# A STUDY TO COMPARE THE EFFECTS OF DISCOVERY TEACHING AND EXPOSITORY TEACHING UPON THE ACHIEVEMENT AND RETENTION LEVELS OF MATRICULATION GRADE-TEN STUDENTS BEING TAUGHT SOME ASPECTS OF THE QUADRILATERAL



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## ABSTRACT

The purpose of the study was twofold. It compared the effects of two different methods of teaching, namely discovery teaching and expository teaching, on the achievement and retention levels of two groups of students.

A unit of work on some aspects of the quadrilateral was taught to two classes of grade ten matriculation students at Clarenville Integrated High School. Each class consisted of 30 students. One group was taught using the discovery package developed by the investigator; the other group was taught in an expository manner using the textbook.

Three tests were used throughout the study--a pretest, a posttest and a retention test. The posttest was a parallel form of the pretest; the retention test was the same as the pretest. The pretest was administered to each group four days prior to the start of the study. A posttest was administered one day after completion of the instructional unit and the retention test was administered three weeks after the posttest.

From the analysis of covariance using the pretest scores as the covariate, no significant difference in achievement on the posttest was found between those students being taught using the discovery method and those students taught using the expository method. However, a significant difference in retention in favor of the discovery treatment was found.

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## CHAPTER I

## THE PROBLEM

### Rationale for the Study

For years the student saw himself and was seen by teachers as the recipient of information. That is, he sat at his desk, listened to the teacher, read and studied his books. It was felt that after eleven or twelve years of this receiving information he would be adequately educated to take on some meaningful role in society. However, in mathematics education during the late 1960's, this form of teaching, called the teacher-lecture method or expository teaching, was being challenged and, in some instances, replaced by another form of teaching called discovery teaching. Taba (1963) observed: "Learning by discovery is one concept made popular in the new curriculum in mathematics and science" (p. 308). The investigator was a student in high school in Newfoundland at this time and can vividly remember the gradual evolvement of discovery teaching into the mathematics curriculum. The student was no longer to be the passive recipient of knowledge but one quite active in the learning environment. Many educators felt that to improve mathematics achievement there had to be a change in methods of instruction. Hence, their attention was focused on the "how" of teaching mathematics.

At the same time, many modern learning theories influenced educators to look at new methods of instruction. These theories, with the aid of pertinent research, recommended that the discovery method of teaching would enhance a student's understanding of concepts. Davis (1967) of the Madison Project claimed that:

> If the child has discovered concepts himself, has devised techniques himself, and has elaborated a mathematical system himself, he really knows "how" and "why" it works in a profound way that is not possible when the system is handed to him or told to him. (pp. 3-4)

The learning theories of Bruner (1961) and Hendrix (1961) and the ideas and opinions of Taba (1962) in curriculum development also echo the view of Davis.

The Report of the Cambridge Conference on School Mathematics in 1963 stated:

> Mathematics is something people do; it is not something that they receive in a passive sense. . . . Our hope would be to convey continually to the student that every mathematical idea appeared first as the solution of some problem by some person. (pp. 28-29)

Such positive reactions as the above are not without opposition. Ausubel (1961) stated:

> The crucial points at issue, however, are not whether learning by discovery enhances learning, retention and transferability, but (a) whether it does so sufficiently for learners who are capable of learning principles meaningfully without it to warrant the vastly increased expenditure of time it requires; and (b) whether in view of the time-cost consideration the discovery method is a feasible technique for transmitting the substantive content of an intellectual or scientific discipline to cognitively mature students who have already mastered its rudiments and basic vocabulary. (p. 20)

According to Ausubel, most efficient learning in our schools occurs by the reception of material which has been presented in near final form.

Skinner (1968), a pioneer in the development of teaching machines and programmed instruction, criticizes discovery as a "sink or swim" method which evades the school's responsibility for instruction.

He advocates presentations which prompt the students' every response, while leading them gradually and in a very logical way, to achieve the objectives set for them.

In support of the teacher-lecture method, Suchman (1967) stated:

> Didactic teaching is appealing to many teachers because (a) it gives them a sense of control; (b) it appears to be efficient; and (c) it is more satisfying to those who find it difficult to let children grope for understanding by hearing and spouting misconceptions or wrong information. (p. 25)

However, in support of discovery teaching, Suchman wrote:

> What appeals to some teachers, however, may not always serve the best long range interests of the child. Hard-won understanding through the learner's own efforts often has greater depth of meaning for him. (p. 25)

There are many techniques and methods that can be successfully used in teaching mathematics. There are many who claim that one theory of learning and instruction is more adequate than another. We, as educators, must ask ourselves not only what students should learn, but also how they learn it, and thus how we should teach it. Each mathematics teacher should familiarize himself with the various theories and utilize that which he finds most beneficial.

Many studies have compared discovery teaching and expository teaching and the effect of these teaching methods on levels of achievement and retention. Great educators of the past such as Jean-Jacques Rousseau, Maria Montessori and John Dewey advocated teaching through exploring and experimenting with the environment. In the late 1960's, Jerome Bruner was the leading spokesman for those who believed that the discovery method of teaching enhances retention and transfer and motivates further learning.

Cummins (1958) did a classroom study in teaching first-quarter calculus to freshmen. Two tests were designed to test the outcome of the experiment. Test One was for the discovery group and Test A was for the traditionally taught group. The scores on Test One for the discovery group were significantly higher than the scores for the traditional group. This study showed that a group of students taught calculus by an inductive, intuitive method achieved more than a group taught by a traditional, deductive method.

Worthen (1968) in a study of mathematical concepts with fifth- and sixth-grade children found that his discovery group did better on tests of retention than did a group taught concepts by teacher exposition.

Bisio (1971) did a study on the effect of manipulative materials on understanding operations with fractions in grade five. The study questioned whether or not the actual manipulation of concrete objects by the student was any more effective than the teacher manipulation of these objects in a demonstration to the class in their understanding of mathematical concepts. His investigation concluded that the use of these objects in either way could be equally effective.

To the writer's knowledge, no systematic investigations have been carried out in Newfoundland to diagnose whether there are significant differences in achievement and retention in students of matriculation mathematics using the two different modes of instruction. The National Advisory Committee on Mathematics Education (NACOME, 1975) recommended that there should be more intuition and application and less deduction in mathematical instruction. In order to demonstrate the worth of this statement, more accurate and adequate information must be made available. It was the intent of the investigator to conduct an experiment that would provide some of this information.

## Purpose of the Study

The main purpose of the study was to compare the achievement of two grade-ten matriculation classes taught a unit on quadrilaterals. One class was taught by a method whereby each student individually manipulated concrete materials, such as blocks and geoboards, and discovered concepts himself. The other class was taught by a method whereby the student listened to the teacher lecture and watched the teacher as he demonstrated the concepts using the same instructional materials. A thorough description of each teaching method is given in Chapter III of this study under Instructional Treatments.

A second purpose of the study was to compare the retention levels of the same students taught by the two modes of instruction. The test for retention was administered three weeks after the unit of work had been completed.

## Hypotheses

In seeking answers to this problem, the following hypotheses were tested. They are stated in the null form.

1. There is no significant difference in levels of achievement between those students taught using the discovery method and those students taught using the expository method.

2. There is no significant difference in retention between those students taught using the discovery methods and those students

taught using the expository method.

## Limitations of the Study

There were some obvious limitations to the study. It was concerned with only one geographical area of the province. Only students from two grade-ten matriculation classes in one high school were included in the sample. Hence, the generalizability of the study is minimal. The study was concerned with only one specific area of mathematics, namely, quadrilaterals and so generalizations could not be made about the effectiveness of these two methods of instruction with other topics and concepts of mathematics.

Each class was taught by the same classroom teacher. Every possible effort was made by this teacher to do an equally good job with each treatment. However, even though such a concerted effort was made by the teacher, the possibility of teacher bias still existed.

The students in the discovery class were more enthusiastic about geometry than they normally would be. This was because of the novelty of the use of manipulative materials.

The teacher who taught both classes used for the experiment was a regular mathematics teacher of one of these classes. Therefore, one class did not need a period of adjustment to get used to the teacher while the other class did.

The two classes used in the present study were intact classes. A discussion of the selection of the students follows in Chapter III of this report.

## Definition of Terms

This section contains a brief description of the various concepts and variables that were used in this study.

Discovery teaching: The definition used here is that borrowed from Hermann (1969). Discovery teaching is a process through which a student is induced to feel (1) that he sees something--a mathematical fact or a relationship that he had not perceived previously, and (2) that his perception was not directly told, given, or displayed to him by another person. The teacher, if necessary, could help the student to make a discovery or generalization.

Expository teaching: This is the method of teaching whereby the teacher explains to the students the content to be learned. The students listen and are later given the opportunity to practise what has been taught.

Both these methods of teaching are elaborated on in Chapter III.

Matriculation student: The definition used here is that borrowed from the Mathematics Bulletin, Grades 7-11, 1978-79. The student for whom matriculation mathematics is designed is one who has an average general ability in mathematics. The student may have been meeting with a high degree of success throughout his school life. He is one who has a greater degree of success when concepts are presented in a manner which deemphasizes mathematical rigor and emphasizes practical applications and logical reasoning. These students may vary greatly in levels of ability and achievement stretching from low to very high academic competency.

Achievement (immediate): This refers to the grade received on the teacher-made tests administered one day after completion of teaching.

Retention: This refers to the grade received on a second teacher-made test administered three weeks after completion of teaching.

## Outline of the Study

A review of related literature is presented in Chapter II. Chapter III contains a description of the instrument, the procedure followed in conducting the study and the method used in collecting and processing the data. Chapter IV contains the results of the data analysis. Chapter V summarizes the conclusions reached as a result of the study, and contains implications and recommendations for future research in this area.

## CHAPTER II

## REVIEW OF RELATED LITERATURE

In this chapter studies which have been completed on the achievement and retention levels of students taught by the discovery approach or expository approach are reviewed. This related research is dealt with separately under three main headings--achievement, use of materials in activity-oriented approaches, and retention. It should be noted that most of the studies reviewed are not at the grade-ten level because few studies have been done at this level on this particular area of mathematics. Since there is so little relevant research, this constitutes a need for this study.

### Achievement

One of the most extensive studies of the effects of discovery teaching was carried out by Worthen (1968). A total of 538 fifth- and sixth-grade students participated in the study. Tests were used to measure initial learning (achievement) in arithmetic. The experimental sample was comprised of 432 of these pupils who were equally divided among 16 classes. A control group, comprised of 106 pupils in three sixth-grade classes, received both the pretest and posttest but received no special instruction during the six-week period used for the experiment. The concepts of arithmetic used in this study were the following:

(a) notation, addition, and multiplication of integers; (b) the distributive principle of multiplication over addition; and (c) exponential notation and multiplication and division of numbers expressed in exponential notation. Prior knowledge of the selected mathematical concepts was measured by a test administered to both treatment groups in the pretest series. Initial learning was measured by the four subsections of the same test administered at the completion of the corresponding subsection of the instructional materials. It was concluded from the data that the expository treatment was superior to the discovery treatment in producing initial achievement.

Similarly, Ray (1961) proposed to provide additional experimental and applied research evidence as to the relative effect of directed discovery upon initial learning as compared with traditional direct and detailed instruction in situations providing numerous problem solving opportunities. One hundred thirty-five students were sampled at random from all ninth-grade boys enrolled in three junior high schools. The subjects were taught the names and functions of the parts of the micrometer, micrometer and vernier principles involved in reading the tool, and how to manipulate and read the instrument for actual measurement. Using the "direct and detailed" instruction method, the teacher presented the learning material, reviewed important points and solved several problems. Using the "directed discovery" method, the pupil was called upon to be active, to carefully study illustrative material on his own, and contemplate leading questions asked by the teacher. After instruction, tests were administered. Ray found that with regard to initial learning there were no statistically significant differences in achievement between students in either group.

Price (1967) did a study on three classes of tenth-grade mathematics students. One class was taught by using a traditional textbook and featured the teacher lecturing on topics from the text. A second class was taught using materials which were especially prepared to promote student discovery of mathematic concepts and principles. The third class used these same teacher-prepared materials, but also made use of other materials. These three classes were referred to respectively as the control, the discovery and the transfer classes. From the results of the testing program the following conclusions were drawn. The group taught using the discovery method of teaching showed a slight but statistically non-significant gain over the control group in achievement in mathematics and showed a greater increase in mathematical reasoning than did the control group.

Keese (1972) tested for an interaction effect on achievement in mathematics between teaching method (discovery and expository) and creativity level (high and low). Prior to the investigation, models which described specific teacher behavior were developed for the discovery and expository methods of teaching. One teacher taught a unit on sequences and series to two intact eighth-grade classes each containing 31 students for a period of twelve days. He employed the discovery method in one class and the expository in the other. Analysis of variance using the Total Mathematics Scale of the Metropolitan Achievement Test as the dependent variable revealed the groups were comparable in mathematical achievement prior to the experiment. An achievement test constructed for the unit on sequences and series was administered to the students at the end of the experiment. It was concluded that the achievement of students taught by the discovery method was significantly greater than the achievement of students taught by the expository method.

Probably more closely related to the present study than any other study quoted is an investigation by Toney (1968). He compared the achievement in basic mathematical understandings of students who individually manipulated the instructional materials with those who saw only teacher demonstration of the same materials. The fourth-grade students were randomly assigned to two groups. There were equivalent numbers of boys and girls in each group. The investigator taught mathematics to both groups. The same lesson was taught to each group with the presentation being different only in the manner in which the instructional materials were utilized. Tests were administered at the beginning of the study and at the conclusion of the study. An analysis of covariance was used to determine if there were differences in the achievement of the two groups at the conclusion of the study. From the analysis of data it was concluded that although no statistically significant difference was found between the class means on the test for basic mathematical understanding, it was concluded that there was a trend toward greater achievement by the group using the individually manipulated materials.

Even though the results were not statistically significant, certain conclusions were drawn from the above study. For example, it was concluded that the use of individually manipulated materials seems to be a somewhat more effective means for building understanding than does a teacher demonstration. Also, a teacher demonstration of instructional materials seems to promote general mathematical achievement as efficiently as does individual manipulation of the materials by the students.

Similar conclusions were reached by Sobel (1954) in a comparative study of teaching some aspects of algebra to grade-nine students using the two methods of teaching, discovery and expository. He found that mathematics achievement was significantly higher for students who used the discovery method. Cummins (1958) found equivalent results from a similar study of teaching calculus to first-year college students.

Thus far, from some of the quoted studies and experiments one might conclude that the discovery method of teaching produces the lowest results with regards to achievement of students. From other studies quoted the discovery method of teaching produced superior results with respect to the achievement of students. However, from a study done by Smith (1975), it was concluded that there was no difference in the achievement levels of students taught by either method. Smith investigated achievement in mathematics as affected by three different teaching methods: lecture, guided discovery and programmed learning. Sixty randomly chosen students were taught College Mathematics I in three separate classes using the three methods being investigated. All three classes were taught by the same instructor.

Twelve hypotheses were tested. Analysis of covariance was used with the significance level set at .05. Of the twelve hypotheses, the null hypothesis that there is no significant difference in achievement in problem solving among students using the three methods was rejected. The null hypothesis that there is no significant difference in achievement between high achievers and low achievers using either the lecture method or the discovery method was also rejected.

It was concluded that superiority over the lecture method cannot be claimed by either the guided discovery or programmed method, but because the interest generated in class and out of class by the discovery method students was so high and because significantly more progress was demonstrated by those students in problem solving, there is a strong argument in favour of guided discovery.

## Use of Materials in Activity-Oriented Approaches

The discovery approach and the use of materials in activity-oriented approaches are closely interwoven in the current experiment. Therefore, a review of some studies which deal explicitly with materials is included.

Monier (1977) did a study to investigate the effects of an activity approach to teaching geometry in certain high schools of Afghanistan. Students in the activity approach were involved in a learning process using solution keys and practical activities to supplement lecture and textbook presentations. This approach was compared to traditional methods which consisted of lecture, use of a textbook and recitation based on memorization only. The following conclusions were drawn from the analysis of the data. In comparison to the traditional approach, the use of activity approach significantly helped students (1) improve their performance in overall understanding of geometry, (2) achieve higher levels in creative thinking, (3) develop greater ability to explain geometric concepts, (4) improve their ability in solving geometric problems, (5) develop the ability to recall geometric concepts better, and (6) develop greater ability in setting up complete proofs for geometric theorems.

Bring (1971) investigated the effects of varying concrete activities on the achievement of objectives in metric and non-metric geometry by students of grades five and six. One hundred two students were divided into two groups characterized by the amount and type of concrete activities afforded the students during the experiment. It was concluded from the study that students using concrete activities achieved higher than students deprived of these activities.

Gilbert (1974) compared three instructional approaches using manipulative devices in third-grade mathematics. One hundred twenty-four students from two suburban elementary schools received instruction in addition and subtraction of two-digit numbers during a three-week period. Their scores on the pre- and posttests were considered in the analysis of the data. The three treatment groups each received an equal amount of instructional time. Teachers followed the investigator's lessons which were designed to control content and teaching approach. Treatment D (Demonstration) refers to mathematics instruction in which students observe and advise the teacher on how to manipulate the instructional devices. In Treatment I (Individual) each student is provided with a set of manipulative devices and given the opportunity to use it on an individual basis. In Treatment G (Group) students work in groups of four with one set of manipulative devices per group. The devices used by the students during the instructional period were straws, counters and place value sheets, and abaci. The pretest was administered three days prior to the first day of experimental instruction and the posttest, identical to the pretest, was administered on the fifteenth day of the program. The author concluded that students receiving Treatment I within one of the schools scored significantly higher than students in Treatments D or G. Within other schools, no treatment differences could be found.

Carmody (1970) investigated the assumption that the use of concrete and semi-concrete materials can contribute significantly to the learning of mathematics at the elementary school level. Three sixth-grade classes from two parochial schools were randomly assigned to the three experimental approaches--the symbolic approach, the semi-concrete approach, and the concrete approach. Units were written which included topics on number bases, properties of even and odd numbers and divisibility tests based on the decimal representation of numbers. Pretests and posttests were administered. Results of the posttests were analyzed using analysis of covariance with the covariates being IQ scores, mathematics ability scores, mathematics achievement scores and the appropriate pretest scores. Significant differences were found between the Symbolic Group and the Semi-concrete Group in favor of the Semi-concrete Group on numeration and two transfer tests. A significant difference was also found in favor of the Concrete-aids Group over the Symbolic Group on one transfer test. No significant differences were found between the Concrete and the Semi-concrete Groups.

Kuhfitting (1974) investigated the effectiveness of guided discovery and concrete materials on mathematics learning. This method of instruction was compared to an abstract method of learning without the use of concrete materials. Forty seventh-grade students were randomly assigned to one of four groups, with five high and five low ability subjects in each. The methods of teaching were called concrete and abstract on the learning aids dimension and maximal and intermediate guidance on the discovery dimension. The learning task consisted of converting American to old English currency and vice versa, and the learning aids were models of coins constructed by the experimenter. On the discovery dimension the amount of guidance was varied as follows: one group was taught by a carefully structured sequence of questions (intermediate guidance) and the other by careful explanations of the individual steps (maximal guidance). On the learning aids dimension the concrete group was allowed to utilize the coin models as manipulative aids, while only verbal references were made in the abstract group. The content was the same for all groups. Significant differences were found in favor of concrete materials and intermediate guidance for the low ability groups but not for the high ability groups. Also, intermediate guidance with concrete aids showed significantly greater transfer on the retention test than the abstract treatment and no significant differences were found for the maximal guidance subjects on transfer.

Moody, Abell and Bausell (1971) examined the effect of activity-oriented instruction and conventional instruction upon original learning, transfer, and retention of third-grade students. The subjects of the study were 90 third-grade students of lower-middle socio-economic families. None of these students had any previous experience in multiplication. The students were randomly assigned to four treatments: (1) the activity-oriented treatment (A) consisted of multiplication instruction in which all students manipulated the instructional materials, (2) the rote treatment (R) consisted basically of expository instruction with students' responses being limited to the completion of prepared exercise sheets and oral responses to specific questions, (3) the rote-word problem treatment (RW) consisted of the same multiplication instruction as R with the addition of practice in solving multiplication word problems, and (4) the control treatment (C) received instruction in addition. All treatments were administered concurrently for a four-week period. After all tests were completed, no significant differences were found in original learning, transfer of original learning to word problems dealing with multiplication, and retention between the two instructional approaches.

Fennema (1971) investigated the relative effectiveness of a symbolic and a concrete model in learning a selected mathematical principle (multiplication). Subjects were second grade students who measured at or above criterion on a qualifying exam and were randomly assigned to three treatments. One group received instruction in multiplication with a meaningful symbolic model (paper and pencil). The second group received instruction in the principle using a meaningful concrete model involving Cuisenaire rods. The third group served as the control group and did not receive any instruction. It was concluded from the study that there were no significant differences in the overall learning of the mathematical principle between the two methods of instruction. However, children who used the symbolic model performed at a significantly higher level than those who used the concrete model with respect to transfer or extension of the principle.

Trueblood (1967) conducted an experiment to provide evidence on whether students, age 9-11, would achieve more by (1) manipulating visual-tactual aids (T-1) or (2) observing and telling the teacher how to manipulate such devices (T-2). The pupils in each class were randomly assigned to T-1 or T-2. An achievement test designed by the investigator was used to measure pupil achievement. An analysis of covariance was performed on the immediate posttest scores. Mental age was used as the covariate. The results of the covariance analysis was that the pupils taught by T-2 scored higher on the immediate posttest than the pupils taught by T-1. The difference was marginally significant ($p = .10$).

Biggs (1967) reported on a study conducted in 1960 when 87 English primary schools, classified according to their teaching methods, were tested on mechanical arithmetic, problem arithmetic and two concept tests. The teaching methods described were (a) using instructional materials such as cruisenaire; (b) activity method; and (c) traditional. From the results it was concluded that the activity method produced significantly lower results on these tests than the other two teaching methods.

In this section, nearly all of the studies are with elementary grade students and on non-geometry topics. It must not be forgotten that most of the studies reviewed in this report are not at the grade-ten level because few studies have been done at this level on this particular area of mathematics. One of the objectives of the present study was to determine if similar patterns existed at the grade-ten level with a unit on a geometry topic.

## Retention

In this section studies done on retention are reviewed. Before the actual research studies are given, it is interesting to note a claim made by Bruner, a leading proponent of discovery learning. Bruner (1961) claimed that student discovery is an aid to the conservation of memory. He stated that:

> Any organization of information that reduced the aggregate complexity of material by embedding it into a cognitive process a person has constructed for himself will make the material more accessible for retrieval. (p. 32)

The investigator found few studies on retention that were directly related to the present study. However, of those studies reported only Trueblood's study had results which were different from the results of the other studies.

Robertson (1970) did a study to compare the effects of the discovery and expository approach of presenting and teaching selected mathematical principles and relationships to fourth-grade pupils. A total of eight schools, thirteen teachers and 374 pupils were selected for the study. Pupils experiencing the discovery treatment scored significantly higher on the retention tests than did those pupils experiencing the expository treatment.

Worthen (1968) did a study that compared the two methods of task presentation which differed primarily in terms of sequence characteristics. Five hundred thirty-eight fifth- and sixth-grade students were used for the experiment. The discovery learning approach produced significantly higher results on retention tests.

Ray (1961) in a study found similar results to Robertson and

Worthen. Six weeks after instruction, Ray found that with respect to retention of material initially learned, subjects taught by the directed discovery method retained a statistically significantly greater proportion of this learning when compared to students instructed by the direct and detailed method. Ray also found with respect to retention of material initially learned that after one week there was no statistically significant difference in retention between students receiving the different modes of instruction.

Trueblood (1967) found no difference in the levels of retention for groups taught by a method whereby each student manipulated visual-tactual aids (T-1) or taught by a method whereby the students told the teacher how to manipulate such devices (T-2). He concluded from the experiment that students taught by T-2 did not retain significantly more than students taught by T-1. Nevertheless, both T-1 and T-2 resulted in a high degree of retention.

## Summary

From the review of literature presented one might be hesistant to draw any definite and firm conclusions as to the relative effectiveness of either the discovery method of teaching or the expository method of teaching.

With regards to initial learning, the general conclusion reached by the investigator from the studies was that the expository teaching produced results that were significantly better than those results of students who were taught by the discovery method. However, for overall achievement the conclusions reached from the studies were not as definite.

It was concluded from certain of the studies that the discovery method produced significantly superior results, while from other studies it was concluded that the expository method produced superior results. Still further, it was concluded from other of the studies that there were indeed no differences in achievement by students taught using either method of instruction.

Similarly, for retention the conclusions reached were not unanimous in any one direction. It was concluded from the studies by Robertson (1970), Ray (1971), and Worthen (1968) that pupils experiencing the discovery treatment scored significantly higher on retention tests. On the other hand, Trueblood (1967) found no difference in the levels of retention for students taught using either method of instruction. From the studies which investigated the use of materials in an activity-oriented approach, it was found that the results obtained were significantly higher for classes which did not use a materials, hands-on approach.

Again it should be noted that most of the studies reviewed are not at the grade-ten level. Because there is so little relevant research, this constitutes a need for this study. The design of this study is outlined in the following chapter.

# CHAPTER III

## DESIGN AND PROCEDURE

In this chapter a description of the design of the study and the procedures used to carry out the study are presented. It includes a description of the population and sampling, a description of the experimental design, a description of the instructional treatment for the experimental group, a description of the instructional treatment for the control group, and a description of the instructional unit and instructional materials. The experimental variable, the non-experimental variables, the administration of the instrument and the methods of the analysis of the data are also discussed.

### Population and Sampling

The subjects for this study were two intact classes of grade-ten matriculation mathematics students at Clarenville Integrated High School. Each class consisted of 30 students. These students were in those two classes because of the following procedure which is used at this high school for the assignment of students to classes: at the end of each school year, the principal of the school (or vice-principal), the guidance counsellor, and the teachers of these students, meet to determine the placement of the students for the following school year. Basically, each student's academic achievement is the determining factor as to his placement for next year. Hence, the students in the two classes used for the study were homogeneously grouped. The students were then assigned to the two classes in no particular manner (a few students had purposely been assigned to different classes in an attempt to avoid discipline problems). Afterwards each class was randomly assigned to one of the treatment groups. Table 1 gives a breakdown of the sample with respect to method of instruction and sex of students.

Table 1

Group, Treatment, and Sex of Students

| Group | Sex | | Treatment |
|---|---|---|---|
| | Boys | Girls | |
| Experimental | 16 | 14 | Discovery (activity-oriented) |
| Control | 13 | 17 | Expository (teacher lecture) |

<u>Teacher Selection</u>

The two classes used for the present study had been assigned two different mathematics teachers at the beginning of the school year in September. For the study the investigator needed only one of these two teachers. To decide which teacher was to teach both classes, a meeting was held which involved those two teachers and the investigator.

The qualifications of each teacher were basically the same. Each teacher had been teaching at the high school level for the same number of years. However, one teacher held a Master of Education degree from Memorial University while the other teacher held two undergraduate degrees from Memorial University. Since the investigator, in collaboration with those two teachers, felt that the difference in teacher qualifications was not a determining factor as to who taught the two treatments, another avenue was taken. That avenue was the convenience of arranging the two timetables of the teachers involved. Such things considered were the increase in teaching periods, the reduction in teacher preparation periods, and the possibility of loss of lunch periods.

All possibilities being weighed and discussed, it was decided that since both teachers taught one of the two classes concerned, it involved switching two classes such that one teacher would teach the two classes used for the study and the other teacher would have one of the other teacher's classes.

## Experimental Design

Two classes were used for this research study. One class was taught by the expository method. This method of instruction is called Treatment A. This class formed the Control Group. The other class was taught using the discovery approach whereby the students used geoboards, paper-folding and paper-cutting to discover various mathematical concepts. The teacher acted as a guide and assisted the students with any difficulties they had with each lesson. This method of instruction is called Treatment B. This class formed the Experimental Group.

Each class was taught by the same teacher, thereby eliminating any limitation of using two different teachers. Every possible effort was made by this teacher to do an equally good job with each treatment. However, even though such a concerted effort was made by the teacher, the possibility of teacher bias still existed. The instructional package developed for the experimental group by the investigator consisted of eight lessons (see Appendix A). The amount of time spent on each lesson is outlined below in Table 2 (1 period = 40 minutes).

Table 2

Time Spent on Each Lesson of Discovery Package

| Lesson | Periods | Time (min.) |
|---|---|---|
| 1 | 2 | 80 |
| 2 | 2 | 80 |
| 3 | 2 | 80 |
| 4 | 2 | 80 |
| 5 | 2 | 80 |
| Practice exercises for lessons 3,4,5 | 2 | 80 |
| Review of concepts and terms | 2 | 80 |
| 6 | 2 | 80 |
| 7 | 2 | 80 |
| Summary | 1 | 40 |
| Review exercises | 2 | 80 |

In summary, 23 class periods were used for the completion of the package. The same number of class periods were used for the Control Group.

A pretest was given to each class four days prior to the start of the study. The scores on the pretest were used as a covariate to take into account any differences between classes which might have existed prior to the beginning of the study. A posttest was administered one day after the completion of the instructional unit, and the retention test was administered three weeks after the posttest.

The two groups were taught the same content with the same behavioral objectives for each group. The same exercises, assignment and tests were used for both groups. The development and planning of the lessons for both groups received an equal amount of time.

The design for the study may be classified as a Campbell and Stanley (1966) 'Pretest-Posttest Design' with an experimental group and a control group. This design is outlined as follows:

$$\begin{array}{ccccc} R & 0_1 & X_1 & 0_2 & 0_5 \\ R & 0_3 & X_2 & 0_4 & 0_6 \end{array}$$

where R represents random assignment of students to groups and random assignment of treatment to groups.

$X_1$ represents the treatment given to the experimental group.

$X_2$ represents the treatment given to the control group.

$0_1$ represents the pretest given to each group.

$0_2$ represents the posttest given to each group.

$0_5$ represents the retention test given three weeks after the posttest.

## Instructional Treatments

For the experimental group, the instructional unit consisted of a package developed by the investigator. This package consisted of a student manual containing instructions and a series of questions and answers to direct the student towards discovering the various geometric concepts concerning the quadrilateral. To aid the student in making the correct discovery, he used geoboards, paper-cutting and paper-folding where he was instructed in the package to do so.

For the experimental group the teacher began each lesson by telling the students the purpose of the lesson, that is, the teacher read through the behavioral objectives outlined at the beginning of the lesson. The primary role of the teacher with this group was not instruction. Rather the teacher acted as a guide, leading the students through the package. He ensured that the students followed adequately the directions outlined in the package. Otherwise, the students may not have discovered those concepts that they were supposed to discover. With the same purpose in mind, the teacher also ensured that they did satisfactorily and with the necessary precision paper-folding, paper-cutting and the construction of each geometric figure and its properties on the geoboards.

The teacher did not merely allow the students to discover properties themselves. The teacher was a very active participant in the whole learning environment. He did not discover 'for' the students but rather aided them in discovering for themselves. In other words, he was an instigator of discussion.

If any students in the experimental group experienced difficulty the teacher helped resolve their problems. That is, where necessary, the teacher and student worked together in that the teacher asked leading questions for the student to answer or ponder and thereby derive a correct conclusion.

For certain conclusions and generalizations, the teacher encouraged the students to write formal deductive proofs. These proofs came in the practise exercises provided at the end of each lesson. In addition to these practise exercises, one take home assignment was given to the students at the midway point of the unit.

Also, at the end of each lesson the role of the teacher was to tie together the facts and ideas discovered throughout that lesson. He saw to it that the students understood the facts and concepts discovered and in some cases realized their importance and connections with either the previous lessons or those lessons to follow.

The investigator did not have any active role in the actual teaching. The investigator, in consultation with the teacher, decided that it was better not to have the students work in groups of two because it was felt by the teacher that not enough learning was taking place. Instead, the students were made to work individually. The instruction for the control group differed from that of the experimental group basically in the method employed. The role of the teacher with this group was primarily to direct instruction. He, at most times, stood at the front of the class and taught them in a traditional expository manner.

The teacher did the actual demonstrations in class using a geoboard which was considerably larger than those used by the students in Treatment A. The reason for using the larger geoboard was to allow the students to better view the teacher's demonstrations. The teacher also did those demonstrations requiring paper-cutting and paper-folding. It is important to note here that the student was a more passive learner than in the experimental group where the student was more actively engaged in the learning process. In other words, the students in the control group were not actively involved in the actual manipulation of concrete materials as were those students in the experimental group. Where neither paper-cutting, paper-folding nor the use of geoboards satisfactorily explained certain concepts, the teacher used the chalkboard for illustrations and explanations.

Upon completion of each lesson, exercises were assigned. The teacher, as with the experimental group, helped any students who were experiencing difficulty. The teacher also 'patrolled' the classroom to determine if all students were doing their work satisfactorily and with sufficient competence. When all students completed the assigned exercises the teacher corrected the exercises and, where necessary, gave explanations.

At the end of each lesson, the teacher tied together the facts and concepts presented in each lesson. Homework was assigned at the discretion of the teacher.

The behavioral objectives, the practise exercises and the homework assignments were the same for the control group as for the experimental group. The students in the control group were taught the chapter on quadrilaterals in the textbook Modern Basic Geometry (Jurgenson et al., 1970). The instructional package for the experimental group was a parallel form of the chapter presented in the textbook.

## Experimental and Non-Experimental Variables

The experimental variable in this study was the method of instruction. The two methods of instruction were the teacher-lecture method and the discovery method.

The pupils used for this study were not told that they would be part of an experiment. Each class was taught by the same mathematics teacher. However, because the experimental group used materials which they usually do not use and because the unit and each individual lesson was learned differently, the students were more enthusiastic about geometry than they normally would be. This was not true for the control group because they were taught in the teacher lecture method. The teacher was instructed by the investigator to teach both groups with the same enthusiasm and try not to bias himself in favor of either mode of instruction.

The non-experimental variables which might have influenced the outcome of the study were the ability of each student, the time of day, sex and classroom conditions such as heating, lighting and ventilation. However, these variables, except sex and the time of day, were controlled since the students were assigned to both classes and the same classroom was used by each group.

The items on the pretest were selected on the basis of the behavioral objectives for the unit. The items on the posttest and retention test were parallel forms of the items on the pretest. Test familiarity was not a problem because of the time factor involved which eliminated rote memorization of either facts or detail. The test items on all three tests were developed along the lines of Bloom's taxonomy (Bloom, 1973) using items of varying degrees of difficulty and complexity. That is, each test was developed according to the levels of Bloom's taxonomy; some items tested factual knowledge, some items tested the students' comprehension of certain concepts, and more items such as proofs tested the students' analytical abilities. The three tests used for this study are given in Appendix B.

The two hypotheses that were tested in this study were as follows:

1. There is no significant difference in levels of achievement between those students taught using the discovery method and those students taught using the expository method.

2. There is no significant difference in retention between those students taught using the discovery method and those students taught using the expository method.

A pretest was given to each class of students prior to any instruction in quadrilaterals. Immediately following the completion of instruction in quadrilaterals, a posttest was administered. A retention test was administered four weeks after completion of the instruction. Analysis of covariance was used on the posttest and retention test using the scores on the pretest as the covariate. The results of these analyses are presented in the following chapter.

CHAPTER IV

ANALYSIS OF DATA

The purpose of this study was to compare the effects of two different methods of teaching, namely, discovery teaching and expository teaching, upon the achievement and retention levels of two groups of tenth-grade geometry students.

Three tests were used--pretest, posttest, and retention test. The posttest and retention tests were parallel forms of the pretest. The analyses of the achievement data and the retention data were carried out using the analysis of covariance. The pretest scores were used as the covariate.

This chapter reports the data collected during the investigation, the results of testing the hypotheses and the other findings of the study. In presenting the results related to each question, the question or null hypothesis is stated, and the results of the analysis given.

## The Pretest

The means and standard deviations of the pretest for the expository group and the discovery group are presented in Table 3. Although the difference between the means for the two groups was not statistically significant ($t = 1.51$, $df = 58$), it was decided to use the pretest scores as a covariate in subsequent analysis.

Table 3

Means and Standard Deviations of Pretest for Each Treatment Group

| Treatments | Means | Standard Deviations |
|---|---|---|
| Expository | 19.53 | 4.85 |
| Discovery | 17.23 | 6.83 |

QUESTION 1

Are there any differences in achievement on the unit on quadrilaterals between students taught under the expository treatment and students taught under the discovery treatment?

Hypothesis 1

There is no significant difference in levels of achievement between those students taught using the discovery method and those students taught using the expository method.

Results

This hypothesis was tested using analysis of covariance of the achievement scores. The scores on the pretest were used as the covariate. Table 4 gives the means and standard deviations of the posttest for the expository group and the discovery group. Table 5 summarizes the analysis of covariance performed on the appropriate scores of the posttest.

Table 4

Means and Standard Deviations of Posttest for Each Treatment Group

| Treatments | Means | Standard Deviations |
|---|---|---|
| Expository | 56.17 | 12.97 |
| Discovery | 56.67 | 12.20 |

Table 5

Analysis of Covariance for Posttest

| | df | $SS_x$ | SP | $SS_x$ | $df^1$ | $SS^1_x$ | $MS^1_x$ | F |
|---|---|---|---|---|---|---|---|---|
| Between | 1 | 79 | -192 | 469 | 1 | 880 | 870 | 0.62 |
| Within | 58 | 2035 | 1981 | 10014 | 57 | 8085 | 142 | |
| Totals | 59 | 2114 | 1789 | 10479 | 58 | 8965 | | |

An F-ratio of 0.62 is not significant at the .05 level of significance. Hypothesis 1 was not rejected. Thus, no significant difference was found between the mean achievement scores of the two groups.

QUESTION 2

Are there any differences in retention on the unit on quadrilaterials between students taught by the expository treatment and students taught by the discovery treatment?

Hypothesis 2

There is no significant difference in levels of retention between those students taught by the discovery method and those students taught by the expository method.

Results

This hypothesis was tested using analysis of covariance of the retention scores. The scores on the pretest were used as the covariate. Table 6 gives the means and standard deviations of the retention test for the expository group and the discovery group. Table 7 summarizes the analysis of covariance performed on the appropriate scores of the retention test.

Table 6

Means and Standard Deviations of Retention Test for Each Treatment Group

| Treatments | Means | Standard Deviations |
|---|---|---|
| Expository | 38.47 | 13.37 |
| Discovery | 44.03 | 12.90 |

Table 7

Analysis of Covariance for Retention Test

| | df | $SS_x$ | SP | $SS_x$ | $df^1$ | $SS_x^1$ | $MS_x^1$ | F |
|---|---|---|---|---|---|---|---|---|
| Between | 1 | 79 | -17 | 4 | 1 | 85 | 85 | 6.20* |
| Within | 58 | 2035 | 1705 | 9245 | 57 | 7817 | 137 | |
| Totals | 59 | 2114 | 1687 | 9249 | 58 | 7902 | | |

* $p < .05$

An F-ratio of 6.20 is significant at the .05 level of significance. Hypothesis 2 was therefore rejected. A significant difference in levels of retention was found between those students taught by the discovery method and those students taught by the expository method.

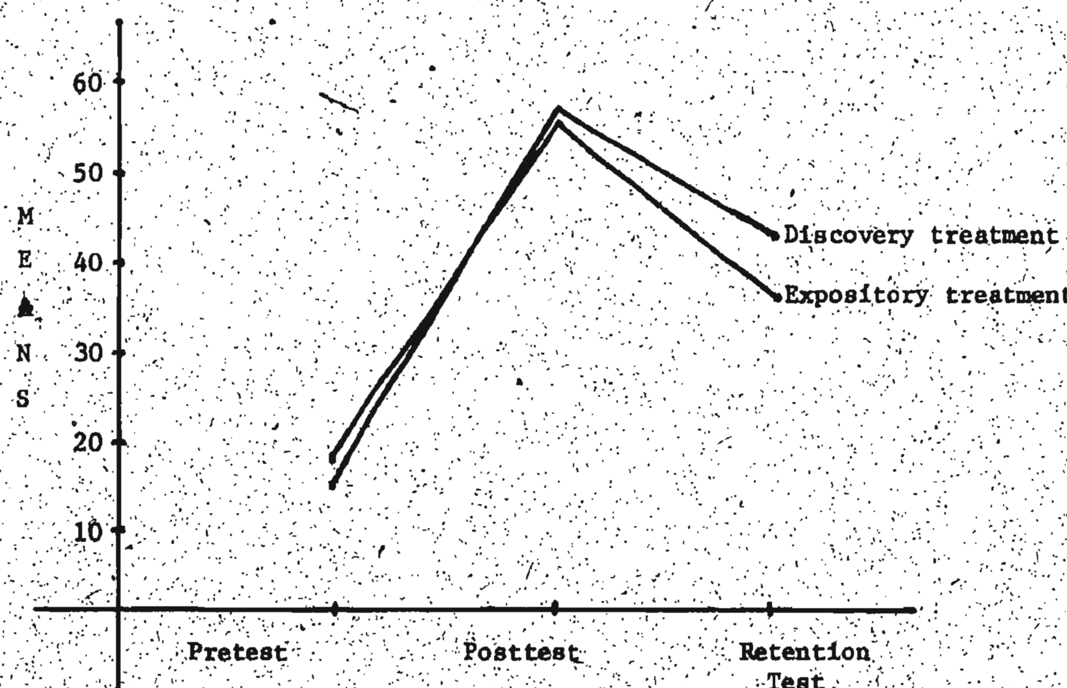


Figure 1. Means of the Tests

Although the students in the discovery group retained more than the students in the expository group, there was a decrease in scores for both groups from the posttest to the retention test. For the expository group, the mean scores dropped from 56.17 on the posttest to 38.47 on the retention test. For the discovery group, the mean scores dropped from 56.67 on the posttest to 44.03 on the retention test. This decrease in mean scores is discussed in Chapter V. The relationship between the means of the three tests is depicted in Figure 1.

Summary

From the analysis of covariance using the pretest scores as the covariate, no significant difference in achievement was found between those students being taught by the discovery method and those students being taught by the expository method.

However, a significant difference in retention in favor of the discovery treatment was found between those students taught using the discovery method and those students taught using the expository method.

## CHAPTER V

## SUMMARY, IMPLICATIONS AND RECOMMENDATIONS

This chapter includes a summary of the study, the implications of the study and recommendations for further investigation.

### Summary

Mathematics educators have searched for the most productive and effective method of teaching mathematics to all groups of students within our school systems. There are many theories as to which method can most effectively help children develop understanding of mathematical concepts. This study used two different methods of instruction--the discovery method and the expository method. The research, as reviewed in Chapter II of this report, is far from conclusive as to which method of instruction is the more effective means of communicating to the child the basic concepts of mathematics.

The present study was designed to investigate the effects of two methods of instruction, namely, discovery teaching and expository teaching, upon the achievement and retention levels of two classes of grade-ten matriculation students being taught some aspects of the quadrilateral.

The population for the study was two intact classes of grade-ten matriculation students at Clarenville Integrated High School. Each class consisted of 30 students. Each of the classes was assigned to one of the two treatment groups.

The content of the unit and take-home assignment was the same for both groups. However, the lesson plans were different and the use of instructional materials was somewhat different. For the group using the discovery treatment, each lesson was a programmed series of questions and answers. Each lesson had previously been written and then individually completed by each student. For the expository group, the teacher taught each lesson using the chalk board and giving an exposition of material rather than have the students discover the ideas and concepts themselves.

The discovery group was characterized by the use of manipulative materials by each student, whereas with the expository group, only the teacher used any manipulative materials.

A pretest was administered to the students in each group four days prior to the beginning of the study. A posttest was administered one day after the completion of the instructional unit, and a retention test was administered three weeks later. During these three weeks no instruction in the concepts of quadrilaterals was given. The posttest was a parallel form of the pretest; the retention test was the same as the pretest. Each test was designed to test the behavioral objectives which are listed at the beginning of each lesson used in the discovery treatment (see Appendix A).

The hypotheses in the study were tested using the statistical technique of analysis of covariance. The level of significance was set at .05. In calculating the analysis of covariance for the posttest and retention test, the scores on the pretest were used as the covariate.

## Conclusions

Based upon the statistical analysis of the data obtained from the instrument used to measure achievement and retention on the unit on quadrilaterals, the following conclusions were drawn:

1. Hypothesis 1 was accepted. An analysis of covariance, with pretest scores as the covariate, showed no significant difference at the .05 level in achievement between treatment groups after the treatments were employed. It was concluded that both groups achieved basically the same.

2. Hypothesis 2 was rejected. An analysis of covariance, with pretest scores as the covariate, showed a significant difference at the .05 level in retention between treatment groups after the treatments were employed. It was concluded that students being taught using the discovery method scored significantly higher than students taught by the expository method.

## Discussion of the Results

There were two major findings drawn from the present study. One was that the discovery method of teaching this aspect of quadrilaterals did not produce a significant difference in achievement immediately after completion of the unit as compared with the expository method of teaching the same material. This finding could be attributed to the fact that the study was conducted over a short time, namely, six weeks. Hence, the superiority of one method of teaching did not have a chance to materialize.

The teacher in this experiment claimed that such facts as definitions of the special quadrilaterals and the statements of certain theorems could best be handled using the expository method. He also concluded that there were certain facts such as the properties of each special quadrilateral which could best be learned using the discovery approach.

The teacher also felt that the study could have produced more significant results had it not taken the students using the discovery approach so long to adapt to this different method of teaching. Basically, all the students were more used to a combination of the discovery approach and expository approach.

The second major finding was the significant difference in retention in favor of the group using the discovery treatment. This result could be interpreted in the light that the novelty effect of a programmed series of questions and answers as used with the discovery approach could have caused the superior results. It is interesting to note that the findings of the present study are very much in agreement with the findings of most of the research studies presented in this report. One of the most extensive studies done on the effects of discovery teaching was by Worthen (1968). It was concluded from the data for his study that the expository treatment was superior to the discovery treatment in producing initial achievement. Also, the discovery learning approach produced significantly higher results on retention tests than did the expository approach, as was the case in the current study.

In relationship to the findings of the experiment, several factors could be discussed as to having a direct bearing on the outcome of the study. One factor was the change from one method of teaching to another. The students involved in the study had been in classes where the teacher basically gave an exposition of facts, concepts, and ideas to be learned. In the discovery treatment class, however, the students individually manipulated materials and were compelled to discover concepts themselves. The students in this class received very little help from the teacher, whereas students in the expository group received instruction in a manner similar to that received prior to the study.

Another factor to be considered is the change of teachers. The students in the expository class had the same teacher as they did before the experiment began. However, the students in the discovery class had a different teacher from the teacher they had before the experiment began. They were now being taught by another teacher whose methods of teaching and conduct were different due to the experimental treatment.

The students in the discovery class were not used to the amount of repetition they were supposed to have done in the package used for the discovery treatment. The students, at the same time, in using the package were supposed to discover certain ideas and concepts which they may have considered obvious. Such factors as these may have caused the students in the discovery class to lose interest in the class and, as a result, adversely affect the results of the study.

If these factors mentioned above had an effect on the results of the study, then the effect would in all probability be of a negative nature upon the discovery treatment. Those factors would not have any effect upon the expository treatment.

In discussing the drop in mean scores from the posttest to the retention test for both groups, two factors could be considered. One factor was that the students used for the experiment were matriculation students. These students may vary greatly in levels of ability and achievement stretching from low to high academic competency. It is possible that many of these students, particularly those with low average ability, would also have high forgetting rates.

Another factor to consider is periodic reinforcement. When the unit on quadrilaterals was completed, there was no mention of quadrilaterals in the interval between the posttest and retention test. Maybe if during this three-week interval, the teacher had one or more class periods of review of the concepts of quadrilaterals, then the students in both groups would perform more favorably on the retention test. The more periods of review the students had, the less they would forget. This must be a consideration in decreasing the drop in grades from the posttest to the retention test.

## Recommendations and Implications

Based upon the findings of the present experiment, the following suggestions for further research have been made:

1. It is recommended that additional research be carried out using a different sample of students being taught the same subject material as was used in this study. The reason for this recommendation would be to see if similar results would be found by using different samples of students.

2. In connection with recommendation 1, it is recommended that

if further research is carried out that more ample time be given to complete the package used for the discovery treatment.

3. It is recommended that additional research be done to determine some method or methods of minimizing the drop in scores from posttest to retention test.

Based upon the findings of the present experiment, the following recommendations for classroom teachers have been made:

1. It is recommended that students, when being taught any mathematical concept using a different instructional approach, should be given ample time to adapt to the different approach.

2. It is recommended that for the purpose of retention of mathematical concepts, the discovery approach be considered.

3. It is recommended that for the purpose of mathematical achievement, the discovery approach and expository approach be considered viable alternatives for instruction. From the comments of the teacher used in the experiment, it was concluded that such facts as definitions and statements of theorems could best be handled using the expository approach. Generalizations and discovering properties of quadrilaterals could best be handled using the discovery approach.

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APPENDICES

With reference to Appendix A and Appendix B,

- all answers in the student package are given in italic print.
- all answers in the practise exercises of the student package are given in italic print and are sometimes placed in parentheses to the right of the problem.
- all answers in Appendix B are given in italic print and (except for the proofs) are placed in parenthesis to the right of the problem.

APPENDIX A

STUDENT PACKAGE ON SOME ASPECTS OF QUADRILATERALS

Table of Contents

# LESSON #1 - STUDENT OBJECTIVES

Upon completion of this lesson the student should be able to do the following correctly, 90% of the time.

1. From a group of n-sided polygons, the student should be able to recognize those polygons which are quadrilaterals.

2. From a group of n-sided polygons, the student should be able to recognize each of the four special quadrilaterals - parallelogram, rectangle, rhombus and square.

3. The student should be able to define quadrilateral, parallelogram, rectangle, rhombus and square.

LESSON #1 - TYPES OF QUADRILATERALS

Let us each take a sheet of paper and fold it in the manner described as follows:

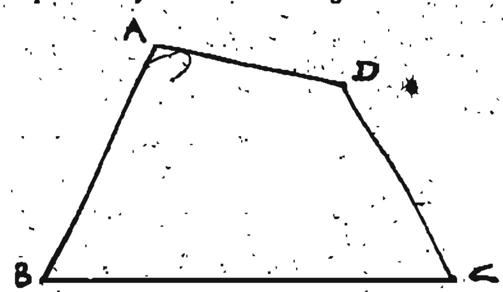
Mark in a counterclockwise manner any four points A,B,C,D (no three points being collinear) on the top half of the sheet. Fold the paper along each of the four segments $\overline{AB}, \overline{BC}, \overline{CD}, \overline{AD}$ as determined by the four points. How many sides has the figure which is formed?

Answer 4

Mark in a counterclockwise manner any four points E,F,G,H (no three points being collinear) on the bottom half of the paper sheet. Arrange the four points in a way different from the arrangement of points A,B,C,D. Fold the paper along each of the four segments $\overline{EF}, \overline{FG}, \overline{GH}, \overline{EH}$ as determined by these four points. How many sides has the figure which is formed?

Answer 4

In either case you probably formed a figure something like the one drawn in this diagram.




Q. How would you define such a figure?

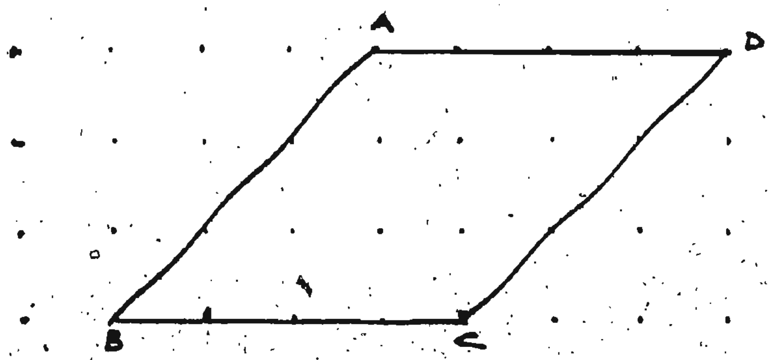
A. *A 4-sided polygon*

Q. What is the name given to such a figure?

A. *Quadrilateral*

As you folded the paper to form the quadrilaterals, no doubt different shapes and sizes of figures were formed. However, there are some quadrilaterals which have standard shapes with their own specific properties. Let's go and discover what these figures are.

On the geoboard you have, using rubber bands, form a figure with both pairs of opposite sides parallel. Your figure should look something like the figure shown in the diagram following. Be certain that all four sides are not congruent and that there are no right angles.




Each student may have different shapes and different sizes of quadrilaterals but the fact which remains the same is that both pairs of opposite sides are parallel.

Q. How would you define this figure?

A. *A quadrilateral with both pairs of opposite sides parallel*

Q. What is the name given to such a figure?

A. *Parallelogram*

Now that you have discovered the general shape of a parallelogram and its definition, next determine if there are any special kinds of parallelograms. Construct on your geoboard four parallelograms of different sizes and different shapes. Be certain that all four sides are not congruent.

Q. What do you notice about the angles in the parallelograms you just made? Are the angles acute angles, obtuse angles or right angles?

A. *Depending on the figure, the angles may be acute, obtuse or right*

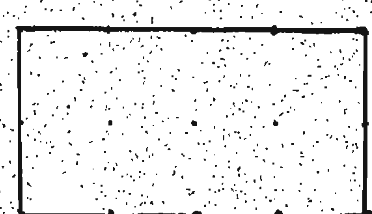
Q. Could these angles be of any size?

A. *For each angle, its measure $x$ must lie in the interval $0 < x < 180$*

Q. Could all four angles be right angles?

A. *Yes*

If so, the figure would look something like this.



Q. Do you consider this a special kind of parallelogram?

A. *Yes*

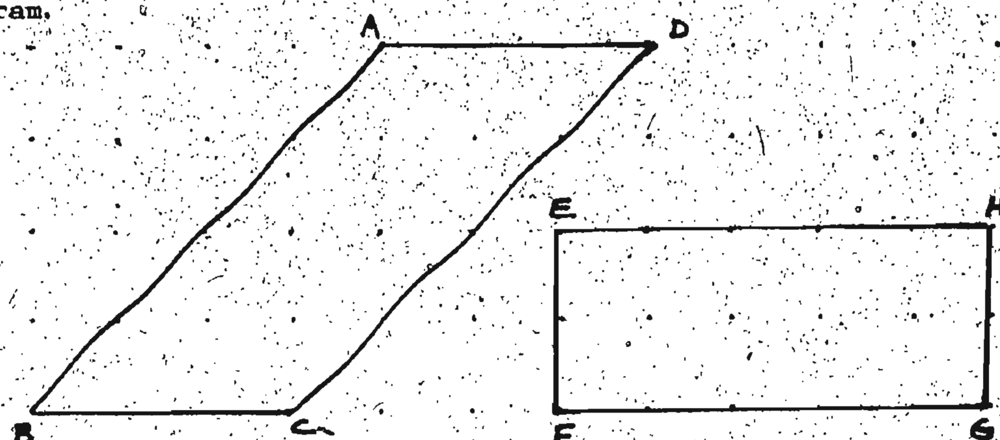
Q. How would you define such a figure?

A. *A parallelogram with four right angles*

Q. What is the name given to such a parallelogram?

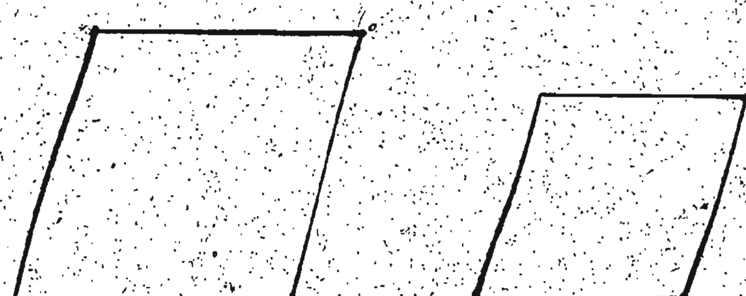
A. *Rectangle*

You must realize at this point that a parallelogram is <u>not necessarily</u> a rectangle but a rectangle is <u>always</u> a parallelogram. In the diagram, parallelogram ABCD is <u>not</u> a rectangle but rectangle EFGH is a parallelogram.




Let us move on to a different kind of parallelogram. On a sheet of paper determine if it is possible for a parallelogram to have all four sides congruent and opposite sides still to be parallel. Do not use figures with right angles.

Your figures should look something like those drawn in the diagram below.



Q. How would you define such a figure?

A. *A parallelogram with all sides congruent*

Q. What is the name given to such a figure?

A. *Rhombus*

Before you move to the next figure, be certain the figures covered so far are clear to you and that you understand each one and their connections.

Q. Is a parallelogram <u>always</u> a rhombus?

A. *No*

Q. Is a rhombus <u>always</u> a parallelogram?

A. *Yes*

Q. Is a rhombus <u>always</u> a rectangle?

A. *No*

Q. Is a rectangle <u>always</u> a rhombus?

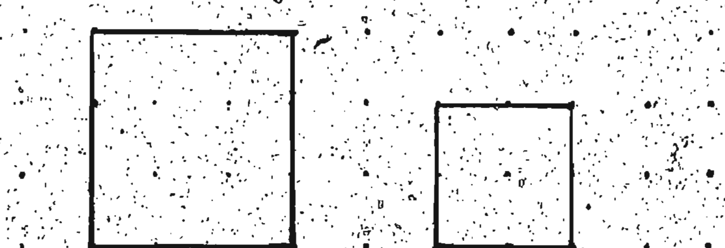
A. *No*

In this special family of parallelograms, there is one other figure to discover.

Q. Use your geoboard to determine if it is possible to have a rhombus with all four angles being right angles.

A. *Yes*

If so, your figures would look something like this.



Q. How would you define such a figure?

A. *A rectangle with all sides congruent or a rhombus with four right angles*

Q. What is the name given to such a figure?

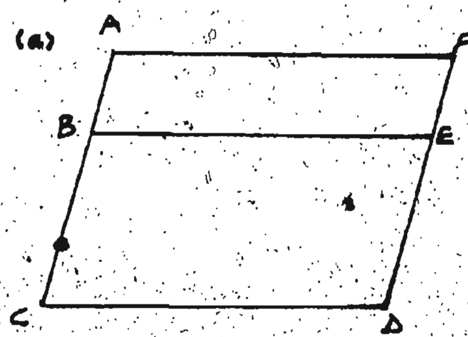
A. *Square*

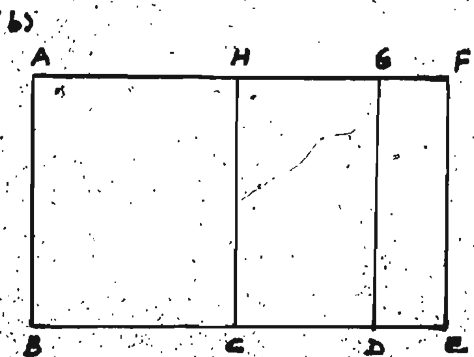
Let us now review the material which we have just covered and write a summation of our conclusions.

1. A quadrilateral is a closed four-sided polygon.
2. A parallelogram is a quadrilateral with both pairs of opposide sides parallel.
3. A rectangle is a parallelogram with four right angles.
4. A rhombus is a parallelogram with all sides congruent.
5. A square is a rectangle with all sides congruent. A square could also be defined as a rhombus with four right angles.

PRACTICE EXERCISES:

1. Use appropriate letters to identify all of the parallelograms in each of the following figures. Assume that if any lines appear to be parallel, they are parallel.




ANSWERS: 1 (a) *ABEF*, *ACDF*, *BCDE*  1 (b) *ABEF*, *ABDG*, *ABCH*, *HCDG*, *HCEF*, *GDEF*

2. Classify each statement as true or false.

   (a) Every rectangle is a parallelogram. *(True)*

   (b) Every rhombus is a parallelogram. *(True)*

   (c) Every square is a parallelogram. *(True)*

   (d) Every square is a rhombus. *(True)*

   (e) Every rhombus is a square *(False)*

3. Complete each of the following with one of the words ALWAYS, SOMETIMES or NEVER.

   (a) A quadrilateral is __?__ a parallelogram. *(Sometimes)*

   (b) A square is __?__ a parallelogram. *(Always)*

   (c) A square is __?__ equilateral. *(Always)*

   (d) A rectangle is __?__ equiangular. *(Always)*

   (e) A rhombus is __?__ a square. *(Sometimes)*

   (f) A parallelogram is __?__ a rhombus. *(Sometimes)*

## LESSON #2 - STUDENT OBJECTIVES

Upon completion of this lesson, the student should be able to do the following correctly, 90% of the time.

1. The student should be able to state and use the fact that the opposite sides of a parallelogram are congruent.

2. The student should be able to state and use the fact that the four sides of a non-square rhombus are congruent.

3. The student should be able to state and use the fact that the four sides of a square are congruent.

4. The student should be able to state and use the fact that the four angles of a non-square rectangle are right angles.

5. The student should be able to state and use the fact that the four angles of a square are right angles.

6. The student should be able to state and use the fact that the opposite angles of a parallelogram are congruent.

7. The student should be able to write formal proofs using the properties of quadrilaterals discovered in Lessons 1 and 2.

LESSON #2 - PROPERTIES OF QUADRILATERALS

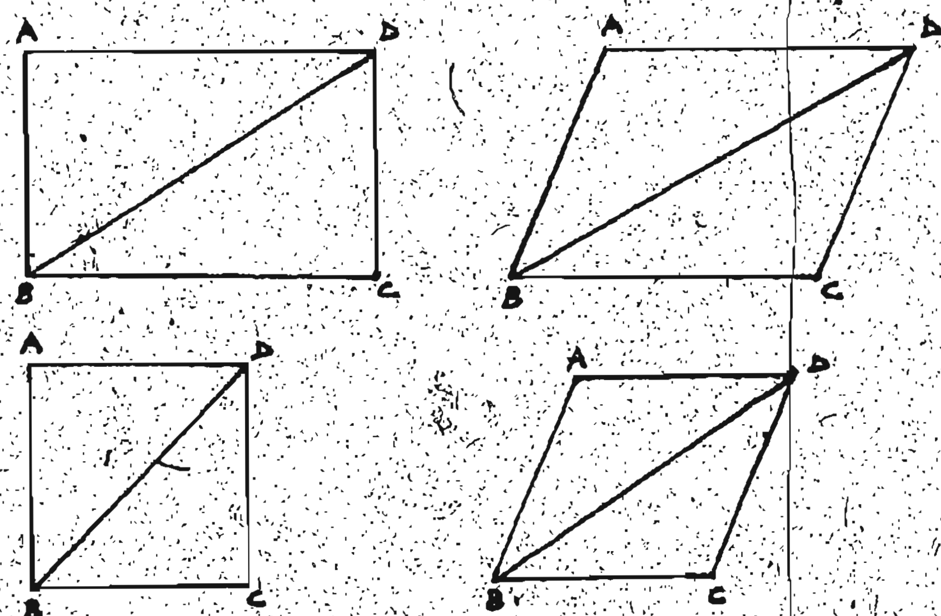
Before we move on to discover the properties of quadrilaterals, recall the conclusions we discovered in Lesson #1:

1. A quadrilateral is a closed four-sided polygon.
2. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
3. A rectangle is a parallelogram with four right angles.
4. A rhombus is a parallelogram with all sides congruent.
5. A square is a rectangle with all sides congruent. A square could also be defined as a rhombus with four right angles.

Let us now move to discover the properties of each of the four quadrilaterals.

On a sheet of paper draw accurately a square, a non-square rhombus, a non-square rectangle and a parallelogram which is neither a rectangle, a square nor a rhombus.

Label each figure ABCD as shown by the figures in the diagram. Join B to D for each figure.




Using scissors cut each figure along $\overline{BD}$.

Place $\overline{AB}$ on $\overline{CD}$ and $\overline{AD}$ on $\overline{BC}$ for each figure.

Q. What did you discover about the opposite sides of each figure?

A. *The opposite sides of each figure are congruent*

Q. Write a general statement about the opposite sides of each figure.

A. *The opposite sides of a parallelogram are congruent*

Q. Do any of these figures have all four sides congruent?

A. *Yes*

Q. If so, what are they?

A. *Square and non-square rhombus*

Q. For each figure that you have drawn on your paper, do any of the angles of any figure appear to be right angles?

A. *Yes*

Q. Which of the figures have all angles being right angles?

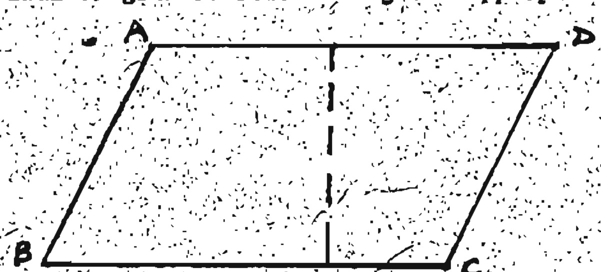
A. *Rectangle and square*

Q. Write a general statement about those figures whose four angles are right angles.

A. *Angles of a square and a non-square rectangle are right angles*

To determine if there is any relationship between the opposite angles of each figure, do the discovery activity for each figure using the exercises provided.

Draw a parallelogram as below using the appropriate block.




NOTE: ∠A and ∠C are opposite angles
∠B and ∠D are opposite angles

Cut parallelogram ABCD along the dotted line. Place ∠B on ∠D and ∠A on ∠C.

Q. Are the opposite angles of this parallelogram congruent?

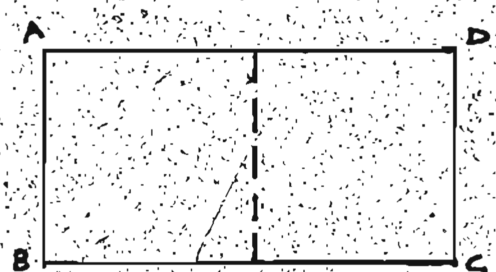
A. *Yes*

Draw at least two other parallelograms of the same shape and repeat the procedure used above.

Q. Write a general statement about the opposite angles of a parallelogram which is neither a square, non-square rhombus nor non-square rectangle.

A. *The opposite angles of a parallelogram which is neither a square, a non-square rhombus, nor a non-square rectangle are congruent*

Draw a non-square rectangle as below using the appropriate block.




Cut rectangle ABCD along the dotted line. Place ∠A on ∠C and ∠B on ∠D.

Q. Are the opposite angles of this non-square rectangle congruent?

A. *Yes*

Draw at least two other non-square rectangles and repeat the procedure used above.

Q. Write a general statement about the opposite angles of a non-square rectangle.

A. *The opposite angles of a non-square rectangle are congruent*

Draw a non-square rhombus as below using the block provided.


Cut rhombus ABCD along the dotted line. Place ∠A on ∠C and ∠B on ∠D.

Q. Are the opposite angles of this non-square rhombus congruent?

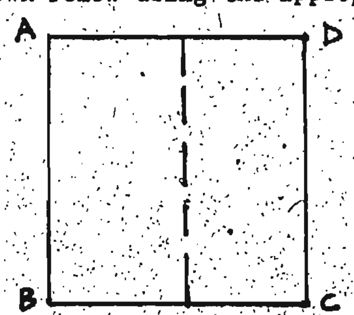
A. *Yes*

Draw at least two other non-square rhombii and repeat the same procedure.

Q. Write a general statement about the opposite angles of a non-square rhombus.

A. *The opposite angles of a non-square rhombus are congruent*

Draw a square as shown below using the appropriate block.




Cut square ABCD along the dotted line. Place ∠A on ∠C and ∠B on ∠D.

Q. Are the opposite angles of this square congruent?

A. *Yes*

Draw at least two other squares and repeat the same procedure.

Q. Write a general statement about the opposite angles of a square.

A. *The opposite angles of a square are congruent*

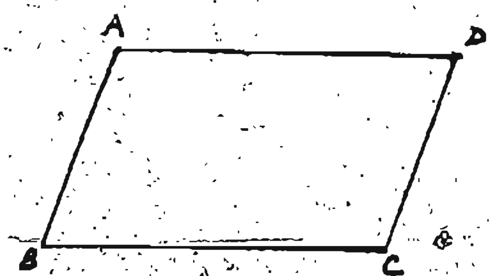
In summary, you should have now discovered the following properties about each quadrilateral.

1. The opposite sides of a parallelogram are congruent.
2. The opposite angles of a parallelogram are congruent.
3. A parallelogram has right angles only when that parallelogram is either a square or a non-square rectangle.
4. A square and a non-square rhombus always have four sides congruent.

PRACTICE EXERCISES:

The quadrilateral shown for exercises 1-3 is a parallelogram. Use appropriate letters to name the following.

1. Two pairs of opposite sides.
2. Two pairs of congruent sides.
3. Two pairs of opposite angles.

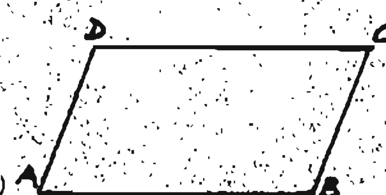



ANSWERS: 1. $\overline{AB}$, $\overline{DC}$ 3. $\angle A$, $\angle C$
$\overline{AD}$, $\overline{BC}$ $\angle B$, $\angle D$
2. $\overline{AB}$, $\overline{DC}$
$\overline{AD}$, $\overline{BC}$

In exercises 4-10, classify each statement as true or false.

4. A rhombus is equilateral. *(True)*
5. A rectangle is equiangular. *(True)*
6. A square is equilateral and equiangular. *(True)*
7. Two consecutive angles of a rectangle are supplementary. *(True)*
8. Two consecutive angles of every parallelogram are supplementary. *(True)*
9. If two angles of a quadrilateral are right angles, the quadrilateral is a rectangle. *(False)*
10. If one angle of a parallelogram is a right angle, the parallelogram is a rectangle. *(True)*

In exercises 11-16, refer to the parallelogram shown. Using the given information, find the indicated measure.




11. AB = 8; DC = ___?___. *(8)*
12. AD = 5; BC = ___?___. *(5)*
13. $m\angle DAB = 80$; $m\angle BCD =$ ___?___. *(80)*
14. $m\angle ADC = 105$; $m\angle ABC =$ ___?___. *(105)*
15. AB = 6 and BC = 4. The sum of the lengths of the four sides of parallelogram ABCD = ___?___. *(20)*
16. DC = 7 and AD = 3. The sum of the lengths of the four sides of parallelogram ABCD = ___?___. *(20)*

In exercises 17, 18 and 19, refer to the figure indicated:

17. Given: Parallelogram ABCD

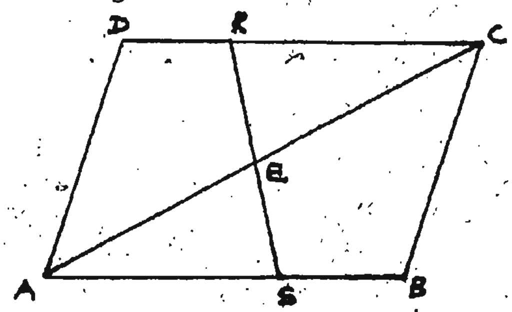
$\overline{AE} \cong \overline{EC}$

Prove: $\overline{RE} \cong \overline{SE}$

18. Given: Parallelogram ABCD

$\overline{RC} \cong \overline{SA}$

Prove: $\overline{AE} \cong \overline{EC}$




*Proof No. 17*

| *Statement* | *Reason* |
| --- | --- |
| 1. $\overline{AE} \cong \overline{EC}$ | 1. *Given* |
| 2. *ABCD is a parallelogram* | 2. *Given* |
| 3. $\overline{AB} \parallel \overline{CD}$ | 3. *Definition of parallelogram* |
| 4. $\angle RCE \cong \angle SAE$ | 4. *If two lines are parallel and cut by a transversal, the alternate interior angles are congruent* |
| 5. $\angle REC \cong \angle SEA$ | 5. *When two lines intersect, the vertical angles formed are contruent* |
| 6. $\triangle REC \cong \triangle SEA$ | 6. *ASA postulate* |
| 7. $\overline{RE} \cong \overline{SE}$ | 7. *Corresponding parts of congruent triangles are congruent* |

*Proof No. 18*

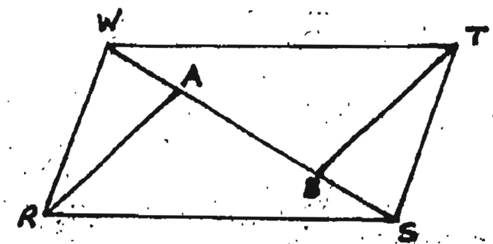
| *Statement* | *Reason* |
| --- | --- |
| 1. $\overline{RC} \cong \overline{SA}$ | 1. *Given* |
| 2. *ABCD is a parallelogram* | 2. *Given* |
| 3. $\overline{AB} \parallel \overline{CD}$ | 3. *Definition of parallelogram* |
| 4. $\angle RCE \cong \angle SAE$ | 4. *If two lines are parallel and cut by a transversal, the alternate interior angles formed are congruent* |
| 5. $\angle CRE \cong \angle ASE$ | 5. *Same as reason 4* |

| | |
|---|---|
| 6. $\Delta CRE \cong \Delta ASE$ | 6. *ASA postulate* |
| 7. $\overline{AE} \cong \overline{EC}$ | 7. *Corresponding parts of congruent triangles are congruent* |

19. Given: Parallelogram RSTW

$\overline{WB} \cong \overline{SA}$

Prove: $\overline{RA} \cong \overline{TB}$




*Proof No. 19*

| *Statement* | *Reason* |
|---|---|
| 1. *RSTW is a parallelogram* | 1. *Given* |
| 2. $\overline{RS} \cong \overline{TW}$ | 2. *Opposite sides of a parallelogram are congruent* |
| 3. $\angle TWB \cong \angle RSA$ | 3. *If two lines are parallel and cut by a transversal, the alternate interior angles formed are congruent* |
| 4. $\overline{WB} \cong \overline{SA}$ | 4. *Given* |
| 5. $\Delta WBT \cong \Delta SAR$ | 5. *SAS postulate* |
| 6. $\overline{RA} \cong \overline{TB}$ | 6. *Corresponding parts of congruent triangles are congruent* |

# LESSON #3 - STUDENT OBJECTIVES

Upon completion of this lesson, the student should be able to do the following correctly, 90% of the time:

1. The student should be able to identify the diagnols of a parallelogram.

2. The student can state and use the fact that the diagonals of a parallelogram are not necessarily congruent.

   (a) The student can state and use in practice exercises (or in problem solving) the fact that the diagonals of a square are congruent.

   (b) The student can state and use in practice exercises (or in problem solving) the fact that the diagonals of a non-square are congruent.

   (c) The student can state and use in practice exercises (or in problem solving) the fact that the diagonals of a non-square rhombus are not congruent.

   (d) The student can state and use the fact that the diagonals of a parallelogram which is neither a square, a non-square rhombus nor non-square rectangle are not congruent.

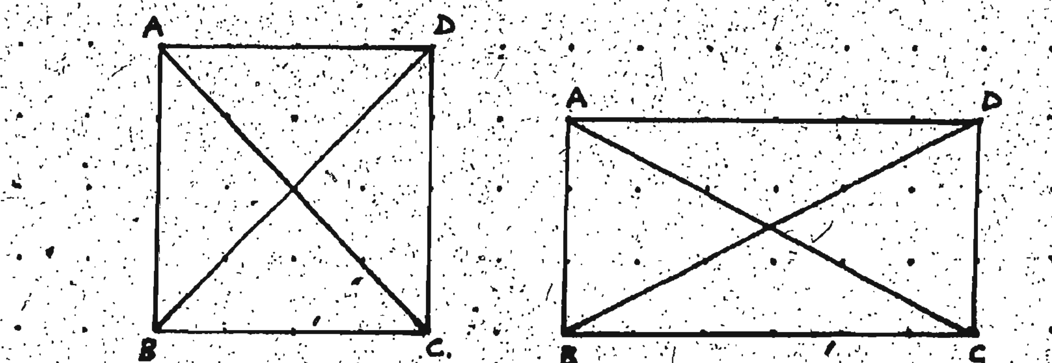
# LESSON #3 - PROPERTIES OF QUADRILATERALS

Before we move on to discover other properties of each quadrilateral, let's review those properties which we already discovered in Lesson #2:

1. The opposite sides of a parallelogram are congruent.
2. The opposite angles of a parallelogram are congruent.
3. A square has four right angles.
4. A non-square rectangle has four right angles.

<u>ACTIVITIES</u>:

The purpose of the next activity is to determine if the diagonals of each specific quadrilateral are congruent. Construct on your geoboard a square and a non-square rectangle similar to those shown in the diagram.




In each figure, join point A to point C and join point B to point D, using rubber bands. AC and BD are called <u>diagonals</u>.

Q. What do you notice about the diagonals of the square?

A. *The diagonals are congruent*

Using rubber bands construct on your geoboard two other squares of different sizes.

Q. What do you notice about the diagonals of these two squares?

A. *The diagonals are congruent*

Q. Write a general statement about the diagonals of a square.

A. *The diagonals of a square are congruent*

Q. What do you notice about the diagonals of the non-square rectangle?

A. *The diagonals of this non-square rectangle are congruent*

Using rubber bands construct on your geoboard two other non-square rectanbles of different sizes.

Q. What do you notice about the diagonals of these two non-square rectangles?

A. *The diagonals of these non-square rectangles are congruent*

Q. Write a general statement about the diagonals of a non-square rectangle.

A. *The diagonals of a non-square rectangle are congruent*

On a sheet of paper draw two parallelograms which are identical. Be certain that no angle is a right angle and that all four sides are not congruent. Label one parallelogram ABCD and the other parallelogram EFGH as shown in the diagram. Draw diagonals $\overline{AC}$ and $\overline{BD}$ of parallelogram ABCD and diagonals $\overline{EG}$ and $\overline{FH}$ of parallelogram EFGH. Cut parallelogram ABCD along $\overline{BD}$. Then place $\overline{BD}$ on $\overline{EG}$ of parallelogram EFGH.


Q. Are these two diagonals congruent?

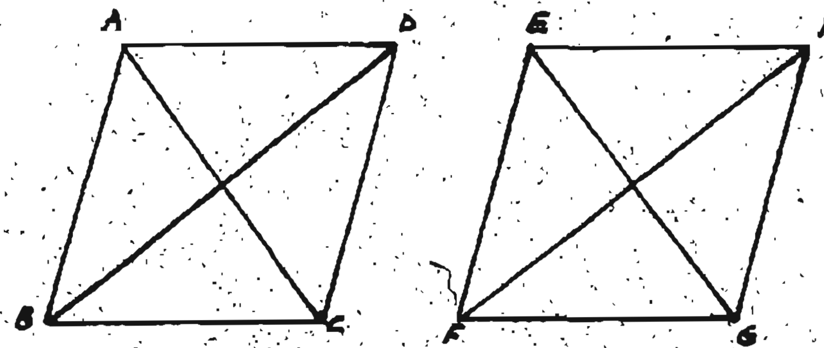
A. *No*

Draw two more parallelograms the same shape as those above and repeat the same procedure as above.

Q. Write a general statement about the diagonals of these parallelograms.

A. *The diagonals of these parallelograms are not congruent*

On another sheet of paper draw two non-square rhombii which are identical. Follow the same procedure as described above for the previous parallelogram. (If necessary, go back and read the section used for the diagonals of a parallelogram.)




Q. Are the two diagonals $\overline{BD}$ and $\overline{EG}$ congruent?

A. *No*

Draw two more non-square rhombii and repeat the same procedure.

Q. Write a general statement about the diagonals of a non-square rhombus.

A. *The diagonals of a non-square rhombus are not congruent*

Q. As for the non-square rectangles, non-square rhombii and those parallelograms which are neither non-square rectangles nor non-square rhombii, repeat the same procedure for the square. Write a general statement about the diagonals of a square.

A. *The diagonals of a square are congruent*

In summary for Lesson #3, we have made the following discoveries:

1. The diagonals of a parallelogram are not necessarily congruent.

   (a) The diagonals of a non-square rhombus are not congruent.

   (b) The diagonals of a non-square rectangle are congruent.

   (c) The diagonals of a square are congruent.

   (d) The diagonals of a parallelogram which is neither a square, non-square rhombus nor non-square rectangle are not congruent.

## LESSON #4 - STUDENT OBJECTIVES

Upon completion of this lesson, the student should be able to do the following correctly, 90% of the time:

1. The student can state and use the fact that the diagonals of a parallelogram always bisect each other.

2. The student can state and use the fact that the diagonals of a parallelogram do not necessarily bisect the opposite angles.

   (a) The student can state and use the fact that the diagonals of a non-square rectangle do not bisect the opposite angles.

   (b) The student can state and use the fact that the diagonals of a non-square rhombus bisect the opposite angles.

   (c) The student can state and use the fact that the diagonals of a square bisect the opposite angles.

   (d) The student can state and use the fact that the diagonals of a parallelogram which is neither a square, a non-square rhombus nor a non-square rectangle do not bisect the opposite angles.

3. The student can state and use the fact that the diagonals of a parallelogram are not necessarily perpendicular.

   (a) The student can state and use the fact that the diagonals of a non-square rectangle are not perpendicular.

   (b) The student can state and use the fact that the diagonals of a non-square rhombus are perpendicular.

   (c) The student can state and use the fact that the diagonals of a square are perpendicular.

   (d) The student can state and use the fact that the diagonals of a parallelogram which is neither a square, a non-square rhombus nor a non-square rectangle are not perpendicular.

## LESSON #4 - PROPERTIES OF QUADRILATERALS

The following discoveries were made in Lesson #3:

1. The diagonals of a parallelogram are not necessarily congruent.

   (a) The diagonals of a non-square rhombus are not congruent.

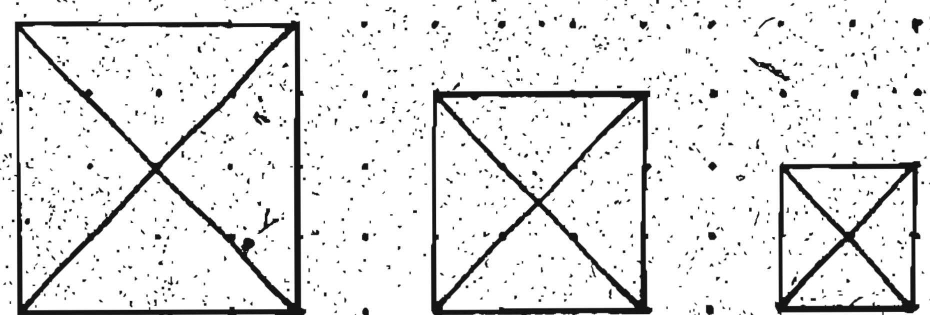
   (b) The diagonals of a non-square rectangle are congruent.

   (c) The diagonals of a square are congruent.

   (d) The diagonals of a parallelogram which is neither a square, non-square rhombus nor non-square rectangle are not congruent.

Now that you have discovered the relationship between the lengths of the diagonals of each quadrilateral, determine whether or not the diagonals of each quadrilateral bisect each other.

On your geoboard, construct three squares of different sizes. Make the diagonals of each square.



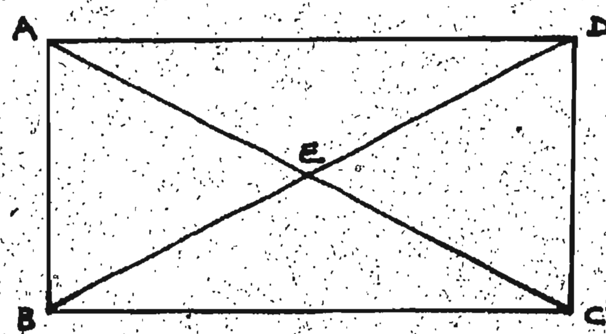
Q. Do the diagonals bisect each other?

A. *Yes*

Q. Write a general statement telling whether or not the diagonals of a square bisect each other.

A. *The diagonals of a square bisect each other*

On another sheet of paper draw a non-square rectangle ABCD with diagonals $\overline{AC}$ and $\overline{BD}$. Label the point of intersection of the diagonals, E. Cut out the four triangles, $\Delta ABE$, $\Delta AED$, $\Delta BEC$, and $\Delta DEC$. Place $\overline{BE}$ along $\overline{ED}$.




Q. Are these segments congruent?

A. *Yes*

Place $\overline{AE}$ along $\overline{EC}$.

Q. Are these segments congruent?

A. *Yes*

Q. Do the diagonals of this non-square rectangle bisect each other?

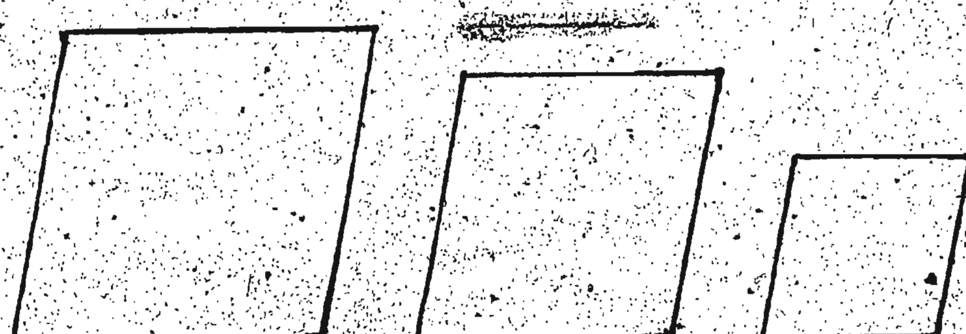
A. *Yes*

Draw two more non-square rectangles of different dizes and repeat the same procedure as above.

Q. Write a general statement telling whether or not the diagonals of a non-square rectangle bisect each other.

A. *The diagonals of a non-square rectangle bisect each other.*

On another sheet of paper draw three non-square rhombii of different sizes. Repeat the same procedure for each rhombus as was used for the non-square rectangles.



Q. Write a general statement telling whether or not the diagonals of a non-square rhombus bisect each other.

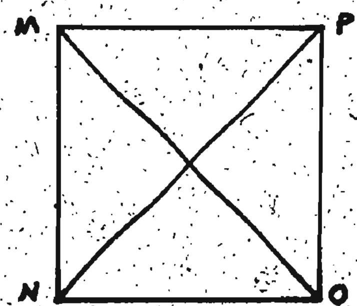
A. *The diagonals of a non-square rhombus bisect each other*

Similarly, draw three parallelograms of different sizes, none of which is a square, a non-square rhombus or a non-square rectangle. Repeat the same procedure for each parallelogram as was used for the non-square rectangles.

Q. Write a general statement telling whether or not the diagonals of such a parallelogram bisect each other.

A. *The diagonals of a parallelogram bisect each other*

Let's now move on to discover if the diagonals of each quadrilateral bisect the opposite angles. On another sheet of paper draw a square and label it MNOP with diagonals $\overline{MO}$ and $\overline{NP}$. Fold the square along diagonal $\overline{MO}$.




Q. Are the two angles $\angle$NMO and $\angle$PMO congruent?

A. *Yes*

Q. Are the two angles $\angle$NOM and $\angle$POM congruent?

A. *Yes*

Q. Therefore, does the diagonal $\overline{MO}$ bisect the opposite angles?

A. *Yes*

Using the same square, fold along diagonal $\overline{NP}$.

Q. Are the two angles $\angle$MPN and $\angle$OPN congruent?

A. *Yes*

Q. Are the two angles $\angle$MNP and $\angle$ONP congruent?

A. *Yes*

Q. Therefore, does the diagonal $\overline{NP}$ bisect the opposite angles?

A. *Yes*

Draw two other squares of different sizes and repeat the same procedure as used for the square.

Q. Write a general statement telling whether or not the diagonals of a square bisect the opposite angles.

A. *The diagonals of a square bisect the opposite angles*

Q. Draw three non-square rectangles and repeat the exact procedure used above to determine if the diagonals bisect the opposite angles. Write a general statement.

A. *The diagonals of a non-square rectangle do not bisect the opposite angles*

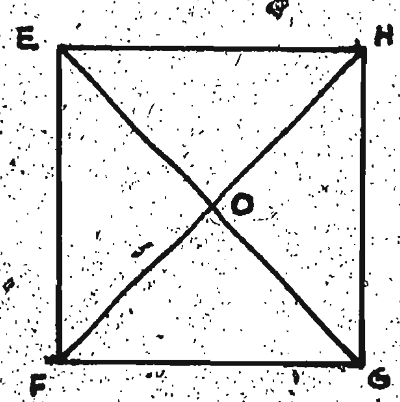
Q. Draw three parallelograms, none of which is a square, a non-square rhombus or a non-square rectangle and repeat the same procedure to determine if the diagonals bisect the opposite angles. Write a general statement.

A. *The diagonals of a parallelogram, which is neither a square, a non-square rhombus, nor a non-square rectangle, do not bisect the opposite angles*

Q. Draw three non-square rhombii and repeat the same procedure to determine if the diagonals bisect the opposite angles. Write a general statement.

A. *The diagonals of a non-square rhombus bisect the opposite angles*

The purpose of the next activity is to discover if the diagonals of each quadrilateral are perpendicular. On another sheet of paper draw a square EFGH with diagonals $\overline{EG}$ and $\overline{FH}$ intersecting at point O. Using your scissors cut out the four triangles, $\Delta EOF$, $\Delta EOH$, $\Delta HOG$ and $\Delta FOG$. Place the four triangles on top of each other remembering to keep vertex O over vertex O.



Q. Are the four triangles congruent? That is, do they lie perfectly on each other?

A. *Yes*

Q. Are the four angles at vertex O congruent?

A. *Yes*

Q. Does this mean that each angle measures $90^{\circ}$?

A. *Yes*

Q. Does this suggest then that the diagonals are perpendicular?

*Yes*

Draw two other squares of different sizes and repeat the same procedure as above.

Q. Write a general statement telling whether or not the diagonals of a square are perpendicular.

A. *The diagonals of a square are perpendicular*

Q. Draw three parallelograms of different sizes none of which is a square, a non-square rhombus or a non-square rectangle. Repeat the same procedure to determine if the diagonals of a parallelogram are perpendicular. Write a general statement.

A. *The diagonals of a parallelogram are not perpendicular*

Q. Draw three non-square rhombii of different sizes and repeat the same procedure to determine if the diagonals of a rhombus are perpendicular. Write a general statement.

A. *The diagonals of a rhombus are perpendicular*

Q. Draw three non-square rectangles of different sizes and repeat the same procedure to determine if the diagonals of a non-square rectangle are perpendicular. Write a general statement.

A. *The diagonals of a non-square rectangle are not perpendicular*

Summary of ideas discovered in Lesson #4:

1. The diagonals of a parallelogram bisect each other.

2. The diagonals of a parallelogram do not necessarily bisect the opposite angles.

(a) The diagonals of a non-square rectangle do not bisect the opposite angles.

(b) The diagonals of a non-square rhombus bisect the opposite angles.

(c) The diagonals of a square bisect the opposite angles.

(d) The diagonals of a parallelogram which is neither a square, a non-square rhombus nor a non-square rectangle do not bisect the opposite angles.

3. The diagonals of a parallelogram are not necessarily perpendicular.

(a) The diagonals of a non-square rectangle are not perpendicular.

(b) The diagonals of a non-square rhombus are perpendicular.

(c) The diagonals of a square are perpendicular.

(d) The diagonals of a parallelogram which is neither a square, a non-square rhombus nor a non-square rectangle are not perpendicular.

# LESSON #5 - STUDENT OBJECTIVES

Upon completion of this lesson, the student should be able to do the following correctly, 90% of the time:

1. The student can state and use the fact that a diagonal separates a parallelogram into two congruent triangles.

2. The student can state and use the fact that the opposite sides of a parallelogram are parallel.

# LESSON #5 - PROPERTIES OF QUADRILATERALS

Let's review first those properties discovered in Lesson #4:

1. The diagonals of a parallelogram bisect each other.

2. The diagonals of a parallelogram do not necessarily bisect the opposite angles.

   (a) The diagonals of a non-square rectangle do not bisect the opposite angles.

   (b) The diagonals of a non-square rhombus bisect the opposite angles.

   (c) The diagonals of a square bisect the opposite angles.

   (d) The diagonals of a parallelogram which is neither a square, a non-square rhombus nor a non-square rectangle do not bisect the opposite angles.

3. The diagonals of a parallelogram are not necessarily perpendicular.

   (a) The diagonals of a non-square rectangle are not perpendicular.

   (b) The diagonals of a non-square rhombus are perpendicular.

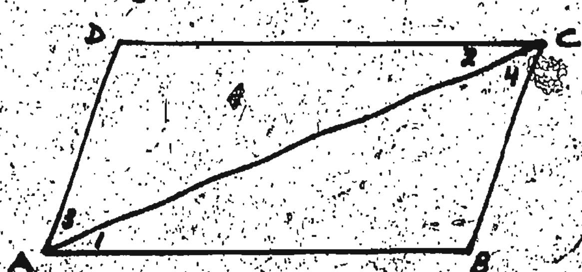
   (c) The diagonals of a square are perpendicular.

   (d) The diagonals of a parallelogram which is neither a square, a non-square rhombus nor a non-square rectangle are not perpendicular.

We now move on to discover if a diagonal of each quadrilateral separates the quadrilateral into two congruent triangles. Let's also discover if the opposite sides of each quadrilateral are parallel. Before we do this experiment we must recall two points:

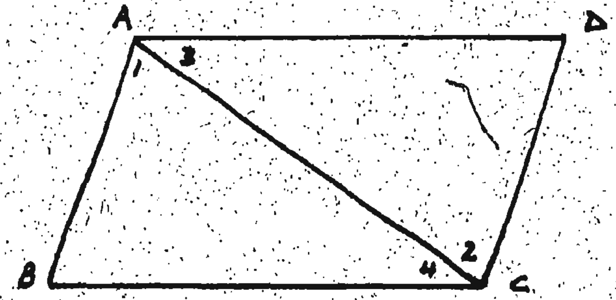
1. Recall that two triangles are congruent if the two triangles can be made to fit perfectly together.

2. Recall that two lines are parallel if the alternate interior angles are congruent. See the diagram.




$\angle 1$ and $\angle 2$ are alternate interior angles.
If $\angle 1 \cong \angle 2$ then $\overline{AB} \parallel \overline{CD}$.

Let's start with the parallelogram which is neither a square, a non-square rhombus nor a non-square rectangle. On a sheet of paper draw three such parallelograms of different sizes and for each parallelogram draw a diagonal as shown below. Also mark the angles as shown.




Cut each such parallelogram ABCD along diagonal $\overline{AC}$.

Q. Can the two triangles of each such parallelogram be made to fit together exactly?

A. *Yes*

Q. Write a general statement stating whether or not a diagonal of such a parallelogram separates the parallelogram into two congruent triangles.

A. *Yes, a diagonal separates a parallelogram into two congruent triangles*

Recall the definition of a parallelogram from Lesson #1. We know that the opposite sides of a parallelogram are parallel. However, now in the following three steps we will prove this.

Q. Is $\angle 1 \cong \angle 2$?

A. *Yes*

Q. Therefore, is $\overline{AB} \parallel \overline{CD}$?

A. *Yes*

Q. Is $\angle 3 \cong \angle 4$?

A. *Yes*

Q. Therefore, is $\overline{BC} \parallel \overline{AD}$?

A. *Yes*

Q. Write a general statement telling whether or not the opposite sides of a parallelogram are parallel.

A. *The opposite sides of a parallelogram are parallel*

Repeat the same procedure for three squares of different sizes.

Q. Write a general statement saying whether or not a diagonal of a square separates the square into two congruent triangles.

A. *A diagonal of a square separates a square into two congruent triangles*

Q. Write a general statement stating whether or not the opposite sides of a square are parallel.

A. *The opposite sides of a square are parallel*

Repeat the same procedure for three non-square rectangles of different sizes.

Q. Write a general statement stating whether or not a diagonal of a non-square rectangle separates the rectangle into two congruent triangles.

A. *A diagonal of a non-square rectangle separates the rectangle into two congruent triangles*

Q. Write a general statement stating whether or not the opposite sides of a non-square rectangle are parallel.

A. *The opposite sides of a non-square rectangle are parallel*

Repeat the same procedure for three non-square rhombii of different sizes.

Q. Write a general statement stating whether or not a diagonal of a non-square rhombus separates the rhombus into two congruent triangles.

A. *The diagonals of a non-square rhombus separates the rhombus into two congruent triangles*

Q. Write a general statement stating whether or not the opposite sides of a non-square rhombus are parallel.

A. *The opposite sides of a non-square rhombus are parallel*

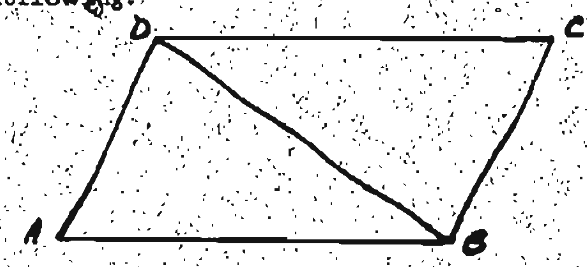
Summary of properties discovered in Lesson #5:

1. A diagonal separates a parallelogram into two congruent triangles.

2. The opposite sides of a parallelogram are parallel.

PRACTICE EXERCISES: Lessons 3, 4, 5

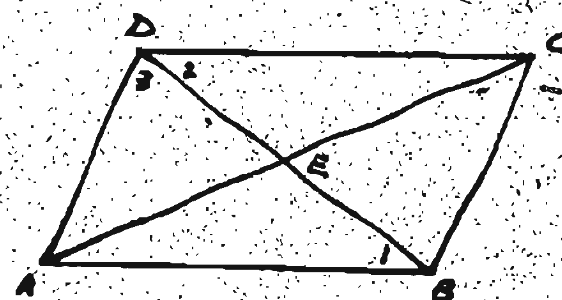
The quadrilateral shown for exercises 1 and 2 is a parallelogram. Use appropriate letters to name the following:




1. A diagonal shown in the figure. *($\overline{DB}$)*

2. A pair of congruent triangles. *($\triangle ABD$, $\triangle CDB$)*

Exercises 3-10 refer to the parallelogram shown. Classify each statement as true or false.




3. $\overline{AB}$ and $\overline{DC}$ are opposite sides. *(True)*

4. $\angle$ DAB and $\angle$ BCD are opposite angles. *(True)*

5. $\overline{AD} \cong \overline{BC}$. *(True)*

6. $\triangle ABD \cong \triangle CDB$. *(True)*

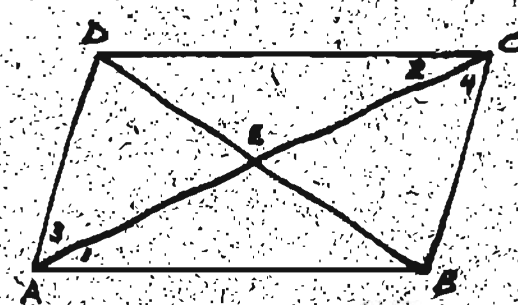
7. $m\angle 1 = m\angle 3$. *(False)*

8. $\angle ABC \cong \angle CDA$. *(True)*

9. AE = DE. *(False)*

10. $\overline{DE} \cong \overline{EB}$. *(True)*

Exercises 11-14 refer to the parallelogram shown. Using the given information, find the indicated measure.



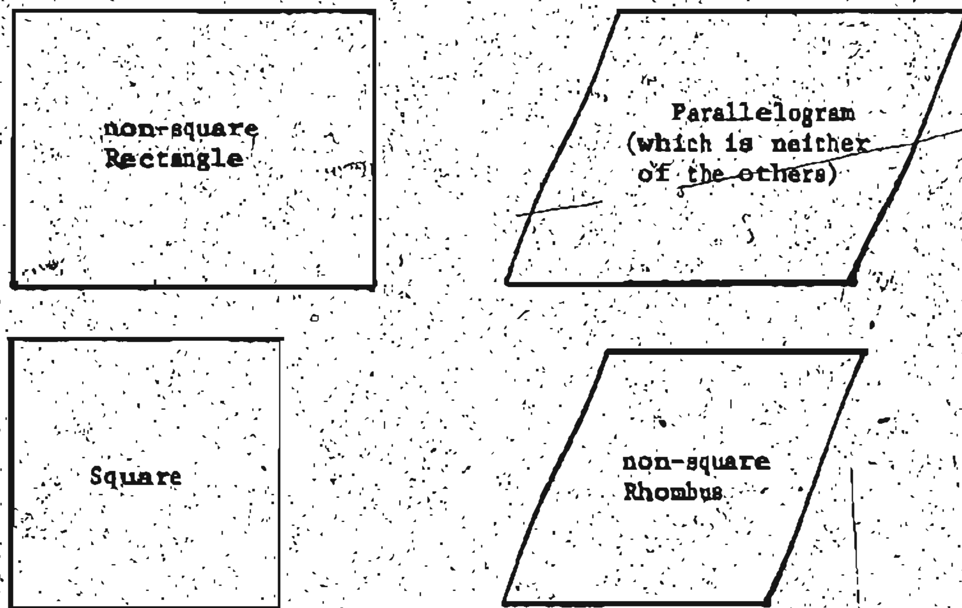

11. $m\angle 1 = 30$ and $m\angle 4 = 50$.
Find $m\angle DCB$. *(80)*

12. $m\angle 3 = 60$ and $m\angle 2 = 15$.
Find $m\angle DAB$. *(75)*

13. $m\angle 1 = 60$ and $m\angle BCD = 82$.
Find $m\angle 4$. *(22)*

14. $m\angle 3 = 58$ and $m\angle BCD = 86$.
Find $m\angle 2$. *(28)*

In exercises 15-22 refer to the quadrilaterals shown. For each exercise state which of the figures satisfy the given condition.




15. Diagonals are congruent. *(non-square rectangle and square)*

16. Diagonals bisect each other. *(all)*

17. Opposite angles are congruent. *(all)*

18. Diagonals are perpendicular to each other. *(non-square rhombus and square)*

19. When a diagonal is drawn, two congruent triangles are formed. *(all)*

20. Diagonals bisect opposite angles. *(non-square rhombus, square)*

21. Diagonals are perpendicular bisectors of each other. *(non-square rhombus and square)*

22. When both diagonals are drawn, eight triangles are formed. *(all)*

In exercises 23-26 draw, if possible, a parallelogram which satisfies the stated condition. If no such parallelogram exists, write NOT POSSIBLE on your paper.

23. Diagonals are congruent but not all sides are congruent. *(rectangle)*

24. Diagonals are perpendicular but do not bisect each other. *(not possible)*

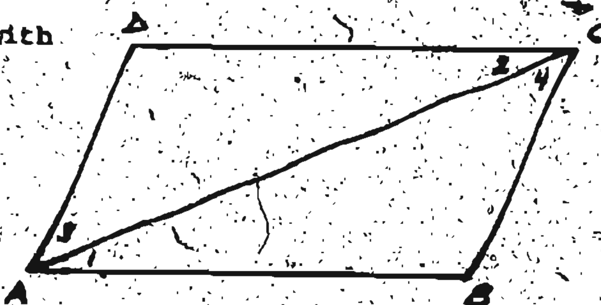
25. Diagonals bisect each other but are not congruent. *(non-square rhombus and parallelogram)*

26. Diagonals are perpendicular bisectors of each other but are not congruent. *(non-square rhombus)*

In exercises 27-31 provide the reasons for the proof of the statement "A diagonal of a parallelogram separates the parallelogram into two congruent triangles."

GIVEN: Parallelogram ABCD with diagonal $\overline{AC}$.

To Prove: $\triangle ABC \cong \triangle CDA$




PROOF:

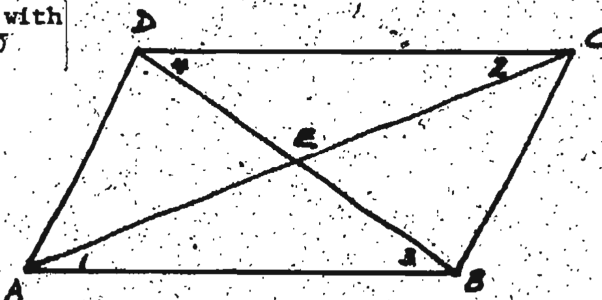
| Statement | Reason |
|---|---|
| 1. ABCD is a parallelogram | 1. Given |
| 2. $\overline{AB} \parallel \overline{DC}$ | 2. Definition of a parallelogram |
| 3. $\angle 1 \cong \angle 2$ | 27. If two parallel lines are cut be a transversal, *the alternate interior angles are congruent* |
| 4. $\overline{AC} \cong \overline{AC}$ | 28. *Congruence of segments is reflexive* |
| 5. $\overline{AD} \parallel \overline{BC}$ | 29. *Opposite sides of a parallelogram are parallel.* |
| 6. $\angle 3 \cong \angle 4$ | 30. *If two parallel lines are cut by a transversal, the alternate interior angles are congruent* |
| 7. $\triangle ABC \cong \triangle CDA$ | 31. *ASA postulate* |

In exercises 32-36 provide the reasons for the proof of the statement, "The diagonals of a parallelogram bisect each other."

GIVEN: Parallelogram ABCD with diagonals $\overline{AC}$ and $\overline{BD}$

To Prove: $\overline{AE} \cong \overline{CE}$

$\overline{BE} \cong \overline{DE}$



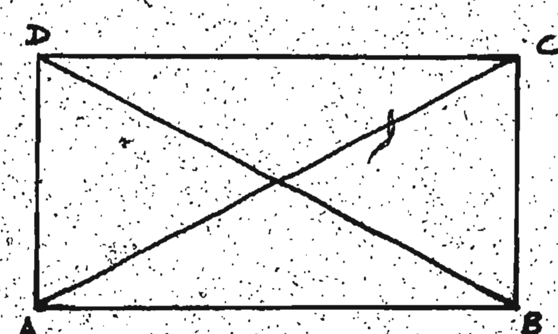

PROOF:

| Statement | Reason |
|---|---|
| 1. ABCD is a parallelogram | 1. Given |
| 2. $\overline{AB} \parallel \overline{DC}$ | 32. *Definition of a parallelogram* |
| 3. $\angle 1 \cong \angle 2$ | 33. *If two parallel lines are cut by a transversal, the alternate interior angles are congruent* |
| 4. $\overline{AB} \cong \overline{CD}$ | 34. *Opposite sides of a parallelogram are congruent* |
| 5. $\angle 3 \cong \angle 4$ | 35. *If two parallel lines are cut by a transversal, the alternate interior angles are congruent* |
| 6. $\triangle AEB \cong \triangle CED$ | 36. *ASA postulate* |
| 7. $\overline{AE} \cong \overline{CE}$; $\overline{BE} \cong \overline{DE}$ | 37. Corresponding parts of congruent triangles are congruent |

In exercises 37-42 provide the reasons for the proof of the statement, "The diagonals of a rectangle are congruent."

GIVEN: Rectangle ABCD with diagonals $\overline{AC}$ and $\overline{BD}$

To Prove: $\overline{AC} \cong \overline{BD}$



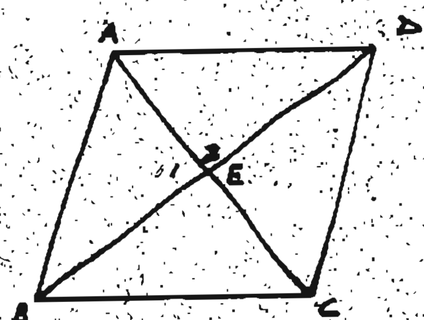
PROOF:

| Statement | Reason |
|---|---|
| 1. Rectangle ABCD with diagonals $\overline{AC}$ and $\overline{BD}$ | 1. Given |
| 2. ABCD is a parallelogram | 2. Definition of a rectangle |
| 3. $\overline{AD} \cong \overline{BC}$ | 37. *Opposite sides of a parallelogram are congruent* |
| 4. $\angle ABC$ and $\angle BAD$ are right angles | 38. *Definition of a rectangle* |
| 5. $\angle ABC \cong \angle BAD$ | 39. *All right angles are congruent* |
| 6. $\overline{AB} \cong \overline{AB}$ | 40. *Congruence of segments is reflexive* |
| 7. $\triangle ABC \cong \triangle BAD$ | 41. *SAS postulate* |
| 8. $\overline{AC} \cong \overline{BD}$ | 42. *Corresponding parts of congruent triangle are congruent* |

In exercises 43-48 provide the reasons for the proof of the statement, "The diagonals of a rhombus are perpendicular."

GIVEN: Rhombus ABCD with diagonals $\overline{AC}$ and $\overline{BD}$

To Prove: $\overline{AC} \perp \overline{BD}$




PROOF:

| Statement | Reason |
|---|---|
| 1. Rhombus ABCD with diagonals $\overline{AC}$ and $\overline{BD}$ | 1. Given |
| 2. $\overline{AB} \cong \overline{AD}$ | 43. *Definition of a rhombus* |
| 3. $\overline{BE} \cong \overline{ED}$ | 44. *Diagonals of a rhombus bisect each other* |

4. $\overline{AE} \cong \overline{AE}$ — 45. *Congruence of segments is reflexive*

5. $\Delta AEB \cong \Delta AED$ — 46. *SSS postulate*

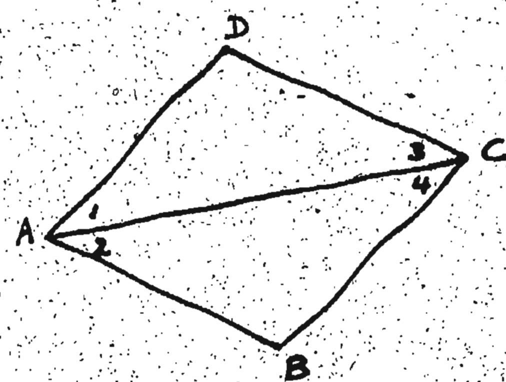
6. $\angle 1 \cong \angle 2$ — 47. *Corresponding parts of congruent triangles are congruent*

7. $\overline{AC} \perp \overline{BD}$ — 48. *If two lines meet to form congruent adjacent angles the lines are perpendicular*

49. Prove the statement, "Each diagonal of a rhombus bisects a pair of opposite angles. Use the information given.

GIVEN: Rhombus ABCD with diagonal $\overline{AC}$

To Prove: $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$

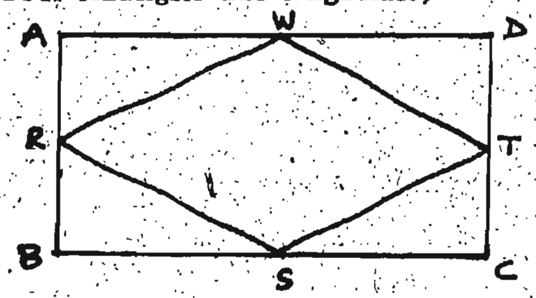



*PROOF:*

| *Statement* | *Reason* |
|---|---|
| 1. *Rhombus ABCD with diagonal* $\overline{AC}$ | 1. *Given* |
| 2. $\overline{AD} \cong \overline{AB}$; $\overline{DC} \cong \overline{BC}$ | 2. *Definition of rhombus* |
| 3. $\overline{AC} \cong \overline{AC}$ | 3. *Congruence of segments is reflexive* |
| 4. $\Delta ADC \cong \Delta ABC$ | 4. *SSS postulate* |
| 5. $\angle 1 \cong \angle 2$<br>$\angle 3 \cong \angle 4$ | 5. *Corresponding parts of congruent triangles are congruent* |

50. GIVEN: Rectangle ABCD; points R, S, T and W are the midpoints of the sides of the rectangle.

To Prove: Quadrilateral RSTW is a rhombus. (Hint: Prove that the four triangles are congruent.)




*PROOF:*

| | *Statement* | | *Reason* |
|---|---|---|---|
| 1. | *ABCD is a rectangle* | 1. | *Given* |
| 2. | $\angle A$, $\angle B$, $\angle C$, $\angle D$ *are right angles* | 2. | *Definition of a rectangle* |
| 3. | $\Delta RBS$, $\Delta TCS$, $\Delta WDT$, *and* $\Delta WAR$ *are right triangles* | 3. | *Definition of right triangle* |
| 4. | $\overline{AD} \cong \overline{BD}$, $\overline{AB} \cong \overline{CD}$ | 4. | *Opposite sides of a rectangle are congruent* |
| 5. | $AD = BC$, $AB = CD$ | 5. | *Definition of congruent segments* |
| 6. | *R, S, T and W are the midpoints of* $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ *and* $\overline{AD}$, *respectively* | 6. | *Given* |
| 7. | $AR = RB = \frac{1}{2} AB$<br>$BS = SC = \frac{1}{2} BC$<br>$CT = TD = \frac{1}{2} CD$<br>$DW = AW = \frac{1}{2} AD$ | 7. | *Definition of midpoint* |
| 8. | $\frac{1}{2} AD = \frac{1}{2} BC$; $\frac{1}{2} AB = \frac{1}{2} CD$ | 8. | *Multiplication property of equality and step 4* |
| 9. | $DW = WA = BS = SC$<br>$AR = RB = CT = TD$ | 9. | *Substitution property of equality* |
| 10. | $\Delta RBS \cong \Delta TCS \cong \Delta TDW \cong \Delta RAW$ | 10. | *LL theorem* |
| 11. | $\overline{RS} \cong \overline{ST} \cong \overline{TW} \cong \overline{WR}$ | 11. | *Corresponding parts of congruent triangles are congruent* |
| 12. | *Quadrilateral RSTW is a rhombus* | 12. | *Definition of a rhombus* |

## REVIEW OF CONCEPTS AND TERMS

At this point, you should have made thirty-six different discoveries regarding the properties of the different quadrilaterals. At the same time, you should have learned six new terms. List each concept discovered and each term learned.

## LESSON #6 - STUDENT OBJECTIVES

Upon completion of this lesson, the student should be able to do the following correctly, 90% of the time:

1. The student should be able to state and use the fact that if both pairs of opposite sides are parallel, the quadrilateral is a parallelogram.

2. The student can state and use the fact that if both pairs of opposite sides are parallel, the quadrilateral is a parallelogram.

3. The student can state and use the fact that if one pair of opposite sides are parallel and congruent, the quadrilateral is a parallelogram.

4. The student can state and use the fact that if both diagonals bisect each other, the quadrilateral is a parallelogram.

5. The student can write a formal proof, using statement and reason, of the statement, "If both pairs of opposite sides of a quadrilateral are congruent, the quadrilateral is a parallelogram."

6. The student can write a formal proof, using statement and reason, of the statement, "If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram."

7. The student can write a formal proof, using statement and reason, of the statement: "If a quadrilateral has one pair of opposite sides that are both parallel and congruent, the quadrilateral is a parallelogram."

8. The student can write a formal proof using the properties of quadilaterals discovered in Lessons #1 through 6.

# LESSON #6 - PROVING A QUADRILATERAL IS A PARALLELOGRAM

In this lesson you shall consider some conditions that will permit us to conclude that certain quadrilaterals are parallelograms. Before you begin the actual discoveries, recall from Lesson #1 that a parallelogram is a quadrilateral with both pairs of opposite sides parallel. Keep this definition in mind as you do the activities.

ACTIVITIES:

On your geoboard construct three quadrilaterals that have one pair of opposite sides that are both parallel and congruent.

Q. Does the fact that a quadrilateral has one pair of opposite sides that are both parallel and congruent always mean that the other pair of opposite sides are always parallel and congruent?

A. *Yes*

Q. Is this sufficient information to prove that a quadrilateral is a parallelogram?

A. *Yes*

Therefore, complete the general statement: "If a quadrilateral has *one pair of opposite sides that are both congruent and parallel,* the quadilateral is a parallelogram."

The next activity is to construct on your geoboard three quadrilaterals that have both pairs of opposite sides congruent.

Q. Does the fact that both pairs of opposite sides are congruent always mean that both pairs of opposite sides are parallel?

A. *Yes*

Q. Is this sufficient information to prove that a quadrilateral is a parallelogram?

A. *Yes*

Therefore, complete the following general statement: "If a quadrilateral has *both pairs of opposite sides congruent,* the quadrilateral is a parallelogram."

Next, on your geoboard make with rubber bands three sets of lines that bisect each other. In each case using these two lines as diagonals, construct three quadrilaterals.

Q. Do the quadrilaterals formed each have both pairs of opposite sides parallel?

A. *Yes*

Q. Isn't each quadrilateral therefore a parallelogram?

A. *Yes*

Complete the following general statement: "If a quadrilateral has *its diagonals bisecting each other*, the quadrilateral is a parallelogram.

In summary, we now have four ways to prove a quadrilateral is a parallelogram:

1. Show both pairs of opposite sides are parallel.
2. Show both pairs of opposite sides are congruent.
3. Show one pair of opposite sides are parallel and congruent.
4. Show the diagonals bisect each other.

PRACTICE EXERCISES:

In exercises 1-6, classify each statement as true or false.

1. Every parallelogram is a quadrilateral. *(True)*

2. Every quadrilateral is a parallelogram. *(False)*

3. If a quadrilateral has one pair of opposite sides parallel, the quadrilateral is a parallelogram. *(False)*

4. If a quadrilateral has one pair of opposite sides parallel and congruent, the quadrilateral is a parallelogram. *(True)*

5. If both pairs of opposite sides of a quadrilateral are congruent, the quadrilateral is a parallelogram. *(True)*

6. The diagonals of every quadrilateral bisect each other. *(False)*

7. State four ways to prove a quadrilateral is a parallelogram.

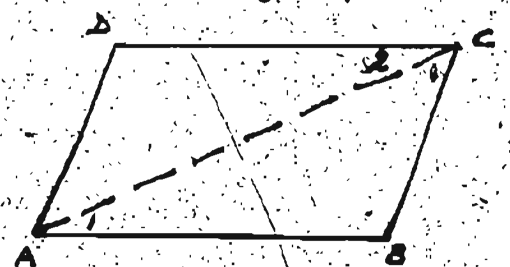
    *(1) Show both pairs of opposite sides are parallel.*

    *(2) Show both pairs of opposite sides are congruent.*

    *(3) Show one pair of opposite sides are parallel and congruent.*

    *(4) Show the diagonals bisect each other.*

In exercises 8-14, provide the reasons for the proof of the statement, "If both pairs of opposite sides of a quadrilateral are congruent, the quadrilateral is a parallelogram."




GIVEN: $\overline{AB} \cong \overline{CD}$

$\overline{BC} \cong \overline{DA}$

To Prove: ABCD is a parallelogram

PROOF:

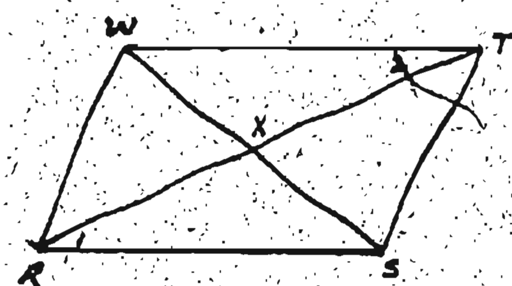
| Statement | Reason |
|---|---|
| 8. $\overline{AB} \cong \overline{CD}$; $\overline{BC} \cong \overline{DA}$ | 8. *Given* |
| 9. Draw $\overline{AC}$ | 9. *Through any two points exactly one line can be drawn* |
| 10. $\overline{AC} \cong \overline{AC}$ | 10. *Congruence of segments is reflexive* |
| 11. $\Delta ABC \cong \Delta CDA$ | 11. *SSS postulate* |

12. $\angle 1 \cong \angle 2$ — 12. *Corresponding parts of congruent triangles are congruent*

13. $\overline{AB} \parallel \overline{DC}$ — 13. *If two lines are cut by a transversal so that the alternate interior angles are congruent, the lines are parallel*

14. ABCD is a parallelogram — 14. *If a quadrilateral has one pair of opposite sides and both congruents are parallel, the quadrilateral is a parallelogram*

In exercises 15-21, provide the reasons for the proof of the statement, "If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram."




GIVEN: $\overline{RX} \cong \overline{TX}$

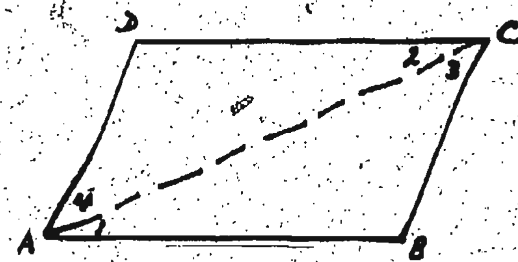
$\overline{WX} \cong \overline{SX}$

To Prove: RSTW is a parallelogram

PROOF:

| Statement | Reason |
| --- | --- |
| 15. $\overline{RX} \cong \overline{TX}$; $\overline{WX} \cong \overline{SX}$ | 15. *Given* |
| 16. $\angle RXS \cong \angle TXW$ | 16. *When two lines intersect, the vertical angles formed are congruent* |
| 17. $\Delta RXS \cong \Delta TXW$ | 17. *SAS postulate* |
| 18. $\overline{RS} = \overline{TW}$ | 18. *Corresponding parts of congruent triangles are congruent* |
| 19. $\angle 1 \cong \angle 2$ | 19. *Corresponding parts of congruent triangles are congruent* |
| 20. $\overline{RS} \parallel \overline{TW}$ | 20. *If two lines are cut by a transversal so that the alternate interior angles are congruent, the lines are parallel* |
| 21. RSTW is a parallelogram | 21. *If a quadrilateral has one pair of opposite sides both congruent and parallel, the quadrilateral is a parallelogram* |

In exercises 22-29, provide those reasons that are omitted for the proof of the statement, "If a quadrilateral has one pair of opposite sides that are both parallel and congruent, the quadrilateral is a parallelogram."




GIVEN: $\overline{AB} \parallel \overline{GD}$

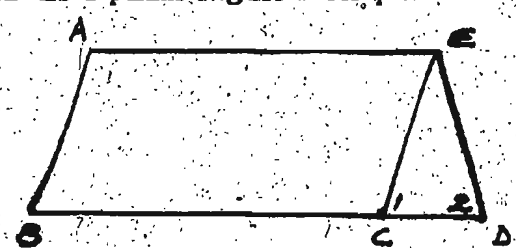
$\overline{AB} \cong \overline{CD}$

To Prove: ABCD is a parallelogram

PROOF:

| | Statement | | Reason |
|---|---|---|---|
| 22. | Draw $\overline{AC}$ | 22. | Through any two different points there is exactly one line |
| 23. | $\overline{AB} \cong \overline{CD}$; $\overline{AB} \parallel \overline{CD}$ | 23. | *Given* |
| 24. | $\angle 1 \cong \angle 2$ | 24. | *If two parallel lines are cut by a transversal the alternate interior angles are congruent* |
| 25. | $\overline{AC} \cong \overline{AC}$ | 25. | *Congruence of segments is reflexive* |
| 26. | $\triangle ABC = \triangle CDA$ | 26. | SAS postulate |
| 27. | $\angle 3 \cong \angle 4$ | 27. | Corresponding parts of congruent triangles are congruent |
| 28. | $\overline{AD} \parallel \overline{BC}$ | 28. | If two lines are cut by a transversal so that *the alternate interior angles are congruent, the lines are parallel* |
| 29. | ABCD is a parallelogram | 29. | Definition of a parallelogram |

In exercises 30 and 31, quadrilateral ABCE is a plane figure with point C on $\overline{BD}$.




30. GIVEN: $\overline{AE} \cong \overline{BC}$

$\overline{AB} \cong \overline{ED}$

$\overline{EC} \cong \overline{ED}$

Prove: ABCE is a parallelogram

PROOF:

| Statement | Reason |
|---|---|
| 1. $\overline{ED} \cong \overline{EC}$; $\overline{AB} \cong \overline{ED}$ | 1. Given |
| 2. $\overline{AB} \cong \overline{EC}$ | 2. Transitive property of congruence |
| 3. $\overline{AE} \cong \overline{BC}$ | 3. Given |
| 4. Quadrilateral ABCE is a parallelogram | 4. If both pairs of opposite sides of a quadrilateral are congruent, the quadrilateral is a parallelogram. |

31. GIVEN: $\overline{AB} \cong \overline{EC}$

$\angle B \cong \angle 1$

Prove: ABCE is a parallelogram

PROOF:

| Statement | Reason |
|---|---|
| 1. $\overline{AB} \cong \overline{EC}$ | 1. Given |
| 2. $\angle B \cong \angle 1$ | 2. Given |
| 3. $\overline{AB} \parallel \overline{EC}$ | 3. If two lines are cut by a transversal so that corresponding angles are congruent, the lines are parallel |
| 5. Quadrilateral ABCE is a parallelogram | 4. If a quadrilateral has one pair of opposite sides that are both parallel and congruent, the quadrilateral is a parallelogram |

## LESSON #7 - STUDENT OBJECTIVES

Upon completion of this lesson, the student should be able to do the following correctly, 90% of the time:

1. The student should be able to draw, using the blocks provided, three parallel lines that are the same distance apart.

2. The student should be able to state and use the fact that if three parallel lines cut off congruent segments on one transversal, they cut off congruent segments on every transversal.

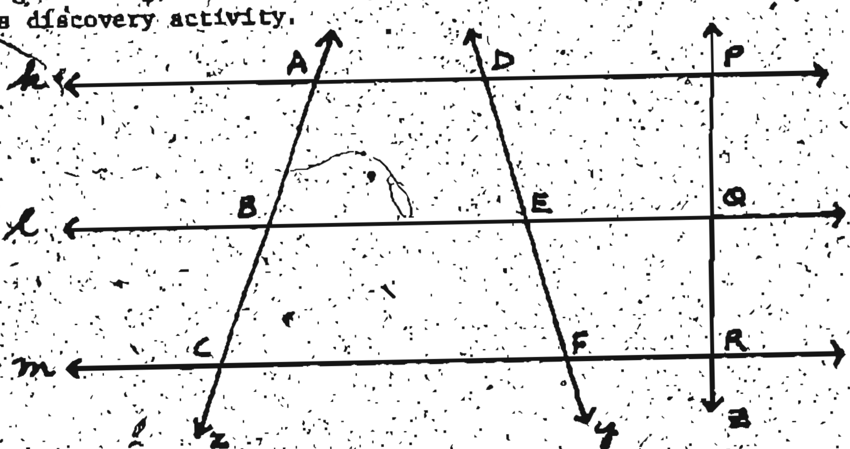
3. The student can state and use the fact that the segment joining the midpoints of two sides of a triangle is parallel to, and is half as long as, the third side.

## LESSON #7 - USING PROPERTIES OF PARALLELOGRAMS

Let's recall the discoveries that were made in Lesson #6: There are four ways to prove a quadrilateral is a parallelogram:

1. Show both pairs of opposite sides are parallel.
2. Show both pairs of opposite sides are congruent.
3. Show one pair of opposite sides are parallel and congruent.
4. Show the diagonals bisect each other.

In the next discovery exercise you will need to construct three parallel lines on a sheet of paper. To do this use one of the blocks. Be certain that all three parallel lines are the same distance apart. Draw a transversal (called line z) which meets each parallel line at right angles. We do this to insure that the two segments on transversal z, $\overline{PQ}$ and $\overline{QR}$, are congruent. Since the parallel lines cut off congruent segments on this transversal, does this necessarily mean that they cut off congruent segments on all other transversals? This is the purpose of this discovery activity.




K || L || M. Lines x,y,z are called transversals. $\overline{PQ} \cong \overline{QR}$.

Use paper folding for this discovery. Fold the paper at point B to determine if $\overline{AB} \cong \overline{BC}$; fold the paper at point E to determine if $\overline{DE} \cong \overline{EF}$. Make certain that the paper is folded in such a manner that $\overline{AB}$ and $\overline{BC}$ lie on each other and also $\overline{DE}$ and $\overline{EF}$ lie on each other.

Q. Is $\overline{AB} \cong \overline{BC}$?

A. *Yes*

Q. Is $\overline{DE} \cong \overline{EF}$?

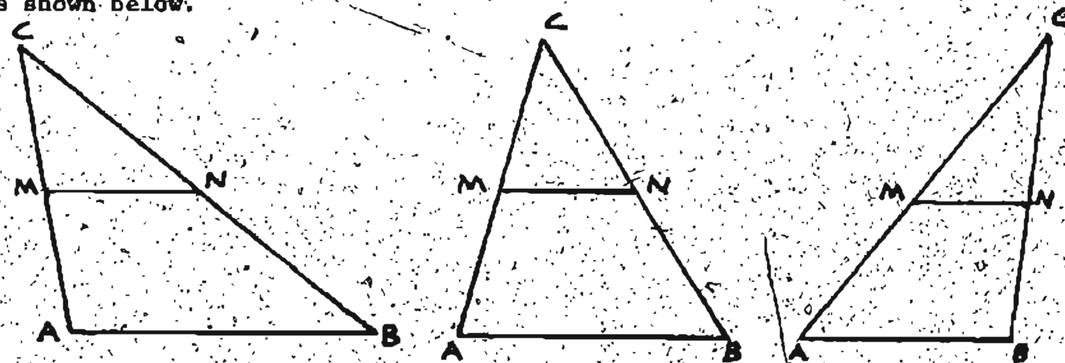
A. *Yes*

Repeat the same procedure for two more sets of three parallel lines.

Q. Therefore, can we conclude that if three parallel lines cut off congruent segments on one transversal, they cut off congruent segments on every transversal?

A. *Yes*

The next discovery exercise involves triangles. We will endeavour to show that the segment joining the midpoints of two sides of a triangle is parallel to, and is half as long as, the third side. On a sheet of paper draw three triangles of different shapes. Determine by trial and error the midpoints of two sides of each triangle and join the two points. That is, fold each segment until the endpoints coincide. The midpoint of each segment is at the point of folding. Label each triangle as shown below.




We now have two considerations for each triangle:

1. Is $\overline{MN} \parallel \overline{AB}$?

2. Is $MN = \frac{1}{2} AB$?

This activity involves paper cutting.

Recall that two lines are parallel if the corresponding angles are congruent. In each figure above $\angle CMN$ and $\angle CAB$ are corresponding angles and $\angle CNM$ and $\angle CBA$ are corresponding angles.

Cut each figure along $\overline{MN}$. Place $\angle CMN$ along $\angle CAB$.

Q. Are they congruent?

A. *Yes*

Q. Therefore, is $\overline{MN} \parallel \overline{AB}$?

A. *Yes*

Q. When $\overline{MN}$ is placed on $\overline{AB}$, is $MN = \frac{1}{2} AB$?

A. *Yes*

Q. Write a general statement concerning the segment joining the midpoints of two sides of a triangle and the third side.

A. *The segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long as the third side.*

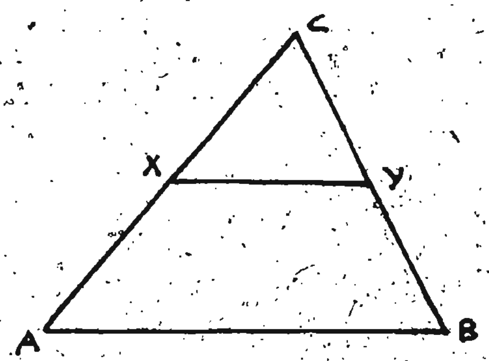
Summary of points discovered in Lesson #7:

1. If three parallel lines cut off congruent segments on one transversal, they cut off congruent segments on every transversal.

2. The segment joining the midpoints of two sides of a triangle is parallel to, and is half as long as, the third side.

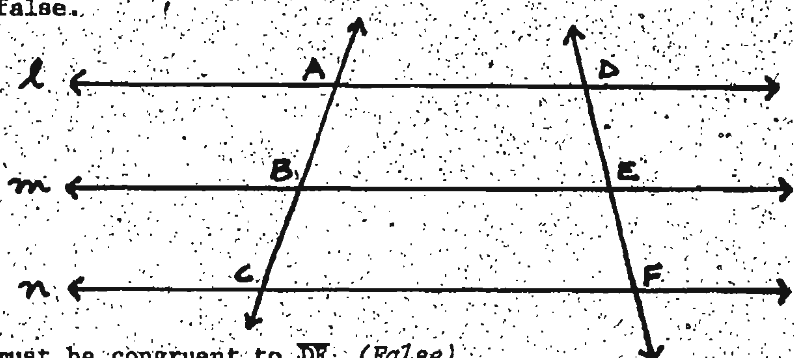
PRACTICE EXERCISES:

In exercises 1-6, X is the midpoint of $\overline{AC}$ and Y is the midpoint of $\overline{BC}$. Classify the statements as true or false.



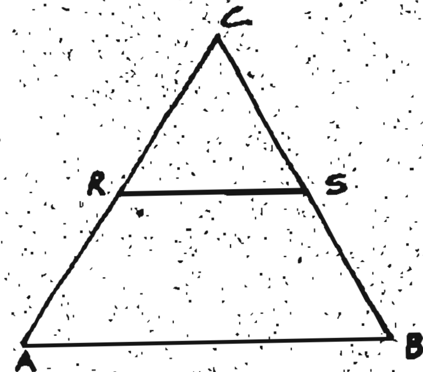

1. $\overline{AX} \cong \overline{XC}$ *(True)*
2. AX = XC *(True)*
3. BY = YC *(True)*
4. XY = $\frac{1}{2}$ AC *(False)*
5. $\overline{XY} \parallel \overline{AB}$ *(True)*
6. When AB = 2, XY = 1 *(True)*

In exercises 7-10, $l \parallel m \parallel n$ and $\overline{AB} \cong \overline{BC}$. Classify the statements as true or false.




7. $\overline{AB}$ must be congruent to $\overline{DE}$ *(False)*
8. DE = EF *(True)*
9. When BC = 5, EF = 5 *(False)*
10. When DE = 6, EF = 6 *(True)*

In exercises 11-18, R is the midpoint of $\overline{AC}$ and S is the midpoint of $\overline{BC}$. Using the information given, find, if possible, the indicated length. If the information is not adequate, write NOT POSSIBLE.




11. AR = 3; RC = __?__ *(3)*

12. BS = 4; AR = __?__ *(not possible)*

13. AB = 6; RS = __?__ *(3)*

14. AB = 9.6; RS = __?__ *(4.8)*

15. RS = 2 2/3; AB = __?__ *(5 1/3)*

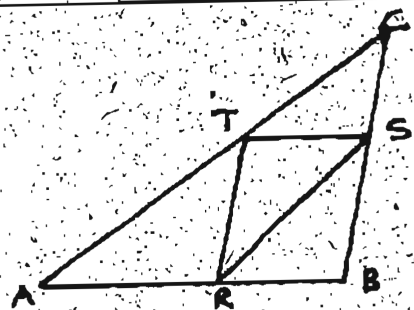
16. RS = 4.1; RC = __?__ *(not possible)*

17. CR = 2; RS = 3; CS = 2. Perimeter of $\Delta ABC$ = __?__ *(14)*

18. AR = 3; RS = 3; CS = 2. Perimeter of $\Delta ABC$ = __?__ *(16)*

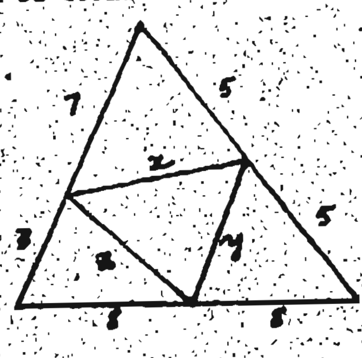
In exercises 19-21, points R,S and T are the midpoints of the sides of $\Delta ABC$. Using the information given, find the indicated lengths.

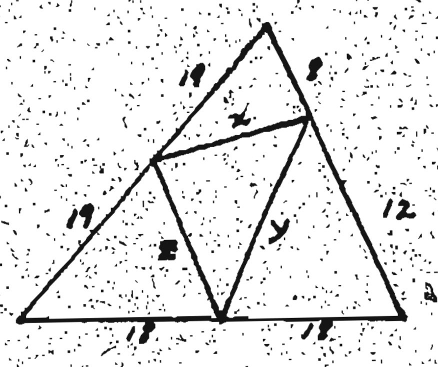
| | AB | BC | AC | ST | RT | RS |
|---|---|---|---|---|---|---|
| 19. | 8 | 10 | 12 | ? *(4)* | ? *(5)* | ? *(5)* |
| 20. | ? *(12)* | ? *(10)* | ? *(14)* | 6 | 5 | 7 |
| 21. | ? *(12)* | 10 | 16 | 6 | ? *(5)* | ? *(8)* |




In exercises 22 and 23, exactly one of the lengths denoted by x,y and z can be found. Find that length.

22.




Answer: *y = 5*

Answer: *z = 10*

# LESSON #8 - STUDENT OBJECTIVES

Upon completion of this lesson, the student should be able to do the following correctly, 90% of the time:

1. The student can recognize a trapezoid among the family of quadrilaterals.
2. The student should be able to define a trapezoid.
3. The student should be able to indicate the bases, legs and median of a trapezoid.
4. The student can state and use the fact that the median of a trapezoid is parallel to the bases and have a length equal to half the sum of the lengths of the bases.

# LESSON #8 - TRAPEZOIDS

Before we move ahead to Lesson #8, let's recall the two ideas discovered in Lesson #7.

1. If three parallel lines cut off congruent segments on one transversal, they cut off congruent segments on every transversal.

2. The segment joining the midpoints of two sides of a triangle is parallel to, and is half as long as, the third side.

In Lesson #1, you discovered four special quadrilaterals - parallelogram, rectangle, rhombus and square. However, there is one other special quadrilateral to discover.

On your geoboard, form three separate quadrilaterals of different sizes which has one and only one pair of parallel sides. Your figures should look something like the figure shown in the diagram.


Note that the two parallel sides do not have to be the same length.

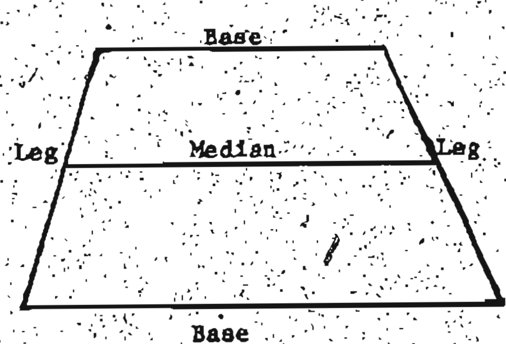
Q. How would you define such a figure?

A. *A quadrilateral with exactly one pair of parallel sides*

Q. What is the name given to such a figure?

A. *Trapezoid*

Now that you have discovered the general shape of a trapezoid, and its definition, let's learn some terms that will be helpful in our discussion of trapezoids.

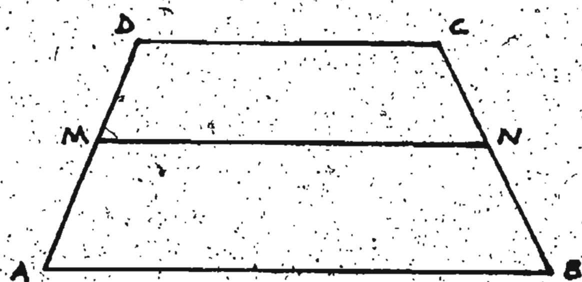



1. The parallel sides are called <u>bases</u>.
2. The non-parallel sides are called <u>legs</u>.
3. If the legs of a trapezoid are congruent, the quadrilateral is called an <u>isoscales trapezoid</u>.
4. The <u>median</u> of a trapezoid is the segment joining the midpoints of the legs.

Next, we must determine two properties of the trapezoid:

1. Is the median of a trapezoid parallel to the bases?

2. Is the length of the median equal to half the sum of the lengths of the bases?




With respect to the figure above we will ask:

1. Is $\overline{MN} \parallel \overline{AB}$? Is $\overline{MN} \parallel \overline{DC}$?

2. Is $MN = \frac{1}{2}(AB + DC)$?

We will use paper cutting for this experiment. On a sheet of paper draw, using the appropriate block, three trapezoids of different sizes and shapes. Draw for each trapezoid its median using trial and error to determine the midpoint of each leg. Follow the same procedure for finding the midpoint of each leg of the trapezoid as was used for finding the midpoints of the sizes of the triangle outlined in Lesson #7.

Cut each figure along median $\overline{MN}$. Place $\overline{MN}$ along $\overline{AB}$.

Q. Is $\angle DMN \cong \angle MAB$?

A. *Yes*

Q. Is $\overline{MN} \parallel \overline{AB}$?

A. *Yes*

Place $\overline{AB}$ along $\overline{MN}$ such that $\angle AMN$ and $\angle MDC$ fit together.

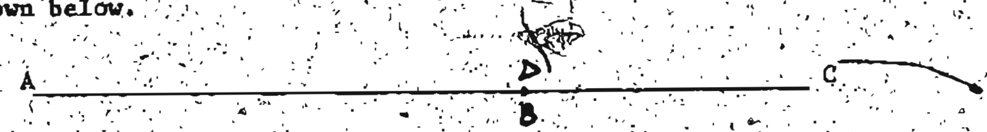
Q. Is $\angle AMN \cong \angle MDC$?

A. *Yes*

Q. Is $\overline{MN} \parallel \overline{DC}$?

A. *Yes*

Next, cut out $\overline{DC}$ and $\overline{AB}$ and place them together to form a longer segment as shown below.




Place $\overline{MN}$ along this segment and by folding determine if $MN = \frac{1}{2}(AB + DC)$

Q. Is $MN = \frac{1}{2}(AB + DC)$?

A. *Yes*

Summary of properties discovered in Lesson #8:

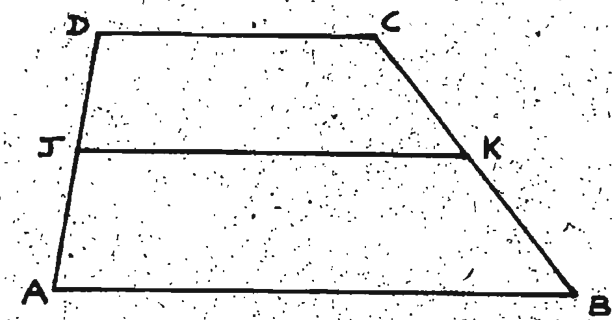
1. A trapezoid is a quadrilateral with exactly one pair of parallel sides.

2. A median of a trapezoid is the segment joining the midpoints of the legs.

3. The median of a trapezoid is parallel to the bases and has a length equal to half the sum of the lengths of the bases.

PRACTICE EXERCISES:

In exercises 1-7, use trapezoid ABCD in which J is the midpoint of leg $\overline{AD}$ and K is the midpoint of $\overline{BC}$.




1. Complete the statement: $\overline{JK}$ = __?__ $\overline{DC}$. (11)

2. If DC = 5, and AB = 9, then JK = __?__. (7)

3. If DC = 8, and JK = 11, then AB = __?__. (14)

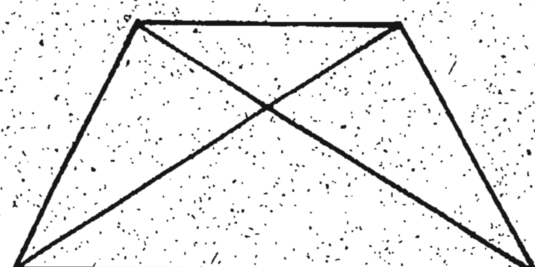
4. If AB = 10, and JK = 8½, then DC = __?__. (7)

5. If DC = n, and AB = n+6, then JK = __?__. (n+3)

6. If DC = 3x, AB = 5x+8 and JK = 6x, then x = __?__. (2)

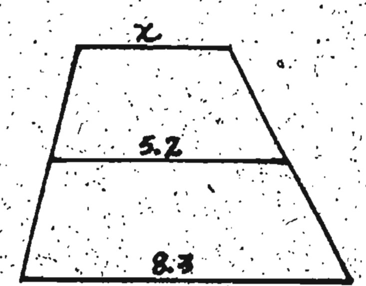
7. Suppose P and Q, not shown in the figure, are the midpoints of $\overline{AJ}$ and $\overline{BK}$. If DC = 9, and AB = 13, then PQ = __?__. (12)

8. Draw a trapezoid which has congruent diagonals.



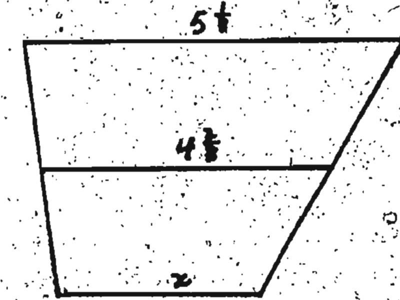
In exercises 9 and 10, the length of a base and the length of a median of a trapezoid are given. Find the value of x, the length of the other base. Show your workings.

9.




Answer: $x = 2.1$

10.




Answer: $x = 4$

## SUMMARY OF TERMS AND GENERALIZATIONS

A. Definitions:

1. A quadrilateral is a closed four-sided polygon.
2. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
3. A rectangle is a parallelogram with four right angles.
4. A rhombus is a parallelogram with all sides congruent.
5. A square is a rectangle with all sides congruent. A square could also be defined as a rhombus with four right angles.
6. A trapezoid is a quadrilateral with exactly one pair of parallel sides.
7. The median of a trapezoid is the segment joining the midpoints of the legs of the trapezoid.

B. The properties of quadrilaterals are summarized in the chart below. A (✓) indicates a figure has the property. The absence of a (✓) indicates a figure does not have the property.

| Property | Parallelogram which is neither of the other three figures | Non-square rectangle | Non-square rhombus | Square |
|---|---|---|---|---|
| 1. Diagonal forms two congruent triangles | ✓ | ✓ | ✓ | ✓ |
| 2. Opposite sides parallel | ✓ | ✓ | ✓ | ✓ |
| 3. Opposite sides congruent | ✓ | ✓ | ✓ | ✓ |
| 4. Opposite angles congruent | ✓ | ✓ | ✓ | ✓ |
| 5. Diagonals bisect each other | ✓ | ✓ | ✓ | ✓ |
| 6. All angles are right angles | | ✓ | | ✓ |
| 7. Diagonals are congruent | | ✓ | | ✓ |
| 8. Diagonals are perpendicular | | | ✓ | ✓ |
| 9. Diagonals bisect opposite angles | | | ✓ | ✓ |
| 10. All sides are congruent | | | ✓ | ✓ |

C. There are four ways to prove a quadrilateral is a parallelogram:

1. Show both pairs of opposite sides are parallel.

2. Show both pairs of opposite sides are congruent.

3. Show one pair of opposite sides are parallel and congruent.

4. Show the diagonals bisect each other.

D. If parallel lines cut off congruent segments on one transversal, they cut off congruent segments on every transversal.

E. The segment joining the midpoint of two sides of a triangle is parallel to, and half as long as, the third side.

F. The median of a trapezoid is parallel to the bases and its length is half the sum of the lengths of the bases.

REVIEW EXERCISES:

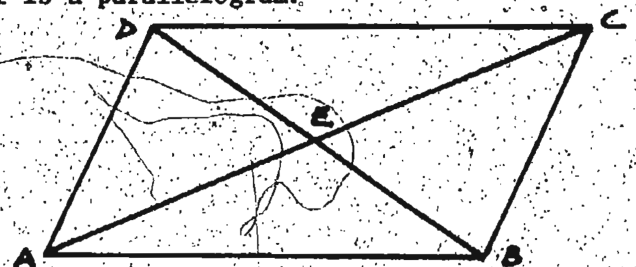
For exercises 1-3, fill in the blanks.

1. A diagonal of a parallelogram separates the parallelogram into two ___?___ triangles. *(congruent)*

2. Opposite ___?___ and opposite ___?___ of a parallelogram are congruent. *(sides; angles)*

3. The diagonals of a parallelogram ___?___ each other. *(bisect)*

Exercises 4-7 refer to quadrilateral ABCD with diagonals $\overline{AC}$ and $\overline{BD}$. Using the information given in each exercise, write the reason why the quadrilateral is a parallelogram.



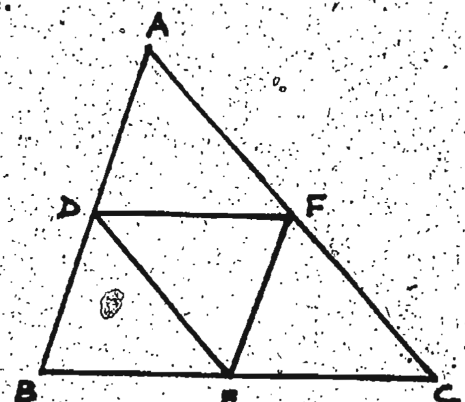

4. $\overline{AB} \cong \overline{DC}$; $\overline{AB} \parallel \overline{DC}$ *(one pair both congruent and parallel)*

5. $\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$ *(both pairs parallel)*

6. AE = EC; BE = ED *(diagonals bisect each other)*

7. $\overline{AB} \cong \overline{DC}$; $\overline{AD} \cong \overline{BC}$ *(both pairs congruent)*

Exercises 8-13 refer to $\Delta ABC$. D,E and F are the midpoints of the sides. Classify each statement as true or false.




8. $\overline{DF} \parallel \overline{BC}$. *(True)*

9. If BC = 12, DF = 6. *(True)*

10. If EF = 5, AD = 5. *(True)*

11. If AC = 14, EF = 7. *(False)*

12. Quadrilateral BDFE is a parallelogram. *(True)*

13. $\Delta DFE \cong \Delta FDA$. *(True)*

In exercises 14-19, correctly complete the following statements by using the most descriptive of the words RECTANGLE, RHOMBUS, or SQUARE.

14. If one angle of a rhombus has a measure of 90, the rhombus is a __?__. *(square)*

15. If a parallelogram is equilateral, the parallelogram is a __?__. *(rhombus)*

16. If one angle of a parallelogram is a right angle, the parallelogram must be a __?__, but may be a __?__. *(rectangle; square)*

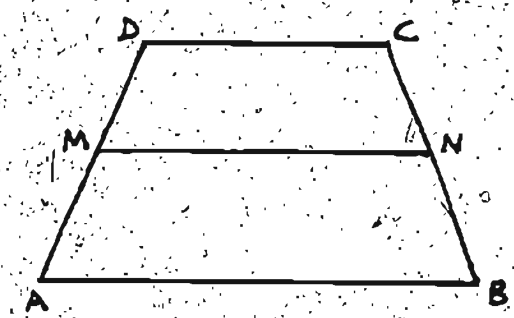
17. An equilateral and equiangular parallelogram is a __?__. *(square)*

18. If the diagonals of a non-square parallelogram are congruent but are not perpendicular, the parallelogram is a __?__. *(rectangle)*

19. If the diagonals of a parallelogram bisect the opposite angles but are not congruent, the parallelogram is a __?__. *(non-square rhombus)*

Exercises 20-23 refer to trapezoid ABCD. Points M and N are the midpoints of $\overline{AD}$ and $\overline{BC}$, respectively.

20. $\overline{MN}$ is called the __?__ of the trapezoid. *(median)*




21. When $\overline{AD} \cong \overline{BC}$, the trapezoid is said to be __?__. *(isosceles)*

22. Find MN when AB = 10, and DC = 6. *(MN = 8)*

23. Find AB when DC = 12, and MN = 17. *(AB = 22)*

Exercises 24-37 refer to the quadrilaterals shown on the following page. For each exercise name all of the figures that satisfy the indicated condition.

24. $\overline{AD} \cong \overline{BC}$. *(all except trapezoid)*

25. $\overline{AD} \parallel \overline{BC}$. *(all except trapezoid)*

26. $\angle A$ is a right angle. *(non-square rectangle, square)*

27. $m\angle A + m\angle B + m\angle C + m\angle D = 360$. *(all)*

28. $m\angle B = m\angle D$. *(all except trapezoid)*

29. $\overline{BC} = \overline{CD}$. *(non-square rhombus, square)*

30. $\overline{AC}$ bisects $\overline{BD}$. *(all except trapezoid)*

31. All sides are congruent. *(square, rhombus)*

32. Exactly one pair of sides are parallel. *(trapezoid)*

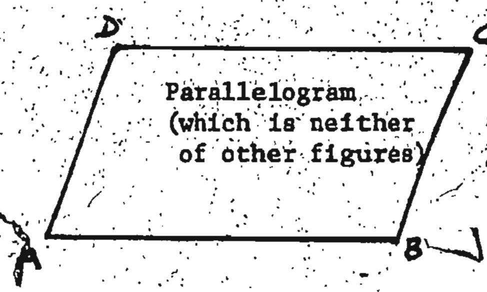
33. All angles are congruent. *(rectangle, square)*

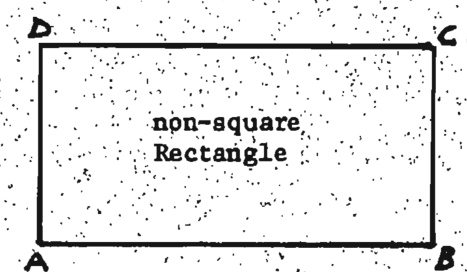
34. $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{CD}$. *(rectangle, square)*

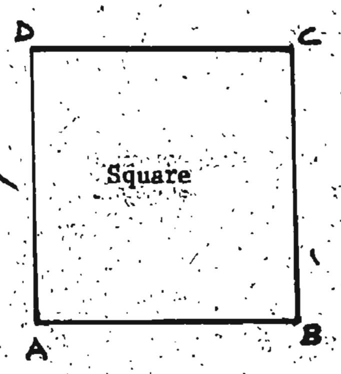
35. $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{CD}$. *(rhombus, square)*

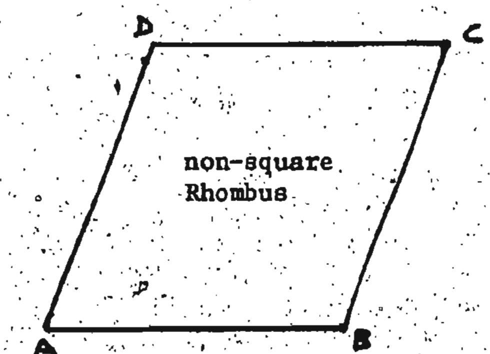
36. When $\overline{AC}$ is drawn, two congruent triangles are formed. *(all except trapezoid)*

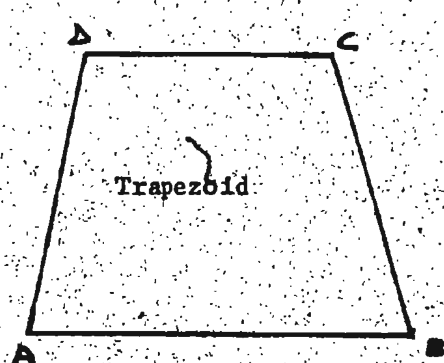
37. A regular polygon. *(square)*



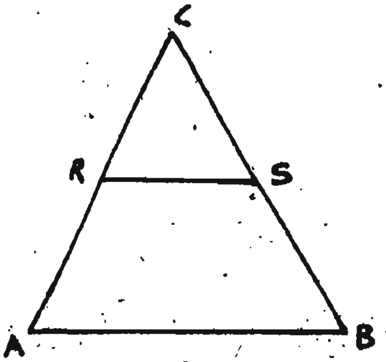



In exercises 38-41, find the value of x. Show your workings.

38.



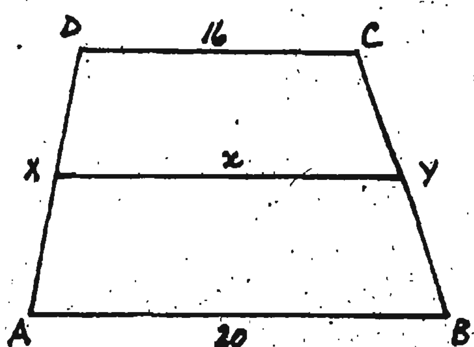

R is the midpoint of $\overline{AC}$.

S is the midpoint of $\overline{BC}$.

AB = 12

Answer: $x = 6$

39.



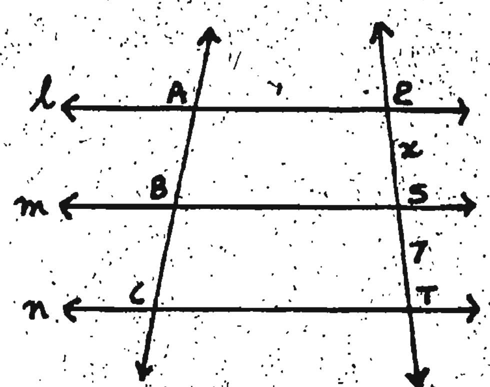

Quadrilateral ABCD is a trapezoid.

$\overline{XY}$ is the median.

AB = 20; CD = 16

Answer: $x = 18$

40.



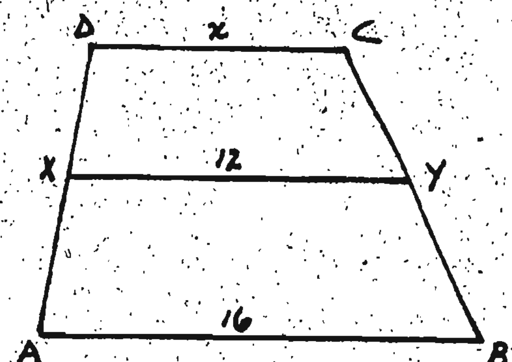

$l \parallel M \parallel n$

$\overline{AB} \cong \overline{BC}$

ST = 7

Answer: $x = 7$

41.




Quadrilateral ABCD is a trapezoid.

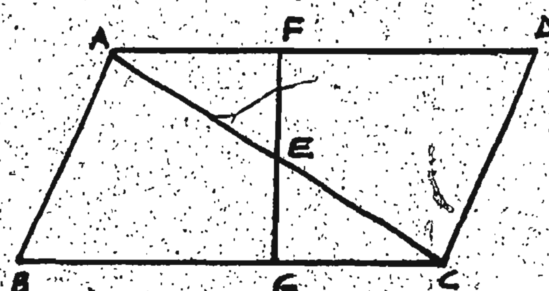
$\overline{XY}$ is the median

XY = 12; AB = 16

Answer: $x = 8$

42. GIVEN: ABCD is a parallelogram
E is the midpoint of $\overline{FG}$

Prove: E is the midpoint of $\overline{AC}$



PROOF:

| | Statement | | Reason |
|---|---|---|---|
| 1. | ABCD is a parallelogram | 1. | Given |
| 2. | $\overline{AD} \parallel \overline{BC}$ | 2. | Definition of a parallelogram |
| 3. | $\angle AFE \cong \angle CGE$ | 3. | If two parallel lines are cut by a transversal, the alternate interior angles are congruent |
| 4. | $\angle FAE \cong \angle GCE$ | 4. | If two parallel lines are cut by a transversal, the alternate interior angles are congruent |
| 5. | E is the midpoint of $\overline{AC}$ | 5. | Given |
| 6. | $FE = GE$ | 6. | Definition of midpoint |
| 7. | $\overline{FE} \cong \overline{GE}$ | 7. | Definition of congruent segments |
| 8. | $\Delta AFE \cong \Delta CGE$ | 8. | AAS theorem |
| 9. | $\overline{AE} \cong \overline{CE}$ | 9. | Corresponding parts of congruent triangles are congruent |
| 10. | $AE = CE$ | 10. | Definition of congruent segments |
| 11. | E is the midpoint of $\overline{AC}$ | 11. | Definition of midpoint |

APPENDIX B

PRETEST, POSTTEST, AND RETENTION TEST

PRETEST

Chapter Test: Quadrilaterals    Geometry 10    November, 1979

You are required to answer all questions. Show all necessary calculations. The value of each question is shown in the margin to the left. TIME: 40 minutes.

Value: 1. For each of the statements below, indicate whether it is true (T) or false (F) by writing T or F in the blank provided for each statement.

(18)

(a) _____ Every rhombus is a square. *(F)*

(b) _____ Every square is a rectangle. *(T)*

(c) _____ Every square is a parallelogram. *(T)*

(d) _____ Every rectangle is a square. *(F)*

(e) _____ Every square is a rhombus. *(T)*

(f) _____ The diagonals of a rhombus bisect each other. *(T)*

(g) _____ The diagonals of a rectangle are perpendicular to each other. *(F)*

(h) _____ The diagonals of a square are perpendicular and bisect each other. *(T)*

(i) _____ If the diagonals of a quadrilateral are perpendicular, the quadrilateral is a rhombus. *(F)*

(14) 2. Would the following conditions for a quadrilateral be sufficient to prove that it is a parallelogram (P)? a rectangle (R)? a rhombus (RH)? a square (S)? a trapezoid (T)? Consider each item separately. Write P, R, RH, S, or T in the blank provided for each statement. For each question there may be more than one answer.

(a) _____ It has two pairs of parallel sides. *(P)*

(b) _____ Three of its angles are right angles. *(R, P)*

(c) _____ It has four sides congruent. *(P, RH)*

(d) _____ Its diagonals are congruent and perpendicular. *(none)*

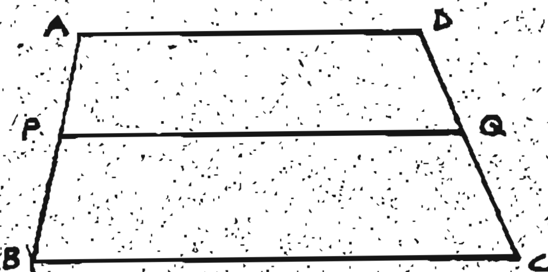
(e) ____ Two sides are parallel. *(none)*

(f) ____ Its diagonals bisect each other. *(P)*

(g) ____ Its diagonals are congruent, are perpendicular, and bisect each other. *(P, R, S)*

3. Quadrilateral ABCD is a trapezoid. $\overline{PQ}$ is a median. If PQ = 9 and BC = 12, find AD. Show your workings. *(AD = 6)*

(6)

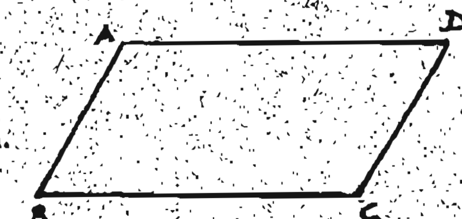



4. Given: $\overline{AB} \parallel \overline{CD}$

$\overline{AB} \cong \overline{CD}$

(14)

Prove: ABCD is a parallelogram. (Do this proof using statement-reason form)

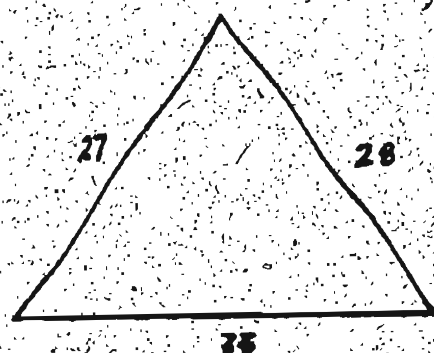



PROOF:

| *Statement* | *Reason* |
| --- | --- |
| 1. *Draw* $\overline{AC}$ | 1. *Through any two different points there is exactly one line* |
| 2. $\overline{AB} \cong \overline{CD}$; $\overline{AB} \parallel \overline{CD}$ | 2. *Given* |
| 3. $\angle BAC \cong \angle DCA$ | 3. *If two parallel lines are cut by a transversal, the alternate interior angles are congruent* |
| 4. $\overline{AC} \cong \overline{AC}$ | 4. *Congruence of segments is reflexive* |
| 5. $\triangle BAC \cong \triangle DCA$ | 5. *SAS postulate* |
| 6. $\angle BCA \cong \angle DAC$ | 6. *Corresponding parts of congruent triangles are congruent* |

| | |
|---|---|
| 7. $\overline{AD} \parallel \overline{BC}$ | 7. *If two lines are cut by a transversal so that alternate interior angles are congruent, the lines are parallel* |
| 8. *ABCD is a parallelogram* | 8. *Definition of a parallelogram* |

(10) 5. Three sides of a triangle measure 28, 33 and 27. The midpoints of these sides are found and the segments determined by them form a new triangle. What is the perimeter of the new triangle formed? Show all workings. *(44)*



POSTTEST

Chapter Test: Quadrilaterals — Geometry 10 — November, 1979

You are required to answer all questions. Show all necessary calculations. The value of each question is shown in the margin to the left. TIME: 40 minutes

Value:

(18) 1. For each of the statements below, indicate whether it is true (T) or false (F) by writing T or F in the blank provided for each statement.

(a) _____ Every rhombus is a square. *(F)*

(b) _____ Every square is a rectangle. *(T)*

(c) _____ Every square is a parallelogram. *(T)*

(d) _____ Every rectangle is a square. *(F)*

(e) _____ Every square is a rhombus. *(T)*

(f) _____ The diagonals of a rhombus bisect each other. *(T)*

(g) _____ The diagonals of a rectangle are perpendicular to each other. *(F)*

(h) _____ The diagonals of a square are perpendicular and bisect each other. *(T)*

(i) _____ If the diagonals of a quadrilateral are perpendicular, the quadrilateral is a rhombus. *(F)*

(14) 2. Would the following conditions for a quadrilateral be <u>sufficient</u> to <u>prove</u> that it is a parallelogram (P)? a rectangle (R)? a rhombus (RH)? a square (S)? a trapezoid (T)? Consider each item separately. Write P, R, RH, S, or T in the blank provided for each statement. For each question there may be more than one answer.

(a) _____ It has two pairs of parallel sides. *(P)*

(b) _____ Three of its angles are right angles. *(R, P)*

(c) _____ It has four sides congruent. *(P, RH)*

(d) _____ Its diagonals are congruent and perpendicular. *(none)*

(e) ______ Two sides are parallel. *(none)*

(f) ______ Its diagonals bisect each other. *(P)*

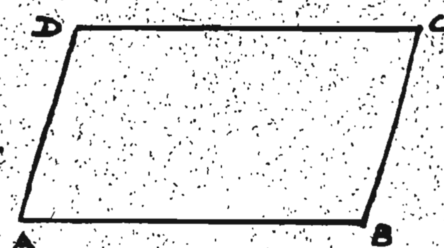
(g) ______ Its diagonals are congruent, are perpendicular, and bisect each other. *(P, R, S)*

(6) 3. Quadrilateral MNOP is a trapezoid. $\overline{XY}$ is a median. If XY = 7 and NO = 12, find MP. Show your workings. *(MP = 2)*


(14) 4. Given: $\overline{AB} \cong \overline{CD}$

$\overline{BC} \cong \overline{DA}$

Prove: ABCD is a parallelogram. (Do this proof using statement-reason form)

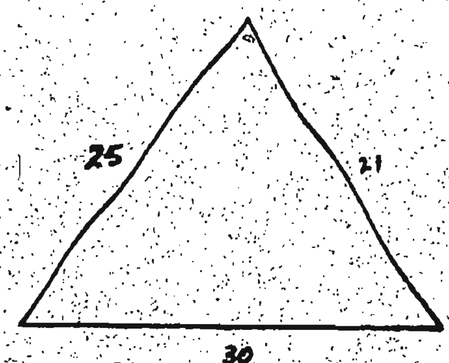



PROOF:

| *Statement* | *Reason* |
|---|---|
| 1. $\overline{AB} \cong \overline{CD}$; $\overline{BC} \cong \overline{DA}$ | 1. *Given* |
| 2. *Draw* $\overline{AC}$ | 2. *Through any two different points, there is exactly one line* |
| 3. $\overline{AC} \cong \overline{AC}$ | 3. *Congruence of segments is reflexive* |
| 4. $\triangle ABC \cong \triangle CDA$ | 4. *SSS postulate* |
| 5. $\angle CAB \cong \angle ACD$ | 5. *Corresponding parts of congruent triangles are congruent* |
| 6. $\overline{AB} \parallel \overline{CD}$ | 6. *If two lines are cut by a transversal so that alternate interior angles are congruent, the lines are parallel.* |

| | | | |
|---|---|---|---|
| 7. | *ABCD is a parallelogram* | 7. | *If a quadrilateral has one pair of opposite sides that are both parallel and congruent, the quadrilateral is a parallelogram* |

(10) 5. Three sides of a triangle measure 30, 25 and 21. The midpoints of these sides are found and the segments determined by them form a new triangle. What is the perimeter of the new triangle formed? Show all workings. *(38)*



## RETENTION TEST

Chapter Test: Quadrilaterals    Geometry 10    November, 1979

You are required to answer all questions. Show all necessary calculations. The value of each question is shown in the margin to the left. TIME: 40 minutes.

Value: 1. For each of the statements below, indicate whether it is true (T) or false (F) by writing T or F in the blank provided for each statement.

(18)

(a) _____ Every rhombus is a square. *(F)*

(b) _____ Every square is a rectangle. *(T)*

(c) _____ Every square is a parallelogram. *(T)*

(d) _____ Every rectangle is a square. *(F)*

(e) _____ Every square is a rhombus. *(T)*

(f) _____ The diagonals of a rhombus bisect each other. *(T)*

(g) _____ The diagonals of a rectangle are perpendicular to each other. *(F)*

(h) _____ The diagonals of a square are perpendicular and bisect each other. *(T)*

(i) _____ If the diagonals of a quadrilateral are perpendicular, the quadrilateral is a rhombus. *(F)*

2. Would the following conditions for a quadrilateral be <u>sufficient</u> to <u>prove</u> that it is a parallelogram (P)? a rectangle (R)? a rhombus (RH)? a square (S)? a trapezoid (T)? Consider each item separately. Write P, R, RH, S, or T in the blank provided for each statement. For each question there may be more than one answer.

(14)

(a) _____ It has two pairs of parallel sides. *(P)*

(b) _____ Three of its angles are right angles. *(R, P)*

(c) _____ It has four sides congruent. *(P, RH)*

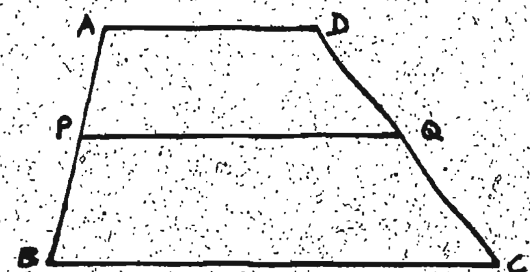
(d) _____ Its diagonals are congruent and perpendicular. *(none)*

(e) _____ Two sides are parallel. *(none)*

(f) _____ Its diagonals bisect each other. *(P)*

(g) _____ Its diagonals are congruent, are perpendicular, and bisect each other. *(P, R, S)*

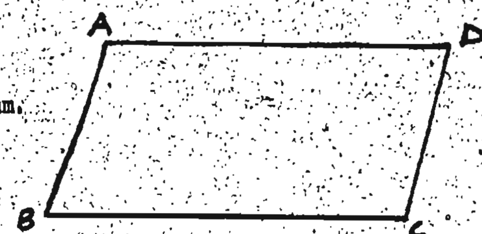
(6) 3. Quadrilateral ABCD is a trapezoid. $\overline{PQ}$ is a median. If PQ = 9 and BC = 12, find AD. Show your workings. *(AD = 6)*




(14) 4. Given: $\overline{AB} \parallel \overline{CD}$

$\overline{AB} \cong \overline{CD}$

Prove: ABCD is a parallelogram. (Do this proof using statement-reason form)



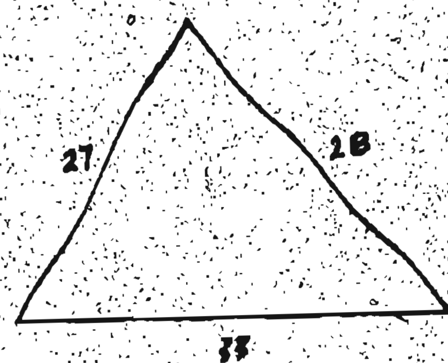

PROOF:

| *Statement* | *Reason* |
|---|---|
| 1. *Draw* $\overline{AC}$ | 1. *Through any two different points there is exactly one line* |
| 2. $\overline{AB} \cong \overline{CD}$; $\overline{AB} \parallel \overline{CD}$ | 2. *Given* |
| 3. $\angle BAC \cong \angle DCA$ | 3. *If two parallel lines are cut by a transversal, the alternate interior angles are congruent* |
| 4. $\overline{AC} \cong \overline{AC}$ | 4. *Congruence of segments is reflexive* |
| 5. $\triangle BAC \cong \triangle DCA$ | 5. *SAS postulate* |
| 6. $\angle BCA \cong \angle DAC$ | 6. *Corresponding parts of congruent triangles are congruent* |

7. $\overline{AD} \parallel \overline{BC}$ — 7. *If two lines are cut by a transversal so that alternate interior angles are congruent, the lines are parallel*

8. *ABCD is a parallelogram* — 8. *Definition of a parallelogram*

(10) 5. Three sides of a triangle measure 28, 33 and 27. The midpoints of these sides are found and the segments determined by them form a new triangle. What is the perimeter of the new triangle formed? Show all workings. *(44)*



# APPENDIX C

## TEST SCORES

EXPERIMENTAL GROUP

Raw Scores

| Pretest | Posttest | Retention Test |
|---|---|---|
| 16 | 55 | 47 |
| 19 | 48 | 40 |
| 15 | 54 | 51 |
| 11 | 61 | 43 |
| 16 | 53 | 46 |
| 18 | 58 | 51 |
| 18 | 57 | 48 |
| 12 | 32 | 20 |
| 38 | 76 | 70 |
| 13 | 38 | 17 |
| 14 | 56 | 41 |
| 15 | 68 | 51 |
| 23 | 82 | 69 |
| 23 | 65 | 49 |
| 11 | 50 | 40 |
| 10 | 63 | 50 |
| 12 | 42 | 28 |
| 32 | 61 | 40 |
| 8 | 39 | 26 |
| 13 | 38 | 21 |
| 11 | 65 | 55 |
| 14 | 68 | 52 |
| 11 | 37 | 28 |
| 28 | 60 | 41 |
| 23 | 58 | 50 |
| 14 | 51 | 39 |
| 18 | 63 | 47 |
| 17 | 60 | 52 |
| 24 | 73 | 57 |
| 20 | 69 | 52 |

CONTROL GROUP

Raw Scores

| Pretest | Posttest | Retention Test |
|---|---|---|
| 15 | 53 | 40 |
| 22 | 80 | 53 |
| 17 | 51 | 30 |
| 22 | 48 | 30 |
| 21 | 80 | 64 |
| 13 | 63 | 51 |
| 27 | 44 | 27 |
| 25 | 60 | 42 |
| 19 | 38 | 18 |
| 23 | 53 | 33 |
| 19 | 56 | 40 |
| 27 | 61 | 50 |
| 18 | 46 | 30 |
| 9 | 63 | 37 |
| 17 | 74 | 53 |
| 14 | 41 | 22 |
| 23 | 66 | 58 |
| 25 | 50 | 40 |
| 14 | 35 | 18 |
| 15 | 30 | 13 |
| 24 | 56 | 41 |
| 20 | 86 | 69 |
| 11 | 50 | 33 |
| 24 | 68 | 43 |
| 23 | 53 | 30 |
| 17 | 56 | 29 |
| 24 | 55 | 47 |
| 20 | 53 | 38 |
| 14 | 59 | 32 |
| 24 | 57 | 43 |