EFFECTS OF NORMAL FORCE, VELOCITY AND LUBRICATION ON ICE FRICTION AT HIGH PRESSURES

by

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ABSTRACT

This thesis focuses on understanding and explaining the behavior of the coefficient of friction for ice against steel in the conditions experienced by ships and offshore structures designed for Polar Regions. Friction tests were executed with a 30°, 25 cm diameter, conical sample of ice and a sandblasted stainless steel plate using a rotational ‘turntable shaped’ apparatus in a large cold room to investigate the effect of the normal force, contact area, pressure velocity and lubrication. Pressures up to 1.4 MPa were measured in the results and the ice was observed to fail by a combination of crushing and abrasion. A design of experiments analysis was performed on the results. This analysis indicated that there was almost no variation in friction coefficient associated with changing pressure, contact area, velocity and lubrication. Using conical ice samples, which changed contact areas during the tests, demonstrated that trends observed in past ice friction studies may not be fully representative of ice interactions with structures since ice pieces found at sea are never of perfect regular shape.
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<td>Friction force</td>
</tr>
<tr>
<td>$F_N$</td>
<td>Normal force</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Coefficient of friction</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Contact angle in between a drop of water and a surface</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$V$</td>
<td>Sliding velocity</td>
</tr>
<tr>
<td>$A$</td>
<td>Apparent area of contact</td>
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<td>$RH$</td>
<td>Relative humidity</td>
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<td>$\lambda$</td>
<td>Thermal conductivity</td>
</tr>
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<td>DAC</td>
<td>Data acquisition and Control System</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Mean Roughness</td>
</tr>
<tr>
<td>$F$</td>
<td>Force</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$a$</td>
<td>Acceleration</td>
</tr>
<tr>
<td>$g$</td>
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</tr>
<tr>
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</tr>
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<td>Converted Output of Friction Sensor</td>
</tr>
<tr>
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<td>Mass of Railing and L-Shaped Part (including Friction Sensor)</td>
</tr>
<tr>
<td>$m_2$</td>
<td>Mass of Forklift Part</td>
</tr>
<tr>
<td>$m_3$</td>
<td>Mass of Camera (including Case)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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</tr>
<tr>
<td>$m_i$</td>
<td>Mass of C-Clamp Holding Camera in Place</td>
</tr>
<tr>
<td>$m_E$</td>
<td>Mass of Extra Parts Not Measured by Bucket Sensor, Excluding Ice Cone</td>
</tr>
<tr>
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</tr>
<tr>
<td>$w_2$</td>
<td>Angular Velocity of Turntable in $\text{rad/s}$</td>
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<tr>
<td>$V_c$</td>
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</tr>
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</tr>
<tr>
<td>$D$</td>
<td>Diameter Turntable</td>
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<tr>
<td>$d_{R,C}$</td>
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<tr>
<td>$A$</td>
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</tr>
<tr>
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</tr>
</tbody>
</table>
\( m_i \) \hspace{1cm} \text{Mass of Ice} \\
\( V_i \) \hspace{1cm} \text{Ice Volume} \\
\( V_{T,I} \) \hspace{1cm} \text{Initial Volume of the Ice in Cone Holder} \\
\( V_{I,cone} \) \hspace{1cm} \text{Volume of Imaginary cone} \\
\( m_{I,cone} \) \hspace{1cm} \text{Mass of Imaginary Cone} \\
\( \rho \) \hspace{1cm} \text{Density of Ice} \\
\( m_{I,R} \) \hspace{1cm} \text{Remaining Ice Mass on Holder} \\
\( F_{I,R} \) \hspace{1cm} \text{Remaining Sample Force} \\
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1. Introduction

Friction is an integral part of daily life. From its importance in tire design, to keep cars on the road, to its effect on bearings or even on individuals shoes as they walk, the effects of friction are important. For a large number of solid materials, the fundamental laws of friction are well understood and widely acknowledged. However for materials sliding on ice, understanding of the physical mechanisms remains contentious. There are competing theories and significant variations in measured friction coefficients. Even the concept of a simple friction coefficient does not hold very well for friction on ice except that it is known that ice exhibits a lower level of friction than most other materials.

Ships and offshore structures designed for Polar Regions interact with ice on a regular basis. Both structures will most often be found crushing, bending and submerging or lifting ice. During all these interactions, there is always contact between ice and material which involves certain levels of sliding and friction. The offshore structure and ship areas that come into contact with ice and that are subject to sliding ice are most often sloped. Robert Frederking, in 2001, showed that friction is an integral part of the total force to which a sloped structure (ship or offshore structure) is subjected. The large portion in the total design force that has to be allotted to friction shows the importance of understanding the behavior of sliding ice and the pertinence of ice friction to the design of ice structures.

Due to its importance in ice structure design and its intriguing inconsistencies with conventional theory, ice friction is the subject of research in this project. The introduction
will go over the history of friction, the main theories developed throughout the years related to the sliding of ice, the factors that have been investigated in relation to influencing ice friction and the experimental methods that have been used as well as the objectives and what will be added to the subject. The main purpose of this work is to measure friction coefficients under conditions when the ice is crushing and changing shape under the sorts of pressures that might be experienced during real life ice-structure interactions. In addition it is hoped to develop a more comprehensive expression for ice friction coefficient that captures some of the previously observed variations associated with changing pressure, contact area and velocity.

1.1. The History of Friction

Friction between materials is a physical mechanism that has been observed well before the specifics defining it were understood. Observation goes back far beyond the findings which led to an equation giving us a way to calculate the friction force. An account of important events relating to friction is discussed in this section.

The use of friction can be traced back to the Neanderthal Age, about 200 000 B.C.. Friction was used by cave men to make fire by rubbing wood on wood or striking flint stones. The first known use of low ice friction was in Scandinavia, 7000 B.C., where rock carvings illustrated the use of sledges to transport heavy goods. Later on, as shown by carvings dating back to 2400 B.C., lubrication was applied by Egyptians to facilitate the sliding of sledges by pouring a liquid in front of them.
The first recognition of friction can be found in *Questiones Mechanicae* by Aristotle, about 2000 years prior to its first quantitative study by Leonardo Da Vinci. Da Vinci studied the influence of the apparent area of contact against the frictional resistance and also studied lubrication. He was furthermore the first to observe and document wear as well as to distinguish between rolling and sliding friction. Da Vinci proposed the 2 laws of friction:

“Friction produces double the amount of effort if the weight be doubled” (friction force proportional to normal force)

“The friction made by the same weight will be of equal resistance at the beginning of its movement although the contact may be of different breadth and length” (friction force independent of contact area)

[Kietzig et al., 2010]

Guillaume Amonton, a French physicist, came to these same conclusions in 1699. His work was however published prior to Da Vinci’s leading to the laws often being referred to as Amonton’s laws. Amonton further observed that roughness was the fundamental cause of friction. The clear distinction between static and dynamic friction in addition to the introduction of the friction coefficient (μ) was then brought forward by Leonard Euler. Charles Augustin Coulomb was later the first to express the force exerted by
friction with the following equation:

\[ F_T = \mu F_N \]  \hspace{1cm} (1)

where \( F_T \) is the force of friction, \( F_N \) the normal force and \( \mu \) the coefficient of friction which was assumed to be independent from velocity at the time. We now know that this only holds if the sliding velocity is not too low or too high.

![Figure 1: Timeline of Friction Studies [Kietzig, Hatzikiriakos & Englezos 2010].](image)

The interest surrounding ice arose only 150 years ago. Michael Faraday brought two ice blocks into contact with each other which instantly bound them to one another by frost.
His conclusion was that the surface of the ice was covered by a liquid-like layer. This discovery precipitated further research in the hopes of understanding ice along with the role that its surface plays in ice friction [Kietzig et al., 2010].

### 1.2. The Different Theories for the Low Friction of Ice

The study of ice friction was only initiated in the last century as mentioned previously. The causes for the slipperiness of ice as well as its surface characteristics became a source of interest which led to a number of theories being brought forth to explain the low friction of ice and, many more questions being raised. The three proposed frictional mechanisms, as well as what led to their suggestion, are discussed below.

As mentioned earlier, Faraday was the first to kindle an interest in the surface characteristics of ice. He discovered a fascinating property of ice, since called “regelation” [Whitney & Smith, 1896]. He showed that two slabs of ice, with flat surfaces, when brought together, would unite, at a room temperature higher than the freezing point. Faraday explained this property by assuming that a small quantity of water, delimited on every side by ice, would have a natural tendency to freeze [Lippincott, 1893]. He maintained that these films, initially on the surfaces of both the pieces, would come to be in the center of the ice once these pieces were pushed together, the solidifying power of ice on water then acting on both sides of this one film and causing it to congeal and freeze the pieces together [Alden, 1883].
The first mechanism or theory proposed to describe the low friction of ice arose from Faraday’s experiments. This mechanism, pressure melting, was however brought forth by James Thomson shortly after Faraday’s publication in 1859. Thomson did not agree with Faraday’s claim that a liquid layer covered the ice. He believed that regelation was due to the pressure between the two ice cubes. His hypothesis, however, was never demonstrated in an experiment [Rosenberg, 2005]. Despite J. Willard Gibbs’s affirmation that Faraday’s conclusions concurred with his own findings in his paper on thermodynamics, Thomson’s supposition prevailed for over a century [Gibbs, 1961]. Pressure melting was likewise referenced in 1886 by John Joly and in 1899 by Osborne Reynolds to explain ice skating. Joly referred to Thomson’s results and maintained that a liquid layer appeared at the surface of the ice, and acted as a lubricating film, due to the extreme pressure the solid was subjected to (as a result of the small contact area of the skating blades) [Joly, 1886]. This film would have the essential characteristics resembling liquid water. Theoretically, at temperatures approaching zero, due to the arrangement of the delimitation in the phase diagram, shown in Figure 2, in between ice Ih and liquid water, this mechanism was plausible. This meant that an increase of the pressure on ice at a constant temperature close to 0 °C would eventually result in a phase change from solid to liquid as it would lower the melting point. However, the amount of pressure needed for this mechanism to develop at temperatures that were not close to 0 °C was not accounted for in Joly’s paper.

As demonstrated by the geophysicist Samuel Colbeck in a series of papers published around 1995, at temperatures lower than -20 °C, the ice would change phase from one solid form to another (see Figure 2) as the pressure was increased with a constant
temperature [Colbeck, 1995]. Moreover, pressures required between – 20 °C and 0 °C to melt the ice were very unlikely to be reached when skating. These facts left unexplained the mechanisms that enable skiing, skating and even bobsleighing at low temperatures, as ice remains slippery even in -30 °C weather. Similarly, how could the concept of a sliding hockey puck be explained as it is not subjected to extreme pressures and still glides on ice?

![Phase Diagram of Water](image)

**Figure 2: Phase Diagram of Water [Rosenberg 2005].**

An alternative to pressure melting as well as the second theory, frictional heating, was proposed by Frank P. Bowden and T. P. Hughes in 1939. Bowden and others experimented on the static and kinematic friction coefficients of both metal and wooden skis on snow. They calculated that pressure was insufficient to cause melting at low
temperatures and their results indicated that metal skis had a higher friction coefficient than wooden ones [Rosenberg, 2005]. They concluded, from these experiments, that frictional heating was responsible for melting the ice. Given that wooden skis have a lower thermal conductivity than metal skis, it was a reasonable assumption. This meant that, due to the friction, wooden skis would not conduct the heat away from the ice as rapidly as metal skis would. This entailed that a good insulating material would slide on ice faster with lower friction than a highly thermal conductive material which was true in general. However, this theory was contradicted by magnesium, a highly thermal conductive material, which gives very low values of friction on both snow and ice [Pounder, 1965]. Colbeck, Najarian and Smith later provided experimental evidence for frictional heating by attaching thermocouples to skate blades and afterwards skis and measuring an increase in temperature with velocity [Colbeck et al., 1997]. This proved that pressure melting was very unlikely the dominant contribution as it is an endothermic process, meaning a decrease in temperature would have been expected [Rosenberg, 2005].

Both theories, pressure melting and frictional heating, shared the assumption that a film of water was formed during the friction process which would act as a lubricant. Elton Roy Pounder, in 1965, proposed to use the wettability index as a means to measure the adhesive properties of the proposed liquid layer on different materials in order to support that layer’s existence. The adhesion of water to any material is maintained by two forces, the cohesive force which keeps all the molecules of the liquid united and the adhesive force, the force with which the molecules of liquid adhere to the surface they come into
contact with. The adhesion force is greater than the cohesion force of the molecules when a material has a high wettability index, water then sticking to the surface and wetting it. A molecule’s cohesive force dominates, when a material has a low wettability index, as the liquid will not want to adhere to the surface and will be more likely to slide on it [Carboni, 2002]. The wettability of a material to water is defined by the contact angle between a drop of water and the material (see Figure 3). Materials with very high wettability indexes, having a contact angle, $\alpha$, near 0°, like aluminum, ski lacquer and nylon, do not have low friction coefficients as they adhere easily to water, which supports the existence of a water film (see Figure 3). Furthermore, Teflon (polytetrafluoroethylene), a highly hydrophobic material (low wettability), has a contact angle of 126° and is probably one of the lowest friction ski coatings known. It is important to note, once again, that magnesium goes against the model and contradicts the hypothesis as it has a very high wettability index but has low friction on snow and ice [Pounder, 1965].

![Figure 3: Contact Angle of a Drop of Water on Different Surfaces](Carboni, 2002)

Thomas H. McConica was the first to propose, in 1950, that the lubricating layer might not be liquid-like but vapor. His supposition arose from extensive tests on numerous
materials, including magnesium. He believed that the water vapor layer would be produced by frictional heating like Bowden’s theory [McConica, 1950]. This assumption accounted for the irregularities with magnesium as it meant that the slider’s thermal conductivity was of much less significance in the friction process, vapor-solid heat transfers being far slower. What's more, according to Pounder (1965), “there is considerable evidence that water vapor or other gases play a role in sliding friction”. It is shown that two surfaces of the same metal, one clean (out-gassed) and the other in its normal state, will render two very different friction coefficients, the normal state surface (with oxide layers containing absorbed gases) producing significantly lower friction. As vapor is sensitive to the pressure and the temperature, this vapor layer would also justify why the friction of ice is dependent of temperature, pressure and many more factors [Pounder, 1965].

C. D. Niven found the assumption of the presence of a liquid-experiment like layer questionable as very cold water, close to freezing, on the relief of pressure or the loss of heat would be less prone to act as a lubricant and would more likely act as an adhesive [Pounder, 1965]. He countered with a new suggestion, in 1959, the final theory, the idea of a disordered surface layer. Niven attributed the low friction of ice to the possibility of a molecular rotation [Niven, 1959]. As the surface H₂O molecules do not have a complete set of hydrogen bonds, they are not entirely locked into place. This molecular setting allows for them to easily rotate along a single bond as presented in Figure 4.
Niven proposed that the single molecule or group of molecules acted as roller bearings on the surface of the ice to reduce friction. This phenomenon, similar to melting, would lead to cooling once the asperities of the material had moved away from a particular spot [Pounder, 1965]. This fall in temperature would then lead to a more rigid-like structure with the possibility for the ice to adhere to the material as in Faraday’s results.

![Figure 4: Disordered Surface Layer [Somorjai, 2008].](image)

1.3. Experimental Apparatus for the Testing of the Friction on Ice

Many articles, journals and book sections have been written to describe and explain various methods of experimentation with ice friction. These experiments, not only conducted in laboratories but also on test sites, offer a diversity of experimental setups. The different apparatus used for the measurement of the friction on ice, the experimental methods as well as their advantages and disadvantages are presented in this section.
1.3.1. Real-Life Experiments

Many studies of ice friction have been driven by sporting activities such as skating, skiing and sledding. Colbeck experimented with real-life ice-skating using an instrumented skate. However, as mentioned in section 1.2, he only measured the temperature under the skates demonstrating that pressure melting was not a viable explanation for the low friction of ice. The coefficient of friction was not measured with his skate [Colbeck et al., 1997].

J. J. de Koning, G. de Groot and G.J. Van Ingen Schenau were the first to attempt real-life tests in order to measure the coefficient of friction from skates. Their speed skates were equipped with strain gauges that allowed for the measure of the push-off, a force resulting from the start of skating, and the friction force during skating. Their experiments were done on many different indoor and outdoor ice rinks with an experienced speed skater [de Koning et al., 1992].

Lutz Hoffman attempted a real-life experiment using a research vessel, the Polarstern, with similar results to K. A. Forland and J. -C. Tattinclaux in 1984. Hoffman managed to measure the tangential and normal forces that the hull was subjected to when the ship was breaking ice using two triaxial force measuring devices installed in pockets on the underside of the ship [Hoffman, 1985]. S. Kivimaa, in 1992, also performed measurements on a ship. Kivimaa used a Finnish coast guard cutter: Uisko. The target
measurements were the ice loads on the ship’s hull, but some friction data was recorded [Mäkinen et al., 1994].

The benefit of these experiments was that they were done in real-life conditions, yet, the control of the many factors and variables that may have influenced the friction of ice was lacking. For de Koning’s tests, different ice rinks may have had different ice making methods in addition to different water quality, and the different locations, different air characteristics (relative humidity, air temperature, air pressure). All these changeable elements may have influenced ice’s friction coefficient. Furthermore, the use of a speed skater added a human factor to the list of variables and could have resulted in changes that are difficult to predict and replicate. The measurements, which are highly dependent on the skater’s performance, would have also varied depending on his skating technique. On different days at different times, his performance may not have been constant resulting in different pressures, velocities and loads [Kietzig et al., 2010]. As with de Koning et al.’s experiment, Hoffman’s and Kivimaa’s tests were carried out in an environment that was hard to control. The atmospheric pressure and temperature, relative humidity as well as the type of ice (first, second year ice, etc.) they were crushing were not discussed [Hoffman, 1985].

1.3.2. Slider Prototypes

Bowden used slider prototypes to measure the friction on snow and ice. His prototype was a sledge cut out from sheets of different materials with rounded edges to facilitate sliding. In this experiment the surface, whether it was snow or ice, was controlled.
However, the method with which the sled was moved to produce kinetic friction was unclear [Bowden, 1953].

Daisuke Kuroiwa as well as Katsutoshi Tusima used a skate-frame with real skate blades for their experiments in measuring the kinetic friction coefficient. The model was accelerated using a catapult, which facilitated the control of the velocity [Kuroiwa, 1977]; [Tusima, Unknown].

D. Slotfeldt-Ellingsen and L. Torgersen as well as K. Itagaki, G.E. Lemieux and N.P. Huber, like Kuroiwa, used automatically accelerated slider prototypes of varying mass and dimension to measure the friction force on ice. They however followed a distinct ice making method to produce the ice used for the tests to further limit the factors that varied [Slotfeldt-Ellingsen & Torgersen, 1983]; [Itagaki et al., 1987].

The use of slider models reduced the unpredictability of certain parameters. The human factor was diminished, as a person did not maneuver the slider. The type of ice used, the velocity and the weight were also controlled. However, one problem came from the fact that the path of the slider could not be completely anticipated. The slider may have diverted from its initial route on the ice over many different launches. This added a certain inconsistency to the experiment that would need to be accounted for [Kietzig et al., 2010].
1.3.3. Linear Apparatus

A variety of linear setups were used by S.J. Jones, H. Kitagawa, K. Izumiyama and H. Shimida., M. Montagnat and E.M. Schulson, S. Ducret, H. Zahouani, A. Midol, P. Lanteri and T.G. Mathia, R. Frederking and A. Barker, S.R. Cho, E.J. Chun, C.S. Yoo, S.Y. Jeong and C.J Lee, B. Lishman, P. Sammonds, D. Feltham and A. Wilchinsky, H. Saeki, T. Ono, N. Nakazawa, M. Sakai and S. Tanaka, L. Karlöf, Axell L. Torgensen and D. Slotfeldt-Ellingsen, and finally B.A. Marmo, J.R. Blackford and C.E. Jeffree. The equipment varied from one experiment to the next but the concept remained the same. They all had a controlled movement of the slider on the ice surface using a specific mechanism. The problem of an unpredictable route for the slider was therefore solved. Additionally, the velocity, the ice making procedure as well as the load remained controlled [Jones et al., 1994; Montagnat & Schulson, 2003; Ducret et al., 2005; Marmo et al., 2005; Frederking & Barker, 2001; Frederking & Barker, 2002; Cho et al., 2011; Karlöf et al., 2005]. Both Montagnat and Schulson as well as Ducret et al. added the regulation of a new factor, the temperature, conducting their experiments in a cold room or freezer unit [Kietzig et al., 2010]. L.E. Raraty and D. Tabor also used this type of model to conduct their experiment. They were however measuring the adhesion and specific shear strength of ice to different metals with varying temperatures and specimen heights and not the friction coefficient [Raraty & Tabor, 1958]. The adhesion is nonetheless intimately linked to the friction force.
An advantage of the linear setup was that the friction on ice occurred on a new ice surface throughout the experiment [Kietzig et al., 2010]. The consistency and nature of the surface did not change or could not be obstructed by broken or melted pieces of ice. The ice, as a factor, remained unaltered. However, a linear setup also required a lot of space and, complex as well as expensive mechanisms.

1.3.4. Rotational Apparatus

A variety of rotational setups were used to measure the friction force on ice. Like the linear setup, rotational devices made for an easy control of the load and the velocity. Controlling the quality of the ice was also achievable. Following a method to produce the ice could also sometimes be simpler as the samples could be somewhat smaller. Contrary to linear devices however, rotational apparatuses were more compact which facilitated the use of a cold box or temperature chambers [Kietzig et al., 2010].

I.I. Kozlov and A.A. Shugai used a rotary viscometer to carry out their experiments. The apparatus used a rotating flat metal ring that slid against a hollow cylinder. The device necessitated the measure of the vertical displacement of the ring as it dug into the cylinder wall, melting it as the ring rotated [Kozlov & Shugai, 1991]. Many more tests made use of a metal disk or ring pushing up against a motionless ice sample. H. Strausky, J.R. Krenn, A. Leitner and F.R. Ausseneqq, D. Buhl, M. Fauve and H. Rhyner, H. Liang, J.M. Martin and T.L. Mogne, and A.-M.Kietzig, S.G. Hatzikiriakos and P. Englezos all succeeded in analyzing the friction of ice using this setup. As mentioned previously, this type of apparatus made for easily controlled factors and minimal variability. But, the
rotating metal part of the setup found itself continuously going over the same ice patch [Strausky et al., 1998; Buhl et al., 2001; Liang et al., 2003; Kietzig et al., 2009]. R.E. Gagnon and J. Mølgaard also used a rotating metal piece for their tests. Their part was however more of a wheel than a plate as the ice sample was pushed up against its outer surface creating the same problems as for the previous setups mentioned (Strausky et al., Buhl et al., Liang et al. and Kietzig et al.’s setups).

D.C.B. Evans, J.F. Nye and K.J. Cheeseman as well as Victor F. Petrenko reversed the previous model so as to have a moving ice sample and a stationary material segment instead. The ice cylinder rotated against the sample, permitting the ice to refreeze when that area turned away from the piece, losing contact with the material [Evans et al., 1975 1976; Petrenko, 1994]. L. Bäurle, D. Szabó, M. Fauve, H. Rhyner and N.D. Spencer, M. H. Liang et al., Bowden and Hughes, A. Mills, C. D. Niven, P. Oksanen and J. Keinonen, Tusima as well as A. Lehtovaara used a similar setup with different variations. The sample was mounted on an arm pushing down on an ice turntable. This permitted an horizontal movement of the piece, allowing it to come in contact with fresh ice every rotation [Bäurle et al., 2006; Liang et al., 2003; Bowden & Hughes, 1939; Oksanen & Keinonen, 1982; Mills, 2008; Niven, 1956; Tusima, 1977; Lehtovaara, 1987]. M. Akkok, C.M.McC. Ettles and S.J. Calabrese, S.J. Calabrese, R. Buxton and G. Marsh and F. Albracht, S. Reichel, V. Winkler and H. Kern’s setup an apparatus opposite to the previous one discussed. They had a rotating sample pushing down on fixed ice. Calabrese et al. used a ring, causing the same problems as Kozlov and Shugai’s apparatus. Akkok et al. used a ball on ice disk model allowing the ice to refreeze after each pass of the ball.
Albracht et al. produced a spiral track for their piece (a pin slider) in order to refrain it from passing twice on the same ice area [Akkok et al., 1987; Calabrese et al., 1980; Albracht et al., 2004].

In fact, to measure the friction on ice with controlled parameters, it is essential to use one of the laboratory apparatuses described above. These techniques ensure good regulation of the velocity, the load, the humidity as well as the air and ice temperature. An established ice making procedure may also add stability to the experiment by limiting undesirable variability in the results [Kietzig et al., 2010]. Experimental mechanisms with samples that go over the same area of ice repeatedly as well as samples that never come into contact with the same ice’s region, whether they are rotating or sliding, are relevant when it comes to modeling ice friction on marine structures. As the movement and displacement of ice is haphazard when cleared by ships or offshore structures, it is hard to model ice friction on these constructions in the laboratory. Furthermore, crushed ice on water in real-life situations does not come in cylindrical or spherical shapes but tends to change shape as an interaction proceeds, starting with a small contact area and increasing to a larger contact.

1.4. Factors that Influence Ice Friction

A large amount of research has already been done for ice friction as reported in section 1.3. The force from sliding on ice is relevant in many different fields. Ice friction is pertinent for winter sports [Bowden & Hughes, 1939; Bowden, 1955; McConica, 1950; Evans et al., 1975; Kuroiwa, 1977; Itagaki et al., 1987; Strausky et al., 1998; Buhl et al.,
2001; Ducret et al., 2005; Karlöf et al., 2005; Bäurle et al., 2006; Kietzig et al., 2009; etc.]. It has also been studied to help with the design of ships and offshore structures in engineering [Calabrese et al., 1980; Saeki et al., 1986; Kishi, Yamauchi, et al., 1994; Mäkinen et al., 1994; Frederking & Barker, 2001; Frederking & Barker, 2002; Lishman et al., 2009; Cho et al., 2011; etc.]. Finally, the friction of ice has likewise been investigated in physics, chemistry and other domains for its interesting characteristics [Niven, 1956; Niven, 1959; Raraty & Tabor, 1958; Tusima, 1977; Goleckit & Jaccardt, 1978; Slotfeldt-Ellingsen & Togersen, 1983; Beeman et al., 1988; Kozlov & Shugai, 1991; Makkonen, 1994; Petrenko, 1994; Jones et al., 1994; Liang et al., 2003; Montagnat & Schulson, 2003; Maeno et al., 2003; Albracht et al., 2004; Marmo et al., 2005; Higgins et al., 2008; Gagnon & Mølgaard, 1989; etc.]. Many, as denoted, have experimented with ice friction and acquired interesting results. Studies have shown that the coefficient of friction for ice varies depending on many different factors. The variety of elements covered in studies as well as their range is remarkable. The results in the variation of ice’s friction coefficient, with respect to many different factors, acquired in different research projects, are further discussed and compared below. It is important to note that some comparison between results were hard to make as the operating conditions were not the same for each experiment.

1.4.1. Temperature $T$

A summary of the operating conditions for the most pertinent experiments yielding data on the effect of temperature upon the friction coefficient of ice is presented in Table 1.
The differences in operating conditions between projects, described in Table 1, may explain the variation between the sets of data that will be discussed subsequently.

Table 1: Difference in Operating Conditions with Respect to the Temperature

<table>
<thead>
<tr>
<th>Reference</th>
<th>Slider Materials</th>
<th>Velocity (m/s)</th>
<th>Normal Force (N)</th>
<th>Contact Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albracht et al., 2004</td>
<td>Cr-Steel pin</td>
<td>0.13</td>
<td>1</td>
<td>~ 2 mm(^2)</td>
</tr>
<tr>
<td>Bäurle et al., 2006</td>
<td>PE Block</td>
<td>3 – 5</td>
<td>52 to 84</td>
<td>2 – 4 – 10 cm(^2)</td>
</tr>
<tr>
<td>Buhl et al., 2001</td>
<td>Polyethylene</td>
<td>5 – 30</td>
<td>5 – 10 – 20</td>
<td>56.25(\pi) cm(^2)</td>
</tr>
<tr>
<td>Bowden &amp; Hughes, 1939</td>
<td>Brass, Ebonite, Ice</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Calabrese et al., 1980</td>
<td>Steel Ring (AISI 1018)</td>
<td>&lt;1</td>
<td>889.6</td>
<td>1235 mm(^2)</td>
</tr>
<tr>
<td>de Koning et al., 1992</td>
<td>Steel Skate Blade</td>
<td>8</td>
<td>700</td>
<td>~400 mm(^2)</td>
</tr>
<tr>
<td>Ducret et al., 2005</td>
<td>UHMW-PE</td>
<td>2500 (\mu)m/s</td>
<td>10</td>
<td>Varies</td>
</tr>
<tr>
<td>Slotfeldt-Ellingsen &amp; Togersen, 1983</td>
<td>HDPE slider rock</td>
<td>0.3</td>
<td>100</td>
<td>15000 mm(^2)</td>
</tr>
<tr>
<td>Saeki et al., 1986</td>
<td>Coated steel</td>
<td>1.3 – 1.5 cm/s</td>
<td>2 kg/cm(^2)</td>
<td>25(\pi) cm(^2)</td>
</tr>
<tr>
<td>Uncoated steel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corroded Steel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tusima, 1977</td>
<td>PA, PE, PTFE, POM, PP, PMMA</td>
<td>(7.4 \times 10^{-5})</td>
<td>4.8</td>
<td>10.24(\pi) mm(^2)</td>
</tr>
<tr>
<td>Unknown</td>
<td>Steel Ball</td>
<td>(7.4 \times 10^{-2})</td>
<td>4.75</td>
<td></td>
</tr>
</tbody>
</table>
Almost all the experiments concerning the temperature appear to display the same trends. All the figures in this section except Figure 12 (Figure 5 to Figure 17) show the same tendencies. The friction coefficient of ice increases steadily as the temperature decreases starting in between -2 °C and -6 °C.

Taking a closer look at Figure 5 to Figure 12 however, the friction coefficient stops decreasing between -2 °C and -6 °C and starts increasing as the temperature approaches the melting point (0 °C). The lowest friction coefficient is therefore achieved between -2 °C and -6 °C. In Figure 12 however, some materials seem to have a decreasing friction coefficient as it gets colder (PA, PE, PTFE and POM). This is highly unusual as they are the only results found that show this trend. It is possible that these materials may see their friction coefficient rising at temperatures lower than around -2 °C and -6 °C and that the increase in friction coefficient at lowering temperatures is not seen as the tests are not
performed at low enough temperatures. It is also possible that the friction coefficient acts differently for these materials due to other factors that will be discussed in the next sections. A difference in results due to operating conditions is however unlikely as Figure 12 also shows two curves (PMMA and PP) displaying the same trends.

Figure 7: Effect of Temperature on the Friction Coefficient with Different Contact Areas [Bäurle et al., 2006].

Figure 8: Effect of Temperature on the Friction Coefficient for Various Experiments [Kietzig et al., 2010].

Figure 9: Effect of Temperature on the Friction Coefficient with Different Normal Forces [Buhl et al., 2001].

Figure 10: Effect of Temperature on the Friction Coefficient Using Various Materials [Albracht et al., 2004].
According to Kietzig et al. in 2010, the phenomenon seen in almost all the figures in this section is closely linked to the thickness of the liquid layer at the surface of the ice. The colder it is, the smaller the liquid layer becomes, thus creating more resistance. And, the warmer it is and the thicker the liquid layer, the more resistance it will yield due to the built-up of capillary bridges. This increase in friction around the melting point may also be explained by the increase in the drag forces from shearing the increasingly large liquid layer [Kietzig et al., 2010]. A balance between the capillary bridges and the thickness of the liquid layer must therefore be achieved to acquire a minimal friction coefficient.

The rise in the friction coefficient of ice around 0 °C is contradicted by Ducret et al.’s, Saeki et al.’s and Bowden and Hughes’s results shown in Figure 14 through Figure 17. The results indicate that the friction coefficient does not increase as the temperature
approaches the melting point. In Ducret et al.’s case (Figure 13), this inconsistency may be due to the fact that Ducret et al. used a rotational setup for his experiments that concluded in the sample rolling over the same ice area for multiple cycles as discussed in section 1.3.4 [Ducret et al., 2005]. This may involve ice on ice friction which may show totally different trends.

![Figure 13: Effect of Temperature on the Friction Coefficient over Many Cycles [Ducret et al., 2005].](image)

In Saeki et al.’s and Bowden & Hughes’s cases (Figure 14 to Figure 17), the curves start at around -2 °C. As the friction coefficient appears to start increasing around that temperature (-2 °C), warming up towards the melting point, this could explain why a rise in the friction coefficient is not observed. We can see in Figure 14 and Figure 15 that the temperature does not affect much the friction coefficient of corroded and coated steel as well as concrete. These figures also show that uncoated or corroded steel seem to produce more friction than coated steel.
The increase in the friction coefficient at very low temperatures is relevant when considering the operating conditions of in-ice structures and ships. These will operate in
much colder climates than the temperatures shown in the figures above, but the
temperature effects may be moderated by proximity to the ocean surface. However, the
use of certain coatings on steels may limit the variation and lower the friction coefficient
as shown by Saeki et al. [Saeki et al., 1986].

1.4.2. Sliding velocity $V$

A summary of the operating conditions for the most pertinent experiments yielding data
on the effect of the velocity on the friction coefficient of ice is presented in Table 2. The
differences in operating conditions between projects, described in Table 2, may explain
the variation between the sets of data that will be discussed subsequently. Bowden was
the first to observe that the friction coefficient of ice decreased with increasing speed.
Evans et al., the first to model ice friction mathematically, confirmed these findings
experimentally and theoretically [Kietzig et al., 2010].

Table 2: Difference in Operating Conditions with Respect to the Velocity

<table>
<thead>
<tr>
<th>Reference</th>
<th>Slider Materials</th>
<th>Temperature °C</th>
<th>Normal Force (N)</th>
<th>Contact Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albracht et al., 2004</td>
<td>Stainless Steel</td>
<td>-7</td>
<td>1 – 2</td>
<td>~ 2 mm²</td>
</tr>
<tr>
<td>Bäurle et al., 2006</td>
<td>PE Block</td>
<td>-10</td>
<td>84</td>
<td>2 – 4 – 10 cm²</td>
</tr>
<tr>
<td>Cho et al., 2011</td>
<td>Paint Steel</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Gliding Steel</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Glass</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Wood</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Rubber</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>de Koning et al., 1992</td>
<td>Steel Skate Blade</td>
<td>-4.6</td>
<td>706</td>
<td>~ 400 mm²</td>
</tr>
<tr>
<td>Study</td>
<td>Material</td>
<td>Stress Level</td>
<td>Surface Condition</td>
<td>Contact Area</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------------</td>
<td>--------------</td>
<td>-------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>Gagnon &amp; Mølgaard, 1989</td>
<td>Steel</td>
<td>-5, -10, -19</td>
<td>50000</td>
<td>0.1225π cm² – 2.25π cm²</td>
</tr>
<tr>
<td>Hoffman, 1985</td>
<td>Ship Hull</td>
<td>-</td>
<td>180 to 2800 KN</td>
<td>1.12 m²</td>
</tr>
<tr>
<td>Jones et al., 1994</td>
<td>Formica Block</td>
<td>~ 0</td>
<td>196.2</td>
<td>15000 mm²</td>
</tr>
<tr>
<td>Kietzig et al., 2009</td>
<td>Polished and Irradiated ASIS 304L</td>
<td>-1.5, -7, -15</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>Kuroiwa, 1977</td>
<td>Waxed/Unwaxed Polyethylene Teflon Perspe</td>
<td>0 to -2.5</td>
<td>2.74</td>
<td>220 mm²</td>
</tr>
<tr>
<td>Lishman et al., 2009</td>
<td>Ice</td>
<td>-10</td>
<td>5000</td>
<td>0.25 m ice thickness</td>
</tr>
<tr>
<td>Marmo et al., 2005</td>
<td>Steel</td>
<td>-3.4, -24.5, -25.1</td>
<td>2.10 – 4.20</td>
<td>~ π 25 mm²</td>
</tr>
<tr>
<td>Saeki et al., 1986</td>
<td>Coated steel Uncoated steel Corroded Steel Concrete</td>
<td>-8</td>
<td>2 kg/cm²</td>
<td>25π cm²</td>
</tr>
<tr>
<td>Tusima, 1977</td>
<td>PA, PE, PTFE, POM, PP</td>
<td>-10</td>
<td>4.8</td>
<td>-</td>
</tr>
<tr>
<td>- Unknown</td>
<td>PMMA, Steel Ball</td>
<td></td>
<td>4.75</td>
<td>10.24π mm²</td>
</tr>
</tbody>
</table>
The results shown in Figure 18 through Figure 34 are contradictory. Figure 18 to Figure 30 will see a decrease in the friction coefficient as the velocity increases. Figure 18 and Figure 19 will see a radical increase in friction as the velocity approaches zero. These are reasonable findings as the static friction coefficient of a material will always be larger than its kinematic coefficient. Figure 20 also supports the trends discussed in section 1.4.1 and shows an increase in the friction coefficient of ice when the temperature decreases below -5 °C. Figure 22 shows that, as in section 1.4.1 with the temperature, the friction coefficient of some materials (mainly painted and polished steel, glass and rubber) will not be affected greatly by the velocity. However, most of the figures, including Figure 19, Figure 21, Figure 24, Figure 26, Figure 29 and Figure 34 have very scattered results, which presents a problem when looking for trends. No other tendencies, aside from the one mentioned previously are noticeable due to the scattered results.

Figure 18: Effect of Velocity on the Friction Coefficient with Different Contact Areas [Bäurle et al., 2006].

Figure 19: Effect of Velocity on the Friction Coefficient [Tusima, Catapult Section, Unknown].
Figure 20: Effect of Velocity on the Friction Coefficient with Different Temperatures [Gagnon & Molgaard, 1989].

Figure 21: Effect of Velocity on the Friction Coefficient for Various Experiments [Marmo et al. 2005].

Figure 22: Effect of Velocity on the Friction Coefficient Using Various Materials [Cho, 2011].
Kietzig et al. explain the decrease in friction coefficient with increasing velocities with frictional melting. The higher the speed, the more melting will be produced increasing lubrication, facilitating sliding. The increase in the friction coefficient, when the velocity increases, can similarly be explained by the increase in the drag forces from the shearing of the thickening liquid layer according to Kietzig et al. [Kietzig et al., 2010]. Figure 31 to Figure 33 show results that contradict the trend. The friction coefficient of the ice will increase with the increase in velocity. The reasons behind this anomaly are unknown but may be related to the differences in operating conditions and variation in other factors.
Figure 25: Effect of Velocity on the Friction Coefficient [Tusima, 1977].

Figure 26: Effect of Velocity on the Friction Coefficient [Saeki et al., 1986].

Figure 27: Effect of Velocity on the Friction Coefficient with Different Surface Roughness at -7 °C [Kietzig et al., 2009].

Figure 28: Effect of Velocity on the Friction Coefficient with Different Surface Roughness at -1.5 °C [Kietzig et al., 2009].
Figure 29: Effect of Velocity on the Friction Coefficient for Various Experiments with Enhanced Lubrication [Kietzig et al., 2010].

Figure 30: Effect of Velocity on the Friction Coefficient [Lishman et al. 2009].

Figure 31: Effect of Velocity on the Friction Coefficient with Different Normal Forces [Albracht et al., 2004].

Figure 32: Effect of Velocity on the Friction Coefficient [Kuroiwa, 1977].
Figure 34 does not show any trend as the results are greatly scattered. However the figure is relevant as it was a real life experiment with an icebreaker that gave these results. It proves that values can be recorded with real life experiments for breaking the ice on ships however scattered they may be. As the speed of advance of an icebreaker in ice waters, or the current guiding pieces of ice, does not usually go over 10 knots, results with speeds reaching over 5.14 m/s are not as pertinent.

1.4.3. Normal force $F_n$

A summary of the operating conditions for the most pertinent experiments yielding data on the effect of the normal force upon the friction coefficient of ice is presented in Table
3. The differences in operating conditions between projects, described in Table 3, may explain the variation between the sets of data that will be discussed next.

**Table 3: Difference in Operating Conditions with Respect to the Normal Force**

<table>
<thead>
<tr>
<th>Reference</th>
<th>Slider Materials</th>
<th>Temperature °C</th>
<th>Velocity m/s</th>
<th>Contact Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albracht et al., 2004</td>
<td>Stainless Steel</td>
<td>-7</td>
<td>0.13</td>
<td>~ 2 mm²</td>
</tr>
<tr>
<td>Buhl et al., 2001</td>
<td>Polyethylene</td>
<td>-5, -10, -15</td>
<td>5 - 30</td>
<td>56.25π cm²</td>
</tr>
<tr>
<td>Mäkinen et al., 1994</td>
<td>Enamel Stainless Steel Inerta 160 3525 A/B</td>
<td>- 0.5 to 5.4</td>
<td>16900π mm² 250000π mm²</td>
<td></td>
</tr>
<tr>
<td>Oksanen &amp; Keinonen, 1982</td>
<td>Ice</td>
<td>-15, -5, -1 0.5</td>
<td>11475 mm²</td>
<td></td>
</tr>
<tr>
<td>Saeki et al., 1986</td>
<td>Coated steel Uncoated steel Corroded Steel Concrete</td>
<td>-8 1.3 - 1.5 cm/s</td>
<td>25π cm²</td>
<td></td>
</tr>
<tr>
<td>Tusima, 1977 Unknown</td>
<td>PA PE PTFE POM PP PMMA Steel Ball</td>
<td>-10</td>
<td>7.4<em>10⁻¹⁵ 7.4</em>10⁻² 10.24π mm²</td>
<td></td>
</tr>
</tbody>
</table>

It is clear from the figures below that the normal force plays an important role in the behaviors of the coefficient of friction of ice. This is just as for conventional friction equations. Figure 35 through Figure 40 show that a growth in the normal force will lower the friction coefficient of ice. This is also supported by some figures shown in sections 1.4.1 and 1.4.2 (Figure 9 and Figure 31). At temperatures approaching the melting point,
the coefficient of friction also does not seem to be influenced as significantly by the normal force as shown in Figure 36, Figure 37 and Figure 38.

Figure 35: Effect of Normal Stress on the Friction Coefficient [Saeki et al., 1986].

Figure 36: Effect of Normal Force on the Friction Coefficient with Different Temperatures [Buhl et al., 2001].

Figure 37: Effect of Normal Force on the Friction Coefficient with Different Temperatures [Oksanen & Keinonen, 1982].

Figure 38: Effect of Normal Force on the Friction Coefficient with Different Temperatures [Albracht et al., 2004].
Akkok et al. showed that the normal force applied on some materials (in particular steel) will have no effect on the friction coefficient [Akkok et al., 1987]. Figure 39 similarly shows that, like in section 1.4.1 with the temperature and in section 1.4.2 with the velocity, the friction coefficient of some materials (mainly Al-Leg and Cr-Ni-Stahl (Stainless Steel)) will not be affected greatly by the normal force. Additionally, Figure 36 to Figure 38 follow the trend discussed in the section on the temperature (section 1.4.1). The friction coefficient lowers as the temperature increases approaching the melting point. Figure 41 seems to contradict the tendency as the coefficient of friction of ice is increasing as the load rises. The reasons behind this anomaly are indeterminate but may be related to the differences in operating conditions and in different factors being altered.
Ice Friction loads on icebreakers, when breaking ice, are stated to be fairly important, up to 2800 kN [Hoffman, 1985] and even higher in concentrated areas. This shows the importance of the understanding of the behavior of the friction coefficient of ice with varying loads.

1.4.4. Apparent Area of Contact $A$

A summary of the operating conditions for the most pertinent experiments yielding data on the effect of the apparent area of contact upon the friction coefficient of ice are presented in Table 4. The differences in operating conditions between projects, described in Table 4, may explain the variation between the sets of data that will be discussed next.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Slider Materials</th>
<th>Temperature °C</th>
<th>Normal Force (N)</th>
<th>Velocity m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bäurle et al., 2006</td>
<td>PE Block</td>
<td>-5</td>
<td>84</td>
<td>3 - 5</td>
</tr>
</tbody>
</table>
The friction coefficient of ice, as shown in Figure 43, increases when the contact area is increased. However, Figure 42 and Figure 44 contradict this trend. Figure 45, contrary to the other figures in this section, shows no differences in friction coefficients between an ice sample having a 0.7cm diameter (Type 1 ice) and a sample with a larger than 3cm diameter (Type 2 ice). As only four figures show the effect of the contact area on the friction in this section, it is hard to determine which tendency will be more likely to occur when doing tests.

Looking at previous sections, Figure 7 in section 1.4.1 and Figure 18 in section 1.4.2, the friction coefficient seems to increase when the contact area is increased. This appears to point towards Figure 43 showing the common tendency. Both Figure 42 and Figure 44 also show a less important dependence of the friction coefficient of ice with the slider’s contact area when this area is larger, even if they show opposite trends.
Figure 42: Effect of Contact Area on the Friction Coefficient [Saeki et al., 1986].

Figure 43: Effect of Contact Area on the Friction Force with Different Normal Forces [Bäurle et al., 2006].

Figure 44: Effect of Contact Area on the Friction Coefficient [Tusima, 1977].
1.4.5. Roughness

Calabrese et al. measured ice’s friction coefficient with respect to the sliding speed for steel with different roughness. His results confirmed that an increase in roughness increased the friction coefficient as well [Calabrese et al., 1980]. Kietzig et al. also conducted experiments investigating the behavior of ice friction with roughness.

According to Kietzig et al.’s findings, shown in Figure 46, the coefficient seems constant for a surface structure with concentric circles [Kietzig et al., 2009]. This is explained by ice’s surface asperities running smoothly in the concentric grooves and therefore causing less interlocking and less friction [Kietzig et al., 2010]. Figure 47 shows the roughness profile of the materials Saeki et al. used in his tests. Figure 16, shown in section 1.4.1 from Saeki et al.’s tests, concurs with Calabrese et al.’s results [Saeki et al., 1986].
Figure 46: Effect of Velocity on the Friction Coefficient with Different Surface Structures [Kietzig et al., 2009].

Figure 47: Typical Surface Irregularity Profiles [Saeki et al., 1986].
Ducret et al. also reported that an increase in roughness increased the coefficient of friction of ice [Ducret et al., 2005]. Marmo et al. believed that the increase in roughness led to a decrease in the liquid layer’s thickness, reducing the lubrication on the ice. Unfortunately, few studies exist that investigated the roughness of a material with respect to ice’s friction coefficient making it hard to compare tendencies [Kietzig et al., 2010]. Mäkinen et al. investigated the effect of potential icebreaker coatings on the friction coefficient of ice with his experimental setup. His findings are shown in Figure 48 [Mäkinen et al., 1994].

Additional experiments should be done to investigate more thoroughly the influence of various degrees of roughness on ice friction. They should also be done with a variety of coatings used on hulls as their roughness is not always the same.
1.4.6. Wettability

As discussed in section 1.2, Pounder, in 1965, proposed to use the wettability index as a means to measure the adhesive properties of the postulated liquid layer. It implied that the more wettable the surface, the higher the friction of the material was on ice. This was supported by Bowden’s experiments [Bowden, 1953]. However, the change in the wettability was only achieved with the change of the material. This meant that the wettability could not be investigated independently from other factors such as the thermal conductivity. Furthermore the friction coefficient of ice may have been influenced as different materials have had different roughness [Kietzig, 2010].

Kietzig et al., in 2009, conducted ice friction experiments with stainless steel sliders, which could be rendered hydrophobic with femtosecond laser irradiation. This controlled the thermal conductivity as the sliders were all of the same material. The irradiated slider however had a rougher surface. Figure 49, demonstrates the importance of the wettability of a material, as a factor influencing the friction coefficient of ice, as the irradiated slider undergoes a large decrease in the friction as its speed increases. As both materials see a large decrease in the friction coefficient, it is harder to understand the use of the wettability factor to reduce ice’s friction. However, taking a closer look at the higher speeds (around 5m/s) in Figure 49, the less wettable surface has the lowest friction. This is also supported in section 1.4.2, in Figure 23, Figure 27 and Figure 28. Polished surfaces seem to have a lower effect on the friction coefficient, keeping it lower and reducing its
variation with different velocities, but, at higher speeds the irradiated surface dominates in low friction coefficients [Kietzig et al., 2009].

![Figure 49: Effect of Wettability on the Friction Coefficient of Ice [Kietzig, 2010].](image)

1.4.7. Relative Humidity RH

The results for Calabrese et al.’s tests on the relative humidity are shown in Figure 50 where the relative humidity influences the friction of ice. The lower percentages of relative humidity render a higher friction coefficient for ice. Higher humidity will increase the lubrication at the material – ice interface lowering the friction. The friction coefficient also seems to remain constant at higher speeds meaning that it would not influence the magnitude of the force icebreakers are subjected to when breaking ice. Unfortunately, no other data exists for this factor. It is therefore hard to compare and conclude with certainty that this factor influences ice friction [Kietzig, 2010].
1.4.8. Thermal Conductivity $\lambda$

Bowden and Hughes conducted tests on skis made of different materials as discussed in section 1.2 [Bowden & Hughes, 1939]. They found that the higher the thermal conductivity of a material, the higher the friction coefficient. These results were however contradicted by magnesium. Itagaki et al.’s tests, with steels of different thermal conductivity, also supported Bowden’s findings [Itagaki et al., 1987]. However, Albracht et al. did not find a significant influence of the thermal conductivity on the friction of ice in his experiments with different materials [Albracht et al., 2004]. Unfortunately, as mentioned in section 1.4.6, it is important to note that the change in materials, in order to vary the thermal conductivity, involves the loss of control of other factors such as roughness and wettability which may vary the results.
1.4.9. Growth Direction of Ice and Presence of Bubbles

Saeki et al. and Tusima investigated the influence of the growth direction and orientation of ice crystals on the coefficient of friction as shown in Table 5.

Table 5: Difference in Operating Conditions with Respect to the Growth Direct of Ice

<table>
<thead>
<tr>
<th>Reference</th>
<th>Slider Materials</th>
<th>Temperature °C</th>
<th>Normal Force (N)</th>
<th>Contact Area</th>
<th>Velocity m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gagnon &amp; Mølgaard, 1989</td>
<td>Steel</td>
<td>-5, -10, -19</td>
<td>50000</td>
<td>0.1225π cm²</td>
<td>0.0595</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>- 2.25π cm²</td>
<td>0.1749</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3994</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8235</td>
</tr>
<tr>
<td>Saeki et al., 1986</td>
<td>Coated steel</td>
<td>-8</td>
<td>2 kg/cm²</td>
<td>25π cm²</td>
<td>2 kg/cm²</td>
</tr>
<tr>
<td></td>
<td>Uncoated steel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Corroded Steel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Concrete</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tusima, 1977</td>
<td>PA</td>
<td>-10</td>
<td>4.8</td>
<td>10.24π mm²</td>
<td>4.8</td>
</tr>
<tr>
<td>Tusima, 1977</td>
<td>PE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unknown</td>
<td>PTFE</td>
<td>-10</td>
<td>-</td>
<td>10.24π mm²</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>POM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PP</td>
<td></td>
<td>4.75</td>
<td></td>
<td>4.75</td>
</tr>
<tr>
<td></td>
<td>PMMA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Steel Ball</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Gagnon and Mølgaard, in 1989, then investigated the influence of the presence of bubbles in the ice on its friction coefficient. Figure 51 through Figure 55 show their findings. At high loads and high velocities, the orientation does not influence the friction of ice greatly according to Figure 51 and Figure 52. However, the friction coefficient along the basal plane seems to vary less and be smaller at small velocities and loads which is not very significant for icebreakers or offshore structures. Figure 53 and Figure 54 do not seem to
show a tendency to influence the friction coefficient of ice and have very scattered results. Figure 55, investigating the influence of the presence of bubbles in ice on its friction coefficient also does not seem to show a trend. Type 1 ice, columnar-grained bubble-free ice and Type 3 ice, bubbly randomly oriented fine-grain ice both appear to have similar scattered results for friction coefficients. This implies that the friction coefficient of ice is very likely not influenced by the presence of bubbles in its structure.

Figure 51: 3D View of Effect of Normal Load and Velocity on the Friction Coefficient Along Prismatic and Basal Plane [Tusima, 1977].
Figure 52: Effect of Direction of Angles along Prism Plane for the Friction Coefficient [Tusima, Unknown].

Figure 53: Effect of Direction of Angles along Basal Plane for the Friction Coefficient [Tusima, Unknown].

Figure 54: Effect of Direction of Angles along Basal Plane for the Friction Coefficient [Tusima, Unknown].
1.4.10. Water in the Sea Ice Interface

A compilation of the operating conditions for the experiments yielding data on the effect of a water sea ice interface on the friction coefficient of ice is presented in Table 6 below.

Table 6: Difference in Operating Conditions with Respect to the Water in Sea Interface

<table>
<thead>
<tr>
<th>Reference</th>
<th>Slider Materials</th>
<th>Temperature °C</th>
<th>Normal Force (N)</th>
<th>Contact Area</th>
<th>Velocity m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saeki et al., 1986</td>
<td>Coated steel</td>
<td>-8</td>
<td>2 kg/cm²</td>
<td>25π cm²</td>
<td>2 kg/cm²</td>
</tr>
<tr>
<td></td>
<td>Uncoated steel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Corroded Steel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Concrete</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slotfeldt-Ellingsen &amp; Torgersen, 1983</td>
<td>HDPE slider</td>
<td>-1.5, -5, -10</td>
<td>100</td>
<td>15000 mm²</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>rock</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Unfortunately not many projects incorporated the investigation of lubrication, with sea water, as an influential factor for ice friction in their research. The data acquired is shown in Figure 56 and Figure 57 below. Both figures show a limited change in the ice friction coefficient with increased lubrication. A maximum change of about 0.05 in between kinetic friction coefficients is observed in Figure 56 (white values) for concrete. The variation is from friction without water in the interface (round data) to friction with water included (square data). All three other materials (uncoated steel, polished steel and zebron) show a minor change in kinetic friction coefficients. The same can be said for the variation in static friction coefficients (black data) in Figure 56. Figure 57 shows that the percentage of water content also does not influence ice friction greatly.

Figure 56: Effects of Sea Water Interface on the Friction Coefficient Using Various Materials [Saeki et al., 1986].

Figure 57: Effect of Liquid Content on the Friction Coefficient Using Various Materials [Slotfeldt-Ellingsen & Torgersen, 1983].
1.4.11. Pertinence of Factors

Many factors were presented and discussed in this section. However, not all of them have a large influence on the friction coefficient of ice.

It is clear that the temperature, the velocity and the normal force influence the friction coefficient of ice significantly as shown in sections 1.4.1, 1.4.2 and 1.4.3. They are also the factors that were studied the most in all the research data found. The apparent area of contact is also relevant as it is directly linked to the normal force applied to ice as shown in section 1.4.4. Unfortunately, these four parameters are almost impossible to control when dealing with ice structures which makes the attempt at reducing the friction forces on the structures difficult. For offshore structures, the outside temperature, the sliding velocity of the drifting ice, its size as well as the force with which it will collide into the structure cannot be controlled. It is the same for icebreakers with the exception of its icebreaking speed which can be gauged. Very low icebreaking speeds can get a ship trapped in ice, and very high speeds can lead to hull damage. Both these results are undesirable and limit the range in velocities when breaking ice. The relative humidity, the growth direction of the ice, the presence of bubbles as well as the water in the sea ice interface are also all environmental factors that cannot be controlled when dealing with ice structures. However, these factors do not influence friction coefficient of ice drastically as shown in sections 1.4.7, 1.4.9 and 1.4.10. They are therefore less pertinent.
The roughness, the wettability and the thermal conductivity are factors that can be controlled as they are directly related to the materials used to build the ice structures. The roughness seems to have an important effect on the friction coefficient as discussed in section 1.4.5. Very low friction coefficients were observed with some ship coatings. The thermal conductivity also seems to have a significant effect on the friction coefficient as presented in section 1.4.8. A material with low conductivity could be combined with a coating of low roughness to achieve minimal friction. However, the effect of the wettability on the friction of ice is too low, as discussed in section 1.4.6, to take it into consideration when choosing materials to build ice structures.

Even if the temperature, the velocity, the normal force and the apparent area of contact are hard to control when operating ice structures, the understanding of their influence on the friction coefficient of ice is important. Better understanding can help with the design of ice structures and with the analysis of expected loads. Pack ice velocities and expected normal forces subjected by the structure can also be determined prior to the design based on environmental factors and statistics. With the understanding of the behavior of ice friction with respect to all these factors, the design parameters could then be used to predict the maximal expected friction force on the structure.

1.5. Additions to the Research

It remains difficult to compare data acquired in different experiments as certain factors are not uniformly controlled. Moreover, other undiscovered factors may still influence the friction of ice. However, the range of data attained shows trends for many leading
parameters in the friction of ice. Furthermore, a better understanding of the behaviors for
the friction force of ice will help improve the calculation of other forces ice structures are
exposed to, such as the force associated with breaking the ice or with moving broken ice,
in order to optimize their design. Figure 58, shows that, as predicted by R. Woolgar and
B. Colbourne, the understanding of the friction force will not only benefit in reducing the
force of friction but also the total pack ice force.

Figure 58: Predicted Pack Ice Force with Respect to the Hull-Ice Friction Coefficient [Woolgar &
Colbourne, 2010].

Reproducing the same environment and ice for many of these tests, in order to replicate
these experiments and compare results is difficult. This is why this research project will
focus on simulating conditions that ice structures are subjected to. The tests realized
within this project will add to the research on ice friction and provide a reference for the
future design of ice structures. The material used, stainless steel with a rough surface will
mimic the roughness of worn coatings on icebreaker hulls while preventing the material from rusting throughout all of the tests. This will limit the change of the roughness of the material induced by changing plates or by the generation or removal of rust. The ice production process for the test samples will yield a standardized ice structure facilitating the reproduction of samples. The sample will be of conical shape to better simulate real ice pieces that are cleared by ice structures. The pressures studied will range from ~ 0 to the crushing pressures of the ice. The velocity will vary under 10 knots to imitate ice crushing and clearing conditions. The temperature will not vary for this report. The temperature will remain well under 0 °C to resemble an Arctic environment. Lubrication with water will also be studied as ice structures interact closely with water. All the factors studied, the ice making procedure, setup used as well as the working conditions will further be discussed in sections 2 and 3.

The overall purpose of the configuration of this research will be to provide measurements of the friction coefficient under conditions that mimic the conditions of real life interactions. Conical samples under high or increasing normal forces will simulate interactions where the contact starts off small and increases as the interaction proceeds. This will provide dynamically changing contact area (and pressure) during the course of a measurement. The objective is to gain insight into how the friction changes under conditions where the ice is undergoing deformation as it might in a real-life ice-structure interaction.
2. Experimental Setup and Testing Materials

Section 1.3 showed the variety of experimental methods and devices used in the past for the study of ice friction. In this section, the experimental apparatus assembled for this research on the friction of ice, the materials tested as well as the reasons behind their use will be discussed.

2.1. Apparatus Choice

As discussed in previous sections, a diversity of experimental setups has been designed in the past for the measurement of the friction on ice. None however were perfect as they all had their advantages and disadvantages. Due to the space restrictions in the available cold room as well as the desire to run relatively long-duration experiments with increasing pressures, the use of a rotational device was chosen for this research. The device used for testing included a turntable, a flat circular plate resting on top of the turntable, a designed measuring apparatus attached to this turntable, as well as a data acquisition and control system including a camera as shown in Figure 59.

2.1.1. The Turntable

The turntable used for this project was previously employed as an ice sample shaper for other experiments. It was built to shape 1 m diameter ice cylinders into conical samples. This turntable consists of a 1.156m diameter recessed disc driven by an AC motor with reduction gear turning at an unregulated speed averaging 20.5 rpm. Its disc is set on a table to which an arm holding a large blade is attached. The turntable’s arm is pinned at one extremity and can be brought down, in a circular motion, with the help of a
mechanism utilizing a jack (see Figure 60). This turntable was used in this research as the rotational part of the complete experimental setup. This reduced the construction time due to extra parts needing to be built in addition to limiting costs.

Figure 59: Experimental Setup Including Camera (Top Left), Data Acquisition and Control System (Bottom Left) and Apparatus (Right).

Figure 60: Turntable Arm and Pin.
2.1.2. The Flat Circular Plate

In order to have a uniform controlled surface on which to compress the ice, a thin flat metallic circular plate, designed and built specifically for this project, was attached inside the disc as shown in Figure 61. It was of similar radius as to fit tightly inside the recessed disc of the turntable. It was tightly fitted as to maximize the locations along the turntable’s disc radius where an ice sample could be compressed, therefore, increasing the choices in test velocities. It also had attachments at opposite ends so as to be driven by the motor alongside the disc as shown in Figure 61.

![Figure 61: Recessed Turntable Disk with Stainless Steel Plate and Plate Attachment.](image)

2.1.3. The Normal Force Apparatus

An extra mechanism was needed in order to push an ice sample against the rotating turntable’s disc and to measure both normal and frictional forces. The device was designed especially for this project and is shown in Figure 62.
The apparatus consists of an attachment, shown in Figure 63, which can be secured to any of the holes along the turntable’s arm to facilitate the change of velocity for each test. As the shaper’s arm is offset from the disc’s center, this attachment also aligns the device with the middle of the circle. This attachment similarly serves as a linear bearing holder.

Figure 62: Device Attached to Turntable.

The bearing (see bearing 1 in Figure 62 and the only bearing shown in Figure 63) is screwed to the attachment which allows a lengthy linear carriage, and therefore the device, to move freely vertically while restraining it horizontally (see linear carriage in Figure 62). The device was designed to move up and down without restrictions as the ice
sample gets shorter as it abrades on the circular plate. It also allows for the device to be easily lifted when the sample needs to be changed. There is a large bucket secured on top of the linear carriage used to receive steel pebbles (see bucket in Figure 62).

![Figure 63: Device Attachment.](image)

This container serves as a varying dead weight normal force for the friction device, as steel pebbles can be added to it, and has a capacity of over 440 pounds combining the weight of both tub and steel pebbles. It also has a handle attached to it to allow a crane to lift it when needed. A sensor is located in between the container and the vertical linear carriage in order to monitor the change in normal force as the experiment proceeds.

The bottom part of the setup secures the ice sample. It is attached to the vertical linear carriage with brackets and contains an L-shaped part and a forklift-shaped part as shown in Figure 64. A horizontal linear carriage is attached to the bottom of the L-shaped part, the part also holding a sensor. The forklift-shaped part holds the ice sample during the experiments (see Figure 65). It is attached to a second bearing (see Bearing 2 in Figure
that permits it to slide horizontally towards the sensor along the flat linear carriage. As the ice is compressed against the rotating flat plate, it is driven sideways along the bearing railing and presses the forklift-shaped part against the sensor. This allows the sensor to measure the lateral resistance (friction force) during the process.

Figure 64: Bottom Part of Setup Including Forklift-Shaped Part (Red Circle) and L-Shaped Part (Purple Circle).

Figure 65: Bottom Setup Holding an Ice Sample with Bearing 2 (Red Circle) and Sensor (Green Circle).

In order to record the tests and to be able to review them in the future, a camera is set up to film the experiments at 60 fps. The depth with which the conical ice sample has been abraded can be measured with a ruler that has been installed on the vertical linear carriage (see Figure 66). This enables the calculation of the contact area of the ice sample at any time. It also allows for the measurement of the weight loss due to the destruction of the ice at the interface. To produce a clear video, lights are installed to illuminate the contact area and the ruler (see Figure 67).
2.1.4. Data Acquisition and Control System

A data acquisition and control system was required to record the data from the 2 sensors during the test (sensors discussed in section 2.1.3). The sampling rate for data acquisition is set at 100 values per second on each channel. The rotational velocity of the turntable was measured, with a separate device, four times during each test in order to assure a constant angular speed. The shaper’s disc’s velocity was monitored using a hand held tachometer and the results recorded manually.

2.2. Materials Used

The type of friction (dry, lubricated, mixed, etc.) achieved at an ice interface remains unclear. Yet, the fact remains that at least two surfaces are required. As noted by Frederic P. Miller (2010), “friction is the force resisting the relative lateral (tangential) motion of solid surfaces, fluid layers, or material elements in contact”. For this research, the friction tests are performed using a metal plate and an ice sample.


### 2.2.1. Metal

The choice of the contact material presented many possibilities. The flat circular plate was made out of stainless steel. This material was chosen because of its resistance to corrosion as it would come into contact with both ice and water. Corrosion was undesirable as it would have changed the surface characteristics of the plate over many tests. The replacement of the plate, once rust appeared, was also unwanted as many plates would add costs to the project. The substitution of the plate also negated the control of the surface characteristics of the metal surface over numerous tests as different plates may have had dissimilar surfaces.

The surface of the flat circular plate was finished using shot blasting in order to even out its coarseness over the entire surface. Shot blasting the plate with glass also served to increase its face’s roughness. An even and rougher surface was desirable to resemble ice structure conditions as components of these structures, coming into contact with ice, are usually fouled and rarely very smooth. A rougher surface also implied higher friction, as discussed in section 1.4.5.

The roughness of surface finishes range from 50 µm (rough) to 0.012 µm (smooth) as shown in Figure 68 [Black, Kohser and Degarmo, 2003]. The mean roughness \( R_a \) of the sandblasted steel plate used in this research, measured with a Taylor-Hobson ‘Surtronic 3’ device, courtesy of the CNRC in St. John’s, was 1.7 µm. This implied that the roughness of the steel plate was midrange overall. In comparison, polished steel had a \( R_a \)
of 0.24 \( \mu m \) and sandblasted aluminum, a 1.5 \( \mu m \) mean roughness. Shot blasting being an abrasive process [Kalpakjian & Schmid, 2009], the \( R_a \) of the plate was in the coarser end of the roughness achievable, when using these surface finish techniques (see Figure 68).

<table>
<thead>
<tr>
<th>( R_a ) ( \mu m )</th>
<th>50</th>
<th>25</th>
<th>12.5</th>
<th>6.3</th>
<th>3.2</th>
<th>1.6</th>
<th>.8</th>
<th>.4</th>
<th>.2</th>
<th>.1</th>
<th>.05</th>
<th>.025</th>
<th>.012</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_a ) ( \mu m )</td>
<td>200</td>
<td>100</td>
<td>50</td>
<td>25</td>
<td>12.5</td>
<td>6.3</td>
<td>3.2</td>
<td>1.6</td>
<td>.8</td>
<td>.4</td>
<td>.2</td>
<td>.1</td>
<td>.05</td>
</tr>
</tbody>
</table>

**Figure 68: Surface Finishes of Manufacturing Processes [Black, 2003].**
2.2.2. Ice

Many ice samples were also required to perform these ice friction tests as each experiment destroyed a sample preventing it from being recycled and reused. The ice structure of the sample and its mechanical properties had to be replicated to produce many identical samples in order to promote consistency in between each test throughout the research project. The type and structure of the ice used for the research was therefore important because: “Ice forces depend on mechanical properties, which depend on physical properties and structure, which in turn depend on the ice formation and growth process” [Colbourne & Daley, 2012].

The types of ice, occurring in nature and found at sea, which interact with ice structures, are listed from thinnest to thickest: New Ice, Nilas, Young Ice, First-year Ice, Old Ice and Iceberg Ice. Their microstructure depends on their temperature history and the application of stress during their formation [Colbourne & Daley, 2012]. Figure 2, the water phase diagram, shown in section 1.2 highlights typical ice structures with relation to pressure and temperature. Many other factors however influence the final structure of naturally occurring ice and manmade ice.

Unfortunately, as sea and iceberg ice are hard to produce and replicate in laboratory and to standardize the structure, this research utilized laboratory made poly-crystalline freshwater ice samples instead of naturally occurring ice. The method used to produce these ice samples is discussed in a later section (section 3.2).
The differences in structures between first-year ice, old ice and iceberg ice compared to the sampled ice are shown in Figure 69 through Figure 72. It is important to note that the
different engineering properties and structures of ice found at sea are very scattered as they sustain different environmental conditions depending on their site of origin.
3. Method

The procedures followed in any research to acquire data and results are essential as they provide guidelines for the experiments. Consistently using the same method for each test helps limit the effects of unknown factors that may have an influence on the data. As many undetermined factors may still have an effect on the friction of ice, standard procedures for each test needed to be followed. In this section, the factors studied as well as the method and procedures followed to produce both ice samples and to perform the tests are discussed.

3.1. Factors Studied

The factors that were studied in this research, as mentioned in section 1.5, were the velocity, the starting normal force, the change in normal force and the lubrication. The contact area, and consequently the pressure, was also monitored indirectly as the cone’s contact surface increased as it was crushed and abraded in the experiments.

3.1.1. Velocity

Three different velocities were used during the experiments. As the turntable had a constant rotational velocity of approximately 20.5 rpm, as mentioned in section 2.1.1, the options available for velocities were limited. The available velocities were dependent on the positions of the holes on the big shaper arm and on the attachment. The farther the sample was placed from the center of the plate, the faster the linear velocity. The angle of the shaper arm was also important. The angle chosen was optimal for the positioning of the apparatus but not measured. The jack, controlling the angle of the arm, was left
untouched throughout the experiments. A line was drawn on the jack marking its extension for that shaper’s arm angle. This provided a marker to check that the angle of the shaper arm was always the same.

The first speed, (Velocity 1), was the highest one that could be achieved with the restrictions mentioned with the center of the sample placed 0.37 meters from the center of the plate. This provided a linear velocity at the middle of the ice sample of 0.8 m/s. The second velocity was the slowest that could be achieved with the restrictions of the experimental setup. The center of the sample was set at a radius of 0.22 m from the center of the turntable. The speed achieved at the center of the sample was 0.473 m/s. The third speed chosen was in between the first two. The sample was located at a distance of 0.295 meters from the center of the stainless steel plate. The velocity to which the center of the ice sample was subjected to in this intermediate position was 0.63 m/s. The calculations done to determine the three velocities are explained in the calculations section (section 4.1.4).

3.1.2. Starting Normal Force

A tub was installed on the apparatus to serve as a varying dead weight normal force with a maximum capacity of 440 pounds (including the weight of both pebbles and tub), as described in section 2.1.3. The starting weights therefore needed to be lower than the maximum as the option of adding weight was also available. Starting at the maximum capacity would consequently have negated the possibility of increasing the weight. Four different starting masses were selected for the research: 0, 50, 100 and 150 kg. These
translated into starting normal forces of 156.96 N, 490.5 N, 981 N and 1471.5 N (see calculations in section 4.1.2)

The different starting normal forces, combining the weight of both pebbles and tub, were measured by the DAC, prior to the start of the experiment, using the sensor located under the tub. It is important to note that the 0 kg starting mass is actually the weight of the tub with no pebbles in it. However, the right amount of pebbles was added to the tub, combining both the weight of the tub and the weight of the pebbles to produce the other starting weights.

Another consideration was that the tub was not the only force pressing on the sample. As the sample was pushed downwards with gravity, the sample of ice itself and the apparatus holding it also pressed it downwards. The added force of the sample itself, the apparatus and the camera were also included in the normal force calculation (see section 4.1.5.4).

### 3.1.3. Change in Normal Force

The option of changing the normal force during the course of an experiment was another chosen factor in this research. If the normal force needed to remain constant, no pebbles were added to the tub. If an increase in normal forces was required, pebbles were added to the tub throughout the test. A water pitcher was utilized to transfer the pebbles from a bin to the tub as consistently as possible throughout the experiment. The DAC was used to monitor the changes in the weight of the tub during the experiment.
An added complication in the test was the crushing and abrading of the ice sample. Whether pebbles were added or not, the ice sample lost bits and pieces of ice and/or melted as it rubbed on the stainless steel plate. This meant that the normal force on the ice sample diminished a little as the test proceeded. This loss of mass was considered in the final calculations as well.

3.1.4. Lubrication

The option of adding water to the metal-ice interface was also chosen as a factor in these experiments. Water was dispensed on the spinning metal plate in front of the ice sample. The water was then driven by the rotating motion of the plate in the metal-ice interface. A constant volume of 2L of water was emptied from a pitcher onto the plate for each lubricated test.

3.1.5. Design of Experiments

Once each factor with its options were determined and defined, a test plan was developed. A list of tests was created using the options available for each factor:

- Velocity: 1, 2, 3
- Starting Normal Force: 156.96 N, 490.5 N, 981 N and 1471.5 N
- Changing Normal Force: Yes, No
- Lubrication: Yes, No

One option from each of the four factors was selected per test using all the possibilities available to determine the appropriate number of tests. 48 unique tests were planned. A numbered list of each test with the options used for each factor is shown in Table 7.
Table 7: Test Numbers with Changes for Different Factors

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Initial Sand Weight</th>
<th>Lubrication</th>
<th>Pressure Change</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>kg</td>
<td>(y/n)</td>
<td>(y/n)</td>
<td>#</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>yes</td>
<td>yes</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>yes</td>
<td>yes</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>yes</td>
<td>yes</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
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<td>no</td>
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</tr>
<tr>
<td>5</td>
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<td>no</td>
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<td>yes</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
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<td>no</td>
<td>1</td>
</tr>
<tr>
<td>29</td>
<td>100</td>
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<td>30</td>
<td>100</td>
<td>yes</td>
<td>no</td>
<td>3</td>
</tr>
<tr>
<td>31</td>
<td>100</td>
<td>no</td>
<td>yes</td>
<td>1</td>
</tr>
</tbody>
</table>
Because the change in velocities for each test was complicated and time consuming, the tests were not fully randomized. A list containing the dates of each test and its conditions is shown in Appendix A.

The exact length of the tests depended on the options for each factor used. When neither lubrication nor change in pressure was required, the tests ran for 5 min or until the cone ceased abrading. If an increase in pressure was necessary, the test ended when the combined weight of both pebbles and tub reached 440 pounds. If lubrication was part of the test, the test ended when the pitcher of water was empty unless the maximum weight, in the case of a change in pressure being part of the experiment, was not reached. In that

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>100</td>
<td>no</td>
<td>yes</td>
<td>2</td>
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<tr>
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<td>3</td>
</tr>
<tr>
<td>34</td>
<td>100</td>
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<td>1</td>
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<td>35</td>
<td>100</td>
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<td>2</td>
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<tr>
<td>36</td>
<td>100</td>
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<tr>
<td>37</td>
<td>150</td>
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<td>yes</td>
<td>1</td>
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<td>38</td>
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<td>yes</td>
<td>yes</td>
<td>2</td>
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<td>39</td>
<td>150</td>
<td>yes</td>
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<td>no</td>
<td>2</td>
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<td>42</td>
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<td>yes</td>
<td>no</td>
<td>3</td>
</tr>
<tr>
<td>43</td>
<td>150</td>
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<td>1</td>
</tr>
<tr>
<td>44</td>
<td>150</td>
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<td>45</td>
<td>150</td>
<td>no</td>
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<td>3</td>
</tr>
<tr>
<td>46</td>
<td>150</td>
<td>no</td>
<td>no</td>
<td>1</td>
</tr>
<tr>
<td>47</td>
<td>150</td>
<td>no</td>
<td>no</td>
<td>2</td>
</tr>
<tr>
<td>48</td>
<td>150</td>
<td>no</td>
<td>no</td>
<td>3</td>
</tr>
</tbody>
</table>
case, the test continued on, even if water ran out, until the combined weight of both pebbles and tub reached 440 pounds.

3.2. Ice Sample Making Procedure

The ice samples used in this research were all prepared in the laboratory. This helped control the structure of the ice. It was also a necessity as the ice samples needed to be in a regular and repeatable shape (conical) and held in a holder (ice ring) as shown in Figure 73.

Figure 73: Ice Ring Holding an Ice Sample.

The equipment available and the time needed to prepare ice samples limited the number of tests that could be achieved per week. Only eight ice holders (ice rings) were available to freeze ice samples. This meant that once eight ice samples were prepared, they needed to be used in experiments before new samples could be frozen. A weekly schedule was
prepared to make best use of the time available. The schedule followed is shown in Table 8. A test day schedule is presented in Appendix B.

As shown in Table 8, procedures to make ice samples included water preparation, ice sample preparation and finally ice sample shaping. The steps for each part of the process are discussed next.

### 3.2.1. Water Preparation

Water was prepared in a specific way as to help produce identical samples for each experiment. Tap water could not be used to freeze ice samples as its air and mineral contents change. The tap water therefore needed to be distilled, deionized and finally deaerated to produce pure water. Uncontaminated water was also needed in the freezing process of the ice as to limit the number of impurities in the sample, which would have weakened it. The distillation, deionization and deaeration procedures are described in “Steps2: Manual of Laboratory Procedures” by Andrew Manuel.

### 3.2.2. Ice Sample Preparation

Once the water was prepared, the samples were prepared for the freezing process. The samples were frozen in buckets that were inserted on the ice rings (ice holders) and then

Table 8: Weekly Schedule for Sample Making and Testing

<table>
<thead>
<tr>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samples</td>
<td>Shaping</td>
<td>Ice</td>
<td>Testing</td>
<td>Testing</td>
<td>Preparing</td>
<td>Samples</td>
</tr>
<tr>
<td>Freezing</td>
<td>Samples</td>
<td>Settling</td>
<td></td>
<td>Ice</td>
<td>Freezing</td>
<td>Samples</td>
</tr>
</tbody>
</table>
held in two conventional household freezers. The steps to prepare the ice samples included making ice seeds, mixing and finally freezing the ice. These procedures are described extensively in the “Steps2: Manual of Laboratory Procedures” written by Andrew Manuel in 2012. The buckets were not filled prior to them being placed in the insulators as in the manual by Andrew Manuel written in 2012. The final product of the ice sample preparation is shown in Figure 74.

![Figure 74: Ice Seeds and Water Flush Against Ice Holder [Manuel, 2012].](image)

**3.2.3. Ice Sample Shaping**

The ice samples prepared and frozen following the previous procedures made cylindrical ice samples. Once the samples were thoroughly solidified, they were molded into the wanted conical shape. A shaper designed at the Memorial University of Newfoundland was used. Shaping a cone with this device allowed for a wide variety of angles for this cone as the apparatus is adjustable. However, all the ice samples used in this research had a 30° angle. To shape the ice samples, the procedures of the “Steps2: Manual of
Laboratory Procedures” written by Andrew Manuel were followed. The shaper used for the shaping of all the samples as well as key components are shown in Figure 75.

![Shaper including Jack (circled in red), Blade (circled in black), Ice Sample (circled in green) and Plug (circled in orange) [Manuel, 2012].](image)

3.3. Testing Procedures

Every week, once eight samples were prepared, eight experiments were performed. As the schedule shows in section 3.2, these tests were scheduled on Wednesdays and Thursdays. A total of 48 different tests were performed over a period of 6 weeks. 9 tests were also redone over a 7th and 8th week to observe the uniformity in the data and results. Due to some delays and impediments in the research however, the tests were not always performed on Wednesdays and Thursdays nor were they executed 6 weeks in a row. (See Appendix A for details.)
On testing days, the procedures were separated into four categories: the steps before and after all experiments were done and the steps during and in between experiments. The procedures for each of the four categories are summarized below.

### 3.3.1. Before Tests Preparations

- Weigh all shaped ice sample without bucket.
- Move all samples from cold room to reefer with buckets covering them.
- Place stainless steel plate on turntable and screw on.
- Hook up DAC to sensors and turn on.
- Turn on light setup.

### 3.3.2. In Between Test Preparations

- Turn off turntable.
- Stop camera.
- Remove camera.
- Stop DAC.
- Note the end measurement of cone depth.
- Lift weight bucket up with crane.
- Remove ice sample from setup.
- Take picture of crushed surface of ice sample.
- Check that attachment is horizontal.
- Lower apparatus (without ice sample) using crane.
• Add or remove necessary pebbles from tub to reach desired starting weight using pitcher. (Depends on starting weight of next experiment)
• Weigh tub.
• Lift apparatus up with crane.
• Clean and scrape excess ice on metal plate using plastic car ice scraper.
• Note room temperature.
• Measure sample height.
• Put sample on setup.
• Lower apparatus slowly until tip of sample touches plate.
• Note the start measurement of cone depth.
• Install Camera.
• Turn on camera.
• Start recording data on DAC.
• Measure water temperature if necessary.
• Turn on turntable and lower apparati simultaneously so that the entire weight of the setup is on the sample.

3.3.3. During Tests Preparations

• Record velocity of turntable at four different times with tachometer.
• Scrape ice accumulation on turntable with plastic ice scraper for car windows.
• Add water with water pitcher if necessary.
• Vacuum excess water if necessary.
• Add pebbles with pitcher if necessary.

3.3.4. After Tests Preparations

• Follow first eight steps of ‘in between’ tests preparations.
• Remove samples from holder rings.
• Remove stainless steel plate from turntable and reefer and clean.
• Check for damage to plate and apparatus.
• Unhook sensors from DAC and put away.
• Put all equipment away.
• Turn off light setup.
• Save data and pictures recorded during the day.
• Fill in rest of test sheets.
4. Results

Once all of the 48 tests were completed, the data were analyzed in order to provide the desired results. In this section, details of the calculations that were carried out, as well as the results from those calculations are presented.

4.1. Preliminary Data Analysis

The output of the DAC provided two time series waves (columns) of data measured in pound force (lbf): the ‘bucket sensor’ data and the ‘friction force sensor’ data (the sensors are discussed in section 2.1.3). A measure from both these sensors was recorded every 1/100 of a second. The size of the data per test varied depending on the length of that experiment.

The analysis of the data was done with the program Igor Pro (version 6.32A) as a large amount of data was recorded per test. The two waves of data from each experiment were loaded individually in the program. Then, four user-defined functions were executed in a specific order to calculate the desired results so as to plot the resulting graphs for each test. These functions, written in Igor itself, were used in order to analyze the data consistently. The calculation steps, for each experiment, involved removing excess data, converting the weights from the DAC output, calculating the extra force on the ice (which was not recorded by the sensor), converting rotational velocities, calculating the contact area of the sample on the plate, calculating the lost ice volume as well as its mass, and, finally, calculating the true normal force on the ice and its friction coefficient. A description of each of the user-defined function is given in the following sections.
4.1.1. Removing Excess Data

The first step in the analysis of the data was to remove irrelevant parts of the time series. As the recording of the data was started prior to the start of the experiment and stopped, sometimes, after the experiment was completed, excess data needed to be cropped. The first user-defined function did just that. A choice of two functions was available depending on the appearance of the data: ‘analysisf’ or ‘analysis’. The difference between the two functions was that the first started cropping from the start of the data whereas the second, from its end. Both their inputs were the same. They required the starting weight of the experiment and whether the weight varied. The options for both inputs as well as the order in which to type them are shown in Table 9 below.

Table 9: Order and Input Options for ‘Analysif’ and ‘Analysis’

<table>
<thead>
<tr>
<th>Analysisf (chwe, stwe) or Analysis (chwe, stwe)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Change in Weight</strong></td>
</tr>
<tr>
<td>Variable : chwe</td>
</tr>
<tr>
<td>0 : No Change</td>
</tr>
<tr>
<td>1 : Change</td>
</tr>
<tr>
<td><strong>Starting Weight</strong></td>
</tr>
<tr>
<td>Variable : stwe</td>
</tr>
<tr>
<td>0 : 0 kg</td>
</tr>
<tr>
<td>50 : 50 kg</td>
</tr>
<tr>
<td>100 : 100 kg</td>
</tr>
<tr>
<td>150 : 150 kg</td>
</tr>
</tbody>
</table>

4.1.2. Weight Conversion of DAC Output

Both functions, discussed in the previous section, then converted the weight recorded of the bucket (in lbf) to a force (in N) \( (F_{N,B}, \text{Bucket Force}) \). Equation 2 shows the formula used

\[
1 \text{ lbf} = 4.44822162 \text{ N} \tag{2}
\]
An example of the calculations follows.

\[ 1 \text{ lbf} = 4.44822162 \text{ N} \]

\[ 110 \text{ lbf} = x \]

\[ x = 493.042 \text{ N} \]

\[ F_{NB} = 493.042 \text{ N} \]

The output of the friction sensor (in lbf) was converted to Newtons (N) \( (F) \). Equation 2 was used for this calculation as well. Please refer to the previous example for calculations.

### 4.1.3. Extra Force Summation

The extra mass, \( m_E \) in kg, which was not measured by the bucket sensor, excluding the weight of the ice cone itself, was then tabulated. The calculation did not change in between tests and is shown below.

Mass of Railing and L-Shaped Part (including Friction Sensor) : \( m_1 = 8.669 \text{ kg} \)

Mass of Forklift Part : \( m_2 = 5.673 \text{ kg} \)

Mass of Camera (including Case) : \( m_3 = 0.397 \text{ kg} \)

Mass of C-Clamp Holding Camera in Place : \( m_4 = 0.198 \text{ kg} \)

\[ m_E = m_1 + m_2 + m_3 + m_4 \]

\[ m_E = 8.669 \text{ kg} + 5.673 \text{ kg} + 0.397 \text{ kg} + 0.198 \text{ kg} \]

\[ m_E = 14.937 \text{ kg} \]
The parts mentioned in the above calculation are discussed in a prior section (section 2.1.3). The extra force $F_{N,E}$ in $N$ was then calculated using Equation 3

$$F = ma \quad (3)$$

where $F$ is the force in $N$, $m$ the mass in $kg$ and $a$ the acceleration in $m/s^2$. An example follows.

$$F_{N,E} = m_E \times g$$

$$F_{N,E} = 14.937 \, kg \times 9.81 \, \frac{m}{s^2}$$

$$F_{N,E} = 146.53 \, N$$

where $F_{N,E}$ is the force resulting from the extra mass in $N$, $m_E$ the extra mass in $kg$ and $g$ the gravitational acceleration $9.82 \, m/s^2$.

**4.1.4. Rotational Velocity Conversion**

The second user-defined function was used to convert the four velocities recorded by the tachometer to an average linear velocity applied to the ice sample. The function ‘velocities’ required the velocity number for the test analyzed as well as the four recordings of the rotational velocity, measured by the tachometer. The options for the inputs as well as the order in which to type them are shown in Table 10 next.

The rotational velocities measured by the tachometer first needed to be converted into $rad/s$. Equations 4 and 5 show the formulas. An example of the computation follows.
\[
\frac{1 \text{ rotation}}{1 \text{ min}} = \frac{1 \text{ rotation}}{60 \text{ sec}}
\]  
(4)

\[
1 \text{ rotation} = 2\pi \text{ rad}
\]  
(5)

Table 10: Order and Input Options for ‘Velocities’

<table>
<thead>
<tr>
<th>Velocity Number</th>
<th>Rotational Velocity 1</th>
<th>Rotational Velocity 2</th>
<th>Rotational Velocity 3</th>
<th>Rotational Velocity 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable : speed</td>
<td>Variable : w1</td>
<td>Variable : w2</td>
<td>Variable : w3</td>
<td>Variable : w4</td>
</tr>
<tr>
<td>1 : Speed #1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 : Speed #2</td>
<td>Measure in rpm</td>
<td>Measure in rpm</td>
<td>Measure in rpm</td>
<td>Measure in rpm</td>
</tr>
<tr>
<td>3 : Speed #3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{500 \text{ rotations}}{1 \text{ min}} = \frac{500 \text{ rotations}}{60 \text{ sec}}
\]

\[
x = 8.333 \frac{\text{rotations}}{s}
\]

\[
\frac{1 \text{ rotation}}{8.333 \text{ rotation}} = \frac{2\pi \text{ rad}}{w_1}
\]

\[
w_1 = 52.4 \frac{\text{rad}}{s}
\]
where \( w_1 \) is the angular velocity of the tachometer in rad/s. Once the angular velocity of the tachometer, in rad/s, was measured, the angular velocity of the turntable is computed using Equation 6

\[
V_c = w_1 \left( \frac{d}{2} \right) = w_2 \left( \frac{D}{2} \right) \tag{6}
\]

where \( V_c \) is the velocity at the point of contact in m/s, \( w_1 \) the angular velocity of the tachometer in rad/s, \( d \), its diameter in m, \( w_2 \), the angular velocity of the turntable in rad/s and \( D \), its diameter in m. Figure 76 shows the point of contact as well as the dimensions for both turntable and tachometer.

![Figure 76: Point of Contact and Dimensions for Tachometer and Turntable.](image)

An example demonstrating the calculation of the angular velocity of the turntable is shown below.

\[
w_1 \left( \frac{d}{2} \right) = w_2 \left( \frac{D}{2} \right)
\]
The velocity, to which the ice sample was subjected to, was dependent of the velocity number of the experiment as three different velocities were used in the research. Each of the three velocities used had a different radius of rotation. The values of the different radii for the study are shown in Figure 77 and discussed in a previous section (section 3.1.1).

![Figure 77: Radiuses for Different Velocity Numbers.](image)

An example of the calculations done, with velocity number 1 using Equation 6 is shown next.

\[
V_c = w_1 \left( \frac{d}{2} \right) = w_2 \left( \frac{D}{2} \right)
\]
\[ V_c = w_2 \times r_1 \]

\[ V_c = 2.15 \frac{rad}{s} \times 0.37 \, m \]

\[ V_c = 0.8 \frac{m}{s} \]

For a given test, four angular velocity readings were measured using the handheld tachometer. They were recorded to assure consistency of the angular rotation of the turntable throughout an experiment. Once the four velocities were converted to turntable velocities in \( m/s \), using the previous example, the average velocity for each experiment was added up. Equation 7 shows the formula used

\[ V_{av} = \frac{V_1 + V_2 + V_3 + V_4}{4} \]

(7)

where \( V_{av} \) is the average velocity of a given test and \( V_1, V_2, V_3 \) and \( V_4 \), the four velocities calculated in \( m/s \) using the recorded tachometer angular velocities. An example for the calculation ensues. This next calculation was done using velocity number 1 and 500 rpm for all tachometer angular velocities.

\[ V_{av} = \frac{0.0189 \frac{m}{s} + 0.0189 \frac{m}{s} + 0.0189 \frac{m}{s} + 0.0189 \frac{m}{s}}{4} \]

\[ V_{av} = 0.0189 \frac{m}{s} \]
4.1.5. True Normal Force on Ice Sample

As the friction force was already computed, from section 4.1.2, the only value needed to calculate the friction coefficient was the true normal force $F_N$. The true normal force was a summation of the bucket force $F_{N,B}$ (calculated in section 4.1.2), the extra force $F_{N,E}$ (calculated in section 4.1.3) and the ice sample force $F_{I,R}$ which had yet to be calculated. The third user-defined function calculated the ice sample mass/force, the true normal force as well as the friction coefficient of the sample for all output values of the DAC in each test. The function ‘samplearea’ required the measures for the ice ring mass as well as the cone mass (once shaped including the ring). The options for the input are shown in Table 11 below.

Table 11: Order and Input Options for ‘Samplearea’

<table>
<thead>
<tr>
<th>Samplearea (ringweight, coneweight)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ice Ring Weight</strong></td>
</tr>
<tr>
<td>Variable: ringweight</td>
</tr>
<tr>
<td>Measure in kg</td>
</tr>
</tbody>
</table>

4.1.5.1. Calculation of Contact Area of Sample on Plate

The first step to calculating the true normal force was to calculate the mass that was lost due to the abrading of the ice sample. This loss varied throughout the test. To be able to compute these values, the change in contact area during the experiment needed to be established.
The contact area of the cone could not be monitored directly by the DAC. Using the video recordings for each test, four contact diameters as well as their respective ice ring diameters were scaled from the video image measured at specific times. The measures that needed to be recorded are shown in Figure 78. As the original dimensions of the cone were known, as shown in Figure 79, a scale could be determined to calculate the real ice contact diameter.

Figure 78: Video Contact and Ice Diameter.
One of the purposes of the third user-defined function was to calculate these four real contact diameters. Equation 8 shows the formula employed

\[
\frac{d_{R,I}}{d_{R,C}} = \frac{d_{V,I}}{d_{V,C}}
\]

(8)

where \(d_{R,I}\) is the real ice diameter, \(d_{R,C}\) the real ice contact diameter, \(d_{V,C}\) the video ice diameter and \(d_{V,I}\) the video ice contact diameter. All the values are in \(m\) and are shown in Figure 78 and Figure 79. An example for the computation follows.

\[
\frac{0.25\ m}{d_{R,C}} = \frac{0.131\ m}{0.04\ m}
\]

\[
d_{R,C} = \frac{0.25\ m \times 0.04\ m}{0.131\ m} = 0.076\ m
\]
Once those four real ice contact diameters, $d_{R,C}$, were computed, the function calculated the contact areas $A_C$ using Equation 9. A sample calculation is shown after Equation 9

$$A = \pi (r)^2$$

$$A_C = \pi \left( \frac{d_{R,C}}{2} \right)^2$$

$$A_C = \pi \left( \frac{0.076 \text{ m}}{2} \right)^2$$

$$A_C = 0.0045 \text{ m}^2$$

where $A$ is the apparent area of contact in $\text{m}^2$ and $r$, the radius of this area in $\text{m}$. A Gauss curve fit was plotted with those four calculated areas using Igor to measure the change in contact area throughout the experiment. Equation 10 shows the equation of the curve fit of the contact area

$$y = y_0 + A \times \exp\left(-\left(\frac{x - x_0}{w}\right)^2\right)$$

where $y_0$, $A$, $x_0$ and $w$ are coefficients specific to that curve and $y$ the contact area $A_C$ in $\text{m}^2$ at time $x$ in $\text{sec}$. Particular coefficients were computed by the program Igor for each experiment. A real ice contact radius $r_{R,C}$ curve was also plotted using Equation 9.

### 4.1.5.2. Volume of Crushed Ice Measurement

Once the real ice contact radius $r_{R,C}$ and contact area $A_C$ were calculated for all the output values of the DAC, the volume of ice lost could be computed. The imaginary shape of the lost ice was a smaller $30^\circ$ cone with the calculated contact area as a base as shown in Figure 80.
The function ‘samplearea’ was used to calculate the contact height $h_C$ using Equation 11 and the imaginary (lost) cone volume ($V_{I,C}$) using Equation 12

$$\tan(30^\circ) = \frac{h_C}{r_{R,C}} \quad (11)$$

$$V_{I,C} = \left(\frac{1}{3}\right) \times A_C \times h_C \quad (12)$$

where $V_{I,C}$ is the imaginary volume of the lost ice in m$^3$ and where both $h_C$ (Contact height) and $r_{R,C}$ (Real ice contact radius) are in m while the contact area $A_C$ is in m$^2$. An example of the calculation follows.

$$h_C = \tan(30^\circ) \times r_{R,C}$$

$$h_C = \frac{0.076 \, m}{2}$$

$$h_C = 0.022 \, m$$
\[ V_{l,c} = \left( \frac{1}{3} \right) \times A_c \times h_c \]

\[ V_{l,c} = \left( \frac{1}{3} \right) \times 0.0045 \, \text{m}^2 \times 0.022 \, \text{m} \]

\[ V_{l,c} = 0.00033 \, \text{m}^3 \]

### 4.1.5.3. Lost Ice Mass Calculation

To get the imaginary cone’s mass, the density of the ice needed to be calculated using Equation 13

\[ \rho = \frac{m_i}{V_i} \quad (13) \]

where \( \rho \) is the density in \( \text{kg/m}^3 \), \( m_i \) the mass of ice in kg, and \( V_i \) the ice volume in \( \text{m}^3 \). The mass of the ice in the sample \( m_{\text{ice}} \), when intact, was calculated subtracting the mass of the ice holder (ice ring), \( m_{\text{holder}} \), from the mass of the ice sample with the ice holder, \( m_{\text{sample}} \), as shown in Equation 14. All the values in Equation 14 are in kg. An example of the calculation is done below.

\[ m_{\text{ice}} = m_{\text{sample}} - m_{\text{holder}} \quad (14) \]

\[ m_{\text{ice}} = 7.808 \, \text{kg} - 3.901 \, \text{kg} \]

\[ m_{\text{ice}} = 7.808 \, \text{kg} - 3.901 \, \text{kg} \]

\[ m_{\text{ice}} = 3.907 \, \text{kg} \]
The volume of the ice $V_{T,I}$ at the start of the experiment was calculated using the original volume of the cone as shown in Equation 15. This volume never changed as all the ice holder rings were identical.

$$V_{T,I} = V_{\text{Cone}} + V_{\text{Cylinder}}$$ (15)

where the different volumes are shown in Figure 81 and are in $m^3$.

Figure 81: Section of Ice Sample with Cone Ice Volume (Grey) and Cylinder Ice Volume (Green)
Where the Ice Ring is in Black.

An example follows, using the dimensions mentioned in Figure 79.

$$V_{T,I} = V_{\text{Cone}} + V_{\text{Cylinder}}$$

$$V_{T,I} = \left(\frac{1}{3}\right) A_c \times H_{\text{cone}} + A_c \times H$$

$$V_{T,I} = \left(\frac{1}{3}\right) \times \pi \left(\frac{0.26}{2}\right)^2 \times \tan(30^o) \times \frac{0.26}{2} + \pi \left(\frac{0.26}{2}\right)^2 \times 0.0511 \text{ m}$$

$$V_{T,I} = 0.00404 \text{ m}^3$$
Once the ice volume $V_{i,I}$ was calculated, the density of the ice, using the full ice mass and volume, was compiled with Equation 13. This was done for each experiment as the ice rings and ice samples varied in mass. An example of the calculation follows.

\[ \rho = \frac{m_i}{V_i} \]

\[ \rho = \frac{3.907 \, kg}{0.00404 \, m^3} \]

\[ \rho = 966.76 \frac{kg}{m^3} \]

The mass of the imaginary ice cone $m_{i,cone}$, or lost ice, was then calculated using the same formula. An example follows.

\[ \rho = \frac{m_{i,cone}}{V_{i,cone}} \]

\[ 966.76 \frac{kg}{m^3} = \frac{m_{i,cone}}{0.00033 \, m^3} \]

\[ m_{i,cone} = 0.319 \, kg \]

where $V_{i,cone}$ is the volume of the imaginary cone in $m^3$, $m_{i,cone}$ the mass of that imaginary cone in kg and $\rho$ the density of the ice in $kg/m^3$. The remaining sample’s mass, $m_{i,R}$, could finally be plotted using Equation 16. A sample calculation follows.

\[ m_{i,R} = m_{ice} - m_{i,cone} \]  \hspace{1cm} (16)
\[ m_{I,R} = 7.808\, kg - 0.319\, kg \]

\[ m_{I,R} = 7.489\, kg \]

The remaining sample mass \( m_{I,R} \) was computed as a changing variable through the whole test. It was then converted to a force \((F_{I,R}, \text{remaining sample force})\) using Equation 3. An example follows.

\[ F = ma \]

\[ F_{I,R} = 7.489\, kg \times 9.81 \frac{m}{s^2} \]

\[ F_{I,R} = 73.47\, N \]

### 4.1.5.4. True Normal Force Summation

Once the remaining sample force, \( F_{I,R} \), was added up, the total weight or true normal force \( F_N \), to which the ice sample was subjected to, could be plotted using Equation 17. An example follows.

\[ F_N = F_{N,B} + F_{N,E} + F_{I,R} \tag{17} \]

where the bucket force \( F_{N,B} \) and extra force \( F_{N,E} \) where calculated in sections 4.1.2 and 4.1.3 respectively and where all the units are \( N \).

\[ F_N = F_{N,B} + F_{N,E} + F_{I,R} \]

\[ F_N = 493.042\, N + 146.53\, N + 73.47\, N \]
The pressure was calculated with Equation 18. A sample calculation follows.

\[ P = \frac{F_N}{A_C} \]  \hspace{1cm} (18)

where \( P \) is the pressure in Pa and where the true normal force \( F_N \) is in N and the contact area \( A_C \) in \( m^2 \).

\[ P = \frac{(F_N)}{A_C} \]

\[ P = 713.04 \text{ N}/0.0045 \text{ m}^2 \]

\[ P = 158.45 \text{ kPa} \]

### 4.1.6. Ice Sample Friction Coefficient

The last task of the third user-defined function, ‘samplearea’, was to calculate the friction coefficient for each of the outputted values. It was computed using Equation 19. An example follows.

\[ \mu = \frac{F_f}{F_N} \]  \hspace{1cm} (19)

where \( F_f \) is the friction force found in section 4.1.2 from the output of the friction sensor in N, \( F_N \) the true normal force found in section 4.1.5.4 in N, and \( \mu \), the dimensionless friction coefficient.

\[ \mu = \frac{F_f}{F_N} \]
\[ \mu = \frac{5.96 \, N}{713.04 \, N} \]
\[ \mu = 0.0084 \]

### 4.1.7. Plotting the Graphs

The fourth and last user-defined function, ‘graphdrawing’, did not calculate values. Its sole purpose was to plot the resulting graphs from all the previously calculated waves. The function ‘graphdrawing’ required whether the test was lubricated or not. The options for the input are shown in Table 12 below.

**Table 12: Order and Input Options for ‘Graphdrawing’**

<table>
<thead>
<tr>
<th>Graphdrawing (lube)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lubrication</td>
</tr>
<tr>
<td>Variable : lube</td>
</tr>
<tr>
<td>0 : No lubrication</td>
</tr>
<tr>
<td>1 : Lubrication</td>
</tr>
</tbody>
</table>

### 4.2. Results

Like most experiments done with ice, the data gathered throughout this research was slightly scattered and varied. Some trends were visible in the data, however, in some cases, one or more tests using similar factors contradicted the trends leading to expected uncertainties. The product of this research is divided into four categories: non-lubricated experiments (fixed normal force); non-lubricated experiments (increasing normal force);
lubricated experiments (fixed normal force); and, lubricated experiments (increasing normal forces).

- Non-lubricated experiments (fixed normal force): refer to section 4.2.1.1.
- Non-lubricated experiments (increasing normal force): refer to section 4.2.1.2.
- Lubricated experiments (fixed normal force): refer to section 4.2.2.1.
- Lubricated experiments (increasing normal forces): refer to section 4.2.2.2.

It is important to note that all the plots that are presented in the following sections have been plotted on a common scale to make comparisons between cases easier. It is also important to note that each single experiment is specific to a particular category. None of the numbered tests correspond to multiple categories.

4.2.1. Non-Lubricated Experiments

Dry (non-lubricated) experiments involve friction tests executed using ice on stainless steel with nothing added to the surface of the steel plate. This category of tests encompasses all the tests completed in this research without added water. Please refer to section 3.1.5 for a list of all the tests performed. Most previous research that has been done on ice friction involved dry tests as identified in section 1.4.

4.2.1.1. Non-Lubricated Experiments with Fixed Normal Force

Table 13 presents all the tests from the research that fall into the non-lubricated fixed normal force category.
Table 13: List of Non-Lubricated Experiments with Fixed Normal Force.

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Initial Sand Weight</th>
<th>Lubrication</th>
<th>Pressure Change</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
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<td>(y/n)</td>
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</tr>
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<td>34</td>
<td>100</td>
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<tr>
<td>48</td>
<td>150</td>
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</table>

### 4.2.1.1.1. Contact Area vs. Time

The range of different contact areas observed in non-lubricated tests with fixed normal forces was quite small. All dry tests with constant normal forces had an area increase through time that resembles Figure 82. The slope of the contact area curve was larger at the beginning of the tests, but reduced to zero eventually. A ‘steady state’ was observed when the contact area stopped increasing. No inconsistencies in the contact area evolution were observed in the tests in this group.
4.2.1.1.2. Friction Force vs. Normal Force

The ranges of friction forces measured in these experiments were dependent on the starting normal force of the test. The downward force did not vary throughout each experiment showing expected consistent normal forces. Higher friction forces and a wider range of scatter were observed in higher starting weight tests as may be seen by comparing Test 36 in Figure 83 and Test 46 in Figure 84.
4.2.1.1.3. Friction Coefficient vs. Contact Area and Pressure

The friction coefficients, observed in this grouping of tests, were small and did not fluctuate significantly. Friction coefficients, no larger than 0.05, were measured. The results presented in Figure 85 and Figure 86 for Test 34 are representative of typical results for this category. Slightly higher friction coefficients were observed at lower velocities and at higher starting weights. The friction coefficient with respect to the pressure remained very level throughout the test as with the friction coefficient with respect to the contact area. The friction coefficient development along the contact area was harder to discern as the changes in area were very small in this category of tests.
Tests number 11, 35 and 48 showed inconsistencies when compared with other experiments in this category. Some of the results are shown in Figure 87 to Figure 90.

Tests 11 and 35, as mentioned previously, were completed at the same velocity (velocity 2). However Test 11 had a starting mass of 0 kg while Test 35 started at 100 kg. Test 48 had nothing in common with the other tests showing inconsistencies as neither its velocity (velocity 3) nor its starting mass (150 kg) matched. The friction coefficients observed in tests 11 and 35, contrary to the other numbered tests in this category, were much higher and reached values of 0.1 to 0.15. Test 48 displayed an unusual trend as well but remained in the 0 to 0.05 range.
Figure 87: Friction Coefficient vs. Contact Area of Test 11.

Figure 88: Friction Coefficient vs. Pressure of Test 11.

Figure 89: Friction Coefficient vs. Contact Area of Test 35.

Figure 90: Friction Coefficient vs. Pressure of Test 35.
4.2.1.2. Non-Lubricated Experiments with Increasing Normal Force

Table 14 presents the 12 numbered tests from the research that fall into the non-lubricated increasing normal force category.

Table 14: List of Non-Lubricated Experiments with Increasing Normal Force.

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Initial Sand Weight</th>
<th>Lubrication</th>
<th>Pressure Change</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>kg</td>
<td>(y/n)</td>
<td>(y/n)</td>
<td>#</td>
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<td>yes</td>
<td>1</td>
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<td>8</td>
<td>0</td>
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<td>yes</td>
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<td>9</td>
<td>0</td>
<td>no</td>
<td>yes</td>
<td>3</td>
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<td>yes</td>
<td>1</td>
</tr>
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<td>20</td>
<td>50</td>
<td>no</td>
<td>yes</td>
<td>2</td>
</tr>
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<td>21</td>
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<td>yes</td>
<td>3</td>
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<tr>
<td>31</td>
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<td>no</td>
<td>yes</td>
<td>1</td>
</tr>
<tr>
<td>32</td>
<td>100</td>
<td>no</td>
<td>yes</td>
<td>2</td>
</tr>
<tr>
<td>33</td>
<td>100</td>
<td>no</td>
<td>yes</td>
<td>3</td>
</tr>
<tr>
<td>43</td>
<td>150</td>
<td>no</td>
<td>yes</td>
<td>1</td>
</tr>
<tr>
<td>44</td>
<td>150</td>
<td>no</td>
<td>yes</td>
<td>2</td>
</tr>
<tr>
<td>45</td>
<td>150</td>
<td>no</td>
<td>yes</td>
<td>3</td>
</tr>
</tbody>
</table>

4.2.1.2.1. Contact Area vs. Time

The range of contact areas observed in non-lubricated tests with increasing normal forces was different than the previous category. No common trends were observed but the tests fell into two categories, as defined by the nature of the contact area development as shown in Figure 91 and Figure 92. Some dry tests with rising normal forces had an area increase through time that resembled the previous category as shown in Figure 91. However, some tests did not appear to have a ‘steady state’ as shown in Figure 92, even if the duration of the test was greater.
The ranges of friction forces measured in these experiments were dependent on the starting normal force as in the preceding category. Similar maximum friction forces were observed in all experiments as all the tests in this set ended with similar normal forces. Consequently, higher starting normal force experiments began with higher friction forces. No clear differences were noted in between tests at different velocities. The progression of the friction force with an increasing normal force for most experiments presented like Figure 93 and Figure 94. The friction force increased moderately but steadily throughout the tests in proportion to the normal force. Tests 8 and 32 however exhibited different trends. Figure 95 and Figure 96 show the friction force versus the normal force for both tests in which the friction force was not linearly related to the normal force.
Figure 93: Friction Force vs. Normal Force of Test 20.

Figure 94: Friction Force vs. Normal Force of Test 43.

Figure 95: Friction Force vs. Normal Force of Test 8.

Figure 96: Friction Force vs. Normal Force of Test 32.
4.2.1.2.3. Friction Coefficient vs. Contact Area and Pressure

The friction coefficients observed in this category of tests were also small and did not vary significantly. Friction coefficients, no larger than 0.05, were measured. Results such as those presented in Figure 97 and Figure 98 are typical. The friction coefficient with respect to the pressure stayed approximately constant throughout. No clear effects of the velocity or the starting normal force on the friction coefficient were observed in this category. The friction coefficient with respect to the area did not increase for some of the tests, as may be seen for the sample data for Test 33 shown in Figure 97. In some cases a slight growth was observed as the area increased, for example see Figure 99. This effect was very slight and is not considered further here.

Figure 97: Friction Coefficient vs. Contact Area of Test 33.

Figure 98: Friction Coefficient vs. Pressure of Test 33.
4.2.1.3. Summary of Results for Non-Lubricated Experiments

Fixed Normal Force

- Range of contact areas measured was quite small.
- All tests had a plot of the area increase through time where the slope of the contact area curve was larger at the beginning of the tests, but reduced to zero eventually. A ‘steady state’ was observed when the contact area stopped increasing.
- Expected consistent normal forces detected with increasing friction force. Higher friction forces and a wider range of scatter were observed in higher starting weight.
- Friction coefficients, no larger than 0.05, were measured.

Figure 99: Friction Coefficient vs. Contact Area of Test 7.
Slightly higher friction coefficients were observed at lower velocities and at higher starting weights.

Tests number 11, 35 and 48 showed inconsistencies.

**Increasing Normal Force**

- Two different types of area increase through time curve attained. First type similar to the results from the non-lubricated fixed normal force category. Second type did not appear to reach a ‘steady state’.
- Higher starting normal force experiments began with higher friction forces.
- No clear differences were noted in between tests at different velocities.
- The friction force increased moderately but steadily throughout the tests in proportion to the normal force.
- Friction coefficients, no larger than 0.05, were measured.
- Tests 8 and 32 showed inconsistencies.

### 4.2.2. Lubricated Experiments

Wet (lubricated) experiments correspond to the friction tests executed with ice on stainless steel, with (room temperature) water added to the surface. This was done in order to incorporate water at the ice-steel interface. Room temperature water was used to provide a longer test period before the lubricating water started to freeze. This category of tests encompasses all tests completed in this research program that used added water or ‘lubrication’. Please refer to section 3.1.5 for a list of all the tests performed. Not many
previous researchers have experimented with this factor in ice friction as discussed in section 1.4.10.

4.2.2.1. **Lubricated Experiments with Fixed Normal Force**

Table 15 presents all the numbered tests from the research that fall into the lubricated fixed normal force category. There are a total of 12 tests corresponding to this category.

**Table 15: List of Lubricated Experiments with Fixed Normal Force.**

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Initial Sand Weight</th>
<th>Lubrication</th>
<th>Pressure Change</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>kg</td>
<td>(y/n)</td>
<td>(y/n)</td>
<td>#</td>
</tr>
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<td>1</td>
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<td>17</td>
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<td>18</td>
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<td>28</td>
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<tr>
<td>42</td>
<td>150</td>
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<td>3</td>
</tr>
</tbody>
</table>

4.2.2.1.1. **Contact Area vs. Time**

The range of different contact areas observed in lubricated tests with fixed normal forces was quite diverse. Half of the wet tests with constant normal forces had an area increase through time that resembled dry tests experiments with no increase in normal force (see Figure 100). The other half of the experiments abraded quickly and increased in area.
drastically and achieved a ‘steady state’ early in the test as shown in Figure 101. Finally, Test 6 increased drastically in area but did not achieve a ‘steady state’ (see Figure 102). However these last experiments lasted a shorter period of time than most in this category.

Figure 100: Change of Area Through Time of Test 41 (Dry Test Behavior).

Figure 101: Change of Area Through Time of Test 28 (‘Steady-State’ Behavior).
4.2.2.1.2. Friction Force vs. Normal Force

The ranges of friction forces measured in these experiments were dependent on the starting normal force of the test. As seen in Figure 103, the downward force did not vary throughout each experiment as indicated by the constant normal forces. Higher peak friction forces were observed for tests with a higher starting weight as shown in Figure 103 and Figure 104. No variations in the friction force were detected between similar tests at different velocities. In all cases, the peak frictional force was proportional to the normal force.
4.2.2.1.3. Friction Coefficient vs. Contact Area and Pressure

The friction coefficients, observed in this grouping of tests, were much larger than those in the dry categories and sometimes fluctuated significantly. Friction coefficients, as high as 0.19, were measured. Very low friction coefficients were also encountered in this type of test. Two different types of results presented as shown in Figure 105 and Figure 106. The results of these tests seemed to be linked to the appearance of their area curve plotted in time. The trends like the one shown in Figure 105 were observed to correspond with area curves more like the one presented in Figure 102 where the curve increased drastically in the beginning of the experiment reaching large area values or when very large contact areas were reached with a lower slope. Development of the friction coefficient similar to the one displayed in Figure 106 were observed with area curves that
did not achieve a very large contact area but had a fairly long ‘steady state’ portion (see Figure 100). Clear differences between friction coefficients at different velocities or starting weights were not observed. The friction coefficient with respect to the pressure did not remain constant throughout the tests. Most experiments had a low friction coefficient at high pressures and a rise and fall at lower pressures as in Figure 107 and Figure 108. However, sometimes the friction coefficient was not at its lowest point at larger pressures. It appeared to be mid-range or max range while a jump and/or fall at lower pressures was still observed (see Figure 109 and Figure 110). This second phenomenon seemed to happen more often when larger contact areas were achieved in a test.

Figure 105: Friction Coefficient vs. Contact Area of Test 42.

Figure 106: Friction Coefficient vs. Contact Area of Test 5.
Figure 107: Friction Coefficient vs. Pressure of Test 42.

Figure 108: Friction Coefficient vs. Pressure of Test 5.

Figure 109: Friction Coefficient vs. Pressure of Test 17.

Figure 110: Friction Coefficient vs. Pressure of Test 6.
Test 4 and 40 showed trends different from the above behavior. The results of Test 4 are shown in Figure 111. This experiment seemed to have many fluctuations in friction coefficient throughout the test which was not observed in any of the other experiments in this category. The area curve as well as the friction coefficient versus pressure curve did not seem abnormal for this test. Test 40 appeared to have a very scattered jump in friction coefficient in the beginning of the test.

![Figure 111: Friction Coefficient vs. Contact Area of Test 4.](image)

### 4.2.2.2. Lubricated Experiments with Increasing Normal Force

Table 16 presents all the numbered tests from the research that fall into the lubricated increasing normal force category. There are a total of 12 tests corresponding to this category.
The range of different contact areas observed in lubricated tests with increasing normal forces was quite diverse as in the previous category. One of the wet tests (Test 1) with increasing normal force had an area increase through time that resembled dry test experiments with no increase in normal force (see Figure 112). Nine out of twelve of the experiments in this group, however, increased in contact area quickly and settled into a ‘steady state’ eventually as shown in Figure 113. Finally, Test 37 and Test 39 increased drastically in area but did not develop into a ‘steady state’ (see Figure 114).

Table 16: List of Lubricated Experiments with Increasing Normal Force.

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Initial Sand Weight</th>
<th>Lubrication</th>
<th>Pressure Change</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>kg</td>
<td>(y/n)</td>
<td>(y/n)</td>
<td>#</td>
</tr>
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<td>yes</td>
<td>1</td>
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<td>39</td>
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</table>

4.2.2.2.1. Friction Coefficient vs. Contact Area

The range of different contact areas observed in lubricated tests with increasing normal forces was quite diverse as in the previous category. One of the wet tests (Test 1) with increasing normal force had an area increase through time that resembled dry test experiments with no increase in normal force (see Figure 112). Nine out of twelve of the experiments in this group, however, increased in contact area quickly and settled into a ‘steady state’ eventually as shown in Figure 113. Finally, Test 37 and Test 39 increased drastically in area but did not develop into a ‘steady state’ (see Figure 114).
Figure 112: Change of Area Through Time of Test 1.

Figure 113: Change of Area Through Time of Test 2.
4.2.2.2.2. Friction Force vs. Normal Force

The ranges of friction forces measured in these experiments did not appear to be as dependent on the starting normal force as in the preceding category. Ten out of the twelve tests in this category exhibited a behavior as depicted in Figure 115 for the progression of the friction force with an increasing normal force. Friction forces appeared to reach a higher point for tests starting with a normal force of 1471.5 N. No correlation was however observed for lower starting normal force tests. Velocity did not appear to show any distinct effect either. The most frequently observed friction force behavior for the experiments discussed in this group was much like the dry tests category. A mild increase in friction force was found to occur while the normal force increased. Unlike the dry tests, for some experiments, a jump in friction was observed at either the beginning or in the middle of the experiments. This type of discontinuity was observed throughout the
lubricated test group. An exception to this trend, Test 39 (see Figure 116), exhibited a relatively constant friction force throughout.

Figure 115: Friction Force vs. Normal Force of Test 15.

Figure 116: Friction Force vs. Normal Force of Test 39.
4.2.2.2.3. Friction Coefficient vs. Contact Area and Pressure

The friction coefficients, observed in this grouping of tests were much larger than the dry categories and sometimes fluctuated significantly. Friction coefficients, as high as 0.25, were measured. Very low friction coefficients were also encountered in this group of tests. Two different types of results presented as shown in Figure 117 and Figure 118.

**Figure 117: Friction Coefficient vs. Contact Area of Test 1.**

**Figure 118: Friction Coefficient vs. Contact Area of Test 26.**

The results of these lubricated tests seemed to be linked to their respective area vs. time curve as with the lubricated experiments with fixed normal force. Trends like the one displayed in Figure 118 were observed to correspond with area curves similar to the one in Figure 113 where the curve increased quickly in the beginning of the experiment and reached a large steady-state area value. Developments of the friction coefficient like in Figure 117 were observed with area curves that did not achieve a very large contact area but had a fairly long ‘steady state’ portion (see Figure 113). Clear differences between
friction coefficients at diverse velocities or starting weights were not detected. The friction coefficient versus area curves for Tests 2 and 38 contradicted the previous notes for this category as shown in Figure 119 and Figure 120. These tests did provide good data for the range of friction coefficients however.

![Figure 119: Friction Coefficient vs. Contact Area of Test 38.](image1)

![Figure 120: Friction Coefficient vs. Contact Area of Test 2.](image2)

The friction coefficient with respect to the pressure did not remain constant throughout the tests. Most experiments had a low friction coefficient at high pressures and exhibited a peak friction coefficient in the lower pressure ranges as depicted in Figure 121 and Figure 122. However, sometimes the friction coefficient was not at its lowest point at larger pressures. It appeared to be at mid-range while a fall at lower pressures was still observed for one of the experiments in this category (see Figure 123). The friction
Coefficient versus pressure curve of test 39 did not follow the trends of this category as shown in Figure 124.

Figure 121: Friction Coefficient vs. Pressure of Test 1.

Figure 122: Friction Coefficient vs. Pressure of Test 26.

Figure 123: Friction Coefficient vs. Pressure of Test 14.
4.2.2.3. Summary of Results for Lubricated Experiments

Fixed Normal Force

- Half of the tests had an area increase through time plot that resembled the ones attained in dry experiments with fixed normal forces. The other half of the experiments abraded quickly, increased in area drastically and achieved a ‘steady state’ early. Test 6 increased drastically in area but did not achieve a ‘steady state’.
- Expected consistent normal forces detected with increasing friction forces.
- Higher friction forces and a wider range of scatter were observed in higher starting weight.

Figure 124: Friction Coefficient vs. Pressure of Test 39.
The friction coefficients were much larger than those in the dry categories and sometimes fluctuated significantly.

Friction coefficients, as high as 0.19, were measured.

Slightly higher friction coefficients were observed at lower velocities and at higher starting weights.

The results of these tests seemed to be linked to the appearance of their area curve plotted in time.

Most experiments had a low friction coefficient at high pressures and a rise and fall at lower pressures.

Test 4 and 40 showed inconsistencies.

**Increasing Normal Force**

One of the wet tests (Test 1) had an area increase through time that resembled dry test experiments with fixed normal forces. Nine out of twelve of the experiments increased in contact area quickly and settled into a ‘steady state’. Test 37 and Test 39 increased drastically in area but did not develop into a ‘steady state’.

The ranges of friction forces did not appear to be as dependent on the starting normal force as in the lubricated fixed normal force category.

Friction forces appeared to reach a higher point for tests starting with a normal force of 1471.5 N.

No correlation was observed for lower starting normal force tests.

Velocity did not appear to show any distinct effect.
The friction force increased moderately but steadily throughout the tests in proportion to the normal force.

Unlike the dry tests, a jump in friction was observed at either the beginning or in the middle of the experiments. This type of discontinuity was observed throughout the lubricated test group.

The friction coefficients were much larger than the dry categories and sometimes fluctuated significantly.

Friction coefficients, as high as 0.25, were measured.

Most experiments had a low friction coefficient at high pressures and exhibited a peak friction coefficient in the lower pressure ranges.

Tests 2, 38 and 39 showed inconsistencies.
5. Discussion

In this section, a design of experiments analysis is presented and the results presented in section 4.2 are discussed following the same sets of organizational categories as presented in that section. The significance of the results as well as any ambiguities are discussed. Attempts at explaining the reasons behind the trends and discrepancies in the data will also be presented. A discussion on the similarities and differences in between this study and past research is then given.

5.1. Design of Experiments

A design of experiments analysis was done with different calculated responses in order to define the interactions in between the four factors studied (see section 3.1 for a discussion of the factors studied) in this research. Six responses were calculated. The first two responses were the maximum friction coefficient achieved and the contact area at which it was reached for each experiment. The second set of responses was an average of the friction coefficient in the ‘unsteady state’ of the contact area curve (when the area was increasing) with the average area during this period of time for every test. Finally, the third set of responses was the average of the friction coefficient in the ‘steady state’ of the contact area curve (when the contact area was no longer increasing) with the average area during this period of time for each experiment.

It is important to note that the tests throughout the research were not fully randomized, as changing velocities in between experiments was quite a complex and time consuming
practice. The results of the design of experiments will be discussed next and are separated in between sets of responses (friction coefficient with its area).

5.1.1. **Maximum Friction Coefficient and its Contact Area**

There are 3 assumptions about the probability distribution of the output when the Analysis of variance (ANOVA) model is used:

- The independence of cases
- The normality
- The equality

The Residuals vs. Run graph showed that the independence of cases assumption was maintained for both the friction coefficient and its contact area as shown in Figure 125 and Figure 126. This graph displayed whether or not the run number had an impact on the outputs. This assumption of the model simplified the statistical analysis conducted. This confirmed that the outcome was independent of the run sequence. However, as the experiments were not always done in that run sequence, this pattern of behavior may vary.

The Normal Plot of Residuals graph showed that the normality assumption was maintained for both the friction coefficient and its contact area as shown in Figure 127 and Figure 128. As seen in the plot, the residuals followed an approximate linear trend. This supported the assumption that the data followed a normal distribution. Two runs
seemed to diverge in the maximum friction coefficient’s figure. Those two runs, Tests 11 and 38, were identified in the section 4.2 as being atypical. Similarly, two runs deviated from the typical response in the maximum friction coefficients vs. contact area data. These two runs, Tests 2 and 40, were also identified in section 4.2 as being out of place. This divergence therefore appeared normal.

![Figure 125](image1) ![Figure 126](image2)

**Figure 125:** Graph of Residuals vs. Run for the Maximum Friction Coefficient.  
**Figure 126:** Graph of Residuals vs. Run for the Maximum Friction Coefficient Contact Area.

The Residuals vs. Predicted graph demonstrated that the equality assumption was maintained as displayed in Figure 129 and Figure 130. The equality assumption meant that the variance of the data in groups should be the same. One outlier was observed in the maximum friction coefficient contact area plot given in Figure 130. That run, Test 28, was not identified in section 4.2 as being out of place. The divergence is probably due to the fact that the maximum friction coefficient contact area is larger than most tests in this
research. The variance could therefore be assumed to be constant. This assumption is supported by data shown in Figure 129 and Figure 130.

**Figure 127**: Graph of Normal Plot of Residuals for the Maximum Friction Coefficient.

**Figure 128**: Graph of Normal Plot of Residuals for the Maximum Friction Coefficient Contact Area.

**Figure 129**: Graph of Residuals vs. Predicted for the Maximum Friction Coefficient.

**Figure 130**: Graph of Residuals vs. Predicted for the Maximum Friction Coefficient Contact Area.
The Analysis of Variance table obtained using the Design Expert software for both the maximum friction coefficient and its contact area are shown in Table 17 and Table 18.

Table 17: Analysis of Variance Table resulting from Design-Expert software for the Maximum Friction Coefficient.

<table>
<thead>
<tr>
<th>Source</th>
<th>Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>Value</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>1.107E-003</td>
<td>24</td>
<td>4.853E-003</td>
<td>5.02</td>
<td>0.0002</td>
<td>significant</td>
</tr>
<tr>
<td>A-Initial We</td>
<td>1.619E-003</td>
<td>1</td>
<td>0.10</td>
<td>107.24</td>
<td>&lt; 0.0001</td>
<td></td>
</tr>
<tr>
<td>B-Lubricant</td>
<td>2.336E-003</td>
<td>2</td>
<td>1.366E-003</td>
<td>1.41</td>
<td>0.2665</td>
<td></td>
</tr>
<tr>
<td>C-Pressure</td>
<td>4.995E-004</td>
<td>3</td>
<td>1.665E-004</td>
<td>0.51</td>
<td>0.6771</td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>2.548E-003</td>
<td>3</td>
<td>7.514E-004</td>
<td>0.77</td>
<td>0.5217</td>
<td></td>
</tr>
<tr>
<td>AD</td>
<td>4.692E-004</td>
<td>6</td>
<td>8.178E-004</td>
<td>0.80</td>
<td>0.5776</td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td>6.159E-005</td>
<td>1</td>
<td>6.159E-005</td>
<td>0.063</td>
<td>0.8036</td>
<td></td>
</tr>
<tr>
<td>BD</td>
<td>6.990E-005</td>
<td>2</td>
<td>4.348E-005</td>
<td>0.045</td>
<td>0.9564</td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>1.505E-003</td>
<td>2</td>
<td>7.524E-004</td>
<td>0.77</td>
<td>0.4714</td>
<td></td>
</tr>
<tr>
<td>Residual</td>
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<td>22</td>
<td>9.725E-004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>0.14</td>
<td>46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 18: Analysis of Variance Table resulting from Design-Expert software for the Maximum Friction Coefficient Contact Area.

<table>
<thead>
<tr>
<th>Source</th>
<th>Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>Value</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>1.221E-004</td>
<td>7</td>
<td>1.745E-005</td>
<td>2.40</td>
<td>0.0383</td>
<td>significant</td>
</tr>
<tr>
<td>A-Initial We</td>
<td>2.412E-005</td>
<td>3</td>
<td>8.039E-006</td>
<td>1.11</td>
<td>0.3582</td>
<td></td>
</tr>
<tr>
<td>B-Lubricant</td>
<td>3.362E-005</td>
<td>1</td>
<td>3.362E-005</td>
<td>4.63</td>
<td>0.0370</td>
<td></td>
</tr>
<tr>
<td>C-Pressure</td>
<td>1.152E-006</td>
<td>1</td>
<td>1.152E-006</td>
<td>0.16</td>
<td>0.6927</td>
<td></td>
</tr>
<tr>
<td>D-Velocity</td>
<td>6.191E-005</td>
<td>2</td>
<td>3.096E-005</td>
<td>4.26</td>
<td>0.0212</td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>2.035E-004</td>
<td>39</td>
<td>7.260E-006</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>4.055E-004</td>
<td>46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The p-value, in the Analysis of Variances (ANOVA) (see Table 17), showed that for the maximum friction coefficient, the lubrication (p-value: <0.0001, significant) influenced the response (maximum friction coefficient) much more significantly than the other did factors (p-value: >0.2103, not significant). This statement was supported by the Half-Normal plot as shown in Figure 131. The p-value, in the Analysis of Variances (ANOVA) (see Table 18), showed that, for the maximum friction coefficient contact area, the lubrication (p-value: 0.0378, significant) as well as the velocity (p-value: 0.0212, significant) influenced the response (maximum friction coefficient area) much more significantly than the other factors did (p-value: >0.3582, not significant). This statement is supported by the Half-Normal plot shown in Figure 132. The large influence of the lubrication, shown in the design of experiments, on the maximum friction coefficient is discussed further in section 5.3.

Figure 131: Half-Normal Plot of Maximum Friction Coefficient.

Figure 132: Half-Normal Plot of Maximum Friction Coefficient Contact Area.
No clear interaction effect in between the initial weight, the change in weight or the velocity and the lubrication on the maximum friction coefficient as determined from the Analysis of Variance were observed in the interaction graphs. Two other Designs of Experiment analyses were done separating the lubricated from the non-lubricated tests. These results showed that the influence of the other factors, when non-lubricated or lubricated, was not shadowed by this large increase in friction coefficient. In fact, none of the three remaining factors seemed to have a significant effect on the maximum friction coefficient as for the design of experiments incorporating all results. The interaction effects in the factorial model for the maximum friction coefficient contact area were not considered as their effects were minimal.

Figure 133, shows consistent trends in maximum friction coefficient contact areas for the velocity and the initial weight. This was observed in all interaction graphs for the maximum friction coefficient contact area. Higher contact areas for maximum friction coefficients were seen at higher velocities and diminished when the velocity decreased. However, the smallest contact area was seen at the midrange velocity and not the smallest velocity. Higher contact areas for maximum friction coefficients were seen at higher starting weights and diminished when the starting normal force decreased. The effect of the velocity was also observed from the design of experiments excluding non-lubricated tests. A similar effect was observed as shown in Figure 134. No studies were done on the behavior of the contact area with different factors.
Figure 133: Interaction Graph of All Factors for the Maximum Friction Coefficient Contact Area.

Figure 134: Effect of Velocity on Maximum Friction Coefficient Contact Area from Lubricated Design of Experiments.
5.1.2. ‘Unsteady State’ Friction Coefficient and its Contact Area

The independence of cases assumption was maintained observing the Residuals vs. Run graph for both the ‘unsteady state’ friction coefficient and its contact area as shown in Figure 135 and Figure 136 below. The normality assumption was maintained as shown from the Normal Plot of Residuals in Figure 137 and Figure 138.

As seen in these plots, the residuals followed an approximate linear trend. One run seemed to diverge in the ‘unsteady state’ friction coefficients figure. That run, Test 38, was mentioned in the results as being out of place and was also out of place in the last section. No runs seemed to diverge in the ‘unsteady state’ friction coefficients contact area figure. This divergence therefore appeared normal. The equality assumption was maintained observing the Residuals vs. Predicted graph shown in Figure 139 and Figure
140. One run seemed to diverge in the ‘unsteady state’ friction coefficients figure. That run, Test 38, was mentioned in section 4.2 as being outside the exhibited trends. No run seemed to diverge in the ‘unsteady state’ friction coefficients contact area figure.

Figure 137: Graph of Normal Plot of Residuals for the ‘Unsteady State’ Friction Coefficient.

Figure 138: Graph of Normal Plot of Residuals for the ‘Unsteady State’ Friction Coefficient Contact Area.

Figure 139: Graph of Residuals vs. Predicted for the ‘Unsteady State’ Friction Coefficient.

Figure 140: Graph of Residuals vs. Predicted for the ‘Unsteady State’ Friction Coefficient Contact Area.
The Analysis of Variance table obtained using the Design Expert software for both the ‘unsteady state’ friction coefficient and its contact area are shown in Table 19 and Table 20 below. The p-value from the Analysis of Variances (ANOVA) (Table 19) showed that, for the ‘unsteady state’ friction coefficient, the lubrication (p-value: <0.0001, significant) influenced the response (‘unsteady state’ friction coefficient) much more significantly than the other factors did (p-value: >0.1196, not significant). The interaction in between starting and changing weight (p-value: 0.0509, significant) seemed to have a larger influence as well but this statement was not supported by the Half-Normal plot as shown in Figure 141. The p-value from the Analysis of Variances (ANOVA) in Table 20 demonstrated that, for the ‘unsteady state’ friction coefficient contact area, the lubrication (p-value: <0.0001, significant) as well as the initial weight (p-value: 0.0932, significant) influenced the response (‘unsteady state’ friction coefficient area) much more significantly than the other factors did (p-value: >0.1460, not significant). The Half-Normal plot, as shown in Figure 142, showed however that the initial weight did not yield a significant effect.
Table 19: Analysis of Variance Table resulting from Design-Expert software for the ‘Unsteady State’ Friction Coefficient.

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Value</th>
<th>Prob &gt; F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.065</td>
<td>24</td>
<td>2.723E-093</td>
<td>7.05</td>
<td>&lt; 0.0001</td>
<td>significant</td>
<td></td>
</tr>
<tr>
<td>A=Initial We</td>
<td>3.660E-004</td>
<td>3</td>
<td>1.227E-004</td>
<td>0.32</td>
<td>0.6126</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B=Lubricant</td>
<td>0.056</td>
<td>1</td>
<td>0.056</td>
<td>144.41</td>
<td>&lt; 0.0001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C=Pressure</td>
<td>5.44E-005</td>
<td>1</td>
<td>5.44E-005</td>
<td>0.14</td>
<td>0.7198</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D=Velocity</td>
<td>1.81E+003</td>
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<td>9.051E-004</td>
<td>2.34</td>
<td>0.1196</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>1.167E-003</td>
<td>3</td>
<td>3.892E-004</td>
<td>1.01</td>
<td>0.4083</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AC</td>
<td>3.512E-003</td>
<td>3</td>
<td>1.171E-003</td>
<td>3.03</td>
<td>0.0509</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AD</td>
<td>7.999E-004</td>
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<td>1.333E-004</td>
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<td></td>
</tr>
<tr>
<td>BC</td>
<td>3.958E-005</td>
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<td>3.958E-005</td>
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<td>0.7503</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BD</td>
<td>7.676E-004</td>
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<td>3.833E-004</td>
<td>0.99</td>
<td>0.3864</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>1.237E-003</td>
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<td>6.167E-004</td>
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<td>0.2243</td>
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<tr>
<td>Residual</td>
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<td>3.864E-004</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
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<td>46</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 20: Analysis of Variance Table resulting from Design-Expert software for the ‘Unsteady State’ Friction Coefficient Contact Area.

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Value</th>
<th>Prob &gt; F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>4.386E-004</td>
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<td>1.019E-005</td>
<td>4.33</td>
<td>0.0005</td>
<td>significant</td>
<td></td>
</tr>
<tr>
<td>A=Initial We</td>
<td>3.953E-005</td>
<td>3</td>
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<td>2.42</td>
<td>0.0932</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B=Lubricant</td>
<td>3.024E-004</td>
<td>1</td>
<td>3.024E-004</td>
<td>7.94</td>
<td>&lt; 0.0001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C=Pressure</td>
<td>1.185E-006</td>
<td>1</td>
<td>1.185E-006</td>
<td>0.28</td>
<td>0.6006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D=Velocity</td>
<td>1.765E-005</td>
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<td>8.828E-005</td>
<td>2.10</td>
<td>0.1460</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>2.047E-005</td>
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<td>6.822E-005</td>
<td>1.62</td>
<td>0.2120</td>
<td></td>
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</tr>
<tr>
<td>AC</td>
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<td>0.2834</td>
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<td>9.796E-007</td>
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<td></td>
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<td>5.107E-005</td>
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<td>0.3159</td>
<td></td>
<td></td>
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<tr>
<td>CD</td>
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<td>2.237E-005</td>
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<td></td>
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<tr>
<td>Residual</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>5.291E-004</td>
<td>46</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
As in the previous section, a large influence of the lubrication was observed in the design of experiments as well as the results (section 4.2.2) and will be discussed in section 5.3. Two other designs of experiment analyses were done separating the lubricated from the non-lubricated tests as mentioned in the previous section. This showed that the influence of the other factors, when non-lubricated or lubricated, did not result in a large increase in friction coefficient. The three remaining factors, in fact, seemed to have no effect on the ‘unsteady state’ friction coefficient or its contact area as for the design of experiments incorporating all results. The interaction graphs, for the ‘unsteady state’ friction coefficient, showed no clear interaction effect in between the initial weight, the change in weight or the velocity and the lubrication on the ‘unsteady state’ friction coefficient as determined from the Analysis of Variance. The interaction graphs for the ‘unsteady state’ friction coefficient contact area also showed no clear interaction effects as in the Analysis of Variance.
5.1.3. ‘Steady State’ Friction Coefficient and its Contact Area

The independence of cases assumption was maintained observing the Residuals vs. Run graph for both the ‘steady state’ friction coefficient and its contact area as shown in Figure 143 and Figure 144 below. The normality assumption was maintained observing the Normal Plot of Residuals graph shown in Figure 145 and Figure 146. As seen in the plot, the residuals followed an approximate linear trend.

The equality assumption was maintained observing the Residuals vs. Predicted graph shown in Figure 147 and Figure 148. No runs seemed to diverge in the ‘steady state’ friction coefficients figure. No runs also seemed to diverge in the ‘steady state’ friction coefficients contact area figure.
Figure 145: Graph of Normal Plot of Residuals for the ‘Steady State’ Friction Coefficient.

Figure 146: Graph of Normal Plot of Residuals for the ‘Steady State’ Friction Coefficient Contact Area.

Figure 147: Graph of Residuals vs. Predicted for the ‘Steady State’ Friction Coefficient.

Figure 148: Graph of Residuals vs. Predicted for the ‘Steady State’ Friction Coefficient Contact Area.
The Analysis of Variance table obtained using the Design Expert software for both the ‘steady state’ friction coefficient and its contact area are shown in Table 21 and Table 22. It is important to note that many tests needed to be ignored in this analysis as many experiments did not reach a ‘steady state’. The p-value from the Analysis of Variances (ANOVA) (in Table 21) showed that, for the ‘steady state’ friction coefficient, the lubrication (p-value: 0.0228, significant) influenced the response (‘steady state’ friction coefficient) much more significantly than the other factor did (p-value: 0.1740, not significant). The Half-Normal plot as shown in Figure 149 supported this observation.

The p-value from the Analysis of Variances (ANOVA) in Table 22 demonstrated that, for the ‘steady state’ friction coefficient contact area, the lubrication (p-value: <0.0001, significant) as well as the initial weight (p-value: 0.0711, significant) influenced the response (‘steady state’ friction coefficient area) much more significantly than the other factors did (p-value: >0.2301, not significant). The Half-Normal plot, shown in Figure 150, showed that the initial weight did not yield a significant effect.
Table 21: Analysis of Variance Table resulting from Design-Expert software for the ‘Steady State’ Friction Coefficient.

**ANOVA for selected factorial model**

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F Value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.058</td>
<td>4</td>
<td>0.015</td>
<td>3.25</td>
<td>0.0236</td>
</tr>
<tr>
<td>A-Initial We</td>
<td>0.033</td>
<td>3</td>
<td>0.011</td>
<td>2.43</td>
<td>0.0831</td>
</tr>
<tr>
<td>B-Lubricant</td>
<td>0.024</td>
<td>1</td>
<td>0.024</td>
<td>5.44</td>
<td>0.0260</td>
</tr>
<tr>
<td>Residual</td>
<td>0.15</td>
<td>33</td>
<td>4.469E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>0.21</td>
<td>37</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 22: Analysis of Variance Table resulting from Design-Expert software for the ‘Steady State’ Friction Coefficient Contact Area.

**ANOVA for selected factorial model**

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F Value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>8.475E-004</td>
<td>24</td>
<td>3.531E-005</td>
<td>3.58</td>
<td>0.0101</td>
</tr>
<tr>
<td>A-Initial We</td>
<td>8.781E-005</td>
<td>3</td>
<td>2.927E-005</td>
<td>2.97</td>
<td>0.0711</td>
</tr>
<tr>
<td>B-Lubricant</td>
<td>4.698E-004</td>
<td>1</td>
<td>4.698E-004</td>
<td>47.64</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>C-Pressure</td>
<td>6.506E-009</td>
<td>1</td>
<td>6.506E-009</td>
<td>8.625E-004</td>
<td>0.9770</td>
</tr>
<tr>
<td>D-Velocity</td>
<td>2.039E-005</td>
<td>2</td>
<td>1.019E-005</td>
<td>1.03</td>
<td>0.5837</td>
</tr>
<tr>
<td>AB</td>
<td>3.021E-005</td>
<td>3</td>
<td>1.007E-005</td>
<td>1.02</td>
<td>0.4152</td>
</tr>
<tr>
<td>AC</td>
<td>1.047E-005</td>
<td>3</td>
<td>3.491E-006</td>
<td>0.35</td>
<td>0.7870</td>
</tr>
<tr>
<td>AD</td>
<td>5.991E-005</td>
<td>6</td>
<td>9.996E-006</td>
<td>1.01</td>
<td>0.4594</td>
</tr>
<tr>
<td>BC</td>
<td>1.097E-006</td>
<td>1</td>
<td>1.097E-006</td>
<td>0.11</td>
<td>0.7440</td>
</tr>
<tr>
<td>BD</td>
<td>3.251E-005</td>
<td>2</td>
<td>1.626E-005</td>
<td>1.65</td>
<td>0.2301</td>
</tr>
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<td>CD</td>
<td>1.235E-005</td>
<td>2</td>
<td>6.174E-006</td>
<td>0.63</td>
<td>0.5501</td>
</tr>
<tr>
<td>Residual</td>
<td>1.282E-004</td>
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<td>9.861E-006</td>
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<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>0.757E-004</td>
<td>37</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
A non-lubricated and lubricated design of experiments analysis was also done to separate these categories of tests. These analyses showed that the influence of the other factors was not associated with a large increase in friction coefficient. In fact, as for the design of experiments incorporating all results, none of the three remaining factors, excluding the lubrication, seemed to have an effect on the 'steady state' friction coefficient. No interactions were kept in the 'steady state' friction coefficient model, as shown in Table 21 so as to have a significant model. The interaction graphs for the 'steady state' friction coefficient showed no clear interaction effect in between the initial weight, the change in weight or the velocity and the lubrication on the 'steady state' friction coefficient contact area as determined from the Analysis of Variance. However, the lubricated design of experiments found that the 'steady state' friction coefficient contact area was influenced by the initial weight of the experiment (see Figure 151). The effect seems unusual as the contact areas, seen in 'steady state' friction coefficient averages, are larger at 50 kg and
100 kg. Tests 4, 8, 32, 35 and 39 which showed inconsistencies in the results were not flagged by the design of experiments analysis.

Figure 151: Effect of Initial Weight on ‘Steady’ Friction Coefficient Contact Area from Lubricated Design of Experiments.

5.2. Non-Lubricated Experiments

5.2.1. Non-Lubricated Experiments with Fixed Normal Force

The results obtained in this category of experiments clearly show that the friction coefficient, in dry environments when the normal force is constant, is not much influenced by changes in area or pressure. As the normal force did not increase in these tests, the area was inversely proportional to the pressure.
Higher friction coefficients at lower velocities were observed in this category which followed the results found in previous research as discussed in section 1.4.2 and shown in Figure 18 to Figure 34. These figures showed a decrease in friction coefficients at lower velocities. The values of the friction coefficient in this category, also agreed with previous studies performed. Most studies found that at velocities similar to the ones used in this research (0.5 m/s – 0.8 m/s), friction coefficients of about 0.03 to 0.06 were measured. Tusima (1977) considered much lower speeds, and their results did not seem to match the results of this group of tests. Most of the other researchers had higher velocity ranges, which may account for the similarity in the results.

The friction coefficient of ice did not behave as expected when considering the variation in area. This category of tests showed very little variation in friction coefficients as the
area increased. This was not the case in past research as discussed in section 1.4.4. This may be explained by the fact that this research incorporated a dynamic change in area through time in experiments due to the shape of the ice sample instead of different fixed contact areas per tests.

This category seemed to reveal a slight increase in friction coefficient with an increase in normal force. This increase also contradicted previous studies as expected behaviors were a decrease in friction coefficients with an increase of normal force (see section 1.4.3). Some dissimilarity in between the literature and the results were observed as mentioned previously. In addition to the reasons stated, experimental conditions in past studies did not always quite match the uncontrolled factors which included the temperature, humidity, and plate temperature. This may also contribute to some of the differences.

Tests number 11 and 35 showed inconsistencies with the results of this research as shown in Figure 154 to Figure 157. The friction coefficients observed in Tests 11 and 35, contrary to the other numbered tests in this category, were much higher and reached 0.1 to 0.15. Both experiments were done at -10°C temperatures, as the rest of the category. However both were the first tests done on the day they were completed. A warmer and less icy plate may account for the discrepancies as warmer temperatures, closer to the freezing point, show higher friction. It is however important to note that some other tests in this category were also done first on the day they were completed and did not show a higher friction coefficient.
Figure 154: Friction Coefficient vs. Contact Area of Test 11.

Figure 155: Friction Coefficient vs. Pressure of Test 11.

Figure 156: Friction Coefficient vs. Contact Area of Test 35.

Figure 157: Friction Coefficient vs. Pressure of Test 35.
5.2.2. Non-Lubricated Experiments with Increasing Normal Force

The results obtained in this category of experiments clearly showed that the friction coefficient, in dry environments when the normal force was increasing, is not much influenced by the area change.

![Figure 158: Friction Coefficient vs. Contact Area of Test 33.](image)

![Figure 159: Friction Coefficient vs. Pressure of Test 33.](image)

Higher friction coefficients were observed at higher velocities in this category which did not follow the results found in previous research as discussed in section 1.4.2. The values of the friction coefficient in this category, compared to previous studies performed, also matched. Most studies found that at velocities similar to the ones used in this research (0.5 m/s – 0.8 m/s), friction coefficients of about 0.03 to 0.06 were measured. Results from Tusima (1977) did not match the results of this group of tests. His research however also showed a decrease in friction coefficient with declining speeds as in most other previous studies. Most of the other researches had much higher velocity ranges which
may account for the similarity in the results. As the non-lubricated experiments with fixed normal force category followed the same trends in velocity changes and area changes for the most part, perhaps the varying normal force, combined with different velocities, influences differently the friction coefficient of ice.

The friction coefficient of ice also did not behave as expected when considering the variation in area. This category of tests showed very little variation in friction coefficients as the area increased. It occasionally showed an increase in friction coefficient with an increase in area as shown in Figure 160. This was not the case in past research as mentioned in section 1.4.4. Expected trends were a decrease in friction coefficients with an increase of area. This may be explained by the fact that this research incorporated a change in area through time in experiments due to the shape of the ice sample instead of different fixed contact areas per tests. This could also be related to the increase in normal force as the area increased. As the normal force did not remain stable, increasing forces were applied to increasing contact areas in time making it hard to analyze the influence of the contact area on the friction coefficient independently.

This non-lubricated category with increasing normal force seemed to reveal a slight increase in friction coefficient with an increase in normal force. As mentioned earlier, the contact area changed as the normal force varied making it hard to assess the influence of the normal force on the friction coefficient separately. However, the area, for most of the tests, reached a ‘steady state’ early, and ceased increasing in diameter while the normal force kept growing. All the tests showing an increase of the friction coefficient seemed to
increase only when the area was increasing in this category. Looking at where the area ceased increasing (maximum area of the test) on the figures, no large variation in friction coefficients (ranges), whether it was an increase or a decrease, was observed. These also contradicted previous studies as expected behaviors were a decrease in friction coefficient with an increase of normal force (see section 1.4.3).

Some dissimilarity in between the literature and the results were observed as mentioned previously. One possible explanation for the discrepancies is that the past studies did not always have the same match uncontrolled temperature, humidity, plate temperature which may contribute to some of the differences.

Tests number 8 and 32 showed inconsistencies with the results of this research as shown in Figure 161 and Figure 162. The friction coefficients, observed in Tests 8 and 32, were not different from the other results in this category. However, Test 8 seemed to be
influenced more by the normal force as discussed previously and Test 32 showed a larger unsteady variation of friction coefficient with the increase of contact area. Both experiments were done at -10 °C temperatures, as the rest of the category. However both were the second tests done on the day they were completed. A warmer and less icy plate may account for the discrepancies. It is however important to note that some other tests in this category were also done second or first on the day they were completed and did not show inconsistencies. Both tests also reached smaller maximum contact areas than most in this category. Some other tests however, had similar maximum contact areas and did not present with these irregularities.

5.2.3. Fixed vs. Increasing Normal Force

When comparing results for both the fixed and increasing normal force categories, we see approximately the same range of friction coefficients for both types of tests. This implies
that even when much higher normal forces are achieved the friction coefficient does not go over 0.05. Higher normal forces also do not seem to give particularly lower friction coefficients in this research compared to past studies which is supported by the design of experiments analysis. It is important to note that some of the tests with increasing normal forces were much longer in duration. This is due to the fact that at lower starting normal forces, more time was required to reach the maximum normal force.

5.3. Lubricated Experiments

5.3.1. Lubricated Experiments with Fixed Normal Force
The results obtained in this category of experiments clearly showed that the friction coefficient, in this type of wet environment when the normal force is constant, is very much influenced by some factor. As the normal force did not increase in these tests, the pressure was inversely proportional to the contact area.

No clear variations between different velocities were observed in this category. Looking back at prior research done with lubrication, no study was completed to investigate the variation in friction coefficients at different velocities, areas or normal forces. It was therefore hard to tell if these results were unusual. In fact, the only study done with lubrication was done with sea water and showed no clear effect of the water on the friction coefficient as discussed in section 1.4.10.

The range of friction coefficients for these types of tests was much larger than in any dry tests performed (except in a few cases). Looking at the video recordings of the tests, a
film of water seemed to collect around the contact area of the ice samples (see Figure 163). It was as if the water, poured on the plate, reached the sample to then be driven to the edges of the ice contact area in order to continue its rotational path determined by the plate. The extra force needed to push the water to the edges of the contact area may have accounted for the increase in the measured friction force and thus the apparent friction coefficients throughout the test. Following this reasoning, as the water poured on the plate ran out eventually, even if sometimes the tests continued, a lowering of friction coefficients would be expected, usually at maximum contact areas as they were reached faster in this category of tests. This did in fact happen for most tests. The friction coefficients in turn stayed low until the water started being dispensed for the tests where the water was not poured right at the start. Taking a closer look at the friction coefficient variation in time beside the area, the friction coefficient seemed to drop usually around the same time for different tests, possibly signifying the running out of water. This implies that the water, sometimes bypassing the ice sample may have had an influence on the friction coefficient.
Taking a closer look at the different types of results presented in section 4.2.2.1, shown in Figure 164 and Figure 165, the appearance of the curve also seemed dependent of the area curve. The friction coefficient appeared to be linked to the ‘unsteady and steady states’ of the area curve. The friction coefficient in Figure 164 reached a maximum value at smaller contact area. This was to be expected as the range of contact areas in these was smaller. However, the friction coefficients in these tests seemed to increase drastically at the beginning of the ‘unsteady’ phase of the area curve and settled back down to a range closer to the ones observed in dry tests when the ‘steady state’ of the area curve was reached. Friction coefficients such as the one in Figure 165 behaved the same. A decrease, when the ‘steady state’ of the area curve was reached, was always seen (not seen in Figure 165 as the ‘steady state’ is reached at a higher area, not displayed in the figure) unless no ‘steady state’, in the area curve, was achieved as shown in Figure 166.

Figure 163: Water Film Collected Around Contact Area of Ice Sample.
and Figure 167. These results may show that a change in the area or an ‘unsteady state’, throughout a friction experiment, increases the friction coefficient drastically. However as the ‘steady state’ was usually attained around the time the water ran out, it is hard to say. These results are therefore a bit misleading and do not quite show if the lubrication does influence the friction coefficient in this study.

Figure 164: Friction Coefficient vs. Contact Area of Test 5.

Figure 165: Friction Coefficient vs. Contact Area of Test 42.
5.3.2. Lubricated Experiments with Increasing Normal Force

The results obtained in this category of experiments clearly showed that the friction coefficient, in this type of wet environment when the normal force is increasing, is also very much influenced by some factor. No clear variations between different velocities and dissimilar normal forces were observed in this category. Looking back at prior research, with lubrication, as mentioned in the previous section, no study was completed to investigate the variation in friction coefficients at different velocities, areas or normal forces (see section 1.4.10). It was therefore difficult to tell if these results were unusual.

The range of friction coefficients for these types of tests was much larger than in the dry tests performed (except for the abnormal ones) as in the previous section. The same phenomenon, with the water film around the contact area, happened during these experiments as well. Taking a closer look at the friction coefficient variation in time...
beside the area, the friction coefficient seemed to drop usually around the same time for
different tests maybe signifying the running out of water as in the other lubricated
category. This seems to imply that the water, sometimes bypassing the ice sample may
have had a similar influence on the friction coefficient as in the previous cases with the
constant normal force.

Taking a closer look at the results presented in section 4.2.2.2, shown in Figure 168, the
appearance of the curve, as in the previous group also seemed dependent of the area
curve. The friction coefficient appeared to be linked to the ‘unsteady and steady states’ of
the area. For the most part, friction coefficient presented like Figure 168. The friction
coefficients in these tests seemed to increase drastically at the beginning of the ‘unsteady’
phase of the area curve and settled back down to a range closer to the ones observed in
dry tests when the ‘steady state’ of the area curve was reached. A decrease, when the
‘steady state’ of the area curve was reached, was always seen unless no ‘steady state’ was
achieved as shown in Figure 169 and Figure 170. These results may show that a change in
the area or an ‘unsteady state’, throughout a friction experiment, increases the friction
coefficient drastically. However as the ‘steady state’ was usually attained around the time
the water ran out, it is hard to say. Tests number 2 and 38 showed inconsistencies but for
no apparent reason.
Figure 168: Friction Coefficient vs. Contact Area of Test 26.

Figure 169: Change of Area Through Time of Test 39.

Figure 170: Friction Coefficient vs. Contact Area of Test 39.
5.3.3. Fixed vs. Increasing Normal Force

When comparing results for both the fixed and increasing normal force categories, we see approximately the same range of friction coefficients for both types of tests. This implies that even when much higher normal forces are achieved the friction coefficient does not go over 0.2 when lubricated. However, this high friction coefficient seen may be due to the film of water that collected around the sample throughout the test. In that case, friction coefficients no larger than the ones seen in the dry tests should have been observed following the findings discussed in section 1.4.10. This could not be clarified by the design of experiments analysis in section 5.1. As ice interacting with ice structures will not be influenced by lubrication in this way (water surrounds the ice), more tests with an even layer of water on the metal surface as well as tests where the water does not run out should be done to clear up these findings and clearly observe how the normal force, area and velocity influences the ice friction. It is important to note that some of the tests with increasing normal forces were much longer in duration as in the dry categories. This is due to the fact that at lower starting normal forces more time was required to reach the maximum normal force. Experimental error, discussed next, may have accounted for the discrepancies in these two tests.

5.4. Discussion of Experimental Error

The reasons for inconsistencies between results in identical categories, as well as when compared to past studies, were quite hard to pinpoint. A discussion of possible experimental error is discussed below to determine if error may be a contributing factor to these observations.
Increasing normal force tests did not have a steady, or automated, increase in normal force as the steel pebbles were transferred from a tub to the experimental bucket with a pitcher by hand. This created normal force profiles which looked more like steps that were not always even. This may have increased the friction force.

For lubricated tests, as mentioned in the last section, water ran out eventually. The rate of water poured on the surface of the plate was also hard to control as the water was hand poured using a pitcher. Both these factors may have contributed to some scatter or irregularity in results.

In addition to these factors, environmental factors due to the variation in temperature were encountered. Tests done first in the morning or in the afternoon were done on a warmer plate as it was brought into the cold room from the laboratory. This was necessary as at the end of the day, plates were quite icy and needed to be defrosted. Tests done later in the day however were performed on the same plate as the deformed ice or poured water froze and made a ring on the plate on which the ice sample grinded. A bumpier ice ring was sometimes encountered when the ice sample froze to the plate once the test was completed before being removed. Scraping the ice was done in between tests to remove excess ice however this method was not perfect. This potential source of variation in experiment could however be relevant to ice structures as they become icy at times. However no significant differences were observed between first tests and later tests done under the same nominal conditions. This potential error could have been avoided by
removing the plate in between each experiment and letting it defrost, to then relay it on the turntable and wait for it to get cold.

The experimental error encountered in the experiments may explain the scatter and some inconsistencies in the data at times. It is however not a significant factor in the observed results as it does not change the fundamental outcome of the research.
6. Conclusion

To conclude, the main purpose of this work, to measure friction coefficients under conditions when the ice was crushing and abrading under the sorts of pressures that might be experienced during real life ice-structure interactions, was achieved. Pressures up to 1.4 MPa were measured in the results and the ice was observed in all tests to be crushing and abrading in a way that increased the contact area for at least some part of each test.

However, the results obtained in the research did not make it possible to develop a more comprehensive expression for the ice friction coefficient that captured some of the previously observed variations associated with changing pressure, contact area and velocity. The results of this study did not agree with past research done on the subject. Lubrication was found to increase the friction coefficient significantly contrary to past findings. This increase was believed to be due to an extra force produced by the water movement around the ice. Dry friction coefficient ranges were observed after a similar time, in each lubricated test, indicating that water stopped being poured on the steel plate. This meant that lubrication did not influence the friction coefficient of ice. The velocity was found to increase friction coefficients in some cases and decrease it in some others. The increase in normal forces seemed to increase the friction coefficient contrary to the findings of past studies.

The design of experiments analysis done with the results indicated that the factors, other than lubrication, did not have significant effects on the measured friction coefficients. No
effects of interacting factors were found to influence the friction coefficient of ice in the
design of experiments analysis completed. Additionally, no trends were observed in the
effects of the three other factors (Velocity, Changing Weight and Starting Weight) on the
friction coefficient of ice.

The results of this study indicate that under realistic crushing and abrading conditions, ice
friction coefficients are generally in the range of 0.03 to 0.05 and the measured friction
coefficients are statistically independent of pressure, contact area and velocity. Changes
in the nature of the friction coefficient were observed in individual tests between cases
where the contact area was dynamically changing and where the contact area had
achieved a ‘steady state’. However these changes did not appear to relate to any of the
other variables.

The results obtained in this study support the fact that, as recognized, ice is a very
complex material. Using conical ice samples which changed the contact areas throughout
the tests, demonstrated that trends observed in past ice friction studies may be difficult to
apply to ice interactions with ice pieces, found at sea, since these are never of perfectly
regular shape. Despite these challenges, previous ice friction studies remain relevant in
domains like winter sports where there is no deformation of the ice or snow during ice-
material interactions.

Finally, tests realized within this project, although preliminary in nature, do provide a
reference for the future design of ice structures and an insight on the direction to take for
future ice friction studies. These results indicate that friction coefficients can be assumed
to be essentially constant in terms of pressure, contact area and velocity, as the classical
friction laws indicate, for ice structure and ice interactions.

An established ice-steel friction coefficient that is known to be effectively constant across
the range of pressures covered in these experiments, which represent the pressures likely
in real life interactions, provides a validated design figure that can be used in analysis
supporting ship and offshore structure designs. Also the methodology developed for these
experiments can, with some practical improvements, be used to develop friction
coefficients for ice with other materials.

It is suggested that the experimental procedures developed for this research might be
improved based on the experiences of this study and further work done to explore the
effects of friction while ice is failing against a structure. In particular the effects of
dynamically changing pressures and areas in the contact zone would be a good subject for
a follow-on study.
Bibliography


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MC CONICA, T.H., 1950. *Sliding on Ice and Snow*. Department of the Army, Office of the Quartermaster General, Military Planning Division, Research and Development Branch, Environmental Protection Section.


Appendices

Appendix A: Experiment List

This appendix shows on which day and in which order each experiment was done. It also shows the temperature of the cold room at the time of the experiment.

Table 23: Experiment List

<table>
<thead>
<tr>
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<th>Date Completed</th>
<th>Order</th>
<th>Cold Room Temperature (°C)</th>
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</thead>
<tbody>
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<td>2(^{nd})</td>
<td>-10.3</td>
</tr>
<tr>
<td>2</td>
<td>April 23(^{rd})</td>
<td>4(^{th})</td>
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</tr>
<tr>
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<td>May 24(^{th})</td>
<td>2(^{nd})</td>
<td>-10.3</td>
</tr>
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</tr>
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</tr>
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</tr>
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<td>1(^{st})</td>
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Appendix B: Test Day Schedule

This appendix presents roughly the daily schedule followed on a test day.

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Figure 171: Test Day Schedule.
Appendix C: Experiment Graphs

This appendix presents all the relevant data, measured and calculated, displayed in graphs, for each experiment. For every test, the friction coefficient versus the area is presented on the top left corner of the page, the area and pressure with respect to time is shown on the top right corner of the page, the friction coefficient with respect to the pressure is shown on the bottom left corner of the page and finally the friction force versus the normal force is shown on the bottom right corner of the page.
Figure 172: Test 1.
Figure 173: Test 2.
Figure 174: Test 3.
Figure 175: Test 4.
Figure 176: Test 5.
Figure 177: Test 6.
Figure 178: Test 7.
Figure 179: Test 8.
Figure 180: Test 9.
Figure 181: Test 10.
Figure A2: Test 11.
Figure 183: Test 12.
Figure 184: Test 13.
Figure 185: Test 14.
Figure 186: Test 15.
Figure 187: Test 16.
Figure 188: Test 17.
Figure 190: Test 19.
Figure 191: Test 20.
Figure 192: Test 21.
Figure 193: Test 22.
Figure 194: Test 23.
Figure 195: Test 24.
Figure 196: Test 25.
Figure 197: Test 26.
Figure 198: Test 27.
Figure 199: Test 28.
Figure 200: Test 30.
Figure 201: Test 31.
Figure 202: Test 32.
Figure 203: Test 33.
Figure 204: Test 34.
Figure 203: Test 35.
Figure 206: Test 36.
Figure 207: Test 37.
Figure 208: Test 38.
Figure 209: Test 39.
Figure 210: Test 40.
Figure 211: Test 41.
Figure 213: Test 43.
Figure 215: Test 45.
Figure 216: Test 46.
Figure 217: Test 47.
Figure 218: Test 48.