AN OPERATIONAL METHODOLOGY FOR OPTIMIZING THE SIZE AND LOCATION OF SCHOOLS WITHIN AN ESTABLISHED NETWORK: A CASE STUDY OF NORTH CENTRAL NEWFOUNDLAND

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An Operational Methodology for Optimizing the Size and Location of Schools Within an Established Network: A Case Study of North Central Newfoundland.

by

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Abstract

The thesis examines a location-allocation problem in the public sector. The problem is to simultaneously locate school facilities on a network and determine the assignment of students to these facilities so that the total costs of operating a socially acceptable system are minimized. The theory is that cost and quality vary with the size and location of facilities within the system. Given the principles outlined in theory, the objective of location-allocation analysis is to aid the decision maker in defining an optimum solution to the problem.

The methodology for locating schools and allocating students evolved from the linear programming solutions to the P-Median and Mini-max problems. The objectives of the design are extended to also consider modifications to existing school systems, the minimization of facility and transportation costs over time and the impact of changes in the configuration of various sized facilities on the quality of the educational system. This leads to the incorporation of an investment constraint, the consideration of the location and value of existing facilities, a procedure to minimize system costs under the design constraints and the derivation of a facility production function.

Acknowledgements

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Chapter I: Approaches to Optimizing the Size and Location of Public Facilities

This thesis develops an operational methodology for optimizing the size and location of school facilities on a regional transportation network. The solution design is an extension of established linear programming methods for solving the problem of optimally locating facilities on a network (ReVelle and Swain, 1970) under investment (Rojeski and ReVelle, 1970) and time (Toregas, 1971) constraints.

The optimum size and location of school facilities on a regional transportation network is defined by that configuration of student to facility assignments which allows the decision maker to meet his quality of service objectives at a minimum cost over time. The problem in this thesis is to isolate the information and mechanisms necessary to systematically define this configuration.

I - A. Theory of Public Service Location

In the context of public service facilities, such as schools, firestations, libraries and welfare offices, traditional location theory, with its emphasis on corporate welfare or profit maximization is inappropriate. Theoretically all consumers (ie. taxpayers) pay for both supply costs (production) and demand costs (transportation) in obtaining public services.

Consumer welfare is achieved by creating a system of facilities which minimizes the aggregate cost of serving all consumers given that a constant quality of service is maintained. The problem can be summarised as the extent to which public services should be centralised (to maximize operating economies) or decentralized (to maximize locational efficiencies).

The following principles are of particular importance in relating facility size and location to the cost and quality of public facility systems.

(1) Principle of Scale Economies. The cost per capita of providing a public facility will tend to decrease as the size of the facility and the number of participants increases until a size is reached where structural and/or operational technology becomes rarefied. Economies of scale are most likely to exist in facilities where there are indivisibilities in the basic factors in the production of the service.

(2) Principle of Transportation Costs. If the size of a facility is increased, the geographic service region of the facility will likely expand, increasing per capita and aggregate transportation costs to the facility. In the provision of a public good or service transportation costs are often born by the consumer. The private sector reflects consumer transportation costs in competitive pricing mechanisms.

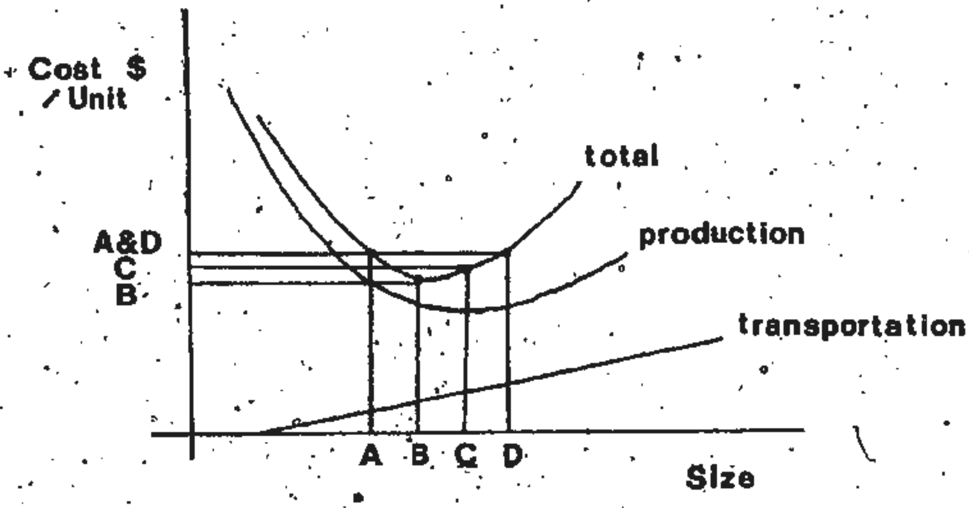
among different locations. In the public sector, they must be considered as part of the overall production costs to be minimized.

(3) Principle of Social Distance. The participation of an individual in services provided at a public facility will diminish as travel time and/or travel cost increase and as individuals lose identity with the service provided. By definition public goods are required to serve the entire (relevant) population rather than the particular percentage which allows profits to be maximized. However, in a space economy, not all consumers can have the same access to service. Consequently, it may be desirable to influence the location of public facilities by an equity constraint such as some socially acceptable limit to consumer accessibility.

Using hypothetical cost data (Table 1.1) the postulated effects of variations in the number and size of facilities on per capita facility, transportation and total costs have been illustrated (Fig. 1.).

Table 1.1: Hypothetical Cost Data for Various Numbers and Sizes of Facilities

System	Facilities	Size	Facility Cost Per Capita	Transport Cost Per Capita	Total Cost Per Capita	Total Served
A	10	100	180	30	210	1000
B	8	125	150	40	190	1000
C	6	166	145	50	195	1000
D	4	250	150	60	210	1000

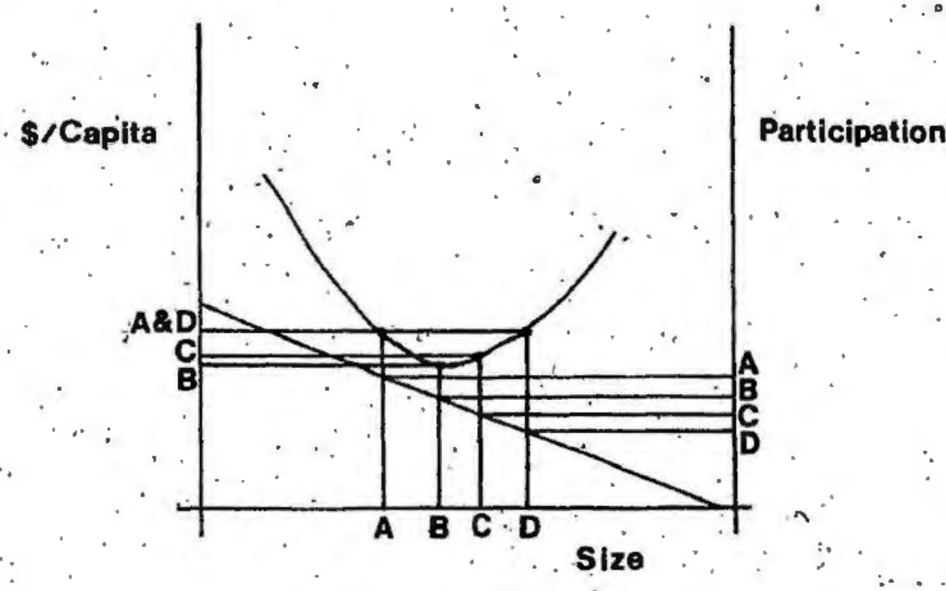


source: Kàdas, 1963, p.198

FIGURE 1 FACILITY COST-SIZE RELATIONSHIP

When the system is served by larger facilities, the facility cost per capita declines. At the same time, transportation cost per capita increases. Facility costs are minimized when there are six facilities at a size of 166 serving the system. However, between facility size B (125) and C (166), the marginal decrease in costs due to scale economies is exceeded by the marginal increase in costs caused by further travel. Thus, total costs are minimized when the system has eight facilities at a size of 125 each.

A second diagram (Fig. 2) depicts the relationship between costs and social distance. Given the total cost curve from Figure 1, the impact of the different systems on consumer participation is demonstrated.



source: Isard, 1960, p.528

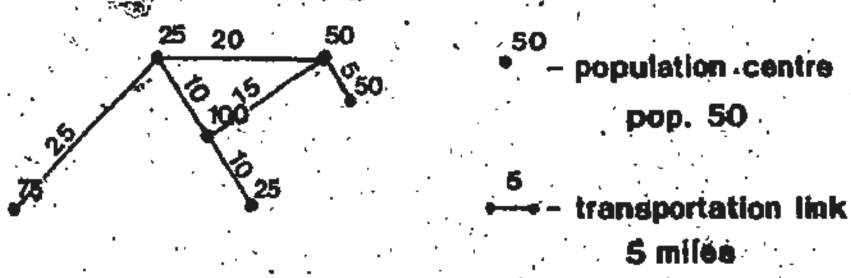
FIGURE 2 COST, DISTANCE AND PARTICIPATION

Just as costs decrease due to scale economies, when a system of fewer and larger facilities is created, participation rates diminish due to increased travel time, higher participant travel cost and a loss of identity with the services of moredistant facilities. The major assumption in this relationship is that the quality of service provided in each alternative size of facility is the same. It is, therefore, important in an analysis of the ability to participate, to identify the independent effect, if any, of facility size on service quality.

(4) Principle of Change. Changes in the spatial structure of the basic elements of supply (facilities and transportation modes) and demand (participants) must be taken into account when locating fixed facilities with long life spans or investment horizons. Minimizing transportation costs, for example, may produce a least cost system in the short run. However, further economies may be achieved in the long run by creating new facilities and/or altering the scale of existing facilities (Parr and Denike, 1970, p. 585). A change in the distribution or density of the individuals being served or a non-uniform change in transport costs as a result of new or improved transportation links will generally obviate an optimum configuration of facilities which was previously derived on static assumptions (Kádas, 1963, p. 199). Temporal instability in facility systems can also be caused by progressive obsclescence and depreciation in existing facilities, from the redundant location of successive generations of these facilities or from an admixture of these possibilities (Scott, 1970, p. 101). In practice when the principle of change is

considered, "the best that can be obtained is a series of sequential local optimization problems each defined over a short run period". (Scott, 1970 p. 104). Indeed, one of the most difficult problems in public facility planning is to achieve an optimum trade-off among scale economies, transportation costs and social distance over time.

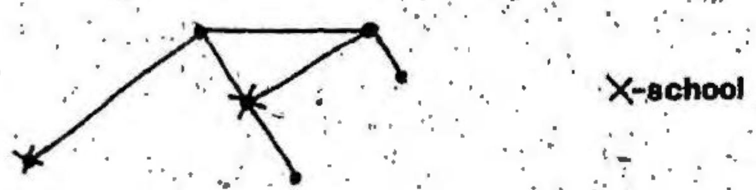
Furthermore, if one principle is allowed to dominate the others in the decision making process, or if only one of these principles is considered, different locational solutions will result. Consider the problem of locating schools on a network of six communities of varying sizes connected by transportation links of varying distances.



If transportation costs are minimized the solution is a school in every community:

If operating and construction costs per capita are minimized, to maximize economies of scale, the solution is one school, assigned to, for example, the largest community:

The solution which minimizes the total costs per capita assigns schools to two locations:



However, if the maximum travel distance to school which is socially acceptable to the communities is ten miles then at least three schools have to be assigned. The most economic solution still satisfying this social constraint is given by the following solution:

Hypothetical costs for the preceding systems are summarized below:

Solution	Transport Cost	School Cost	Total Cost
#1 Transport Cost	0	$105/Person ($34,125)	$34,125
#2 School Cost	$2/Person/Mile ($9,750)	$75/Person ($24,375)	$34,125
#3 Least Cost	$2/Person/Mile ($4,500)	$85/Person ($27,625)	$32,125
#4 Socio-Econ	$2/Person/Mile ($2,000)	$95/Person ($30,875)	$32,875

Solution #4 maximizes the effectiveness of the school system in relation to the principles considered. This, of course, only holds true in the short run because of the forces of change which are constantly influencing the population of communities; the effective distance between them and the operating economies of schools. If one of these forces cause a facility to become obsolete or a location to become redundant, a further analysis would have to examine the costs of relocation or replacement before recommending an alternative solution to maximize the effectiveness of the system.

I-B. Location-Allocation Models

In the sixties rapid advances were made in the application of computerized mathematical techniques to central facility location. Location-allocation models minimize some function of distance in order to achieve a cost or quality of service objective and define optimum systems of central facilities.

Location-allocation analysis had early beginnings with Alfred Weber (1909) who considered the optimum location of a factory on a transportation plane between two resources and a single market. The objective in his theory was to minimize transport costs by locating the factory in a strategic position which would trade-off the costs of transporting two different raw materials from their sources to the factory and one finished product from the factory to the market.

A mathematical expression of Weber's problem was familiarized in the literature by Kuehn and Kuenne (1962). These authors examined the problem in continuous space with the objective to find the location which minimized the sum of the weighted Euclidean distances to that location from various demand centres. Their mathematical solutions to this problem were exact.

Cooper (1964) considered the more complex problem of searching continuous space for more than one location to service the requirements of various demand centres while still minimising the sum of the weighted distances. Given a set of demand centres partitioned according to the number of locations to be found, Cooper has generated a heuristic process which alternates between two rules; (1) find the location within each partition which minimizes the sum of the weighted distances to that location from the set of demand centres, then (2) repartition the demand centres by assigning each centre to its closest facility location. The final solution is obtained when the partitioning process stabilizes. Although solutions generated in this manner are good, there is no certainty that they are exact.

Several exact algorithms to this basic location-allocation problem have recently been developed by Kuenne and Soland (1971) and Ostresh (1973). Their solution processes call on the tree searching method known as branch and bound. The branching process partitions the set of all solutions into smaller subsets until all demand points are assigned to centres, thus creating a feasible solution. While, the branching process ensures the complete enumeration of all feasible solutions, bounding streamlines the solution process so that complete enumeration is not necessary. Ostresh's solution process uses three bounds. The first tests each solution for geometric possibility ie., because any group must be assigned to a nearest centre, any group assigned to the same centre must be spatially disjoint from any other. The second tests each solution for shape ie., each centre must be located at the minimum point of its set of demand points. The third bound tests each solution against the previous best. ie., any solution which doesn't minimize distance better than the previous best is ruled out.

Since tree searching methods have only small data handling capabilities, the heuristic methods are still widely acknowledged in solving the multiple location Weberian problem.

The basic location-allocation problem can be greatly simplified by defining it in discrete space. The modified problem is to minimize the sum of the weighted distances given that the locations of supply and demand points are restricted to network nodes and transportation, to network links.

Hakimi (1964, 1965) provided the two underlying theorems of what is known as the P-Median problem; (1) there is a point on the graph which minimizes the sum of the weighted shortest distances from all nodes to that point which is itself a node of the graph, and (2) there is a set of p points, consisting entirely of nodes of the graph which minimizes, the sum of the weighted distances to the closest of any p points on the graph.

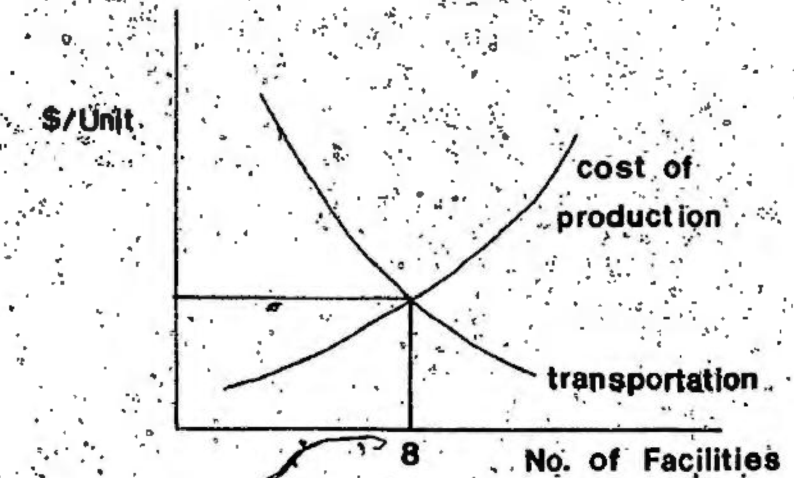
Maranzana (1964) developed a heuristic process to solve the multiple location problem on a network. Much like Cooper's algorithm for continuous space, Maranzana's algorithm alternates between two rules; (1) demand nodes are partitioned into groups by assigning them to the closest of the preselected facility nodes, then (2) for each group the median node is found and designated the new facility location. The final solution is achieved when the groups stabilize.

Teitz and Bart (1968) improved on this method using successive node substitution. In their process a number of nodes are predesignated as centres. A node which is not a centre is substituted for each that is a centre, the one where the greatest improvement is made is designated the centre. An iteration is complete when each non-centre is allowed the opportunity to be substituted for any one of the centres. The final solution is achieved when no improvement can be made in an iteration.

ReVelle and Swain (1970) have shown that an exact solution to the multiple location problem in discrete space can be found by setting the problem up as a linear program. The linear program design to be solved by the simplex method is defined by an objective function to minimize the sum of the weighted distances and constrained so ~~that~~; each node must fully assign, each node only assigns to a self-assigning node, and the number of self-assigning nodes is limited to the number of facilities desired.

The commonly known linear programming transportation algorithm is another variation of a discrete space problem. Given a number of central facilities whose locations and capacities are known and a number of demand points, the problem is to minimize transportation costs. In describing the spatial non-efficiency of high school hinterlands, Yeates (1963) was among the first users of the transportation algorithm to geography. Gould and Leinbach (1966) first used it in a heuristic process to define optimum patient hospital systems in Guatemala. Perhaps the most interesting and generally applicable use of the transportation algorithm was by Goodchild and Massam (1969). They solve the multiple location problem with capacitated facilities in much the same way as Cooper treated the problem with non-capacitated centres. Their heuristic process alternates on two rules; (1) the location of the median point is found for the demand in each prepartitioned space, then (2) the transportation algorithm is used to assign surpluaes from a capacitated centre in one location to the deficit of another while minimizing transportation costs. The space is thus repartitioned and one iteration is complete. This process continues until no improvements can be made.

As explained in Section I-A of this chapter, the optimum location-allocation system does not depend solely on the minimization of transportation costs. In many cases economic optimization is dependent on the number and size of facilities as well as their disposition in relation to demand. Tornqvist (1971) has developed an alternative search procedure for the multiple location problem in continuous space, which resembles Teitz and Barts' approach to discrete space. Briefly, each of a predetermined number of facilities is given a location and a search step is defined. A facility is moved one step in four different directions from its source. The facility either remains in its current location if transportation costs cannot be reduced by moving to one of the four new locations or moves to the location which minimizes transportation costs. In one iteration this procedure is repeated for each facility. The final solution is obtained when no moves are made in one complete iteration. In his location-allocation problem for cement block factories in Sweden, Tornqvist generates several solutions for different numbers of factories and develops a trade-off curve between the rising per unit cost of production and the declining per unit cost of transportation with each additional factory serving a constant demand (Fig. 3).



source: Abler, Adams, & Gould, 1971, p.549

FIGURE 3 PRODUCTION – TRANSPORTATION COST TRADEOFF

The number of factories is optimum when the combined production and transportation costs are minimum. This is denoted by the intersection of the two curves in Fig. 3. The optimum location of these factories is determined in Tornqvist's search process. The same tradeoff curve, however, could be developed in conjunction with any of the previous multiple location solution processes.

The fixed charges problem defines a set of models which have incorporated facility costs into their design to minimize total costs in warehouse and plant location. The number of facilities in the solution to a fixed charges problem will reflect the economies of scale which are obtainable over a given demand surface. Several heuristic processes (Kuehn and Hamburger, 1963 and Feldman, Lehrer and Ray, 1966) and exact tree searching methods (Efroymson and Ray, 1966 and Spielburg, 1969) have been developed to solve this problem.

An exact solution to the fixed charges problem for public facilities has been provided by Rojeski and ReVelle (1970) in their extention of the linear programming approach to the multiple location problem. Their objective function to minimize the sum of the weighted distances is constrained by a budget which ultimately determines the number of facilities to be located.

The mini-max problem defines a completely different approach to location analysis for public facilities. The mini-max problem is based on the premise that the quality of service diminishes with distance from the source. This is particularly apparent in the location of firestations. The problem is to minimize the number of facilities required to bring all demand points within a maximum allowable distance of a facility. A noteable method for solving this problem using linear programming has been developed by Toregas (1971).

The linear programming approach and its implications for the school location-allocation problem will be discussed in greater detail in the development of the solution design in Chapter II.

The unsolved problem of optimum facility location over time is given scant coverage in the literature.

Scott (1970) and Teitz (1968) conclude that little can be done but obtain a series of sequential local optimization problems connected by a minimum cost path.

I - C. Format

The remaining chapters of this thesis develop and apply an operational methodology for optimizing the size and location of schools in North-Central Newfoundland. Chapter II introduces the optimization objectives and discusses the development and application of the linear programming methodology. Chapter III describes the study area and data sources used in the analysis. In Chapter IV study area data is interfaced with the methodological requirements. Finally, in Chapter V several solutions to the study area problem are generated and examined.

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Chapter II: A Linear Programming Model for the School Location-Allocation Problem

The methodology developed in this chapter focuses on four basic objectives:

1) to minimize fixed and variable facility costs within the school system over a given investment period.

2) to minimize student transportation costs throughout the school system over the same investment period.

3) to locate schools within an acceptable (the range which may be defined by both student and educator) maximum travel time of all students.

4) to ensure that recommended changes in the size and location of schools within the system do not compromise the system's potential to maximize the quality of its educational environment.

Since the most common problem in facility planning is to make adjustments to existing school facility systems, the methodology put forward considers the location, size and value of existing facilities and evaluates them against the four objectives.

In order to account for facility depreciation and continued locational redundancy, the methodology examines the present value for the continuing costs of school replacement, operation and transportation over a given investment period. Other elements of change such as shifts in population and improvements to transportation are the subject of planning projections and can be accounted for by restructuring data inputs.

The solution generated through the application of the linear programming design defines the modifications to the existing location of schools and allocation of students which limit the maximum time travelled by students at minimum facility and transportation costs to the school system over the investment period.

The impact of the changes in school size and location on the quality of education is isolated in an educational production function.

II-A. Linear Programming Location-Allocation Models

1) P-Median Problem

ReVelle (1968) introduced a linear programming design to solve the multiple facility location-allocation problem in discrete space. The problem, known as the P-Median problem, is to locate a specific number of facilities on a network so as to minimize average participant travel time. Hakimi (1964, 1965) proved that if all participant demand originates at nodes on the network, there is an optimum solution to this problem where all facilities are at nodes.

Thus, ReVelle focuses on the set of problems where population centers are identified as nodes on a transportation network. His objective function is to minimize the average distance travelled by participants while allowing complete variability in the size and location of each facility. ReVelle and Swain (1970, p. 31) set up the problem as follows:

"Let a_i = the population of the i^{th} community, $i = 1,2...,n$. The people in the i^{th} community are to be assigned to one and only one center; that is, the assignment cannot be partial. The center may be in the community itself or in one of the other (n-1) communities. The number of centers is m, and each center has a cluster of communities assigned to it. The shortest distance from community i to community j is d_{ij}. Variables are defined as

$$X_{ij} = \begin{cases} 0 \text{ if community } i \text{ does not assign to community } j. \\ 1 \text{ if community } i \text{ does assign to community } j. \end{cases}$$

The people-miles from node i to node j is $a_i \cdot d_{ij}$. The objective function to be minimized is

$$Z = \sum_{j=1}^{n} \sum_{i=1}^{n} a_i \cdot d_{ij} \cdot X_{ij}$$

Three types of constraints are required. The first type demands that each community be fully assigned.

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad i = 1,2,\ldots,n$$

The second type restricts assignment to only those communities which assign to themselves:

$$X_{jj} \geq X_{ij} \qquad \begin{array}{l} i = 1,2,\ldots,n \\ j = 1,2,\ldots,n \\ i \neq j \end{array}$$

Finally, the third type of constraint fixes the number of central facilities and thus the number of communities which may assign to themselves.

$$\sum_{i=1}^{n} X_{ii} = m$$

where m = number of central facilities."

Revelle and Swain (1970 p. 33) prove that limiting assignments X_{ij} to 0 or 1 does not destroy the possibility of achieving an optimum solution:

"The proof is by contradiction. Assume the optimal solution has been reached and that some x_{ik} is fractional. Since $\sum_{j=1}^{n} X_{ij} = 1$, there must be at least one other community r within the set of n communities, for which x_{ir} is fractional. Unless both r and k are equally distant from i, it will be less costly for community i to assign its population to the closest community, indicating that the optimal solution is not yet achieved. Thus, at the optimum, x_{ij} is zero or one. If several communities which are designated as centers are equally distant from community i, then fractional x_{ij}'s are possible at the optimum, but such solutions are alternate optima and can obviously be replaced in which i is assigned to only one community."

2) P-Median Problem modified with an Investment Constraint.

In a more recent article Rojeski and ReVelle (1970, p. 34) remove the need to specify the number of facilities (m) by introducing two constraints:

a) An investment constraint,

$$\overbrace{\sum_{j=1}^{n} f_j X_{jj}}^{A} + \overbrace{\sum_{j=1}^{n} b_j \sum_{i=1}^{n} a_i X_{ij}}^{B} \leq C$$

which limits the total fixed costs A plus the total variable cost B to a resource level, C.

where f_j = fixed cost of opening facility j

b_j = variable cost expansion coefficient of facility j.

It should also be noted that the fixed and variable cost of facilities may vary among locations.

b) A closest center constraint,

$$X_{ij} \geq X_{jj} - X_{ii} \text{ for adjacent } i\text{-}j \text{ pairs}$$

which limits assignments to closest facilities. In practise the closest center constraint is employed only when an initial solution has been obtained and a community is not assigned to its closest facility.

The objective function of the Rojeski/ReVelle design is to minimize the average distance travelled to facilities given the constraining budget. The authors feel that the resource or budget level is a more meaningful constraint than the number of facilities because it relates better to the actual decision making process. They also point to the usefulness of knowing the tradeoff relationship between the minimization of average distance travelled and the budget level.

3) Mini-Max Problem.

Toregas (1971) focused on the mini-max problem where the objective is to minimize the maximum distance that any one user has to travel to reach a facility. In respect to public facility location, Toregas considered this to be an advance on the P-Median problem because the concept of average distance travelled by the general public has no real meaning to the individual. He also felt that planners and decision makers would relate better to a facility plan objective which was to contain maximum travel distances rather than minimize an obscure mathematical average.

Toregas' design approaches the mini-max problem in reverse. Given a specified maximal service distance, the objective function minimizes the number of facilities. The design is given as follows (Toregas, 1971, p. 21):

"

$I = (1,2,\ldots,m)$ = set of nodes demanding service

$J = (1,2,\ldots,n)$ = set of nodes which are potential facility sites

d_{ij} = the shortest distance (time) from node i to node j.

S = the maximal service distance

X_j = 1, if a facility is located at j, 0 otherwise.

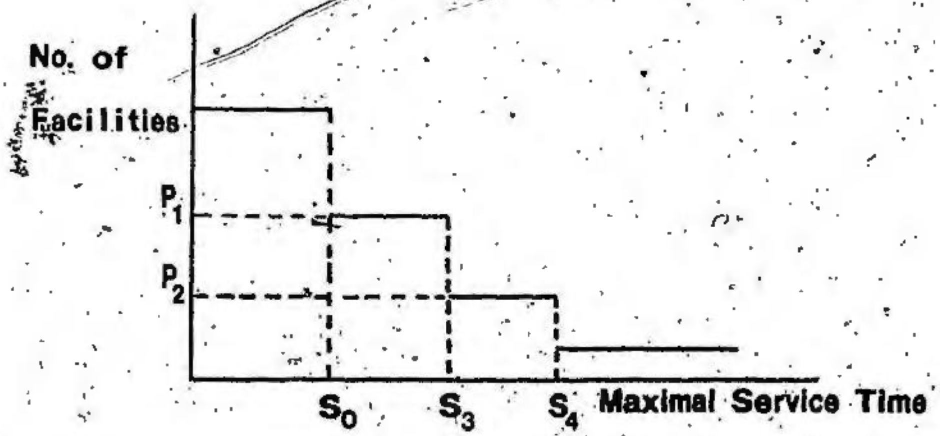
A service subset Ni is defined for node i as the set of nodes j which are candidates for the siting of a facility to serve node i, each member of this set j must be no more than S units away from node i, that is:

$$N_i = (j \in J \mid d_{ji} \leq S) \text{ for all } i \in I$$

Note: S may be different for each location i. We may now structure the basic model:

$$\text{Minimize } \sum_{j \in J} X_j \text{ subject to: } \sum_{j \in N_i} X_j = 1 \quad i \in I$$
$$X_j = (0,1) \; j \in J$$ "

Toregas (1971, p. 22) produces what he terms a "cost effectiveness curve" to identify the optimal solution to the mini-max problem (Fig. 4).



source: Toregas,1971, p.22

FIGURE 4 COST EFFECTIVENESS CURVE

Given P_2 facilities, the minimum maximum distance that any one user would have to travel in order to reach a facility is S_3. To improve on this, the number of facilities would have to be increased to P_1.

II-B The Linear Programming Design

The following design combines the methods of ReVelle, Rojeski, Swain and Toregas in order to identify the minimum investment level C required to provide for a given maximum student travel time S. In addition, the design is further extended to consider the value of existing facilities to the school system over time. The objective function of the model is to minimise the total time travelled by all students within the system, given that no individual student will have a travel time which exceeds the maximum time constraint and that total facility costs within the system over a specified investment period do not exceed the investment or budget constraint.

The objective function, which may be solved with the simplex method of linear programming, is

$$\text{MIN} \sum_{i=1}^{n} \sum_{j \in N_i} a_i d_{ij} X_{ij}$$

subject to the following constraints:

1) The first constraint ensures that all students from each community are fully assigned to one school and that at least one school is within the maximum travel time of each community.

$$\sum_{j \in N_i} X_{ij} = 1 \qquad i = 1, 2, \ldots, n$$

For example, when $i = 1$, N_1 is the set of communities where it is possible to locate a school j to serve community 1 within the maximum travel time and community 1 can assign to one and only one of the j school locations in N_1.

2) The second constraint states that a community may only assign to a self-assigned community (ie. one with a school).

$$X_{jj} \geq X_{ij} \qquad i = 1, 2, \ldots, n \quad j \in N_i,\ i \neq j$$

For example, if x_{jj}=1, then $_j$ is a potential location for an assignment of students $x_{ij} = 1$ from community $_i$, given that $_j$ belongs to the subset N_i. If $x_{jj} = 0$, then no assignment is possible.

3) The third constraint limits the sum of all costs associated with the construction and operation of school facilities in the system over a given investment period to a value within a budget level C.

$$\overbrace{\sum_{j=1}^{n} f_j x_{jj}}^{A} + \overbrace{\sum_{j=1}^{n} b_j \sum_{i=1}^{n} a_i x_{ij}}^{B} \leq C$$

where f_j is the constant or fixed cost of a school facility $_j$.

b_j is the variable cost coefficient related to the size of school $_j$.

$\sum_{i=1}^{n} a_i x_{ij}$ — is an index of the size of school $_j$ related to the number of students (a_i) assigned to the school.

If a school were not located in community $_j$, x_{jj} would equal zero and $\sum_{i=1}^{n} a_i x_{ij}$ would equal zero so that no cost would be attributed to that community under this constraint.

If a school facility were located in community $_j$, x_{jj} would equal 1, part A of the equation would equal f_j

and part B would equal b_j times the number of students from all assigning communities

By summing the costs for each school j, the total cost of the system is compared to the budget level C for each potential solution generated during the optimizing procedure.

4) The fourth constraint restricts communities to assigning to their closest school.

$$X_{ij} \geq X_{jj} - X_{ii} \quad \text{for adjacent } i\text{-}j \text{ pairs}$$

For example, if $x_{ij} = 0$ and j is the closest school to community i (ie. $x_{jj} = 1$) and i does not self-assign (ie. $x_{ii} = 0$), the constraint is violated. This constraint is only employed if a community i is assigned to other than its closest school j in the initial solution.

5) The fifth constraint restricts assignments to non-negative values.

$$X_{ij} \geq 0 \qquad \begin{array}{l} i = 1, 2, \ldots, n \\ j \in N_i \end{array}$$

6) An optional constraint can be added to stipulate which community locations must have a facility and which are not suitable.

$x_{ii}=1$ if community $_i$ is to have a facility
$x_{ii}=0$ if community $_i$ is not to be considered for a facility.

As mentioned in the introduction to this chapter, a common planning application of the location-allocation problem appears in the need to identify optimum changes to existing school facility systems. For this reason constraint 3 is modified to account for the value of existing schools.

$$\sum_{j=1}^{n} (f_j - P_j)\, x_{jj} + \sum_{j=1}^{n} b_j \sum_{i=1}^{n} a_i\, x_{ij} \le C$$

Where P_j is the value of the existing school at $_j$.

The value of an existing school P_j is only credited if the school is used in the final solution (ie. $x_{jj} = 1$). A discussion of the amount credited if only part of the capacity of the school is used will be provided in the actual application of this design in Chapter V.

Given a specific budget level C, computer generated solutions will generally not contain all zero-one x_{ij} assignment variables. Rojeski and ReVelle (1970, p. 359) have found, however, that by varying the budget slightly from the given level, fractional assignments ($x_{ij} \neq 1$ or 0) and schools ($x_{jj} \neq 1$ or 0) can be eliminated. This method will also be discussed and demonstrated by example in Chapter V.

The solution generated by this basic linear programming design minimizes the total time travelled by students within the budget, maximum travel time and locational constraints.

II - C. Application of the Model

The final objective in the application of this model is to determine the modifications to the existing location-allocation system which will limit the maximum student travel time to a socially acceptable level at a minimum total cost over the investment period without compromising the quality of education provided at the schools. The steps required to achieve this objective follow.

1. Minimize Facility Costs

The first step is to establish the minimum present value cost necessary to construct and operate schools within the system for the given investment period while still satisfying the maximum travel time constraint. The first computer run determines if the maximum travel time constraint can be met within a predetermined budget level. If there are ample resources in the budget to meet this constraint, the minimum cost level can be determined by rerunning the problem at decreasing budget levels until the linear programming solution becomes infeasible. Conversely, if it is not possible to meet the constraint within the given level, the model is rerun several times at increasing budget levels until the solution becomes feasible. At the margin of feasibility a location-allocation solution which minimizes the total time travelled while respecting both the maximum time and minimum budget constraints will have been found.

Because of the incorporation of the P_j value in the budget constraint, this solution defines the modifications to the existing location of schools and allocation of students which minimize costs. No new school facility will be created unless the maximum travel time constraint is exceeded and/or no existing facility abandoned unless there are greater savings in amalgamation.

2. Determine Transportation Costs

The second step in defining the least cost solution is to determine the present value cost of transportation for the given investment period. Transportation costs are assumed to be a function of the total travel time for each school and therefore can be calculated from student-school assignments as follows:

$$TC = f\left(\sum_{i=1}^{n} a_i d_{ij} x_{ij}\right)$$

3. Minimize Total Costs.

The third step combines the facility budget level and transportation costs to determine the total present value cost to the school system for the investment period. If the budget allocated to the construction and operation of schools were increased above the minimum level, there could be an increase in the number of facilities and subsequently a reduction in total travel time and transportation costs. Since the decrease in transportation costs could offset the increase in the budget, the total school system costs should be examined for several alternative budget levels. Combined facility and transportation costs can be identified at increasing budget levels starting at the minimum until an upward trend appears in the total. A hypothetical total cost curve is constructed in figure 5 for alternative budgetary solutions.

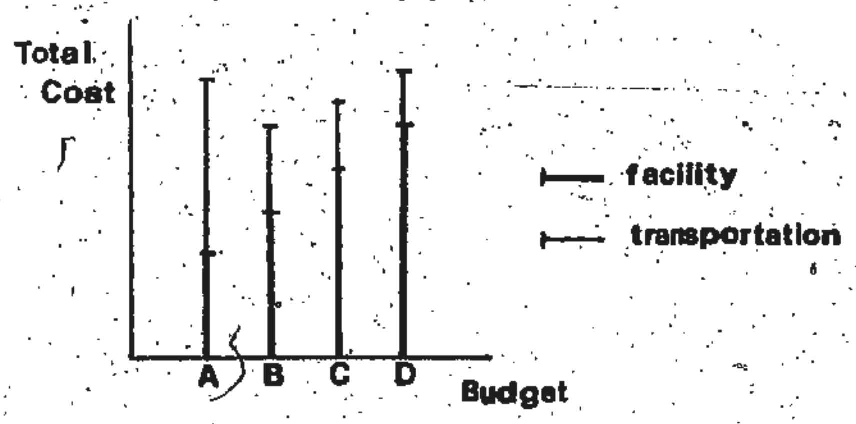


FIGURE 5 TOTAL COST CURVE FOR BUDGET ALTERNATIVES

Budget B in figure 5 would contribute the location-allocation system which minimizes total costs over the investment period within the maximum travel time and locational constraints given.

4. Cost/Social Distance Tradeoff.

The fourth step is to compare the minimized cost for several maximum travel time constraints. Toregas (1971, p.4) indicates that ...

> "It would be quite difficult for the decision maker to decide on the particular level of this maximized service distance which would best represent the needs and desires of his area."

Thus, by generating several solutions, a tradeoff curve can be developed which will compare the value of the maximum travel time against the funding levels necessary to implement them. (Fig. 6).

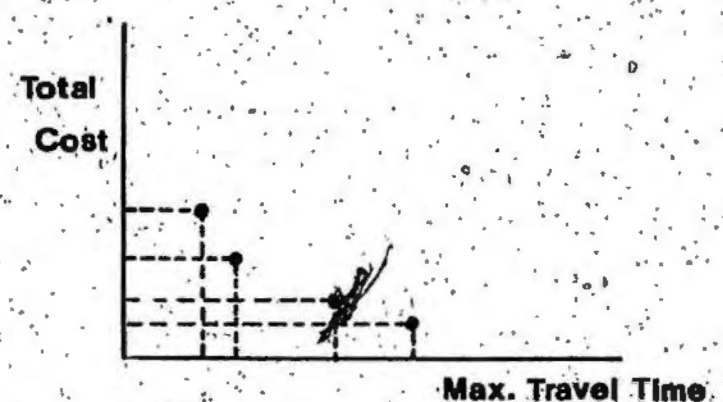


FIGURE 6 COST-TRAVEL TIME TRADEOFF CURVE

5. Impact on the Quality of Educational Services.

The final step is to evaluate each solution on the trade-off curve to ensure that the recommended changes in the size and location of schools within the system do not compromise the system's potential to maximize the education quality.

In this study the evaluation of the impact of changes in size and location on quality is accomplished through the derivation of an educational production function. To produce this function a surrogate measure of quality is regressed against an array of school input variables including size and location factors.

Assuming that size and location are important factors in the determination of school quality, the impact of changes in these factors can be estimated by isolating the unit rate of change in the dependent variable (quality) caused by the unit changes in the independent size and location variables. This can be achieved by limiting the range of variables considered in the multiple regression to those associated with size and location or by holding those variables not associated with size and location statistically constant and concentrating on the rate of change caused by the partial regression (b) coefficients of the size and location variables.

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Chapter III: An Existing School Location-Allocation System in North Central Newfoundland

This chapter describes the system of schools to be examined and the sources of the data required to operationalize the linear programming model.

III - A. Study Area

The system of schools to be examined covers an area designated as North-Central Newfoundland (Fig. 7). In Newfoundland, secondary educational responsibility is denominational. This study investigates only those schools and students which are under the administration of the Integrated Educational Committee of Newfoundland. Although there are five districts encompassed by the study area (Fig.7), it is assumed that for the purposes of facility planning they can be treated as one.

Students are located in 86 communities within the study area. Of these communities, those with a student population under 20 (minimum classroom size) or a total population of under 250 (size necessary to provide minimum ammenities to the school) were considered impractical locations for potential schools. Thus, 21 communities out of 86 were dismissed as potential facility sites. Appendix I lists the various communities in the study area, their total population, student population and locational suitability. Communities are only listed

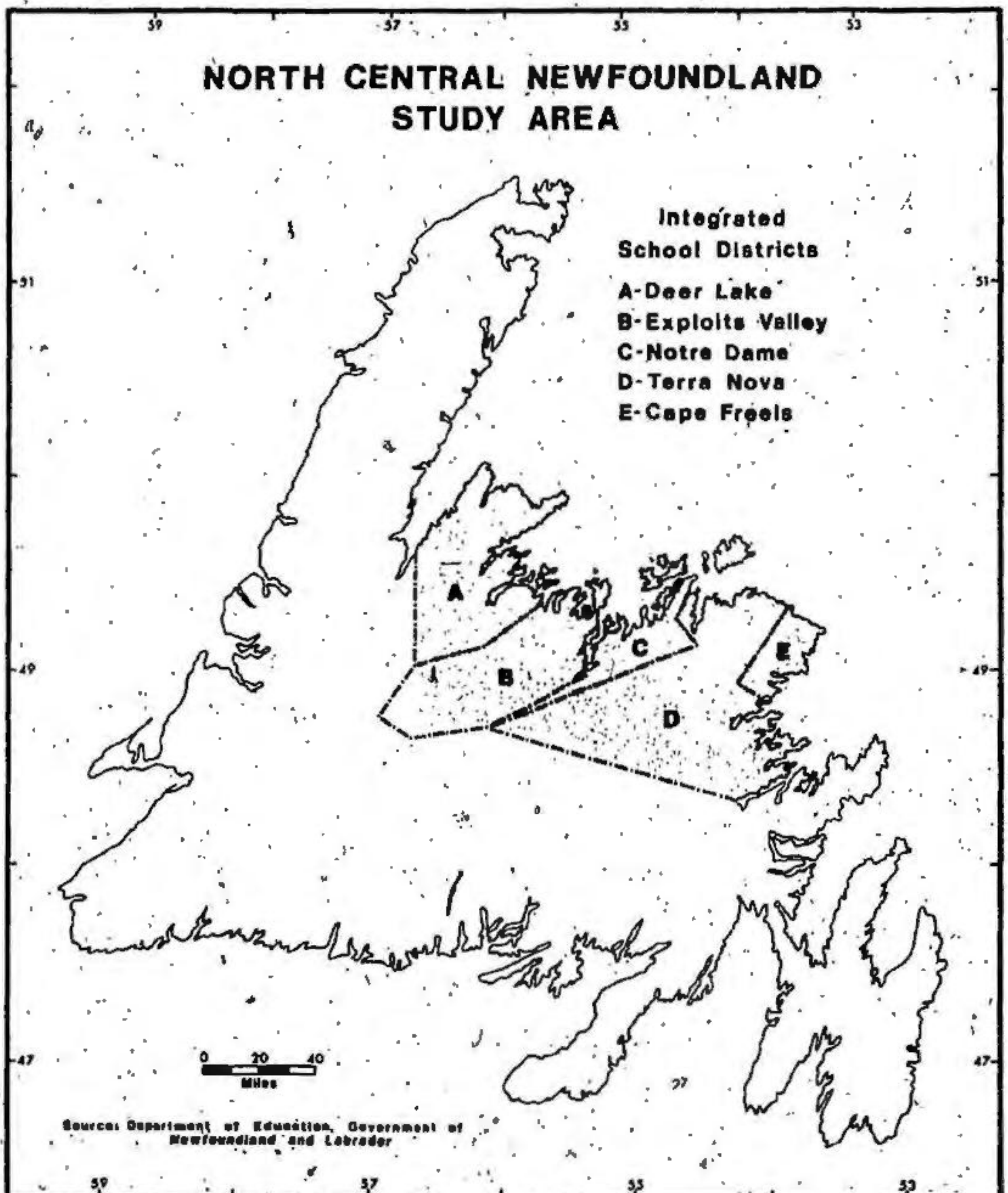
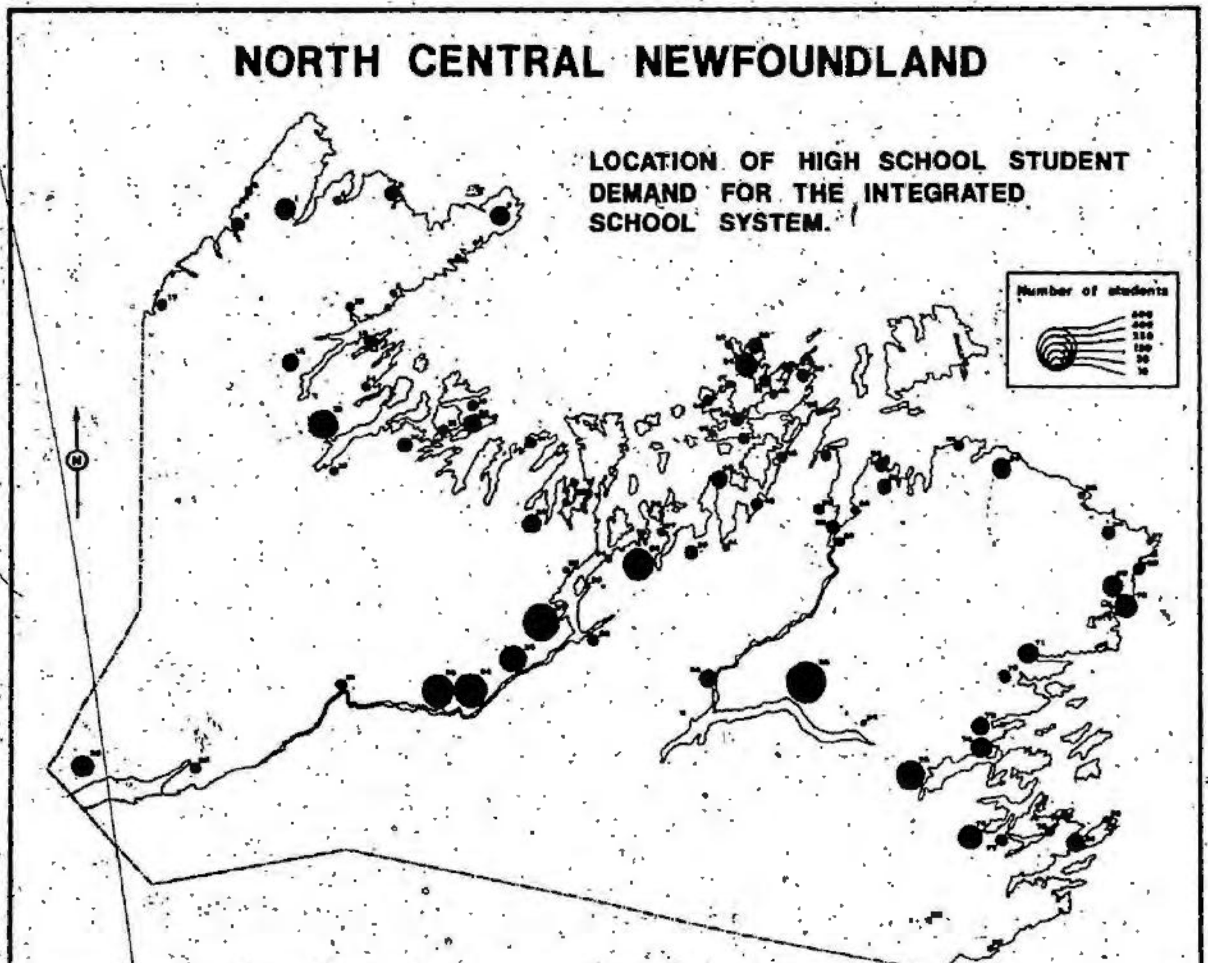

NORTH CENTRAL NEWFOUNDLAND
STUDY AREA
Integrated
School Districts
A-Deer Lake
B-Exploits Valley
C-Notre Dame
D-Terra Nova
E-Cape Freels
A
B
C
D
E
59
57
55
53
51
49
47
0 20 40
Miles
Source: Department of Education, Government of Newfoundland and Labrador

FIGURE 7

if their 1973 high school student population is greater than one. Figure 8 shows the distribution of student population by community over the study area. The population trends shown in Figure 9 indicate a stabilization in total population and student population over the study area. Because of this stability, erratic changes in the distribution and volume of student demand are not anticipated in the short run.

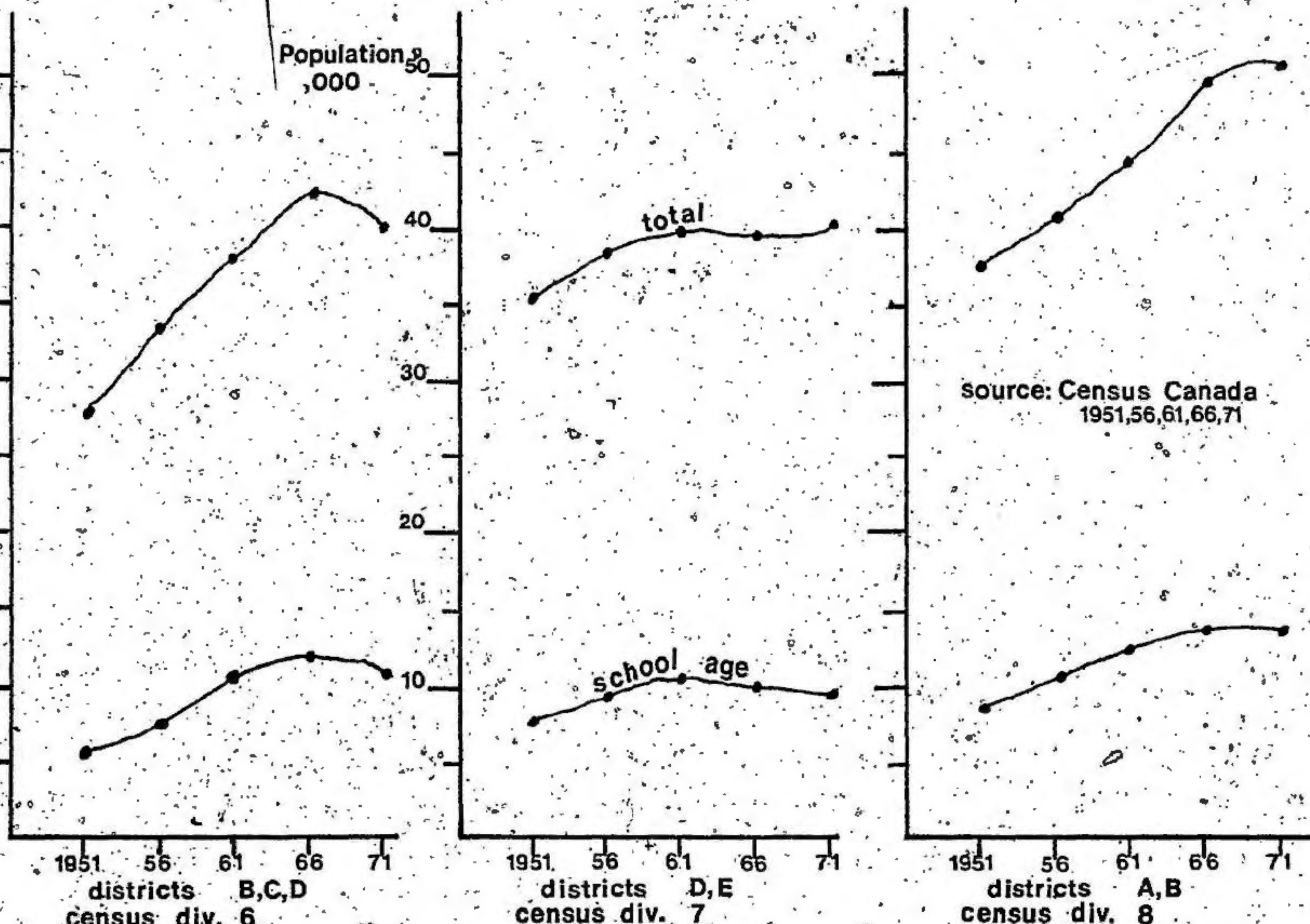
Each designated community node was assumed to have an internal travel time of 10 minutes. A network linking the designated nodes was constructed to represent the travel time between communities by school bus. The estimates of the travel time by bus corresponding with each link in the road network were provided by school principals in response to the questionnaire in Appendix II. On routes where there were no existing bus trips, the travel time was estimated on the basis of a regional rate per mile. The travel time matrix was developed directly from this data. It was assumed that there would be no major changes to the road network in the forseeable future. Forecast improvements could have been input into the travel time matrix by creating new links or reducing the travel time on existing links. The completed travel time network is illustrated in figure 10.

FIGURE 8



Source: The Newfoundland and Labrador Schools Directory, Department of Education, Government of Newfoundland and Labrador, 1972-73.

FIGURE 9



POPULATION TRENDS IN NORTH CENTRAL NEWFOUNDLAND

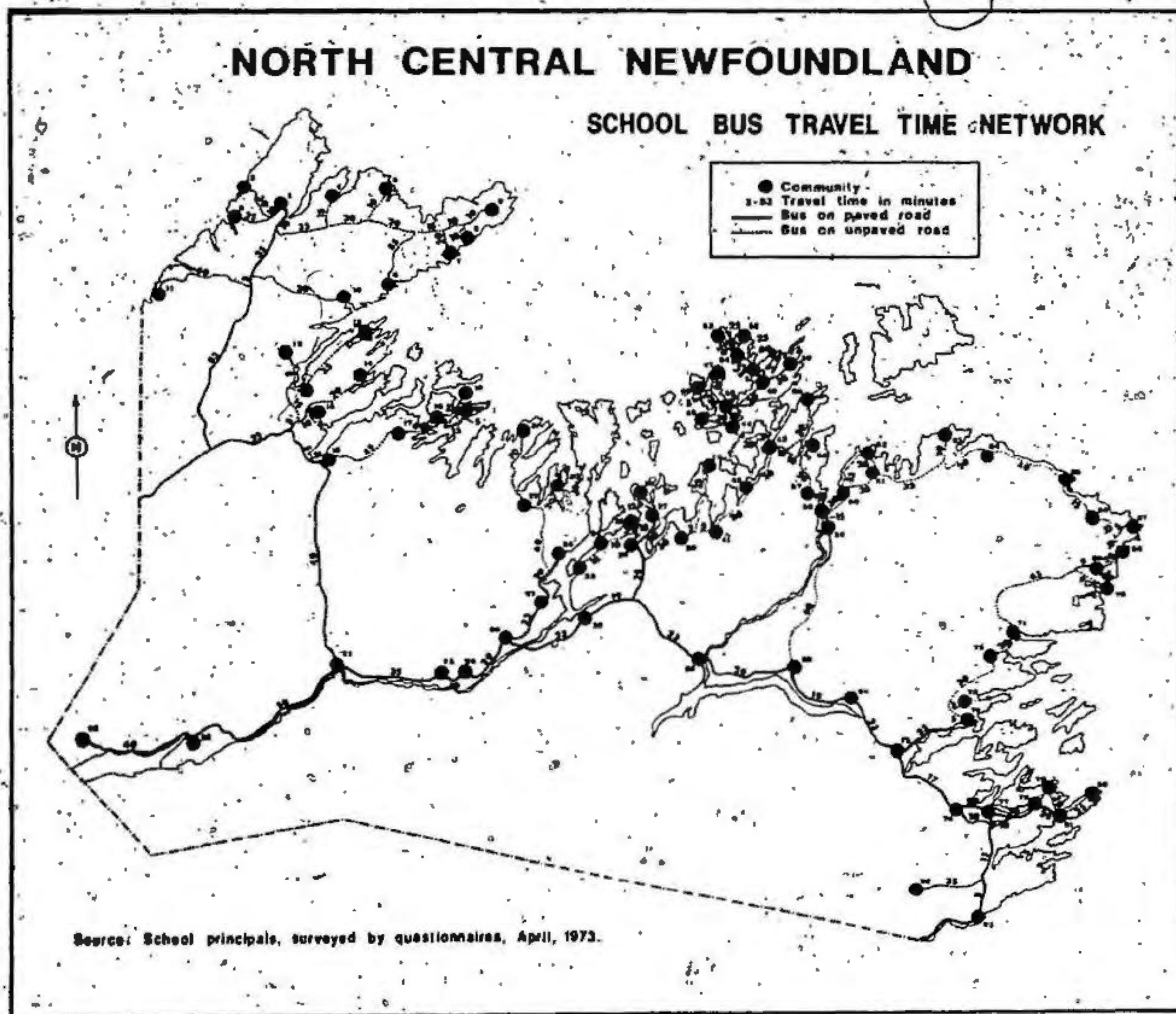

NORTH CENTRAL NEWFOUNDLAND
SCHOOL BUS TRAVEL TIME NETWORK
Community
Travel time in minutes
Bus on paved road
Bus on unpaved road
Source: School principals, surveyed by questionnaires, April, 1973.

FIGURE 10

Figure 11 shows the existing configuration of school facilities and student trips in the study area.

III - B Data Sources

The variety of sources of study area data required to operationalize the linear programming model follow.

1) Population by Community.

a. Source - Statistics Canada, Population, Province of Newfoundland and Labrador - Incorporated cities, towns, villages, and unincorporated communities, 1971.

b. Purpose - to identify those communities within the study area which are of sufficient size to be potential school locations.

2) Student population by school.

a. Source - Government of Newfoundland and Labrador, The Newfoundland and Labrador Schools Directory, 1972-73.

b. Purpose - to identify the size and location of existing Integrated high schools in North-Central Newfoundland.

3) Student population by community.

a. Source - Questionnaire, shown in Appendix II, sent to school principals within the study area in April, 1973.

b. Purpose - to identify the spatial distribution of student demand within the study area.

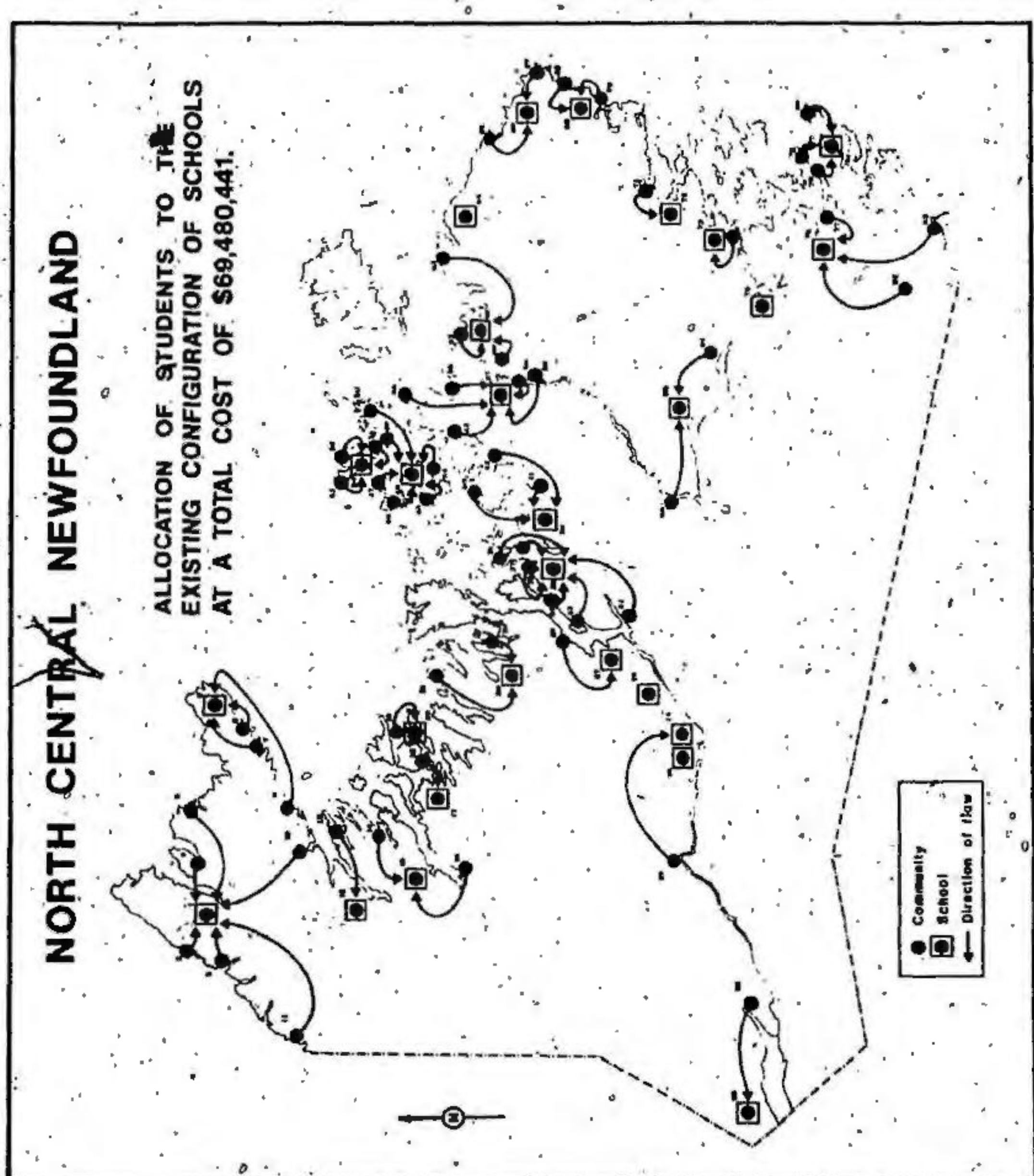

NORTH CENTRAL NEWFOUNDLAND
ALLOCATION OF STUDENTS TO THE
EXISTING CONFIGURATION OF SCHOOLS
AT A TOTAL COST OF $69,480,441.
Community
School
Direction of flow

FIGURE 11

4) Student and total population trends by census subdivision.

a. Source - <u>Census Canada</u>, 1951, 1956, 1961, 1966, 1971

b. Purpose - to identify trends in the volume and distribution of student demand which might impact on facility planning in the study area.

5) Travel time on transportation network links via school bus.

a. Source - Questionnaire, shown in Appendix II, sent to school principals within the study area in April, 1973.

b. Purpose - to obtain an approximation of current bus travel times on the existing network which can be related to the maximum travel time objective and transportation costs.

6) Maximum travel time objective. Given the study area in question, a maximum travel time of 30 minutes is considered an optimum objective in the provision of educational services.

a. Source - Interview with the Executive Secretary of the Integrated Education Committee on March 28, 1973. This informed judgement was taken in lieu of a complete survey of both educators and students.

b. Purpose - to measure the performance of the existing system and to gain an objective or starting point for the cost-social distance tradeoff.

7) Bus transportation costs by school.

a. Source - Questionnaire, shown in Appendix III, sent to the school boards in April, 1973.

b. Purpose - to relate transportation costs for each school in the study area to student travel time on buses.

8) Student travel time on buses by school.

a. Source - Questionnaire, shown in Appendix II, sent to school principals within the study area in April, 1973.

b. Purpose - to relate student bus travel for each school to the transportation costs recorded for the same schools.

9) School construction cost index for North-Central Newfoundland. The index is based on a cost per square foot multiplier. The base cost for Toronto in 1961 allowing 80 miles for the transportation of manpower and materials is $11.58 per square foot. The chronological readjustment multiplier to 1973 is 2.109. The geographical adjustment multiplier for Cornerbrook, Newfoundland is .8027. For contracts less than 10,000 square feet, the plus factor is 10% for a complete cost of $21.57 per square foot and for contracts over 10,000, the plus factor is 6% for a

complete cost of $20.79 per square foot.

a. Source - Director General, Technical Services and Special Projects Division, D.R.E.E., School Construction Canada-Cost Evaluation Index, prepared by CERTEC Inc., Montreal, 1973.

b. Purpose - to determine the cost of replacement and the value of Integrated high schools in North-Central Newfoundland, as well as the cost of constructing new facilities in the area.

10) Square footage of existing Integrated high schools in North-Central Newfoundland.

a. Source - Space inventory and floor plans, Department of Education, Gov't of Newfoundland and Labrador.

b. Purpose - to be used in conjunction with the construction cost index to determine the cost of school replacement.

11) Original construction dates for the Integrated high schools in North-Central Newfoundland.

a. Source - Facility profiles from the data base for the Report on Schools in Newfoundland, prepared for the Dept. of Education by P. Warren and B. Fisher.

b. Purpose - used in the estimation of the depreciated value of existing schools in the study area.

12) Operation and maintenance costs for Integrated high schools ranging in size from 40 to 1000 students.

a. Source - Questionnaire, shown in Appendix III, sent to all Integrated school boards in April, 1973. This questionnaire was designed to be compatible with the cost data collection format used by the Department of Education, A Reporting Manual for School Boards, Government of Newfoundland and Labrador, January, 1973.

b. Purpose - to determine the annual expenses which are attributed to operating various sized schools in Newfoundland.

III - C. An Educational Production Function for Integrated Schools in Newfoundland.

As in past studies of educational production functions (Bowles, 1970 and Burkhead, Fox and Holland, 1967) multiple regression was used to estimate changes in the quality of output accounted for by various factor inputs to the educational system.

Since no uniform achievement or aptitude testing exists in Newfoundland, the average grade 11 score by school was accepted as the best available measure of the quality of school output. Because these scores were arrived at using two evaluation procedures; shared evaluation, 50% by the school and 50% by the province and provincial evaluation, 100% by the province, the type of evaluation by school was added to the regression analysis as a zero-one input variable.

Other input variables, purported to effect the quality of education (eg, Bowles, 1970, Burkhead, Fox and Holland, 1967, Dawson, 1969 and Kiesling, 1967) and used in this analysis are listed below.

	Type	Variable	Source
a.	Input-socio-economic	% urbanization	by census sub-division, 1966, Statistics Canada.
		Average level of education	by census sub-division, Copes, 1972, p. 57
		Average income per family	by census division 1961, Statistics Canada.

Type	Variable	Source
b. Input-school Environment	Enrollment	Schools Directory, Dept. of Education, 1972-73.
	Cost/Student	Questionaire, Appendix IV.
	Enroll/capacity	Warren/Fisher Report, 1973*
	% change in enrollment in 1970-72	Schools Directory, Dept. of Education, 1970-71, 1972-73.
	Library capacity	Warren/Fisher Report, 1973*
	Instructional spaces	Warren/Fisher Report, 1973*
	Age of Structure	Warren/Fisher Report, 1973*
	Pupils/teacher	Warren/Fisher Report, 1973*
	Overall Facility rating	Warren/Fisher Report, 1973*
c. Output-quality	Average score on Grade 11 Provincial exams.	by school, 1972 scores, Dept. of Education

* The data used in the analysis is not found in the body of the Warren-Fisher Report on Schools but was actually contained on computer cards which were used to compile the report.

The regression analysis was based on data compiled from a sample of 50 Integrated High Schools in Newfoundland. Of the variables regressed against the grade 11 score, the following were entered into the regression equation at an F value greater than 1(Table 3.1).

Table 3.1: Educational Production Function

Variable	Coefficient (b)	RSQ
Evaluation System	37.30127	.31
Age of Structure	-3.21296	.35
Average Level of Education	15.57461	.40
Cost/Pupil	0.00444	.43
Enroll/Capacity	18.23474	.46
Average Family Income	-0.01568	.49
Enrollment	0.12548	.50
Instructional Spaces	-2.57128	.52
Constant (a)	301.42017	

The regression equation has a correlation coefficient of .7199 and explains .5182 of the variance in grade 11 scores. Possible transformations were considered for all data. However, scattergrams of each independent variable plotted against the dependent variable revealed that no transformations would lead to a better fitting relationship.

Although the level of explanation is low at .518, the regression equation can be used to estimate changes in quality since no other data is available which would lead to a better estimate. The level of explanation achieved compares to the following studies:

	Output Variable	Explanation Achieved	Reference
1.	Verbal Achievement Black Students	.30	(Bowles, 1970, p. 22)
2.	Achievement Test Score	.343	(Kiesling, 1967, p. 356-367)
3.	Change in Reading Score	.711	(Katzman, 1971).

Enrollment, average family income by school location, average level of education by school location and average cost are identified as the variables within the regression model which will change with and could be measured for each alternative solution generated by the linear programming model. The impact of changes in any of the size and location variables on the quality of education at the various schools can be measured by multiplying the variable's unit change by its b coefficient.

References

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Chapter IV: Costs and Performance for the Existing School System in N. C. Newfoundland

This chapter provides the interface between study area data and methodology in describing the cost and performance of the existing school location-allocation system.

IV - A. Investment Period

The average life or investment period for a school facility was assumed to be twenty-five years since the ages of all operating schools in the system were within this time frame. Twenty-five years can reasonably be considered an appropriate time for replacement or extensive redesign and renovation.

In order to account for facility depreciation and continued locational redundancy, the continuing costs of school replacement, operation and transportation over the twenty-five year period are discounted to a common present value.

IV - B. Value of Existing School Facilities (P_j)

The value of each school in the existing system is calculated on the basis of its present cost to replace minus the discounted cost of actual replacement required during the 25 year investment period. Existing school replacement costs are calculated using the Department of Regional Economic Expansion's school construction cost index of $21.57 per square foot for contracts less than 10,000 sq. ft. and $20.79 per sq. ft. for contracts over 10,000 sq. ft. (see Chapter III-B-9). The following example illustrates the computations used to calculate the value of the school facility at Point Leamington.

Value of Point Leamington High School.

1) Total Square Feet 13,461

Construction Cost Index x 20.79

Cost to Replace $ 279,864

2) Facilities Constructed — Date

4 rooms 1966

3 rooms 1970

3) Expected functional life of improvements - 25 years.

4) Replacements during Investment Period.

Facility	Time	Cost	Cost Attributed to I.P.
4 rooms	18 years	4/7(279,864)=159,522	7/25(159,522)=44,666
3 rooms	22 years	3/7(279,864)=120,341	3/25(120,341)=14,441

5) Present Value Cost of Replacement

Discount Rate = 5%

4 rooms $44,666/(1.05)^{18}$ = $18,560

3 rooms $14,441/(1.05)^{22}$ = $ 4,937

6) Value of Existing School

Cost to Replace	$279,864
less Present Value Cost of Replacement	23,497
Value	$256,367

Assumptions:

Item 3: It is assumed that schools will have a functional life of no longer than 25 years (the investment period) before they are replaced or extensively remodelled.

Item 4: Costs at the year of replacement are assumed to be equivalent to the present day cost to replace. In future applications, it is recommended that construction cost be subjected to an escalation factor.

Costs are assumed to be in direct proportion to the physical portion of the building in need of replacement.

The costs attributed to the investment period are assumed to be in direct proportion to the portion of the replacements' functional economic life which passes during the investment period.

Item 5: The discounting formula used to calculate present value cost (PVC) is

$$PVC = \frac{A_n}{(i+1)^n}$$

where An - is the amount to be discounted from the n^{th} year.

$(i + 1)^n$ - is the discount factor or one plus the interest rate $_i$ all to the $_n$th power.

The discount rate of interest of 5% may be considered low depending on the opportunity cost appropriate to investments in education in Newfoundland. A higher rate would further reduce the impact of future costs on present values.

Item 6: In future applications it is recommended that the Present Value Cost of Replacement be calculated by discounting the construction costs incurred in the year of replacement and subtracting the discounted residual or depreciated value of the new structure at the end of the investment period. This method is superior because it values the cost of replacement as of the date of replacement (ie. today's costs escalated to that date and discounts this cost and the residual value as they occur.)

IV - C. Development of the Budget Constraint

1. Background to Facility Size-Cost Relationships

Alesch and Dougharty (1971) conducted a study into the feasibility of determining the extent to which differences in the size of facilities affect the unit cost of producing public services. They concluded that ..." If the research effort is vigorous, it is very likely that it is possible to learn whether the unit costs of production and size are related. It is highly unlikely however, that the analyst could define a "most efficient" size for providing very complex services". (1971, p. 16)

Most authors investigating the relationship between size and cost for public facilities have attempted to define the "most efficient" size of a facility. Generally they have hypothesized that the long run average cost curve would be parabolic in shape, commencing with economies of scale, following with an area of constant returns to scale (the area of the "most efficient" sized facility) and ending up with diseconomies of scale. Most studies (eg. Dawson, 1969 and 1972, Hirsch, 1959, Katzman, 1971, and Riew, 1968) use multiple regression analysis to test this hypothesis. Katzman (1971) has produced the closest fit in his study of educational services. His regression estimate of the average cost curve is given by:

$$AC = \text{constant} + a_1\ (\text{enrollment}) + a_2\ (\text{enrollment})^2 + a_3\ (\text{capacity}) + a_4\ (\text{capacity})^2 + a_5\ (\text{capacity}\cdot\text{enrollment})$$

He found that only coefficients a_1 and a_2 were significantly different from zero and that they explained 53% of the variance (Katzman, 1971, p. 87). A point of agreement among the various studies is that average cost is not simply a function of facility size but also the quality of service conditions, physical inputs and the state of technology.

In his study of the Ontario school system, Dawson (1969) gave a quality weighting to each of the various sized schools in his sample. Economies of scale are likely to exist in school facilities due to the indivisibility of the basic factors in the production of education, that is the classroom, gymnasium, teacher, principal and various technical teaching aids. However, as suggested by Alesch and Dougharty, Dawson was unable to produce an estimate of the long run average cost curve or define the "most efficient" size of a school because of the complexities demonstrated in his data on existing educational services. He did, however, have success in defining a least squares estimate of the total cost curve. Given the comparative shape of the hypothesized total cost and average cost curves (Fig. 12), deviations in the sample data are likely to make it more difficult to produce an estimate of the average cost curve than an estimate of the total cost curve.

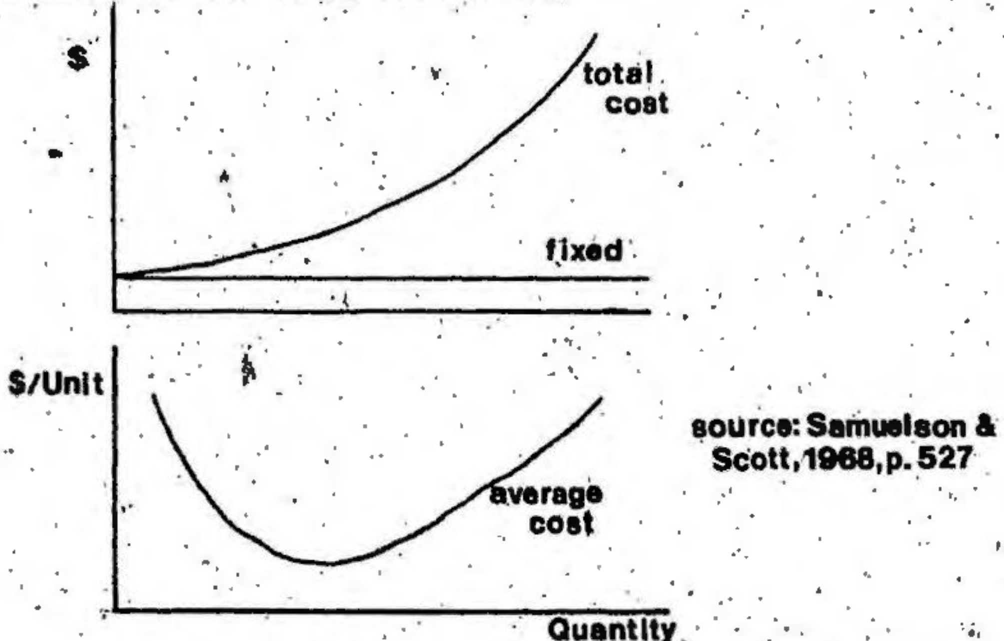

FIGURE 12 TOTAL AND AVERAGE COST CURVES

Dawson (1972, p. 307) found increasing returns to scale or constant returns to scale for almost all types of school facilities examined. Hirsch (1959, pp. 232-3) and Dawson (1969, pp. 11 and 48) suggest that in the case of most public services, diseconomies of scale should not be expected from data on existing systems because diseconomies would lead to the opening of new facilities and the consideration of the social distance between participant and facility would limit the maximum size of any one facility.

Although Dawson's formulation (1972, p. 307), has some nonlinear components, the total cost curve as shown on figure 12 lends itself to a fairly accurate linear estimation. The availability of a linear estimate is critical in the linear programming design where all constraints must be linear.

2. Budget Constraint

The budget constraint is given:

$$\sum_{j=1}^{n} (f_j - P_j) x_{jj} + \sum_{j=1}^{n} b_j \sum_{i=1}^{n} a_i x_{ij} \leq C$$

Where f_j - is the constant or fixed cost of school facility j

b_j - is the variable cost coefficient related to the size of school j.

The procedure used to set up this constraint is to:

a. compute the total present value cost of constructing as new and operating the various sized schools in the study area over the investment horizon.

b. relate the total cost to the size of each school (measured by the enrollment) using linear regression analysis.

c. fit the regression equation into the constraint design, the a-intercept or constant as f_j and the b - coefficient or slope as b_j.

3. Preparation of the Budget Constraint from Sample Data on Existing Schools.

a. Construction Costs

Since the linear programming design identifies the size of a school by the number of students assigned $\left(\sum_{i=1}^{n} a_i X_{ij}\right)$, it is necessary to be able to estimate construction costs from enrollment figures. Replacement costs or as new construction costs were regressed against enrollment for the sample of schools in the study area (see Chapter III-B-9 & 10). Figure 13 shows the resulting distribution of points around the regression line. The level of explanation (R^2) is low at .536. Significant deviations from the regression line are generally the result of enrollment being substantially below school capacity. The most noteable example of this is Buchans' Integrated High

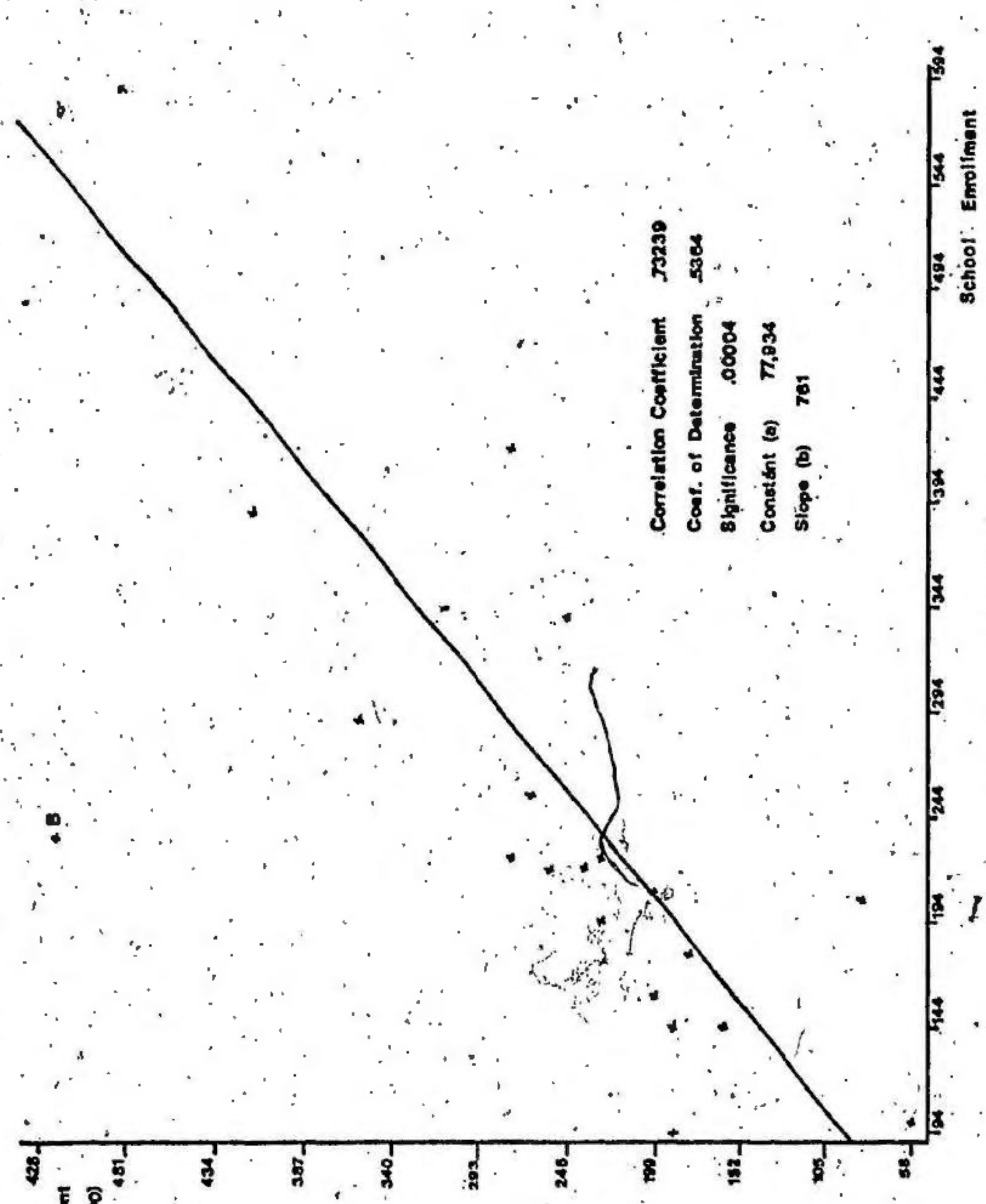

FIGURE 13 REPLACEMENT COST - SCHOOL SIZE RELATIONSHIP

FIGURE 13

School (point B in figure 13). The regression estimate (77,934 + 761 (enrollment)) of construction costs, therefore, allows for an enrollment/capacity ratio of less than 1. It is, however, accepted as the best average estimate of construction costs from enrollment for this analysis. In future applications it is recommended that the construction costs be regressed against enrollment capacity for each school.

b. Operation and Maintenance Costs

On the basis of the data collected from the school boards (see Chapter III - B - 12), the total present value cost of operation and maintenance over the twenty-five year investment period was calculated for each school in a sample of 50. Since costs were assumed to remain level over the period, the formula for the present value of an annuity (PVa) was used:

$$PVa = \frac{a}{i} \left(1 - \frac{1}{(1+i)^n}\right)$$

where a - was the annual operation and maintenance costs.

i - is the discount rate of interest - 5%

$(1+i)^n$ - is the discount factor

n - is the number of consecutive time periods the annuity lasts - 25 years.

A more accurate assumption recommended for future analysis would allow escalation in costs over time. In this problem, however, since the low discount rate of 5% gives substantial weight to future costs, the level cost assumption may not be seriously out of place.

c) Total Costs

The replacement cost of each of the 50 schools in the sample was estimated using the regression equation previously shown in figure 13. It was assumed that the total present value cost of each school was equal to its replacement cost plus its present value cost of operation and maintenance. Inherent in this assumption is that the costs of operation and maintenance do not vary with the age of a school since all costs were arrived at from data on existing buildings.

To facilitate compatibility with the budget constraint design;

$$\sum_{j=1}^{n} (f_j - P_j) X_{jj} + \sum_{j=1}^{n} b_j \sum_{i=1}^{n} a_i X_{ij} \leq C$$

total present value costs must be expressed as a linear function of student enrollment $\left(\sum_{i=1}^{n} a_i X_{ij} \right)$.

The total present value cost of each of the 50 schools was, therefore, regressed against its corresponding student enrollment. Figure 14 shows the resulting distribution of points around the regression line. The level of explanation (R^2) is excellent at .96 and so

FIGURE 14

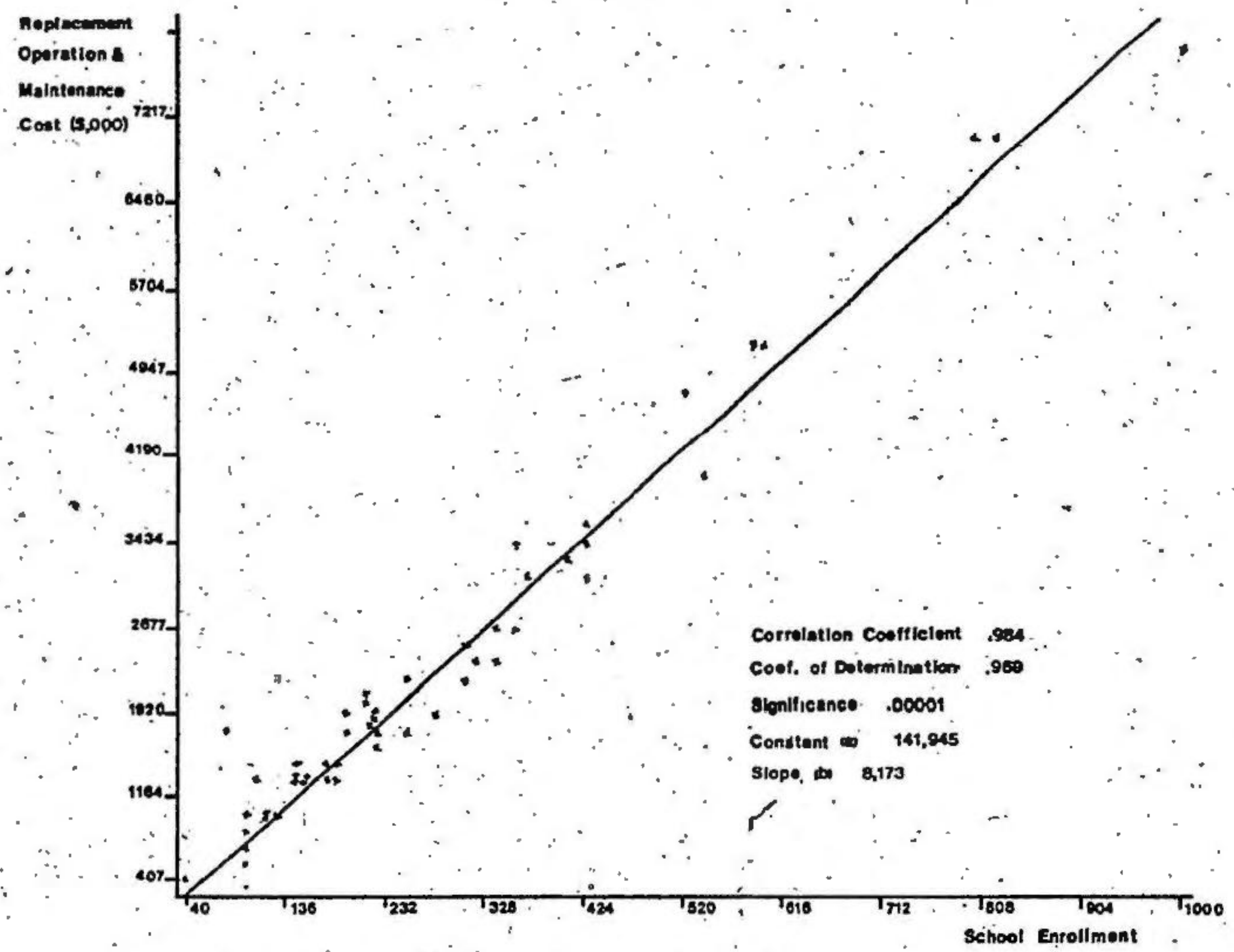

FIGURE 14 TOTAL REPLACEMENT, OPERATION AND MAINTENANCE COST-SCHOOL SIZE RELATIONSHIP

the regression equation (141,945 + 8,173 (enrollment)) was substituted into the budget constraint for f_j and b_j. The P_j values were input into the constraint for communities with existing schools.

The completed constraint will estimate the total present value cost as new of all the various sized schools generated within the linear programming solutions. When the model places a school in the location of an existing school, the estimated cost as new will be reduced by the value of this school. The model sums the cost of all schools within the system and ensures that this sum is within the budget level C for the final solution.

IV - D. Transportation Costs

After the solution which minimizes the cost of replacement, operation and maintenance has been generated, several alternative solutions with increased budget levels are tested to ensure that increases in transportation costs caused by having fewer and larger schools under a lower budget, do not actually exceed the savings achieved by lowering the budget. While the possibility of this happening is limited because of the distance constraint already in place, it must be considered in the overall approach to total cost minimization.

In the objective function of the linear programming design;

$$\text{MIN} \sum_{i=1}^{n} \sum_{j \in N_i} a_i \, d_{ij} \, X_{ij}$$

the notation $a_i\ d_{ij}\ x_{ij}$ designates the amount of student time from community i to j (ie. the number of students multiplied by the time-distance between community and facility). To facilitate compatibility with this design the relationship between the cost of transportation and student travel time is identified from cost data collected from the school boards (see Chapter III - B - 7) and the bus travel data collected from school principals (see Chapter III - B - 8). Complete data was collected for a sample of 20 facilities.

Current annual transportation costs were treated as an annuity over the twenty-five year investment period to arrive at the present value cost of transportation at each school. The discount rate of 5 per cent was used in the computations.

The present value cost of providing transportation over the investment period was regressed against the total student travel time for each facility. The resulting distribution of data points around the regression line is shown in Figure 15. With a level of explanation (R^2) of .755, the regression equation (-17736 + 63 (Total Travel Time to School in Minutes)) was accepted as the

FIGURE 15

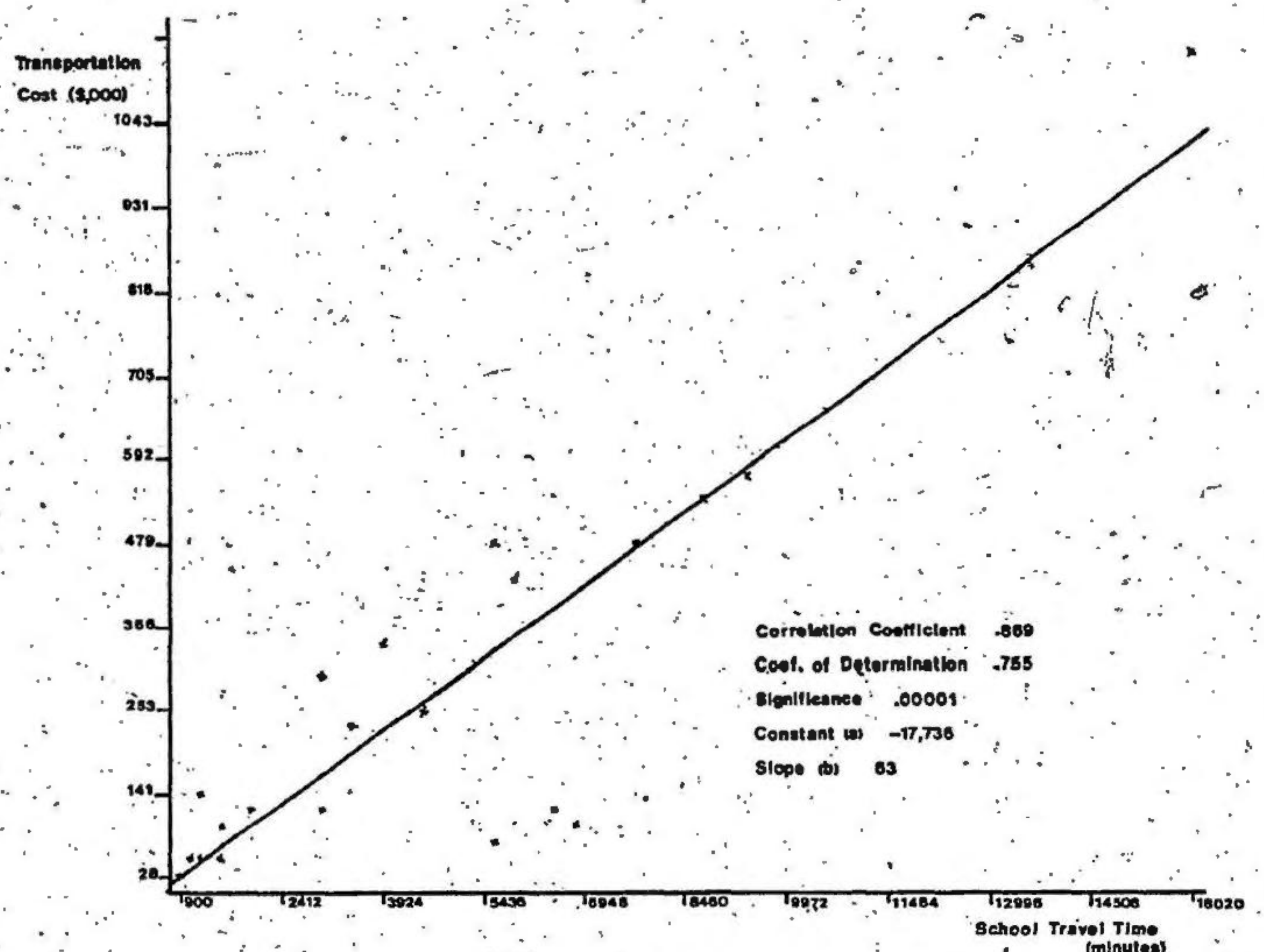

FIGURE 15 TRANSPORTATION COST/TRAVEL TIME RELATIONSHIP

best estimate of transportation costs. The threshold before any transportation costs are incurred was found to be 28 students travelling 10 minutes (thus accounting for the negative constant).

IV.- E. Performance of the Existing School System

1. Cost

The total present value cost of the existing system (Fig. 11, p. 48) is estimated at $69,480,441. This figure is based on the equations presented in this chapter for estimating the present value cost of replacement, operation and maintenance, and transportation allowing a credit for the value of the existing twenty-seven schools. A breakdown of the existing system's cost components is given in Table 4.1.

2. Travel time objective.

Table 4.2 identifies those communities whose students have a travel time exceeding the 30 minute travel time objective. The performance of the investment in the existing system is summarized as follows:

28% of the communities beyond 30 minutes of a school
11% of the students beyond 30 minutes of a school
11.5% of the communities beyond 45 minutes of a school
4.5% of the students beyond 45 minutes of a school
6% of the communities beyond 60 minutes of a school
3.1% of the students beyond 60 minutes of a school

TABLE 4.1: BREAKDOWN OF COSTS BY SCHOOL IN THE EXISTING SYSTEM

NO.	COMMUNITY NAME	NUMBER OF STUDENTS ASSIGNED	REPLACEMENT OPERATION AND MAINTENANCE COSTS ($)	STUDENT TRAVEL TIME (POP. - MINUTES)	TRANSPORT COSTS ($)	LESS VALUE OF EXISTING FACILITIES ($)	TOTAL ($)
1	Baie Verte	453	3,844,542	19,695	1,231,518	- 243,675	4,832,385
9	Lascie - Seal Cove	179	1,605,003	3,860	227,104	- 165,817	1,666,290
12	Rattling Hr. - King's Pt.	160	1,449,706	3,400	197,926	- 49,989	1,597,643
15	Springdale	396	3,378,652	4,856	290,280	- 432,330	3,236,602
17	Robert's Arm	134	1,237,195	1,555	80,898	- 145,102	1,172,991
20	Triton - Jim's Cv. - Card's Hr.	158	1,433,359	2,020	110,393	- 191,596	1,352,156
23	Buchan's	232	2,038,198	3,790	222,664	- 98,599	2,162,263
24	Grand Falls	483	4,089,747	6,615	401,853	- 297,847	4,193,753
25	Windsor	393	3,354,132	3,930	231,544	- 128,929	3,456,747
26	Bishop's Falls	288	2,495,914	2,880	164,942	- 148,589	2,512,267
27	Botwood - Peterview	524	4,424,860	5,410	325,420	- 85,878	4,664,402
29	Point - Leamington	207	1,833,861	5,740	346,352	- 256,367	1,923,846
36	Lewisporte	495	4,187,829	7,879	482,029	- 431,366	4,238,492
39	Campbellton	247	2,160,801	5,610	338,106	-246,973	2,251,934
48	Chanleport - Carter's Cv. - Virgin Arm - Fairbank	295	2,553,129	8,615	528,713	- 198,168	2,883,674
54	Twillingate - Bluff Head Cv.	350	3,002,671	5,910	357,135	- 299,317	3,060,489
57	Victoria Cv.	252	2,201,668	8,085	495,096	- 203,580	2,493,184
61	Carmanville	215	1,899,249	5,160	309,563	- 189,561	2,019,251
64	Musgrave Hr. - Doting Cv.	121	1,130,940	1,210	59,014	- 147,803	1,042,151
66	Lumsden	101	967,470	1,235	60,600	- 117,007	911,063
69	Pound Cove - Wesleyfield - Brookfield	391	3,337,785	6,880	418,662	- 321,027	3,435,420
72	Trinity	202	1,792,993	3,530	206,172	- 185,707	1,813,458
73	Wellington	284	2,463,220	2,840	162,405	- 169,955	2,455,670
75	Middle Hr. - Dark Cv. - Gambo	307	2,651,211	3,070	176,994	- 199,181	2,629,024
76	Glovertown	301	2,683,905	4,340	257,550	- 149,899	2,791,556
81	Eastport - Happy Adventure	207	1,833,861	2,990	171,920	- 133,773	1,872,008
85	Gander	718	6,010,519	8,410	515,710	- 389,279	6,136,950
	Totals		70,062,420		5,045,335	5,627,314	69,480,441

TABLE 4.2: COMMUNITIES OUTSIDE THE 30 MINUTE TRAVEL TIME OBJECTIVE

NO.	COMMUNITY NAME	NO. OF STUDENTS	TRAVEL TIME
2	Seal Cove	67	40 min
3	Wild Cove	18	45 min
4	Ming's Bight	25	1 hr
5	Pacquet - Woodstock	92	1 hr 15 min
6	Nipper's Hr.	20	1 hr 30 min
7	Snook's Arm - Round Hr.	13	40 min
10	Burlington - Smith's Hr.	31	1 hr 15 min
11	Westport - Purbeck's Cove	41	1 hr 30 min
13	Harry's Hr. - Jackson's Cove	60	40 min
21	Badger	51	45 min
22	Millertown Buchans Jct.	49	40 min
31	Leading Tickles	57	1 hr 10 min
32	Norris Arm	49	40 min
33	Laurenceton	12	40 min
38	Little Burnt Bay	5	35 min
43	Boyd's Cove	38	50 min
45	Cottle's Island	15	45 min
47	Tizzard's Hr.	19	35 min
50	Herring Neck - Too Good Arm - Fake's Arm and Cobb's Arm	60	1 hr
55	Port Albert	18	55 min
59	Gander Bay South	31	45 min
60	Davidsville - Maine Pt.	35	35 min
63	Aspen Cove - Ladle Cove	43	45 min
82	Terra Nova	10	1 hr

In the following chapter the linear programming model will be used to define the modifications to the existing locations of schools and allocations of students which will improve the performance of the school system at a minimum cost over the investment horizon without compromising education quality.

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Chapter V: Optimum Size and Location of Schools in North-Central Newfoundland

This chapter assembles the basic inputs and constraints and applies the linear programming model to the school location-allocation problem.

V - A. Data Assembly

Because of the large amount of data processing involved in setting up the variables included in the objective function and constraints, it is worthwhile to standardize the process in a short computer program. The program presented in Appendix IV was used to interface the data with a linear programming package from the ICES library.* The program converts the travel time matrix (d_{ij}), student demand (a_i), existing facility value (P_j) and school size-cost function (f, and b_j) into a form which is compatible with the problem design. It eliminates all X_{ij} pairs where the travel time between them is greater than the specified maximum S and all X_{jj} variables where j has been excluded from potential school locations.

* In this analysis the ICES (Integrated Civil Engineering Systems) package was used because of its availability at Memorial University. In August 1973, IBM's MPS linear programming package was being mounted for use at the University. Because of its advanced capabilities in large matrix manipulation, it is recommended for potential users.

Although ReVelle and Swain (1970, p. 35) suggest that constraint 2 ($X_{jj} \geq X_{ij}$) should be only included when needed, the program produces a full set of $X_{jj} \geq X_{ij}$ constraints for the first computer run, because the minimum cost solution occurs at the threshold of a non-feasible solution.

V - B. 30 Minute Solution

1. Initial Solution

The first problem is to define the modifications to the existing location of schools and allocation of students which minimize the costs of locating schools within 30 minutes of all students. Five of the communities in the study area could not be served under the design constraints, ie. the communities were too small to justify a facility and were not within 30 minutes of a community which did have such a potential. The unserved communities are:

3. Wild Cove
7. Snook's Arm - Round Hr.
33. Lawrenceton
55. Port Albert
82. Terra Nova

These communities are included at the end of the analysis when they are assigned to their closest facility and costs are subsequently adjusted.

In order to judge the initial level C for the budget constraint the value of existing schools ($5,627,314) was subtracted from the estimated costs of replacement, operation and maintenance ($70,062,420) in the existing system (see Table 4.1, p.74, for cost breakdown). The result is $64.4 M. The budget was then increased in increments of $1 M from this level until a feasible solution was found at $66.4 M. The budget level was then decreased by $100,000 at a time until an infeasible solution was found at $65.9 M. The minimum budget occured just above the level of infeasibility at $66 M. Figure 16 illustrates the location-allocation solution generated by the linear programming model for the minimum budget level of $66 M.

2. Budget Variation

Because the X_{ij} allocations and X_{jj} schools were not all zero (0) or one (1) at $66 M., it was necessary to vary the budget slightly to achieve the location-allocation system shown in figure 16. A partial assignment of .764651 occurred from community 40 to school 39, X_{4039} and community 40 partially (.235349) self-assigned, X_{4040}. To compute the amount by which the budget had to be varied, the following two alternatives were considered: either community 40 completely self assigns ($X_{4040} = 1$ and $X_{4039} = 0$) or it assigns completely to school 39 ($X_{4040} = 0$ and $X_{4039} = 1$).

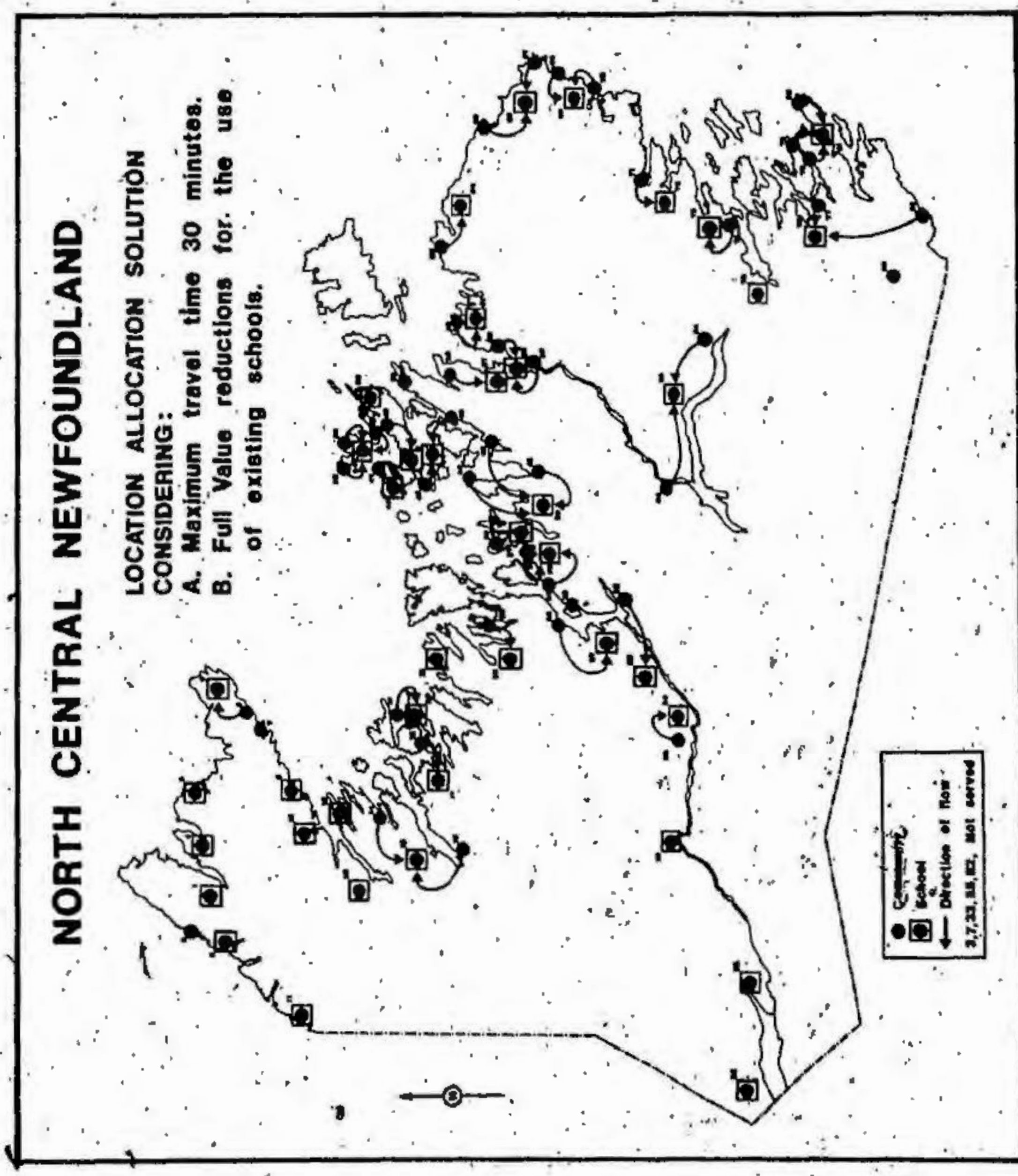

NORTH CENTRAL NEWFOUNDLAND
LOCATION ALLOCATION SOLUTION
CONSIDERING:
A. Maximum travel time 30 minutes.
B. Full Value reductions for the use of existing schools.
Community
School
Direction of flow

FIGURE 16

In alternative one, the capacity of the school at 40 would be expanded and the school at 39 would be reduced. This would require an increase in costs of:

* PLUS 141,946 (.764651) + 8,173.5 (.764651 x (the number of students at 40 = 97))

MINUS 8,173.5 (.764651 x (the number of students assigned to 39 from 40 = 97))

EQUALS 141.946 (.764651) = $108,488

In alternative two, there would be no school at 40 and the school at 39 would be expanded. This would result in a decrease in the budget required of:

* MINUS 141,946 (.235349) + 8173.5 (.235349 x (the number of students at 40 = 97))

PLUS 8,173.5 (.235349 x (the number of students assigned to 39 = 97))

EQUALS-141,946 (.235349) = -$33,407

The location-allocation solution shown in figure 16 was based on alternative two of the budget variations because it yielded the minimum budget requirement of ($66,000,000 - 33,407) $65,970,943.

* Note: Equation used is from the budget constraint (Chapter IV - C) where f_j = 141,946 and b_j = 8,173.5.

Rojeski and ReVelle (1970, p. 359-360) discuss the method of adjusting the budget level to obtain integer solutions:

> "This is an unusual solution to the location of central facilities in the sense that it is not an approximate solution to the problem. It is an exact solution, it is the structure that is approximate. This is in contrast to heuristic procedures which are commonly approximate solutions to exact structures.
>
> The drawback is a lack of sensitivity or fine tunning; between two levels of funding both of which produce integer solutions. There may be other integer solutions which are not located".

This lack of sensitivity is not actually a problem in the acceptability of the minimum cost solution generated by the model. For example, in this problem it was determined that no feasible solution exists at \$65.9 M., therefore any alternative integer solutions if existent could be no more than .001 or 0.1% lower than the generated solution at \$65.97 M.

3. Proportional Reductions

When this solution was generated the value of existing schools (P_j) had been integrated into the design in order that the solution would reflect possible cost savings realized through the use of these facilities.

The solid line in figure 17 represents a hypothetical total cost-size function at location j. The value of the existing school at j (P_j) is a constant value in the budget constraint, independent of the size ($\sum_{i=1}^{n} a_i X_{ij}$) of the school created at j in the final solution. When the value of the existing school is introduced, the size-cost function shifts downward by the amount P_j (see dotted line in figure 17).

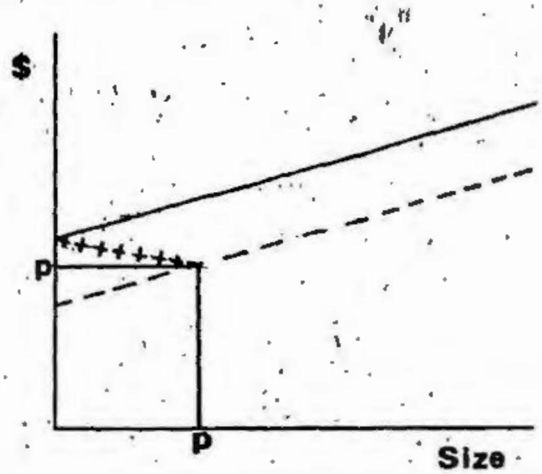


FIGURE 17 TOTAL COST FUNCTION WITH PROPORTIONAL REDUCTIONS

In actuality, unless a school is being used to its full capacity (p represents the size of facility P_j in figure 17), the cost savings to the system should not be equivalent to the school's full value. Because of the unknown effect of unused capacity on the cost of operating and maintaining a school and the unknown salvage value of unused capacity throughout the system, it was assumed that when the size-cost relationship is below full capacity (p) of the existing school (P_j) there is a reduction in savings proportional to unused capacity (see crossed line in figure 17).

At the minimum budget level of $65.97 M. several schools had unused capacity. Table 5.1 indicates the proportional reductions which were implimented for each of these schools. The total savings due to existing schools was reduced from $5,498,385 to $4,804,019. The minimum budget level required for replacement, operation and maintenance had to be increased from $65,970,943 to $66,665,329.

To find out whether the proportional as opposed to full reductions would result in the generation of an alternative location-allocation solution, the problem was rerun through the linear programming model with the following changes:

1) the $-P_j$ value for each facility listed in Table 5.1 was changed to the proportional reduction value.
2) the budget level constraint is increased to $66,690,000 to allow for the potential loss in savings.

The new solution, again achieved through budget variation, is shown in figure 18. Note that because the full reduction was not allocated for the school at community 48, students at 48 were assigned to the school at 44 and the students at 49 to 50. The location-allocation solution shown in figure 18 was considered

Table 5.1: Proportional Reductions in 30 Minute Solution

No.	School Location	Students/Capacity	Full Value Reduction - P_j (\$)	Proportional Reduction (\$)
1.	Baie Verte	179/453	243,675	97,470
4.	La Scie Shoe Cove	146/179	165,817	135,970
12.	Rattling Bk. King's Pt.	100/160	49,989	31,493
23.	Buchan's	183/232	98,599	77,893
29.	Point Leamington	160/207	256,367	197,403
36.	Lewisporte	398/495	431,366	345,093
48.	Carter's Cove Fairbank Chanceport Virgin Arm	103/295	198,168	69,359
57.	Victoria Cove	89/252	203,580	71,253
61.	Carmanville	137/215	189,561	121,319
76.	Glovertown	291/301	149,899	145,402

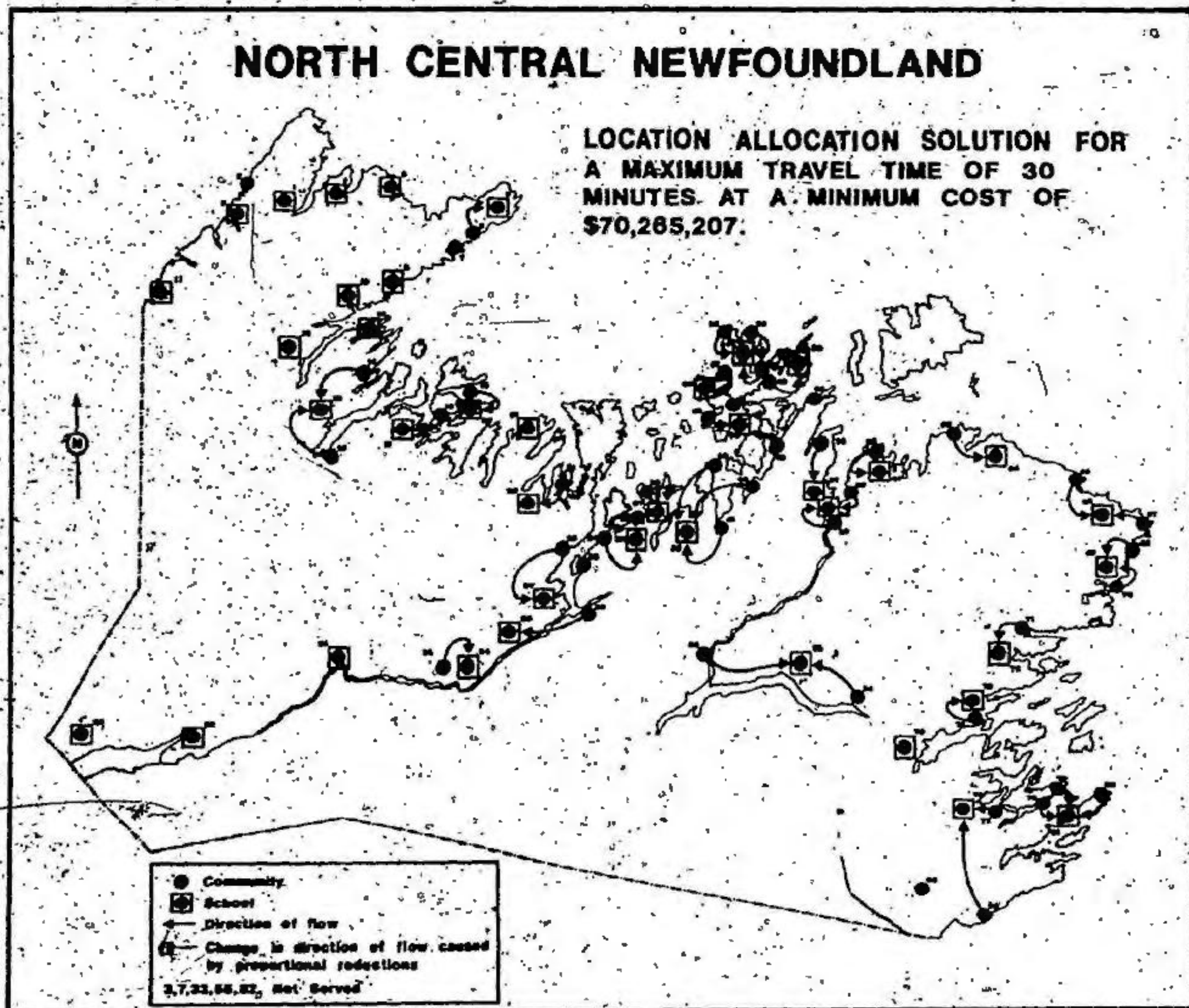

NORTH CENTRAL NEWFOUNDLAND
LOCATION ALLOCATION SOLUTION FOR A MAXIMUM TRAVEL TIME OF 30 MINUTES AT A MINIMUM COST OF $70,265,207.
Community
School
Direction of flow
Change in direction of flow caused by proportional reductions

FIGURE 18

to be the final solution since no further proportional reductions were possible. The minimized cost of replacement, operation and maintenance under the 30 minute maximum time constraint, as calculated for this final solution, is $66,592,742.

4. Minimum Total Costs

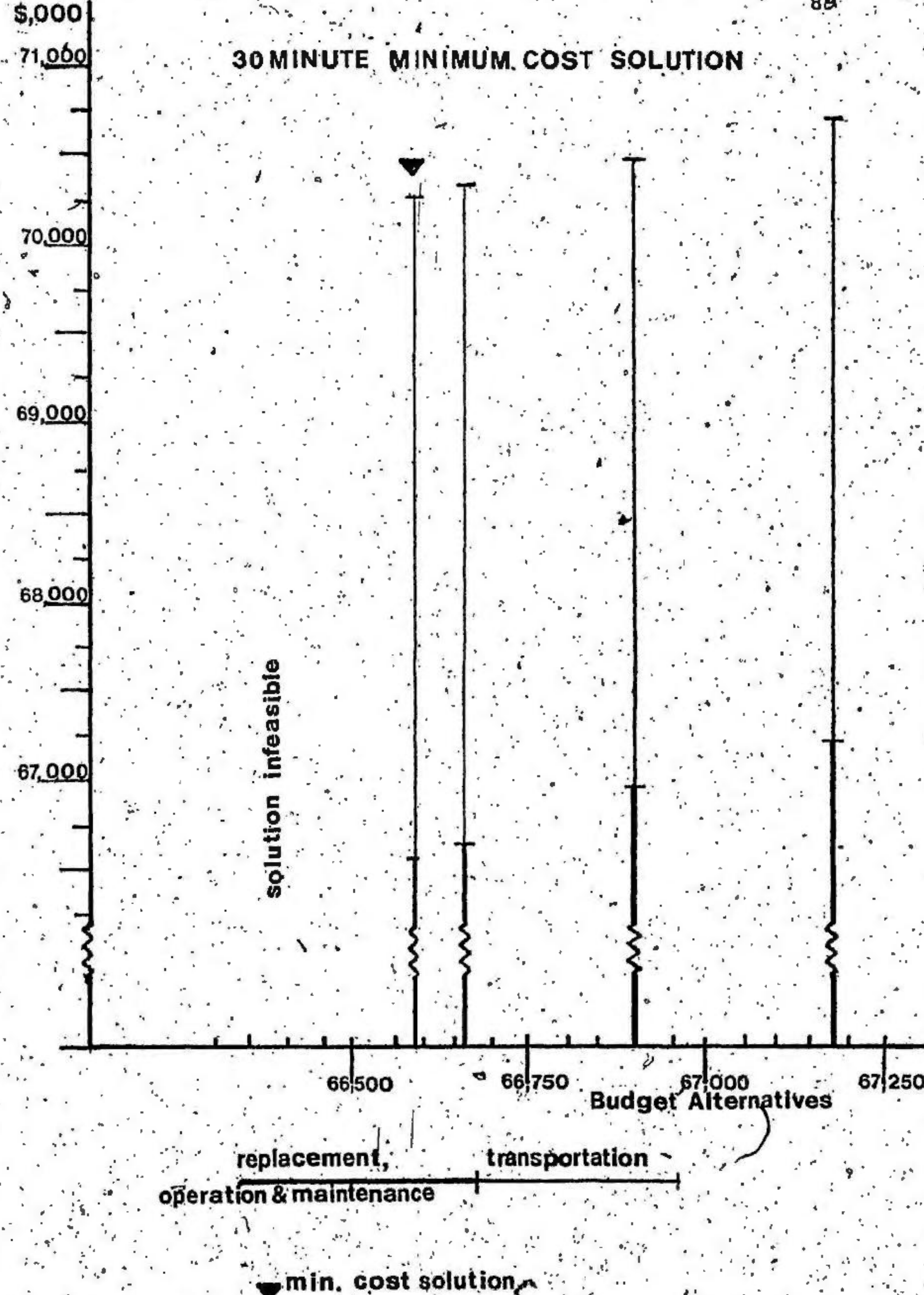
The transportation costs for each school generated in the final solution was estimated using the regression equation (-7736 + 63 (Total Student Travel Time)), see Chapter IV- D. The sum of these costs, the total transportation cost of the school system, is $3,672,465.

To identify the minimum total cost solution, the school facility costs and transportation costs were combined for several budget alternatives. A cost breakdown for each of the alternatives is given in Table 5.2. Figure 19 displays the alternatives graphically. All budget alternatives reflect proportional reductions. At no time when the budget level is increased do savings in transportation costs exceed the additional facility costs. Therefore, the minimum cost solution for the 30 minute time constraint is contributed through the minimum budget alternative (see figure 18) with total transportation and facility costs of $70,256,207.

TABLE 5.2: BUDGET CONSTRAINT ALTERNATIVES (30 MINUTE SOLUTION)

FULL REDUCTION CONSTRAINT	PROPORTIONAL REDUCTION CONSTRAINT	TRANSPORT COST ($)	REPLACEMENT OPERATION AND MAINTENANCE ($)	FULL REDUCTION ($)	PROPORTIONAL REDUCTION ($)	R. O. & M. MINUS FULL RED. ($)	R. O. & M. MINUS PROP. RED. ($)	TOTAL COST FULL RED. ($)	TOTAL COST PROP. RED. ($)
66,250,000		3,503,039	71,753,240	5,498,385	4,563,238	66,254,835	67,190,002	69,757,874	70,693,041
66,150,000		3,588,746	71,611,294	5,498,385	4,707,700	66,112,889	66,903,594	69,701,635	70,492,320
66,000,000		3,658,903	71,469,348	5,498,385	4,804,019	65,970,943	66,665,329	69,629,846	70,324,232
65,900,000		Solution Infeasible							
66,000,000	66,600,000	3,672,465	71,327,402		4,734,660		66,592,742		70,265,207*

* Lowest budget constraint and minimum cost solution.



$,000
30 MINUTE MINIMUM COST SOLUTION
71,000
70,000
69,000
68,000
67,000
solution infeasible
66,500
66,750
67,000
67,250
Budget Alternatives
replacement, operation & maintenance
transportation
min. cost solution

FIGURE 19

The extra cost of providing for unserved demand is calculated on the basis of transportation to and educational services from the closest facility. Table 5.3 shows the assignments and cost for the 30 minute solution.

The complete cost of the final solution is therefore calculated at $71,059,690. Recalling that the cost of the existing system was $69,480,441, the additional cost of providing educational services within 30 minutes of all but 6% of the communities and 1% of the students is estimated at $1,579,249. The cost to amortize this amount over the 25 year investment period at 5% is (1,579,249 x 0.070952) $112,050 per annum.

V - C. 45 and 60 Minute Solutions.

The decision to accept the 30 minute solution may well be based on whether funds are in fact available. Consequently, it is advisable to identify the cost savings which could be realized by increasing the maximum travel time constraint. In this analysis further solutions were generated for 45 and 60 minute travel time constraints.

1. 45 ~~Minute~~ Solution.

Table 5.4 lists the budgetary alternatives considered and the corresponding costs for the maximum travel time constraint of 45 minutes. Figure 20 displays the alter-

Table 5.3: 30 Minute Solution - Additional Cost for Unserved Demand

Assigning Community	Facility Location	Travel Time	Transport Cost $	Education Cost $	Total Cost $
Wild Cove (3)	Bare Verte (1)	45 min.	51,030	147,114	198,144
Snook's Arm Round Hr. (7)	La Scie Shoe Cv. (9)	40 min.	32,760	106,249	134,009
Lawrenceton (33)	Lewisporte (36)	40 min.	30,240	98,076	128,316
Port Albert (55)	Victoria Cv. (57)	55 min.	62,370	147,114	209,484
Terra Nova (82)	Glovertown (76)	1 hr.	37,800	81,730	119,530
			Grand Total		$ 794,483.

TABLE 5.4: BUDGET CONSTRAINT ALTERNATIVES (45 MINUTE SOLUTION)

FULL REDUCTION CONSTRAINT	PROPORTIONAL REDUCTION CONSTRAINT	TRANSPORT COST ($)	REPLACEMENT OPERATION AND MAINTENANCE ($)	FULL REDUCTION ($)	PROPORTIONAL REDUCTION ($)	R. O. & M. MINUS FULL RED. ($)	R. O. & M. MINUS PROP. RED. ($)	TOTAL COST FULL RED. ($)	TOTAL COST PROP. RED. ($)
65,500,000		4,108,776	71,116,243	5,627,314	5,094,554	65,488,929	66,021,689	69,597,705	70,130,465
65,350,000		4,182,007	70,974,306	5,627,314	5,190,612	65,346,992	65,783,694	69,528,999	69,965,701
65,300,000		4,200,450	70,832,360	5,498,385	5,094,446	65,333,975	65,737,914	69,534,425	69,938,364
65,200,000		Solution Infeasible							
	65,750,000	4,200,450	70,832,360		5,094,446		65,737,914		69,938,364*

* Lowest budget constraint and minimum cost solution

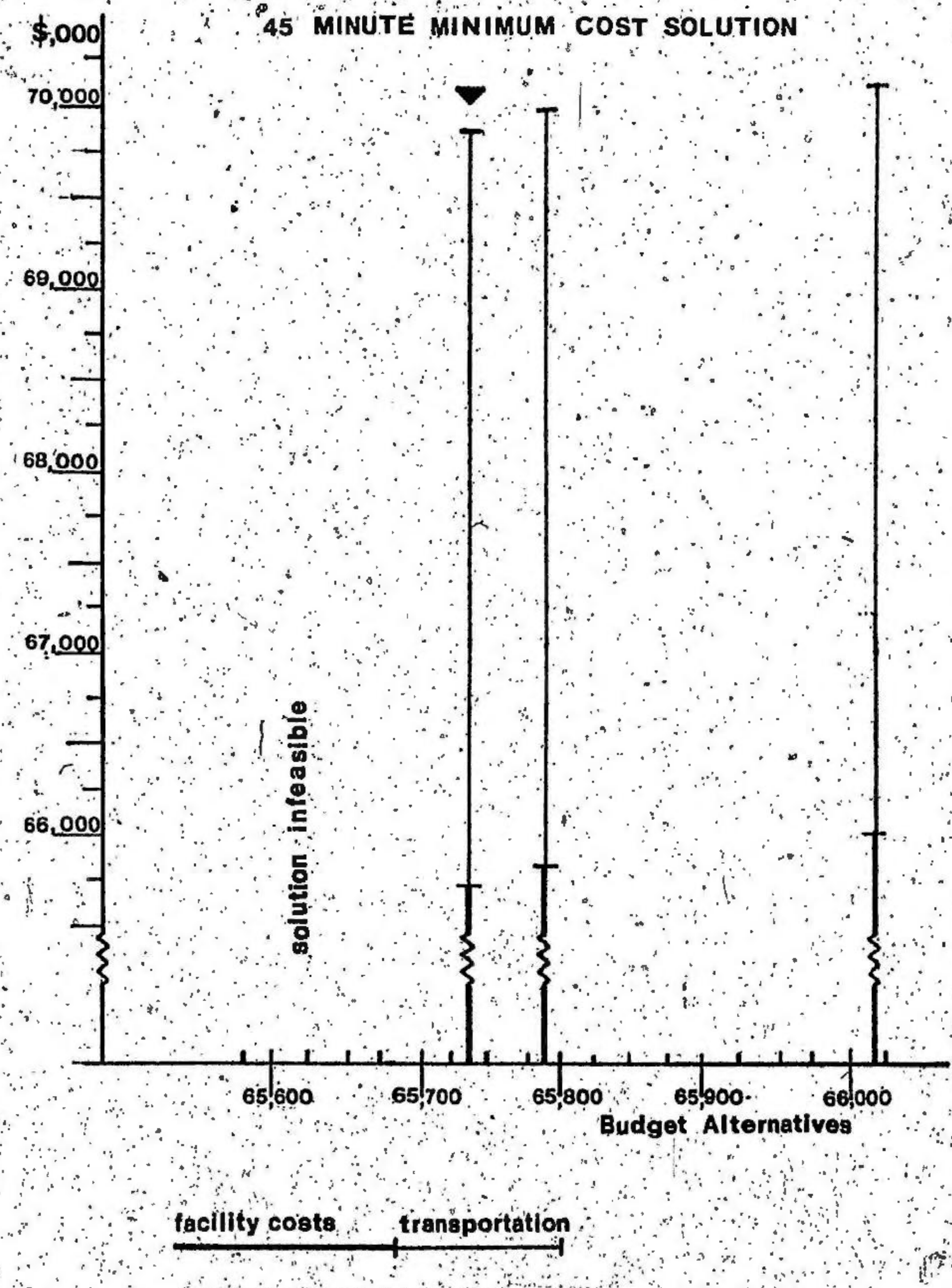

45 MINUTE MINIMUM COST SOLUTION
$,000
70,000
69,000
68,000
67,000
66,000
solution infeasible
65,600
65,700
65,800
65,900
66,000
Budget Alternatives
facility costs
transportation
minimum cost solution

FIGURE 20

natives graphically.

As in the case of the 30 minute constraint, the minimum cost solution (Fig. 21) was contributed through the lowest budget alternative. In the minimum cost solution for 45 minutes, the cost of replacement, operation and maintenance less the value of existing facilities is $65,737,914. Transportation costs are $4,200,450 for a total cost of $69,938,364.

The extra cost of providing for unserved demand at community 82 is $119,530 (community 82 is assigned to the closest facility 76). The complete cost of the solution is therefore calculated at $70,057,894. The cost of providing educational services within 45 minutes of all but 1.1% of the communities or 0.1% of the students is $1,001,796 less than the cost to provide the services within 30 minutes, an annual savings of $71,078. (cost to ammortize $1,001,796 over 25 years at 5%).

2. 60 Minute Solution

Table 5.5 lists the budgetary alternatives considered and corresponding costs for the maximum travel time constraint of 60 minutes. Figure 22 displays the alternatives graphically.

As with the previous constraints, the minimum cost solution (Fig. 23) was contributed through the lowest

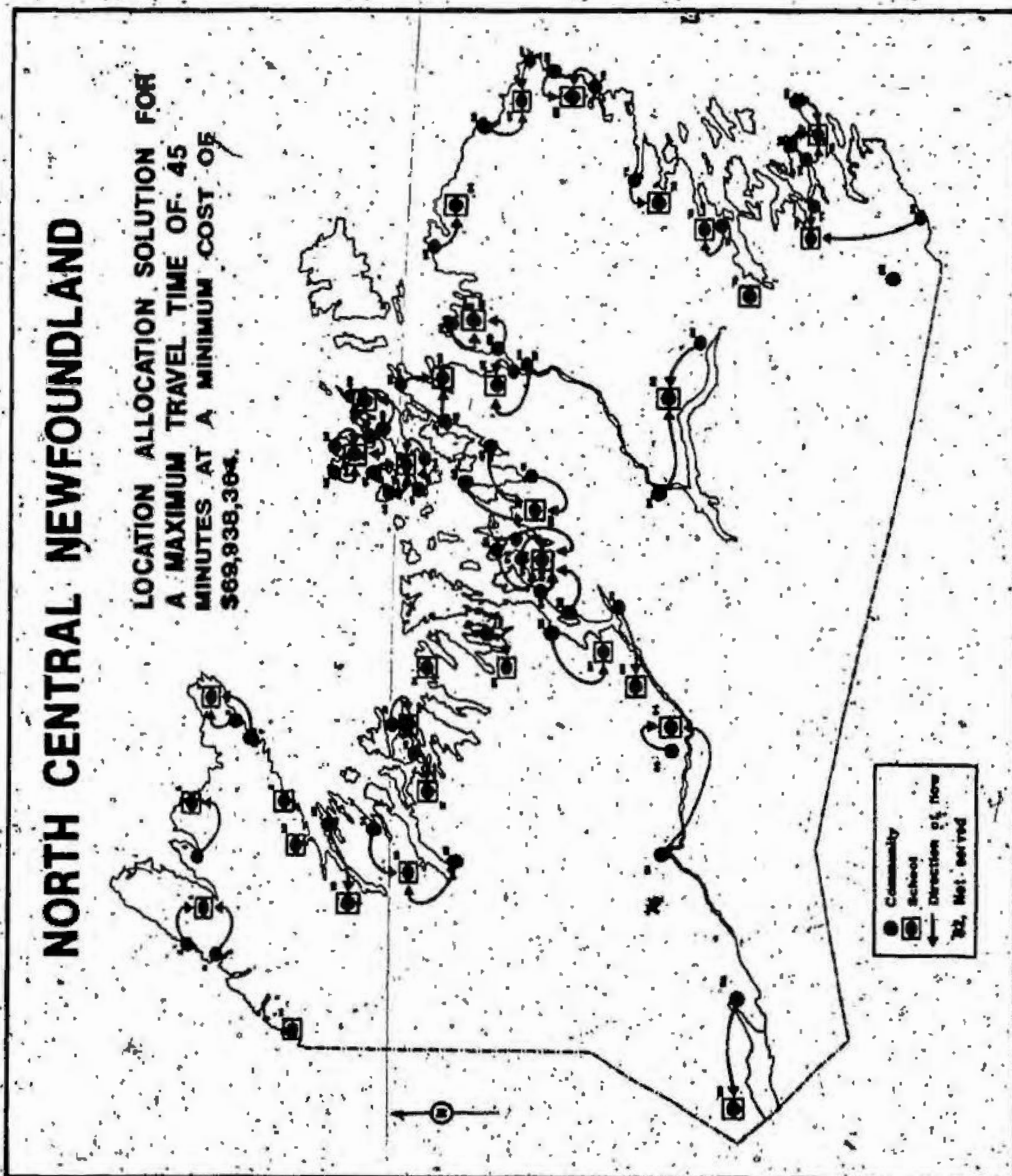

NORTH CENTRAL NEWFOUNDLAND
LOCATION ALLOCATION SOLUTION FOR A MAXIMUM TRAVEL TIME OF 45 MINUTES AT A MINIMUM COST OF $69,938,364.
Community
School
Direction of flow
32, Not served

FIGURE 21

TABLE 5.5: BUDGET CONSTRAINT ALTERNATIVES (60 MINUTE SOLUTION)

FULL REDUCTION CONSTRAINT	PROPORTIONAL REDUCTION CONSTRAINT	TRANSPORT COST ($)	REPLACEMENT OPERATION AND MAINTENANCE ($)	FULL REDUCTION ($)	PROPORTIONAL REDUCTION ($)	R. O. & M. MINUS FULL RED. ($)	R. O. & M. MINUS PROP. RED. ($)	TOTAL COST FULL RED. ($)	TOTAL COST PROP. RED. ($)
65,200,000		4,365,355	70,772,149	5,627,314	5,333,955	65,144,835	65,438,194	69,910,190	69,803,549
65,100,000		4,383,788	70,630,203	5,498,385	5,237,789	65,131,818	65,392,414	69,515,606	69,776,202
65,050,000		Solution Infeasible							
	65,400,000	4,383,788	70,630,203		5,237,789		65,392,414	69,515,606	69,776,202*

* Lowest budget constraint and minimum cost solution.

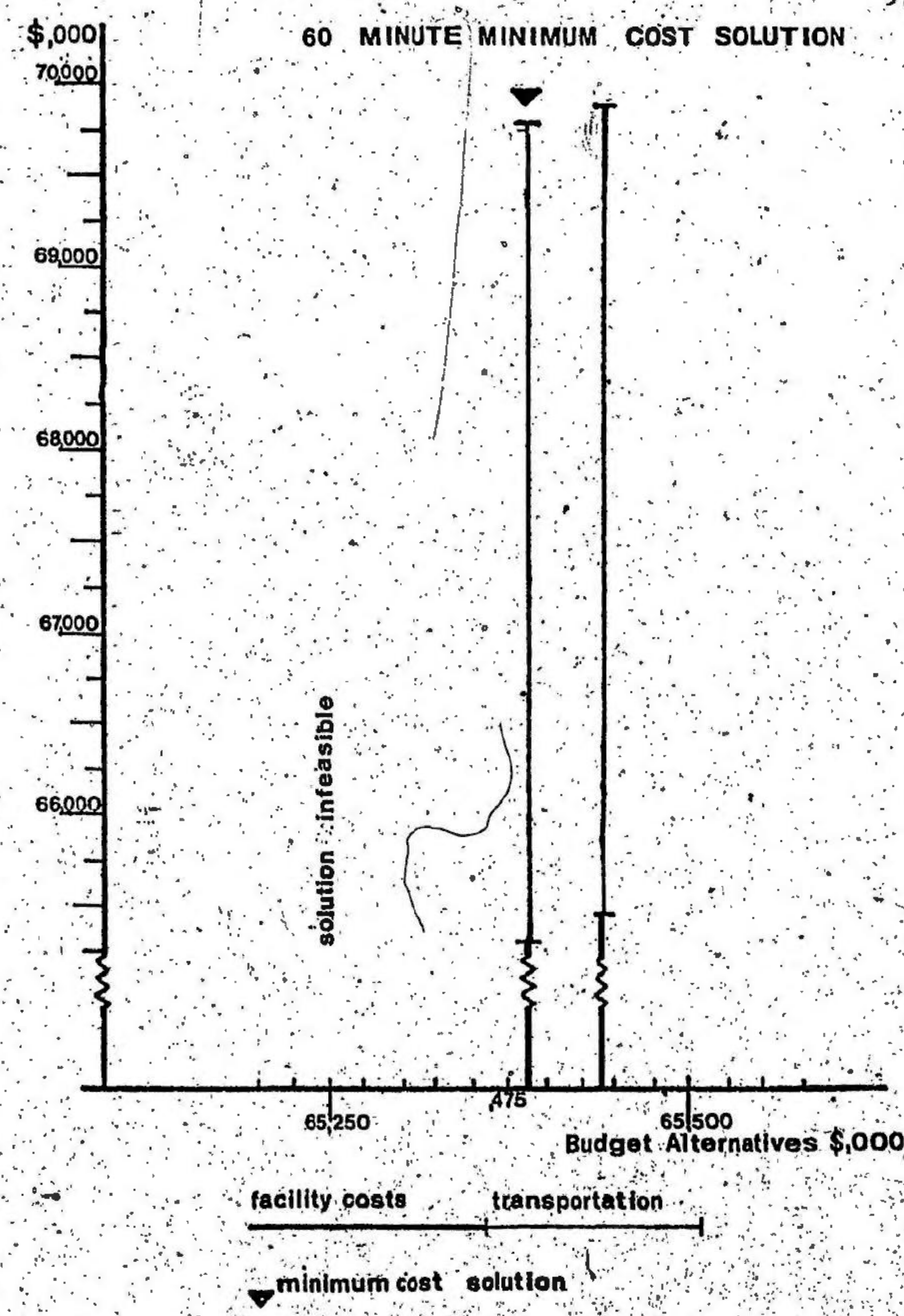
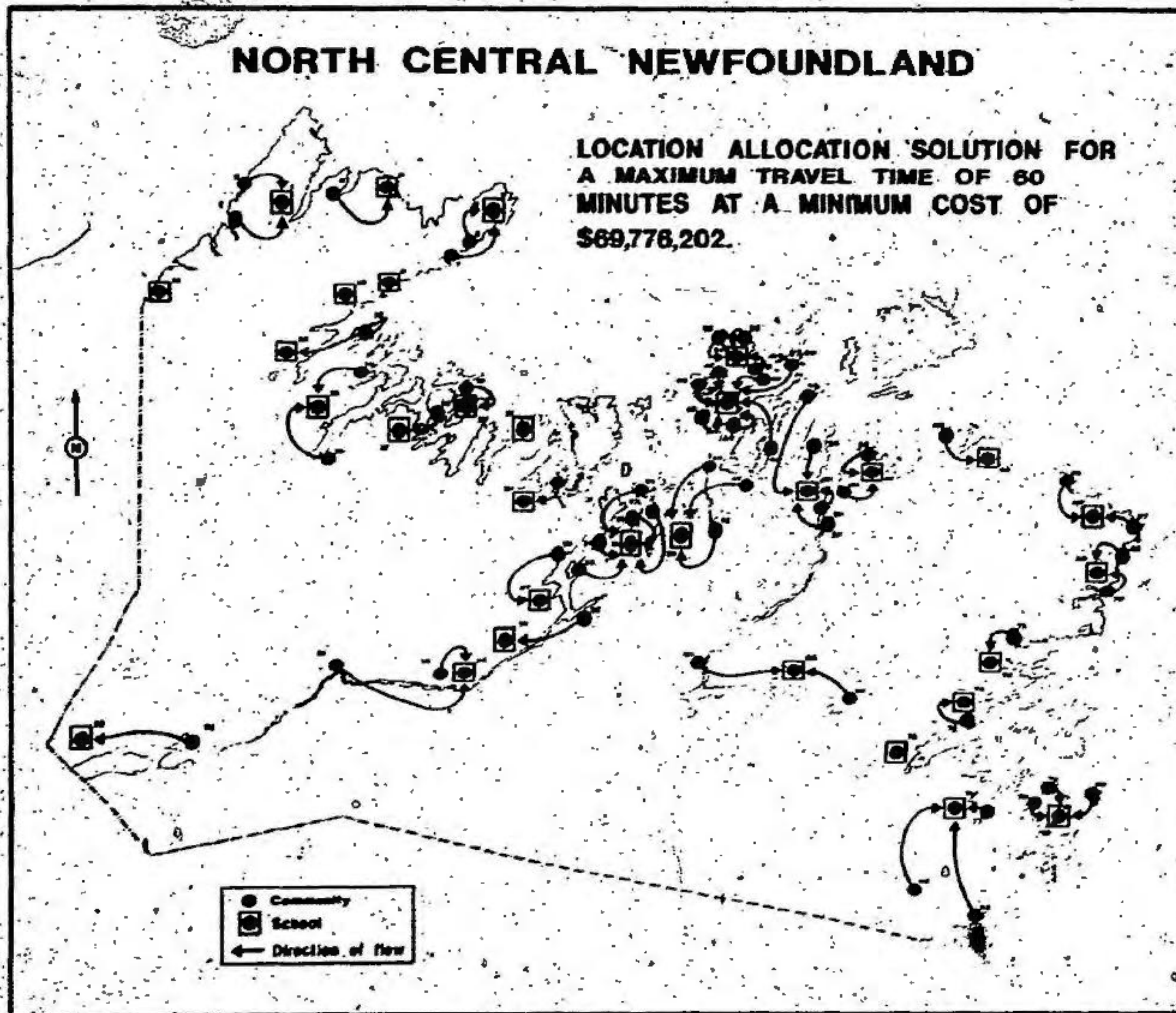

$,000
60 MINUTE MINIMUM COST SOLUTION
70,000
69,000
68,000
67,000
66,000
solution infeasible
475
65,250
65,500
Budget Alternatives $,000
facility costs
transportation
minimum cost solution

FIGURE 22

FIGURE 23


NORTH CENTRAL NEWFOUNDLAND
LOCATION ALLOCATION SOLUTION FOR A MAXIMUM TRAVEL TIME OF 60 MINUTES AT A MINIMUM COST OF $69,776,202.
Community
School
Direction of flow

budget alternative. The minimum cost location-allocation solution within the 60 minute constraint has a replacement, operation and maintenance (less the value of existing schools) budget of $65,737,914 and transportation costs of $4,383,788 for a total cost of $69,776,202. There are no communities which can not be served within 60 minutes travel time of a potential facility.

The costs of providing educational services within 60 minutes of all communities are only $277,328 more than the costs of the existing system or an additional cost of $19,676 (cost to amortize $277,328 over 25 years at 5%) per annum.

V - D. Time-Cost Tradeoff

The tradeoff between the minimized total cost of operating the school system and the maximum student travel time can be displayed as in figure 24.

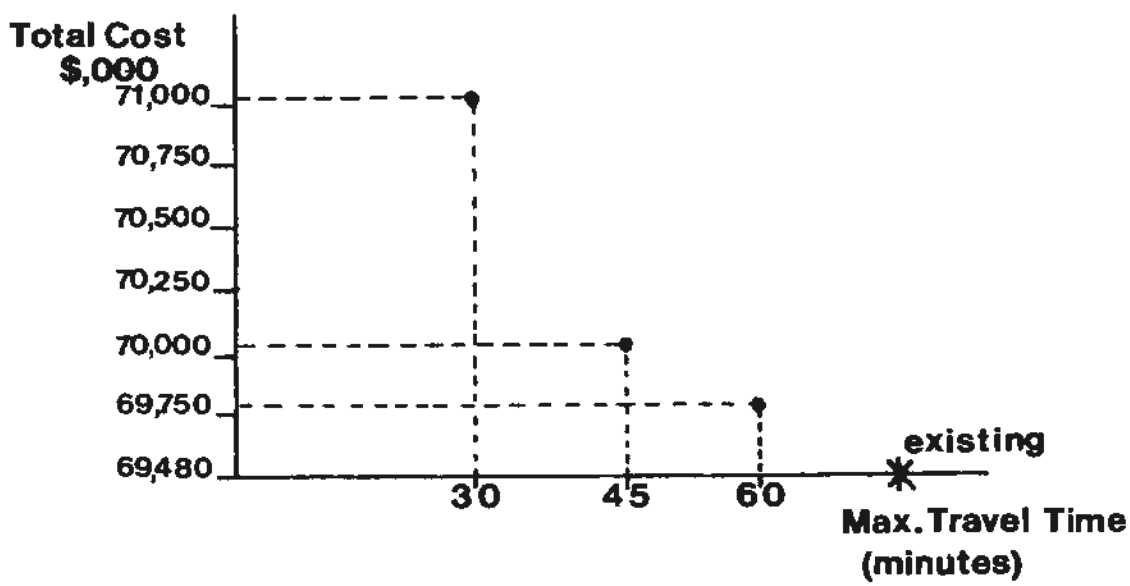


FIGURE 24 TIME-COST TRADEOFF CURVE

Figure 24 indicates that more stringent restriction on travel times common to the existing system will bring accelarating increases in total costs. Table 5.6 facilitates a further understanding of the performance of each time-cost solution. Of particular note is that a 2% increase in present value costs would resolve a 30 minute service objective which is not being met for 11% of the students and 28% of the communities in North-Central Newfoundland.

V - E. Changes in the Educational Environment

The impact of changes in the size and location of the various schools in the existing system on the quality of education is assessed using the educational production function derived in Chapter III - C. Recall that grade 11 scores were used as a surrogate measure of the quality of a school's educational output. The standard deviation of these scores over the total population of integrated high schools in Newfoundland is 39.5 points. A significant change in the quality of education is assumed to be created when unit changes in the variables of the production function produce an increase or decrease in the estimate of the quality of a school's educational output in excess of 39.5 units. Using this criteria, the location-allocation

Table 5.6: Breakdown of Maximal Service Time Performance

Total Number of Students - 7805

	Over 30 Min.		Over 45 Min.		Over 60 Min.		Total Cost
	% stud't	% Comm.	% stud't	% Comm.	% stud't	% Comm.	
Existing System	11%	28%	4.5%	11.5%	3.1%	6%	$69,480,441
60 Min. Solution	6.9%	20.9%	1%	3.4%	-	-	$69,757,769
45 Min. Solution	5.9%	18.6%	.1%	1.1%	-	-	$70,057,894
30 Min. Solution	.9%*	5.8%*	.4%*	2.3%*	-	-	$71,059,690

* Students who were not within the maximum time of a potential facility location.

solutions for each travel time constraint were compared to the existing system.

No significant changes in quality were found except in the case of Badger (community 21) in the 30 minute solution which dropped 45.9 points. In the existing system, Badger assigned to a large school in Grand Falls (community 24). In the 30 minute solution Badger self-assigns.

Table 5.7 shows the changes in the average quality of schools over the whole system created by the changes in the size and location of schools in each solution.

Table 5.7: Changes in the Quality of the School System

	Solution	Points attributable to size and location	Average Score	Change in Average Score
1.	Existing System	161.89	401.89	
2.	60 Minute Sol.	159.01	399.01	-2.88
3.	45 Minute Sol.	156.76	396.76	-2.25
4.	30 Minute Sol.	153.91	393.91	-2.85

Although an overall downward trend is identified (Table 5.7), it fails to have a significant impact on individual schools with the noted exception. If the 30 minute solution were to be accepted further consideration could be given to whether or not to maintain the present link between students at Badger and the school at Grandfalls. Otherwise, it is concluded that any of the solutions are equally desirable in terms of the quality of education received.

V - F. Summary

The solutions generated by the linear programming model in this chapter have determined the location of schools and the allocation of students which minimize costs for various time constraints.

Such analysis and the decision making aids which result are of considerable value to educational planners and policy makers. The analysis:

(1) gives several yardsticks (maximal travel time, quality of education and minimized cost objectives) by which the existing system can be assessed.

(2) indicates the nature and cost of changes necessary to increase the effectiveness of the existing system.

(3) helps in the determination of fiscal and service priorities for the school system by displaying the tradeoff relationship between these two elements.

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Chapter VI: Conclusions

This thesis has developed an operational methology for optimizing the size and location of school facilities on a network. The methodology for locating schools and allocating students evolved from Toregas' Mini-max problem. The objectives of Toregas' design were extended to also consider modifications to existing facility systems, the minimization of facility and transportation costs over time and the impact of changes in the configuration of various sized facilities on the quality of the system. This led to the incorporation of an investment constraint, the consideration of the location and value of existing facilities, a procedure to find the minimum possible system costs under linear programming design constraints and the derivation of a facility production function.

VI - A. Extension of Methodology

Each feature of the methodology has merit which is not limited to the analysis of the school location problem. The cost minimization and evaluation of existing facilities elements are generally applicable to all facility planning problems. The maximum travel time concept, while of paramount importance in planning emergency facility systems, has also been found to be an objective in the location of such central services as libraries, post offices and health and welfare offices. The measurement of quality in a production function should be used when it is known that the quality of the system is influenced by the size and location of facilities within the system.

Within the thesis several points have been made as to further refinement of the analysis. Since the approach is based on the fixed and constraining linear programming methodology, it is not expected to be a part of future breakthroughs in location-allocation analysis. Because the approach can be applied with some elementary knowledge of linear programming, its value is in use and its future is in refinements to the operational methodology.

VI - B. Planning Implications

The linear programming design and methodology developed in the preceding chapters generates several decision making aids which integrate cost minimization and maximal travel time objectives without compromising educational quality. The process allows the decision maker to evaluate his own priorties and achieve an optimum solution in respect to his objectives.

The location-allocation solutions do not optimize these objectives in the longer term. However, over a 25 year investment period several complicating factors have been incorporated and stabilized through the consideration of all costs over the investment period and the elimination of school locations in smaller centres which are subject to decline. The methodology is sensitive to changes in the age and value of existing schools, in the distribution of students, in the technology of school construction and operation, and in the travel times on the transportation network. Revised solutions should therefore be generated at regular intervals within the planning period. In successive applications, the methodology is able to react to obsolescence and changes in the spatial structure of demand to further minimize costs or meet new social objectives. In the absence of erratic change, a program utilizing this methodology should approximate the least cost path towards satisfying social objectives in school facility planning.

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Appendix I: Population, Students and Locational Suitability for Communities in N. C. Newfoundland.

Appendix I: Population, Students and Locational Suitability for Communities in N. C. Newfoundland.

No.	Community Name	1971 Population	1973 No. of Students	Suitability
1.	Baie Verte	2397	179	1
2.	Seal Cove	698	67	1
3.	Wild Cove	172	18	0
4.	Ming's Bight	378	25	1
5.	Pacquet-Woodstock	717	92	1
6.	Nipper's Hr.	275	20	1
7.	Snooks Arm-Round Hr.	374	13	0
*8.	Tilt Cove	50	8	0
9.	Lascie-Shoe Cove	1466	138	1
10.	Burlington-Smith's Hr.	893	31	1
11.	Westport-Purbeck's Cove	490	41	1
12.	Rattling Brook-King's Pt.	823	100	1
13.	Harry's Hr.-Jackson's Cove	490	60	1
14.	Little Bay-St. Patrick	784	28	1
15.	Springdale	3224	340	1
16.	South Brook	802	28	1
17.	Robert's Arm	1044	91	1
18.	Pilley's Island	495	43	1
19.	Brighton	272	44	1
20	Triton-Jim's Cove-Card's Hr.	1497	114	1

No.	Community Name	1971 Population	1973 No. of Students	Suitability
21.	Badger	1187	51	1
22.	Millertown-Buchan's Jt.	616	49	1
23.	Buchan's	1907	183	1
24.	Grand Falls	1677	432	1
25.	Windsor	6644	393	1
26.	Bishop's Falls	4133	288	1
27.	Botwood-Peterview	5068	507	1
28.	Northern Arm-Philps Head	424	17	0
29.	Point Leamington	940	145	1
30.	Paradise	116	15	0
31.	Leading Tickles	405	57	1
32.	Norris Arm	1191	49	1
33.	Laurenceton	304	12	0
34.	Brown's Arm-Porterville	309	21	0
35.	Stanhope-	257	13	0
36.	Lewisporte	3175	364	1
37.	Embree-	814	31	1
38.	Little Burnt Bay-	509	5	0
39.	Campbellton	730	72	1
40.	Comfort Cove-Newstead	776	97	1
41.	Loon Bay-	145	18	0
42.	Birchy Bay-	580	60	1
43.	Boyd's Cove-	210	38	0

No.	Community Name	1971 Population	1973 No. of Students	Suitability
44.	Summerford-	839	57	1
45.	Cottle's Island-	433	15	0
46.	Whales Gulch -Bridgeport-Morten's Hr.-	701	41	1
47.	Tizzard's Hr.	178	18.	0
48.	Chanceport-Carter's Cove Virgin Arm-Fair Bank-	1130	75	1 1
49.	Indian Cove-Merritt Hr. Newville-Hillgrade-	1446	28	1
50.	Herring Neck-Too Good Arm Pike's Arm-Cobb's Arm-	510	60	1
51.	Little Hr.-Purcells Hr. Kettle Cove-Manuel's Cove	610	54	1
52.	Durrell	270	91	1
53.	Crow Head	510	24	1
54.	Twillingate-Bluff Head Cove	1608	181	1
55.	Port Albert	133	18	1
56.	Horwood-Stoneville	878	42	1
57.	Victoria Cove-	258	47	1
58.	Wings Pt.-Clarke's Head	641	76	1
*59.	Gander Bay South	200	31	0
60.	Davidsville-Main Pt.	457	35	1
61.	Carmanville	839	74	1
62.	Noggin Cove-Fredrickton	669	63	1

No.	Community Name	1971 Population	1973 No. of Students	Suitability
63.	Aspen Cove-Ladle Cove	470	43	1
64.	Musgrave Hr.-Doting Cove	1232	121	1
65.	Deadman's Bay-	219	21	0
66.	Lumsden-	630	68	1
67.	Cape Freels-	123	12	0
68.	Newtown-Templeman	563	60	1
69.	Pound Cove-Wesleyville Brookfield-	1142	154	1
70.	Valley Field-Badger's Quay -Pool's Is.-	1457	177	1
71.	Indian Bay-Wareham -Centreville-	1116	151	1
72.	Trinity-	577	51	1
73.	Wellington-	930	120	1
74.	Hare Bay-	1485	164	1
75.	Middle Brook-Darkcove -Gambo-	2586	307	1
76.	Glovertown-	1915	228	1
77.	Traytown	344	43	1
78.	Sandringham	223	30	0
79.	St. Chad's-Burnside	301	26	1
80.	Salvage	227	26	1

No.	Community Name	1971 Population	1973 No. of Students	Suitability
81.	Eastport-Sandy Cove -Happy Adventure-	1058	127	1
*82.	Terra Nova	50	10	0
83.	Charlottetown-	309	20	0
84.	Benton-	198	10	0
85.	Gander-	7748	593	1
86.	Glenwood-Appleton-	1326	115	1

* Statistics Canada Population Estimate Not Available

Source - The Newfoundland and Labrador Schools Directory, Government of Newfoundland and Labrador, 1972-73.
- Questionnaire as per Appendix II, sent to school principals April, 1973.
- Population, Province of Newfoundland, incorporated cities, towns, villages and unincorporated communities statistics Canada, 1971.

Appendix II: Questionnaire and Covering Letters Sent to School Principals in April, 1973

P. O. Box 6116

Phone 726-6790-1-2-3

Integrated Education Committee

Floor 6
Royal Trust Building
Water Street
St. John's - - - Newfoundland

Executive Secretary: C. C. HATCHER, B.A. (Ed.), M.Ed.
Executive Officer: W. C. WOODLAND, B.Sc., B.A. (Ed.)
Executive Officer: REV. A. B. LEGROW, B.A., B.D.
Administrative Officer: D. WADLAND

March 29, 1973

Mr
Principal,
Integrated School,

Dear Sir:

Mr. David Naphtali, a student at Memorial University, is presently doing a study to determine the optimal size and location of Secondary Schools in Newfoundland. He will be forwarding shortly a questionnaire to you requesting specific information necessary for this study. I would ask you to give him every co-operation in obtaining the information required.

Kind regards,

Yours truly,

C. C. HATCHER,
EXECUTIVE SECRETARY.

CCH:llr

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
St. John's, Newfoundland, Canada

Department of Geography

Dear Sir,

I am presently working on a graduate thesis project funded by Central Mortgage and Housing Corporation to design a method which determines the optimal size and location of secondary schools in Newfoundland. One of the objectives of the project is to apply the methodological design to the system of Integrated secondary schools in central Newfoundland. The data requested in the enclosed questionnaire will be used to locate student demand sources, establish cost-distance relationships and determine student travel time constraints. The accuracy and completeness of your answers to the questionnaire are important. With your co-operation this project could represent a substantial contribution to the art of educational planning in Newfoundland.

Please answer each question as directed. If you feel it is necessary to qualify any of your answers, use the space provided for comments at the end of the form.

QUESTIONNAIRE ENCLOSED

Please return the completed questionnaire as soon as possible in the envelope provided (postage is prepaid).

Thank you for your assistance.

Yours sincerely,

David Naphtali.

DN/sf
Encl.

SCHOOL:

LOCATION:

I. List the names of the communities served by your school, the number of students coming from each community, the number of students who travel by bus from each community and estimate their travel time to school (one way).

Name of Community	# Students	# Students travelling by bus	Travel Time
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN
			__HRS__MIN

** if additional space is required, use the back of this form.

(con't p.2)

II. How many buses in each of the following capacity categories service your school?

under 20 passengers ____________

21 - 35 passengers ____________

36 - 50 passengers ____________

over 50 passengers ____________

III. Comments (if any):

____

____

____

____

____

____

____

____

____

____

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____

Principal ____________ Date ____________

Appendix III: Questionnaire and Covering Letters Sent to School Boards in April, 1973.

P. O. Box 6116

Phone . 726-6700-1-2-3

Integrated Education Committee

Floor 6
Royal Trust Building
Water Street
St. John's - - - Newfoundland

Executive Secretary: C. C. HATCHER, B.A. (Ed.), M.Ed.
Executive Officer: W. C. WOODLAND, B.Sc., B.A. (Ed.)
Executive Officer: REV. A. B. LEGROW, B.A., B.D.
Administrative Officer: D. WADLAND

MEMORANDUM TO SCHOOL BOARD SUPERINTENDENTS

Mr. David Naphtali, a student at Memorial University, is presently making a study to determine the optimal size and location of Secondary Schools in Newfoundland. This in fact is the basis of his thesis. I have discussed this with him and I consider it the case of worthwhile research which could be of benefit to all of us. I would ask your professional staff and your Board to give Mr. Naphtali every co-operation in the completion of the questionnaires which he will be forwarding to you.

Kind regards,

C. C. HATCHER,
EXECUTIVE SECRETARY.

March 29, 1973

CCH:llr

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
St. John's, Newfoundland, Canada

Department of Geography

Dear Sir,

I am presently working on a graduate thesis project funded by Central Mortgage and Housing Corporation to design a method which determines the optimal size and location of secondary schools in Newfoundland. The data requested on the enclosed questionnaires will be used to establish the relationship between the cost and the size and quality of secondary schools in the province. The accuracy and completeness of your answers to these questionnaires are important. With your co-operation this project could represent a substantial contribution to the art of educational planning in Newfoundland.

Each questionnaire form in this envelope pertains to ~~a central~~ or regional high school within your district. The name and location of the school are given at the head of each form.

Please complete each form as directed. If you feel it is necessary to qualify any of your answers, use the space provided for comments in section C.

FORMS ARE ENCLOSED

Please return the completed forms as soon as possible in the envelopes provided (postage is prepaid).

Thank you for your assistance.

Yours sincerely,

David Naphtali.

DN/sf
Encl.

N.B. The information you supply on this form will be used in strict confidence.

SCHOOL:

LOCATION:

Section A, specify the annual wages and salaries paid to the following:

i) School Administrative Staff (this should include the salaries of the clerical assistants employed for this school) $________

ii) School Teaching Staff (this should include the salaries of the principal, vice-principal and complete teaching staff) $________

iii) Custodial and Maintenance Staff (this should include the wages of all persons permanently employed for this school in a maintenance or custodial occupation;—see next section for contracted work) $________

Section B, specify annual expenditures on the following:

i) Instructional Materials, Desks, Furniture and Equipment (this figure should be based on an estimate of the average annual purchases for this school) $________

(con't p. 2)

ii) Fuel and Utilities (light, heat, water, telephone, etc.) $______

iii) Maintenance Supplies (mops, buckets, paint, soap, light bulbs, etc.) $______

iv) Contract Services (this figure should be based on an estimate of the average annual expenditure on contract work, i.e., work by persons not permanently employed for this school on such things as repairs, painting, grounds keeping, etc.) $______

v) Insurance Cost (this should include insurance on the building, furniture, and equipment) $______

vi) Pupil Transportation (estimate the annual expenditures on board owned and contracted transportation equipment and services which are directly attributable to this school) $______

Section C. Comments (if any):

____

____

____

____

____

____

____

Business Manager Date

Appendix IV: L. P. Interface Program

The following program was used to generate data sets for the ICES linear programming package. Data sets were generated for maximum time constraints of 30 minutes, 45 minutes and 60 minutes. The information needed and then corresponding definitions in the program are:

INFORMATION	PROGRAM DEFINITION
A. Travel time matrix -	D(I,J)
B. Designation of unsuitable communities; 1 if suitable, 0 if not -	LDEV(I)
C. Student population by community (a_i)	POP(I)
D. Value of existing facilities (P_j) -	SALV(I)
E. Maximum travel time constraint (S) -	DMAX
F. Replacement, operation and maintenance cost relationship	
a-intercept (f_j) -	F(J)
b-Slope (b_j) -	R
G. Budget constraint -	BUD
The program will print and punch several constraints in decreasing increments -	BINC
for a specified number of times -	NINC

As it stands the program will provide a print-out and a card set for the generated data.

```
FORTRAN IV G LEVEL  21                  MAIN                DATE = 73222          2

      DIMENSION POP(86),LDEV(86),D(86,86)
      DIMENSION SALV(86),KR(86)
      READ(5,1) NX,DMAX,FR,R,BUD,BINC,NINC
    1 FORMAT(I3,F2.0,F10.4,F8.4,F10.2,F10.2,I3)
      READ(5,2) (SALV(I),I=1,NX)
    2 FORMAT(6(13F6.0/),8F6.0)
      DO 4 I=1,NX
      READ(5,5) (D(I,J),J=1,NX),POP(I),LDEV(I)
    5 FORMAT(40F2.0/40F2.0/6F2.0,F5.0,I1)
    4 CONTINUE
      KT=0
      DO 6 I=1,NX
      DO 7 J=1,NX
      IF(LDEV(J).NE.1) GO TO 8
      IF(D(I,J).EQ.0.0) GO TO 8
      IF(D(I,J).LE.DMAX) GO TO 9
    8 D(I,J)=-1.0
      GO TO 7
    9 KT=KT+1
      D(I,J)=D(I,J)*POP(I)
    7 CONTINUE
    6 CONTINUE
      NXC=0
      DO 10 I=1,NX
      DO 11 J=1,NX
      IF(D(I,J).NE.-1.0) GO TO 10
   11 CONTINUE
      WRITE(6,12) I
   12 FORMAT(' ','COMMUNITY',1X,I2,1X,'IS RESIDUAL TO THE SYSTEM')
      NXC=NXC+1
   10 CONTINUE
      DO 66 I=1,NX
      KR(I)=-1
      DO 67 J=1,NX
      IF(D(J,I).EQ.-1.0) GO TO 67
      KR(I)=KR(I)+1
   67 CONTINUE
   66 CONTINUE
      IKR=0
      DO 13 I=1,NX
      IF(KR(I).EQ.-1) GO TO 13
      IKR=IKR+KR(I)
   13 CONTINUE
      KON=NX-NXC+IKR+1
      KVAR=KT
      KSLA=IKR+1
      NKON=KON+1
      WRITE(7,14) NKON,KVAR,KSLA
```

```
   14 FORMAT('LP',1X,'ROW',1X,I3,1X,'COL',1X,I3,1X,'SLACK',1X,I3)
      WRITE(6,15) NKON,KVAR,KSLA
   15 FORMAT(' ','LP',1X,'ROW',1X,I3,1X,'COL',1X,I3,1X,'SLACK',1X,I3)
      WRITE(7,16)
   16 FORMAT('PRINT RHS')
      WRITE(6,17)
   17 FORMAT(' ','PRINT RHS')
      KVAR=NX-NXC
      DO 18 I=1,KVAR
      WRITE(7,19) I
   19 FORMAT(9X,I3,9X,'1,0')
      WRITE(6,20) I
   20 FORMAT(' ',9X,I3,9X,'1,0')
   18 CONTINUE
      KT=NX-NXC+1
      KP=KON-1
      DO 21 I=KT,KP
      WRITE(7,22) I
   22 FORMAT(4X,'G',4X,I3,9X,'0,0')
      WRITE(6,23) I
   23 FORMAT(' ',4X,'G',4X,I3,9X,'0,0')
   21 CONTINUE
      DO 24 I=1,NINC
      WRITE(7,25) KON,BUD
   25 FORMAT(4X,'L',4X,I3,F10,0)
      WRITE(6,26) KON,BUD
   26 FORMAT(' ',4X,'L',4X,I3,F10.0)
      BUD=BUD-BINC
   24 CONTINUE
      WRITE(7,27)
   27 FORMAT('PRINT MATRIX')
      WRITE(6,28)
   28 FORMAT(' ','PRINT MATRIX')
      KPX=NX-NXC
      DO 29 I=1,NX
      IF(KR(I).EQ.-1) GO TO 29
      IF(D(I,I).EQ.-1.0) GO TO 29
      WRITE(7,30) I,I,D(I,I)
   30 FORMAT(4X,I2,I2,3X,'0',F10.0)
      WRITE(6,31) I,I,D(I,I)
   31 FORMAT(' ',4X,I2,I2,3X,'0',F10.0)
      NXC=0
      DO 70 K=1,I
      DO 71 IC=1,NX
      IF(D(K,IC).NE.-1.0) GO TO 70
   71 CONTINUE
      NXC=NXC+1
   70 CONTINUE
```

```
      NI=I-NXC
      WRITE(7,32) I,I,NI
   32 FORMAT(4X,I2,I2,1X,I3,9X,'1.0')
      WRITE(6,33) I,I,NI
   33 FORMAT(' ',4X,I2,I2,1X,I3,9X,'1.0')
      IF(KR(I).EQ.0) GO TO 81
      KPX=KPX+KR(I)
      KRX=KPX-(KR(I)-1)
      DO 34 K=KRX,KPX
      WRITE(7,35) I,I,K
   35 FORMAT(4X,I2,I2,1X,I3,9X,'1.0')
      WRITE(6,36) I,I,K
   36 FORMAT(' ',4X,I2,I2,1X,I3,9X,'1.0')
   34 CONTINUE
   81 OR=(POP(I)*R)+(FR-SALV(I))
      WRITE(7,37) I,I,KON,OR
   37 FORMAT(4X,I2,I2,1X,I3,F10.0)
      WRITE(6,38) I,I,KON,OR
   38 FORMAT(' ',4X,I2,I2,1X,I3,F10.0)
      L=-1
      NJ=0
      DO 39 J=1,NX
      NXC=0
      DO 68 K=1,J
      DO 69 JC=1,NX
      IF(D(K,JC).NE.-1.0) GO TO 68
   69 CONTINUE
      NXC=NXC+1
   68 CONTINUE
      IF(D(J,I).EQ.-1.0) GO TO 39
      IF(I.EQ.J) GO TO 39
      L=L+1
      WRITE(7,40) J,I,D(J,I)
   40 FORMAT(4X,I2,I2,3X,'0',F10.0)
      WRITE(6,41) J,I,D(J,I)
   41 FORMAT(' ',4X,I2,I2,3X,'0',F10.0)
      NF=J-NXC
      WRITE(7,42) J,I,NF
   42 FORMAT(4X,I2,I2,1X,I3,9X,'1.0')
      WRITE(6,43) J,I,NF
   43 FORMAT(' ',4X,I2,I2,1X,I3,9X,'1.0')
      KSX=KRX+L
      WRITE(7,44) J,I,KSX
   44 FORMAT(4X,I2,I2,1X,I3,8X,'-1.0')
      WRITE(6,45) J,I,KSX
   45 FORMAT(' ',4X,I2,I2,1X,I3,8X,'-1.0')
      OR=POP(J)*R
      WRITE(7,46) J,I,KON,OR
```

```
FORTRAN IV G LEVEL  21                  MAIN                 DATE = 73222

        46 FORMAT(4X,I2,I2,1X,I3,F10.0)
           WRITE(6,47) J,I,KON,OR
        47 FORMAT(' ',4X,I2,I2,1X,I3,F10.0)
        39 CONTINUE
        29 CONTINUE
           STOP
           END
```