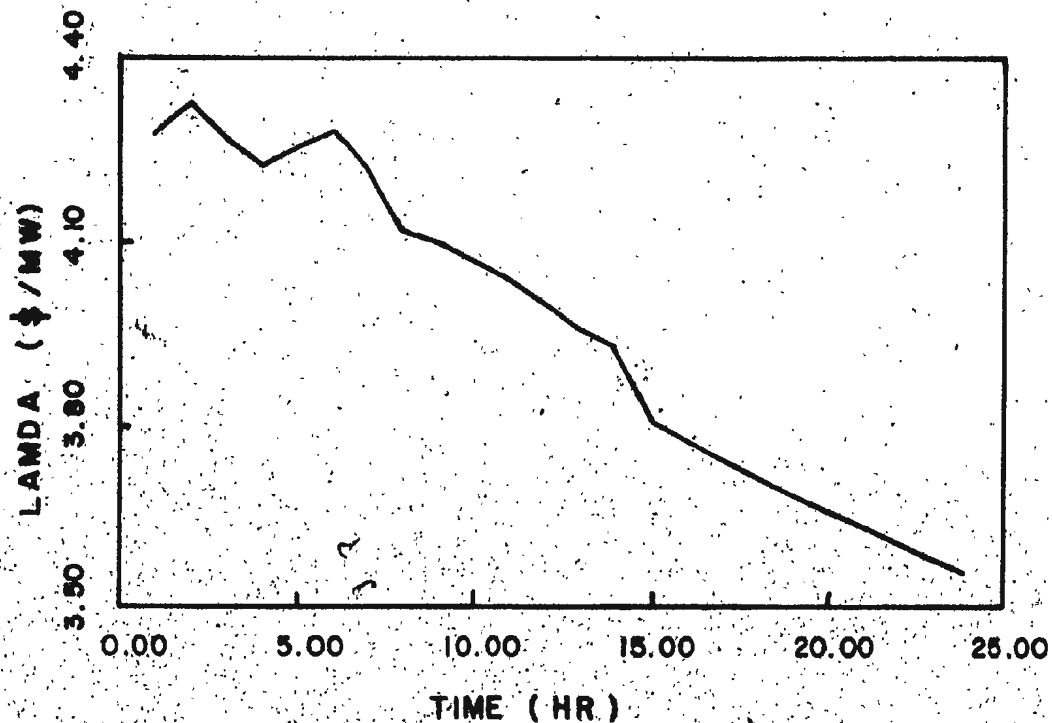


FIGURE 4.4-5. Variation in $\lambda(t)$.

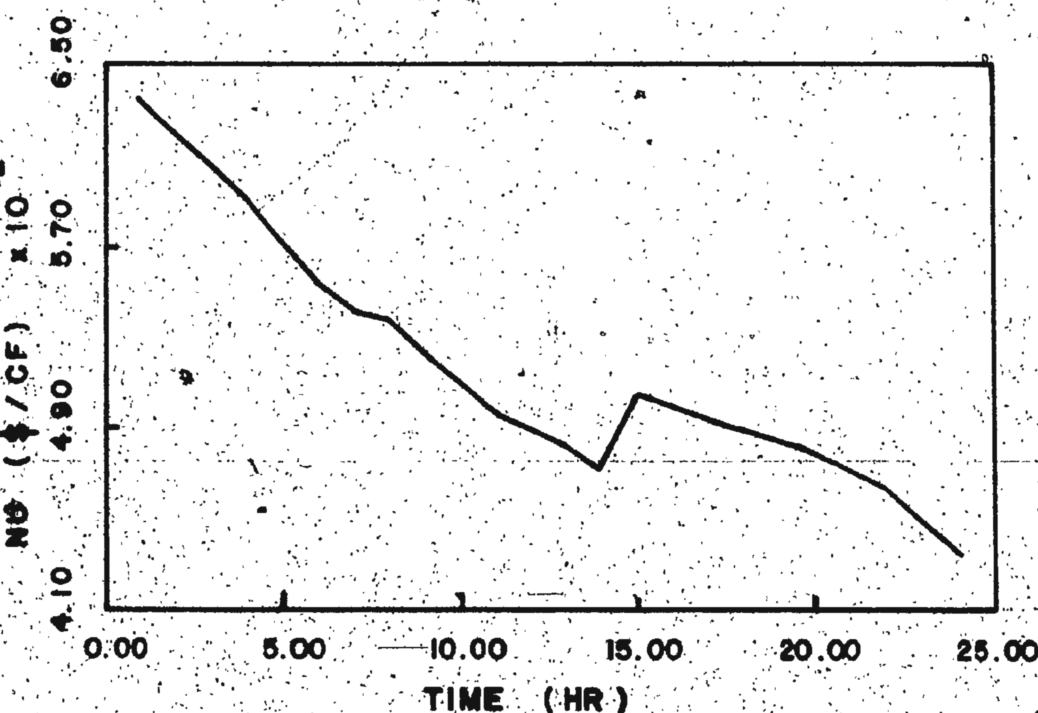


FIGURE 4.4-6. Variation in $\nu(t)$.

TABLE 4.4-2. VARIATIONS IN $\lambda(t)$ AND $\nu(t)$.

TIME PERIOD HR	NU PLANT NO. 1 $/CF	NU PLANT NO. 2 $/CF	NU PLANT NO. 3 $/CF	NU PLANT NO. 4 $/CF	NU PLANT NO. 5 $/CF	LAMBDA $/MW
1	0.06329	0.06331	0.06326	0.06328	0.06330	4.279
2	0.06169	0.06171	0.06167	0.06169	0.06171	4.327
3	0.06030	0.06032	0.06027	0.06029	0.06031	4.266
4	0.05891	0.05893	0.05889	0.05891	0.05892	4.222
5	0.05713	0.05715	0.05711	0.05713	0.05715	4.250
6	0.05515	0.05517	0.05514	0.05516	0.05517	4.282
7	0.05404	0.05406	0.05402	0.05404	0.05406	4.215
8	0.05353	0.05355	0.05351	0.05353	0.05355	4.114
9	0.05207	0.05209	0.05205	0.05207	0.05209	4.093
10	0.05084	0.05086	0.05082	0.05084	0.05086	4.061
11	0.04951	0.04953	0.04950	0.04952	0.04953	4.034
12	0.04883	0.04884	0.04881	0.04883	0.04884	3.989

TABLE 4.4-2. VARIATIONS IN $\lambda(t)$ AND $\nu(t)$. CONT'D.

TIME PERIOD HR	NU PLANT NO. 1 S/CF	NU PLANT NO. 2 S/CF	NU PLANT NO. 3 S/CF	NU PLANT NO. 4 S/CF	NU PLANT NO. 5 S/CF	LAMBDA $/MW
13	0.04807	0.04808	0.04805	0.04806	0.04908	3.951
14	0.04704	0.04706	0.04702	0.04704	0.04705	3.921
15	0.05052	0.05054	0.05050	0.05051	0.05052	3.800
16	0.04991	0.04993	0.04988	0.04989	0.04990	3.767
17	0.04927	0.04929	0.04925	0.04926	0.04926	3.736
18	0.04891	0.04894	0.04889	0.04890	0.04891	3.703
19	0.04835	0.04837	0.04833	0.04834	0.04835	3.674
20	0.04772	0.04774	0.04770	0.04770	0.04771	3.648
21	0.04701	0.04703	0.04699	0.04699	0.04700	3.622
22	0.04616	0.04617	0.04614	0.04614	0.04615	3.597
23	0.04478	0.04480	0.04477	0.04478	0.04478	3.570
24	0.04333	0.04335	0.04332	0.04333	0.04334	3.539

In Figure 4.4-7, the net head is presented. This curve also performs as expected. Table 4.4-3 gives the tabulated results. Actual computer printout is found in Appendix C.

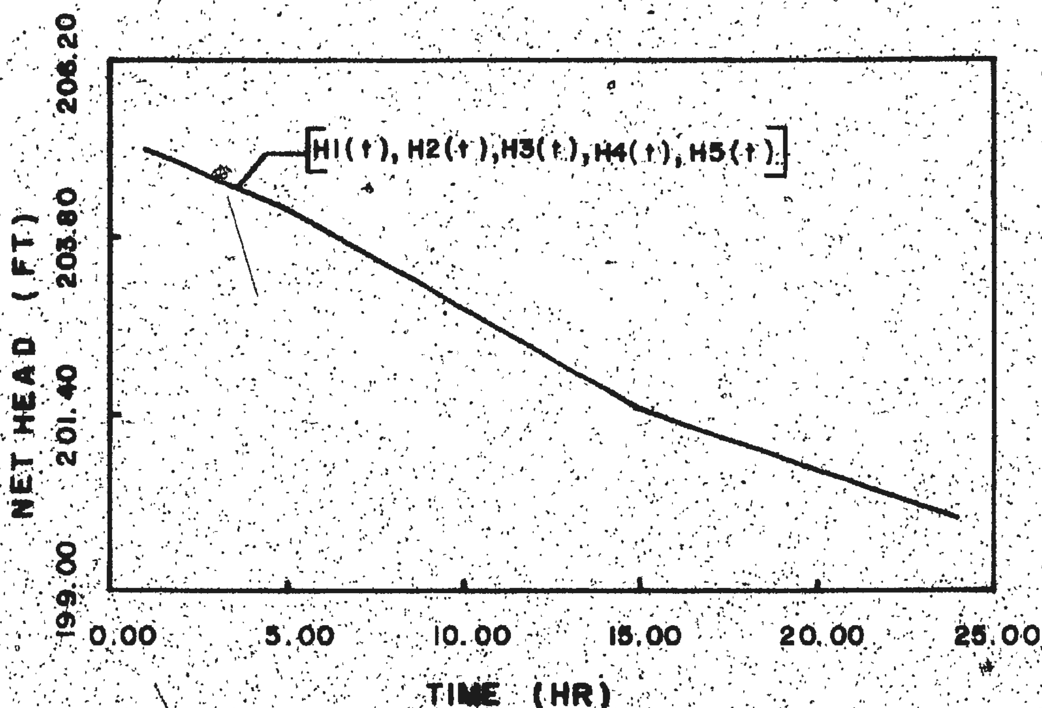


FIGURE 4.4-7. Variation in h(t).

TABLE 4.4-3

VARIATION IN h(t). TABULATED RESULTS.

TIME PERIOD HR	NET HEAD PLANT NO. 1 FT	NET HEAD PLANT NO. 2 FT	NET HEAD PLANT NO. 3 FT	NET HEAD PLANT NO. 4 FT	NET HEAD PLANT NO. 5 FT
1	205.00	205.00	205.00	205.00	205.00
2	204.81	204.81	204.81	204.81	204.81
3	204.59	204.59	204.59	204.59	204.59
4	204.38	204.38	204.38	204.38	204.38
5	204.16	204.16	204.16	204.16	204.16
6	203.91	203.91	203.91	203.91	203.91
7	203.62	203.62	203.62	203.62	203.62
8	203.34	203.34	203.34	203.34	203.34
9	203.09	203.09	203.09	203.09	203.09
10	202.82	202.82	202.82	202.82	202.82
11	202.55	202.55	202.55	202.55	202.55
12	202.26	202.26	202.26	202.26	202.26
13	201.98	201.98	201.98	201.98	201.99
14	201.71	201.71	201.71	201.71	201.71
15	201.43	201.43	201.43	201.43	201.43
16	201.26	201.26	201.26	201.26	201.26
17	201.09	201.09	201.10	201.10	201.10
18	200.93	200.93	200.93	200.93	200.93
19	200.77	200.77	200.77	200.77	200.77
20	200.62	200.62	200.62	200.62	200.62
21	200.46	200.46	200.46	200.46	200.46
22	200.30	200.30	200.30	200.30	200.30
23	200.13	200.13	200.13	200.13	200.13
24	199.95	199.95	199.95	199.95	199.95

CHAPTER V

COORDINATION EQUATIONS FOR ECONOMIC OPERATION OF POWER SYSTEMS WITH HYDRO PLANTS ON THE SAME STREAM

5.1 INTRODUCTION

A newly developed set of coordination equations for electric power systems with hydro plants on the same stream is presented. Time delay of flow between plants is taken into account. The resulting equations represent a natural extension of the pioneering Kron-Ricard equations from which the useful concepts of water-worth can be obtained easily.

This preserves the intuitive form of the coordination equations and provides insights into the physical meaning of the variables concerned.

5.2 FORMULATION

The system considered is assumed to contain one thermal plant whose active power generation is denoted by P_1 and a hydro subsystem. The objective of the economic scheduling problem is to minimize the thermal cost represented by F over the optimization interval $[o,T]$.

$$J = \int_o^T F[P_1(t)]\, dt \qquad (5.1)$$

The function F is assumed available. The optimization is to be carried out while satisfying the hydro subsystem constraints as well as the electric network constraints. For simplicity we assume that only one constraint is used which requires an active power balance.

$$f_D(t) = \sum_{i=1}^{N} P_i(t) - P_L(t) - P_D(t) = 0 \quad . \tag{5.2}$$

The system demand, P_D, is assumed given for the optimization interval. Power losses are denoted by P_L. The number of system plants is denoted by N.

The power system considered is assumed to contain three hydro plants on the same stream. This is the minimum number required for a general formulation. The plants are ordered as follows:

Plant 2 up-stream plant

Plant 3 intermediate plant

Plant 4 down-stream plant .

It is assumed that the performance characteristic of each hydro plant is modeled by the Glimn-Kirchmayer model

$$f_{q_i}(t) = q_i - K_i \psi_i(h_i) \phi_i(P_i) = 0 \quad . \tag{5.3}$$

The rate of water discharge in m^3/sec is denoted by q_i, the effective head in meters is denoted by h_i, and P_i denotes the active power generation in MW. The functions ψ and ϕ are given by

$$\psi(h) = a_0 + a_1 h + a_2 h^2$$

$$\phi(P) = b_0 + b_1 P + b_2 P^2$$

The model assumes the availability of the parameters K, a_0, a_1, a_2, b_0, b_1, and b_2 for each plant.

Each of the hydro plants is assumed to draw water from a vertical sided reservoir whose surface area is s in m^2. The reservoir's natural water inflow is denoted i(t) in m^3/s). A forecast of i(t) is assumed available over the optimization interval [o,T]. The continuity equation for reservoir 2, at the up-stream plant is thus

$$f_{h_2}(t) = s_2 h_2(t) - s_2 h_2(0) - \int_o^t i_2(Z)\, dZ$$

$$+ \int_o^t q_2(Z)\, dZ = 0 \quad . \tag{5.4}$$

As the intermediate reservoir, the inflow is composed of a natural component $i_3(t)$ as well as that due to the controlled discharge q_2 of the up-stream plant delayed by τ_{23} hours. As a result, the continuity equation for reservoir three is expressed as

$$f_{h_3}(t) = s_3 h_3(t) - s_3 h_3(0) - \int_o^t i_3(Z)\, dZ$$

$$+ \int_o^t [q_3(Z) - q_2(Z-\tau_{23})]\, dZ = 0 \quad . \tag{5.5}$$

In a similar way, the continuity equation for reservoir four is given by

$$f_{h_4}(t) = s_4 h_4(t) - s_4 h_4(0) - \int_o^t i_4(Z)\, dZ$$

$$+ \int_o^t [q_4(Z) - q_3(Z-\tau_{34})]\, dZ = 0 \quad . \tag{5.6}$$

The water draw-down constraints over the optimization interval for the system are given by

$$J_{b_i} = \int_o^t q_i(Z)\, dZ - v_i = 0 \quad . \tag{5.7}$$

The volumes v_2, v_3, v_4 are assumed available and correspond to hydraulic requirements such as navigational, irrigation or other constraints.

5.3 DIRECT OPTIMALITY CONDITIONS

The optimal operation strategy is described by a set of optimality conditions obtained by forming an augmented objective functional as indicated in Appendix D. This takes the form

$$J_A = \int_o^T I_A(t)\, dt \quad . \tag{5.8}$$

Optimality is attained by setting the derivatives of I_A with respect to control variables to equal zero. We choose initially to have both discharge, q, and head, h, to be control variables in addition to the active power generations.

Differentiating with respect to the thermal active power generation we obtain

$$\frac{\partial I_A}{\partial P_1} = \frac{\partial F}{\partial P_1} + \lambda(t) \left[\frac{\partial P_L}{\partial P_1} - 1 \right] = 0 \quad . \tag{5.9}$$

This is the familiar all-thermal, equal incremental cost expression. Differentiating with respect to the hydro active power generation we get

$$\frac{\partial I_A}{\partial P_i} = \lambda(t) \left[\frac{\partial P_L}{\partial P_i} - 1 \right] - m_i(t) \frac{\partial q_i}{\partial P_i} = 0 \quad . \tag{5.10}$$

Taking the derivatives of I_A with respect to h_i, we obtain

$$\frac{\partial I_A}{\partial h_i} = s_i n_i(t) - m_i(t) \frac{\partial q_i}{\partial h_f} = 0 \quad . \tag{5.11}$$

Taking the derivatives with respect to each of the q_i's we get

$$\frac{\partial I_A}{\partial q_2} = \nu_{o_2} + m_2(t) + [n_2(T) - n_2(t)] - N_3(t) = 0 \tag{5.12}$$

$$\frac{\partial I_A}{\partial q_3} = \nu_{o_3} + m_3(t) + [n_3(T) - n_3(t)] - N_4(t) = 0 \tag{5.13}$$

$$\frac{\partial I_A}{\partial q_4} = \nu_{o_4} + m_4(t) + [n_4(T) - n_4(t)] = 0 \quad . \tag{5.14}$$

Equations (5.9) through (5.14) together with (5.2) through (5.7) constitute the optimality conditions necessary for solving the problem.

5.4 COORDINATION EQUATIONS

The direct optimality conditions obtained above can be reduced to a form that represents a direct extension of the well-known Kron-Ricard's equations by simply eliminating the functions $m_i(t)$ and $n_i(t)$. The algebraic details of the process are given in Appendix E. The resulting equations consist of the following.

For the thermal plant we have from Equation (5.9),

$$L_1 \frac{\partial F}{\partial P_1} = \lambda(t) \quad . \tag{5.15}$$

For the up-stream plant

$$L_2 \nu_2(t) \frac{\partial q_2}{\partial P_2} = \lambda(t) \tag{5.16}$$

where

$$\nu_2(t) = \nu_2 \phi_2(t,o) + \int_o^t \phi_2(t,\sigma)\, u_2(\sigma)\, d\sigma \quad . \tag{5.17}$$

For the intermediate plant

$$L_3 \nu_3(t) \frac{\partial q_3}{\partial P_3} = \lambda(t) \tag{5.18}$$

where

$$\nu_3(t) = \nu_3 \phi_3(t,o) + \int_o^t \phi_3(t,\sigma)\, u_3(\sigma)\, d\sigma \quad . \tag{5.19}$$

For the down-stream plant

$$L_4 \nu_4(t) \frac{\partial q_4}{\partial P_4} = \lambda(t) \tag{5.20}$$

where

$$\nu_4(t) = \nu_4 \phi_4(t,o) \quad . \tag{5.21}$$

In the above

$$\phi_i(t,t_o) = \exp\{\int_{t_o}^t M_i(Z)\, dZ\} \tag{5.22}$$

$$M_i(Z) = \frac{1}{s_i} \frac{\partial q_i}{\partial h_i} \tag{5.23}$$

$$u_i(\sigma) = \dot{m}_{i+1}(t + \tau_{i(i+1)}) \qquad 0 \leq t \leq T - \tau_{i(i+1)}$$
$$= 0 \qquad T - \tau_{34} < t \leq T \tag{5.24}$$

$$m_4(t) = -\nu_4 \exp\left[\int_o^t M_4(\sigma)\, d\sigma\right] \tag{5.25}$$

$$m_3(t) = -\nu_3\phi_3(t,o) - \int_o^t \phi_3(t,\sigma)\, u_3(\sigma)\, d\sigma \tag{5.26}$$

$$m_2(t) = -\nu_2'\phi_2(t,o) - \int_o^t \phi_3(t,\sigma)\, u_2(\sigma)\, d\sigma \tag{5.27}$$

As is common in conventional theory, the loss penalty factors are given by

$$L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \tag{5.28}$$

5.5 WATER-WORTH

The functions $\nu_i(t)$ in Equations (5.16), (5.18), and (5.20) represent the water-worth functions for the hydro plants considered. In the classical theory for economic operation for fixed head hydro plants these functions are constant and represent the conversion factors necessary to convert the incremental discharge into an equivalent incremental cost. When hydro plants are of the variable head type, the water-worth is no longer a constant for each plant (for the given allowable volume of water available). Instead, the water-worth increases over the optimization interval as long as discharge exceeds the natural inflow into the reservoir. This concept has been discovered by Kron and Ricard and illustrated vividly by Glimn and Kirchmayer for plants that are hydraulically isolated.

The coordination equations derived here for systems with plants on the same stream provide us with the basis for obtaining water-worth functions $\nu_i(t)$ defined by Equations (5.17), (5.19), and (5.21). Inspection of each of the equations reveals that the water-worth is made of two components. The first is the coupling-free term $\nu_i \phi_i(t,o)$ while the second component pertains to coupling and represents a penalty for discharge arriving at a plant further down-stream. Note that for the down-stream plant only the coupling-free term exists.

CHAPTER VI

CONCLUSIONS AND FUTURE WORK

6.1 CONCLUSIONS

An algorithm based on the Newton-Raphson iterative technique is developed for application to the problem of optimal economic operation of variable-head hydro electric power systems. When this algorithm is applied to several test systems, the results show that it is successful in obtaining the optimal hydro-thermal dispatching schedule.

The computer evaluation of the behavior of the variables under varying system constraint conditions is consistent with the predetermined analytical observations. In other words, the variables react as predicted to changes in the system.

The amount of computer storage and computational time required for the successful solution of the problem is significantly reduced by exploiting the sparsity of the Jacabian matrix. As is pointed out in Section 6.2, even further savings are realizable through this process.

The coordination equations for hydraulically coupled variable-head hydro-thermal electric power systems are developed for several configurations of systems.

6.2. FUTURE WORK

As in any type of research work there is always one more detail that could be accomplished before the topic is brought to a close. The following items constitute the details which form possible starting points for any continuing work on this topic.

It is shown in the preceeding chapters how exploitation of the sparsity of the Jacobian matrix results in reduced core space and computer time. As mentioned, it is possible to further exploit this sparsity and obtain more savings. This additional work should be directed towards performing the matrix manipulations on an elemental basis. This will require further partitioning of the submatrices detailed in Chapter two and taking advantage of the sparsity of the block diagonal matrix $\underline{J}_A$.

Another path would be to work towards improving the methods used to obtain the initial estimates of the variables. Such an improvement will increase the overall efficiency of the program and will also reduce the computational time required for solution.

As another point, the optimality equations for hydraulically-coupled hydro plants in hydro-thermal systems should be implemented. As well, the program could be expanded to include pumped storage units.

As a last item, an indepth sensitivity analysis of the program should be performed to determine, if any, the limitations of the program that are not obvious through the testing that has been performed.

CHAPTER VII

BIBLIOGRAPHY AND REFERENCES

1. Arismunander, A. (1960), "General Equations for Short Range Optimization of a Combined Hydro-Thermal Electric System", M.Sc. Thesis, Univ. of British Columbia.

2. Arismunander, A. and Noakes, F. (1962), "General Time-Dependent Equations for Short Range Optimization of Hydro-Thermal Electric Systems", AIEE Trans. 82, Part III, 88-93.

3. Bedford, F.W. and Dwivedi, T.D. (1970), "Vector Calculus", McGraw-Hill Book Co., New York.

4. Bernholtz, B. and Graham, L.J. (1962), "Hydro-Thermal Economic Scheduling, Part I, Solution by Incremental Programming", AIEE Trans. 79, Part III, 921-932.

5. Bernholtz, B. and Graham, L.J. (1962a), "Hydro-Thermal Economic Scheduling, Part II, Extension of The Basic Theory", AIEE Trans. 80, Part III, 1089-1096.

6. Bernholtz, B. and Graham, L.J. (1962b), "Hydro-Thermal Economic Scheduling, Part III, Scheduling the Thermal Subsystem Using Constrained Steepest Descent", AIEE Trans. 80, Part III, 1096-1105.

7. Bernholtz, B. and Graham, L.J. (1962c), "Hydro-Thermal Economic Scheduling, Part IV, A Continuous Procedure for Maximizing the Weighted Output of a Hydro-Electric Generating Station", AIEE Trans. 80, Part III, 1105-1107.

8. Bonaert, A.P., El-Abiad, and Koivo, A.J. (1971), "Optimal Scheduling of Hydro-Thermal Power Systems by a Decomposition Technique Using Pertubations", IEEE, 7th PICA Conf. Proc., Boston, Mass., May, 23-26.

9. Carey, J.J. (1954), "Short Range Load Allocation Hydro-Thermal Electric System", AIEE Trans. 73, Part III-b, 1260-1267.

10. Chandler, W.G., Dandeno, P.L., Glimn, A.F., and Kirchmayer, L.K. (1953), "Short Range Economic Operation of a Combined Thermal and Hydro-Electric Power System", AIEE Trans. 72, Part III, 1057-1065.

11. Converse, A.O. (1970), "Optimization", Holt, Rinehart, and Winston, Inc., New York.

12. Cypser, R.J. (1954), "Computer Search for Economical Operation of a Hydro-Thermal Electric System", AIEE Trans. 73, Part III-B, 1260-1267.

13. Dahlin, E.B. (1964), "Theoretical and Computational Aspects of Optimal Principles With Special Application to Power System Operation", Doctoral Dissertation, Univ. of Pennsylvania, Philadelphia, Penn.

14. Dahlin, E.B. and Shen, D.W.C. (1966), "Optimal Solution to the Hydro-Steam Dispatch Problem for Certain Practical Systems", IEEE Trans., PAS-85, No. 5.

15. Dandeno, P.L. (1961), "Hydro-Thermal Economic Scheduling-Computational Experience With Coordination Equations", AIEE Trans. 80, Part III, 1219-1228

16. Drake, J.H., Kirchmayer, L.K., Mayall, R.B., and Wood, H. (1962), "Optimum Operation of a Hydro-Thermal System", AIEE Trans. 82, Part III, 242-250.

17. Elgerd, O.I. (1971), "Electric Energy Systems Theory: An Introduction", McGraw-Hill, New York.

18. El-Hawary, and Christensen, G.S. (1977), "Optimum Scheduling of Power Systems Using Functional Analysis, Short Paper", IEEE Control Syst. Soc. Trans. AC-17, 518-522.

19. El-Hawary, M.E. and Christensen, G.S. (1979), "Optimal Economic Operation of Electric Power Systems", Academic Press, New York, Mathematics in Science and Engineering, Vol. 142.

20. El-Hawary, M.E. and Landrigan, J.K. (1982), "Optimum Operation of Fixed-Head Hydro-Thermal Electric Power Systems: Powell's Hybrid Method Versus Newton-Raphson Method", IEEE Trans., PAS-101, No. 3.

21. Glimn, A.F. and Kirchmayer, L.K. (1958), "Economic Operation of Variable-Head Hydro-Electric Plants", AIEE Trans. 77, Part III, 1070-1079.

22. Hammond, R.H., Rogers, W.B. and Houck, B. Jr. (1976), "Introduction to Fortran IV", McGraw-Hill Book Co., New York.

23. Hano, I., Tamura, Y. and Narita, S. (1966), "An Application of the Maximum Principle to the Most Economical Operation of Power Systems", IEEE Trans., PAS-85, No. 5.

24. Kreyszig, E. (1972), "Advanced Engineering Mathematics", 3rd Edition, John Wiley and Sons, Inc., New York.

25. Nanda, J. and Bijwe, P.R. (1981), "Optimal Hydro-Thermal Scheduling With Cascaded Plants Using Progressive Optimality Algorithm", IEEE Trans. PAS-100, No. 4, Apr., 2093-2099.

26. Protter, M.H. and Morrey, C.B. Jr. (1970), "College Calculus With Analytic Geometry", 2nd Edition, Addison Wesley Pub. Co., Reading, Mass.

27. Ricard, J. (1940), "The Determination of Optimum Operating Schedule For Inter-Connected Hydro and Thermal Stations", p. 167, Revue Generale de Electricite, Paris, France,

28. Rule, W.P., Finkenaur, R.G. and Patrick, F.G. (1973), "Fortran IV Programming", Prindle, Weber and Schmidt, Inc., Boston, Mass.

29. Soares, S., Lyra, C. and Tavares, H. (1980), "Optimal Generation Scheduling of Hydro-Thermal Power Systems", IEEE Trans., PAS-99, No. 3.

30. Stagg, G.W. and El-Abiad, A.H. (1968), "Computer Methods in Power System Analysis", McGraw-Hill, New York.

31. Watters, J. (1968), "Fortran Programming", Heinemann Educational Books Ltd., Pitman Press, England.

APPENDIX A

APPENDIX A

FIRST-ORDER PARTIAL DERIVATIVES

This appendix lists the first order partial derivatives which form the elements of the Jacobian matrix.

The partial derivatives of $f_{s_i}(t_k)$ are obtained on the basis of Equation (3.34) as

$$\frac{\partial f_{s_i}(t_k)}{\partial P_{s_i}(t_k)} = 2\gamma_{s_i} + 2 \sum_{j=1}^{N_s} B_{i_j} \lambda(t_k) \quad , \quad (i=1 \rightarrow N_s) \tag{A.1}$$

$$\frac{\partial f_{s_i}(t_k)}{\partial P_{h_i}(t_k)} = 2 \sum_{j=1}^{N_h} B_{i_j} \lambda(t_k) \quad , \quad (i=1 \rightarrow N_s) \tag{A.2}$$

$$\frac{\partial f_{s_i}(t_k)}{\partial \lambda(t_k)} = C_i + 2 \sum_{j=1}^{N_g} B_{i_j} P_j(t) \quad , \quad (i=1 \rightarrow N_s) \tag{A.3}$$

$$\frac{\partial f_{s_i}(t_k)}{\partial h_i(t_k)} = 0 \tag{A.4}$$

$$\frac{\partial f_{s_i}(t_k)}{\partial \nu_{o_i}} = 0 \tag{A.5}$$

The partial derivatives of $f_{h_i}(t_k)$ are obtained on the basis of Equation (3.35) as

$$\frac{\partial f_{h_i}(t_k)}{\partial P_{s_i}(t_k)} = 2 \sum_{j=1}^{N_s} B_{i_j} \lambda(t_k) \quad , \quad (i=1 \to N_h) \tag{A.6}$$

$$\frac{\partial f_{h_i}(t_k)}{\partial P_{h_i}(t_k)} = \{K\nu_i(t_k)\ \psi_i(h)\ [2\gamma_{h_i} + \frac{K\ t_k}{s_i}(a_{1_i} + 2a_{2_i}(t_k))x$$

$$(\beta_{h_i} + 2\gamma_{h_i} P_{h_i}(t_k)^2] + 2 \sum_{j=1}^{N_h} B_{i_j} \lambda(t_k)\} \quad , \quad (i=1 \to N_h) \tag{A.7}$$

$$\frac{\partial f_{h_i}(t_k)}{\partial \lambda(t_k)} = C_i + 2 \sum_{j=1}^{N_g} B_{j_i} P_j(t) \quad , \quad (i=1 \to N_h) \tag{A.8}$$

$$\frac{\partial f_{h_i}(t_k)}{\partial h_i(t_k)} = K\nu_i(t_k)\ (\beta_{h_i} + 2\lambda_{h_i} P_{h_i}(t_k)\ x$$

$$[a_{1_i} + 2a_{2_i}(h_i(t_k) + \frac{K\ t_k}{s_i}(\phi_i(P_h)\ \psi_i(h))] \quad , \quad (i=1 \to N_h) \tag{A.9}$$

$$\frac{\partial f_{h_i}(t_k)}{\partial \nu_{o_i}} = [\frac{\nu_i(t_k)}{\nu_{o_i}}]\ [K\ \psi_i(h)]\ x\ (\beta_{h_i} + 2\gamma_{h_i} P_{h_i}(t_k)) \quad , \quad (i=1 \to N_h) \tag{A.10}$$

The partial derivatives of $f_D(t_k)$ are obtained on the basis of Equation (3.36) as

$$\frac{\partial f_D(t_k)}{\partial P_{s_i}(t_k)} = \sum_{i=1}^{N_s} C_i + 2 \sum_{j=1}^{N_g} B_{i_j} P_j(t) \quad , \quad (i=1 \to N_s) \tag{A.11}$$

$$\frac{\partial f_D(t_k)}{\partial P_{h_i}(t_k)} = \sum_{i=1}^{N_h} C_i + 2 \sum_{j=1}^{N_g} B_{ij} P_j(t) , \quad (i=1 \rightarrow N_h) \tag{A.12}$$

$$\frac{\partial f_D(t_k)}{\partial \lambda(t_k)} = 0 \tag{A.13}$$

$$\frac{\partial f_D(t_k)}{\partial h_i(t_k)} = 0 \tag{A.14}$$

$$\frac{\partial f_D(t_k)}{\partial v_{o_i}} = 0 . \tag{A.15}$$

The partial derivatives of $f_{i_i}(t_k)$ are obtained on the basis of Equation (3.37) as

$$\frac{\partial f_{i_i}(t_k)}{\partial P_{s_i}(t_k)} = 0 \tag{A.16}$$

$$\frac{\partial f_{i_i}(t_k)}{\partial P_{h_i}(t_k)} = - \frac{\Delta}{s_i} K \psi_i(h) (\beta_{h_i} + 2\gamma_{h_i} P_{h_i}(t_k)) , \quad (i=1 \rightarrow N_h) \tag{A.17}$$

$$\frac{\partial f_{i_i}(t_k)}{\partial \lambda(t_k)} = 0 \tag{A.18}$$

$$\frac{\partial f_{i_i}(t_k)}{\partial h_i(t_k)} = 1 - \frac{\Delta}{s_i} K \phi_i(P_h) \times (a_{1_i} + a_{2_i} h_i(t_k)) , \qquad (i=1 \rightarrow N_h) \qquad (A.19)$$

$$\frac{\partial f_{i_i}(t_k)}{\partial h_i(t_k+1)} = -1 \qquad (A.20)$$

$$\frac{\partial f_{i_i}(t_k)}{\partial \nu_{o_i}} = 0 \qquad (A.21)$$

The partial derivatives of f_{b_i} are obtained on the basis of Equation (3.38) as

$$\frac{\partial f_{b_i}}{\partial P_{s_i}(t_k)} = 0 \qquad (A.22)$$

$$\frac{\partial f_{b_i}}{\partial P_{h_i}(t_k)} = K\Delta\psi_i(h) \times (\beta_{h_i} + 2\lambda_{h_i} P_{h_i}(t_k) , \qquad (i=1 \rightarrow N_h) \qquad (A.23)$$

$$\frac{\partial f_{b_i}}{\partial \lambda(t_k)} = 0 \qquad (A.24)$$

$$\frac{\partial f_{b_i}}{\partial h_i(t_k)} = K\Delta\phi_i(P_h) \times (a_{1_i} + 2a_{2_i} h_i(t)) , \qquad (i=1 \rightarrow N_h) \qquad (A.25)$$

$$\frac{\partial f_{b_i}}{\partial \nu_{o_i}} = 0 \qquad \text{(A.26)}$$

APPENDIX B

APPENDIX B

THE COMPUTER PROGRAM

B-1 INTRODUCTION

The computer program presented herein solves for the optimum hydrothermal dispatch schedule. The algorithm, described in the text, revolves around the Newton-Raphson technique where the solution is obtained by iterative techniques.

The program logic is detailed in the flowchart presented in Figure B-1. The program itself is written in fortran language and has an approximate storage requirement of 1500 program lines.

The sections which follow detail the variables used in the calculations, a full listing of the computer program, a description of the matrix inversion routine, and some basic data on the computer.

B-2 PROGRAM VARIABLE DEFINITION LIST

The following list contains the definitions and dimensions of the terms used in the program. They are cataloged according to their function within the program.

Control Terms

N	number of discrete time intervals into which the study period is divided
NUMS	number of thermal plants in the system
NUMH	number of hydro plants in the system

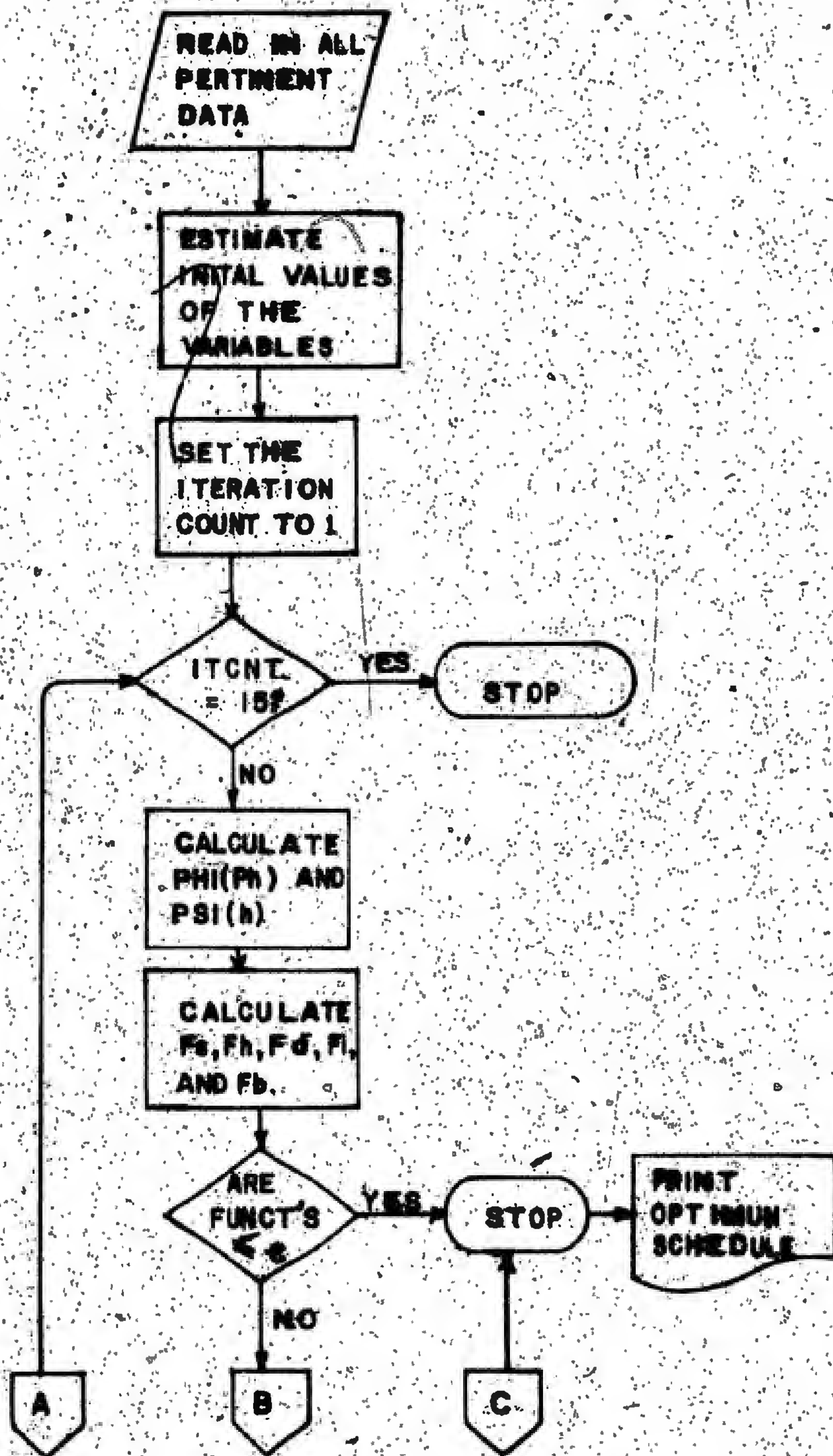


FIGURE B-1. Algorithm Flowchart.

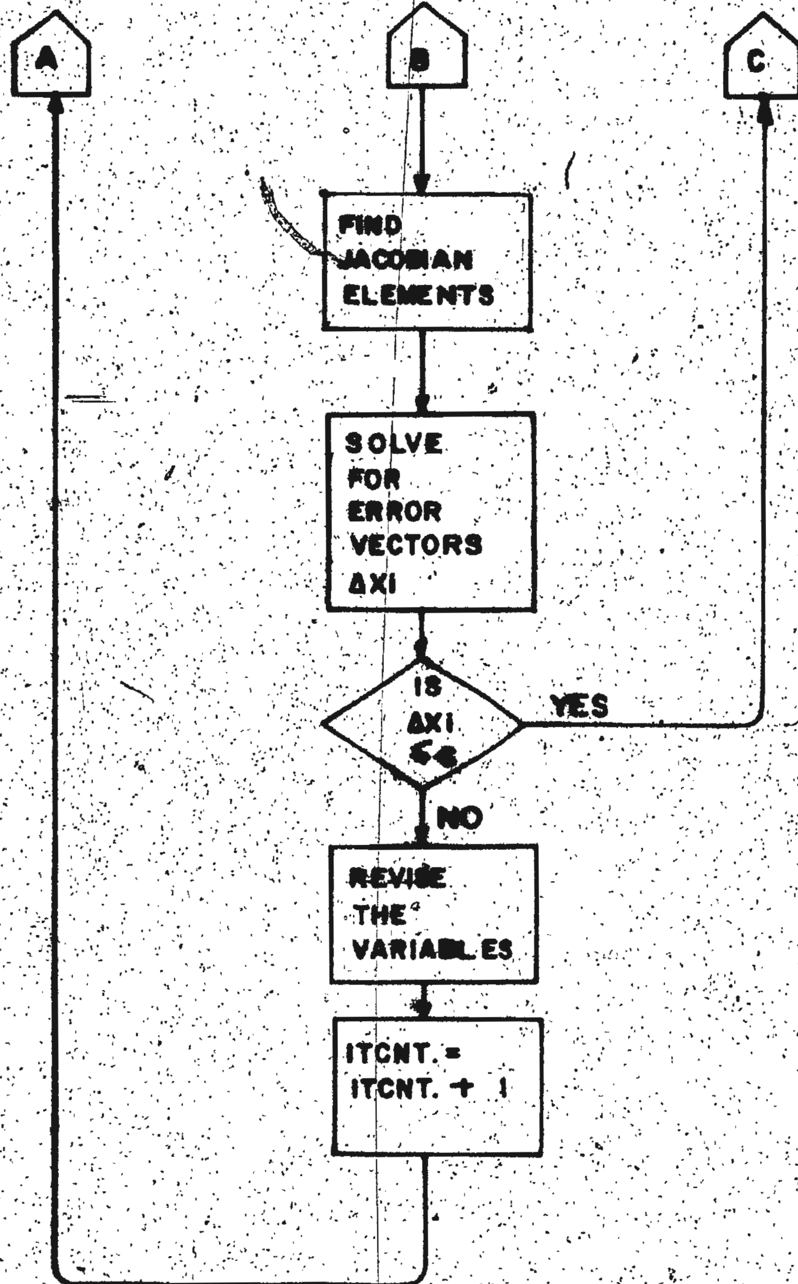


FIGURE B-1 (Cont'd). Algorithm Flowchart.

ITCNT	number of iterations completed by the program.

Variable Dimension Values

LAP	IR
LAPO	IC
LYO	IIR
LYT	IIC
NH	IRR
NHA	IIC
NS	NSA
MA	IA

Optimization Variables

Ps(NUMS,N)	generated thermal power for plant NUMS at time period N.
Ph(NUMH,N)	generated hydro power for plant NUMH at time period N.
h(NUMH,N)	the reservoir level of plant NUMH at time N.
LAM(N)	the incremental cost of power $\lambda(t)$ for time instant N.
NEW(NUMH)	the water worth coefficient ν_o for the reservoir of plant NUMH.

Quadratic Model Coefficients

ALPHAH(NUMH)	α_h for plant NUMH.
BETAH(NUMH)	β_h for plant NUMH.
GAMAH(NUMH)	γ_h for plant NUMH.
BETAS(NUMS)	β_s for plant NUMS.

GAMAS(NUMS) λ_s for plant NUMS.

AZ(NUMH) a_o for plant NUMH.

AO(NUMH) a_1 for plant NUMH.

AT(NUMH) a_2 for plant NUMH.

Transmission Loss Coefficients

BSS BSH BHS BHH CH CS KLO	These transmission losses are for interconnections and other modes. S refers to thermal and H refers to hydro.

System Constants and Data

PD(N) Power demand at time N.

K(NUMH) System's constant factor for plant NUMH.

DELT(I) Length of time subintervals I.

Partial Derivatives (see Appendix A)

DFBH(NUMH,N) $\frac{\partial[F_b]}{\partial P_h(t)}$

DFBD(NUMH,N) $\frac{\partial[F_b]}{\partial h(t)}$

DFIH(NUMH,N) $\frac{\partial[F_i]}{\partial P_h(t)}$

DFID(NUMH,N) $\frac{\partial[F_i]}{\partial h(t)}$

DFIDP(NUMH,N)	$\frac{\partial[F_i]}{\partial h(t+1)}$
DFDS(NUMS,N)	$\frac{\partial[F_d]}{\partial P_s(t)}$
DFDH(NUMH,N)	$\frac{\partial[F_d]}{\partial P_h(t)}$
DFHS(NUMH,N)	$\frac{\partial[F_h]}{\partial P_s(t)}$
DFHH(NUMH,N)	$\frac{\partial[F_h]}{\partial P_h(t)}$
DFHL(NUMH,N)	$\frac{\partial[F_h]}{\partial \lambda(t)}$
DFHD(NUMH,N)	$\frac{\partial[F_h]}{\partial H(t)}$
DFHN(NUMH,N)	$\frac{\partial[F_h]}{\partial \nu_o}$
DFSS(NUMS,N)	$\frac{\partial[F_s]}{\partial P_s(t)}$
DFSH(NUMH,N)	$\frac{\partial[F_s]}{\partial \lambda(t)}$

<u>Jacobian Submatrices</u>

AP(LAP,LAP) APO(LAPO,LAPO)	Submatrices of JA.
AINV(LYO,LYO)	Inverse of matric JA.
BP(LYO,LYT) CP(LYT,LYO) DP(LYT,LYT)	Submatrices of J. See text for further reference.

Reservoir Data

FLO(NUMH,N)	Natural inflow to the reservoir of plant NUMH.
EPCE(NUMH,N)	$\psi(t)$ - quadratic model of reservoir variation.
S(NUMH)	Reservoir surface area for plant NUMH.
B(NUMH)	Available amount of water from reservoir of plant NUMH.

System Output

LOSS(N)	Transmission losses.
NU(t)	Water worth coefficient $\nu(t)$ determined as
	$\nu_o(t)$ exp[Mt] (NEW(NUMH) * P(NUMH,N) .
P(NUMH,N)	Time variable portion of NU(t).
Q(NUMH,N)	Hydro plant discharge.
FCST(NUMS)	Thermal plant daily fuel cost.

Function Variables

PHI(NUMH,N)	$\phi(P_h)$ - quadratic model for the hydro plant performance.
FS(NUMS,N), FH(NUMH,N), FI(NUMH,N), FD(N), FB(NUMH)	System equations. See text for further discussion and explanation.
YO(LYO), YT(LYO)	Vectors containing values of the functions for iteration i.
XO(LYO), XT(LYO)	Vectors containing the error vectors generated by the program.

Variables, Vectors and Matrices used to Determine the Maximum Relative Error

FO(LYO)	DECX1(LYO)
FOO(LYO)	DELX2(LYT)
FT(LYT)	TESTX
FTT(LYT)	

Vectors and Matrices used to Determine the Error Vectors

R1(LYT,LYO)	R5(LYT)
R2(LYT,LYT)	R6(LYO)
R3(LYT,LYT)	R7(LYO)
R4(LYT)	

General Purpose (dummy) Variables

F1	F11	TF
F6	F12	CBA
F2	AC	CC
F7	SUM	FA
C	FF1	AA
Z	FF2	FB
X1	FF3	AB
X2	WW	FC
X3	WX	EE
F3	WY	FF
F8	WZ	P
F4	FF4	TEMP1
F9	FF6	ABC

F5	E	FAC
F10	BP	

B-3 THE COMPUTER PROGRAM LISTING

The following pages present the listing of the computer program in full. The dimensions shown are for the first test system of one thermal and one hydro plant for a 24-hour time period.

```
C###########################################################
C#################         S T A R T          ##############
C#################           O F              ##############
C#################       P R O G R A M        ##############
C###########################################################
C###########################################################
C                                                          C
C                                                          C
C                                                          C
C                                                          C
C**********************************************************
C*****************          M A I N          **************
C*****************       P R O G R A M       **************
C**********************************************************
C
C          THIS PROGRAM SOLVES FOR THE
C          OPTIMUM HYDRO THERMAL DISPATCH
C          SCHEDULE FOR ELECTRIC POWER
C          SYSTEMS CONTAINING VARIABLE
C          HEAD RESEVOIRS.
C
C          THE METHOD UTILIZES THE WELL
C          KNOWN NEWTON-RAPHSON METHOD
C          TO OBTAIN THE SOLUTION
C          THROUGH ITTERITIVE TECHNIQUES.
C
C          THE PROGRAM IS SUBDIVIDED INTO
C          VARIOUS SECTIONS AS FOLLOWS:
C
C          THE MAIN PROGRAM IS RESPONSIBLE
C          FOR ALL ARRAY AND DATA MANIPULATION
C          AND CONTROL.THE PROGRAM ALSO
C          KEEPS TRACK OF THE NUMBER
C          OF ITERATIONS AND ENSURES THAT
C          ALL PROGRAM TOLERANCES ARE
C          MET.
C
C          SUBROUTINES RONE AND RTWO ARE
C          RESPONSIBLE FOR THE TRANSFER OF
C          DATA FROM THE DATA FILE TO THE
C          MAIN PROGRAM AND THE OTHER
C          SUBROUTINES.
C
C          SUBROUTINE THREE CALCULATES THE
C          VALUE OF PHI(T) WHICH IS THE
C          QUADRATIC MODEL OF THE HYDRO
C          PLANT'S PERFORMANCE.
C
C          SUBROUTINE FOUR FINDS THE VALUE
```

```
C          OF EPCE(T). THIS QUADRATIC
C          EQUATION MODELS THE RESEVIOR
C          VARIATIONS.
C
C          SUBROUTINE T IS RESPONSIBLE FOR
C          CALCULATING THE VALUES OF THE
C          SYSTEM EQUATIONS THAT ARE TO BE
C          MINIMIZED. ONCE THESE VALUES
C          ARE DETERMINED,THE SUBROUTINE
C          THEN POSITIONS THEM IN TWO
C          ARRAYS YO AND YT. THESE AND
C          THIER STRUCTURES ARE FURTHER
C          DESCRIBED IN THE TEXT.
C
C          SUBROUTINE JAC CALCULATES THE
C          THE VALUES OF THE PARTIAL
C          DERIVATIVES OF THE SYSTEM
C          EQUATIONS AND POSITIONS THEM
C          WITHIN THE SUBMATRICES AS
C          DESCRIBED IN THE TEXT.IT
C          ALSO DETERMINES THE INVERSE
C          OF THE BLOCK DIAGONAL MATRIX
C          NAMED ' JA '.
C
C          SUBROUTINE GUESS IS THE
C          SUBROUTINE WHICH GENERATES
C          THE INITAL ESTIMATES THAT
C          ARE USED IN THE SOLUTION
C          OF THE OPTMAL STRATEGY.
C
C          SUBROUTINES MINVRD AND
C          SUBMXD ARE THE COMMERICALLY
C          OBTAINED SUBROUTINES WHICH
C          TOGETHER FIND THE INVERSE
C          OF THE MATRICES.
C
C          SUBROUTINES X1MULT AND X2MULT
C          ARE SUBROUTINES DEVELOPED
C          TO HANDLE THE MATRIX
C          MULTIPLICATION FOUND IN
C          THE MAIN PROGRAM.
C
C-----------------------------------------------------
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PS(1,24),PH(1,24),LAM(24),PD(24)
      DIMENSION H(1,24),FLO(1,24),DELT(24)
      DIMENSION PHI(1,24),EPCE(1,24),FB(1)
      DIMENSION FS(1,24),FH(1,24),FI(1,24),FD(24)
      DIMENSION IR(2),IC(2),IIR(23),IIC(23)
      DIMENSION IRR(25),ICC(25),BHS(1,1)
```

```
      DIMENSION LOSS(24),NUMBEH(1),NUMBES(1)
      DIMENSION ALPHAH(1),BETAH(1),GAMAH(1)
      DIMENSION AZ(1),AO(1),AT(1),CH(1)
      DIMENSION BETAS(1),GAMAS(1),CS(1)
      DIMENSION BSS(1,1),BHH(1,1),BSH(1,1)
      DIMENSION NEW(1),S(1),K(1),FCST(1)
      DIMENSION Q(1,24),P(1,24),NU(1,24)
      DIMENSION B(1),DELX1(71),DELX2(25)
      DIMENSION DFBH(1,24),DFBD(1,24),
     1 DFIH(1,24),DFID(1,24),DFIDP(1,24),
     1 DFDS(1,24),DFDH(1,24),DFHS(1,24),
     1 DFHH(1,24),DFHL(1,24),DFHD(1,24),
     4 DFHN(1,24),DFSS(1,24),DFSH(1,24),
     5 DFSL(1,24)
      DIMENSION AP(2,2),APO(23,23)
      DIMENSION YO(71),YT(25)
      DIMENSION AINV(71,71),BP(71,25)
      DIMENSION CP(25,71),DP(25,25)
      DIMENSION R1(25,71),R2(25,25)
      DIMENSION R3(25,25),R4(25)
      DIMENSION R5(25),R6(71),R7(71)
      DIMENSION XO(71),XT(71)
      DIMENSION FOO(71),FTT(25)
      DIMENSION FO(71),FT(25)
      REAL*8 NU,KLO,LAM,K,NEW.LOSS
      OPEN(UNIT=10,NAME='STORE.DAT',ACCESS='SEQUENTIAL',
     1 TYPE='OLD',DISPOSE='SAVE',FORM='FORMATTED')
      OPEN(UNIT=6,NAME='OUT.DAT',TYPE='NEW',DISPOSE='SAVE'
C-----------------------------------------------------
C
C            READ IN THE CONTROL TERMS
C
C-----------------------------------------------------
C
C            N-----NUMBER OF DISCRETE INTERVALS
C            NUMH--NUMBER OF HYDRO PLANTS
C            NUMS--NUMBER OF THERMAL PLANTS
C
C-----------------------------------------------------
      READ(10,123)N,NUMH,NUMS
123   FORMAT(3I10)
C-----------------------------------------------------
C
C            READ IN THE VARIABLES
C-----------------------------------------------------
C            THESE VARIABLES ARE THE COEFFICIENTS
C            FOR THE QUADRATIC MODELS OF: THE FUEL
C            COSTS (BETAS,GAMAS),THE HYDRO PLANTS
C            PERFORMANCE CHARACTERISTICS (ALPHAH,
C            BETAH,GAMAH),AND THE RESEVOIR VARIATIONS
```

```
C            AZ,AO,AT). ALSO INCLUDED ARE THE
C            SYSTEM'S PROPORTIONALITY CONSTANT (K),
C            THE RESEVOIR AREA (S) AND THE AVAILABLE
C            AMOUNT OF WATER (B). THE TRANSMISSION
C            LOSS COEFFICIENTS (BSS,BSH,BHS,BHH,CS,
C            CH,KLO) ARE READ IN AS WELL.
C-------------------------------------------------------------
      CALL RONE(ALPHAH,BETAH,GAMAH,AZ,AO,AT,
     1 BETAS,GAMAS,CS,CH,BSS,BSH,BHH,N,KLO,K,S,NUMH,
     2 NUMS,B,BHS)
C-------------------------------------------------------------
C
C          WRITE OUT THE VARIABLES
C
C-------------------------------------------------------------
      WRITE(6,1000)
1000  FORMAT(1H1,///)
      WRITE(6,1001)NUMS
1001  FORMAT(/,10X,'NUMS='I10)
      WRITE(6,1002)NUMH
1002  FORMAT(/,10X,'NUMH='I10)
      DO 6001 NH=1,NUMH
      WRITE(6,1003)ALPHAH(NH)
1003  FORMAT(/,10X,'ALPHAH=',E30.20)
      WRITE(6,1004)BETAH(NH)
1004  FORMAT(/,10X,'BETAH=',E30.20)
      WRITE(6,1005)GAMAH(NH)
1005  FORMAT(/,10X,'GAMAH=',E30.20)
      WRITE(6,1006)AZ(NH)
1006  FORMAT(/,10X,'AZ(A-ZERO)=',E30.20)
      WRITE(6,1007)AO(NH)
1007  FORMAT(/,10X,'AO(A-ONE)=',E30.20)
      WRITE(6,1008)AT(NH)
1008  FORMAT(/,10X,'AT(NH)=',E30.20)
      WRITE(6,1009)CH(NH)
1009  FORMAT(/,10X,'CH=',E30.20)
      WRITE(6,1010)BHH(NH,NH)
1010  FORMAT(/,10X,'BHH=',E30.20)
      WRITE(6,1021)B(NH)
1021  FORMAT(/,10X,'B=',E30.20)
      WRITE(6,1020)S(NH)
1020  FORMAT(/,10X,'S=',E30.20)
6001  CONTINUE
      DO 6002 NS=1,NUMS
      WRITE(6,1011)BETAS(NS)
1011  FORMAT(/,10X,'BETAS=',E30.20)
      WRITE(6,1012)GAMAS(NS)
1012  FORMAT(/,10X,'GAMAS=',E30.20)
      WRITE(6,1013)CS(NS)
1013  FORMAT(/,10X,'CS=',E30.20)
```

```
C
C        ESTIMATE THE INITAL VALUES
C
C-----------------------------------------------
C
C       THESE INITAL VALUES ARE THE INITAL
C       GUESSES USED IN THE ITERITIVE
C       PROCESS OF THE NEWTON-RAPHSON
C       METHOD.
C
C       PS----THERMAL POWER
C       PH----HYDRO POWER
C       LAM---THE INCREMENTAL COST OF
C             POWER COEFFICIENT.
C       DELT--THE LENGTH OF THE DISCRETE
C             INTERVAL
C       PHI---THE QUADRATIC MODEL FOR
C             THE HYDRO PLANT PERFORMANCE
C             CHARACTERIC
C       EPCE--THE QUADRATIC MODEL FOR
C             RESEVOIR VARIATION
C       Q-----THE BIQUADRATIC MODEL FOR
C             THE RESEVOIR DISCHARGE
C
C-----------------------------------------------
       CALL GUESS(PS,PH,LAM,H,PD,DELT,AZ,AO,AT,
     1 B,S,K,N,ALPHAH,BETAH,GAMAH,KLO,CS,CH,BSS,
     2 BSH,BHH,BETAS,GAMAS,NEW,NUMS,NUMH,PHI,EPCE,
     3 Q,BHS)
C-----------------------------------------------
C
C        BEGIN THE ITERATION COUNT
C        AND CHECK TO SEE THAT THE
C        THE NUMBER OF ITERATIONS
C        DOES NOT EXCEED THE PRESET
C        LIMIT.
C
C-----------------------------------------------
       ITCNT=1
902    CONTINUE
       IF(ITCNT.EQ.15)GOTO 4080
C
C-----------------------------------------------
C
C        REVISE FO AND FT
C
C-----------------------------------------------
C
C       THESE VECTORS ARE USED IN
C       THE CALCULATION OF THE
```

```
C        MAXIMUM RELATIVE ERROR
C
C-----------------------------------------------------
         L=1
         DO 9077 I=1,N
         DO 9078 NS=1,NUMS
         FO(L)=PS(NS,I)
9078     CONTINUE
         FO(L)=LAM(I)
9077     L=L+1
         DO 9079 I=2,N
         DO 9079 NH=1,NUMH
         FO(L)=H(NH,I)
9079     L=L+1
C
         L=1
         DO 9080 I=1,N
         DO 9080 NH=1,NUMH
         FT(L)=PH(NH,I)
9080     L=L+1
         DO 9081 NH=1,NUMH
         FT(L)=NEW(NH)
9081     L=L+1
C-----------------------------------------------------
C
C         CALCULATE THE VALUES
C         OF PHI(T) AND EPCE(T)
C-----------------------------------------------------
C         THESE ARE THE QUADRATIC MODELS
C         OF THE HYDRO PLANT'S PERFORMANCE
C         AND THE RESEVOIR VARIATION
C         RESPECTIVELY.
C-----------------------------------------------------
         CALL THREE(ALPHAH,BETAH,GAMAH,PH,PHI,N,NUMH)
         CALL FOUR(AZ,AO,AT,H,EPCE,N,NUMH)
C-----------------------------------------------------
C
C         SET UP THE VECTORS YO AND YT
C         AND DETERMINE IF THEY
C         ARE WITHIN THE ALLOWABLE
C         LIMIT.
C
C-----------------------------------------------------
C
C         THESE VECTORS ARE DESCRIBED IN
C         IN THE TEXT. BRIEFLY,THEY ARE
C         THE FUNCTIONS WHICH ARE TO BE
C         MINIMIZED.ONCE THE VALUE OF
C         THE FUNCTIONS ARE CALCULATED
C         THEY ARE THEN COMPARED TO A
```

```
C          MINIMUM CITERION VALUE WHICH
C          IS THE TEST FOR CONVERGENCE.
C
C-----------------------------------------------------------
         CALL T(FS,FH,FD,EI,FB,PS,PH,LAM,H,EPCE,
     1 PHI,K,DELT,PD,KLO,YO,YT,BETAS,GAMAS,GAMAH,
     2 BETAH,CH,CS,BSS,BHS,BSH,BHH,B,S,FLO,N,NEW,
     3 AO,AT,NUMS,NUMH,P,Q,LYO,LYT)
C
C
         DO 4000 I=1,LYO
         IF(DABS(YO(I)).GT.1.E-5) GOTO 4020
         IF(I.EQ.LYO) GOTO 4010
4000     CONTINUE
4010     DO 4011 I=1,LYT
         IF(DABS(YT(I)).GT.1.E-5) GOTO 4020
         IF(I.EQ.LYT) GOTO 4030
4011     CONTINUE
C-------------------------------------------------------
C
C          DETERMINE THE ELEMENTS OF THE
C          JACOBIAN MATRIX AND SET UP THE
C          PARTITIONED MATRICIES.
C
C-------------------------------------------------------
4020     CONTINUE
         CALL JAC(ALPHAH,BETAH,GAMAH,GAMAS,EPCE,LAM,
     1 DELT,PHI,BSS,BHS,BSH,BHH,CS,CH,N,K,S,H,PH,
     2 PS,PD,KLO,AO,AT,NEW,NUMS,NUMH,P,Q,IR,IC,IIR,IIC,
     3 NU,LYO,LYT,DFSS,DFSH,DFSL,DFHS,DFHH,DFHL,
     4 DFHD,DFHN,DFDS,DFDH,DFIH,DFID,DFBH,DFBD,
     5 DFIDP,LAP,LAPO,AINV,BP,CP,DP,AP,APO)
C-------------------------------------------------------
C
C          CALCULATE THE DIFFERENCE ELEMENT
C          VECTORS AND DETERMINE IF THEY
C          ARE WITHIN THE TOLERANCE LIMIT.
C
C-------------------------------------------------------
C
         CALL X1MULT(CP,AINV,R1,LYT,LYO,LYO)
C
         CALL X1MULT(R1,BP,R2,LYT,LYO,LYT)
C
         DO 9000 I=1,LYT
         DO 9000 J=1,LYT
         R3(I,J)=DP(I,J)-R2(I,J)
9000     CONTINUE
C
C
```

```
      MA=LYT
      IA=LYT
      CALL MINVRD(R3,IA,MA,DET,IER,IRR,ICC)
C
C
      CALL X2MULT(R1,YO,R4,LYT,LYO)
C
      DO 9001 I=1,LYT
      R5(I)=YT(I)-R4(I)
9001  CONTINUE
C
C
      CALL X2MULT(R3,R5,XT,LYT,LYT)
C
C
      CALL X2MULT(BP,XT,R6,LYO,LYT)
C
C
      DO 9002 I=1,LYO
      R7(I)=YO(I)-R6(I)
9002  CONTINUE
C
C
      CALL X2MULT(AINV,R7,XO,LYO,LYO)
C
C-----------------------------------------------------------
C
C              REVISE THE VARIABLES
C
C-----------------------------------------------------------
      L=1
      DO 3000 I=1,N
      DO 3001 NS=1,NUMS
      PS(NS,I)=PS(NS,I)-XO(L)
3001  L=L+1
      LAM(I)=LAM(I)-XO(L)
3000  L=L+1
C
C
      DO 3002 I=2,N
      DO 3002 NH=1,NUMH
      H(NH,I)=H(NH,I)-XO(L)
3002  L=L+1
C
C
      L=1
      DO 3003 I=1,N
      DO 3003 NH=1,NUMH
      PH(NH,I)=PH(NH,I)-XT(L)
3003  L=L+1
```

```
C
C
      DO 3004 NH=1,NUMH
      NEW(NH)=NEW(NH)-XT(L)
      L=L+1
3004  CONTINUE
C-----------------------------------------------------
C
C          SET UP THE NEW
C        VECTORS FOO AND FTT
C-----------------------------------------------------
C          THESE VECTORS ARE USED IN
C          THE CALCULATION OF THE
C          MAXIMUM RELATIVE ERROR
C-----------------------------------------------------
      L=1
      DO 9070 I=1,N
      DO 9071 NS=1,NUMS
      FOO(L)=PS(NS,I)
9071  L=L+1
      FOO(L)=LAM(I)
9070  L=L+1
      DO 9072 I=2,N
      DO 9072 NH=1,NUMH
      FOO(L)=H(NH,I)
9072  L=L+1
C
C
      L=1
      DO 9073 I=1,N
      DO 9073 NH=1,NUMH
      FTT(L)=PH(NH,I)
9073  L=L+1
      DO 9074 NH=1,NUMH
      FTT(L)=NEW(NH)
9074  L=L+1
C----------------------------------------------------
C           CALCULATE THE RELATIVE
C           ERROR AND DETERMINE IF
C           IT IS WITHIN THE
C           ALLOWABLE LIMITS.
C----------------------------------------------------
      DO 9015 I=1,LYO
      DELX1(I)=(FO(I)-FOO(I))/FO(I)
9015  CONTINUE
      DO 9016 I=1,LYT
      DELX2(I)=(FT(I)-FTT(I))/FT(I)
9016  CONTINUE
C
C
```

```
      TESTX=DABS(DELX1(1))
      DO 5105 NN=2,LYO
      IF(DABS(DELX1(NN)).GT.TESTX) TESTX=
     1 DABS(DELX1(NN))
5105  CONTINUE
      DO 5106 NN=1,LYT
      IF(DABS(DELX2(NN)).GT.TESTX) TESTX=
     1 DABS(DELX2(NN))
5106  CONTINUE
      WRITE(6,5107)ITCNT,TESTX
5107  FORMAT(/,10X,'THE MAXIMUM RELATIVE ERROR FOR'
     1 ,/,10X,'ITERATION NUMBER',I3,/,10X,' IS ',
     2 E20.10,/)
C
C
      DO 9010 I=1,LYO
      IF(DABS(DELX1(I)).GT.1.E-5) GOTO 4070
      IF (I.EQ.LYO)GOTO 9011
9010  CONTINUE
9011  DO 9012 I=1,LYT
      IF(DABS(DELX2(I)).GT.1.E-5) GOTO 4070
      IF(I.EQ.LYT) GOTO 4030
9012  CONTINUE
C---------------------------------------------
C
C      REVISE THE ITERATION COUNT
C
C---------------------------------------------
4070  CONTINUE
      ITCNT=ITCNT+1
      GOTO 902
C---------------------------------------------
C
C      CALCULATE THE THERMAL
C      FUEL COSTS AND THE SYSTEM
C      LOSSES.
C
C---------------------------------------------
4030  CONTINUE
C
C
      FCST(NS)=0.
      DO 4435 I=1,N
      DO 4435 NS=1,NUMS
      FCST(NS)=FCST(NS)+(1+BETAS(NS)*PS(NS,I)+
     1 (GAMAS(NS)*PS(NS,I)**2.))
4435  CONTINUE
C
C
      DO 4445 I=1,N
```

```
      SUMM=0.
      DO 4448 NS=1,NUMS
      SUMM=SUMM+PS(NS,I)
4448  CONTINUE
      DO 4449 NH=1,NUMH
      SUMM=SUMM+PH(NH,I)
4449  CONTINUE
      LOSS(I)=SUMM-PD(I)
4445  CONTINUE
      GOTO 4081
C%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C%%%%%%%%%%%%%%%%                        %%%%%%%%%%%%%%%
C%%%%%%%%%%%%%%%%      S T A R T         %%%%%%%%%%%%%%%
C%%%%%%%%%%%%%%%%         O F            %%%%%%%%%%%%%%%
C%%%%%%%%%%%%%%%%      P R I N T         %%%%%%%%%%%%%%%
C%%%%%%%%%%%%%%%%    S E C T I O N       %%%%%%%%%%%%%%%
C%%%%%%%%%%%%%%%%                        %%%%%%%%%%%%%%%
C%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C
C
C       START HERE FOR NON-COVERGING SYSTEM
C
C
4080  WRITE(6,5000)
5000  FORMAT(1H1)
      WRITE(6,5001)
5001  FORMAT(10X,'THE PROGRAM IS EXITING DUE TO',
     1 ' NON-CONVERGENCE')
      GOTO 9999
C
C
C       START HERE FOR CONVEGING SYSTEM
C
C
4081  WRITE(6,5050)
5050  FORMAT(1H1)
      WRITE(6,5051)
5051  FORMAT(20X,'-----------------------------------',
     1'------------')
      WRITE(6,5052)ITCNT
5052  FORMAT(15X,'THE SOLUTION WAS OBTAINED IN',I3,
     1 'ITERATIONS')
      WRITE(6,5051)
      WRITE(6,5050)
C
      DO 5110 NH=1,NUMH
5110  NUMBEH(NH)=NH
      DO 5111 NS=1,NUMS
5111  NUMBES(NS)=NS
C
```

```
C
C
      WRITE(6,5056)
5056  FORMAT(30X,'P A G E  1',/////////////)
      WRITE(6,5051)
      WRITE(6,5057)
5057  FORMAT(25X,'TIME',1X,<NUMS>(3X,'THERMAL'))
      WRITE(6,5058)
5058  FORMAT(24X,'PERIOD',<NUMS>(4X,'PLANT'))
      WRITE(6,5059)(NUMBES(NS),NS=1,NUMS)
5059  FORMAT(30X,<NUMS>(5X,'NO:',I2))
      WRITE(6,5060)
5060  FORMAT(25X,'(HR)',1X,<NUMS>(5X,'MW',3X))
      WRITE(6,5051)
      DO 5061 I=1,N
      WRITE(6,5062)I,(PS(NS,I),NS=1,NUMS)
5062  FORMAT(26X,I2,2X,<NUMS>(3X,F7.2))
5061  CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)
C
      WRITE(6,5063)
5063  FORMAT(30X,'P A G E  2',//////////////)
      WRITE(6,5051)
      WRITE(6,5064)
5064  FORMAT(25X,'TIME',1X,<NUMH>(4X,'HYDRO',1X))
      WRITE(6,5065)
5065  FORMAT(24X,'PERIOD',<NUMH>(4X,'PLANT'))
      WRITE(6,5066)(NUMBEH(NH),NH=1,NUMH)
5066  FORMAT(30X,<NUMH>(5X,'NO:',I2))
      WRITE(6,5067)
5067  FORMAT(25X,'(HR)',1X,<NUMH>(5X,'MW',3X))
      WRITE(6,5051)
      DO 5068 I=1,N
      WRITE(6,5069)I,(PH(NH,I),NH=1,NUMH)
5069  FORMAT(26X,I2,2X,<NUMH>(3X,F7.2))
5068  CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)
C
      WRITE(6,5070)
5070  FORMAT(30X,'P A G E  3',//////////////)
      WRITE(6,5051)
      WRITE(6,5071)
5071  FORMAT(25X,'TIME',1X,<NUMH>(3X,'NET',2X))
      WRITE(6,5072)
5072  FORMAT(30X,<NUMH>(3X,'HEAD',1X))
      WRITE(6,5073)
5073  FORMAT(24X,'PERIOD',<NUMH>(3X,'PLANT'))
```

```
      WRITE(6,5074)(NUMBEH(NH),NH=1,NUMH)
5074  FORMAT(30X,<NUMH>(3X,'NO:',I2))
      WRITE(6,5075)
5075  FORMAT(26X,'HR',2X,<NUMH>(3X,'FT',3X))
      WRITE(6,5051)
      DO 5076 I=1,N
      WRITE(6,5077)I,(H(NH,I),NH=1,NUMH)
5077  FORMAT(26X,I2,2X,<NUMH>(2X,F6.2))
5076  CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)
C
      WRITE(6,5078)
5078  FORMAT(30X,'P A G E  4',//////////////)
      WRITE(6,5051)
      WRITE(6,5079)
5079  FORMAT(25X,'TIME',5X,'POWER'5X,'POWER',5X,'LAMBDA')
      WRITE(6,5080)
5080  FORMAT(24X,'PERIOD',4X,'DEMAND',4X,'LOSSES')
      WRITE(6,5081)
5081  FORMAT(26X,'HR',8X,'MW',8X,'MW',7X,'$/MW')
      WRITE(6,5051)
      DO 5082 I=1,N
      WRITE(6,5083)I,PD(I),LOSS(I),LAM(I)
5083  FORMAT(26X,I2,5X,F7.2,3X,F7.2,4X,F6.3)
5082  CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)
C
      WRITE(6,5084)
5084  FORMAT(30X,'P A G E  5',//////////////)
      WRITE(6,5051)
      WRITE(6,5085)
5085  FORMAT(25X,'TIME'1X,<NUMH>(4X,'NU',3X))
      WRITE(6,5086)
5086  FORMAT(24X,'PERIOD',<NUMH>(3X,'PLANT',1X))
      WRITE(6,5087)(NUMBEH(NH),NH=1,NUMH)
5087  FORMAT(30X,<NUMH>(3X,'NO:',I2,1X))
      WRITE(6,5088)
5088  FORMAT(26X,'HR',2X,<NUMH>(4X,'$/CF',1X))
      WRITE(6,5051)
      DO 5089 I=1,N
      WRITE(6,5090)I,(NU(NH,I),NH=1,NUMH)
5090  FORMAT(26X,I2,2X,<NUMH>(2X,F7.5))
5089  CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)
C
      WRITE(6,5092)
5092  FORMAT(30X,'P A G E  6',//////////////)
```

```
      WRITE(6,5051)
      WRITE(6,5093)
5093  FORMAT(25X,'TIME',1X,<NUMH>(3X,'RESEVIOR'))
      WRITE(6,5094)
5094  FORMAT(26X,'HR',2X,<NUMH>(4X,'INFLOW',1X))
      WRITE(6,5095)
5095  FORMAT(30X,<NUMH>(5X,'PLANT',1X))
      WRITE(6,5096)(NUMBEH(NH),NH=1,NUMH)
5096  FORMAT(30X,<NUMH>(5X,'NO:',I2,1X))
      WRITE(6,5097)
5097  FORMAT(26X,'HR',2X,<NUMH>(5X,'CFS',3X))
      WRITE(6,5051)
      DO 5098 I=1,N
      WRITE(6,5099)I,(FLO(NH,I),NH=1,NUMH)
5099  FORMAT(26X,I2,2X,<NUMH>(3X,F8.1))
5098  CONTINUE
      WRITE(6,5051)
      WRITE(6,5050)
C
      WRITE(6,5100)
5100  FORMAT(30X,'P A G E  7',///////////////)
      WRITE(6,5051)
      WRITE(6,5101)
5101  FORMAT(24X,'THE VALUE OF THE COST FUNCTIONS FOR '
     1 ,/,20X,'THE THERMAL GENERATING PLANTS IN ($/DAY) ',
     2 'ARE',//)
      WRITE(6,5051)
      DO 5103 NS=1,NUMS
      WRITE(6,5102)NUMBES(NS),FCST(NS)
5102  FORMAT(/,24X,'PLANT NO:',I3,5X,'FUEL COST ',
     1 F8.2)
5103  CONTINUE
      WRITE(6,5104)
5104  FORMAT(/)
      WRITE(6,5051)
      WRITE(6,5050)
C%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C%%%%%%%%%%%%                              %%%%%%%%%%%%%%
C%%%%%%%%%%%%   E N D   O F   P R I N T    %%%%%%%%%%%%%%
C%%%%%%%%%%%%       S E C T I O N          %%%%%%%%%%%%%%
C%%%%%%%%%%%%                              %%%%%%%%%%%%%%
C%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
      GOTO 9999
9999  CONTINUE
      CLOSE(UNIT=10)
      CLOSE(UNIT=6)
      STOP
      END
C
C**********************************************************
```

```
C**************   S U B R O U T I N E   **************
C**************           R O N E        **************
C*****************************************************
C
      SUBROUTINE RONE(ALPHAH,BETAH,GAMAH,AZ,AO,AT,
     1 BETAS,GAMAS,CS,CH,BSS,BSH,BHH,N,KLO,K,S,NUMH,
     2 NUMS,B,BHS)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION K(NUMH),ALPHAH(NUMH)
      DIMENSION BETAH(NUMH),GAMAH(NUMH)
      DIMENSION AZ(NUMH),AO(NUMH),AT(NUMH)
      DIMENSION BHH(NUMH,NUMH),S(NUMH),B(NUMH)
      DIMENSION BETAS(NUMS),GAMAS(NUMS),BHS(NUMH,NUMS)
      DIMENSION CS(NUMS),BSS(NUMS,NUMS),BSH(NUMS,NUMH)
      DIMENSION CH(NUMH)
      REAL*8 KLO,K
C
      DO 100 NH=1,NUMH
      READ(10,111)ALPHAH(NH),BETAH(NH),GAMAH(NH)
100   CONTINUE
C
      DO 200 NH=1,NUMH
      READ(10,111)AZ(NH),AO(NH),AT(NH)
111   FORMAT(3E10.5)
200   CONTINUE
C
      DO 201 NH=1,NUMH
      READ(10,1)CH(NH),S(NH)
1     FORMAT(2E10.5)
201   CONTINUE
C
      DO 202 NH=1,NUMH
      READ(10,2)B(NH),K(NH)
2     FORMAT(2E10.5)
202   CONTINUE
C
      DO 101 NS=1,NUMS
      READ(10,3)BETAS(NS),GAMAS(NS),CS(NS)
3     FORMAT(3E10.5)
101   CONTINUE
C
      READ(10,5)KLO
5     FORMAT(E10.5)
C
      DO 102 NSA=1,NUMS
      DO 102 NSB=1,NUMS
      READ(10,6)BSS(NSA,NSB)
6     FORMAT(E10.5)
102   CONTINUE
C
```

```
      DO 104 NS=1,NUMS
      DO 104 NH=1,NUMH
      READ(10,8)BSH(NS,NH)
8     FORMAT(E10.5)
104   CONTINUE
C
      DO 105 NH=1,NUMH
      DO 105 NS=1,NUMS
      READ(10,9)BHS(NH,NS)
9     FORMAT(E10.5)
105   CONTINUE
C
      DO 103 NHA=1,NUMH
      DO 103 NHB=1,NUMH
      READ(10,7)BHH(NHA,NHB)
7     FORMAT(E10.5)
103   CONTINUE
C
      RETURN
      END
C
C*****************************************************************
C**************   S U B R O U T I N E   **************
C**************          R T W O          **************
C*****************************************************************
C
      SUBROUTINE RTWO(PD,FLO,DELT,N,H,NUMH)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION DELT(N),PD(N)
      DIMENSION FLO(NUMH,N),H(NUMH,N)
      DO 100 I=1,N
      DO 100 NH=1,NUMH
      READ(10,1)FLO(NH,I)
1     FORMAT(E10.5)
100   CONTINUE
      I=1
      DO 101 NH=1,NUMH
      READ(10,2)H(NH,I)
2     FORMAT(E10.5)
101   CONTINUE
      DO 102 I=1,N
      READ(10,3)PD(I),DELT(I)
3     FORMAT(2E10.5)
      WRITE(6,4002)I,PD(I)
4002  FORMAT(5X,'I=',I3,'  PD=',E20.10)
      WRITE(6,4003)I,DELT(I)
4003  FORMAT(5X,'I=',I3,'  DELT=',E20.10)
102   CONTINUE
      RETURN
      END
```

```
C
C*****************************************************************
C**********       S U B R O U T I N E          *************
C**********            T H R E E               *************
C*****************************************************************
C
      SUBROUTINE THREE(ALPHAH,BETAH,GAMAH,PH,
     1 PHI,N,NUMH,PD)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ALPHAH(NUMH),BETAH(NUMH),GAMAH(NUMH)
      DIMENSION PD(NUMH),PHI(NUMH,N)
      DIMENSION PH(NUMH,N)
      DO 1 I=1,N
      DO 1 NH=1,NUMH
      PHI(NH,I)=ALPHAH(NH)+BETAH(NH)*PH(NH,I)+
     1 GAMAH(NH)*PH(NH,I)**2.
    1 CONTINUE
      RETURN
      END
C
C*****************************************************************
C**********       S U B R O U T I N E          *************
C**********             F O U R                *************
C*****************************************************************
C
      SUBROUTINE FOUR(AZ,AO,AT,H,EPCE,N,NUMH)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION EPCE(NUMH,N),H(NUMH,N+1)
      DIMENSION AZ(NUMH),AO(NUMH),AT(NUMH)
      DO 1 I=1,N
      DO 1 NH=1,NUMH
      EPCE(NH,I)=AZ(NH)+AO(NH)*H(NH,I)+
     1 AT(NH)*H(NH,I)**2.
    1 CONTINUE
      RETURN
      END
C
C*****************************************************************
C**********       S U B R O U T I N E          *************
C**********              " T "                 *************
C*****************************************************************
C
     SUBROUTINE T(FS,FH,FD,FI,FB,PS,PH,LAM,H,EPCE,
     1 PHI,K,DELT,PD,KLO,YO,YT,BETAS,GAMAS,GAMAH,BETAH,
     2 CH,CS,BSS,BHS,BSH,BHH,B,S,FLO,N,NEW,AO,AT,NUMS,
     3 NUMH,P,Q,LYO,LYT)
     IMPLICIT  REAL*8 (A-H,O-Z)
     DIMENSION FS(NUMS,N),FH(NUMH,N),FD(N),S(NUMH)
     DIMENSION FI(NUMH,N),PS(NUMS,N),PH(NUMH,N)
     DIMENSION LAM(N),H(NUMH,N),PD(N),BSS(NUMS,NUMS)
```

```
      DIMENSION PHI(NUMH,N),BSH(NUMS,NUMH),BHS(NUMH,NUMS)
      DIMENSION EPCE(NUMH,N),Q(NUMH,N),BHH(NUMH,NUMH)
      DIMENSION BETAS(NUMS)
      DIMENSION AO(NUMH),AT(NUMH)
      DIMENSION GAMAH(NUMH),BETAH(NUMH)
      DIMENSION P(NUMH,N),GAMAS(NUMS)
      DIMENSION FLO(NUMH,N),DELT(N),K(NUMH)
      DIMENSION NEW(NUMH)
      DIMENSION FB(NUMH),B(NUMH),CS(NUMS),CH(NUMH)
      DIMENSION YO(LYO),YT(LYT)
      REAL*8 KLO,K,LAM,NEW
C
C       CALCULATE FS(T)
C
      DO 1 I=1,N
      DO 1 NS=1,NUMS
      F1=0.
      DO 2 NH=1,NUMH
2     F1=F1+(2.*BSH(NS,NH)*PH(NH,I))
      F6=0.
      DO 15 NSA=1,NUMS
15    F6=F6+2.*BSS(NS,NSA)*PS(NSA,I)
      FS(NS,I)=(BETAS(NS)+(2.*GAMAS(NS)*PS(NS,I)
     1 LAM(I)*(CS(NS)+F6+F1)
1     CONTINUE
C
C     CALCULATE Q(T)
C
      DO 3 I=1,N
      DO 3 NH=1,NUMH
      Q(NH,I)=K(NH)*EPCE(NH,I)*PHI(NH,I)
3     CONTINUE
C
C     CALCULATE FH(T)
C
      DO 4 I=1,N
      DO 4 NH=1,NUMH
      F2=0.
      DO 5 NS=1,NUMS
5     F2=F2+(2.*BHS(NH,NS)*PS(NS,I))
      P(NH,I)=(K(NH)/S(NH))*0.00012913*
     1 PHI(NH,I)*(AO(NH)+2.*AT(NH)*H(NH,I))
     2 *DELT(I)*(I-1)
      F7=0.
      DO 16 NHA=1,NUMH
16    F7=F7+2.*BHH(NH,NHA)*PH(NHA,I)
      C=CH(NH)+F2+F7
      Z=2.-(((2.*GAMAH(NH))+(BETAH(NH)*PH(NH,I))
     1 PHI(NH,I))
      FH(NH,I)=(NEW(NH)*DEXP(P(NH,I))*Z*
```

```
     1 (Q(NH,I)/PH(NH,I)))+C*LAM(I)
4      CONTINUE
C
C      CALCULATE FI(T)
C
       J=N-1
       DO 6 I=1,J
       DO 6 NH=1,NUMH
       X1=H(NH,I)-H(NH,I+1)
       X2=(DELT(I)/S(NH))*0.00012913
       X3=FLO(NH,I)-Q(NH,I)
       FI(NH,I)=X1+X2*X3
6      CONTINUE
C
C      CALCULATE FD(T)
C
       DO 10 I=1,N
       F3=0.
       DO 7 NS=1,NUMS
7      F3=F3+(CS(NS)*PS(NS,I))
       F8=0.
       JJJ=NUMS-1
       DO 17 NS=1,JJJ
       DO 17 NSA=2,NUMS
17     F8=F8+BSS(NS,NSA)*(PS(NS,I)*PS(NSA,I))
       F4=0.
       DO 8 NH=1,NUMH
8      F4=F4+(CH(NH)*PH(NH,I))
       F9=0.
       JJ=NUMH-1
       DO 18 NH=1,JJ
       DO 18 NHA=2,NUMH
18     F9=F9+BHH(NH,NHA)*(PH(NH,I)*PH(NHA,I))
       F5=0.
       DO 9 NS=1,NUMS
       DO 9 NH=1,NUMH
9      F5=F5+(BHS(NH,NS)*PS(NS,I)
     1 *PH(NH,I))
       F10=0.
       DO 19 NH=1,NUMH
       DO 19 NS=1,NUMS
19     F10=F10+(BSH(NS,NH)*PH(NH,I)*PS(NS,I))
C
       F11=0.
       DO 20 NH=1,NUMH
20     F11=F11+BHH(NH,NH)*PH(NH,I)**2
C
       F12=0.
       DO 21 NS=1,NUMS
21     F12=F12+BSS(NS,NS)*PS(NS,I)**2
```

```
C
      FD(I)=KLO+PD(I)+F3+F4+F5+F8+F9+F11+F12+F10
10    CONTINUE
C
C     CALCULATE FB(T)
C
      DO 11 NH=1,NUMH
      SUM=0.
      DO 12 I=1,N
      AC=Q(NH,I)*DELT(I)*3600.
      SUM=SUM+AC
12    CONTINUE
      FB(NH)=B(NH)-SUM
11    CONTINUE
C""""""""""""""""""""""""""""""""""""""""""""""""""
C
C      SET UP THE VECTOR YO
C
C""""""""""""""""""""""""""""""""""""""""""""""""""
       L=1
       DO 1001 I=1,N
       DO 1000 NS=1,NUMS
       YO(L)=FS(NS,I)
1000   L=L+1
       YO(L)=FD(I)
1001   L=L+1
       J=N-1
       DO 1002 I=1,J
       DO 1003 NH=1,NUMH
       YO(L)=FI(NH,I)
1003   L=L+1
1002   CONTINUE
C""""""""""""""""""""""""""""""""""""""""""""""""""
C
C       SET UP THE VECTOR YT
C
C""""""""""""""""""""""""""""""""""""""""""""""""""
       L=1
       DO 1004 I=1,N
       DO 1005 NH=1,NUMH
       YT(L)=FH(NH,I)
1005   L=L+1
1004   CONTINUE
       DO 1006 NH=1,NUMH
       YT(L)=FB(NH)
1006   L=L+1
C
C
C
C
```

```
      RETURN
      END
C
C*****************************************************************
C*****************************************************************
C**********      S U B R O U T I N E         *********************
C**********              J A C               *********************
C*****************************************************************
C*****************************************************************
C
      SUBROUTINE JAC(ALPHAH,BETAH,GAMAH,GAMAS,EPCE,LAM,
     1 DELT,PHI,BSS,BHS,BSH,BHH,CS,CH,N,K,S,H,PH,
     2 PS,PD,KLO,AO,AT,NEW,NUMS,NUMH,P,Q,IR,IC,IIR,IIC,
     3 NU,LYO,LYT,DFSS,DFSH,DFSL,DFHS,DFHH,DFHL,
     4 DFHD,DFHN,DFDS,DFDH,DFIH,DFID,DFBH,DFBD,
     5 DFIDP,LAP,LAPO,AINV,BP,CP,DP,AP,APO)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ALPHAH(NUMH),BETAH(NUMH),GAMAH(NUMH)
      DIMENSION GAMAS(NUMS),EPCE(NUMH,N),LAM(N)
      DIMENSION DELT(N),PHI(NUMH,N),BSS(NUMS,NUMS)
      DIMENSION BSH(NUMH,NUMS),BHH(NUMH,NUMH),CS(NUMS)
      DIMENSION CH(NUMH),K(NUMH),S(NUMH),H(NUMH,N)
      DIMENSION PH(NUMH,N),PS(NUMS,N),PD(N),AO(NUMH)
      DIMENSION AT(NUMH),IR(LAP),IC(LAP)
      DIMENSION IIR(LAPO),IIC(LAPO),BHS(NUMH,NUMS)
      DIMENSION P(NUMH,N),Q(NUMH,N)
      DIMENSION NU(NUMH,N),NEW(NUMH)
      DIMENSION DFBH(NUMH,N),DFBD(NUMH,N),
     1 DFIH(NUMH,N),DFID(NUMH,N),DFIDP(NUMH,N),
     2 DFDS(NUMS,N),DFDH(NUMH,N),DFHS(NUMH,N),
     3 DFHH(NUMH,N), DFHL(NUMH,N),DFHD(NUMH,N),
     4 DFHN(NUMH,N),DFSS(NUMS,N),DFSH(NUMS,N),
     5 DFSL(NUMS,N),AP(LAP,LAP),APO(LAPO,LAPO)
      DIMENSION AINV(LYO,LYO),BP(LYO,LYT)
      DIMENSION CP(LYT,LYO),DP(LYT,LYT)
      REAL*8 KLO,K,LAM,NEW,NU
C
C
C      CALCULATE THE ELEMENTS OF THE
C      JACOBIAN MATRIX.
C
C
      DO 100 I=1,N
C
      DO 101 NS=1,NUMS
      FF1=0.
      DO 10 NSA=1,NUMS
10    FF1=FF1+BSS(NS,NSA)*LAM(I)
      DFSS(NS,I)=2.*(GAMAS(NS)+FF1)
101   CONTINUE
```

```
C
      IF(NUMS.EQ.NUMH)GOTO 90
      IF(NUMS.LT.NUMH)GOTO 91
      IF(NUMS.GT.NUMH)GOTO 92
C
90    DO 102 NS=1,NUMS
      F1=0.
      DO 103 NH=1,NUMH
103   F1=F1+(2.*BSH(NS,NH)*LAM(I))
      DFSH(NS,I)=F1
102   CONTINUE
C
      DO 106 NH=1,NUMH
      DO 107 NS=1,NUMS
107   F3=F3+(2.*BHS(NH,NS)*LAM(I))
      DFHS(NH,I)=F3
106   CONTINUE
C
      GOTO93
C
91    DO 1020 NS =1,NUMS
      DO 1030 NH=1,NUMH
      F1=0.
1030  F1=F1+(2.*BSH(NS,NH)*LAM(I))
1020  DFSH(NS,I)=F1
C
      DO 1060 NH=1,NUMS
      F3=0.
      DO 1070 NS=1,NUMS
1070  F3=F3+(2.*BHS(NH,NS)*LAM(I))
1060  DFHS(NH,I)=F3
C
      GOTO 93
C
92    DO 1021 NS=1,NUMH
      F1=0.
      DO 1031 NH=1,NUMH
1031  F1=F1+(2.*BSH(NS,NH)*LAM(I))
1021  DFSH(NS,I)=F1
C
      DO 1061 NH=1,NUMH
      F3=0.
      DO 1071 NS=1,NUMH
1071  F3=F3+(2.*BHS(NH,NS)*LAM(I))
1061  DFHS(NH,I)=F3
C
      GOTO 93
C
93    DO 104 NS=1,NUMS
```

```
      FF2=0.
      DO 11 NSA=1,NUMS
11    FF2=FF2+2.*BSS(NS,NSA)*PS(NSA,I)
      F2=0.
      DO 105 NH=1,NUMH
105   F2=F2+(2.*BSH(NS,NH)*PH(NH,I))
      DFSL(NS,I)=CS(NS)+FF2+F2
104   CONTINUE
C
      DO 108 NH=1,NUMH
      FF3=0.
      DO 12 NHA=1,NUMH
12    FF3=FF3+2.*BHH(NH,NHA)
      FF3=FF3*LAM(I)
      NU(NH,I)=NEW(NH)*DEXP(P(NH,I))
      WW=(BETAH(NH)+2.*GAMAH(NH)*PH(NH,I))
      WX=(AO(NH)+2.*AT(NH)*H(NH,I))
      WY=(K(NH)*DELT(I))/S(NH)
      WZ=2.*GAMAH(NH)
      DFHH(NH,I)=K(NH)*NU(NH,I)*EPCE(NH,I)
     1 *(WZ+WY*WX*(WW**2.)*0.00012913)+FF3
108   CONTINUE
C
      DO 109 NH=1,NUMH
      FF4=0.
      DO 13 NHA=1,NUMH
13    FF4=FF4+2.*BHH(NH,NHA)*PH(NHA,I)
      F4=0.
      DO 110 NS=1,NUMS
110   F4=F4+(2.*BHS(NH,NS)*PS(NS,I))
      DFHL(NH,I)=CH(NH)+F4+FF4
109   CONTINUE
C
      DO 112 NH=1,NUMH
      DFHN(NH,I)=(NU(NH,I)/NEW(NH))*
     1 (2.*GAMAH(NH)*PH(NH,I)+BETAH(NH))
     2 *K(NH)*EPCE(NH,I)
112   CONTINUE
C
      DO 113 NS=1,NUMS
      FF5=0.
      DO 14NSA=1,NUMS
14    FF5=FF5+2.*BSS(NS,NSA)*PS(NSA,I)
      F5=0.
      DO 114 NH=1,NUMH
114   F5=F5+(BSH(NS,NH)*PH(NH,I))
      DFDS(NS,I)=CS(NS)+FF5+F5
113   CONTINUE
C
      DO 115 NH=1,NUMH
```

```
      FF6=0.
      DO 15 NHA=1,NUMH
15    FF6=FF6+2.*BHH(NH,NHA)*PH(NHA,I)
      F6=0.
      DO 116 NS=1,NUMS
116   F6=F6+(BHS(NH,NS)*PS(NS,I))
      DFDH(NH,I)=CH(NH)+FF6+F6
115   CONTINUE
C
      DO 117 NH=1,NUMH
      DFIH(NH,I)=-K(NH)*EPCE(NH,I)*
     1 (BETAH(NH)+2.*GAMAH(NH)*PH(NH,I))
     2 *(DELT(I)/S(NH))*0.00012913
117   CONTINUE
C
      DO 118 NH=1,NUMH
      DFID(NH,I)=1.-(K(NH)*PHI(NH,I)*
     1 (AO(NH)+2.*AT(NH)*H(NH,I))*
     2 (DELT(I)/S(NH)))*0.00012913
118   CONTINUE
C
      DO 119 NH=1,NUMH
      DFIDP(NH,I)=-1.
119   CONTINUE
      DO 120 NH=1,NUMH
      DFBH(NH,I)=-3600.*K(NH)*DELT(I)
     1 *EPCE(NH,I)*(BETAH(NH)+2.*GAMAH(NH)
     2 *PH(NH,I))
120   CONTINUE
C
      DO 121 NH=1,NUMH
      DFBD(NH,I)=-3600.*K(NH)*DELT(I)*
     1 PHI(NH,I)*(AO(NH)+2.*AT(NH)*H(NH,I))
121   CONTINUE
100   CONTINUE
C
C
      DO 111 I=2,N
      DO 111 NH=1,NUMH
      E=AO(NH)+(2.*AT(NH)*(H(NH,I)+
     1 (K(NH)/S(NH))*DELT(I)*0.00012913*
     2 PHI(NH,I)*EPCE(NH,I)))
      DFHD(NH,I)=K(NH)*NU(NH,I)*E*
     1 (BETAH(NH)+2.*GAMAH(NH)*PH(NH,I))
111   CONTINUE
C
C*****************************************************
C
C               SET UP THE MATRACIES
C               A(I) AND CONSTRUCT
```

```
C                THE INVERSE OF JA
C
C********************************************************
        LL=1
        MM=1
        DO 500 I=1,N
        DO 499 II=1,LAP
        DO 499 J=1,LAP
        AP(II,J)=0.
499     CONTINUE
        L=1
        M=1
        DO 501 NS=1,NUMS
        AP(L,M)=DFSS(NS,I)
        L=L+1
501     M=L
        L=NUMS+1
        M=1
        DO 502 NS=1,NUMS
        AP(L,M)=DFDS(NS,I)
502     M=M+1
        M=NUMS+1
        L=1
        DO 503 NS=1,NUMS
        AP(L,M)=DFSL(NS,I)
503     L=L+1
        DO 517 III=1,LAP
        DO 517 JJJ=1,LAP
517     CONTINUE
C
C       FIND THE INVERSE
C
        MA=LAP
        IA=LAP
        CALL MINVRD(AP,IA,MA,DET,IER,IR,IC)
C
C       POSITION THE BLOCK IN THE
C       MATRIX AINV
C
        MM=LL
        DO 504 II=1,LAP
        DO 505 J=1,LAP
        AINV(LL,MM)=AP(II,J)
505     MM=MM+1
        LL=LL+1
504     MM=MM-LAP
        LL=LAP*I+1
500     CONTINUE
C
C
```

```
C       SET THE I'TH + 1 BLOCK
C
C
        DO 509 I=1,LAPO
        DO 509 J=1,LAPO
        APO(I,J)=0.
509     CONTINUE
C
C
        L=1
        M=1
        J=N-1
        DO 510 I=1,J
        DO 511 NH=1,NUMH
        APO(L,M)=DFIDP(NH,I)
        L=L+1
511     M=M+1
510     CONTINUE
        IF(N.EQ.1)GOTO 516
        M=1
        L=NUMH+1
        J=N-1
        DO 512 I=2,J
        DO 513 NH=1,NUMH
        APO(L,M)=DFID(NH,I)
        L=L+1
513     M=M+1
512     CONTINUE
C
C       INVERT THE BLOCK
C
C
        MA=LAPO
        IA=LAPO
        CALL MINVRD(APO,IA,MA,DET,IER,IIR,IIC)
C
C
C       POSITION THE INVERSE
C       WITHIN THE MATRIX
C       AINV
C
C
516     LL=(LAP*N)+1
        MM=LL
        DO 514 I=1,LAPO
        DO 515 J=1,LAPO
        AINV(LL,MM)=APO(I,J)
515     MM=MM+1
        LL=LL+1
514     MM=MM-LAPO
```

```
C
C
C""""""""""""""""""""""""""""""""""""""""""""""""""""""""""
C
C         SET UP THE MATRIX " JB "
C
C""""""""""""""""""""""""""""""""""""""""""""""""""""""""""
      DO 519 I=1,LYO
      DO 519 J=1,LYT
      BP(I,J)=0.
519   CONTINUE
C
C
      IF(NUMS.LE.NUMH)GOTO520
      IF (NUMS.GT.NUMH) GOTO 525
C
C
520   M=1
      L=1
      DO 521 I=1,N
      DO 522 NS=1,NUMS
      BP(L,M)=DFSH(NS,I)
      M=M+1
522   L=L+1
      L=((NUMS+1)*I)+1
521   M=(NUMH*I)+1
      GOTO 530
C
C
C
525   M=1
      L=1
      DO 526 I=1,N
      DO 527 NS=1,NUMH
      BP(L,M)=DFSH(NS,I)
      M=M+1
527   L=L+1
      M=(NUM*I)+1
526   L=((NUMS+1)*I)+1
C
C
C
530   M=1
      L=NUMS+1
      DO 531 I=1,N
      DO 532 NH=1,NUMH
      BP(L,M)=DFDH(NH,I)
532   M=M+1
531   L=L+NUMS+1
C
```

```
C
C
      L=(NUMS*N)+1+N
      M=1
      J=N-1
      DO 533 I=1,J
      DO 533 NH=1,NUMH
      BP(L,M)=DFIH(NH,I)
      L=L+1
533   M=M+1
C""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""
C
C          SET UP THE MATRIX " JC "
C
C""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""
      DO 535 I=1,LYT
      DO 535 J=1,LYO
      CP(I,J)=0.
535   CONTINUE
C
C
      IF(NUMS.GE.NUMH) GOTO 540
      IF(NUMS.LT.NUMH) GOTO 545
C
C
540   M=1
      L=1
      DO 541 I=1,N
      DO 542 NH=1,NUMH
      CP(L,M)=DFHS(NH,I)
      L=L+1
542   M=M+1
      L=(NUMH*I)+1
541   M=((NUMS+1)*I)+1
      GOTO 550
C
C
545   M=1
      L=1
      DO 546 I=1,N
      DO 547 NH=1,NUMS
      CP(L,M)=DFHS(NH,I)
      L=L+1
547   M=M+1
      L=(NUMH*I)+1
546   M=((NUMS+1)*I)+1
C
C
550   M=NUMS+1
      L=1
```

```
      DO 551 I=1,N
      DO 552 NH=1,NUMH
      CP(L,M)=DFHL(NH,I)
552   L=L+1
551   M=M+NUMS+1
C
      L=NUMH+1
      M=(NUMS*N)+1+N
      DO 553 I=2,N
      DO 553 NH=1,NUMH
      CP(L,M)=DFHD(NH,I)
      L=L+1
553   M=M+1
C
C
      M=(NUMS*N)+1+N
      DO 555 I=2,N
      L=(NUMH*N)+1
      DO 554 NH=1,NUMH
      CP(L,M)=DFBD(NH,I)
      L=L+1
554   M=M+1
555   CONTINUE
C"""""""""""""""""""""""""""""""""""""""""""""""""""
C
C     SET UP THE MATRIX " JD "
C
C"""""""""""""""""""""""""""""""""""""""""""""""""""
      DO 556 I=1,LYT
      DO 556 J=1,LYT
      DP (I,J)=0.
556   CONTINUE
      L=1
      M=1
      DO 560 I=1,N
      DO 561 NH=1,NUMH
      DP(L,M)=DFHH(NH,I)
      L=L+1
561   M=M+1
560   CONTINUE
C
C
C
      L=1
      DO 562 I=1,N
      M=(NUMH*N)+1
      DO 563 NH=1,NUMH
      DP(L,M)=DFHN(NH,I)
      L=L+1
563   M=M+1
```

```
562     CONTINUE
C
C
C
        M=1
        DO 564 I=1,N
        L=(NUMH*N)+1
        DO 565 NH=1,NUMH
        DP(L,M)=DFBH(NH,I)
        L=L+1
565     M=M+1
564     CONTINUE
C
        RETURN
        END
C
C*****************************************************************
C***************     S U B R O U T I N E      ***************
C***************         G U E S S            ***************
C*****************************************************************
C
        SUBROUTINE GUESS(PS,PH,LAM,H,PD,DELT,AZ,AO,AT,B,
     1 S,K,N,ALPHAH,BETAH,GAMAH,KLO,CS,CH,BSS,BSH,BHH,
     2 BETAS,GAMAS,NEW,NUMS,NUMH,PHI,EPCE,Q,BHS)
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION PS(NUMS,N),PH(NUMH,N),LAM(N),H(NUMH,N)
        DIMENSION PD(N),EPCE(NUMH,N),PHI(NUMH,N),Q(NUMH,N)
        DIMENSION DELT(N),AZ(NUMH),AO(NUMH),AT(NUMH),B(NUMH)
        DIMENSION S(NUMH),K(NUMH),ALPHAH(NUMH),BHS(NUMH,NUMS
        DIMENSION BETAH(NUMH),GAMAH(NUMH),CS(NUMS),CH(NUMH)
        DIMENSION BSS(NUMS,NUMS),BSH(NUMS,NUMH),BETAS(NUMS)
        DIMENSION GAMAS(NUMS),BHH(NUMH,NUMH),NEW(NUMH)
        REAL*8 LAM,NEW,K,KLO
C
        TF=0.
        DO 10 I=1,N
            TF=TF+DELT(I)
10      CONTINUE
        I=1
        DO 11 NH=1,NUMH
        EPCE(NH,I)=AZ(NH)+(AO(NH)*H(NH,I))+
     1 (AT(NH)*H(NH,I)**2.)
        BP=B(NH)/(TF*K(NH)*EPCE(NH,I)*3600.)
        XX=DSQRT((BETAH(NH)**2.)-(4.*GAMAH(NH)
     1 *(ALPHAH(NH)-BP)))
        PH(NH,I)=(-BETAH(NH)+XX)/(2.*GAMAH(NH))
11      CONTINUE
        CBA=0.
        DO 777 NH=1,NUMH
```

```
777    CBA=CBA+PH(NH,I)
       DO 110 NS=1,NUMS
       CC=1.-(CBA/PD(I))
       PS(NS,I)=CC*(PD(I)/NUMS)
110    CONTINUE
C
C
       NS=1
       FA=0.
       DO 30 NSA=1,NUMS
30     FA=FA+2.*BSS(NS,NSA)*PS(NSA,I)
       AA=BETAS(NS)+(2.*GAMAS(NS)*PS(NS,I))
       DO 13 NH=1,NUMH
       FB=0.
       DO 31 NSA=1,NUMH
31     FB=FB+2.*BSH(NSA,NH)*PH(NH,I)
       AB=CS(NS)+FA+FB
13     CONTINUE
       LAM(I)=(-AA/AB)
C
C
       DO 14 NH=1,NUMH
       PHI(NH,I)=(ALPHAH(NH)+(BETAH(NH)*PH(NH,I)
     1 ))+(GAMAH(NH)*PH(NH,I)**2.)
14     CONTINUE
       DO 15 NH=1,NUMH
       FC=0.
       DO 32 NHA=1,NUMH
32     FC=FC+2.*BHH(NH,NHA)*PH(NH,I)
       Q(NH,I)=K(NH)*EPCE(NH,I)*PHI(NH,I)
       EE=((2.*ALPHAH(NH))+(BETAH(NH)*
     1 PH(NH,I)))/PHI(NH,I)
       FF=Q(NH,I)/PH(NH,I)
       P=(PHI(NH,I)/S(NH))*(AO(NH)+(2.*AT(NH)*
     1 H(NH,I)))*DELT(I)*0.00012913
       TEMP1=CH(NH)+FC
       ABC=0.0
       DO 16 NS=1,NUMS
       ABC=ABC+2.0*BHS(NH,NS)*PS(NS,I)
16     CONTINUE
       ABC=ABC+TEMP1
       NEW(NH)=(-LAM(I)*ABC)/((2.-EE)*FF
     1 *DEXP(P))
15     CONTINUE
       DO 20 NS=1,NUMS
       DO 20 I=2,N
       FAC=PD(I)/PD(1)
       PS(NS,I)=PS(NS,1)*FAC
20     CONTINUE
       DO 22 NH=1,NUMH
```

```
      DO 22 I=2,N
      FAC=PD(I)/PD(1)
      EPCE(NH,I)=EPCE(NH,1)*FAC
      PHI(NH,I)=PHI(NH,1)*FAC
      Q(NH,I)=Q(NH,1)*FAC
      PH(NH,I)=PH(NH,1)*FAC
      H(NH,I)=H(NH,I-1)
22    CONTINUE
      DO 24 I=2,N
      FAC=PD(I)/PD(1)
      LAM(I)=LAM(1)*FAC
24    CONTINUE
      RETURN
      END
C
C*************************************************************
C************  S U B R O U T I N E  *******************
C************       M I N V R D      *******************
C*************************************************************
C
      SUBROUTINE MINVRD(A,IA,MA,DETA,IER,IR,IC)
      REAL*8 A(IA,IA),DETA,PIV,PIV1,TEMP
      DIMENSION IR(MA),IC(MA)
      IER=0
      DETA=1.
      DO 1 I=1,MA
      IR(I)=0
1     IC(I)=0
      DO 123 IJKL=1,MA
2     CALL SUBMXD(A,IA,IA,MA,MA,IR,IC,I,J)
      PIV=A(I,J)
      DETA=PIV
      IF(PIV.EQ.0.0D0) GOTO 17
      IR(I)=J
      IC(J)=(I)
      PIV=1.D0/PIV
      DO 5 K=1,MA
5     A(I,K)=A(I,K)*PIV
      A(I,J)=PIV
      DO 9 K=1,MA
      IF(K.EQ.I) GOTO 9
      PIV1=A(K,J)
6     DO 8 L=1,MA
8     A(K,L)=A(K,L)-PIV1*A(I,L)
      A(K,J)=PIV1
9     CONTINUE
      PIV1=A(I,J)
      DO 11 K=1,MA
11    A(K,J)=-PIV*A(K,J)
      A(I,J)=PIV1
```

```
123    CONTINUE
12     DO 16 I=1,MA
       K=IC(I)
       M=IR(I)
       IF(K.EQ.I)GOTO 16
       DETA=-DETA
       DO 14 L=1,MA
       TEMP=A(K,L)
       A(K,L)=A(I,L)
14     A(I,L)=TEMP
       DO 15 L=1,MA
       TEMP=A(L,M)
       A(L,M)=A(L,I)
15     A(L,I)=TEMP
       IC(M)=K
       IR(K)=M
16     CONTINUE
       RETURN
17     IER=1
       RETURN
       END
C
C****************************************************************
C***********      S U B R O U T I N E       *************
C***********          S U B M X D           *************
C****************************************************************
C
       SUBROUTINE SUBMXD(A,IA,JA,MA,NA,IR,IC,I,J)
       REAL*8 A(IA,JA),TEST,X,DABS
       DIMENSION IR(MA),IC(NA)
       I=0
       J=0
       TEST=0.D0
       DO 5 K=1,MA
       IF(IR(K).NE.0) GOTO 5
       DO 4 L=1,NA
       IF (IC(L).NE.0) GOTO 4
       X=DABS(A(K,L))
       IF(X.LT.TEST) GOTO 4
       I=K
       J=L
       TEST=X
4      CONTINUE
5      CONTINUE
       RETURN
       END
C
C****************************************************************
C***********      S U B R O U T I N E       *************
C***********          X 1 M U L T           *************
```

```
C*********************************************************
C
      SUBROUTINE X1MULT(AM,BM,RM,NA,NAMB,MB)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AM(NA,NAMB),BM(NAMB,MB),RM(NA,MB)
      DO 30 I=1,NA
      DO 20 J=1,MB
      RM(I,J)=0.
      DO 10 K=1,NAMB
      RM(I,J)=RM(I,J)+AM(I,K)*BM(K,J)
10    CONTINUE
20    CONTINUE
30    CONTINUE
      RETURN
      END
C
C*********************************************************
C***********     S U B R O U T I N E      *************
C***********          X 2 M U L T         *************
C*********************************************************
C
      SUBROUTINE X2MULT(AM,BM,RM,NA,MB)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AM(NA,MB),BM(MB),RM(NA)
      DO 30 I=1,NA
      RM(I)=0.
      DO 20 K=1,MB
      RM(I)=RM(I)+AM(I,K)*BM(K)
20    CONTINUE
30    CONTINUE
      RETURN
      END
C
C
C
C#########################################################
C#########################################################
C#############       E N D    O F       ################
C#############       P R O G R A M      ################
C#########################################################
C#########################################################
```

B-4 MATRIX INVERSION ROUTINE

The subroutines MINVRD and SUBMXD found in the listing are developed by the University of Waterloo of Waterloo and is available as part of a scientific package. This matrix inversion routine is available in either single (MINVRS) or double (MINVRD) persicision.

The main purpose of the program is to compute the inverse of a matrix by the direct method.

The subroutine is called in the following way:

CALL MINVRD (A, IA, DET, IER, IR, IC)

where

A is dimensioned with absolute size IA by IA and the portion of matrix being used by the subprogram is represented by MA.

IR and IC are dimensioned to MA - they are work vectors.

IER is an error parameter usually set at 0 but equals 1 if A is singular.

The accuracy of the routine is dependent upon the size of the given matrix. The round-off error is minimized by searching for the largest pivotal element at each stage of the process. The greatest accuracy is 13 digits with MINVRD, and 4 for MINVRS.

The routine takes the matrix stored in A and computes the inverse of it by the direct method. It does this by searching for the largest pivotal element at each stage of the procedure. The result is stored in position A. The determinant $|A|$ is also calculated and is stored in DET. The matrix A is destroyed.

The core requirements for MINVRD is 2688 Bytes for the object code.

B-5 COMPUTER SPECIFICATIONS

The computer used in this research belongs to Memorial University of Newfoundland. The equipment and software is produced by Digital Equipment of Canada. The hardware of the system is the VAX 11/780 computer and peripheral support equipment. The software is Version 3.0, VAX/VMS and in particular the language used for the program was VAX-11 Fortran (Fortran 77).

The computer has 4.0 Mb of main storage and 706 Mb of disk storage. In addition, the tape drive units are capable of operating on 1600 or 6250 bpi.

APPENDIX C

APPENDIX C

COMPUTER PRINTOUT

C-1 INTRODUCTION

This appendix presents the actual form of the computer printout from the program.

It has been manually transferred to a word processing device to obtain an output of suitable quality. Otherwise, there are no alterations.

ECONOMIC DISPATCH SCHEDULE

TEST SYSTEM NO: 1

COMPUTATIONAL RESULTS

THE SOLUTION WAS OBTAINED IN 7 ITERATIONS

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 1
IS 0.9003693759E+00

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 2
IS 0.9790824988E-01

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 3
IS 0.1397912817E-01

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 4
IS 0.2885521034E-02

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 5
IS 0.6953570515E-03

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 6
IS 0.1929799536E-03

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 7
IS 0.5060176689E-04

TIME PERIOD (HR)	THERMAL PLANT NO: 1 MW
1	180.65
2	196.63
3	186.69
4	181.01
5	193.96
6	197.59
7	195.28
8	180.72
9	160.20
10	156.18
11	163.58
12	153.00
13	144.90
14	139.31
15	89.29
16	83.63
17	78.63
18	70.35
19	66.75
20	62.76
21	59.45
22	57.03
23	56.59
24	55.75

TIME PERIOD (HR)	HYDRO PLANT NO: 1 MW
1	542.42
2	572.19
3	567.33
4	568.14
5	598.18
6	614.39
7	620.83
8	603.34
9	571.50
10	573.92
11	604.71
12	588.53
13	577.85
14	573.77
15	427.89
16	422.96
17	419.54
18	403.98
19	404.67
20	404.52
21	405.01
22	411.14
23	430.98
24	453.69

TIME PERIOD HR	NET HEAD PLANT NO: 1 FT
1	205.00
2	204.81
3	204.60
4	204.38
5	204.15
6	203.90
7	203.62
8	203.32
9	203.04
10	202.78
11	202.51
12	202.21
13	201.91
14	201.62
15	201.32
16	201.17
17	201.02
18	200.86
19	200.72
20	200.58
21	200.44
22	200.29
23	200.13
24	199.95

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	LAMBDA $/MW
1	681.00	42.07	3.784
2	722.00	46.82	3.880
3	708.00	46.03	3.820
4	703.00	46.16	3.786
5	741.00	51.17	3.864
6	758.00	53.98	3.886
7	761.00	55.12	3.872
8	732.00	52.05	3.784
9	685.00	46.71	3.661
10	683.00	47.10	3.667
11	716.00	52.29	3.682
12	692.00	49.53	3.618
13	675.00	47.75	3.569
14	666.00	47.08	3.536
15	491.00	26.18	3.236
16	481.00	25.58	3.202
17	473.00	25.17	3.172
18	451.00	23.34	3.122
19	448.00	23.42	3.100
20	443.00	23.28	3.077
21	441.00	23.46	3.057
22	444.00	24.17	3.042
23	461.00	26.56	3.040
24	480.00	29.43	3.034

TIME PERIOD HR	NU PLANT NO: 1 $/CF
1	0.04812
2	0.04698
3	0.04589
4	0.04481
5	0.04347
6	0.04219
7	0.04102
8	0.04025
9	0.03983
10	0.03888
11	0.03734
12	0.03681
13	0.03622
14	0.03551
15	0.03854
16	0.03809
17	0.03761
18	0.03751
19	0.03696
20	0.03647
21	0.03591
22	0.03520
23	0.03400
24	0.03267

TIME HR	RESEVIOR INFLOW PLANT NO: 1
HR	CFS
1	12000.0
2	12000.0
3	12000.0
4	12000.0
5	12000.0
6	12000.0
7	12000.0
8	12000.0
9	12000.0
10	12000.0
11	12000.0
12	12000.0
13	12000.0
14	12000.0
15	12000.0
16	12000.0
17	12000.0
18	12000.0
19	12000.0
20	12000.0
21	12000.0
22	12000.0
23	12000.0
24	12000.0

THE VALUE OF THE COST FUNCTIONS FOR
THE THERMAL GENERATING PLANTS IN ($/DAY) ARE

PLANT NO: 1 FUEL COST 9844.70

ECONOMIC DISPATCH SCHEDULE

TEST SYSTEM NO:2

COMPUTATIONAL RESULTS

THE SOLUTION WAS OBTAINED IN 13 ITERATIONS

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 1
IS 0.5500958074E+00

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 2
IS 0.5925722180E-01

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 3
IS 0.1329054893E-01

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 4
IS 0.6463546968E-02

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 5
IS 0.3478227069E-02

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 6
IS 0.2019128443E-02

```
THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 7
 IS 0.1204589334E-02

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 8
 IS 0.7322724724E-03

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 9
 IS 0.4512366086E-03

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 10
 IS 0.2809106443E-03

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 11
 IS 0.1762657394E-03

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 12
 IS 0.1113009484E-03

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 13
 IS 0.7063917420E-04
```

TIME PERIOD (HR)	THERMAL PLANT NO: 1 MW	THERMAL PLANT NO: 2 MW
1	188.99	183.61
2	207.02	201.65
3	198.75	193.38
4	194.82	189.45
5	210.31	204.95
6	216.19	210.83
7	215.88	210.52
8	202.29	196.92
9	181.97	176.59
10	179.73	174.35
11	190.55	185.18
12	180.62	175.23
13	173.44	168.05
14	169.17	163.78
15	110.46	105.03
16	105.88	100.45
17	102.06	96.62
18	94.02	88.58
19	91.86	86.42
20	89.17	83.73
21	87.37	81.93
22	86.91	81.46
23	89.80	84.35
24	93.79	88.35

TIME PERIOD (HR)	HYDRO PLANT NO: 1 MW	HYDRO PLANT NO: 2 MW
1	560.26	511.42
2	588.05	538.14
3	580.83	531.71
4	579.28	530.72
5	607.54	556.23
6	621.60	568.92
7	625.48	573.02
8	604.22	557.17
9	567.74	529.95
10	567.31	530.94
11	597.17	553.98
12	576.13	541.41
13	560.92	533.24
14	552.81	530.14
15	392.55	421.39
16	384.64	417.06
17	378.25	414.05
18	358.94	401.98
19	356.84	402.22
20	352.70	401.21
21	351.30	402.18
22	354.86	406.40
23	373.60	419.35
24	402.14	436.03

TIME PERIOD HR	NET HEAD PLANT NO: 1 FT	NET HEAD PLANT NO: 2 FT
1	205.00	206.00
2	204.71	205.88
3	204.40	205.75
4	204.08	205.62
5	203.75	205.48
6	203.39	205.34
7	203.01	205.18
8	202.61	205.01
9	202.22	204.86
10	201.86	204.72
11	201.50	204.57
12	201.09	204.41
13	200.70	204.26
14	200.31	204.11
15	199.93	203.96
16	199.70	203.86
17	199.48	203.77
18	199.26	203.68
19	199.05	203.60
20	198.85	203.51
21	198.65	203.43
22	198.44	203.34
23	198.23	203.25
24	198.00	203.15

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	LAMBDA $/MW
1	1362.00	82.29	3.834
2	1444.00	90.86	3.942
3	1416.00	88.67	3.893
4	1406.00	88.26	3.869
5	1482.00	97.02	3.962
6	1516.00	101.54	3.997
7	1522.00	102.90	3.995
8	1462.00	96.60	3.914
9	1370.00	86.25	3.792
10	1366.00	86.33	3.778
11	1432.00	94.88	3.843
12	1384.00	89.38	3.784
13	1350.00	85.65	3.741
14	1332.00	83.89	3.715
15	982.00	47.43	3.363
16	962.00	46.03	3.335
17	946.00	44.97	3.312
18	902.00	41.53	3.264
19	896.00	41.34	3.251
20	886.00	40.81	3.235
21	882.00	40.78	3.224
22	881.00	41.63	3.221
23	922.00	45.11	3.239
24	960.00	50.31	3.263

TIME PERIOD HR	NU PLANT NO: 1 $/CF	NU PLANT NO: 2 $/CF
1	0.04769	0.04684
2	0.04651	0.04624
3	0.04541	0.04566
4	0.04433	0.04509
5	0.04300	0.04438
6	0.04175	0.04370
7	0.04063	0.04307
8	0.03993	0.04263
9	0.03961	0.04236
10	0.03874	0.04183
11	0.03727	0.04101
12	0.03688	0.04066
13	0.03643	0.04027
14	0.03586	0.03983
15	0.03921	0.04116
16	0.03889	0.04086
17	0.03856	0.04055
18	0.03861	0.04042
19	0.03822	0.04007
20	0.03970	0.03976
21	0.03751	0.03940
22	0.03697	0.03897
23	0.03591	0.03834
24	0.03447	0.03760

TIME HR	RESEVIOR INFLOW PLANT NO: 1	RESEVIOR INFLOW PLANT NO: 2
HR	CFS	CFS
1	5500.0	11000.0
2	5500.0	11000.0
3	5500.0	11000.0
4	5500.0	11000.0
5	5500.0	11000.0
6	5500.0	11000.0
7	5500.0	11000.0
8	5500.0	11000.0
9	5500.0	11000.0
10	5500.0	11000.0
11	5500.0	11000.0
12	5500.0	11000.0
13	5500.0	11000.0
14	5500.0	11000.0
15	5500.0	11000.0
16	5500.0	11000.0
17	5500.0	11000.0
18	5500.0	11000.0
19	5500.0	11000.0
20	5500.0	11000.0
21	5500.0	11000.0
22	5500.0	11000.0
23	5500.0	11000.0
24	5500.0	11000.0

THE VALUE OF THE COST FUNCTIONS FOR
THE THERMAL GENERATING PLANTS IN ($/DAY) ARE

PLANT NO: 1 FUEL COST 11764.85

PLANT NO: 2 FUEL COST 11413.87

ECONOMIC DISPATCH SCHEDULE

TEST SYSTEM NO: 3

COMPUTATIONAL RESULTS

THE SOLUTION WAS OBTAINED IN 7 ITERATIONS

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 1
IS 0.8491699406E+00

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 2
IS 0.1134254453E+00

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 3
IS 0.1611677021E-01

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 4
IS 0.3041146059E-02

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 5
IS 0.6151115247E-03

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 6
IS 0.1286717119E-03

THE MAXIMUM RELATIVE ERROR FOR
ITERATION NUMBER 7
IS 0.2744106656E-04

TIME PERIOD (HR)	THERMAL PLANT NO: 1 MW	THERMAL PLANT NO: 2 MW
1	263.76	268.84
2	271.83	276.90
3	261.53	266.61
4	254.26	259.34
5	258.93	264.01
6	264.21	269.29
7	253.08	258.16
8	236.22	241.30
9	232.83	237.91
10	227.37	232.46
11	222.90	227.99
12	215.37	220.47
13	209.04	214.13
14	204.13	209.23
15	183.91	189.02
16	178.40	183.51
17	173.29	178.40
18	167.71	172.82
19	162.92	168.03
20	158.45	163.57
21	154.19	159.31
22	150.03	155.14
23	145.49	150.61
24	140.40	145.51

TIME HR	HYDRO PLANT NO: 1 MW	HYDRO PLANT NO: 2 MW	HYDRO PLANT NO: 3 MW	HYDRO PLANT NO:4 MW	HYDRO PLANT NO: 5 MW
1	544.87	544.87	544.87	544.87	544.87
2	571.12	571.12	571.12	571.12	571.12
3	565.29	565.29	565.29	565.29	565.29
4	564.57	564.57	564.57	564.57	564.57
5	590.19	590.19	590.19	590.19	590.19
6	620.72	620.72	620.72	620.72	620.72
7	606.78	606.78	606.78	606.78	606.78
8	572.39	572.39	572.39	572.39	572.39
9	581.18	581.18	581.18	581.18	581.18
10	581.99	581.99	581.99	581.99	581.99
11	586.90	586.90	586.90	586.90	586.90
12	572.74	572.74	572.74	572.74	572.74
13	563.22	563.22	563.22	563.22	563.22
14	562.77	562.77	562.77	562.77	562.77
15	441.64	441.64	441.64	441.64	441.64
16	436.74	436.74	436.74	436.74	436.74
17	433.11	433.11	433.11	433.11	433.11
18	423.25	423.25	423.25	423.25	423.25
19	419.39	419.39	419.39	419.39	419.39
20	417.75	417.75	417.75	417.75	417.75
21	418.06	418.06	418.06	418.06	418.06
22	421.60	418.06	418.06	418.06	418.06
23	435.90	435.90	435.90	435.90	435.90
24	451.47	451.47	451.47	451.47	451.47

TIME HR	NET HEAD PLANT NO: 1 FT	NET HEAD PLANT NO: 2 FT	NET HEAD PLANT NO: 3 FT	NET HEAD PLANT NO:4 FT	NET HEAD PLANT NO: 5 FT
1	205.00	205.00	205.00	205.00	205.00
2	204.81	204.81	204.81	204.81	204.81
3	204.59	204.59	204.59	204.59	204.59
4	204.38	204.38	204.38	204.38	204.38
5	204.16	204.16	204.16	204.16	204.16
6	203.91	203.91	203.91	203.91	203.91
7	203.62	203.62	203.62	203.62	203.62
8	203.34	203.34	203.34	203.34	203.34
9	203.09	203.09	203.09	203.09	203.09
10	202.82	202.82	202.82	202.82	202.82
11	202.55	202.55	202.55	202.55	202.55
12	202.26	202.26	202.26	202.26	202.26
13	201.98	201.98	201.98	201.98	201.98
14	201.71	201.71	201.71	201.71	201.71
15	201.43	201.43	201.43	201.43	201.43
16	201.26	201.26	201.26	201.26	201.26
17	201.10	201.10	201.10	201.10	201.10
18	200.93	200.93	200.93	200.93	200.93
19	200.77	200.77	200.77	200.77	200.77
20	200.62	200.62	200.62	200.62	200.62
21	200.46	200.46	200.46	200.46	200.46
22	200.30	200.30	200.30	200.30	200.30
23	200.13	200.13	200.13	200.13	200.13
24	199.95	199.95	199.95	199.95	199.95

TIME PERIOD HR	POWER DEMAND MW	POWER LOSSES MW	LAMBDA $/MW
1	3223.00	38.96	4.283
2	3367.00	37.31	4.331
3	3318.00	36.56	4.269
4	3300.00	36.46	4.226
5	3434.00	39.85	4.254
6	3593.00	44.08	4.285
7	3503.00	42.12	4.218
8	3302.00	37.48	4.117
9	3338.00	38.64	4.097
10	3331.00	38.75	4.064
11	3346.00	39.41	4.037
12	3262.00	37.53	3.992
13	3203.00	36.29	3.954
14	3191.00	36.23	3.925
15	2559.00	22.32	3.803
16	2524.00	21.82	3.770
17	2496.00	21.46	3.740
18	2437.00	20.51	3.706
19	2408.00	20.13	3.678
20	2391.00	19.97	3.651
21	2384.00	20.00	3.625
22	2393.00	20.34	3.600
23	2454.00	21.74	3.573
24	2520.00	23.32	3.542

TIME HR	NU PLANT NO: 1 $/CF	NU PLANT NO: 2 $/CF	NU PLANT NO: 3 $/CF	NU PLANT NO:4 $/CF	NU PLANT NO: 5 $/CF
1	0.06332	0.06332	0.06332	0.06332	0.06332
2	0.06173	0.06173	0.06173	0.06173	0.06173
3	0.06033	0.06033	0.06033	0.06033	0.06033
4	0.05894	0.05894	0.05894	0.05894	0.05894
5	0.05716	0.05716	0.05716	0.05716	0.05716
6	0.05519	0.05519	0.05519	0.05519	0.05519
7	0.05407	0.05407	0.05407	0.05407	0.05407
8	0.05356	0.05356	0.05356	0.05356	0.05356
9	0.05210	0.05210	0.05210	0.05210	0.05210
10	0.05087	0.05087	0.05087	0.05087	0.05087
11	0.04954	0.04954	0.04954	0.04954	0.04954
12	0.04886	0.04886	0.04886	0.04886	0.04886
13	0.04809	0.04809	0.04809	0.04809	0.04809
14	0.04706	0.04706	0.04706	0.04706	0.04706
15	0.05054	0.05054	0.05054	0.05054	0.05054
16	0.04992	0.04992	0.04992	0.04992	0.04992
17	0.04929	0.04929	0.04929	0.04929	0.04929
18	0.04893	0.04893	0.04893	0.04893	0.04893
19	0.04837	0.04837	0.04837	0.04837	0.04837
20	0.04773	0.04773	0.04773	0.04773	0.04773
21	0.04702	0.04702	0.04702	0.04702	0.04702
22	0.04617	0.04617	0.04617	0.04617	0.04617
23	0.04480	0.04480	0.04480	0.04480	0.04480
24	0.04335	0.04335	0.04335	0.04335	0.04335

TIME HR	RESEVIOR INFLOW PLANT NO: 1 CFS	RESEVIOR INFLOW PLANT NO: 2 CFS	RESEVIOR INFLOW PLANT NO: 3 CFS	RESEVIOR INFLOW PLANT NO:4 CFS	RESEVIOR INFLOW PLANT NO: 5 CFS
1	12000.0	12000.0	12000.0	12000.0	12000.0
2	12000.0	12000.0	12000.0	12000.0	12000.0
3	12000.0	12000.0	12000.0	12000.0	12000.0
4	12000.0	12000.0	12000.0	12000.0	12000.0
5	12000.0	12000.0	12000.0	12000.0	12000.0
6	12000.0	12000.0	12000.0	12000.0	12000.0
7	12000.0	12000.0	12000.0	12000.0	12000.0
8	12000.0	12000.0	12000.0	12000.0	12000.0
9	12000.0	12000.0	12000.0	12000.0	12000.0
10	12000.0	12000.0	12000.0	12000.0	12000.0
11	12000.0	12000.0	12000.0	12000.0	12000.0
12	12000.0	12000.0	12000.0	12000.0	12000.0
13	12000.0	12000.0	12000.0	12000.0	12000.0
14	12000.0	12000.0	12000.0	12000.0	12000.0
15	12000.0	12000.0	12000.0	12000.0	12000.0
16	12000.0	12000.0	12000.0	12000.0	12000.0
17	12000.0	12000.0	12000.0	12000.0	12000.0
18	12000.0	12000.0	12000.0	12000.0	12000.0
19	12000.0	12000.0	12000.0	12000.0	12000.0
20	12000.0	12000.0	12000.0	12000.0	12000.0
21	12000.0	12000.0	12000.0	12000.0	12000.0
22	12000.0	12000.0	12000.0	12000.0	12000.0
23	12000.0	12000.0	12000.0	12000.0	12000.0
24	12000.0	12000.0	12000.0	12000.0	12000.0

THE VALUE OF THE COST FUNCTIONS FOR
THE THERMAL GENERATING PLANTS IN ($/DAY) ARE

PLANT NO: 1 FUEL COST 16747.22

PLANT NO: 2 FUEL COST 17074.42

APPENDIX D

APPENDIX D

THE AUGMENTED OBJECTIVE FUNCTIONAL

The first step in the optimization process requires that each constraint f_k be paired with an appropriate multiplier function to form the produce F_k. The proper balance equation is paired with the multiplier function $\lambda(t)$, to form

$$F_D(t) = \lambda(t)\ f_D(t) \quad .$$

The discharge models f_{q_i}, are paired with the multiplier functions $m_i(t)$, to form

$$F_{q_i}(t) = m_i(t)\ f_{q_i}(t) \quad . \tag{D.1}$$

Finally, the continuity equations $f_{h_i}(t)$ are paired with the multiplier function $\dot{n}_i(t)$, to form

$$F_{h_i}(t) = \dot{n}_i(t)\ f_{h_i}(t) \quad . \tag{D.2}$$

The inclusion of the water draw-down constraint requires constant multipliers ν_{o_i} to pair with j_{b_i} to form J_{b_i} according to

$$J_{b_i} = \nu_{o_i}\ j_{b_i} \quad . \tag{D.3}$$

This follows since j_{b_i} represent definite integrals and not time functions.

The next step is to form the individual contributions J_k to the original objective by first defining:

$$J_D = \int_o^T F_D(t)\, dt \qquad (D.4)$$

$$J_{q_i} = \int_o^T F_{q_i}(t)\, dt \qquad (D.5)$$

$$J_{h_i} = \int_o^T f_{h_i}(t)\, dt \qquad (D.6)$$

The augmented objective functional is thus given by

$$J_A = J + J_D + \sum_{i=2}^{4} (J_{q_i} + J_{h_i} + J_{b_i}) \qquad (D.7)$$

Let us note here that all components of J_A are explicit functions of the control variables, with the exception of J_{h_i} which needs some transformation.

Consider, for example, the function $f_{h_2}(t)$ where the integral of the control variable $q_2(t)$ appears. The part of J_{h_2} corresponding to this term is transformed as follows

$$\int_o^T n_2(t) \int_o^t q_2(Z)\, dZ\, dt = \int_o^T [n_2(T) - n_2(t)]\, q_2(t)\, dt \qquad (D.8)$$

The corresponding terms in f_{h_3} and f_{h_4} are treated similarily. The last term in both f_{h_3} and f_{h_4} needs special treatment as well

$$\int_o^T \dot{n}_3(t) \int_o^t q_2(Z-\tau_{23})\, dZ\, dt =$$

$$+ \int_{-\tau_{23}}^{o} [n_3(T) - n_3(t+\tau_{23})]\, q_2(t)\, dt$$

$$+ \int_o^{T-\tau_{23}} [n_3(T) - n_3(t+\tau_{23})]\, q_2(t)\, dt \quad . \tag{D.9}$$

With the above mentioned transformations, we can thus write the relevant elements of the augmented functional J_{h_i} as:

$$J_{h_2} = \int_o^T \{s_2\dot{n}_2(t)\; h_2(t) + [n_2(T) - n_2(t)]\; q_2(t)\; dt \tag{D.10}$$

$$J_{h_3} = \int_o^T \{s_3\dot{n}_3(t)\; h_3(t) + [n_3(T) - n_3(t)]\; q_3(t)$$

$$- N_3(t)\; q_2(2)\}\; dt \tag{D.11}$$

$$J_{h_4} = \int_o^T \{s_4\dot{n}_4(t)\; h_4(t) + [n_3(T) - n_4(t)]\; q_4(t)$$

$$- N_4(t)\; q_3(t)\}\; dt \quad . \tag{D.12}$$

In the above we define

$$N_3(t) = n_3(T) - n_3(t+\tau_{23}) \qquad 0 \leq t \leq T - \tau_{23}$$

$$= 0 \qquad T - \tau_{23} < t \leq T \tag{D.13}$$

$$N_4(t) = n_4(T) - n_4(t+\tau_{34}) \qquad 0 \leq t \leq T - \tau_{34}$$

$$= 0 \qquad T - \tau_{34} < t \leq T \quad . \qquad \text{(D.14)}$$

In writing J_h elements we drop terms that are independent of the control variables.

APPENDIX E

APPENDIX E

REDUCING THE HYDRO OPTIMALITY CONDITIONS

Let us start with the down-stream plant optimality conditions (5.14) and (5.11), the derivatives of (5.14) with respect to time gives us

$$\dot{m}_4(t) = \dot{n}_4(t) \ . \tag{E.1}$$

Define

$$M_4(t) = \frac{1}{s_4} \frac{\partial q_4}{\partial h_4} \ . \tag{E.2}$$

Thus, Equations (5.11) and (E.1) combine to give the first order differential equation

$$\dot{m}_4(t) = m_4(t)\ M_4(t) \ . \tag{E.3}$$

The solution of (E.3) is

$$m_4(t) = m_4(0)\ \exp\{\int_o^t M_4(t)\ dt\} \ . \tag{E.4}$$

From Equation (5.14), we have

$$m_4(0) = n_4(0) - n_4(T) - \nu_{o_4} \ . \tag{E.5}$$

To conform with the classical theory, we let

$$\nu_4 = - m_4(0) \ . \tag{E.6}$$

As a result, the optimality equation (5.10) for plant 4 is

$$\nu_4 \exp\{\int_o^t M_4(t)\ dt\} \frac{\partial q_4}{\partial P_4} = \lambda(t) \left[1 - \frac{\partial P_L}{\partial P_4}\right] . \tag{E.7}$$

This is exactly Kron-Ricards equation for uncoupled hydro plants.

Consider next the intermediate plant, number 3, Equation (5.13) upon differentiation gives

$$\dot{m}_3(t) = \dot{n}_3(t) = \dot{N}_4(t) . \tag{E.8}$$

From (D.14), we get

$$\begin{aligned} \dot{m}_3(t) &= \dot{n}_3(t) - \dot{n}_4\ (t + \tau_{34}) \\ & \qquad\qquad 0 \le t \le T - \tau_{34} \\ &= \dot{n}_3(t) \qquad\qquad T - \tau_{34} < t \le T . \end{aligned} \tag{E.9}$$

Using (E.1), we further obtain

$$\begin{aligned} \dot{m}_3(t) &= \dot{n}_3(t) - \dot{m}_4\ (t + \tau_{34}) \\ & \qquad\qquad 0 \le t \le T - \tau_{34} \\ &= \dot{n}_3(t) \qquad\qquad T - \tau_{34} < t \le T . \end{aligned}$$

Equation (5.11) for plant 3, is written as

$$\dot{n}_3(t) = m_3(t)\ M_3(t) \tag{E.10}$$

where

$$M_3(t) = \frac{1}{s_3} \frac{\partial q_3}{\partial h_3} . \tag{E.11}$$

Using Equation (E.9) in (E.10) we thus write

$$\dot{m}_3(t) = m_3(t)\ M_3(t) - u_3(t) \quad . \tag{E.12}$$

In Equation (E.14) we have

$$u_3(t) = \dot{m}_4(t + \tau_{34}) \qquad 0 \leq t \leq T - \tau_{34}$$
$$= 0 \qquad T - \tau_{34} < t \leq T \quad .$$

On the basis of Equation (E.13), we have Equation (E.9) written as

$$\dot{m}_3(t) = \dot{n}_3(t) - u_3(t) \quad . \tag{E.14}$$

Equation (E.12) is an inhomogeneous linear equation whose solution is given by

$$m_3(t) = \phi_3(t,o)\ m_3(0) - {}_o\int^t \phi_3(t,\sigma)\ u_3(\sigma)\ d\sigma \tag{E.15}$$

where

$$\phi_3(t,t_o) = \exp\{{}_{t_o}\int^t M_3(Z)\ dZ\} \tag{E.16}$$

To conform with conventional theory let

$$\nu_3 = - m_3(0) \quad . \tag{E.17}$$

As a result, the optimality condition (5.10) for plant 3, is written as

$$- \{\nu_3 \phi_3(t,o) + {}_o\int^t \phi_3(t,\sigma)\ u_3(\sigma)\ d\sigma\} \frac{\partial q_3}{\partial P_3} = \lambda(t)\ [1 - \frac{\partial P_L}{\partial P_3}] \tag{E.18}$$

In a similar fashion we can arrive at the following relation for plant 2

$$\{\nu_2 \phi_2(t,\sigma) + \int_0^t \phi_2(t,\sigma)\ u_2(\sigma)^* d\sigma\} \frac{\partial q_2}{\partial P_2} = \lambda(t) \left[1 - \frac{\partial P_L}{\partial P_2}\right] \qquad (E.19)$$