A REPORT ON THE DEVELOPMENT OF A TEACHER HANDBOOK FOR THE UTILIZATION OF VARIOUS INSTRUCTIONAL MATERIALS IN THE TEACHING OF SECONDARY MATHEMATICS

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A REPORT ON

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by

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ABSTRACT

This report contains a resources handbook written for teachers of secondary mathematics. The project started with a suggestion to the writer by a university professor that a resources handbook for secondary mathematics was needed. The writer, consequently, made a search of the library and all available reference aids to determine what resources handbooks were available to teachers of secondary mathematics. The search indicated that no suitable resources handbook was available. The writer, therefore, decided to write one.

The contents of the handbook has been divided into eleven chapters. The chapters discuss films, filmstrips, the overhead projector, games, puzzles and recreational problems, projects and models. One chapter is devoted to miscellaneous items. There is an annotated bibliography of recommended books and other aids and a directory of publishers.

The handbook aims to provide general background information and specific information for each topic discussed. The handbook was evaluated informally while in the process of preparation. This was followed by a formal evaluation using a questionnaire submitted to experienced secondary mathematics teachers. A few additions were made on the basis of suggestions from the formal

evaluation.

The writer recommends that other handbooks be written for secondary mathematics. Such handbooks should, however, be restricted to one or two topics. These topics could then be considered in considerable detail.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

BACKGROUND TO THE PROJECT

Developments in Mathematics Curriculum

For two decades the mathematics curriculum has been the subject of considerable study throughout Canada and The United States. Large sums of money have been spent on the research and the development of new mathematics programs. The new programs were designed to make the study of mathematics a more meaningful and interesting experience for students. The new programs, however, have not lessened the need for teachers to incorporate a variety of motivational techniques into their teaching.

Difficulty in mastering any subject in school is related to the interest that a student has in that subject. Butler, Wren and Banks (1970) have this to say:

> It may be taken as axiomatic that students will work most diligently and most effectively at tasks in which they are genuinely interested. To create and maintain interest becomes, therefore, one of the most important tasks of the teacher of secondary school mathematics. (p. 137)

They add that this task is also one of the most difficult problems the teacher has to deal with.

In maintaining interest, it is well to remember that students tend to remain interested in those things that they understand completely and in those things that bring them most success. It must be remembered also, that success should not be achieved by diluting the subject matter, nor should concern for complete understanding overlook the intellectual stretching necessary to mathematical growth. The work, therefore, should also aim to present a continual challenge to the student.

Teachers can seek to motivate and maintain greater student interest and growth by making use of a variety of multisensory aids. Multisensory aids include many different things. They include the use of films, filmstrips, pictures, transparencies, charts and models. They include also, such things as reference books, projects, displays and computing instruments.

Student interest can sometimes be generated by various other activities. Such activities are not necessarily confined to the regular classroom setting. Some students may be interested in preparing research papers on the lives of famous mathematicians or on some historical topic in mathematics. Mathematics clubs, participation in field trips or in mathematical contests are other activities that may capture the interest of other students.

Sobel (1975) points out that "research seems to indicate that when mental ability is held constant, differences in achievement are the result of differences in motivation". (p. 479). Teachers of mathematics, therefore, should consider using a variety of aids and activities to generate student interest and motivation.

Learning Resources in Mathematics

Teaching aids in mathematics are not new. Good mathematics teachers have always made use of the support materials and resources in the school and in the community. Within the past twenty-five years, however, there has been a rapid and substantial expansion in the development and use of various instructional aids in the teaching of mathematics. Berger (1973) points out that this change was ushered in by "a 1945 report to the Board of Directors of The National Council of Teachers of Mathematics by the Committee on Multi-sensory Aids" (p. v). Subsequently, this report was published as the eighteenth yearbook under the title Multi-Sensory Aids in the Teaching of Mathematics.

Hildebrandt in the introduction to the eighteenth yearbook states:

> It is hoped that the report will be followed in a few years by one showing improvements and changes which have kept pace with the progress of such aids in the world around us. (p. viii)

This hope was realized in 1973. In that year The National Council of Teachers of Mathematics published its thirty-fourth yearbook entitled Instructional Aids in Mathematics. The book contains information concerning the growing spectrum of available instructional aids, suggestions for selecting and evaluating materials, and some guidance in the use of these aids. The book attempts to provide information to teachers at all grade levels of mathematics.

Books dealing with the teaching of secondary mathematics make frequent references to the expanding use of multisensory aids and activities by mathematics teachers. Butler, Wren and Banks (1970) cite three reasons for the increased use of multisensory aids by secondary mathematics teachers. They state:

> One reason is that more teachers have come to see the usefulness of films, filmstrips, and transparencies than ever before, and have learned how to use projection equipment successfully. Another reason is that the projection equipment and facilities for its use have greatly improved. A third is that the number and accessibility of films, filmstrips, and transparencies have increased enormously, and their quality has also improved. (p. 148)

Johnson (1971), a leading mathematics educator in the area of instructional materials in the mathematics classroom, states:

> Instructional materials are as essential for the mathematics teacher as spices are for the chef. They are the necessary extra ingredients that make teaching and learning mathematics a pleasant, satisfying experience. Models, pamphlets, films, and diagrams give to a mathematics lesson breadth and depth that would be difficult to obtain in any other way. (p. 349)

Johnson and Rising (1967), in their book entitled <u>Guidelines for Teaching Mathematics</u>, devote six chapters to a discussion of the use of multisensory aids and activities in the teaching of mathematics.

The two professional journals published by the National Council of Teachers of Mathematics, The Mathematics Teacher and The Arithmetic Teacher, carry frequent articles relating to the use of multisensory aids and activities in mathematics teaching.

Research studies have demonstrated the effectiveness of instructional media when the medium is carefully selected and used. Moldstad (1974) writes that:

> Twenty years of decision-oriented media research have produced significant evidence to justify the following claims when instructional technology is carefully selected and used.
>
> 1. Significantly greater learning often results when media are integrated into the traditional instructional program.
>
> 2. Equal amounts of learning are often accomplished in significantly less time using instructional technology.
>
> 3. Multimedia instructional programs based upon a "systems approach" frequently facilitate student learning more effectively than traditional instruction.
>
> 4. Multimedia and/or audio tutorial instructional programs are usually preferred by students when compared with traditional instruction. (p. 390)

Moldstad (1974) refers to a study by Chance (1960), who studied the effect that the use of 200 specially prepared transparencies would have on student learning.

When the instructional approach (transparencies plus current practice) was compared with the traditional lecture - discussion approach on identical content, the researchers arrived at four conclusions as follows:

> 1. The group having the added use of the transparencies did significantly better on mean final course examination scores and final course grades (at .05 level of confidence).
> 2. The three faculty members unanimously agreed on the desirability of using these transparencies in their teaching.
> 3. Use of the transparencies resulted in an average savings of 15 minutes per class period.
> 4. Students reported overwhelming preference for instruction using transparencies. (p. 392)

Other more recent studies support the use of various educational media in teaching mathematics. Sherrill (1971) found that achievement with word problems in mathematics was improved by the presence of a pictorial representation of the problem situation. Gray (1973) found that a laboratory program contributed to the improvement in attitude and achievement of mathematically deficient students. Purser (1973) concluded that the use of manipulative activities was feasible. Blazek (1971) found that students used library materials to a greater extent when the teacher referred to them in teaching. The literature and experimental studies, then, support the idea that various media and discovery activities can be effective in mathematics teaching.

CHAPTER II

NEEDS ASSESSMENT

Statement of Needs

The need for a resources handbook for teachers of secondary mathematics was brought to the writer's attention by a professor at Memorial University of Newfoundland. The professor suggested to the writer that a handbook which called attention to the use of films, filmstrips, the overhead projector, puzzle problems, games and other activities in the teaching of secondary mathematics might be useful to mathematics teachers.

Subsequently, the writer and a fellow student cooperated in compiling a questionnaire to gather some information regarding resource materials that secondary mathematics teachers had in their schools and the use that teachers made of these resource materials. The questionnaire (Appendix B) was mailed to 60 secondary mathematics teachers throughout the province of Newfoundland and Labrador. Forty-five questionnaires were returned. To the question, "Do you have access to any professional material pertaining to the teaching of your specific subject area?", all respondents answered "yes". However, none listed the titles of any specific resource handbooks except <u>The Mathematics Teacher</u> and <u>The Arithmetic Teacher</u>.

To the question "Do you think that there is a need for more emphasis on resources based teaching in this subject?", 39 answered "yes" and six answered "no". A review of the additional comments for the question indicated that respondents would make more use of various resource materials if suitable items were readily available to them.

The replies to question 8 gave some indication of how often various materials were being used. The responses for a number of the materials are shown in Table 1. Responses for the materials omitted from the table showed that these items were not used by any of the respondents.

Table 1

Use of Resource Material

n = 45 Materials	Use A	B	C	D
Motion picture, 16 mm	0	0	5	40
Filmstrips	0	2	5	38
Tapes	0	2	5	38
Transparencies	0	4	6	35
Books	45	0	0	0
Periodicals	0	0	8	37
Newspapers	0	0	7	38
Models	0	0	12	33
Posters	0	0	2	43
Pictures	0	0	9	36
Charts	0	0	16	29
Simulation games	0	0	3	42
Programmed instruction	0	0	10	35

Note. A indicates "weekly"; B indicates "once a month"; C indicates "2 - 3 times a year"; D indicates "nil".

The responses, as shown in the table, indicated that respondents made use of most of these materials two or three times per year or never used them at all. A few respondents said that they used filmstrips, tapes and transparencies once a month. All 45 of the respondents said that they made weekly use of books, most likely textbooks, not supplementary books. The responses for tapes would seem to suggest that they were used with filmstrips, since the responses were, in all cases, the same.

To gather more information regarding the need of a handbook for secondary mathematics, the writer interviewed twenty secondary mathematics teachers. None of the teachers interviewed had been sent the questionnaire. The interviews confirmed the fact that secondary mathematics teachers had in their schools few resources beyond the textbook. All of the teachers interviewed expressed the view that a handbook for secondary mathematics would be useful.

Alternative Solutions

Having established the fact that a handbook for secondary mathematics was needed, the writer was faced with two solutions to fill that need. The first, and obvious solution, was to carry out a search to determine what resource handbooks were available to secondary mathematics teachers and then to pass this information along to them. If no suitable handbook could be found, the problem could be solved by preparing a suitable handbook.

Summary of Available Materials

The writer's search for resource handbooks written for secondary mathematics teachers began in The Education Library of Memorial University. The card catalogue was searched and the titles of all books found that appeared to relate to the kind of book sought were recorded. Each book was later examined for content. The writer next searched the curriculum reference aids. In addition the back issues of The Mathematics Teacher for five years were searched.

The search revealed few books with specific daily suggestions and ideas useful to secondary mathematics teachers. A few books were found that did contain some useful information for secondary mathematics. The eighteenth and thirty-fourth yearbooks published by The National Council of Teachers of Mathematics were found to contain much excellent information relating to various aspects of secondary mathematics. Teaching Mathematics: A Sourcebook of Aids, Activities, and Strategies by Sobel and Maletsky (1975) is an excellent book with many useful ideas and suggestions for secondary mathematics teachers. A Mathematics Laboratory Handbook for Secondary Schools by Krulik (1972) contains a small number of useful exercises. These, then, were the only books found which could be considered as resource handbooks with any worthwhile information for secondary mathematics teachers.

The search of the curriculum reference aids and the back issues of The Mathematics Teacher failed to produce any titles of books devoted to teaching suggestions for secondary mathematics.

Rationale for Development of Handbook

All of the books found proved to be unsatisfactory for the kind of handbook the writer believed would be most useful to secondary mathematics teachers. The eighteenth yearbook of NCTM, while it does have one section of very useful information, is devoted primarily to academic articles. The thirty-fourth yearbook of NCTM is a very good general reference textbook with many excellent ideas and much information useful to the secondary mathematics teacher, but it, too, was considered unsatisfactory. Sobel and Maletsky (1975) have written an excellent handbook, but it fails to mention some of the things which the writer believes should be included. Krulik (1972) is too limited in topics covered to meet the need.

On the basis of personal experience in teaching secondary mathematics for fifteen years, and from the information obtained in the twenty interviews with secondary mathematics teachers, the writer was convinced that the best solution to the needs for a handbook would be to write one. The writer has done this. Every effort has been made to produce a handbook to meet the needs of secondary mathematics teachers as the writer has come to see that need.

CHAPTER III

ANALYSIS OF USERS AND OF HANDBOOK PURPOSES

Intended Users

The handbook was prepared for the secondary mathematics curriculum in Newfoundland and Labrador. The intended primary users, therefore, were the secondary mathematics teachers of that province. The content and presentation of materials in the handbook, however, should be appropriate for the secondary mathematics curriculum of other provinces as well.

Conditions of Use

Full use of the ideas and activities in the handbook are open to all secondary mathematics teachers. The ideas and activities presented do not require extensive training in methods courses; nor do they require an abundance of equipment. The section devoted to the use of the overhead projector assumes a basic knowledge of the preparation of transparencies. Some suggestions in the section devoted to projects require adequate library facilities. In preparing the handbook, however, the writer was conscious of the fact that many secondary mathematics teachers in this province would not have access to well stocked resource centres or to adequate budgets. Consequently, the emphasis in the handbook is on practical ideas and suggestions that require no special training, no special facilities, and limited financial resources.

Purposes of Handbook

In general terms the purposes of the handbook are:

1. To provide a number of specific ideas, activities and suggestions for teaching secondary mathematics.

2. To provide some specific information relevant to the use of films, filmstrips and the overhead projector in teaching secondary mathematics.

3. To provide some specific information relevant to the use of a number of activities in teaching secondary mathematics.

4. To provide an annotated bibliography of books and materials recommended for secondary mathematics.

5. To provide a directory of publishers and their Canadian agents for the items in the bibliography.

The particular purposes for the individual chapters of the handbook are included in Chapter IV of this report.

CHAPTER IV

PROCEDURES AND EVALUATION

Preparation of the Handbook

It was decided to organize the contents into eleven chapters. Each chapter was prepared with one or more particular purposes in mind.

Chapter Purposes

The particular purposes for the individual chapters are the following.

For chapter one:

1. To point out the support of research regarding the use of various instructional materials in teaching secondary mathematics.

2. To call attention to the need for the handbook and to state the two major objectives of the handbook.

For chapter two:

1. To point out the usefulness of 16 mm films in teaching secondary mathematics.

2. To point out the limited supply of secondary mathematics films presently available in this province and to suggest what teachers can do to increase this supply.

3. To make a number of specific suggestions regarding the use of mathematics films in the classroom.

For chapter three:

1. To point out the advantage of 35 mm film-strips, call attention to the available supply of filmstrips, and to make specific suggestions regarding their use in the classroom.

For chapter four:

1. To point out a number of specific uses for the overhead projector in teaching secondary mathematics.

For chapter five:

1. To identify a number of roles for mathematical games in secondary mathematics.

2. To enumerate a number of points concerning the use of mathematical games.

3. To describe a number of appropriate games for teaching secondary mathematics.

For chapter six:

1. To enumerate a number of suggestions for using puzzle and recreational problems in teaching secondary mathematics.

2. To provide a number of appropriate puzzle and recreational problems together with solutions.

For chapter seven:

1. To discuss the purposes of mathematical projects, and make some suggestions for developing student interest in projects.

2. To provide a number of specific suggestions for appropriate student projects.

For chapter eight:

1. To point out a number of uses for models.

2. To provide descriptions for a number of appropriate models for teaching secondary mathematics.

For chapter nine:

1. To provide a number of miscellaneous items appropriate for secondary mathematics.

For chapter ten:

1. To provide an annotated bibliography of books, films, filmstrips and materials recommended for secondary mathematics.

For chapter eleven:

1. To provide a directory of publishers and their Canadian agents for the items in the bibliography.

Scope and Limitations of the Handbook

The handbook was designed to focus attention on a number of aids, ideas, activities and materials suitable for use in the teaching of secondary mathematics. In particular, the handbook discusses the use of 16 mm films, 35 mm filmstrips, and the overhead projector. In addition, there are chapters devoted to mathematical games, puzzle and recreational problems, projects, and mathematical models. There is an annotated bibliography of recommended books, films and filmstrips and a directory of publishers with their Canadian agents.

Because the handbook was designed to focus attention on a variety of aids, ideas, and activities, it became necessary to limit the space devoted to each chapter. The writer has, therefore, tried to include aids, ideas, and activities that may have application to more than one grade level. The aids, ideas, and activities are likewise limited to those that are practical to almost any classroom situation. The suggestions are those that require relatively small cost to implement. In some cases the ideas and activities will serve as a model of the kind of things that can be done.

The nature of the handbook placed certain limitations on the evaluation of the individual items. It has not been possible, for example, due to time and expense, to have each item field tested by a random selection of teachers. The evaluation of the handbook has been based, therefore, upon the assessments of a small, but qualified, number of teachers and other resource persons.

Organization of the Handbook

Each of the eleven chapters was divided into a number of sections. It was decided to begin each chapter with a general discussion of information related to the topic of the chapter. Each chapter was then completed with a number of specific suggestions or examples of the kinds of items discussed. The completed handbook comprises Appendix A of this report.

Gathering of the Materials

The materials for the handbook came from the writer's own experience, gained in teaching secondary mathematics for fifteen years, and from various publications found in The Education Library at Memorial University of Newfoundland. All the university holdings of volumes of The Mathematics Teacher published by The National Council of Teachers of Mathematics were searched. Of the many items found, each was carefully considered and some selections were chosen for inclusion in the handbook. The items included are listed in the acknowledgements section. Items selected and included from other publications are likewise listed in the same section of the handbook.

In preparing chapter two, dealing with 16 mm films, and chapter three, dealing with 35 mm filmstrips, the writer made use of information found in the catalogue published by The Department of Education, Division of Instruction. In addition, several personal visits were made to the Division to gain first hand information.

Observations made by the writer regarding what is available in 16 mm films and 35 mm filmstrips were based on a search of reference sources, publishers' catalogues and reviews found in professional publications at The Memorial University Library.

A large part of the section of recommended books and materials, chapter ten, was compiled with the help of references found in the library. Two reference sources were particularly helpful. These were Mathematics Library, Elementary and Junior High School by Hardgrove and Miller (1973) and The AAAS Science Book List, 3rd Edition (1970). Other selections for chapter ten were made from bibliographies found in yearbooks published by The National Council of Teachers of Mathematics. Some other selections were found in bibliographies contained in various other books dealing with the teaching of mathematics at the secondary level. Selections were made also from favourable reviews found in other professional publications. The 16 mm films were selected from AAAS Science Film Catalog (1975).

Chapter eleven, the directory of publishers and their

Canadian agents, was compiled partly from reference sources and partly from information sheets supplied to the writer as part of a graduate course that dealt with the selection of materials for schools.

Writing of the Handbook

After the individual chapter purposes were written down and the materials collected, the actual writing of the handbook began. A general discussion of information relating to the chapter topic was first written. This was followed by the writing and editing of the individual items.

Informal Evaluation and Revisions

The handbook was subjected to two informal evaluations and revisions. The first draft of the handbook was prepared and given to a small number of fellow teachers for their observations and suggestions. The writer then prepared a second draft of the handbook with various revisions based on the observations and suggestions made by fellow teachers. The second draft was submitted to the writer's supervisor for evaluation. The supervisor in turn suggested some changes and revisions, in particular with respect to chapters two and three. The writer then rewrote these two chapters and made a number of other minor revisions to other chapters. The third draft of the handbook was then submitted to the supervisor who approved it for field testing.

Formal Evaluation and Revisions

Preparation of the Questionnaire

In consultation with the supervisor, it was decided to prepare a questionnaire which used a five point scale to evaluate how well the purposes stated for each chapter had been achieved. In addition, it was decided to include a section of general information pertaining to the respondents and a third section asking respondents to give specific comments regarding the strong or weak points of the chapters or sections of chapters.

The writer then drew up the questionnaire (Appendix C) and, after obtaining approval from the supervisor, prepared the questionnaire for mailing.

Submission of the Handbook and Questionnaire

Thirty copies of the handbook were prepared for mailing. One copy of the handbook and two copies of the questionnaire were mailed to twenty-four secondary schools throughout The Province of Newfoundland and Labrador. The handbook and questionnaires were sent to the school principals. A covering letter was included, asking the principals to pass along the handbook to two members of the school staff for completion. A letter to respondents was attached to the questionnaire. Four questionnaires were sent to school board supervisors and two questionnaires were submitted to instructors at Memorial University who were involved in secondary mathematics methods courses. A copy of the questionnaire is included in Appendix C of this report. Copies of the letter to principals and the letter to respondents are included in Appendix D of this report.

Results of the Questionnaire

A total of fifty-four questionnaires were sent out. Forty-eight were sent to schools, four were sent to school board supervisors and two were sent to instructors at Memorial University. A total of twenty questionnaires was returned. Nineteen were returned from schools and one was returned from a school board supervisor.

The experience of teachers responding ranged from two years experience to twenty-five years experience. Eighteen teachers had six or more years experience. All teachers held a grade five or higher teaching certificate. The school board supervisor had a grade six teaching certificate and sixteen years of experience in teaching secondary mathematics. A summary of information on respondents is given in Tables 2 and 3.

Table 2

Teaching Experience of Respondents

Years of Experience	1-5	6-10	11-15	16-20	21-25
Number of Teachers	1	11	5	1	2

Table 3

Teaching Certificate of Respondents

Certificate	11 or below	111	1V	V	V1	V11
Number of Teachers	0	0	0	4	11	5

Section B of the questionnaire asked the respondents to rate the handbook, chapter by chapter, on the purposes stated for that chapter. Respondents were asked to rate each purpose on a five point scale from poor to excellent according to the degree to which they felt the purposes had been achieved for the chapter.

Respondents were asked to indicate their rating by circling the appropriate number. The tabulated results are shown in Tables 4, 5, 6 and 7.

The rating for the tables is as follows:
1 represents poor; 2 represents fair; 3 represents good; 4 represents very good; 5 represents excellent.

Table 4

Respondents' Ratings for Chapter one

n = 20 objective	rating 1	2	3	4	5	mean
1. Research support for instructional materials	2	6	5	5	2	3
2. Need and major objectives of handbook	0	3	2	11	4	3.8

It can be seen that eight of the twenty respondents felt that the references to the support of research was poorly or only fairly achieved. Some of these respondents commented that one reference was not enough. It should be noted, however, that twelve respondents felt that the one reference in the introduction was satisfactory. Fifteen of the twenty respondents rated objective two as very good or excellent. Such a rating would seem to suggest that respondents felt a strong need for the handbook. The mean for objective one was 3 while that for objective 2 was 3.8.

Table 5

Respondents' Ratings for Chapters two, three and four

n = 20 objective	rating 1	2	3	4	5	mean
Chapter 2: Films						
1. Usefulness in secondary mathematics	0	1	3	10	6	4.0
2. Supply and what teachers can do	0	1	6	9	4	3.8
3. Specific suggestions	0	1	3	11	5	4.0
Chapter 3: Filmstrips						
1. Advantages, supply and suggestions for use	0	1	3	11	5	4.0
Chapter 4: The Overhead Projector						
1. Specific uses in the classroom	0	1	0	5	14	4.6

It can be seen that a high percentage of respondents rated the objectives for these chapters as good, very good or excellent. The overall mean rating was 4. A number of respondents expressed a desire for specific information regarding titles of films and filmstrips available. Chapter four was rated quite highly. From the comments it was interesting to note that many teachers said that they were unable to make much use of the overhead projector. In most cases, one or two machines had to be shared among many more teachers. The trouble of making sure a machine was available when needed was considered as just too much an inconvenience.

Table 6

Respondents' Ratings for Chapters
five, six, seven, eight and nine

n = 20 objective	rating 1	 2	 3	 4	 5	mean
Chapter 5: Games						
1. Roles in secondary mathematics	0	1	7	8	4	3.8
2. Points concerning	0	0	10	5	5	3.8
3. Appropriate games	0	3	7	5	5	3.6
Chapter 6: Problems						
1. Suggested uses	0	1	6	7	6	3.9
2. Appropriate problems	0	2	1	10	7	4.0
Chapter 7: Projects						
1. Purposes of and student interest	0	1	10	5	4	3.6
2. Appropriate projects	0	0	11	5	4	3.6
Chapter 8: Models						
1. Suggested uses	0	0	4	10	6	4.0
2. Appropriate models	0	1	2	11	6	4.0
Chapter 9: Miscellaneous						
1. Appropriate items	0	1	2	9	8	4.2

These chapters were well received by respondents. This can be seen from the fact that the mean ratings for chapters five, six and seven are close to 4 while the means for chapters eight and nine are 4 and 4.2. A small number of respondents said that they did not agree with the use of games at the secondary level. The lower rating given by these respondents may have been influenced by this feeling. Only a few respondents gave a rating of fair. Some respondents made the comment that time to cover the prescribed course was a problem, so there would be very limited time available for such activities as games, problems, projects or models. Some respondents pointed out that adequate library facilities were a problem in doing written projects. Such comments concur with those the writer has pointed out in the handbook. The section of miscellaneous items received favourable comments from respondents.

Table 7

Respondents' Ratings for Chapters ten and eleven

n = 20 objective	rating 1	2	3	4	5	mean
Chapter 10: Bibliography	0	0	3	9	8	4.3
Chapter 11: Directory	0	0	3	9	8	4.3

These two chapters were very well received by all respondents. There would seem to be a great need for information of this kind. Since the two chapters are so closely related, it was not surprising that the ratings for the two were the same.

Revisions

The respondents made only a few suggestions for revisions to the handbook. In the final draft of the handbook revisions were made to chapters one, seven and ten. As a number of respondents had expressed dissatisfaction with the small amount of research described in chapter one, revisions were made to overcome this deficiency. Several respondents said that the list of topics for projects in chapter seven was a little brief. One respondent sent along a list of other topics that might be included. The writer decided to add all of these topics as an additional section entitled "Other Topics for Projects". Chapter ten was enlarged by adding a section devoted to films and filmstrips. These were added partly to satisfy the request made by some respondents, who suggested that such information would be helpful.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

On the basis of research in preparing this handbook and from comments and ratings given in the questionnaire the writer makes the following conclusions:

1. The various purposes stated for the handbook generally have been successfully achieved.
2. The various purposes stated for the individual chapters have been successfully achieved.
3. Teachers of secondary mathematics, while recognizing the worth of films, filmstrips and the overhead projector, make little use of these at the present time because of the very limited supply available to them for use.
4. Teachers of secondary mathematics seem generally to agree that the use of games, puzzle problems, and projects can be beneficial but claim that pressure of time to cover work puts severe limits upon the amount of time that can be used for these kinds of activities. Projects, in particular, must be limited because library resources are just not available in many schools.
5. There is a very real need for research pertaining

to teacher and pupil attitudes regarding the use of films, filmstrips, games, puzzle problems, and projects in secondary mathematics teaching.

Recommendations

The writer recommends:

1. That other handbooks of this kind be prepared in other subject areas.
2. That other handbooks in secondary mathematics be prepared. Such handbooks might consider taking one or two of the chapter topics and treating them in more detail than the present writer has been able to do.
3. That more research be carried out to determine the attitudes of students and teachers towards current teaching practices in secondary mathematics. In particular the research should seek to determine whether or not too much emphasis is presently being placed upon the use of the textbook and the chalkboard.
4. That all teachers of secondary mathematics make every effort to insure that the mathematics section of school libraries is adequately stocked.
5. That teachers of secondary mathematics, through personal request and through The Mathematics Council of The Newfoundland Teachers Association, make recommendations to The Division of Instruction

of The Department of Education that it make a determined effort over the next five years to provide secondary mathematics teachers with a wider selection of current films.

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APPENDIX A

A RESOURCES HANDBOOK

FOR SECONDARY MATHEMATICS

by

BENJAMIN BRUSHETT

TABLE OF CONTENTS

CHAPTER 1

GENERAL INTRODUCTION

Research indicates that the wise use of instructional materials such as films, filmstrips, models and transparencies are quite effective in the instructional process. This fact is as true in mathematics as it is true in any other subject. Johnson (1971), a leading mathematics educator in the area of instructional materials in the mathematics classroom, claims that:

> Instructional materials are as essential for the mathematics teacher as spices are for the chef. They are the necessary extra ingredients that make teaching and learning mathematics a pleasant, satisfying experience. Models, pamphlets, films and diagrams give to a mathematics lesson breadth and depth that would be difficult to obtain in any other way. (p. 349)

Many research studies have been done which demonstrate the effectiveness of instructional media when the medium is carefully selected and used. Moldstad (1974) writes that:

> Twenty years of decision-oriented media research have produced significant evidence to justify the following claims when instructional technology is carefully selected and used.
>
> 1. Significantly greater learning often results when media are integrated into the traditional instructional program.

2. Equal amounts of learning are often accomplished in significantly less time using instructional technology.

3. Multimedia instructional programs based upon a "systems approach" frequently facilitate student learning more effectively than traditional instruction.

4. Multimedia and/or audio tutorial instructional programs are usually preferred by students when compared with traditional instruction. (p. 390)

Moldstad (1974) refers to a study by Chance (1960), who studied what effect the use of 200 specially prepared transparencies would have on student learning. When the instructional approach (transparencies plus current practice) was compared with the traditional lecture - discussion approach on identical content, the researchers arrived at four conclusions as follows:

1. The group having the added use of the transparencies did significantly better on mean final course examination scores and final course grades (at .05 level of confidence).

2. The three faculty members unanimously agreed on the desirability of using these transparencies in their teaching.

3. Use of the transparencies resulted in an average savings of 15 minutes per class period.

4. Students reported overwhelming preference for instruction using transparencies. (p. 392)

Other, more recent studies support the use of various educational media in teaching mathematics. Sherrill

(1971) found that achievement with word problems in mathematics was improved by the presence of a pictorial representation of the problem situation. Gray (1973) found that a laboratory program contributed to the improvement in attitude and achievement of mathematically deficient students. Purser (1973) concluded that the use of manipulative activities was feasible. Blazek (1971) found that students used library materials to a greater extent when the teacher referred to them in teaching. The literature and experimental studies, then, support the idea that various media and discovery activities can be effective in mathematics teaching.

Secondary mathematics teachers have had few resource handbooks available to them. The Thirty-Fourth Yearbook published by The National Council of Teachers of Mathematics contains many useful ideas that the secondary mathematics teacher can adopt, but it is not a handbook of specific ideas and activities. The writer was able to locate only one such general resources handbook. It is entitled Mathematics: A Sourcebook of Aids, Activities, and Strategies. This book is listed in the bibliography section. The writer recommends it highly to all secondary mathematics teachers.

The writer has attempted, in this handbook, to bring together a variety of useful ideas and activities and to provide some other general information helpful to busy teachers. In particular, the handbook attempts to achieve two objectives. They are:

1. To provide secondary mathematics teachers with some specific information relevant to secondary mathematics.

2. To provide secondary mathematics teachers with a number of specific ideas, activities, and suggestions useful in teaching secondary mathematics.

The writer sincerely hopes that teachers of secondary mathematics will find the ideas, activities, suggestions and information of practical use in their teaching.

CHAPTER 2

16 MM FILMS

Films are recognized today as an effective instructional aid in the classroom. As recognition of this fact, considerable sums of money are being spent each year by departments of government to provide films on a free loan basis to schools. Some subject areas -- mainly social studies and the sciences -- have made wide use of films. This is not true, however, for mathematics at the secondary level. This may be due to the fact that mathematics is largely concerned with abstract ideas. As a result mathematics teachers have traditionally thought the textbook and the chalkboard to be adequate for mathematics instruction.

There are areas of mathematics, however, that are difficult or impractical to present in a textbook. The same is true for teacher presentation at the chalkboard. Such areas as the practical uses of mathematics, the integration of mathematics with other subjects, drawings of complex three-dimensional effects, and dynamic ideas that depend on motion are seldom, if ever, dealt with in textbooks. Through the medium of film, all of these areas, and others as well, can be effectively presented.

Mathematics teachers should, then, give careful consideration to the use of films in instruction.

Why Use 16 MM Films

Some concepts and information essential to a good grounding in mathematics are difficult to present in textbooks or to illustrate at the chalkboard. Films can be used effectively to bridge this gap in the instructional process. Johnson and Rising (1967), two noted mathematics educators, point out that films can be used to attain the following goals.

1. Visualize abstract ideas so that they have meaning.

2. Illustrate applications of mathematics in our world. Although field trips are best for seeing mathematics in action, they require time and community resources that are often not available.

3. Bring to the classroom important first hand accounts of new activities in mathematics.

4. Build favorable attitudes toward and interest in mathematics. The uniqueness of mathematics — its power, its elegance, its artistic side — is seldom dealt with in textbooks.

5. Present the history of mathematics and other enrichment topics.

6. Illustrate the discovery of relationships or principles.

7. Present dynamic ideas that depend on motion. The use of animation or slow motion can give dramatic illustrations of mathematical ideas.

8. Correlate mathematics with other subjects by

presenting supplementary materials.

9. Provide complex drawings of three-dimensional effects. These are very difficult to do with drawings in textbooks or on the chalkboard.

10. Teach how to solve problems.

11. Introduce a new subject or unit.

12. Summarize or review units within a course. (p. 235)

What is Available in Mathematics Films

The high cost of films makes it impossible for individual schools to purchase films. Films are, therefore, usually purchased by some centralized body. The centralized body then makes the films available to schools for limited periods on either a rental or free loan basis. The latter service is available to teachers in this province through The Department of Education, Division of Instruction.

At present, the number of mathematics films available in this province, through The Department of Education, for use at the secondary school level is not large. The current film catalogue issued by The Division of Instruction lists approximately 50 titles. In recent years, however, the number of good quality mathematics films being produced has been steadily increasing. It seems reasonable to expect that some of these will be added to the present supply at The Department of Education. The number added will depend, no doubt, upon the demands made by mathematics teachers.

What can be done to Increase the Number of Mathematics Films Available

If good quality mathematics films are being produced, mathematics teachers should seek to keep informed about them. Teachers can do this by reading the reviews of new films. Such reviews can be found in the two professional magazines, The Arithmetic Teacher and The Mathematics Teacher. Other publications, such as Canadian Materials and The AAAS Science Film Catalog, carry useful information as well. Teachers could pass along their recommendations for film purchases to the body supplying the film service.

Some Suggestions Regarding use of Mathematics Films

There is no ONE way which is always the best way to present a film. The following suggestions may, therefore, prove helpful.

1. Prepare students for what will be seen in the film. Teachers should carefully preview the film to see how the content and treatment relate to what students have studied or are about to study. Differences in vocabulary, for example, should be discussed. Students might be instructed to view the first showing for overall general impressions of the points the film is trying to make. A discussion of these points could then follow.

2. Provide for a second or third showing of the film. Students should be asked to view the film keeping one or two purposes in mind. Follow the viewing with questions and discussion on the purposes.

3. Maintain an effective learning situation.

   a. Remember that attention, purpose, motivation, responsibility and thinking are just as essential when using film as they are at any other time. Hold students responsible for learning from the film. Nonattentive students should be treated the same as they would be treated in the regular classroom situation.

   b. Don't hesitate to stop the film and comment on what was just shown. Reverse the film and reshow if deemed necessary. Teacher comments can be made during the showing if the sound in the film does not interfere. On some occasions the sound might be turned off and the teacher provide the commentary.

   c. If possible, show the film in sufficient light so that students may take notes. If this is impossible, stopping the film for short intervals to make notes has its advantages.

   d. Encourage students to explore any unknown territories suggested in the film. One way to do this is to ask students if there are any unanswered questions in the film. If so, a discussion of these could arouse the interest of some students to explore them further.

4. Make an effort to measure the positive and negative effects of the film. Did the film, for example, make any change in student attitude? Did the film broaden the knowledge of students? Were some points clarified by the film? The teacher can gather information on these, and other questions, by observing the students, by discussions and by having the students do written work requiring them to demonstrate an understanding of the film.

5. Keep a card file of each film used. Lined cards measuring 5 by 7 would be helpful here. Record all necessary information required for reordering the film. Record also a synopsis of the film's content and the points students are expected to get from viewing the film. Additional information regarding positive and negative effects, vocabulary, etc., can also be recorded. These cards may be time consuming to prepare but they will prove to be timesavers in the long run.

CHAPTER 3

35 MM FILMSTRIPS

The 35 MM filmstrip is now widely used by teachers. Its use in teaching mathematics can be very helpful. The filmstrip brings together in a logical sequence small units of mathematics. Filmstrips have a number of unique advantages which make them useful to the mathematics teacher.

Some Advantages of 35 MM Filmstrips

1. They are relatively cheap to buy. This means that individual schools are able to build up collections. When filmstrips are available in the school the teacher can plan a lesson using the filmstrip with the certainty that the filmstrip will be there when it is needed.
2. They are particularly well suited to many topics in mathematics.
3. The teacher can always be in complete control of the rate of presentation of the pictures. It is easy to advance or rewind quickly to desired sections. Audio tapes that sometimes accompany filmstrips can be used or omitted, as the teacher wishes.

4. The filmstrip often permits the teacher to show the class a large number of pictures, diagrams or facts quickly. Such filmstrips are quite useful for review purposes.
5. The filmstrip can be viewed on individual viewing machines in the resource centre. Slower students can use filmstrips over and over again. In addition, students who miss class presentations can use the filmstrip as a help to catch up on work missed.
6. Filmstrips sometimes deal with such topics as the applications of mathematics and the history of mathematics. Such filmstrips are a helpful supplement to the textbook and helpful for enrichment and motivation.
7. Filmstrips often come complete with a teacher guide containing a synopsis of the filmstrips, background information, lesson plans, vocabulary lists and follow up classroom activities.

<u>What is Available in 35 MM Filmstrips</u>

Individual schools and district resource centres wishing to build up collections of mathematics filmstrips will find that many good items are available for purchase. New items are being produced as well. Most distributors will send items to prospective customers for preview purposes. Hence items can be carefully previewed and judged suitable or otherwise before a committment to purchase is made.

The Department of Education, Division of Instruction, has available a small number of titles suitable for secondary mathematics. These are available, free of charge to any teacher requesting them.

Information on new titles as they become available can be found in professional publications, various reference sources and, of course, distributor catalogs. The Arithmetic Teacher and The Mathematics Teacher have sections devoted to reviews of new products and materials.

Suggestions for use of 35 MM Filmstrips

The suggestions already made regarding the use of 16 MM films are equally applicable to 35 MM filmstrips.

1. Prepare the student for what is to be seen in the filmstrip.
2. Show the filmstrip more than once.
3. Maintain an effective learning situation.
4. Try to assess student reaction to the filmstrip.
5. Keep a reference card file for each filmstrip.

CHAPTER 4

THE OVERHEAD PROJECTOR

Few technological developments have had more of an impact on teaching than has the overhead projector. Few have offered more advantages to the teacher. The simplicity of operation is perhaps its major advantage. One merely places a transparency on the stage of the machine and turns the machine on. The projector allows the teacher to control effectively the attention of the class at any time, by simply turning the machine on or off. While using the overhead projector the teacher is facing the class and thus can maintain the eye-to-eye contact that builds good rapport with students.

There is usually no need to adjust classroom lighting unless direct sunlight is shining on the projection surface. A screen, while it is recommended, is not absolutely necessary. A light colored wall serves quite well as a projection surface.

The horizontal stage of the projector is easy to use. A sheet of clear acetate is placed on the stage of the projector. The teacher then writes on the sheet of acetate with a grease pencil or felt pen. The pens come in a variety of colors and are of two types, those that make permanent markings and those that make markings easily removed with a cloth and water. A pen marker should be tried in advance of taking it into the classroom for use. Some markers "bead up" or run. Others smear easily. Felt pens which have been especially manufactured for use with overhead projectors are recommended for best results.

Many companies are in the business of producing commercial transparencies for use in the mathematics classroom. These products are usually well done, the artwork is good and the mathematics accurate. Two factors, however, militate against the extensive use of these transparencies. First, they are quite expensive and, secondly, they usually come in sets. Teachers often find that only a few in the set are useful for an individual class. The best transparencies, therefore, are those produced by the teacher to fit the needs of the classes taught. This does not mean commercially prepared transparencies should never be purchased. It does mean that they should be purchased only on "an approval" basis. This will avoid costly expenditures for sets of transparencies that may gather dust on a shelf.

Purpose of the Section

The purpose of this section of the handbook is to suggest some specific ways in which the mathematics teachers can use the overhead projector as an aid to effective teaching.

Suggested Uses of the Overhead Projector

1. The overhead projector can be used for the daily discussion of homework. Used in this way it can be a real time-saver and get the class started quickly. Each student can be given a sheet of acetate to place in a notebook. Assign different students to prepare the homework problems on the sheet of acetate. During class the students can then present the solutions to the class for discussion. Through discussion, the entire class can learn from the errors and strengths of fellow students' work. The sheets of acetate can be easily cleaned with a soft cloth and a little water.
2. The teacher can use the projector to show the solution to many types of problems. The complete solution to the problem can be carefully and neatly prepared on a transparency and then presented to the class, one step at a time. This is easily done by using a sheet of paper to cover any desired part of the solution. This method also leaves the teacher free to direct attention to any part of the solution. After a full

solution has been discussed, the full solution is there for students to copy or study. The transparency can then be filed away for any future use the teacher may wish to make of it.

3. Review of basic facts and concepts can be quickly done if these have been placed on transparencies. On any given day, a few minutes of the class period can be used to have students read the information on transparencies. It is a well established fact that the more often we see a statement or a diagram the greater is the chance we will remember that information.
4. Individual items for a short quiz can be written on a transparency. This can save the time often taken up with preparing stencils and duplicating. A few items neatly printed or typed on a transparency can serve the same purpose as duplicated hand out sheets.
5. For many lessons in mathematics a base transparency can be used over and over. There is no need to mark on any prepared transparency to call attention to details or explain things. Simply place a second sheet of clear acetate over the prepared master and make any markings on the top sheet of acetate. The top sheet can be easily cleaned for reuse. For example, the teacher may want to

discuss with the class the ideas of base and height of a triangle. The various types of triangles could be neatly drawn on the base transparency. By placing a second sheet over the master the teacher could indicate the relationship between base and height for the various triangles.

6. In teaching congruence of two triangles the overhead projector can be effectively utilized by using triangles of different colors. It would be easy to demonstrate the idea of superimposing one over the other to illustrate the SAS, SSS, and AAS properties of congruency.
7. The various plane geometric figures such as the rectangle, parallelogram, square, triangle etc., can be discussed effectively by using the stage of the overhead projector. Narrow strips of acetate of different colors could be used to form the various shapes. The strips can be marked off into units of length if desired. Three strips of lengths 3, 4 and 5 units respectively can be used to form the right triangle. Many of the properties of plane figures, as well as statements of theorems and corollaries can be physically demonstrated using strips of acetate.

8. Work with graphs is very well suited to the use of the overhead projector. A master transparency of graph paper can be used in many ways. By placing the master transparency on the stage of the projector an image can be projected on the chalkboard. This would be particularly helpful when having students do sample work on the chalkboard. Erasing the student's work from the chalkboard would leave the graph line there for another student to use. Producing neat and accurate graphs on the chalkboard can be time consuming. By using the overhead projector and a master sheet of graph paper the task is not very time consuming and the work can be much more accurate and neat. If a second sheet of clear acetate is placed over the master transparency on the projector stage the teacher can work on this sheet without marking on the master copy. The projector can be used to demonstrate effectively the solution set to systems of equations and inequalities. The solution set for the individual equations or inequalities in the system is first drawn on separate sheets of acetate. The sheets are then placed one over the other to display the solution set to the system.

9. The overhead projector can be an effective teaching aid in teaching students how to locate information from tables of numbers. The teacher can simply prepare a section of the tables on a master transparency. Care must be taken to make sure that the numbers on the transparency are large enough for easy reading when projected. Merely duplicating the table unto the transparency is likely to be unsatisfactory because figures in printed tables are usually small. Consequently, the section of the tables should be done by hand or with the use of a typewriter that has characters of the proper size for making transparencies. Projecting a section of the table unto the screen makes it easy for all students to follow the teacher as the correct way to look up information in the tables is demonstrated.

10. The technique of number nine applies equally well in teaching students how to make correct readings from the ruler and protractor. Plastic copies of these project well. Measurement seatwork exercises can be placed on the stage of the projector. The teacher then uses the ruler or protractor to make the required measurement.

The placement and reading are both then projected on the screen for students to see.

11. The overhead projector is effective and useful in demonstrating the basic straight edge and compass constructions of geometry. Often when the teacher performs these at the chalkboard the student may not have a clear view of each step as it is being done. Proper use of the stage of the overhead projector can eliminate this difficulty.

12. The main points of the lesson which the teacher would ordinarily write on the chalkboard can be placed on a transparency, and the various points uncovered at the appropriate time. Only the particular point, formula, or term being discussed should be displayed. To do this, cover the sections of the transparency with pieces of light cardboard using masking tape for a hinge. At the appropriate time any piece of cardboard can be lifted to reveal the desired section. The items when covered can be easily identified by writing a duplicate on the cardboard covering the item. The student's attention is then focused on the point under discussion.

13. Copies of diagrams from the student's textbook

can be transferred to an overhead transparency for class discussion. Again care must be taken regarding the letters or other symbols. Depending on what the teacher wishes to do, the diagram can be made with the letters or symbols omitted, or these may be hand printed in suitable size type.

14. In discussing topics concerning banking, copies of various forms used by customers can be hand drawn on transparencies.

15. In geometry, discussions involving two - column proofs can be conducted using prepared transparencies. Either the statements or reasons in the proof can be omitted and added as students supply them. Again, the statements may be supplied and the reasons required. Teachers who have more than one class can show the transparencies produced in one class to other classes.

16. To file transparencies use a paper punch and file the transparencies in a large ring binder. The binder is convenient for taking along to class. The desired transparencies can then be taken out, used, and replaced immediately in the proper place.

CHAPTER 5

MATHEMATICAL GAMES

Learning in mathematics can sometimes be stimulated with amusement and pleasure. Games are often considered to be merely a recreational activity but if properly used they can serve to facilitate learning in the classroom.

Many types of individual and group activities are essential in learning mathematics. The uniqueness and activity of a game can sometimes serve to break up the monotony of drill while at the same time accomplishing the same desired end. Teachers can try to capitalize on student interest in the activity and competition of games and use this interest to promote the learning of mathematical skills.

Two keys to learning mathematics are meaningful experiences and practical applications. These two keys need to be combined with a variety of systematic practice and drill. Since practice helps to build accuracy, efficiency, and retention, it is an essential part of the learning process. The practice needs to be purposeful and to be administered in the right amounts and at the right times. A systematic program of practice can be stimulated and enhanced by the use of mathematical games.

Purpose of the Section

The purpose of this section is to identify the roles of mathematical games, to suggest a number of points to remember about the use of mathematical games in the classroom, and to describe a small number of games appropriate to secondary mathematics.

Roles of Mathematical Games

1. Games can be used to make practice periods pleasant and successful.
2. Games can be used to teach vocabulary, definitions, rules and formulas.
3. Games can be useful in providing for individual differences in that students can select and play games according to needs and interests.
4. Games help to foster desirable attitudes towards mathematics. The informality, the competition, and the satisfaction of contributing to a group activity make games a satisfying experience. The student who finds enjoyment and satisfaction in the mathematics classroom is likely to develop a favorable attitude towards the subject.
5. Games offer the opportunity for student-teacher planning and student leadership. The organizing, planning and directing of classroom games can be done largely by students. Games by their very

nature are unique in the variety of opportunities they provide for student directed activities.

Four Points to Remember about using Mathematical Games

1. Develop a game according to the needs of the class. The game should make a contribution to learning mathematics. The items used in the game should be closely related to the regular class work. Major emphasis should be on the learning of mathematical concepts and skills rather than on the pleasure of playing the game.
2. Use the game at the proper time. For a game to make the maximum contribution, it should be used at the time when the ideas or skills are being taught and reviewed. A game should be played also for a relatively short time so that students do not lose interest.
3. Plan and organize the game carefully so that the informality and the excitement of the setting do not defeat the purpose. Have all materials at hand so that the game can proceed in an orderly fashion. Keep the rules simple. Use a few practice plays to get started, then expect work to proceed in an orderly manner. Undue noisy behavior should not be tolerated. Coaching should

never be allowed. The loss of points for breaking this rule will usually be sufficient to maintain proper behavior. Avoid the choosing of team members by pupil captains so that low ability pupils will not be embarrassed by being last choice.

4. Try to emphasize the responsibility of learning something from the game. This can be done by follow-up activities such as discussions, readings, or tests.

Appropriate Games for Learning Mathematics

The appropriate games are limited only by the ingenuity of teachers and students. Most lessons involve practice that can be converted into a learning game. A play or turn will usually involve working a problem, making an association, giving a definition, stating a formula or identifying some property or characteristic. Quiz games, board games, and card games can be successfully used.

While many mathematical games are produced commercially, the writer believes that the best games are those invented by the teacher and students to fit the needs of the particular class.

Materials needed for making Mathematical Games

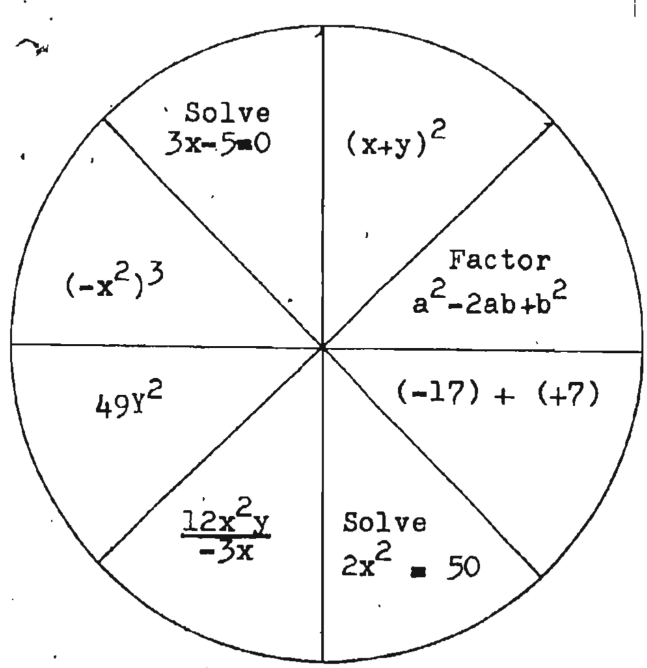
Games can be made from simple, inexpensive materials such as 3 by 5 cards, pieces of cardboard or heavy paper and pieces of plywood. A supply of markers in various colors is useful as well. Drawing original designs on the backs of cards can sometimes be a worthwhile geometry assignment. Calendar numbers are always available to be cut and glued to cards.

Some Specific Games

Spin It

This game fits many topics. The game is made by drawing a circular region on a large sheet of heavy paper. The paper can be taped to a wall or the chalkboard. The circular region is then divided into a suitable number of sections. Items placed in the sections may be similar or they may be mixed. A pointer is mounted at the centre.

To play the game choose two teams. A person is appointed to keep scores and act as a moderator. Players take turns spinning the pointer. The player then has to solve the problem in a given time limit. Players may or may not be permitted to use pencil and paper. This will depend upon the ability of the players and the difficulty of the problems. A successful solution or answer earns five points for the team. If a player on the team cannot correctly solve the item or answer the question, the opportunity to solve the item or answer the question is passed to the player's opponent on the other team. Should the player's opponent fail to give the correct solution, play is returned to the next member on the team which selected the item. If this player still fails to give the correct solution, play is passed again to the next member of the opposite team. Play continues in this way until some member of one of the teams manages to provide the correct solution. The player whose turn is next then spins for the second item. If the pointer stops on an item which has already been answered, that player should spin again until the pointer stops at an item not yet used. The person who is moderator keeps a record of items as they are used. To make this easy to do, each item can be assigned a number. Play continues for a set time or until an agreed team score has been earned.



<u>Pick It</u>

The rules of the games are the same as for Spin It except the individual items are written on cards which can then be drawn from a suitable container. An alternate method would be to mix the cards well and then take them in turn. As an item is used it is placed to one side.

<u>Count Off</u>

Items for this game should be written on separate cards. The object of the game is to count by adding or subtracting a given number. The numbers used may be integers, fractions or decimals. Pen and paper may be permitted or not permitted depending upon the ability of the students and the difficulty of the calculations. Any convenient number may be used as a beginning number. Students playing should be arranged in rows or in a circle. When a player makes a mistake, that player moves to the front of the row or to the beginning of the circle and the other players shift back by one position. The winner is the player in the last position at the end of the game. When the leader reads an item from the cards, the players respond in turn until some player makes a mistake. Play then continues beginning at the next player and with a different item.

Mathematical Bingo

Materials Needed. A number of 3 by 5 call cards, one for each item to be used. A set of player cards. Each player card can be cut from heavy paper or light cardboard. Each player card is divided into 25 cells by drawing vertical and horizontal lines. Cell sizes will vary depending on the nature of the items used.

Making the game. Assign 5 problems to each student in the class. The problems can be written on slips of paper. Answers to the problems can be supplied or the students may be asked to work out the answers. Answers to all problems should be carefully checked. Have the students write each problem on a 3 by 5 call card. Next hand out one player card to each student. Have each student then write the answers to the problems he has on the player card. Answers are to be written in on the player card using a random order. Have students write the word "free" in the centre cell. Now have students exchange the player cards. Once again instruct each student to write in the answers to the problems he has. Exchange player cards three more times. Each time have students write in the answers to the problems held. This will complete one class set of player cards. Additional sets of player cards can be made in a similar way.

Playing the game. The game is played like ordinary bingo using the call cards and player cards. The caller for the game selects a call card and reads out the problem or writes it on the chalkboard. An alternate method would be to have the call cards made on acetate. Items could then be placed on the stage of the overhead projector. Each student playing then individually solves the problem and covers the answer if it appears on the player card held. The caller keeps a record of the items called and the correct matching answers. Answers can be written on the back of each call card. If call cards are made on acetate the answers can be placed on masking tape near the bottom of the card. A time limit suitable to the difficulty of the problem should be allowed for solving the problem and covering the answer on the player card. The first player to cover five cells in a vertical, horizontal, or diagonal line wins. Play can continue until several winners are found.

<u>Suitable topics</u>. The game lends itself to all levels of difficulty and to many topics. In algebra the call cards could be equations, evaluation items, radicals, operations with signed numbers, or simple word problems, to name just a few. In geometry call cards could deal with definitions, area, volume, angles, etc.

CHAPTER 6

PUZZLE AND RECREATIONAL PROBLEMS

Students of mathematics enjoy puzzle problems. Although these are often thought of as recreational, other worthwhile benefits can accrue from their use in the mathematics classroom. For example, they may stimulate intellectual curiosity, develop abilities in space perception, and modes of thinking.

Using Puzzle Problems in the Classroom

1. A weekly puzzle problem could be posted on the classroom bulletin board and students encouraged to submit written solutions. At the end of the week the names of students submitting correct solutions could be posted. Perhaps some small award could be given to the student submitting the first correct solution or to the student submitting the greatest number of correct solutions for the month.
2. Puzzles can be written on 3 by 5 index cards and the cards kept in a suitable container. Students who have finished their assignment for a period might be permitted to draw a card from the container and work at a solution while the other students finish their work.

3. The last period before the Christmas or Easter break could be devoted to solving puzzle problems.
4. Occasionally, part of the regular class period, or a whole class period, could be taken to deal with puzzle problems. Puzzle problems related to work currently being studied or work recently studied would be especially useful for these times. Students who have successfully solved a puzzle problem could present the solution to the class. Students could be encouraged to write their own puzzle problems and test them on fellow students or the teacher.

A Selection of Puzzle Problems

Over the years teachers can gather a supply of puzzles that have proved to be interesting to students. The writer has included here a small number gleaned from various sources. Teachers should consult the bibliography section of the handbook for some books of puzzles. Solutions are given in the section following the problems.

Building a House

Two men decide to build three houses, all exactly alike. The first man built his house in four weeks. It took the second man twelve weeks to build his house. If both men work together at their respective rates, how long will it take them to build the third house?

Which Salary would you Choose

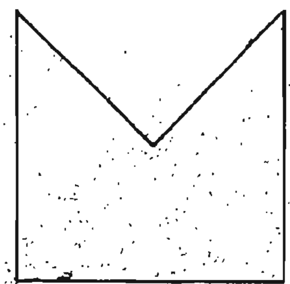
Two men started working in the same office at the same salary. After a period of time they both decided to ask for a raise. When they presented their request to their boss, he answered them as follows: "You both have done exceptional work and deserve the raises which I am about to offer you. I will give you your choice of a $100 raise every six months, or a raise of $300 every year, either raise to commence as of this date." Which would be the better raise? Show why your answer is correct.

The Carpenter Problem

A carpenter, building a house, came across a square hole in the floor. The hole was exactly 2 cm. square. The only material he had on hand to repair the hole was a circular broom handle 2 cm. in diameter, and of course a hammer and saw. He was able to repair the hole. How did he manage to do this?

Dividing a Figure

Divide the figure below into eight parts such that all are the same size and shape. It can be done so keep trying.



The Horse-Trader Problem

A horse-trader complained to his friend that the day had been a very poor one. He had sold two horses for $75 each. On the first horse he made a profit of 30%; and on the second he took a loss of 30%. "That doesn't seem to be any loss at all", asserted the friend. "Your profit and loss balance each other." "On the contrary", said the horse-trader, "I lost almost $15". Who was right? Show why.

Cost of Animal

A farmer bought pigeons for 50 cents each, pigs for $5 each and calves for $10 each. If he paid $100 for 100 animals, how many of each did he buy?

The Man Selling Oranges

A man was selling oranges on the street. To his first customer he sold half his oranges and two more; to his second customer he sold half of what was left and two more. After the third customer bought half of what was left and two more, one orange remained. How many oranges did the man have to start with?

The Clock Problem

A man has two clocks. One clock does not run at all; the other loses four minutes every day. Which clock is more often correct? Explain.

The Cube Problem

Suppose you have a 3 cm. cube which is painted red. Answer the following questions about the cube.

a. How many cuts are required to divide the cube into one cm. cubes?

b. How many cubes would there be?

c. How many cubes would have four red faces?

d. How many cubes would have three red faces?

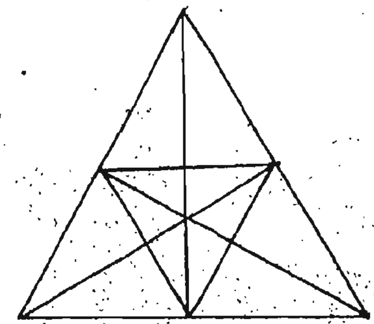
e. How many cubes would have two red faces?

f. How many cubes would have one red face?

g. How many cubes would have no red faces?

Count the Triangles

Here is a problem that requires both a system of attack and patience. Simply count the number of triangles in the figure. The triangles include triangles of all shapes and sizes, triangles within triangles, and triangles overlapping triangles.



The Traffic Officer and the Speeder

A traffic officer brought a speeder before the judge. "How do you know this man was speeding?" asked the judge. "I followed him," replied the officer. "In fact I followed him for ten km. The first five km he drove at the rate of 80 km/h and the second five km he drove at the rate of 40 km/h. So you see, he averaged 60 km/h over the ten km distance, which is 5 km over our 55 km/h speed limit. I am therefore charging this man with driving a distance of 10 km at an average speed of 60 km/h." "Since you state your charge in this manner," replied the judge, "I will have to dismiss the case. If what you say is true, this man did not even average 55 km/h for the ten km." Who was correct?

Working for the Horse

A boy agreed to work one year for $240 and a horse. At the end of seven months the boy quit. He received $100 and the horse. What was the value of the horse?

Water and Wine

A glass is half full of wine and another three times as large is one-fourth full of wine. The glasses are filled with water and mixed together. What part of the mixture is wine?

Filling the Reservoir

One pipe can fill a reservoir in 10 days, another can fill it in 16 days, and a third can empty it in 20 days. How long will it take to fill the reservoir if all the pipes are left open?

How Much is the Bottle

Rich Mr. Vanderford bought a bottle of very old French brandy in a liquor store. The price was $45. When the store owner handed him the wrapped bottle he asked Mr. Vanderford to do him a favor. He would like to have the old bottle back to put on display in his store window. He would be willing to pay for the empty bottle. "How much?" asked Mr. Vanderford. "Well," the store owner replied, "the full bottle cost $45 and the brandy cost $40 more than the empty bottle. The empty bottle is ....." "Five dollars," interrupted Mr. Vanderford who thought he knew his figures better than the store owner. "Sorry, sir, you can't figure," replied the store owner. What is the correct answer?

The Snail Problem

A snail is at the bottom of a well 20 metres deep. Every day the snail climbs 7 metres and every night it slips back 2 metres. In how many days will the snail climb out of the well?

Cats and Mice

If 5 cats can catch 5 mice in 5 minutes, how many cats are required to catch 100 mice in 100 minutes?

The Phonograph Record

A phonograph record has a total diameter of 30 cm. The recording itself leaves an outer margin of 2 cm; the diameter of the unused centre of the record is 10 cm. There is an average of 45 grooves to each centimetre. How far does the needle travel when the record is played?

The Chain Problem

A man has five chains of three links each. He wants to make a single chain by cutting links and welding them again. How should he proceed if he wants to make the smallest number of possible cuts?

What is the Average Rate

A man drives to work at the rate of 40 km/h and returns along the same route at the rate of 80 km/h. What is his average rate of speed for the trip?

Integers Problem

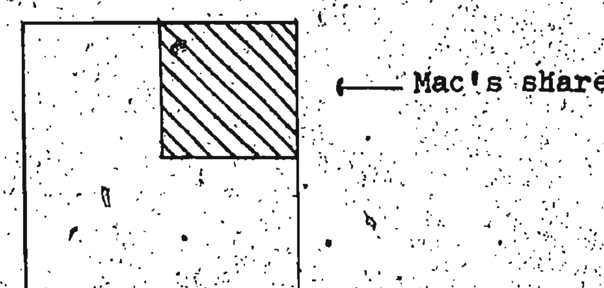
Show that no pair of integers satisfies the equation $3X + 6Y = 95$.

Handshake Problem

There are 8 men in a room. Each one shakes hands with each of the others once. How many handshakes are there?

Dividing the Birthday Cake

Mac, the ship's cook, baked himself a square birthday cake. Four of his pals noticed the cake and demanded a share. "Ok," answered the wily Mac, as he cut out a quarter section for himself, "the rest is yours if you can cut it such that all four pieces are exactly the same shape and size". The four pals were soon enjoying a share of the cake and they did not make a single diagonal cut. How did they cut the cake?



SOLUTIONS TO PROBLEMS

Building a House

Divide the job (1) by the fractional part of the job both men can do in a week $1 \div (\frac{1}{4} + 1/12)$. Together the men will build the house in three weeks.

Which Salary would you Choose

The better salary would be $100 raise every six months. Here's proof. Assume each man is receiving $2000 a year or $1000 every six months. The salaries for three years are shown.

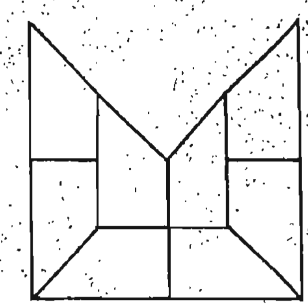
Year	$100 raise every six months.	$300 raise every year.
	$1100 $1200	
1st	$2300	$2300
	$1300 $1400	
2nd	$2700	$2600
	$1500 $1600	
3rd	$3100	$2900

The Carpenter Problem

The carpenter sawed 2 cm off the circular handle, split it transversely, and pounded it into the hole with the flat side towards the surface.

Dividing a Figure:

The solution looks like this:



The Horse-Trader Problem:

The horse trader was correct. He made a profit of $17.31 on one horse and took a loss of $32.14 on the other for an overall loss of $14.83.

Cost of Animal:

The farmer had 90 pigeons, 9 pigs, and 1 calf.

The Man Selling Oranges:

The man started with 36 oranges.

The Clock Problem:

The clock that doesn't run at all is correct twice a day the other only twice a year.

The Cube Problem:

a. 6 b. 27 c. 0 d. 8 e. 12 f. 6 g. 1

Count the Triangles:

There are 47 triangles.

The Traffic Officer and the Speeder:

The judge was correct. The average speed for the 10 km was 53 and one third km per hour.

Working for the Horse:

At the end of 7 months the boy had earned $140 and 7/12 of the horse. He received $100 and the horse. If 5/12 of the horse is worth $40, the whole horse must be worth $96.

Water and Wine:

The total amount of wine is one and 1/4 times the small glass. The total amount of the mixture is 4 times the small glass. Thus, 5/16 part of the mixture is wine.

Filling the Reservoir:

The part filled in one day is $1/10 + 1/16 - 1/20$ or $9/80$. Hence it will take $(1 \div 9/80)$ or 8 and 8/9 days.

How Much is the Bottle:

The empty bottle costs $2.50 and the wine $42.50. The equation $b + (b + 40) = 45$.

The Snail Problem:

Three days and 5/7 of the daylight hours of the fourth day. At the end of the third day the snail is 15 metres up, so that at the rate of 7 metres for the day it will reach the rim within 5/7 of the daylight hours.

Cats and Mice:

5, of course, unless they get tired before the job is done.

The Phonograph Record:

The number of grooves per centimetre has nothing to do with the answer. The needle is stationary except for the movement towards the centre of the disc. Hence the distance travelled is $15 - (5 + 2) = 8$ cm.

The Chain Problem:

Cut all three links of one length of chain and use them to weld together the remaining four lengths.

What is the Average Rate:

$$\text{Av. Rate} = \frac{\text{Total Distance}}{\text{Total Time}}$$

If distance travelled to work is d km, the time would be d/40. Time to return is d/80. So that the average rate is $2d \div (d/40 + d/80) = 53$ and 1/3 km/h.

Integers Problem:

Since the left side is divisible by 3 and the right side is not, there can be no integer that satisfies the equation.

Handshake Problem:

Represent the men as A, B, C, D, E, F, G, H. The handshakes can then easily be represented as follows. We see that there are 28.

AB, AC, AD, AE, AF, AG, AH

BC, BD, BE, BF, BG, BH

CD, CE, CF, CG, CH

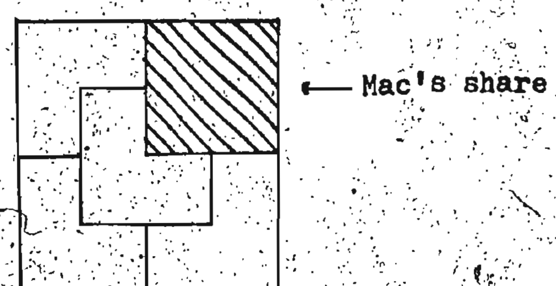
DE, DF, DG, DH

EF, EG, EH

FG, FH

GH

Dividing the Birthday Cake:



CHAPTER 7

MATHEMATICAL PROJECTS

Projects serve two purposes in mathematics. They provide a challenge to the student to delve into some particular topic beyond the requirements of the regular work, to the extent of the student's ability. They give a student a glimpse of the fascinating study of the history of mathematics, so that mathematicians of the past become living people.

Developing Student Interest in Projects

Success in the development of student interest in mathematical projects will depend in large measure on the teacher. The teacher can seek to generate student interest by throwing out a casual remark, an anecodote, or a reference to something of historical value in connection with a topic being studied. In addition, the teacher will have to show interest, enthusiasm, and a readiness to give time and individual guidance to students who take up the challenge of a project. This does not mean doing the research for a student. It does mean providing a list of topics from which students may get ideas. It means discussing topics generally with the whole class and in more detail with individual students who express an interest in a topic. It means guiding the student in planning and developing a project. It means providing adequate resource materials, or directing students to them.

Forms of Mathematical Projects

The mathematical project can take a number of forms. It can consist of an oral or written report, done on some book, person, or topic. It can consist of building some model designed to illustrate an idea in mathematics. It can be a slide-tape, or television production. It can be the preparation of a number of overhead projectuals relating to topics in mathematics. It can be the preparation of a wall chart of mathematical terms, formulas, or geometrical figures. It can be compiling a collection of optical illusions, mathematical cross puzzles, or word problems. It can be the preparing of a display.

Library Resources Important

Library resources are essential in the research of topics. Where these are lacking, mathematics teachers can make an effort to insure that the mathematics section is gradually built up. Bibliographies of resource materials available inside and outside the school can be prepared and distributed to students. Teachers can make a point of browsing through magazines and newspapers for articles of mathematical interest. A file of such materials can be started and added to periodically. Publications of business, industry and government are other good sources of materials. Students who produce projects of excellent quality can be encouraged to contribute them to the school library or resource centre.

Some Suggested Projects

1. Build a transit or clinometer. (Plans for these projects can be found in chapter 8).
2. Have students select some topic of mathematics and design a model which illustrates the principles of the topic chosen.
3. Provide the student with a set of basic instructions for some model and have the student construct the model. (Chapter 8 of this handbook has some plans.)
4. Have students devise a mathematical game or crossword puzzle.
5. Have students prepare a collection of mathematical tricks, anedotes, or cartoons.
6. Have students prepare a scrapbook of clippings of charts, tables, or graphs from such sources as daily newspapers, magazines, and reports. Such a project should help a student to gain a better appreciation and understanding of the wide use made of mathematics in preparing and presenting information.

7. Have students prepare a scrapbook collection of designs using mathematics. Many items of consumer goods use mathematical designs. It would be best to have students confine their attention to one type of goods. Wall coverings, floor coverings, fabrics, and furniture coverings are good sources.
8. Have students prepare a written research paper on the life and work of some well known mathematician.
9. Provide the opportunity for a student or a small group of students to accept responsibility for the presentation of some topic to the class.
10. Have a small group of students plan and set up a display to illustrate some mathematical concept.

<u>Other Topics for Projects</u>

1. Extending the Pythagorean Theorem.
2. The Golden Ratio or Rectangle.
3. The Nine Point Circle.
4. The Historical Development of π. (I Kings 7:23).
5. Compare the Sum of the Angles in a Triangle in Euclidian and Non-Euclidian Geometries.
6. Euler's Line.
7. Polyhedrons and Euler's Formula: $F + V - E = 2$.
8. Various Ways to Find the Area of a Triangle.
9. Comparison of the Volumes of a Sphere, Cone and

Cylinder with the same Radius and where the Height equals the Diameter.

10. Various Proofs of Pythagoras.
11. Different Types of Geometry.
12. The Three Problems of Antiquity.
13. The Properties of a Kite.
14. Circular Geoboards.
15. The Square Root Spiral.
16. Geometric Factoring.
17. The Origin of Geometry.
18. Mobiles.
19. Geometric Designs.
20. Geometric Illusions.
21. Geometry of Snowflakes.
22. Geometry of Shells.
23. Geometry of Crystals.
24. Graphic Designs.
25. The Square Geoboard.
26. Practical Applications of Geometry.
27. Spherical Geometry.
28. Topology.
29. The Complete Quadrilateral.
30. The Parallel Postulate.
31. Circular Regions.
32. Geometry and Architecture.

33. Geodesic Domes (greenhouses).
34. Uses of Parallel Lines - Navigation, Sports, Carpentry, Electricity, Surveying, and other fields.
35. Tessellations.
36. Stewart's Theorem, Morley's Theorem, Ptolemy's Theorem, Ceva's Theorem, Simpson's Theorem.
37. Pythagoras and Music.
38. The Contributions to Geometry of Rene Descartes, Apollonius, or any other mathematician.
39. Heron's Formula for the Area of a Triangle.
40. Brahmagupta's Formula for the Area of a Cyclic Quadrilateral.
41. Platonic Solids.
42. Euclid's Elements Compared to Student's Present Textbook.
43. Euclid's Classification of Quadrilaterals.
44. Orthocenter, Centroid, Incenter, Circumcenter.
45. The Trifoil, Quartefoil, Cinquefoil, etc.
46. The Reuleaux Triangle.
47. The Pantograph.
48. The Baculum or Cross-staff.
49. The Saccheri Quadrilateral.
50. Hyperbolic and Elliptical Geometries.

CHAPTER 8

MODELS

Models can help students to think. Models may be drawings, concrete devices or mathematical formulas. Whatever their form, models help provide a basis to help students solve some problem or discover some new idea. Models can serve as a link between the thought process and the reality to abstract ideas and can encourage creative thinking.

Students should be alerted to the inherent dangers in the use of physical models to represent abstract ideas. Consider, for example, the term "circle". Unless care is taken, the student may come to think of a circle as a disc or circular region. The model can lay the foundation for learning an abstract concept, but the student must be reminded that the concrete representation does not give a complete conception of the abstract idea.

Models, of course, are unnecessary for some students. Some talented students are quite capable of, and enjoy, discovering generalizations. For other students, even the models are meaningless, and may be of little help to them in building an understanding of a concept.

Some uses of models

1. Models may be used as demonstration aids to add meaning to a concept. In discussing area, volume, measurement and a ratio, for example, a demonstration model can be helpful.

2. Students may be given some model with directions to carry out some laboratory experiment designed to lead to the discovery of some new ideas and relations.

3. Models may be used as practice devices through which the student builds accuracy, understanding and efficiency. Paper folding exercises, for example, provide useful exercises in illustrating relationships between line segments. A number line may be a device for practicing the addition and subtraction of signed numbers. The use of mathematical tables is another example.

4. Models may be used as measuring instruments for studying mathematical applications inside and outside the classroom. A homemade transit, or clinometer can be used to help measure indirectly the height of a building, or the distance across a pond. The volume of irregular solids can be measured using liquid displacement. The pythagorean theorm can be shown to be true by building squares on the sides of a model

triangle.

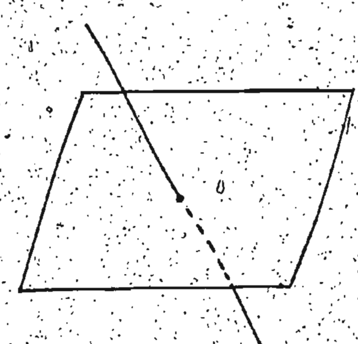
5. The building of models may be interesting and challenging projects for independent student work. A student can be given the opportunity to plan, construct and demonstrate a model which illustrates some mathematical concept.

6. Models may be used as enrichment devices to present ideas which are not in the textbook. Moebius strips or hexaflexagons, for example, suggest some of the curiosities of topology.

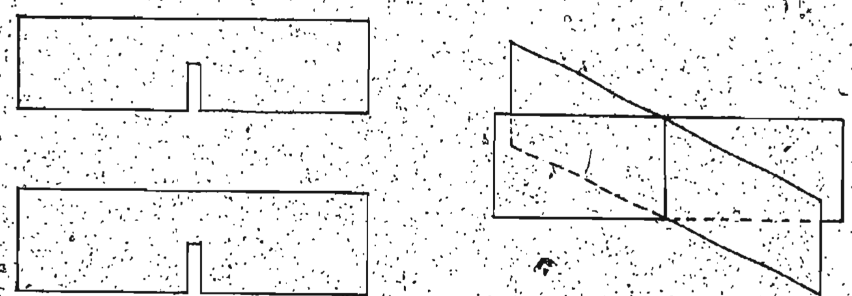
Some Specific Models

Basic Geometrical Facts

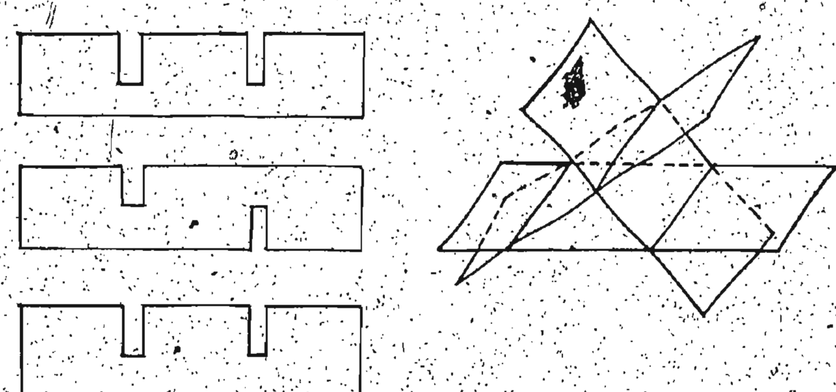
1. A straight line intersects a plane in a point. This fact is easy to represent if a piece of cardboard represents the plane and a stiff piece of wire is used to represent the line.



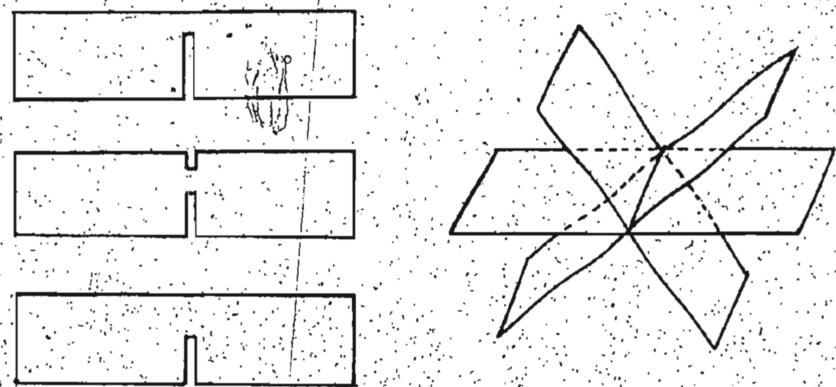
2. Two planes intersect in a straight line. This fact can be illustrated by cutting two pieces of cardboard halfway through and then fitting the pieces together to form the intersecting planes.



3. Three planes may intersect in three parallel lines. To illustrate this, cut three pieces of cardboard halfway through and fit the pieces together to form the intersecting planes.



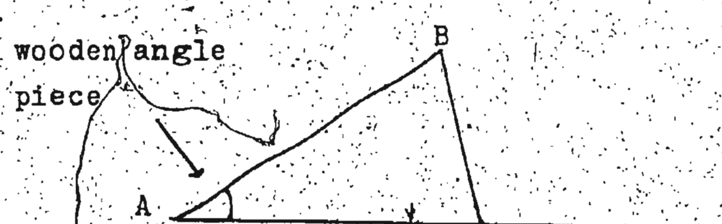
4. Three planes may intersect in a straight line. To illustrate this fact, cut three pieces of cardboard as shown and fit them together.



5. Each of three planes may be perpendicular to the other two. This fact is readily illustrated by directing the attention of students to a corner of the classroom.

Other Models for Plane Geometry

Triangles. Select three strips of wood so that the length of the two shorter pieces is equal to the length of the longer strip. Fasten the strips with small metal hinges at points A and B. The wooden angle piece is held in place by the weight of the two upper strips.

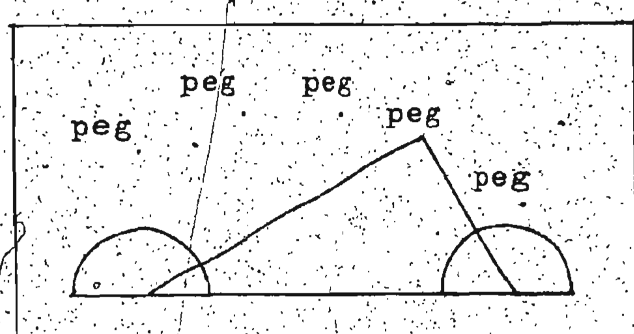


This model is useful to illustrate:

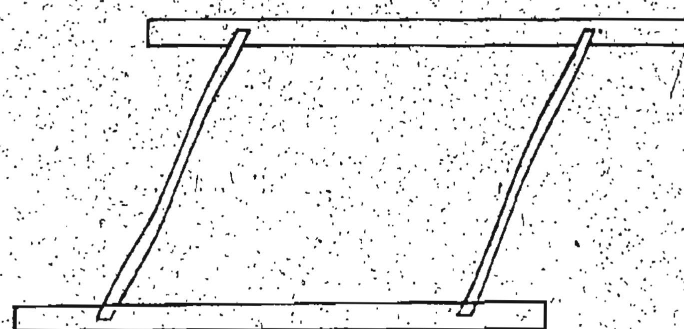
(a) The various shapes of triangles, acute, right-angled, and obtuse.

(b) The fact that the sum of the two shorter sides of a triangle must be greater than the longest side.

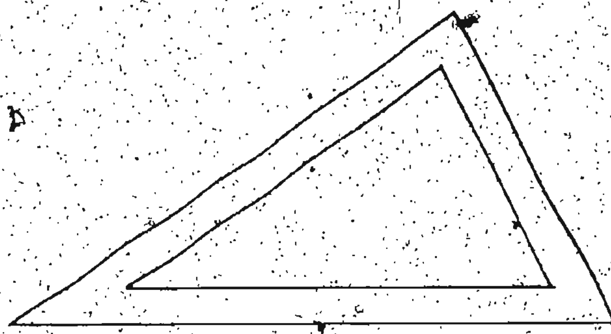
<u>Exterior angle of a triangle</u>. The exterior angle of a triangle is greater than either of the non-adjacent interior angles. To demonstrate this, two protractors can be fastened at each end of the base of a triangle that has been drawn on a piece of painted plywood measuring about 16 inches by 27 inches. An elastic cord leading from holes at each end of the base can be looped over pegs to locate the vertices of various triangles. Readings on the protractors when the vertex is in various positions show the constant relation between an exterior angle of a triangle and one of its non-adjacent interior angles, and also the changing relation between the exterior angle and its adjacent interior angle.



The parallel ruler. This useful model for use at the chalkboard is made from two halves of a meter stick or two whole meter sticks if preferred. The pieces are connected by links of wood or metal fastened to the sticks by round head screws. The model is handy for drawing neat parallels and various parallelograms. It is a constant reminder to the students of the theorem: If both pairs of opposite sides of a quadrilateral are equal, then it is a parallelogram.



Similar triangles chalkboard device. Various shaped similar triangles can be made from plywood as illustrated below. Cut out a triangle having a base of 18 inches and a height of 14 inches. Then cut out from this a second triangle whose sides are parallel to and $1\frac{1}{2}$ inches from the original triangle. Sand the edges and you have a convenient model for drawing quickly, neat similar triangles. Other models can easily be made using any desirable shapes and dimensions.



<u>Quadrilateral models</u>. These can be made from narrow strips of wood and round head bolts. If desired, the measurements of sides can be shown by marking units of length on the strips with a marker. Many facts of plane geometry can easily and forcefully be demonstrated with such simple models. If small screw eyes are attached at the vertices and elastic cord are used, the possibilities for use are greatly increased. A few suggestions follow. The list is not meant to be complete.

1. Connect four strips of unequal length to form the general quadrilateral. The following may be noted:
   - a. The figure is not rigid,
   - b. The diagonals are unequal,
   - c. The diagonals do not bisect each other,
   - d. The angles of the figure are not bisected by the diagonals,
   - e. Cord joining the mid points of the sides will form a parallelogram.

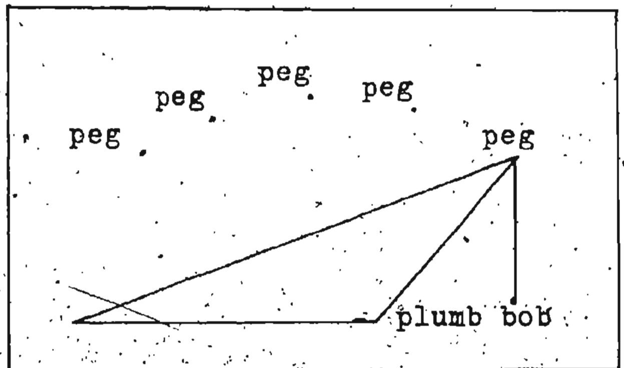
2. Connect four strips to form a rectangle. Note that:

a. The diagonals are equal and bisect each other but the angles of the rectangle are not bisected,

b. If the shape is changed to a general parallelogram then the diagonals still bisect each other but the angles of the figure are still not bisected. Observe also the congruent triangles in both the figures.

3. Take two unequal strips and fasten them at their mid points. An elastic cord stretched around the ends assumes the shape of a parallelogram. As the angle between the strip is changed note what happens to the diagonals with respect to bisecting opposite angles of the parallelogram. With two strips of equal length the properties of the square and rhombus can be demonstrated.

Triangle model. This model is useful to show the position of the altitude for various shaped triangles. The base of a triangle is drawn on a piece of plywood measuring about 12 inches by 17 inches. Narrow elastic, leading from holes at the ends of the base, can be looped over pegs to form the other two sides. Various shaped triangles can be formed. When the model is held in a vertical position, a plumb bob suspended from the pegs shows the position of the altitude of the triangle with respect to the base.



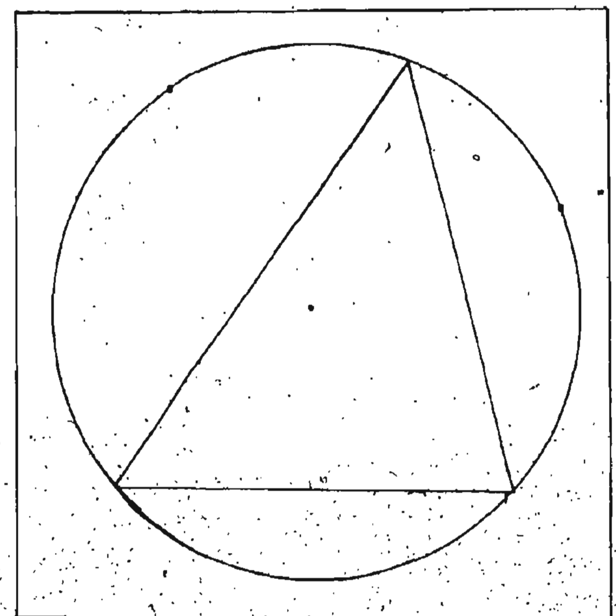
<u>A circle model</u>. Draw a circle of 12 inches diameter on a piece of plywood. Mark the centre and a chord. Narrow elastic leads from the ends of the chord to pegs located at desired positions on the circumference to form various triangles. The position of the centre of the circle in the various triangles is demonstrated. This model will also demonstrate the following:

a. An angle inscribed in a segment whose arc is less than a semicircle is obtuse,

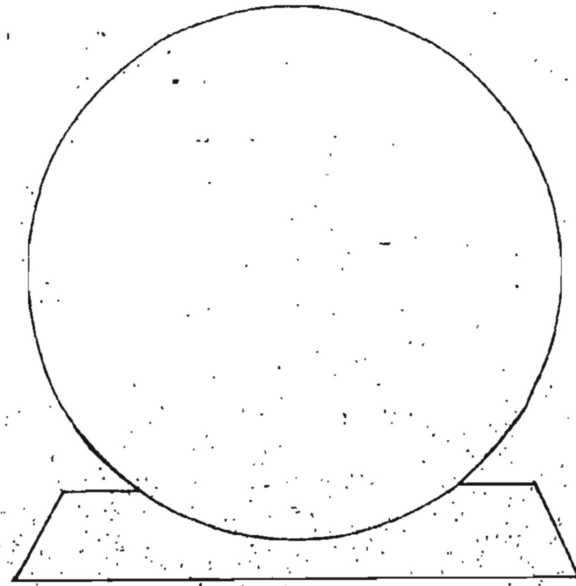
b. An angle inscribed in a segment whose arc is more than a semicircle is acute,

c. The median to the hypotenuse of a right triangle is one-half the hypotenuse,

d. Angles inscribed in the segment are equal.



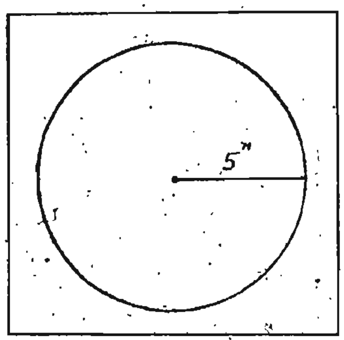
Another circle model. A model to illustrate theorems relating to chords, arcs, central angles and inscribed angles of a circle can be made by forming a circle from a piece of metal or plastic moulding. The edge of the moulding should be such that it has a slot in which a nail head can slide. Two color elastic bands are used, one color to represent the dotted construction lines of a figure and the other to show the usual sides of the figure. The elastic bands are hooked over the nails which are bent as shown in the diagram.



Plans for Building a Transit

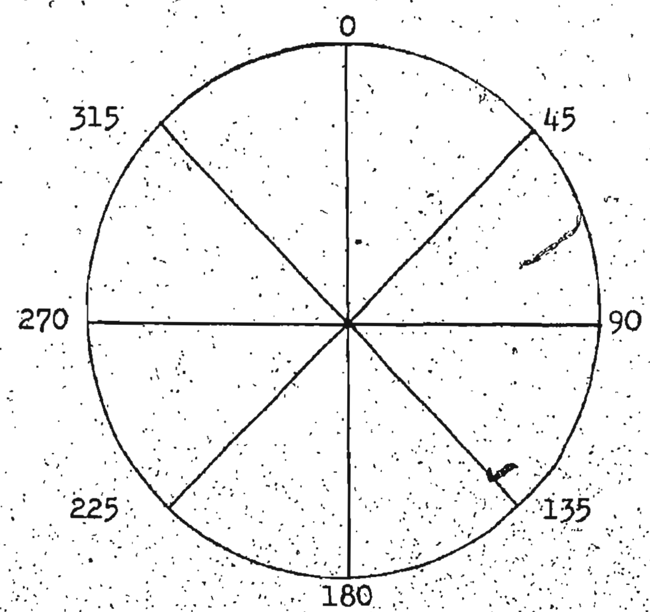
A very worthwhile project might be to have students build and use a homemade model of the transit. The following steps can be followed.

Step 1. The table top, shown in the diagram can be made from a piece of plywood about one foot square and about ½ inch in thickness. Draw a circle of 5 inch radius on the plywood and drill a 3/4 inch hole at the centre of the circle.

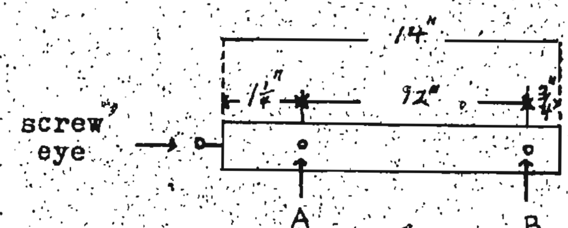


Step 2. Mount the table top on three legs. The legs can be made from broom handles, or leg fittings may be obtained from some discarded table at home. The legs should be mounted at points corresponding to the vertices of an equilateral triangle.

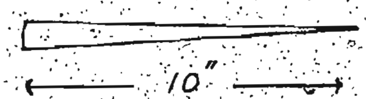
Step 3. Construct the horizontal protractor on good quality cardboard and glue or tack it to the circle on the table top. Remember to cut out the hole at the centre. If desired, the protractor can be drawn directly on the table top. See diagram.



Step 4. Next make the dowel stick from a piece of wood 3/4 inch in diameter and about 14 inches long. Attach a screw eye in one end and drill two holes of ¼ inch in diameter at points A and B as shown in the diagram.

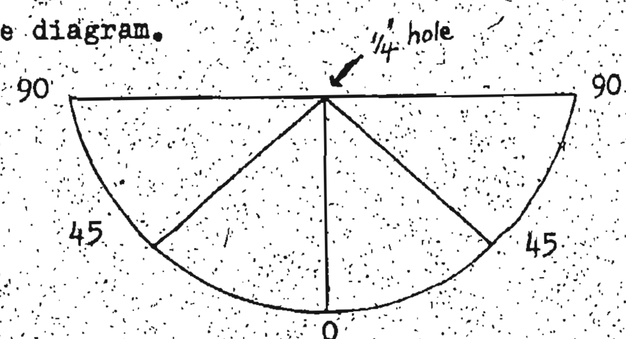


Step 5. Make the pointer for the horizontal protractor from a dowel stick ¼ inch in diameter and sharpened at one end. The pointer fits into the ¼ inch hole drilled in the dowel stick at point A. See diagram for step 4.

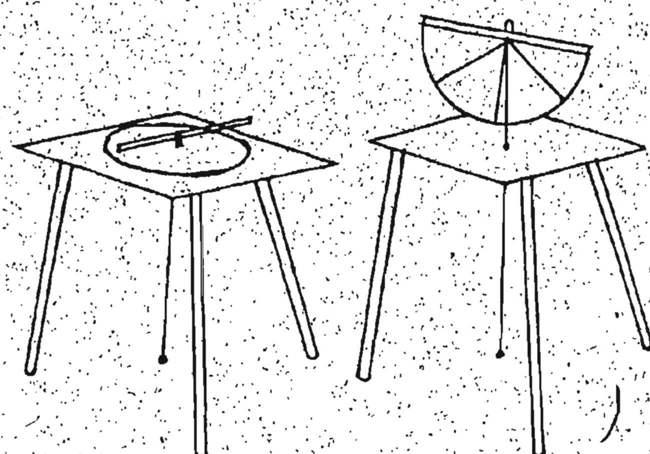


Step 6. Push the screw eye end of the dowel stick through the table top and attach the pointer at point A. Next attach a plumb bob to the screw eye with a piece of string and the transit is ready for measuring horizontal angles.

Step 7. Next make the vertical protractor. The protractor should be drawn on good quality cardboard and then glued or tacked to a thin piece of plywood. Drill a ¼ inch hole at the centre of the straight edge for attaching the protractor to the dowel stick. See diagram.



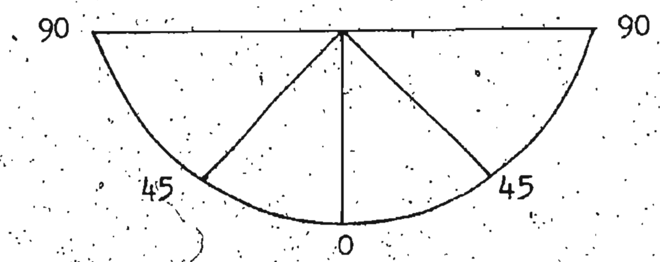
Step 8. Push a short section of ½ inch dowel stick through the 3/4 inch dowel stick at the point B. Now mount the vertical protractor on the short section of ½ inch dowel stick. Finally attach a short plumb bob to the short piece of ½ inch dowel stick and the transit is complete for measuring both horizontal and vertical angles. See diagram of completed transit.



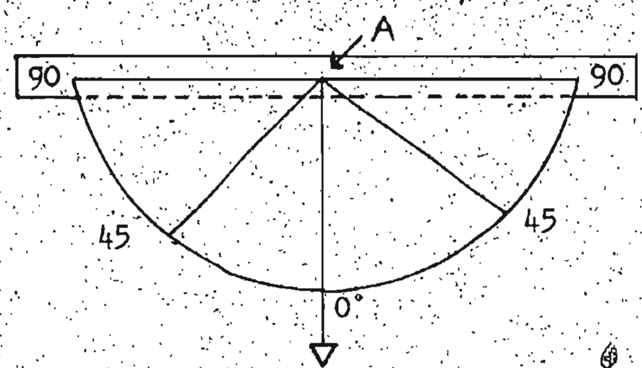
A Clinometer

It is easy to have students make models of hand held clinometers for measuring angles of elevation and depression.

Step 1. Copy a protractor on a piece of stiff cardboard, but put the numbering of the degrees from 0 degrees on the middle to 90 degrees at either end.



Step 2. Take a strip of wood or stiff cardboard with a straight top edge, and attach the protractor to it so that the base line of the protractor is parallel to the top edge of the cardboard.



Step 3. Fix a piece of fine string to the protractor so that the string hangs from point A. Tie a small weight to the other end of the string and the clinometer is ready for use to measure angles of elevation and angles of depression.

<u>How to use the Clinometer</u>

Have the students work in pairs, one student to use the clinometer and the other to note and record the reading. Instruct students to measure first the height of their eye level. Demonstrate the correct way to hold the clinometer. The clinometer should be held at eye level. Sight along the top edge of the piece of cardboard, and tilt the edge up or down until you are looking straight at some object. The angle of elevation or depression is then read while the clinometer is held in position. After a little practice students will soon learn to hold the model with a steady hand. Problems can be solved by making scale drawings or by using measurements and the trigonometric ratios.

CHAPTER 9

MISCELLANEOUS

This section contains a number of items which the mathematics teacher will be able to use from time to time. The items, hopefully, will serve to suggest other ideas which individual teachers will develop further.

Graphing Pictures

Picture possibilities are limited only by the inventiveness of the teacher and the students. Cartoon-type pictures are likely to be more appealing and are easier to adapt to graphing. Almost any simple line drawing, however, can be graphed. It is also easy to assign coordinates to spell out messages. Some examples are, "Merry Xmas", "Happy Easter" and "Be My Valentine".

Method

Duplicate the list of points and announce to the class that lines connecting the points serially will produce a picture or a message. A few items to get you started follow.

The Teacher.

X	Y	X	Y	X	Y
15	-20	-5	30	-15	11
-2	-25	-14	20	-15	13
-25	-40	-6	21	-27	-1

0	-32	-4	19	-25	-2
-5	-30	-6	20	-13	-1
15	-25	-14	20	-11	-7
15	-15	-15	14	-2	-5
20	20	-19	14	-8	-13
15	30	-10	12	0	-15
				-2,	-25

Note. Read the points from top to bottom of each column starting on the left. First point is (15, -20) next point is (-2, -25) and so on to last point which is (-2, -25).

The Kangaroo.

X	Y	X	Y	X	Y
-14	14	30	6	-1	0
-15	11	20	3	-5	0
-17	10	12	4	-5	-4
-11	8	1	-4	-4	-7
2	13	0	-10	-6	-5
17	8	-11	-17	-7	0
26	8	-11	-13	-10	0
37	15	-2	-9	-18	-6
				-21	4
				-22	5
				-20	9
				-14	14

The Cat.

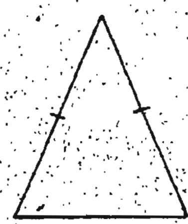
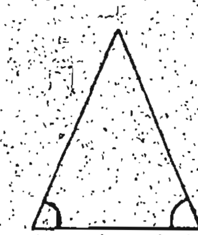
X	Y	X	Y	X	Y	X	Y
-9	0	7	-6	11	-4	-14	6
-11	0	9	-8	11	1	-13	5
-11	-4	8	-12	13	3	-14	4
-9	-6	7	-12	13	7	-15	5
-9	-12	6	-13	7	11	-15	12
-10	-12	7	-14	3	12	-12	7
-11	-13	9	-14	-1	12	-8	7
-10	-14	10	-12	-1	11	-5	12
-8	-14	11	-9	3	7	-5	5
-7	-13	14	-12	7	5	-6	4
-7	-9	13	-12	9	3	-7	5
-5	-12	12	-13	9	0	-6	6
-6	-13	13	-14	-9	0	-5	5
-5	-14	15	-14	-10	1	-5	3
-3	-14	16	-12	-11	0	-9	0
-2	-12	12	-7	-15	3		
-7	-6	12	-5	-15	5		

Symbolized Theorems

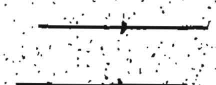
The following simple technique may prove helpful in the teaching of geometry. Students may find this technique a helpful device in stating reasons for geometric proofs. Basically the technique is to state the theorem or postulate or corollary in the "if --------, then --------" form.

Some examples follow.

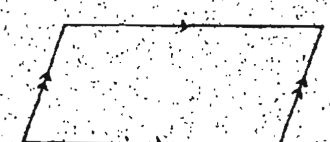
1. If two sides of a triangle are congruent, then the angles opposite these sides are congruent.

If  then 

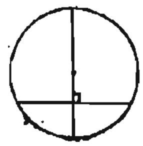
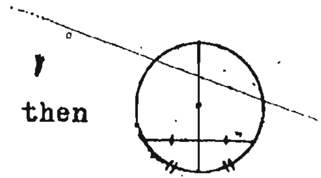
2. If two lines are parallel to each other, then the alternate interior angles are congruent.

If  then 

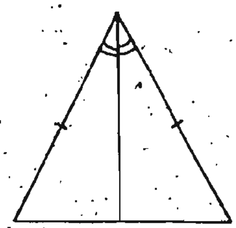
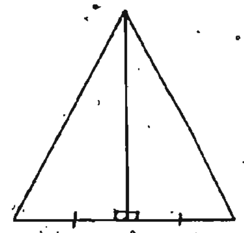
3. If a quadrilateral is a parallelogram, then the opposite sides are congruent.

If  then 

4. If a line passes through the centre of a circle and is perpendicular to a chord of the circle, then it bisects the chord and its arc.

If  then 

5. If a line bisects the vertex angle of an isosceles triangle, it is perpendicular to the base and bisects it.

If  then 

__What Am I__

Here is an idea for review of some unit or units of work. Organize the following activity along the line of a quiz contest or charades. The item to be guessed is described by a series of statements, beginning with some very general description and proceeding to a precise description. The more information required to identify the item the lower the score. The items can be made up by the teacher, or better still, have students submit items as a project. Another suggestion is to have the class divided into two or more teams. The teams then compose the items to ask of the opposing team. Some sample items follow.

Fraction.

a. I am usually thought of as part of a whole. (10 points)

b. My appearance may be changed by division or multiplication without changing my value. (8 points)

c. Sometimes I am less than one. (6 points)

d. Sometimes I am called a ratio. (4 points)

e. Sometimes I am called a per cent. (2 points)

f. Sometimes I am a decimal. (1 point)

Rectangle.

a. I am a geometric figure.

b. I am present in every room.

c. Sometimes I am a square.

d. My opposite sides are parallel.

e. My area is length times width.

f. My four corners are right angles.

Formula.

a. I am a way of expressing relationships.

b. I am used to make predictions.

c. I can be translated into a sentence.

d. I can change my appearance by the multiplication axiom without changing my value.

e. I am an expression of equality made up of symbols, variables, and numbers.

Point.

a. I am a location in space.

b. I have no dimensions.

c. I am usually represented by a small black dot.

d. I am the intersection of two lines.

e. Two of me determine a straight line.

Per Cent.

a. I am used to advertise a sale.

b. I am used very often in a bank.

c. I am really a fraction.

d. I am changed to a decimal before I am used.

e. I am so many out of one hundred.

Euclid.

a. I wrote a famous mathematics book.

b. I lived about 300 B. C.

c. My book is a best seller.

d. My book was written in Greek but has been translated into many languages.

e. My book is about geometric theorems.

Angle.

a. I have no width or length but still can be measured.

b. I am a simple geometric figure.

c. I have only two sides.

d. I have one vertex.

e. I am measured in degrees.

Exponent.

a. In multiplication problems I am added.

b. Sometimes I am called a logarithm.

c. I am a short way to show multiplication.

d. I am usually written small.

e. I tell how many times a number is taken as a factor.

Parabola.

a. I am the locus of points equidistant from a given point and a given line.

b. I am a conic section.

c. I am a curve shaped like the path a ball travels when it is thrown into the air.

d. I am the graph of $y = x^2$.

Zero.

a. Sometimes I am not considered to be a number.

b. I am the last of our ten digits to be invented.

c. I am often used to locate the decimal point.

d. I am not used as a divisor.

e. Sometimes I am called a cipher.

Drills for Signed Numbers

Some teachers may find the following drill exercises time saving as well as a novel way to provide drill where needed. Note that the student works towards a definite and immediate goal in that by applying the correct procedure and knowledge of signed numbers the student will arrive at an accurate solution in the square marked Z.

										Y	
	-5	-3	4	-2	1	-1	0	5	-7	-8	
	-8	2	3	7	-3	0	5	-5	6	7	
	9	-4	-5	8	-2	5	-5	-7	-9	-10	
	4	-3	2	0	4	-1	8	6	2	22	
	1	-1	7	-1	8	2	-2	0	5	19	
	-3	-4	-6	0	-3	1	0	-9	-10	-34	
	7	5	-1	6	-8	3	2	4	-4	14	
	-1	0	6	-7	-4	9	-7	8	3	7	
	6	9	-1	-3	3	-2	0	-6	4	10	
X	10	1	9	8	-4	16	1	-4	-10	27	Z
										Z	

<u>Exercise 1</u>. Duplicate the table but omit the numbers from column Y and row X. The student is directed to add the signed numbers horizontally and place answers in column Y. Then to add vertically and place answers in row X. Finally total column Y and row X to Z.

<u>Exercise 2</u>. Use the same table but omit some of the numbers other than those in column Y and row X and have the student find the missing numbers.

Exercise 3. The same grid can be used to provide multiplication drill. To do this identify the columns as a, b, c, d, ... i, and the rows as j, k, l, ... r. Then hundreds of combinations may be formulated by employing various systems such as the following.

(a) (b) could mean to multiply each value in column a by each value in column b. Thus (5) (-3), (5) (2), (5) (-4), etc., then (-8) (-3), (-8) (2), etc.

Other systems are (a) (c); (a) (d); ------ (h) (i)

and (j) (k); (j) (l); ------ (j) (r)

also (a + b); (c + d); (m-n) (j-k); etc.

Exercise 4. Division combinations can be formulated in a similar manner to that suggested in exercise 3.

Exercise 5. The same grid can be used to provide practice drill in subtraction. The following change in numbers obtained to go in column Y will be (filling in from top to bottom) -2, -23, 28, -14, -17, 28, 0, -9, 2, 3, and for row X (filling in from left to right) -20, -7, -1, -12, 6, -18, -1, 14, -4, 3. Direct students to subtract the signed numbers horizontally to column Y and vertically to row X. Then subtract in column Y and row X to a final result in Z.

Mathematical Terms in Everyday Expressions

The following is a partial list of mathematical terms used in everyday expressions. Teachers and pupils can add to the list. Enjoyable activities can be built around these. Students will enjoy completion quizzes based on this information. Symbols may be placed beside the blanks to be completed as an aid to recollection of the correct term. Such a quiz can incorporate spelling and vocabulary as well as knowledge of mathematical terms. The writer leaves it to the teacher to decide where a blank is to be used.

Going around in circles	Wage scale
A vicious circle	He's square
Human equation	Eternal triangle
To go off on a tangent	Ice cream cone
Fraction of a second	Master's degree
Cube steaks	100 per cent right
Square dance	Absolutely wrong
Round steak	Batting average
Volume of business	Average Joe
Sphere of influence	The great divide
Circle of friends	Every inch a sailor
Prime roast of beef	Every inch a king
Acute appendicitis	Ceiling zero
Equal rights	Exponent of justice

Square knot

Zero hour

Countdown

Hot rod

Square deal

Double date

Pizza Pie

Times are tough

Tea for two

Straight and narrow way

Negative attitude

Positive attitude

Pentagon building

Sum and substance

X marks the spot

Capacity audience

Telegraph

Set an example

Cheaper by the dozen

Plus fours

Crossing the Bar

Joan of Arc

Rule of thumb

Baby formula

First degree murder

The great unknown

Out of all proportion

Lost chord

Welcome sign

Divide and conquer

Line of least resistance

Point of order

Rule the roost

Hard times

Octagon box

Split second

Safety in numbers

Literal translation

First sign of Spring

Constant as the North Star

Take an interest in

Integral part

A cat has nine lives

Friday the thirteenth

The three muskateers

2 is company, 3 is a crowd

One's better half

At the 11th hour

Find all the angles	The dirty thirties
Intelligence quotient	The gay nineties
The Golden Rule	The ten commandments
20,000 leagues under the sea	The seven wonders of the world
The night has 1000 eyes the day but one	One in the hand is worth two in the bush
1492 Columbus sailed the ocean blue	Louis XIV furniture
The seven hills of Rome	

A Matching Quiz of Geometrical Terms

U	1. A broken angle	A.	Secant
F	2. Place where people are sent for committing crimes	B.	Transformation
W	3. A beast	C.	Decagon
C	4. A lost phonograph record	D.	Polygon
Q	5. A clever angle	E.	Pi
R	6. What one old farmer said to another	F.	Prism
V	7. An angle that is never wrong	G.	Postulate
M	8. Used to tie up packages	H.	Sphere
S	9. That man does not talk plainly	I.	Geometry
L	10. What girls like to find at the beach	J.	Sector
K	11. They voted "yes" on tractors for Cuba	K.	Protractor
E	12. Printer's dessert	L.	Tangent

H	13.	A sharp weapon	M.	Chord
I	14.	What little acorns say when they grow up	N.	Centre
Y	15.	The one in charge	O.	Coincide
G	16.	What the forgetful professor did with the letter he carried for a week before mailing	P.	Hypotenuse
N	17.	What the husband did when his mother-in-law wanted to go home	Q.	Acute angle
O	18.	What a person should do when it rains	R.	Loci
T	19.	The way the poet wrote his love letters	S.	Ellipse
D	20.	A dead parrot	T.	Inverse
			U.	Rectangle
			V.	Right angle
			W.	Line
			X.	Hexagon
			Y.	Ruler
			Z.	Triangle

<u>The Geometrical Romance of Miss Poly Hedron</u>

Have students fill in the words that are underlined. The first letter of each word might be given as a clue. An alternate method would be to give no clue but to give the answers in a jumbled list and have students pick the correct words to fill the blanks. In any event the piece should be useful for general review or for a change in the regular classroom routine.

Miss Poly Hedron was acute girl. She had a sweet face but sum called her a round number because she had a figure of good proportions. Her dress was always stylish to a high degree. One day she met Mr. Octagon at her office in the business sphere. Mr. Gon was very tall and angular but had an erect bearing. It was a case of love at first sight.

He called on Miss Poly on consecutive evenings and they would converse about their common interests at length. He said many complimentary things to her and when he left he said that he would return in thirty days. She thought it would be very romantic to correspond with him when he was at a distance. She was sure it would terminate after some courtship, in a proposition that they would be married and have an extended honeymoon. On their return they would construct a house adjacent to her mother's on a fashionable square. Time elapsed, however, with no letter from Mr. Gon. Miss Poly was sad; lines appeared in her face; she refused to eat pi, and every now and then she heaved a loci. One day while out working she felt inclined to sit down at the base of a pine tree and gather cones. Just at this point a messenger brought a telegram inscribed with her name.

It announced that Mr. Gon had left her social circle and had gone to prism because his radical political views were opposite to those of the ruler of his sector. Her father said Mr. Gon was a tangent who had always been a locus.

CHAPTER 10

BIBLIOGRAPHY

The following is a list of books recommended for mathematics at the secondary school level. The prices may not be current. This information should be useful, however, to teachers when placing orders. A directory of Canadian distributors is provided in a section following this bibliography.

Alder, Irving. Mathematics: The story of numbers, symbols, and space. Golden, 1960, 55 pp., $3.95.
A brief treatment of such topics as number, symbols, regular polygon, mathematics in nature, mathematics in music and probability, to show the scope of mathematics.

-------------. Logic for beginners. Day, 1964, 158 pp., illus., $4.50.
Examines how to draw a conclusion from what is known and how to arrive at reasons for a known conclusion. Games, jokes and puzzles are used as illustrations. Brain teasers with answers.

-------------. Time in your life. Day, 1955, 127 pp., $3.75.
Man's reliance upon the sun, moon and stars as indicators of time; early and modern time pieces; the history of the calendar; time zones; and other important influences of time in our lives are considered.

-------------. Reading in mathematics. NCTM, 1972, 182 pp., $5.00.
Writing old and new, showing mathematics in use. Accumulated expressly for junior and senior high school student pleasure.

------------. Magic house of numbers. Day, 1957, 128 pp., illus., $3.75.
Mathematical curiosities and riddles hold interest, but more than mere entertainment makes this primer an excellent introduction to the whole basis of our number system.

Asimov, Isaac. Breakthroughs in science. Houghton, 1960, 197 pp., $4.50.
Brief, terse biographies of people famous for their contributions toward progress in knowledge. Among these are Archimedes, Galilee, Newton and Einstein.

------------. Science, numbers, and I. Doubleday, 1968, 226 pp., $4.95.
An interesting series of essays on a variety of subjects. Those in the section on "number" deal with the ratios in musical scales and harmony, the statistics of great rivers and cities, antipodal points on the earth, the paradoxes of time created by travelling across the date line, and number mysticism.

------------. Realm of numbers. Houghton, 1959, 200 pp., $3.50.
For the reader with a background in simple mathematics acquired in elementary school, this is an introduction of mathematical principles and their evolution. The discussion begins with finger counting and progresses through square root, logarithms, rational and irrational numbers and on to infinity.

Bakst, Aaron. Mathematical puzzles and pastimes. Van Nostrand, 1954, 206 pp., $6.95.
Elementary mathematics presented with a light touch through a large variety of mathematical recreations with solutions.

Barnard, D. St. P. A book of mathematical and reasoning problems; fifty brain teasers. Van Nostrand, 1963, 109 pp., $2.50.
A collection of fifty reasoning problems. Special sections provide leads or hints for solutions. The solutions generally stress the mathematical nature of the reasoning involved.

-------------. One hundred braintwisters. Van Nostrand, 1967, 168 pp., $3.95.
A continuation of the collection in the author's fifty braintwisters. Also includes leads and solutions.

Barr, Stephen. A miscellany of puzzles, mathematical and otherwise. Crowell, 1965, 164 pp., $3.50.
A varied and delightful collection of 62 puzzles to be solved by folding or cutting paper; with pencil and paper; with the use of arithmetic, algebra, or geometry; or by common sense. Answers that provide insight into method are included.

Beckhard, Arthur. Albert Einstein. Putnam, 1959, 126 pp., $3.50.
The life of the great mathematician-scientist who played an important part in creating the atomic age, discussed simply and effectively.

Bell, Eric T. Men of mathematics. Simon, 1937, 592 pp., $7.95.
Biographies of men who have invented and influenced mathematics.

Bendick, Jeanne. How much and how many. McGraw, 1947, 188 pp., $3.25.
This story gives both the historical background and modern applications of weights and measures. It is an excellent source book.

Bergamini, David, and the Editors of Life. Mathematics. Time, 1970, 200 pp., $5.70.
Survey of the history of some aspects of mathematics, including its role in art and nature. Chapters are devoted to the calculus, probability and chance, topology and outstanding mathematicians.

Degrazia, Joseph. Math is fun. Emerson, 1965, 159 pp., $4.95.
A very good book. Collection of 196 puzzle type problems complete with solutions. The majority of the problems should appeal to alert high school students.

Diggins, Julia E. String, straight-edge and shadow. The story of geometry. Viking, 1965, 160 pp., $4.50.
The story of how man's practical need to measure and his curiosity about order in the universe led to the development of geometry.

Dudeney, Henry E. Amusements in mathematics. Rev. ed. Dover, 1958, 258 pp., $2.50.
About 400 puzzles, problems, paradoxes and brain-teasers, both old and new classified under such topics as arithmetic, algebra, geometry and games.

Emmett, E. R. Brain puzzler's delight. Emerson, 1970, 254 pp., $6.00.
One hundred puzzles of varying types that are arranged in order of difficulty. Solutions included.

Gardner, Martin. Perplexing puzzles and tantalizing teasers. Simon, 1969, 95 pp., illus., $3.95.
Ridiculous riddles, unusual puzzles, and tricky questions that challenge and entertain. All are interestingly presented in text and drawing. Solutions included.

Geometric Design Posters. A set of two posters. One poster contains 28 designs from Line Designs. The second poster contains 78 figures from Creative Constructions. Available from Creative Publications.

Glenn, William H., and Johnson, Donovan H. Exploring mathematics on your own. Doubleday, 1960, 303 pp., illus., $4.95.
A book of recreational mathematics that includes an introduction to recent developments as well as a consideration of classical concepts.

Halacy, Dan. Charles Babbage-Father of the computer. Macmillan, 1970, 170 pp., $4.95.
A concise but well-told story of the life of a man who conceived, designed and began building in 1820 a calculating machine that could have advanced the computer age by 100 years. His machine could be programmed in advance, had a large memory of numbers, did all work automatically and printed out answers.

Heafford, Philip. The math entertainer. Emerson, 1959, 176 pp., $4.95.
A set of fifty entertaining quizzes with answers.

Hess, Adrien L. Mathematical projects handbook. NCTM, 1977, 48 pp., $2.00.
Useful guide for junior and senior high school teachers and students in choosing and developing projects. It has an extensive bibliography.

Hoghen, Lancelot. The wonderful world of mathematics. Doubleday, 1968, 69 pp., illus., $3.95.
The growth and development of mathematics through the ages described in story and pictures.

Holt, Michael and Marjoram, D. T. E. Mathematics in a changing world. Walker, 1973, 293 pp., $10.00.
The book brings clearly to mind the fact that mathematics is a part of every facet of life in today's society. The application of mathematical models to many of the current societal problems is the major thrust of the book.

Hunter, J. A. H., and Madachy, Joseph S. Mathematical diversions. Van Nostrand, 1963, 178 pp., $4.95.
A collection of almost a hundred problems classified under such headings as paradoxes, mystic arrays, topological delights and teasers. Solutions included.

Johnson, Donovan A. Excursions in outdoor measurement. Walch, 1974, 121 pp.
A treasury of ideas to interest students in measurement, the tools of measurement and the mathematics needed for successful measurement. Suggestions are complete for making many of the instruments unavailable in the average classroom.

Judd, Wallace. Games, tricks and puzzles for a hand calculator. Dymax, 1974, 91 pp.
Contains an assortment of trick calculations, messages spelled upside down, number facts and relationships, puzzles, games and problems. The format is designed to challenge the reader to work out the tricks and puzzles. The book is easy to read, with many diagrams, and should appeal to students.

Kline, Morris. Mathematics and the physical world. Crowell, 1959, 482 pp., $6.00.
Mathematics is viewed in its relationship to the physical sciences as the basic tool of research scientists. The role of mathematics in the study of nature is surveyed in all phases of the subject from arithmetic to calculus.

Kondo, Herbert. Albert Einstein and the theory of relativity. Watts, 1969, 182 pp., $4.50.
A biography that chronicles the most important scientific advancements with personal events in the life of Mr. Einstein.

Lloyd, Sam. The eighth book of tan. Dover, 1968, 52 pp., $1.25.
A collection of 700 tangrams with solutions. The introduction contains some history of tangrams and other puzzles.

Lenchner, George. The overhead projector in the mathematics classroom. NCTM, 1974, 31 pp., $1.80.
Emphasizes applications in teaching mathematics and describes techniques for making effective use of the aid. Color illustrations and bibliography.

Margenare, James and Sentlowitz, Michael. How to study mathematics. NCTM, 1977, 32 pp., $1.50.
A readable and appealing self-help for the struggling but earnest junior or senior high school student; contains captivating cartoons and a list of diagnoses and prescriptions.

Menninger, K. W. Mathematics in your world. Viking, 1962, 291 pp., illus., $5.00.
Not a mathematics book but a narrative account of the role that mathematics plays in human everyday life. This is for the student and average laymen who would like a little "recreation" with his learning.

Muir, Jane. Of men and numbers: The story of the great mathematicians. Dodd, 1961, 249 pp., illus., $3.50.
Contains succinct, informative biographical sketches of 12 of the more famous mathematicians, emphasizing their contributions to the pure science rather than personal anecdotes. Portrayed are Pythagoras, Euclid, Archimedes, Descartes, Cardano, Pascal, Newton, Euler, Gauss, Lobatchevski, Galois and Cantor.

Razzell, Arthur G. and Watts, K. G. O. Probability: The science of chance. Doubleday, 1967, 47 pp., $3.25.
A well-illustrated text that uses interesting situations to show how even chance events fall into patterns and how these patterns form the basis for predictions. Some exercises are suggested.

Reid, Constance. From zero to infinity: What makes numbers interesting. Crowell, 1965, 145 pp., illus., $3.95.
Each of the ten digits, zero through nine, is discussed individually, showing its historical development, its unique characteristics, and its particular usefulness in the everyday world.

Rogers, James T. Story of mathematics for young people. Pantheon, 1966, 127 pp., $4.95.
The text and numerous pictures telescope the history of mathematics into an understandable visual story. The accomplishments of great mathematicians in relation to the culture in which they lived is a highlight of this easy-to-read book.

Rusch, Richard B. Man's marvelous computer: The next quarter century. Simon, 1970, 128 pp., $4.95.
Explains the history and operation of computers, and describes some of their current accomplishments and predicts some future uses.

Salkind, Charles T. and Earl, James M. The MAA problem book 111. Random, 1973, 186 pp., $2.12.
Contains contest problems based on the regular high school curriculum and calls for no more than intermediate algebra and plane geometry. Problems call for a certain amount of insight and ingenuity.

Schaaf, William L. Mathematics in use, as seen on postage stamps. NCTM, 1976, $2.00.
This is a set of eight colorful posters, each portraying international stamps representing a particular use of mathematics. Each is 37 cm by 28 cm.

―――――――. The high school mathematics library. NCTM, 1976, 80 pp., $3.00.
Sixth edition of popular bibliography reflects new trends and needs, with a section on metrication and wider coverage for computers, data processing, geometry, expository and recreational mathematics, the mathematically gifted and professional books for teachers.

Seymour, D. and Schadler, R. Creative constructions. Creative, 1974.
Creative constructions is a design and construction book to complement Line Designs. It contains more than 250 designs all constructed with straight-edge and a compass.

Seymour, D., Silvey, L., and Snider, J. Line Designs. Creative, 1974.
Line design is an 80 page construction and design book to complement activities from Creative Constructions. The book contains more than 100 designs all made with straight lines using the technique commonly known as "curve stitching".

Silverberg, Robert. Clocks for the ages-how scientists date the past. Macmillan, 1971, 238 pp., $5.95.
History of the development of techniques of measurement and of the units of measure used by scientist to date ancient artifacts and natural objects such as rocks.

Sobel, M. A. & Maletshy, E. M. A sourcebook of aids, activities, and strategies. Prentice - Hall, Inc., 1975, 235 pp.
The title is descriptive of the contents.

Steinhaus, H. Mathematical Snapshots. Oxford, 1969, 266 pp., $7.50.
Discussions and illustrations of some applications of mathematics are included. Models of many of the objects described can be made and studied.

Vergara, William C. Mathematics in everyday things. Harper, 1959, 301 pp., $5.95.
Through questions and answers the author relates the principles of mathematics to everyday objects and occurrences. He also discusses some mathematical concepts that are interesting in their own right. Among the questions are "which rectangle is the most beautiful?" "Is the number 13 unlucky?" "What is the principle behind whispering galleries?" and "How fast do meteors travel?"

Walters, Helen B. Wernher Von Braun; Rocket engineer. Macmillan, 1964, 185 pp., $4.95.
The biography of a rocket engineer who from boyhood was driven by an insatiable curiosity to explore space. His difficulties and successes with mathematics and physics are detailed.

Winter, John. String sculpture. Creative, 1972.
String sculpture explains the fundamentals of creating string sculpture designs formed on geometric outlines. It proceeds from simple two-dimensional designs through complex three-dimensional figures employing intersecting planes. The book is a natural extension of Line Design.

The following is a list of some of the many films currently available, which are appropriate for secondary mathematics. Consult chapter 11 for the name and address of the producer.

How Man Learned to Count. CCM, B & W, 30 min.

Features Bil Baird and his puppets. A trip through history to see how arithmetic developed from the cave man to the present. Demonstrates the counting systems of Egypt, Rome, Phoenicia, Carthage and India and how they contributed to our modern number system.

How's Chance. CCM, B & W, 30 min.

Explains how to arrive at a probable total by taking a representative sampling of the whole and projecting the percentages. Demonstrates how mathematics is used in business, industry and government to determine vital advance predictions of probable totals.

Mysterious "X". CCM, B & W, 30 min.

Bil Baird and his puppets demonstrate everyday applications of algebra and show how the great scientists use its terms to explain the phenomena of nature.

Sine Language. CCM, B & W, 30 min.

Bil Baird and his puppets throw some light on the study of trigonometry, by showing some of the everyday uses of trigonometry and its importance in navigation, land surveying and electrical currents.

What's the Angle? CCM, B & W, 30 min.

The fundamental concepts of plane and solid geometry are explained. Shows how this science was used and developed by the Egyptians, Chinese and Greeks and how this science is used today in architecture, construction, surveying and navigation.

Inequality. TLF, B & W, 20 min.

This film looks at a graphic presentation of Olympic championship performances and poses such questions as the following. How should we plot the performance of other finalists? Where would we seek world record performances?

Algebra. WARD, 13 min.

This is a pre-algebra film which discusses the use of symbols to express and develop mathematical statements of relationships. It introduces the concept of algebraic equations and gives an example of the simplification of an equation to obtain a clearer statement of relationships.

Developing the General Equation of a Circle. BFA, 12 min.

This film, partially designed by computers, shows the relationships between right triangles and circles where the radius of the circle is the hypotenuse of the triangle. The development of this relationship through a series of visually exciting sequences leads to the discovery of the general equation of the circle.

Geometric Concepts. WARD, 10 min.

Introduces the concept of point, line, line segment, ray, angle, triangle, simple closed curve, inside and outside, plane, and polyhedron. The five regular solids are formed from patterns in the plane and are rotated into various positions by computer animation.

A Geometry Lesson. CCM, B & W, 13 min.

This prize winning film relates principles of geometry to everyday existence by showing the geometric bases of natural forms and by indicating how geometry functions in art and science.

Possibly So, Pythagoras! IFB, 14 min.

Investigates this theorem through inductive experimentation as well as through formal deductive proof.

By means of motion picture animation the student can watch triangles continually change.

<u>Similar Triangles in Use</u>. IFB, 11 min.

The practical value of knowing that corresponding sides of similar triangles are proportional is clearly presented. The film shows how measurement of triangles is used to determine the distances between things we cannot easily reach. The use of the surveyor's quadrant and sextant is shown.

The following is a short list of the many filmstrips currently available and appropriate for secondary mathematics. The professional magazine <u>The Mathematics Teacher</u> carries reviews of filmstrips as a regular feature. Consult chapter 11 for the address of distributors.

<u>The Aha! Box</u>. Scientific American, Inc., 1977, $135.

Set of six color filmstrips with accompanying audio-tape cassettes. There is an extensive teacher's guide. The problems will serve as a fine resource to classroom teachers concerned with problem-solving and related strategies.

<u>Math for the Young Consumer</u>. Educational Activities, Inc., 1976, $56.

Set of four color filmstrips with accompanying audiotape cassettes. The set has practical lessons in consumer mathematics. Actual brand products are shown and compared. Teacher's guide.

<u>The Metric System</u>. Harper and Row, $125.

Set of six color filmstrips with accompanying audiotape cassettes. Each filmstrip is a take off on a well-known movie. Sound is good. Teacher's guide.

<u>Donald in Mathmagic Land</u>. Walt Disney Educational Media Company, 1976, $71.

Set of four color filmstrips with accompanying audiotape cassettes. The filmstrips are an adaptation from the movie of the same name. Animation, narration and pace are excellent. Teacher's guide.

<u>History of Measurement</u>. Walt Disney Educational Media Company, 1976, $88.

Set of five color filmstrips with accompanying audiotape cassettes. This is a motivational set of filmstrips, presented in the usual high-quality Disney style. Teacher's guide.

CHAPTER 11

A DIRECTORY OF PUBLISHERS AND CANADIAN AGENTS

The writer advises ordering from the Canadian agent where possible. For convenience, the name and address of the Canadian agent is given immediately following the address of the publisher. Agencies are changed frequently, but addresses were correct at the time this section of the book was prepared.

Crowell	Thomas Y. Crowell Co. 201 Park Ave., South New York, N. Y. 10003
	The Copp Clark Publishing Co., Ltd. 517 Wellington St., West Toronto 2B, Ontario
Creative	Creative Publications P. O. Box 10328 Palo Alto CA 94303
	Edu-Media Limited 1 Adam Street P. O. Box 1240 Kitchener, Ontario N2G 4H1
Day	The John Day Co. Inc. 200 Madison Ave. New York, N. Y. 10016
	Longmans Canada Limited 55 Barber Greene Road Don Mills, Ontario

Dodd	Dodd, Mead & Co. 79 Madison Ave. New York, N. Y. 10016 Dodd, Mead & Co., (Canada) Ltd. 25 Hollinger Road Toronto 16, Ontario
Doubleday	Doubleday & Co., Inc. 501 Franklin Ave. Garden City, N. Y. 11530 Doubleday Publishers 105 Bond Street Toronto 2, Ontario
Dover	Dover Publications, Inc. 180 Varick Street New York, N. Y. 10014 General Publishing Co., Ltd. 30 Lesmill Road Don Mills, Ontario
Dymax	Dymax P. O. Box 310 Menlo Park CA 94025 USA
Emerson	Emerson Books, Inc. 251 W. 19th Street New York, N. Y. 10011 Smithers & Bonellie Ltd. 56 Esplanade Street Toronto 1, Ontario
Golden	Golden Press Inc. 150 Parish Dr. Wayne, N. J. 07470 The Copp Clark Publishing Co., Ltd. 517 Wellington St., West Toronto 2B, Ontario

Harper	Harper & Row, Publishers 49 E. 33d Street New York, N. Y. 10016
	The Copp Clark Publishing Co., Ltd. (Address listed previously)
Houghton	Houghton Mifflin Co. 2 Park Street Boston, Mass. 02107
	Thomas Nelson & Sons (Canada) Ltd. 81 Curlew Drive Don Mills, Ontario
McGraw	McGraw-Hill Book Company 205 West 42nd Street New York, N. Y. 10036
	McGraw-Hill Ryerson Limited 330 Progress Avenue Scarborough, Ontario
MacMillan	The MacMillan Co. of Canada Ltd. 70 Bond Street Toronto 2, Ontario
NCTM	National Council Of Teachers of Mathematics 1906 Association Drive Reston, Virginia 22091
Oxford	Oxford University Press Inc. 16-00 Pollett Dr. Fair Lawn, N. J. 07410
	Oxford University Press 70 Wynford Drive Don Mills, Ontario
Pantheon	Pantheon Books, Inc. 33 W. 60th Street New York, N. Y. 10023

	Random House of Canada Ltd. 10 Vulcan Street Rexdale, Ontario
Prentice	Prentice-Hall, Inc., Rte. 9W Englewood Cliffs New Jersey. 07632
	Prentice-Hall of Canada Ltd. 1870 Birchmount Road Scarborough, Ontario
Putnam	G. P. Putman's Sons 200 Madison Avenue New York, N. Y. 10016
	Longmans Canada Limited 55 Barber Greene Road Don Mills, Ontario
Random	Random House, Inc. 30 W. 60th Street New York, N. Y. 10023
	Random House of Canada Ltd. 10 Vulcan Street Rexdale, Ontario
Simon	Simon & Schuster, Inc. 630 Fifth Avenue New York, N. Y. 10020
	The Copp Clark Publishing Co., Ltd. (Address already given)
Time	Time-Life Books Time & Life Bldg. New York, N. Y. 10020
	General Learning Corporation 115 Nugget Avenue Agincourt, Ontario

Van Nostrand	D. Van Nostrand Co. 120 Alexander Street Princeton, N. J. 08540
	McClelland and Stewart Ltd. 25 Hollinger Road Toronto 13, Ontario
Viking	Viking Press 625 Madison Avenue New York, N. Y. 10022
	The MacMillan Co., of Canada Ltd. 70 Bond Street Toronto 2, Ontario
Walch	J. Weston Walch Portland, Maine
Walker	Frank R. Walker Co. 5030 N. Harlem Avenue Chicago, Ill. 60656
Watts	Franklin Watts 575 Lexington Avenue New York, N. Y. 10022

Producer/Distributor Directory for Films

BFA	BFA Educational Media 2211 Michigan Avenue Santa Monica, California 90404
CCM	See MACM
IFB	International Film Bureau 332 South Michigan Avenue Chicago, Illinois 60604

MACM	Macmillan Films 34 MacQuesten Parkway South Mount Vernon, New York 10550
TLF	Time Life Films 100 Eisenhower Drive Paramus, New Jersey 07652
WARD	Wards Natural Science Establishment, Inc. Modern Learning Aids Division P. O. Box 1712 Rochester, New York 14603

Filmstrip Distributor Directory

Scientific American, Inc.
415 Madison Ave.
New York, N. Y. 10017

Educational Activities, Inc.
Freeport, N. Y. 11520

Harper and Row
2500 Crawford Ave.
Evanston, Ill. 60201

Walt Disney Educational Media Company
500 South Buena Vista Street
Burbank, CA 91521

ACKNOWLEDGEMENTS

Puzzle problems #1 - #16. From Yes, math can be fun. Portland, Maine: J. Weston Walch, 1960, by L. G. Brandes.

Puzzle problems #17 - #20. From Math is fun. New York: Emerson Books, Inc., 1965, by J. Degrazia.

Puzzle problem #21. From Enrichment mathematics for the grades, 27th yearbook. Washington: NCTM, 1963.

The kangaroo, The cat. From The mathematics teacher. Vol., 52, pp. 41-43.

Symbolized theorems. From The mathematics teacher. Vol., 52, p. 107.

What am I. From The mathematics teacher. Vol., 52, pp. 130-131.

Drills for signed numbers. From The mathematics teacher. Vol., 52, pp. 247-249.

Mathematical terms in everyday expressions. From The mathematics teacher. Vol., 50, p. 517.

A matching quiz of geometrical terms. From Creative geometry experiments. Troy, Michigan: Midwest Publications Co., Inc., 1970, by D. A. Backeye.

The geometrical romance of miss poly hedron. From <u>Games for learning mathematics</u>. Portland, Maine: J. Weston Walch, 1960 by D. A. Johnson.

Other Topics for projects. Suggested by Wilbert Boone, Beothuk Collegiate, Baie Verte, Newfoundland.

REFERENCES

AAAS Science Film Catalog. New York & London: R. R. Bowker Company, 1975.

Blazek, R. D. Teacher utilization of nonrequired library materials in mathematics and the effect on pupil use. (Doctoral dissertation, university of Illinois at Urbana-Champaign, 1971). Dissertation Abstracts International, 1971, 32A, 994.

Gray, T. M. A field study of mathematics laboratory development in Youngstown, Ohio (Doctoral dissertation, University of Pittsburg, 1973). Dissertation Abstracts International, 1973, 34B, 1184 - 1185.

Hardgrove, C. E., & Miller, H. F. Mathematics Library Elementary and Junior High School. Reston, Virginia: The National Council of Teachers of Mathematics, 1973.

Johnson, D. A. Instructional materials in the mathematics classroom. In D. B. Aichele & R. E. Rays (Eds.), Readings in secondary school mathematics. Boston: Prindle, Weber & Schmidt, Inc., 1971.

Johnson, D. A., & Rising, G. R. Guidelines for teaching mathematics. Belmont, California: Wadworth Publishing Company, Inc., 1967.

Moldstad, J. A. Selective review of research studies showing media effectiveness: a primer for media directors. AV Communication Review, 1974, 22, 387-407.

Purser, J. D. The relation of manipulative activities, achievement and retention, in a seventh-grade mathematics class: an exploratory study (Doctoral dissertation, University of Georgia, 1973). Dissertation Abstracts International, 1973, 34A, 3255-3256.

Sherrill, J. M. The effects of differing presentations of mathematical word problems upon the achievement of tenth grade students (Doctoral dissertation, The University of Texas at Austin, 1970). Dissertation Abstracts International, 1971, 31A, 3427.

Sobel, M. S., & Maletsky, E. M. Teaching mathematics: A sourcebook of aids, activities and strategies. Toronto: Prentice-Hall of Canada, Ltd., 1975.

The AAAS Science Book List, 3rd Edition. Baltimore, Maryland: George W. King Printing Co., 1970.

APPENDIX B

RESOURCE UTILIZATION QUESTIONNAIRE

RESOURCE UTILIZATION QUESTIONNAIRE

1. Name (optional) ______________________________

2. Name of school (optional) ______________________

3. School address (optional) ______________________

____

4. Number of years experience in teaching this subject

____

5. May I quote you? yes no

6. Do you have access to any professional material pertaining to the teaching of your specific subject area?

yes no

If yes, please list a. ______________

b. ______________

c. ______________

7. Do you think that there is a need for more emphasis on resources based teaching in this subject?

yes no

additional comments ______________________

____

____

8. The following is a partial list of resource based materials. Use the appropriate letter of the legend to indicate your use of each:

LEGEND

Use	How often used	Letter indication
Constant	weekly	A
Occasional	once a month	B
Seldom	2-3 times a year	C
Never	nil	D

Materials	Letter	Materials	Letter
Motion picture, 8 mm	____	Vertical file	____
Motion picture, 16 mm	____	Newspapers	____
Filmstrips	____	Study prints	____
Slides	____	Models	____
Records	____	Posters	____
Tapes	____	Pictures	____
Television	____	Charts	____
Radio	____	Globes	____
Transparencies	____	Simulation games	____
Books	____	Programmed instruction	____
Periodicals	____	Others	____

9. Could you please describe briefly some of the specific ways in which you used the items checked A and B in question 8.

Television ______________________________

____

____

Radio ______________________________

____

____

Transparencies ______________________________

____

____

Books ______________________________

____

____

Periodicals ______________________________

____

____

Vertical file ______________________________

____

____

Newspapers ______________________________

____

____

Motion picture, 8 mm ______________________________

____

Motion picture, 16 mm ______________________

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Filmstrips ______________________

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Slides ______________________

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Records ______________________

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Tapes ______________________

____

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Study prints ______________________

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Models ______________________

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Posters ______________________

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Pictures ______________________

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Charts ____

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Globes ____

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Simulation games ______________________________________

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____

Programmed instruction ________________________________

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____

Others ____

____

____

Any additional comments?

I appreciate your time and effort in completing this questionnaire. Thank You.

APPENDIX C

FORMAL EVALUATION QUESTIONNAIRE

FORMAL EVALUATION QUESTIONNAIRE

QUESTIONNAIRE

This questionnaire is divided into THREE sections. Please complete each section to the best of your ability.

SECTION A: GENERAL INFORMATION

1. Are you presently teaching secondary mathematics?

   Yes ____ No ____

2. If you answered NO to question one, have you taught secondary mathematics? How long?

   Yes ____, ____ Years. No ____

3. Total number of years teaching secondary mathematics.

   ____ Years.

4. Teaching certificate held. Grade II or below ____

   Grade III ____ Grade IV ____ Grade V ____

   Grade VI ____ Grade VII ____

SECTION B: RATING OF CHAPTER PURPOSES

In this section you are requested to rate each of the chapters of the handbook on the purposes stated for that chapter. Your rating will indicate the degree to which you feel the purposes have been achieved. You are asked to rate each item on a "five-point" scale as follows:

poor	fair	good	very good	excellent
1	2	3	4	5

Please indicate your rating for each item by circling your choice.

Chapter 1. Introduction

1. To point out the support of research regarding the use of various instructional materials in teaching secondary mathematics. 1 2 3 4 5
2. To call attention to the need for the handbook and to state the two major objectives of the handbook. 1 2 3 4 5

Chapter 2. Films

1. To point out the usefulness of 16 mm films in teaching mathematics. 1 2 3 4 5
2. To point out the limited supply of secondary mathematics films presently available in this province and to suggest what teachers can do to increase this supply. 1 2 3 4 5

3. To make a number of specific suggestions regarding the use of films in the classroom. 1 2 3 4 5

Chapter 3. Filmstrips

1. To point out the advantages of 35 mm filmstrips, call attention to the available supply of filmstrips and to make specific suggestions regarding their classroom use. 1 2 3 4 5

Chapter 4. The Overhead Projector

1. To point out a number of specific uses for the overhead projector in the classroom. 1 2 3 4 5

Chapter 5. Mathematical Games

1. To identify a number of roles for mathematical games in secondary mathematics. 1 2 3 4 5
2. To enumerate a number of points concerning the use of mathematical games. 1 2 3 4 5
3. To describe a number of appropriate games for secondary mathematics. 1 2 3 4 5

Chapter 6. Puzzle and Recreational Problems

1. To enumerate a number of suggestions for the use of puzzle and recreational problems in secondary mathematics. 1 2 3 4 5
2. To provide a number of appropriate puzzle and recreational problems. 1 2 3 4 5

Chapter 7. Mathematical Projects

1. To discuss the purposes of mathematical projects and to make some suggestions for developing student interest in projects. 1 2 3 4 5
2. To provide a number of specific suggestions for appropriate student projects. 1 2 3 4 5

Chapter 8. Models

1. To point out a number of uses for models. 1 2 3 4 5
2. To provide descriptions for a number of appropriate models for secondary mathematics. 1 2 3 4 5

Chapter 9. Miscellaneous

1. To provide a number of miscellaneous items appropriate for secondary mathematics. 1 2 3 4 5

Chapter 10. Bibliography

1. To provide an annotated bibliography of books and materials recommended for secondary mathematics. 1 2 3 4 5

Chapter 11. Directory of Publishers and Agents

1. To provide a directory of publishers and their Canadian agents for the items recommended in the bibliography. 1 2 3 4 5

SECTION C: COMMENTS AND SUGGESTIONS FOR IMPROVEMENT

In this section you are requested to please give your specific comments regarding the strong points OR weak points of each of the chapters or section of the chapter. Kindly include also any suggestions you may have for the improvement of chapters or chapter sections. If space allotted is insufficient, please use the blank side of the sheet for additional comments.

Chapter 1. Introduction
Comments/Suggestions

Chapter 2. Films

Comments/Suggestions

Chapter 3. Filmstrips

Comments/Suggestions

Chapter 4. The Overhead Projector

Comments/Suggestions

Chapter 5. Mathematical Games

Comments/Suggestions

Chapter 6. Puzzle and Recreational Problems

Comments/Suggestions

Chapter 7. Mathematical Projects

Comments/Suggestions

Chapter 8. Models

Comments/Suggestions

Chapter 9. Miscellaneous

Comments/Suggestions

Chapter 10 and 11. Bibliography and Directory

Comments/Suggestions

APPENDIX D

CORRESPONDENCE

LETTER TO RESPONDENTS

P. O. Box 18
Creston, Nfld.
AOE 1KO
March 9, 1979

Dear

I would greatly appreciate it, if you could find the time over the next two weeks to read the enclosed handbook and complete the questionnaire.

I am preparing this handbook to complete the requirements for the Master's Degree in Learning Resources. I hope to complete this requirement over the next few months. You will understand, then, why I would like to have the questionnaire returned promptly.

If you should have any item that you would be willing to have me include in the handbook, I would be happy to give it careful consideration. Credit for any item or items used will be recognized in the handbook. In the meantime, the handbook is yours to keep and I hope it will be helpful to you in your work.

I thank you, most sincerely, for your help.

Yours very truly,

Benjamin Brushett

sp
Enclosure

LETTER TO PRINCIPALS

P. O. Box 18
Creston, Nfld.
AOE 1KO
March 9, 1979

Dear

I am enclosing a copy of my handbook entitled "A Resources Handbook For Secondary Mathematics" together with two copies of a questionnaire. It would be most helpful to me if you would please pass the handbook and questionnaires along to the head of the mathematics department in your school. I am anxious to get information from as many teachers as possible. Two members of your staff could complete the questionnaires independently or one questionnaire could be completed collectively by your staff. If the questionnaire is completed collectively it would be helpful if each member of your staff taking part could supply answers to questions 3 and 4 of section A of the questionnaire.

I have prepared this handbook as part of the requirement for the Master's Degree in Learning Resources. I am making every effort to complete this requirement over the next few months. You will understand, then, how important your help and that of your staff is to me at this time.

My sincere thanks to you for your help and cooperation.

Yours truly,

Benjamin Brushett

sp
Enclosures