TEACHING MULTIPLICATION OF WHOLE NUMBERS IN THE ATLANTIC PROVINCES EDUCATIONAL FOUNDATION MATHEMATICS CURRICULUM: A RESOURCE FOR ELEMENTARY TEACHERS

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Teaching Multiplication of Whole Numbers in the Atlantic Provinces Educational Foundation Mathematics Curriculum: A Resource for Elementary Teachers

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ABSTRACT

A tremendous shift in the conceptualization of teaching elementary mathematics has been re-popularized in the past ten years. This shift has recently influenced our Newfoundland and Labrador curriculum, largely through the efforts of the *Atlantic Provinces Education Foundation* (APEF). APEF directions are based on the changes advocated by the *National Council of Teachers of Mathematics* (NCTM), primarily in the *Curriculum and Evaluation Standards for School Mathematics* (1989).

Teachers in Newfoundland and Labrador are currently in transition with their teaching roles in delivering the new mathematics curriculum. This project, designed for elementary teachers, addresses the teaching of multiplication of whole numbers in the APEF curriculum. Specifically, the project explores alternate algorithms and examines procedural and conceptual understanding in the teaching of multiplication in today's elementary classroom. In particular, changing conceptions of what matters for students to learn demand increased attention to alternate forms of task presentation and student response: oral, written, and model. Expectations of students' conceptual knowledge are broadened to consider language and number sense. Potential changes in assessment, reflecting the advocated shift, are then offered. Finally, an extensive list of sample tasks, pertinent to multiplication of whole numbers, is made available. It is the intention that this project will serve not only as positive food for thought in a changing conceptualization of teaching mathematics in general, but also as a resource for teachers of grades four to six, and perhaps at either side of this indicated range, in teaching the multiplication of whole numbers in the new APEF Mathematics Curriculum.

INTRODUCTION

The Department of Education is in the process of implementing new programs in mathematics education for Newfoundland and Labrador. To date, the new *Atlantic Provinces Education Foundation* (APEF) Mathematics Curriculum for Primary and Elementary has been fully implemented in grades kindergarten, one and two. The new program is being implemented in grade three in this current school year, 2002-2003. It is projected that the new curriculum will reach grades four through six in subsequent years, respectively. For the purpose of this project, it is worthy to note that "elementary" in Newfoundland and Labrador is defined to begin in grade four, with students seeing their final elementary year in grade six.

As with any new ideology of teaching, educators must take time to reflect on suggested changes. As one of these teachers, I feel we have to consider our own ideologies to identify what we believe to be the fundamentals of teaching elementary mathematics. It is also our responsibility to stay current with the latest teaching ideas. Ball (1996) suggests that "our challenge is to experiment, study, reflect on, and reformulate our hypotheses" (p.500). The challenge is for educators to conceptualize for themselves the meaning of the new directions and the implications for the classroom.

It is this challenge that is the driving force behind this project. There are many questions, which accompany such a fundamental shift in teaching practices. From where is this new direction of teaching mathematics coming? What does it mean to teach for conceptual understanding versus procedural understanding? How does this affect the activity and the overall atmosphere in the mathematics classroom? If there is such a change in the doing of mathematics, surely there is a corresponding change in assessment. What implications exist for me as a mathematics educator?

This project begins with a review of literature, primarily pre-service teacher textbooks, reflecting the practices promoted through the *National Council of Teachers of Mathematics* (NCTM). In presenting such information, my intent is to provide teachers with the opportunity to become familiar with the fundamental changes in teaching elementary mathematics. The findings of this literature review are an integral part of such a challenge. It is also the intention that the project assist teachers in their efforts to adjust to the new expectations of them as mathematics educators in Newfoundland and Labrador, at the same time raising awareness of the direction of the APEF Mathematics Curriculum. It is important to note that teachers of grades four, five, and six are currently following the old program and generally have little or no exposure to the new curriculum advocated in the APEF mathematics program. This project is intended to provide the elementary teachers of Newfoundland and Labrador with exposure to the nature of the new program and what it means to teach mathematics from this perspective. Teachers are also students in this regard.

The practical component of the project examines teaching the multiplication of whole numbers with the new APEF vision in mind, and is meant to be a resource that my fellow elementary mathematics educators can use. Additional literature findings are offered where necessary. Initially, there is a look at an example of the standard multiplication algorithm. This is followed by several alternate algorithms for multiplication of whole numbers, as a reference. The majority of the practical component looks at the scope of multiplication of whole numbers through three lenses: 1) procedural knowledge; 2) conceptual knowledge; and 3) appropriate assessment. Each perspective is elaborated upon, identifying critical information and sample tasks to illustrate a new face of teaching multiplication, which we are expected to simulate in our classrooms.

Structurally, this document exists in two distinct sections: 1) the Literature Review and 2) the Project itself. Please be aware that the literature review is primarily of an informative nature, with an intent to provide a context for the second, more practical component. The information is meant to answer questions about where this new way of teaching is coming from and what it means in a somewhat theoretical sense. It is a framework for the elementary teacher to understand the upcoming changes in teaching elementary mathematics and is a survey of information from methods' text authors and teacher educators. As these are more secondary resources, the information is not primarily intended for the research community but rather for the elementary teaching community of Newfoundland and Labrador.

The second part of this document is meant to be an illustration of what it means to do mathematics according to the vision of the NCTM and the APEF. It is intended that a teacher in grade four, five, or six, could literally have the project detached and given to him/her as a resource to provide some insight into the changes that are occurring in teaching elementary mathematics, and more specifically in this case, multiplication of whole numbers. However, it is important that teachers be cognizant that the literature review is prepared to provide a context for a deeper understanding of the project examples.

LITERATURE REVIEW

New Directions

From where is this new conceptualization of teaching mathematics coming? It is the position of the NCTM that "we live in a time of extraordinary and accelerating change" (NCTM, 2000, p.4). "The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater" (NCTM, 2000, p.4). The NCTM see mathematics having a distinct role in everyday life, in the workplace, in the scientific and technical community, and as part of cultural heritage. Additionally, there is the document *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (1989). This document is considered to be one of the most influential works in restructuring mathematics education in the United States. It identifies that "several factors - growth of technology, increased applications, impact of computers, and expansion of mathematics itself - have combined in the past quarter century to extend greatly both the scope and the application of the mathematical sciences" (National Research Council, 1989, p.4). The authors suggest that "in tomorrow's world, the best opportunities for jobs and advancement will go to those prepared to cope confidently with quantitative, scientific, and technological issues. Mathematical power provides the key to these opportunities" (National Research Council, 1989, p.12). Van de Walle (2001) recognizes the perspective of employers within our changing society: "Today's employers are searching for the ability as well as the confidence to solve problems that have never been encountered before. Children need to see themselves learning to reason and learning to solve problems, not just learning skills" (Van de Walle, 2001, p.4). It has been said that "Today's schools labor under the legacy of a structure designed for the industrial age misapplied to educate children for the information age" (National Research Council, 1989, p.11).

American national concerns arise from students showing a basic lack of understanding in mathematics. American students' performance levels are, at best, mediocre in international comparison. In response to such performance and reports such as *Everybody Counts*, the drive has been to better develop students' conceptual understanding of mathematics, as well as improve students' overall problem-solving abilities. American research and recommendations characteristically find their way into the teaching world in Canada, and eventually into Newfoundland and Labrador. This holds true in the evident use of the *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Professional Standards for Teaching Mathematics* (1991) as a framework and guide by the APEF in its development of the new mathematics program.

Newfoundland and Labrador, in conjunction with the other Atlantic provinces, has developed a document, *Foundations for the Atlantic Canada Mathematics Curriculum,* outlining a new focus for teaching mathematics. The *National Council of Teachers of Mathematics* (NCTM) has provided the root of these new directions with many recommendations for change. Kennedy and Tipps (1997) and Van de Walle (2001) present five major goals for students, which are a part of the NCTM's vision of the mathematics classroom: Students should 1) learn to value mathematics; 2) become confident in their ability to do mathematics; 3) become mathematical problem solvers; 4) learn to communicate mathematics; and 5) learn to reason mathematically.

The *Professional Standards for Teaching Mathematics* (1991) have been grouped into four major categories focusing on instruction and teachers: standards for 1) teaching mathematics; 2) evaluation of teaching; 3) professional development of teachers of mathematics; and 4) support and development of mathematics teachers and teaching. These standards see teachers as "key agents of change in the classroom" (Van de Walle, 2001, p.9).

Generally, there is a call for a shift from teacher-centered to child-centered approaches to instruction. Consequently, the standards identify five major shifts in the classroom environment, that are the responsibility of the teacher, to foster learning experiences that empower our students to develop mathematically (NCTM, 1991). Firstly, it is important to develop classrooms as math communities rather than individuals in the classroom doing math. Secondly, rather than the teacher being the sole authority for correct answers, logic and mathematical evidence should be used as verification. Thirdly, a shift is needed away from memorizing procedures and rote memorization toward mathematical reasoning. Next, it is necessary to move away from an emphasis on mechanistic findings of answers toward conjecturing, inventing, and problem solving. Finally, a shift toward connecting mathematics, its ideas and its applications and away from seeing it as isolated concepts and procedures, is essential.

Further to this, in teaching mathematics it is a professional responsibility to provide authentic learning experiences for students. The standards for teaching mathematics are arranged into four categories. Teachers need to provide worthwhile mathematical tasks, encourage discourse among students and between students and teachers, provide for an environment in which learning will be enhanced, and analyze continually both teaching and learning (NCTM, 1991).

Kennedy and Tipps (1997) identify three premises for learning mathematics in this context: 1) knowing mathematics is doing mathematics; 2) the use of mathematics has become broader; and 3) tools of technology have changed ways in which data are stored, analyzed, transmitted and used. It is our responsibility, as teachers, to reflect upon these standards and premises as we attempt to deliver a stimulating mathematics program to our students.

"Our contemporary world demands a kind of mathematical knowledge that is very different from that required in the past" (Heddens and Speer, 1995, p.2). Riedesel (1996) suggests that fostering mathematical growth in a contemporary world necessitates changes in both curricular content and instructional style. Mathematics is seen as "seeking solutions, not just memorizing procedures; exploring patterns, not just memorizing formulas; and formulating conjectures, not just doing exercises" (Riedesel, 1996, p.15). Hyde (1989) and Ball (1997) have suggested that too often what passes for conceptual understanding in our classrooms is, mistakenly, the memorized manipulation of symbols, devoid of meaning; it is the narrowed learning experiences where it is more likely for students to get the correct answers. "Understanding is not an all-or-nothing proposition" (Ball, 1997, p.735). Rather "understanding is variable and not nearly as stable or eternally consistent as we pretend" (Ball, 1997, p.736). In order to promote increased understanding of mathematics among our students, teachers have to illustrate, via active learning, the various meanings of mathematical concepts. Teaching for meaning has not been in the forefront of traditional mathematics instruction. It is, however, highly advocated in the new APEF Foundations of teaching elementary mathematics.

Teaching For Meaning

Despite early efforts to teach mathematics with an emphasis on understanding, classroom teaching of mathematics has continued to focus on lower level learning, i.e. calculations, computational drill, and rote memorization of facts; that is, the focus has continued to be *procedural* or *instrumental knowledge* - completing the process of the work. "The *Principles and Standards* document makes it very clear that there is a time and place for drill and practice, but it should never come before understanding" (Van de Walle, 2001, p.17). The current emphasis is on *conceptual* or *relational* knowledge - understanding mathematical concepts; that is, mathematics is taught for meaning.

Skemp (1978) identifies four major benefits of relational understanding: 1) It is more adaptable to new tasks; 2) It is easier to remember; 3) Relational knowledge can be effective as a goal in itself; and 4) Relational schemas are organic in quality; that is, they act as an agent of their own growth. According to Van de Walle (2001), relational understanding is intrinsically rewarding and helps with learning new concepts and procedures. By understanding concepts, memory is enhanced and in fact, there is less to remember. Relational understanding improves problem-solving abilities, is self-generative, and improves attitudes and beliefs. For example, teaching students that multiplication can be used to compare, to solve combination or rate problems, or to efficiently handle repeated addition contributes to the benefits as listed above. The same holds true when students understand the properties of multiplication and how to use them. Seeing multiplication in its many different roles enhances the aforementioned benefits of relational understanding. Rote memorization of basic multiplication facts does not offer the same results. Interestingly, Riedesel (1996) suggests that "when we teach mathematics without meaning ... we force [our students] to memorize numerous unconnected bits of information. We lead them to believe that math doesn't make sense. We undermine their self-confidence and motivation to learn by creating feelings of confusion and helplessness" (Riedesel, 1996, p.67).

"Mathematics becomes useful to a student only when it has been developed through a personal intellectual engagement that creates new understanding" (National Research Council, p.6). As stated in a Chinese proverb: "Tell me, I'll forget. Show me, I may remember. Involve me, and I'll understand." What, then, is understanding? "Understanding can be defined as a measure of the quality and quantity of connections that an idea has with existing ideas" (Van de Walle, 2001, p.28). Despite what we may think, children generally do not give haphazard answers. Their responses make sense in their own worlds. Teachers must be cautious to ensure that students are engaging in reflective activity to construct new knowledge accurately.

As part of the cognitive theory of learning, Wittrock (1989) suggests that children must be mentally active in order to generate learning. "The study of mathematics should be a stimulating endeavor that enables students to build from sets of individual experience and that expands their abilities to think mathematically" (Heddens and Speer, 1995, p.12). This represents a shift from the traditional method of telling children what to do and then having them learn through drill and practice. Simply stated, it is through doing the mathematics themselves that students encounter and develop the concepts. However, in the new program, there is more to "doing" mathematics than this.

"Doing" Mathematics

"Children will become confident 'doers' of mathematics only if mathematics makes sense to them and if they believe in their ability to make sense of it" (Trafton and Claus, 1994, p.21, in Van de Walle, 2001, p.xv). "Exploring, using and applying mathematics must always be at the heart of learning the subject" (Haylock and McDougall, 1999, p.1).

Such statements provoke reflection regarding the traditional approach to teaching elementary mathematics. Most of today's educators learned mathematics in a traditional classroom where the teacher was the source of all knowledge, and where instruction was generally procedural, followed by drill and practice, usually with paper and pencil. If manipulatives, such as base ten blocks, counters, and colour tiles, were used at all, the teacher outlined what was to be done and the students followed the procedure step by step. Students' attention was on teacher's directions and not the mathematical ideas. Their focus was on getting the right answers, relying on the teacher to determine the correctness of the response.

This rejuvenated conceptualization of teaching elementary mathematics sees the child as an active participant in learning. Teachers pose problems using various means of presenting tasks (i.e. oral, written, and model). The focus is on students actively trying to figure things out, testing ideas and making conjectures, and developing reasons and offering explanations, also in various oral, written, or modeled forms. Students are constructing their own knowledge. For example, Cognitively Guided Instruction (CGI) is one of the recent models of instruction and intervention that researchers are developing. In line with the cognitive view of learning where children make sense of new knowledge in light of existing knowledge and beliefs, the basic premise of CGI is that you start with what children know and then build on that knowledge by allowing access for participation in advanced higher-order activities. "The message of CGI is that when teachers begin listening to children they come to realize how much more the children know than they recognized previously. They come to realize that children have a lot of mathematical knowledge on which to build" (Hankes, 1996, p.5). Students of CGI have on average made a grade level gain in achievement and have reported being more confident and better able to understand mathematics. Basically, CGI has the potential to fall in line with the open-ended broad-based nature of mathematical activities consistent with this current view of teaching elementary mathematics.

Another point of interest is the "no answer book" notion; that is, rather than waiting for the teacher to tell them what to do and how to do it, students do not rely on the teacher. This promotes attention on mathematical ideas rather than on the teacher's directions. Children's thinking becomes the primary source of knowledge. Multiple solutions and various algorithms are advocated. Van de Walle (2001) identifies verbs such as *explore*, *investigate*, *solve*, *justify*, *conjecture*, as the verbs of "doing" math as opposed to the actions of traditional math such as *memorize, copy, add, subtract, multiply, divide*, etc.. The latter mathematical verbs generally focus children's thoughts on following the given directions - a somewhat passive role - whereas the former mathematical verbs more likely attract children's thoughts to the mathematical ideas and to engaging in active participation of their mathematical learning. For example, asking children to explore what happens to any number every time they multiply by ten or by one hundred or by one thousand and then asking them to communicate and justify what they find, has more potential to lead children to identifying and understanding products of the powers of ten instead of simply telling them to "just know" that they are to add the same number of zeros that are in ten, one hundred, or one thousand to the other factor to get the answer. The authors of *Everybody Counts* state that "Mathematics today involves far more than calculation; clarification of the problem, deduction of consequences, formulation of alternatives, and development of appropriate tools are as much a part of the modern mathematician's craft as are solving equations or providing answers" (National Research Council, 1989, p.5).

Though less emphasis has been placed on efficiency in procedural knowledge as the ultimate goal, there is a value in recognizing the traditional and even more so, alternate algorithms. "Children should be actively involved in devising their own algorithms for solving multiplication and division problems. Research indicates that no single algorithm is the right algorithm to teach" (Kouba and Franklin, 1995, p.576).

An *algorithm* is a *"finite, step-by-step procedure for accomplishing a task that we wish to complete"* (Usiskin in Randolph and Sherman, 2001, p.480). Holmes (1995) says, "The study of standard algorithms is made more meaningful if children have opportunities to explore extended algorithms. Extended or developmental algorithms show each step in the procedure and help children grasp the reasoning behind the algorithms" (p.216). It is maintained that these algorithms need to be considered tools for solving real-life problems. Heddens and Speer (1995) suggest that the extended form of our algorithm be used to help children develop an understanding of the multiplication of larger numbers. They feel students should have the opportunity to apply their knowledge of the basic structure and of the algorithm to two-digit by one-digit and two-digit multiplication. When they are ready to develop procedural skill efficiency, the authors suggest the standard algorithm. This is consistent with the position of Kennedy and Tipps. "Care must be taken when children are taught to use algorithms for multiplication… so the algorithms' meanings and applications are clear" (Kennedy and Tipps, 1994, p.378).

Further to this, many feel that considering the standard algorithm to be the "rule" of multiplication is a great disservice to our students. Koller Caliandro (2000) alludes to Kamii and Livingston's (1994) elaboration to say that though algorithms provide a certain security of producing correct answers, students tend to function like machines. "Their thinking remains blocked and paralyzed by the program" (Kamii and Livingston in Koller Caliandro, 2000, p.423-424). "Activities using these (alternative) algorithms help students with a range of mathematical abilities expand their perceptions of mathematics from that of a rule-based and single-answer discipline to one that involves multiple approaches and personal constructs" (Simonsen and Teppo, 1999, p.519).

Not only are the students affected, but so too are other pivotal people in the elementary classroom, namely teachers. "In addition to making mathematics more meaningful for children, many teachers reported that they found teaching to be more exciting when more emphasis is placed on discovering and sharing procedures than on memorizing and practicing traditional algorithms" (Carroll and Porter, 1997, p.370).

Riedesel (1996) advocates that we experiment with children's methods and then discuss the various approaches to identify a most effective method. Though the children may fixate on the so-called "best way", it is important to encourage experimentation with the various approaches. Koller Caliandro (2000) cites a profound statement by Kamii and Livingston which supports the appropriate use of alternate algorithms as a vehicle for deeper understanding of multiplication and mathematics in general. "Children's first methods are admittedly inefficient. However, if they are free to do their own thinking, they invent increasingly efficient procedures just as our ancestors did. By trying to bypass the constructive process, we prevent them from making sense of arithmetic" (Koller Caliandro, 2000, p.424). Students can apply their own aptitudes. Randolph and Sherman (2001) reiterate that "students develop their own understanding of and skills in, arithmetic operations, enhancing their decision-making and critical-thinking skills" (p.484) by engaging with alternate algorithms. However, despite such an emphasis, teachers should be cautious not to let the notion that students invent these alternate algorithms on their own become the ultimate goal. More often than not, it is necessary that these be taught.

Another important factor to consider is that students get to see mathematics as a "process" rather than just "questions and answers" when algorithms - standard or alternate - are being used. “Although accuracy in arithmetic is essential, the process by which the solution is obtained, as well as the child’s experience throughout the process, is of no less importance” (Bonsangne, Gannon, and Watson, 2000, p.311). In the students’ presentation of such work, teachers can see a more comprehensive view of what a student is thinking, enabling diagnostic and appropriate planning procedures. There is the concern that using such methods takes a lot of time. However, it is said that the benefits of saving time on practicing the rules, and increased understanding, and motivation make it worth the effort.

For students and teachers to engage in this new way of “doing” mathematics, it means they have to take risks. In particular, students have to present their ideas and thoughts for other people to see, something that is somewhat unfamiliar in most math classrooms today. It is critical that we, as teachers, provide a safe, non-judgmental environment where everybody listens to one another and respects what is said, whether the answer is correct or incorrect. Students have to know that they will not be ridiculed. Van de Walle (2001) refers to creating a “spirit of inquiry, trust, and expectation” (p.17). Instead of math class being a collection of individuals, it should be a community of learners committed to exploring and understanding various aspects of mathematics. To complete such a task successfully takes time and requires effort from everyone in the class. Such is the nature of “doing” mathematics under the new directions of elementary mathematics. This notion gives birth to a changing role of problem solving and questioning in teaching and learning mathematics.

The Role Of Problem Solving

"At one time teachers treated 'problem solving' as a topic, like addition, geometry, or measurement" (Kennedy and Tipps, 1997, p.8). Though this seems to be changing, albeit slowly, problem solving is beginning to be viewed in much broader context. It is seen as the essence of mathematics. From this perspective, children should learn mathematics through problem solving. "In teaching via problem solving, problems are valued not only as a purpose for learning mathematics but also as a primary means of doing so" (Schroeder and Lester in Kennedy and Tipps, 1997, p.10).

Rather than problem solving being seen as a distinct topic of instruction where the problem is separate from the learning, the problem is meant to engage the students in making sense of the key concepts to be learned. Researchers and teacher educators (Holmes, 1995; Kennedy and Tipps, 1997; Lepper and Hodell, 1989; Meece, 1991; Riedesel, 1996;Van de Walle, 2001) identify several reasons to support this notion. "As a *means* for teaching mathematics, problems enable students to construct mathematical ideas" (Holmes, 1995, p.2). Children develop their understanding, confidence, and self-worth. "Natural curiosity is a powerful teacher, especially for mathematics" (National Research Council, 1989, p.43). Often children, through their own sense of wonder and curiosity, will pose their own problems, which are much better contexts for learning because, as Lepper and Hodell (1989) and Meece (1991) suggest, such a vested interest in curriculum increases motivation and self-confidence in students.

Problems used to introduce strategies should involve students in more than just applying known procedures. "Open-ended problems, sometimes called process problems, give students opportunities to apply diverse procedures to solve them" (Kennedy and Tipps, 1997, p.11). The idea of using various techniques to find multiple solutions involves many children in a single activity and provides a sense of intrigue. For the child, there is a sense of discovery that there is more than one possible method of finding one or more solutions. Problem solving develops mathematical power; it promotes "doing" mathematics. Students' attention shifts to ideas and sense-making rather than correct answers. Problem solving has also been known to provide enjoyment for the students engaged in it.

Considering the new role of problem solving in the classroom, it is inevitable that the teacher adopt a new role as well. Marilyn Burns (2000) provides an adequate description:

> A problem solving curriculum, however, requires a different role from the teacher. Rather than directing a lesson, the teacher needs to provide time for students to grapple with problems, search for strategies and solutions on their own, and learn to evaluate their own results. Although the teacher needs to be very much present, the primary focus in the class needs to be on the students' thinking processes. (p.29)

A role such as this demands a change in the teacher's use of appropriate questioning techniques. Sullivan and Clarke (1991) maintain that "good questions have three features: the students are required to do more than simply remember a strategy to answer them; the students can learn in the process of answering the question; and the questions have several acceptable answers" (p.14). They also reiterate that questions should be suitable for all abilities. For example, asking children how many different ways they can show a product of twenty-four is preferable to asking children what is eight times three, four times six, or two times twelve.

If these are the advocated changes for a more meaningful mathematical learning experience for children, then assessment has to reflect such changes. It is unfair to teach one way and assess in a way totally different from how the children have been taught and how they have learned.

Changes In Assessment

Given the changes in goals and instruction in a new dynamic view of teaching elementary mathematics, it should follow that assessment will reflect such changes. The NCTM has also established *Assessment Standards* for school mathematics. The *Standards* define *assessment* as "the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward mathematics and of making inferences from that evidence for a variety of purposes" (Assessment Standards, 1995, in Van de Walle, 2001, p.62). Not synonymous with testing, measurement, or evaluation, assessment is more than a collection of data.

Baroody and Coslick (1998) and Van de Walle (2001) reiterate the various purposes of assessment. Assessment may be used to monitor student progress, make instructional decisions, evaluate or monitor student growth in mathematical achievement, and/or evaluate programs. All this should be in the interest of promoting student growth, improving instruction, recognizing accomplishment, and modifying programs. "At its heart, it should be a means for helping teachers better understand their students so that they can effectively guide learning" (Baroody and Coslick, 1998, p.3-18). In recognizing these purposes, Baroody and Coslick (1998) highlight appropriate assessment in terms of making connections and applying existing knowledge to a new task. "If understanding is defined in terms of connections, then it follows that the degree of understanding can be gauged by the number, accuracy, and the strength of a student's connections, and [secondly] transfer (application of existing knowledge to a new task) provides strong evidence of understanding" (Baroody and Coslick, 1998, p.3-22). In determining what it is we should assess, it is important to keep in mind the student goals from the NCTM's vision of the mathematics classroom, as previously discussed. Hence, concepts and procedures, mathematical processes, problem solving, and mathematical disposition all have a place in mathematics assessment. Instruction and assessment should be combined in alignment with the *Standards*.

The APEF Mathematics Curriculum explicitly advocates the following examples of alternate forms of assessment: 1) Performance; 2) Paper and Pencil; 3) Interview; 4) Portfolio; and 5) Presentation. Such techniques are among those promoted in methods' books of Souviney (1994), Riedesel (1996), and Van de Walle (2001).

A *performance task* "involves gauging children's dispositions, strategies, or understandings by actually observing them performing a task and by analyzing their performance" (Baroody and Coslick, 1998, p.3-27). *Paper and pencil tasks* are tasks which provide opportunity for students to communicate their thinking and clarify their thoughts about mathematics while helping to reinforce what they already know and understand. These may be in the form of computation with standard and alternative algorithms, paper and pencil tests, or student writing journals. According to Van de Walle (2001), an *interview* is a "one on one discussion with a child to help you see how she is thinking about a particular subject, what processes she uses in solving problems, or what attitudes and beliefs she may have" (p.80). In this process, teachers must be nonjudgmental, be good listeners, and try to put the child at ease by using strategies such as talking at the child's level and not interrupting. "A *portfolio* is an assemblage of many types of student work selected with both student and teacher input, designed to provide a holistic view of some aspect of mathematics that may not be evident from examination of any single entry" (Van de Walle, 2001, p.79). A *presentation* is a significant task requiring extended effort, which explores a mathematical concept from one in-depth perspective or from many perspectives. This in-depth or varied perspective is presented to the class or to a group other than classmates. The presentation of what the students have discovered helps prepare children to feel more confident about their selected topic and about their ability to communicate their understandings to others. This type of communication in mathematics is receiving increased awareness and importance. The relative unfamiliarity of teachers with these techniques poses an obstacle to more authentic and more accurate assessment of our students. Familiarity with an increased bank of assessment techniques will assist teachers in overcoming this obstacle to appropriate assessment. This project will provide some specific examples of these alternate forms of assessment as required by the new provincial curriculum, via the medium of the operation of multiplication of whole numbers.

Pause For Thought

Reflecting on the information drawn from perspectives of current teacher educators and writers of pre-service teachers' textbooks, several points come to mind regarding the practicality of implementation of such a conceptualization of teaching elementary mathematics. Just how will this ideology of teaching fit in the everyday operation of a classroom in Newfoundland and Labrador? There are many facets of the realities of teaching which must be considered. Ball (1996) identifies the challenges of incomplete knowledge, competing commitments, and of anticipating, interpreting and responding to students as sources of uncertainty to solid implementation of this kind of teaching. "We also need to recognize that any change in teaching behavior takes time and effort" (Hyde, 1989, p.226).

Inappropriate professional development, where teachers are expected to effectively implement new teaching approaches following a one day intensive workshop, is a major concern. The decline in professional development opportunities combined with the reduction of mathematics consultants at the district level has reduced ongoing support in efforts to assist teachers in truly understanding and effectively implementing the recently re-popularized methods of teaching elementary mathematics. Without such support, teachers' conceptualization of what it means to teach this way may be skewed or lack proper clarification. Hyde (1989) says that "schools need to establish ways for teachers to participate in continual dialogues about teaching practice" (Hyde, 1989, p.229).

Heddens and Speer (1995) share an interesting perspective that the success of the whole program hinges on inter-grade level communication. "A student's image of what mathematics is, how it is best learned, and how it relates to their world is not something formed within a given school year - it develops over time" (Heddens and Speer, 1995, p.13). If it is the case that teachers are without a similar vision, inter-grade communication poses another challenge.

Conclusion

"Teaching in today's world is tremendously complex. Any successful program must go beyond the limits of mathematics pedagogy and content to reflect the real world in which teaching takes place" (Troutman and Lichtenberg, 1995, p.xvi). "A fundamental shift in conceptualization and emphasis is occurring, one that has dramatic implications for teaching. Administrators and policy makers must understand the delicate balances that are required between mathematical concepts and reasoning, conceptual and procedural knowledge, developmental instruction and practice, and related issues" (Hyde, 1989, p.224). Though Schifter (1996) referred specifically to the practice and principles of constructivism in the mathematics classroom, I feel the article commentary holds true for teaching in general; "Teachers ... expecting to develop a finished repertoire of behaviors that, once achieved, will become routine will be disappointed." "There is no point of arrival, but rather a path that leads on to further growth and change" (Schifter, 1996, p.499).

Souviney (1994), provides an interesting thought which reiterates the need for ongoing professional development and reflection of our role as teachers:

> The teacher must understand the learning process, select curriculum materials, organize a safe classroom environment, and provide individuals with instructional support at the appropriate teaching moment. The task is complex. However, a teacher who is familiar with the variety of effective instructional practices will be more successful in satisfying the wide range of student needs found in the typical elementary classroom." (p.47)

To provide pertinent information to develop this variety of instructional practices is one of the goals of this project.

Being that we work with such a diverse population of children in our teaching, methods will grow and evolve accordingly with this diversity in mind. The variety of children's ideas will serve as wonderful resources for discussion and learning for everyone involved, thus fostering a positive growth and extended level of understanding of mathematical concepts. It is the role of the mathematics educator to provide authentic learning opportunities in meaningful ways for the children that we teach, fostering a positive learning environment reflective of the NCTM standards. Such should be our commitment to our students.

The focus of the practical component of this project is to help teachers become familiar with these changes. It can also be used as a resource geared toward teaching multiplication of whole numbers in the elementary grades.

CONTEXT FOR THE PROJECT

Pertinent to the mathematics curriculum, this project focuses on the notion that elementary teachers in Newfoundland and Labrador (those of grades four to six), will be faced with a transition period in the very near future in teaching elementary mathematics. As previously mentioned, the new APEF Mathematics Curriculum has been and is being implemented in Newfoundland and Labrador as far as grade three, and is expected to continue into the following grades in subsequent years. In developing this project, it is assumed that the students will be entering grade four having completed the APEF curriculum in grades kindergarten to three. Thus this work is more intended to benefit teachers of grades four, five, and six, experiencing the new program for the first time, rather than the students specifically, as the students will have already spent their previous years following the new program.

Furthermore, the following information is developed with cognizance of elementary teachers who work in classrooms with students of varying degrees of mathematical abilities. The information and examples, reflective of the APEF conceptualization of teaching elementary mathematics, are meant for elementary teachers throughout Newfoundland and Labrador to peruse and implement in their classrooms.

One particular content strand is that of developing operational sense and applying operational principles. Pertinent to this, the operation of multiplication of whole numbers, in terms of the changes necessary to reflect this new conceptualization of teaching elementary mathematics, is the focus of the project. To start, there is a brief look at an example of a standard multiplication algorithm. Then some alternate algorithms are presented as a reference and as tools to support the value of alternate algorithms as discussed in the literature review. The remainder of the practical portion of this project looks at the scope of multiplication of whole numbers through three lenses: 1) procedural knowledge; 2) conceptual knowledge; and 3) appropriate assessment. Each of these is briefly revisited in order to facilitate understanding of these concepts.

A major point concerning procedural knowledge lies with the need for increased attention to alternate forms of presentation of tasks and student response to the tasks: *oral*, *written*, and *model*. In an attempt to make clear the subtle differences which exist here, these alternate forms of procedural knowledge are illustrated in table format using a common multiplication example. Additional examples are also listed.

In educating children about multiplication of whole numbers, there are certain concepts of which children need to demonstrate understanding. Getting at that understanding is often the challenge. Sample questions and tasks are presented which indicate an appropriate level of conceptual knowledge a child should have about the multiplication of whole numbers. This would suggest development, not only of the properties and meanings of multiplication, but also of the language and number sense associated with multiplication of whole numbers.

Finally, appropriate assessment is a key factor in rounding out authentic, meaningful learning experiences for our students. Alternate forms of assessment, as advocated by the APEF Mathematics Curriculum are described with tables of applicable tasks included. Throughout the project, the sample tasks have been generated from my experiences as a teacher and as a student. They come from university classes in the graduate program (i.e. research, class notes, course texts, and sharing with the professor and with classmates), from school resources (i.e. student texts, teacher guides and handbooks, curriculum guides, and sharing with colleagues and students), while others are completely original. It is not intended that these represent the sole tasks to be used by teachers. Nor is it intended that use of these examples by teachers is mandatory. However, it is intended that they be available to teachers to explore as evolving pedagogical tools for teaching multiplication of whole numbers in elementary mathematics. It is the intention of this project to help support the required shift in teaching and the teachers' corresponding transitions.

Being that Newfoundland and Labrador has only implemented the new APEF mathematics program in grades Kindergarten, 1, 2, and 3 over the past four years, the focus has continued to be on lower level learning in grades 4-6. Therefore the majority of learning tends to be procedural in nature versus dealing with a child's prior conceptual experience. From this, in Newfoundland and Labrador, a child's formal experience with multiplication of whole numbers, generally commences with brief exposure to basic facts in primary grades followed by further development of the operation in the subsequent elementary grades. Consequently, it is important to note that the value of this resource increases when used in the earlier stages of students' formal experiences with multiplication of whole numbers. Though some information is applicable in grade three, it may be most ideal if used in the grade four setting. Establishing a sound base in multiplication as a mathematical foundation is important. Thus this information has potential for use in grades five and six and perhaps even beyond, for those children experiencing difficulties with conceptual understanding of multiplication. Children's conceptual knowledge, really knowing what it is they are doing in mathematics and why they are doing it, has become the emphasis.

The material presented here can be used by teachers as an independent resource. However, it can also serve to supplement other resources, such as curriculum guides, student texts, and corresponding teacher guides.

ALGORITHMS FOR MULTIPLICATION

The standard algorithm is presented to facilitate the illustration of the differences in the alternate algorithms. Though there are numerous alternate algorithms, some are considered, by some educators and students alike, to be less helpful in the classroom than others. Of the four alternate algorithms presented, the first and second tend to be most applicable for use in the elementary classroom. The others are purely informational for teachers, and may serve as an extension for some students. Though algorithms are seen as procedural in nature, they have been included in the hopes that teachers would no longer feel chained to the standard algorithm only and that they would see some potential to foster a child's understanding of why the standard algorithm is what it is, versus simply calculating a product in the traditional fashion. A common multiplication example is used throughout to help clarify the difference in algorithms.

☛ Multiply: 45 x 12

STANDARD:

```
   1
  45
 x12
----
  90
+450
----
 540
```

- ➢ Multiply 5 by 2 to get 10. Record the 0 and carry the 1 (which represents 1 group of 10). Multiply 4 by 2 to get 8 (representing 8 tens) and add the group of 10 that was carried over for a total of 9 groups of 10. Thus the 90.
- ➢ Move to the second addition line, under the 90. Write the 0 in the ones place, under the 0 of the 90 above it. Multiply 1 by 5 to get 5. Record the 5 (representing 5 tens). Multiply 1 by 4 to get 4. Record the 4 (representing 400). Thus the 450.
- ➢ Add the 90 and the 450 for the result: 540.

<u>ALTERNATE:</u>

1) Partial Product:

➢ Multiply in parts.
Break up the first number to multiply by parts of the second number as illustrated in A) and B)

A)

$$\begin{array}{lr} & 45 \\ & \underline{\times\ 12} \\ (40 \times 10) & 400 \\ (40 \times\ 2) & 80 \\ (5 \times 10) & 50 \\ (5 \times\ 2) & \underline{10} \end{array}$$

OR

B)

$$\begin{array}{rr} 45 & 45 \\ \underline{\times\ 2} & \underline{\times\ 10} \\ 90 & 450 \end{array}$$

➢ Add the partial products to find the final product.

A)

$$\begin{array}{r} 45 \\ \underline{\times\ 12} \\ 400 \\ 80 \\ 50 \\ \underline{+\ 10} \\ 540 \end{array}$$

OR

B)

$$\begin{array}{rcrcr} 45 & & 45 & & 45 \\ \underline{\times\ 2} & & \underline{\times\ 10} & & \underline{\times\ 12} \\ 90 & + & 450 & = & 540 \end{array}$$

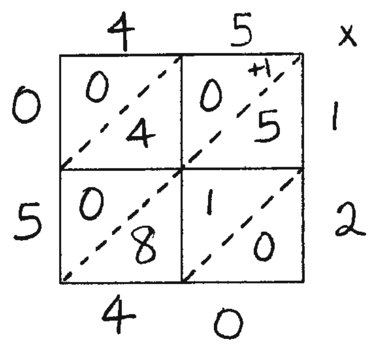
(Area Model): The same process is applied using an area model.
(not required to be drawn to scale)
The parts of the factors are written in this model as opposed to remaining abstract as in the partial products method.
Base ten blocks can also be used to illustrate this model.

40	5	
400	50	10
80	10	2

$400 + 50 + 80 + 10 = 540$

$45 \times 12 = 540$

2) Lattice Method:



45 x 12 = 540

- Draw a grid as shown.
- Write one factor across the top of the grid, being careful to place a single digit atop one block on the grid.
- Write the second factor down the right side of the grid in the same fashion.
- Calculate the separate products, writing each product as a two-digit number (i.e. 5 x 1 = 05). Write the tens digit at the top of the diagonal line and the ones digit at the bottom of the diagonal. Single digit products are written with 0 in the tens place.
- Proceed to add the numbers in each diagonal, beginning in the lower-right diagonal "column".
- Proceed with the addition along diagonals from right to left, regrouping the tens digit to the next diagonal row, if necessary.
- The final product is determined by reading from the upper-left corner, down around through to the lower-right corner.

Note: This method can be extended to multiplying numbers beyond 2-digit by 2-digit by extending the grid to accommodate the digits of the factors. The same process is used. Placement of the numbers along the side or along the top inconsequential.

3) Russian Peasant Method:

	12	x	45	
	6	x	90	
(odd)	3	x	180*	
(odd)	1	x	360*	180 + 360 = 540
			540	12 x 45 = 540

- Use two columns of numbers, one for each factor.
- To get each number in the left hand column, divide the preceding number by 2. Ignore the remainder.
- To get each number in the right hand column, multiply the preceding number by 2.
- Stop when you get to a one in the left hand column.
- Identify all the numbers in the left hand column that are odd numbers.
- Note the numbers in the right hand column which correspond to these odd numbers in the left hand column.
- The sum of these numbers in the right hand column is the final product.
- It is interesting to compare the result when the method is repeated for 45 x 12.

(odd)	45	x	12*	
	22	x	24	
(odd)	11	x	48*	
(odd)	5	x	96*	12 + 48 + 96 + 384 =
540				
	2	x	192	
(odd)	1	x	384*	
			540	12 x 45 = 540

4) Egyptian/ Doubling:

	1	x	45	=	45
	2	x	45	=	90
	*4	x	45	=	180*
	*8	x	45	=	360*
	16	x	45	=	720
➡	12	x	45	=	540

- Begin with one group of either 12 or 45. In general, picking the larger number is preferable.
- Start with one group of 45.
- Continue to double the number of groups until there are at least 12 groups of 45. That is, stop when you reach/pass 12.
- Identify the digits in the first column with a sum equal to 12.
- Add the corresponding products to find your final product.

  <u>NOTE:</u> In this example, you are looking to find 12 groups of 45.

- This can also be completed by starting with one group of 12 and doubling until you can find 45 groups of 12. However, when you compare the result, the findings are interesting and support the notion that starting with the larger number is less cumbersome.

Teachers are recommended to try additional examples to ensure their own level of comfort with these alternate algorithms prior to engaging them in the classroom. For example, the product 28 x 67 = 1876 can be verified using the various algorithms shown.

Alternatives to the traditional algorithms enhance students' conceptual and skill development at their own levels of understanding and decision-making. Additional benefits are those that are not easily measured, such as enjoyment, feelings of success, and building students' overall outlook of multiplication and mathematics, in general, into a positive one.

THREE LENSES ON THE MULTIPLICATION OF WHOLE NUMBERS

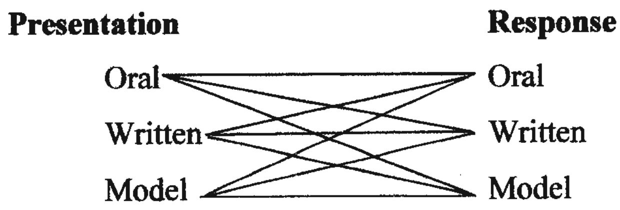
The scope of this section looks at multiplication of whole numbers from three perspectives: 1) procedural knowledge; 2) conceptual knowledge; and 3) appropriate assessment. Each perspective will be briefly revisited theoretically, followed by applicable task examples pertinent to teaching the APEF curriculum.

Procedural Knowledge

Procedural knowledge, also known as *instrumental understanding*, "is an important component of mathematical learning. It includes such things as conventions, symbolisms, routines, and manipulative procedures" (Liedtke, 1998, p.4). The process of completing the work has traditionally featured calculations according to a standard algorithm, which we are all expected to master. Traditional development of procedural knowledge fosters an atmosphere of information giving, (i.e. Here's how you do it.) followed by drill and practice for mastery. This is most often a pencil and paper task. Today, teachers must develop an openness to the non-traditional algorithms with the acknowledgement that basic ability with the traditional algorithm is to be developed at some point, usually later rather than sooner.

Not only must teachers consider the use of alternate algorithms, but they must also consider the alternate forms of presentation and response. Teaching practices can reflect any combination thereof.

Alternate Forms



It is important to note that mathematics educators are traditionally most familiar with a written presentation requesting a written student response. We must be open to exploring new and various forms of procedural knowledge in mathematics. The following pages illustrate these alternate forms by looking at the same multiplication task via the various combinations of presentation and response. Though some differences are quite subtle, it remains that the tasks are, in fact, different.

ORAL PRESENTATION with VARIOUS RESPONSES
(Teacher's instructions are given orally.)

☞ Sample: Calculate: 45 x 12

Teacher	Response Type	Student
Can you tell me the product of 45 and 12? Follow-up: Can you tell me how you got that answer?	**ORAL**	Student responds orally, preferably using mental math to find the answer. Ans.: ➡ 540 Follow-up: 45 times 2, well that's 90. 45 times 10 - that's 450. 450 plus 90 equals 540. So 45 times 12 is 540.
Showing all your workings, would you please write down on your paper what 45 times 12 is?	**WRITTEN**	Student writes: $\begin{array}{r} {}^{1}\\ 45 \\ \underline{\times\ \ 12} \\ 90 \\ \underline{+\ 450} \\ 540 \end{array}$ *Note: Alternative algorithms are acceptable here
Using a model of your choice, represent 45 x 12.	**MODEL** ***Note:** With larger numbers, this may become quite cumbersome and/or require a lot of manipulatives	Student models using choice of manipulative. i.e. a) grouping with unit blocks/ or objects of choice b) base ten blocks/grid

WRITTEN PRESENTATION with VARIOUS RESPONSES
(Teacher's instructions are written. Teacher writes on chalkboard, worksheet, or on medium of choice.)

☞ Sample: Calculate: 45 x 12

Teacher	Response Type	Student
Tell me the product of 45 and 12, and how you got it for your answer?	**ORAL**	Student responds orally, preferably using mental math to find the answer. Ans.: ➡ 540 <u>Follow-up:</u> 45 times 2, well that's 90. 45 times 10 - that's 450. 450 plus 90 equals 540. So 45 times 12 is 540.
Showing all your workings, solve 45 times 12 on your paper. Often, there are no instructions. A student sees: 45 x 12	**WRITTEN**	Student writes: 1 45 x 12 90 + 450 540 * <u>Note:</u> Alternative algorithms are acceptable here
Using a model of your choice, represent 45 x 12.	**MODEL**	Student models using choice of manipulative. i.e. a) grouping with unit blocks b) base ten blocks/grid

MODEL PRESENTATION with VARIOUS RESPONSES
(Teacher presents task in model form, using manipulative of choice.)

☛ Sample: Calculate: 45 x 12

Teacher	Response Type	Student
Will you please tell me what mathematical statement this model represents? <u>Follow-up:</u> How do you know that?	**ORAL**	Student responds orally, preferably using mental math to find the answer. Ans.: ➡ 540 Student explains according to model presented.
Write down what mathematical statement this model represents.	**WRITTEN**	Student writes: $\begin{array}{r} {\scriptstyle 1} \\ 45 \\ \underline{\times\ \ 12} \\ 90 \\ \underline{+450} \\ 540 \end{array}$ * <u>Note:</u> Alternative algorithms are acceptable here
I've used unit cubes to show 23 x 16. Now, would you please model 45 x 12?	**MODEL**	Student models using choice of manipulative. i.e. a) grouping with unit blocks b) base ten blocks/grid

Procedural knowledge is basically computation. The following are additional examples of tasks for procedural knowledge and can be adapted to the presentation and response forms of the teacher's choice, as previously suggested.

SAMPLE PROCEDURAL TASKS

- ✓ How many blocks would be in a train built from 4 sets of 6 blocks?
- ✓ Calculate the area of a rectangle which measures 6 metres by 7 metres.
- ✓ On your paper, can you please write how you would multiply 26 by 53?
- ✓ How would you multiply 50 x 37?
- ✓ Use the pattern blocks to solve 12 x 13.
- ✓ Solve: $\begin{array}{r} 35 \\ \underline{\times\ 64} \end{array}$
- ✓ Find the product. 267 x 5 = □

Conceptual Knowledge

Conceptual knowledge, also known as *relational understanding*, "consists of relationships constructed internally and connected to existing ideas or relationships" (Liedtke, 1998, p.5). Understanding mathematical concepts is becoming the focus of mathematics teaching and learning. It is no longer acceptable for a student to simply be able to compute. It is necessary that a child understand what it is he or she is doing and why. This section on conceptual knowledge reflects what it is children should be encouraged to understand about the operation of multiplication. As a teacher, I am basically asking the student to tell me what he/she knows. Based on what he/she knows, a student should be able to express what else it is that he/she knows. Pertinent to these types of tasks, one should continue to be cognizant of the oral, written, and model combinations in form with respect to presentations and responses.

☞ Consider the following student responses to the same question:

What is the product of 45 x 12 and how did you find it?

Response #1	Response #2
If: 45 x 10 = 450	If: 50 x 12 = 600
and: 45 x 2 = 90	and: 5 x 12 = 60
Then: 45 x 12 = 540	Then: 45 x 12 = 540
By adding the two products together.	By subtracting the 2nd product from the first.

Response # 1 demonstrates that the student understands and can apply the distributive the distributive property of multiplication over addition.

Response # 2 demonstrates that the student understands and can apply property of multiplication over subtraction.

Based on the child's response to such questions, the teacher can remark what the child knows about the properties of multiplication (See Appendix A), as well as what he/she knows about the relationship of multiplication to division. The following table indicates typical questions associated with conceptual knowledge a child should have of multiplication of whole numbers.

CONCEPT		SAMPLE QUESTIONS
Commutative/Order Property	✓	If 45 x 12 = 540, can you tell me what 12 x 45 is?
Associative Property	✓	Can you make any adjustments to help yourself complete this multiplication sentence, 45 x 12 x 2?
Distributive Property over Addition	✓	Investigate how knowing that 12 x 5 = 60 and 12 x 20 = 240 helps you find the product of 12 x 25?
Distributive Property over Subtraction		Explore how knowing the fact that 30 x 24 = 720 helps you solve 28 x 24.
Multiplicative Property of Zero	✓	What can you tell me about any factor multiplied by 0?
Identity Property of Multiplication	✓	What number multiplied by 58 results in 58?
Multiplication by Powers of 10	✓	Explore what happens when you multiply a number by 10? by 100? by 1000? Is there a pattern? If yes, describe it. Is this true for any number? What does this tell you?
Relationship of Multiplication and Division	✓	If 45 x 12 = 540, what is 540 ÷ 12? What is 540 ÷ 45? Does a relationship exist here? Explain.

It is important to note that conceptual knowledge should not be confined only to these properties of multiplication. Firstly, students should have a certain degree of familiarity with the language associated with multiplication of whole numbers. What is a factor? What is a product? How are these related? Is there more than one way to indicate what the symbol "x" represents in a multiplication statement? Secondly, the various meanings of multiplication and how the operation can be used vastly contributes to understanding of whole number multiplication. Students should be familiar and comfortable with these (See Appendix B). Finally, and perhaps more importantly, development of a student's number sense, or his/her intuition about numbers, is essential.

According to Liedtke (1998, p.14), number sense includes ideas such as:

- understanding the different meanings of numbers;
- being able to illustrate relationships between numbers in a concrete or a semi-concrete way;
- understanding the relative magnitude of numbers;
- being able to estimate or check the reasonableness of calculated answers; and
- being flexible in numerical situations.

Teachers must be able to ask appropriate questions to solicit student responses that can indicate the development of number sense or lack thereof. Interpreting the student response is also critical to establishing a student's conceptual knowledge. The following are some sample questions related to the development of number sense in multiplication of whole numbers.

SAMPLE QUESTIONS	
✓	What do you think of when I say "product" ?
✓	Use these cubes to show 6 x 3. If that is 6 x 3, show me 3 x 6.
✓	Make up a word problem for 5 x 8.
✓	If 8 x 15 = 120, what is 9 x 15? 80 x 15?
✓	Without calculating, which product is greater, 24 x 18 or 37 x 21? How do you know?
✓	Tell how you know 3 x 2012 is greater than 6000.
✓	Tell how the first product helps you find the second product. $\begin{array}{r} 70 \\ \underline{\times\ 4} \\ 280 \end{array}$ $\begin{array}{r} 68 \\ \underline{\times\ 4} \end{array}$

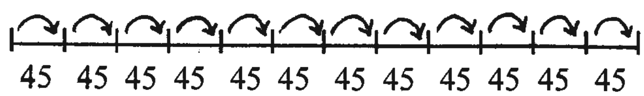
Appropriate Assessment

As discussed in the literature review, it should follow that assessment reflects the changes in goals and instruction in this new dynamic view of teaching elementary mathematics. The APEF Mathematics Curriculum advocates the following examples of alternate forms of assessment: 1) Performance; 2) Paper and Pencil; 3) Interview; 4) Portfolio; and 5) Presentation.

These alternate assessment forms are illustrated in a two-pronged approach, using tasks pertinent to the multiplication of whole numbers. Firstly, to emphasize the differences between the types of assessment, a common multiplication example is explored via the various forms of assessment. Secondly, to further clarify a particular type of assessment, different examples of each type of assessment are offered. It is also important to note that these tasks may also serve as instructional tasks and are not limited to assessment only. Regardless of form or function, these tasks should be familiar to the student prior to being used for assessment purposes.

At this point, I feel it necessary to reiterate that elementary teachers in Newfoundland and Labrador, generally have not yet been exposed to tasks of this nature, as they are still following the old program. The tasks presented here would be totally different than those found in the current (old) curriculum guide, as they fall more in line with the new program reflective of the APEF, which has yet to be implemented in grades four, five, and six.

☞ 45 x 12

TYPE OF ASSESSMENT	SAMPLE TASK
Performance	✓ Ask students to use counters or base ten blocks to show 45 x 12. ✓ Have students construct 45 using base ten blocks. Ask if the number were to be repeated 11 more times, what would the total be. Have them record the appropriate multiplication sentence.
Paper and Pencil	✓ Ask students to write, in their journals, two different ways to think about and illustrate 45 x 12. ✓ Determine the missing digits. $\begin{array}{r} 4\square \\ \times\ \square 2 \\ \hline 5\square 0 \end{array}$
Interview	✓ Present the following picture to the student.  Ask the following questions: 1) What multiplication picture does this show? 2) How does knowing 45 x 6 help you find 45 x 12? ✓ Tell the student that to compute 45 x 12, Sherri first said "12 x 4 = 48". Ask: "What do think Sherri would say next?".
Portfolio	✓ Pose and answer three problems that can be solved using the multiplication sentence 45 x 12. Show more than one way each problem may be solved, one of which doesn't involve multiplying. ✓ Show as many different ways that you can to compute 45 x 12.

TYPE OF ASSESSMENT (continued)	SAMPLE TASK (continued)
Presentation	✓ Have students create a problem representing 45 x 12 and present to the class to solve. ✓ Choose one way to solve 45 x 12 and present it to the class. Be sure to explain what you did and why you chose to do it. Ask the class if there is another way to solve the problem.

To conclude the practical portion of the project, various examples of each type of alternate assessment tasks are listed in the following tables. Again, these are not meant to be required tasks for teaching multiplication of whole numbers. They are merely suggested tasks available to teachers to help them become familiar with what it means to teach multiplication in the new APEF curriculum.

SAMPLE PERFORMANCE TASKS

✓ Ask the students to use unit cubes to illustrate a specific multiplication fact such as 7 x 8. (*can substitute multiplication fact of your choice).

✓ Ask students to use base ten materials to determine how far a cross-country skier can ski in 6 hours, if the skier skis 12 km/hr.

✓ For multiplication facts for x4 , practice the double and double again strategy by putting students in pairs and having them take turns asking each other x4 facts. Students should attempt to provide answers by doubling and doubling again.

i.e. 4 x 6	2 x 6 = 12	
	2 x 12 = 24	Therefore: 4 x 6 = 24

✓ Target Game: Students are to fill in the ☐ .

3 x ☐	➔ 26	☐ are left over
6 x ☐	➔ 34	☐ are left over
9 x ☐	➔ 87	☐ are left over

✓ Ask the students to use grid paper to draw a picture of 8 x 14. Ask them to partition the array in a particular way that might be helpful, in the multiplication procedure, for them or other students, in calculating 8 x 14.

✓ Ask a student to draw a graph to show the 5 times table.

✓ Ask the students to use the digits 4, 5, 6, and 7 to make the largest (or smallest) possible product for ☐☐ x ☐☐.
Ask if there is a better arrangement of these four digits to create a multiplication sentence which yields an even larger (or smaller) possible product.

✓ Ask a student to use a manipulative of his/her choice to explain the order property (or any other designated property) of multiplication.

✓ Use a 10 by 10 grid paper to show me 8 x 9.

✓ Mr. Doolittle and his class of 24 students were preparing Christmas parcels to donate to underprivileged families. Every person prepared 2 boxes each. They all decided it would be nice to put ribbons on the parcels. One parcel required 30 cm of ribbon. Determine how much ribbon the class would need to complete the Christmas Giving Project.

SAMPLE PENCIL & PAPER TASKS

✓ Answer the following by:

1) doing the easiest ones first; circle them O and tell why they were the easiest
2) doing the next easiest ones and put a rectangle □ around them; explain why you thought these were the next easiest ones
3) doing the ones you think are the next easiest and put a triangle △ around these; explain your choice
4) doing any remaining problems; leave as they are and tell why these were your last choice

9	7	4	7	5	4	9	8	6	8	2
x 9	x 5	x 8	x 8	x 3	x 4	x 5	x 9	x 4	x 5	x 9

✓ Ask the students to use the digits 7, 8, and 9 in 3 different ways to write 3 different multiplication sentences. Then solve each. □□ x □ =

✓ Determine the maximum number of people a bus can legally hold if there are 44 seats on the bus and transportation regulations state that only 3 people are permitted to sit in one seat.

✓ Tell the students that the "Sandwich Shop" sells white and whole wheat bread only and has ham, turkey, chicken, and roast beef to go on the customer's choice of bread. Given this information, ask the students to illustrate the number of different choices of sandwiches they can have at the "Sandwich Shop".

Extension: If the customer can choose only one of pickles, carrots, or potato chips to go with the sandwich, determine how many different choices the customer would have now.

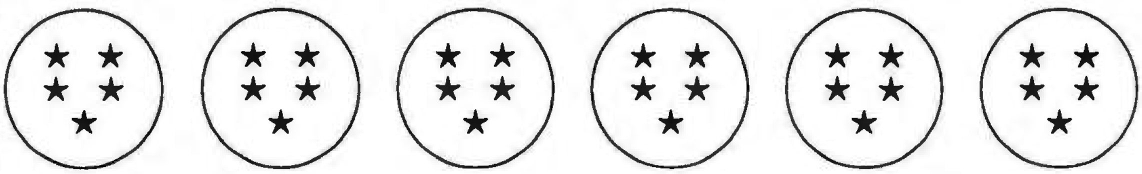
✓ Ask the students to determine the missing digits in the following and similar multiplication sentences. NOTE: One of these is not possible. Can you find which one? Explain why this one is your choice.

□5	□43	□6□	□6
x 6	x 5	x 4	x □
27□	47□□	1468	185

SAMPLE INTERVIEW TASKS

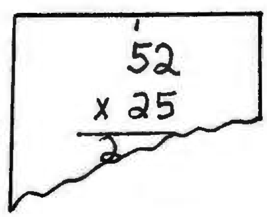
✓ Ask: How would you go about multiplying 7 x 8? Why would you choose this way? Is there another way you could do it?

✓ Ask: Can you tell me what this model shows?
Can you rearrange the total number of stars to make a different multiplication sentence?



✓ Ask: Can you explain to me how knowing 6 x 6 can help you figure out 12 x 6.

✓ Show a student a partial answer and explain that this student's response was accidentally torn off. Ask the student how he/she would finish out the given problem.



✓ Tell the student that when asked to calculate 16 x 5, two students answered correctly but in different ways. Thomas said: "That's 50 plus 30."
Michael said: "That's 8 x 10."
Can you explain how Thomas and Michael got their answers? How would you do this problem?

✓ Louisa accidentally multiplied 7 x 34, instead of 8 x 34. If she correctly found the product of 7 x 34 to be 238. Explain how Louisa can find a correct result without having to start over.

✓ Is the product of 5 x 257 greater or less than 1500? Explain how you can tell without having to do the actual multiplication.

✓ ***Pondering Pencils:*** Tell students that you have 7 boxes of pencils with 12 pencils in each, and 3 other boxes with 5 pencils in each. Ask them to describe at least 2 ways to find the total number of pencils and explain which way they would prefer to use and why.

SAMPLE PORTFOLIO TASKS

✓ Explain how you would use multiplication to help you find the perimeter of a square? of a rectangle?

✓ Have students create a bank of word problems for multiplication and solve them. i.e. 1 per day. At the end of each week, the student chooses their favourite word problem to pose to the class.

✓ ***Homerun Trot:*** In softball, the distance between each base is 60 metres. On Sarah's team, 8 people hit 2 homeruns each. Altogether, how many feet did these 8 people run in their homerun trot? Find the solution in 2 different ways. Explain which you prefer and why?

✓ Write a polite note to Aaron telling him why you knew his answer was wrong before you multiplied it out. Aaron's answer:

$$\begin{array}{r} 226 \\ \underline{\times 5} \\ 930 \end{array}$$

✓ Great Big Sea has hired you to design the stage for their next concert. The stage needs to be rectangular and have an area of 320 square metres.
How many different stages can you design?
Which do you think would be the best and why?

✓ You are performing your own comedy act. Decide how many seats you are going to have available. Assuming the seats can only be arranged in a rectangle, show how many different ways you can arrange your seating plan.

SAMPLE PRESENTATION TASKS

- ✓ Ask students to prepare a report on an upcoming community event where they are responsible for the seating arrangement. They are in charge of seating no less than 100 but no more than 200 people. Present your project to the class as if it were a proposal to the community council. Explain how multiplication can be used to help you create the seating possibilities and why you decided on the arrangement that you did.

- ✓ Ask students to prepare a report on how much floor covering materials and what it would cost to re-floor 3 rooms of their choice in their house. Get 3 different price quotes from the local businesses to find the best price. Present to the class. Show how multiplication can help find possibilities.

- ✓ Ask the students to poll their family members to find out how many hours of sleep each person usually gets in one night. Using multiplication, prepare a presentation on how many hours sleep each person would get

  a) per week?
  b) per month?
  c) per year?

  What would be the total number of hours slept by all in your household for the same 4 time periods (day, week, month, year)?

- ✓ Ask a student to determine the total number of hours spent in organized physical activity by students in the class by using multiplication.

  i.e. playing hockey in the local Minor Hockey program
  bowling in the Youth Bowling program
  training in the local gymnastics program
  swimming in lessons and/or swim team programs

- ✓ Mrs. Gardenfare hired two students to plant her 150 flower bulbs in her rectangular flower bed in her back yard. Devise several different proposals to show Mrs. Gardenfare.

- ✓ You are an expert dietitian. You have taken on the job of planning one week's menu of three balanced meals per day for a family of four: two adults, a six year old boy and a thirteen year old girl. Consult the latest Canada Food Guide recommendations for nutritional requirements for such a family. Write your menu for the week. Using grocery ads from newspapers, determine the cost per week for your recommended menu.

(Adapted from Kulm, 1994, p.46)

APPENDIX A

Properties Of Multiplication

PROPERTY	STATEMENT
Commutative/Order Property	Two factors can be multiplied in any order and it does not affect the product. Example: 6 x 4 = 24 OR 4 x 6 = 24
Associative Property	When multiplying more than two factors, you can group any two factors together first, and then multiply by the remaining factors. The product will be the same. Example: 2 x 3 x 5 (2 x 3) x 5 OR 2 x (3 x 5) 6 x 5 2 x 15 30 30
Distributive Property of Multiplication over Addition	The product of a number and a sum can be expressed as a sum of two products. Example: 4 x (10+ 5) = (4 x 10) + (4 x 5) Thus helping students solve 4 x 15.
Distributive Property of Multiplication over Subtraction	The product of a number and a difference can be expressed as a difference of two products. Example: 7 x (20 - 2) = (7 x 20) - (7 x 2) Thus helping students solve 7 x 18.
Multiplicative Property of Zero	When two factors are multiplied and one of the factors is zero, then the product is zero. Example: 5 x 0 = 0, 67 x 0 = 0, 139 x 0 = 0
Identity Property of Multiplication	Whenever you multiply a whole number by one, the product will be that whole number. Example: 4 x 1 = 4, 23 x 1 = 23, 567 x 1 = 567

APPENDIX B

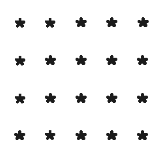
Meanings Of Multiplication

MEANINGS OF MULTIPLICATION

1) As a process of repeated addition: A special case of addition where all the addends are of equal size and are added together over and over or repeatedly.

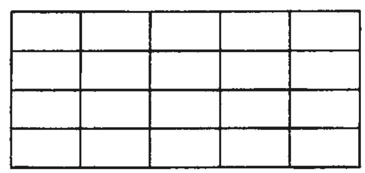
Ex. $4 + 4 + 4 + 4 + 4 = 20$ $4 \times 5 = 20$

2) As an array: An array is an arrangement of objects or symbols into orderly columns and rows. You can read this vertically then horizontally, or vice versa, to get the factors. The product of these factors then represents the total. Often this is in a rectangle or grid form.



$4 \times 5 = 20$

array of objects

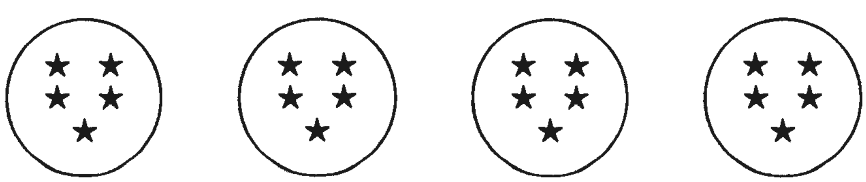


$4 \times 5 = 20$

rectangular array

3) ***As a collection of equal groups:*** This multiplication meaning is often the most commonly understood. It can be seen as joining a number of equal collections – so many groups with an equal number of objects in each group.

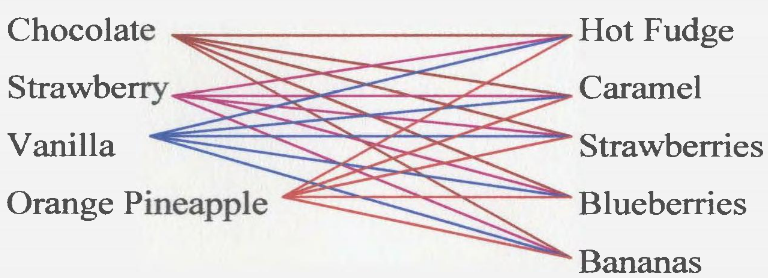
		# of objects		
<u># of groups</u>		<u>in each group</u>		<u>total #</u>
factor	x	factor	=	product



4 x 5 = 20

4) As a Cartesian product or "Combinations": Also known as *"cross product"* or *"combinations"*, this meaning is understood by matching two different elements or variables. This is sometimes a little bit difficult. The best way for us to think of it is *combinations of factors.*

Ex. You have four kinds of ice-cream and five possible toppings. How many possible combinations can you get?

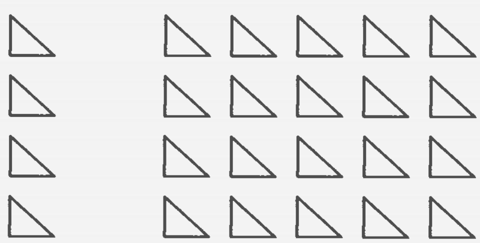


Tree Diagram: When all the lines are counted, there are 20.

4 flavours x 5 toppings = 20

5) As a ratio: Each triangle is really a representation of 5.

For every one triangle there are really 5.

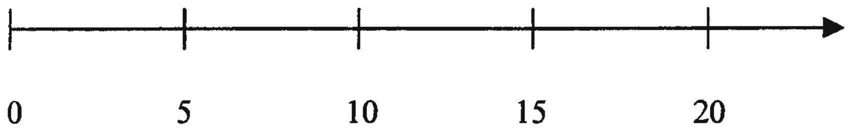


4 x 5 = 20

6) As a mapping: A similar concept to addition, multiplication maps or assigns a pair of whole numbers to a specific whole number. The difference being the multiplication operation finds "the product" to be the unique whole number as a result of the mapping.

Ex. $(4,5) \xrightarrow{x} (20)$

7) As a number line:



$$4 \times 5 = 20$$

This represents moving 5 spaces 4 times on a number line for a total of 20 spaces – another way of understanding multiplication.

8) As a rate: This meaning involves finding a total, given a number of items and a rate affecting them. Ex. It takes a painter 5 hours to paint 1 door. How many hours does it take to paint 4 doors?

$$4 \times 5 = 20$$

9) As a comparison: This simply is used to help compare. Ex. Julie has saved \$5. Deanne has saved 4 times as much. How much has Deanne saved?

$$4 \times \$5 = \$20$$

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