A DIAGNOSIS OF SOME MATHEMATICS DIFFICULTIES OF GRADES SEVEN AND EIGHT STUDENTS IN SPECIAL EDUCATION CLASSES

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A DIAGNOSIS OF SOME MATHEMATICS DIFFICULTIES OF GRADES SEVEN AND EIGHT STUDENTS IN SPECIAL EDUCATION CLASSES

by

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ABSTRACT

The primary purpose of this study was to investigate the strengths and weaknesses of the mathematical performance of special education students and average students (as defined by the teacher) at the grade seven and eight level. Two questions to be explored were:

(1) Are there common errors among students in a special education class?

(2) Are there errors which are common to both the regular class and the special education class?

The subjects of the study were 51 junior high students representing three schools located in Central Newfoundland. Thirty of these were special education students who ranged in age from 12 to 18, and the regular class numbered 21. A test devised by the investigator, and administered to all students individually, served as a means of data collection. During the testing interview the student was asked to explain the regrouping process in addition and subtraction of whole numbers, and his reasoning in computation when such was not clear to the investigator. Before the test began, the researcher asked the student to "think out loud" and he was recorded on a cassette tape. Manipulative devices such as beads, an abacus, number lines, place value holders and a fraction kit, were provided for students who needed to use them.

Analysis of the students' responses indicated that the regular class was superior to the special education group in every category of the test. The obvious deficiencies in the special education group were in the

sections on division of whole numbers, decimals and fractions. However, certain errors emerged as common to both groups. Several students in both groups used manipulative devices in their calculations, and more than half of each class were unable to explain the regrouping process in addition and subtraction of whole numbers.

An important observation arising from the study was the researcher's feeling that not only should the special education mathematics program be different from the regular program, but it should be adapted to meet the individual needs of the student. The investigator felt there was a necessity for three different types of programs: the regular program, a remedial program and an activity-learning approach. An important recommendation which arose from this research was that a study be conducted within the special education classes to determine which program or method of instruction might be best suited to the student and his needs.

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CHAPTER I

BACKGROUND TO THE PROBLEM

Introduction

Individual Differences

Education has long recognized that grouping children in grades according to chronological age does not assure homogeneity of groups in any other characteristics (Kirk, 1962). Within every grade you will find a wide range of individual differences in children. Thus, every classroom teacher should organize his instructional methods and assignments to meet the needs of students who may vary from one to three grades above or below the grade in which they are placed.

But, as Johnson (1963) said, ". . . instruction is still organized and applied so as to be of the greatest value to the central or major group of children . . . (p. 59)." The educational emphasis has been such as to provide primarily for the mass or group, rather than the individual. Despite the fact that educators and psychologists have been emphasizing the characteristics and needs of the individual during the past two or three decades, curricula are still planned and practiced for the group (Johnson, 1963). It seems that specific methods of instruction are designed or selected on the basis of the general learning characteristics of the large middle section of the class.

Educators committed to the goals of universal and quality instruction have attempted to reduce school failure and improve quality by trying to create classrooms that were homogeneous. For example, grouping children according to I.Q. has been only one of the many ways teachers attempted to achieve this homogeneity, and, as early as 1916, Terman wrote: "Not only in the case of retarded or exceptionally bright children, but with many others also, intelligence tests can aid in correctly placing the child in school (p. 16)."

Efforts to decrease heterogeneity by removing various groups of children from the mainstream curriculum have continued. Groups of children labelled "mentally retarded," "emotionally disturbed," etc., have been removed from the regular class to decrease the range of individual differences and thereby lighten the load of teachers and insure teaching success (Farrald & Schamber, 1973, p. 7).

However, any group of children, no matter how carefully selected, is characterized by great variability along many dimensions. Besides the differences in physical growth, there are diversities in social maturity, environment, and socioeconomic factors. The investigator felt that this was especially true within the confines of a special education class.

Nature of Special Education

Kirk (1962) has stated that special education is not a total program which is entirely different from the education of the ordinary child, and refers only to those aspects which are unique. Yet another researcher, Deno (1971), has said that special classes and the tendency to meet the problem of heterogeneity via segregation are fed by the natural tendency of any organization to get rid of what makes its goals difficult. A similar viewpoint was given by Anderson (1971) who wrote that segregated classrooms evolved when "regular teachers in regular classrooms who could not cope with the irregular behavior of children dissatisfied with their learning environment shifted them to special schools and classrooms, convincing themselves that the isolation was for the student's own good . . . (p. 7)."

Special education should involve meeting the needs of children. This should not be considered as a hollow cliché, but a mandatory concept; and in the exceptional child, educators such as Cruickshank and Johnson (1958) felt the concept of individual differences reached its epitome. For this reason, classes of special education customarily are smaller than is generally the rule in school; always the basic concept of meeting the individual's needs and his differences is present.

However, placement in any type of educational situation must be followed by continuous evaluation. This was supported by Cruickshank (1958) who stated that the status of all exceptional children changes. Since there is frequent and often rather rapid growth and change in the status of the child, it follows that there must be frequent reappraisal of the child's educational placement. If this is neglected some children will remain in special education facilities, who have ceased being exceptional children.

Purpose of the Study

The major purpose of the study was to investigate the mathematical strengths and weaknesses of a special education class as compared to a "regular" class; this was determined by a diagnostic instrument. Two questions were also explored:

1. Are there common errors among the students within a special education class?

2. Are there errors which are common to both the "regular" class and the special education class?

Significance of the Study

Special education, its quality, kind and amount, must be dependent upon the growth pattern of the child in relation to his peers and the discrepancies in growth within himself (Kirk, 1962). Grzynkowicz (1971, p. 76) has also said that it would be beneficial if the special education teacher were able to determine what problems the child was having and initiated some form of remediation. Cruickshank (1958, p. 77) has maintained that all children with any degree of differences require a complete assessment of that difference in order to arrive at a logical decision as to how to modify the educational situation to the child's best advantage.

Nevertheless, to the writer's knowledge, no systematic investigations have been carried out in Newfoundland to diagnose the mathematical problems of the students in a regular class or a special education class. In order to remedy any undesirable situations, adequate and accurate information must be available. It is hoped that this study will provide some of this information.

Definitions

This section contains a brief description of each of the variables used in the study.

Diagnostic Testing: A technique through which the teacher assesses what each student "knows" in order to find a starting point for further learning.

Diagnosis: Diagnosis, as determined by this study, involved (1) observing the student at work; and (2) interviewing the student, i.e. having the student think aloud and tell the steps he used in solving a problem.

Special Education Students: They are those students in the special education classes who were selected for the study. The placement of these students was dependent upon the procedures used by the school board to select pupils for these classes. The Department of Education (Newfoundland) defines special education students as students "who, for mental and physical causes, are unable to benefit from regular classroom instruction." Generally, students are placed in special education classes on the basis that their intelligence quotient is between 50 to 80.

Exceptional Children: Students who are in a special education class. They may be very bright, or, at the other extreme, very dull.

Regular Class Students: They are students of "average" ability as judged by their homeroom teacher.

Diagnostic Test: The diagnostic test covered operations on the whole numbers, fractions and decimals. It was given on an individual basis.

Delimitations

There were several delimitations to this study. It dealt with only one geographical area of the province, and only students from three special education classes were diagnosed. Also, no attempt was made to exhaust all the socioeconomic and environmental factors associated with the students' backgrounds. The study was further limited by the fact that teachers involved with the special education classes varied with respect to academic qualifications and personality.

Outline of the Study

A review of the related literature is presented in Chapter II. Chapter III contains the procedures followed in conducting the study, and the method used in collecting and processing the data. The results of the data analysis are discussed in Chapter IV. The final chapter summarizes the conclusions reached as a result of the study, and contains some implications for further research.

CHAPTER II

REVIEW OF RELATED LITERATURE

Background

The Importance of a Testing Program

A comprehensive testing program should start during the first year of the child's school experience and continue throughout his life. Johnson (1963) has suggested that this would ensure the recognition of educational problems at an early date and provide the administration and teaching personnel with an opportunity to correct them before they became acute.

The Teacher as Diagnostician

As early as 1935, Willis Clarke (1935, p. 138) wrote that if teachers are to assist in the direction of educational experiences so that individual differences and developmental needs of pupils are to be cared for, a much more extensive testing program will be required. In fact he said that the use of diagnostic tests should be considered one of the professional techniques of the teacher. This idea has been further supported by contemporary educators such as Farrald (1973), who felt that the regular classroom teacher, although an educational generalist, is the leader of the diagnostic and prescriptive teaching process. Farrald (1973, p. 3) has written that the classroom teacher constitutes the only resource available to schools in sufficient numbers to allow educational institutions to provide comprehensive diagnostic and prescriptive teaching services to all children. Differential treatment should be an outgrowth of the diagnostic process, and the further one removes the diagnostic process from the individual most responsible for a given child's learning, the less potent the diagnostic process becomes in terms of directing appropriate intervention. Stanford (1966) has warned teachers who interpret test results to keep in mind that a diagnosis looks to the future; its purposes are to provide a sound basis for planning future instruction. It is obviously true that many of the problems of adjustment and success in the various grades are due to the fact that students lack certain skills or facts which are presumed to be known.

Looking at the idea of the teacher diagnostician from another aspect, Bronder (1973) explained: "The teacher in a diagnostic teaching session can provide the human interaction missing in many individualized programs, and can function as a diagnostician of a student's affective needs as well as cognitive growth."

Reisman (1972) also commented that while teaching and testing usually concentrated on the cognitive domain, attitudes and emotions seemed to be directly involved in learning mathematics.

The Advantages of Diagnostic Teaching

Diagnostic teaching is appropriate for all children: the gifted, the average, the slow learners, the children who excel in mathematics, the children who always are troubled by endeavors in mathematics (Reisman, 1972). The diagnostic strategy would enable one to create new teaching sequences which are more effective with some children, and to allow one to identify strong and weak areas in mathematics for one child or for the whole class. Greene and Buswell (1930) felt that teachers who used diagnostic tests soon became familiar with the types of abilities and skills necessary to learn arithmetic. They discovered early the processes that were particularly difficult and those that were relatively easy, so that by observation and insight in teaching they were able to make use of preventive measures which rendered unnecessary some remedial measures that otherwise would be necessary. For example, a teacher might learn by diagnostic testing that pupils able in most phases of multiplication have difficulty with an intermediate zero in the multiplier.

Survey Tests as Compared to Diagnostic Tests

Survey tests in arithmetic are useful in giving the grade level at which a pupil performs and in furnishing information as to which of the fundamental processes causes him the most trouble. Such a test, for example, might show that a pupil was "up to par" in all processes except division, but it would not show which type of division problems he could and could not do (Blair, 1956). This idea was supported by Caldwell (1965) who found that standardized tests only sample student skills in each of the grades, and should not be considered a complete measure of all learning that occurred or needed to occur in the classroom. Other researchers (Rappaport, 1959; Gray, 1966) have commented that standardized tests measure skills, whereas no test has been devised that measures "understandings." Gray's concept was that "conventional tests of speed and accuracy, including most standardized arithmetic achievement tests, do not provide the type of information which is so necessary for an evaluation of the outcomes of any of the new arithmetic programs (Gray, 1966, p. 191)." Brownell (1956), too, has asserted that standardized tests rarely, if ever, provide means to assess understanding of arithmetical ideas and procedures.

However, the emphasis today has shifted from computational techniques to a thorough understanding of the mathematical ideas behind them. The need for children who can think and reason with mathematical ideas has taken precedence. That is not to imply that computation should be neglected, but rather that understanding ideas should come first (Report of the Royal Commission on Education and Youth, p. 153).

Related Research

Cases Emerging from Analysis of Survey and Diagnostic Tests

Brueckner (1969, p. 348) reported that four kinds of cases emerged from the results of survey and diagnostic tests that were analyzed:

Type 1: Cases whose performance was at or above the level that could ordinarily be expected of children of their ability and grade level.

Type 2: Cases of simple retardation whose performance was somewhat below the normal level but for whom the regular program was probably adequate. The deficiency usually responded readily to carefully directed instruction.

Type 3: Specific disability cases, such as a child who for some reason had a marked weakness in subtraction. There was always some interfering habit or an ineffective approach in this type of ability, and a remedial program based on a systematic diagnosis of the difficulty was necessary.

Type 4: Complex disability cases included the more complicated, subtle kinds of weakness. Such children were often normal in intelligence. Even though they might be severely retarded in arithmetic, they might be capable in other areas such as reading. These children had developed blockings, tensions and faulty attitudes that made them ineffective learners of arithmetic.

These four categories should be regarded as descriptive designations and there was no clear line of demarcation among them.

Advantages of Oral Diagnosis

The literature showed many studies on diagnosing errors in written work, but relatively few examples of oral diagnosis. Bronder (1973, p. 41) noted two such studies by Burge in 1934 and by Brownell and Watson in 1936. They reported that the use of an interview technique was more reliable in ascertaining errors than was a test. Francis Lankford's recent experience in a study indicated that knowledge of a pupil's thinking as he computes, could be determined by carefully conducted individual interviews (Lankford, 1974). Gray (1966) remarked on the fact that the individual interview seemed to give fairly sound evidence of the varying levels of understanding that exist among children. Smith and Neisworth (1969) have also reported that "having the child explain aloud his reasoning methods will provide promising clues (p. 157)."

Beatty, Madden and Gardner (1966) seemed to agree with this viewpoint when they stated:

> In diagnosis one is interested in specific responses, rather than generalized scores. . . . Interferences from performance on a test must be considered in relation to information gained from listening to what a pupil says as he reasons out ideas, observing how he works . . . (p. 21).

Beatty et al. (1966) also gave a useful guideline to the teacher in analyzing the work of pupils: "Ask pupils to explain to you why they did what they did when errors have been made. Discussing items with pupils is an excellent method for identifying the real difficulties encountered by the pupils (p. 21)."

Bronder (1973) mentioned a more recent study by Clever. Clever proposed a model in which the diagnostic teacher interacted face-to-face with a small group of students to assess a level of mastery on specific content items and to use the information gathered to prepare an educational prescription. This study concluded that significantly greater gains in level of mastery on specific content items were made by students for whom the teacher received a diagnosis than for those for whom the teacher received no assessment.

Blair (1956) reiterated the fact that the technique of having the pupil "speak out" while working his problems is very important. Errors could be discovered in this way which would be entirely overlooked by a mere examination of the test after the pupil had completed it. Rather than pencil and paper tests, Brownell (1956) advocated such methods of evaluation as insightful observation of pupils at work, pupils' oral reports, questioning of students, and observing them as they make errors.

One can scarcely overemphasize the importance of discovering the mental processes which lie behind pupils' methods of work. Therefore, as Green and Buswell (1930, p. 275) implied, when serious difficulties are encountered by pupils, the only final solution to the trouble is a detailed analysis of how the difficulties were produced, followed by an attempt to improve the pupil's work by some change in the methods involved.

Factors Involved in Arithmetic Deficiency

Beilin (1971) has written that when the mathematical idea to be learned depends on a level of thought beyond that which the child possesses, the idea is either partially learned or learned with much difficulty, and his grip on the idea is tenuous. Bernstein (1959) reported a study by Grace Fernald (1943) in which she listed the causative factors involved in arithmetic deficiency:

1. Mental deficiency.
2. Reading disability - (a) unable to read problems; (b) lack of what may be loosely described as background, such as key word meanings.
3. Lack of number concept - (a) skill in the basic tables of fundamentals; (b) problem-solving.
4. Blocking of adjustment by ideational or habitual factors or by emotional response (Bernstein, 1959, p. 186).

A further summary of the causes of backwardness in arithmetic was listed under three classifications by Schonell and Schonell (1957, p. 73). They were the following:

A. Environmental causes of backwardness.
   - (1) Paucity of pre-school experience.
   - (2) Two early commencement of number with dull pupils.
   - (3) Discontinuity:
     - (a) Between school and school.
     - (b) Too rapid promotion.
   - (4) Teaching methods:
     - (a) Over-explanation of processes with duller pupils.
     - (b) Over-emphasis of mechanical work.
     - (c) A too-extensive syllabus.
     - (d) Commencing a new step before the previous one is mastered.
     - (e) Bad grading of examples and an endeavor to teach two similar but not identical types of examples in the same

lesson.

B. Intellectual causes of backwardness.

(1) Deficiency in general intelligence.

(2) Weak memory for numbers.

(3) Weakness in concentration.

C. Emotional causes of backwardness.

(1) Psychological effects of failure.

(2) Tempermental disabilities:

(a) The impulsive child.

(b) The nervous child.

(c) The unsympathetic teacher.

Kenneth Lovell (1971, p. 14) cited four different factors which attribute to the difference in children's thinking:

1. Biological factors.
2. Factors which resulted from the process of socialization.
3. Factors which related to schooling and education and to cultural transmission generally.
4. Factors of self or auto-regulation. It is the reflection of the child and his own co-ordinating activities in factors two and three above, made possible also by one, which is so important in the advancement of thinking skills.

These factors, too, contributed to the various arithmetical deficiencies found in students.

Various studies (Erickson, 1958; Hildreth, 1936; Plank, 1950) have examined and found a positive relationship between intelligence and arithmetic, and attitudes and arithmetic. Alvin and Helen Rose (1961, p. 56) examined the interrelationships of intelligence, sibling position and sociocultural background with the success and failure of 456 children in arithmetic. They found a significant relationship between I.Q. and success or failure; there was also greater significance between I.Q. and arithmetic performance when the children are involved in a homogeneous classroom situation than a learning situation that is socioculturally heterogeneous. A more recent study by Noel (1970, p. 69) found that the relationship between I.Q. and problem solving is significantly greater for girls than for boys.

Types of Errors in Arithmetic

There were also certain areas of mathematical errors which emerged as common to all studies:

1. Errors in the use of zero, both as a placeholder in multiplication and division, and other errors, such as 8 x 0 = 8.
2. Errors in borrowing in all kinds of subtraction.
3. Errors in understanding and the use of the decimal point in all four fundamental operations.
4. Errors in carrying in multiplication and division.
5. Errors in the tables of fundamental facts (Bernstein, 1959, p. 193).

Brueckner, Grossnickle and Reckzeh (1961, p. 490) have discovered that the sources of difficulty in number operations were the following:

1. Lack of understanding of the number system and of the ways in which it operates in computational procedures.
2. Lack of knowledge of the basic number facts leading to guessing and random incorrect responses.
3. Lack of understanding of the meaning of number operations, and of the various steps involved in solutions.

4. Inability to perform computations with reasonable speed and accuracy.

5. The use of inefficient unsystematic procedures in making computations.

Keys (1973) has maintained that often it was not the algorithms that created problems, but rather the inability to perform the fundamental skills that were required to use the algorithm. The need to start where the pupil is has been voiced many times in many places. Robert Smith (1973), in research involving 323 students, concluded that:

> A close examination of various computation algorithms reveal that basic principles of place value underlie many of the processes. For example, in addition and subtraction with regrouping, the pupil learns to name tens as ones, ones as tens, and so on, in order to facilitate the computational process. Comprehension of such renaming requires a basic understanding of place value concepts (p. 1).

Ruddell (1959) investigated the level of difficulty found in division and discovered that "it /division/ is complicated because it involves many of the other processes: addition, multiplication and subtraction, and also includes principles unique to division alone (p. 97)." Ruddell recorded five understandings required for division:

1. Division is a special case of subtraction.

2. Division is the reverse of multiplication.

3. An understanding of the Hindu-Arabic decimal system of notation and the place value of number is essential to understanding the division of whole numbers.

4. Divisor-dividend-quotient relationships hold generalizations essential for complete understanding of the division process.

5. Addition-subtraction-multiplication within the division process must be understood.

For illustrative purposes, the list of the most frequent errors in operation with decimals made by a group of 168 sixth, seventh, and eighth graders was given by Brueckner et al. (1961, p. 485.) This is given in Table 1.

Gus Buswell and Lenore John have devised a list of the ineffective work habits and errors in the four fundamental operations of whole numbers exhibited by 106 eighth-grade pupils, as is illustrated in Table 2.

Lankford, in his 1974 study, noted the responses and the pupil's rationale for obtaining each answer. One such example is given in Table 3.

Summary

On the basis of the data secured through a systematic case study a statement can be made of what the nature and underlying causes of the deficiency seem to be and the kinds of remedial measures that should be undertaken. Treatment cannot be effective unless it is guided by the results of a diagnosis. The methods of diagnosis should be adopted and applied in the study of the work of any pupil whose work is seriously deficient (Brueckner et al.). Unless his faults are known, remedial instruction cannot be effectively planned.

Table 1

Errors Made by Sixth-, Seventh-, and Eighth-Grade Pupils in the Addition, Subtraction, Multiplication, and Division of Decimals (Adapted from Brueckner)

Difficulty	Frequency of Error
Addition of decimals:	
Errors in placing decimal point	275
Weakness in number combinations	128
Misplacing whole numbers	34
Carrying difficulties	31
Inability to add fractions and decimals	23
Subtraction of decimals:	
Borrowing difficulties	221
Misplacing decimal number in subtrahend	74
Weakness in subtraction facts	50
Confuses subtraction with addition	45
Decimal point omitted	17
Multiplication of decimals:	
Misplacing decimal point	631
Errors in multiplication	365
Omitting decimal point	119
Failure to prefix zero	87
Inability to multiply decimal and fraction	62

....continued

Table 1 (continued)

Difficulty	Frequency of Error
Division of decimals:	
Decimal point misplaced	1436
Errors in division	376
Decimal point omitted	356
Failure to reduce remainder to decimal	172
Failure to prefix zero in quotient	163

NOTE: The size of the numbers in the frequency of error column surpasses the number of pupils that were involved because each pupil was given a quantity of examples to compute.

Table 2

Ineffective Work Habits and Errors Exhibited by 106 Eighth-Grade Pupils (Adapted from Buswell and John)

Habit or Error	Frequency
Addition:	
Errors in combinations	94
Added carried number last	76
Added carried number irregularly	55
Irregular procedure in column	50
Grouped two or more numbers	43
Retraced work after partly done	32
Carried wrong number	30
Split numbers	27
Dropped back one or more tens	22
Forgot to add carried number	19
Subtraction:	
Errors in combinations	63
Did not allow for having borrowed	48
Error in reading	35
Deducted from minuend when borrowing was not necessary	19
Said example backward	18
Deducted two from minuend after borrowing	12
Counting	10
Error due to minuend and subtrahend digits being same	6

....continued

Table 2 (continued)

Habit or Error	Frequency
Used minuend or subtrahend as remainder	5
Multiplication:	
Error in adding the carried number	69
Used multiplicand as multiplier	56
Errors in multiplication combinations	46
Carried a wrong number	44
Wrote rows of zeros	43
Errors in addition	42
Error in single zero combinations, zero as multiplier	42
Errors in reading	35
Forgot to carry	27
Error in position of partial products	23
Division:	
Errors in subtraction	79
Errors in multiplication	79
Found quotient by trial multiplication	57
Used long-division form for short division	52
Errors in division combinations	41
Omitted digit in dividend	38
Omitted final remainder	30
Used remainder larger than divisor	29
Used short-division form for long division	29
Omitted zero resulting from another digit	27

Table 3

Examples of Wrong Answers for 3/4 - 1/2 = ____, and the Pupil's Rationale for Obtaining Each Answer

Answers	Pupil's Rationale
2/2 or 1	"3 minus 1 equals 2, and 4 minus 2 equals 2."
2/2	Chose 2 as the common denominator because "need to find number that will go into 4 because the bottom number has to be the same as this." (Points to the "2" in 1/2.) Then, "3 minus 1 equals 2."
Incomplete	Wrote 2/4 for 3/4 from "4 goes into 4, 1 time, and 3 minus 1 equals 2." Wrote 2/4 for 1/2 from "2 goes into 4, 2 times, and 2 times 1 equals 2." Couldn't go further.
Incomplete	Rewrote the exercise vertically, then "You have to make 4 and 2 even. 2 won't go into 3 evenly, 8 won't go, 9 won't go into 4. Try 12. 3 divided by 12 goes 4 times, 4 divided by 12 goes 3 times 1 divided by 12, it will go 1 time; 1 goes into 2, 1 time, and 1 left over. You have to try another number." Stopped.
0/4	Chose 4 as the common denominator, then "4 into 4, 1 time; 1 times 3 equals 3. So 3/4 equals 3/4. 2 goes into 4, 3 times; 3 times 1 equals 3, so 1/2 equals 3/4. 3 minus 3 equals 0."
5/4	"1/2 equals 2/4 and 3/4 equals 3/4; then 3 plus 2 equals 5. When you subtract, you don't subtract, you add the opposite."
9/8	"Have to make the 3 a 13; have to make 1 a 10." Then, "13 minus 4 equals 9, and 10 minus 2 equals 8."
1/1	"3 subtract 4 is 1." for the numerator; "1 subtract 2 leaves 1," for the denominator.
0/0	"3 won't go into 1." Wrote 0 for the numerator. "4 won't go into 2." Wrote 0 for the denominator.
1 2/8	Chose 8 as the common denominator. Wrote 3/4 as 12/8 from "3 times 4 equals 12" and 1/2 as 2/8 from "1 times 2 equals 2." Then 12/8 - 2/8 = 10/8 = 1 2/8.

....continued

Table 3 (continued)

Answers	Pupil's Rationale
0/8	Chose 8 as the common denominator. Wrote 3/4 as 1/8 from "8 will go into 4, 2 times, and 2 will go into 3, 1 time." Wrote 1/2 as 1/8 from "8 will go into 2, 4 times; 4 will go into 1, 1 time." 1/8 - 1/8 = 0/8.
1/2	Wrote 3/4 as 3/4 and 1/2 as 1/4. Then 3/4 - 1/4 = 2/4 = 1/2.
Incomplete	First chose 8 as the common denominator. Wrote 1/8 for 3/4 from "4 goes into 8, 2 times, and 3 from 2 is 1." Wrote 3/8 for 1/2 from "8 goes into 2, 4 times, and 4 take away 1 is 3." Then "That ain't gonna work because you wan't take 3 from 1." Tried a common denominator of 16. By the same process, got 1/16 for 3/4 and 7/16 for 1/2. Still could not subtract 7 from 1 so gave up.
2/4	Chose 4 as the common denominator. Wrote 3/4 for 3/4 and 1/4 for 1/2. Then 3/4 - 1/4 = 2/4.
2/4	Chose 4 as the common denominator. Then 3 - 1 = 2.
0/4	Chose 4 as the common denominator. Wrote 4/4 for 3/4 from "4 goes into 4, 1 time; 1 plus 3 is 4." Wrote 5/4 for 1/2 from "2 goes into 4, 2 times, plus the 1 is 5. Can't take 5 from 4, so I borrow 1 from the denominator, make if 5/4." Then 5/4 - 5/4 = 0/4.
22	"2 take away 4 is 2. 1 take away 3 is 2."
2/6	Chose 6 for the denominator, then "3 minus 1 equals 2" for the numerator. Then, "2 will go into 6, 4 times, and 4 will go into 6 with 2 left over."
4/8	Wrote 6/8 for 3/4 from "4 times 2 equals 8, so 3 times 2 equals 6." Wrote 2/8 for 1/2 from "1 times 2 equals 2." Reasoned that because he multiplied the 3 of 3/4 by 2 he must use the same number here. Then 6/8 - 2/8 = 4/8.
11	Three over four leaves 1; 1 from 1/2 leaves 1.
1/2	Chose 4 as the common denominator. Wrote $\overline{4} - \overline{4}$. "Put my 3 here, minus 1/4; that would be 2/4 or 1/2."
1/4	Wrote 4/4 for 3/4 from "4 goes into 4, 1 time; 3 plus 1 equals 4." Wrote 3/4 for 1/2 from "2 will go into 4, 2 times; 2 plus 1 equals 3." 4 - 3 = 1, "bring down the 4."

CHAPTER III

DESIGN OF THE STUDY

Introduction

This chapter presents a description of the design of the study. It includes information about the following: the instrument, the pilot study, the sample, the procedure used in conducting the study, and the method of collecting and analyzing the data.

The Instrument

Several diagnostic instruments were reviewed by the investigator for selection purposes. Since the results of these did not give pertinent information to the researcher, it was decided to devise another diagnostic instrument which would be given on an individual basis. However, the questions contained in the standardized tests were used as a basis for developing the instrument used in this study. Also, a few sections of the test were presented in three ways according to Bruner's enactive, ikonic and symbolic levels. If a student was unable to answer a question symbolically, he was shown a picture illustrating the question; if this did not succeed, manipulative materials such as place value sticks, an abacus, a fraction kit and beads were provided. The addition and subtraction of whole numbers and the addition of fractions were presented in this way. The student was asked to think aloud as he solved the problems. In this way the researcher could question the student about his method of calculations or ask him to explain the algorithm. This idea was advocated by Beatty, Madden and Gardner (1956), who felt that items and patterns of response should be discussed with the students.

Before the test was constructed, a set of behavioral objectives was listed and was categorized along Bloom's taxonomy by the researcher in conjunction with a specialist in mathematics education. The objectives were as follows:

Properties:

1. The student will apply the commutative property of multiplication, the associative property of addition, and the distributive property of multiplication over addition by supplying the missing factors in illustrating the properties.

Application/Analysis

2. The student will demonstrate a knowledge of place value by interpreting the value of the tens place in a three-digit numeral.

Comprehension

3. The student will demonstrate the meaning of additive identity and multiplicative identity in examples where they have to supply the correct identity.

Application

Addition of Whole Numbers:

1. The student will compute the sum of two two-digit numbers: (a) without regrouping, (b) with regrouping in the tens place only, (c) with regrouping in the hundreds place only, (d) with regrouping in the tens and hundreds place.

Application

2. The student will solve simple word problems involving dollars and cents.

Analysis

3. The student will compute the sum of two-digit numbers and explain the renaming and regrouping that is necessary.

Analysis

4. The student will supply the missing digits in a two-digit sum, where part of the answer is given.

Analysis/Synthesis

Subtraction of Whole Numbers:

1. The student will compute the difference of two whole numbers, less than 10,000 with no renaming.

Application

2. The student should be able to subtract numbers between 100 and 1000 with renaming hundreds as tens and tens as ones.

Application

3. The student should be able to estimate a difference by rounding to the nearest ten or hundred.

Analysis

4. The student should compute the difference where as many as three renamings are necessary.

Application

Multiplication of Whole Numbers:

1. The student should name the product of one and any whole number; of zero and any whole number.

Comprehension

2. The student should be able to multiply any two whole numbers

up to 999.

Application

3. The student should be able to solve simple word problems.

Analysis/Synthesis

Division of Whole Numbers:

1. The student should be able to divide any number by one; divide zero by any number.

Comprehension

2. The pupil will divide with a divisor between ten and 100; he will divide by a number less than 100 where the dividend is less than 10,000.

Application

3. The pupil should be able to divide to find the greatest multiple of the divisor that is less than the dividend and then name the remainder.

Application

Decimals:

1. The student should be able to write a decimal equivalent to a fraction; to write a fraction equivalent to a decimal.

Application

2. The pupil will compare two decimals in order to tell which is the greater or lesser number.

Evaluation

3. The pupil will add and subtract using decimals. (See addition and subtraction of whole numbers.)

Application

Fractions:

1. The student will write a fraction indicating which portion of

a set is shaded.

Application

2. The student will solve equations of the type, 3/4 = n/12.

Analysis

3. The student will add and subtract with fractions having the same denominator.

Application

4. The student will add and subtract fractions by finding a common denominator.

Application

5. The student should be able to add and subtract using mixed numbers where renaming is necessary.

Application

6. The student should be able to find a product using fractions.

Application

The test itself consisted of questions involving the four fundamental operations of whole numbers, addition, subtraction and multiplication of decimals and fractions, and rewriting decimals as fractions and fractions as decimals. A copy of this test can be found in Appendix B. Students were asked to think aloud and to explain their steps. For example, if a student said "carry," he was asked to explain what that meant. The student was asked to explain the algorithm on which he was working.

The Pilot Study

In order to check on (1) the student's ability to perform the necessary calculations, (2) the maximum amount of time required by pupils to complete the test, and (3) to determine if the instrument was adequate as a device to collect the data necessary for this research, a pilot study was carried out on a small number of special education students in St. John's. The responses were not analyzed in a manner for presentation, but they were examined in an effort to ascertain if modifications were necessary. The results of this study indicated that the test in its original form could be used and that the study could be continued as planned.

Sampling

Two groups of students were drawn for the study from the Central Newfoundland areas of Botwood, Grand Falls and Baie Verte. There were a number of reasons for selecting this particular area, some of which were:

1. A request for assistance had come from within the school system from one particular school board.
2. This particular area is a diverse one which enabled the researcher to gather information on students from various cultural, social and economic backgrounds.
3. It provided an opportunity to conduct research in a rural setting, which the researcher felt necessary in Newfoundland.

The groups have been described in the following paragraphs:

A. Special Education Class. This group consisted of 13 boys and 17 girls who were between the ages of 12 and 18. The makeup was dependent upon the procedures used by the school board to select students for these classes. For some students it was their first year in a special education class, and others had been there for four years. The group was at the Junior High Level.

B. Regular Class. This group consisted of eight boys and 13 girls between the ages of 12 and 14. No attempt was made by the investigator to match them chronologically or otherwise with the other group. The only stipulation was that they be "average" students, and from then on the selection process was left to the teacher's judgment. In each case, these students were selected from the "B" classes of the seventh or eighth grade.

Procedure

The interviews were conducted in a room (usually that reserved for guidance) where a single pupil and the interviewer could work undisturbed. The pupil was given a set of computational exercises, and asked to do them as he usually did, but to "think out loud as you compute." The exercises were as described in the section under "Instruments." Although the average length of the interview was 30 minutes, some students finished in 15 minutes while others required 45 minutes. The pupil was not hurried; he simply completed as many exercises as he could. A verbatim record of each interview was made on a cassette tape.

Analysis of Data

The data from the completed tests were tabulated by the investigator. Descriptive statistics were used and a report on the strengths and weaknesses for each student was made and placed on file cards. The results are presented in tabular form in Chapter IV.

CHAPTER IV

ANALYSIS OF DATA

Introduction

In this chapter the data relevant to the study are presented and analyzed. The chapter is divided into two sections. Section one contains a description and discussion of the errors and student habits that were observed during the test. The analysis of the data is given in the second section. The investigator felt that if one-third or more of any group made errors in any particular section or question of the test, she could reasonably assume that the students were weak or deficient in that area.

Section I: Description of Errors and Student Habits

Number Properties

The first part of the test presented the number properties, which were written horizontally (i.e. $5 + \square = 5$). Table 4 indicates the number of people who computed these incorrectly. It is evident from data in Table 4 that the special education students are very weak in the knowledge of number properties. However, it should be noted that even the presentation of the questions made it difficult for them. It seemed to the investigator that these students did not fully comprehend the meaning of the "equals" sign in the equation form of the question. For example, in the property $5 + \square = 5$, many students said, "five plus five equals ten,"

Table 4

Errors in Properties

Properties	Special Education		Regular Class	
	Number	Percent	Number	Percent
I. Additive Identity	14	46.6%	0	0%
II. Multiplication Identity	16	53.3%	2	9.5%
III. Commutative (Multiplication)	17	56.7%	2	9.5%
IV. Place Value	17	56.7%	2	9.5%
V. Distributive	27	90%	11	52.4%
VI. Associative (Addition)	14	46.6%	1	4.8%

and gave ten as their answer. Also, in the question $9 \times \square = 9$, one student said, "If I knew how much nine times nine were, I'd know the answer." This idea of adding or multiplying each number was prevalent in the other four examples.

The percentage of people who made errors in the distributive property was high for both groups. It should be noted here that two out of the three students in the special education group who did get this one correct were doing grade 10 mathematics.

In the following sections of the test, the investigator noted student habits, as well as errors, as they solved the questions. The habits were used as aids in computation, but some of them were rather cumbersome.

Addition of Whole Numbers

The second section dealt with the addition of whole numbers. There were 13 different errors or habits observed in the special education group, and seven for the regular group. They have been included in Tables 5 and 6.

Tables 5 and 6 indicate that many of the errors were made by only one person or by several people in only one instance. For convenience and for the hope of giving an accurate picture of this study, these lists of errors and habits have been summarized in Table 7. Future reference to all types of errors for other sections are listed in Appendix C.

Discussion. Table 7 indicated that the two groups did not compute equally well in addition, and twenty percent more of the special education group used manipulative materials than did the regular group. Both groups had difficulty in the explanation of "carried," and some of them seemed

Table 5

All Errors/Habits for Special Education Group in the Addition of Whole Numbers

Errors/Habits	Number of People who Made Them
I. Understanding of the regrouping process in the algorithm. When a student said, "carry the one," the investigator asked, "What do you mean when you say, "carry the one"? The different responses were:	
(a) The one is one out of ten.	1
(b) It's just one.	9
(c) Always said it was one ten, no matter if it were hundreds, etc.	1
(d) A one is carried because there's not room enough to put down both numbers.	1
(e) You're supposed to carry the one.	1
(f) Not sure - In this case, students said, "The one is one ten, no it's one hundred, no it's a ten...." They didn't know the meaning.	8
(g) The one comes from ten.	1
II. A. Errors in combination (e.g. 8 + 5 = 12).	10
B. Errors in combination with 0 (e.g. 1 + 0 = 0).	5
III. Used manipulative devices to add:	
(a) counted on fingers	10
(b) tapped out answers	1
(c) used beads to count	2
IV. Forgot to include carried number	6
V. Multiplied the ones digit and then added (e.g. 13 + 34 = 52).	2

....continued

Table 5 (continued)

	Errors/Habits	Number of People who Made Them
VI.	Subtracted instead of adding the carried number.	1
VII.	Multiplied instead of adding.	1
VIII.	When the question was written horizontally, the student added each digit (e.g. 57 + 8 = 20).	1
IX.	Wrote answer backwards.	1
X.	Added the same digit in two columns (e.g. 57 + 8 = 145).	2
XI.	Subtracted instead of adding.	1
XII.	Omitted the word problem.	2
XIII.	Could not do question: "Supply the missing numbers 43☐ + 2☐8 = ☐98."	
	(a) Added the other numbers in the column.	6
	(b) Put zero in each blank.	1
	(c) Added horizontally.	1
	(d) Omitted it.	1

Table 6

All Errors/Habits for Regular Group in the Addition of Whole Numbers

	Errors/Habits	Number of People
I.	Understanding of the regrouping process in the algorithm. The responses were:	
	(a) The one is a ten.	4
	(b) It's just one.	6
	(c) Not sure.	3
II.	Errors in combination	6
III.	Used manipulative devices to add:	
	(a) Used beads to count.	2
	(b) Counted on fingers.	3
IV.	Multiplied instead of adding.	1
V.	Forgot to add carried number	2
VI.	Omitted the question: "Supply the missing numbers. $43\square + 2\square 8 = \square 98$.	1
VII.	Got the correct answers, but wrote the incorrect one.	1

Table 7

Summary of Errors/Habits in Addition of Whole Numbers

	Special Education		Regular Class	
	Number	Percent	Number	Percent
I. Understanding of the algorithm; meaning of the carried one:				
(a) It's ten.	1	3.33%	4	19%
(b) Just one.	9	30%	6	28.5%
(c) Not sure.	8	26.6%	3	14.3%
(d) Others.	4	13.3%	0	0%
Total	18	73.23%	13	61.8%
II. Incorrect combinations with zero; i.e. 1 + 0 = 0	5	16.7%	0	0%
III. Used manipulative devices such as counting on fingers, using beads, and tapping out answer.	13	43.3%	5	23.8%
IV. Could not do question on "Supply the missing numbers."				
(a) Added other numbers in the column.	6	20%	0	0%
(b) Put zero in each blank.	1	3.3%	0	0%
(c) Added across.	1	3.3%	0	0%
(d) Omitted it.	1	3.3%	1	4.76%
Total	9	29.9%	1	4.76%

to be puzzled as to why the researcher would ask the meaning of the carried one when it was very evident (to them) that it was "just one." There were several errors made in combinations, but, on the whole, only three students made mistakes in each example. These students, from the special education class, were judged by the researcher to be lacking in the skills necessary to compute accurately.

Subtraction of Whole Numbers

Similar results were obtained in this phase of the study. The students were again asked to explain the regrouping process in terms of what was meant by borrowing the one. Table 8 showed that some of the errors and habits were similar to those of addition.

Discussion. The number "zero" posed several difficulties to at least five students in both addition and subtraction. For instance, they would say that 0 - 6 = 6, 5 - 0 = 0, and in the addition section, 1 + 0 = 0. Even though manipulative devices were provided, the students saw no need to use them in those examples. One student drew circles on his paper and crossed out the number to be subtracted. This method, however accurate, proved to be awkward and time consuming.

The subtraction process itself seemed to be purely mechanical in many cases, as it was in Lankford's (1974) study. For example, in the question, 800 - 60, a special education student in thinking out loud, said: "This one here is zero, 6 from 0, well you can't do, so you borrow 1 from 8, make that 7, put up 1, you have 10, 6,7,8,9,10, that's 4, 7 comes down." Here the student counted to find 10 - 6. In another example, 834 - 590, a student in a regular class reasoned thus: "Four from 0 is 4, 3 from 9 you can't do, so borrow from the 8, make it 7, put down 1; 13

Table 8

Summary of Errors/Habits in Subtraction of Whole Numbers

	Special Education		Regular Class	
	Number	Percent	Number	Percent
I. Understanding the regrouping process in terms of "borrowing" a number:				
(a) Could not explain "borrowing."	5	15.7%	7	33.3%
(b) Said it was making a number bigger.	5	16.7%	2	9.5%
(c) Regrouped when not necessary.	6	20%	1	4.8%
Total	16	53.4%	10	47.6%
II. (a) Errors in combination (i.e. 12 - 3 = 8)	4	13.3%	0	0%
(b) Errors in combination with zero (i.e. 4 - 0 = 0, 0 - 6 = 6).	6	20%	3	14.3%
III. Subtracted smaller number from larger.	3	10%	2	9.5%
IV. Read example backwards but got the correct answer (i.e. 7 - 5, the student read this as 5 minus 7).	3	10%	2	9.5%

from 9 is 4, 7 from 5 is 2." In this case, the student read the individual columns backwards, even though the correct answer was obtained. These two examples were typical practices of several students.

There was one question in this part of the test involving estimation. It was interesting to note that only one person out of 51 made the correct response. The rest of the students merely subtracted the question (52 - 28) without rounding off the numbers. The investigator asked a few students to guess at the answer, but even in this case they subtracted the numbers and gave 24 as their answer.

Pictures illustrating the subtraction process were provided for students who had difficulty in this operation at the symbolic level. These pictures, which were used with one student, are illustrated in Diagram 1.

The other errors that were made in this section have been placed in Appendix C.

Multiplication of Whole Numbers

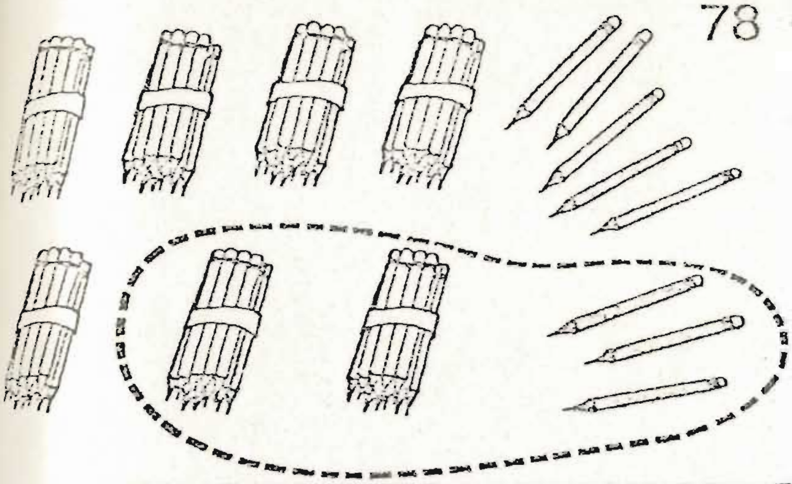
Errors in this section of the test are summarized in Table 9.

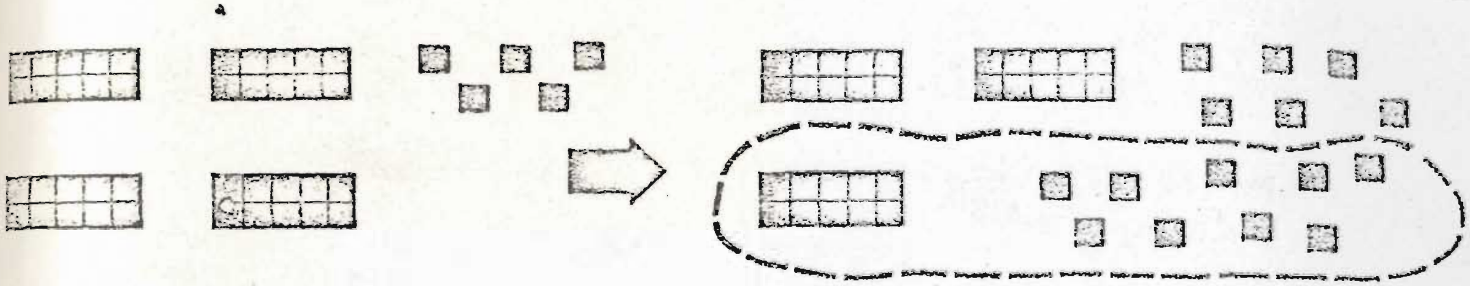
Discussion. The percentage of the fourth observation was rather high because the eight students in the first category were not considered. Two people confused the partial product when the multiplier had three digits. For example, the workings of one student were the following:

321	In multiplying by the tens digit, this pupil
x 130	said, 3 ones are 3, 3 twos are 6, 3 threes are
000	9, but he wrote 3.
363	When he multiplied by the hundreds digit, the
323	student said 3 ones are 3, 3 twos are 2, 3 threes
35930	are 3.

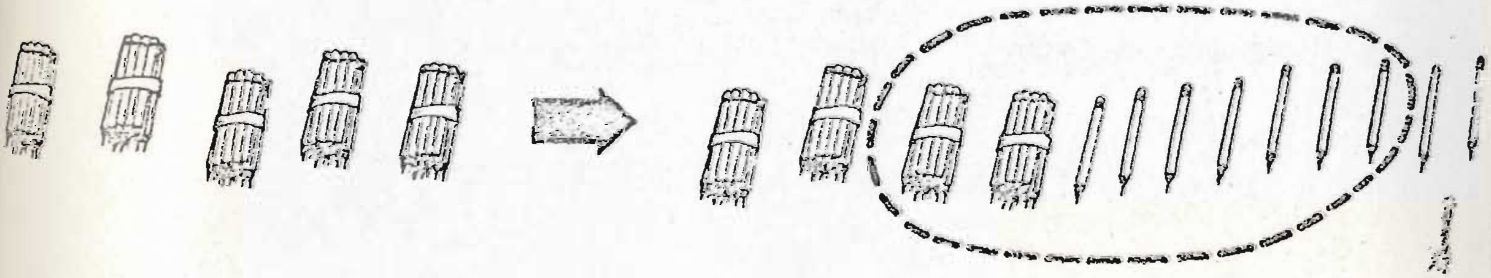
Diagram I

$78 - 23 =$





$$\begin{array}{r} 45 \\ -\,19 \\ \hline \end{array}$$



$$\begin{array}{r} 50 \\ -\,27 \\ \hline \end{array}$$

Ikonic Representation For Subtraction

Of Whole Numbers

Table 9

Summary of Errors/Habits in Multiplication
of Whole Numbers

	Special Education		Regular Class	
	Number	Percent	Number	Percent
1. Couldn't multiply with a two-digit multiplier.	8	26.7%	0	0%
2. Wrote rows of zeros.	14	66.7%	18	85.8%
3. Error in position of partial product.	4	13.3%	2	9.5%
4. Couldn't give a reason for the position of partial product.	5 (out of 23)	23%	7	33.3%

The eight people in the special education group who couldn't multiply with a two-digit multiplier were from one particular class, and most of them attempted the examples. However, a common occurrence was the multiplication of the ones digit by the ones digit, the tens by tens, and hundreds by hundreds. The researcher provided a dot table similar to the illustration in Diagram 2 for those who had difficulty with the multiplication combinations. The purpose of this table was to illustrate multiplication at Bruner's ikonic level, but it was discovered that most students found it confusing and did not know how to use it. In most cases, therefore, the multiplication tables were supplied to the students who could not work without them.

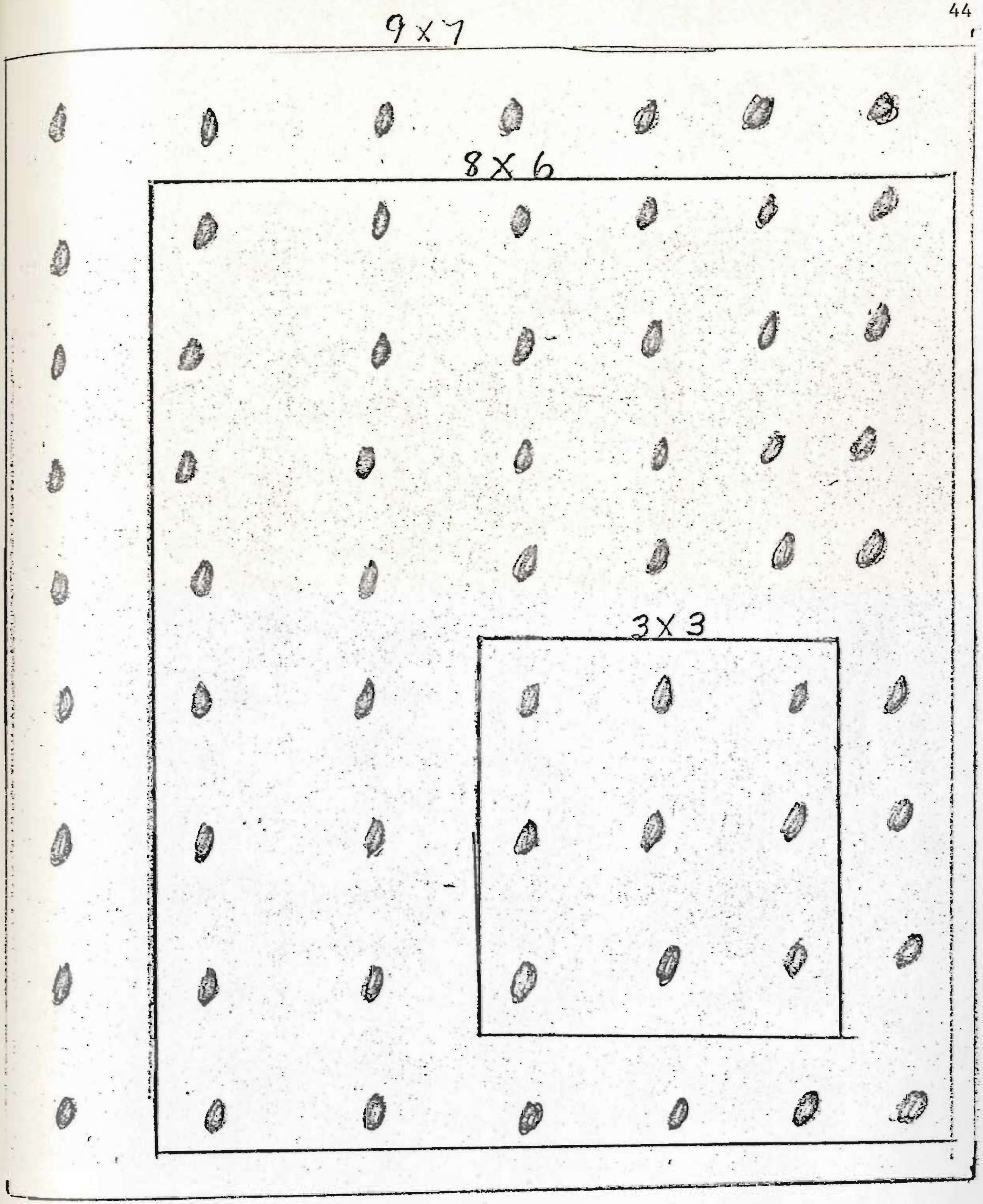
Division of Whole Numbers

Division was the weakest of the four fundamental operations on whole numbers. The special education group had 17 different types of errors, and the regular group 14. Both groups used trial multiplication to some extent to find solutions, and some counted to get division combinations. The rest of the errors are summarized in Table 10.

Discussion. The first error mentioned here was common to both groups. It came as a result of the question, $8\overline{)1624}$; the students who made this error gave an answer of 23. To determine if this was merely a careless omission of zero, the investigator gave each of these students another example, $3\overline{)1521}$, and the answer given by all students was 57.

As Table 10 indicates, five students could divide only if the numbers were included in the multiplication tables. An example of this was shown in the question, $4\overline{)76}$. The answer given was, "Can't be done because 4 only goes up to 48 on the tables," or a similar answer, "76

Diagram 2



Dot Table For Multiplication of Whole Numbers

Table 10

Summary of Errors/Habits in Division of Whole Numbers

	Special Education		Regular Class	
	Number	Percent	Number	Percent
1. Omitted zero resulting from another digit (i.e. 8)1624.	11	36.7%	11	52.4%
2. Used trial multiplication.	9	30%	15	71.4%
3. Used long division form for short division.	9	30%	1	4.8%
4. Errors in division combinations.	4	13.3%	1	4.8%
5. Could divide only if numbers were in the multiplication tables.	5	16.7%	0	0%
6. Used remainder larger than divisor.	0	0%	4	19%
7. Used digits of the divisor separately (63)126 = 6)12 + 3)6 = 22).	3	10%	0	0%

is not on the times tables." One student said, "If they had a decimal, I'd be able to do them," and the person did manage to do better than she had done after the investigator put a decimal point in two questions. The investigator found out later that the class had been working with division of decimals.

One student's method of dividing was rather interesting and is illustrated here:

```
    216
  ------
  8)1624
    16
    --
    02
     0
    --
     04
     32
     --
     2R
```

The pupil said 8 into 16 is 2, then multiplied 8 x 2 to get 16; he subtracted 16 from 16 and got 0, brought down 2; then he said 8 into 2, 8 2's are 16; he put 16 in the quotient, brought down the 4 and multiplied 4 x 8 to get 32. The student had a combination of multiplication and division here, as he had in other examples.

Another person divided the dividend by itself in four examples receiving one or 11 for the answer. The researchers gave the student 12 beads and asked him to share them among six boys; the student completed this (manipulating the beads), and was given another example, 15 beads among three boys, which he worked out successfully. However, when these examples were written symbolically, the student could not do them.

The investigator also noticed that many students relied on the multiplication tables for division combinations.

Decimals

There were very few mistakes in the section on decimals that were not previously mentioned in the tables on addition and subtraction of whole numbers. Again, there were several students who could not manipulate the numbers when they were written horizontally; that problem resembled the first part of the test dealing with the properties. These students rewrote the questions in a vertical position. The results have been summarized in Table 11. Though there was only one question each in multiplication and subtraction, the errors were also presented in tabular form.

Discussion. Some students who computed the addition of decimals correctly, did so mechanically. They added the numbers and put in the "dot." The decimal point as such had no meaning to them as they had not worked with decimals in their math program prior to this testing. One student added each digit in the sum. For example, he said that 2.4 + 0.2 was 8. The subtraction question was 3.2 - .4. Three people in the regular class were confused as to which number was larger, 3.2 or .4. Consequently, when they rewrote the problem vertically, they subtracted 3.2 from .4.

There were two items in which the student had to insert the correct sign of less than, greater than or equals. The first of these, .864 ○ .684 presented no difficulty; when asked why they inserted a "greater than" sign most students said 86 was bigger than 68. One student who was very deficient in mathematics said .864 = .684 because there were "the same numbers in each of them." The second example, .060 ○ .06, posed more difficulty. Only seven percent of the special education group and approximately 40 percent of the regular group solved it correctly. The majority of students gave "greater than" for the answer because "sixty is greater than six." One person gave an example to explain his reasoning: ".060 would be like 60 pieces out of a pie and .06 would be only six pieces. Therefore .060 $\overset{>}{\bigcirc}$ .06."

The investigator realized that the test presented too few decimal questions on which to give an adequate diagnosis. Nevertheless, the fact

Table 11

Summary of Errors/Habits in Decimals

	Special Education		Regular Class	
	Number	Percent	Number	Percent
Addition				
I. Omitted questions.	3	10%	0	0%
II. Omitted decimal point.	5	16.7%	0	0%
III. Subtracted instead of added.	1	3.3%	0	0%
IV. Decimal point in the wrong place.	0	0%	1	4.8%
Subtraction				
I. Omitted question.	4	13.3%	0	0%
II. Added instead of subtracted.	1	3.3%	0	0%
III. Subtracted minuend from the subtrahend.	0	0%	3	14.3%
Multiplication				
I. Omitted question.	14	46.7%	0	0%
II. Omitted decimal point.	2	6.67%	1	4.8%
III. Decimal point in wrong place.	0	0%	2	9.5%
Signs ($<$, $>$, $=$)				
I. Omitted question.	12	40%	0	0%
II. Said either $.060 < .06$, or $.060 > .06$.	16	53.3%	13	61.9%

that many students were unable to solve them gave evidence of a weakness in this area and the need for a better foundation. Two questions on the test required the student to rewrite fractions as decimals and vice versa. These were omitted by 73 percent of the special education group, and by approximately 35 percent of the other group. Thus, any analysis given here would be very limited because of the numbers involved. The percentages given in Tables 12 and 13 have been inflated because they were calculated on the number of people in each group who actually attempted the questions.

There were no outstanding errors in the decimals to fractions section, but in the reverse operation there were a few which are explained.

Five of the students could rewrite fractions as decimals only if the denominator was ten. For instance they said 3/10 was .3, 6/10 was .6, 8/10 was .8, but were unable to rewrite 2/5 as .4. Three students merely placed a decimal point between the numerator and denominator; for example, they said 2/5 was the same as 2.5. These students were uncertain if this was either the correct answer or the proper method.

One student had an unusual method of solving this kind of problem, but it only worked when the denominator was ten, as is illustrated in Diagram 3.

Fractions

The questions on the final section of the diagnostic instrument were varied; they included two items where the student was required to name the fractions represented by pictures, two involving addition, one subtraction, one multiplication and one on equivalent fractions. There were 10 students from the special education group who omitted the entire section; thus the analysis for this group was rather limited because of the numbers involved.

Diagram 3

Change these fractions to decimals

(a) $\frac{3}{10}$ .30

(b) 2/5 .10

Student's reasoning:

Three multiplied by ten is thirty.

Put a decimal point before 30.

Two multiplied by five is 10.

Put a decimal point before 10.

Student Example for Rewriting Fractions as Decimals

Table 12

Summary of Errors/Habits in Rewriting Decimals as Fractions

	Special Education		Regular Class	
	Number (out of 8)	Percent	Number (out of 11)	Percent
I. Had 100 as the denominator in each.	1	12.5%	2	18.2%
II. Left in decimal point	0	0%	3	27.2%

Table 13

Summary of Errors/Habits in Rewriting Fractions as Decimals

	Special Education		Regular Class	
	Number (out of 8)	Percent	Number (out of 11)	Percent
I. Could do if the denominator was 10.	1	12.5%	4	28.5%
II. Put a decimal point between the numerator and denominator	1	12.5%	2	14.2%

Many of the students made errors in the addition of fractions and in this case the investigator provided diagrams to see if the student could add at the ikonic level. The results of the student's symbolic operations are presented in Table 14.

Discussion. Only six students out of the thirty in the special education group could add at the symbolic level. The others needed the various pictures that have been reproduced in Diagrams 4 and 5. A fraction kit was also provided; this gave the student a chance to use pieces of cardboard to make either circles or rectangles, and it gave the investigator an opportunity to ask questions as to how much of a circle remained when a few pieces were taken away. The important point here was that the student manipulated the pieces themselves.

The pictures were used in the following manner: if a student gave an incorrect answer in adding 3/5 + 1/5 and 1/4 + 3/8, he was shown a chart, as in Diagram 4, and asked to give an answer from looking at the picture. Then he was asked 1/6 + 1/3 and so on. It was not necessarily in this order and not all the pictures were used with all students. If a student made a mistake with this he was given the fraction kit where he used pieces to represent fractions. Many times when a student was at this enactive level he did not know how to either name or write a fraction, and the researcher had to be careful to distinguish between diagnostic teaching and diagnostic testing. Several students became confused in adding 1/6 + 1/3. One person in particular gave two answers: 2/9 if he were looking at the numbers on the picture, and 1/2 if he was looking at the picture. Nine out of the 21 regular class students were also shown pictures to help them add.

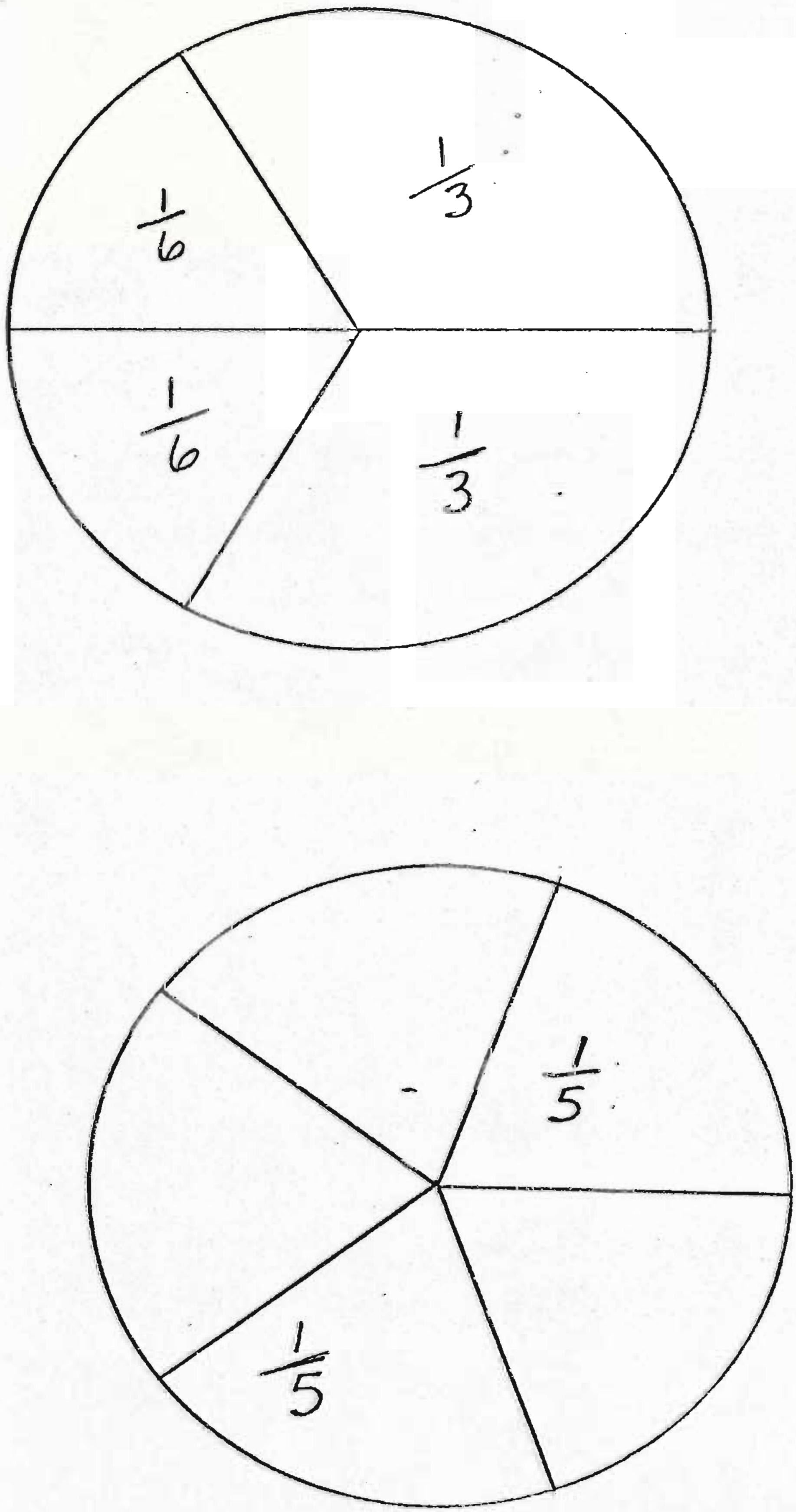
As an example of the type of answers students gave in adding frac-

Table 14

Summary of Errors/Habits in Addition and Multiplication of Fractions

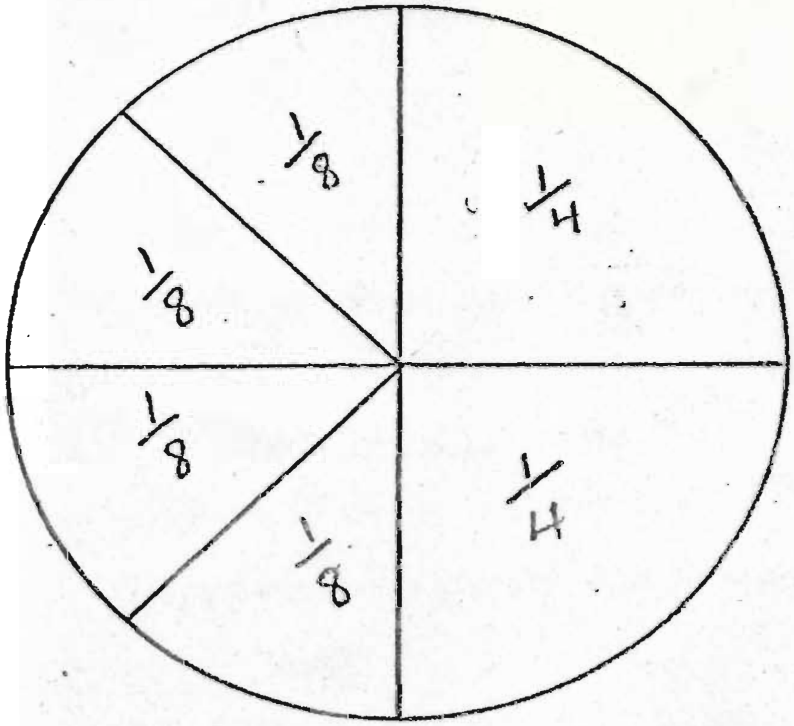
	Special Education		Regular Class	
	Number (out of 20)	Percent	Number (out of 21)	Percent
Addition				
I. Added the denominator.	10	50%	5	23.8%
II. Didn't get a common denominator.	10	50%	5	23.8%
III. Added each digit (example 3/5 + 1/5 = 14)	2	10%	0	0%
Multiplication	(out of 14)		(out of 16)	
I. Left out whole number (i.e. omitted the "2" in 2 3/5)	6	42.8%	3	18.7%
II. Added instead of multiplying.	3	21.4%	7	43.7%
III. Inverted the multiplier.	1	7.1%	0	0%

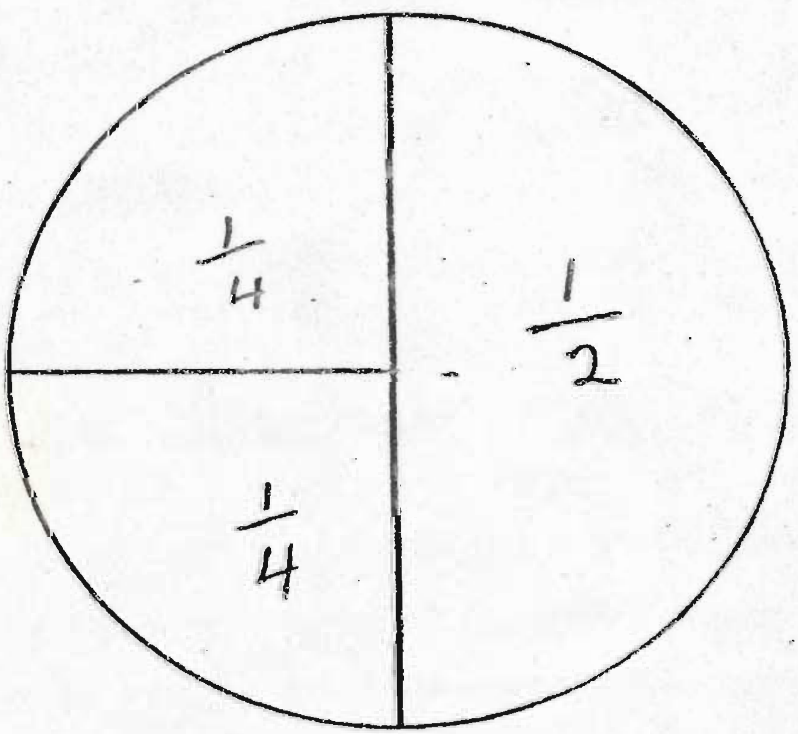
Diagram 4



Ikonic Representation For Fractions.

Diagram 5





Ikonic Representation For Fractions

tions has been set forth in Table 15. Several students could not complete the equivalence relationship, even by using the picture in Diagram 6. A common answer given was three since three of the circles were not shaded. The multiplication of the mixed numeral and proper fraction was poorly done. Only five of the special education group and eight in the regular group completed it. A common mistake here was that most students would omit the "two" in 2 3/5: they lacked the skills necessary to change the mixed numeral to an improper fraction.

Nine people in the special education group attempted the subtraction exercise, and six different answers were given; four people were correct. Sixteen of the regular group completed the question and there were seven different answers; nine people were correct.

Section II: Analysis and Conclusion

Introductory Comments

A final summary of the student deficiencies in the entire test has been presented in Table 16. The figures in this table have been based on the number of students who have shown weaknesses (errors in 50 percent of the questions) in each category.

On the whole, the regular class group was superior to the special education group in every section of the test. However, there were individuals in the special education group who performed far better than individuals in the regular class. The major deficiencies in the special education group were in the sections on division of whole numbers, decimals and fractions. The regular group had difficulty with division questions and the decimals. The percentages for those who were unable to change fractions to decimals and vice versa were quite high, but it must be remembered that

Table 15

Examples of Wrong Answers for 3/5 + 1/5
and Pupil's Rationale

Wrong Answers	Rationale
1. 3/10	1. 3 ones are 3, 5 plus 5 is 10.
2. 4/10	2. 3 plus 1 is 4, 5 plus 5 is 10.
3. 3/5	3. 3 ones are 3 over 5.
4. 14	4. 3 plus 5 is 8, 1 plus 5 is 6, 8 plus 6 is 14.
5. 4/15	5. 3 plus 1 is 4, 5 plus 5 is 15.
6. 86	6. A. 3 plus 5 is 8, 1 plus 5 is 6, so 86.
	B. Same as 31 plus 55 = 86.

Diagram 6

$$\frac{4}{6} = \frac{\square}{9}$$

Ikonic Representation For Equivalent Fractions

Table 16

Weaknesses in Each Section
Summary Table

		Special Education		Regular Class	
		Number	Percent	Number	Percent
1.	Properties	16	53.3%	1	4.8%
2.	Addition	3	10%	0	0%
3.	Subtraction	7	23.3%	0	0%
4.	Division	15	50%	1	4.8%
5.	Division of $8\overline{)1624}$ type	4	13.3%	10	47.6%
6.	Multiplication	9	30%	0	0%
7.	Fraction to decimals	2	25%*	6	42.8%*
8.	Decimals to fractions	1	12.5%*	5	45.4%*
9.	Addition and subtraction of decimals	10	33.3%	1	4.8%
10.	Fractions	24	80%	8	38%
11.	Multiplication of fractions	1	3.3%	5	23.8%

*These percentages were based on the number of people who actually tried to do the questions. The figures seem to indicate that the special education group was on a par with the regular group in these examples; however, more students in the latter group attempted the questions.

only the regular class had worked with decimals in their curriculum.

For the category on "Properties" a student was classified as deficient if he had incorrect answers on 50 percent or more of the test items in that category. The section on division had to be subdivided because there were some people who could not divide at all (i.e. they had made mistakes in each example); but there were others who were unable to do the questions, $8\overline{)1624}$ or $3\overline{)1521}$. The number in this latter group have not been included with the former. Similarly the section on fractions was subdivided and could be interpreted in this way: 80 percent of the special education group and 38 percent of the regular group were deficient or unable to compute the addition or subtraction of fractions. One out of the remaining six students of the special education group, and five out of the remaining thirteen students of the regular group were unable to multiply. However, these figures are for the symbolic level of operating with fractions. If the number of people who could add using diagrams or pictures were included, the number would drop considerably (see Table 17).

Bruner's Levels

Manipulative devices and pictures representing Bruner's ikonic and enactive stages were illustrated for the addition and subtraction of whole numbers; a dot table was provided for multiplication, and several other devices were used in the addition of fractions and equivalent fractions. However, most students were not familiar with the material. For example, those students who had difficulty in adding or multiplying whole numbers had no idea about how to use an abacus, a number line, a place value box or the dot table. The only items they did use well in this instance were the beads and the materials used in adding fractions. Table 17 indicates

Table 17

The Number of Students at Bruner's 3 Stages of Cognitive Development for Addition of Fractions

	Symbolic		Ikonic		Enactive	
	Number	Percent	Number	Percent	Number	Percent
Special Education	6	20%	16	53.3%	8	26.6%
Regular Class	14	66.6%	7	33.3%	0	0%

the number of students at Bruner's three stages of development for the addition of fractions.

In the special education class, eight students lacked the skills necessary to name or write a fraction; but, given a fraction kit, they could tell the number of pieces left in a circle after some had been removed. Sixteen of them could add fractions by looking at pictures and six of them were able to add the numbers directly. In the regular class only seven needed to use pictures in adding, and there was no one at the enactive stage.

The results in the realm of Bruner's three stages were limited since the only area where students recognized ikonic representation was fractions.

If the students had been given instruction by the investigator in the use of the various devices, the results may have indicated that the students were also at the ikonic stages in other categories of the test. The purpose of the study, however, was not for instruction, but to gather information to make an adequate diagnosis.

Nature of the Difficulties

A comparison of both groups revealed very little difference in the types of errors made in the four fundamental operations on whole numbers. However, a number of students displayed difficulty in each operation; that is, they had either committed several types of errors or had a mistake in each question. The operation of division was most difficult for the special education group with approximately 50 percent either omitting the section completely or attacking it with no comprehension. About two-thirds of the same group also showed difficulty with at least one other fundamental operation. The section on rewriting decimals as fractions and vice versa was omitted by most students in both groups (see Table 16). However, this topic was not covered with these students.

Analysis of Hypotheses

The first question this study hoped to answer was: "Are there common problems among the students within the special education class?" Throughout the previous discussion it was evident that there were. The "significant" errors among this group have been summarized as:

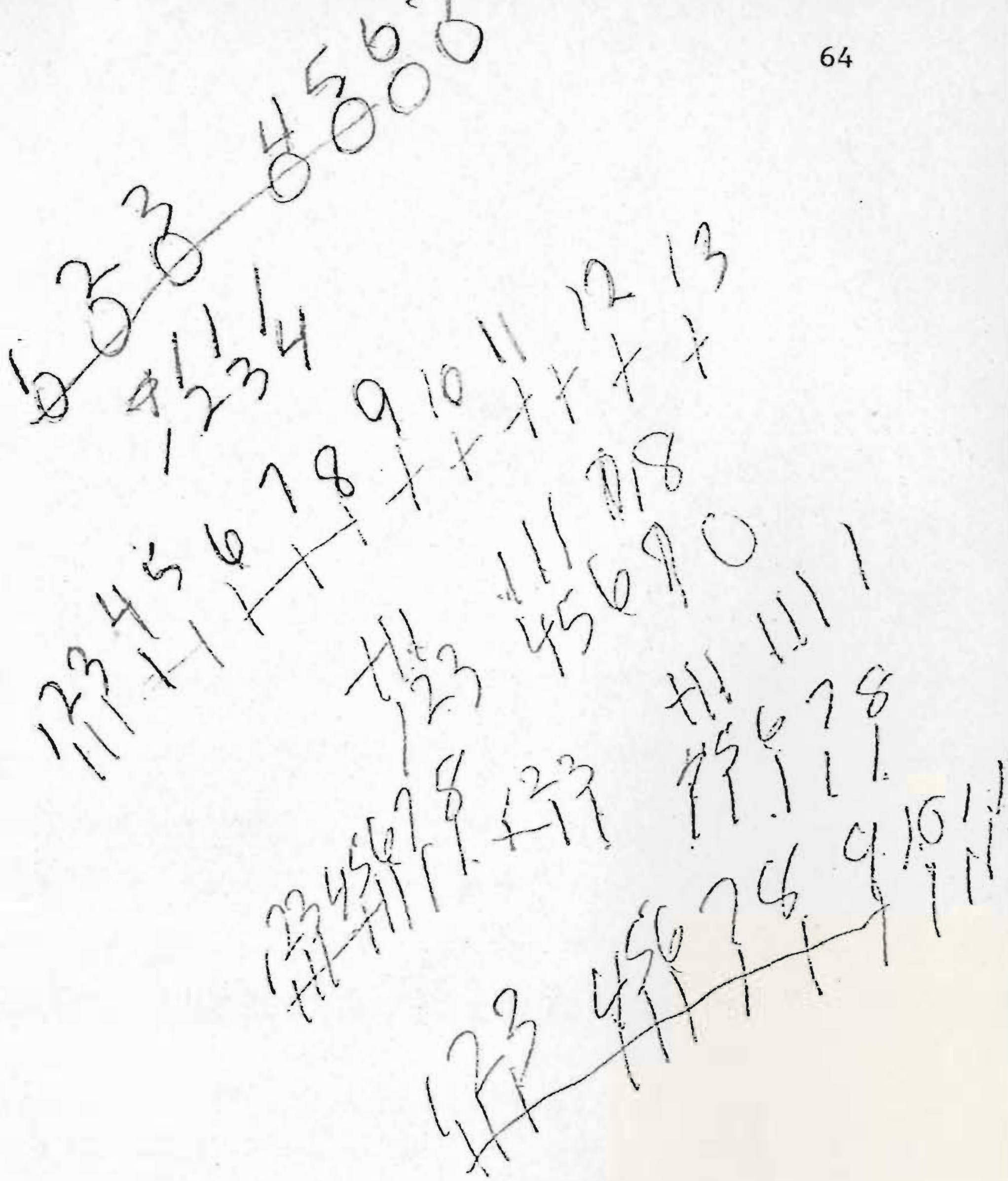
A. Estimation -- This was the one question on the test that no one in the special education group did correctly. Also, there was no attempt made to estimate any computations in other questions before solving them.

B. Properties -- Table 4 indicated that over half of the special education group were weak in this area, with the greatest number of errors having been made in the distributive property. However, not only the properties, but the method of presentation was difficult for the students who were not accustomed to working with the equation form of the question.

C. Addition and subtraction -- A majority of students could not explain the regrouping process involved in these operations. Also, various forms of manipulative devices, some of which were cumbersome, were utilized by many students. An example of this is illustrated in Diagram 7. A small percentage of the group had difficulty with combinations involving zero. These pupils invariably gave answers of zero to such questions as $4 - 0$ and $1 + 0$.

D. Multiplication and division -- Though these are inverse operations,

```
   4 7 13
 − 1 5 2
 -------
   3 2 2
```



```
   7 8 3
 −   2 3
 -------
   5 6 0
```

```
   8 13 4
 − 5  9 0
 --------
   3  4 0
```

Student Example For Subtraction Of Whole Numbers more students had difficulty with division as compared to multiplication. Most students could multiply a one digit number by a two digit number, but there were eight students who were unable to compute a two digit number by a two digit number. Students relied on the multiplication tables for combinations in both operations and often found division combinations by trial multiplication (see Diagram 8). The most common error in division was the omission of zero in one example, $8\overline{)1624}$.

E. Decimals -- Too few students had worked with decimals in their mathematics program to give an indication of patterns of errors. However, only one person was able to insert the correct sign ($<$, $>$, $=$) between .060 and .06.

F. Fractions -- All of the students could work at some stage of Bruner's cognitive development in working with fractions (see Table 17).

G. Concept of zero -- Some students consistently made errors in all operations involving zero.

The second question this study wished to answer was: "Are there problems which are common to both the regular class and the special education class?" Tables 4 and 7-14 indicate that there were, and they have been classified as follows:

A. Estimation -- Only one person in both groups did this correctly.

B. Properties -- More than half of each group lacked the skill to complete the distributive property.

C. Addition and subtraction -- The regular group, as well as the special education class, manipulated materials to add and subtract whole numbers and decimals. Students were not forced to use these

```
   203
9 ) 1624
    16
    024
     24
      0R
```

```
 32      32      32
×26     ×30     ×31
        960      32
        960    326
              ×2 1992
                  32
 32               64
×20               22
 00                    32
                      ×23
 64                   196
                  32   64
                  ×  4836
                   128
```

```
      2
63 ) 126
     126
      0R
```

```
      31
32 ) 992
     992
      0R
```

```
 32      32
×10     ×12
 06      64
 32      32

 32      32
×15     ×25
 160     160
  32      64
         800
```

Student Example For Division Of

Whole Numbers.

devices; they were there for their advantage if they needed them. A majority of both groups were unable to explain the regrouping process in these two operations.

D. Multiplication and division -- Both groups relied on trial multiplication to find division combinations, and almost half of each class omitted the zero in the question, $8\overline{)1624}$. Approximately 10 percent of both groups erred in the position of partial products.

E. Decimals -- The only question in the decimals which was comparable in any way was that on inserting the correct sign ($<$, $>$, $=$).
More than half of each group answered them incorrectly.

F. Fractions -- There were several errors (failing to get a denominator or adding denominators) in the symbolic stage of addition. However, Table 17 indicates that one-third of the regular class and one-half of the special education group were operating at the ikonic stage in this category.

The two questions explored above led the investigator to feel that the differences within the special education class required the need for different programs. There seemed to be a lack of structure within the special education program. Some students were doing mathematics "on sheets," some were using textbooks from grades four to seven, and a few others were doing grade ten mathematics. Thus, the researcher has recommended three types of programs:

<u>Type A</u>: An activity learning approach to be used with those students who find it easy to work with manipulative devices and materials. This would be geared to lead students from the enactive to the symbolic stages.

<u>Type B</u>: A special education program whereby certain students would go

to the regular class for mathematics following an adequate diagnosis in the skills necessary for that particular grade level.

Type C: A remedial mathematics program designed to give slow students a "catching up" period; this would allow him to proceed to either the regular class or the special education class for mathematics.

Other recommendations based on this study will be presented in Chapter Five.

CHAPTER V

SUMMARY, CONCLUSIONS AND IMPLICATIONS FOR FURTHER STUDY

The purpose of this chapter is to present a summary of the study: the problem investigated, the methodology employed, and the results obtained. Recommendations for further research are also included.

The Problem

This study was designed to diagnose the mathematical strengths and weaknesses of a special education class as compared to a "regular" class at the Junior High level. It was hoped that it would provide information to the teachers, who must organize their instructional methods to accommodate individual needs of students. Its significance rested on the value it might have as an aid in assessing the mathematical competencies of these students, and thus arriving at a logical decision as to how to modify the educational situation to the students' best advantage.

Instrumentation and Methodology

The instrument used in this study was a test, devised by the investigator and given to the students on an individual basis. Each pupil was asked to "think out loud" as he completed the questions and he was recorded on a cassette tape. In addition to the test, the investigator occasionally asked the students to explain an algorithm, or if it was not clear how he computed, to explain his reasoning. A pilot study was conducted on a small group of special education students to determine if the test would yield the information required for the study, and to check on the amount of time needed to complete it. No revisions were made as a result of this study.

The sample consisted of 30 special education students, whose ages ranged from 12 to 18, and 21 "regular" class students. They were between 12 and 14 years of age. The term "regular" student was defined to be an average student, and the investigator depended on the teachers' judgment for the selection of this group. In most instances they were in the "B" class of grades seven or eight.

The data collection took place during the first two weeks of June, 1974. The data from the test were processed by the investigator and descriptive statistics were compiled.

Limitations

The degree to which one can generalize in any one piece of research is limited when the research is conducted in a specific geographical location. Other factors, such as I.Q., socioeconomic status and teacher qualifications affected the results of this study. In addition, use of the individual interview make replication of the study difficult. While the above limitations were important from a theoretical point of view, it was felt that there was a need to undertake this preliminary study to help form the basis for a more extensive examination of the problem.

Conclusions

Examination of the descriptive statistics led to the following conclusions:

I. There were certain errors that were common to a majority of students in the special education group. Students had difficulty in explaining the regrouping process in the addition and subtraction of whole numbers. Many pupils in this group were able to answer questions through the use of enactive and ikonic materials, especially in the area of addition of fractions.

II. There were errors and habits which were common to both the regular class and the special education group. These included explaining the regrouping process, dividing by the use of trial multiplication (see Diagram 8), and errors in combinations with zero.

III. The differences within the special education class seemed to indicate the need for different mathematics programs. The investigator has suggested three types: (a) an activity learning approach; (b) a remedial or "catching up" program; (c) a program whereby the student would attend the regular class for his mathematics lesson. Each of these programs would follow a thorough diagnosis of the student's mathematical skills.

IV. Though observation and talks to the special education students were not part of this study, it seemed apparent to the researcher that a basic problem of the learning process is the mental attitude attached to being in a special education class.

Implications and Suggestions for Further Research

As a result of the study, the investigator has made the following recommendations for further research:

1. A study should be conducted whereby all special education students

should be diagnosed, with a view to returning to regular classrooms all those who are capable of performing adequately there.

2. The researcher felt (1) that the results of this study indicated differences in the special education program, and (2) that three separate mathematics programs should be implemented in special education. Thus it is recommended that three pilot studies, based on three different mathematics programs, be undertaken. During this implementation, a formative evaluation procedure should be introduced to constantly update and revise the program.
3. This study was concerned with mathematical strengths and weaknesses. The researcher recommends that a similar study be conducted to consider the relationship of such factors as teachers' qualifications, socioeconomic status, I.Q. and specific learning problems of special education students. This diagnostic study could also be extended to other subject areas of the curriculum.
4. There seemed to be a lack of structure within the mathematics program in the special education class, and also a lack of criteria in relation to the placement of special education students. Two recommendations follow from this:
   A) It is recommended that a study be initiated to determine the program or method of instruction best suited to the special education student and his needs.
   B) It is suggested that a study be undertaken within Newfoundland to determine if school boards use the criterion for placement of children in a special education class, as set by the Department of Education (Newfoundland).

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APPENDIX A

CORRESPONDENCE

293 Freshwater Road
St. John's, Nfld.
May 4th., 1974

Mr. Walter Cull
Exploits Valley Integrated School Board
Grand Falls, Nfld.

Dear Sir:

As part of the requirements for the M. Ed. program in Education Curriculum, I am conducting a study of the mathematical performance of special education students, as determined by a diagnostic test. The test will be given on an individual basis.

Since this instrument is geared to students who are at the Grades 7 and 8 level, or between the ages of 12 and 16, I am asking your permission to be able to give it in Memorial Academy.

I thank you in anticipation of your co-operation. Without it, this study will not be possible.

Yours truly
Sharon Basha.

Exploits Valley Integrated School Board

P.O. Box 70 – Postal Code A2A 2J3
GRAND FALLS, NEWFOUNDLAND
Telephone Nos. 489-2168-69 or 489-6271

From the office of

Superintendent

May 10, 1974.

Miss Sharon Basha,
293 Freshwater Road,
Apartment 95,
ST. JOHN'S, Nfld.

Dear Miss Basha:

Pursuant to our recent conversation this is to advice you that permission is hereby granted for you to conduct a study involving thirty-five Special Education students at the Grades V11 or V111 level in the Botwood schools.

Yours very truly,

W. A. Cull,
Superintendent of Education.

WAC/ra.

EXPLOITS VALLEY INTEGRATED SCHOOL DISTRICT

MEMORIAL ACADEMY

Water Street, Botwood, Nfld.

May 21,1974

Miss. Sharon Basha
293 Freshwater Road
Apt 95
St. John's, Nfld.

Dear Miss. Basha:

Of course we will be most delighted to have you conduct a study among our Special Education students.

Enclosed you will find the names and ages of our fifteen students who make up our two opportunity classes.

When may we expect you?

Yours truly,

A.M. Bessey
Principal

AMB/gl

APPENDIX B

COPY OF THE TEST

$5 + \square = 5$

$\square \times 12 = 12$

$58 \times \square = 36 \times 58$

$$456 = 400 + \square + 6$$

$$(4 \times 10) + (4 \times 5) = 4 \times \square$$

$$(3 + 6) + 8 = 3 + (6 + \square)$$

$$\begin{array}{r} 13 \\ +\ 34 \\ \hline \end{array}$$

$$\begin{array}{r} 470 \\ +\ 181 \\ \hline \end{array}$$

$$\begin{array}{r} 64 \\ +\ 16 \\ \hline \end{array}$$

$57 + 8 =$

$$\begin{array}{r} 75 \\ +\ 86 \\ \hline \end{array}$$

Jack had $1.55. He earned $1.75. How much money does Jack have?

Supply the missing numbers.

$$\begin{array}{r} 4\ 3\ \square \\ +\ 2\ \square\ 8 \\ \hline \square\ 9\ 8 \end{array}$$

SUBTRACTION

$$\begin{array}{r} 473 \\ -\ 152 \\ \hline \end{array}$$

$$\begin{array}{r} 783 \\ -\ 23 \\ \hline \end{array}$$

$$\begin{array}{r} 834 \\ -\ 590 \\ \hline \end{array}$$

$$\begin{array}{r} 800 \\ -\ \ 60 \\ \hline \end{array}$$

ESTIMATE 72 - 48 =

$$\begin{array}{r} 962 \\ -\ 603 \\ \hline \end{array}$$

$3 \overline{) \, 63}$

$4 \overline{) \, 76}$

$9 \overline{) \, 89}$

$8\overline{)1624}$

$63\overline{)\ 126}$

$32\overline{)\ 992}$

$$\begin{array}{r} 14 \\ \times\ \ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 67 \\ \times\ 40 \\ \hline \end{array}$$

$$\begin{array}{r} 32 \\ \times\ 16 \\ \hline \end{array}$$

$$\begin{array}{r} 212 \\ \times\ \ 13 \\ \hline \end{array}$$

$$\begin{array}{r} 508 \\ \times\ \ \ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 110 \\ \times\ \ 15 \\ \hline \end{array}$$

$$\begin{array}{r} 958 \\ \times\ \ 5 \\ \hline \end{array}$$

BRENDA SAVED 5¢ EACH DAY FOR 7 DAYS. HOW MUCH DID SHE SAVE?

$$\begin{array}{r} 321 \\ \times 130 \\ \hline \end{array}$$

CHANGE THESE FRACTIONS TO DECIMALS.

(A) $\frac{3}{10}$

(B) $\frac{2}{5}$

CHANGE THESE DECIMALS TO FRACTIONS:

(A) .25

(B) .009

ADD: 2.4 + 0.2

ADD: 3.1 + 12.0

SUBTRACT: 3.2 - .4

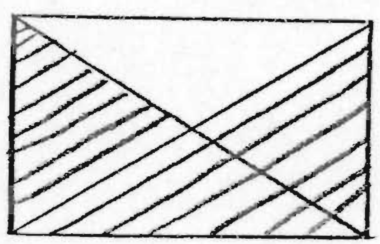
MULTIPLY: $4 \times .2$

GIVE THE CORRECT SIGN ($<$, $>$, $=$) FOR EACH:

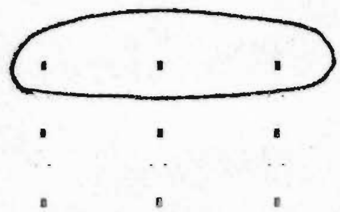
(A) .864 $\quad\quad$ .684

(B) .060 $\quad\quad$ .06

WHAT FRACTIONAL PART IS REPRESENTED BY THE SHADED PART OF THE FIGURE BELOW?



WHAT FRACTIONAL PART OF THE DOTS IS ENCLOSED IN THE FIGURE?



ADD $\frac{3}{5} + \frac{1}{5}$

ADD $\frac{1}{4} + \frac{3}{8}$

SUBTRACT

$$\begin{array}{r} 3\frac{3}{4} \\ -\ 1\frac{5}{8} \\ \hline \end{array}$$

MULTIPLY: $2\frac{3}{5} \quad \times \quad \frac{5}{6}$

FIND N IN THE EQUATION:

$$\frac{3}{4} = \frac{N}{8}$$

APPENDIX C

LIST OF ERRORS IN ALL THE SECTIONS

Errors/Habits in Subtraction
by Special Education

		Number of Students
1.	Added instead of subtracted.	1
2.	Subtracted smaller number from larger.	3
3.	Used same digit in two columns.	3
4.	Used manipulative devices to subtract:	
	(a) Used beads to count.	2
	(b) Drew circles and crossed them out to subtract.	2
	(c) Use of pictures.	1
5.	Errors in combination.	4
6.	Errors in combination with zero; i.e. 4 - 0 = 0; 5 - 0 = 0; 0 - 6 = 0.	6
7.	Read example backwards but got correct answer.	2
8.	Understanding of the algorithm in explaining regrouping process:	
	(a) Couldn't explain borrowing process.	5
	(b) Explained the "one" in terms of making a number bigger.	4
	(c) Borrowed when not necessary.	6
	(d) Did not allow for having borrowed.	3

Errors/Habits in Subtraction
by Regular Class

		Number of Students
I.	Added instead of subtracted.	2
II.	Read the example backwards, but got the right answer.	3
III.	Errors in combination with zero.	3
IV.	Subtracted the smaller from the larger.	1
V.	Borrowed when not necessary.	1
VI.	Regrouping process:	
	(a) Couldn't explain what borrowing meant.	9

Errors/Habits in Division
by Regular Class

		Number of Students
1.	Wrote answer as decimal.	3
2.	Found quotient by adding.	2
3.	Found quotient by trial multiplication.	15
4.	Omitted zero resulting from another digit.	11
5.	Error in multiplication combination.	4
6.	Included extra 0 in the answer (i.e. $32\overline{)992} = 301$).	1
7.	Used long division form for short division.	10
8.	Error in subtraction.	1
9.	Used remainder larger than divisor.	4
10.	Neglected to use remainder within the question.	2
11.	Error in division combinations.	1
12.	Used short division form for long division.	2
13.	Counted to get division combination (i.e. $8\overline{)24}$ = 2 x 8 = 16, 17, 18, 29, 20, 21, 22, 23, 24). Thus the student got an answer of 3.	1

Errors/Habits in Division by
Special Education Class

	Number of Students
1. Omitted zero resulting from another digit.	11
2. Error in multiplication.	3
3. Used long division form for short division.	9
4. Found quotient by trial multiplication.	9
5. Found quotient by adding.	4
6. Multiplied instead of dividing.	1
7. Divided dividend by itself.	1
8. Used remainder without new dividend figure.	1
9. Used short division form for long division.	1
10. Used remainder larger than divisor.	1
11. Left a remainder larger than divisor.	1
12. Used digits of the divisor separately.	3
13. Error in division combinations.	4
14. Added instead of divided.	1
15. Found correct answer but wrote an incorrect one.	3
16. Errors in subtraction.	1

Errors/Habits in Multiplication
by Regular Class

		Number of Students
I.	Wrote rows of zeros.	18
II.	Error in combination.	2
III.	Confused product when the multiplier had two or more digits.	2
IV.	Based unknown combination on a familiar one.	1
V.	Error in position of partial products.	2
VI.	Got the right answer; wrote an incorrect one.	1

Errors/Habits in Multiplication
by Special Education Class

		Number of Students
I.	Wrote rows of zeros.	14
II.	Error in combination.	3
III.	Couldn't multiply with a two digit multiplier.	8
IV.	Multiplied the carried number.	1
V.	Error in position of partial products.	4
VI.	Got the correct combination from a known one (i.e. 5 x 7 = 5 x 5 + 10).	1
VII.	Added instead of multiplying.	1
VIII.	Counted to get multiplication combinations.	1
IX.	Didn't add the carried number.	1

Errors/Habits in Decimals by
Special Education Class

		Number of Students
Multiplication		
I.	Omitted the question.	14
II.	Incorrect combination.	1
III.	Omitted the decimal point.	2
Signs, $<$, $>$, =		
I.	Omitted the questions	12
II.	Said that .060 was either greater than or less than .06.	16
Addition/Subtraction		
I.	Omitted the questions.	3
II.	Added each digit (i.e. 2.4 + 0.2 = 8).	1
III.	Omitted decimal point.	5
IV.	Subtracted instead of added.	1
V.	Added instead of subtracted.	1

Errors/Habits in Decimals
by Regular Class

	Number of Students
Multiplication	
I. Decimal point in wrong place.	2
II. Omitted the decimal point.	1
III. Included extra zero.	1
Signs, <, >, =	
I. Said .060 was either greater than or less than .06.	13
II. Wrote that .864 = .684.	1
Addition/Subtraction	
I. Decimal point in wrong place.	1
II. Wrote answer as a negative.	1
III. Rewrote 3.2 - .4 as .4 - 3.2. (These students got an answer of 3.2.	3

Errors/Habits in Fractions
by Regular Class

		Number of Students
I.	Multiplied the numerator.	1
II.	Wrote larger number as the denominator.	3
III.	Added the denominator.	5
IV.	Didn't find a common denominator.	3
V.	Left out the "2" in multiplication (i.e. 2 3/5 x 5/6 = 15/30.	3
VI.	Rewrote mixed number incorrectly.	2
VII.	Cross-multiplied the numerator and denominator.	1
VIII.	Omitted the multiplication question.	5
IX.	Added instead of subtracted.	2
X.	Subtracted the smaller from the larger number (i.e. 3 3/4 - 1 5/8 = 2 2/8).	1

Errors/Habits in Fractions by
Special Education Class

		Number of Students
I.	Omitted entire section.	10
II.	Added the denominators.	10
III.	Didn't find a common denominator.	10
IV.	Multiplied the numerators.	1
V.	Added each digit of the fractions (i.e. 3/5 + 1/5 = 14).	2
VI.	Didn't include "2" in multiplication.	6
VII.	Added instead of multiplying.	3
VIII.	Inverted the multiplier in multiplication.	1
IX.	Subtracted the smaller number from the larger.	2

Errors/Habits in Rewriting Fractions as Decimals and Vice Versa by Regular Class

	Number of Students
Fractions to Decimals	
I. Omitted the section.	7
II. Multiplied the numerator by denominator (i.e. 2/5 = 2(5) = .10.	1
III. Wrote decimal point between the numerator and denominator.	2
IV. Multiplied the numerator by .01.	1
V. Could do only if the denominator was 10.	4
Decimals to Fractions	
I. Omitted the section.	10
II. Multiplied by .01.	1
III. Left in decimal point.	3
IV. Multiplied both examples by 1/100.	2

Errors/Habits in Rewriting Fractions as Decimals and Vice Versa by Special Education Class

	Number of Students
<u>Fractions to Decimals</u>	
I. Omitted the section.	22
II. Wrote 0 after the decimal point (i.e. 2/5 = .04).	1
III. Could do only if the denominator was 10.	1
IV. Wrote decimal point between the numerator and denominator (i.e. 2/5 = 2.5).	1
<u>Decimals to Fractions</u>	
I. Omitted the section.	22
II. Included extra zeros in the fraction (i.e. .009 = 900/1000).	1
III. Had 100 as the denominator in both examples.	1

APPENDIX D

RELATED LITERATURE

RELATED LITERATURE

The following textbooks in mathematics were analyzed in preparing the objectives and the test used in this study.

Eicholz, R. E., & O'Daffer, P. G. Elementary school mathematics, Book 3 (2nd ed.). Ontario: Addison-Wesley, 1969.

Eicholz, R. E., & O'Daffer, P. G. Elementary school mathematics, Book 4 (2nd ed.). Ontario: Addison-Wesley, 1969.

Eicholz, R. E., & O'Daffer, P. G. Elementary school mathematics, Book 5 (2nd ed.). Ontario: Addison-Wesley, 1969.

Eicholz, R. E., & O'Daffer, P. G. Elementary school mathematics, Book 6 (2nd ed.). Ontario: Addison-Wesley, 1969.

Keedy, M. L. and others. Exploring elementary mathematics, 5. Toronto: Holt, Rinehart & Winston, 1970.

Keedy, M. L. and others. Exploring elementary mathematics, 6. Toronto: Holt, Rinehart & Winston, 1970.

Keedy, M. L. and others. Exploring modern mathematics, Book 1. Toronto: Holt, Rinehart & Winston, 1965.

Keedy, M. L. and others. Exploring modern mathematics, Book 2. Toronto: Holt, Rhinehart & Winston, 1966.

O'Daffer and others. Success with mathematics, 1. Ontario: Addison-Wesley, 1972.

O'Daffer and others. Success with mathematics, 2. Ontario: Addison-Wesley, 1972.