AN INVESTIGATION OF THE RELATIONSHIP BETWEEN COGNITIVE DEVELOPMENT AND THE ABILITY TO SOLVE TWO-STEP VERBAL PROBLEMS IN ELEMENTARY SCHOOL CHILDREN

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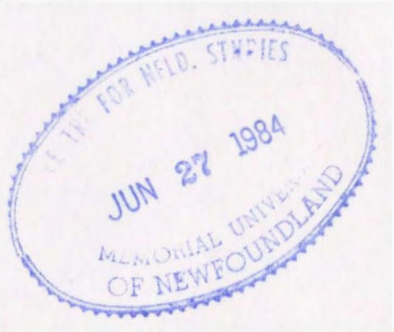
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AN INVESTIGATION OF THE RELATIONSHIP BETWEEN COGNITIVE DEVELOPMENT AND THE ABILITY TO SOLVE TWO-STEP VERBAL PROBLEMS IN ELEMENTARY SCHOOL CHILDREN

by

© Walter F. Ryan, B.Sc., B.Ed.

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ABSTRACT

Purpose

The major purpose of this study was to determine whether the Piagetan cognitive development stage of the student affects the ability to solve mathematical two-step verbal problems. Four questions were explored:

1) Does a student's cognitive development stage affect his ability to solve two-step verbal problems?
2) Does there exist a relationship between the sex of the student and his ability to solve two-step verbal problems?
3) Does the sex of the student influence his cognitive development stage?
4) Does reading ability affect a student's ability to solve two-step verbal problems?

Sample

The sample for the study consisted of 40 grade seven students. These students were randomly selected from the total population of grade seven students attending three suburban elementary schools in metropolitan St. John's.

Method and Procedures

The sample students were administered the investigator's Verbal Problem Test and the comprehension

section of the Gates-MacGinite Reading Test, Level E, Form 1, as a group activity. Each individual student was then interviewed in an attempt to determine his formal reasoning ability on four Piagetan tasks.

Each hypothesis was tested for significance at the .01 level.

Results and Conclusions

Analysis of the results indicated that Piagetan cognitive development stage and reading comprehension ability are significant factors in mathematical two-step verbal problem solving performance. The sex of the student was not a factor in determining either the student's cognitive development stage or his two-step verbal problem solving performance.

The major conclusion of this study was that students in the concrete stage of development have more difficulty with two-step verbal problems than students in the formal stage of development.

ACKNOWLEDGEMENTS

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BACKGROUND TO THE PROBLEM

Introduction

Teaching students to solve problems has been recognized by many educators as one of the major roles of mathematics teaching. In this connection, the National Council of Supervisors of Mathematics (1978) asserted that learning to solve problems is the principal reason for studying mathematics and the National Council of Teachers of Mathematics (1980) recommended that, "problem solving be the focus of school mathematics in the 1980's." (p. 1).

But the term 'problem solving' encompasses a wide assortment of problems. These problems range from the traditional verbal problem in textbooks, to nonroutine problems which are not solvable by a direct application of an algorithm, to problems which apply mathematics to the real world.

The most basic of these three different types of problems and the one that students encounter most often in their school work is the traditional verbal problem. Verbal problems are introduced to students in grade one

and thereafter are reintroduced at each succeeding grade level. In this way, the basic skills required for verbal problem solving are continually explored and practised.

Verbal Problems

The important role given to verbal problems in mathematics curriculum throughout the primary and elementary grades would lead one to infer that by the late elementary grades students would have gained some proficiency at solving verbal problems. But the results of the second National Assessment of Educational Progress in which 9, 13, and 17 year old students were tested have demonstrated that students in the late elementary grades do not possess adequate verbal problem solving skills. In this assessment students performed adequately only on single step verbal problems which required the application of a single arithmetic operation. Student performance on multistep verbal problems and on verbal problems involving irrelevant data left much to be desired.

Research which attempts to discern the causes of this difficulty which students have with verbal problems is voluminous and findings by researchers prompted various proposed problem areas. Among these problem areas are the following.

1. Student's inability to read the problem (Martin, 1964).

2. Student's inability to do the necessary computations (Knifong & Hilton, 1976).

3. Student's inability to translate from the English to mathematical expressions (Clements, 1980).

4. Difficulties underlying the syntax and structure of the verbal problems (Jerman & Rees, 1972).

These research findings have enabled educators to gain a clearer understanding of the inherent difficulties that students may encounter when solving verbal problems. Unfortunately, these findings have not led to the implementation of changes which have resulted in marked improvements in the verbal problem solving scores of students.

A different approach to understanding the difficulties that students have with verbal problems appears necessary. A developmental approach is worthy of consideration. In this approach, the difficulties that students encounter when solving verbal problems are not perceived as lying within the problems but rather are seen as a consequence of the student's present inability to handle the situation presented in the problem.

Piagetan Logic

According to Inhelder and Piaget (1958/1958) children pass through four stages in their development of logical

thinking. These stages are:

Stage I - Sensori-motor

Stage II - Pre-operational

Stage III - Concrete operations

Stage IV - Formal operations

One of the main characteristics of the formal operation stage as contrasted with the concrete operation stage is the ability to think formally; that is, the ability of interpropositional thinking (Flavell, 1963). According to Flavell:

> Concrete operations are intrapropositional since they go to make the content of individual propositions, whereas formal operations are interpropositional, since they involve the logical relation among the propositions thus formed. (p. 206)

It may be that as the complexity of the verbal problem increases the ability to think interpropositionally helps a person to perceive the relationships that exist between the quantities that are contained in the problem statements. Thus attempts to teach students who have not attained the formal operation stage how to solve verbal problems which require formal reasoning ability will not prove to be very successful.

This study attempted to correlate student's performance on two-step mathematical verbal problems with their performance on Piagetan formal reasoning tasks.

Purpose of the Study

The major purpose of this study was to investigate the following question. Does a student's cognitive development stage affect his ability to solve two-step verbal problems?

Three minor questions were also investigated in this study.

1) Does there exist a relationship between the sex of the student and his ability to solve two-step verbal problems?

2) Does the sex of the student influence his cognitive development stage?

3) Does reading ability affect a student's ability to solve two-step verbal problems?

Based on these questions the following null hypotheses were formulated and tested in this study.

H_{o1}: There is no significant relationship between the ability to solve two-step verbal problems and the cognitive development stage of the student.

H_{o2}: There is no significant relationship between the sex of the student and the student's ability to solve two-step verbal problems.

H_{o3}: There is no significant relationship between the sex of the student and the student's cognitive development stage.

H_{o4}: There is no significant relationship between reading comprehension and the ability to solve two-step verbal problems.

Need for the Study

Many areas of mathematics instruction culminate in applying and solving verbal problems. Therefore, it is important to understand the factors which contribute to the underlying difficulties that students have with verbal problems. Caldwell and Goldin (1979), Days, Wheatley, and Kulm (1979), and Grady (1976) have shown that students who operate at the formal stage have less difficulty solving verbal problems than students who operate at the concrete stage. However, most research on the attainment of the formal operation stage (Dale, 1970; Friot, 1970; Tisher, 1971) have found that the formal operation stage is not attained by the majority of students until the last years of high school. Furthermore, there is some evidence (Graybill, 1974) which suggests that boys attain the formal operation stage at an earlier age than girls. This apparent earlier attainment of the formal operation stage by boys may be one of the contributing factors to the apparent superior performance of boys on verbal problems (Armstrong, 1981).

If these research findings can be substantiated, especially the relationship between cognitive development and verbal problem solving, then implications can be generated for the teaching of verbal problems at the late elementary school level. For example, placement of

students, nature of instructional strategies, and placement and sequencing of verbal problem topics may have to be altered. If the possibility of learning how to solve certain types of verbal problems is hindered or restricted by lack of acquistion of the necessary cognitive development stage, then the teaching of this type of verbal problem may need to be postponed and its place in the curriculum sequence changed. Also, teachers may need to use models and play acting to explain methods of verbal problem solving to students who are operating at the concrete stage, however these methods of explanation may not need to be used with students who are operating at the formal stage.

Definitions of Terms

Mathematical verbal problem. A verbal problem in mathematics refers to a written or printed word description of a quantitative situation about which a question is raised.

One-step verbal problem. A mathematical verbal problem requiring the application of one arithmetic operation in order to obtain a solution.

Two-step verbal problem. A mathematical verbal problem requiring the application of two arithmetic operations in order to obtain a solution.

Verbal problem solving ability. The trait measured by the investigator's Verbal Problem Test.

Reading comprehension ability. The trait measured by the comprehension section of the Gates-MacGinite Reading Test, Level E, Form 1.

Delimitations

Certain delimitations of this study are noted in this section. The study dealt with only one geographical area of the province, and only students from three schools were investigated. There was no attempt to exhaust all the socioeconomic and environmental factors associated with students' backgrounds. The study was further limited by the selection of a particular set of multistep verbal problems, namely two-step problems.

Outline of the Study

A review of the related literature is presented in Chapter II. Chapter III contains the procedures followed in conducting the study, and the method used in collecting and processing the data. The results of the data analysis are discussed in Chapter IV. The final chapter summarizes the conclusions reached as a result of the study, and

contains implications and recommendations for further research.

CHAPTER II

REVIEW OF RELATED LITERATURE

Introduction

The literature on the research areas of verbal problem solving, Piagetan studies and reading ability is extensive. This review treats only those studies which are directly related to the present study. The review is presented in the following order: (1) Theoretical basis for Piagetan research; (2) Research related to the age of attainment of formal operations; (3) Research related to the sex factor in attainment of the formal operation stage; (4) Research on the relationship between Piagetan cognitive development stage and achievement; (5) Research on the relationship between the sex of the student and verbal problem solving ability; and (6) Research on the relationship between reading ability and verbal problem solving ability.

Theoretical Basis for Piagetan Research

In Piagetan theory there are four stages of growth which characterize intellectual development: the sensori-motor stage, the pre-operational stage, the concrete operational stage, and the formal operational stage.

The sensori-motor stage lasts from birth to approximately two years of age. During this initial stage of development the child progresses from the reflex level of the newborn infant to the relatively coherent organization of sensori-motor action of the two year old. It should be noted, however, that this organization of sensori-motor action involves simple perceptual and motor adjustment to things rather than symbolic manipulations of them.

The pre-operational stage lasts from approximately two years of age to seven years of age. In this second stage of development the child makes his first attempts to come to grips with the new world of symbols. During this stage the child's thought processes evolve towards operational thought. But, as yet, the thought processes are not operational. It is only the last two stages, the concrete stage and the formal stage, that are operational.

Inhelder and Piaget (1958/1958) define an operation as a "a reversible internalizable action which is bound up

with others in an integrated structure" (p. xiv). In other words:

> An operation is a type of action. It can be carried out either directly, in the manipulation of objects, or internally, when it is categories or (in the case of formal logic) propositions which are manipulated. Roughly, an operation is a means for mentally transforming data about the real world so that they can be organized and used selectively in the solution of problems. An operation differs from simple action or goal directed behavior in that it is internalized and reversible. (Inhelder & Piaget, 1958/1958, p. xiii)

In the stage of concrete operations the child can only organize and classify information about objects or actions which are immediately present. Whereas, in the stage of formal operations, the child can reason about the potential as well as the real.

The child in the stage of formal operations has thought processes available to him which are not available to the child in the stage of concrete operations. Among these thought processes are the ability of hypothetico-deductive thinking, the ability of abstract thought, and the ability of formal thought.

It is the presence of these extra thought processes that may permit a student in the stage of formal operations to handle situations in verbal problems that may be incomprehensible to a child in the stage of concrete operations.

Research Related to the Age of Attainment of Formal Operations

Inhelder and Piaget (1958/1958) stated that the thought processes characteristic of the formal operation stage develop in students between the ages of 11 and 15 years with the equilibrium point being reached at an age of 15 years. Most first year students in grade seven are 12 or 13 years old which means that if they are similar to the students studied by Inhelder and Piaget they should be at the transition between the concrete stage of development and the formal stage of development.

However, there has been a large number of studies recently which have demonstrated that many students do not attain the formal operation stage until a later date. For example, Lovell (1961) in an extensive replication of Inhelder and Piaget's (1958/1958) work on the formal stage obtained results which supported Piaget's main stages of development, but the results did show that the transition ages of the stages were later than those claimed by Inhelder and Piaget. The group studied by Lovell was comprised of 200 subjects ranging in age from 8 to 18, together with 10 college students and 3 adults. Ten Piagetan tasks were employed to determine a subject's cognitive development stage.

Other researchers, in contrast to Lovell's approach of studying a wide range of age groups in one study, have concentrated on one particular age group in a study. The

results from these studies are generally supportive of the findings of Lovell. For example, Tisher (1971) in an attempt to validate a paper and pencil test, the purpose of which was to distinguish between concrete operational and formal operational students, tested 232 junior high school students. The paper and pencil test consisted of items which were based on four Piagetan tasks. For validation purposes, 57 of the students who completed the paper and pencil test were randomly selected and tested on three Piagetan tasks using an interview technique. In the age group from 13 years 5 months to 14 years 9 months only 25 percent were classified as operating at the formal stage. In the older age group of 15 and 16 year olds 40 percent were classified as formal thinkers.

While Tisher investigated the performance of high school students on Piagetan formal reasoning tasks, other researchers have studied older age groups. In one study, Juraschek (1975) investigated the performance of 131 prospective elementary teachers, 19 mathematics student teachers and 11 honors calculus students on three Piagetan tasks. Each subject was assigned a score of one, two, three, or four on each task depending upon whether he was classified as early concrete, late concrete, early formal, or late formal. The scores for each subject from the three tasks were then added to determine their cognitive development stage. Most of the student teachers and the

honors calculus students were classified as formal thinkers, but only 52 percent of the prospective elementary teachers were classified as formal thinkers.

McKinnon and Renner (1971) obtained results from their study which were supportive of the findings of Juraschek. In their study, Mckinnon and Renner investigated the performance of 131 college freshmen on five Piagetan tasks. They found that almost 75 percent of the college freshmen to be either partially or completely concrete operational.

In contrast to the approach of employing several Piagetan tasks to gain a broad perspective of the child's stage of development, some researchers have attempted to study only one aspect of formal operation stage thought. In one study, Dale (1970) attempted to investigate the age of attainment of combinatorial reasoning ability. In his study, Dale tested 200 children ranging in age from 6 years to 16 years on Inhelder and Piaget's colorless chemical liquid experiment. The procedure followed by Dale in the testing process was an interview technique similar to the one employed by Inhelder and Piaget. In the 11 and 12 year old age group only 10 percent of the students were able to completely solve the experiment, while in the 15 year old age group only 25 percent of the students were able to completely solve the experiment.

In a study which is of particular relevance to the present study, Karplus and Peterson (1970) investigated

the age of attainment of the understanding of the concept of proportion. In their study, Karplus and Peterson tested students from grade 4 to grade 12 on their grasp of the proportion concept. They used an instrument which consisted of two similar stickmen on a chart. The measurement of the two figures was demonstrated to the students using paper clips. The chart was then removed and the students were asked to respond to measurement and prediction questions about the stickmen. The results of the investigation demonstrated that successful proportional reasoning was not achieved until the last years of high school. This result is important because many of the multistep arithmetic verbal problems require proportional reasoning in the solutions.

The results of a study by Lawson and Wollman (1980) lends special significance to the findings of the study by Karplus and Peterson. In their study, Lawson and Wollman attempted to determine whether instruction on the concept of proportion would be beneficial to only those students who are in the formal stage of development. The sample for the study was 12 males and 18 females from a seventh grade class. The researchers classified the students as either early concrete, late concrete, or early formal on the basis of a pretest containing three Piagetan tasks. The students were instructed during 10 class periods on methods of solving proportion problems. After completion

of the training sessions, the students were administered a posttest on proportions. The results of the posttest indicated that the students classified as early concrete had little or no success with the proportion problems, the students classified as late concrete had only limited success with the proportion problems, and the students classified as early formal had fairly good success solving the proportion problems. When the results of this study by Lawson and Wollman and the results of the study by Karplus and Peterson (1970) are considered, then one may infer that attempts to teach the methods of solution to multistep verbal problems requiring proportional reasoning will prove fruitless for the vast majority of the students in the upper elementary grades.

In summary, many of the studies conducted on the formal operation stage have shown that the age of attainment of the formal stage to be later than the period of 11 to 15 years as postulated by Inhelder and Piaget. Of special importance to this study is the finding that students do not attain understanding of the concept of proportion until the last years of high school.

Research Related to the Sex Factor in Attainment of Formal Operation Stage

Inhelder and Piaget (1958/1958) did not differentiate between the sexes in the attainment of the formal operation stage and the research in this area of sex differences is inconclusive. One body of research indicates that boys attain the formal operation stage at a significantly earlier age than girls, while the other body of research tends to indicate that there is no difference between the sexes in the attainment of the formal operation stage. The present investigator is not aware of research which shows that girls attain the formal stage at a significantly earlier age than boys.

Dulit (1972) used two Piagetan tasks to investigate the cognitive development stage of groups of subjects. The four groups tested were: Average younger adolescents, age 14; Average older adolescents, age 16 and 17; Gifted older adolescents, age 16 and 17; and Average adults, age 20 to 55. The results for each group favored boys over girls in the attainment of the formal operation stage. For the three older groups, the percentages of males operating at the formal level were two to four times as great as those for females.

Graybill (1974) obtained results supportive of the findings of Dulit when he investigated sex differences in

the transition from concrete to formal operational thinking. He administered four Piagetan tasks to pairs of 9, 11, 13, and 15 year old boys and girls. Each student was interviewed individually on each task. From the results of his study Graybill concluded that boys attained the formal operation stage at a significantly earlier age than girls.

Other researchers (Field & Cropley, 1969; Dale, 1970) in their investigations of the formal operation stage, obtained results which favored boys over girls in the attainment of the formal stage. Dale commenting on his results stated:

> The finding that boys were more successful than girls in solving the problem was unexpected in view of the absence of mention of sex differences by Inhelder and Piaget (1958). This difference merits investigation. (p. 285)

In contrast to the findings of Graybill (1974) and Dulit (1972), other researchers have found no significant sex differences in the attainment of the formal operation stage. For example, Saarni (1973) in a study attempting to compare problem solving performance among formal operational, transitional, and concrete operational students with the effect of relative field independence administered three Piagetan tasks and two productive thinking problems to 64 middle class students ranging in age from 10 years 9 months to 15 years 1 month. No

significant sex differences were demonstrated on the Piagetan tasks.

In agreement with the findings of Saarni are the results of a study by Ball and Sayre (1972), a study by Friot (1970), and a study by Tisher (1971), all of which showed no significant relationship between the sex of the student and the attainment of the formal operation stage.

Research on the Relationship between Cognitive Development Stage and Achievement

The present study sought to investigate the relationship between Piagetan cognitive development stage and a particular aspect of achievement: namely, arithmetic verbal problem solving performance. Thus, the following discussion is of particular relevance.

The relationship between Piagetan cognitive development stage and achievement has been studied for students from primary school to college. At all levels a significant relationship has been found to exist.

At the primary level, Vaidya and Chansky (1980) conducted a study to investigate the relationship between performance on Piagetan cognitive development tasks, cognitive style, and mathematics achievement. Thirty four students, 17 males and 17 females, from each of the grades two, three, and four comprised the sample for the study.

These students were administered three tests: Stanford Achievement Test to ascertain mathematics achievement; Children's Embedded Figure Test to differentiate between field independent students and field dependent students; and Conservation Test Battery to determine cognitive development level. A statistical analysis of the results showed that cognitive level was a significant factor in determining mathematics achievement at each grade level.

At the junior high level, Weeks (1973) conducted a study to investigate the effect that sex, socio-economic status, scholastic aptitude, and school achievement had on the formal operational attainment of students. Weeks administered the Piagetan conservation of volume task, a logical reasoning task, and a task of the understanding of the correlative to 190 grade seven students, 195 grade eight students, and 175 grade nine students. Analysis of the results showed that a significant relationship existed between achievement and each of the three reasoning tasks at each grade level.

At the junior high and senior high levels, Ball and Sayre (1972) conducted a study to investigate the relationship between Piagetan cognitive development stage and science achievement. The researchers administered five Piagetan tasks to 419 students, comprised of grades eight, nine and ten biology students, and grade eleven chemistry students. To be classified as operating at the

formal level students had to display formal thinking on at least four of the five Piagetan tasks. The results showed that at each grade level a significant relationship existed between scholastic achievement in the science courses and the students' achievement on the Piagetan tasks.

The study by Juraschek (1975) investigated the relationship between Piagetan cognitive development stage and achievement at the college level. In his study, Juraschek found that the correlation between performance on the Piagetan tasks and mathematics scores was .68, which was significant at an alpha level of .01.

Researchers have also investigated the effect that cognitive development stage has on problem solving performance. A study by Caldwell and Goldin (1979) investigated the relative difficulties for elementary school students of four types of verbal problems: abstract factual, abstract hypothetical, concrete factual, and concrete hypothetical. The investigators hypothesized that:

> The adolescent at the stage of formal operational thought can construct systems and theories and can draw conclusions from pure hypothesis as well as from observations. Two of the main characteristics of this stage, as contrasted with the stage of concrete operations which precedes it, are the ability to deal with abstract situations and the capacity of thinking in a hypothetical-deductive manner (Johannot, 1947; Piaget, 1968). Thus a strictly developmental model would suggest that for elementary school children, concrete problems and factual problems should be less difficult than abstract problems and hypothetical

> problems respectively. For older subjects the differences should tend to disappear. (p. 323)

A verbal problem test, consisting of five problems of each type, and a computational test were administered to the sample, 399 students distributed among grades four, five, and six. Analysis of the results led the researchers to conclude:

> The findings of this study confirm that for elementary school children concrete verbal problems are substantially less difficult than abstract ones, when other relevant variables are controlled. The result still holds when attention is limited to children who possess definite competency in the appropriate computational skills. This tends to reinforce our notions of cognitive developmental consequences for problem solving. (p. 334)

The results of the study by Saarni (1973) support the notion of Caldwell and Goldin of cognitive developmental consequences for problem solving. In his study Saarni, after an analysis of the results, concluded that:

> The results obtained in this study indicate that individuals classified as formal operational (or transitional) were generally more competent problem solvers on the productive thinking problems than those classified as concrete operational. (p. 342)

Grady (1976) in an investigation closely related to this study found that the cognitive level accounted for more of the variance in the problem scores of first year algebra students than any variable considered except I.Q. But the relationship between problem solving performance and cognitive level did not prove to be statistically significant.

However, the investigator did note that with a larger sample than the 33 first year algebra students used in the study a statistically significant result for problem solving performance and cognitive level was very possible. Grady also found that formal operational students used more means-end heuristics in solving the verbal problems than the concrete operational students.

In an attempt to extend the findings of Grady (1976), Days, Wheatley, and Kulm (1979) investigated the differences in problem solving processes used by concrete operational and formal operational students. In addition, they attempted to discern the effect that the problem structure, either simple or complex, had on the processes the students used to solve the problems. Twenty nine concrete operationals and 29 formal operational students were selected as a sample for the study. Each of the selected students was required to attempt solutions of four simple structure and four complex structure problems. The researchers employed a thinking aloud technique and audiotaped the verbalizations of the students. The students verbalizations were subjected to a protocol-coding scheme. Analysis of the results showed that formal operational students employed a wider variety of problem solving processes and strategies compared to the concrete operational students. For concrete operational students there was no significant difference in difficulty between

simple structure and complex structure problems. Concrete operational students scored relatively low on both sets of problems. Formal operational students found the complex structure problems considerably more difficult than the simple structure problems. But on both sets of problems the formal operational students scored significantly better than the concrete operational students.

Research on the Relationship between Sex of the Student and Verbal Problem Solving Ability

In the vast majority of local, national, and international mathematics assessments boys have outperformed girls on verbal problems, especially multistep verbal problems, at all grade levels.

In the 1979-80 California mathematics assessment, comparison by sex of performance in the grade three level of the different skill areas tested led researchers to conclude that:

> There is a general difficulty level, however, that is, easy computation items are easy application items and hard computation questions are hard application questions. When the test data are examined by sex, however, the pattern is broken; girls outperform boys on basic skills and computation in multiplication, but boys outperform girls in application of basic facts and of multiplication. ("Student Achievement in California Schools: 1979-80 Annual Report", 1980, p. 213)

Comparisons by sex at the grade six level was

performed on the 1977-78 results of the annual California mathematics assessment. These comparisons lend further credence to the notion that boys are better problem solvers than girls, as can be evidenced from the remarks of the investigators:

> In the content areas of application of whole numbers and fractions, boys obtained higher scores than girls.... A closer look at the individual questions in decimal applications showed that while girls did better on problems that required a single computation or only a one-step analysis, boys did better on problems that required more complex reasoning. Often these problems required an intermediate computation (two-step analysis). This pattern seemed to be borne out in the area of application of whole numbers and decimals.... It is particularly interesting to note that the relative performance of boys and girls did not seem to be related to the context of the problem. ("Student Achievement in California Schools: 1977-78 Annual Report", 1978, p. 98)

In the National Assessment of Educational Progress (NAEP) mathematics assessment in the United States, 9, 13, and 17 year old students are tested for their competency in a wide range of mathematical skills. In the second national assessment conducted in 1977-78, boys outperformed girls at each age level on multistep verbal problems. For example, in the number and numeration skill area, boys aged nine years averaged 2.25 more correct solutions on multistep verbal problems compared to girls, 3.40 more correct solutions at age 13, and 4.97 more correct solutions at age 17.

Further support for boys superior performance on multistep verbal problems was obtained from the results of

the International Comparative Study of Achievement in mathematics held in 1967. Postlewaite (1971) in an analysis of these results by sex noted that, "In each population boys scored higher than girls even when other factors had been held constant. This was true for verbal and computational problems as well as total score" (p. 87).

Additional support for boys superior performance on verbal problems was obtained by Armstrong (1981) from her analysis of the results of two mathematics projects, Women in Mathematics Project and the second NAEP mathematics assessment. From her findings, Armstrong concluded that:

> Even when differences in participation are taken into account, men at nearly every level of participation have an advantage in solving typical one- and two-step word problems. (pp. 369-370)

Research on the Relationship between Reading Ability and Verbal Problem Solving Ability

That a relationship exists between a student's reading ability and his verbal problem solving ability seems obvious. Researchers have been interested in this relationship between reading ability and verbal problem solving ability for a long period of time, as can be evidenced by Monroe and Englehart's (1931) review of the studies for the first quarter of the century. More recent reviews of the research literature between reading ability and verbal problem solving ability found that correlations of this

relationship range from .40 to .86 (Aiken, 1967; & Martin, 1963). Other researchers (Dodson, 1972, & Kilpatrick, 1967) have shown that reading ability when taken together with I.Q. and computational ability adds significantly to the systematic variance accounted for in the problem solving scores of students.

Since a significant relationship between reading ability and verbal problem solving performance has been well established, some researchers have turned their attention to attempting to determine what facets of reading are important to verbal problem solving. Linville (1976) conducted a study to determine whether the difficulty of verbal problems in arithmetic is significantly affected by variation in syntax and vocabulary of the problem statement. Verbal problems were constructed with easy syntax, simple sentences only, or difficult syntax, complex sentences, combined with easy vocabulary or difficult vocabulary. Thus four verbal problem tests were constructed, with all the problems on the same test having the same degree of syntax and level of vocabulary. One of the four tests was then randomly given to each of the 408 fourth grade students who comprised the sample for the study. From the results obtained the researcher concluded:

> 1. Syntax and vocabulary level can both be determiners of difficulty in verbal arithmetic problems. Vocabulary level could be more crucial in determining success than syntax....

4. Pupils who have scored high in reading achievement can be expected to experience greater success in solving verbal arithmetic problems than pupils who scored low in reading achievement. (p. 157)

Several studies (Jerman & Mirman, 1974, Jerman & Rees, 1972, & Searle, Lorton, & Suppes, 1974) have used a linear regression model to study the effect of a multitude of different syntax variables on verbal problem difficulty. These studies have shown that the syntactic complexity of arithmetic verbal problems is a definite contributor to verbal problem difficulty.

Summary of Related Literature

A review of the research related to sex of the student, Piagetan cognitive development stage, reading ability, and verbal problem solving ability reveals that in certain of the relationships definite patterns have been shown to exist, while in other relationships the evidence is often contradictory and therefore inconclusive.

Piagetan theory has distinguished between the capabilities of the child in the concrete stage of development and the child in the formal stage of development. Of importance to this study is the fact that in Piagetan theory the formal operational student is seen as possessing extra thought processes that a concrete operational student does not have.

Inhelder and Piaget (1958/1958) had hypothesized that the thought processes characteristic of the formal stage develop in students between the ages of 11 to 15, but most research studies have shown that the age of attainment of the formal stage to be much later. Additionally, it has been shown that students do not understand the concept of proportion until the last years of high school.

In the research on how the sex of the student is related to the attainment of the formal stage, two sets of opposing findings have developed. In one set of findings boys have been found to attain the stage of formal operations at a significantly earlier age than girls, while in the other set of research no difference between the sexes in the attainment of the formal stage has been found.

Researchers have generally found that a relationship exists between Piagetan cognitive development stage and achievement. In terms of problem solving, formal operational students appear to be more competent problem solvers than concrete operational students.

In many of the mathematics assessments that have been conducted male students have outperformed female students on verbal problems, especially multistep verbal problems. This has been true for local, national, and international mathematics assessments.

Reading ability has been shown to be one of several significant factors which affect verbal problem solving

performance. Two of the facets of reading which help to contribute to this relationship between reading and verbal problem solving have been shown to be vocabulary and syntax.

In this study, the investigator is attempting to determine whether a significant relationship exists between Piagetan cognitive development stage and verbal problem solving performance. Whereas the majority of the research on this relationship has used algebraic verbal problems in their investigations, this study will concentrate on arithmetic verbal problems. The procedures followed in conducting the study and the methods used in collecting and processing the data are outlined in Chapter III. In Chapter IV, the results of the data analysis are discussed. In Chapter V, the conclusions reached as a result of this study are summarized, and implications and recommendations for further research are made.

CHAPTER III

DESIGN OF THE STUDY

Introduction

This chapter details aspects of the design of the study. It includes descriptions of the development of the verbal problem test, the piloting of the verbal problem tests, the development of the Piagetan tasks, the Gates-MacGinite Reading Test, the sample, the procedures used in conducting the study, and the method of collecting and analyzing the data.

Development of the Verbal Problem Test

Two verbal problem tests were constructed for piloting. One test consisted of 12 one-step verbal problems, while the other test contained eight two-step verbal problems. The investigator decided to pilot both one- and two-step verbal problems in order to determine the degree of difficulty each of these types of verbal problems posed for the grade seven student. In this way, a decision on the inclusion of either one- or two-step

verbal problems or a mixture of both types could be made for the final study.

The verbal problems chosen for inclusion on the pilot tests were selected after a careful analysis of types of verbal problems contained in four elementary arithmetic textbooks, listed in appendix E. The selected problems were typical of the ones that grade seven students might reasonably be expected to have mastered.

The problems on the pilot tests were designed so that they contained only whole numbers in their statements and also had whole number solutions. The problems were constructed in this way to eliminate any unneccesary computational difficulties that students might encounter in the solution process.

The two pilot tests are included in appendix B with the correct solutions to the problems on the pilot tests in appendix C.

Piloting of the Verbal Problem Tests

The two verbal problem tests were piloted with two grade seven classes, a total of 72 students. The classes for the pilot study were selected from a suburban St. John's elementary school. This type of school was chosen because its students had similar backgrounds to the students of the schools to be used in the final study.

TABLE 1

Percentage of Correct Solutions to Problems on the One-step Verbal Problem Pilot Test

Number of Problem	Percentage of Correct Solutions
1	100
2	100
3	100
4	93.8
5	100
6	93.8
7	100
8	100
9	96.9
10	96.9
11	100
12	90.6

The results of pilot test one, as noted in Table 1, showed that students had little difficulty in solving the one-step verbal problems. Therefore, it was decided that

this type of problem would not be of statistical value to the final study.

However, it was decided to include two of the one-step verbal problems, namely verbal problem number 1 and verbal problem number 11, at the beginning of the verbal problem test for the final study. These two one-step verbal problems were not to be used in the evaluation in the final study, but they would, hopefully, act as motivators. It was felt that most, if not all, students would be able to solve these two one-step verbal problems and this was verified as is indicated in the scores on the pilot test. The investigator reasoned that, in this way, each student in the final study would feel a sense of accomplishment at the beginning of the verbal problem test. Therefore, this should lessen the feeling of frustration of those students who are unable to solve the first one or two two-step verbal problems and thereby lead to a greater effort being put into the attempted solutions of the remaining two-step verbal problems.

Since the purpose of the verbal problem test for the final study was to discriminate among the problem solving abilities of students, it was decided to include two-step verbal problems numbered 2, 3, 4, 7, and 8 from pilot test

two on the final verbal problem test. The percentage of correct solutions to these problems on the pilot study ranged from 46.9% to 65.6%, as can be seen from Table 2. The average percentage of correct solutions to these problems on the pilot study was 56.25%.

TABLE 2

Percentage of Correct Solutions to Problems on the Two-step Verbal Problem Pilot Test

Number of Problem	Percentage of Correct Solutions
1	87.5
2	46.9
3	65.6
4	65.6
5	87.5
6	90.6
7	56.3
8	46.9

Thus the verbal problem test for the final study consisted of two one-step verbal problems and five two-step verbal problems. The solutions to the two one-step verbal

problems were not to be scored.

The final study verbal problem test is included in appendix D.

Development of Piagetan Tasks

Piagetan literature from the 1920's to the 1970's was studied in order to decide upon appropriate Piagetan tasks to use in the study to differentiate between formal operational and concrete operational thinkers. After carefully weighing the advantages and disadvantages of numerous possible tasks the following four were selected for inclusion in the study.

(1) Conservation of Volume using Two Identically Shaped Cylinders of Different Weights (Piaget, 1941/1974, p. 234).

(2) Logical Inference from a Verbal Statement (Piaget, 1928/1966, p. 87).

(3) Equilibrium in the Balance (Inhelder & Piaget, 1958/1958, pp. 164-181).

(4) Distinguish Form from Content in a Verbal Statement (Piaget, 1953/1957, pp. 18-19).

A brief description of the theory behind each of these Piagetan tasks will be presented now.

(1) Conservation of Volume using Two Identically Shaped Cylinders of Different Weights.

In relation to this task, Inhelder and Piaget (1958/1958) stated that:

> At this first formal level we can observe, in contrast, an important reworking of the operations and the explanations. The conservation of volume is finally acquired and the volume is finally distinguished from the quantity and the weight. (p. 141)

Furthermore, Inhelder and Piaget (1958/1958) stated that:

> Without a doubt the reason for this is that, in contrast, to simple forms of conservation, which the subject masters by simple additive compensations, the conservation of volume throughout changes of form presupposes the ability to handle proportions. However, we shall see ... why the concept of proportion does not itself appear before the formal level, when it arises in connection with certain general properties of the group structures characteristic of propositional operations. (p. 36)

(2) Logical Inference from a Verbal Statement.

Piaget (1928/1966) stated that only those students who are formal thinkers are able to determine the correct answer to a statement requiring logical inferences and thereafter are able to explain why the selected answer is the correct one.

(3) Equilibrium in the Balance.

According to Inhelder and Piaget (1958/1958) it is only at the stage of formal operations, stage III, that students discover the law of levers, $W/W' = L/L'$.

(4) Distinguish Form from Content in a Verbal Statement.

When given a statement which contains a contradiction within it, Piaget (1953/1957) stated that the concrete operational child critizes the data contained in the verbal statement while the formal operational child

accepts the data as given and brings out the contradiction contained within the statement.

A pilot study was conducted with eight children in order that the investigator could refine his interview technique with the Piagetan tasks.

Gates-MacGinite Reading Test

The Gates-MacGinite Reading Tests are a series of tests which provide information on the progress of students in two important areas of reading: vocabulary development and reading comprehension. The investigator decided to use the reading comprehension section of Level E, Form 1, of these tests to measure student's reading ability.

The comprehension section of the tests measures the student's ability to read complete prose passages with understanding. The questions asked are of two types: literal questions and inferential questions. A literal question asks about something that is explicitly stated in the passage. An inferential question asks about something that is only implied in the passage.

The reliability coefficients provided in the manual for the comprehension section of the tests ranged from .85 to .92. These values were calculated using the Kuder-Richardson Formula 20. The value of the reliability coefficient for the test used in this study was .88.

Content validity of the comprehension section of the tests is based on a proportional emphasis on different areas of reading material. To determine whether the comprehension section of the test has content validity for the students to be tested the curriculum studied by the students has to be looked at carefully. Thus, content validity is based on the judgement of the examiner.

Sample

Forty grade seven students were selected for the study. These forty students were randomly selected from the total population of 221 grade seven students attending three suburban elementary schools in the St. John's area.

The forty students selected for the study ranged in age from 12 years 3 months to 14 years 7 months.

Procedures

The verbal problem test developed for this study and the reading comprehension section of the Gates MacGinite Reading Test, Level E, Form 1, were administered by the investigator as a group activity in each of the three schools. After completion of these two tests, the students in each school were interviewed individually on the Piagetan tasks.

Upon entering the interview room, each student was given a brief overview of the study. They were told that the investigator was interested in attempting to discern the possible causes of difficulty that students encounter when solving verbal problems. They were told that the purpose of the verbal problem test was to determine whether they, as individual students, experienced difficulty with verbal problems. They were told that the purpose of the reading test was to determine whether reading difficulties were a main cause of verbal problem difficulty. The students were then asked if they had any questions about the study which they would like to ask. Finally, the students were told that during the interview session the investigator was interested in analyzing the thinking processes of each student. Thus, all comments and questions by the student were welcome.

The first task administered to each student was the 'Conservation of Volume using Two Identically Shaped Cylinders of Different Weights' task. Each student was shown two metal cylinders, a copper cylinder and an aluminium cylinder, that were the same size but different weights. It was indicated to each student that the two cylinders were the same size. Each student was then instructed to take a cylinder in each hand. The student was asked if there was a difference in weight between the two cylinders. Each student was next presented with two

identical cylinders partially filled with water and was allowed to adjust the water levels until he or she was confident that the two cylinders contained the same amount of water. The student was then asked whether the heavy cylinder, copper cylinder, would push the water level up more, or whether the lighter cylinder, aluminium cylinder, would push the water level up more, or whether the two cylinders would push the levels up the same. A successful prediction with a correct explanation gave the student a score of one on this task. Otherwise, the student was assigned a score of zero on this task.

The second task administered to each student was the 'Logical Inference from a Verbal Statement' task. Each student was given a card with the following problem on it:

> There are three girls: Mary, Jennie, and Sally. Mary is lighter than Sally, Mary is darker than Jennie. Who is darkest of all? (Piaget, 1928/1966, p. 87)

Each student was instructed to read the problem and to determine the answer to the question. After the answer was supplied by the student, he or she was asked why he or she had chosen that person. If the student was able to give the correct answer and also give an acceptable explanation then he or she was assigned a score of one on this task. Otherwise, the student was given a score of zero on this task.

The third task administered to each student was the 'Equilibrium in the Balance' task. The student was first shown the balance apparatus, which consisted of a balance arm with spokes on it at equal intervals. Weights of 10 grams and 5 grams were supplied to the student. A weight of 10 grams was hung 4 units of length from the fulcrum. The student was asked to hang another 10 gram weight on the opposite arm to achieve a balance. Following completion of this part of the task, the student's 10 gram weight was removed from the balance. The student was then given two 5 gram weights and was asked to place them on the lever to achieve a balance. Next, the two 5 gram weights were removed, and the student was given one 5 gram weight and was asked to hang it on the lever to achieve a balance. After the student had selected the location for the 5 gram weight, he or she was asked to explain the selection. Those students who made a correct prediction on the last part of this task and then were able to give an explanation based on proportional reasoning were given a score of one on this task. Otherwise, the student received a score of zero on this task.

The final Piagetan task administered to each individual student was the 'Distinguish Form from Content in a Verbal Statement' task. Each student was given a card with the following statement on it:

> "I am glad that I do not like liver, because if I liked it, I would always be eating it, and I hate eating things I dislike." (Gorman, 1972, p. 43)

Each student was instructed to read the statement and express his opinion about the statement. No guidance was given on the part of the investigator as to the expected opinion. If the student was able to perceive the contradiction in the statement then he or she was given a score of one on this task. Otherwise, the student was assigned a score of zero on this task.

Procedures for Data Analysis

After completion of the study, the verbal problem tests were scored and the students assigned a score from zero to five depending on the number of correct solutions to the two-step verbal problems. The scores for the four Piagetan tasks were added so that each student received a score from zero to four on the Piagetan tasks. Raw scores on the comprehension tests were converted to t-scores for data analysis purposes.

To test hypotheses 1 and 4 the Pearson product-moment correlation coefficient was calculated and tested for significance at the .01 level.

To test hypotheses 2 and 3 the point-biserial correlation coefficient was calculated and tested for significance at the .01 level.

In this chapter, the procedures used in conducting the study and the method used in collecting and processing the data have been outlined. In Chapter IV, the results of the data analysis are discussed. In Chapter V, the conclusions reached as a result of the study are summarized, and implications and recommendations for further research are made.

CHAPTER IV

ANALYSIS OF DATA

Introduction

The major purpose of this study was to determine if the Piagetan cognitive development stage affects the ability to solve verbal problems. Consequently, the major hypothesis in this study concerned the relationship between Piagetan cognitive development stage and the ability to solve two-step verbal problems. A secondary major hypothesis concerned the relationship of reading comprehension to problem solving ability. Additionally, two minor hypotheses concerned the relationship of sex to Piagetan cognitive development stage and to verbal problem solving ability.

Forty students were randomly selected from the total population of grade seven students attending three elementary schools. The students were first administered the reading comprehension test and the verbal problem test. After completion of these two tests, individual students were interviewed to discern their level of thought on four Piagetan tasks.

Presentation and Analysis of Results

The major hypothesis to be tested in this study was:

H_{01}: There is no significant relationship between the ability to solve two-step verbal problems and the cognitive development stage of the student.

To test this hypothesis, the investigator's Verbal Problem Test (VPT) and the Piagetan Task Test (PTT) were administered to the forty students. The scores on the VPT corresponded to the number of correct solutions to the two-step verbal problems, while the scores on the PTT corresponded to the number of tasks on which the student displayed formal reasoning. The mean and standard deviation of the scores obtained on these two tests along with the Pearson product-moment correlation coefficient between the two groups of test scores are contained in Table 3.

The Pearson product-moment correlation coefficient between the scores on the VPT and the scores on the PTT was calculated to be .72. As noted in Table 3, this value proved to be significantly different from a value of zero at an alpha level of .01. Consequently, null hypothesis 1 was rejected.

TABLE 3

Mean and Standard Deviation of
and Pearson Product-moment Correlation Coefficient between
Student Scores on VPT and PTT

Test	Mean	Standard Deviation	r_{xy}	z
VPT	2.93	1.46	.72	5.52*
PTT	1.58	1.24		

*z-value is significant at an alpha level of .01.

The second hypothesis to be tested in this study was:

H_{o2}: There is no significant relationship between the sex of the student and the student's ability to solve two-step verbal problems.

In the study sample, there were 22 boys and 18 girls. The mean and standard deviation of the scores received by males and females on the VPT and the point-biserial correlation coefficient between these two sets of scores are contained in Table 4.

The point-biserial correlation coefficient between the sex of the student and the problem solving performance as measured by the VPT was calculated to be .012 in favor of females. As noted in Table 4, this value

did not prove to be significantly different from a value of zero at an alpha level of .01. Consequently, null hypothesis 2 was accepted.

TABLE 4

Mean and Standard Deviation of and Point-biserial Correlation Coefficient between Male and Female Scores on VPT

Sex	Mean	Standard Deviation	r_{pb}	z
Male (N = 22)	2.91	1.63	.012	.074*
Female (N = 18)	2.94	1.26		

*z-value is not significant at an alpha level of .01.

The third hypothesis to be tested in this study was:

H_{03}: There is no significant relationship between the sex of the student and the student's cognitive development stage.

The mean and standard deviation of the scores received by males and females on the PTT and the point-biserial correlation coefficient between these two sets of scores are contained in Table 5.

The point-biserial correlation coefficient between the

sex of the student and the formal reasoning performance on the PTT was calculated to be .014 in favor of males. As noted in Table 5, this value did not prove to be significantly different from a value of zero at an alpha level of .01. Consequently, null hypothesis 3 was accepted.

TABLE 5

Mean and Standard Deviation of and Point-biserial Correlation Coefficient between Male and Female Scores on PTT

Sex	Mean	Standard Deviation	r_{pb}	z
Male (N = 22)	1.59	1.30	.014	.086*
Female (N = 18)	1.56	1.20		

*z-value is not significant at an alpha level of .01.

The final hypothesis to be tested in this study was:

H_{o4}: There is no significant relationship between reading comprehension and the ability to solve two-step verbal problems.

To test this hypothesis, the reading comprehension section of the Gates-MacGinite Reading Test, Level E, Form 1, was administered to the forty students. The raw

scores obtained on the comprehension test were converted to t-scores for statistical purposes. The mean and standard deviation of the t-scores obtained on the reading comprehension test and of the scores students obtained on the VPT are contained in Table 6. Table 6 also contains the Pearson product-moment correlation coefficient between these two sets of scores.

The Pearson product-moment correlation coefficient between the reading comprehension test scores and the VPT scores was calculated to be .44. As noted in Table 6, this value proved to be significantly different from a value of zero at an alpha level of .01. Consequently, null hypothesis 4 was rejected.

TABLE 6

Mean and Standard Deviation of and Pearson Product-moment Correlation Coefficient between Reading Test t-scores and VPT Scores

Test	Mean	Standard Deviation	r_{xy}	z
Reading	49.63	8.22	.44	2.87*
VPT	2.93	1.46		

*z-value is significant at an alpha level of .01.

Discussion and Additional Findings

The correlation coefficient calculated for the relationship between verbal problem solving ability as measured by the VPT and formal reasoning ability on the Piagetan tasks was .72. In other words, this relationship accounted for 49% of the variance in the problem solving scores of the sample students. This relationship proved to be significant at an alpha level of .01. There are several reservations one has to consider when trying to interpret this result. First, very few of the students who participated in the study would be classified as being in the formal stage of development. If one were to classify only those students who displayed formal reasoning on at least three of the Piagetan tasks as operating in the formal stage of development, then a total of nine students, five boys and four girls, would have satisfied this criterion. This represented only 23.5% of the total student sample used in the study. One should note, however, that the lowest score obtained by those nine formal operational students on the VPT was 3 and that the mean score for those nine students on the VPT was 4.4. This result did demonstrate that those students who were classified as operating in the formal stage of development had very little difficulty with the two-step verbal problems on the VPT.

However, if one were to classify only those students who scored zero or one on the Piagetan tasks as being in the concrete stage of development, then a total of 22 students, 13 boys and 9 girls, would have satisfied this criterion. This result represented 55% of the total student sample used in the study. That is, over one-half of the students who participated in this study were still in the concrete stage of development. The highest score obtained by those 22 concrete operational students on the VPT was 4 and the mean score for those 22 students on the VPT was 2.04. This result did demonstrate that those students who were classified as operating in the concrete stage of development successfully solved some of the two-step verbal problems on the VPT, but they were unable to display mastery of this type of problem. To determine whether a pattern existed in the concrete operational student's ability to solve certain of the two-step verbal problems, an item analysis was performed on the test results of the concrete operational student's performance on the VPT. The results obtained are contained in Table 7.

As can be seen from Table 7, only problem number 5 was mastered at an acceptable level by the population of students who were classified as being in the concrete stage of development. Problem number 3 proved to be the most difficult for this group of students with only 13.6% of

the students who were classified as operating in the concrete stage of development obtaining the correct solution. All other two-step verbal problems on the VPT were solved successfully by less than one half of the students who were classified as operating in the concrete stage of development.

TABLE 7

Percentage of Correct Solutions by Item Obtained by those Students Classified as Concrete on the Two-step Verbal Problems on the VPT

Item Number	Percentage of Correct Solutions
3	13.6%
4	36.4%
5	72.7%
6	27.3%
7	40.9%

Summary

The four hypotheses tested in the study were:

H_{o1}: There is no significant relationship between the ability to solve two-step verbal problems and the cognitive development stage of the student.

H_{o2}: There is no significant relationship between the sex of the student and the student's ability to solve two-step verbal problems.

H_{o3}: There is no significant relationship between the sex of the student and the student's cognitive development stage.

H_{o4}: There is no significant relationship between reading comprehension and the ability to solve two-step verbal problems.

Analysis of the results of the study led to acceptance of null hypotheses 2 and 3 and rejection of null hypotheses 1 and 4. Two additional findings of interest were that the majority of students who participated in the study appeared to be still in the concrete stage of development and that those students who were classified as being in the concrete stage of development displayed inadequate mastery of four of the five two-step verbal problems on the VPT.

In this chapter, the results of the study have been presented and analyzed. In Chapter V, the conclusions reached as a result of this study are summarized, and implications and recommendations for further research are made.

CHAPTER V

SUMMARY, RESULTS, LIMITATIONS, CONCLUSIONS, IMPLICATIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

Summary

The major purpose of this study was to discern whether the Piagetan cognitive development stage of the student affects the ability to solve mathematical verbal problems. The study also attempted to determine whether reading comprehension ability was related to verbal problem solving ability. Two minor relationships involving sex and verbal problem solving ability, and sex and student's cognitive development stage were also investigated.

A verbal problem test was developed for the study. Two verbal problem tests were piloted to determine appropriate items to include on the verbal problem test for the final study. Pilot test 1 contained only one-step verbal problems, while pilot test 2 contained only two-step verbal problems. From the results of the pilot, only two-step verbal problems were deemed appropriate for statistical analysis in the final study. Therefore, the

verbal problem test for the final study was composed of two one-step verbal problems and five two-step verbal problems. The two one-step verbal problems were not scored since their sole purpose was to act as motivators.

Four Piagetan tasks were selected as measures of formal reasoning ability. The comprehension section of the Gates-MacGinite Reading Test, Level E, Form 1, was used as a measure of reading ability.

Forty students were randomly selected from the total population of grade seven students attending three elementary schools in the St. John's area. At the beginning of the first day that the investigator spent in each school, the selected students in that school were administered the reading test and the verbal problem test. After completion of these two tests, the selected students in the school were interviewed individually on the Piagetan tasks.

All tests were scored by the investigator. On the investigator's verbal problem test, the student's score corresponded to the number of correct solutions that he or she obtained on the five two-step verbal problems. On the Piagetan tasks, the student's score corresponded to the number of tasks on which the student displayed formal reasoning. The raw scores that the students received on the reading test were converted to t-scores for statistical purposes.

Four hypotheses were tested in the study. They were:

H_{o1}: There is no significant relationship between the ability to solve two-step verbal problems and the cognitive development stage of the student.

H_{o2}: There is no significant relationship between the sex of the student and the student's ability to solve two-step verbal problems.

H_{o3}: There is no significant relationship between the sex of the student and the student's cognitive development stage.

H_{o4}: There is no significant relationship between reading comprehension and the ability to solve two-step verbal problems.

Hypotheses 1 and 4 were tested using the Pearson product-moment correlation coefficient which was tested by a Z-test for significance at the .01 level. Hypotheses 2 and 3 were tested using the point-biserial correlation coefficient which was tested by a T-test for significance at the .01 level.

Results of Data Analysis

The analysis of the data collected in the study gave the following results:

1. Null hypothesis 1 was rejected. The obtained value of the Pearson product-moment correlation coefficient, .72, between Piagetan cognitive development stage and the

two-step verbal problem solving ability proved to be significant at a level of significance of .01.

2. Null hypothesis 2 was accepted. No significant relationship existed between the sex of the student and two-step verbal problem solving ability. The calculated value of the point-biserial correlation coefficient was only .012.

3. Null hypothesis 3 was accepted. No significant relationship existed between the sex of the student and the Piagetan cognitive development stage. The calculated value for the point-biserial correlation coefficient was only .014.

4. Null hypothesis 4 was rejected. The obtained value of the Pearson product-moment correlation coefficient, .44, between reading ability and two-step verbal problem solving ability proved to be significant at a level of significance of .01.

In summary, this study found that for the population of grade seven students tested, the cognitive development stage of the student and the student's reading comprehension ability were significant factors in the student's ability to solve two-step verbal problems. These results support the findings of Saarni (1973), Days, Wheatley, and Kulm (1979), and Caldwell and Goldin (1979) who found that formal operational students were generally better problem solvers than concrete operational students.

Additionally, these results support the findings of Aiken (1972) and Martin (1964) who found that reading ability was a significant factor in mathematical achievement.

This study also found that the sex of the student was not a significant factor in the attainment of the formal operation stage of development. This result is not in agreement with the findings of Graybill (1974), Dulit (1972), or Dale (1970) who found that boys attain the formal operation stage at a significant earlier age than girls. But the result is in agreement with the findings of Saarni (1973), Ball and Sayre (1972), Friot (1970), and Tisher (1971) who found no significant difference in the attainment of the formal operation stage between boys and girls. Since the study did not attempt to measure the percentage of males and females who had attained the formal operation stage at different age levels, caution must be exercised in interpreting this result. Additionally, the number of students in this study who had attained the formal operation stage was very minimal.

Finally, this study found that the sex of the student was not a significant factor in two-step verbal problem solving ability. This result contradicts one of the major findings obtained from analyzing local, national, and international mathematics assessments. But Fennema (1977) and Meyer (1978) have proposed that the apparent superior mathematical performance by boys compared to girls is

influenced by the differential treatment received by boys and girls during the instructional process. Additionally, these two researchers have noted that girls do not appear to be as highly motivated in mathematics as boys. Since the results of this study do not support the hypothesis that boys outperform girls on multistep verbal problems, then these factors identified by Fennema and Meyer may not be present in the classes used in this study. However, this is only a supposition since no attempt was made in this study to measure these factors.

Limitations

Certain limitations of this study are noted in this section. First, any generalization to other populations of students not sampled in this study must take the following considerations into account. The sample of students used in the study was small, 40 students, from one geographical area of the province, St. John's. All three schools which participated in the study were located in the suburban sections of St. John's and consequently, the majority of students attending these three schools were from the middle socio-economic class.

Problem solving ability, formal reasoning ability, and reading ability were limited in this study to the traits measured by the tests used in the study in

ascertaining them. There was no claim made that the skills necessary to solve two-step verbal problems were identical to the skills necessary to solve other multistep verbal problems.

Finally, the relatively short length of the verbal problem test, five two-step verbal problems, and the Piagetan task test, four tasks, were major limitations of the study.

Conclusions

The following conclusions seem to be justified on the basis of the data collected, correlation coefficients calculated, and the tests of significance run:

1. Students who are still in the concrete stage of development have more difficulty with two-step verbal problems than students in the formal stage of development.

2. The sex of the student does not affect either the student's cognitive development stage or his ability to solve two-step verbal problems.

3. A student's reading ability affects his ability to solve two-step verbal problems.

Implications

The results of this study when taken together with the results of other research studies dealing with problem solving ability and/or cognitive development stage contain certain implications for educators and teachers at the elementary school level.

First, and in the opinion of this investigator, the most important observation is the relatively small percentage of students in the upper elementary grades who have attained the formal stage of development. This implies that for the vast majority of students in the upper elementary grades, methods and strategies of instruction must be of a concrete nature. That is, teachers must employ methods of instruction, such as modelling and play acting, which are conducive to students in the concrete stage of development.

The observation that the majority of students in the upper elementary grades are still in the concrete stage of development poses implications for the sequencing and placement of types of multistep verbal problems. Thus, a thorough analysis of types of multistep verbal problems should be undertaken and recommendations should be made as to the proper sequencing and placement of these different types. For example, multistep verbal problems requiring proportional reasoning should be delayed until late high

school in order that students would have acquired the necessary formal thought processes necessary to understand this type of problem.

Finally, it is important to note that a student's reading ability affects his verbal problem solving ability. In light of this finding, it may be necessary for classroom teachers of mathematics to instruct students on how to properly read mathematical verbal problems.

Recommendations for Further Research

As a result of the study, it is recommended that:

1. Studies similar to the present study, but using three-step or other variants of multistep verbal problems, be conducted at the seventh grade level to determine whether there exists a general relationship between cognitive development and multistep verbal problem solving ability.

2. Studies similar to the present study be conducted with students in the grades in junior and senior high school to try and determine if the attainment of the formal operation stage of development by a larger proportion of the students leads to a higher multistep verbal problem solving performance in these grades.

3. Studies be undertaken to try and determine if all the new thought processes available to the formal operational student are equally important in determining problem solving ability.

4. Studies be conducted in the upper elementary grades to try and determine if sex is a factor in other aspects of mathematical achievement.

REFERENCES

Aiken, L. R. Language factors in learning mathematics. Mathematics Education Reports. Columbus, Ohio, 1967. (ERIC Document Reproduction Service No. ED 068 340)

Armstrong, J. Achievement and participation of women in mathematics: results of two national surveys. Journal for Research in Mathematics Education, 1981, 12, 356-372.

Ball, D., & Sayre, S. Relationship between student Piagetan cognitive development and achievement in science. (Doctoral dissertation, University of North Colorado, 1972). Dissertation Abstracts International, 1972, 33, 1069A-1070A. (University Microfilms No. 72-23, 794)

Caldwell, J., & Goldin, G. Variables affecting word problem difficulty in elementary school mathematics. Journal for Research in Mathematics Education, 1979, 10, 135-146.

Clements, M. A. Analyzing children's errors on written mathematical tasks. Educational Studies in Mathematics, 1980, 11, 1-21.

Dale, L. The growth of systematic thinking: replication and analysis of Piaget's first chemical experiment. Australian Journal of Psychology, 1970, 22, 277-286.

Days, H., Wheatley, G., & Kulm, G. Problem structure, cognitive level, and problem solving performance. Journal for Research in Mathematics Education, 1979, 10, 135-146.

Dodson, J. Characteristics of successful insightful problem solvers. NLSMA Report No. 31. Stanford, California: School Mathematics Study Group, 1972.

Dulit, E. Adolescent thinking a la Piaget: the formal stage. Journal of Youth and Adolescence, 1972, 1, 281-301.

Fennema, E. Sex-related differences in mathematics achievement: myths, realities and related factors. Washington, D.C.: National Institute of Education, 1977. (ERIC Document Reproduction Service No. ED 160 445)

Field, T., & Cropley, A. Cognitive style and science achievement. Journal of Research in Science Teaching, 1969, 6, 2-10.

Flavell, J. The developmental psychology of Jean Piaget. New York: Litton Educational Publishing, Inc., 1963.

Friot, E. The relationship between an inquiry teaching approach and intellectual development. (Doctoral dissertation, University of Oklahoma, 1970). Dissertation Abstracts International, 1971, 31, 5872A. (University Microfilms No. 73-32, 215)

Grady, M. Problem solving in algebra as related to Piagetan levels of thought. (Doctoral dissertation, University of Texas at Austin, 1975). Dissertation Abstracts International, 1976, 36, 6587A. (University Microfilms No. 76-8, 034)

Graybill, L. A. A study of sex differences in the transition from concrete to formal thinking patterns. (Doctoral dissertation, Rutgers University, 1973). Dissertation Abstracts International, 1974, 34, 3988A. (University Microfilms No. 73-32, 215)

Inhelder, B., & Piaget, J. (The growth of logical thinking from childhood to adolescence) (A. Parsons and S. Milgram, trans.) New York: Basic Books, Inc., Publishers, 1958. (French version originally published, 1958.)

Jerman, M., & Mirman, S. Linguistic and computational variables in problem solving in elementary mathematics. Educational Studies in Mathematics, 1974, 5, 317-362.

Jerman, M., & Rees, R. Predicting the relative difficulty of verbal arithmetic problems. Educational Studies in Mathematics, 1972, 4, 306-323.

Juraschek, W. The performance of prospective teachers in certain Piagetan tasks. (Doctoral dissertation, University of Texas at Austin, 1974). Dissertation Abstracts International, 1975, 35, 5989A-5990A. (University Microfilms No. 75-4, 400)

Karplus, R., & Peterson, R. Intellectual development beyond elementary school II: ratio, a survey. School Science and Mathematics, 1970, 70, 813-820.

Kilpatrick, J. Analyzing the solution of word problems in mathematics: an exploratory study. Unpublished doctoral dissertation, Stanford University, 1967.

Knifong, J., & Holtan, B. An analysis of children's written solutions to word problems. Journal for Research in Mathematics Education, 1976, 7, 106-112.

Lawson, A., & Wollman, W. Developmental level and learning to solve problems of proportionality in the classroom. School Science and Mathematics, 1980, 80, 69-75.

Linville, W. Syntax, vocabulary, and the verbal arithmetic problem. School Science and Mathematics, 1976, 76, 152-157.

Lovell, K. A follow-up study of Inhelder and Piaget's the growth of logical thinking. British Journal of Psychology, 1961, 52, 143-153.

Martin, M. Reading comprehension, abstract verbal reasoning, and computation as factors in arithmetic problem solving. (Doctoral dissertation, State University of Iowa, 1964). Dissertation Abstracts, 1964, 24, 4547-4548. (University Microfilms No. 64-3395)

McKinnon, J., & Renner, J. Are colleges concerned with intellectual development. American Journal of Physics, 1971, 39, 1047-1052.

Meyer, R. A. Sex-related differences in mathematical problem solving performance and intellectual abilities. Michigan: Western Michigan University, 1978. (ERIC Document Reproduction Service No. ED 166 023)

Monroe, W. S., & Engelhart, M. D. A critical summary of research relating to the teaching of arithmetic. University of Illinois Bulletin 29, Bureau of Educational Research Bulletin No. 58. Urbana: University of Illinois, 1931, 81-88.

National Council of Supervisors of Mathematics. Position statement on basic skills. Mathematics Teacher, 1978, 71, 147-152.

National Council of Teachers of Mathematics. An agenda for action. Reston, Virginia: NCTM, 1980.

Piaget, J. (Logic and psychology) (W. Mays, trans.) New York: Basic Books, Inc., Publishers, 1957. (French version originally published, 1953.)

Piaget, J. (Judgement and reasoning in the child) (C. Ogden, Ed. and M. Warden, trans.) Totowa, New Jersey: Littlefield, Adams & Co., 1966. (French version originally published, 1928.)

Piaget, J. (The child's construction of quantities) (A. Pomerans, trans.) London: Routledge & Kegan Paul, Ltd., 1974. (French version originally published, 1941.)

Postlewaite, T. International Association for the Evaluation of Educational Achievement (IEA) - the mathematics study. Journal for Research in Mathematics Education, 1971, 2, 69-103.

Saarni, C. Piagetan operations and field independence as factors in children's problem solving performance. Child Development, 1973, 44, 338-345.

Searle, B., Lorton, P., Jr., & Suppes, P. Structural variables affecting CAI performance on arithmetic word problems of disadvantaged and deaf students. Educational Studies in Mathematics, 1973, 5, 371-384.

Student achievement in California schools: 1977-78 annual report. Sacremento, California: California State Department of Education, 1978. (ERIC Document Reproduction Service No. ED 167 610)

Student achievement in California schools: 1979-80 annual report. Sacremento, California: California State Department of Education, 1980. (ERIC Document Reproduction Service No. ED 195 559)

Tisher, R. A Piagetan questionnaire applied to pupils in a secondary school. Child Development, 1971, 42, 1633-1636.

Vaidya, S., & Chansky, N. Cognitive development and cognitive style as factors in mathematical achievement. Journal of Educational Psychology, 1980, 73, 326-330.

Weeks, R. The relationship of grade, sex, socio-economic status, scholastic aptitude, and school achievement to formal operations attainment in a group of junior high school students. (Doctoral dissertation, Kent State University, 1973). Dissertation Abstracts International, 1973, 34, 2405A. (University Microfilms No. 73-27, 261)

APPENDIX A

CORRESPONDENCE

TELEPHONE 753-8530

Roman Catholic School Board for St. John's

BELVEDERE
BONAVENTURE AVENUE
ST. JOHN'S, NEWFOUNDLAND
A1C 3Z4

72

January 4, 1981

Dear Principal

This will introduce to you Mr. Walter Ryan who is on a Master's program at the University. He is interested in doing a project dealing with Word Problems - Piaget's Levels of Thought with a random sample of Grade Seven students.

The Board has no objection to this study and I am sure Mr. Ryan will appreciate your cooperation in allowing some of your students to be involved.

Sincerely yours,

Brother A. F. Brennan
Superintendent

AFB/md

25 Fourth Street
Mount Pearl, Nfld.
A1N 2B4
March 22, 1982

Principal

Dear

This is to confirm the undertaking of the previously discussed study in your school. My records show that I will be conducting my testing in your school on April and , 1982. If these dates are no longer appropriate please contact me by telephoning 364-3867.

I have enclosed a list of students in your school that have been selected for the study. If any of the selected students have left your school please inform me of such.

The basic setup of the study is as follows: On the first hour and one half of day one I will administer a problem solving test and a reading comprehension test to the entire group of selected students in your school. In the subsequent time on the two testing days I will be administering four Piagetan tasks to each individual student.

Thank you for your co-operation; it is greatly appreciated.

Walter Ryan

APPENDIX B

COPY OF THE VERBAL PROBLEM PILOT TESTS

VERBAL PROBLEM PILOT TEST 1

DIRECTIONS: Solve each of the following problems in the space provided. Show all your calculations.

1. In Hal's school there are 456 students in the fourth grade and 373 in the fifth grade. How many students are in the two grades?

2. A rope is 150 meters long. A piece 112 meters long is cut off. How long is the remaining piece?

3. Marie earned $140 giving haircuts. She charges $7 per haircut. How many haircuts did Marie do?

4. There are 26 books in an encyclopedia set. Each book weighs 830 grams. How much does the whole set weigh?

5. St. George's has 783 registered voters. In the last election only 387 people voted. How many registered voters did not vote?

6. Flat Bay has 970 people. An average citizen uses about 65 liters of water each day. How many liters of water are used by all the citizens of Flat Bay in one day?

7. A runway at an airport is 372 meters long. The airport plans to increase its length by 123 meters. How long will the runway be after it has been lengthened?

8. Main Street is 240 meters long. The street is marked off into sections which are each 8 meters long. A tree is going to be planted in each section. How many trees will be needed?

9. Denise earns $12 on each wig she sells. She sold 132 wigs. How much money did she earn from the sale of the wigs?

10. The eighth grade presented a class play. The first performance 172 people attended. The second performance 136 people attended. How many people saw the play?

11. Mark has 360 baseballs to pack into boxes. Each box can hold 12 baseballs. How many boxes does Mark need?

12. A loaded truck and trailer weighs 272 metric tons. Without the load, the truck and trailer weighs 105 metric tons. How much does the load weigh?

VERBAL PROBLEM PILOT TEST 2

DIRECTIONS: Solve each of the following problems in the space provided. Show all your calculations.

1. Last year pollution killed 1500 fish in Kingston lake. Sewage was blamed for 700 dead fish and industry for 600 dead fish. How many fish were killed by other pollution causes?

2. A tank contains 250 liters of milk. One liter fills 3 small milk cartons. 60 small milk cartons are filled from the tank. How many liters of milk are left in the tank?

3. 900 kilograms of newspaper can be made from about 18 trees. City News needs 2700 kilograms of newspaper. How many trees must be used to make this amount of newspaper?

4. Suppose each person who walks through City Park throws 2 candy bar wrappers on the ground. Every hour 65 people walk through the park. After 12 hours, how many bar wrappers are all over the park?

5. Joe's dress shop receives its dresses in cartons. In the last carton there were 140 dresses. 6 of the dresses were torn and therefore returned. The rest of the dresses were sold at $12 each. How much money was received from the sale of the dresses?

6. Pat's garage pays its head mechanic $21,385 annually. The 6 assistant mechanics are paid $15,328 annually each. How much should the garage budget for mechanics salaries?

7. Jim's chalk company has 36,000 sticks of chalk. 24 sticks of chalk are placed in a box and 30 boxes are placed in a carton. How many cartons are needed?

8. A high school service club is planning to wrap 54 Christmas gifts for a children's hospital. Each gift requires 12 meters of ribbon. Rolls of ribbon contain 36 meters. How many rolls of ribbon will be needed?

APPENDIX C

CORRECT RESPONSES TO TEST ITEMS ON VERBAL PROBLEM PILOT TESTS

CORRECT RESPONSES TO ITEMS ON

VERBAL PROBLEM PILOT TEST 1

Item	Correct Response
1	829
2	38 meters
3	20
4	21, 580 grams
5	396
6	63, 050 liters
7	495 meters
8	30
9	$1584
10	308
11	30
12	167 metric tons

CORRECT RESPONSES TO ITEMS ON

VERBAL PROBLEM PILOT TEST 2

Item	Correct Response
1	200
2	230 liters
3	54
4	1560
5	$1608
6	$113, 353
7	50
8	18

APPENDIX D

FINAL STUDY VERBAL PROBLEM TEST

VERBAL PROBLEM TEST

DIRECTIONS: Solve each of the following problems in the space provided. Show all your calculations.

1. In Hal's school there are 456 students in the fourth grade and 373 in the fifth grade. How many students are in the two grades?

2. Mark has 360 baseballs to pack into boxes. Each box holds 12 baseballs. How many boxes does Mark need?

3. A tank contains 250 liters of milk. One liter fills 3 small milk cartons. 60 small milk cartons are filled from the tank. How many liters of milk are left in the tank?

4. 900 kilograms of newspaper can be made from 18 trees. City News needs 2700 kilograms of newspaper. How many trees must be used to make this amount of newspaper?

5. Suppose each person who walks through City Park throws 2 candy bar wrappers on the ground. Every hour 65 people walk through the park. After 12 hours, how many bar wrappers are all over the park?

6. Jim's chalk company has 36,000 sticks of chalk. 24 sticks of chalk are placed in a box and 30 boxes are placed in a carton. How many cartons are needed?

7. A high school service club is planning to wrap 54 Christmas gifts for a children's hospital. Each gift requires 12 meters of ribbon. Rolls of ribbon contain 36 meters. How many rolls of ribbon will be needed?

APPENDIX E

TEXTBOOK REFERENCES

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The following elementary textbooks were analyzed in preparing the mathematical verbal problems used in this study.

Anderson, P., Bittinger, M., Keedy, M., & Smith, S. General mathematics. Don Mills, Ontario: Addison-Wesley Publishing Company, 1980.

Bumfield, C., Eicholz, R., FLeenor, C., O'Daffer, P., & Sharks, M. School mathematics I. Don Mills, Ontario: Addison-Wesley Publishing Company, 1971.

Denholm, R., Hankins, D., Herrick, M., & Vojtko, G. Mathematics for individual achievement, level 7. Boston: Houghton Mufflin Company, 1975.

Dilley, C., Jackson, A., & Rucker, W. Heath elementary mathematics, book 4. Toronto: D. C. Heath Canada Ltd., 1973.