A STUDY IN THE PIAGETIAN COGNITIVE LEVELS OF DEVELOPMENT IN GRADE 9 STUDENTS

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A STUDY IN THE PIAGETIAN COGNITIVE LEVELS OF DEVELOPMENT IN GRADE 9 STUDENTS

An Internship Project

Presented to

The Department of Curriculum and Instruction

Memorial University of Newfoundland

In Partial Fulfillment

of the Requirements for the Degree

Master of Education

by

Montford L. Pritchett

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ABSTRACT

A STUDY IN THE PIAGETIAN COGNITIVE LEVELS
OF DEVELOPMENT IN GRADE 9 STUDENTS

by

Montford L. Pritchett

It was the purpose of this study to determine the Piagetian cognitive levels of development at which Grade 9 students are working.

A review of the literature indicated that there were four distinct Piagetian stages of cognitive development:

1. Sensori-motor stage (0 to 2 years old),
2. Pre-operational stage (2 to 7 years old),
3. Concrete operation stage (7 to 11 or 12 years old),
4. Formal operation stage (beginning at 11 or 12 years old).

The following five tasks were chosen in an attempt to provide answers to the problem: Conservation of Matter, Conservation of Weight, Conservation of Volume, the Oscillations of the Pendulum, and Combinations of Colorless Chemicals.

The investigator and four fellow teachers observed each of the 30 subjects individually as they investigated the tasks. The sample was randomly selected from the 200 Grade 9 students in Queen Elizabeth Regional High School, Foxtrap.

Each student was classified into a minimum level of cognitive development utilizing the first three tasks and then he was classified into a level of cognitive development on the basis of each of the last two tasks. The percentages of students, working at the different levels of cognitive development, were determined. Levels of development were given numerical values. The performances of the three Mathematical groups, Honours, Matriculation, and Pre-Vocational, were compared on the Piagetian tasks. The implications of the study were considered.

The major findings and conclusions of the study were as follows:

1. Only approximately one-third of the students were working at the level of formal reasoning and approximately two-thirds of the students were working at the concrete operational level of reasoning.

2. With respect to performance on the Piagetian tasks, there existed a reasonably high correlation between the streams Honours and Pre-Vocational Mathematics ($r = 0.811$), a moderate correlation between the streams Matriculation and Pre-Vocational Mathematics ($r = 0.466$), and a low correlation between the streams Honours and Matriculation Mathematics ($r = 0.302$).

3. There were significant differences in performance on the Piagetian tasks among the three groups, but this proved to exist only between the Honours Mathematics group and the Pre-Vocational Mathematics group.

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CHAPTER I

THE PROBLEM

Introduction

Over the past few years, the development of the 'New Mathematics' has seen many changes made in the mathematics curriculum. The Cambridge Conference on School Mathematics (1966) has advocated that more topics be placed into the High School, which were previously taught at the university level. However, mathematics educators in Newfoundland are presently advocating that not all children are adequately prepared to cope with the content and rigour of such high school mathematics courses. It becomes necessary to first look at the characteristics and abilities of the students who are enrolled in such courses.

Bruner (1967) writes:

> Instruction is, after all, an effort to assist or to shape growth. In devising instruction for the young, one would be ill advised indeed to ignore what is known about growth, its constraints and opportunities. And a theory of instruction is in effect a theory of how growth and development are assisted by diverse means [p. 1].

Most models of instructional development and curriculum development advocate similarly that the first step in such development is the assessment of student characteristics. In discussing studies of learners and learning processes, Taba (1962) states:

> Information from these studies serves an additional function of helping to determine what is feasible at any one point of development, or the appropriate level on which these outcomes are attainable. Thus the studies of developmental sequence should indicate what responses of intellectual, emotional, or social maturity can be attained by students at different age levels with different abilities and varying patterns of social learning [p. 194].

Hunt (1961) notes that Piaget and his colleagues have studied the development of intellectual functions and logic in children for more than thirty years. Hunt writes that "... Piaget's work contributes in highly varied ways not only to the theory of development, whence it derived but to the general theory of behavior, and, at least partially, to neuropsychology [p. 305]." So Piaget's work would seem to be a reasonable one to choose in a study concerning children's development. Even Bloom (1971) states: "Piaget's framework makes it possible for the teacher to know what a 4-year-old has to master in order to progress to the level of a normal 7-year-old and eventually to that of an adolescent [p. 297]."

Statement of the Problem

This study was undertaken in an attempt to determine the Piagetian cognitive levels of development at which Grade 9 students are working. To achieve this goal, conclusions were reached concerning the following questions:

1. What proportion of Grade 9 students appear to be working at i) the concrete operational stage, and ii) the formal operational stage?

2. Is there any correlation between this cognitive level of development and the student's placement in a high school mathematics stream, in this case, Honours and Matriculation Mathematics, as defined by the Newfoundland Department of Education, and the locally defined Pre-Vocational Mathematics?

3. Are there significant differences in performance in the Piagetian tasks among the three high school mathematics streams?

Significance of the Study

In other parts of Canada and in many parts of the world, studies, such as Lovell (1961) and Renner and Stafford (1972), have been carried out to determine the cognitive levels of development of school children. In some geographical areas, it has been determined that a student's cognitive development lags behind such development of a student in other geographical areas. The researcher could find no evidence to indicate that such a study has been carried out and recorded in Newfoundland. Such a study could provide means of comparing students in this province with students elsewhere.

According to Piaget, children progress from the concrete operational stage to the formal operational stage at about age eleven or twelve years old. On the basis of his finding, it could be argued that most students in Grade 9, who are on the average fourteen or fifteen years old,

should be functioning at the formal operational level of thought. This study has attempted to discern if these students were actually working at that level of reasoning.

Presently in Newfoundland, high school mathematics has been developed along a tri-level streaming approach. In each grade it is possible to place a student into one of three different streams. Is there any correlation between a student's placement into one of these streams and his level of cognitive maturity? One would be inclined to believe that a person's placement in a stream would be as a consequence of his ability to think, but this may not necessarily be the case.

CHAPTER II

REVIEW OF RELEVANT LITERATURE

Introduction

To review Jean Piaget's literature is an overwhelming task. His writing over the years has been prolific. According to Flavell (1968), he has written over 25 books and over 150 journal articles in a period of more than 40 years. Therefore, this chapter will only deal with the recurring themes and most significant notions of his writings. It will be necessary to define those terms which he frequently uses, to review his conception of a child's development of knowledge, to look closely at the different stages of development which Piaget describes, and to examine some of the research related to the topic under study.

The Foundation of Piaget's Theory

The following quotation is an example of Piaget's writing. To comprehend statements such as this one requires an understanding of the basic terms of his theory. Piaget (1960) writes:

> Every response whether it be an act directed towards the outside world or an act internalized as thought, takes the form of an adaptation, or better of a re-adaptation. The individual acts only if he experiences a need, i.e. if the equilibrium between the environ-

> ment and the organism is momentarily upset, and action tends to re-establish the equilibrium, i.e. to adapt the organism (Claparede) [p. 4].

Phillips (1969) notes that three terms which are found frequently in Piaget's literature are function, structure, and content. Phillips states that "the basic underlying idea is that functions remain invariant but that structures change systematically as the child develops. This change in structure is development [p. 7]." Flavell (1968) thinks that by content Piaget refers to the uninterpreted behavioral reactions, by function he refers to the intellect's endeavour to relate the old and the new meaningfully (it is the activity of the intellect that is characteristic of the development of all ages), and by structure he refers to the organizational ability of the intellect. Structure is created by function and inferred by content. Flavell (1968) summarizes by stating that "function permits cognitive progress; content is the behavior that informs us that functioning has occurred; and structure is inferred organizations that explain why a particular content appeared [p. 18]."

One of the most fundamental terms of Piaget's theory is the term operation, which Piaget (1964) describes as central for understanding the development of knowledge. He thinks that to know an object is not merely to look at it and make a mental image but it is to act on it. Piaget contends that to know an object is to modify, to transform the object, and to understand the process of this transform-

ation and as a consequence to understand the way the object is constructed. Consequently Piaget (1964) describes an operation as "an interiorized action which modifies the object of knowledge [p. 176]." Thus an operation becomes the essence of knowledge. An operation is not only an interiorized action, but it is also a reversible one - that is, it can take place in both directions, as in the case of addition and subtraction, for example. Piaget further notes that an operation cannot be isolated but it only makes sense in terms of a total 'structure'. Piaget (1950) contends that the fundamental problem of empiricist theories of "mental experiment" is that they attempt to look at an operation as one single act. He perceives the systems which are noticeable in thought as "groupings" which are analogous to groups in Mathematics. Thus his "grouping" constitutes a well-defined structure whose operational rules make up the logic of wholes which translates into an axiomatic or formal pattern the actual work of the mind when it reaches the operational level of its development, that is, its form of final equilibrium. Piaget refers to these cognitive structures as schemata.

Flavell (1968) defines schemata as cognitive structures which refer to a class of similar action sequences that are strong, bounded totalities in which the elements of behavior are tightly interrelated. A schema is labeled by the behavior sequence to which it refers. Thus Piaget speaks of the schema of sucking, the schema of hearing, the schema of sight, and so on. An action sequence, if it is to be a schema, must have

certain cohesiveness, and must maintain its identity as a quasi-stable repeatable unit. Piaget (1952) sees a schema as "... an ensemble of sensori-motor elements mutually dependent or unable to function without each other [p. 244]." The same schema in different subjects will not be exactly alike but it will share common features. Flavell notes that schemata, being structures, are both created and modified by intellectual functioning and are far from being static and inert.

It is now necessary to return to another term which was utilized in discussing structure - equilibrium. Before this term can be explained, the notions of assimilation and accommodation will be discussed. An understanding of these two terms should lead to a better knowledge of what Piaget means by equilibrium.

Piaget's concept of function has already been described. Phillips (1969) describes two basic functions: organization and adaptation. Flavell (1968) notes that these two properties or invariants hold true at all levels of development. Every act is organized and Phillips (1969) contends that the dynamic aspect of organization is adaptation. Flavell (1968) notes that adaptation is seen as the interplay of two processes: assimilation and accommodation. He contends that assimilation occurs when the child acts on an environmental object in relation to his previous experience with some similar object and imposes some of his own conceptions on it, whereas in accommodation new activities

are incorporated into the child's bank of knowledge in response to the demands of the environment. Mink (1964) provides the example of a child who has a schema for pulling. If the child wishes to reach an object on the far edge of a rug and does so by pulling the mat, Piaget would say that the behavior was assimilated into the existing cognitive structure. He also gives an example of a girl who has a lifting schema and can lift a bag of groceries. Yet if the same girl wishes to lift a 50-lb. boulder for the first time, she must accommodate by building the behavior sequences involved in rigging some kind of pulley system. Thus the existing cognitive structure is changed to cope with the new problem situation. Adaptation is a balance of the terms assimilation and accommodation. To summarize, Mink (1964) states that if one can take into an existing behavioral pattern, one assimilates, but if one must change his pattern of response, one accommodates.

This leads directly to what Piaget (1964) refers to as the principal factor in the development from one set of structures to another: equilibration or equilibrium. Piaget (1950) states that "... the equilibrium of operational thought is in no way a state of rest, but a system of balancing interchanges, alterations which are being continually compensated by others [p. 40]." Flavell (1968) notes that through interaction with the environment data is assimilated which creates a disequilibrium within the organism whereby existing schemata cannot cope with the new data. Through the process of accom-

modation the new data is incorporated into the organism to create a higher level of equilibrium. It is through this process of striving for equilibrium that cognitive development takes place. Piaget (1964) sees equilibration as a process of self-regulation and notes that he uses this term in the sense of processes with feedback and feedforward, or processes which regulate themselves by a progressive compensation of systems. This process of equilibration takes the form of successful levels of equilibrium. There exists a sequence of levels such that it is not possible to attain the second level unless equilibrium of the first level is attained. The equilibrium of the third level can only become possible when equilibrium of the second level has been reached and so on.

It is only now that one can make sense of Piaget's concept of intelligence. Piaget (1950) sees intelligence as "the state of equilibrium towards which tend all the successive adaptations of a sensori-motor and cognitive nature, as well as all assimilatory and accommodatory interactions between the organism and the environment [p. 11]." He perceives intelligence, which is the most plastic and at the same time most durable structural equilibrium of behavior, as essentially a system of living and acting operations. He sees structure as a particular form of equilibrium, more or less stable within its restricted field and losing its stability or reaching the limits of it's field. Since these structures form different levels, they are to be regarded as succeeding one another according to a law of development,

such that each one brings about a more inclusive and stable equilibrium for the processes that emerge from the preceding level. Intelligence to Piaget, then, is only a generic term to indicate the superior forms of organization or equilibrium of cognitive structurings.

Piaget's Concept of a Child's Development of Knowledge

Shulman (1970) points out that Piaget considers the child as a developing organism passing through biologically determined cognitive stages. These stages are age-related, although wide variations in cultures or environments will yield differences in individual rates of development. Shulman notes that it is possible to characterize certain growth periods by the formal structures most useful for describing the child's cognitive functioning during that time span. These growth periods define for Piaget the major stages of intellectual growth.

Hunt (1961) suggests the following underlying themes of Piaget's work:

1. Continuous and progressive changes take place in the structures of behavior and thought in the developing child.

2. Successive structures make their appearance in a fixed order.

3. The nature of accommodation (adaptive change to outer circumstances) suggests that the rate of development

is, to a considerable degree, a function of the child's encounters with his environment.

4. Thought processes are conceived to originate through a process of internalizing actions. Intelligence increases as thought processes are loosened from their basis in perception and action and thereby become reversible, transitive, associative, and so on.

5. A close relationship exists between thought processes and properties of formal logic.

These themes provide a rather good résumé of Piaget's work. The four main stages in the development of the child's mental structures according to Piaget are generally accepted as follows (Copeland, 1970):

1. Sensori-motor stage (approximately 0 to 2 years old),

2. Pre-operational stage (approximately 2 years to 7 years old),

3. Concrete operation stage (approximately 7 years to 11 or 12 years old),

4. Formal operation stage (beginning at 11 or 12 years old).

These stages will be discussed in detail later.

It is worthwhile noting at this point that Piaget sees a distinct difference between learning and the development of knowledge. Piaget (1964) views the development of knowledge as a process which concerns the totality of the structures of knowledge. However, learning is provided by situations as

opposed to being spontaneous; it is also limited, to a single problem or structure. So Piaget believes that development explains learning. Piaget describes four factors which, he believes, explain the development from one structure to another:

1. maturation which is a continuation of embryogenesis,

2. the role of experience of the effects of the physical environment on the structure of intelligence,

3. social transmission in the broad sense (linguistic transmission, education, and so on), and

4. equilibrium.

Piaget (1964) notes that the first factor, maturation, must certainly play an indispensable role in the transition from one set of structures to another, but this factor does not explain everything since the average age at which the stages appear varies a great deal from one society to another. Although the order of the appearance of the stages does remain constant, the chronological ages at which these stages appear may vary greatly.

With respect to the second factor, Copeland (1970) points out that there are two types of experiences in the development of a child's knowledge: the physical experience and the logical mathematics experience. He describes the act of weighing two objects to determine if they have the same weight as a physical experience. The logical mathematics experience comes not from the object or objects themselves

but from the action of the learner on the objects. Copeland's example depicts a child counting a certain number of pebbles in different arrangements and determining that the total is always the same. Piaget contends that these experiences are necessary before there can be operations. Again he notes that this does not totally explain development. He illustrates this by noting that children acquire conservation of substance before they acquire conservation of weight or conservation of volume. He argues that conservation of substance cannot be acquired by experience.

The third factor, Copeland (1970) writes, involves the imparting of knowledge by language. He states that this factor is important, but only when the child possesses a 'structure' that allows him to understand the language being used. Here the child almost becomes lost because the teacher may not realize that the child does not possess this 'structure'. This time Copeland gives the example of the child who may see the other children in his family as his brothers and sisters, but he does not see himself as a brother of the other children in the family. He can only think in one direction.

It is the fourth factor which Piaget refers to as the fundamental one. Copeland (1970) provides an example of equilibration. Having passed through the necessary stages of maturation, the child is shown two balls of clay the same size. One is flattened. He is asked which has more substance, the flat piece or the round piece. Then he restores the flattened piece to its original shape and sees that they are the same.

It is this type of reversibility that leads him to a stage of equilibrium, whereby he realizes that changes in shape do not transform the amount of substance. Through assimilation and accommodation the process of equilibration is carried out.

Since the factors affecting the changes from one structure to another have been discussed, the stages of development will now be described.

The Stages of Development

The first two stages of development are not very significant to this study. Therefore, they are not discussed in very much detail. If students at the Grade 9 level are not working at a formal operational level, then one could expect them to be working at the concrete operational stage. Thus only these two stages are described in detail.

The first stage, commonly referred to as the sensori-motor stage, is generally considered to last the first two years of life. Piaget (1964) views this stage as the period in which the practical knowledge is developed which constitutes the substructure of later representational knowledge. The example which he gives is that of the construction of the schema of a permanent object. At first, an object has no permanence for the infant. Once the object is out of sight, it no longer exists and no attempt is made to find it again. After the infant is a few months old, the infant attempts to locate the object again after he has first seen it. This he

does by localizing it spatially. During this period the child develops a series of structures, such as the permanence of an object which leads to the construction of sensori-motor space, and other constructions such as temporal succession and causality.

During the pre-operational stage, Piaget (1964) observes that there are the beginnings of language, of the symbolic function, and therefore of thought or representation. The sensori-motor activities are not immediately translated into operations as Piaget has defined them. There is as yet no conservation which is the psychological evidence of the presence of reversible action. Piaget contends that a child at the pre-operational stage will think that there is more liquid in a tall, thin glass than in a short, wide glass. This absence of reversibility is perhaps the most important single characteristic of pre-operational thought.

By symbolic thought Piaget (1950) is referring to the ability to differentiate between a signifier (a word, an image, and so on) and a significate (a perceptually absent event) and to use one to call forth or refer to the other. Flavell (1968) contends that Piaget believes that with the growth and refinement of the capacity to imitate, the child is eventually able to make internal imitations as well as external, visible ones. He is able to recall imitations made in the past. This internal imitation takes the form of an image and this image constitutes the first signifier. Eventually the child can use the image signifiers as anticipative outlines of future action.

Piaget (1972) states that the age of seven to eight years, on the average, marks a decisive turning point in the development of conceptual tools. The interiorized actions which the child possesses up to now acquire the status of operations with respect to reversible transformations modifying certain variables and conserving others as invariants. The reason why they are called concrete operations, Piaget (1964) notes, is because they operate on objects, and not yet on verbally expressed hypotheses. At this level there exists the operations of classification, ordering, the construction of the idea of number, spatial and temporal operations, and the fundamental operations of elementary mathematics, of elementary geometry, and even of elementary physics.

So if both the pre-operational stage and the stage of concrete operations utilize representational thought, what is the difference? Flavell (1968) recognizes the difference when he describes the concrete operational child as one who behaves in a variety of tasks as though an integrated assimilatory organization were functioning in equilibrium or in balance with a discriminative, accommodatory mechanism.

Hunt and Sullivan (1974) contend that it is necessary to outline five specific operations in formulating the properties of concrete operations:

1. Combinativity: an operation in which two classes may be combined into one comprehensive class that embraces them both.

2. Reversibility: every logical or mathematical

operation is reversible in that there is an opposite operation that cancels it (for example, 3 + 4 = 7 can be reversed to 7 - 4 = 3).

3. Associativity: an operation combining several classes without regard to grouping, for example, (a + b) + c = a + (b + c).

4. Identity: an operation that can be nullified by combining it with its opposite (for example, $^{+}A - A = 0$).

5. Tautology (or special identities): an operation related to logical classifications. Here repetition of a proposition, classification, or relation leaves them unchanged.

The child is unable to accomplish these operations prior to the concrete operational stage.

Copeland (1970) gives an example of reversibility. A child is shown two identical glasses containing the same amount of water. The water from one glass is poured into a taller glass with a smaller diameter. In the pre-operational stage, the child will answer that the taller glass has the more liquid while in the concrete operational stage, the child will answer that they contain the same amount. Piaget would say that the child has the concept of conservation of amount and realizes that the process can be reversed.

For the property of combinativity Hunt and Sullivan (1970) provide an appropriate example. Twenty beads are placed in a box. The child acknowledges that they are made of wood (comprising class B). Seventeen beads were brown and

were designated subclass A; three were white and were designated subclass A1. Does the child understand that A + A1 = B? When asked if there were more wooden beads or brown beads, the pre-operational child replies brown while the concrete operational child replies wooden. However, the pre-operational child is able to discern that all the brown ones are made of wood, that if the brown ones are taken away the white ones are left, and that if all the wooden ones are taken away, there are none left. The pre-operational child is able to centre his whole attention on all class B or on the subclass A or A1, but he is unable to handle both simultaneously. He fails to grasp the logical and mathematical truth that the whole is equal to the sum of its parts, whereas the concrete operational child can grasp this concept. Although the child at this stage understands the problem in an experiment, he is unable to do so in a verbal test.

Thus, the concrete operational child is beginning to extend his thought from the actual toward the potential. Flavell (1968) notes that there are several important limitations of concrete operations:

1. The structuring and organizing activity of the concrete operations is oriented toward concrete things and events in the immediate present. Although there is some movement toward the potential, it is of limited scope and consists mostly of simple generalizations of existing structures to new content. He does not consider all possible eventualities at the onset and then attempt to discover which

of these really occur.

2. The concrete operational child has to eliminate the various physical properties of objects and events one by one because his cognitive instruments are insufficiently "formal", insufficiently detached, and insufficiently dissociated from the subject matter they relate to so that they may permit a content-free structuring.

3. The various concrete operational systems exist as separate islands of organization during this stage. They do not interlock to form a simple, integrated system - a system by which the child can pass from one substructure to another in the course of a single problem. The various cognitive structures, adequate though they may be in their own separable domains, fail to combine into the unified whole necessary to manage certain complex tasks.

Flavell (1968) contends that concrete operational children possess both inversion or negation and reciprocity, the two kinds of reversible operations. These children do not possess a total structure which permits them to coordinate the two and solve multivariable problems which require this kind of coordination. Negation refers to class groupings whereas reciprocity is found in relation groupings.

Piaget (1964) believes that the operations of the concrete operational child are surpassed by the operations of the child who has reached the fourth stage of development. This is the level which he refers to as the formal or hypothetic-deductive stage because the child can now reason

on hypotheses and not only on operations. Copeland (1970) states that such a child begins to classify, order and enumerate in the verbal proposition form of deductive logic. Such a person can operate with the form of an argument and ignore its empirical content.

Flavell (1968) states that the most important property of formal operational thought concerns the real versus the possible. Piaget and Inhelder (1958) believe that in formal thought there is a reversal of the direction of thinking between reality and possibility in the subject's method of approach. They contend that possibility no longer appears as an extension of an empirical situation or of actions actually performed. It is reality that is now secondary to possibility. Flavell (1968) notes that the formal operational child or adolescent begins his consideration of the problem by determining all the possible relations that could hold true in the data and then attempts, through a combination of experimentation and logical analysis, to find out which of these possible relations hold true. Reality is considered to be a special subset within the totality of things which the data would admit as hypotheses. Thus conclusions are rigorously deduced from premises whose truth is regarded only as hypothetical at first.

Flavell (1968) lists three characteristics of formal thought as implied by the above consideration:

1. A cognitive strategy which tries to determine reality within the context of possibility is fundamentally

hypothetic-deductive in nature. To try to discover the real among the possible implies that one first considers the possible as a set of hypotheses to be successfully confirmed.

2. Formal thinking is, above all, propositional thinking. The important entities which the adolescent manipulates in his reasoning are no longer the raw data themselves but assertions or propositions that contain the data. Formal operations are really operations performed upon the results of prior concrete operations. Flavell notes that Piaget refers to formal operations as second degree operations or operations of the second power.

3. The adolescent subjects all the individual variables to a combinatorial analysis, a method which guarantees that the possible will be exhaustively manipulated until he is left with the real.

As with the concrete operational child, the adolescent possesses both inversion and reciprocity, but on a more sophisticated level. He is able to utilize these experimental methods in multivariable cases. Flavell (1968) notes that inversion applies to the ability to eliminate variables. What the formal operational child possesses is the ability to determine the interaction of variables. Reciprocity refers to the ability to hold a factor's effect constant while a second factor is being varied. Piaget and Inhelder (1958) supply an example of an experiment where the problem is to study the separate effects of the kind of metal and the length on the flexibility of a rod. The concrete operational child

is unable to cope with this problem whereas the adolescent is able to deal with this problem by neutralizing or controlling the variable length. He chooses two different kinds of metals of the same length.

Piaget and Inhelder (1958) contend that two discoveries are found at the beginning of the formal stage:

1. factors can be separated out by neutralization as well as by exclusion, and

2. a factor can be eliminated not only for the purpose of analyzing its own role but with a view toward analyzing the variations of other associated factors.

Piaget and Inhelder (1958) also believe that the form of equilibrium at this stage is the inevitable result of the earlier mental development and for this very reason it can be considered final in relation to later stages (adult thinking). From stage to stage, equilibrium of operations at each new plateau is both more stable and covers a more extensive field than the previous one. This general form of equilibrium can be conceived of as final in the sense that it is not modified during the life of the individual although it may be integrated into larger systems.

In attempting to explain the adolescent's new capacity to orient himself toward what is abstract and not immediately present, which is an indispensable instrument in his adaptation to the adult social framework, and as a result his most immediate and most deeply experienced concern, Piaget and Inhelder (1958) contend that this capacity is the most direct

and simplest manifestation of formal thinking. Formal thinking is both thinking about thought and a reversal of relations between what is real and what is possible. These are the two characteristics which the adolescent uses to build his ideals in adapting to society. The adolescent is both capable of reflective thinking and of escaping the concrete present toward the realm of the abstract and the possible. Formal structures not organized by themselves are only later applied as adaptive instruments. The two processes, structural development and everyday application, both belong to the same reality. Logic is not isolated from life but it is no more than the expression of operational coordinations essential to action.

Some Related Research

Piaget and Inhelder (1958) believe that the transition to the stage of formal reasoning commences at 11 or 12 years of age. Most first year students in Grade 9 are fourteen or fifteen years old, which means that they should be working at the level of formal reasoning according to Piaget and Inhelder. In their research they contend that a problem is solved by children of a certain age when three-quarters of the children of this age respond correctly.

Recently, there have been a number of studies conducted which seem to indicate that the formal operation stage may not be reached by a large proportion of students until a

later date. In replicating the experiments of Piaget and Inhelder on 200 British subjects, most of whom were between the ages of eight and eighteen years of age, Lovell (1961) concludes that it is only rarely that average to bright junior school children attain the stage of formal thinking, that the ablest of the secondary modern and comprehensive school pupils reach the stage of formal thought, but not all the older grammar school pupils always do so. However, he concludes that the least able of secondary modern and comprehensive school pupils certainly remain at a low level of logical thought and many of them do not seem to pass beyond the concrete stage of thinking. Jackson (1965) reached similar findings. Of 44 fifteen-year-old English children of subnormal intelligence, with intelligence quotients between 60 and 80, 24 were working at the concrete level of cognitive development and 20 were working at a pre-operational level of development. Of the 48 fifteen-year-old children of normal intelligence, with intelligence quotients between 90 and 110, 24 were working at the formal operational level and 24 were working at the concrete operational stage. While discussing the experiences of researchers in Leeds concerning aspects of proportionality and probability which Piaget contends require formal operational thought, Lovell (1970) notes that the ages at which the cognitive stages were reached were older than Piaget proposes.

There are a number of American studies which have reached similar conclusions. Renner and Stafford (1972)

conclude that 423 of 588 students from Grades 7 to 12, 72%, are working at the concrete operational stage. In this study conducted in Oklahoma schools, they state that 78 of 94 students tested at the Grade 9 level were working at the concrete operational level. Lawson and Renner (1974) note that of 143 college students, selected from over 300 freshman students in a private university in Oklahoma, 51% were classified as concrete operational. Nordland, Lawson, and Kahle (1974) conclude that only 13.2% of 506 disadvantaged students from a predominantly black urban senior high school were reasoning at a formal operational level. The majority were found to be concrete thinkers. Kohlberg and Gilligan (1971) cite a study by Kuhn, Lenger, and Kohlberg where the percentages of 265 persons at various ages showing clear formal operational reasoning at the pendulum task are as follows: age ten to fifteen - 45%, age sixteen to twenty - 53%, age twenty-one to thirty - 65%, and age forty-five to fifty - 57%. They conclude that these figures indicate that it is not until age 21 to 30 that a clear majority (65%) attain formal reasoning. Elkind (1961) questions the age at which conservation of volume appears. His experiment indicates a later age for this attainment. Tower and Wheatley (1971) found only 61% of college students demonstrated understanding of abstract concepts of volume.

As most researchers do agree, Piaget (1964) contends that the ordering of the stages of cognitive development is constant but the ages at which the stages are reached may vary

from one society to another. He gives the example of two Canadian psychologists, Monique Laurendeau and Father Adrien Pinard, who performed studies in Quebec, Canada. They discerned that students in Montreal acquired the cognitive stages of development at approximately the same age as those tested by Piaget. When they applied similar tests to students in Martinique, they found a time delay of four years. The present study has attempted to find out if Grade 9 students in a particular high school are working at the formal operational stage. There is much evidence to indicate that they need not be working at this level.

In a study conducted on early-adolescents in Grades 5 and 7, Keating (1975) classifies the students by psychometric testing as bright and average. When these students were evaluated on Piagetian tasks, Keating notes that the bright group evidenced formal operations far more frequently than the average groups of the same age. Similarly DeVries (1974) was able to find a relationship between the brighter students and their performance on Piagetian tasks, but between the other groups of lower intelligence, DeVries was not able to establish the same relationship. In a study on male adolescents of ages 12, 14, and 16, Youdin (1966) indicates that the development of formal thought in adolescence is an interaction of age and intelligence.

In the present study, a relationship was sought between a student's cognitive development and his placement in a particular mathematics stream. The underlying assumption

here is that students of above average ability have been placed in the Honours stream, students of average ability have been placed in the Matriculation stream, and students of below average ability have been placed in the Pre-Vocational Mathematics stream. Consequently it could be projected that such placement is a result of one's brightness and a relationship between placement and cognitive development was sought.

CHAPTER III

PROCEDURES IN THE STUDY

This chapter will describe the activities used to classify students according to the Piagetian cognitive levels of development, the procedures followed in carrying out the study, and the limitations of the study.

Activities Used for Classifying Students into Cognitive Levels of Development

It was decided that five activities would be used to classify students into the different levels of cognitive development. The first three activities were used to determine if the student had acquired the prerequisite skills in the concrete operational stage of development. This breakdown of classification will be described in more detail in the next section. In describing the five activities or tasks, the materials used, the procedure which was followed, and the stage at which a student can be expected to perform this task or portions of the task, will be outlined.

These five activities were used to classify the student according to the following accepted scheme as Piaget and Inhelder (1958) use regularly throughout their work:

I: not reached the concrete operational stage.

IIA: transition into the concrete operational stage.

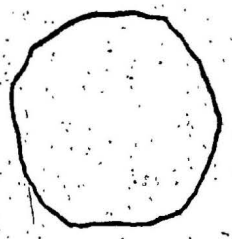
IIB: working at the concrete operational stage.

IIIA: transition into the formal operational stage.

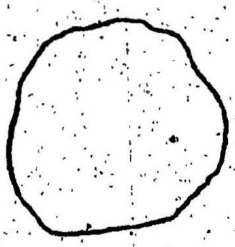
IIIB: working at the formal operational stage.

Task 1. Conservation of Solid Amount

The student should be able to perform this task at age 7 or 8 - the early part of the concrete operational stage. Two pieces of clay of different colors, for example red and blue, were rolled into balls of equal size.



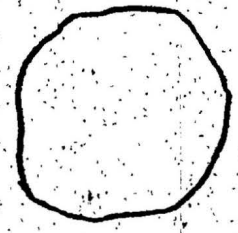
Red Ball



Blue Ball

The student was asked if he agreed that they had the same amount of substance in them. If he did not agree, then he was asked to fix them so that they had the same amount. When the student agreed that the balls had the same amount, one of the balls was distorted. Then the student was asked whether there was more substance in the red ball than the blue ball, whether there was more substance in the blue ball than the red ball, or whether they had the same amount.

Red Ball Distorted



Blue Ball Remains The Same

Task 2. Conservation of Weight

The student should be able to perform this task at age 9 or 10 - the middle part of the concrete operational stage. The format of this task is similar to that for the first activity. Two pieces of clay of different colors, for example red and blue, were rolled into balls of equal size. The student was allowed to add or subtract until he agreed that the two balls weighed exactly the same. One ball was distorted. After the ball was distorted, the student did not touch the ball again. Then the student was asked whether the red ball was heavier than the blue one, whether the blue ball was heavier than the red one, or if they both weighed the same.

Task 3. Conservation of Volume

According to Piaget, this ability is usually developed towards the end of the concrete operational period. The student was presented with two identical beakers containing equal amounts of water. When the student was convinced that the two amounts were equal, or when he adjusted the two levels of water until they were equal, he was then asked if the distorted ball of clay (from the previous task) would push the level up more, if the nondistorted ball of clay would push the level up more, or if the two amounts of clay would push the levels up equally. Successful completion of the task placed the student's level at the minimum of IIB.

Task 4. The Oscillations of a Pendulum

The material used in this task consisted of a string which could be shortened or lengthened from a stand, and a set of three weights. The variables which the student had to consider were the length of the string, the weight of the pendulum bob, the height of the release point, and the force of the push given by the subject when he released the pendulum.

The student was required to observe the pendulum while it was swinging and to vary the factors noted above. He had to isolate the one variable, the length of the string, which affected the frequency of the oscillations of the pendulum. The following provides some indication of the behavior of a student which could be expected at the categories I, IIA, IIB, IIIA, and IIIB as defined previously. These are guidelines which were taken from the section which Piaget and Inhelder (1958) wrote on the oscillations of a pendulum.

Stage I. Students at this stage have no organized manner of attacking the problem. Their expectations of what should happen prohibit them from observing what is actually happening. The subject is unable to disassociate the impetus which he gives the pendulum from the motion which is independent of his action.

Stage IIA. At this point the student can order the lengths, elevations and pushes but he is apt to conclude that more than one variable is relevant to the problem. He will most likely determine that the push has no effect upon the

natural rate of the pendulum. He will conclude that several variables affect the frequency of the oscillations because he varies several conditions simultaneously. The student is unable to do ordering with weights yet.

Stage IIB. This subject can do an accurate ordering of the effects of weight, but he still cannot always separate the variables. The subjects still lack a formal combinatorial system. They still vary several factors simultaneously so that, when they see a change in conditions occur, they believe that it is the result of each one of the factors.

Stage IIIA. At this stage the child is able to separate the variables, but he does not do it in a systematic manner. The formal operations are present but they are not sufficiently organized to properly manipulate the combinations involved. The tendency still exists to vary two variables simultaneously. The student is able to determine the true implications but he may still be unable to eliminate the false ones.

Stage IIIB. Now the subject can isolate variables by varying one factor while holding all others constant. He is able to exclude the one variable which plays the causal role. A complex combinatorial system is now in operation.

Task 5. Combinations of Colorless Chemical Bodies

Four similar containers held colorless, odorless liquids: 1) diluted sulphuric acid, 2) water, 3) hydrogen peroxide, and 4) thiosulphate. A smaller container, labeled

g, contained potassium iodide. The students used droppers and test tubes for mixing the solutions. Each student had five droppers, one by each of the liquids 1, 2, 3, 4, and g. A test tube containing 1) diluted sulphuric acid, and 3) hydrogen peroxide, was presented to the student. The subject had not seen this mixture placed into the tube. While the student watched, several drops of g - potassium iodide - were added to the test tube with a dropper. The liquid turned yellow. The student was then asked to reproduce this color. The following indicates the behavior of a student which could be expected at the stages I, IIA, IIB, IIIA, and IIIB as defined previously. These are guidelines which were taken from the section which Piaget and Inhelder (1958) wrote on combinations of colorless chemical bodies.

Stage I. At this stage the students randomly associate two elements at a time and explain the results in prelogical causality.

Stage IIA. At this stage the subjects multiply all the factors 1 to 4 by g.

Stage IIB. Now a preliminary attempt at combination is observed but only by trial and error. It is unsystematic.

Stage IIIA. At this level there are two things which can be noted: 1) a systematic method in the use of n-by-n combinations, and 2) an understanding that the color is due to combinations.

Stage IIIB. The difference in IIIA and IIIB is that the combinations and explanations appear in a more systematic fashion.

The Sample Studied

The sample consisted of 30 students in Grade 9 at Queen Elizabeth Regional High School in Foxtrap. They were chosen randomly from the 200 students in the six classes of students in Grade 9 at the time. It was found that the sample was very willing to cooperate and the 30 students in the sample were obtained from the first 32 students who were randomly chosen for the study. There were 14 males and 16 females, who were all 14 or 15 years old. Their average age was 14 years and 10 months.

A Summary of the Procedure Followed

Four of the author's fellow teachers and the author were actually engaged in carrying out the study. These four teachers were well qualified to aid in the study. Two of the teachers had previously completed Master of Education programmes. Three were high school science teachers, two of whom had honors undergraduate degrees. The fourth teacher taught Mathematics and had his graduate degree completed in the Mathematics area of Curriculum and Instruction.

These interviewers of students in the sample met and discussed the techniques to be used in the carrying out of the study. Each of these teachers was given guidelines for carrying out the study. These guidelines can be found in Appendix A. At this meeting such items were discussed as

keeping the room out of limit from homeroom students after school was finished, filling out the student response sheet, remembering to mix the mixture for Task 5 before interviewing a student, and the type of questions which the interviewer could ask. It was agreed that no question would be asked which would give away a solution to the experiment. But such questions as "Is there anything else you can do?", "What did you determine by doing this?", "What affects the rate of the swinging of the pendulum?", were acceptable questions. It was also decided that if the student determined the correct mixture for Task 5 quickly, then he would be asked if he could find other mixtures which would achieve the same result. This meeting, just prior to the start of the study, was initiated for the purpose of consistency in carrying out the study.

It took one school week from Monday to Friday to carry out the study. In that time, each interviewer was supposed to interview six students. However, the author and another interviewer tested eight students each, the third interviewer did six and the final two tested four students each. The last two teachers had commitments arise during the week which meant that they could not carry out all their interviews in one week. So that all the students could be interviewed in one week, the author and another teacher interviewed two extra students each. Each teacher was able to interview two students each afternoon. Each student was interviewed individually and each interview lasted between

twenty-five and thirty-five minutes. Each subject was exposed to the five tasks as outlined previously. His behavior was observed and recorded on student response sheets as can be found in Appendix B.

The materials for the task were prepared at lunchtime and it took ten to fifteen minutes to distribute the materials to the five different locations after school. This gave the students a break from the normal routine of the day before the interviews commenced.

Students were not told that they were to be interviewed until that afternoon when their interviews were scheduled. This limited the amount of communication and the students' curiosity concerning the study. Similarly, students from the same class were tested on the same day to the greatest possible extent. This also limited the amount of communication about the study.

Limitations of the Study

Since this study was undertaken in only one high school in Newfoundland, the generalizability of the study is limited. One can only say that the population of the sample is the number of Grade 9 students in that school. As well, the students at the school are doing a Pre-Vocational Mathematics course in lieu of the Basic Mathematics course offered provincially. Thus when the relationship between the cognitive levels of development of the students and their

placement in the different mathematics streams are considered, the results cannot be readily applied to other high schools in Newfoundland which are not doing this course.

Piaget's studies can only be considered quasi-experimental. Usually the exact nature of the questioning arises from the situation and the rapport established between the interviewer and the interviewee. There are no rigid control situations. In Piaget's case, the results were not exposed to rigorous data analysis. Often he did not give the number of students who had been studied and the exact procedures which he had followed. However, it is hoped that the extraneous variables were more rigidly controlled and the procedures of the present study are well outlined.

Since the students were tested in a one-week period, the history bias should be at a minimum. In such a short period of time it is unlikely that a student, who was tested toward the end of the study, would have made the transition from one cognitive stage to another during the period of testing. The transition from one stage to another is usually gradual. In order that the testing period be short, five experimenters were used in the study. To accomplish a consistent approach to interviewing students, a meeting was held to discuss the Piagetian methods of interviewing students.

If students were aware that they were going to be tested individually, then there existed the possibility that the students who had been subjected to the classification procedures would discuss the activities involved with those

who were going to be interviewed. This was overcome to a certain degree in two ways: i) most of the students from one class were tested at the same time, and ii) students were not notified that they were involved in the study until the afternoon in which they were to be tested. It is also worth noting that it is unlikely that students would acquire the techniques for solving the last two tasks by discussing these problems with other students. These techniques for solving the last two tasks were observed and used for classifying students.

CHAPTER IV

RESULTS OF THE STUDY

In the last chapter, the procedures for carrying out the study were described. In this chapter, the results of these procedures will be analyzed in an attempt to present answers to the questions which were asked in Chapter 1. However, before the analysis of data can be presented, it is necessary to first consider what behavior was evident for categorizing students into the cognitive stages of development and how the data is to be treated so that it can be evaluated.

Categorization of Students

In the following section of this chapter, Table 1 describes the results which were obtained when the students were categorized into various stages of cognitive development. The present section will attempt to clarify how these results were obtained, using the classification scheme on pages 29 and 30. When a student is named Student 3, for example, this refers to the student by that number in Table 1. The present section is necessary at this time to indicate how the results were obtained.

The first three tasks presented no problems for purposes of categorization. The student could be categorized

on the basis of whether he answered the questions correctly or incorrectly. These first three tasks only categorize a student at his minimum level of performance. For example, the first task enabled the investigator to know that a subject could cope with what is expected of a person thinking at a pre-operational level of thought. This does not mean that this is the maximum level of cognitive development which the subject has achieved. Similarly, a correct response to the third task only indicates that a student has reached the concrete operational stage as his minimum level of cognitive development. The fourth and fifth tasks require close observation of a student before the student can be categorized. Some examples of student behavior which helped the researcher categorize the students are now presented. The researcher found that no student exhibited behavior which could be described as pre-operational.

How students coped with Task 4, the oscillations of the pendulum, is first described utilizing the guidelines of the behavior expected for this task outlined in the last chapter. Student 4, who was classified as IIA on Task 4, placed the larger weight on a long string, the medium weight on a medium-length string, and the small weight on a short string. In each case this subject observed the motion of the pendulum. It was obvious that this student did not order the weights and varied two factors simultaneously. Student 1, who was classified as IIB, did on occasions vary more than one factor. But at first he attempted to vary the three

weights on one string while holding the variables push and height constant. However, as he progressed further into the experiment, he was inclined to vary two variables simultaneously. Neither student came up with an appropriate conclusion. Since Student 4 varied both weight and length simultaneously, she concluded that both these factors affected the rate of swinging of the pendulum. She did not vary the height of release. Student 1 concluded: "the smaller the weight, the faster the pendulum swings, and the shorter the length, the faster the pendulum swings." Student 5, who was classified as IIIA, varied one weight on all three strings, he varied the three weights on the same string, and then he varied the height at which he released the pendulum and the push which he gave the pendulum. He held the other three variables constant while determining the effect of one variable. However, this student concluded that the weight and the length affected the rate of the pendulum. He could not exclude the irrelevant variables, although he did systematically find the correct variable.

The students' approach to Task 5, combinations of colorless chemicals is now discussed utilizing the guidelines of expected behavior for this task outlined in the last chapter. Student 3, who was classified as IIA, first tried 1 x g, 4 x g, 3 x g and 2 x g. Then, after a time, she tried two combinations and g. These attempts were unsystematic. She did find the correct combination, but when she was asked if there was another combination which would give the same

effect, she went through the two combinations with g again. She was only able to find the same combination again and then she did not realize that it was the same combination which she had found before. When asked if there were other combinations which she could have tried, she said she did not know if there were any. Student 7, who was classified as IIB, started very systematically. He tried 1 x g, 2 x g, 3 x g, 4 x g, 1 x 2 x g, 3 x 4 x g, 3 x 4 x g and 1 x 3 x g. By the time he got the combination 1 x 3 x g, he was mixed up in his approach and struggling to ascertain the two combinations and g. These attempts were not very systematic. However, when asked what else he could do to find another correct combination, he did say he could try three combinations and g. Student 13, who was classified as IIIA, was extremely systematic in his approach. He tried 1 x 2 x 3 x 4 x g, 2 x 3 x 4 x g, and 2 x 3 x 1 x g. At this point he achieved the color which was a little lighter because of the effect of 2(water) which diluted it somewhat. He then tried 1 x 3 x 4 x g and then 1 x 3 x g. When asked what else he could have tried if this had not been the correct mixture, he replied that he could have tried all the three mixtures and g, all the two mixtures and g, and each solution and g. This researcher found several examples where the subjects did not try the combinations 1 x g, 2 x g, 3 x g, and 4 x g first. Although Piaget and Inhelder (1958) contend that students attempt these combinations first, Lovell (1961) also found that this was not always so.

Procedures of Data Collection

Once the students were categorized as indicated in the previous section, they were put in alphabetical order and numbered from 1 to 30. Table 1 shows the age of each student and how he was categorized. The first three tasks were treated together. For example, Student 1 got the answer to Task 1 correct, the answer to Task 2 correct, but he got the answer to Task 3 incorrect. The first three tasks place him at a cognitive level of IIA, but Tasks 4 and 5 both place him at a level of IIB. This is not inconsistent because a correct response to Task 3 really places the student at the end of the concrete operational stage. Student 2 answered all of the first three tasks correctly. This places that student at IIB, but this is really the maximum level of cognitive development at which the first three tasks can place the student. Although this subject was considered IIB on both Tasks 4 and 5, he could have scored higher on these two tasks. Student 5 is an example of a subject who also answered the first three responses correctly and was categorized as IIIA on Tasks 4 and 5.

For purposes of data analysis, the five stages of cognitive development as outlined on page 32, will be considered as follows: I - 1, IIA - 2, IIB - 3, IIIA - 4, and IIIB - 5. These stages are not really ranked in an ordinal scale, since cognitive development represents a continuous growth. Although the measuring instrument which was used was

TABLE 1

The Cognitive Levels of Development with Respect to the Different Piagetian Tasks

Student	Age Yr.(Mo.)	Cognitive Levels of Development Task 1	Task 2	Task 3	Task 4	Task 5
1	15 (3)	c*	IIA	x**	IIB	IIB
2	15 (6)	c	c	IIB	IIB	IIB
3	15 (7)	I	x	x	IIA	IIA
4	14 (7)	c	IIA	x	IIA	IIB
5	15 (3)	c	c	IIB	IIIA	IIIA
6	14 (6)	c	c	IIB	IIB	IIB
7	15 (3)	c	IIA	x	IIA	IIB
8	14 (8)	c	IIA	x	IIIA	IIIA
9	14 (11)	c	c	IIB	IIB	IIB
10	15 (3)	c	IIA	x	IIA	IIB
11	14 (4)	c	c	IIB	IIIA	IIIA
12	14 (10)	c	c	IIB	IIB	IIB
13	14 (7)	c	IIA	x	IIIA	IIIA
14	14 (10)	c	IIA	x	IIB	IIB
15	14 (10)	c	c	IIB	IIIA	IIB
16	15 (0)	I	x	x	IIA	IIA
17	14 (5)	c	c	IIB	IIIA	IIIA
18	14 (6)	I	x	IIB	IIA	IIA
19	14 (5)	c	IIA	x	IIB	IIB
20	14 (7)	c	IIA	x	IIIA	IIIA
21	15 (9)	I	x	x	IIA	IIA
22	14 (11)	c	IIA	x	IIB	IIB
23	14 (7)	c	IIA	x	IIIA	IIIA
24	14 (7)	c	c	IIB	IIB	IIB
25	14 (8)	c	c	IIB	IIIA	IIIA
26	15 (1)	c	IIA	x	IIB	IIA
27	14 (9)	c	c	IIB	IIIA	IIIA
28	15 (2)	c	IIA	x	IIB	IIB
29	14 (10)	c	c	IIB	IIIA	IIIA
30	14 (7)	c	c	IIB	IIB	IIB

*The letter c is used in the table to represent responds correctly.

**The letter x is used in the table to represent responds incorrectly.

not sensitive enough to measure between 2 and 3, it is still reasonable to think that a person who scores a 2 on Task 4 and a 3 on Task 5 could be considered 2.5. This would indicate that such a person is between the level of transition into the concrete operational stage and the level of being fully concrete operational. Lovell (1961) actually uses more such categories in breaking down the stages of cognitive development. In answering questions 2 and 3 which were asked in Chapter 1, the average of the measures for Task 4 and Task 5 will be used.

Similarly, to ascertain if there are any significant differences in performance in the Piagetian tasks among the three high school mathematics streams, the groups will be considered as:

Group 1 - Honours Mathematics,

Group 2 - Matriculation Mathematics,

Group 3 - Pre-Vocational Mathematics.

Data Analysis

To answer question 1, requires a breakdown in the number of students working at the various stages of cognitive development. Table 2 depicts the numbers of students working at each stage for the different tasks. Note the large number of students working at the concrete operational stage. But Table 3 really answers the question. For Task 4, it was found that 63.3% of the students were performing at the

TABLE 2

The Number of Students in Each Level of Cognitive Development According to the Piagetian Tasks

Piagetian Task(s)	Stage of Cognitive Development I	IIA	IIB	IIIA	IIIB	Total
Tasks 1, 2, 3	3	13	14	-	-	30
Task 4	-	7	12	11	-	30
Task 5	-	6	14	10	-	30

TABLE 3

Percentage of Students in Each Stage of Cognitive Development According to Task 4 and Task 5

Stage of Cognitive Development	Percentage Task 4	Task 5
Pre-operational	-	-
Concrete Operational	63.3	66.7
Formal Operational	36.7	33.3

concrete operational stage of cognitive development and for Task 5, it was found that 66.7% of the students were working at this level. Thus approximately two-thirds of these students can be said to be working at the concrete operational level of thought. As a matter of interest, Table 4 was drawn up to show the percentages of students in the three groups Honours, Matriculation, and Pre-Vocational Mathematics who are working at the different stages of cognitive development. To determine the percentages in the various stages, the average of the performances in Task 4 and Task 5 was used. There is a decline in the percentages of students at the formal operational stage of development from Honours to Matriculation to Pre-Vocational and an increase in the percentages of students at the concrete operational stage of development.

TABLE 4

Percentages of Students in the Different Streams Working at the Different Stages of Cognitive Development

Stages of Cognitive Development	Mathematics Streams		
	Honours	Matriculation	Pre-Vocational
Concrete Operational	33.3	64.7	100
Formal Operational	66.7	35.3	0
Total	100.0	100.0	100

To answer question 2, which asks if there is any correlation between the placement of students in the three groups,

which is considered nominal measurement, and performance on the Piagetian tasks, which is considered interval measurement, the point-biserial correlation coefficient was used. Table 5 shows the information used to calculate this correlation coefficient. Since the calculation of the correlation coefficient implies the use of nominal-dichotomous measures, three coefficients were calculated to determine the correlation i) between placement in the two streams Honours and Matriculation Mathematics and performance on the Piagetian tasks, ii) between placement in the two streams Honours and Pre-Vocational Mathematics and performance on the Piagetian tasks, and iii) between placement in the two streams Matriculation and Pre-Vocational Mathematics and performance on the Piagetian tasks. As shown in Table 5, the correlation coefficients were i) 0.302, ii) 0.811, and iii) 0.466, respectively. Thus the conclusion which was reached is that there exists a reasonably high correlation between placement in the streams Honours and Pre-Vocational Mathematics and performance on the Piagetian tasks. The correlation between performance on the Piagetian tasks and placement in the streams Honours and Matriculation is low and the correlation between performance on the Piagetian tasks and placement in the streams Matriculation and Pre-Vocational Mathematics is moderate.

To answer question 3, a one-way analysis of variance was calculated with respect to the three groups and their performance on the Piagetian tasks. Their performance on the Piagetian tasks was again considered the average of their

TABLE 5

The Point-Biserial Correlation Coefficients and the Information Required for Their Calculation

Data	Groups Compared		
	Matriculation-Honours	Pre-Vocational-Honours	Pre-Vocational-Matriculation
n_0	17	7	7
n_1	6	6	17
n	23	13	24
$\bar{X}._0$	3.21	2.50	2.50
$\bar{X}._1$	3.67	3.67	3.21
$\bar{X}.$	3.33	3.04	3.00
s_x	0.684	0.749	0.707
r_{pb}	0.302	0.811	0.466

Symbolism:

n_0 - number of students in one group.

n_1 - number of students in the other group.

n - $n_0 + n_1$.

$\bar{X}._0$ - mean of students in n_0 group on performance of the Piagetian tasks.

$\bar{X}._1$ - mean of students in n_1 group on performance of the Piagetian tasks.

$\bar{X}.$ - mean of all n scores on performance of the Piagetian tasks.

s_x - the standard deviation of all n scores on performance of the Piagetian tasks.

r_{pb} - the point-biserial correlation coefficient.

performance on Task 4 and Task 5, when the stages were given the measures of 1, 2, 3, 4, and 5 as indicated in the last section. Table 6 shows the results which were derived from this calculation.

TABLE 6

Sums of Squares, Degrees of Freedom, Mean Squares, F-Ratio, Probability for the One-Factor Analysis of Variance

Source of Variation	SS	df	MS	F	p
Between	4.60	2	2.30	6.00	<.01
Within	10.36	27	.38		

Since the probability of such a F-ratio occurring is less than .01, this would indicate that there are significant differences in performance on the Piagetian tasks among the three high school streams. At this point, it would be interesting to find out where the significant differences occur. To do this, a Scheffé test was conducted (Glass & Stanley, 1970).

Consider the contrast c among the means m_1, m_2, and m_3 where $c = c_1m_1 + c_2m_2 + c_3m_3$, $m_1 = 3.67$, $m_2 = 3.21$, $m_3 = 2.5$, and $C_1 + c_2 + c_3 = 0$. For the purposes of this Scheffé test, the contrasts which will be calculated are shown in Table 7. The standard error of each contrast is then calculated. Table 8 depicts the standard errors of the contrasts, and the ratios of the contrasts (in Table 7) to the standard

TABLE 7

Values of Contrasts

Values of c_1	c_2	c_3	Contrast	Value of Contrast
1	-1	0	$m_1 - m_2$	0.46
1	0	-1	$m_1 - m_3$	1.17
0	1	-1	$m_2 - m_3$	0.71

TABLE 8

Contrasts, Standard Errors of Contrasts, Ratios of Contrasts to Standard Errors of Contrasts

Contrast	Standard Error of Contrast	Ratio of Contrast to Standard Error of Contrast
$m_1 - m_2$	0.293	1.57
$m_1 - m_3$	0.343	3.41*
$m_2 - m_3$	0.277	2.56

*Significant at the .01 level of significance.

errors of the contrasts. In order for this ratio to be significant at the .05 level of significance, it must be greater than or equal to 2.59. If this ratio is to be significant at the .01 level of significance, it must be greater than or equal to 3.31.

The only significant difference in the level of performance on the Piagetian tasks is between the two streams Honours Mathematics and Pre-Vocational Mathematics. There were no significant differences between the other two pairs of groups although the comparison between the Matriculation Mathematics group and the Pre-Vocational Mathematics group barely failed to be significant at the .05 level of significance.

CHAPTER V

SUMMARY, CONCLUSIONS AND IMPLICATIONS

Summary

Initially, the problem was stated. Basically, there were three questions to be answered:

1. What proportion of Grade 9 students appear to be working at i) the concrete operational stage and ii) the formal operational stage?

2. Is there any correlation between this cognitive level of development and the student's placement in the high school mathematics streams, in this case, Honours, Matriculation or Pre-Vocational Mathematics, as defined by the Department of Education of Newfoundland?

3. Are there significant differences in performance in the Piagetian tasks among the three high school mathematics streams?

The literature was reviewed with special emphasis placed on the last two of the four stages of cognitive development:

1. Sensori-motor stage,
2. Pre-operational stage,
3. Concrete operations stage,
4. Formal operations stage.

Some of the research studies indicated that students did not

always reach the formal operations stage at the age stated by Piaget.

Five tasks were selected to determine the level of cognitive development of students: Conservation of Matter, Conservation of Weight, Conservation of Volume, the Oscillations of the Pendulum, and Combinations of Colorless Chemicals. Five experimenters conducted the study in one week. There were 30 students randomly selected for the study from the exactly 200 students in Grade 9 at Queen Elizabeth Regional High School. Each student was observed individually by an experimenter as he attempted to perform each of the five tasks correctly. These observations were noted on Student Response Sheets.

The main limitation of the study is that it was conducted in only one high school. Therefore, the population of the study was only considered to be the number of Grade 9 students in that particular school. Some of these students were doing a Pre-Vocational Mathematics course rather than the Basic Mathematics course offered provincially. Thus the generalizability of the study is limited.

Conclusions

Once the data was compiled, it was found that only approximately one-third of the students were working at the level of formal operations and that approximately two-thirds of the students were working at the concrete operational

level of reasoning.

To determine the correlation between placement in the three groups of students and their performance on the Piagetian tasks, the point-biserial correlation coefficient was calculated three times in a pairwise manner using the three groups. The results indicated that there was a reasonably high correlation between placement in the Honours Mathematics stream and the Pre-Vocational stream and performance on the Piagetian tasks ($r = 0.811$), that there was a moderate correlation between placement in the streams Matriculation Mathematics and Pre-Vocational Mathematics and performance on the Piagetian tasks ($r = 0.466$), and that there was a low correlation between placement in the streams Honours Mathematics and Matriculation Mathematics and performance on the Piagetian tasks ($r = 0.302$).

A one-way analysis of variance was conducted which discerned that there were significant differences in performance on the Piagetian tasks among the three groups at the .01 level of significance. The results of the Scheffé test indicated that this significant difference in performance on the Piagetian tasks occurred between the Honours Mathematics group and the Pre-Vocational group. The differences in performance between the other two pairs of groups were not significant.

Implications

The results of this study would seem to imply that the majority of students in Grade 9 are not working at the

formal operational stage of cognitive development. There are a number of topics in Grade 9 Mathematics which require formal operations, not the least of which is deductive reasoning in Geometry. How does a student working at the concrete operational level of thought cope with these mathematical concepts which require more advanced reasoning? Is the learning of such topics rote or does the learning of such topics increase the rate of transition from the stage of concrete operations to the stage of formal operations?

If two-thirds of the students in Grade 9 are working at the concrete operational stage of cognitive development, are there means by which they may be taught so that their level of cognitive development can be advanced? Why is there a delay in their growth of logical reasoning? After conducting a survey on some Piagetian-related studies, Farrell (1969) states:

> The studies reported here strengthen the previous description of adolescent thought. Some flexibility as to the age at which formal operations begin to develop and that at which the structure is in equilibrium is necessitated by the conclusion of various researchers. Factors of culture, type of formal education, innate ability, and others must be recognized as variables which can promote or deter intellectual development [p. 15].

There are a number of factors which can retard intellectual growth, including cultural ones. It may be possible that there are factors of culture peculiar to this particular community, and perhaps to Newfoundland, which have retarded growth.

As the findings indicated, there only existed a high correlation between students' placement in the high ability

group and the low ability group and their performance on the Piagetian tasks. Only medium and low correlations existed between the other two pairs of groups: between the medium ability and the low ability group and between the high ability group and the medium ability group. This would seem to imply that the distinctions made between students in the top and the middle groups and between students in the middle and the bottom groups are not so easily made. There could be errors in the choice of students for these groups or there could be other factors which must be considered, such as student work habits, the amount of time which students spend studying at home, and the home environment of the student.

Thus some of the implications from the study would appear to be: a large proportion of students may not be ready for topics in certain subjects which require formal operational thought; there would seem to be a need for developing and utilizing methods for advancing the development of cognitive thought; there could be a need for the inclusion of more topics, which require concrete operational thought, in the secondary schools; if culture is the reason for the retardation of logical development, the problem could be a larger social one, and students may be placed into mathematical groups utilizing other factors than cognitive development.

The investigator has posed a number of questions, which this study has attempted to answer. Other questions have arisen from the results of this study. There are questions which could be subjected to further investigation.

BIBLIOGRAPHY

Bloom, B.S., J.T. Hastings, and G.F. Madaus. Handbook on Formative and Summative Evaluation of Student Learning. New York: McGraw-Hill Book Co., 1971.

Bruner, Jerome S. Toward a Theory of Instruction. Cambridge, Mass.: The Belnap Press of Harvard University Press, 1967.

Bruner, Jerome S., and others. Studies in Cognitive Growth: A Collaboration at the Center for Cognitive Studies. New York: John Wiley and Sons, Inc., 1967a.

Cambridge Conference on School Mathematics. Report of the 1963 Conference. Goals for Mathematics. Boston: Houghton Mifflin Co., 1966.

Copeland, Richard W. How Children Learn Mathematics. New York: The MacMillan Co., 1970.

DeVries, Rheta. "Relationship among Piagetian, I.Q., and Achievement Assessments." Child Development 45 (No. 3, 1974), 746-756.

Ducksworth, Eleanor. "Piaget Rediscovered." Journal of Research in Science Teaching 2 (No. 3, 1964), 196-203.

Elkind, David. "Quantity Conceptions in Junior and Senior High School Students." Child Development 32 (Sept., 1961), 551-560.

Farrell, Margaret A. "The Formal Stage: A Review of Research." Journal of Research and Development in Education 3 (Fall, 1969), 111-118.

Flavell, J.H. The Developmental Psychology of Jean Piaget. Princeton, New Jersey: D. Van Nostrand Co., Inc., 1968.

Glass, Gene and Julian Stanley. Statistical Methods in Education and Psychology. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1970.

Hunt, David E. and Edmund Sullivan. Between Psychology and Education. Illinois: The Dryden Press, 1974.

Hunt, J.M. Intelligence and Experience. New York: Ronald Press, 1961.

Innskeep, James E. "Building a Case for the Application of Piaget's Theory and Research in the Classroom." The Arithmetic Teacher 19 (April, 1972), 255-260.

Jackson, Stephen. "The Growth of Logical Thinking in Normal and Sub-Normal Children." British Journal of Educational Psychology 35 (Feb., 1965), 255-258.

Karplus, R. and E. Karplus. "Intellectual Development Beyond Elementary School I: Deductive Logic." School Science and Mathematics 70 (May, 1970), 398-406.

Keating, Daniel. "Precocious Cognitive Development at the Level of Formal Operations." Child Development 46 (No. 1, 1975), 276-280.

Kohlberg, Lawrence and Carol Gilligan. "The Adolescent as a Philosopher: The Discovery of the Self in a Post-Conventional World." Daedus 100 (Fall, 1971), 1051-1084.

Lawson, E., J.H. Nordland, and Alfred Devito. "Piagetian Formal Operational Tasks: A Crossover Study of Learning Effects and Reliability." Science Education 58 (No. 2, 1974), 267-276.

Lawson, A.E. and J.W. Renner. "A Quantitative Analysis of Responses to Piagetian Tasks and Its Implications for Curriculum." Science Education 58 (No. 4, 1974), 544-559.

Lovell, Kenneth. "A Follow-up Study of Inhelder and Piaget's The Growth of Logical Thinking." British Journal of Psychology 52 (No. 2, 1961), 143-153.

Lovell, Kenneth. "Developmental Process in Thought." The Journal of Experimental Education 37 (Fall, 1968), 14-21.

Lovell, Kenneth. An Introduction to Human Development. London: MacMillan and Co. Ltd., 1968a.

Lovell, Kenneth. "Proportionality and Probability." Piagetian Cognitive Development and Mathematics Education. Edited by Myron F. Rosskopf, Leslie P. Steffe, and Stanley Taback. Washington: National Council of Teachers of Mathematics, 1971.

Lunzer, E.A. The Regulation of Behavior. New York: American Elsevier Publishing Co., Inc., 1968.

Mecke, Gunter and Viola Mecke. "The Development of Formal Thought as Shown by Explanations of the Oscillations of a Pendulum: A Replication Study." Adolescence 6 (Summer, 1971), 219-228.

Mink, Oscar G. "Experience and Cognitive Structure." Journal of Research in Science Teaching 2 (No. 3, 1964), 196-203.

Nordland, Floyd H., Anton Lawson, and Jane B. Kahle. "A Study of Levels of Concrete and Formal Reasoning in

Disadvantaged Junior and Senior High School Science Students." Science Education 58 (No. 4, 1974), 569-575.

Phillips, John L. The Origins of Intellect: Piaget's Theory. San Francisco: W.H. Freeman and Company, 1969.

Piaget, Jean. The Psychology of Intelligence. London: Routledge and Kejan Paul Ltd., 1950.

Piaget, Jean. The Origins of Intelligence in Children. New York: International University Press, 1952.

Piaget, Jean. "Development and Learning." Journal of Research in Science Teaching 2 (No. 3, 1964), 176-186.

Piaget, Jean. The Principles of Genetic Epistemology. New York: Basic Books, Inc., 1972.

Piaget, Jean and Barbel Inhelder. The Growth of Logical Thinking from Childhood to Adolescence. New York: Basic Books Inc., 1958.

Renner, John and Don Stafford. Teaching Science in the Secondary School. New York: Harper and Row, Publishers, 1972.

Rosskopf, Myron J. "Mathematics and Cognitive Development in Children." Theory into Practice 10 (April, 1971), 124-128.

Rosskopf, Myron J. "Piagetian Research and the School Mathematics Program." The Arithmetic Teacher 19 (April, 1972), 309-314.

Sawada, Daiyo. "Piaget and Pedagogy: Fundamental Relationships." The Arithmetic Teacher 19 (April, 1972), 293-301.

Shulman, Lee S. "Psychology and Education." Mathematics Education. The Sixty-ninth Yearbook of the National Society for the Study of Education. Chicago: University of Chicago Press, 1970.

Taba, Hilda. Curriculum Development: Theory and Practice. New York: Harcourt, Brace, and World, Inc., 1962.

Tower, T.O. and G. Wheatley. "Conservation Concepts in College Students: A Replication and Critique." The Journal of Genetic Psychology 118 (No. 2, 1971), 265-270.

Weaver, J. Fred. "Some Concerns about the Applications of Piaget's Theory and Research to Mathematical Learning

and Instruction." The Arithmetic Teacher 19 (April, 1972), 263-268.

Youdin, Lee William. "Formal Thought in Adolescence as a Function of Intelligence." Child Development 37 (Sept., 1966), 697-708.

APPENDICES

Appendix A

Instructions to Colleagues re Activities for Classifying Students

Task 1.

Material: 2 pieces of clay of different colors.

These pieces of clay are rolled into balls of equal size.

Question to the student: Do these balls contain the same amount?

If the response to the question is negative, ask the student to make each ball contain the same amount. When the student agrees that each ball contains the same amount, distort one ball.

Question to the student: Does the distorted ball or the non-distorted ball contain the more or do they both contain the same amount?

Response: _____ (Record response on response sheet.)

Task 2.

Material: 2 pieces of clay of different colors.

The student is again presented with the two balls of clay as in Task 1. Give them to the student.

Question to the student: Do both balls weigh the same?

If the response is negative, ask the student to work with the balls until he agrees that they weigh the same.

When he agrees, take them back. Distort one ball. (Do not allow the student to touch the balls of clay after they have been distorted.)

Question to the student: Does the distorted ball weigh more than the non-distorted one, or does the non-distorted ball weigh more than the distorted one, or do they both weigh the same?

Response: _____ (Record response on response sheet.)

Task 3.

Materials: 2 pieces of clay from previous task, one distorted and one not distorted, 2 identical beakers of water.

The two beakers are presented to the student containing the same amount of water.

Question to the student: Do the two beakers contain the same amount of water?

If the response is negative, allow the student to adjust the levels of water until he agrees that each beaker contains the same volume of water.

Question to the student: Will the distorted ball of clay push the level of the water up more, will the non-distorted ball of clay push the level up more, or will the two pieces of clay push the levels up the same?

Response: _____ (Record response on response sheet.)

Task 4.

Material: A small stand, string, 3 different-sized weights.

Instructions to the students: Do as many experiments as you need so that you may determine, using different lengths of string and different weights, what makes the pendulum go slow or fast. (Demonstrate the swinging of the pendulum.) The things which you may vary are the length of the string (demonstrate), the weight of the object fastened to the string (demonstrate), the height of the dropping point (demonstrate), and the force of the push which you give the pendulum (demonstrate).

Observations: (Record your observations on the response sheet.)

Is the student able to disassociate the impetus which he gives the pendulum from the motion which is independent of his action? _____

Does the student order the lengths of the pendulum? _____

Does the student order the weights? _____

Is the student able to determine that the height at which the pendulum bob is released has no effect upon the rate of the pendulum? _____

Does the student vary more than one factor at a time? _____

Does the student approach the problem systematically? _____

Does the student isolate variables by varying one factor while holding all others constant? _____

Is the student able to isolate the pertinent variable?

Other observations: _____

Task 5.

Material: 4 similar containers holding 1) diluted sulphuric acid, 2) water, 3) hydrogen peroxide, and 4) thiosulphate, a smaller container, labeled g, containing potassium iodide, a beaker, droppers, and test tubes.

Before the student has arrived, put 1) diluted sulphuric acid, and 3) hydrogen peroxide into a test tube. Make sure that this is prepared so that the student does not observe the liquids which are mixed.

Show the student the mixture. While the student is watching, add several drops of g (potassium iodide) to the test tube, using an eye dropper.

Instructions to the student: Using the liquids in the containers 1), 2), 3), 4), and g, see if you can reproduce the same color yellow.

If the student finds the correct combination quickly, ask him the following question: Are there other combinations which produce this same color? Thus the investigator will be able to determine his knowledge of combinations.

Observations: (Record your observations on the response sheet.)

Is the student able to consider the four combinations 1 and g, 2 and g, 3 and g, and 4 and g? _____

Does the student only consider these four combinations?

Is there an attempt made at various combinations, but only by trial and error and even then the student is unable to produce the various combinations? _____

Is the student able to produce the various n-by-n combinations, but somewhat unsystematically? _____

Is the student able to produce the various n-by-n combinations systematically? _____

Is the student able to find the correct combination? _____

Other observations: _____

Appendix B

Student Response Sheet

Student's Name: _____ Date of Birth: _____

Age: Yrs.____ Mos. ____

Task 1.

Does the distorted ball or the non-distorted ball contain the more or do both contain the same amount?

_____

Task 2.

Does the distorted ball weigh more than the non-distorted one, or does the non-distorted ball weigh more than the distorted one, or do they both weigh the same? _____

Task 3.

Will the distorted ball of clay push the level of the water up more, will the non-distorted ball of clay push the level up more, or will the two amounts of clay push the levels up the same? _____

Task 4.

Is the student able to disassociate the impetus which he gives the pendulum from the motion which is independent of his action? _____

Does the student order the lengths of the pendulum?

Does the student order the weights?

Is the student able to determine that the height at which the pendulum bob is released has no effect upon the rate of the pendulum?

Does the student vary more than one factor at a time?

Does the student approach the problem systematically?

Does the student isolate variables by varying one factor while holding all others constant?

Is the student able to isolate the pertinent variable?

Other observations:

Task 5.

Is the student able to consider the four combinations 1 and g, 2 and g, 3 and g, and 4 and g?

Does the student only consider these four combinations? _____

Is there any attempt made at various combinations, but only by trial and error and even then the student is unable to produce the various combinations? _____

_____

Is the student able to produce the various n-by-n combinations, but somewhat unsystematically? _____

_____

Is the student able to produce the various n-by-n combinations systematically? _____

_____

Is the student able to find the correct combination?

_____

_____

Other observations:

Appendix C

Related Information

The following references were analyzed in preparing the Piagetian tasks for categorizing students:

Piaget, Jean and Barbel Inhelder. The Growth of Logical Thinking from Childhood to Addlescence. New York: Basic Books, Inc., 1958.

Renner, John and Don Stafford. Teaching Science in the Secondary School. New York: Harper and Row, Publishers, 1972.