ANALYSIS OF RETROSPECTIVE ERROR IN AN ADAPTIVE FRAME WORK FOR VIRTUAL POPULATION ANALYSIS

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ANALYSIS OF RETROSPECTIVE ERROR IN AN ADAPTIVE FRAME WORK FOR VIRTUAL POPULATION ANALYSIS

By

© Harshana Rajakaruna

A thesis submitted to the School of Graduate Studies in partial fulfillment of the requirements for the degree of Master of Science

Department of Biology
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St.John's Newfoundland

Abstract

Retrospective problem (RP) in an adaptive framework for virtual population analysis (ADAPT) is based on the observation that in some fisheries retrospective error in current stock size estimates (RE_S) in successive yearly assessments display a trend rather than a random pattern. I have investigated the likelihood of occurrence of the RP resulting from chance in the presence of random variation of "realistic" (lognormal) errors in "input data" in the use of ADAPT. Both simulation and analytical methods were used. It was found that the RP occurs with high likelihood by chance alone in a fishery. In combination with positive mean and median bias in RE_S, a random-walk property in time series estimates of RE_S that gives false impressions of trends (generated by non-drift random variations of errors in "input data") creates a RP. I have also explored the fundamental causal factor and the casual mechanism of the RP using an explanatory mathematical model. The RE_S was fundamentally explained by a function of temporal difference in ratios of abundance-index and cohort-size, which is ADAPT regression-independent. This explanatory model was based on the finding that in the overall regression, residuals of each age class are added to near zero. The RE_S forms temporal trends caused by changes in the biases of factors that influence the above difference, thereby creating a RP. It was also found that series of positive RE_S generated by temporal increase in the proportion of underestimated catch might not be "true" overestimations. This reality in the ADAPT implies that RP evident in fisheries may also be "illusionary" and may not always require corrective treatment.

Acknowledgements

I have had the opportunity to work with fisheries "numbers of change" and those models that capture them, which greatly fascinated me, thanks to Dr. Yong Chen, my co-supervisor. I would like to thank him for his advice, patience, and the trust he put on me. I would also like to thank my co-supervisor Dr. Paul Snelgrove for his advice and the care he has taken to correct my thesis. Appreciation is also extended to Dr. David Schneider and Dr. Ransom Myers who suggested useful thesis revisions. I must also thank Dr. Randall Peterman for the opportunity he gave me for a valuable discussion. Without the financial support for this research provided by the Fisheries Conservation Chair and the MUN School of Graduate Studies, and through a Natural Sciences and Engineering Research Council of Canada Research Grant to Dr. Chen, this study would have been impossible. I dedicate my thesis to Dr. Hemesiri Kotagama, an economist and a philosopher with whom I had a close association during my earlier education and who was a great inspiration to me.

Table of Contents

List of Tables

List of Figures

List of Abbreviations and Symbols

ψ	Summation of retrospective error in cohort-sizes-at-age (in natural logarithm), $RE_{ln,a,y}$, of all the incomplete cohorts, by the assessment year Y (current year)
Φ	"True" error ("true" underestimation)
∇	Bias in specified natural mortality in recent years
α	Proportion of error in catch in early years
ϕ	Proportions of improvements in research surveying gear in early years
φ	Proportions of improvements in research surveying gear in recent years
χ	A coefficient
∂	Bias in the specified natural mortality in early years
β	Proportion of error in catch in recent years
η	A coefficient
γ_a	Number of incomplete cohorts with the age class a in the assessment year Y
$\nabla \ln Q$	Difference in the summation of retrospective error of estimated log catchability coefficients for year Y
A	Terminal age
ADAPT	Adaptive framework for virtual population analysis
ADAPT	Objective function of the adaptive framework for virtual population analysis
$C_{a,y}$	Catch of fish at age a and year y
CLSMCC	Change of the log-summation of mean year specific catchability coefficients of age-classes between reference estimates of early years (which contains complete cohorts) and recent years (which contains incomplete cohorts)
CPUE	Catch per unit effort
$F_{a,y}$	Fishing mortality at age a and year y

$I_{a,y}$	Observed research survey abundance index for fish at age *a* in year y
$\ln \Gamma_a$	Log-catchability coefficient of the data point at age *a* of the cohorts when they become complete in the fishery in the immediate assessment year
$\ln \Lambda_a$	Log-catchability coefficient of the data point at age *a* of the year added in the immediate assessment year
$\ln Q_{Y,Inc}$	Log-summation of estimated catchability coefficients in the assessment year *Y*
K_a	Year- and age specific catchability coefficients in the complete part of the age class *a*
k_a	Year and age-specific catchability coefficients in the incomplete part of age class *a* after the cohort-sizes-at-ages were adjusted with estimations once those cohorts completed in the fishery (after *A-1* years from the assessment year *Y*)
M	Natural mortality
M_d	Median of $RE_{a,ln}$
m_d	Median of $RE_{c,ln}$
$N_{a,Com}$	Cohort size at age of a cohort complete in the fishery, denoted by *Com*
$N_{a,Inc}$	Cohort-size at age *a* when the cohort is incomplete in the fishery, denoted by *Inc*
$N_{a,y}$	Number of fish at age *a* still alive at the beginning of year *y*
$q_{1,Y}$	Catchability coefficient of the current recruitment calculated after the newly recruited cohort is complete in the fishery
q_a	Catchability coefficient of age class *a*
r	Correlation coefficient
RE	Retrospective error
RE_c	Retrospective error in cohort size estimate; defined as $N_{a,Inc}$-$N_{a,Com}$
$RE_{c,ln}$	Retrospective error in cohort size estimate (in natural logarithms (*ln*)); defined as $\ln N_{a,Inc}$-$\ln N_{a,Com}$
$RE_{ln,c,1}$	RE of the recruitment in the current assessment year
$RE_{ln,c,2}$	RE of the recruitment in the next immediate assessment year

$RE_{ln,s,1}$	RE of the current stock size (in natural logarithm) in the current assessment year
$RE_{ln,s,2}$	RE of the current stock size (in natural logarithm) in the next immediate assessment year
RE_s	(Subscript *s* refers to stock, which consists of a number of cohorts) calculated as the summation of RE_c for a given year
$RE_{s,ln}$	Calculated as the summation of $RE_{c,ln}$ for a given year
RP	Retrospective problem
RVRED	Random variations of "realistic" errors in "input data" (lognormal errors in catch and abundance indices)
TTBEDM	Temporal trends in the errors of "input data" (estimates of commercial catch and abundance indices) and model misspecifications (such as assumption of constant natural mortality and relative vulnerability of age classes or partial recruitment)
TTBRDM	Temporal trends in the bias of the estimates of "input data" and model misspecifications
VPA	Virtual population analysis
W	Year up to which the cohorts are complete
Y	Current year

CHAPTER 1

INTRODUCTION

Virtual population analysis (VPA) is one of the most commonly used methods in stock assessment when catch-at-age data are available (Megrey 1989). Among a wide variety of VPA methods (Pope and Shepherd 1985), Gavaris's (1988) method, known as the adaptive framework (ADAPT), has been used extensively for estimating stock sizes of exploited fish stocks in Atlantic Canada and the northeastern United States. The problem of tuning of virtual population analyses has been addressed for more than a decade, originally using *ad hoc* (statistically not well-defined) methods, and more recently, using statistically based approaches such as ADAPT (Christensen 1996). The ADAPT approach is given in Conser and Powers (1990) and Mohn and Cook (1993). Conser (1993) gives a brief history of ADAPT. Lassen and Medley (2001) gives a detailed account of the current practice. ADAPT is essentially a cohort analysis of catch-at-age data where indices of abundance are included to estimate a relatively small number of parameters.

A problem has been identified in ADAPT (Sinclair et al. 1991; ICES 1991) known as the retrospective problem (RP) (Mohn 1999). The RP is based on the fact that in some fisheries (Rivard and Foy 1987; Gascon 1988; Sinclair 1991; ICES 1991), retrospective error in current stock size estimates in successive yearly assessments as point estimates has generally displayed a positive trend rather than a random pattern (Mohn 1993). These trends in positive REs have been illustrated in retrospective analyses of stocks estimated in the northern cod fisheries in division 2J3KL (Fig. 1.1a) and haddock fisheries in division 4TvW (Fig. 1.1b). Retrospective error (RE) is the difference in the estimates of stock parameters between the period a cohort entered (current estimates) and passed through (reference estimates) a fishery. In other words, RE is the deviation of contemporaneous estimates of stock parameters from subsequent estimates for the same year as more recent years of data are added to the same model (Mohn 1999). Specifically, obtaining a time series of positive (negative) RE in current stock size (fishing mortality) estimates (RE_S) in successive yearly assessments is known as the retrospective problem (RP). Myers et al. (1997) stated that RP existed in fishing mortality as a series of underestimations, in the northern cod fishery.

Trends in RE in stock size estimates have been reproduced by temporal trends in the errors of "input data" (estimates of commercial catch and abundance indices) and model misspecifications (such as assumption of constant natural mortality and relative vulnerability of age classes or partial recruitment) (TTBEDM) initially by Sinclair et al. (1991) and more recently by Mohn (1999). Myers et al. (1997) suggested that increased discarding or underreporting results in consistent underestimation of fishing mortality in retrospection. Error in the estimation of natural mortality may also cause trends in the RE in stock size estimates (Lapointe et al. 1989). However, the existence of TTBEDM might have not always been the likely factor that resulted in RP in all the fisheries that showed trends in positive RE. Mohn (1999) has related the RP to trends in catchability coefficients that are assumed to be constant in the analysis but actually change from year to year, the basis of which was suggested initially by Evans (1996). However, the trends in catchability coefficient are also a result rather than the cause of the RP. The hypothesis

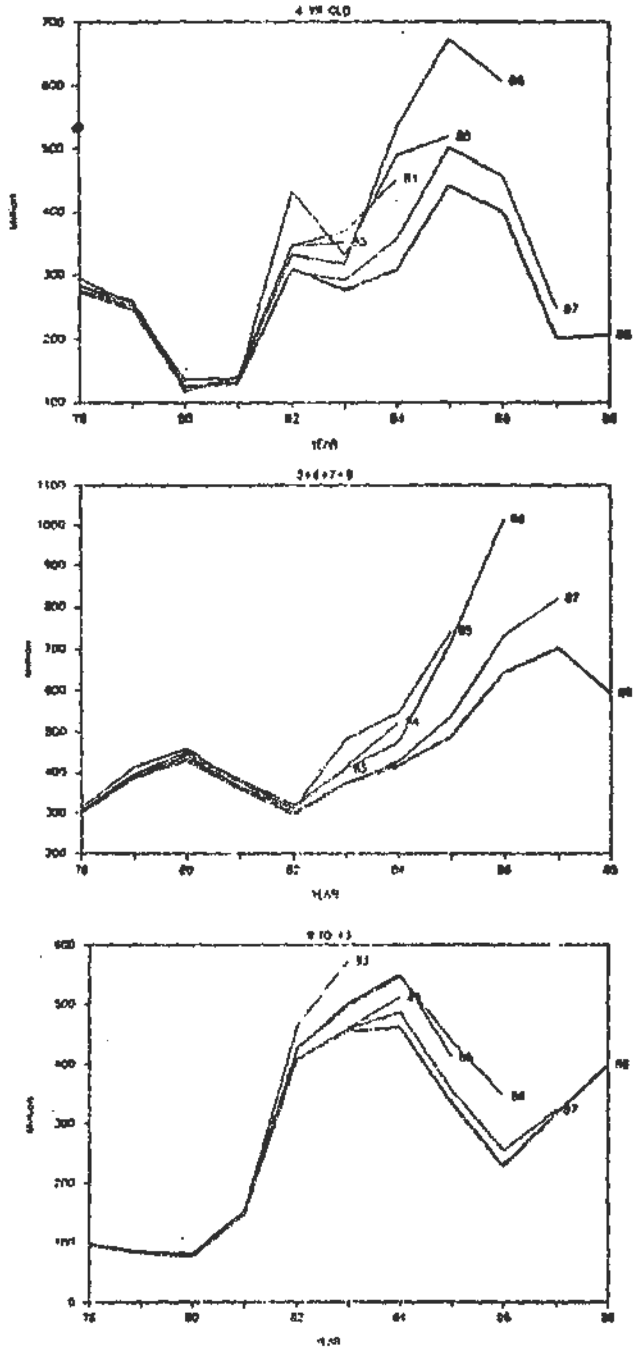


Figure 1.1a

Retrospective analysis of stocks estimated by ADAPT for NAFO division 2J3KL Cod [Reprinted from Sinclair et al. (1991)].

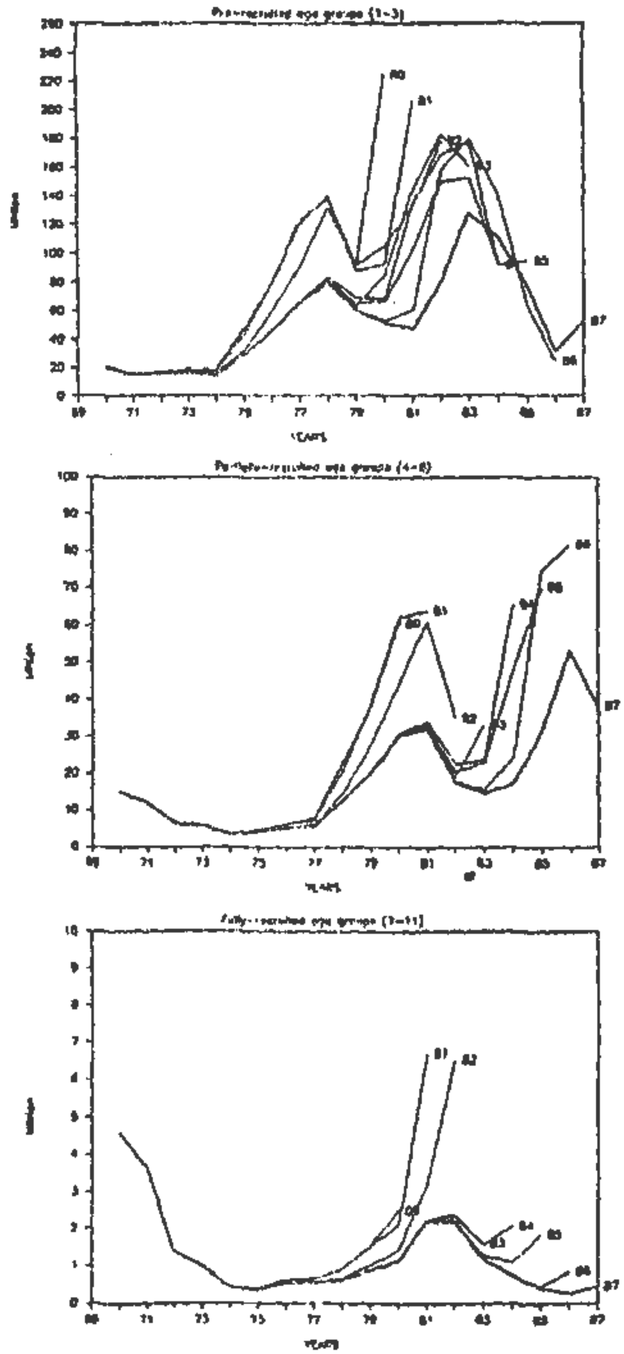


Figure 1.1b

Retrospective analysis of stocks estimated by ADAPT for NAFO division 4TvW Haddock [Reprinted from Sinclair et al. (1991)].

that a RP results from an increasing trend in catchability caused by improved technology or spatial changes of fish stocks (Parma 1993) was also not supported by the patterns of catchability observed in the east Scotian Shelf (ESS) cod fishery, which displayed a RP (Mohn 1999). Although TTBEDM has been demonstrated to be generating a RP, the fundamental and unified causal factor has not been identified in previous studies. Understanding the "true" nature of the RP has been given little emphasis.

Mohn (1999) suggested that failure to correct the RP for stocks with data like ESS cod could lead to catch level advice that would be twice or more the intended level. However, Mohn (1999) has not distinguished the RE from "true" error, convincingly, which Sinclair et al. (1991) had attempted before. Hence, before recommending methods for rectifying the RP, as was the case in Mohn (1999), understanding the "true" nature of RP is vital in stock assessment. To be able to understand the "true" nature of the RE that creates a "true" RP, we need to know how RP occurs. When RP exists and is related to "true" overestimations, the management strategies based on the estimates of stock parameters may become biased towards recommended catch levels that lead to overexploitation of a stock over successive years, not allowing a stock to recover. Extreme cases of consistent overestimation of stock sizes can have disastrous management consequences, as illustrated by the collapse of the northern cod fishery (Hutchings and Myers 1994; Walters and Pearse 1996; Myers et al. 1997). However, for cases in which RP may not be related to "true" overestimations, a fishery may not collapse as a result of an apparent RP. To provide a theoretical foundation to understand the "true" nature of the RP phenomenon, I have explored the mathematical basis of the occurrence of RE related to both the "true" parameter value and the converged value (reference estimates) of stock sizes.

Although, the RP in current stock size estimates is often discussed in terms of trends in positive RE_S, in some stocks such as 3Pn4RS cod (Fig. 1.2), RE_S has also shown negative trends. In some fisheries, for example, SNE yellow tail flounder (Fig. 1.3a), 5Z haddock (Fig. 1.3b), and Irish Sea (VIIa) plaice (Fig. 1.3c), RE_S has also displayed random patterns. Although, trends in and random patterns of RE_S can be reproduced by TTBEDM as shown in Sinclair et al. (1991) and Mohn (1999), there is a likelihood that these patterns could also result solely from random variations of "realistic" errors in "input data" (lognormal errors in catch and abundance indices) (RVRED) with a random walk property in the estimation model for time series data suggesting a chance-occurrence. If a RP is generated by such chance, then RE_S will have positive (or negative) signs for a number of years and negative (or positive) signs for a number of years and will continue to exhibit similar patterns. Because retrospective analyses in previous studies were mostly limited to a small number (six to ten years) of stock assessments, they were inconclusive as to how the trends in RE_S would change when more years of data are added to the same model. The absolute values of positive RE_S are more likely larger than the absolute values of negative RE_S, perhaps because estimated stock sizes are mean biased in the presence of "realistic" errors in "input data" as suggested by Gavaris (1993). However, the apparent temporal trends in positive (more likely large) RE_S may lead one to believe that trends in RE_S are generated solely by TTBEDM rather than by any chance. The trends in negative (more likely small) RE_S have been overlooked in the past analyses, perhaps, because, they do not appear to have

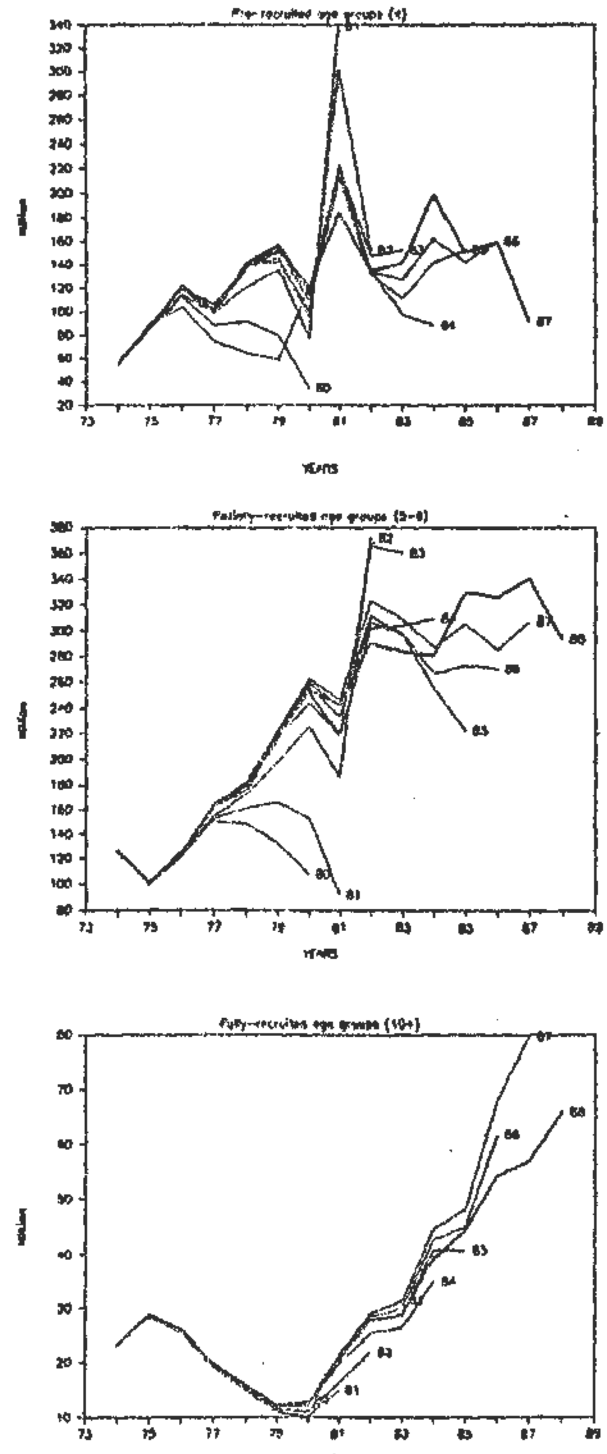


Figure 1.2

Retrospective analysis of stocks estimated by ADAPT for NAFO division 3Pn4RS Cod [Reprinted from Sinclair et al. (1991)].

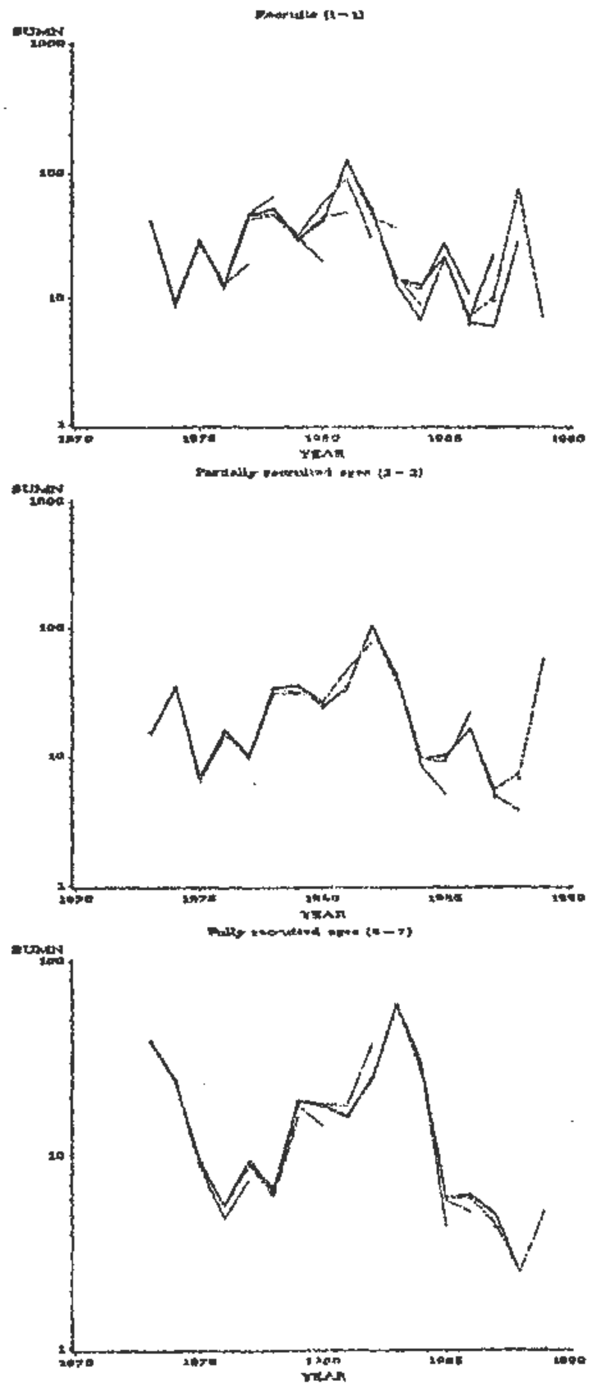


Figure 1.3a

Retrospective analysis of stocks estimated by ADAPT for SNE yellow tail Flounder [Reprinted from ICES (1991)].

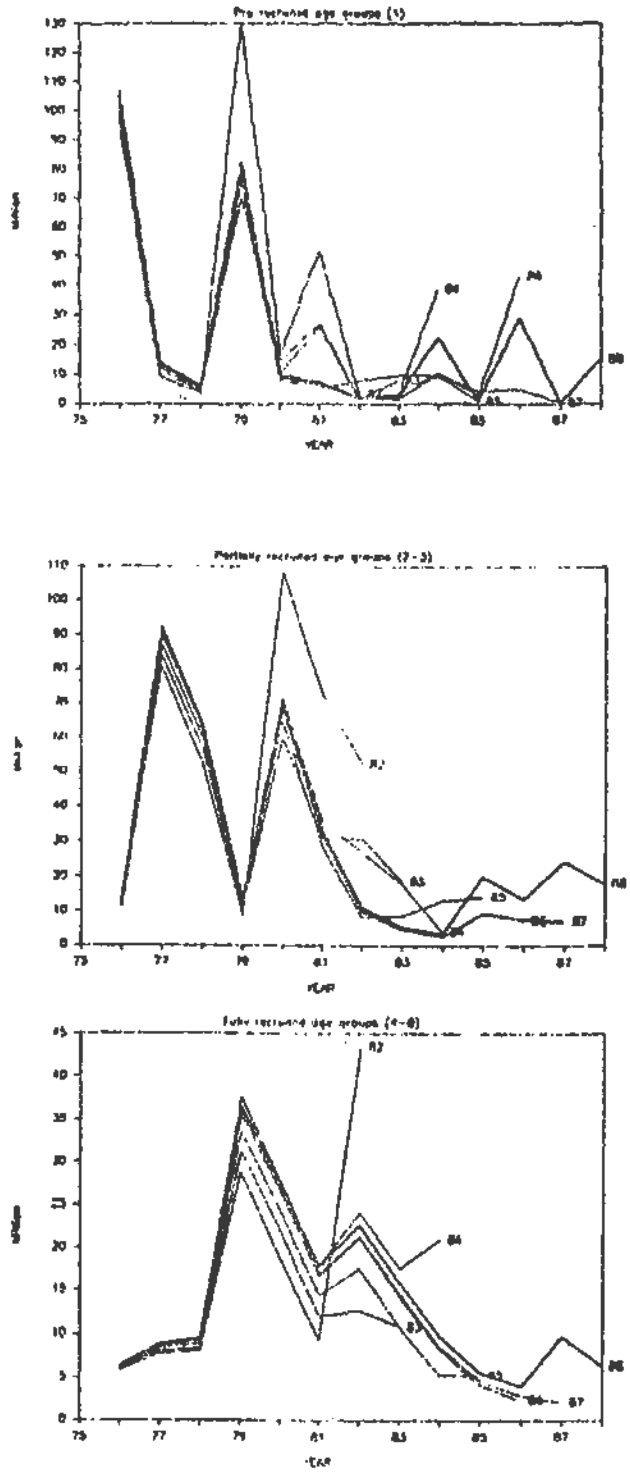


Figure 1.3b

Retrospective analysis of stocks estimated by ADAPT for 5Z Haddock [Reprinted from Sinclair et al. (1991)].

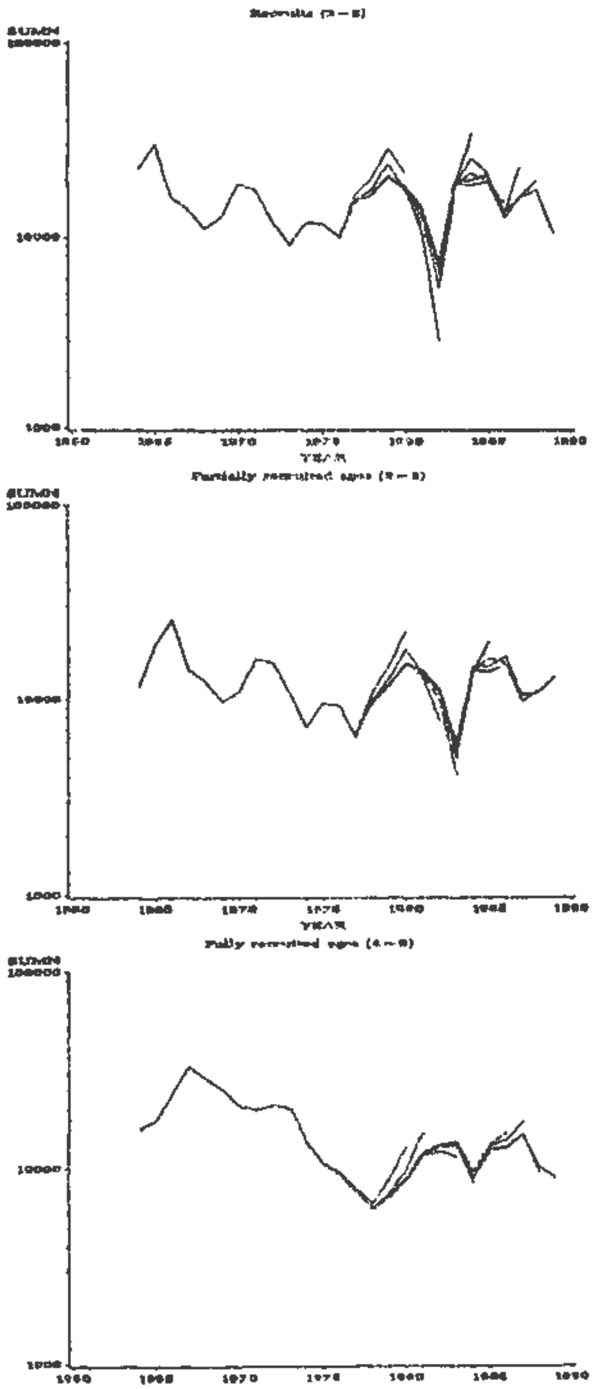


Figure 1.3c

Retrospective analysis of stocks estimated by ADAPT for Irish Sea (VIIa) Plaice [Reprinted from ICES (1991)].

created a problem. I have explored the likelihood and the causal mechanism of the occurrence of RP by chance in the presence of RVRED.

CHAPTER 2

RANDOM OCCURRENCE OF THE RETROSPECTIVE PROBLEM IN ADAPT

2.1 Introduction

ADAPT is a non-linear regression algorithm, in which parameters are estimated by a least squares method that minimizes the sum of squares of differences between the observed and the predicted abundances simultaneously for each age class (Quinn II and Deriso 1999) in complete cohorts, defined as the cohorts that have passed through the fishery, and incomplete cohorts, defined as the cohorts that will continue to contribute to the fishery (Hilborn and Walters 1992). An objective function of ADAPT can be written as

$$(2.1) \quad Minimize \sum_{a=1}^{A} \sum_{y=1}^{Y} \left[\ln\left(I_{a,y}\right) - \ln\left(q_a N_{a,y}\right)\right]^2$$

where, $I_{a,y}$ is the observed research survey abundance index for fish at age a in year y, $N_{a,y}$ is the number of fish at age a still alive at the beginning of year y, A is the terminal age, Y is the current year, and q_a is the catchability coefficient of age class a. Parameter q is assumed to be a constant for each age class. For a cohort in a fishery, $N_{a,y}$ is calculated by Pope's approximation (Quinn II and Deriso 1999), and is written as

$$(2.2) \quad N_{a,y} = N_{a+1,y+1} e^{M} + C_{a,y} e^{\frac{M}{2}}$$

where, M is the natural mortality, and $C_{a,y}$ is the catch of fish at age a and year y. For a complete cohort, cohort size after the terminal age can be assumed zero (Hilborn and Walters 1992), and thus cohort size at any age can be back-calculated with equation (2.2) using only catch-at-age data and M. However, equation (2.2) becomes invalid if the proportion of catch to "true" stock size were small whereas the cohort size after the terminal age has been assumed zero, because, under such scenario in the extreme case, no-catch suggests zero cohort size. Terminal cohort-size is negligible compared with the accumulated catch and other deaths only if the cohort has been recruited for a length of time and catches have been high (Magnusson 1995). In a scenario in which catch has been low, cohort-sizes in complete cohorts are readjusted with a terminal cohort-size assumption, $N_{A,y}$, with an additional component $U_{A,y}$ computed as follows from a terminal fishing mortality assumption.

(2.3) $$N_{A,y} = C_{A,y}e^{\frac{M}{2}} + U_{A,y}$$

where $C_{A,y}$ is the catch at the terminal age, and $U_{A,y} = C_{A,y}e^{\frac{M}{2}}\left(\frac{1}{e^{F_A}-1}\right)$ (see simplified equation for equation (2.3) in Myers and Cadigan 1995).

(2.4) $$F_{a,y} = -\ln\left(\frac{N_{a+1,y+1}}{N_{a,y}}\right) - M$$

Stock sizes at the terminal age in the years prior to the current year are calculated by iteratively recalculating the terminal age $F_{A,y}$ (Patterson and Kirkwood 1995), having assumed that terminal fishing mortality, $F_{A,y}$, is the average of fishing mortalities, $F_{a,y}$, in the age classes of the same year adjacent to the terminal age (Shelton et al.1996). For cohorts completed in the fishery, $F_{A,y}$ are calculated with high weight on the estimates of catches in the terminal ages and less sensitivity to the estimates of cohort sizes in the terminal year. Hence, estimates of those $F_{A,y}$ are near fixed values. Therefore, readjusted cohort sizes at age for a complete cohort ($N_{a,Com}$) are obtained nearly independently of the regression procedure. Hence, the estimates of complete cohort sizes do not change markedly when more years of data are added to the ADAPT model. The above assumption is violated slightly for the complete cohort(s) adjacent to the incomplete cohorts depending on the number of $F_{a,y}$ values averaged for the $F_{A,y}$ assumption. For incomplete cohorts, the cohort sizes in the current year are unknown parameters to be estimated (current estimates), and they become near fixed values (reference estimates) when these cohorts become complete in the fishery. This fixing process leads to the problem known as the RP.

Gavaris (1993) explored the "statistical" bias (mean bias) of parameters estimated with ADAPT using Box's (1971) method, and suggested that the mean bias in cohort-size estimates in ADAPT (Smith and Gavaris 1993) might account for some of the RP. He also suggested that the bias arising from model misspecification might be an important contributory factor. Mean bias in ADAPT was also reported by Myers and Cadigan (1995). Myers et al. (1997) referring to Smith and Gavaris (1993) also suggested that mean bias might result in a RP. I have investigated the likelihood and the causal mechanism of the occurrence of RP in a fishery in the use of ADAPT caused by random variation of "realistic" errors in "input data" (lognormal errors in catch and abundance indices) (RVRED) using a Monte Carlo simulation method (Chen 1996) and further mathematical analyses. These errors are commonly assumed for fisheries data.

2.2 Methods

2.2.1 Simulation Method

Fisheries were simulated based on commercial catch and research survey abundance indices from northern cod fisheries from 1978 to 1991 (Shelton et al. 1996). To examine the trends in RE in a fishery with a large number of cohorts that produce complete sequence from recruitment to completion, a matrix containing 34 complete cohorts was generated from the available cod fishery data by randomly selecting their complete cohorts. It was treated as the base data after the catch and the abundance indices were adjusted for the "true" values calculated by the following method. Figure (2.1) illustrates the corresponding distribution of "true" stock sizes.

ADAPT objective function in equation (2.1) was fitted with the base data to estimate the N (stock size) matrix and F (fishing mortality) matrices. Natural mortality was assumed to be 0.2. Fishing mortality in the terminal age was set equal to the fishing mortality in the age prior to the terminal age. The estimated N and F matrices were regarded as "true" stock sizes and fishing mortality rates, respectively. The "true" catch-at-age matrix was calculated based on the N matrix and F matrix using a catch equation and an exponential survival equation (Hilborn and Walters 1992). "True" total catch in year j (TC_j) was calculated from the "true" catch-at-age matrix as $TC_j = \sum_i TC_{i,j}$. For each year, log-normal errors were assigned for the "true" total catch to yield "observed" total catch for a given year, $C_y = TC_y e^{\varepsilon_y}$, where $\varepsilon_y \in N(0, \sigma^2)$. The coefficient of

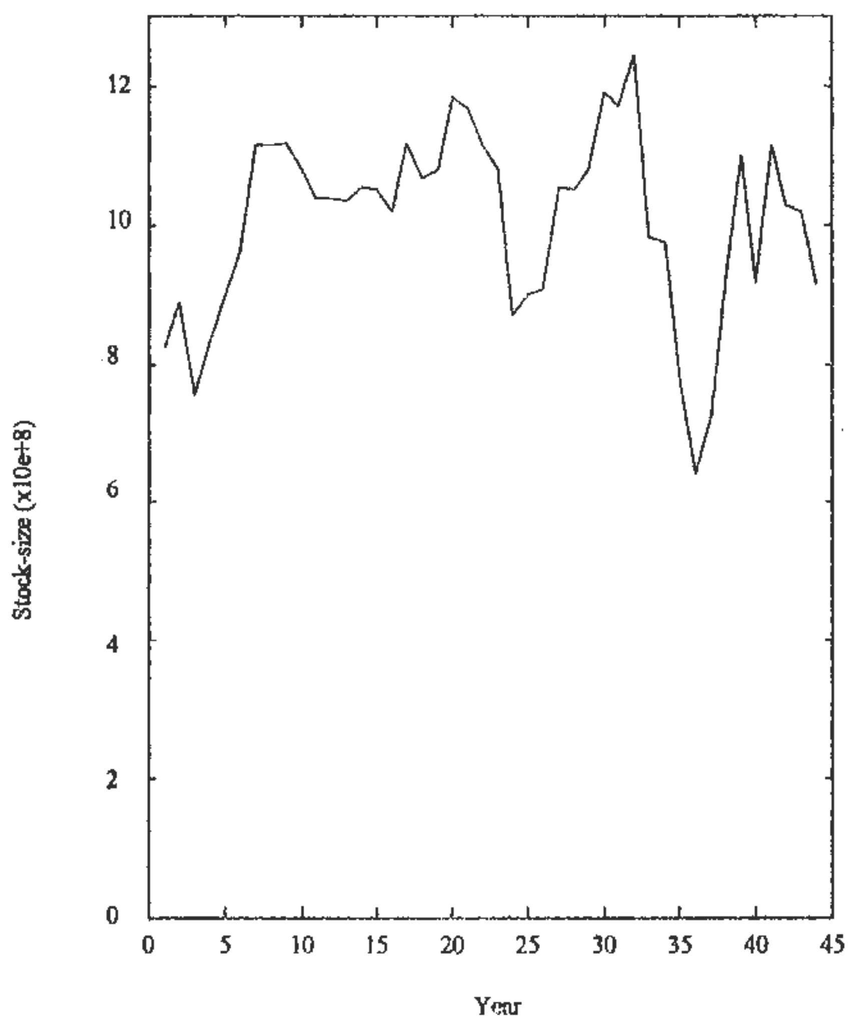


Figure 2.1

"True" stock sizes of the simulated fisheries variation for C was set at 25%, which is an arbitrary but realistic value for many fisheries (Walters 1998). Multinomial errors were assigned to the "true" age composition data to derive "observed" age composition data for a given year using the following procedure: (a) value *u* was randomly drawn from a uniform distribution $U(0,1)$; (b) for a given year, age composition was $p(1), p(2), p(3)\ldots$ calculated from the "true" catch-at-age matrix $p(a) = TC_a / \Sigma TC_a$ ($\Sigma p(a) = 1$), assigning one "fish" to age k if $\sum_{a=1}^{k-1} p(a) < u < \sum_{a=1}^{k} p(a)$; (c) in this instance *(a)* and *(b)* were repeated n times and "fish" in each age group was denoted by $n1, n2, n3\ldots$; (d) $p1=n1/n, p2=n2/n, p3=n3/n,\ldots$ were calculated. These $p1, p2\ldots$ values were the "observed" age composition for the given year.

The variation associated with "observed" age composition was set at a small value by taking the fish sample size n=1000 (Chen 1996). For a given year, "observed" catch-at-age was calculated as the product of the "observed" age composition and the "observed" catch. The above procedure was repeated for each year to obtain an "observed" catch-at-age matrix with the known "true" values of the N and F matrices. After fitting the "true" catch-at-age data, an abundance index was predicted for each age a in each year y, using $I_{a,y} = q_a N_{a,y}$ (where q_i is the estimated catchability coefficient, and $N_{a,y}$ is the "true" stock size), and was treated as the "true" I. "Observed" I for age a and year y was obtained by, $I = I_{a,y} e^{\varepsilon}$, where $\varepsilon_y \in N(0, \sigma^2)$. The standard deviation of the lognormal error was derived from the standard deviation of log residuals of I in the original model fitting. "True" q was the estimate of q at "true" I and "true" C.

2.2.2 Method of Analysis

The occurrence of RP was tested in the presence of lognormal errors in both the catch and the abundance indices. These errors are commonly assumed for fisheries data. The ADAPT algorithm was run for 500 samples from the base data incorporated with errors starting from year 7 through to year 40, thereby obtaining current stock size estimates comprising of 28 completed cohorts. Starting year was chosen on the basis that each age-class contained at least one data point of a completed cohort in the fishery.

RE for any cohort-size-at-age a, denoted by RE_c (subscript c refers to cohort), was calculated as the difference between the cohort-size when the cohort was incomplete in the fishery (current estimate) and the cohort size when the same cohort was complete in the fishery (reference estimate) (i.e. $N_{a,Inc}$-$N_{a,Com}$, where $N_{a,Inc}$ is the cohort-size at age a when the cohort is incomplete (*Inc*), and $N_{a,Com}$ is the cohort-size at age a after the cohort is completed (*Com*) in the fishery. RE_c for estimates of cohort sizes in natural logarithms (*ln*), denoted by $\mathrm{RE}_{c,\ln}$, was defined as ($\ln N_{a,Inc}$-$\ln N_{a,Com}$). Here, $\mathrm{RE}_{c,\ln}$ or $\mathrm{RE}_{s,\ln}$ should not be confused with log of RE_c and log of RE_s. The RE_s (subscript s refers to stock) and $\mathrm{RE}_{s,\ln}$ were calculated as the summation of RE_c and $\mathrm{RE}_{c,\ln}$, respectively, for a given year. The RE was standardized by dividing RE by its corresponding standard deviation. In the standardizing procedure, the RE was not subtracted from the mean RE because the behavior of the mean was one aspect examined in this study.

Median-unbias is defined such that estimator $\hat{\theta}$ is said to be a median-unbiased estimator of θ if $P[\hat{\theta} < \theta] = P[\hat{\theta} > \theta]$ (Bain and Engelhardt 1992). The tendency of the moving average (of the number of samples tending towards infinity) of the medians suggested the expected value of the median, and indicated median bias. The tendency of the moving average (of the number of samples tending towards infinity) of the means suggested the expected value of the mean (Mikhail 1976), and indicated mean bias.

2.3 Results

2.3.1 Statistical Properties of Retrospective Error

Results indicate that the average of both the median and the mean of standardized $RE_{c,ln}$ for 500 sample estimations of each 28 completed cohorts were both distributed near and evenly around zero (Fig. 2.2a). The moving average of both the median and the mean of standardized $RE_{c,ln}$ for 500 sample estimations of 28 completed cohorts approached zero (Fig. 2.3a). Based on the criteria defined for mean and median bias, it can be concluded that both the mean and the median were unbiased estimators of log cohort-size in retrospection. [In other words $RE_{c,ln}$ (i.e. $\ln N_{a,Inci}$-$\ln N_{a,Com}$) was both mean- and median-unbiased]. The average of the median of standardized RE_c for 500 sample estimations of each of the 28 completed cohorts was distributed near and evenly around zero, whereas the average of the mean of standardized RE_c was positive (Fig. 2.2b). The moving average of the median of standardized RE_c for 500 sample estimations of 28 completed cohorts tended towards zero, whereas the moving average of the mean of standardized RE_c tended towards a finitely positive value (Fig. 2.3b). Therefore, it was

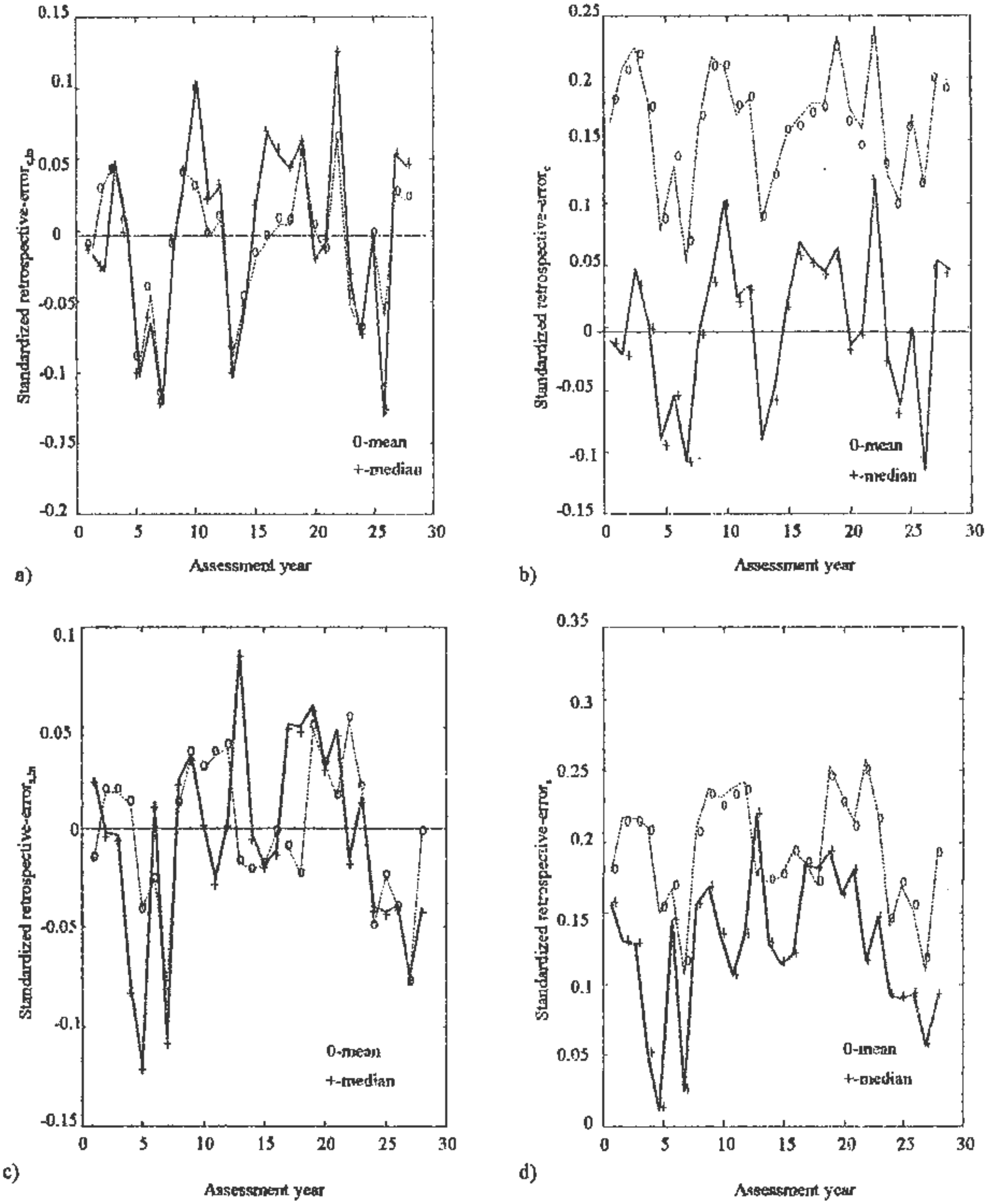


Figure 2.2
Averages of both the means and the medians of standardized (a) $RE_{c,ln}$ (b) RE_c (c) $RE_{s,ln}$ (d) RE_s of 28 completed cohorts in the fishery for 500 samples with RVRED.

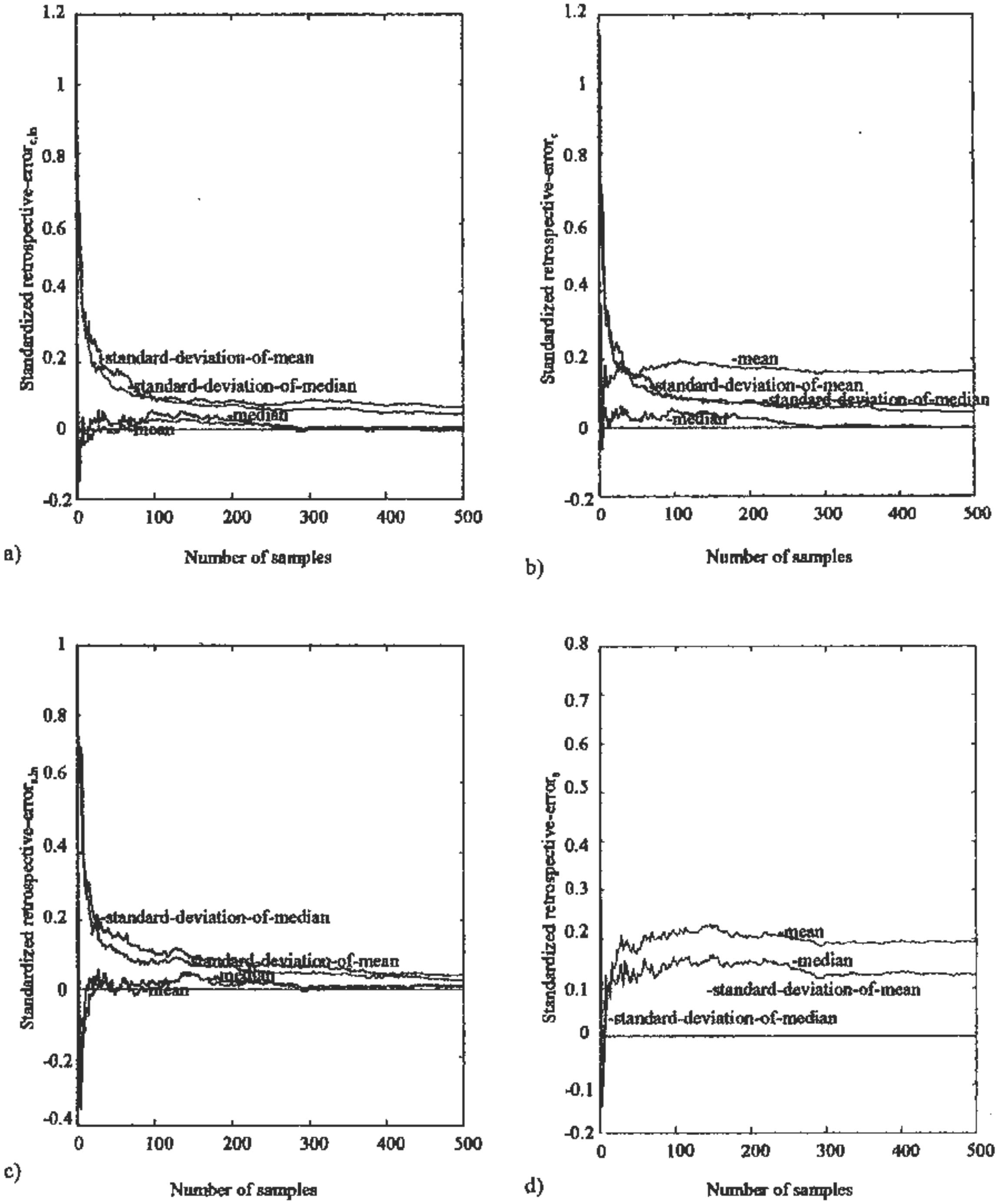


Figure 2.3
Moving average (with increasing number of samples) of both the means and the medians of the standardized (a) $RE_{c,ln}$ (b) RE_c (c) $RE_{s,ln}$ (d) RE_s of 28 completed cohorts in the fishery for 500 samples with RVRED.

concluded that the median was an unbiased estimator of cohort-size in retrospection, whereas the mean was biased.

The results also indicate that the average of both the mean and the median of standardized $RE_{s,ln}$ for 500 sample estimations of 28 yearly assessments were distributed near and evenly around zero (Fig. 2.2c), and the moving average of both the mean and the median of $RE_{s,ln}$ for 500 sample estimations tended towards zero (Fig. 2.3c). The average of both the mean and the median of standardized RE_s for 500 sample estimations of 28 yearly assessments were all positive (Fig. 2.2d), whereas the moving average of both the mean and the median of RE_s for 500 sample estimations tended towards a finite positive value (Fig. 2.3d). Therefore, both the median and the mean were biased estimators of stock size in retrospection, but unbiased estimators of log-stock size in retrospection. However, all time series estimates of RE showed random walk properties. Hence, the medians and means of the estimates also showed random walk properties.

2.3.2 Time Series Patterns of Retrospective Error

The results indicate that the likelihood that an estimate of current stock size in any assessment year will have positive RE_s (as a member of a set of successive positive RE_s) (Fig. 2.4a.1) is greater than that it will have negative RE_s (as a member of a set of successive negative RE_s) (Fig. 2.4c). (Note that in Figures 2.4a.1 and 2.4a.2, the probabilities are given for a limited number of assessment-years because the probabilities for assessment-years both below and above the given range lead to a computation-bias resulting from the limitation in the computation). Positive mean bias in RE_s results in

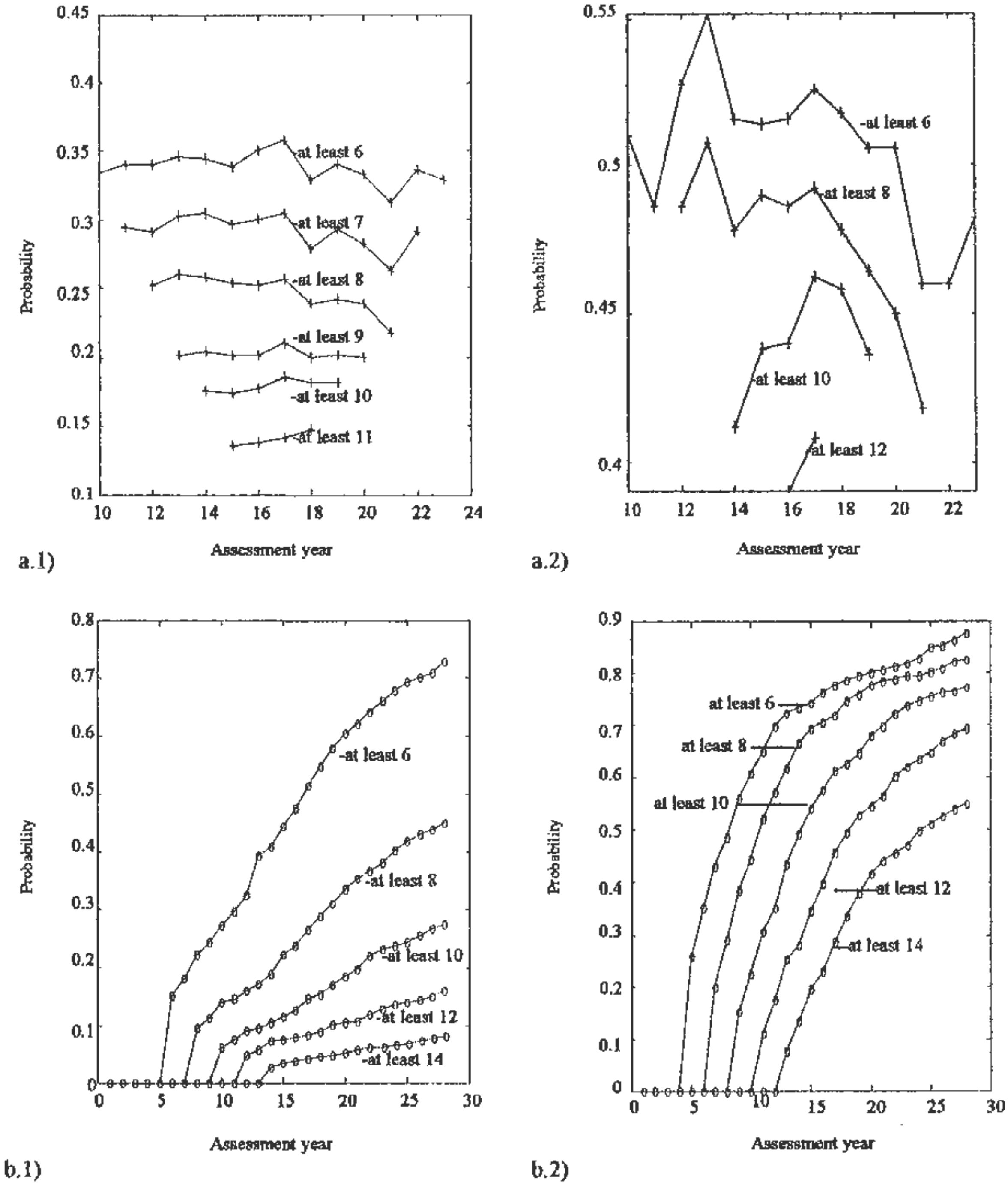
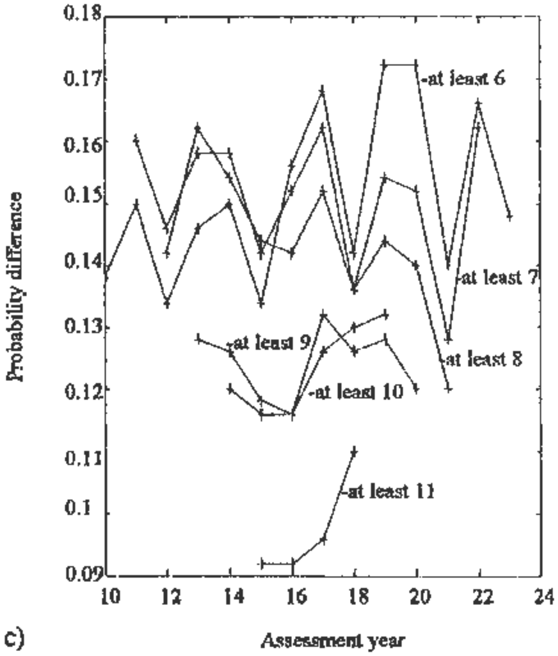
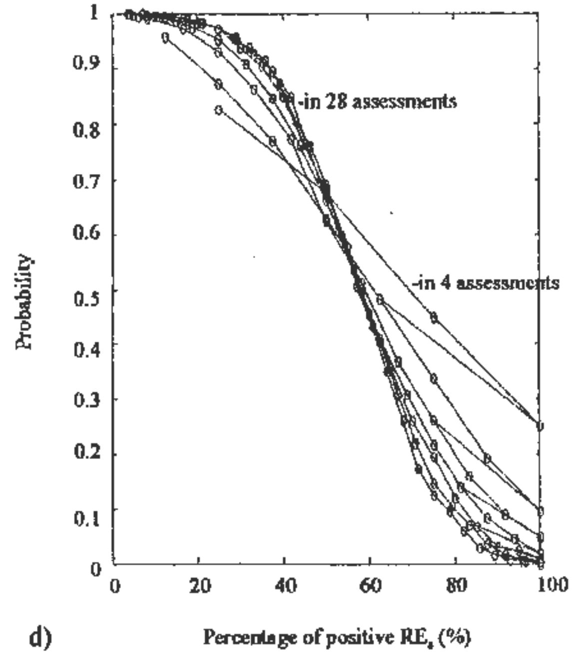


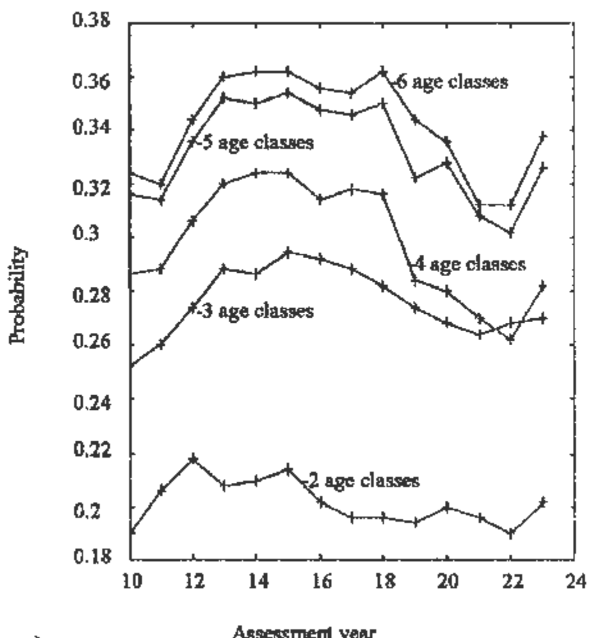
Figure 2.4
Probability that a current stock size estimate in an assessment year will have positive RE_s as a member of a set of successive positive RE_s of at least (a.1) 6 to 14 years, (a.2) 6 to 12 years inclusive of 2 negative RE_s (after the first positive RE_s). Probability of obtaining a series of successive positive RE_s of at least (b.1) 6 to 14 years, (b.2) 8 to 14 years inclusive of 2 negative RE_s (after the first positive RE_s) by an assessment year.



c)



d)



e)

Figure 2.4 (Continued)
(c) Difference between the probabilities that a current stock size estimate in an assessment year will have positive RE_s (as a member of a set of successive positive RE_s of at least 6 to 11 years) and negative RE_s (as a member of a set of successive negative $RE_s$of at least 6 to 11 years). (d) Probability of obtaining a given percentage of positive RE_s within a given number of yearly assessments. (e) Probability that a current stock size estimate in an assessment year will have positive RE_s (as a member of at least 6 successive positive RE_s in yearly assessments) for fisheries with increased number of age classes.

more likely larger magnitudes for trends in positive RE_s than those for trends in negative RE_s. The likelihood that an estimate of current stock size in any assessment year will have positive RE_s (as a member of a set of at least 6 successive positive RE_s) was between 30% and 35% (Fig. 2.4a.1). This value increased to up to 55% when time series of positive RE_s have been considered with an inclusion of at most 2 negative RE_s within the series of positive RE_s (Fig. 2.4a.2). Arbitrary negative RE_s within a series of positive RE_s is common in fisheries which have been categorized as displaying the RP. From Figure (2.4b.1), it can be inferred that there is 30-35% likelihood that the fishery will have positive RE_s in the current stock size in at least 6 successive assessment years within the first 10 years of assessments. Figure (2.4b.1) also indicates that this probability would increase by as much as 50% within the first 16 years of assessments. Similarly, within the first 20 years of assessments, there is a 20% chance that the fishery will have at least 10 years of successive positive RE_s. Figure (2.4b.2) depicts that these values increase markedly with the inclusion of at most 2 negative RE_s within the series of positive RE_s. For example, within the first 15 years of assessments, there is a 70% chance that the fishery will have at least 8 years of successive positive RE_s inclusive of at most 2 negative RE_s. From Figure (2.4d) it can be inferred that there is a 50% likelihood that the fishery will have at least 60% positive RE_s in yearly assessments, irrespective of their RE patterns. These probabilities would likely increase in fisheries that have larger number of age classes (Fig. 2.4e). Thus, when the number of age classes in a fishery is high, the probability of obtaining a number of successive positive RE_s will also be high. However, RE_c did not display long temporal trends, and varied randomly between positive and negative values more frequently than was the case for RE_a as was seen in Figure (2.2b) compared with Figure (2.2d) for their mean values of the medians.

2.4 Discussion

2.4.1 Analysis of Statistical Properties of Retrospective Error

Gavaris (1993), with reference to Ratkowsky (1990), suggested that ADAPT models which are parameterized in natural logarithms are close-to-linear regression models, so that cohort-sizes estimated by ADAPT in natural logarithmic scale, $\ln N_{a,Inc}$, are mean-unbiased. Given that least squares estimators are unbiased and minimum variance linear estimators for linear models (Fox, 1997), the ADAPT model with cohort-sizes parameterized in natural logarithm will be close to linearity, and will yield normally distributed likelihood estimates with negligible bias and small standard variation (Gavaris 1993). Based on Gavaris' (1993) results and their statistical foundation, let $E(\ln N_{a,Inc}) = \mu$ such that (s.t.) Error($\ln N_{a,Inc}$) ~N(0,σ), where μ is the mean of $\ln N_{a,Inc}$ in the sampling distribution. $\ln N_{a,Com}$ is mean-unbiased, because $N_{a,Com}$ is determined by equations (2.2) and (2.4) from catch-at-age data with log normal errors. In other words,

$$(2.5) \qquad E(\ln N_{a,Com}) = \ln\left(\left(\sum_{j=a}^{A-1} e^{\frac{(2j+1-2a)M}{2}} E(C_j)\right) + e^{A-a}E(U_A)\right) = \mu \quad \text{for any } N_{a,Com}$$

where $E(U_{A,y}) = e^{\frac{3M}{2}} E\left(\frac{C_{A,y}C_{A,y+1}}{C_{A-1,y}}\right)$ given the assumption that fishing mortality in the terminal age is equal to the fishing mortality in the age immediate to the terminal age of the same year. (See Lindgren (1993) for the mathematical properties of the expected value used here). Hence, $E(\ln N_{a,Com})$=E($\ln N_{a,Inc}$). Thus, E($\ln N_{aIncr}$-$\ln N_{a,Com}$)=0 because $E(\ln N_{a,Inc})$-$E(\ln N_{a,Com})$= $E(\ln N_{aInc}$-$\ln N_{a,Com})$ (Lindgren 1993). In other words, $RE_{c,ln}$ is mean-unbiased. Furthermore, when the error in catch is log-normal, $\ln N_{a,Com}$ follows a normal distribution (Gavaris 1993). This can be written as if $Error(\ln N_{a,Com})$~N(0,σ), then $\ln N_{a,Com}$ +$Error(\ln N_{a,Com})$~N($\ln N_{a,Com}$,σ) where $\ln N_{a,Com}$ is the true value. (Hereafter $\ln N_{a,Com}$ stands for the true value with the errors). $RE_{c,ln}$ also follows a normal distribution with its mean zero [because E($\ln N_{a,Inc}$-$\ln N_{a,Com}$)=0]. Thus, the median is also intrinsically unbiased. However, for an in depth mathematical analysis, the integration of c.d.f (cumulative density function) of $RE_{c,ln}$ can be derived for a definite integral from infinity to median, m_d, of $RE_{c,ln}$. This quantity is equal to 0.5 (see Dudewicz 1988), expressed mathematically as,

$$\int_{\infty}^{m_d} \frac{1}{\sqrt{2\pi}\sigma RE_{c,\ln}} e^{-\frac{1}{2}\left(\frac{\ln R_{c,\ln}}{\sigma}\right)^2} dRE_{c,\ln} = \frac{1}{2}$$

After substituting $\frac{\ln RE_{c,\ln}}{\sigma} = u$,

$$(2.6) \quad \int_{\infty}^{e^{\sigma u}} \frac{1}{\sqrt{2\pi\sigma}} e^{-u^2/2} du = \frac{1}{2}$$

Although, this equation cannot be solved algebraically for m_d (see Dudewicz 1988), the integral will approach 1 for a definite integral from negative infinity to positive infinity, because it is a normal distribution for random variable u with mean zero. Thus, half of the area under the curve can be taken as a definite integral from infinity to zero and from this, $e^{\sigma u} = 0$, i.e m_d=0. In other words $Median(\ln N_{a,Inc}$-$\ln N_{a,Com})$=0. This derivation confirms that $RE_{c,\ln}$ is median-unbiased. Furthermore, from $Median(\ln N_{a,Inc}-\ln N_{a,Com})=0$,

we obtain $Median(\frac{\ln N_{a,Inc}}{\ln N_{a,Com}})=0$; thus $Median(\frac{N_{a,Inc}}{N_{a,Com}})=1$. Therefore, $Median(N_{a,Inc}-N_{a,Inc})=0$ given that $N_{a,Inc}$ and $N_{a,Com}$ are independent and continuous, and hence, RE_c is median-unbiased. When the median or the middle value of $RE_{c,\ln}$ is zero, the median of RE_c also becomes zero, because log inverse transformation does not change the order of the RE with respect to their magnitudes. If $E(\ln N_{a,Inc}-\ln N_{a,Com})=0$, and the error of $\ln(\frac{N_{a,Inc}}{N_{a,Com}})$ is normally distributed, then $E(\frac{N_{a,Inc}}{N_{a,Com}})=e^{\frac{\sigma^2}{2}}>1$ (Sinha and Kale 1980).

Thus, $E(N_{a,Inc}-N_{a,Com})>0$, given that $N_{a,Inc}$ and $N_{a,Com}$ are independent and continuous. This confirms that RE_c is mean biased.

Let $RE_{s,\ln}$ be $\sum_{a=1}^{A} RE_{c,\ln,a}$. For the mean of $RE_{s,\ln}$,

$$(2.7)\quad E(RE_{s,\ln})=\sum_{a=1}^{A} E(RE_{c,\ln,a})=0$$

(i.e. $RE_{s,\ln}$ is mean-unbiased). For the median of $RE_{s,\ln}$, M_d,

$$(2.8)\quad \int_{\infty}^{M_d} \frac{1}{\sqrt{2\pi}\sigma\prod_{a=1}^{A} RE_{c,\ln,a}} e^{-\frac{1}{2}\left(\frac{\sum_{a=1}^{A}\ln R_{c,\ln,a}}{\sigma}\right)^2} d\prod_{a=1}^{A} RE_{c,\ln,a}=\frac{1}{2}$$

After substituting $\frac{\sum_{a=1}^{A}\ln RE_{c,\ln,a}}{\sigma}=u$,

$$(2.9)\quad \int_{\infty}^{e^{\sigma u}} \frac{1}{\sqrt{2\pi}\sigma} e^{-u^2/2} du=\frac{1}{2}$$

Then, according to a similar argument given in the analysis of median-bias of $RE_{c,ln}$, M_d=0 [i.e, $Median(\sum_{a=1}^{A} \ln RE_{c,\ln,a})$=0]. Thus $RE_{s,ln}$ is median-unbiased. Furthermore,

(2.10) $\ln(Median(\prod_{a=1}^{A} RE_{c,a})) = 0$

Thus $Median(\sum_{a=1}^{A} RE_{c,a})$=1, confirming that RE_s is both mean- and median- biased (because $Median(\sum_{a=1}^{A} RE_{c,a}) > 0$). These results were confirmed by a simulation that mimicked the foundation of RE_s estimations. In this case, simulation of $RE_{c,a} \in LN(m_d = 0, \mu > 0)$ for a=1:A yielded $\sum_{a=1}^{A} RE_{c,a} \in LN(M_d > 0, \mu > 0)$. *LN* stands for log-normal.

2.4.2 Analysis of Time Series Patterns of Retrospective Error

Positive median bias in RE_s does not necessarily explain why RE_s form temporal trends. Mathematical investigation in Chapter (3) shows that the temporal difference (change) in the log-summation of mean year-specific-catchability-coefficients of age-classes (CLSMCC) between the reference estimates of early (in complete cohorts) and recent (in incomplete cohorts) years, which are ADAPT regression independent, forms the RE in current stock size estimates (for stock sizes in natural logarithm). The correlation between CLSMCC and $RE_{s,ln}$ is a result of an approximation of the ADAPT regression model (equation 2.1) to a simplified close-to-linear least squares regression model, and has been used as an explanatory model of the formation of trends in RE_s. Detailed derivation of this approximation will be given in Chapter (3). Only the relevant equations for the examination of the formation of temporal trends in RE_s that result from RVRED will be presented here.

The following derivation (in Chapter 3) was based on the finding that sum of residuals of each age class of the overall ADAPT regression are near zero.

$$\text{(2.11)}\quad \text{CLSMCC}= \sum_{a=1}^{A-1} \gamma_a[Mean(K_a) - Mean(k_a)]$$

where, γ_a is the number of incomplete cohorts with the age class a in the assessment year Y, K_a is the year and age-specific catchability coefficients in the complete part of the age class a, and k_a is the year and age-specific catchability coefficients in the incomplete part of age class a, after the cohort sizes-at-ages were adjusted with estimations, once those cohorts completed in the fishery (after A-1 years from the assessment year Y). i.e.

$Mean(K_a) = \dfrac{1}{(Y-\gamma_a)} \displaystyle\sum_{y=1}^{Y-\gamma_a} \ln\left(\frac{I_{a,y}}{N_{a,y}}\right)$ and, $Mean(k_a) = \dfrac{1}{\gamma_a} \displaystyle\sum_{y=Y-\gamma_a+1}^{Y} \ln\left(\frac{I_{a,y}}{N_{a,y}}\right)$ where, Y is the current year, and y is any year starting from 1 to Y. $I_{a,y}$ is the abundance index of age a and year y. $N_{a,y}$ is the stock size of age a and year y. Thus, from equation (2.11),

$$\text{(2.12)}\quad RE_{\ln,S} = -\frac{1}{\xi} \sum_{a=1}^{A-1} \gamma_a[Mean(K_a) - Mean(\kappa_a)]$$

where, $\xi = \left(\displaystyle\sum_{i=1}^{A-a} \frac{(A-i)}{(A-1)} \eta_i\right)$, which is dependent on the number of age classes and the proportion of catch-at-age (or partial recruitment) specific to a fishery. η is a time independent coefficient. Some examples of model fitting of equation (2.12) with the data from the simulated fishery are given in Figure (2.5). Equation (2.12) was validated using the simulated data (see Fig. 3.1c in Chapter 3). The correlation coefficient was greater than 0.9 for the correlation between the two sides of equation (2.12), for ξ=2.46, for the assessment years after the estimations matrices had an equal number of complete and incomplete cohorts (see Fig. 3.2b in Chapter 3). The models in equation (2.12) well explained the RE patterns in the estimations for the haddock fishery in division 4x (see Fig. 3.3 in Chapter 3).

The following derivation (in Chapter 3) was based on the finding that the residual at the estimated current recruitment in the overall regression was near zero. The above fact in combination with the fact that the sum of residuals in the recruitment age class in the overall regression was near zero, leads to,

(2.13) $RE_{\ln,1,Y} = \ln q_{1,Y} - MeanK_a$

where, $q_{1,Y}$ is the catchability coefficient of the current recruitment, calculated after the newly recruited cohort is complete in the fishery. Equation (2.13) was validated using the simulated data (see Fig. 3.1d in Chapter 3). The correlation coefficient was greater than 0.94 for the correlation between the two sides of the equation for the assessment years after the estimation matrices had an equal number of complete and incomplete cohorts (see Fig. 3.2c in Chapter 3). Some examples of model fitting of equation (2.12) with the data from the simulated fishery are given in Figure (2.5). The models in equation (2.13) explained well the RE patterns in the estimations in the haddock fishery in division 4x (see Fig. 3.3 in Chapter 3).

From equation (2.12), $RE_{\ln,a,2}$, the RE of the current stock size (for the stock sizes in natural logarithm) in the next immediate assessment year, can been written as a

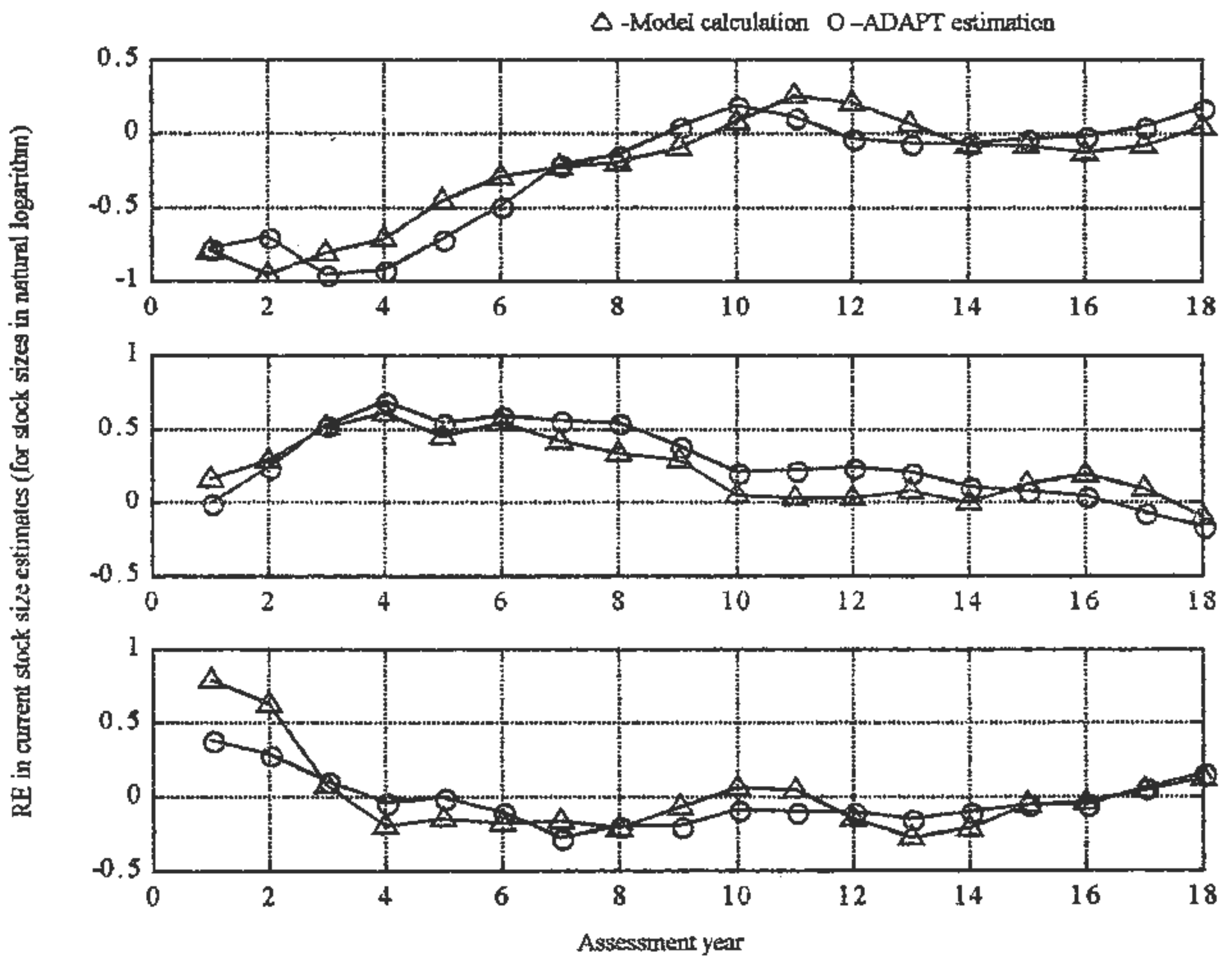


Figure 2.5a

Examples of fitting equation (2.12) with ADAPT for time series data from the simulated fisheries.

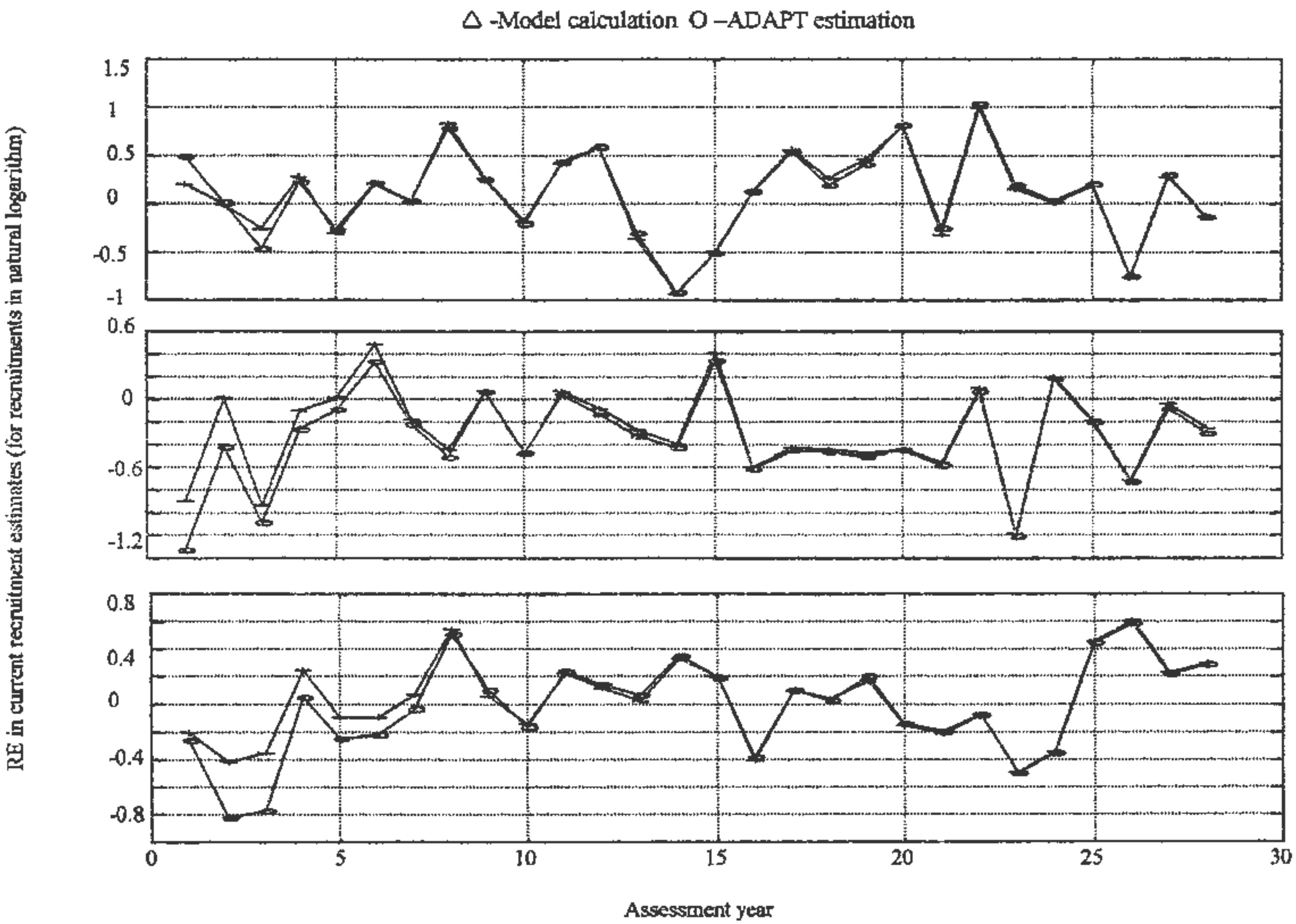


Figure 2.5b

Examples of fitting equation (2.13) with ADAPT for time series data from the simulated fisheries.

function of $RE_{ln,s,1}$, the RE of the current stock size (for the stock sizes in natural logarithm) in the current assessment year as,

(2.14) $\Re_{\ln,S,2} = \Re_{\ln,S,1} + \hbar$

where $\hbar \approx \frac{1}{\xi}\sum_{a=1}^{A-1}\left[\frac{\gamma_a}{Y-\gamma_a+1}\left(\ln\Gamma_a - Mean(K_{a(})\right) + \left(\ln\Lambda_a - \ln\Gamma_a\right)\right]$, where

$\ln\Gamma_a = \ln\left(\frac{I_{a,Y+1-A}}{N_{a,Y+1-A}}\right)$ and, $\ln\Lambda_a = \ln\left(\frac{I_{a,Y+1}}{N_{a,Y+1}}\right)$. In this instance, $\ln\Gamma_a$ is the log-catchability coefficient of the data point at age *a* of the cohorts when they become complete in the fishery in the immediate assessment year, and $\ln\Lambda_a$ is the log-catchability coefficient of the data point at age *a* of the year added in the immediate assessment year. Equation (2.14) was validated with data from the simulated fisheries (see Fig. 2.6 for some examples of model-fitting). The RE calculated from the model was correlated with the RE estimated by ADAPT, and obtained r=0.86 for calculations started with matrices which contained at least an equal number of complete and incomplete cohorts. Based on the assumption of stationarity in the time series model that age specific catchability coefficients are time invariant, so that their mean and variance are constant. i.e $\ln\Gamma_a \sim N(q, \sigma_q)$ and $\ln\Lambda_a \sim N(q,\sigma_q)$ s.t. $Mean(K_a) \sim N(q, \sigma_q)$, and $Mean(k_a) \sim N(q, \sigma_q)$. Therefore, $RE_{ln,s,x} \sim N(0,\sigma_{s,1})$ (for x=1 to 2) and $\hbar \sim N(0,\sigma_{s,2})$. Furthermore, $\ln\Gamma_a \sim N(q, \sigma_q)$ and $\ln\Lambda_a \sim N(q,\sigma_q)$ (for the reference estimates of stock sizes that are used for their calculations) are statistically independent variables. Hence, equation (2.14) behaves as a random walk model (see Weiss 1994) for RE of current stock size estimates (for stock sizes in natural logarithm) of each yearly assessment (see Fig. 2.7a for random walk

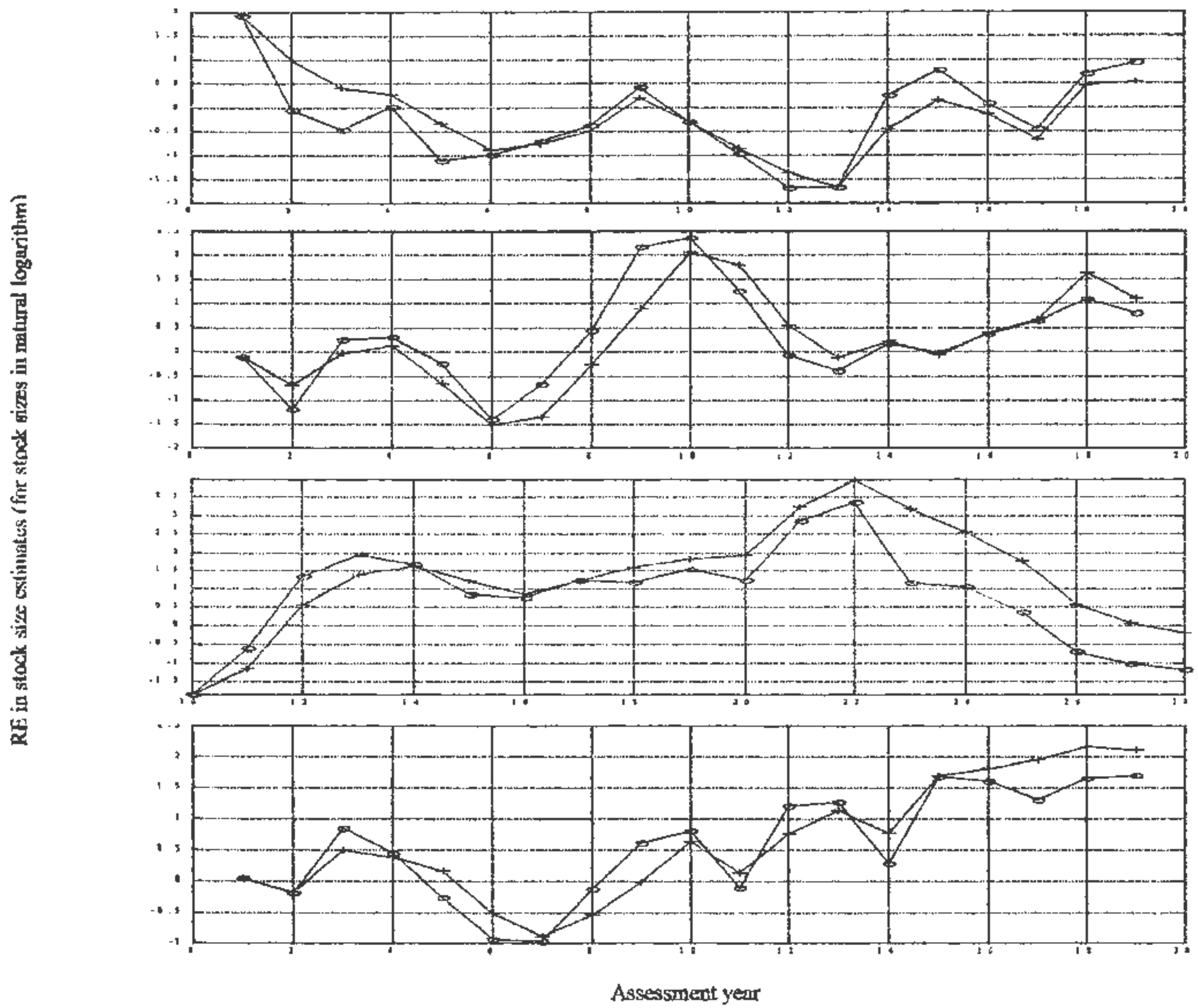


Figure 2.6

Examples of fitting equation (2.14) with ADAPT for data from the simulated fisheries. (+) Model calculation. (o) ADAPT estimations

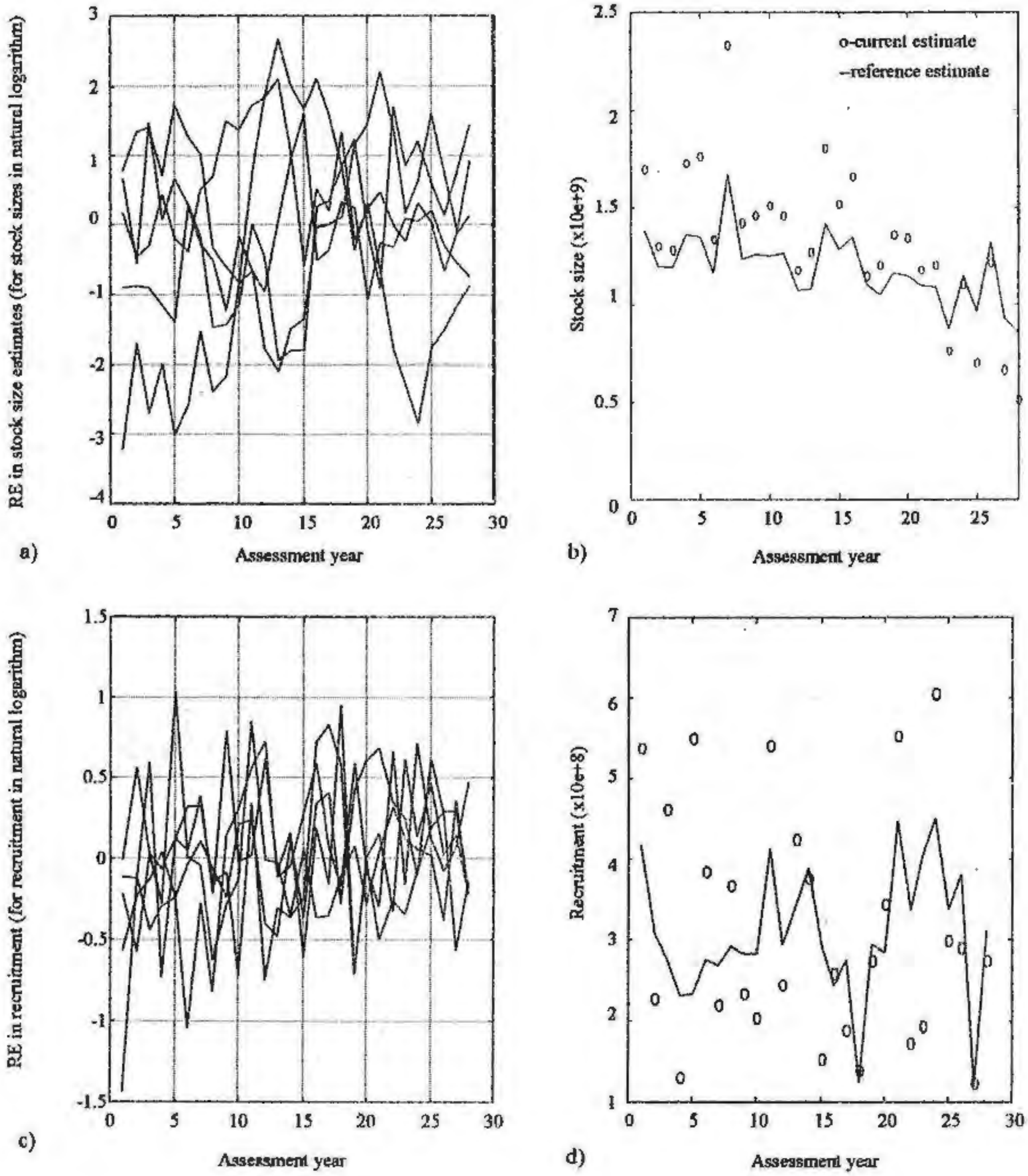


Figure 2.7

Examples of random walks in the estimates of (a) $RE_{s,ln}$ and (c) $RE_{c,ln}$ in recruitment (of 5 arbitrary model simulations) resulted from RVRED. Examples of (b) RE_s and (d) RE_c in recruitment resulted from RVRED.

trends). Because the expected value of the change-factor, $E(\hbar)$, is zero, the random walk model behaves as no-drift [when variance of $\ln \Gamma_a$ equals the variance of $\ln \Lambda_a$, and the variance of $\ln \Gamma_a$ equals the variance of $Mean(K_a)$, according to the model assumption]. Equation (2.14) holds the properties of aperiodicity and recurrence about the expected values (see Weiss 1994). Hence, trajectories in RE tend to lie on the same side of the expected value for long time intervals, aperiodically, rather than being scattered about the expected value. This random walk property in equation (2.14) could give a false impression of a time trend in $RE_{ln,s}$ depending on which section of the series we happen to look at (see Brown et al. 1994). In contrast to Mohn's (1999) claim that non-stationarity has caused the RP, this study demonstrates that false trends in RE could occur even without the violation of the assumption of the stationarity in the model. This mechanism implies that over short periods, trends in $RE_{ln,s}$ may be dominated by random variation of errors in "input data" with the estimates having the property of a random walk model. Figure (2.8) depicts some examples of the behavior in the components of equation (2.14) when RVRED forms time series patterns in RE. Although the examples in Figures (2.8a.1 and 2.8a.2) show trends in RE for longer periods, the corresponding random components of the random walk model, which is $\hbar$, does not show any trends rather vary randomly between positive and negative values along the time series. However, the cumulative $\hbar$ along the time series, which is the factor that determines the trends in RE, will have similar signs for a time period depending on the presence of a mean gradient in the first and second components in equation (2.14) between early and recent years, although year specific $\hbar$ will show random variations. This property is the

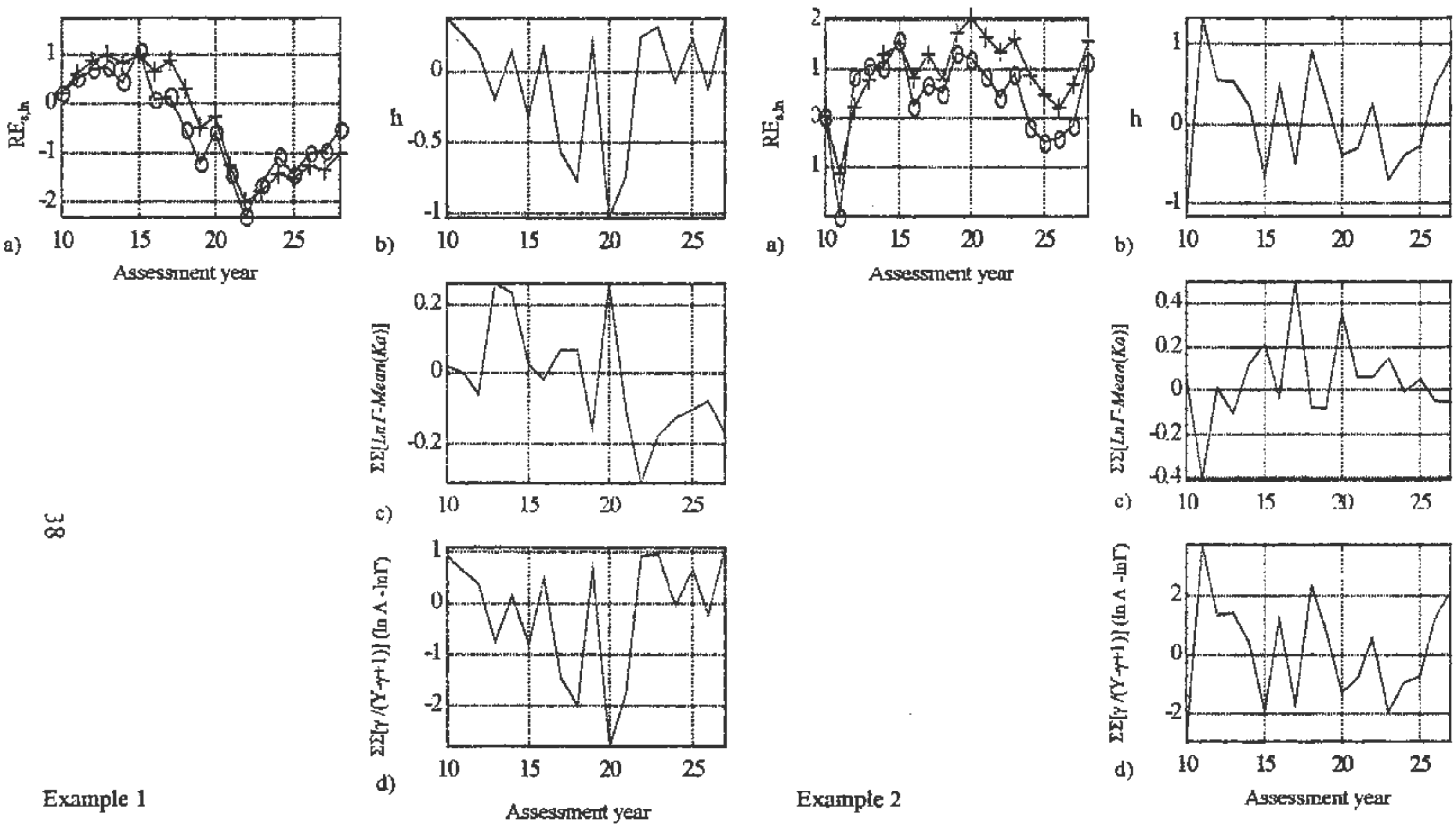


Figure 2.8

(a) Examples of fitting equation (2.14) with ADAPT for data from the simulated fisheries. + Model calculation. o ADAPT estimations. (b) Random variation in the additive component in the random walk model in equation (2.14). (c) Random variation in the first component within the additive component. (d) Random variation in the second component within the additive component.

mechanism that underlies in random walk models. Because RE_s is both positively median and mean biased, RE_s may remain for even longer periods on the positive side of the expected value, more likely with higher values, than on the negative side, more likely with small values. Therefore, this random mechanism may generate RE patterns that are prominent for positive RE. However, expected value of h can also be non-zero as a result of a difference in the expected values of $\ln \Gamma_a$ and $\ln \Lambda_a$ caused by non-random factors, for example, those discussed in Mohn (1999). This phenomenon could cause a drift in the RE for a time period until h becomes unbiased (i.e the expected values of $\ln \Gamma_a$ and $\ln \Lambda_a$ equal $Mean(K_a)$). The RE in yearly assessments may continue to generate similar signs depending on the continuity of the gradient between the expected values of $\ln \Gamma_a$ and $\ln \Lambda_a$ and $Mean(K_a)$. These patterns will be examined in Chapter (3) in more detail. Furthermore, magnitudes of RE caused by RVRED were comparable with those of empirical assessments given in ICES (1991) and Sinclair et al. (1991). The example in Figure (2.7b) enables both the pattern and the magnitude of RE to be compared with those of empirical assessments in Figures (1.1-1.3).

From equation (2.13), RE_{ln} in the current recruitment, $RE_{ln,c,2}$, the RE of the recruitment in the next immediate assessment year, can be written as a function of $RE_{ln,c,1}$, the RE_{ln} of the recruitment in the current assessment year.

(2.15) $\Re_{ln,c,2} \approx \Re_{ln,c,1} + \lambda$

where, $\lambda = \left[\frac{1}{(Y+1)} \left[\ln \Gamma_a - Mean(K_a) \right] + \left(\ln q_{1,Y} - \ln q_{1,Y+1} \right) \right]$, s.t. $\ln q_{1,Y} \sim N(q, \sigma_q)$ and $\ln q_{1,Y+1} \sim N(q, \sigma_q)$ and $Mean(K_a) \sim N(q, \sigma_q)$ by model assumption. Hence, $RE_{c,ln}$ will also behave as a random walk model but with recursion more frequently than that in $RE_{s,ln}$ (see Fig. 2.7b for some examples). The model in equation (2.15) was validated with data from the simulated fishery (see Fig. 2.9 for some examples of model-fittings). The RE calculated from the model was correlated with the RE estimated by ADAPT, and obtained r=0.99.

This analysis demonstrates that RP can occur by chance alone in the presence of random variation of "realistic" errors in "input data" as a result of a random walk property in the estimation model with the likelihood increasing with the increased number of age classes in a fishery. The hypothesis of random occurrence of RP is also consistent with the fact that RP has appeared more prevalent in ADAPT than that in other estimation methods (ICES 1991), suggesting that ADAPT model error structure might have caused the RP. Parma's (1993) hypothesis that "RP results from an increase in trend in catchability caused by improved technology or spatial changes of fish stock", was not supported by the patterns observed in the cod fishery in the eastern Scotian Shelf for which catchability was not expected to change over time (Mohn, 1999) suggesting those RE patterns might have occurred by random walk trends caused by random error variations.

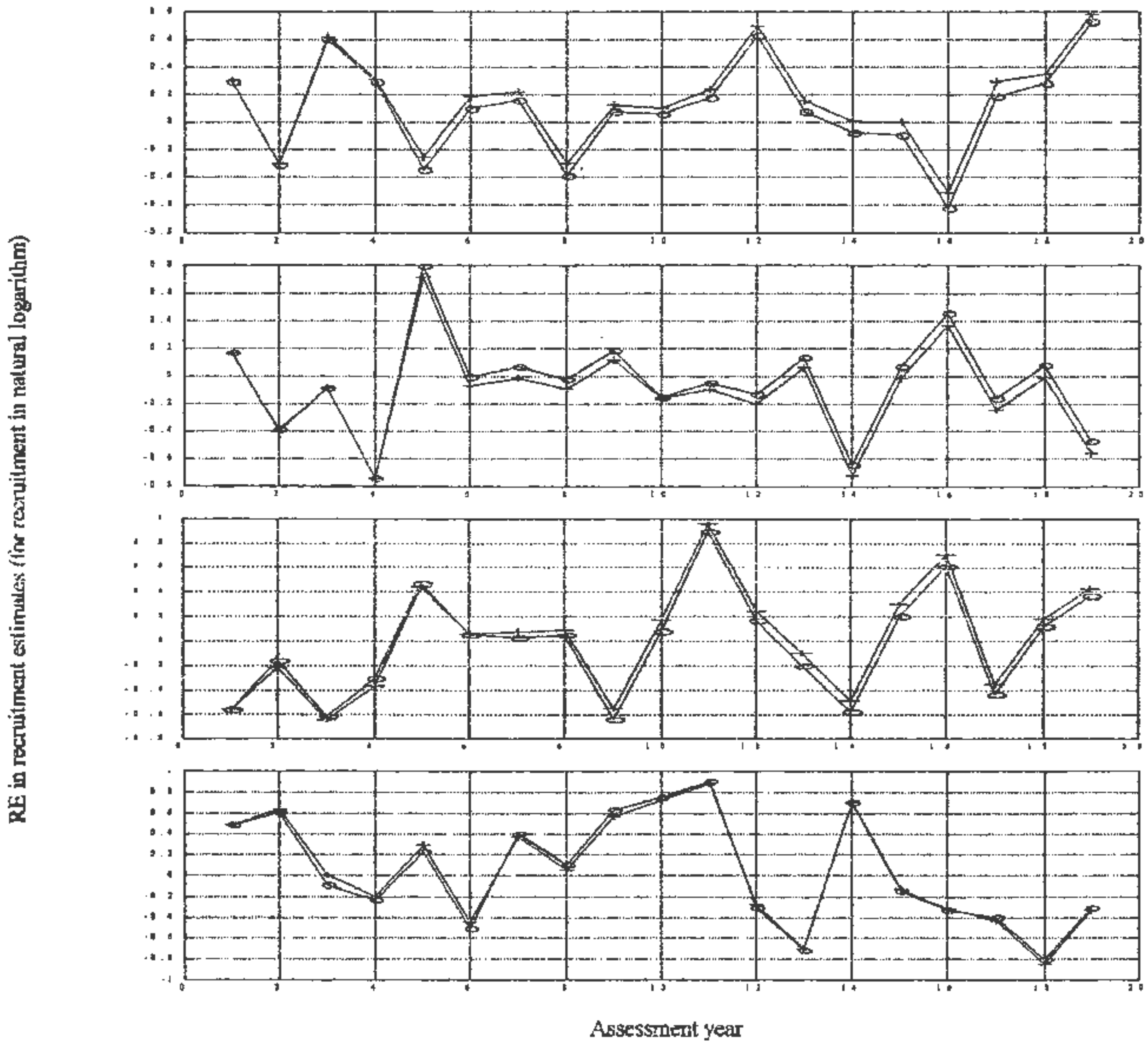


Figure 2.9

Examples of fitting equation (2.14) with ADAPT for data from the simulated fisheries. (+) Model calculation. (o) ADAPT estimations

CHAPTER 3

THE FUNDAMENTAL AND UNIFIED CAUSAL FACTOR AND THE CAUSAL MECHANISM OF RETROSPECTIVE ERROR IN ADAPT

3.1 Introduction

The previous study (Chapter 2) has indicated that RP can occur by chance alone in the presence of random variations of "realistic" errors in "input data". The likelihood of obtaining longer time series of positive RE_s increases with increased number of age classes in a fishery. It was further suggested that cohort size estimates were less likely to form RE patterns caused by random variations.

Although causal factors of the RP suggested by Mohn (1999) and Sinclair (1991) are valid from a pragmatic perspective, the fundamental and unified causal factor and the underlying causal mechanism cannot be fully understood until their mathematical basis is examined. Exploring how RP occurs is important to be able to understand the "true" nature of the RP, or whether the respective problem is "real" (whether RP is a real problem). Previous studies have not addressed this question at great depth. In this study, I have employed an analytical approach to develop an explanatory model to examine the fundamental causal mechanism of RE that creates the RP. The model was validated using data from the simulated fishery in the previous study (Chapter 2). The model explained well the RP in a haddock fishery in division 4x. Also examined were the theoretical possibilities of the formation of temporal trends in RE caused by temporal trends in the biases of factors that influence model-variables. The explanatory model was further extended to analyze RE patterns related to "true" error patterns.

3.2 Analytical Model

The definitions of notations found in this chapter are given under section (2.2.2). In searching for an analytical solution to the objective function of ADAPT (equation 2.1), the normal equations of equation (2.1) were derived as a set of partial derivatives with respect to parameters approximating zero, and the equations were solved simultaneously for the parameters (see Neter et al. 1989). The partial derivative of equation (2.1) with respect to age (a) specific catchability coefficients (q_a) approximating zero, can be written as, $\frac{\partial(ADAPT)}{\partial \ln q_a}=0$ where $ADAPT$ stands for the objective function (equation 2.1).

Thus, for any age-class $a=1:A$,

$$(3.1)\quad Y \ln q_a = \sum_{y=W+1}^{Y} \ln \frac{I_{a,y}}{N_{a,y,Inc}} + \sum_{y=1}^{W} \ln \frac{I_{a,y}}{N_{a,y,Com}}$$

where $N_{a,y,Inc} = N_{a+(Y-y),Y}e^{M(Y-y)} + \sum_{k=1}^{Y-y} C_{a+(Y-y)-k,Y-k}e^{\frac{M}{2}(2k-1)}$ are the cohort sizes of age a in year y for the incomplete cohorts, and

$N_{a,y,Com} = C_{A,y+(A-a)}e^{\frac{(A-a)M}{2}}\left(\frac{1}{e^{F_{A,y}}-1}\right) + \sum_{k=y+A}^{y} C_{k-y,k}e^{\frac{M}{2}[2(y+A-k)-1]}$ are the cohort sizes for the complete cohorts. Y is the current year, and W is the year up to which the cohorts are complete. Therefore, in the right hand side of equation (3.1), both $N_{a,y,Inc}$ and $N_{a,y,Com}$ are considered to be constants in deriving the partial derivative with respect to q_a. Equation

(3.1) is identical to the sum of residuals, $\sum_{a,y} \varepsilon_{a,y}$, of a close to linear regression model of the form **Y**=m**X**+c+ε, ε~N(0,σ), with **Y**=ln(**I**), m=1, **X**=ln(**N**) (where **N** are the parameters for incomplete cohorts, and observations for complete cohorts), and c=ln(q) with $\sum_{a,y} \varepsilon_{a,y} = 0$.

The partial derivatives of the ADAPT objective function with respect to current stock-size parameters, $N_{a,Y}$, approximating zero, can be written as $\frac{\partial(ADAPT)}{\partial \ln N_{a,Y}} = 0$.

Thus,

(3.2) $\ln\frac{I_{a,Y}}{q_a N_{a,Y}} + 2\sum_{y=Y-1}^{Y-a}\left(\ln\frac{I_{a+1-y+Y,y}}{q_{a+1+y+Y} N_{a+1-y+Y,Y}}\frac{\partial \ln N_{a+1-y+Y,y}}{\partial \ln N_{a,Y}}\right)=0$ for all $N_{a,Y}$ for all a for any incomplete cohort starting from (a,Y) coordinates of the estimation matrix backwards.

Where $\frac{\partial \ln N_{a-x,y-x}}{\partial \ln N_{a,y}} = \frac{\partial \ln(N_{a,y}e^{xM} + \sum_{k=1}^{x} C_{a-k,y-k}e^{M(k-\frac{1}{2})})}{\partial \ln N_{a,y}}$ for any x-years backward in a cohort, where, C is catch. The partial derivative $\frac{\partial \ln N_{a-x,y-x}}{\partial \ln N_{a,y}}$ can be expanded as

$$(3.3) \quad \frac{\partial \ln N_{a-x,y-x}}{\partial \ln N_{a,y}} = \frac{\partial \ln N_{a,y}e^{xM} + \partial \ln(1 + \frac{\sum_{k=1}^{x} C_{a-k,y-k}e^{M(k-\frac{1}{2})}}{N_{a,y}e^{xM}})}{\partial \ln N_{a,y}}$$

The theoretical problem facing a linear approximation to the equation (2.1) is that the series of both equations (3.1) and (3.2) cannot be solved simultaneously for q and N. The series of equation (3.2) are not linear in N, and hence there may be multiple solutions to equation (2.1). ADAPT uses a numerical, non-linear search procedure for solving the normal equations iteratively. To avoid obtaining a local minimum to the sum of normal equations, because multiple solutions to equation (2.1) may be present, and to obtain a global minimum, the search procedure is repeated with different initial values assigned to the variables in the normal equations. The non-linear solution to equation (2.1) needs to satisfy each normal equation because the global minimum for equation (2.1) is consistent with satisfying the normal equations. The simulation results in Chapter 2 indicated that the sum of residuals along each age class approximated zero in every ADAPT assessment, together with the total sum of residuals near zero at the global minimum. This is the same as satisfying equation (3.1) with the solution of the non-linear approximation to equation (2.1). Results also indicated that the sum of residuals along incomplete cohorts, with a close to 1, yielded a similar result, approximating zero. Confirming this fact, equation (3.3) shows that when equation (3.2) corresponds to a cohort close to the current recruitment, the partial derivative $\frac{\partial \ln N_{a-x,y-x}}{\partial \ln N_{a,y}}$ tends to e^{xM} because the catch contributing to the cohort size is less than the value of parameter N in the current cohort size of the particular cohort, with equation (3.3) taking the form of the equation of the sum of log residuals through the cohort, approximating zero. As a special case of the equation (3.2), from equation (3.3),

$$(3.4) \quad \ln \frac{I_{1,y}}{q_1 N_{1,y}} = 0$$

for current recruitment. This equation implies that the residual in the current stock recruitment estimate is zero. For the analysis of retrospective error, the facts that are required are that the sum of residuals through each age class approximates zero (equation 3.1), and the residual of the current stock recruitment approximates zero (equation 3.4). Patterson and Kirkwood (1995) have stated that ADAPT has dealt with computational problems such as analysis being bound-constrained or finishing on false minima. They further stated that this computational problem is a "cost" of using the ADAPT method. I suggest that a more reliable estimate from ADAPT is obtained if equation (3.4) is incorporated as a parallel criterion for reaching the global minimum. This may be achieved by obtaining the local minimum that gives the lowest value for $\ln \frac{I_{1,Y}}{q_1 N_{1,Y}}$ near zero (relative to other residuals) as a solution near the global minimum rather than subjectively deciding a local minimum upon an indeterministic lower value for the sum of squares residuals of the overall model. However, the lowest value for $\ln \frac{I_{1,Y}}{q_1 N_{1,Y}}$ always coincided with the lowest value for the sum of squares residuals, but an arbitrary lower value for the sum of squares residuals of the overall model did not always guarantee that ADAPT reached the global minimum. By contrast, finding the lowest value for the objective-function, based only upon the least sum of squares of residuals, becomes even more difficult when data matrices become larger and produce multitudinal solutions as local minima.

From equations (3.1),

$$(3.5) \quad \ln Q_{Y,Inc} = \frac{1}{Y} \sum_{a=1}^{A-1} \sum_{y=1}^{Y} \ln \frac{I_{a,y}}{N_{a,y}} \quad \text{s.t.}$$

$\ln Q_{Y,Inc}$ is the log-summation of catchability coefficients in the assessment year Y. Let,

$$(3.6)\quad \psi = \sum_{a=1}^{A-1}\sum_{y=Y}^{Y-a} RE_{\ln,a,y} \text{ where } RE_{\ln,a,y} = (\ln N_{a,y,Inc} - \ln N_{a,y,Com})$$

Where ψ is the summation of retrospective error in cohort-sizes-at-age (for cohort sizes in natural logarithm), $RE_{\ln,a,y}$, of all the incomplete cohorts, by the assessment year Y (the current year). $RE_{\ln,a,y}$ is given by the subtraction of ADAPT estimated cohort-sizes-at-age in incomplete cohorts by year Y, ($lnN_{a,y,Inc}$), from cohort-sizes-at-age computed after those cohorts become complete in the fishery (after (A-1) years from year Y), by a near deterministic method using equation (2.2) and (2.4), ($lnN_{a,y,Com}$).

$N_{a,y,Inc} = N_{a+Y-y,Y,Inc} e^{M(Y-y)} + \upsilon_{a,y}$ and $N_{a,y,com} = N_{a+Y-y,Y,Com} e^{M(Y-y)} + \upsilon_{a,y}$ where

$\upsilon_{a,y} = \sum_{j=(Y-y)+a-1}^{a} \sum_{k=Y-1}^{y} C_{j,k} e^{\frac{M}{2}(2[Y-k]-1)}$ is the cohort-size-at-age contributed by the catch-at-age components of a cohort. $N_{Y-y,Y,Inc}$ is the cohort size in the current year estimated in the assessment year Y.

$N_{a+Y-y,Y,Com} = C_{A,y+(A-a)} e^{\frac{(A-a)M}{2}} \left(\frac{1}{e^{F_{A,y}}-1}\right) + \sum_{j=A-1}^{Y-y} \sum_{k=A-a+y-1}^{Y} C_{j,k} e^{\frac{M}{2}[2(A-j)-1]}$ is the size of a cohort in the current year in the assessment year Y, calculated after the cohort is complete in the fishery.

After the cohort-sizes-at-ages in the incomplete cohorts are determined by the estimates from the assessment after (A-1) years from year Y, catchability coefficients for the assessment year Y are readjusted by similar calculations. This approach will yield an equation similar to equation (3.5) for $\ln Q_{Y,Com}$. By subtracting equation (3.5) from the equation obtained above for $\ln Q_{Y,Com}$, an equation for ψ can be obtained as,

$$(3.7)\quad \psi = -Y\nabla \ln Q$$

where, $\nabla \ln Q = (\ln Q_{Y,Inc} - \ln Q_{Y,Com})$, i.e. $\nabla \ln Q$ is the difference in the summation of RE of log catchability coefficients for year Y. By simplifying the right hand side (R.H.S) of equation (3.7),

$$(3.8) \quad \psi = -\sum_{a=1}^{A-1} \gamma_a [Mean(K_a) - Mean(k_a)]$$

where $Mean(K_a) = \frac{1}{(Y-\gamma_a)} \sum_{y=1}^{Y-\gamma_a} \ln\left(\frac{I_{a,y}}{N_{a,y}}\right)$ and $Mean(k_a) = \frac{1}{\gamma_a} \sum_{y=Y-\gamma_a+1}^{Y} \ln\left(\frac{I_{a,y}}{N_{a,y}}\right)$. γ_a is the number of incomplete cohorts that belong to the age class a in the assessment year Y, K_a is the year and age-specific catchability coefficients in the complete part of the age class a, and k_a is the year and age-specific catchability coefficient in the incomplete part of the age class a after the cohort-sizes-at-ages are adjusted with estimations once those cohorts are completed in the fishery (after $A-1$ years from the assessment year Y). Estimated catchability coefficient in the assessment year Y was approximated by $Mean(K_a)$.

From equation (3.6), $RE_{\ln,a,y}$ was written as a function of RE of the estimated current cohort-sizes-at-age of the particular cohort in the year Y.

$$(3.9) \quad RE_{\ln,a,y} = \ln\left(\frac{N_{Y-y,Y,Inc}e^{M(Y-y)} + \upsilon_{a,y}}{N_{Y-y,Y,Com}e^{M(Y-y)} + \upsilon_{a,y}}\right) \text{ for any } a \text{ and } y$$

Equation (3.6) can also be written as either

$$(3.10) \quad RE_{\ln,a,y} = \ln\left(\frac{\frac{N_{Y-y,Y,Inc}e^{M(Y-y)}}{\upsilon_{a,y}} + 1}{\frac{N_{Y-y,Y,Com}e^{M(Y-y)}}{\upsilon_{a,y}} + 1}\right) \text{ for any } a \text{ and } y$$

or

$$(3.11) \quad RE_{\ln,a,y} = \ln\left(\frac{\dfrac{\upsilon_{a,y}}{N_{Y-y,Y,Inc}e^{M(Y-y)}}+1}{\dfrac{\upsilon_{a,y}}{N_{Y-y,Y,Com}e^{M(Y-y)}}+1}\right)+\ln\left(\frac{N_{Y-y,Y,Inc}}{N_{Y-y,Y,Com}}\right) \text{ for any } a \text{ and } y$$

where, $\ln\left(\dfrac{N_{Y-y,Y,Inc}}{N_{Y-y,Y,Com}}\right)$ is the RE of the estimated cohort-size-at-age in the current year Y.

In the triangle of incomplete cohorts in the cohort-sizes-at age matrix, cohort-sizes-at-age are much greater than $\upsilon_{a,y}$ towards the cohorts-at-age in the current year, and the opposite is true in the opposite direction. Equations (3.10) and (3.11) show that under the above scenarios, $RE_{\ln,a,y}$ of the same cohort in years close to the current year approximate the $RE_{\ln,a,y}$ of the same cohort in the current year, while $RE_{\ln,a,y}$ of a number of years away from the current year approximates zero. This was apparent in the case analyses of RE (e.g. see Fig 1.1-1.3). Equation (3.11) showed that the first component of the R.H.S always takes the opposite sign to the latter component, which is $RE_{\ln,a,y}$ of the particular cohort in the current year. Therefore, $RE_{\ln,a,y}$ in the current year is always greater than $RE_{\ln,a,y}$ of the same cohort in the previous years as assessed by the current year. This pattern was apparent in the case analyses of RE (e.g. see Fig. 1.1-1.3). Because the first component of the R.H.S of equation (3.11) is a function of the second component, $RE_{\ln,a,y}$ was approximated as,

$$(3.12) \quad RE_{\ln,a,y} = \bar{\eta}_{a,y} \ln\left(\frac{N_{Y-y,Y,Inc}}{N_{Y-y,Y,Com}}\right)$$

where $\bar{\eta}_{a,y} = \prod_{k=1}^{Y-y+a-1} \eta_k$. In this case η_k<=1 are coefficients specific to the year *k*, with η_k

decreasing with an increase in *k*, and assuming that the ratio $\frac{V}{N_{Y-y,Y,Com}e^{M(Y-y)}}$ is

approximately the same for each age class. Hence, η_k is a negative exponential discrete function starting with the value 1 for *k*=1, and decreasing asymptotically towards zero for increasing *k*. It was assumed that $RE_{ln,a,Y}$, the RE of the cohort sizes for the current year, is approximately the same for all the age classes given that the results of the estimates are in natural logarithms. These assumptions enable a linear approximation to $RE_{ln,s}$ from $RE_{ln,a,y}$ as

(3.13) $$\sum_{a=1}^{A-1}\sum_{y=Y}^{Y-a} RE_{ln,a,y} = \xi RE_{ln,s}$$

where, $\xi = \left(\sum_{i=1}^{A-a}\frac{(A-i)}{(A-1)}\eta_i\right)$. This coefficient is dependent on the number of age classes

and the proportion of catch-at-age specific to a fishery. Substituting equation (3.13) in equation (3.8),

(3.14) $$RE_{ln,S} = -\frac{1}{\xi}\sum_{a=1}^{A-1}\gamma_a[Mean(K_a) - Mean(\kappa_a)]$$

To examine $RE_{ln,c}$ in the newly recruited cohort size estimates, from equation

(3.4), $\ln N_{1,Y} = \ln(\frac{I_{1,Y}}{q_1})$. Thus,

(3.15) $$RE_{ln,1,Y} = \ln(\frac{I_{1,Y}}{q_1 N_{1,Y,Com}})$$

where $N_{1,Y,C}$ is the cohort size estimated as a function of catch after the cohort is completed in the fishery. Equation (3.15) can also be written as

$$(3.16) \quad RE_{\ln,1,Y} = \ln(\frac{q_{1,Y}}{q_1})$$

Where, q_1 is the estimated catchability coefficient of age-class 1 in the assessment year Y, and $q_{1,Y}$ is the catchability coefficient of the current recruitment calculated after the newly recruited cohort is complete in the fishery. Lnq_1 can be approximated with $Mean(K_a)$. Thus, from equation (3.16),

$$(3.17) \quad RE_{\ln,1,Y} = \ln q_{1,Y} - MeanK_a$$

3.3 Methods

In order to validate the explanatory analytical model that was developed to examine the theoretical basis of the formation of RE, data from the simulated fisheries in Chapter (2) were used. As an example, the model was tested with data from haddock fisheries in division 4X (Hurley et al. 2000) from 1970 to 1999 for age classes 2 to 10. These assessments contained 10 cohorts each with full sequence from recruitment to completion in the fishery.

3.4 Results

Equations (3.8), (3.13), (3.14), and (3.17) were confirmed using data from the simulated fisheries (Fig. 3.1) and obtained r>0.92, r>0.90, r>0.94, respectively, for the

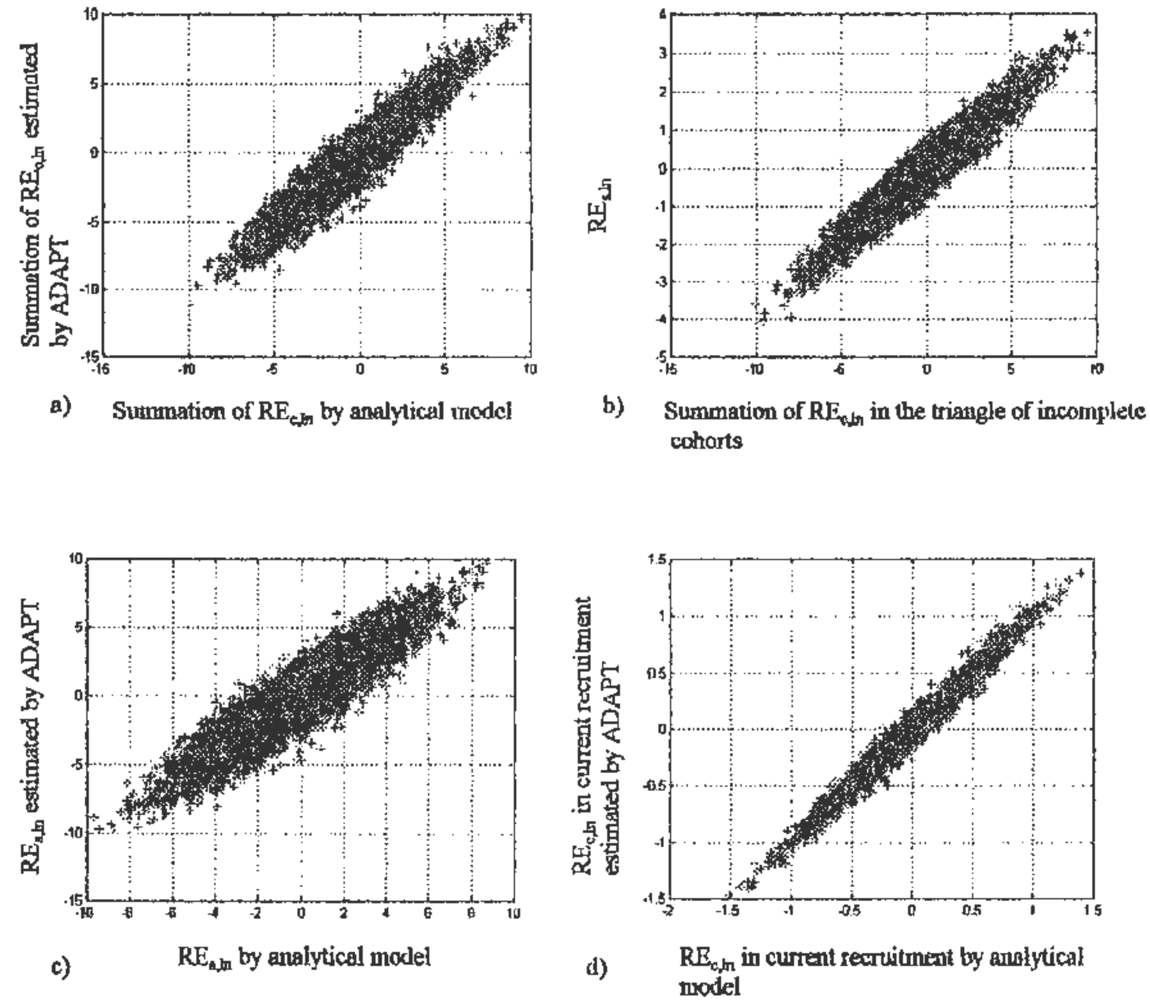


Figure 3.1
(a) Correlation between the summation of $RE_{ln,c}$ (in the triangle of incomplete cohorts) calculated from the analytical model [equation (3.8)] and that estimated from ADAPT for data from the simulated fishery.
(b) Correlation between the $RE_{ln,s}$ of current stock sizes and the summation of $RE_{ln,c}$ of cohort sizes in the triangle of incomplete cohorts. Coefficient ξ in equation (3.13) was 2.46.
(c) Correlation between the $RE_{ln,s}$ obtained from the analytical model [equation (3.14)] and that estimated from ADAPT for data from the simulated fishery.
(d) Correlation between the $RE_{ln,c}$ of current recruitment obtained from the analytical model [equation (3.17)] and that estimated from ADAPT for data from the simulated fishery.

correlations between the two sides of the corresponding equations for the assessment years after the estimation matrices had an equal number of complete and incomplete cohorts (Fig. 3.2). When the estimation matrices had more incomplete cohorts than complete cohorts, the correlations were weaker (Fig. 3.2). The residuals of the above correlations were a result of the approximations made in the analysis. Coefficient ξ in equation (3.14) (Fig. 3.2b) was 2.46. Furthermore, Figure (2.5) in Chapter (2) depicts examples of model fitting with time series data from the simulated fishery for equations (3.14 and 3.17).

The RE in current stock size estimates and current recruitment estimates of haddock fishery in division 4X were well explained by models in equation (3.14) and (3.17) (Fig. 3.3) with correlation coefficients 0.86 and 0.99, respectively, for ξ equals 2.46. In other words, RE in current stock size estimates (for stock sizes in natural logarithm) can be explained by the difference in the mean log ratios of abundance-index-at-age to cohort-size-at-age between the reference estimates in early and recent years. Similarly, RE in current recruitment estimate can be explained by the difference in the mean log ratio of abundance-index-at-age to cohort-size-at-age of age class 1 between the reference estimates in earlier and recent years. Hence, models in equation (3.14) and (3.17) can be used to explain the mechanism of RE that is generated by factors that influence the difference in ratios in the models. Temporal biases of factors that influence the difference in the ratios of the model could generate successive RE that creates the RP.

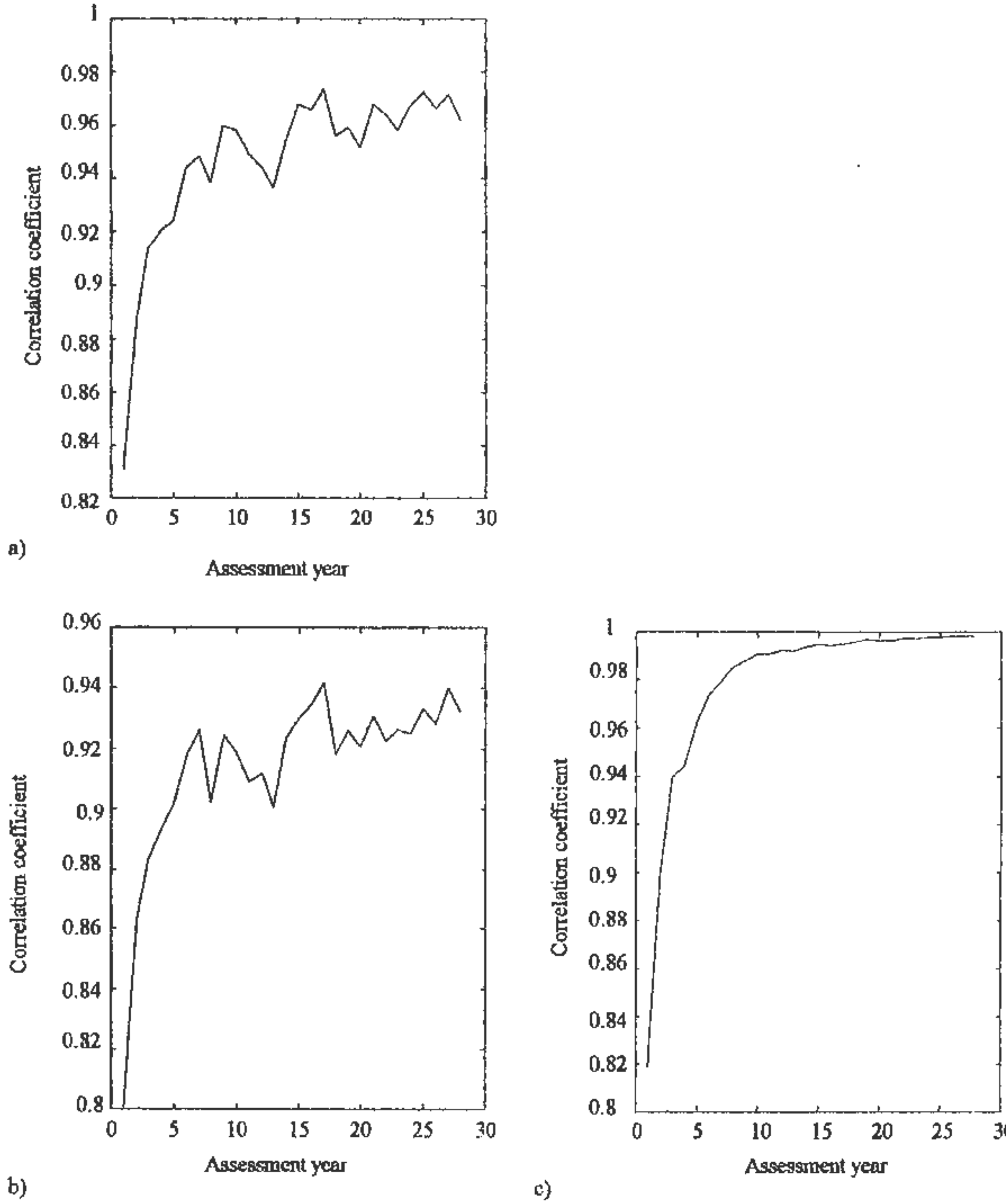
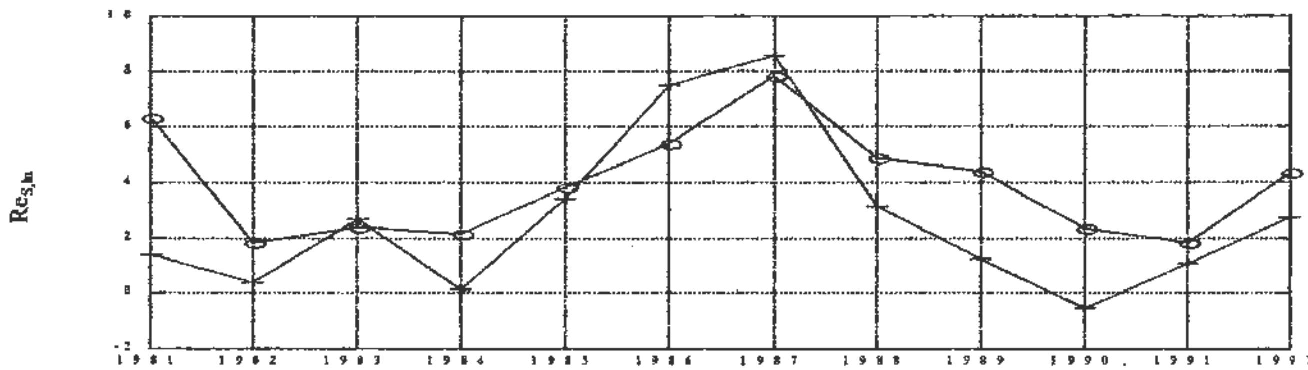
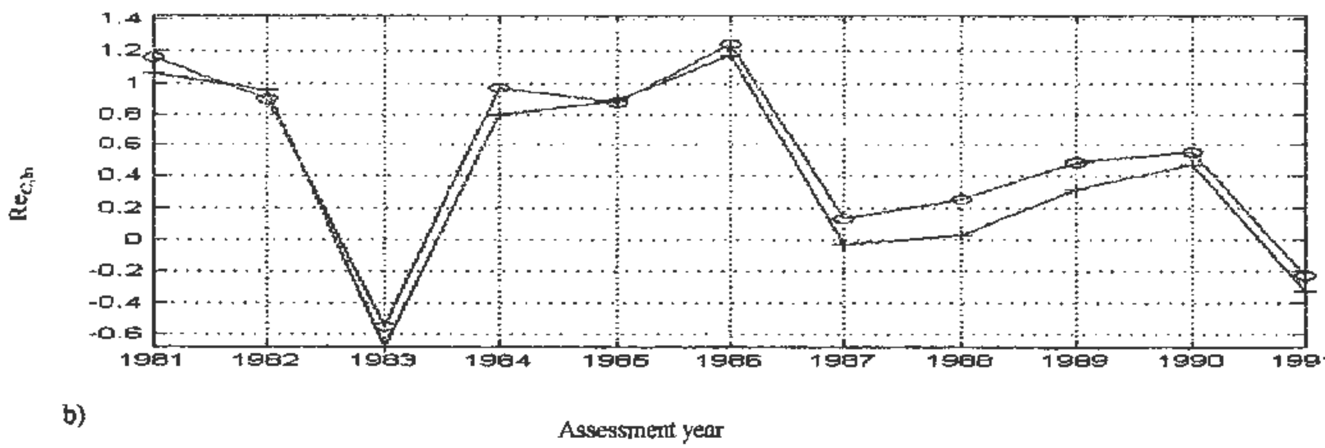


Figure 3.2

Coefficients of correlations in (a) equation (3.8) (b) equation (3.14) and (c) equation (3.17) with increasing number of years of data [correspond to Figures (3.1a) (3.1c) and (3.1d)].



a)



b)

Figure 3.3

Fitting (a) equation (3.17) and (b) equation (3.14) with ADAPT for RE in current stock size estimates ($r=0.8584$) (for stock sizes in natural logarithm) and current recruitment estimates ($r=0.9921$) (for cohort sizes in natural logarithm) respectively, for data from 4X haddock fishery from year 1970 to 1999. (+) ADAPT estimation (O) model calculation

3.5 Discussion

ADAPT assumes that the difference between the mean log-ratios in equation (3.14) is unbiased with mean equal to 0, because the expected values of both mean ratios are the same. This assumption arises because ADAPT assumes that catchability coefficients are time invariant. However, according to equation (3.14), temporal trends in the bias of the estimates of "input data" and model misspecifications (TTBRDM) could generate a difference in the mean ratios and thereby generate a temporal trend in $RE_{\bar{s},\ln}$ creating a RP.

3.5.1 Retrospective Error by Temporal Trends in Bias of the Estimates of Catch

For a detailed mathematical analysis of RP caused by TTBEDM, from equation (3.14), let

$$(3.18)\quad Mean(\tilde{K}_a) = \frac{1}{(Y-\gamma_a)} \sum_{y=1}^{Y-\gamma_a} \ln\left(\frac{I_{a,y}}{N_{a,y} - \alpha N_{a,y}} \right)$$

$$(3.19)\quad Mean(K_a) = \frac{1}{(Y-\gamma_a)} \sum_{y=1}^{Y-\gamma_a} \ln\left(\frac{I_{a,y}}{N_{a,y}} \right)$$

$$(3.20)\quad Mean(\bar{k}_a) = \frac{1}{\gamma_a} \sum_{y=Y-\gamma_a+1}^{Y} \ln\left(\frac{I_{a,y}}{N_{a,y} - \beta N_{a,y}} \right)$$

$$(3.21)\quad Mean(k_a) = \frac{1}{\gamma_a} \sum_{y Y-\gamma_a+1}^{Y} \ln\left(\frac{I_{a,y}}{N_{a,y}} \right)$$

where it was assumed that cohort sizes estimated deterministically using catch data, is subject to an error proportional to the "true" cohort size with coefficient α for the early years and coefficient β for the recent years. Subtracting equation (3.15) from equation (3.17), and substituting in equation (3.14),

$$RE_{\ln,S} = -\frac{1}{\xi}\sum_{a=1}^{A-1}\gamma_a[Mean(K_a) - Mean(\kappa_a) + \ln(1-\beta) - \ln(1-\alpha)]$$

Here, $Mean(K_a) = Mean(k_a)$, because the model assumes $E(K_a)$=$E(k_a)$ when cohort-size-at-age is "true". Therefore, $E(\mathrm{RE}_{\ln,s})$=0. Hence,

$$\text{(3.22)} \quad RE_{\ln,S} = \left(\frac{(A-1)[\ln(1-\alpha) - \ln(1-\beta)]}{\xi}\right)\sum_{a=1}^{A-1}\gamma_a$$

From equation (3.22), it can be inferred that when the proportion of error in catch in early years, α, is less than that in recent years, β, the log-stock size in the current year is overestimated in retrospection, and vice versa. These patterns of RE were demonstrated by Sinclair et al. (1991) and Mohn (1999) in simulation studies (Fig. 3.4 and Fig. 3.5a). For example, proportional errors α and β can be attributed to the discarded catch, which is usually excluded from catch estimates and under reported catch. The previous study (Chapter 2) indicated that RE_s is both mean and median biased, in contrast to $\mathrm{RE}_{s,\ln}$. Hence, the positive bias of RE_s generated by the model structure could decrease the expected degree of underestimation in RE_s. With time, depending on whether the gradient between $Mean(K_a)$ and $Mean(k_a)$ increases, remains the same, or decreases, the stock sizes in the current year tend to be biased retrospectively, more, equal, or less in successive assessments. That is, when the factors that influence the difference between the mean log-ratios in equation (3.14) continue to remain the same

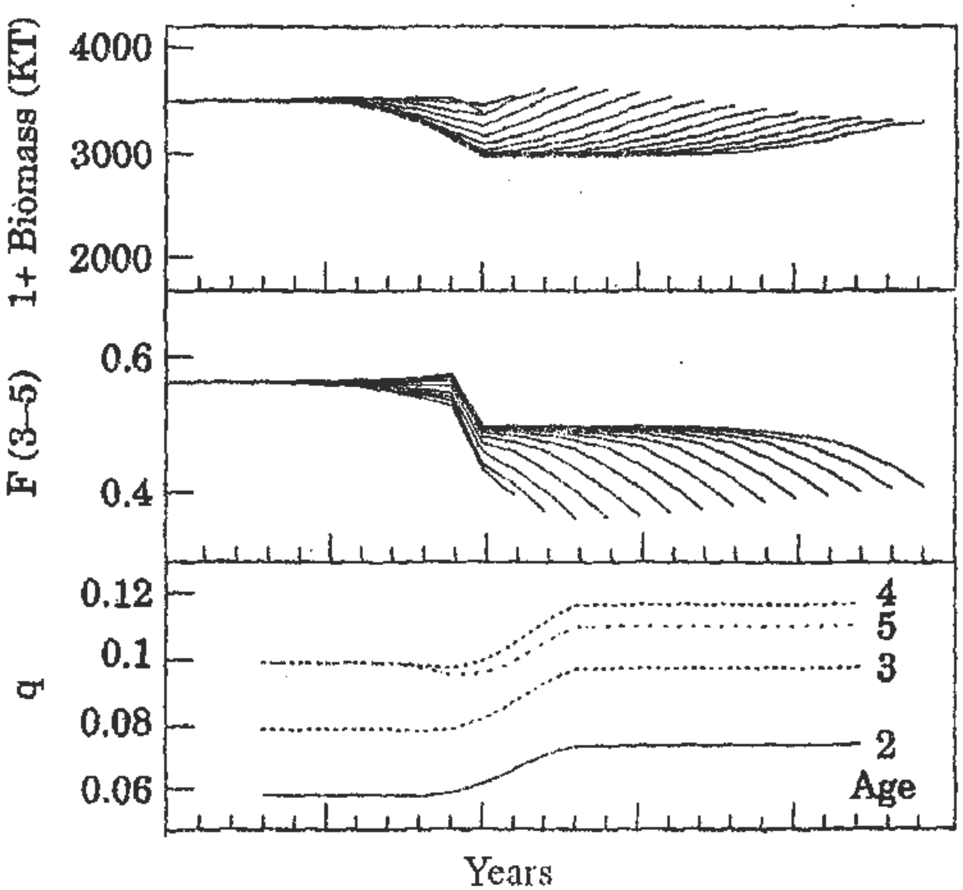


Figure 3.4

Retrospective analysis of discarded catch perturbation simulation experiment (in which catch was subjected to discarding in recent years) [Reprinted from Mohn (1999)].

a)

b)

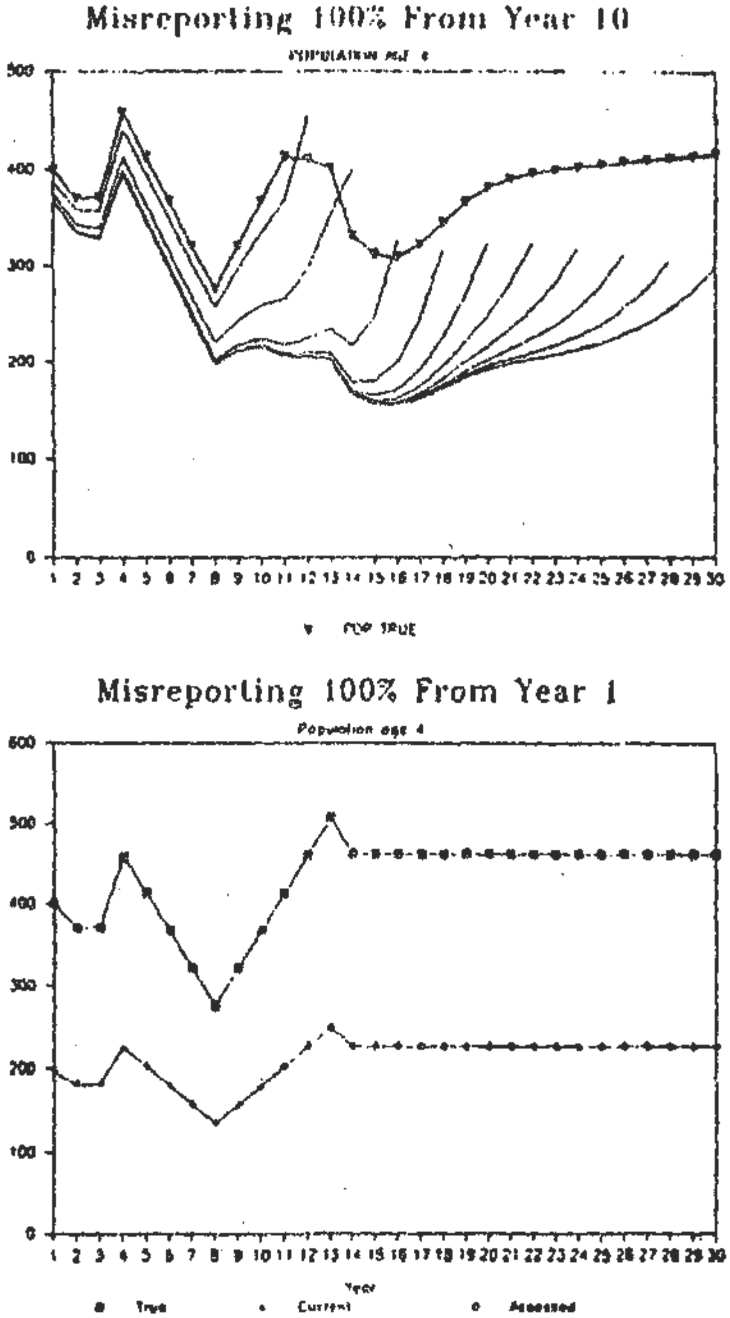


Figure 3.5

Stock size estimates for two simulated misreporting situations. (a) Only half of the catch was reported after 10 years. (b) Only half the catch was reported for the entire time series [Reprinted from Sinclair et al. (1991)].

temporally, the difference between the mean ratios will also tend to be the same (either positive or negative accordingly), generating temporal trends in RE that create the RP. There is a high likelihood that successive assessments will yield the same sign for the RE, because the moving means in equation (3.14) vary only by fractions between successive assessments.

To further explain the correlated RE, from equation (3.14), $RE_{ln,s,2}$, the RE of the current stock size in the next immediate assessment year, can be written as a function of $RE_{ln,s,1}$, the RE of the current stock size of the current assessment year.

(3.23) $RE_{\ln,S,2} \approx RE_{\ln,S,1} + \hbar$

where, $\hbar = \frac{1}{\xi}\sum_{a=1}^{A-1}\left[\frac{\gamma_a}{(Y-\gamma_a+1)}\left[\ln\Gamma_a - Mean(K_a)\right] + \left(\ln\Gamma_a - \ln\Lambda_a\right)\right]$,

$\ln\Gamma_a = \ln\left(\frac{I_{a,Y+1-A}}{N_{a,Y+1-A}}\right)$ and $\ln\Gamma_a = \ln\left(\frac{I_{a,Y+1}}{N_{a,Y+1}}\right)$. Here, $\ln\Gamma_a$ is the log-catchability coefficient of the data point at age *a* of the cohorts that become complete in the fishery in the next immediate assessment year and $\ln\Lambda_a$ is the log-catchability coefficient of the data point at age *a* of the year added in the next immediate assessment year. According to the model assumption that year specific catchability coefficients are time invariant, $\ln\Gamma_a \sim N(q, \sigma_q)$, $\ln\Lambda_a \sim N(q,\sigma_q)$, $Mean(K_a) \sim N(q, \sigma_q)$, and $Mean(k_a) \sim N(q, \sigma_q)$, then, $RE_{ln,s,1} \sim N(0,\sigma_{s,1})$ and $\hbar \sim N(0,\sigma_{s,1})$. However, $\hbar$ becomes biased as a result of TTBEDMs, and may continue to generate $\hbar$ with similar sign depending on the continuity of maintaining a similar, or higher gradient with similar direction in the difference in log-ratios of equation (3.14). This pattern will lead to $RE_{s,ln}$ of more likely similar signs in successive current stock size estimations. This random walk model has been discussed in Chapter (2) in detail.

Thus, from equations (3.22), it can be argued that if the catch underestimation problem becomes worse in recent years of a fishery, possibly as a result of new management plans, the current stock size is more likely to be overestimated in retrospection in successive assessments. That is, when $\alpha < \beta < 1$ in equation (3.22), then $RE_{s,ln} > 0$. $RE_{s,ln}$ can also have successive negative values, and stock size in the current year can therefore be underestimated in retrospection in successive years if catch is increasingly overestimated in recent years or the catch underestimation problem is more serious in early years compared with that of recent years. As indicated above, the time-series patterns described for $RE_{s,ln}$ may show a small constant shift towards the positive side after they are transformed to RE_s. Therefore, the underestimation patterns of current log-stock sizes estimated by ADAPT may be overshadowed by the positive bias of RE in the transformed stock sizes. Furthermore, from equation (3.22), when $\alpha = \beta > 0$ (when the proportion of discarding is consistent over the time series), then $RE_{s,ln}$ disappears. These patterns of RE were demonstrated by Sinclair et al. (1991) in a simulation study (Fig. 3.5b). According to the Figure (3.5b), in the study by Sinclair et al. (1991), current and reference estimates do not display a divergence, yet both estimates were consistently "truly" underestimated.

3.5.2 Retrospective Error by Temporal Trends in Bias of the Estimates of Abundance Indices

The same line of reasoning can be used to analyze the effect on RE of trends in year and age-specific catchabilities along years caused by technological advancements. ADAPT often uses catch per unit effort (CPUE) obtained from research surveys rather than from commercial fisheries as an index of fish abundance. Both the abundance indices input to the ADAPT model and the "true" cohort sizes are independent of CPUE (which corresponds to the efficiency) in commercial fishing gear and the accuracy of catch data. An increase in CPUE caused by improved fishing efficiency of commercial fishing gear, without a similar increase in "true" cohort sizes, can skew the empirical probability equation of catch-at-age of a cohort more towards older age classes. This increased CPUE cannot influence ADAPT estimation of current stock sizes, however, because it cannot generate bias either in catch estimates or in abundance indices input to the ADAPT model. A change in the abundance indices estimated in surveys, notwithstanding a similar trend in "true" cohort sizes, can only bias the abundance indices input to the ADAPT model. From equation (3.14), let

$$(3.24)\quad Mean(\overline{K}_{\phi}) = \frac{1}{(Y-\gamma_a)}\sum_{y=1}^{Y-\gamma_a}\ln\left(\frac{(1+\phi)I_{a,y}}{N_{a,y}}\right)$$

$$(3.25)\quad Mean(\overline{k}_{\varphi}) = \frac{1}{\gamma_a}\sum_{y=Y-\gamma_a+1}^{Y}\ln\left(\frac{(1+\varphi)I_{a,y}}{N_{a,y}}\right)$$

where $N_{a,y}$ is the "true" cohort size at age a and year y, and ϕ and φ are the proportions of improvements in research survey gear in early years and recent years, respectively (i.e.

having assumed that recent gear has not been standardized with respect to the early gear). Thus from equation (3.11), we obtain

$$(3.26)\quad RE_{\ln,S} = \left(\frac{(A-1)[\ln(1+\varphi)-\ln(1+\phi)]}{\xi}\right)\sum_{a=1}^{A-1}\gamma_a$$

According to equation (3.26) when $\varphi > \phi$, then $RE_{\ln,s} > 0$ (i.e. when the surveying gear is improved over time), current stock size tends to be overestimated in retrospection. Mohn (1999) has demonstrated the pattern of RE resulted from the above scenario in a simulation study (Fig. 3.6a). The pattern of RE displayed in his study was a result of the condition that $\varphi > \phi$. Furthermore, when the trend in the bias of the estimates of abundance index changes direction RE pattern changes its direction (Fig. 3.6b). The change displayed in that simulation study was a result of the change of the condition from $\varphi < \phi$ to $\varphi > \phi$. Equation (3.26) shows that when $\varphi = \phi$ (>0) (when research survey CPUE is standardized over the years as an index of abundance, yet is consistently biased throughout the years), ADAPT cannot create $RE_{\ln,s}$ although it may estimate consistently and reliably biased stock sizes. Increase in CPUE in commercial fishing gear that results from improved technology cannot influence RE as long as the catch is reported accurately.

3.5.3 Retrospective Error by Misspecification of Natural Mortality

The same method of analysis discussed above can be used to examine the impact of the temporal differences in the "true" natural mortality on RE. From Pope's (1972) equation, for a complete cohort in the fishery,

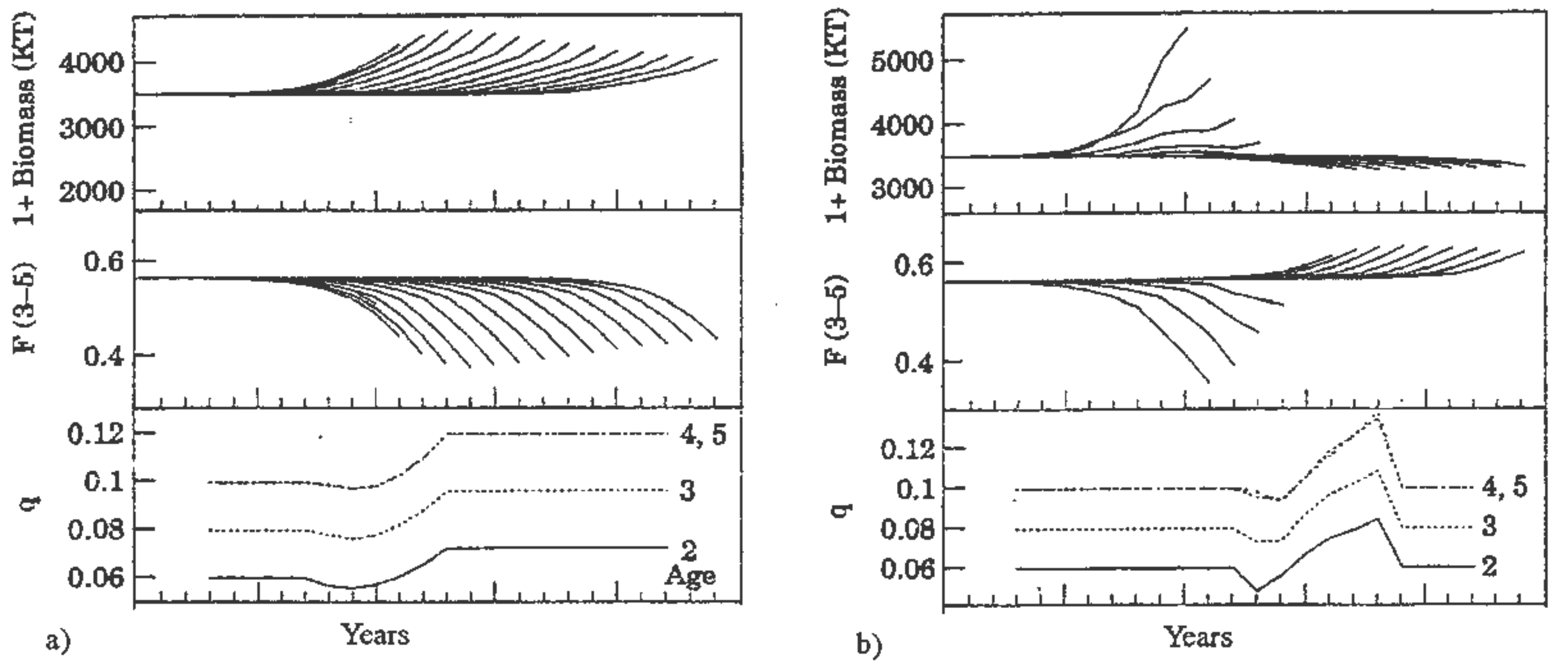


Figure 3.6

Retrospective analysis of abundance indices perturbation simulation experiment (a) in which "research survey vessel effect" or the catchability of research survey vessels was increased in recent years, (b) in which the catchability of research survey vessels was changed from increase to decrease along the years [Reprinted from Mohn (1999)].

(3.27) $N_{a,y,Com} = \sum_{j=A}^{a} \sum_{k=y+(A-a)}^{y} C_{j,k} e^{\frac{M}{2}[2(A-j)-1]}$

where, M is the specified natural mortality. Let ∂ be the bias in the specified natural mortality in early years. Thus, the R.H.S of equation (3.27) can be written as

(3.28) $\sum_{j=A}^{a} \sum_{k=y+(A-a)}^{y} C_{j,k} e^{\frac{M+\partial}{2}[2(A-j)-1]} = e^{\partial}\left(\sum_{j=A}^{a} \sum_{k=y+(A-a)}^{y} C_{j,k} e^{\frac{M}{2}[2(A-j)-1]} \right)$

Thus, from equation (3.11)

(3.29) $Mean(\overline{\overline{K_{\partial}}}) = \frac{1}{(A-\gamma_a)} \sum_{y=1}^{Y-\gamma_a} \ln\left(\frac{I_{a,y}}{e^{\partial} N_{a,y}} \right)$

(3.30) $Mean(\overline{\overline{k_{\nabla}}}) = \frac{1}{\gamma_a} \sum_{y=Y-\gamma_a+1}^{Y} \ln\left(\frac{I_{a,y}}{e^{\nabla} N_{a,y}} \right)$

where, ∇ is the bias in specified natural mortality in recent years. Thus ∂+M and ∇+M represent the "true" natural mortality in the particular time periods. Substituting (3.29) and (3.30) in equation (3.14),

(3.31) $RE_{\ln,S} = \left(\frac{(A-1)[(\partial+M)-(\nabla+M)]}{\xi} \right) \sum_{a=1}^{A-1} \gamma_a$

According to equation (3.31), when the "true" natural mortality does not show temporal trends, i.e. when $\partial = \nabla >< 0$, RE disappears, notwithstanding that the specified natural mortality is consistently underestimated or overestimated over time. Furthermore, if the "true" natural mortality is decreasing or increasing over time (i.e. when $\partial > \nabla$ or $\partial < \nabla$, respectively), then RE>0 or RE<0, respectively, regardless of whether M is consistently underestimated or overestimated temporally. The above relation holds true even for different M values for different age-classes. The conditional basis presented here was supported by Lapointe et al. (1989) in a simulation analysis. If "true" natural mortality varies across different age classes, even if specified as an age-independent constant, then cohort sizes estimated by Pope's (1972) equation can be biased, depending on both the catch proportions among the age classes of a given cohort, and the pattern of age-specific biases in the specified natural mortality. Hence, temporal differences in catch proportions along cohorts, for example, caused by an increase in CPUE in commercial fishing gear, could generate RE that is dependent on the patterns of age-specific biases in specified natural mortality. A RE pattern that was demonstrated in a simulation study by Mohn (1999) (Fig. 3.7), in which he has perturbed the "specified" natural mortality (*M*) instead of the "true" natural mortality. This scenario is not found in fisheries, which displayed RP, yet can be explained similarly by the model in equation (3.31). In reality, only "true" natural mortality changes without notice, diverging from the specified natural mortality.

3.5.4 Retrospective Error in Fishing Mortality

Furthermore, the factors that influence RE in stock size estimations also hold true for RE in fishing mortality. From equation (2.4), RE in fishing mortality, RE_{Fa}, can be written as

(3.32) $$RE_{F_a} = -[\ln(N_{a+1,y+1,Inc}) - \ln(N_{a+1,y+1,Com})] + [\ln(N_{a,y,Inc}) - \ln(N_{a,y,Com})]$$

Equation (3.32) can be written as

(3.33) $$RE_{F_a} = -[RE_{N_{\ln,a+1,y+1}} - RE_{N_{\ln,a,y}}]$$

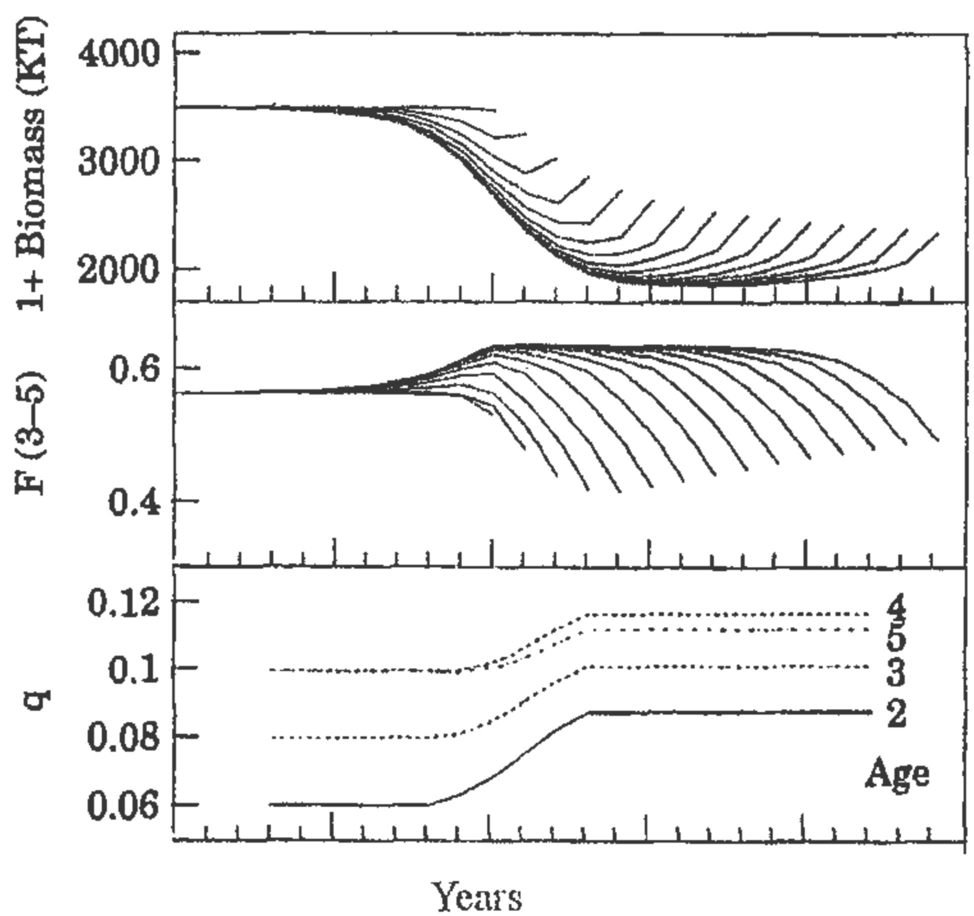


Figure 3.7

Retrospective analysis of specified natural mortality perturbation simulation experiment (in which specified natural mortality was decreased in recent years) [Reprinted from Mohn (1999)].

where $RE_{N_{\ln,a+1,y+1}} = \ln\left(\frac{N_{a+1,y+1,Inc}}{N_{a+1,y+1,Com}}\right)$ and $RE_{N_{\ln,a,y}} = \ln\left(\frac{N_{a+1,y+1,Inc}e^{M} + f_1(C)}{N_{a+1,y+1,Com}e^{M} + f_1(C)}\right)$

where $f_1(C)$ is a function of catch. Thus, in equation (3.33), $RE_{Nln,a+1,y+1} > RE_{Nln,a,y}$ in all cases. Therefore, RE_{Fa} is always opposite in sign to RE in cohort size estimations, and hence, behaves similar to the RE of the related cohort-size-at-age. Thus, RE_{Fa} depends on the same factors that influence the RE of cohort size estimations. Hence, fishing mortality will display the RP with negative retrospective error, as shown in Myers et al. (1997) (Fig. 3.8).

3.5.5 Comparison of Retrospective Error with "True" Error

Equation (3.17) is a special case of equation (3.14). Therefore, all the facts that hold for equation (3.14) will also hold for equation (3.17). This pattern implies that TTBEDMs that affect RE in stock size estimation will also affect RE in cohort size estimation of recruitment, again generating a RP. According to equation (3.17), factors that are variable across age classes will not generate RE, because those factors do not generate differential time-series biases. Biases across age classes could not form trends in RE as long as they are temporally consistent. Hence, a temporal trend in partial recruitment could generate a RP, because it may be a result of a differential time-series bias in CPUE-at-age. This causal relation was demonstrated by Mohn (1999) in a simulation study that assumed a change in the selectivity by change in fishing gear over the years (Fig. 3.9). Nonetheless, the decision on how to set the terminal age and the corresponding terminal cohort sizes may not result in RE in stock size estimations, as

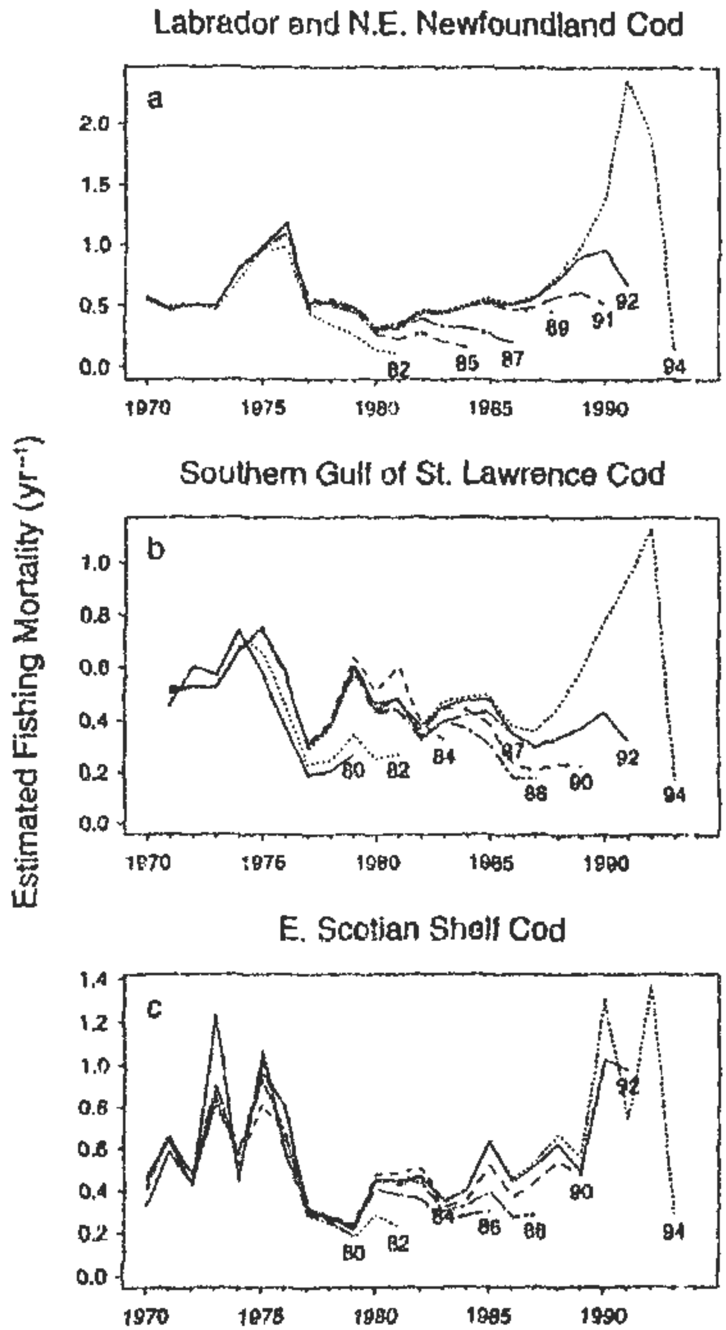


Figure 3.8

Retrospective analysis of fishing mortality in northern cod fishery [Reprinted from Myers et al. (1997)].

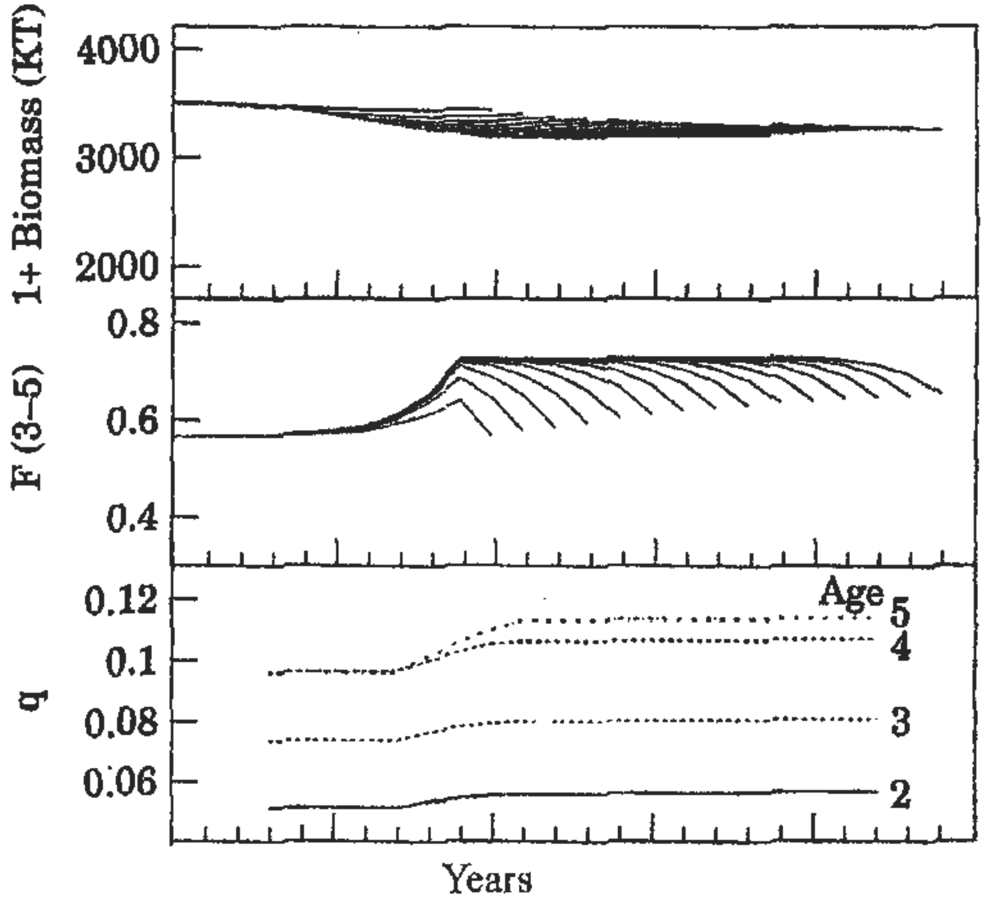


Figure 3.9

Retrospective analysis of a partial recruitment perturbation simulation experiment [Reprinted from Mohn (1999)].

long as the decision is consistent with all the yearly assessments, although it may result in "true" errors. However, the RE in stock size estimations may be more important than the absolute error in terms of the fisheries management, because the management parameters need to be estimated reliably more so than accurately when using adaptive methods.

However, for the scenario in which catch was increasingly underestimated over time, positive RE are not "real" overestimations of current stock sizes. This can be explained as follows, and has been demonstrated by Sinclair et al. (1991) in a simulation study (Fig. 3.5a). From the definition of RE, we obtain a current stock size estimate by adding a reference estimate (stock size of a particular year after all the cohorts contained in it complete the fishery) to the RE as follows,

$$(3.34)\quad N_{s,\ln,Inc} = N_{s,\ln,Com} + RE_{s,\ln}$$

In a fishery where catch is increasingly underestimated over time, $N_{s,ln,Com}$ will also be increasingly underestimated. Let $\underline{N}_{s,ln,Com}$ be the "true" stock size with "true" error ("true" underestimation) Φ. Then equation (3.34) can be written as,

$$(3.35)\quad N_{s,\ln,Inc} = \underline{N}_{s,\ln,Com} + \left(RE_{s,\ln} - \Phi\right)$$

Equation (3.35) implies that when catch is increasingly underestimated (i.e. RE becomes positive), RE no longer becomes an error, because RE decreases the true error from the current stock size estimate. In other words, if there is an increasing trend in catch-underestimation over time, then current stock size estimates will be more accurate than their reference estimates, which is contradictory to the definition of RE. However, when the underestimation of catch in recent years remains the same for a long period, RE decreases, because α approaches β in equation (3.22). When RE decreases over time, from equation (3.34), $N_{s,ln,Inc}$ will have a large "true" error. This pattern implies that low

RE or even the absence RE or RE patterns does not always guarantee that current stock size estimates are more accurate, and vice versa. Therefore, RP generated by TTBEDMs in ADAPT as a result of temporally increasing catch-underestimation may not contribute to successive "true" overestimation of current stock size ("true" underestimation of fishing mortality) and may not contribute to a collapse of a fishery. This conclusion is contrary to the general expectation (e.g. Myers et al. 1997). Myers et al. (1997) hypothesis that increased discarded or underreported catch underestimates the fishing mortality so that the management decision based on those parameters leads to the collapse of fisheries (eg. the northern cod fishery) is refuted by the findings presented here. Under such scenario, according to findings presented here, Myers et al. (1997), reference estimates of fishing mortality become overestimated rather than current estimates of fishing mortality become underestimated. However, in a similar basis, an increased aggregation of fish, thereby increased CPUE, with decline of stocks (Hutching and Myers 1994), may result in a series of positive RE_s with "true" overestimation of stock sizes. Similarly, increase in catchability in RSV that results from improved fishing gear may generate a series of positive RE, which are "true" overestimations. Table (3.1) summarizes all the possible scenarios of obtaining "true" error patterns in current stock size estimations. Positive RE generated by temporally decreasing "true" natural mortality are also "true" overestimations. It is possible that the so-called "retrospective problem" may not always be an actual problem, but rather an "illusion". Lapointe et al. (1989) also has argued that some apparent trends (or lack of them) in stock sizes estimated using virtual populations analysis might be artifacts, and therefore, inferences must be made cautiously.

Table 3.1 Time series patterns of RE in stock size estimates (errors in current estimates related to their reference estimates) compared with patterns of "true" errors in stock size estimates (errors in current estimates related to their "true" stock size).

Factors	**Type of RE pattern generated**	**"True" overestimations**	**"False" overestimations**	**"True" underestimations**	**"False" underestimations**	**"True" neutral**	**"False" neutral**
Increased underestimated catch	***Positive***		✓				
Decreased underestimated catch	*Negative*			✓			
Uniformly underestimated catch	*Neutral*						✓
Increased overestimated catch	*Negative*				✓		
Decreased overestimated catch	***Positive***	✓					
Uniformly overestimated catch	*Neutral*						✓
Increased true natural mortality (>M)	*Negative*			✓			
Decreased true natural mortality (>M)	***Positive***	*Condl.*					
Increased true natural mortality (<M)	*Negative*			*Condl.*			
Decreased true natural mortality (<M)	***Positive***	✓					
Increased true natural mortality (=M at beginning)	*Negative*			✓			
Decreased true natural mortality (=M at beginning)	***Positive***	✓					
Increased CPUE due to improved RSV-FG (the "vessel effect")	***Positive***	✓					
Increased fish aggregation with a decline of the stock	***Positive***	✓					

Note: M-specified natural mortality, RSV-research survey vessels, C-commercial, FG-fishing gear, Condl-Conditional.

3.5.6 Differentiating Retrospective Problem Occur by Chance and TTBEDM

Randomly occurring RP can be distinguished from RP occurred by TTBEDMs by examining RE patterns in the current stock recruitment estimates. According to the analysis, current recruitment estimates form RE patterns, which are consistent with that of current stock size estimates when the causal factors are TTBEDMs. However, current recruitment estimates are less likely to form RE patterns resulting from RVRED (as suggested in Chapter 2). Thus, according to the criterion that RE in current recruitment estimates form or do not form trends along with trends in RE in current stock size estimates, we could identify whether a RE pattern we may observe in a fishery stock assessments using ADAPT was caused by nonrandom factors or by random factors. Once we identify the "true" cause, we may obtain a more accurate estimate of the current stock size by reassessing the past stock sizes or the present stock sizes incorporating the likely "true" error into the model. The conformity of the model-explanations of the mechanism of generating patterns of RE with results of previous simulation studies indicates that the fundamental mechanism for generating a RE is the result of a temporal difference of mean log-ratios of abundance-index-at-age to cohort-size-at-age in a fishery between reference estimates of early and late years. The RE could form temporal trends caused by changes in the biases of factors that influence the above differences, thereby creating a RP. However, caution is needed in deciding whether a RP displayed in a fishery is "real" when making management decisions based on estimated parameters.

3.5.7 A Graphical Explanatory Model

Mechanism of RE can be summarized in a graphical format (see Fig 4.1 in Summary), and can be used to theorize the effects of trends in the biases of the estimates of "input data" on RE. The Table (3.1) may also be explained by the graphical model. Thus, equation (3.5) can be written as,

$$(3.18)\quad \sum_{a=1}^{A-1}\sum_{y=1}^{Y}\ln I_{a,y} = \sum_{a=1}^{A-1}\sum_{y=1}^{Y}\ln N_{a,y} + Y\ln Q_Y$$

which can be written again as a regression function of the form **Y**=m**X**+c+ε where

Y=$\sum_{a=1}^{A-1}\ln I_{a,y}$ for y=1 to Y, and **X**=$\sum_{a=1}^{A-1}\ln N_{a,y}$ for y=1 to Y, and m=1 and c=YlnQ_Y which is the log summation of catchability coefficients across age classes. ε~N(0,σ). In this model, the vector **Y** is known. The value *c* is a constant for a given assessment. However, some values of vector **X** are parameters to be estimated. This simplified form of ADAPT objective function meets the properties of a simple linear least squares regression model for given values of **X** that we have discussed in detail under the section that contained analytical derivations (Chapter 3). Hence the parameters contain in **X** are estimated with errors in the corresponding **I** values the same or less than the errors in **I** values correspond to those near fixed values of **X**. This compound model has been discussed in the beginning of this chapter as a system of parallel processing simple linear least squares regression models along each age class (or partial regression of the overall model with respect to catchability coefficient in each age class). Residuals in **I** that correspond to the estimates in **X** are a result of the partial regression of the overall model with respect to current cohort sizes in each cohort. This underlying mechanism implies that log-summation of catchability coefficients is the entity that is exclusively estimated, and the estimated current stock size is a near "measurement" for a given value of log-summation of abundance indices across age classes, rather than a statistical "estimation" by definition. Small residuals for **I** correspond to current stock size estimates have been ignored. Thus, the following example shows that when cohorts pass through a fishery, the "estimates" of **X** become fixed values so that the difference that they create between the current "estimates" and the reference "estimates" (near ADAPT regression independent values) become the RE. Hence, a trend in the log-ratio of abundance indices to "reference" stock size estimates (not necessarily "true" values) creates the RP.

Figure (3.10) demonstrates some examples of plotting data from the simulated fisheries for an assessment of current stock size for two different scenarios. (a) A fishery that displays trends in negative $RE_{s,ln}$ and (b) a fishery that displays trends in positive $RE_{s,ln}$. In both cases, data corresponds to the reference estimates of **X**. The Figure shows that current estimates of stock sizes (for stock sizes in natural logarithm) in recent years are estimated at the point where the log summation of abundance index intercepts the regression line corresponds to early years of data. However, when those recent years of data show a trend in their yearly log-summation of catchability coefficients across age classes for individual years in their reference estimates, their current estimates generally tend to produce a series of RE. Positive or negative trends in RE in stock size estimates occur when the log-summation of catchability coefficients across age classes for individual years in their reference estimates increase or decrease in general. This fact is even more accurate for the estimated current recruitment because the residual in the

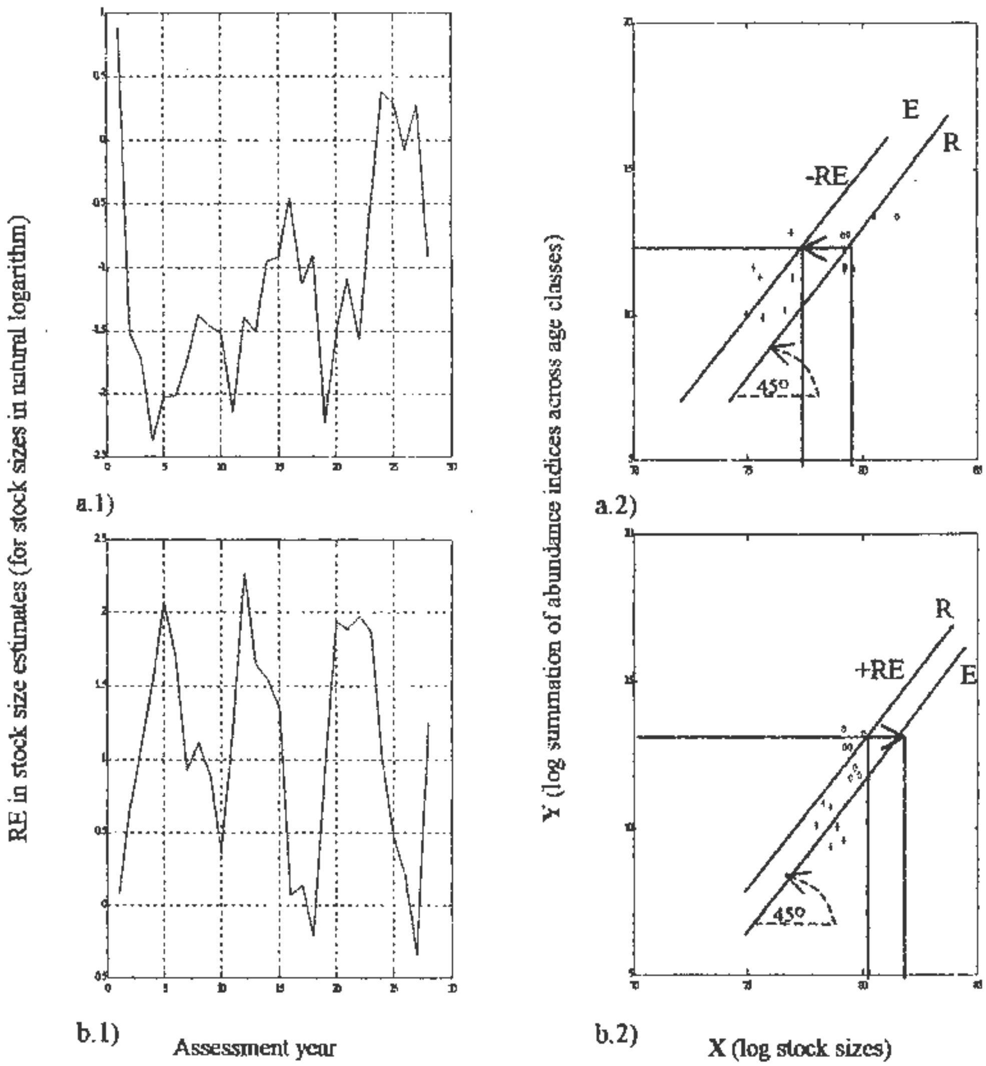


Figure 3.10

Examples of (a.1) negative trend and (b.1) positive trend in RE of stock size estimates (for stock sizes in natural logarithm) from assessment year 7 through to 14 of the simulated fisheries in the presence of RVRED. (a.2 and b.2) Formation of RE resulted from mean shift in catchability coefficients corresponding to data points in (a.1) and (b.1).

(+) Reference estimates of stock sizes in early years. (o) Reference estimates of stock sizes in recent years. (E) Regressioo line for regression with only early years of data. (R) Regression line for regression with only recent years of data. In both scenarios (a) and (b), regression line for the overall regression in the assessment year 14 will lie between the given two regression lines for each case.

abundance index corresponding to the current recruitment estimate is near zero. This simplification of the regression mechanism of the ADAPT was used to develop a graphical model in order to simplify and theorize the mechanism of $RE_{s,ln}$ (see Fig. 4.1 in Summary).

SUMMARY

An adaptive framework for virtual population analysis (ADAPT) (Gavaris 1988) is extensively used for estimating stock-sizes of exploited fish stocks in Atlantic Canada and the northeastern United States. A major issue in ADAPT is the retrospective problem (Mohn, 1999). Obtaining time-series of positive (negative) retrospective error in current stock size (fishing mortality) estimates in successive years is known as the retrospective problem (RP). Retrospective error (RE) is the deviation of contemporaneous estimates of stock parameters from subsequent estimates for the same year as more recent years of data are added to the same model (Mohn 1999). Specifically, RE is the difference in the estimates of stock parameters between the time period of a cohort entered (current estimates) and passed through (reference estimates) a fishery. Understanding the causal factors, the causal mechanism of obtaining RP and the "true" nature of RP are important because overestimation of current stock sizes (or underestimation of fishing mortality) over subsequent years may result in management decisions that lead to the collapse of a fishery.

In the first study (Chapter 2), I have investigated the occurrence of RP in a fishery in the use of ADAPT by chance in the presence of random variation of "realistic" errors in "input data" (lognormal errors in catch and research survey abundance indices) (RVRED). It was found that there is a high likelihood that RP could occur by chance alone with the likelihood increasing with increased number of age classes in a fishery. The magnitude of the RE that resulted by chance is as high as the magnitude of the RE that resulted from estimates of empirical assessments and their patterns were also similar.

The RE in current cohort size estimates (RE_C) were mean-biased but median-unbiased in the presence of RVRED. Addition of mean biased but median unbiased estimates of RE_C across age classes results in a mean and median biased estimate of RE of current stock size (RE_S). Time series of RE_S is found to have the properties of a random walk model, which gives a false impression of temporal trends generated by non-drift random variations of errors in "input data" to the model. These randomly generated trends of RE_S were more likely to be positive as a result of positive median bias RE_S and were large in magnitude as a result of positive mean bias of RE_S. Thus, a time series of positive RE_S caused by RVRED could be easily misinterpreted as those caused by temporal trends in the bias of the estimates of "input data" and model misspecifications (TTBEDM). However, the likelihood of a random occurrence of time series trends of RE_C was minimal.

In the second study (Chapter 3), I have employed an analytical and a simulation approach to examine the fundamental and the unified cause and the mechanism of RE_S that creates the RP. An explanatory model was developed based on the finding that in ADAPT, residuals of cohort-size-at-age estimates in each age class from the overall regression add to near zero, and the residual of the current recruitment estimate is also near zero at the global minimum. It was found that estimates of RE of current stock size (for stock sizes in natural logarithm) in a fishery can be fundamentally explained by the difference in mean log-ratios of abundance-index-at-age to cohort-size-at-age between reference estimates in early and recent years, which are ADAPT regression independent. The RE could form temporal trends caused by temporal trends in the biases of factors that influence this difference, thereby creating the RP. Correlation between the RE_S and the change of catchability coefficients (Mohn 1999) is also a result of simplifying the overall non-linear regression model to a set of parallel processing partial linear least squares regression models of each age class. This parallel processing extends even to a set of partial regression models of each incomplete cohort which results in obtaining a near zero value for the residual of current recruitment estimate. A near zero value for the residual of the current recruitment estimate can be used as a parallel criterion for reaching the global minimum.

Among the many factors that could theoretically generate RP, a more likely factor may be a temporal increase in the proportion of underestimated catch. However, the series of positive REs generated by the above factor may not be "true" overestimations. Technological advancements in commercial fishing gear or temporally consistent biases in the estimated catch and specified natural mortality may not generate the difference that creates a RP. Only temporal decrease in overestimated catch, decrease in "true" natural mortality, increase in the aggregation of fish with a decline of stock, and increase in catchability of RSV fishing gear could generate RP with "true" overestimations. Therefore, it may not be advisable to correct for RP unless the "true" nature of the pattern of RE has been identified.

Findings described in Chapter (3) can be summarized in a graphical format (Fig 4.1), and can be used to theorize the effects of trends in the biases of the estimates of "input data" on RE in the current stock size estimates. The graphical model in Figure (4.1) depicts that, if area A contains data points of early years for which cohort sizes are only marginally underestimated as a result of underestimated catch, and area B contains data points added to the model in recent years for which cohort sizes are more

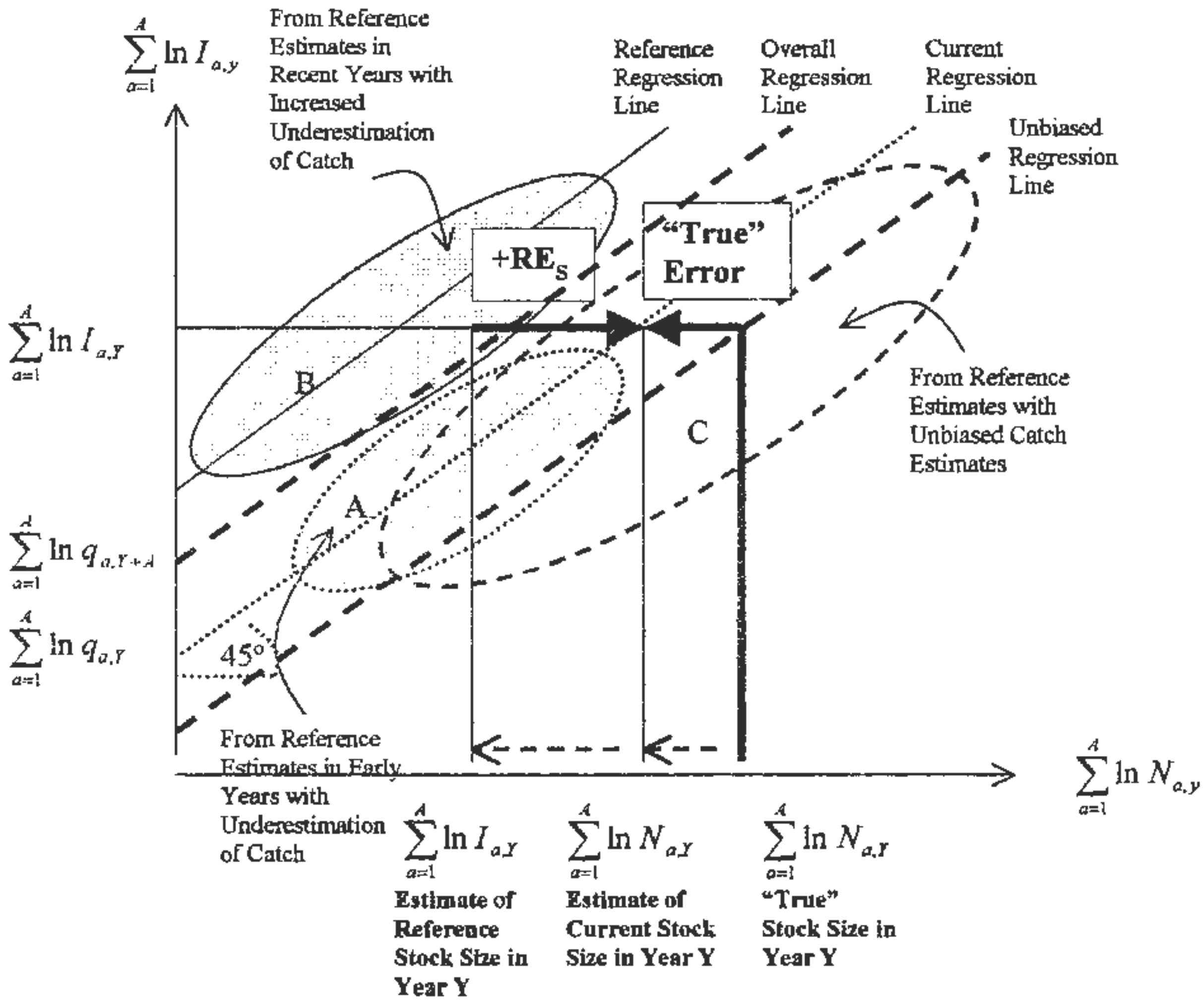


Figure 4.1
Retrospective error in comparison with "true" error of a current stock size estimate (for stock sizes in natural logarithm) in the presence of an increased trend in underestimation of catch.

underestimated than those in area A, and if the trends remains for a number of year, then the current stock sizes are more likely to be overestimated compared with the reference estimates. Yet, if the "true" cohort sizes lie in area C, the current estimate is much more likely to be closer to the "true" value than the reference estimate. The Figure (4.1) illustrates that when catch is underestimated more in recent years than that in early years, then the current estimates are more accurate than the reference estimates, and the RP that occurs under such scenario is "illusionary" rather than "real". This argument was supported by the results (seemFig. 3.5a) of a simulation study by Sinclair et al. (1991). Causal factors of the RP that occurs under different scenarios are given in Table (3.1). Those scenarios under which the RP occurs were discussed in Sinclair et al. (1991) and Mohn (1999). Fundamentally, ADAPT estimates log-catchability coefficients by simply averaging year and age specific log catchability coefficients along age classes. This averaging causes the RP when "input data" are differentially biased temporally. On the one hand, if the estimates of "input data" in early years were biased, then irrespective of how accurately the "input data" were estimated in recent years, the current stock size estimates are always biased. On the other hand, if the estimates of "input data" in early years were accurate, then irrespective of how biased the "input data" estimates were for recent years, the current stock size estimates remain near unbiased for a number of years (see Fig. 3.5a). Thus, if there is prior knowledge that catches have been underestimated more in recent years than in early years (for example, with the introduction of new management strategies), then ignoring the RP may be a wiser management strategy than trying to minimize RP. Furthermore, consistently biased estimates of "input data" do not form trends in RE. Hence, the absence of the RP in a fishery does not necessarily mean that estimates of current stock sizes are accurate. More importantly, more years of data added to the model do not essentially improve the accuracy of the estimation although they improve their reliability.

In conclusion, the RP in ADAPT could occur randomly as a result of a random-walk property of the estimation model in the presence of random variations of "realistic errors" in "input data" and non-randomly as a result of temporal trends in biases of the estimates of "input data" and model misspecifications. Because the magnitudes of RE generated by RVRED are as high as those for RE observed in the empirical assessments, and there is a high likelihood that RP may occur randomly, RP encountered in empirical studies may be a result of chance as well as a result of TTBEDMs. Thus, this study concludes that the RP in reality may not merely be a data problem, but instead may be an intrinsic problem with the structure of the model. However, had RP been caused by TTBEDMs, the resulting RP might not have always been "real". Therefore, in contrast to Mohn's (1999) claim, I suggest that failure to correct the RP may not always lead to catch-level advice that advises catches greater than the intended level. Instead, correcting the RP may sometimes lead to catch-level advice greater than the intended level, depending on whether the TTBEDMs meet the specific conditions discussed in Table (3.1). In circumstances where the RP occurs as a result of TTBEDMs, the RP may be treated by Bayesian and Fuzzy logic methods given the availability of prior knowledge of possible TTBEDMs.

The RP in stock size estimations that occurs by chance (as a result of RVRED) can be distinguished from that occurs by non-random factors (TTBEDMs) by the absence or the presence of the RP in cohort size estimations, specifically by trends in RE in recruitment estimates. If RE in cohort size estimates in yearly assessments were randomly distributed, then a RP may not be attributed to TTBEDM. According to this criterion, the RP seen in 4VsW cod (Fig. 4.2a) and 4X Haddock (Fig. 4.2b) fisheries might have occurred by chance rather than nonrandom factors, given that they did not show distinguishable RE patterns in current recruitment estimates, but instead were distributed randomly over the years. By contrast, the RP that occurred in 2J3KL cod (see Fig. 1.1a) and 4TvW haddock fisheries (see Fig. 1.1b) might have been caused by nonrandom factors, given that both data sets showed distinguishable parallel RE patterns in current recruitment estimates over the years.

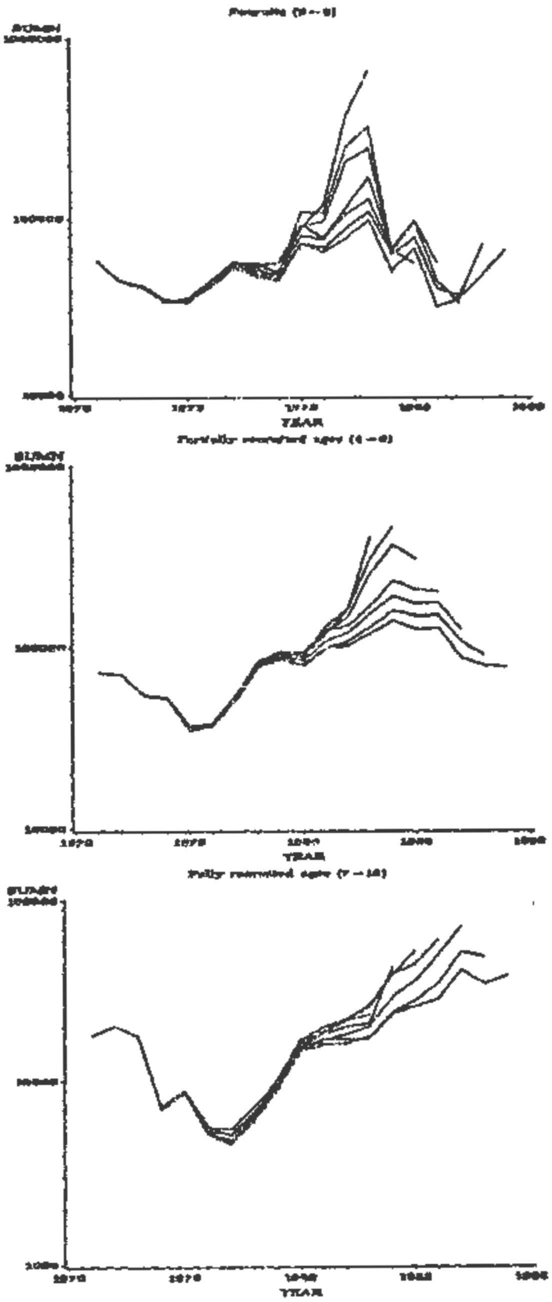


Figure 4.2a

Retrospective analysis of stocks estimated by ADAPT for 4VsW Cod [Reprinted from Sinclair et al. (1991)]

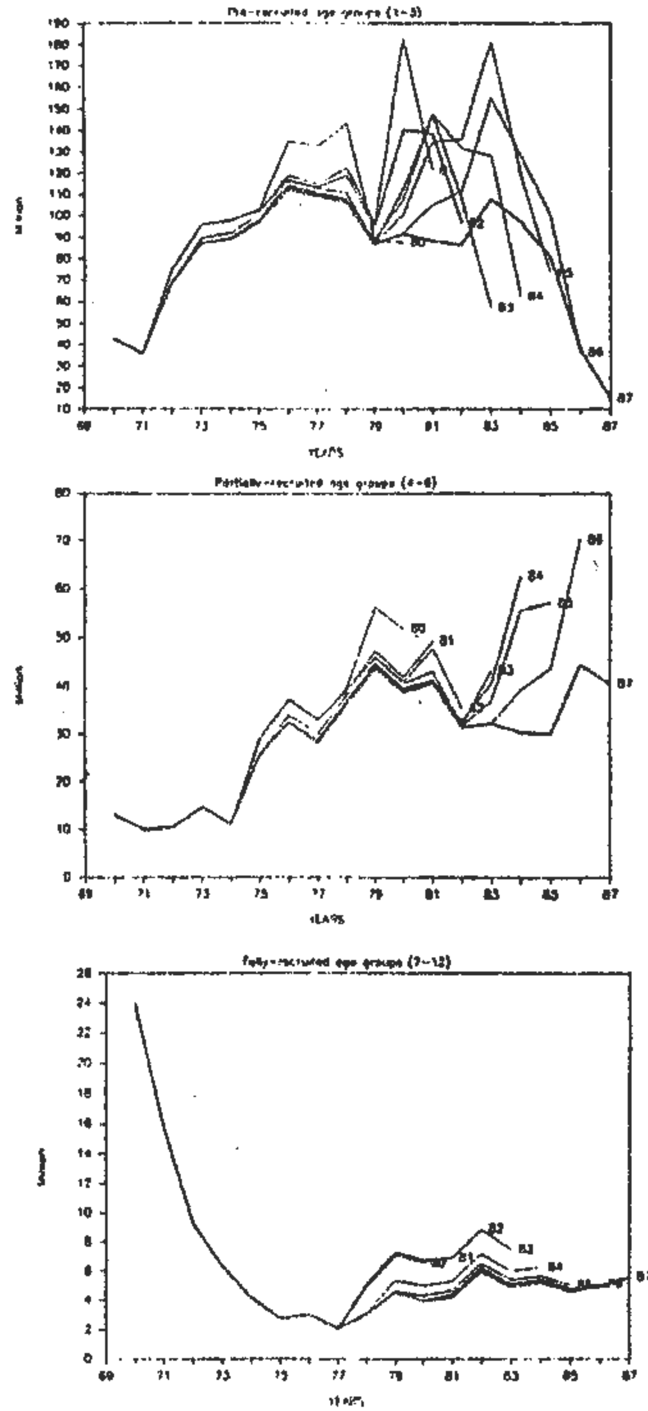


Figure 4.2b

Retrospective analysis of stocks estimated by ADAPT for 4X Haddock [Reprinted from Sinclair et al. (1991)].

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