APPLICATION OF OPTIMIZATION METHODS FOR POWER SYSTEM ECONOMIC OPERATION AND TRANSFER CAPABILITY EVALUATION

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Application of Optimization Methods for Power System Economic Operation and Transfer Capability Evaluation

By

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Abstract

Power system economic operation and transmission transfer capability continue to gain importance due to the continuous increase of power system capacities and complex interconnections. The evaluations of economic operation and transmission transfer capability have many applications in power system planning, operation and real time control. This work deals with the applications of optimization methods in these two areas of power systems. In this thesis, different optimization techniques are discussed. Optimal power flow, security constrained optimal power flow, transmission transfer capability, and transmission transfer capability during contingencies are evaluated for different power systems. Simultaneous transfer capability is also evaluated by the optimization methods. The results are presented to show the performance and application of the optimization methods. Sequential Quadratic Programming is selected and implemented as an optimization method for carrying out this research. Matlab Optimization Toolbox is used as a tool to implement the optimization methods. Case studies using different power system models presented in this these show that the optimization methods are efficient methods for power system optimization problems and satisfy the requirements of modem power systems, to meet the requirements of economic operation and transfer capability.

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Contents

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List of Abbreviations and Symbols

BFGS : Broyden, Fletcher, Goldfarb and Shanon formula

E : Voltage

EMS : Energy management system

EPRI : Electric Power Research Institute

F : Cost function

GE : Greater than type inequality constraints

GRG : Generalized reduced gradient method

GSF : Generation shift factor

H : Henssian matrix

I : Current

LE : Less than type inequality constraints

LODF : Line outage distribution factor

IP : Integer programming

LP : Linear programming

NERC : North American Electric Reliability Council

NLP : Nonlinear programming

OPF : Optimal power flow

OTDF : Outage transfer distribution factor

P : Active power flow

PTDF : Power transfer distribution factor

p.u. : Per unit

Q : Reactive power flow

QP : Quadratic programming

RTS : Reliability Test System

SCOPF : Security constrained optimal power flow

SLP : Sequential linear programming

SP : Separable programming

SQP : Sequential quadratic programming

STC : Simultaneous Transfer Capability

TTC : Transmission Transfer Capability

Y : Bus admittance matrix

θ : Impedance angle

$/hr : Dollar per hour

Chapter 1

INTRODUCTION

1.1 Motivation

Operating an interconnected power system is a challenging task. The utilities have to ensure that the security and reliability of the power systems are maintained at acceptable levels. These must be achieved at the lowest possible cost. Optimal power flow (OPF) is used to determine an optimal operating condition for power systems while considering the limitations of the equipments and other operating constraints [1]. Security constrained optimal power flow (SCOPF) has the same goals as OPF, but includes constraints under possible contingencies as well [2]. Accurate and efficient methods are required to solve OPF and SCOPF problems.

Due to environmental and other constraints, power utilities are required to transfer power form existing power sources to meet the demand of loads instead of building new generation and transmission facilities. Some power companies prefer to purchase power from other companies to reduce the overall operating cost. Transmission transfer capability (TTC), the maximum additional active power that can be transferred between two parts of power systems, must be known for power system planners and operators [3]. The ability of a transmission network to allow the possible and reliable transactions at the same time is simultaneous transfer capability (STC) [4]. STC evaluation is one of the basic works for power utilities transaction operation and planning. Accurate and efficient methods are demanded for TTC and STC calculation in power systems.

Optimization methods have been commonly used for power system problems. In additional to achieving economic operating conditions, these methods have the potential to be effectively used for power system transfer capability evaluation. This is the motivation for the research presented in this thesis. During the initial phase of the research, OPF and SCOPF problems are considered. The thesis then focuses on the application of optimization methods for transfer capability evaluation. Case studies using different power system models are presented through the thesis.

1.2 A Brief Overview of the Research in This Area

Optimal power flow (OPF) and transmission transfer capability (TTC) problems are nonlinear constrained optimization problem. A wide variety of optimization techniques have been applied for OPF problems. These techniques can be classified as:

- Linear programming (LP)
- Quadratic programming (QP)
- Newton-based methods
- Nonlinear programming (NLP)

Linear programming (LP) treats problems with linear objective functions and constraints. Since OPF problems are nonlinear optimization problems, the objective functions and constraints should be linearized to enable a LP solution. LP is a mature technique, the calculation speed is very fast and it can easily handle large power system OPF problems. Since it is a linear approach, it is only accurate near the setting points [5].

Quadratic programming (QP) treats problems with quadratic objective functions and linear constraints. The objective function and constraints of OPF problems have to be converted to quadratic problem formulation if QP is directly applied. Newton-based methods are favored for their high convergence speed. They are powerful tools for unconstrained optimization problems. However forming the Hessian matrix for Newton approach is very complex.

Nonlinear programming is designed for nonlinear optimization problems. It includes Augmented Lagrangian methods, interior point methods, successive linear programming, sequential quadratic programming, generalized reduced gradient methods and sequential convex programming, etc. Each method has its advantages and disadvantages. No method is suitable for all kinds of optimization problems. Sequential quadratic programming and interior point methods are commonly used for OPF calculation.

The two basic methods for TTC evaluation are power distribution factor methods and repeated load flow. Power distribution factors methods is favored for its fast TTC calculation. However, it is based on DC load flow and can only consider transmission line thermal limits. Voltage limits and the affect of reactive power should also be taken into account in modern power systems. It is a linear approach and does not meet the requirements of modern power systems [6]. Repeated load flow obtains TTC by repeatedly calculating the full AC load flow at each step until a certain limit is reached. It includes line limits, voltages limits and reactive power flow and it is accurate. However, calculating load flow repeatedly is not very efficient [7].

The development of OPF and TTC has used the progress in optimization techniques and advances in computer technology. Powerful and practical numerical optimization methods for power system engineering and operation can ensure the best electrical and financial performance of power system. Optimization methods have considerable applications to power systems. They have and will save hundreds of millions of dollars annually and ensure that power systems operate reliably and securely [8].

1.3 Organization of this Thesis

The thesis is organized as follows:

A discussion of optimization techniques is presented in Chapter 2. The definition and classification of optimization problems are given. The optimal conditions and optimal algorithm are introduced. The classification, theories and features of general optimization techniques are presented. It mainly described Sequential Quadric Programming (SQP), which is applied as the optimization technique used in the studies presented in this thesis.

Chapter 3 focuses on optimal power flow (OPF). Economic dispatch and security constrained optimal power flow (SCOPF) are explained. The algorithms for OPF and SCOPF are presented. The minimum cost problem of OPF and SCOPF is considered as an example to discuss the application of the optimization techniques in OPF and SCOPF evaluation.

Transmission transfer capability (TTC) and the requirement for TTC evaluation are introduced in Chapter 4. Two methods for TTC calculation: distribution factors and repeated load flow are explained. The advantages and disadvantages of these two methods are discussed. The limitations of these two methods for present power systems require a new technique for TTC evaluation.

Chapter 5 discusses the application of the optimization technique in TTC evaluation and is devoted to the calculation and simulation results of the optimization technique. TTC from single bus to single bus and from single area to single area studies are presented. TTC evaluation during contingencies is considered. The definition of simultaneous transfer capability (STC) is introduced in this chapter as well. The STC calculation algorithm and the effects of STC to other power transfer paths are presented. The optimization technique is also applied in STC evaluation.

Chapter 6 gives the conclusions of the thesis, highlights the contribution of this research and suggestions for future research are given.

Chapter 2

OPTIMIZATION TECHNIQUES

2.1 Introduction

Optimization problems arise in many areas of life and work. In architecture field, the building is expected to be built with minimum cost within limited time and satisfy the customer's requirements. In industry, device drives are expected to be operated to yield maximum efficiency with minimum losses. In business field, the markets are expected to run economically and optimally. Optimization problems have become more and more challenging. However, with the capability of the present generation computer, many optimization techniques can be efficiently implemented.

This chapter provides an overview of the optimization problems and discusses many of the available methods to solve optimization problems. The research presented in the later part of the research using an optimization package, which is based on Sequential Quadratic Programming method. This method is discussed in detail. A brief comparison of the various optimization techniques is also presented.

2.2 Definition of the Optimization Problem

The purpose of an optimization problem is to adjust (or control) some variables in order to minimize or maximize a certain objective function under certain conditions. There are three common features in these problems. First, all these problems have a goal that should be achieved. Second, there are some conditions that must be satisfied. Finally, there are some variables that can meet the goals and requirements if they are chosen properly. The three steps to form an optimization problem are: selecting optimization variables, choosing an objective function and identifying a set of constraints. The objective function is the general goal of the problem. It can be a single objective function or multiple objective functions. Constraints represent the conditions that should be satisfied [9].

Most of the optimization problems can be expressed in the following form:

Find a vector of optimization variables, $x=(x_1,x_2,\ldots,x_n)^T$ in order to minimize an objective function, f(x)

Subject to:

$g_i(x) \leq 0$	i = 1, 2,...,m	Inequality constraints (Less than type, LE)
$h_i(x) = 0$	i = 1, 2,...,p	Equality constraints (EQ)
$x_{iL} \leq x_i \leq x_{iU}$	i = 1, 2,...,n	Bounds on optimization variables

A maximization problem can be converted to a minimization problem by multiplying a negative sign to the objective function. By the same manner, the greater than type inequality constraints (GE) can be converted to less than type inequality constraints (LE). Hence, minimization problems and less than type inequality constraints can respectively represent optimization problems and inequality constraints. They are the common forms used for the studying of optimization techniques.

Equality constraints represent the relationships between the optimization variables.

For a general optimization problem, the number of equality constraints p must be less than the number of optimization variables n. If p equals n, then the problem can be solved by the equality constraints, and it is not an optimization problem. On the other hand, if p is larger than n, some of the constraints must be dependent on others and they are redundant. Since the inequality constraints do not represent a specific relationship between the optimization variables, there is no limit on the number of inequality constraints. The bound constraints are in fact inequality constraints. Some optimization techniques take advantage of their simple forms, and in some cases they are separated from other constraints.

2.3 Classification of the Optimization Problem

Based on the linear or nonlinear characteristics of the objective function and constraints, optimization problems can be classified into linear programming (LP), nonlinear programming (NLP) and quadratic programming (QP) problems.

The standard linear programming problem can be stated as:

$$\begin{aligned} &\text{Minimize:} \quad c^T x \\ &\text{Subject to:} \quad \begin{pmatrix} Ax = b \\ x \geq 0 \end{pmatrix} \end{aligned} \tag{2.1}$$

Where: c is a vector of objective coefficients

A is a matrix of constraint coefficients

b is a vector of right-hand sides of constraints

If a quadratic term $\frac{1}{2}x^T Qx$ is added in the objective function, then the linear problem changes to standard Quadratic Problem:

$$\text{Minimize: } \quad c^T x + \frac{1}{2} x^T Q x$$
$$\text{Subject to: } \quad \begin{pmatrix} Ax = b \\ x \geq 0 \end{pmatrix} \tag{2.2}$$

Where: c is a vector of objective coefficients

A is a matrix of constraint coefficients

b is a vector of right-hand sides of constraints

Q is a symmetric matrix

From above equations, it is seen that similar to linear programming, the constraints of standard quadratic programming are linear too. The only difference between standard linear and quadratic programming is that there is a quadratic term in the objective function of quadratic programming problem. Although the difference between the formulations of these two programming looks small, the methods used to solve these two programming problems are completely different. Many nonlinear programming studies are based on linear programming and quadratic programming. Nonlinear programming problem becomes challenging when the size of problem increases.

Based on whether constraints are present or not, optimization problem can be classified into unconstrained optimization problem and constrained optimization problem. Unconstrained problems only need to find the minimum of the objective function. Constrained problems have to find not only the minimum of the objective function, but the solution must also satisfy the problem's constraints.

2.4 Local and Global Optimization

Global minimum is defined as for all $x \in S$ when $x^* \in S$ if $f(x^*) \leq f(x)$, and then x^* is global minimum. Local minimum is defined as for all feasible x in neighborhood of x^* when $x^* \in S$ and if $f(x^*) \le f(x)$, then x^* is local minimum. Figure 2.1 illustrates the points of local and global minimum. Finding or even verifying, global minimum is difficult. Most optimization methods are designed to find local minimum, which may or may not be the global minimum.

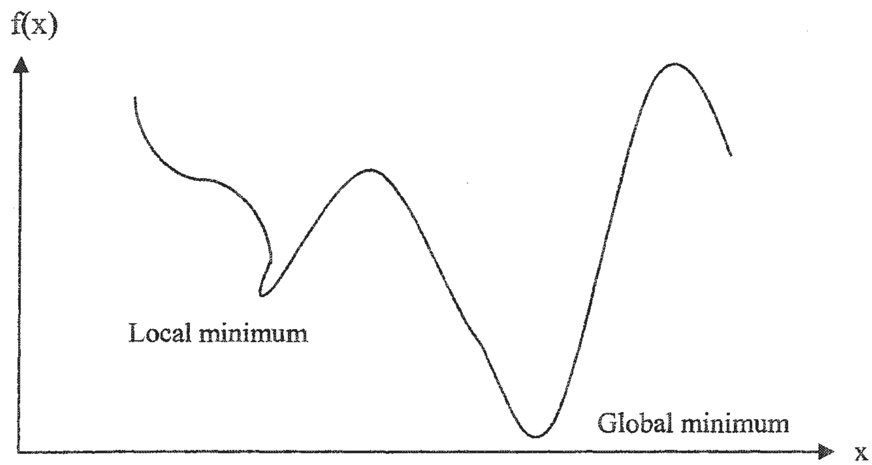

Fig.2.1 Illustration of local minimum and global minimum

2.5 Simple Criteria to Find the Minimum and Maximum

For a given function, to find the minimum and/or maximum, and to distinguish the minimum and maximum are very important for optimization problem study. The minimum and/or maximum can be detected by the gradients of the function. Whether the point located is the minimum or maximum can be judged by the second derivative of the function.

2.5.1 First Derivative Test

Based on calculus, it is known that the gradients at the limit points of a function equal to zero. Figure 2.2 shows that in the left neighborhood of the local minimum the first derivative of the function is negative and in the right neighborhood of the local minimum the first derivative of the function is positive. Figure 2.3 shows that in the left neighborhood of the local maximum the first derivative of the function is positive and in the right neighborhood of the local maximum the first derivative of the function is negative. Second derivative test is required to specifically know whether the optimum point is at the maximum or the minimum of the function.

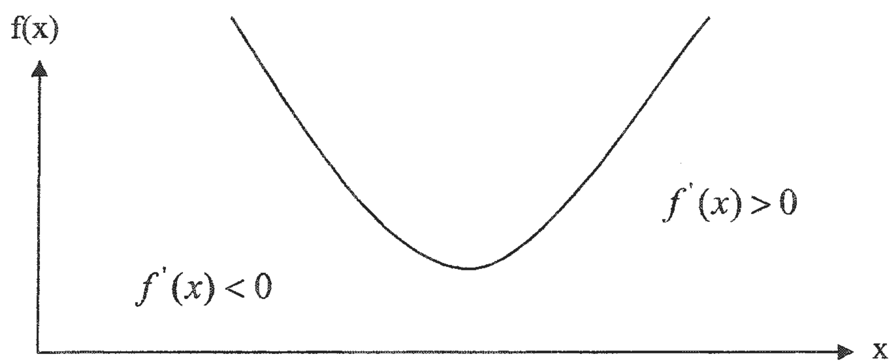

Fig.2.2 Local minimum

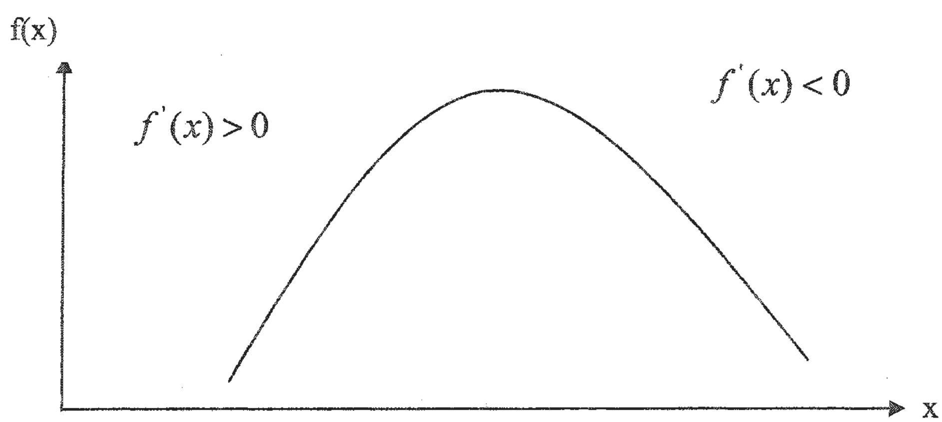

Fig.2.3 Local maximum

2.5.2 Second Derivative Test

The value of the square sub-matrix of the matrix is the principal minor. For example: the first principal minor A_1 is the first diagonal element of the matrix **A**; the second principal minor A_2 is the value of the $2 \otimes 2$ matrix that consisted from the first two rows and columns. The sign of a $n \otimes n$ matrix A can be determined by its principal minors Ai, i = 1, …, n:

1. Positive definite when $A_i > 0 \quad \mathrm{i} = 1, \ldots, \mathrm{n}$
2. Positive semidefinite when $A_i \geq 0 \quad \mathrm{i} = 1, \ldots, \mathrm{n}$
3. Negative definite when $\left\{ \begin{array}{ll} A_i < 0 & \mathrm{i} = 1, 3, 5, \ldots \text{(odd indices)} \\ A_i > 0 & \mathrm{i} = 2, 4, 6, \ldots \text{(even indices)} \end{array} \right.$
4. Negative semidefinite when $\left\{ \begin{array}{ll} A_i \leq 0 & \mathrm{i} = 1, 3, 5, \ldots \text{(odd indices)} \\ A_i \geq 0 & \mathrm{i} = 2, 4, 6, \ldots \text{(even indices)} \end{array} \right.$
5. Indefinite if none of the above cases.

Hessian matrix H is the second partial derivative of the function $\nabla^2 f(x)$. It can be stated as:

$$\nabla^2 f(x) = \left(\frac{\partial^2}{\partial x_i \partial x_j} f(x) \right) \qquad \text{i,j=1,2,…,n} \qquad (2.3)$$

Hessian matrix $\nabla^2 f(x)$ can be used to detect the optimum points by:

1. x_k is at least a local minimum if $\nabla^2 f(x)$ is positive definite
2. x_k is at least a local maximum if $\nabla^2 f(x)$ is negative definite
3. x_k is inflection point if $\nabla^2 f(x)$ is indefinite

4. x_k cannot be decided only by Hessian if $\nabla^2 f(x)$ is semidefinite

2.6 Optimality Condition

To analyze the optimization problem, it is convenient to add a positive slack variable so as to convert the inequality constraints to equality constraints and combine the objective function and constraints into one function. The linear combination of objective and constraint function for deriving optimality conditions and optimization algorithms is known as Lagrangian function. The Lagrangian function is formulated as:

$$L(x,u,v,s) = f(x) + \sum_{i=1}^{m} u_i\left[g_i(x) + s_i^2\right] + \sum_{i=1}^{p} v_i h_i(x) \tag{2.4}$$

Where: u is a vector of the Lagrange multiplier for inequality constraints

v is a vector of the Lagrange multiplier for equality constraints

s is a vector of slack variable

Based on the Lagrangian function, the complete set of necessary conditions for optimizing a constrained minimization problem, known as Karush-kuhn-Tucker (KT) conditions is given as:

1. Gradient conditions: $\nabla f(x^*) + \sum_{i=1}^{m} u_i \nabla g_i(x^*) + \sum_{i=1}^{p} v_i \nabla h_i(x^*) = 0$ (2.5)

2. Regularity condition: x^* must be one of the solutions of the gradient condition.

3. Constraints: $g_i(x) + s_i^2 = 0 \quad i = 1\ldots,m$; $h_i(x) = 0 \quad i = 1\ldots,p$ (2.6)

4. Complementary slackness conditions: $u_i s_i = 0$, i = 1, 2…m (2.7)

5. Feasibility conditions: $s_i^2 \geq 0$, i = 1, 2…m (2.8)

6. Sign of inequality multipliers: $u_i \geq 0$, i = 1, 2...m (2.9)

KT condition can be used to solve optimization problems. The equality conditions are used to obtain the results. The inequality conditions are used to verify the results. The optimization methods aim to find the solution vector x^*, which satisfies the above conditions.

If the number of variables in the objective function f(x) is n and the number of equality constraints h(x) and inequality constraints g(x) are p and m respectively, then the total number of unknown variables in Lagrangian function (2.4) including the Lagrange multipliers for inequality and equality constraints and slack variables is $n+2\otimes m+P$. From gradient conditions (2.5), n equality equations will be obtained. From equality constraints conditions (2.6), $m+p$ equality equations will be obtained. It is obvious that either u_i or s_i is zero, so m equality equations will be obtained from complementary slackness conditions (2.7). Total the conditions 2, 3 and 4 give $n+2\otimes m+P$ equality equations. Then the $n+2\otimes m+P$ unknown variables can be solved by the $n+2\otimes m+P$ equality equations. The results under different assumptions (either u_i or s_i is zero) are called KT points. All the conditions must be checked out by conditions 5 and 6 to determine whether the KT points are feasible or not. The function value f(x) should also be calculated to determine the value of the objective function.

2.7 Optimization Algorithm

The basic idea in solving an optimization problem is to start from an initial point to approach the solution iteratively. Every iteration of a numerical optimization method can be expressed as the combination of the descent direction d^k and the step length α_k. It can be given as [10]:

$$x^{k+1} = x^k + \alpha_k d^k \qquad k=0, 1,... \tag{2.10}$$

x^0 is the initial point and can be chosen arbitrarily. At each iteration, d^k and α_k should be chosen to satisfy $f(x^{k+1})<f(x^k)$.

2.7.1 Unconstrained Optimization Techniques

Combining step length calculation techniques and descent direction strategies can solve Unconstrainted Optimization problems. A suitable step length should be computed along the descent direction to achieve a "sufficient" decrease to the solution [11]. Search methods calculate the step length along the search direction. They include analytical line search, equal interval search, section search, golden section search, quadratic interpolation method and approximate line search based on Armijo's rule [9].

The over all performance of unconstrained optimization techniques are largely determined by the convergence direction [12]. The techniques to calculate descent direction are:

- Steepest descent: $d^k = -\nabla f(x^k)$ (2.11)
- Conjugate gradient: $d^k = -\nabla f(x^k) + \beta d^{k-1}$ (2.12)
- Modified Newton: $$f(x^{k+1}) \approx f(x^k) + \nabla f(x^k)^T d^k + \frac{1}{2}(d^k)^T H(x^k) d^k$$ $$d^k = -[H(x^k)]^{-1} \nabla f(x^k)$$ (2.13)
- Quasi-Newton: $d^k = -Q^k \nabla f(x^k)$ (2.14)

$\nabla f(x)$ is the first partial derivative of the function, which is known as Jacobian [13].

$$\nabla f(x) = \frac{\partial}{\partial x_i} f(x) \tag{2.15}$$

$H(x)$ is the Hessian matrix of the function

Q is an updated matrix which approximates the inverse of Hessian matrix

β is a scalar that is determined according to the type of conjugate gradient

To solve unconstrained optimization problem, first start from the given initial values, find the descent direction of the function, then calculate the step length to approach the direction and update the start values for the next iteration from equation (2.10). This process is repeated until the differences between the current values and the solutions are within a specified convergence tolerance. At present, the most powerful technique for unconstrained optimization problem is based on Quasi-Newton update (2.14).

2.8 General Nonlinear Programming Methods

Corresponding to the classification of optimization problems, optimization techniques can be classified as linear programming, quadratic programming and nonlinear programming. Linear programming is considered as a reliable and robust technique for a wide range of linear optimization problems [1]. There are many efficient and robust algorithms for solving linear problems. Simplex method, interior point methods and successive linear programming are common methods for such problems [8]. The main methods for quadratic programming are Primal Affine Scaling method (PAS), Primal Dual method and Active Set method [9].

Nonlinear programming is an extension of linear programming, when linear objective function or constraints are replaced by nonlinear ones. For this reason, many optimization techniques for nonlinear programming problems are based on linear programming techniques. Constrained problems can be solved using unconstrained techniques by forming Lagrangian function. The algorithms for linear programming and quadratic programming problem provide some possibilities in solving nonlinear programming problems. Some nonlinear programming problems can be solved by the methods for linear programming problem by linearizing both the objective function and constraints. Some nonlinear programming problems can also be solved by generating quadratic programming intermediate problems.

2.8.1 Optimization Techniques for Nonlinear Programming Problems

The methods for nonlinear programming problem can be classified by direct and indirect methods. The basic idea for the indirect method is to convert the constrained problem to an unconstrained optimization problem by forming Lagrangian function and solve it by unconstrained optimization techniques. A penalty function maybe added to the Lagrangian function to force the constraints to be satisfied if one or more constraints are violated. Augmented Lagrangian Penalty method is an example of such methods. Using Taylor series, the linearization around the current points to approximate the objective and constraint function can be achieved.

Most direct methods build a sub-problem using linear approach or quadratic approach in order to approximate the objective function and constraint functions around the current points. The convergence direction is obtained by solving this sub optimization problem. The next point is reached by combining the direction and the step length generated by search methods. Most direct methods for solving nonlinear problems include some linearization. Sequential Linear Programming is the representative of linearization methods and Sequential Quadratic Programming (SQP) is one of the most popular direct methods.

2.8.2 Basic Sequential Quadratic Programming (Basic SQP)

Based on linearization methods, which linearize the objective function and constraints by Taylor series to build a sub-problem, Basic Sequential Quadratic Programming adds a quadratic term $\frac{1}{2}d^T d$ in the linearized objective function and keeps the constraint linear. This then forms a standard quadratic programming sub-problem.

The standard Basic Sequential Quadratic Programming sub-problem can be stated as:

$$\begin{aligned} &\text{Minimize} \quad f(x^k)+\nabla f(x^k)^T d+\frac{1}{2}d^T d \\ &\text{Subject to} \quad hi(x^k)+\nabla hi(x^k)^T d=0 \qquad i=1,2,\ldots p \\ &\qquad\qquad\quad gi(x^k)+\nabla hi(x^k)^T d\le 0 \qquad i=1,2,\ldots m \end{aligned} \tag{2.16}$$

The assumptions of Basic Sequential Quadratic Programming are that the problems are not too big, the function and gradients can be evaluated with sufficiently high precision and the problems are smooth and well scaled. The advantages of this method are that the quadratic programming sub-problem is convex and the convergence speed in the neighborhood of a solution is superior. But the quadratic programming sub-problem can be infeasible even though the original problem is feasible.

2.8.3 Refined Sequential Quadratic Programming (SQP)

Refined Sequential Quadratic Programming uses a more general quadratic term $\frac{1}{2}d^T Hd$ instead of $\frac{1}{2}d^T d$ to improve the direction generated by quadratic programming sub-problem. For safeguard, a factor r is added to overcome the possibility of the infeasibility of quadratic programming sub-problem. The complete form of Refined Sequential Quadratic Programming can be stated as:

$$\begin{aligned} &\text{Minimize} \quad f(x^k)+\nabla f(x^k)^T d+\frac{1}{2}d^T Hd \\ &\text{Subject to} \quad \bar{r}h_i(x^k)+\nabla h_i(x^k)^T d=0 \qquad i=1,2,\ldots p \\ &\qquad\qquad\quad r_i g_i(x^k)+\nabla h_i(x^k)^T d=<0 \qquad i=1,2,\ldots m \end{aligned} \tag{2.17}$$

$$\text{where} \quad H=\nabla^2 f+\sum_{i=1}^{m}u_i\nabla^2 g_i+\sum_{i=1}^{P}v_i\nabla^2 h_i \tag{2.18}$$

$\bar{r}=0.9$ and $r_i=0.9$ if $g_i(x)>0$. Otherwise it is 1

u_i and v_i are Lagrange multiplier

The overall approach to solve optimization problem by Basic Sequential Quadratic Programming and Refined Sequential Quadratic Programming are the same.

However, obtaining Hessian matrix (H) of the Lagrangian function requires second derivatives of the function and this is difficult for large system. Since H is not always a positive definite matrix, this will make the problem infeasible. To overcome these disadvantages, many methods are used to update Hessian matrix, H instead of second derivatives. Broyden, Fletcher, Goldfarb and Shanon (BFGS) formula is one of the methods used to approximate the Hessian matrix [10]. Quasi-Newton method also uses BFGS formula to approximate the inverse of the Hessian matrix in unconstrained optimization problems.

There are many other approaches for nonlinear constrained problem such as

Generalized Reduced Gradient Method (GRG) and Sequential Convex Programming.

The research focused later in this thesis use optimization toolbox of Matlab [14]. This toolbox uses Sequential Quadratic Programming and linear search for constrained nonlinear problems. A simple example of basic sequential quadratic programming is illustrated below.

2.8.4 An Example of Basic Sequential Quadratic Programming

Minimize $f = \frac{1}{2}x_1 + x_1 x_2$

Subject to: $\begin{pmatrix} x_1 x_2 - 10 = 0 \\ -2x_1^2 + x_2 / 3 \le 0 \end{pmatrix}$

Initial guess: (1,9)

$$\nabla f = \left(\frac{1}{2} + x_2 \quad x_1\right)$$

First derivative: $\nabla h = (x_2 \quad x_1)$

$$\nabla g = (-4x_1 \quad \frac{1}{3})$$

------------Iteration 1--

Current points (1,9)

Function values: $\begin{pmatrix} f = 9.5 \\ h = -1 \\ g = 1 \end{pmatrix}$

$$\nabla f = (9.5 \quad 1)$$

Gradients: $\nabla h = (9 \quad 1)$

$$\nabla g = (-4 \quad 0.3333)$$

Quadratic sub-problem

Minimize $9.5d_1 + d_2 + \frac{1}{2}(d_1^2 + d_2^2)$

Subject to: $\begin{pmatrix} -1 + 9d_1 + d_2 = 0 \\ 1 - 4d_1 + 0.3333d_2 \le 0 \end{pmatrix}$

Direction d = (0.1905, -0.7143)

Using linear search to get the step length α = 1

New points (1.1905, 8.2857)

-----------Iteration 2---

Current points: (1.1905, 8.2857)

Function values: $\begin{pmatrix} f = 10.4594 \\ h = -0.1359 \\ g = -0.0727 \end{pmatrix}$

$$\nabla f = (8.7857 \;\; 1.1905)$$
Gradients: $\nabla h = (8.2857 \;\; 1.1905)$
$$\nabla g = (-4.762 \;\; 0.3333)$$

Quadratic sub-problem:

Minimize $8.7857d_1 + 1.1905d_2 + \frac{1}{2}(d_1^2 + d_2^2)$

Subject to: $\begin{pmatrix} -0.1359 + 8.2857d_1 + 1.1905d_2 = 0 \\ -0.0727 - 4.762d_1 + 0.3333d_2 \le 0 \end{pmatrix}$

Direction d = (0.006, 0.0727)

Using linear search to get the step length α = 1

New points (1.1965, 8.3584)

-----------Iteration 3---

Current points (1.1965, 8.3584)

Function values: $\begin{pmatrix} f = 10.9587 \\ h = 4.588e-4 \\ g = -0.0769 \end{pmatrix}$

$$\nabla f = (8.8584\ \ 1.1965)$$
Gradients: $\nabla h = (8.3584\ \ 1.1965)$
$$\nabla g = (-4.786\ \ 0.3333)$$

Quadratic sub-problem

Minimize $8.8584d_1 + 1.1965d_2 + \frac{1}{2}(d_1^2 + d_2^2)$

Subject to: $\begin{pmatrix} 8.3584d_1 + 1.1965d_2 = 0 \\ -0.0769 - 4.786d_1 + 0.3333d_2 \le 0 \end{pmatrix}$

Direction d = (-0.01, 0.701)

Using linear search to get the step length α = 1

New points (1.1865, 8.4285)

------------Iteration 4--

Current points (1.1865, 8.4285)

Function values: $\begin{pmatrix} f = 10.9534 \\ h = 1.2141e-4 \\ g = -0.0059 \end{pmatrix}$

$$\nabla f = (8.9285\ \ 1.1865)$$
Gradients: $\nabla h = (8.4285\ \ 1.1865)$
$$\nabla g = (-4.7458\ \ 0.3333)$$

Quadratic sub-problem

Minimize $8.9285d_1 + 1.1865d_2 + \frac{1}{2}(d_1^2 + d_2^2)$

Subject to: $\begin{pmatrix} 8.4285d_1 + 1.1865d_2 = 0 \\ -0.0059 - 4.7458d_1 + 0.3333d_2 \le 0 \end{pmatrix}$

Direction d = (-0.0008, 0.0059)

Using linear search to get the step length $\alpha = 1$

New points (1.1856311, 8.434326653)

Function values: $\begin{pmatrix} f = 10.9528 \\ h = 9.88e-8 \\ g = 0 \end{pmatrix}$

The solution (1.1856, 8.4343) is found successfully after 4 iterations; the maximum violation is 9.88e-8, which it is less than the tolerance. The minimum of objective function value is 10.5928.

The solution is illustrated in figure 2.4. The shallow part is the feasible area for inequality constraints. The minimum point is on the line of equality constraint and inside the feasible area of inequality constraints. Hence it is a feasible solution.

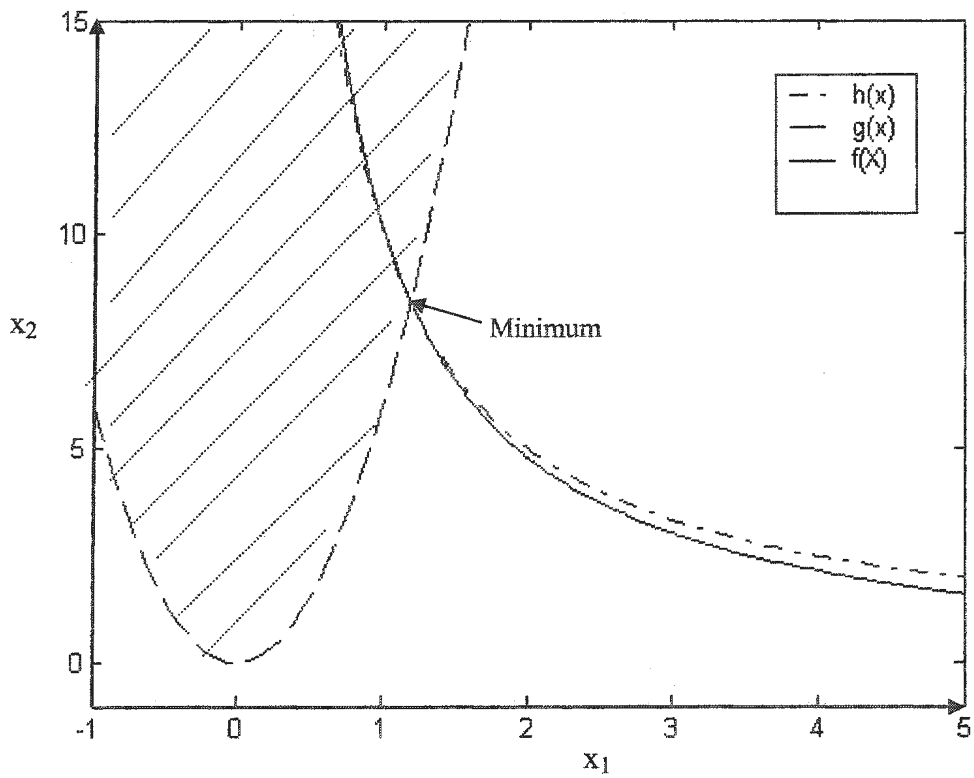

Fig.2.4 Graphical solution

2.9 Large Scale Optimization

An optimization problem is denoted to be large, if it cannot be solved by any standard method because of storage requirements, round-off errors, or excessive computing times. However, large-scale problems often have some special structures, such as sparse Jacobian or Hessian matrix. Exploring these properties to make problems simple is an important and challenging part for solving large-scale optimization problems.

Basically, all the optimization techniques lead to the solution of large linear system. For large nonlinear system, the efficiency of nonlinear techniques depends on how the problem is formulated and how the sub-problem is defined. Sequential Linear Programming (SLP), Interior Point, Sequential Quadratic Programming (SQP) and Sequential Convex Programming are good for large-scale optimization problem. Dynamic programming, Lagrangian relaxation, decomposition and normalization are the common optimization techniques that are used to simplify the large-scale problem or improve the convergence speed of the problem [15].

2.10 Conclusions

For nonlinear constrained optimization problems, the main methods are linear programming (LP) based methods and non-linear programming (NLP) based methods. Linear programming based methods linearize the objective function and constraints. They are robust techniques for some nonlinear optimization problems, but it is an approximate approach. Sequential Quadratic Programming (SQP) converts the objective function and constraints to the standard quadratic programming sub-problem. The direction to the optimal points is obtained by solving this Quadratic sub-problem. Because it is a quadratic approach, SQP is more accurate than linear programming method. Newton method is a very powerful and fast computational method for unconstrainted optimization problem. But for constrainted optimization problems, it needs to form the Lagrangian function and calculate the Hessian and gradient matrices of the Lagrangian function. To build the Hessian or modified Hessian matrix is time-consuming and complicated. Augmented Lagrangian Penalty function method is generally used, but the condition number of the Hessian matrix of the function tends to be infinity when the penalty parameter μ becomes too large.

Sequential Quadratic Programming (SQP) is considered the best direct method for solving general nonlinear constrained problem and it is a "state of art". They are extensions of linear programming and standard algorithms to solve general smooth nonlinear optimization problems with constraints.

This chapter has presented an overview of different optimization techniques. A simple numerical example is used to illustrate the performance of basic sequential quadratic programming. In the later part of this thesis, instead of developing a program to implement an optimization algorithm, the features of Matlab Optimization toolbox are effectively applied for different power system problems.

Chapter 3

OPTIMAL POWER FLOW

3.1 Introduction

Some of the main goals of power system operation are economy, security and quality of power supply. It is the responsibility of the utility to ensure that the customers receive continuous power supply at constant frequency and at nearly constant voltage. Economic operation of power system is related to achieving this goal with the lowest possible cost. A secure power system is one where the power system continuous to operate even after some contingencies (like generator or transmission line outages). Energy Management Systems (EMS) which monitor and control the operation of power system include software tools that implement Optimal Power Flow (OPF) and Security Constrained Optimal Power Flow (SCOPF). This chapter presents the formulations of OPF and SCOPF problems. As discussed in the previous chapter, Matlab Optimization toolbox is used to solve these problems. Before introducing the OPF problems, the well-known Economic Dispatch problem is briefly discussed. In all these problems, the goal is limited to minimizing the total fuel cost of generation in a power system having thermal generating units. Case studies using a sample 7-bus power system and the IEEE 30-bus power system are presented in detail. In the final part of this chapter, a summary of the OPF and SCOPF problems is given.

3.2 Economic Dispatch Problem

Economic Dispatch (ED) problem is one of the earliest power system optimization problems. The objective of basic economic dispatch is to minimize the total fuel cost of generators in a power system, while satisfying the power balance and limits of generation units [16].

The objective function is the total fuel cost of generators, and can be stated as:

$$\text{Minimize } F = \sum_{i=1}^{n} F_i(P_i) \tag{3.1}$$

Subject to:

The inequality constraints are the generation limits. They can be expressed as:

$$P_{i\min} \le P_i \le P_{i\max} \qquad i=1, 2\ldots, n \tag{3.2}$$

If the system is considered as a lossless system, in which the sum of the power generated equals the sum of the received load and the losses are neglected, then the equality constraints can be expressed as:

$$\sum_{i=1}^{n} P_i = P_D \tag{3.3}$$

If the system is not considered as a lossless system, the equality constraints can be expressed as:

$$\sum_{i=1}^{n} P_i = P_D + P_L \tag{3.4}$$

Where: n is number of units in the system

F_i is fuel cost function of generating unit 'i'

P_i is generation of unit 'i'

P_D is total load demand

P_L is total loss in the system

The solution of the ED problem can be determined easily. For example, in a system, if the losses are ignored, at the minimum cost the following equations are satisfied [17]:

$$\lambda = \frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}} = ... = \frac{dF_n}{dP_{Gn}} \tag{3.5}$$

On the other hand, if the losses are considered, at the minimum cost the following equations are satisfied:

$$\lambda = L_1 \frac{dF_1}{dP_{G1}} = L_2 \frac{dF_2}{dP_{G2}} = ... = L_n \frac{dF_n}{dP_{Gn}} \tag{3.6}$$

Where: L_i is penalty factor of plant i, which is stated as:

$$L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}} \tag{3.7}$$

P_L is total loss in the system

n is the number of generation units

3.3 Optimal Power Flow

The concept of optimal power flow (OPF) was introduced as an extension of conventional economic dispatch in the beginning of 1960s. OPF problem was defined to determine the optimal setting for control variables in a power network subject to various constraints [1]. With the increase of power system size, interconnections, and environment considerations, OPF plays an important role in today's Energy Management

Systems.

OPF problems aim to schedule the controls of power systems to optimize a certain objective function while satisfying a set of physical and operational constraints imposed by equipment limitations and security requirements [18]. Since the relationships between power system parameters are nonlinear, and OPF problems should meet the requirements of systems and equipments limitations, OPF problems are constrained nonlinear optimization problems [9]. They aim at optimizing a certain objective function, which depends on a set of state variables by adjusting a set of control variables, while satisfying a set of physical and operational constraints imposed by equipment limitations and security requirements [19].

In mathematical terms OPF problems can be reduced to a set of nonlinear equations, which can be stated as [20]:

$$\text{minimize} \qquad F(x,u) \tag{3.8}$$

Subject to equality constraints:

$$g(x, u) = 0 \tag{3.9}$$

and inequality constraints:

$$h(x,u) \le 0 \tag{3.10}$$

With the Variable limitations:

$$u_{min} \le u \le u_{max} \tag{3.11}$$

$$x_{min} \le x \le x_{max} \tag{3.12}$$

F is a scalar function that represents the power system's optimization goal.

g are equality constraints.

h is a vector of functions corresponding to the system physical and operating limits.

x is the vector of dependent variables.

u is the vector of control variables.

Some of the general goals of OPF problems are active power cost minimization, active power loss minimization, control shift minimization and minimization of the number of power system control variables.

The dependent variables present the states of the system and can be adjusted by control variables. They contain bus voltage magnitudes and phase angles, reactive output of generators, and fixed parameters such as the reference bus angle, non-controlled active and reactive loads, fixed bus voltages, line parameters, and other state variables. Control variables can be influenced by the operator by using engineering rules to offer more controls for handling violation, controlling effectiveness, improving stability and optimal performance. They consist of real and reactive power generation, phase shifter angles, net interchange, active load and reactive load shedding, DC transmission line flows, controllable voltage settings, LTC transformer tap settings and line switching. State variables describe the states of the system that usually come as a part of the solution of the OPF.

The equality constraints contain power flow equations, which specify the generation and load balance for a particular operating condition. The inequality constraints are more complex. They contain limits on control variables and operating limits. Limits on control variables include generator output limits, transformer tap limits, and shunt capacitor ranges. Operating limits include line and transformer flow limits, active power and reactive power interchange limits, active power and reactive power reserve margin (fixed and dynamic) limits, voltage and angle limits. Compared with ED, OPF considers additional constraints like generator reactive power limits, voltage limits and transmission line thermal limits.

3.4 Optimal Power Flow Calculation Algorithm

OPF algorithms are designed to find an AC power flow solution, which optimizes a performance function, such as fuel cost or network losses, while at the same time keep the system operating within the equipment limits and system requirements. For example, when system losses are minimized, an optimal schedule of generators power output, transformer tap settings and controllable voltage settings are determined which produce the minimum operating loss while at the same time avoiding any violation [12].

OPF requires solving a set of nonlinear equations describing optimal operation of a power system. The difficulty of solving OPF is dependent on system size and the number of control variables [21]. Different optimization methods have been used to solve the OPF problem. Mainly these are linear programming (LP) based methods, nonlinear programming (NLP) based methods, integer programming (IP) based methods, and separable programming (SP) methods. Most constraints are functions of only a small number of variables and hence the Jacobian and Hessian matrices are sparse. There is a large benefit in considering the problems that involve sparse matrices. Hence the optimization methods, which contain sparse matrix techniques, can be efficiently used [22].

Newton based method is very powerful with fast convergence for OPF, but to build Hessian matrix is very time consuming and it may generate problems for constrained problem. Linear programming method can easily handle objective function and constraints by linearization. At present, extending sequential quadratic programming technique or combining sequential quadratic techniques with other methods, are researched and applied for power system optimization.

3.5 OPF Minimum Cost Problem

Different objectives have been considered in OPF problems. However, this chapter focuses on minimizing the total fuel cost of the generators.

3.5.1 Objective Function

The objective function is the minimum cost of generating power. This is considered as the total fuel cost. The cost function typically uses cubic cost model, which is expressed as:

$$F = \sum_{i=1}^{n} (\gamma_i + \alpha_i P_{Gi} + \beta_i P_{Gi}^{\ 2}) \tag{3.13}$$

Where: F is the total fuel cost, dollars per hour ($/hr)

P_{Gi} is the output active power of Unit i, megawatts (MW)

α_i, β_i, γ_i are the cost coefficients of generators.

3.5.2 Variables

The nodal voltages (magnitude and phase) are the main state variables. The outputs of generating units, generator voltages, transformer tap setting, etc, are the main control variables.

3.5.3 Equality Constraints

The equality constraints contain the basic power flow equations.

$$\sum_{i=1}^{m} P_i = 0 \tag{3.14}$$

$$\sum_{i=1}^{m} Q_i = 0 \tag{3.15}$$

P_i and Q_i is the power flow at bus i

m is the number of buses

Equations (3.14) and (3.15) are usually expressed in terms of the bus voltage magnitudes, bus voltage angles and the elements of the bus admittance matrix [17].

3.5.4 Inequality Constraints

Only steady state power system operation is considered in the study presented in this chapter. The inequality constraints contain limits on control variables: generator output limits, transformer taps limits, and shunt capacitor range, and operating limits: thermal limits of line and transformer flows, limits of node voltage (magnitude and angle).

$$P_{Gi_{\min}} \leq P_{Gi} \leq P_{Gi_{\max}} \qquad i = 1, 2, 3\ldots n \tag{3.16}$$

$$P_{ij_{\min}} \leq P_{ij} \leq P_{ij_{\max}} \qquad i, j = 1, 2, 3\ldots m \tag{3.17}$$

$$Q_{ij\,\min} \le Q_{ij} \le Q_{ij\,\max} \qquad i, j = 1, 2, 3\ldots m \qquad (3.18)$$

$$V_{i\,\min} \le V_i \le V_{i\,\max} \qquad i = 1, 2, 3\ldots m \qquad (3.19)$$

$$C_{i\,\min} \le C_i \le C_{i\,\max} \qquad i = 1, 2, 3\ldots k \qquad (3.20)$$

Where:

P_{Gi} is generation power at bus i.

V_i is the voltage at bus i.

P_{ij} and Q_{ij} are the power flow between bus i and j.

C_i is the control variable.

n is the number of generators.

m is the number of buses.

k is the number of control variables.

Equations (3.16), (3.17), (3.18) and (3.19) are usually expressed in terms of the magnitudes and angles of bus voltage, the elements of the bus admittance matrix and the system and equipment limits.

3.6 Method for Solving OPF Problems

Sequential Quadratic Programming has been used to solve OPF problems in this research. Matlab optimization toolbox has been used in programming. The objective function and constraints equations are written in different '*m*' files to be the function files. Matlab optimization toolbox uses '*fmincon*' as the command to call and solve the functions in the main program.

3.7 Security Constrained Optimal Power Flow

Power system should be operated in such a way that power is delivered reliably. While the system is operated reliably, the system should be operated most economically. Power system security means the ability of power systems to withstand sudden disturbance such as short circuits or unanticipated loss of system components [8]. An operationally "secure" power system is one with low probability of blackout or equipment damage [2]. Security Constrained Optimal Power Flow (SCOPF) is defined as an optimal power flow considering contingencies of certain transmission and generation facilities. Programs, which can make control adjustments to the base or pre-contingency operation to prevent violation in the post-contingency conditions, are called "SCOPF".

A power system is typically classified into many security levels. A security level 1 is a system where all load is supplied, no operating limits are violated and no limit violations occur in the event of contingencies. Security level 2 is a system where all load is supplied no operating limits are violated and any violation caused by a contingency can be corrected by appropriate control action without loss of load [23]. Under any contingency and in the normal system condition, there should be no violation in system conditions and limits. With the system in the correctively secure state, whenever a contingency takes place, the predetermined rescheduling results should be implemented to ensure that the systems operate without violations. Only security level 1 is considered in the study presented in this thesis.

3.8 Security Constrained Optimal Power Flow Calculation Algorithm

SCOPF ensures power systems survive from the transient state after a disturbance and move into an acceptable steady state condition where all power system components operate without any limits violation. SCOPF requires solving a set of nonlinear equations to describe optimal and secure operation of a power system. OPF problems consider only the constraints in the normal operating condition of the power system. SCOPF problems consider the constraints in normal operating condition as well as during contingencies. These constraints can be very large even for medium size networks. This makes the SCOPF problems very challenging.

Only static security assessment [24] is considered in this study. The three steps of security assessment of a power system consist of contingency selection, contingency evaluation and preventive/ corrective control action [25]. Contingency analysis and special OPF are the two main steps for SCOPF computation. Contingency analysis models the effect of different contingencies on the power systems. Specific techniques are used to identify contingencies in the order of their impacts on the power system operation [2]. The order of which outage constraints should be imposed may be used to simplify the SCOPF problems [26]. These methods are included in the overall scheme of SCOPF computations.

Power systems security monitoring calculations are carried out in power system planning and operation [27]. The power flow dispatch set points can be used as starting points for OPF and SCOPF problems, and the OPF results can be used as starting points for SCOPF. SCOPF problems can be solved by starting at the solution of OPF and to check whether there is any outage constraint violation under this solution. If there is no limit violation, the SCOPF is obtained by the OPF results. If there are limit violations under outages, the complete security constraints should be added [28].

In the study presented in this thesis, Matlab Optimization toolbox is used to implement SCOPF algorithm.

3.9 Minimum Cost Problem for SCOPF

The goal is to obtain the lowest possible fuel cost while satisfying operating constraints under normal conditions as well as under specified contingencies.

3.9.1 Objective Function

The objective function is the same as that for the OPF problem.

$$F = \sum_{i=1}^{n} (\gamma_i + \alpha_i P_{Gi} + \beta_i P_{Gi}^{\ 2}) \tag{3.21}$$

Where: F is the total fuel cost, dollars per hour ($/hr)

P_{Gi} is the output active power of Unit i, megawatts (MW)

α_i, β_i, γ_i are the cost coefficients of generators.

3.9.2 Variables

Similar to the variables for OPF problem, the nodal voltages (magnitude and phase) are the main state variables. The outputs of generating units, generator voltages, transformer tap setting, etc, are the main control variables. In addition to that, these variables exist both in base conditions and post contingencies.

3.9.3 Equality Constraints

Each outage case is characterized by a new set of equations. In SCOPF problems the constraints in normal operating condition and the constraints in the event of specific outages should be considered. The equality constraints contain the power flow equations in normal state and contingency states, which can be expressed as:

$$\sum_{i=1}^{m} P_i = 0 \quad \sum_{i=1}^{m} Q_i = 0 \tag{3.22}$$

$$\sum_{i=1}^{m} P_i' = 0 \quad \sum_{i=1}^{m} Q_i' = 0 \tag{3.23}$$

Where:

P_i and Q_i is the power flow at bus i in normal state

P_i' and Q_i' is the power flow at bus i in contingency states

m is the number of buses

3.9.4 Inequality Constraints

The power system should operate without violation in normal condition and after contingencies. Both the limits under normal operating conditions and after specified contingencies should be satisfied.

The inequality constraints of normal state can be expressed as:

$$P_{Gi_{\min}} \le P_{Gi} \le P_{Gi_{\max}} \qquad i = 1, 2, 3 \ldots n \tag{3.24}$$

$$P_{ij\,\min} \leq P_{ij} \leq P_{ij\,\max} \qquad i, j = 1, 2, 3 \ldots m \qquad (3.25)$$

$$Q_{ij\,\min} \leq Q_{ij} \leq Q_{ij\,\max} \qquad i, j = 1, 2, 3 \ldots m \qquad (3.26)$$

$$V_{i\,\min} \leq V_{i} \leq V_{i\,\max} \qquad i = 1, 2, 3 \ldots m \qquad (3.27)$$

$$C_{i\,\min} \leq C_{i} \leq C_{i\,\max} \qquad i = 1, 2, 3 \ldots k \qquad (3.28)$$

The inequality constraints of contingency states can be expressed as:

$$P_{G\,slack\min}^{'} \leq P_{G\,slack}^{'} \leq P_{G\,slack\max}^{'} \qquad (3.29)$$

$$P_{ij\,\min} \leq P_{ij}^{'} \leq P_{ij\,\max} \qquad i, j = 1, 2, 3 \ldots m \qquad (3.30)$$

$$Q_{ij\,\min} \leq Q_{ij}^{'} \leq Q_{ij\,\max} \qquad i, j = 1, 2, 3 \ldots m \qquad (3.31)$$

$$V_{i\,\min} \leq V_{i}^{'} \leq V_{i\,\max} \qquad i = 1, 2, 3 \ldots m \qquad (3.32)$$

$$C_{i\,\min} \leq C_{i}^{'} \leq C_{i\,\max} \qquad i =1, 2, 3 \ldots k \qquad (3.33)$$

Where :

P_{Gi} is generation power at bus i in normal conditions

$P_{G}'_{slack}$ is generation power of slack bus in contingency states

V_i is the voltage at bus i in normal conditions

V_i' is the voltage at bus i in contingency states

P_{ij} and Q_{ij} are the power flow between bus i and j in normal condition

P_{ij}' and Q_{ij}' are the power flow between bus i and j in contingency states

C_i is the control variables in normal condition

C_i' is the control variables in contingency states

n and m are respectively the number of generators and buses k is the number of control variables

The number of contingencies will determine the size of the constraints of SCOPF problem. If there are n contingencies are considered, the number of SCOPF constraints is almost $n \oplus 1$ times of the number of OPF constraints.

3.10 Method for Solving SCOPF Problems

Similar to OPF problems, Matlab optimization toolbox has been used in programming. The objective function and constraints equations are written in different Matlab '*m*' files as the function files. They are called by the main program. Sequential Quadratic Programming is used by Matlab optimization toolbox for nonlinear constrained problems.

3.11 Case Study of a 7-Bus System

The data of the 7-bus system considered in this study is given in Appendix 1. Table 3.1 shows the summary of this system. Figure 3.1 illustrates the basic operating condition of this system using PowerWorld Simulator [29]. The system is divided into three areas: top area, left area and right area. The total load is 760.0 MW and 130.0 Mvar. Bus 7 is considered as the slack bus in this study.

Table 3.1 Summary of the 7-bus power system

Buses	7	
Generators	5	
Loads	6	
Lines	11	
Generation	765.2 MW	110.7 Mvar
Load	760.0 MW	130.0 Mvar

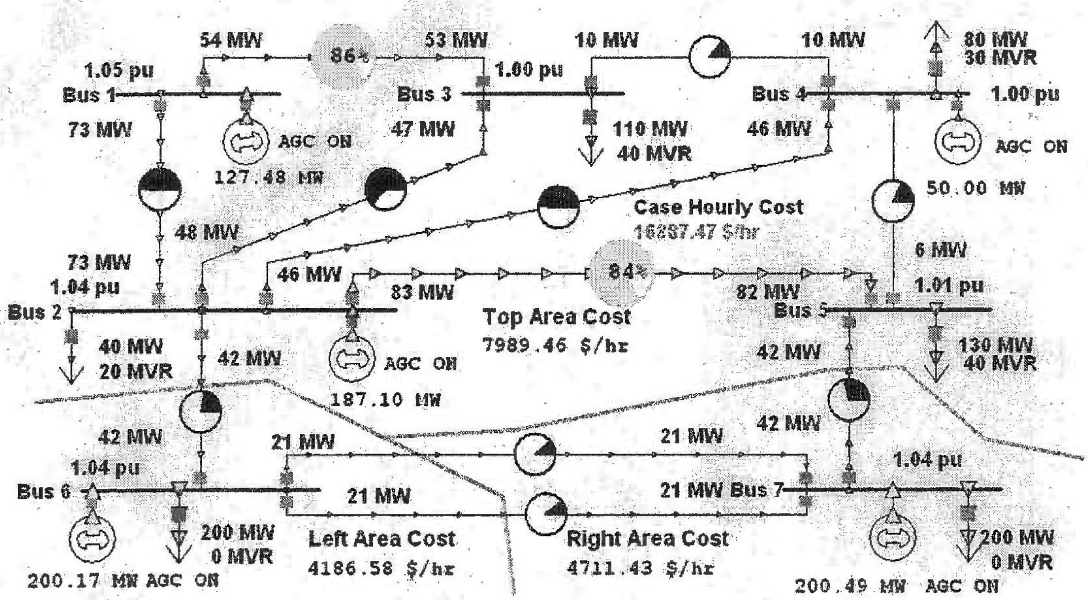

Fig.3.1 Base case operating conditions of the 7-bus system

Cubic cost model is used in this study. Table 3.2 gives the cost coefficients of the five generators in this seven-bus system. The generation cost function (F)of each area is:

$$F\_top = 2.04*(373.5 + 7.62*G_1 + 0.002*G_1^2) + 2.06*(403.61 + 7.519 *G_2 + 0.0014*G_2^2) + 2.09*(253.24 + 7.836*G_4 + 0.0013*G_4^2) \quad (3.34)$$

$$F\_left = 2.14*(388.93 + 7.573*G_6 + 0.0013*G_6^2) \quad (3.35)$$

$$F\_right = 2.57*(194.28 + 7.771*G_7 + 0.0019*G_7^{\,2}) \tag{3.36}$$

If OPF is needed to be calculated for the complete system, the three areas should be considered as a single area. The total generation cost function is the objective function of the OPF problem:

$$\begin{aligned} F\_top = {} & 2.04*(373.5 + 7.62*G_1 + 0.002*G_1^{\,2}) + 2.06*(403.61 \\ & + 7.519*G_2 + 0.0014*G_2^{\,2}) + 2.09*(253.24 + 7.836*G_4 \\ & + 0.0013*G_4^{\,2}) + 2.14*(388.93 + 7.573*G_6 + 0.0013*G_6^{\,2}) \\ & + 2.57*(194.28 + 7.771*G_7 + 0.0019*G_7^{\,2}) \end{aligned} \tag{3.37}$$

Where: G_1, G_2, G_4, G_6 and G_7 are the output active power of generator 1, 2, 4, 6 and 7 respectively.

Table 3.2 Cost coefficients of generators in the 7-bus power system

Cost coefficients	Gen.1	Gen.2	Gen.4	Gen.6	Gen.7
α_i	373.5	403.61	253.24	388.93	194.28
β_i	7.62	7.519	7.836	7.573	7.771
γ_i	0.002	0.0014	0.0013	0.0013	0.0019
Fuel cost	2.04	2.06	2.09	2.14	2.57

3.11.1 Optimal Power Flow

The hourly cost of the base case and economical dispatch are calculated. Then the hourly cost of OPF without considering transmission line thermal limits is calculated. Finally the hourly cost of OPF with transmission thermal limits is calculated. The results are shown in Table 3.3. Table 3.3 illustrates that the economic dispatch has the lowest cost and the base case has the highest cost. The cost of OPF is less than that of the base case, but it is more expensive than that of OPF without considering any transmission line thermal limits. Figure 3.2, Figure 3.3 and Figure 3.4 respectively illustrate these three cases: OPF, OPF without line limits constraints and economic dispatch as obtained in PowerWorld Simulator. As seen in these figures, overloads occur in economic dispatch and OPF without considering line thermal limits. These two cases are not desirable in power system operation. Figure 3.2 shows that line 2-5 reaches its thermal limit. Therefore overload will occur in line 2-5 if we try to decrease the cost more while keeping the same load demand. The OPF results presented in Table 3.3 are obtained using Matlab and verified using PowerWorld Simulator.

Table.3.3 Hourly cost of the 7-bus power system

Study Cases	Gen.1 (MW)	Gen.2 (MW)	Gen.4 (MW)	Gen.6 (MW)	Gen.7 (MW)	Cost ($/hr)
Base Case	127.48	187.1	50	200.17	200.49	16887.47
OPF (no line thermal limits)	215.83	290.0	110.0	150.0	10.83	16267.83
OPF	100.0	150.0	183.06	220.0	112.77	16547.54
Economical Dispatch	197.34	288.68	127.17	163.32	-0.35	16223.59

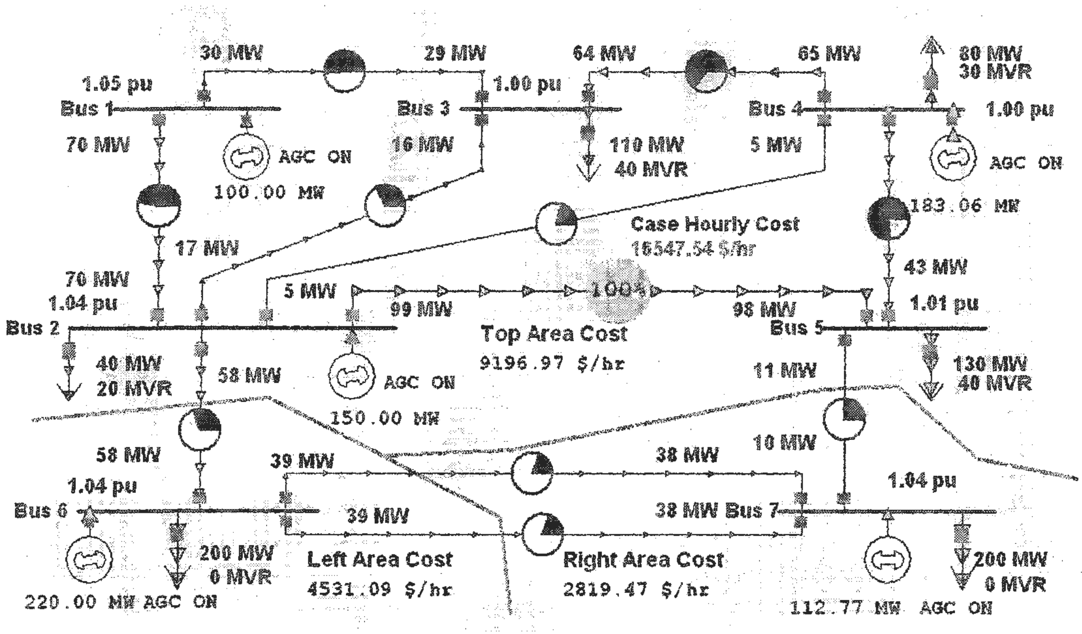

Fig.3.2 OPF results of the 7-bus power system

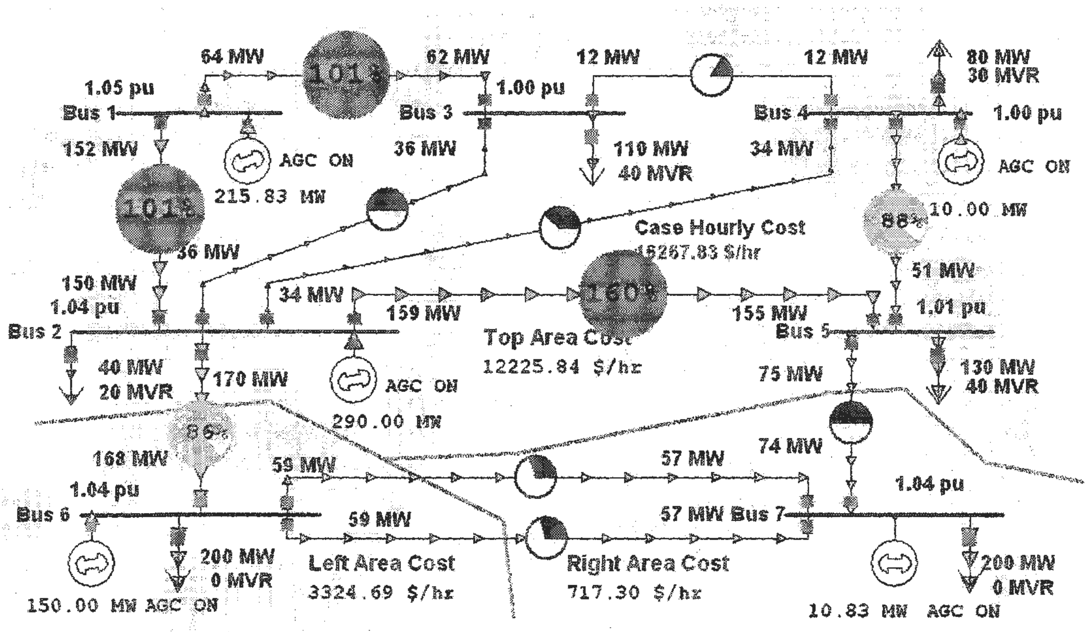

Fig.3.3 OPF results of the 7-bus system without line thermal limit constraints

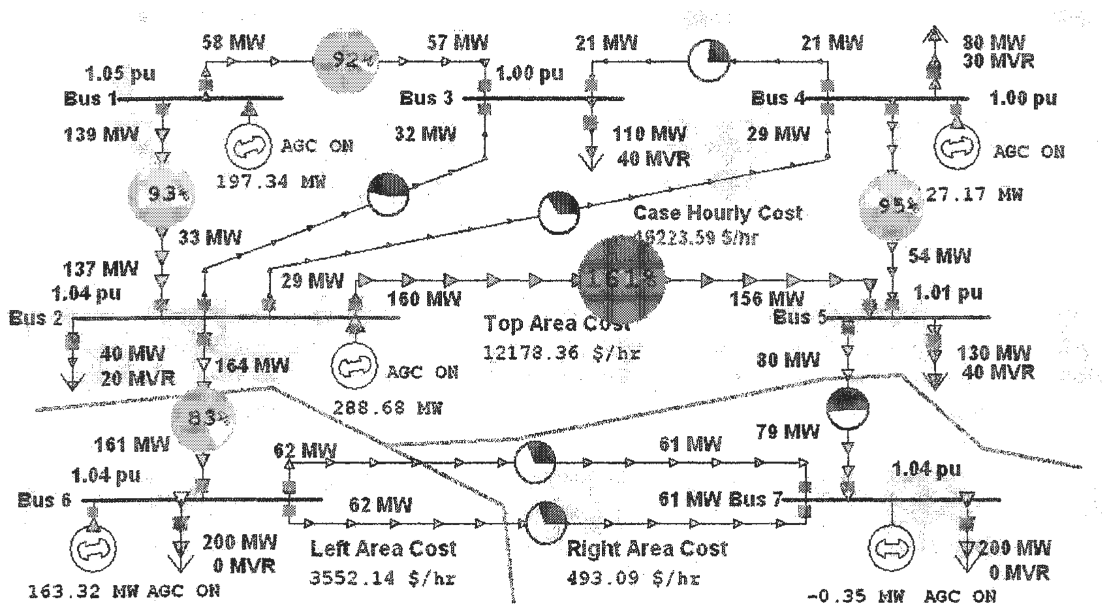

Fig.3.4 Economic dispatch results of the 7-bus system

3.11.2 Security Constrained Optimal Power Flow

The SCOPF study has focused on single transmission line outage only for the 7-bus system. In the optimization problems, the constraints considering transmission line outage are included. Table 3.4 gives the result of SCOPF during different single line outages. It illustrates that in order to keep the system to operate securely and optimally during individual single line outage, the generation output of each generator and cost may change. The summary of cost data presented in Table 3.4 shows that the cost for SCOPF is higher than the cost of 16547.54 $/hr for normal OPF. Consider line 2-5 outage as an example. Figure 3.5 and Figure 3.6 separately illustrate the system, which is secure during line 2-5 outage, in normal operation and during line 2-5 outage. These two figures show that both in normal condition and during line 2-5 outage, the system can operate without any violation. Figure 3.6 shows that line 4-5 reach its thermal limit during line

2-5 outage. Therefore overload will occur in line 4-5 if the cost is reduced more while keeping the same load demand. The results presented in Table 3.4 are obtained by Matlab and verified by PowerWorld Simulator.

Table 3.4 SCOPF with one line outage of the 7-bus power system

Outages	G1(MW)	G.2(MW)	G.4(MW)	G6(MW)	G.7 (MW)	Cost($/hr)
Line 2-5	100	172.84	81.63	288.89	122.8	16610
Line 3-4	100	150	162.18	241.03	112.5	16548
Line 2-6	100	150	106.26	297.96	111.9	16583
Line 5-7	100	150	195.08	207.97	112.9	16551
Line 4-5	100	155.21	72.6	291.32	146.4	16717
Line 6-7	100	150	184.69	179.56	150.6	16683

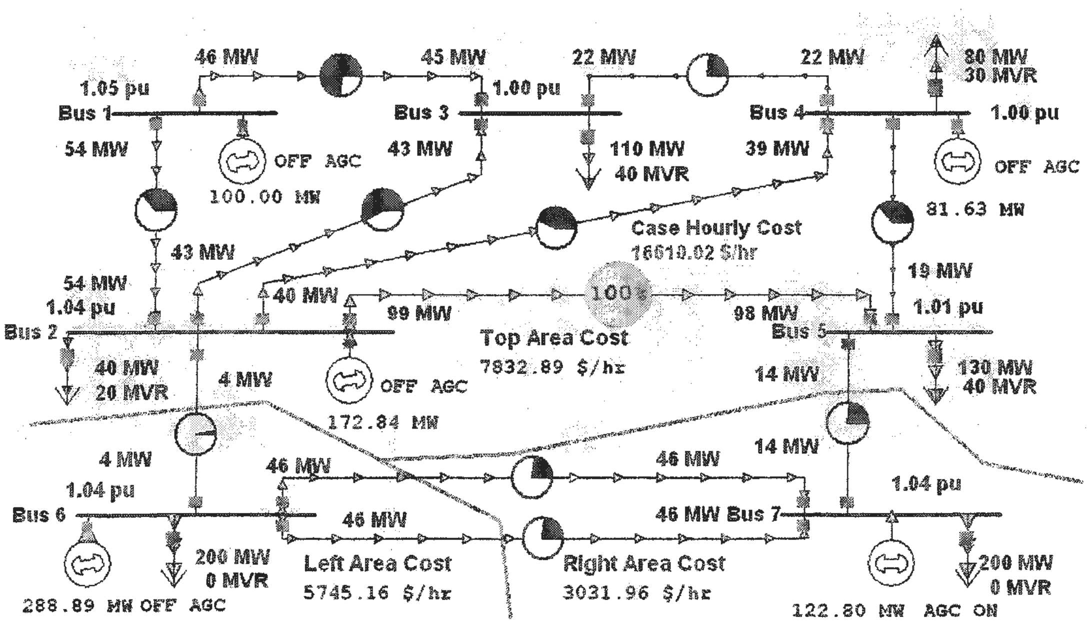

Fig. 3.5 SCOPF results of the 7-bus system in normal operating condition

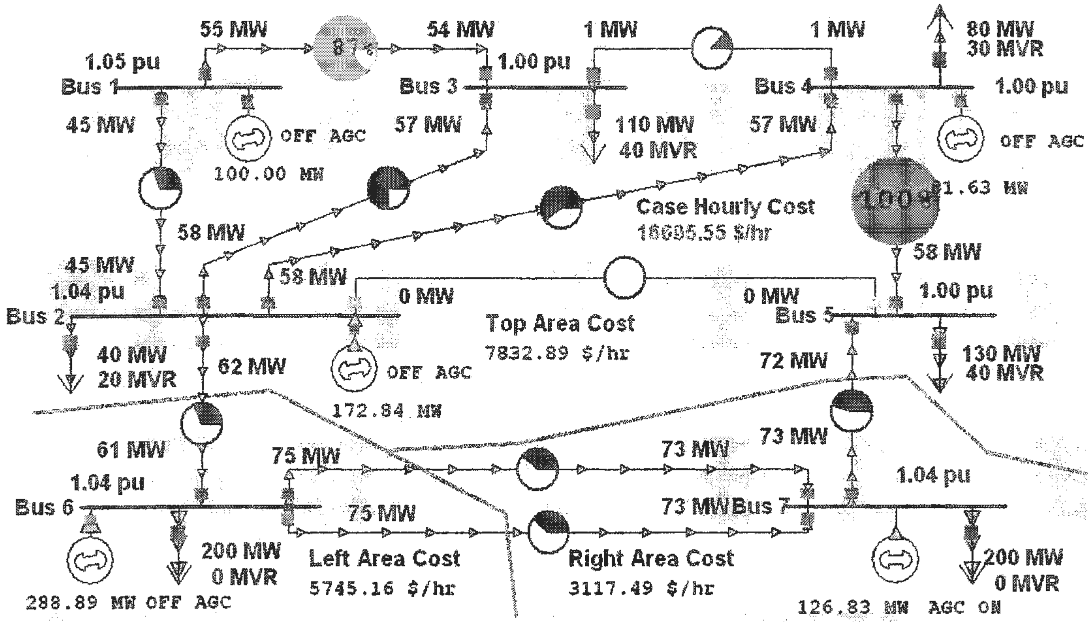

Fig. 3.6 SCOPF results of the 7-bus system during line 2-5 outage

3.12 Case Study of the 30-Bus System

The data of the 30-bus system considered in this study are given in Appendix 2. Table 3.5 shows the summary of this system. Figure 3.7 illustrates the basic operating condition of this system in PowerWorld Simulator. The system is divided into three areas and the total load is 237MW and 77.6Mvar. Bus 1 is considered as slack bus in this study.

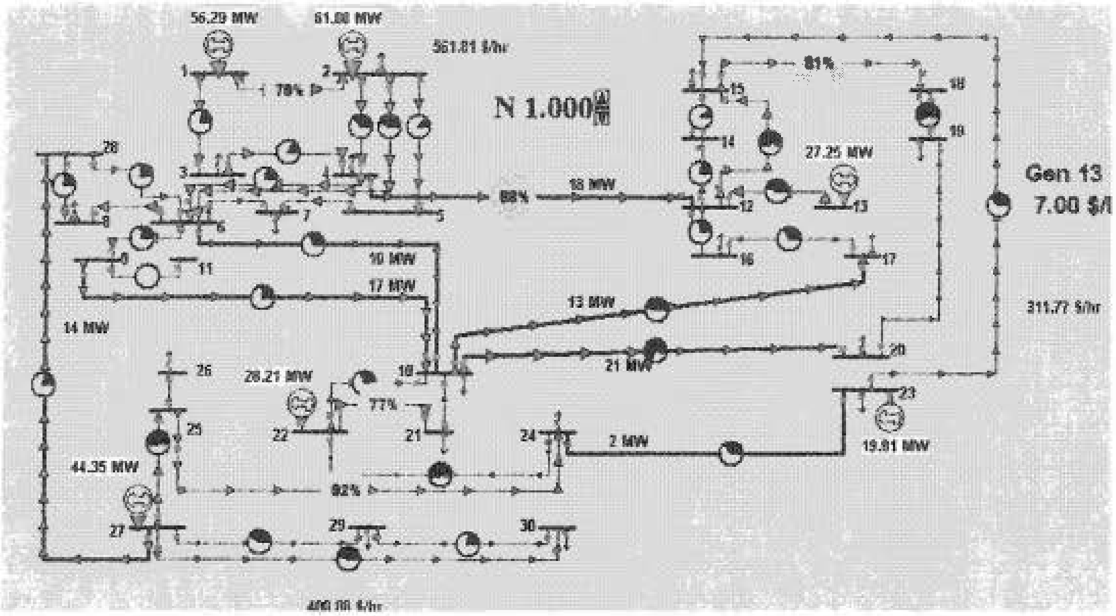

Fig.3.7. Base case operating condition of the 30-bus power system

Table 3.5 Summary of the 30-bus power system

Buses	30	
Generators	6	
Loads	21	
Switched Shunt	2	
Lines	41	
Generation	232.3 MW	77.3 Mvar
Load	236.9 MW	76.9 Mvar

Cubic cost model is used in this study. Table 3.6 gives the cost coefficients of the six generators in this 30-bus power system. The generation cost function of each area is:

$$F\_1 = (3.26*G_1 + 0.0326*G_{12}) + (2.8*G_2 + 0.028*G_2^2) \tag{3.38}$$

$$F\_2 = 7*G_{13} + (5.1*G_{23} + 0.051*G_{23}^2) \tag{3.39}$$

$$F\_3 = (4.5*G_{22} + 0.045*G_{22}^2) + (3.72*G_{27} + 0.0372*G_{27}^2) \quad (3.40)$$

The three areas should be considered as a single area to calculated OPF for the complete system. The objective function of the OPF problem is the total generation cost function (F):

$$\begin{aligned} F = {} & (3.26*G_1 + 0.0326*G_{12}) + (2.8*G_2 + 0.028*G_2^2) \\ & + 7*G_{13} + (5.1*G_{23} + 0.051*G_{23}^2) \\ & + (4.5*G_{22} + 0.045*G_{22}^2) + (3.72*G_{27} + 0.0372*G_{27}^2) \end{aligned} \quad (3.41)$$

Where: G_1, G_2, G_{13}, G_{22}, G_{23} and G_{27} are respectively the output active power of generator 1, 2, 13, 22, 23 and 27.

Table 3.6 Cost coefficients of generators in the 30-bus power system

Cost coefficients	Gen.1	Gen.2	Gen.13	Gen.22	Gen.23	Gen.27
α_i	0	0	0	0	0	0
β_i	3.26	2.8	7	4.5	5.1	3.72
γ_i	0.002	0.028	0	0.045	0.051	0.0372
Fuel cost	1	1	1	1	1	1

3.12.1 Optimal Power Flow

The hourly cost of the base case and complete system OPF are calculated. The results are shown in Table 3.7 and Table 3.8. Table 3.8 illustrates that the cost of OPF for the complete system is less than the total cost of the base case. However, the cost of each area at the OPF settings is not always less than the cost of each area in base case.

Figure 3.8 illustrates OPF results of this 30-bus power system in PowerWorld Simulator. It shows that line 4-12 reaches its thermal limit. Therefore overload will occur on line 4-12 if we try to decrease the cost more while keeping the same load demand. The results presented in Table 3.7 and Table 3.8 are obtained using Matlab and verified using PowerWorld Simulator.

Table 3.7 Power schedule of the 30-bus power system at OPF

	Gen.1 (MW)	Gen.2 (MW)	Gen.13 (MW)	Gen.22 (MW)	Gen.23 (MW)	Gen.27 (MW)
OPF	52.87	70.87	24.05	28.63	19.75	40.76
Base case	56.29	61.0	27.25	28.21	19.81	44.35

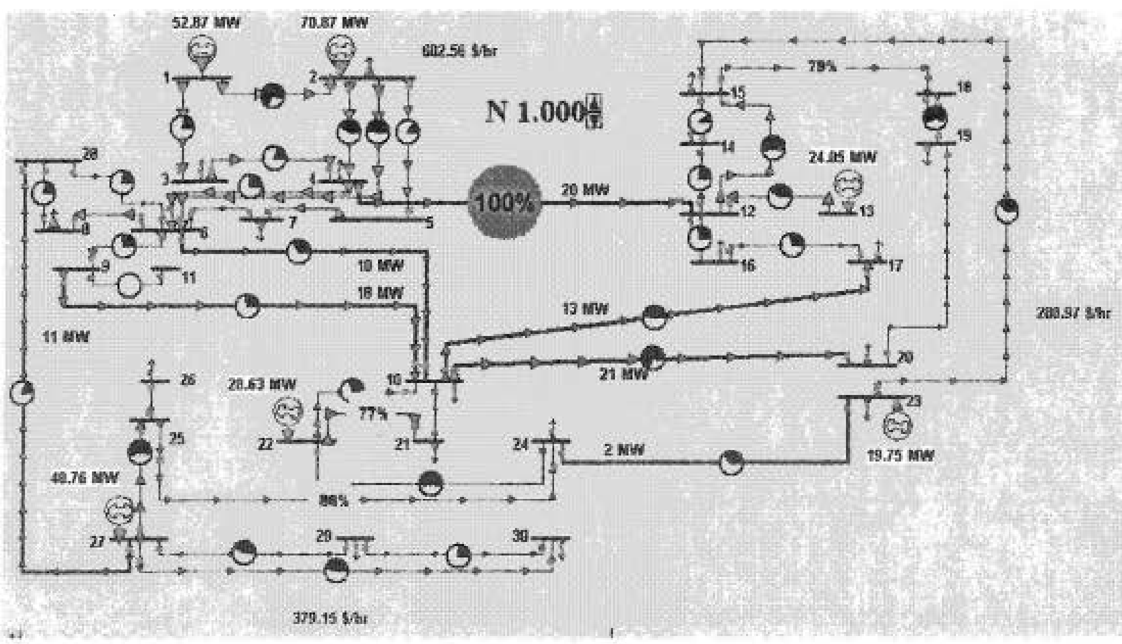

Fig.3.8. OPF results of the 30-bus power system

Table 3.8 Hourly cost of each area of the 30-bus system at OPF

	Area 1 ($/hr)	Area 2 ($/hr)	Area 3 ($/hr)	Total ($/hr)
OPF	602.37	289	379.12	1270.5
Base Case	561.81	311.77	400.88	1274.5

3.12.2 Security Constrained Optimal Power Flow

For the 30-bus system, the SCOPF study has focused on single transmission line outage only.

The constraints considering transmission line outage are included in the optimization problems. Consider line 4-12 outage as an example, the results of SCOPF is given in Table 3.9. It shows that the output generation power of slack bus changed and those of other generators are fixed in normal operation and during this outage. As seen in Table 3.9, the cost of SCOPF in normal operation is higher that of normal OPF, and the cost of SCOPF in normal operation is less than the cost during the outage of line 4-12.

Figure 3.9 and Figure 3.10 separately illustrate the SCOPF results in normal operation and during line 4-12 outage in PowerWorld simulator. These two figures show that both in normal condition and during line 4-12 outage, the system can operate without any violation. Figure 3.10 shows that line 24-25 reach its thermal limit during line 4-12 outage. The results presented in Table 3.9 are obtained using Matlab and verified using PowerWorld Simulator.

Table 3.9 SCOPF results with one transmission line outage of the 30-bus system

	Gen.1 (MW)	Gen.2 (MW)	Gen.13 (MW)	Gen.22 (MW)	Gen.23 (MW)	Gen.27 (MW)	Cost ($/hr)
Normal condition	51.99	69.83	27.86	28.77	20.05	38.28	1270.5
Line 4-12 Outages	52.51	69.83	27.86	28.77	20.05	38.28	1273.9

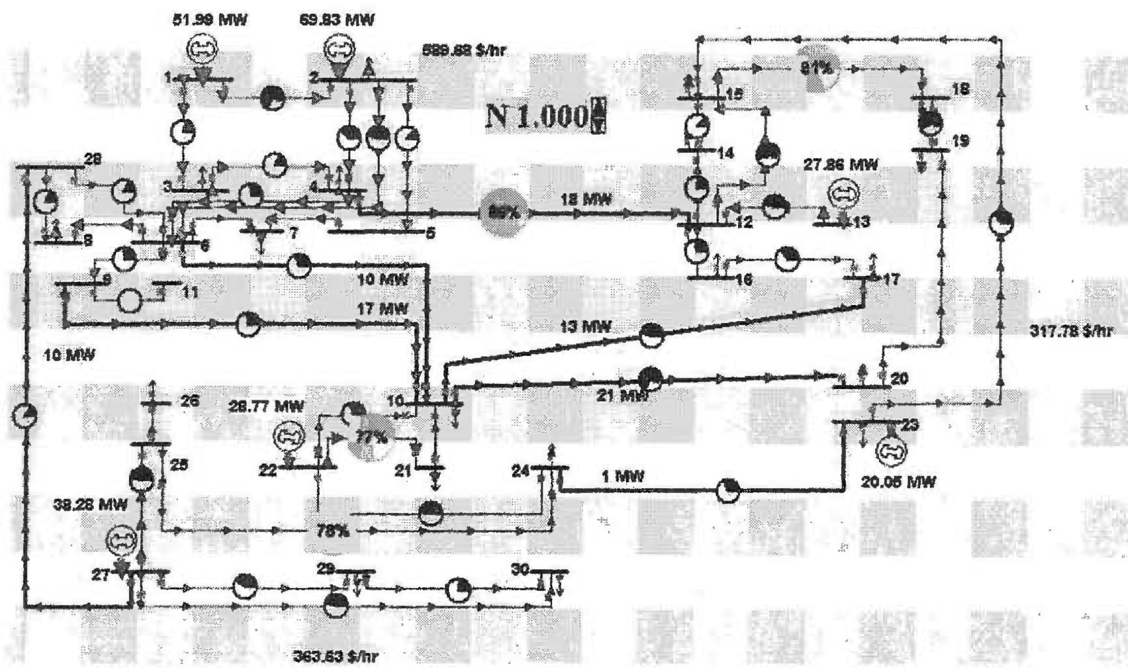

Fig. 3.9 SCOPF results of the 30-bus system in normal operating condition

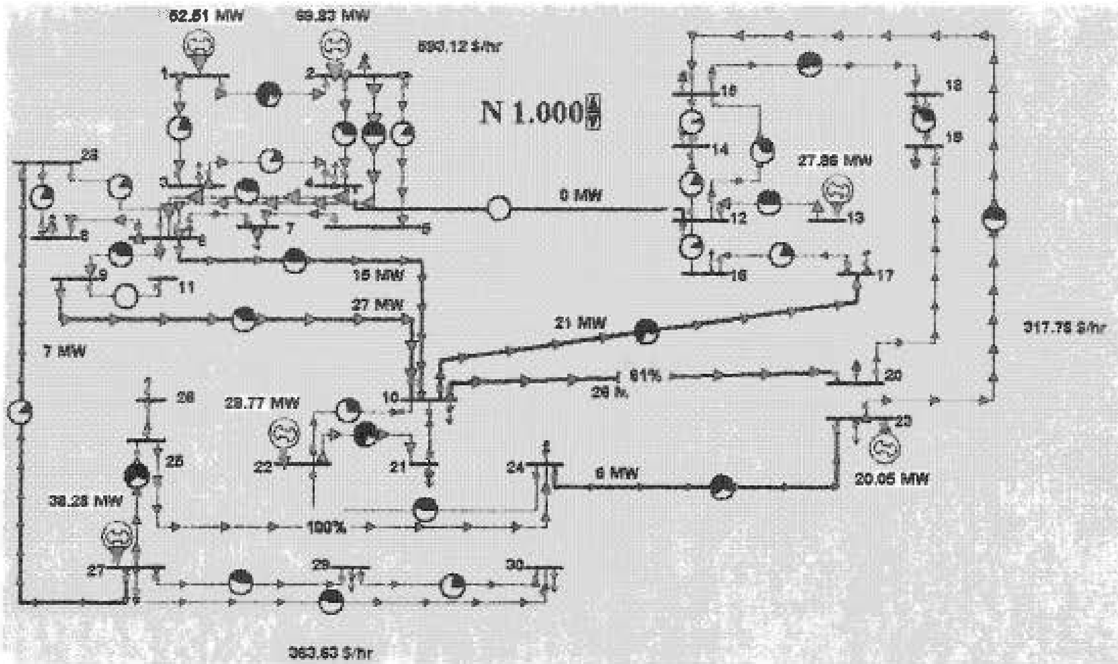

Fig. 3.10 SCOPF results of the 30-bus system during line 4-12 outage

3.13 Conclusions

This chapter has applied a nonlinear programming based optimization technique to solve OPF and SCOPF problems. As seen from the case studies, the fuel cost of a power system at OPF settings is less than that of a power system under normal condition. Although the fuel cost of a power system at OPF settings is more than that of a power system at Economic Dispatch settings, OPF is more desirable than Economic Dispatch for power systems. Transmission line thermal limits and other limits may be violated in Economic Dispatch. The fuel cost of a power system at SCOPF settings is more than that of a power system at OPF settings. However, the system can be operated safely even during contingencies. Since it is difficult to completely prevent contingencies in power system, the effect of possible contingencies must be considered. SCOPF can ensure that the power system operates economically both in normal condition and during contingencies.

The study presented in this chapter shows that the nonlinear programming based optimization techniques can handle OPF and SCOPF problems efficiently. SCOPF problems are much more challenging than OPF problems due to the large amount of constraints. With the improvement of computer capabilities, nonlinear programming based optimization methods can implement OPF and SCOPF problems efficiently and accurately.

Chapter 4

TRANSMISSION TRANSFER CAPABILITY

4.1 Introduction

The traditional optimal power flow (OPF) problems consider minimum cost and minimum loss operations. During the last few years, there has been a significant change in the operation of power systems. As a result of the utility deregulation, there is a major change in the nature of power flow over the interconnected power systems. The focus of OPF analysis has shifted to a more economic, flexible and controllable operation. Maximum power transfer between regions, elimination of loop flows or setting the active power flow along desired paths are some of the new possible goals under new conditions [30].

With the introduction of competition in the utility industry, it is possible for customers to buy less expensive electrical energy from remote locations. As a result, it is important to know how much power can be transferred safely across the system without violating any operating limits. Transmission Transfer Capability (TTC) is the measure of the ability of interconnected electrical power systems to reliably move or transfer electric power from one area to another area by way of all transmission lines (or paths) between those areas under specified system conditions [31].

This chapter introduces the definition of TTC and the characteristics of TTC. It also introduces the purpose and requirements of TTC computation. Two basic methods for TTC evaluation, repeated load flow and distribution factor methods are described in this chapter. Case studies of these two methods using a sample 7-bus power system and the New England 39-bus power system are presented in detail. In the final part of this chapter, a summary of these two methods is given.

4.2 Definition of Transmission Transfer Capability

The definitions of TTC in North American Electric Reliability Council (NERC)'s May 1995 *Transmission Transfer Capability* document [31] is " Transfer capability is the measure of the ability of interconnected electric systems to reliably move or transfer electric power from one area to another area by way of all transmission lines (or paths) between those areas under specified system conditions. The units of transfer capability are in term of electric power, generally expressed in megawatts (MW). In this context, area refers to the configuration of generating stations, switching stations, substations, and connecting transmission lines that may define an individual electric system, power pool, control area, sub region, or region". In some cases, TTC may be better defined by a series of equations than by a single number, particularly when determining TTC values for two or more parallel or interacting paths [32].

In general, TTC is the maximum amount of additional active power that can be transferred between two parts of a power system. Maximum means that a violation will occur if more than this amount of active power flow is added between these two areas [3]. Additional means that TTC is the amount of active power above the base case power flow and its value does not include the existing power flow.

4.3 Characteristics of TTC

Transfer capability is directional in nature. The transfer capability from area A to area B is not generally equal to the transfer capability from area B to area A. The points of electric power injection, the direction of transfer across the network and the points of delivery are the main factors used to determine TTC. TTC is a variable quantity, which depends upon operating conditions in the near term and forecasted conditions in the long term. System conditions, parallel flows and critical contingencies are the main points that affect TTC [33]. TTC will change when the following events happen: base power flow model changes; transmission outage schedule changes; a new facility is added or removed. Base system conditions include load demand, generation dispatch, and system configuration. Base scheduled transfer is the basic conditions for TTC analysis. TTC must recognize time-variant power flow conditions and the effects of simultaneous transfer / parallel path flow from a reliability viewpoint.

TTC represents the reliability limit of a transmission path at any specified point in time. TTC can be used as an indicator to know how far stability limits are from the operating points in terms of Megawatts. It is also a measure of the network capability for further commercial activity over and above existing committed use [34].

4.4 Purpose of TTC Evaluation

TTC plays an important role in both the planning and operation of power systems. TTC can be considered as an indicator of power system security and is very useful for real time security analysis, because it can be used to estimate the ability of a power system or an interconnection system to remain secure following facilities outages [35].

TTC can be used as to measure transmission strength and it is very useful in comparing and evaluating alternate transmission system configurations. A system with large transfer capability can be considered more robust and flexible than the system with limited transfer capability.

Under normal operating condition, the purpose of TTC evaluation is to determine the quantity of additional power transfer possible while maintaining power system security. TTC can be used to compare the relative merits of planned transmission improvements. During the facilities outages, the purpose of TTC evaluation is to determine the quantity of lost power that can be replaced by the potential reserves. It also can be used to evaluate the real-time ability of the interconnected transmission system to transfer electric power between areas. Evaluation of TTC is very important for both power system planning and operation.

4.5 TTC Evaluation

TTC is limited by thermal limits, voltage limits and stability limits. The maximum amount of electrical current that flows through the transmission path and electrical facility should be limited to prevent the damage caused by overheating. System voltage must be maintained within specified limits. The power system must be able to withstand disturbances that occur during normal operating condition.

Power flows obey the laws of physics, not the requirements of contracts. Among the parallel paths, those with lower impedance will attract more power flow than those with higher impedance. TTC is related to the system configuration, like generation schedule, consumption pattern etc. [36]. TTC calculations depend on the points of electric power injection, the direction of transfer across the network and the points of power extraction. TTC should be evaluated considering possible contingencies as well. Base case is the starting point of TTC computation. During contingencies or during transmission lines out of service for maintenance, TTC values might change depending on the outage. The calculation of TTC is generally based on computer simulation of the operation of the interconnected transmission network under a specific set of assumed operating conditions [37]. There are many methods for TTC evaluation. Two of the widely used methods are the distribution factors method and repeated load flow method. These two methods and case studies using these two methods are presented in the following sections.

4.6 Distribution Factors

Use of the distribution factors is one of the widely used methods for calculating transfer capability in power systems [27]. Distribution factors method is simple and fast since it does not involve any iterative technique. Distribution factors include power transfer distribution factor (PTDF), line outage distribution factor (LODF) and outage transfer distribution factor (OTDF) [29]. They are widely used in TTC calculation under base case loading conditions and under outage conditions. Distribution factors measure the effects of a power transfer change or an outage has on the remaining part of the system. They are expressed in percent of the change in power. Distribution factors method uses linear approximations for updating and estimating TTC. It is a linear method and commonly is computed with DC load flow. DC load flow is the basis for calculating distribution factors. DC load flow is discussed in the following section.

4.6.1 DC Load Flow

DC load flow only considers the active power, ignores reactive power and assumes the voltage magnitudes of all buses are 1 *pu*. DC load flow model ignores the resistances of transmission line and transformer. It is a lossless model and the sum of the total real power generation equals the total load real power [38].

The complex power flow in each bus of a power system can be stated as:

$$P_i + jQ_i = E_i I_i^* \tag{4.1}$$

Where $$I_i = \sum_{k=1}^{N} Y_{ik} E_k \tag{4.2}$$

Substituting equation (4.2) into equation (4.1), the power flow equation can be expressed as:

$$\begin{aligned} P_i + jQ_i &= E_i \sum_{k=1}^{N} Y_{ik}^* E_k^* = \sum_{k=1}^{N} |E_i||E_k|(G_{ik} - jB_{ik})e^{j(\theta_i - \theta_k)} \\ &= \sum_{k=1}^{N} \{|E_i||E_k|[G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)] \\ &\quad + j[|E_i||E_k|[G_{ik}\sin(\theta_i - \theta_k) - B_{ik}\cos(\theta_i - \theta_k)]]\} \end{aligned} \tag{4.3}$$

Where: P_i and Q_i are the real and reactive power at bus i respectively.

θ_i, θ_k are the phase angles at buses i and k respectively

$|E_i|$, $|E_j|$ are the bus voltage magnitudes respectively

$G_{ik}+jB_{ik} = Y_{ik}$ is the i, k element in the Y matrix (bus admittance matrix) of the power system

Considering the active power and assuming the voltage magnitudes for all buses to be 1 *pu*, equation (4.3) becomes:

$$P_i = \sum_{k=1}^{N} \{[G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)] \quad (4.4)$$

The first derivative of equation (4.4) is:

$$\frac{\partial P_i}{\partial \theta_k} = G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k) \quad (4.5)$$

Since θ_i-$\theta_k \cong 0$, $\cos(\theta_i$-$\theta_k) \cong 1$; The resistances of transmission line and transformer are ignored, $G_{ik} \cong 0$; equation (4.5) changes to:

$$\frac{\partial P_i}{\partial \theta_k} = -B_{ik} \quad (4.6)$$

The power flow adjustment equation is:

$$\Delta P_i = \sum_{k=1}^{N} (\frac{\partial P_i}{\partial \theta_k} \Delta \theta_k) = \sum_{k=1}^{N} (-B_{ik} \Delta \theta_k) \quad (4.7)$$

This can be expressed in matrix form as:

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} -B_{11} & -B_{12} & \cdots \\ -B_{21} & -B_{22} & \cdots \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \vdots \end{bmatrix} \quad (4.8)$$

Since the resistance in each transmission line is neglected,

$$-B_{ik} = -\frac{1}{x_{ik}} \quad (4.9)$$

Equation (4.8) can be written as:

$$\Delta P = [B] \Delta \theta \quad (4.10)$$

Where: B matrix is the imaginary part of the Y-bus with all the "shunt terms" removed.

From equation (4.10), the change of phase angle can be obtained as:

$$\Delta\theta = [X]\Delta P \quad (4.11)$$

X matrix is the inverse of B matrix and it is the standard matrix used in DC load flow.

DC load flow ignores the resistances in transmission lines, reactive and complex power flows and assumes voltage magnitude is 1 *pu*. The power flowing in transmission line *n* to *m* can be expressed as:

$$\begin{aligned} P_{nm} &= E_n I_{nm}^* = |E_n| e^{j\theta_n} \left(\frac{|E_n| e^{j\theta_n} - |E_m| e^{j\theta_m}}{jx_{nm}}\right)^* = \frac{1 - e^{j(\theta_n - \theta_m)}}{jx_{nm}} \\ &= \frac{1 - \cos(\theta_n - \theta_m) + j\sin(\theta_n - \theta_m)}{jx_{nm}} \end{aligned} \quad (4.12)$$

Since $\theta_n - \theta m \cong 0$, $\cos(\theta_i - \theta_k) \cong 1$ and $\sin(\theta_i - \theta_k) \cong \theta_n - \theta m$, the power flow in transmission line *n* to *m* is:

$$P_{nm} = \frac{1}{x_{nm}}(\theta_n - \theta_m) \quad (4.13)$$

DC load flow gives no indication of reactive power flow, complex power flows and voltage magnitude changes. It is only useful for calculating active flows on transmission lines and transformers. DC load flow calculation is very fast compared to the standard iterative load flow methods.

4.6.2 Generation Shift factors (GSFs)

Generation Shift factor (GSF) or adjustment factor expresses the change in power flowing on a particular transmission facility that is caused by increasing generation at a specific bus. They are expressed in percent (up to 100%) of the power change in a specific bus. Since it represents the linear relation between the bus injection power change and the power flow change in transmission lines, it can be considered as a sensitivity factor. GSF is only applied in a source-sink pair. The power injected at the source must be matched by the power removed at the sink. The slack bus is specified as the power sink in GSF computation.

DC load flow is applied in GSF calculation. To calculate the GSF for the generator at bus *n*, we inject 1 *pu* power at bus *n* to be the source bus and set the slack bus as the sink bus to absorb the increased power. ΔP is a vector, which has 1 in row *n* and –1 in reference row and has zeros in other rows. From equation (4.11), the phase angles changes are obtained from the following equation:

$$\Delta\theta = [X]\begin{bmatrix} \vdots \\ +1 \\ \vdots \\ -1 \end{bmatrix} \begin{matrix} \\ \text{..................... row n} \\ \\ \text{.......reference row} \end{matrix} \tag{4.14}$$

Here the 1 in row *n* represents that there is 1 *pu* power increase in bus *n* and the –1 in reference row represents that there is 1 *pu* power decrease in reference bus to compensate for the increased power in bus *n*. Zeros in other rows of the ΔP vector indicate that the injection power is kept constant in other buses.

The ith row of Δθ vector (4.14) is:

$$\Delta\theta_i = X_{in} * 1 + X_{i,slack} * (-1) = X_{in} - X_{i,slack} \tag{4.15}$$

The sensitivity factor that represents the effect of the power in line *i-j* caused by injection power change in bus *n* is calculated by the following equation:

$$GSF_{n,ij} = \frac{dP_{ij}}{dP_n} = \frac{d}{dP_n}\left[\frac{1}{x_{ij}}(\theta_i - \theta_j)\right] = \frac{1}{x_{ij}}\left(\frac{d\theta_i}{dP_n} - \frac{d\theta_j}{dP_n}\right) = \frac{1}{x_{ij}}(X_{in} - X_{jn}) \tag{4.16}$$

Where: X_{in} is the ith element of the Δθ vector in equation (4.14).

X_{jn} is the jth element of the $\Delta\theta$ vector in equation (4.14).

x_{ij} is the line reactance of line *i-j*.

Since TTC is the maximum additional power transfer from area *n* to area *m* without violation, at the TTC setting, there will be a transmission line, which reaches its thermal limit. Consider that the transmission line *i-j* reaches its thermal limit when the additional power transfer from *n* to *m* reaches the maximum value, TTC. Then the following equation is obtained:

$$P_{ij}^{basecase} + GSF_{n,ij} * TTC_{n,slack} = P_{ij}^{limit} \tag{4.17}$$

The TTC from bus *n* to slack bus is:

$$TTC_{n,slack} = \frac{P_{ij}^{limit} - P_{ij}^{basecase}}{GSF_{n,ij}} \tag{4.18}$$

Transmission line *i-j* reaches its thermal limit when the transaction from bus *n* to slack bus achieves its maximum.

4.6.3 Power Transfer Distribution Factors (PTDFs)

While the power increases in a specific transmission path, the power flows in other transmission paths will also change, but in different rates. These different rates are called Power Transfer Distribution Factors (PTDFs). PTDFs measure the change in the electrical loadings on system facilities due to a change in the electric power transfer from one area to another area. They are expressed in percent (up to 100%) of the power change in specific transmission line.

PTDF is the fraction of the amount of transaction from one area to another area that flow over a given transmission line [27]. It determines the linear impact of a transfer (or change in power injection) on the element of power system. It is also a sensitivity factor since it represents the linear relationship between the transaction change and the power flow change in other lines. Consider the PTDF on transmission line *i-j* caused by the power change in transmission path *n-m* as an example. It can be expressed as:

$$PTDF_{ij,nm} = \frac{\Delta P_{ij}}{\Delta P_{nm}} \tag{4.19}$$

Similar to GSF calculation model, the power sources and power sinks must be specified in PTDFs calculation model.

Similar to equation (4.14), the vector of $\Delta\theta$ is:

$$\Delta\theta = [X]\begin{bmatrix} \vdots \\ +1 \\ \vdots \\ -1 \end{bmatrix} \begin{matrix} \\ \text{.................... row n} \\ \\ \text{.......................row m} \end{matrix} \tag{4.20}$$

The ith row of $\Delta\theta$ vector equation (4.20) is:

$$\Delta\theta_i = X_{in} * 1 + X_{im} * (-1) = X_{in} - X_{im} \tag{4.21}$$

Then the sensitivity of the power change in line *i-j* to the power change in transmission path *n-m* can be calculated by the following equation:

$$\begin{aligned} PTDF_{ij,nm} &= \frac{dP_{ij}}{dP_{nm}} = \frac{d}{dP_{nm}}\left[\frac{1}{x_{ij}}(\theta_i - \theta_j)\right] = \frac{1}{x_{ij}}\left(\frac{d\theta_i}{dP_{nm}} - \frac{d\theta_j}{dP_{nm}}\right) \\ &= \frac{1}{x_{ij}}\left(\frac{\Delta\theta_i}{\Delta P_n} - \frac{\Delta\theta_j}{\Delta P_n}\right) = \frac{1}{x_{ij}}(\Delta\theta_i - \Delta\theta_j) = \frac{1}{x_{ij}}(X_{in} - X_{im} - X_{jn} + X_{jm}) \end{aligned} \tag{4.22}$$

Where: x_{ij} is the line reactance for line connecting zone *i* and zone *j*

X_{im} is the value in ith row and mth column of matrix X

From equations (4.16) and (4.22), the relationship between PTDF and GSF is obtained:

$$PTDF_{ij,nm} = \frac{1}{x_{ij}}[(X_{in} - X_{jn}) - (X_{im} - X_{jm})] = GSF_{n,ij} - GSF_{m,ij} \tag{4.23}$$

Equation (4.23) shows that PTDF can be obtained from GSF.

GSFs are very closely related to PTDFs. The differences between them is that for a lossless system, the source and sink bus pair can be any bus in PTDFs, but they only can be generation bus and slack bus in GSFs. GSF can be considered as a special case of PTDF, which sets slack bus as the power sink. GSFs for all generation buses can be obtained from the PTDFs calculation formulation by considering the slack bus as the sink bus.

4.6.3.1 Using PTDF to Calculate TTC in Normal Operating Condition

Under normal operating condition, PTDF can be used to calculate TTC, where the change in power flow through the line *i-j* given as ΔP_{ij}^{new} that is due to the change in power flow from area *n* to *m* given as P_{nm}^{new}. The change in power flow through the line *i-j* under such conditions is evaluated as:

$$\Delta P_{ij}^{new} = PTDF_{ij,nm} * P_{nm}^{new} \tag{4.24}$$

Where: $PTDF_{ij,nm}$ is the PTDF on line *i-j*, which is caused by the power change on line *n-m*.

The power flow between the given line *i-j* P_{ij}^{new} after considering transaction change P_{nm}^{new} is:

$$P_{ij}^{new} = P_{ij}^{basecase} + \Delta P_{ij}^{new} \tag{4.25}$$

Where: $P_{ij}^{basecase}$ is the base case power flows between transmission line *i-j*.

To calculate TTC using PTDFs, since the maximum additional power transfer from area *n* to area *m* is TTC_{nm}, the power change in line *i-j*, which is caused by the TTC from area *n* to area *m*, is $PTDF_{ij,nm} * TTC_{nm}$. At this point, there will be a transmission line, which reaches its thermal limit. Assume that transmission line *i-j* reaches its thermal limit first, which is $P_{ij}^{new} = P_{ij}^{limit}$. The power in line *i-j* can be expressed as:

$$P_{ij}^{basecase} + PTDF_{ij,nm} * TTC_{nm} = P_{ij}^{limit} \tag{4.26}$$

TTC can be obtained from:

$$TTC_{nm} = \frac{P_{ij}^{limit} - P_{ij}^{basecase}}{PTDF_{ij,nm}} \tag{4.27}$$

4.6.3.2 Using PTDF to Calculate TTC with Outages

When PTDF is used to calculate TTC during outages, it considers the system with outage as a new system. TTC and PTDF of the "new" system can be calculated by the same method discussed in section 4.6.3.1, except that the X matrix of the system should be modified and power flow during outage should be calculated again. This method is not efficient if more than one outage occurs separately in the system. At each outage, the X matrix should be modified to update PTDF and power flow and TTC should be calculated repeatedly.

4.6.4 Line Outage Distribution Factors (LODFs)

Line outage distribution factor (LODF) measures the redistribution of electric power on remaining system facilities as a result of an outage of a single system facility or element. It expresses how a change of a line's status affects the power flow on other lines in the power system. LODF is expressed in percent of the pre-contingency electrical power of the outaged facility on other transmission lines.

For example, $LOPF_{ij,nm}$ gives the percent of the pre-outage flow on line *n-m*, P_{nm} that will flows on line *i-j* after the line *n-m* outage. The definition of LODF can be expressed as:

$$LODF_{ij,nm} = \frac{\Delta P_{ij}}{P_{nm}} \tag{4.28}$$

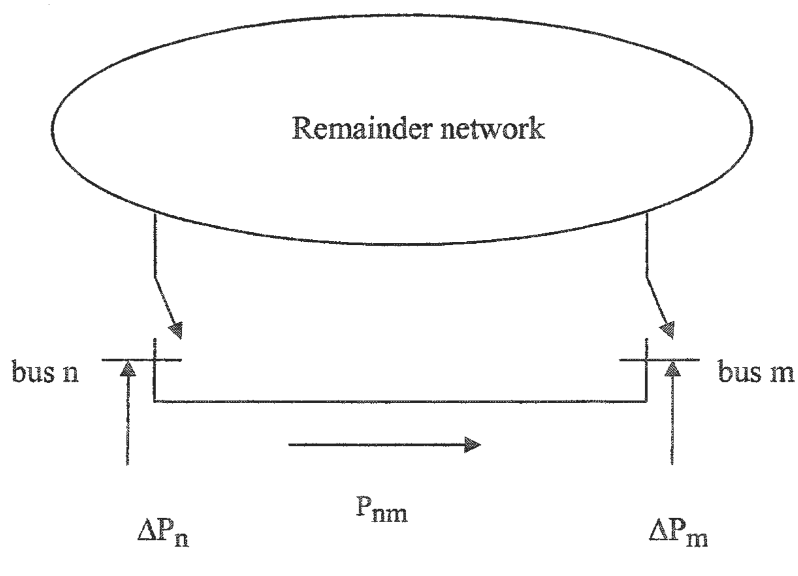

Fig.4.1. Line n-m outage model

As shown in Figure 4.1, to model the line *n-m* outage, two injection powers are added to the system. If there is P_{nm} power flowing from bus *n* to bus *m* in pre-outage, one injection power ΔP_n is added to bus *n* and another injection power ΔP_m is added to bus *m* to make the currents between the remainder network to bus *n* and bus *m* zero. Effectively the line *n-m* is isolated from the remainder of the network. This simulates the line *n-m* outage. The injection power $\Delta P_n = - \Delta P_m$, and the new power flow from bus *n* to bus *m* is $P_{nm}^{new} = \Delta P_n$. The vector of ΔP is:

$$\Delta P = \begin{bmatrix} \vdots \\ \Delta P_n \\ \vdots \\ \Delta P_m \end{bmatrix} \tag{4.29}$$

From equations (4.11) and (4.29),

$$\Delta\theta_n = X_{nn}\Delta P_n + X_{nm}\Delta P_m = (X_{nn} - X_{nm})\Delta P_n \tag{4.30}$$

$$\Delta\theta_m = X_{mn}\Delta P_n + X_{mm}\Delta P_m = (X_{nm} - X_{mm})\Delta P_n \tag{4.31}$$

The new power flow from bus *n* to *m* is:

$$P_{nm}^{new} = \frac{1}{x_{nm}}(\theta_n^{new} - \theta_m^{new}) = \frac{1}{x_{nm}}(\theta_n - \theta_m) + \frac{1}{x_{nm}}(\Delta\theta_n - \Delta\theta_m) \tag{4.32}$$

$$P_{nm}^{new} = P_{nm} + \frac{1}{x_{nm}}(X_{nn} + X_{mm} - 2X_{nm})\Delta P_n = \Delta P_n \tag{4.33}$$

Then:

$$\Delta P_n = \frac{x_{nm}}{x_{nm} - (X_{nn} + X_{mm} - 2X_{nm})} P_{nm} \tag{4.34}$$

From equations (4.11) and (4.29),

$$\Delta\theta_i = X_{in}\Delta P_n + X_{im}\Delta P_m = (X_{in} - X_{im})\Delta P_n \tag{4.35}$$

$$\Delta\theta_j = X_{jn}\Delta P_n + X_{jm}\Delta P_m = (X_{jn} - X_{jm})\Delta P_n \tag{4.36}$$

From equations (4.34), (4.35) and (4.36), the sensitivity of power change in line *i-j* that is caused by line *n-m* outage is:

$$LODF_{ij,nm} = \frac{\Delta P_{ij}}{P_{nm}} = \frac{\frac{1}{x_{ij}}(\Delta\theta_i - \Delta\theta_j)}{P_{nm}} = \frac{(X_{in} - X_{im} - X_{jn} + X_{jm})\Delta P_n}{x_{ij} P_{nm}}$$
$$= \frac{x_{nm}(X_{in} - X_{jn} - X_{im} + X_{jm})}{x_{ij}[x_{nm} - (X_{nn} + X_{mm} - 2X_{nm})} \tag{4.37}$$

The power change in line *i-j* caused by line *n-m* outage is:

$$\Delta P_{ij}^{outage} = LODF_{ij,nm} * P_{nm} \tag{4.38}$$

The outage power flow in line *i-j* after line *n-m* outage is

$$P_{ij}^{outage} = P_{ij} + LODF_{ij,nm} * P_{nm} \tag{4.39}$$

LODF can be used to evaluate the effect of outages on power systems. It is very useful for power system security analysis. It is also used in TTC calculations.

4.6.5 Outage Transfer Distribution Factors (OTDFs)

The PTDF values that include outage elements are actually Outage Transfer Distribution Factors (OTDFs). It is the percent of transfer that will flow on a branch after an outage occurs. It is the PTDF with a specific system facility removed from service (outage).

The relationships between these factors can be summarized as: PTDF represents the percent of power change in one transaction shows on the other facilities of the system; LODF represents the percent of the pre-outage power flow of the outaged facility shows on the remaining facilities of the system; OTDF is similar to PTDF. PTDF is used for the pre-outage system, but OTDF is used for the post-outage configuration of the system.

OTDF provides a linear approach of the effect of post outage power flow change in transaction *n-m* on other facilities. OTDF is a function of PTDF and LODF. For one single line outage, the vector of OTDF value for line *i-j* during line *n-m* outage can be obtained from:

$$OTDF_{ij} = PTDF_{ij} + LODF_{ij,nm} * PTDF_{nm} \tag{4.40}$$

During line *n-m* outage, when transaction *q-p* achieves it maximum value, TTC, if line *i-j* arrive its thermal limit. The OTDF of line *i-j* during line *n-m* outage caused by line *q-p* transaction is:

$$OTDF_{ij,qp} = PTDF_{ij,qp} + LODF_{ij,nm} * PTDF_{nm,qp} \tag{4.41}$$

4.6.5.1 Using LODF, OTDF to Calculate TTC with Outages

OTDF can be used to calculate TTC during outages. Similar to (4.25), TTC of transaction *q-p* using LODF and OTDF is:

$$P_{ij}^{outage} + TTC_{qp} * OTDF_{ij,qp} = P_{ij}^{\lim it} \tag{4.42}$$

$$TTC_{qp} = \frac{P_{ij}^{\lim it} - P_{ij}^{outage}}{OTDF_{ij,qp}} \tag{4.43}$$

Using LODF and OTDF to calculate TTC with outages is as simple as using PTDF to calculate TTC in normal operating condition. The X matrix of the power system need not be modified at each outage, and the power flow after outages can be obtained from LODF instead of repeating the power flow calculations. This can greatly reduce the computation time and memory requirements.

4.7 Case Studies Using Distribution Factors

The 7-bus power system used in the OPF study presented in Chapter 3 and the 39-bus power system are used in this study. TTC has been determined using the different factors presented in the previous sections.

4.7.1 Base Case of the 7-Bus Power System

The data of the 7-bus power system considered in this study is given in Appendix 1. The base case loading conditions are shown in Figure 4.2. Transmission line resistances are considered negligible to make the system appear as a lossless system and to simplify the formulation of the B and X matrices of the power system for distribution factors calculation.

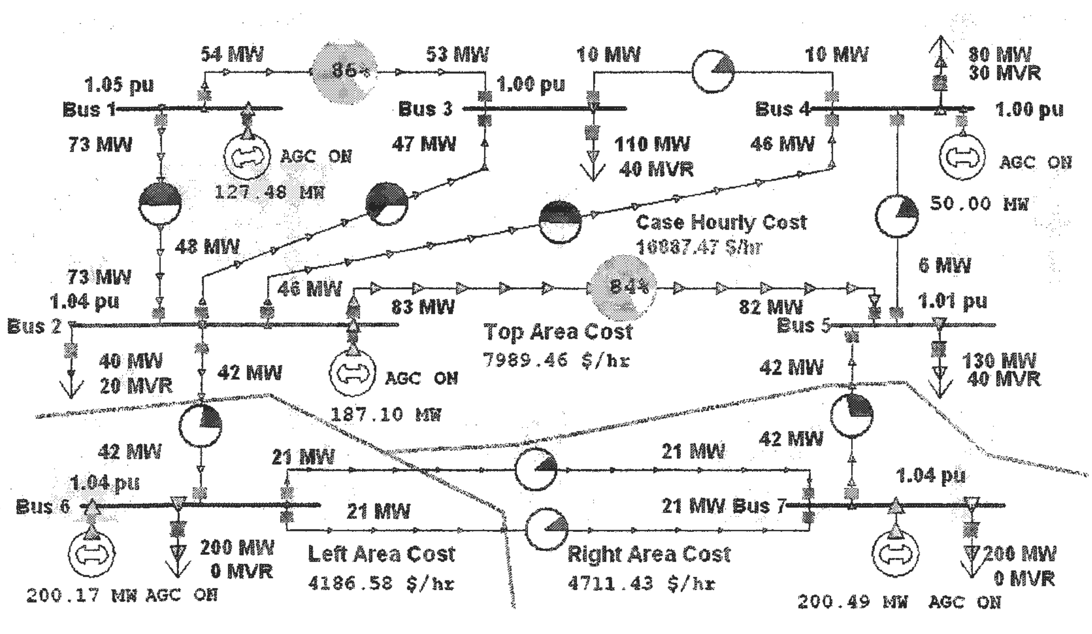

Fig.4.2 Base case loading conditions of the 7-bus power system

4.7.2. Generation Shift Factors (GSFs)

The GSF for the 7-bus system is given in Table 4.1. Bus 7 is considered as the slack bus for this study.

Table 4.1 GSFs of the 7-bus power system

Line	Bus 1 (%)	Bus2 (%)	Bus4 (%)	Bus6 (%)
1-2	81.08	-3.14	13.62	-2.10
1-3	18.92	3.14	-13.62	2.10
2-3	-1.80	5.24	-22.70	3.49
2-4	1.05	6.64	-28.75	4.42
2-5	37.89	39.99	26.72	26.66
2-6	43.94	44.99	38.36	-36.67
3-4	17.11	8.38	-36.32	5.59
4-5	18.16	15.02	34.92	10.01
5-7	56.05	55.01	61.64	36.67
6-7	21.97	22.50	19.18	31.66
6-7	21.97	22.50	19.18	31.66

As an example, the third column of Table 4.3 gives the GSFs for all transmission lines due to the power injected at bus 2. The GSF value for line 2-5 is 39.99% in this condition. If 100 MW active power is injected at bus 2, 39.99MW active power will flow on line 2-5. From this column, it can be observed that line 5-7 has the largest GSF value 55.01%, line 2-6 has the second largest GSF value 44.99% and line 2-5 has the third largest GSF value 39.99%. According to the power flow solution, line 2-5 has the second largest power flow percentage of its thermal limit (84%) under normal operating condition, while the power flow percentage of their thermal limits for line 5-7 and line 2-6 are small. This indicates that line 2-5 has less power flow margin. Hence, line 2-5 will reach its thermal limit first when the power injected at bus 2 is increasing. The base case power flow from bus 2 to 5 is 83.25 MW. The thermal limit of line 2-5 is 100MVA and its active power limit is assumed to be 100 MW as well. To calculate TTC from bus 2 to slack bus 7, from equation (4.17), the following equation is obtained:

$$| 83.25 + TTC_{27} * 0.3999 | = 100 \tag{4.44}$$

This gives TTC from bus 2 to slack bus 7 as 41.89 MW

This TTC result can be verified by PowerWorld Simulator by adding this amount of active power (41.89 MW) to the generator output at bus 2. Figure 4.3 shows the PowerWorld simulation of the 7-bus power system after considering the TTC from bus 2 to 7. It illustrates that line 2-5 reaches its thermal limit after the generation at bus 2 is increased by 41.89 MW. Bus 7, being the slack bus reduces its generation by a similar amount. The results shown in Table 4.1 and TTC value are obtained by Matlab program, and they are verified by PowerWorld Simulator.

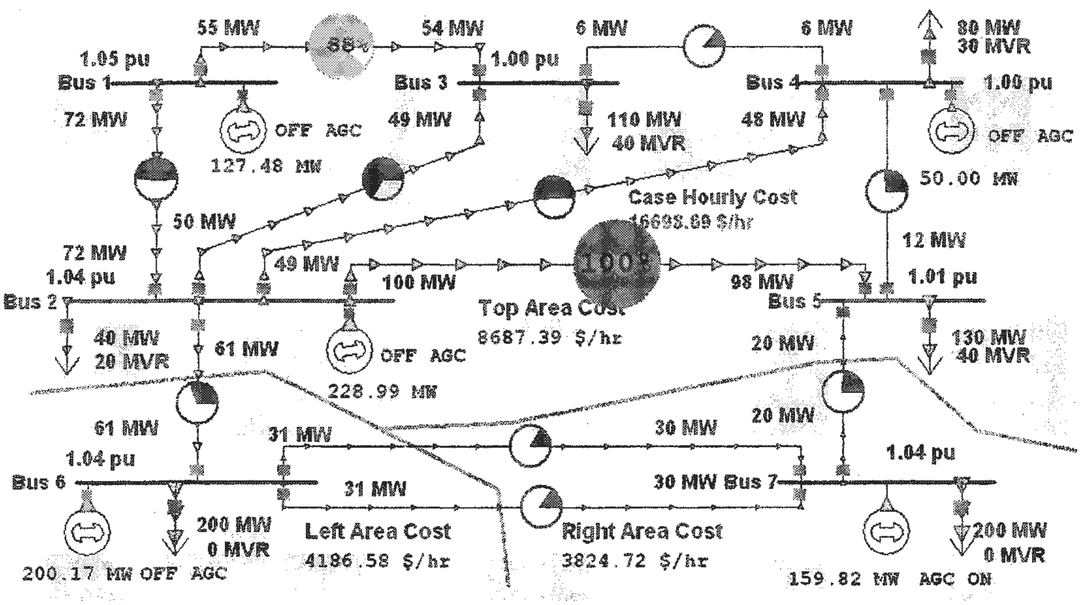

Fig.4.3 TTC from bus 2 to 7 of the 7-bus power system

4.7.3 Power Transfer Distribution Factors (PTDFs)

Using the X matrix obtained from 4.7.1 and equation (4.22), PTDF can be calculated. The PTDFs for the 7-bus power system is given in Appendix 6. Bus 7 is considered as the slack bus in this study. The last column of Appendix 6 gives the PTDF for each transmission line due to the power flow change from bus 6 to 7.

The seventh column of Appendix 6 gives the percent of power flow change between bus 2 to 6 flowing on the other transmission lines. The PTDF for line 2-5 is 13.33% in this condition. When there is 100 MW power flow increment from bus 2 to 6, there will be 13.33 MW power flow increment in line 2-5. It can be observed that the largest PTDF value 81.67% is in line 2-6, the second largest PTDF value 13.33% is in line 2-5. Under normal operating condition, transmission line 1-3 has the largest power flow percentage of its thermal limit (86%) and transmission line 2-5 has the second largest power flow percentage of its thermal limit (84%), which indicates that these two transmission lines have less power flow margin compared to the other transmission lines. The seventh column of Appendix 6 shows that PTDF value is very small in line 1-3 (1.05%) compared with those in line 2-6 (81.67%) and line 2-5 (13.33%). Hence, line 2-5 and line 2-6 are likely to reach their thermal limits when power flow from bus 2 to 6 is increased.

Consider that the line 2-5 to reaches its thermal limits first. The line 2 to 5 base case power flow is 83.25 MW. The thermal limit of line 2-5 is 100MVA and its active power limit is assumed to be 100 MW as well. To calculate TTC from bus 2 to bus 6, from equation (4.26), the following equation is obtained.

$$| 83.25 + \Delta P_{26}^{max} * 13.33\% | = 100 \tag{4.45}$$

ΔP_{26}^{max} is 125.66 MW under this assumption.

Consider that the line 2-6 to reach its thermal limits first. The line 2 to 6 base case power flow is 42.23 MW. The thermal limit of line 2-6 is 200MVA and its active power limit is assumed to be 200 MW as well. To calculate TTC from bus 2 to bus 6, from equation (4.26), the following equation is obtained.

$$| 42.23 + \Delta P_{26}^{max} * 81.67\% | = 200 \tag{4.46}$$

ΔP_{26}^{max} is 193.18 MW under this assumption.

The above calculations show that TTC from bus 2 to 6 has a value of 125.66 MW. Adding this amount of active power (125.66 MW) to the generator output at bus 2, and to the load at bus 6, this TTC result can be verified by PowerWorld Simulator. Figure 4.4 shows the Power World simulation of the 7-bus system under the above conditions. It illustrates that line 2 to 5 reaches its thermal limit after adding 125.66 MW active power flow from bus 2 to 6. The results shown in Appendix 6 and TTC values are obtained by Matlab program, and they are verified by PowerWorld Simulator.

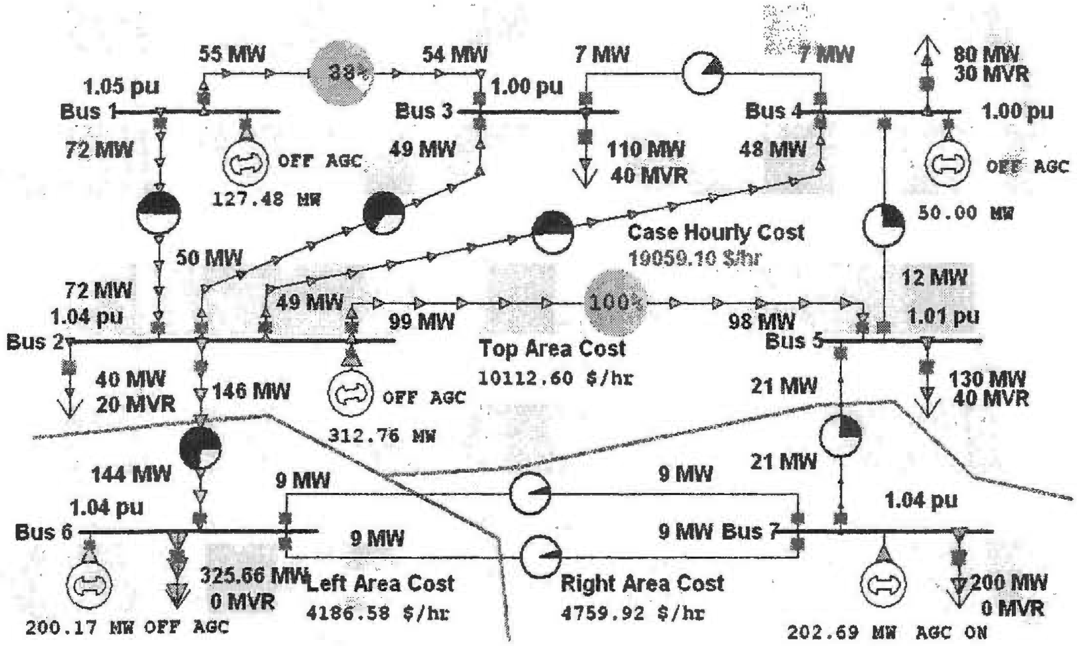

Fig.4.4 TTC from bus 2 to 6 of the 7-bus power system

4.7.3.1 TTC Calculation during Outage by PTDF

Considering line 4-5 outage as an example, the PTDFs for transmission lines due to power flow change from bus 2 to 6 is shown in Appendix 7. Actually they are the OTDFs for the system with this outage. Appendix 7 shows that only the power flow from line 2-5, line 2-6 and line 5-7 will increase if the power flow from line 2-6 increases during line 4-5 outage. Their PTDFs are 16.67%, 83.33% and 16.67% respectively. From power flow solution during line 4-5 outage, line 5-7 has larger power flow margin than those of line 2-5 and line 2-6. So line 2-5 or line 2-6 will reach their thermal limits first when power from bus 2 to 6 is increasing during line 4-5 outage.

Consider that the line 2-5 reaches its limit first. The power flow from bus 2 to 5 during line 4-5 outage is 87.35 MW, which is obtained from power flow solution. Similar to equation (4.45), the following equation is obtained:

$$| 87.35 + \Delta P_{26}^{max} * 16.67\% | = 100 \quad (4.47)$$

ΔP_{26}^{max} is 75.88 MW in this assumption

Consider that the line 2-6 reaches its limit first. The power flow from bus 2 to 6 during line 4-5 outage is 44.3 MW, which is obtained from power flow solution. Similar to equation (4.46), the following equation is obtained:

$$| 44.3 + \Delta P_{26}^{max} * 83.33\% | = 200 \quad (4.48)$$

ΔP_{26}^{max} is 186.85 MW in this assumption

Thus, TTC from bus 2 to 6 is 75.88MW during line 4-5 outage.

4.7.4 Line Outage Distribution Factors (LODFs)

Appendix 8 gives the LODFs of the 7-bus system with different line outages. The ninth column of Appendix 8 gives the LODF for each transmission line during line 4-5 outage. It is observed that that line 2-5 has the maximum LODF value (66.67%). When line 4-5 opens, 66.67% of the pre-outage power flow on line 4-5 will be shown on line 2-5. At the same time the power flow percentage of its thermal limit for line 2-5 is 84%, so its power flow margin is 16%, which is small. These indicate that while the power flow from bus 2 to 6 is increasing, line 2-5 reaches its thermal limit first during line 4-5 outage.

From power flow solution under base case, the pre-outage power flow in line 4-5 is

6.01MW. Then power flow in line 2-5 during line 4-5 outage is:

$$83.25 + 6.01 * 66.67\% = 87.35 \text{ MW} \tag{4.49}$$

4.7.5 Outage Transfer Distribution Factors (OTDFs)

From 4.7.4, line 2-5 is critical for TTC form bus 2 to 6 evaluation during line 4-5 outage. It will reach its thermal limit first while the power from bus 2 to 6 is increasing during line 4-5 outage. From equation (4.41), the OTDF in line 2-5 which is caused by line 4-5 outage can be obtained from following relation:

$$\begin{aligned} OTDF_{25,45} &= PTDF_{25,26} + LODF_{25,45} * PTDF_{26,45} \\ &= 13.3296 + 0.6667*5.0059 \qquad (4.50) \\ &= 16.67\% \end{aligned}$$

That means during line 4-5 outage, if there is 100 MW power flow increment on line 2-6, 16.67 MW power flow will be added on line 2-5. From 4.7.4, the power flow in line 2 to 5 is 87.35 MW during line 4-5 outage. From equation (4.42), the TTC from bus 2 to 6 can be obtained from:

$$| 87.35 + TTC_{26} * 16.67\% | = 100 \tag{4.51}$$

Hence, TTC from bus 2 to 6 during line 4-5 outage is 75.88 MW

The expression of equation (4.51) is the same as that of equation (4.47), but some of the values have different meaning or come from different sources. Here 87.35MW in equation (4.51) is the post-outage power flow and it is calculated by LODF and the pre-outage power flow. The post-outage power flow in equation (4.47) comes from evaluating power flow again by a complete power flow calculation, which may be a set of nonlinear equations if an AC model is applied or a set of linear equations if DC model is applied. The above description obviously shows that LODF and OTDF are very powerful tools in TTC computation during outages. However, PTDF is not efficient to calculate TTC during outages.

In the case of line 4-5 outage, TTC from bus 2 to 6 has a value of 75.88 MW. Adding this amount of active power (75.88 MW) to the generator output at bus 2, and to the load at bus 6, this TTC result can be verified by PowerWorld Simulator. Figure 4.5 shows the PowerWorld simulation of the 7-bus power system under the above conditions. It illustrates that line 2-5 reaches its thermal limit after adding 75.88 MW power flow from bus 2 to 6 during the outage of line 4-5. The results shown in Appendix 8 and TTC value are obtained by Matlab program, and they are verified by PowerWorld Simulator.

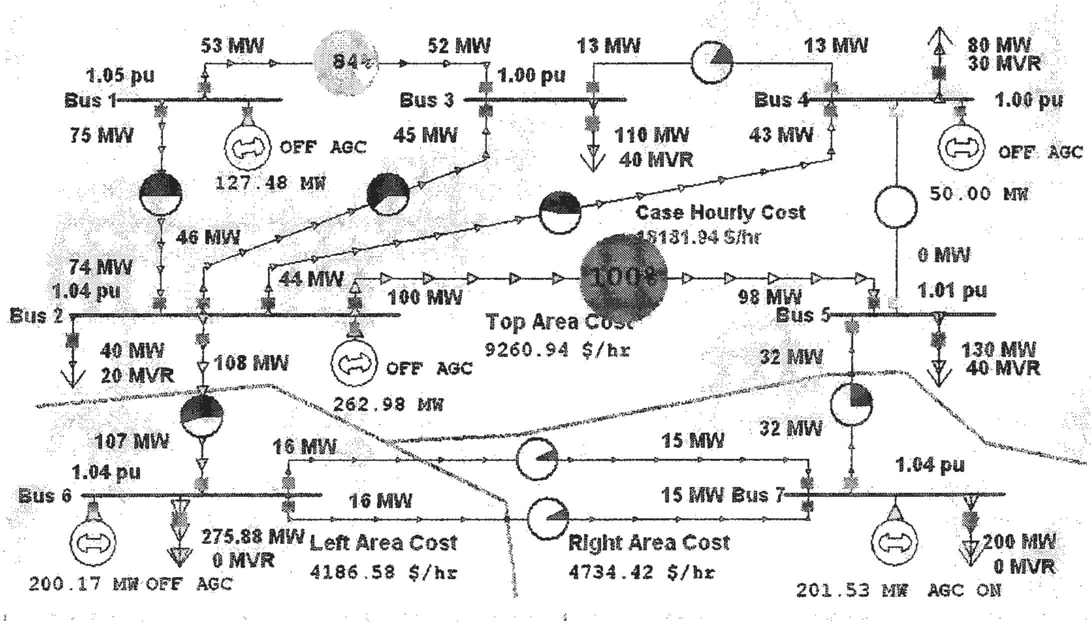

Fig.4.5 TTC from bus 2 to 6 during line 4-5 outage of the 7-bus system

4.7.6 Base Case of the 39-Bus System

The data of the New England 39-bus system considered in this study is taken from TTC calculator [39]. The data are given in Appendix 3. TTC from bus to bus in normal operating condition using PTDF is considered in this study. Fig.4.6 shows the single line diagram of the 39-bus system. Transmission line resistances are considered negligible to make the system appear as a lossless system to simplify the formulation of the B and X matrices of the system for distribution factors calculation.

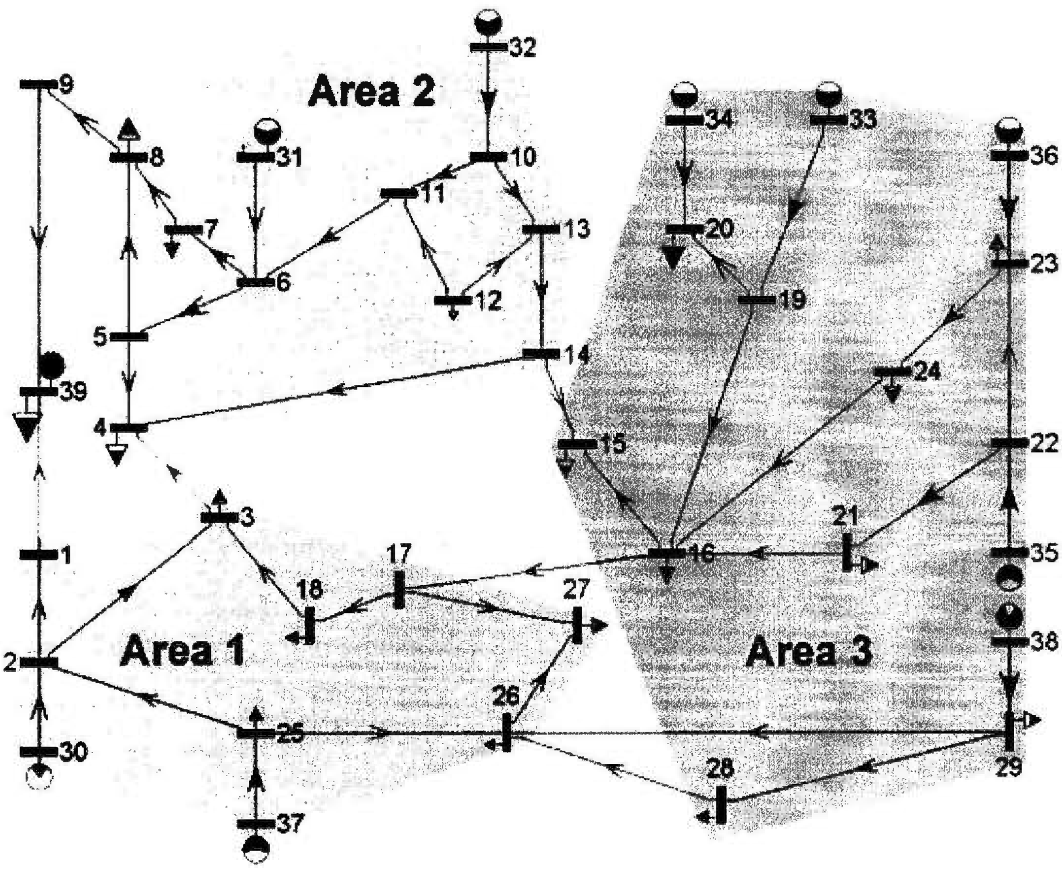

Fig.4.6 Single line diagram of the 39-bus power system

4.7.6.1. TTC Calculation from Bus 36 to 38

Appendix 9 gives the PTDF for each transmission line when the power flow from bus 36 to bus 38 is changing. It can be observed that the largest PTDF value is in line 29-38 (100%), the second largest PTDF absolute value is in line 23-36 (100%) and the third largest PTDF value is in line 16-17 (81.14%). The power flow and power flow percentages of these three lines are –824.45 MW, 33%; -558.34 MW, 22% and 206.5 MW, 36% respectively. The PTDF value for line 29-38 is positive but the power flow from line 29 to 38 is negative. The power flow from bus 29 to bus 38 will decrease when the power from bus 36 to bus 38 increases. Hence, line 16-17 or line 23-36 will reach its thermal limits first when the power from bus 36 to bus 38 increases.

Consider that the line 16-17 reach its thermal limit first when the power flow from bus 36 to 38 is increased. Its active power limit is assumed to be 600 MW. From equation (4.26), the following relations are valid:

$$| 206.5 + \Delta P_{36\text{-}38}^{max} * 81.14\ \% | = 600 \qquad (4.52)$$

$\Delta P_{36\text{-}38}^{max}$ is 484.96 MW

Consider that the line 23-36 reach its thermal limit first when the power flow from bus 36 to 38 is increased. Its active power limit is assumed to be 2500 MW. From equation (4.26), the following relations are valid:

$$|-558.34 - \Delta P_{36\text{-}38}^{max} * 100\% | = 2500 \qquad (4.53)$$

$\Delta P_{36\text{-}38}^{max}$ is 1941.66 MW

The TTC from bus 36 to 38 is 484.96 MW. The PTDFs in Appendix 9 and TTC result are obtained from Matlab program, and they are verified by PowerWorld Simulator.

4.7.6.2 TTC Calculation from Bus 32 to 39

Appendix 10 gives the PTDF for each transmission line when power flow from bus 32 to 39 is changing. Following the method discussed in the previous section, TTC from bus 32 to bus 39 is determined as 227.99 MW. Transmission line 6-11 reaches its limit under this condition.

4.8 Repeated Load Flow

The simplest method of TTC calculation is repeated load flow method. To evaluate TTC from area A to area B, first the power flow of base case is solved. Then the power generation in area A and the corresponding load in area B are increased by a acceptable small step, and the power flow of each step is calculated until one system limit is reached. At this point, the maximum transfer power increment gives the TTC.

The basic steps of repeated load flow method to calculate TTC are:

- Define the initial conditions
- Calculate TTC based on the initial conditions
- Apply system changes to the initial conditions
- Recalculate TTC

Repeated load flow can be very accurate if each step of power increment is made small enough, but this method is not efficient since every solution has to be checked for violation of limits. For example, if 10 iterations are needed for TTC calculation and there are 120 contingencies in each iteration, then 1200 power flow solutions are required. If it requires 30 seconds to solve each power flow for a typical power system, it will need 10 hours to calculate one TTC, considering all the contingencies. However, to compare the results presented earlier, few TTC case studies are done using repeated power flow for the 7-bus power system.

4.9 Case Studies Using Repeated Load Flow

To calculate TTC from bus 2 to 6, the generation power in bus 2 is increased in steps of 0.1MW. In addition, the load in bus 6 is increased at the same step. The power flow is calculated at each step until one transmission line reaches its thermal limit. Here the TTC from bus 2 to 6 is found to be128 MW. Figure 4.7 shows the PowerWorld simulation of the 7-bus system under the above condition. It illustrates that line 2-5 reaches its thermal limit after the generation at bus 2 increased by 128 MW and the load at bus 6 increased by 128 MW. This result is obtained from PowerWorld Simulator.

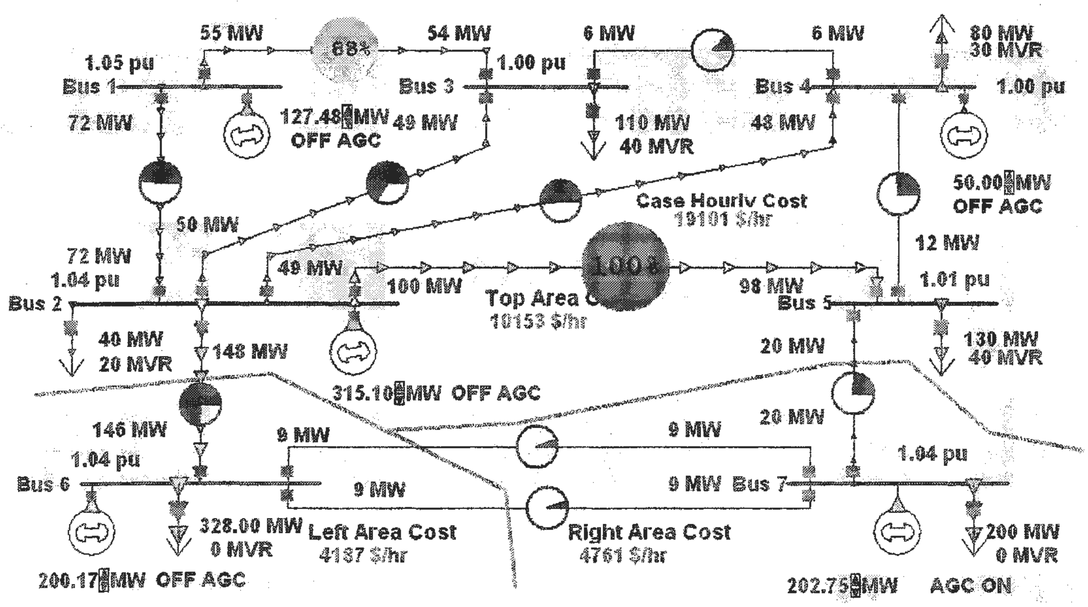

Fig.4.7 TTC from bus 2 to 6 of the 7-bus power system

TTC from bus 2 to 6 is also calculated considering the outage of line between bus 4 and bus 5. Using repeated power flow this TTC is estimated to be 78.2 MW. The simulation for this case using the PowerWorld Simulator is shown in Figure 4.8. This result indicates that when the power system is week (due to the outage of a transmission line), the TTC is reduced significantly.

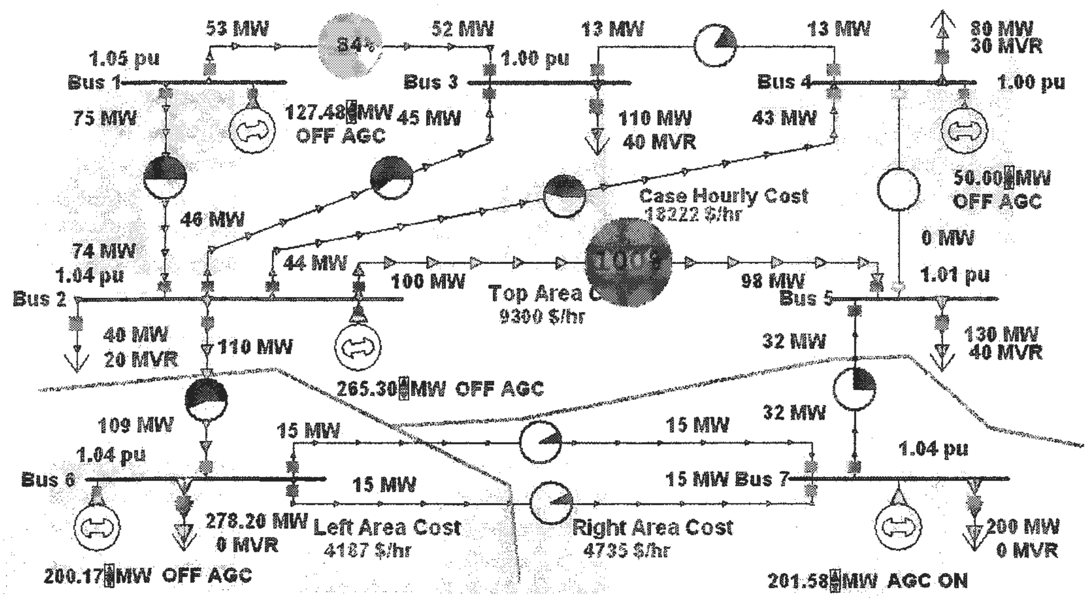

Fig. 4.8 TTC from bus 2 to 6 during line 4-5 outage of the 7-bus system

4.10 Conclusions

The studies presented in this chapter illustrate the effectiveness of the two methods for TTC calculations. Methods based on the DC load flow model are computationally very efficient and are efficient to monitor the transmission systems for contingencies and their system effects. But they are not accurate and cannot monitor voltage and reactive power in power systems. Distribution factors for TTC calculation use DC load flow, which are simple and very efficient to calculate TTC especially during outages. Methods based on AC load flow formulation are computationally slower than the methods based on DC load flow in TTC calculation. Repeated AC load flow method gives accurate results but it is not efficient for large power systems.

The following chapter will focus on applying optimization techniques in TTC evaluation. This technique has the potential to overcome the limitations of the two methods presented in this chapter.

Chapter 5

TRANSFER CAPABILITY EVALUATION USING OPTIMIZATION METHODS

5.1 Introduction

During recent years, the interconnections of power transmission networks have greatly increased and as a result the transactions between different power transmission networks occur frequently. The amount of transferred power has grown at a higher rate than the speed of generation increase [40]. Load growth does not easily stimulate the addition of power transmission facilities as much as before due to environmental and other constraints [41], modern power systems are interconnected to share their resources for the purpose of achieving optimal and reliable operating conditions and assisting each other in emergencies [42]. Transmission Transfer Capability (TTC) is the measure of the strength of a network and it is important for power systems planners to evaluate how far the power system is from the stability limits [43]. Evaluation of TTC has become one of the primary concerns for power systems operation and planning.

The studies presented in this chapter use optimization methods for TTC evaluation. Non-simultaneous and simultaneous transfer capabilities are evaluated under basic operating conditions and during contingencies and TTC from bus to bus and from area to area are calculated. The power systems used in this case studies include the sample 7-bus power system and the New England 39-bus power systems studied in chapter 4 as well as the IEEE Reliability Test 24-bus Power System and IEEE Reliability Test 72-bus Power System.

5.2 Summary of TTC Evaluation Methods

Traditionally, TTC is calculated by power transfer distribution factor methods, which are based on DC power flow models [6]. DC power flow models can consider thermal limits and have been widely used due to their high computation speed [44]. However, DC power flow models cannot consider system voltage limits and reactive power flow, which have played important roles in transfer capability. Distribution factor methods no longer meet the requirements of the modern power systems operation and planning.

AC models are applied more often in modern power systems operation and planning studies due to the increased reactive power influences caused by the unusual heavy transaction in the deregulated environments [45]. Repeated load flow method was used as one of the main methods for TTC calculation [46]. This method is based on full AC power flow solutions, which accurately determines the effects of reactive power flows, voltage limits and the line flows [7]. However, the disadvantage of this method is the long execution time due to repeated load flow calculation until certain systems or equipment limits are reached.

Electric Power Research Institute (EPRI) developed a software package to solve TTC using a method, which is based on optimal power flow (OPF) solutions. The method maximizes the specified power transfer by changing control such as: tap-changers and switching variables, subject to the power flow equilibrium and limit constraints [47].

Since TTC problems are optimization problems and the objective functions are to maximize the total generation supplied and load demanded between areas, TTC can be obtained by optimization techniques. The optimization formulations are based on OPF, which re-dispatches power generation and load distribution of specified sources and sinks as well as reactive power generation to avoid any overload or other violations. For the research presented in this thesis, Sequential Quadratic Programming (SQP) method is selected as the optimization technique since it is recently developed and has proven to be an effective method for constrained nonlinear programming.

5.3 Non-simultaneous Transfer Capability

To maintain system reliability while serving a wide range of transmission transactions, TTC for critical transmission paths should be continuously computed and updated following system condition changes [48]. TTC is a function of the strength of the transmission network and is affected by contingencies, reactive power, load distributions and generation locations. Many sophisticated techniques in power system analysis such as optimal power flow, contingency analysis and voltage stability analysis should be combined to solve this problem. To initialize a TTC study, the power transfers from a single source to a single sink are only considered [49].

5.4 TTC Evaluation under Base Case Operating Condition

TTC is related to the power system operating conditions. The solution of the base case is the starting point for TTC evaluation. Normally base transfers represent electrical power transfers between areas that are modeled under base case. This refers to the additional amount of electric power that can be transferred above the base level with all facilities in service and without any violation. A point to point transfer from area A to area B is specified by increasing power at the source bus in area A and reducing power at the sink bus in area B [50]. The formulations of TTC from single area to single area are given below:

The objective function is to calculate the maximum active power increment that can be transferred between two areas. This equals to the minimum of the negative value of this function.

$$T = -\sum_{i=1}^{ag} \Delta T_i \tag{5.1}$$

Similar to the equations of OPF equality constraints, the equality constraints of TTC problems are the power balance at each node:

$$\sum_{i=1}^{m} P_i = 0 \qquad i = 1, 2, 3 \ldots m \tag{5.2}$$

$$\sum_{i=1}^{m} Q_i = 0 \qquad i = 1, 2, 3 \ldots m \tag{5.3}$$

Similar to the equations of OPF inequality constraints, the inequality constraints are the generation limits, power flow limits, bus voltage limits and control variable limits:

$$P_{Gi\min} \le P_{Gi} \le P_{Gi\max} \qquad i = 1, 2, 3 \ldots n \tag{5.4}$$

$$P_{ij\min} \le P_{ij} \le P_{ij\max} \qquad i, j = 1, 2, 3 \ldots m \tag{5.5}$$

$$Q_{ij\min} \le Q_{ij} \le Q_{ij\max} \qquad i, j = 1, 2, 3 \ldots m \tag{5.6}$$

$$V_{i\min} \le V_i \le V_{i\max} \qquad i = 1, 2, 3 \ldots m \tag{5.7}$$

$$C_{i\min} \le C_i \le C_{i\max} \qquad i = 1, 2, 3\ldots k \qquad (5.8)$$

Where:

$$\sum_{i=1}^{ag} \Delta T_i = \sum_{j=1}^{bl} \Delta TL_j \qquad (5.9)$$

$$P_{Gai} = P_{Gai}' + \Delta T_i \qquad i = 1, 2,\ldots ag \qquad (5.10)$$

$$P_{Lbj} = P_{Lbj}' + \Delta TL_j \qquad j = 1, 2,\ldots bl \qquad (5.11)$$

- T is the TTC from area A to B

ΔT_i is the additional active power transfer from generation bus i in area A to area B

ΔTL_j is the active load increment at bus j in area B

P_{Gai}' is the generation active power at bus i in area A before considering TTC

P_{Gai} is the generation active power at bus i in area A after considering TTC

P_{Lbj}' is the active load at bus j in area B before considering TTC

P_{Lbj} is the active load at bus j in area B after considering TTC

P_{Gi} is the generation power at bus i

V_i is the voltage at bus i

P_i and Q_i are the power flows at bus i

P_{ij} and Q_{ij} are the power flows from bus i to j

C_i is the control variable

ag is the number of generators in area A

bl is the number of loads in area B

n and m are the number of generators and buses in the power system

k is the number of control variables in the power system

Equation (5.1) shows that the total generation active power increment is TTC.

Equation (5.9) shows that the total generation active power increment in the source area A equals to the total active load increment in the sink area B. The constraints make the results meet the requirements of system conditions and equipment limits.

Matlab provides an efficient platform for development of application programs. The objective functions and constraints are written in separate '*m*' files. After initial variables are set, the command '*fmincon*' is used in the main program to call these functions to perform the calculation.

5.5 Case Studies

The sample 7-bus power system and the 39-bus power systems used in TTC study presented in Chapter 4 and IEEE Reliability Test 24-bus and 72-bus Systems are used in this study. Matlab optimization toolbox is used in TTC calculation in the study presented in this chapter. Sequential Quadratic Programming is used by Matlab optimization toolbox for nonlinear constrained optimization problems.

5.5.1 TTC from Single Bus to Single Bus

To evaluate TTC from single bus to single bus, the generation power at the sending bus is increased and the same amount of active power is increased at the load of receiving bus, and then the maximum generation power increment in this transmission path without any system limit violation is calculated. The formulations of TTC from bus x to bus y are stated as followings:

The objective function is stated as:

$$T = -\Delta T_{xy} \tag{5.12}$$

The equality constraints are the power balance at each node:

$$\sum_{i=1}^{m} P_i = 0 \qquad i = 1, 2, 3\ldots m \tag{5.13}$$

$$\sum_{i=1}^{m} Q_i = 0 \qquad i = 1, 2, 3\ldots m \tag{5.14}$$

The inequality constraints include followings:

$$P_{Gi\min} \le P_{Gi} \le P_{Gi\max} \qquad i = 1, 2, 3\ldots n \tag{5.15}$$

$$P_{ij\min} \le P_{ij} \le P_{ij\max} \qquad i, j = 1, 2, 3\ldots m \tag{5.16}$$

$$Q_{ij\min} \le Q_{ij} \le Q_{ij\max} \qquad i, j = 1, 2, 3\ldots m \tag{5.17}$$

$$V_{i\min} \le V_i \le V_{i\max} \qquad i = 1, 2, 3\ldots m \tag{5.18}$$

$$C_{i\min} \le C_i \le C_{i\max} \qquad i = 1, 2, 3\ldots k \tag{5.19}$$

Where: $P_{Gx} = P_{Gx}' + \Delta T_{xy}$ (5.20)

$$P_{Ly} = P_{Ly}' + \Delta T_{xy} \tag{5.21}$$

ΔT_{xy} is the TTC from bus x to bus y

P_{Gx}' is the generation active power at bus x before considering TTC

P_{Gx} is the generation active power at bus x after considering TTC

P_{Ly}' is the active load at bus y before considering TTC

P_{Ly} is the active load at bus y after considering TTC

P_{Gi} is the generation power at bus i

V_i is the voltage at bus i

P_i and Q_i are the power flows at bus i

P_{ij} and Q_{ij} are the power flows from bus i to j

C_i is the control variable

n and m are the number of generators and buses in the power system

k is the number of control variables in the power system

Equation (5.20) and equation (5.21) shows that the generation power increment at sending bus x is the same as the load increment at receiving bus y, which is the TTC from bus x to bus y.

Similar to the OPF calculation, the objective functions and constraints are written in separate '*m*' files. A main program is used to call these functions to perform the required calculations.

5.5.1.1 7-Bus Power System

The data of this 7-bus System are given in Appendix 1. The base case loading condition of the 7-bus power system is shown in Figure 5.1. Here bus 7 is considered as the slack bus. The active generation power at bus 2 and bus 7 are 187.10MW and 200.49MW. The active load at bus 6 is 200MW.

Consider TTC from bus 2 to bus 7 as an example, the TTC from bus 2 to bus 7 under base case is 41.51MW obtained from Matlab program. This amount of generation active power is added to the bus 2 and this TTC result can be verified by PowerWorld Simulator as shown in Figure 5.2. Figure 5.2 illustrates that in case of TTC from bus 2 to 7, line 2-5 reaches its limits, all facilities are in service and no violation occurs.

Consider TTC from bus 2 to bus 6 as another example, the TTC from bus 2 to bus 6 under base loading conditions is 128.06MW. After this amount of active power is added to the generator output at bus 2 and to the load at bus 6, this TTC can be verified by PowerWorld Simulator as shown in Figure 5.3. Figure 5.3 illustrates that in the case of TTC from bus 2 to 6, line 2-5 reaches its limits, all facilities are in service and no violation occurs.

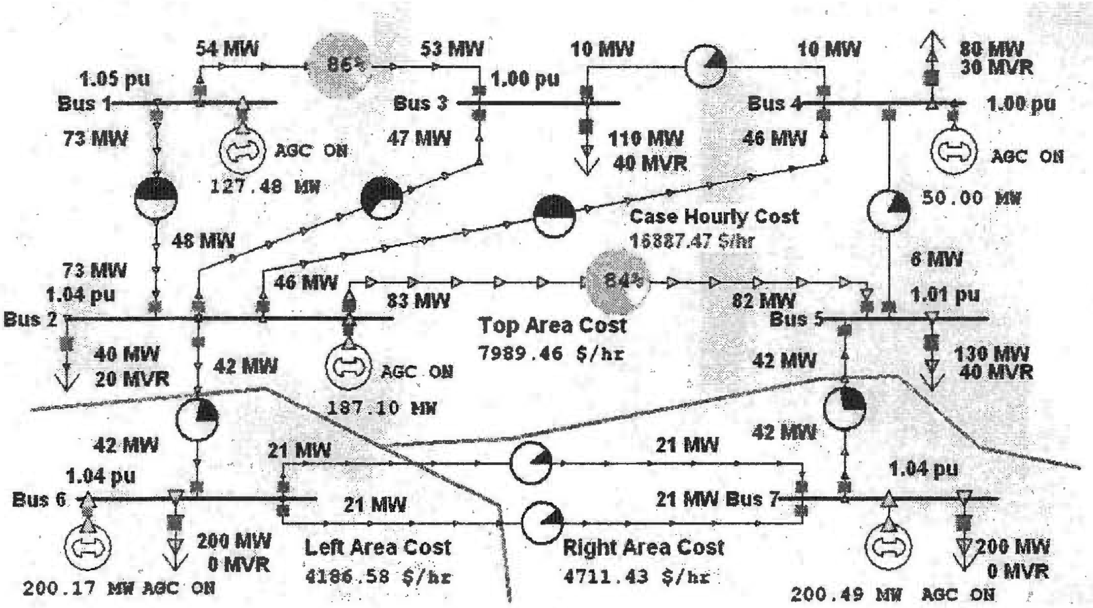

Fig. 5.1 Base case operating conditions of the 7-bus system

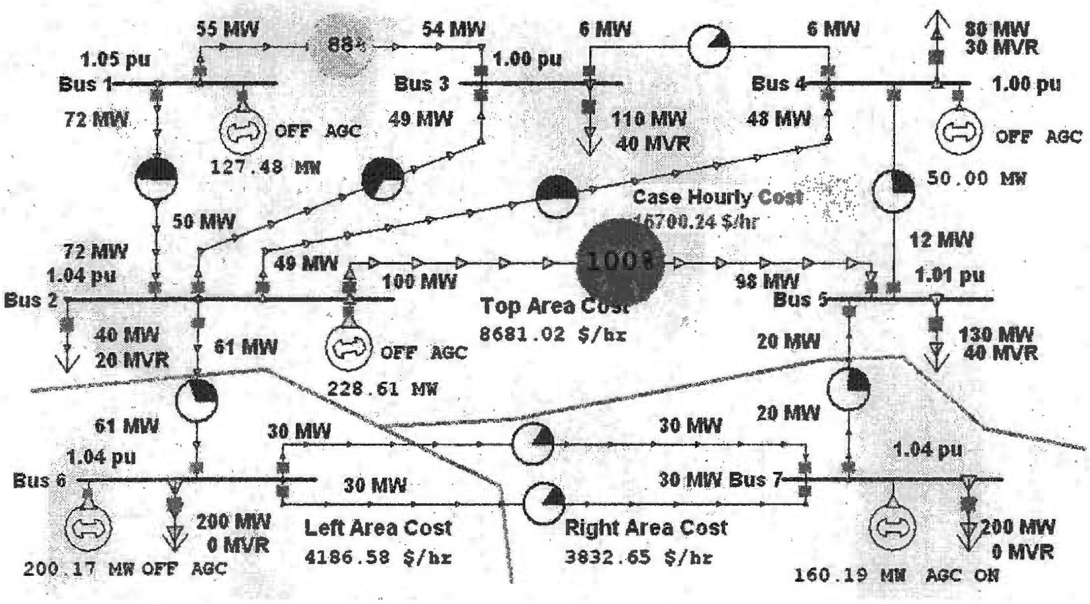

Fig. 5.2 TTC from bus 2 to 7 of the 7-bus power system

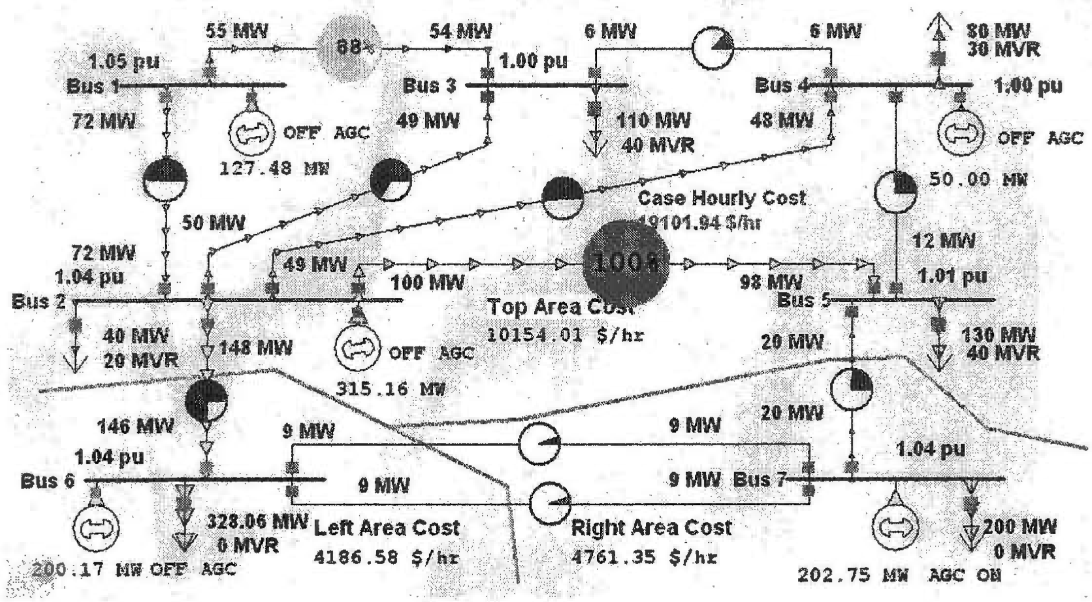

Fig. 5.3 TTC from bus 2 to 6 of the 7-bus power system

5.5.1.2 39-Bus Power System

The data of this 39-bus power system are given in Appendix 3. Figure 5.4 gives the single line diagram of the 39-bus power system. Bus 31 is considered as the slack bus in this study. Table 5.1 gives the some of the single bus to bus TTC results of the 39-bus system. Consider TTC from bus 36 to bus 38 as an example. Table 5.1 shows that after 484.25 MW active power is added in the generation output at bus 36 and the load at bus 38, line 16-17 reaches its thermal limit. Hence 484.25 MW is the TTC from bus 36 to 38. These TTC results in Table 5.1 are obtained from Matlab program and are verified by PowerWorld Simulator.

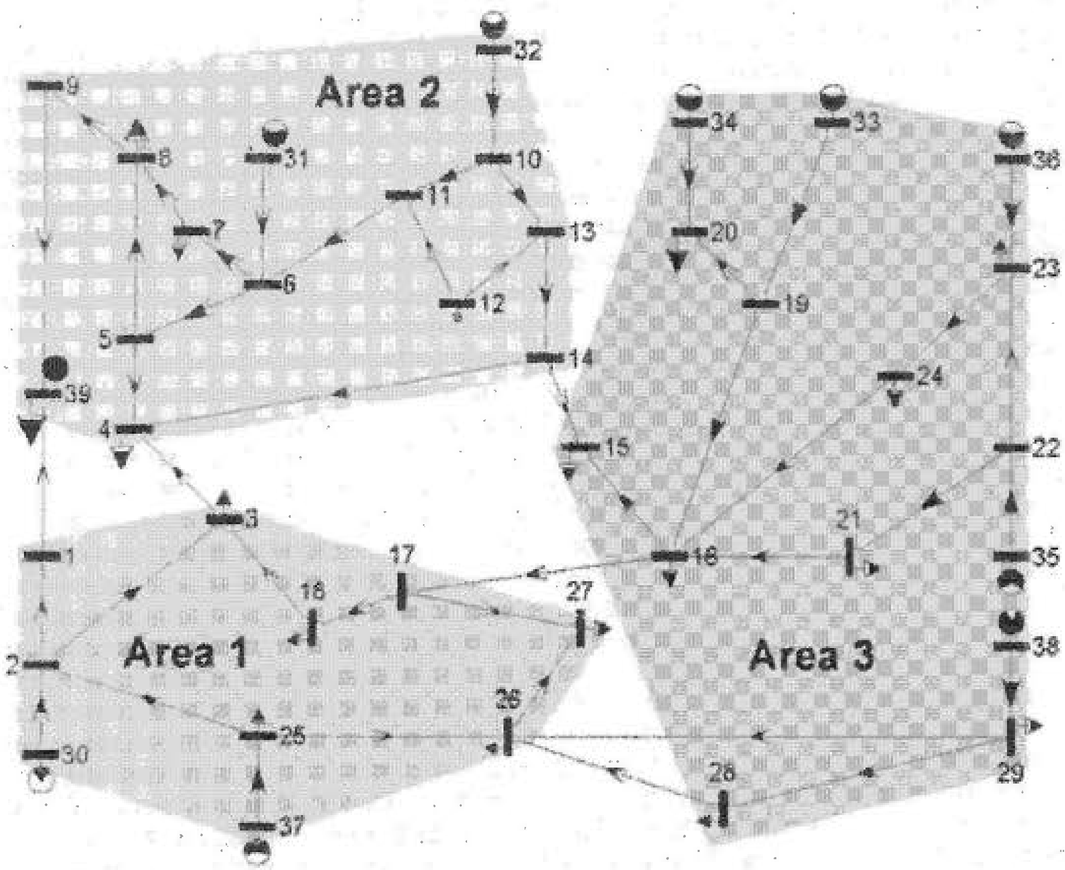

Fig.5.4 Single line diagram of the 39-bus power system

Table 5.1 TTC single bus to bus results of the 39-bus system

From bus	To bus	TTC (MW)	Limiting Facilities
36	38	484.25	Line 16-17
32	39	229.6	Line 6-11

5.5.1.3 IEEE Reliability Test 24-Bus Power System

The data of this IEEE Reliability Test 24-bus Power System (RTS) are given in Appendix 4. Figure 5.5 gives the single line diagram of the 24-bus power system. Table 5.2 and Table 5.4 give the different generation schedules of the 24-bus power system.

Table 5.3 and Table 5.5 give the TTC from single bus to single bus results of the 24-bus power system under these two different generation schedules. Consider TTC from bus 21 to bus 16 as an example. For generation schedule 1 the TTC is 620.72 MW and line 16-17 reaches its limit in this case. For generation schedule 2 the TTC reduces to 494.18 MW and line 16-17 reaches its limit in this case. The TTC results in these two generation schedules are different. This shows that TTC will change if the base case operating conditions change. The TTC results in Table 5.3 and Table 5.5 are obtained from Matlab program and they are verified by PowerWorld Simulator.

Table 5.2. Generation schedule 1 of the RTS 24-bus power system

Generator Bus	1	2	7	13	15	16	18	21	22	23
Gen. Power (MW)	152	152	240	472	155	155	400	400	102.4	660

Table 5.3. TTC of the RTS 24-bus power system under generation schedule 1

From bus	To bus	TTC (MW)	Limiting Facilities
23	15	747.91	Line 15-16
21	16	620.72	Line 16-17
1	9	351.66	Line 1-2

Table 5.4. Generation schedule 2 of the RTS 24-bus power system

Generator Bus	1	2	7	13	15	16	18	21	22	23
Gen. Power (MW)	172	172	240	285.3	215	155	400	400	195.5	660

Table 5.5. TTC of the RTS 24-bus power system under generation schedule 2

From bus	To bus	TTC (MW)	Limiting Facilities
23	15	850.31	Line 15-16
21	16	494.18	Line 16-17
1	9	346.25	Line 1-2

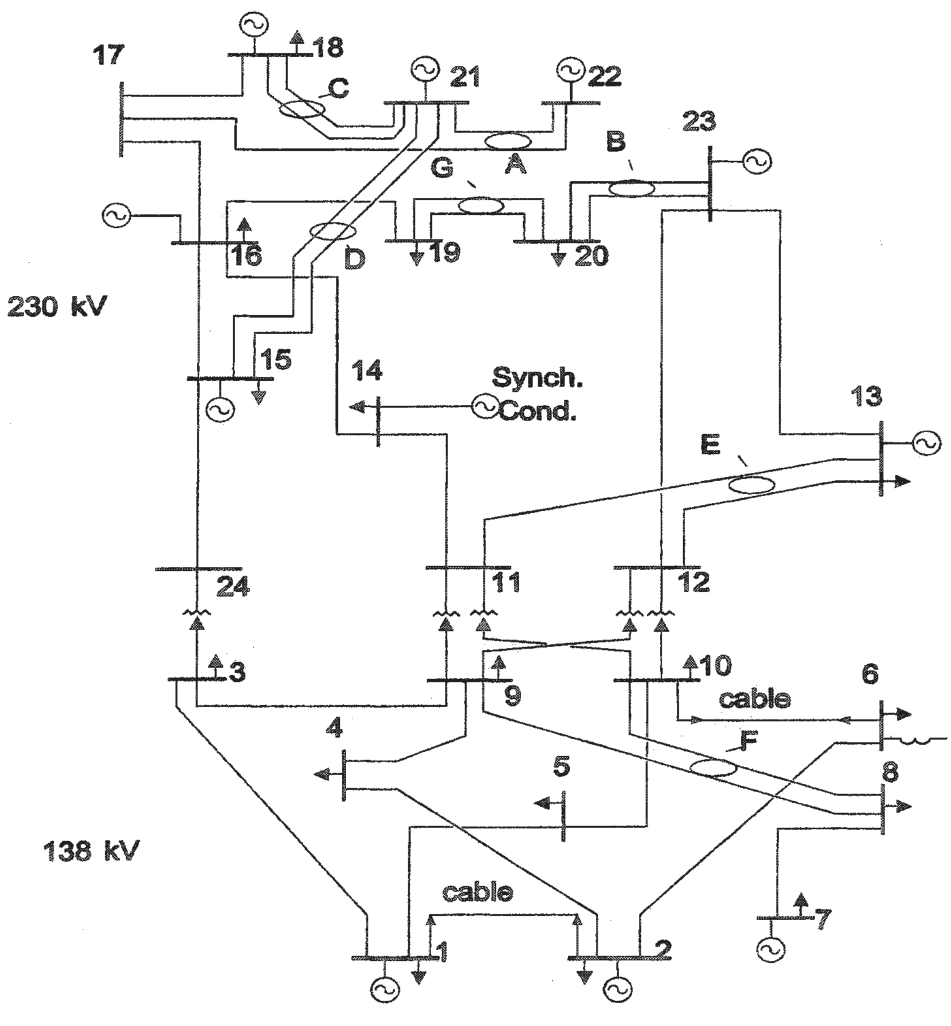

Fig.5.5. Single line diagram of the IEEE RTS 24-bus power system

5.5.2 TTC from Single Area to Single Area

The area in TTC studies presents the configuration of generating stations, switching stations, substations and connecting transmission lines that may define an individual electric system [31]. Similar to the TTC evaluation from single bus to single bus, to evaluate TTC from single area to single area, first the generation power is increased in the sending area and the same total amount of active power is increased at the loads of receiving area, then the maximum generation power increment in the sending area with no violation occurs in the system is calculated. The maximum generation power increment in sending area or the maximum active load increment in receiving area is TTC. It will be dispatched at different rates to the generator buses in the sending area and the load in receiving area. These TTC evaluation formulations are given in the section 5.4.

5.5.2.1 7-Bus Power System

There are three areas (top area, left area and right area) in this 7-bus power system. Table 5.6 gives the power increment distribution of TTC from different area to area for the 7-bus system. For example, from Matlab program, the TTC from top area to left area of the 7-bus power system is obtained as 202.64MW. This amount of active power is distributed to the generator buses in sending area and the load buses in receiving area according to the calculation results in Table 5.6. The generation power at bus 1 in top area increases 62.7MW, the generation power at bus 2 in top area decreases 10.07MW and the generation power at bus 4 in top area increases 150MW. The total generation power increment in top area is 202.64MW. The total load increment is also 202.64MW. This TTC result can be verified by PowerWorld Simulator. Figure 5.6 illustrates the TTC results from top area to left area of the 7-bus system in PowerWorld. In this case, line 1-2 and line 2-6 reach their thermal limits.

Table 5.6 7-bus power system single area to single area TTC

From Area – to Area	Power Increment (MW)				TTC (MW)	Limiting facilities
	Gen.1	Gen.2	Gen.4	Gen.6		
Top – Left	62.7	-10.07	150	0	202.63	Line 2-5
Top – Right	–24.82	-37	150	0	88.18	Line 2-5
Left – Right	0	0	0	62.09	62.09	Line 2-5

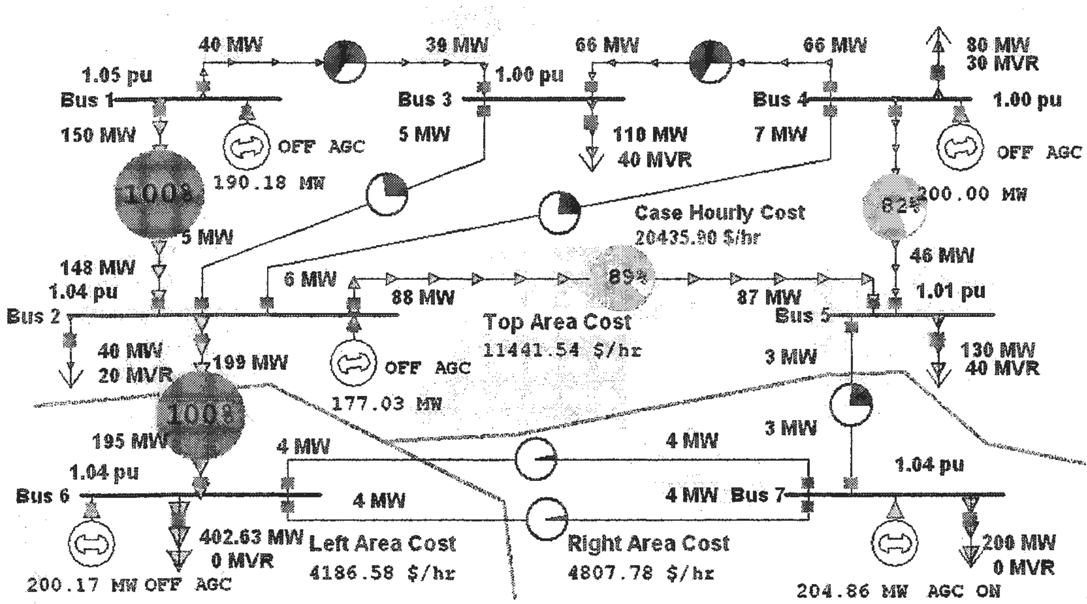

Fig. 5.6 TTC from top area to left area of the 7-bus power system

5.5.2.2 39-Bus Power System

There are three areas (area 1, area 2 and area 3) in the 39-bus power system. The TTC from single area to single area results of the 39-bus power system are given in Table 5.7. Consider TTC from area 1 to area 3 as an example, it is 697.8 MW, and line 16-15 will reach its thermal limit when this TTC result is applied to the system.

Table 5.7 39-bus power system single area to single area TTC

From area	To area	TTC (MW)	Limiting Facilities
1	3	697.8	Line 16-15
1	2	615.15	Line 2-25
3	2	990.53	Line 4-14

5.5.2.3 IEEE Reliability Test 72-Bus Power System

The IEEE Reliability Test 72-bus System is also used in this study. The data of this 72-bus System are obtained from a report prepared by the Reliability Test System Task Force and are given in Appendix 5 [51]. The single line diagram of the 72-bus System is shown in Appendix 5, Figure A.5. Figure 5.7 shows the simplified diagram of the interconnection of this 72-bus System. There are three tie lines between area 1 and area 2, one tie line between area 1 and area 3 and one tie line between area 2 and area 3. Table 5.7 gives the TTC results from single area to single area of the 72-bus system. Consider TTC from area 1 to area 2 as an example. The TTC from area 1 to area 2 is 2195.7 MW and tie line 107-203 (between area 1 and area 2) reaches its thermal limits in this case.

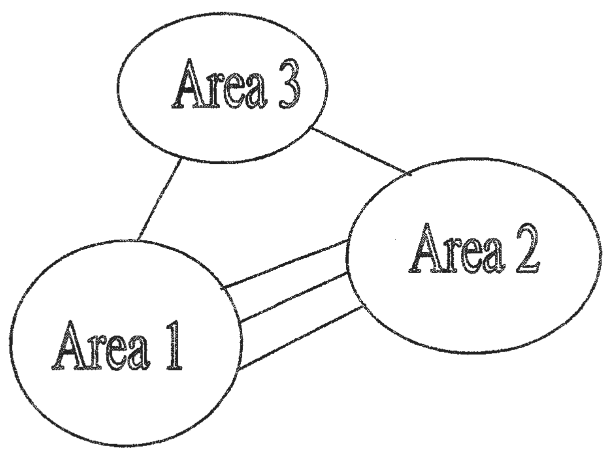

Fig.5.7 Simplified diagram of the RTS 72-bus system interconnection

Table 5.8 TTC from single area to single area of RTS 72-bus power system

From Area	To Area	TTC (MW)	Limiting Facilities
1	2	2195.7	Line 107-203
1	3	1610.1	Line 121-323
2	3	1645.6	Line 318-223

5.6 TTC Evaluation during Outages

It is desirable that the operating condition of a power system is acceptable even after an outage of a transmission line or a transformer. Hence, contingencies must also be considered in TTC evaluation. Not only the constraints under base case conditions, but also the constraints in the event of outages should be included. TTC value under outages is less than or equal to that under base case loading conditions.

Consider only one specified single component outage as an example. The objective function of TTC from area A to area B during one outage is sated as:

$$T = -\sum_{i=1}^{ag} \Delta T_i \qquad (5.22)$$

Similar to equality constraints equations of security constrained optimal power flow (SCOPF), the equality constraints are the power balance at each node in base case operating condition:

$$\sum_{i=1}^{m} P_i = 0 \qquad i = 1, 2, 3\ldots m \qquad (5.23)$$

$$\sum_{i=1}^{m} Q_i = 0 \qquad i = 1, 2, 3\ldots m \qquad (5.24)$$

The power balance at each node in contingency states also must be included:

$$\sum_{i=1}^{m} P_i' = 0 \qquad i = 1, 2, 3 \ldots m \qquad (5.25)$$

$$\sum_{i=1}^{m} Q_i' = 0 \qquad i = 1, 2, 3 \ldots m \qquad (5.26)$$

The inequality constraints are the generation limits, the power flow limits, the bus voltage limits and control variable limits in base case operating conditions:

$$P_{Gi\min} \le P_{Gi} \le P_{Gi\max} \qquad i = 1, 2, 3 \ldots n \qquad (5.27)$$

$$P_{ij\min} \le P_{ij} \le P_{ij\max} \qquad i, j = 1, 2, 3 \ldots m \qquad (5.28)$$

$$Q_{ij\min} \le Q_{ij} \le Q_{ij\max} \qquad i, j = 1, 2, 3 \ldots m \qquad (5.29)$$

$$V_{i\min} \le V_i \le V_{i\max} \qquad i = 1, 2, 3 \ldots m \qquad (5.30)$$

$$C_{i\min} \le C_i \le C_{i\max} \qquad i = 1, 2, 3 \ldots k \qquad (5.31)$$

In addition, the generation limits, the power flow limits, the bus voltage limits and control variable limits in contingency states are also included:

$$P_{Gi\min} \le P_{Gi}' \le P_{Gi\max} \qquad i = 1, 2, 3 \ldots n \qquad (5.32)$$

$$P_{ij\min} \le P_{ij}' \le P_{ij\max} \qquad i, j = 1, 2, 3 \ldots m \qquad (5.33)$$

$$Q_{ij\min} \le Q_{ij}' \le Q_{ij\max} \qquad i, j = 1, 2, 3 \ldots m \qquad (5.34)$$

$$V_{i\min} \le V_i' \le V_{i\max} \qquad i = 1, 2, 3 \ldots m \qquad (5.35)$$

$$C_{i\min} \le C_i' \le C_{i\max} \qquad i = 1, 2, 3 \ldots k \qquad (5.36)$$

Where:

$$\sum_{i=1}^{ag} \Delta T_i = \sum_{j=1}^{bl} \Delta TL_j \qquad (5.37)$$

$$P_{Gai} = P_{Gai}'' + \Delta T_i = P_{Gai}' \qquad i = 1, 2, 3 \ldots ag \qquad (5.38)$$

$$P_{Lb\,j} = P_{Lb\,j}'' + \Delta TL_j = P_{Lb\,j}' \qquad j = 1, 2, 3 \ldots bl \tag{5.39}$$

- T is the TTC from area A to B

ΔT_i is the additional power transfer from generation bus i in area A to area B

ΔTL_j is the load increment at load bus j in area B

P_{Gai}'' is the active generation power at bus i in area A in basic operating conditions before considering TTC

P_{Gai} is the active generation power at bus i in area A in basic operating conditions after considering TTC

P_{Gai}' is the active generation power at bus i in area A in contingency states after considering TTC

P_{Lbj}'' is the active load at bus j in area B in basic operating conditions before considering TTC

P_{Lbj} is the active load at bus j in area B in basic operating conditions after considering TTC

P_{Lbj}' is the active load in area B bus j in contingency states after considering TTC

P_{Gi} is the generation power at bus i in base case operating conditions

P_{Gi}' is the generation power at bus i in contingency states

P_i and Q_i is the power flow at bus i in base case operating conditions

P_i' and Q_i' is the power flow at bus i in contingency states

P_{ij} and Q_{ij} are the power flow from bus i to bus j in base case operating conditions

P_{ij}' and Q_{ij}' are the power flow from bus i to bus j in contingency states

V_i is the voltage at bus i in base case operating conditions

V_i' is the voltage at bus i in contingency states

C_i is the control variable in base case operating conditions

C_i' is the control variable in contingency states

ag is the number of generators in area A

bl is the number of loads in area B

n is the number of generators the power system

m is the number of buses in the power system

k is the number of control variables in the power system

Equations (5.38) and (5.39) show that the active generation power in sending area and the active load in receiving area keep constant after contingencies happened to ensure a fixed amount of power transferred between areas in normal condition and during contingencies.

The number of contingencies will determine the size of the TTC problem. If there are n contingencies are considered, the number of constraints of TTC problem considering outages is approximate $n \oplus 1$ times of the number of constraints of TTC problem without considering any contingencies.

In order to evaluate the TTC that maintains acceptable operating condition for the power system under normal operating conditions and during any single component outage, all the constraints of all different outages should be considered. The desired TTC value is the minimum value of all TTC results considering one specified single component outage. With this amount of additional power transfer, the power system can be operated without any violation under base case operating conditions and during any single component outage.

5.7 Case Studies

Only single line outage is considered for the 7-bus power system and the 24-bus power system in this study. The results are given in the following sections.

5.7.1 7-Bus Power System

In the base case operating conditions of the 7-bus system, one or more violations will occur in the system during transmission line 1-2, or 2-3, or 2-4, or 2-6, or 5-7 outages. Hence, these lines outages are not considered in this study. Table 5.9 shows the TTC from bus 2 to bus 6 under single transmission line outages. It can be observed that different outages have different effects on power system TTC values. In general, the TTC value with outages is less than the TTC value without outages. Consider line 4-5 outage as an example. The TTC from bus 2 to bus 6 is 78.26 MW. With this amount of generation power increment, the system can be operated without violations under base case and during line 4-5 outage. The TTC results shown in Table 5.9 are obtained from Matlab program and are verified by PowerWorld Simulator. Figure 5.8 illustrates the TTC from bus 2 to bus 6 during line 4-5 outage. Figure 5.8 shows that during line 4-5 outage, line 2-5 reaches its thermal limit after adding 78.26 MW active power to the generation output at bus 2 and to the load at bus 6.

Table 5.9 TTC from bus 2 to bus 6 under one single transmission line outage

Outaged transmission line	TTC (MW)	Limiting facility
line 1-3	75.69	Line 2-5
line 2-5	91.09	Line 2-5
line 3-4	128.06	Line 2-5
line 4-5	78.26	Line 2-5
line 6-7	131.2	Line 2-5

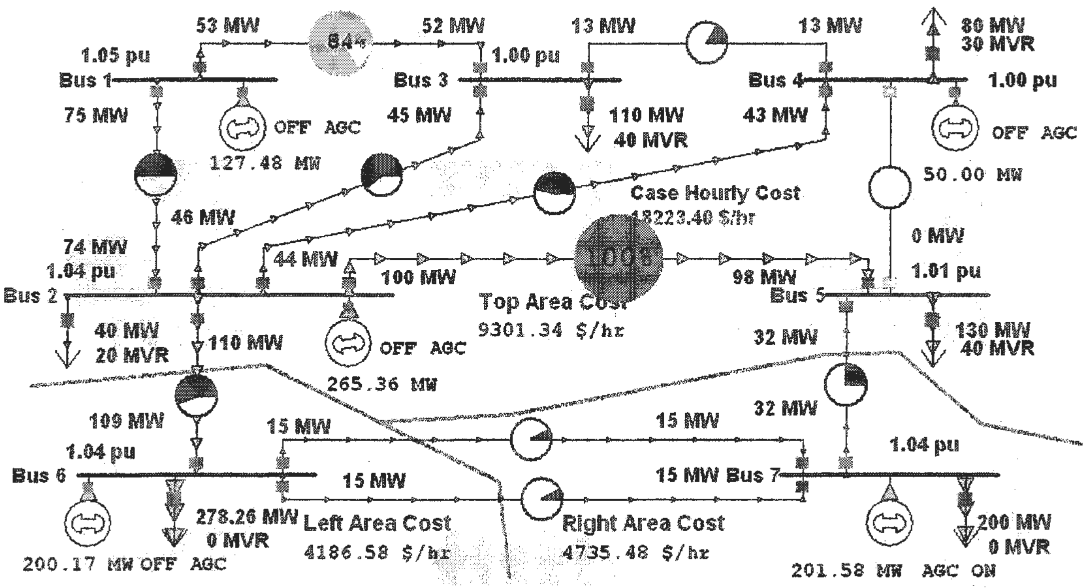

Fig.5.8 TTC from bus 2 to bus 6 during line 4-5 outage

Table 5.9 shows that the minimum TTC value shown is 75.69MW corresponding to line 1-3 outage. Thus 75.69 MW is the TTC that no violation will occurs during any single line outage. Figure 5.9 illustrates the TTC from bus 2 to bus 6 results during line 1-3 outage in PowerWorld Simulator. Figure 5.9 shows that with the outage of line 1-3, TTC from bus 2 to 6 is 75.69 MW and line 2-5 reaches its thermal limit in this case.

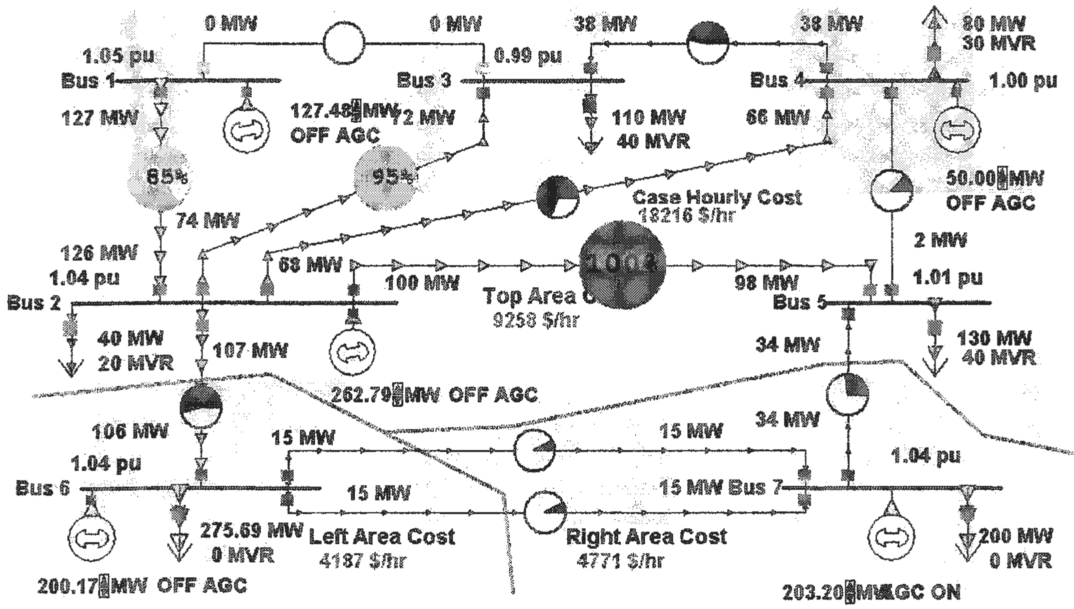

Fig.5.9 TTC from bus 2 to 6 during line 1-3 outage

5.7.2 IEEE 24-Bus Power System

Table 5.10 gives the TTC from bus 1 to bus 9 of the IEEE RTS 24-bus system considering some specified single transmission line outages. Consider line 17-22 outage as an example. The TTC from bus 1 to bus 9 is 346.25MW, and line 1-3 reaches its thermal limit in this case. It can be observed from Table 5.10 that TTC results from bus 1 to bus 9 do not change during line 17-22, line 11-13, line 9-11 or line 14-16 outages. The reason is that the network of this IEEE RTS 24-bus system is strong. Hence, some specified single line outages would not have significant effects on some specified TTC. The TTC results shown in Table 5.10 are obtained from Matlab program and are verified by PowerWorld Simulator.

Table 5.10 TTC from bus 1 to 9 with one transmission line outage of 24-bus system

Outaged Line	TTC (MW)	Limiting facilities
Line 17-22	346.25	Line 1-2
Line 11-13	346.25	Line 1-2
Line 9-11	346.25	Line 1-2
Line 14-16	346.25	Line 1-2
Line 5-10	305.06	Bus voltage
Line 1-3	286	Bus voltage

5.8 Simultaneous Transfer Capability

Recently, the increase of competition in power system has caused the power transfers among utilities to increase. In general, energy transactions between electric power companies are used to reduce operating cost [52]. Many power companies are involved in more than one transaction at the same time [4]. Simultaneous Transfer Capability (STC) refers to the maximum interchange of power between several pairs of power companies using the same network at the same time [53]. It is defined as the ability of a transmission network to allow the reliable movement of electric power from areas of supply to areas of demand. STC is the maximum simultaneous interchange capability of a power system, which is the amount of power that can be interchanged between any power companies without exceeding continuous facility loading capabilities when all facilities are in services. STC involves more than two sources or more than two sinks. As important as TTC in power systems, STC is an issue of concern for both power system planner and operators.

5.9 Simultaneous Transfer Capability Evaluation

Similar to TTC evaluation, the key aspect in calculating the maximum simultaneous transfer capability (STC) is the physical and operational limitations of the transmission system, such as circuit rating and bus voltage level. Another important issue concerning STC evaluation is the impact of control optimization, including rescheduling of generators active power, adjustments on terminal voltages, tap changes on transformers, etc. [54]. STC evaluation is a complex task. The problem resolves itself into the simultaneous import of power into one company or into a few companies from a number of other companies.

In STC studies, a power system is divided into three areas: study area, transfer participating areas and external areas. Power transactions will be classified into feasible and unfeasible. Feasible transactions can be accommodated without violating the system and equipments constraints. Unfeasible transactions can not be accommodated fully without violating the system and equipments constraints. The order of transactions schedule can determine the feasible or unfeasible of other transactions [55].

Interconnections between areas called interfaces or corridors usually are several transmission lines. Power systems usually have thousand of transmission lines but only a few lines may bound the safe region. The interconnection congestions normally are the main limitations of power exchanges to the network [56]. Instead of imposing limits of all transmission lines limitations, it is also possible only to impose the constraints of the corridors. Since it will greatly reduce the size of the constraints to improve the computation speed, the use of corridor or interface limits is quite common.

5.10 Algorithm for Simultaneous Transfer Capability Evaluation

STC is also calculated by the optimization technique used for TTC evaluation. The objective function may be the maximum of the sum of the simultaneous transfer capability or the individual transfer capability under some firm transactions. The constraints are based on optimal power flow or security constrained optimal power flow.

Consider the calculation of the maximum total simultaneous power transactions between areas as an example. The objective function can be stated as:

$$ST = -\sum_{i=1}^{Ns} \Delta T_i \tag{5.40}$$

The equality constraints are the power balance at each node:

$$\sum_{i=1}^{m} P_i = 0 \qquad i = 1, 2, 3 \ldots m \tag{5.41}$$

$$\sum_{i=1}^{m} Q_i = 0 \qquad i = 1, 2, 3 \ldots m \tag{5.42}$$

The inequality constraints are the generation limits, the power flow limits, the bus voltage limits and control variable limits:

$$P_{Gi\min} \le P_{Gi} \le P_{Gi\max} \qquad i = 1, 2, 3 \ldots n \tag{5.43}$$

$$P_{ij\min} \le P_{ij} \le P_{ij\max} \qquad i, j = 1, 2, 3 \ldots m \tag{5.44}$$

$$Q_{ij\min} \le Q_{ij} \le Q_{ij\max} \qquad i, j = 1, 2, 3 \ldots m \tag{5.45}$$

$$V_{i\min} \le V_i \le V_{i\max} \qquad i = 1, 2, 3 \ldots m \tag{5.46}$$

$$C_{i\min} \le C_i \le C_{i\max} \qquad i = 1, 2, 3 \ldots k \tag{5.47}$$

Where:

$$\sum_{i=1}^{Ns} \Delta T_i = \sum_{j=1}^{Nr} \Delta TL_j \tag{5.48}$$

$$P_{GS\,i} = P_{GS\,i}' + \Delta T_i \qquad i = 1, 2, \ldots Ns \tag{5.49}$$

$$P_{LR\,j} = P_{LR\,j}' + \Delta TL_j \qquad j = 1, 2, \ldots Nr \tag{5.50}$$

- ST is the STC

ΔT_i is the additional power transfer from sending areas generation bus i to receiving areas

ΔTL_j is the active load increment at bus j in receiving areas

P_{GSi}' is the active generation power at bus i in sending areas before considering TTC

P_{GSi} is the active generation power at bus i in sending areas after considering TTC

P_{LRj}' is the active load at bus j in receiving area before considering TTC

P_{LRj} is the active load at bus j in receiving area after considering TTC

P_{Gi} is the generation power at bus i

V_i is the voltage at bus i

P_i and Q_i are the power flows at bus i

P_{ij} and Q_{ij} are the power flows from bus i to j

C_i is the control variable

Ns is the number of generation buses in sending areas

Nr is the number of loads in receiving areas

n and m are the number of generators and buses in the power system

k is the number of control variables in power system

Equation (5.48) shows that the total active generation power increment in the sending areas equals to the total active load increment in the receiving areas.

5.11 Case Studies

The 39-bus power system used in section 5.5.1.2 and the IEEE Reliability Test 72-bus power system used in section 5.5.2.3 are used to demonstrate the performance of this method. STC is only evaluated with all facilities in service in these studies.

5.11.1 39-Bus Power System

From the study presented in section 5.5.2.2, the TTC from area 1 to area 2 is 697.8 MW and the TTC from area 1 to area 3 is 615.15 MW. The sum of these two individual TTC is 1312.95 MW, while the STC from area 1 to area 2 and from area 1 to area 3 is obtained as 1060.96 MW. In general, the power that can be imported simultaneously is less than the sum of the individual transfer capabilities. Table 5.11 gives the STC of the 39-bus system under different power transfer ratios. P13 is stated as the power transfer from area 1 to area 3, P12 is stated as the power transfer from area 1 to area 2 and P13/P12 is stated as the power transfer ratio between these two transactions. For example, at the maximum simultaneously power transfer, the power transfer from area 1 to area 3 stated as P13 is 862.08 MW, and the power transfer from area 1 to area 2 stated as P12 is 198.88 MW. The power transfer ratio is 4.4347. In this case, it is can be observed that the power transfer from area 1 to area 3 during simultaneous transfer is 862.08 MW, which is larger than the individual TTC from area 1 to area 3 697.8MW. The reason is that the loads both in area 3 and area 2 are simultaneously adjustable when this STC is applied in the system. However, only the loads in area 3 are adjustable when the individual TTC from area 1 to area 3 is applied. The more areas for load adjustment make it possible to increase the power transferred from area 1 to 3.

Table 5.11. STC of the 39-bus system under different transaction ratios

P13/P12	P13 (MW)	P12 (MW)	STC from area 1 to 2 and from area 1 to 3 (MW)
0.01	9.93	993.2	1003.13
2	707.17	353.59	1060.76
3	795.69	265.23	1060.92
4.4347	862.08	198.88	1060.96
6	909.35	151.56	1060.91
8	943.04	117.88	1060.92
20	1010.32	50.52	1060.84
30	1026.55	34.22	1060.77
100	1050.2	10.5	1060.72

Consider the transfer capability from area 1 to area 2 while the transaction from area 1 to area 3 is fixed. Table 5.12 gives the STC under these cases. P13 is stated as the power transferred from area 1 to area 3. TTC12 is stated as the TTC from area 1 to area 2. Consider the power transferred from area 1 to area 3 is fixed at 697.8MW as an example. The maximum power can be transferred from area 1 to area 2 is 360.38MW. The total STC is 1057.88 MW in this case.

Consider the transfer capability from area 1 to area 2 while the transaction from area 1 to area 3 is fixed. Table 5.13 gives the STC under these cases. P12 is stated as the power transferred from area 1 to area 2. TTC13 is stated as the TTC from area 1 to area 3.

Consider the power transferred from area 1 to area 2 is fixed at 1000.02 MW as an example. The maximum power can be transferred from area 1 to area 3 is 0 MW. There is no additional power can be transferred from area 1 to area 3 in this case. The total STC is 1000.02MW.

Table 5.12. STC of the 39-bus system under a fixed transaction from area 1 to area 3

Fixed P13 (MW)	TTC12 (MW)	STC (MW)
697.8	360.08	1057.88
1060.71	0	1060.71

Table 5.13. STC of the 39-bus system under a fixed transaction from area 1 to area 2

Fixed P12 (MW)	TTC13 (MW)	STC (MW)
615.15	432.81	1048.8
1000.02	0	1000.02

5.11.2 IEEE Reliability Test 72-Bus Power System

The transfer capability from bus to bus with the fixed transactions is studied here. Table 5.14 gives the individual TTC values of the 72-bus system. Consider TTC from bus 123 to bus 115 as an example. This individual TTC is 855.27 MW and line 115-116 reaches its thermal limit in this case. Table 5.15 gives these STC values under one fixed 700 MW transaction from bus 123 to 115. Consider TTC from bus 123 to bus 115 as an example. This TTC value is greatly reduced to 155.27MW and line 115-116 reaches its thermal limit in this case. In addition, a 300MW fixed transaction from bus 101 to 209 is added to the system, Table 5.16 gives these STC values under these two fixed transactions. Consider TTC from bus 123 to bus 115 as an example. This TTC value is slightly increased to 168.64 MW and line 115-116 reaches its thermal limit in this condition. In addition to these two transactions, an 800 MW transaction from bus 121 to 316 is added to the system, Table 5.17 gives these STC values under these three fixed transactions. Consider TTC from bus 123 to bus 115 as an example. This TTC value is reduced to 74.21 MW and it is line 316-317 instead of line 115-116 that reaches its thermal limit in this condition. It can be observed that TTC values may be reduced or increased by the effects of other transactions. The limiting facilities may be changed as well.

Table 5.14 Individual TTC of the 72-bus system

From bus	To bus	TTC (MW)	Limiting facility
123	115	855.27	Line 115-116
101	209	391.79	Line 101-102
121	316	836.3	Line 316-317
213	314	332.08	Line 314-316

Table 5.15 STC of the 72-bus system with one fixed transaction

From bus	To bus	TTC (MW)	Limiting facility
123	115	155.27	Line 115-116
101	209	406.36	Line 101-102
121	316	770.36	Line 316-317
213	314	315.37	Line 314-316

Table 5.16 STC of the 72-bus system with two fixed transactions

From bus	To bus	TTC (MW)	Limiting facility
123	115	168.64	Line 115-116
101	209	106.36	Line 101-102
121	316	806.62	Line 316-317
213	314	321.53	Line 314-316

Table 5.17 STC of the 72-bus system with three fixed transactions

From bus	To bus	TTC (MW)	Limiting facility
123	115	74.21	Line 316-317
101	209	97.2	Line 101-102, 316-317
121	316	6.62	Line 316-317
213	314	5.64	Line 316-317

5.12 Conclusions

Optimization techniques provide efficient methods for TTC evaluation in power systems. It is based on full AC load flow considering thermal limits, voltage limits, control variable limits and the effects of reactive power. It is an accurate method for TTC evaluation. The case studies presented in this chapter show the efficient performance of this method. It can evaluate TTC and STC for a particular power system. It can evaluate TTC and STC under base case operating condition and during contingencies. This method will be useful to the utilities in identifying constraining facilities for bulk power transfer. Utilities can also use this approach in evaluating the economics of different alternative for power transfers. From a planning perspective, utilities can make the needed investments considering long term energy transaction contracts. The proposed method can be easily implemented considering the developments in optimization methods and the significant enhancements in the capability of present generation computers.

Chapter 6

CONCLUSIONS

6.1 Summary of the Research and Contributions of the Thesis

Efficient applications of optimization methods in power systems are presented in this thesis. The optimization method presented in this thesis has been very successful in achieving the goals of obtaining economic operation and evaluating transmission transfer capability. Minimization of system cost while maintaining power system security and maximization of transfer capability while maintaining no violations in power system are accomplished by the implementation of optimization methods to the power systems. The optimization method has proven to be very efficient for solving power systems economic operation and transfer capability problems.

Economical operations and transmission transfer capability are very important for power systems planning and operation. Optimal Power Flow achieves economic operation of the power system under different constraints. TTC evaluations measure the strength of transmission path for power transactions. This research will be used to operate power systems, which are capable to satisfy the continuous growing demand for electrical energy while reducing the operation cost. The outages of transmission line are considered in OPF and TTC problems. This helps to operate and design power systems with the lowest possibilities of system blackout and equipments damaged.

At present, simultaneous power transactions occur very often between power utilities with the help of interconnections. By simultaneous power transactions, power utilities reduce operation cost and get power support from other utilities. In this thesis, Simultaneous transfer capability is efficiently evaluated by the optimization method as well.

This thesis has considered three key problems for power systems: economy, security and transmission transfer capability. The optimization method presented in this thesis overcomes some of the disadvantages of other methods. The results shown in the studies presented in this thesis illustrate that the performance of this method is accurate and efficient. This research is expected to be useful for power utilities.

6.2 Suggestions for Future Research

Optimization methods have wide applications in power systems. The research presented in this thesis can be extended in the following areas:

- Not only minimization of cost, the optimization techniques can be applied in many different OPF objective problems, such as: minimum active power loss, minimum number of controls rescheduled, minimum control shift, etc.
- Additional research can be carried out in SCOPF considering all random contingencies. Contingency studies and contingency selections should be included to avoid the large number of constraints, which will increase when the number of contingencies increases.
- Transient stability is one of the main concerns for power system planning and operation. Transient stability can also be considered in OPF and TTC evaluation

through the optimization techniques.

- To implement the different methods considered in this research, Matlab has been used as the simulation platform. It may be possible consider other optimization packages as well, and compare the computational advantages of the different software tools.

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Appendix 1 Data of the 7-Bus Power System

Appendix 1 contains the information of the 7-bus power system discussed in the thesis. The single line diagram of the system is shown in Figure A.1. The generation details of the system are shown in Table A.1. The line characteristics of the system are shown in Table A.2. Line parameters use 100MVA base.

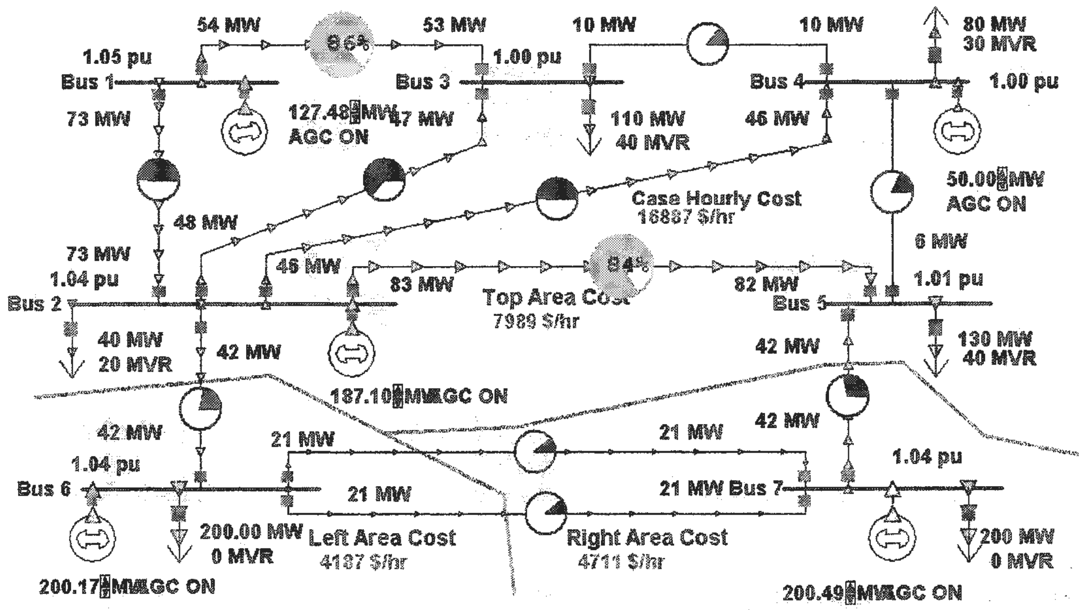

Fig.A.1 Single line diagram of the 7-bus power system

Table A.1 Generation details of the 7-bus power system

Bus	Generation		Limits (MW)	
	(MW)	(Mvar)	Min	Max
1	127.48	18.02	100.0	400.0
2	187.1	46.02	150.0	500.0
4	50.0	14.57	50.0	200.0
6	200.17	-6.58	150.0	500.0
7	200.49	38.64	0	600.0

Table A.2 Line characteristics of the 7-bus power system

From Bus	To Bus	Circuit	Resistance (p.u.)	Reactance (p.u.)	Line charging (p.u.)	Line limit (MVA)
1	2	1	0.01	0.06	0.06	150
1	3	1	0.04	0.24	0.05	65
2	3	1	0.03	0.18	0.04	80
2	4	1	0.03	0.18	0.04	100
2	5	1	0.02	0.12	0.03	100
2	6	1	0.01	0.06	0.05	200
3	4	1	0.005	0.03	0.02	100
4	5	1	0.04	0.24	0.05	60
7	5	1	0.01	0.06	0.04	200
6	7	1	0.04	0.24	0.05	200
6	7	2	0.04	0.24	0.05	200

Appendix 2 Data of the 30-Bus Power System

Appendix 2 contains the information of the 30-bus power system discussed in the thesis. The single line diagram of the system is shown in Figure A.2. The generation details are shown in Table A.3.

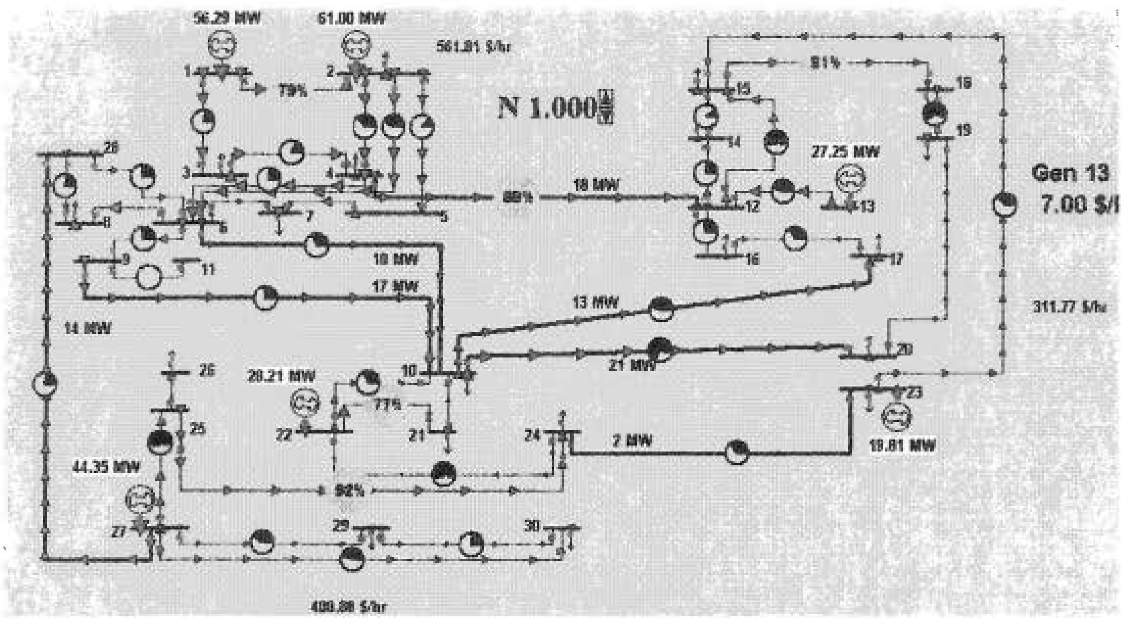

Figure A.2 Single line diagram of the 30-bus power system

Table A.3. Generators details of the 30-bus power system

Bus	Generation		Limits (MW)		Limits (Mvar)	
	(MW)	(Mvar)	Min	Max	Min	Max
1	56.29	-11.18	10.0	85.0	-99999.0	99999.0
2	61.0	24.11	10.0	115.0	-20.0	100.0
13	27.25	10.28	0	35.0	-15.0	45.0
22	28.21	35.5	5.0	45.0	-15.0	63.0
23	19.81	14.35	5.0	50.0	-10.0	40.0
27	44.35	3.8	5.0	65.0	-15.0	49.0

Appendix 3 Data of the 39-Bus Power System

Appendix 3 contains the information of the 39-bus power system discussed in the thesis. The single line diagram of the system is shown in Figure A.3. The summary of the 39-bus power system is shown Table A.4. The generation details are shown in Table A.5.

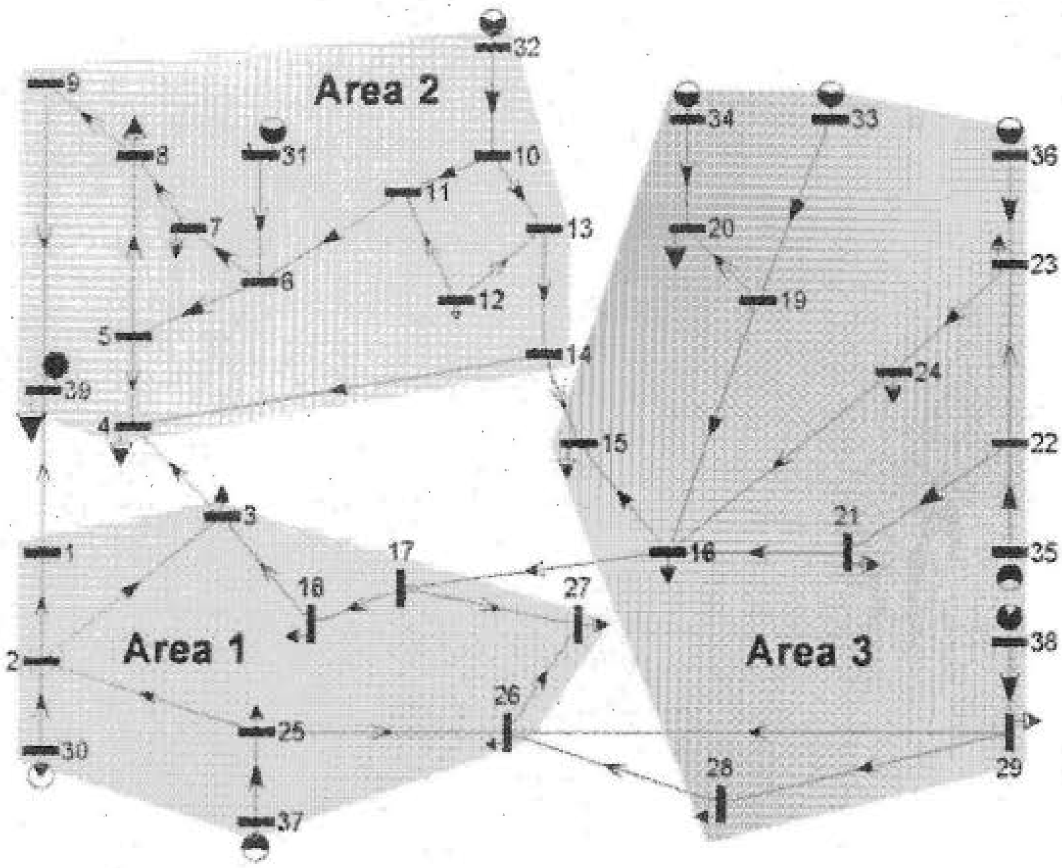

Figure A.3 Single line diagram of the 39-bus power system

Table A.4 Summary of the 39-bus power system

Buses	39	
Generators	10	
Loads	19	
Lines	48	
Areas	3	
Interfaces	1	
Generation	6150.5 MW	1408.9 Mvar
Load	6197.7 MW	1449.4 Mvar

Table A.5 Generators details of the 39-bus power system

Bus	Generation		Limits (MW)	
	(MW)	(Mvar)	Min	Max
30	250.0	85.8	-10000.0	10000.0
31	577.65	236.02	-10000.0	10000.0
32	650.0	234.79	-10000.0	10000.0
33	632.0	140.3	-10000.0	10000.0
34	508.0	134.03	-10000.0	10000.0
35	650.0	193.95	-10000.0	10000.0
36	560.0	132.36	-10000.0	10000.0
37	540.0	36.49	-10000.0	10000.0
38	830.0	65.57	-10000.0	10000.0
39	1000.0	190.06	-10000.0	10000.0

Appendix 4 Data of the IEEE Reliability Test 24-Bus Power System

Appendix 4 contains the information of the IEEE Reliability Test 24-bus power system discussed in the thesis. The single line diagram is shown in Figure A.4. The summary of the 24-bus power system is shown Table A.6. The generation details are shown in Table A.7.

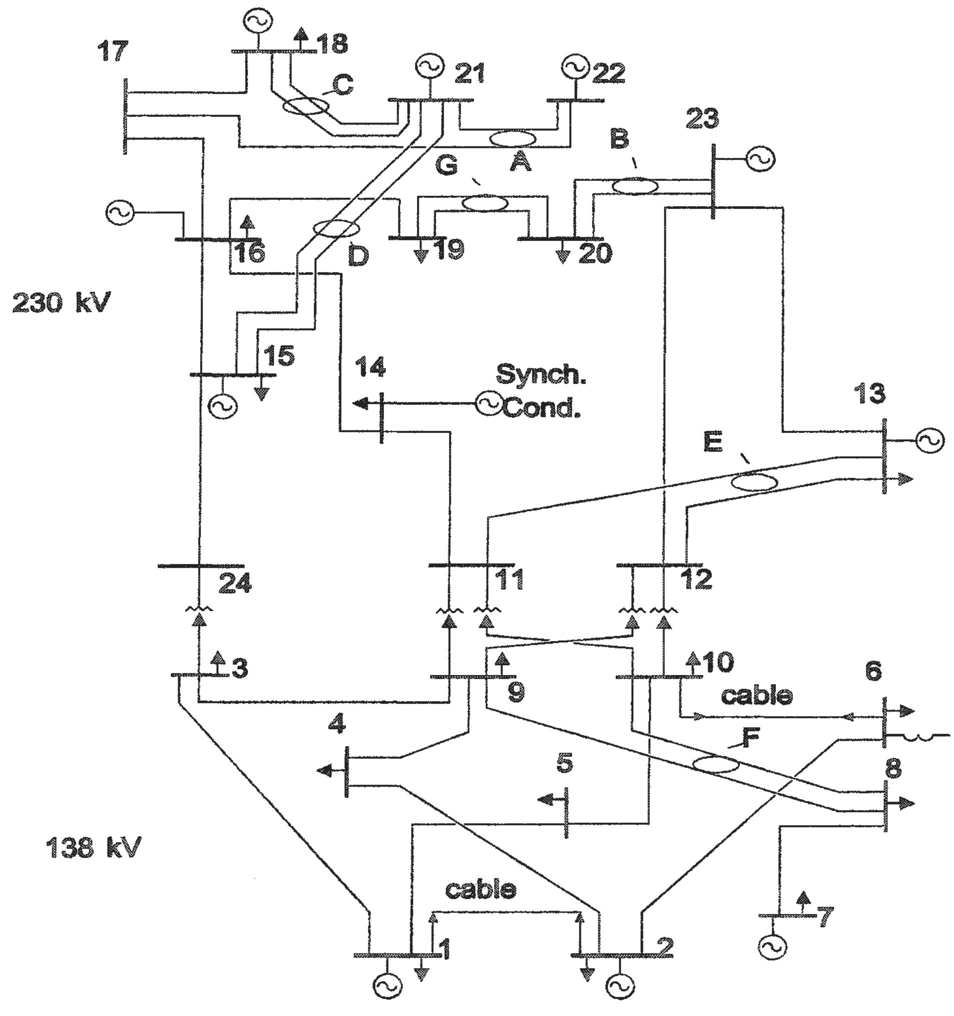

Fig.A.4 Single line diagram of the IEEE RTS 24-bus power system

Table A.6 Summary of the 24-bus power system

Buses	24	
Generators	10	
Loads	17	
Lines	38	
Switch shunt	1	
Areas	1	
Interfaces	1	
Generation	2850 MW	580 Mvar
Load	2887.5 MW	513.1 Mvar
Shunts	0 MW	95.8 Mvar

Table A.7 Generators details of the 24-bus power system

Bus	Generation		Limits (Mvar)	
	(MW)	(Mvar)	Min	Max
1	152.0	37.79	-50.0	60.0
2	152.0	31.36	-50.0	60.0
7	240.0	67.62	0	120.0
13	472.0	108.27	0	160.0
15	155.0	80.0	-50.0	80.0
16	155.0	80.0	-50.0	80.0
18	400.0	74.27	-50.0	200.0
21	400.0	-5.89	-50.0	200.0
22	101.55	-26.47	-900.0	800.0
23	660.0	40.11	-125.0	310.0

Appendix 5 Data of the IEEE Reliability Test 72-Bus Power System

Appendix 5 contains the information of the IEEE Reliability 72-bus power system discussed in the thesis. The summary of the 24-bus power system is shown Table A.8. The generation details are shown in Table A.9.

Table A.8 Summary of the 72-bus power system

Buses	72	
Generators	30	
Loads	51	
Lines	119	
Switch shunt	3	
Areas	3	
Interfaces	0	
Generation	8550MW	1740 Mvar
Load	8662.7 MW	1462 Mvar
Shunts	112.72 MW	-564.96 Mvar

Table A.9 Generators details of the 72-bus power system

Bus	Generation		Limits (Mvar)	
	(MW)	(Mvar)	Min	Max
101	152.0	37.79	-50.0	60.0
102	152.0	31.36	-50.0	60.0
107	240.0	67.62	0	120.0
113	472.0	108.27	0	160.0
115	155.0	80.0	-50.0	80.0
116	155.0	80.0	-50.0	80.0
118	400.0	74.27	-50.0	200.0
121	400.0	-5.89	-50.0	200.0
122	101.55	-26.47	-900.0	800.0
123	660.0	40.11	-125.0	310.0
201	152.0	37.79	-50.0	60.0
202	152.0	31.36	-50.0	60.0
207	240.0	67.62	0	120.0
213	472.0	108.27	0	160.0
215	155.0	80.0	-50.0	80.0
216	155.0	80.0	-50.0	80.0
218	400.0	74.27	-50.0	200.0
221	400.0	-5.89	-50.0	200.0
222	101.55	-26.47	-900.0	800.0
223	660.0	40.11	-125.0	310.0

301	152.0	37.79	-50.0	60.0
302	152.0	31.36	-50.0	60.0
307	240.0	67.62	0	120.0
313	472.0	108.27	0	160.0
315	155.0	80.0	-50.0	80.0
316	155.0	80.0	-50.0	80.0
318	400.0	74.27	-50.0	200.0
321	400.0	-5.89	-50.0	200.0
322	101.55	-26.47	-900.0	800.0
323	660.0	40.11	-125.0	310.0

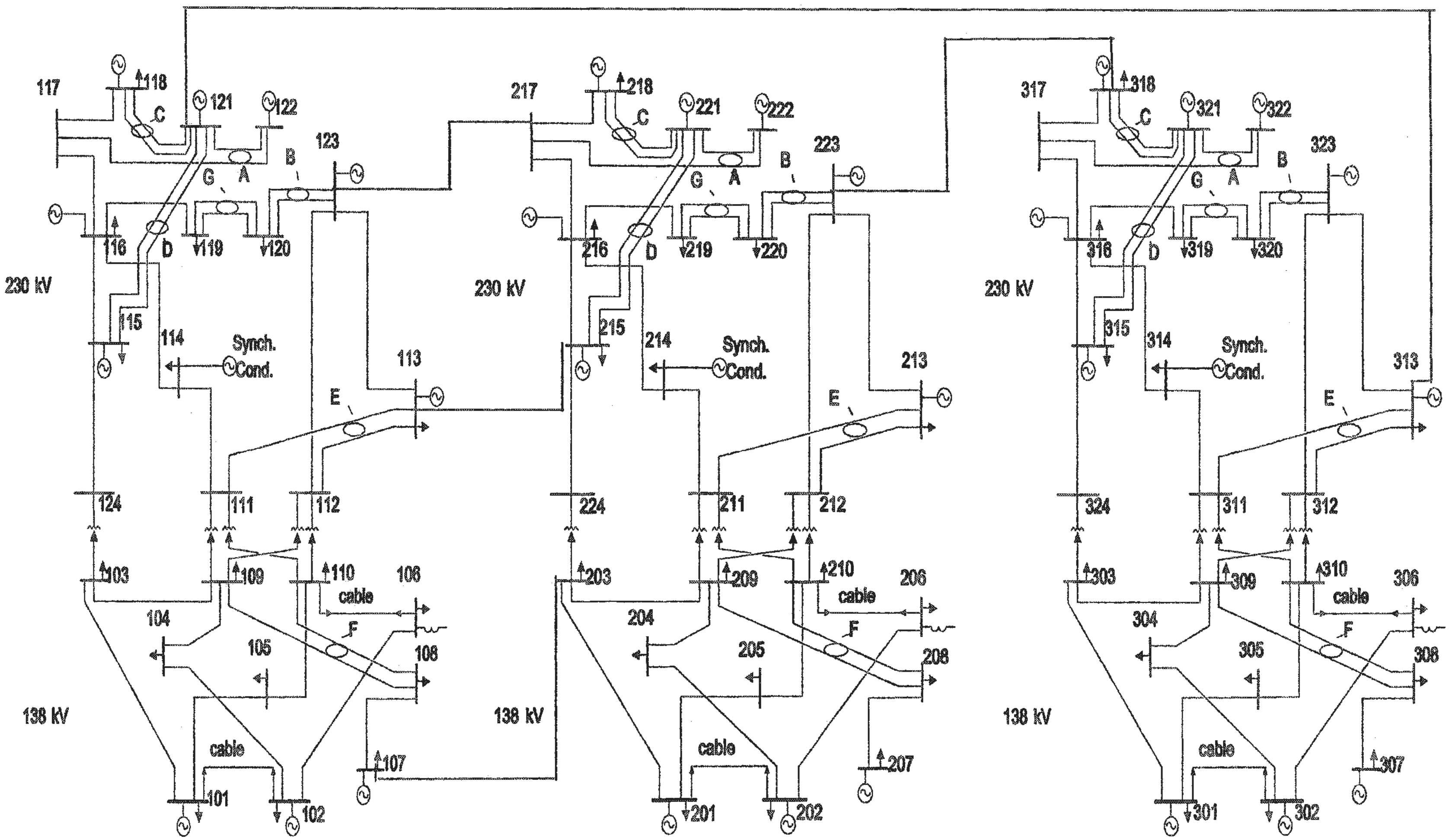

Fig. A.5 Single line diagram of the 72-Bus Power System

Appendix 6 PTDFs of the 7-Bus Power System

Appendix 6 contains the PTDFs of the 7-bus power system discuss in the thesis and shows them in Table A.10.

Table A.10 The PTDFs of the 7-bus power system

Line	Power Changed line (%)									
	1-2	1-3	2-3	2-4	2-5	2-6	3-4	4-5	5-7	6-7
1-2	84.23	63.10	-21.13	-16.76	-4.19	-1.05	4.37	12.57	1.05	-2.10
1-3	15.77	36.90	21.13	16.76	4.19	1.05	-4.37	-12.57	-1.05	2.10
2-3	-7.04	28.17	35.22	27.94	6.98	1.75	-7.28	-20.95	-1.75	3.49
2-4	-5.59	22.35	27.94	35.39	8.85	2.21	7.45	-26.54	-2.21	4.42
2-5	-2.10	8.38	10.48	13.27	53.32	13.33	2.79	40.05	-13.33	26.66
2-6	-1.05	4.19	5.24	6.64	26.66	81.67	1.4	20.02	18.34	-36.67
3-4	8.73	-34.92	-43.66	44.70	11.18	2.80	88.36	-33.53	-2.79	5.59
4-5	3.14	-12.57	-15.72	-19.91	20.02	5.01	-4.19	39.93	-5.01	10.01
5-7	1.05	-4.19	-5.24	-6.64	-26.66	-18.33	-1.4	-20.02	81.66	36.67
6-7	-0.52	2.10	2.62	3.32	13.33	-9.17	0.7	10.01	9.17	31.66
6-7	-0.52	2.10	2.62	3.32	13.33	-9.17	0.7	10.01	9.17	31.66

Appendix 7 PTDFs of the 7-bus System with Line 4-5 Open

Appendix 7 contains the PTDFs with line 4-5 open considering transaction from bus 2 to 6 of the 7-bus power system discuss in the thesis and shows them in Table A.11.

Table A.11 TDFs with line 4-5 open considering transaction from bus 2 to 6

Bus	Bus	From (%)	To (%)
1	2	0	0
1	3	0	0
2	3	0	0
2	4	0	0
2	5	16.67	-16.67
2	6	83.33	-83.33
3	4	0	0
4	5	0	0
5	7	16.67	-16.67
6	7	-8.33	8.33
6	7	-8.33	8.33

Appendix 8 LODFs of the 7-Bus Power System

Appendix 8 contains the LODFs of the 7-bus power system discuss in the thesis and shows them in Table A.12.

Table A.12 LODFs of the 7-Bus Power System

Line	Outage line (%)									
	1-2	1-3	2-3	2-4	2-5	2-6	3-4	4-5	5-7	6-7
1-2		100	-32.6	-25.9	-9.0	-5.7	37.5	20.93	5.7	-3.1
1-3	100		32.6	25.9	9.0	5.7	-37.5	-20.93	-5.7	3.1
2-3	-44.6	44.6		43.2	15.0	9.5	-62.5	-34.88	-9.5	5.1
2-4	-35.4	35.4	43.1		19.0	12.1	64.0	-44.19	-12.1	6.5
2-5	-13.3	13.3	16.2	20.5		72.7	24.0	66.67	-72.7	39.0
2-6	-6.6	6.6	8.1	10.3	57.1		12.0	33.33	100	-53.7
3-4	55.4	-55.4	-67.4	69.2	23.9	15.2		-55.81	-15.2	8.2
4-5	19.9	-19.9	-24.3	-30.8	42.9	27.3	-36.0		-27.3	14.7
5-7	6.6	-6.6	8.1	10.3	57.1	100	12.0	-33.33		53.7
6-7	-3.3	3.3	4.0	5.1	28.6	50	6.0	16.67	50	
6-7	-3.3	3.3	4.0	5.1	28.6	50	6.0	16.67	50	46.3

Appendix 9 PTDFs of the 39-bus system

Considering Line 36-38 as the Transaction

Appendix 9 contains PTDFs of the 39-bus system discuss in the thesis considering line 36-38 as the transaction and shows them in Table A.13.

Table A.13 PTDFs of the 39-bus system considering line 36-38 as the transaction

Line	1-2	1-39	1-39	2-3	2-25	2-30	3-4
PTDF	5.41	-2.71	-2.71	-28.26	33.6734	0	-13.45
Line	3-18	4-5	4-14	5-6	5-8	6-7	6-11
PTDF	-14.81	-0.46	-12.99	-3.13	2.66	2.75	-5.87
Line	6-31	6-31	7-8	8-9	9-39	10-11	10-13
PTDF	0	0	2.75	5.41	5.41	5.35	-5.35
Line	10-32	12-11	12-13	13-14	14-15	15-16	16-17
PTDF	0	0.53	-0.53	-5.87	-18.86	-18.86	81.14
Line	16-19	16-21	16-24	17-18	17-27	19-20	19-33
PTDF	0	-52.44	-47.56	14.81	66.33	0	0
Line	20-34	21-22	22-23	22-35	23-24	23-36	25-26
PTDF	0	-52.44	-52.44	0	47.56	-100	33.67
Line	25-37	26-27	26-28	26-29	28-29	29-38	
PTDF	0	66.32	50	50	50	100	

Appendix 10 PTDFs of the 39-bus System

Considering Line 32-39 as the Transaction

Appendix 10 contains PTDFs of the 39-bus system discuss in the thesis considering line 32-39 as the transaction and shows them in Table A.14.

Table A.14 PTDFs of the 39-bus system considering line 32-39 as the transaction

Line	1-2	1-39	1-39	2-3	2-25	2-30	3-4
PTDF	-42.2	21.1	21.1	-33.08	-9.11	0	-25.43
Line	3-18	4-5	4-14	5-6	5-8	6-7	6-11
PTDF	-7.66	-2.66	-22.77	-31.31	28.65	29.15	-60.46
Line	6-31	6-31	7-8	8-9	9-39	10-11	10-13
PTDF	0	0	29.15	57.80	57.80	59.52	40.48
Line	10-32	12-11	12-13	13-14	14-15	15-16	16-17
PTDF	-100	0.94	-0.94	39.54	16.77	16.77	16.77
Line	16-19	16-21	16-24	17-18	17-27	19-20	19-33
PTDF	0	0	0	7.66	9.11	0	0
Line	20-34	21-22	22-23	22-35	23-24	23-36	25-26
PTDF	0	0	0	0	0	0	-9.11
Line	25-37	26-27	26-28	26-29	28-29	29-38	
PTDF	0	-9.11	0	0	0	0	