# FINITE ELEMENT ANALYSIS OF DYNAMIC STRUCTURE-MEDIUM INTERACTION WITH SOME REFERENCE TO UNDERGROUND NUCLEAR REACTOR CONTAINMENTS



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by

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To my wife

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ABSTRACT

A finite element solution is developed for the problem of time-history response of reinforced underground cavity subjected to dynamic disturbances of the underground environment. The cavity can be of any shape, reinforced by either rock bolts or any elastic liner, with bending stiffness taken into consideration. Available methods that can solve the cavity problem are examined and an available computer programme (plane stress) modified. Introduction of a new reinforcing plate element necessitated development of two new subroutines and the extension of a third one along with certain modifications in the other subroutines and the main programme for matching requirements. The modifications enable the determination of displacements and the internal forces -- time histories of the liner.

A quantitative study of the following parameters that affect the response of the cavity reinforcement and the surrounding medium is carried out: 1) cavity reinforcement, 2) cavity shape, 3) isolation of the entire structure from the surrounding medium by a soft, energy absorbing material and 4) properties of the filling material in the cut-and-cover structural type. The modified computer programme has applications to problems outside the field of two-dimensional rock-structure interaction such as the dynamic analysis of beams, plane frames and arches.

A finite element model is developed to simulate the dynamic analysis of infinite space. The results of the study indicate that reinforcing the cavity by a steel liner decreases the stresses in the medium

by about 10% while the use of a rock bolting pattern with about 80% of the amount of steel required in a reinforcing liner decreases the stresses around the cavity by 25% and more. The horseshoe shape proves to be the best among the various shapes considered decreasing the stresses by 10-15%. Large reductions (of the order of 80%) in shell (liner) membrane forces and bending moments are reached by isolating the entire structure from the surrounding medium by a soft, energy absorbing material, which agree with the results from another investigation. It is also pointed out that a proper selection of the properties of the filling material in the cut-and-cover structure can reduce the internal forces in the structure and the stresses in the adjacent medium. It is shown that a significant reduction cannot be achieved by a single property variation but only through a proper combination of different properties (Figs. 33 to 36). The results obtained from the new model indicate the need for further modelling work in the solution of earthquake interaction problems for underground cavities.

# CHAPTER I

# INTRODUCTION

## 1.1 General Remarks

The problem of analysing and designing underground unlined or lined cavities of various shapes to resist dynamic disturbances caused by nuclear explosions or earthquakes is of considerable importance. The applications could be in nuclear power plants, hydro-electric power complexes (e.g. Churchill Falls), oil and natural gas chambers, and tunnels in the mining and transportation industries.

Engineers are restricted to a certain extent by a lack of adequate literature in this area. Even for static loads, much of the design practice today is of a semi-empirical nature (Benson and Kierans (6)).

## 1.2 Statement of the Problem

The purpose of this investigation is to determine the elastic response of reinforced and unreinforced underground cavities of various shapes when subjected to ground motion resulting from nuclear blasts or earthquakes.

The time dependent variations in the displacement, velocity and acceleration amplitudes associated with representative points of the rock-structure system are obtained.

## 1.3 Layout

This section outlines the areas covered in this thesis.

The second Chapter is a brief literature review of material related to Rock-Structure Interaction in the following fields: rock mechanics; static analysis numerical or experimental methods; dynamic analysis of structures including both discrete structures and continua, numerical solutions of the equations of motion, wave propagation in solids, experimental techniques for analysing soil-structure interaction problems, solutions based on numerical methods different from the finite element method, finite element solutions of dynamic problems including wave propagation problems, existing techniques for modelling an infinite underground system by the finite element method.

The third Chapter presents the practical side of the problem including siting criteria, geotechnical data requirements, configuration, different methods of reinforcing a cavity, and design concepts.

Chapter four presents the general solution of the previously stated problem (section 1.2) including the formulation of a computer programme to give the time-history of the displacements and stresses in the cavity reinforcement (rock bolting or elastic liner) and the surrounding medium. Factors affecting the response of the reinforced cavity were investigated. The parameters considered are: i) cavity reinforcement (rock bolting and elastic liner), ii) cavity shape, iii) isolation material between the structure and the surrounding medium, and iv) filling material in the cut-and-cover structures. To formulate the general solution, a new plate element was introduced for simulating the cavity liner together with two new subroutines for calculating the time-history for displacements and stresses in the new plate elements. Modification of the other subroutines and the main programme, originally prepared by Blakey (8), was carried out. The programme was also modified to solve both plane strain and plane stress-problems as the original programme was based on plane stress analysis. A new model for determining transient response to earthquake excitation is presented.

The fifth and the last Chapter describes the verification of the results obtained and checking of the correctness of the new subroutines used in modifying Ref.'s (8) computer programme. The results of the different shapes are compared with that of the circular shape previously analysed by Ref. 8 while those of the isolated structure are compared with the results of Costantino and Marino (15). A comparison between the results obtained by the new model and those of Kuhlemeyer (33) and Castellani (9) is presented. Discussion and conclusions are presented at the end.

A description of the computer programme is given in Appendix A. Appendix B presents the damping formulation in terms of critical damping. A numerical example of the dynamic analysis of underground cavity using the new model is presented in Appendix C.

## 1.4 Notation

| | |
|---|---|
| a | outer radius of the foam, and compressional wave velocity |
| b | length along the discontinuity halfway toward both adjacent nodes, and the sear wave velocity |
| $b\sigma_r$ and $b\tau_{r\theta}$ | radial (normal) and tangential viscus stresses respectively |
| $[\bar{B}]$ | transformation matrix |
| $C_1$ and $C_2$ | coefficients |
| $C_c$ | critical damping |

| | |
|---|---|
| $C_i$ | seismic velocity |
| $Cl_i$ | the compressional wave-length for the frequency $\omega i$, $Cl_i = 2a\pi/\omega i$ |
| $[C]$ | damping matrix |
| $E$ | modulus of elasticity |
| $f_n$ | average normal force over length b |
| $f_t$ | tangential load per unit length |
| H/L | roof, rise to span ratio of a horseshoe shape |
| H'/L' | wall, rise to span ratio of a horseshoe shape |
| $K_{mv}$ | factor between 0 and 1 |
| $[K]$ | stiffness matrix |
| $[\bar{K}]$ | stiffness matrix in the global coordinates |
| L | radial penetration of rock bolt |
| $m_{nv}$ | virtual mass of the fluid |
| $[M]$ | mass matrix |
| n | displacement mode number |
| $P_{ni}$ | force fector at the discontinuity node i in the n-direction |
| $\{P\}$ | exciting load vector |
| $\{P\}_o$ | zero force vector |
| $\{P\}_R$ | applied load submatrix |
| R | distance source-boundary |
| S | span of the underground opening |
| $Sl_i$ | shear wave-length for frequency $\omega_i$ |
| $S_o$ | cohesive shear strength |
| $[S]_R$ | reduced stiffness matrix |
| $[SS]$ | system stiffness matrix |

| | |
|---|---|
| $T_n$ | period of vibration in the n-th mode |
| $T_o$ | tensile strength of the medium |
| $\{\ddot{U}\}$, $\{\dot{U}\}$ and $\{U\}$ | acceleration, velocity and displacement vectors respectively |
| $\{U\}_R$ | horizontal and vertical displacement vectors |
| $\{U\}_o$ | submatrix of the unknown rotations |
| $Y$ | slip criterion |
| $\alpha$ | angle of inclination of the frame element w.r.t. the horizontal global axis |
| $\sigma_r$ | theoretical radial stress |
| $\rho$ | medium density |
| $\tau_{r\theta}$ | theoretical tangential stress |
| $\nu$ | Poisson ratio |
| $\mu$ | the coefficient of friction |
| $\Delta_{mn}$ | $K_{mv}\ m_{vn}$ |
| $\xi_m$ | critical damping ratio |
| $\omega_n$ | the circular frequency of the n-th mode |

CHAPTER II

REVIEW OF LITERATURE

## 2.1 General

The general underground cavity problem can be analysed using the finite element method and/or any other numerical approach, such as the lumped parameter method (or soil-spring method or compliance method) using equivalent frequency-independent foundation springs, dashpots and masses. Agrawal (2) has indicated the finite element to be superior for deep embedments, inhomogeneous soil strata and flexible foundations.

## 2.2 History of Rock Mechanics

While the medium could be soil or rock this investigation is restricted to rock media of RQD 75%.

The history of rock mechanics goes back to the second half of the last century when the first attempts in investigating the mechanical and physical properties were carried out. However, most researchers regard 1950 as the year in which systematic research into rock mechanics began in the U.S.A. The history is now well documented in a number of texts, e.g. Széchy (55), Obert and Duvall (41), and Jaeger (27) and (28). Refs. 20 and 47 include some of the recent papers which cope with the theoretical, laboratory, and field researches in rock mechanics.

## 2.3 Analysis

### 2.3.1 Static Analysis

Until 1950, virtually all investigations for evaluating stresses around underground cavities were based on exact analytical methods derived from the mathematical theory of elasticity. As a result, the shapes of cavities, which could be properly analysed, were limited to those expressible by simple equations such as circular and elliptical tunnels and spherical and spheroidal cavities. Furthermore, the rock or soil medium was normally assumed to be continuous, homogeneous, isotropic and elastic. For shapes which are not geometrically simple, experimental techniques such as photoelasticity and geotechnical modelling were used for analysis. However, the solution was time consuming and expensive.

With the advent of the high-speed digital computer and the rapid development of numerical methods of analysis, the finite element method (FEM) proves to be one of the most powerful numerical techniques for the stress analysis of complex structural systems. The first published computer programme for an underground cavity analysis using the FEM was that of Wilson (60) in 1963. Sigvaldason, Benson and Thompson (54) have given a brief formulation of the finite element method with typical examples to demonstrate the versatility of the method in analysing underground cavities. Extensive static finite element analyses have already been carried out for noncircular cavities in rock for the Churchill Falls project (6). Yu and Coates (62) developed a finite element programme to simulate stress distributions around typical, irregularly-shaped mining openings. The simulation takes into account the actual non-homogeneous mechanical properties of the rock and the possibility that the stresses are produced by gravity and tectonic forces. Watson, Kammer, Langley, Selzer and Beck (58) studied a horseshoe shaped containment for variations of required arch and wall thicknesses as functions of rock quality, seismic loading, rise-span ratio and span were determined. The study included the determination of variations in thickness for different concrete strengths and percentages of reinforcing steel, and a comparison of the required thicknesses for flat and parabolic arch walls. Kulhawy (35) stated that "as of this writing (1974), there have been no published journal articles, to the best of the writer's knowledge, which have established criteria for appropriate modelling of underground openings using the FEM." One of the purposes of this study was to establish some criteria for modelling underground openings in rock by the FEM. For the sake of clarity and simplicity, these studies were limited to plane strain analyses of homogeneous, linear elastic rock masses. A simulation of the incremental excavation was presented. Ref. 35 concludes that:

> 1. Arbitrary linear strain quadratic elements are most appropriate. 2. Incremental excavation modeling should be employed for all field cases except when full face excavation occurs or when the rock is treated as a homogeneous, linear elastic mass. 3. Incremental excavation can be modeled very effectively using the techniques described herein. 4. A minimum of 125-150 elements should suffice for analyses of simple structures in homogeneous rock where there is a plane of symmetry and only one-half of the system need be analyzed. 5. Boundaries of the finite element mesh should be located at least 6 radii away from the centre of the opening to insure that the computed stresses and displacements are within an error of less than 10 per cent of the theoretical. For more accurate results, the boundaries must be extended further.

### 2.3.2 Dynamic Analysis

#### 2.3.2.1 Methods Other Than the Finite Element Method

Gilbert (22) studied the stresses in the medium using a "high frequency" approximation similar to that in geometrical optics. The method does not allow the solution to be carried out for sufficient time to obtain the maximum stresses. Pao (43) considered the stresses around a hole in a thin plate when enveloped by a plane stress wave varying harmonically in time and space behind the wave front, i.e. the plane stress problem corresponding to the plane strain problem treated herein. In the analysis presented by Baron and Parnes (5) the displacements at the boundary of an unlined cavity were obtained by an integral transform approach. The solution of the transformed equations was expressed using Hankel functions and the evaluation of the inverse transform needed considerable computational effort. Values of displacements obtained were then used as influence coefficients in determining the displacements of the shell.

Paul (45) analysed the effect of a plane stress wave incident on an unlined cylindrical cavity in an elastic medium. The reflected and diffracted waves were described in terms of displacement potentials which represent outgoing shear (s) and dilatational (P) waves. A method was developed for determining the values of these potentials. Logcher (36) used the same method as that of Ref. 5 and extended it to include stresses away from the boundary. Also, possible failure mechanisms based solely on the elastic stress state were discussed. Results were presented in the form of contour maps of principal stresses at each time interval. The orientations of possible failure planes were also presented. Ref. 45 described a different method for solving the partial differential equations which has several advantages over the approach of Ref. 5. The solution involves superposition of the stress field of an incoming plane step wave and the stress field corresponding to waves which diverge from a line source. In order to satisfy the boundary conditions, the sum of the stresses due to the diverging wave and the incident stress, expanded in Fourier series, were set equal to zero at a fixed radius from the line source. In general this results in two linear integral equations, one from the radial stress condition and one from the shear stress condition, which must be solved simultaneously for two unknown functions appearing in the integrands of the integrals. The integral equations were solved numerically in order to determine these unknown functions. A Fortran programme was used for solving the simultaneous integral equations and computing the various stresses at the boundary and away from it.

The results obtained from the computer were checked by solving the same basic equations by another method called the short-time solution, which is only valid when the variable in the Taylor expansions, time, is small. The short-time solution was used not only to check the accuracy of the machine solution for short-times, but also to provide the initial values of the unknown functions involved in the two linear integral equations which were obtained from the boundary conditions as stated above. It also determined the analytic character of the solution at short times, and gave insights useful in developing the integration process. As a further check on the computer solution, the static stresses on the boundaries and in the medium were compared with those obtained from the machine solution at very long times.

Yoshihara (61) studied the interaction of plane elastic waves with a thin, hollow, cylindrical shell embedded in an elastic medium and obtained the response of the shell to a plane stress wave whose wave front travels in a direction perpendicular to the cylinder axis and envelops the shell. The shell was assumed to be elastic, isotropic, homogeneous, and of infinite length surrounded by an elastic, isotropic, and homogeneous medium whose motions conform to the ordinary theory of elasticity.

The response of the shell was studied by expressing the two components of displacement, radial and tangential, in terms of Fourier Series, each term of which represents a mode. The equations of motion of the shell in vacuo were derived from expressions giving the strain and kinetic energies due to generalized external forces. Forces on the shell result from the stresses in the medium at the boundary. Stresses in the medium were taken to be the sum of the stresses due to the incoming stress wave expressed in terms of Fourier Series whose coefficients are known, and those due to the reflected and diffracted effects expressed in terms of a pair of displacement potentials representing the effects of waves diverging from the axis of the shell. The two pairs of coupled integro-differential equations in terms of the generalized coordinates of the shell and the displacement potentials were solved mode-wise by a step-by-step, iterative integration technique using the Newmark-Beta Method, to obtain the values of the potential functions, the accelerations, velocities, and displacements of the shell. The shell stresses were found from the displacements, and those in the medium from the potential functions.

Although the equations account for an infinite number of modes, only the first three modes were considered in detail. The computed solution was compared with values obtained from a series expansion of the equations, which is valid for short times, and with the static solution based on the theory of elasticity to which the general solution should approach asymptotically.

Mow (37, 38) solved the same problem by the method of undetermined parameters. The dynamic and static results were presented for the case of a harmonically varying incident wave. Ref. 37 reviewed the methods of dynamic analysis and through an attempted correlation concluded that the dynamic stress concentration factors for cylindrical shapes underground are about 10 to 20 percent larger than the static ones. Also, the effect of lining the cavity was studied. A time requirement equal to nearly three transit times (one transit time being the time for the stress wave to traverse the opening, which is a function of the wave velocity and the diameter of the opening) was indicated for the stresses to reach its peak values. The investigation did not refer to radial stresses as these tend to be zero for the unlined case in the region near the opening and increase in the lined case where the liner provides support in the radial direction through bending of the shell and increases stiffness of the opening near the cavity.

The exact solution of Ref. 37 is not satisfactory for the general case of a rock opening, where the elastic properties can vary through different regions of the continuum. Additionally, for openings which may be rock-bolted, it would appear that no analytical solution can be formulated to be sufficiently general to cover the random location and varying loading due to rock bolting.

2.3.2.2 Finite Element Method (FEM)

Costantino (12 to 15) presented the use of numerical (finite element method) and experimental techniques in studying the dynamic response of buried cylinders. The influence of different variables on the displacements and stresses in the shell and the surrounding medium was studied. The concept of isolating the entire structure from the surrounding medium for structural safety was investigated. The study included the behaviour of a foam-isolated, thin, cylindrical shell embedded in Ottawa sand during the passage of a plane pressure pulse to develop a theoretical and/or semi-empirical method of predicting the response of structures encompassed with energy absorbing materials. This enabled the determination of changes introduced by an encompassing layer of isolation material (closed cell, polyurethane foam) in the response characteristics of a cylindrical shell tunnel model buried in soil subjected to a plane pressure pulse. The results of the study indicated the beneficial effect of the isolation layer in reducing the shell membrane forces and bending moments.

In the solution presented by Refs. 12 and 13 which determined the "free-field" motions at a given point in rock media to define the input loading on a ring structure, the true interaction phenomenon between the ring and the rock was not considered as the effect of the ring on the rock displacements was not accounted for. Furthermore, according to Ref. 8, instabilities appeared in the numerical method and the resulting computer code required large amounts of computational time involving complex computer systems, data manipulation via tape drives and other devices. This produced unsatisfactory results.

Ref. 8 presented a solution to the problem of time-varying response of reinforced circular cavity in rock subjected to blast excitation using the finite element method for modelling and the Newmark Beta Method for solving the resulting equations of motion. The computer programme presented by Ref. 8 gives the time-history for stresses and displacements of the rock media and reinforcement of the circular cavity. Ref. 8 studied the effects of changing many variables in a typical finite element formulation, such as mesh size, element shape and size, and time step of integration. The stability of the linear acceleration method as applied to multi-degree of freedom systems with regard to step size was also investigated. A table summarizing several recommendations of the time interval for the different methods of numerical integration was also presented. However, the solution was restricted to circular shapes. Although Ref. 8 considered the mesh size and the element shape and size, these variables were not related to the type of load excitation. Based on the work of several investigators, including his own, Kuhlemeyer (34) pointed out that "a maximum element length equal to one-eighth of the wavelength of the slowest body wave propagating in the elastic material is recommended (based upon experience) for analysis of two-or three-dimensional layered media."

2.3.2.3 Experimental Techniques

The experimental work on buried cylinders has been reviewed by Kierans, Reddy and Heale (31). To include the effect of medium-structure interaction, Costantino and Vey (16) suggested a virtual mass of the buried shell to be:

$$\Delta m_n = k_{mv} \, m_{vn}$$

where $m_{vn}$ = virtual mass of a fluid

$$= \rho a \frac{n}{n^2+1}$$

in which $\rho$ = medium density

$a$ = outer radius of foam

and $n$ = displacement mode number

$k_{mv}$ = fraction between 0 and 1

This is similar to the well-known added mass concept used in hydrodynamic interaction where the added mass is taken equal to that of the displaced water, see for example Ref. 48.

Ref. 16 concluded that crushable foams can significantly reduce the loads on the primary structure. Costantino and Marino (15) pointed out that large reductions in the shell membrane forces and bending moments are produced when isolating the entire structure from the surrounding medium by a soft, energy absorbing layer.

## 2.4 Boundary Conditions in Transient Response Problems

The rigid boundary conditions used by Ref. 8 can only be used for analysing blast type of loading as the stress field due to the incident wave will be unaffected by reflected wave effects only for the first few milliseconds of the load application period. This is the limitation of this type of boundary in obtaining the response due to an earthquake type of loading with a duration of 30 seconds. Although the total energy released even by a large nuclear blast is small compared with that released by a moderate-sized earthquake and blasting vibrations contain only a limited range of principal frequencies compared to those of earthquakes, the general character of blast-excited ground motion is sufficiently similar to earthquake-excited ground motion. This enables large blasts to provide a useful technique for studying the structural response to earthquakes.

To overcome the problem of reflections at rigid boundaries, which modify the stress and displacement in the medium and the structure, Ref. 33 used energy absorbing boundaries. The basic purpose of this absorbing or viscous boundary is to control the ratios between the transmitted energies of the reflected and incident waves. These energies can be computed from the amplitudes of the reflected and incident waves respectively as indicated by Ref. 33.

In the one-dimensional case the direction of wave propagation is known, which simplifies the problem of wave energy absorption. For complicated two-dimensional mixed boundary value problems neither wave directions nor the relative magnitudes of energy associated with the various types of elastic waves are known. This makes it impossible to develop an exact energy absorbing boundary.

Ref. 9 used the same technique as that of Ref. 33 and examined the effectiveness of the artificial boundary conditions developed by Ref. 33. Ref. 9 evaluated the errors involved in the boundary conditions defined by Ref. 33 in analysing plane body waves. The errors were found to be proportional to the ratio of the wave length to the distance between the source of excitation and the artificial boundary according to the following expressions:

For compressional waves

$$\left|\frac{\sigma_r - {}^b\sigma_r}{{}^b\sigma_r}\right|_w = \frac{cl_i}{R}\,\frac{1-3\nu}{2\pi(1-\nu)} \qquad (2.1)$$

For shear waves

$$\left|\frac{\tau_{r\theta} - {}^b\tau_{r\theta}}{{}^b\tau_{r\theta}}\right|_w = \frac{sl_i}{2\pi R} \qquad (2.2)$$

where (see Fig. 1):

$\sigma_r$ and $\tau_{r\theta}$ are the theoretical stress along the direction r within an infinite space.

${}^b\sigma_r$ and ${}^b\tau_{r\theta}$ are respectively normal and tangent viscous stresses used by Ref. 33.

$cl_i$ and $cl_i$ are the compressional and shear wave-lengths for the frequency $w_i$, $cl_i = 2\pi a/w_i$ and $sl_i = 2\pi b/w_i$; a and b are the P and S wave-velocities.

$\nu$ is the Poisson ratio.

R is the distance source-boundary.

In the two large and small models shown in Fig. 2 investigated by Ref. 9, the following conclusions were drawn: 1) simulation of infinite space is more accurate with the large model; 2) in both models the errors increase at the end of the excitation, which is governed by the short frequency content of the exciting pulse.

A new model for simulating an infinite space will be presented in this thesis in Chapter four. Using this model the transient response problems can be analysed.

## 2.5 Wave Propagation in Solids

The basic theory of wave propagation in solids has been documented by Kolsky (32) and Timoshenko (56), and recently (1973) in the two texts of Achenbach (1) and Pao and Mow (44). It has been also explained and briefly summarized by Refs. 8 and 3. The application of the FEM in wave propagation problems has been studied extensively by Costantino (17), Ref. 33, Ref. 34, and recently by Bahar (4).

### 2.5.1 General

Impact (or impulsive) loads are characterized by an almost instantaneous rise in magnitude followed by a rapid decrease, the duration of the impact load being of the order of a few milliseconds or microseconds. Most frequently the load is applied over only a small area of the body.

Structures subjected to impact loads behave very differently than when subjected to conventional loads. Under conventional loading, each part of the loaded body influences stresses and deformations that occur in other parts of the body. Under impact, at the commencement the remote portions of the body remain undisturbed and deformations produced in the vicinity of the loading are propagated through the body in the form of elastic waves. If the dimensions of the body are large, the time taken by the waves to traverse the body becomes of practical importance and should be considered.

## 2.5.2 Wave Propagation in the Presence of Discontinuties

The theory of wave propagation in homogeneous materials has been adequately described by a number of references stated in section 2.5 This section considers the influence of discontinuties, such as a cavity, an underground structure, or discontinuity planes in the medium due to geologic surface of weakness or defects in the rock mass joints (fractures), faults, unconformities, bending planes, or metamorphic foliation.

The behaviour of the cavity is dependent upon the "free-field" ground motion modified by the presence of the cavity. Structural response is also a function of the relative magnitudes of the effective duration of the stress wave compared with the natural period of vibration of the structure. Both these considerations could be unimportant if the stiffness of the structure could exactly match that of the removed rock and the stress wave could pass through undiminished. But this is not a practical solution to the problem.

Wang and Voight (57) described a two-dimensional discrete element (three noded triangular) method for the solution of rock mass stress analysis problems. Geological defects were included in the analysis-modes of displacement involving slip and separation along discrete discontinuity surfaces. Dual nodal points were utilized for systems of non-intersecting planar discontinuities (e.g. discontinuity traces 'A' and 'B' of Fig. 3). The general equation of equilibrium was then written in the normal manner after adding a new force vector to the ordinary one (including initial and body and external forces). The new force vector accounts for the forces at nodal points along the discontinuity. The solution was obtained by allowing two degrees of freedom at each discontinuity node and applying the Coulomb-Navier Criterion with tension 'cut off' (Fig. 5). According to this the criterion for an average normal force, $f_n$, acting over length b, can be expressed in the form:

$$f_n = \frac{P_{ni}}{b} \leqslant T_o \tag{2.3}$$

Slip will occur if

$$Y = |f_t| - S_o - \mu f_n = 0 \tag{2.4}$$

where

$P_{ni}$ is the force vector at the discontinuity node i in the n-direction (see Fig. 4).

$T_o$ is the tensile strength of the medium at node i.

$f_t$ is the tangential force per unit length.

$S_o$ is the cohesive shear strength.

$\mu$ is the coefficient of friction.

### 2.5.3 Finite Element Solution for Wave Propagation in Layered Media

#### 2.5.3.1 General

The principal advantage of the finite element analysis in this investigation lies in the arbitrariness with which the element shape, size, and properties may be specified. No special consideration is required in the use of the associated computer code which makes it a very powerful method in analysing irregular shapes of underground openings located in layered soil deposits, taking into account the coupling between adjacent openings in analysing a cavity group.

In all discrete approaches (finite element, lumped mass, etc.), the governing partial differential equations of motion of the continuum are reduced to a finite set of ordinary differential equations with respect to time which describe the motion of a finite number of points (or lumped masses) spaced over the free field. As the finite element method is well established (Zienkiewicz (63) and Desai (19)), comments will be restricted to some particular features related to dynamic analysis.

When using the finite element method in modelling, Ref. 34 recommended a maximum element length equal to one-eighth of the wave length of the slowest body wave propagating in elastic material for the analysis of two- or three-dimensional layered media. The equations of motion defining the dynamic behaviour of the finite element model are given by:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{P\} \quad (2.5)$$

in which $[M]$, $[C]$, and $[K]$ are the mass, damping, and stiffness matrices and $\{\ddot{U}\}$, $\{\dot{U}\}$, $\{U\}$, and $\{P\}$ are the acceleration, velocity, displacement, and exciting force vectors respectively.

The stiffness matrices for various types of elements and their assembly can be found in a number of references. Lumped and consistent mass matrices have also been studied thoroughly. The damping matrix can be a simple diagonal matrix as used by Ref. 33. On the other hand, it can be symmetric if expressed in terms of the consistent mass and stiffness matrices. If a lumped mass matrix is to be used, the damping matrix will be a diagonal one as used by Ref. 8 and can be expressed in the form:

$$[C] = C_1[M] + C_2[K] \quad (2.6)$$

where $C_1$ and $C_2$ are constants to be chosen to provide the desired effect. This makes the computation much simpler and the next question is to relate the constants $C_1$ and $C_2$ to the critical damping $C_c$ or the ratio ($\xi$) of damping C to the critical damping $C_c$. This is done, as shown in Appendix B, and the constants $C_1$ and $C_2$ are

$$C_1 = \xi_m \omega_m$$

$$C_2 = \xi_m / \omega_m$$

where $\xi_m$ is the damping ratio and $\omega_m$ is the angular frequency ($\sqrt{\frac{K}{M}}$) for the m-th mode. In practice, any desired damping ratio may be selected merely by using one of the constants $C_1$ or $C_2$ and setting the other equal to zero.

### 2.5.3.2 Solution of the Equations of Dynamic Equilibrium

Eqn. 2.5 can be solved by modal superposition or, as shown here, by direct integration (step-by-step). The initial conditions for Eqn 2.5 involve the displacement and velocity at the time, $t = 0$; therefore, the direct integration procedure requires at least this amount of information at the previous time, $t_n$, to predict the state of motion at the current time, $t_{n+1}$. Also, the value of the acceleration at time, $t_n$, or the state of motion at earlier time, $t_{n-1}$, is required to start the integration.

When the characteristics of Eqn. 2.5 are defined in terms of the natural frequencies and mode shapes, the frequency spectrum ranges from the lowest, or fundamental, frequency to an infinite limit point in continuous systems. For discretized systems, previously stated in section 2.5.3.1, Nickell (40) indicated that as the infinite limit point no longer exists, a frequency that corresponds to the most rapidly varying mode shape is to be considered instead of that at the infinite limit point. This frequency is called the "cut-off frequency." If the discretized system is excited by forcing functions contain frequencies higher than the cut-off frequency as might be encountered in wave propagation problems, noise (random spatial response) is generated in the cut-off modal response.

2.5.3.3 Layered Media

Recently Ref. 4 developed a method using a transfer matrix approach to the elastodynamics of layered media. The method is based on the matrix formulation of the Laplace transformed one-dimensional wave equation. The matrix formulation shows that the need for matching interface conditions explicitly is avoided by imposing the continuity of the state vector across each interface. This is accomplished through the continued multiplication of transfer matrices of the various layers as interfaces are traversed. Therefore, the size of the matrix does not exceed two by two, and is independent of the number of layers contained in the medium. In contrast, the classical formulation of the problem necessitates the simultaneous solution of 2n linear algebraic equations in 2n unknowns. (n is the number of layers.)

2.6 Soil-Structure Interaction

The subject has been studied in considerable detail by Allgood (3), Ref. 2, Clough (10), Whitman (59), and Ruse and Dawkins (51). The last reference presented a three-dimensional finite element analysis of soil-structure interaction. The 3-D analysis was also recommended by Hadjian (25) in order to obtain a general solution for the soil-structure interaction problem. Ref. 8 presented a brief summary for the problem of rock-structure interaction and the problem was solved using linear strain rectangular (LSR) and constant strain triangular (CST) elements without considering the rock and the structural response separately. Ref. 3 summarized the available knowledge on soil-structure interaction outlining the major areas of work.

The application of the FEM to earth-structure interaction was presented by Ref. 10 with numerous examples of simple retaining structures, temporary earth support systems, partially buried continuous wall and base slab structures, underground support systems (cavities supported by rock bolts and openings supported by concrete linings).

CHAPTER III

PRACTICAL APPLICATIONS

3.1 Why Underground?

The following advantages of underground siting were summarized from Reddy and Kierans (49).

1. Protection against natural and man-caused damage to structures.
2. Improvement in plant configuration by the ability to design three-dimensionally as opposed to surface structure two-dimensionality.
3. Reduction of the seismic motion when compared with that at the base of a surface structure.
4. Separation of component facilities such as containment structure and turbine plant structure.
5. Avoiding the risk of an undetected hidden fault when using the current exploratory techniques for the location of an underground site.
6. Three-dimensional support capability provided by the surrounding medium to functional structures such as a turbine-generator system etc.
7. Reduction of power transmission costs and construction periods by location close to load centres.
8. Savings in buildings, substructures and foundations.
9. Use of excavated rock as construction material.

3.2 Siting Criteria and Structural Selection

The broad siting guidelines have been reported by Ref. 58.

3.3 Geotechnical Data Requirements

Geotechnical investigations including field and laboratory tests and field surveying are well documented in texts on Rock Mechanics previously listed in Chapter two (section 2.2). Ref. 54 reviewed the geotechnical considerations in analysing underground openings in rock. Kierans, Reddy, and Heale (30) presented the required geotechnical data for the design of nuclear power plant facilities.

From the above literature it can be stated that the main steps that have to be followed in an underground investigation are: detailed geological maping, air photo-interpretation for major structural features, surface test pitting for surface geology, bore hole core drilling in sufficient detail to delineate any faults which may intersect proposed cavities, logging of all cores, compression test of rock samples, hydraulic pressure testing for water permeability, plate-jack testing to determine in-situ rock characteristics, and historic data on seismic loading experience for the general area.

3.4 Seismic Loading

Ref. 58 indicated that although the ground motions (accelerations, velocities, and displacements) occurring at some depth below the surface are less than those at the surface due to the attenuation of surface wave amplitudes away from the surface, since the wave lengths associated with earthquake frequencies are much higher than the probable depths of burial for underground facilities, significant attenuation may not be observed. Ref. 30 pointed out that due to soil layers, the seismic loading on an underground structure is not affected by the amplification of body (P and S) and surface (L) waves and that the magnitude of the seismic loading can be specified by response spectra or by actual accelograms. However, for low intensity stress waves travelling through the soil medium, there is no shear in the medium and the seismic velocity is that due to compressional waves only. The seismic velocity in this case can be approximated by: $C_1^2 = E/\rho$, where $C_1$ is the seismic velocity, E is the modulus of elasticity of the medium and $\rho$ is the medium density. In designing for this type of loading, the stiffness of the underground structures (tunnel liners, large diameter sewer pipes; installations of hydroelectric and nuclear power plants) is a very important factor in the induced moments and shears. It is, therefore, highly desirable to design with the largest possible flexibility compatible with the load to be carried. Furthermore, the joints along the axis of the tunnel should be designed to transmit the shear forces induced in them by the ground motion. Flexibility will permit the structures to follow the displacements of the medium and hence reduce the seismic forces to a minimum.

## 3.5 Configuration and Embedment

The layouts and the configurations of typical underground nuclear power plants, hydro-electric power complexes and tunnels in mining and transportation, are well documented in the literature. The experience in those fields recommends the deep locations of the facilities in order to provide additional protection from harmful radiation in the event of a seismic incident or a major internal accident.

### 3.6 Factors Affecting the Size of Underground Openings

Ref. 58 indicates that while limitations on the cavity size are primarily based on experience rather than on theoretical foundations, a first approximation can be made on the basis of elasticity theory if the excavation is for a flat roof cavity in massive sedimentary rock. However, the size and spacing of the cavities should allow conventional equipment to be used. To avoid the doubling effect of displacement amplitudes on the structure which occurs upon reflection of body waves at the earth's surface, Glass (23) indicates that the cavity should be located deeper than about one-quarter wave length from the surface.

### 3.7 Structural Types

The classification of primary underground containments and descriptions given by Ref. 30 are used in this section. The main types are included in Fig. 6.

i) Cut-and-Cover Structures. This type of structure is suitable for shallow locations in unloose soils to minimize the cost of retaining structures in the construction process and is only convenient in low intensity population areas. As the structure supports the fill this type is not economical in rock where the cavity is almost self-supporting and requires only slight additional reinforcement (rock bolting or elastic liner). However, a proper selection of the type of filling material with respect to the original properties of the medium can reduce the straining actions in the structure and protect it from the high pressures associated with nuclear blasts or earthquake excitations. This is explained in more detail in the last two chapters in sections 4.4, 4.4.2, 4.4.3, and 5.4.

ii) Unlined Cavity. When reinforced by properly designed rock bolting patterns, an unlined cavity develops the required adequate structural integrity for low-intensity seismic excitation. A procedure for superposition of the post-tensioned bolted rock stress field on the induced stress field to obtain the final state of stress has to be carried out.

iii) <u>Lined Cavity</u>. A lined cavity is required for high intensity seismic areas and for coping with hydrological conditions. Linings, in certain conditions, may reduce stresses to the order of 30% (Ref. 61). In the finite element analysis presented herein, the liner will be represented by thin shell and plate elements to simulate the curved and straight parts of the cavity configuration. The thickness of the liner is assumed to be small to permit the expression of all stress components in terms of a function which describes the deflection of the middle surface (Flügge (21)).

iv) <u>Lined Cavity with Annular Filling</u>. This type of construction protects the structure against possible large ground motions (Ref. 15 and Dawkins (18)). Sections 4.4, 4.4.2, and 5.4 in this thesis include studies of this type and the obtained results.

v) <u>Multiple Cavities</u>. The multiple cavities presented in Fig. 6 should be investigated with respect to final configuration and the relative locations. The investigations will lead to the optimum layout in which a minimization of tension stress zones could be achieved. This is done through a trial and error procedure. Interaction in the system complicates the exact dynamic analysis. However, preliminary design, as stated in Ref. 30, may be carried out by ignoring the structural effects of the separating media in the analysis. The final design will require more extensive and refined analysis.

## 3.8 Different Methods for Reinforcing the Cavity

The cavity may be reinforced either by rock bolting or lining. The modified computer programme presented in Appendix A can analyse any type of cavity reinforcement.

i) Rock Bolting. Rock bolting represents the cheapest method of reinforcing underground cavities. It is convenient in areas of low intensity seismic excitation. The types of rock bolts, design and their utilisation and installation along with the required field tests have been presented by Refs. 41 and 28, Goodman and Ewoldsen (24) and Coates and Sage (11). Rock anchors can also be used to i) increase the vertical load capacity of a pillar by compressing it horizontally, ii) stabilizing slopes including loose blocks and iii) in preventing slips at joints or faults. Rock bolts are installed in the inner surface of an excavation carved in an initially stressed body (Jerry (29)). At the time of installation the stresses around the excavation approach the final values for an unlined tunnel. However, Ref. 24 stated that "depending on the severity of the initial stress field and the manner of excavation, the pre-bolting stress state may approach the applicable elastic solution" and conclude that only average rock bolt pressure and not the length and spacing of bolts seems to affect the efficiency of the reinforcement. They also formulated a computer programme to determine the global stresses and plot the influence regions from which the costs of each trial design can be calculated. The solution is three-dimensional, and arbitrary joint orientations can be considered. An important part of the design and installation of a rock anchor is the calculation of the extension expected on tensioning, a loss of 10% of the original rock anchor load was reported by Ref. 11. Benson, Conlon, and Merritt (7) concluded that the required length of arch rock bolts is not affected by the span of the arch (see Fig. 7).

The principal advantages of rock bolting outlined in Ref. 30 are: "(a) increase of interlayer shear strength in laminated or jointed rock, (b) radial confinement around the gallery, and (c) passive action in preventing large deformations from destroying the keying action of joint blocks." Ref. 30 suggests the consideration of the following points in the installation of rock bolts:

> The spacings should be such that relationship between the structural stability of the cavity's rock bolted roof and walls and the possibility of inter-bolt rock separation is optimized as to cost. Rock bolts should be anchored within the zones of induced compressive stress surrounding the cavity. The sequences of bolting should be consistent with the excavation pattern -- typical rock bolt lengths in the crown ranged from 0.2 to 0.4 span and on the side walls 0.1 to 0.5 height, and bolts are to be tensioned to two-thirds of their yield strength.

ii) Reinforcing Liner. The liner could be steel and/or reinforced concrete, or prestressed concrete. The computer programme of Ref. 8 was modified to include a new plate element that simulates together with the shell element of Ref. 8 an elastic liner with bending stiffness for any irregular shaped cavity. Ref. 18 described different types of liners together with a discrete model of the liner-packing system for determination of the liner response. The total displacement was considered to consist of two parts: one corresponding to the rigid body translation in which the cavity retains its original shape and the second part corresponding to distortion of the original boundary. It was observed that the rigid body motions are much larger than those corresponding to distortion. A simple single-degree-of-freedom system was suggested to study the rigid body displacement of the liner.

### 3.9 Design Analysis of an Underground Cavity

Ref. 30 describes the design procedure of a cavity located in rock or rock-like material with a Rock Quality Designation (RQD) 75%. The following sections review the preliminary and final design procedures.

#### 3.9.1 Preliminary Design

i) Based on the theory of elasticity and/or field tests, determine the in-situ stress state with no cavity in the medium.

ii) From the project requirements and the geotechnical data, decide the configuration and embedment of the cavity.

iii) With the finite element model shown in Fig. 8, considering the in-situ stresses and introducing the required cavity, determine the induced-stress state. Several trials based on changing the shape and the relative location of the cavities (if more than one) may be carried out to determine the optimum solution where a minimization of the tension zones around the cavity is achieved.

iv) Based on the geotechnical data and the stress state (obtained above), establish the structural type. In the case of a cavity reinforced with rock bolts, determine the rock bolting pattern as indicated in section 3.8.

v) For multiple cavities, ignore the separating media and consider an enveloping opening.

#### 3.9.2 Final Design

i) Having obtained the initial stress state in the medium after excavating the cavity, a finite element time-history analysis should be carried out. A description of the computer programme is presented in

Appendix A.

ii) Superpose the dynamic stresses obtained from the time-history analysis on the static stresses also obtained from a finite element analysis to obtain the final true stress state (Sheha (53)).

CHAPTER IV

ANALYSIS AND RESULTS

## 4.1 General

This chapter presents the solution of the problem including the formulation of a computer programme to give the time history of displacements and stresses in the cavity reinforcement and the surrounding medium. The cavity could be of any shape and the reinforcement could be rock bolting or any elastic liner. Only one active reinforcement is considered at a time and when both rock bolting and elastic liner are used, the latter is ignored as a reinforcing support and assumed to only protect the cavity from radiation and moisture. The solution allows for layered media and the presence of a soft, energy absorbing packing material placed in the annulus between the face of the cavity and the structural liner. The different parameters that affect the response are investigated. A new model is presented which can simulate the dynamic analysis of infinite space problems.

## 4.2 Solution for an Unlined and Lined Cavity of Any Shape

For low-intensity seismic areas and high strength rock without discontinuities, rock bolting without lining represents a safe and economical reinforcement. Later in section 4.3.1 a numerical example for reinforcing the cavity by rock bolts is presented. For poor rocks (low strength and discontinuities) and soil deposits, rock bolting cannot provide the required structural integrity which makes the use of elastic (or rigid) liner essential.

Although the bending stiffness of the elastic liner of the cavity was considered in the solution in Ref. 8, the programme was suitable only for circular openings. Also, the solution was adequate only for blast-excited ground motion, since a true picture of the response could be obtained only for the first few milliseconds. The time-history of displacements and stresses is correct only until the exciting stress wave is reflected at the rigid boundary. This is why attention was focussed on these two problems and a new plate element included in modifying the computer programme of Ref. 8 to simulate together with the shell element formulated by Ref. 8 elastic liners of any shape. Also, a new finite model is constructed for the dynamic modelling of an infinite space. This model has a displaced, elastic, energy absorbing boundary and is presented in detail later in section 4.6

### 4.2.1 Plate System - Stiffness Matrix

To complete the cavity reinforcement stiffness formulation, the plate element must be integrated into the general system of the structure. This involves the introduction of the plate element stiffness matrix (6x6) into the overall structural system to be compatible with the shell elements of Ref. 8. Both plate and shell elements are taken as two-dimensional (plane) elements for the plane strain (or stress) analysis of the whole system. The elements, therefore, can be termed frame and circular elements.

The plate element stiffness matrix [K] presented by Przemieniecki (46) is used. Based on the orientation of the element with respect to the global coordinates (X and Y) as shown in Fig. 9, the transformation matrix is written in the following form:

$$
\left[\bar{B}\right] = \begin{bmatrix} -\text{Cos}\alpha & -\text{Sin}\alpha & 0 & 0 & 0 & 0 \\ \text{Sin}\alpha & -\text{Cos}\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\text{Cos}\alpha & -\text{Sin}\alpha & 0 \\ 0 & 0 & 0 & \text{Sin}\alpha & -\text{Cos}\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.1}
$$

Hence, the plate element stiffness matrix in the global coordinates $\left[\bar{K}\right]$ can be obtained from the relationship

$$
\left[\bar{K}\right] = \left[\bar{B}\right]^T \left[K\right]\left[\bar{B}\right] \tag{4.2}
$$

This is computed for each element in subroutine FRAMES developed by the author. Assembly of the individual element stiffness matrices to form the total stiffness matrix of the plate system including the rotational degrees of freedom at each node $\left[SS\right]$ is performed in the first part of subroutine FRAME also developed by the author in modifying the programme of Ref. 8. Since the elements representing the rock media are allowed only two degrees of freedom at each node, to assemble the system (the structure and the surrounding medium) into an integrated interactive one, the stiffness matrix of the structure $\left[SS\right]$ must be condensed to simulate the two-dimensional, linear loading at each node.

### 4.2.2 Reduction of the Plate Element Stiffness Matrix

The plate system stiffness matrix $\left[SS\right]$ formulated as a result of assembling each of the plate element matrices is of the order NSE3 by NSE3 (where NSE3 = 3* number of nodal points of the system), i.e. two orthogonal linear displacements and one rotation at each node. In the case studied here, the forces are applied at these nodes as a result of the displacements in the rock media. It is, therefore, sufficient to define the plate system coordinates in the directions in which loading occurs. Thus, the plate stiffness matrix must be modified to move all rotational terms to the last rows and columns, resulting in the following force-displacement relationship:

$$\begin{Bmatrix} \{P\}_R \\ \{P\}_o \end{Bmatrix} = \begin{bmatrix} [S]_{11} & [S]_{12} \\ [S]_{21} & [S]_{22} \end{bmatrix} \begin{Bmatrix} \{U\}_R \\ \{U\}_o \end{Bmatrix} \qquad (4.3)$$

The matrix $\{P\}_R$ represents the applied forces on the shell from the rock media and is a vector of the order NSE2 (NSE2 = twice the number of nodal points in the system). The plate stiffness matrix is partitioned accordingly, and the displacement vector $\{U\}_R$ represents the horizontal and vertical displacements at each node. The submatrix $\{U\}_o$ contains the unknown rotations that occur at each node. The submatrix $\{P\}_o$ is a null matrix, since there are no moments imposed at the nodes by the rock media. The required reduced stiffness matrix is defined as $[S]_R$ in the equation

$$\{P\}_R = [S]_R \{U\}_R \qquad (4.4)$$

Proceeding in the same way as Ref. 8 in deriving the reduced shell element stiffness matrix, the required reduced plate element stiffness matrix is evaluated from the following expression:

$$S_R = [S]_{11} - [S]_{12}[S]_{22}^{-1} [S]_{21} \qquad (4.5)$$

Formulation of the $[S]_R$ matrix following Eqn. 4.5 and its assembly into the total system stiffness matrix (including the rock media) are accomplished in subroutine FRAME.

#### 4.2.3 Computation of Displacements and Stresses

After assembling the overall system (the medium and the reinforcement) stiffness matrix allowing two degrees of freedom at each node, both horizontal and vertical displacements at each node are calculated for each time step of integration using the Newmark Beta Method. This is accomplished mainly in subroutine RECURS (2) and the MAIN programme. Stresses in the medium are then calculated in subroutines RECTS and TRIST as will be described later in section 4.2.5. Stresses in the shell and plate elements of the elastic liner are evaluated in the form of straining actions (normal, shearing forces and bending moments) at each node. This is accomplished in subroutine SHELLS which has been extended by the author to provide the solution for the new plate element as follows:

After formulating the reduced plate system stiffness matrix from Eqn. 4.5 and assembling it into the overall stiffness matrix as given in the previous section, Eqn. 4.4 is solved for the horizontal and vertical displacements at each node, i.e. the displacement vector $\{U\}_R$ is obtained. From Eqn. 4.3 the unknown rotations at each node of the plate elements, $\{U\}_o$ are evaluated in terms of $\{U\}_R$ as follows:

$$\begin{aligned} \{U\}_o &= -[S]_{22}^{-1}\,[S]_{21}\,\{U\}_R \\ &= -\,[TST]\,\{U\}_R \end{aligned} \qquad (4.6)$$

The multiplication of the first two matrices on the right hand side of Eqn. 4.6 $[TST]$ is computed and the values stored in subroutine FRAME called from SHELLS. After calculating all the displacements (horizontal, vertical and rotation), $\{U\}$, subroutine FRAMES is then called from SHELLS to construct the individual plate element stiffness matrix with all rotational terms moved to the last two rows and columns $[K]$. Hence, for each element, the straining actions $\{P\}$ are calculated according to the relationship:

$$\{P\} = [K]\,\{U\} \qquad (4.7)$$

In $\{P\}$ the first, second and the fifth elements are the normal, shearing force and bending moments respectively at the first node of the element while the other elements represent those at the second node.

4.2.4 Application

The formulation of the general system stiffness matrix (rock media plus cavity reinforcement) is essential to a consistent formulation of the proposed solution. The importance of such a solution is noticed when cavity reinforcement is required in high-intensity seismic areas. In such a case rock bolting is not adequate and an elastic liner is essential to provide the required reinforcement for the excited cavity.

A rigid liner can cause damage in the surrounding media depending on the relative motion of the rock media and the rigid body motion of the liner. Ref. 59 pointed out that in the case of embedment in soil, if the structure moves more than the medium, then the medium exerts a restraining effect upon the structure, but if the structure moves less than the medium, the medium exerts a driving effect on the structure.

4.2.5 Description of the Computer Programme

Appendix A presents the flow charts. After the input of data and several checks for completeness and consistency, the MAIN programme calls subroutine ASSEMB for assembly of the stiffness matrix for various types of elements BAR (for rock bolting), TRIAN, RECT (for rock media), SHELL and FRAME (for cavity liner). Subroutines SHELL and FRAME assemble the total liner stiffness matrix. Each of the arch components are calculated in ARC and the straight components in FRAMES. LOAD is then called and the equations are solved by matrix recursion. Displacements are calculated in RECURS (2), while in SHELLS, RECTS, and TRIST, stresses in the cavity liner (shell and plate elements) and medium (rectangular and triangular elements) are calculated for each time step of integration. Stresses in the rock bolts are also calculated in RECTS.

## 4.3 The Effect of Cavity Reinforcement on the Response

Rock bolting and elastic steel liner were investigated as cavity reinforcement. Each kind was used in reinforcing the same cavity (84 in.) outlined in Figs. 10 and 11 and excited by a 1 psi step pulse of infinite duration as shown in the same figures.

Since the compressional wave velocity for the assumed medium is approximately 150 inches per millisecond, only the time-history up to the first 3 m-sec. was determined to avoid the effect of the reflected waves from the used rigid boundaries. This is also valid for the other investigations in the following sections of this chapter.

### 4.3.1 Rock Bolting

The semi-circular roof of the 84 inch span cavity presented in Fig. 10 was reinforced by eleven 30 inches steel rock bolts 1/2 inch diameter @ 11 inches apart. Each of the vertical walls was reinforced by four 20 inches steel bolts 1/2 inch diameter @ 10 inches apart. Steel and rock properties are presented in the same figure. The model consists of 119 nodal points and 147 elements and is laterally excited by the step pulse defined in the previous section. Results are presented in Figs. 12 and 13.

### 4.3.2 Elastic Liner

A one inch thick steel liner was used to reinforce the same cavity described in the previous section. A model of 100 nodes and 120 elements was used (Fig. 14) and excited laterally by the loading function described in section 4.3. Fig. 15 presents the results.

## 4.4 The Effect of the Shape of the Cavity on the Response

For the same dynamic loading as that described in section 4.3, the shape effect has been investigated through different configurations outlined in the four Figs. 16, 17, 18 and 19. Only the horseshoe shape was subjected to a parametric study. Three elements at different locations for each of the cavity configurations were examined. Figs. 20, 21 and 22 present a comparative study of the shape effect on the time-history of the stresses in the three elements.

### 4.4.1 Horseshoe Shaped Cavity

The horseshoe shape has been recommended as one of the most suitable shapes for underground cavities (Refs. 55 and 58). In deciding the best relative dimensions of the horseshoe shape, the following parameters were investigated:

i) roof, rise to span ratio H/L

ii) wall, rise to span ratio H'/L', see Fig. 23.

All cases were investigated using the same model shown in Fig. 19 subjected to the same dynamic load. The results are presented in Figs. 24, 25 and 26.

### 4.4.2 Other Shapes

In hydroelectric power complexes, the geometry of the complex (transformer gallery, power house, and surge chamber) is an assemblage of irregular shapes (Fig. 27). The relative location of the cavities in the cavity group should be investigated to obtain the optimum solution based on a minimization of tension zones in between the cavities. The modified computer programme presented in Appendix A can easily handle such a problem for short time dynamic analysis, since the rigid boundary has not been modified in the programme of Ref. 8. Later in this chapter, the standard structural programme STRUDL-II is used in analysing the displaced, elastic, energy absorbing boundary used in the proposed finite dynamic model which simulates the dynamic analysis of infinite space.

## 4.5 Isolated Structures

At the present time considerable effort is being directed toward the design of structures capable of withstanding the extremely high pressures and accelerations associated with nuclear blasts or earthquake excitations.

The idea is to isolate the structure from the surrounding medium using a soft, elastic, energy absorbing material between the liner of the cavity and the medium. This isolating material is easily deformed to absorb the energy produced by the exciting load, i.e. acting almost like a rubber ring protecting the structure from any disturbances in the surrounding medium. On the other hand, a crushable material could be used in isolating the entire structure providing the required protection. In this case when the stresses in the isolating material reach the crushing strength, no more load will be transmitted to the structure through the medium due to the arching effect of the crushed material. This indicates the need for further work to account for the elimination of the cracked and crushed elements from the system.

### 4.5.1 The Effect of Isolation on the Response

To study the effect of isolating the structure entirely from the surrounding medium by soft, energy absorbing material, the model shown in

Fig. 28 was considered. The model consists of a steel cylindrical shell 84 inches in diameter and one inch thick isolated from the surrounding medium by closed cell polyurethan foam of @ 20 inches thickness. The properties of the three materials (steel, isolation material and rock) are presented in Fig. 28. The study shows a significant reduction in the straining actions of the structure together with a considerable reduction in the stress state of the surrounding medium. Figs. 29, 30 and 31 illustrate the isolation effect on membrane forces and bending moments in the structure and stresses in the medium.

### 4.5.2 The Effect of Layered Media on the Response

This study would be of particular interest for the cut-and-cover structural type. The investigation carried out for the model shown in Fig. 32 may provide the answer to the question: Whether it would be better to use a filling material stiffer or softer than that of the medium? In answering the question, six different filling materials were investigated through a comparative study of the internal forces in the structure and the stresses in two elements, one in front of the fill (EL. No. 106), the other behind (EL. No. 150). Both normal (density $\rho$ = 0.0002 lb. sec.$^2$ in.$^{-4}$) and heavy (density $\rho$ = 0.0005) concrete were used each as filling material of three different qualities, i.e. (E = 2, 3, 6 and 3, 7, 10x10$^6$ psi respectively). Results are presented in Figs. 33 to 36 indicating that a proper combination of the different properties of the filling material significantly reduces the internal forces in the structure and the stresses in the adjacent medium.

## 4.6 New Finite Dynamic Model to Simulate an Infinite Space

In static problems, the boundaries of a continua are selected based on the consideration that movement at a point on that boundary will have little effect at points moderately far removed (Saint-Venant's principle). For dynamic analysis no such criterion exists since the disturbing wave will reflect from the boundary and return to the point of interest in perhaps less time than required for study of the stress-time history of the point. This is why with the rigid boundaries of Ref. 8, earthquake excitation of an approximate duration of 30 seconds cannot be analysed.

Ref. 33 developed an approximate energy absorbing viscous boundary shown in Fig. 37 in which the infinite half space is modelled to a finite region bounded by a free surface, acted on by the following normal and tangential viscous forces:

$$b\sigma_n = C_1 \rho a \dot{\bar{U}} \times \bar{n} \qquad (4.8)$$

$$b\tau_n = C_2 \rho b \dot{\bar{U}} \times \bar{t} \qquad (4.9)$$

where $\dot{\bar{U}}$ is the velocity vector of the particles at the boundary surface; $\bar{n}$ and $\bar{t}$ are the unit normal and tangential vectors; a and b are the compressional and shear wave velocities; $C_1$ and $C_2$ are coefficients controlling the magnitude of the reflected wave amplitudes, i.e. the percentage of energy absorption. For body waves, $C_1 = C_2 = 1$ leads to the best energy absorption. Almost perfect absorption is obtained in the range where the incident angle of the body waves is greater than $30^o$. However, since the direction of all elastic waves is not known in the general case, it will not be possible to simulate an exact boundary for the absorption of the energy associated with the incident waves.

Based on a displaced energy absorbing boundary rather than a viscous one, the infinite boundary problem is solved herein using the model presented in Fig. 38 in which the boundary absorbs strain energy instead of the kinetic energy absorbed by the viscous boundary defined in Eqns. 4.8 and 4.9. In considering the loss of energy during the motions, i.e. damping in the system, Ref. 33 considered the damping by a diagonal matrix composed of the boundary dashpot constants multiplied by the appropriate area over which they are applied; all terms in the damping matrix are zero except for those terms corresponding to the boundary nodal points. In the proposed model presented in Fig. 38, the energy is dissipated in the structure (not presented in the figure) as structural damping and in the soil as material damping. Also, the extent of the finite element mesh and the energy absorbing displaced boundary takes into account the radiation or spatial damping. Radiation damping is extremely important in problems of vibrating foundations but of minor importance in studies of earthquake response (Seed, Lysmer and Hwang (52)). The model presented in Fig. 38 is based on a simple idea, that is placing an elastic energy absorbing boundary consisting of a system of X and Y elastic springs at each supporting node on the boundary. These springs will allow a time varying displacement at the boundary equal to that of the free-field solution. The evaluation of the appropriate values of spring constants to represent the deformation characteristics of the soil or the rock media is very important in obtaining a true picture of the response of the system to any dynamic excitation. For linear (elastic) systems, approximate values of the spring constants can be evaluated based on the soil resistance to lateral and vertical loads which is a function of the deflection of the artificial boundary defined by the relationship:

$$p = E_s U \tag{4.10}$$

where $p$ = soil reaction, lb/in of the boundary length

$U$ = deflection of the boundary, in.

$E_s$ = soil modulus, psi

The modulus is not a unique property of the soil; it is a function of the strength and the stiffness of the soil, but it is also influenced significantly by such variables as depth below the soil surface, load magnitude and manner of load application. In most cases the soil modulus tends to increase with depth (Ross (50)). However, the Young's modulus, $E_s$, of the soil can be determined dynamically, according to Ref. 8, by determining the frequency of induced longitudinal vibrations in a specimen and substituting these values and the density of the rock into the expressions for the longitudinal velocity and the modulus of elasticity. These expressions are

$$a = 2fL \tag{4.11}$$

$$E_s = a^2 \rho \tag{4.12}$$

where $a$ = longitudinal wave velocity

$f$ = fundamental longitudinal frequency

$L$ = length of specimen

$E_s$ = Young's modulus

and $\rho$ = density of rock

According to Ref. 8, the ratio of the dynamic to static values of Young's modulus varies between 0.85 and 2.9 and more compact the rock, the more closely will the static and dynamic values agree. It seems reasonable, from the literature on the dynamic analysis of laterally loaded piles and from the investigations of Ref. 50 on determining the appropriate values of the spring constants, to assume that $E_s$ varies linearly in shallow depths according to the relationship:

$$E_s = ky \qquad (4.13)$$

where k = the modulus of subgrade reaction of the soil, $lb/in^3$

y = the depth below the ground surface, in.

and in deep locations, a constant value of $E_s$ can be used. For nonlinear analysis the same principle of generating a displaced boundary to overcome the problem of wave reflection from the rigid boundaries is applicable. By using elementary wave theory, the time-history of displacements on the postulated boundary can be determined for a free-field with no cavity in the medium. At each time step, the displacements at the boundaries are used as boundary conditions to solve the resulting equations of motion of the system.

Instead of modifying the rigid boundary used in the computer programme of Ref. 8, the standard structural programme STRUDL-II was used in analysing the example illustrated in Fig. 38 to test the proposed artificial boundary for an overall system damping ratio of 5%. The exciting load, medium properties, and the time increment of integration are the same as those of Ref. 9 presented in Fig. 2. Fig. 39 presents the results of the analysis compared with those of Refs. 33 and 9.

# CHAPTER V

# DISCUSSION AND CONCLUSIONS

## 5.1 Presentation of Results

The results will be presented in separate sets, each set representing one of the problems investigated. The cases considered are: 1) cavity reinforcement, 2) cavity shape, 3) isolation of the structure from the surrounding medium by a soft, energy absorbing material, 4) the filling material in the cut-and-cover structure, and 5) boundary conditions to simulate infinite space for dynamic analysis. The figures of the results are presented at the end of this chapter.

## 5.2 Cavity Reinforcement

The results presented in Figs. 12 and 13 of the rock bolted cavity described in the previous chapter indicate a considerable reduction of about 25% in the stress field around the cavity due to the rock bolting reinforcement pattern described in section 4.3.1 and a general reduction in the displacements of the cavity boundary. The time-histories for stresses in the rock bolted cavity shown in Figs. 10 and 11 indicate most of the reinforcing bolts of the vertical walls to be in compression while those reinforcing the roof are in tension. This indicates the need for a study of the amount of prestressing required for rock anchors (prestressed bolts) reinforcement.

The use of an elastic liner generally reduces the stresses in the medium especially the tangential stresses around the opening. However, since the liner provides a support in the radial direction, it increases the stresses in the radial direction which are supposed to vanish in the case of no lining. Ref. 61 indicated that under favourable conditions, tunnel linings generally reduce stresses in the medium by about 30% or more. Fig. 15 presents the results of the study.

From the results obtained with the two kinds of reinforcement, the rock bolting with about 80% of the amount of steel used in the elastic liner shows a higher reduction (three times or more) in the stresses around the cavity and in the rest of the medium than in the case of the elastic liner. However, displacements were reduced in the lined case more than the rock bolted one.

5.3 Cavity Shape

Four different shapes were investigated: i) circular, ii) semi-circular roof with vertical walls, iii) flat circular roof with long vertical walls, and iv) horseshoe (Figs. 16 to 19). To compare the shape effects on displacements and stresses in the structure and the surrounding medium, the area of the opening was kept almost constant in the four cases. This is the reason for considering the same model as that studied in Ref. 8 for the circular shape and with slight modifications in the other three shapes. The modelling was based on a 100 node and 120 element finite domain. The results indicated the horseshoe shape to be the best from the viewpoint of stress in the medium. Therefore, more attention was devoted to the horseshoe shape in the parametric study described in the previous chapter. The results are illustrated in Figs. 20, 21 and 22 for the four shapes and in Figs. 24, 25 and 26 for the different configurations of the horseshoe shape. The results show a decrease in the bending moment at the crown of the horseshoe liner with increase of the rise to span ratio of the roof, H/L, and the corresponding decrease of the rise to the span ratio of the walls, H'/L' (shape no. 3, Fig. 23). Also, the examination of the time-history of the normal forces at the crown section of the three configurations of the horseshoe shape outlined in Fig. 23 indicates that the maximum normal force decreases with the increasing H/L ratios and corresponding decreasing of H'/L' ratios. This suggests that the high-horseshoe shape is the best from the viewpoint of the internal forces in the lining (the maximum bending moment in shape no. 3 was reduced to almost one-half that in shape no. 1 and the maximum normal forces were reduced to almost one-fourth of those for other shapes). However, stresses in the adjacent medium were higher for the high-horseshoe shape than those for flatter shapes as shown in Fig. 26.

5.4 The Medium

The medium adjacent to the structure was investigated for two different cases to study its effect on the internal forces in the liner and the stresses in the rest of the extended medium. The two cases described in sections 4.5.1 and 4.5.2 and outlined in Figs. 28 and 32 were investigated for a step pulse excitation defined in section 4.3. In the first case, i.e. the entirely isolated structure, a reduction as high as 80% in the shell (structure) membrane forces and bending moments was obtained using closed cell polyurethane foam ( $\rho = 6x10^{-6}$ lb.sec$^2$/in$^4$, $\mu = 0.3$ and $E = 1400$ psi) of thickness about one-fourth of the shell diameter (Figs. 29 and 30). The annular material absorbs the energy produced by the excitation causing this significant reduction of straining actions in the structure. Also, since it allows for more displacements in the adjacent medium, the stresses in the medium were reduced by 10-15% (Fig. 31). In the second case, i.e. the cut-and-cover structure, the results presented in Figs. 33 and 34 indicate that neither the density ($\rho$) of the filling material nor its elasticity (E) as separate values can greatly affect the internal forces in the structure but a certain combination between the values of the two properties can produce the maximum reduction in the straining actions in the structure, i.e. protect the structure from the dynamic disturbances. This is clear from Figs. 33 and 34 in which a concrete filling material, whose density, $\rho$, = $2 \times 10^{-4}$ lb.sec$^2$/in$^4$ and a modulus of elasticity, E, = $6 \times 10^6$ psi, leads to almost the same results as those of a heavy concrete whose $\rho = 5 \times 10^{-4}$ and $E = 10 \times 10^6$. Also, Figs. 33 and 34 show that for the same material density, the internal forces in the structure can be reduced significantly with the increase of the elasticity of the filling material. On the other hand, the increase of the elasticity generally increases the stresses in the adjacent medium. However, dense materials having a high modulus of elasticity increase the stress values of the medium in front of the fill (between the approaching wave front and the structure) and decrease those behind (Figs. 35 and 36). In this case the fill acts as a fender protecting the medium behind.

### 5.5. Boundary Conditions to Simulate Infinite Space

The postulated boundaries of the physical model described in the previous chapter (section 4.6) were used in analysing the same example as that considered by Ref. 9 (Fig. 2). Because of the limitations on computer time which exceeded more than one hour CPU time to analyse the problem with the same finite element mesh used by Ref. 9, a coarse mesh outlined in Fig. 38 was used. The constants of the elastic springs at each supporting node were approximately evaluated as the product of the modulus of elasticity and the distance halfway toward both adjacent nodes. The solution is based on plane strain analysis in which the rectangular elements (PSR) and triangular elements (CSTG) have two degrees of freedom at each node. The results are presented in Fig. 39. Although the peak value of the response using the proposed model is the same as that obtained by using the boundary conditions of Ref. 33, the time for maximum response is not the same. From the review of literature and particularly the results presented in Ref. 52 considering the effect of soil modulus and spring stiffness on results of interaction spring analyses, it appears that the time shift in the maximum response illustrated in Fig. 39 could be related to the evaluation of the spring constants. This indicates the need for further work to simulate dynamically excited infinite space. From the general equation of dynamic equilibrium (Eqn. 2.5), it appears that in future work the boundary conditions should represent the mass, damping and stiffness matrices to include the effect of the accelerations, velocities, and displacements of the artificial boundaries. This could be achieved by considering variable damping in the different elements of the system. One of the approaches for considering this type of damping is the use of Rayleigh damping matrix of the form presented in Chapter two and Eqn. 1 in Appendix B for each element. The damping formulation in the modified computer programme is based on the same equation and needs a modification to permit the inclusion of variable damping in each element. Since this damping is a frequency-dependent (see Eqns. 5 and 7 in Appendix B), very high damping can occur at high frequencies causing (according to Ref. 52) a serious limitation in the analysis of structures containing high-frequency equipment such as nuclear power plants. However, for other types of structures this phenomenon can be of little importance. In general, the use of the finite element method in analysing high-frequency response problems is limited to the cut-off frequency of the discrete system.

## 5.6 Verification of the Results

After deciding the modification of the computer programme of Ref. 8 to suit the general dynamic solution of underground reinforced cavities of any shape, the solution of Ref. 8 was examined. Ref. 8 studied the stability of the method of the numerical integration (Newmark Beta Method) used in the programme and verified the results obtained by comparing them with those of the theoretical solution of Ref. 37. The two problems presented by Ref. 8 as an application to underground lined cylindrical cavities were solved as a check on the correctness of the available programme. After modifying the programme to include a new reinforcing element, the problems considered in Ref. 8 were solved again and the same results were obtained proving the correctness of the modifications in some of the subroutines and the main programme. The problem outlined in Fig. 14, including 6 shell elements and 8 new plate elements in modelling the elastic liner of the cavity was then analysed. The results were compatible with those of the similar problem solved by Ref. 8 and illustrated in Fig. 16.

A study of various parameters affecting the stresses and displacements in the structure and the surrounding medium showed good agreement with certain values obtained by previous investigators, particularly with those of Ref. 15 in studying a structure surrounded by an isolated, soft, energy absorbing material. The results of the analysed problem presented in Fig. 38 using the new artificial, elastic, energy absorbing boundary were compared with those obtained by Ref. 9 using the viscous boundary of Ref. 33. The peak value of the response of the top node on the postulated boundary was almost the same with a shift of about 0.05 seconds. The results have been discussed previously in section 5.5.

## 5.7 Conclusions

The following conclusions are drawn from this study:

1. Rock bolting reinforcement, whenever feasible, provides the most efficient and economic solution reducing the stresses around the unreinforced cavity by 25% and more.

2. Generally speaking, the cavity liner reduces the stresses in the medium. In this study a reduction of the order of 10% was obtained.

3. The shape of the cavity affects the response of the medium and the structure (liner). The horseshoe shape proves to be the best, with a reduction of the order of 10% in the stress state of the medium compared to a circular cavity or semi-circular roof and vertical wall cavity.

4. A reduction as high as 80% in the membrane forces and bending moments was caused by isolating the entire structure from the surrounding medium by a soft, energy absorbing material. The stresses in the medium were reduced by about 10-15%.

5. The properties of the filling material in the cut-and-cover structure affect the time-history of the internal forces in the structure and the stresses in the adjacent medium. It appears that a proper selection of the combined properties, the density and the elasticity, of the filling material can lead to a significant reduction in the straining actions of the structure and the stresses in the surrounding medium.

6. Further work should be done to simulate an adequate finite

boundary for the dynamic analysis of infinite space problems. It seems that the formulation of this boundary should include the mass, damping and stiffness matrices of the equations of the dynamic equilibrium of the discrete system.

5.8 Contributions

1. Solutions for dynamically excited underground reinforced cavities of any shape.

2. Assembly of the plate element system stiffness matrix after condensing it to include only two degrees of freedom at each nodal point to match the elements of the adjacent medium into the overall system (the cavity reinforcement and the medium) stiffness matrix.

3. Modification of the computer programme of Ref. 8 to include a new reinforcing element through the development of two new subroutines and the modifications of the other subroutines and the main programme to account for the new element.

4. Development of a new model for simulating a dynamically excited infinite space obviating the complications of wave reflection.

5. Study of the effects of different parameters (i.e. cavity reinforcement, cavity shape, isolating material between the structure and the medium and filling material in the cut-and-cover structure) on the time-histories of the internal forces in the reinforcing liner and the stresses in the reinforcing rock bolts and the medium.

5.9 Summary and Recommendations

A general formulation has been presented for a reinforced underground cavity subjected to dynamic excitation. The cavity can be of any shape reinforced by rock bolts, an elastic liner or a combination of both.

A new straight element with bending stiffness has been set up for the reinforcing elastic liner of the opening consistent with the two-force coordinate system of the finite element continuum. Two new subroutines were developed along with modifications in the other subroutines and the main programme of Ref. 8, particularly subroutines SHELLS, SHELL and ASSEMB, to account for the new element. The influences of the various parameters listed in section 5.1 on the response of the underground reinforced cavity were investigated. The modified computer programme has applications outside the immediate problem of the underground cavity such as the dynamic analysis of beams, plane frames and arches. A new model for the dynamic analysis of underground structures surrounded by an infinite medium is presented.

The work can be extended to the following areas:

1. Nonlinear or inelastic analysis in terms of stiffness properties. A computer code for calculation of the time-history of multi-mass systems with consideration of nonlinearities has been constructed by Österle (42).

2. The analysis of the crushable material used as annular filling between the structure and the medium by considering the failure of elements in which stresses reach the ultimate strength of the material (Ref. 63 has considered such elements).

3. Formulation of the damping matrix on an individual element basis (variable damping). This can provide a good approach for the simulation of a finite artificial boundary in modelling an infinite space which is dynamically excited. Furthermore, according to Ref. 52, the material damping of the soil, in case of strong shaking, will be much larger than that in the structure and consideration of the variable damping becomes

important.

4. Approximate modal analysis for nonlinear systems (Newmark (39)).

5. Inclusion of a capability for node separation in the case of fracture of the medium or separation of the cavity liner from the medium. Ref. 57 described a two-dimensional discrete element method for static stress analysis problems in the presence of discontinuities.

6. Extension of the numerical technique used in the original computer programme of Ref. 8 (Newmark Beta Method) as one of the single-point predictor-corrector methods, to multi-point predictor-corrector formulae for numerical integration. This is applicable to elastic and inelastic systems with several advantages in accuracy and computational time especially in elastic systems where the stiffness matrix remains constant. Humar and Wright (26) presented one such multi-point method and examined its truncation error and stability. However, for cases with large number of degrees of freedom, the stability criterion may be too restrictive.

7. Application of the transfer matrix approach for layered media (Ref. 4). The main advantage of the method is that the size of the overall transfer matrix is independent of the number of layers, and always remains two by two.

8. Extension to three-dimensional analysis.

9. Structure-medium modelling problems for earthquake excitation.

FIGURES

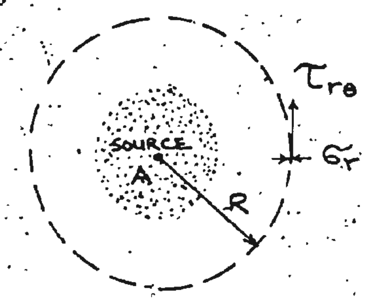



FIG. 1. FINITE DISTURBED REGION (REF. 9)

DATA FOR THE EXAMPLE SHOWN BELOW

TIME INCREMENT = 0.005 SEC.

MODEL: 198 NODAL POINTS

160 ELEMENTS

MEDIUM: $\rho = 2.$

$E = 10^4\ t/m^2$

$\nu = 0.15$

LOAD :

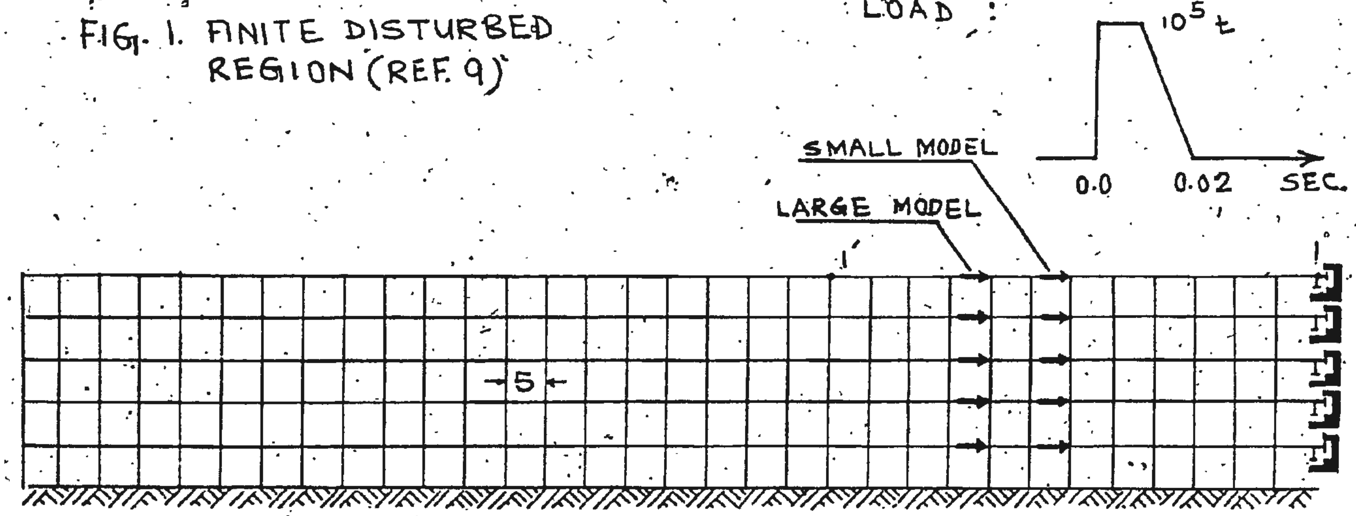



FIG. 2. FINITE ELEMENT MODEL OF A SOFT LAYER RESTING UPON ROCK (REF. 9)

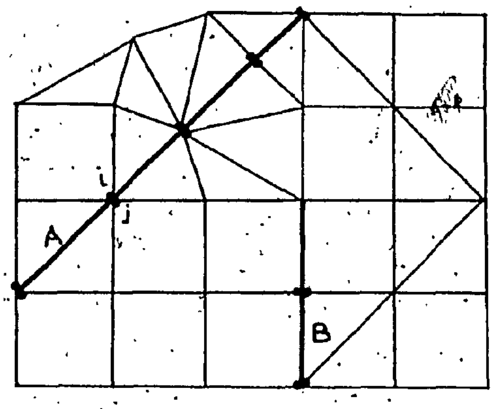



FIG. 3. EXAMPLE FINITE ELEMENT MESH CONTAINING DISCONTINUITY TRACES A AND B (REF. 57)

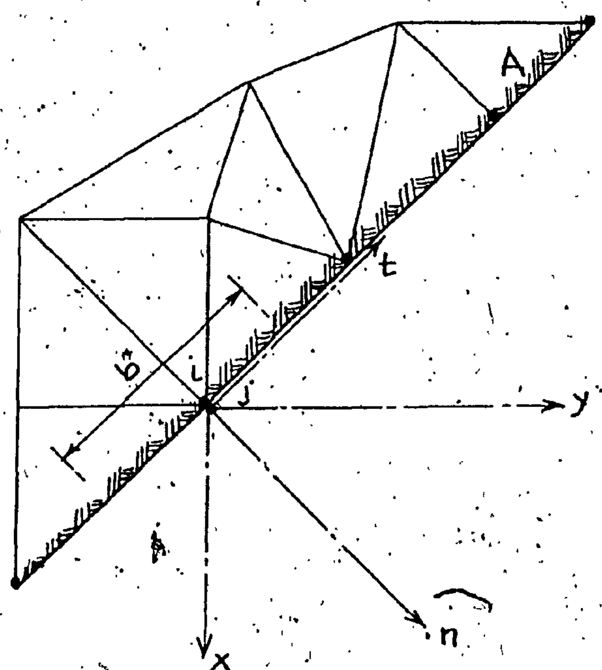



FIG. 4: EXPANDED VIEW OF DISCONTINUITY A FROM FIGURE 3. (REF. 57)

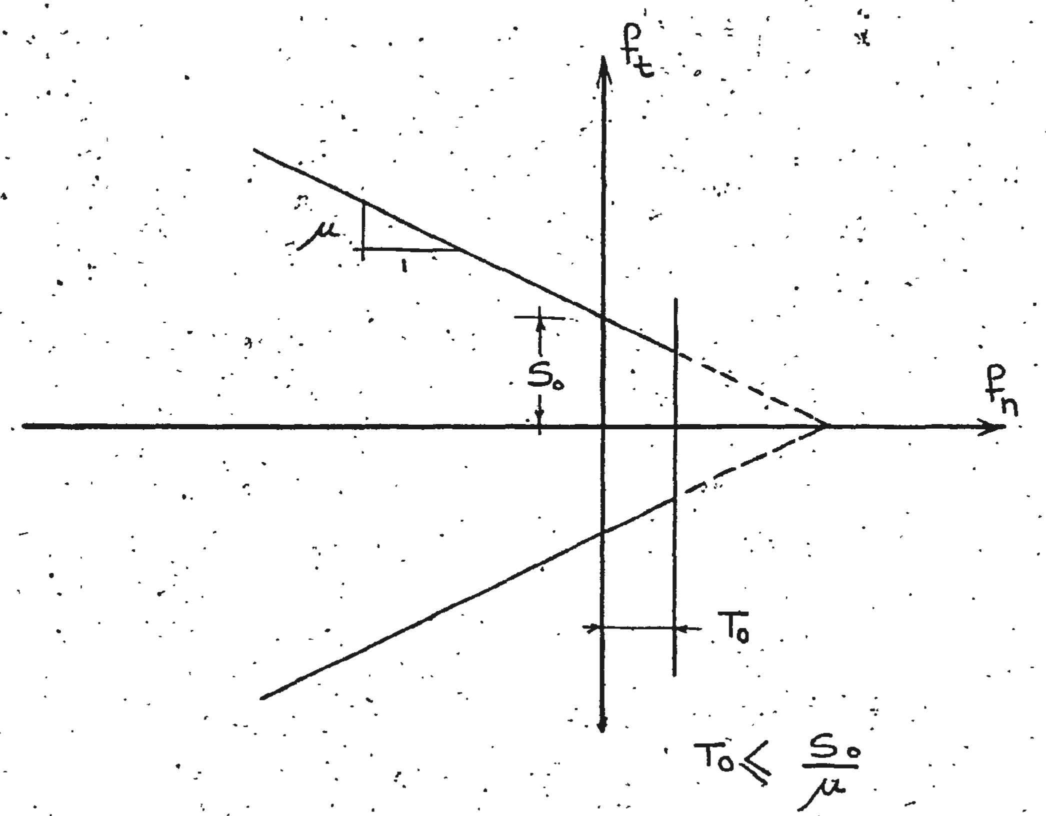



FIG. 5. COULOMB-NAVIER CRITERION WITH TENSION 'CUTOFF' (REF. 57)

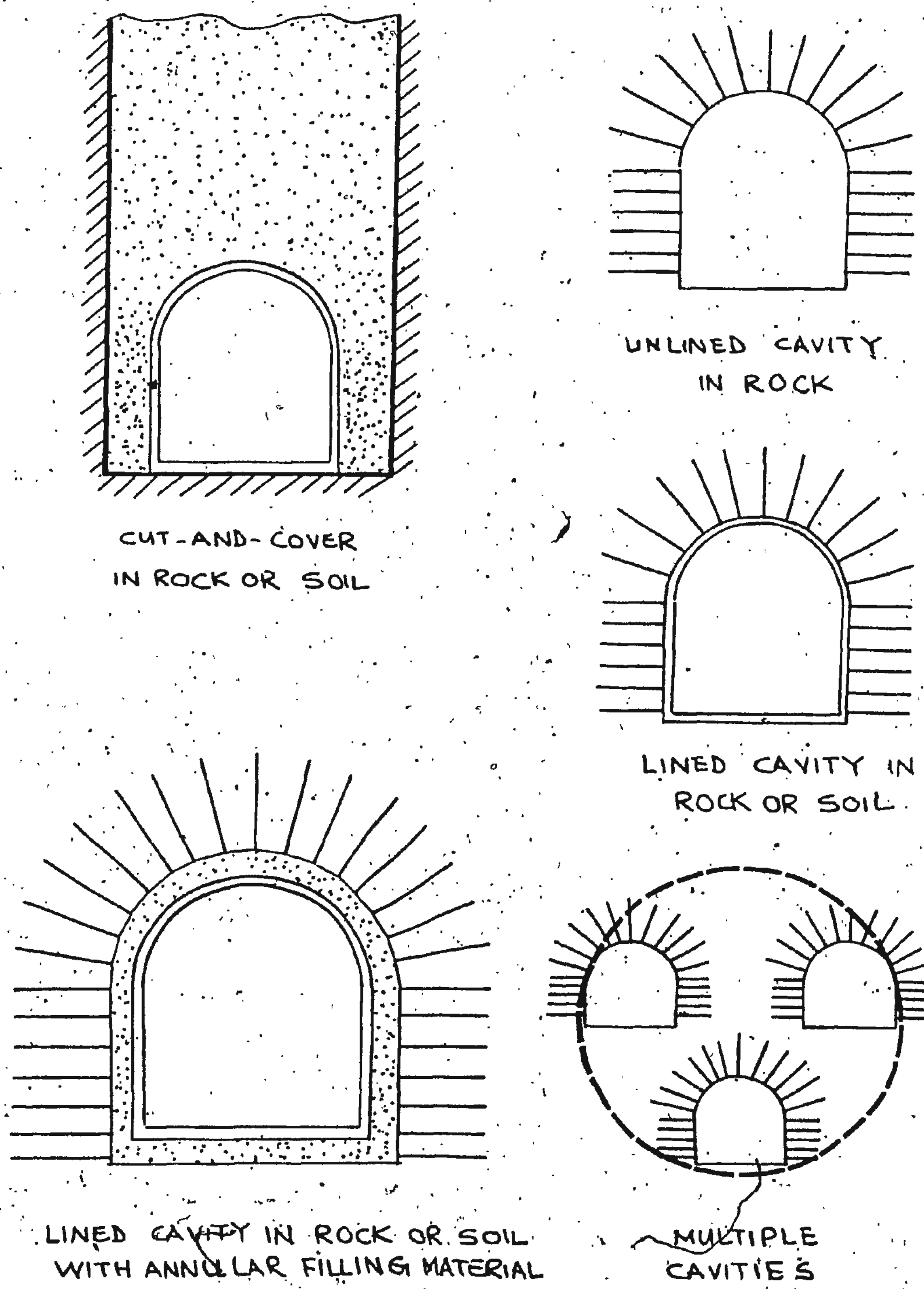



FIG. 6. UNDERGROUND CAVITIES (REF. 31)

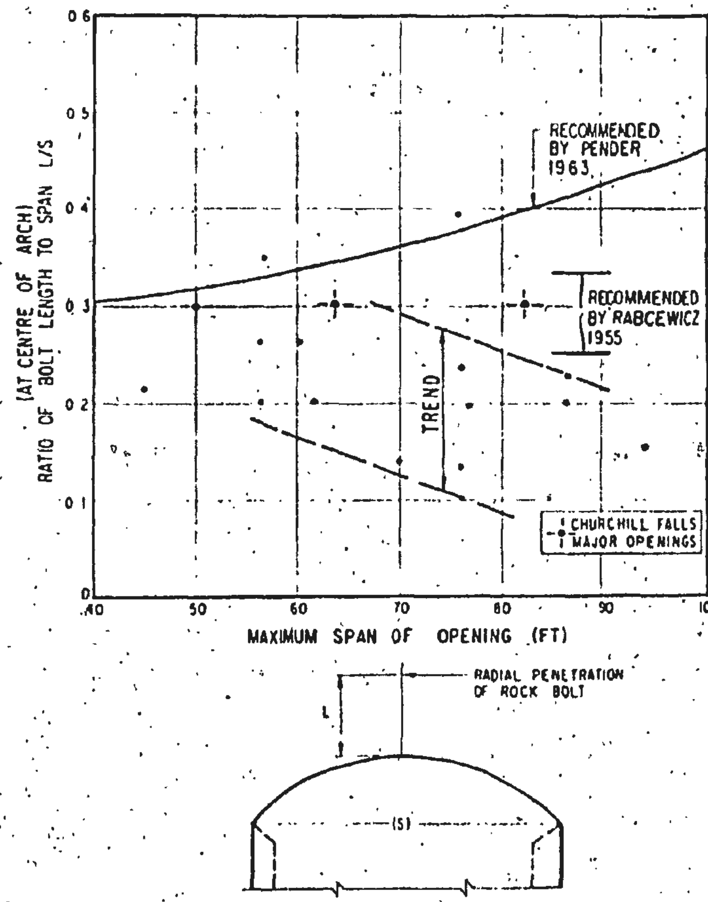



FIG. 7. ARCH BOLTING FOR PRECEDENT OPENINGS (REF. 7)

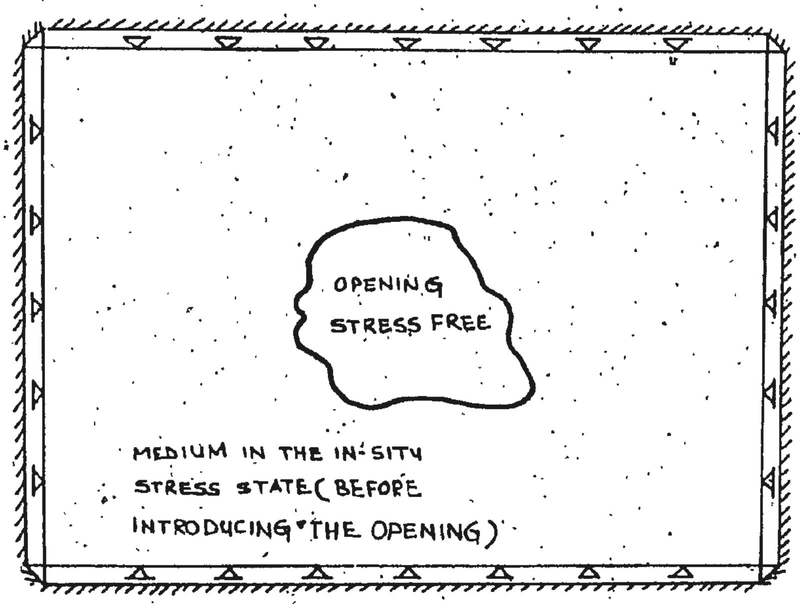



FIG. 8. BOUNDARY CONDITIONS OF FINITE ELEMENT SOLUTION FOR THE INDUCED STRESS STATE

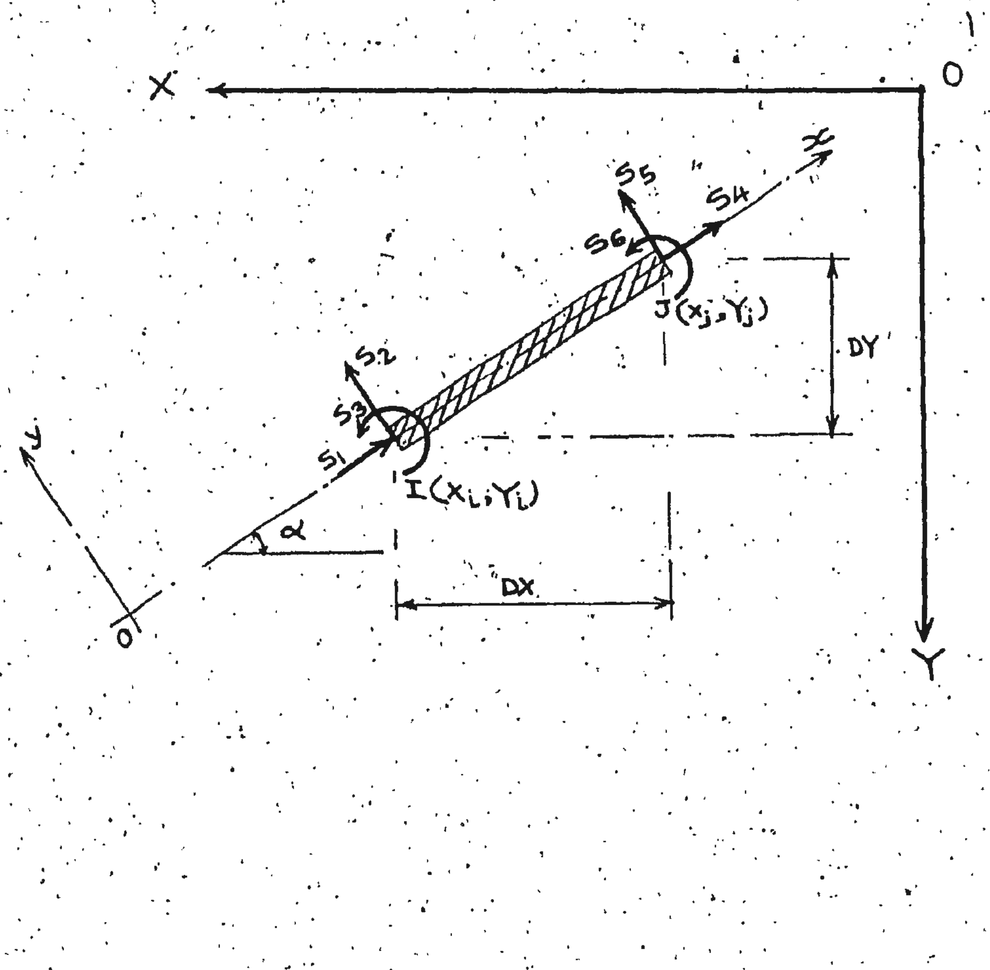
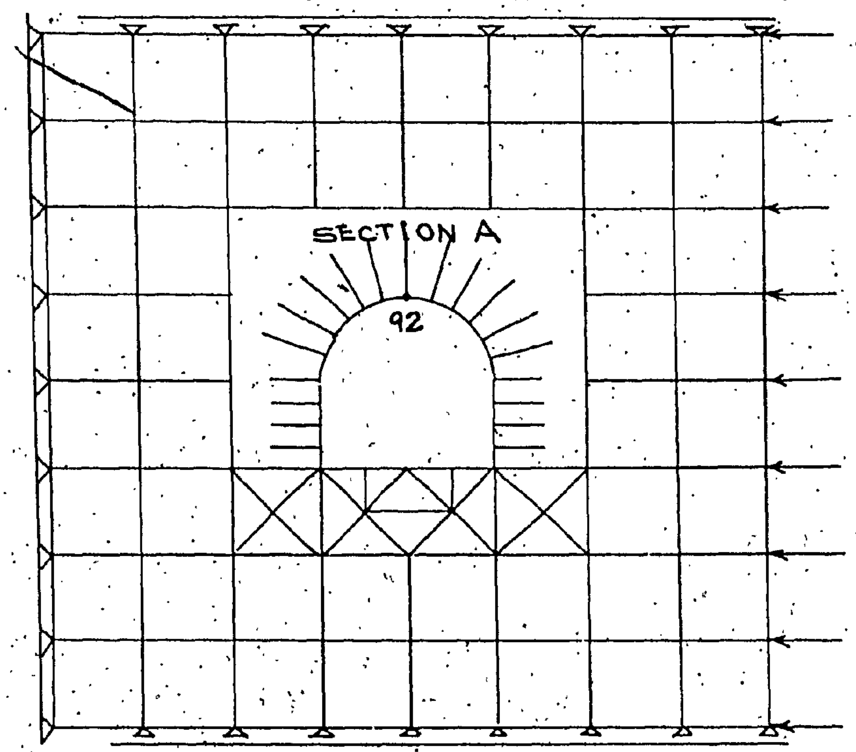



FIG. 9. PLATE ELEMENT IN THE GLOBAL SYSTEM.




MODEL; PLANE STRAIN ANALYSIS
119 NODAL POINTS
147 ELEMENTS INCLUDING 19 BAR EL. REPRESENTING THE ROCK BOLTING REINFORCEMENT

MATERIALS:
i) ROCK MEDIA : $\rho = 0.00025$ , $E = 5 \times 10^6$, $\nu = 0.3$
ii) ROCK BOLTS : $\rho = 0.000735$, $E = 29 \times 10^6$

UNITS : INCH, LB., SEC.

FIG. 10. ROCK BOLTED CAVITY

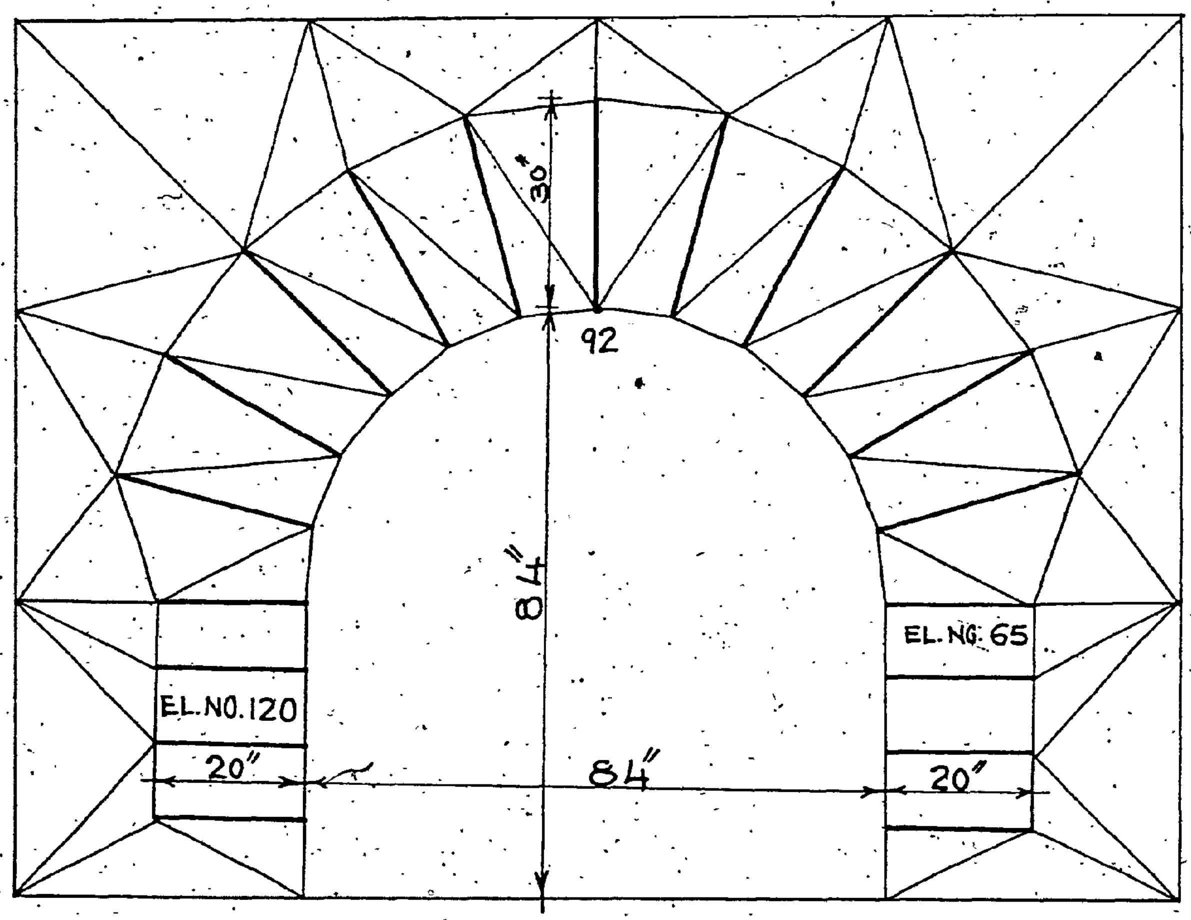



FIG. 11. DETAILED VIEW OF SECTION A IN FIG.10

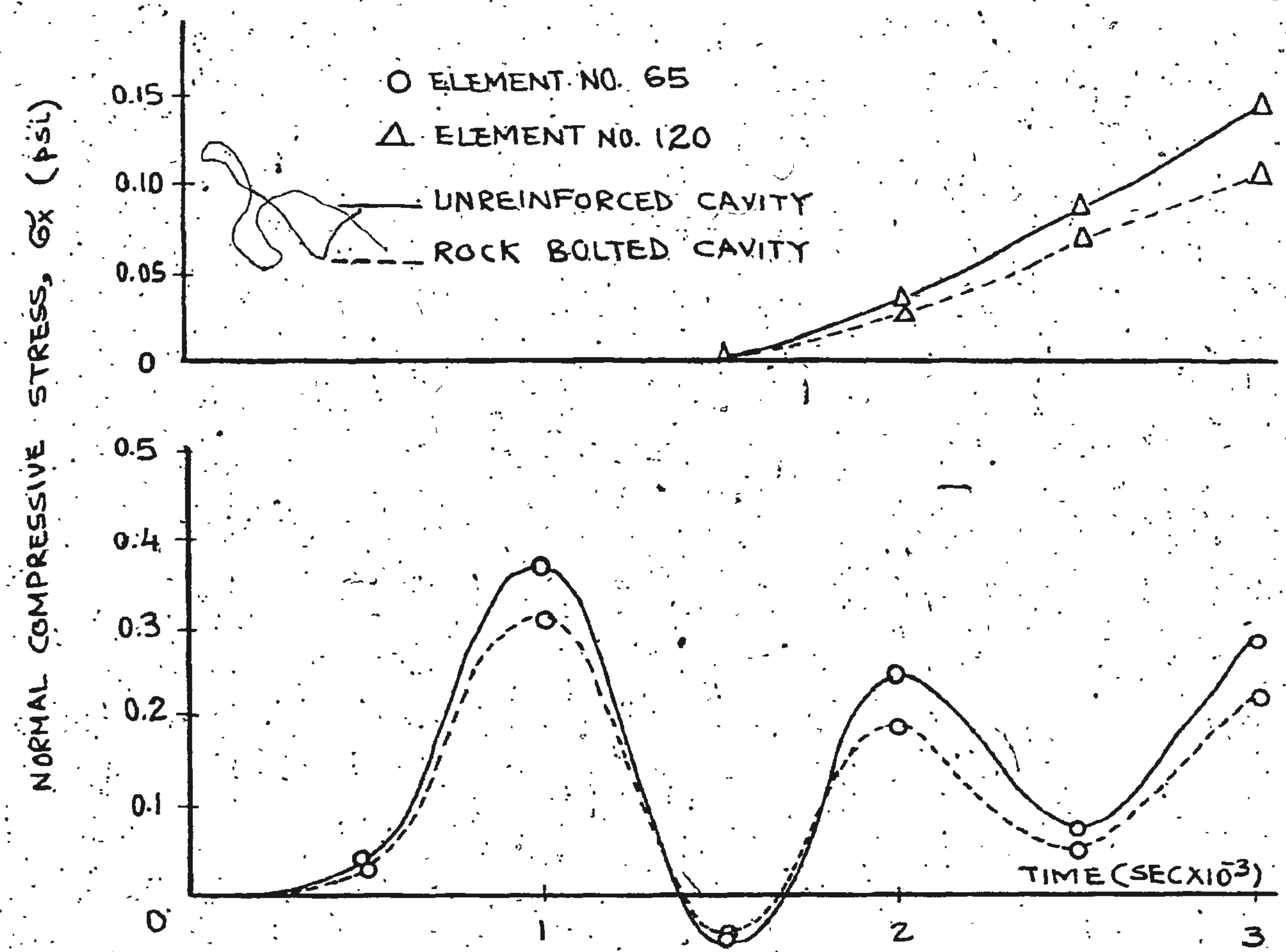



FIG. 12. STRESS VS. TIME, ELEMENTS NO. 65 AND 120 — ROCK BOLTED CAVITY

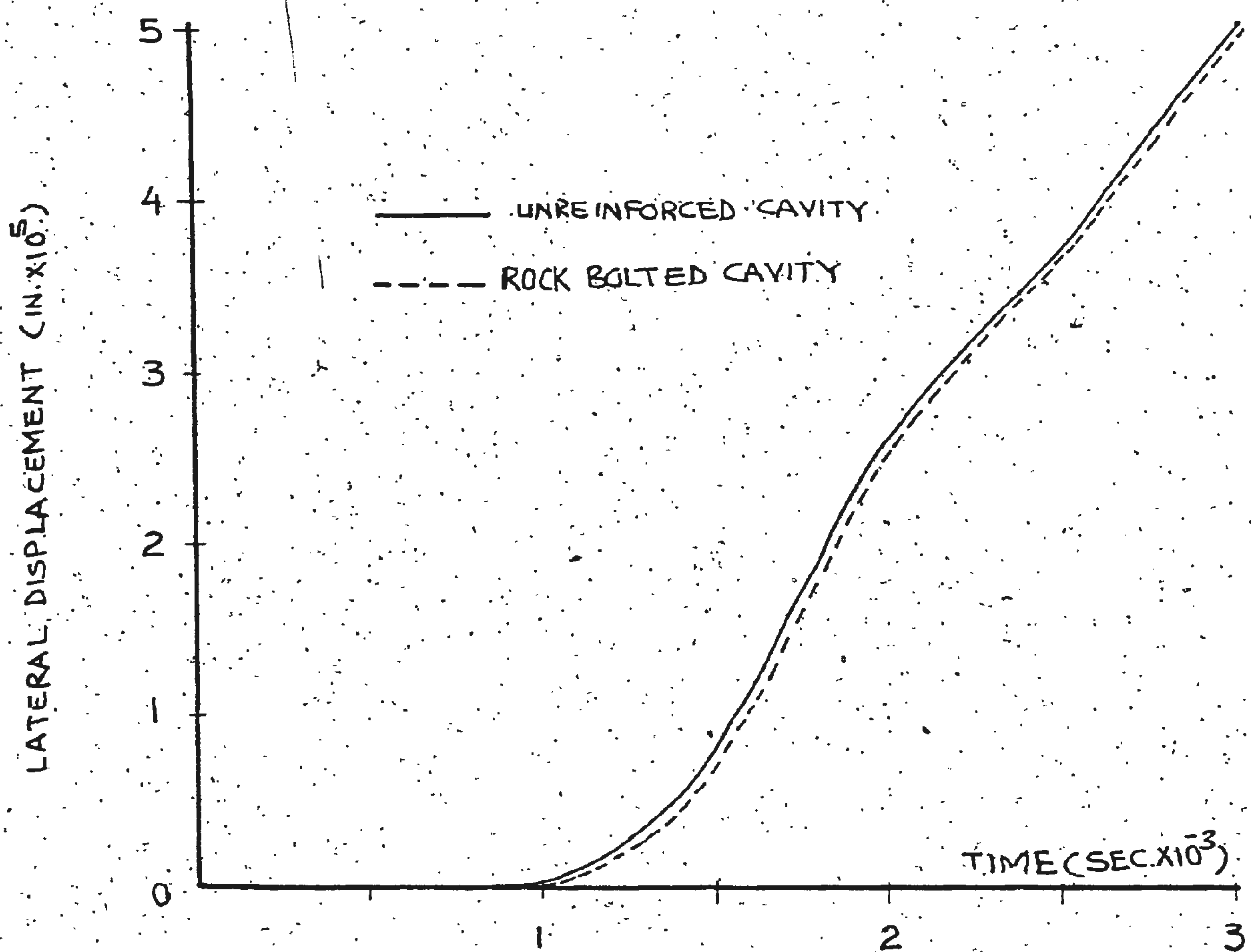



FIG. 13. DISPLACEMENT VS. TIME, NODE NO. 92 – ROCK BOLTED CAVITY

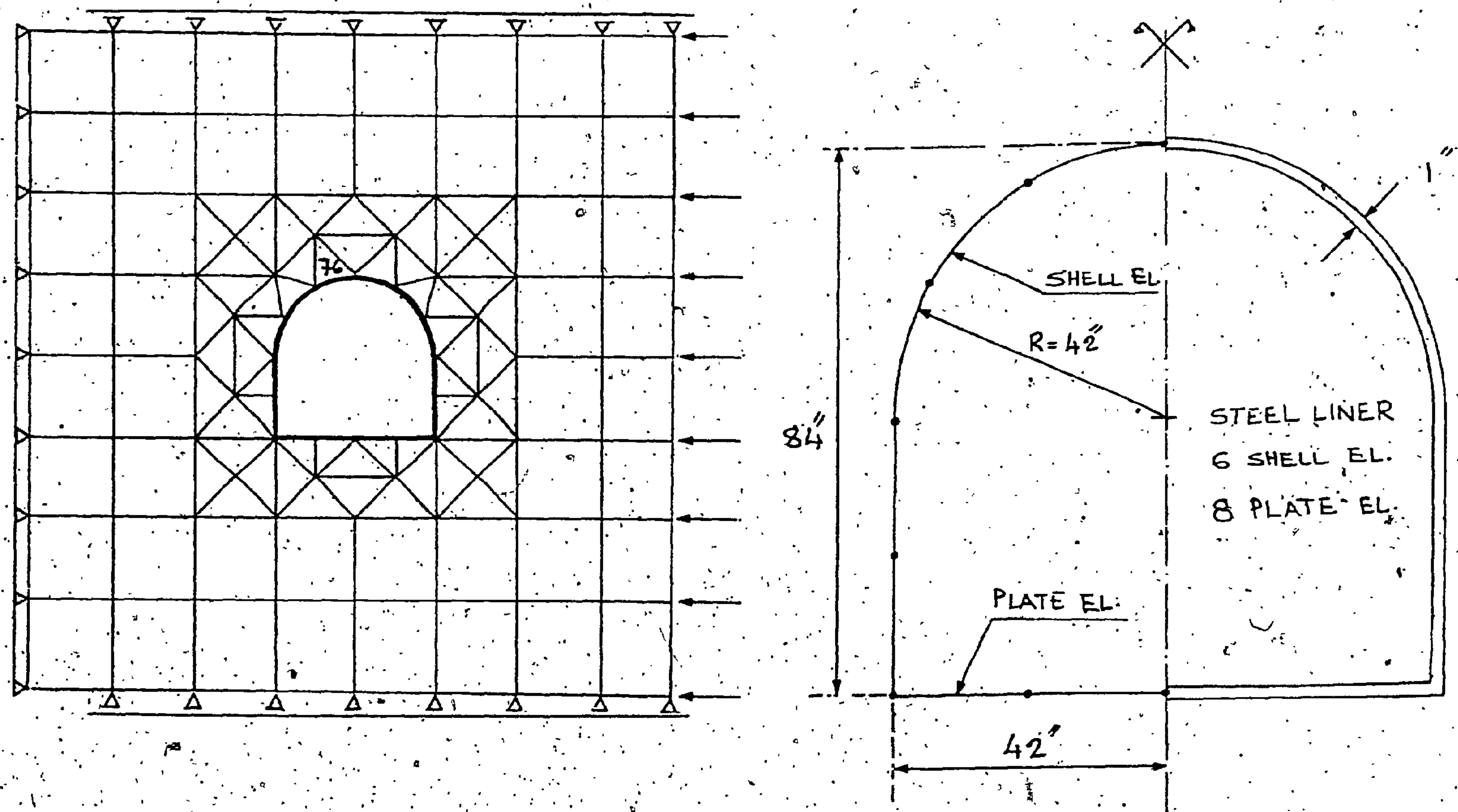



FIG. 14. LINED CAVITY

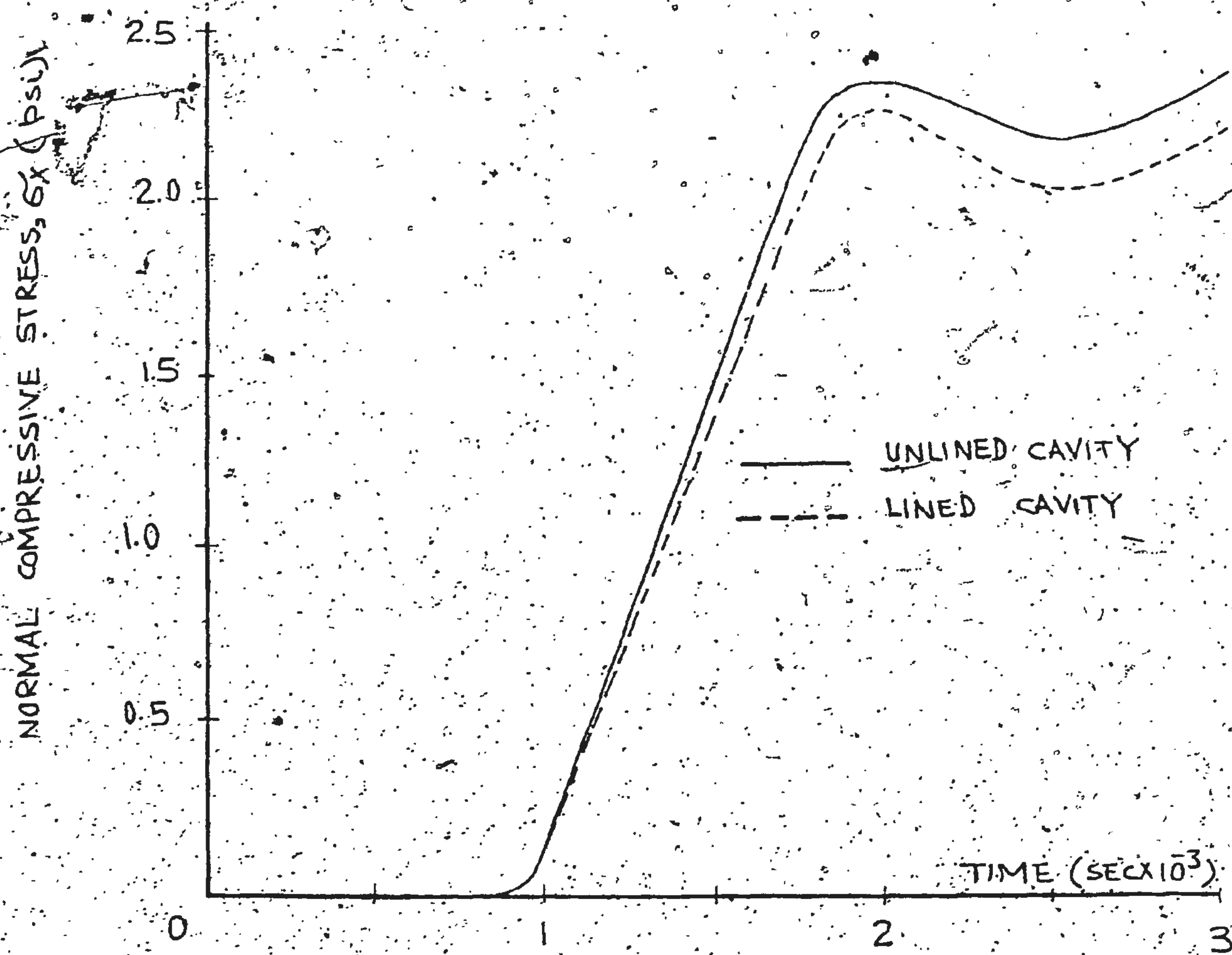



FIG. 15. STRESS VS. TIME, ELEMENT NO. 76 - LINED CAVITY

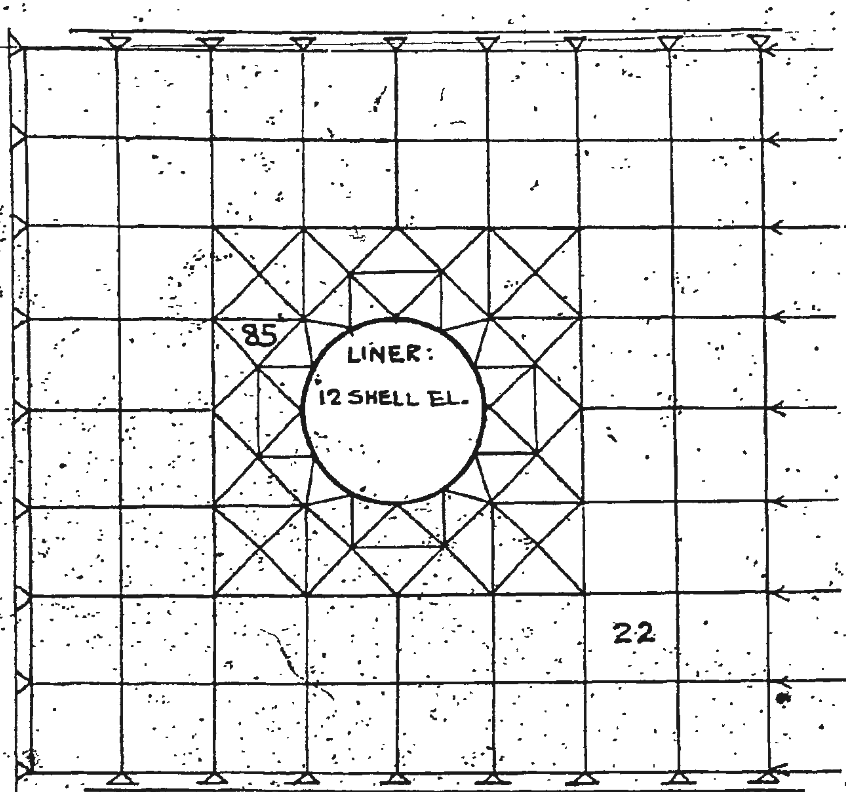



FIG. 16. CIRCULAR CAVITY
(SHAPE NO. 1)

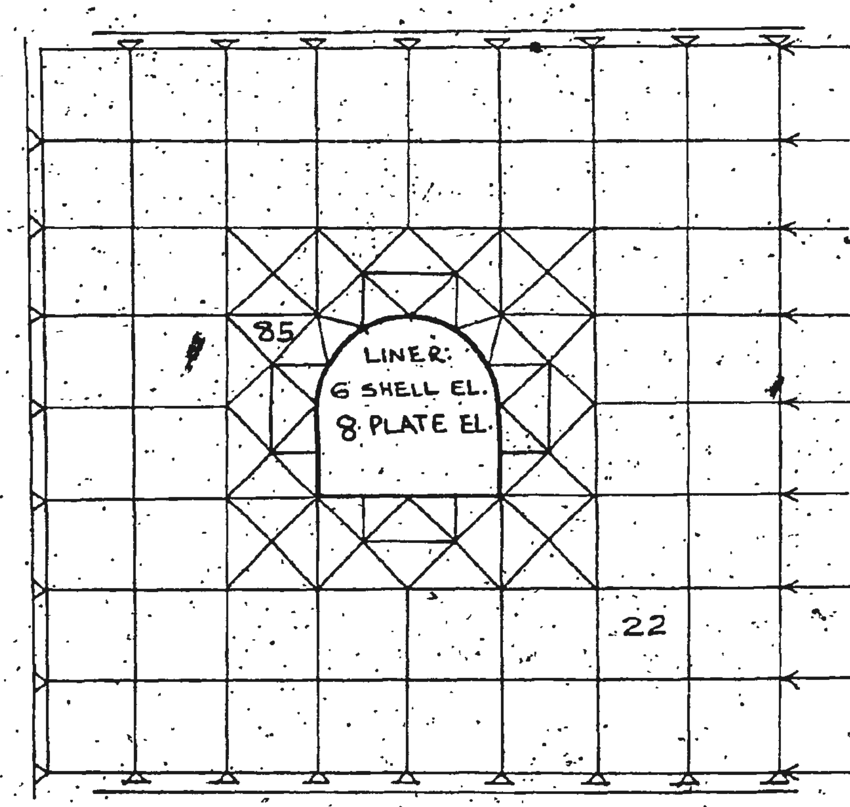



FIG. 17 CAVITY WITH SEMI-CIRCULAR ROOF AND VERTICAL WALLS
(SHAPE NO. 2)

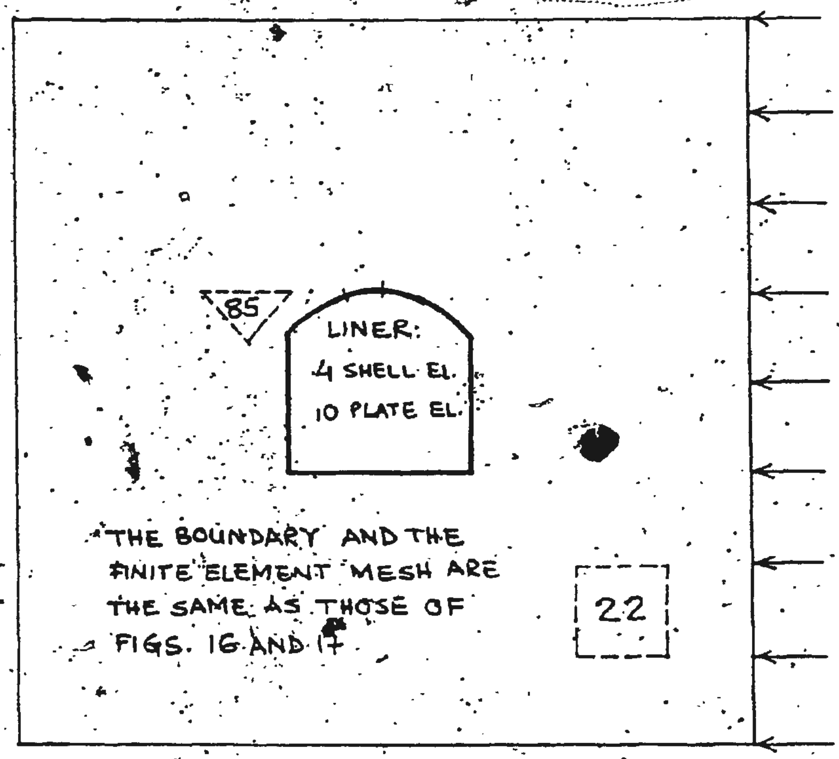



FIG. 18. CAVITY WITH FLAT ROOF AND LONG VERTICAL WALLS (SHAPE NO. 3)

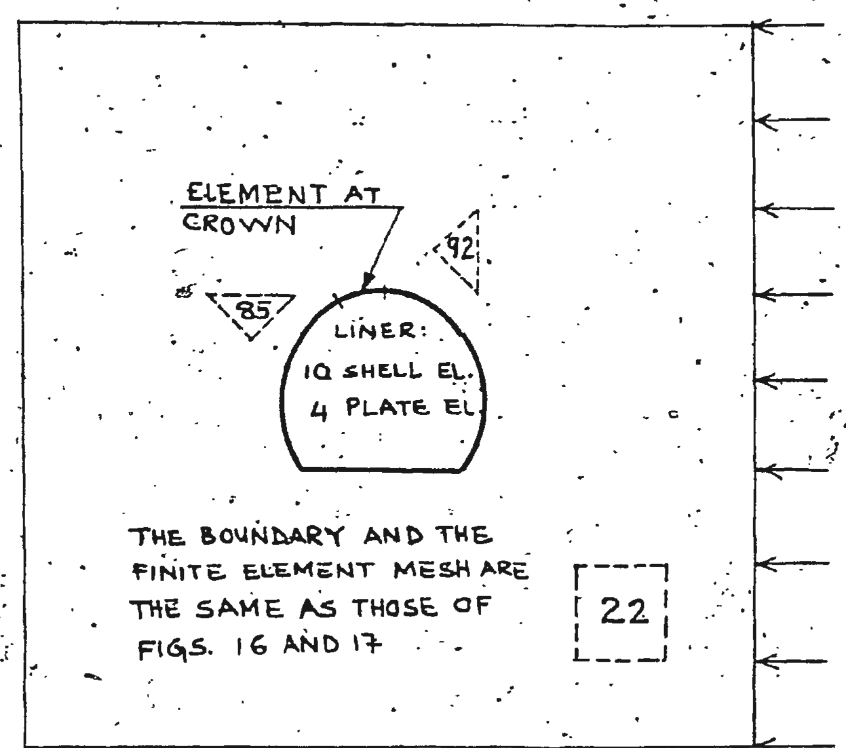



FIG. 19. HORSESHOE SHAPED CAVITY (SHAPE NO. 4).

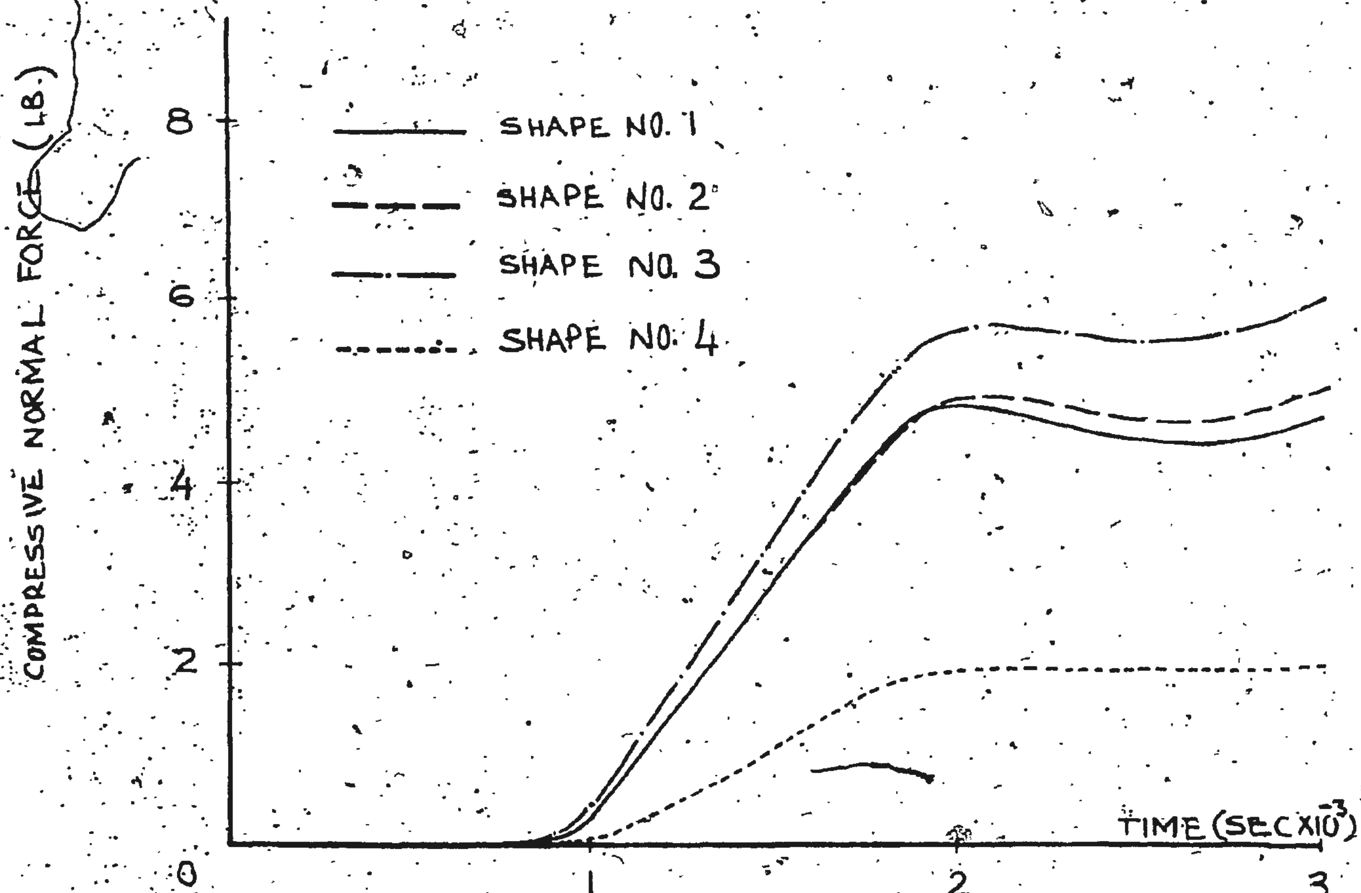



FIG. 20. NORMAL FORCE VS. TIME, ELEMENT AT CROWN

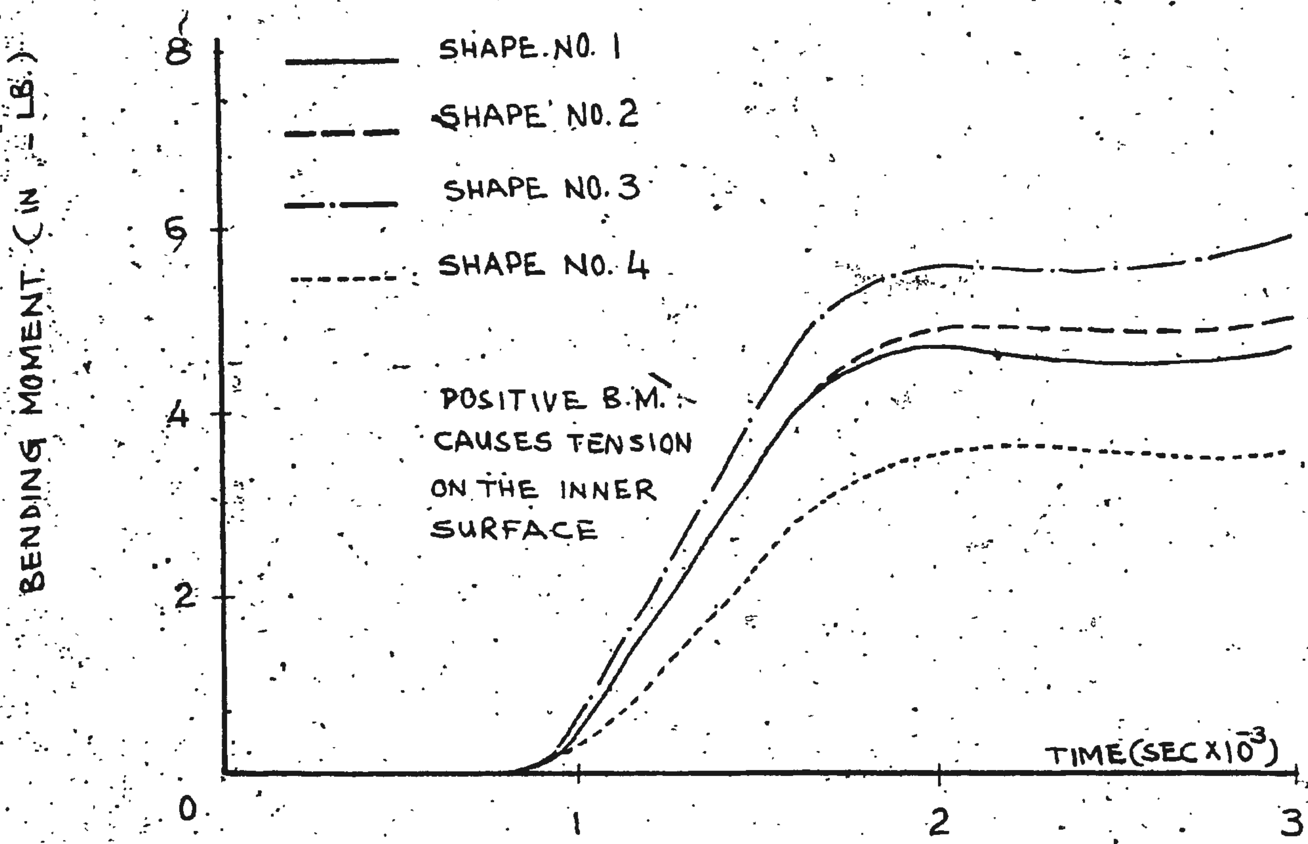



FIG. 21. BENDING MOMENT VS. TIME, ELEMENT AT CROWN

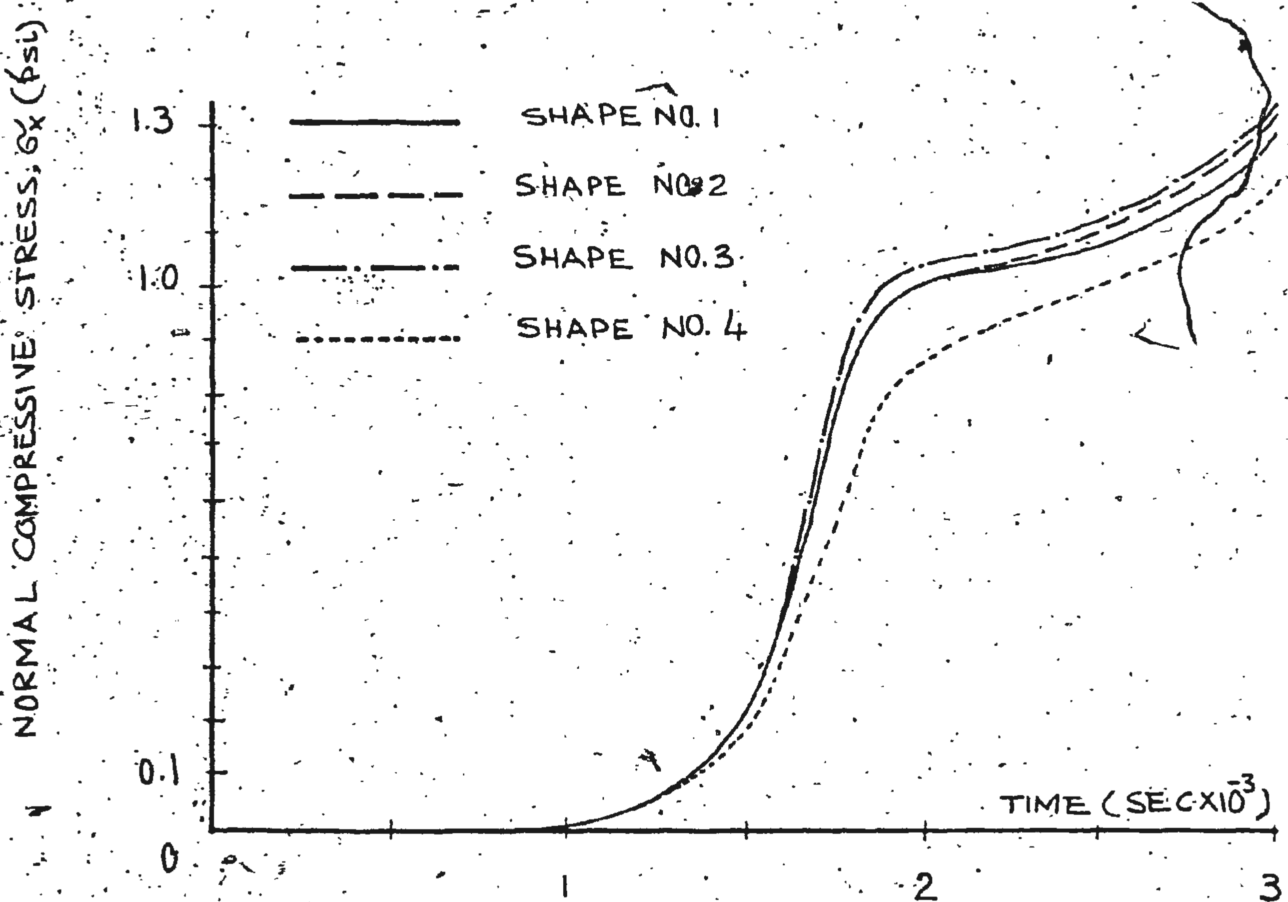



FIG. 22 STRESS VS. TIME, ELEMENT NO. 85

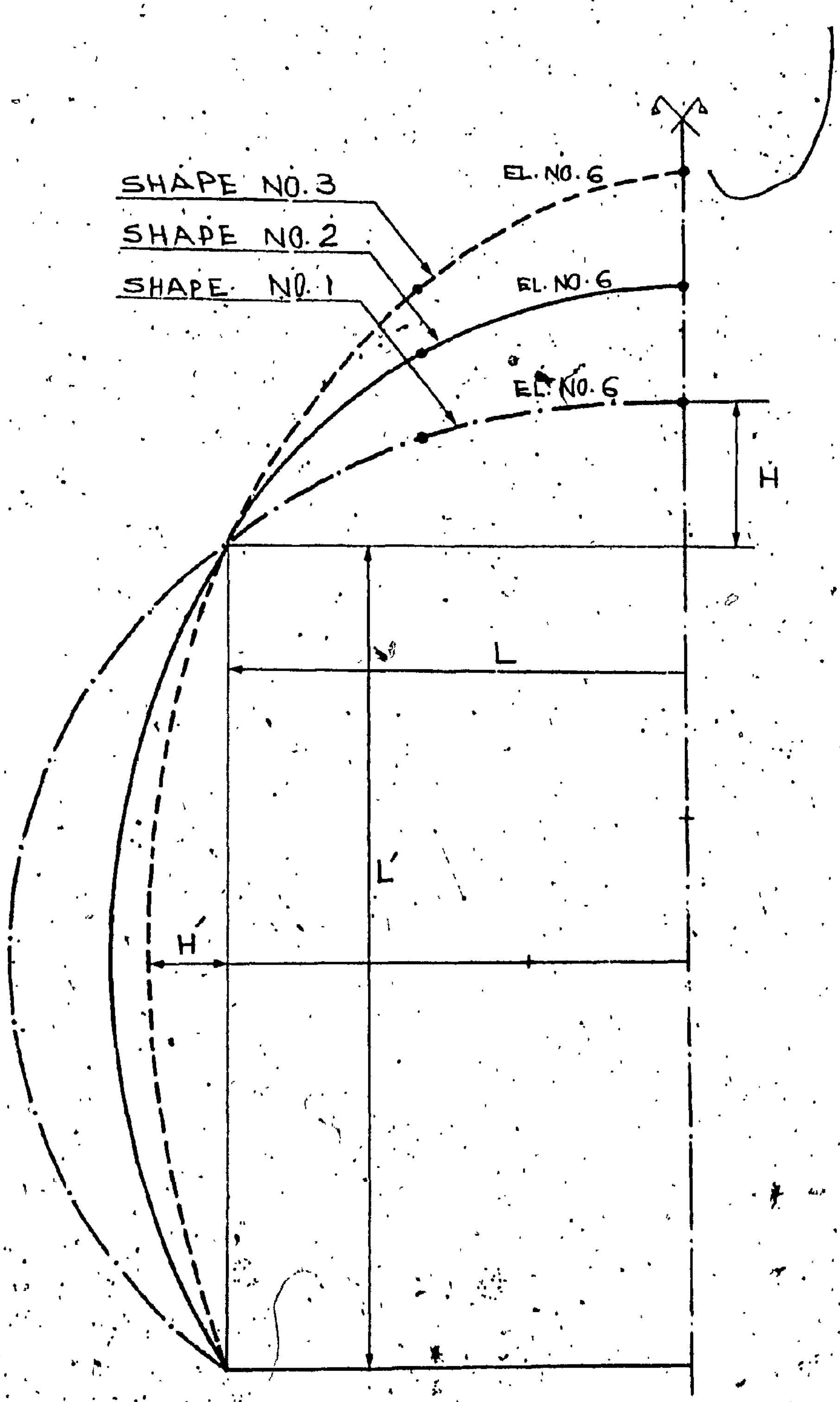



FIG. 23. DIFFERENT HORSESHOE SHAPES

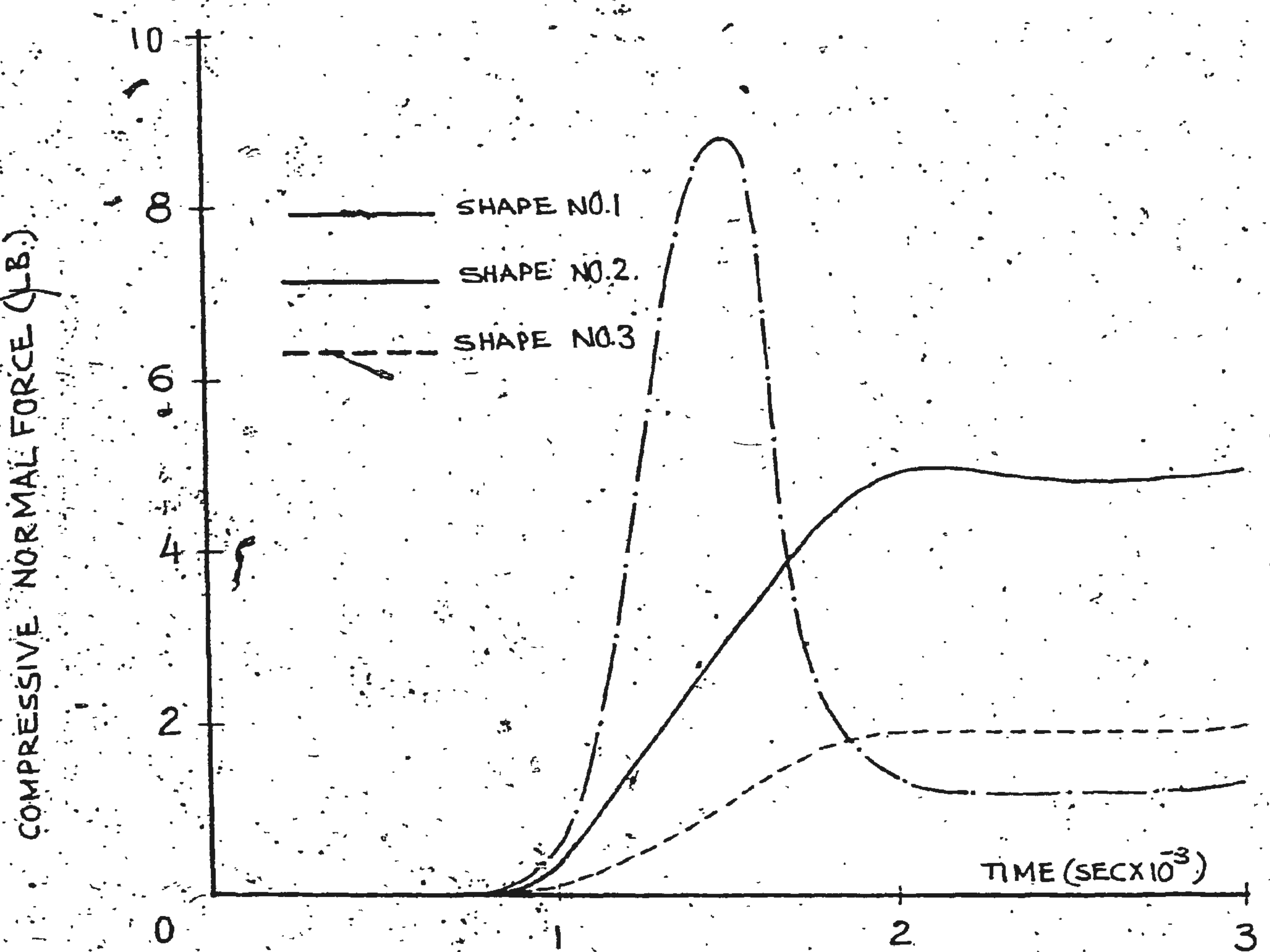



FIG. 24 NORMAL FORCE VS. TIME, ELEMENT NO. 6 - FIG. 23

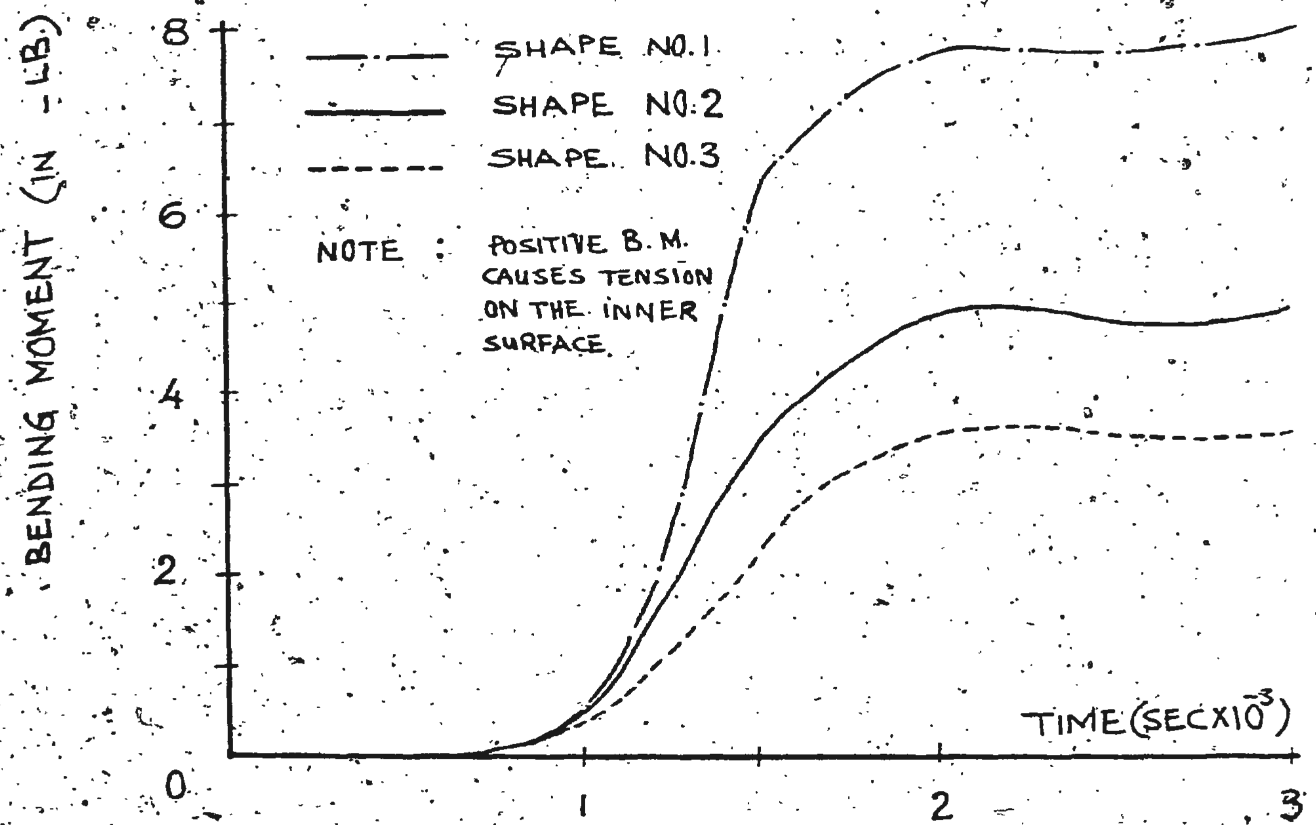



FIG. 25 BENDING MOMENT VS. TIME, ELEMENT NO. 6

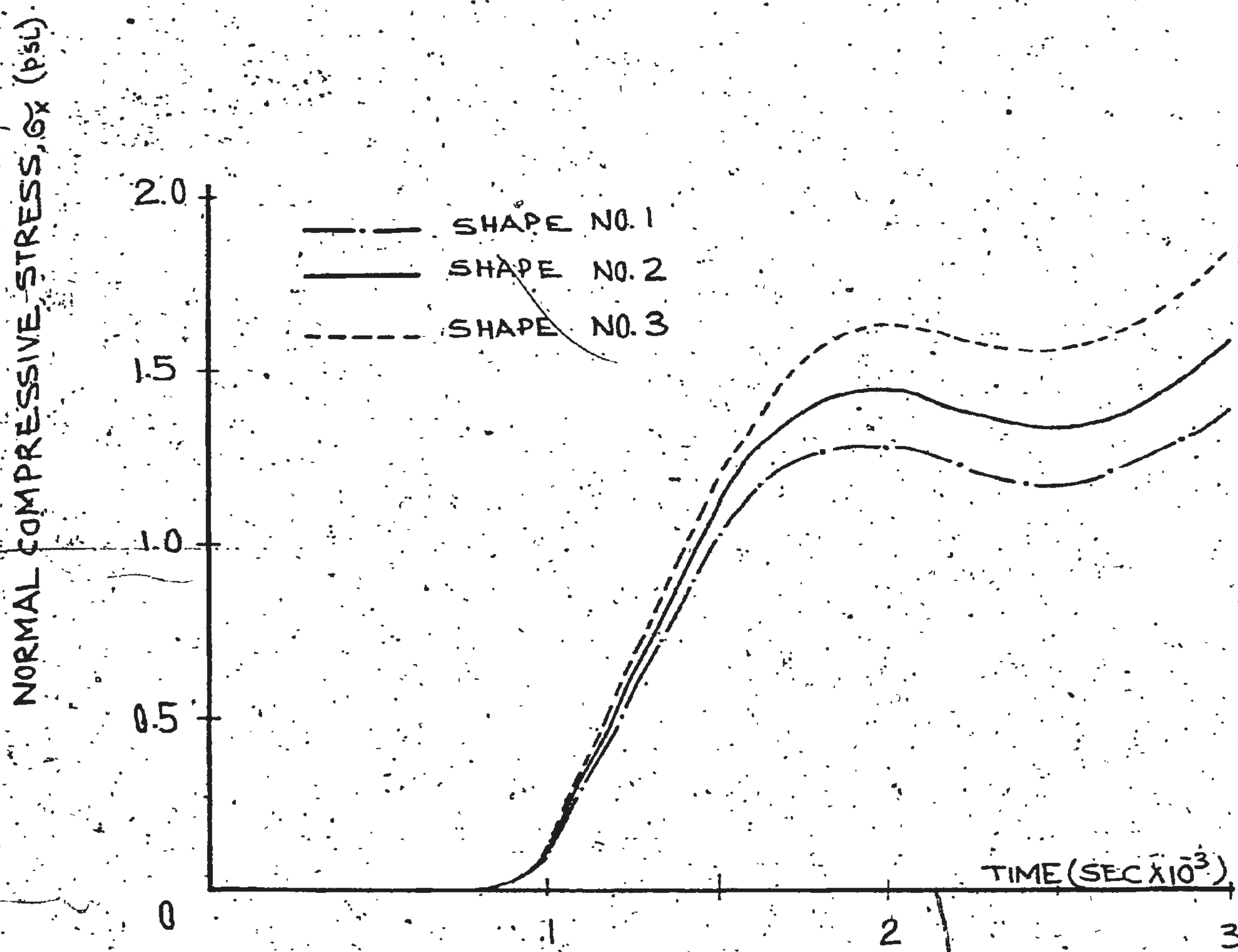



FIG. 26. STRESS VS. TIME, ELEMENT NO. 92 - FIG. 19

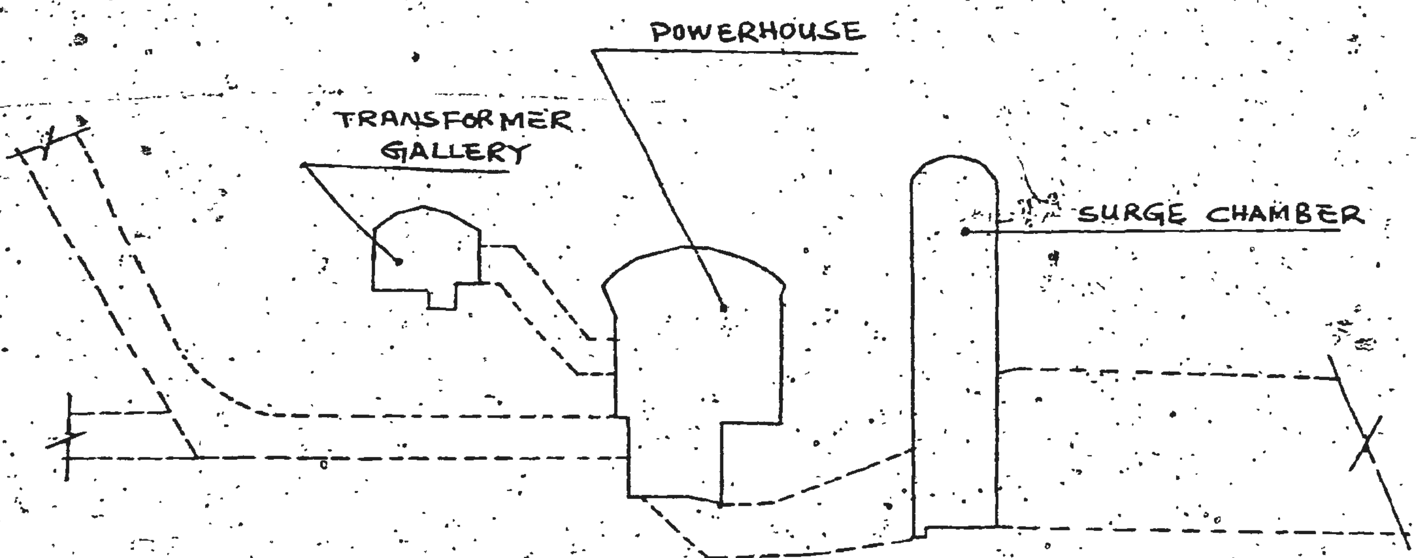
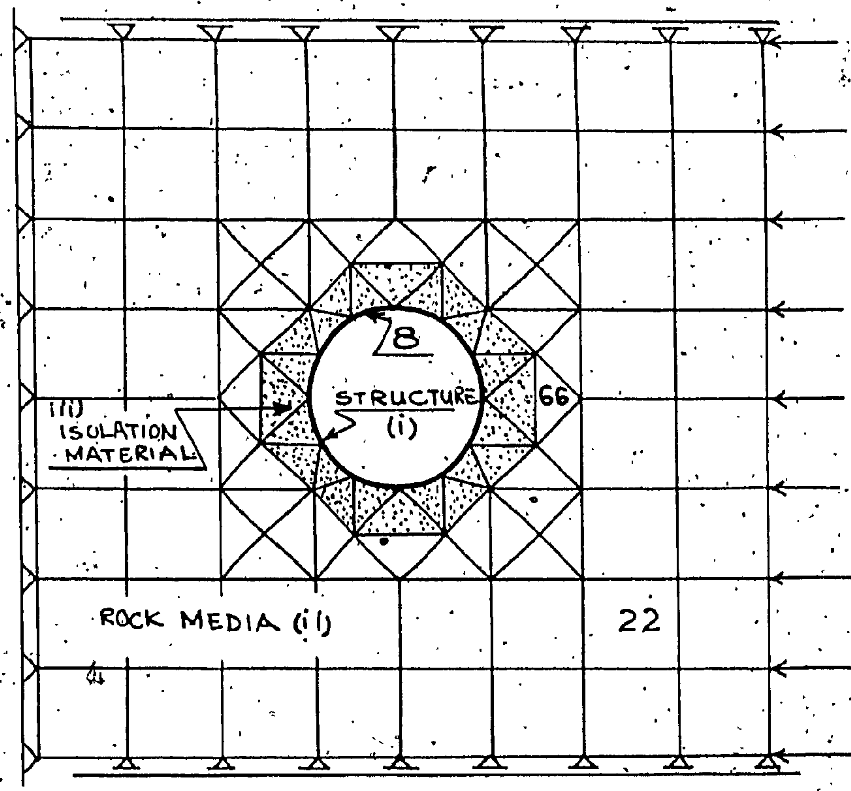



FIG. 27. UNDERGROUND HYDRO-ELECTRIC POWER COMPLEX - (REF 54)




MODEL: PLANE STRAIN ANALYSIS
100 N.P., 120 EL. INCLUDING
12 SHELL EL.

MATERIALS: i) STEEL : $\rho = 0.000735$, $\nu = 0.3$, $E = 29 \times 10^6$
ii) ROCK : $\rho = 0.00025$, $\nu = 0.3$, $E = 5 \times 10^6$
iii) CLOSED CELL POLYURETHANE FOAM : $\rho = 6 \times 10^{-6}$, $\nu = 0.3$, $E = 1400$

UNITS: INCH, LB., SEC.

FIG. 28. ISOLATED STRUCTURE

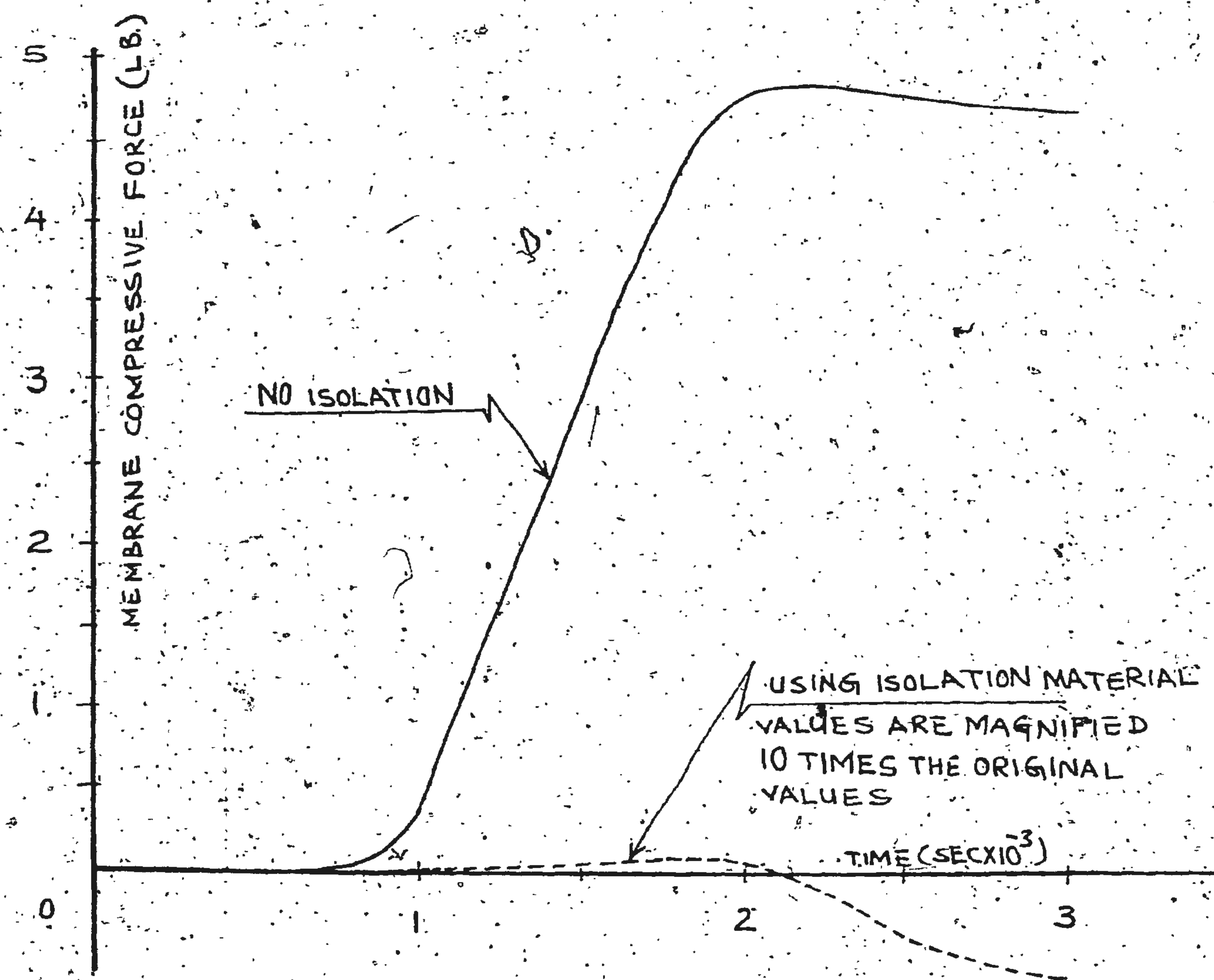



FIG. 29. MEMBRANE FORCE VS. TIME, ELEMENT NO. 8 - ISOLATED STRUCTURE.

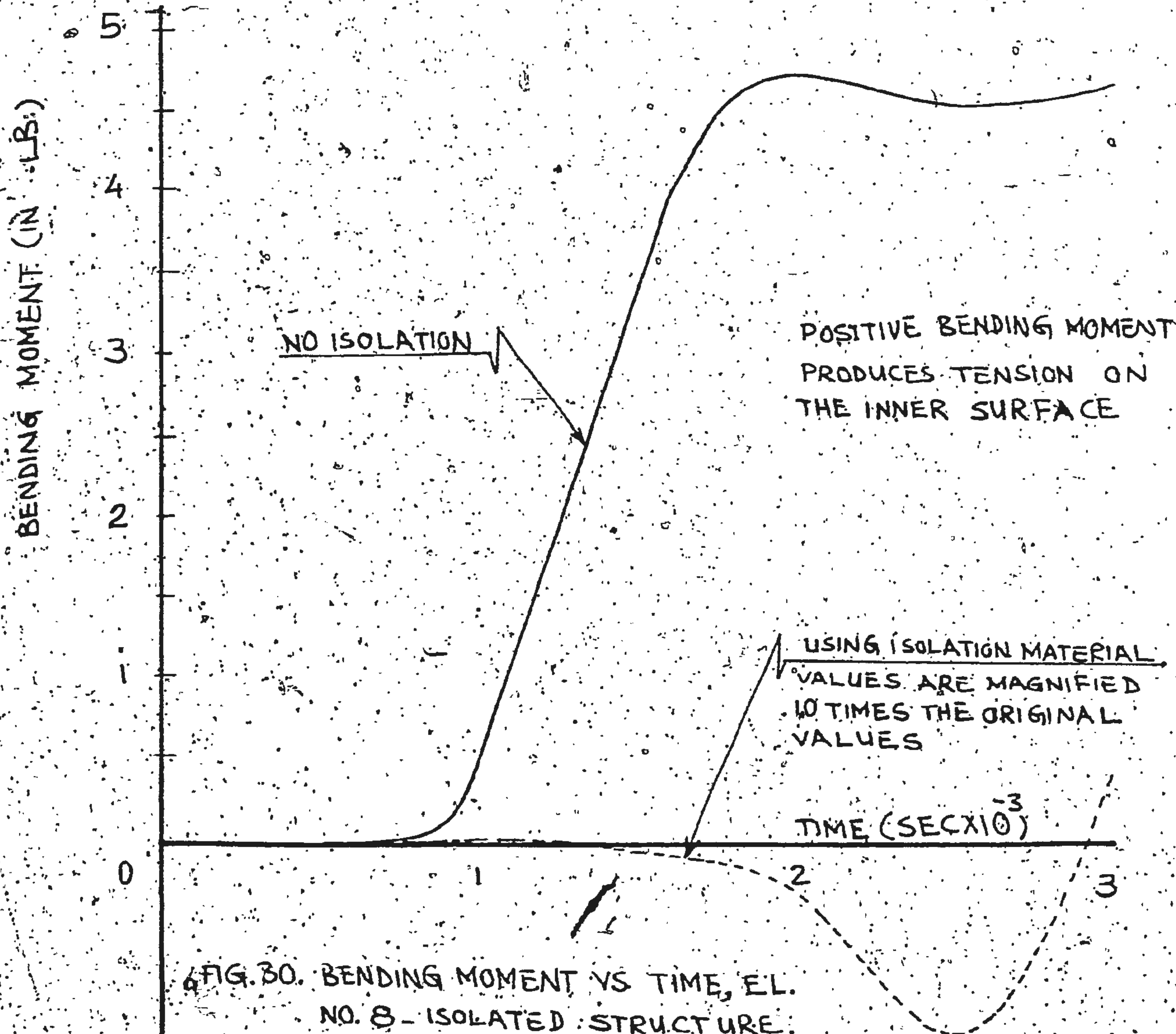



FIG. 30. BENDING MOMENT VS TIME, EL. NO. 8 - ISOLATED STRUCTURE.

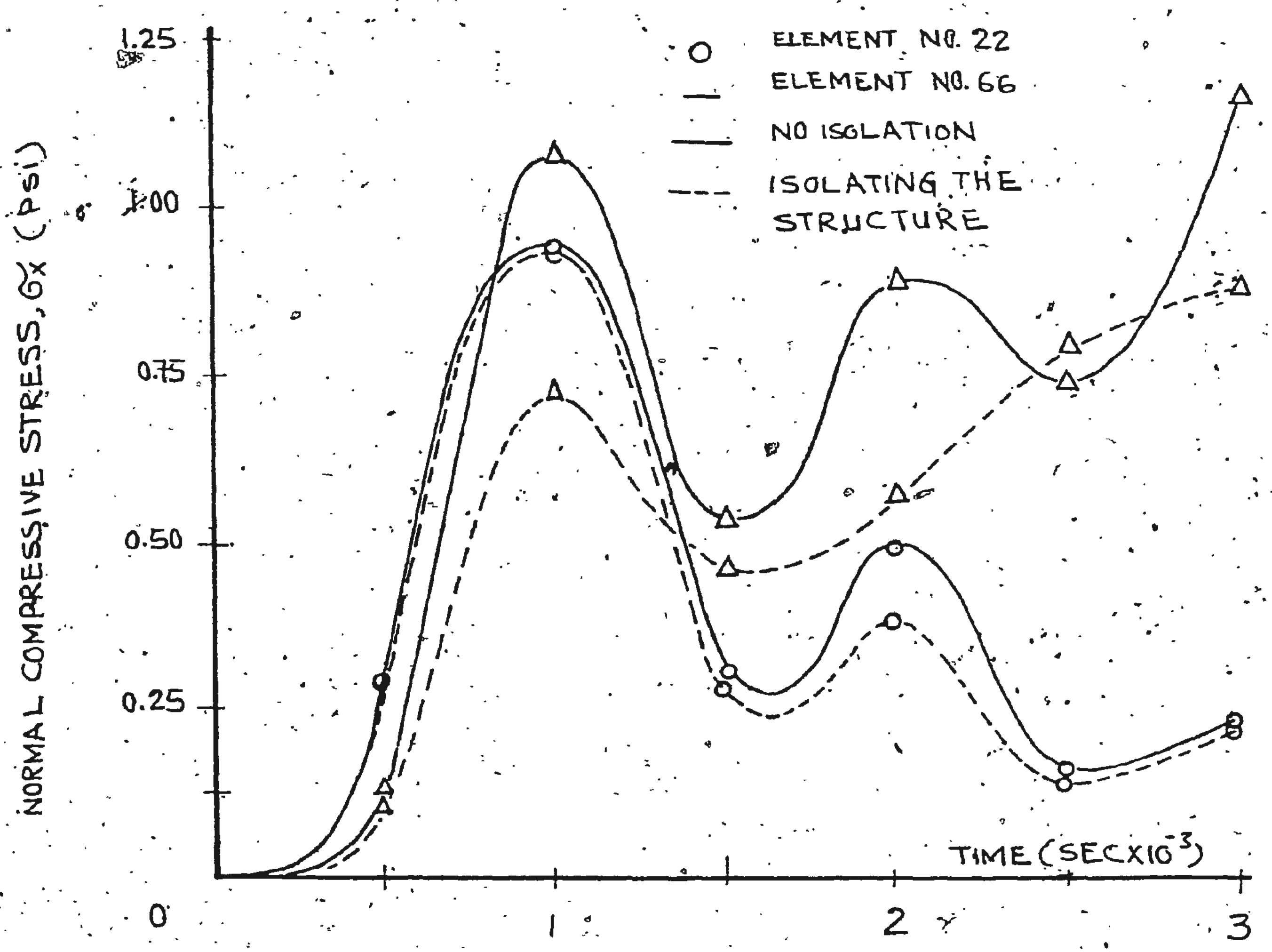



FIG. 31. STRESS VS. TIME, ELEMENTS NO. 22 AND 66 – ISOLATED STRUCTURE.

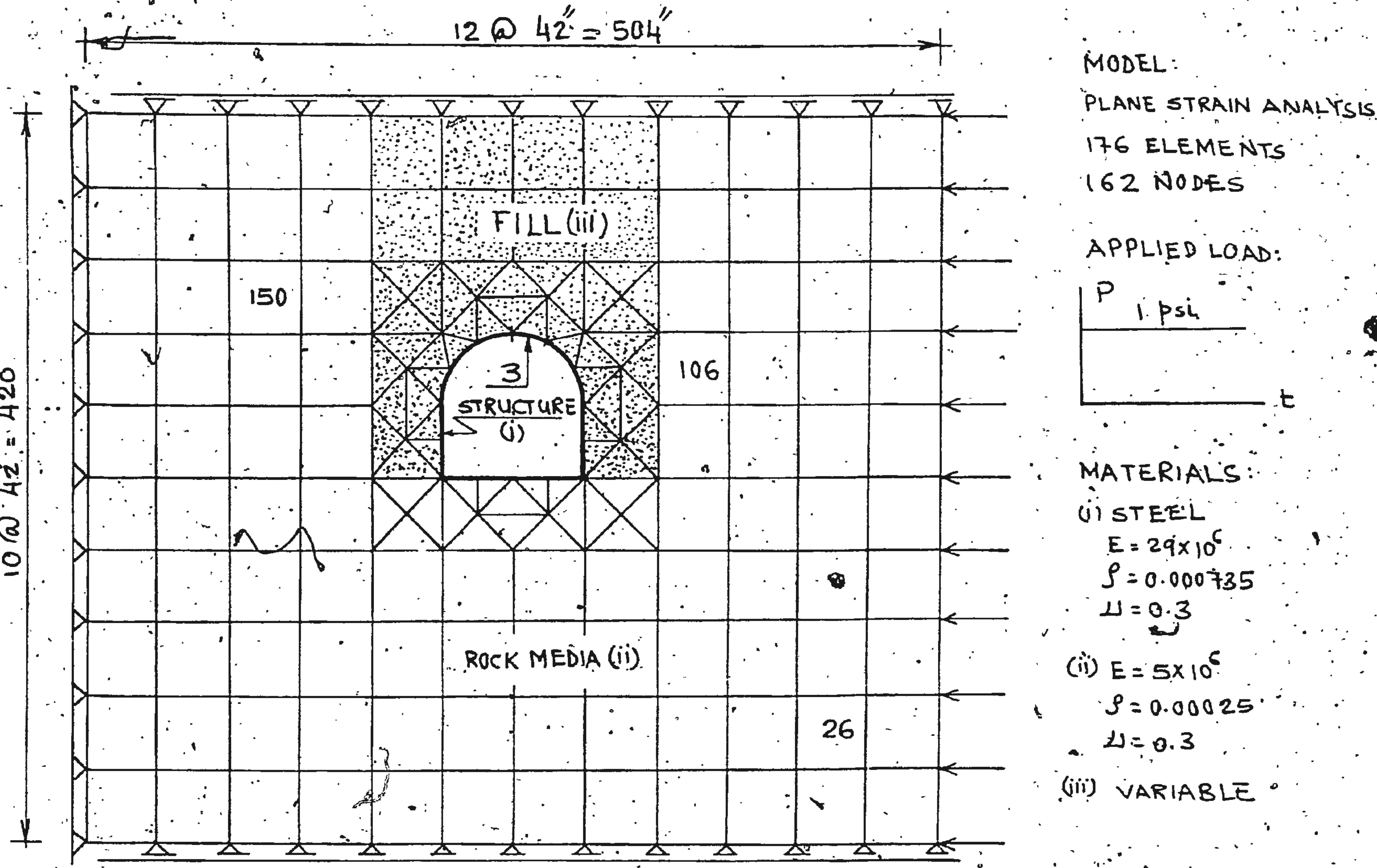



FIG. 32. MODEL USED FOR CUT-AND-COVER STRUCTURE

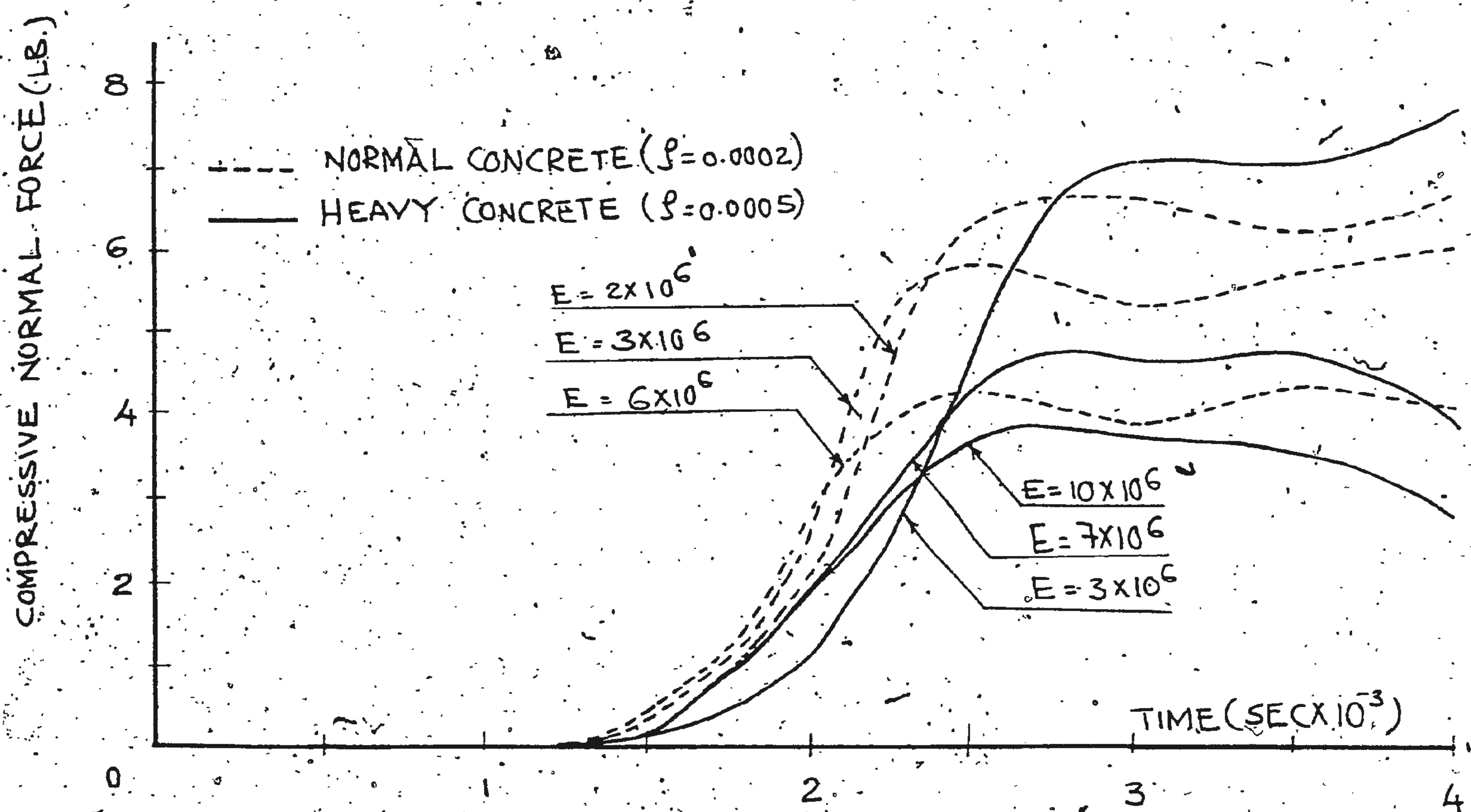



FIG. 33. NORMAL FORCE VS. TIME, ELEMENT NO. 3 - CUT-AND-COVER STRUCTURE.

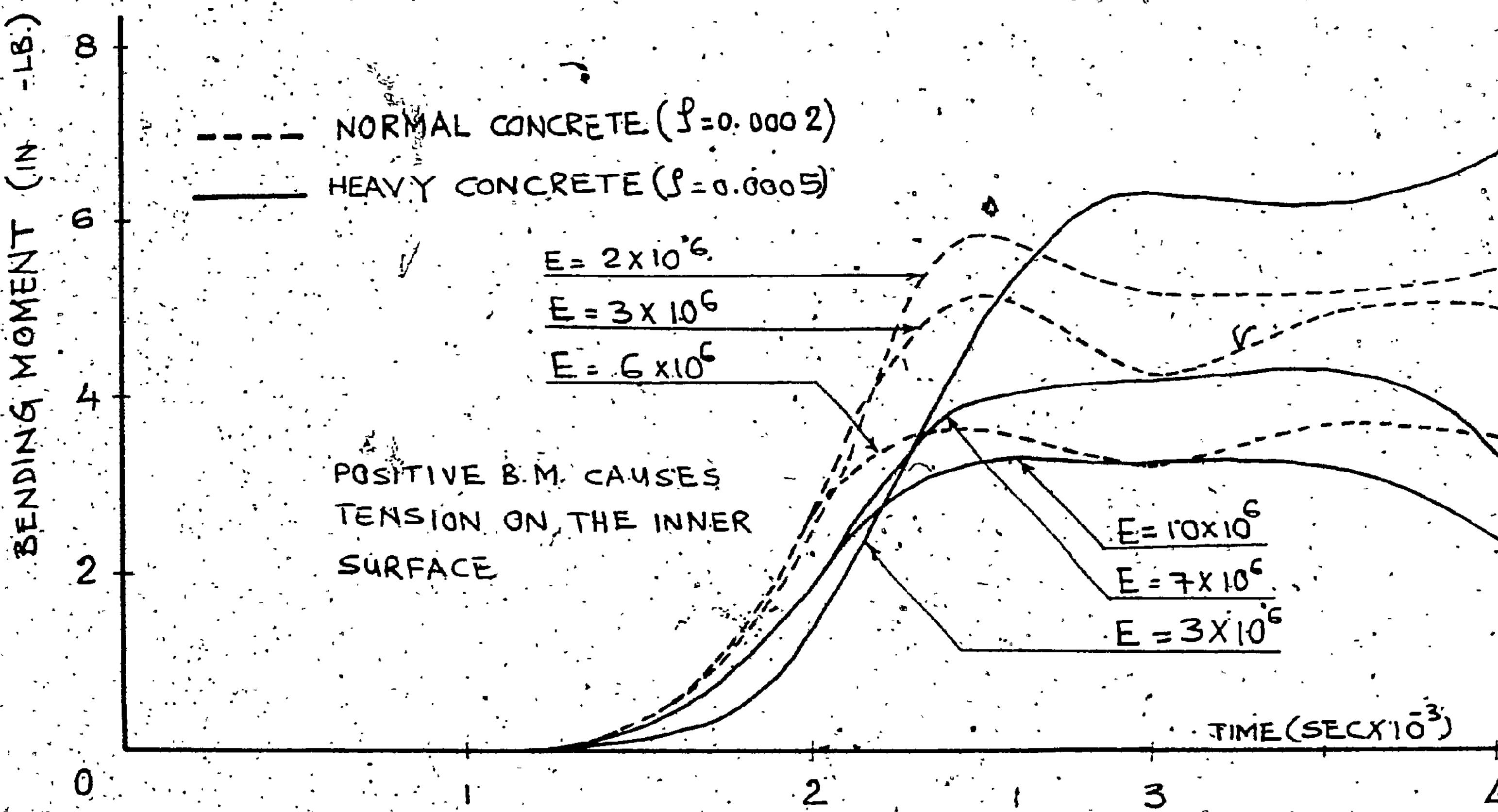



FIG. 34. BENDING MOMENT VS. TIME, ELEMENT NO. 3 - CUT- AND-
COVER STRUCTURE.

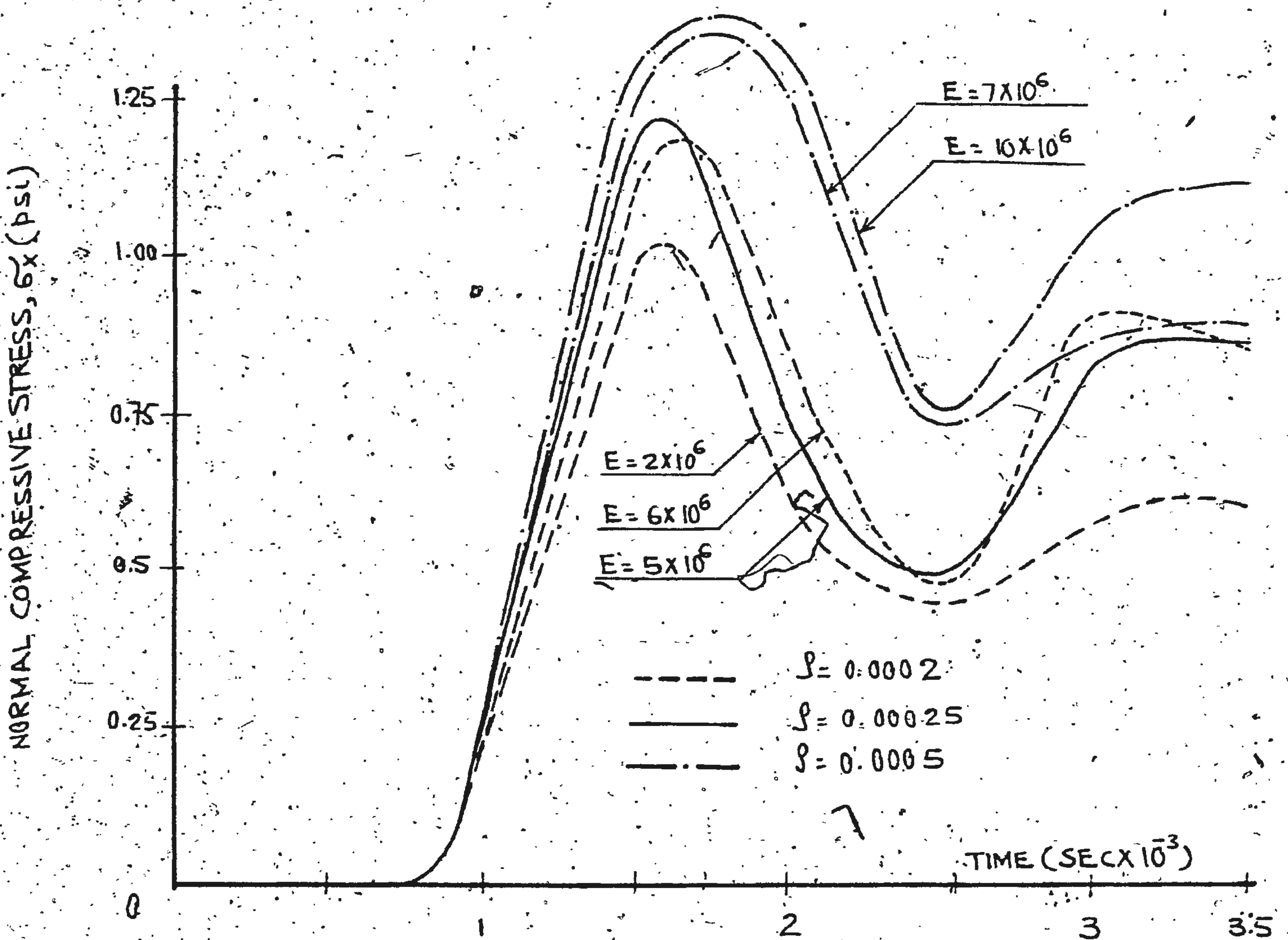



FIG. 35. STRESS VS. TIME, ELEMENT NO. 106 – CUT-AND-COVER STRUCTURE

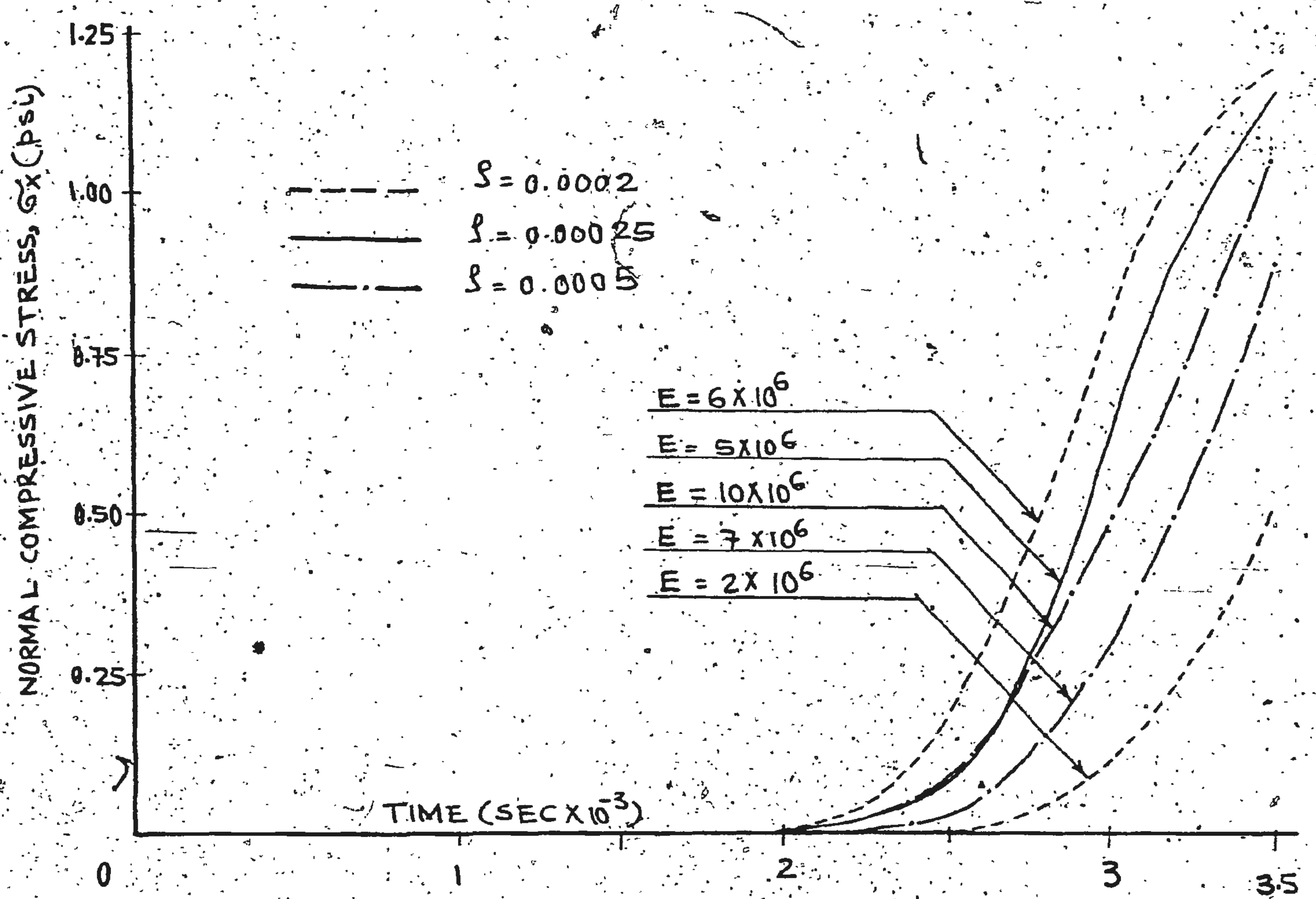



FIG. 36. STRESS VS. TIME, ELEMENT NO. 150 - CUT-AND-COVER STRUCTURE

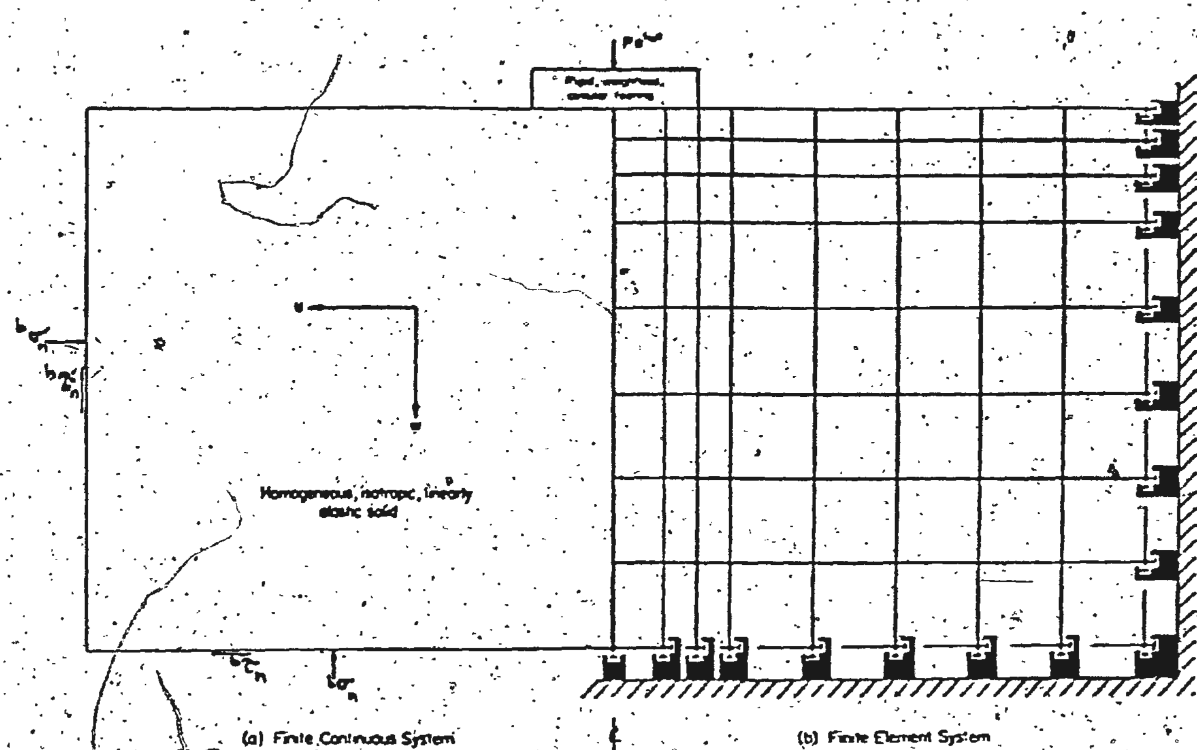
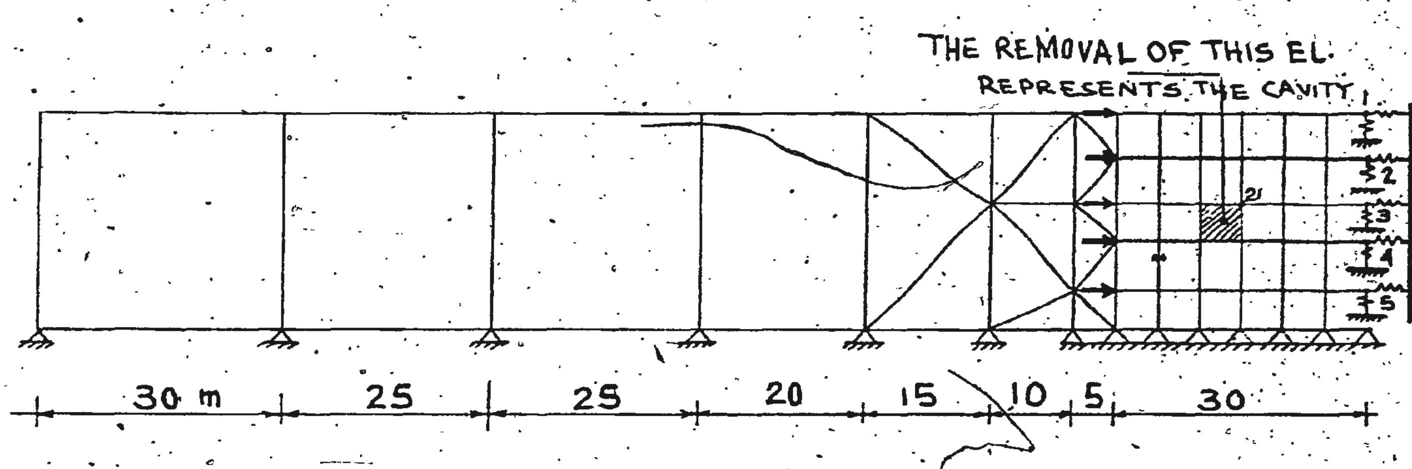



FIG. 37 FINITE MODELS OF A VERTICALLY EXCITED RIGID FOOTING (REF. 33)




MODEL:

PLANE STRAIN ANALYSIS
59 NODAL POINTS (N.P.)
50 ELEMENTS

SPRING CONSTANTS = $2.5 \times 10^4$ t/m
FOR N.P. 1 AND $5 \times 10^4$ FOR N.P. 2–5

MATERIAL:

$\rho = 2.$
$E = 10^4$ t/m²
$\nu = 0.15$

LOAD:

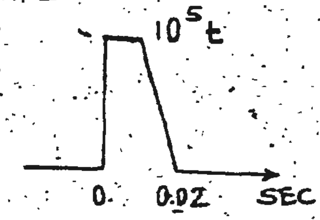



FIG. 38 NEW FINITE DYNAMIC MODEL

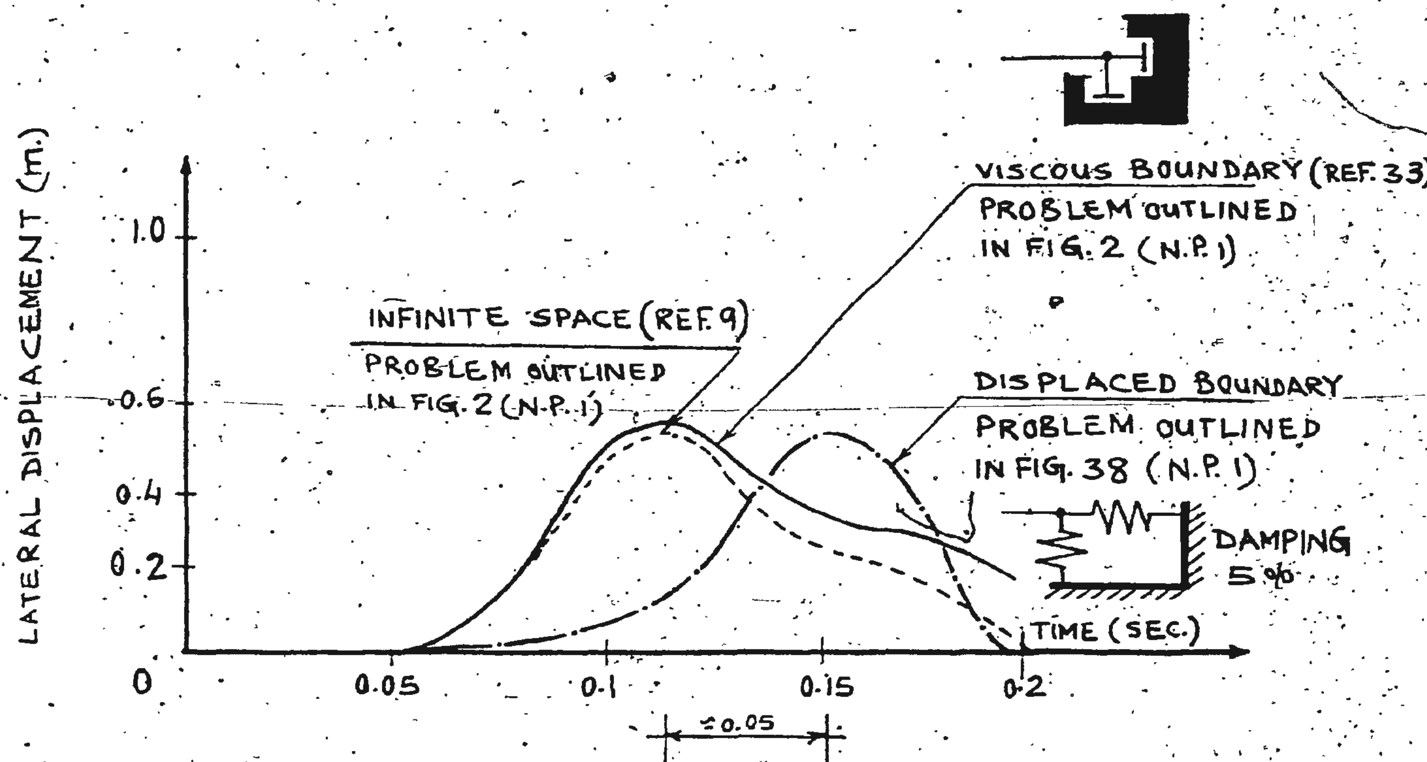



FIG. 39 DISPLACEMENT VS. TIME, NODE NO. 1 – VISCOUS AND DISPLACED BOUNDARIES (FIGS. 2 AND 38)

# APPENDIX A

## FLOW CHART OF THE MODIFIED COMPUTER PROGRAMME

APPENDIX A

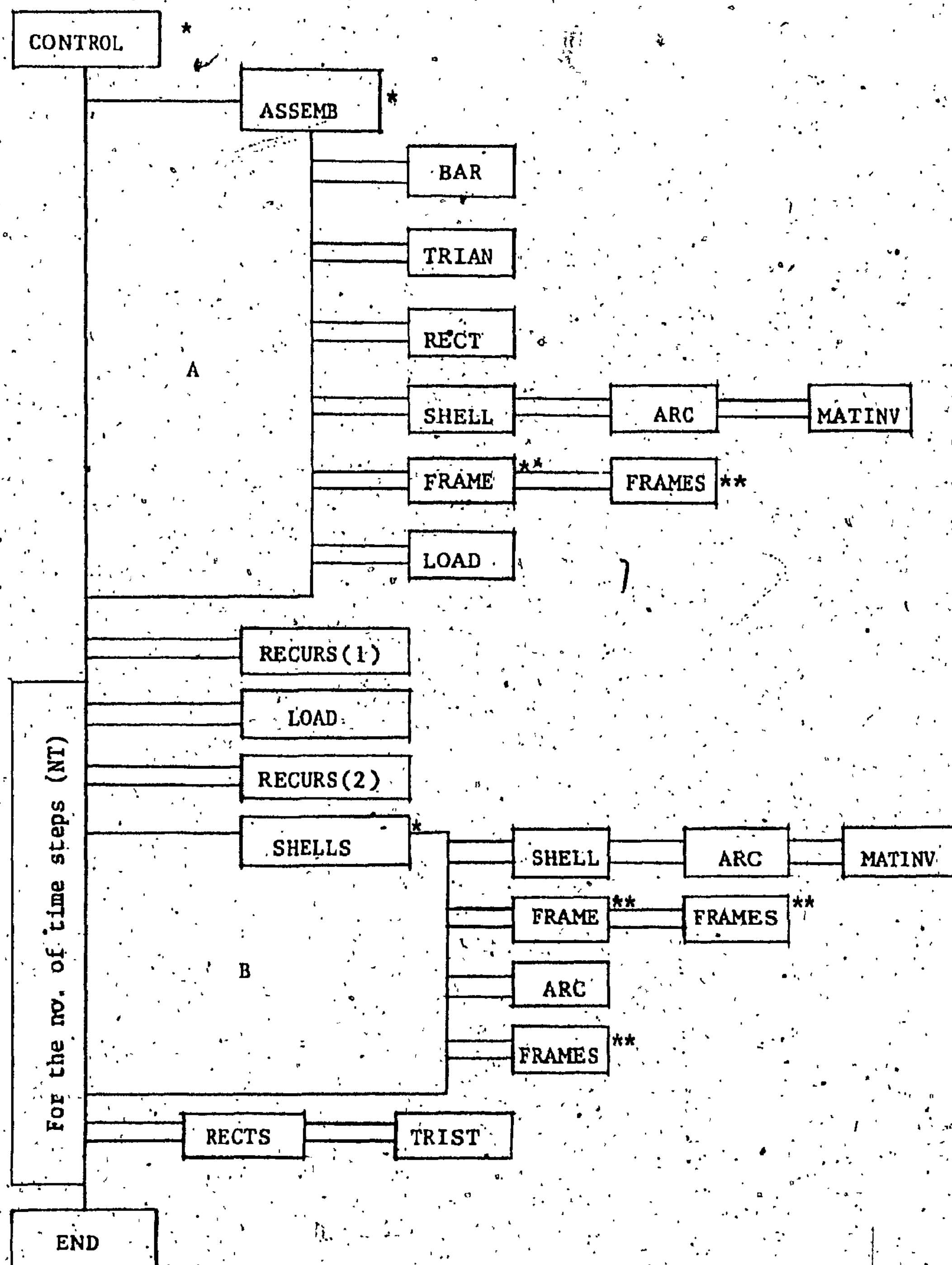



Fig. 40. Flow Chart of the Modified Programme

* Modified subroutines

** New subroutines

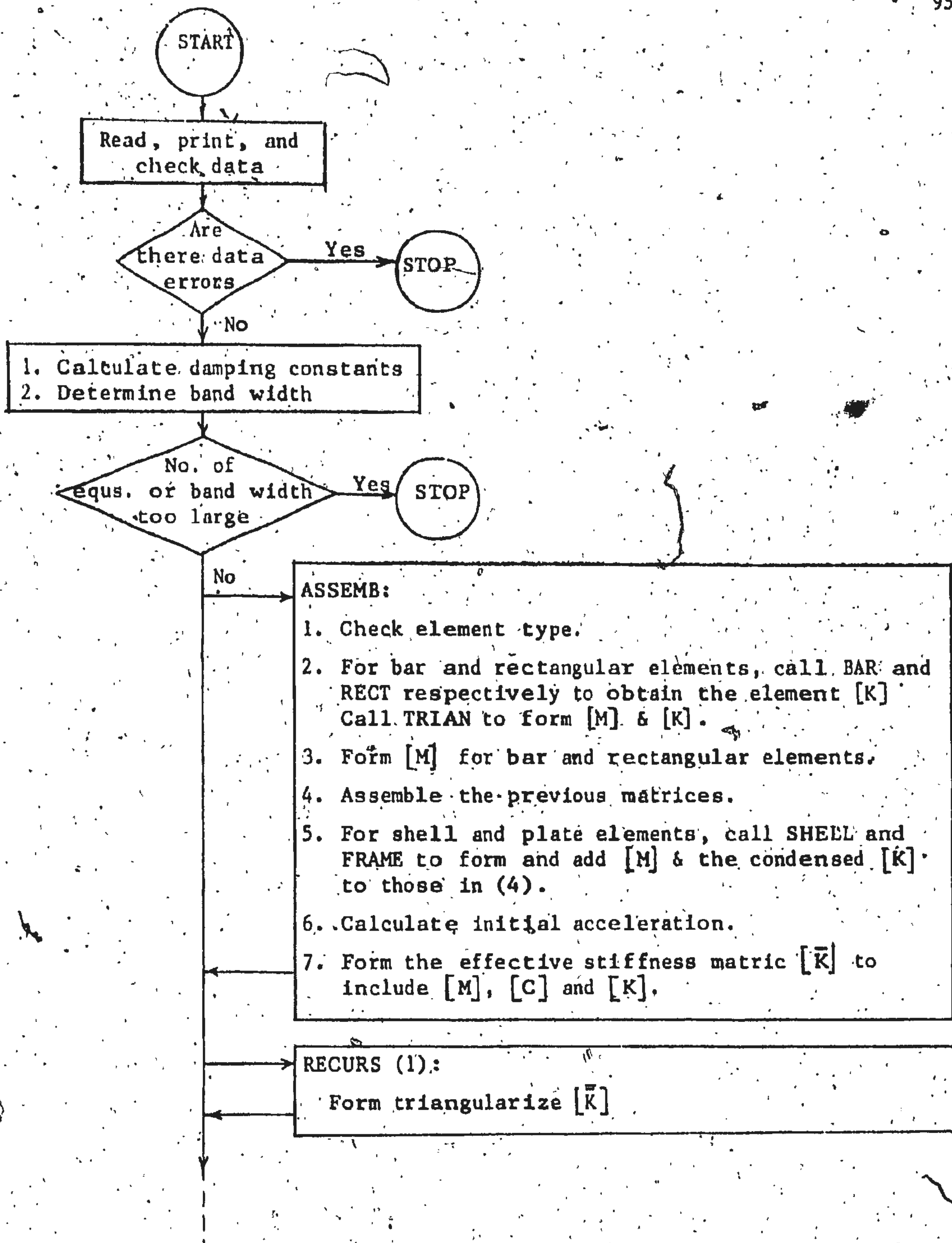



Fig. 40. Cont. Section A

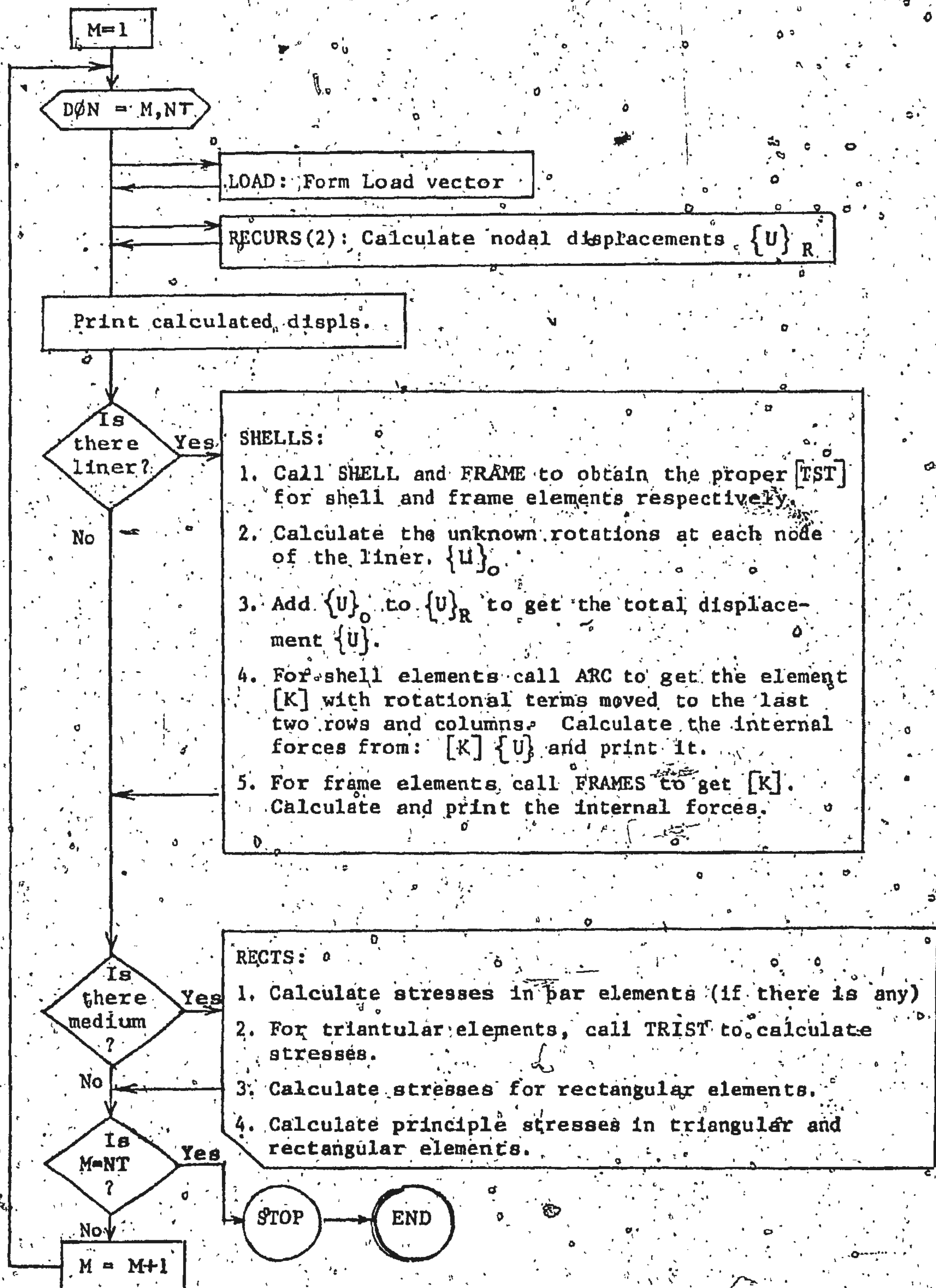



Fig. 40. Cont. Section B

## APPENDIX B

## DAMPING OF THE SYSTEM

# APPENDIX B

## DAMPING OF THE SYSTEM

As stated in Chapter Two, the damping matrix $\left[C\right]$ could be written in the form

$$\left[C\right] = C_1\left[M\right] + C_2\left[K\right] \qquad (1)$$

where $\left[M\right]$ and $\left[K\right]$ are the mass and stiffness matrices of the system and $C_1$ and $C_2$ are constants chosen to provide the desired damping effect. These constants can be related to the critical damping $C_c$ or to the critical damping ratio $\xi$ in the following manner

$$C_c = 2\sqrt{KM} \qquad (2)$$

$$\xi = \frac{C}{C_c} \qquad (3)$$

Introducing Eqns. 2 and 3 into Eqn. 1 gives

$$= \frac{C_1 M}{2\sqrt{KM}} + \frac{C_2 K}{2\sqrt{KM}}$$

$$= \frac{C_1}{2}\sqrt{\frac{M}{K}} + \frac{C_2}{2}\sqrt{\frac{K}{M}} \qquad (4)$$

i.e. the critical damping ratio $\xi_n$ for the n-th mode is

$$\xi_n = \frac{C_1}{2\omega_n} + \frac{C_2\omega_n}{2} \qquad (5)$$

where ${}_n$ is the angular frequency for the n-th mode.

It is clear from Eqn. 5 that $\xi_n$ has some minimum value corresponding to a certain value of $\xi_n$. Setting the derivative of $\xi_n$ with respect to $\omega_n$ equal to zero gives

$$\omega_n = \sqrt{\frac{C_1}{C_2}} \qquad (6)$$

Using Eqn. 5 specifying the percentage contributions from the two terms to obtain the desired damping ratio, the constants $C_1$ and $C_2$ can be determined as follows:

Assuming equal contribution from the two terms, gives

$$C_1 = \xi_n \omega_n$$
$$C_2 = \xi_n / \omega_n \qquad (7)$$

or, in terms of the period of the system

$$C_1 = 2\pi \xi_n / T_n$$
$$C_2 = \xi_n T_n / 2\pi \qquad (8)$$

## APPENDIX C

## DYNAMIC RESPONSE OF UNDERGROUND CAVITY

# APPENDIX C

## DYNAMIC RESPONSE OF UNDERGROUND CAVITY

The underground cavity shown in Fig. 38 is excited by the dynamic loading shown. The elastic, displaced, energy absorbing boundary is used to simulate the infinite extension of the medium. Fig. 41 presents a comparison of the responses of node number 21 in Fig. 38 for an absorbing boundary and a rigid one.

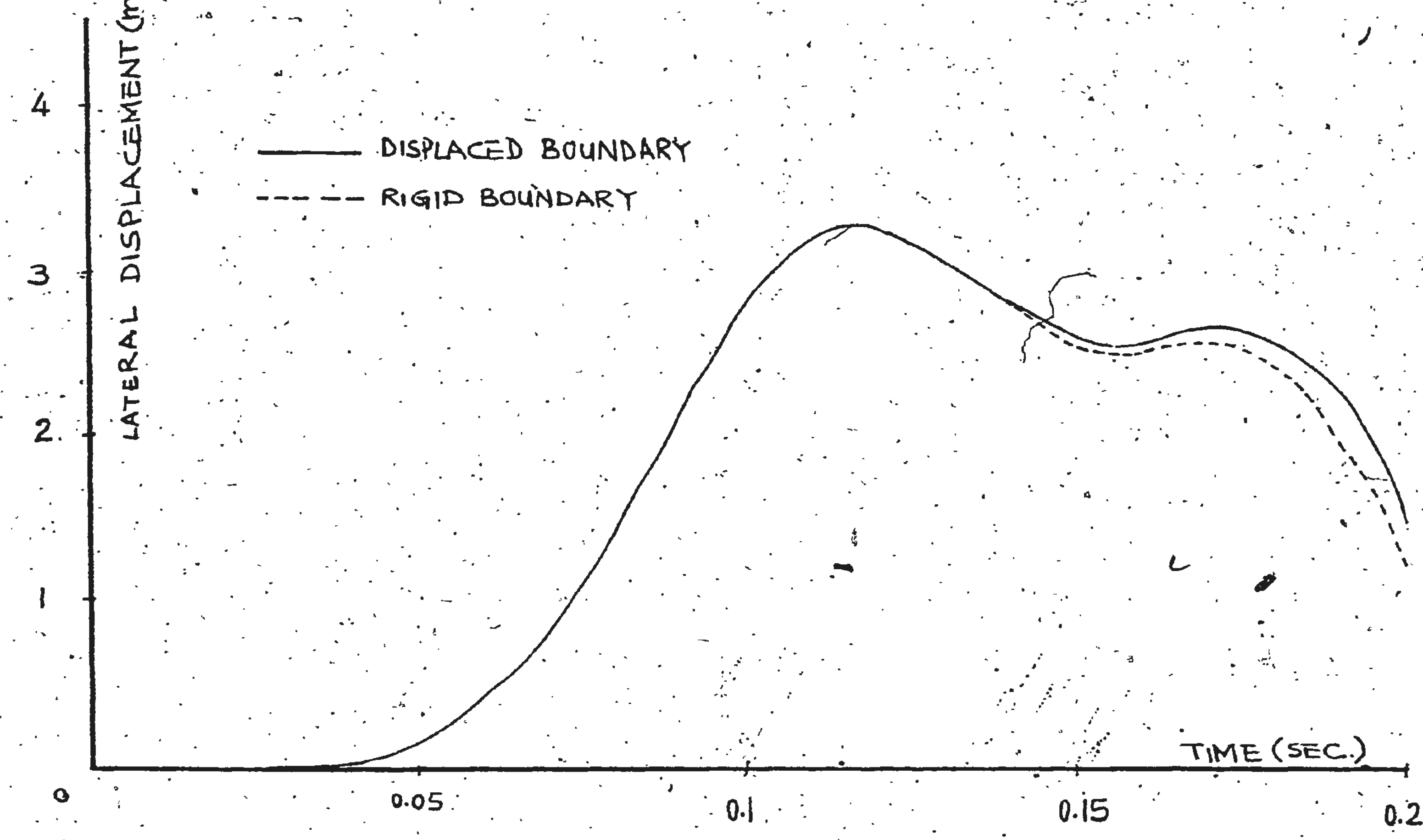



FIG. 41. RESPONSE OF UNDERGROUND CAVITY, NODE NO. 21 - FIG. 38

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