THE OBJECTIVES OF DEDUCTIVE GEOMETRY IN NEWFOUNDLAND SECONDARY SCHOOLS AS PERCEIVED BY CONCERNED GROUPS

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A Thesis

Presented to

The Department of Curriculum and Instruction

Memorial University of Newfoundland

In Partial Fulfillment

of the Requirements for the Degree

Master of Education

by

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August 1973

ABSTRACT

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It was the purpose of this study (1) to construct an instrument for determining how different individuals perceive the objectives of deductive geometry; and (2) to determine how concerned groups perceive the objectives of deductive geometry in the secondary school.

An initial list of 78 objectives was obtained from an analysis of literature and a survey of current textbooks at the secondary school level. The initial list of objectives was edited and revised to 35 items. Suggestions were elicited from a panel of mathematics educators on clarity, comprehensiveness and compactness. The final form of the instrument consisted of 35 possible objectives of deductive geometry in the secondary school.

The list of 35 objectives, each on a separate card, and a 5 point scale for rating the importance of each objective was submitted by mail to 85 individuals identified as being members of one of the following groups: (1) Geometry teachers in Newfoundland secondary schools, and (2) Mathematics educators in universities in Canada and the United States.

Mean ratings were computed for each item as perceived

by each group. These were used to rank the 35 items in order of importance for each group. Comparisons were made between groups to determine whether or not agreement existed on the important and non-important items and other general conclusions were drawn. Implications of the study were considered.

The major findings and conclusions of the study were as follows:

1. Geometry teachers in Newfoundland schools did not agree with mathematics educators on the important objectives of deductive geometry.

2. Geometry teachers in Newfoundland schools agreed with mathematics educators on the non-important objectives of deductive geometry.

3. In general, geometry teachers seemed to put more stress on those objectives which are at a low taxonomic level while mathematics educators stressed those at higher levels.

4. Both geometry teachers and mathematics educators considered the rote memorization of theorems to be non-important.

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CHAPTER I

THE PROBLEM

INTRODUCTION

The past few years have been marked by innovations and changes in the content and teaching methods of mathematics in the secondary school. The "new math" and the "revolution in mathematics" have become part of the language of the past decade. In spite of these exciting changes, however, it has proved difficult to obtain widespread agreement concerning changes in geometry. Although there seems to be general agreement that geometry must continue to hold a place in the high school curriculum, and that traditional thinking on the subject must be modified, there continues to be much debate on just what should form the basis of a high school geometry programme, and what exactly should be accomplished by such a course.

On the first of these questions; namely, what should form the basis of a programme, there have been numerous proposals that many traditional topics and approaches be jettisoned in favour of newer perspectives. The suggested changes[1] have included approaching geometry through such

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[1]Howard F. Fehr, Frank M. Eccles, and Bruce Meserve, "What Should Become of the High School Geometry Course? The Mathematics Teacher, LXV (February, 1972), 102.

means as transformations, vectors and coordinates. Nevertheless, the treatment of geometry in the high school today is remarkably similar to the Euclidean model set down more than twenty-three centuries ago. Fehr[2] describes the present course as "... traditional Euclidean synthetic geometry of 2- and 3-space, modified by an introduction of ruler and protractor axioms." In short, the terms "Euclidean geometry" and "deductive geometry" are still synonymous to most teachers and educators. In Newfoundland secondary schools all of the present geometry course in Grade Nine and much of the course in Grade Ten is Euclidean.[3] The present study restricts itself solely to a Euclidean approach to high school geometry and the question of whether or not other approaches might be better was not considered.

On the question of what should be accomplished by a course in deductive geometry there is a diversity of views. Meserve[4] felt that while the objectives of algebra are basically to develop the properties of the fields of rationals, reals and complex numbers, the changing role of geometry remains "... a source of confusion to many teachers

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[2]Howard F. Fehr, "The Present Year-long Course in Euclidean Geometry Must Go," The Mathematics Teacher, LXV (February, 1972), 102.

[3]Newfoundland and Labrador Department of Education, Programme of Studies, 1972-73, Grade I-XI, p. 38.

[4]Bruce E. Meserve, "Geometry in the United States," Geometry in the Secondary School, National Council of Teachers of Mathematics Conference (Washington: National Council of Teachers of Mathematics, 1967), p. 2.

and administrators, a challenge to all who are alert to the needs of their students." Allendoerfer,[5] too, expressed a sense of confusion over geometry when he stated: "To some it is the study of geometric figures, while in the minds of others it is almost identified with a method of proof."

With such a lack of consensus, the task of developing a realistic set of objectives for the deductive geometry course in the secondary school is, at best, a difficult one. Many divergent factors must be considered in arriving at such a set. Obviously it will depend upon the elementary school geometry program and must be related to the nature of the learner and his needs in society. Consequently, no one set of objectives is likely to be acceptable for all groups of students and certainly not for every individual. Another consideration is the level of generality of the statements of objectives. Such general statements as, "To acquire the skills and knowledge for good citizenship" serve only the purpose of keeping certain ideals about education in a democracy well in mind.[6] In order to be useful to the teacher, objectives are required in terms of specific behaviors to be obtained by the student. As Taba[7] states:

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[5]Carl B. Allendoerfer, "The Dilemma in Geometry," The Mathematics Teacher, LXII (March, 1969), 165.

[6]Robert N. Harding, "The Objectives of Mathematics Education in Secondary Schools as Perceived by Various Concerned Groups" (unpublished Doctor's dissertation, University of Nebraska, 1968), p. 6.

[7]Hilda Taba, Curriculum Development: Theory and Practice (New York: Harcourt, Brace and World, Inc., 1962), p. 228.

> A platform of general objectives, no matter how well defined is still an inadequate guide for the specific aspects of curriculum, such as the selection of content and experiences for particular units on a particular grade level. These general objectives need to be translated into more specific ones.

Such specific objectives are useful in planning content and sequence of a course of study and are required in order to evaluate the products of learning.

The strongest influence on statements of objectives for geometry is presently wielded by organized groups of scholars and mathematics educators. Yet it is the classroom teacher who ultimately determines the importance of various objectives and the nature of classroom experiences to be provided to meet the objectives. There are indications that classroom teachers have traditionally been unaware of recognized goals and have often taught in a manner which has not reflected current thinking on the subject. As far back as 1930 Betz[8] wrote:

> Unfortunately, the teachers of geometry, who should have been its most enthusiastic and successful exponents, have only too often been its worst enemies by their lack of acquaintance with its history and its distinctive characteristics, and by their apparent inability to formulate and to realize the immediate and ultimate objectives of the subject.

Quast[9] cited a more recent example of the same problem:

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[8] William Betz, "The Transfer of Training With Particular Reference to Geometry," <u>The Teaching of Geometry</u>, Fifth Yearbook of the National Council of Teachers of Mathematics (New York: Bureau of Publications, Teachers College, Columbia University, 1930), p. 151.

[9] William G. Quast, "Geometry in the High Schools of the United States: An Historical Analysis from 1890 to 1966," (unpublished Doctor's dissertation, Rutgers, The State University, 1968), p. 330.

> ... today the idea of student discovery is widely accepted as a means to develop insight and understanding of geometric principles, yet many teachers fail to achieve these goals by emphaszing the memorization of propositions.

The conclusion to be drawn is that while the formulation of goals is essential for a sound geometry program, of equal importance is the degree to which such goals are transmitted to the teachers of the program.

THE PURPOSE OF THE STUDY

It was the purpose of this study (1) to construct an instrument for determining how different individuals perceive the objectives of deductive geometry; and (2) to determine how concerned groups perceive the objectives of deductive geometry in the secondary school.

Some Specific Questions

The following were the type of questions to be answered in the study:

1. Do geometry teachers and mathematics educators agree on the important objectives of deductive geometry?

2. Do geometry teachers and mathematics educators agree on the non-important objectives of deductive geometry?

3. What trends can be determined from the way teachers and educators perceive the objectives of deductive geometry?

SIGNIFICANCE OF THE STUDY

Johnson and Rising[10] cite the lack of clear goals for teaching mathematics as one of the most important unresolved issues in mathematics. This is just as much of an issue in geometry as in any other area of mathematics. Yet one of the primary decisions of a geometry teacher must be in regard to the goals he sets up for the course. The experiences he provides for the pupils and the final evaluation should be in reference to the objectives he has determined. Today's geometry teacher, in Newfoundland as elsewhere, is challenged to present modern geometry as a subject that is livlier, freer and more informal, yet no less precise.[11] The author feels that teachers are prevented from fully meeting this challenge by a confusion of aims resulting from the failure of reformers to transmit reasonable goals to the geometry classroom teacher. Many students are still taught" ...as though the subject can best be learned and understood by committing facts to memory."[12] The new geometry recognizes that forcing students to recall a given number of theorems and facts day after day does not

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[10]Donovan A. Johnson and Gerald R. Rising, Guidelines for Teaching Mathematics (Belmont, California: Wadsworth Publishing Company, 1967), p. 382.

[11]Helen R. Pearson and James R. Smart, Geometry (Boston: Ginn and Company, 1971), p. v.

[12]Quast, op. cit., p. 340.

necessarily produce understanding of geometry.[13]

The significance of this study, then, lies in the need for determining to what extent the goals of deductive geometry have been clarified by present day educators and to what extent they have been transmitted to teachers in Newfoundland schools. According to Adler[14] such are "...the questions we must ask ourselves to measure the extent of our success during the seventies." Admittedly, such information is a small step toward achieving changes in the geometry classroom, but it is a necessary prelude to achieving a curriculum based upon clearly stated objectives.

DEFINITION OF TERMS

1. Deductive Geometry:

Deductive geometry is a formal structure of undefined terms, definitions, postulates based upon assumptions, and theorems which can be proved reasonably and logically through the use of these assumptions.

2. Mathematics Educator:

The term "mathematics educator" is used to refer to faculty members of the Mathematics Education Department or Mathematics Department of recognized universities.

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[13]Pearson and Smart, op. cit., p. v.

[14]Irving Adler, "Criteria of Success in the Seventies," The Mathematics Teacher, LXV (January, 1972), 41.

3. Geometry Teacher:

The term "geometry teacher" is used to refer to any teacher who taught at least one class in Grade Nine or Ten geometry in Newfoundland in the school year 1972-73.

CHAPTER II

REVIEW OF RELATED LITERATURE

This chapter will trace the major changes that have occured in the thinking on Euclidean geometry over the past seventy years or so. The review will thus be a summary of the changes in emphasis, the differences in opinion, the additions, deletions and reorganization of objectives during that time. Such an historical perspective is necessary to become acquainted with the traditional objectives of deductive geometry, some of which may still be emphasized by classroom teachers, as well as to illustrate how present-day objectives have evolved from those of the past.

Objectives Prior to 1920

The objectives and teaching methods of deductive geometry at the turn of the present century generally reflected a belief in mental discipline as the goal of all mathematics. Brooks[1] stated the mental discipline thesis as follows:

> The mind is cultivated by the activities of its faculties... Mental exercise is thus the law of mental development. As the muscle grows strong by use, so any faculty of the mind is developed by its use and exercise. An inactive mind, like an unused muscle, becomes weak

[1]Edward Brooks, "Mental Science and Methods of Mental Culture," Readings in the History of Mathematics Education, eds. J. K. Bidwell and R. G. Clason (Washington: National Council of Teachers of Mathematics, 1970), p. 84.

> and unskillful... let the mind remain inactive and it acquires a mental flabbiness that unfits it for any severe or prolonged activity.

The dominating value of geometry under such a philosophy was seen as an exercise in logic, a means of mental training, based on the belief that it gave formal power which could be applied to other fields.[2] Consequently, the main activities were to memorize and reproduce theorems and their proofs. This is verified in the report of the Committee of Ten which recommended that ample opportunity for recitation should be provided and that all proofs that were not formally perfect be rejected. It stressed elegance in both oral and written proofs and considered the lack of "oral elegance" a flaw that made the recitation of proofs practically valueless. This, it added, "...prevents the discipline for which this exercise is chiefly prized."[3]

Gradually it was being recognized, however, that mere learning of theorems would not develop greater reasoning power or increase ability to apply the pattern of reasoning outside geometry. The whole idea of mental discipline as a theory of instruction was being questioned. The emphasis in Euclidean geometry was shifting to training

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[2] R. E. M. Wong, "The Status and Direction of Geometry for Teachers" (unpublished Doctor's dissertation, University of Michigan, 1968), p. 9.

[3] Alan R. Osborne and F. Joe Crosswhite, "Forces and Issues Related to Curriculum and Instruction, 7-12," A History of Mathematics in the United States and Canada, Thirty-Second Yearbook of the National Council of Teachers of Mathematics (Washington: National Council of Teachers of Mathematics, 1970), p. 166.

in the method of attacking original exercises and discovering proof. Thus, while mental discipline as a viable theory of education, and drill as a procedure lasted to some extent into the early 1900's, the three step process of "state a rule, give an example, practice" was beginning to show signs of yielding to inductive reasoning and discovery teaching processes.[4]

1920 to 1940

The first significant report on mathematics education in this period was published in 1923 by the National Committee on Mathematical Requirements.[5] This Committee provided an extensive discussion of the aims of instruction in mathematics. Three categories of aims were utilized: (i) practical or utilitarian aims, (ii) disciplinary aims, and (iii) cultural aims. The practical aims to be achieved through the study of geometry were: familiarity with geometric forms common in nature, industry and life; knowledge of the properties and relations of these forms; knowledge of the spatial relationships and; development of

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[4]Philip S. Jones and Arthur F. Coxford, Jr., "Mathematics in the Evolving Schools," A History of Mathematics Education in the United States and Canada, Thirty-Second Yearbook of the National Council of Teachers of Mathematics (Washington: National Council of Teachers of Mathematics, 1970), p. 32.

[5]The National Committee on Mathematical Requirements, "The Reorganization of Mathematics in Secondary Education," Readings in the History of Mathematics Education, eds. J. K. Bidwell and R. G. Clason (Washington: National Council of Teachers of Mathematics, 1970), p. 84.

spatial imagination. Properties and relations were considered to include congruences, similarity, triangle angle sums, the Pythagorean relation, areas and volumes. Disciplinary aims were concerned with developing correct habits and attitudes and with the development of the ability to think clearly with quantitative concepts. Appreciation of geometric form, logical reasoning and "the power of thought, the magic of the mind,"[6] were the stated cultural goals. Such a statement of objectives is significant in that it proposes a compromise position between the idea of mental discipline and geometry for practical use. It, therefore, abandoned extreme positions and said that, with proper restrictions, general mental discipline is a valid aim for geometry.[7]

Among the recognized leaders in mathematics education at that time, Reeve did extensive work in defining the objectives of mathematics teaching. He defined the general objectives of mathematics in terms of establishing certain habits of action, thinking, moral conduct and character, and in terms of creating ideals of simple language, accurate reasoning, original thought, and reliable information.[8]

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[6]Ibid.

[7]W. G. Quast, "Geometry in the High Schools of the United States: An Historical Analysis from 1890 to 1966" (unpublished Doctor's dissertation, Rutgers, The State University, 1968), p. 117.

[8]W. D. Reeve, "Objectives in the Teaching of Mathematics," The Mathematics Teacher, XVIII (November, 1925), pp. 385-89.

General objectives of geometry, as enumerated by Reeve, included understanding of the need for proof, the difference between proof and intuition, and the meaning of deductive terminology. Desired behaviors included logical thinking, critical attitude, neatness and accuracy. Appreciation of rigorous thinking, as well as the aesthetic values of geometry were also listed as general objectives. Reeve also itemized two hundred thirty-five specific bits of knowledge or abilities the student should master in the study of the subject.[9]

During the 20's and early 30's mathematics courses as a part of the school curriculum were coming under heavy attack. Many progressives and other educators were advocating maximum application to daily life and consequently the removal of much mathematics from the curriculum entirely. Because much of this criticism was focused particularly on geometry, mathematics educators were forced to defend its position.[10] As a result numerous attempts were made at this time to formulate objectives for geometry that would clearly justify its place in the school curriculum.

A primary argument used in the defense of geometry was its logical nature, which was unlike that of any other

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[9]W. D. Reeve, "Objectives in Teaching Demonstrative Geometry," The Mathematics Teacher, XX (December, 1927), 433-50.

[10]W. D. Reeve, "Attacks on Mathematics and How to Meet Them," The Place of Mathematics in Modern Education, Eleventh Yearbook of the National Council of Teachers of Mathematics (Washington: National Council of Teachers of Mathematics, 1936), pp. 1-21.

subject in the secondary school.[11] Allen[12] summarized the argument as follows:

> At the risk of becoming a welcome target for the utilitarian educator we shall make the claim for demonstrative geometry primarily as an exercise in logic, as a means of mental training, and as a medium for developing high ideals of accuracy and truth. In opposition to those who would train for specific utility only, we have sufficient evidence safely to maintain that there is far greater educational value in the power to think through a new problem for one's self than in acquiring rote knowledge of time-honored facts.

While such an argument retained some implication of mental discipline, the important goal was training in deductive thinking. Reeve further supports the same idea:[13]

> If demonstrative geometry is not taught in order to enable the pupil to have the satisfaction of proving something, to train him in deductive thinking, to give him the power to prove his own statements, then it is not worth teaching at all.

Betz[14] contended that the development of deductive thinking ability not only justified geometry but made it absolutely essential. "It offers," he argued, "the simplest and most convenient introduction to postulational thinking which has yet been devised."

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[11]Quast, op. cit., p. 142.

[12]Gertrude Allen, "Objectives in Teaching of Mathematics in Secondary Schools," The Mathematics Teacher, XVI (February, 1923), 75, cited by Quast, op. cit., p. 142.

[13]W. D. Reeve, "The Teaching of Geometry," The Teaching of Geometry, Fifth Yearbook of the National Council of Teachers of Mathematics (Washington: National Council of Teachers of Mathematics, 1930), p. 13.

[14]William Betz, "The Transfer of Training With Particular Reference to Geometry," ibid., p. 151.

In view of later thinking along similar lines, it is necessary to point out that most people in this period saw the goal of developing logical thinking important for transfer to non-mathematical situations. This is illustrated, for example, by the report of the Committee on the Function of Mathematics in General Education[15] which stated that many teachers of mathematics believed that:

> ... the rigor of the proof in this field sets a standard which careful demonstration in other fields of thought may well attempt to emulate, and that students should therefore learn geometry in order to learn to reason with equal rigor in other fields.

Obviously, the Committee saw notions of proof to be broader than the restrictive mathematical view of deduction and felt that the student should have experiences with the application of deduction outside the field of mathematics. Christofferson[16] went even further in this direction in stating:

> Geometry achieves its highest possibilities if, in addition to its direct and practical usefulness, it can develop the power to think clearly in geometric situations and to use the same discrimination in non-geometric situations; if it can develop the power to generalize with caution from specific cases and to realize the force and all inclusiveness of deductive statements.

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[15]Commission on Secondary School Curriculum of the Progressive Education Association, Mathematics in General Education, Report of the Committee on the Function of Mathematics in General Education (New York: D. Appleton-Century Company, Inc., 1940), p. 188.

[16]H. C. Christofferson, Geometry Professionalized for Teachers (Oxford, Ohio: Published by the Author, 1933), p. 28, cited by F. B. Allen, "Teaching for Generalization in Geometry," The Mathematics Teacher, XLII (June, 1950), 245.

He seems to have felt that only when an attempt is made to generalize to non-mathematical situations are the full potentialities of the subject realized.

It is noticeable that in these expressed aims of deductive geometry little reference was made to the facts of geometry. The subject was not being justified on the basis of giving students control of useful geometric knowledge.[17] Wuest[18] advanced the idea that problem solving was the chief objective and knowledge of geometric fact was a minor goal; hence analytic thinking should be stressed. Webb[19] advocated rigorous logic established by using a few non-independent assumptions in the teaching of geometry. Stroup[20] urged an intuitive approach, leading eventually to formal proof, without too much concern for rigor. Hall[21] emphasized the application of geometric methods of thinking to life situations as an important outcome to be sought. These and

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[17]R. N. Harding, "The Objectives of Mathematics Education in Secondary Schools as Perceived by Various Concerned Groups" (unpublished Doctor's dissertation, University of Nebraska, 1968), p. 23.

[18]Alma M. Wuest, "Analysis Versus Synthesis," The Mathematics Teacher, XX (January, 1927), 46-49, cited by Harding, Ibid., p. 24.

[19]H. E. Webb, "'Elementary Geometry' and the 'Foundations'," The Mathematics Teacher, XIX (January, 1926), 1-12, cited by Harding, op. cit., p. 24.

[20]P. Stroup, "When is a Proof not a Proof," The Mathematics Teacher, XIX (December, 1926), 499-505, cited by Harding, op. cit., p. 24.

[21]E. L. Hall, "Applying Geometric Methods of Thinking to Life Situations," The Mathematics Teacher, XXXI (December, 1938), 379-84, cited by Harding, op. cit., p. 24.

other similar arguments were probably best summed up by Fawcett[22] when he concluded:

> The consensus of opinion therefore seems to be that the most important values to be achieved from the study of demonstrative geometry are an acquaintance with the "nature of proof" and a familiarity with "postulational thinking."

Towards the end of the period under study, however, the stated objectives of geometry began to reflect the social conditions of the time and the imminence of the War, resulting in greater consideration of practical aims. As Kinsella contended: "When many people did not have enough to eat, education had to justify itself in practical terms."[23] The end result was a period when social utility was a major factor in determining what was taught. This is reflected in the goals suggested by such people as Breslich[24] who cites objectives like acquisition of geometric knowledge and ability to use facts and principles and; the development of drawing skills, spatial imagination and an appreciation of geometry as a science.

There is some evidence to indicate that the way teachers perceived geometry objectives in the 20's and 30's

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[22] Harold P. Fawcett, The Nature of Proof, Thirteenth Yearbook of the National Council of Teachers of Mathematics (Washington: National Council of Teachers of Mathematics, 1938), p. 6.

[23] John J. Kinsella, Secondary School Mathematics, (New York: The Center for Applied Research in Education, Inc. 1965), p. 11.

[24] E. R. Breslich, "The Nature and Place of Objectives in Teaching Geometry," The Mathematics Teacher, XXXI (November, 1938), 307-15, cited by Harding, op. cit., p. 24.

did not exactly coincide with that of educators of the time. Fawcett[25] reported the existence of a communication gap when he wrote:

> While allegiance is paid verbally to those large general objectives related to the nature of proof, actual classroom practice indicates that the major emphasis is placed on a body of theorems to be learned rather than on the method by which these theorems are established. The pupil feels that these theorems are important in themselves and in his earnest effort to 'know' them he resorts to memorization.

Concrete evidence of the discrepancy between educators and teachers was reported by Shibli[26] who published the results of a questionnaire responded to by 181 teachers in secondary education. In the study participants were asked to rank twenty given aims of teaching geometry, selecting what they considered the seven most important. The aim "to make clear the process of deductive thinking" received little support from teachers. This would indicate that while mathematics educators considered the value of geometry to be in postulational thinking, practicing teachers placed little emphasis on this aspect of the subject.[27]

Further study of Shibli's aims showed that the most frequently cited goals were concerned with developing clear thinking, precise expression, the ability to analyze a

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[25]Fawcett, op. cit., p. 1.

[26]J. Shibli, Recent Developments in the Teaching of Geometry (State College, Pennsylvania: J. Shibli, Publisher, 1932), pp. 213-16, cited by Quast, op. cit., p. 168.

[27]Quast, op. cit., p. 168.

complex situation into simpler parts, an inquiring or questioning attitude of mind, and mental habits and attitudes that are needed in life situations. Thus it would appear that traditional habits of mental discipline and transfer of training were still considered significant by teachers.

In summary, the evolution of objectives of deductive geometry, and indeed of all mathematics, in the 1920's and 1930's centred primarily on the issue of the usefulness of mathematics. Consequently, even though the most frequently cited aim of geometry was the development of critical thinking ability in the student, it is significant to note that most educators seemed to value such ability strictly for transfer to non-mathematical situations. This position was to be the subject of considerable debate in the next two decades.

The Dawn of Reform[28] (1940-55)

Two reports which sought to define the place of mathematics in education were published in the early 1940's. The more widely known of these was the report of the Joint Commission of the Mathematical Association of America, Inc., and the National Council of Teachers of Mathematics. The Commission was originally appointed in 1933 by the Mathematical Association of America to study the place of mathematics in secondary education, and was later

[28]J. H. Hlavaty, "Capsule History of the NCTM.," The Mathematics Teacher, LXIII (February, 1970), 141.

incorporated into the Joint Commission[29]

The Joint Commission expressed the view that a high level of understanding of both inductive and deductive processes was considered necessary, and the ability to make applications desirable. It proposed that the student have "conscious experience" with both inductive and deductive reasoning. Other behaviors recommended were: habitually seeking to identify the inductive or deductive nature of a problem, seeking to discover and remove ambiguity in the use of terms, understanding the relations between assumptions and conclusions, and the ability to judge the validity of reasoning.[30] In the field of geometric form the Commission identified three abilities: (i) to recall and apply fundamental metric relations, (ii) to recognize spatial relations, and (iii) to recognize and state functional relations between area and volume as the dimensions of a figure are changed.[31]

The second report appearing in the 1940's was the report of the Committee on Post War Plans. It was created by the National Council of the Teachers of Mathematics and issued two reports; the first in 1944 and the second in 1945. This Committee was working at a time when the primary

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[29]Joint Commission of the Mathematical Association of America, Inc., and the National Council of Teachers of Mathematics, The Place of Mathematics in Secondary Education, Fifteenth Yearbook of the National Council of Teachers of Mathematics (New York: Bureau of Publication, Teachers College, Columbia University, 1940), pp. 6-7.

[30]Ibid., p. 24

[31]Ibid., p. 62-69.

concern was the war effort and the problems connected with manpower. It therefore expressed a bias toward teaching mathematics from a utilitarian point of view. In all of its reports the aim stressed was development of functional competence and mathematical power, with attention given to useful applications.[32] The checklist of necessary mathematical competencies published by the Committee stressed the skills for "dealing with the problems of everyday life."[33]

Despite the movement for practical aims by such groups as the Committee on Post War Plans, many people continued to cite understanding of deductive thinking and postulational systems as primary goals. Many still saw deductive proof important for transfer to non-mathematical situations. Van Waynen[34] argued: "Geometry is the ideal vehicle for teaching the simple pattern of clear thinking and its application to the vital problems of everyday life." Kinney and Purdy[35] contended that it was through the transfer of deductive reasoning that the student demonstrated his understanding of geometry:

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[32]Commission on Post War Plans, "Second Report," The Mathematics Teacher, XXXVIII (April, 1945), 195-221, cited by Harding, op. cit., p. 37.

[33]W. D. Reeve (ed.), "Guidance Report of the Committee on Post War Plans," The Mathematics Teacher, XL (November, 1947), 315.

[34]M. Van Waynen, "What Kind of Geometry Shall We Teach?" The Mathematics Teacher, XLIII (January, 1950), 3.

[35]Lucien B. Kinney and C. Richard Purdy, Teaching Mathematics in the Secondary School (New York: Rinehart and Company, Inc., 1952), p. 100.

> When thoroughly understood, similar logical processes may be transferred to social and personal problems, to explore their applicability and the readjustments that are necessary.

A similar argument had earlier been advanced by Smith:[36]

> ... according to present day thinking, students who understand the meaning of proof, who are able to formulate proofs, and who are capable of making intelligent criticisms of alleged proofs in geometry can use much of this knowledge in other fields.

Significantly, however, Smith later conceded that such transfer as an objective was subsidiary to the largest objective of all - developing the ability to understand, make and criticize deductive proofs.[37]

This latter opinion was typical of a definite change in emphasis that was occuring in the goals of geometry in the 40's and 50's. Aims other than the development of logical thinking began to be stated, and there began to grow the idea that an understanding of postulational thinking was vital to the understanding of mathematics. It was thus important in its own right and not just for possible transfer to other areas. As early as 1942, Mallory and Fehr[38] had protested that:

> Another movement that has weakened the mathematics program is the introduction of an excessive amount of 'Reasoning in Life Situations' into the subject of

[36] R. R. Smith, "On the Teaching of Geometry," The Mathematics Teacher, XLII (January, 1949), 57.

[37] Ibid., p. 59.

[38] Virgil S. Mallory and Howard F. Fehr, "Mathematical Education in War Times," The Mathematics Teacher, XXXV (November, 1942), 292, cited by J. Wilson, "Trends in Geometry," The Mathematics Teacher, XLVI (January, 1952), 68.

> geometry. In many cases this has resulted in befuddled thinking and a lack of knowledge of plane geometry...

Attention was focussed more and more on mathematics as a discipline in the early 50's with the growing awareness of the need for highly trained mathematicians in industry, defense and space programs. The new emphasis was being placed on meaning and understanding; so much so, that in 1955 Kinsella[39] charged that some schools were neglecting the development of skills.

Most of the other aims put forward for deductive geometry in this period stressed knowledge of geometrical facts and their practical applications. As Wilson[40] stated, much emphasis was placed on "... information concerning the facts and principles of space, including information which serves as a backdrop for the appreciation of the mechanical life of today."

<u>The Era of Reform[41] (1955-)</u>

The years since 1955 have seen unprecedented change in the teaching of mathematics. It has been an age of "new" and "modern" mathematics when schools have made radical changes in the courses being offered. In 1962 Moise[42] was

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[39]Kinsella, op. cit., p. 72.

[40]J. Wilson, "Trends in Geometry," <u>The Mathematics Teacher</u>, XLVI (January, 1952), 68.

[41]Quast, op. cit., p. 225.

[42]Edwin Moise, "The New Mathematics Programs," <u>Revolution in Teaching: New Theory, Technology, and Curricula</u>, eds. Alfred de Grazia and David A. Sohn (New York: Bantam Books, 1962), p. 171.

able to state:

> ... the sheer volume of new programs and experimental text materials is by now overwhelming. The publications of the School Mathematics Study Group (SMSG), now fill over three linear feet on a bookshelf high enough to hold them in a vertical position; the production of the University of Illinois Committee on School Mathematics (UICSM) are quantitatively less, but still impressive; and if we add these to the Maryland group, the Ball State group and others, it is plain that a full critical survey of this literature would require a book.

Unfortunately, geometry was the area of mathematics least affected by the reform movement.[43] Although some changes did occur in the teaching of geometry, traditional ideas and procedures persisted throughout the 50's. Quast[44] cited poorly prepared teachers who were unable to discern the means of implementing stated goals as a major cause of this failure to change. Allendoerfer[45] summarized the commonly held objectives of the late 50's when he listed the three main goals of deductive geometry as (i) to teach some of the important facts about geometry as such, such as the properties of triangles, circles, parallel lines and the like; (ii) to teach the deductive method as it is applied to mathematical reasoning, and thus to give students a first taste of the nature of mathematical proof; and (iii) to teach logical reasoning per se and to show the students how it can be

[43] Quast, op. cit., p. 237.

[44] Ibid.

[45] Carl B. Allendoerfer, "Deductive Methods in Mathematics," Insights into Modern Mathematics, Twenty-Third Yearbook of the National Council of Teachers of Mathematics (Washington: National Council of Teachers of Mathematics, 1957), p. 65.

applied in non-mathematical situations. Allendoerfer felt that to achieve these objectives was too much to expect from one course and consequently the geometry offered suffered from watering down and a lack of popularity.[46] It did indicate clearly, however, that teachers were interested in developing deductive reasoning for transfer in mathematics as well as to other areas.

One of the most significant reports in recent years in mathematics education was the report of the Commission on Mathematics. The section of the report devoted to geometry emphasized teaching geometry for its own sake, rather than for application to non-mathematical situations. It gave three main objectives for the inclusion of geometry in the high school curriculum: (i) the acquisition of information about geometric figures in the plane and space; (ii) the development of understanding of the deductive method as a way of thinking and a reasonable skill in applying this method to mathematical situations; and (iii) the provision of opportunities for original and creative thinking by students.[47] The Commission took the position that it is a disservice to the student and to mathematics for geometry to be presented as though its study would enable a student to solve a substantial number of his life problems by deductive

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[46]Ibid.

[47]Commission on Mathematics, <u>Program for College Preparatory Mathematics</u> (New York: College Entrance Examination Board, 1959), pp. 22-23.

reasoning.[48]

With some reservations, the three objectives put forward by the Commission have received widespread acceptance up to the present day. Fawcett,[49] citing a careful review of literature related to school geometry, concluded that in the first two objectives the Commission was undoubtedly reflecting the position of the profession. The third, however, he questioned, since "... provision of opportunities for original and creative thinking is not a function of the subject, but the responsibility of the teacher."

Allendoerfer[50] was in general agreement with these objectives but suggested that two more should be added. He included (i) integration of geometric ideas with other parts of mathematics, and (ii) an understanding of the basic facts about geometric transformations.

Adler[51] called attention to goals of geometry that put less emphasis on deductive reasoning. He listed the following as being the most significant goals of geometry: (i) exploration of relationships among geometric facts previously learned; (ii) introduction to the role of transformations of space in the study of geometry; (iii) mastery of

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[48]Ibid.

[49]Harold B. Fawcett, "The Geometric Continuum," The Mathematics Teacher, LXIII (May, 1970), 44.

[50]Carl B. Allendoerfer, "The Dilemma in Geometry," The Mathematics Teacher, LXII (March, 1969), 165.

[51]Irving Adler, "What Shall We Teach in High School Geometry?" The Mathematics Teacher, LXI (March, 1968), 228.

a variety of techniques; (iv) development of critical thinking; and (v) development of an understanding of the nature of a mathematical model. Although he cited many good reasons for stressing axiomatic-deductive reasoning in the tenth grade, Adler contended that ". . . unfortunately, during the last decade there has been a tendency to stress deductive reasoning more and more while the need for it has become less and less."[52]

Fehr,[53] too, recently cautioned against overstressing logical reasoning in geometry when he stated: "We should not identify geometric thinking with logical thinking, for the latter is the domain of all mathematics." He listed three primary objectives: (i) to know what geometric thinking is, what it studies, and how it devises its method to do this study; (ii) to transmit important information about space; and (iii) to develop a high degree of skill in geometric problem solving.

One of the more prominent aspects of mathematics teaching in the past few years has been the attention given to structure. Many educators have seen the development in students of a feeling for the structure of mathematics as a major objective to be achieved. Rosskopf[54] argued: "If we

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[52]Ibid.

[53]Howard F. Fehr, "The Present Year-long Course in Euclidean Geometry Must Go," The Mathematics Teacher, LXV (February, 1972), 102.

[54]Myron F. Rosskopf, "Modern Emphases in the Teaching of Geometry," The Mathematics Teacher, L (April, 1957), 273.

want to teach secondary mathematics in a more modern spirit, then these modern concepts of axiomatic structure and relationships between structures must be reflected in our teaching." On the subject of structure, Buck[55] stated:

> ... the geometry course should be designed to reveal, rather than to conceal, the structure of the subject. For example, a course which introduces postulates in great gobs, and lumps together all the theorems about parallels, is designed for ease in memorization rather than for clearness of understanding. Insofar as possible the significance of postulates should be explored and the theorems should be grouped according to the assumption under which they are proved.

Butler et al[56] saw emphasis of structure as the main objective of geometry:

> ... the course will aim mainly at giving the students deeper insights into how geometry may be structured; how a large body of geometric facts and relations can be made to grow by logical processes from a few simple statements made at the outset.

This implies that the students gain some feeling for the meaning of implication and for the roles of undefined terms, definitions, postulates and theorems in the deductive process. It also implies that geometry teachers should emphasize elements of mathematical structure that are found not only in geometry but are also common to other branches of mathematics.[57]

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[55]Charles Buck, "What Should High School Geometry Be?" <u>The Mathematics Teacher</u>, LXI (May, 1968), 469.

[56]Charles H. Butler, F. L. Wren and J. H. Banks, <u>The Teaching of Secondary School Mathematics</u> (5th ed.; New York: McGraw-Hill Book Company, 1970), p. 390.

[57]Ibid.

Summary

Gradual, but continual change has marked the objectives of deductive geometry in the secondary school. In general terms this change could almost be considered as a variation in relative importance of practical and intellectual objectives. In the early part of the present century the primary aims were basically utilitarian and many felt that geometry disciplined and trained the mind to think logically in other areas. Although the theory of mental discipline gradually lost support, the idea of transfer of training to other areas persisted throughout the 20's and 30's and to a lesser extent through the 40's and 50's. In recent years the aims of geometry have evolved to become more mathematical in content.

It should be pointed out here that while the review of literature has been very wide in scope it is none the less applicable to the Newfoundland geometry curriculum. In geometry, as in all of mathematics, this province has been a part of the North American trend in curriculum development over the past two decades. The subject matter presently taught here, as elsewhere, reflects the reform movement of the 60's and the work of the various study groups of that time. This does not in any sense imply that the study is universal in nature since schools and teachers and a variety of other parochial factors are unique to one particular area. Thus any conclusions drawn would have to be restricted to the area under study.

CHAPTER III

INSTRUMENT CONSTRUCTION AND THE GROUPS STUDIED

In Chapter I it was proposed to answer questions concerning how two groups of individuals perceived the objectives of deductive geometry. In order to answer these questions an instrument was constructed which consisted of a list of objectives and this was used to obtain information from mathematics educators in selected universities and from geometry teachers in secondary schools throughout the province of Newfoundland.

This chapter gives a description of how the final list of objectives was obtained, how the sample was selected and how the survey of teachers and mathematics educators was carried out.

CONSTRUCTING THE INSTRUMENT

The Initial Form

The literature pertinent to the objectives of deductive geometry was surveyed and analyzed. This analysis served to provide a framework of general categories within which to set the more specific goals of instruction. These general categories are ones that have received varying emphasis at different periods of time in the teaching of deductive geometry and thus they provided a satisfactory basis for enumerating the possible objectives of deductive geometry at the present time. The four categories were:

1. The basic terms and manipulative skills of deductive geometry. (By manipulative skills is meant the ability to use geometric instruments in making simple drawings.)

2. The structure of deductive geometry.

3. Proofs in deductive geometry.

4. Applications of deductive geometry, both practical and to other areas of mathematics.

Using the categories stated above and by analyzing a selection of current textbooks on deductive geometry, a comprehensive list of behavioral objectives was prepared. The criteria used in writing these objectives were:

1. That they be expressed in terms of behavior expected from the student rather than oriented toward the teacher. In other words, they expressed what the student should be able to do after studying deductive geometry rather than what the teacher should teach in a particular course.

2. That they be specifically stated to clearly infer the desired behavior expected of the student.

The latter criterion presented one obvious problem, that of balancing the level of generality of each item with the desired length of the list. It was intended to make the list comprehensive yet the nature of the study restricted its length. An effort was made, therefore, to establish a level of generality which would preserve meaning and avoid ambiguity as much as possible, and at the same time ensure a degree of compactness. It was recognized that efforts in this direction could never be totally successful as different levels of meaning are attached to many words by different individuals. To facilitate clarity as much as possible an example was supplied with all but a few of the objectives.

A preliminary list of very specific objectives contained seventy-eight items. After careful study the list was found to contain many repetitions and many ambiguous statements which necessitated the elimination of several items. In addition it was found that by raising the level of generality slightly, as many as three items could sometimes be combined into one. The initial editing produced a list of thirty-five possible objectives of deductive geometry.

It should be pointed out that it was not intended that all of these objectives be necessarily the best or most desirable ones. Rather it was intended that they should be a reflection of what is implied in the literature as being the goals of instruction of deductive geometry held by secondary school teachers and other educators at different periods of time.

Final List of Objectives

The initial list of objectives, together with a proposed five point scale of importance was submitted to a panel of mathematics educators at Memorial University for validation. They were asked to comment, if necessary, on the following:

1. Comprehensiveness - Have important items been omitted?

2. Compactness - Can some items justifiably be omitted? Should they be shortened? Should two or more objectives be combined?

3. Clarity - Are the basic meanings clear? How can clarity be improved?

4. Instructions accompanying the instrument - Do the instructions clearly indicate what is required?

The individuals contacted felt that very few changes were necessary in the initial list. The changes that were made resulted from questions about the meaning of certain objectives or objections to certain words or phrases within objectives, and the misuse of examples to illustrate the meanings of certain items.

The final list of objectives consisted of thirty-five statements to be evaluated. The final list of objectives, together with the instructions and recording sheet can be found in Appendix A.

Final Form of the Instrument

Each of the thirty-five objectives in the final list was reproduced by off-set printing process on a 2 X 4" card. Cards were used rather than booklet form because in the rating process they would allow maximum flexibility on the part of the subjects by providing the option of changing an initial rating. The groups under study were asked to make use of this particular feature of the instrument by arranging the cards into groups and ensuring that the final arrangement really reflected their thinking on deductive geometry. Only then would the arrangement be recorded on the Recording Sheet.

In summary, the instrument consisted of thirty-five objectives of deductive geometry each on a separate card, together with appropriate instructions and a recording sheet.

THE GROUPS STUDIED

Two groups of individuals were identified for use in the study. These were mathematics educators in selected universities and geometry teachers in secondary schools in Newfoundland.

The list of mathematics educators numbered 25 and were arbitrarily selected from universities in Canada and the United States.

Geometry teachers were obtained by randomly selecting schools in Newfoundland and using information obtained from the Department of Education to obtain the identity and address of the geometry teacher or teachers in each school.[1] Only teachers of Grades Nine or Ten geometry were used in the study since it is in these courses that deductive processes are most emphasized[2]. The roster of geometry teachers numbered 60.

Packets containing the objective cards, instructions and recording sheet, together with a covering letter were mailed to teachers in late October, 1972, and to mathematics educators in December, 1972. For both groups follow-up letters were necessary and these were sent approximately one month after the original inquiry. Copies of all letters to both groups, together with the names of universities contacted, are found in Appendix B.

A reliability study was carried out on one of the two groups under study; namely, the geometry teachers. It was assumed that reliability in the group of mathematics educators would be at least as high as for this group. The reliability was determined by administering the instrument a second time to a subgroup of the original sample. The time lapse between the first and second survey was approximately 2 months.

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[1]Government of Newfoundland and Labrador, The Newfoundland and Labrador Schools Directory, 1972-73, (St. John's: Government of Newfoundland and Labrador, 1972).

[2]Newfoundland and Labrador Department of Education, Programme of Studies, 1972-73, Grade I-XI, (St. John's: Government of Newfoundland and Labrador, 1972), p. 38.

ANALYSIS OF DATA

The chapter which follows gives a description of how the data gathered from geometry teachers and mathematics educators was analyzed. The analysis was done primarily with a view to examining the extent of agreement between the two groups. To accomplish this, mean ratings of importance were calculated for each item as perceived by each group. These were used to rank the 35 items in order of importance for each group. Comparisons were made between groups to determine whether or not agreement existed on the important and non-important items and other general conclusions were drawn.

CHAPTER IV

RESULTS OF THE STUDY

Chapter III described how an instrument was constructed to determine how different individuals perceive the objectives of deductive geometry. It also described how that instrument was used in a small survey on two groups of people closely connected with the teaching of deductive geometry in the secondary school. This chapter outlines the results of that survey and presents answers to the questions posed in Chapter I.

RESPONSE TO THE SURVEY

A total of 85 inquiries were distributed and 63 (74 percent) useable responses were obtained and used in the calculations. These were broken down fairly evenly on a percentage basis between the two groups as illustrated in the table below.

Table 1

Response to the Survey

Group	Number of Inquiries Made	Number of Responses	Percent
Mathematics Educators	25	19	76
Geometry Teachers	60	44	73
Totals	85	63	74

TREATMENT OF RESPONSES

Each individual who responded to the inquiry was asked to rate each of the thirty-five objectives on a five point scale of importance. Rating "1" indicated that the objective was considered extremely important, rating "5" indicated unimportance, and the other three ratings represented unspecified intermediate points along the continuum. Respondents were informed that there was no limit on the number of items that they could place in any one rating category. (See Appendix A). Results of teacher and educator ratings are shown in Tables 2 and 3 respectively.

As indicated in Table 1, the size of the sample in both the group of mathematics educators and geometry teachers was limited. This was done because a major portion of the study was devoted to constructing the instrument and that necessitated limiting the scope of the survey which followed. Because of the sample size no attempt was made to do an extensive analysis of the data gathered. Rather, very general trends only were pointed out and used to draw implications and make suggestions for further study. (See Chapter V).

The data gathered was analyzed with a view to determining the extent to which there was agreement between groups on the important and non-important items. To study the objectives which were rated as important and non-important the mean rating and rank for each item was determined

Table 2

Results of Teacher Ratings

Item	Distribution 1	2	3	4	5
1	26	10	5	2	1
2	16	7	10	3	8
3	24	5	9	2	4
4	9	20	9	4	2
5	14	20	9	1	0
6	13	21	8	1	1
7	12	17	10	4	1
8	16	14	4	5	5
9	1	6	6	11	20
10	5	7	9	14	9
11	19	13	7	3	2
12	20	13	5	3	3
13	15	9	6	8	6
14	17	7	10	6	4
15	18	15	9	1	1
16	5	8	3	9	19
17	5	13	10	7	9
18	3	4	4	7	26
19	7	11	8	9	9
20	19	12	10	3	0
21	23	12	6	2	1
22	6	8	15	8	7
23	3	18	18	8	7
24	13	18	10	2	1
25	11	14	5	9	5
26	15	16	4	6	3
27	6	9	13	8	8
28	13	8	7	9	7
29	12	11	10	6	5
30	3	13	11	9	8
31	17	6	10	6	5
32	5	1	7	14	17
33	28	12	2	1	1
34	19	9	10	4	2
35	18	5	5	10	6

Table 3

Results of Educator Ratings

Item	Distribution 1	2	3	4	5
1	3	7	3	4	2
2	3	5	4	3	4
3	2	7	4	3	3
4	3	6	4	5	1
5	5	7	4	3	0
6	3	8	4	4	0
7	3	6	3	5	2
8	3	5	3	7	1
9	0	1	1	2	15
10	4	2	9	3	1
11	12	5	2	0	0
12	13	4	1	0	1
13	8	5	3	3	0
14	12	3	1	3	0
15	5	3	9	2	0
16	2	0	5	7	5
17	5	7	3	2	2
18	0	0	3	5	11
19	8	6	3	2	0
20	4	7	7	0	1
21	3	6	5	2	3
22	2	6	7	4	0
23	2	6	5	4	2
24	3	6	6	2	2
25	11	2	4	1	1
26	11	6	1	1	0
27	4	5	7	2	1
28	7	4	4	3	1
29	5	8	4	2	0
30	1	4	3	5	6
31	11	2	4	2	0
32	1	3	6	2	7
33	4	4	7	3	1
34	5	6	6	2	0
35	3	1	1	5	9

for each of the two groups. These results made it possible to examine what type of item was rated very high and very low by each group, and to what extent there was agreement on these items between the two groups.

RELIABILITY

As stated previously (Chapter III) a reliability study was conducted on the group of geometry teachers. A subgroup of 12 teachers was asked to evaluate the objective items a second time, two months after the first inquiry. The correlation coefficient between first and second ratings was calculated for each of the 12 respondents. These were transformed using Fisher's Z and the mean obtained. The mean was then transformed back to give a reliability coefficient of .79 .

EXTENT OF AGREEMENT BETWEEN GROUPS

To gain some insight into the extent to which agreement existed between the two groups mean ratings were calculated and used to rank each of the 35 items. Table 4 presents the mean ratings and the rank of each item for each of the groups under study.

Since the items were rated on a 5 point scale of importance with 1 being the most important and 5 being the least important, the items could be graded as follows:

Items with mean rating 1 - 2 ......Important

Items with mean rating 2 - 2.5 ....Trend toward importance

Table 4

Mean Ratings and Ranks

Item	Teachers		Educators	
	Mean Rating	Rank	Mean Rating	Rank
1	1.68	2	2.73	21=*
2	2.54	19	3.00	29
3	2.02	10	2.61	27
4	2.31	16	2.73	21=
5	1.93	5=	2.26	10=
6	2.00	8	2.47	16
7	2.20	13	2.84	25
8	2.29	15	2.89	27
9	3.97	34	4.63	35
10	3.34	30	2.73	23
11	2.00	8	1.47	1
12	2.00	8	1.52	2
13	2.56	21	2.05	8
14	2.38	17	1.73	4
15	1.90	4	2.42	14=
16	3.65	32	3.68	32
17	3.04	26	2.42	14=
18	4.11	35	4.42	34
19	3.04	26	1.94	7
20	1.93	5=	2.31	12=
21	1.81	3	2.78	24
22	3.04	26	2.68	19=
23	3.63	31	2.89	27
24	2.09	11	2.68	19=
25	2.61	23	1.89	6
26	2.22	14	1.57	3
27	3.06	28	2.52	17
28	2.75	24	2.31	12=
29	2.56	21	2.15	9
30	3.13	29	3.57	30=
31	2.45	18	1.84	5
32	3.84	33	3.57	30=
33	1.52	1	2.63	18
34	2.11	12	2.26	10=
35	2.56	21	3.84	33

*The symbol "=" is used in the table instead of .5 .

Items with mean rating 2.5 - 3.5 .. Neutral

Items with mean rating 3.5 - 4 .... Trend toward non-importance

Items with mean rating 4 - 5 ...... Non-important

As a justification of this classification it must be pointed out that if the items had been rated randomly a normal distribution would be expected to result, with a mean rating of 3. Thus the probability is very low that a random distribution would yield a mean rating of 2 or less, or of 4 or more. Indeed, a study of Table 2 would reveal that for those items which had a mean rating of 2 or less (important range) a minimum of 70 percent of the teachers and 68 percent of the educators placed the items in the first two categories of the rating scale. For the items which had mean ratings in the non-important range (4 - 5) a minimum of 75 percent of the teachers and 84 percent of the educators placed the items in the last two categories of the rating scale.

Using the above classification it can be observed from Table 4 that very few of the items were rated as non-important by either group. In the group of geometry teachers only item 18 - Without using references reproduce a complete proof of Pythagoras' Theorem - was considered non-important. In the group of mathematics educators item 18 again and item 9 - Make neat drawings to represent three dimensional figures - fell into the same category. Both the non-important and the trend toward non-important ranges combined contained only 5 items as rated by geometry teachers and 6 as rated by mathematics educators.

Geometry teachers rated 9 items (1, 5, 6, 11, 12, 15, 20, 21, 23) in the important range as compared to 7 items (11, 12, 14, 19, 25, 26, 31) by the other group. Altogether out of the 35 items evaluated, geometry teachers considered 18 items to be either important or tend toward importance, while the comparable number for mathematics educators was 16.

Group Comparison on Items

The degree of agreement between groups can be illustrated by comparing the items ranked at both ends of the scale. A comparison of those items occupying the middle ranks would not be meaningful since the neutral rating has emerged in some cases from a disagreement within groups, with some rating the item high, others low, hence the mean in the centre, and in other cases from widespread neutral ratings on the part of individuals within a group.

Tables 5 and 6 show the items occupying the upper 10 ranks for each of the groups concerned and the comparable ranks of the same items for the other group. Comparing Tables 5 and 6 it can be readily observed that in the upper extreme range, for example ranks 1 to 5, there are no items in common whatsoever. Even in the first 10 ranks only three items - Item 5: State a set of conditions under which two or more triangles are congruent; Item 11: Make reasonable conjectures; and Item 12: Recognize false assumptions - are common to both tables as having been ranked in the upper 10 by both groups. There did not, therefore, seem to be any

Table 5

Analysis of Items Ranked Highest by Teachers

Rank	Item as Rated by Teachers	Comparable Rank of Same Item by Educators
1	33 - Apply geometry in real-life*	18
2	1 - Define basic geometric terms	21
3	21 - Provide complete proofs	24
4	15 - Justify simple conclusions	14=
5	5 - Conditions of triangle congruency	10
6	20 - Give reasons for steps in proofs	12=
7	6 - Conditions of triangle similarity	16
8	11 - Make reasonable conjectures	1
9	12 - Recognize false assumptions	2
10	3 - Perform basic constructions	27

*For complete statement of items see Appendix A.

Table 6

Analysis of Items Ranked Highest by Educators

Rank	Item as Rated by Educators	Comparable Rank of Same Item by Teachers
1	11 - Make reasonable conjectures*	8
2	12 - Recognize false assumptions	8
3	26 - Explain the statement of a theorem	14
4	14 - Draw conclusions from statements	17
5	31 - Describe the structure of geometry	18
6	25 - Identify erroneous statements	23
7	19 - Disprove simple propositions	26
8	13 - Distinguish between inductive and deductive proof	21
9	29 - Distinguish between postulates and theorems	21
10	5 - Conditions of triangle congruency	5
	34 - Apply geometry to mathematics	12

*For complete statement of items see Appendix A.

significant agreement between the two groups on the more important objectives of deductive geometry.

At the other end of the scale, however, a somewhat different result was apparent. Table 7 shows the 5 items rated by teachers in the non-important or trend toward non-importance range and the corresponding items for educators. From Table 7 it can be observed that in the bottom 5 ranks there was very strong agreement, with 4 items (9, 16, 18, 32) being placed there by both groups. The only exceptions were items 23 and 35. Item 23 which was ranked 31 by teachers was ranked 27 by educators. Item 35 which was ranked 33 by educators was ranked 21 by teachers.

It would seem then, in summing up this section, that the two groups were in basic agreement on picking out those objectives that are superfluous to a deductive geometry course but could not agree on selecting the objectives which should be emphasized most in such a course.

Table 7

Group Comparison on Least Important Items

Rank	Item as rated by teachers	Item as rated by educators
31	23#	32*
32	16*	16*
33	32*	35##
34	9*	18*
35	18*	9*

*Item rated in bottom 5 ranks by both groups.

#Item 23 was ranked 27 by educators.

##Item 35 was ranked 21 by teachers.

ANALYSIS OF TRENDS AND SELECTED ITEMS

In examining the objectives as they were evaluated by the two groups under study a few trends became apparent, the first of these being probably the most important.

1. Although no attempt was made to classify the objectives taxonomically, there is some evidence to suggest that teachers have placed more emphasis on those items which are on the lower levels of the taxonomy[1] and mathematics educators have stressed those items which are at higher levels. In the upper ranked items teachers have included many of the objectives stressing the basic geometric terms and manipulative skills (Items 1-9). These objectives involve only the recall of previously learned material and as such occupy a low taxonomic level. On the other hand, most of the objectives assigned to the upper ranks by educators are those that refer to some facet of proof and as such generally represent a higher intellectual level.

Table 8 shows a breakdown by categories (Chapter III) of the 16 items rated by educators in the range of importance or trend toward importance and the corresponding items by geometry teachers. There it can be seen that 13 of these items as ranked by educators refer to deductive proof. (4 of the 5 'proof' objectives excluded emphasize rote memorization and as such are on a low taxonomic level.) In comparison the

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[1]Benjamin S. Bloom (ed.), <u>The Taxonomy of Educational Objectives, Handbook I: Cognitive Domain</u> (New York: David McKay Company Inc., 1965), pp. 62-200.

Table 8

Categories* of Upper Ranked Items

Ranks	Category of items assigned by Educators	Category of items assigned by Teachers
1	P	A
2	P	B
3	P	P
4	P	P
5	S	B
6	P	P
7	P	B
8	P	P
9	P	P
10	B	B
11	A	P
12	P	A
13	P	B
14	P	P
15	P	B
16	P	B

*Abbreviations of categories (See Chapter III) are as follows:
B - Basic terms and manipulative skills
S - Structure of deductive geometry
P - Proof
A - Applications

Summary of Table 8

	B	S	P	A
Number of items placed by educators	1	1	13	1
Number of items placed by teachers	7	0	7	2

table shows that the corresponding ranks by teachers include only 7 items pertaining to proof, but all except 2 (Items 2 and 9) of those items which refer to the basic skills. Most of the basic skills objectives were assigned by educators to ranks below 24 (lower one-third).

2. As Table 9 indicates, those items which stress rote memorization of theorems were ranked very low by both groups. This is particularly significant as far as the teachers group is concerned since the traditional criticism of the teaching of deductive geometry in the secondary school has been the over emphasis on memorizing theorems.[2] Item 16- Without using references reproduce the complete proof of any theorem after being given a reasonable time to study it - was ranked 32 by geometry teachers. The same trend was further evidenced by Item 17 which was ranked 26, Item 23 which was ranked 31, and perhaps most significantly, Item 18 - Without using references reproduce a complete proof of Pythagoras' Theorem - which was ranked 35, and so was considered by geometry teachers to be the least important of all items evaluated.

3. Teachers almost unanimously rated the practical applications of deductive geometry very highly, resulting in an overall rank of number 1 for that item. This is in contrast to the educators who agreed that the subject is not

[2]R. E. M. Wong, "The Status and Direction of Geometry for Teachers" (unpublished Doctor's dissertation, University of Michigan, 1968), p. 9.

Table 9

Ranks of Rote Memorization Items

Item and Description*	Rank by Teachers	Rank by Educators
16 - Reproduce complete theorems	32	32
17 - Reproduce outlines of proofs	26	14
18 - Reproduce complete proof of Pythagoras' Theorem	35	34
23 - Reproduce an outline of Pythagoras' Theorem	31	27

*For complete statement of items see Appendix A.

one of the more important ones (Table 4). The high rating by teachers is in spite of a lack of emphasis by current textbooks (Appendix C) and raises some conjecture on the degree to which teachers emphasize in the classroom the objectives they would rate as important.

CHAPTER V

SUMMARY, CONCLUSIONS AND IMPLICATIONS

SUMMARY

It was the purpose of this study (1) to construct an instrument for determining how different individuals perceive the objectives of deductive geometry; and (2) to determine how concerned groups perceive the objectives of deductive geometry in the secondary school. Some questions of the following type were to be answered:

1. Do geometry teachers and mathematics educators agree on the important objectives of deductive geometry?

2. Do geometry teachers and mathematics educators agree on the non-important objectives of deductive geometry?

An analysis of the literature pertaining to deductive geometry revealed four main areas of emphasis and a survey of current textbooks of the secondary school level (Appendix C) produced an initial list of 78 specific objectives.

The initial list of objectives was edited and revised to 35 items. Suggestions were elicited from a panel of mathematics educators on comprehensiveness, compactness and clarity, as well as the instructions accompanying the objectives. The final form of the instrument consisted of 35 possible objectives of deductive geometry in the secondary school.

The list of 35 objectives, each on a separate card, and a 5 point scale for rating the importance of each item were submitted by mail to 85 individuals identified as being members of one of the following groups:

1. Mathematics educators
2. Geometry teachers

Replies were obtained from 65 individuals in selected universities in Canada and the United States and in secondary schools throughout the province of Newfoundland.

Mean ratings of importance were computed for each item as perceived by each group. These were used to rank the 35 items evaluated in order of importance for each group. Comparisons were made between groups to determine whether or not agreement existed on the important and non-important items and other general conclusions were drawn.

Limitation of the Study

As stated previously the major limitation of the study was the size of the sample investigated. This permitted only very general statistical description of the information obtained and only the most obvious trends and conclusions were pointed out.

CONCLUSIONS

When the two groups were compared on the items rated to be most important, there was no agreement. In the items assigned to the upper 5 ranks there were no items at all common to both groups. In the upper 10 ranks 3 items only appeared from both groups.

When the two groups were compared on the items rated to be least important there was significant agreement. In the items assigned to the bottom 5 ranks there was agreement on 4 out of the 5.

A brief analysis of the objectives as rated by both groups found that generally teachers put more emphasis on those objectives at the lower levels of taxonomy while mathematics educators stressed those items which would occupy the higher levels. The major exception to this was rote memorization of theorems which was rated very low by teachers as well as educators.

In summary, then, the major conclusions drawn from the study could be enumerated as follows:

1. Geometry teachers in Newfoundland schools did not agree with mathematics educators on the important objectives of deductive geometry.

2. Geometry teachers in Newfoundland schools agreed with mathematics educators on the non-important objectives of deductive geometry.

3. In general geometry teachers seemed to put more stress on those objectives which are at a low taxonomic level while mathematics educators stressed those at higher levels.

4. Both geometry teachers and mathematics educators considered the rote memorization of theorems to be non-important.

IMPLICATIONS AND SUGGESTIONS FOR FURTHER STUDY

The results of this study would seem to imply that in one particular area of mathematics, deductive geometry, any course based upon the objectives considered important by an individual, or by a small group of like-minded individuals, will quite possibly lack balance as to the objectives perceived important by other groups of individuals. This suggests a possibility for further investigation on a more extensive level, involving more groups and a wider range of individuals. Consideration could be given to including such additional groups as skilled people not directly concerned with the secondary school curriculum such as scientists, engineers and mathematicians; secondary school students; parents of secondary school mathematics students; and provincial curriculum planning committee.

This study also points to the need for investigating the effects of various external factors on how geometry teachers view the objectives of deductive geometry. Using mathematics educators as a basis for comparison, it could be determined if teacher-educator agreement was affected by the following teacher variables: number of years of university training, number of years of teaching experience, the study of Euclidean geometry courses at university, and the percentage of teaching time spent in teaching mathematics.

Differences of opinion between educators and teachers as defined in this study may be due to a number of factors, including lack of knowledge or appreciation of wide varying situations. In any case it must be recognized that these individuals occupy positions in their respective institutions where they can influence what is to be taught in any given course. This study points to the advisability of continued collaboration among university educators and secondary school teachers, not only in preparing instructional materials for high school geometry and determining the content of mathematics courses for teachers, but in examining the total preparation program for geometry teachers. Because of his deep interest in, and intimate knowledge of his own specialized area, each should have a unique contribution to make in this determination.

BIBLIOGRAPHY

BOOKS

Bidwell, J. K., and R. G. Clason (eds.). Readings in the History of Mathematics Education. Washington: National Council of Teachers of Mathematics, 1970.

Bloom, Benjamin S. (ed.). Taxonomy of Educational Goals, Handbook I: Cognitive Domain. New York: David McKay Company, 1956.

Butler, C. H., F. L. Wren, and J. H. Banks. The Teaching of Secondary School Mathematics. 5th. ed. New York: McGraw Hill Book Company, 1970.

Christofferson, H. C. Geometry Professionalized for Teachers. Oxford, Ohio: Published by the Author, 1933.

Commission on Mathematics, College Entrance Examination Board. Programs for College Preparatory Mathematics. Princeton, New Jersey: Educational Testing Service, 1959.

Commission on Secondary School Curriculum of the Progressive Education Association. Mathematics in General Education. Report of the Committee on the Function of Mathematics in General Education. New York: Appleton-Century-Crofts, Inc., 1940.

Edwards, Allen L. Statistical Analysis. 3d ed. New York: Holt, Rinehart and Winston, Inc., 1969.

Fawcett, Harold P. The Nature of Proof. National Council of Teachers of Mathematics, Thirteenth Yearbook. New York: Bureau of Publications, Teachers College, Columbia University, 1938.

Geometry in the Secondary School. Papers Presented at a Joint Session of the Mathematics Association of America and the National Council of Teachers of Mathematics. Washington, D.C.: National Council of Teachers of Mathematics, 1967.

Government of Newfoundland and Labrador. The Newfoundland and Labrador Schools Directory, 1972-73. St. John's: Government of Newfoundland and Labrador, 1972.

Gronlund, Norman E. Stating Behavioural Objectives for Classroom Instruction. New York: The MacMillan Company, 1970.

Johnson, Donovan A., and Gerald R. Rising. Guidelines for Teaching Mathematics. Belmont, California: Wadsworth Publishing Company, 1967.

Joint Commission of the Mathematical Association of America, Inc., and the National Council of Teachers of Mathematics. The Place of Mathematics in Secondary Education. The National Council of Teachers of Mathematics, Fifteenth Yearbook. New York: Bureau of Publications, Teachers College, Columbia University, 1940.

Keedy, Mervin L., and others. Exploring Geometry. New York: Holt, Rinehart and Winston, Inc., 1967.

Kinney, Lucien B., and C. Richard Purdy. Teaching Mathematics in the Secondary School. New York: Rinehart and Company, Inc., 1952.

Kinsella, John J. Secondary School Mathematics. New York: The Center for Applied Research in Education, Inc., 1965.

Mager, Robert F. Preparing Instructional Objectives. Palo Alto, California: Fearon Publishers, Inc., 1962.

Moise, Edwin E., and Floyd E. Downs, Jr. Geometry. Menlo Park, California: Addison-Wesley Publishing Company, 1971.

National Council of Teachers of Mathematics. A History of Mathematics Education in the United States and Canada. Thirty-Second Yearbook. Washington, D.C.: National Council of Teachers of Mathematics, 1959.

________. Insights into Modern Mathematics. Twenty-Third Yearbook. Washington, D.C.: National Council of Teachers of Mathematics, 1957.

________. The Place of Mathematics in Modern Education. Eleventh Yearbook. Washington, D.C.: National Council of Teachers of Mathematics, 1936.

________. The Teaching of Geometry. Fifth Yearbook. New York: Bureau of Publications, Teachers College, Columbia University, 1930.

Newfoundland and Labrador Department of Education. Programme of Studies, 1972-73, Grade I-XI. St. John's: Government of Newfoundland and Labrador, 1972.

Pearson, Helen R., and James R. Smart. Geometry. Boston: Ginn and Company, 1971.

Shibli, J. Recent Developments in the Teaching of Geometry. State College, Pennsylvania: J. Shibli, Publisher, 1932.

Sohn, David A., and Alfred de Grazia (eds.). Revolution in Teaching: New Theory, Technology and Curricula. New York: Bantam Books, 1962.

Taba, Hilda. Curriculum Development: Theory and Practice. New York: Harcourt, Brace and World, Inc., 1962.

Wilcox, Marie S. Geometry, A Modern Approach. Menlo Park, California: Addison-Wesley Publishing Company, 1968.

Willoughby, Stephen S. Contemporary Teaching of Secondary School Mathematics. New York: John Wiley and Sons, Inc., 1967.

PERIODICALS

Adler, Irving. "Criteria of Success in the Seventies," The Mathematics Teacher, LXV (January, 1972), 33-41.

________. "What Shall We Teach in High School Geometry?" The Mathematics Teacher, LXI (March, 1968), 226-38.

Allen, F. B. "Teaching for Generalization in Geometry," The Mathematics Teacher, XLII (June, 1950), 245-51.

Allen, Gertrude. "Objectives of Teaching Mathematics in Secondary Schools," The Mathematics Teacher, XVI (February, 1923), 65-77.

Allendoerfer, Carl B. "The Dilemma in Geometry," The Mathematics Teacher, LXII (March, 1969), 165-69.

________. "The Second Revolution in Mathematics," The Mathematics Teacher, LVIII (December, 1965), 690-95.

Breslich, E. R. "The Nature and Place of Objectives in Teaching Geometry," The Mathematics Teacher, XXXI (November, 1938), 307-15.

Brown, Kenneth E. "Why Teach Geometry?" The Mathematics Teacher, XLIII (March, 1950), 103-06.

Buck, Charles. "What Should High School Geometry Be?" The Mathematics Teacher, LXI (May, 1968), 466-71.

Commission on Post War Plans. "Guidance Report," The Mathematics Teacher, XL (October, 1947), 315-39.

Fawcett, Harold P. "The Geometric Continuum," The Mathematics Teacher, LXIII (May, 1970), 411-20.

Fehr, Howard F. "New Thinking in Mathematical Education," The Mathematics Teacher, LIII (October, 1960), 424-29.

________. "The Present Year-long Course in Euclidean Geometry Must Go," The Mathematics Teacher, LXV (February, 1972), 102, 151-54.

________, Frank M. Eccles, and Bruce Meserve. "What Should Become of the High School Geometry Course?" The Mathematics Teacher, LXV (February, 1972), 102.

Hall, E. L. "Applying Geometric Methods of Thinking to Life Situations," The Mathematics Teacher, XXXI (December, 1938), 379-84.

Hlvaty, J. H. "Capsule History of the NCTM.," The Mathematics Teacher, LXIII (February, 1970), 135-46.

Mallory, Virgil S., and Howard F. Fehr. "Mathematical Education in War Times," The Mathematics Teacher, XXXV (November, 1942).

Massimiano, Carmen C. "The Influence of the Study of Plane Geometry on Critical Thinking," The Mathematics Teacher, XLIX (February, 1956), 151-53.

Meserve, Bruce E. "Euclidean and Other Geometries," The Mathematics Teacher, LX (January, 1967), 2-11.

________. "The Evolution of Geometry," The Mathematics Teacher, XLIX (May, 1956), 372-82.

________. "New Trends in Algebra and Geometry," The Mathematics Teacher, LV (October, 1962), 452-61.

Northrop, E. P. "Modern Mathematics and the Secondary School," The Mathematics Teacher, XLVIII (October, 1955), 386-93.

Reeve, W. D. "Objectives in Teaching Demonstrative Geometry," The Mathematics Teacher, XX (December, 1927), 433-50.

________. "Objectives in the Teaching of Mathematics," The Mathematics Teacher, XVIII (November, 1925), 385-89.

________ (ed.). "Guidance Report of the Committee on Post War Plans," The Mathematics Teacher, XL (November, 1947), 315-339.

Rosskopf, Myron F., and Robert M. Exner. "Modern Emphases in the Teaching of Geometry," The Mathematics Teacher, L (April, 1957), 272-79.

Smith, R. R. "On the Teaching of Geometry," The Mathematics Teacher, XLII (January, 1949), 56-60.

Stroup, P. "When is a Proof Not a Proof," <u>The Mathematics Teacher</u>, XIX (December, 1926), 499-505.

Van Waynen, M. "What Kind of Geometry Shall We Teach?" <u>The Mathematics Teacher</u>, XLIII (January, 1950), 3-11.

Webb, H. E. "'Elementary Geometry' and the 'Foundations'," <u>The Mathematics Teacher</u>, XIX (January, 1926), 1-12.

Wilson, J. T. "Trends in Geometry," <u>The Mathematics Teacher</u>, XLVI (January, 1953), 67-70.

Wuest, Alma. "Analysis versus Synthesis," <u>The Mathematics Teacher</u>, XX (January, 1927), 46-49.

UNPUBLISHED MATERIAL

Harding, Robert N. "The Objectives of Mathematics Education in Secondary Schools as Perceived by Various Concerned Groups." Unpublished Doctor's dissertation, University of Nebraska, 1968.

Quast, William G. "Geometry in the High Schools of the United States: An Historical Analysis from 1890 to 1966." Unpublished Doctor's dissertation, Rutgers, The State University, 1968.

Wong, Ruth E. M. "Status and Direction of Geometry for Teachers." Unpublished Doctor's dissertation, University of Michigan, 1968.

APPENDICES

APPENDIX A

FINAL LIST OF OBJECTIVES AND RELATED MATERIAL

List of Objectives

1. Define the basic geometric terms.

   Example: An angle is defined as ...

2. Use precise terminology in written and oral work.

   Example: Incorrect reference to "line" when "line segment" is intended.

3. Use straightedge and compass to perform the basic geometric constructions.

   Example: To construct a perpendicular line from a point on the line.

4. Differentiate between the properties of quadrilaterals.

   Example: What is the difference between a square and a rectangle?

5. State a set of conditions under which two or more triangles are congruent.

   Example: Triangles are congruent if ...

6. State a set of conditions under which two or more triangles are similar.

   Example: Triangles are similar if ...

7. State all the conditions under which two or more lines are parallel.

   Example: Lines are parallel if and only if ...

8. State Pythagoras' Theorem.

Example: For any right triangle, $a^2 + b^2 = c^2$, where a and b are measures of the legs and c is the measure of the hypotenuse.

9. Make neat drawings to represent three dimensional figures.

Example: Make a neat drawing of a tetrahedron.

10. Write a conditional sentence.

Example: If x is greater than y, then y is less than x.

11. Make reasonable conjectures.

Example: If AB is the longest side of triangle ABC, then the largest angle is ...

12. Recognize false assumptions.

Example: Assumptions such as corresponding angles being congruent when lines are not parallel.

13. Distinguish between inductive and deductive reasoning.

Example: The difference between inductive and deductive reasoning is ...

14. Apply the methods of inductive and deductive reasoning in order to state conclusions from given statements.

Example: Barking dogs do not bite; My dog barks. What is the conclusion?

15. State the definitions, axioms, and postulates used to justify a simple conclusion.

Example:

If $AB \cong CD$, show why $AC \cong BD$.

16. Without using references, reproduce the complete proof of any theorem after being given a reasonable time to study it.

Example: Without using references write out the proof of: "The sum of the measures of the angles of a triangle is 180."

17. Without using references, construct an outline of the proof of a particular theorem.

Example: Without giving details, outline the proof of the following ...

18. Without using references reproduce a complete proof of Pythagoras' Theorem.

19. Disprove simple propositions through the use of counter-examples.

Example: Prove that all equilateral triangles are not congruent.

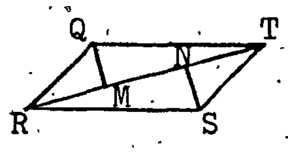
20. Write the reasons for the various steps in a given proof.

Example:

STATEMENT	REASON
$\overline{AC} \cong \overline{DF}$; $\overline{BC} \cong \overline{EF}$	?
$\angle C \cong \angle F$	?
$\triangle ABC \cong \triangle DEF$	?

21. Provide complete proofs of given statements using definitions, axioms, postulates, theorems and corollaries previously proved.

Example:



Given: QRST is a parallelogram; RM $\cong$ NT

Prove: QM $\cong$ SN

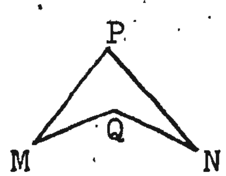
22. Prove that a compass and straightedge construction is correct.

Example: Write a justification for the construction of the perpendicular bisector of a segment.

23. Construct an outline of the proof of Pythagoras' Theorem without filling in the details.

24. Perform constructions on figures in order to prove some conclusion.

Example:



Given: PM = PN
QM = QN
Prove: $\angle M \cong \angle N$

25. Identify erroneous statements in a given proof.

Example: Identify such erroneous statements as:
$\triangle ABC \cong \triangle DEF$ (SSA)

26. Without giving the proof, illustrate and explain the statement of any particular theorem when given the statement.

Example: With the aid of a drawing, explain what is stated in the following: If a line is tangent to a circle, then it is perpendicular to the radius at the point of tangency.

27. Explain the various steps in the proof of Pythagoras' Theorem if given a completed proof.

28. Explain why some terms are left undefined in deductive geometry.

Example: The term "point" is left undefined because ...

29. Distinguish between postulates and theorems.

Example: The difference between a postulate and a theorem is ...

30. Reproduce selected postulates.

Example: Without referring to a book, state three postulates concerning lines and planes.

31. Describe the basic structure of postulational geometry.

Example: Explain the relationship between undefined terms, postulates, definitions and theorems.

32. State the purpose of a lemma.

33. Name some practical uses for geometry in real-life situations.

Example: Use Pythagoras' Theorem to solve physical problems.

34. Apply geometric ideas to other areas of mathematics.

Example: Use deductive methods to prove that for any real number x, $-(-x) = x$

35. Formulate an answer to the question: "Why do I need to study deductive geometry?"

Instructions Accompanying Objectives

INSTRUCTIONS FOR SORTING OBJECTIVE CARDS

Each of the enclosed cards contains one possible objective of deductive geometry. You are kindly asked to sort the 35 cards into 5 groups, ranging from Group 1, which contains what you feel are the very important objectives of deductive geometry, to Group 5 which you feel are the unimportant objectives. The Groups 2, 3, 4 will thus contain groups of objectives which are perceived in decreasing order of importance. In short, Group 1 are the most important, Group 2 are slightly less important, and so on to Group 5 which are the unimportant objectives.

Please feel free to place as many objectives as you wish in any one group, as well as leave any group empty.

The idea of placing the objectives on cards was to allow you the maximum flexibility in changing your initial ratings. It is hoped, therefore, that you will satisfy yourself that your final rating really reflects how you perceive the objectives of deductive geometry.

When you have sorted the cards to your satisfaction record the number shown on each card in the appropriate column on the Recording Sheet.

Please return the Recording Sheet in the enclosed envelope.

Thank you very much for your consideration.

Return Sheet

RECORDING SHEET FOR RATING OF OBJECTIVES

After you have sorted the cards into the five groups place the number on each card in the appropriate column below. For example, if you place objectives numbered 2, 4, 6, 8, 10 in group 3, then these numbers should be recorded in column 3 below.

Please remember that a rating of "1" indicates that you consider the objective to be very important; a rating of "5" indicates unimportance. Ratings "2", "3", and "4" represent points along the continuum.

VERY IMPORTANT UNIMPORTANT

1	2	3	4	5

APPENDIX B

CORRESPONDENCE

Letter of Validation

Dear ________,

Enclosed you will find the list of objectives, instructions and scoresheet which I intend to use in my thesis: "The Objectives of Deductive Geometry in Newfoundland Secondary Schools as Perceived by Concerned Groups." The purpose of submitting this to you is to ask your assistance in the validation of these objectives.

Concerning the validation would you comment if necessary on the following:

1. Comprehensiveness - Have important objectives been omitted?
2. Compactness - Can some items justifiably be ommitted? Should they be shortened? Should two or more objectives be combined?
3. Clarity - Are the basic meanings clear? How can clarity be improved?
4. Instructions - Do the instructions clearly indicate what is required?

In addition, would you please sort the objectives as outlined in the instructions. Your sorting will not in any way be part of the thesis but I would just like to get some idea of items on which there might be wide disagreement.

Thank-you for your cooperation.

Sincerely,

Letter to Geometry Teachers

Dear Teacher,

What different people see as the important objectives of deductive geometry in the high school is the subject of a thesis I am writing for the Department of Curriculum and Instruction at Memorial University. To do this I need the opinions of geometry teachers such as yourself, who ultimately decide what should or should not be emphasized in the classroom.

To get the opinions of teachers I have drawn up a list of 35 objectives of deductive geometry which can be rated in terms of importance or non-importance. The objectives are not based specifically on any particular geometry course but they do generally reflect the deductive geometry content as presently covered in our Grade 9 and 10 courses.

I realize that your participation in this study will be an extra burden in an already busy schedule. However, if you can possibly spare the few minutes required to sort the cards as outlined in the instructions, it would be greatly appreciated. Please note that there are no right or wrong ways to sort the cards; rather the object is to see to what extent our geometry teachers can agree with each other.

There is no need for you to identify yourself in any way if you do not want to. Any comments you might want to make on the list of objectives or on the study itself would be welcome. They would not be a part of the thesis but would be valuable in shaping my own thinking on the subject.

Many thanks for your cooperation.

Sincerely,

Letter to Mathematics Educators

Dear ________,

Your name was suggested to me by _______ of Memorial University, Newfoundland, Canada, as a person who might be helpful to me in a Master's thesis I am presently writing for the Department of Curriculum and Instruction at Memorial University. The topic of the proposed study is "The Objectives of Deductive Geometry in Newfoundland Secondary Schools as Perceived by Concerned Groups." One of the groups whose opinions I am surveying is educators in selected universities in Canada and the United States.

To get the opinion of educators I have drawn up a list of 35 possible objectives of deductive geometry which can be rated in terms of importance or non-importance. Please note that the question of whether Euclidean Geometry or some other geometry should be taught in the secondary school is not a consideration of the study.

I realize that your participation in this study will be an extra burden in an already busy schedule; however, if you can possibly spare the few minutes required to sort the cards as outlined in the instructions, it would be deeply appreciated.

Many thanks for your cooperation.

Sincerely,

Follow-up Letter to both Groups

Dear ________,

Some time ago I sent you material requesting your assistance in classifying the objectives of deductive geometry. I apologize for bothering you this one last time, but this is just a reminder in case you intended to respond to that inquiry but just forgot to do so. Your assistance would be very much appreciated. If you have already responded, I thank you very kindly.

Sincerely,

Letter for Reliability

Dear Geometry Teacher,

You might recall that some time ago I sent you the same set of cards you now find enclosed, and you very kindly sorted them for me. I hesitate to take advantage of your good nature, but I am now asking if you would please sort them again. The reason for this is that in the study which I am doing I have to demonstrate the reliability of the instrument being used. This can only be done by asking you to repeat the procedure so that I can compare the results of both sortings and see to what extent they agree.

I apologize for having to bother you again in this way and would certainly not do so if there were any other means of establishing reliability. In any event, whether you find the time to respond to this inquiry or not, I thank you very kindly for your assistance in the past.

Sincerely,

APPENDIX C

RELATED INFORMATION

Textbooks

The following textbooks in deductive geometry on the secondary school level were analyzed in preparing the list of objectives used in the study:

Keedy, Mervin L., and others. Exploring Geometry. New York: Holt, Rinehart and Winston, Inc., 1967.

Moise, Edwin E., and Floyd E. Downs Jr. Geometry. Menlo Park, California: Addison-Wesley Publishing Company, 1971.

Pearson, Helen R., and James R. Smart. Geometry. Boston: Ginn and Company, 1971.

Wilcox, Marie S. Geometry, A Modern Approach. Menlo Park, California: Addison-Wesley Publishing Company, 1968.

Universities Represented in the Survey

Mathematics educators from the following universities responded to the survey:

Boston University
Boston, Massachusetts

Kent State University
Kent, Ohio

Memorial University of Newfoundland
St. John's, Newfoundland

University of Alberta
Edmonton, Alberta

University of British Columbia
Vancouver, British Columbia

University of Texas
Austin, Texas