SOIL-STRUCTURE INTERACTION IN LATERALLY LOADED PILES

MOHAMED NABEEL ABDEL-SALAM ABDEL-HAI ABDEL-SALAM

SOIL-STRUCTURE INTERACTION IN LATERALLY LOADED PILES

by

MOHAMED NABEEL ABDEL-SALAM ABDEL-HAI ABDEL-SALAM, B.Sc. (Eng.)

A Thesis Submitted in Partial Fulfillment

of the Requirements for the Degree of

MASTER OF ENGINEERING

Faculty of Engineering and Applied Science

Memorial University of Newfoundland

July 1979

St. John's Newfoundland

ABSTRACT

Lateral loading on piles is a common occurrence in pile-supported structures such as multistoried buildings, large-span frames and arches and electrical transmission towers. A reference to these can be found in classical geotechnical literature. With the expansion of offshore oil and gas production activities into deeper waters, the demand is ever increasing for designing huge structures supported on large piles. Although such piles are also laterally loaded piles, the environmental lateral loads are several order magnitudes larger than that on comparable land-based structures. The ocean soil itself is unique by virtue of its environmental location. The soil - structure interaction problem under these conditions has attracted the attention of researchers during the past two decades.

Different methods of analysis are available at present for solving the laterally loaded pile problem. A comparison between two theoretical methods, the finite difference and the finite element, is presented here. In this process, a computer program was developed to generate the p-y curves which relate the soil resistance to the pile deflection, in order to idealize the soil - structure interaction. Soil nonlinearity is considered in both methods with the aid of these curves. This p-y curve concept was extended to layered soil systems. New computer programs were developed in addition to adapting an existing one and a comparison was made of the results obtained by the finite difference and the finite element methods. Full scale test results in soft clay, stiff clay and layered soil systems reported in publications were compared with the theoretical solutions.

A parametric analysis was done to establish the variables which are critical and significant in the behavior of the laterally loaded piles. The effects of 1) pile loading, 2) pile properties, 3) soil properties and 4) soil layering on the maximum deflection and the maximum positive and negative bending moments were examined. Two types of soils, soft clay and stiff clay, representing typical offshore soils were considered in the analysis. Both free and fixed head piles, representing two extreme types of end conditions, were studied. The CALCOMP graph plotter was used to generate all the graphs for the parametric studies.

Conclusions and recommendations for future research are presented.

ACKNOWLEDGEMENTS

The author is deeply indebted to his supervisor, Dr. T.R. Chari, Associate Professor of Engineering, for his excellent guidance, encouragement, and continuous support with the provision of extensive reference material during the course of the research and for his careful review of the manuscript.

Thanks are due to Dr. R. T. Dempster, Dean of Engineering, for the facilities provided. The fellowship support given by the Memorial University of Newfoundland is gratefully acknowledged. Special thanks are due to Dean F.A. Aldrich, Dean of the School of Graduate Studies for his constant encouragement. Part-time research assistance offered by Dr. T.R. Chari funded through a NSERC grant # G0213 is gratefully acknowledged.

Mr. G. Somerton of NLCS (Newfoundland and Labrador Computer Services) provided valuable help in the familiarization with the CALCOMP plotter and the computerized plotting. The cooperation of Miss Marilyn Hoyles in typing this thesis within a short period of time is acknowledged with thanks.

This study at the Memorial University would not have been possible without the leave of absence granted by the General Organization for Housing, Building and Planning Research, Ministry of Housing, Egypt, which is gratefully acknowledged.

Finally, the author wishes to specially thank his wife for her great help in typing parts of the draft and the final thesis, and for her constant encouragement, understanding and patience.

TABLE OF CONTENTS

CHAPTER Page

LIST OF TABLES

Table | Page

Table		Page

LIST OF FIGURES

Figure | Page

Figure | Page

Figure		Page

Figure | Page

Figure		Page

NOTATIONS

The symbols listed below and used in this thesis generally conform to those suggested by the American Society of Civil Engineers (Nomenclature for Soil Mechanics, Journal of the Soil Mechanics and Foundations Division, June 1962) and the Canadian Geotechnical Journal . They are also defined when they first appear in the text. SI Units are used throughout.

A	- the static matrix
B	- the deformation matrix
c , C	- soil undrained shear strength
C1	- soil undrained shear strength for the soft clay layer in a layered soil system
C2	- soil undrained shear strength for the stiff clay layer in a layered soil system
d , D	- pile diameter , width of pile
e	- internal element displacements vector
E_s	- soil modulus , secant modulus of soil reaction, coefficient of subgrade reaction
EI	- pile flexural rigidity
F	- internal element forces vector
FD	- finite difference
FE	- finite element
GAMA	- soil effective unit weight
GAMA1	- soil effective unit weight for the soft clay layer in a layered soil system
GAMA2	- soil effective unit weight for the stiff clay layer in a layered soil system
h	- pile increment length , element length

H	- horizontal load at the top of the pile
k_1, k_2, k_3	- constant values
K_i	- soil spring constant
L	- pile length
L1	- depth of the upper soft clay layer in a layered soil system
M	- pile bending moment
M(MAX), M_{max+}	- maximum positive bending moment in the pile
M(MIN), M_{max-}	- maximum negative bending moment in the pile
MT	- external moment at the top of the pile
n	- number of elements in the finite element method
N_p	- ultimate lateral soil resistance coefficient
NF	- number of F entries
NR	- number of P entries
p, p_i	- soil resistance per unit length of pile
p_u	- ultimate soil resistance per unit length of pile
P	- external nodal loads vector
P_x	- axial load on pile
p-y curves	- curves relating soil resistance to pile deflection
R_i	- pile flexural rigidity at point i
s	- slope of the elastic curve
S	- the stiffness matrix
ST	- slope of the elastic curve at the top of the pile
V	- shear force

V	- vertical load at the top of the pile
x, x_1, x_2, x_3, x_4	- depth below ground surface
X	- external nodal displacement vector
y , y_i , Y	- pile deflection
y_{50}	- deflection at one-half the ultimate soil resistance
$Y(MAX)$, Y_{max}	- maximum pile deflection
γ'	- soil effective unit weight
ϵ_{50}	- the strain corresponding to one-half the maximum principal stress difference

SYMBOLS

DEG	- degree
ft	- feet
in	- inch
kN	- kilonewton
m	- meter
mm	- millimeter
MN	- meganewton ($= 10^6$ newton)

CHAPTER I

INTRODUCTION

1.1 General

Piles must often support substantial lateral loads as well as axial loads. Examples of pile-supported structures which must support large lateral loads include marine structures, large advertising signs, and transmission towers.

The design of axially loaded piles is at present done satisfactorily using bearing capacity theories. But the problem of laterally loaded piles is more complex because of the type of its interaction with soil. The problem is further complicated due to the nonlinear soil response.

With the advent of computers and facilities for high speed iterative processing, analysis of laterally loaded piles has received considerable attention in recent years. A valid solution to the problem must insure both compatibility and equilibrium for the soil, the pile and the superstructure. With the methods of solution currently available, the most important but difficult part of the problem is to express the soil resistance characteristics. Theoretical methods available at present include different variations of finite difference and finite element analysis.

Published literature also contains the results of full scale tests on laterally loaded piles. Such tests are invariably expensive to conduct. Any theoretical method has to be supported by experimental evidence before the theory can find wide acceptance, and this is particularly important in any soil - structure interaction problem. With this objective in view, published full scale test data were used in this thesis in an evaluation of the methods of analysis.

Three main factors are commonly believed to affect the behavior of a laterally loaded pile; 1) the pile loading , 2) the pile properties and 3) the soil properties. Each of these parameters has an influence on both the deflection and the bending moment in the pile along its length. The bending moment computation is required to determine the size of the foundation and the deflection is important with regard to the serviceability of the superstructure. Methods evaluated here compute these two responses and the influence of the different variables on such computations.

1.2 Scope of the Present Investigation

The purpose of this investigation is two fold:

1. To present a comparative study between the finite difference and the finite element approaches for analyzing the laterally loaded piles. Computer programs will be developed including the effects of soil nonlinearity and soil layering. Results will be compared with published full scale tests data.
2. The variables which are critical and significant in the behavior of the laterally loaded piles will be established. The effects of 1) the pile loading, 2) the pile properties, 3) the soil properties and 4) the soil layering, on the maximum deflection and the maximum positive and negative bending moments will be examined. Two types of soils, soft clay and stiff clay which are common in the offshore environment will be considered in the analysis. Both free and fixed head piles will be studied.

CHAPTER II

REVIEW OF LITERATURE

2.1 General

The various environmental factors which cause high lateral forces on pile foundations can be grouped under earthquakes, wind and wave forces, differential water pressure and lateral earth pressure.

In this chapter the different methods of analysis of the laterally loaded piles will be briefly discussed. The soil response which is the key element in predicting the behavior of the laterally loaded piles is discussed. Previous work on the parametric studies of the problem is also reviewed.

2.2 Examples of Laterally Loaded Piles

Offshore structures used for oil drilling are often subjected to heavy wave and wind forces. The piles supporting these structures have to resist lateral forces up to 3300 kN per pile (Loftin, 1976). Bridge piers and coastal structures such as dolphins are also subjected to a similar type of lateral loads but not necessarily of the same magnitude.

There are a number of other examples in which structures, and hence the foundations are subjected to lateral loading. These include tall buildings subjected to wind loads, large-span rigid frames and arches designed to resist heavy horizontal forces, road signs subjected to wind forces, guard rail posts where impacting vehicles deliver the lateral load, pile anchors used to support bulkheads, piles used to retain slopes and to increase the stability (Broms, 1972), and electrical transmission towers subjected to large lateral forces (Radhakrishna et al, 1977).

Another interesting application of laterally loaded piles is the behavior of pile groups which include both vertical and batter piles. Even when these pile groups are subjected to only axial loads, the deformation of the group causes high lateral earth pressures resulting in lateral loads on the individual piles (Broms, 1972).

2.3 Methods of Analysis of the Laterally Loaded Pile Problem

The methods of predicting the behavior of laterally loaded piles can be divided into two main categories, namely: 1) elastic methods and 2) subgrade reaction methods (Broms, 1972). Elastic methods assume that the soil surrounding the pile reacts as an elastic half-space and conforms to the theories of elasticity (Focht and Koch, 1973), while the subgrade reaction methods are based on the assumption that the soil mass resisting the lateral forces is replaced by a series of springs acting independently of one another (Radhakrishna et al, 1977). Both methods will be briefly discussed.

2.3.1 Elastic Methods

The lateral deflection of pile foundations can be calculated from Mindlin's equations (Mindlin, 1936) assuming that the soil can be replaced by an ideal, elastic and isotropic material with a constant modulus of elasticity and a constant Poisson's ratio (Broms, 1972).

Spillers and Stoll (1964) were probably the first to use Mindlin's equation for calculation of the deflections of a laterally loaded pile. The behavior was analyzed by replacing the lateral earth pressure along the pile by a series of point loads. Douglas and Davis (1964) have calculated from Mindlin's equation the pressure distribution, the lateral displacement and the rotation of laterally loaded vertical plates. Poulos (1971) extended this work for single piles. The laterally loaded piles were replaced by thin rectangular strips with the same width and total length as the pile. These strips were divided into a number of segments and the lateral earth pressure on each segment was assumed to be constant. Elastic solutions were also applied by Lee (1975) to full-scale foundation tests. Banerjee and Davies (1978) proposed a generalized elastic analysis of the working load response of single piles embedded in a soil whose modulus of elasticity increases linearly with depth.

As long as the soil response to an applied lateral load remains within the elastic range, an elastic approach can be used to determine the behavior of the pile. Unfortunately, the near-surface soil around most laterally loaded piles is generally strained well into the plastic range so that direct application of an elastic half-space procedure is not valid (Focht and Koch, 1973).

2.3.2 Subgrade Reaction Methods

These methods are based on a simplified load-deflection response model for the soil surrounding the pile. It is assumed that the soil resistance p on a pile is related to the pile lateral deflection y according to the equation

$$p = - E_s \ y \qquad [1]$$

where E_s is the soil modulus or the coefficient of subgrade reaction. The pile is regarded as supported laterally by a series of independently acting mechanical springs. It should be noted that the concept of the coefficient of subgrade reaction does not take into account the continuity of the soil mass (Broms, 1972).

The governing differential equation for the soil-pile system according to the subgrade reaction approach in case of a pile supporting a horizontal load is given by (Hetenyi, 1946)

$$EI \frac{d^4y}{d\,x^4} - p = 0 \qquad [2]$$

where

y = lateral deflection of the pile at point x along the pile length

p = soil reaction per unit length of the pile

EI = pile flexural rigidity

substitution of equation [1] into equation [2] yields

$$EI \frac{d^4y}{d\,x^4} + E_s\, y = 0 \qquad [3]$$

Solutions to equation [3] have been presented by many authors. Closed form solutions for simple boundary conditions are presented by Hetenyi (1946). The finite difference approach was discussed by several authors; Palmer and Thompson (1948), Glesser (1954), Focht and McClelland (1955), Howe (1955), Reese and Matlock (1956), Reese and Ginzbarg (1958), Matlock and Reese (1960), Bowles (1968), Reese and Manoliu (1973) and Reese (1975b, 1977). They analyzed the laterally loaded pile problem for any variation of soil modulus with depth, for nonlinear load-displacement relationships and for nonuniform pile sections. A brief description of the contribution of each author is presented in a subsequent section.

Broms (1964) presented methods for the calculation of the ultimate lateral resistance and lateral deflections at working loads of both free and fixed head single piles. Lateral deflections at working loads have been calculated using the subgrade reaction theory and taking into account edge effects both at the ground surface and the bottom of the pile.

The lateral deflection can also be calculated by the methods of successive approximations as proposed by Muzas (1972) by using coefficient of subgrade reaction which is either constant or increases exponentially with depth. A similar approach has also been used by Mustafayev et al (1972). The results can be expressed in a non-dimensional form for the cases where the coefficient of subgrade reaction is constant or increases linearly with depth.

Mori (1964), Reddy and Valsangkar (1970) and Madhav et al (1971) have solved the differential equation [3] for elasto-plastic soils when the load - deformation relationship consists of two straight lines. Reddy and Valsangkar (1970) presented the results in a non-dimensional form for the cases when the coefficient of subgrade reaction below the plastic surface zone is either constant or increases linearly with depth.

Bowles (1972, 1974) analyzed the laterally loaded pile problem using the stiffness or displacement method of the finite element approach. He considered the coefficient of subgrade reaction as a function of the depth x according to the equation

$$E_s = k_1 + k_2 x^{k_3} \qquad [4]$$

where k_1, k_2, k_3 are constants that can have any arbitrary values in the analysis.

2.4 Soil Response in the Subgrade Reaction Methods

The modulus of subgrade reaction has an important application in the analysis of the laterally loaded pile problem and extensive studies have been made on this soil modulus. Terzaghi (1955) has shown that the coefficient of subgrade reaction for cohesionless soils can be expected to increase linearly with depth, while it is essentially constant with depth for preloaded cohesive soils. Matlock and Reese (1960) have discussed the application of the finite difference method when the coefficient of subgrade reaction is a power function or a polynomial.

Early applications of the subgrade reaction theory assumed that the soil mass resisting the lateral forces is replaced by a series of linear, independently acting springs. Ignoring the nonlinearity of the soil response may lead to conservative linear predictions in variance with actual behavior (Ismael and Klym, 1978) . A more promising approach to account for the soil nonlinearity is the p-y curves concept which will be discussed in the following sections.

2.4.1 The p-y Curves Concept

The p-y curves are generally used to characterize the soil resistance - deformation relationship for the laterally loaded piles. These curves are related to the strength - deformation characteristics of the soil as obtained from conventional soil tests. The use of p-y curves in the solution to the problem of the laterally loaded piles was first proposed by McClelland and Focht (1958) . Based on the analysis of field test data, they proposed a linear conversion of the scales for nonlinear laboratory stress-strain curves to produce correspondingly shaped p-y curves for laterally loaded piles. The current methods for constructing p-y curves are empirically derived from the results of lateral load tests on instrumented piles. The use of empirical methods for design purposes is justifiable when the design conditions approximate those upon which the method is based. The methods presently used for constructing p-y curves for clays are based on the results of lateral load tests on instrumented piles carried out by Matlock (1970) for soft clays and by Reese et al (1975) for stiff clays. The important difference between Matlock's (1970) soft clay method and Reese et al (1975) stiff clay method is that the former is for a strain hardening soil and the latter is for a strain softening soil. The resistance-deformation behavior of a strain hardening soil is characterized by increased resistance with increasing deformation, while for a strain softening soil, the resistance reaches a peak value and then drops to some lower value at larger deformations (Stevens and Audibert, 1979). The use of the p-y curves in geotechnical engineering has been described by McClelland and Focht (1958), Matlock and Reese (1960), Matlock (1970), API (1974), Reese et al (1974, 1975), Reese and Welch (1975), and Lee and Gilbert (1979).

2.4.1.1 <u>Basic Idea of p-y Curves Concept</u>

The concept of p-y curves can be explained by referring to figure 1 (Reese and Cox, 1969). Figure 1a shows the section through a deep foundation at some depth x_1 below the ground surface. Figure 1b shows the possible soil-stress distribution in the soil around the foundation after it has been installed and before it has been loaded laterally. The deflection of the foundation through a distance y_1 as shown in figure 1c, generates unbalanced soil stresses presumably as indicated

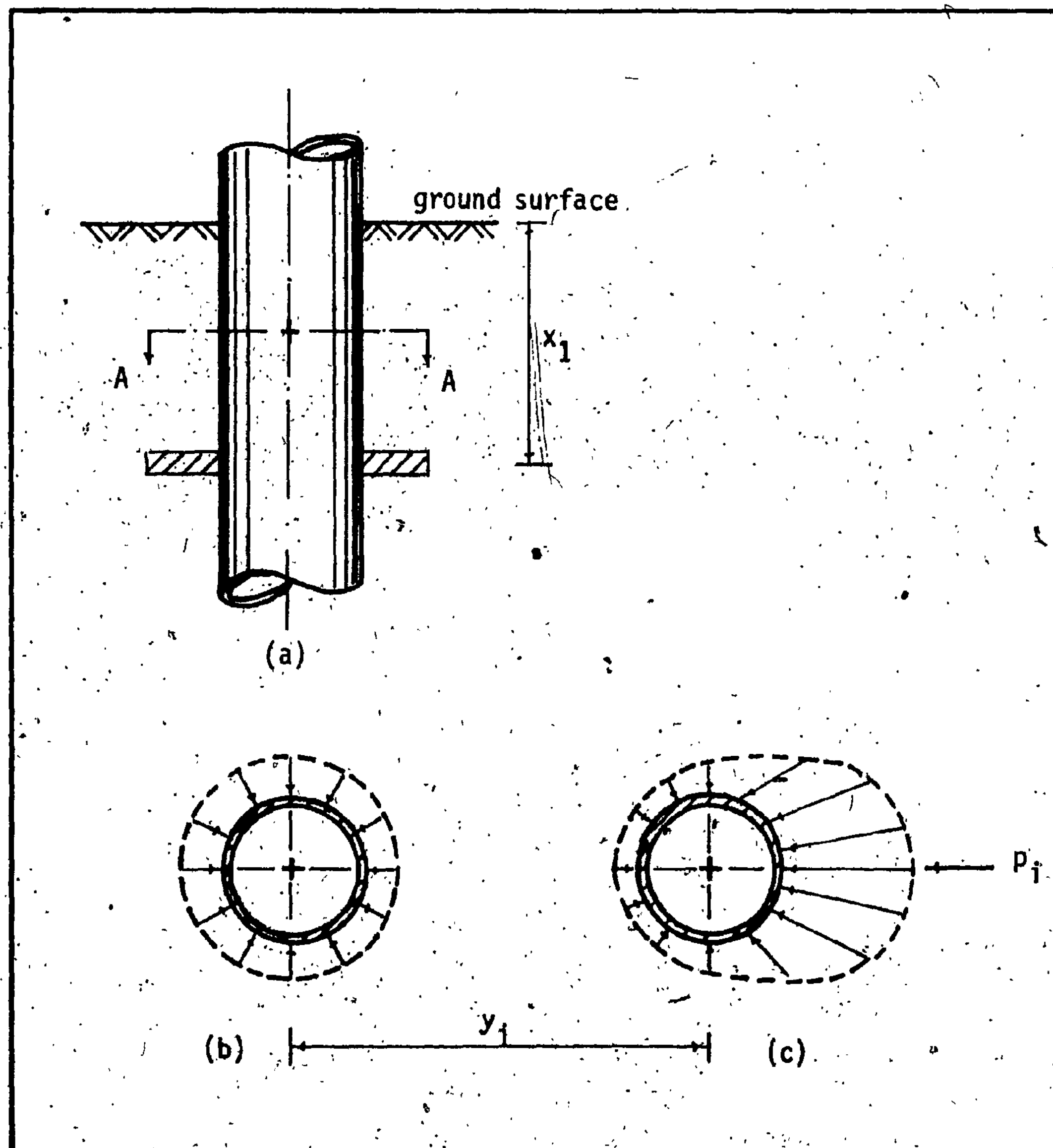


Figure 1 Graphical definition of p and y: (a) side view; (b) A-A, earth pressure distribution prior to lateral loading; (c) A-A, earth pressure distribution after lateral loading (Reese and Cox, 1969)

in the figure. Integration of the soil stresses yeilds an unbalanced force p_i per unit length of the pile. The deflection of the pile could generate a soil resistance parallel to the axis of the pile. However, it is assumed that such soil resistance would be quite small and it can be ignored in the analysis.

For the solution of the laterally loaded pile problem, it is necessary to predict a set of p-y curves, such as those shown in figure 2. This set of curves would seem to imply that the behavior of the soil at a particular depth is independent of the soil behavior at all other depths. That assumption of course is not strictly true. However, experiments seem to indicate that for the patterns of pile deflections that can occur in practice, the soil reaction at a point is dependent essentially on the pile deflection at that point and not on pile deflections above and below. Thus for purposes of the analysis, the soil can be removed and replaced by a set of discrete mechanisms with load-deflection characteristics of a type shown in figure 2.

2.4.1.2 Factors Affecting the p-y Curves

The proper form of the p-y relation is influenced by a number of factors including 1) natural variation of soil properties with depth, 2) the general form of the pile deflection, 3) the corresponding state of stress and strain throughout the affected soil zone, 4) depth below the ground surface, 5) the pile diameter and 6) the rate, sequence and history of cyclic loadings.

2.5 Parametric Studies

The parameters which affect the behavior of laterally loaded piles are: 1) the pile loading, 2) the pile properties and 3) the soil properties.

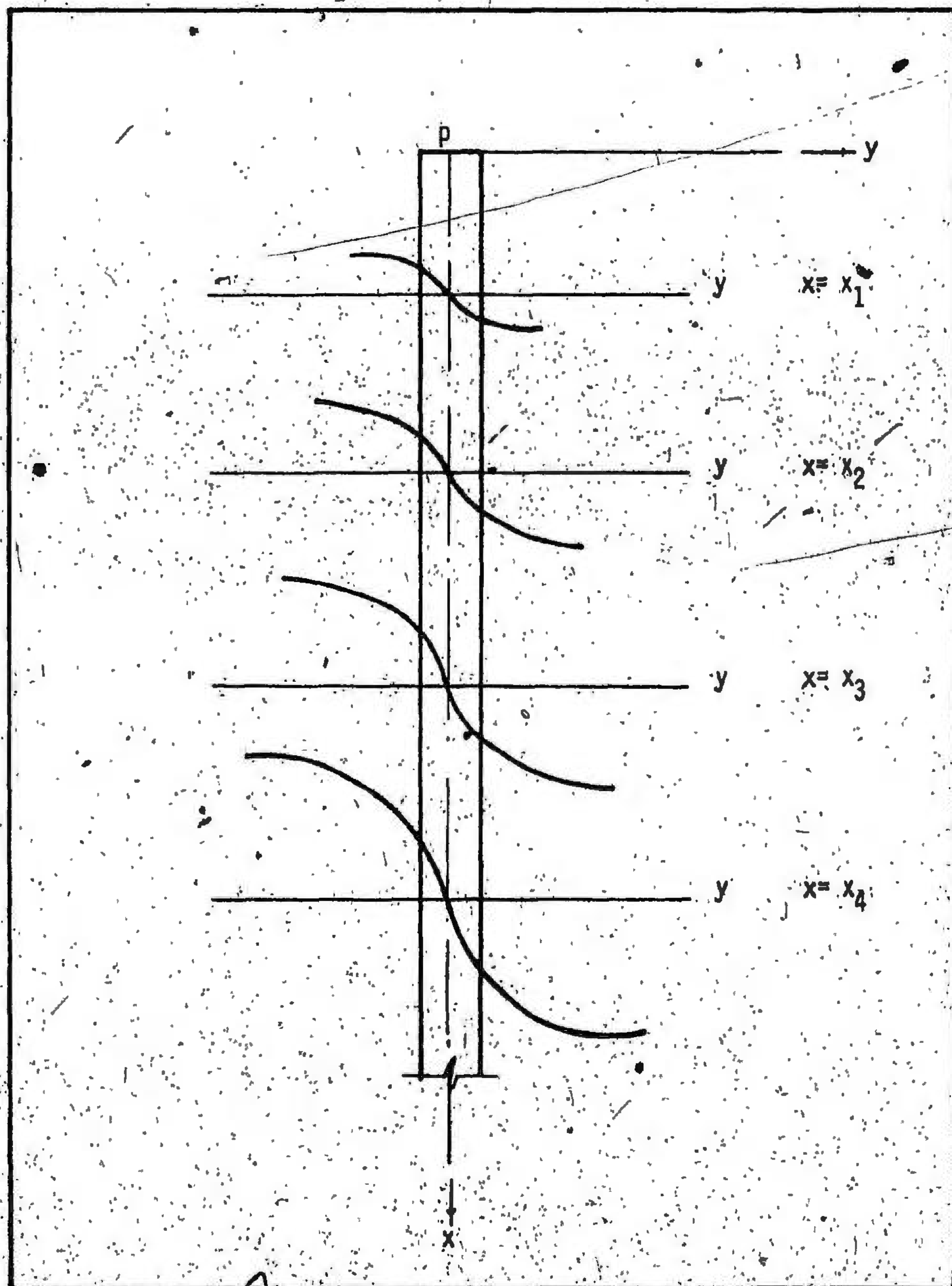


Figure 2 Set of p-y curves (Reese and Cox, 1969)

The effect of the pile loading has received much attention by many researchers. Most of those studies have been done to investigate the effect of increasing lateral loads on the behavior of the piles. The studies were generally conducted after full scale tests. Matlock (1970) performed his tests on piles embedded in soft clays, while Reese et al (1974, 1975) studied the behavior of laterally loaded piles in sand and in stiff clays. Ismael and Klym (1978) studied the behavior of laterally loaded rigid piers in layered cohesive soils. The effect of the vertical load on the behavior of the laterally loaded piles was presented by Zuhkov and Balov (1978) based on both theoretical and experimental analysis. The effect of the external moment in the case of a free head pile embedded in sand was theoretically studied by Palmer and Brown (1954) based on the experimental work of Mason and Bishop (1954).

Palmer and Brown (1954) presented a theoretical study on the effect of the pile properties on the maximum deflection and maximum bending moment in a free head laterally loaded pile embedded in sand.

To the author's knowledge, no parametric analysis has been done to study the effect of the engineering properties, such as the soil shear strength and unit weight, on the behavior of laterally loaded piles.

Davisson and Gill (1963) studied the effect of the surface layer in a layered system of soil on the pile-soil system. They concluded that the surface layer exerts an overwhelming influence on the behavior of the pile.

2.6 Summary

Examples of laterally loaded piles have been mentioned. A review of the different methods of analysis has been presented. The soil response in the laterally loaded pile problem was discussed. Previous work on the parametric studies of the problem was briefly reviewed. Additional literature is presented in conjunction with the discussion in chapters III and IV.

CHAPTER III

COMPARISON OF FINITE DIFFERENCE AND FINITE ELEMENT METHODS

3.1 General

Two of the widely used approaches in analyzing laterally loaded piles are the finite difference and the finite element methods. The difficult problem in both is the idealization and representation of the soil surrounding the pile. To account for the soil nonlinearity, the p-y curves concept is generally accepted as the best approach (API, 1974; Reese and Welch, 1975). A computer program was developed in this research to generate the p-y curves for a pile embedded in 1) soft clay below water surface, 2) stiff clay below water surface and 3) stiff clay above water surface. This program is capable of being used for any layered system made up of these broad types of soils. The program can generate the curves for short term static loading. The computer program to solve the laterally loaded piles problem using the finite difference method described by Reese and Manoliu (1973) and Reese (1975b,1977) was modified to include the p-y curves generation capability.

Bowles (1972,1974) has suggested a concept in the use of the finite element method for solving the laterally loaded pile problem. Using this approach and taking into account the soil nonlinearity, a computer program was developed as a part of this research.

A comparison between the finite difference and the finite element approaches and a comparison with the results of published full scale tests are presented.

3.2 Soil Response

As already discussed, the p-y curves approach will be introduced in the analysis to account for the soil nonlinearity. The soil will be removed and replaced by a set of discrete mechanisms with load - deflection characteristics of a type shown in figure 2.

In this analysis the short term static loading will be studied. Short term static loading can furnish a basis for finally evaluating the behavior of the pile under cyclic loading (Matlock, 1970). This aspect is very relevant to offshore foundations where loads such as those due to waves tend to be cyclic. Another form of analysis is the dynamic analysis which is not within the scope of this work. In case of long term static loading both creep and visco-elastic effects should be introduced. This is also not considered in this research work. The analysis presented here gives the basis for both dynamic and long term static analyses.

3.2.1 p-y Curves for Soft Clays

Recommended procedures for developing p-y curves for piles in soft clay below water surface have been proposed by Matlock (1970). The procedures are semi-empirical and rely heavily on pile load tests performed on a 323.85 mm (12.75 in) diameter pile driven into soft to medium silty clays at two different test sites. A simplified summary of the procedures was presented by Reese (1975a) for short term static loading as follows:

1. Obtain the best estimate of the variation of the soil shear strength and effective unit weight with depth. Also obtain the value of ε_{50}, the strain corresponding to one-half the maximum principal stress difference. If no values of ε_{50} are available, typical values

suggested by Skempton (1951) are to be used. These values are given in table 1.

Table 1 Typical values of ε_{50}, the strain corresponding to one-half the maximum pricipal stress difference (Skempton,1951).

Consistency of Clay	ε_{50}
soft	0.020
medium	0.010
stiff	0.005

2. Compute the ultimate soil resistance per unit length of the pile p_u, at the required depth as follows

$$p_u = N_p \cdot c \cdot d \qquad [5]$$

where

d = width of pile

c = undrained shear strength at the required depth

N_p = ultimate lateral soil resistance coefficient

For great depths where sufficient confinement exists, N_p = 9 is suggested by Matlock (1970). Near the surface the soil in front of the pile is not well confined and as the pile deflects, this soil is pushed up and away from the pile. For this type of failure mode, Matlock (1970) suggested $N_p \approx 3$ at the surface which increases with depth as follows

$$N_p = 3 + \frac{\gamma'}{c} x + \frac{0.5}{d} x \qquad [6]$$

where

x = depth from ground surface to p-y curve

γ' = average effective unit weight from ground surface to p-y curve

The value of p_u is computed at each depth where a p-y curve is desired, based on the shear strength at that depth.

3. Compute the deflection y_{50}, at one-half the ultimate soil resistance from the equation

$$y_{50} = 2.5 \ \varepsilon_{50} \ d \qquad [7]$$

4. Points describing the p-y curves are now computed from the relationship

$$p/p_u = 0.5 \ (y/y_{50})^{1/3} \qquad [8]$$

The value of p remains constant beyond $y = 8\ y_{50}$.

3.2.2 p-y Curves for Stiff Clays

Reese and Welch (1975) presented procedures for developing p-y curves for stiff clays above water surface. This procedure will not be discussed here in detail since the main concern is offshore piles where the soil is below water surface. Development of p-y curves for stiff clays below water surface is given by Reese et al (1975) based on the results of tests conducted on two 609.60 mm (24 in) and one 152.40 mm (6 in) diameter piles driven into stiff clays. Using an approach similar to that used by Matlock (1970) for soft clays, an empirical correlation of the experimental data, soil behavior and foundation engineering concepts was formulated by Reese et al (1975) into a design procedure to construct the p-y curves. That procedure, for short term static loading, can be summarized as follows:

1. Obtain values for undrained soil shear strength and effective unit weight from ground surface to the required depth x .
2. The ultimate lateral soil resistance p_u, per unit length of pile is determined from the smaller value given by the following equations:

$$p_u = 2\,c\,d + \gamma'\,d\,x + 2.83\,c\,x \qquad [9]$$

$$p_u = 11\,c\,d \qquad [10]$$

3. The p-y curves may be defined in five different sections as described in detail by Reese et al (1975).

3.2.3 p-y Curves for a Layered Soil

Extending the above procedures, a method was evolved for constructing the p-y curves in a layered system. The p-y curve was assumed to be primarily influenced by the type of the soil for which it is generated and a suitable computer program PYCURV was developed. Layered systems composed of various combinations of soft clays below water surface, stiff clays below water surface and stiff clays above water surface were considered. This program can handle up to 5 layers and as many as 25 p-y curves can be constructed along the pile length. The values of the undrained soil shear strength, the soil effective unit weight and the strain corresponding to one-half the maximum principal stress difference ε_{50}, should be defined for each layer. For the analysis, at least two p-y curves are required in each layer. This program can generate equi-distant p-y curves in each layer, with the first at the ground surface and the last at the bottom of the pile. There will be two p-y curves at the interface of any two adjacent layers; one representing the upper layer and the other for the lower one.

The zone of importance in the lateral behavior of piles is the upper 6 to 10 m (20 to 30 ft) (Reese and Welch, 1975). Below that zone the p-y curves are identical because the ultimate lateral soil resistance becomes constant in a given soil layer. The program developed here also enables one to analyze a given single soil system efficiently by considering an upper zone as an imaginary layer and developing as many p-y curves as are required at small intervals in order to increase the accuracy of the solution. The lower zone can be considered as another layer with the same properties and a smaller number of p-y curves can be generated to adequately define the remaining soil.

The computer program developed was introduced as a subroutine PYCURV , in both the finite difference and the finite element approaches. Details of this subroutine are included in the listing of the finite/element program given in appendix C. The results obtained and a comparison with the full scale tests will be discussed in section 3.5 .

3.2.4 Comparison of Results

Measured and computed families of p-y curves for a static test on 609.60 mm (24 in) and 152.40 mm (6in) diameter piles are shown in figures 3 and 4 . The measured values are the results of lateral load tests conducted by Reese et al (1975) on piles driven into stiff clays. The computed values are the output of the computer program already discussed where the soil was divided into four layers of stiff clay to satisfy the variation in the undrained soil shear strength with depth at the site. The general shape of the p-y curves in figures 3 and 4 is similar for both the measured and the computed curves. For any particular depth the initial part of the curve is relatively steep. The ultimate soil resistance shown in the p-y curve corresponds to a particular deflection. Beyond this deflection there is a reduction in soil resistance with continued deflection. With regard to the family of curves in figures 3 and 4 , the slope of the initial part of the curve increases with depth as does the magnitude of the ultimate resistance. It can be noticed that while the

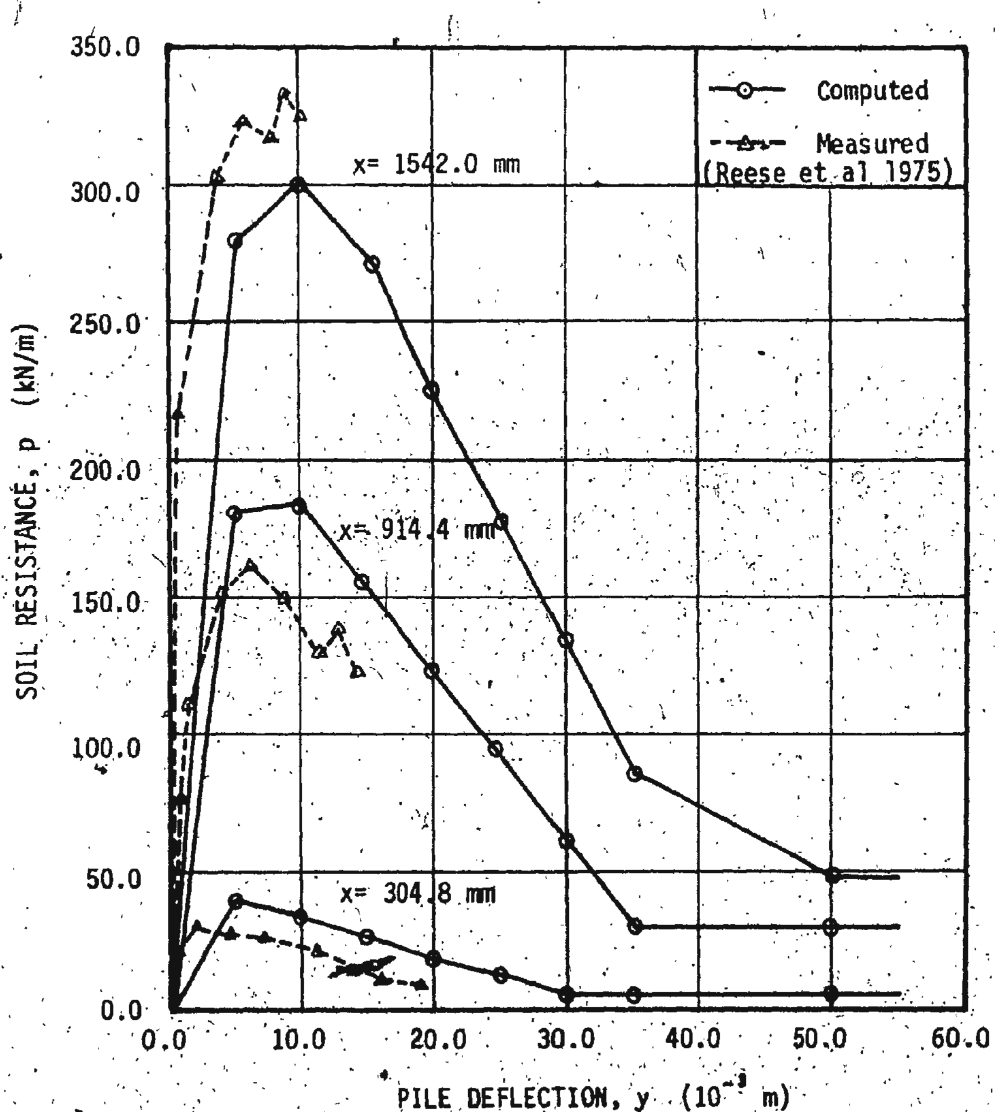


Figure 3 Comparison of computed and measured p-y curves for static test on 609.60 mm diameter pile

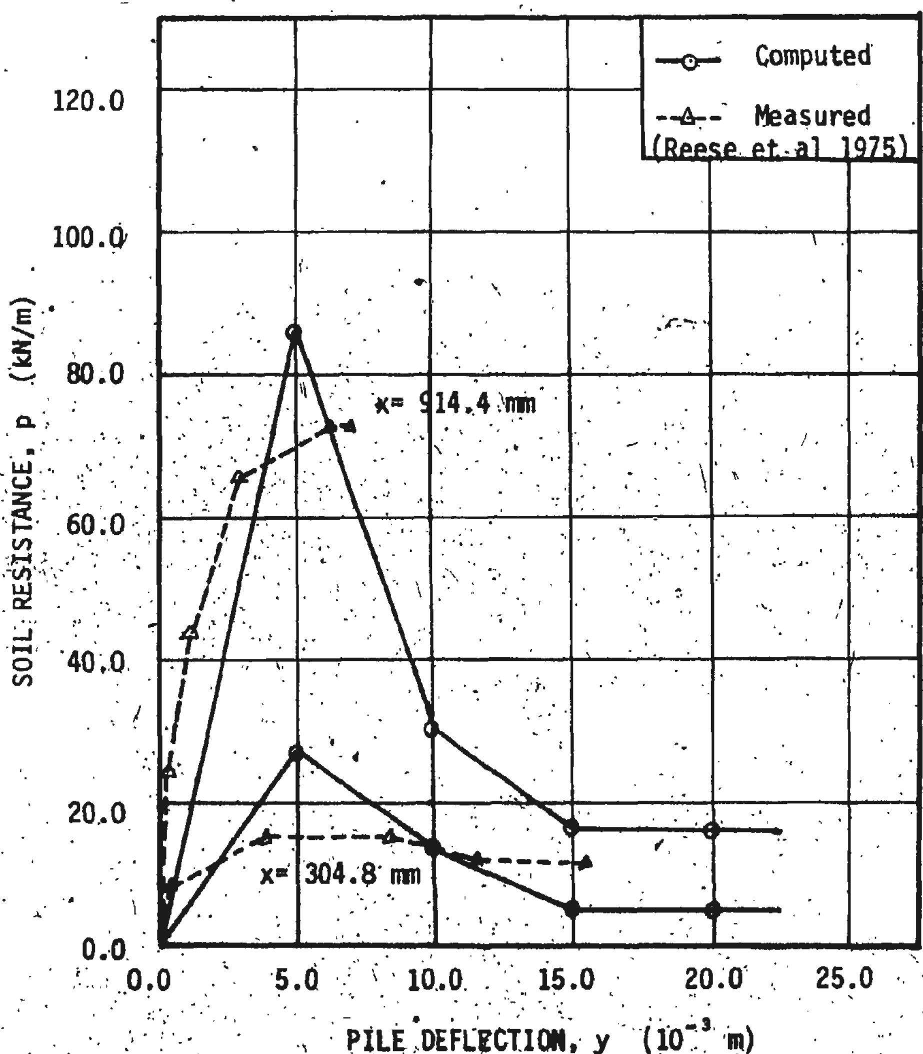


Figure 4 Comparison of computed and measured p-y curves for static test on 152.40 mm diameter pile

measured curves are described up to a deflection of 0.02 m , the computed curves are extrapolated to higher values of pile deflections. It can also be seen that the ultimate soil resistance in the case of a 609.60 mm diameter pile is significantly higher than that for a 152.40 mm diameter pile. The small irregularities in the shape of the measured curves are due to experimental problems (Reese et al, 1975) .

Figure 5 shows a predicted family of the computed p-y curves for Sabine soft clay for short term static loading. In that site the soil undrained shear strength was 14.5 kN/m^2 in the upper region of significance to the analysis (Matlock, 1970) .

To check the accuracy of the computed p-y curves, they were used as an input to a computer program (Reese and Manoliu, 1973 ; Reese, 1975b, 1977) that yields the pile behavior under lateral loading. There was good agreement between the experimental and the computed results as will be discussed in section 3.5 .

3.3 <u>The Finite Difference Method</u>

The use of the finite difference approach in analyzing the laterally loaded pile problem was first suggested by Palmer and Thompson (1948) as a method of solution for free head piles (the different end conditions of pile heads will be discussed subsequently). The mechanics of this solution was considerably simplified by Gleser (1954) and modified by Focht and McClelland (1955) . Howe (1955) set up the finite difference equation solution in an electronic computer which significantly reduced the time required for the solution. A further extension was made by Reese and Matlock (1956) to enable the introduction of moment and shear as the boundary conditions and to produce a set of non-dimensional

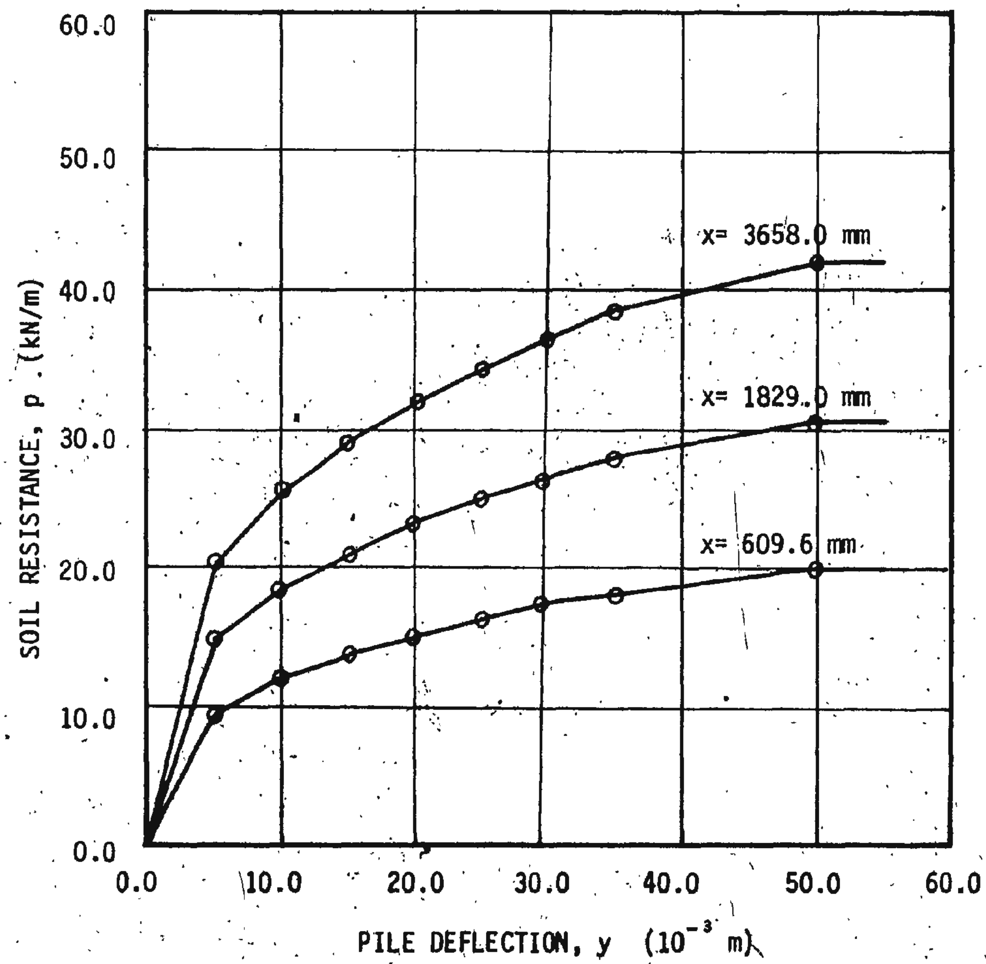


Figure 5 Predicted family of p-y curves for Sabine soft clay for short term static loading

solutions for the problem. A computer program was developed by Reese and Ginzbarg (1958) in which the flexural rigidity of the pile may be changed abruptly at points along the length of the pile. The method was generalized by Matlock and Reese (1960). Reese and Manoliu (1973) developed a computer program which uses successive difference equations based on repeated reference to the p-y curves. The soil modulus was determined at increments along the pile such that there was both compatibility and equilibrium for the soil, the pile and the superstructure. That program has the advantage of analyzing laterally loaded piles subjected to both horizontal and vertical loading with different boundary conditions. Details of the program are documented by Reese (1975b, 1977).

3.3.1 The Governing Equation

The differential equation to be solved is derived on the assumption that the pile is a linearly elastic beam and that the soil reaction may be represented as a line load (Hetenyi, 1946):

$$EI \frac{d^4 y}{d x^4} + P_x \frac{d^2 y}{d x^2} - p = 0 \qquad [11]$$

where

y = lateral deflection of the pile at point x along the pile length

p = soil reaction per unit length of the pile

P_x = axial load on pile

EI = pile flexural rigidity.

Other beam formulae useful in the analysis are :

$$EI \frac{d^3 y}{d x^3} = V \qquad [12a]$$

$$EI \frac{d^2 y}{d x^2} = M \qquad [12b]$$

and

$$EI \frac{d y}{d x} = s \qquad [12c]$$

in which

v = shear force

M = bending moment

s = slope of the elastic curve

For convenience in solving equation [11] , a secant modulus of soil reaction , E_s , is often used :

$$E_s = - \frac{p}{y} \qquad [13]$$

3.3.2 Solution of the Governing Differential Equation

It is desired to express the governing differential equation [11] in a finite difference form. This is accomplished by discretizing the pile length using a constant increment h , as shown in figure 6 . Substituting equation [12b] into equation [11] and using equation [13]

$$\frac{d^2 M}{d x^2} + P_x \frac{d^2 y}{d x^2} + E_s \, y = 0 \qquad [14]$$

The differential equation in the finite difference form at point m is

$$y_{m-2} \quad R_{m-1}$$

$$+ \quad y_{m-1} \, (-2 \, R_{m-1} - 2 \, R_m + P_x \, h^2 \,)$$

$$+ \quad y_m \quad (\quad R_{m-1} + 4 \, R_m + R_{m+1} - 2 \, P_x \, h^2 + E_{s_m} \, h^4 \,)$$

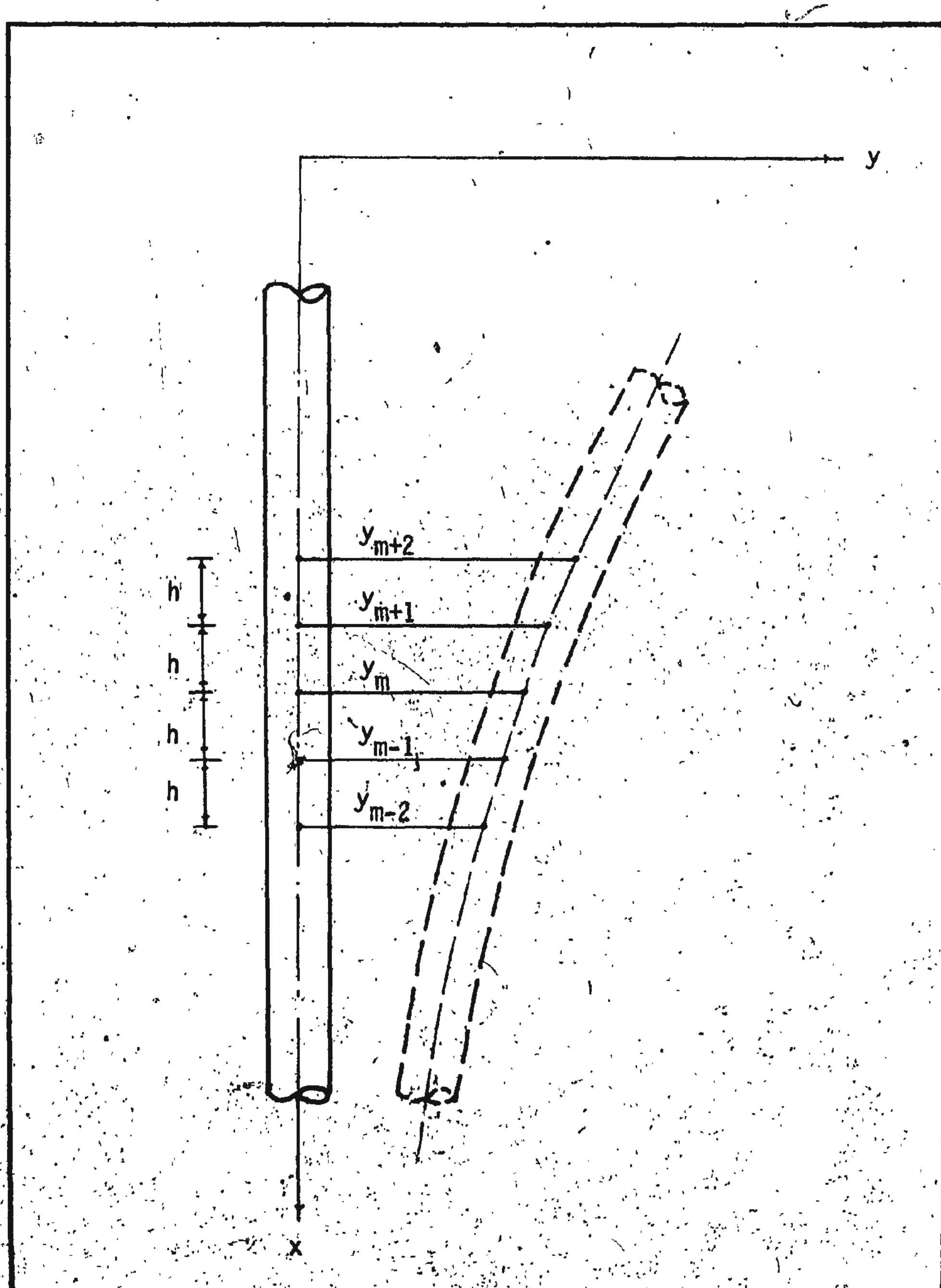


Figure 6 Representation of deflected pile (Reese, 1975b)

$$+ \quad y_{m+1} \quad (-2\,R_m \quad - \quad 2\,R_{m+1} \quad + \quad P_x\, h^2\,)$$

$$+ \quad y_{m+2} \qquad R_{m+1} \;=\; 0 \qquad [15]$$

in which R_i is the pile flexural rigidity at point i :

$$R_i \;=\; E_i I_i \qquad [16]$$

It should be noted that the axial force P_x , which produced compression is assumed to be positive and it acts through the pile axis without any eccentricity.

Two imaginary points at the top and two imaginary points at the bottom are added as a device for introducing the boundary conditions. Applying the boundary conditions to the top and bottom of the pile, equation [15] can be solved (Gleser, 1954) .

3.3.3. The Computer Program

The computer program developed by Reese and Manoliu (1973) and extended by Reese (1975b, 1977) uses successive difference equations based on repeated reference to the p-y curves. This program was further modified in the present research to allow for the generation of the p-y curves for various soil systems as already described in section 3.2.3. In addition to the flexibility of generating the p-y curves, the program allows for step changes in the pile flexural rigidity. Three sets of boundary conditions are considered at the top of the pile: 1) A pile carrying a known lateral load H , and moment MT ; this case will be referred to as free head pile where the pile head is free to rotate and translate, 2) A pile carrying a known lateral load H, with a defined slope of the elastic curve at the top ST ; if ST equals zero, this case will be equivalent to the conventional fixed head pile where the pile head is free to translate but fixed against rotation, and 3) A pile carrying a known lateral load H , with a defined rotational restraint constant MT/ST, at the top. In addition, an axial load may be specified in all these cases.

A flow chart for the entire computer program is given in appendix C. Details of the subroutines developed for generating the p-y curves are similar in both the finite difference and finite element computer programs, and they are included in the listing given in appendix C.

3.3.3.1 Computer Core Storage Requirement

The modified program requires about 256 K of storage to compile and about 76 K of storage to execute one problem on the FORTRAN H complier of an IBM 370/158 machine. It is to be noted that the program uses double precision arithmatic.

3.3.3.2 Running Time

The central processor unit (CPU) time of the program cannot be defined precisely because of the fact that the program solves nonlinear problems and the CPU time depends on the magnitude of the problem. However, for the types of the examples illustrated in this thesis, the CPU time was less than 2 seconds for each problem.

3.3.4 Sample Problem

A sample problem was chosen to demonstrate the use of the modified finite difference program. A free head pipe pile of 0.05 m wall thickness was assumed to be filled with concrete and driven in a soft clay soil below water surface. The pile is subjected to a horizontal load and an external moment. The input data given below is chosen as typically representing offshore piles in template structures .

a) pile loading:

horizontal load , H	=	500.00	kN
*vertical load , V	=	0.00	kN
external moment , MT	=	1000.00	kN m

b) pile properties:

pile length , L	=	30.00	m
pile diameter , D	=	1.00	m
pile flexural rigidity , EI	=	2710.00	MN m^2

c) soil properties :

soil undrained shear strength , C	=	25.00	kN/m^2
soil effective unit weight , GAMA	=	6.00	kN/m^3
strain corresponding to 1/2 the maximum principal stress difference , ε_{50}	=	0.02	

The pile was divided into 30 increments each 1.0 m length. The upper 10.0 m of the soil was considered as an imaginary layer in which 11 p-y curves were generated 1.0 m apart. The rest of the soil was considered as another layer with the same properties, but only 2 p-y curves were generated in that layer.

The output of the computer program shows that the p-y curves become identical below a depth of 9.0 m . The maximum deflection occurs at the ground surface and is equal to 100.99 mm , whereas the maximum positive bending moment occurs at a depth of 7.0 m below the ground surface and is equal to 2934.90 kN m.. The soil modulus E_s , increases with depth

*The vertical load is not zero for offshore structures, but it is neglected here so that the loading conditions for the two programs are identical.

until it has a constant value of 10444.00 kN/m^2 below a depth of 11.0 m. The soil resistance p , has a maximum value of 82.035 kN/m at a depth of 7.0 m. A partial computer output is shown in figure 7 and plotted in figure 8.

3.4 The Finite Element Method

The use of the finite element scheme in analyzing laterally loaded piles started in the early 70s. Approximate two-dimensional idealization assuming plane strain conditions for complex pile foundations (Desai et al, 1974) and plane stress idealization for determination of p-y curves (Yegian and Wright, 1973) have been used. Bowles (1972, 1974) analyzed the laterally loaded pile problem using the stiffness or displacement method of analysis. Ruser and Dawkins (1972) and Wittke (1974) have considered the use of three dimensional finite element methods for laterally loaded piles. Desai and Appel (1976) developed a finite element procedure that can allow for nonlinear behavior of soils, nonlinear interaction effects and simultaneous application of axial and lateral loads. Kuhlemeyer (1979) presented a formulation for finite element analysis of laterally loaded piles in which the beam bending aspect of the problem was properly and efficiently considered.

The stiffness method which was presented by Bowles (1972, 1974) was chosen in this thesis to analyze the laterally loaded pile problem because of its simplicity. A computer program was developed in the present research for this approach. The program takes into account the soil nonlinearity by repeated reference to the p-y curves which are generated in the program. A discussion of the method is presented in section

X (M)	Y (M)	M (KN M)	ES (KN/M2)	P (KN/M)
0.0	0.10079D+00	0.10000D+04	0.46915D+03	-0.47379D+02
0.10000D+01	0.87973D-01	0.14763D+04	0.63627D+03	-0.56975D+02
0.20000D+01	0.75561D-01	0.18966D+04	0.83998D+03	-0.63419D+02
0.30000D+01	0.63729D-01	0.22536D+04	0.10969D+04	-0.69991D+02
0.40000D+01	0.52788D-01	0.25406D+04	0.14315D+04	-0.75573D+02
0.50000D+01	0.42785D-01	0.27520D+04	0.18517D+04	-0.79227D+02
0.60000D+01	0.33798D-01	0.28842D+04	0.24135D+04	-0.81572D+02
0.70000D+01	0.25874D-01	0.29349D+04	0.31705D+04	-0.82035D+02
0.80000D+01	0.19034D-01	0.29035D+04	0.42394D+04	-0.80692D+02
0.90000D+01	0.13265D-01	0.27914D+04	0.54275D+04	-0.71996D+02
0.10000D+02	0.85261D-02	0.26073D+04	0.72458D+04	-0.61778D+02
0.11000D+02	0.47493D-02	0.23615D+04	0.10444D+05	-0.49599D+02
0.12000D+02	0.18430D-02	0.20660D+04	0.10444D+05	-0.19256D+02
0.13000D+02	-0.29922D-03	0.17513D+04	0.10444D+05	0.31249D+01
0.14000D+02	-0.17960D-02	0.14397D+04	0.10444D+05	0.18757D+02
0.15000D+02	-0.27616D-02	0.11469D+04	0.10444D+05	0.28841D+02
0.16000D+02	-0.33040D-02	0.88288D+03	0.10444D+05	0.34506D+02
0.17000D+02	-0.35206D-02	0.65339D+03	0.10444D+05	0.36767D+02
0.18000D+02	-0.34961D-02	0.46067D+03	0.10444D+05	0.36511D+02
0.19000D+02	-0.33016D-02	0.30447D+03	0.10444D+05	0.34480D+02
0.20000D+02	-0.29947D-02	0.18274D+03	0.10444D+05	0.31275D+02
0.21000D+02	-0.26204D-02	0.92291D+02	0.10444D+05	0.27367D+02
0.22000D+02	-0.22121D-02	0.29206D+02	0.10444D+05	0.23102D+02
0.23000D+02	-0.17930D-02	-0.10776D+02	0.10444D+05	0.18725D+02
0.24000D+02	-0.13778D-02	-0.32033D+02	0.10444D+05	0.14389D+02
0.25000D+02	-0.97448D-03	-0.38902D+02	0.10444D+05	0.10177D+02
0.26000D+02	-0.58551D-03	-0.35593D+02	0.10444D+05	0.61149D+01
0.27000D+02	-0.20960D-03	-0.26169D+02	0.10444D+05	0.21898D+01
0.28000D+02	0.15650D-03	-0.14556D+02	0.10444D+05	-0.16344D+01
0.29000D+02	0.51731D-03	-0.85765D+01	0.10444D+05	-0.54025D+01
0.30000D+02	0.87643D-03	0.0	0.10444D+05	-0.91530D+01

Figure 7 Partial computer output for the sample problem using the finite difference program

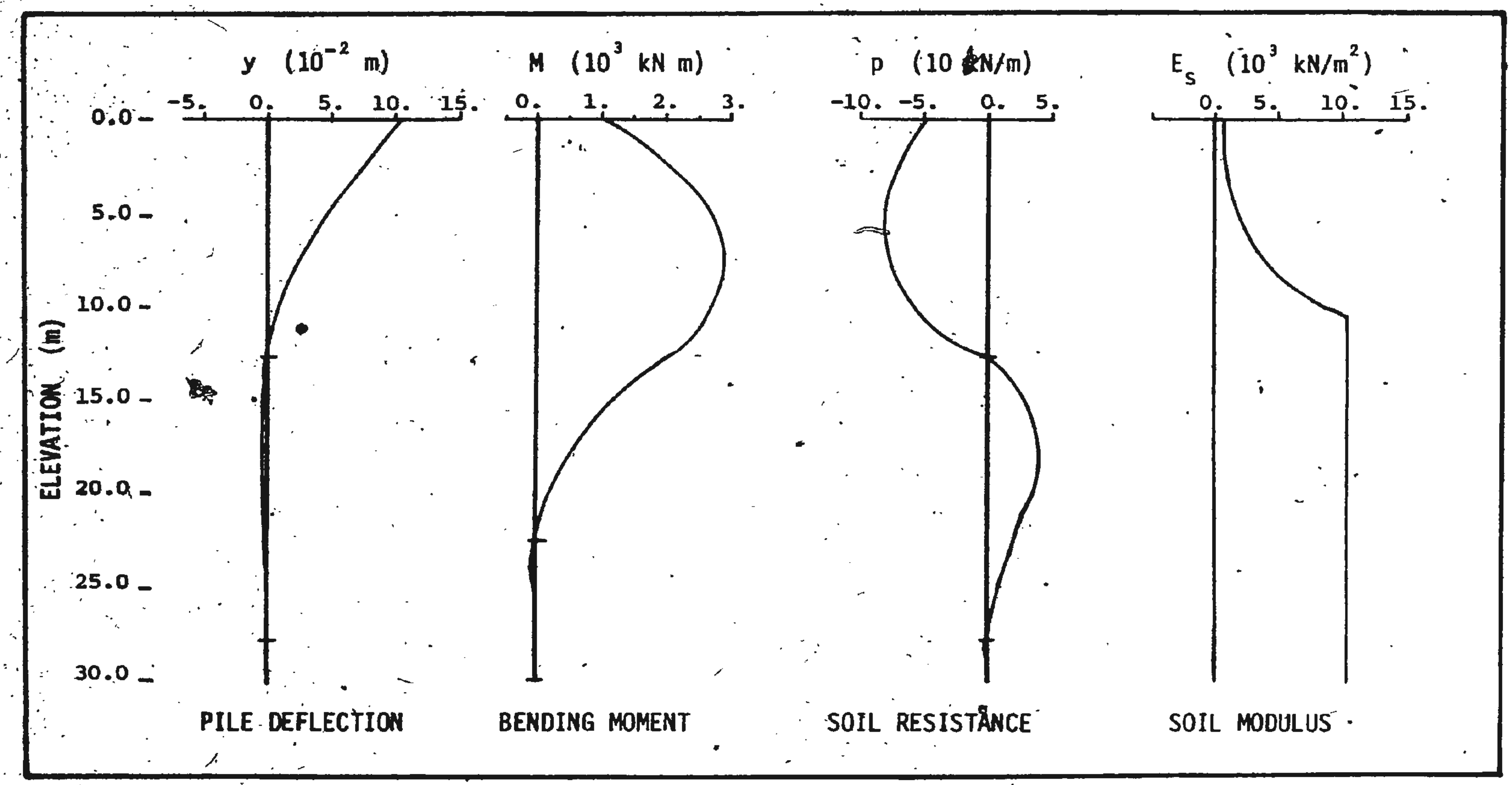


Figure 8 Output results for the sample problem using the finite difference program

3.4.1 and the effect of the soil nonlinearity is discussed in section 3.4.2 . A brief description of the program along with a sample problem are presented in sections 3.4.3 and 3.4.4 . The matrix notation of Wang (1970) will be used in the analysis.

3.4.1 The Stiffness Method

The fundamental equations of the stiffness method of the finite element approach are as follows:

$$P = A\ F \qquad [17]$$

$$e = B\ X = A^T\ X \qquad [18]$$

$$F = S\ e \qquad [19]$$

where A is the static matrix of size NP x NF acting as a bridge between external joint or nodal loads P and internal element forces (acting at the nodes) F. The connection between the internal element displacements e and the external nodal displacements X is the deformation matrix B which can be shown to be the transpose of the static matrix A (Laursen 1969; Wang, 1970) . The stiffness matrix S of size NF x NF is the relationship between the internal element forces F and the resulting element displacements e.The NP and NF symbols are the number of P and F entries respectively.

Substitution of equation [18] into equation [19] yields,

$$F = [S\ A^T]\ X \qquad [20]$$

and substitution of equation [20] into equation [17] gives,

$$P = [A\ S\ A^T]\ X \qquad [21]$$

The P vector is the applied load vector, X is the unknown displacement vector, and $[A\ S\ A^T]$ is a square matrix of size NP x NP which can be inverted to give,

$$X = [A \; S \; A^T]^{-1} \; P \qquad [22]$$

With the computed X vector, the desired internal forces at the selected nodes F , are obtained from equation [20].

Figure 9 shows the pile-soil system and the corresponding analytical model. It also shows the coding which is used in building the A , S and P matrices in case of a free head pile. Details of constructing both the static and the stiffness matrices are presented by Bowles (1974).

3.4.2 <u>Soil Response</u>

The soil effect on the pile in the stiffness method approach appears in the stiffness matrix S, as a soil spring constant K_i , where

$$F_i = K_i \; e_i \qquad [23]$$

and i varies from (2n+1) to (3n+1), where n is the number of elements.

The method chosen to determine the values of the soil spring constant is based on using a second degree parabolic curve to describe the variation of the soil modulus with depth (Newmark, 1942). This method is also valid for a constant or linearly varying soil modulus. This gives nodal soil spring constants as follows:

$$K_i = \frac{h}{12} \left[E_{s_{(i-1)}} + 10\, E_{s_{(i)}} + E_{s_{(i+1)}} \right] \qquad [24a]$$

where

h = element length

$E_{s_{(i)}}$ = soil modulus at node i , where i varies from (2n+2) to (3n)

At the top of the pile, the soil spring constant becomes

$$K_{2n+1} = \frac{h}{24} \left[7\, E_{s_{(2n+1)}} + 6\, E_{s_{(2n+2)}} - E_{s_{(2n+3)}} \right] \qquad [24b]$$

and at the bottom of the pile we have

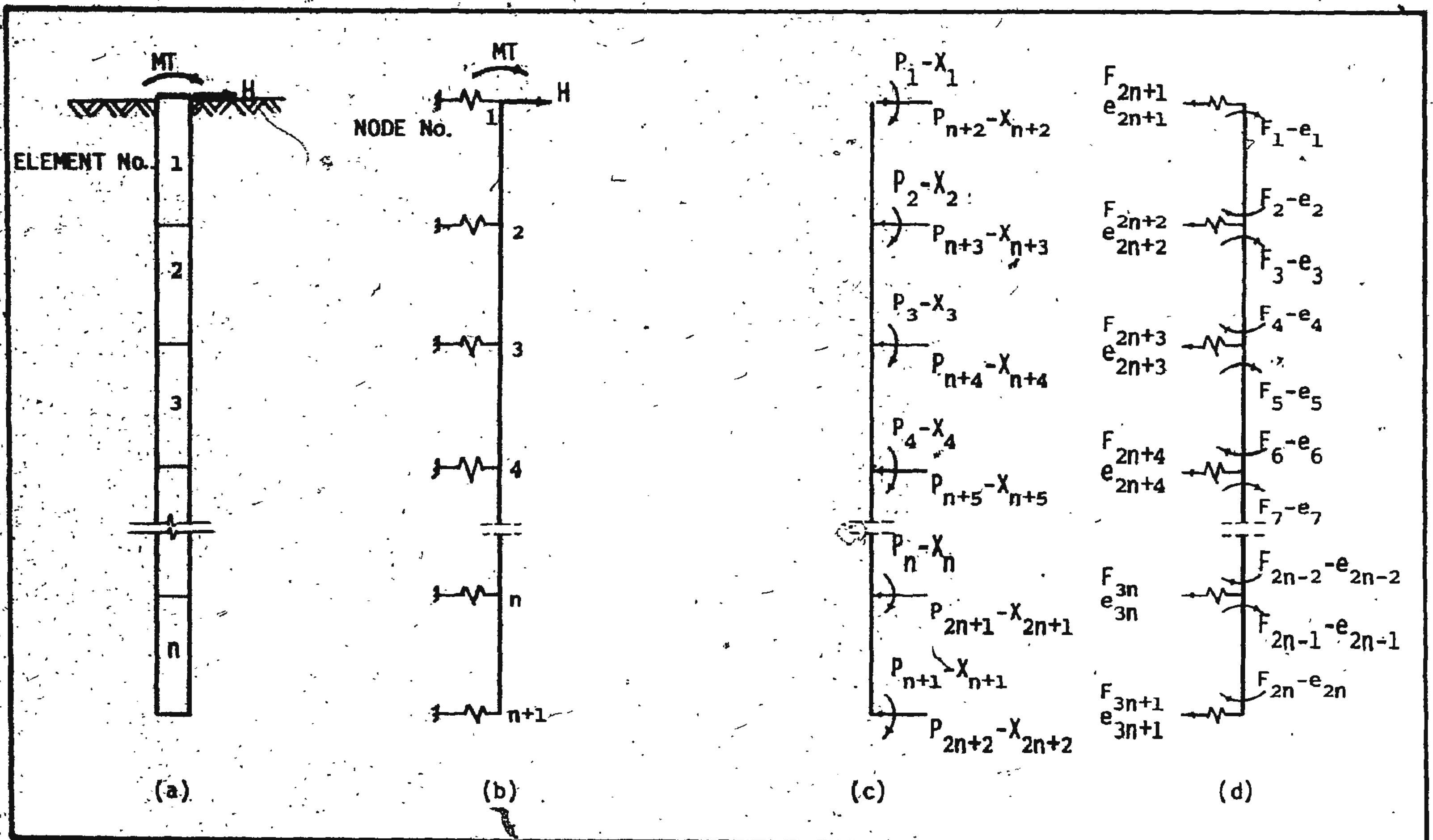


Figure 9 Illustration of the stiffness method coding for a free head pile: (a) pile-soil system, (b) analytical model, (c) P-X coding, (d) F-e coding

$$K_{3n+1} = \frac{h}{24} \left[7 E_{s(3n+1)} + 6 E_{s(3n)} - E_{s(3n-1)} \right] \qquad [24c]$$

A reduction in the first soil spring constant at the top of the pile is recommended (Bowles, 1972,1974 ; Stevens and Audibert, 1979) to account for surface disturbance, tension cracks, seasonal changes and lack of confinement of soil. 50% reduction was used by Bowles (1972,1974) . A study was made in this research to determine the optimal percentage of reduction in the soil spring constant at the top of the pile such that the pile deflection at the ground surface obtained by the finite element method is identical with that obtained using the finite difference approach. The study shows that the relation between the percentage of reduction and the pile deflection at the ground surface is linear, and a reduction of 20% is recommended for a free head pile embedded in soft clay. Results are shown in figure 10.

The soil nonlinearity is taken into consideration by developing as many p-y curves as required along the pile length. Initial values for the soil modulus are assumed at the nodal points, and the finite element procedure is performed to determine the pile deflections. Revised values for the soil modulus are then obtained with reference to the p-y curves and the finite element procedure is repeated using these revised values to get updated values for the pile deflections. The above procedure is repeated until the solution converges.

3.4.3 The Computer Program

The computer program using the finite element approach was developed in its entirety. The program uses successive cycles of the finite element

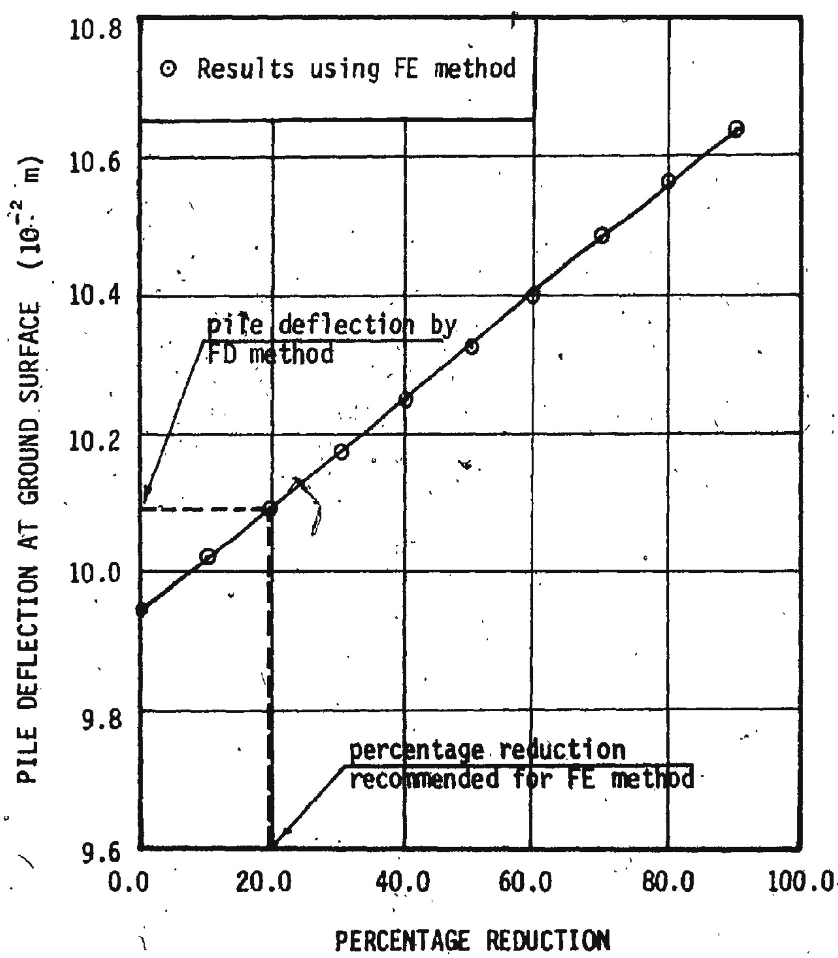


Figure 10 Percentage reduction in the soil spring constant at the top of the pile

procedure calculations based on repeated reference to the p-y curves to determine the values of the soil modulus at the nodal points which insure both compatibility and equilibrium for the soil, the pile and the superstructure. The program can generate a set of p-y curves in a layered soil system similar to those previously described in the finite difference method in section 3.3.3 . A reduction of 20% in the soil spring constant at the top of the pile is used. Step changes in the pile flexural rigidity can be introduced at any depth provided that the change occurs at a nodal point. As many as 30 elements can be used in the program. This program in its present form is capable of analyzing only free head piles where a lateral load H and an external moment MT are specified at the top of the pile without any axial load.

This program is not so flexible as the finite difference program in that any change in the end conditions would require major changes in the program. Only one type of end condition has been chosen for this analysis with a view to evaluating the comparative merits of the finite element and finite difference methods. A listing of the entire finite element program is given in appendix C.

3.4.3.1 Computer Core Storage Requirement

The finite element program requires about 256 K of storage to compile. 216 K of storage will be required to execute one problem on the FORTRAN H compiler of an IBM 370/158 machine as compared to 76 K for the finite difference method. It should be noted that the program uses double precision arithmetic.

3.4.3.2 Running Time

The CPU time of the program cannot be defined precisely because the program solves nonlinear problems and the CPU time depends on the magnitude of the problem. However, for the types of the examples illustrated in this thesis the CPU time was about 85 seconds for each problem as compared to 2 seconds for the finite difference method.

3.4.4 Sample Problem

The example that was chosen earlier for the finite difference program was solved by the finite element method. The pile loading, pile properties and soil properties are given in section 3.3.4 . The pile was divided into 30 elements of 1.0 m length each. The soil was analyzed in a similar way as mentioned in section 3.3.4 and an identical set of p-y curves were obtained.

The output of the computer program shows that the maximum deflection at the ground surface is equal to 100.99 mm , while the maximum positive bending moment occurs at a depth of 7.0 m below the ground surface and is equal to 2945.40 kN m. The soil modulus E_s increases with depth until it reaches a constant value of 10444.00 kN/m^2 below a depth of 11.0 m. The soil resistance p has a maximum value of 82.735 kN/m at a depth of 7.0 m. A partial computer output is shown in figure 11 and plotted in figure 12. It should be noted that the signs of both the deflection y and the soil resistance p are opposite to those obtained using the finite difference program because the coordinate systems are opposite in the finite element method. A comparative statement showing the salient values obtained by both methods is given in table 2.

3.5 Comparative Study

A comparative study between the finite difference and the finite

X (M)	Y (M)	M (KN M)	ES (KN/M2)	P (KN/M)
0.0	-0.10099D+00	0.10000D+04	0.46915D+03	0.21079D+02
0.10000D+01	-0.87984D-01	0.14789D+04	0.63623D+03	0.56246D+02
0.20000D+01	-0.75518D-01	0.19916D+04	0.83986D+03	0.63759D+02
0.30000D+01	-0.63751D-01	0.22645D+04	0.10966D+04	0.70323D+02
0.40000D+01	-0.52813D-01	0.25891D+04	0.14312D+04	0.75959D+02
0.50000D+01	-0.42011D-01	0.27617D+04	0.18519D+04	0.79754D+02
0.60000D+01	-0.33824D-01	0.28946D+04	0.24124D+04	0.82145D+02
0.70000D+01	-0.25899D-01	0.29454D+04	0.31686D+04	0.82735D+02
0.80000D+01	-0.19056D-01	0.29134D+04	0.42363D+04	0.84916D+02
0.90000D+01	-0.13283D-01	0.28005D+04	0.54228D+04	0.72751D+02
0.10000D+02	-0.85396D-02	0.26148D+04	0.72389D+04	0.62805D+02
0.11000D+02	-0.47568D-02	0.23664D+04	0.10444D+05	0.48407D+02
0.12000D+02	-0.18442D-02	0.20695D+04	0.10444D+05	0.19260D+02
0.13000D+02	0.30596D-03	0.17534D+04	0.10444D+05	-0.31954D+01
0.14000D+02	0.18089D-02	0.14405D+04	0.10444D+05	-0.18891D+02
0.15000D+02	0.27791D-02	0.11464D+04	0.10444D+05	-0.29024D+02
0.16000D+02	0.33245D-02	0.88143D+03	0.10444D+05	-0.34720D+02
0.17000D+02	0.35426D-02	0.65115D+03	0.10444D+05	-0.36997D+02
0.18000D+02	0.35180D-02	0.45786D+03	0.10444D+05	-0.36741D+02
0.19000D+02	0.33223D-02	0.30132D+03	0.10444D+05	-0.34697D+02
0.20000D+02	0.30132D-02	0.17946D+03	0.10444D+05	-0.31469D+02
0.21000D+02	0.26360D-02	0.89883D+02	0.10444D+05	-0.27529D+02
0.22000D+02	0.22242D-02	0.26231D+02	0.10444D+05	-0.23229D+02
0.23000D+02	0.18013D-02	-0.13393D+02	0.10444D+05	-0.18812D+02
0.24000D+02	0.13822D-02	-0.39284D+02	0.10444D+05	-0.14435D+02
0.25000D+02	0.97482D-03	-0.40581D+02	0.10444D+05	-0.10181D+02
0.26000D+02	0.58179D-03	-0.36776D+02	0.10444D+05	-0.60760D+01
0.27000D+02	0.20196D-03	-0.26896D+02	0.10444D+05	-0.21092D+01
0.28000D+02	-0.16888D-03	-0.14907D+02	0.10444D+05	0.17553D+01
0.29000D+02	-0.53259D-03	-0.46728D+01	0.10444D+05	0.55613D+01
0.30000D+02	-0.89487D-03	-0.90949D-12	0.10444D+05	0.45728D+01

Figure 11 Partial computer output for the sample problem using the finite element program

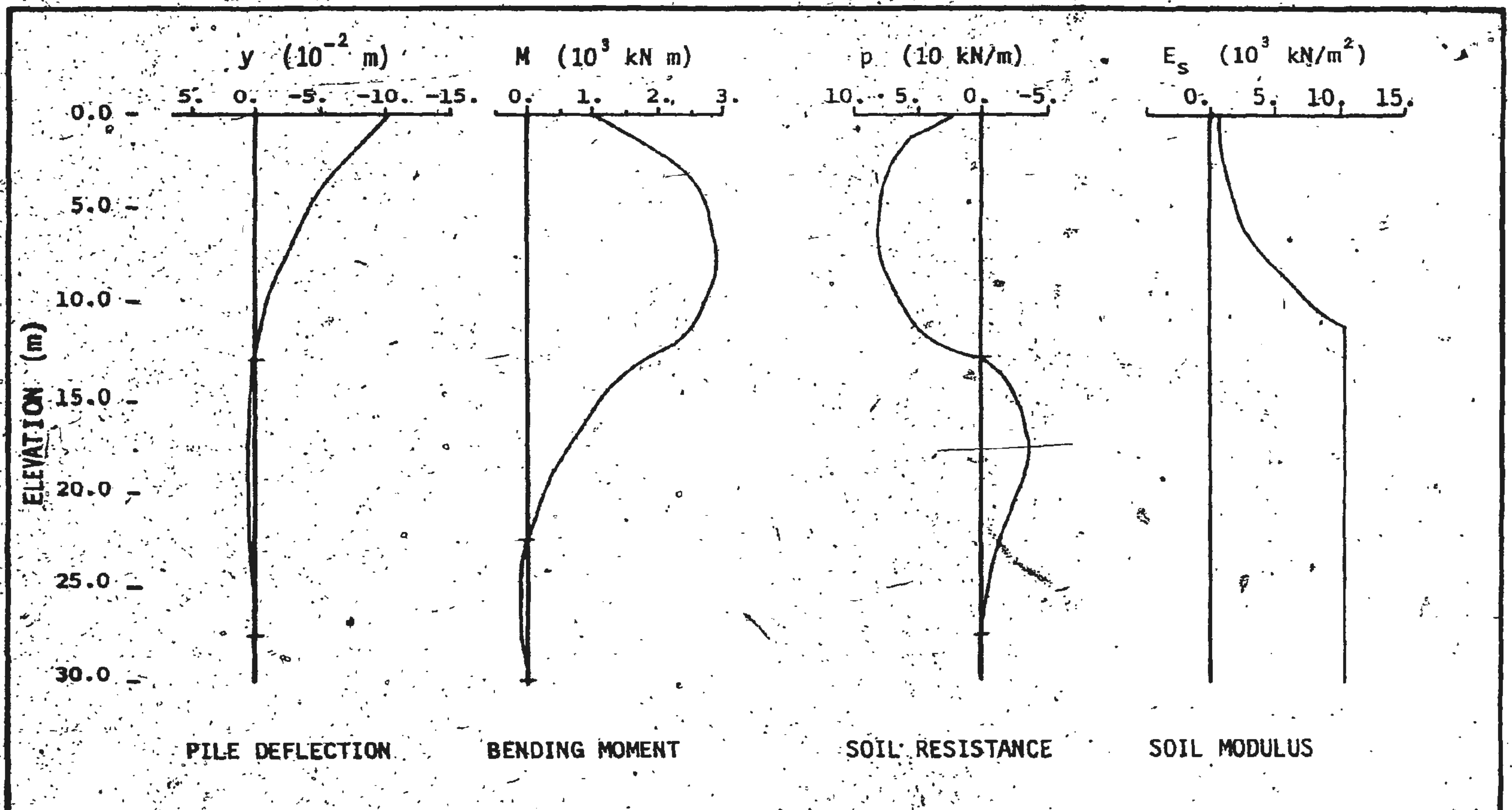


Figure 12 Output results for the sample problem using the finite element program

Table 2 Comparison between the two outputs of the finite difference and the finite element programs for solving the sample problem.

Method	Finite Difference	Finite Element
Maximum deflection, mm	100.99	100.99
Maximum bending moment, kN m	2934.90	2945.40
Depth to the maximum bending moment, m	7.00	7.00
Maximum value of soil modulus, kN/m^2	10444.00	10444.00
Maximum soil resistance, kN/m	82.035	82.735
Depth to the maximum soil resistance, m	7.00	7.00

element approaches in solving the laterally loaded pile problem will be presented here. A comparison between the two theoretical schemes and published results of full scale tests in different types of soils will be discussed.

3.5.1 Comparison Between the Finite Difference and the Finite Element Methods

The two methods presented in sections 3.3 and 3.4 insure both compatibility and equilibrium for the soil, the pile and the superstructure under the effect of lateral loading. The finite difference approach is based on expressing the governing differential equation [11] in a difference equation form [15] and extending it to sequential points along the pile length with the aid of two imaginary points at the top of the pile and another two at the bottom to represent the boundary conditions. On the other hand , the finite element approach is based on discretizing the pile into a finite number of elements and expressing the external nodal forces in terms of the external nodal displacements with the aid of both the static and the stiffness matrices (equation [21]), then inverting the relationship (equation [22]), to obtain the solution. In both the computer programs soil nonlinearity is taken into consideration by repeated reference to the p-y curves along the pile length which are generated in the programs.

3.5.1.1 Computer Core Storage Requirement and Running Time

Both the programs require about 256 K of storage to compile, but the finite difference program requires only 76 K of storage to execute one problem on the FORTRAN H compiler while the finite element program requires 216 K of storage to execute the same problem on the same compiler . Also the first program solves one problem in less than 2 seconds of CPU time while the latter one solves the same problem in about 85 seconds.

The reason for the large storage and the greater time requirement for the second program is that it involves large matrices and the process of inverting such matrices requires both storage and CPU time. This problem can be overcome by using more powerful computer techniques in dealing with these matrices and taking advantage of the symmetrical, banded or sparce matrices. Such modifications would form part of a separate project on computer techniques.

3.5.1.2 Number of Elements

While the finite difference program can use as many as 150 increments along the pile length, the finite element program allows only up to 30 elements. The maximum allowable number of elements which is 30 in the finite element program proved to be sufficient to yield good results when these results are compared with the experimental values as shown in subsequent sections. However, this number can be increased without a significant increase in both the computer core storage and the CPU time by using more powerful computer techniques as previously mentioned in section 3.5.1.1 . Moreover, long piles with pile properties and loadings of the order discussed in this thesis, can be analyzed based on the upper 15 to 20 m of the length as the pile length is not a critical factor in the analysis (Bowles, 1972, 1974) . This fact makes the maximum number of elements specified in the finite element program sufficient enough to obtain good results.

3.5.1.3 Boundary Conditions

The finite difference program can deal with piles with different degrees of head fixity, while the finite element program as developed here is capable of analyzing only free head piles. It is to be noted that the latter program can be modified to allow for other types of end conditions. The advantage of the finite difference approach over the finite element one is that it is capable of dealing with both horizontal and vertical loading, while the latter can handle only lateral loading.

3.5.2 Field Test Results

A number of full scale tests on laterally loaded piles have been conducted during the last few years. The results obtained from three different sites have been chosen to present a comparison between the full scale tests and the finite difference and the finite element analyses.

3.5.2.1 Laterally Loaded Piles in Stiff Clay

Reese et al (1975) conducted a series of full scale tests on two 609.60 mm (24 in) and one 168.30 mm (6 in) diameter piles driven in stiff clay below water table. The pile and soil properties reported in these tests were input into the finite difference and the finite element programs. The computed and measured moment curves for the 609.60 mm diameter pile subjected to an increasing set of horizontal static loads are shown in figures 13 to 16. Computed and measured values of maximum bending moment as a function of the lateral load for static loading are shown in figures 17 and 19 for the 609.60 mm and the 168.30 mm diameter piles respectively. Curves in fugures 18 and 20 show the comparison between computed and measured values of deflection at groundline as a function of the lateral static loading for the 609.60 mm and 168.30 mm diameter piles respectively. A good agreement is observed

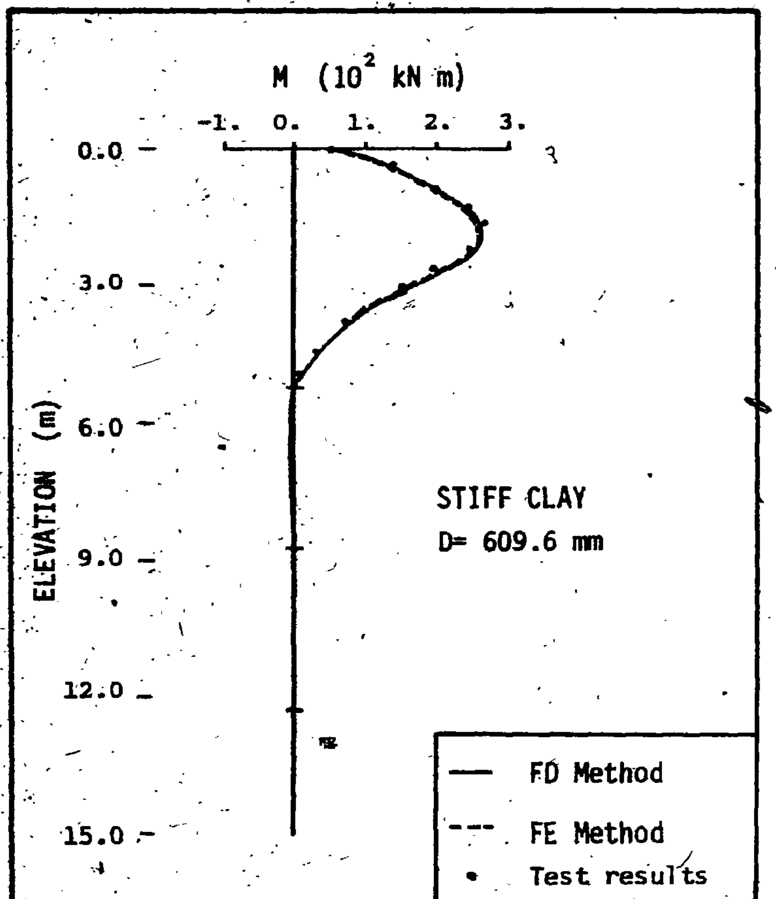


Figure 13 Computed moment curves and test results, H = 180.0 kN

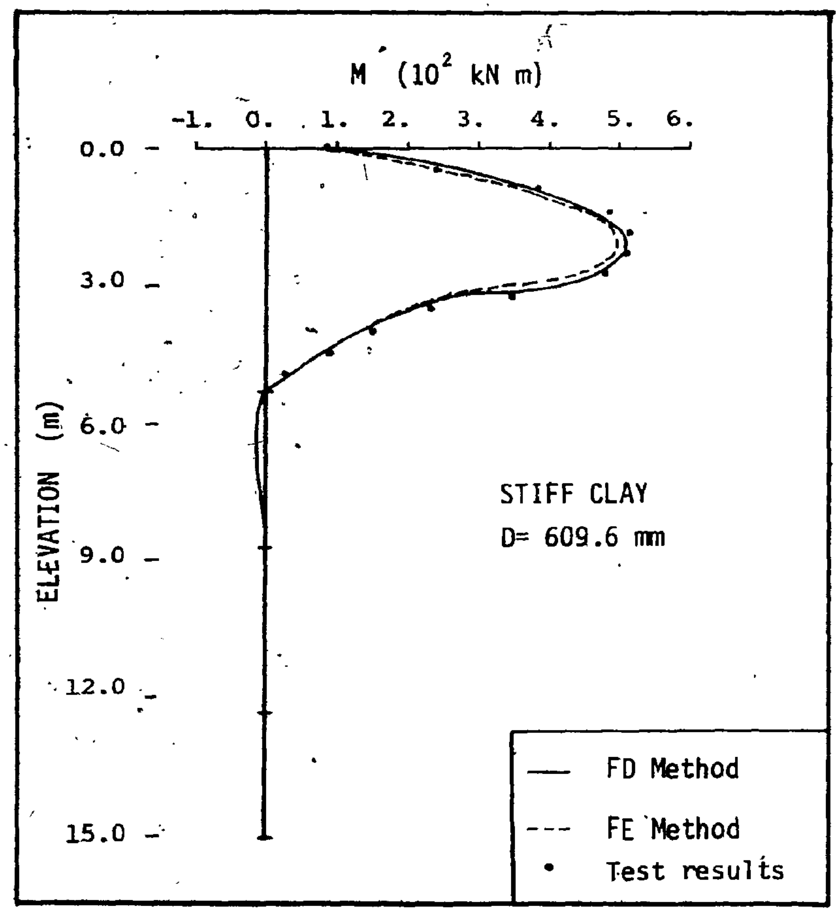


Figure 14 Computed moment curves and test results, H = 318.0 kN

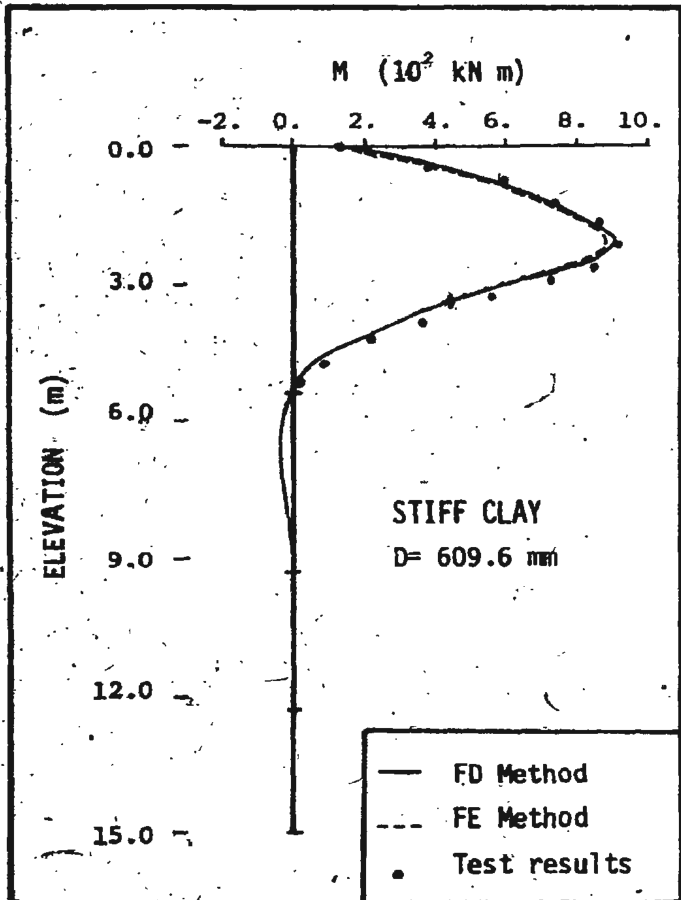


Figure 15 Computed moment curves and test results, H = 486.0 kN

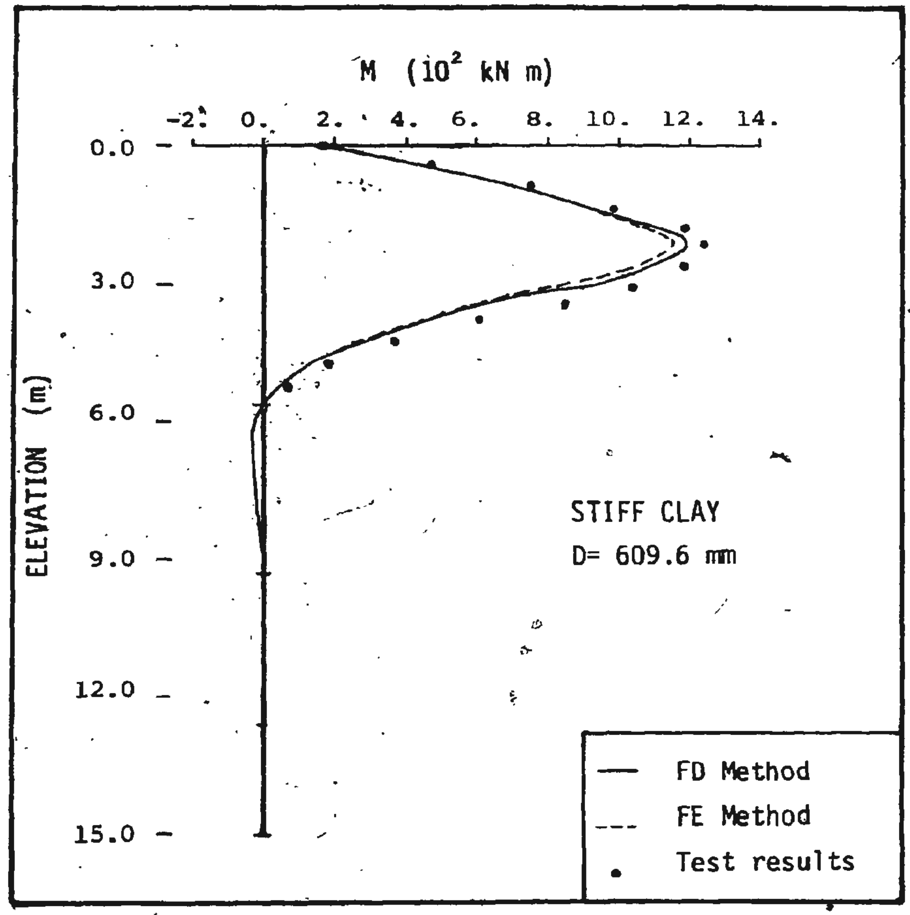


Figure 16 Computed moment curves and test results, H = 606.0 kN

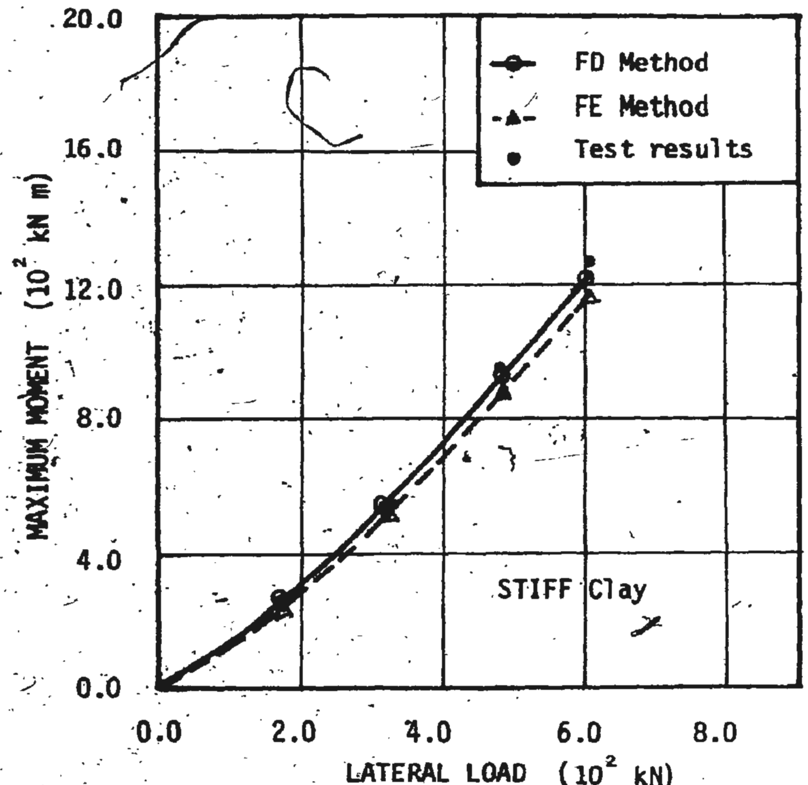


Figure 17 Computed and measured values of maximum bending moment, D = 609.60 mm

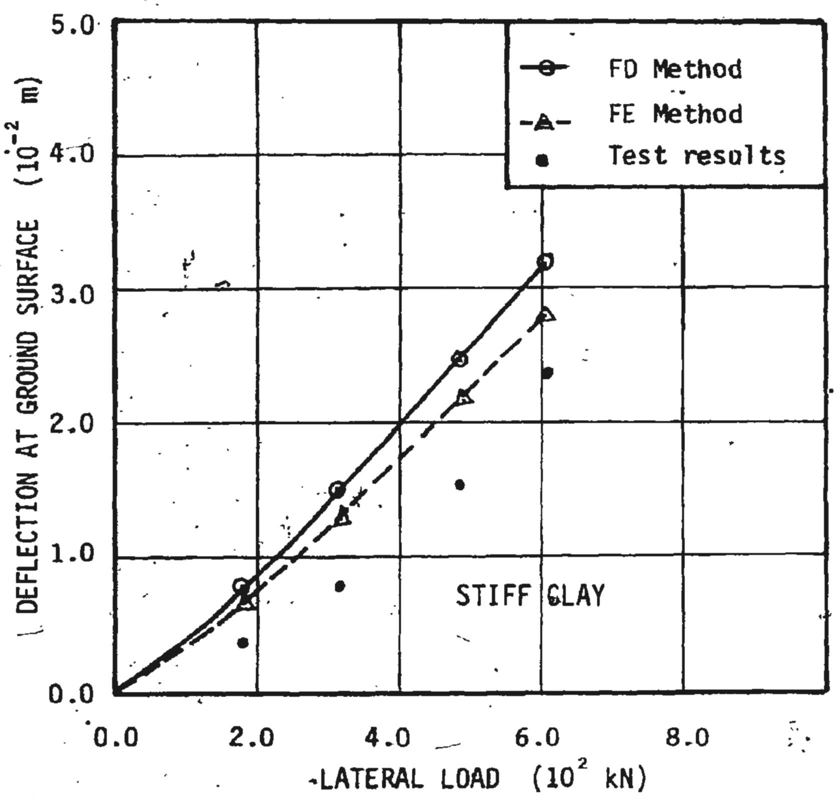


Figure 18 Computed and measured values of deflection at ground surface, D = 609.60 mm

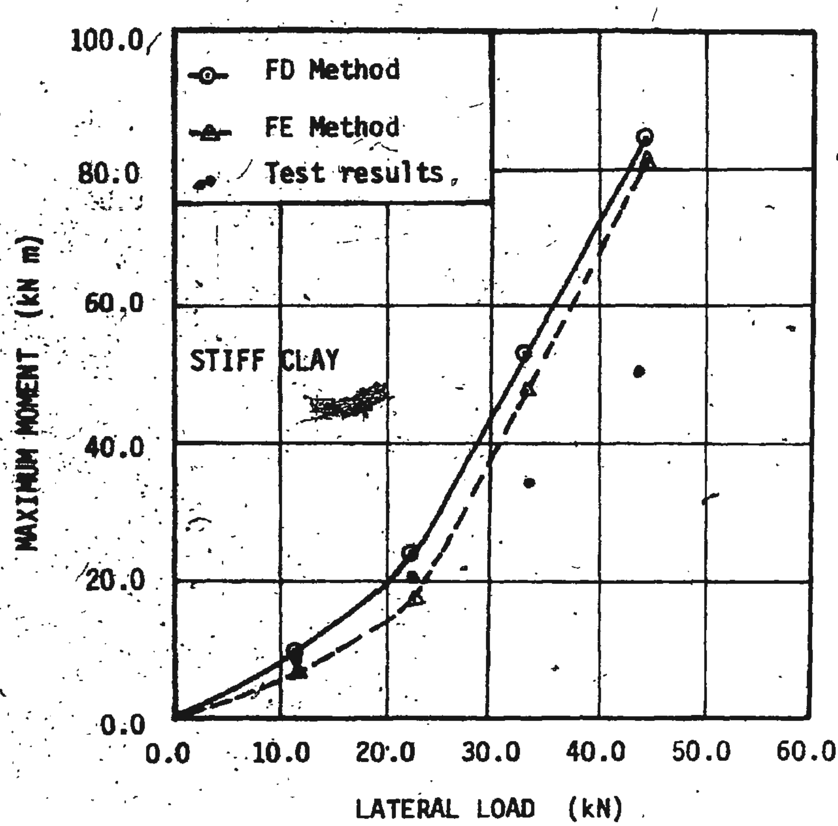


Figure 19 Computed and measured values of maximum bending moment, D = 168.30 mm

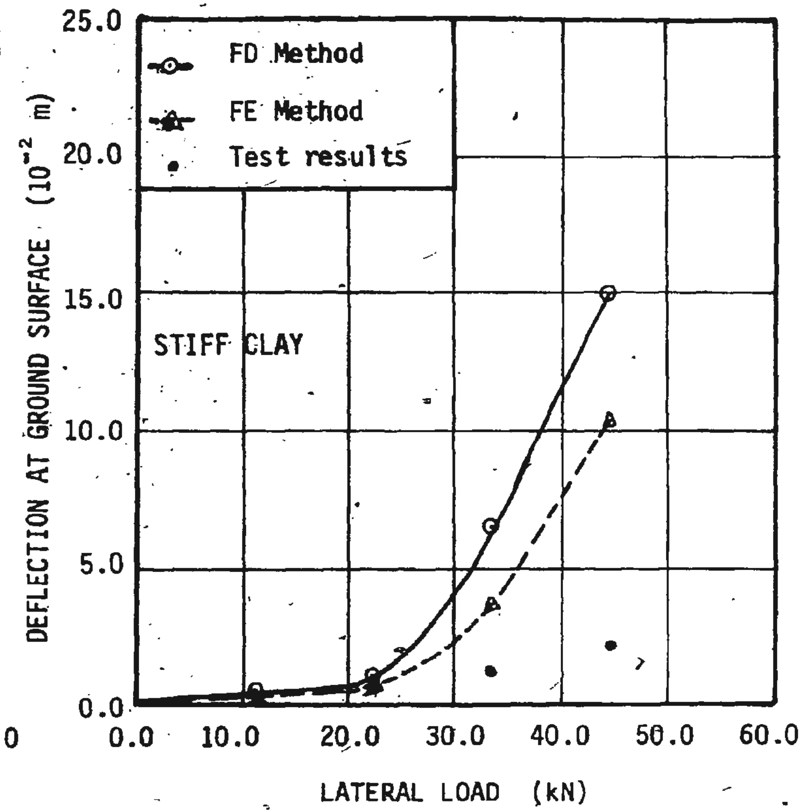


Figure 20 Computed and measured values of deflection at ground surface, D = 168.30 mm

between the computed and measured bending moments. The agreement is not satisfactory for the deflection at groundline, especially in the case of the 168.30 mm diameter pile. The poor agreement is attributed to the sensitivity of the deflection to the soil modulus which is heavily influenced by the derived p-y curves.

3.5.2.2 Laterally Loaded Piles in Soft Clay

The results of the full scale tests conducted by Matlock (1970) on a laterally loaded pile driven in a soft clay layer below water surface at Sabine site, Texas were compared with those computed using the two theoretical methods. The computed and measured moment curves for different static load increments are shown in figures 21 to 24. Here also, comparison indicates a good agreement between the computed and measured bending moments. The pile deflections were not reported in the test and no comparison could therefore be made.

3.5.2.3 Laterally Loaded Piles in a Layered Soil System

The efficiency of the two computer programs in dealing with a layered soil system was briefly stated earlier and presented further in this section. Full scale tests by Ismael and Klym (1978) are the basis of the comparison between computed and measured results. The test was conducted on a 1.5 m diameter rigid concrete pier with a total length of 12.0 m . The soil profile at the site generally consists of a layered system of stiff silty clays. Both the pile and soil properties were introduced in the computer programs and the output computed deflections are compared with the measured values in figure 25. Results indicate that both the programs do not show good correlation, but the correlation with the finite element method is slightly better. This deviation be-

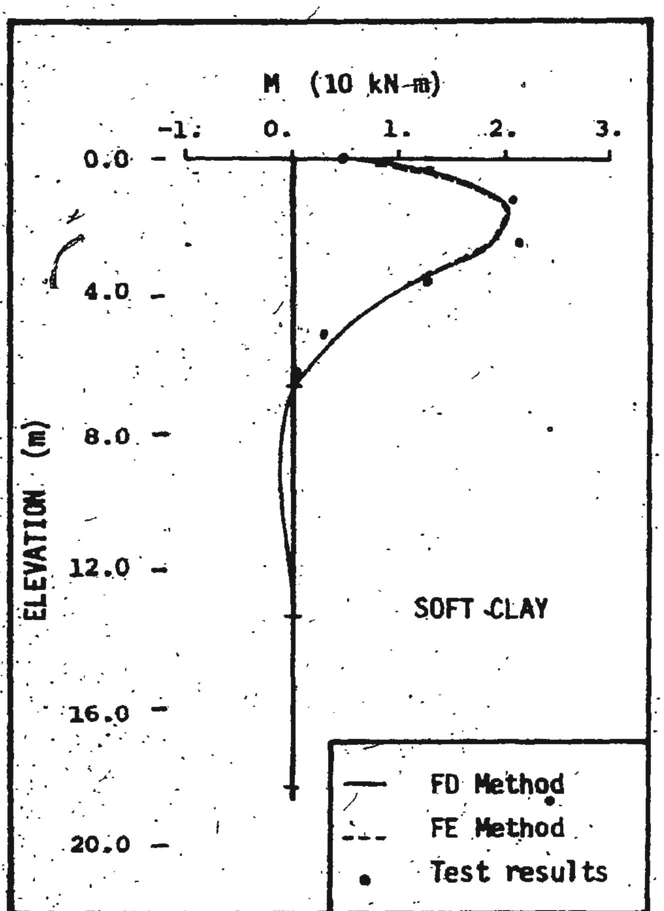


Figure 21 Computed moment curves and test results, H = 17.8 kN

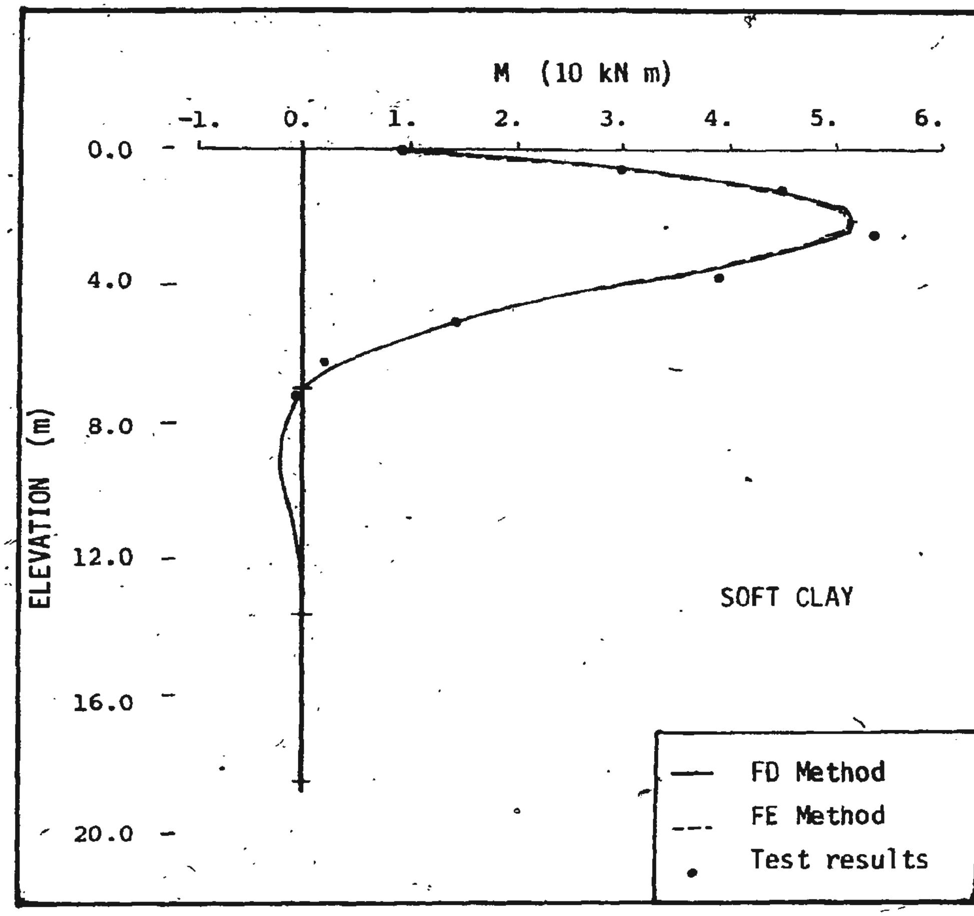


Figure 22 Computed moment curves and test results, H = 35.6 kN

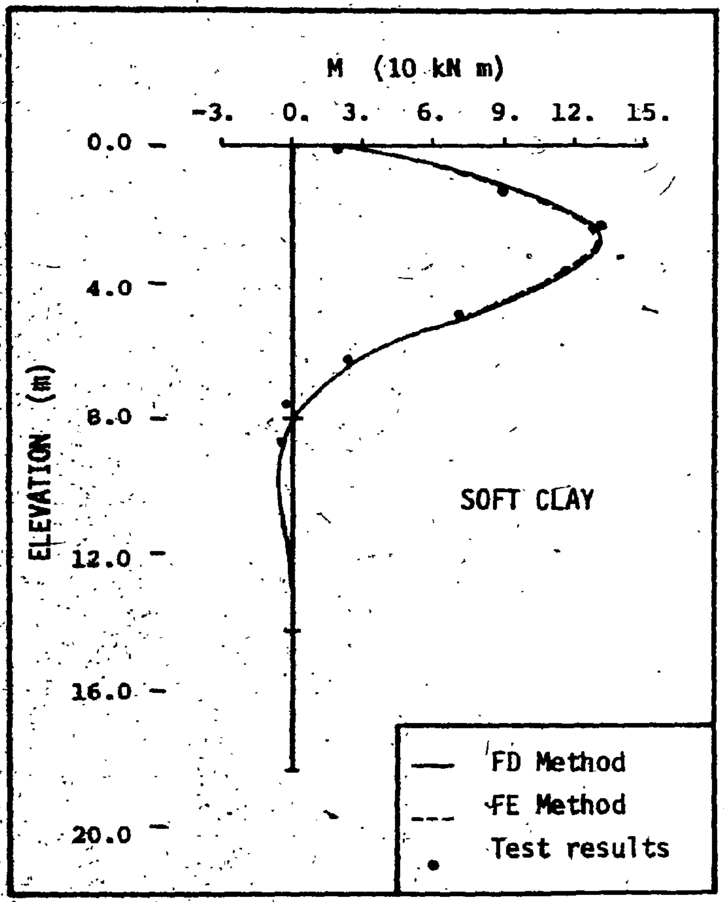


Figure 23 Computed moment curves and test results, H = 71.2 kN

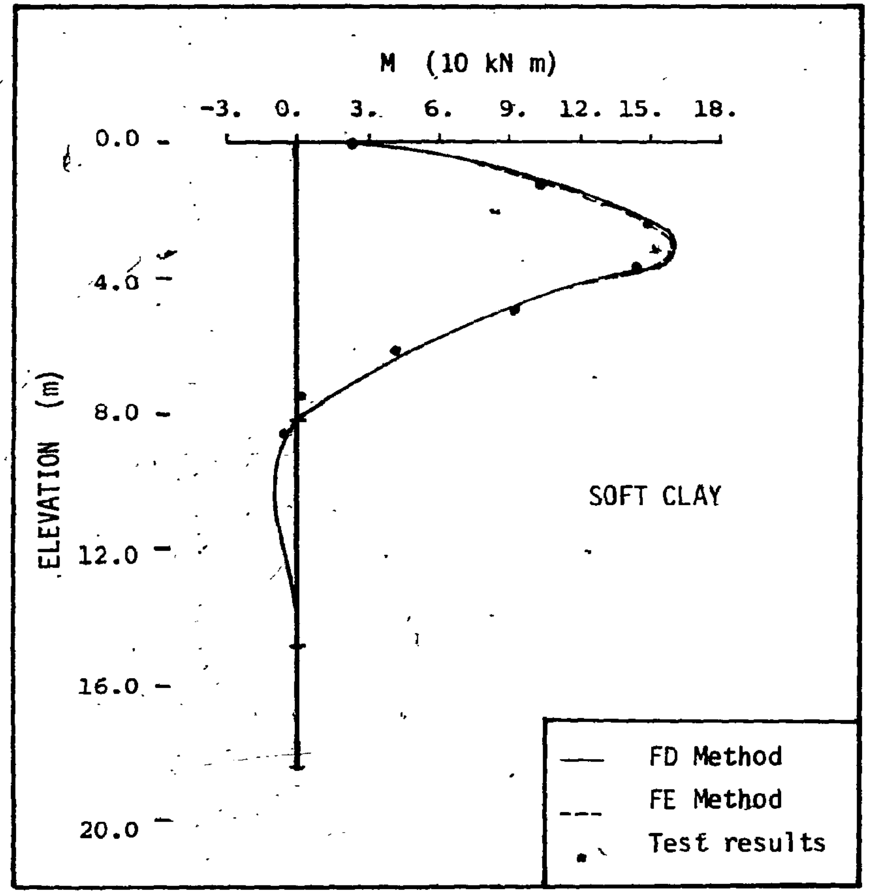


Figure 24 Computed moment curves and test results, H = 80.1 kN

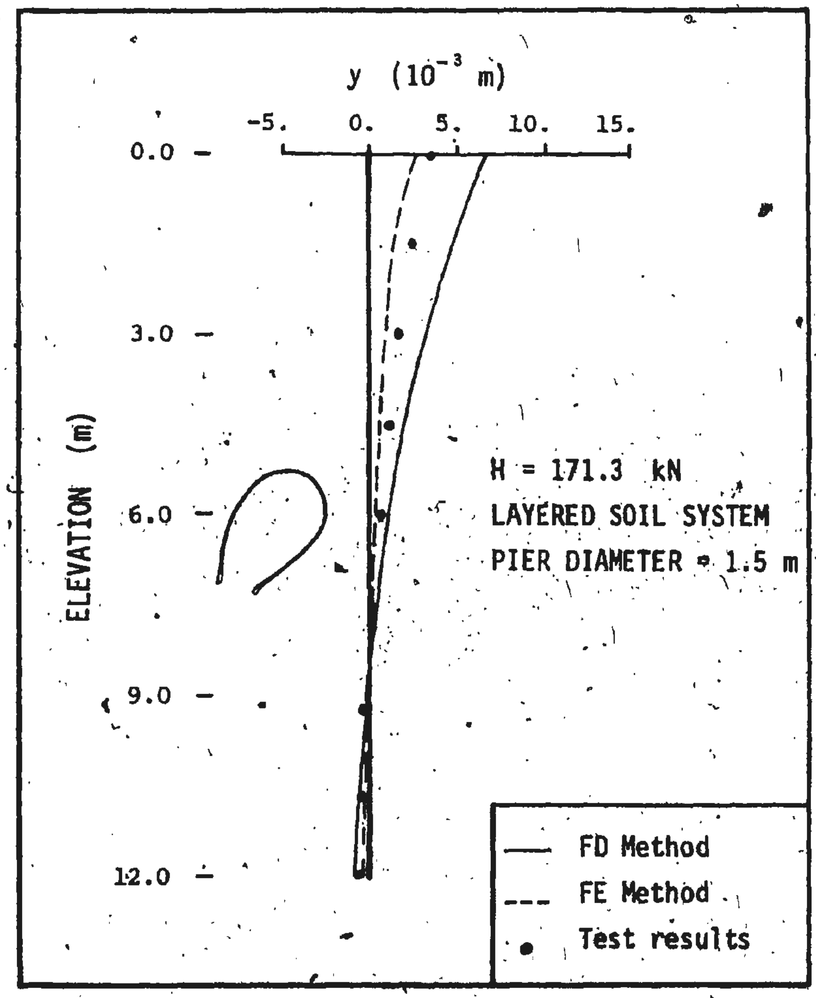


Figure 25 Computed deflection curves and test results

tween the theoretical and experimental results is attributable to the effect of the pier rigidity on the computed p-y curves. Also the differential equation [11] is applicable to the case of rigid piers only with a degree of approximation.

3.6 Summary

Both the finite difference and the finite element approaches in analyzing the laterally loaded pile problem were briefly discussed. A computer program using the finite difference method was modified to generate the p-y curves along the pile length, while another program using the finite element method was developed. A comparison between the two theoretical schemes was presented. Methods of predicting the p-y curves in different types of soils were extended for a layered soil system. Generally, the results obtained showed satisfactory agreement compared with the field results. Conclusions drawn from this study are presented in chapter V.

CHAPTER IV

PARAMETRIC ANALYSIS

4.1 General

In the process of analyzing laterally loaded piles, a number of variables were considered in determining the static response. These include the pile loading and the properties of the piles as well as those of the supporting soil. As a part of this study, an analysis of these variables was made to determine their relative importance.

Parametric studies were made for 1) a free head pile in soft clay below water surface, 2) a free head pile in stiff clay below water surface, 3) a fixed head pile in soft clay below water surface, and 4) a fixed head pile in stiff clay below water surface. A free head pile is a pile with its head free to translate and rotate. A fixed head pile is a pile with its head free to translate but fixed against rotation. These two types of piles were chosen as representative of two extreme situations that can occur in practice. It is to be realized that there is no ideal free head or fixed head piles that would be found in practice, since there will always be some amount of rotation. Such actual situations would be intermediate between the two extreme cases discussed here. For each of the four cases, a study was made to determine the effect of a) the horizontal load, b) the vertical load, c) the external moment, d) the pile length, e) the pile diameter, f) the pile flexural rigidity, g) the soil undrained shear strength and h) the soil effective unit weight. The effect of each of these variables on the maximum deflection and the maximum positive and negative bending moments was studied. Also, for a layered soil composed of soft clay below water surface underlain by stiff clay, a study was made to determine the effect of the soft clay layer depth on the maximum deflection and the maximum positive and negative bending moments in both free and fixed head piles.

An algorithm was introduced in the finite difference program already discussed to vary the different parameters, one at a time, and the corresponding pile deflection and bending moment were obtained. In each case, the magnitude and position of the maximum deflection and the maximum positive and negative bending moments were obtained. Separate plotting subroutines were developed to plot both the pile deflection and bending moment along the pile length and the variation of the maximum deflection and the maximum positive and negative bending moments as a function of any desired variable. Partial listing of the plotting subroutines is included in appendix C. A CALCOMP graph plotter was used to generate all the graphs.

4.2 Practical Range of Input Data

In order to do the parametric studies, a practical range had to be determined for 1) the pile loading , 2) the pile properties, and 3) the soil properties. Table 3 shows the pile properties and the corresponding pile loading for some typical offshore structures. Table 4 shows the soil properties for ocean sediments in both the Atlantic and the Pacific oceans.

4.2.1 Pile Loading

Some offshore piles in the Bass Strait off the Coast of Australia support horizontal loads up to 3300 kN (Loftin, 1976), while some others in the North Sea are subjected to horizontal loads as low as 325 kN (O'Neill and Ghazzaly, 1977). The vertical loads vary between 23000 kN (McClelland, 1974) and 1850 kN (O'Neill and Ghazzaly, 1977). The external moment at the top of the pile ranges from 9800 kN m (Focht and Koch, 1973) to 2300 kN m (O'Neill and Ghazzaly, 1977).

Table 3 Pile properties and pile loading for some typical offshore structures.

Reference	Type and location of the structure	Pile length (m)	Pile diameter (m)	Wall thickness (m)	Horizontal load (kN)	Vertical load (kN)	External moment (kN m)
Focht and Koch (1973)	Typical North Sea offshore structure	76.0	1.22	0.0635	2000.0		9800.0
McClelland (1974)	Drilling platform, Gulf of Mexico.	91.0	1.22		1100.0	18000.0	
	Drilling platform, Cook Inlet, Alaska.	28.0	0.76		900.0	18000.0	
	Drilling platform, North Sea.	73.0	1.37	0.051 - 0.064	2100.0	23000.0	
	Oil storage structure, Persian Gulf.	28.0	0.91			5400.0	
Loftin (1976)	Drilling and production platform, Bass Strait off the coast of Australia.	137.0	1.22 - 1.52	0.025 - 0.051	3300.0	22000.0	
O'Neill and Ghazzaly (1977)	Typical North Sea platform structure.	52.0	1.07	0.025 - 0.051	325.0	1850.0	2300.0
		52.0	1.07	0.025 - 0.051	325.0	2500.0	2300.0
	Typical Gulf of Mexico structure.	30.0	1.11 - 1.42	0.051 - 0.076	1300.0	5700.0	7800.0
		30.0	1.11 - 1.42	0.051 - 0.076	1300.0	7500.0	7600.0

Table 4 Soil properties for ocean sediments in Atlantic and Pacific Oceans .

Reference	Location	Undrained shear strength (kN/m^2)	Saturated unit weight (kN/m^3)
Keller (1969) Noorany and Gizienski (1970) Fukuoka and Nakase (1973)	Atlantic Ocean	less than 3.5 to 23.0	14.75 to 17.00
	Pacific Ocean	less than 3.5 to 17.5	12.25 to 14.75

4.2.2 Pile Properties

Most of the offshore piles are steel pipe piles filled with concrete. The pile lengths vary between 137 m for a deepwater fixed platform in Bass Strait off the coast of Australia (Loftin, 1976) and 28 m for a drilling platform in Cook Inlet, Alaska (McClelland, 1974). The pile diameters range from 1.52 m (Loftin, 1976) to 0.76 m (McClelland, 1974), while the wall thickness varies between 76 mm and 25 mm.

4.2.3 Soil Properties

4.2.3.1 Soil Undrained Shear Strength

Average shear strength values in the Atlantic ocean range from less than 3.5 kN/m^2 to 14.0 kN/m^2, for the upper few feet of sea-floor sediments. Sediments with a shear strength of 3.5 kN/m^2 to 7.0 kN/m^2 appear to predominate in the North Atlantic basin. In contrast to the North Atlantic, large portions of the North Pacific sea-floor are covered with sediments whose average shear strength is less than 3.5 kN/m^2. However there are some areas in the Pacific ocean where the shear strength values range from 14.0 kN/m^2 to 17.5 kN/m^2 (Keller, 1969; Fukuoka and Nakase, 1973). Higher values for the shear strength, as high as 23.0 kN/m^2, in the Atlantic ocean have also been reported (Noorany and Gizienski, 1970).

A clay sample is considered to be stiff, very stiff, or hard if the undrained shear strength is higher than 50.0 kN/m^2, and it is considered to be very soft, soft, or medium if the undrained shear strength is less than 50.0 kN/m^2 (Terzaghi and Peck, 1948). In this analysis if the undrained shear strength is less than 50.0 kN/m^2, the clay will be considered as soft clay, and if it is higher than this value, the clay will be referred to as a stiff clay.

Since this study concerns with offshore piles, the stiff clay layer below water surface will be simply referred to as stiff clay, and the soft clay layer below water surface as soft clay.

4.2.3.2 Soil Unit Weight

For the upper few feet of the sea-floor in both the Atlantic and the Pacific oceans, the saturated unit weight of sediments is found to range from 11.50 kN/m^3 to 19.50 kN/m^3, but more frequently varies from 12.25 kN/m^3 to 17.00 kN/m^3. The value of the saturated unit weights is generally lower in the North Pacific, (12.25 kN/m^3 to 14.75 kN/m^3) compared to the North Atlantic, (14.75 kN/m^3 to 17.00 kN/m^3) (Keller, 1969 ; Fukuoka and Nakase, 1973).

4.2.4 Input Data

From a brief survey of the practical range of the different parameters affecting the laterally loaded offshore piles, values were chosen for the parametric analysis. The standard values used in the parametric studies and the range of variation considered for each variable are tabulated in table 5. Each of these parameters was varied within its range while all other variables were kept constant. The results will be discussed in the following sections.

4.3 Parametric Studies

Each of the parameters identified in table 5 was varied in seven increments within its range while all other variables were kept constant. The parameter effect was studied for the four cases; two pile types (free head and fixed head) and two soil types (soft clay and stiff clay). In each case, plots of the deflection and the bending moment along the

Table 5 Standard values and range of variation considered for each variable in the parametric studies.

Variable		Standard value	Range of variation
Pile loading :			
horizontal load	, kN	800.00	100.00 to 1150.00
vertical load	, kN	8000.00	0.00 to 18000.00
external moment	, kN m	0.0	0.00 to 2400.00
Pile properties :			
Pile length	, m	60.00	24.00 to 96.00
pile diameter	, m	1.00	0.75 to 2.25
pile wall thickness	, m	0.05	
pile flexural rigidity (steel pipe pile filled with concrete)	, MN m^2	2710.00	2000.00 to 20000.00
Soil properties :			
(a) soft clay			
undrained shear strength	, kN/m^2	25.00	15.00 to 45.00
effective unit weight	, kN/m^3	6.00	2.00 to 8.00
(b) stiff clay			
undrained shear strength	, kN/m^2	100.00	55.00 to 205.00
effective unit weight	, kN/m^3	8.00	5.00 to 11.00

pile were obtained for the two extreme values of the parameter. Because of the scale limitations, only the points where these curves cross the longitudinal axis of the pile are shown in the lower portions. These are points of zero deflection and zero bending moment. The plot showing the effect of the parameter on the maximum deflection and the maximum positive and negative bending moments was finally drawn. Magnitudes and positions of the maximum deflection , Y_{max} , the maximum positive bending moment, M_{max+} and the maximum negative bending moment , M_{max-} for each increment were tabulated.

The parametric study effectively involves nine variables , with four types of soil-pile combinations. A total of 102 graphs and 32 tables were obtained after the analysis. Only three visuals are presented in the main text of the thesis for each parameter, and the remaining are serially arranged in an appendix, but referred and discussed in the main text. A similar method is adopted for the tabulated results.

4.3.1 Pile Loading

4.3.1.1 Effect of the Horizontal Load

Typical graphs showing the effect of the horizontal load are shown in figures 26 to 28. Tabulated results are summarized in table 6. Both the graphs and the table are for a free head pile in soft clay. Similar graphs and tables for a free head pile in stiff clay and a fixed head pile in the two types of soil are given in appendix A (tables 16 to 18) and appendix B (figures 60 to 68).

In case of free head piles, Y_{max} occurs at the ground surface and increases in magnitude with increasing horizontal load. However, the resulting bending moment for the two types of soil is slightly different, In case of stiff clay, the location of M_{max+} is almost independent of the horizontal load, while it shifts down along the pile axis in case of soft clay. A shift as much as 5.0 m was observed for the range of loading considered here. In both types of soil M_{max+} significantly increases with the increase of the horizontal load. The ratio of M_{max+} to M_{max-} is higher in case of stiff clay.

For fixed head piles, both Y_{max} and M_{max-} occur at the ground surface and increase with the increasing horizontal load. In case of stiff clay, the deflection is very small, where its maximum value equals to only 5.90 mm corresponding to a horizontal load of 1150.0 kN . For both types of soil M_{max-} is higher than M_{max+} by three to four times.

An interesting phenomenon noticed here is that the ratio of M_{max-} to M_{max+} for fixed head piles is much smaller than M_{max+} to M_{max-} in case of free head piles. Obviously this means that the design of fixed head piles will be more economical in terms of optimization compared to a free head pile. Also in case of stiff clay, the position of M_{max+} is nearer to the ground surface and both the deflection and the bending moment vanish very rapidly along the pile axis, while in soft clay such a phenomenon is not noticed. This implies that an upper shorter portion of the pile is subjected to significant bending stresses in case of stiff clay.

4.3.1.2 <u>Effect of the Vertical Load</u>

Typical graphs showing the effect of the vertical load are shown in figures 29 to 31. Tabulated results are summarized in table 7. Both the graphs and the table are for a free head pile in soft clay. Similar graphs and tables for a free head pile in stiff clay and a fixed head pile in the two types of soil are given in appendix A (tables 19 to 21) and appendix B (figures 69 to 77) .

Results show that the increase of the vertical load causes a corresponding increase in the magnitudes of Y_{max} , M_{max+} and M_{max-} for both free and fixed head piles in the two types of soil. This agrees with the experimental and theoretical studies presented by Zuhkov and Balov (1978) for laterally loaded piles subjected to vertical loading. However, this increase appears to be significant only in the case of a free head pile in soft clay, but is not appreciable in the other three cases. It can also be noticed that the position of M_{max+} and M_{max-} is nearly independent of the vertical load especially for fixed head piles.

The magnitude of Y_{max} is significant only for a free head pile in soft clay, while it is almost negligible even for higher vertical loads, in the case of fixed head pile in stiff clay. Also, for piles driven in soft clays, a longer upper portion of the pile is subjected to appreciable bending stresses compared to the case of stiff clays.

4.3.1.3 Effect of the External Moment

Typical graphs showing the effect of the external moment are shown in figures 32 to 34. Tabulated results are summarized in table 8. Both the graphs and the table are for a free head pile in soft clay. Similar graphs and table for a free head pile in stiff clay are given in appendix A (table 22) and appendix B (figures 78 to 80).

In both types of soil, Y_{max} , M_{max+} and M_{max-} increase almost linearly with the increase of the external moment, and the magnitude of increase in M_{max+} is approximately the same as the increase in the external moment. But in case of soft clay there is a small shift in the depth to M_{max+} with an increase in the external moment, while that depth is independent

(Text continued on page 78)

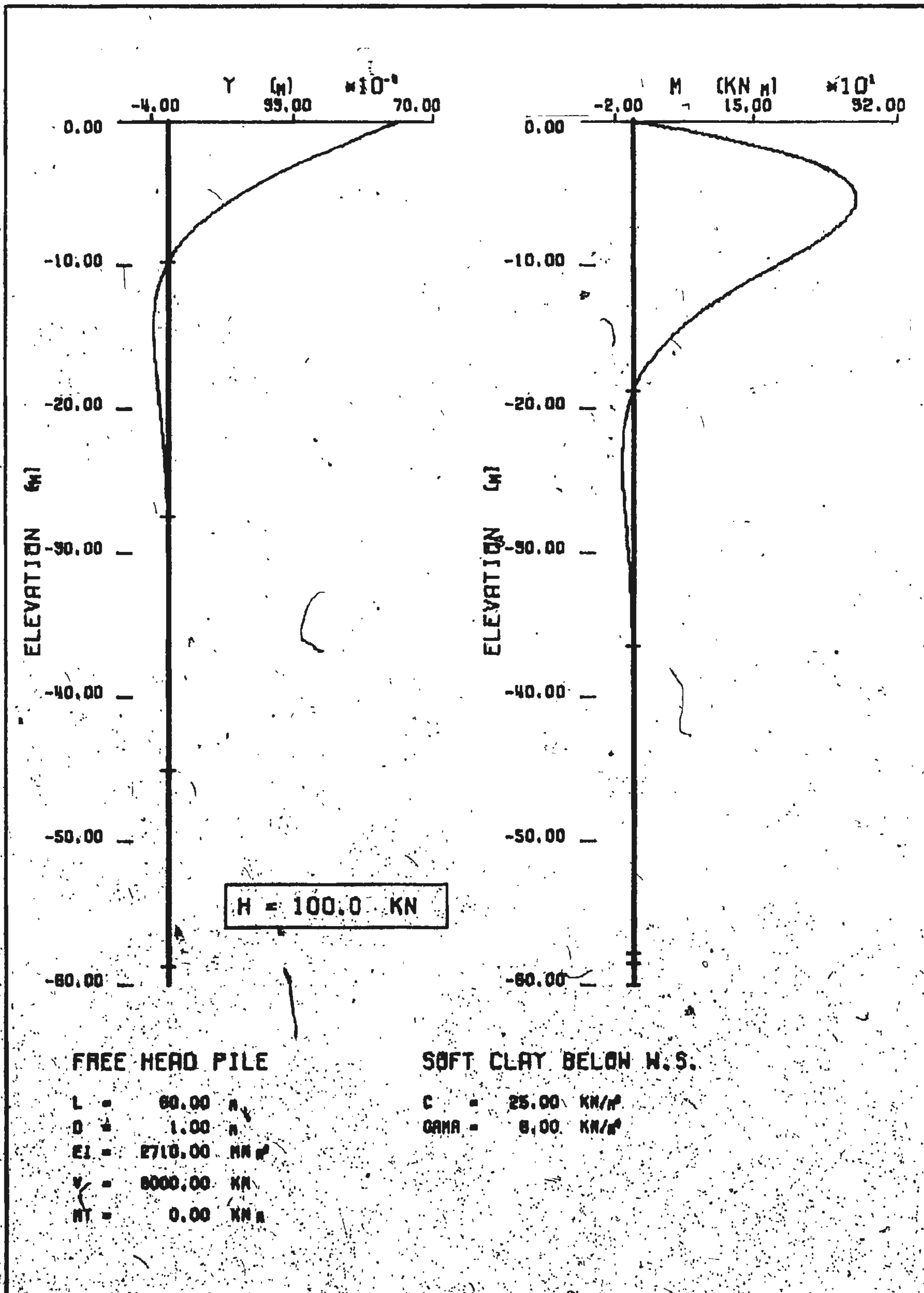


Figure 26 Deflection and bending moment for a free head pile in soft clay, H = 100.0 kN.

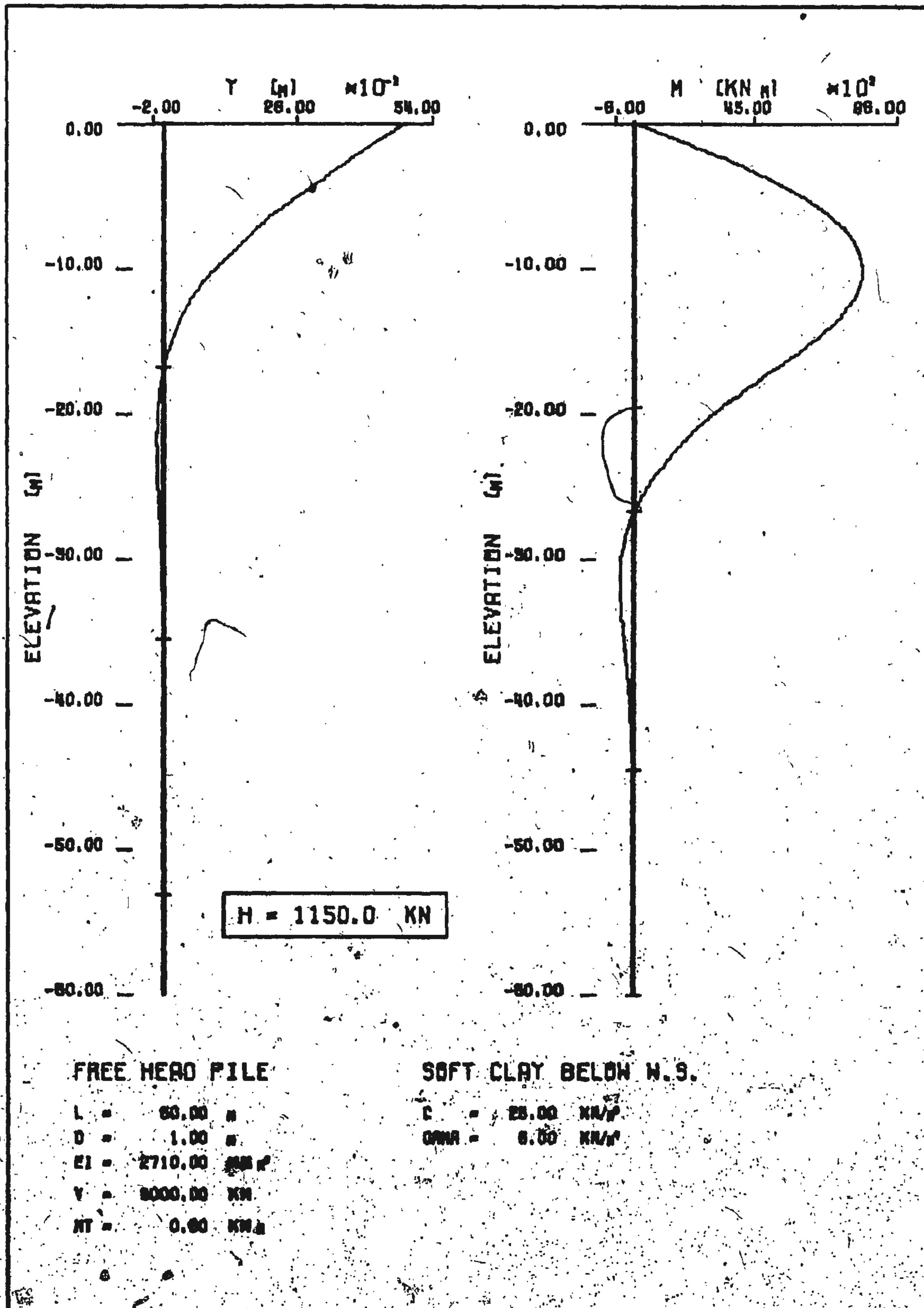


Figure 27 Deflection and bending moment for a free head pile in soft clay, H = 1150.0 kN.

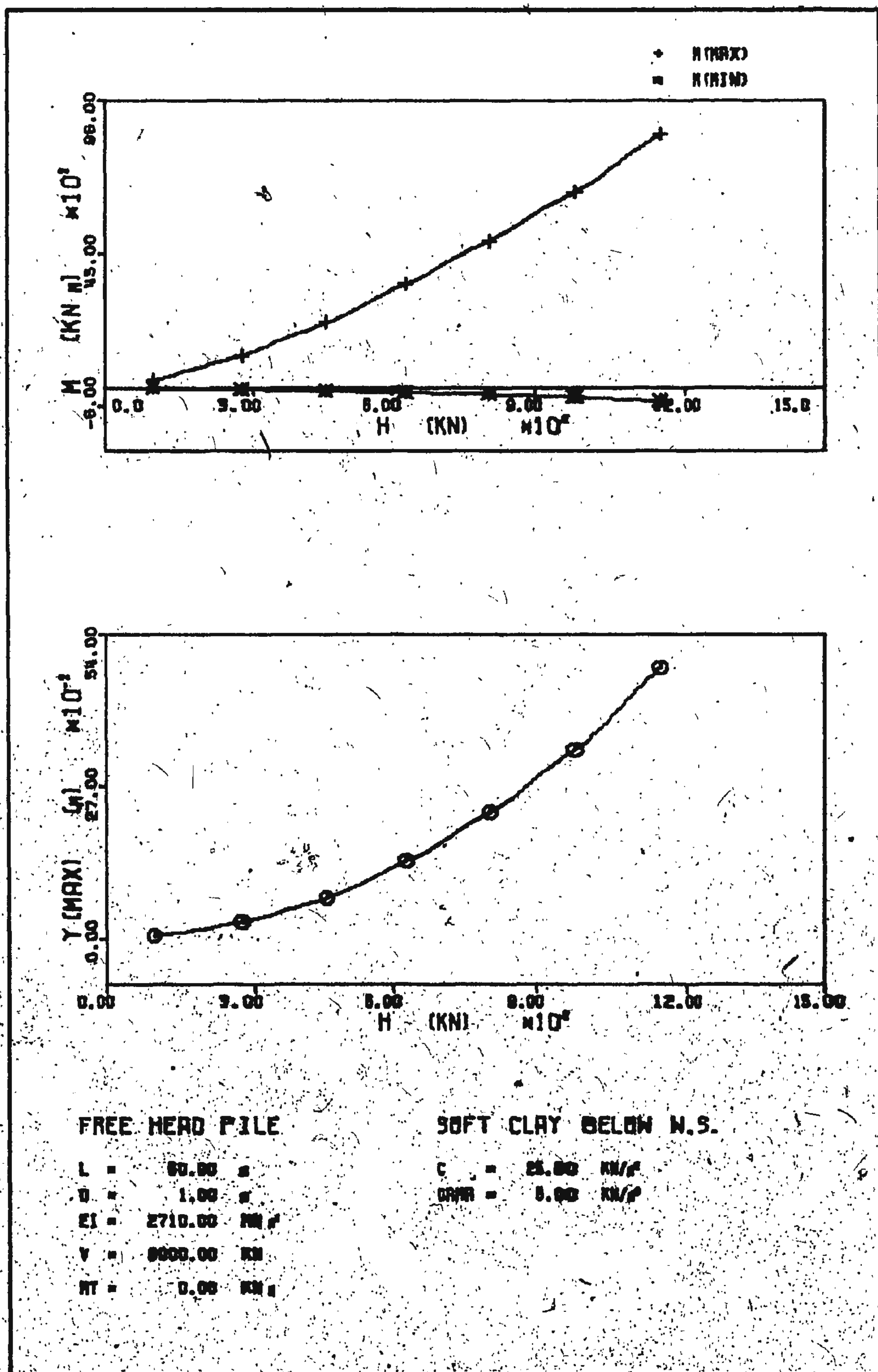


Figure 28 Effect of the horizontal load on the maximum deflection and the maximum bending moment for a free head pile in soft clay.

Table 6 Effect of the horizontal load on the maximum deflection and the maximum bending moment for a free head pile in soft clay .

H (kN)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
100	0.0061	275.5	5.0	- 12.5	23.0
275	0.0291	1093.6	7.0	- 50.0	24.0
450	0.0727	2187.4	8.0	- 98.4	26.0
625	0.1375	3469.4	8.0	- 150.4	27.0
800	0.2241	4933.0	9.0	- 216.0	29.0
975	0.3351	6548.5	10.0	- 317.1	30.0
1150	0.4804	8472.5	10.0	- 446.0	31.0

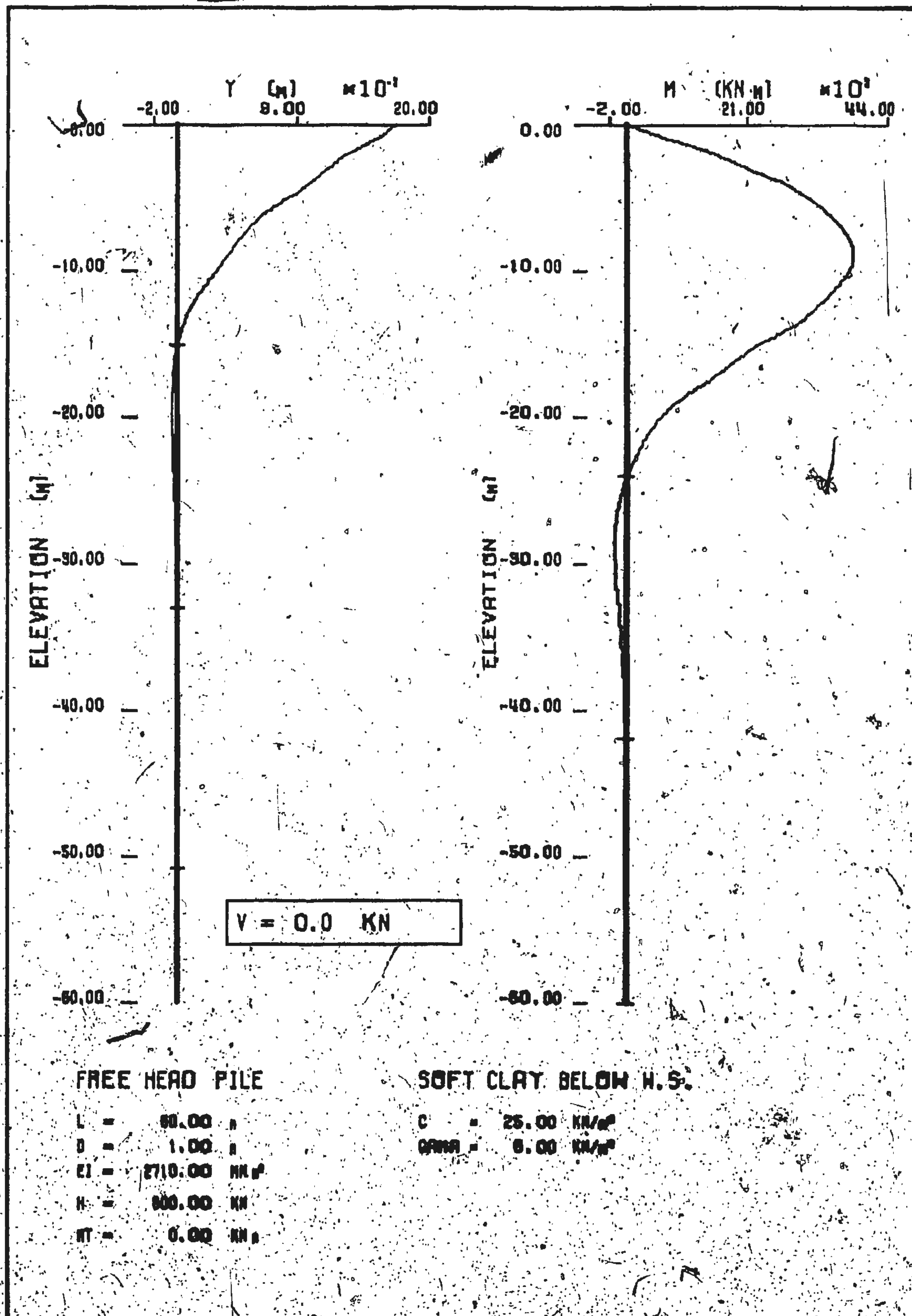


Figure 29 Deflection and bending moment for a free head pile in soft clay, V = 0.0 kN.

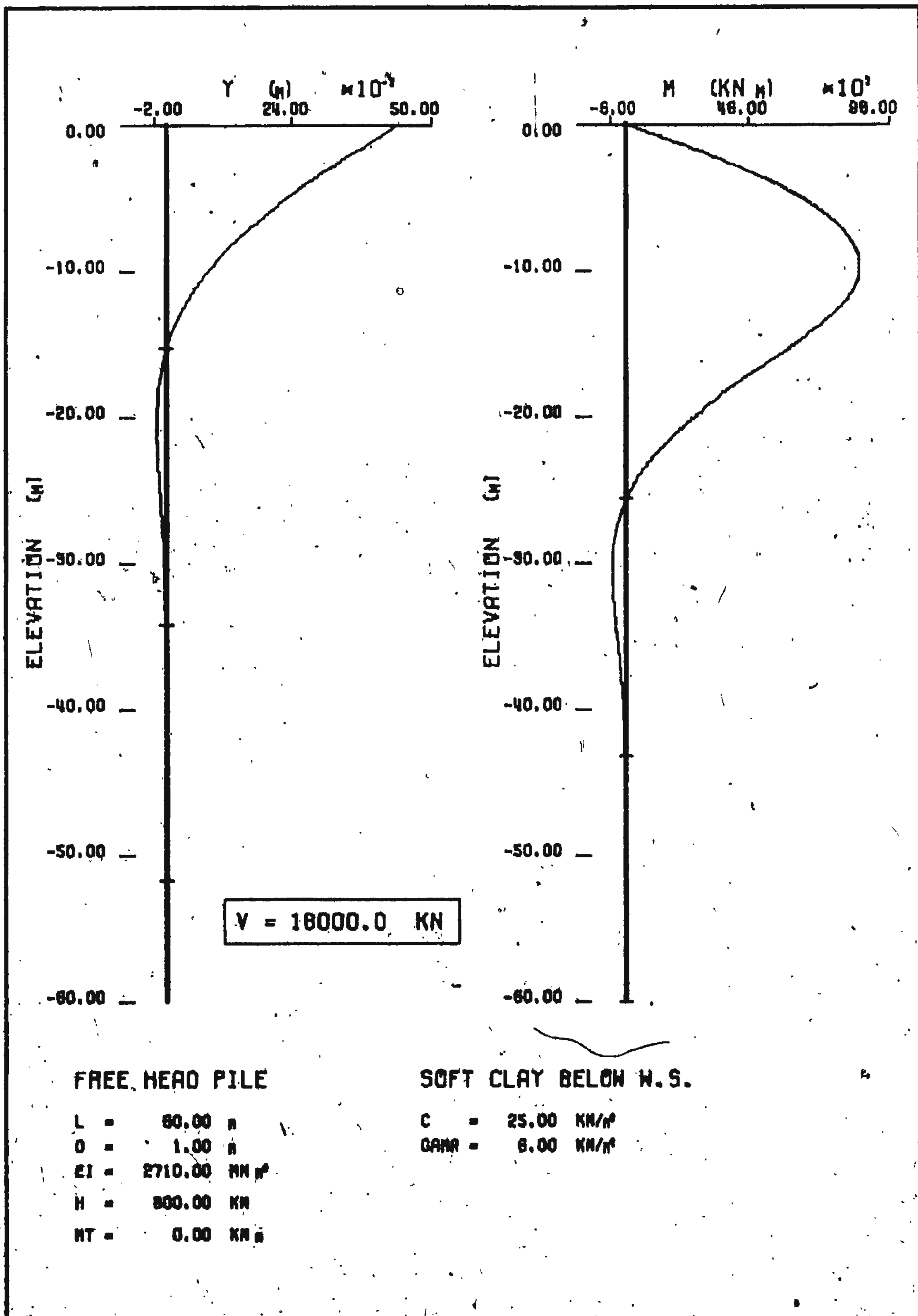


Figure 30 Deflection and bending moment for a free pile in soft clay, V = 18000.0 kN.

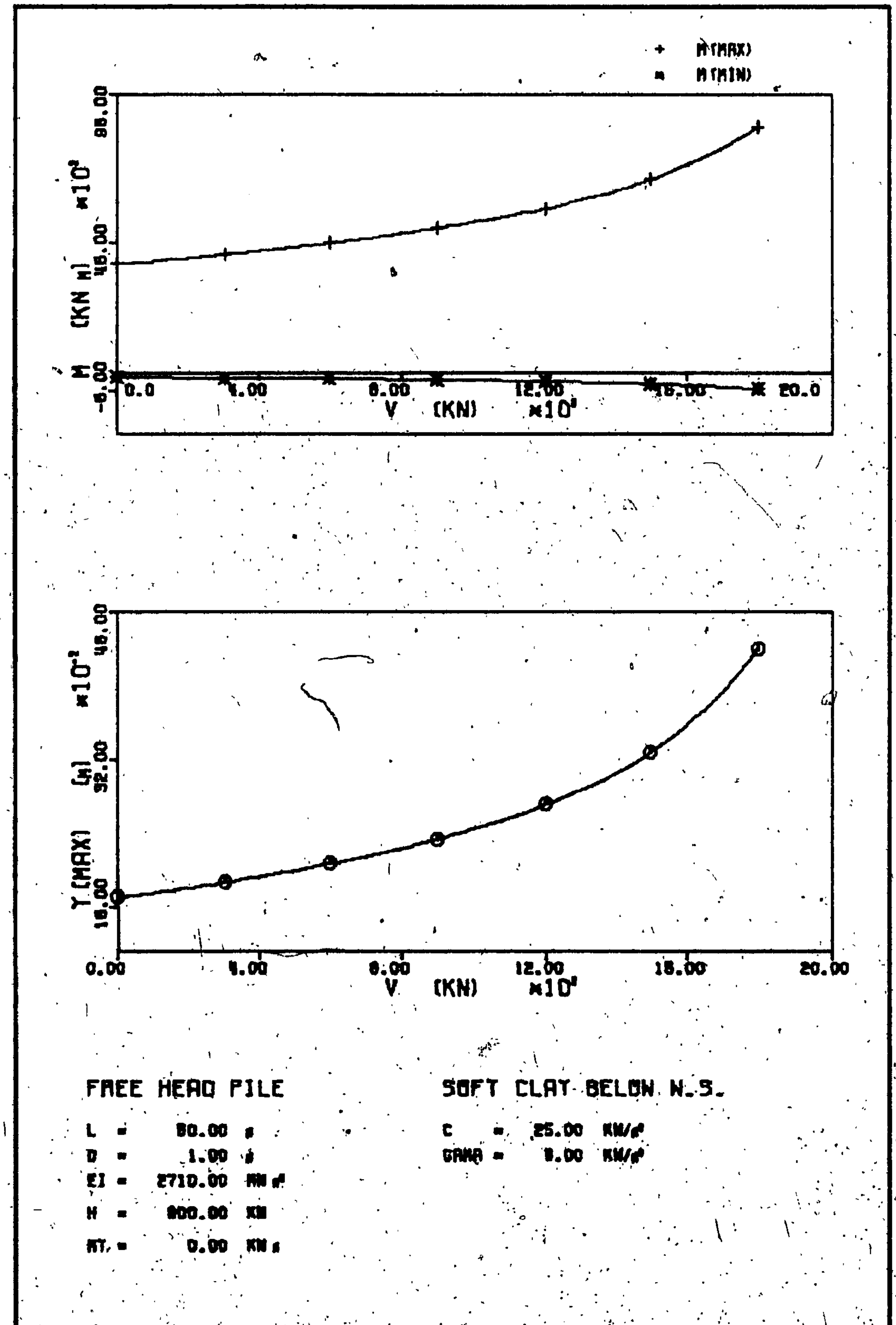


Figure 31 Effect of the vertical load on the maximum deflection and the maximum bending moment for a free head pile in soft clay.

Table 7 Effect of the vertical load on the maximum deflection and the maximum bending moment for a free head pile in soft clay .

V (kN)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0	0.1704	3850.7	9.0	- 150.1	28.0
3000	0.1867	4188.2	9.0	- 167.4	29.0
6000	0.2072	4600.3	9.0	- 191.7	29.0
9000	0.2339	5122.4	9.0	- 230.5	29.0
12000	0.2712	5817.8	9.0	- 289.6	29.0
15000	0.3280	6821.7	9.0	- 378.2	29.0
18000	0.4398	8665.4	10.0	- 558.2	30.0

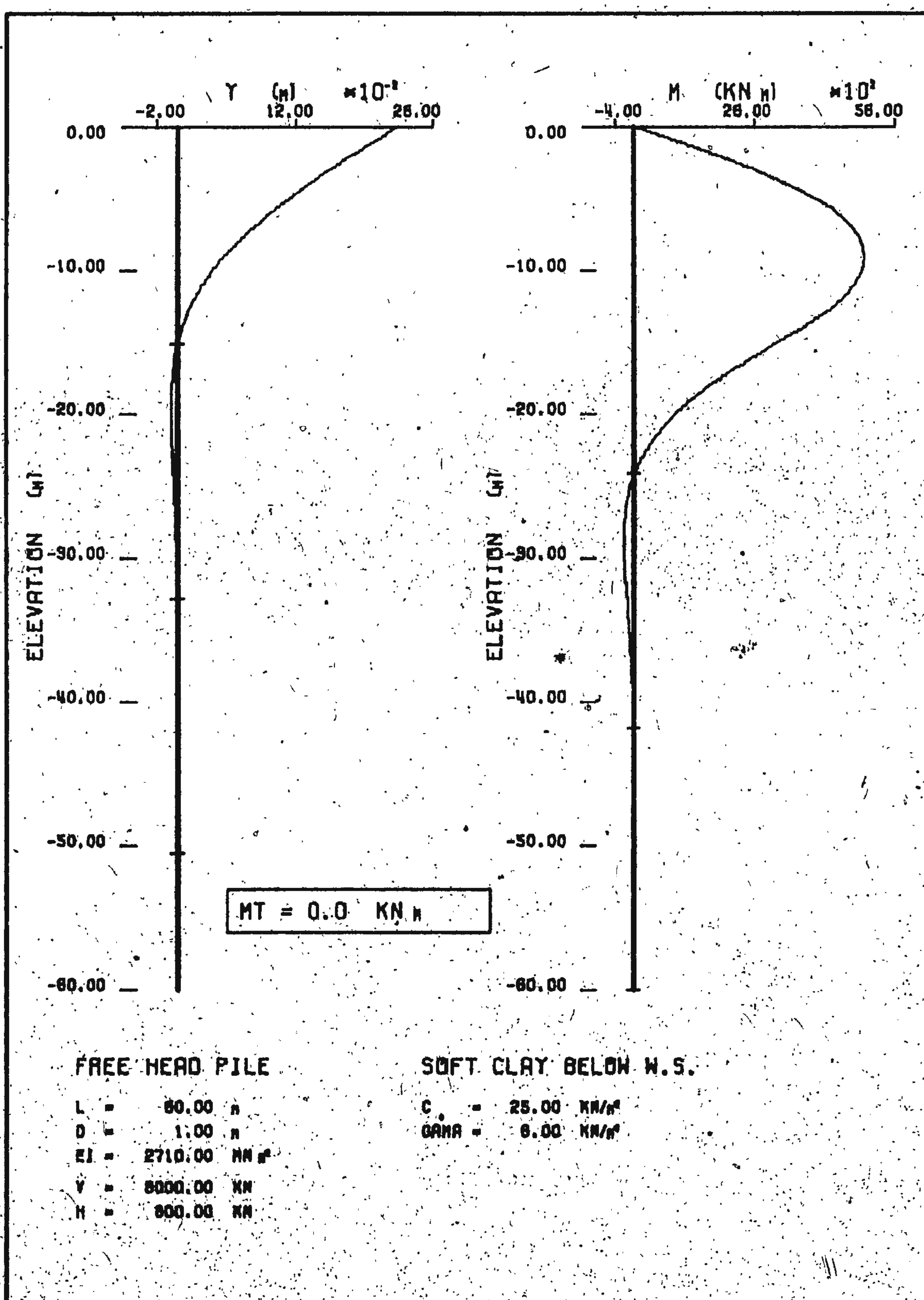


Figure 32 Deflection and bending moment for a free head pile in soft clay, MT = 0.0 kN.m.

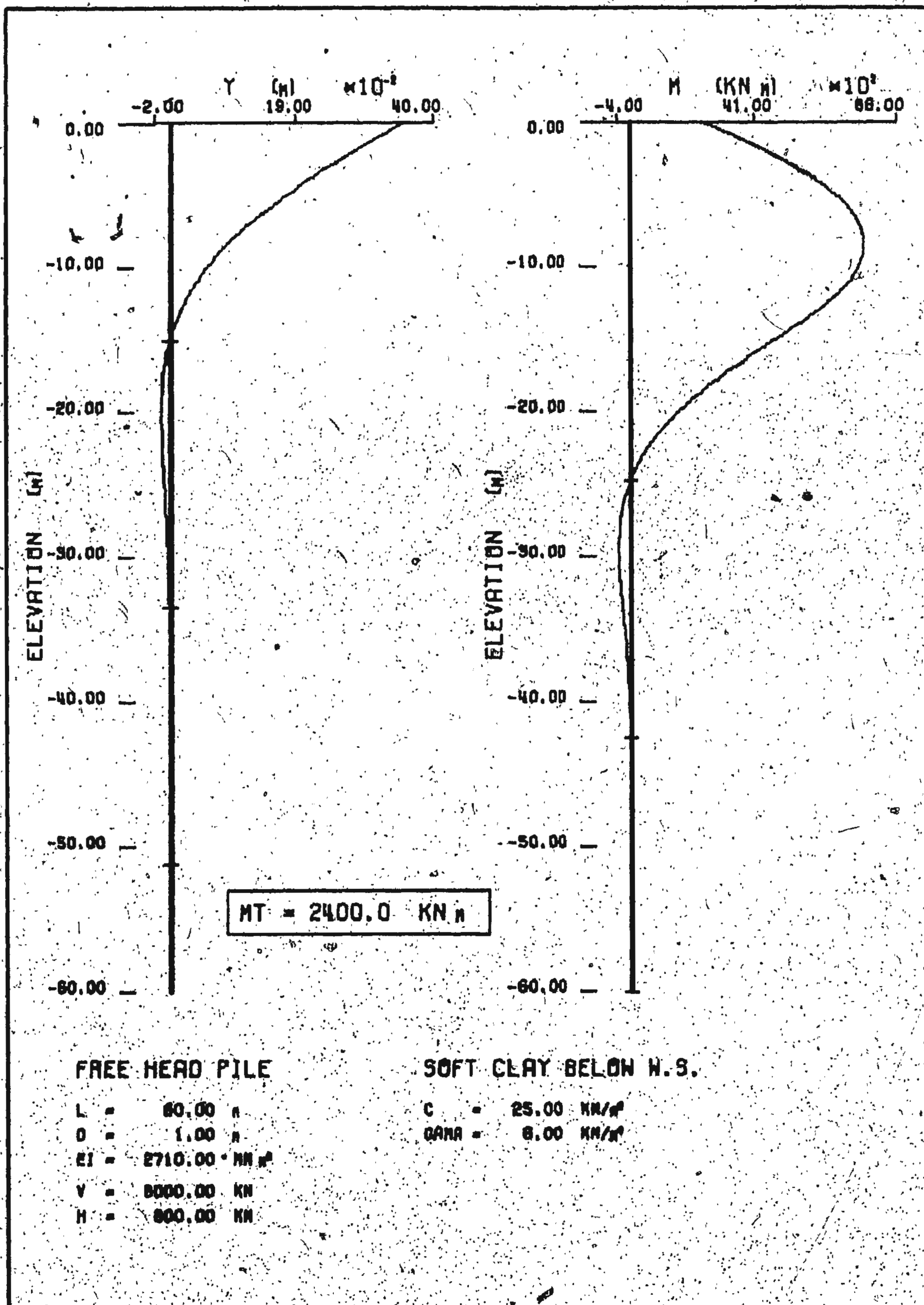


Figure 33 Deflection and bending moment for a free head pile in soft clay, MT = 2400.0 kN m.

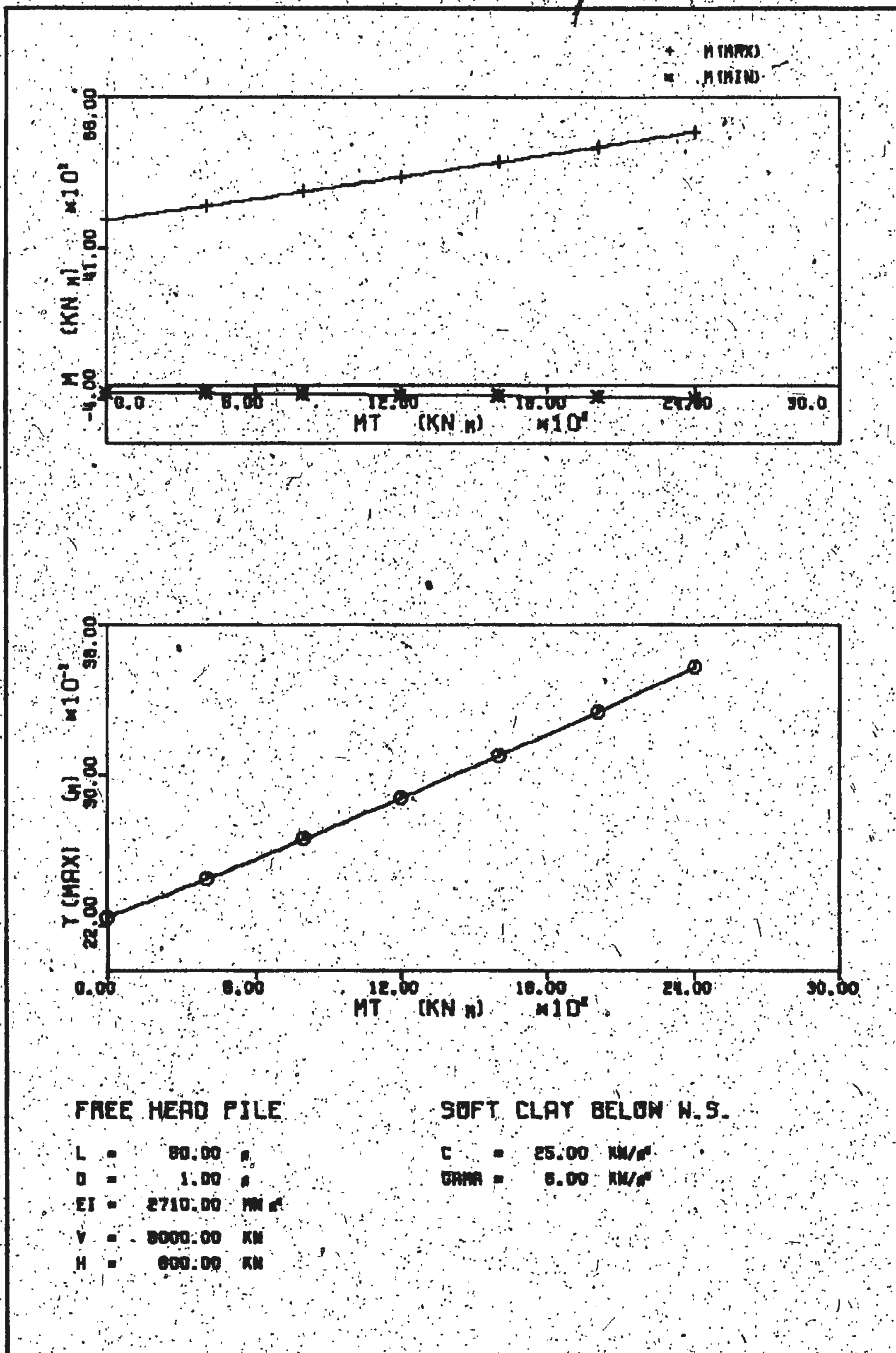


Figure 34 Effect of the external moment on the maximum deflection and the maximum bending moment for a free head pile in soft clay.

Table 8 Effect of the external moment on the maximum deflection and the maximum bending moment for a free head pile in soft clay.

MT (kN m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0	0.2241	4933.0	9.0	- 216.0	29.0
400	0.2447	5358.6	9.0	- 240.5	29.0
800	0.2660	5790.9	9.0	- 268.0	29.0
1200	0.2878	6226.4	9.0	- 296.0	29.0
1600	0.3103	6667.7	9.0	- 324.4	29.0
2000	0.3334	7117.7	8.0	- 353.1	29.0
2400	0.3572	7581.6	8.0	- 382.7	29.0

of the external moment in case of stiff clay. Similar conclusions were drawn by Palmer and Brown (1954) for a free head pile in sand, based on both theoretical and experimental work. It can also be noticed that the position of M_{max+} is nearer to the ground surface in stiff clay than soft clay for all values of the external moment.

4.3.2 Pile Properties

4.3.2.1 Effect of Pile Length

Typical graphs showing the effect of the pile length are shown in figures 35 to 37. Tabulated results are summarized in table 9. Both the graphs and the table are for a free head pile in soft clay. Similar graphs and tables for a free head pile in stiff clay and a fixed head pile in the two types of soil are given in appendix A (tables 23 to 25) and appendix B (figures 81 to 89).

Results indicate that both the magnitude and location of Y_{max}, M_{max+} and M_{max-} are independent of the pile length for all the four cases. However, for short piles in soft clay, when the pile length approaches a value which causes a failure of the pile-soil system, a disproportional increase in Y_{max} is noticed compared to the change in the pile length. The failure in such a case is likely to be by a complete overturning. For higher values of pile lengths, the curve of Y_{max} becomes asymptotic to a constant value indicating that the pile response is independent of any further increase in the pile length. Hence as far as the lateral resistance is considered it is a waste of material to use piles longer than a specific length which insures stability for the pile-soil system. It should be noted that there may be situations where vertical bearing capacity considerations may necessitate piles of greater lengths, and thus the final length would be determined by the two different design criteria.

A comparison between figures 35 and 36 shows that there are fewer reversals in the sign of both the deflection and the bending moment curves for shorter piles. This may be due to less fixity at the ends of relatively short piles.

4.3.2.2 Effect of Pile Flexural Rigidity

Typical graphs showing the effect of the pile flexural rigidity are shown in figures 38 to 40. Tabulated results are summarized in table 10. Both the graphs and the table are for a free head pile in soft clay. Similar graphs and tables for a free head pile in stiff clay and a fixed head pile in the two types of soil are given in appendix A (tables 26 to 28) and appendix B (figures 90 to 98).

For free head piles, Y_{max} decreases with the increase of the pile flexural rigidity. The rate of change in Y_{max} is more sensitive in the lower range of flexural rigidity values. The behavior of the moment curves is slightly different in the two types of soil. In case of soft clay, with the increase in the values of flexural rigidity the magnitudes of both M_{max+} and M_{max-} slightly decrease initially and then they start increasing with further increase in the flexural rigidity. On the other hand, in case of stiff clay this initial decrease in the moment is not observed. This decrease is very small, less than 5% and occurs only for a free head pile in soft clay. This may be an anomaly due to the compatibility requirement between the pile and the soil response. It is felt that this anomaly might require further detailed investigation. For both types of soil, the locations of both M_{max+} and M_{max-} shift down along the pile axis with increasing flexural rigidity, which means that a longer portion of the pile is subjected to significant bending stresses for higher values of the pile flexural rigidity.

For fixed head piles in the two types of soil, both the deflection and the moment responses are similar to the case of a free head pile in stiff clay. However, the magnitude of Y_{max} in case of a fixed head pile in stiff clay is relatively small.

The downward movement of the location of M_{max+} and M_{max-} with increasing flexural rigidity can also be interpreted in another way. The flexural rigidity was varied by changing the thickness of the steel in the pipe piles. Thus, with decreasing the flexural rigidity, the pile approaches an equivalent precast concrete pile. Hence in the case of a precast concrete pile the point of maximum moment will be nearer to the ground surface compared to a steel pile of the same size and length driven in a similar type of soil and subjected to the same loadings. But the magnitude of the maximum moment in the steel pile will exceed that of the concrete pile.

Palmer and Brown (1954) obtained similar results for free head piles in sand. For a 750% increase in the flexural rigidity, the magnitude of Y_{max} almost diminished and a lowering of 0.50 m in the location of M_{max+} was observed.

4.3.2.3 <u>Effect of Pile Diameter</u>

Typical graphs showing the effect of the pile diameter are shown in figures 41 to 43 . Tabulated results are summarized in table 11 . Both the graphs and the table are for a free head pile in soft clay. Similar graphs and tables for a free head pile in stiff clay and a fixed head pile in the two types of soil are given in appendix A (tables 29 to 31) and appendix B (figures 99 to 107). Here the pile diameter was varied while all other variables, including the flexural rigidity, were kept constant.

For free head piles, the magnitudes of Y_{max} and M_{max+} decrease with an increase in the pile diameter, with higher rate of decrease for smaller values of the diameter. But in case of soft clay the location of $M_{max\pm}$ slightly shifts upwards with the increase of the pile diameter. On the other hand, the location is independent of the pile diameter in case of stiff clays. Reversals in the direction of the deflection and the bending moment curves are seen to be more for larger diameters in comparison with smaller diameters. Similar results were reported by Palmer and Brown (1954) for free head piles embedded in sandy soils.

For fixed head piles, the magnitudes of both Y_{max} and M_{max-} decrease with increasing pile diameter, but relatively small values of Y_{max} are obtained in case of a fixed head pile in stiff clay.

4.3.2.4 Effect of Pile Diameter and Corresponding Flexural Rigidity

Typical graphs showing the effect of the pile diameter and the corresponding flexural rigidity are shown in figures 44 to 46. In this case, the flexural rigidity was changed by changing the diameter, with a constant thickness of pile material. Tabulated results are summarized in table 12. Both the graphs and the table are for a free head pile in soft clay. Similar graphs and tables for a free head pile in stiff clay and a fixed head pile in both types of soil are given in appendix A (tables 32 to 34) and appendix B (figures 108 to 116).

For free head piles, Y_{max} decreases with increasing pile diameter.

(Text continued on page 98)

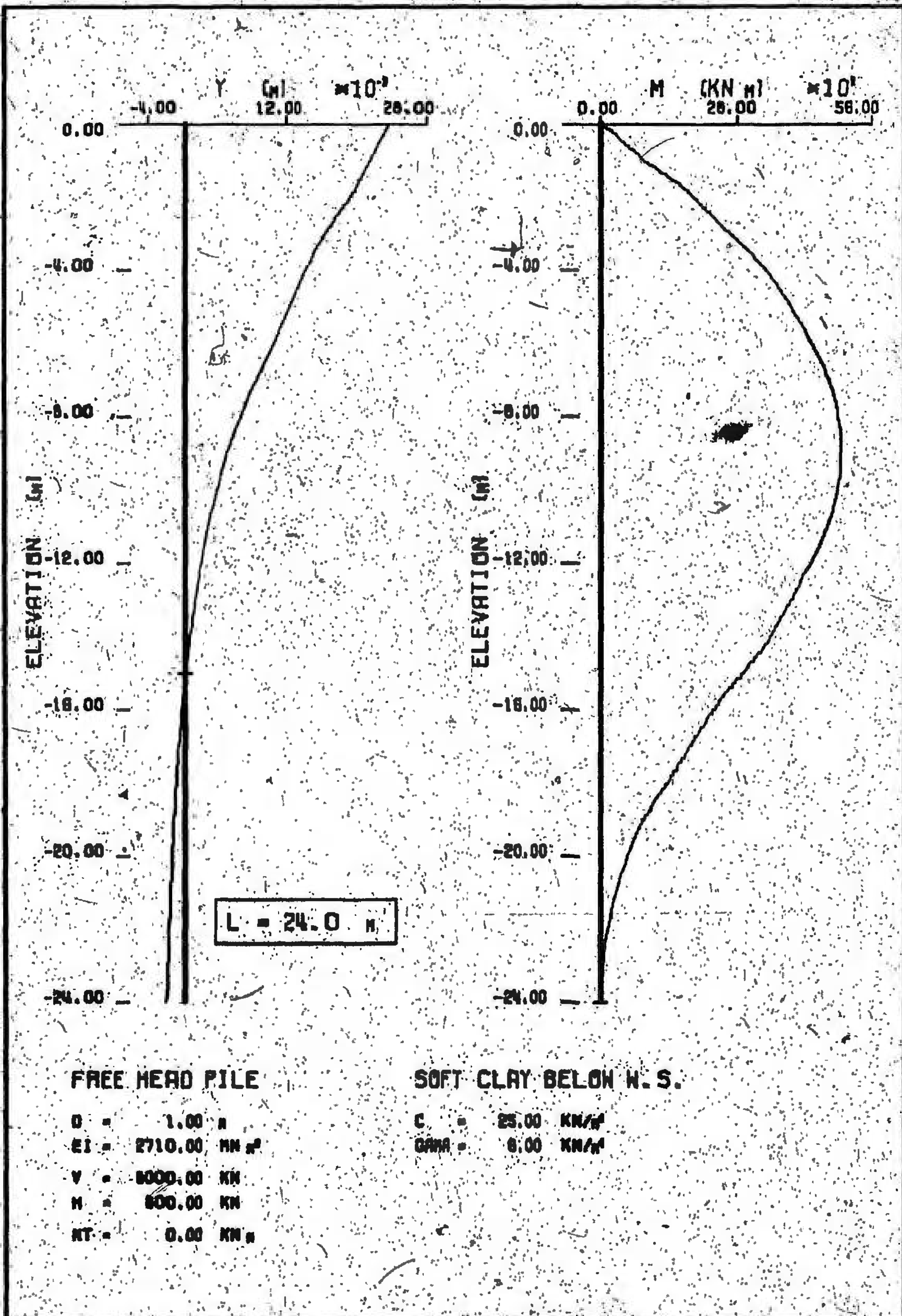


Figure 36 Deflection and bending moment for a free head pile in soft clay, L = 24.0 m.

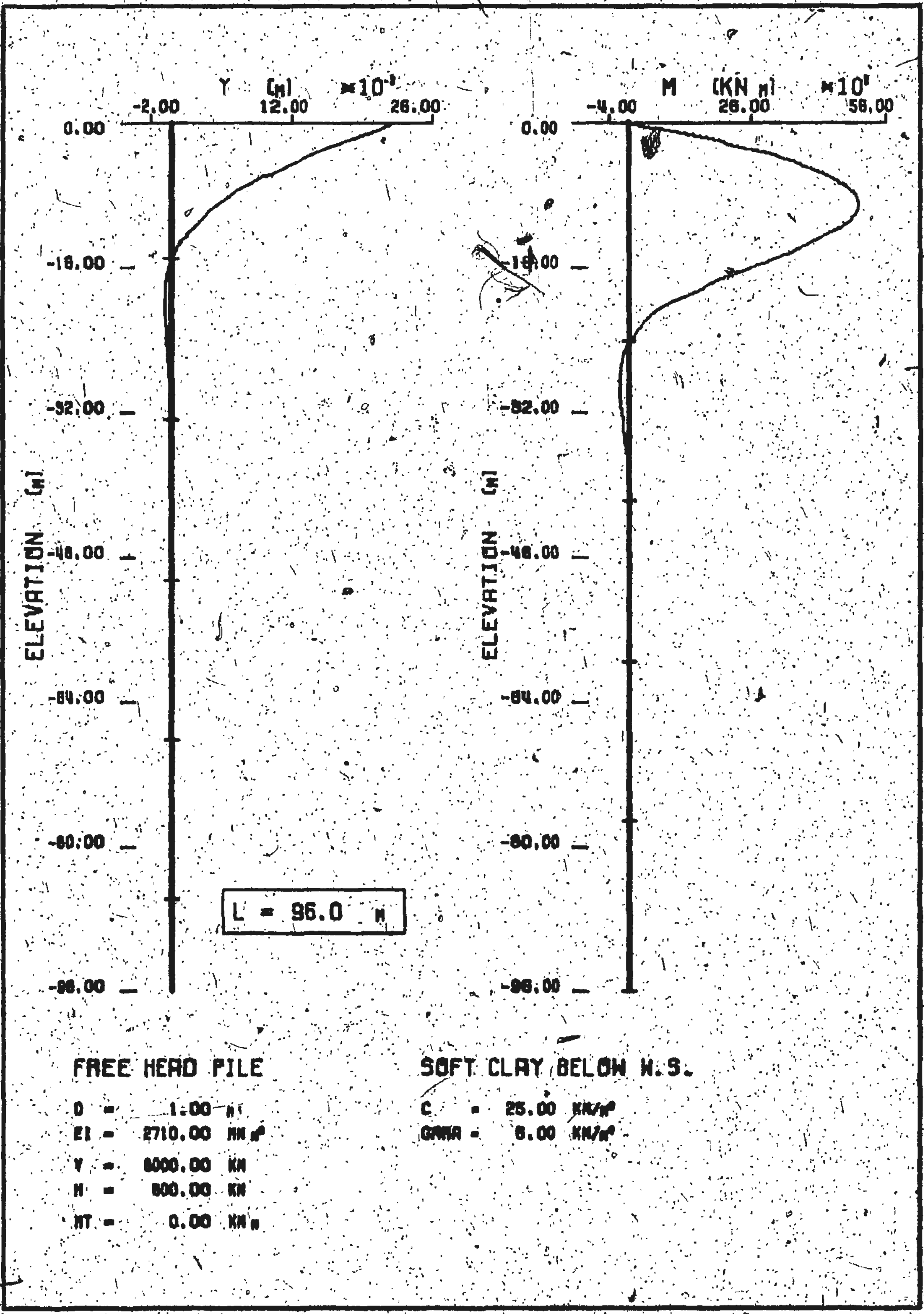


Figure 36 Deflection and bending moment for a free head pile in soft clay, L = 96.0 m.

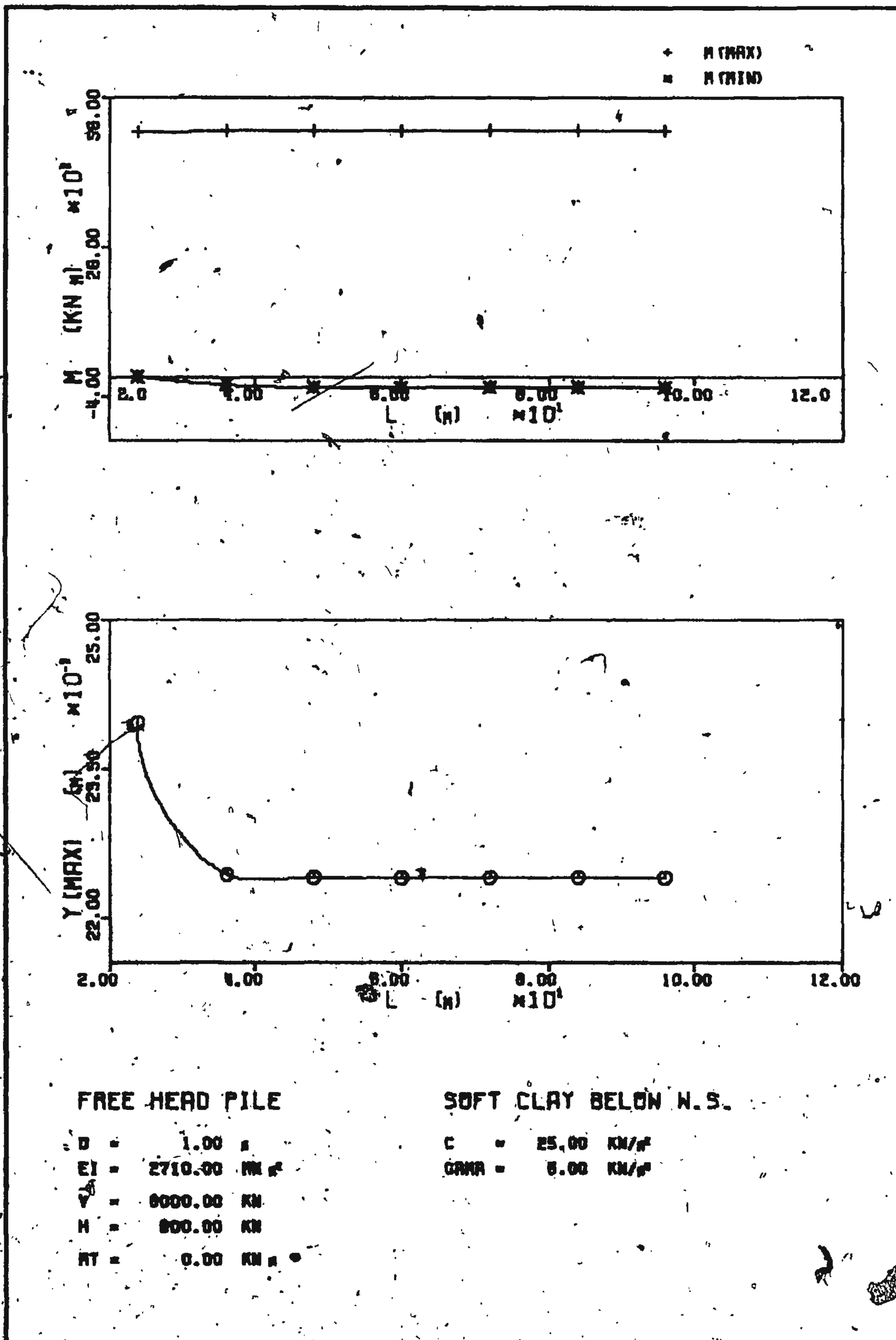


Figure 37 Effect of the pile length on the maximum deflection and the maximum bending moment for a free head pile in soft clay.

Table 9 Effect of the pile length on the maximum deflection and the maximum bending moment for a free head pile in soft clay .

L (m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
24.0	0.2397	4903.5	9.0	0.0	24.0
36.0	0.2243	4932.9	9.0	- 165.4	28.0
48.0	0.2241	4933.0	9.0	- 215.6	29.0
60.0	0.2241	4933.0	9.0	- 216.0	29.0
72.0	0.2241	4933.0	9.0	- 216.0	29.0
84.0	0.2241	4933.0	9.0	- 216.0	29.0
96.0	0.2241	4933.0	9.0	- 216.0	29.0

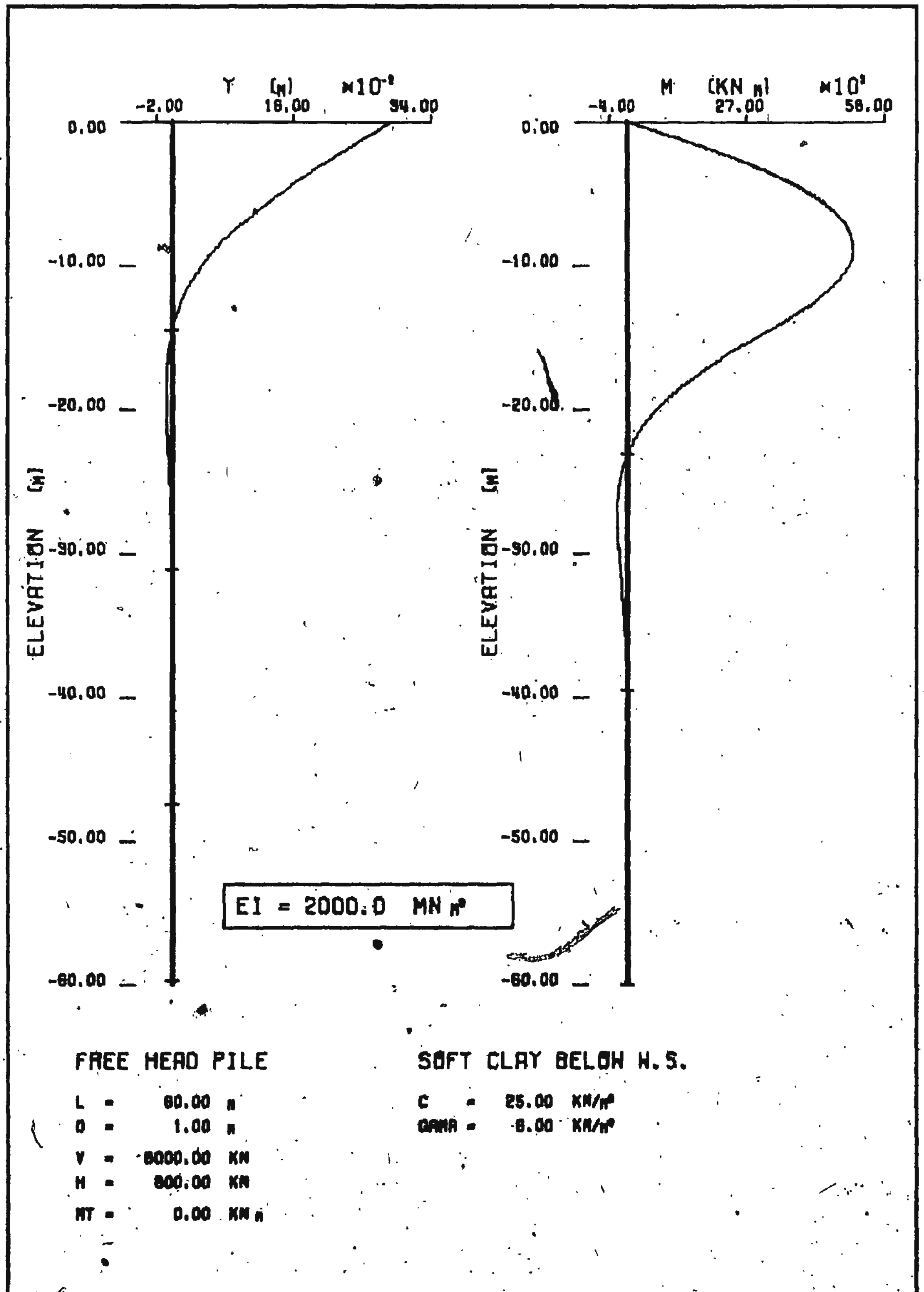


Figure 38 Deflection and bending moment for a free head pile in soft clay, EI = 2000.0 MN m^2.

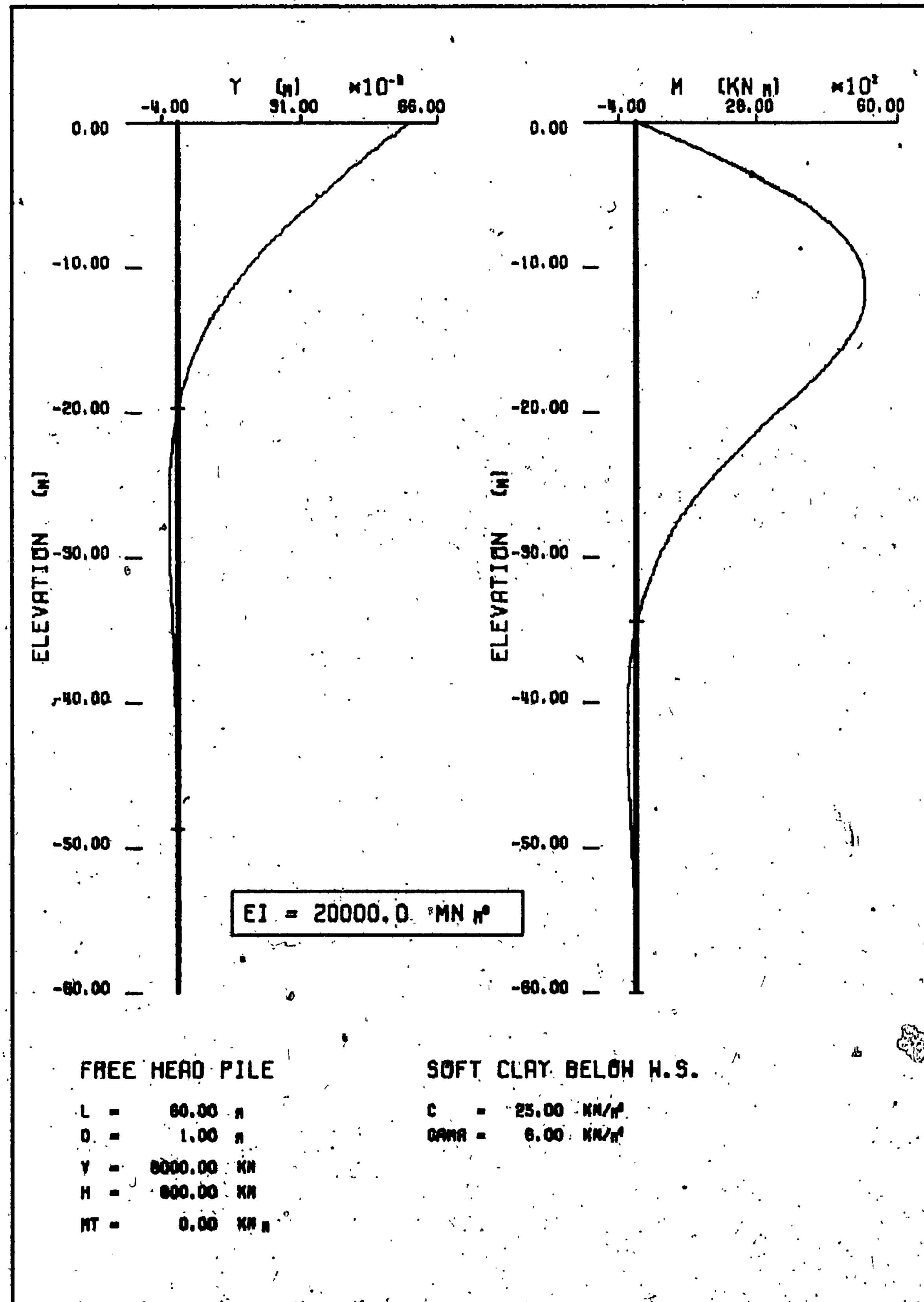


Figure 39 Deflection and bending moment for a free head pile in soft clay, EI = 20000.0 MN m^2.

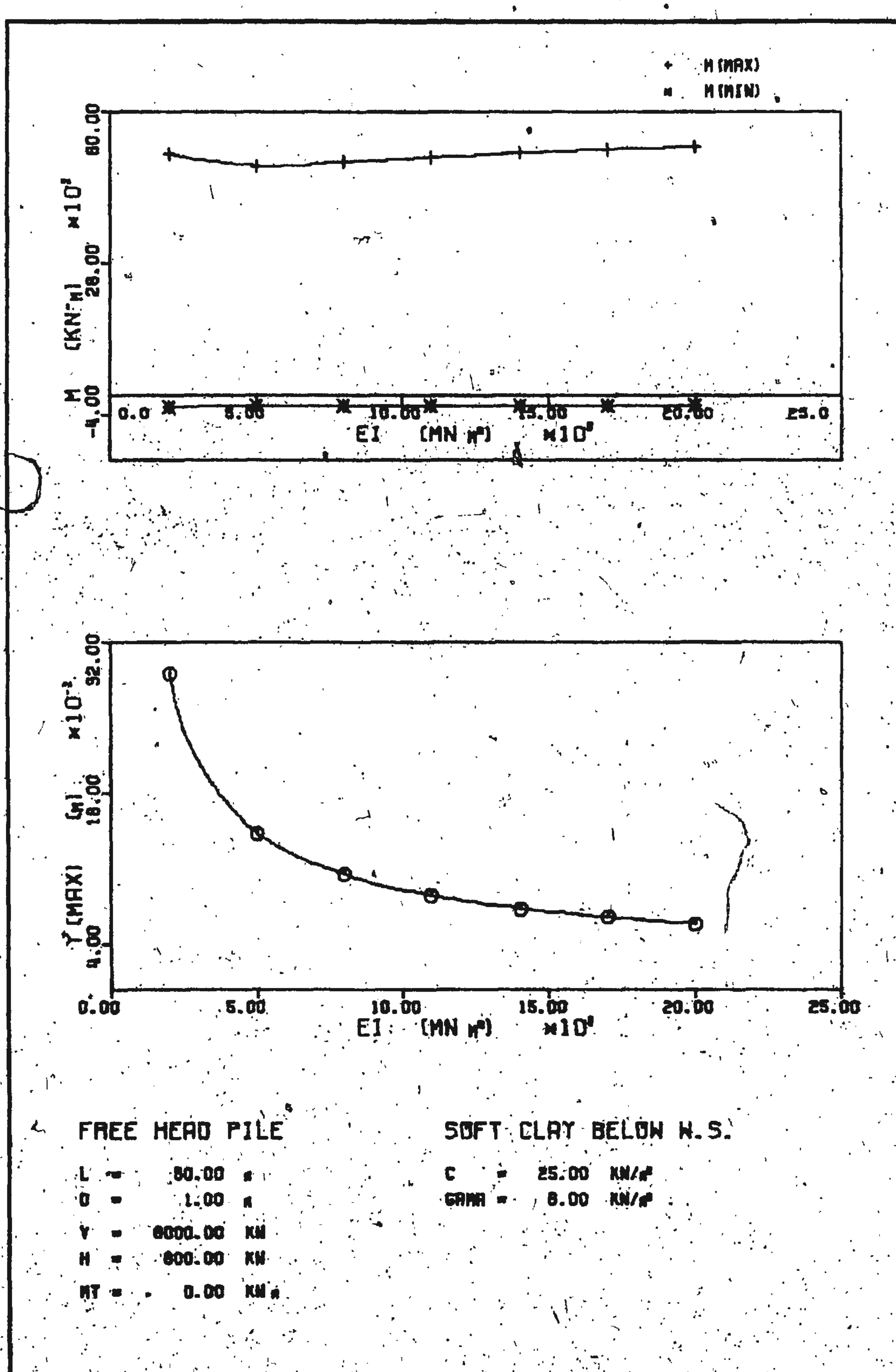


Figure 40. Effect of the pile flexural rigidity on the maximum deflection and the maximum bending moment for a free head pile in soft clay.

Table 10 Effect of the pile flexural rigidity on the maximum deflection and the maximum bending moment for a free head pile in soft clay .

EI ($MN\ m^2$)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
2000	0.2906	5093.1	9.0	- 241.0	27.0
5000	0.1423	4861.7	10.0	- 206.4	32.0
8000	0.1038	4948.4	10.0	- 211.6	35.0
11000	0.0847	5043.3	11.0	- 216.9	37.0
14000	0.0729	5133.0	11.0	- 220.3	39.0
17000	0.0647	5206.6	11.0	- 221.3	40.0
20000	0.0586	5270.7	12.0	- 219.3	41.0

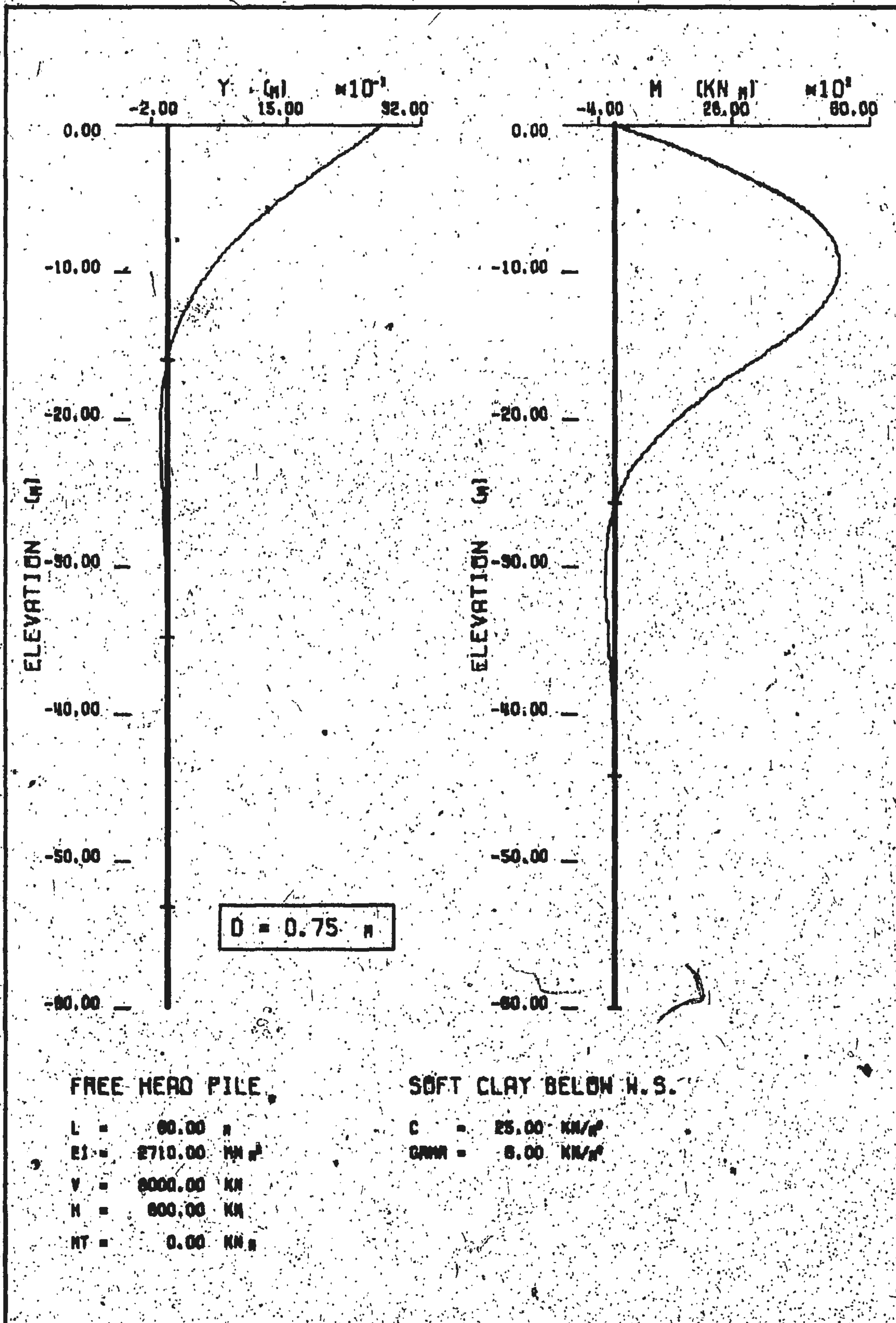


Figure 41 Deflection and bending moment for a free head pile in soft clay, D = 0.75 m.

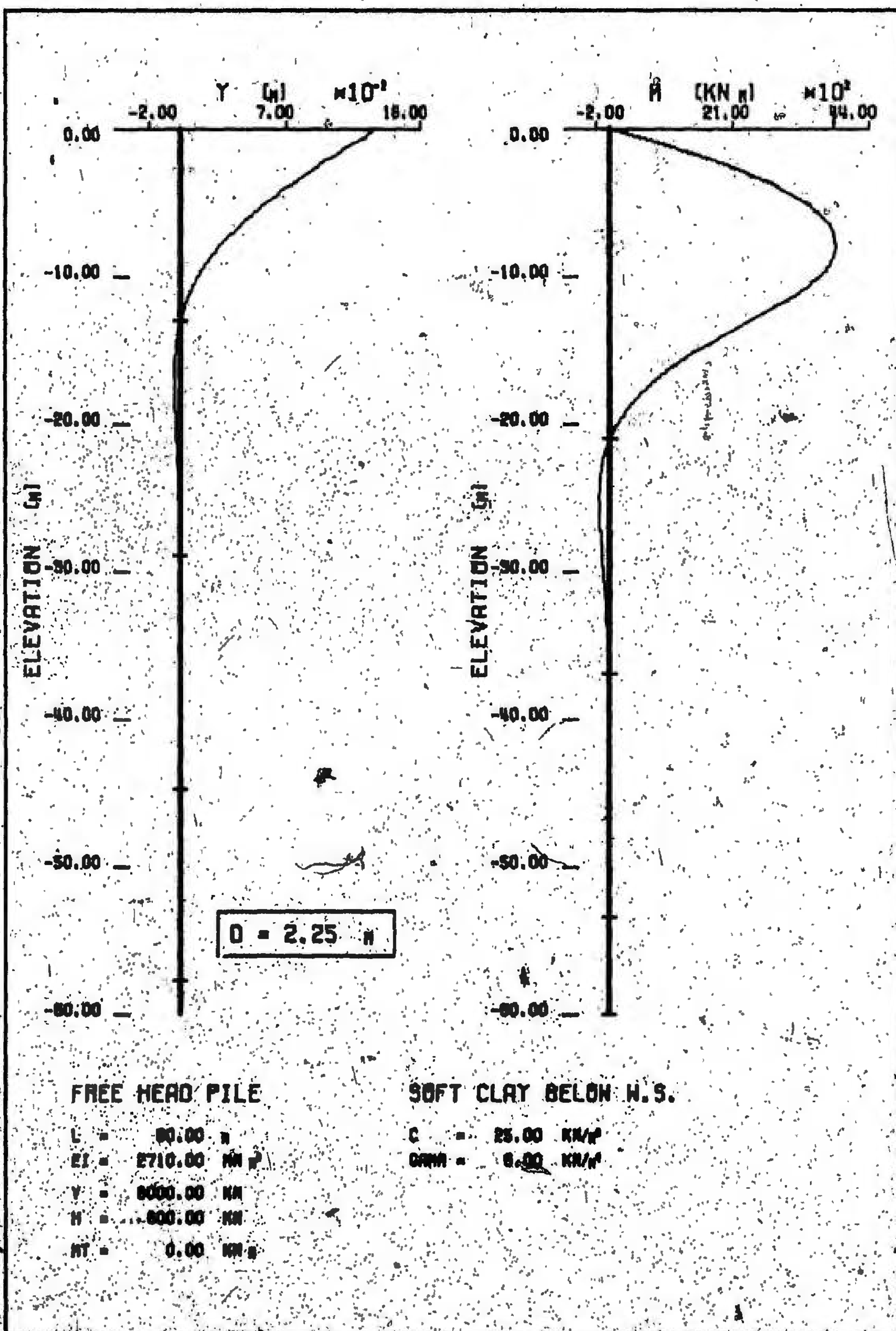


Figure 42 Deflection and bending moment for a free head pile in soft clay, D = 2.25 m.

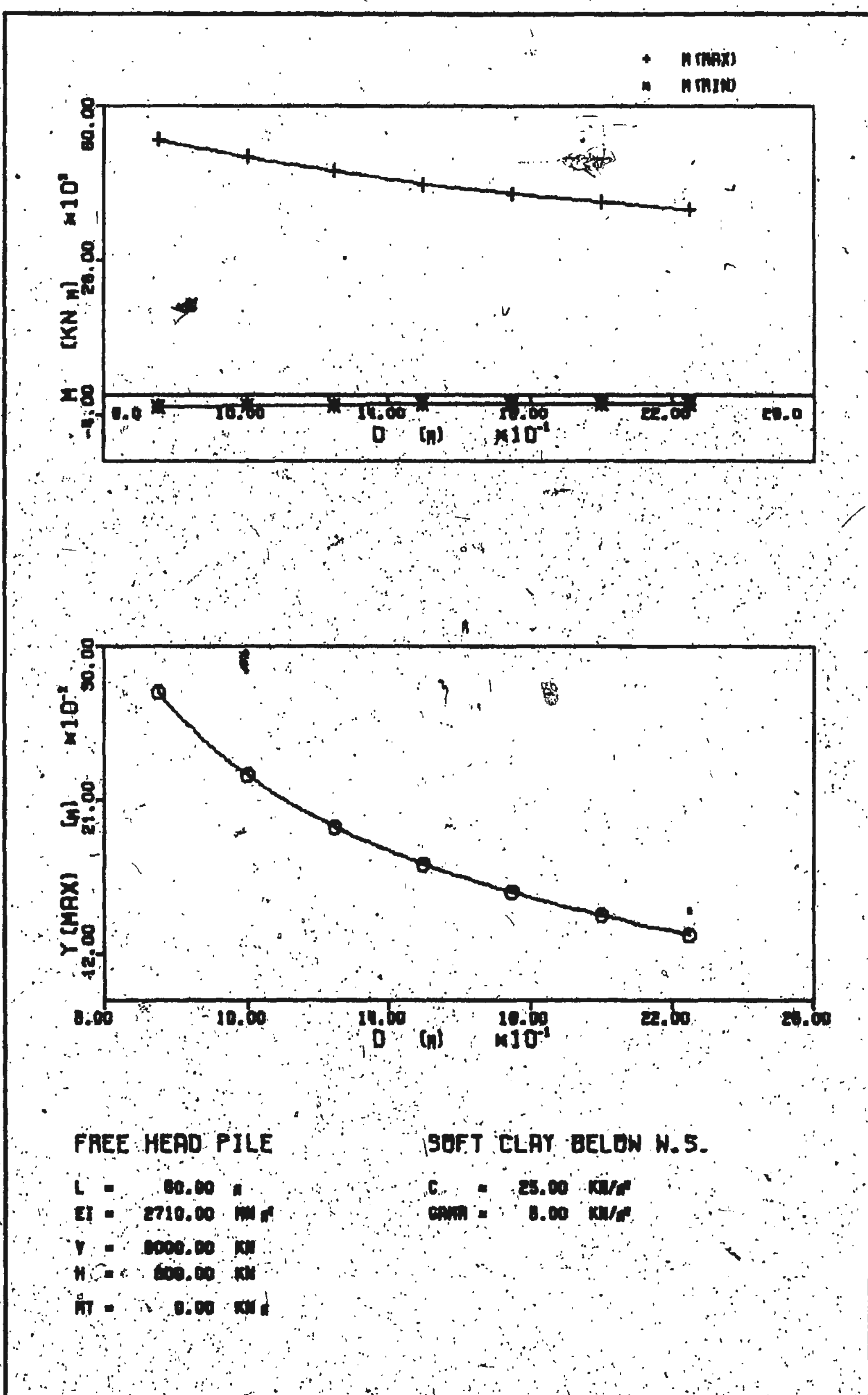


Figure 43 Effect of the pile diameter on the maximum deflection and the maximum bending moment for a free head pile in soft clay.

Table 11 Effect of the pile diameter on the maximum deflection and the maximum bending moment for a free head pile in soft clay .

D (m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0.75	0.2731	5295.0	9.0	- 249.3	30.0
1.00	0.2241	4933.0	9.0	- 216.0	29.0
1.25	0.1937	4643.0	9.0	- 196.3	27.0
1.50	0.1725	4384.0	9.0	- 186.0	27.0
1.75	0.1558	4184.6	8.0	- 179.5	26.0
2.00	0.1422	4006.7	8.0	- 170.9	26.0
2.25	0.1306	3847.1	8.0	- 166.0	25.0

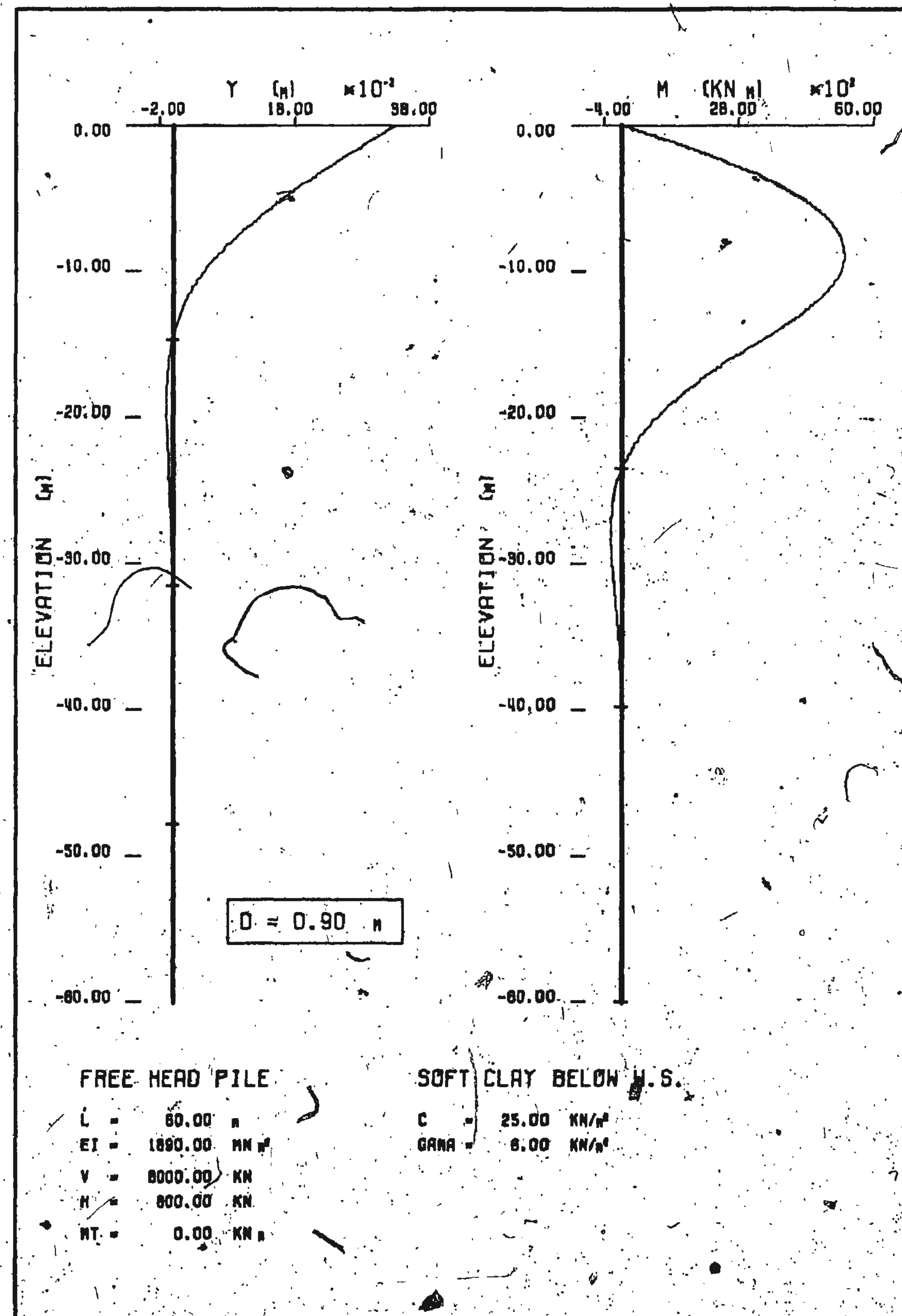


Figure 44 Deflection and bending moment for a free head pile in soft clay, $D = 0.90$ m , $EI = 1890.0$ MN m^2.

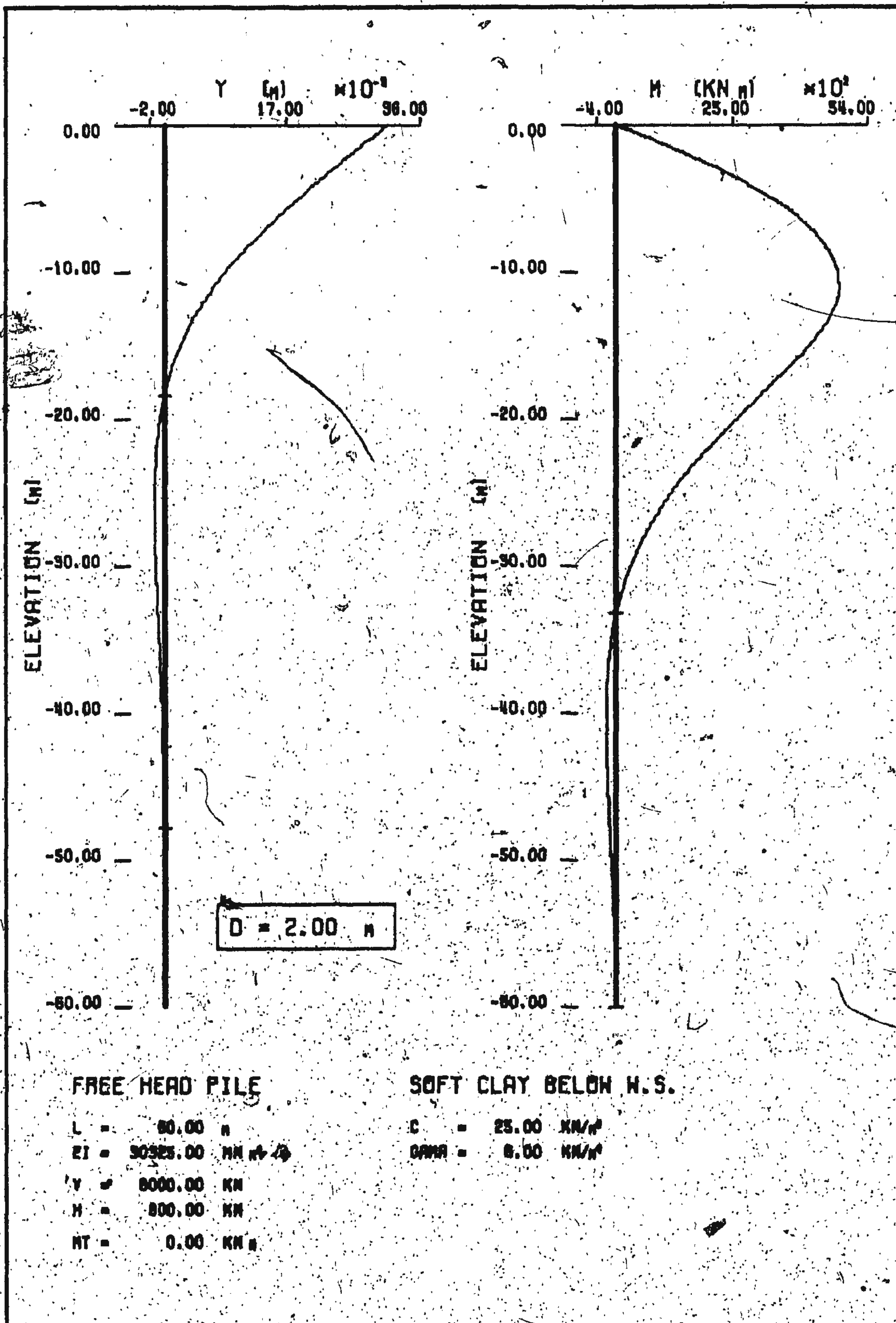


Figure 45 Deflection and bending moment for a free head pile in soft clay, D = 2.00 m , EI = 30325.0 MN m^2.

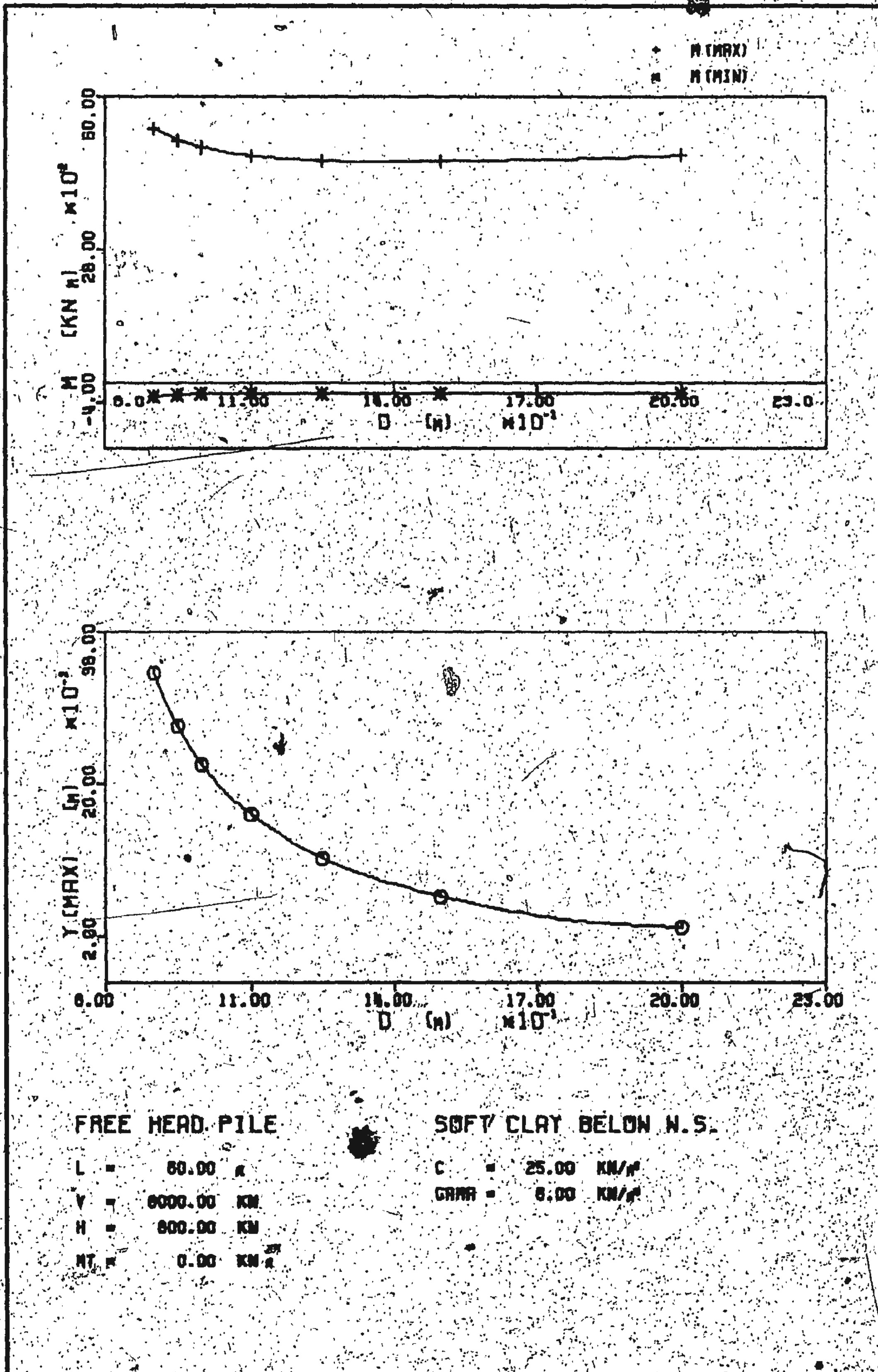


Figure 46 Effect of the pile diameter and the corresponding flexural rigidity on the maximum deflection and the maximum bending moment for a free head pile in soft clay.

Table 12 Effect of the pile diameter and the corresponding flexural rigidity on the maximum deflection and the maximum bending moment for a free head pile in soft clay.

D (m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0.90	0.3305	5315.6	9.0	- 263.7	28.0
0.95	0.2687	5085.3	9.0	- 236.8	28.0
1.00	0.2241	4933.0	9.0	- 216.0	29.0
1.10	0.1642	4757.9	9.0	- 201.5	30.0
1.25	0.1118	4653.5	10.0	- 198.7	32.0
1.50	0.0669	4652.7	10.0	- 200.9	35.0
2.00	0.0313	4778.2	11.0	- 205.2	40.0

However, in soft clay the magnitude of M_{max+} starts decreasing with the increase of the pile diameter, then it slightly increases with further increase in the diameter. For stiff clay, that magnitude of M_{max+} slightly decreases for lower range of increasing diameter, then a significant increase is noticed with continued increase in diameter. In both soils, the depth to M_{max+} moves down with the increase of the pile diameter. The pile response in this study appears to be similar to the case in which only the flexural rigidity was varied (section 4.3.2.2) rather than the case where the pile diameter alone (section 4.3.2.3) was increased. This implies that the pile flexural rigidity has a greater influence on the pile response than the diameter of the pile alone.

For fixed head piles also, the deflection and the bending moment response are similar to the case where flexural rigidity was the variable. The magnitude of Y_{max} decreases while that of M_{max-} increases corresponding to an increase in the pile diameter. Deflections in case of fixed head piles are generally smaller than those for free head piles in the same type of soil.

4.3.3 Soil Properties

4.3.3.1 Effect of Soil Undrained Shear Strength

Typical graphs showing the effect of the soil undrained shear strength are shown in figures 47 to 49. Tabulated results are summarized in table 13. Both the graphs and the table are for a free head pile in soft clay. Similar graphs and tables for the other cases are given in appendix A (tables 35 to 37) and appendix B (figures 117 to 125).

For both types of piles in soft clay, the magnitudes of Y_{max}, M_{max+} and M_{max-} significantly decrease with an increase in the undrained soil shear strength. The position of M_{max+} moves upward along the pile axis with increasing shear strength. The portion of the pile which is effectively under bending is shortened with increasing soil strength.

The pile response in case of stiff clay is similar to that in soft clay for both types of piles. However, the magnitudes of Y_{max}, M_{max+} and M_{max-} are smaller in case of stiff clay. Also, reversal in the sign of both the deflection and the bending moment curves is more pronounced in stiffer clays. For both types of piles the position of M_{max+} is nearer to the ground surface in case of stiff clay.

4.3.3.2 Effect of Soil Unit Weight

Typical graphs showing the effect of the soil unit weight are shown in figures 50 to 52. Tabulated results are summarized in table 14. Both the graphs and the table are for a free head pile in soft Clay. Similar graphs and tables for the other three cases are given in appendix A (tables 38 to 40) and appendix B (figures 126 to 134).

In case of free head piles, both Y_{max} and M_{max+} slightly decrease with an increase in the soil unit weight. This decrease is almost linear and it is more pronounced in the case of soft clay. A slight upward move in the position of M_{max+} is noticed in case of soft clays, while the position does not appear to be affected by an increase in the soil unit weight for stiff clay.

Similar behavior is noticed for Y_{max} and M_{max-} in case of fixed head piles in both types of soil, but with smaller magnitudes of deflections.

4.4. Layered System of Soil

In most practical offshore sites the soil properties vary with the depth below the sea-bed. Generally the soil undrained shear strength

(Text continued on page 108)

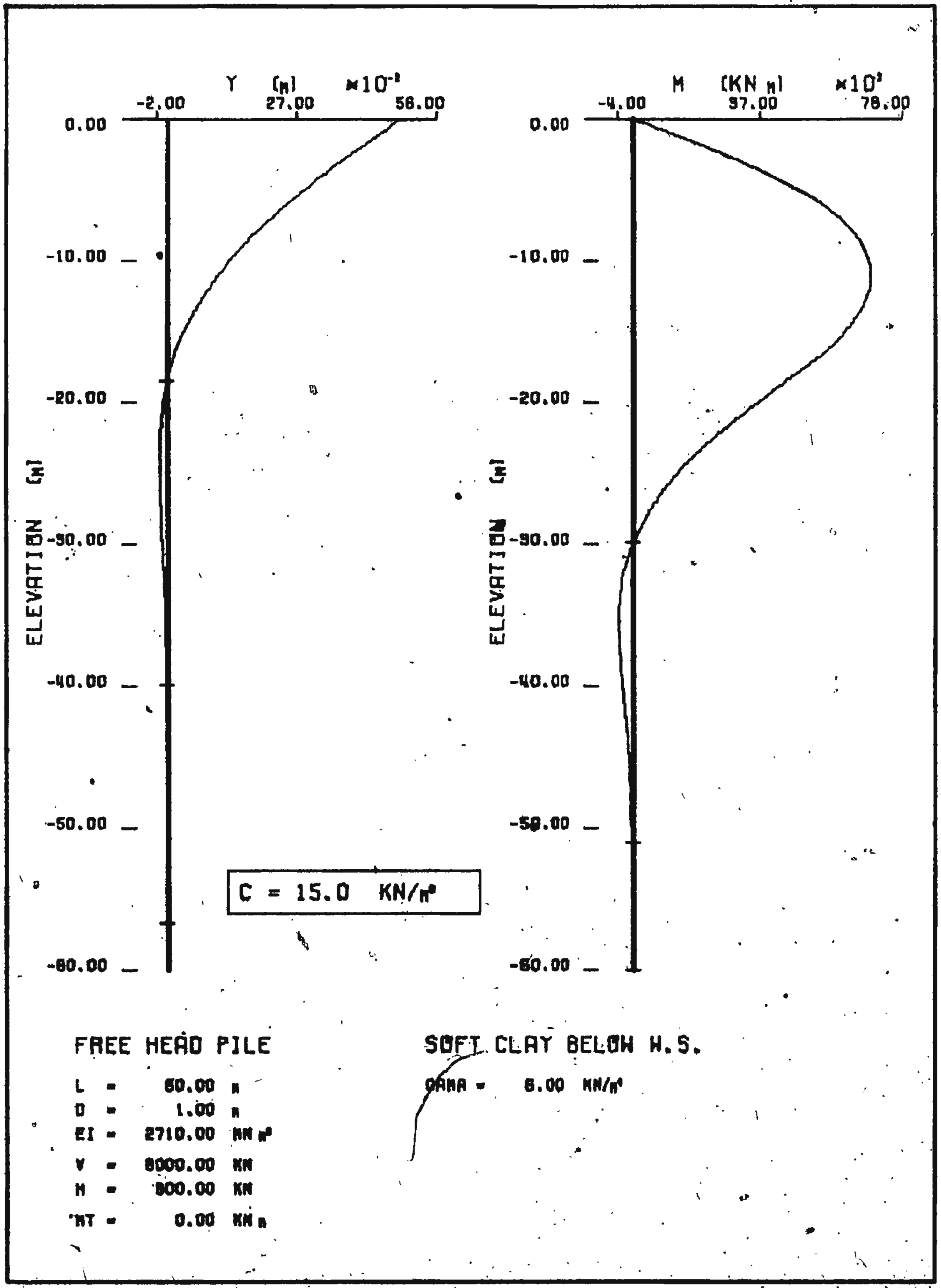


Figure 47 Deflection and bending moment for a free head pile in soft clay, C = 15.0 kN/m^2.

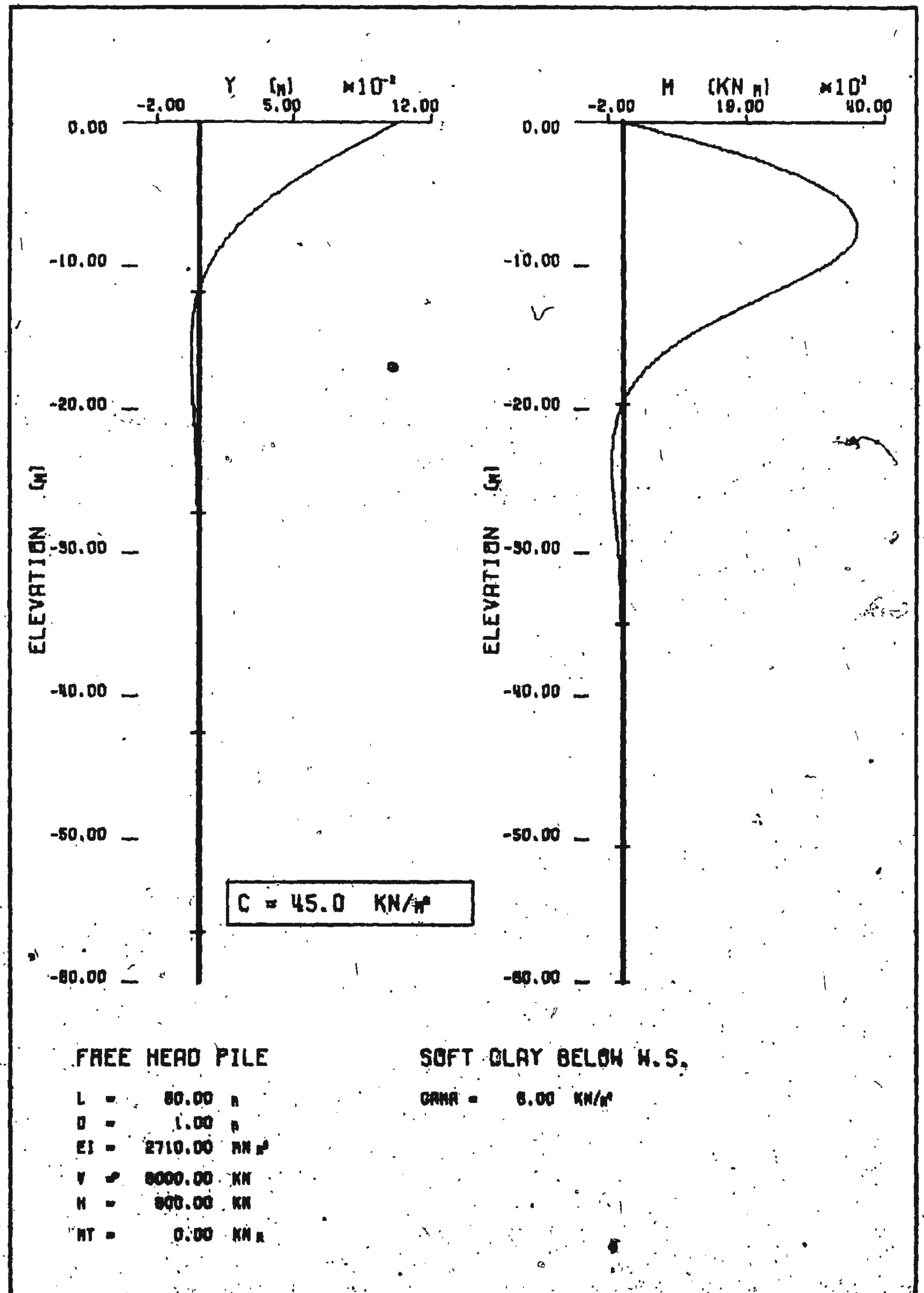


Figure 48 Deflection and bending moment for a free head pile in soft clay, C = 45.0 kN/m².

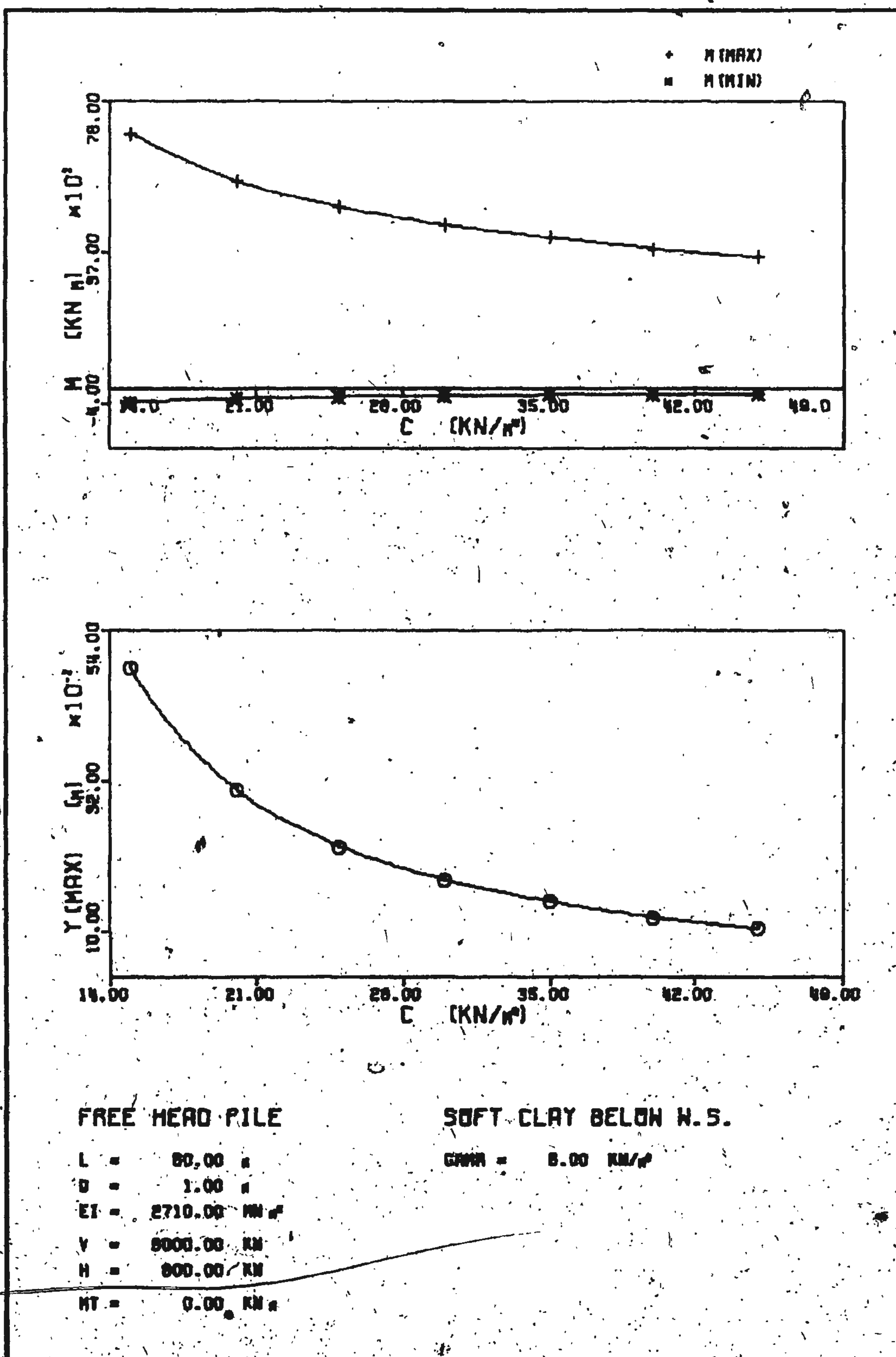


Figure 49. Effect of the soil undrained shear strength on the maximum deflection and maximum bending moment for a free head pile in soft clay.

Table 13 Effect of the soil undrained shear strength on the maximum deflection and the maximum bending moment for a free head pile in soft clay .

C [illegible]^2)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
15.0	0.4856	6906.6	11.0	- 388.1	35.0
20.0	0.3068	5619.3	10.0	- 270.7	31.0
25.0	0.2241	4933.0	9.0	- 216.0	29.0
30.0	0.1757	4441.6	8.0	- 188.8	27.0
35.0	0.1437	4094.7	8.0	- 173.6	26.0
40.0	0.1209	3796.0	8.0	- 162.5	24.0
45.0	0.1040	3562.6	7.0	- 152.4	24.0

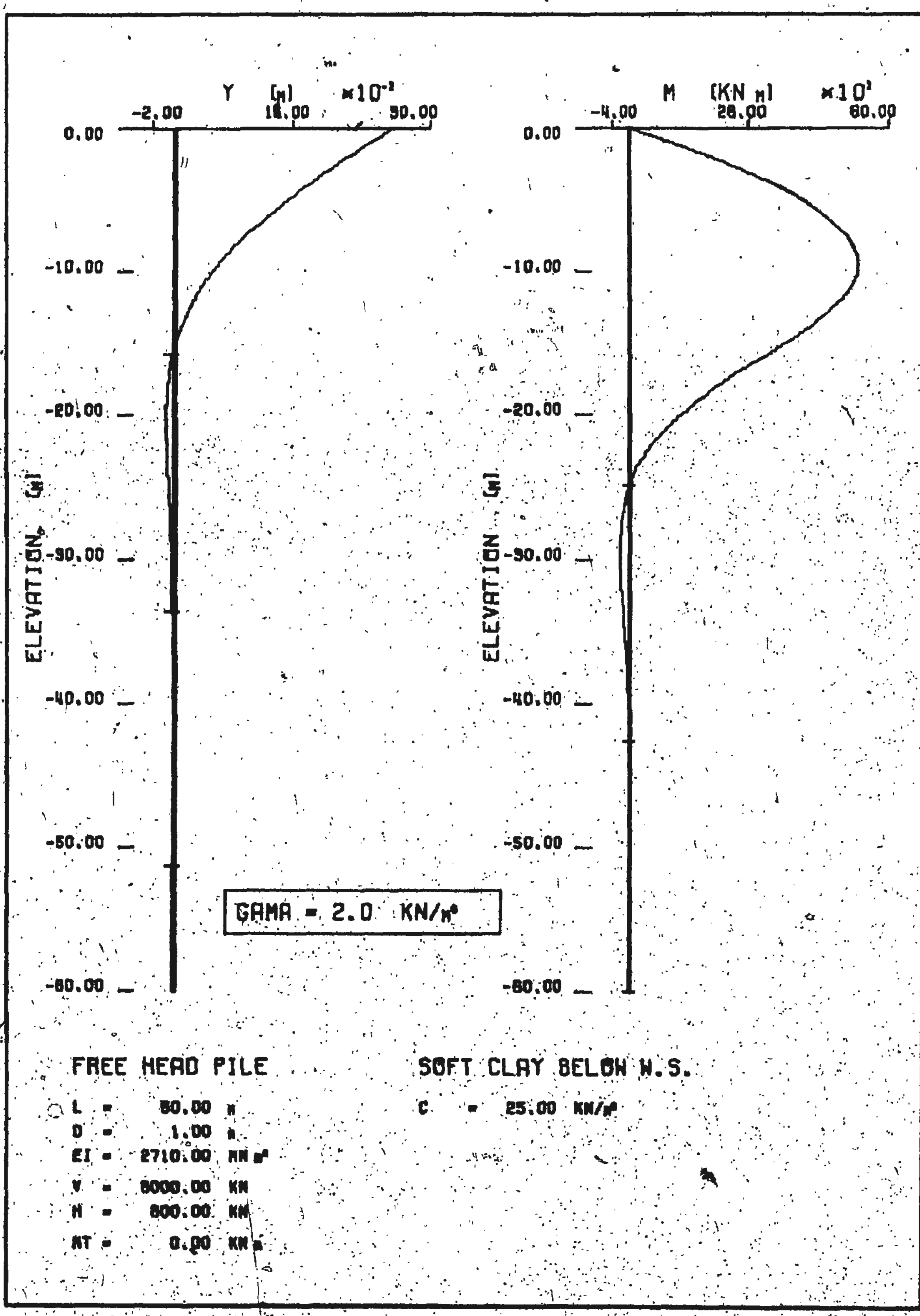


Figure 50 Deflection and bending moment for a free head pile in soft clay, $\gamma' = 2.0$ kN/m³.

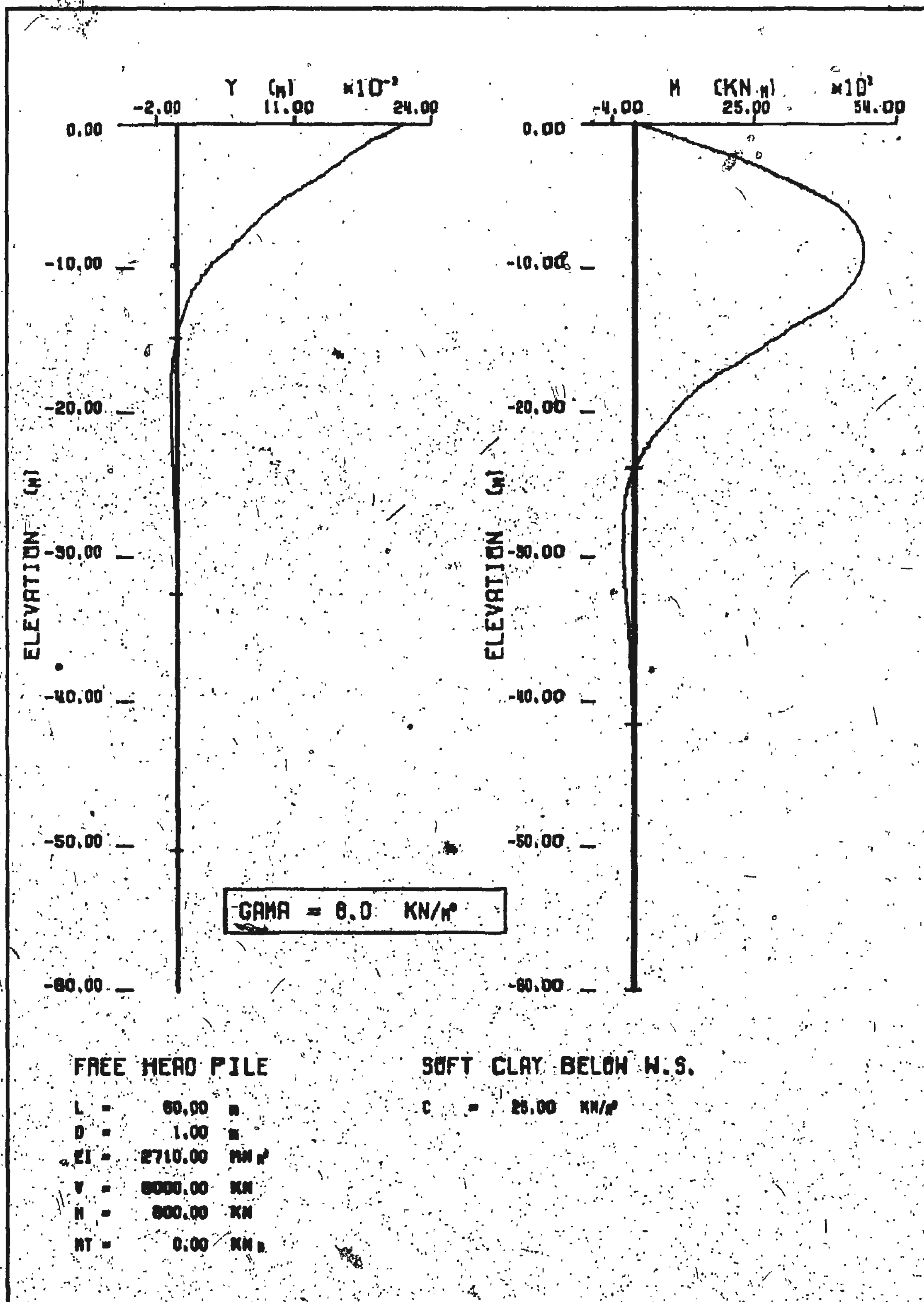


Figure 51 Deflection and bending moment for a free head pile in soft clay, $\gamma' = 8.0$ kN/m³.

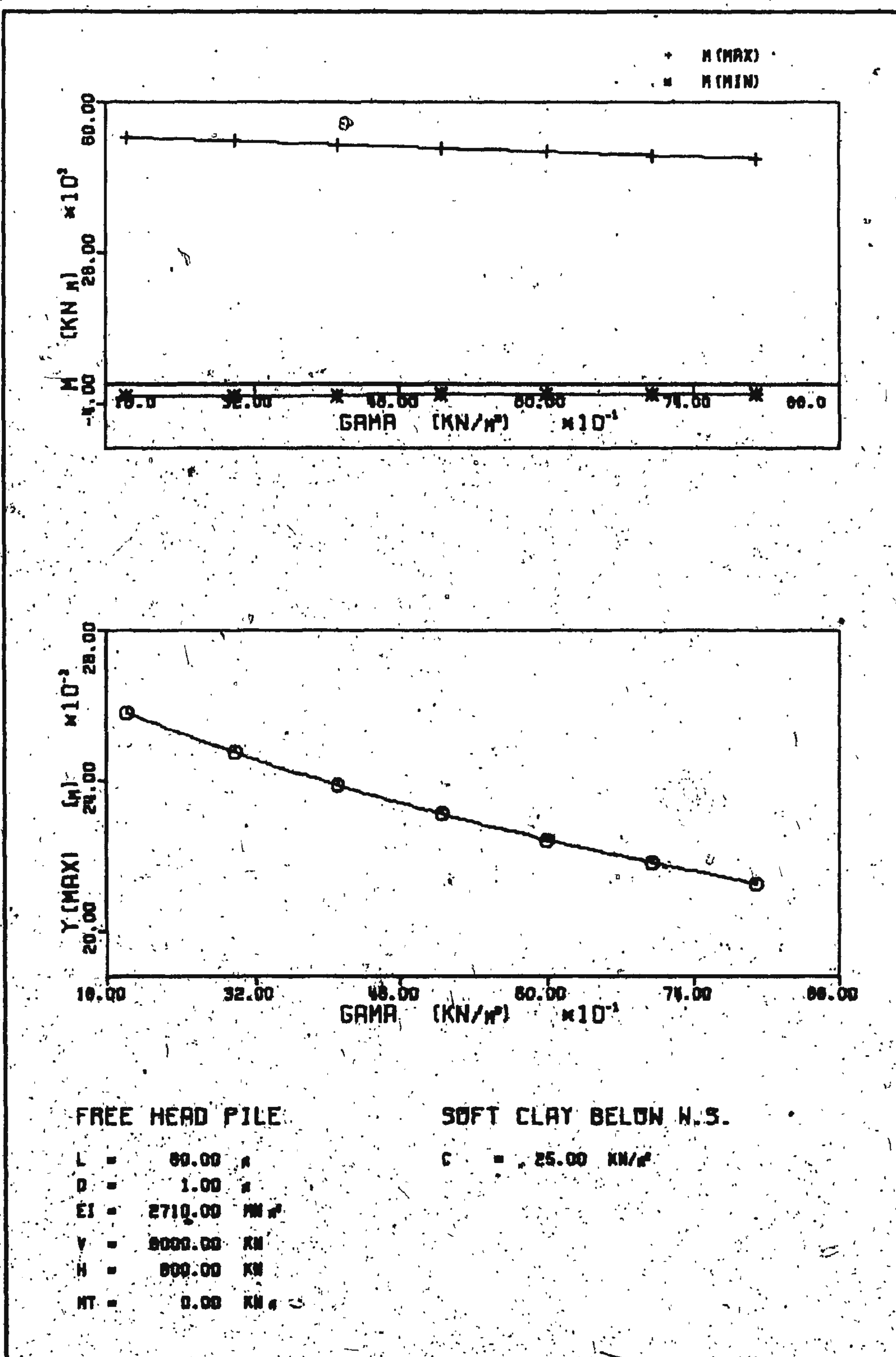


Figure 52 Effect of the soil unit weight on the maximum deflection and the maximum bending moment for a free head pile in soft clay.

Table 14 Effect of the soil unit weight on the maximum deflection and the maximum bending moment for a free head pile in soft clay .

γ' (kN/m^3)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
2.0	0.2579	5235.0	10.0	- 234.5	29.0
3.0	0.2474	5155.2	9.0	- 229.6	29.0
4.0	0.2387	5080.0	9.0	- 225.3	29.0
5.0	0.2310	5006.0	9.0	- 221.2	29.0
6.0	0.2241	4933.0	9.0	- 216.0	29.0
7.0	0.2181	4864.0	9.0	- 211.4	28.0
8.0	0.2125	4798.2	9.0	- 208.2	28.0

has lower values for the upper few meters of the ocean floor, then increases for deeper soil strata (Agarwal et al, 1977; Mahmood and Hough, 1978). For some deep-sea sediments stiffer clay underlain by softer layers has been reported (Silva et al, 1976). This phenomenon rarely occurs on the continental shelves where most offshore structures are constructed. Conditions obtaining in deep-sea sediments will not be discussed here. The problem of soil layering was idealized by assuming two layers of clay; a soft clay layer underlain by a stiff clay. The study was made for 1) a free head pile, and 2) a fixed head pile, both driven into the layered soil system. The standard values in table 5 for the pile loading, pile properties and the soil properties were used in the analysis.

In this study the depth of the upper soft clay layer was varied between 0.0 m and 60.0 m (the pile length) in ten increments. Six plots for the deflection and the bending moment along the pile were obtained for each of the two piles. The plot showing the effect of the depth of the soft clay layer on the maximum deflection and the maximum positive and negative bending moments was finally drawn for the two types of piles. Magnitudes and positions of the maximum deflection and the maximum positive and negative bending moments for each increment were also tabulated.

The graphs and the table in case of a free head pile are presented in the main text of the thesis, and for fixed head pile they are arranged at the end of appendices A and B.

4.4.1 Effect of the Soft Clay Layer

Typical graphs showing the effect of the soft clay layer in a layered soil system are shown in figures 53 to 59. Tabulated results are summarized in table 15. Both the graphs and the table are for a free head pile.

For both types of piles Y_{max} increases rapidly and significantly when the soft clay layer depth increases form 0.0 m to 15.0 m, then for a further increase in depth the magnitude of Y_{max} is not appreciably affected. The magnitude of Y_{max} for a free head pile is about 4.5 times that for a fixed head pile. Y_{max} always occurs at the ground surface independent of the depth of the soft clay layer while the position of M_{max+} moves down along the pile axis for lower values of the soft clay layer depth.

In the case of free head piles, the magnitude of M_{max+} which governs the design of the pile, increases very quickly corresponding to an increase in the depth of the soft clay layer from 0.0 m to 10.0 m, then for a further increase it slightly decreases and reaches a constant value. However for fixed head piles, the magnitude of M_{max-} which governs the design, increases very rapidly when the depth of the soft clay layer increases from 0.0 m to 15.0 m, then it almost remains constant and is independent of any further increase in the depth of the soft clay stratum.

According to the above analysis, it can be concluded that the surface layer exerts a controlling influence on the behavior of a soil-pile system subjected to lateral loading. This is in agreement with similar conclusions drawn by Davisson and Gill (1963).

4.5 Summary

Parametric studies were made for 1) a free head pile in soft clay, 2) a free head pile in stiff clay, 3) a fixed head pile in soft clay and

(Text continued on page 118)

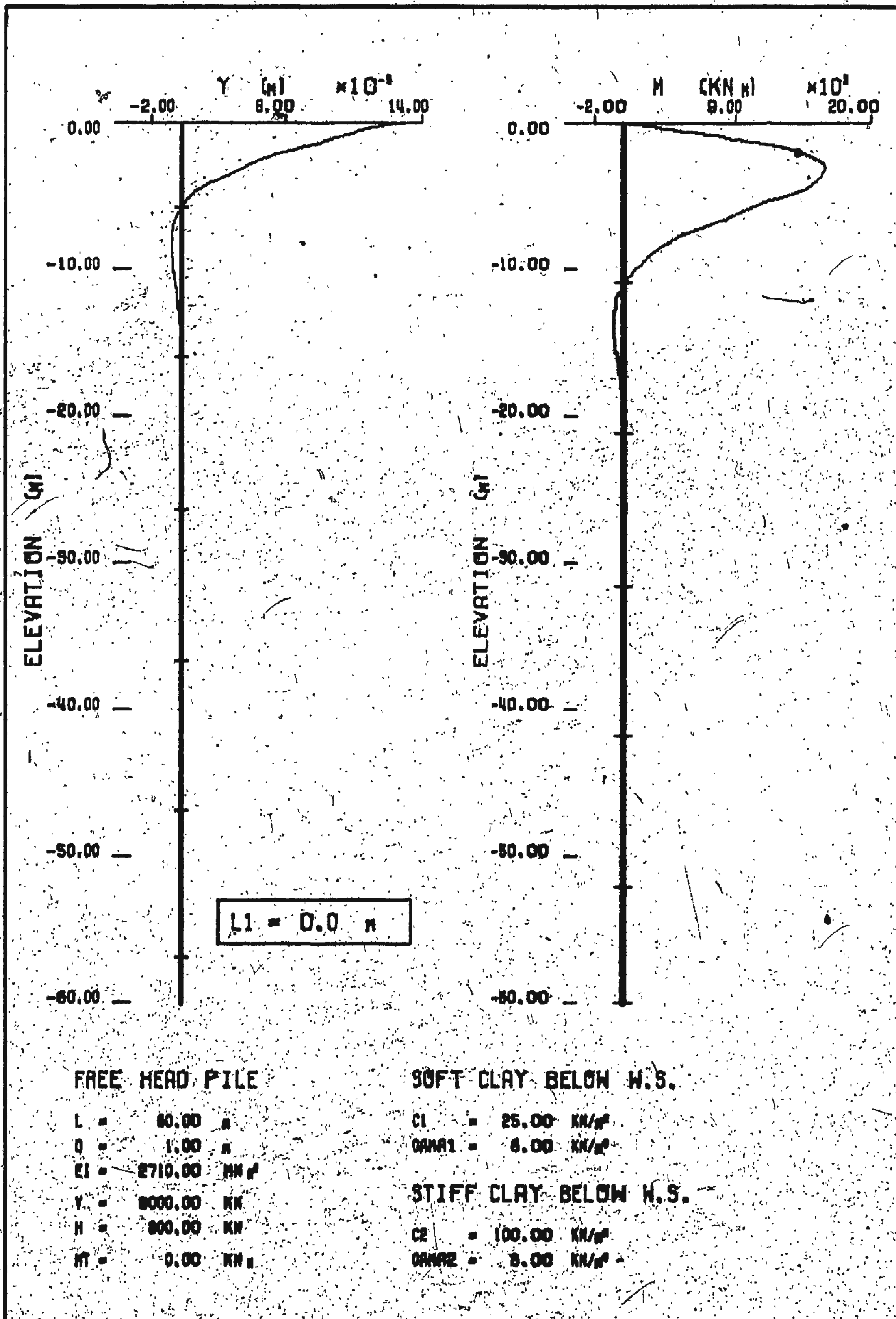


Figure 53 Deflection and bending moment for a free head pile in a layered soil system, L1 = 0.0 m.

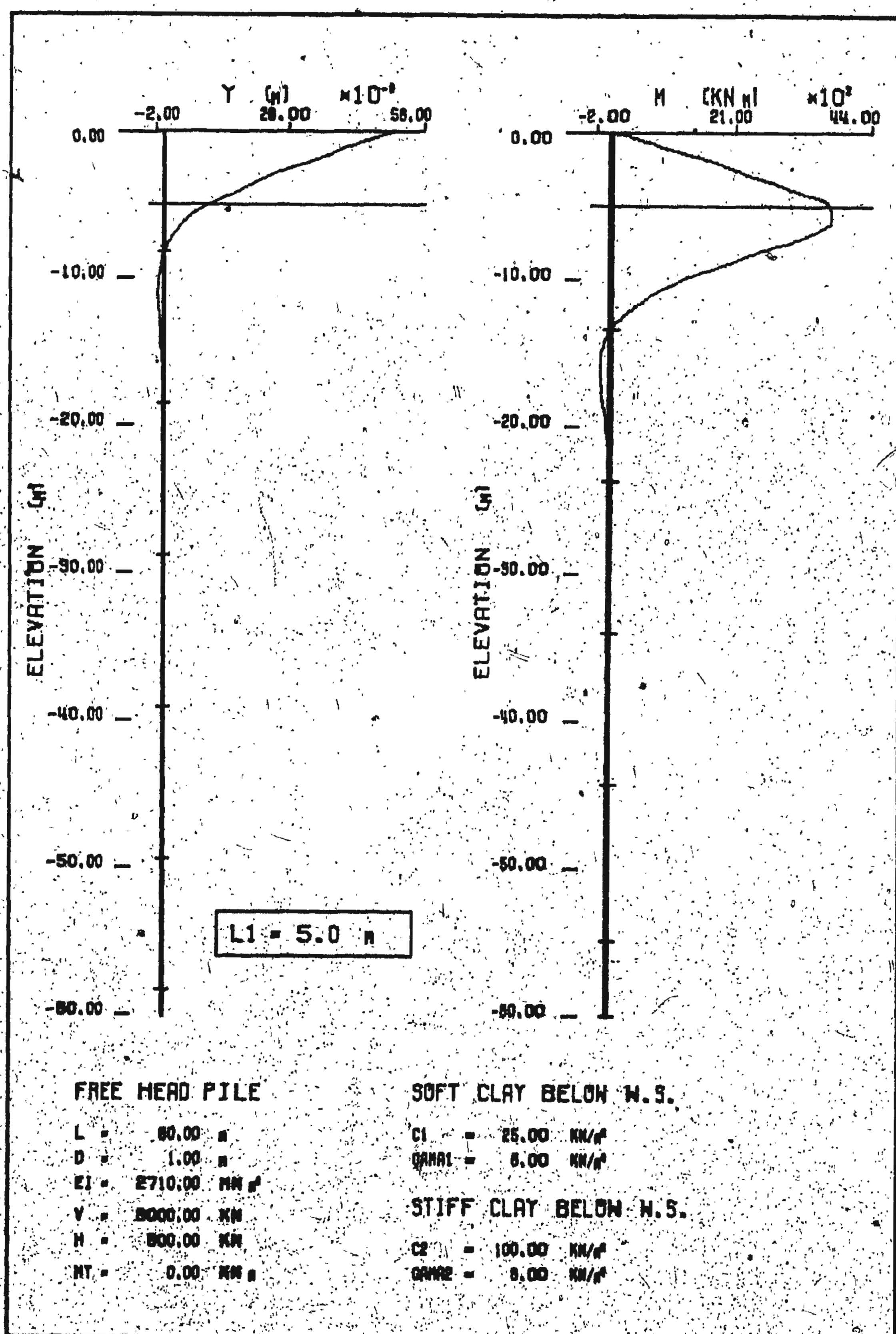


Figure 54 Deflection and bending moment for a free head pile in a layered soil system, L1 = 5.0 m.

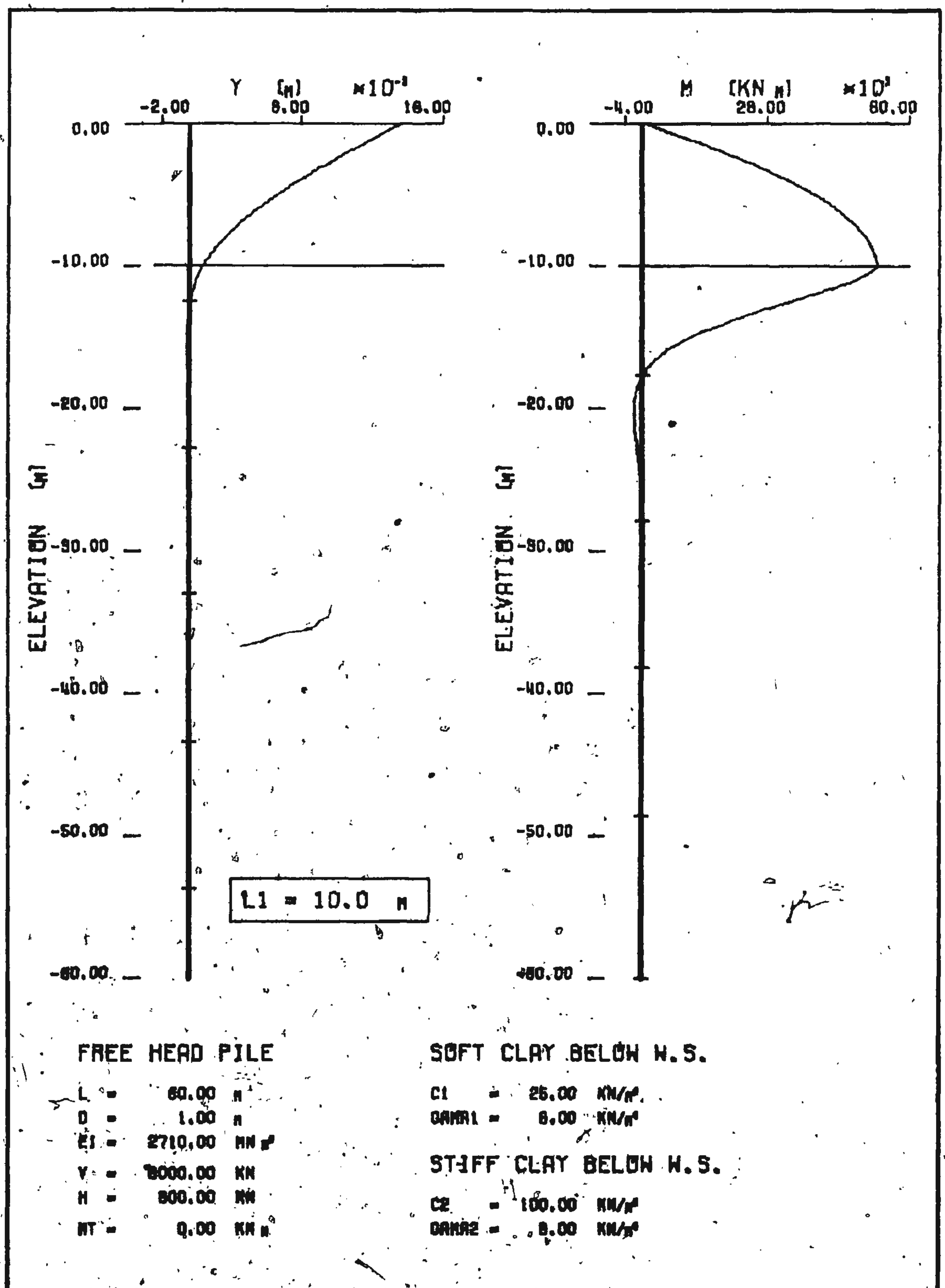


Figure 55 Deflection and bending moment for a free head pile in a layered soil system, L1 = 10.0 m.

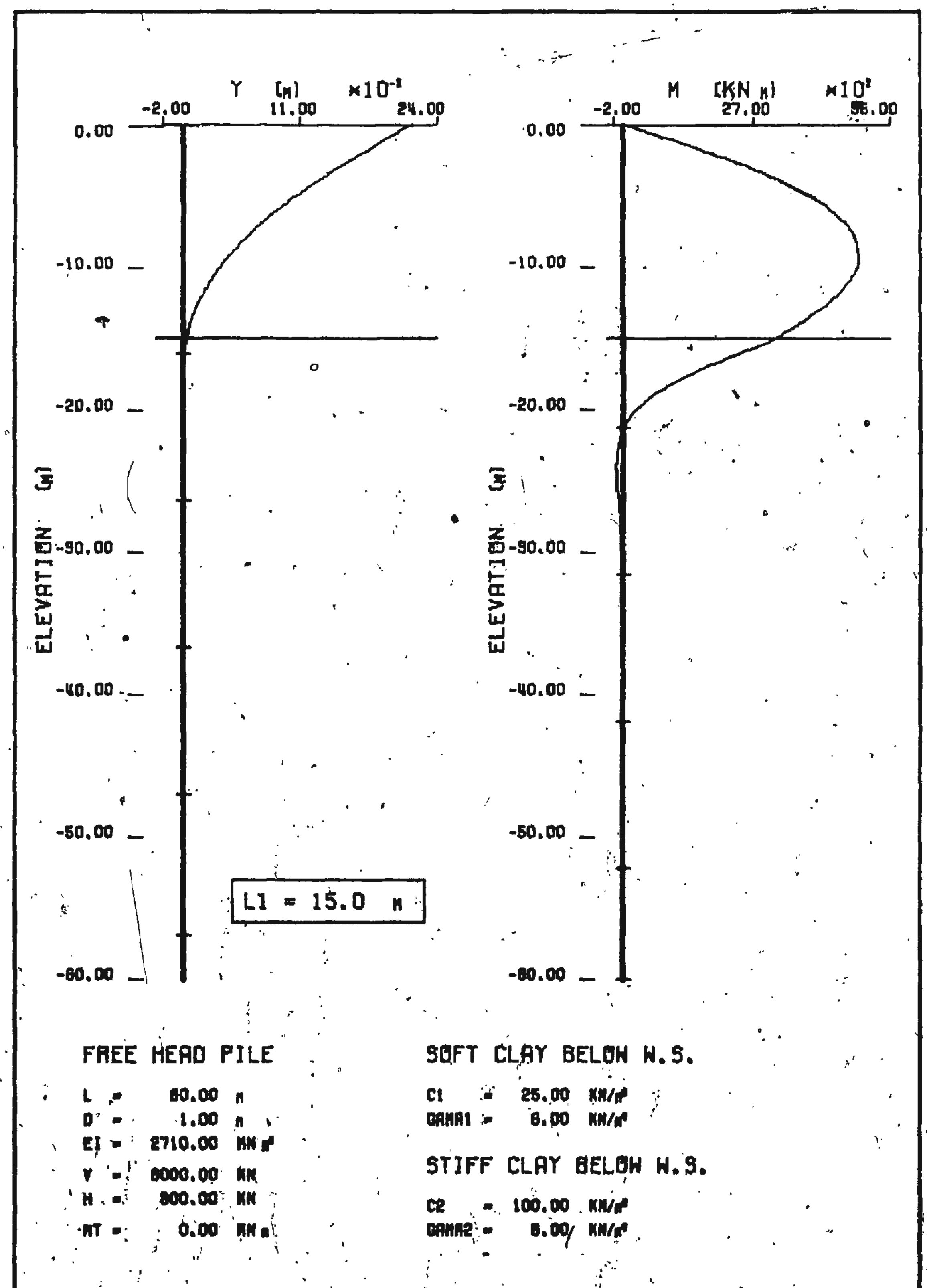


Figure 56 Deflection and bending moment for a free head pile in a layered soil system, L1 = 15.0 m.

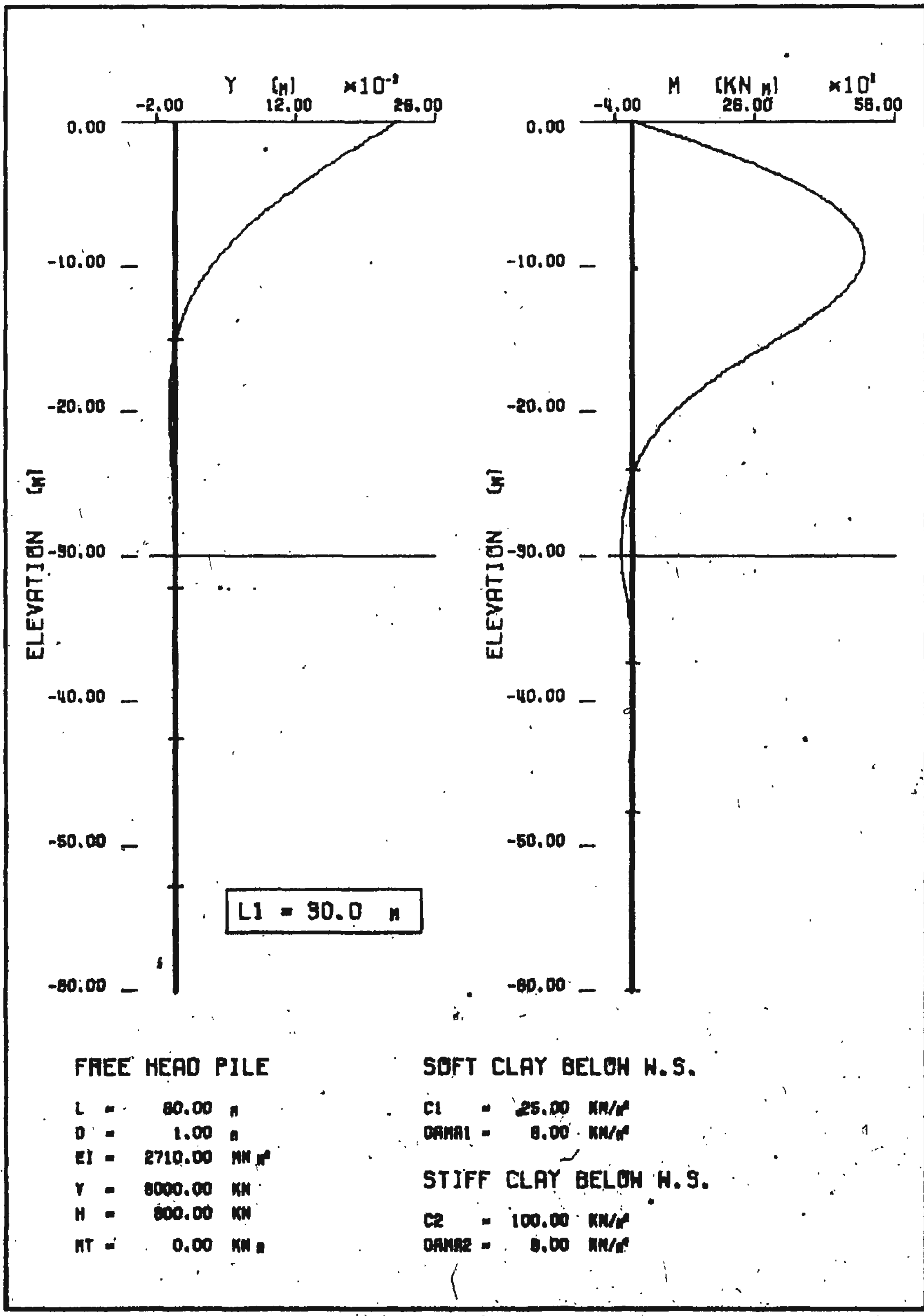


Figure 57 Deflection and bending moment for a free head pile in a layered soil system, L1 = 30.0 m.

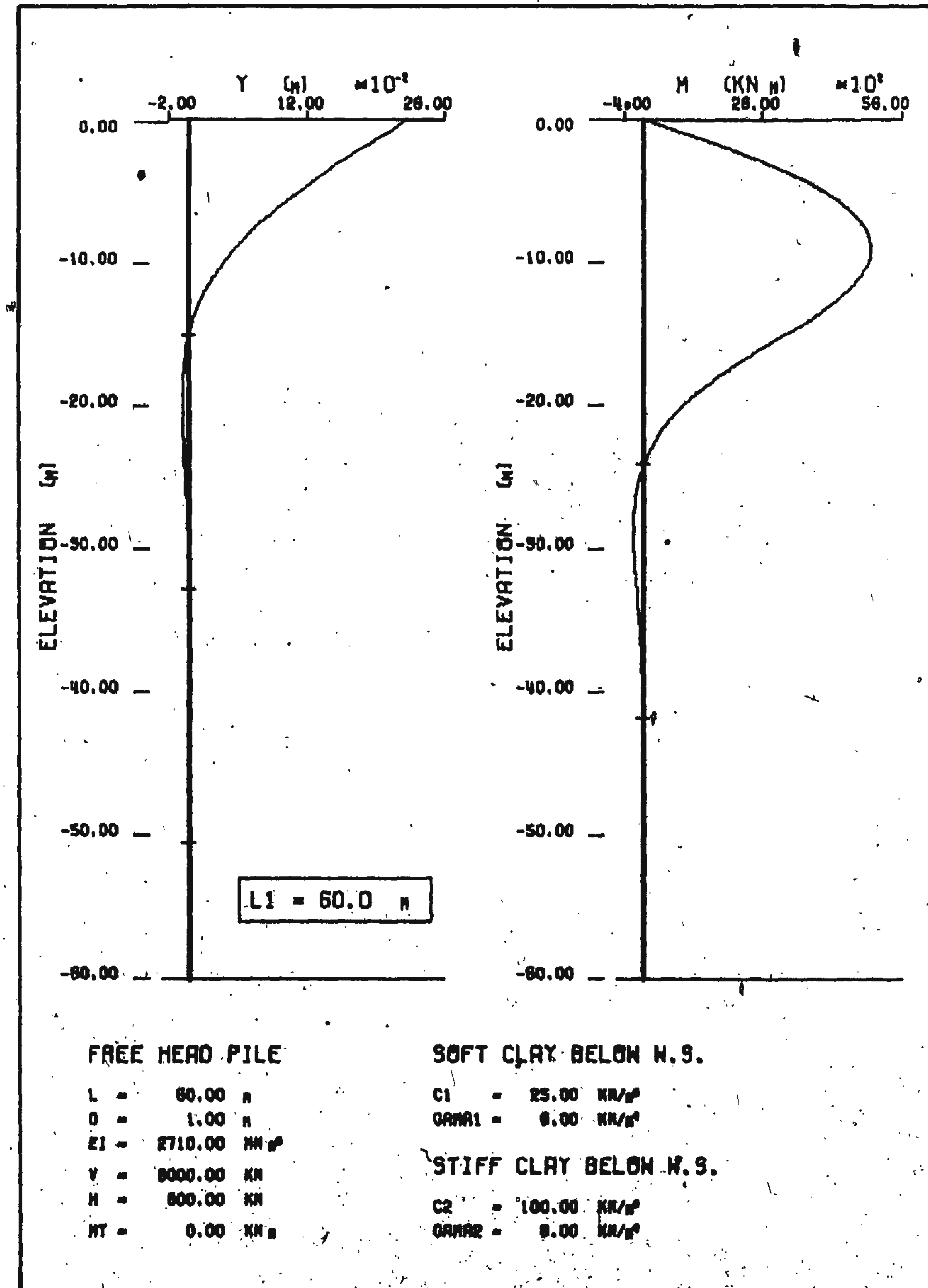


Figure 58 Deflection and bending moment for a free head pile in a layered soil system, L1 = 60.0 m.

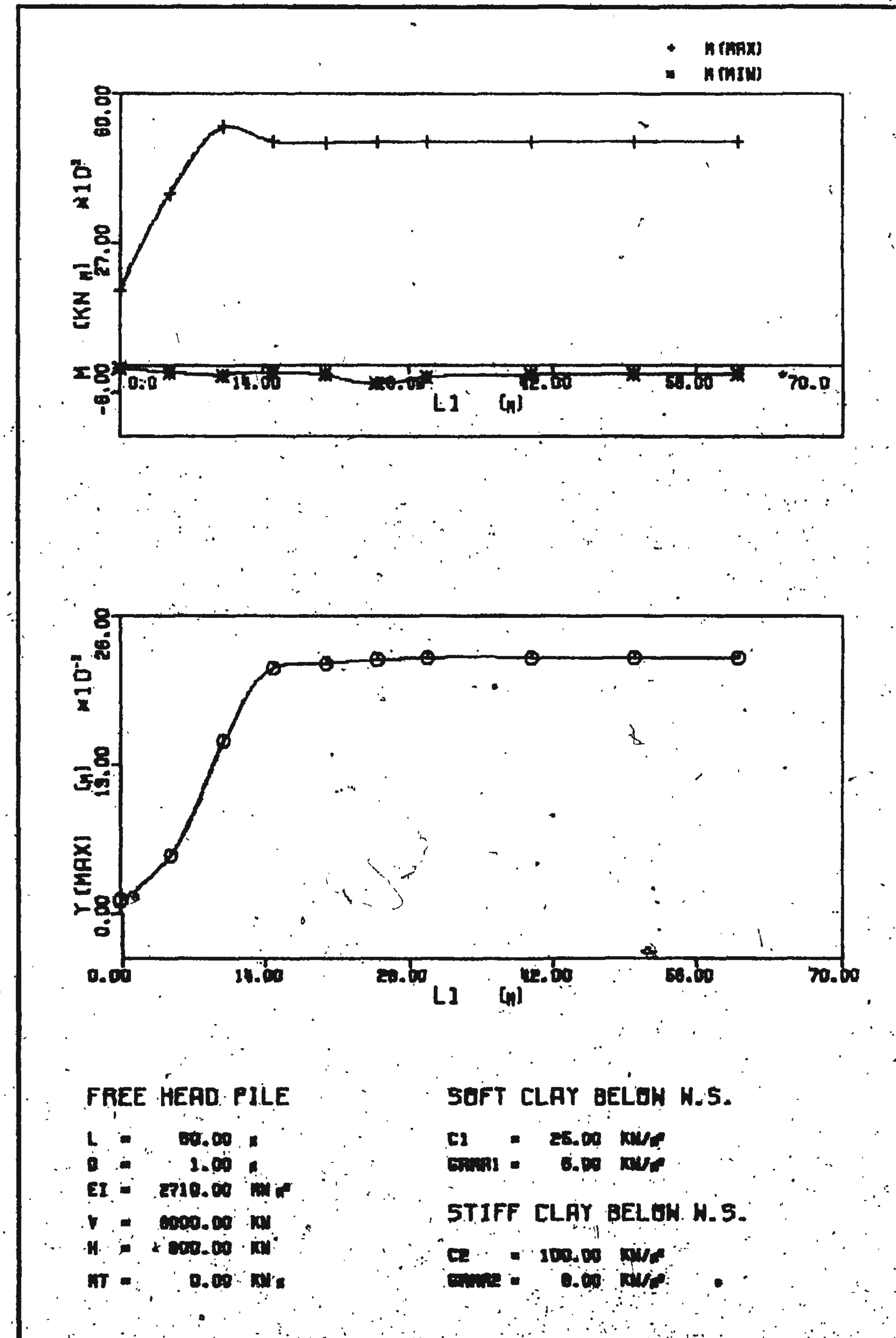


Figure 59 Effect of the soft clay layer depth on the maximum deflection and the maximum bending moment for a free head pile in a layered soil system.

Table 15 Effect of the depth of the soft clay layer on the maximum deflection and the maximum bending moment for a free head pile in a layered soil system .

L1 (m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0.0	0.0121	1658.6	3.0	- 68.7	13.0
5.0	0.0504	3788.9	6.0	- 161.8	16.0
10.0	0.1503	5265.6	10.0	- 222.0	20.0
15.0	0.2149	4939.3	9.0	- 157.9	24.0
20.0	0.2187	4926.8	9.0	- 216.8	24.0
25.0	0.2220	4935.5	9.0	- 418.3	26.0
30.0	0.2239	4933.4	9.0	- 277.6	30.0
40.0	0.2241	4933.0	9.0	- 220.2	29.0
50.0	0.2241	4933.0	9.0	- 216.0	29.0
60.0	0.2241	4933.0	9.0	- 216.0	29.0

4) a fixed head pile in stiff clay. The effects of 1) pile loading, 2) pile properties and 3) soil properties, on the maximum deflection and the maximum bending moment in the pile were examined. Another study was made to determine the effect of soil layering on the maximum deflection and the maximum moment for both free and fixed head piles driven in a layered soil system. Variations within practical ranges of the parameters were studied. The finite difference method was used in the parametric analysis and the CALCOMP graph plotter was used to generate all the graphs.

In general, results showed good agreement with similar published analyses.

CHAPTER V

SUMMARY AND CONCLUSIONS

The laterally loaded pile problem has received considerable attention in recent years. In the research reported here, a comparison between two theoretical methods of analysis, the finite difference and the finite element methods, has been presented. Soil nonlinearity was considered by adopting the p-y curves concept. A comparison was made of the results obtained by the two theoretical methods. Full scale test results reported in publications were compared with the theoretical solutions. A parametric analysis was done to establish the relative significance of the variables influencing the behavior of laterally loaded piles. The effects of 1) pile loading, 2) pile properties, 3) soil properties and 4) soil layering, on the maximum deflection and the maximum bending moment were examined. Two types of soils, soft clay and stiff clay, were considered in the analysis. Both free and fixed head piles were studied. The CALCOMP graph plotter was used to generate all the graphs for the parametric analysis.

The following conclusions are drawn from the present investigation :

(a) Comparative Study

1. The soil nonlinearity can be efficiently represented in terms of the p-y curves. These curves can be generated in a computer program as was demonstrated. The methods presented here for predicting the p-y curves in different types of soil show good agreement with reported field data.
2. The method of constructing the p-y curves developed in this investigation lends itself easily to the analysis of layered soils.
3. A comparison of the finite difference and the finite element programs shows that the core storage and CPU time requirement are more for the

finite element method.

4. A reduction of 20% in the first soil spring constant at the top of the pile is recommended when analyzing free head piles in soft clays using the finite element method.
5. About 30 elements will be sufficient to yield good results in the finite element program.
6. Comparison between the theoretical moments and the full scale tests results are generally agreeable, but the agreement in the computed and measured deflections is not satisfactory.

(b) Parametric Analysis

1. Y_{max} always occurs at the ground surface.
2. Y_{max} increases with the increase of the lateral load, the vertical load and the external moment. It decreases with the increase of the pile diameter, the pile flexural rigidity, the soil undrained shear strength and the soil unit weight, while it is not affected by the pile length.
3. The parameters which significantly affect the magnitude of Y_{max} are the horizontal load, the vertical load (in case of free head piles in soft clay), the external moment, the pile diameter (in case of free head piles in soft clay), the pile flexural rigidity and the soil undrained shear strength.
4. The value of Y_{max} is generally significant in case of free head piles in soft clay while it is very small in case of fixed head piles in stiff clay.
5. In stiff clays, failure occurs corresponding to relatively small values of the pile deflection. It may thus be stated that the behavior of stiff clay tends to be brittle.
6. The value of M_{max+} in case of free head piles and that of M_{max-} for fixed head piles increase corresponding to an increase in the horizontal load, the vertical load, the external moment (for free head piles), and

the pile flexural rigidity, while they decrease with the increase of the pile diameter and the soil undrained shear strength. They are not affected significantly by the soil unit weight. Pile length also does not influence M_{max+} and M_{max-} greatly.

7. The position of M_{max+} moves downwards due to increase in the horizontal load and the pile flexural rigidity, and it moves upwards corresponding to increase of the external moment, the pile diameter and the soil undrained shear strength. Both the upward and downward shifts are within a small range. The position of M_{max+} is not affected by the vertical load, the pile length and the soil unit weight.

8. In general, the position of M_{max+} is nearer to the ground surface in case of stiff clay than soft clay.

9. The ratio of M_{max+} to M_{max-} in case of free head piles is higher than the reciprocal ratio in case of fixed head piles. Obviously this means that the design of fixed head piles will be more economical in terms of optimization compared to free head piles.

10. There are more reversals in the sign of the deflection and the bending moment curves when the soil-pile system becomes more stable due to longer piles, larger diameters or stiffer soil.

11. The results obtained from this investigation agree with those obtained by Palmer and Brown (1954) for a free head pile embedded in sand, Matlock (1970) for laterally loaded piles in soft clay, Reese et al (1975) in case of stiff clay, and Zuhkov and Balov (1978) for axially and laterally loaded piles.

12. In case of a layered soil system composed of a soft clay layer underlain by a stiffer one, only the smaller depths of the soft clay layer affect Y_{max}, M_{max+}, and M_{max-}, but when the depth increases the pile behaves

as if it is fully embedded in soft clay without any influence from the stiff clay layer. This agrees with the conclusions of Davisson and Gill (1963) that the surface layer has a controlling influence on the behavior of a laterally loaded pile in a layered soil system.

13. The conclusions reported here are restricted to single piles driven in clay soil layers below water surface.

Recommendations for Future Research

a) In this study, only short-term static loading was considered. Long-term static loading, taking into account creep and visco-elastic effects, and also dynamic behavior are to be investigated.

b) Effect of the soil layering can be studied for different layered soil systems, and a comparison with more field data can give a better evaluation of the method presented here.

c) Modifications to the finite element program presented here can be made to allow for different end conditions and to reduce both storage and CPU time requirement.

d) A three-dimensional finite element approach which considers both soil and pile elements, the nonlinear soil behavior, and the nonlinear interaction effects can be compared with the methods presented here as a different approach for analyzing laterally loaded piles.

e) Laboratory experiments, and full scale tests if possible, should be conducted to support the theoretical analysis.

f) An interesting area of study and an extension of this research, is the effect of lateral loads on pile groups. Such studies should be undertaken if the interpretation of the behaviour of pile groups in the field is to be meaningful and of practical use.

REFERENCES

1. Agarwal, S.L., Malhotra, A.K., and Banerjee, R. (1977) , "Engineering properties of calcareous soils affecting the design of deep penetration piles for offshore structures" , paper # OTC 2792, Proceedings 9th Offshore Technology Conference , Houston , Texas.

2. "API recommended practice for planning, designing and constructing fixed offshore platforms" , report RP-2A , American Petroleum Institute, Jan. , 1974.

3. Banerjee, P.K. , and Davies, T.G. (1978) , "The behavior of axially and laterally loaded single piles embedded in nonhomogeneous soils", Geotechnique, Vol. 28 , No. 3 , pp 309-326 .

4. Bowles, J.E. (1968) , "Foundation analysis and design" , Chapter 9 , McGraw Hill , New York.

5. Bowles, J.E. (1972) , "Finite element analysis of lateral piles" , Proceedings Conference Finite Element Method in Civil Engineering, McGill University, Montreal , pp 677-700.

6. Bowles, J.E. (1974) , "Analytical and computer methods in foundation engineering" , Chapter 9 , McGraw Hill , New York.

7. Broms, B.B. (1964) , "Lateral resistance of piles in cohesive soils" , Proceedings, ASCE , journal of Soil Mechanics and Foundation Engineering, Vol. 90 , SM2 , pp 27-63.

8. Broms, B.B. (1972) , "Stability of flexible structures (piles and piles groups) : general report" , Proceedings of the 5th European Conference on Soil Mechanics and Foundation Engineering , Madrid, pp 239-269.

9. Davisson, M.T. , and Gill, H.L. (1963) , "Laterally loaded piles in a layered soil system" , Proceedings, ASCE , journal of the Soil Mechanics and Foundation Division, Vol. 89 , SM3, pp 63-94.

10. Desai, C.S. , and Appel, G.C. (1976) , "3-D analysis of laterally loaded structures" , Proceedings of 2nd International Conference on Numerical Methods in Geomechanics, Blacksburg , Virginia .

11. Desai, C.S. , Johnson, L.D. , and Hargett, C.M. (1974) "Analysis of pile supported gravity lock" , Proceedings , ASCE , journal of the Geotechnical Engineering Division, Vol. 100, GT9.

12. Douglas, D.J. , and Davis, E.H. (1964) , "The movement of buried footings due to moment and horizontal load and the movement of anchor plates" , Geotechnique, Vol. 14 , No. 2 , pp 115-132.

13. Focht, J.A. , and Koch, K.J. (1973) , "Rational analysis of the lateral performance of offshore pile groups" , paper # OTC 1896 , Proceedings 5th Offshore Technology Conference, Houston , Texas .

14. Focht, J.A. , and McClelland, B. (1955) , "Analysis of laterally loaded piles by difference equation solution", The Texas Engineer , Texas Section , ASCE .

15. Fukuoka, M. , and Nakase, A. (1973) , "Problems of soil mechanics of the ocean floor" , Proceedings 8th International Conference on Soil Mechanics and Foundation Engineering , Vol. 42 , Moscow, pp 205-222 .

16. Gleser, S.M. (1954) , "Lateral load tests on vertical fixed head and free head piles" , Special Technical Publication 154, ASTM, pp 75-93 .

17. Hetenyi, M. (1946) , "Beams on elastic foundation" , University of Michigan Press, Ann Arbor, Mich.

18. Howe, R.J. (1955) , " A numerical method for predicting the behavior of laterally loaded piling" , EPR Publication 412 , Shell Development Company , Houston , Texas .

19. Ismael, N.F. , and Klym, T.W. (1978) , "Behavior of rigid piers in layered cohesive soils" , Proceedings, ASCE , journal of the Geotechnical Engineering Division , GT8 , pp 1061-1074 .

20. Keller, G.H. (1969) , "Engineering properties of some sea-floor deposits", Proceedings, ASCE , journal of the Soil Mechanics and Foundation Division, SM6 , pp 1379-1392 .

21. Kuhlemeyer, R.L. (1979) , "Static and dynamic laterally loaded piles" , Proceedings, ASCE, journal of the Geotechnical Engineering Division, GT2 , pp 289-304 .

22. Laursen, H.I. (1969) , "Structural analysis" , Chapter 14, McGraw Hill , New York .

23. Lee, C.F. (1975) , "Analytical solutions for the load-deflection response of laterally loaded piers" , Ontario Hydro Research Division, Report No. 75-402-H , Toronto .

24. Lee, P.Y. , and Gilbert, L.W. (1979) , "Behavior of laterally loaded pile in very soft clay" , paper # OTC 3401 , Proceedings 11th Offshore Technology Conference, Houston , Texas .

25. Loftin, T.D. (1976) , "Feasibility of a fixed platform for 1300 feet of water" , paper # OTC 2689 , Proceedings 8th Offshore Technology Conference , Houston , Texas .

26. Madhav, M.R. , Kameswara Rao, N.S. , and Madhavan, K. (1971) , "Laterally loaded piles in elastoplastic soil" , Soils and Foundation, Vol. 11 No. 2 , Tokyo .

27. Mahmood, A. , and Hough, J.C. (1978) , "Seabottom soil properties and acoustic characteristics" , paper # OTC 3208 , Proceedings 10th Offshore Technology Conference , Houston , Texas .

28. Mason, H.G. , and Bishop, J.A. (1954) , "Measurement of earth pressure and deflection along the embedded portion of a 40-ft steel pile" , Supplement to Symposium on Lateral Load Tests on Piles, ASTM .

29. Matlock, H. (1970) , "Correlations for design of laterally loaded piles in soft clay" , paper # OTC 1204 , Proceedings 2nd Offshore Technology Conference , Houston , Texas .

30. Matlock, H. , and Reese, L.C. (1960) , "Generalized solutions for laterally loaded piles" , Proceedings , ASCE , journal of the Soil Mechanics and Foundation Division , Vol. 86 , SM5 , pp 63-91 .

31. McClelland, B. (1974) , "Design of deep penetration piles for ocean structures" , Proceedings , ASCE , journal of the Geotechnical Engineering Division , Vol. 100 , GT7 , pp 709-747 .

32. McClelland, B. , and Focht, J.A. (1958) , "Soil modulus for laterally loaded piles" , Transactions , ASCE , Vol. 123 , paper No. 2954 , pp 1049-1086 .

33. Mindlin, R.D. (1936) , "Force at the point in the interior at a semi-infinite soild" , Physics No. 5 , pp 195-202 .

34. Mori, H. (1964) , "The behavior of pipe piles under vertical and horizontal load" , Proceedings Symposium on Bearing Capacity of Piles , New Delhi .

35. Mustafayev, A.A. , Mamedov, K.M. , and Ismailov, B.G. (1972) , "Pipe filling piles of sea oilfield structures and methods of calculation for lateral forces" , Proceedings 5th European Conference on Soil Mechanics and Foundation Engineering , Madrid, pp 381-386 .

36. Muzas, F. (1972) , "Contribution to the study of laterally loaded piles" , Proceedings 5th European Conference on Soil Mechanics and Foundation Engineering , Madrid , pp 387-396 .

37. Newmark, N.M. (1942) , "Numerical procedure for computing deflections, moments, and buckling loads" , Proceedings , ASCE , Vol. 68 , No. 5 , pp 697 .

38. Noorany, I. , and Gizienski, S.F. (1970) , "Engineering properties of submarine soils : state-of-the-art review" , Proceedings , ASCE , journal of the Soil Mechanics and Foundation Division , SM5 , pp 1735-1762 .

39. O'Neill, M.W. , and Ghazzaly, O.I. (1977) , "Analysis of three-dimensional pile groups with nonlinear soil response and pile-soil-pile interaction" , paper # OTC 2838 , Proceedings 9th Offshore Technology Conference, Houston , Texas .

40. Palmer, L.A. , and Brown, P.P. (1954) , "Analysis of pressure, deflection, moment, and shear by the method of difference equations" , Supplement to Symposium on Lateral Load Tests on Piles , ASTM .

41. Palmer, L.A. , and Thompson, J.B. (1948) , "The earth pressures and deflections along the embedded lengths of piles subjected to lateral thrust" , Proceedings 2nd International Conference on Soil Mechanics and Foundation Engineering, Vol. V , pp 156-161 .

42. Poulos, H.G. (1971) , "Behavior of laterally loaded piles : I-Single piles" , Proceedings, ASCE , journal of the Soil Mechanics and Foundation Division , Vol. 97 , SM5 , pp 711-732 .

43. Radhakrishna, H.S. , Lee, C.F. , and Klym, T.W. (1977) , "Laterally loaded piers in glacial till" , 30th Canadian Geotechnical Conference , Saskatoon , pp IV13-IV32.

44. Reddy, A.S. , and Valsangkar, A.J. (1970) , "Generalized solutions for laterally loaded piles on elasto-plastic ground" , Soils and Foundation , Vol. 10 , No. 3 , Tokyo , pp 66-80 .

45. Reese, L.C. (1975a) , "Laterally loaded piles" , GESA Report No. D-75-14, UCCC Report No. 75-14 , Geotechnical Engineering Software Activity, University of Colorado Computing Center, Boulder, Colorado .

46. Reese, L.C. (1975b) , "Analysis of laterally loaded piles-Software documentation-" , GESA Report No. D-75-7 , UCCC Report No. 75-10, Department of Civil Engineering, University of Texas at Austin, Texas .

47. Reese, L.C. (1977) , "Laterally loaded piles : program documentation", Proceedings , ASCE , journal of the Geotechnical Engineering Division, Vol. 103 , GT4 , pp 287-305 .

48. Reese, L.C. , and Cox, W.R. (1969) , "Soil behavior from analysis of tests of uninstrumented piles under lateral loading" , Performance of Deep Foundations , ASTM , Special Technical Publication No. 444 , pp 160-176 .

49. Reese, L.C. , Cox, W.R. , and Koop, F.D. (1974) , "Analysis of laterally loaded piles in sand" , paper # OTC 2080 , Proceedings 6th Offshore Technology Conference , Houston, Texas .

50. Reese, L.C. , Cox, W.R. , and Koop, F.D. (1975) , "Field testing and analysis of laterally loaded piles in stiff clay" , paper # OTC 2312 , Proceedings 7th Offshore Technology Conference, Houston, Texas.

51. Reese, L.C. , and Ginzbarg, A.S. (1958) , "Difference equation method for laterally loaded piles with abrupt changes in flexural rigidity" , EPR Memo. Report 39, Shell Development Company , Houston, Texas .

52. Reese , L.C. , Manoliu, I.(1973) , "Analysis of laterally loaded piles by computer" , Buletinul Stiintific , Al Institutului de Constructii Bucuresti , Anul XVI , NR. 1 , pp 35-70 .

53. Reese, L.C. , and Matlock, H. (1956) , "Non-dimensional solutions for laterally loaded piles with soil modulus assumed proportional to depth" , Proceedings 8th Texas Conference on Soil Mechanics and Foundation Engineering , Austin , Texas .

54. Reese, L.C. , and Welch, R.C. (1975) , "Lateral loading of deep foundations in stiff clay" , Proceedings, ASCE , journal of the Geotechnical Engineering Division , Vol. 101 , GT7 , pp 633-649 .

55. Ruser, J.R. , and Dawkins, W.P. (1972) , "3D finite element analysis of soil-structure interaction" , Proceedings , Symposium on Application of Finite Element Methods in Geotechnical Engineering , Vicksburg .

56. Silva, A.J. , Hollister, C.D. , Laine, E.P. , and Beverly, B.E. (1976) , "Geotechnical properties of deep sea sediments : Bermuda Rise" , Marine Geotechnology, Vol. 1 , No. 3 , pp 195-232 .

57. Skempton, A.W. (1951) , "The bearing capacity of clays" , Proceedings, Building Research Congress, Division I , London .

58. Spillers, W.R. , and Stoll, R.D. (1964) , "Lateral response of piles" , Proceedings, ASCE , journal of the Soil Mechanics and Foundation Division , Vol. 90 , SM6 , pp 1-9 .

59. Stevens, J.B. , and Audibert, J.M.E. (1979) , "Re-examination of p-y curve formulations" , paper # OTC 3402 , Proceedings 11th Offshore Technology Conference , Houston, Texas .

60. Terzaghi, K. (1955) , "Evaluation of coefficients of subgrade reaction", Geotechnique, Vol. 5 , No. 4 , pp 297-326 .

61. Terzaghi, K. , and Peck, R.B. (1948) , "Soil mechanics in engineering practice" , John Wiley & Sons , New York , PP 30 .

62. Wang, C.K. (1970) , "Matrix method of structural analysis" , 2nd. Ed. , International Textbook Co. , Scranton , Pa. .

63. Wittke, W. (1974) , "Bemessung von horizontal belastetem grossbohrpfahlen nach der methods finite elements" , Der Bauirgenieur , Vol. 449 .

64. Yegian, M. , and Wright, S.G. (1973) , "Lateral soil resistance-displacement relationships for pile foundations in soft clays" , paper # OTC 1893 , Proceedings 5th Offshore Technology Conference , Houston, Texas.

65. Zhukov, N.V. , and Balov, I.L. (1978) , "Investigation of the effect of a vertical load on horizontal displacements and resistance of horizontally loaded pile columns" , Osnov. Fund. Mech. Grunt. 20 , No. 1 , pp 11-14 .

APPENDIX A

TABLES

SHOWING THE RESULTS OF THE PARAMETRIC ANALYSIS

Table 16 Effect of the horizontal load on the maximum deflection and the maximum bending moment for a free head pile in stiff clay .

H (kN)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
100	0.0012	177.4	3.0	- 7.4	13.0
275	0.0034	487.9	3.0	- 20.4	13.0
450	0.0057	806.8	3.0	- 33.7	13.0
625	0.0085	1194.5	3.0	- 49.8	13.0
800	0.0121	1658.6	3.0	- 68.7	13.0
975	0.0168	2191.5	3.0	- 91.8	14.0
1150	0.0237	2835.4	4.0	- 121.3	14.0

Table 17 Effect of the horizontal load on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay .

H (kN)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
100	0.0026	87.1	9.0	- 360.2	0.0
275	0.0079	263.5	10.0	-1054.0	0.0
450	0.0172	548.0	10.0	-1963.6	0.0
625	0.0306	881.7	11.0	-2999.7	0.0
800	0.0480	1241.1	12.0	-4134.9	0.0
975	0.0697	1624.4	13.0	-5363.5	0.0
1150	0.0950	2021.8	14.0	-6653.7	0.0

Table 18 Effect of the horizontal load on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay .

H (kN)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
100	0.0005	51.8	5.0	- 216.5	0.0
275	0.0014	142.4	5.0	- 595.5	0.0
450	0.0023	233.1	5.0	- 974.4	0.0
625	0.0032	323.7	5.0	-1353.4	0.0
800	0.0041	414.3	5.0	-1732.3	0.0
975	0.0049	505.0	5.0	-2111.3	0.0
1150	0.0059	607.1	5.0	-2526.4	0.0

Table 19 Effect of the vertical load on the maximum deflection and the maximum bending moment for a free head pile in stiff clay .

V (kN)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0	0.0117	1594.5	3.0	- 64.2	14.0
3000	0.0119	1617.9	3.0	- 65.8	13.0
6000	0.0120	1642.1	3.0	- 67.5	13.0
9000	0.0122	1667.0	3.0	- 69.2	13.0
12000	0.0124	1692.6	3.0	- 71.0	13.0
15000	0.0126	1719.0	3.0	- 72.9	13.0
18000	0.0127	1746.3	3.0	- 74.8	13.0

Table 20 Effect of the vertical load on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay.

V (kN)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0	0.0454	1163.9	12.0	-3975.4	0.0
3000	0.0463	1192.0	12.0	-4033.1	0.0
6000	0.0473	1221.2	12.0	-4093.3	0.0
9000	0.0484	1251.3	12.0	-4156.2	0.0
12000	0.0495	1283.0	12.0	-4222.2	0.0
15000	0.0506	1316.3	12.0	-4291.5	0.0
18000	0.0519	1351.7	12.0	-4365.0	0.0

Table 21 Effect of the vertical load on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay.

V (kN)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0	0.00403	410.1	5.0	-1721.6	0.0
3000	0.00404	411.7	5.0	-1725.6	0.0
6000	0.00405	413.3	5.0	-1729.6	0.0
9000	0.00406	414.9	5.0	-1733.7	0.0
12000	0.00407	416.5	5.0	-1737.7	0.0
15000	0.00408	418.1	5.0	-1741.8	0.0
18000	0.00409	419.8	5.0	-1745.9	0.0

Table 22 Effect of the external moment on the maximum deflection and the maximum bending moment for a free head pile in stiff clay .

MT (kN m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0	0.0121	1658.6	3.0	- 68.7	13.0
400	0.0146	2060.6	3.0	- 86.0	13.0
800	0.0171	2478.6	3.0	- 104.0	13.0
1200	0.0200	2897.2	3.0	- 121.7	13.0
1600	0.0234	3329.5	3.0	- 139.4	13.0
2000	0.0270	3773.3	3.0	- 157.5	13.0
2400	0.0307	4215.2	3.0	- 175.5	13.0

Table 23 Effect of the pile length on the maximum deflection and the maximum bending moment for a free head pile in stiff clay .

L (m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
24.0	0.0121	1658.6	3.0	- 68.5	13.0
36.0	0.0121	1658.6	3.0	- 68.7	13.0
48.0	0.0121	1658.6	3.0	- 68.7	13.0
60.0	0.0121	1658.6	3.0	- 68.7	13.0
72.0	0.0121	1658.6	3.0	- 68.7	13.0
84.0	0.0121	1658.6	3.0	- 68.7	13.0
96.0	0.0121	1658.6	3.0	- 68.7	13.0

Table 24 Effect of the pile length on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay .

L (m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
24.0	0.0486	1209.9	12.0	-4138.5	0.0
36.0	0.0480	1240.3	12.0	-4135.3	0.0
48.0	0.0480	1241.1	12.0	-4134.9	0.0
60.0	0.0480	1241.1	12.0	-4134.9	0.0
72.0	0.0480	1241.1	12.0	-4134.9	0.0
84.0	0.0480	1241.1	12.0	-4134.9	0.0
96.0	0.0480	1241.1	12.0	-4134.9	0.0

Table 25. Effect of the pile length on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay .

L (m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
24.0	0.0041	414.3	5.0	-1732.3	0.0
36.0	0.0041	414.3	5.0	-1732.3	0.0
48.0	0.0041	414.3	5.0	-1732.3	0.0
60.0	0.0041	414.3	5.0	-1732.3	0.0
72.0	0.0041	414.3	5.0	-1732.3	0.0
84.0	0.0041	414.3	5.0	-1732.3	0.0
96.0	0.0041	414.3	5.0	-1732.3	0.0

Table 26 Effect of the pile flexural rigidity on the maximum deflection and the maximum bending moment for a free head pile in stiff clay .

EI ($MN\ m^2$)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
2000	0.0147	1665.0	3.0	- 68.8	13.0
5000	0.0086	1674.2	3.0	- 71.1	15.0
8000	0.0069	1762.4	4.0	- 75.1	17.0
11000	0.0060	1835.1	4.0	- 77.8	19.0
14000	0.0055	1903.9	4.0	- 81.3	20.0
17000	0.0051	1957.0	4.0	- 84.0	21.0
20000	0.0049	2013.9	5.0	- 86.1	22.0

Table 27 Effect of the pile flexural rigidity on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay .

EI ($MN\ m^2$)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
2000	0.0579	1218.3	12.0	-4004.0	0.0
5000	0.0334	1282.3	13.0	-4443.2	0.0
8000	0.0255	1318.2	14.0	-4715.9	0.0
11000	0.0214	1338.6	15.0	-4917.4	0.0
14000	0.0188	1358.1	15.0	-5079.3	0.0
17000	0.0169	1369.6	16.0	-5214.9	0.0
20000	0.0155	1382.2	16.0	-5332.1	0.0

Table 28 Effect of the pile flexural rigidity on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay .

EI ($MN\ m^2$)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
2000	0.0045	396.2	5.0	-1628.2	0.0
5000	0.0033	459.4	6.0	-1964.4	0.0
8000	0.0028	495.9	7.0	-2165.3	0.0
11000	0.0025	522.8	7.0	-2314.1	0.0
14000	0.0023	547.2	8.0	-2434.3	0.0
17000	0.0022	567.4	8.0	-2536.0	0.0
20000	0.0021	580.0	8.0	-2624.7	0.0

Table 29 Effect of the pile diameter on the maximum deflection and the maximum bending moment for a free head pile in stiff clay .

D (m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0.75	0.0131	1686.0	3.0	- 71.8	14.0
1.00	0.0121	1658.6	3.0	- 68.7	13.0
1.25	0.0119	1637.8	3.0	- 68.6	13.0
1.50	0.0117	1602.9	3.0	- 66.9	13.0
1.75	0.0114	1562.6	3.0	- 64.3	13.0
2.00	0.0111	1523.7	3.0	- 61.5	13.0
2.25	0.0109	1484.7	3.0	- 59.0	13.0

Table 30 Effect of the pile diameter on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay .

D	Y_{max}	M_{max+}	Depth to M_{max+}	M_{max-}	Depth to M_{max-}
(m)	(m)	(kN m)	(m)	(kN m)	(m)
0.75	0.0569	1286.8	13.0	-4370.1	0.0
1.00	0.0480	1241.1	12.0	-4134.9	0.0
1.25	0.0425	1198.1	12.0	-3961.9	0.0
1.50	0.0386	1158.9	11.0	-3817.9	0.0
1.75	0.0355	1115.2	11.0	-3692.8	0.0
2.00	0.0329	1069.3	11.0	-3583.0	0.0
2.25	0.0307	1028.2	10.0	-3484.3	0.0

Table 31 Effect of the pile diameter on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay .

D	Y_{max}	M_{max+}	Depth to M_{max+}	M_{max-}	Depth to M_{max-}
(m)	(m)	(kN m)	(m)	(kN m)	(m)
0.75	0.00428	396.3	5.0	-1748.6	0.0
1.00	0.00406	414.3	5.0	-1732.3	0.0
1.25	0.00399	429.4	5.0	-1723.9	0.0
1.50	0.00396	434.9	5.0	-1712.0	0.0
1.75	0.00395	434.5	5.0	-1698.9	0.0
2.00	0.00393	429.7	5.0	-1685.3	0.0
2.25	0.00392	423.9	5.0	-1671.5	0.0

Table 32 Effect of the pile diameter and the corresponding flexural rigidity on the maximum deflection and the maximum bending moment for a free head pile in stiff clay .

D	Y_{max}	M_{max+}	Depth to M_{max+}	M_{max-}	Depth to M_{max-}
(m)	(m)	(kN m)	(m)	(kN m)	(m)
0.90	0.0158	1689.9	3.0	- 70.2	13.0
0.95	0.0136	1664.2	3.0	- 70.2	13.0
1.00	0.0121	1658.6	3.0	- 68.7	13.0
1.10	0.0099	1655.0	3.0	- 69.9	14.0
1.25	0.0079	1702.6	4.0	- 72.2	16.0
1.50	0.0058	1829.6	4.0	- 77.8	18.0
2.00	0.0040	2201.9	5.0	- 93.8	22.0

Table 33 Effect of the pile diameter and the corresponding flexural rigidity on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay .

D	Y_{max}	M_{max+}	Depth to M_{max+}	M_{max-}	Depth to M_{max-}
(m)	(m)	(kN m)	(m)	(kN m)	(m)
0.90	0.0639	1237.0	12.0	-4067.2	0.0
0.95	0.0550	1238.2	12.0	-4097.5	0.0
1.00	0.0480	1241.1	12.0	-4134.9	0.0
1.10	0.0374	1243.1	12.0	-4217.6	0.0
1.25	0.0270	1253.7	13.0	-4346.8	0.0
1.50	0.0173	1245.9	14.0	-4564.8	0.0
2.00	0.0090	1220.5	16.0	-4995.0	0.0

Table 34 Effect of the pile diameter and the corresponding flexural rigidity on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay .

D (m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0.90	0.0047	387.6	5.0	-1615.9	0.0
0.95	0.0044	402.3	5.0	-1674.2	0.0
1.00	0.0041	414.3	5.0	-1732.3	0.0
1.10	0.0036	440.5	6.0	-1850.4	0.0
1.25	0.0030	487.4	6.0	-2023.4	0.0
1.50	0.0024	556.4	7.0	-2305.9	0.0
2.00	0.0017	691.8	9.0	-2859.6	0.0

Table 35 Effect of the soil undrained shear strength on the maximum deflection and the maximum bending moment for a free head pile in stiff clay .

C (kN/m^2)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
55.0	0.0255	2247.6	4.0	- 96.6	16.0
80.0	0.0154	1816.8	3.0	- 77.6	14.0
105.0	0.0115	1622.1	3.0	- 67.9	13.0
130.0	0.0093	1481.6	3.0	- 60.4	12.0
155.0	0.0078	1363.0	3.0	- 56.5	12.0
180.0	0.0069	1284.1	3.0	- 52.2	11.0
205.0	0.0063	1238.6	3.0	- 51.6	11.0

Table 36 Effect of the soil undrained shear strength on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay .

C (kN/m^2)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
15.0	0.0889	1518.9	15.0	-5099.4	0.0
20.0	0.0625	1351.8	13.0	-4529.0	0.0
25.0	0.0480	1241.1	12.0	-4134.9	0.0
30.0	0.0390	1152.2	11.0	-3843.9	0.0
35.0	0.0328	1074.5	11.0	-3611.4	0.0
40.0	0.0283	1019.9	10.0	-3421.0	0.0
45.0	0.0248	955.8	10.0	-3259.1	0.0

Table 37 Effect of the soil undrained shear strength on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay .

C (kN/m^2)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
55.0	0.0064	477.1	6.0	-2031.4	0.0
80.0	0.0048	425.5	6.0	-1821.0	0.0
105.0	0.0039	412.0	5.0	-1713.4	0.0
130.0	0.0034	397.8	5.0	-1633.2	0.0
155.0	0.0030	382.1	5.0	-1572.3	0.0
180.0	0.0027	367.3	4.0	-1524.2	0.0
205.0	0.0025	366.0	4.0	-1484.0	0.0

Table 38 Effect of the soil unit weight on the maximum deflection and the maximum bending moment for a free head pile in stiff clay .

γ' (kN/m³)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
5.0	0.01222	1663.3	3.0	- 68.8	13.0
6.0	0.01219	1661.7	3.0	- 68.7	13.0
7.0	0.01217	1660.2	3.0	- 68.7	13.0
8.0	0.01214	1658.6	3.0	- 68.7	13.0
9.0	0.01211	1657.0	3.0	- 68.6	13.0
10.0	0.01209	1655.5	3.0	- 68.6	13.0
11.0	0.01206	1653.9	3.0	- 68.5	13.0

Table 39 Effect of the soil unit weight on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay .

γ' (kN/m³)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
2.0	0.0541	1319.5	13.0	-4311.0	0.0
3.0	0.0522	1299.1	12.0	-4260.5	0.0
4.0	0.0506	1278.9	12.0	-4215.0	0.0
5.0	0.0493	1259.7	12.0	-4174.1	0.0
6.0	0.0480	1241.1	12.0	-4134.9	0.0
7.0	0.0469	1222.5	12.0	-4099.5	0.0
8.0	0.0459	1204.2	12.0	-4065.8	0.0

Table 40 Effect of the soil unit weight on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay .

γ' (kN/m^3)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
5.0	0.004075	415.5	5.0	-1734.8	0.0
6.0	0.004070	415.1	5.0	-1734.0	0.0
7.0	0.004065	414.7	5.0	-1733.2	0.0
8.0	0.004060	414.3	5.0	-1732.3	0.0
9.0	0.004055	414.0	5.0	-1731.5	0.0
10.0	0.004050	413.6	5.0	-1730.7	0.0
11.0	0.004044	413.2	5.0	-1729.8	0.0

Table 41 Effect of the depth of the soft clay layer on the maximum deflection and the maximum bending moment for a fixed head pile in a layered soil system .

L1 (m)	Y_{max} (m)	M_{max+} (kN m)	Depth to M_{max+} (m)	M_{max-} (kN m)	Depth to M_{max-} (m)
0.0	0.0041	414.3	5.0	-1732.3	0.0
5.0	0.0119	1053.0	6.0	-2881.4	0.0
10.0	0.0330	1746.4	10.0	-3904.9	0.0
15.0	0.0465	1321.1	12.0	-4119.5	0.0
20.0	0.0476	1268.4	12.0	-4114.8	0.0
25.0	0.0478	1252.9	12.0	-4130.5	0.0
30.0	0.0480	1242.3	12.0	-4134.8	0.0
40.0	0.0480	1241.2	12.0	-4134.9	0.0
50.0	0.0480	1241.1	12.0	-4134.9	0.0
60.0	0.0480	1241.1	12.0	-4134.9	0.0

APPENDIX B

FIGURES

SHOWING THE RESULTS OF THE PARAMETRIC ANALYSIS

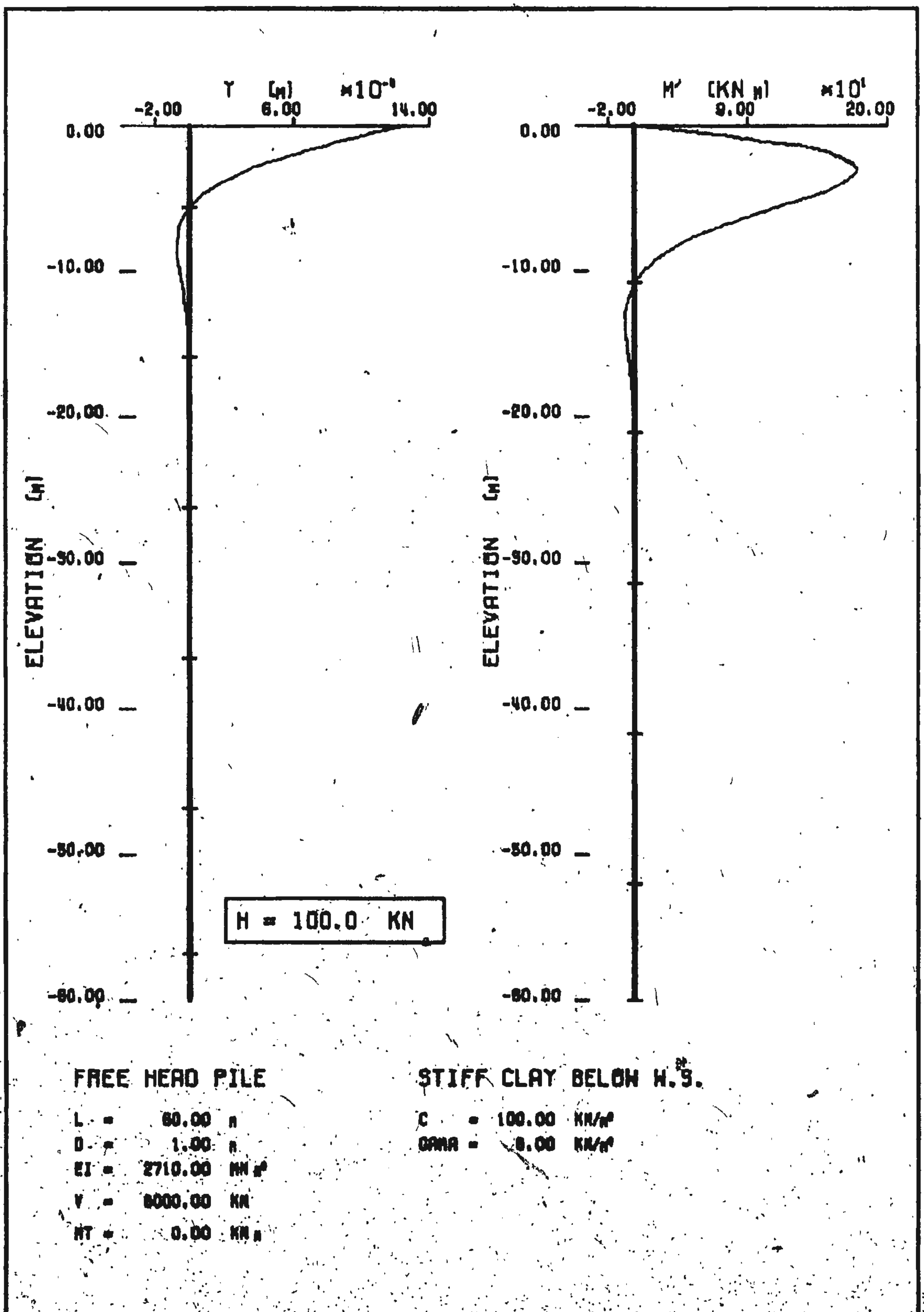


Figure 60 Deflection and bending moment for a free head pile in stiff clay, H = 100.0 kN.

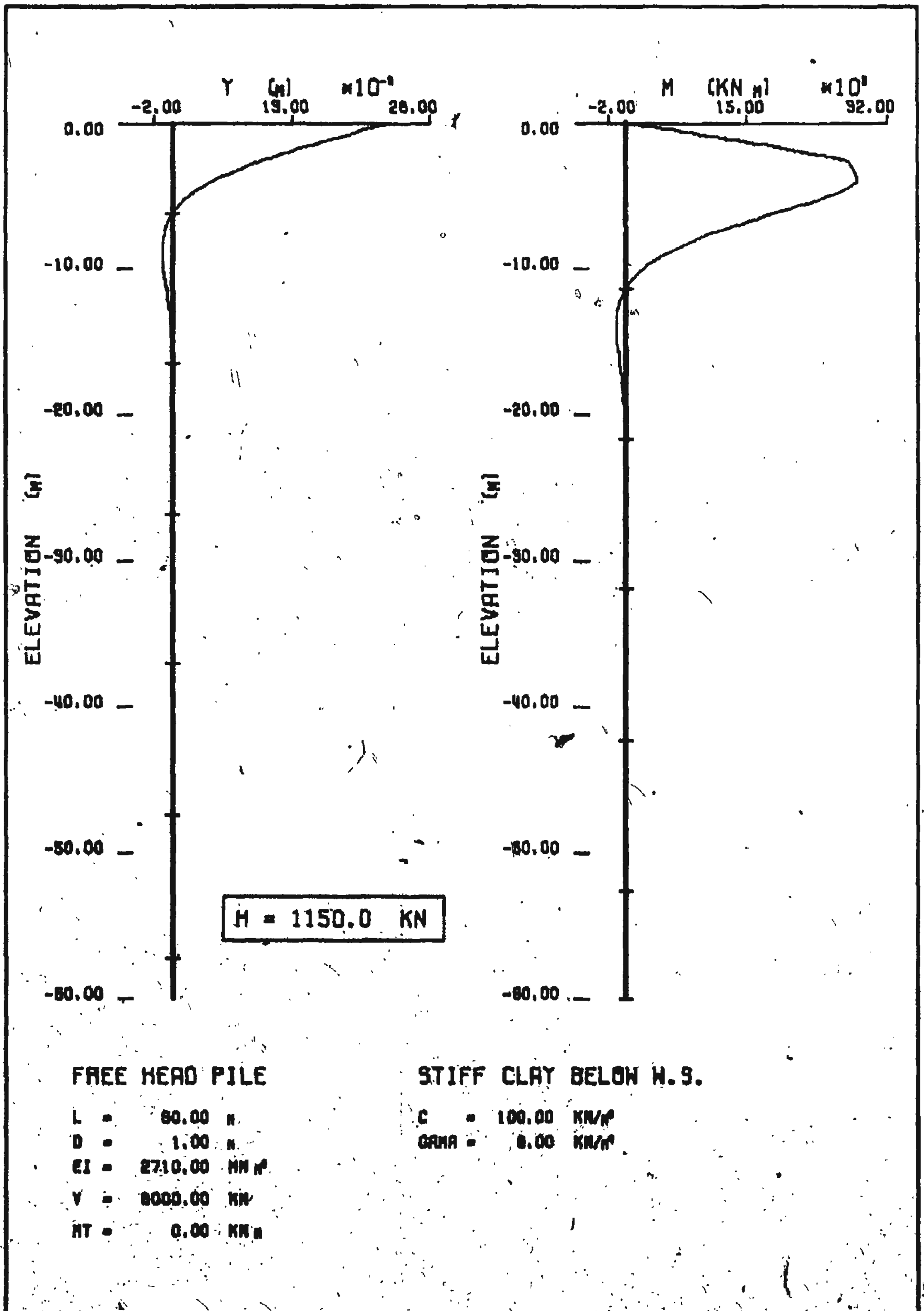


Figure 61 Deflection and bending moment for a free head pile in stiff clay, H = 1150.0 kN.

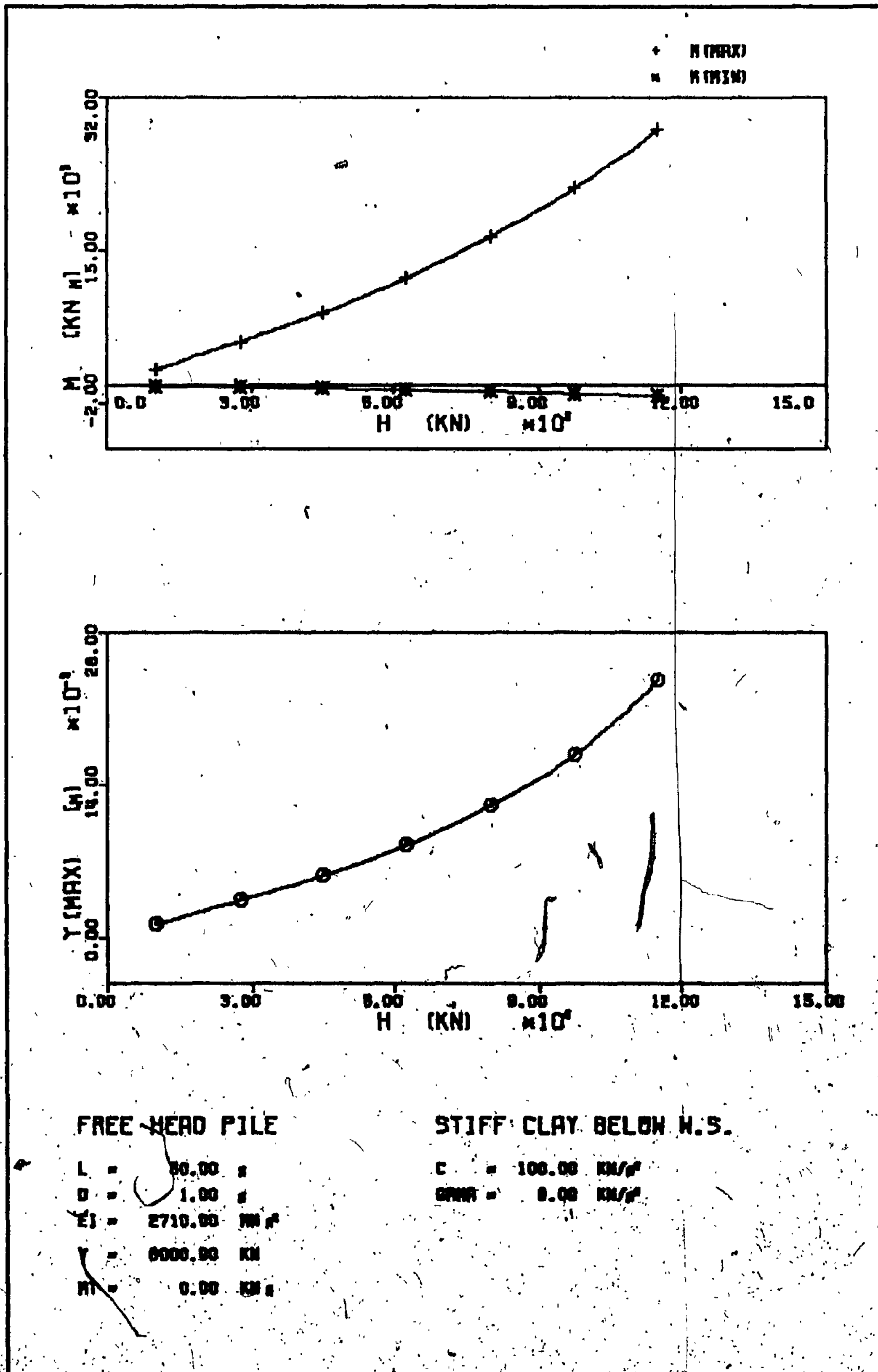


Figure 62 Effect of the horizontal load on the maximum deflection and the maximum bending moment for a free head pile in stiff clay.

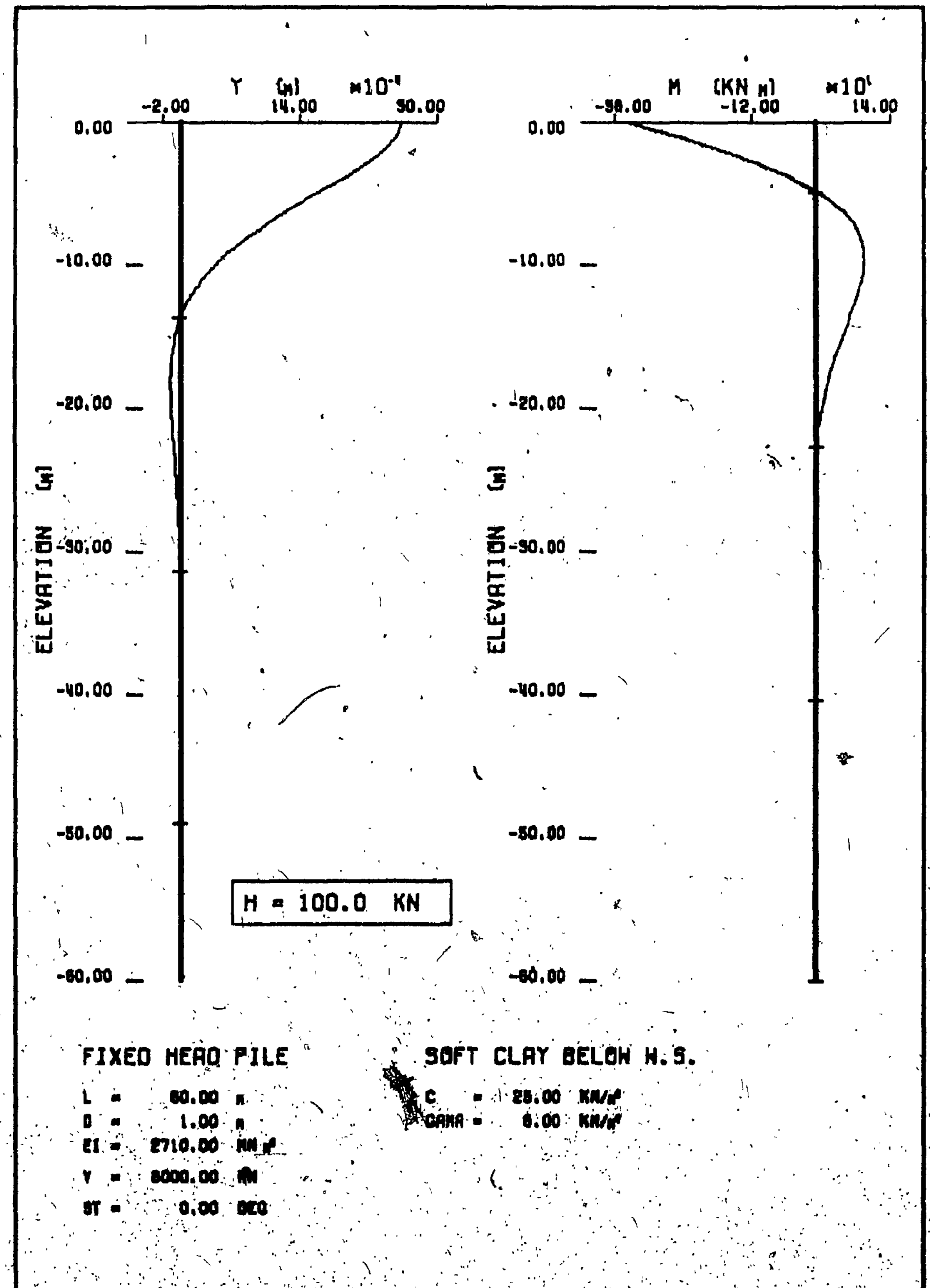


Figure 63 Deflection and bending moment for a fixed head pile in soft clay, H = 100.0 kN.

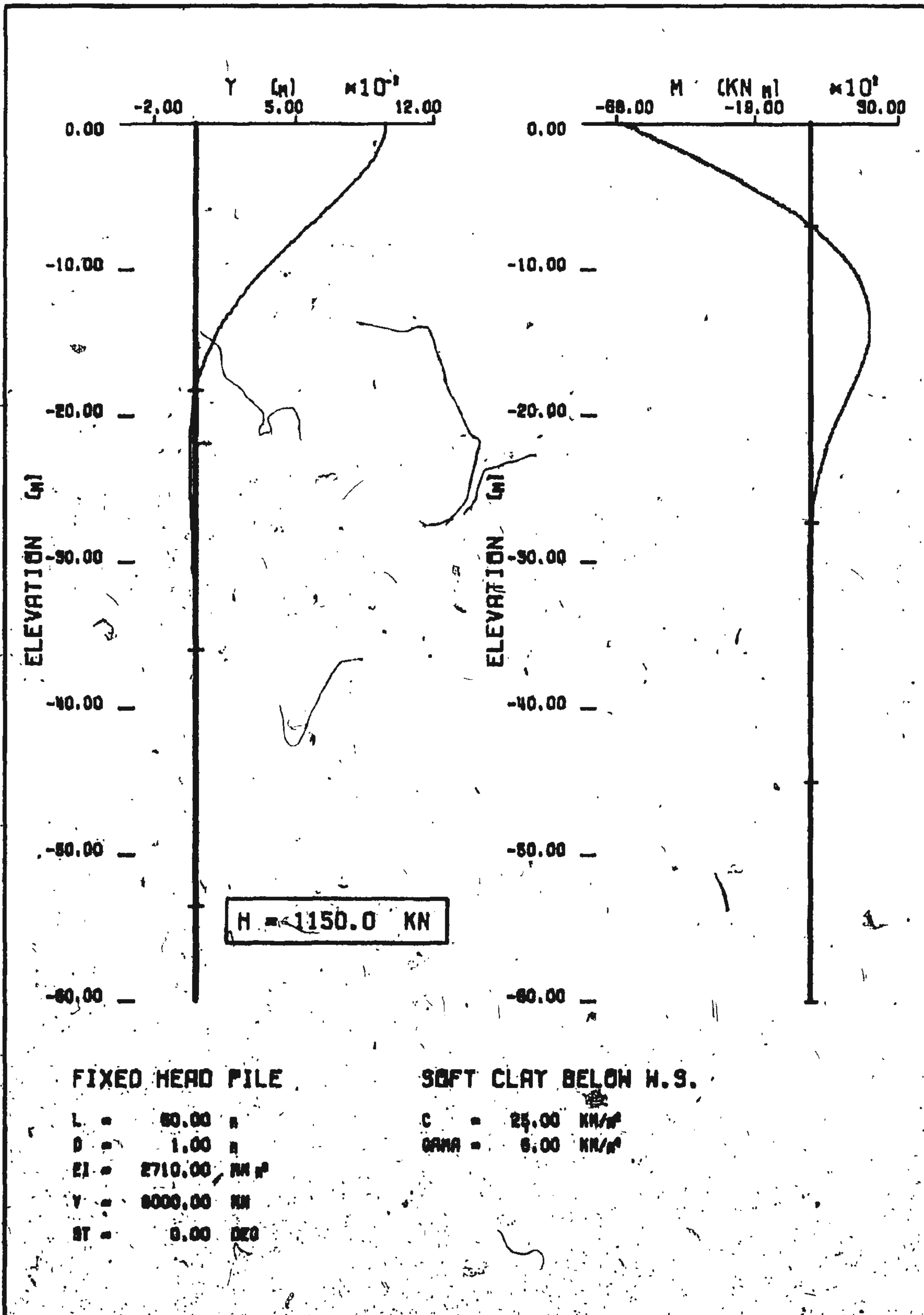


Figure 64 Deflection and bending moment for a fixed head pile in soft clay, H = 1150.0 kN.

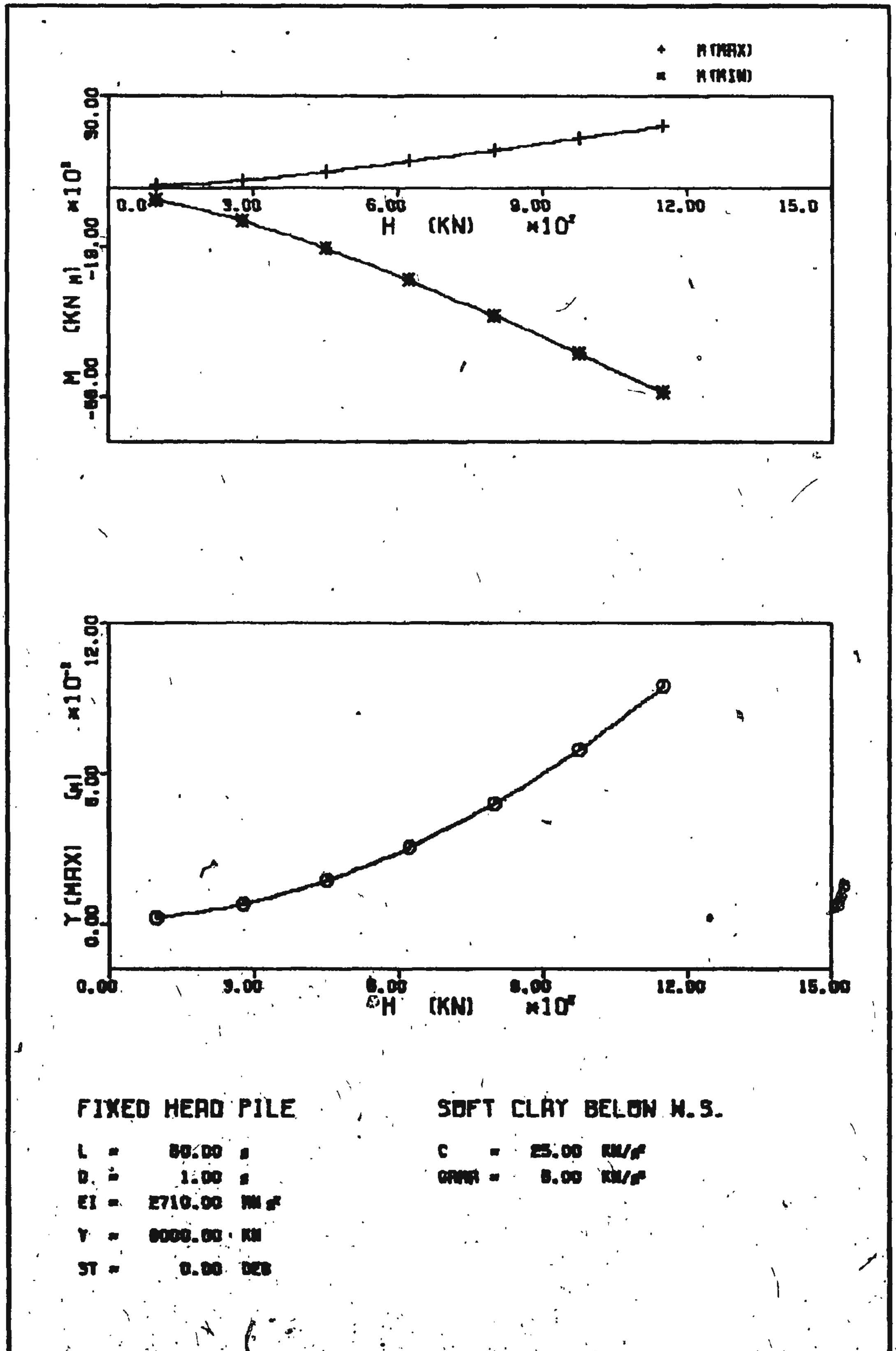


Figure 65 Effect of the horizontal load on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay.

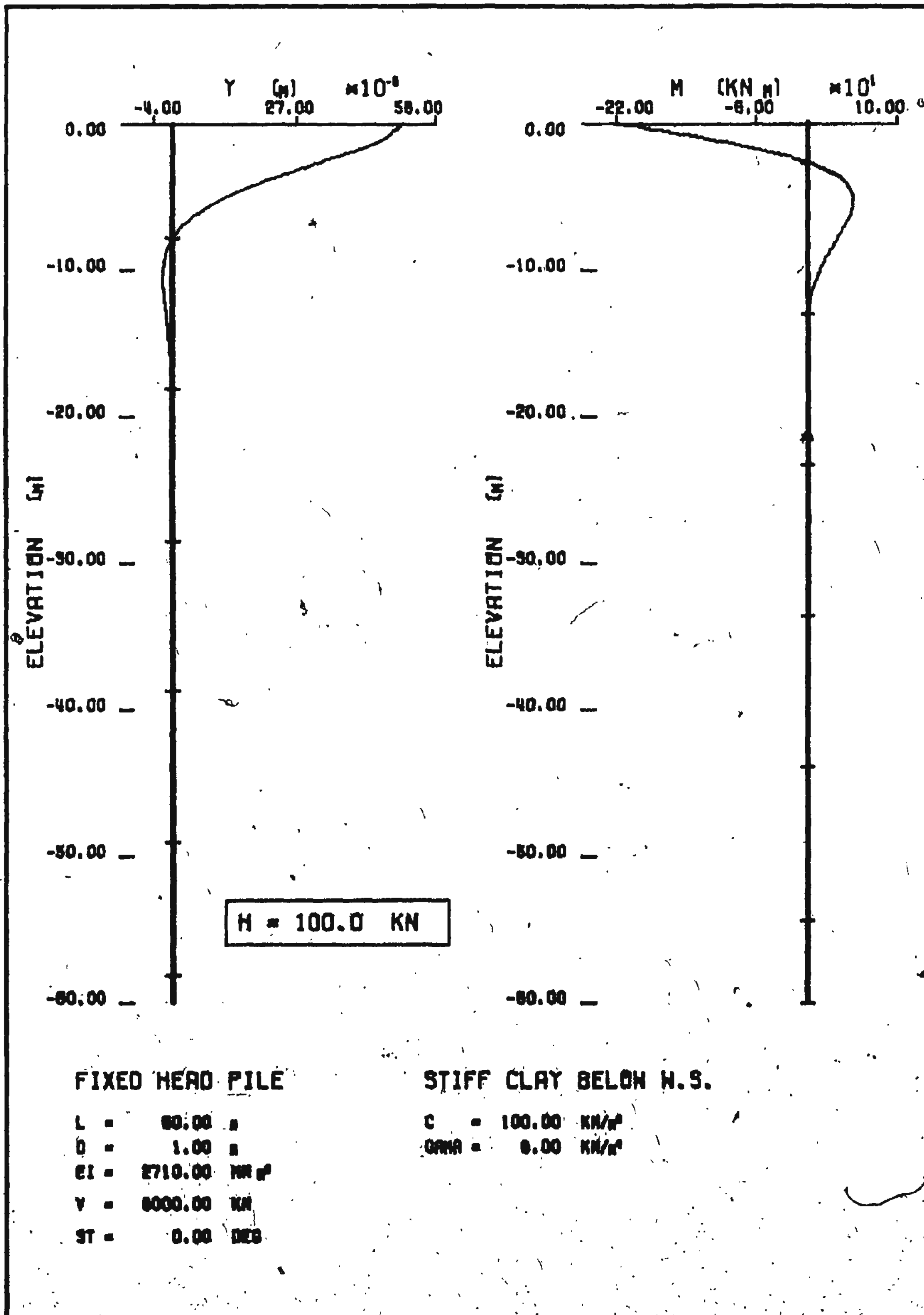


Figure 66 Deflection and bending moment for a fixed head pile in stiff clay, H = 100.0 kN.

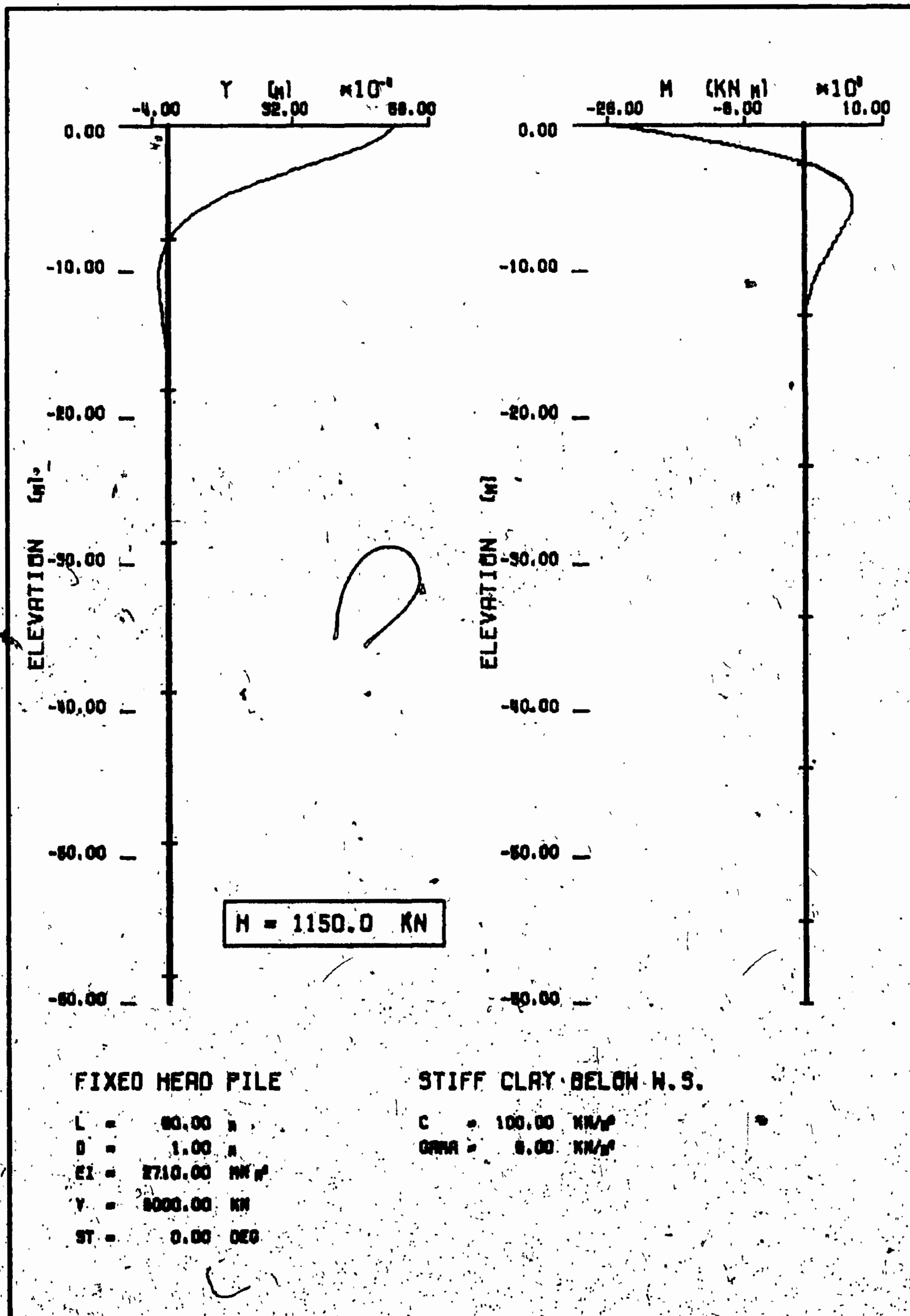


Figure 67 Deflection and bending moment for a fixed head pile in stiff clay, H = 1150.0 kN.

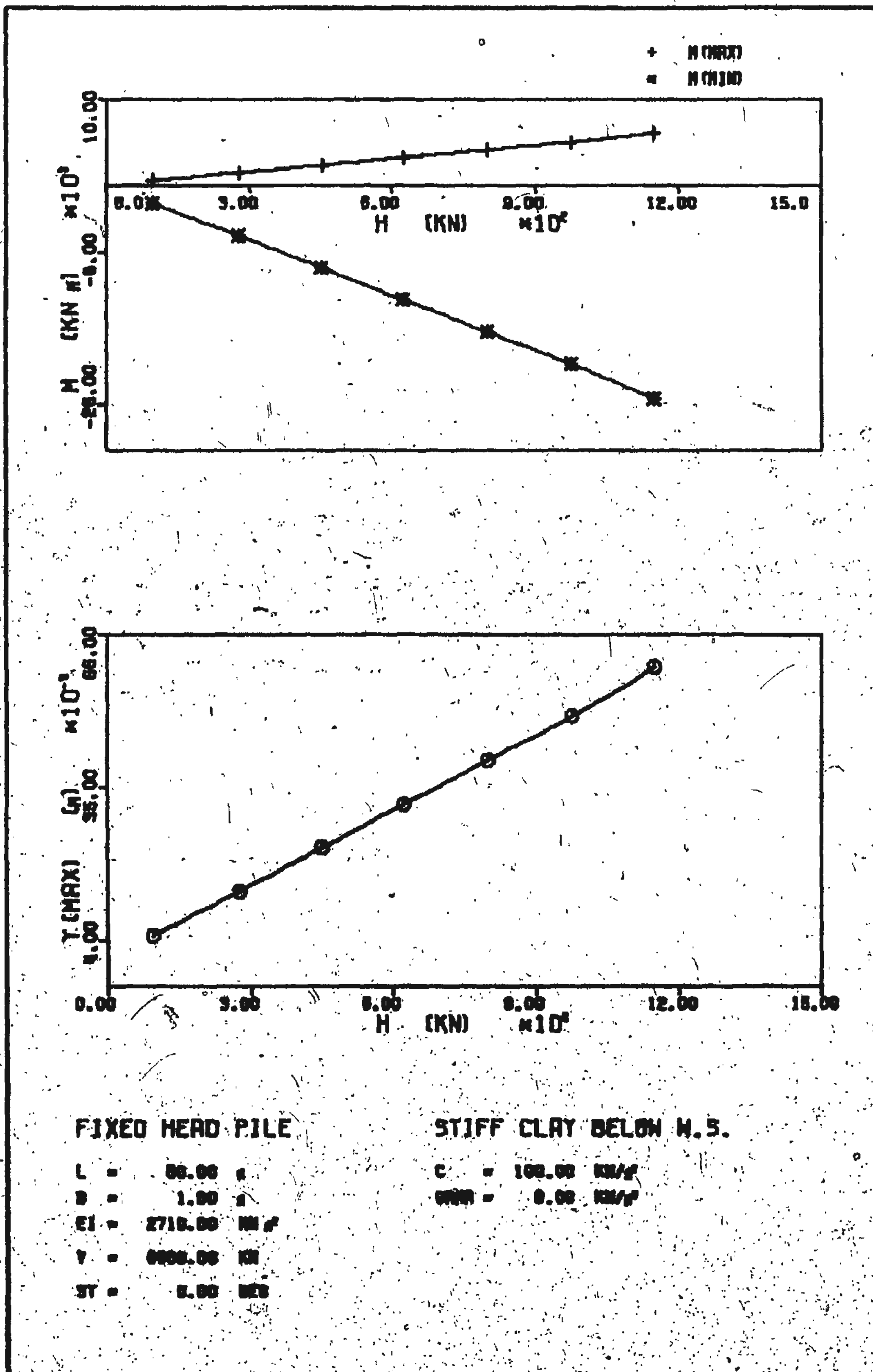


Figure 68 Effect of the horizontal load on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay.

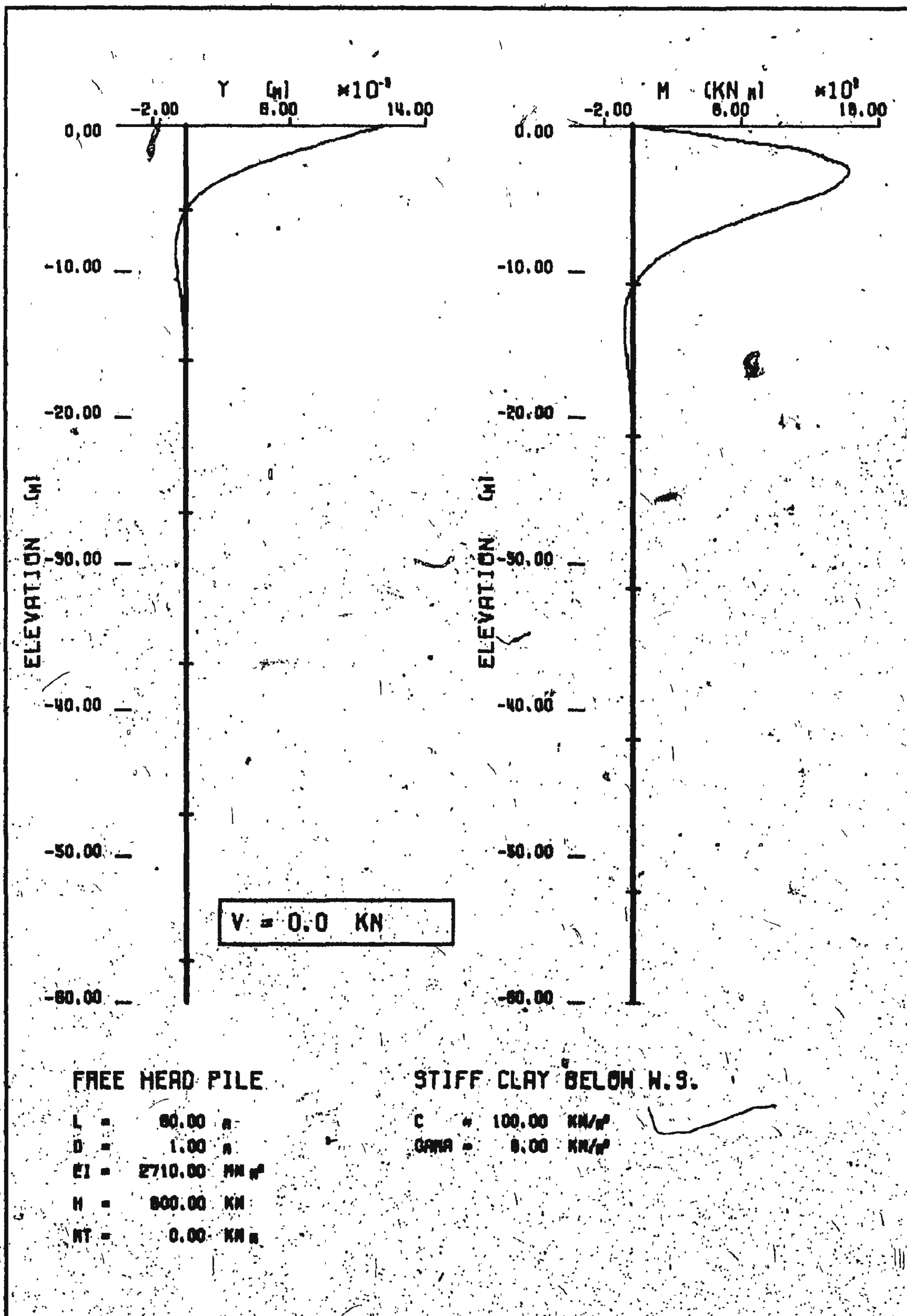


Figure 69 Deflection and bending moment for a free head pile in stiff clay, V = 0.0 kN.

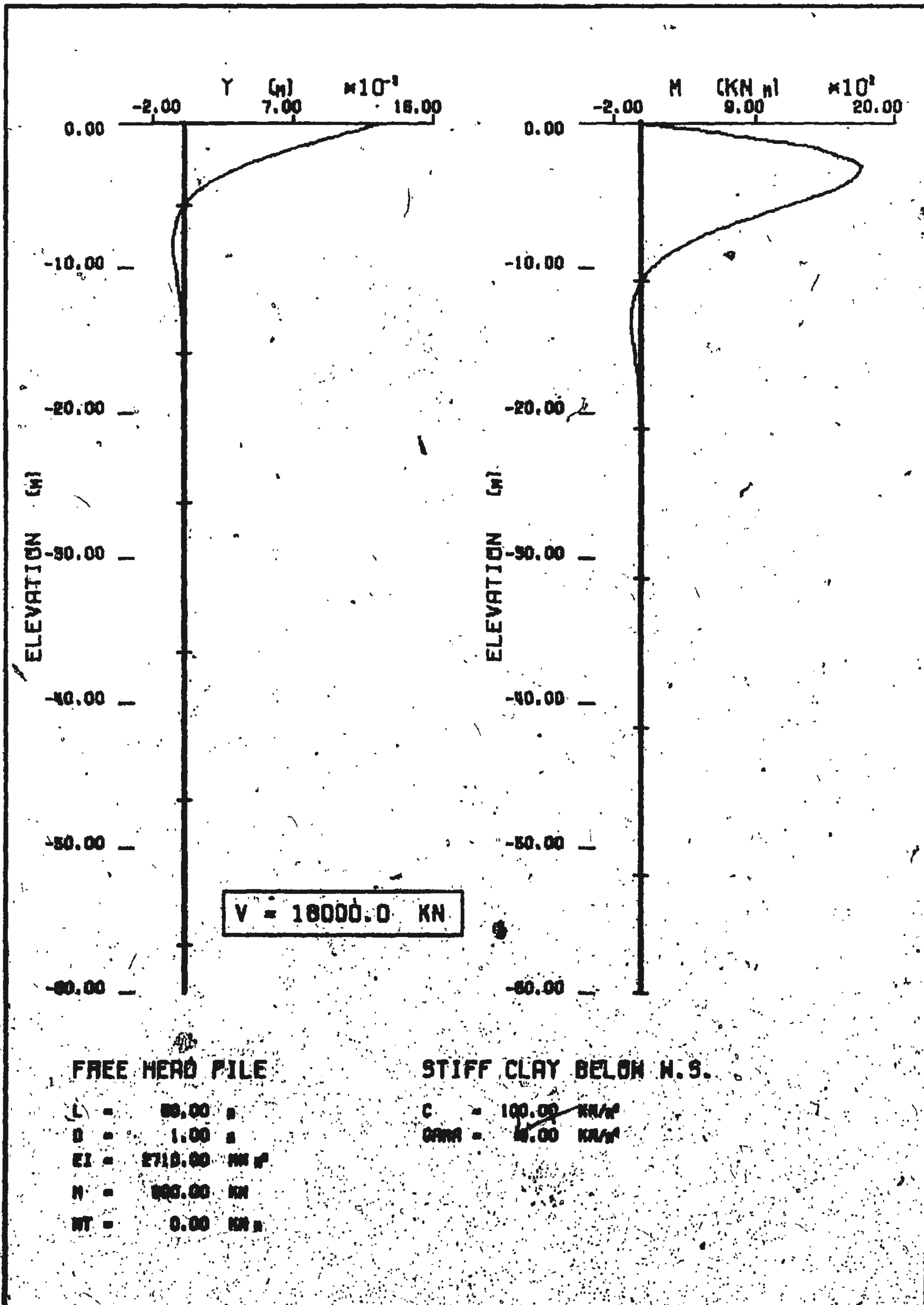


Figure 70 Deflection and bending moment for a free head pile in stiff clay, V = 18000.0 kN.

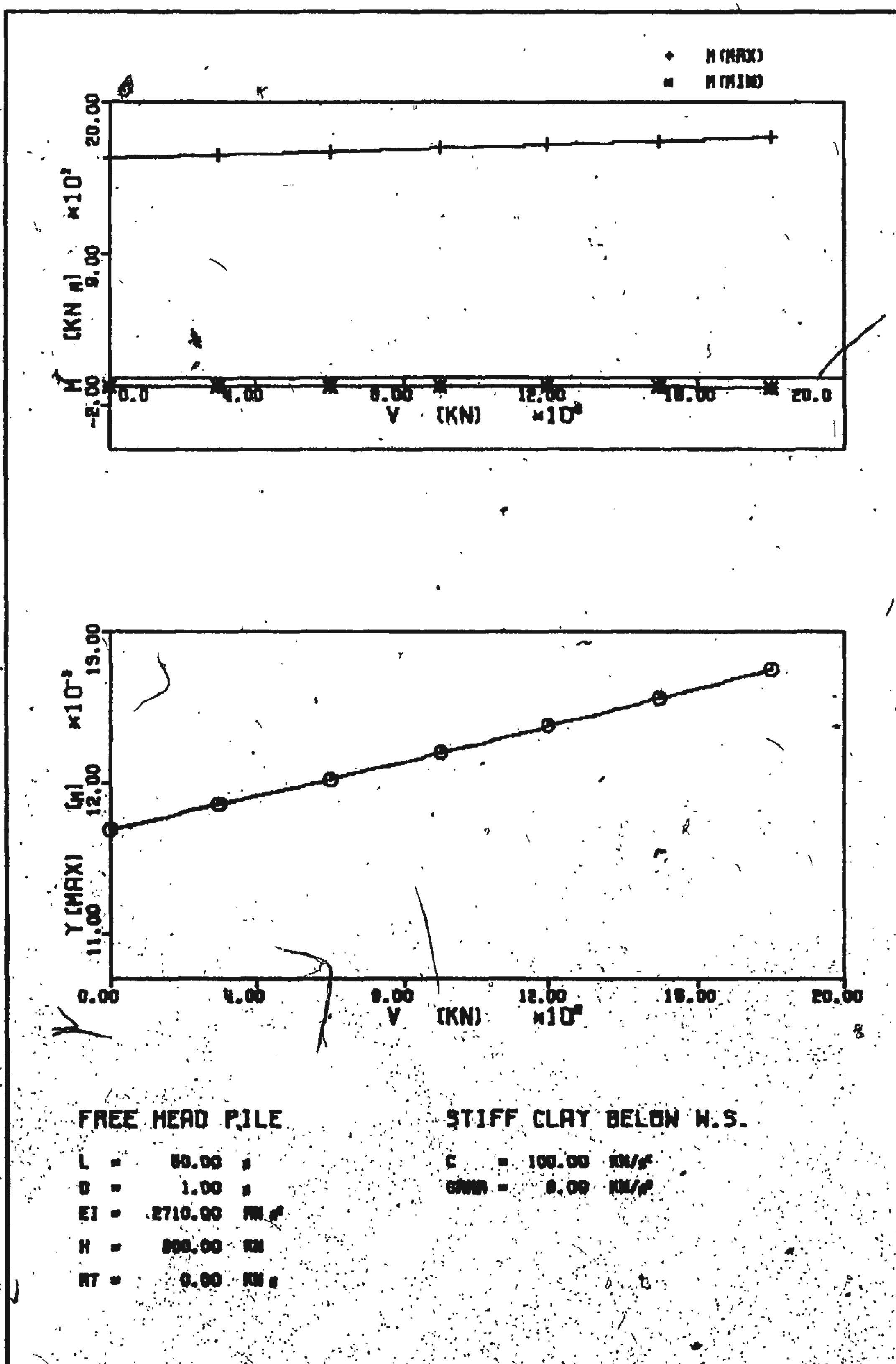


Figure 71 Effect of the vertical load on the maximum deflection and the maximum bending moment for a free head pile in stiff clay.

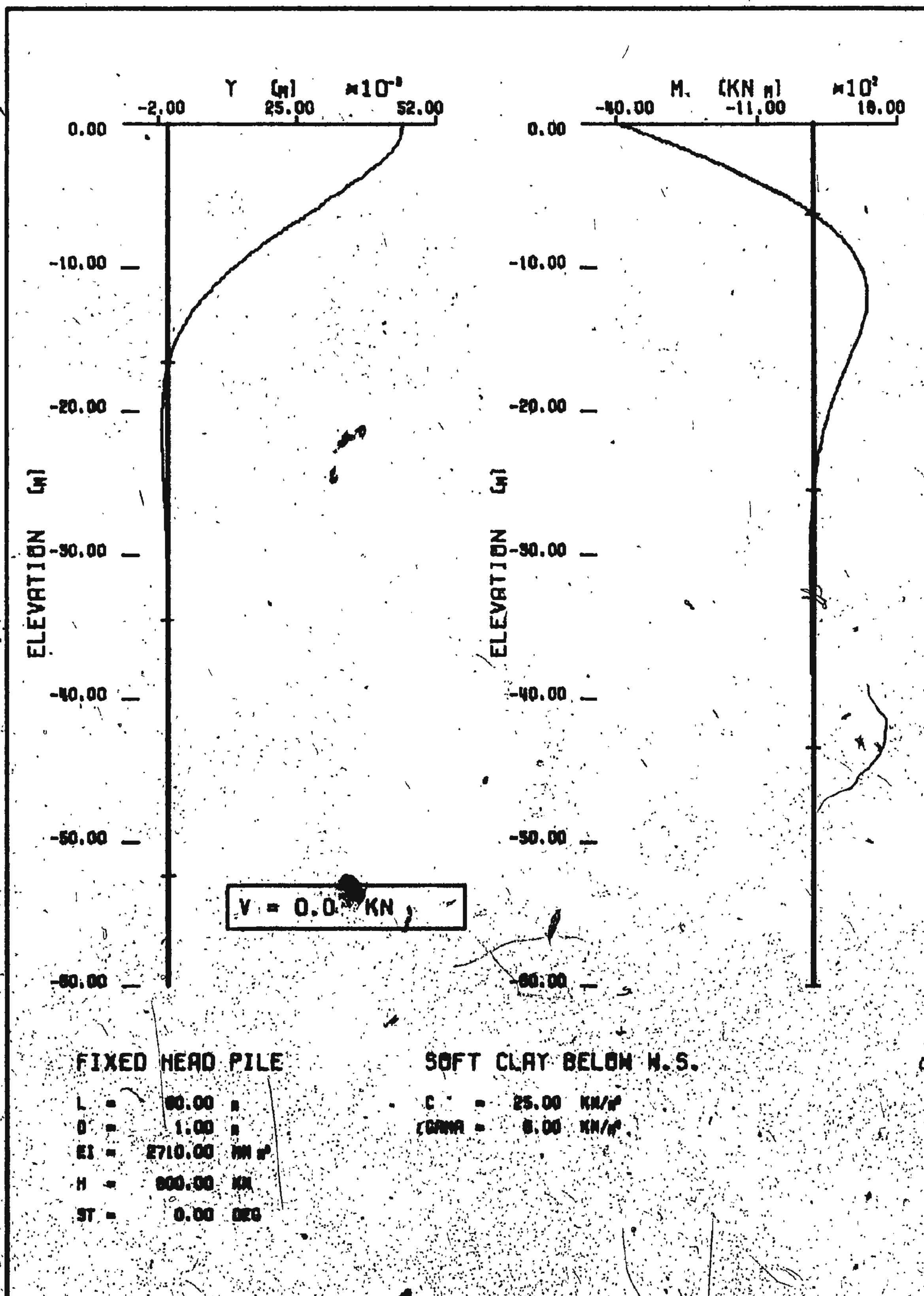


Figure 72. Deflection and bending moment for a fixed head pile in soft clay, V = 0.0 kN.

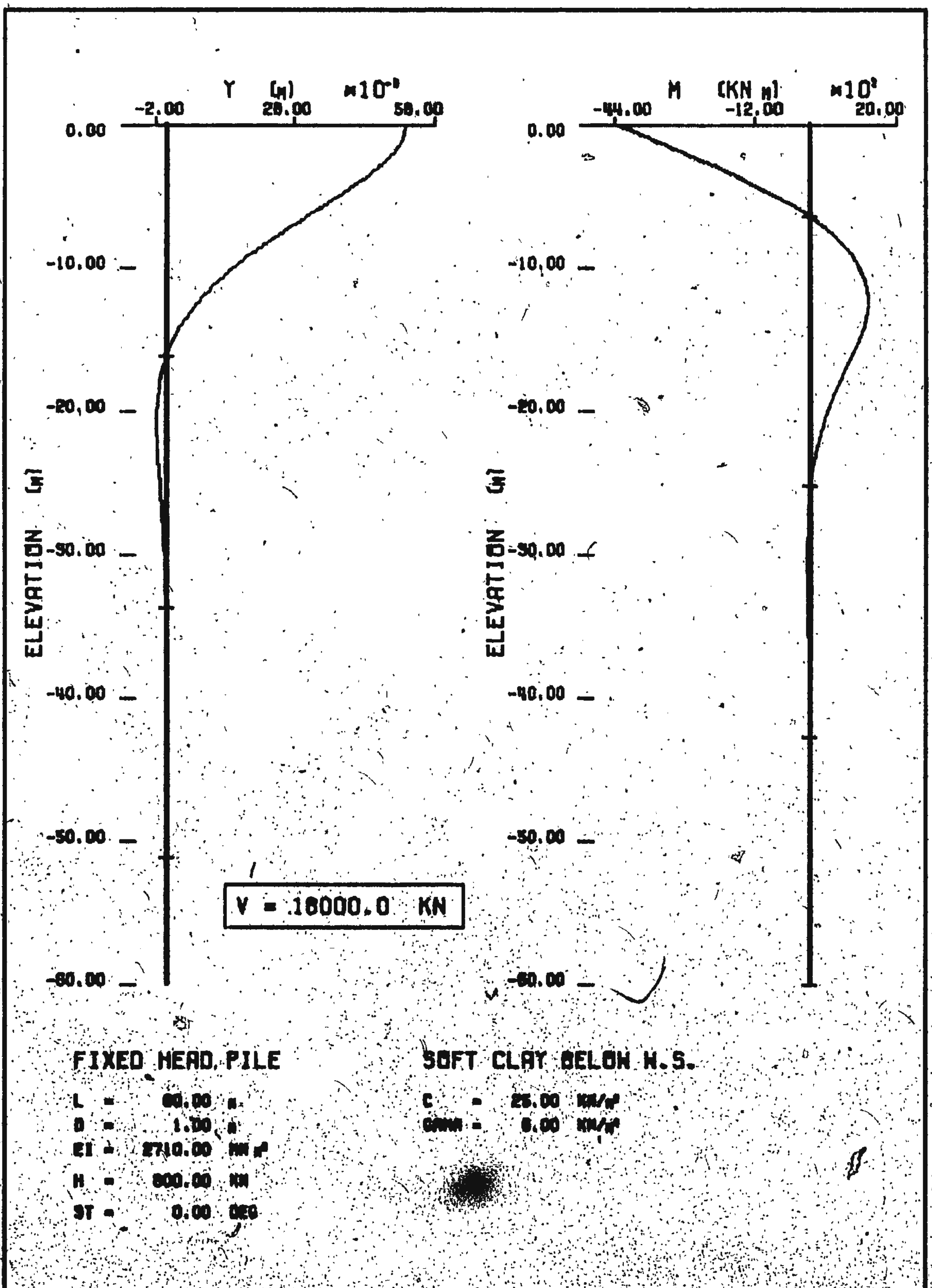


Figure 73. Deflection and bending moment for a fixed head pile in soft clay, V = 18000.0 kN.

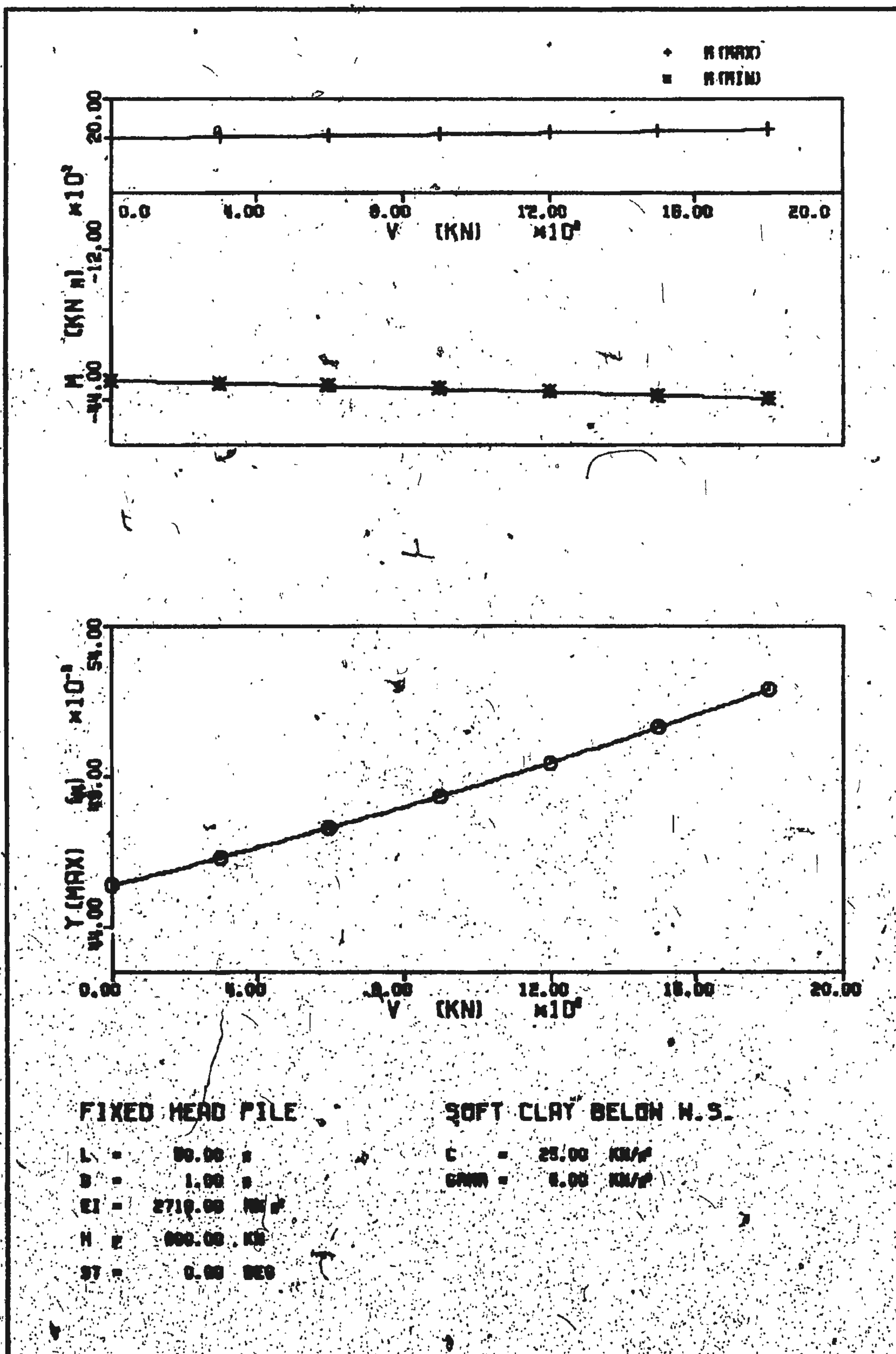


Figure 74 Effect of the vertical load on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay.

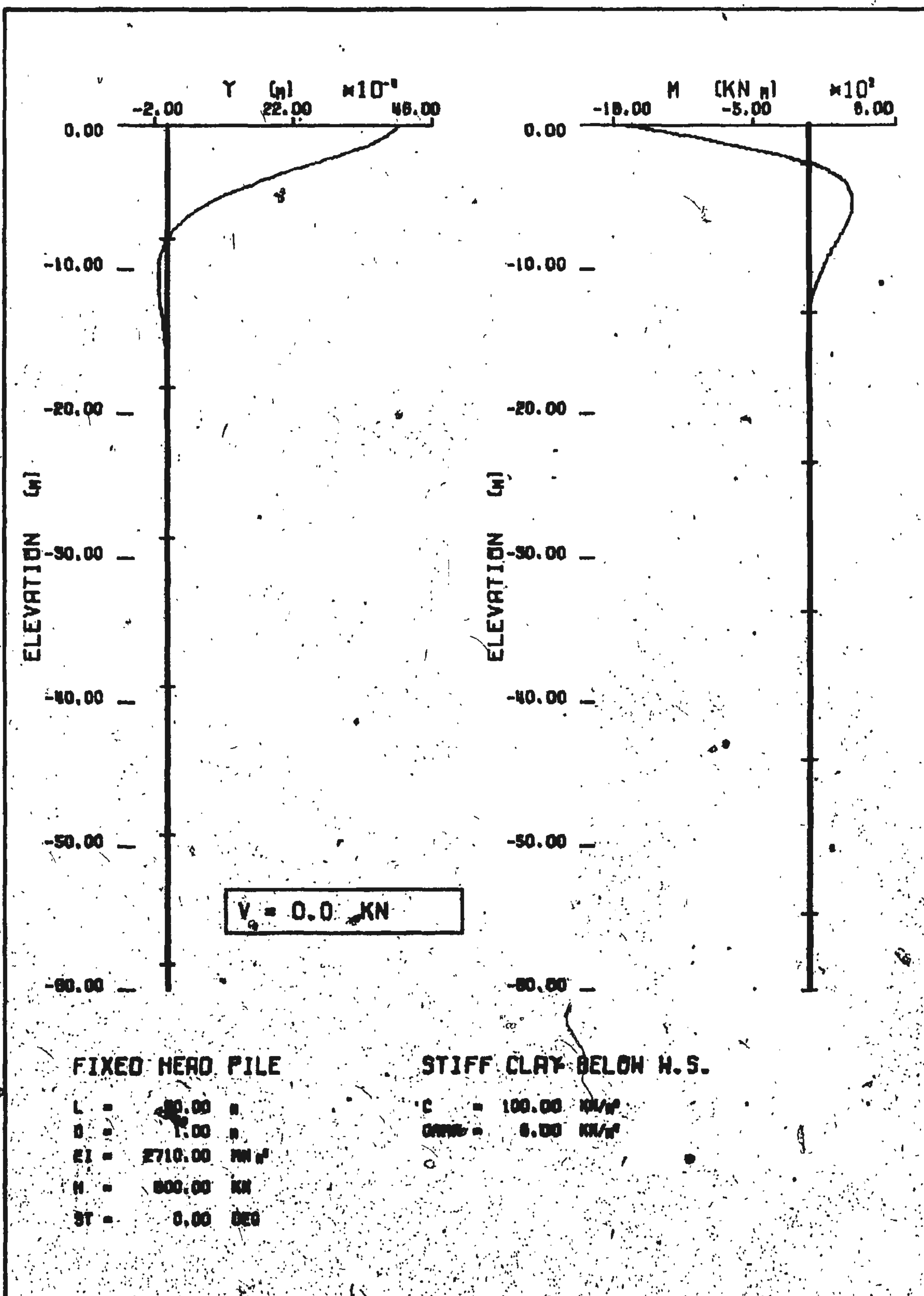


Figure 75 Deflection and bending moment for a fixed head pile in stiff clay, V = 0.0 kN.

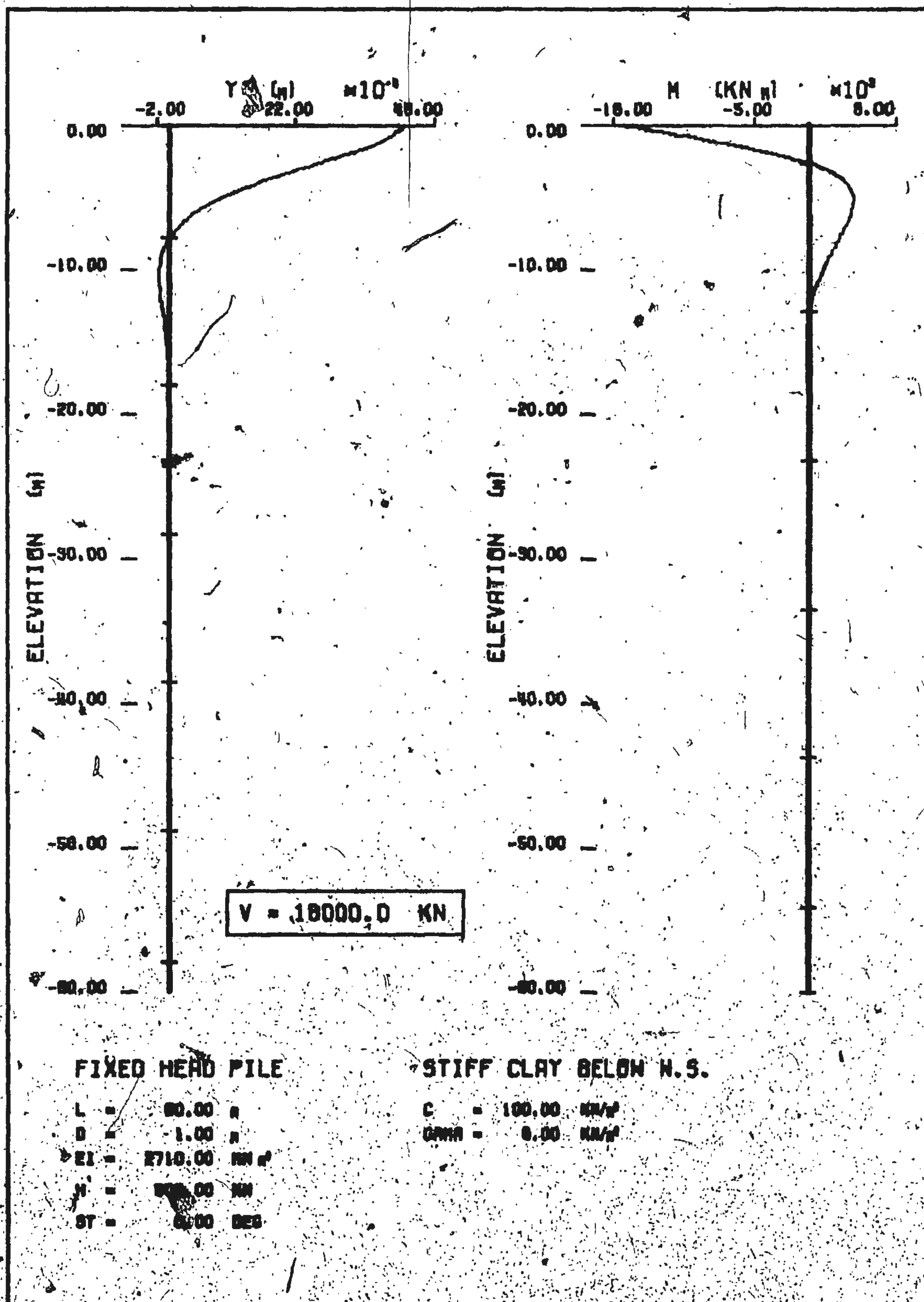


Figure 76 Deflection and bending moment for a fixed head pile in stiff clay, V = 18000.0 kN.

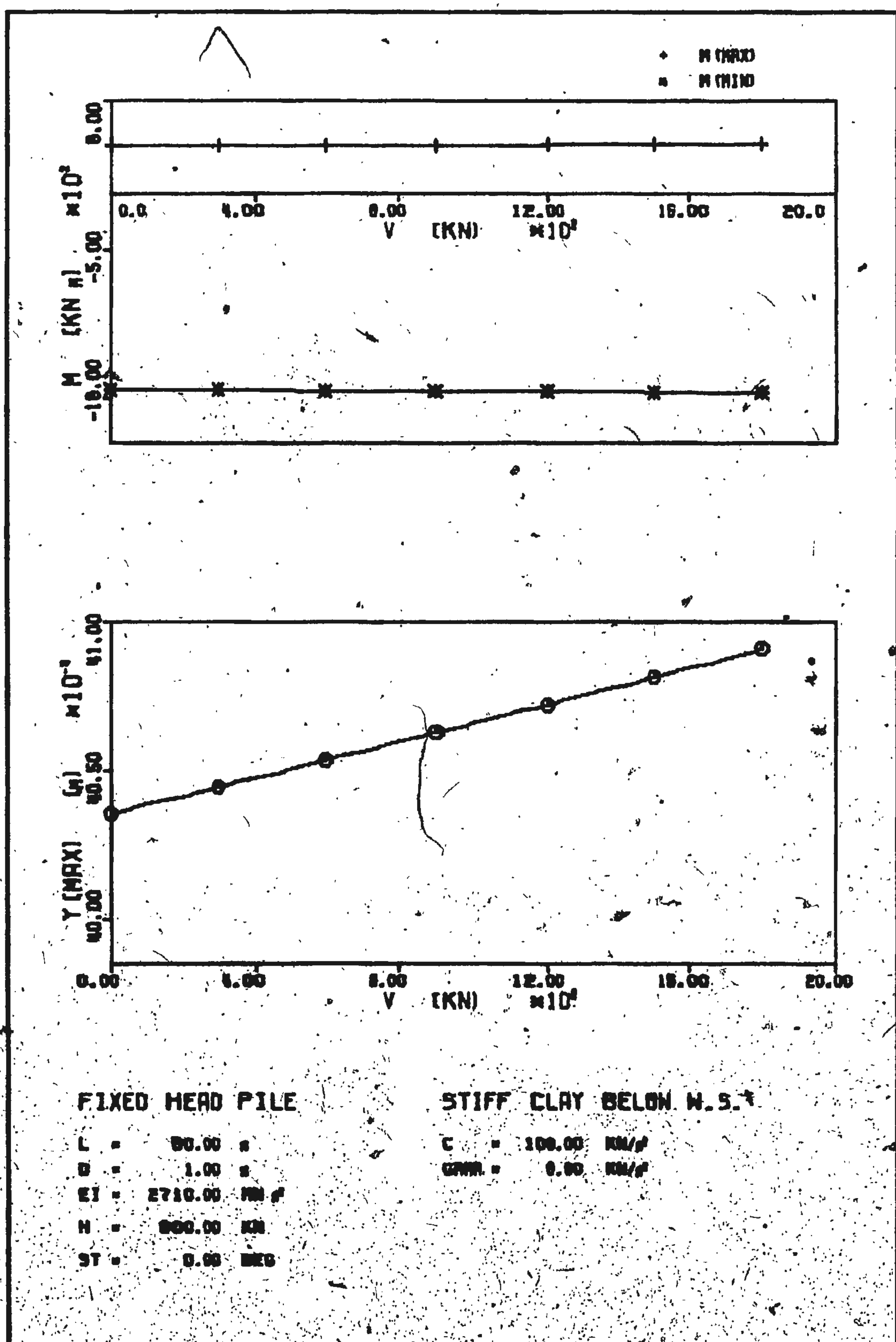


Figure 77 Effect of the vertical load on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay.

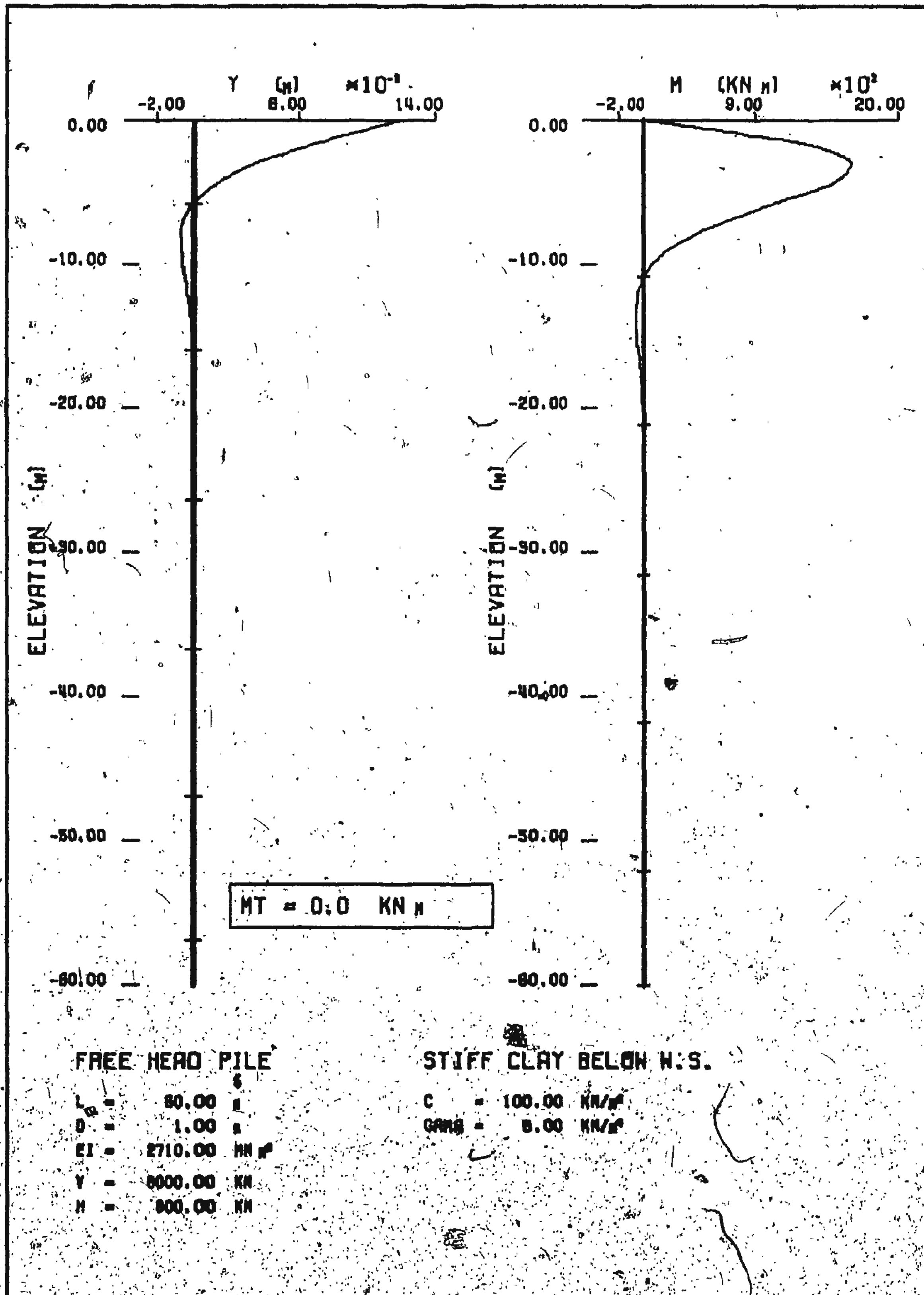


Figure 78 Deflection and bending moment for a free head pile in stiff clay, MT = 0.0 kN m.

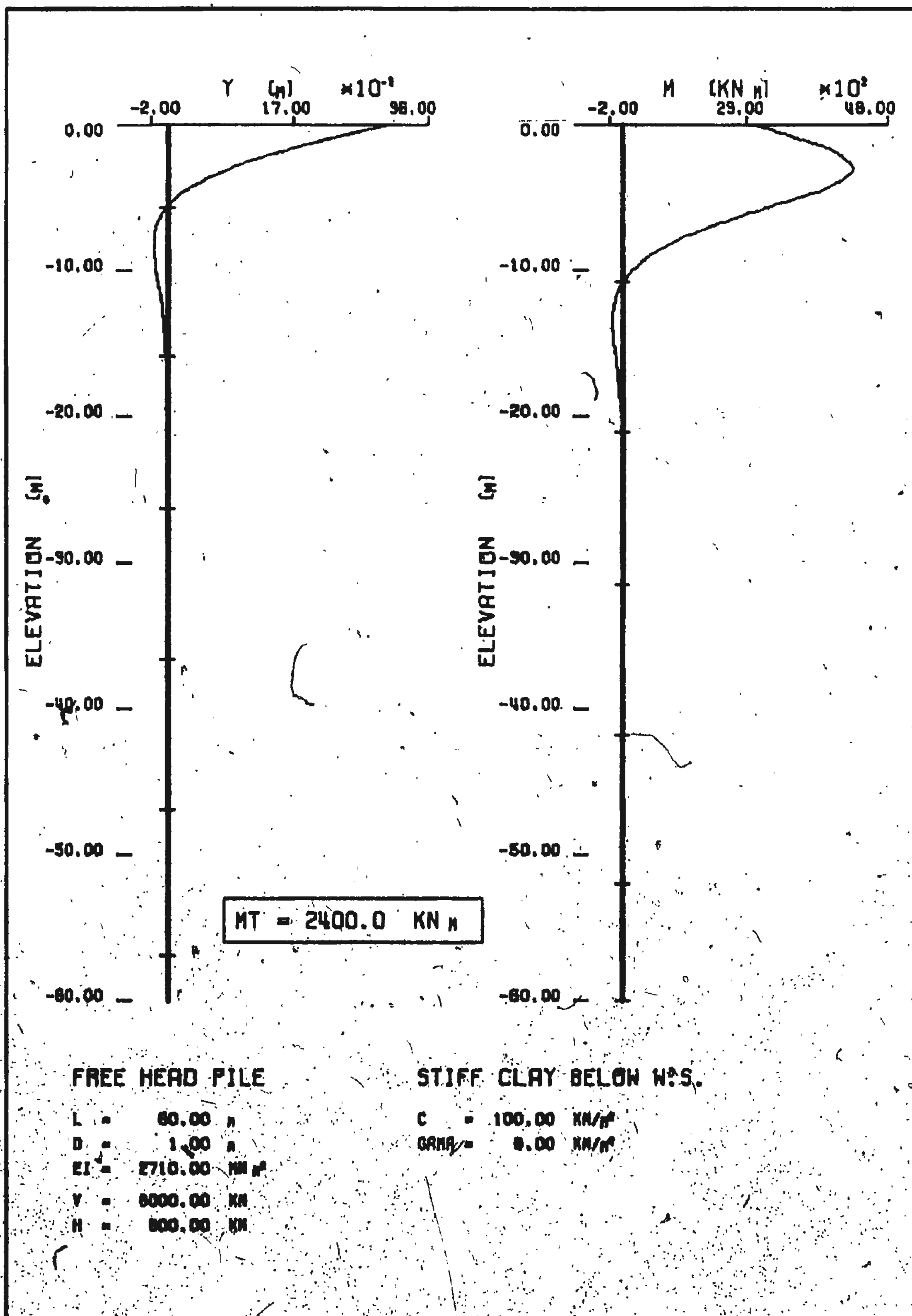
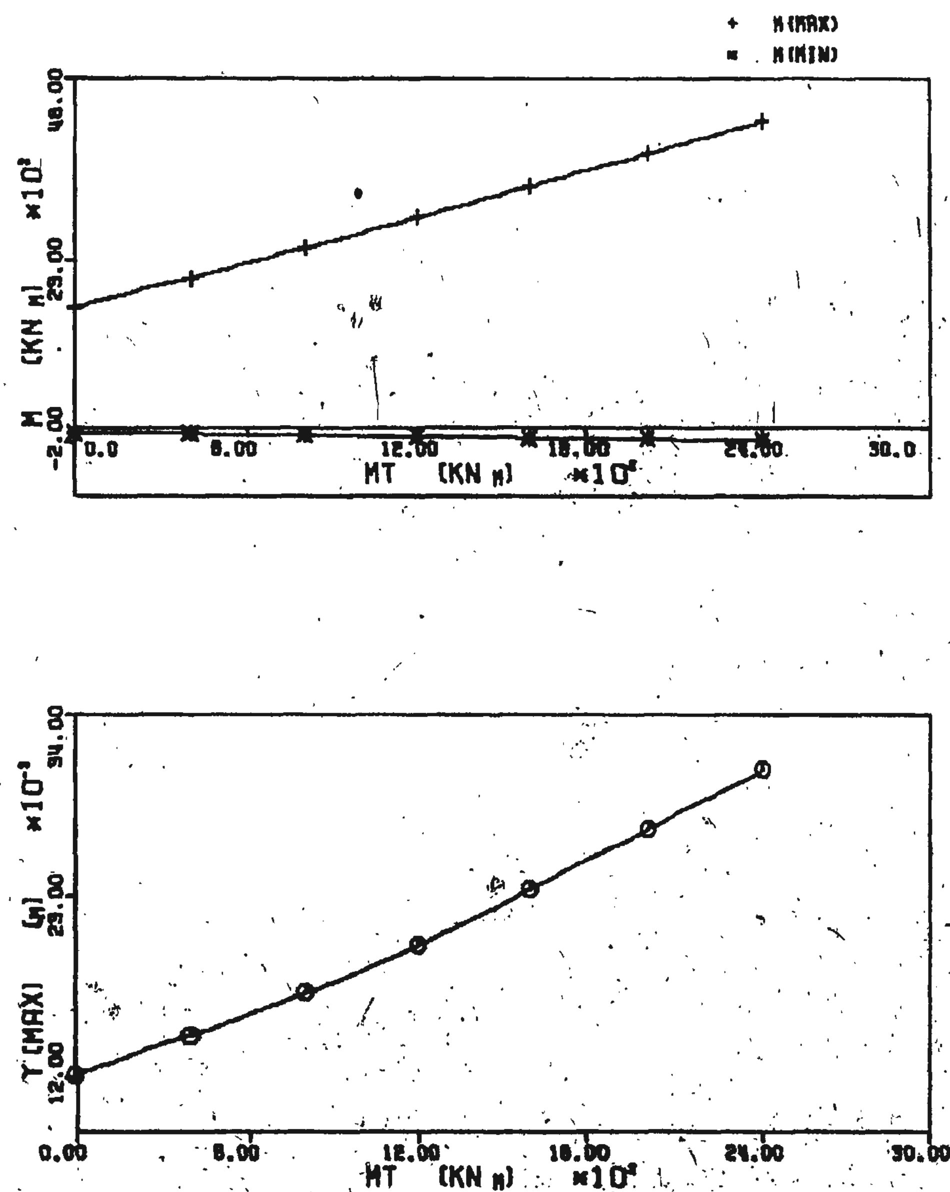


Figure 79. Deflection and bending moment for a free head pile in stiff clay, MT = 2400.0 kN m.



FREE HEAD PILE

L = 80.00 M
D = 1.00 M
EI = 2710.00 MN M²
V = 8000.00 KN
H = 800.00 KN

STIFF CLAY BELOW W.S.

C = 100.00 KN/M²
GAMA = 8.00 KN/M³

Figure 80 Effect of the external moment on the maximum deflection and the maximum bending moment for a free head pile in stiff clay.

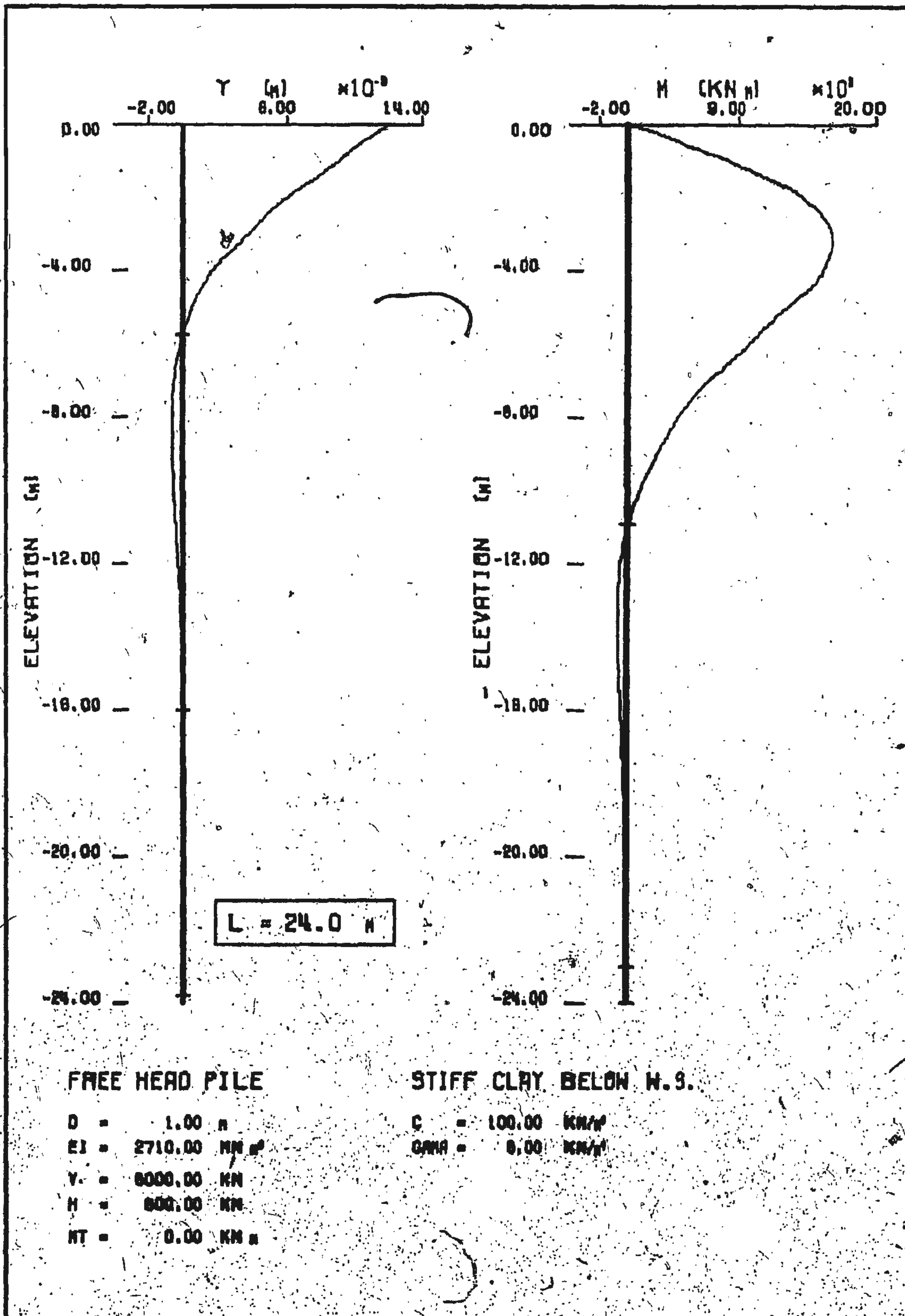


Figure 81 Deflection and bending moment for a free head pile in stiff clay, L = 24.0 m.

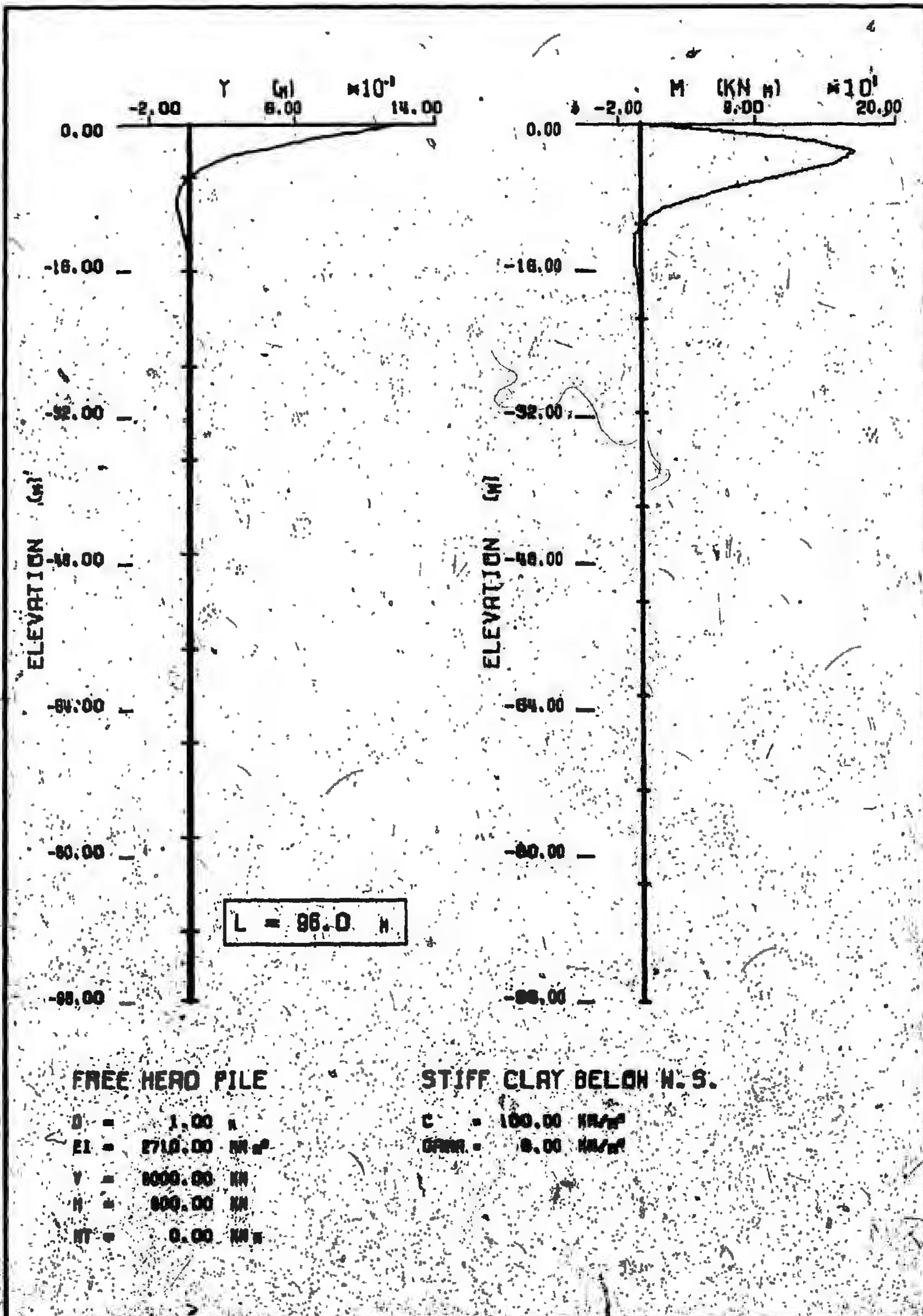


Figure 82 Deflection and bending moment for a free head pile in stiff clay, L = 96.0 m.

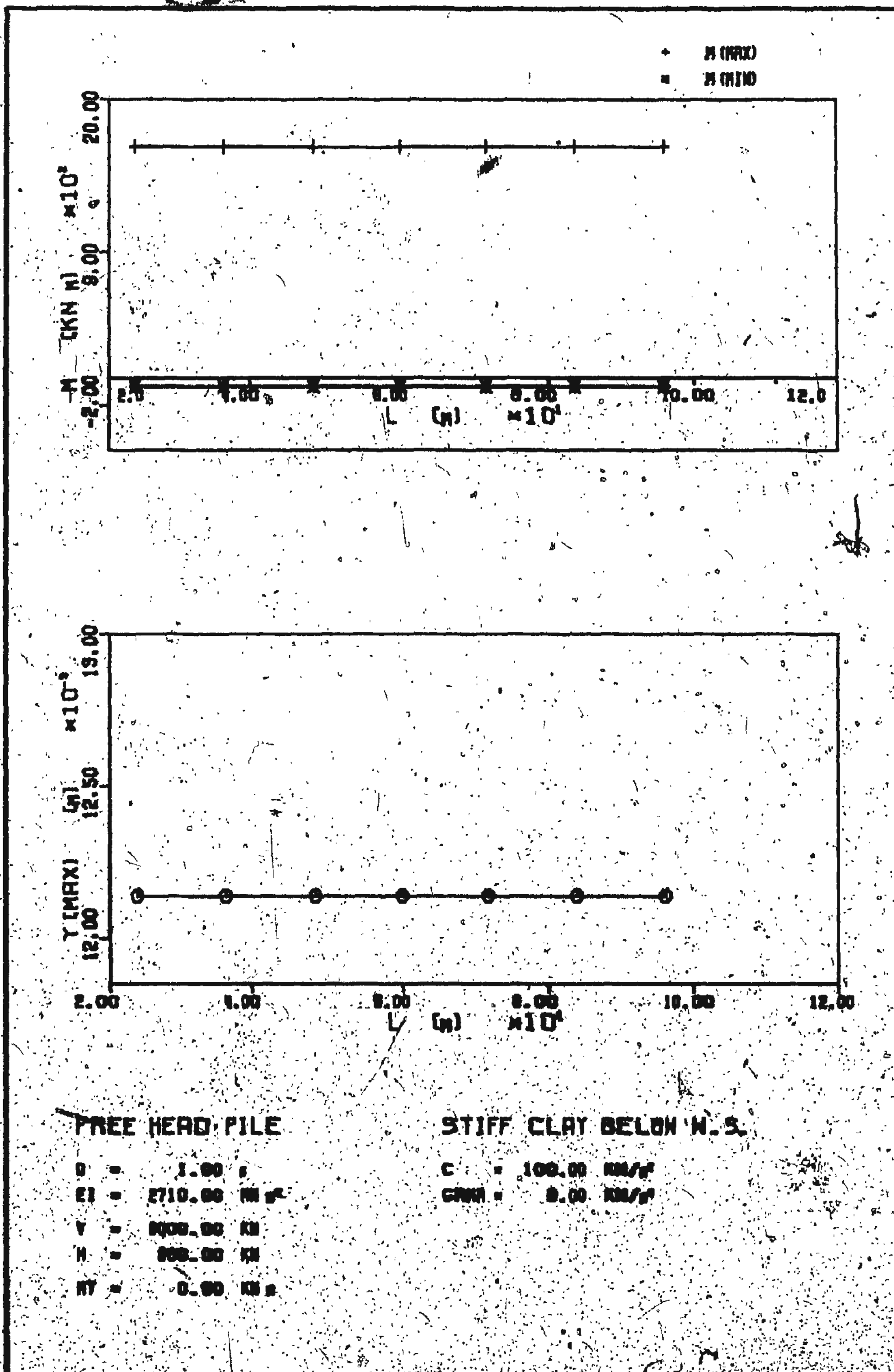


Figure 83 Effect of the pile length on the maximum deflection and the maximum bending moment for a free head pile in stiff clay.

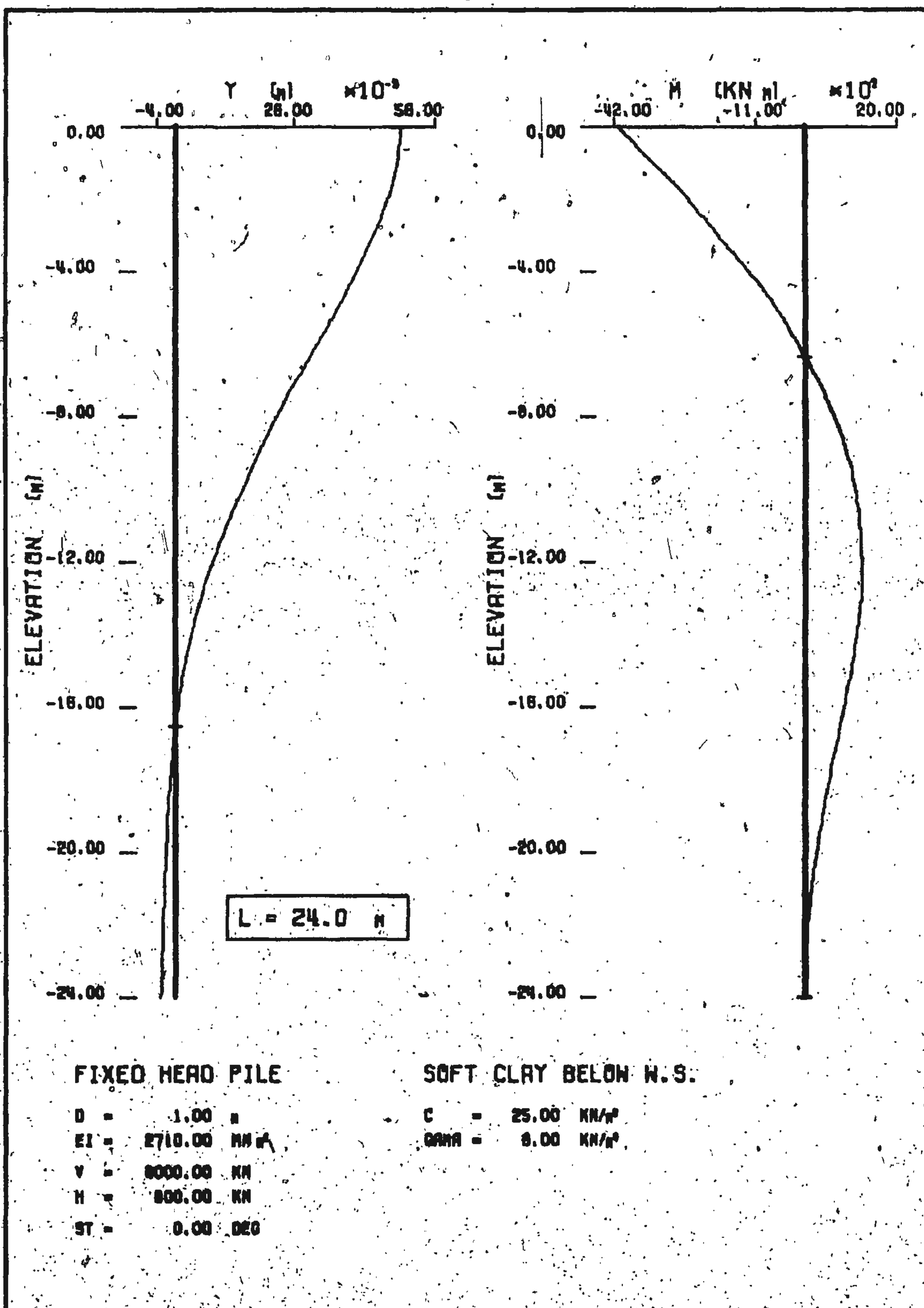


Figure 84 Deflection and bending moment for a fixed head pile in soft clay, L = 24.0 m.

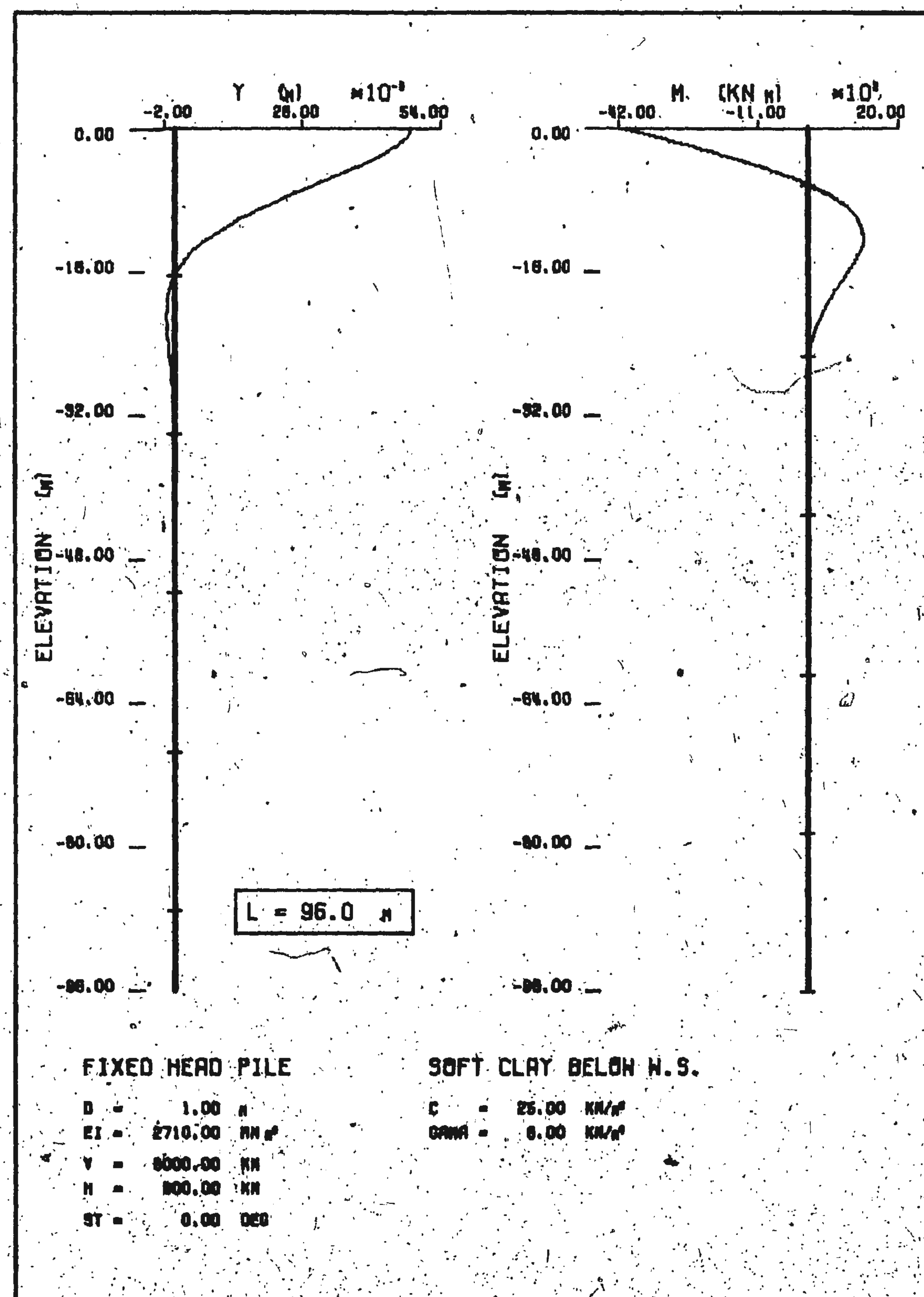


Figure 85 Deflection and bending moment for a fixed head pile in soft clay, L = 96.0 m.

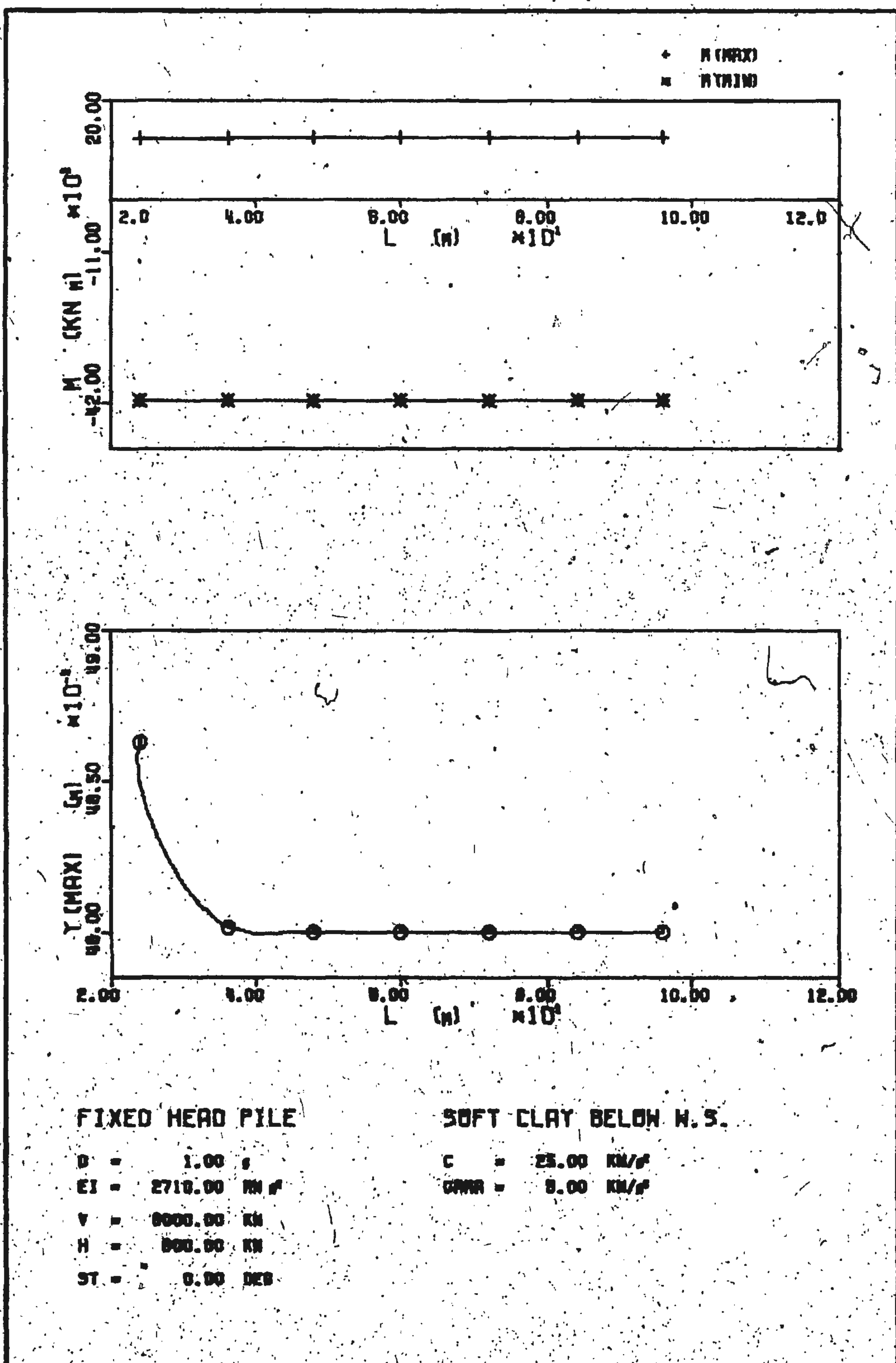


Figure 86 Effect of the pile length on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay.

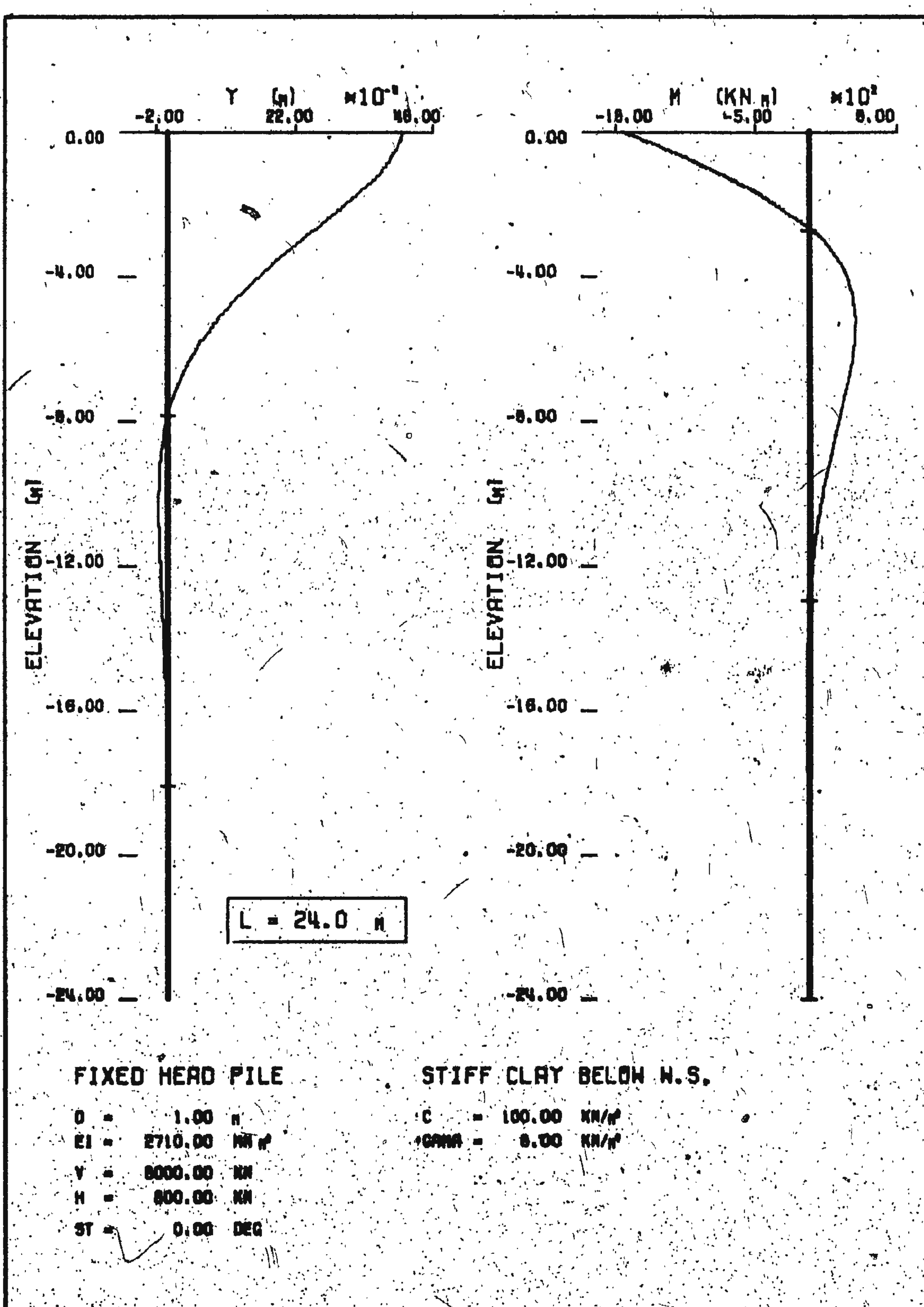


Figure 87 Deflection and bending moment for a fixed head pile in stiff clay, L = 24.0 m.

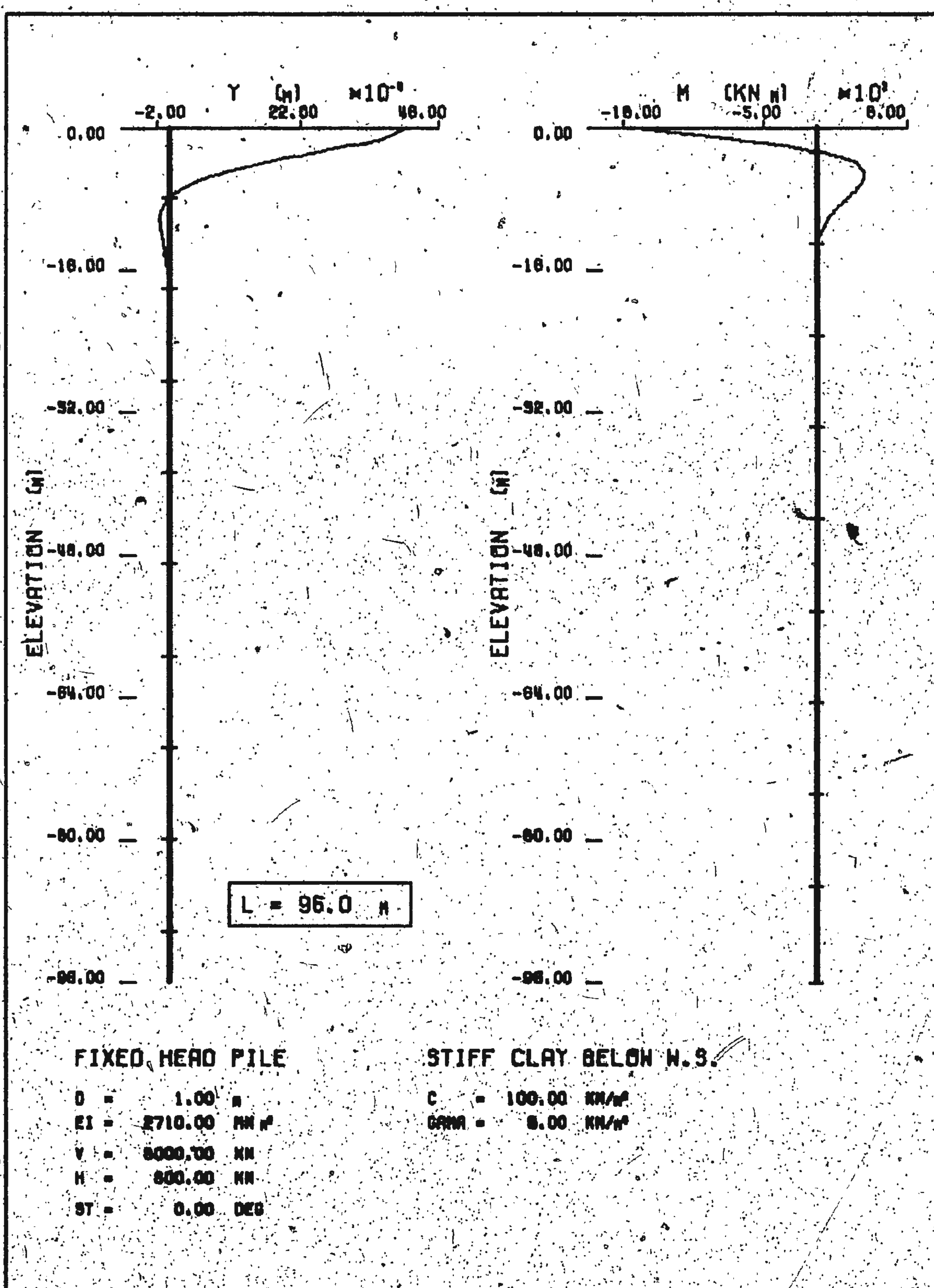


Figure 88. Deflection and bending moment for a fixed head pile in stiff clay, L = 96.0 m.

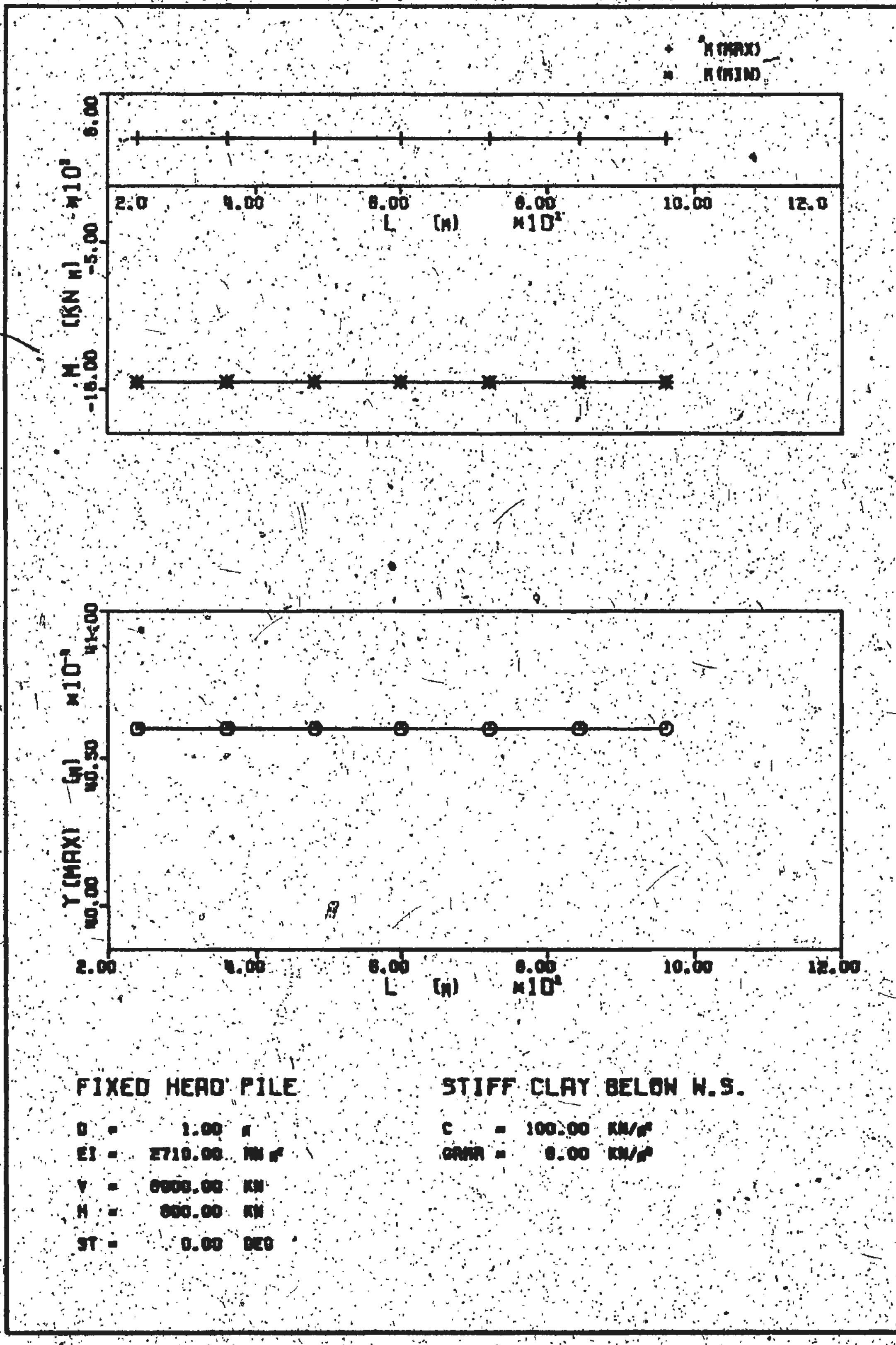


Figure 89 Effect of the pile length on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay.

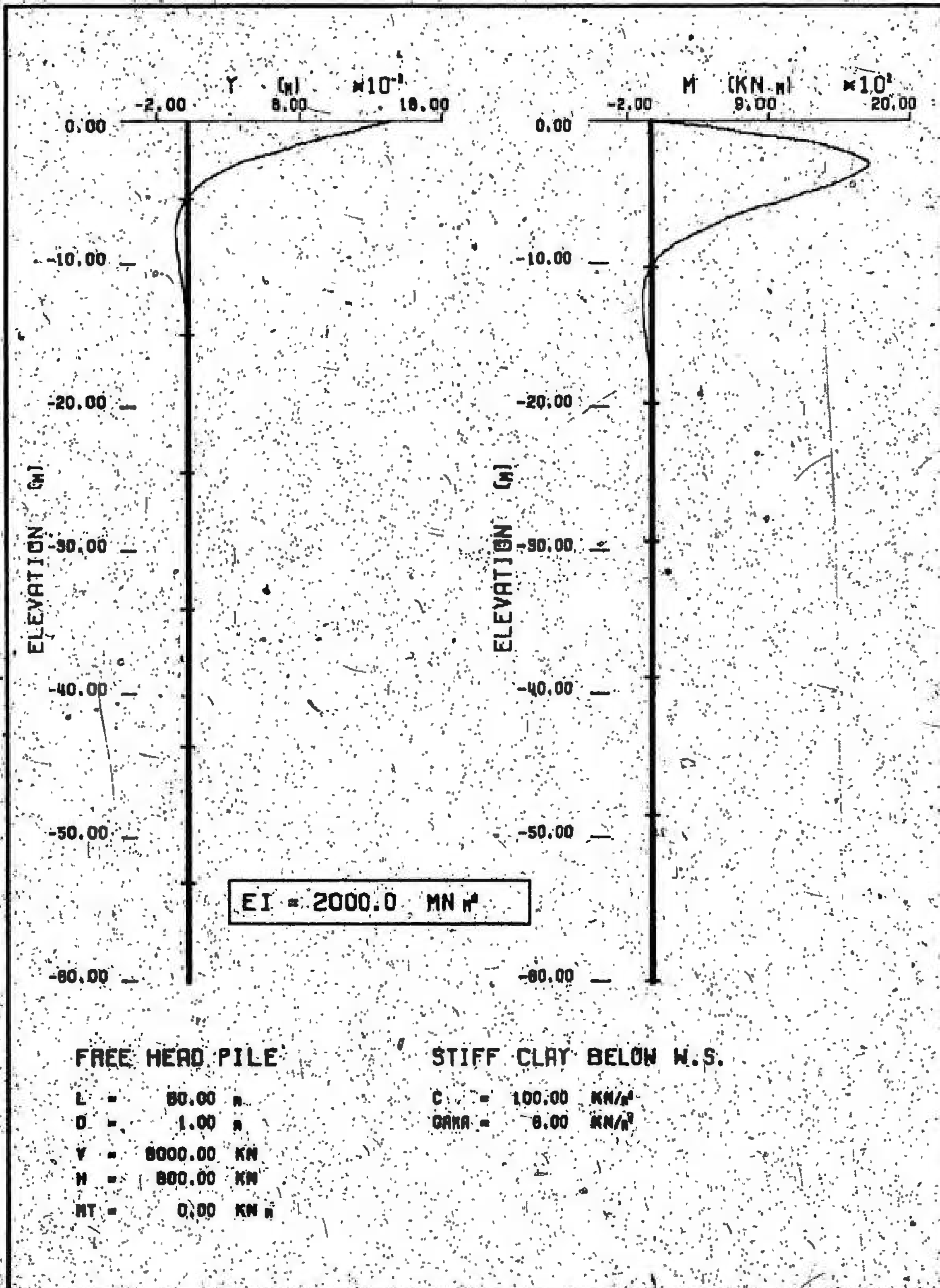


Figure 90 Deflection and bending moment for a free head pile in stiff clay, EI = 2000.0 MN m^2.

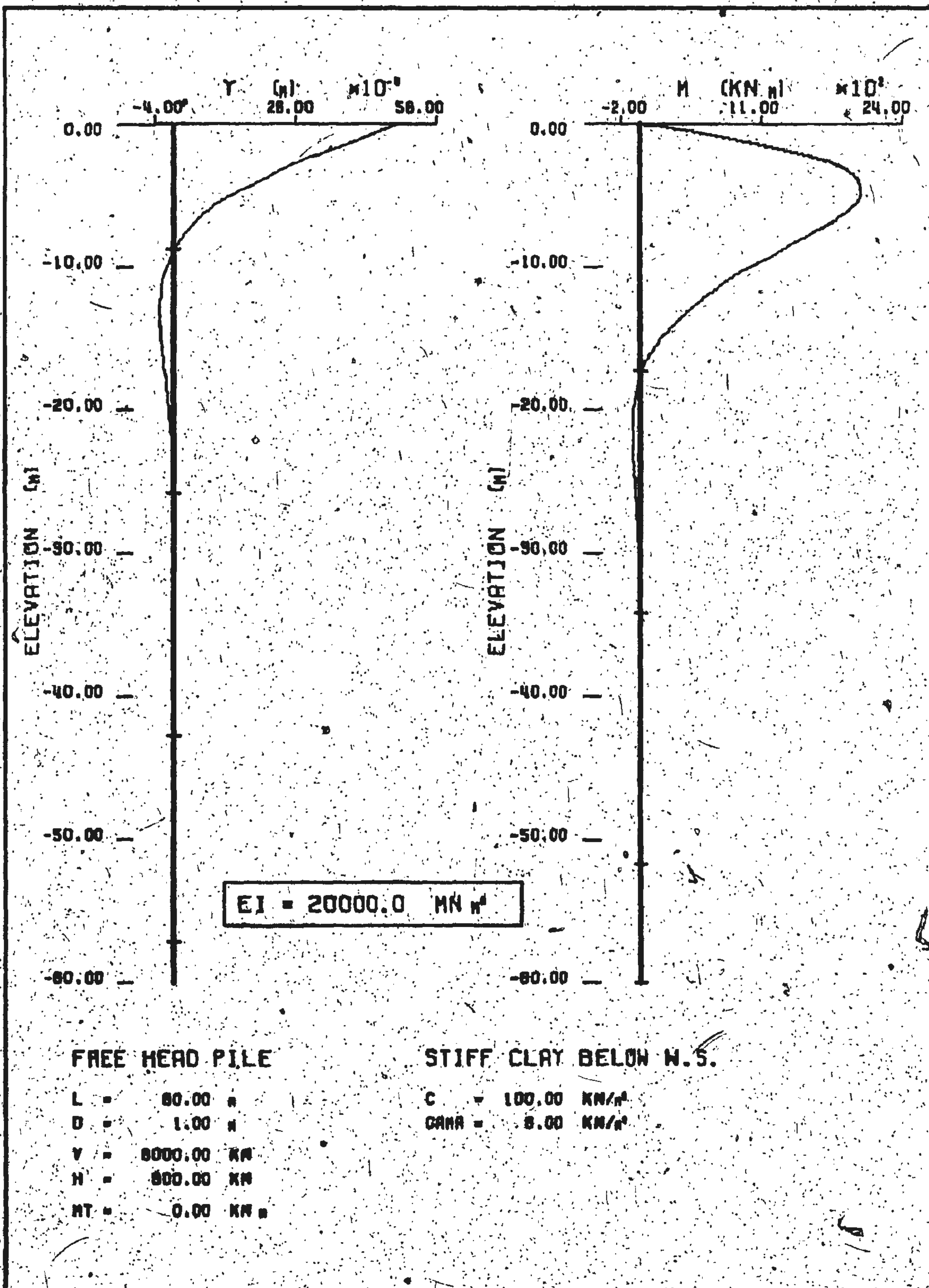


Figure 91 Deflection and bending moment for a free head pile in stiff clay, EI = 20000.0 MN m^2.

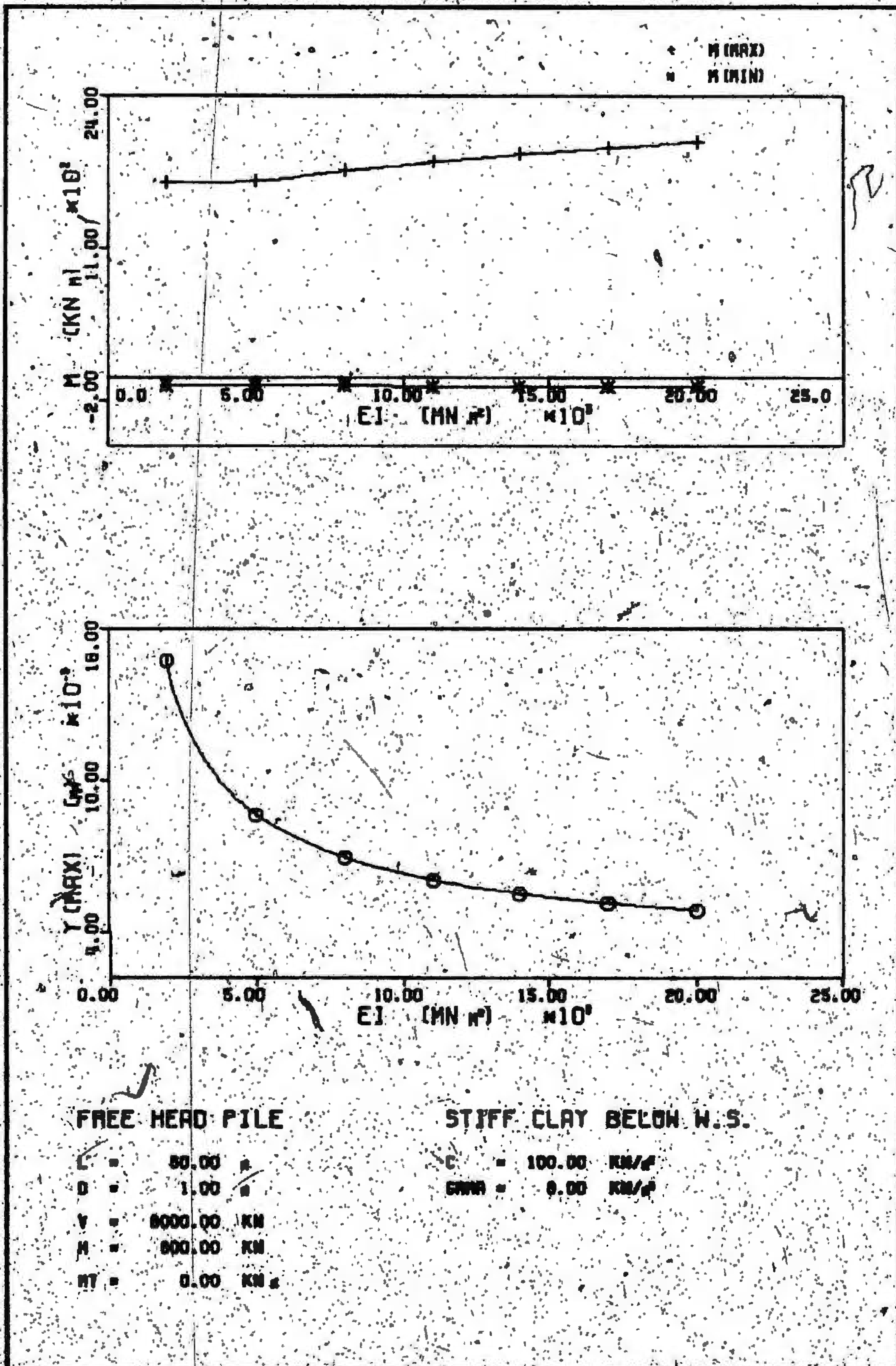


Figure 92 Effect of the pile flexural rigidity on the maximum deflection and the maximum bending moment for a free head pile in stiff clay.

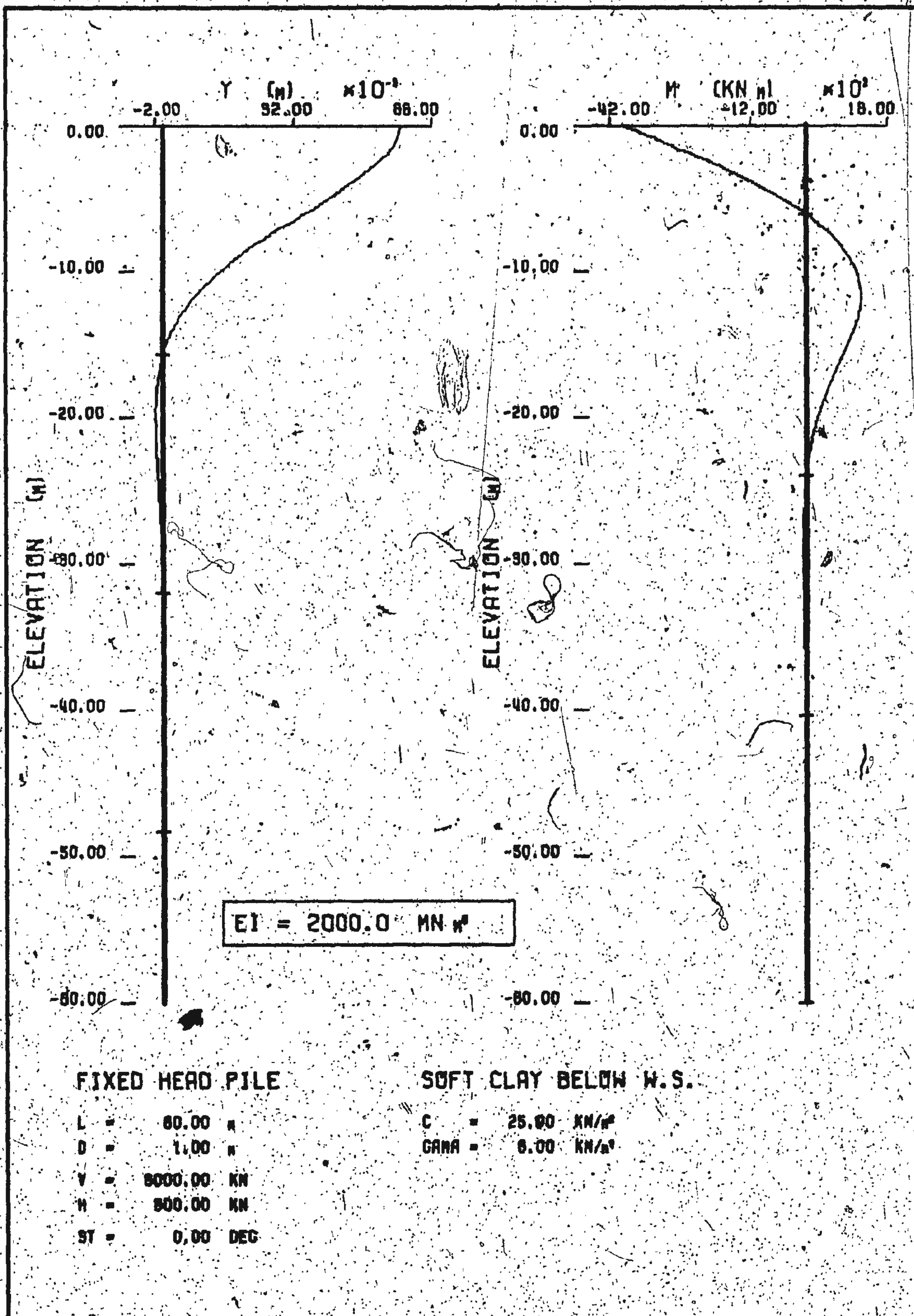


Figure 93 Deflection and bending moment for a fixed head pile in soft clay, EI = 2000.0 MN m^2.

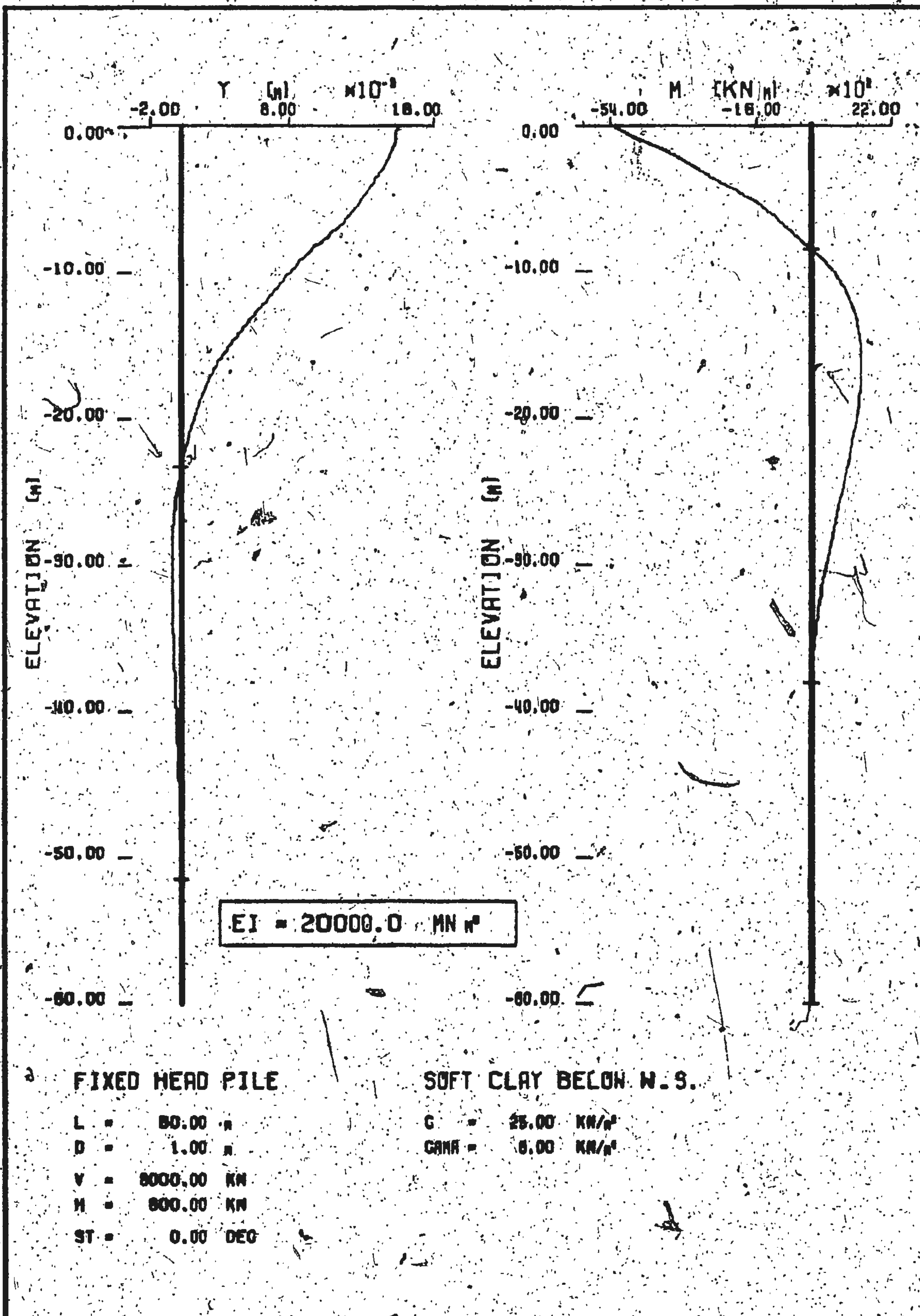


Figure 94 Deflection and bending moment for a fixed head pile in soft clay. EI = 20000.0 MN m^2.

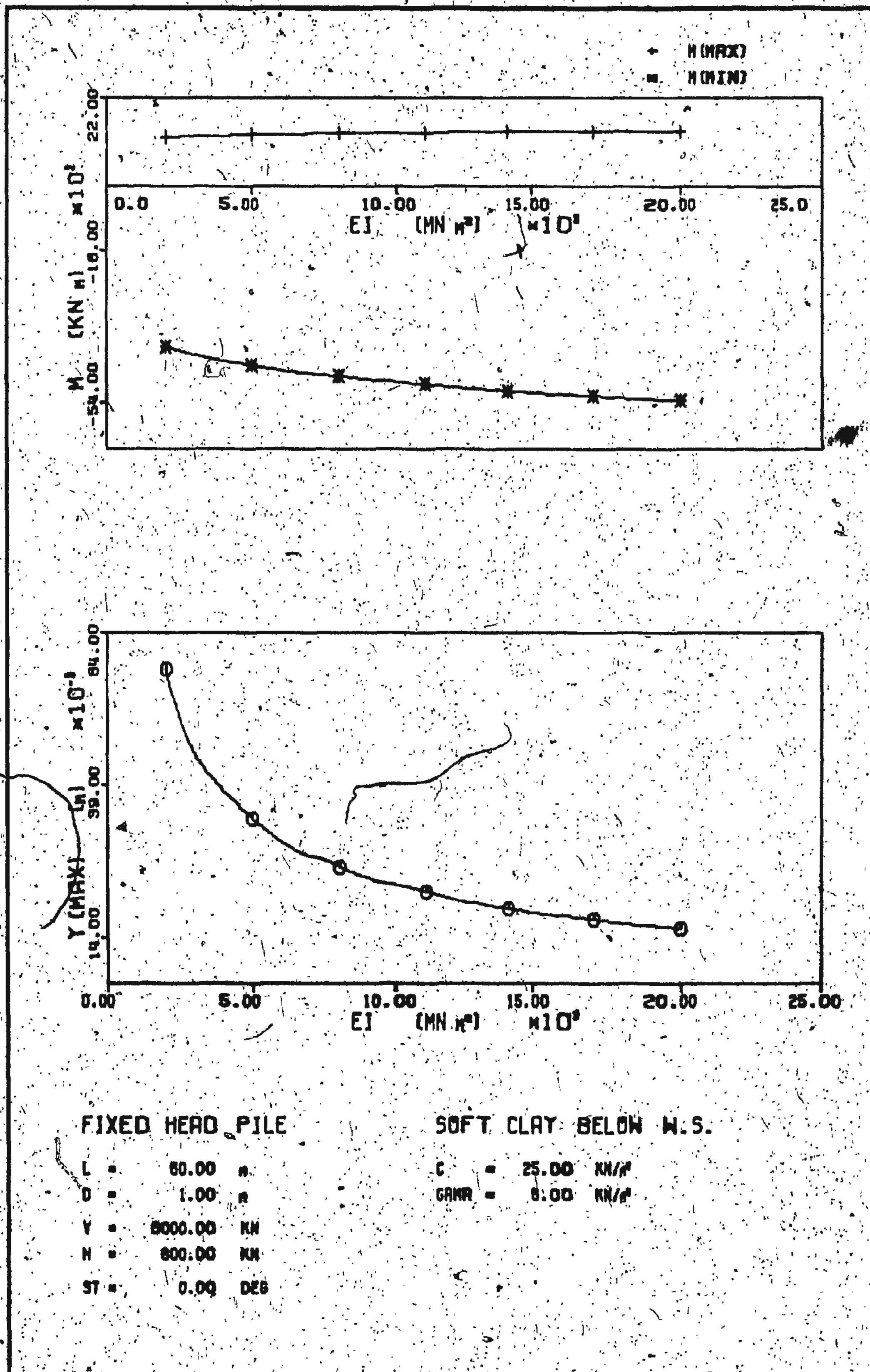


Figure 95 Effect of the pile flexural rigidity on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay.

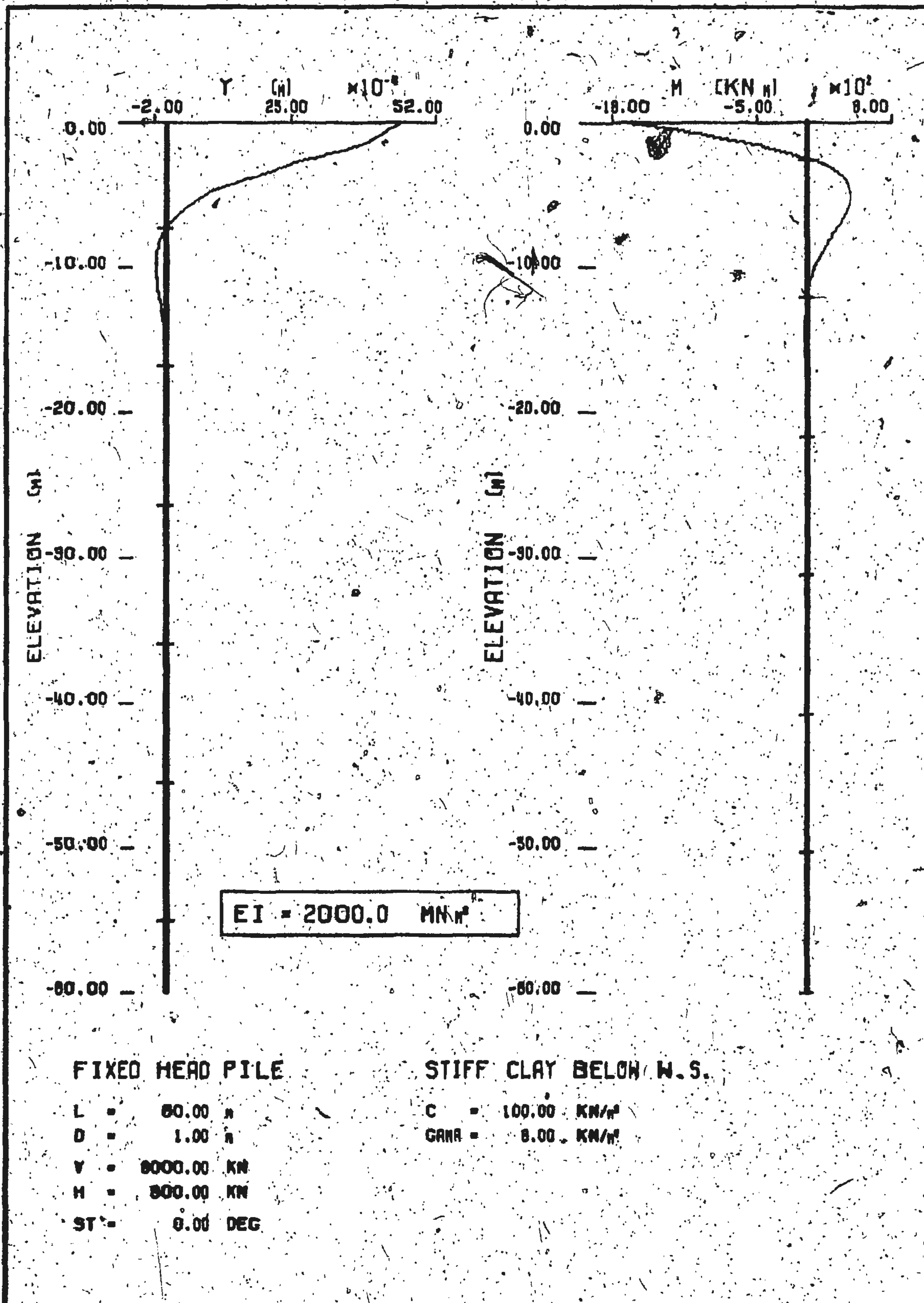


Figure 96 Deflection and bending moment for a fixed head pile in stiff clay, EI = 2000.0 MN m².

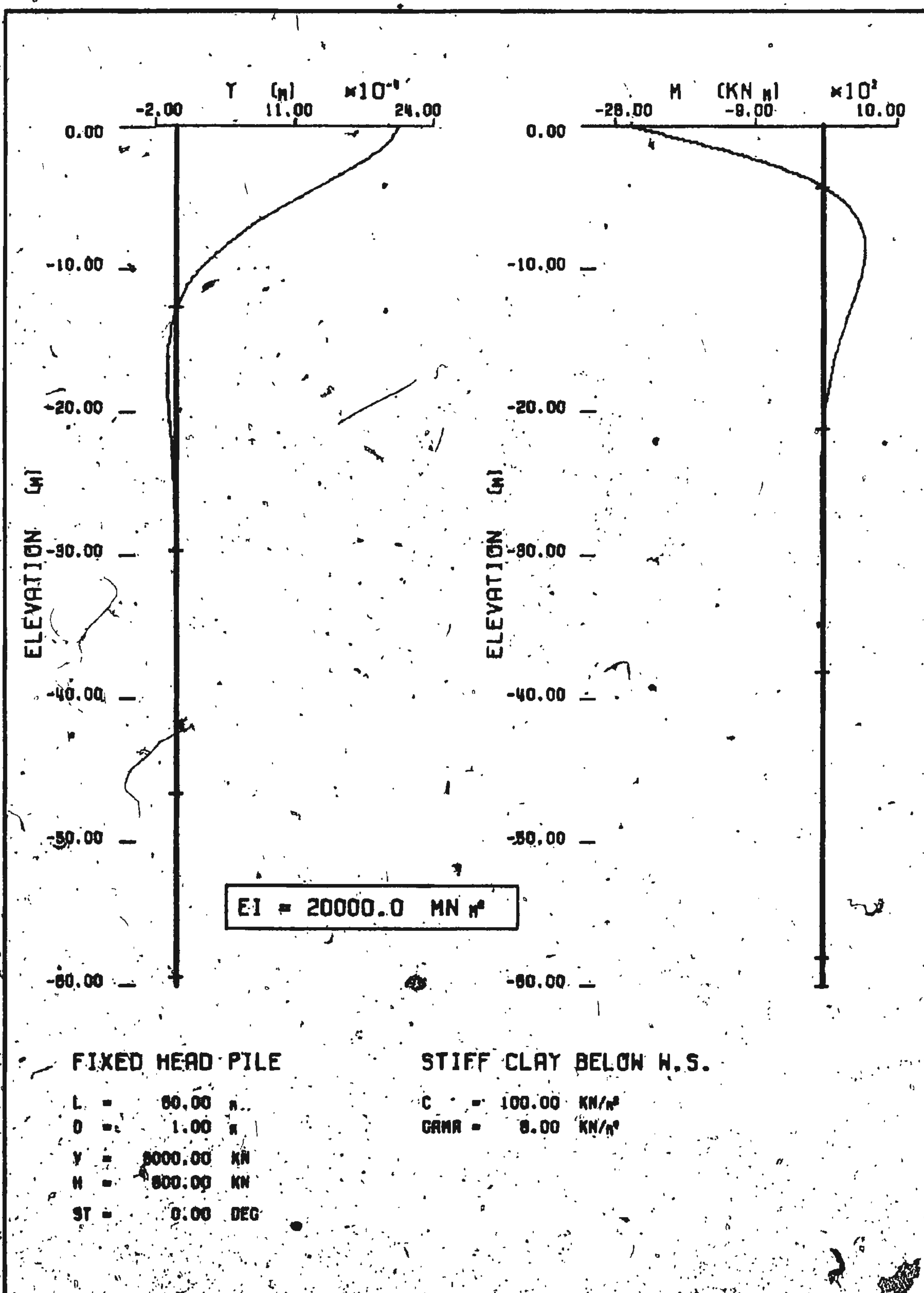


Figure 97 Deflection and bending moment for a fixed head pile in stiff clay, EI = 20000.0 MN m².

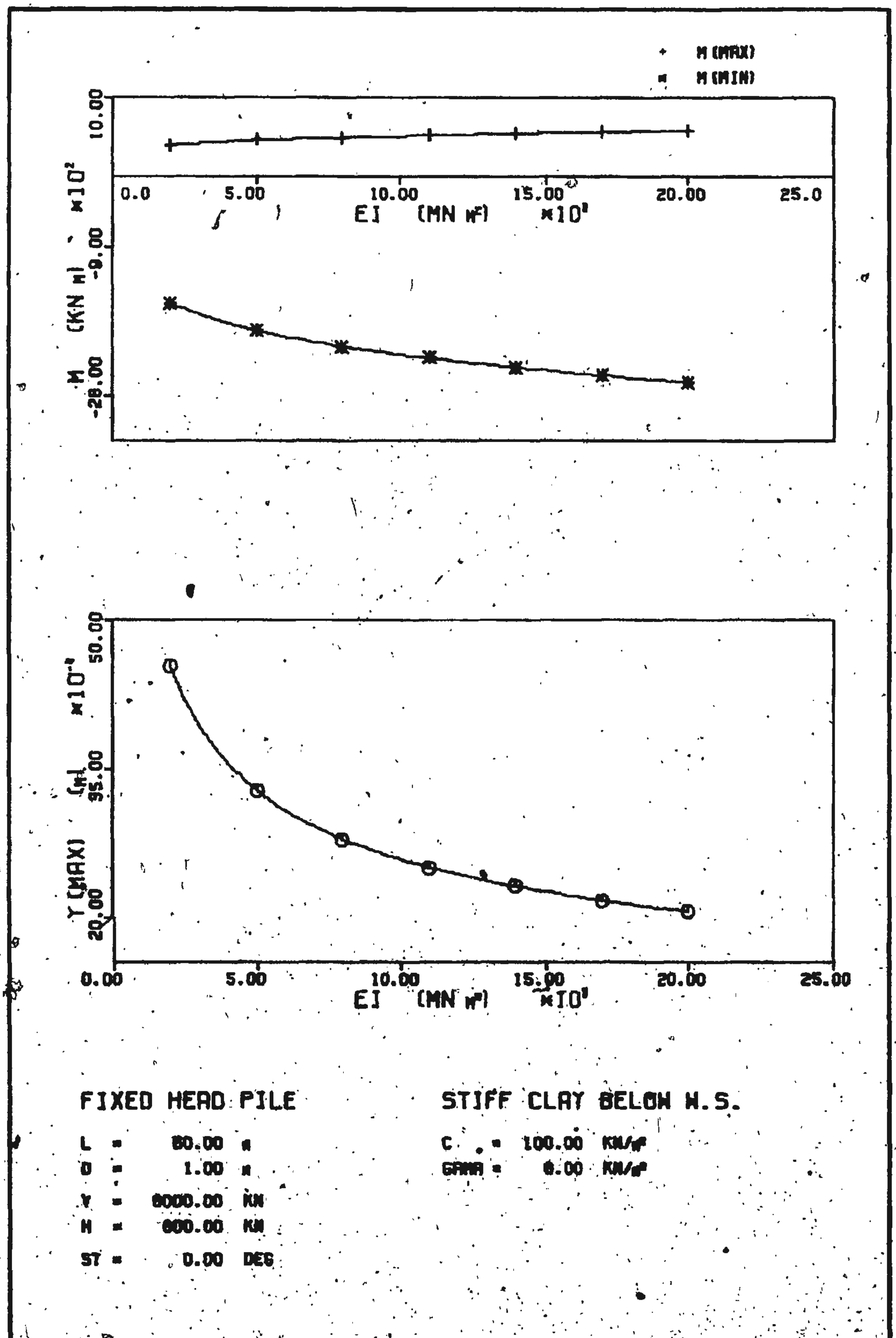


Figure 98 Effect of the pile flexural rigidity on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay.

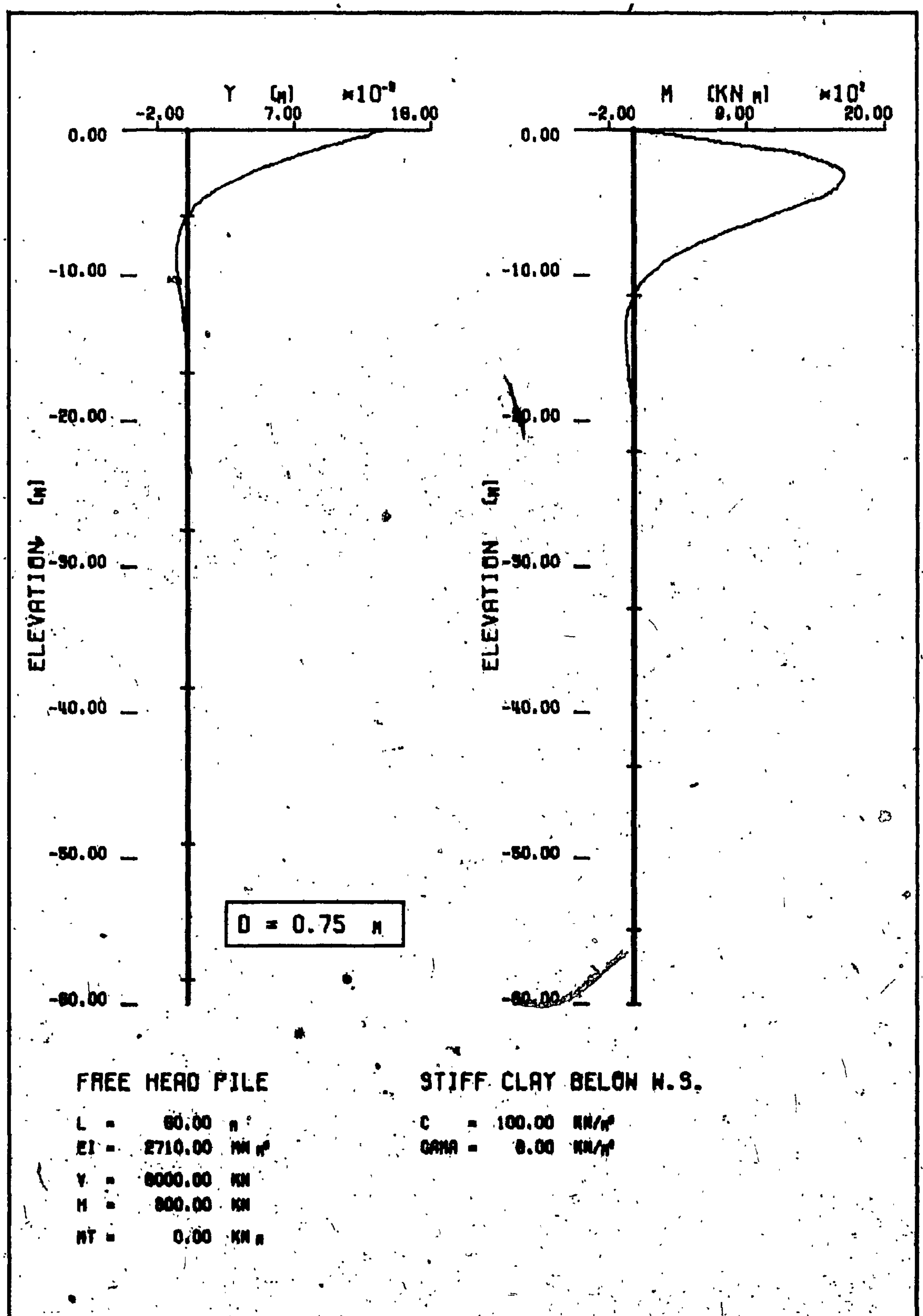


Figure 99: Deflection and bending moment for a free head pile in stiff clay, D = 0.75 m.

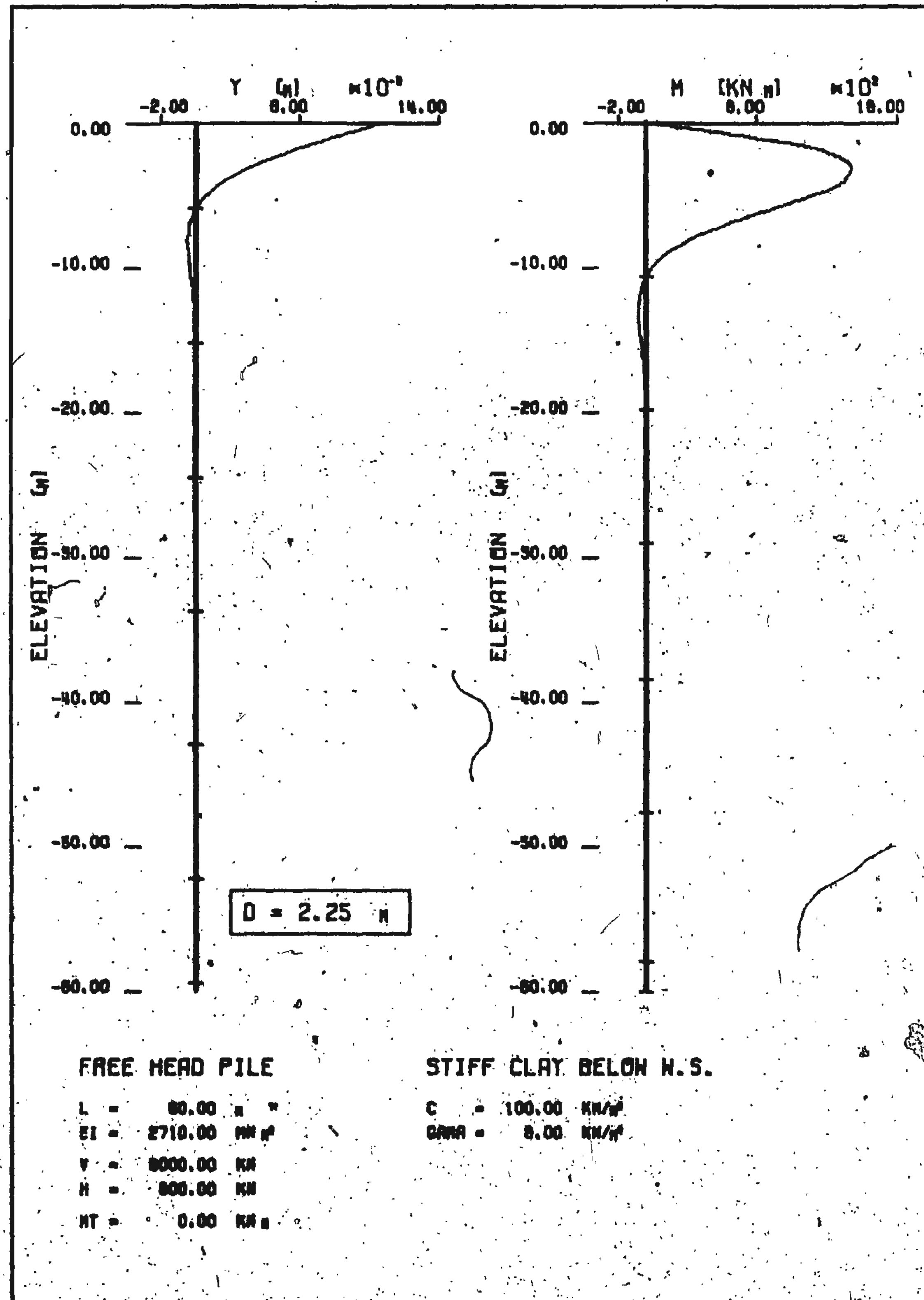


Figure 100 Deflection and bending moment for a free head pile in stiff clay, D = 2.25 m.

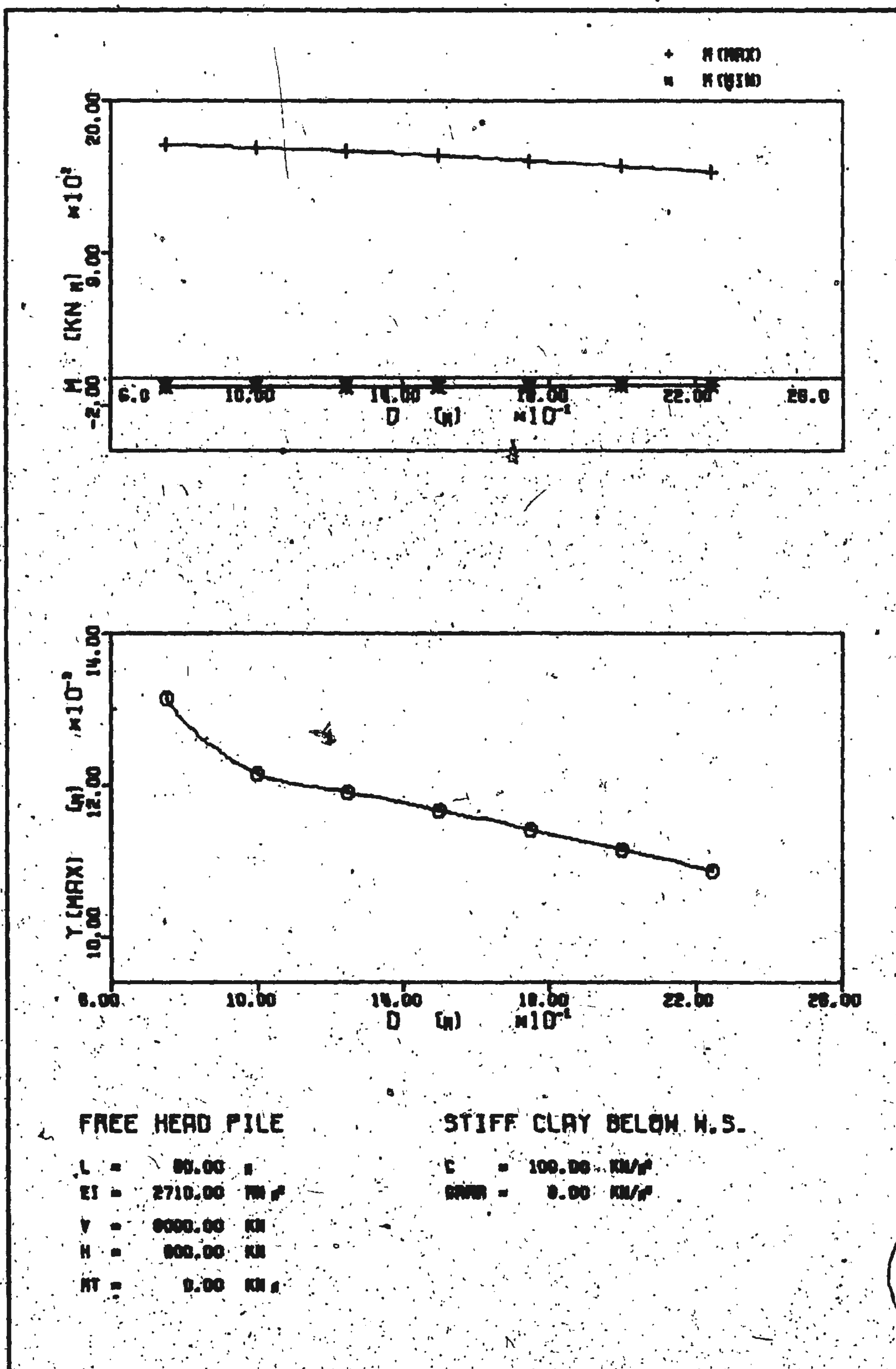


Figure 101 Effect on the pile diameter on the maximum deflection and the maximum bending moment for a free head pile in stiff clay.

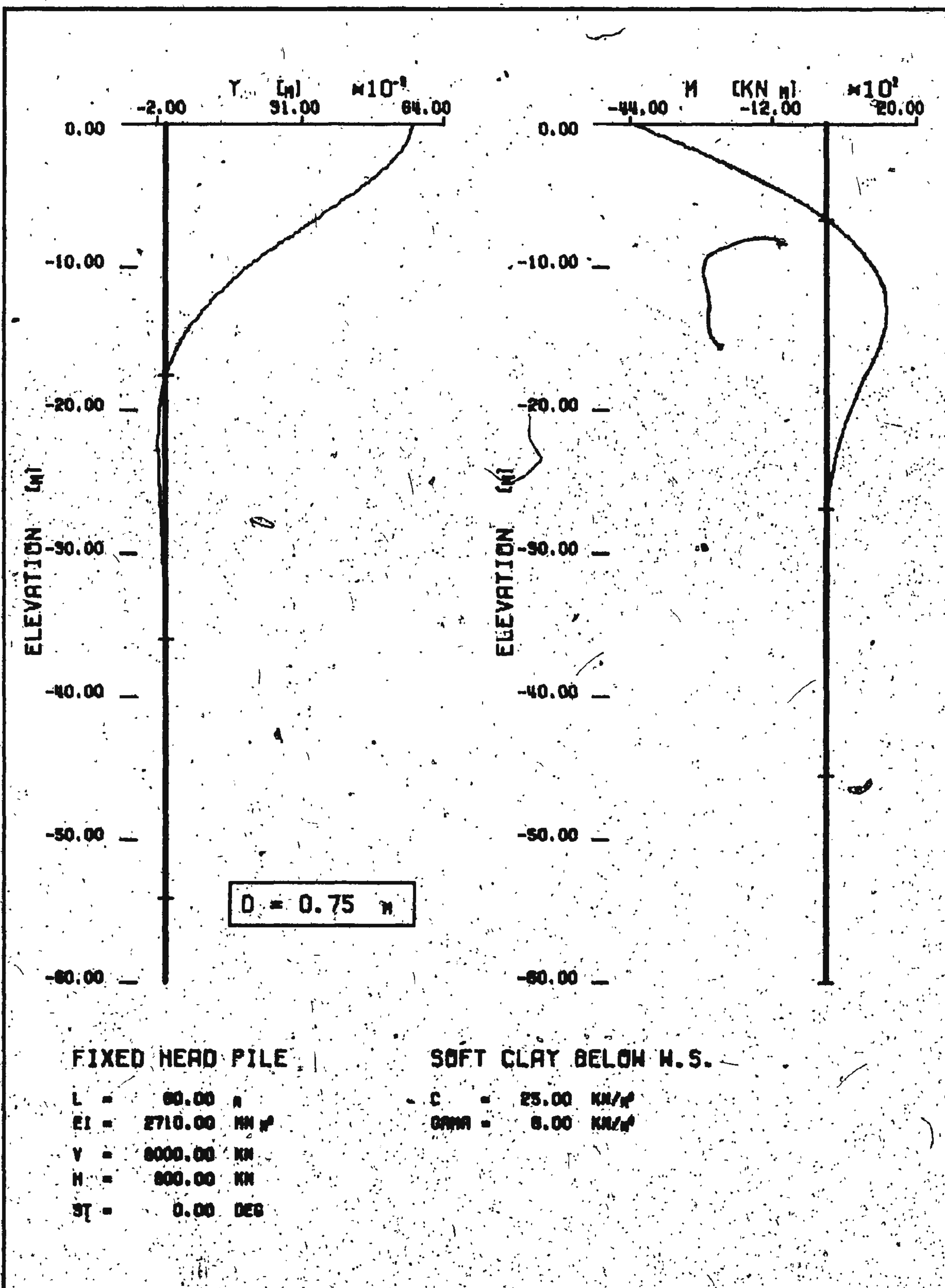


Figure 102 Deflection and bending moment for a fixed head pile in soft clay, D = 0.75 m.

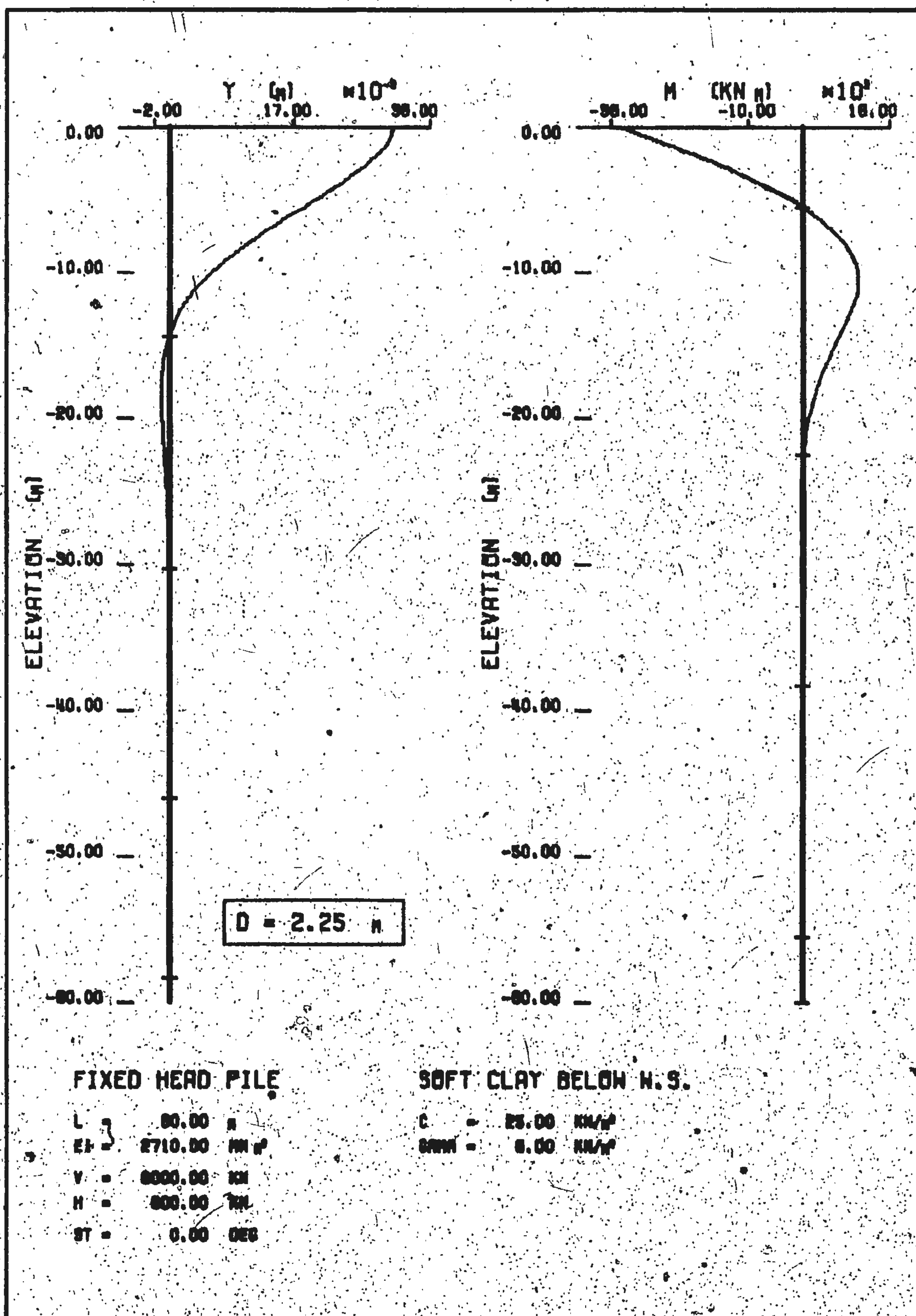


Figure 103 Deflection and bending moment for a fixed head pile in soft clay, D = 2.25 m.

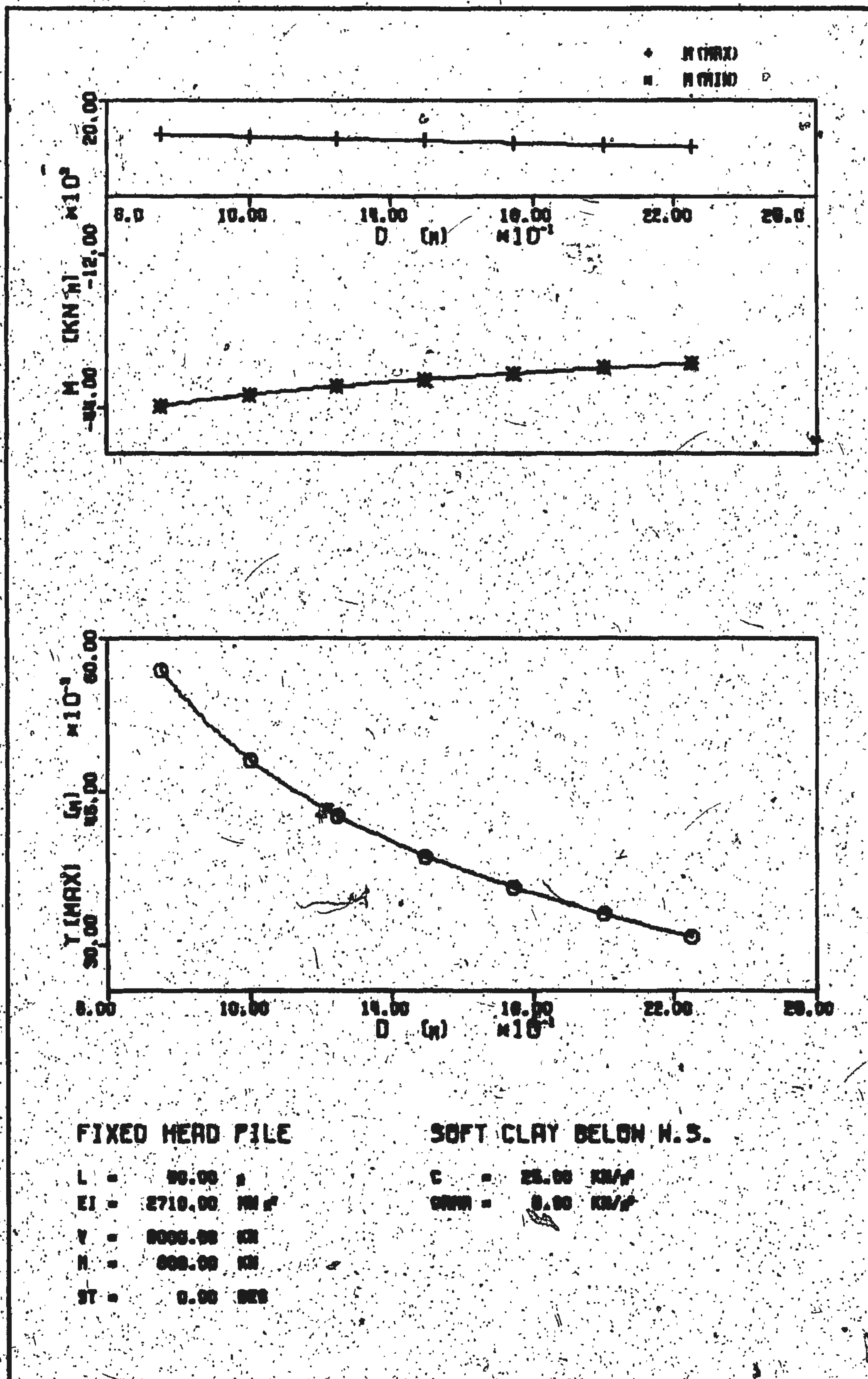


Figure 104. Effect of the pile diameter on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay.

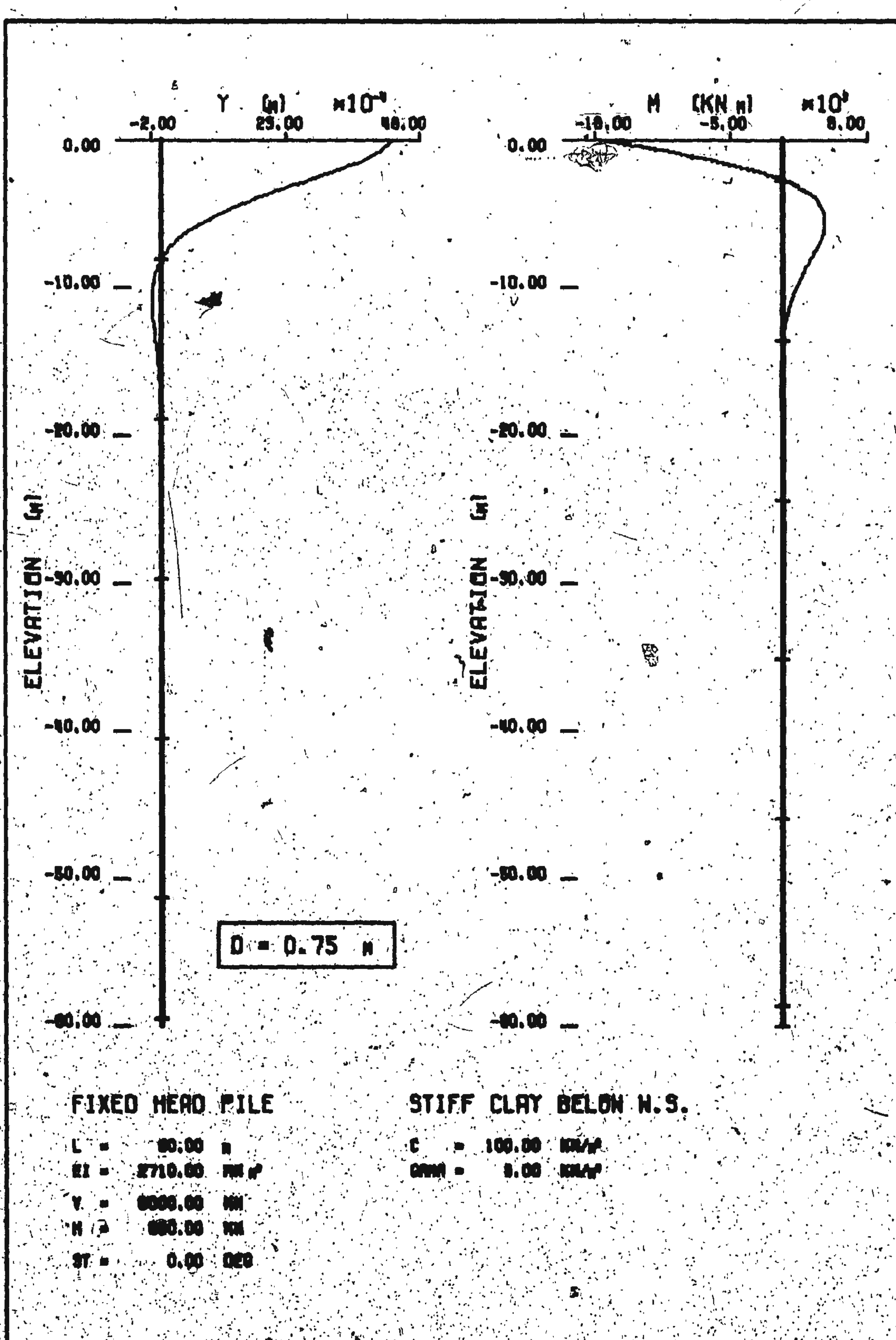


Figure 105 Deflection and bending moment for a fixed head pile in stiff clay. D = 0.75 m.

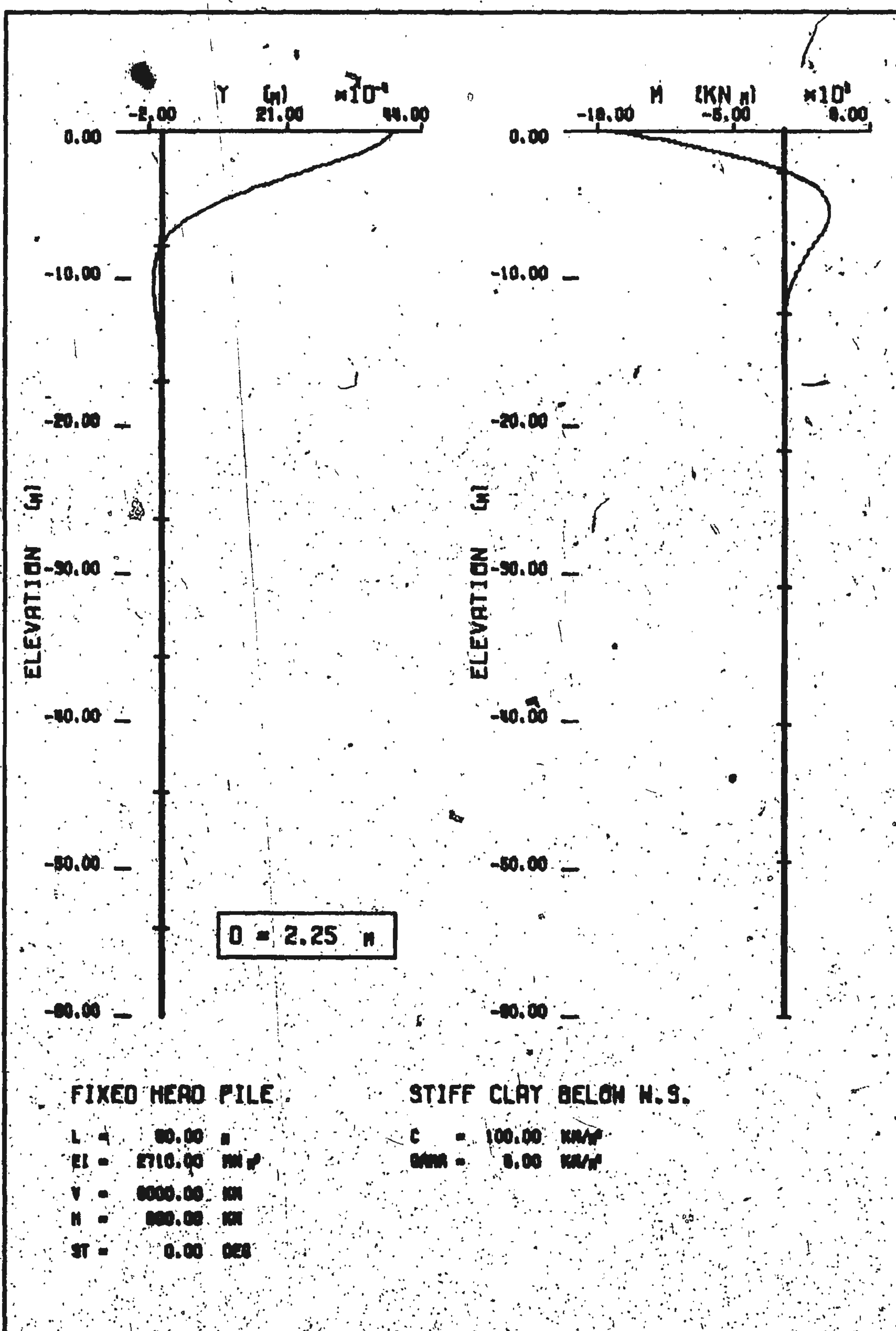


Figure 106 Deflection and bending moment for a fixed head pile in stiff clay, D = 2.25 m.

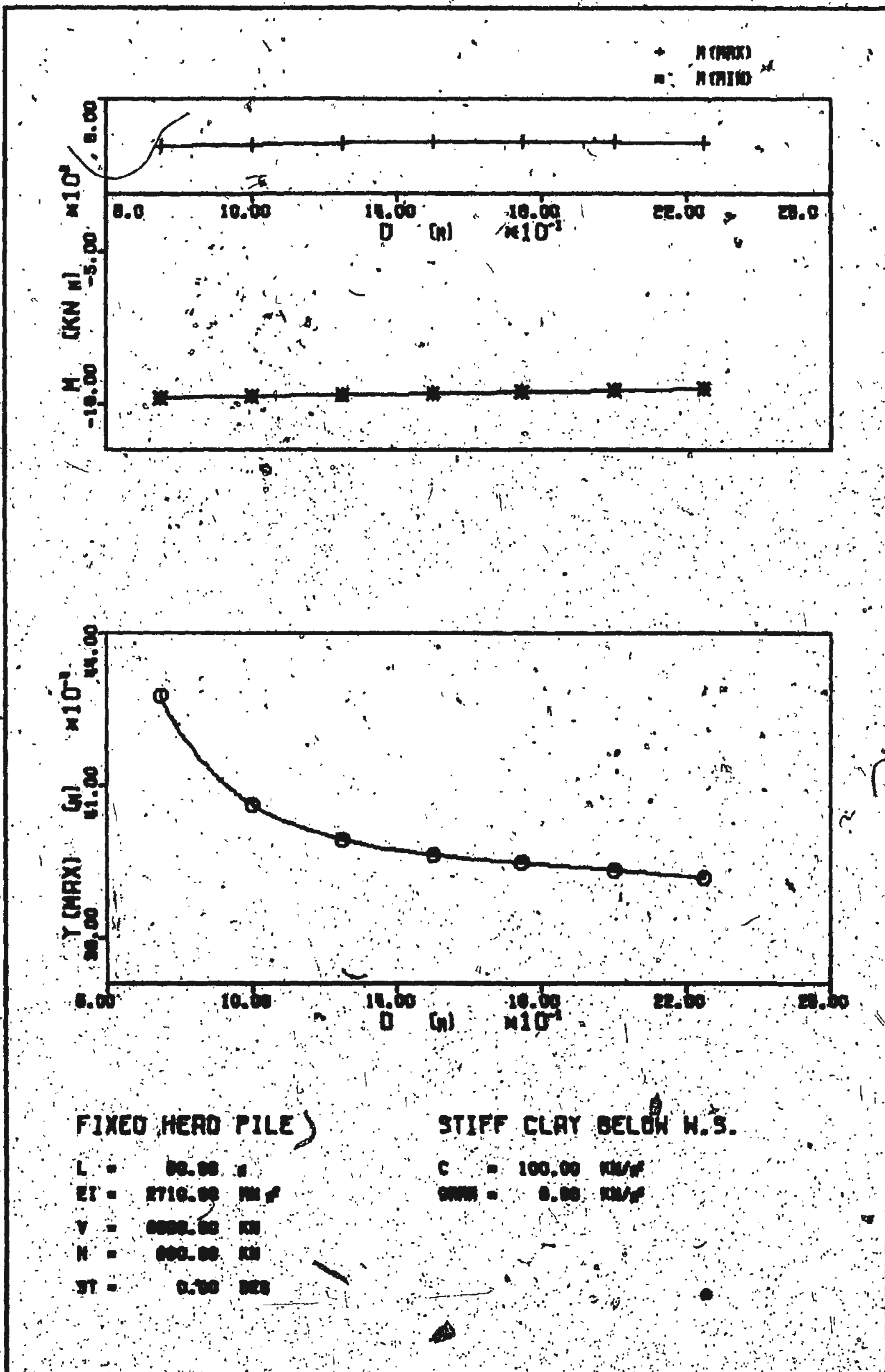


Figure 107. Effect on the pile diameter on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay.

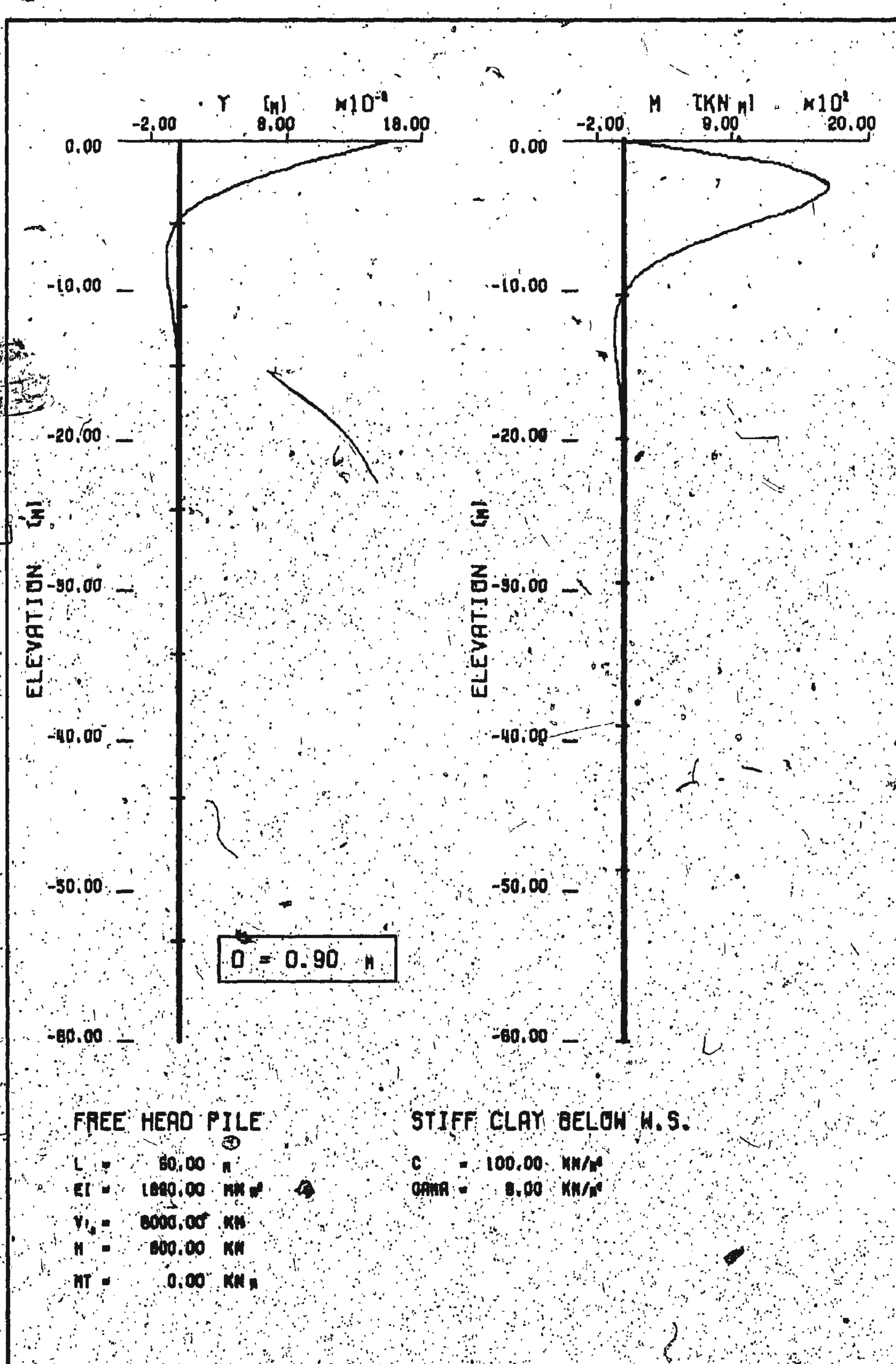


Figure 108 Deflection and bending moment for a free head pile in stiff clay, D = 0.90 m , EI = 1890.0 MN m^2.

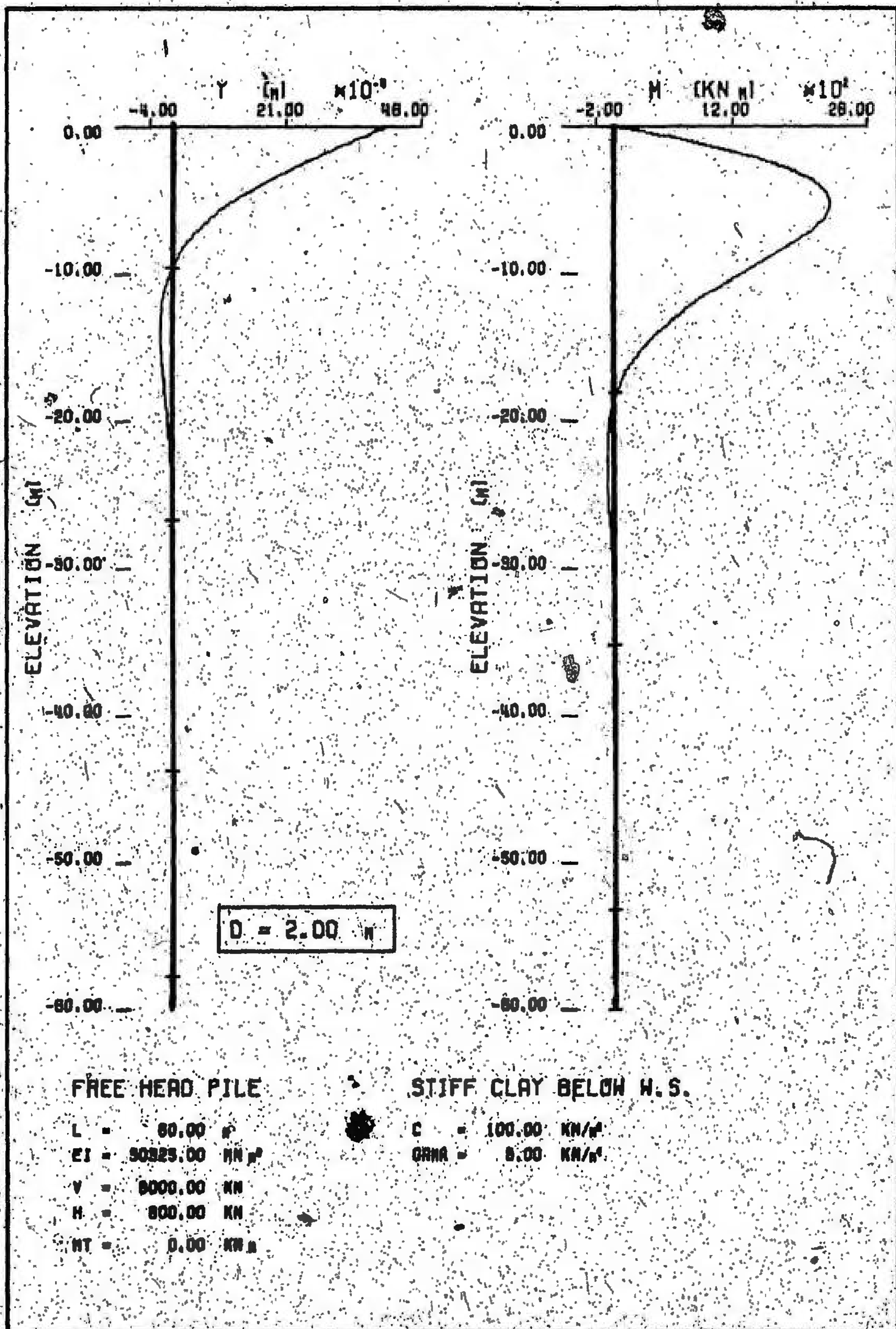


Figure 109 Deflection and bending moment for a free head pile in stiff clay, D = 2.00 m , EI = 30325.0 MN m^2.

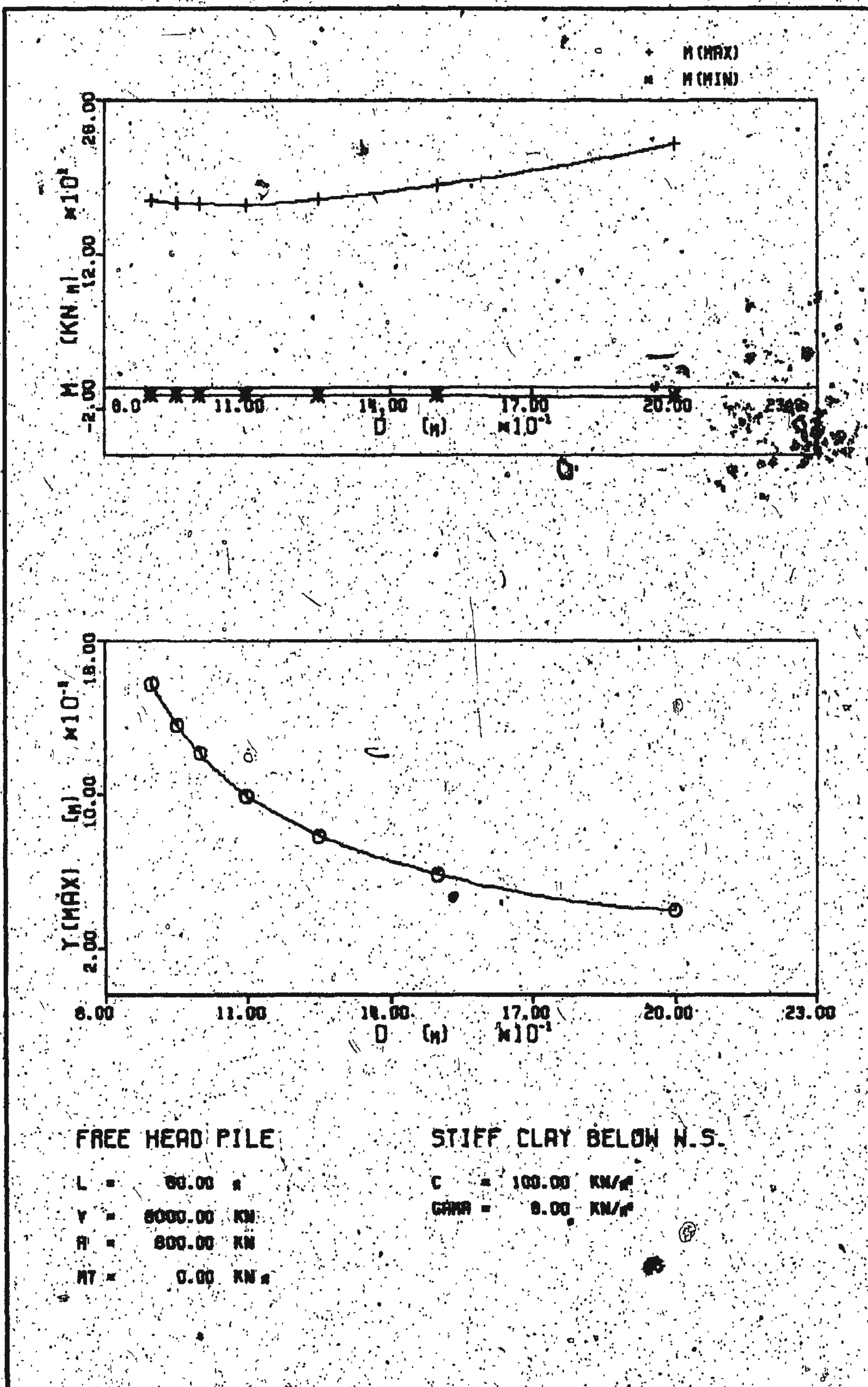


Figure 110. Effect of the pile diameter and the corresponding flexural rigidity on the maximum deflection and the maximum bending moment for a free head pile in stiff clay.

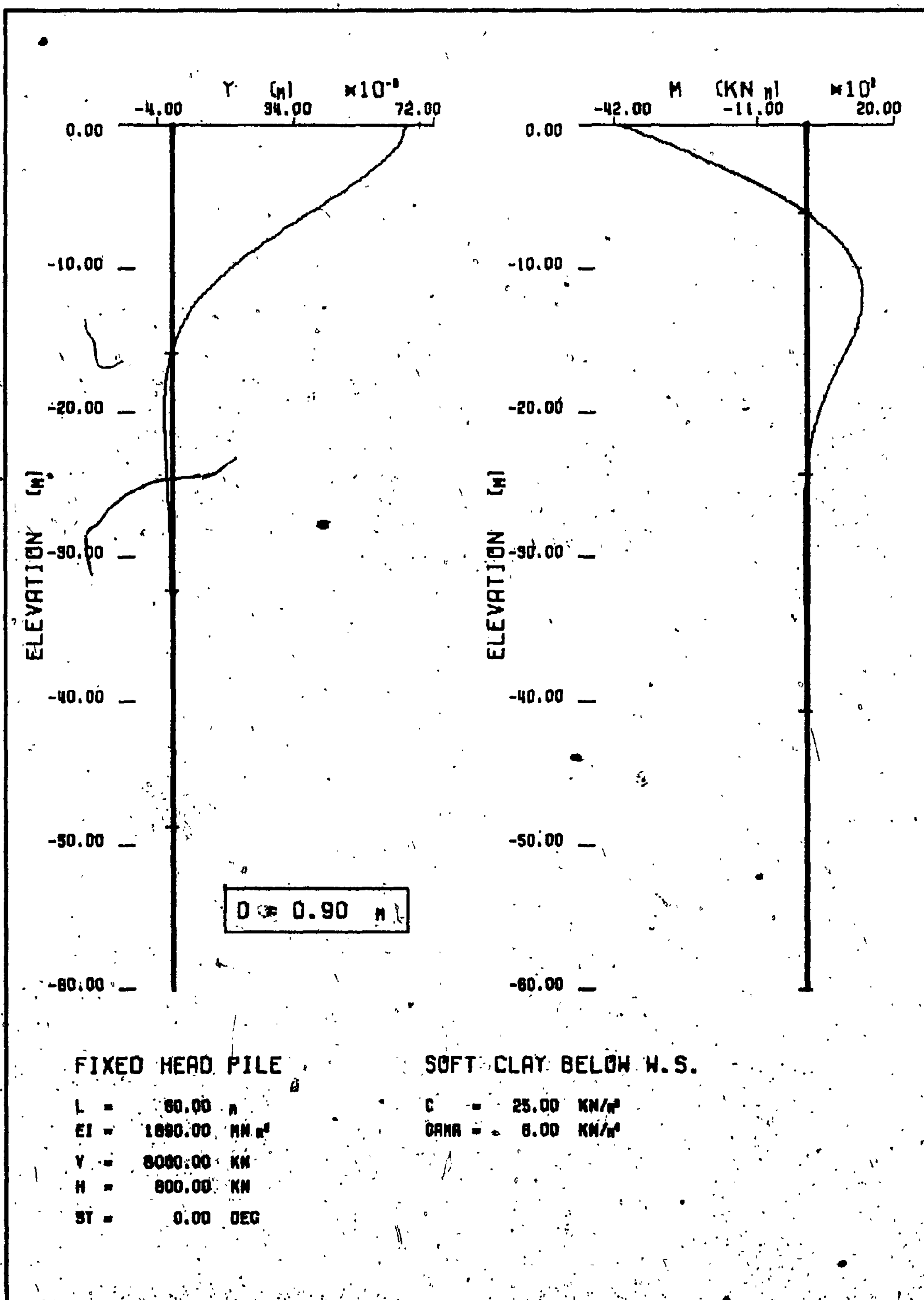


Figure 111. Deflection and bending moment for a fixed head pile in soft clay, D = 0.90 m , EI = 1890.0 MN m^2.

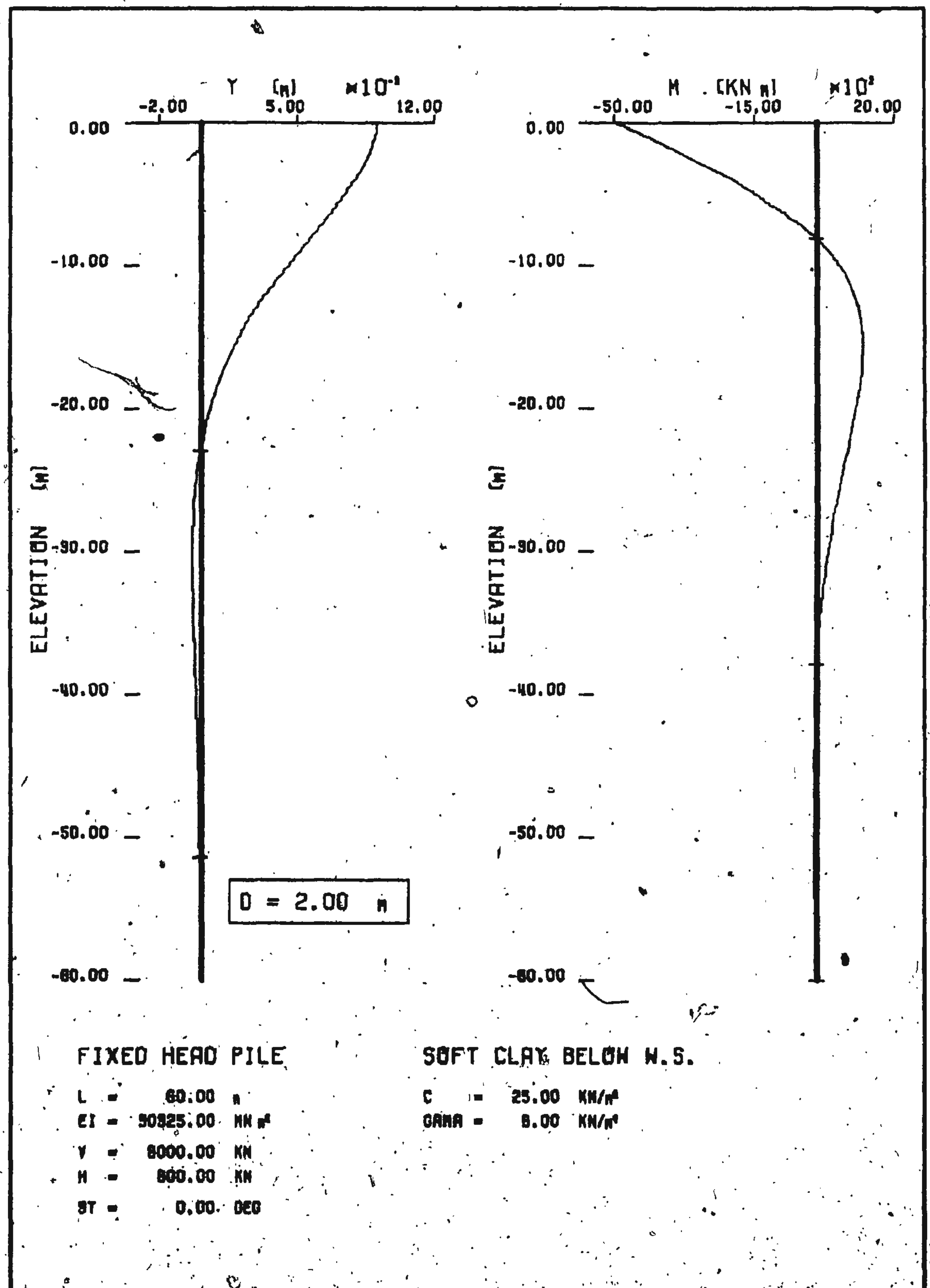


Figure 112 Deflection and bending moment for a fixed head pile in soft clay, $D = 2.00$ m, $EI = 30325.0$ MN m^2

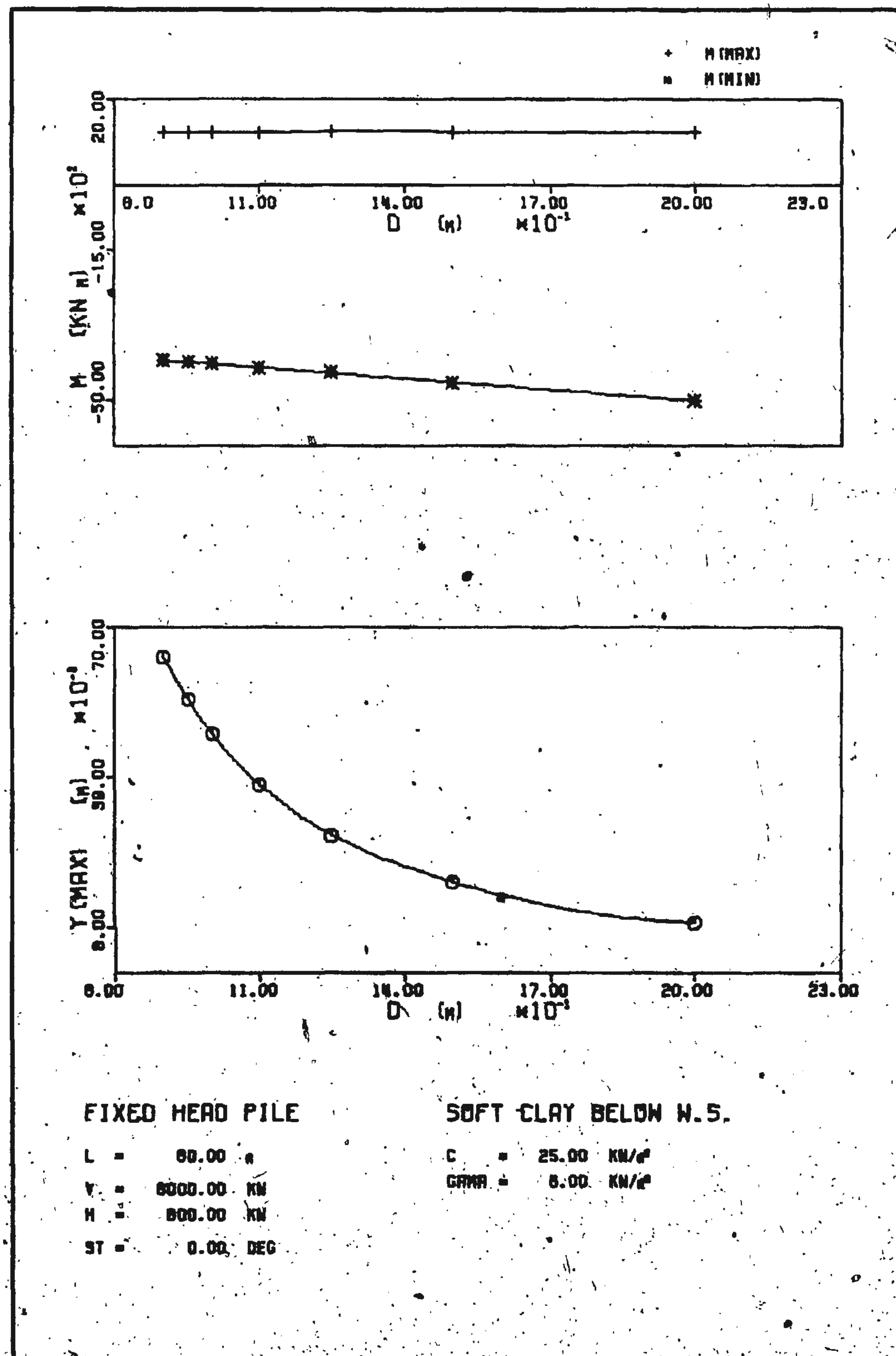


Figure 113 Effect of the pile diameter and the corresponding flexural rigidity on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay.

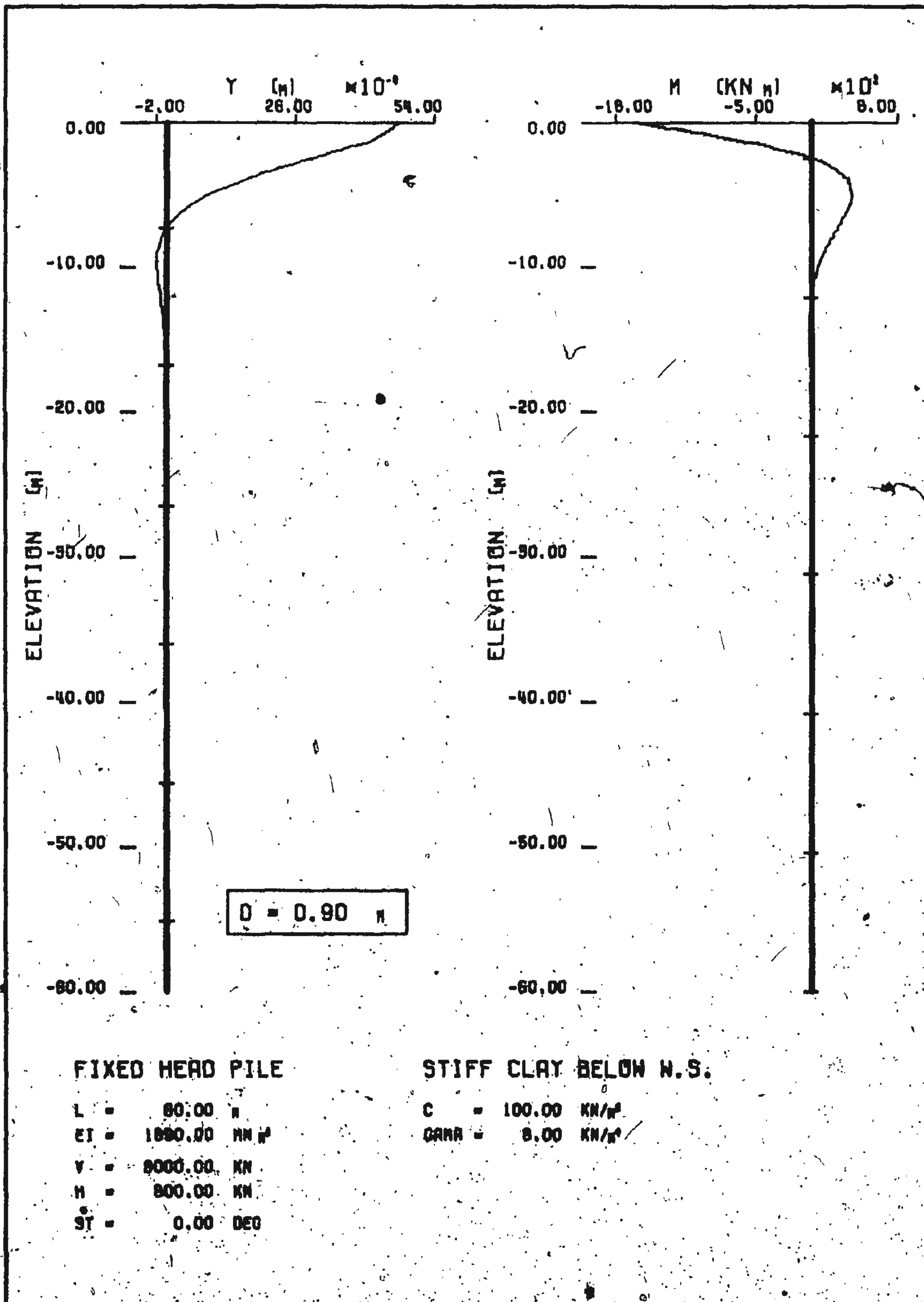


Figure 114 Deflection and bending moment for a fixed head pile in stiff clay, D = 0.90 m , EI = 1890.0 MN m^2.

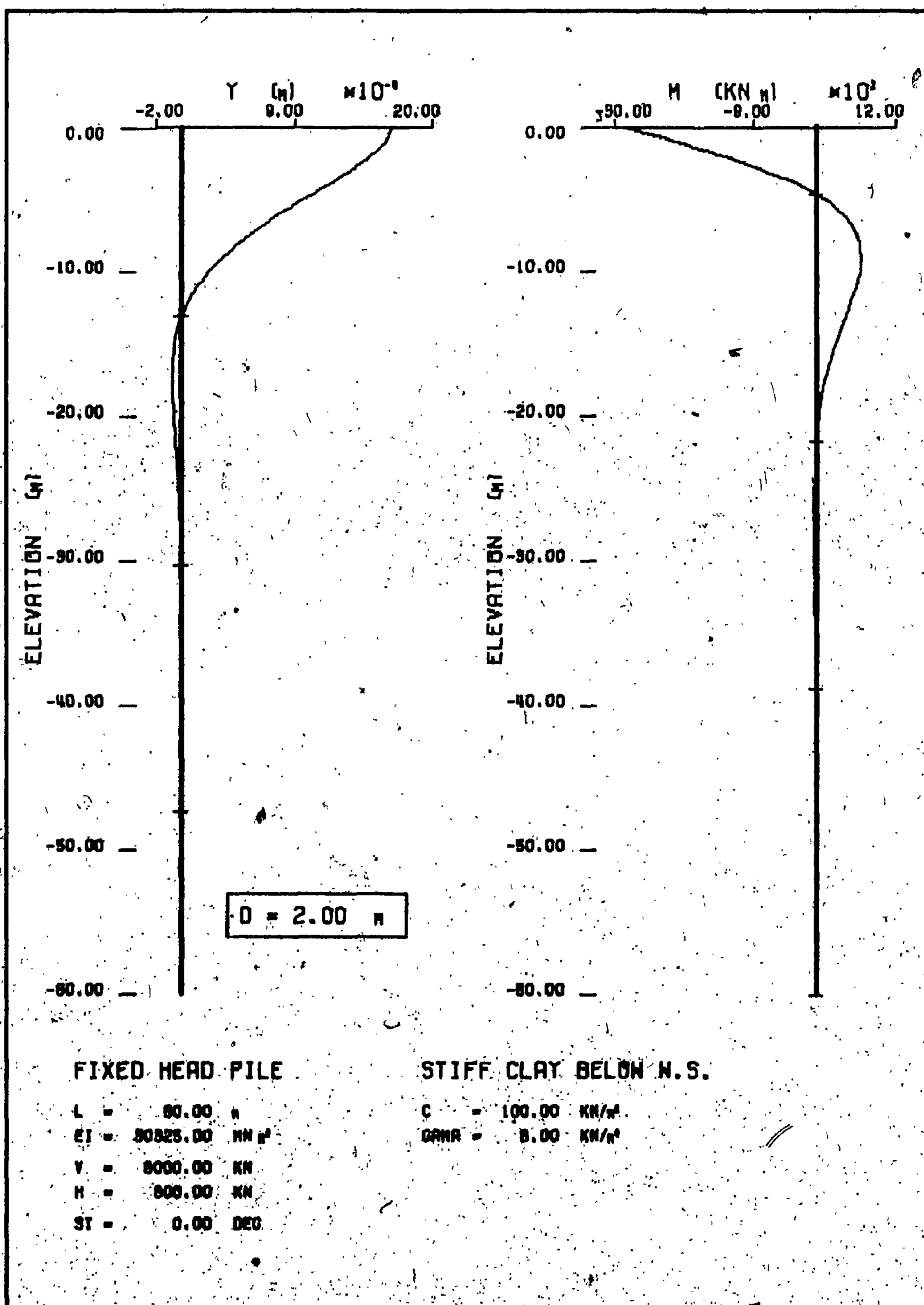


Figure 115 Deflection and bending moment for a fixed head pile in stiff clay, D = 2.00 m , EI = 30325.0 MN m^2.

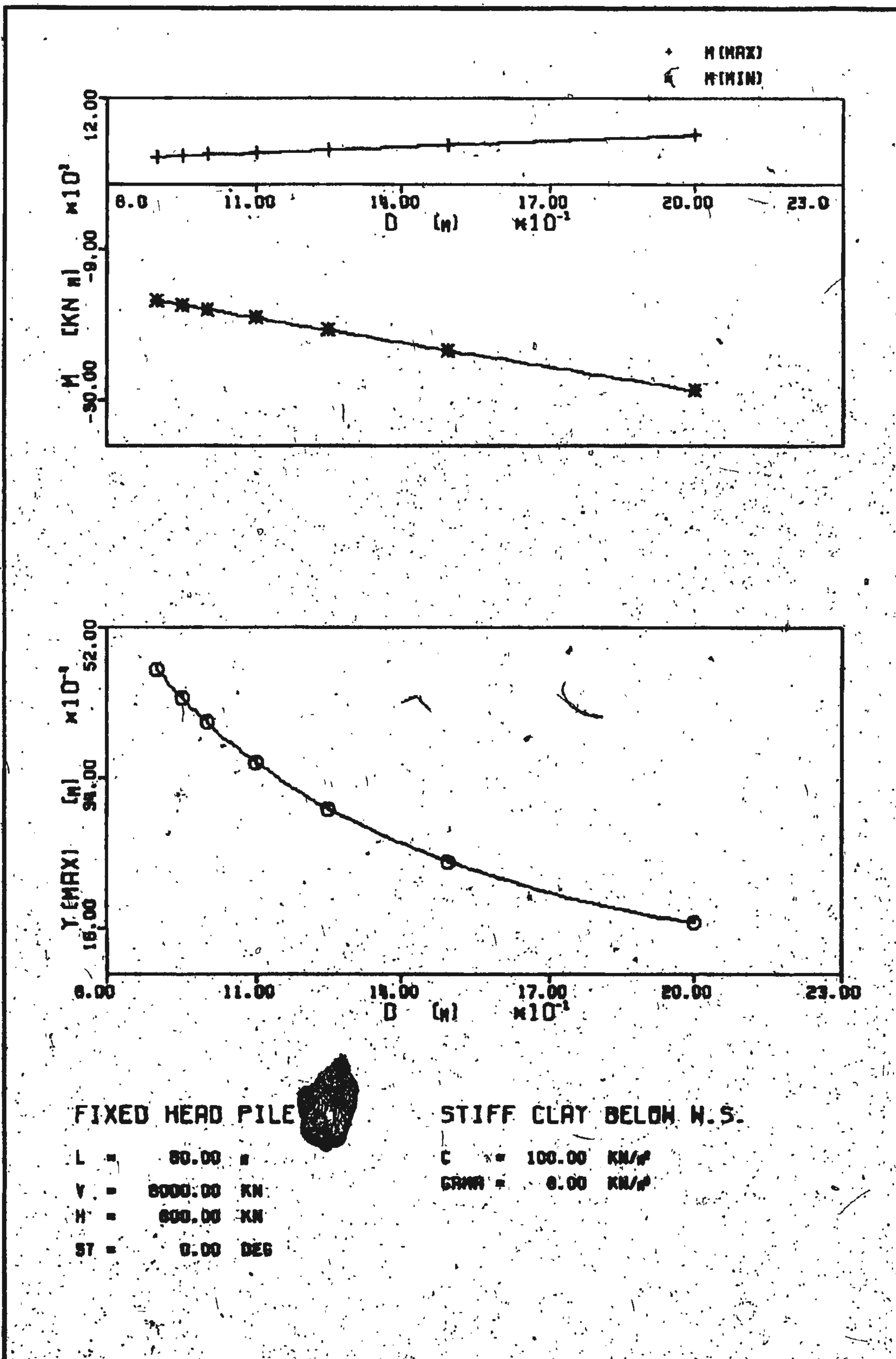


Figure 116 Effect of the pile diameter and the corresponding flexural rigidity on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay.

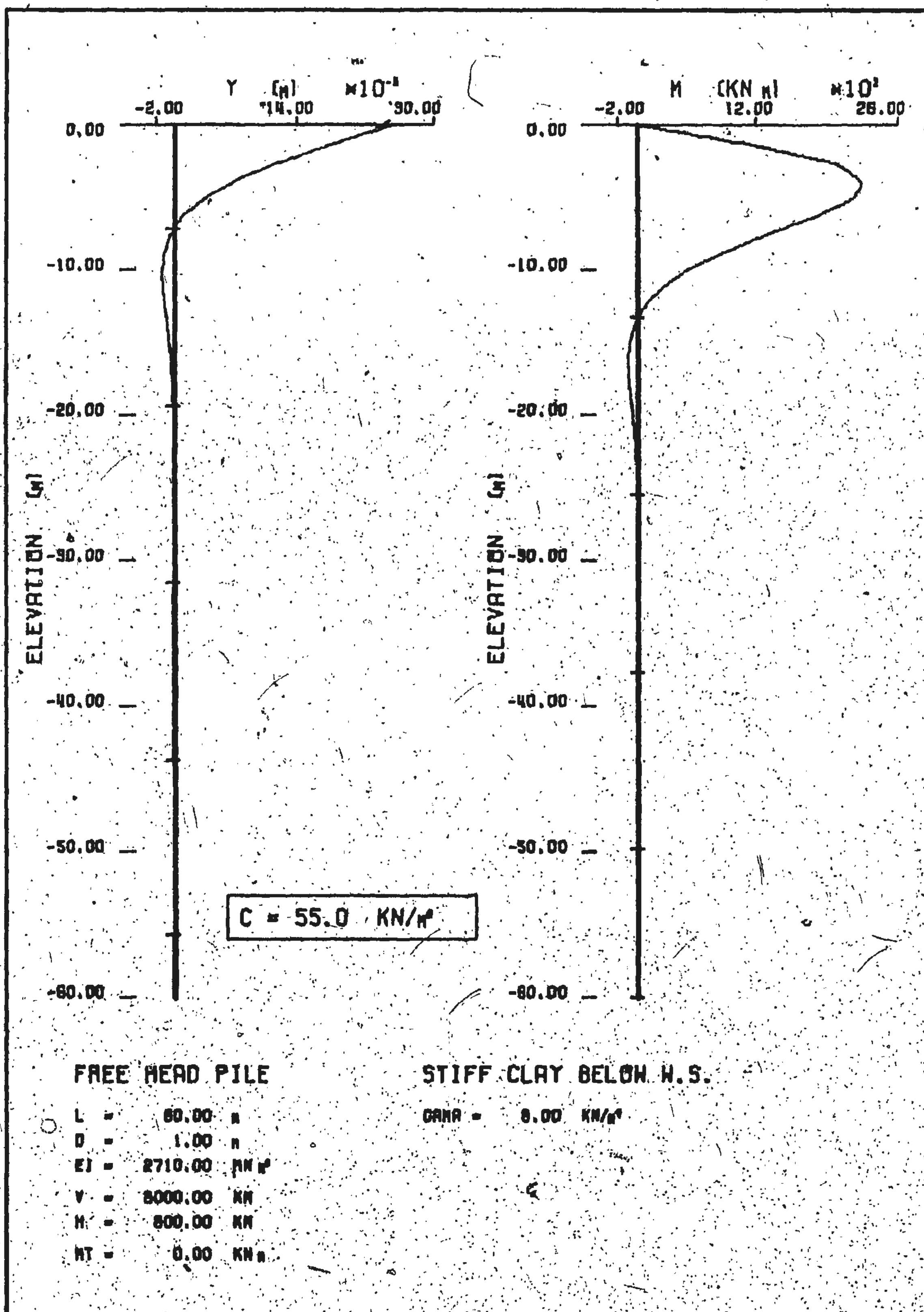


Figure 117 Deflection and bending moment for a free head pile in stiff clay, C = 55.0 kN/m².

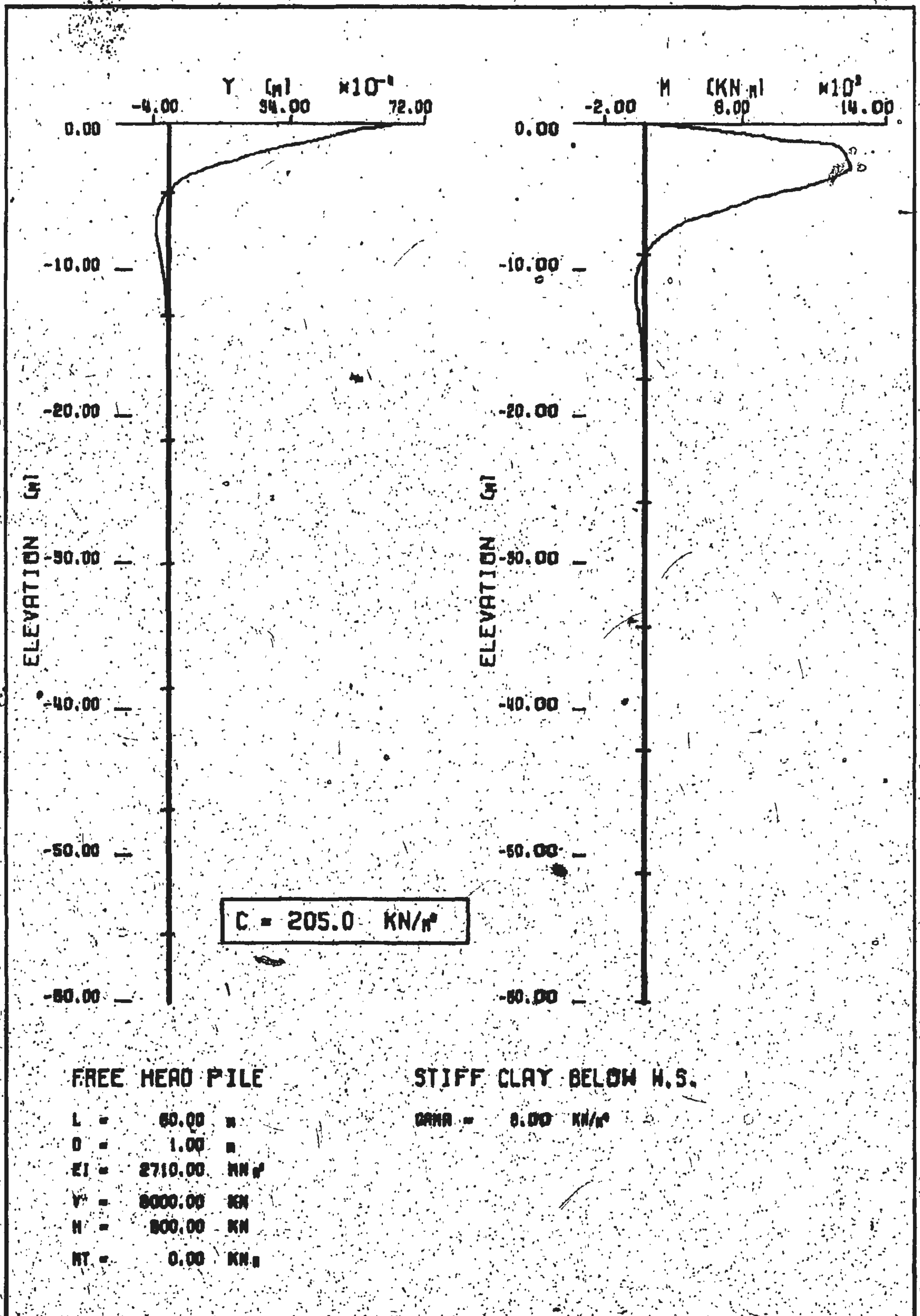


Figure 118 Deflection and bending moment for a free head pile in stiff clay, C = 205.0 kN/m^2.

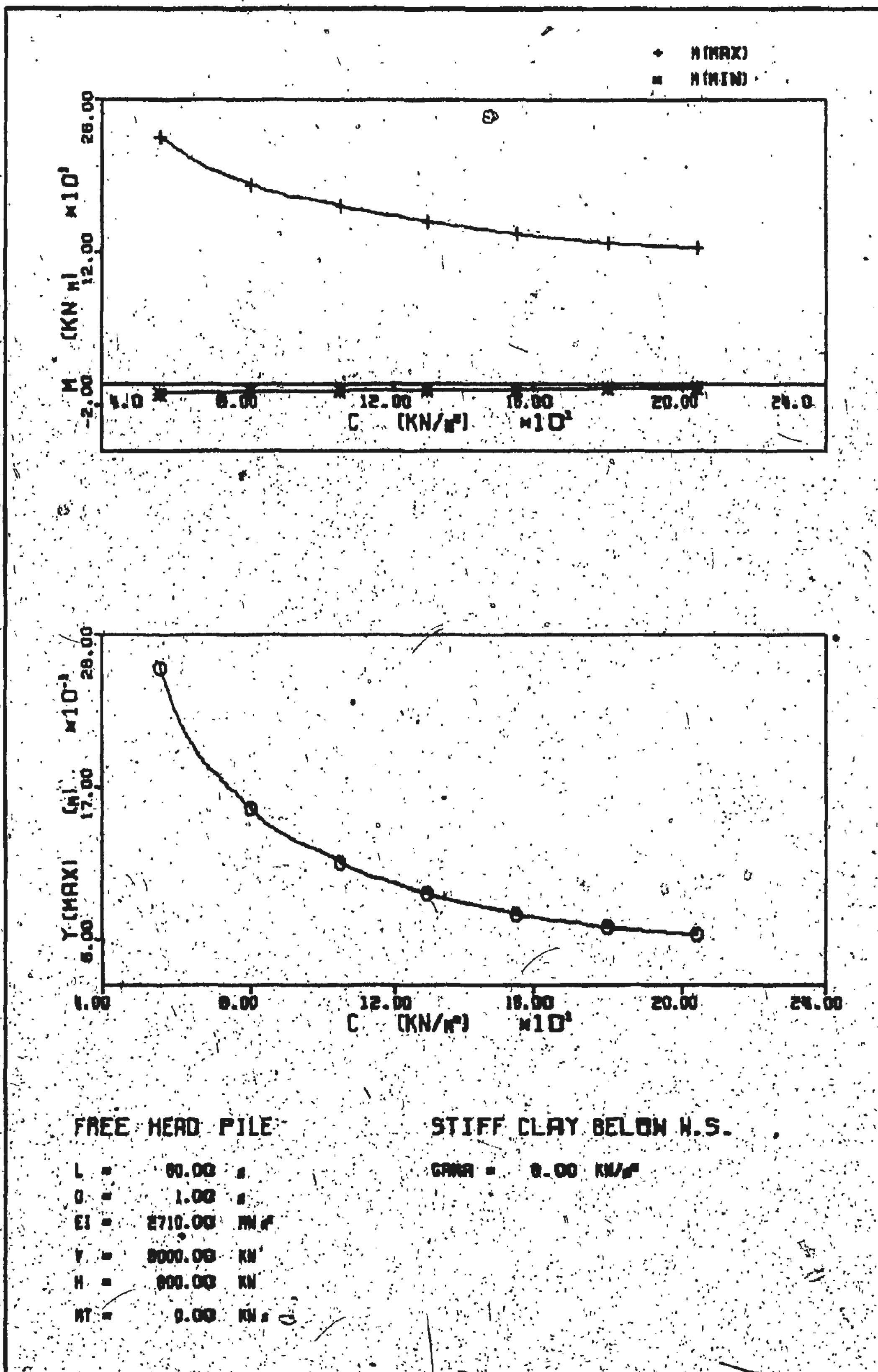


Figure 119 Effect of the soil undrained shear strength on the maximum deflection and the maximum bending moment for a free head pile in stiff clay.

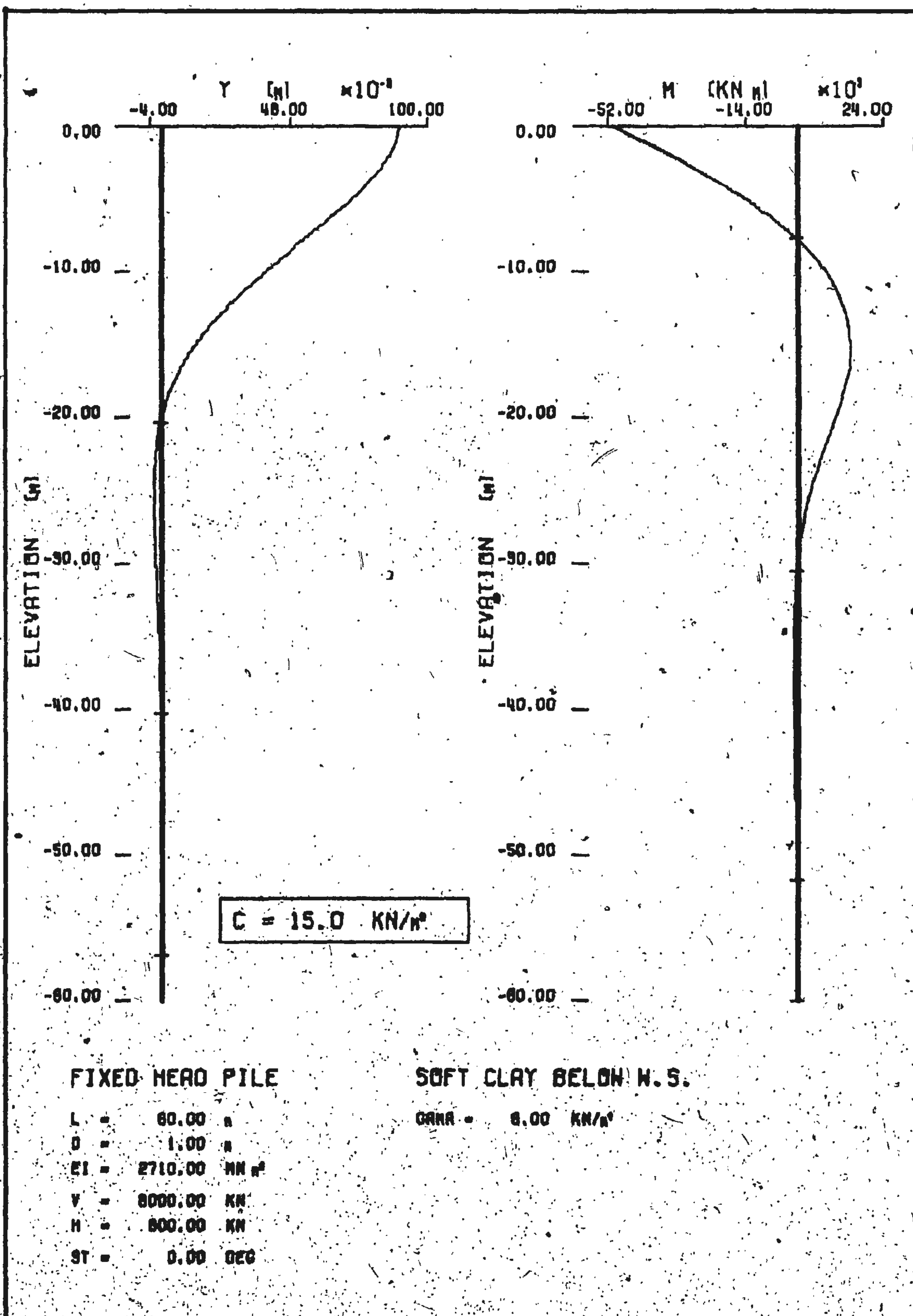


Figure 120 Deflection and bending moment for a fixed head pile in soft clay, C = 15.0 kN/m².

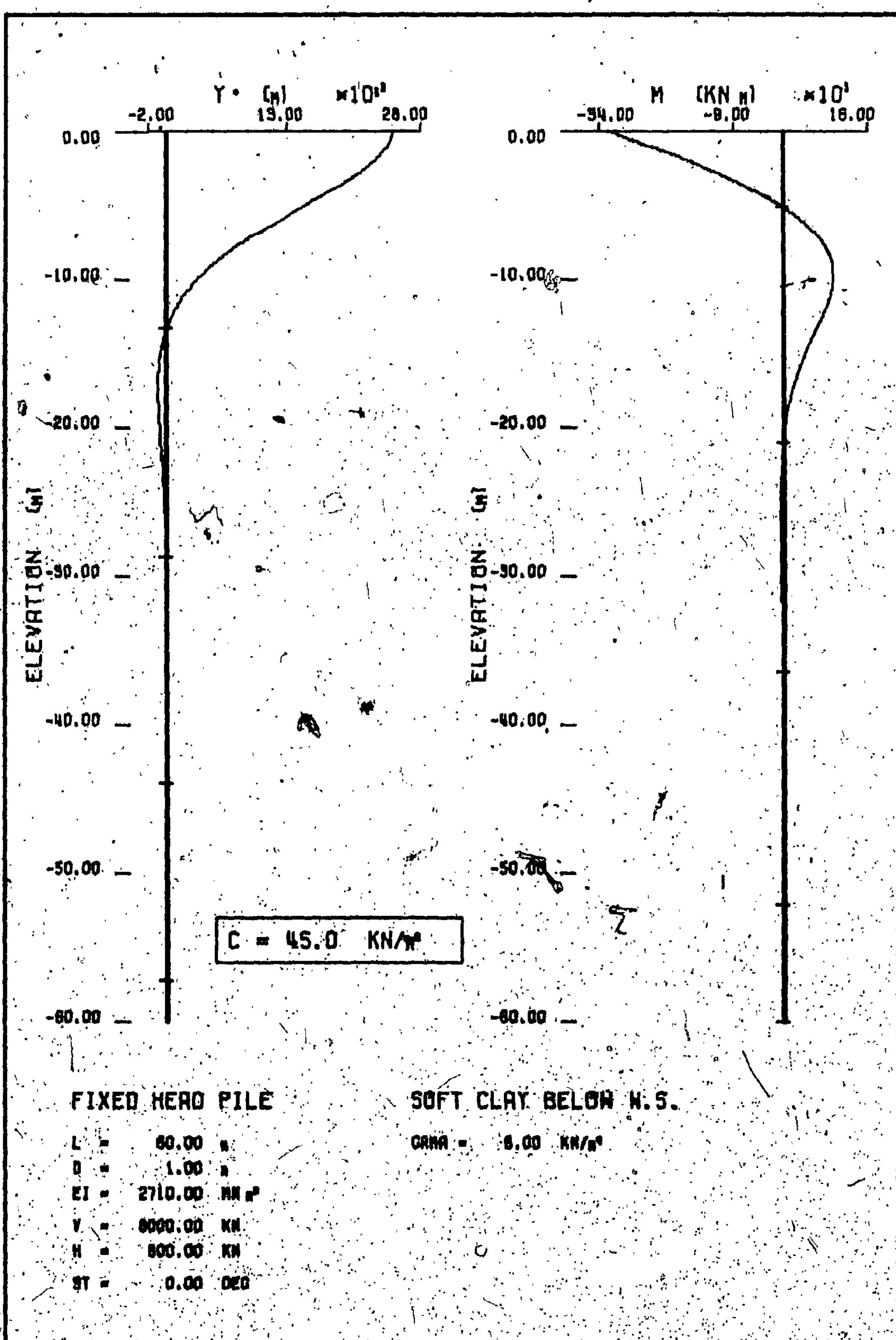


Figure 121 Deflection and bending moment for a fixed head pile in soft clay, C = 45.0 kN/m².

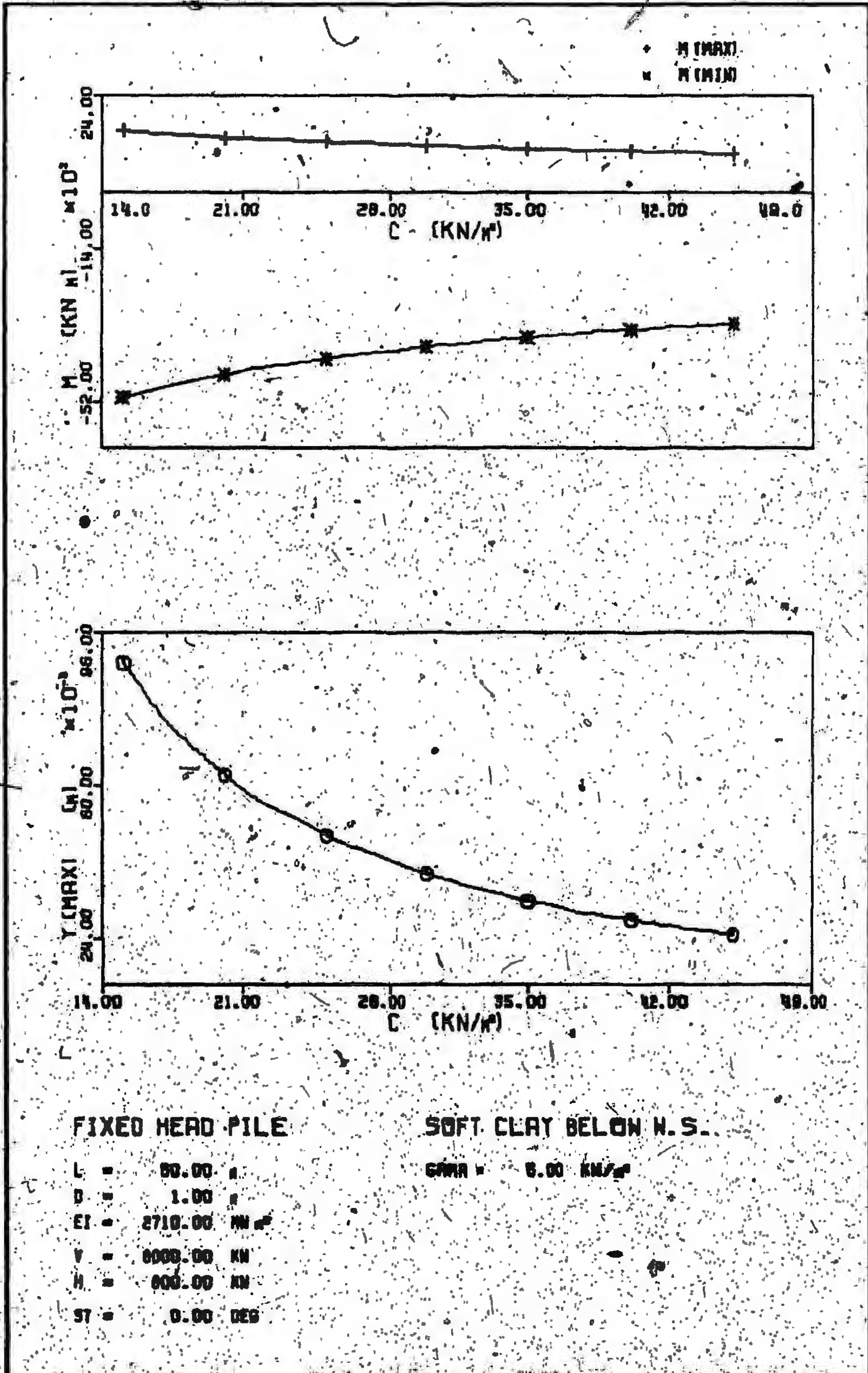


Figure I22. Effect of the soil undrained shear strength on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay.

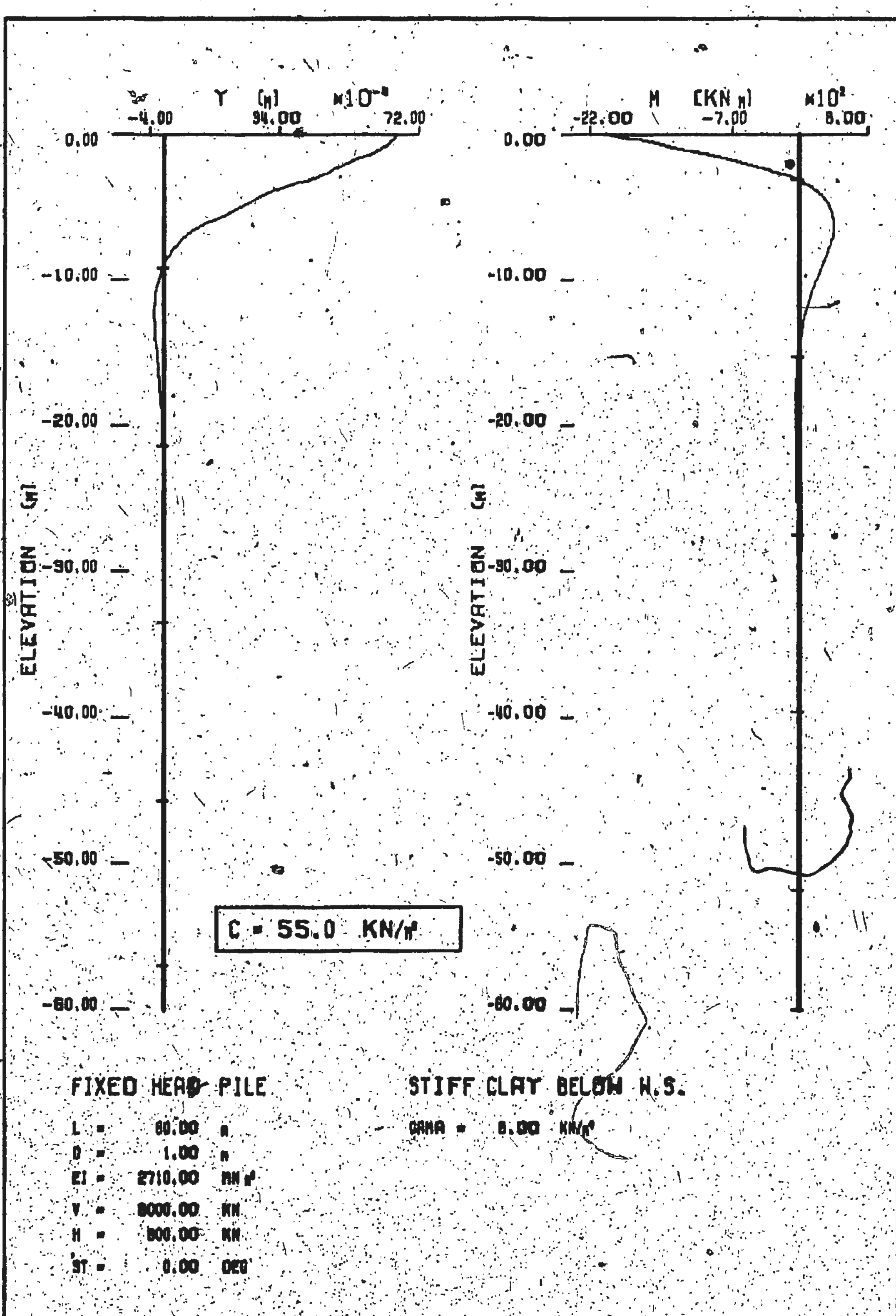


Figure 123 Deflection and bending moment for a fixed head pile in stiff clay, C = 55.0 kN/m^2.

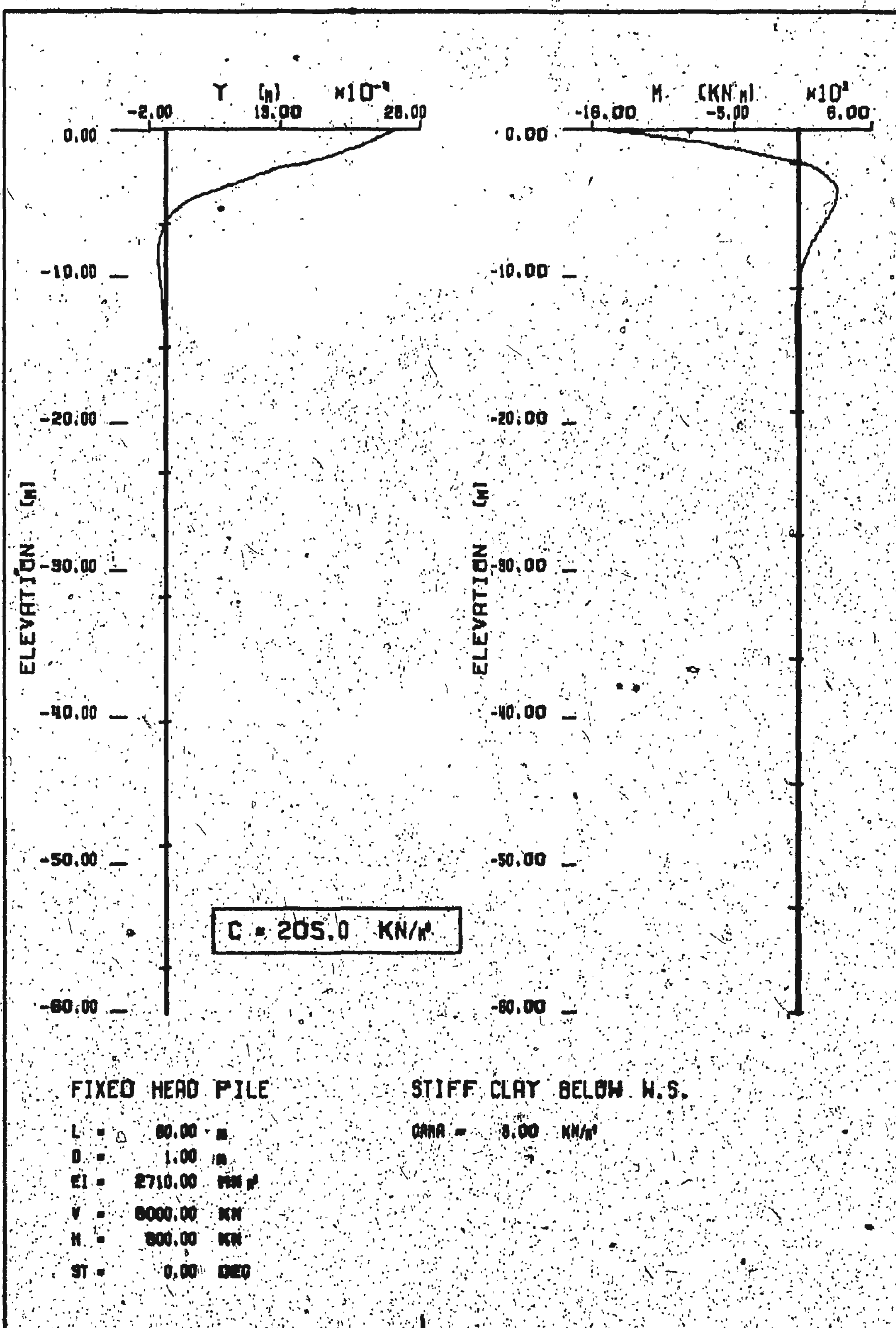


Figure 124 Deflection and bending moment for a fixed head pile in stiff clay, C = 205.0 kN/m².

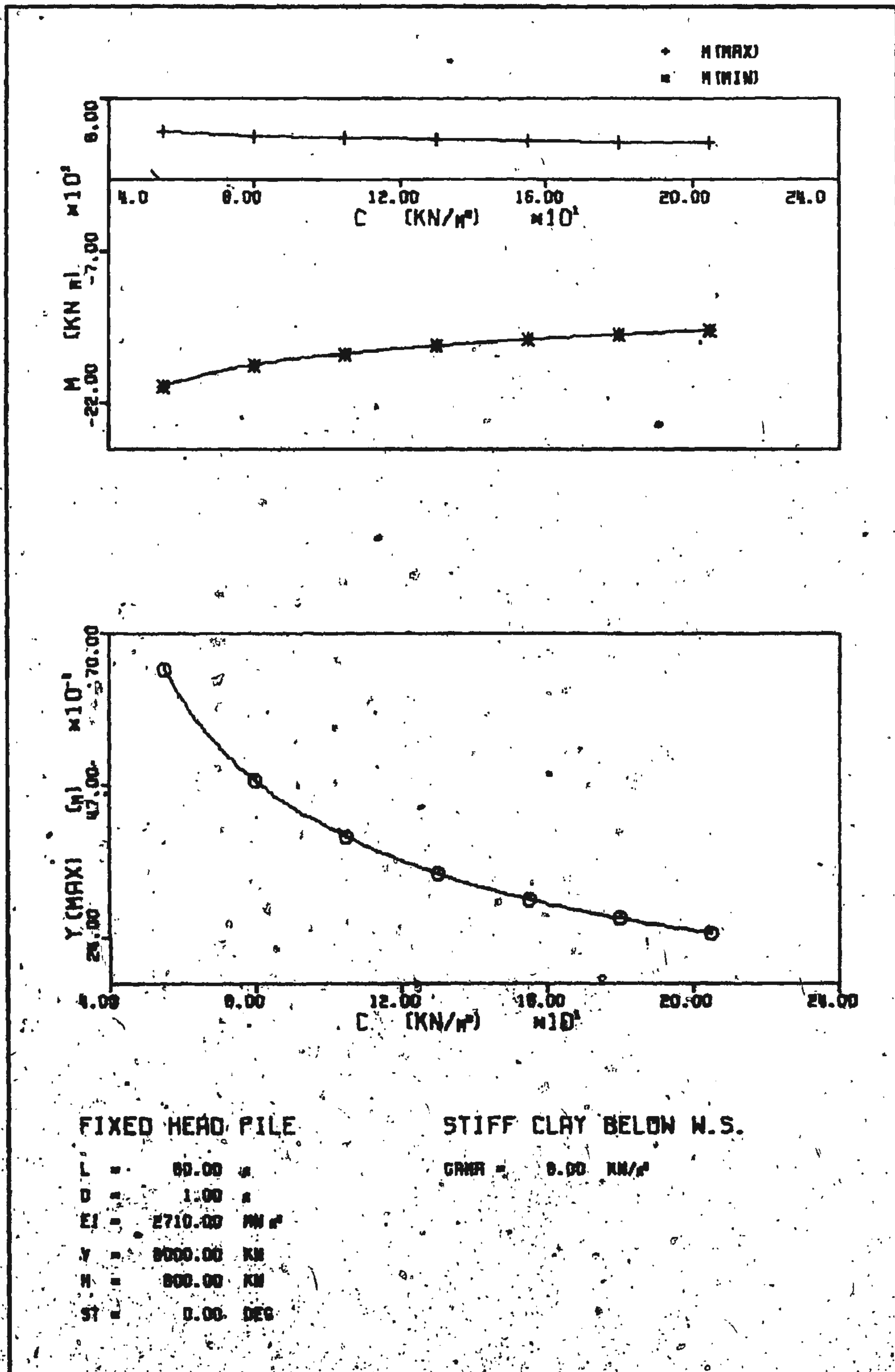


Figure 125 Effect of the soil undrained shear strength on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay.

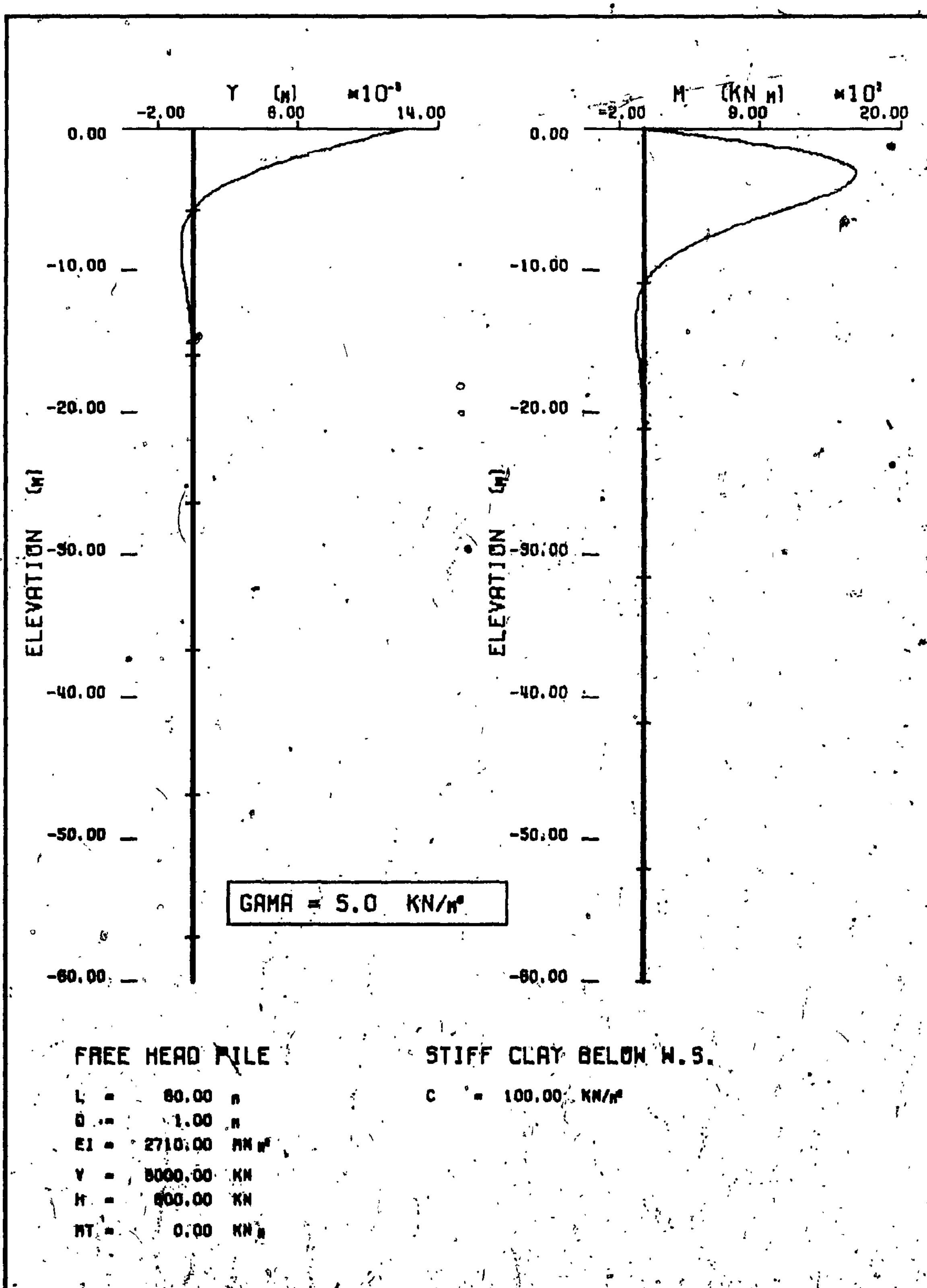


Figure 126. Deflection and bending moment for a free head pile in stiff clay, $\gamma' = 5.0$ kN/m³.

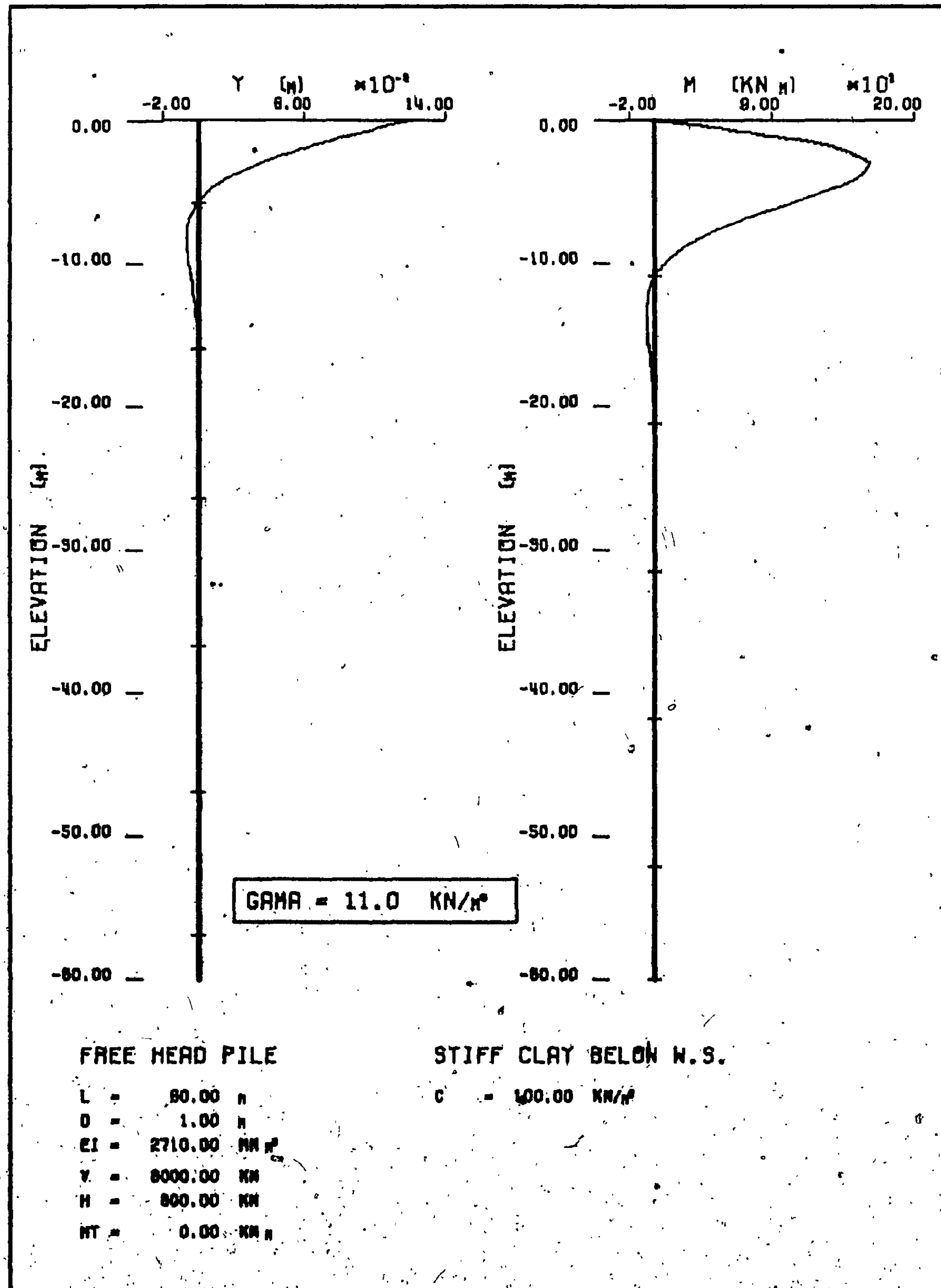


Figure 127 Deflection and bending moment for a free head pile in stiff clay, $\gamma' = 11.0$ kN/m^3.

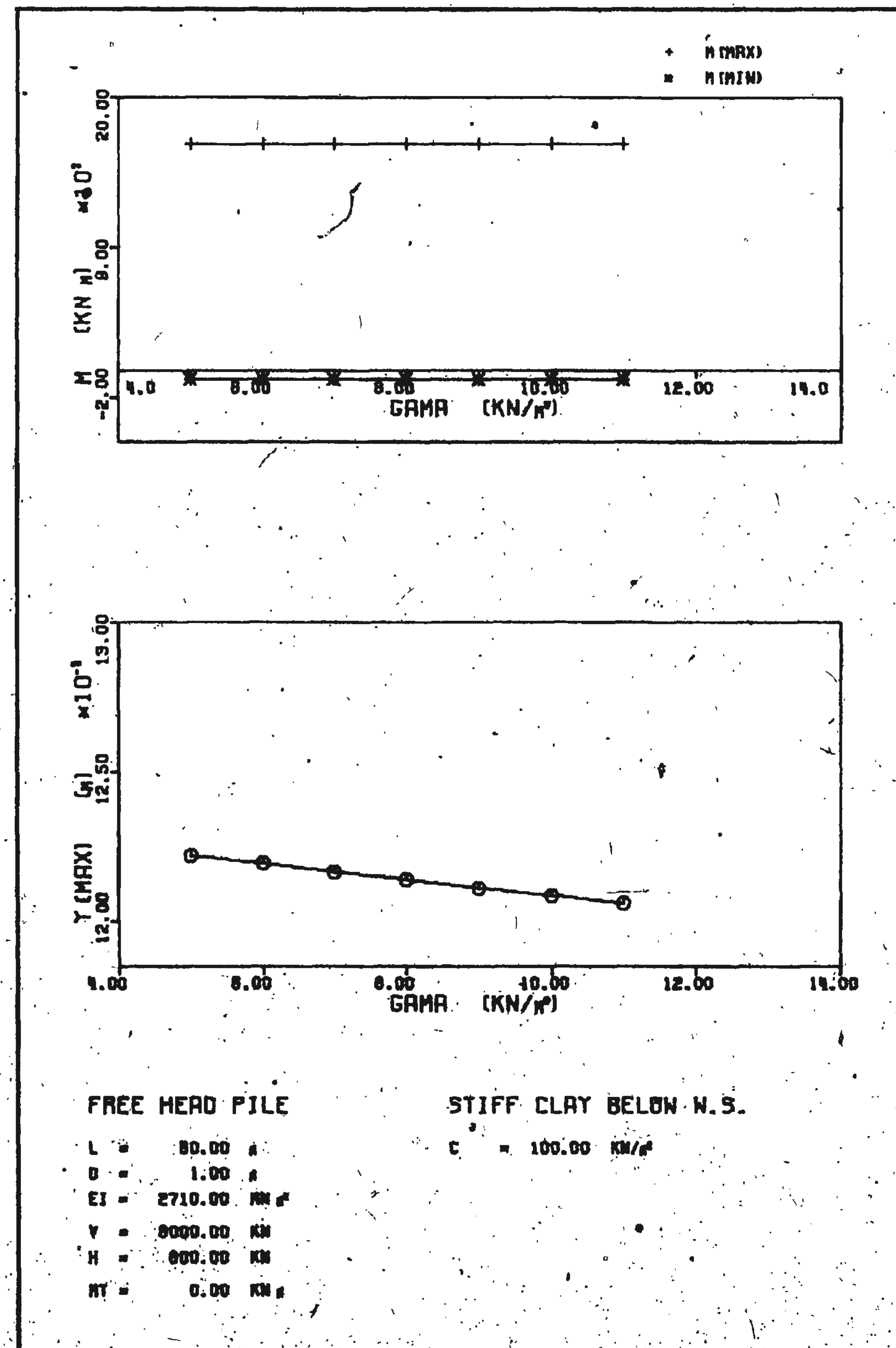


Figure 128 Effect of the soil unit weight on the maximum deflection and the maximum bending moment for a free head pile in stiff clay.

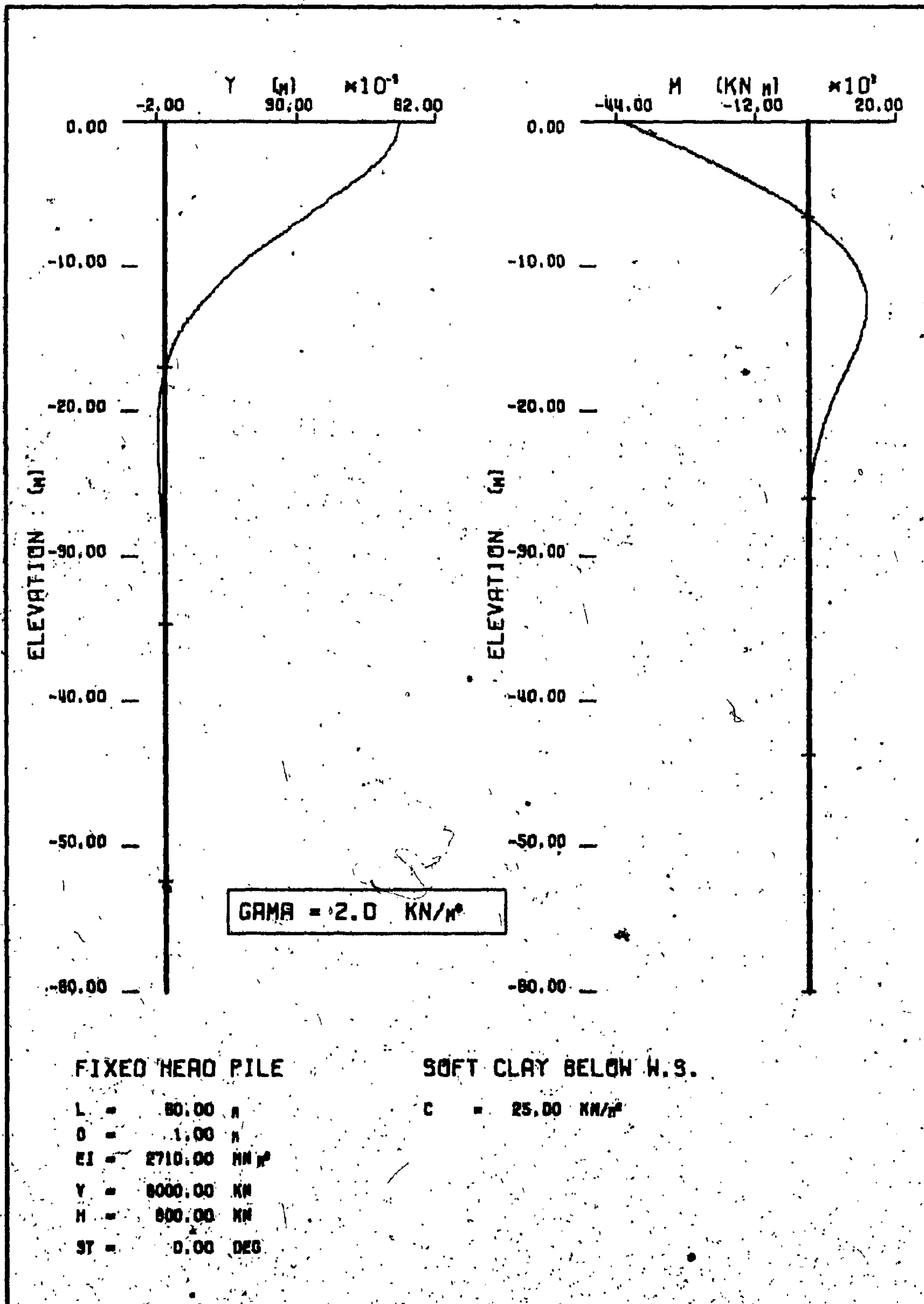


Figure 129 Deflection and bending moment for a fixed head pile in soft clay, $\gamma' = 2.0$ kN/m^3.

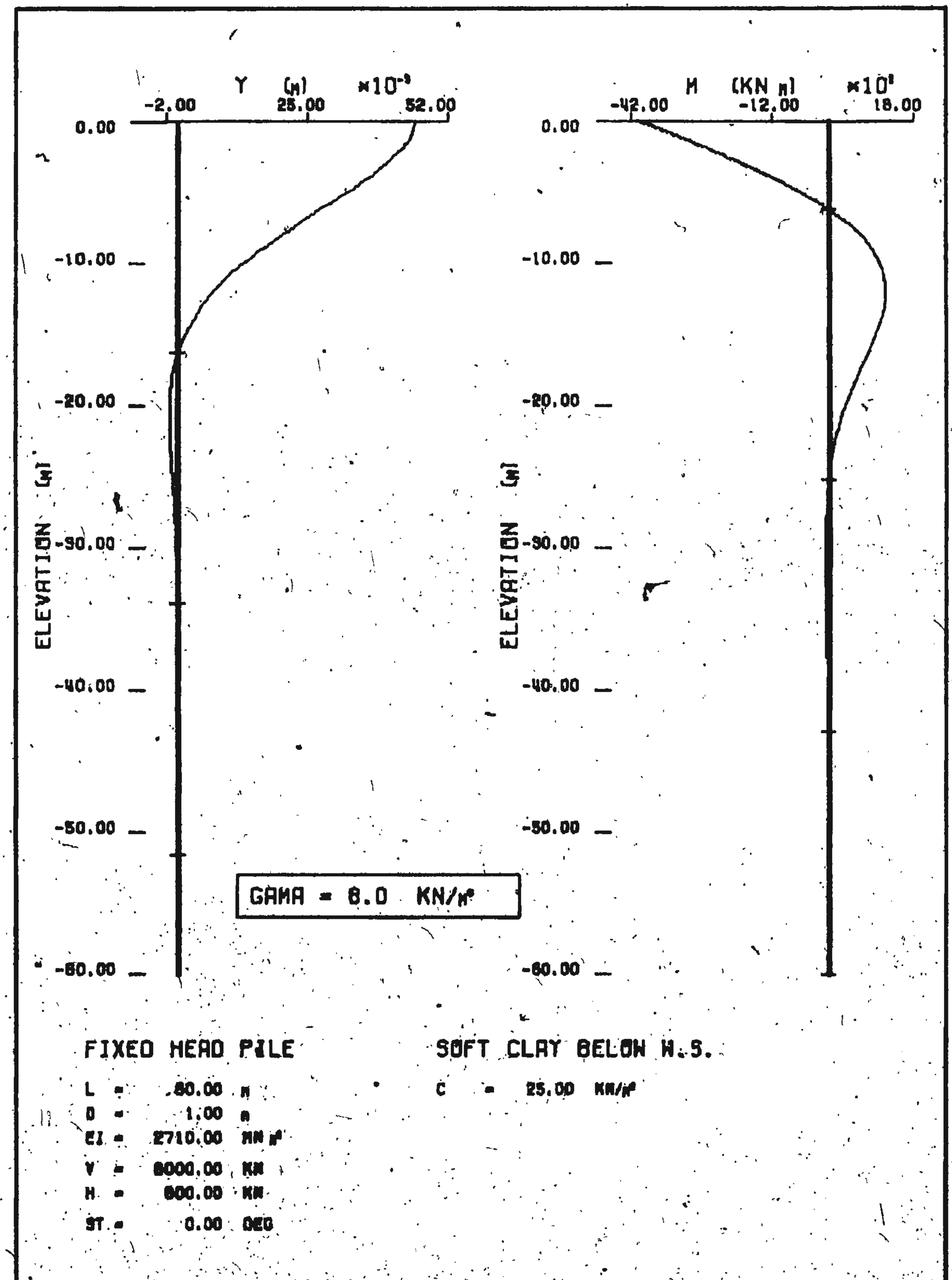


Figure 130 Deflection and bending moment for a fixed head pile in soft clay, $\gamma' = 8.0$ kN/m^3.

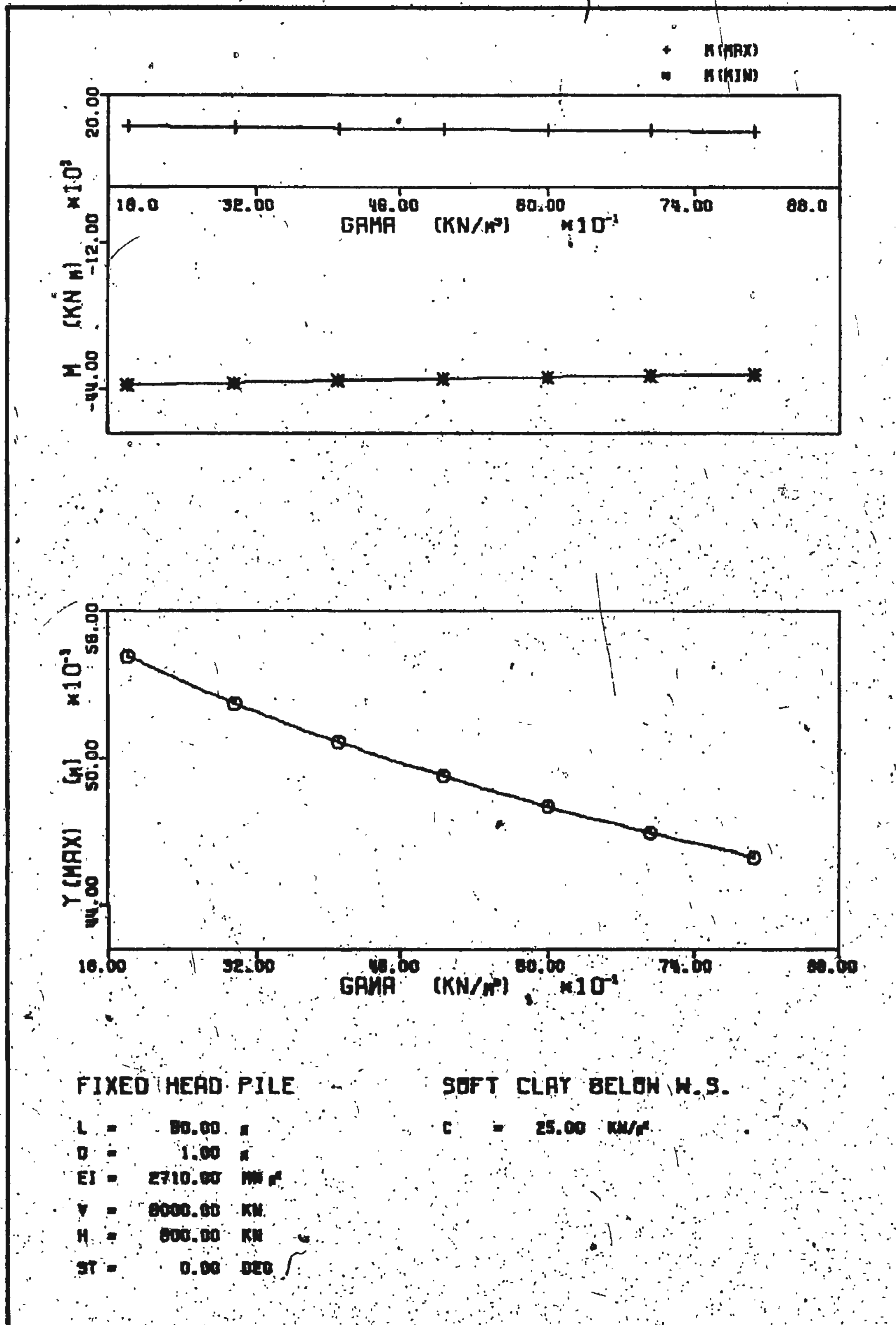


Figure 131 Effect of the soil unit weight on the maximum deflection and the maximum bending moment for a fixed head pile in soft clay.

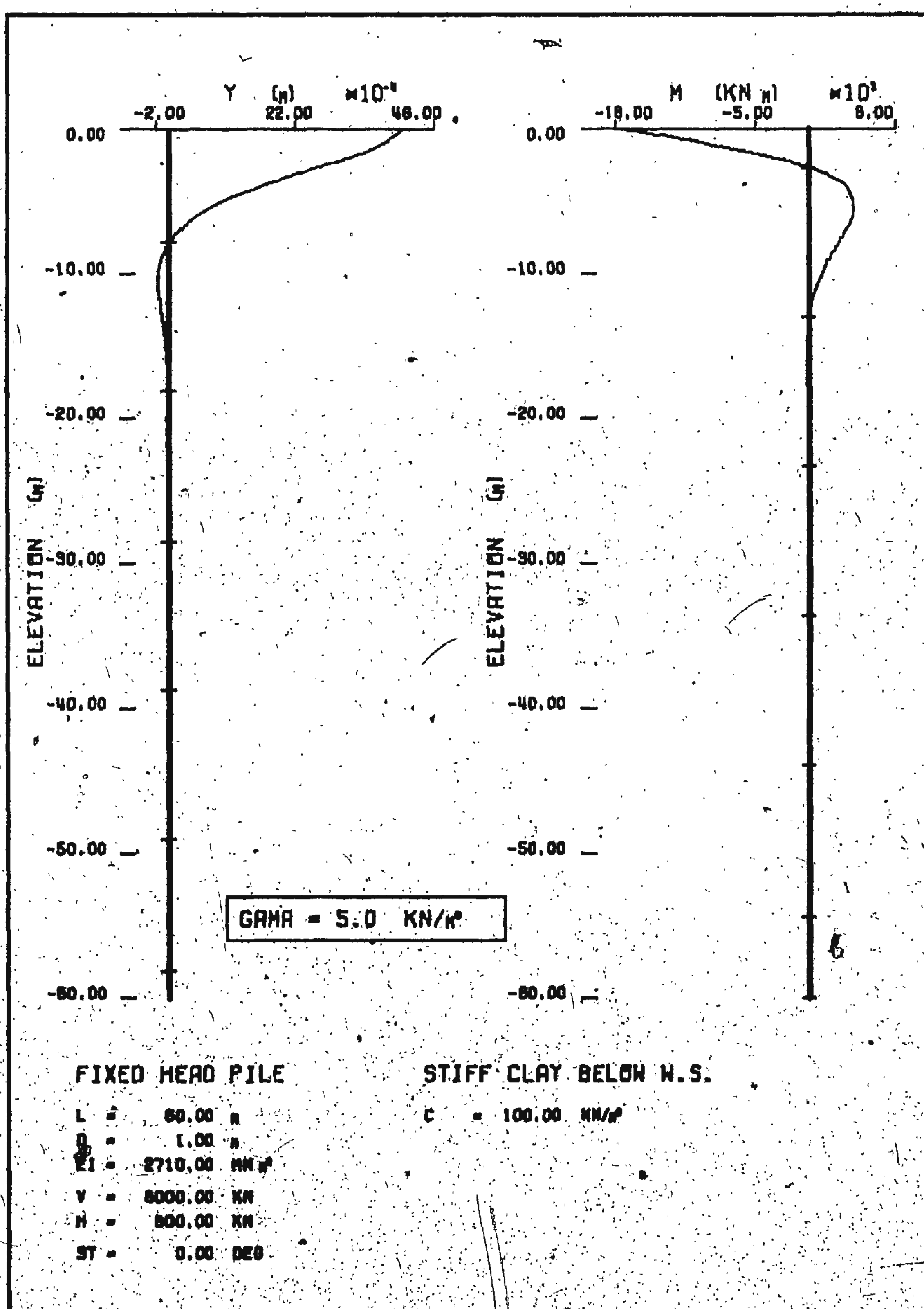


Figure 132 Deflection and bending moment for a fixed head pile in stiff clay, $\gamma' = 5.0$ kN/m³.

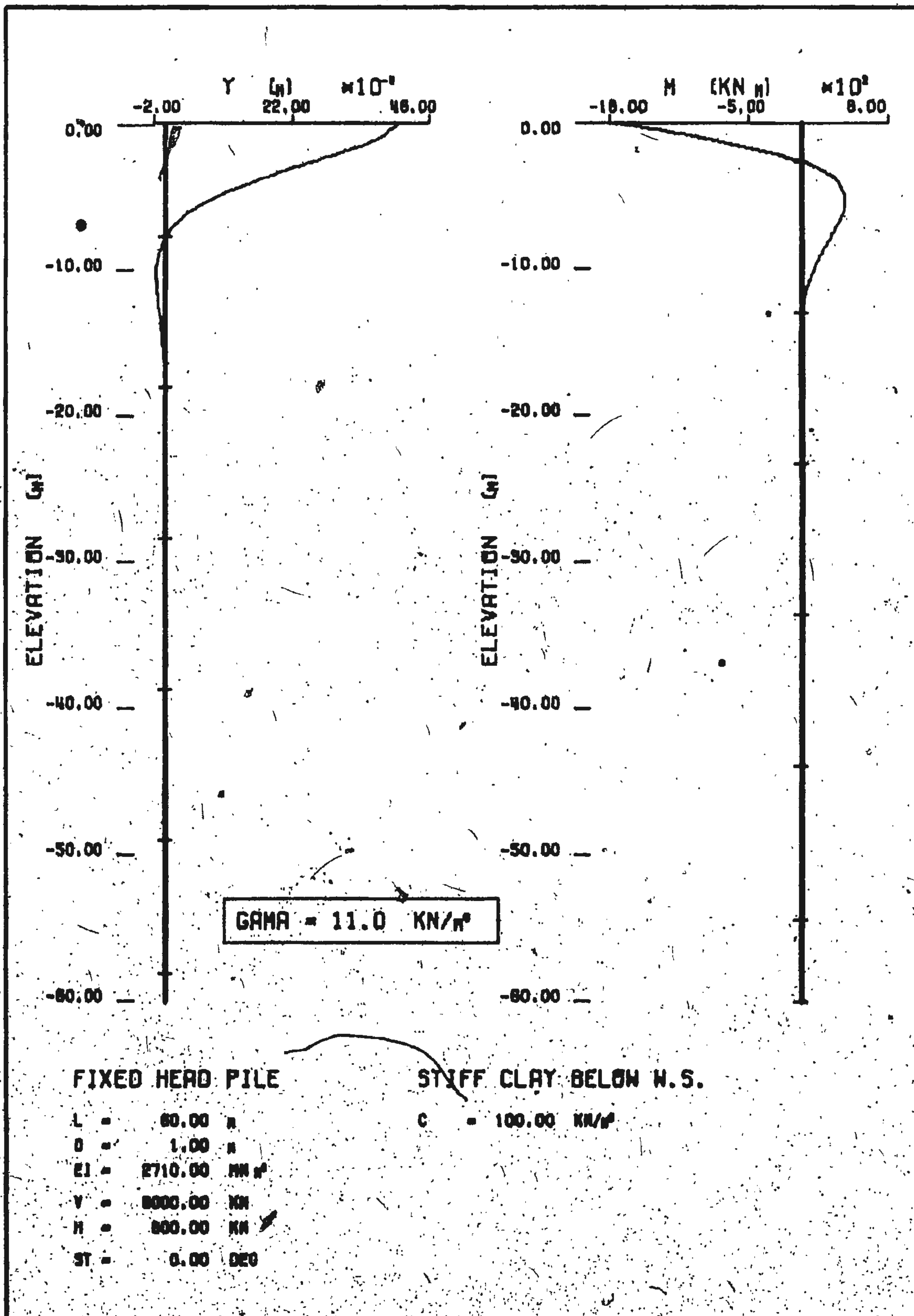


Figure 133 Deflection and bending moment for a fixed head pile in stiff clay, $\gamma' = 11.0$ kN/m^3.

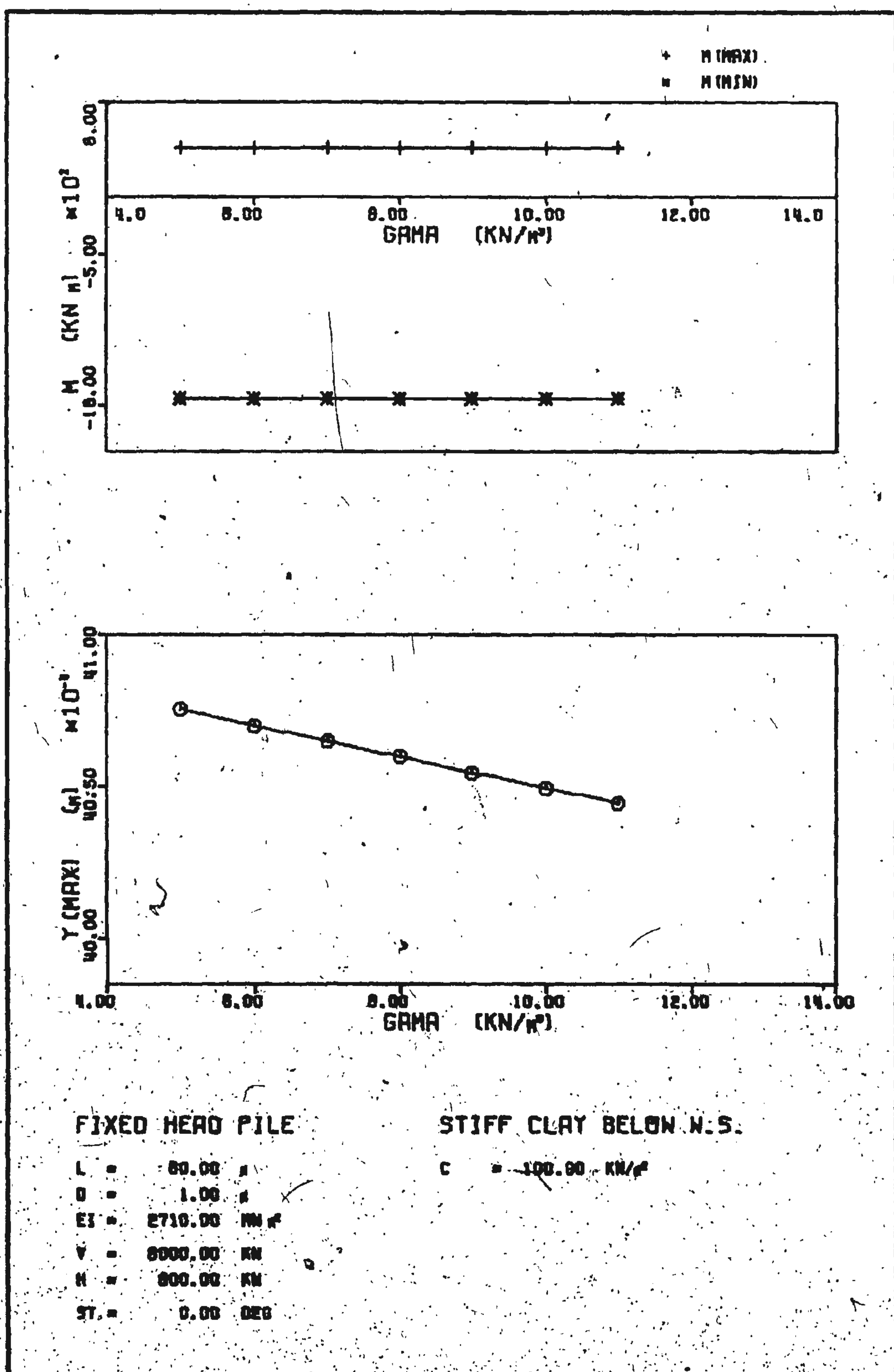


Figure 134 Effect of the soil unit weight on the maximum deflection and the maximum bending moment for a fixed head pile in stiff clay.

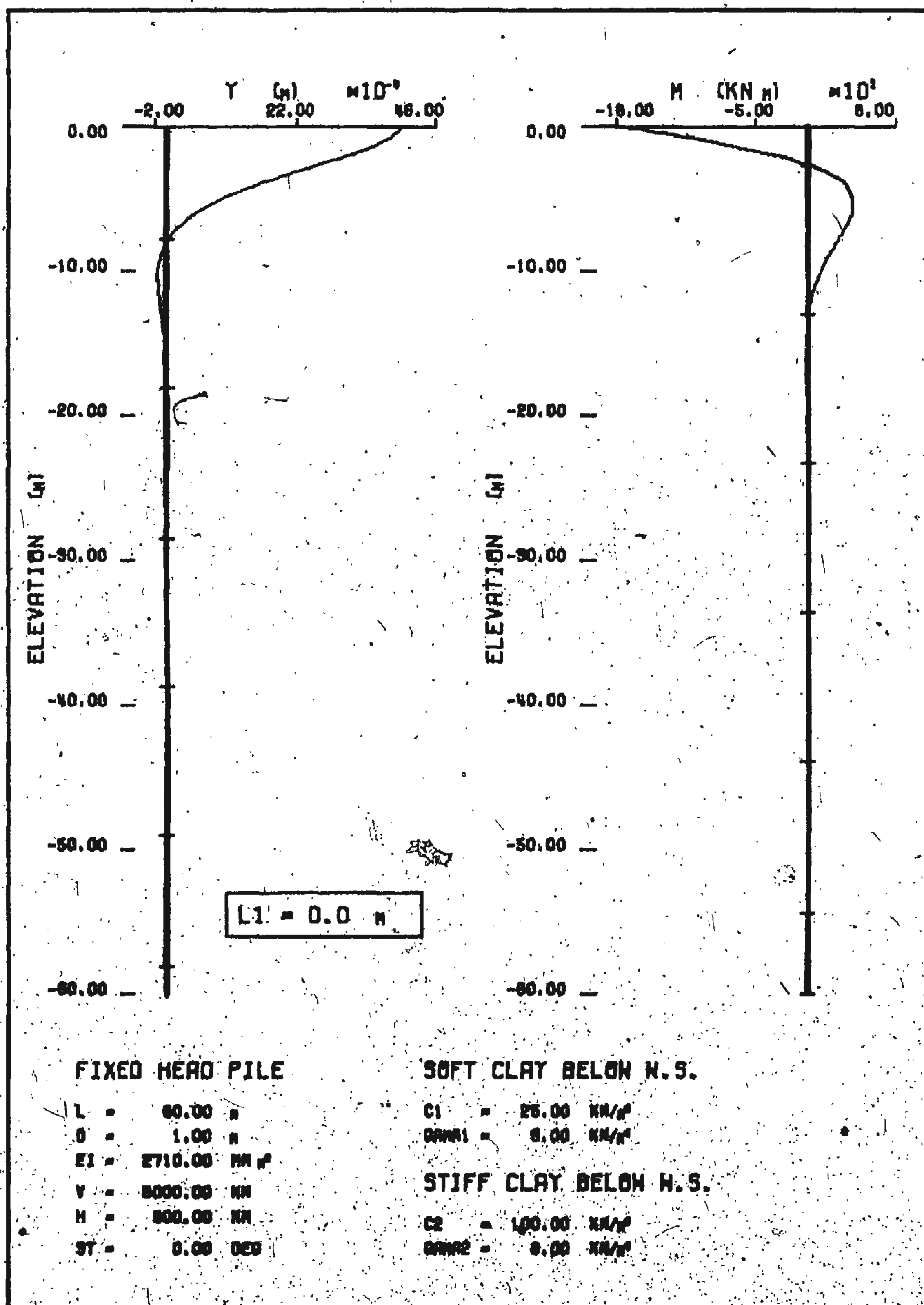


Figure 135 Deflection and bending moment for a fixed head pile in a layered soil system, L1 = 0.0 m.

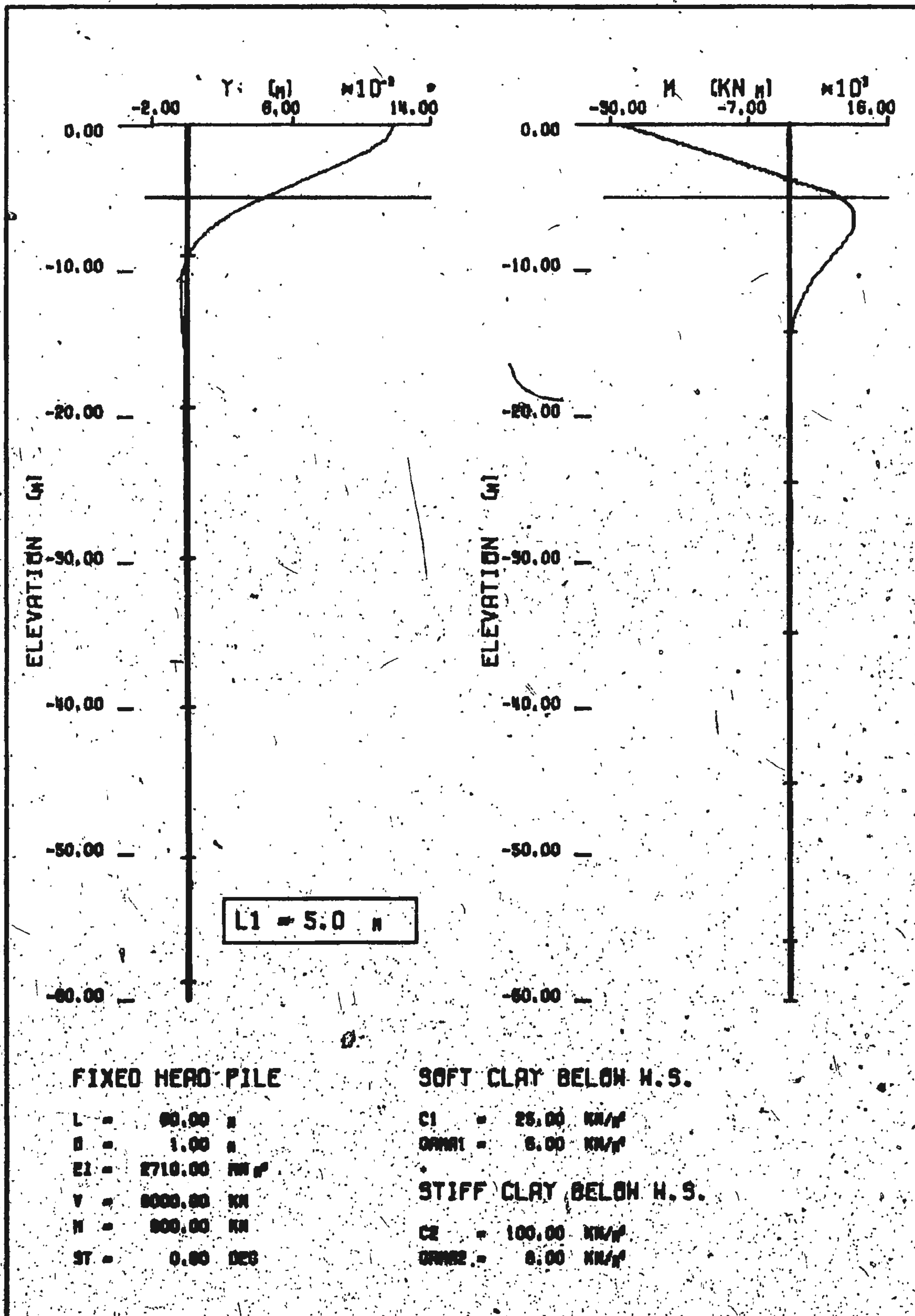


Figure 136 Deflection and bending moment for a fixed head pile in a layered soil system, L1 = 5.0 m.

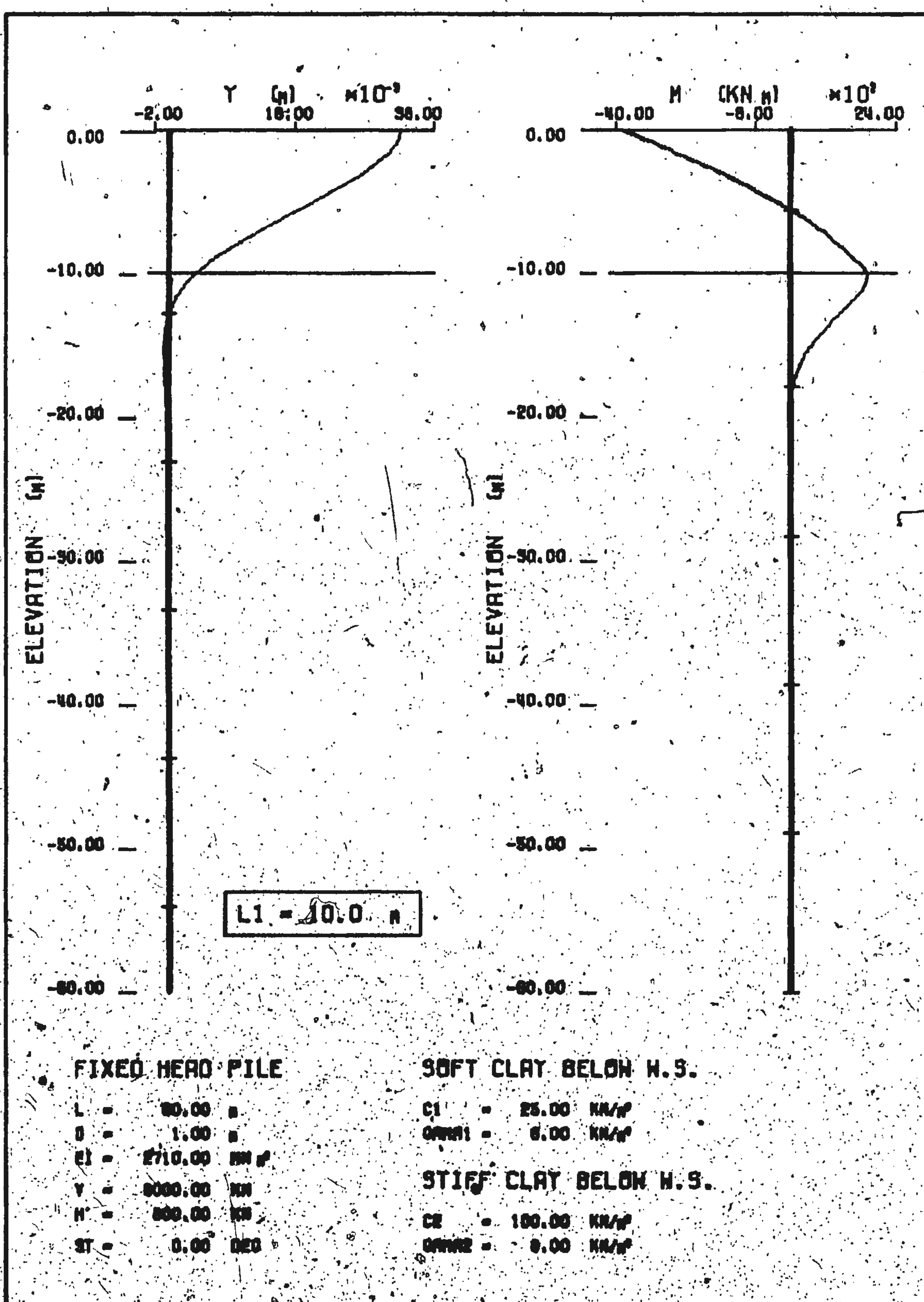


Figure 137 Deflection and bending moment for a fixed head pile in a layered soil system, L1 = 10.0 m

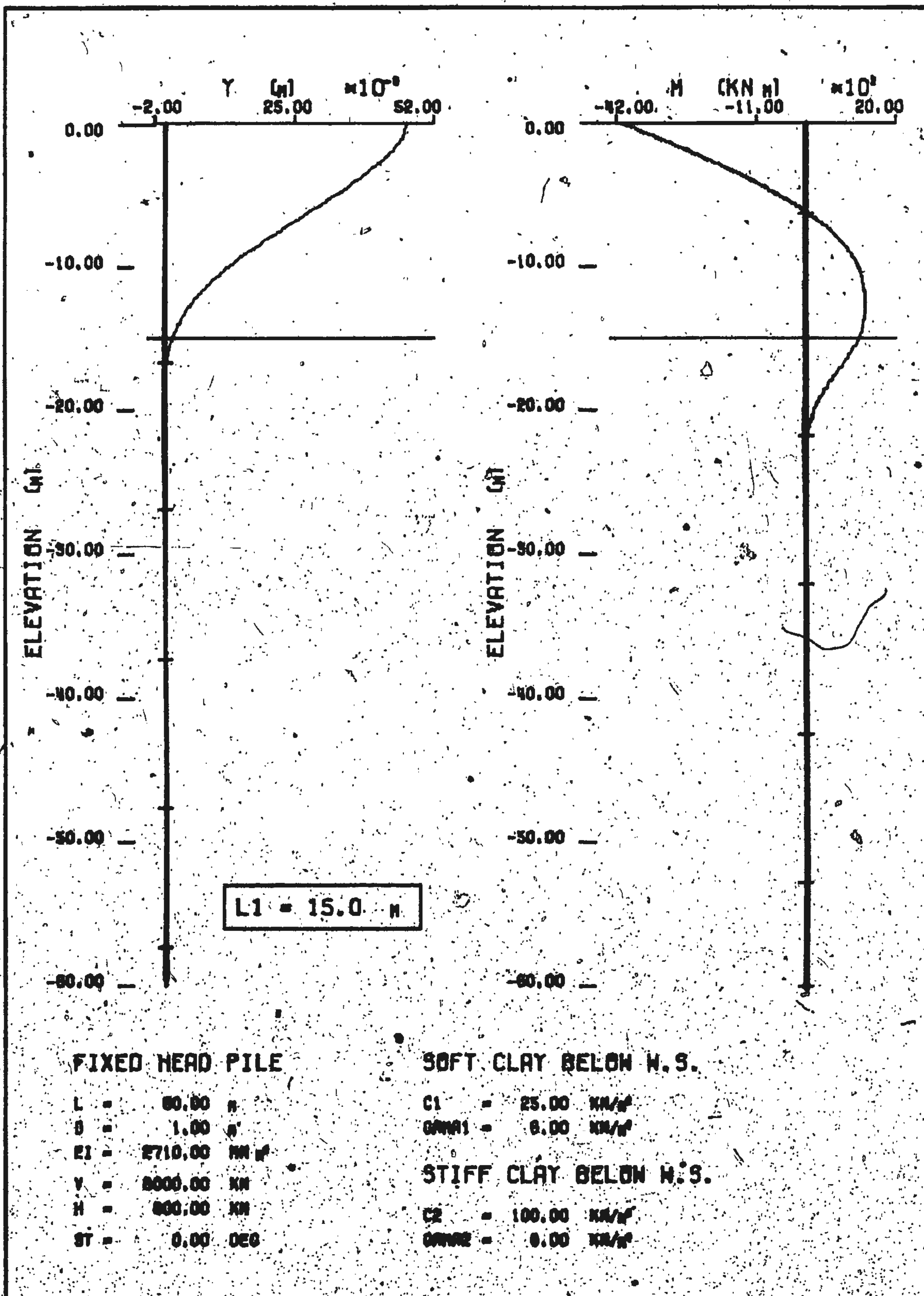


Figure 138 Deflection and bending moment for a fixed head pile in a layered soil system, L1 = 15.0 m.

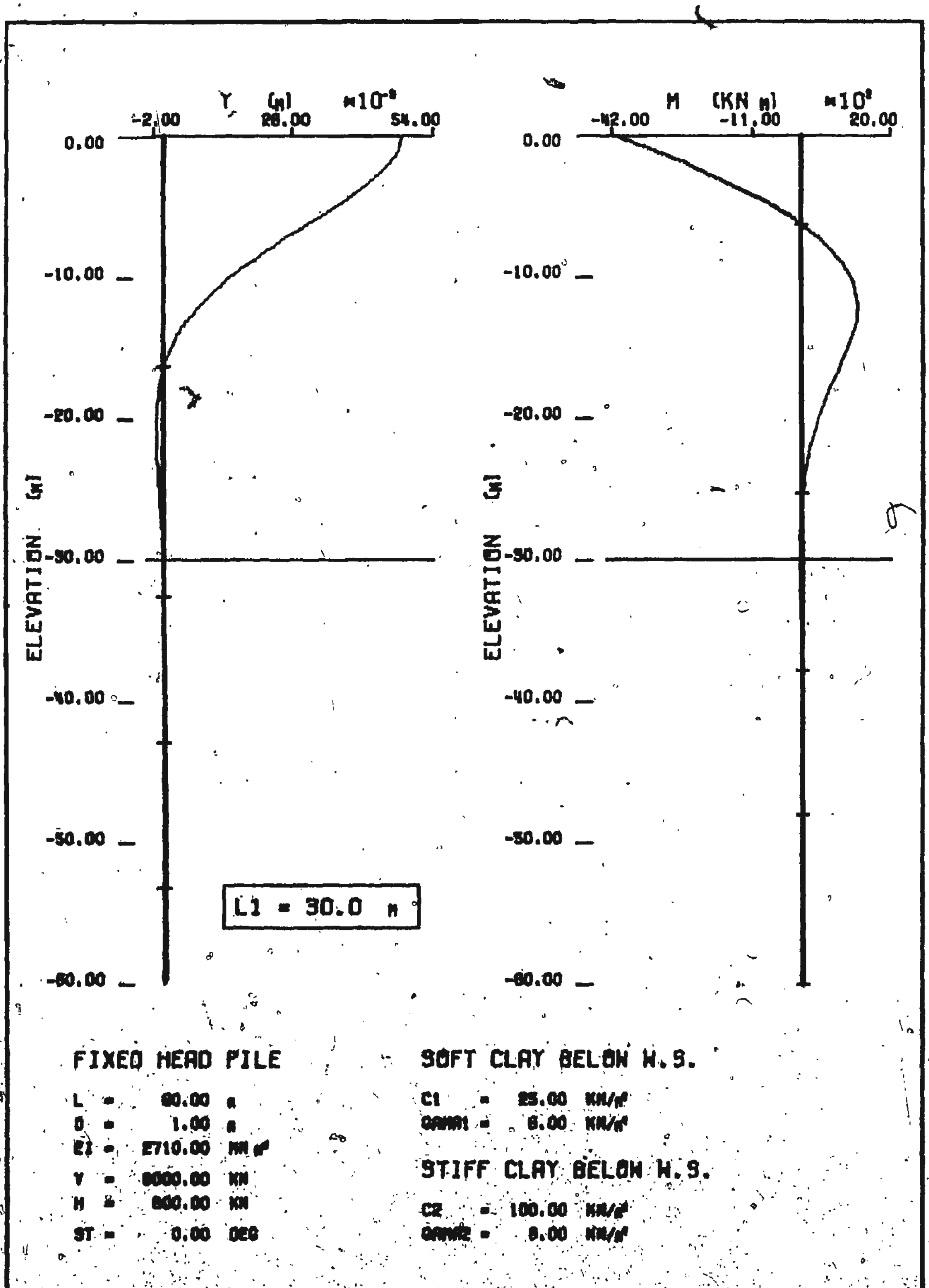


Figure 139 Deflection and bending moment for a fixed head pile in a layered soil system, L1 = 30.0 m.

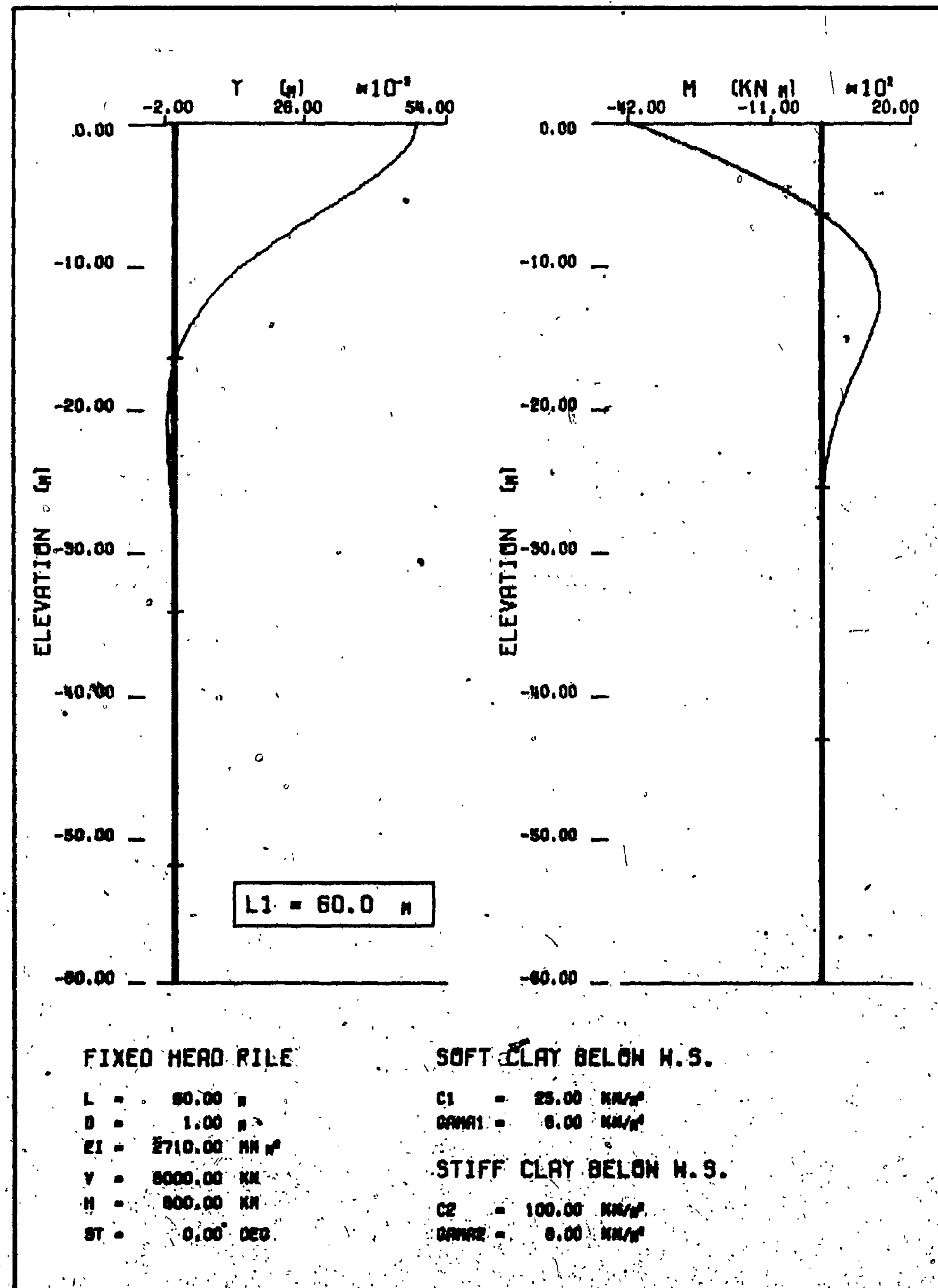


Figure 140 Deflection and bending moment for a fixed head pile in a layered soil system, L1 = 60.0 m.

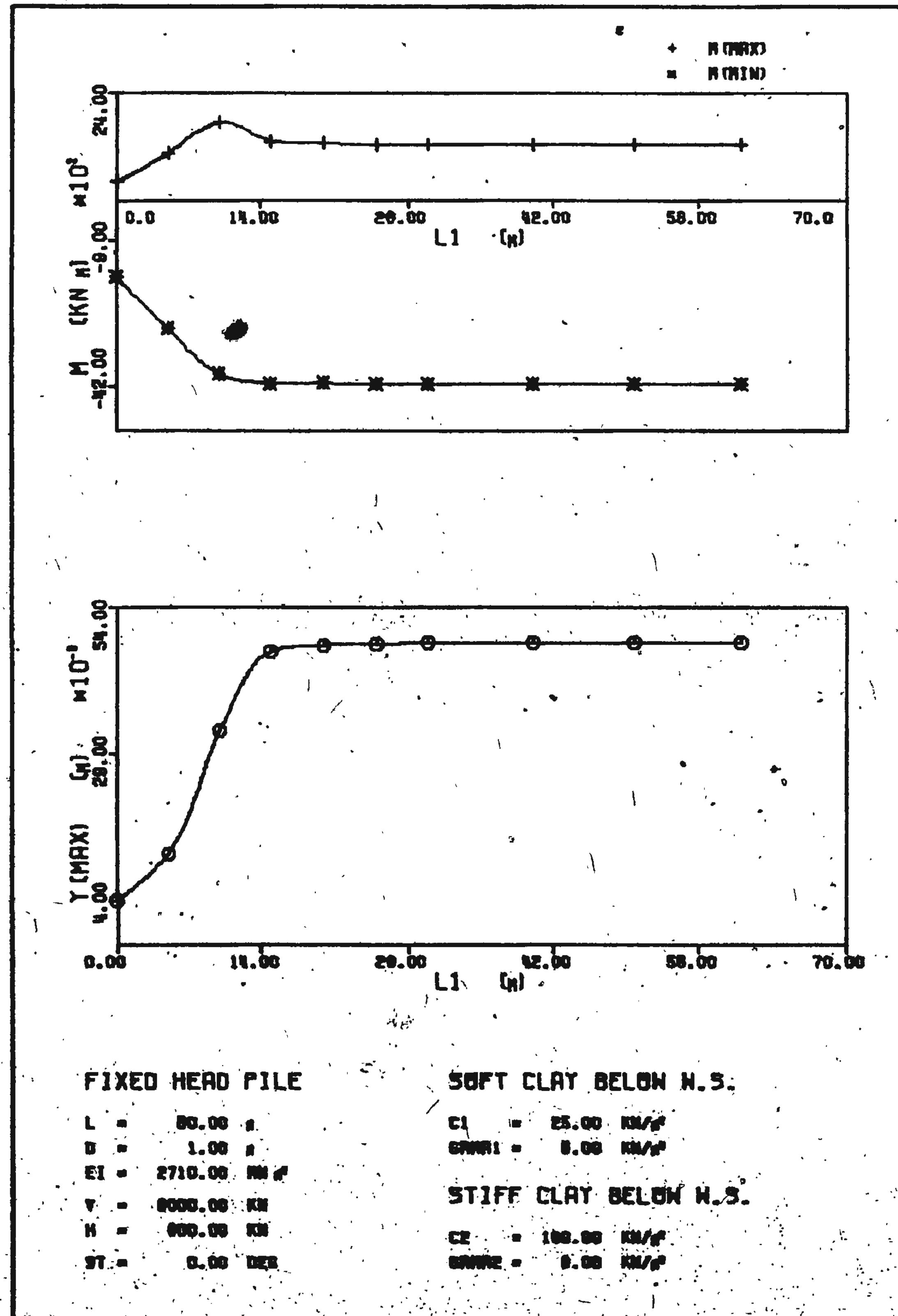
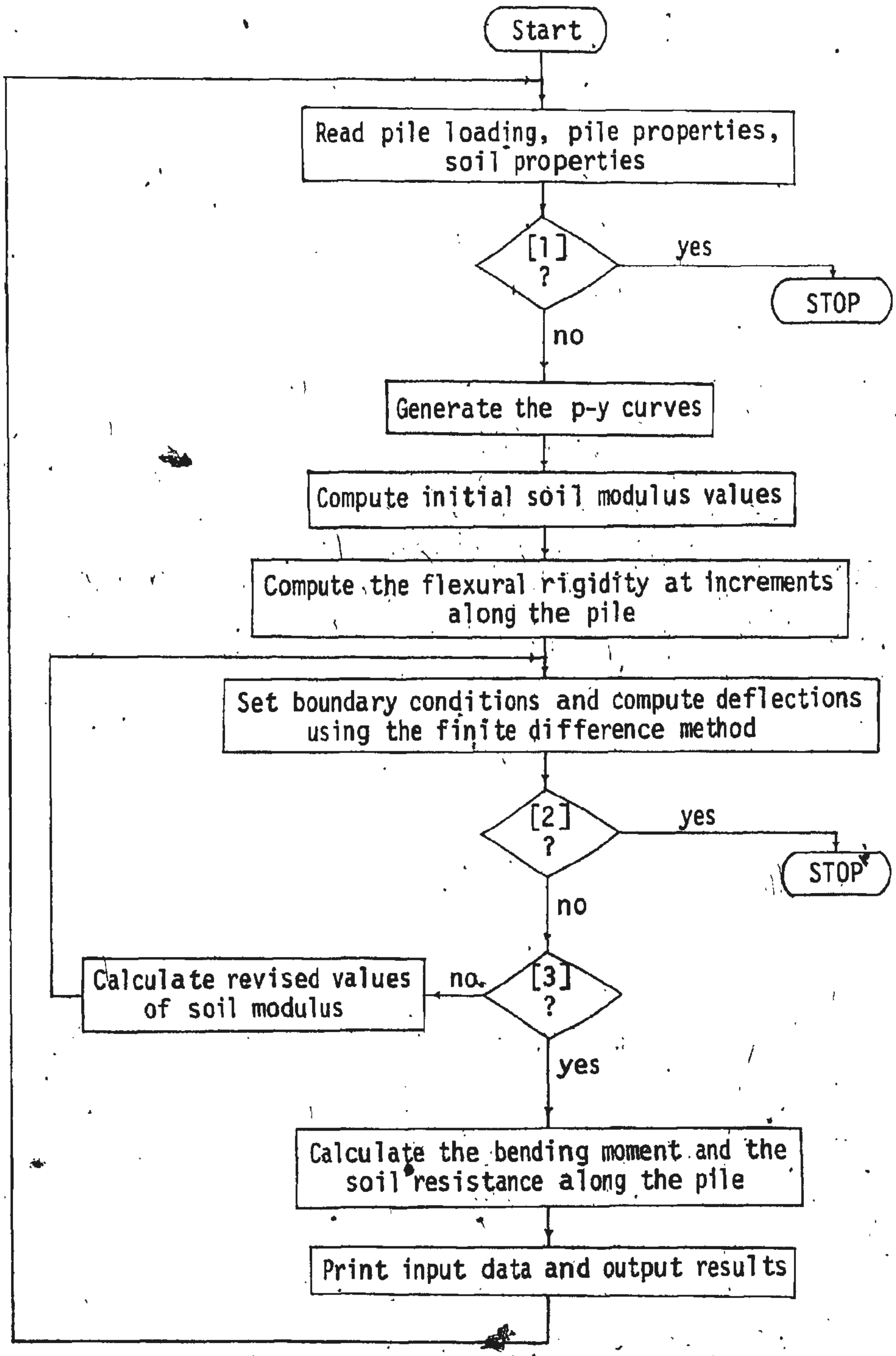


Figure 141 Effect of the soft clay layer depth on the maximum deflection and the maximum bending moment for a fixed head pile in a layered soil system.

APPENDIX C

COMPUTER PROGRAMS

C.I FLOW CHART FOR THE FINITE DIFFERENCE PROGRAM



[1] Termination
[2] Greater than 150 iterations or Excessive deflection
[3] Deflection within iteration tolerance

C.II FINITE ELEMENT PROGRAM (INCLUDING THE SUBROUTINES FOR GENERATING THE p-y CURVES)

```
C
C
C
C
C     *********************************************************************
C     *                                                                   *
C     *                                                                   *
C     *                       FINITE ELEMENT SOLUTION                     *
C     *                                                                   *
C     *                                 FOR                               *
C     *                                                                   *
C     *                      A LATERALLY LOADED PILE                      *
C     *                                                                   *
C     *                                                                   *
C     *********************************************************************
C
C
      IMPLICIT REAL * 8 (A-H , O-Z)
      REAL * 4 XA(31),Y1(31),Y2(31)
      INTEGER TITLE(20)
C
      DIMENSION  A(62,91),S(91,2),SAT(91,62),ASAT(62,62)
      DIMENSION  P(62),X(62),F(91)
      DIMENSION  SMOD(31),DISP(31),SOILR(31),BM(31),ELEV(31)
      DIMENSION  ASAT1(62,62),WKAREA(62)
      DIMENSION  EIP(3),DISTEI(3),NLELM(3)
C
      COMMON /BLOC01/ PL,PD,DEPTH(5),C(5),GAMA(5),EPS50(5),NXL(5),
     1                LKODE(5),NL
      COMMON /BLOC02/ NX,NUM,XX(25),YY(25,25),PP(25,25)
C
C
  100 CONTINUE
      READ (5,5,END=440) (TITLE(I),I=1,20)
      READ (5,6) PL,PD,NELM,NEWPY
      READ (5,10) BC1,BC2
      READ (5,15) NEI
      DO 105 I=1,NEI
  105 READ (5,6) EIP(I),DISTEI(I)
      IF (NEWPY .EQ. 0) GO TO 141
      READ (5,15) NL
      IF (NL .EQ. 1) GO TO 120
      NLM1 = NL - 1
      READ (5,10) (DEPTH(I) ,I=1,NLM1)
      SD = 0.0
      DO 110 I=1,NLM1
  110 SD = SD + DEPTH(I)
      DEPTH(NL) = PL - SD
      GO TO 130
  120 DEPTH(NL) = PL
  130 DO 140 I=1,NL
  140 READ (5,10) C(I),GAMA(I),EPS50(I),NXL(I),LKODE(I)
      CALL PYCURV
C
```

```
C
  141 CONTINUE
      WRITE  (6,20) (TITLE(I), I=1,20)
      WRITE  (7,5) (TITLE(I), I=1,20)
      WRITE  (7,25) PL,PD,BC1,BC2
      WRITE  (6,30)
C
C
      M = 3 * NELM + 1
      N = 2 * NELM + 2
      NELM1  = NELM + 1
      NELM2  = NELM + 2
      NELM3  = NELM + 3
      NELMD  = NELM + NELM
      NELMD1  = NELMD + 1
      NM1  =  N - 1
      TOL  =  0.100D-04
      H = PL / NELM
C
C
C     COMPUTE INITIAL SOIL MODULUS VALUES
C
      DO 145 I=1,NELM1
  145 SMOD(I) = (I-1) * H * 100.
C
C     BUILD  A MATRIX
C
      DO 150 I=1,N
      DO 150 J=1,M
  150 A(I,J)  = 0.0
      A(1,1)  = 1.0
      A(NELM1,NELMD) = 1.0
      DO 160 I=2,NELM
      JS = 2 * I - 2
      JS1 = JS + 1
      DO 160 J=JS,JS1
  160 A(I,J) = 1.0
      J = NELMD1
      DO 170 I=NELM2,N
      A(I,J) = -1.0
  170 J = J + 1
      Z = 1.0 / H
      JJ = 1
      DO 180 I=NELM2,NM1
      DO 180 J=1,2
      A(I,JJ) = Z
  180 JJ = JJ + 1
      JJ = 1
      DO 190 I=NELM3,N
      DO 190 J=1,2
      A(I,JJ) = -Z
  190 JJ = JJ + 1
C
```

```
C       BUILD  P MATRIX
C
        DO 200 I=1 ,N
  200 P(I)  = 0.0
        P(1)  = BC2
        P(NELM2) =  BC1
C
C       BUILD  S MATRIX  IN TWO  COLUMNS
C
        DO 210 I=1,M
        DO 210 J=1,2
  210 S(I,J) = 0.0
        DO 211 I=1,NEI
  211 NLELM(I) = DISTEI(I) /  H
        NNS  = 1
        DO 221 K=1,NEI
        IF (K .NE. 1) NNS=2*NLELM(K-1) + 1
        NNF  =  2 * NLELM(K)
        EI  =  EIP(K)
        S1  =  4.0 * EI / H
        S2  =  2.0 * EI /  H
        DO 220 I=NNS,NNF
        S(I,1) = S1
        S(I,2) = S2
        IF (I/2*2  .NE. I ) GO TO 220
        S(I,1) = S2
        S(I,2) = S1
  220 CONTINUE
  221 CONTINUE
        ITERNO = 1
C
  225 CONTINUE
        S(NELMD1,1) = (H/24.)  * (7.0*SMOD( 1)  + 6.0*SMOD(2)  -  SMOD(3))
       1                                                              * 0.80
        S(M,1) = (H/24.)  * (7.0*SMOD(NELM1 )  + 6.0*SMOD(NELM1-1)
       1                          -  SMOD(NELM1-2))
        NELMD2 = 2  * NELM + 2
        MM1  = M -  1
        DO 230 I=NELMD2,MM1
        J =  I  - NELMD
  230 S(I,1) = (H/12.)  * (SMOD(J-1) + 10.0*SMOD(J)  +  SMOD( J+1))
        IF (ITERNO .GT. 1) GO TO 255
C
C       BUILD  SAT MATRIX
C
        DO 240 I=1,M
        DO 240 J=1,N
  240 SAT(I,J) = 0.0
        DO 250 I=1,NELMD
        II = I
        IF (I/2*2  .EQ. I) II = I - 1
        DO 250 J=1,N
  250 SAT(I,J) =  S(I,1)*A(J,II)  +  S(I,2)*A(J,II+1)
```

```
C
  255 CONTINUE
      J = NELM2
      DO 260 I=NELMD1,M
      SAT(I,J) = -S(I,1)
  260 J = J + 1
C
C     BUILD  ASAT MATRIX
C
      DO 270 I=1,N
      DO 270 J=1,N
      ENTRY = 0.0
      DO 265 K=1,M
  265 ENTRY = ENTRY + A(I,K) * SAT(K,J)
  270 ASAT(I,J) = ENTRY
C
C     INVERT ASAT MATRIX
C
      CALL LINV1F (ASAT,N,62,ASAT1,0,WKAREA,IER)
      DO 275 I=1,N
      DO 275 J=1,N
  275 ASAT(I,J) = ASAT1(I,J)
C
C     COMPUTE X MATRIX
C
      DO 330 I=1,N
      X(I) = 0.0
      DO 330 K=1,N
  330 X(I) = X(I) + ASAT(I,K) * P(K)
C
C     COMPUTE PILE DISPLACEMENTS
C
      DO 340 I=1,NELM1
  340 DISP(I) = X(NELM1+I)
      IF (ITERNO .NE. 1) GO TO 350
      YTOPO = DISP(1)
      GO TO 360
  350 YTOPN = DISP(1)
      IF (DABS(YTOPO-YTOPN) .LE. TOL) GO TO 370
      YTOPO = YTOPN
  360 WRITE (6,35) ITERNO,YTOPO
      CALL SOILM (H,NELM,DISP,SMOD)
      ITERNO = ITERNO + 1
      IF (ITERNO .LE. 100) GO TO 225
      WRITE (6,40)
      GO TO 440
C
C     COMPUTE F MATRIX
C
  370 DO 380 I=1,M
      F(I) = 0.0
      DO 380 K=1,N
  380 F(I) = F(I) + SAT(I,K) * X(K)
```

```
C
C     COMPUTE  SOIL RESISTANCE & BENDING MOMENT ALONG  THE PILE
C
      DO 390 I=1,NELM1
      II = NELMD + I
  390 SOILR(I) = F(II)
      DO 400 I=1,NELM
      II = 2 * I - 1
  400 BM(I) = F(II)
      BM(NELM1) = -F(NELMD)
C
      DO 410 I=1,NELM1
  410 ELEV(I) = (I-1) * H
C
C
C     PRINT OUTPUT
C
      WRITE (6,45)
      WRITE (6,50) BC1
      WRITE (6,51) BC2
      WRITE (6,52) PD,PL,H,NELM,TUL
      WRITE(6,55) NL
      WRITE (6,56)
      DO 415 I=1,NL
  415 WRITE (6,57) I,DEPTH(I),C(I),GAMA(I),EPS50(I),NXL(I),LKODE(I)
C
      WRITE (6,60)
      WRITE (6,65)
      DO 420 J=1,NX
      WRITE (6,66) XX(J),YY(1,J),PP(1,J)
      DO 420 K=2,NUM
      WRITE (6,67) YY(K,J),PP(K,J)
  420 CONTINUE
C
      WRITE (6,70)
      DO 430 I=1,NELM1
  430 WRITE (6,75) ELEV(I),DISP(I),BM(I),SMOD(I),SOILR(I)
C
      DO 435 J=1,NELM1
      XA(J) = ELEV(J)
      Y1(J) = -DISP(J)
      Y2(J) = BM(J)
  435 CONTINUE
      WRITE (7,15) NELM1
      WRITE (7,80) (XA(I) ,I=1,NELM1)
      WRITE (7,80) (Y1(I) ,I=1,NELM1)
      WRITE (7,80) (Y2(I) ,I=1,NELM1)
C
      GO TO 100
C
C
    5 FORMAT (20A4)
    6 FORMAT (2015.6,2I5)
```

```
10 FORMAT (3D15.6,2I5)
15 FORMAT (16I5)
20 FORMAT ('1',,5(/),20A4)
25 FORMAT (6D13.5)
30 FORMAT(' ',5(/),15X,'ITERATION INFORMATION'/15X,21(1H-)////15X,
  1        'ITER. NO.',11X,'YT (M)'/15X,9(1H-),11X,6(1H-)/)
35 FORMAT(17X,I3,10X,D15.7)
40 FORMAT ('1',5(/),10X,'100 ITER NOT ENOUGH')
45 FORMAT('1',3(/),30X,'LATERALLY LOADED PILE PROGRAM'/30X,29(1H-),
  1        5(/),10X,'INPUT INFORMATION'/10X,17(1H-))
50 FORMAT(' ',3(/),15X,'HORIZONTAL LOAD     H    =',D13.5,'   KN'/)
51 FORMAT (            15X,'MOMENT AT TOP       MT   =',D13.5,'   KN M'/ )
52 FORMAT (            15X,'PILE DIAMETER       D    =',D13.5,'   M'//
  2                    15X,'PILE LENGTH         L    =',D13.5,'   M'//
  3                    15X,'INCREMENT LENGTH         =',D13.5,'   M'//
  4                    15X,'NO. OF INCREMENTS        =',I6//
  5                    15X,'ITER. TOLERANCE          =',D13.5,'   M'//)
55 FORMAT(' ',3(/),15X,'NO. OF LAYERS',10X,'=',I5//)
56 FORMAT(' ',2(/),15X,'PROPERTIES OF EACH LAYER'/15X,24(1H-)///
  1        2(/),8X,'LAYER',9X,'DEPTH (M)',13X,'C (KN/M2)',12X,
  2        'GAMA (KN/M3)',13X,'EPS50',14X,'NXL',7X,'LKODE'/
  3        8X,5(1H-),9X,9(1H-),13X,9(1H-),12X,12(1H-),13X,5(1H-),
  4        14X,3(1H-),7X,5(1H-)/)
57 FORMAT(10X,I1,4(10X,D12.5),10X,I2,10X,I1)
C
60 FORMAT('1',5(/),10X,'OUTPUT INFORMATION'/10X,18(1H-)//)
65 FORMAT(' ',5(/),10X,'DEPTH TO P-Y CURVE (M)',8X,'Y (M)',16X,
  1        'P (KN/M)'/10X,22(1H-),8X,5(1H-),16X,8(1H-))
66 FORMAT (' ',5X/5X,3(10X,D12.5))
67 FORMAT(27X,2(10X,D12.5))
C
70 FORMAT(' ',5(/),13X,'X (M)',17X,'Y (M)',16X,'M (KN M)',13X,
  1        'ES (KN/M2)',13X,'P (KN/M)'/13X,5(1H-),
  2        17X,5(1H-),16X,8(1H-),13X,10(1H-),13X,8(1H-)/)
75 FORMAT(5(10X,D12.5))
80 FORMAT(6E13.5)
C
C
C
440 STOP
    END
```

```
C
C
C
      SUBROUTINE PYCURV
C
C     ******************************************************************
C     *                                                                *
C     *                                                                *
C     *THIS SUBROUTINE COMPUTES P-Y CURVES FOR A PILE IN A LAYERED SOIL*
C     *                                                                *
C     *      SOIL LAYERS WHICH ARE CONSIDERED :                        *
C     *                STIFF CLAY ABOVE WATER SURFACE                  *
C     *                STIFF CLAY BELOW WATER SURFACE                  *
C     *                SOFT  CLAY BELOW WATER SURFACE                  *
C     *                                                                *
C     *                                                                *
C     *     PL    =  PILE LENGTH                                       *
C     *     PD    =  PILE DIAMETER                                     *
C     *     NL    =  NUMBER OF SOIL LAYERS                             *
C     *     D     =  DEPTH OF A SOIL LAYER                             *
C     *     C     =  SHEAR STRENGTH OF A SOIL LAYER                    *
C     *     GAMA  =  EFFECTIVE UNIT WT. OF A SOIL LAYER                *
C     *     EPS50 =  STRAIN CORRES. TO 1/2 MAX. PRINCIPAL STRESS DIFF. *
C     *              OF A SOIL LAYER                                   *
C     *     NXL   =  NUMBER OF P-Y CURVES IN A SOIL LAYER              *
C     *     KODE  =  CODE OF A SOIL LAYER :                            *
C     *                STIFF CLAY ABOVE WATER SURFACE     KODE = 1     *
C     *                STIFF CLAY BELOW WATER SURFACE     KODE = 2     *
C     *                SOFT  CLAY BELOW WATER SURFACE     KODE = 3     *
C     *                                                                *
C     *                                                                *
C     ******************************************************************
C
C
      IMPLICIT REAL * 8 (A-H , O-Z)
      DIMENSION NXC( 5),DX(5),Y(25)
      DIMENSION D(5) , C(5), GAMA(5)
      DIMENSION XOPDT(9), AT(9),CT(5),SKT(5),EPSCT(5)
      COMMON /BLOC01/ PLS,PDS,DS(5),CS(5),GAMAS(5),
     1                EPS50(5),NXL(5),KODE(5),NL
      COMMON /BLOC02/ NX,NUM,X(25),YM(25,25),PP(25,25)
      COMMON /BLOC03/ C1,C2
      EXTERNAL F
C
      DATA XOPDT / 0.00D 00,0.50D 00,1.00D 00,1.50D 00,2.00D 00,
     1          2.50D 00,3.00D 00,3.50D 00,4.00D 00 /
      DATA AT / 0.200D 00,0.350D 00,0.450D 00,0.506D 00,0.550D 00,
     1        0.572D 00,0.588D 00,0.595D 00,0.600D 00 /
      DATA CT / 0.070D 02,0.105D 02,0.210D 02,0.420D 02,0.560D 02 /
      DATA SKT / 0.0333D 04,0.0500D 04,0.1000D 04,0.2000D 04,0.2666D 04/
      DATA EPSCT / 0.008D 00,0.007D 00,0.005D 00,0.004D 00,0.0035D 00 /
C
C
C
```

```
      CIM = 254.D-04
      CLBKN = 4448.D-06
      CMI = 1.0 / CIM
      CKNLB = 1.0 / CLBKN
      PL = PLS * CMI
      PD = PDS * CMI
      DO 100 I=1,NL
      D(I) = DS(I) * CMI
      C(I) = CS(I) * (CKNLB/(CMI*CMI))
      GAMA(I) = GAMAS(I) * (CKNLB / (CMI*CMI*CMI))
  100 CONTINUE
C
C     ------------------------------------------------------------------
C     DETERMINE THE LOCATIONS OF  P-Y CURVES  X(I)
C     ------------------------------------------------------------------
C
      NXC(1) = NXL(1)
      IF (NL .EQ. 1) GO TO 600
      DO 500 I=2,NL
      NXC(I) = NXC(I-1) + NXL(I)
  500 CONTINUE
  600 CONTINUE
      NX = NXC(NL)
      IF (NX .GT. 25) GO TO 2400
      DO 700 I=1,NL
      DX(I) = D(I) / (NXL(I)-1)
  700 CONTINUE
      X(1) = 0.0
      K = 1
      NXM1 = NX - 1
      DO 800 I=1,NXM1
      IF (I .EQ. NXC(K)) GO TO 750
      X(I+1) = X(I) + DX(K)
      GO TO 800
  750 CONTINUE
      K = K+1
      X(I+1) = X(I)
  800 CONTINUE
C
C     ------------------------------------------------------------------
C     DETERMINE THE PILE DEFLECTIONS  Y(I)
C     ------------------------------------------------------------------
C
      NUM = 18
      CMI = 1.0 / 254.D-4
      I1 = 8
      I2 = 15
      I1P1 = I1 + 1
      I1P2 = I1 + 2
      I2P1 = I2 + 1
      I2P2 = I2 + 2
      YI1P1 = .05 * CMI
      YI2P1 = .50 * CMI
```

```
      DY1 = 0.006 * CMI
      DY2 = 0.050 * CMI
      DY3 = 0.250 * CMI
      Y(1) = 0.0
      DO 900 I=2,I1
      Y(I) = Y(I-1) + DY1
  900 CONTINUE
      Y(I1P1) = YI1P1
      DO 950 I=I1P2,I2
      Y(I) = Y(I-1)+DY2
  950 CONTINUE
      Y(I2P1) = YI2P1
      DO 1000 I=I2P2,NUM
      Y(I) = Y(I-1)+DY3
 1000 CONTINUE
C
C     -----------------------------------------------------------
C     CALCULATE THE SOIL RESISTANCE  P(I)
C     -----------------------------------------------------------
C
      KK = 1
      DO 2300 K=1,NX
      IF (K .GT. NXC(KK)) KK=KK+1
      DO 1100 J=1,NUM
      YM(J,K) = Y(J)
 1100 CONTINUE
      IF (KK .EQ. 1) GO TO 1300
      DC = 0.0
      GAMADC = 0.0
      KKM1 = KK-1
      DO 1200 I=1,KKM1
      DC = DC + D(I)
      GAMADC = GAMADC + GAMA(I)*D(I)
 1200 CONTINUE
      GAMAV = (GAMADC+GAMA(KK)*(X(K)-DC)) / X(K)
      GO TO 1400
 1300 CONTINUE
      GAMAV = GAMA(1)
 1400 CONTINUE
      NN = KODE(KK)
      GO TO (1500,1700,1900) , NN
C
C     -----------------------------------------------------------
C     STIFF CLAY ABOVE WATER SURFACE
C     -----------------------------------------------------------
C
 1500 CONTINUE
      PU1 = (3.0+(GAMAV/C(KK))*X(K) +(0.5/PD)*X(K)) *C(KK)*PD
      PU2 = 9.0 * C(KK) *PD
      PU = PU1
      IF (PU2 .LT. PU1) PU = PU2
      Y50 = 2.5 * EPS50(KK) * PD
      DO 1600 J=1,NUM
```

```
      PP(J,K) = (0.5 * (YM(J,K)/Y50)**(1./4.)) * PU
      IF(YM(J,K) .GT. (16.0*Y50)) PP(J,K) = PU
 1600 CONTINUE
      GO TO 2100
C
C     ------------------------------------------------------------------
C     STIFF CLAY BELOW WATER SURFACE
C     ------------------------------------------------------------------
C
 1700 CONTINUE
      PU1 = 2.0*C(KK)*PD + GAMAV*PD*X(K) + 2.83*C(KK)*X(K)
      PU2 = 11.0 * C(KK) * PD
      PU = PU1
      IF (PU2 .LT. PU1) PU = PU2
      XOPD = X(K) / PD
      A = 0.60
      IF (XOPD .LE. 4.0) CALL YINTPL (XOPDT,AT,9,9,XOPD,A)
      CALL YINTPL (CT,SKT,5,5,C(KK),SK)
      CALL YINTPL (CT,EPSCT,5,5,C(KK),EPSC)
      YC = EPSC * PD
      C1 = SK * X(K)
      C2 = 0.5 * PU / DSQRT(YC)
      Y2 = A * YC
      CALL CHECK (F,0.0D 0,Y2,ROOT)
      Y1 = ROOT
      Y3 = 6.0 * Y2
      Y4 = 18.0 * Y2
      DO 1800 J=1,NUM
      IF (YM(J,K) .LT. Y1)
     1        PP(J,K) = C1 * YM(J,K)
      IF (YM(J,K) .GE. Y1   .AND.  YM(J,K) .LT. Y2)
     1        PP(J,K) = C2 * DSQRT(YM(J,K))
      IF (YM(J,K) .GE. Y2   .AND.  YM(J,K) .LT. Y3)
     1        PP(J,K) = C2 * DSQRT(YM(J,K)) - 0.055*PU*((YM(J,K)-Y2)/Y2)**
     2                                                                1.25
      IF (YM(J,K) .GE. Y3   .AND.  YM(J,K) .LT. Y4)
     1        PP(J,K) = C2 * DSQRT(Y3) - 0.411*PU - (0.0625*PU/YC)
     2                                             *(YM(J,K)-Y3)
      IF (YM(J,K) .GE. Y4)
     1        PP(J,K) = C2 * DSQRT(Y3) - 0.411*PU - 0.75*PU*A
      IF ( PP(J,K) .LT. 0.0 )  PP(J,K) = 0.0
 1800 CONTINUE
      GO TO 2100
C
C     ------------------------------------------------------------------
C     SOFT CLAY BELOW WATER SURFACE
C     ------------------------------------------------------------------
C
 1900 CONTINUE
      PU1 = (3.0+(GAMAV/C(KK))*X(K) +(0.5/PD)*X(K)) *C(KK)*PD
      PU2 = 9.0 * C(KK) *PD
      PU = PU1
      IF (PU2 .LT. PU1) PU = PU2
```

```
      Y50 = 2.5 * EPS50(KK) * PD
      DO 2000 J=1,NUM
      PP(J,K) = (0.5 * (YM(J,K)/Y50)**(1./3.)) * PU
      IF(YM(J,K) .GT. (8.0*Y50)) PP(J,K) = PU
 2000 CONTINUE
C
 2100 CONTINUE
      X(K) = X(K) * CIM
      DO 2200 J=1,NUM
      YM(J,K) = YM(J,K) * CIM
      PP(J,K) = PP(J,K) * (CLBKN/CIM)
 2200 CONTINUE
 2300 CONTINUE
      GO TO 2500
 2400 CONTINUE
      WRITE(6,10) NX
      STOP
 2500 CONTINUE
      RETURN
C
C
   10 FORMAT('1',5(/),10X,'NX = ',I3,10X,'GT  25')
C
      END
C
C
C
      SUBROUTINE YINTPL (X,Y,NDIM,N,XBAR,YBAR)
C
C     ***********************************************************************
C     *                                                                     *
C     *           TO INTERPOLATE FOR YBAR CURRES. TO XBAR                   *
C     *         ( A SET OF TABULATED VALUES FOR X&Y IS KNOWN )              *
C     *                                                                     *
C     ***********************************************************************
C
      IMPLICIT REAL * 8 (A-H , O-Z)
      DIMENSION X(NDIM),Y(NDIM)
C
      IF (XBAR .LT. X(1)  .OR.  XBAR .GT. X(N)) GO TO 250
      DO 100 I=1,N
      IF (XBAR .GE. X(I)) GO TO 100
      L = I
      GO TO 200
  100 CONTINUE
      YBAR = Y(N)
      GO TO 300
  200 CONTINUE
      SLOPE = (Y(L)-Y(L-1)) / (X(L)-X(L-1))
      YBAR = Y(L-1) + SLOPE*(XBAR-X(L-1))
      GO TO 300
  250 CONTINUE
      WRITE(6,10) XBAR
```

```
      STOP
  300 CONTINUE
      RETURN
C
   10 FORMAT(' ',5(/),20X,'XBAR = ',D13.5,10X,'OUT OF  RANGE'//)
C
      END
C
C
C
      SUBROUTINE CHECK (F,A,B,ROOT)
C
C     ****************************************************************
C     *                                                              *
C     *      TO CHECK FOR THE INTERSECTION OF THE FUNCTIONS F1 & F2  *
C     *              ( F = F1-F2 ) WITHIN A GIVEN RANGE A,B          *
C     *                                                              *
C     ****************************************************************
C
      IMPLICIT REAL * 8 (A-H , O-Z)
      COMMON /BLOC03/ C1,C2
C
      N = 50
      DX = (B-A) / N
      X = A+DX
      YO = F(X)
      NM1 = N-1
      DO 100 I=1,NM1
      X = X+DX
      IF (I .EQ. NM1) X = B
      YN = F(X)
      IF (YO*YN .LE. 0.0) GO TO 200
      YO = YN
  100 CONTINUE
      ROOT = 0.0
      GO TO 300
  200 CONTINUE
      ROOT = X - DX/2.0
  300 CONTINUE
      RETURN
      END
C
C
C
      FUNCTION F(X)
C
C     ****************************************************************
C     *                                                              *
C     *               TO COMPUTE THE FUNCTION F = F1 - F2            *
C     *                                                              *
C     ****************************************************************
C
      IMPLICIT REAL * 8 (A-H , O-Z)
```

```
      COMMON /BLUC03/ C1,C2
C
      F1(X) = C1 * X
      F2(X) = C2 * DSQRT(X)
      F = F1(X) - F2(X)
      RETURN
      END
```

```
C
C
C
      SUBROUTINE SOILM (H,NELM,Y,ES)
C     ******************************************
C     * CALCULATE REVISED VALUES OF SOIL MODULUS *
C     ******************************************
C
C
C
C
      IMPLICIT REAL * 8 (A-H , O-Z)
      DIMENSION Z(31), YR(25), PR(25)
      DIMENSION Y(31),ES(31)
C
C
      COMMON /BLOC02/ NX,NUM,X(25),YM(25,25),PP(25,25)
C
C
C ************************************************************************
C SET DEFLECTION VALUES EQUAL TO Z(I)
C ************************************************************************
      N1 = NELM + 1
      DO 1 I=1,N1
      Z(I) = DABS(Y(I))
    1 CONTINUE
C
C
C ************************************************************************
C STATION DEPTH EQUAL TO A P-Y CURVE DEPTH
C ************************************************************************
C     ...INTERPOLATE SOIL RESISTANCE BETWEEN DEFLECTIONS ON A P-Y CURVE
      B=0.0
      DO 31 I=1,N1
         DIST=B*H
         K=1
    2    IF (DIST-X(K)) 12,3,10
    3    DO 4 JJ=1,NUM
            YR(JJ)=YM(JJ,K)
            PR(JJ)=PP(JJ,K)
    4    CONTINUE
         JJ=1
    5    IF (Z(I)-YR(JJ)) 9,6,7
    6    PC=PR(JJ)
         GO TO 27
    7    IF (JJ-NUM) 8,6,6
    8    JJ=JJ+1
         GO TO 5
    9    TERM1=PR(JJ-1)
         TERM2=((PR(JJ)-PR(JJ-1))/(YR(JJ)-YR(JJ-1)))*(Z(I)-YR(JJ-1))
         PC=TERM1+TERM2
         GO TO 27
   10    IF (K-NX) 11,3,3
   11    K=K+1
```

```
      GO TO 2
C
C
C ***********************************************************************
C STATION DEPTH NOT EQUAL TO A P-Y CURVE DEPTH
C ***********************************************************************
C       ...INTERPOLATE RESIST. BETWEEN DEFLECTIONS ON ADJACENT CURVE
   12   DO 13 JJ=1,NUM
           YR(JJ)=YM(JJ,K-1)
           PR(JJ)=PP(JJ,K-1)
   13   CONTINUE
        JJ=1
   14   IF (Z(I)-YR(JJ)) 18,15,16
   15   PA=PR(JJ)
        GO TO 19
   16   IF (JJ-NUM) 17,15,15
   17   JJ=JJ+1
        GO TO 14
   18   TERM3=PR(JJ-1)
        TERM4=((PR(JJ)-PR(JJ-1))/(YR(JJ)-YR(JJ-1)))*(Z(I)-YR(JJ-1))
        PA=TERM3+TERM4
C
C       ...INTERPOLATE RESIST. BETWEEN DEFLECTIONS ON ADJACENT CURVE
   19   DO 20 JJ=1,NUM
           YR(JJ)=YM(JJ,K)
           PR(JJ)=PP(JJ,K)
   20   CONTINUE
        JJ=1
   21   IF (Z(I)-YR(JJ)) 25,22,23
   22   PB=PR(JJ)
        GO TO 26
   23   IF (JJ-NUM) 24,22,22
   24   JJ=JJ+1
        GO TO 21
   25   TERM5=PR(JJ-1)
        TERM6=((PR(JJ)-PR(JJ-1))/(YR(JJ)-YR(JJ-1)))*(Z(I)-YR(JJ-1))
        PB=TERM5+TERM6
C
C
C ***********************************************************************
C INTERPOLATE SOIL RESISTANCE BETWEEN ADJACENT P-Y CURVES AT DEFLECTION
C ***********************************************************************
   26   PC=PA+((PB-PA)/(X(K)-X(K-1)))*(DIST-X(K-1))
C
C
C ***********************************************************************
C SET REVISED SOIL MODULUS VALUES
C ***********************************************************************
   27   IF (Z(I)) 28,29,28
   28   ES(I)=PC/Z(I)
        GO TO 30
   29   ES(I)=0.0
   30   B=B+1.0
```

```
31 CONTINUE
   RETURN
   END
```

C.III PARTIAL LISTING OF PLOTTING SUBROUTINES USED IN THE PARAMETRIC ANALYSIS GRAPHS'

```
C
C
C
      SUBROUTINE PLOTNN
C
C     ******************************************************************
C     TO PLOT THE RESULTS
C     ******************************************************************
C
C
      DIMENSION IBUF(1000)
      DIMENSION X(152),Y1(152),Y2(152)
      DIMENSION V(22),Y1ABMX(22),Y2MAX(22),Y2MIN(22),BIGARR(42)
      DIMENSION Y01(20),Y02(20),PL(20)
      DIMENSION XCROS(100)
C
      NDC = 152
      REWIND 10
      CALL PLOTS (IBUF,1000,6)
C
      READ (10) NOPC,NOPP
      PHT1 = 11.0 * NOPP
      PWD1 = 8.5
      CALL PLOT (4.0,1.0,-3)
      CALL PLOT (PHT1,0.0,2)
      CALL PLOT (PHT1,PWD1,3)
      CALL PLOT (0.0,PWD1,2)
      CALL PLOT (0.0,0.0,2)
      NOPPM1 = NOPP - 1
      DO 100 I=1,NOPPM1
      PHT11 = 11.0 * I
      CALL PLOT (PHT11,PWD1,3)
      CALL PLOT (PHT11,0.0,2)
  100 CONTINUE
      PHT11 = PHT11 + 2.5
      DO 200 I=1,2
      PHT11 = PHT11 + 8.5
      PWD11 = 9.6
      PWD12 = -0.8
      CALL PLOT (PHT11,PWD11,3)
      CALL PLOT (PHT11,PWD12,2)
  200 CONTINUE
      CALL PLOT (1.0,1.3,-3)
C
      PHT2 = 9.0
      PWD2 = 6.6
      DO 300 I=1,NOPP
      CALL PLOT (PHT2,0.0,2)
      CALL PLOT (PWT2,PWD2,2)
      CALL PLOT (0.0,PWD2,2)
      CALL PLOT (0.0,0.0,2)
      IF (I .EQ. NOPP) GO TO 400
      CALL PLOT (11.0,0.0,-3)
  300 CONTINUE
```

```
  400 CONTINUL
      CALL PLOT (11.5,-1.5,-3)
      PHT2 = 9.0
      PWD2 = 6.1
      CALL PLOT (PWD2,0.0,2)
      CALL PLOT (PWD2,PHT2,2)
      CALL PLOT (0.0,PHT2,2)
      CALL PLUT (0.0,0.0,2)
      XD = -(PHT1+0.5) + 0.8
      YO = 1.5 + 1.07
      CALL PLOT (XO,YO,-3)
C
      READ (10) NOHA,KODEP,KODES
      DO 500 II=1,NOPP
      READ (10) X,Y1,Y2,N,VV
      XAX = 6.0
      YAX = 1.8
      XAXR = 6.0
      YAXR = 2.0
C
      CALL MNDLEX (X,NDC,N,XAX,XAXR,X(N+1),X(N+2),IEXPX)
      CALL MNDLEX (Y1,NDC,N,YAX,YAXR,Y1(N+1),Y1(N+2),IEXPY1)
      CALL MNDLEX (Y2,NDC,N,YAX,YAXR,Y2(N+1),Y2(N+2),IEXPY2)
      Y01(II) = -( Y1(N+1) / Y1(N+2) )
      Y02(II) = -( Y2(N+1) / Y2(N+2) )
      PL(II) = X(N) / X(N+2)
C
      CALL XAXIS (IEXPX,XAXR,X(N+1),X(N+2))
      CALL YAXIS (IEXPY1,YAXR,Y1(N+1),Y1(N+2))
      CALL Y1TITL (IEXPY1,YAXR)
      CALL FLINE (X,Y1,-N,1,0,0)
      CALL CROSNG (X,Y1,NDC,N,XCROS,NXCROS)
      CALL PLOTXO (XCROS,NXCROS,X(N+1),X(N+2),Y01(II))
C
      CALL PLOT (0.0,3.3,-3)
      CALL XAXIS (IEXPX,XAXR,X(N+1),X(N+2))
      CALL YAXIS (IEXPY2,YAXR,Y2(N+1),Y2(N+2))
      CALL Y2TITL (IEXPY2,YAXR)
      CALL FLINE (X,Y2,-N,1,0,0)
      CALL CROSNG (X,Y2,NDC,N,XCROS,NXCROS)
      CALL PLOTXO (XCROS,NXCROS,X(N+1),X(N+2),Y02(II))
C
      CALL PLOT (5.6,-2.8,-3)
      IF (NOHA .EQ. 1) CALL SQRH  (VV)
      IF (NOHA .EQ. 2) CALL SQRV  (VV)
      IF (NOHA .EQ. 3) CALL SQRMT (VV)
      IF (NOHA .EQ. 4) CALL SQRL  (VV)
      IF (NOHA .EQ. 5) CALL SQREI (VV)
      IF (NOHA .EQ. 6) CALL SQRD  (VV)
      IF (NOHA .EQ. 7) CALL SQRD  (VV)
      IF (NOHA .EQ. 8) CALL SQRC  (VV)
      IF (NOHA .EQ. 9) CALL SQRG  (VV)
C
```

```
      CALL PLOT (2.2,-1.07,-3)
      CALL VTITLS (NOHA,KODEP,KODES,II)
C
      IF (II .EQ. NUPP) GO TO 600
      CALL PLOT (3.2,0.57,-3)
  500 CONTINUE
C
C
  600 CONTINUE
      CALL PLOT (3.6,0.9,-3)
      READ (10) V,Y1ABMX,Y2MAX,Y2MIN
      VAX = 4.5
      YAX = 1.8
      VAXR = 5.0
      YAXR = 2.0
C
      DO 700 I=1,NOPC
      BIGARR(I) = Y2MAX(I)
      BIGARR(NOPC+I) = Y2MIN(I)
  700 CONTINUE
C
      NOPC2 = NOPC + NOPC
      CALL MNDLEX (V,22,NOPC,VAX,VAXR,V(NOPC+1),V(NOPC+2),IEXPV)
      CALL MNDLEX (Y1ABMX,22,NOPC,YAX,YAXR,Y1ABMX(NOPC+1),Y1ABMX(NOPC+2)
     1            ,IEXPD)
      CALL MNDLEX (BIGARR,42,NOPC2,YAX,YAXR,
     1            BIGARR(NOPC2+1),BIGARR(NOPC2+2),IEXPM)
C
      DO 800 I=1,2
      Y2MAX(NOPC+I) = BIGARR(NOPC2+1)
      Y2MIN(NOPC+I) = BIGARR(NOPC2+1)
  800 CONTINUE
C
      CALL VAXIS (IEXPV,VAXR,V(NOPC+1),V(NOPC+2))
      IF (NOHA .EQ. 1) CALL TITLH  (IEXPV,VAXR,-0.3)
      IF (NOHA .EQ. 2) CALL TITLV  (IEXPV,VAXR,-0.3)
      IF (NOHA .EQ. 3) CALL TITLMT (IEXPV,VAXR,-0.3)
      IF (NOHA .EQ. 4) CALL TITLL  (IEXPV,VAXR,-0.3)
      IF (NOHA .EQ. 5) CALL TITLEI (IEXPV,VAXR,-0.3)
      IF (NOHA .EQ. 6) CALL TITLD  (IEXPV,VAXR,-0.3)
      IF (NOHA .EQ. 7) CALL TITLD  (IEXPV,VAXR,-0.3)
      IF (NOHA .EQ. 8) CALL TITLC  (IEXPV,VAXR,-0.3)
      IF (NOHA .EQ. 9) CALL TITLG  (IEXPV,VAXR,-0.3)
      CALL YAXIS (IEXPD,YAXR,Y1ABMX(NOPC+1),Y1ABMX(NOPC+2))
      CALL DTITL (IEXPD,YAXR)
      CALL FLINE (V,Y1ABMX,-NOPC,1,1,1)
      CALL PLOT (VAXR,-0.3,3)
      CALL PLOT (VAXR,YAXR,2)
      CALL PLOT (0.0,YAXR,2)
      CALL PLOT (0.0,3.5,-3)
C
      Y0 = -(BIGARR(NOPC2+1)) / (BIGARR(NOPC2+2))
      CALL VAXISM (IEXPV,VAXR,V(NOPC+1),V(NOPC+2),Y0)
```

```
      IF (NOHA .EQ. 1) CALL TITLH  (IEXPV,VAXR,YO)
      IF (NOHA .EQ. 2) CALL TITLV  (IEXPV,VAXR,YO)
      IF (NOHA .EQ. 3) CALL TITLMT (IEXPV,VAXR,YO)
      IF (NOHA .EQ. 4) CALL TITLL  (IEXPV,VAXR,YO)
      IF (NOHA .EQ. 5) CALL TITLEI (IEXPV,VAXR,YO)
      IF (NOHA .EQ. 6) CALL TITLD  (IEXPV,VAXR,YO)
      IF (NOHA .EQ. 7) CALL TITLD  (IEXPV,VAXR,YO)
      IF (NOHA .EQ. 8) CALL TITLC  (IEXPV,VAXR,YO)
      IF (NOHA .EQ. 9) CALL TITLG  (IEXPV,VAXR,YO)
      CALL YAXIS (IEXPM,YAXR,BIGARR(NOPC2+1),BIGARR(NOPC2+2))
      CALL MTITL (IEXPM,YAXR)
      CALL FLINE (V,Y2MAX,-NOPC,1,1,3)
      CALL FLINE (V,Y2MIN,-NOPC,1,1,11)
      CALL PLOT (0.0,0.0,3)
      CALL PLOT (0.0,-0.3,2)
      CALL PLOT (VAXR,-0.3,2)
      CALL PLOT (VAXR,YAXR,2)
      CALL PLOT (0.0,YAXR,2)
C
      CALL PLOT (3.8,2.1,-3)
      CALL SYMBLS
C
      CALL PLOT (-4.0,-8.1,-3)
      CALL HTITLS (NOHA,KODEP,KODES)
C
      XN = -(PHT1+5.5)
      YN = 2.17
      CALL PLOT (XN,YN,-3).
      CALL PLOT (2.5,0.0,-2)
      CALL PLOT (2.8,0.0,-3)
      CALL PILE (Y01,Y02,PL,20,NOPP)
C
      RETURN
      END
```

```
C
C
C
      SUBROUTINE MNDLEX (V,ND,N,VAX,VAXR,VMN,DELV,IEXP)
C
C     ******************************************************************
C     TO GET  VMIN , DELV , EXP
C     ******************************************************************
C
      DIMENSION V(ND)
      VMX = V(1)
      VMN = V(1)
      DO 200 I=2,N
      IF (VMX .GE. V(I)) GO TO 100
      VMX = V(I)
  100 IF (VMN .LE. V(I)) GO TO 200
      VMN = V(I)
  200 CONTINUE
      IF (VMX .EQ. VMN) VMN = 0.9 * VMX
      DELV = (VMX-VMN) / VAX
      CALL VEXP(VMN,DELV,VAXR,IEXP)
      CALL MNDLM (VMN,DELV,VAXR,IEXP)
      RETURN
      END
C
C
C
      SUBROUTINE VEXP (VMN,DELV,VAXR,IEXP)
C
C     ******************************************************************
C     TO GET THE VALUE OF THE EXP CORRES. TO SCALE VALUES 10.0 TO 99.99
C     ******************************************************************
C
      VMNR = VMN
      VMXR = VMNR + DELV*VAXR
      ABMX = ABS(VMNR)
      IF (ABS(VMXR) .GT. ABS(VMNR)) ABMX = ABS(VMXR)
      IEXP = 0
      IF (ABMX .GE. 100.0) GO TO 100
      IF (ABMX .LT. 10.0) GO TO 200
      GO TO 300
  100 CONTINUE
      ABMX = ABMX / 10.0
      IEXP = IEXP + 1
      IF (ABMX .GE. 100.0) GO TO 100
      GO TO 300
  200 CONTINUE
      ABMX = ABMX * 10.0
      IEXP = IEXP - 1
      IF (ABMX .LT. 10.0) GO TO 200
  300 RETURN
      END
C
C
```

```
C
      SUBROUTINE MNDLM(VMN,DELV,VAXR,IEXP)
C
C     ************************************************************************
C     TO GET ROUNDED SCALE VALUES FOR VMIN , DELV
C     ************************************************************************
C
      VMNS = VMN / (10.**IEXP)
      DELVS = DELV / (10.**IEXP)
      VMXS = VMNS + DELVS*VAXR
      RR = 1.0D-10
      IF (VMNS) 100,200,300
  100 VMNSR = VMNS - RR
      GO TO 400
  200 VMNSR = VMNS
      GO TO 400
  300 VMNSR = VMNS + RR
  400 DELVSR = DELVS + RR
      IF (DELVSR .LT. 2.0) GO TO 500
      DEN = 1.0
      VMNSM = (IFIX(VMNSR)/2) * 2
      IF (VMNSR .LT. 0.0  .AND.  ABS(VMNSM-VMNS) .GT. RR)
     1                           VMNSM = VMNSM - 2.0
      GO TO 600
  500 DEN = 2.0
      VMNSM = IFIX(VMNSR)
      IF (VMNSR .LT. 0.0  .AND.  ABS(VMNSM-VMNS) .GT. RR)
     1                           VMNSM = VMNSM - I
  600 D = VMXS - VMNSM
      DR = D - RR
      DO 700 I=1,400
      DELVSM = I / DEN
      IF (DR .LE. VAXR*DELVSM) GO TO 800
  700 CONTINUE
      STOP
  800 VMN = VMNSM * (10.**IEXP)
      DELV = DELVSM * (10.**IEXP)
      RETURN
      END
```

```
C
C
C
      SUBROUTINE XAXIS (IEXPX,XAXR,XMN,DELX)
C
C     ******************************************************************
C     TO DRAW THE X-AXIS (ELEVATION)
C     ******************************************************************
C
      XPAGE = 0.0
      YPAGE = 0.0
      XMNS = XMN / (10.0**IEXPX)
      DELXS = DELX / (10.0**IEXPX)
      FPN = XMNS
      CALL PLOT (XPAGE,YPAGE-0.25,2)
      CALL NUMBER (XPAGE+0.07,YPAGE-0.63,0.07,FPN,90.0,2)
      II = XAXR
      DO 100 I=1,II
      XPAGE = XPAGE + 1.0
      FPN = FPN - DELXS
      CALL NUMBER (XPAGE,YPAGE-0.77,0.07,FPN,90.0,2)
      CALL PLOT (XPAGE,YPAGE-0.25,3)
      CALL PLOT (XPAGE,YPAGE-0.15,2)
  100 CONTINUE
C
      IF (IEXPX .EQ. 0) GO TO 200
      VEXP = IEXPX
      XDIS = (XAXR/2.0) - 0.9
      CALL SYMBOL (XPAGE-XDIS,YPAGE-0.82,0.1,'ELEVATION  (',180.0,12)
      CALL SYMBOL (999.,999.,0.07,'M',180.0,1)
      CALL NUMBER (999.,YPAGE-0.89,0.05,VEXP,180.0,-1)
      CALL SYMBOL (999.,999.,0.1,')   *10',180.0,7)
      GO TO 300
  200 CONTINUE
      XDIS = (XAXR/2.0) - 0.71
      CALL SYMBOL (XPAGE-XDIS,YPAGE-0.82,0.1,'ELEVATION  (',180.0,12)
      CALL SYMBOL (999.,999.,0.07,'M',180.0,1)
      CALL SYMBOL (999.,999.,0.1,')',180.0,1)
  300 CONTINUE
      XPAGE = 0.0
      CALL PLOT (XPAGE,YPAGE,3)
      RETURN
      END
C
C
C
      SUBROUTINE YAXIS (IEXPY,YAXR,YMN,DELY)
C
C     ******************************************************************
C     TO DRAW THE Y-AXIS
C     ******************************************************************
C
      XPAGE = 0.0
      YPAGE = 0.0
```

```
      YMNS = YMN / (10.0**IEXPY)
      DELYS = DELY / (10.0**IEXPY)
      FPN = YMNS
      CALL PLOT (XPAGE-0.05,YPAGE,2)
      CALL NUMBER (XPAGE-0.08,YPAGE-0.14,0.07,FPN,90.0,2)
      II = YAXR
      DO 100 I=1,II
      CALL PLOT (XPAGE,YPAGE,3)
      YPAGE = YPAGE + 1.0
      FPN = FPN + DELYS
      CALL PLOT (XPAGE,YPAGE,2)
      CALL PLOT (XPAGE-0.05,YPAGE,2)
      IF (I .EQ. II) GO TO 200
      CALL NUMBER (XPAGE-0.08,YPAGE-0.21,0.07,FPN,90.0,2)
  100 CONTINUE
  200 CONTINUE
      CALL NUMBER (XPAGE-0.08,YPAGE-0.28,0.07,FPN,90.0,2)
      YPAGE = 0.0
      CALL PLOT (XPAGE,YPAGE,3)
      RETURN
      END
```

```
C
C
C
      SUBROUTINE Y1TITL (IEXPY1,YAXR)
C
C     ******************************************************************
C     TO WRITE THE DEFLECTION TITLE  Y
C     ******************************************************************
C
      IF (IEXPY1 .EQ. 0) GO TO 100
      VEXP = IEXPY1
      YDIS = (YAXR/2.0) - 0.5
      CALL SYMBOL (-0.2,YDIS,0.1,'Y  (',90.0,4)
      CALL SYMBOL (999.,999.,0.07,'M',90.0,1)
      CALL SYMBOL (999.,999.,0.1,')   *10',90.0,7)
      CALL NUMBER (-0.27,999.,0.05,VEXP,90.0,-1)
      GO TO 200
C
  100 CONTINUE
      YDIS = (YAXR/2.0) - 0.2
      CALL SYMBOL (-0.2,YDIS,0.1,'Y  (',90.0,4)
      CALL SYMBOL (999.,999.,0.07,'M',90.0,1)
      CALL SYMBOL (999.,999.,0.1,')',90.0,1)
  200 CONTINUE
      CALL PLOT (0.0,0.0,3)
      RETURN
      END
C
C
C
      SUBROUTINE Y2TITL (IEXPY2,YAXR)
C
C     ******************************************************************
C     TO WRITE THE MOMENT TITLE  M
C     ******************************************************************
C
      IF (IEXPY2 .EQ. 0) GO TO 100
      VEXP = IEXPY2
      YDIS = (YAXR/2.0) - 0.6
      CALL SYMBOL (-0.2,YDIS,0.1,'M  (KN',90.0,6)
      CALL SYMBOL (999.,999.,0.07,' M',90.0,2)
      CALL SYMBOL (999.,999.,0.1,')   *10',90.0,7)
      CALL NUMBER (-0.27,999.,0.05,VEXP,90.0,-1)
      GO TO 200
C
  100 CONTINUE
      YDIS = (YAXR/2.0) - 0.4
      CALL SYMBOL (-0.2,YDIS,0.1,'M  (KN',90.0,6)
      CALL SYMBOL (999.,999.,0.07,' M',90.0,2)
      CALL SYMBOL (999.,999.,0.1,')',90.0,1)
  200 CONTINUE
      CALL PLOT (0.0,0.0,3)
      RETURN
      END
```

```
C
C
C
C
      SUBROUTINE PILE (Y01,Y02,PL,ND,N)
C
C     ******************************************************************
C     TO DRAW THE PILE
C     ******************************************************************
C
      DIMENSION Y01(ND),Y02(ND),PL(ND)
      DO 100 JJ=1,N
      CALL PLOT (0.0,Y01(JJ),3)
      CALL PLOT (PL(JJ),Y01(JJ),2)
      YN = 3.3 + Y02(JJ)
      CALL PLOT (0.0,YN,3)
      CALL PLOT (PL(JJ),YN,2)
      CALL PLOT (11.0,0.0,-3)
  100 CONTINUE
      CALL PLOT (10.0,0.0,999)
      RETURN
      END
C
C
C
C
      SUBROUTINE CROSNG (X,Y,ND,N,XCROS,NXCROS)
C
C     ******************************************************************
C     TO CALCULATE THE POINTS OF ZERO DEFL. OR B.M.
C     ******************************************************************
C
      DIMENSION X(ND) , Y(ND)
      DIMENSION XCROS(100)
      DEL = 1. E-6
      NXCROS = 0
      Y(1) = Y(1) + DEL
      I = 1
    5 IF (Y(I) * Y(I+1)) 10,20,30
   10 NXCROS = NXCROS + 1
      XCROS(NXCROS) = X(I) + Y(I) * (X(I+1) - X(I)) / (Y(I) - Y(I+1))
      GO TO 30
   20 NXCROS = NXCROS + 1
      XCROS(NXCROS) = X(I+1)
      IF (Y(I) .LT. 0.0) Y(I+1) = Y(I+1) + DEL
      IF (Y(I) .GT. 0.0) Y(I+1) = Y(I+1) - DEL
   30 I = I + 1
      IF (I .NE. N) GO TO 5
      RETURN
      END
C
C
C
C
```

```
C
      SUBROUTINE PLOTXO (XCROS,NXCROS,XMN,DELX,YO)
C
C     ******************************************************************
C     TO PLOT THE POINTS OF ZERO DEFL. OR B.M.
C     ******************************************************************
C
      DIMENSION XCROS(100)
      DO 100 I=1,NXCROS
      XPAGE = (XCROS(I) - XMN) / DELX
      CALL SYMBOL (XPAGE,YO,0.1,13,0.0,-1)
  100 CONTINUE
      RETURN
      END
```