THE FREEZING OF FISH FILLETS OF IRREGULAR GEOMETRY WITH APPLICATION TO COMMERCIAL PRACTICE

ROBERT JOHN GILL

INFORMATION TO USERS

THIS DISSERTATION HAS BEEN MICROFILMED EXACTLY AS RECEIVED

This copy was produced from a microfiche copy of the original document. The quality of the copy is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

PLEASE NOTE: Some pages may have indistinct print. Filmed as received.

Canadian Theses Division
Cataloguing Branch
National Library of Canada
Ottawa, Canada K1A 0N4

AVIS AUX USAGERS

LA THESE A ETE MICROFILMEE TELLE QUE NOUS L'AVONS RECUE

Cette copie a été faite à partir d'une microfiche du document original. La qualité de la copie dépend grandement de la qualité de la thèse soumise pour le microfimage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

NOTA BENE: La qualité d'impression de certaines pages peut laisser à désirer. Microfilmée telle que nous l'avons reçue.

Division des thèses canadiennes
Direction du catalogage
Bibliothèque nationale du Canada
Ottawa, Canada K1A 0N4

THE FREEZING OF FISH FILLETS OF IRREGULAR GEOMETRY WITH APPLICATION TO COMMERCIAL PRACTICE

by

© Robert John Gill, B.Sc.

A Thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Engineering

Department of Engineering and Applied Science
Memorial University of Newfoundland

March 1976

St. John's Newfoundland

ABSTRACT

Experiments conducted on a rectangular slab of fish muscle with heat transfer through one surface result in freezing times which are adequately predicted by the Nagaoka, et al. modification to the Plank solution for the freezing time of an infinite slab. Neumann's exact solution to the Fourier heat conduction equation results in freezing times which progressively overestimate experimental times with increasing Biot number. Using the Nagaoka, et al. formulation, formulae are obtained for the freezing time, in terms of weight, of irregularly shaped commercial fish fillets. With the aid of such relationships, an analysis is made of a commercial freezing process with a view to optimizing the production rate.

ACKNOWLEDGEMENTS

The author would like to express his thanks to the following people:

1. Dr. G.R. Peters for his advice and assistance throughout his supervision of this study.
2. Technicians D. Tilley and T. Laaning for their ready assistance during the experimental stages.
3. Mrs. D. Strange for typing the thesis.
4. The management of National Sea Products of St. John's for their time and cooperation.

TABLE OF CONTENTS

CHAPTER PAGE

LIST OF TABLES

LIST OF FIGURES

FIGURE NO.		PAGE

CHAPTER I

INTRODUCTION

From early times, man has preserved foodstuffs by freezing naturally in areas which have a sufficiently low ambient temperature. During the latter half of the nineteenth century, mechanical refrigeration was developed with the subsequent use of "sharp freezing," actually a relatively slow process in which food is frozen in cold stores through natural air convection. Freezing of fruits began in the early part of this century and of vegetables about 1930, by which time it had been found that "quick freezing" led to a sizeable improvement in quality (1).[1] In recent years, extremely fast freezing rates have been obtained with the use of cryogenics such as liquid nitrogen and carbon dioxide.

Although an increased freezing rate leads to a better quality product within certain limits, it is rather difficult to determine an optimum rate of freezing. Liquid nitrogen may result in a short freezing time, but the economics of this method of freezing may render it unfeasible. In addition, rapid freezing rates may have an adverse effect on the quality of the product due to rupturing caused by temperature-induced

---

[1]Numbers in parenthesis refer to references at end of thesis.

mechanical stresses. The final judgment of quality comes in the eating of the product and, in this regard, it has been determined that no noticeable improvement is gained in eating quality beyond a certain freezing rate (2). It is, however, possible to detect changes in the visual quality of the product over a broader range of freezing rates, and features such as texture and appearance will certainly affect the market value of the commodity. Longer freezing times, for example, result in larger ice crystals and a coarse appearance, giving the impression of freezing damage and poor quality.

For the purpose of freezing, food may be regarded as a complex combination of organic and inorganic substances, often in solution in a large quantity of water, contained within a basic structure of microscopic cells (3). The greater portion of the water is frozen out of solution in most foodstuffs at -5°C, although it has been shown that a significant fraction remains as bound water in the liquid state at temperatures as low as -40°C (11 per cent in the case of haddock muscle) (4). Sussman has measured non-frozen water in cod (81 per cent moisture content) using nuclear magnetic resonance. At -20°C, he found about 8 per cent water unfrozen and suggested that all water is frozen at -67°C (5).

A typical temperature-time curve for a slab of fish muscle is shown in Figure 1. Unlike a pure substance, the inorganic salt solution has no single freezing temperature

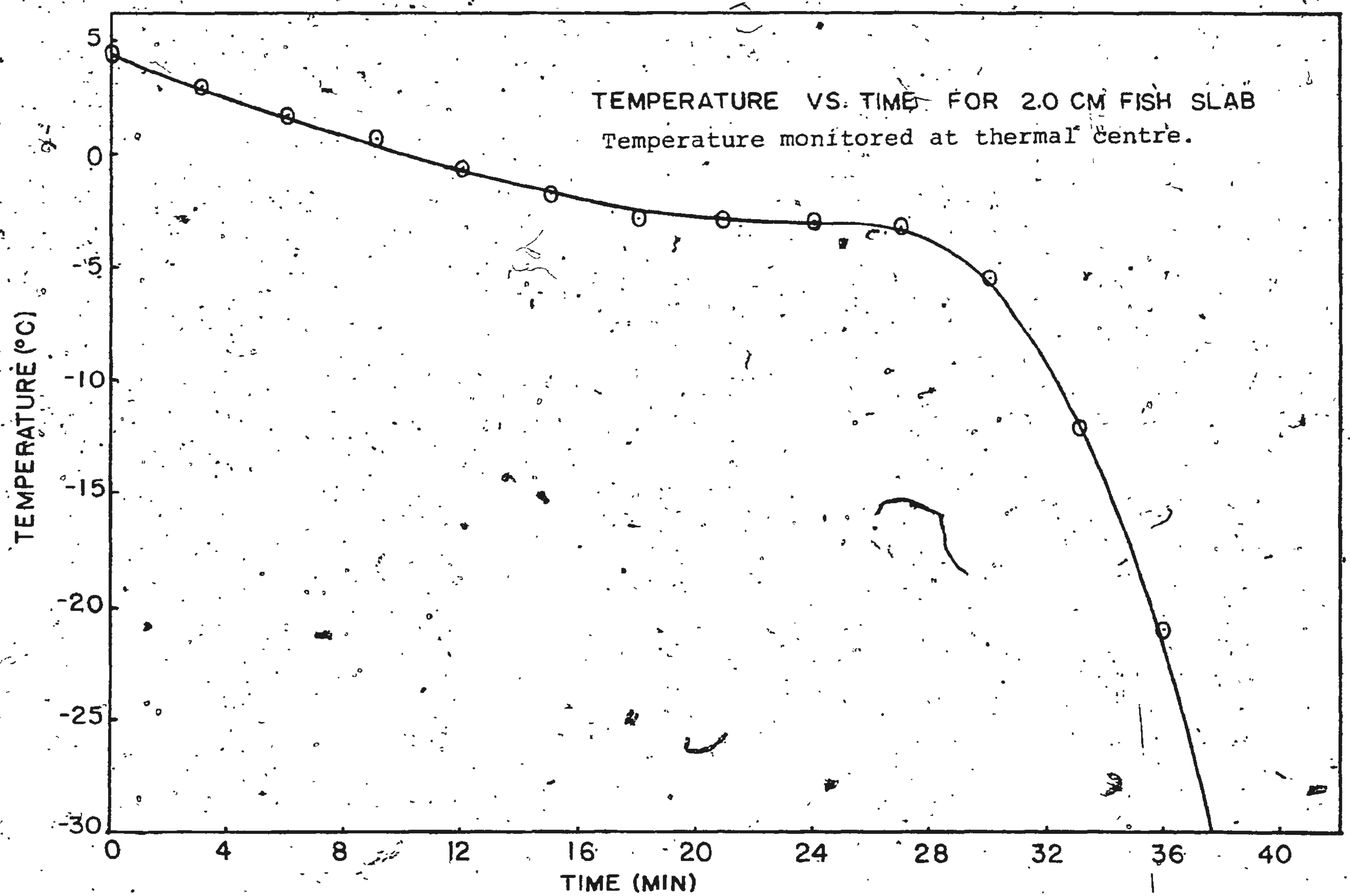

Figure 1. Freezing curve of fish slab. Medium temperature = -39°C.

but instead a range of temperatures over which most of the water is frozen. The temperature zone 0^{o} to $-5^{o}C$, which corresponds with the removal of the major part of the latent heat of fusion, is often called the "thermal arrest zone." As the temperature of the product passes through this zone, the inorganic salt solution progressively separates into crystals of pure water ice and a more concentrated solution. The size and location of the ice crystals is determined by the rate of freezing, small crystals being representative of a quickly frozen product, large crystals, the result of slower freezing. The time required for the thermal centre (i.e. the location which cools the slowest) to pass through this "zone of maximum crystal formation" may be regarded as the freezing time of the product. It has been suggested that a body may be regarded as being "quick-frozen" if this period is two hours or less (6). A second, broader definition of freezing time, known as the "effective freezing time," is that time which is required to reduce the temperature of the thermal centre from an initial, prescribed temperature, through the thermal arrest zone, to a final temperature (7). The latter definition of freezing time is more accommodating than the former as it includes initial and final temperatures which are outside the thermal arrest zone, a situation which usually occurs in the practical case. For this reason, the "effective freezing time" shall be used exclusively throughout this work.

Jason and Jowitt (8) state that it is desirable to bring the product through the thermal arrest zone in the shortest possible time as it is usually within this pronounced change-of-phase region that the quality of the product is most susceptible to changes in freezing rate. They argue that the rates of many irreversible spoilage reactions vary with the salt concentrations as well as with the reaction rate constants. With a decrease in temperature, the former increase while the latter decrease. The combination of these two factors leads to a peak reaction rate which usually occurs in the thermal arrest zone.

A further factor necessitating a short freezing time is that "drip losses," due to dissolution of the solid cellular food material, upon thawing of the product, increase with increasing freezing time. Such losses may amount to as much as 10 per cent of the original weight of the product (8).

It is very difficult mathematically to accurately predict the freezing times of foods due to the temperature dependence of the thermodynamic properties and the lack of a distinct phase change temperature. Since the concentration of the components in the water-ice mixture changes as freezing progresses, a gradual depression of the freezing point temperature results (as mentioned in the discussion of Figure 1), thus necessitating the choosing of an average freezing temperature for calculation purposes. In addition, because the thermodynamic properties such as thermal conductivity, specific heat and density also vary with temperature, it is again necessary to choose average values of these properties for each of the frozen and thawed phases. For many practical purposes this may be sufficient. However, this temperature dependence of the thermodynamic properties means that an exact analytical solution to the general heat conduction equation is generally very complex and may be impossible to obtain for certain boundary conditions.

To accommodate such variation in properties, numerical methods have been used extensively. A number of authors have reported on the use of such methods (primarily using the finite difference technique) to determine the freezing times of food products usually in the form of regular shapes such as slabs, cylinders and spheres (9, 10, 11, 12). While most such solutions have been based on the assumptions that (i) the initial temperature is the same throughout the body to be frozen, (ii) the cooling medium temperature is constant, and (iii) the phase change occurs at a constant average temperature, some writers have dispensed with one or another of these assumptions. A complicated series solution to the freezing problem has been obtained by Westphal (13) assuming that the medium temperature is an arbitrary function of time. A few writers have developed solutions for the case in which the phase change occurs over a range of temperatures (14, 15, 16).

There is very little data, however, based on actual heat transfer experiments to validate any such solutions, be they analytical or numerical. The various assumptions imposed on the derivation of many of the theoretical formulae restrict their use. One alternative is to obtain semi-theoretical formulations which are based on a combination of experimental data and theoretical formulations. Such solutions may have fewer limitations since they are obtained with less dependence on various assumptions regarding heat transfer and thermodynamic properties.

One of the objectives of this work is to investigate such solutions to determine their applicability to freezing fish fillets as found in commercial practice. Freezing times are first determined experimentally for rectangular slabs of fish muscle, in contact through the lower surface with a refrigerated plate, and are compared with computed times from available formulae.

In the industrial situation where irregularly-shaped fish fillets are often frozen as individual units prior to packaging (known commonly as IQF, or Individually Quick Frozen), it is desirable to be able to predict the freezing time in terms of the weight of the product as this is the most easily measured parameter. To this end, empirical relations expressing freezing time in terms of weight are developed for two species of fish, and an analysis is made of the production of IQF products using such relationships. In a similar manner, such an approach could be extended to other fish species or to other foodstuffs that are individually frozen. Such relationships have obvious merit not only in determining the rate of operation of the freezing process, but also in determining the optimum throughput of product in freezer systems.

CHAPTER 2

BACKGROUND THEORY AND LITERATURE REVIEW

2.1. Neumann's Solution

The general Fourier heat conduction equation may be written as:

$$\rho s \frac{\partial T}{\partial \theta} = \operatorname{div}(k \operatorname{grad} T).$$

For heat flow in one dimension this equation becomes:

$$\frac{\partial T}{\partial \theta} = \alpha \frac{\partial^2 T}{\partial x^2}. \qquad 2.1.1$$

where, T is temperature,

θ is time,

x is distance measured from the "thermal centre",

α is diffusivity,

and $\alpha = \dfrac{k}{\rho s}$,

where, k is thermal conductivity,

ρ is density,

s is specific heat.

With the assumption that k, ρ and s are temperature independent, Neumann (17) has derived an exact, analytical solution for the case of change of phase in a semi-infinite slab subject to the following conditions:

1. If L_f is the latent heat of fusion and the solid-liquid interface is at $X(\theta)$, then $T_1 = T_2 = T_f$ at $x = X(\theta)$,

where subscripts 1 and 2 refer to the liquid (thawed) and solid (frozen) phases respectively, and T_f is the phase change temperature.

2. If region $x > X(\theta)$ contains liquid at temperature T_1 and $x < X(\theta)$ contains solid at temperature T_2, then when the solid-liquid interface moves a distance dX, a quantity of heat $L_f \rho dX$ per unit area will be released and must be removed by conduction. This may be written as:

$$k_2 \frac{\partial T_2}{\partial x} - k_1 \frac{\partial T_1}{\partial x} = L_f \rho \frac{dX}{d\theta}$$

where, L_f is the latent heat of fusion.

3. $T_1 \rightarrow T_i$ as $x \rightarrow \infty$ where T_i is the initial constant temperature of the liquid phase.

4. $T_2 = 0$ at $x = 0$.

Condition 4 implies that the temperature of the solid or frozen phase attains the temperature of the cooling medium (i.e. zero degrees) immediately upon contact; in other words, the cooling medium surface heat transfer coefficient is infinite. Allowance may be made for a medium temperature other than zero degrees, and a correction factor may be considered for a finite heat transfer coefficient.

When equation 2.1.1 is solved subject to conditions 1-4, the temperatures in the thawed and frozen regions are given by the following equations:

$$T_1 = T_i - \frac{T_i - T_f}{\operatorname{erfc} \lambda (\alpha_2/\alpha_1)} \operatorname{erfc} \frac{x}{2(\alpha_1 \theta)^{\frac{1}{2}}} \qquad 2.1.2$$

$$T_2 = \frac{T_f}{\text{erf}\lambda} \text{erf} \frac{x}{2(\alpha_2\theta)^{\frac{1}{2}}} \qquad 2.1.3$$

where λ is a numerical constant determined by trial and error from

$$\frac{e^{-\lambda^2}}{\text{erf}\lambda} - (\alpha_2/\alpha_1)^{\frac{1}{2}} \frac{k_1(T_i-T_f)e^{-\frac{\alpha_2}{\alpha_1}\lambda^2}}{k_2\ T_f\ \text{erfc}\lambda(\alpha_2/\alpha_1)^{\frac{1}{2}}} = \frac{\lambda\Pi^{\frac{1}{2}}L_f}{s_2T_f}. \qquad 2.1.4$$

As can be seen from equations 2.1.2 and 2.1.3, there is a second power dependence of θ, the freezing time, upon the distance from the thermal centre, or more clearly, upon the thickness of the product.

The chief deficiency in this solution is that of the assumption of an infinite surface heat transfer coefficient. In most applications of food freezing, this will not be the case, as for example with air blast and contact plate freezers. The approximation to an infinite heat transfer coefficient may be valid in the case of immersion freezing of foods in refrigerant sprays at low temperatures.

Charm (18) has proposed a correction factor to be used when the heat transfer coefficient is finite. He suggests adding an equivalent thickness, t_e, of unfrozen "starting" material to the normal thickness 't'. The equivalent thickness is given as

$t_e = \frac{k_1}{h}$ where 'k_1' is the thermal conductivity of the unfrozen material, and 'h' is the freezing medium surface heat transfer coefficient.

Hence the fictional thickness is

$$t' = \frac{k_1}{h} + t$$

and the time to freeze a semi-infinite slab of thickness t, having a surface heat transfer coefficient h would be equal to the time to freeze a semi-infinite slab of thickness t' with an infinite heat transfer coefficient. However, according to Cowell (19), the use of this modified thickness results in overestimated freezing times when the Biot number ($Bi = \frac{ht}{k}$) is less than one. This matter is discussed again in Chapter 3 of this work.

2.2. Plank's Solution

The problem of the freezing of an infinite slab (linear flow) was solved by Plank (20) by taking an energy balance over the slab. Two assumptions were made in the derivation: (i) the temperature of the material to be frozen was initially at the freezing point throughout, and (ii) the heat transfer from a freezing front in the sample to the cooling medium was in the steady state. The solution made no attempt to deal with pre-freezing and sub-freezing (tempering) zones.

Plank's formula may be applied in situations in which the initial and final temperatures are close to the freezing temperature; that is, essentially in the thermal arrest zone. In following sections, modifications to this basic formula shall be considered; such modifications enable the solution to be used over a broader range of temperatures. It is a simple matter to derive Plank's equation for the case of an infinite slab which is insulated on its top surface and exposed to a cooling medium on its lower surface as shown in Figure 2.

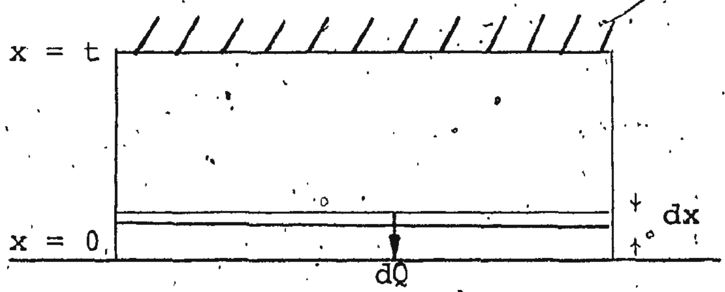

Figure 2. Heat transfer in one direction from an infinite slab.

The initial temperature is the same throughout the slab and is equal to the phase change temperature. Only latent heat is to be extracted from the slab since its final temperature at the end of the process will also be equal to the phase change temperature. As film 'dx' freezes, an amount of heat dQ is liberated, and the solid-liquid interface progresses upwards from plane $x = 0$ to plane $x = t$.

The quantity of heat removed from dx is given by

$$dQ = - L_f \rho A dx \qquad 2.2.1$$

where L_f and ρ are as defined in Section 2.1,

and A is the lower surface area of the film.

This amount of heat must be transferred through the frozen part and to the outside medium. This transfer is governed by Fourier's law:

$$dQ = -UA\Delta T \, d\theta \qquad 2.2.2$$

where $\Delta T = T_f - T_m$,

T_f is phase change temperature,

T_m is cooling medium temperature,

and U is the overall conductance of the frozen material of thickness 'x' and conductivity 'k' and of the surface conductance which includes the surface heat transfer coefficient 'h',

$$U = \frac{1}{\frac{1}{h} + \frac{x}{k}} \qquad 2.2.3$$

Combining equations 2.2.2 and 2.2.3:

$$dQ = -\frac{A(T_f - T_m)\,d\theta}{\frac{1}{h} + \frac{x}{k}} \qquad 2.2.4$$

Combining 2.2.1 and 2.2.4:

$$d\theta = \frac{L_f \rho}{(T_f - T_m)}\left(\frac{1}{h} + \frac{x}{k}\right)dx.$$

Upon integration over θ and x, the expression for freezing time becomes:

$$\theta = \frac{L_f \rho}{(T_f - T_m)}\left(\frac{t}{h} + \frac{t^2}{2k}\right). \qquad 2.2.5$$

2.3. Modifications to Plank's Solution

2.3.1. Modification of Nagaoka, Hotani, and Takagi

Nagaoka, et al. (21) have modified Plank's basic formula to include an initial temperature above, and a final temperature below, the phase change temperature by considering the total heat to be removed instead of only the latent heat of fusion. Thus, when replacing L_f in Plank's equation by Z, the total of sensible and latent heat, the resulting time, θ, will denote the total time required for the material to cover the entire temperature range. However, the result will be slightly inaccurate for the following reasons (22):

(i) The thermal conductivity is lower in the thawed state than in the frozen state (see Figure 3a, Chapter 3).

(ii) During the sensible cooling period prior to freezing, the density is slightly higher than it is for the frozen state (see Figure 3b, Chapter 3).

(iii) The mean temperature difference between the thawed state and the medium is greater than the difference between the phase change temperature and the medium temperature.

To a certain extent, these errors tend to cancel each other as some of them enter into the numerator and others into the denominator of equation 2.2.5. The additional error introduced by considering the tempering zone is probably rather insignificant as a vast percentage of the water, as established earlier, has frozen out of the material at temperatures close to the phase change temperature.

To accommodate these errors, Nagaoka, et al. introduced a correction factor which is incorporated into equation 2.2.5. The correction factor 'E' varies with the temperature difference in the pre-freezing zone $(T_1 - T_f)$ and is given by (22):

$$E = 1 + 0.0080\ (T_1 - T_f) \text{ for Celsius temperatures,}$$

$$E = 1 + 0.00445\ (T_1 - T_f) \text{ for Fahrenheit temperatures.}$$

Rewriting equation 2.2.5 with this modification gives:

$$\theta = \frac{EZ\rho}{(T_f - T_m)} \left(\frac{t}{h} + \frac{t^2}{2k}\right) \qquad 2.3.1$$

2.3.2. Mott's Modification

A modification to Plank's formula (equation 2.2.5) by Mott (23) has been developed through the dimensional analysis of experimental data on freezing time. In this procedure, a functional relationship among three dimensionless groups is utilized for the calculation of freezing time. The relationship is written as:

$$S = \frac{B + 1}{G} \qquad 2.3.2$$

where S is a shape factor given by

$$S = A(t/V)$$

$$B = \frac{ht}{2k}$$

$$G = \frac{\theta h (T_f - T_m)}{\rho H t} \qquad 2.3.3$$

where A is area,

V is volume,

t is thickness for an infinite slab

(t = radius for a cylinder and sphere),

H is an enthalpy difference based on some datum.

All other symbols have their usual meaning.

The two extreme values for the shape factor, as given by Mott, are S = 1 for an infinite slab since $\frac{A}{V} = \frac{1}{t}$ and S = 3 for a sphere $(\frac{A}{V} = \frac{3}{r})$.

It is a simple matter to show that equation 2.3.2 is basically equation 2.5.2 with the latent heat term replaced by the enthalpy term, or equation 2.3.1 without the correction factor and modified slightly by the insertion of the enthalpy difference term.

Equation 2.3.2 rearranged becomes:

$$G = \frac{B + 1}{S} = \frac{\frac{ht}{2k} + 1}{S} \qquad 2.3.4$$

Also, equation 2.3.3 rearranged becomes:

$$\theta = \frac{G \rho H t}{h(T_f - T_m)} \qquad 2.3.5$$

Combining equations 2.3.4 and 2.3.5,

$$\theta = \frac{(\frac{ht}{2k} + 1)\rho H t}{S\, h(T_f - T_m)}$$

or,

$$\theta = \frac{(\frac{1}{h} + \frac{t}{2k})\rho H t}{S(T_f - T_m)} .$$

For an infinite slab, $S = 1$

and $$\theta = \frac{\rho H}{(T_f - T_m)} \left(\frac{t}{h} + \frac{t^2}{2k}\right).$$

The enthalpy difference 'H' has simply replaced the latent heat term L_f in Plank's basic expression.

From Bakal's discussion (24) of Mott's method, one would infer that the relationship among the dimensionless groups in equation 2.3.2 represents a linear dependence only of freezing time upon thickness. However, it is obvious that the dependence is also upon the second power of thickness.

CHAPTER 3

PRELIMINARY EXPERIMENTS WITH RECTANGULAR SLABS OF FISH MUSCLE

3.1. Introduction

An attempt to experimentally validate the analyses of Chapter 2 should be made so that the feasibility of such solutions might be determined for application in practical situations. There is a dearth of supportive heat transfer data for such solutions and as a result, there is confusion regarding the validity of these formulations (25, 26).

Of the three thermodynamic properties--thermal conductivity, density and specific heat--required for the calculation of freezing times, thermal conductivity is probably the least well investigated and known. The equation that has been used extensively to predict the thermal conductivity of frozen foods and of mixtures of two or more components is Eucken's adaptation of Maxwell's equation for the conductivity of a heterogeneous substance composed of small spheres of one substance dispersed in another (27):

$$k = k_c \left[\frac{1 - 2 \dfrac{k_c - k_d}{2k_c + k_d} b}{1 + \dfrac{k_c - k_d}{2k_c + k_d} b} \right]$$

where k is the conductivity of the mixture,

k_c is conductivity of the continuous phase,

k_d is conductivity of dispersed phase,

b is relative volume of the dispersed phase.

The equation assumes a low concentration of the dispersed phase so that heat flow through one particle is not affected by heat flow through another.

Long (3) has carried out thermal conductivity measurements of cod muscle and compared them with values computed from the Euchen equation. He found reasonable agreement between the experimental and theoretical values. Figure 3a shows the results of this work. The average conductivity of fish in the frozen state is about three and one half times that of the thawed state.

The change in density of cod muscle has also been studied by Long (3). From Figure 3b, it appears that the density decreases by about 10 per cent upon freezing, a small amount which would not be critical in most calculations of freezing time.

The average specific heats of foodstuffs in the thawed and frozen states have been found to be accurately predicted for most cases of food freezing (28) by the Siebel formula which expresses the specific heat in terms of the moisture content of the food. For the thawed state, the specific heat is given by (26) $S_1 = 0.008a + 0.20$, and for the frozen state, $S_2 = 0.003a + 0.20$ where 'a' is the

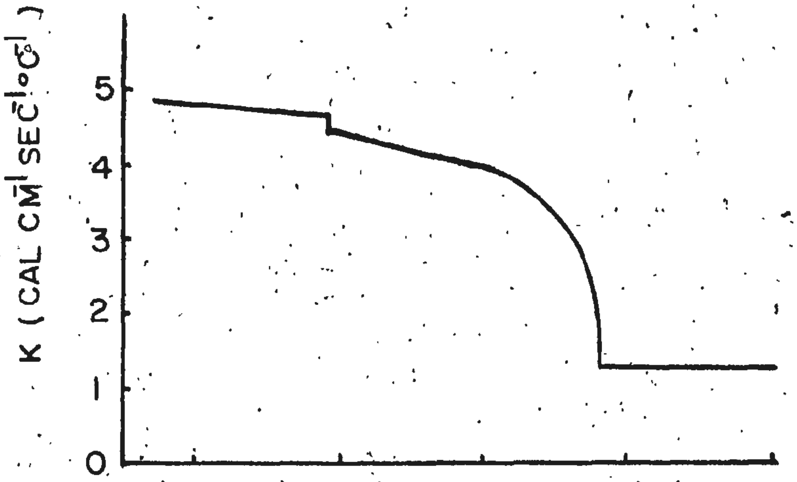

Figure 3a. Thermal conductivity vs. temperature (after Long, 1954).

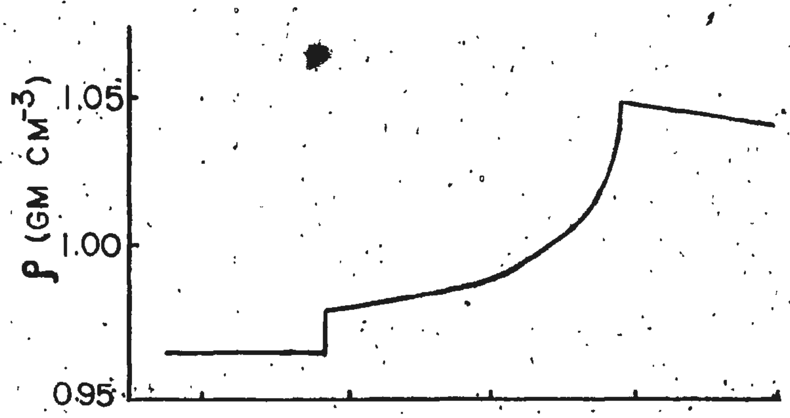

Figure 3b. Density vs. temperature (after Long, 1954).

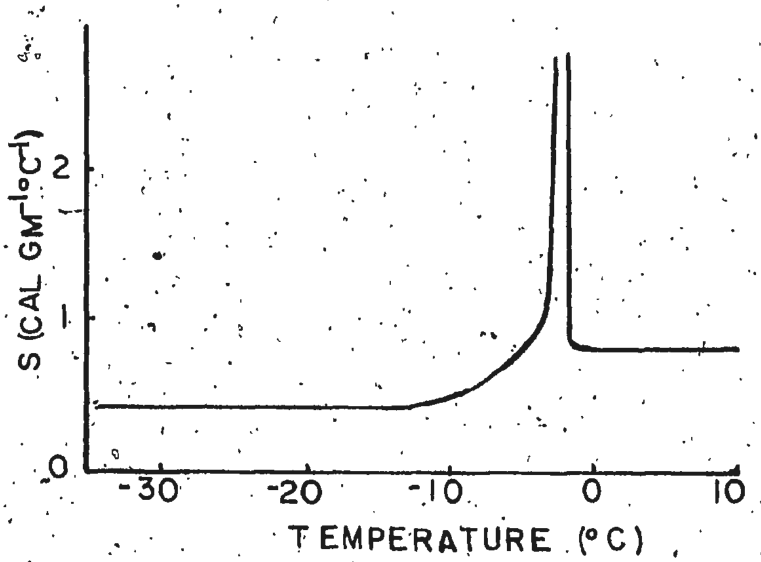

Figure 3c. Specific heat vs. temperature (after Jason and Jowitt, 1969).

moisture content in per cent and 0.20 is the specific heat of the solid constituents of the substance. Using the data of Riedel, Jason and Jowitt (8) have produced Figure 3c which shows the variation of specific heat with temperature. While the temperature is in the thermal arrest zone, the specific heat tends to infinity. In the pre-freezing and tempering zones, the specific heat shows only minor dependence on temperature.

A fourth property required to compute freezing time--the latent heat of fusion--is usually obtained by multiplying the latent heat of fusion of water by the moisture content of the food. This method has been shown to give reasonable results for the latent heats of foodstuffs in the thermal arrest zone (28).

In the marketplace, where a high standard of quality is imperative, the producer must be concerned with maintaining a high degree of predictability, control and economy. Traditional methods of using experienced operators to manually adjust process conditions based on visual observations are inadequate in view of the present complexities of food processing. However, before a rational approach can be taken towards such problems, valid relationships involving the parameters of a production process must be established. To this end, preliminary experiments are conducted on rectangular slabs of fish muscle to determine freezing times in terms of the thicknesses of the slabs. Later, work is carried out on irregularly shaped commercial fish fillets, examples of which are shown in Figures 11 and 12 of Chapter 4.

Experimental freezing times obtained for the rectangular slabs are compared with times computed from Neumann's and Nagaoka et al. solutions using thermodynamic properties obtained from the literature. Both solutions can accommodate pre-freezing and tempering zones, and with modification, Neumann's formula may be used for the case of a finite heat transfer coefficient. The problem of non-zero medium temperature may be easily surmounted by adding to each temperature involved the number of degrees by which the medium temperature is different from zero. A description of the measurement of the surface heat transfer coefficient is given in the following section.

3.2. Measurement of the Heat Transfer Coefficient of the Refrigerated Plate

3.2.1. Theoretical Development

In order to compute freezing time, the surface heat transfer coefficient of the plate must be determined. A method based on the approach of Ball and Olson (29) and used by others (25, 30) affords a determination of the heat transfer coefficient by considering transient heat conduction in an infinite solid such as an insulated rectangular block of ice which is in contact with the cooling medium.

The equation which enables one to determine the heat transfer coefficient is determined from the Fourier transient heat conduction equation (2.1.1) subject to the following boundary conditions for heat flow in one direction in an infinite slab (31):

(i) at $x = 0$, $\frac{\partial T}{\partial x} = 0$ (i.e., one surface insulated),

(ii) at $x = t$, $-(\frac{\partial T}{\partial x}) = \frac{h}{k}(T-T_m)$

where T_m is the medium temperature,

(iii) at $\theta = 0$, $T = T_0$ (initial condition) θ is time.

The solution to the Fourier equation subject to these conditions is:

$$\frac{T-T_m}{T_0-T_m} = \sum_{i=1}^{\infty} \frac{2 \sin\beta_i}{\beta_i + \sin\beta_i \cos\beta_i} \cos\beta_i \frac{x}{t} e^{-\frac{\beta_i^2}{t^2}\alpha\theta} \qquad 3.2.1$$

where β_i is obtained from the root equation

$Bi = \beta_i \tan\beta_i$ in which $Bi = \frac{ht}{k}$ is the Biot number. For high Fourier numbers ($F_0 = \frac{\alpha\theta}{t^2}$), the series converges rapidly, and a first term approximation may be written as:

$$\log\frac{T-T_m}{T_0-T_m} = -\frac{\beta_1^2\alpha\theta}{2.303t^2} + \log\left[\frac{2 \sin\beta_1}{\beta_1 + \sin\beta_1 \cos\beta_1} \cos\beta_1 \frac{x}{t}\right]$$

or,

$$\frac{T-T_m}{T_0-T_m} = -\frac{\theta}{f} + \log j$$

where $f = \frac{2.303}{\alpha\lambda_1^2}$ $\qquad$ 3.2.2

and j is the quantity in square brackets. In equation 3.2.2

$$\lambda_1 = \frac{\beta_1}{t}$$

If $\frac{T-T_m}{T_0-T_m}$ is plotted against θ on semi-log paper, 'f' is the reciprocal slope of the asymptote to the cooling curve, or the time required for the asymptote to traverse one log cycle. 'j' is the intercept value (or lag factor) which is obtained by extrapolating the asymptote to the $\theta = 0$ axis.

3.2.2. Experimental Description

The refrigeration system consisted of a horizontal plate (16" x 30" x 3/4") connected to a unit consisting of a compressor, receiver and air-cooled condenser. The refrigerant was dichlorodifluoromethane (Refrigerant 12). The plate was enclosed by a one-quarter inch plywood cover which rested on the mounting platform and provided an air space of approximately two inches above the plate. The inside of the cover was insulated with one inch polystyrene. A general view of the refrigeration system is shown in Figure 4.

A rectangular block of ice measuring 15.0 x 10.0 x 2.0 cm. was used in determining the heat transfer coefficient. This block was insulated around the edges with 6.0 cm. of polystyrene and on the top surface with 2.0 cm. of polystyrene which contacted the lower insulated surface of the cover over the refrigerated plate. Before the block was frozen, an iron-constantan thermocouple had been carefully positioned

Figure 4. Refrigeration equipment.

on the surface of the water in an aluminum tray. Once frozen, the block of ice was removed from the tray and transferred to the insulated frame. It was then allowed to reach an equilibrium temperature a few degrees below the freezing point in a refrigerator before it was transferred to the refrigerated plate. The ice surface temperature was then monitored on a digital output temperature recorder. This experiment was carried out three times and gave results which were in very good agreement.

It should be noted that it was impossible with the set-up to directly determine the temperature of the cooling medium, i.e., the refrigerant in the evaporator coils. However, as the steel plate on which the sample is placed is thin, it is to be expected that the surface temperature of the plate is very close to the temperature of the refrigerant, and it is this temperature that was monitored during the experiments and used as the cooling medium temperature.

3.2.3. Experimental Results and Calculations

A representative cooling curve of the block of ice is shown in Figure 5. From three plots of log $\frac{T-T_m}{T_0-T_m}$ against time, the mean value of 'f' was found to be 22.5 minutes. Table 1 summarizes the results of the three experiments and Figure 6 shows the log $\frac{T-T_m}{T_0-T_m}$ vs. θ plot for experiment 1.

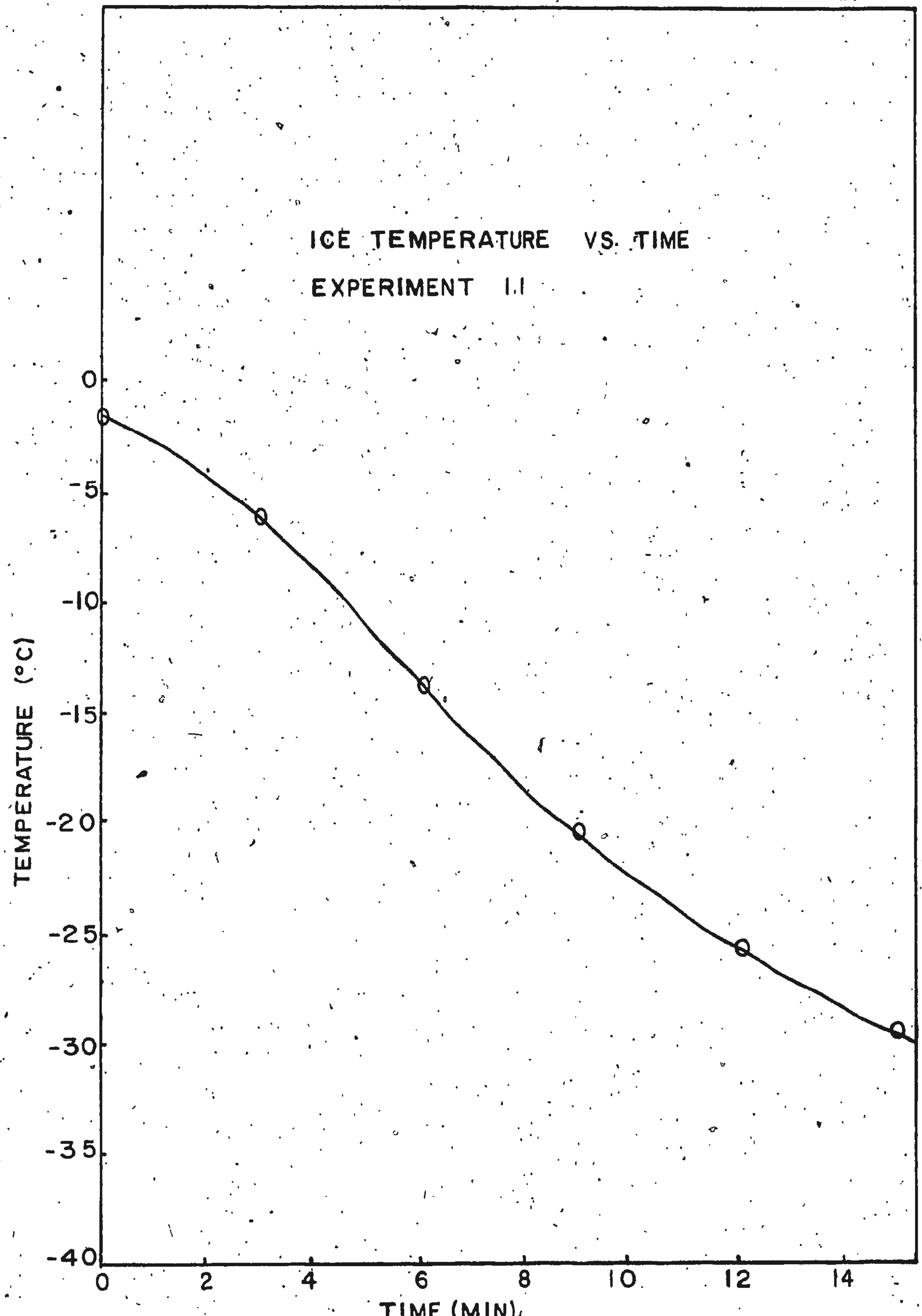

Figure 5. Cooling curve of ice slab (linear plot).

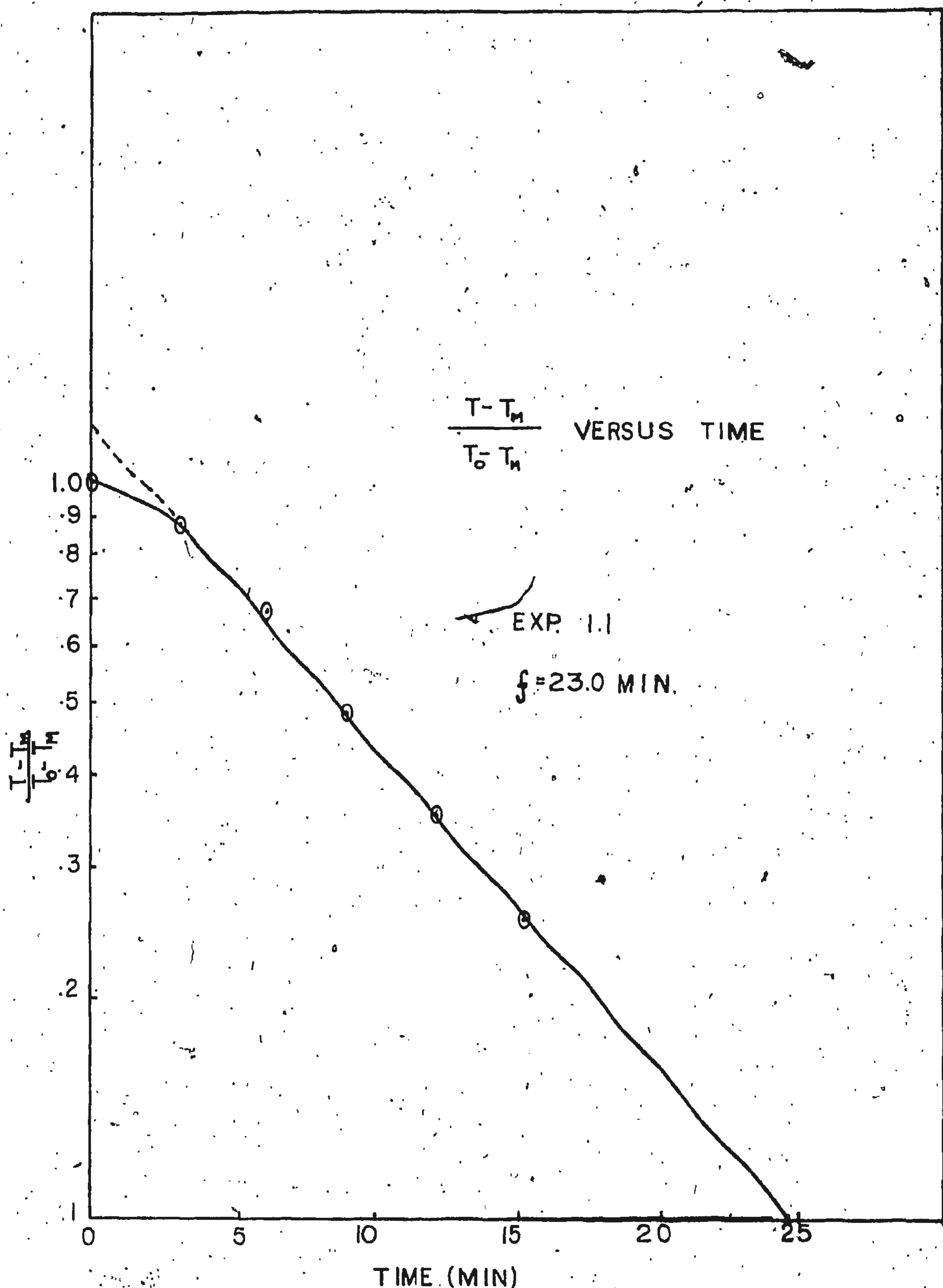

Figure 6. Cooling curve of ice slab (logarithmic plot).

TABLE 1

'f' VALUES FOR ICE SLAB

Experiment No.	Average Plate Temp. (°C)	f (Min.)
1.1	-39	23.0
1.2	-39	22.4
1.3	-39	22.2

By using equation 3.2.2, λ_1^2 may now be determined. The thermal diffusivity of ice may be found from the following average data of thermal conductivity, density and specific heat of ice (32):

$k = 4.8 \times 10^{-3}$ cal/cm-sec-°C,

$\rho = 0.92$ gm/cm^3,

$s = 0.49$ cal/gm-°C,

$\alpha = k/\rho s = 1.06 \times 10^{-2}$ cm^2/sec.

Therefore $\lambda_1^2 = 0.161$ cm^{-1}.

However, λ_1 is a function of h, the heat transfer coefficient, which is obtained by trial and error according to the following steps (25):

(i) Assume a value for 'h'.

(ii) Evaluate the Biot number ($Bi = \frac{ht}{k}$) for the ice sample.

(iii) Find β_1 corresponding to Bi from Figure 7.

(iv) Solve $\beta_1/t = \lambda_1$ and compare λ_1^2 thus obtained with the values obtained from equation 3.2.2.

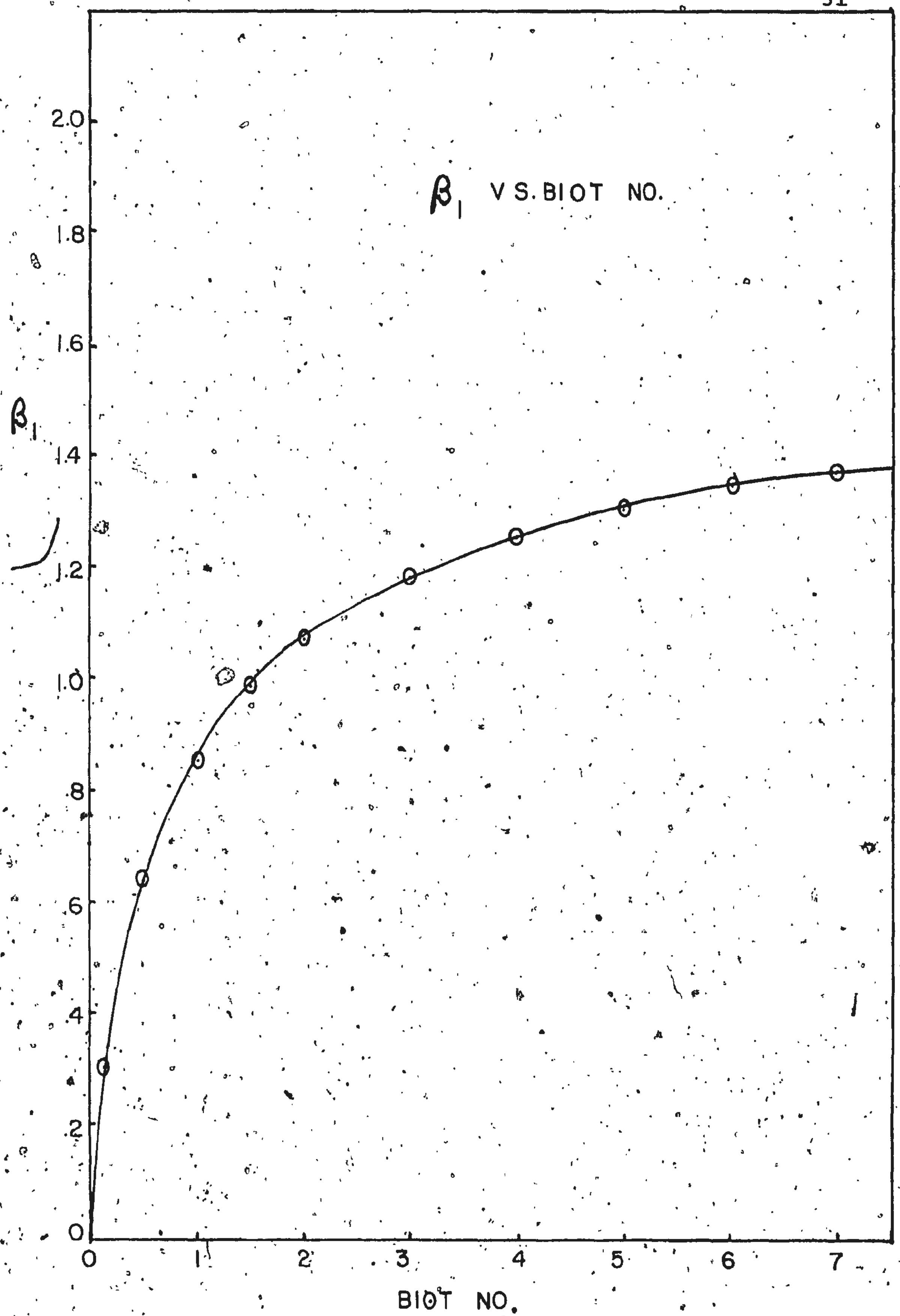

Figure 7. Graph to determine β_1.

(v) Repeat with other values of 'h' and plot values of 'h' against calculated values of λ_1^2 as shown in Figure 8.

From Figure 8, the heat transfer coefficient is found to be 2.04×10^{-3} cal/cm^2-sec-^{o}C. Values given in the literature for plate freezers are in the range 1.36 to 2.04×10^{-3} (26).

3.3. The Freezing Times of Fish Muscle Slabs

3.3.1. Experimental Description

The refrigeration system has been described in Section 3.2.2. A three-quarter inch plywood frame with inside dimensions of 10.0 x 6.0 x 4.0 cm. was insulated with 6.0 cm. of polystyrene; frame and insulation were glued to a sheet of 20-guage aluminum which rested on the refrigerated plate while a fish sample was being frozen. The aluminum prevented the fish slab from sticking to the plate while having no measurable effect on the heat transfer characteristics. The insulation around the periphery was intended to reduce heat flow through the edges to a very small amount. Thermocouples positioned around the periphery and at the geometric centre of the slab (all at the same depth) showed that the temperature was uniform over the slab throughout the freezing period.

The thermal centre was found to be at the top surface of the slab. An iron-constantan (Type J) thermocouple consisting of 0.005 inch diameter wires was fixed to the surface

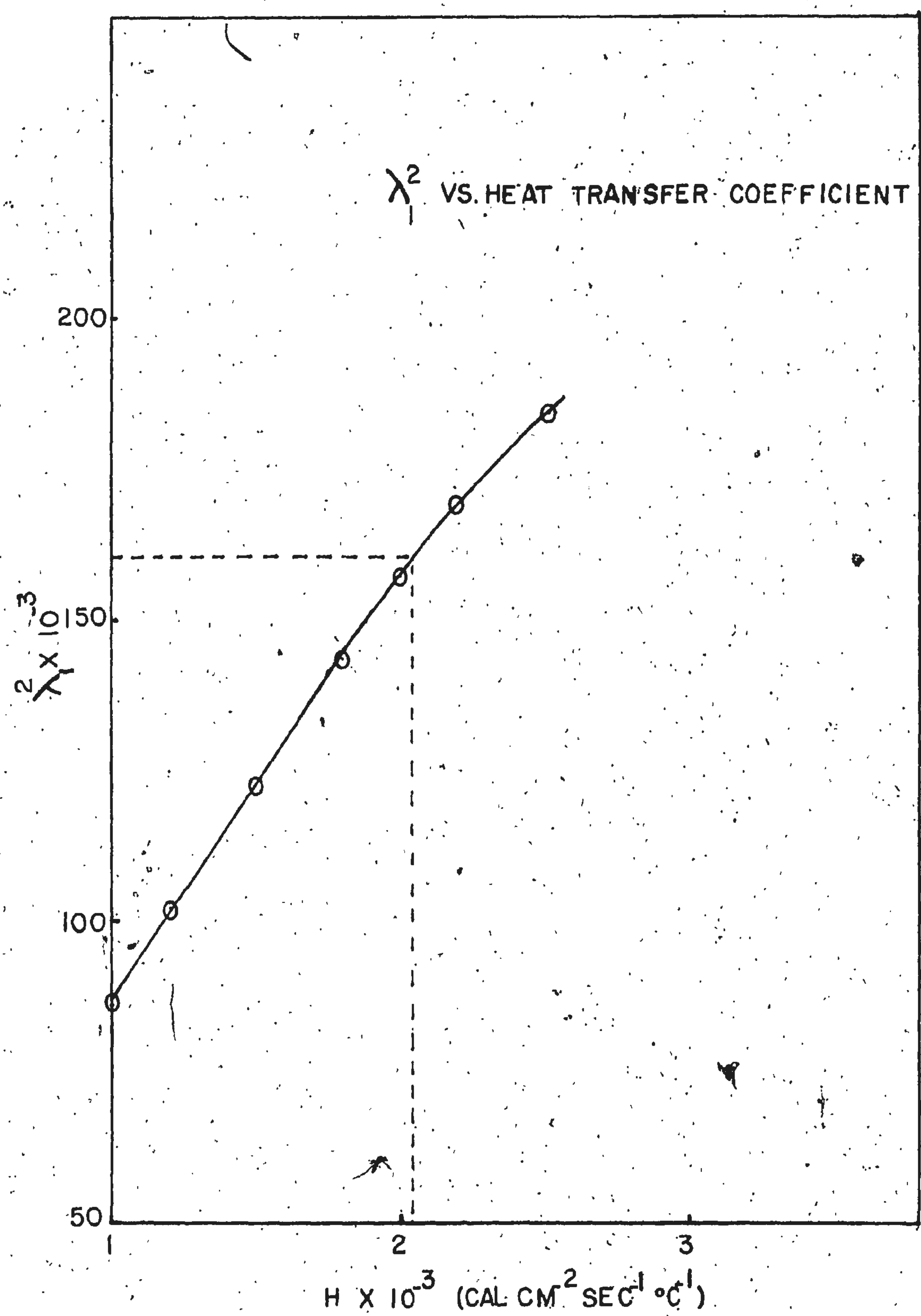

Figure 8. Graph to determine the heat transfer coefficient.

of the slab at its geometric centre, and the temperature was monitored as described in Section 3.2.2. For this experiment, the recorder was fitted with a manual selector to enable other temperatures (in other parts of the slab, on the plate, etc.) to be recorded as well.

The fish samples were cut from commercially frozen one-pound cod blocks using stainless steel frames as guides when cutting. These frames had inside dimensions of 10.0 cm. x 6.0 cm. with thicknesses ranging from 0.5 cm. to 4.0 cm. in 0.5 cm. increments. The samples were allowed to thaw in these frames after which each was transferred to the insulated wooden frame.

The thermocouple was then fixed to the surface of the slab and the latter was allowed to equilibrate to a temperature of 40°F (4.4°C) in a refrigerator, whereupon it was quickly transferred to the refrigerated plate. A stop watch was started and the temperature recorded at regular intervals until the temperature reached a few degrees below 0°F (-17.8°C). This procedure was repeated for each sample from 0.5 cm. to 4.0 cm. thickness (a total of eight samples). Graphs were plotted to determine the time for the temperature to decrease from 40°F (4.4°C) to 0°F (-17.8°C). The "effective freezing times" were then compared with times computed from Neumann's and Nagaoka et al's. formulations.

3.3.2. Computation of Freezing Time by Neumann's Formula

The temperature at the thermal centre of the fish slab is given by $T_2 = \frac{T_f}{\text{erf}\lambda} \text{ erf } \frac{t}{2(\alpha_2\theta)^{\frac{1}{2}}}$ 3.3.1

where T_f is the phase change temperature,

t is the thickness of the slab,

α_2 is the diffusivity of the frozen phase.

Using $t' = \frac{k_1}{h} + t$ to accommodate a finite heat transfer coefficient, the formula becomes

$$T_2 = \frac{T_f}{\text{erf}\lambda} \text{ erf } \frac{t'}{2(\alpha_2\theta)^{\frac{1}{2}}} \qquad 3.3.2$$

The following data are used in the computation of the freezing times of the slabs of cod muscle:

(i) experimental initial temperature: $T_1 = 40^{\circ}F = 4.4^{\circ}C$,

(ii) experimental final temperature: $T_2 = 0^{\circ}F = -17.8^{\circ}C$.

(iii) average phase change temperature (26): $T_f = 28^{\circ}F = -2.2^{\circ}C$,

(iv) latent heat of fusion (26): $L_f = 64$ cal/gm,

(v) experimental medium temperature: $T_m = -39^{\circ}C$,

(vi) experimental heat transfer coefficient:

$$h = 2.04 \times 10^{-3} \frac{\text{cal}}{\text{cm}^2\text{-sec-}^{\circ}\text{C}},$$

(vii) specific heat of thawed phase (8): $s_1 = 0.84 \frac{\text{cal}}{\text{gm-}^{\circ}\text{C}}$,

(viii) specific heat of frozen phase (8): $s_2 = 0.44 \frac{\text{cal}}{\text{gm-}^{\circ}\text{C}}$,

(ix) average conductivity of thawed phase (3):

$$k_1 = 1.3 \times 10^{-3} \frac{\text{cal}}{\text{cm-sec-}^{\circ}\text{C}},$$

(x) average conductivity of frozen phase (3):

$$k_2 = 4.2 \times 10^{-3} \text{ cal/cm-sec-}^{\circ}\text{C},$$

(xi) density of thawed phase (3): $\rho_1' = 1.05$ gm/cm3,

(xii) density of frozen phase (3): $\rho_2 = 0.98$ gm/cm3.

Since the medium temperature must be zero degrees, all temperatures are increased by 39 degrees.

λ may be determined by plotting the left and right-hand sides of equation 2.1.4 of Chapter 2 with the above data inserted. From Figure 9, $\lambda = 0.32$.

Equation 3.3.2 may now be written (with substitutions and evaluations) as:

$$\frac{\text{erf } t'}{0.208\theta^{1/2}} = 0.20$$

$$\theta = 11.9\, t'^2$$

$$= 11.9\,(t + \frac{k_1}{h})^2$$

$$= 11.9\,(t + 0.66)^2$$

where the effective freezing time, θ, is in minutes.

3.3.3. Computation of Freezing Times by Nagaoka et al's. Formula

Nagaoka et al's. modification to Plank's formula with the correction factor for temperatures on the Celsius scale is:

$$\theta = \left[1 + 0.0080(T_1 - T_f)\right] \cdot \frac{Z\rho}{(T_f - T_m)} \left(\frac{t}{h} + \frac{t^2}{2k}\right)$$

where Z is the total heat (sensible and latent) to be

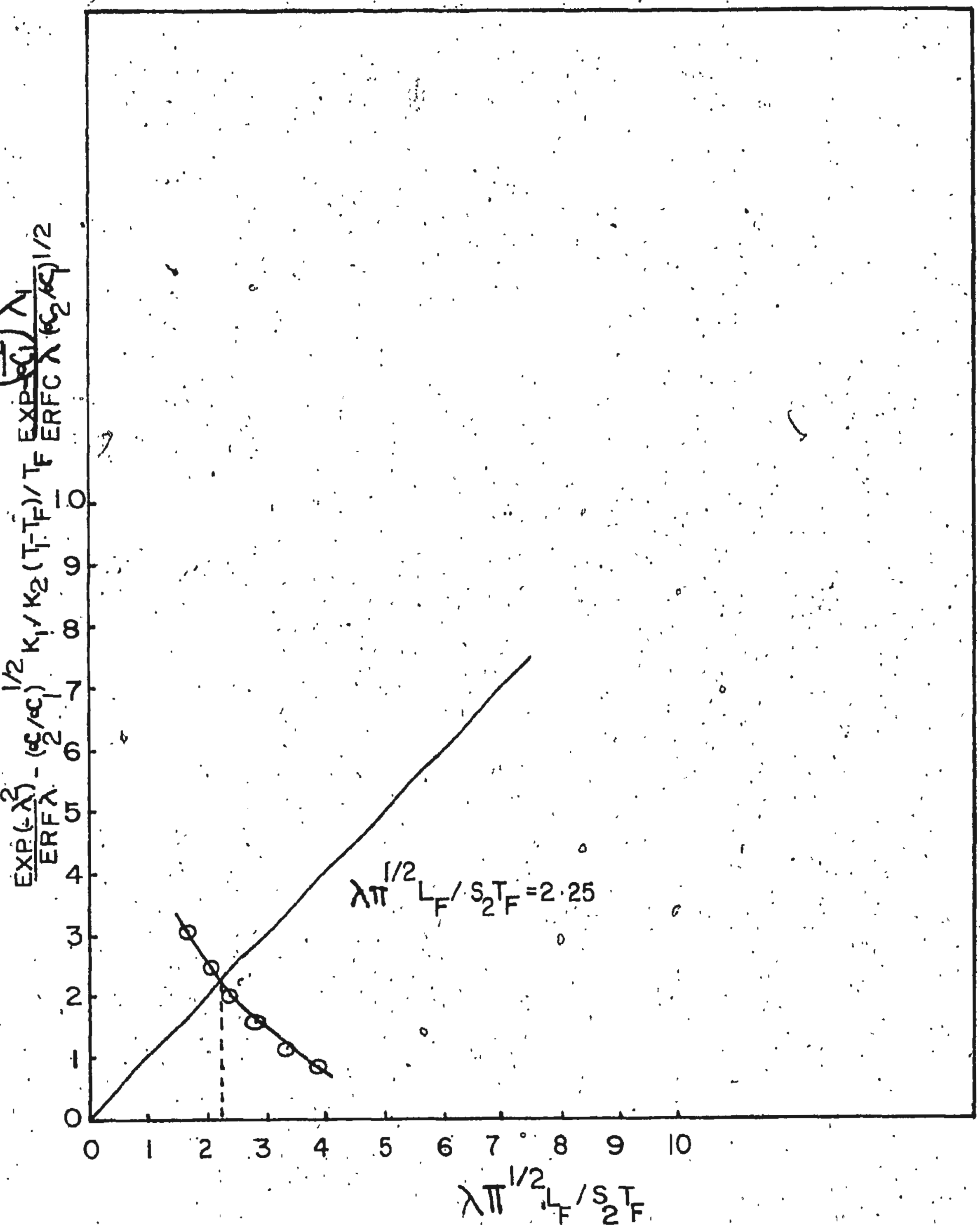

Figure 9. Graph to determine λ from equation 2.1.4.

removed from the slab to reduce the temperature from an initial to a final value,

i.e. $Z = s_1(T_1-T_f) + L_f + s_2(T_f-T_2)$.

Upon substitution of the data listed in Section 3.3.2, the formula may be reduced to the form:

$$\theta = 17.8\ t + 4.3\ t^2.$$

3.3.4. Discussion of Experimental and Computed Results

The results of experiments with slabs of cod muscle of eight different thicknesses from 0.5 cm. to 4.0 cm. are shown in Table 2 and a plot of freezing time against thickness is shown in Figure 10. For comparison purposes, computed freezing times from Neumann's and Nagaoka et al's. formulations are also plotted against slab thickness. The graphs clearly show the disparity between the results from Neumann's theory and those computed from Nagaoka et al's. modified formula. While the latter agree favourably with experimental freezing times, the former's overestimation of freezing times becomes progressively worse with increasing thickness. It follows that, in addition to overestimating freezing times for Biot numbers less than one as Cowell (19) mentioned, the Neumann solution, with the adaptation for a finite heat transfer coefficient, also overestimates even more markedly for increasing Biot numbers. It is clearly evident that the freezing time dependence on thickness is a

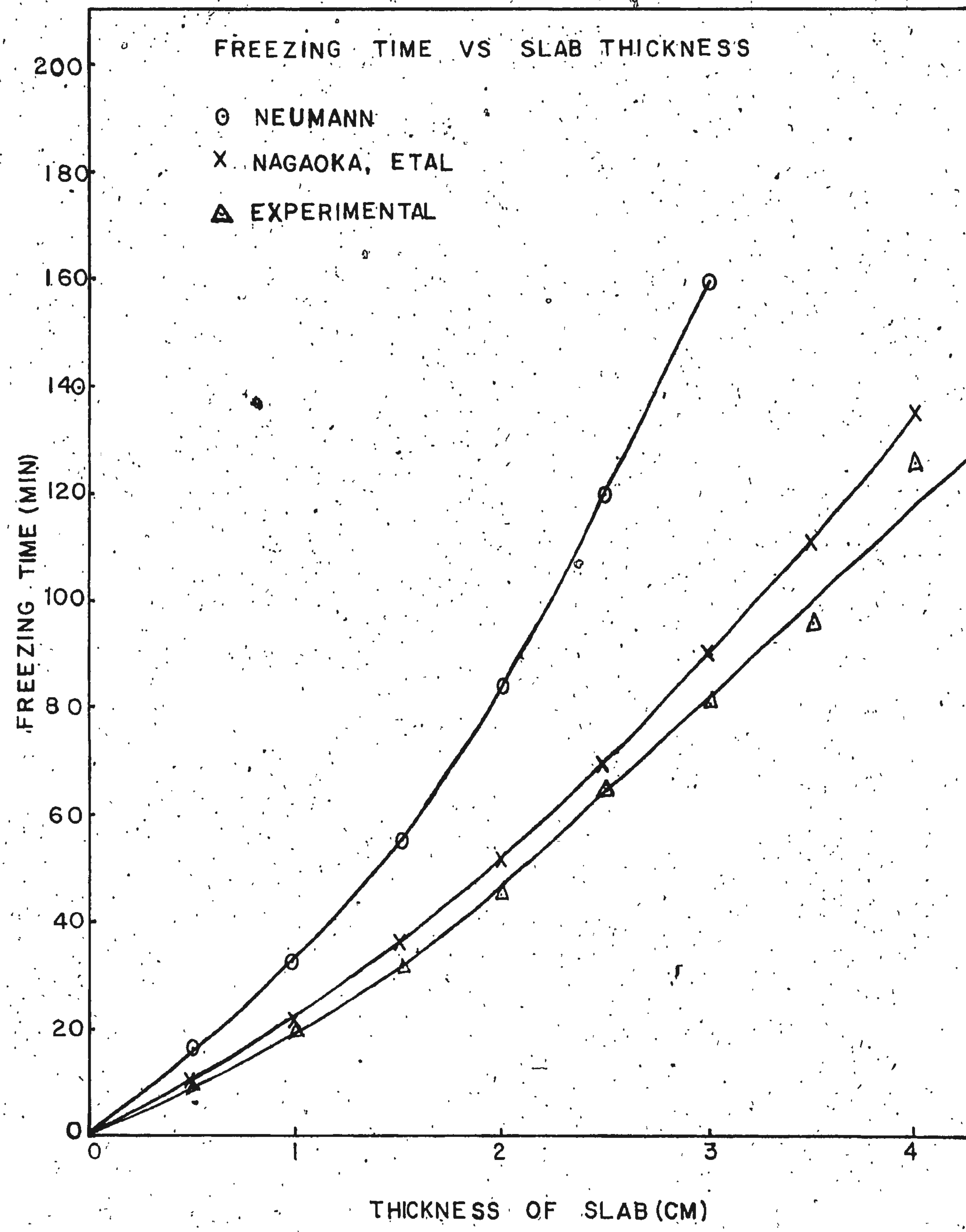

Figure 10. Experimental and computed freezing times for fish muscle slab.

relationship incorporating first and second power terms, with the linear term predominating for finite values of the heat transfer coefficient and small thicknesses.

TABLE 2

FREEZING TIMES OF FISH SLAB

Slab Thickness (cm.)	Experimental Effective Freezing Time (Min.)
0.5	9.6
1.0	19.8
1.5	31.1
2.0	45.0
2.5	65.0
3.0	80.5
3.5	95.5
4.0	126.5

The differences in the results from the Nagaoka et al. formulation and experimental times as shown in Figure 10 increase only slightly with increasing slab thickness. This would seem to indicate, not unexpectedly, the increasing importance of edge losses as the approximation to an infinite slab becomes less accurate with thicker slabs. However, for food products which can be considered as infinite slabs, the formulation would appear to be reasonably accurate in the prediction of food freezing times and as such should be a valuable tool for aid in the design of freezing equipment and in the determination of throughput rates of freezing processes.

CHAPTER 4

FREEZING TIMES OF COMMERCIAL FISH FILLETS

4.1. Introduction

It has been shown that experimental freezing times for infinite slabs of fish muscle may be predicted by equation 2.3.1. The second part of this work is concerned with the application of this equation to the freezing of commercial fish fillets. Examples of such fillets are the ocean perch (Sebastes marinus) and flounder (Hippoglossoides platessoides) fillets shown in Figures 11 and 12.

It is immediately evident that the shape of such fillets is not that of a rectangular slab, the degree of deviation from such a regular configuration being dependent upon the species. The length and width of a particular fillet are not constant, and the critical dimension of thickness varies throughout the entire length of the fillet rising to a maximum value at some location which is the thermal centre of the fillet. Although it is rather difficult to say precisely how the heat transfer takes place in such an irregular shape, the ratios of maximum length and width to maximum thickness are usually large enough to ensure that the major part of the heat removed from the thermal centre is transferred in the downward direction (for a fillet being frozen from the lower surface).

(a)

(b)

Figure 11. Ocean perch fillet.
(a) plan view, (b) profile.

(a)

(b)

Figure 12. Flounder fillet.
(a) plan view, (b) profile.

Jason and Jowitt (8) suggest that, in the case of a large heat transfer coefficient, a rectangular slab may be considered infinite if the shorter side length is six or seven times the thickness. Long (3) suggests a ratio of length to thickness of fifteen for the case of freezing rectangular slabs in an air blast freezer for which the heat transfer coefficient is typically small and in the order of 0.5 cal/cm^2-sec-^{o}C (26). The heat transfer coefficient determined in Chapter 3 of this work is approximately four times this value and indicates that the former ratio (8) is more applicable to this case.

From an observation of Figures 11 and 12, it is to be expected that the approximation to an infinite slab is more accurate for the flounder fillet than for the perch fillet due to the more extended shape of the former. The extent to which each conforms to the infinite slab approximation is later determined by establishing a relationship between the regular slab and irregular fillet configurations for each species.

One might expect that such a relationship could possibly be affected by heat transfer into or from the air layer above the fillet. To determine the effect of this air layer on the freezing time, the experiments with the rectangular slabs of Chapter 3 were repeated with the top surface insulation removed. The resulting freezing times, together with those from Table 2, page 40, for comparison purposes, are shown in Figure 13. It can be seen that this air layer has negligible influence on the freezing time and essentially acts as

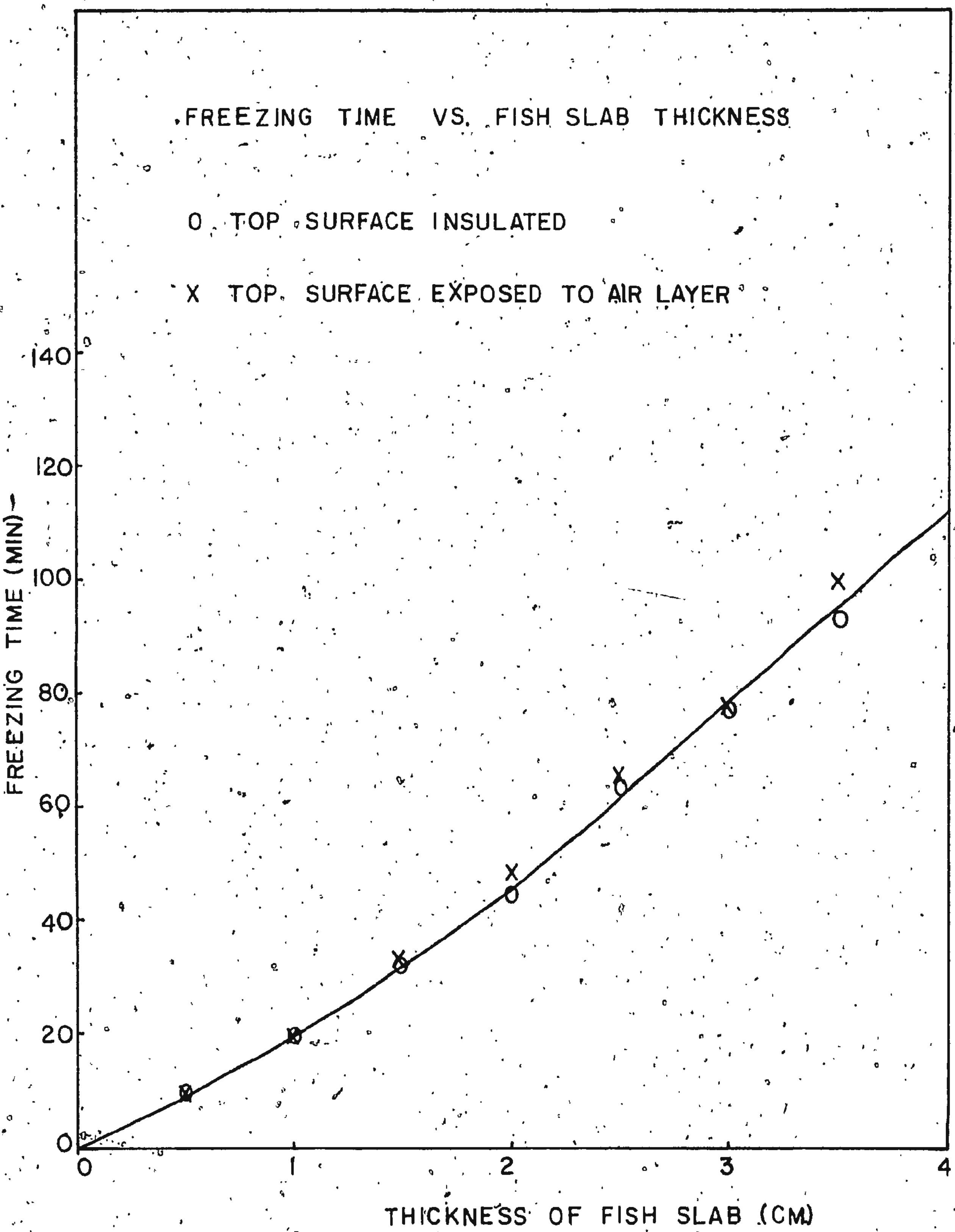

Figure 13. Fish slab freezing curve with insulated and exposed top surface.

an insulating barrier over the top surface of the sample being frozen.

In the commercial situation, the weight of the fillets is easily measured, and it would be advantageous, in terms of efficient operation, to know the freezing time in terms of the weight. The following section proposes a method by which the freezing time can be expressed in terms of the weight of the commercial fillet.

4.2. The Freezing Time-Weight Relationship

In order to obtain an expression (based on equation 2.3.1) for freezing time in terms of the fillet weight, it is necessary to determine the thickness of an infinite slab in terms of the weight of the irregularly-shaped fillet. One method of approaching this problem is to first determine a relationship between the maximum thickness of the fillet and the thickness of an equivalent infinite slab (i.e., the thickness that results in the same freezing time), and, second, to determine a relationship between the maximum thickness and weight of the fillet. By combining the two, a third relationship is obtained for the infinite slab thickness in terms of the fillet weight.

A suggested relationship between fillet maximum thickness and equivalent infinite slab thickness is

$$t = c_1 t_{max}^{\gamma} \qquad 4.2.1$$

where t_{max} is the fillet maximum thickness,

t is the thickness of an equivalent infinite slab,

c and γ are constants determined by the fillet configuration.

A relationship between fillet maximum thickness and weight may be determined by observing that, for a rectangular slab,

$$t = c_2 W^{1/3}$$

where $c_2 = (\frac{1}{\rho ab})^{1/3}$; ρ is density, a and b are length/thickness and width/thickness.

It is to be expected that for the irregularly shaped fillet,

$$t_{max} = c_2 W^{\alpha} \qquad 4.2.2$$

where α is a constant.

Combining equations 4.2.1 and 4.2.2,

$$t = c_1 c_2^{\gamma} W^{\alpha\gamma}$$

or,

$$t = c_1 c_2^{\gamma} W^{\beta} \qquad 4.2.3$$

where $\beta = \alpha\gamma$.

Combining equations 4.2.3 and 2.3.1

$$\theta = \frac{EZ\rho}{(T_f - T_m)}\left(\frac{c_1 c_2^{\gamma}}{h} W^{\beta} + \frac{c_1^2 c_2^{2\gamma} W^{2\beta}}{2k}\right)$$

For a given set of heat transfer conditions,

$$\theta = K_1 W^{\beta} + K_2 W^{2\beta} \qquad 4.2.4$$

where $K_1 = \dfrac{EZ\rho c_1 c_2^{\gamma}}{(T_f - T_m)h}$,

and $K_2 = \dfrac{EZ\rho c_1^2 c_2^{2\gamma}}{(T_f - T_m)2k}$,

ρ, k, T_f are constants for the fillet species,

E, Z, T_m, h are determined by process conditions,

c_1, c_2, γ, β are determined experimentally for a specific species.

The advantage of this approach to the problem of determining freezing time in terms of fillet weight over, for example, an empirical fit of a curve to experimental data will now be obvious. Suppose that an empirical fit of freezing time against fillet weight yielded an equation of the form $\theta = K_1' W^{\psi} + K_2' W^{n}$. From such an equation, unlike a formulation based on equation 2.3.1, one would obtain no information on heat transfer conditions or, more important, information on process conditions and temperatures, a matter which is of paramount importance to allow flexibility in an operation and yet to predict the result of such flexibility.

4.3. Experimental Description

Experiments were carried out with two species of fish that are commonly frozen commercially by the IQF (Individually Quick Frozen) method. (This method is described more fully in Chapter 5). Samples of ocean perch and flounder fillets were collected from the filleting line in a commercial installation. These samples were a good representation of the weight range experienced in the industry. For each fillet, the weight and maximum thickness were measured. The measurement of maximum thickness provided some difficulty as the softness of the fresh muscle provided little resistance to the calipers used in the measurement. It was found that first freezing the sample resulted in hardened surfaces which provided more resistance and therefore eliminated inconsistencies due to different finger pressures on the calipers. The contribution to the overall error in the thickness measurement due to the expansion of the muscle upon freezing was negligible.

The method of measurement of freezing time for each fillet was similar to that described for the infinite slab. The fillet was first allowed to reach an equilibrium temperature of 40°F (4.4°C) whereupon it was transferred to the refrigerated plate. In this experiment, the fillet was not placed in an insulated frame as some similarity with the industrial process was desired. The time required for the temperature to drop from 40°F (4.4°C) to 0°F (-17.8°C) (i.e. the effective freezing time) was obtained in the manner described in Chapter 3, and the times so obtained were plotted against the maximum thickness and weight of the fillets. The details of the procedure used to find the values of the constants in equation 4.2.4 are described in Section 4.4.

4.4. Experimental Results

4.4.1. Ocean Perch (Sebastes marinus)

The results of experiments with ocean perch are summarized in Table 3.

TABLE 3

OCEAN PERCH DATA

Weight of Fillet (gm.)	Maximum Thickness (cm.)	Time to Freeze (Min.) from 40°F (4.4°C) to 0°F (-17.8°C)
107.0 ± 0.5	1.6 ± 0.1	27.0 ± 0.5
123.0	1.8	29.0
66.0	1.7	23.5
106.0	1.7	29.0
144.0	1.9	33.0
55.0	1.6	23.0
77.0	1.7	29.0
125.0	1.9	35.0
69.0	1.7	25.5
200.0	2.4	47.0
179.0	2.3	41.0
95.5	1.9	33.0
137.0	2.0	36.0
118.0	1.9	34.0
116.5	1.9	34.0
122.0	1.8	28.0
110.0	1.8	28.5
87.0	1.8	26.0
114.5	1.8	30.0
113.5	1.9	35.0
98.0	1.8	29.0
82.5	1.6	25.5
94.0	1.7	29.0
160.0	2.0	37.5
55.0	1.6	22.0
30.5	1.3	14.0
50.0	1.5	18.0
84.0	1.8	28.0
61.0	1.7	25.0
186.0	2.2	45.0
190.0	2.4	52.5

In Figure 14, the experimental freezing time is plotted against the maximum thickness of the fillet. Assuming that each maximum thickness is the constant thickness of an infinite slab, a second plot is made by calculating freezing times from equation 2.3.1. It is now a simple matter to determine a maximum thickness which will result in the same freezing time as the equivalent infinite slab. Table 4 has been constructed by selecting various freezing times and determining from Figure 14 the corresponding maximum thicknesses of the fillets along with the thicknesses of the equivalent infinite slabs.

TABLE 4

MAXIMUM FILLET AND EQUIVALENT SLAB THICKNESSES

Freezing Time (Min.)	Fillet Maximum Thickness (cm.)	Equivalent Slab Thickness (cm.)
25	1.66	1.11
30	1.81	1.29
35	1.45	1.96
40	1.61	2.11
45	2.26	1.77
50	1.92	2.41

From equation 4.2.1, we obtain:

$$\ln t_{max} = \frac{1}{\gamma} \ln t - \frac{1}{\gamma} \ln c_1.$$

A plot of $\ln t_{max}$ against $\ln t$ results in a straight line with slope $\frac{1}{\gamma}$ and intercept $-\frac{1}{\gamma} \ln c_1$. From Figure 15 $\gamma = 1.63$ and $c_1 = 0.484$. Hence the relationship between the

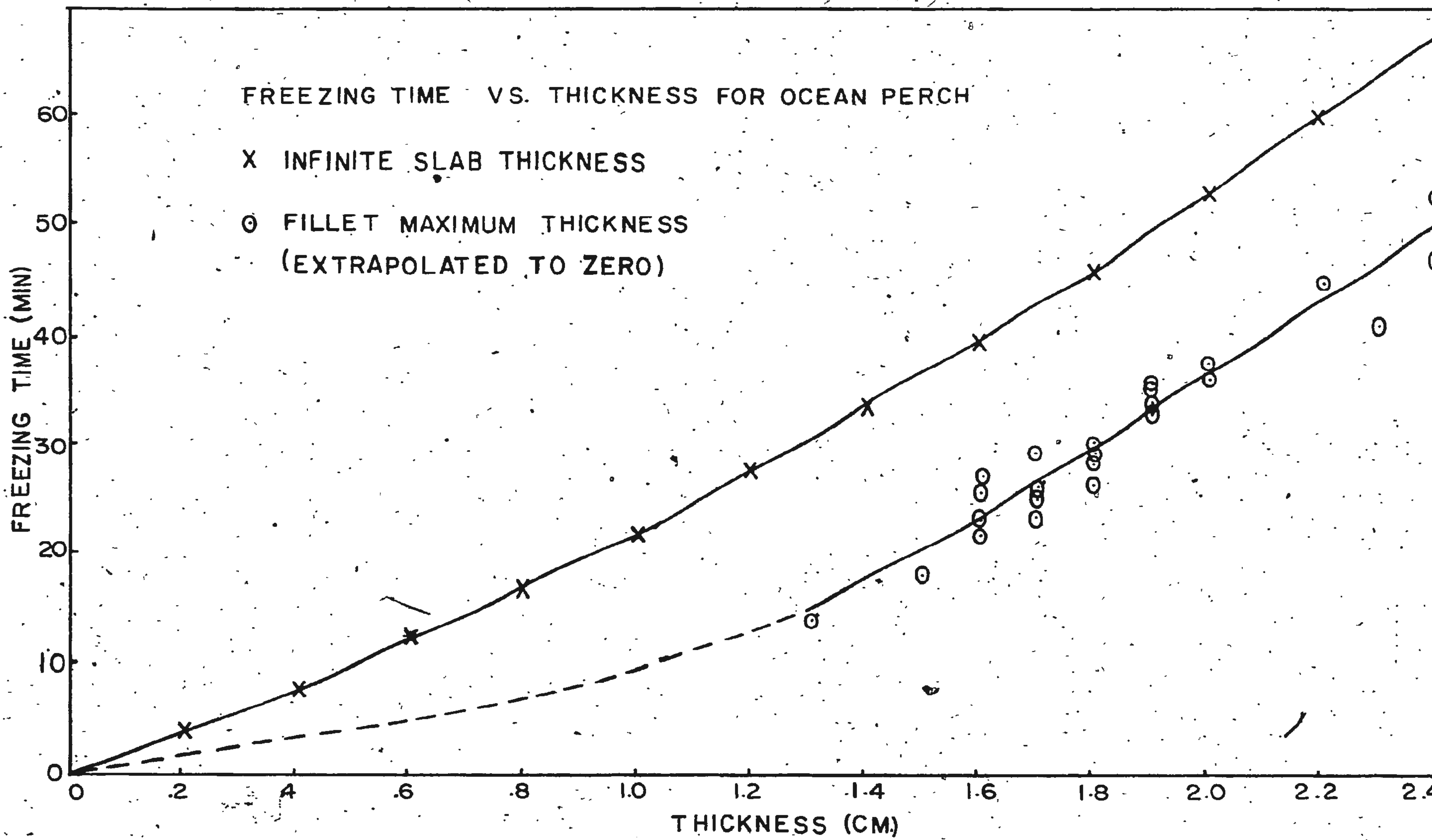

Figure 14. Freezing time curves for ocean perch.

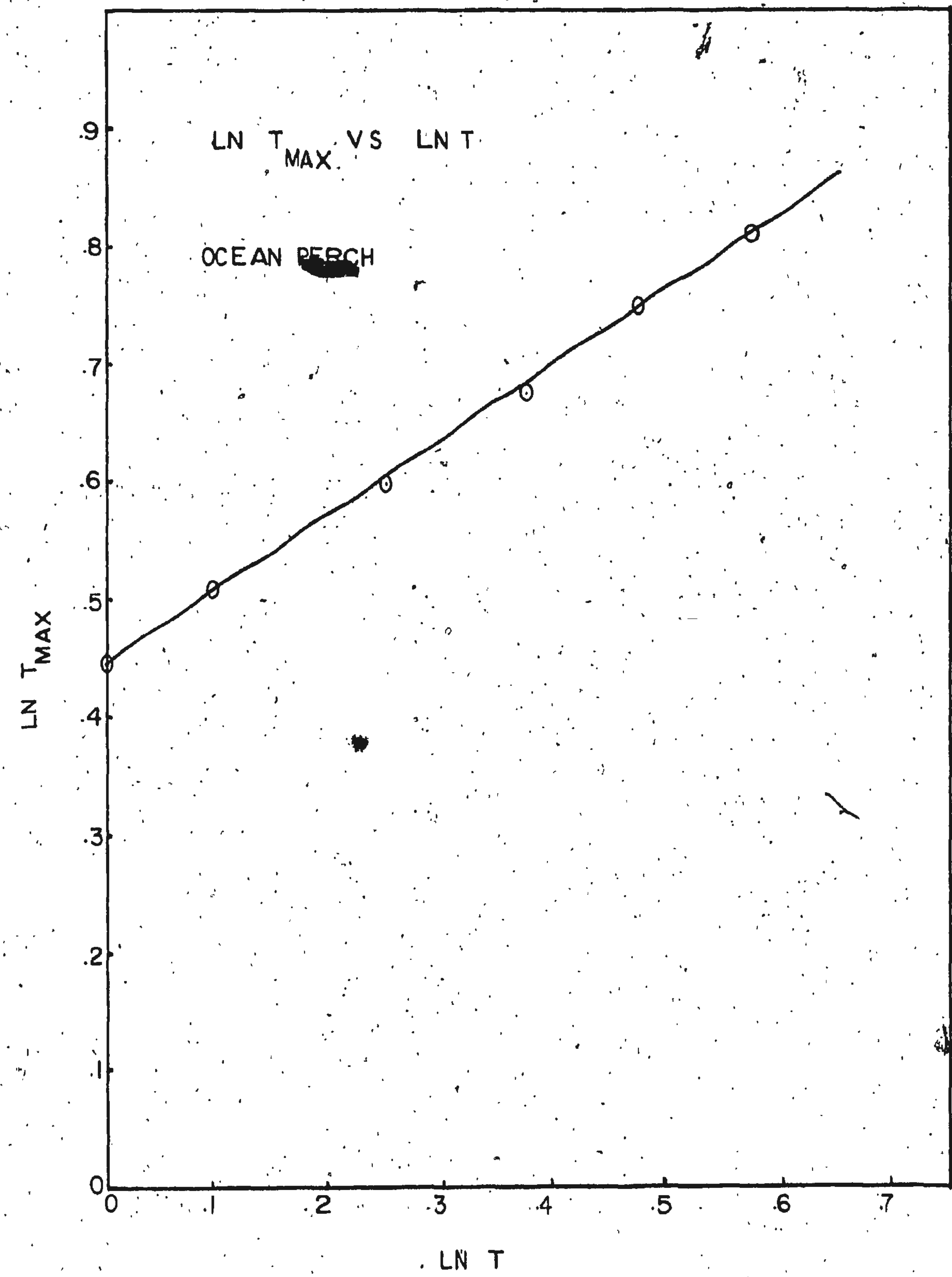

Figure 15. Log_e of maximum fillet thickness versus $\log_e$ of infinite slab thickness for ocean perch fillets.

maximum fillet thickness of ocean perch and the thickness of the equivalent infinite slab may be written as $t = 0.484\ t_{max}^{1.63}$. In Figure 16, a plot is made of this relationship showing computed values of equivalent slab thicknesses up to a maximum fillet thickness of 1.3 cm. and experimentally determined values from Figure 14 up to a maximum fillet thickness of 2.2 cm. The graph shows that the equation fits the experimental points well, and the computed points for thicknesses below those used experimentally have been added for completeness.

The relationship between fillet weight and maximum thickness has been established by equation 4.2.2. By plotting ln W against ln t_{max}, the constants α and c_2 may be determined. The curve of Figure 17 has been fitted to the data by the least squares method, from which $\alpha = 0.298$ and $c_2 = 0.459$. Thus the relationship between maximum thickness and fillet weight for ocean perch is:

$$t_{max} = 0.459W^{0.298}$$

The required relationship between fillet weight and freezing time expressed by equation 4.2.4 may now be determined. The data which allow the calculation of the enthalpy term, Z, have been obtained from the literature and are identical to the data for cod muscle used in Chapter 3. In fact, these data will give good results for most fish species; the exceptions are those species that have a high oil content (e.g. salmon, herring and mackeral) which reduces the water content and results in lower values of

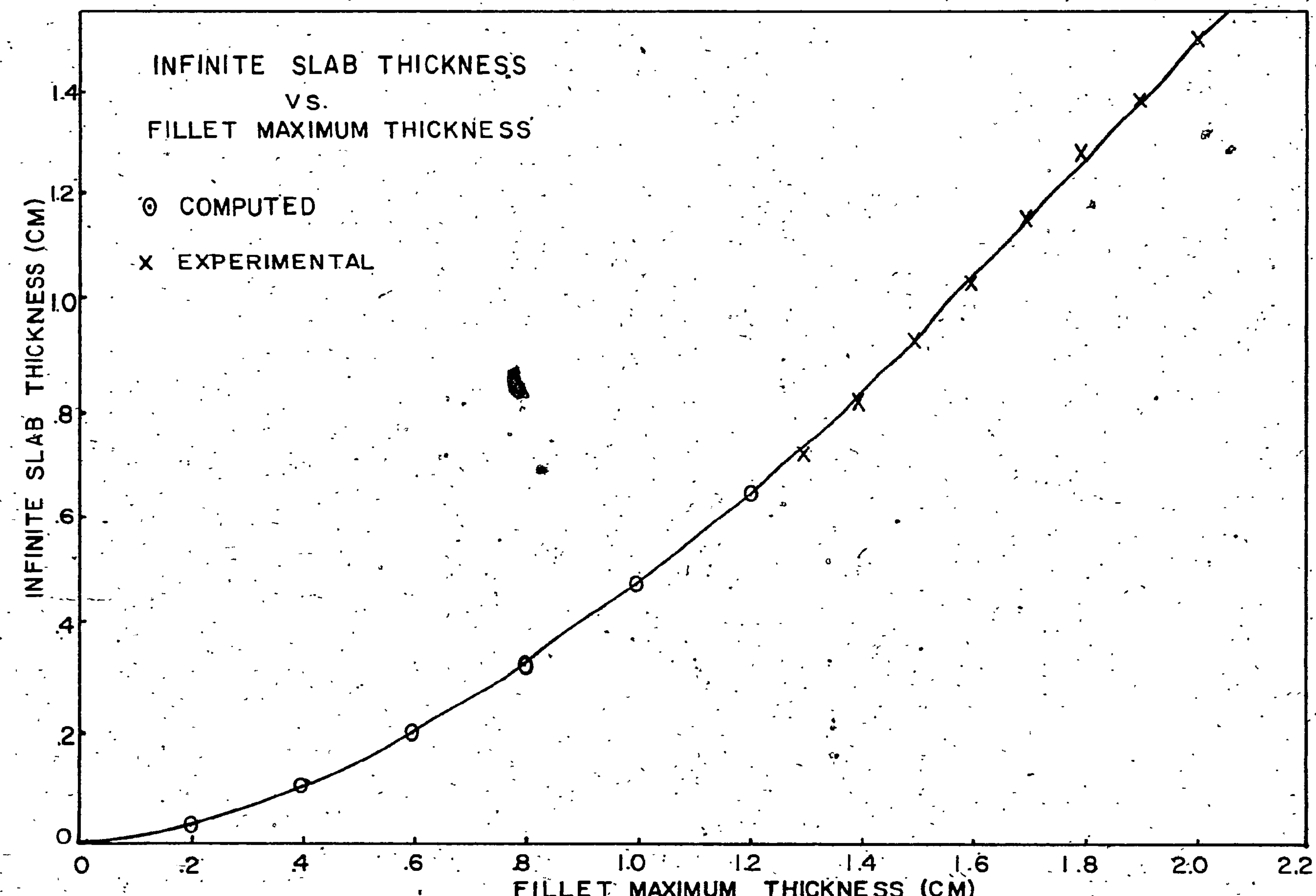

Figure 16. Infinite slab thickness versus fillet maximum thickness for ocean perch.

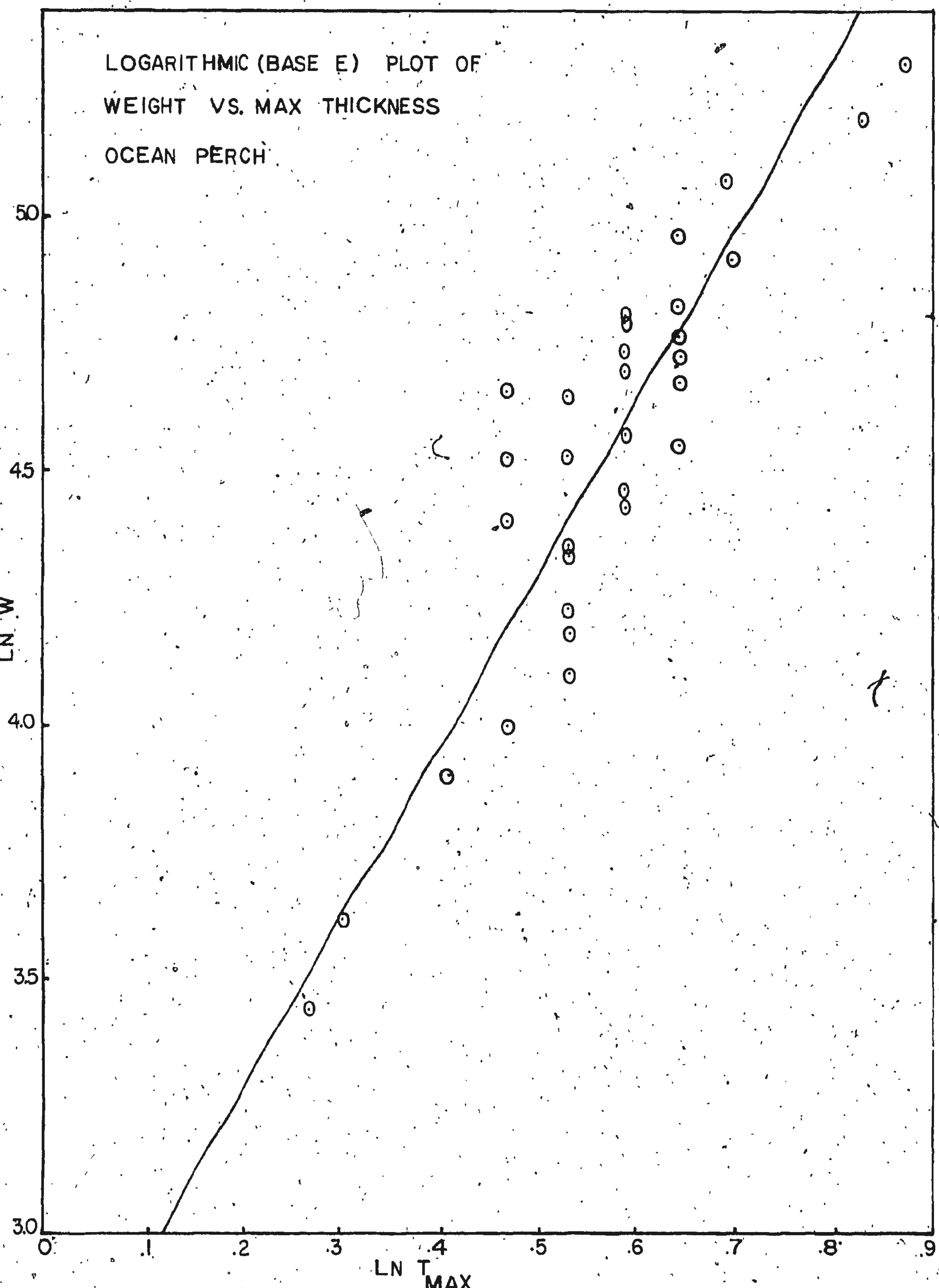

Figure 17. Log_e of weight versus $\log_e$ of fillet maximum thickness.

the specific and latent heat terms. Species such as cod, perch, flounder, hake and haddock all have moisture contents of 80 per cent ±2 per cent, while salmon, herring, etc., have moisture contents of 60 per cent ±3 per cent (26).

Values of the constants and data for ocean perch are:

$T_1 = 4.4^\circ C$ $\quad T_f = -2.2^\circ C$ $\quad T_m = -39^\circ C$

$T_2 = -17.8^\circ C$

$s_1 = 0.84$ cal/gm-$^\circ$C $\quad s_2 = 0.44$ cal/gm-$^\circ$C $\quad L_f = 64$ cal/gm.

$\alpha = 0.298$, $\gamma = 1.63$ $\quad \beta = \alpha\gamma = 0.486.$

$c_1 = 0.484.$

$c_2 = 0.459$ and $\frac{EZ\rho}{T_f - T_m} = 0.036.$

$$\text{Therefore, } K_1 = \frac{EZ\rho}{T_f - T_m} \cdot \frac{c_1 c_2^\gamma}{h} = 2.40$$

$$\text{and } K_2 = \frac{EZ\rho}{T_f - T_m} \cdot \frac{c_1^2 c_2^{2\gamma}}{2k} = 0.077.$$

Hence, the relationship between freezing time and fillet weight for ocean perch may be written as:

$$\theta = 2.40W^{0.486} + 0.077W^{0.972}.$$

This relationship is plotted in Figure 18 which also shows the experimental freezing times in terms of the fillet weights.

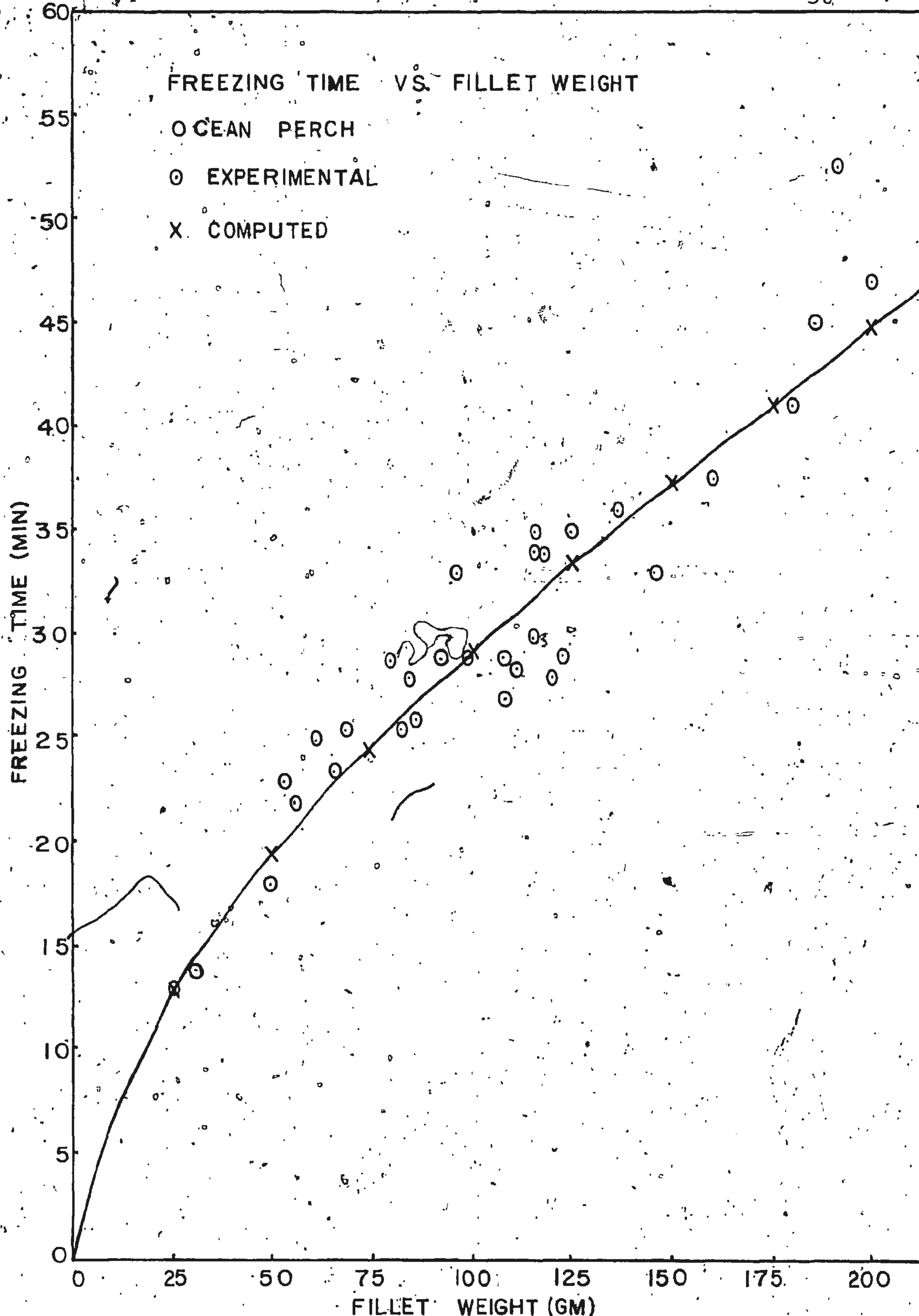

Figure 18. Freezing time versus weight for ocean perch fillets.

4.4.2. Flounder (Hippoglossoides platessoides)

Experiments conducted with flounder fillets were similar to those described in Section 4.4.1. The results of these experiments are shown in Table 5.

TABLE 5

FLOUNDER DATA

Weight of Fillet (gm.)	Maximum Thickness (cm.)	Time to Freeze from 40°F (4.4°C) to 0°F (-18°C) (Min.)
161 ± 0.5	1.8 ± 0.1	35.0 ± 0.5
56.5	1.0	21.5
121.5	1.3	23.0
68.5	1.2	22.5
117.0	1.3	22.5
179.5	1.4	30.0
86.0	1.6	24.5
79.5	1.2	19.5
290.0	1.8	49.0
80.0	1.2	19.0
288.0	1.7	54.5
204.0	1.5	33.0
157.0	1.6	32.0
86.5	1.4	25.0
131.0	1.5	30.0
260.0	1.9	50.0
224.0	1.7	46.0
45.0	1.2	17.0
128.0	1.2	28.0

Graphs similar to those of Section 4.4.1 were plotted for flounder, and from these graphs the various constants required in equation 4.2.4 were determined. From Figure 19 the maximum fillet thickness and thickness of an equivalent infinite slab were determined. With these values, a plot of

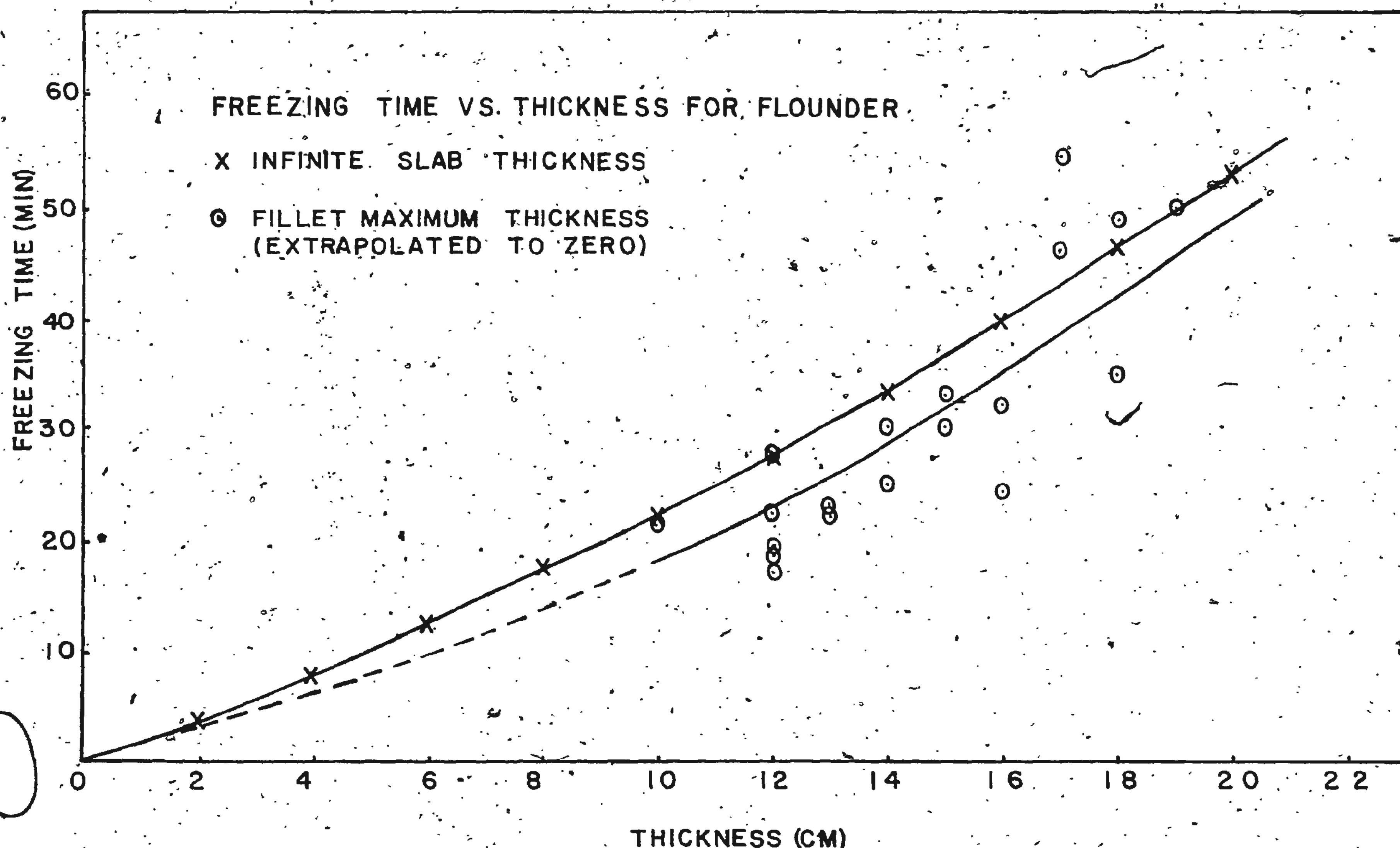

Figure 19. Freezing time curves for flounder fillets.

$\ln t_{max}$ against $\ln t$ was made and is shown in Figure 20. The values of constants γ and c_1 resulting from this plot are $\gamma = 1.17$ and $c_1 = 0.839$. A plot by the least squares method of $\ln W$ against $\ln t_{max}$ in Figure 21 resulted in values of the constants of $\alpha = 0.276$ and $c_2 = 0.346$. Upon substitution of these constants and the appropriate data into equation 4.2.4, the effective freezing time in terms of the fillet weight for flounder is:

$$\theta = 4.28W^{0.323} + 0.25W^{0.646}.$$

This relationship is plotted in Figure 22 which also shows the experimental freezing time against fillet weight.

4.5. Discussion of Results

It is evident from examination of Figures 18 and 22 that the deviation of experimental freezing times from times calculated with the aid of equation 4.2.4 is more pronounced for flounder than for perch. This may be attributed to the greater irregularity in flounder fillets due to extensive trimming which may be necessary during the filleting operation. Such trimming, which is less extensive in the case of perch, entails the removal of muscle in order to eliminate blood spots, parasites and jelly. As a result, the relationship between weight and maximum thickness is not as definitive as it would otherwise be. This, in turn, leads to a lower reliability in the equation relating freezing time and weight of the fillet. Notwithstanding these anomalies,

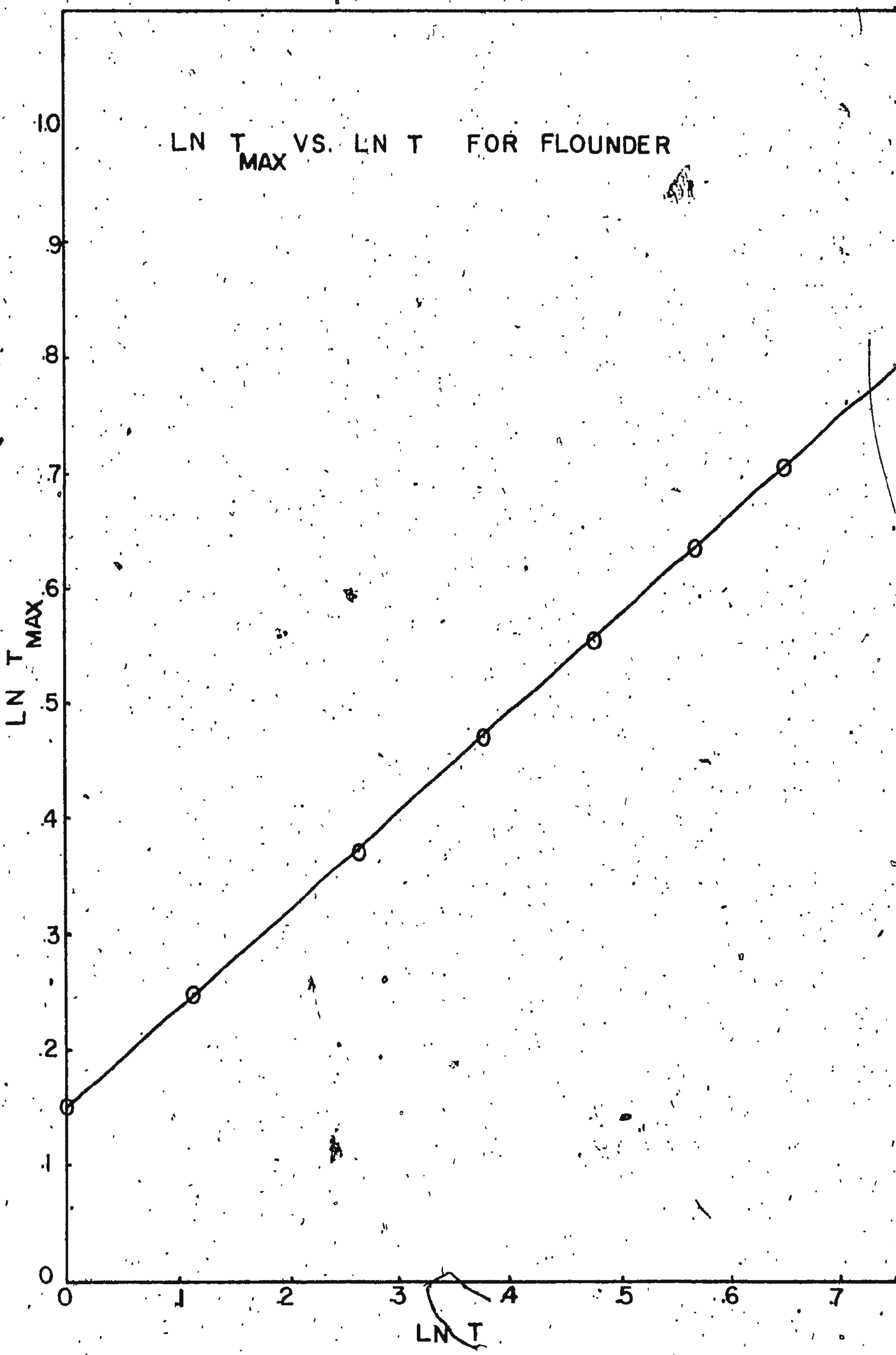

Figure 20. Log_e of fillet maximum thickness versus $\log_e$ of infinite slab thickness for flounder.

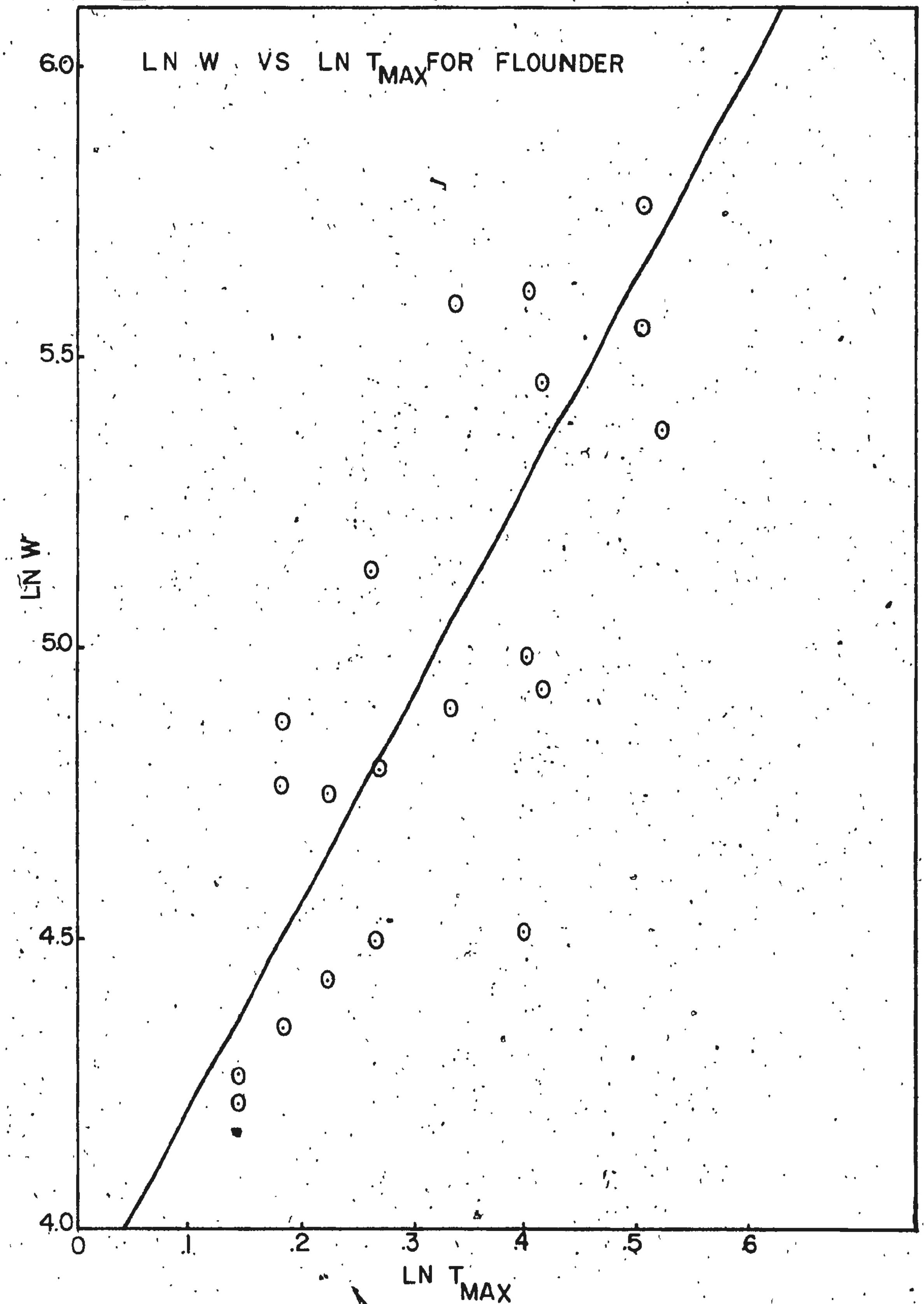

Figure 21. Log_e of weight versus $\log_e$ of fillet maximum thickness for flounder.

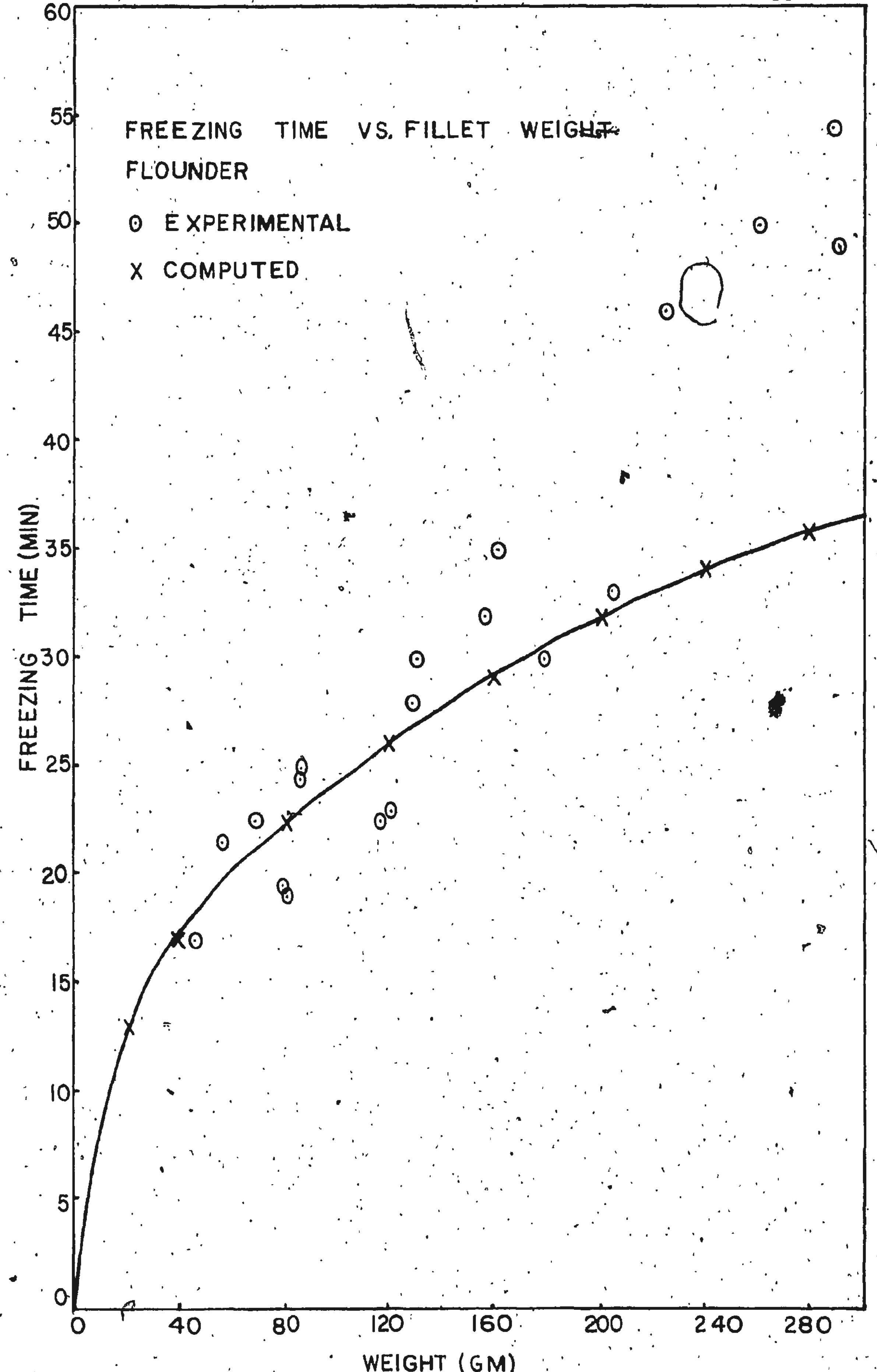

Figure 22. Freezing time versus weight for flounder fillets.

however, the prediction of freezing times for flounder fillets agrees favourably with experimental times for the major percentage of the weight distribution given in Table 6, Chapter 5.

The prediction of freezing times by equation 4.2.4 for perch fillets gives results in good agreement with experimental freezing times over the range of fillets tested. Some comparison of the reliability of the freezing time-weight relationship may be made for each species by determining the standard error of the calculated freezing time. For perch the error is found to be ±2.6 minutes while for flounder this error extends to ±7.4 minutes. If only flounder fillet weights below 220 grams are considered, the standard error in the calculated freezing time decreases to ±2.8 minutes. It will be later seen in Chapter 5 that in a typical flounder fillet distribution, only about 1 per cent of the fillets weigh more than 200 grams.

The freezing time-weight relationship for these two species of fish are specific representations of the more general semi-theoretical formula (equation 4.2.4) which should find application in many situations where the freezing time is required in terms of the weight of the product being frozen. It would then be a matter of determining the values of the various constants in this equation, some of which are set by the process conditions and others by the configuration of the product being frozen. For regular bodies such as slabs, cylinders and spheres, such constants may be written down directly; for irregular bodies, other approaches are possible. The body may be approximated by a regular form on the basis of geometry so that the dimensions of one may be immediately expressed in terms of the other; this is an obviously expedient method where feasible. However, with objects which show considerable irregularity, such relationships, if at all possible, may be very complex. One alternative then is to use the approach of this work in which an equivalent regular shape for the irregular object is determined on the basis of actual experimentation into the freezing time of such objects.

4.6. Extension to Other Initial Temperatures

In equation 4.2.4, K_1 and K_2 contain the correction factor 'E' of Nagaoka et al's. formulation and 'Z', the enthalpy term, both of which are functions of the initial temperature of the body. 'Z' also depends on the final temperature of the frozen body but to a much lesser degree as the specific heat in the frozen phase is approximately half that of the thawed phase. In addition, heat removal is at a faster rate in the frozen zone than the thawed zone because of the increase in thermal conductivity (as established in Chapter 3). For these reasons, equal changes in the initial and final temperatures of the body will contribute differently to the overall freezing time of the body, the variation of freezing time being less pronounced with final temperature than with initial temperature. Because of this, the final temperature has been arbitrarily kept constant at $-17.8^{o}C$ ($0^{o}F$) while the initial temperature assumes different values. The resulting plots of equation 4.2.4 for perch and flounder are shown in Figures 23 and 24 respectively. These charts enable one to quickly determine the freezing time of a sample of specific weight for a prescribed initial temperature.

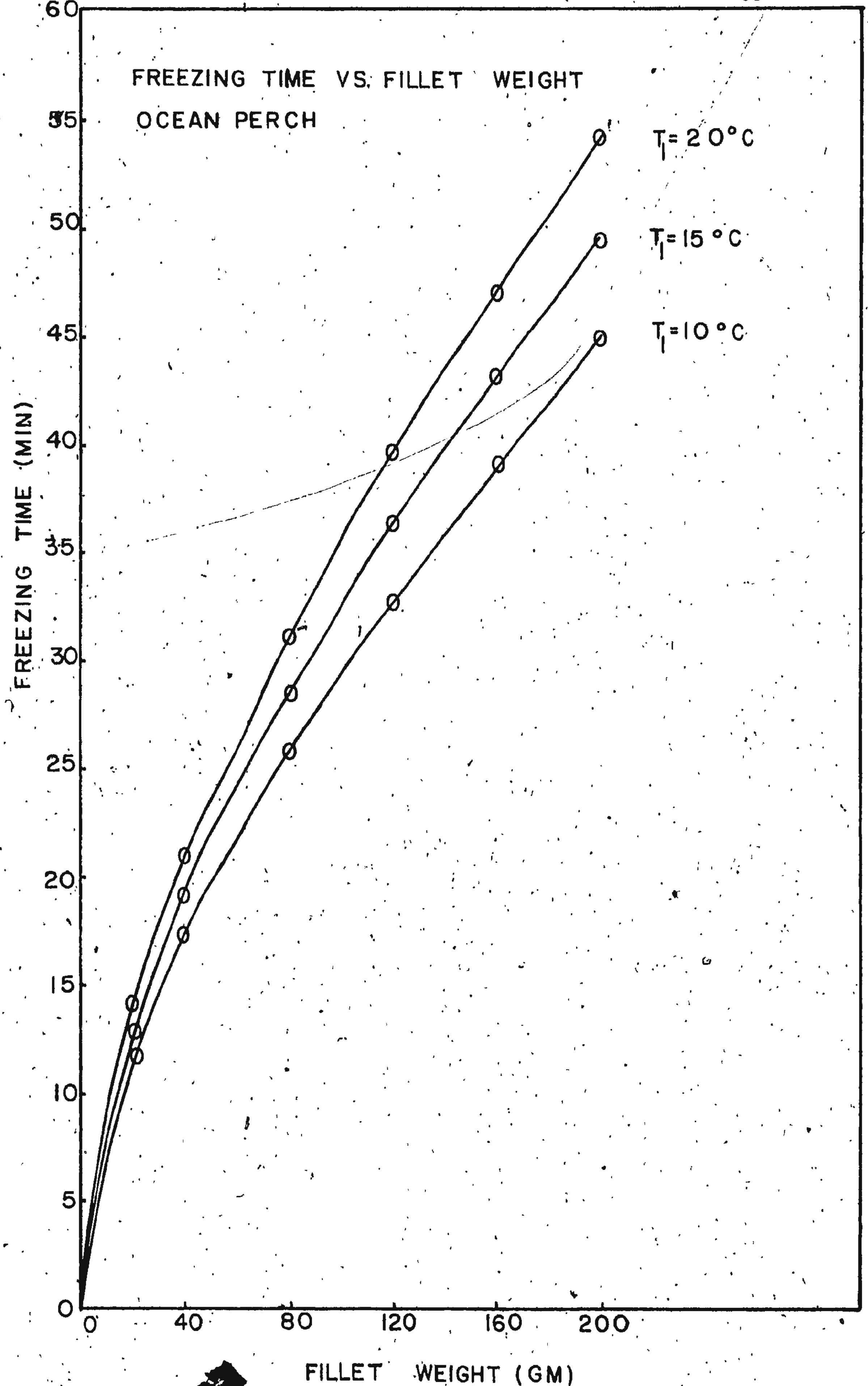

Figure 23. Freezing time curves for different initial temperatures of ocean perch fillets.

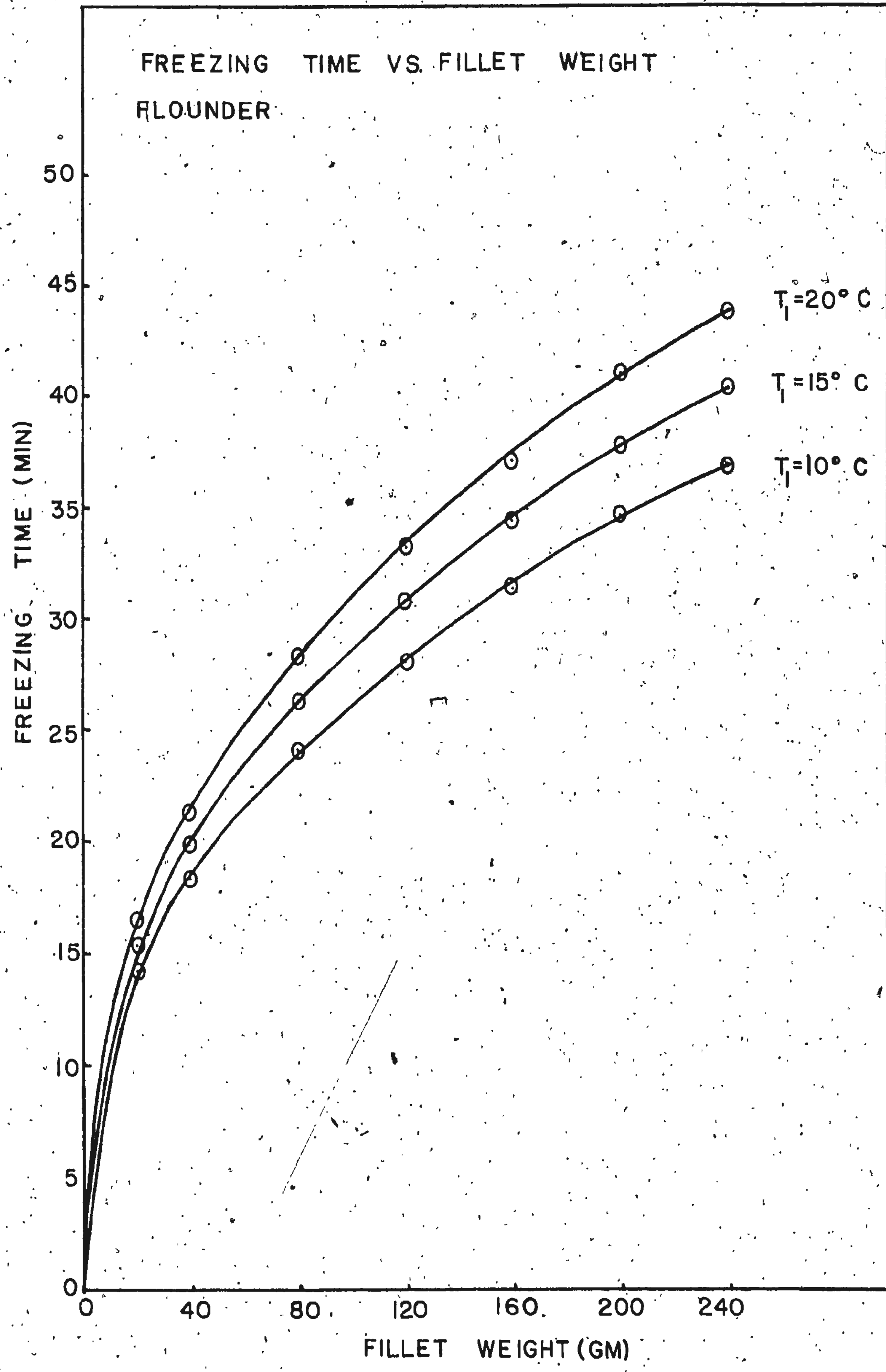

Figure 24. Freezing time curves for different initial temperatures of flounder fillets.

CHAPTER 5

THE APPLICATION TO COMMERCIAL IQF PROCESSES

5.1. A Description of IQF Systems

In IQF (Individually Quick Frozen) processes, foodstuffs are frozen as separate units prior to packaging. The absence of a packaging material means that higher heat transfer rates may occur with a subsequent decrease in the freezing time of the product.

There are many types of IQF systems, but the majority consist of a continuous belt on which the product to be frozen rests. The product may be cooled from the lower surface by circulating refrigerant beneath the belt or from the top surface by directly spraying the product with liquid refrigerant. A third possibility is that of spraying the product from below through a meshed belt. Other designs are the immersion freezer in which the product is immersed in liquid refrigerant, and the rotating double-walled drum in which the refrigerant circulates between the walls and removes heat from the product which adheres to the surface of the drum (33).

5.2. Production Theory of IQF Systems

Fish fillets of the type described in Chapter 4 are often frozen in IQF systems. It has been seen that such fillets extend over a broad range of weights, and assuming that every fillet is to pass through a prescribed temperature range, it is obvious that the process time will be determined by the largest weight present in the input. It would appear to be advantageous to have the fillets sorted before entering the freezing system and divided so that all fillets below a certain weight could be processed at a faster rate and all those above that weight processed at the previous rate. For the purposes of this development, the fillet input is arbitrarily split in half. With a two unit set-up, each unit would contain half of the total number of fillets and be operated at a rate determined by the maximum weight present in each of the new weight ranges. For a one unit set-up the inputs could be run consecutively.

The total load passing through the process may be expressed as:

$$\text{Load} = \text{number of fillets} \times \text{mean fillet weight}$$

$$= \frac{\text{Effective surface area}}{\text{Area per fillet}} \times \text{mean fillet weight.}$$

For a rectangular slab,

$$\text{Load} = \frac{\text{Effective surface area}}{\text{Area per fillet}} \times \rho \bar{l}\bar{w}\bar{t}$$

where $\bar{l}$, $\bar{w}$, and $\bar{t}$ are the dimensions of the mean fillet slab.

$$\text{Load} = \text{Effective surface area} \times \rho \bar{t}.$$

The loading density is,

$$D = a\bar{t} \qquad 5.2.1$$

where 'a' is approximately equal to the density of the fillet, but is also affected by the number of voids on the freezing surface.

For an irregularly shaped fillet, the loading density becomes (using equation 4.2.1),

$$D = ac_1\bar{t}^{\gamma}_{max}. \qquad 5.2.2$$

The production rate for a single process may be written as,

$$Q = \frac{DA}{\theta} = \frac{ac_1A\bar{t}^{\gamma}_{max}}{\theta} \qquad 5.2.3$$

where θ is the process time,

A is the effective surface area of the freezer.

Now t_{max} and fillet weight, W, are related by equation 4.2.2 and for the mean fillet weight,

$$\bar{t}_{max} = c_2\bar{W}^{\alpha}. \qquad 5.2.4$$

Combining equations 5.2.3 and 5.2.4,

$$Q = \frac{aAc_1c_2^{\gamma}\bar{W}^{\beta}}{\theta} \qquad 5.2.5$$

where $\beta = \alpha\gamma$.

The process time θ based on the maximum fillet weight is obtained from equation 4.2.4:

$$\theta = K_1W_m^{\beta} + K_2W_m^{2\beta}. \qquad 5.2.6$$

Combining equations 5.2.5 and 5.2.6 results in the following expression for the production rate in units of mass per unit time:

$$Q = \frac{\bar{W}^{\beta}}{K_3W_m^{\beta} + K_4W_m^{2\beta}} \qquad 5.2.7$$

where $K_3 = \dfrac{K_1}{aAc_1c_2^{\gamma}}$ and $K_4 = \dfrac{K_2}{aAc_1c_2^{\gamma}}$.

A comparison may now be made between production rates based on different weight ranges by investigating the production rate ratio:

$$\frac{Q}{Q_1} = \left(\frac{\bar{W}}{\bar{W}}\right)^{\beta} \frac{K_1 W_{m_1}^{\beta} + K_2 W_{m_1}^{2\beta}}{K_1 W_m^{\beta} + K_2 W_m^{2\beta}} \qquad 5.2.8$$

5.3. Special Cases of the Theory

5.3.1. High Heat Transfer Coefficient

In some IQF systems, such as the liquid refrigerant spray and immersion systems, the heat transfer coefficient is high enough to render the effect of the term $K_1 W^{\beta}$ in equation 5.2.8 negligible since $K_1 \to 0$ for large h.

Equation 5.2.8 may then be approximated by,

$\frac{Q}{Q_1} = (\bar{W}/\bar{W}_1)^{\beta} \; (W_{m_1}/W_m)^{2\beta}$ thus making a determination of K_2 unnecessary.

5.3.2. Low Heat Transfer Coefficient

If the heat transfer coefficient is sufficiently small as, for example, in air blast freezers, then $K_1 >> K_2$ and the approximation to equation 5.2.8 is,

$$\frac{Q}{Q_1} = (\bar{W}\; W_{m_1}/\bar{W}_1\; W_m)^{\beta}.$$

5.3.3. Fillet in the Shape of a Rectangular Slab

It has been established that for an irregular fillet,

$$K_1 = \frac{EZ\rho c_1 c_2^{\gamma}}{(T_f - T_m)h} \quad \text{and} \quad K_2 = \frac{EZ\rho c_1^2 c_2^{2\gamma}}{(T_f - T_m)2k}.$$

For a rectangular slab, $t = t_{max}$; hence, $c = \gamma = 1$ and $\alpha = 1/3$ as shown in Chapter 4, Section 2. Thus,

$$K_1 = \frac{EZ\rho c_2}{(T_f - T_m)h} \quad \text{and} \quad K_2 = \frac{EZ\rho c_2^2}{(T_f - T_m)2k}.$$

Rewriting equation 5.2.8 with $\beta = \alpha\gamma = 1/3$ results in the form,

$$\frac{Q}{Q_1} = (\bar{W}/\bar{W}_1)^{1/3} \left(\frac{K_1 W_{m_1}^{1/3} + K_2 W_{m_1}^{2/3}}{K_1 W_m^{1/3} + K_2 W_m^{2/3}} \right).$$

Use of this form may give reasonably accurate results in situations where the IQF product approximates a rectangular slab. It can be seen, for example, that by examining the values of β obtained in Chapter 4 for perch and flounder fillets ($\beta_p = 0.486$, $\beta_f = 0.323$), one would expect this form of the production rate ratio to give more accurate results for flounder than for perch.

Similar approximations to those made in Sections 5.3.1 and 5.3.2 may be made for the rectangular slab.

If $h \to \infty$, $\dfrac{Q}{Q_1} = (\bar{W}/\bar{W}_1)^{1/3} \; (W_{m_1}/W_m)^{2/3}$.

If $h \to 0$, $\dfrac{Q}{Q_1} = (\bar{W}\, W_{m_1}/\bar{W}_1\, W_m)^{1/3}$.

5.4. Application of the Production Theory

5.4.1. Example 1: Flounder (Hippoglossoides platessoides) fillets

The advantage of dividing inputs to IQF systems may be shown by considering a typical distribution of flounder fillets as shown in Table 6.[1]

TABLE 6

FLOUNDER FILLET DISTRIBUTION

Fillet Weight (gm.)	Percentage
50	3
80	14
110	34
140	32
170	12
200	4
230	1

Mean weight $\bar{W}_1 = 126$ gm.
Maximum weight $W_{m_1} = 230$ gm.

When divided in half,

for lower half, $W_m = 110$ gm.

$\bar{W} = 98$ gm.

for upper half, $W_m = 230$ gm.

$\bar{W} = 153$ gm.

The data to be used in equation 5.2.8 has been given in Chapter 4, Section 4.

For the lower half, the ratio of production rates is found to be,

$$\frac{Q}{Q_1} = \left(\frac{98}{126}\right)^{0.323} \left[\frac{4.28(230)^{0.323} + 0.25(230)^{0.646}}{4.28(110)^{0.323} + 0.25(110)^{0.646}}\right]$$

$= 1.24$ or an increase in production rate of 24 per cent.

[1]The distribution was obtained from measurements at a local fish processing plant.

For the upper half of the fillet input,

$$\frac{Q}{Q_1} = \left(\frac{153}{126}\right)^{0.323} \left[\frac{4.28(230)^{0.323} + 0.25(230)^{0.646}}{4.28(230)^{0.323} + 0.25(230)^{0.646}}\right]$$

= 1.06 or an increase in production rate of 6 per cent.

The overall production rate with the unsorted input was $2Q_1$. With the input divided as described the production will be $1.24Q + 1.06Q_1 = 2.30Q_1$ for an overall increase in production rate of 15.0 per cent.

The process time for the upper half will remain the same as for the original total input. The lower half will be run at a faster rate. The ratio of the new process time to the old process time for the lower half of the input may be found from,

$$\frac{\theta}{\theta_1} = \frac{K_1 W_m^{\beta} + K_2 W_m^{2\beta}}{K_1 W_{m_1}^{\beta} + K_2 W_{m_1}^{2\beta}}.$$

Upon substitution of the appropriate data, $\frac{\theta}{\theta_1} = 0.74$. The new process time is thus 74 per cent of the previous process time for the lower half of the input.

5.4.2. Example 2: Ocean Perch (Sebastes marinus)

The procedure may be repeated with ocean perch fillets whose typical distribution is shown in Table 7.

TABLE 7

OCEAN PERCH FILLET DISTRIBUTION

Fillet Weight (gm.)	Percentage
20	3
50	21
80	30
110	27
140	6
170	9
200	3

Mean weight $\bar{W}_1$ = 95 gm.

Maximum weight W_{m_1} = 200 gm.

When divided in half, for lower half, W_m = 80 gm.

$\bar{W}$ = 64 gm.

for upper half, W_m = 200 gm.

$\bar{W}$ = 125 gm.

Repeating the procedure of Section 5.4.1 with the appropriate data for ocean perch fillets, the following production rate ratios are determined:

For the lower half of the input $\frac{Q}{Q_1} = 1.44$,

or an increase in production rate of 44 per cent.

For the upper half, $\frac{Q}{Q_1} = 1.14$,

or an increase in production rate of 14 per cent.

The overall increase in production rate for the perch fillets is 29 per cent. This sizeable increase in production rate as compared to that for the flounder fillets is due mainly to the increase in production rate of the lower half of the input. The maximum weight for the lower half of the perch fillets is 80 gm. while that for the flounder fillets is 126 gm. When the ratio of maximum weight of the unsorted input to the maximum weight of the lower half of the sorted input is found for each species, it will be noticed that the ratio for perch is considerably larger. This factor coupled with the larger β-factor for perch accounts for the marked difference in the increases in production rates for the two species.

The new process time for the lower half of the perch fillets is found to be 58 per cent of the previous time with unsorted input.

5.5. Conclusion

From the foregoing analysis, the theoretical advantage of splitting the input to IQF systems has been clearly established. From observations of local IQF processes, the indication is that presorting is not done; instead the process rate is set on the basis of a high percentage of the total weight of the distribution, 80 per cent being a common figure. This means that the thermal centres of 20 per cent of the input do not reach the final prescribed temperature and, consequently, that fraction may not be frozen adequately and may be of inferior quality.

The split in this analysis has been arbitrarily chosen to be at the median of the input and need not be the optimum split either from a theoretical or practical point of view. In the practical situation, the concern is that all units are operating uniformly and are being utilized equally. This may pose a problem when units are being run at different rates leading to completion of the processes at different times. In the theoretical development, a continuous supply of fillets has been assumed with each input batch exhibiting identical distribution characteristics. Such assumptions may not be borne out in practice. Despite such possible discrepancies in the theoretical development and practical situation, the analysis clearly shows that considerable advantage is to be gained by dividing the input to IQF systems. The calculations have been based on a specific set of heat transfer and process conditions, but the results are indicative of what might be expected with different IQF processes experiencing various operating conditions.

REFERENCES

REFERENCES

1. Brennan, J.G., Butters, J.R., Cowell, N.D., and Lilly, A.E.V. Food Engineering Operations. Elsevier Pub. Co. Ltd., 1969, p. 290.

2. Dyer, W.J. "Speed of Freezing and Quality of Frozen Fish," Fish Inspection and Quality Control, edited by Keruzer, R. FAO pub. Fishing News (Books) Limited, 1971.

3. Long, R.A.K. "The Cooling and Freezing of Fish - Some Thermodynamic Aspects," Ph.D. thesis, University of Aberdeen, 1954.

4. Charm, S.E., and Moody, P. "Bound Water in Haddock Muscle," Ashrae J. V.8, N.6, 1966.

5. Sussman, M.V. "A Study of the Behavior of Water in Frozen Fish Tissue using Nuclear Magnetic Resonance and X-Ray Powder Diffraction Techniques." Report to U.S. Bureau of Commercial Fisheries, Technological Laboratories, Gloucester, Mass. Tufts University, 1965, cited from Charm, S.E. and Moody P. "Bound Water in Haddock Muscle," Ashrae J. V.8, N.6, 1966.

6. Anon. "Code of Practice and Recommendations for the Handling of Quick Frozen Foods" (National Association of Frozen Food Producers: undated) cited from Brennan, J.G., et al. Food Engineering Operations, Elsevier Pub. Co. Ltd., 1969, p. 283.

7. Anon. "Recommendations for the Processing and Handling of Frozen Food" (Int. Inst. of Refrig., 1964), cited from Brennan, J.G. et al. Food Engineering Operations, Elsevier Pub. Co. Ltd., 1969, p. 283.

8. Jason, A.C., and Jowitt, R. "Physical Properties of Foodstuffs in Relation to Engineering Design." Torry Memoir 340, reprinted from Dechema-Monographien, V.63, 1969, p. 21.

9. Albasiny, E.L. "The Solution of Non-Linear Heat Conduction Problems on the Pilot Ace." Proc. I.E.E. 103, Part B, Suppl. 1, No. 4, 1956, cited from Jason, A.C., and Jowitt, R. "Physical Properties of Foodstuffs in Relation to Engineering Design." Torry Memoir 340, reprinted from Dechema-Mono. V. 63, 1969, p. 21.

10. Charm, S.E., Brand, D.H., and Baker, D.W. "A Simple Method for Estimating the Freezing and Thawing Times of Cylinders and Slabs," Ashrae J. V. 14, N. 11, 1972.

11. Lambourne, A. "The Temperature Distribution in a Slab on Freezing and Thawing," unpublished paper, Torry Research Station, Aberdeen, 1970.

12. Tarnawski, W. "Mathematical Model of Frozen Consumption Products." Int. J. Heat Mass Transfer, V. 19, N. 1, 1976.

13. Westphal, K.O. "Series Solution of Freezing Problem with Fixed Surface Radiating into a Medium of Arbitrary Varying Temperature." Int. J. Heat Mass Transfer, V. 10, 1967, p. 195.

14. Keller, G.J., and Ballard, J.H. "Predicting Temperature Changes in Frozen Liquids." Ind. Eng. Chem. V. 48, N. 2, 1956.

15. Tien, R.H., and Geiger, G.E. "A Heat Transfer Analysis of the Solidification of Binary Eutectic Systems." J. Heat Transfer, V. 9, N. 8, 1967.

16. Bakal, A., and Hayakawa, K. 1970. "Heat Conduction in an Infinite Slab during Freezing or Defrosting when the Phase Change occurs in a Range of Temperatures." Presented at the annual meeting of the Inst. of Food Tech., San Francisco, 1970, cited from Bakal, A., and Hayakawa, K. "Heat Transfer during Freezing and Thawing of Foods." Advances in Food Research, V. 20, Academic Press, 1973, p. 230.

17. Carslaw, H.S., and Jaeger, J.C. Conduction of Heat in Solids. Claredon Press, Oxford, 1959, p. 282.

18. Charm, S.E. Fundamentals of Food Engineering. 2nd edition, AVI Publishing Co., 1971, p. 238.

19. Cowell, N.D. "The Calculation of Food Freezing Times," Proceedings of Twelfth International Conference of Refrigeration. Madrid, 1967.

20. Plank, R.Z. Ges. Kalte-Ind. V. 20, 1913, p. 109 (in German), cited from Ede, A.J. "The Calculation of the Rate of Freezing and Thawing of Foodstuffs." Mod. Refrig., V. 52, N. 3, 1949.

21. Nagaoka, J., Hotani, S., and Takagi, S. "Experiments on the Freezing of Fish in an Airblast Freezer," Proceedings of Ninth International Congress of Refrigeration, V. 2, N. 4, Paris, 1955.

22. Levy, F.L. "Calculating the Freezing Time of Fish in Airblast Freezers." J. of Refrigeration, V. 1. N. 3, 1958.

23. Mott, L.F. "The Prediction of Product Freezing Time." Aust. Refrig., Air Cond. Heat, V. 18, 1964, p. 16, cited from Bakal, A., and Hayakawa, K. "Heat Transfer during Freezing and Thawing of Foods." Advances in Food Research, V. 20, Academic Press, 1973, p. 247.

24. Bakal, A., and Hayakawa, K. "Heat Transfer during Freezing and Thawing of Foods." Advances in Food Research, V. 20, Academic Press, 1973, p. 248.

25. Baker, D.W., and Charm, S.E. "Freezing Rates of Seafoods as Influenced by Heat Transfer Coefficients of Liquid Refrigerants." Freezing and Irradiation of Fish, edited by Kreuzer, R., FAO Pub. Fishing News (Books) Limited, 1967.

26. Slavin, J.W. "Freezing Seafood - Now and in the Future." Ashrae J., V. 6, N. 5, 1964.

27. Van den Berg, L., and Lentz, C.P. "Effect of Composition on Thermal Conductivity of Fresh and Frozen Foods." Can. Inst. Food Sci. Technol. J., V. 8 N. 2, 1975.

28. Woolrich, W.R. "Specific and Latent Heat of Foods in the Freezing Zone." Ashrae J. V. 8, N. 4, 1966.

29. Ball, C.O., and Olson, F.C.W. Sterilization in Food Technology, McGraw-Hill, 1957.

30. Pflug, I.J., Blaisdell, J.L., and Kopelman, J. "Developing Temperature-Time Curves for Objects that can be approximated by a Sphere, Infinite Plate, or Infinite Cylinder." Ashrae Trans. V. 71, N. 1935, Part I, 1965.

31. Kreith, F. Principles of Heat Transfer. Int. Textbook Co., 1963, p. 130.

32. Ashrae Applications Handbook, 1974, p. 24.13.

33. Rasmussen, C.L., and Olson, R.L. "Freezing Methods as Related to Cost and Quality." Food Technology, V. 26, N. 12, 1972.