AN ANALYSIS OF CONTENT OBJECTIVES OF JUNIOR HIGH SCHOOL MATHEMATICS BASED ON PERCEPTIONS OF A SELECTED NUMBER OF SEVENTH AND EIGHTH GRADE TEACHERS

JOHN CHARLES CHIPMAN

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by

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A Report submitted in partial fulfillment of the
requirements for the degree of
Master of Education

Department of Education
Memorial University of Newfoundland

April 1976

St. John's Newfoundland

ACKNOWLEDGEMENTS

The writer gratefully acknowledges the assistance of all those people who made possible the final completion of this report. I would like to express my appreciation to the teachers who took the time from a busy schedule to read and complete the questionnaire. Special thanks to Dr. Alex Brace of Memorial University who was always ready, day or night, to offer advice and encouragement, and to Mrs. Ruth Spurrell who proof-read and typed the final manuscript. Very special apprecia-tion to my family whose patience and encouragement helped me see it through.

J.C.

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LIST OF ABBREVIATIONS

1. NCTM -- National Council of Teachers of Mathematics.
2. GCMP -- Greater Cleveland Mathematics Program.
3. SSMCIS -- The Secondary School Mathematics Curriculum Improvement Study.
4. NAEP -- National Assessment of Educational Progress.
5. SMSG -- School Mathematics Study Group.
6. UICSM -- University of Illinois Committee on School Mathematics.

CHAPTER I

INTRODUCTION

During the 1950's, particularly in the United States, tremendous changes began to take place in mathematics education. These changes were generated by increasing demands being placed on mathematics by a society that was becoming increasingly more technically and scientifically based. More and more graduates of high school were entering universities and other post-secondary institutions which required higher levels of competency than had previously been the case. In addition, the new technology of the computer age required that the average citizen be better able to understand and interpret the workings of the new society of which he was a part. The emphasis on mathematics and the natural sciences was given added emphasis by the realization in the late 1950's that the technical supremacy of the United States was being challenged, as witnessed by the launching of Sputnik I in 1957.

The revolution, spurred on by the injection of massive amounts of federal funds, brought about drastic changes in content and method in mathematics education. As a result of the revolutionary work of the University of Illinois Committee On School Mathematics, the School Mathematics Study Group, the Commission on Mathematics, and others, new concepts found their way into the classroom. There was a massive influx of teachers back to university for upgrading, and writers and publishing companies reaped profits with new textbooks and instructional materials.

The new curriculum was a mathematician's delight. There was a great deal of emphasis on structure and rigor. No longer was it sufficient to know "how"; now it was required to know "why". Topics were introduced in primary and elementary grades which were previously reserved for high school, and, in turn, high school students were presented with materials previously reserved for university. Algebra was given increased emphasis, probability and statistics were introduced, and there were some, such as Howard Fehr, who even advocated the introduction of calculus in the junior high school.

The revolution continued through the 1960's and even gained impetus as publishing companies flooded the market with new textbooks, and mathematicians propagated the new mathematics much to the dismay of most parents, many students, and even some teachers. But the revolution was not without its critics. Many prominent mathematicians and educators deplored the emphasis on rigor and structure, arguing that it favoured the more capable college-bound student. Most significant among the critics were the teachers themselves who saw the difficulties that many students were experiencing and witnessed a decline in competencies, particularly in basic skills.

As a result of the criticism and concern, a counter-revolution began to take form in the late 1960's and early 1970's. The change is attested to by Herbert J. Greenberg of the University of Denver, who, in an address to the 11th. Annual Northwest Mathematics Conference in 1972, said:

> In the late 1960's the pendulum began to swing again, not back to the old mathematics, but away from the extremes of the new mathematics and toward a kind of middle ground that acknowledged the need for computational skills as well as applications of mathematics.[1]

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[1]Herbert J. Greenberg, "The Objectives of Mathematics Education," Mathematics Teacher, LXVII (November, 1974), p.639.

Evidence of concern is also given by recent attempts at reassessment of the aims and objectives of mathematics education with ensuing adjustment and refinement of curriculum. Several notable studies have been or are being conducted, particularly in the United States, to attempt to determine for what reasons mathematics should be taught and, consequently, what mathematics. Foremost among these studies are:

1. The Committee on Basic Mathematical Competencies and Skills of NCTM.

2. The National Longitudinal Study of Mathematical Abilities.

3. National Assessment of Educational Progress.

4. The School Mathematics Study Group.

The latter group is a perrenial study group in mathematics education which is continually revising and updating its programmes.

Other evidence is the obvious change in content in most recent textbooks. There is in those texts an obvious move to a more basic approach with increased emphasis on basic skills and concepts. On the local scene this trend is reflected in the adoption of the most recent textbooks. In grades seven and eight the revised (1971) editions of Exploring Modern Mathematics are without the rigor of the previous (1967) editions. In high school in this current year the emphasis on a more general approach to mathematics is reflected in the choice of textbooks which are, supposedly, more suited to the large middle group of students.

It is reasonable to assume that the aims and objectives of any educational programme would be determined by the needs of society, in general, and by the academic and technical sectors of society, in particular. The determination and enlistment of these aims and objectives is usually left to a number of select groups or individuals of high academic and professional standing whose deliberations and specifications are incorporated into the curriculum by the various writing teams and publishing companies. When these new publications, properly scrutinized, are placed in the hands of the teachers, the content is usually followed religiously but the degree of emphasis which teachers place on the various aspects of a programme determine, to a great extent, the degree to which desired objectives are attained. Of primary importance, therefore, is a reasonable perception on the part of the teacher as to what is important in mathematics. It is the intention of this project to investigate the aims and objectives of mathematics education with particular emphasis on teachers' perceptions of the relative importance of a selected number of specific content-oriented objectives.

PURPOSE OF THE PROJECT

It was stated in the introduction that there has been growing concern in recent years with the current state of mathematics. It is not that mathematics education has deteriorated. Rather, it appears that the quality of mathematics and mathematics instruction has improved. What is causing the concern is the revolutionary changes that took place in a relatively short period of time. Many of the changes were abrupt and drastic and as a result there was a great deal of confusion. The confusion, in many instances, resulted from teachers' misinterpretation of the purposes for the introduction of the many new topics in mathematics. So, we found teachers trying to <u>teach</u> understanding, teaching other numeration systems and other bases for mastery, considering the fundamental properties as ends in themselves, and generally missing the true spirit of the revolution completely.

Many assumed that it was no longer necessary to be concerned with basic skills but this was a false assumption as it was never specified by any curriculum revision group that this should be the case. In fact, the mathematics curriculum reform groups of the 1950's and 1960's, without exception, were careful to spell out the importance of maintaining skills. Not one of them proposed a lessening of skills.[2] What, then, is the cause of the problem? It is the writer's opinion that a basic cause of many problems in mathematics education is a lack of awareness on the part of teachers of the aims and objectives of mathematics education and an ignorance or misunderstanding of the purpose for the introduction of the many new topics in mathematics.

It is the purpose of this project to construct a comprehensive list of basic content objectives for junior high school mathematics and to present this list to a selected group of grade seven and eight mathematics teachers for their evaluation. The object is to determine the teachers' perceptions of the degree of importance of these objectives. Of particular importance is the degree of emphasis that teachers place on computational versus structural aspects of mathematics as this appears to be a perrenial problem difficult to resolve.

JUSTIFICATION FOR THE PROJECT

The idea for this project came from an examination of materials from the Committee on Basic Mathematical Competencies and Skills of NCTM. The object of this committee was to draw up a list of basic competencies and skills which would serve as a guide for curriculum

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[2] Glyn K. Wooldridge, "Some Comments on Computation," Teaching Mathematics, III (February, 1976), p.4.

developers. Other studies of similar nature have been conducted for the same purpose. Many individuals, (notably Max S. Bell, Sol Weiss, and Herbert J. Greenberg), are concerned with the state of mathematics and make proposals for revision and adjustment which will make mathematics more suitable to the needs of the majority of students in today's society.

Of greater significance to the writer are the comments and opinions of many colleagues in mathematics education. Generally, teachers do not profess to experience any considerable degree of difficulty in primary and low elementary grades not does there appear to be any great difficulty on the part of the majority of students. However, as students move up the academic ladder, especially at the junior high school level, an increasingly greater proportion seem to experience difficulty; more students appear to be turned off from mathematics. This attitude is often carried over into high school and, so, it is not uncommon to find that many students, when given the opportunity, elect not to do mathematics - a once favoured subject.

In addition, one hears the constant complaint of high school mathematics and science teachers that students are seriously deficient in basic mathematical competencies and skills. Similarily, the university and technical schools are dissatisfied with the mathematical abilities of high school graduates coming to them.

The junior high school level was chosen because of its crucial position in the overall mathematics curriculum. Junior high school is the bridge between the elementary mathematics of lower grades and the more specialized mathematics of high school. At this level it would be reasonable to assume that the majority of students have mastered the basic skills and concepts necessary for further study in mathematics. Also, at this level students are moving from the concrete and semi-concrete to the abstract level of mathematics. More emphasis is placed on the structural aspects of mathematics and the program is more integrated. In short, at this point the basics should be consolidated and the foundation laid for more formal studies in mathematics.

LIMITATIONS OF THE PROJECT

This study is by no means an exhaustive study of the aims and objectives of mathematics. Though consideration is given to the aims and objectives of mathematics in general, emphasis is on the content objectives with particular reference to the junior high school level. An attempt has been made to construct a comprehensive list of content objectives. Initially, a list of 156 objectives was constructed. This list was later condensed to a more workable list of 115 objectives. Still, the length of the list imposed several restrictions. The writer acknowledges the following limitations and restrictions of this study.

1. The study is confined to a rather localized area comprised of the Integrated and Roman Catholic schools in the area from Brigus to Victoria in Conception Bay, an area with a total school population in excess of 7000 students.

2. The study is confined to junior high school and only grade seven and eight teachers are involved in the survey.

3. The sample of teachers is pre-selective. No attempt was made to randomize the sample, rather, every grade seven and eight mathematics teacher was involved.

4. There is no attempt to use sophisticated statistical techniques. The chief statistical measure used is the arithmetic mean. The writer realizes the limitations of such a measure and interprets it merely as an indicator. No attempt is made to make absolute judgements.

5. An attempt was made to determine teachers' perceptions on the degree of difficulty that they perceive students to have with respect to each objective. This proved to be impossible. Since this factor was of secondary consideration, it is not reported on herein.

CHAPTER II

THE AIMS AND OBJECTIVES OF MATHEMATICS

THE PRE-WORLD WAR II PERIOD

In the distant past the main emphasis in mathematics in primary and elementary school was on computation, or what was commonly called arithmetic, with any treatment of other branches of mathematics (e.g. geometry, algebra, and trigonometry) reserved for the final years of study in high school. Thus, mathematics was treated as a series of segmented subjects.

Developments during the early part of the 20th century led to an increased interest in the role of mathematics in society. One of the major factors contributing to this interest was the Great Depression of the late 1920's and early 1930's. Among the important investigations conducted during this era was that of the Joint Commission to Study the Place of Mathematics in Secondary Education (1933). According to Butler, the Commission attempted "to define the place of mathematics in the modern education program and then organize a mathematical curriculum for grades 7 to 14 in terms of the major mathematical fields which would provide for continuity of development and flexibility of administration."[3] The program was based upon an assumed normal mathematical ability of a pupil who had completed grade six. As a definition of normal mathematical ability the Commission enumerated the following points:

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[3]Charles H. Butler, F. Lynwood Wren, and J. Houston Banks, The Teaching of Secondary Mathematics, (New York: McGraw-Hill Book Company, 1970), p. 23.

1. A familiarity with the basic concepts, the processes, and the vocabulary of arithmetic.

2. Understanding of the significance of the different positions that a given digit may occupy in a number, including the case of a decimal fraction.

3. A mastery of the basic number combinations in addition, subtraction, multiplication, and division.

4. Reasonable skill in computing with integers, common fractions, and decimal fractions.

5. An acquaintance with the principal units of measurement, and their use in everyday life situations.

6. The ability to solve simple problems involving computation and units of measurement.

7. The ability to recognize, to name, and to sketch such common geometric figures as the rectangle, the square, the circle, the triangle, the rectangular solid, the sphere, the cylinder, and the cube.

8. The habit of estimating and checking results.[4]

It is apparent from this list that the emphasis was placed on the computational aspect of mathematics and its application in the utilitarian sense. This was in keeping with the philosophy of the times when education was seen as a means of solving the social and economic problems of the day. Conspicuous by its absence is any mention of some of the more formal aspects of mathematics such as structure and reasoning. No consideration is given to topics related to algebra.

About the same time (1932), The Progressive Education Association established a committee on mathematics and its report in 1938 enumerated the functions of mathematics in terms of four "basic aspects of living".

1. Personal living.

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[4] Ibid., quoting from Joint Commission of the Mathematical Association of America, Inc. and the National Council of Teachers of Mathematics, "The Place of Mathematics in Secondary Education," Fifteenth Yearbook (Washington, D.C., National Council of Teachers of Mathematics, 1940), p. 23.

2. Immediate personal-social relationships.

3. Social-civic relationships.

4. Economic relationships.[5]

Here, as in the previous instance, the emphasis is on the social and economic aspects of mathematics. Obviously, the role of mathematics was interpreted as satisfying the needs of the people with respect to these four basic aspects of living. Skills and applications suited to situations encountered in daily life would determine the content of mathematics programmes.

During this period mathematics was given unprecedented attention, possibly because of its role in economics and its importance in consumer related situations. But the attention was too soon diverted to another sphere, as the United States and its allies found themselves engulfed in the second major conflict of the century. The economic revival overshadowed the experiences of the depression and the emphasis on education shifted to training inductees into the military.

THE POST WORLD WAR II PERIOD

The Second World War had a great effect on mathematics education in an unsuspected way. Serious deficiencies in mathematics were discovered among inductees in war-training programmes. Consequently, the Commission on Post War Plans was established by the National Council of Teachers of Mathematics in 1944 to plan mathematics programmes. One of its main theses was that "the school should

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[5]Ibid, quoting from Commission on Secondary School Curriculum of the Progressive Education Association, "Mathematics in General Education", Report of the Committee on the Function of Mathematics in General Education, (New York: Appleton-Century-Crofts, Inc., 1940), p.25.

guarantee functional competence in mathematics to all who can possibly achieve it".[6] The approach was a general one with the emphasis on an opportunity for all. The situation in the thirties and the revelations of the forties gave added emphasis to efforts to improve mathematics education. There was increased concern about a general rather than specialized education. The major problem as far as mathematics was concerned was to determine those aspects of mathematical knowledge which would be comprehensible to and significant for every individual who was capable of participating in the education process. The Commission, in its report, made the following proposal:

> ... whatever the ability of an educable person may be, the general education programme in mathematics must provide him with a background of skills and information to enable him to compute with facility; to understand, appreciate, and construct a valid argument; to recognize and analyze a problem situation; to discriminate between known and unknown elements; to distinguish between relevant and irrelevant data; to recognize basic relationships; to detect fundamental differences, restrictions, and possibilities; to make intelligent guesses and estimates; and to evaluate and interpret results.[7]

Butler had a similar philosophy. Writing at a later date he said:

> The fact that this programme should be designed for the general student, and not for the student interested in specialization, does not imply that it should be a watered down treatment of computational techniques or a memorization of formulas or rules. There are mathematical concepts and procedures which are of importance to the educated individual. It would seem that in a technological age, no person is well-informed without at least some fundamental knowledge of the nature of proof, the basic concepts of the structure of our number system, algebraic and geometric structures, the nature of measurement, the concepts of relation and function, and basic statistical measures.[8]

Butler does not envisage a general mathematics programme as one for the slower group of students but as one for every citizen, or almost every citizen. He advocates a slower paced mathematics programme

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[6] Ibid, p.29.

[7] Ibid, quoting from Report of the Commission on Mathematics (New York: College Entrance Examination Board, 1959), p.11

[8] Ibid, p. 31.

for students of very low-level competence. He also recommends an enriched programme for the most capable students, but the most important fact is that he does not envisage different programmes. He sees the differentiation not in subject matter but in interpretation, pace, and enrichment. He sees a two-track programme which will, on the one hand, cater to the user of mathematics, and, on the other hand, will cater to the student who wishes to pursue more advanced work in mathematics. He advocates a common core for all levels. This is what he means by "general mathematics", a unifying body of concepts and processes for all. Butler lists the following objectives of general mathematics:

Mathematics should make provision for,

1. Competence in basic skills and understandings for dealing with number and form.
2. Habits of effective thinking - a broad term involving analytical, critical, and postulational thinking, as well as reasoning by analogies and the development of intellectual curiosity.
3. Communication of thought through symbolic notation and graphs.
4. Development of the ability to distinguish between relevant and irrelevant data.
5. Development of the ability to make relevant judgements through discrimination of values.
6. Development of intellectual independence.
7. Development of aesthetic appreciation and expression.
8. Cultural advancement through a realization of the significance of mathematics in its own right and in relation to the total physical and social structure.[9]

The changes in mathematics education that took place in the post-Sputnik era were chiefly changes in content; the fundamental objectives of mathematics education were basically unchanged. True,

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[9] Ibid, p.43.

There was a greater degree of emphasis on mathematics for the more academically inclined, with more concentration on the more formal aspects of mathematics. Yet, many mathematicians recognized the need for a broad general programme in mathematics which would be suited to the needs of the majority. One such mathematician is Leroy G. Callahan, who said:

> It has been fashionable of late to downgrade, if not in fact to denigrate, such direct, immediate, and utilitarian objectives in the mathematics programme. Yet reports of the death of the social utilitarian objectives in mathematics for general education may be somewhat premature. The development of salable skills in the contemporary market place, the development of intelligent consumer skills, the development of quantitative skills needed to enable one to enjoy increasing leisure-time activities may have their legitimate place in the development of the "good life" for students in general education.[10]

Callahan outlines two levels of objectives of mathematics. His emphasis is on the socializing or humanizing aspects of mathematics. He defines Level I objectives as CONTENT objectives, those which enable man to use mathematics as a tool to aid him "to know and translate more accurately his objective world."[11] He defines Level II objectives as FORM objectives. These include the logical reasoning associated with mathematics which is thought of as "an indispensible study in the general education of man."[12] Inherent in this categorization are three broad objectives of mathematics in general education - the utilitarian contribution, the speculative or thinking contribution, and the affective or humanizing contribution.

Another mathematician who considers mathematics from the general or liberal-arts point of view is Irving Allen Dodes. He emphasizes

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[10] Leroy G. Callahan, "Mathematics in General Education - Changes Constants, Concerns", Educational Leadership, (Washington, D.C.:NEA, May, 1970), p.827.

[11] Ibid, p.828.

[12] Ibid, p.829.

the cultural aspect of mathematics, "designed to enable the citizen to understand and appreciate the mechanism and background of his environment."[13] However, he recognizes the need for certain skills necessary for "the adequate participation of a citizen in his technological, commercial, and industrial society."[14] He also says that mathematics should "open the eyes of the students to the beauty and wonder of mathematics without attempting to make the student into a half mathematician."[15]

Howard Fehr, a noted mathematician and mathematics educator, takes a different point of view, but he does recognize the need for mathematics for the mass of the student population. He says that "the main virtue of mathematics in modern society is the fact that it aids the non-mathematician, the applier, to do his job with greater efficiency and insight."[16] He sees mathematics as the basis for all technological research and technical training and recognizes three basic levels of mathematics needs:

1. For the professional mathematician.
2. For the scientist and technologist.
3. For the semi-professional technician and tradesman.

Fehr separates mathematics into two categories - with respect to TRAINING and with respect to EDUCATION. He distinguishes between the two as follows:

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[13] Irving Allen Dodes, "Some Comments on General Mathematics," Perspectives on Secondary Mathematics Education, ed. Jerry A McIntosh (New York: Prentice-Hall Inc., 1971), p. 155.

[14] Ibid, p. 156

[15] Ibid, p. 157.

[16] Howard F. Fehr, "Mathematical Education for a Scientific, Technological, and Industrial Society," Mathematics Teacher, LXI (November, 1968), p. 665.

> Training is conceived as constricting the mind to think, and the body to operate, in a given way. Education, on the other hand, is conceived as liberating the mind, freeing it to ask questions, to seek other solutions, to look for new relations, to adapt one's thinking and self to new conditions."[17]

In this respect, he indicates that it is the role of the school to educate and the role of the vocational school or technical institution to train. He thereby discounts the inclusion of technical or consumer-type mathematics in the school curriculum. His emphasis appears to be on the future study aspect of mathematics. He makes no reference to the social or cultural contribution of mathematics. His opinion appears to be that everybody should study "honest" mathematics, not just arithmetic. To quote Fehr:

> The mass of the people should study mathematics in the same meaningful and similarly structured, (though not so rigorous), manner, adjusted to their rate of learning, to their mental ability to make abstractions and deal with complex ideas, and to their need for many concrete examples and applications of any mathematical concept to be learned. They should study the same mathematics - but not so much, not so abstract, and at a much slower rate."[18]

Fehr's point of view is worthy of further consideration. His viewpoint is that all branches of mathematics have certain unifying concepts and processes. Some of these such as sets, relations, algebraic structures, and mappings are common to all branches and serve as the unifying force. He therefore envisages the mathematics programme as an integrated program drawing on all branches of mathematics with an emphasis on the development of the intellect but with provision for the informational and skill dimension.

Indicative of what Fehr would include in a mathematics programme is the outline of course content for grades seven and eight of the

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[17] Ibid, p. 669.

[18] Ibid, p. 670.

SSMCIS programme, of which Fehr was director. The following is a list of topics:

Course I (Grade 7)

1. Finite number systems.
2. Sets and operations.
3. Mathematical mappings.
4. Integers and addition.
5. Probability and statistics.
6. Multiplication of integers.
7. Lattice points in a plane.
8. Sets and relations.
9. Transformations of the plane.
10. Segments, angles, isometries.
11. Elementary number theory.
12. The rational number.
13. Some applications of rational numbers.
14. Algorithms and their graphs.

Course II (Grade 8)

1. Mathematical language and proof.
2. groups.
3. An introduction to axiomatic affine geometry.
4. Fields.
5. The real number system.
6. Co-ordinate geometry.
7. Real functions.
8. Descriptive statistics.
9. Transformations of the plane, isometries.

10. Length, area, and volume.[19]

The above list of content indicates a high level programme with concentration on the more formal aspects of mathematics. With the exception of the last item in Course II, there is no emphasis on the computational aspects of mathematics. Very little attention is given to the utilitarian value of mathematics. The overall SSMCIS programme is given in Appendix C but some topics are worthy of note as an indication of mathematical content at various levels. As an example, we find algebra of matrices and circular functions in grade nine; computer programming and vector spaces in grade ten; differential and integral calculus in grade eleven. There is no doubt that such a programme would provide problems for many students regardless of pace and presentation.

MORE RECENT DEVELOPMENTS

As was mentioned previously, the emphasis began to change in the latter part of the 1960's. One of the groups that began to revise its programmes was the School Mathematics Study Group, beginning in 1966. It hoped to develop a curriculum "that will provide students with a clear understanding of the nature of mathematical applications and of the variety of ways in which mathematics can be useful in our society."[20] A panel met in March, 1966 to make preliminary plans and from their discussions two broad principles emerged:

1. The initial segment of the secondary school mathematics curriculum should be devoted to those mathematical concepts which all citizens should know in order to function satisfactorily in our rapidly expanding technological society.

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[19] Howard F. Fehr, "The Secondary School Mathematics Curriculum Improvement Study: A Unified Mathematics Program," The Mathematics Teacher, LXVII (January, 1974) p.31.

[20] W. Eugene Ferguson, "The Junior High School Mathematics Program - Past, Present, and Future," Mathematics Teacher, LXIII (May, 1970), p. 387.

2. The exposition of this mathematics for the average to slow-moving student will need to be developed if the project is to be a success.[21]

The panel recognized two basic groups of students and, while suggesting the same basic programme for both, realized that the pace would vary. They envisaged a single programme for junior high school with variations in content in senior high school.

Following the preliminary deliberations a committee of 20 teachers and mathematicians met in the summer of 1966 to begin to write detailed outlines of proposed materials for the junior high school grades. While it is inconvenient to reproduce all of their recommendations here, the following are representative of the features of the seven - nine sequence:

1. An attempt is made to fuse arithmetic, algebra, and geometry.
2. Geometry is presented in a concrete, intuitive, descriptive way.
3. One, two, and three dimensional geometry will be treated.
4. Co-ordinate geometry will appear as appropriate in helping describe sets of points algebraically. Solution sets of algebraic equations will be interpreted or described geometrically.
5. The process of model building in applied mathematics will be developed at appropriate places in the seven - nine sequence.
6. Some relaxation in the present stress on structure may be noticeable, but structure is still definitely one of the unifying themes.
7. Topics related to computers and their use will be introduced, (e.g., flow-charting of mathematical algorithms.).
8. The concept of function will be considered early and will be used in many different types of mathematical content whenever possible.
9. The concept of a vector (displacement) appears in grades eight and nine.

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[21] Ibid, p. 387.

10. Probability appears in grades seven and eight with some statistics in grade nine.

11. Numeration systems will get little treatment because it will be assumed that this topic will have been covered in elementary school.

12. The set concept and set notation will be used whenever convenient, but it will not be overplayed.

13. Notation and terminology introduced in these grades will be com-patible with present day usage in mathematics.[22]

Subsequent to these meetings, materials were developed for grades seven and eight but, to the time of writing of the article quoted, these materials were not available for general publication. It was the aim of the committee to develop a junior high school programme which would lead to an up-to-date high school programme. However, their long-range plan was to develop a unified K - 12 programme. The committee was not strictly concerned with content; they also gave consideration to supplementary materials, instructional processes, student placement, and learning difficulties.

At about the same time (1964) the National Assessment of Educational Progress (NAEP) was begun in the United States. A survey was conducted in ten subject areas of the elementary and secondary school curriculum. One of the subject areas was mathematics and in the initial stages general objectives for mathematics education were determined. As reported by Foremen and Mehrens, they are as follows:

1. Recall and/or recognition of definitions, facts and symbols.
2. Perform mathematical manipulations.
3. Understand mathematical concepts and processes.
4. Solve mathematical problems - social, technical, and academic.

[22] Ibid, p. 388

5. Use mathematics and mathematical reasoning to analyze problem situations, define problems, formulate hypotheses, make decisions, and verify results.

6. Appreciate and use mathematics.[23]

The results of the survey are not completely known at this time. Preliminary reports indicate a decrease in basic computational proficiency. At present, objectives are being redefined and it is intended to reassess the population in 1975-1976.

Over the past half century one of the major forces at work in the development of mathematics curricula has been the National Council of Teachers of Mathematics. Cognizant of the growing concern about the state of mathematics, and recognizing its responsibility in the matter, NCTM appointed an ad hoc committee in March, 1970 "to draw up a list of basic mathematical competencies, skills, and attitudes essential for enlightened citizenship in contemporary society."[24] The Committee viewed mathematics in three basic aspects:

1. Mathematics as a tool for effective citizenship and personal living.

2. Mathematics as a tool for the functioning of the technological world.

3. Mathematics as a system in its own right.[25]

The first of these is for everyman, the average citizen; the second is for the scientist, the engineer, or technologist who uses mathematics as a tool; the third is for the professional mathematician. The committee sees a basic mathematics programme as one which will satisfy the needs of all three groups. For the first group the

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[23] Dale I. Foreman and William A. Mehrens, "National Assessment in Mathematics," Mathematics Teacher, LXIV (March, 1971), p. 141.

[24] E.L. Edwards, Eugene D. Nichols, Glyn H. Sharpe, "Mathematical Competencies and Skills Essential for Enlightened Citizenship," A Report of the Committee on Basic Mathematical Competencies and Skills, NCTM, Mathematics Teacher, LXV, (November, 1972) p. 671.

[25] Ibid, p. 672.

committee listed 48 items of content in 10 categories. The complete list is presented in Appendix C. The Following categories are considered and are presented here as an indication of content:

1. Numbers and numerals.
2. Operations and properties.
3. Mathematical sentences.
4. Geometry.
5. Measurement.
6. Relations and functions.
7. Probability and statistics.
8. Graphing.
9. Mathematical reasoning.
10. Business and consumer mathematics.[26]

It is important to note that the objectives referred to here and presented in detail in the Appendix are not to be considered as the minimum competencies required of all citizens since the committee cautions that many will not be capable of attaining all of them.

In addition to considering basic skills and competencies the committee investigated the broader aspects of mathematics education. For the mathematically inclined the committee proposed several principles concerning the structure of mathematics:

1. The deductive nature of mathematics.
2. Be able to carry through a consistent argument.
3. Be able to differentiate between a valid argument and an invalid one.
4. Be familiar with the basic properties of operations on numbers.
5. Be able to verify whether or not a given system possesses given properties.

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[26] Ibid, p. 673 - 674.

6. Be able to recognize that various concepts and operations are related to each other.

7. Be able to perceive patterns displayed in sequence.[27]

The committee emphasizes the fact that educators must be fully aware of the changing nature of society and that any programme in mathematics must be such that it makes provision for the changes that are so rapidly occurring. Specific reference is made to the ready access to cheap pocket calculators and the affect these will have on computational skills. Reference is also made to the constantly increasing bombardment of statistics, facts, and figures. In the conclusion of its report the committee states that "It is also the hope of the National Council of Teachers of Mathematics that professional groups concerned with mathematical education for all citizens will constantly strive to interpret the factors influencing change, seeing these in relation to their implications for the mathematics curriculum."[28]

One well-known mathematics educator who has taken a firm stand on the role of mathematics is Max S. Bell. In an address to the 50th Annual Meeting of the National Council of Teachers of Mathematics in 1973 he made several statements of significance to this discussion. The statements are background assumptions for a proposal for a mathematics programme for "everyman." They are:

1. A sound mathematical base well beyond mere calculation skills is useful or essential for more and more people in their working lives.

2. The school mathematics experience is a clear failure for at least a majority of people and perhaps a large majority.

3. Adequate terminal outcomes for everyman must also provide a durable basis for additional and more specialized learning of mathematics, statistics and so on.

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[27]Ibid, p. 675.

[28]Ibid, p. 677.

4. The reforms of the 1960's have been very useful in defining a more valid content for all school mathematics and in giving a much better mathematics experience than formerly to those who complete a college preparatory mathematics sequence. For everyman, however, they have had relatively little effect.

5. The key to an adequate mathematics experience for everyman lies in the years before high school, and probably in primary grades.

6. Specification of mathematics-learning outcomes by exhaustive listing of behaviorial objectives has so far been largely a dead end.

7. The widespread identification of mathematics as a natural focus for programming, computer-assisted instruction, accountability schemes, and various distortions of individualized learning has also proved to be a dead end. Again, this may be so, not because they are inherently bad ideas, but because they have concentrated mainly on skill and drill methods and outcomes.

8. The widespread availability of cheap electronic calculators will have a profound effect and must move us very soon to re-evaluate many of our current practices in the teaching of school mathematics.[29]

While Bell does not explicitly say that computational skills should be down-graded, he does say that the emphasis should be on "what addition means, where it is appropriately used, judging reasonableness of answers, (if only to detect mistakes in button punching), interpretation of results, and much else."[30] In keeping with this philosophy, Bell gives a detailed list of what is "really" wanted as a minimum residue for everyman from the school mathematics experience. The complete list is presented in Appendix C. The following are the broad concepts and skills considered:

1. The main uses of numbers (without calculations).

2. Efficient and informed use of computational algorithms.

3. Relations such as equal, equivalent, less or greater, congruent, similar, parallel, perpendicular, subset, etc.

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[29] Max S. Bell, "What Does Everyman Really Need from School Mathematics," Mathematics Teacher, LXVII (March, 1974), p. 196.

[30] Ibid, p. 198.

4. Fundamental measurement concepts.

5. Confident, ready, and informed use of estimates and approximations.

6. Links between "the world of mathematics" and "the world of reality".

7. Uses of variables.

8. Correspondences, mappings, functions, transformations.

9. Basic logic.

10. "Chance", fundamental probability ideas, descriptive statistics.

11. Geometric relations in the plane and space.

12. Interpretation of informational graphs.

13. Computer uses (e.g., flow charting).[31]

Bell does not consider his list as being absolute or exhaustive and admits that it is a personal view not based on formal research. He does, however, emphasize the importance of such a list as "a guide for teachers at all levels, as a guide to better and more imaginative evaluation, to formulate the content of teacher-training programmes and to give teachers perspective on what needs doing in school mathematics learning experiences."[32]

Up to this point consideration has been given to mathematics for the average student or for the majority of students, with some reference to the more talented students and those of lesser ability. It is reasonable to assume that students of high capability can attain an acceptable level of mastery of basic skills and concepts, but the slower group tends to have great difficulty. There is a great deal of controversy concerning what mathematics this group should study - should it be a programme

[31]Ibid, p. 199

[32]Ibid, p. 201

of computational arithmetic; should it be built around the mathematics these students will need as citizens, workers, and consumers; or should it be the same mathematics that all other students will study but presented in a different way and at a different pace? The question as to what should be taught to the slower group of students was posed to 200 leading mathematics educators in the United States in 1968. They were asked to rate 47 topics with respect to their acceptance in a mathematics programme for slow learners. The complete list of topics is presented in Appendix C. As an indication of content, the following topic groupings were considered:

1. Whole and rational numbers.
2. Real numbers.
3. Number theory.
4. Intuitive geometry.
5. Measurement.
6. Logic.
7. Managing income.[33]

An examination of the results of the survey shows an emphasis on topics oriented towards computation. Acceptance of such modern topics as permutations, topology, probability, linear programming, and computer mathematics was denied or, at least, undecided. The group surveyed was also undecided about logic and proof, and consumer mathematics. There was no doubt about the acceptance of such topics as basic operations and properties, number theory, intuitive geometry, and measurement. While the results of the study are not to be taken too much for granted, they do give an idea of the opinions of a significant group of educators.

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[33]Sol Weiss, "What Mathematics Shall We Teach the Slow Learner?" Mathematics Teacher, LXII (November, 1969), p. 572.

A similar study was conducted in 1972. In the study 260 teachers in a selected number of vocational schools in the United States were asked to rank 66 basic skills essential for success in different vocational specialties. The highest ranked items were those associated with fundamental operations. The first 10 items in order of importance were:

1. Addition of whole numbers.
2. Subtraction of whole numbers.
3. Multiplication of whole numbers.
4. Division of whole numbers.
5. Reading and writing decimals.
6. Addition of fractions.
7. Subtraction of fractions.
8. Rounding off decimals.
9. Addition of decimals.
10. Subtraction of decimals.[34]

As might be expected from a survey of a group of this nature, the emphasis would be on the very basic elements. They would tend to be more concerned about technique since, as Fehr would say, they are involved in training rather than educating.

The foregoing discussion indicates that there are two schools of thought with respect to mathematics in the schools, particularly at the junior high school level. One group advocates a relaxation of rigor in mathematics with increased emphasis on establishing a broader base for all students. This group is more concerned with the fundamentals

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[34]Albert P. Shulte, "Teacher Perception of Basic Mathematical Skills Needed in Secondary Vocational Education," Mathematics Teacher, LXVI, (January, 1973), p. 63.

of mathematics, particularly as they relate to applications to and interpretation of the social, technical, and economic aspects of society. The other group is more concerned with mathematics as an entity in itself, with emphasis on mathematical structures and those other aspects of mathematics which appeal to mathematicians. They would include in a mathematics programme in junior high school and senior high school such topics as are normally reserved for post-secondary institutions.

Even though there is disagreement as to what should be taught and why, there are some common elements which all agree, with varying degrees of emphasis, should be included in a mathematics programme in junior and senior high school. In general, a good mathematics programme should provide for:

1. Reasonable skill in computing with whole numbers, integers, common fractions, and decimals. Though, in some instances, there would not be any emphasis on the teaching of these skills, still it would be considered as essential that students be proficient in this aspect of mathematics.

2. An understanding and appreciation of the properties of operations on numbers with, on the one hand, an emphasis on their use in computational algorithms, and, on the other hand, an emphasis on the application of the fundamental properties in mathematical structures and the deductive process.

3. An understanding of the nature of measurement; a familiarity with the basic units of measurement and their sub-units; and an ability to estimate and make reasonably accurate measurements using standard measuring devices. (Including measurement of plane angles.)

4. The ability to distinguish between valid and invalid arguments, between important and unimportant information; to construct a valid argument. (There are some who would include the construction of a logical proof.)

5. The ability to estimate with a reasonable degree of accuracy and to make reasonable predictions.

6. An understanding of the fundamental concepts and relationships in two and three dimensional geometry, including the ability to identify and reproduce basic geometric figures. (Some would include the calculation of perimeter, area, and volume of these figures.)

7. The ability to perceive relationships among fundamental mathematical operations and concepts; to recognize similarities and differences.

8. The ability to record information in graphical form and to interpret information recorded by statistical means.

9. The ability to apply mathematics in the solution of problems in consumer-related situations.

10. An appreciation of the role and significance of mathematics in our society.

Though the above list is general and incomplete, it is an indication of the trend in mathematics education today. Probably the most notable feature of the latest programmes in mathematics is the increasing emphasis on providing for the broadest possible programme for the largest number of people. Now the emphasis has shifted from the subject to the learner.

CHAPTER III

DESIGN OF THE PROJECT

As stated earlier, the purpose of this project is to construct a list of basic content objectives for junior high school mathematics and to present the list to a selected group of mathematics teachers for their evaluation. The first step in the preparation of the list of objectives involved a survey of available literature on the objectives of mathematics education. This survey is reported on in the previous chapter. The survey is not strictly confined to junior high school; rather, the emphasis is on terminal outcomes of school mathematics programmes with specific reference to junior high school when possible and appropriate. The second step involved a survey of available commercially produced programmes in junior high school mathematics. The programmes surveyed are listed in Appendix D. The third step involved the compilation of a list of 156 objectives. This list was revised and reduced to a more workable list of 115 objectives. The objectives were listed in developmental order in 9 categories as follows:

1. Structure.
2. Numbers and numeration.
3. Computation.
4. Geometry.
5. Algebra.
6. Measurement.
7. Functions and graphs.
8. Logic and proof.

9. Miscellaneous topics:

   a) set theory.

   b) probability and statistics.

   c) consumer mathematics.

This list of objectives provided the basis for the questionnaire to junior high school teachers.

POPULATION AND SAMPLE

The sample of junior high school teachers was selected from schools in the Avalon North Integrated School District and the Roman Catholic School District for Conception Bay North. The area surveyed was confined to the region from Brigus to Victoria in Conception Bay and included a total of 11 schools. The sample was selected on recommendation of the supervisory staffs of the above districts, considering the academic qualifications and, particularly, the teaching experience of the teachers. An effort was made to ensure that the respondents had a basic familiarity with junior high school mathematics programmes. So that teachers could see the totality of the current programme the survey was conducted near the end of the 1974-75 school year. Further details on the respondents are given in Chapter IV.

PREPARATION OF THE INSTRUMENT

The questionnaire was prepared in three sections, as follows:

A. General information on respondents.

B. Rating of broad content areas. Here respondents were asked to rate the eleven categories with respect to degree of importance in the junior high school programme.

C. Rating of content objectives. This was the main part of the questionnaire. The respondents were asked to rate each item with regard to relative degree of importance considering,

1. its importance for the majority of students.

2. its importance as a prerequisite for future study in mathematics.

3. its importance with respect to its vocational and social application.

4. the relative amount of emphasis that should be placed on it.

The items were arbitrarily listed in random order on the questionnaire so as not to create concept clusters which might influence ratings on topics in relation to topics of the same concept orientation. Items were not arranged in categories. An auxiliary function of the questionnaire was to ask teachers to rate each item on the basis of degree of difficulty that they perceived students to have with that item. As was previously stated, this proved to be an impossible task and it is not reported on herein.

The rating scale was a simple five-point scale constructed as follows:

NOT IMPORTANT				VERY IMPORTANT
1	2	3	4	5

Respondents were asked to rate every item whether it was included in current programmes or not. A rating on an item not included in current programmes would be considered as an indication of whether it should or should not be included. Respondents were asked to give supplementary information with respect to the textbook being used. They were asked to rate the textbook with regard to degree of difficulty and suitability for the average student. They were also asked to indicate the major strengths and weaknesses of the textbook being used.

ADMINISTRATION OF THE INSTRUMENT

The questionnaire, with covering letter, was delivered to each of the respondents and a brief discussion was held with each concerning the nature and purpose of the project. Respondents were asked to complete the questionnaire and return in sealed envelope within a period of two weeks. Respondents were specifically asked not to sign the questionnaire or leave any identifying marks. In general, the response to the survey was good. All except two members of the sample group returned the questionnaire within the expected time period. Questionnaires were never received from these two as they left the area at the end of the school year.

ANALYSIS OF THE DATA

Upon receipt of the questionnaires responses on each item were combined in the form of a frequency distribution on the five-point scale. A rank ordering of items was prepared on the basis of the mean rating on each item. The mean rating was determined by assigning a weight to each point on the scale, determining the aggregate weighted rating and dividing this by the number of responses on each item. The weighting was assigned on the following basis:

1. A weight of 1 for a rating of 1.
2. A weight of 2 for a rating of 2.

etc.

Assuming open intervals on the scale, ratings were computed to two decimal places. This was necessary in order to provide a reasonable degree of discrimination on such a large number of items.

On the basis of mean rating, the objectives were ranked in order of importance. The table showing the ranking of objectives is shown in Appendix B. To facilitate analysis of date it was necessary to consolidate the list of objectives. This was done by combining closely related objectives. As an example, items 75 and 80 were combined as one item since both were associated with making accurate measurements. Likewise, items 18 and 19, dealing with bases other than ten, were combined as one item. Many items could not be combined in this way since they were independent of any others.

The mean rating was chosen as the chief statistical measure in this study because of the nature of the project. There is no attempt to compare with other studies or with other factors. The study is in the form of a survey and the mean is often used as a method to establish ranking of items in surveys of this nature. Such a measure was used by Sol Weiss (1969) in his survey of 200 leading mathematics educators in the United States, reported on in this paper.

The writer realizes the inadequacies and limitations of such an analysis. It is important to realize that results and conclusions of the survey are not to be taken literally or interpreted in absolute terms but should be considered merely as indications of the perceptions of a restricted number of teachers of mathematics.

CHAPTER IV

RESULTS OF THE PROJECT

INFORMATION ON THE SAMPLE

The questionnaire, containing 115 objectives, was given to 22 grade seven and eight mathematics teachers in 11 schools. Respondents were asked to supply information concerning years of experience and academic and professional training. The basic information is given in Table 1 below. The average number of years of teaching experience was 13.2 years with a minimum of 1 year experience and a maximum of 33 years experience. The average number of years of experience teaching grade seven and/or eight mathematics was 11.6 years with a minimum of 1 year experience and a maximum of 33 years experience. However, these figures are slightly misleading because of the one person with 33 years experience. In fact, the average teacher had fewer number of years experience in both instances. The average number of years of professional (university) training was 5.3 years with 3 of the respondents having completed graduate programmes. Though the majority of the respondents had done fewer than 5 mathematics courses, the average number of mathematics courses completed was 5.8. This is because of the fact that one of the respondents had completed 20 courses and two others had completed in excess of 12 courses. It is more reasonable to say that the average respondent had done 3 or 4 mathematics courses. The average number of methods courses done was 1.9 but this again is deceiving since 10 of the respondents had done less than 2 courses. Two respondents had never done a methods course.

The information presented here would tend to indicate that the

TABLE 1

EXPERIENCE AND PROFESSIONAL TRAINING
OF RESPONDENTS

	Number of Years of Teaching Experience				
	1 - 3	4 - 6	7 - 9	10 - 12	13 or more
Number of Respondents	1	2	7	4	6

	Number of Years Teaching Grade Seven and/or Eight Mathematics				
	1 - 3	4 - 6	7 - 9	10 - 12	13 or more
Number of Respondents	3	5	2	6	4

	Number of Years of Professional Training				
	1 - 3	4	5	6	7
Number of Respondents	0	5	8	4	3

	Number of Mathematics Courses Completed[a]				
	1 - 3	4 - 6	7 - 9	10 - 12	13 or more
Number of Respondents	10	3	2	2	3

	Number of Methods Courses Completed[b]				
	None	1	2	3	4
Number of Respondents	2	8	6	2	2

[a]Semester courses

[b]Semester courses in the theory and practices of teaching mathematics.

sample group is comprised of a reasonably competent group of teachers if years of experience, and academic and professional training are any indication. Certainly, it would be reasonable to assume that these teachers should have an awareness of mathematics programmes in grades seven and eight and, also, some perception of what is or is not important for the average student at that level.

RESULTS OF THE SURVEY

The detailed results of the survey are considered to be too lengthy to present in context and are presented for reference in Appendix B. Initially, respondents were asked to rate the 11 basic content areas of junior high school mathematics programmes with respect to order of importance. The results of their ratings are presented in TABLE 2.

TABLE 2

BANK ORDERING OF BASIC CONTENT AREAS OF JUNIOR HIGH SCHOOL MATHEMATICS

Rank	Item	Mean Rating
1	Computation	4.60
2	Numbers and Numeration	4.33
3	Measurement	4.07
4	Consumer Mathematics	3.87
5	Geometry	3.67
6	The Structure of Mathematics	3.60
7	Algebra	3.33
8	Logic and Proof	2.53
8	Set Theory and Notation	2.53
10	Probability and Statistics	2.33
10	Functions and Graphs	2.33

It is not particularly surprising that they place the greatest amount of emphasis on computation. In spite of the fact that current mathematics programmes are not supposed to emphasize computation, it appears that the majority of teachers do so. One very noticable point about the ratings is the difference between the ratings for the seventh and eighth items. Here there is a greater difference than between any other two adjacent items. This may result from the fact that the last four items are not greatly emphasized in current programmes. As an example, the unit on probability and statistics, although contained in the textbooks, is not covered under the present syllabus. The five highest rated items are normally considered as traditional topics in mathematics for junior high school. The remaining items refer to the more formal topics in mathematics which are recent additions to mathematics curriculum. The exceptions are items 7 and 8, which are also traditional topics but were usually reserved for the later portion of junior high school or for high school mathematics programmes. One surprising result is the relatively high rating given to a somewhat controversial topic - the structure of mathematics.

A more detailed account of the results is given in Table 3. This table contains 55 objectives consolidated from the list of 115 objectives contained in Appendix B. The objectives are ranked in order of importance as perceived by the teachers surveyed based on the mean of the ratings given by these teachers. In some instances several items have the same rating, hence, the same rank. This results from the large number of items in the questionnaire.

A quick perusal of Table 3 indicates a concentration on lower level objectives. The highest rated items are on the comprehension or application levels. Higher level objectives are rated lowest. This would tend to indicate that teachers emphasize the fundamental skills or the "doing" aspects of mathematics.

In Section B of the questionnaire teachers were asked to rate 11 categories of junior high school mathematics. As reported in Table 2, the highest rated item was computation. The ratings on the specific items in Section C, as reported in Table 3, tends to corroborate this fact. Of the 10 highest ranked items in Table 3, 7 are specifically related to computation. The fundamental operations on numbers are given particular emphasis, with operations on whole numbers, operations on integers, operations on rationals (decimal form), and operations on rationals (fractional form) being rated 1st, 4th, 4th, and 8th respectively. Other computational oriented objectives which received a high rating are: computation involving percent (2nd); finding averages (3rd); computing perimeter, area, and volume (8th); and square roots (21st). Other objectives which involve the application of fundamental operations were also highly rated; these include computing discounts and net prices on purchases (13th), and problems related to banking (24th). The lowest rated item of a computational nature involves use of exponential and scientific notation (39th).

The second highest rated category, as reported in Table 2, is numbers and numeration. The ratings given on specific items do not support this, however. Several items are given relatively high ratings with the highest rated item being place value (4th). Other items which received high ratings are prime factorization (14th), classification of rationals (21st), and rounding off numbers (28th). The remaining items in this category are given relatively low ratings, for example, prime and composite numbers (44th), density and completeness properties (49th), other bases (54th), and numeration systems (55th). In general

TABLE 3

CONSOLIDATED LIST OF BASIC CONTENT OBJECTIVES FOR JUNIOR HIGH SCHOOL MATHEMATICS RANKED ACCORDING TO MEAN RATING

Rank	Description	Mean Rating
1	Fundamental operations on whole numbers	4.93
2	Computation involving percent	4.67
3	Average of two or more numbers	4.60
4	Fundamental operations on integers	4.53
4	Fundamental operations involving decimals	4.53
4	Place value of a given digit	4.53
4	Perform accurate measurements	4.53
8	Fundamental operations on rational numbers expressed in the form of common fractions	4.40
8	Equivalent fractions	4.40
8	Computation of perimeter, area, and volume given the formulas	4.40
11	Expressing common fractions in decimal and percentage form and vice versa	4.37
12	Perform basic geometric constructions	4.33
13	Compute discounts and net price on purchases	4.20
14	Understanding and application of prime factorization	4.00
14	Application of the Pythagorean Principle	4.00
16	Understanding and application of the fundamental properties of operations on numbers	3.93
17	Familiarity with basic geometric relationships	3.87
18	Identification and definition of basic geometric figures	3.78

TABLE 3 - Continued

Rank	Description	Mean Rating
19	Solution of simple equations and inequalities	3.73
19	Interpretation of graphs	3.73
21	Computation of square roots	3.67
21	Classification of rational numbers	3.67
23	Using scales and indirect measurement	3.63
24	Solving problems related to banking	3.60
24	Conversion from Imperial to Metric System of Measurement and vice versa	3.60
26	Conversion from one unit of measurement to another, within each system	3.47
27	Explain the inverse relationship between operations and between concepts	3.40
28	Rounding off numbers to specified number of digits	3.33
28	Solving problems related to insurance	3.33
30	Determine the absolute value of a given rational number	3.27
31	Explain and illustrate the concept of similarity of geometric figures	3.23
31	Use inductive and deductive reasoning in problem situations	3.23
33	Graph a relation or function using ordered pairs	3.20
34	Maintain a budget and bank account record	3.20
35	Represent data in statistical form	3.17
36	Solving problems involving taxes	3.13
36	Explain and illustrate the concept of congruency of geometric figures	3.13

TABLE 3 - Continued

Rank	Description	Mean Rating
38	Solve mathematical problems by algebraic means	3.10
39	Use exponential and scientific notation	3.07
39	Interpret statistical graphs	3.07
39	Determine the degree of accuracy or precision of a given measurement	3.07
42	Estimate measurements	3.03
42	State and apply the properties of equality and inequality	3.03
44	Define and identify prime and composite numbers	2.93
45	Perform fundamental operations on polynomials	2.83
46	State basic relationships and perform operations on sets	2.80
47	Make accurate predictions and perform simple experiments in probability	2.73
48	Factor polynomials of second degree	2.67
49	Explain the Density and Completeness properties	2.63
50	Evaluate functions for given elements of the domain	2.53
51	Test validity of statements using truth tables	2.50
52	List the basic properties of a number system	2.47
53	Construct simple algebraic and geometric proofs	2.40
54	Convert to and compute in bases other than base ten	2.03
55	Write numerals in numeration systems other than Hindu-Arabic	1.40

this category was not given the prominence it received in Section B, as indicated in Table 2. In fact, the overall rating based on mean ratings as reported in Table 4 is significantly lower. Based on combined mean ratings, numbers and numeration is ranked in 6th position as compared with 2nd position in table 2.

TABLE 4

RANK ORDERING OF CONTENT AREAS OF JUNIOR HIGH SCHOOL MATHEMATICS BASED ON COMBINED MEAN RATINGS

Content Area	Mean Rating	Rank
Computation	4.05	1
Measurement	3.98	2
Geometry	3.76	3
Consumer Mathematics	3.81	4
Structure	3.34	5
Numbers and Numeration	3.10	6
Algebra	3.03	7
Logic and Proof	3.00	8
Probability and Statistics	2.98	9
Functions and Graphs	2.89	10
Set Theory	2.80	11

Measurement is rated 3rd in table 2. The ratings on specific items, as shown in table 3, tends to corroborate this rating. Individual items were rated 4th, 23rd, 24th, 26th and 42nd with the emphasis on performing accurate measurements. Surprisingly, the lowest rated item in this category was estimating measurements.

Consumer related mathematics is rated 4th in Table 2. In the individual item ratings objectives related to consumer mathematics were ranked 13th, 24th, 28th, 34th, and 36th. Overall, consumer mathematics was ranked 4th in importance, maintaining the same position as reported in Table 2.

The 5th highest ranked category in Table 2 is Geometry. In the specific item ratings geometry was ranked 3rd overall. Specific items were rated as follows: computation of perimeter, area, and volume (8th); geometric constructions (12th); Pythagorean Principle (14th); basic relationships (17th); basic figures (18th); similarity (31st); and congruence (36th).

The structure of mathematics is rated 6th in Table 2. Based on specific item ratings, structure was ranked 5th. The highest ranked item in this category was fundamental properties of operations (16th), followed by inverse relationships (27th). Other items in this category were ranked relatively low.

The remaining categories are ranked lower with specific items being rated relatively low. Algebra is ranked 7th with the highest rated item being solving simple equations and inequalities (19th), and the lowest rated item being operations on polynomials (48th). Logic and proof is ranked 8th overall with the highest rated item being formal proofs (53rd). Specific items in the three remaining categories received very low ratings.

The results of this study tend to indicate a fair degree of consistency on the part of teachers in rating important content areas in junior high school mathematics. The emphasis is on computation, measurement, geometry, and the other more traditional and practical aspects of mathematics such as algebra and consumer mathematics. There is less emphasis on the more formal aspects of mathematics such as set theory, functions, logic and probability and statistics.

CHAPTER V

SUMMARY AND CONCLUSIONS

This study has attempted to determine a comprehensive list of basic content objectives for junior high school mathematics and to determine the relative importance of these by surveying a selected group of grade seven and eight mathematics teachers. Initially, a list of objectives was constructed from a detailed survey of mathematics programmes and curriculum outlines. A questionnaire was constructed from these objectives and presented to 22 teachers who were asked to rate each objective with respect to degree of importance in a junior high school mathematics programme. Of the 22 teachers surveyed 20 completed and returned the questionnaire. Analysis of the results was based on the mean rating of specific objectives and, on the basis of these ratings, items were ranked in order of importance as perceived by the teachers surveyed. Because of the number of items, it was necessary to consolidate the list of objectives to a more workable list. The results presented in the preceeding chapter are based on this consolidated list.

As was earlier indicated, it was not the intention of this study to present absolute judgements but rather to present an evaluation of teachers' perceptions with respect to the objectives listed in Appendix B. Consequently, any results are not to be taken literally but as indications of what might be the case. The following appear to be the major considerations as a result of this study:

(1) Teachers tended to emphasize the computational aspects of mathematics with particular emphasis on the fundamental operations on number.

(2) Measurement was given a prominent position with emphasis on the ability to perform direct measurement rather than on estimation or indirect measurement. This is significant in light of the recent emphasis that measurement is receiving in our schools.

(3) Geometry was also rated very highly. However, the emphasis is again on fundamental concepts with such topics as congruence being rated relatively low and formal proof being virtually rejected for inclusion in a junior high school mathematics programme.

(4) Consumer related mathematics is given prominence in spite of the fact that it has been given very little attention in junior high school mathematics in recent years.

(5) One surprising result is the relatively high rating given to structural aspects of mathematics. However, the aspects of structure which were rated highest were those related to the computational algorithms.

(6) Algebra was not rated highly. Probably this is a reflection of the de-emphasis of algebra at this level in the last two or three years.

(7) Functions and graphs was rated low, but some aspects such as reading and constructing graphs were considered as being important.

(8) Logic and proof was rated relatively low but inductive and deductive reasoning were given relatively high ratings. Formal proof was one of the lowest rated of all items on the questionnaire.

(9) Probability and statistics were not considered as being im-

portant. This is probably because of the fact that most teachers have little experience with these topics.

(10) Some of the "new math" topics were virtually rejected. Topics such as sets and set notation, functions, truth tables, number systems, other bases, and numeration systems were the lowest rated of all items.

In general, teachers tended to emphasize the lower level, "doing" type objectives. There tended to be more emphasis on traditional topics than on other topics. Topics which receive a relatively high degree of attention in present texts tended to be rated higher than those which are not covered in present programmes. It might be fair to say that teachers tended to emphasize arithmetic rather than mathematics.

DISCUSSION

The most obvious result of this study is the concentration of attention on the computational aspects of mathematics. This may be the result of a reaction to the more formal mathematics programmes of recent years. Many teachers have expressed dissatisfaction with some of the topics in "new" mathematics and, consequently, have tended to de-emphasize, if not completely omit, any treatment of these topics. Such reaction could be a result of teachers not having an adequate understanding of these topics or could be because of their misinterpretation of the purpose for the inclusion of these topics in mathematics programmes in the first place. Another possible reason for the emphasis on computational oriented topics may be the previous experience of teachers. Most teachers tend to concentrate on a textbook and, consequently, do not have much exposure to mathematics beyond the textbook they are teaching. As a result, their perceptions of what they <u>should</u> teach are greatly influenced by what they <u>are</u> teaching. In short, they tend to emphasize the familiar.

It is difficult to accept the fact that teachers believe that computation should be given the emphasis it has been given here. Certainly, fundamental concepts and skills are important. It is desirable that every student achieve a reasonable degree of proficiency with regard to basic skills and that they have a reasonable understanding of basic concepts, but it is questionable whether they should receive the degree of emphasis in junior high school as is indicated herein. Junior high school should provide more opportunities for the application of fundamental concepts and skills learned in previous grades and also provide opportunities for the development of skills and mastery of concepts necessary in high school and beyond. Any junior high school mathematics programme should have a fair degree of emphasis on pre-algebra concepts, as algebra is the major component of mathematics in high school for the majority of students and is the foundation to most branches of higher mathematics. One of the most serious problems in mathematics today is the gap between grade eight and grade nine. The majority of students entering grade nine are not adequately prepared for a formal algebra course and this may be the result of a lack of exposure to algebra in previous grades.

The emphasis on measurement is very timely. Currently, a great deal of attention is being given to measurement in the schools. At times it appears as if measurement never existed before. In the past measurement was indirect, and the student was a passive participant. Now, with the introduction of the Metric System of Measurement, measurement has been given a prominence it had never received before. Significantly, it is not the product but the process which is causing the interest. Hopefully, it will have a therapeutic affect and teachers will adopt the process in other areas of mathematics.

The emphasis on consumer oriented mathematics is in keeping with present emphasis on consumerism, particularly in the media. The crucial question is whether such mathematics is really mathematics or just "psuedo-economics". In any event, much of the difficulty experienced is with the non-mathematical concepts. From a mathematical point of view, its true value may lie in the fact that such mathematics provides an opportunity for the application of basic skills and for the re-inforcement of these skills. In spite of the emphasis on consumer mathematics, probability and statistics are not given the prominence they deserve. In this age of the mass media, when people are being bombarded with information, every citizen should have a minimum understanding of the basis for the compilation of statistics and the significance of statistics as it affects them. Interpretation of information has to be a prime objective.

One factor that causes some concern is the lack of emphasis on logic and proof. It would not be wise to recommend formal proof for every student in junior high school but, certainly, every student should be exposed to the nature of proof in simple inductive and deductive processes. The very nature of the learning process itself requires that attention be given to inductive and deductive reasoning. It is quite possible that many of the problems experienced by students in mathematics may be a result of too little attention being paid to these processes. There may be too much showing and telling, and not enough seeking and enquiring.

It is the opinion of the writer that a mathematics programme in junior high school should be an integrated, non-specialized programme.

Mathematics should go beyond the realms of arithmetic but must also recognize the need for proficiency in basic skills and concepts, and for the maintenance and re-inforcement of these. Students should be exposed to the fundamental concepts and skills related to geometry and algebra, and to those social and economic oriented topics which rightly belong in a mathematics programme. But mathematics should be more than that; it should give attention to those skills which enable a student to work independently, relying on his own ability and experiences. While it is difficult to make specific recommendations as a consequence of this study, the following opinions are offered for consideration:

(1) It is the opinion of the writer that we tend to treat school as being segmented or compartmentalized into several distinct and separate groupings of grades. For example, we have K - 3, 4 - 6, 7 - 8, and 9 - 11 treated as "units" in our school systems. Consequently, we find separations between these groupings from an administrative and curriculum point of view. Usually there is little communication between teachers at different levels and, consequently, lack of continuity. For this reason, teachers at a particular level, or in a particular grade, are not aware of what goes on at a lower or higher level. One of the most serious deficiencies is the lack of awareness of curriculum content at other levels. Therefore, there should, and must, be some effort to make teachers more aware of the total programme in the schools. This could be accomplished by establishing a policy of periodic rotation of teachers from grade to grade. Many schools have independently established this policy and, if teachers comments are of any significance, it appears

to be worth the effort and inconvenience. Another way that this problem could be alleviated is by a concentrated inservice programme wherein the emphasis is on the total programme rather than on specifics at individual grade levels. This approach has been taken in The Avalon North Integrated School District in Special Education. Teacher committees have been formed from preliminary meetings of all Special Education teachers and these committees have been given the task of determining "terminal outcomes" of a mathematics programme for students in Special Education. After proper deliberations, a list of objectives will be presented to all teachers for consideration and evaluation. After this is done, topics will be allocated to the various levels as deemed appropriate. Of course, consideration will be given to more than content but the main aim will be to familiarize all teachers with the total programme.

(2) The junior high school mathematics programme should be an integrated, non-specialized programme from grade seven to grade nine with specialization beginning in grade ten. This programme could be considered as terminal for many students, with mathematics not considered as a required subject in grade ten and grade eleven. The writer feels very strongly that mathematics should be an elective in these grades and that entrance requirements of technical and vocational schools should be adjusted to consider the real abilities of applicants as required by their chosen field.

(3) Serious consideration should be given to computation and its place in the junior high school mathematics programme. What

affect is the year-by-year repetition of "more of the same" mathematics having on students? What significance does the cheap electronic calculator have for mathematics in junior high school? Should we not be more concerned with the real problems that students face? It is the writer's opinion that if a student is not reasonably proficient in fundamentals by junior high school, further emphasis is not going to serve much purpose except to "turn off" already frustrated students.

(4) More attention should be given to consumer oriented mathematics (but not too much) and this should be considered as an avenue for maintaining and reinforcing skills. Also, probability and statistics should be treated in junior high school, with particular emphasis on interpretation of statistical information.

The foregoing is a presentation of the perceptions of a selected group of grade seven and eight mathematics teachers. While it is rather informal, it does give an indication of where they place the emphasis in mathematics. This information is of particular interest to the writer as a guide for inservice programmes and curriculum development. It is difficult to make absolute judgements or draw precise conclusions, but the trend is obvious. In spite of much talk about "new" mathematics and in spite of intensive pre-service and inservice programmes, the emphasis still lies where it always has been – on the fundamentals, the "practical" aspects of mathematics. But this is just one group of teachers whose orientation lies in that direction. It would be interesting to assess the perceptions of teachers in senior high school with respect to these objectives and compare the results of both groups. I would expect that there would be significant differences.

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APPENDIX - A

Copy of Questionnaire With Accompanying Letter

Bay Roberts, C.B.,
June 4th, 1975.

Dear Teacher:

I am presently involved in a study related to mathematics at the Seventh and Eighth Grade levels. This study is concerned with the objectives of mathematics at these levels and is being conducted as partial fulfillment of the requirements for the Master of Education programme at Memorial University of Newfoundland. From preliminary investigations I have compiled a list of 115 specific objectives. These objectives are concerned with basic skills and concepts normally considered at the grade levels referred to above. The objectives are not based on any specific textbook series, but represent a broad spectrum of topics from a variety of programmes.

I would appreciate your examining the enclosed questionnaire and completing it as per instructions thereon. You will notice that you are being asked to rate the objectives in two respects, namely:

A. With respect to the degree of importance of the objective.
B. With respect to the degree of difficulty that you perceive students to have in attaining this objective.

Concerning the first rating, you are requested to give a rating for the objective even though it may not be directly related to content of current mathematics programmes at the above mentioned levels. Concerning the second rating, you are requested to give a rating where the objective is related to content normally completed by students under current programmes. Provision is made for a non-applicable rating where this is not the case.

Anticipating your co-operation, I sincerely appreciate your assistance in this study and hope to contact you personally in the near future.

Yours sincerely,

John C. Chipman

QUESTIONNAIRE

This questionnaire is divided into THREE sections. Please complete each section to the best of your ability.

SECTION "A": GENERAL INFORMATION

1. In what grade(s) are you now teaching mathematics? ____________.
2. How many grades are there in the classroom in which you are now teaching Grade Seven and/or Eight mathematics? ____________.
3. Number of years teaching mathematics in Grades Seven and/or Eight. (Include this current year.) ____________.
4. Total number of years teaching experience. ____________.
5. Number of University courses completed in mathematics. ____________.
6. Number of years of academic training. ____________.
7. Number of University courses completed in mathematics education. (Methods courses.) ____________.
8. Teaching certificate held. ____________.
9. Degree(s) held. ____________.

SECTION "B":

In this section you are requested to rate the following broad content areas of mathematics with regard to the degree of importance that you perceive it to have at the Grade Seven and Eight levels. Should the content not be included in current programs, your rating will indicate your opinion as to whether it should be included.

You are required to rate each item on a "five point" scale as follows:

NOT IMPORTANT				VERY IMPORTANT
1	2	3	4	5

The rating that you give to each individual item will indicate the relative degree of importance that you attach to that item. Please give a rating for each item by circling your choice.

CATEGORY		RATING				
I.	The structure of mathematics.	1	2	3	4	5
II.	Numbers and Numeration.	1	2	3	4	5
III.	Computation.	1	2	3	4	5
IV.	Geometry.	1	2	3	4	5
V.	Algebra	1	2	3	4	5
VI.	Measurement.	1	2	3	4	5
VII.	Functions and graphs.	1	2	3	4	5
VIII.	Logic and proof.	1	2	3	4	5
IX.	Set theory and notation.	1	2	3	4	5
X.	Probability and statistics	1	2	3	4	5
XI.	Consumer mathematics.	1	2	3	4	5

SECTION "C": This section is the most important of this questionnaire. It contains a list of 115 objectives related to basic skills and concepts normally considered in Grade Seven and Eight mathematics programs. In determining your rating for each individual item please consider the following guidelines:

1. Your rating should reflect your perception of importance and difficulty for the majority of students.
2. Your rating should reflect your perception of the relative degree of importance of each objective as a prerequisite for future study in mathematics.
3. Your rating should reflect your perception of the relative importance of each objective with respect to its vocational and social application.
4. Your rating should reflect your perception of the relative amount of emphasis that should be placed on each objective in classroom instruction.

Please consider these guidelines when determining your rating. You are asked to rate each item in two aspects, namely:

A. Degree of Importance.

You are required to rate each Item on a "five point scale" as follows:

NOT IMPORTANT				VERY IMPORTANT
1	2	3	4	5

B. Degree of difficulty.

The rating should be given on a "five point scale" with provision for a "non-applicable" rating in cases where the objective is not covered in content in current programs.

The rating scale is as follows:

NOT APPLICABLE	NOT DIFFICULT				VERY DIFFICULT
0	1	2	3	4	5

ITEM		RATINGS					
1. Add, subtract, multiply, and divide whole numbers.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
2. Determine the place value of a given digit in a given numeral.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
3. Identify the fundamental properties of addition and multiplication. (e.g. commutative property, inverses.)	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
4. Classify a number as being whole, natural, integral, rational, irrational, or real.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
5. Add, subtract, multiply, and divide integers.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
6. Illustrate the fundamental properties of addition and multiplication.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
7. Use the fundamental properties of addition and multiplication with respect to these operations in computation.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
8. Round off numbers to the nearest ten, hundred, etc.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
9. Recognize relationships among lines. (e.g. parallel, perpendicular, etc.)	A.		1	2	3	4	5
	B.	0	1	2	3	4	5

ITEM		RATINGS					
10. Define and identify basic geometric figures. (e.g. ray, segment, angle, etc.)	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
11. Add, subtract, multiply, and divide rational numbers expressed in fractional form.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
12. Add, subtract, multiply, and divide rational numbers expressed in decimal form.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
13. Explain the inverse relationship between addition and subtraction, and between multiplication and division.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
14. Express a given number to a specified number of significant digits.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
15. Find the square root of a given positive rational number.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
16. Classify angles on the basis of degree measure. (e.g. acute, obtuse, etc.)	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
17. Define and illustrate relationships between angles. (e.g. adjacent, supplementary, etc.)	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
18. Explain the "division by zero" rule. (i.e. explain why division by zero is undefined.)	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
19. Distinguish between prime and composite numbers.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
20. Find the absolute value of a given rational number.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
21. Classify polygons according to their basic properties.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
22. Write the prime factorization of a given positive integer.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
23. Use prime factorization to determine the G.C.F. and L.C.M. of two or more positive integers.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
24. Write equivalent fractions for a given fraction.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
25. Express a fraction as a decimal and vice versa.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
26. Express a fraction as a percent and vice versa.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5
27. List in order the prime numbers less than 100.	A.		1	2	3	4	5
	B.	0	1	2	3	4	5

ITEM		RATINGS
28. Write numerals in numeration systems other than the Hindu-Arabic. (e.g. Roman or Egyptian).	A. B.	1 2 3 4 5 0 1 2 3 4 5
29. Write a given numeral in a base other than the decimal. (e.g. in base Two or Five.)	A. B.	1 2 3 4 5 0 1 2 3 4 5
30. Add and subtract in bases other than the decimal	A. B.	1 2 3 4 5 0 1 2 3 4 5
31. State and illustrate the properties of equality and inequality.	A. B.	1 2 3 4 5 0 1 2 3 4 5
32. State and illustrate the reflexive, symmetric and transitive properties of a relation.	A. B.	1 2 3 4 5 0 1 2 3 4 5
33. Order a series of rational numbers from smallest to largest and vice versa.	A. B.	1 2 3 4 5 0 1 2 3 4 5
34. Explain the Density Property of Rational Numbers.	A. B.	1 2 3 4 5 0 1 2 3 4 5
35. Find a given percent of a given number.	A. B.	1 2 3 4 5 0 1 2 3 4 5
36. Find what percent one number is of another.	A. B.	1 2 3 4 5 0 1 2 3 4 5
37. Find a number when a percent of it is given.	A. B.	1 2 3 4 5 0 1 2 3 4 5
38. Explain and illustrate the concept of congruency of segments and angles.	A. B.	1 2 3 4 5 0 1 2 3 4 5
39. State and explain the minimum conditions necessary for congruency of triangles.	A. B.	1 2 3 4 5 0 1 2 3 4 5
40. Use the properties of congruency to solve simple computational problems involving plane geometric figures.	A. B.	1 2 3 4 5 0 1 2 3 4 5
41. Write a given positive integer in exponential notation.	A. B.	1 2 3 4 5 0 1 2 3 4 5
42. Express a given rational number in scientific notation.	A. B.	1 2 3 4 5 0 1 2 3 4 5
43. Compute products and quotients using exponential notation.	A. B.	1 2 3 4 5 0 1 2 3 4 5
44. Compute products and quotients using scientific notation.	A. B.	1 2 3 4 5 0 1 2 3 4 5
45. Explain and illustrate the concept of similarity of triangles and other polygons.	A. B.	1 2 3 4 5 0 1 2 3 4 5

ITEM		RATINGS
46.	Use the concept and properties of similarity to solve simple computational problems involving plane geometric figures.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
47.	Find the average of two or more numbers.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
48.	Use ratio and proportion to solve problems.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
49.	State the formulas for finding the perimeter and area of plane geometric figures including the circle.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
50.	Given the formula compute the perimeter and area of a plane geometric figure.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
51.	Use standard geometric instruments to perform basic geometric construction. (e.g. bisect an angle.)	A. 1 2 3 4 5 B. 0 1 2 3 4 5
52.	State and apply the Pythagorean Principal in simple computational problems involving the right triangle.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
53.	Given the formula, determine the volume of a given geometric solid.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
54.	List the requirements for a number system.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
55.	Distinguish between variable and constant.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
56.	Distinguish between closed and open sentences.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
57.	Write simple relations involving and from verbal descriptions.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
58.	Solve simple First Degree equations using the addition and multiplication properties of equality.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
59.	Solve simple First Degree inequalities using the addition and multiplication properties of inequality.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
60.	State the degree of a given polynomial.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
61.	Write a given polynomial in ascending or descending order.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
62.	Define and illustrate function and relation.	A. 1 2 3 4 5 B. 0 1 2 3 4 5

ITEM		RATINGS
63. Distinguish between function and relation by definition and/or example.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
64. Distinguish between domain and range of a relation or function.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
65. Distinguish between independent and dependent variable.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
66. Add polynomials of given degree.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
67. State the additive inverse of a given polynomial.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
68. Subtract polynomials using the additive inverse principle.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
69. Write an equation or relation in standard linear form from graphical representation or tabular information.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
70. Graph a relation or function given in standard form using ordered pairs.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
71. Interpret required information from graphical representation.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
72. Evaluate a function for a given value of the domain.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
73. Find the product of two polynomials.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
74. Find the quotient of two polynomials.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
75. Factor polynomials of the second degree.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
76. Solve equations by factoring, using the "zero-products" principle.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
77. Solve conjunctions of equations (two variables) using the substitution or the addition method.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
78. Write an algebraic description of a given mathematical problem.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
79. Solve mathematical problems by algebraic means.	A.	1 2 3 4 5
	B.	0 1 2 3 4 5
80. Prove simple algebraic properties. (Example: Prove the cancellation property of addition.)	A.	1 2 3 4 5
	B.	0 1 2 3 4 5

ITEM		RATINGS
81.	Draw logical conclusions from simple numerical data. (Example: number patterns.)	A. 1 2 3 4 5 B. 0 1 2 3 4 5
82.	Use inductive reasoning to arrive at logical conclusions.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
83.	Use deductive reasoning to arrive at logical conclusions.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
84.	Apply generalized statements to specific examples or situations.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
85.	State the converse of a given statement.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
86.	Use counter-examples to test the validity of statements.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
87.	Test the validity of logical implications using truth tables.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
88.	Estimate measurements of length, weight, etc.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
89.	Read a scale accurately.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
90.	Use standard measuring instruments to measure accurately. (Including angle measure.)	A. 1 2 3 4 5 B. 0 1 2 3 4 5
91.	Choose the most appropriate scale for making a particular measurement.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
92.	Convert from one unit of measurement to another. (e.g. Convert from inches to yards.)	A. 1 2 3 4 5 B. 0 1 2 3 4 5
93.	Convert from the British to the Metric System and vice versa.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
94.	Estimate measures in the Metric System.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
95.	Measure accurately in the Metric System.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
96.	Determine the relative degree of precision or accuracy of two or more measures.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
97.	Use scales to determine actual dimensions from scale drawings and from maps.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
98.	Use indirect measurement involving similar triangles.	A. 1 2 3 4 5 B. 0 1 2 3 4 5
99.	Find the union or intersection of two or more sets.	A. 1 2 3 4 5 B. 0 1 2 3 4 5

	ITEM		RATINGS
100.	Distinguish between finite and infinite sets.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
101.	Explain the Completeness Property of Real Numbers.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
102.	Distinguish between Equal and Equivalent sets.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
103.	Perform simple probability experiments. (e.g. tossing a coin.)	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
104.	Determine the probability of the occurence of a particular event in simple probability experiments.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
105.	Represent statistical data in graphical form from numerical information.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
106.	Read statistical graphs to determine prescribed information.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
107.	Arrange statistical information in the form of a frequency distribution.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
108.	Determine the mean, the median, and the mode for given numerical data.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
109.	Use prescribed guidelines to prepare a budget.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
110.	Compute Bank Interest and Proceeds of a Promissory Note.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
111.	Maintain a record of a bank account from given information.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
112.	Compute carrying charges and total installment price on time payment purchases.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
113.	Compute premiums payable for home, auto, and life insurance for given principal amounts from tables.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
114.	Compute income tax payable from given information.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5
115.	Compute sales price on discount purchases.	A.	1 2 3 4 5
		B.	0 1 2 3 4 5

APPENDIX B

RANK ORDERING OF CONTENT OBJECTIVES
BASED ON MEAN RATING

RANK	DESCRIPTION	MEAN RATING
1	Add, subtract, multiply, and divide whole numbers	4.93
2	Find what percent one number is of another	4.73
	Measure accurately in the Metric System	4.73
4	Find a given percent of a given number	4.67
5	Find a number when a percent of it is given	4.60
	Find the average of two or more numbers	4.60
	Given the formula, compute the perimeter and/or area of a given geometric figure	4.60
8	Determine the place value of a given digit in a given numeral	4.53
	Add, subtract, multiply, and divide integers	4.53
	Add, subtract, multiply, and divide using decimals	4.53
	State the formula for finding the area or perimeter of a plane geometric figure	4.53
12	Express a fraction as a decimal and vice versa	4.47
	Estimate measurements of length, weight, and capacity in the British Imperial System	4.47
	Convert from one unit of measurement to another	4.47
	Estimate measurements in the Metric System	4.47
16	Add, subtract, multiply, and divide rational numbers in the form of common fractions	4.40
	Write equivalent fractions for a given fraction	4.40
	Express a common fraction as a percent and vice versa	4.40
19	Use standard instruments to perform basic geometric constructions	4.33
	Use standard measuring instruments to measure accurately	4.33

APPENDIX B — Continued

RANK	DESCRIPTION	MEAN RATING
21	Write simple relations involving , , and from verbal descriptions	4.27
	Read a scale accurately	4.27
23	Compute sales prices on discount purchases	4.20
24	Use fundamental properties with respect to the operations in computation	4.13
	Recognize relationships among lines (e.g. parallel)	4.13
26	Write the prime factorization of a given integer	4.07
	Given the formula, determine the volume of a given solid	4.07
28	Round off numbers to the nearest ten, hundred, etc.	4.00
	Define and identify basic geometric figures	4.00
	State and apply the Pythagorean Principle in simple computational problems involving the right triangle	4.00
31	Identify the fundamental properties of addition and multiplication	3.93
	Use prime factorization to find the G.C.D. and L.C.M. of two or more positive integers	3.93
	Order a series of rational numbers from smaller to larger and vice versa	3.93
	Classify angles by kind on the basis of degree measure	3.93
35	Choose the most appropriate scale for making a measurement	3.80
	Convert from the British Imperial to the Metric System of measurement and vice versa	3.80
37	Illustrate the fundamental properties of addition and multiplication.	3.73
	Explain and illustrate the concept of congruency of segments and angles	3.73
	Distinguish between variable and constant	3.73
	Solve simple first degree equations	3.73

APPENDIX B - Continued

RANK	DESCRIPTION	MEAN RATING
	Interpret required information from graphs	3.73
42	Classify a number as being natural, whole, integral, etc.	3.67
	Find the square root of a given positive rational number	3.67
	Maintain a record of a bank account from given information	3.67
45	Distinguish between prime and composite numbers	3.60
	Define and illustrate relationships between angles	3.60
47	Write a given number in exponential notation	3.53
	Use scales to determine actual dimensions	3.53
	Graph a relation or function	3.53
	Represent statistical data in graphical form	3.53
	Compute bank interest	3.53
	Compute carrying charges on installment purchases	3.53
53	Use analogy to draw conclusions	3.47
	Use inductive reasoning to arrive at logical conclusions	3.47
55	Explain the inverse relationship between addition and subtraction, multiplication and division	3.40
	State and illustrate the properties of equality and inequality	3.40
	Use ratio and proportion to solve problems	3.40
	Classify polygons according to their basic properties	3.40
	Use deductive reasoning to arrive at logical conclusions	3.40
60	Explain and illustrate the concept of similarity	3.33
	Compute insurance premiums	3.33
62	Find the absolute value of a given rational number	3.27

APPENDIX B — Continued

RANK	DESCRIPTION	MEAN RATING
63	Draw logical conclusions from simple numerical data	3.20
	Use prescribed guidelines to compile a budget	3.20
65	Express a given rational number in scientific notation	3.13
	Use similarity to solve computational problems	3.13
	Solve simple first degree inequalities	3.13
	Solve mathematical problems by algebraic means	3.13
	Distinguish between dependent and independent variable	3.13
	State the converse of a given statement	3.13
	Read statistical graphs to determine prescribed information	3.13
	Compute income tax payable from given information	3.13
73	Write an algebraic description of a given mathematical problem	3.07
	Determine the relative degree of accuracy or precision of two or more measures	3.07
75	Explain teh division by zero rule	3.00
	Determine the mean, median, and mode for given statistical data	3.00
77	Explain and illustrate the minimum conditions necessary for congruency of triangles	2.93
	Multiply polynomials	2.93
	Divide polynomials	2.93
	Solve conjunctions of equations	2.93
	Use counter-examples to test validity of statements	2.93
82	Compute products and quotients using exponential notation	2.87
	Add polynomials of given degree	2.87
	Solve equations by factoring	2.87

APPENDIX B - Continued

RANK	DESCRIPTION	MEAN RATING
	Use indirect measurement involving similar triangles	2.87
	Write an equation or relation in standard form from graphical representation or tables	2.87
87	Compute products and quotients using scientific notation	2.80
	Distinguish between closed and open sentences	2.80
	Subtract polynomials using the additive inverse principle	2.80
	Find the union and intersection of two or more sets	2.80
	Distinguish between finite and infinite sets	2.80
	Distinguish between equal and equivalent sets	2.80
	Determine the probability of the occurence of a particular event in simple probability experiments	2.80
	Arrange statistical information in the form of a frequency distribution	2.80
95	Explain the Density Property of Rational Numbers	2.73
	Use congruency to solve simple computational problems	2.73
	Evaluate a function for a given element of the domain	2.73
98	State and illustrate the reflexive, symmetric, and transitive properties of a relation	2.67
	Express a given number to a specified number of significant digits	2.67
	Write a given polynomial in ascending or descending order	2.67
	State the additive inverse of a given polynomial	2.67
	Perform simple probability experiments	2.67
103	State the degree of a given polynomial	2.60
104	Explain the completeness property of real numbers	2.53
	Define and illustrate function and relation	2.53
	Distinguish between function and relation	2.53

APPENDIX B - Continued

RANK	DESCRIPTION	MEAN RATING
107	List the requirements for a number system	2.47
	Factor polynomials	2.47
109	Prove simple algebraic properties	2.40
110	List in order the prime numbers less than 100	2.27
111	Add and subtract in bases other than base ten	2.20
112	Distinguish between domain and range of a function	2.13
113.	Test the validity of logical statements using truth tables	2.07
114	Write a given numeral in bases other than base ten	1.87
115.	Write numerals in numeration systems other than the Hindu-Arabic	1.40

APPENDIX C

SAMPLES OF CURRICULUM OUTLINES SURVEYED

I

LIST OF OBJECTIVES

NCTM COMMITTEE ON BASIC SKILLS AND COMPETENCIES[1]

1. Numbers and numerals:

   a) Express a rational number using decimal notation
   b) List the first ten multiples of 2 through 12
   c) Use the whole numbers in problem solving
   d) Recognize the digit, its place value, and the number represented through billions
   e) Describe a given positive rational number using decimal, percent, or fractional notation
   f) Convert to Roman numerals from decimal numerals and conversely (e.g. date translation)
   g) Represent very large and very small numbers using scientific notation

2. Operations and properties:

   a) Write equivalent fractions for given fractions such as 1/2, 2/3 and 3/5
   b) Use the standard algorithms for the operations of arithmetic on positive rational numbers
   c) Recognize and use properties of operations (grouping, order, etc.) and properties of certain numbers with respect to operations (a.1 = a; a + 0 = a; etc.)
   d) Solve addition, subtraction, multiplication, and division problems involving fractions
   e) Solve problems involving percent
   f) Peform arithmetic operations with measures
   g) Estimate results
   h) Judge the reasonableness of answers to computational problems

3. Mathematical sentences:

   a) Construct a mathematical sentence from a given verbal problem
   b) Solve simple linear equations such as:
      $a + 3 = 12; \quad 16 - n = 4$
   c) Translate mathematical sentences into verbal problems

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[1]E.L. Edwards, Eugene D. Nichols, Glyn H. Sharpe, "Mathematical Competencies and Skills Essential for Enlightened Citizenship," A Report of the Committee on Basic Mathematical Competencies and Skills, NCTM, Mathematics Teacher, LXV, (November, 1972) pp. 673-674.

4. Geometry:

   a) Recognize horizontal lines, vertical lines, parallel lines, perpendicular lines, and intersecting lines
   b) Classify simple plane figures by recognizing their properties
   c) Compute perimeters of polygons
   d) Compute the areas of rectangles, triangles, and circles
   e) Be familiar with the concepts of similarity and congruence of triangles

5. Measurement:

   a) Apply measures of length, area, volume (dry or liquid), weight, time, money, and temperature
   b) Use units of length, area, mass, and volume in making measurements
   c) Use standard measuring devices to measure length, area, volume, time, and temperature
   d) Round off measurements to the nearest given unit of the measuring device (ruler, protractor, therometer, etc.) used
   e) Read maps and estimate distances between locations

6. Relations and functions:

   a) Interpret information from graphical representation of a function
   b) Apply the concepts of ratio and proportion to construct scale drawings and determine percent and other relations
   c) Write simple sentences showing the relations , , = , and for two given numbers

7. Probability and statistics:

   a) Determine the mean, median, and mode for given statistical data
   b) Analyze and solve simple probability problems such as tossing coins or drawing one red marble from a set containing one red marble and four white marbles
   c) Estimate answers to computational problems
   d) Recognize the techniques used in making predictions and estimates from samples

8. Graphing:

   a) Determine measures of real objects from scale drawings
   b) Construct scale drawings of simple objects
   c) Construct graphs indicating relationships of two variables from given sets of data
   d) Interpret information from graphs and tables

9. Mathematical reasoning:

   a) Produce counter-examples to test validity of statements
   b) Detect and describe flaws and fallacies in advertising and propoganda where statistical data and inferences are employed
   c) Gather and present data to support an inference or argument

10. Business and consumer mathematics:

   a) Maintain personal bank records
   b) Plan a budget including record keeping of personal expenses
   c) Apply simple interest formulas to installment buying.
   d) Estimate the real cost of an article
   e) Compute taxes and investment returns
   f) Use the necessary mathematical skills to appraise insurance and retirement benefits.

II

SSMCIS UNIFIED MATHEMATICS CURRICULUM[2]

COURSE I: (Grade Seven)

1. Finite Number Systems
2. Sets and Operations
3. Mathematical Mappings
4. Integers and Addition
5. Probability and Statistics
6. Multiplication of Integers
7. Lattice Points in a Plane
8. Sets and Relations
9. Transformations of the Plane
10. Segments, Angles, Isometries
11. Elementary Number Theory
12. The Rational Number
13. Some Applications of the Rational Number
14. Algorithms and Their Graphs

COURSE II: (Grade Eight)

1. Mathematical Language and Proof
2. Groups
3. An Introduction to Axiomatic Affine Geometry
4. Fields
5. The Real Number System
6. Coordinate Geometry
7. Real Functions
8. Descriptive Statistics
9. Transformations of the Plane, Isometries
10. Length, Area, and Volume

Appendix A - Mass Points

[2]Howard F. Fehr, "The Secondary School Mathematics Curriculum Improvement Study: A Unified Mathematics Program," The Mathematics Teacher, LXVII (January, 1974), p. 31.

APPENDIX C - Continued

COURSE III: (Grade Nine)

1. Introduction to Matrices
2. Linear Equations and Matrices
3. Algebra of Matrices
4. Graphs and Functions
5. Combinatorics
6. Probability
7. Polynomials and Rational Functions
8. Circular Functions
9. Informal Space Geometry

COURSE IV: (Grade Ten)

1. Programming in BASIC
2. Quadratic Equations and Complex Numbers
3. Circular Functions II
4. Conditional Probability and Random Variables
5. Algebra of Vectors
6. Linear Programming
7. Sequences and Series
8. Exponential and Logarithmetic Functions
9. Vector Spaces and Subspaces

COURSE V: (Grade Eleven)

1. Introduction to Continuity
2. More about Continuity
3. Limits
4. Linear Approximations and Derivatives
5. Properties of Derivatives
6. Further Study of the Derivative
7. Linear Mappings and Linear Programming
8. Probability: Expectation and Markov Chains
9. Integration

COURSE VI: (Grade Twelve)

1. Infinity
2. Conics
3. Circular Functions - Analytic Properties
4. Exponential and Logarithmic Functions - Analytic Properties
5. Integration Techniques and Applications
6. Probability: Infinite Outcome

BOOKLET:

A. Introduction to Statistical Inference.
B. Determinants, Matrices and Eigenvalues.
C. Algebraic Structures, Extensions, and Homomorphisms.
D. An Introduction to Differential Equations.
E. Geometry Mappings and Transformations.

III

PROPOSED PROGRAMME FOR "MATHEMATICS FOR EVERYMAN"[3]

1. The main uses of numbers (without calculation):

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[3]Max S. Bell, "What Does Everyman Really Need From School Mathematics," Mathematics Teacher, LXVII, (March, 1974), p. 199.

1.1 Counting
1.2 Measuring
1.3 Coordinate systems
1.4 Ordering
1.5 Indexing
1.6 Identification numbers, codes
1.7 Ratios

2. Efficient and informed use of computational algorithms:

2.1 Intelligent use of mechanical aids to calculation

3. Relations such as equal, equivalent, less or greater, congruent, similar, parallel, perpendicular, subset, etc:

3.1 Existence of many equivalence classes
3.2 Flexible selection and use of appropriate elements from equivalence classes (e.g., fractions, equations, etc.)

4. Fundamental measure concepts:

4.1 "Measure functions" as a unifying concept
4.2 Practical problems: role of "unit"; instrumentation; closeness of approximation
4.3 Pervasive role of measures in applications
4.4 Derived measures via formulas and other mathematical models

5. Confident, ready, and informed use of estimates and approximations:

5.1 "Number Sense"
5.2 Rapid and accurate calculation with one and two digit numbers
5.3 Appropriate calculation via positive and negative powers of ten
5.4 Order of magnitude
5.5 Guess and verify proceedures; recursive processes
5.6 "Measure sense"
5.7 Use of appropriate ratios
5.8 Rules of Thumb; rough conversions (e.g. "a pint is a pound"); standard modules
5.9 Awareness of reasonable cost or amount in a variety of situations

6. Links between "the world of mathematics" and "the world of reality":

6.1 Via building and using "mathematical models"
6.2 Via concrete "embodiments" of mathematical ideas

7. Uses of variables:

7.1 In formulas
7.2 In equations
7.3 In functions
7.4 For stating axioms and properties
7.5 As parameters

APPENDIX C - Continued

8. Correspondences, mappings, functions, transformations:

    8.1 Inputs, outputs, appropriateness of these for a given situation
    8.2 Composition ("If this happens, and then that, what is the combined result?")
    8.3 Use of representational and coordinate graphs

9. Basic logic:

    9.1 "Starting points"; agreements (axioms); and primitives (undefined words)
    9.2 Consequences of altering axioms (rules)
    9.3 Arbitrariness of definitions; need for precise definition
    9.4 Quantifiers (all, some, there exists, etc.)
    9.5 Putting together a logical argument

10. "Chance", fundamental probability ideas, descriptive statistics:

    10.1 Prediction of mass behavious vs unpredictability of single events
    10.2 Representative sampling from populations
    10.3 Description via arithmetic mean, median, standard deviation

11. Geometric relations in plane and space:

    11.1 Visual sensitivity
    11.2 Standard geometry properties and their applications
    11.3 Projections from three to two dimensions

12. Interpretation of informational graphs:

    12.1 Appropriate scales, labels, etc.
    12.2 Alertness to misleading message

13. Computer uses:

    13.1 Capabilities and limitations
    13.2 "Flow chart" organization of problems for communication with computer

IV

ACCEPTANCE INDEX AND RECOMMENDATIONS FOR TOPICS FOR A MATHEMATICS PROGRAMME FOR SLOW LEARNERS[4]

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[4]Sol Weiss, "What Mathematics Shall We Teach The Slow Learner?" Mathematics Teacher, LXII (November, 1969), p. 572. (From a survey of 200 leading mathematics educators in the United States.)

APPENDIX C - Continued

Topic	No (%)	Yes (%)	Index	Recommendation
Whole and rational numbers				
1. Operations	1.3	97.4	4.9	Yes
2. Properties	8.9	88.4	4.6	Yes
3. Negative rational numbers	11.6	77.4	4.2	Yes
Real numbers				
4. Operations	20.0	65.2	3.9	Yes
5. Properties	25.8	52.9	3.5	Yes
6. Systems of numeration	15.5	58.7	3.8	Yes
7. Sets	18.1	58.1	3.9	Yes
8. Ratio and percent	3.9	89.0	4.6	Yes
Number theory				
9. Primes	7.7	74.2	4.3	Yes
10. Divisibility	11.0	71.0	4.1	Yes
11. Highest common factor	12.3	68.4	4.0	Yes
12. Lowest common multiple	12.3	71.0	4.1	Yes
13. Clock arithmetic	20.6	53.5	3.6	Yes
14. Nonmetric geometry	12.3	63.2	4.0	Yes
Intuitive geometry				
15. Congruence	5.8	81.9	4.4	Yes
16. Similarity	3.9	81.9	4.4	Yes
17. Basic constructions	6.5	81.9	4.4	Yes
18. Symmetry	7.1	68.4	4.1	Yes
19. Trigonometric ratios	36.8	35.5	3.0	Undecided
Measurement				
20. Linear	0.6	95.9	4.8	Yes
21. Square	0.6	94.8	4.8	Yes
22. Cubic	1.3	89.0	4.7	Yes
23. Pythagorean theorem	4.5	78.7	4.3	Yes
24. Formulas	3.2	85.2	4.5	Yes
25. Equations	3.2	90.3	4.5	Yes
26. Inequalities	11.0	63.2	3.9	Yes
27. Graphs and statistics	5.2	81.9	4.3	Yes
28. Permutations & combinations	42.6	27.1	2.7	Undecided
29. Probability	33.5	39.0	3.2	Undecided
30. Vectors	59.4	18.7	2.2	No
31. Coordinate geometry	34.8	43.2	3.0	Undecided
32. Linear programming	64.5	11.0	1.9	No
Logic				
33. Proof	49.0	29.7	2.8	Undecided
34. Deductive reasoning	41.9	37.4	2.9	Undecided
35. Truth tables	59.4	15.5	2.2	No
36. History of mathematics	23.2	51.6	3.5	Yes

APPENDIX C - Continued

IV

	Topic	No (%)	Yes (%)	Index	Recommendation
37.	Slide rule	29.0	42.6	3.2	Undecided
38.	Computer mathematics	40.6	24.5	2.6	Undecided
39.	Computing earnings	29.7	45.8	3.2	Undecided
40.	Handling money and accounts	22.6	61.9	3.7	Yes
Managing income					
41.	Budgets	32.3	43.2	3.2	Undecided
42.	Installment buying	29.7	42.6	3.3	Undecided
43.	Buying a home	40.0	42.6	2.8	Undecided
44.	Buying a car	32.3	49.6	3.2	Undecided
45.	Insurance	37.4	36.8	3.0	Undecided
46.	Taxation	37.4	38.1	3.0	Undecided
47.	Measuring instruments and devices (how to use)	5.8	67.7	4.3	Yes

APPENDIX C.—Continued

V

RANK ORDERING OF BASIC MATHEMATICAL SKILLS ESSENTIAL FOR SUCCESS IN DIFFERENT VOCATIONAL SPECIALTIES[5]

RANK	PERCENT	NUMBER	SKILL
1	95.0	247	Addition of whole numbers
2.5	94.6	246	Subtraction of whole numbers
2.5	94.6	246	Multiplication of whole numbers
4	93.0	242	Division of whole numbers
5	86.9	226	Reading and writing decimals
6	85.0	221	Addition of fractions
7	83.5	216	Subtraction of fractions
8	80.0	208	Rounding off decimals
10	79.6	207	Addition of decimals
10	79.6	207	Subtraction of decimals
10	79.6	207	Multiplication of fractions
12.5	78.4	204	Meaning of percentage
12.5	78.4	204	Changing common fractions to decimals
14.5	77.6	202	Reducing fractions to lowest terms
14.5	77.6	202	Multiplication of decimals
16	76.9	200	Division of fractions
17	76.1	198	Reading a rule
18	75.3	196	Rounding numbers
19.5	74.6	194	Changing decimals to common fractions
19.5	74.6	194	Decimal equivalents
21	73.8	192	Division of decimals
22	72.3	188	Improper fractions or mixed numbers
23	71.5	186	Subtraction of mixed numbers
24	71.1	185	Addition of mixed numbers
25.5	70.3	183	Averages
25.5	70.3	183	Reading large numbers
27	69.2	180	Comparing fractions
28.5	68.8	179	Multiplication of mixed numbers
28.5	68.8	179	Ratio and proportion
30	68.4	178	Changing percents to decimals
31.5	68.0	177	Reducing mixed numbers to simplest form
31.5	68.0	177	Finding a percent of a number
33	67.3	175	Division of mixed numbers
34	65.0	169	Multiplying whole numbers and decimals by 10, 100, etc.
35	64.6	168	Dividing whole numbers and decimals by 10, 100, etc.
37	63.4	165	Changing decimals to percent
37	63.4	165	Finding what part one number is of another

[5]Albert P. Shulte, "Teacher Perception of Basic Mathematical Skills Needed in Secondary Vocational Education," Mathematics Teacher, LXVI, (January, 1973), p. 63. (From a survey of 260 teachers in selected Vocational Schools in the United States.)

APPENDIX C - Continued

V

RANK	PERCENT	NUMBER	SKILL
37	63.4	165	Measuring angles
39	60.0	156	Squaring a number
40	59.2	154	Finding a number when a percent of it is known
41	58.0	151	Using pi ( )
42	55.0	143	Square root
44	54.6	142	Area of a square
44	54.6	142	Changing percents to common fractions
44	54.6	142	Circumference using c = d
46	54.2	141	Area of a circle using A = r^2
48	53.4	139	Changing common fractions to percents
48	53.4	139	Finding what percent one number is of another
48	53.4	139	Simple algebraic equation with one unknown
50	52.6	137	Finding a number when a fractional part of it is known
51	52.3	136	Area of a rectangle
52	51.9	135	Liquid measure
53	51.1	133	Area of circle
54.5	50.3	131	Acute and obtuse angles
54.5	50.3	131	Micrometer reading
56	46.5	121	Pythagorean Theorem - Finding length of unknown side of right triangle
57	44.6	116	Temperature conversion
58	43.4	113	Area of a triangle
59	42.6	111	Volume of a cylinder
60	42.3	110	Volume of a cube
61	41.5	108	Volume of a rectangular solid
62	39.6	103	Area of a parallelogram
63	34.6	90	Area of a trapezoid
64	31.9	83	Volume of a sphere
65	25.7	67	Volume of a pyramid
66	25.3	66	Volume of a cone

APPENDIX D

LIST OF TEXTBOOKS SURVEYED

1. Ebos, Frank and Bob Robinson. Math Is, Books I and II, Don Mills, Ont.: Thomas Nelson and Sons (Canada) Ltd., 1975.

2. Eicholz, Robert E. et al. School Mathematics, Books I and II, Don Mills, Ont.: Addison Wesley Publishing Company, 1967.

3. Fleenor, Charles R. et al. Success With Mathematics, Books I and II, Don Mills, Ont.: Addison Wesley Publishing Company, 1972.

4. Fleenor, Charles R., Robert E. Eicholz, and Phares O'Daffer. School Mathematics, Books I and II, Don Mills, Ont.: Addison Wesley Publishing Company, 1974.

5. Keedy, Mervin L., Richard E. Johnson, and Patricia L. Johnson. Exploring Modern Mathematics, Books I and II, Toronto: Holt Rinehard and Winston, Inc., 1965.

6. ______. Exploring Modern Mathematics, Books I and II, Toronto: Holt Rinehard, and Winston, Inc., 1968.

7. ______. Exploring Modern Mathematics, Books I and II, Toronto: Holt Rinehard, and Winston, Inc., 1971.

8. Peters, Max et al. Exploring Mathematics - Insights and Skills, Books I, II, and III, New York: Globe Book Company, Inc., 1974.

9. Van Engen, Henry et al. Seeing Through Mathematics, Books I, II and III, Toronto: W.J. Gage Ltd., 1967.