# USER PAIRING AND NETWORK PERFORMANCE OPTIMIZATION IN COOPERATIVE WIRELESS NETWORK CODING

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**User Pairing and Network Performance Optimization in Cooperative Wireless Network Coding**

by

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## Abstract

In today's wireless networks, diversity is regarded as an efficient and established means to combat multipath fading. Moreover, user cooperation has emerged lately as an elegant technique to achieve spatial diversity over wireless channels, where the installation of multiple antennas on handheld, battery-powered, mobile terminals is often impractical. Recently, the application of network coding in cooperative wireless networks has gained increasing interest with its potential to further boost the network performance, such as in terms of the achievable throughput. With network coding, the relaying nodes are allowed to linearly combine packets from multiple source nodes, and then forward the combined packets for better resource utilization.

We propose mutual user pairing in amulti-user infrastructure-based network-coded cooperative wireless networkto realize network coding, in the absence of dedicated relay nodes. We propose an optimal user pairing algorithm, and tailor it to maximize the network capacity. Next, we develop heuristic pairing algorithms which approach the optimal performance at a reduced complexity.Performance analysis is conducted in terms of the average capacity per user, average outage probability per user, and user-fairness.

For energy-constrained network-coded cooperative networks, we subsequently address the problem of transmission power minimization. A joint optimization problem is formulated and solved to find the pairing which maximizes the network capacity, and minimizes the transmission power, such that certain performance constraints in terms of the average capacity per user or average outage probability per user are satisfied.

**Acknowledgements**

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Finally, I would like to thank my friends and family.

Talha Rasheed

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# List of Symbols

| | |
|---|---|
| $k$ | Relay assignment index in two-way relay channels |
| $\Theta$ | Exhaustive set containing all possible relay assignment permutations |
| $N_R$ | Number of relay nodes |
| $N_{users}$ | Number of users to be paired in an infrastructure-based network |
| $y$ | Received information symbols at destination node |
| $x$ | Transmitted source information symbols |
| $h$ | Channel coefficient integrating the effect of path-loss and Rayleigh fading |
| $n$ | Additive white Gaussian noise at the receiver |
| $N_0$ | Power spectral density of $n$ |
| $C_{sum}$ | Sum network capacity |
| $P$ | Transmit power |
| $\Gamma$ | Average signal-to-noise ratio |
| $\gamma$ | Instantaneous signal-to-noise ratio |
| $R$ | Packet code rate in case of point-to-point transmission |
| $\alpha$ | Rate allocation factor between direct transmission and network coding phase |
| $\overline{P}$ | Outage probability |
| $\Pi$ | Exhaustive set containing all possible user-pairings in network-coded network |
| $\mathcal{P}$ | User-pairing set in network-coded network |
| $\mathcal{G}$ | General, undirected weighted graph |
| $V$ | Vertices in $\mathcal{G}$ representing the users to be paired |
| $E$ | Set of edges in $\mathcal{G}$ |
| $w$ | Weights assigned to edges, $E$ |
| $\mathcal{M}$ | Matching in $\mathcal{G}$ |
| $\Phi_{out}$ | Average outage probability function |
| $\Phi_{cap}$ | Average network capacity function |
| $\varepsilon$ | Tolerance for the bisection optimization |

# List of Abbreviations

| | |
|---|---|
| MIMO | Multiple-Input Multiple-Output |
| S | Source node |
| D | Destination node |
| R | Relay node |
| MRC | Maximal Ratio Combining |
| EGC | Equal-Gain Combining |
| SC | Selection Combining |
| TDMA | Time-Division Multiple Access |
| XOR | Exclusive-OR |
| SNR | Signal-to-Noise Ratio |
| ORA | Optimal Relay Assignment |
| CSI | Channel State Information |
| SER | Symbol Error rate |
| BS | Base Station |
| BER | Bit Error Rate |
| PER | Packet Error Rate |

# Chapter 1

# Introduction

In modern wireless communication networks, there is a consistently growing demand for higher data rates, improved service quality, better coverage area, and shorter processing times. The impediments to achieving these goals are primarily the limited available channel bandwidth and the dynamic nature of the wireless channels. In addition, wireless channels are unpredictable, owing to the effects of small and large scale fading [1]. The small scale fading, usually simply termed as *fading* is often the most detrimental. In a wireless medium, multiple copies of the transmitted signal, resulting from the random scattering of the electromagnetic wave from the surrounding objects arrive at the receiver. These copies arrive at the receiver having undergone different channels, and thus arrive with different gains, phase shifts, and delays. The multiple copies interfere at the receiver and can add in a constructive or destructive fashion, which results in the amplification or the attenuation of the received signal. In case of attenuation, the signal is said to have undergone fading. This may result in the unsuccessful reception of the transmitted signal, as the receiver may not be able to distinguish the received signal from thermal noise [2]-[3].

### 1.1 Diversity in Wireless Networks

In wireless communication systems, *diversity* is regarded as an efficient and established means to combat the small scale fading. It is the technique by which multiple copies of the transmitted signal can be received over independently faded channels at the receiver and combined. In case one or more copies of the signal are affected by severe fading, the receiver can still detect the signal from the other copies. The term *diversity gain* is used to quantify the number of independently faded copies of the transmitted signal at the receiver. In practice, independent channels can be achieved primarily in three physical domains: time, frequency, and space. Diversity could also be achieved in other forms such as space-time diversity and cooperative diversity [4].

Time diversity could be achieved by transmitting the same signal multiple times, in different time slots. These time slots should be separated at least by the coherence time of the channel such that it is made sure that the channels at these time slots are independent. The drawback of time diversity is the decreased data rate and increased latency. Frequency diversity can be achieved by transmitting multiple copies of the same signal in different frequency bands. The frequency separation should be enough to guarantee channel independence. However, more spectrum is required to achieve frequency diversity. Finally, space diversity is achieved by sending and/or receiving the signal over multiple antennas, separated well enough, such that the channels are independent. Spatial diversity on the other hand neither causes increased latency, nor decreases the bandwidth efficiency, and therefore has attracted extensive interest from industry and research community in recent years. Communication systems employing multiple transmit and/or receive antennas are called Multiple-Input Multiple-Output (MIMO) systems. It is important to situate the multiple transmit and/or receive antennas sufficiently far apart (usually more than half a wavelength) such that the fading over the channels between any pair of transmit and receive antennas is statistically independent.

Although the gains associated with the use of multiple antennas in MIMO systems, such as improved channel capacity, higher throughput, better error performance, and energy efficiency, are very well established, there are certain limitations associated with their practical deployment. For instance, installing multiple antennas can often be impractical owing to the additional resource overhead, such as in terms of space for installing multiple antennas, or power. This is particularly true for mobile terminals, and these limitations on the installation of multiple antennas make the achievement of transmit diversity (from the end-user's perspective) impractical.

To overcome these drawbacks, distributed nodes in a wireless network can cooperate and intelligently share their antennas to form the so-called virtual antenna arrays. This form of user cooperation has emerged lately as an elegant technique to achieve spatial diversity over wireless channels, such as in the form of cooperative diversity, which exploits the broadcasting nature of the wireless medium [5]. The notion itself stems from the classical relaying model with intelligent antenna sharing and signal combining at the receiver to realize spatial diversity. In cooperative transmission, users can utilize their time, frequency, and/or other resources to share their antennas to form virtual antenna arrays and emulate the operation of a MIMO system. Besides retaining the benefits innate toMIMO systems, cooperative diversity brings about few more, such as

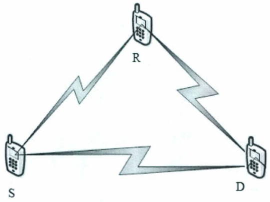



Fig.1.1.A typical cooperative wireless network.

improved energy efficiency, and has been widely shown to achieve remarkable performance gains in wireless networks [4], [6].

### 1.2 Overview of Cooperative Transmission Protocols

Fig. 1.1 shows a typical cooperative transmission network which consists of a source node (S) transmitting to a destination node (D) with the assistance of a relay node (R). The cooperative transmission consists of two phases. During the first phase, the source node transmits its message to the destination (D). Due to the broadcasting nature of the wireless medium, this message is overheard at the relay node (R). In the second phase, the relay node then forwards the overheard packet (after necessary processing) to the destination over an orthogonal channel. The destination then combines the two copies of the same packet received from the source and the relay over the two phases using any of the combining techniques such as Maximum-Ratio Combining (MRC), Equal-Gain

Combining (EGC), or Selection Combining (SC). This way, spatial diversity is achieved, as the two copies of the same packet are received over potentially uncorrelated channels.

The protocols for cooperative transmission can be broadly categorized on the basis of a number of options. These could be the relaying strategy, relaying behaviour in case of a decoding failure, and the type of coding employed in the second phase. For instance, some of the common relaying strategies are [4]:

- *Amplify-and-Forward:* In this type of relaying strategy, the relay node simply amplifies the received message from the source and forwards it to the destination. Amplify-and-Forward achieves the full diversity gain. However, the disadvantage of this protocol is that the forwarded message is a noisy version of the original message, as the noise added at the relay node is also amplified.
- *Decode-and-Forward:* With Decode-and-Forward relaying, the relay node first decodes the message received from the source, re-encodes it, and forwards the source message to the destination. Decode-and-Forward performs better in case of good source-relay channels, i.e., when the outage probability over the source-relay link is low, whereas Amplify-and-Forward performs better when the source-relay channels are of poor quality.
- *Compress-and-Forward:* In this protocol, the relaying node digitizes and compresses the message received from the source in order to decrease the redundancy. The compressed message is then re-encoded and forwarded to the destination. The destination then combines the packets from the source and relay.

Some other relaying strategies include demodulate-and-forward and quantize-and-forward. Moreover, the relaying protocols can also be *static* and *adaptive*[4]. In static protocols the relay node would always forward the source's packet, irrespective of whether it was received successfully or not. On the other hand, protocols could also be adaptive, such that the relay forwards the source's message only if it decoded the message correctly to avoid error propagation.

### 1.3Introduction to Network Coding

Network coding was first introduced in [7] for wireline networks. The central notion behind network coding is to allow the network nodes to combine the information packets from multiple sources before transmission, instead of simply relaying/forwarding them as in classical routing. In effect, the intermediate nodes in the network between the source and destination (such as relays and routers) can perform coding of the packets to achieve the multicast capacity of the network graph. This is demonstrated in Fig. 1.2 which shows a classic "butterfly" network. It is assumed that the source S wants to multicast two bits *a* and *b* to two sinks D1 and D2 simultaneously, with each link having a capacity of 1 bps. With traditional routing, each of the intermediate nodes will simply forward a copy of the packet they receive. The shaded node can forward *a* or *b*. This will make it impossible to achieve the multicast capacity of 2 bps. However, with network coding, the intermediate relay node (which is shaded) can perform coding, which is a bit-wise XOR operation, on *a* and *b* and multicast over the two outgoing links. This way, D1 receives $a$ and $a + b$, and can recover $b$ as $b = a+ (a+b)$. In the same manner, D2receives $b$ and $a + b$ and can hence recover $a$. Both D1 and D2 therefore receive at 2 bps,

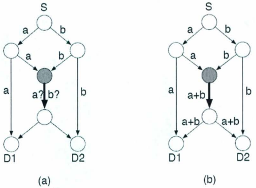


Fig.1.2. Butterfly network [1].

and thus achieve the multicast capacity.

The utility of network coding in multicast wireline networks was first demonstrated in [1]. Ever since, itis extended to various wireless applications [1]. In fact, wireless packet networks tend to be naturally suited for network coding owing to the special characteristics of the wireless links, such as theirbroadcasting nature and unreliability,for which network coding itself is a natural solution. Moreover, combined with the fact that protocol design for wireless communication is much more flexible than for the wireline case, network coding seems an ideal means to achieve remarkable performance gains in wireless networks.

Owing to the simplicity and the potential of network coding, the wireless communication research community has expended significant interest and effort to utilize it in a variety of applications in wireless networks. These range from opportunistic routing in mesh networks to distributed storage in sensor networks [8]. Network coding for wireless networks isessentially a coding strategy for the decode-and-forward

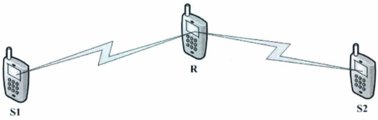



Fig.1.3. Two sources S1 and S2 communicating with the help of relay node R.

cooperative transmission protocol. With network coding, the relay node, after decoding, is allowed to perform further processing of the source's packet before forwarding it to the destination. The application of network coding in cooperative wireless networks has recently gained increasing interest[9], with its potential to significantly boost the network throughput and performance. A typical example of network coding in wireless networks is depicted in Fig. 1.3. The network consists of two sources S1 and S2swapping their packets with the help of the relay node R,over orthogonal channels. Assuming Time Division Multiple Access (TDMA), S1 transmits its packet first, followed by S2in the first phase. Meanwhile, the relay node R overhears both these transmissions, and combines the two packets, for instance using the bit-wise XOR operation, and then broadcasts the combined packet in the second phase which helps both source nodes S1 and S2 to achieve diversity gain.

Another network coding scenario is presented in Fig. 1.4, where the network consists of two sources S1 and S2, transmitting to a common destination (D) with the help of the relay node (R). The sources S1 and S2 send their respective information packets to the destination node (D) over orthogonal channels during the first phase. These packets

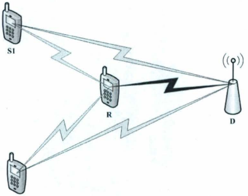


Fig. 1.4. A typical wireless network with two sources transmitting to a common destination with the assistance of a common relay node.

are also overheard at the relay node (R). The relay decodes the two information packets, and can subsequently combine the two packets, for instance using the bit-wise XOR operation. It then forwards the combined packet in the second phase which helps both sources S1 and S2 to achieve diversity gain. Assuming TDMA, a total of three time slots are required with network coding, whereas in case of traditional routing, the number of required time slots are four to achieve a diversity order of two for both nodes. This directly results in a 25 percent throughput improvement.

The application of network coding to wireless networks promises to change many aspects of networking. In effect, network coding deviates from the classical networking approach where wireless networks are treated as physical means of data transportation, allowing for data manipulation within the network. The application of network coding in wireless networks has been studied in a variety of settings, including the cases of (a) two sources transmitting to a common destination[10]-[13], as is depicted in Fig.1.4. This case is an important building block for numerous manifestations of wireless communication, such as the infrastructure-based cellular networks,

(b) multi-cast [14]-[15], where network coding is employed at the intermediary nodes in the network to improve the throughput for information dissemination, and

(c) for two-way relay channels [16]-[19], for instance in ad hoc networks, where the intermediary nodes in the network serve as relays by forwarding the network coded packets for the source-destination pairs.

### 1.4 Relay Selection in Cooperative Wireless Networks

The design criterion which greatly impacts the performance of cooperative networks, both without and with networkcoding is the proper relay selection [16]. As user cooperation and intelligent relay selection can significantly boost the network throughput with antenna sharing, an improperly selected relay can however deteriorate the system performance.

#### 1.4.1 Literature Review of Relay Selection Schemes in Cooperative Networks

Directed by the significance of relay selection in cooperative networks, the problem of relay selection/assignment is receiving extensive interest from the research community. The array of proposed solutions fall mainly into two categories: infrastructure–oriented protocols which usually comprise of optimal solutions (often based on exhaustive searches), and sub-optimal implementation–oriented heuristicsolutions. In this section, we survey some of the most conspicuous and representative publications in this area from the literature.

The authors in [20] address the issue of joint optimization of relay selection and power allocation to maximize the average network capacity. They first propose an optimal solution for the joint optimization problem. However, to alleviate thecomplexity, they separate the joint optimization problem into the sub-problem of single best relay selection with uniform power distribution between the source and relay nodes, and then optimal power allocation for the chosen source-relay pair. A so called "semi-distributed" algorithm is then proposed for a network environment with multiple source-destination pairs where each relay node individually decides on its suitability to act as a relay, and the final decision is made by the central entity. It has been shown that the sub-optimal algorithm with reduced computational complexity can provide comparable performance to that of the optimal scheme, which is based on exhaustive search. The authors consider the system model as shown in Fig. 1.5 [20].

The network consists of multiple source and dedicated relay nodes, and a single destination node. The relays are assumed to operate in the Amplify-and-Forward mode. For finding the optimal solution for a single source, the set of feasible relay nodes (i.e., the ones which can provide better capacity performance than direct transmission) are searched for, and the one which maximizes (1.1) is selected as relay,

$$i = \arg \max_{j=1,\ldots,N_R} \left\{ \frac{|h_{sj}|^2 |h_{jd}|^2 \, SNR}{(|h_{sj}| + |h_{jd}|)\, SNR + 1} \right\}, \tag{1.1}$$

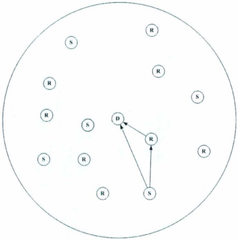



Fig.1.5. System model of a wireless network [20].

where *SNR* is the Signal-to-Noise ratio at the transmitter, $h_{sj}$ and $h_{jd}$ are the channel coefficients from the source to relay $j$, and relay $j$ to destination respectively, and $N_R$ is the number of relays. The channel coefficients integrate the multipath fading and the propagation path-loss. If none of the potential relay nodes offer an increased capacity over direct transmission, i.e., if the set of feasible relay nodesisempty, thesource node goes with direct transmission. The authors then find an optimal solution for power allocation to further improve the performance after relay selection.

Following this optimal solution, the authors propose a semi-distributed relay selection scheme for a network environment which comprises multiple source and relay nodes, under the assumption of equal power allocation between a pair of source and relay. The algorithm is divided into two steps:feasible set generation, and relay node allocation. In the first phase, the nodes transmit hand-shaking packets before actual data transmission to allow the relay nodes to estimate the channel gains from the source and destination nodes. All relay nodes can hence decide on their feasibility (this happens in a distributed fashion), and report their respective indices to the destination. The destination can then perform the relay node allocation from the feasible set by randomly picking a relay node and assigning it to one of the source nodes. Thissub-optimal scheme with lesscomputational complexityisdemonstratedtoachievenear-optimal performance.

The authors of [21] propose the so-called Optimal Relay Assignment (ORA) algorithm for a network environment with multiple source and relay nodes. The objective is to maximize the minimum capacity among the pairs of source and destination nodes. The notable features of this algorithm are (i) guarantee of optimality, (ii) polynomial time complexity, and (iii) final capacity of every source-destination pair is more than that achievable with direct transmission. In the proposed scheme, a source-destination pair is assigned at most one relay, and a single relay node can assist at most one source-destination pair. After an initial "random" relay node assignment, the solution is adjusted in each iteration to achieve a greater value of the objective function (the minimum capacity among all source-destination pairs). In particular, the source node with the lowest capacity is identified and a better relay node for it is searched. However, in case the "better" relay is pre-assigned to some other source, another relay for that other source node is searched for, and so on. Hence within a single iteration, there are two possibilities: (i) a better solution (i.e., a higher value of the objective function) is found, and the algorithm moves on to the next iteration, or (ii) a better solution could not be found, and the algorithm terminates. The algorithm is shown to run in a polynomial time; also, it is argued that in case of a non-optimal solution, the algorithm would keep on iterating, and would terminate only in case the assignment solution is optimal. The optimality of the algorithm is also formally proven.

In [22], the authors consider relay selection in a multiple-access network with a single base station to extend the coverage area using cooperation. The authors derive the optimal relay locations based on two cases, i.e., if the destination uses packets from the relay as well as the source MRC for detection, or only the packet from the relay node. In the former case, the optimal (normalized, wr.t. to the distance between source and destination) relay location (along the line joining the source and destination) from the destination is shown to be

$$x^* = \frac{1}{1+0.5^{\left(1/p-1\right)}}, \tag{1.2}$$

where $p$ is the path loss exponent. In case $p \geq 2$, an interesting observation is that the optimal relay location is closer to the source node. In the case of no-MRC at the receiver, the optimal relay position is shown to be at the mid-point between the source and destination along the line joining the source and destination. The authors then propose a simple distributed algorithm – nearest neighbour routing, in which the relay nearest to the source node can be selected as the helper. Though far from optimal, it is very easy to implement in a distributed fashion.

### 1.4.2 Literature Review of Relay Selection Schemes in Cooperative Networks employing Network Coding

Network coding has recently been studied extensively for cooperative wireless networks as the combining of data at intermediate relay nodes can further improve the

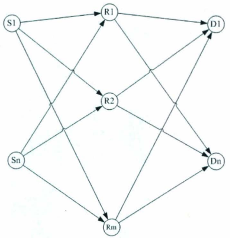



Fig.1.6. A cooperative network with n communication pairs and m relays [23]-[24]

network throughput as well as robustness. In particular, the two-way relay channel model has received the most interest as it could be regarded as the basic building module of many wireless networks. Relay selection in network coding environments is particularly interesting as more than one source nodes have to be involved in the relay selection process asopposedto justone inconventionalcooperative networks. In this section, some of the most representative schemes from the literature addressing relay selection/assignment in cooperative networks with network coding are surveyed.

In [23] the authors consider the system model as shown in Fig. 1.6.The number of relay nodes is assumed to be greater or equal to the number of communicating pairs, and the direct link between the pairs is ignored. Moreover, only asingle relay is assigned to every pair. For ease of comprehension, it is assumedthat one of the nodes in the communicating pair is the Source (S) and the other one is the destination (D). In the first timeslot, the node S transmits its packet which is received and decoded at the selected relay. Similarly, in the second timeslot, node D transmits its packet and it is received and decoded at the relay node. The relay then XORs the two packets and broadcasts the network coded packet which is then heard by both S and D (thereby saving one timeslot compared with traditional relaying using TDMA for instance). The authors then propose an optimal and a sub-optimal scheme for best-relay selection. They consider the channel coefficients over the two links, i.e., the source-relay and relay-destination, and assume that the weaker of the two coefficients will dominate the end-to-end performance. The proposed optimal relay assignment criterion is such that the minimum channel coefficient over the two links is maximized. For the optimal solution, all possible assignment permutations are considered (which are $P_m^{N_R}/N_R$, where $P$ represents permutations, in case of $N_R$ relays and $m$ pairs). If $\Theta$ denotes the set consisting of all possible permutations, the index of the optimal assignment, $k^*$, is given by

$$k^* = \arg\max \max_{k=1,\ldots,P_m^{N_R}} |h|^2{}_{k,\min}, \tag{1.3}$$

where $|h|_{k,\min}$ is the weakest source-relay or relay-destination sub-channel. The authors then propose a sub-optimal scheme by exploiting the correlation within the elements of set $\Theta$. The set $\Theta$ is partitioned into $P_m^{N_R}/N_R$, smaller subsets. The subsets containing correlated elements are not searched for, hence reducing the number of permutations over which the search is run.

In [17], the authors propose analog network coding using differential modulation over two-way relay channels, such that the Channel State Information (CSI) is not requiredto be known at the source, destination, or the relay nodes, and is therefore

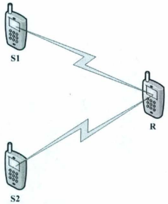


Fig.1.7.Two sources transmitting to a common destination; the relay overhears the transmissions [11].

estimated. Only a single pair of sources is considered in the model with multiple intermediary relay nodes. An optimal relay selection criterion is proposed; the relay which minimizes the estimated sum Symbol Error Rate (SER) of the two sources is selected, according to

$$k^* = \min_{k=1,\ldots,N_R} \left\{ SER_{1,k}(h_{1,k}, h_{2,k}) + SER_{2,k}(h_{1,k}, h_{2,k}) \right\}, \tag{1.4}$$

where $SER_{1,k}(h_{1,k}, h_{2,k})$ and $SER_{2,k}$ are the estimated Symbol Error Rates for Source 1 and Source 2, respectively, for relay $k$, $h_{1,k}$ is the channel coefficient from Source 1 to relay $k$, and $h_{2,k}$ is the channel coefficient from Source 2 to relay $k$.

The best-relay selection is carried out by only one source; hence the decision making node has to calculate the SER for the other source node. The authors then propose a simple sub-optimal relay selection scheme, in which the relay which minimizes the maximum estimated SER of the two sources is selected, i.e.,

$$k = \min_{k=1,\ldots,N_R} \max\left\{SER_{1,k}(h_{1,k}, h_{2,k}), SER_{2,k}(h_{1,k}, h_{2,k})\right\}, \tag{1.5}$$

The sub-optimal min-max scheme is demonstrated to perform very close to the optimal solution, especially as the number of available relay nodes increases.

A multiple-access scenario as depicted in Fig. 1.7 is considered in [11]. The two sources transmit their respective packets to the base station (BS) in the first phase, which comprises two timeslots. These packets are also overheard at the intermediate nodes. In the second phase (i.e., the network coding phase), the selected relay combines the decoded packets from the sources in the first phase and relays the network coded packet to the BS. A single transmission from the relay thus helps both sources to achieve diversity gain. For relay selection, the authors propose a rather unappealing solution of exhaustive search for the best relay (in terms of maximization of the sum capacity of the two nodes). This scheme is infeasible for network environments which usually comprise multiple relay nodes; development of implementation-oriented solutions is an extremely interesting and worth-while area for future investigation.

In the works on cooperative wireless network coding surveyed in this section, and within others from the literature, the relays are assumed to be dedicated, i.e., they transmit nothing for themselves when relaying. In practice this translates to the fact that the relaying node cannot transmit for itself while it is helping another user. A possibility is for the network provider to deploy stand-alone dedicated nodes to act as relays. In effect, the assumption of dedicated relay nodes places additional constraints on wireless terminals, or necessitates additional infrastructure from the service provider to support the network.

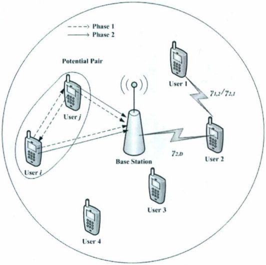



Fig. 1.8. Cooperative wireless network.

Moreover, in the case of multiple-access networks (i.e., the case of multiple sources transmitting to a common destination, such as a base station in [25]), truly multi-user environments are not considered. The number of sources in the network is limited to two, and the issue of scalability to real-world multiuser networks is not addressed. Moreover, the assumption of the presence of dedicated relays in the network is maintained.

## 1.5 Thesis Motivation and Contributions

In perspective of the outlined limitations of related works, we are motivated to address the problem of partner selection (pairing) in a truly multi-user environment, where users employnetworkcodingto transmit to acommon destination (e.g. a base station in a cellular environment), in the absence of dedicated relay nodes. This is an important communicationscenario, and to the best of our knowledge, the problem of *mutual user pairing* in such multi-user environments has not been addressed previously in the literature. In the absence of dedicated relay nodes, and as shown in Fig. 1.8, users are considered to *mutually pair* among themselves to realize network coding. The pairing should be performed to optimize certain system performance metrics, such as network capacity, outage probability, and/or fairness. Nodes constituting a pair periodically swap the roles of source and relay for the mutual benefit of achieving diversity gain.

Our objectives are:

(a) to address the problem of mutual user pairing in a multiuser environment, such as to optimize certain system performance parameters, and

(b) in conjunction with the user pairing schemes, to address the transmission power optimization, with constraints on certain network performance metrics.

The *major contributions* of this thesis are summarized as follows:

1. We formulate and solve an optimization problem to obtain the user pairing which optimizes system performance metrics. We tailor our algorithm to maximize the network capacity, but this can also be used to optimize the outage probability, user-fairness, or other performance metrics.
2. The optimality of the algorithm is verified; however, to address the computational complexity, we then propose implementation-oriented heuristic user pairing algorithms. The heuristic schemes are designed to approach the optimal performance at a significantly reduced complexity. We propose

algorithms which address average network capacity, average outage probability, and user-fairness. The performance of the optimal and heuristic algorithms is investigated through extensive simulations.

3. Once the problem of user pairing is solved, we next address the issue of power minimization, and solve a joint optimization problem. We perform user pairing to maximize the total network capacity, and minimize the transmission power per user, such that certain network performance constraint, such as in terms of the average capacity or average outage probability, is satisfied.

**List of Publications:**

Our work, during the course of this thesis has resulted in the following publications:

- T. Rasheed, M. H. Ahmed, and O. A. Dobre, "User-Pairing for Capacity Maximization in Cooperative Wireless Network Coding," submitted to *IEEE ICC 2012*.

- T. Rasheed, M. H. Ahmed, O. A. Dobre, and M. Saad, "Optimal User-Pairing in Cooperative Wireless Network Coding with Constrained Power Minimization," accepted to *IEEE RWS 2012*.

- T. Rasheed, Y. P. Chen, O. A. Dobre, and M. H. Ahmed, "Medium Access Control in Wireless Sensor Networks: Contemporary Design Issues and Future Research Directions," in Proc. *IEEE NECEC 2010*.

- T. Rasheed, M. H. Ahmed, and O. A. Dobre, "Cooperative Communication for Cognitive Radio Networks," in Proc. *IEEE NECEC 2010*.

- T. Rasheed, M. H. Ahmed, and O. A. Dobre, "Relay Selection Schemes for Cooperative Communication and Network Coding: A Survey, " in Proc. *IEEE NECEC* 2010.

### 1.6 Organization of the Thesis

The rest of the thesis is organized as follows. In Chapter 2 we lay out the system model, and then compute the capacity and outage probability for the network-coded cooperation under consideration. Chapter 3 describes the pairing algorithms to realize network coding. We propose various optimal and heuristic pairing schemes which address network performance parameters, such as capacity, outage probability, and user-fairness. In Chapter 4, we perform power minimization, and solve the joint optimization problem to minimize the transmission power, while meeting certain constraints on the network performance. Performance analysis of the proposed algorithms is conducted in Chapter 5, with extensive simulations. Scenarios are highlighted as to when certain (pairing and joint/constrained optimization) algorithms are preferable over others. Chapter 6 summarizes the findings of this thesis, outlines the main conclusions, and finallypresents recommendations for possible future research directions.

# Chapter 2

# Capacity and Outage Probability Analysis of Network-Coded Cooperation

In this chapter, we outline the system and signal model for the network-coded cooperation. We subsequently perform the capacity and outage analysis of the network-coded cooperation by presenting the capacity and outage probability expressions. For sufficiently large packet length, the outage probability demonstrates a lower bound on the packet error rate [26]. Throughout the analysis, we assume perfectly orthogonal channels, exhibiting quasi-static (i.e. block) Rayleigh fading, and half-duplex transmissions.

Section 2.1 outlines the system and the signal model. The network-coded cooperation scenario under consideration is presented in Section 2.2. Subsequently, the capacity and outage probability analysis is performed in Section 2.3.

## 2.1 System Model

The system model of the network coded cooperation considered in this work is shown in Fig. 2.1. We consider a single cell with an even number of users ($N_{users}$). Nodes areuniformly and randomly distributed over the entire cell and are assumed to be equippedwith single antennas. We assume no dedicated relay nodes inthe cell. Usersstrategicallypair among themselves, and periodically swap the roles of the source

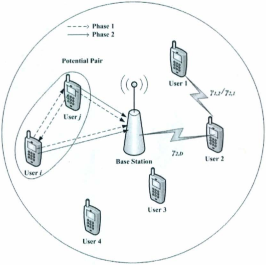



Fig.2.1. System model under consideration.Dotted and solid lines represent source- and network-coded packet transmissions respectively.

andrelay to realize network coding, and achieve spatial diversity. Nodes constituting a pair first broadcast their respective packets to the base station, and also overhear each other's transmissions. In case of a successful detection of the partner's packet, a network-coded packet is subsequently transmitted by the overhearing node, which helps both nodes in the pair to achieve diversity gain.

The received signal at the relay or destination nodes is given by

$$y[m] = h[m]x[m] + n[m] \tag{2.1}$$

where $x[m]$ is the transmitted signal, $h[m]$ is the channel coefficient which integrates the effect of path loss and frequency non-selective Rayleigh fading, and $m$ is the time index. The term $n[m]$ is the zero-mean additive white Gaussian noise (AWGN) with

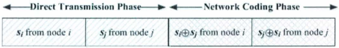



Fig. 2.2. Packets transmitted by the paired nodes $i$ and $j$ in the two phases. In case of inter-user transmission failure, an individual packet is transmitted by the relaying node in the network coding phase.

power spectral density ($N_0$), capturing the effect ofthermal noise at the receiver.

We model the inter-user and user-destination channels as non-ideal (i.e. noisy with Rayleigh fading). Thus, a node constituting a pair sometimes may not be able to detect the packet of its partner, and as a result, it may not always forward the network-coded packet to help its partner. The network-coded packet transmission and detection of a pair of nodes follow the model proposed in [27]. The communication with the common destination (such as a base station or access point) is performed over two phases, and each phase consists of two orthogonal channels (we assume Time Division Multiple Access (TDMA) in this work). This model is depicted in Fig. 2.2, where it is assumed that nodes $i$ and $j$ constitute a pair, where $i, j \in \{1, \ldots, N_{users}\}$, and $i \neq j$. The node $i$ transmits its packet to the base station in the first time slot during the first phase, i.e., the direct transmission phase, while node $j$ overhears. Subsequently, node $j$ transmits its packet in the second time slot while node $i$ overhears. This is followed by the second, orthe network coding phase of transmission[1]. Now, if node $i$ had decoded its partner's packet in the previous phase, it would combine it with its ownpacket, and send the network coded packet to the base station. Otherwise, node $i$ would send an additional packet for itself.

[1] The terms "first phase" and "direct transmission" phase, and "second phase" and "network coding phase" are used interchangeably in the context.

Meanwhile, node $j$ does the same in the second time slot of the second phase. At the base station, the two independently faded network coded packets arecombined using any of the well-known combining techniques, such as Selection Combining (SC), Equal-Gain Combining (EGC), or maxim ratio combining (MRC) [3]. This packet is then jointly decoded with the packets received in the first phase to recover the information bits. A maximum diversity order of two for each user can therefore be achieved. This concludes the two phases of communication with the base station.

The energy allocation is non-equal but symmetric (with respect to the two phases), i.e., individual nodes within the pair may use different transmission powers in a single phase, but the transmission power of a particular node is equal in the two phases. Cyclic redundancy checks are assumed to detect decoding errors at the receiving nodes. Moreover, incorporating an additional flag bit in the packets transmitted in the second phase helps the base station determine the success of inter-user transmissions, and hence the nature of the packets received in the second phase.

Noteworthy is the fact that we assume no dedicated relays in the cell, as the relay nodes also transmit for themselves when relaying. Moreover, since users transmit over orthogonal channels, there is no same-cell interference. All channels, i.e. inter-user and source-destination, are assumed to be spatially independent, frequency flat Rayleigh fading, with pure AWGN. We assume block fading, such that all channels remain constant during the two phases. The signal model for the two-phase network coded cooperation scenario is formally presented next.

### 2.1.1 Signal Model

In the first phase, the source node $i$ transmits $L/2$ symbols, and therefore the time index $m=1,\ldots,L/2$. For the source-to-destination transmission, the symbols receivedat the destinations are given by

$$y_{i,D}[m]=h_{iD}[m]s_i[m]+n_D[m], \tag{2.2}$$

where $s_i[m]$ are the transmitted source information symbols, $n_D[m]$ is the AWGN noise at the receiver, and the channel coefficient $(h_{i,D}[m])$ captures the effect of path loss and frequency non-selective Rayleigh fading. We assume perfect channel state information at all receivers, i.e., the channel coefficients are perfectly estimated, and that perfect synchronization exists between nodes which perform coherent detection. The channel coefficient is assumed to be constantover the two phases (including $2L$ symbols), and the dependency of $h$ on time $m$ is henceforth dropped. The received symbols at node $j$ are

$$y_{i,j}[m]=h_{i,j}s_i[m]+n_j[m], \tag{2.3}$$

where $n_j[m]$ is the AWGN noise at node $j$, and $h_{i,j}$ is the coefficient of the channel from node $i$ to node $j$. Similarly, for $m=L/2+1,\ldots,L$, node $j$ (now assuming the role of source) sends its packet to the base station, which is overheard by $i$. The received symbols at $D$ and $i$ are given respectively as

$$y_{j,D}[m]=h_{j,D}s_j[m-L/2]+n_D[m], \tag{2.4}$$

and

$$y_{j,i}[m]=h_{j,i}s_j[m-L/2]+n_i[m], \tag{2.5}$$

where $s_j[m]$ are symbols transmitted by node $j$, $n_i[m]$ is the noise at node $i$, and $h_{j,D}$ and $h_{j,i}$ are the coefficients of the channel between $j$ and $D$, and $j$ and $i$, respectively. In the second phase of transmission, $i$ and $j$ transmit for $m = L+1,...,3L/2$ and $m = 3L/2+1,...,2L$, respectively. The received symbols at $D$ from $i$ and $j$ are given respectively by

$$y_{i,D}[m] = h_{i,D}(s_i[m-L] \oplus s_j[m-L]) + n_D[m], \tag{2.6}$$

and

$$y_{j,D}[m] = h_{j,D}(s_i[m-3L/2] \oplus s_j[m-3L/2]) + n_D[m], \tag{2.7}$$

where '$\oplus$' denotes the bit-wise XOR operator.

In case the partner does not decode the source's packet, it transmits additional symbols for itself during the second phase of transmission.

**2.2 Capacity and Outage Analysis of the Network Coded Cooperation**

In wireless communication, the dynamic and time-varying nature of the fading channels makes the design of communication systems extremely challenging. An efficient means to combat the effects of time-varying fading over wireless channels is through the use of spatial diversity. In this work we consider network-coded cooperation as a cooperative transmission approach to realize spatial diversity. We consider mutual user pairing, where users strategically pair, and swap the roles of source and relay to realize network coding and achieve spatial diversity. The relay nodes are not dedicated, i.e., they transmit for their partner, as well as for themselves when relaying.

The inter-source and source-destination channel capacities for nodes $i$ and $j$ are functions of the corresponding channel coefficients, and, therefore they are random variables. Moreover, an outage over a link is defined as the event of throughput falling below a target information rate. We use the outage probability at a certain rate as a metric of the packet error rate (PER) for the block-based transmissions under consideration [28]. The inter-source channels are modeled as non-ideal (due to noise and fading), and successful decoding at the relay is not guaranteed. This translates to the fact that the relay forwards a network coded packet in the second phase only if it decoded its partner's packet correctly. Otherwise, it transmits its own packet only. Hence, the average throughput of the pair depends on the success of inter-source transmissions, which must first be determined.

### 2.2.1 Direct Transmission Phase

In the direct transmission phase, nodes $i$ and $j$ sequentially broadcast their respective packets, containing $k$ information bits, to the base station and also overhear each other's transmissions. The inter-source information theoretic channel capacity for node $i$ is $C_{i,j} = \log_2(1+\gamma_{i,j})$ [bits/sec/Hz], where $\gamma_{i,j} = |h_{i,j}|^2 P_i / N_0$ is the instantaneous SNR of the inter-source link, with $P_i$ as the transmit power. An outage occurs whenever $C_{i,j} < 2R$, where $R$ is the packet information rate in case of the point-to-point transmission. For Rayleigh fading, the outage probability for node $i$ is given as [27]

$$\overline{P}_{i,j} = 1 - \exp(-\frac{2^{2R}-1}{\Gamma_{i,j}}), \tag{2.8}$$

where $\Gamma_{i,j}$ is the average SNR of the inter-source link. The outage probability for node $j$ can similarly be calculated by replacing $\Gamma_{i,j}$ by $\Gamma_{j,i}$ in (2.8).

### 2.2.2 Network Coding Phase

The success of inter-source packet transmissions can lead to the following four distinct cases [27]:

**Case A:** When both nodes $i$ and $j$ forming a pair decode each other's packets, they both transmit the network-coded packet in the second phase, which results in a full cooperation scenario, for that pair.

**Case B:** When none of the two nodes decode each other's packet, they send additional packets for themselves in the second phase, and the system returns to a non-cooperative scenario, for that pair of packets.

**Case C:** When only node $j$ decodes $i$, and not vice-versa, only node $j$ transmits the network-coded packet in the second phase (which helps both nodes), whereas node $i$ repeats its own packet.

**Case D:** When only $i$ decodes $j$'s packet, and not vice-versa, only node $i$ transmits the network-coded packet in the second phase (which helps both nodes), whereas node $j$ repeats its own packet.

We consider maximum ratio combining (MRC) at the destination, which forms the combined packet by the weighted sum of the received packets over the two phases. To determine the channel capacity and outage probability for the four possible cases, parts of the packets from nodes $i$ and $j$ which are used for decoding at the destination should be identified. In this subsection, we perform the capacity and outage analysis for the four possible cases for node $i$ only. A similar approach holds for node $j$. The underlying assumption is that nodes $i$ and $j$ constitute a pair, and mutually cooperate to realize network coding. The algorithms for user pairing in a multiuser environment will formally be presented in the following chapter.

**Case A:** Both nodes $i$ and $j$ comprising the pair decode each other's packets in the direct transmission phase. Each node transmits the network coded packet $(s_i \oplus s_j)$ in the network coding phase. For decoding, a packet $[s_i, (s_i \oplus s_j)']$ of length $N$ is formed, where the prime denoted the MRC. As this packet contains $2k$ information bits, its code rate is $\frac{2k}{N} = 2R.$ The two parts of this packet are essentially received over parallel channels whose capacities add together. The outage event for node $i$ is [27]

$$C_{i,D} = \alpha \log_2(1+\gamma_{i,D}) + (1-\alpha)\log_2(1+(\gamma_{i,D}+\gamma_{j,D})) < 2R, \tag{2.9}$$

where $\alpha$ is the fraction of time allocated to the first phase. From the perspective of capacity, the effect of MRC at the receiver is reflected by the addition of the two received SNRs (as in the second term in 2.9). The outage probability of the event in (2.9) is approximated as (the derivation is shown in Appendix A)

$$\overline{P}_{i,D} \approx 2\frac{(2^{2R/\alpha}-1)^2}{\Gamma_{i,D}\Gamma_{j,D}}(1-\alpha)^2. \tag{2.10}$$

This represents the outage probability given the occurrence of Case A. The probability of occurrence of Case A is given by the product of probabilities of successful decoding at nodes $i$ and $j$ which can be computed from (2.8). Defining the overall outage probability as $\overline{P}_{i,D,A}$ where 'A' indicates the case, we get

$$\overline{P}_{i,D,A} = (1-\overline{P}_{i,j}).(1-\overline{P}_{j,i}).\overline{P}_{i,D}. \tag{2.11}$$

**Case B:**Neither of the two nodes $i$ and $j$ constituting the pair decode each other's packets. Each source node transmits additional packets for itself. At the destination, a packet $[s_i, s_j]$ is formed whose code rate is $R$. The outage event in this case is [27]

$$C_{i,D} = \alpha \log_2(1+\gamma_{i,D}) + (1-\alpha)\log_2(1+\gamma_{i,D}) < R. \tag{2.12}$$

where the two terms in (2.12) come from the contributions to the total capacity from the two phases, respectively. Following the same approach as in Case A, the outage probability is approximated as

$$\overline{P}_{i,D,B} \approx \overline{P}_{i,j}.\overline{P}_{j,i}.\left[\frac{2^R-1}{\Gamma_{i,D}}\right]. \tag{2.13}$$

**Case C:**Only $j$ can correctly decode $i$'s packet, but not vice versa. In this case, node $j$ helps $i$, but $i$ transmits for itself during the network coding phase. The $(2-\alpha)N$ and code rate of $2R/(2-\alpha)$. The outage event for node $i$ in this case is information symbols of $i$ are decoded from the packet $[s_i, (s_i \oplus s_j), s_i]$ with length of

$$C_{i,D} = \alpha \log_2(1+\gamma_{i,D}) + (1-\alpha)\log_2[(1+\gamma_{i,D})(1+\gamma_{j,D})] < 2R/(2-\alpha). \tag{2.14}$$

Following the same approach as in Case A, the outage probability is computed as

$$\overline{P}_{i,D,C} \approx \frac{1}{2}(1-\overline{P}_{i,j}).\overline{P}_{j,i}.\frac{(2^{2R/(2-\alpha)}-1)^2}{\Gamma_{i,D}\Gamma_{j,D}}. \tag{2.15}$$

**Case D:** Only node $i$ can correctly decode node $j$'s packet and not vice versa. In this case $i$ helps $j$, but $j$ transmits for itself during the network coding phase. To decode $i$'s information symbols, a packet $[s_i,(s_i \oplus s_j)]$ of code rate $2R$ is formed at the destination. The outage event for this case is

$$C_{i,D} = \alpha \log_2(1+\gamma_{i,D}) + (1-\alpha)\log_2(1+\gamma_{i,D}) < 2R, \tag{2.16}$$

and following the same approach as Case A, the outage probability is approximated as

$$\overline{P}_{i,D,D} \approx \overline{P}_{i,j}.(1-\overline{P}_{j,i}).\left[\frac{2^{2R}-1}{\Gamma_{i,D}}\right]. \tag{2.17}$$

The total outage probability is the sum of the outage probabilities for the four cases, i.e.

$$\overline{P}_T = \overline{P}_{i,D,A} + \overline{P}_{i,D,B} + \overline{P}_{i,D,C} + \overline{P}_{i,D,D} \tag{2.18}$$

## 2.3 Conclusion

In this Chapter, we presented the signal and system model for the network-coded cooperation under consideration. We presented and capacity and outage probability analysis for a pair of nodes, considering non-ideal inter-user channels. In the next chapter, we address the challenging problem of the mutual pairing of users in the multi-user cellular environment. More specifically, we propose and present optimal and heuristic user-pairing strategies to address various network performance metrics, such as average capacity, average outage probability, and user-fairness.

# Chapter 3

# User Pairing in Network-Coded Cooperative Wireless Networks

### 3.1. Mutual User Pairing to Realize Network Coding

We address the problem of the mutual pairing of users, or partner selection in a multi-user network-coded cooperative wireless network, to achieve spatial diversity. As outlined in Chapter 2, users, having data to transmit, mutually pair among themselves to realize network coding, while transmitting to a common destination. This could be an access point in a wireless local area network or a base station in a cellular environment. Two nodes constituting a pair periodically swap the roles of source and relay for the mutual benefit of achieving diversity gain. Hence, only users with data to transmit participate into cooperation, and idle users are not engaged. This system model is depicted in Fig. 3.1.

Transmission to a common destination in a wireless network is an important communication scenario, and to the best of our knowledge, the problem of mutual user pairing in such multi-user environments has not been addressed previously in the literature.

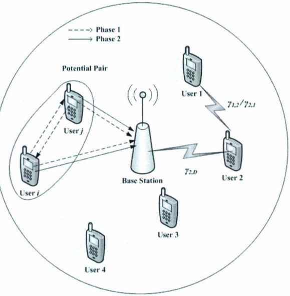



Fig. 3.1. The system model. Dotted and solid lines represent source- and network-coded-packet transmissions, respectively.

### 3.2 User Pairing to Optimize System Performance

As shown in Fig. 3.1, users strategically pair among themselves to realize spatial diversity. For this network-coded cooperation scenario under consideration, the user pairing strategy directly impacts the overall network performance. Moreover, the user pairing can be performed to optimize certain network performance metrics, such as maximizing the total network capacity, minimizing the outage probability, and/or maximizing the per-user throughput fairness.

In this chapter, we first formulate and solve an optimization problem (using the maximum weighted matching algorithm) to obtain the user pairing which yields the maximum achievable total network throughput. In order to facilitate the pairing process, we subsequently propose implementation-oriented heuristic algorithms which approach the optimal performance at a reduced computational complexity. In particular, we propose max-max pairing to maximize the network capacity at a significantly reduced complexity. Moreover, max-min pairing algorithm is proposed to minimize the outage probability, with a very low complexity.

### 3.2.1 Optimal User Pairing $\mathbb{P}^*$ to Maximize Network Capacity

We formulate and solve the problem of determining the optimal user-pairing $\mathbb{P}^*$ which maximizes the total network capacity. We have the set of all possible pairing sets $\Pi$, such that every set $\mathbb{P} \in \Pi$ is the pairing containing $N_{users}/2$ disjoint user pairs. Each pairing $\mathbb{P}$ is therefore a symmetric mapping of elements from the set $\mathbb{X} \in \{1,2,...,N_{users}\}$ to the set $\mathbb{Y} \in \{1,2,...,N_{users}\}$, with the restriction of an element from $\mathbb{X}$ not being mapped to the same element in $\mathbb{Y}$. The goal is to find the optimal pairing $\mathbb{P}^*$ that maximizes the total network capacity given by:

$$C_{sum} = \sum_i C_i. \tag{3.1}$$

Therefore,

$$\mathbb{P}^* = \arg\max_{\mathbb{P}\in\Pi} C_{sum}(\mathbb{P}). \tag{3.2}$$

At first glance, this looks like the problem of maximum weighted matching (i.e., pairing) in bipartite graphs, and any of the assignment algorithms, such as the well-known Hungarian algorithm [29], seems as acandidate solution. However, asit was observed, a

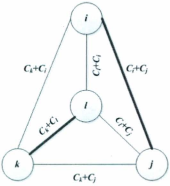



Fig. 3.2. A potential matching in the weighted, undirected graph; the edges drawn with thick lines are part of the matching.

weight matrix $\mathbf{W}$, with zeros on the main diagonal and symmetric entries, $[\mathbf{W}]_{i,j} = [\mathbf{W}]_{j,i} = C_{i,D} + C_{j,D}$, where $C_{i,D}$ and $C_{j,D}$ are the source-destination channel capacities for $i$ and $j$, respectively, and $[\mathbf{W}]_{i,j}$ and $[\mathbf{W}]_{j,i}$ describe the weight of the assignment of node $i$ to $j$, and node $j$ to $i$, respectively (where $i$ and $j$ constitute a potential pair), did not always lead to a symmetric assignment. To find the optimal solution, we therefore model this problem as maximum weighted matching in general graphs.

We construct a weighted undirected graph $\mathcal{G} = (V, E)$, where the vertices $V$ are the users to be paired, connected by the set of edges $E$. Furthermore, $|V| = N_{users}$ and $|E| = N_{users}(N_{users} - 1)/2$ (as the graph is fully connected), where $|.|$ denotes the cardinality of the set. Each edge $(i, j)$ has an associated weight $w_{i,j} = C_{i,D} + C_{j,D}$. The goal is to find the matching (i.e., pairing) with the maximum total weight. This maximum weighted matching covers all the vertices in the graph, and each vertex is connected only to a single edge. Moreover, each edge in the graph connects two distinct vertices. One such potential matching for a weighted graph with four nodes is shown in Fig. 3.2. It is noteworthy that the edge with the maximum weight may not be a part of the maximum weighted matching.

When the number of users to be paired is large, the problem of finding the optimal pairing (i.e., the matching with the maximum total weight) is clearly far from trivial, whereas an exhaustive search is prohibitively expensive. To solve this pairing problem, we use Jack Edmond's maximum weighted matching algorithm for general graphs, which is described in [30]. In the following, we present a succinct description of the algorithm, and the reader is referred to [30] for more details.

The idea is to start with an empty pairing, and then, during each stage, to find an augmenting path in the graph which yields the maximum increase in weight. The *blossoms method* is used for finding the *augmenting paths*in the graph. To explain this problem of maximum weighted matching in general graphs, we clarify some terms from graph theory. A *matching* in a graph is a set of edges, such that no two edges share a common vertex. A sample matching in a non-fully connected graph, consisting of 8 vertices is shown in Fig. 3.3. Furthermore, a vertex in the graph withrespect to a matching $\mathcal{M}$ is *free* if none of the edges in the matching are incident on thatvertex.An *alternating path*inthe graph with respect to the matching $\mathcal{M}$ is such that its edges alternately belong to the matching $\mathcal{M}$, and not to the matching $\mathcal{M}$. Moreover, an *augmenting path* is an alternating path between free vertices.

The matching $\mathcal{M}$ is *not* maximum matching if and only if there is an augmenting path with respect to $\mathcal{M}$. We search for the augmenting paths in the graph by performing

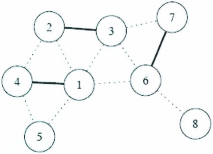



Fig. 3.3. The solid lines show the edges forming a matching.

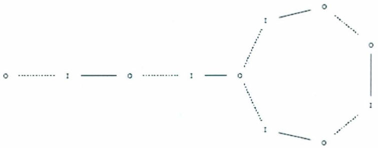
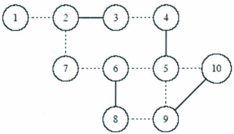


Fig. 3.4. A cycle of inner and outer vertices.

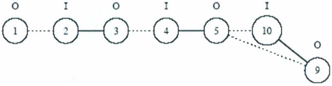
a breadth-first search starting from free vertices. We call an edge in the matching as 'solid' and an edge not in the matching at 'dotted'. To search for the augmenting path from a free vertex, we build a tree of alternating paths. The root, as well as all the vertices which are at an even distance from the root are called 'inner vertices'. If we run into a free inner vertex, then an augmenting path to that vertex can be constructed.

The step of building the tree is based on scanning an outer vertex, $v$. Each solid edge $(v, w)$, where $w$ is not already in the tree, is added to the tree. Vertex $w$ is designated an 'inner', and the solid edge $(w, x)$, which is unique, and is incident with $w$ is added to the tree, and $x$ is labeled as 'outer'.

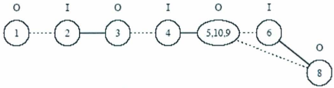
During the process of scanning the outer vertex $v$, if we encounter an edge $(v, w)$, in which $w$ is outer, we then form a cycle as in Fig. 3.4. In this case, we contract the cycle to form a super-vertex, called a *blossom*, and continue so on. Moreover, if we encounter a free vertex, then an augmenting path can be constructed from the root to that vertex. We show this with an example. Consider the following graph:




Starting with a breadth-first search from vertex 1, we see cycle 5 – 10 – 9 in the following graph.



A blossom is formed by shrinking vertices 5, 10, and 9, and the search is continued.




We then shrink (5, 10, 9), 6, 8 into a single vertex.


We hence find an augmenting path in the shrunk graph. By unshrinking, the following augmneting path in the original graph can be found.


We start with an empty pairing, and during each stage find an augmenting path in the graph which leads to the maximum increase in weight. The algorithm solves the pairing problem in $O(N^3)$ time, and avoids the need for an exhaustive search. Moreover, if the number of users to be paired is large, the set of users can be split into randomly chosen smaller groups to reduce the complexity of the algorithm, while however compromising the performance.

### 3.3 Heuristic User Pairing Algorithms – Approaching Optimal Performance

In this section, we propose computationally simpler heuristic user-pairing schemes to simplify the pairing process. In particular, we propose max-max pairing to maximize the total network capacity. Moreover, max-min pairing is proposed to minimize the average outage probability.

### 3.3.1 Max-max pairing

This algorithm pairs users with the objective of approaching the optimal capacity at a much reduced computational complexity. A weight matrix $\mathbf{W}$ with zeros on main diagonal, and symmetric entries $[\mathbf{W}]_{i,j} = [\mathbf{W}]_{j,i} = C_{i,D} + C_{j,D}$ is established, where $i$ and $j$ are potential pairs. The $O(N^3)$ algorithm is formally presented in the following:

a) Initialize an empty pairing $\mathcal{P}$,

b)Select the largest element from $\mathbf{W}$, for instance $[\mathbf{W}]_{i,j}$, and form the pair by augmenting $\mathcal{P}$ with $i$ and $j$,

c) Update $\mathbf{W}$ by removing the rows and columns corresponding to the pair formed in (b),

d) Continue from (b) until $\mathcal{P}$ is complete and all nodes have been paired.

Max-max pairing has the same big $O$ complexity as the optimal pairing, which depicts that it scales similarly to the changes in input size, as the optimal pairing. However, max-max pairing is significantly computationally simpler than the optimal pairing, as it requires simpler computations. This is also reflected in the average simulation times which are referred to in Chapter 5.

### 3.3.2 Max-min pairing

This heuristic algorithm is designed to address the system outage probability. We start with the weakest user (in terms of the SNR to the BS) in the cell and pair it with the user having the strongest of the weaker of source-relay and relay-destination links, since the outage performance is always determined by the weaker of the two links [31], and continue so on for other users. The algorithm has complexity of $O(N^2)$, and is formally presented as follows:

a) Initialize an empty pairing $\mathcal{P}$,

b) Select a node $i$ with the lowest $\gamma_{i,D}$ and pair it with $j$ with $\max[\min(\gamma_{i,j}, \gamma_{j,D})]$,

c) Augment the pairing $\mathcal{P}$ with the pair formed in (b), and update the set of eligible nodes.

d) Continue from (b) until $\mathcal{P}$ is complete and all nodes have been paired.

Apparently, max-min pairing is computationally efficient because it involves cheap computations. This is also reflected by the simulation times as stated in Chapter 5.

### 3.3.3 Random pairing

Pairing users randomly is the most straight-forward strategy, and is the simplest to implement in practice. From the set of eligible users, two randomly chosen nodes are paired. $\mathcal{P}$ is augmented, the set of eligible users is updated, and the algorithm repeats until all users have been paired. Although random selection is not an effective way of pairing, we include it here for comparison purposes.

## 3.4 Conclusion

In this chapter, we considered the problem of mutual user pairing in network-coded cooperative networks. We proposed an optimal pairing algorithm, and tailored it to maximize the network capacity. We subsequently proposed computationally simpler heuristic pairing algorithms. In particular, we proposed the max-max pairing with the objective of maximizing the network capacity. Moreover, we proposed the max-min pairing to minimize the outage probability.

The performance analysis of the proposed optimal and heuristic algorithms is presented in Chapter 5,where these are compared in terms of average capacity, average outage probability, and user-fairness. The suitability of these algorithms, in view of varying system performance requirements is also discussed.

# Chapter 4

# Power Minimization: Joint & Constrained Optimization

In energy-constrained wireless networks, the design of energy efficient protocols is imperative. For the network-coded cooperation scenario under consideration, we have emphasized that the gains associated with cooperation and network coding are the improved throughput and outage performance, brought about by the achieved spatial diversity. However, for energy constrained wireless networks such as sensor and cellular networks, where minimizing the energy consumption is one of the objectives, these performance gains can be traded-off with energy savings, and can therefore result in significantly improved battery lifetimes.

In this chapter, we consider power minimization, and solve a joint optimization problem. In the joint optimization problem, we perform user pairing to maximize the total network capacity, and minimize the transmission power per user, such that certain network performance constraint in terms of the average outage probability per user, or the average capacity per user is satisfied. We use the maximum weighted matching algorithm (as described in Chapter 3, Section 3.2.1) to obtain the optimal user pairing which leads to the maximum total network capacity. Subsequently, we use the bisection optimization

[32], to solve for the minimum transmission power per user, such that the given constraint on the average capacity per user, or on the average outage probability per user is satisfied.

### 4.1 Power Minimization: Joint Optimization of Power and Capacity

We first find the optimal user pairing $\mathcal{P}^*$ which maximizes the total network capacity

$$C_{sum} = \sum_i C_i. \tag{4.1}$$

Therefore,

$$\mathcal{P}^* = \arg\max_{\mathcal{P}\in\Pi} C_{sum}(\mathcal{P}), \tag{4.2}$$

where $\Pi$ is the set of all possible user pairing sets, such that every set $\mathcal{P}\in\Pi$ is the pairing containing $N_{users}/2$ disjoint user pairs. The maximum weighted matching algorithm is used to solve the problem of determining the optimal user pairing which leads to the maximum total network capacity. We construct a weighted undirected graph $\mathcal{G}=(V,E)$, where the vertices $V$ are the users to be paired, i.e., $i,j\in\{1,...,N_{users}\}$, $i\neq j$, connected by theset of edges $E$. Furthermore, $|V|=N_{users}$ and $|E|=N_{users}(N_{users}-1)/2$ (as the graph is fully connected), where $|\cdot|$ denotes the cardinality of the set. Each edge $(i,j)$ has an associated weight $w_{i,j}=C_{i,D}+C_{j,D}$. The pairing is obtained from the maximum weighted matching algorithm (explained in Section 3.2.1, Chapter 3).

Afterdetermining the optimal capacity pairing $\mathcal{P}_{cap}^{*}$, we use it further, and perform power minimization using the bisection optimization method [32], such that the network performance constraint is met. Equal power allocation is assumed for all users. The bisection method, sometimes also referred to as the binary search algorithm, can be used to locate the root of a continuous function by enclosing it in an initial search interval, and then successively halving it, such that the root stays enclosed within the new interval [32].

### 4.1.1 Power Minimization & Capacity Maximization, with a Constraint on Average Outage Probability per User

Given the performance constraint in terms of the average outage probability per user, i.e.,

$$\Phi_{out}(P) \le \Phi_{out\_th}, \tag{4.3}$$

where $\Phi_{out}(P)$ is the average outage probability per user, which is a monotonically decreasing function of the transmission power per user, $P$, and $\Phi_{out\_th}$ is the maximum acceptable average outage probability per user. The optimal transmission power per user, $P_{min}^{*}$, i.e., the minimum power which meets thisconstraint on outage probability satisfies

$$\Phi_{out\_th} - \Phi_{out}(P_{min}^{*}) = 0. \tag{4.4}$$

We use the bisection method to solve this constrained optimization problem. To find $P_{min}^{*}$, we locate the root of the function

$$F(P) = \Phi_{out\_th} - \Phi_{out}(P). \tag{4.5}$$

An upper and lower bound on the transmission power define the initial search interval $[P_l, P_u]$, such that it contains the root of *F(P)*, i.e., $P_{min}^*$. The function *F(P)* will have opposite signs at the endpoints of this search interval, as the root is contained within this interval. This search interval is halved in subsequent iterations, and the value of either $P_l$ or $P_u$ (whichever is farther from the root) is updated, and assigned the value equal to the mid-point of the interval in the previous iteration. This is done such that the root stays trapped within the new interval, i.e., *F(P)* still has opposite signs on the new end points. The bisection method converges to the actual root with a predefined tolerance, $\varepsilon$. The algorithm for outage probability–constrained power minimization is formallyexpressed as:

(1) Choose the initial values for $P_l$ and $P_u$, such that the root lies within $[P_l, P_u]$,

(2) Set the transmission power to $P = P_l + (P_u - P_l)/2$, i.e., the mid-point of the search interval,

(3) Obtain the new optimal capacity pairing $\mathcal{R}_{cap}^*$ (using the maximum weighted matching algorithm) for the current transmission power $P$,

(4) *If* $F(P) = 0$, **exit**

*Else if* $(P_u - P_l) < \varepsilon$ AND $F(P) > 0$, **exit**

*Else if* $F(P) \cdot F(P_l) > 0$, then $P_l = P$

*Else* $P_u = P$

**go to step (2)**.

### 4.1.2 Power Minimization & Capacity Maximization, with a Constraint on Average Capacity per User

Given the performance constraint in terms of the average capacity per user, i.e.,

$$\Phi_{cap}(P) \geq \Phi_{cap\_th}, \tag{4.6}$$

where $\Phi_{cap}(P)$ is the average capacity per user, which is a monotonically increasing function of the transmission power per user, $P$, and $\Phi_{cap\_th}$ is the minimum acceptable average capacity per user. The optimal transmission power, $P_{min}^{*}$, i.e., the minimum power per user which meets this constraint on average capacity per user satisfies the equation

$$\Phi_{cap\_th} - \Phi_{cap}(P_{min}^{*}) = 0. \tag{4.7}$$

We use the bisection method to solve this constrained optimization problem. To find $P_{min}^{*}$, we locate the root of the function

$$F(P) = \Phi_{cap\_th} - \Phi_{cap}(P). \tag{4.8}$$

The algorithm for capacity – constrained power minimization is formallyexpressed as:

(1) Choose the initial values for $P_l$ and $P_u$, such that $[P_l, P_u]$ contains the root of *F(P)*,

(2) Set the transmission power to the mid-point of the search interval, i.e., $P = P_l + (P_u - P_l)/2$,

(3) Obtain the new optimal capacity pairing $\mathcal{P}_{cap}^{*}$ for the current transmission power $P$, using the maximum weighted matching algorithm,

(4) *If* $F(P) = 0$, **exit**

*Else if* $(P_u - P_l) < \varepsilon$ AND $F(P) < 0$, **exit**

*Else if* $F(P) \cdot F(P_l) > 0$, then $P_l = P$

*Else* $P_u = P$

**go to step (2)**.

### 4.2 Conclusion

In this Chapter, we considered the problem of power minimization for energy constrained wireless networks. For the network-coded cooperation scenario under consideration, we presented a joint optimization algorithm which maximizes the total network capacity, and minimizes the transmission power per user, while meeting the constraint on the network performance in terms of the average capacity per user, or the average outage probability per user.

# Chapter 5

# Performance Analysis and Simulation Results

In this chapter, we present the simulation results and performance analysis for the network-coded cooperation frameworkconsidered in this thesis. We first show the performance analysis for the problem of user pairing to maximize the total network capacity, given fixed transmission power. The proposed algorithms are evaluated and compared in terms of the average capacity per user, average outage probability per user, and the per-user throughput fairness.

Performance analysis for the joint optimization problem for power minimization is subsequently presented. The algorithms are evaluated in terms of the average transmission power per user, average capacity per user, average outage probability per user, and the per-user throughput fairness.

The simulation setup is as follows. We use the exponential path-loss model [33] with a reference distance of 1 m, and path-loss exponent of 3.5. The inter-source and uplink channel bandwidth is 10 MHz. The antennas at the mobile stations and the base station are modeled as having absolute gains of 6 and 20 dBi, respectively. The information rate $R = 0.25$ bps/Hz, and the users are uniformly and randomly distributed over a cell of radius 1 km, with the base station situated at the center. Equal power allocation is assumed for all users.

### 5.1 User Pairing for Capacity Maximization: Fixed Power Allocation

In this section, we present the simulation results for the optimal and heuristic user pairing algorithms, which we proposed in Chapter 3, to maximize the cell capacity. The results are averaged over $10^3$randomly generated location sets, and $10^3$randomly generated Rayleigh channel samples per location. All users use a fixed transmission power of 1 Watt. In Fig. 5.1, the average capacity per user is shown versusthe number of users, for the four pairing schemes. As expected, the optimal pairing algorithm, based on the maximum weighted matching, and designed to maximize the cell capacity, yields the maximum throughput per user for all number of users ($N_{users}$), and is therefore used as the benchmark for the heuristic schemes. The optimality of the algorithm was also verified through extensive comparisons with the exhaustive search pairing. From the proposed heuristic pairing algorithms, max-max pairing achieves the closest capacity to the optimal pairing. For $N_{users} = 30$ and 40 for instance, the max-max pairing is shy of the optimal pairing by 6.03 and 6.12 percent, respectively. This performance is achieved approximately four times faster when compared with the optimal pairing in terms of the average simulation times. Weighing the performance degradation against the relative complexities of the two algorithms, max-max pairing emerges as a very good choice for practical implementation. On the other hand, the max-min pairing algorithm is inferior to max-max pairing, and performs worse than random pairing in terms of the average

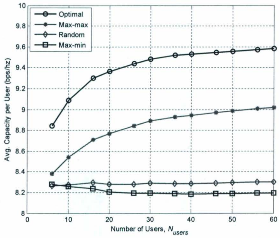



Fig. 5.1. Average capacity per user versus the number of paired users in the cell for the proposed pairing algorithms.

capacity per user. This is anticipated, as max-min pairing is designed to address the outage probability by pairing the strongest user in the cell (in terms of the source-destination SNR) with the weakest one, and the second strongest with the second weakest one etc., which leads to a lower value of average capacity per user.

Though the optimal pairing scheme is designed to maximize the network throughput, it also achieves the best outage performance. Moreover,theoutage performance oriented max-min pairing algorithm matches the optimal algorithm in terms of the average outage probability per user, as they both demonstrate zero outage for all values of $N_{users}$. When compared with the optimal pairing, the max-min pairing achieves this performance approximately forty times faster, as reflected by the average simulation times. Results for the average outage probability per user for the max-max pairing and

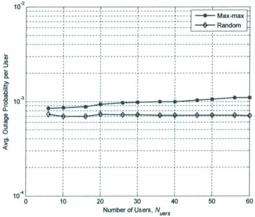



Fig. 5.2. Average outage probability per user versus the number of paired users in the cell for the max-max and random pairing algorithms.

random pairing are depicted in Fig. 5.2. Max-max pairing is observed to perform worse than random pairing for all $N_{users}$. This is owing to the aggressive nature of max-max pairing, which leads to a greater variance and spread within pairs (in terms of throughput), and therefore results in relatively high average outage probability per user.

Fairness performance, measured in terms of the per-user throughput Jain's fairness index, which is defined as $J = \left( \sum_{i=1}^{N_{users}} C_{i,D} \right)^2 \Big/ \left( N_{users} \cdot \sum_{i=1}^{N_{users}} C_{i,D}{}^2 \right)$, is depicted in Fig. 5.3. The optimal pairing demonstrates the best fairness performance and achieves the maximum value of Jain's fairness index, which is around 0.98. This is because the Jain's fairness index is averaged over all location sets, and provides a measure of the long-term fairness. The performance of the heuristic schemes is worse than optimal pairing as both max-max and max-min pairing lead to a greater spread and variance within pairs (in terms of throughput), which leads to lower fairness. The max-max pairing leads to a slightly

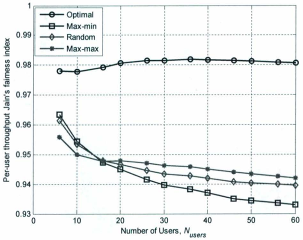



Fig. 5.3. Per-user throughput Jain's fairness index versus the number of paired users in the cell for the proposed pairing algorithms.

better per-user throughput fairness than max-min pairing for most values of $N_{users}$, as max-max pairing is designed to maximize the throughput for pairing users.

### 5.2 Power Minimization: Joint Optimization of Power and Capacity

We herein present the results for power minimization, given certain network performance constraint. The performance constraint is in terms of the average outage probability per user, or the average capacity per user. The pairing is performed to maximize the total network capacity, using the optimal capacity pairing algorithm, outlined in Section 5.1. The power is subsequently minimized using the bisection optimization, such that the network performance constraint is satisfied. The results presented herein are averaged

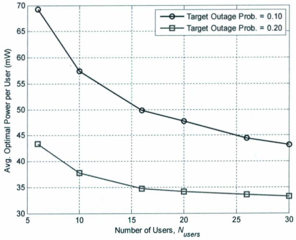



Fig. 5.4. Optimal (minimum) power allocation per user versus the number of paired users in the cell, to meet the constraint on maximum average outage probability per user.

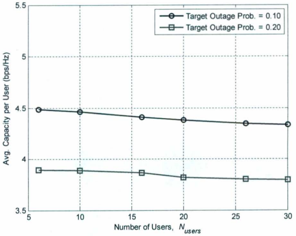



Fig. 5.5. Average capacity per user versus the number of paired users in the cell. The constraint is in terms of the maximum average outage probability per user.

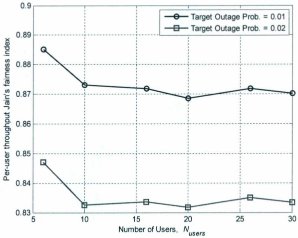



Fig. 5.6. Per-user throughput Jain's fairness index versus the number of paired users in the cell. The constraint is in terms of the maximum average outage probability per user.

over $10^2$ randomly generated location sets and $10^3$ randomly generated Rayleigh channel samples per location.

### 5.2.1 Power Minimization and Capacity Maximization, with a Constraint on Average Outage Probability per User

In Fig. 5.4, the results for optimal power allocation per user (i.e., power minimization) arepresented to meet the network performance constraint of the average outage probability per user of 0.10 and 0.20, with the latter requiring lower power (because of the inverse relationship of transmit power and outage probability). As it is observed, the optimal power decreases monotonically with the number of pairing users. As the number of users increase, the pairing opportunities improve, which allows the threshold outage probability to be achieved with lower power.

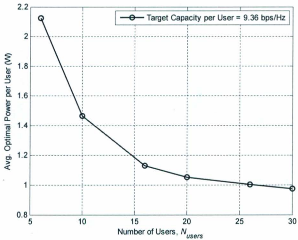



Fig. 5.7. Minimum average transmission power per user versus the number of pairing users. Target capacity per user = 9.36 bps/Hz.

Fig. 5.5 shows the results for average capacity per user versus the number of pairing users. A lower value of outage constraint leads to a higher average capacity, and vice versa, because of the inverse relationship between outage probability and capacity. It is noteworthy that the capacity for a certain outage constraint is steady, as anticipated. However, with a fixed transmission power (i.e., without power minimization), the capacity increases monotonically with the number of users as the pairing opportunities improve.

Results for the per-user throughput Jain's fairness index versus the number of pairing users are presented in Fig. 5.6. For a lower valueof the targetaverage outage probability (meaning thereby a higher average capacity), the Jain's fairness index is higher. This is expected as the variations in the capacity for different users, relative to (a higher value of) average capacity are lower, leading to a higher value of the fairness index.

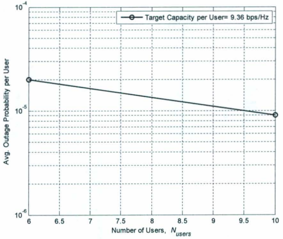



Fig. 5.8. Average outage probability per user versus the number of pairing users. Target capacity per user = 9.36 bps/Hz.

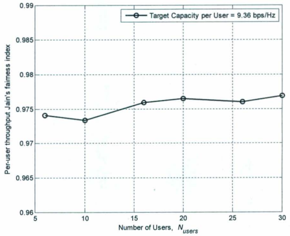



Fig. 5.9. Per-user throughput Jain's fairness index versus the number of pairing users. Target capacity per user = 9.36 bps/Hz.

### 5.2.2 Power Minimization and Capacity Maximization, with a Constraint on Average Capacity per User

Fig. 5.7 shows the results for optimal power allocation (i.e., power minimization) against the number of pairing users, to achieve the threshold average capacity. The value of the threshold capacity is chosen as 9.36 bps/Hz, which is the value achieved with optimal capacity pairing, for a fixed transmission power of 1 Watt, for $N_{users} = 20$ (refer to Fig. 5.1). As expected, the optimal power decreases monotonically with increasing the number of pairing users, or, in other words, with improving the pairing opportunities. An interesting point on the curve is for $N_{users} = 20$, where the optimal power is approximately 1.05 Watts. This point is consistent with the results observed in Fig. 5.1 in Section 5.1, where a fixed power of 1 Watt produced an average capacity of 9.36 bps/Hz, for optimal capacity pairing, for $N_{users} = 20$. The subtle discrepancy is owing to the tolerance of the bisection optimization. The bisection optimization converges to the solution (for optimal power), which can be greater than the true value by as much as a predefined tolerance.

Results for the average outage probability per user are depicted in Fig. 5.8. The outage probability is zero for $N_{users} > 10$, and is therefore not plotted on the logarithmic scale. The outage probability diminishes to zero as the pairing opportunities improve with the increasing number of users.

The per-user throughput Jain's fairness index is shown in Fig. 5.9. For a single channel realization at a particular location set, only the *average capacity per user* should meet the threshold value, as different users in the cell achieve varying capacity. This means that user-fairness for a particular channel realization may not be high. However, the fairness index plotted in Fig. 5.9 is averaged over the location sets, which provides a good measure of the long-term user-fairness. The average per-user throughput fairness is steady, and is close to unity.

### 5.4 Conclusions

In this chapter, we present the simulation results and performance analysis for theproposed framework for the network-coded cooperation in this thesis. We present the results for the algorithms to maximize the total network capacity, with a fixed transmission power. It is observed that the optimal pairing algorithm achieves the best performance in terms of the average capacity per user, average outage probability per user, and the per-user throughput fairness. Of the heuristic algorithms, the max-max pairing approaches the optimal capacity, and demonstrates good fairness, whereas the max-min pairing algorithm matches the optimal pairing in terms of the average outage probability per user.

We then consider joint optimization as we perform power minimization and capacity maximization, given network performance constraints in terms of the average outage probability per user, or average capacity per user. It is observed that the average optimal power per user required to meet the performance constraint decreases monotonically with the number of pairing users, as the pairing opportunities improve.

# Chapter 6

# Conclusions and Future Work

Our novel work presented in this Thesis paves the way towards a practical implementation of network coding in infrastructure-based cooperative wireless networks. The major contributions, conclusions, and future research directions are presented in the following sections.

## 6.1 Contributions of the Thesis

Our key contributions in this Thesis are enumerated as follows:

a) Realization of network coding in infrastructure-based cooperative wireless networks through mutual user pairing, in the absence of dedicated relay nodes,

b) Devising of an optimal mutual user pairing algorithm. In this work, we tailor the optimal pairing algorithm to maximize the network capacity,

c) Designing of the heuristic max-max pairing algorithm to approach the optimal capacity at a significantly reduced computational complexity,

d) Designing of the heuristic max-min pairing algorithm to minimize the outage probability at a reduced complexity, and

e) Capacity maximization and power minimization through joint optimization for energy-constrained network-coded cooperative wireless networks, given network performance constraint in terms of the average capacity or average outage probability.

These are summarized in the next sections.

### 6.1.1 Mutual User Pairing in Infrastructure-based Network-Coded Cooperative Wireless Networks

The design criterion which greatly impacts the performance of cooperative networks is proper relay selection. One of the contributions of our work is addressing the problem of mutual user pairing in an infrastructure-based network-coded cooperative wireless network, where users having data to transmit mutually pair among themselves to realize network coding. We consider a truly multi-user environment, and assume no dedicated relays in the cell. Two nodes constituting a pair periodically swap the roles of the source and relay to mutually achieve spatial diversity. The inter-user channels are modeled as non-ideal (noisy with Rayleigh fading). Conditioned on the successful detection of the source's packet, a network-coded packet is formed at the relay by a linear combination of its own packet and the source's packet. This underlines the significance of the quality of source-relay channel for the performance of network-coded cooperation. A single transmission of this network-coded packet therefore helps both nodes to achieve diversity gain. We assume spatially independent, frequency flat Rayleigh fading channels, with additive white Gaussian noise (AWGN), exhibiting block fading.

### 6.1.2 Optimal User Pairing to Maximize Network Capacity

Our next objective it to perform user pairing to optimize certain network performance metrics, such as average capacity, average outage probability, and/or user-fairness. We propose an optimal user pairing algorithm and tailor it to maximize the network capacity. This is based on the Jack Edmond's maximum weighted matching algorithm in general graphs [30]. We construct a weighted graph where the vertices represent the users to be paired, connected by the edges with weight equal to the sum of the capacities of connected vertices, given that they pair with each other.

This optimal capacity pairing algorithm demonstrates the highest average capacity, lowest average outage probability, and the highest per-user throughput fairness. For networks with smaller number of users and where pairing complexity is not the foremost concern, the optimal pairing is most favourable. The optimality of the algorithm is verified through extensive comparisons with the exhaustive search pairing. The average optimal capacity per user, with a fixed transmission power, increases monotonically with the number of pairing users, as the pairing opportunities improve.

### 6.1.3 Max-max Pairing: Approaching the Optimal Capacity

We subsequently propose heuristic algorithms, designed to approach the optimal performance at a reduced computational complexity. In particular, we first propose max-max pairing to maximize the capacity. It was demonstrated that max-max pairing approaches the optimal capacity (within ~7 percent of optimal capacity for the range of number of users considered in simulations), and exhibits excellent average per-user throughput Jain's fairness index of more than 0.94 for all number of users. The average simulation time of the max-max algorithm was four times lesser than that of the optimal capacity pairing algorithm. Max-max pairing is therefore an excellent choice when high throughput and fairness are desirable, at a reduced computational complexity. However, due to the aggressive nature of max-max pairing to maximize the capacity, the spread among the pairs (in terms of capacity), for a single channel realization is higher, which leads to a higher average outage probability per user.

### 6.1.4 Max-min Pairing: Minimizing the Outage Probability

We then propose max-min pairing algorithm to minimize the outage probability. The max-min pairing matches the optimal pairing in terms of the average outage probability per user, as they both demonstrate zero outage for all channel realizations considered in our simulations. The operation of max-min pairing underlines the fact that the outage performance is dominated by the weaker of the source-relay and relay-destination links. However, since max-min pairing pairs the weakest user in the cell with the strongest user, and the second weaker with the second strongest etc., it demonstrates a lower average capacity per user. Moreover, the max-min pairing is forty times faster than the optimal capacity pairing in terms of the average simulation time. Max-min pairing is therefore preferable for scenarios where the average outage probability is of vital concern with a reduced computational complexity.

### 6.1.5 Power Minimization: Joint Optimization of Power and Capacity

Our next objective is to trade-off the achieved performance gains, in terms of improved throughput and outage performance for power minimization; this is vital for energy-constrained wireless networks, such as sensor and cellular networks. We solve a joint optimization problem to perform capacity maximization and constrained power minimization, given the network performance constraint in terms of the average capacity per user or the average outage probability per user. We use the maximum weighted matching algorithm to obtain the user pairing which maximizes the network capacity. We subsequently use the bisection optimization to obtain the minimum transmission power which meets the network performance constraint. The optimal (i.e., minimum) transmission power to meet the given constraint decreases monotonically with the increase in the number of pairing users. As the number of pairing users increase, the pairing opportunities improve, which allows the performance constraint to be achieved with lower transmission power.

### 6.2 Recommendations for future research

Our novel work on infrastructure-based network coded cooperative networks paves the way towards a practical deployment. Owing to the novelty of this work, there are a number of off shooting research directions.

We consider equal power allocation to all users in the cell. Relaxation of this condition, and consideration of non-equal transmit power is an important future consideration. Moreover, optimization of the rate and power allocation between the first, i.e., direct, and second, i.e., the network coding phases of transmission, depending on the inter-source and source-destination channel states is an intriguing problem for investigation. Furthermore, the design and incorporation of network-channel codes into the considered framework, which can enhance the performance is an interesting problem for future consideration.

**Appendix A.**

We present herein the derivation of Equation 2.10. The outage event for node $i$ is

$$C_{i,D} = \alpha \log_2(1+\gamma_{i,D}) + (1-\alpha)\log_2(1+(\gamma_{i,D}+\gamma_{j,D})) < 2R. \tag{A.1}$$

The probability of outage is

$$P_{i,D} = P\{(1+\gamma_{i,D})^{\alpha}(1+\gamma_{i,D}+\gamma_{j,D})^{1-\alpha} < 2^{2R}\}, \tag{A.2}$$

$$P_{i,D} = \iint_A \frac{1}{\Gamma_{i,D}\Gamma_{j,D}} \exp\left(-\frac{\gamma_{i,D}}{\Gamma_{i,D}} - \frac{\gamma_{j,D}}{\Gamma_{j,D}}\right) d\gamma_{i,D} d\gamma_{j,D}, \tag{A.3}$$

where $A \equiv \{(1+\gamma_{i,D})^{\alpha}(1+\gamma_{i,D}+\gamma_{j,D})^{1-\alpha} < 2^{2R}\}$. Extracting $\gamma_{i,D}$ and $\gamma_{j,D}$ from $A$, and using Taylor's series in two variable, we get the ranges for $\gamma_{i,D}$ and $\gamma_{j,D}$ as

$$0 < \gamma_{j,D} < [2^{2R/(1-\alpha)} - 1] - [2^{2R/(1-\alpha)} + 1]\gamma_{i,D}, \tag{A.4}$$

$$0 < \gamma_{i,D} < [2^{2R/(1-\alpha)} - 1](1-\alpha). \tag{A.5}$$

Using these ranges to solves the integral, we get the outage probability as

$$\overline{P}_{i,D} \approx 2.\frac{(2^{2R/\alpha}-1)^2}{\Gamma_{i,D}\Gamma_{j,D}}(1-\alpha)^2. \tag{A.6}$$

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