ACTIVITY-ORIENTED INSTRUCTION VERSUS TRADITIONAL TEXTBOOK METHOD IN TENTH GRADE GEOMETRY: A COMPARATIVE STUDY

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Activity-Oriented Instruction Versus Traditional Textbook Method in Tenth Grade Geometry: A Comparative Study

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ABSTRACT

The study was conducted to determine whether a significant difference existed between the effects of two methods of instruction, namely activity-oriented and traditional textbook. The two dependent variables examined were achievement and attitudes.

A unit of work on the geometry of the circle was taught to two groups of 12 students randomly assigned from a general grade ten class at Buchans Public High School. One group was taught using the textbook; the other group used an instructional package devised by the investigator.

Parallel forms of a criterion referenced test were administered as a pretest and posttest in order to measure the achievement variable. Attitudes were measured by issuing a pretest and posttest attitudes questionnaire.

The analysis of the results showed a significant difference in achievement on the posttest, only when the Canadian Test of Basic Skills scores were used as a covariate. A chi square analysis of the attitudes data showed a significant difference between treatments on the posttest but no significant difference on the pretest. No significant correlation between attitudes and achievement was found to exist on the pretest scores for control group, experimental group or combined group and the control group's posttest scores. A significant correlation did exist for the experimental and control group's posttest scores.

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BACKGROUND TO THE PROBLEM

Introduction

General Mathematics

Educators have recognized that students need various types of mathematics to conform with their needs, interests and abilities. Trying to meet these needs has become a difficult task. The current mathematics program in Newfoundland high schools (the Tri-Level System) has been designed to try to meet the needs of students with various mathematical abilities.

In a panel discussion on the Tri-Level program at the provincial Mathematics Teachers Conference held at Grand Falls (November 18-20, 1976), most teachers appeared to be satisfied with the honors and matriculation streams; but there were indications of much dissatisfaction with the basic program. It is possible that this could be attributed to the fact that according to Morris Kline (1973) the new mathematics movement in the 60's completely neglected the needs of the general student. Thus, a major problem that still remains is that of providing a meaningful curriculum for the slow learner. Even though the student failed to learn the fundamentals of arithmetic in the elementary grades, we still tend to organize a curriculum for him that repeats the same material in the same way in the junior and senior high schools. This same dull routine and unimaginative content fails to produce skills and also kills any interests these students may have had for mathematics (Sobel, 1967).

Weiss (1969) contends that there is no agreement as to what course should be offered to the terminal student. Some people believe that the only mathematics these students need is computational arithmetic; others believe the programs for the low-achiever should be built around what he needs as a citizen to function adequately in society. Other educators contend that the program for slow learners should be the same as that for other students, but should be taught in a different manner. For example, the same content may be presented at a lower level, and students may be given the opportunity to develop mathematical concepts through intuition and discovery.

Many curriculum developers and other educators feel that it is not what we teach, but how we teach it that is important. According to Bloom (1968), about 90% of all students can master what we have to teach them; but it is the task of instruction to find the means whereby our students can perform such mastery. Dunn (1976) supports this view in saying, ". . . that what constitutes the subject matter of study is unimportant in itself, but that the vital thing is the activity which it stimulates (p. 109)." However, it is the opinion of the investigator that careful consideration must be given to the needs and the characteristics of the general student in order to determine an adequate program of instruction for him. Taba (1962) has emphasized that the first step in any curriculum design is an assessment of needs.

<u>Nature of General Students</u>

Slow learners have been defined in terms of IQ range, mathematical achievement, teacher grades, reading level, and various combinations of these, or in psychological terms, deficient in cognitive functioning. But Schultz (1972) argues, "that slow learning can result from deficient affective functioning as well as cognitive functioning (p. 1)." There is no confined set of criteria that defines a slow learner. Kurtz and Spiker (1976) suggest that not all slow learners meet the technical definition of an IQ in the 80-90 range; many of these children have average or better intelligence scores. They claim that the group labelled slow learners is still a heterogeneous group, and that each child is unique. Not all of the children with mathematical disability possess the same learning deficiences.

Some common affective characteristics of slow learners are insecurity, poor self image, a need for recognition, approval and status, lacking school and social skills, have short term goals, seek immediate gratification, culturally disadvantaged, and slow physical learning style (Sobel, 1967; Schultz, 1972). Rehabilitation of the affective domain is a primary objective of curriculum programs for slow learners. Schultz (1972) indicates that it is the teacher who, as the manager of instruction, must provide the necessary learning experiences to meet the needs of the low achiever. Cronbach (1967) reviewed several studies which investigated the affective domain in relation to cognitive learning. He found that the low motivated student functions best when short term goals are defined, a maximum of explanation and guidance are provided, and feedback occurs at short intervals.

Instructional Methods and Materials

Dr. Norman France (1967) stressed that the teacher, not the content nor the method, really demonstrates the effectiveness of what is being done. Since the general students have so many different characteristics, the teacher must be flexible in the classroom and be aware of and be able to implement the various instructional techniques. Collins (1972) stated that since slow learners have special needs, they require appropriate styles of teaching. A similar viewpoint was expressed by Johnson (1967) who wrote that materials and methods are as essential for the mathematics teacher as spices are for the chef.

Morris Kline (1976) argues that the new mathematics movement had little or no impact in many schools simply because much of the new material is still presented in the traditional expository manner, and many textbooks actually present traditional material with a sprinkling of new-math terminology, symbolism and jargon. Polya (1963), an advocate of the heuristic and discovery approach, said that the problem with high school textbooks is that they contain, almost exclusively, merely routine examples that illustrate and offer practice in application of just one isolated rule.

What is needed is more exploration and discovery with concrete materials, films, games and other media; and applications of concepts acquired through activity oriented and intuitive sessions. Bruner (1960) stated that intuitive thinking is important in order to establish understanding of the material before using more traditional methods of education and proof. The media centre or intuitive practice in the classroom opens new approaches to learning and makes it possible for the child to become an explorer of knowledge (Ward, 1968).

Purpose of the Study

The major purpose of the study was to compare achievement and attitudes of a group of grade X students (experimental group) taught a unit of work on the geometry of the circle, using an instructional package designed by the researcher with a group of students (control group) using the textbook "Modern Basic Geometry" (Jurgensen et al., 1973). The experimental group worked in an activity oriented setting; whereas the control group were taught in the conventional expository textbook manner. The basic problem explored was: Do general students who study through an activity oriented approach obtain higher grades and develop more positive attitudes toward geometry than their counterparts who are taught by the conventional textbook and deductive expository method?

In seeking answers to this problem, the researcher attempted to test the following hypotheses:

1. General students who learn geometry intuitively obtain higher grades on a criterion referenced test than students who use the textbook method.
2. Students who study geometry using the inductive exploratory approach acquire more positive attitudes than students who are exposed to the conventional textbook approach.
3. Students who show indications of more positive attitudes and interests toward mathematics obtain higher grades on achievement tests.

Justification for the Study

With the Tri-Level program that is currently being used in Newfoundland, many of the smaller schools find they are able to cope with only two streams. Because of the size of the school, the ability of the students, the number of teaching periods per teacher per week, etc., many of the smaller schools offer only the matriculation and basic streams. As a result, fifty percent or more of the students in these schools are placed in the basic stream. According to the St. John's Evening Telegram (April 14, 1977, p. 2), the basic mathematics course is being taught to 100% of the pupils in some schools. Teachers argue that students cannot do the matriculation mathematics, especially the geometry course. Provincial statistics for the school year 1976-77 show that 27% of grade 11 pupils are enrolled in the basic program.

The basic program consists mainly of a repeat of elementary and junior high school topics. The Evening Telegram (April 14, 1977) quotes Mr. L. Ryan, a delegate at the Newfoundland Teachers' Association's Annual General Meeting, as stating that, "a pass in grade 11 basic mathematics is equivalent to average grade 8." Such dull topics presented in the traditional manner neither enhances the student's mathematical abilities nor stimulates his interests, but makes the disinterested and low motivated student even more disillusioned with it all (Sobel, 1967). Maybe if we could adjust our methods of instruction to a style that is stimulating and flexible to the needs of the low achiever, he may be enlightened mathematically. He may develop more positive attitudes toward mathematics and school, and probably obtain higher academic grades.

To the writer's knowledge, no systematic investigations have been carried out in Newfoundland to diagnose the difference in achievement and attitudes of general students using two different methods of instruction. The National Advisory Committee on Mathematics Education (1975) recommended that there should be more intuition and application, and less deduction in mathematical instruction. In order to confirm the worth of this statement, more accurate and adequate information must be made available. It was the intent of the investigator to conduct an experiment that would attempt to provide some of this information.

Definition of Terms

This section contains the meanings of the various concepts and variables used in the study.

Abstraction: consists of the extraction of what is common to a number of different situations in the formation of a class, the end being a realization of the attributes which make elements eligible or not for membership in the class.

Concrete: consists of experimenting with and reasoning about various material things, patterns, problems, etc.

Affective Functioning: possessing or lacking the various social, moral and other affective behaviors expected of normal students.

Cognitive Functioning: the ability to achieve academically on various tests and assignments.

General Student or Low Achiever: He is a student who (1) is achieving below his assigned grade level; (2) is not mentally retarded; and (3) has no serious emotional problems.

Regular Student: He is a student who is able to obtain average or higher academic grades and adjusts well to the school environment.

Expository Teaching: It is the traditional lecture by the teacher, while the student listens and later is given the opportunity to practise what has been taught.

Intuitive (activity-oriented) Teaching: The student is given the opportunity to experiment with various materials and is guided towards an insight. When the insight is found, he forms a generalization, and then puts his theory into practice in order to anchor the acquired concepts. The teacher acts as an organizer of learning experiences and a consultant in the classroom instead of a dispenser of knowledge.

Tri-Level System: This is a mathematics program consisting of three basic streams, namely: -

1. Honors stream: The students are able to proceed quickly because of high ability and are well prepared (Dodes, 1967).
2. Matriculation stream: Students have less ability, are less prepared, or require more motivation. The course varies in pace and depth (Dodes, 1967).
3. Basic stream: The student is of low ability, poorly prepared, low interest level, or miscellaneous difficulties. The course consists of a repeat of many elementary topics (fractions, decimals, percentage, etc.) with some consumer mathematics and probability (Dodes, 1967; Henderson et al., 1975).

Criterion Referenced Test: It is a test aimed at evaluating a student's mastery of a specific content area and a set percentage of correct responses constitute mastery of the content area. It differs from a norm referenced test in that criterion referenced test results are compared with the accepted number of correct responses that constitutes mastery whereas norm referenced test results are compared with the mean of a population sampling which is often referred to as the national norm.

Limitations of the Study

A study of this nature has a number of limitations. Since the experiment was conducted over a short period of time, the results obtained may not compare with other results obtained if the study was carried out over a longer time period. Thus, the long term effects cannot be determined. The fact that the content of the unit tends to lend itself to discovery learning, indicates that any conclusions drawn from the results may not apply to other areas of the discipline or other disciplines.

The sampling population of the experiment was an intact class; thus the sample is probably not representative of the larger population of general mathematics students. The generalizability of the obtained results was restricted to essentially the sample population.

Another weakness of the study may be that of experimental bias. Although the investigator, who taught both methods of instruction, tried to control for experimental bias, subconsciously the researcher's bias may have come through in his teaching.

Outline of the Study

A review of related literature is presented in Chapter II. Chapter III contains a description of the instrument, the procedure followed in conducting the study, and the method used in collecting and processing the data. The results of the data analysis are discussed in Chapter IV. The final chapter summarizes the conclusions reached as a result of the study, and contains implications and recommendations for future research in this area.

Chapter II

REVIEW OF RELATED LITERATURE

This section deals with a review of learning theories related to activity oriented learning and instruction, some methods of instruction, various studies that have attempted to determine the effects of intuitive learning on mathematical achievement and affective attitudes, and the role of attitudes in mathematics learning.

Learning Mathematics

Views on methods of teaching and instruction have changed tremendously during the past two or three decades. This is a result of profound learning theories put forward by people like Gagne, Piaget, Bruner, Dienes and Ausubel. Piaget believes that action rather than perception is the primary source of knowledge. The child goes through four developmental stages from infancy to adulthood. Case (1973) says that, according to Piaget, one stage is built upon another; thus educators should be promoting a curriculum aimed at providing activities to assist elementary school children in representing number, space and time.

Kline (1973) proposes that a thorough understanding of the concrete must precede the abstract. Children should learn mathematics intuitively since mathematics was discovered by analogies to real life situations. It took mathematicians a thousand years to discover negative numbers and another thousand to accept it. It is natural, then, that students will experience difficulty in mastering such concepts, and should be provided learning opportunitites that will facilitate understanding them. Students must gradually accustom themselves to the new concepts with the assistance of the teacher.

Bruner (1960) describes four basic criteria essential in the process of education.

1. Structure -- is to learn how things are related. Mastery of fundamental ideas not only involves grasping of general principles but also the development of an attitude toward learning and inquiry, toward guessing and hunches, and toward the possibility of solving problems on one's own.
2. Readiness for learning -- any subject can be taught in some intellectually honest form to any child at any stage of development. The intellectual development of children responds to influences from the environment, and in the case of the child it is namely the school.
3. Intuitive thinking -- it is important to establish intuitive understanding of material before employing a more formal treatment.
4. Motives of learning -- the environment at which the teacher is the centre can help motivate the student in his development. The structure of knowledge of learning and the specialist in any discipline must design a curriculum that is true to its structure.

Being an advocate of the discovery approach, Bruner (1961) points out four advantages of discovery learning:

1. It increases one's ability to assemble material sensibly.
2. It gives one the pleasure of finding out for oneself.

3. It provides learning of attitudes and activities that go with inquiry.
4. It creates the effect of making material more readily accessible in memory.

Dienes (1963) defines the learning of mathematics as the apprehension of relationships between concepts connected with numbers and with their applications to problems arising in the real world. Children's early concepts are much more constructively than analytically directed; thus children are not capable of abstract thought or analytical thinking until age eleven or twelve. Dienes contends that before a child can be operational with any concept he must complete a three step learning cycle.

1. Preliminary games -- corresponds to Piaget's first stage. It has little or no structure; it has the haphazard appearance of play.
2. Structured games — has directionality built into it, corresponding to the structure of the concepts whose formation it is intended to precede. Here the student will work towards an insight. When the insight occurs he is ready for the next stage, but if the indicated direction is not picked up he needs more preliminary games.
3. Practice game -- it helps anchor the new insight into the child's experience and it could act as preliminary games for later concepts.

Every part of the learning cycle should be developed for all prequisite concepts before any new concepts are attempted. Children should have structured materials to experiment with, to allow manipulative play to do its job, and should be allowed to make their own constructions from these manipulations. The role of the teacher is to provide an opportunity for the child to be exposed to these stages. Group discussion is good for introducing and summing up topics; but in between, creative and constructive activity must be done actively by the child.

In contrast to Bruner and Dienes, Ausubel (1965) feels that discovery learning is worthless except when the student is in the concrete mode, and too often children tend to jump to the wrong conclusions. Like Ausubel, Cronbach (1966) claims that one of the major disadvantages of discovery learning is the length of time required. Before a student can become a critical thinker, which is one of the ultimate goals of mathematics instruction, he must be able to respond actively and meaningfully to expository teaching (Ausubel, 1965).

In a review of discovery learning, Bittinger (1968) concluded that the amount of discovery used should be a function of the grade level in which it is used and worked in progressively through the grades.

Role of Attitudes

According to Neale (1969), positive attitudes toward mathematics are thought to play an important role in causing students to learn mathematics. Logically one would expect a student who likes mathematics to obtain higher grades than a student, similar in ability, who has a dislike for the discipline. Hence, certain attitudes toward mathematics have become important objectives of mathematics instruction.

Like Anttonen (1967) and Ryan (1968), Neale and Proshek (1967) found convincing evidence that attitude toward learning mathematics becomes increasingly unfavourable as students progress through school.

In testing the part played by attitudes in causing students to learn mathematics, Anttonen (1967), Ryan (1968) and Husen (1967) found that such a role was slight. Results from all three studies indicated a correlation of .20-.40 between the two variables, despite the fact that there were substantial differences in instruments and populations used.

Keane (1972) tested 679 students of 32 teachers selected for their positive or negative preference for teaching mathematics. The results of the attitude tests showed no relation between student attitude and achievement, and teacher attitude has no effect on student achievement. O'Rielly (1976) reports on the Project for the International Evaluation of Educational Achievement in Mathematics. Data was collected from 48 ninth and tenth grade classes in twelve schools in Ontario. Correlation coefficients calculated on the data showed that achievement is directly related to the attitude that mathematics is important but difficult to learn. Neale (1969) concluded that children learn mathematics for a number of reasons which include attitude, desire to be good students, desire to gain adult approval and because they are required to learn it. To isolate any one factor as a significant factor in causing students to learn mathematics is a grave mistake.

Instructional Methods

According to Bloom (1968), about 90% of all students can master what we have to teach them, but it is the task of instruction to provide the means whereby our students can perform such mastery. With the growth of learning theories, various methods of instruction have been devised and tested. Young (1968) describes six basic types of mathematics instruction methods.

1. Synthetic -- proceeds from the known to the unknown, it puts together known truths and perceives a truth that is unknown. Ex.: traditional statements of deductive proofs.
2. Analytic -- traces out a path from the unknown to the known. It pulls apart the statement under question into simple statements.
3. Deductive -- proceeds from the general to the particular.
4. Inductive -- proceeds from the particular to the general.
5. Heuristic method -- the pupil is to be a discoverer, not a passive recipient of knowledge.
6. Laboratory method -- proposes that the experimental origin of mathematics be fully recognized; it aims at maintaining order of instruction from the concrete to the abstract.

The following discussion in this section will concentrate largely on the effects of two methods of instruction (expository-deductive versus discovery-activity oriented) on mathematical achievement and affective attitudes. Keese (1972) used a research design to look at the interaction between two methods of teaching (expository versus discovery) and two creativity levels (high and low). Two groups of 13-15 year olds were taught a unit of material on sequences and series for twelve days using the discovery method with one group and the expository method with the other. Prior to this, students had been classified as of high or low creativity using the Torrence tests, and the four resulting groups had been checked for mathematical comparability. After the teaching unit, students were given an achievement test. The results indicated no interaction of teaching method with creativity level nor any significant difference in achievement between high and low creativity, but indicated that those taught by the discovery method had achieved considerably more than the others. The implication seemed to be that the method of teaching was the more important variable in mathematics achievement.

In a comparative study of teaching some aspects of algebra to grade nine students using two methods of teaching, namely, discovery and expository, Sobel (1954) found that mathematics achievement was significantly higher for students who used the discovery method. Cummings (1958) found similar results from a similar study of teaching calculus to first year college students.

Price (1967) used three classes of tenth grade general mathematics students to determine whether the use of a discovery approach would affect student achievement in and attitudes toward mathematics. One class was taught by traditional textbook method which was essentially deductive in nature. A second group was taught using materials prepared to promote student discovery of mathematical concepts, and a third group used the same teacher-prepared materials plus other materials specifically designed to aid a transfer of mathematical thinking to real world problems. Each group was given a pretest and posttest to evaluate mathematical achievement, reasoning ability, critical thinking-ability and attitude towards mathematics in general. From the results, Price concluded that both the discovery and transfer groups showed a slight but statistically nonsignificant gain over the control group in mathematics achievement. Both of these groups also showed a positive attitude change towards mathematics while the traditional textbook group showed a negative change.

Nichols (1973) conducted an experiment to compare two methods of instruction in multiplication and division for third grade pupils. The experimental group used manipulative materials and pupil discovery whereas the control group used abstractions and semi-concrete materials combined with teacher exploration and exposition. An analysis of covariance on the collected data showed a significant difference favoring students using manipulative materials and discovery in both arithmetic understanding and favorable attitudes.

DeVenney (1972) describes a similar experiment conducted during 1965-68 by the School Mathematics Study Group (SMSG). Fifteen schools, ten experimental and five control, in South San Francisco, participated in allowing seventh grade classes to be tested at the beginning of the school year. The students involved in the study lacked organization in their approach to learning and were immature compared to other seventh graders. Attention and interest span were exceptionally short. They held negative attitudes toward math and school in general and exhibited inferiority attitudes with regard to their ability to succeed in mathematics.

The Standard Achievement Test Intermediate II in arithmetic computation and applications was used as pretest and posttest for grade 7, and the advanced Test for Junior High School Students was used as posttest for grade 8. The following results were observed:

1. Thirteen percent of experimental and ten percent of control classes were absent from classes eleven percent of the time, but there were no significant differences on gains in computation and application between absentees and non-absentees.
2. Students in the experimental classes displayed substantial losses over the summer in computation but showed substantial gain in applications.
3. On the SAT computation scale, at the end of grade 7, control

classes showed a mean gain of 1.7 years while the experimental classes showed a mean gain of 1.2 years.

4. On a scale of mathematical concepts constructed by SMSG, students in the experimental classes showed significantly greater gains than those in control classes.
5. Psychological scales were used to measure student attitudes toward mathematics. Experimental classes developed highly positive attitudes toward mathematics whereas pupils in the control classes showed no evidence of such reversal of attitudes.

Wood (1976) found no significant differences in achievement or attitude of 219 students (102 experimental and 117 control) used in an experiment conducted at Ames Senior High School, Iowa, during the 1974-75 school year. The control group was taught in the formal deductive manner while the experimental group studied the same content in an informal discovery approach.

At Bloomington Middle School, Indiana, Smith (1974) used 82 students from grades 6-8 to determine the effects of laboratory instruction upon achievement and attitudes of an experimental group as compared with a control group. Both control and experimental groups were taught by the same instructor. The results of the study showed no significant difference between experimental and control groups on achievement or attitudes by grade level or collectively.

However, Biggs (1967) reported on a study conducted in 1960 when 87 English primary schools, classified according to their teaching methods, were tested on mechanical arithmetic, problem arithmetic and two concept tests. The teaching methods described were (a) using structural materials such as cuissenaire, (b) practical environmental activity method; and (c) traditional. The results indicated fairly clearly that method (b) produced the worst results on these tests.

Richards and Balton (1971) administered a large battery of tests of intelligence, mathematical abilities, divergent thinking and attitudes to school subjects to a sample of 265 eleven-year-old children. The sample came from three schools which the researchers claim were similar in social class, intelligence and time devoted to mathematics teaching. The three schools were deliberately chosen as examples of schools in which discovery teaching of mathematics, traditional teaching of mathematics and a system somewhere in between these poles were used. The results indicated that discovery methods of teaching are significantly less efficient with regard to mathematical achievement than the other two methods.

As Dunn (1976) suggests, there is considerable disagreement about the effects of discovery or activity-oriented learning on mathematics achievement, but a lot of these discrepancies could be attributed to weaknesses or limitations in research design, evaluation instruments, and statistical techniques. Weiner (1975), in a review of research on discovery learning, lists a number of common errors made in educational research. Some of these weaknesses are:

1. Not specifying a priori alpha level.
2. Not reporting power levels for significance tests.
3. Failure to use random selection procedures.
4. Failure to adequately control teacher variable.
5. Not defining key terms in the study.
6. Employing measuring instruments of questionable reliability and validity.

7. The use of pooling indiscriminate data from different groups.
8. Failure to use appropriate experimental units of analysis.
9. Using multiple t-tests for detecting differences between three or more groups.
10. Failure to use multivariate tests for studies employing more than one criterion variable.
11. Misuse of ANCOVA when a covariate is influenced by the treatment.
12. Failure to control novelty effects.

Instructional Aids

According to Kinder (1973), a simple definition of an instructional aid is "anything a teacher finds useful in designing and implementing instruction." With the knowledge explosion of postwar years and the rapid growth of technology, the educational media are beginning to play a major role in the process of education. Modern media seem to complement many of the recent learning theories. Gillett (1973) claims that media provide stimuli for creative activities as well as vehicles for production of original works, and can increase the realism, dynamics, emotionalism of information and the student's motivation to learn.

Johnson (1967) similizes, instructional materials are as essential for the mathematics teacher as spices are for the chef. Instructional aids give breadth and depth to a mathematics lesson and add meaning and interest to verbal instruction. The successful teacher must use concrete experiences for each student to develop a new concept; he connects these to experiences and ideas the student has already dealt with through problem solving. Finally he completes the structure by making it a mathematical system.

Many mathematics educators are now advocating a mathematics lab or mini-lab in the classroom to supplement the mathematics program. Sweet (1967) defends a mathematics lab in that the approach demands each student's participation and allows him to work at his own pace, leaving the instructor more time for individual assistance. The laboratory is usually conducted informally for more student interaction and hence enhances the sharing of ideas with one another.

Howard (1970), in a study of "Teaching Mathematics to the Culturally Deprived and Academically Retarded Rural Children," found that the use of mathematical models and instruments served as a motivational tool for teachers and pupils in arousing and maintaining interests in mathematics. In a comparative study of two methods of teaching arithmetic to inner city junior high school students, Schippert (1965) found that inner city pupils who manipulated actual models and presentations of mathematical principles showed significantly higher achievement on measures of skills than pupils taught by expository methods. This approach also provided concrete meaning to abstract concepts at early levels.

Scott (1972) described an experiment conducted in Chicago public schools, where mathematics laboratories were set up in eight schools using the proverb:

I hear, and I forget
I see, and I remember
I do, and I understand

as a basic rationale for the move. The objectives of the program were:

1. Provide many materials that appeal to the various senses.
2. Provide meaningful experiences.
3. Establish an atmosphere of trust and respect.
4. Develop a positive attitude toward learning.

5. Provide materials and experiences to meet the needs and abilities of average and below average and advanced pupils.

This was not a mathematics program in itself but a supplement to the regular classroom instruction. As a result, all teachers observed favorable behavior and willingness of pupils to participate in lab activities and transfer from lab activities to the classroom.

Maffei (1976) outlines a general mathematics program tried at Dreker High School, Columbia, South Carolina. Most students showed indications of low reading levels; thus the curriculum was less keyed to reading directions and comprehension, and geared more to understanding the concepts of mathematics through the use of problems and physical models. After three months the same diagnostic test was used in retesting the students. There was an average of 25% increase in the number of problems scored correct and some classes registered as much as 40% increase. Affectively, discipline problems were noticeably lower, and slow students were beginning to develop some positive self-concepts of their ability to do mathematics. Because they were treated as mature independent individuals rather than as children needing supervision, students seemed to enjoy working in the centre.

Summary

From the review of literature one cannot draw any definite conclusions. However, much of the material indicates that educators' views of instruction are changing from that where the teacher is the dominant figure in the classroom to one where the students play a more active role in the teaching-learning environment.

Many educators contend that students' attitudes toward the discipline play a major role in determining academic achievement. However, a number of studies that have attempted to determine correlations between achievement and attitudes have provided conflicting results. Even where significant correlations were obtained, it was difficult to determine whether positive attitudes caused high academic achievement or vice versa.

Research conducted for the purpose of investigating the effects of various methods of instruction have also provided conflicting results. Some experiments, such as those reported by Sobel (1954), Price (1967) and Nichols (1973) showed strong evidence in support of discovery learning, whereas others reported by Biggs (1967), and Richards and Balton (1971) showed strong evidence contrary to this hypothesis.

When interpreting results of experimental studies, one must always consider the limitations of the studies.

Keeping in mind the issues mentioned in the above review of research literature, the researcher investigated the influence of two methods of teaching (expository and activity-oriented) on mathematical achievement and the attitudes of two groups of grade X students. The effects of teaching methods cannot be distinctively predicted from the evidence provided by the research literature. For this reason, the investigator attempted to obtain some results that would either support or reject hypothesized effects of discovery and expository methods of instruction.

Chapter III

EXPERIMENTAL DESIGN

This chapter presents a description of the study design and the procedure followed in conducting the study. The content of the chapter consists of a discussion of the sampling procedure, treatment and instrumentation, pilot studies, hypotheses and analysis of the data.

Sampling and Procedures

The experiment was a two group pretest-posttest study conducted in the central Newfoundland town of Buchans. The subjects of the experiment were an intact class of grade X low achievers in mathematics. All students in the class were operating below their assigned grade level. According to results of the Canadian Test of Basic Skills issued in the ninth month of the grade eight academic year (May, 1975), the mean grade level of all students was approximately seven years three and one-half months. Table 1 gives a more detailed breakdown of the grade levels of each group in concepts, problems and total mathematical ability.

Table 1

Results of Canadian Test of Basic Skills

	Mean Mathematics Grade Level		
	Concepts	Problems	Total
Experimental Group	7 yrs. ½ mos.	7 yrs. ½ mos.	7 yrs. ½ mos.
Control Group	7 yrs. 7½ mos.	7 yrs. 6.4 mos.	7 yrs. 7 mos.
Whole Class	7 yrs. 4 mos.	7 yrs. 3 mos.	7 yrs. 3½ mos.

After assigning each of the twenty-four members of the class a number from 01 to 24, a table of random numbers was used (Glass & Stanley, 1970) in designating each student to either an experimental or control group. Before the students were informed of which group they were in, an attitudes pretest and achievement pretest were administered to all students. Table 2 shows a breakdown of the sample with respect to method of instruction and sex of students.

Table 2

Group, Treatment and Sex of Students

Group	Sex: Boys	Sex: Girls	Treatment
Experimental	6	6	Activity-oriented
Control	6	6	Traditional textbook

The experimental group, consisting of six boys and six girls, was assigned the activity-oriented discovery method of instruction. Each student was provided an instruction workbook accompanied by solid objects, geoboards and overhead transparencies. The students were advised to work alone or in groups of two, and to use the accompanying media materials to aid them in completing the lessons. The teacher acted as a guide and also assisted in helping students with any difficulties they might have incurred. The students completed six lessons in twelve forty-minute class periods. At the end of the instructional period, an attitudes posttest and achievement posttest were administered to all students.

The control group also consisted of six boys and six girls. They were assigned the traditional textbook-expository method of instruction.

Each student used the textbook Modern Basic Geometry (Jurgenson, Maier & Donnelly, 1973) where the teacher usually lectured on a topic and then students completed practice exercises. These students studied the same content material as the experimental, which was completed in twelve forty-minute class periods. The achievement posttest and attitudes posttest were administered at the termination of the instructional period.

Treatment

The major purpose of the study was to examine the effects of activity-oriented instruction on students' attitudes and achievement as opposed to the effects of the traditional textbook method of instruction. Since no adequate facilities were available, necessary materials were produced by the researcher with the assistance and guidance of various resource personnel at the Department of Curriculum and Instruction, Memorial University.

An instructional package consisting of a written student handbook, a teacher's handbook, solid objects (circles), overhead transparencies, and geoboards was produced to enable students to acquire various geometric concepts through the procedure of guided intuitive processes. The content covered in the package paralleled that in the textbook Modern Basic Geometry (Jurgenson et al., 1970). The student or group of students was guided through setting up and observing a series of patterns on a geoboard from which he and his partners formulated generalizations and concepts. To anchor the acquired concepts, a set of drill exercises was provided at the end of each of the six lessons. In addition to drill exercises, home assignments were designed to facilitate research of available related material and applications of the concepts in question to real world problems. Appendices D and E contain the materials produced for the study.

In constructing the instructional package the following instructional goals were followed:

1. To provide an opportunity for students to make generalizations and conclusions by manipulating physical materials and models on geoboards.
2. To provide exercises that reinforce the generalizations and concepts that have been made.
3. To provide enrichment activities and projects for all students, and more advanced projects for brighter students.
4. To encourage students to avail of resource material in doing projects.
5. To motivate interest in, and understanding of, the geometry of the circle through the hands-on-approach.
6. To apply the acquired concepts to real-life situations and problems.

In addition to these guidelines, each lesson was designed to correspond to a statement of student objectives with provision for entertaining unexpected outcomes. These student or behavioral objectives were listed and categorized along Wilson's model (Bloom, 1971) and Bloom's taxonomy (Bloom, 1973) by the researcher in conjunction with Dr. R. Connelly, a specialist in mathematics education, and Dr. D. Carl, a specialist in instructional development. The behavioral objectives are listed at the beginning of each lesson of the student handbook (see Appendix D).

Instrumentation

Achievement

The achievement pretest and posttest were parallel forms of a 28 item criterion referenced test designed to evaluate the behavioral objectives. In order to evaluate all of the objectives, questions of various types were used in the test (see Table 3 for the breakdown of the nature of the test items). To determine the validity and reliability of the test, each test item was compared with its corresponding behavioral objective by a content specialist and an instructional development specialist at Memorial University.

Table 3

Item Breakdown of Criterion Referenced Test

Type of Item	Item Nos.	Value	Sub-Total Value
Multiple choice	1-10	10	
	15-20	6	
	23	1	17
Fill in the blanks	11-14 21,22	12 3,4	19
Written answers	24-26	6	6
Problem solving	27,28	8	8
			Total - 50

The pretest was administered to all students before they were assigned to their respective groups. After each group was given twelve forty-minute periods of instruction, both groups were issued the posttest. Every student was given ample time to complete his responses to all questions. Students were given a grade out of a possible 50 marks which was then converted to a percentage.

Attitudes

The attitude pretest and posttest were parallel forms of a fifteen item questionnaire designed by the researcher. The items were selected from Wilson's model of attitude testing and modified to correspond to the related discipline. The questionnaire is printed in Appendix B.

Prior to completing the questionnaire, the students were informed that their answers to the questions would be held confidential and were asked to express their honest feelings in their responses to the questions. The students responded to each question by checking any one of five letters (a, b, c, d, e) which were coded as: strongly agree, agree, undecided, disagree, and strongly disagree, respectively. For each item, a score of 1-high was assigned to either of the positive responses and 0-low to either of the two negative responses. No score was assigned to an "undecided" response. The maximum score one could obtain was fifteen.

Each item on the test was categorized as either interests, attitudes or opinions, and also into the four affective taxonomic levels of receiving, responding, valuing and organization. Table 4 gives a more complete breakdown of the item taxonomy levels and the attitude, interests or opinions category.

Pilot Studies

Instructional Package

During the development of the instructional package, a formative evaluation was conducted in March, 1976, in the form of a pilot study with six students who were classified by their teacher and principal as

Table 4

Item Breakdown of Attitudes Test

		Item Nos.	No. of Items
Affective Categories	Interest	1,9,10,13,14,15	6
	Attitude	2,3,4,5,6,7	6
	Opinion	8,11,12	3
Affective Taxonomy Levels	Receiving	9,11,12,13	4
	Responding	2,3,6	3
	Valuing	4,5,10,14	4
	Organization	1,7,8,15	4

low achievers in geometry. The purpose of this study was mainly to examine the wording of the instructions to ensure that students could understand and follow the instructions.

Later in March, 1976, a pilot study of the criterion referenced test, involving 30 students who were assumed to have attained the objectives of the instruction, was conducted. The purpose of the pilot study was to examine the following:

1. The time period required for all students to complete the test.
2. The wording of the questions such that students knew what the tasks required.
3. To determine if the questions adequately tested the objectives they were designed to test.

On the basis of these two pilot studies and feedback received from resource personnel, the final modifications in the instructional package and testing instruments were made.

Hypotheses

The following are statements of the main hypotheses written in the null form.

1. There is no significant difference in scores between experimental and control groups as indicated by the mean scores on the pretest.
2. There is no significant difference between experimental and control groups as indicated by the means of the gain scores from pretest to posttest.
3. There is no significant difference in mathematical achievement between treatment groups.
4. There is no significant difference between treatment groups as indicated by the responses on the attitudes pretest.
5. There is no significant difference between treatment groups as indicated by the responses on the attitudes posttest.
6. There is no significant difference in attitude changes from pretest to posttest.
7. There is no significant relationship between attitudes and achievement for experimental group, control group or combined group.

Analyses

A report on the analyses of the accumulated data is presented in Chapter IV. To determine any significant difference in achievement, analyses of variance and covariance, using pretest scores and CTBS scores as covariates, were carried out on the achievement data. A chi square analysis was carried out on the attitude test results, and the point biserial correlation formula was used to determine any significant relationship between attitudes and mathematical achievement.

Chapter IV

RESULTS OF THE EXPERIMENT

In this chapter the analyses of the data relevant to the study are presented. The analyses of the achievement data were carried out using the t-test, ANOVA and ANCOVA. To analyze the attitudes data, the chi square test was used, and the point biserial correlation formula was used to test any relationship between attitudes toward mathematics and achievement in mathematics.

Achievement Data Analyses

The criterion referenced test was administered as the achievement pretest, and upon completion of the treatments the achievement test was administered as the posttest. Both the pretest and the posttest were parallel forms designed to measure the attainment of the behavioral objectives. Table 5 lists the pretest, posttest and gain score achievement mean scores of the two treatment groups.

Table 5

Pretest, Posttest and Gain Score Means for Each Treatment Group

Treatment	Mean Scores		Mean Gain Score
	Pretest	Posttest	
Activity oriented	16.83	54.50	38.50
Traditional textbook	16.33	47.50	31.23

Pretest

An administration of the t-test was carried out on the pretest data and the gain scores. Table 6 gives the calculated t-values and the probability levels for testing equal means of the pretest and the gain score data. A probability level of .05 or better was considered significant. The derived t-value of .18 showed no significant difference between group achievement on the pretest, and a t-value of 1.154 with p = .13 also showed no significant difference between groups in achievement gains at the levels considered.

Table 6

Results of the t-tests

Source	Group Means		t-value	Probability Level
	Activity-oriented	Traditional Textbook		
Pretest	16.83	16.33	.18	.43
Gain Scores	38.50	31.23	1.154	.13

Posttest

The results of a Canadian Test of Basic Skills (CTBS), administered during the ninth month of the grade eight academic year (May, 1975), were used as a covariate in the analyses of the achievement data in order to isolate any interference of mathematical skills from the final achievement. Table 7 shows the CTBS, posttest and adjusted achievement means for each treatment group. The CTBS scores are given in grade levels instead of percentage.

Table 7

CTBS, Posttest and Adjusted Achievement Mean Scores for Each Treatment Group

Treatment	Mean Scores		Adjusted Mean
	CTBS	Posttest	
Activity-oriented	69.90*	54.50	58.35
Traditional textbook	77.60+	47.50	42.38

*Reads - Grade level of 6 years 9.9 months.

+Reads - Grade level of 7 years 7.6 months.

A one-factor analysis of variance and an adjusted analysis of variance of the achievement scores for the two treatment groups were carried out. Table 8 gives the results of the analysis of variance of the posttest achievement scores and the probability level. The derived F-ratio of 0.889 with $p > .25$ showed no significant difference between treatment groups. A probability level of .05 or better was considered significant.

To determine any significant difference between adjusted means, where CTBS scores were used as the covariate, an F-ratio of 27.82, significant at the .001 alpha level, was obtained. Table 9 contains the adjusted mean square, the F-ratio and the probability level.

An analysis of the adjusted mean was also performed using the pretest as the covariate. Table 10 gives the pretest, posttest and adjusted means for each treatment where the pretest was the covariate. The calculated F-ratio of .90 with $p > .25$ indicated that there was no significant difference between adjusted means (see Table 11).

Table 8

Results of ANOVA on Achievement Data

Source	df	Mean Square	F-ratio	Probability Level
Group	1	294		
Within	22	330.82	.889	$p > .25$

Table 9

Results of ANCOVA, Using CTBS as Covariate

Source	df	Mean Square	F-ratio	Probability Level
Group	1	6026.47		
Within	21	216.66	27.82	$p < .001$

Table 10

Pretest, Posttest and Adjusted Mean Achievement Scores for Each Treatment Group

Treatment	Mean Scores		Adjusted Means
	Pretest	Posttest	
Activity oriented	16.83	54.50	54.21
Traditional textbook	16.33	47.50	47.21

Table 11

Results of ANCOVA, Using Pretest as Covariate

Source	df	Mean Square	F-ratio	Probability Level
Group	1	247.08		
Within	21	274.07	.90	$p > .25$

Summary of Achievement Results

From the obtained values for t of .18, there was no significant difference between treatment groups on the pretest and a t value of 1.154 showed no significant difference in the means of the gain scores for the two treatment groups.

The derived F-ratios of .889 for the one way analysis of variance and .90 for the analysis of covariance, using the pretest as the covariate, indicated no significant difference in mathematical achievement between treatments. However, a F-ratio of 27.82, with $p < .001$ obtained from the analysis of covariance using CTBS as the covariate, was evidence of a significant difference between treatment groups, in favour of the activity-oriented group.

Conclusions

The analyses of the achievement results were used to test the first three hypotheses.

Hypothesis 1. There is no significant difference in achievement between experimental and control groups as indicated by the pretest mean scores. Based on the results of the t-test on the pretest scores, indicating no significant difference between the achievement means for treatment groups, the author accepted hypothesis one.

Hypothesis 2. There is no significant difference between experimental and control groups as indicated by the gain scores from pretest to posttest. The results of the t-test performed on the gain scores showed no significant difference between the mean gain scores for treatment groups. Thus hypothesis two was also accepted.

Hypothesis 3. There is no significant difference in mathematical achievement between treatment groups. On the basis of the ANOVA and ANCOVA (pretest as covariate) results, hypothesis three was accepted. According to the ANCOVA analysis where CTBS was the covariate, there was a significant difference in mathematical achievement at the .001 alpha level in favour of the activity-oriented treatment group. From this the investigator concluded that mathematical skills played a significant role in determining student achievement; hence, hypothesis three was rejected in favour of the activity-oriented method of instruction.

Attitudes Data Analysis

Pretest

Prior to treatment, the fifteen item attitudes questionnaire was administered to all students as the pretest. An item analysis was done on the test whereby each response was graded 0 or 1. Zero meant the student responded to the question negatively and 1 meant the student responded positively. Appendix C contains a frequency table of the number of positive and negative responses to each item for each group.

A chi square analysis compared the observed frequencies with the expected frequencies between treatment groups. Table 12 is a two-dimensional crossbreak of the observed and expected frequencies on the pretest. The calculated chi square of .113 with p = .73, also contained in the table, showed no significant difference in attitude between treatment groups. A probability of .05 or better was considered significant.

Table 12

Calculation of χ^2, Attitudes Pretest Data

	0(-ve)	1(+ve)	
Activity-oriented	63.5762[a] 65 1.4238[b]	96.4238 95 -1.4238	160
Traditional textbook	56.4238 55 -1.4238	85.5762 87 1.4238	142
	120	182	302

a $\frac{160 \times 120}{302} = 63.5762$; etc.

b 65 - 63.5762 = 1.4238; etc.

$\chi^2 = .113$, $p = .73$

Posttest

The posttest, which was a parallel form of the pretest, was administered to all students upon completion of the treatments. An item analysis of the test was conducted in the same manner as for the pretest (see Appendix C). The calculated chi square of 10.467, shown in Table 13, was significant at the .01 alpha level, which implies a significant difference in attitudes between treatment groups in favor of the activity-oriented treatment.

Table 13

Calculation of χ^2, Attitudes Posttest Data

	0(-ve)	1(+ve)	
Activity-oriented	44.6179[a] 32 -12.6179[b]	113.3821 126 12.6179	158
Traditional textbook	40.3821 53 12.6179	102.6179 90 -12.6179	143
	85	216	301

a. $\frac{158 \times 85}{301} = 44.6179$; etc.

b. $32 - 44.6179 = -12.6179$; etc.

$\chi^2 = 10.467$, $p < .01$

Another chi square analysis of the data was conducted to determine any significant difference in attitude changes from pretest to posttest for each group. Tables 14 and 15 give the obtained and expected frequencies, and the calculated chi square ratios for comparing attitude changes from pretest to posttest for each treatment group. A chi square

Table 14

Calculation of χ^2, Attitude Changes for Activity-oriented Treatment Group

	0(-ve)	1(+ve)	
Pretest	48.8050 65 16.1950	111.1950 95 -16.1950	160
Posttest	48.1950 32 -16.1950	109.8050 126 16.1950	158
	97	221	318

$\chi^2 = 15.563,\ p < .001$

Table 15

Calculation of χ^2, Attitude Changes for Traditional Textbook Treatment Group

	0(-ve)	1(+ve)	
Pretest	53.1805 55 1.1895	88.1895 87 -1.1895	142
Posttest	54.1895 53 -1.1895	88.8105 90 1.1895	143
	108	177	285

$\chi^2 = .084$, $p = .78$

ratio of 15.563 was indicative of a significant change in attitudes for the activity-oriented treatment group, but a chi square ratio of .084 indicated no such change for the traditional textbook treatment group.

Summary of Attitudes Analyses

According to the results of the chi square analysis on the pretest data, there was no significant difference between treatment groups, but a chi square of 10.467 showed a significant difference ($p < .01$) in attitudes on the posttest in favour of the activity-oriented treatment. In comparing attitude changes from pretest to posttest, a chi square value of 15.563 showed a significant change ($p < .001$) for the activity-oriented group, but a chi square value of .084 showed no such change for the traditional textbook treatment group.

Conclusions

The results obtained from the chi square analyses of the attitudes data were used to test hypotheses four, five and six.

Hypothesis 4. There is no significant difference between treatment groups as indicated by responses on the attitudes pretest. On the basis of the obtained chi square value of .113, which was not significant at the .05 level, hypothesis four was accepted.

Hypothesis 5. There is no significant difference between treatment groups as indicated by the responses on the attitudes posttest. The obtained chi square value of 10.467 was found to be significant with $p < .01$; thus hypothesis five was rejected in favour of the activity-oriented treatment group.

Hypothesis 6. There is no significant difference in attitude changes from pretest to posttest. A chi square value of 15.563, significant at the .001 alpha level, was indicative of a significant change in attitudes, from pretest to posttest for the experimental group. However, a chi square of .084, not significant at the .05 level, showed there was no significant change in attitudes for the control group. Thus, hypothesis six was rejected in favor of the activity-oriented treatment group, but it was accepted for the traditional textbook treatment group.

Attitudes-Achievement Relationship

Pretest

To determine any linear relationship between students' attitudes and mathematical achievement, the point biserial correlation formula was used. If a student scored less than 8 on the attitudes test, he was considered to possess low attitudes and he was then given a dichotomous grade of 0. If he obtained 8 or more, he was given a grade of 1.

Pretest

The pretest achievement means of the group scoring 0 and the group scoring 1 on the attitudes pretest were calculated for each treatment group. Table 16 contains the number of students with low and high attitudes and the respective pretest achievement means.

A point biserial correlation coefficient was calculated for each treatment, and the combined group. All calculated coefficients of $r_{pb} = .07$ for the experimental group, $r_{pb} = .41$ for the control group, and $r_{pb} = .12$ for the combined group showed no significant linear relationship between attitudes and mathematical achievement. The derived correlation coefficients, which were considered significant at the .05 alpha level are listed in Table 17.

Table 16

Students with High and Low Attitudes, and Respective Pretest Achievement Means

Treatment Group	No. of Students		Achievement Means	
	Low	High	Low	High
Activity-oriented	6	6	16.30	17.30
Traditional Textbook	7	5	14.07	19.60
Combined Group	13	11	15.08	18.37

Table 17

Point Biserial Correlation Coefficients for Pretest Data

Treatment Group	Calculated r_{pb}	Significance
Activity-oriented	.07	NS*
Traditional Textbook	.41	NS
Combined Group	.12	NS

*NS -- Not significant.

Posttest

Table 18 contains the number of students with low and high attitudes on the posttest, and the posttest achievement means of the group scoring 0 and the group scoring 1 on the attitudes posttest. In determining any relationship between attitudes and achievement, the point biserial correlation coefficients were calculated for each treatment group, and the combined group, and the calculated coefficients are written in Table 19.

Table 18

Students with High and Low Attitudes, and Respective Posttest Achievement Means

Treatment Group	No. of Students		Achievement Means	
	Low	High	Low	High
Activity-oriented	2	10	31.00	59.20
Traditional Textbook	6	6	39.30	55.70
Combined Group	8	16	37.25	57.88

Table 19

Point Biserial Correlation Coefficients for Posttest Data

Treatment Group	Calculated r_{pb}	Significance
Activity-oriented	.60	S*
Traditional Textbook	.45	NS
Combined Group	.54	S**

*Significant at the .05 level.

**Significant at the .01 level.

The derived r_{pb} of .45, not significant at the .05 alpha level, was evidence of no linear relationship between attitudes and achievement for the traditional textbook treatment group. However, the calculated $r_{pb} = .60$ ($p < .05$) for the activity-oriented treatment group and $r_{pb} = .54$ ($p < .01$) for the combined group indicated that there was a significant positive relationship between attitudes and achievement.

Summary of Relationship Between Attitudes and Achievement

On the basis of the three correlation coefficients calculated on the pretest data, namely, $r_{pb} = .07$, $r_{pb} = .41$, and $r_{pb} = .12$, no significant relationship between attitudes and achievement was found to exist, for either the experimental group, control group or combined group. The two correlation coefficients of .60 for the experimental group and .54 for the combined group calculated on the posttest data showed a significant positive linear relationship between attitudes and mathematical achievement. A coefficient of .45 derived from the control group's posttest data did not show any significant relationship between attitudes and achievement.

Conclusions

Using the results of the point biserial correlation formula, hypothesis seven was tested for significance.

Hypothesis seven. There is no significant relationship between attitudes and achievement for experimental group, control group or combined group. On the basis of the results, hypothesis seven was accepted for the pretest attitudes and achievement scores of the activity-oriented group, the traditional textbook group and the combined group. Hypothesis seven was rejected for the attitudes and achievement posttest scores of the activity-oriented group and the combined group. From the analysis of the data for these groups a significant linear relationship was found to exist between mathematics attitudes and achievement. In an analysis of the control group's posttest scores no significant relationship was found to exist between attitudes and achievement; thus hypothesis seven was accepted for the control group.

Summary

An analysis of the results of testing the seven hypotheses associated with the major purposes of the study are contained in this chapter.

As outlined in Chaper I, the purpose of the study was to examine the effects on mathematical achievement, attitudes and attitude-achievement relationships of activity-oriented instruction as compared with the effects of traditional textbook instruction on the same variables.

From the analysis of variance and analysis of covariance using the pretest scores as the covariate, no significant difference was found between the effects of the treatments on achievement in mathematics. However, an analysis of covariance, using the CTBS scores as the covariate, showed a significant difference between treatment in favour of the activity-oriented treatment.

From a chi square analysis, the traditional textbook treatment produced no significant change in attitudes, but a significant change in attitudes resulted from the activity-oriented treatment. The analysis also showed a significant difference in attitudes between treatment groups on the posttest.

No significant relationship between mathematics achievement and attitudes, using the pretest scores, was found for the activity-oriented group, the traditional textbook group, and the combined group. The attitude and achievement scores on the posttest showed a significant relationship between attitudes and achievement for the activity-oriented group and the combined group, but no significant relationship between the two variables for the traditional textbook group.

The implications and recommendations arising from these results will be discussed in Chapter V.

Chapter V

SUMMARY DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

Summary of the Experiment

This study was designed to investigate the effects of two methods of instruction, namely activity-oriented and traditional textbook, on mathematics attitudes and achievement. The experiment was carried out using both discovery and expository learning situations. In order to gather the required data, an intact low achieving grade ten class was selected and the treatments were employed.

Sample

The sample of 24 grade ten students from Buchans Intergrated High School was used for the experiment and the analyses purposes. The 24 students were randomly assigned to either the activity-oriented treatment or the traditional textbook treatment. It was assumed that this class was typical of grade ten low achievers in mathematics. The mean grade level determined by the Canadian Test of Basic Skills administered during the ninth month of the grade eight academic year was seven years and four months.

Instruments

A mathematics achievement pretest and posttest (criterion referenced test, Appendix A) was constructed for the study. The tests, which consisted of 28 items each, were parallel forms designed to test each of the behavioral objectives, which are listed at the beginning of each lesson in the student manual (Appendix D). The pretest was administered to the whole class prior to being assigned to treatments and at the termination of the treatments the posttest was administered.

An attitudes pretest and posttest (Appendix B) was also constructed for the study. This instrument, which consisted of 15 items, was administered as a pretest and posttest to evaluate general attitudes towards geometry and mathematics in general. The questions covered the four taxonomic levels of receiving, responding, organizing and valuing.

In the analysis of mathematical achievements the pretest was used as the covariate. The chi square was used to analyze the attitudes data and the point biserial correlation formula was used to analyze the relationship between mathematics attitudes and achievement.

Conclusions

The summary of the findings presented are based on the results of testing the seven hypotheses.

In mathematics achievement, no significant difference was found to exist between the activity-oriented treatment group and the traditional textbook treatment group prior to treatments. An analysis of covariance, with pretest as the covariate, showed no significant difference in achievement between treatment groups after the treatments were employed. However, an analysis of covariance using CTBS as the covariate found that the activity-oriented treatment produced a significant difference in achievement over the traditional textbook treatment.

A chi square analysis of mathematics attitudes of students in the activity-oriented treatment and the traditional textbook treatment showed no significant difference between treatment groups prior to treatments, but after treatment a significant difference between groups was found to exist in favor of the activity-oriented group. A chi square analysis of changes in attitudes for each group found no significant change for the traditional textbook group, but a significant change in attitude was found for the activity-oriented treatment group.

No significant relationship between attitudes and achievement in mathematics was found to exist on the pretest scores for the activity-oriented group, the traditional textbook group and the combined group. Using the posttest scores, a significant relationship between mathematics attitudes and achievement was found for the activity-oriented group and the combined group, but the traditional textbook strategy produced no significant relationship.

Implications Resulting from the Findings

The findings of the achievement analysis indicated that the activity-oriented method of instruction produced a significant difference in achievement when the effects of mathematical skills were cancelled out. Without using the CTBS as the covariate neither strategy of instruction was more successful than the other.

It is evident from the findings that activity-oriented instruction is just as effective as, if not more effective, than the traditional textbook method of instruction. This result must be interpreted in light of the limitation of the short time duration of the study, and the fact that the content does lend itself to discovery learning. Other areas of the discipline or other disciplines may not be as susceptible to activity-oriented or discovery techniques of instruction.

The findings for the attitude data did show a significant difference in mathematics attitudes in favor of the activity-oriented treatment group. The chi square analyses indicated a significant difference in attitudes between treatment groups on the posttest and a significant change in attitudes for the activity-oriented group, but no significant change in mathematics attitudes for the traditional textbook group. This result must be interpreted in light of the limitation that the manipulation of solid objects and geoboards could have been a novelty, hence could produce a significant attitude change over a short period of time. However, much of the literature shows that significant attitude changes do result from activity-oriented instructional strategies.

Although a significant relationship between mathematics attitudes and achievement scores was found for the activity-oriented group and the combined group, it was difficult to say whether the attitude changes caused the students to achieve more or the increased achievement caused significant attitude changes. Neale (1969) hypothesized that rather than good attitudes causing students to learn, it might be that learning causes favourable attitudes.

The generalizability of this study is limited in that the sampling population was an intact class and was not a random representation of the larger population of grade ten general mathematics students. The limitations of the study must be considered in any interpretation of the findings of the investigation.

Recommendations for Further Research

This study analyzed mathematics and achievement scores of an intact class of grade ten students diagnosed as low achievers in mathematics. Each member of the class was randomly assigned to one of two treatment groups. Thus a study with a more randomized selection of subjects would provide more detailed information for determining effects of the instructional strategies on the mathematics achievement and attitudes of high, medium and low achievers.

Since this study compared only two methods of instruction, a second recommendation is that the effects of other instructional strategies on mathematics achievement and attitudes be studied. Using other instructional methods and material, one could probably control the novelty effects of the geoboard. Another aspect of such a study would be to determine any existing relationship between attitudes and achievement in mathematics using varied instructional procedures. Marsh (1976) found a significant relationship between attitude and achievement scores for the teacher-directed group, overall teacher-directed strategy, and overall mediated and non-mediated strategies on the pretest scores; and for these same groups plus student-directed mediated and student-directed groups, and overall student-directed strategy on the posttest.

When determining the effects of instruction on mathematics attitudes and achievement, the time period of this study is another limitation that should be considered. A fourth recommendation is that a more extensive study on a larger scale be conducted to assess long-term effects of varied instructional techniques on mathematics achievement and attitudes.

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Appendix A

Criterion Referenced Test

Criterion Referenced Test

Instructions: In each of the following check the correct answer or fill in the blanks where necessary.

1. The accepted fractional and decimal approximations for Pi are:

   (a) 22/7 and 3.41 (c) 22/7 and 3.14

   (b) 7 1/3 and 4.13 (d) 3 1/7 and 4.13

2. If the radius of a circle is 14 cm, then the circumference is approximately:

   (a) 88 cm (c) 44 cm

   (b) 28 cm (d) 7 cm

3. If the diameter of a circle is 28 cm, then the circumference is approximately:

   (a) 44 cm (c) 88 cm

   (b) 56 cm (d) 14 cm

4. The formula for finding the area of a circle is:

   (a) $A = \frac{1}{2}bh$ (c) $A = 2\pi r$

   (b) $A = \pi r^2$ (d) $A = \pi d$

5. The formula for finding the circumference of a circle is:

   (a) $C = \pi d$ (c) $C = \pi \times d$

   (b) $C = \pi \times 2r$ (d) $C = 2\pi r$

   (e) All of the above

6. If the radius of circle B is greater than the radius of circle A, and the radius of circle C is greater than the radius of circle B, which one of the following statements is true?

   (a) Circumference of C is greater than B.

   (b) Circumference of A is greater than B.

   (c) Circumference of B is greater than either A or C.

   (d) Circumference of A is greater than C.

7. Which of the following best describes the characteristics of a circle that distinguishes it from other plane geometric shapes?

(a) A circle is a round figure.

(b) A circle is a closed figure with no corners.

(c) A circle is a closed curve with all points on the circumference the same distance from its centre.

(d) A circle is a closed figure with a centre.

8. If the diameter of a circle is 17 cm, its area is:

(a) 226.87 cm^2 (c) 53.38 cm^2

(b) 907.46 cm^2 (d) 106.76 cm^2

For questions 9-16 use Figure 1.

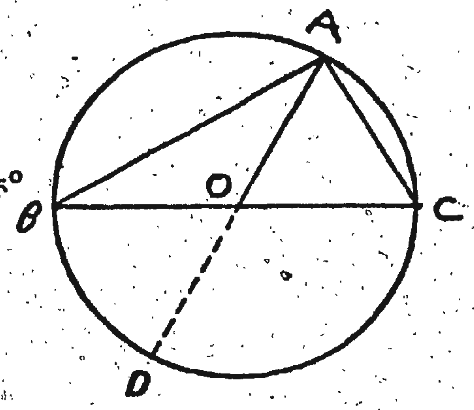


Fig. 1

9. If the m$\widehat{AC}$ = $37\frac{1}{2}^\circ$, what is m$\angle$AOC?

(a) 74° (c) $37\frac{1}{2}^\circ$

(b) 75° (d) 18.75°

10. If the m$\angle$AOB = 152°, then the m$\widehat{AB}$ is:

(a) 76° (c) 304°

(b) 152° (d) 90°

11. If the m$\angle$AOC = 68°, then the m$\angle$ABC is:

(a) 68° (c) 112°

(b) 136° (d) 34°

12. If, in Figure 1, m$\widehat{BD}$ = 31°, classify each of the following as either acute, obtuse, or right angles.

$\angle$ ABC ____________

$\angle$ BAC ____________

$\angle$ DOC ____________

$\angle$ AOC ____________

13. If, in Figure 1, the m$\widehat{AB}$ = 112°, find each of the following:

$m\widehat{AC}$ = ________°

$m\angle AOB$ = ________°

$m\angle AOC$ = ________°

$m\widehat{BD}$ = ________°

$m\angle ABC$ = ________°

$m\angle DAC$ = ________°

14. Referring to Figure 1, name an inscribed angle intercepting the arc $\widehat{AB}$.

____

15. In Figure 1, $\overline{BC}$ is a diameter of the circle. The $m\angle A$ is:

(a) 180°   (c) 45°

(b) 90°   (d) cannot be determined

16. Whenever $\overline{BC}$ is any diameter, which of the following statements is always true?

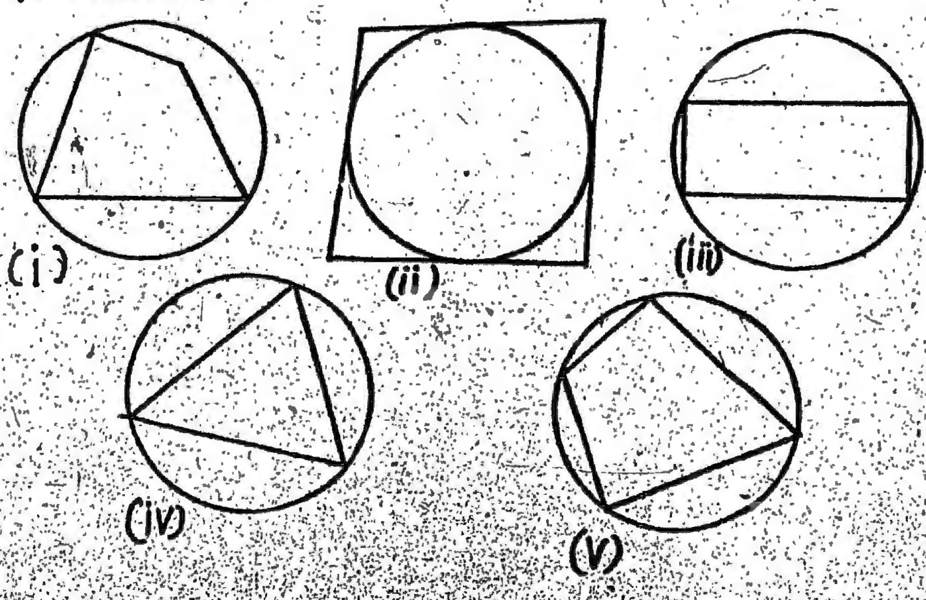
(a) Angle C is always 45°.

(b) Angle A is always a right angle.

(c) Angle B is always a right angle.

(d) Angle C is always a right angle.

17. Which of the following diagrams fit the definition of an inscribed quadrilateral?



(a) i and iii (d) ii and v

(b) iii and iv (e) iii and ii

(c) iii and v (f) i and v

For questions 18–20 use Figure 2.

18. In Figure 2, the chords $\overline{ST}$ and $\overline{PQ}$ intersect at point X. If $m\overset{\frown}{PT} = 89^\circ$ and $m\overset{\frown}{SQ} = 103^\circ$, then the m∠PXT is:

(a) 103° (c) 89°

(b) 96° (d) 192°

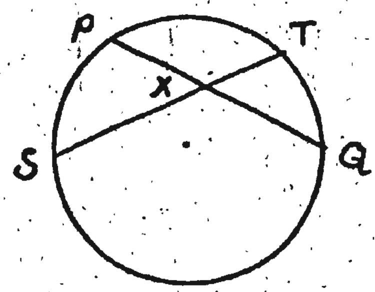


Fig. 2

19. If the $m\overset{\frown}{SP} = 76^\circ$ and $m\overset{\frown}{SQ} = 88^\circ$, then the m∠SXQ is:

(a) 96° (c) 88°

(b) 76° (d) 192°

20. If the m∠PXT = 105° and $m\overset{\frown}{PT} = 100^\circ$, then the $m\overset{\frown}{SQ}$ is:

(a) 205° (c) 210°

(b) 50° (d) 110°

21. In Figure 3, the $m\overset{\frown}{BC} = 72^\circ$, $m\overset{\frown}{DC} = 98^\circ$, and mABC = $\overset{\frown}{182}^\circ$. Find each of the following:

m∠ABC = ______

m∠DCB = ______

m∠ABC = ______

m∠DAB = ______

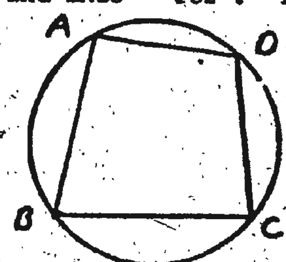


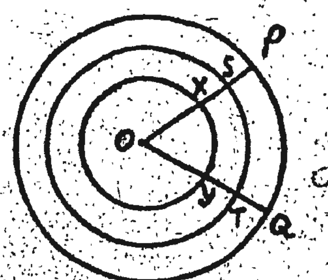
Fig. 3

22. In the diagram, O is the centre of each circle and $m\overset{\frown}{ST}$ is 45°. Find each of the following:

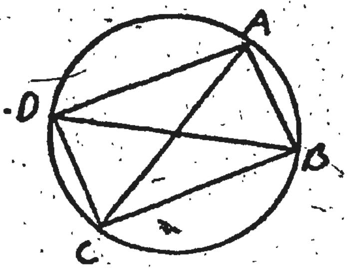
$m\overset{\frown}{PQ}$ = ______

m∠XOY = ______

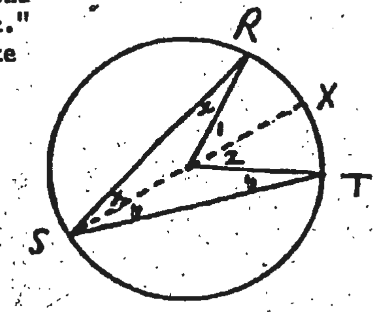
$m\overset{\frown}{XY}$ = ______



23\. This diagram is an illustration where the diagonals AC and BD of an inscribed quadrilateral ABCD are diameters of the circle. Is the following statement true or false? "Whenever the diagonals are diameters of the circle, the inscribed quadrilateral will always be either a square or a rectangle. (True / False)



24\. The following is a proof of the statement: "The measure of an inscribed angle is one-half the measure of its intercepted arc." Which step or steps are incorrect? Make the necessary corrections.



Given: A circle with centre O.

∠RST is an inscribed angle.

To prove: $m\angle RST = \frac{1}{2}\, m\overset{\frown}{RT}$

Draw auxillary lines $\overline{OR}$ and $\overline{OT}$.

Let $m\angle OSR = x$, and $m\angle OST = y$.

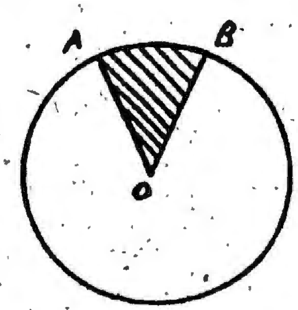
Statements	Reasons
1. $\overline{OR} \cong \overline{OS}$	1. equal radii
2. ∠ ORS = ∠ OSR	2. ∠ s opp. ≅ sides are ≅
3. $m\angle ORS = x$	3. substitution
4. $\overline{OT} \cong \overline{OT}$	4. equal radii
5. $\angle OTS \cong \angle OST$	5. same as 2
6. $m\angle OTS = y$	6. substitution
7. $m\angle 1 = x + x = 2x$	7. Defn. of exterior ∠ and addition property
8. $m\angle 2 = y + y = 2y$	8. angles of a triangle = 180°
9. $m\angle 1 + m\angle 2 = m\overset{\frown}{RX} + m\overset{\frown}{XT}$	9. Defn. of inscribed ∠ 's.
10. $2x + 2y = m\overset{\frown}{RXT}$	10. subst. & addition prop.
11. $x + y = \frac{1}{2} m\overset{\frown}{RXT}$	11. division prop.
12. $m\angle RST = \frac{1}{2} mRXT$	12. subst.

25\. Describe in one sentence your understanding of 'Pi' as it relates to a circle.

26. Write in your own words the definition of a circle.

27. A real estate agent has the option of buying a rectangular building lot 1,000 feet wide and 1,500 feet long at 16¢ per square foot; or a circular lot 1,500 feet in diameter at a rate of 15¢ per square foot. Which do you consider the better buy? Show why.

28. Given the radius of the circle shown below is 6cm and $m\widehat{AB} = 45^{\circ}$, find the area of the unshaded section.



Appendix B

Attitude Questionnaire

Attitude Questionnaire

Instructions: For each question, circle the letter which best corresponds to your response to the question. All students are requested to express their true and honest feelings; no one but the person administering the test will see your answers.

Key:

a - strongly agree | d - disagree
b - agree | e - strongly disagree
c - don't know

1.	The subject I enjoy least is geometry.	a	b	c	d	e
2.	Geometry is boring.	a	b	c	d	e
3.	Geometry is fun.	a	b	c	d	e
4.	Geometry is a useful subject.	a	b	c	d	e
5.	Geometry is more of a game than it is hard work.	a	b	c	d	e
6.	Geometry is so difficult, it takes the fun out of it.	a	b	c	d	e
7.	Geometry is easier for me than any other subject.	a	b	c	d	e
8.	There are not enough practical problems in my geometry course.	a	b	c	d	e
9.	I feel I am capable of learning the basic principles of geometry.	a	b	c	d	e
10.	I would like to pursue a career in mathematics.	a	b	c	d	e
11.	I am proud of my mathematics school work.	a	b	c	d	e
12.	I am discouraged with my mathematics school work.	a	b	c	d	e
13.	I would like to study more geometry this year and next.	a	b	c	d	e
14.	Outside of school, I would like to apply what I learned in school.	a	b	c	d	e
15.	I like working with angles of triangles and circles better than reading a story about Brazil.	a	b	c	d	e

Appendix C

Attitude Item Analysis

Frequency Table for Attitudes
Pretest Item Analysis

Item Number	Activity-oriented Method		Traditional Textbook Method	
	Negative 0	Positive 1	Negative 0	Positive 1
1	4	7	0	10
2	3	9	3	8
3	5	7	5	6
4	2	7	1	5
5	4	6	2	8
6	4	8	4	7
7	9	2	8	4
8	4	5	2	3
9	1	9	1	8
10	8	2	6	1
11	5	4	9	3
12	5	7	5	6
13	6	5	2	5
14	3	8	4	6
15	2	9	3	7
Total	65	95	55	87

Frequency Table for Attitudes
Posttest Item Analysis

Item Number	Activity-oriented Method		Traditional Textbook Method	
	Negative 0	Positive 1	Negative 0	Positive 1
1	1	10	2	10
2	0	12	1	9
3	0	11	6	6
4	1	9	2	6
5	2	10	0	10
6	1	11	2	9
7	5	5	9	3
8	2	5	3	4
9	1	10	1	6
10	5	1	6	1
11	4	7	7	4
12	4	8	4	6
13	4	8	3	3
14	2	8	3	6
15	0	11	4	7
Total	32	126	53	90

Appendix D

Student Instruction Manual

on

Some Aspects of the Circle

Table of Contents

Lesson #1 -- Discovering Pi

Student Objectives

Upon completion of this lesson the student should be able to do the following correctly, 90% of the time.

1. The student should know the fractional and decimal approximations of Pi and in one written sentence describe the meaning of Pi.

2. The student should be able to write the relationship between the circumference and the diameter of a circle in the form of a general formula, namely, $C = \pi d$.

3. Given the measure of the radius or diameter of a circle, the student should be able to compute the circumference by applying the formula: $C = \pi d$.

4. Given the radii, diameters, or any combination of the two for two or more different circles, the student should be able to name, without using the formula $C = \pi d$, which circle has the greatest circumference. The student should be able to explain in one or two written sentences why he chose this answer.

A Class Discussion

Using circle #1, let us introduce a couple of Pin Men ($P.M_1$ and $P.M_2$) to run from point A to point B.

i.e. $P.M_1$ runs across the diameter

$P.M_2$ runs along the edge of the circle

Q. Which Pin Man will arrive at point B first?

A.

Q. Write a general statement about the shortest distance between two points.

A.

Suppose $P.M_2$ wants to challenge $P.M_1$ again. $P.M_1$ decides to run to B and then back to A, and $P.M_2$ runs right around the circle.

Q. Who will win this time? Why?

A.

Q. What is your conclusion about the outside of a circle and the diameter?

A.

Now suppose P.M_1 starts from A, runs to B, returns to A and then back to B, while P.M_2 runs around the circle, i.e. from A to A.

Q. Who do you think will win the third race? Why?

A.

If you are not sure about the last answer, let us try this exercise.

Activity Exercise

Using a piece of string or a measuring tape, measure the distance around each of the four circles in front of you, and also measure the diameter of each. Record your results.

Q. What do you notice?

A.

Q. How many diameters make up an outside?

A.

Q. Is it exactly three times?

A.

You have probably discovered by now that the outside of the circle is three times and a bit times the diameter. No one has ever found exactly what that bit is. Many great mathematicians have calculated the number to as many as 35 decimal places, and recently modern computers have calculated it to as many as 10,000 places after the decimal.

This strange fellow is so important, he has been given a name. He is not even English. He is Greek and his name is Pi; he looks like this (π). He has an approximate value equal to 3 1/7 or 3.14.

Q. Suppose you know that the diameter of a circle is 14 cm, can you find the outside? What is the measure?

A.

Q. Can you write a general formula for finding the outside of a circle?

A.

Definition: The mathematical word for the outside of a circle is circumference. The formula for finding the circumference is:

Circumference = Pi x Diameter

or $C = \pi d$

Practice Exercises

1. Using the formula $C = \pi d$ where $\pi = 3\ 1/7$, find each of the following:

   (a) d = 14 cm, find C (b) d = 3 1/2 cm, find C

   (c) C = 44 cm, find d (d) C = 66 cm, find d

   (e) r = 3 1/2 cm, find C (f) r = 17.5 cm, find C

2. Repeat exercise #1 using $\pi = 3.14$.

3. Mr. Barnes, a farmer, wishes to fence a circular pasture which is 70 metres in diameter. How many metres of fencing material will he need to do the job? How much will it cost if fencing costs $1.25 per metre?

4. Mr. Jones, another farmer, has a pasture which is 50 metres in radius. Using the information given in #3, find which pasture will cost more to fence. How much more?

5. If John wishes to obtain his bicycle on the opposite side of a circular field, what route would you suggest that he take? Explain.

6. For an enrichment activity, see: <u>The Story of Mathematics</u> by James T. Rogers, 1970, pages 55-56.

Lesson #2 -- Defining a Circle

Student Objectives

Upon completion of this lesson, the student should be able to perform the following correctly, 90% of the time.

1. Given a circle, the student should be able to identify its unique characteristics that distinguish it from other geometric closed figures.

2. Given the formula for the area of a circle and the measure of the diameter or radius of a circle, the student should be able to calculate the area.

3. By applying the formulae for area and circumference, the student should be able to solve examples of real life problems similar to the following.

   A real estate agent has the option of buying a rectangular building lot 1,000 feet wide and 1,500 feet long at 16¢ per square foot, or a circular lot 1,500 feet in diameter at a rate of 15¢ per square foot. Which choice do you consider to be the better buy? (Students may use a calculator to perform the computational algorithms.)

4. Students should be able to write a formal definition of a circle.

Class Discussion (Use overheads #3-6)

Q. Write your own definition of a circle.

A.

Q. Is a circle a straight line? See overhead #3.

A.

Q. Is it curved line like this? See overhead #3.

A.

Q. So we agree it is a closed curve; this is a closed curve, but is it a circle? See overhead #4.

A.

Q. You will now probably define a circle as a closed curve with no kinks. This is a closed curve with no kinks, but would you call it a circle? Why? See overhead #5.

A.

Q. Using the information gathered in the discussion, write another definition of a circle. Is it different from the first definition?

A.

Compare the definition you have just written with this one.

Definition: A CIRCLE is a plane figure bounded by a closed curve and all points on the curve are the same distance from a fixed point called the centre.

Extra for experts:

The formula for finding the area of a circle is: $A = \pi r^2$. To derive this formula, see: Mathematics in the Secondary School Classroom: Selected Readings by Gerald R. Rising and Richard A. Wilson, 1972, pages 80-81.

Practice Exercises

1. Given $A = \pi r^2$, find the area of a circle if:

   (a) $r = 7$ cm (b) $r = 3.2$ cm

   (c) $d = 10$ cm (d) $r = 1.5$ cm

   (Use the appropriate value for π)

2. If the diameter of a circular flower garden is 14 metres and 5 tulips are planted in every square metre of the garden, how many tulips can be planted in the garden?

3. Draw a circle including each of the following components, and label each component.

   (a) Radius $\overline{OP}$ (b) Diameter $\overline{AB}$

   (c) Arc $\overset{\frown}{XY}$ (d) Tangent $\overleftrightarrow{RST}$

   (e) Chord $\overline{MN}$ (f) Central angle AOB

   (g) Inscribed angle AYB

4. (a) A farmer has the option of buying a rectangular piece of farm land 2,000 metres wide and 3,000 metres long at 2½¢ per square metre, or a circular piece of farm land 1,390 metres in radius at the same rate of 2½¢ per square metre. If you were the farmer, what choice would you make? Justify.

   (b) Supp[illegible] the farmer wanted to fence in his land and fencing cost $2.[illegible]tre, find the cost of fencing each piece of land. Wha[illegible] land would you buy now? Explain why.

A Review of Concepts and Terms

1. There are ________° of arc in a circle.
2. The sum of the angle measures of a triangle is ________°.
3. The angles opposite congruent sides of a triangle are ________.
4. There are ________° of arc in a semicircle.
5. The number of degrees in a right angle is ________.

See overheads 7 and 8 for a review of the following terms.

Central angle	Radius
Chord	Diameter
Secant	Circumference
Tangent	Semicircle
Inscribed angle	Arc

Lesson #3 -- Central Angles

How to Make Your Own Circular Geoboard

Materials:

30 cm square piece of ½ inch plywood

15 - 3/4 inch screws or brass nails

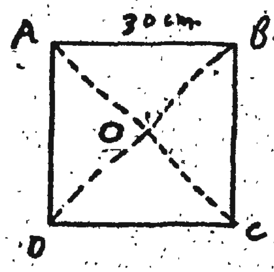
set of compasses of at least 15 cm radium

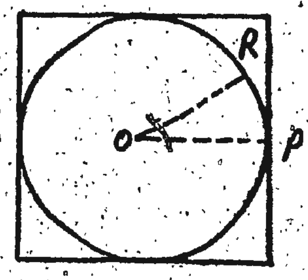
protractor

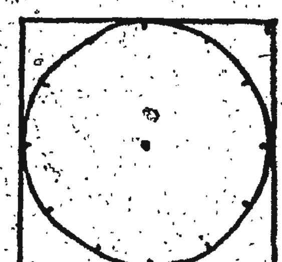
felt marking pen and an ordinary lead pencil

one mètre stick

elastic bands (at least a dozen per student)







Procedure:

1. To find the centre of the board, draw in pencil the diagonals $\overline{AC}$ and $\overline{BD}$ to intersect at O.
2. With compass tip at O and radius equal $\overline{OP}$, draw a circle and erase lines $\overline{AC}$ and $\overline{BD}$.
3. Draw faintly in pencil, radius OP; and with protractor draw angle ROP = 30°.
4. Erase lines $\overline{OR}$ and $\overline{OP}$. With compass tip at R and radius equal RP, draw points on the circle to form consecutive arcs, each equal to RP.
5. At each point, place a 3/4 inch screw about ¼ inch into the wood. Also place a screw at the centre.

Your circular geoboard is now complete. Use elastic bands to form diagrams and patterns.

Student Objectives

Upon completion of this lesson, the students should be able to do the following correctly, 90% of the time.

1. Given a diagram containing various labelled parts of a circle, the student should be able to name a central angle by writing the three letters which identify the angle.

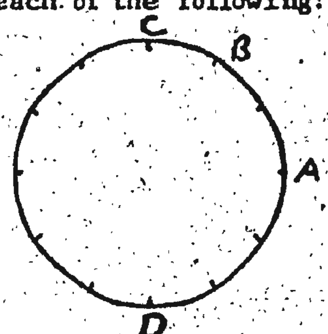
2. When given the measure of an arc intercepted by a central angle, the student should be able to write the measure in degrees of the central angle, and vice versa.

3. The student should be able to compute the measure in degrees of various angles and arcs in a given diagram when specific information is given about a designated angle(s) and/or arc(s). The student will do so by using his knowledge of angles in a triangle and central angles as related to arcs of a circle.

4. Given that angle A is a central angle of one of three concentric circles and the measure of angle A = 72°, the student should be able to determine the measure of each arc of each circle intercepted by angle A.

Class Activity

1. Set up your geoboard as shown in the diagram. Find the measure of each of the following:



(a) $\widehat{AC}$ = ______

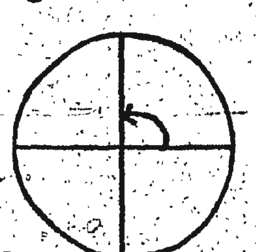
(b) $\widehat{BC}$ = ______

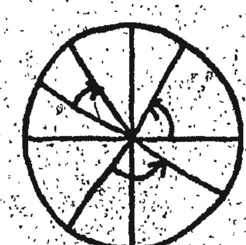
(c) $\widehat{ADC}$ = ______

(d) $\widehat{DAB}$ = ______

(e) $\widehat{DC}$ = ______

2. Use your geoboard to set up the central angles as shown in the diagram. Discuss the measure of each angle with your partner.





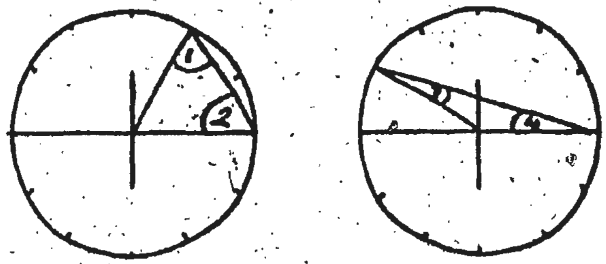
3. On your geoboard construct central angles of:

(a) 30° (b) 60° (c) 90°

(d) 180° (e) 240°

4. Write a generalization of the relationship between the central angles and the arc it intercepts.

5. Set up the following angles on your geoboard.



Q. What can be said about the measure of:

(a) angle 1 and angle 2?

(b) angle 3 and angle 4?

6. Use your geoboard to construct a triangle that has one vertex at the centre of the circle and its other two vertices on the boundary of the circle. Exchange geoboards with someone near you and find the degree measure of each angle of his/her triangle while he/she does the same with yours.

Practice Exercises

In Figure 1, BC is a diameter. Find the following:

1. $m\angle 1 = 40^\circ$, $m\widehat{AC} =$ ________.
2. $m\angle 2 = 28^\circ$, $m\angle 3 =$ ________.
3. $m\angle 3 = 25^\circ$, $m\angle 1 =$ ________.
4. $m\widehat{AC} = 36^\circ$, $m\angle 1 =$ ________.
5. $m\angle 1 = 42^\circ$, $m\angle 2 =$ ________.

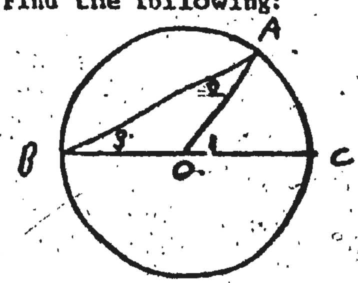


Fig. 1

6. In Figure 2, $\overline{AB}$ is a diameter.

   If $m\widehat{AC} = 50^\circ$, $m\angle ACB =$ ________.

   (Hint: Draw $\overline{OC}$. Find $m\angle ACO$ and $m\angle BCO$.)

7. If $m\widehat{BC} = 125^\circ$ in Figure 2, then $m\angle ACB =$ ________.

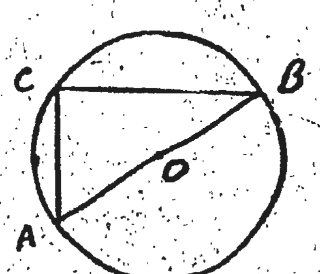


Fig. 2

In Figure 3, $\overline{WU}$ is a diameter. Find each of the following:

8. $m\angle UOX = 40^\circ$, $m\widehat{UX} =$ ________.

9. $m\angle WOU$ = ________.

10. $m\widehat{UXW}$ = ________.

11. $m\widehat{VW}$ = 135°, $m\angle VOU$ = ________.

12. $m\widehat{WXV}$ = 228°, $m\angle VOW$ = ________.

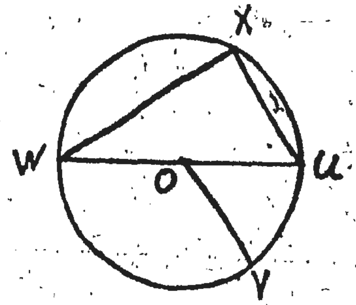


Fig. 3

Lesson #4a -- Inscribed Angles

Student Objectives

Upon completion of this lesson, the students should be able to perform the following correctly, 90% of the time.

1. The student should be able to recognize an inscribed angle constructed in a circle by writing the three letters which identify the angle.

2. When given the measure of an arc intercepted by an inscribed angle, the student should be able to write the measure of the inscribed angle in degrees, and vice versa.

3. Given the measure of a central angle, the student should be able to write the measure of an inscribed angle in degrees, if the inscribed angle intercepts the same arc as the central angle.

4. Given an angle inscribed in a semicircle, the student should be able to write a generalization about the measure of all angles inscribed in a semicircle.

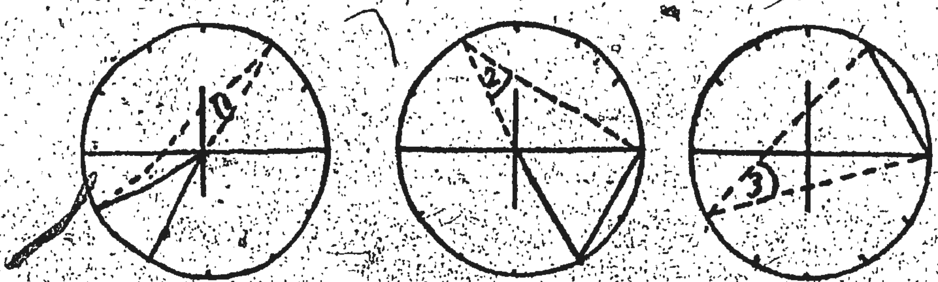
5. Given the measure of a number of labelled arcs, the student should be able to decide whether the corresponding inscribed angles are acute, obtuse or right angles by writing the appropriate name for each given arc.

6. Given the measure of labelled angles and/or arcs in a diagram, the student will have to use his knowledge of arcs, central angles, inscribed angles and angles in a semicircle to calculate the measures of various unknown angles and arcs in the diagram.

7. The student should be able to write a formal proof or criticize a given proof of the statement: "The measure of an inscribed angle is equal to one-half the measure of its intercepted arc."

Class Activity

1. Using your geoboard, set up the angles as shown below. Using your knowledge of the angles of a triangle, find the measures of angles 1, 2, and 3.

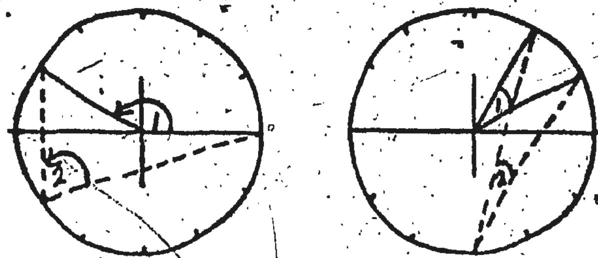


Is there a relationship between the measure of an inscribed angle and the arc it intercepts? Explain.

2. Set up these angles on your geoboard, and using the conclusion you have made in ex. 1, find the measures of angles 1 and 2.

Write a general conclusion about the relationship between the inscribed angle of a circle and the arc it intercepts.

3. Discuss the degree measure of angles 1 and 2 for each diagram below. Record your results on the chart.



(a) (b)

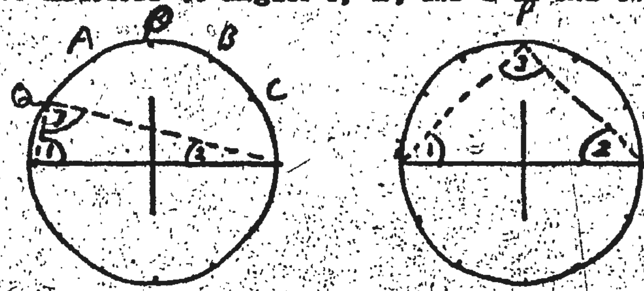
	a	b
∠1		
∠2		

What is the relationship between the measure of an inscribed angle and a central angle which intercepts the same arc?

Lesson #4b — Angles in a Semicircle

1. Set up the following angles on your geoboard and discuss with your partner the degree measure of angles 1, 2, and 3. What happened to the measures of the angles from diagram (a) to (b)?

2. Use the figure in number 2(a) and move Q to points A, P, B, and C. Record your measures of angles 1, 2, and 3 in the chart below.



(a) (b)

What happens to angles 1, 2, and 3?

Did you notice anything unique about the measure of angle 3?

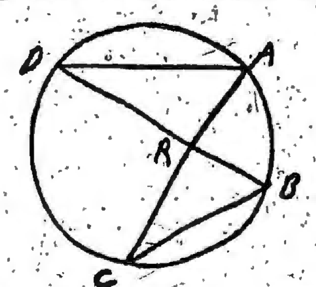
Write a generalization about the measure of the angle inscribed in a semicircle.

We will write the proof of this statement in a later exercise.

	Q	A	P	B	C
1					
2					
3					

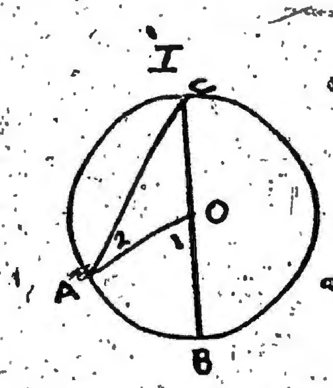
Practice Exercises

Given: $\overline{AC}$, $\overline{BD}$, $\overline{DA}$, and $\overline{BC}$ are chords of a circle



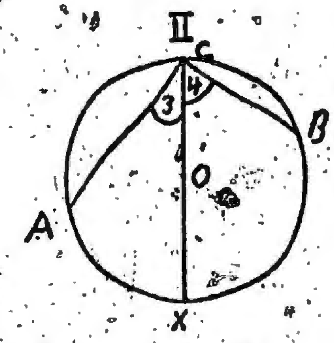
1. If $m\overset{\frown}{AB} = 96^\circ$, $m\angle D$ = ________, and $m\angle C$ = ________.
2. If $m\overset{\frown}{DC} = 118^\circ$, $m\angle A$ = ________, and $m\angle B$ = ________.
3. If $m\angle D = 44^\circ$, $m\overset{\frown}{AB}$ = ________, and $m\angle C$ = ________.
4. If $m\overset{\frown}{DC} = 179^\circ$ and $m\overset{\frown}{AB} = 68^\circ$, $m\angle ARD$ = ________.
5. Using the information given in #4, find the measure of angle BRC.
6. Is $\angle D = \angle C$? Is $\angle A = \angle B$? Show why.
7. Suppose $m\overset{\frown}{AB} = m\overset{\frown}{CD}$, what could you conclude about angles A, B, C, and D?
8. If $m\overset{\frown}{AB} = m\overset{\frown}{CD}$ and $m\overset{\frown}{AB} + m\overset{\frown}{CD} = 180^\circ$, what is the measure of $\angle A$?
9. Theorem: The measure of an inscribed angle is equal to half the measure of its intercepted arc.

   Prove: Each of the following three cases.



$m\angle ACB = \frac{1}{2} m\overset{\frown}{AB}$

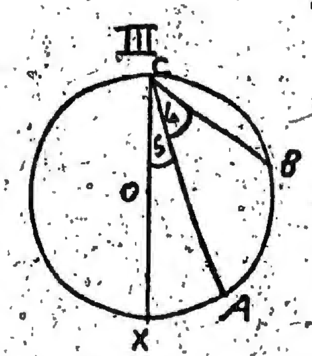
Hint: $m\angle ACB + m\angle 2 = \angle 1$.



$m\angle ACB = \frac{1}{2} m\overset{\frown}{AB}$

Hint: Apply case I and use the assmpt.

$m\overset{\frown}{AB} = m\overset{\frown}{AX} + m\overset{\frown}{XB}$



$m\angle ACB = \frac{1}{2} m\overset{\frown}{AB}$

Hint: Apply case I and use the assmpt.

$m\overset{\frown}{AB} = m\overset{\frown}{XB} - m\overset{\frown}{XA}$

Lesson #5 -- Inscribed Quadrilaterals

Student Objectives

Upon completion of this lesson, the students should be able to perform the following correctly, 90% of the time.

1. Given an inscribed quadrilateral and sufficient information about the measures of intercepted arcs, the students should be able to state the measure of each of the interior angles of the quadrilateral.

2. The student should be able to show by a logical step-by-step reasoning why the sum of the measures of any pair of opposite angles in an inscribed quadrilateral is always equal to 180^{o}.

3. Given a number of diagrams containing a quadrilateral and a circle in each illustration, the student should be able to select that diagram(s) which fit(s) the definition of an inscribed quadrilateral.

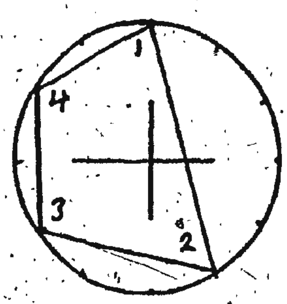
4. Given that two of the diagonals of an inscribed quadrilateral are actually diameters of the circle, the student should be able to answer the following question correctly: "The inscribed quadrilateral must be either a rectangle or a square." (True / False) Why?

Question for Thought:

Why does a cat usually curl into a ball shape when she is sleeping?

Class Activity

1. Using your geoboard, set up a quadrilateral similar to the one shown below. Record the degree measure of each of the four interior angles.



Angle 1 = ________

Angle 2 = ________

Angle 3 = ________

Angle 4 = ________

$\angle 1 + \angle 3 =$ ________

$\angle 2 + \angle 4 =$ ________

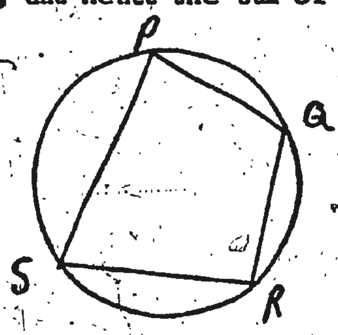
$\angle 1 + \angle 2 + \angle 3 + \angle 4 =$ ________

2. Repeat exercise 1 several times, and each time use a different quadrilateral which has its vertices on the circumference of the circle. Record your results and discuss them with your partner.

	A	B	C	D	E	F
∠1						
∠2						
∠3						
∠4						
∠1 + ∠3						
∠2 + ∠4						
∠1 + ∠2 + ∠3 + ∠4						

3. Write a general conclusion of what you have discovered about the angles of a quadrilateral inscribed in a circle.

4. Write a formal proof for the general case, i.e. "If PQRS is any quadrilateral inscribed in a circle, prove that m∠P + m∠R = 180° and m∠Q + m∠S = 180°, and hence the sum of all four angles = 360°."



Proof:

Enrichment Exercise

Repeat the procedure in exercise 1 and 2 for a 5-sided, 6-sided, 7-sided polygon and so on.

How many degrees would you expect for the total of the degree measures of the interior angles of a 20-sided polygon? Explain.

Write a general formula for finding the sum of the measures of the interior angles of an n-sided polygon.

Lesson #6 — Angles Formed by Intersecting Chords

Student Objectives

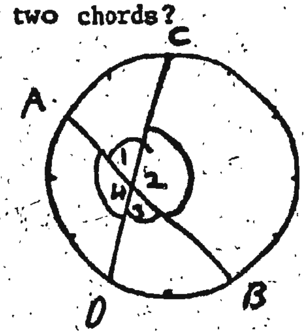
Upon completion of this lesson, the students should be able to perform the following correctly, 90% of the time.

1. Given two chords of a circle that intersect and the measure of at least two intercepted arcs, the student should be able to calculate the measure of the angles formed by the intersecting chords and vice versa.

2. Given the measure of an angle formed by two intersecting chords, the measure of one of the intercepted arcs and using the relationship $m\angle x = \frac{1}{2}(a + b)$ where x is the given angle, and a and b are the two corresponding intercepted arcs, the student should be able to calculate the measure of the other intercepted arc.

Class Activity

1. Using your geoboard, set up a figure as shown below. Can you find a way to determine the degree measure of the angles formed by the intersection of any two chords?



Procedure:

(a) Find the measure of angle 1. (Hint: Draw $\overline{AC}$ and find $m\angle A$ and $m\angle C$, then find $m\angle 1$)

(b) Compare the measure of angle 1 with the $m\widehat{AC} + m\widehat{BD}$.

(c) Find the measure of angle 2.

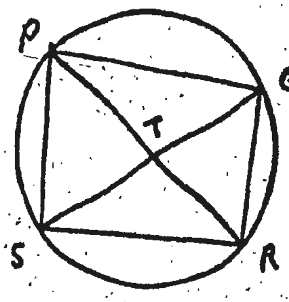
(d) Compare the measure of angle 2 with the $m\widehat{AD} + m\widehat{BC}$.

(e) Repeat the procedure to find the $m\angle 3$ and $m\angle 4$.

(f) Write a general conclusion.

Practice Exercises

In the diagram, PQRS is an inscribed quadrilateral and $\overline{PR}$ and $\overline{QS}$ intersect at point T. If $m\angle P = 78^{\circ}$, $m\widehat{SR} = 89^{\circ}$, and $m\widehat{PQ} = 76^{\circ}$, find each of the following:



1. $m\angle SRQ$ = ________
2. $m\angle PQR$ = ________
3. $m\angle PSR$ = ________
4. $m\widehat{PS}$ = ________
5. $m\angle PTQ$ = ________
6. $m\angle QTR$ = ________
7. $m\angle PRS$ = ________

Complete the following using the above diagram.

8. What is the sum of the four angles with vertices at T? ________
9. There are four triangles that have T as one of its vertices. How many degrees of angle are in each triangle? ________
10. The number of degrees in all four triangles = ________
11. Is the $m\angle P + m\angle Q + m\angle R + m\angle S$ the same as the answer obtained in number 10? If not explain.
12. Therefore, $m\angle P + m\angle Q + m\angle R + m\angle S$ = ______ - ______

= ____________

Unit Review

Summary of Conclusions and Generalizations

1. The measure of any central angle of a circle is equal to the measure of its intercepted arc.
2. The measure of any inscribed angle of a circle is always one-half the measure of its intercepted arc, or one-half the measure of the central angle intercepting the same arc.
3. The measure of any angle inscribed in a semicircle is always a right angle.
4. The sum of the measures of the angles of an inscribed quadrilateral is 360°, and the sum of the measures of a pair of opposite angles is 180°.
5. The measure of an angle formed by intersecting chords in a circle is one-half the sum of the measures of the intercepted arcs.

Review Exercises

Fill in the blanks.

1. When a quadrilateral is inscribed in a circle, the sum of the measures of a pair of opposite angles = ____________
2. If a radius of a circle is 9 1/5 inches long, a diameter of the same circle is _______ inches long.
3. If points T and V lie on a circle, $\overline{TV}$ must be a _________ (radius, diameter or chord) of the circle.
4. $\overline{EF}$ and $\overline{GH}$ are chords of a circle. $\overline{EF}$ is further from the centre than $\overline{GH}$. Can $\overline{EF}$ be congruent to $\overline{GH}$? __________
5. The shortest distance between any two points on a plane surface is a __________
6. Use letters and symbols to name examples of the following:

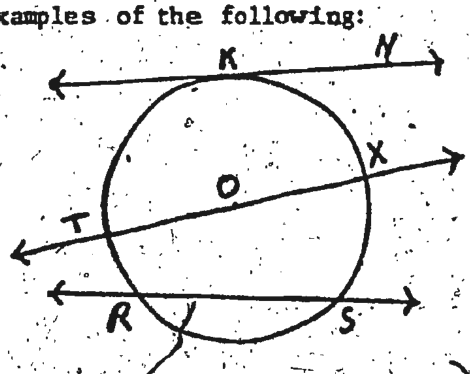
   (a) Diameter ____

   (b) Two chords ____ & ____

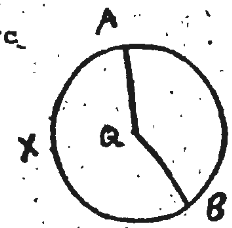
   (c) Two secants ____ & ____

   (d) Tangent ____

   (e) Radius ____



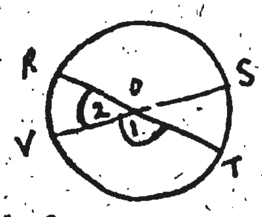
In exercises 7 and 8, $\overset{\frown}{AXB}$ is a major arc of the circle, and $m\overset{\frown}{AXB} = 215°$.



7. $m\overset{\frown}{AB}$ = ________

8. $m\angle AQB$ = ________

In exercises 9 and 10, $\overline{RT}$ and $\overline{VS}$ are diameters of the circle.

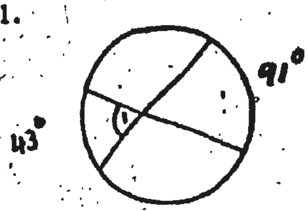


9. If $m\angle 1 = 138°$, then $m\overset{\frown}{RS}$ = ________

10. If $m\overset{\frown}{VT} = 141°$, then $m\angle 2$ = ________

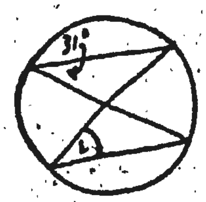
In exercises 11-15, find the measure of angle 1.

11.



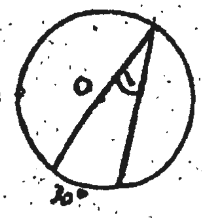
$m\angle 1$ = ________

12.



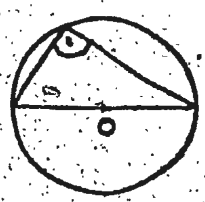
$m\angle 1$ = ________

13.

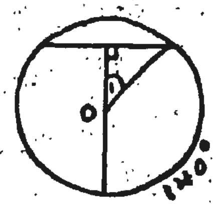


$m\angle 1$ = ________

14.



15.



$m\angle 1$ = ________ $m\angle 1$ = ________

In the diagram, $\overline{EF}$ and $\overline{HG}$ are chords of the circle that intersect at point P.

16. If $m\overset{\frown}{EG} = 120°$ and $m\overset{\frown}{GF} = 60°$,

then $m\angle 1$ = ________

17. If $m\overset{\frown}{EG} = 66°$ and $m\overset{\frown}{HF} = 106°$,

then $m\angle 2$ = ________

18. If $m\widehat{EH} = 126^\circ$, $m\widehat{HF} = 108^\circ$, and $m\widehat{GF} = 56^\circ$, then $m\angle 2$ = ________

19. If $m\widehat{EH} = 130^\circ$ and $m\angle 1 = 95^\circ$, then $m\widehat{GF}$ = ________

20. Suppose $\overline{EF}$ and $\overline{HG}$ intersect at the centre, what would be $m\widehat{HFG}$?

21. Find the circumference of a circle that has a radius of 32 feet.

22. Find the area of a semicircle whose radius is 12 centimetres.

23. Prove: "If a central angle POQ = 180°, then an inscribed angle intercepting the same arc is a right angle."

Appendix E

A Teacher's Guide to:

An Activity-Oriented Approach to

Teaching Certain Aspects of the Circle

Table of Contents

Part I: Teaching Tips

Introductory Remarks to Teachers

The purpose and objective of this instructional package is to motivate interest in and increase understanding of certain concepts of the geometry of the circle for the general mathematics students. The success of the method of instruction proposed in this package will be largely determined by the role the teacher plays in organizing, planning, implementing, and expanding the individual lessons. It is suggested that the teacher allow all students to take the leading role in discovering various concepts. The duty of the teacher is to assist the students in following instructions, supply additional information where needed, provide extra follow-up exercises, and use any motivational techniques he has available, such as class demonstrations and newspaper clippings. Every teacher should conduct the lessons such that the anticipated behaviors are attained.

As students complete each lesson, they should be encouraged to work at home assignments, especially those that involve the utilization of research material.

This manual contains some suggestions the teacher may use in the classroom, samples of home assignments, sample tests, and solutions to practice exercises.

Lesson #1 -- Discovering Pi

Suggestions for Teachers

This lesson should be introduced with a discussion between students and teacher about the distance around a circle and the distance along the diameter. Such a discussion could follow the format outlined in the activity exercise throughout the lesson. If the teacher feels this exercise is too elementary, then he may generate an alternative discussion.

All students should be given the opportunity to measure the circumference and diameter of circular objects of various sizes, calculate the ratio C/d, and then compare their results. (Recording all students' results on the chalkboard and calculating the average may be of interest.)

The discussion should continue by introducing Pi and its approximate fractional and decimal values. Next, encourage students to conjecture a general formula for finding the circumference of a circle.

When working with solid objects, the students should work in groups of two. If the teacher feels a review of terms such as radius, diameter, etc. is needed, then he may do so with the aid of overhead number seven.

Next, the students should be required to complete the practice exercises and when finished be encouraged to attempt the enrichment activity.

NOTE: The accompanying overheads should be viewed by students after some discussion or interaction has taken place.

Solutions to Practice Exercises

1. (a) 44 cm (b) 11 cm (c) 14 cm (d) 21 cm (e) 22 cm (f) 110 cm

2. (a) 43.96 cm (b) 10.99 cm (c) 14.01 cm (d) 21.02 cm (e) 21.98 cm (f) 109.9 cm

3. C = 219.8 metres; cost = $274.75

Mr. Jones' pasture will cost more. It costs $392.50.

$392.50 - $274.75 = $117.75 more than Mr. Barnes'.

4. Walk along the diameter since the shortest distance between two points is a straight line.

5. See: Story of Mathematics by J. T. Rogers, pages 55-56.

Lesson #2 -- Defining a Circle

Suggestions to Teachers

Before using the overheads (numbers 3-6), the teacher may request each student to write his own definition of a circle and compare it later with the final definition. Using the overheads, the lesson is to be a total interaction between students and teacher in deciding the unique characteristics of a circle, and later putting them all together into one formal definition.

The derivation of the formula for finding the area of a circle is fairly difficult; thus after reviewing the concept of area, the students may be given the computational formula and asked to apply it. However, if the teacher feels some of his students are capable, he may encourage them to attempt the exercise entitled "Extra for Experts."

All students should complete the practice exercises, and an interesting discussion could develop from question #4. To summarize lessons 1 and 2, review the overheads #1–7, and discuss the various points being made.

Solutions to Practice Exercises

1. (a) 154 cm^2 (b) 32.15 cm^2 (c) 78.5 cm^2 (d) 7.07 cm^2

2. A = 153.86 m^2 No. of tulips = 769

3. Answers may vary

4. rectangular piece costs $150,000

   circular piece costs $151,699.85

   cost to fence rectangle = $25,000

   cost to fence circle = $21,823

A Motivational Demonstration

If you find your class is dull and the students appear to be bored, try this:

Materials: (i) a ball of string

(ii) a piece of string 2 metres long

Procedure: Request two students to assist you, or three volunteers where you give the instructions.

Ask two students to stand in the middle of the classroom. While one student holds the ball of string, the other holds the end and both students are directed to walk away from each other, imagining that they are walking along the equator of the earth. Ask the class the following questions.

Q. What will happen if they continue walking along the equator?

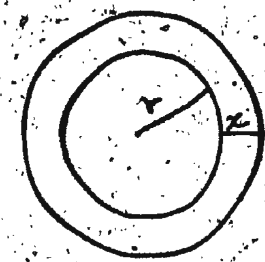
A. They will meet, and the string will be equal to the length of the equator if held tightly along the surface.

Now suppose the third student cuts the string at the centre and joins in two metres of string.

Q. How far above the surface of the earth will the string be if it is held uniform along the circumference? That is, find x in the diagram.

A. Record students' guesses and then prove the answer.

Proof:



Let the small circle (0_1) represent the string before the 2 metres was added, and let the large circle (0_2) represent the string after the 2 metres was added.

Circumference of 0_1 is:

$$C_{0_1} = 2\pi r$$

Circumference of 0_2 is:

$$C_{0_2} = 2\pi (r + x)$$

Also: $C_{0_2} = C_{0_1} + 2$ metres

$$C_{0_1} + 2 = 2\pi (r + x)$$

$$2\pi r + 2 = 2\pi r + 2\pi x$$

$$2 = 2\pi x$$

$$x = 1/\pi = .318 \text{ metres}$$

$$= 32 \text{ centimeters}$$

Lesson #3 -- Central Angles

Suggestions to Teachers

A brief discussion on defining central angles and measures of arc is in order for introducing the lesson. If the students have had no experience with the geoboard, then a demonstration of some things which can be done on the geoboard is essential before continuing with the lesson.

The students should work in groups of two with a geoboard for each group. They should be permitted to work on their own by following the instructions in the manual. The role of the teacher is to assist and guide any students that are having problems. If students are able to follow the instructions without any difficulty, then the overheads may be left for the summary discussion at the end of the lesson.

All students must complete the practice exercises.

Solutions to Practice Exercises

(1) $m\widehat{AC} = 40^\circ$ (2) $m\angle 2 = 28^\circ$ (3) $m\angle 1 = 50^\circ$

(4) $m\angle 1 = 36^\circ$ (5) $m\angle 2 = 21^\circ$ (6) $m\angle ACB = 90^\circ$

(7) $m\angle ACB = 90^\circ$ (8) $m\widehat{UX} = 40^\circ$ (9) $m\angle WOU = 180^\circ$

(10) $m\angle VOU = 45^\circ$ (11) $m\widehat{UXW} = 180^\circ$ (12) $m\angle VOW = 132^\circ$

Lesson #4 -- Inscribed Angles

Suggestions to Teachers

A discussion on the definitions of inscribed angles and chords with a review of previous work is suggested as an introduction to this lesson. Students should next be allowed to work in groups of two, following the instructions in the manual. The corresponding overheads may be used if necessary.

Upon completion of part A, students may complete the appropriate practice exercises, or may continue with part B of the lesson and then complete the practice exercises.

Using overhead number 10, summarize the lesson by discussion the generalizations made in the lesson.

Solutions to Practice Exercises

(1) $m\angle D = 48^\circ$, $m\angle C = 48^\circ$ (2) $m\angle A = 59^\circ$, $m\angle B = 59^\circ$

(3) $m\widehat{AB} = 88^\circ$, $m\angle C = 44^\circ$ (4) $m\angle ARD = 56\frac{1}{2}^\circ$

(5) $m\angle BRC = 56\frac{1}{2}^\circ$ (6) Yes, they intercept the same arc.

(7) $\angle A \cong \angle B \cong \angle C \cong \angle D$ (8) $\angle A = 45^\circ$

(9) Case I

1. $m\angle ACB + m\angle 2 = m\angle 1$	1. defn. of exterior angles
2. $m\angle 1 = m\widehat{AB}$	2. prop. of central angles
3. $m\angle ACB = m\angle 2$	3. $\angle$'s opp. $\cong$ sides are $\cong$

4. $m\angle ACB + m\angle ACB = m\widehat{AB}$	4. subst. 2 & 3 in step 1
5. $2m\angle ACB = m\widehat{AB}$	5. addition prop.
6. $m\angle ACB = \frac{1}{2}m\widehat{AB}$	6. division prop.

Case II

1. $m\angle ACX = \frac{1}{2}m\widehat{AX}$	1. from case I
2. $m\angle BCX = \frac{1}{2}m\widehat{BX}$	2. from case I
3. $m\angle ACB = m\angle ACX + m\angle BCX$	3. angle addition thm.
4. $m\widehat{AB} = m\widehat{AX} + m\widehat{BX}$	4. addition of arcs
5. $m\angle ACX + m\angle BCX = 1m\widehat{AX} + \frac{1}{2}m\widehat{BX}$	5. addition of steps 1 & 2
6. $m\angle ACB = \frac{1}{2}(m\widehat{AX} + m\widehat{BX})$	6. subst. step 3 in 5
7. $m\angle ACB = \frac{1}{2}m\widehat{AB}$	7. subst. step 4 in 6

Case III

1. $m\angle ACX = \frac{1}{2}m\widehat{AX}$	1. from case I
2. $m\angle BCX = \frac{1}{2}m\widehat{BX}$	2. from case I
3. $m\angle ACB = m\angle BCX - m\angle ACX$	3. inverse of angle add.
4. $m\widehat{AB} = m\widehat{BX} - m\widehat{AX}$	4. inverse of add. of arcs
5. $m\angle BCX - m\angle ACX = \frac{1}{2}m\widehat{BX} - \frac{1}{2}mAX$	5. subst. steps 1 & 2
6. $m\angle ACB = \frac{1}{2}(m\widehat{BX} - m\widehat{AX})$	6. subst. step 3 in 5
7. $m\angle ACB = \frac{1}{2}m\widehat{AB}$	7. subst. step 4 in 6

(10) To prove $m\angle ACB = 90^\circ$.

1. $\overline{AB}$ is a diameter	given
2. $m\widehat{AB} = 180^\circ$	2. from defn. of diameter
3. $\angle$ ACB is an inscribed angle	3. defn. of inscribed $\angle$
4. $m\angle ACB = \frac{1}{2}m\widehat{AB}$	4. prop. of inscribed $\angle$
5. $m\angle ACB = 90^\circ$	5. mult. from steps 2 & 4

Lesson #5 -- Inscribed Quadrilaterals

Suggestions to Teachers

Introduce the lesson with a discussion on inscribed quadrilaterals, and then follow the same procedure used in lessons 3 and 4. The teacher may request the students to complete those exercises in lesson 6 that are related to lesson 5, or they may complete lesson 6 first.

Depending on the abilities of the class, the teacher may organize the Enrichment Exercise into a separate lesson. If the class has been doing well throughout the unit and appears to be interested, the developer suggests that the teacher expand on this exercise until the students arrive at a general formula for finding the total number of degrees of angle in any n-sided inscribed polygon. That is:

$$\text{no. of degrees} = (n - 2) \times 180^\circ$$

Lesson #6 -- Angles Formed by Intersecting Chords

Suggestions to Teachers

Follow the same procedure used in lessons 3-5; but first clarify what is meant by the phrase "angles formed by intersecting chords."

Solutions to Practice Exercises

1. $m\angle SRQ = 202^\circ$
2. $m\angle PQR = 108\frac{1}{2}^\circ$
3. $m\angle PSR = 71\frac{1}{2}^\circ$
4. $m\widehat{PS} = 128^\circ$
5. $m\angle PTQ = 82\frac{1}{2}^\circ$
6. $m\angle QTR = 97\frac{1}{2}^\circ$
7. $m\angle PRS = 64^\circ$
8. 360°
9. 180°
10. $4 \times 180^\circ = 720^\circ$
11. No, because the sum of angles P, Q, R, and S does not include the central angles.
12. $720^\circ - 360^\circ = 360^\circ$

Solutions to Unit Review Exercises

1. 180° 2. 4.6 inches 3. chord 4. no 5. straight line

6. (a) $\overline{TX}$ (b) $\overline{RS}$, $\overline{TX}$ (c) $\overline{TX}$, $\overline{RS}$ (d) $\overline{KN}$ (e) $\overline{OX}$ or $\overline{OT}$

7. $m\overset{\frown}{AB} = 145^\circ$
8. $m\angle AQB = 145^\circ$
9. $m\overset{\frown}{RS} = 138^\circ$
10. $m\angle 2 = 39^\circ$
11. $m\angle 1 = 67^\circ$
12. $m\angle 1 = 31^\circ$
13. $m\angle 1 = 25^\circ$
14. $m\angle 1 = 90^\circ$
15. $m\angle 1 = 40^\circ$
16. $m\angle 1 = 90^\circ$
17. $m\angle 2 = 86^\circ$
18. $m\angle 2 = 89^\circ$
19. $m\overset{\frown}{GF} = 60^\circ$
20. $m\overset{\frown}{HFG} = 180^\circ$
21. 200.96 $ft.^2$
22. 226.08 cm^2

Conclusion of Unit

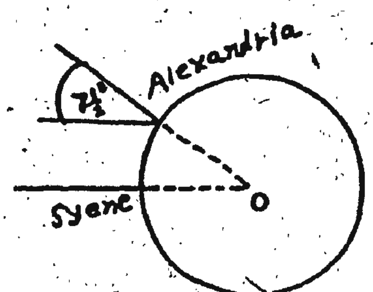
One lesson should be devoted to reviewing the material covered in the work unit. Students should be given the opportunity to complete the review exercises and ask questions before they are required to write the final evaluation test.

How the students are to be evaluated is entirely at the discretion of the individual teacher. The following section of this manual contains some samples of assignments and exam questions.

Part II: Samples of Assignments and Examination Questions

Assignment #1

Below is a diagram of the information available to the Greek Mathematician, Eratosthenes (275-194 B.C.) when he computed the circumference of the earth. If the distance from Syene to Alexandria was about 500 miles, and the indicated angle was about $7\frac{1}{2}^{\circ}$, find the circumference of the earth by studying the works of Erastosthenes in any of the listed references. Write a report of your findings.



Graza, Vivian Shaw. A Survey of Mathematics: Elementary Concepts and Their Historical Development. New York: Holt, Rinehart and Winston, 1968, p. 173.

Harris, Esther. The How and Why Wonderbook of Mathematics. New York: Highland, Grosset and Dunlop, 1961, p. 27.

Lancelot, Hogben. The Wonderful World of Mathematics. Garden City, New York: Doubleday and Co. Inc., 1955, p. 160.

Assignment #2

Review the reference material listed below and describe in your own words how Archimedes discovered the value of Pi.

Benson, Frances. Famous Mathematicians. New York: J. B. Lippincott Co., 1966, pp. 16-17.

Graza, Vivian Shaw. A Survey of Mathematics: Elementary Concepts and Their Historical Development. New York: Holt, Rinehart and Winston, 1968, p. 174.

Lancelot, Hogben. The Wonderful World of Mathematics. New York: Doubleday and Co. Inc., 1955, p. 37.

Rogers, James T. The Story of Mathematics. Leicester, Great Britain: Brockhampton Press, 1968, p. 46.

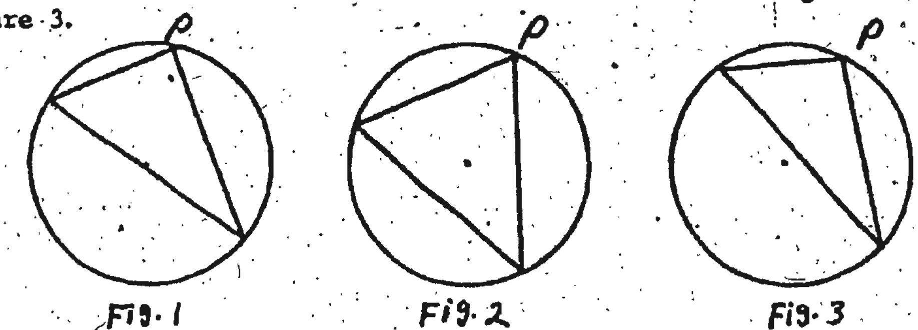
Assignment #3

In each of the following, find the measure of angle P, and give an explanation for your answer.

1. Make a triangle with one side containing the centre of the circle. See Figure 1.

2. What if the centre of the circle lies inside of the triangle? See Figure 2.

3. What if the centre of the circle lies outside of the triangle? See Figure 3.



Complete each of the following by underlining the correct answer.

4. If one of the sides of an inscribed triangle contains the centre of the circle, then the angle at the circumference will always be (acute, right, or obtuse). Explain why this is always true.

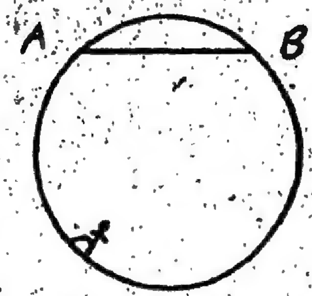
5. If the centre of the circle lies inside of the inscribed triangle, then the angle at the circumference will always be (acute, right, or obtuse). Explain why.

6. If the centre of the circle lies outside of the inscribed triangle, then the angle at the circumference will always be (acute, right, or obtuse). Explain why.

Assignment #4

1. If your principal asked you to make a stop sign for the school traffic patrol committee, describe how you would make the sign from a piece of plywood that is 4 ft. by 4 ft. You may use diagrams drawn in pencil to illustrate the procedure.

2. Suppose in the diagram following, $\overline{AB}$ represents the crossbar between the goal posts on a football field, and a kicker wishes to kick a football through the goal posts. Explain why it would be mathematically possible to kick from any point on the circle without increasing the difficulty of the angle.



3. A bicycle speedometer can be installed on a bicycle with 24 or 26 inch tires. If each of these bikes is ridden at the same speed, will the speedometers register the same speed for each? Explain.

4. Copy the following diagram. R shows the starting position of a spoke in a wheel. If the wheel turns 495°, show the new position of the spoke by drawing it on your sketch. Explain.