FUZZY METHODOLOGY FOR PREDICTION OF OCCUPATIONAL ACCIDENT RATE

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By

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ABSTRACT

An occupational accident is defined as an unexpected and unplanned occurrence arising out of or in connection with work, resulting in personal injury, disease or death. The human cost of occupational accidents is vast and the economic burden of poor occupational health and safety practices is staggering, resulting in the loss of billions of dollars annually.

Over the last 40 years, occupational safety has been regulated under various national legislative schemes to ensure a balanced approach to workplace health and safety issues and to minimize hazards and reduce risk in the workplace. Model development in the research of accidents is considered to be the most effective way of studying the occupational accident issue, providing a proactive approach to address occupational concerns.

The majority of research directed towards occupational accidents is qualitative and relies on the opinions of experts in the ranking of risk. A key component in many occupational accident models lies in the derivation of qualitative data obtained through a survey of safety experts to propose graded or ranked causes of accidents. The subjective nature of expert opinion or judgements introduces a degree of uncertainty within the analytical process. This work focuses on the development of a fuzzy methodology which is aimed to enhance the effectiveness of accident models by providing a mathematical tool to account for vagueness and uncertainty associated with expert judgements and opinions and to capture this uncertainty within the analysis. The novelty of the proposed methodology lies in an approach that embraces uncertainty as an inseparable element of the system. The proposed methodology recognizes that uncertainty

plays a role in decision making and uses fuzzy set theory to account for and minimize uncertainty associated with the subjective nature of expert opinions. The fuzzy methodology will be incorporated into a predictive model developed to predict the frequency of occupational accidents and associated costs within the oil and gas industry.

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INTRODUCTION

1.1 OCCUPATIONAL ACCIDENT ANALYSIS

An occupational accident is defined as an unexpected and unplanned occurrence arising out of or in connection with work, resulting in personal injury, disease or death. According to the International Labour Office (ILO), over 337 million accidents occur on the job site annually, resulting in 6300 fatalities per day and more than 2.3 million fatalities per year.

Safety and health conditions vary considerably between countries, economic sectors and social groups. The human cost of occupational accidents is vast and takes a particularly heavy toll in developing countries. The ILO estimates that the economic burden of poor occupational health and safety practices is at 4% of the global Gross Domestic Product (GDP) each year. In a study conducted by the UK's Health and Safety Executive (HSE), it was established that up to £31.8 billion ($50.4 billion CAD) was lost in 2001/02 due to accidents at work and work-related illnesses (http://www.hse.gov.uk/statistics/pdf/costs.pdf).

Many accident models have been developed in an attempt to address the occupational accident issue. The effectiveness of models in the research of accidents is considered to be the most suitable way of studying the occupational accident issue (Attwood *et al.,* 2006b). Early accident models were largely qualitative in nature relying on opinions and case studies to propose graded or ranked causes of accidents. Some accident models adopted a statistical approach to study relationships between factors affecting occupational accidents while others provided a vehicle to produce improvements in areas of the working environment (Attwood *et al.*, 2006b). In an attempt to provide a holistic view of the occupational accident issue, Daryl Attwood *et. al* developed a quantitative model to predict the frequency of occupational accidents and their associated costs. Attwood's model allows operators to optimize management decisions, provides stakeholders with a tool to predict accident frequency under their specific regime and offers the capability to compare predicted safety improvements resulting from changes in various safety elements.

1.2 OVERVIEW OF ATTWOOD'S PREDICTIVE MODEL

Attwood's model takes a quantitative, holistic approach to predict the frequency of occupational accidents and includes the identification of constituent factors affecting accidents and the determination of their interrelationships. The model consists of three fundamental layers; (1) Direct Layer; (2) Corporate Layer; and (3) External Layer. The direct layer consists of five main components considered to directly affect the frequency of occupational accidents which include an individual's behaviour and capabilities, weather, safety design and personal protective equipment. Individual behaviour is divided into attitude and motivation. Individual capability is divided into mental and physical. Mental capability is further divided into knowledge and intelligence while physical capability is sub-divided into coordination, fitness and lack of fatigue. The corporate layer includes corporate safety culture, safety training programs and safety procedures. The external elements include the value placed on human life, commodity price, shareholder's pressure and royalty regime. The basic premise of Attwood's model states that worker's behaviours are influenced by corporate culture, their environment and procedures are controlled corporately and corporate decisions and actions are influenced by external elements.

Attwood's model uses a modified reliability network to model the accident process. The overall safety system can be subdivided into sub-systems or sub-sets which can be configured in a 'series' or 'parallel' set-up. The direct layer's five main elements are connected in a series formation while the direct element subsets, for example attitude and motivation, are connected in a parallel arrangement. The reliability of the overall system is calculated from the direct elements. The model accounts for the fact that not all elements affect overall safety performance equally. The strength or relative importance of the five main elements directly affecting accidents are quantified using information gained from a panel of experts within the industry. These relative importance values are then used within the mathematical model. Matrices of influence coefficients were generated from the panel of safety experts to rank each external and corporate element's level of influence on the corporate and direct level factors, respectively. This is consistent with the model's philosophy that external elements affect corporate decisions and actions and these, in turn, influence factors directly affecting the accident process.

Within the model, overall system reliability is a function of the direct layer component's reliabilities. The corporate element reliabilities can be determined from external element values therefore predictions can be made on a basis of a complete set of direct, corporate or external element reliabilities. Once the system reliability has been calculated, the expected accident rate is calculated, usually to obtain the number of accidents per year. The model also provides a method to evaluate cost savings associated with accident frequency reduction. The cost element is determined by multiplying the cost of an accident by the expected number of accidents.

1.3 MODELING UNCERTAINTY

Attwood's predictive model uses quantitative data derived from a survey of safety experts to rank each component's effect on safety within the direct, corporate and external layers. The subjective nature of expert judgements and opinions introduces a degree of uncertainty within the model. This level of uncertainty attached to the integration of subjective evaluations is a concern when analyzing systems through model development. In analyzing the frequency of occupational accidents it is important to determine how uncertainty should be included in the assessment model.

Fuzzy set theory (FST) provides a useful tool to address the uncertainty associated with the subjectivity of expert opinions and to propagate uncertainty through the model. Its purpose is to allow one to better model phenomena that exhibit a certain kind of uncertainty, degree-vagueness (Smithson & Verkuilen, 2006). "The utility of FST in model development has been seen in its ability to more appropriately represent the human-inferencing process and to provide a more user-friendly interface through the use of natural language" (Zadeh, 1996). Attwood's predictive model was used to illustrate the use of FST to address uncertainty because it offers a comprehensive, logical framework and provides deductive analysis to the occupational accident issue.

Lotfi A. Zadeh introduced the term "computing with words" to explain the notion of reasoning linguistically rather than with numerical quantities. "Humans use natural language as a means of computing and reasoning, arriving at conclusions expressed as words from premises expressed in natural language" (Zadeh, 1996). The use of natural language or linguistic variables within the framework of models may be a necessity when available information is too imprecise to justify the use of numbers. The use of linguistic variables within Attwood's predictive model allows experts to rank the importance of factors in a natural way, providing a more user-friendly approach to the analysis process. The linguistic variables offer an intuitive meaning which is particularly useful when relying on a panel of experts, while fuzzy numbers are used for the "internal implementation of reasoning mechanisms" (Baroni *et al.*, 1998). The incorporation of fuzzy set theory within the model provides a useful tool to account for the uncertainty associated with expert judgements and opinions and reduces the ambiguity and imprecision arising out of the subjectivity of this data.

1.4 FUZZY SET THEORY

Fuzzy set theory is a mathematical framework to account for fuzziness or uncertainty. The term fuzziness is used to describe an uncertain state in which the transition between the state of concern and its complement is gradual hence it is difficult to make a sharp distinction (Kikuchi, 1998). Fuzzy set theory is an extension of classical set theory and was first introduced by Lotfi A. Zadeh, a mathematician and computer scientist of Iranian Azeri origin. Zadeh introduced the concept of a fuzzy set which is a set whose boundary is not sharply defined. This concept contrasts with the classical concept of a set, a crisp set, whose boundary is required to be precise. That is, a crisp set is a collection of things for which it is known whether any given thing is inside or outside the set. The boundaries of classical sets are precise therefore a set membership is determined with complete certainty. "A fuzzy set is based on a classical set, but it adds one more element: a numerical degree of membership of an object in the set, ranging from 0 to 1" (Smithson & Verkuilen, 2006). Contrary to classical crisp sets, fuzzy sets do not have sharp boundaries therefore a member of a fuzzy set may belong to the set to a greater or lesser degree.

"One of the principal motivations for introducing fuzzy sets is to represent imprecise concepts" (Klir, 1997). Virtually all human activities involve reasoning based on vague concepts and incomplete information therefore FST plays a key role in bridging the gap between imprecise concepts which are used to describe reality and precise classical mathematics. An individual's membership in a fuzzy set is a matter of degree therefore the degree of membership of an individual in a fuzzy set expresses the degree of compatibility of the individual with the concept represented by the fuzzy set (Klir, 1997). It is therefore important in each application of FST to construct appropriate fuzzy sets and their associated membership functions that adequately capture the intended meaning of the concept being analyzed.

A major contribution of FST is its capability of representing vague data and modeling uncertainty. FST has been used to model systems that are hard to define precisely, incorporating imprecision and subjectivity into the model formulation and solution process. "FST represents an attractive tool to aid research in areas where the dynamics of the decision environment limit the specification of model objectives, constraints and the precise management of model parameters" (Kahraman, 2006). Since 1965, FST has proven to be a powerful tool for representing quantitatively and manipulating the imprecision of decision-making problems in engineering, business, medicine, manufacturing among many other industrial sectors.

1.4.1 DEFINITIONS AND CONCEPTS OF FST

- **Fuzzy Number:** A fuzzy number is described in terms of a number word and a linguistic modifier, such as *approximately*, *nearly* or *around*. A fuzzy number is used when quantifiable phenomena cannot be characterized in terms of absolutely precise numbers.

- **Fuzzy Logic:** Fuzzy logic is viewed as a system of concepts, principles and methods for dealing with modes of reasoning that are approximate rather than exact.
- **Linguistic Variables:** A linguistic variable is a verbal quantifier denoted by a full name, such as "*several*" or "*extremely unlikely*" which carries an intuitive meaning to describe a vague concept. Linguistic variables can be made precise using FST by creating a fuzzy number defined on the interval [0, 1].
- **Membership Functions:** A membership function is an index of "sethood" that measures the degree to which an object x with property A is a member of a particular defined set. It assigns to each element x of X a number, $A(x)$, in the closed unit interval [0, 1] that characterizes the degree of membership of x in A. Membership functions are functions of the form:

$$\mathrm{A}: \mathrm{X} \rightarrow [0,1]$$

- **Fuzzy Union:** Fuzzy union is defined as the maximum degree of membership in the sets. Membership in the union of $X \cup Y$ may be written as $m_{X\cup Y} = \max(m_X, m_Y)$.
- **Fuzzy Intersection:** Fuzzy intersection is defined as the minimum degree of membership in the sets. Membership in the intersection of $X \cap Y$ may be written as $m_{X\cap Y} = \min(m_X, m_Y)$.
- **α – Cuts of Fuzzy Sets:** The α – cut of a fuzzy set A is the crisp set ${}^{\alpha}A$ that contains all the elements of the universal set X whose membership degrees in A are *greater than or equal to* the specified value of α. It is a means of restricting membership degrees that are greater than or equal to some chosen value α in [0, 1].

- **Extension Principle:** The extension principle is a principle for fuzzifying crisp functions. It is a method of extending point-to-point operations to fuzzy sets and is the basic tool for the development of fuzzy arithmetic.
- **Aggregation of Fuzzy Numbers:** When using multiple experts to rank the importance or influence of particular elements with linguistic variables, it is necessary to aggregate their opinions in order to achieve a more reliable assessment. There are many methods available to aggregate expert's opinions such as the arithmetic averaging operation, fuzzy preference relations and max-min Delphi method.
- **Arithmetic Averaging Operation:** The most commonly used method to aggregate expert's opinions. The arithmetic averaging operation satisfies two characteristics of rational combination: (1) a small variation in any possibility distribution does not produce a noticeable change in the combined possibility distribution; and (2) when experts are equally weighted it can also include weights that contain the relative importance of one expert to another (Huang, 1998).
- **Fuzzy Probability:** Fuzzy probability is a fuzzy number, which is expressed by a fuzzy set and characterized by its membership function μ. It can be represented by a triangular or trapezoidal shape or bell shaped membership function (Cheng, 2000).
- **Fuzzy Error Possibility:** Fuzzy error possibility is essentially a fuzzy probability characterized by a membership function to account for the uncertainties of fuzzy data.
- **Fuzzy Error Factor:** Fuzzy error factor accounts for uncertainties and vagueness associated with fuzzy outcomes and can be calculated from the fuzzy error possibility.

- **Fuzzy Uncertainty Index (FUI):** Fuzzy uncertainty index can measure the amount of uncertainty an event contributes to the final outcome. It is an index used to help the analyzer decide on which fuzzy data to collect so that uncertainties can be lowered.
- **Total Recordable Injury (TRI):** Total Recordable Injury is a group of injuries which include fatalities and lost time injuries, medical aid injuries and restricted work injuries.
- **Total Recordable Injury Rate (TRIR):** Total Recordable Injury Rate is a calculated statistic to track the frequency rate of lost time injuries (LTI), medical aids (MA) and restricted work cases (RWC).

$$TRIR = [(LTI + RWC + MA)\, x\, 200{,}000]/Exposure\ Hours$$

- **Defuzzification:** Defuzzification is the process of combining all fuzzy outputs into a specific composite result. Its purpose is to convert the fuzzy set into a real (crisp) number, that best represents the fuzzy set. Several methods exist for the defuzzification process including centre of area method, centre of maxima method, mean of maxima method and weighted average defuzzify method.

1.5 PROBLEM STATEMENT

Daryl Attwood *et. al* (2006c) developed a holistic, quantitative model to predict the frequency of occupational accidents and their associated costs in the offshore oil and gas industry. The basic premise of Attwood's model states that worker's behaviours are influenced by corporate culture, their workplace environment and procedures are controlled corporately and corporate decisions and actions are influenced by external elements (Attwood *et al.,* 2006c). The frequency of occupational accidents is related to factors having a direct impact on the process such as individual capabilities and behaviours and the quality of personal protective equipment. Many of these direct factors are influenced by decisions made at a corporate level while corporate culture is influenced by external elements such as commodity price and royalty regime. The predictive model consists of three fundamental layers and uses quantitative data derived from a survey of safety experts to rate each factor's effect on safety for the specific environment as compared to the global average. The subjective nature of expert opinions used to derive this quantitative data introduces a degree of uncertainty within the model. The use of a numerical scale to rate each components effect on safety has significant drawbacks, one of which concerns the precision to be ascribed to a numeric value. It is also unnatural for an expert to express a judgement in numerical terms and is generally much easier and more reliable to use linguistic variables, such as *very low*, *low*, *high*, than to choose a number in the real interval 1-10 (Baroni & Guida, 1998).

The use of fuzzy linguistic variables within Attwood's predictive model allows experts to rank each factor in a natural way, providing a more user-friendly, intuitive approach to the analysis process. The linguistic variables are converted into fuzzy numbers which carry more information than a crisp, numerical rating factor and allow the judgemental uncertainties associated with experts' subjective opinions to be properly expressed. These fuzzy numbers are characterized by membership functions which incorporate the uncertainty of the component.

The incorporation of FST into Attwood's predictive model is aimed to enhance the effectiveness of the model by providing a mathematical tool to account for vagueness and uncertainty associated with expert judgements and opinions and to effectively propagate this uncertainty through the model. The proposed methodology recognizes that uncertainty plays a role in decision making and incorporates a fuzzy approach to account for and minimize uncertainty while maintaining the simplicity of Attwood's model.

1.6 OBJECTIVES OF PRESENT WORK

The preceding discussion indicates that fuzzy set theory is a useful tool to account for uncertainty arising from the subjective nature of expert opinions. A key component of Attwood's predictive model lies in the derivation of quantitative data obtained through a survey of safety experts but the model does not determine how this uncertainty should be included in the assessment and analysis process. A methodology for the predictive model has been developed to address this uncertainty and propagate it through the model, providing a more user friendly, intuitive approach to the analysis process.

The objectives of this research are:

- To successfully incorporate fuzzy set theory into the framework of Attwood's predictive model to account for and minimize uncertainty associated with the subjective nature of expert opinions through:
    - the development of a linguistic, qualitative scale of importance to rate each components effect on safety for the specific case, providing an intuitive, user-friendly approach to the analysis process;
    - conversion of linguistic variables into fuzzy numbers through development of membership functions;

  - the aggregation of fuzzy numbers and the calculation of fuzzy error probability and fuzzy error rate;
  - propagation of fuzzy numbers to adjust component reliabilities to determine fuzzy outcomes;
  - the use of fuzzy operations to calculate a crisp numeric output (defuzzification) and estimate the degree of uncertainty each component contributes to the final outcome.
- To analyze the precision and error robustness of the fuzzy approach.

1.7 ORGANIZATION OF THESIS

This thesis is divided into six chapters. The first chapter gives a broad overview of the occupational accident analysis, Attwood's predictive model, fuzzy set theory and its significance within the modeling of uncertainty. Further, it describes FST with some basic definitions and concepts, followed by the objectives of this work.

Chapter 2 presents a detailed description of Attwood's predictive model including why it was chosen for this study, a discussion of specific elements within the model and method of analysis used to predict the frequency and associated costs of occupational accidents. Chapter 3 provides a discussion on FST, including some applications of FST within the industrial sector. This chapter also provides an overview of fuzzy set mathematics which will be used within the proposed methodology. Chapter 4 discusses modeling details and outlines the steps involved to incorporate FST within the predictive model to account for uncertainty. Chapter 5 presents a case study to illustrate the use of FST within Attwood's predictive model. Chapter 6 offers a summary, final concluding remarks and recommendations for future work.

Chapter 2

QUANTITATIVE ANALYSIS OF OCCUPATIONAL ACCIDENTS

2.1 OVERVIEW OF EARLY ACCIDENT ANALYSIS

Occupational accident hazards are associated with everyday work activity and are a major contributor to individual risk. According to industry accident statistics, a workers' potential for injury or death from occupational accidents is at least as high as that associated with major accidents such as fires and explosions (Attwood *et al.*, 2006a). In a study conducted by the UK's Health and Safety Executive (HSE), it was established that in 2001/02 occupational accident failures cost the British economy between £13.1 - £22.2 billion ($20.8 - $35.2 billion CAD) and cost society as a whole between £20.0 - £31.8 billion ($31.7 - $50.4 billion CAD) (http://www.hse.gov.uk/statistics/pdf/costs.pdf). A total of 28.5 million working days were lost in 2009/2010 of which 23.4 million days were lost due to work-related ill health and 5.1 million days were lost due to workplace injury (http://www.hse.gov.uk/statistics/index.htm).

Model development is one way of attempting to understand and positively affect a problem. The effectiveness of models in the study of accidents has been noted by several authors and model development is considered to be the most suitable way of studying the occupational accident issue (Attwood *et al.*, 2006c). Early accident models had made significant progress in the quantification of risks associated with catastrophic events such as fires and explosions but the majority of research directed towards occupational accidents was largely qualitative with opinions and case studies used as input data to propose graded or ranked causes of accidents. Some accident models adopted a statistical approach using historical data to study existing relationships between factors as opposed to offering a predictive model to help guide management decisions. Other models provided a vehicle to produce improvements in specific areas of the working environment but did not adopt a holistic view of the occupational accident problem (Attwood *et al.,* 2006b).

In an attempt to address the occupational accident issue, Daryl Attwood et.al developed a holistic, quantitative model based on reliability techniques and capabilities of predicting occupational accident frequency in the offshore oil and gas industry. One of the main objectives of Attwood's research was to apply a quantitative approach to the prediction of occupational accident frequency which had been largely qualitative in nature.

Attwood's predictive model was selected for this study because of its ability to provide deductive analysis to the occupational accident issue. The model provides a comprehensive, structural framework that offers a logical formulation for predicting occupational accident frequency and highlighting areas that require attention in order to improve overall safety performance, thus allowing operators to optimize management decisions, including the choice of where to allocate monies to improve overall safety.

The model uses quantitative data from published sources that are easily accessible and readily available to the public. The model execution methodology involves a calibration run which uses known accident rates and a predictive mode following adjustments to base case reliabilities. The degree of adjustment is determined using quantifiable comparisons of safety conditions in specific and base cases which are easy to monitor and assess.

2.2 ATTWOOD'S PREDICTIVE MODEL

The basic premise of Attwood's model states that "worker's behaviours are influenced by corporate culture and their workplace environment and procedures are controlled corporately. Furthermore, corporate decisions and actions are influenced by external elements" (Attwood *et al.,* 2006c). The predictive model consists of three fundamental layers - direct layer, corporate layer and external layer. The specific elements of the model are outlined in Figure 2.1.

<table>
<tr><th colspan="2">External Layer</th><th>Corporate Support Layer</th><th colspan="3">Direct Layer</th></tr>
<tr><td colspan="2">Value placed on human life</td><td>Corporate Safety Culture</td><td colspan="2" rowspan="2">Individual Behaviour</td><td>Attitude</td></tr>
<tr><td rowspan="4">Financial Drivers</td><td>Price of Oil</td><td>Safety Training Program</td><td>Motivation</td></tr>
<tr><td>Shareholder Pressure</td><td>Safety Procedures</td><td rowspan="5">Individual Capability</td><td rowspan="2">Mental</td><td>Knowledge</td></tr>
<tr><td rowspan="2">Royalty Regime</td><td rowspan="7"></td><td>Intelligence</td></tr>
<tr><td rowspan="3">Physical</td><td>Coordination</td></tr>
<tr><td colspan="2" rowspan="5"></td><td>Fitness</td></tr>
<tr><td>Lack of fatigue</td></tr>
<tr><td colspan="3">Weather</td></tr>
<tr><td colspan="3">Safety Design</td></tr>
<tr><td colspan="3">Personal Protective Equipment</td></tr>
</table>

FIGURE 2.1 - SPECIFIC ELEMENTS OF MODEL (ATTWOOD *ET AL.,* 2006A)

The model takes a holistic approach to accidents and includes the identification of constituent factors and the determination of their interrelationships. Direct factors affecting the occupational accident issue include human behaviours and capabilities, weather, safety related design and quality of personal protective equipment. Many of these direct factors are influenced by decisions taken at the corporate layer which include corporate safety culture, safety training programs and safety procedures. The model includes an external layer that includes region-based cultural and financial pressures, which are considered to influence corporate actions and decisions, which in turn, directly affect occupational accident frequency. The architecture of the model is shown in Figure 2.2.

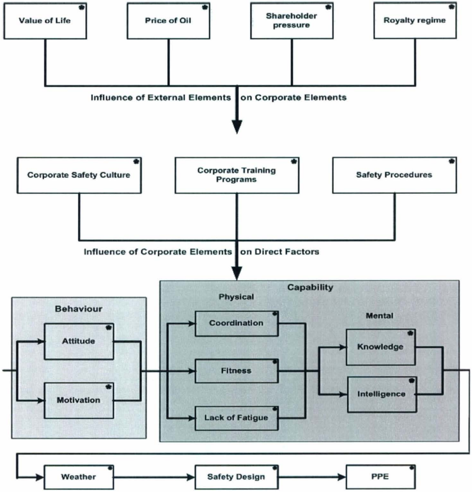


Figure 2.2 - Model Structure (Attwood *et al.*, 2006a)

Each of the three layers (direct, corporate, external) are divided into various components which were chosen based on discussions with offshore oil and gas colleagues, Attwood's personal experience from years of working in the field and a comprehensive literature review. The goals of such a comprehensive review were to understand and critically assess previous approaches to the occupational accident problem, to identify gaps in the occupational accident knowledge and to systematically consider the factors thought by other researchers to affect occupational accident frequency (Attwood *et al.,* 2006a). The following sections outline the components of each of the three layers with a description of each.

2.2.1 DIRECT LAYER

The five components considered to directly affect accident frequency are:

- Worker's behaviours
- Worker's capabilities
- Weather conditions
- Safety Related Design
- Personal Protective Equipment

Behaviours are personal choices which are influenced by one's attitude and motivation. Attitude is a person's perspective toward a specific target. Most attitudes are the result of either direct experience or observational learning. Motivation is the set of reasons that determines one to engage in a particular behaviour (Attwood *et al.,* 2006a).

Capabilities are the abilities to perform actions and are divided into mental and physical. Mental capabilities are of two categories, knowledge-based and intelligence-based. The knowledge-based component comprises the safety related information retained by the worker following training sessions. The intelligence-based component allows the worker to cope with safety issues not specifically covered by training and procedures. The physical capabilities associated with avoiding occupational accidents are considered to be good coordination, a reasonable degree of fitness and lack of fatigue (Attwood *et al.,* 2006a).

Weather conditions can directly affect the likelihood of accidents by creating hazardous working conditions that often lead to accidents on the job-site. Inclement weather conditions and extreme temperatures can also affect worker concentration, increasing the likelihood of accidents. The optimization of safety related design can reduce accident frequency. Non-slip walkways and visible warning signs are examples of measures taken to improve workplace safety. Personal protective equipment (PPE), including safety boots, hard hat and safety glasses can provide protection of the individual when working and prevent a serious injury from occurring.

2.2.2 CORPORATE LAYER

The second fundamental layer is the safety related support provided by the company, comprised of:

- Corporate Safety Culture
- Safety Training Programs
- Safety Procedures

Corporate culture is the moral, social and behavioural norms of an organization, based on the beliefs, attitudes and perceptions of its employees. Most companies expend considerable effort in developing a strong, positive safety culture in an attempt to create a healthy, safe environment. Safety training programs such as accident investigation, emergency preparedness and hazard management, provide the basic structure of an effective health and safety system while safety procedures, such as measurement, accountability, planning and organization, help incorporate the safety training programs into a successful safety system.

2.2.3 EXTERNAL LAYER

The third fundamental layer is referred to as the external layer which consists of societal pressures such as the value placed on human life and financial drivers such as commodity price, shareholder pressure and royalty regime. Attwood's model supports the belief that fundamental change requires improvement, at least, at the corporate level, which is, in turn, driven by external factors (Attwood *et al.,* 2006a).

Societal expectations differ throughout the world and the associated forces affect an organizations safety results. Some regions place a higher value on a human life than others. Financial pressures originate from several sources, including price of commodities, corporate shareholder pressure and royalty regime. When the commodity price is low, there is an increased pressure to 'cut corners' everywhere and this includes the quality of safety programmes enacted by operators (Attwood *et al.,* 2006a).

Shareholders collectively own the company and have potential to profit if the company performs well but also have the potential to lose if the company performs poorly. Therefore shareholders often exert a degree of pressure on company directors and management to improve performance and maximize profits. Royalty regime is the system of governance over revenues and profits and it is largely region-specific. Royalty regime may be designed to recognize inherent risks and provide arrangements for future projects.

2.3 METHOD OF ANALYSIS

Attwood's model uses a modified reliability network to model the accident process. The concept of using a reliability network originated with the recognition that ' similar to a physical engineering system, safety programme success depends on the reliability of individual components' (Attwood *et al.,* 2006a). Individual components of a safety programme perform at different levels of reliability and system improvements are usually enacted by making improvements to these components.

The overall safety system can be subdivided into sub-systems or sub-sets which can be configured in a 'series' or 'parallel' set-up as outlined in Figure 2.3. With a series configuration, the reliability of the sub-set is the product of component reliabilities with sub-set reliability always less than that of the least reliable component. This corresponds to the concept that for some sub-sets of the safety system, all elements must be operating relatively efficiently to produce a satisfactory result. In a series configuration, the weakest sub-set controls the performance of the system. A failure of any component results in failure for the entire system. With a parallel set-up, the reliability of the sub-set is calculated by subtracting the product of component probabilities of failure from unity and the reliability of the sub-set is always greater than the most reliable component. This corresponds to the concept that for some sub-sets of the safety system, poor performance in some elements can be compensated for by superior performance by others within the sub-set. In a parallel configuration, the strongest sub-set controls the performance of the system. At least one of the units must succeed for the system to succeed. Units in parallel are also referred to as redundant units. Redundancy is a very important aspect of system design and reliability in that adding redundancy is one of several methods of improving system reliability. In a parallel system, all n units must fail for the system to fail. If unit 1 succeeds or unit 2 succeeds or any of the n units succeeds, the system succeeds.

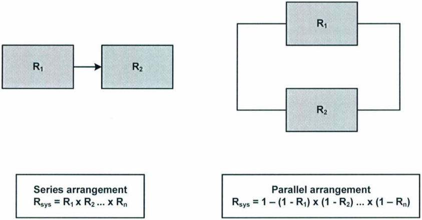


FIGURE 2.3 - SERIES VERSUS PARALLEL SUBSETS (ATTWOOD ET AL., 2006A)

The direct layer elements are connected in a reliability network. The reliability of the overall safety system is calculated from these direct elements. The five main direct elements (behaviour, capabilities, weather, safety design and personal protective equipment) are connected in a series configuration, reflecting the belief that all must work well in an efficient safety programme. The direct element subsets, for example motivation and attitude, are connected in parallel arrangements, reflecting the belief that a degree of compensation is available in the process.

The model accounts for the fact that not all elements affect overall safety performance equally. Consistent with the overall model structure choices, levels of importance have been made on a layer by layer basis. The five main direct elements affecting accidents and their sub-sets were quantified based on a survey of safety experts. The model uses quantitative data derived from these surveys of safety experts to account for the differing relative importance of factors.

Experts are asked to assess, using a 1-10 scale, each direct element's ability to affect occupational accident frequency. Results are then normalized to ensure that the relative importance of each element within each group is extracted in a consistent manner. The resulting 'relative importance' values, as displayed in Figure 2.4, are then used within the mathematical model, utilizing a process of 'strengthening' or 'weakening' individual components in the reliability network, with a likeness to adding redundant units to a physical system. Following the normalisation process, the strengths of components within the following subgroups sum to unity:

- Primary Direct Level - behaviour, capability, weather, safety design, personal protective equipment
- Behavioural Subgroup - attitude, motivation
- Capability Subgroup - mental, physical
- Mental Capability Subgroup - knowledge, intelligence
- Physical Capability Subgroup - coordination, fitness, lack of fatigue

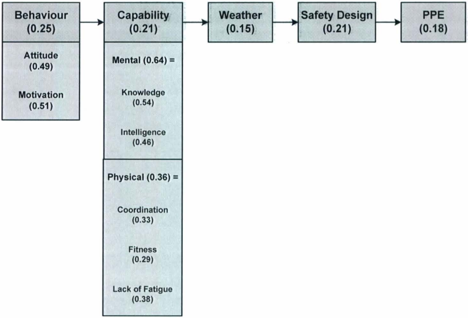


FIGURE 2.4 - ELEMENT STRENGTHS (ATTWOOD *ET AL.*, 2006A)

2.3.1 INFLUENCE AT MODEL INTERFACES

The model philosophy proposes that external elements affect corporate decisions and actions and these, in turn, influence factors directly affecting the accident process. Matrices of 'influence coefficients' were generated for the external-corporate and corporate-direct interfaces. Information gained from a survey of safety experts was used to quantitatively assess each external elements level of influence on corporate factors and each corporate elements level of influence on the direct factors affecting the accident process.

The external-corporate and corporate-direct influencing coefficients, as outlined in Tables 2.1 and 2.2 respectively, are used to adjust lower level (corporate) element reliability whenever the higher level (external) values change. For example, corporate safety culture is influenced by the external factors 'value placed on human life', 'price of oil', 'shareholder pressure' and 'royalty regime'. The reliability of corporate safety culture is automatically increased with increases to either one of the external factor values. The lower (corporate) level reliability is the sum of the products of the external level reliabilities considered to have an effect on the corporate element and the respective influencing coefficients. To illustrate the reliability calculation, assume the initial reliabilities of the external factors to be 0.60, 0.50, 0.40 and 0.60. The reliability for safety training is calculated as shown in Table 2.3. Improvements to any of the direct factors affecting the accident process may be made in isolation of changes in the more senior elements (external, corporate elements).

Table 2.1 - External-corporate influencing coefficients (Attwood *et al.*, 2006a)

	Safety Training	Safety Procedures	Corporate Safety Culture
Value placed on human life	0.43	0.43	0.44
Price of oil	0.18	0.19	0.18
Shareholder pressure	0.27	0.26	0.25
Royalty regime	0.12	0.12	0.13

Table 2.2 - Corporate-direct influencing coefficients (Attwood *et al.*, 2006a)

	Attitude	Motivation	Fitness	Lack of Fatigue	Coordination	Knowledge	Intelligence	Weather	Safety Design	PPE
Training	0.33	0.33	0.34	0.31	0.00	0.36	0.00	0.00	0.31	0.31
Procedures	0.30	0.30	0.30	0.31	0.00	0.30	0.00	0.00	0.32	0.33
Safety culture	0.37	0.37	0.36	0.38	0.00	0.34	0.00	0.00	0.37	0.36

Table 2.3 - Method of external element influence on corporate elements (Attwood *et al.*, 2006a)

Safety Training Reliability	**Component Reliability**	**Influencing coefficient**	**(Component reliability) x (Influencing coefficient)**
Value placed on human life	0.60	0.43	0.26
Price of oil	0.50	0.18	0.09
Shareholder pressure	0.40	0.27	0.11
Royalty regime	0.60	0.12	0.07
Sum of the products = reliability value = (0.60 x 0.43) + (0.50 x 0.18) + (0.40 x 0.27) + (0.60 x 0.12) = 0.53			**0.53**

2.3.2 RELIABILITY CALCULATION

Overall system reliability is a function of the direct layer component reliabilities. Direct layer component reliabilities can be directly input, intentionally over-written or determined from the corporate element reliabilities using the method of influence at the corporate-direct interfaces (sum of the product of corporate reliabilities and corporate-direct influence coefficients). Corporate element reliabilities can, in turn, be determined from external element values, which is consistent with the model's holistic approach to the accident process and the belief that accidents are caused directly at the workplace but are affected by corporate and external elements.

Predictions on the frequency of occupational accidents can be made on the basis of a complete set of direct, corporate or external element reliabilities.

Once component reliabilities have been assigned, system reliability is calculated according to the method based on standard reliability theory as shown below.

$$R_{sys} = (R_b)_{sb} \times (R_c)_{sc} \times (R_w)_{sw} \times (R_{sd})_{ssd} \times (R_{ppe})_{sppe}$$

where:

R_b = *reliability of behaviour*
R_c = *reliability of capability*
R_w = *reliability of weather*
R_{sd} = *reliability of safety design*
R_{ppe} = *reliability of personal protective equipment*
sb = *strength of behaviour*
sc = *strength of capability*
sw = *strength of weather*
ssd = *strength of safety design*
sppe = *strength of personal protective equipment*
Equations to calculate the above reliabilities are outlined in Appendix A.

Once system reliability has been calculated, the expected accident rate (accidents per year) is calculated according to the reliability model as shown below.

$$R(t) = exp\,[-\int_0^t \lambda dt] = e^{-\lambda t}\,,\ t > 0$$

where:

λ = *accident rate*
$R(t)$ = *system reliability*
t = *time*

Taking the natural logarithms of both sides and setting t = 1 we get:

$$\lambda = -\ln(R(t))$$

This approach is based on the assumption of constant failure rate. With applying the philosophy of constant failure rate to offshore occupational accident frequency, Attwood believed the parallel could be drawn that until accident causation became relatively well understood, the accident rate was relatively high. However, evidence exists to confirm that the industry has reached a state of relatively constant accident rate which validates the required constant failure rate assumption (Attwood *et al.,* 2006a).

The model also provides a method to evaluate cost savings associated with accident frequency reductions. The cost element is determined by multiplying the cost of an accident by the expected number of accidents.

2.4 MODEL EXECUTION METHODOLOGY

As noted, the safety system is treated as a modified reliability network. Component reliability values determine overall system reliability, which is used to predict accident frequency. Model development was based on a review of related literature, expert opinion and reliability analysis concepts. Quantitative inputs are required to adjust component reliability for direct, corporate and external factors. The model also uses quantitative data derived from expert opinion surveys to account for the differing relative importance of factors and the influences of external elements on corporate actions and of corporate actions on the direct accident process.

The accident frequency predictive process requires the model to be run in two distinct modes: 1) Calibration Mode where known accident rates are used to determine base case component reliabilities; and 2) Predictive Mode where accident frequency is predicted for a specific case. The model can be used for a variety of purposes such as to compare the number of predicted and actual annual accidents or to compare the predicted and actual lost time incident rate (LTI). It can also be used to compare safety results in an 'ideal' environment to those obtained in a 'worst case' scenario. A discussion of the model execution process is described below.

2.4.1 CALIBRATION MODE

Calibration of the model is required to determine base case component reliabilities because the subsequent predictive model run requires a comparison of specific and base cases. A base case is chosen to be any situation where both safety results and operating conditions are known. The type of accident statistic used for calibration depends on which output statistic is desired. For example, if a particular accident rate in a region or industry is required, then the corresponding global average value of that particular rate is used for calibration. If the expected annual number of a specific kind of accident on an installation having a given POB (person on board) is required, then the global average rate of that type of accident is combined with the POB to determine accident numbers expected had the facility been operating under average safety conditions (Attwood *et al.,* 2006a).

Using the global average accident statistics, an accident rate or output is calculated to calibrate the model for average safety conditions. An iterative process, using the goal seeking function in Microsoft Excel, is used to determine individual component inputs for the model to have predicted this number or rate of annual accidents (Attwood *et al.,* 2006a). Although many combinations of component reliabilities could produce the accident rate or output required for calibration purposes, the absolute values of individual base case component reliabilities are not important. Model execution is based on a quantified comparison of specific and base cases and not a comparison of absolute component reliability values. Therefore, the individual component reliabilities assigned by the calibration process are identical to one another. Once the output (ie. accident rate) is determined using global average accident statistics, a starting reliability is calculated using the reliability equation based on a constant failure rate. This starting reliability is then assigned to each base case component within the external, corporate and direct layers to set the base case for comparison purposes.

2.4.2 COMPONENT RELIABILITY ADJUSTMENTS

The degree of component reliability adjustment is based on the opinion of experts familiar with both base (average global) and specific case safety conditions. The experts assign scores from 1 to 10 for each factor within the external, corporate and direct layers, representing the component's specific case conditions compared with the global average, which is represented by a score of 5. Higher scores (6-10) represent situations superior to the global average safety results while lower scores (1-4) represent situations less favourable to the global average. For the specific case run, adjustments are made to the component reliabilities by using the square of the ratio of the specific case to average case score (5). For example, if the expert panel assigned a score of 6 to personal protective equipment, representing the direct factor's specific case condition, this would produce a reliability increase of $(6/5)^2 = 1.44$ to personal protective equipment. The base case reliability for personal protective equipment is multiplied by the square of the ratio of the specific case to average case score which would result in a reliability increase for higher scores (6-10) and a reliability decrease for lower scores (1-4).

2.4.3 PREDICTIVE MODE

To predict accident frequency for a specific case, the model is run following adjustment of the base case component reliabilities. Accident frequency predictions can be made by directly entering the specific case expert scores for the direct, corporate or external layer components. If direct layer component reliabilities are used, they can be input into the model to calculate overall system reliability and accident frequency rate directly. If corporate layer component reliabilities are used, they can be input into the model and allowed to influence the direct layer values through use of influence coefficients at the corporate-direct interface. Once the direct layer reliabilities are determined, overall system reliability and accident frequency rates can be calculated. If external layer reliabilities are input, they can determine the corporate values which in turn, determine the direct values and overall system reliability and accident frequency rate can be achieved.

2.4.4 COMPARISION OF PREDICTIVE VS. ACTUAL ACCIDENT RATES

As stated previously, data for the calibration portion of the model are publicly available statistics. The type of accident statistic used for calibration depends on which output statistic is desired. The actual accident rate calculated to calibrate the model is then compared to the predicted result. The model offers the capability to compare predicted safety improvements resulting from changes in various safety elements. It also provides a method to evaluate cost savings associated with accident frequency reduction, allowing operators to optimize management decisions, including the choice of where to allocate monies on improving overall safety.

2.5 UNCERTAINTY WITHIN THE ANALYSIS PROCESS

A key component of Attwood's predictive model lies in the derivation of quantitative data obtained through a survey of safety experts to adjust component reliabilities. The subjective judgements and opinions used to rate each components effect on safety for the specific environment introduces a degree of uncertainty within the model. The predictive model does not determine how this uncertainty should be included in the assessment and analysis process. A methodology for the predictive model has been developed to minimize this uncertainty and propagate it through the model, providing an approach that embraces uncertainty as an inseparable element of the system. The application of such a model can help predict the frequency of occupational accidents while recognizing uncertainty and incorporating it within the analysis process by use of a mathematical framework called fuzzy set theory (FST). A major contribution of FST is its capability of representing uncertainty and modeling systems that are hard to define precisely, incorporating imprecision and subjectivity into the model formulation and solution process. The following chapter provides a discussion on FST and the fuzzy mathematics used to address the uncertainty within the predictive model.

Chapter 3

FUZZY SET THEORY

3.1 EMERGENCE OF FUZZY SET THEORY

Fuzziness is the uncertain state in which the transition between the state of concern and its complement is gradual and hence it is difficult to make a sharp distinction (Kikuchi and Pursula, 1998). Fuzziness can be found in many areas of daily life but it is particularly frequent in all areas in which human judgement, evaluation and decisions are important (Zimmerman, 2001).

Fuzzy set theory is a mathematical framework to account for fuzziness or uncertainty. The theory was first introduced by Lotfi A. Zadeh, a mathematician and computer scientist of Iranian Azeri origin. Zadeh was interested in the problems of complex systems and the use of simple models to represent such issues. He published a paper in 1965, introducing the concept of a fuzzy set, describing it as a class of objects with a continuum of grades of membership. He characterised the fuzzy set by a membership function which assigns to each object in the set, a grade of membership ranging between 0 and 1.

Most of the early interest in FST pertained to representing uncertainty in human cognitive processes (Zadeh, 1965). Since 1965, fuzzy set theory has been studied extensively and is now recognized as an important problem modeling and solution technique due to its ability to quantitatively and qualitatively model problems which involve vagueness and imprecision. Fuzzy set theory has proven to be a powerful way of representing quantitatively and manipulating the imprecision of decision-making problems in engineering, business, medicine, manufacturing among many other industrial sectors.

3.2 APPLICATIONS OF FUZZY SET THEORY IN INDUSTRY

A major contribution of fuzzy set theory is its capability of representing vague data and modeling uncertainty. FST has been used to model systems that are hard to define precisely, incorporating imprecision and subjectivity into the model formulation and solution process. Kahraman (2006) identified fuzzy set theory as an attractive tool to aid research in Industrial Engineering (IE) when the dynamics of the decision environment limit the specification of model objectives, constraints and the precise management of model parameters. Zimmerman (1983) concluded that fuzzy set theory can be used as a language to model problems which contain fuzzy phenomena or relationships, as a tool to analyze such models in order to gain better insight into the problem and as an algorithmic tool to make solution procedures more stable or faster.

In the analysis of transportation problems, fuzzy set theory has been used to analyze traffic flow and control, planning, demand analysis, routing and scheduling and pavement management. Kikuchi and Pursula (1998) examined the nature of uncertainty present in transportation planning and explored appropriate mathematical frameworks to account for such uncertainties. Kituchi and Pursula identified two types of uncertainty found in many transportation engineering and planning problems: fuzziness and ambiguity. Fuzziness refers to the uncertainty caused by a lack of definition of words while ambiguity refers to the uncertainty caused by the lack of information about the subject matter. Fuzziness is prevalent in transportation planning due to the descriptive nature of the treatment of problems (Kikuchi and Pursula, 1998).

Fuzzy set theory has been used in propagating imperfect or incomplete information in health risk assessment studies. Health risk is related to an individual's location, activity and behaviour or preferences, as well as the pollutant emission rates and physical, chemical and biological processes involved in the fate and transport of the pollutants. Intrinsic variability and extensive uncertainty exists within health risk assessment studies. A study conducted by Kentel and Aral (2006) provided a review of several available approaches used in decision-making, some of which involved defuzzification techniques, the possibility and necessity measures. The study proposed a risk tolerance measure which could be used in decision making and provided an effective metric for evaluating the acceptability of a fuzzy risk with respect to a crisp compliance criterion.

McCauley-Bell and Badiru (1996) conducted a two-phase research project to develop a fuzzy-rule based system for quantifying and predicting the risk of occupational injury, specifically, cumulative trauma disorders (CTD's) of the forearm and hand. The first phase of research focused on development and representation of linguistic variables to qualify risk levels. The variables were then quantified using fuzzy-set theory, allowing the model to evaluate qualitative and quantitative data. The second phase of research focused on the analytic hierarchy processing (AHP) to assign relative weights to the identified risk factors. A fuzzy rule base was constructed with all of the potential combinations for the given factors. The system provided linguistic risk levels as well as quantified risks in assessing the overall risk of injury.

Mure, Demichela, and Piccinini (2005) developed a method to assess risks of occupational accidents using fuzzy logic. The purpose of the work was to create a methodological instrument that could semi qualitatively assess the risk of occupational accidents for different industrial and site activities and to identify the most efficient intervention measures to reduce risks. The analysis model allowed for an assessment to be made of the level of risk of a work phase and/or a work sector and verification and quantification to be made of the reduction of risks after having adopted preventive and/or protective measures. A priority of interventions could also be established on the basis of the assessed risk levels.

Fuzzy set theory has been applied to the occupational safety risk analysis within the construction industry as a means of accounting for uncertainty. A study by Gurcanli and Mungen (2009) proposed a method for assessment of the risks that workers are exposed to at construction sites by using a fuzzy rule-based safety analysis to deal with uncertainty and insufficient data. By using this approach, historical accident data, subjective judgements of experts and the current safety level of a construction site can be combined. The relevance of the study was linked to the possibility of providing safety scores for the construction sites that could result in work improvement and productivity. The application of the proposed method revealed which safety items and factors were most important in improving workers safety. It also enabled one to decide where to concentrate resources in order to improve the safety of the work environment. The study began with different kinds of knowledge acquisition ways to establish a body of information that could be beneficiary in developing fuzzy linguistic parameters and their associated membership functions to qualify occupational risks on construction sites. The input parameters of the fuzzy system were derived from the raw data and judgement of experts.

In the scope of the study, Gurcanli and Mungen identified, investigated and classified 5239 occupational accidents in the construction industry. By combining the data and subjective judgement of safety experts, Gurcanli and Mungen were able to derive three parameters namely accident likelihood (AL), current safety level (CSL) and accident severity (AS) and utilize the input parameters for the fuzzy rule-based system.

Gurcanli and Mungen argued that the proposed fuzzy rule-based method of analysis is a new approach for construction which can easily incorporate the present characteristics of the site and construction conditions by taking into account the degree of uncertainties of judgements made by safety experts. The study focused on daily, routine safety measures rather than safety management principles and provided a preliminary but innovative approach for safety evaluation on construction sites.

Chang, Tsujimura, Gen and Tozawa (1995) combined composite and comparison methods of analyzing fuzzy numbers into an efficient procedure for solving project scheduling problems. The comparison method eliminates activities that are not on highly critical paths while the composite method determines the most critical path. The fuzzy Delphi method is used to determine the activity time estimates with activity times represented by triangular fuzzy numbers.

3.3 INTRODUCTION TO FUZZY SET THEORY

Forming sets and analyzing relationships is usually the first step toward organizing thoughts and understanding the structure of a complex problem (Klir, 1997). A central assumption of classical set theory states that the boundaries of classical sets are required to be drawn precisely and therefore, set membership is determined with complete certainty (Klir, 1997). The classical notion of a set is crisp meaning that the set is something clear and concise. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition meaning that an element either belongs to a set or does not belong to a set. In binary language, the element is assigned a value of 1 if belonging to a set and a value of 0 otherwise. An analysis based on a crisp set takes place in a rigid frame of a system where a clear demarcation exists between the correct and the incorrect. In classical set theory, it says that an individual must be a member of a set or its compliment but not both (Klir, 1997).

Two important laws of classical set theory are the law of contradiction and the law of the excluded middle. The law of contradiction states that any proposition affirming a fact and denying it at the same time is false. It says that the same individual cannot simultaneously be a member of a set and its compliment. The law of excluded middle is closely related and states that any proposition must be either true or false, but not both. If sets have imprecise boundaries, then the two classically important principles, the laws of contradiction and excluded middle, will no longer be true.

In our daily lives, virtually all human activities involve reasoning based on vague concepts and incomplete information. The stipulation that a statement is either true or false usually cannot be applied. Most sets and propositions are not neatly characterized and exact boundaries cannot be precisely determined. For example, the statement "Jon K is healthy" cannot be evaluated simply by a definite yes or no because we do not have any strict criteria for the clean demarcation between healthy and not healthy (Klir, 1997). A set of healthy people is allowed in classical set theory only if significant simplifying assumptions are made and the partition between healthy and unhealthy people is imposed. Without the imposition of arbitrary partitions or boundaries, a set cannot be defined in terms of classical set theory, a circumstance that has prevented classical mathematics from functioning fully in disciplines dealing with vagueness and other kinds of uncertainty (Klir, 1997). One of the principle motivations for introducing fuzzy set theory was to deal with such uncertainty, bridging the gap between imprecise concepts which are used to describe reality and precise classical mathematics.

3.3.1 FUZZY SETS

A fuzzy set is a mathematical formalism to represent a fuzzy concept. Fuzziness is defined as something vague or uncertain and is inherently associated with our linguistic expression (Kikuchi, 1998). Given a proposition "x is A", fuzziness is the situation that the truth of the proposition cannot be determined because A is not clearly defined therefore the uncertainty under fuzziness is caused by the lack of definition of words.

A fuzzy set is defined as a pair (X, μ) where X is a set and $\mu: X \to [0,1]$. For each $x \in X$, $\mu(x)$ is the grade of membership of x. If $X = \{x_1, \cdots x_n\}$ the fuzzy set (X, μ) can be denoted as:

$$\left\{ {\mu(x_1)}/{x_1} \ldots {\mu(x_n)}/{x_n} \right\}$$

An element mapping to the value 0 means that the member is not included in the fuzzy set where as the value 1 describes a fully included member. Values strictly between 0 and 1 characterize the fuzzy members. The set $\{x \in X|\mu(x) > 0\}$ is called the support of the fuzzy set (X, μ) and the set $\{x \in X|\mu(x) = 1\}$ is called the kernel of the fuzzy set (X, μ).

Fuzzy set theory is an extension of the classical notion of a set since the indicator functions of classical sets are special cases of the membership functions of fuzzy sets, if the fuzzy set only takes values of 0 or 1. In fuzzy set theory, classical bivalent sets are looked upon as special fuzzy sets and are referred to as crisp sets. Fuzzy set theory cannot be considered independent of the classical approach but should be looked upon as complementary to the classical statistical approach when dealing with human perception and decision processes as it provides the mathematical framework to deal with the nature of uncertainty.

3.4 MEASUREMENTS OF FUZZINESS

As previously stated, classical sets may be viewed as special fuzzy sets, called crisp sets, whose membership grades are restricted to 0 and 1 values. Any set that is not crisp involves some degree of fuzziness which results from the imprecision of its boundaries. Klir (1997) states that the less precise the boundary, the more fuzzy the set.

To measure fuzziness means to assign a nonnegative number to each fuzzy set. These numbers must satisfy some requirements that can easily be justified on intuitive grounds as essential for capturing the concept of fuzziness. One requirement is that the measure of fuzziness should be zero for all crisp sets and greater than zero for all other sets. Another requirement is based on our intuition that the sharper the boundary of a fuzzy set, the less fuzzy the set is. The sharpness of a boundary of a fuzzy set is determined by the closeness of its membership grades to the ideal values 0 and 1. The closer the membership grades are to the ideal values, the sharper the boundary.

3.4.1 MEMBERSHIP FUNCTIONS

The characteristic function of a fuzzy set, called a membership function, $\mu A(x)$, can possess a value between 0 and 1 depending on the degree that an element (x) is compatible with the fuzzy notion. This results in grades of membership for sets rather than full or no membership designations. An individual's membership in a fuzzy set may admit some uncertainty therefore its membership is a matter of degree. The degree of membership of an individual in a fuzzy set expresses the degree of compatibility of the individual with the concept represented by the fuzzy set (Klir, 1997).

Each fuzzy set A, is defined in terms of a relevant universal set X, by a membership function which assigns to each element x of X a number, $A(x)$, in the closed unit interval $[0,1]$ that characterizes the degree of membership of x in A. Membership functions are thus functions of the form:

$$A: X \rightarrow [0,1]$$

A fuzzy set has a membership that is not absolute. Fuzzy sets generalize the characteristic classical function in allowing all values between 0 and 1. A fuzzy subset A of X is defined by its membership function, written $A(x)$, whose values can be any number in the interval $[0,1]$. The value of $A(x)$ is called the grade membership of x in fuzzy set A and is often denoted by $\mu(x)$. If $\mu(x)$ is only 0 or 1, then the characteristic function of a crisp, non-fuzzy set A would apply. If

$\mu(x)$ takes on values in between 0 and 1, then x belongs to A if $\mu_A(x) = 1$, x does not belong to A when $\mu_A(x) = 0$ and x is in A with membership $\mu_A(x)$ if $0 < \mu_A(x) < 1$.

Membership functions of fuzzy sets play a central role in fuzzy set theory. In each application of fuzzy set theory, appropriate membership functions must be constructed so that the intended meanings of relevant linguistic terms are captured and the fuzzy set is adequately defined. These meanings are strongly dependent on the context in which the linguistic terms are used. For example, the word young has a different meaning when applied to children or university professors and its meaning is even more varied when applied to different types of objects such as geological formations or trees.

3.4.2 REPRESENTATION OF MEMBERSHIP FUNCTIONS

Each fuzzy set is uniquely defined by a membership function. The most common ways in which membership functions are displayed are through graphical, tabular and list, geometric and analytical representation. Graphical representation is most frequently used and illustrates membership functions whose universal sets are either 1-dimensional or 2-dimensional Euclidean space. For universal sets that are finite, membership functions can be represented by tables, which list all elements in the universal set and the corresponding membership grades. Using tabular representation, the fuzzy set is characterized by a list in which the members of the set are conjoined with the degree of membership in the set.

Geometric representations are most often used to represent membership functions whose universal set X is a finite set. When a universal set is infinite, which is often the case for a set of real numbers, the membership function is often represented in analytic form. For example, the universal set of the fuzzy set '*about 2*' is the set of all real numbers. This kind of fuzzy set, called a fuzzy number, can be represented by an analytic form which describes the shape of this fuzzy number. The fuzzy set, whose graph is shown in Figure 3.1, may capture the concept of '*about 2*'. It can be expressed in the following analytic form:

$$A(x) = \begin{cases} x - 1 & when\ 1 \leq x \leq 2 \\ 3 - x & when\ 2 \leq x \leq 3 \\ 0 & otherwise \end{cases}$$

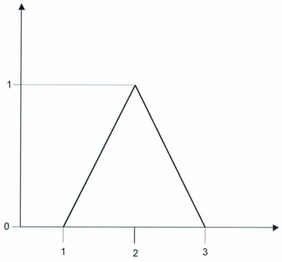


FIGURE 3.1 - A MEMBERSHIP FUNCTION OF *ABOUT 2*

Any symmetric, triangular membership function characterized by the three parameters a, b and c, as shown in Figure 3.2, is defined by the membership function:

$$A(x) = \begin{cases} 0 & x < a \\ \left(\dfrac{(x-a)}{b-a}\right) & when\ a \le x \le b \\ \left(\dfrac{(c-x)}{c-b}\right) & when\ c \le x \le d \\ 0 & x > c \end{cases}$$

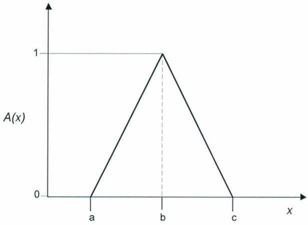


FIGURE 3.2 - GENERIC, SYMMETRIC, AND TRIANGULAR MEMBERSHIP FUNCTION

Another important class of membership functions is trapezoidal shaped, which is captured by the generic graphical representation in Figure 3.3. Each function in this class is characterized by the four parameters a, b, c, and d via the generic form:

$$A(x) = \begin{cases} \left(\dfrac{(a-x)}{a-b}\right) & when\ a \le x \le b \\ 1 & when\ b \le x \le c \\ \left(\dfrac{(d-x)}{d-c}\right) & when\ c \le x \le d \\ 0 & otherwise \end{cases}$$

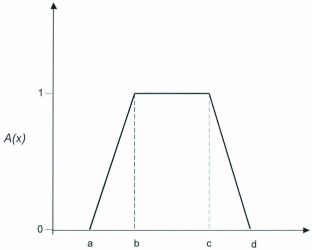


FIGURE 3.3 - TRAPEZOIDAL MEMBERSHIP FUNCTION

3.5 OPERATIONS ON FUZZY SETS

Essential to the application of fuzzy set theory are set operations. They aggregate two concepts that are represented by sets and form a new set representing a new concept, or they process different types of information and generate new information (Kukuchi, 1998). In the case of crisp sets, set operations are binary, defined on either the max or min operations of 0 or 1; hence, they cannot represent the uncertainty perceived when two notions are aggregated. In the case of fuzzy sets, different operators are possible in order to capture and preserve the fuzziness embedded in the original concepts.

The three basic operations on classical sets are complement, union and intersection. While these operations are unique in classical set theory, their extensions in fuzzy set theory are not unique. Distinct operations in each of these classes reflect distinct meanings of the linguistic terms *and*, *not* and *or* when employed in sentences of natural language in different contexts. These special operations on fuzzy sets which are referred to as standard fuzzy operations are the most common operations in practical applications of fuzzy set theory.

3.5.1 STANDARD FUZZY COMPLEMENT

Given a fuzzy set A defined on a universal set X, its complement Ã is another fuzzy set on X that inverts, in some sense, the degrees of membership associated with A. While for each $x \in \mathrm{X}, \mathrm{A}(x)$ expresses the degree to which x belongs to A, $\tilde{\mathrm{A}}(x)$ expresses the degree to which x does not belong to A. The standard fuzzy complement is expressed by the formula:

$$\tilde{A}(x) = 1 - \mathrm{A}(x)$$

for all $x \in \mathrm{X}$.

One consequence of the imprecise boundaries of fuzzy sets is that they overlap with their complement. This is one of the fundamental differences between classical set theory and fuzzy set theory. In classical set theory, sets never overlap with their complements.

3.5.2 STANDARD FUZZY UNION

Consider a universal set X and two fuzzy sets A and B defined on X. The standard fuzzy union of A and B, denoted by A ∪ B, is defined by the membership functions using the formula:

$$(\mathrm{A} \cup \mathrm{B})(x) = max[\mathrm{A}(x), \mathrm{B}(x)]$$

To illustrate, let X be a set of n doctor's patients identified by numbers 1, 2,..., n. Let A denote the fuzzy set of those patients in X having high blood pressure and let B denote the fuzzy set of patients having high fever. If patient 1 has $\mathrm{A} = \text{high blood pressure} = 0.6$ and $\mathrm{B} = \text{high fever} = 0.3$, then the set A ∪ B for patient 1 is expressed by:

$$(\mathrm{A} \cup \mathrm{B})(1) = max[0.6, 0.3] = 0.6$$

Using the fuzzy union equation, one can determine the set A ∪ B of patients in X that have high blood pressure or high fever by taking the maximum value of A or B.

3.5.3 STANDARD FUZZY INTERSECTION

The standard fuzzy intersection, denoted A ∩ B, is defined by the membership functions using the formula:

$$(\mathrm{A} \cap \mathrm{B})(x) = min[\mathrm{A}(x), \mathrm{B}(x)]$$

for all $x \in X$.

Continuing with the doctor's patients example, with patient 1 having A = high blood pressure = 0.6 and B = high fever = 0.3, then the set A ∩ B for patient 1 is expressed by:

$$(\mathrm{A} \cap \mathrm{B})(1) = min[0.6,0.3] = 0.3$$

The standard fuzzy operations do not satisify two laws of their classical counterparts: the law of excluded middle and the law of contradiction. This is a consequence of imprecise boundaries of fuzzy sets.

It can easily be verified that the standard fuzzy operations satisfy all other properties of the corresponding operations in classical set theory. By restricting ourselves to the standard fuzzy operations, the great expressive power of fuzzy set theory is not fully utilized. In particular, the standard fuzzy operations are not capable of expressing the full variety of meanings of the linguistic terms *and*, *not* and *or* when applied to fuzzy concepts of natural language. The standard fuzzy operations have been found adequate in most practical applications of fuzzy set theory.

3.6 FUZZY ALPHA – CUT TECHNIQUE

Fuzzy numbers are numerical approximations described by fuzzy sets and are used when one interprets or perceives information that has potential measurement imprecisions. Fuzzy numbers can be used to represent interval numbers using the alpha (α) cut to represent the set of elements in a fuzzy set that have a degree of membership, $(\mu(x))$, greater than or equal to the α membership value. The α-cut is the set of elements in a fuzzy set having membership $\mu(x)$ which can be represented by:

$$\tilde{A} = \{x \mid x \,\epsilon\, X, and\, \mu(x) \geq \alpha\} = [aL, cR] = [a, c]$$

The alpha (α) cut technique uses fuzzy set theory to represent uncertainty or imprecision in the parameters. Uncertain parameters are considered to be fuzzy numbers with some membership functions. Figure 3.4 shows a parameter X represented as a triangular fuzzy number with support of A_0. The wider the support of the membership function, the higher the uncertainty.

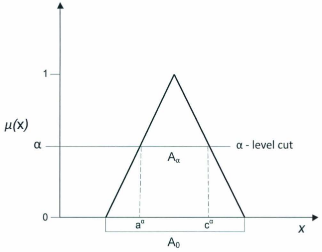


FIGURE 3.4 - TRIANGULAR FUZZY NUMBER WITH SUPPORT A_0 AND α-CUT

The fuzzy set that contains all elements with a membership of $\alpha \in [0, 1]$ and above is called the α–cut of the membership function. At a resolution level of α, it will have support of A_{α}. The higher the value of α, the higher the confidence in the parameter (Li & Vincent, 1995). By defining the interval of confidence at level α, a triangular fuzzy number, defined by the triplet (a, b, c) is characterized as:

$$\forall_{\alpha} = [0,1]:$$

$$\tilde{A}_{\alpha} = [a^{\alpha}, c^{\alpha}] = [(b-a)\alpha + a, c - (c-b)\alpha]$$

The alpha (α) method is based on the extension principle, which implies that functional relationships can be extended to involve fuzzy arguments and can be used to map the dependent variable as a fuzzy set. In simple arithmetic operations, this principle can be used analytically. However, in most practical modeling applications, relationships involve partial differential equations and other complex structures that make analytical application of the principle difficult. Therefore, interval arithmetic is used to carry out the analysis.

3.7 THE EXTENSION PRINCIPLE

The extension principle is a method of extending point-to-point operations to fuzzy sets. It is the basic tool for the development of fuzzy arithmetic as it provides a method for fuzzifying crisp functions. Suppose that f is a point to point mapping function from X to Y and A is a fuzzy set on X defined as:

$$A = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \cdots + \frac{\mu_A(x_n)}{x_n}$$

The extension principle states that the image of fuzzy set A under the mapping f(·) can be expressed as a fuzzy set B:

$$B = f(A) = \frac{\mu_A(x_1)}{y_1} + \frac{\mu_A(x_2)}{y_2} + \cdots + \frac{\mu_A(x_n)}{y_n}$$

where $yi = f(xi)$.

More generally, we have:

$$\mu B(y) = \max_{x=f^{-1}(y)} \mu_A(x)$$

Let $\tilde{I}$ and $\tilde{U}$ be two fuzzy numbers defined in terms of universal sets X and Y respectively. Let the symbol $*$ denote a general arithmetic operation, i.e. $* \equiv \{+,-,\times,\div\}$. An arithmetic operation or mapping between these two fuzzy numbers denoted $\tilde{I} * \tilde{U}$ will be defined in terms of the universal set Z and can be accomplished using the extension principle, by:

$$\mu_{\tilde{I}*\tilde{U}}(z) = \bigvee_{x*y=z} \left(\mu_{\tilde{I}}(x) \wedge \mu_{\tilde{U}}(y)\right)$$

which results in another fuzzy set, the fuzzy number resulting from the arithmetic operation on fuzzy numbers $\tilde{I}$ and $\tilde{U}$.

3.7.1 INTERVAL ANALYSIS IN ARITHMETIC

A fuzzy set can be thought of as a crisp set with ambiguous boundaries. A convex membership function defining a fuzzy set can be described by the interval associated with different levels of α-cuts. A fuzzy set, A, is said to be convex if and only if all of its α-cuts are convex in the classical sense. That is, for each α-cut, A_α, for any $r, s \in A_\alpha$ and any $\lambda \in [0,1]$ then $\lambda r + (1-\lambda)s \in A_\alpha$. Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds:

$$I_1 = [a, b] \quad where\ a \leq b$$

$$I_2 = [c, d] \quad where\ c \leq d$$

When a = b and c = d, these interval numbers degenerate to a scalar real number. Again, using the symbol $*$ to denote a general arithmetic property, the following equation represents another interval.

$$I_1 * I_2 = [a, b] * [c, d]$$

The interval calculation depends on the magnitudes and signs of the elements a, b, c and d. Table 3.1 displays the various combinations of set-theoretic intersection ($\cap$) and set-theoretic union ($\cup$) for the six combinations of these elements given that $(a < b)$ and $(c < d)$ still hold true.

Table 3.1 - Set Operations on Intervals

Cases	Intersection ($\cap$)	Union ($\cup$)
$a > d$	ϕ	$[c, d] \cup [a, b]$
$c > b$	ϕ	$[a, b] \cup [c, d]$
$a > c, b < d$	$[a, b]$	$[c, d]$
$c > a, d < b$	$[c, d]$	$[a, b]$
$a < c < b < d$	$[c, b]$	$[a, d]$
$c < a < d < b$	$[a, d]$	$[c, b]$

Based on the information in Table 3.1, the four arithmetic operations associated with the above equation are:

$$[a, b] + [c, d] = [a + c, b + d]$$

$$[a, b] - [c, d] = [a - d, b - c]$$

$$[a, b] \cdot [c, d] = [min(ac, ad, bc, bd), max(ac, ad, bc, bd)]$$

$$[a, b] \div [c, d] = [a, b] \cdot \left[\frac{1}{d}, \frac{1}{c}\right] \; provided \; that \; 0 \not\subset [c, d]$$

$$\alpha[a, b] = \begin{cases} (\alpha a, \alpha b) \; for \; \alpha > 0 \\ (\alpha b, \alpha a) \; for \; \alpha < 0 \end{cases}$$

Where ac, ad, bc and bd are arithmetic products and $1/d$ and $1/c$ are quotients.

Chapter 4

MODELING OF UNCERTAINTY

4.1 VAGUENESS AND UNCERTAINTY

The development of predictive models for occupational accidents is often fraught by variability and uncertainty associated with the subjective nature of expert opinions and judgements. This level of uncertainty attached to the integration of subjective evaluations is a concern when analyzing systems through model development. Fuzzy set theory (FST) provides a useful tool to address this variability and to propagate uncertainty through the model. The utility of FST in model development has been seen in its ability to more appropriately represent the human-inferencing process and to provide a more user-friendly interface through the use of natural language (Zadeh, 1996).

The term 'computing with words' was introduced by Lotfi A. Zadeh to explain the notion of reasoning linguistically rather than with numerical quantities. "Humans use natural language as a means of computing and reasoning, arriving at conclusions expressed as words from premises expressed in a natural language" (Zadeh, 1996). Words have fuzzy denotations therefore a key aspect of computing with words is that it involves a fusion of natural language and computation with fuzzy variables (Zadeh, 1996).

The use of natural language or linguistic variables within the framework of models may be a necessity when available information is too imprecise to justify the use of numbers or "when dealing with situations which are too complex or ill-defined to be reasonably described by conventional quantitative expression" (Lin, 1997). A linguistic variable is a non-numeric variable whose values are words or sentences in natural or artificial language which are used to facilitate the expression of rules and facts. The use of linguistic variables allows the analyst to assess model parameters in a natural way, providing a more user-friendly approach for analyzing the specific case. Fuzzy set theory provides a useful tool for directly working with linguistic expressions in the modeling and analysis of occupational accidents.

4.2 INCORPORATING FST INTO ATTWOOD'S PREDICTIVE MODEL

Model development includes the identification of constituent factors and a determination of their interrelationships. Attwood conducted a comprehensive literature review of early accident models and a thorough evaluation of the related available statistic accident data to gain an understanding of the major factors contributing to the occupational accident issues. A thorough review of internet sources, company annual reports and open literature offering analysis of offshore occupational accidents was undertaken to understand major factors affecting this issue (Attwood *et al.,* 2006a).

The model execution process is comprised of five stages: Calculation of accidents using global average conditions, calibration run, component reliability adjustment, predictive run and comparison of predictions with estimates of actual numbers of accidents. The first step involves obtaining data to calibrate the model for average conditions. Data for the calibration portion of the model application are publicly available statistics. The type of accident statistic used for calibration depends on which output statistic is desired. If a particular accident rate is required, then the corresponding global average value of that particular rate is used for calibration (Attwood *et al.,* 2006c).The second stage of execution involves calibrating the model in order to set base case component reliabilities. The base case result obtained from step one is used to calibrate the model to a global average accident expectation, allowing all base case component reliabilities to be set.

The third stage of model execution involves component reliability adjustment, where an expert panel assigns scores of 1-10 to each factor within the direct, corporate and external layers. This rating system represents the component's effect on safety within the specific regime being analyzed, compared to the global average, which is represented by a score of 5. Each score represents the component's specific case condition with higher scores representing situations more favourable to safety results while lower scores represent situations less favourable to safety (Attwood *et al.,* 2006c). At this stage of model execution, the author proposes the use of fuzzy set theory (FST) to account for uncertainty associated with the subjective judgements of the expert panel. Incorporating FST to account for judgemental uncertainties associated with experts opinion involves the assignment of linguistic variables to represent each factor's effect on safety or specific case condition, conversion of linguistic variables into a fuzzy numerical range through the development of membership functions, aggregation of fuzzy numbers into one fuzzy variable to represent each factors effect on safety, propagation of these fuzzy numbers throughout Attwood's model to determine the fuzzy outcome or frequency of occupational accidents and the use of a defuzzification technique or fuzzy operations to calculate a crisp numeric output.

4.2.1 FUZZY NUMBERS

The concept of a fuzzy number arises from the fact that many quantifiable phenomena cannot be characterised in terms of absolutely precise numbers (Klir, 1997). A fuzzy number is one which is described in terms of a number word and a linguistic modifier, such as *approximately*, *nearly*, or *around*. There are several geometric mapping functions of fuzzy numbers to represent the linguistic variables, but the most common are triangular and trapezoidal shapes as they are easy to construct and manipulate. Most current applications that employ fuzzy numbers are not significantly affected by the shapes of functions hence it is quite natural to choose simple linear functions, represented by straight lines, as in the case of triangular or trapezoidal (Klir, 1997).

A triangular fuzzy number is a fuzzy number A in X, if its membership function $f_A: X \rightarrow [0,1]$ is:

$$f_A(x) = \begin{cases} \dfrac{(x-a)}{(b-c)} & a \leq x \leq b \\ \dfrac{(c-x)}{(c-b)} & b \leq x \leq c \\ 0 & otherwise \end{cases}$$

when $a \leq b \leq c$.

The triangular fuzzy number can be denoted by $A = (a, b, c)$. The parameter $'b'$ gives the maximal grade of $f_A(x)$ $(ie.\ f_A(b) = 1)$ and is the most probable value of the evaluation data. The parameters $'a'$ and $'c'$ are the lower and upper bounds of the available area for the evaluation data.

A trapezoidal fuzzy number is a fuzzy number A in X, if its membership function $f_A: X \rightarrow [0,1]$ is:

$$f_A(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \dfrac{(d-x)}{(d-c)} & c \leq x \leq d \\ 0 & otherwise \end{cases}$$

when $a \leq b \leq c \leq d$.

The trapezoidal fuzzy number can be denoted by $A = (a, b, c, d)$. The interval $[b, c]$ are the most likely values of $f_A(x)$. The parameters $'a'$ and $'d'$ are the lower and upper bounds of the available area for evaluation.

Triangular fuzzy numbers were selected to represent linguistic variables for Attwood's model. Triangular geometric mapping functions can be easily justified on intuitive grounds as they capture the concept of fuzziness, making it easy for evaluation.

4.2.2 DEVELOPMENT OF LINGUISTIC VARIABLES

The goal of fuzzy linguistic variables is to represent the condition of an attribute at a given interval. In FST, several intervals or ranges may be specified with respective linguistic variables offering the continuum of a given variable (McCauley-Bell, 1996). In the continuum of component importance as it relates to safety results in Attwood's model, the fuzzy linguistic variables are used to assign the relative importance of factors with regards to their overall effect on safety. The ability of FST to offer a natural-language interface and a graded degree of safety rating are the main reasons for utilizing this methodology within this phase of model execution. By introducing a linguistic, qualitative scale of importance to rate component reliabilities, it provides a more natural, user-friendly approach for analyzing the specific case since people are better at qualitative judgement tasks than they are at quantitative estimates, preferring to express with verbal phrases as opposed to numerical estimates.

The objective of this stage of linguistic variable development is to identify ranges where the safety experts could assign a particular linguistic value to an interval. To accomplish this, the entire level of factor existence needs to be partitioned into as many levels as necessary to accurately represent the continuum of the factor. Five linguistic variables are used to rank each factor's importance or effect on overall safety. Descriptions of the five levels of linguistic variables are presented in Table 4.1. Each safety expert can assign linguistic terms, such as 'Low' or 'Very High' for determining a factors effect on safety.

Table 4.1 - Description of Linguistic Variables

Linguistic Variable	Fuzzy Range	Description
Very Low (VL)	0-2	Represents situations having very little to no effect on safety in a specific region. Represents least favourable results on safety.
Low (L)	2-4	Represents situations having a slight or low effect on safety.
Medium (M)	4-6	Represents situations having an average effect on safety.
High (H)	6-8	Represents situations having a high or considerable effect on safety.
Very High (VH)	8-10	Represents situations having a very high effect on safety. Represents most favourable results on safety.

The five linguistic variables are then translated into triangular fuzzy numbers. Triangular fuzzy numbers are utilized to capture the vagueness of the fuzzy linguistic terms and represent the subjective and conflicting assessment of the panel of safety experts (Yang, 2003).

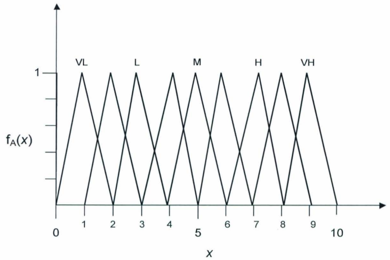


FIGURE 4.1 - TRIANGULAR FUZZY NUMBERS REPRESENTING LINGUISTIC VARIABLES

Experts apply the linguistic terms to rate each component's effect on safety results. Each of the linguistic variables are represented by an individual and overlapping triangular shaped membership function that travels throughout the entire interval [0,1] as shown in Figure 4.1. The membership functions translate the linguistic terms into triangular fuzzy numbers. For example, $\tilde{3}$ represents a factor having a 'low' effect on safety results and $\tilde{9}$ represents a 'very high' effect on safety. Overlapping functions were used to represent ill-defined boundaries between each linguistic variable.

The base of the triangle or support of the membership function represents the range of uncertainty. The wider the support of the membership function, the higher the uncertainty. Taking the linguistic variable 'Low', the base of the triangle can be seen to extend from $\tilde{2}$ to $\tilde{4}$

so 'Low' is defined as every component whose effect on safety is bounded between the fuzzy interval of $\tilde{2}$ and $\tilde{4}$. The upper vertex of the triangle is located just over a vertical line from $\tilde{3}$. In FST, maximal grade of the membership degree to the fuzzy set of 'Low' is 1. The membership degree begins to decline along each side of the vertex until a membership degree of 0 is reached at the base of the triangle at both the lower and upper bounds of $\tilde{2}$ and $\tilde{4}$ respectively. This represents the core of FST in the sense that everything is a question of grade. The remainder of fuzzy sets 'Very Low', 'Medium', 'High' and 'Very High' are described in a similar way.

4.2.3 MEMBERSHIP FUNCTIONS

One of the principal motivations for introducing fuzzy sets is to represent imprecise concepts. A factor's membership in a fuzzy set may admit some uncertainty, therefore its membership is a matter of degree. In Attwood's predictive model, a factor within the direct layer may be a member of the fuzzy set 'High' to the degree to which the factor meets the concept of 'High'. The concept of 'High' represents situations having a considerable or high effect on safety results. Alternatively, the degree of membership of a factor in a fuzzy set expresses the degree of compatibility of the factor with the concept represented by the fuzzy set.

To qualify as a fuzzy number, the membership function must capture an intuitive conception of a set of numbers that are around a given real number, or possibly, around an interval of real numbers. The fuzzy set 'High' extends from the fuzzy interval $\tilde{6}$ to $\tilde{8}$ with the value $\tilde{7}$ corresponding to the maximal grade of the membership function having a value of 1. This represents the most probable value of the evaluation data. The values $\tilde{6}$ and $\tilde{8}$ represent the lower and upper bounds of the evaluation data and are represented by a membership grade of 0. If a direct factor is rated as having a 'High' effect on safety results, the factor's degree of membership depends on its compatibility with the concept represented by 'High'. If the factor is highly compatible to the concept of 'High', its membership value will be closer to 1 whereas if compatibility is low, its membership value will be closer to 0. Each membership value is then represented by a fuzzy number using a numerical approximation system to convert the linguistic term 'High' in terms of its corresponding fuzzy number.

Within this phase of model execution, experts are asked to apply linguistic terms to rate each components effect on safety results. A numerical approximation system was proposed to systematically convert the five linguistic terms to their corresponding fuzzy numbers. Figure 4.1 represents the conversion scale chosen to represent assessments of the experts. The corresponding membership functions of these five linguistic values in triangular fuzzy numbers are illustrated as follows:

$$f_{VL}(x) = \begin{cases} 1 & 0 < x \leq 1 \\ \dfrac{(2-x)}{1} & 1 < x \leq 2 \\ 0 & otherwise \end{cases}$$

$$f_{L}(x) = \begin{cases} \dfrac{(x-2)}{1} & 2 < x \leq 3 \\ \dfrac{(4-x)}{1} & 3 < x \leq 4 \\ 0 & otherwise \end{cases}$$

$$f_{M}(x) = \begin{cases} \dfrac{(x-4)}{1} & 4 < x \leq 5 \\ \dfrac{(6-x)}{1} & 5 < x \leq 6 \\ 0 & otherwise \end{cases}$$

$$f_H(x) = \begin{cases} \frac{(x-6)}{1} & 6 < x \leq 7 \\ \frac{(8-x)}{1} & 7 < x \leq 8 \\ 0 & otherwise \end{cases}$$

$$f_{VH}(x) = \begin{cases} \frac{(x-8)}{1} & 8 < x \leq 9 \\ 1 & 9 < x \leq 10 \\ 0 & otherwise \end{cases}$$

4.2.4 UTILIZING EXPERT DATA

The criteria used to form the expert panel were based on knowledge of specific regions, experience in safety design, project management, offshore surveying and safety consultancy (Attwood *et al.,* 2006c). For the purposes of this work, a weighting factor was not introduced to represent the relative quality of different experts. Thus the opinion of each expert is assigned equal weight in terms of significance or importance.

Each expert is asked to apply one of five natural linguistic expressions (very low, low, medium, high or very high) in rating each factor's effect on safety results. Each linguistic variable is translated into a triangular fuzzy number by use of Figure 4.1. Each linguistic term, A, can be represented as a fuzzy number in the form $\tilde{A} = (a, b, c)$ where $a \leq b \leq c$. The triangular fuzzy numbers and representative fuzzy values for each linguistic variable are displayed in Table 4.2.

Table 4.2 – Triangular Fuzzy Numbers and Fuzzy Values for Linguistic Variables

Linguistic Variable	Description	Fuzzy Number, $\tilde{A}$	Fuzzy Value, $\tilde{A} = (a, b, c)$	Membership Function
Very Low (VL)	Represents situations having very little to no effect on safety in a specific region. Represents least favourable results on safety.	$\tilde{1}$	(0, 1, 2)	$f_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-c)} & a \le x \le b \\ \frac{(c-x)}{(c-b)} & b \le x \le c \\ 0 & otherwise \end{cases}$
Low (L)	Represents situations having a slight or low effect on safety.	$\tilde{3}$	(2, 3, 4)	
Medium (M)	Represents situations having an average effect on safety.	$\tilde{5}$	(4, 5, 6)	
High (H)	Represents situations having a high or considerable effect on safety.	$\tilde{7}$	(6, 7, 8)	
Very High (VH)	Represents situations having a very high effect on safety. Represents most favourable results on safety.	$\tilde{9}$	(8, 9, 10)	

The membership function can be cut horizontally at a finite number of α- confidence levels between 0 and 1 to obtain lower and upper bounds for each confidence interval as displayed in Figure 4.2. For each α-cut of the parameter, the model is run to determine the minimum and maximum possible values of the output. This information is then directly used to construct the corresponding fuzziness (membership functions) of the output which is used as a measure of uncertainty.

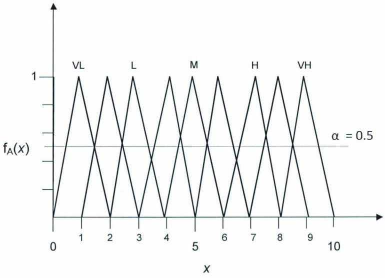


FIGURE 4.2 - TRIANGULAR FUZZY NUMBERS WITH A- CONFIDENCE LEVEL OF 0.5

Due to different opinions of the expert panel, it is necessary to combine or aggregate the opinion of each expert into a single fuzzy number. There are many methods to aggregate fuzzy numbers including mean, median, maximum, minimum and mixed operators. The arithmetic averaging (mean) operation does not produce a noticeable change in the combined possibility distribution when there are small variations in any possibility distribution and it is the most commonly used aggregate method (Huang, 2001). For the present work, the mean operator has been selected to pool expert opinion. Using the mean aggregation method, let:

$$A_{ij} = (a_{ij}, b_{ij}, c_{ij}) \; or \; (a_{ij}, b_{ij}, c_{ij}, d_{ij}), \qquad i = 1,2, \ldots, m; \; j = 1,2, \ldots, n,$$

represent the linguistic expression of the element or event i given by expert j. The average equation for aggregating the n experts' opinions to a single fuzzy number is defined as:

$$M_i = \left(\frac{1}{n}\right) \times (A_{i1} + A_{i2} + A_{i3} + \cdots + A_{in}),$$

$$i = 1, 2, 3, \dots, m$$

where M_i represents the average fuzzy number of the m elements or events. To illustrate the average or mean aggregation method, take 'Medium' (M) and 'High' (H) to be the fuzzy numbers selected by a panel of 2 experts, which are defined as follows:

$$f_M(x) = \begin{cases} \frac{(x-4)}{1} & 4 < x \leq 5 \\ \frac{(6-x)}{1} & 5 < x \leq 6 \\ 0 & otherwise \end{cases}$$

$$f_H(x) = \begin{cases} \frac{(x-6)}{1} & 6 < x \leq 7 \\ \frac{(8-x)}{1} & 7 < x \leq 8 \\ 0 & otherwise \end{cases}$$

Using the $\alpha - cut$ addition and the average aggregate equation:

$$f_{M+H}(z) = \max_{z=x+y} \left(f_M(x) \wedge f_H(y) \right)$$

$$W = \frac{1}{2} \times (M + H)$$

where W represents the average fuzzy number. The $\alpha - cut$ of M and FH are:

$$M_\alpha = [m_1, m_2], \;\; H_\alpha = [h_1, h_2]$$

Which means that at some level, x can be either m_1 or m_2 and y can be either h_1 or h_2. By setting $\alpha = (x-4)/1 \; for \; f_M(x)$, the $\alpha - cut$ obtained for m_1 is:

$$\alpha = (m_1 - 4)/1 \; or \; m_1 = \alpha + 4.$$

Similarly, the other $\alpha - cut$ values obtained are:

$$m_2 = 6 - \alpha;\ h_1 = \alpha + 6;\ h_2 = 8 - \alpha$$

The addition of M and H are computed as:

$$f_{M+H}(z) = \max_{z=x+y}\left(f_M(x) \wedge f_H(y)\right)$$

$$= [(m_1) + (h_1), (m_2) + (h_2)]$$

$$= [(\alpha + 4) + (\alpha + 6), (6 - \alpha) + (8 - \alpha)]$$

$$= [(2\alpha + 10), (14 - 2\alpha)]$$

The average fuzzy number W is computed as follows:

$$W = \frac{1}{2} \times [(2\alpha + 10), (14 - 2\alpha)] = [(\alpha + 5), (7 - \alpha)]$$

Let $W_a = [z_1, z_2] = [(\alpha + 5), (7 - \alpha)]$ then $\alpha = z_1 - 5$ and $\alpha = 7 - z_2$.

Thus, the membership function of the aggregated (average) fuzzy number W is:

$$f_W(x) = \begin{cases} \frac{(z-5)}{1} & 5 < x \leq 6 \\ \frac{(7-z)}{1} & 6 < x \leq 7 \\ 0 & otherwise \end{cases}$$

A fuzzy error factor is calculated to account for the imprecision of data. The error factor is associated with the most possible value of the linguistic variables. The term "error possibility" is essentially a fuzzy probability and is used to obtain a fuzzy error rate for each linguistic expression. The linguistic variables are translated into fuzzy error probabilities by triangular membership functions displayed in Figure 4.3.

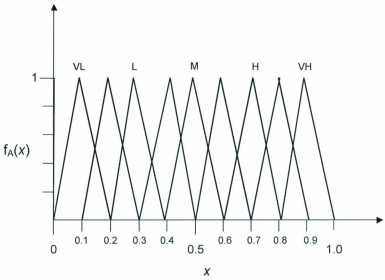


FIGURE 4.3 - TRIANGULAR FUZZY NUMBERS REPRESENT FUZZY ERROR PROBABILITIES

Aggregate experts' opinions can be transformed into one "fuzzy error probability" and the "fuzzy error rate", E_r , can be obtained from the fuzzy error probability using the following equations proposed by Huang *et al.* (2001):

$$E_r = \begin{cases} {}^{1}/_{10^M}, & E_P \neq 0 \\ 0, & E_P = 0 \end{cases}$$

$$M = [{}^{1}/_{E_P} - 1]^{{}^{1}/_{3}} \times 2.301,$$

$$E_r \equiv error\ rate,$$

$$E_P \equiv error\ possibility,$$

To illustrate, suppose 3 members assigned the linguistic variables very high (VH), medium (M) and low (L) to assess a factor's effect on safety. The linguistic variables are translated into fuzzy error probabilities by triangular membership functions displayed in Figure 4.3. The fuzzy probability values associated with very high, medium and low are {(0.8, 0.9, 1.0), (0.4, 0.5, 0.6), (0.2, 0.3, 0.4)} respectively. The aggregated fuzzy error probability is calculated as follows:

$$x_{avg} = \frac{\{(0.8, 0.9, 1.0) + (0.4, 0.5, 0.6) + (0.2, 0.3, 0.4)\}}{3}$$

$$= (0.47, 0.57, 0.67)$$

The aggregated fuzzy error probability (0.47, 0.57, 0.67) is converted into the fuzzy error rate by the following equations:

$$M = [{}^{1}/_{0.47} - 1]^{1/3} \times 2.301 = 2.395$$

$$E_r = \frac{1}{10^{2.395}} = 0.00403$$

$$M = [{}^{1}/_{0.57} - 1]^{1/3} \times 2.301 = 2.095$$

$$E_r = \frac{1}{10^{2.095}} = 0.00809$$

$$M = [{}^{1}/_{0.67} - 1]^{1/3} \times 2.301 = 1.817$$

$$E_r = \frac{1}{10^{1.817}} = 0.0152$$

Table 4.3 displays the corresponding error rate (E_r) of the error probability (E_p).

Table 4.3 - Fuzzy Error Rate (E_r) and Fuzzy Error Probability (E_p)

E_p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
E_r	0.0000164	0.000223	0.000881	0.00232	0.00500	0.00977	0.0184	0.0355	0.0782	1.0

4.2.5 DEFUZZIFICATION TO OBTAIN CRISP NUMERIC OUTPUT

Once each linguistic expression assigned by the expert panel is aggregated into one fuzzy number to rank each components effect on safety, they are directly entered into the model to adjust component reliabilities. The aggregated fuzzy numbers represent location-specific scoring assigned to each factor by the expert panel as they compare the specific situation to the global average. The aggregated fuzzy numbers are used to adjust component reliability to predict the overall system reliability and accident frequency of the specific case by one of the following three scenarios:

1. Adjusting direct layer component reliabilties, inputing the adjusted values into the model to calculate system reliability and accident frequency directly.
2. Adjusting corporate component reliabilities, inputing the adjusted values into the model which influence the direct layer values. Once direct layer values are determined, system reliability and accident frequency are calculated
3. Adjusting external component reliabilities, inputing the adjusted values into the model which influence corporate values and, in turn, the direct values, facilitating system reliability and accident frequency calculations.

The model uses the adjusted component reliabilities to predict overall system reliability and frequency of accidents for the specific case. The final outcome predicted by the model is a fuzzy value for it contains the uncertainties propagated from the ranking of each factor by use of fuzzy numbers. This fuzzy predicted outcome (frequency of occupational accidents) must now be translated into a crisp numeric output through a process of defuzzification.

Defuzzification is the process of combining all fuzzy outputs into a specific composite result. It is the process used to calculate the crisp value of a fuzzy set. "When using multiple inputs, the intent of defuzzification is to translate the obtained linguistic value and membership function into a singular crisp value" (McCauley-Bell, 1996). There are many methods of defuzzification but the three most commonly used methods are Maximum Defuzzification, Weighted Average Defuzzification and the Centroid Defuzzification Technique.

Maximum defuzzification takes the strongest fuzzy output as the result for the system output. It gives the output with the highest membership function. This defuzzification technique is very fast but is only accurate for peaked output. Maximum defuzzification is considered to be a poor method due to the lack of input from other factors (McCauley-Bell, 1996). With the Weighted Average defuzzification technique, the output is obtained by the weighted average of each output of the set of rules stored in the knowledge base of the system. This method is computationally faster and easier and gives fairly accurate results.

The Centroid defuzzification technique, also known as center of gravity (COG) or center of area technique, was developed by Sugeno in 1985 and is the most accepted and commonly used method of defuzzification. The COG technique is considered to be a more effective method than the maximum defuzzification and weighted average techniques because it is very accurate and considers the contribution of all fuzzy outputs and the degree to which each is true. Because of its effectiveness and accuracy in the defuzzification process, the COG technique will be utilized in the present work.

The COG method determines the center of the area of the combined membership functions. Using the membership function $f_W(z)$ for the average fuzzy number, W, the COG of the area under the membership function is calculated as follows:

$$z = \frac{\int_a^c f_W(z) z dz}{\int_a^c f_W dz}$$

where $[a, c]$ is an interval containing the support of f_w.

Chapter 5

CASE STUDY

In order to verify the methodology of FST, a case study is presented in this chapter to illustrate the use of FST within Attwood's predictive model. The methodology used to incorporate FST within the model is outlined in detail in Section 4.2 of Chapter 4. This case study was previously executed by Attwood *et al.* (2006c) to showcase the effectiveness and versatility of the predictive model by comparing the number of predicted and actual annual accidents on a Newfoundland (NL), Canada, based installation. The main purpose of this case study is to construct an easy method to evaluate and minimize uncertainty and integrate it into the framework of Attwood's predictive model by use of FST. Through this proposed methodology, the judgemental uncertainties associated with experts' subjective opinions can be expressed properly by using fuzzy sets and the accident rate can be assessed with better confidence. The subsequent section of this chapter will give a detailed account of the incorporation of FST within the model to address the uncertainty and imprecision arising out of the subjectivity of expert opinion.

5.1 CASE STUDY

A case study has been executed to compare the number of predicted and actual annual accidents on a Newfoundland (NL), Canada, based installation. The main objective of this study is to enhance the effectiveness of the predictive model by using FST as a means of modeling uncertainty. As a methodology, FST incorporates imprecision and subjectivity in the form of expert opinions into the model formulation and solution process, providing a framework to achieve "all the universally recognized advantages of fuzzy representation such as cognitive plausibility and robustness" (Baroni & Guida, 1998). The proposed methodology recognizes that uncertainty plays a role in decision making and uses FST to minimize uncertainty and assess the occupational accident rate with greater confidence through a) the assignment of linguistic variables to represent each factor's effect on safety or specific case condition; b) conversion of linguistic variables into a fuzzy numerical range through the development of membership functions; c) aggregation of fuzzy numbers into one fuzzy variable to represent each factors effect on safety; d) calculation of the fuzzy error probability and fuzzy error rate to estimate the degree of uncertainty each component contributes to the final outcome; e) propagation of fuzzy numbers to adjust component reliabilities to determine the fuzzy outcome or frequency of occupational accidents; f) the use of fuzzy operations to calculate a crisp numeric model output; and g) interpretation of the uncertainties associated with the fuzzy outcomes. The stepwise analysis of the described methodology of FST is presented below.

5.2 ACTUAL VS PREDICTED ACCIDENT RATE ON A NL OIL & GAS PLATFORM

A Newfoundland based 100 POB (persons on board) production installation was chosen as a case study for the model in a paper published by Attwood et al. (2006) entitled "Validation of an Offshore Occupational Accident Frequency Prediction Model – A Practical Demonstration Using Case Studies" Attwood assumed production to be a 24 hour operation which is normal operational procedure on most offshore oil and gas installations, with a split shift scenario for each worker. This means that 50% of workers are "on shift" while 50% of workers are resting. This scenario can also be viewed as if 50% of the POB are working continuously.

Data for the calibration portion of the model application are publicly available (Attwood *et al.*, 2006a). For the purposes of this case study, the expected number of accidents on a 100 POB installation was calculated by combining the 2004 annual average global accident rates, TRIR (total recordable incident rate) available from the OGP database with the total number of people on board. Table 5.1 displays accident rates (events per million hours) for the Newfoundland case study.

Table 5.1 - Accident rates per million hours (Attwood *et al.*, 2006c)

	2000	**2001**	**2002**	**2003**	**2004**	**Average**
Global average (TRIR)	8.84	6.85	5.77	4.87	**6.36**	6.54
Newfoundland (TRIR)	10.16	9.49	8.04	11.45	4.36	8.70
Number of accidents (based on global average TRIR)	3.87	3.00	2.53	2.13	2.79	2.86
Number of accidents (based on Newfoundland TRIR)	4.45	4.16	3.52	5.02	1.91	3.81

Step 1 – Calculate Actual number of accidents under global average conditions

Using the global average (TRIR) for 2004 of 6.36 accidents per million manhours, and assuming 50% of POB are working continuously, the expected number of accidents is calculated as follows:

$$Expected\ accidents$$

$$= 6.36\ accidents/1{,}000{,}000\ manhours \times 100\ persons \times 0.50\ working \times 24\ hours/day \times 365.25\ days/year$$

$$= 2.79$$

Step 2 – Calibration run to determine base case component reliabilities

The model is then run in calibration mode in order to set base case component reliabilities. The expected number of accidents is used to back calculate overall system reliability. Once the output (ie. accident rate = 2.79) is determined using global average accident statistics, a starting reliability is calculated using the reliability equation based on a constant failure rate as outlined below.

$$R(t) = \exp[-\int_0^t \lambda dt] = e^{-\lambda t},\ t > 0$$

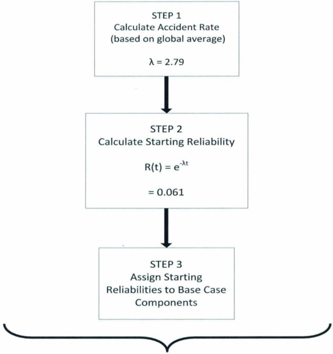
$$= e^{-2.79(1)}$$

$$= 0.061$$

where:

λ = accident rate = 2.79
R(t) = system reliability
t = time = 1

This starting reliability, 0.061, is then assigned to each base case component within the external, corporate and direct layers to set the base case for comparison between actual and predicted results. The expected number of accidents is 2.79 based on the global average and overall system reliability is 0.061 as determined through the calibration run. Figure 5.1 displays systematic steps involved in assignment of base case reliabilities for the calibration run.



External Layer	Base Case Reliability
Price of oil	0.061
Shareholder pressure	0.061
Royalty regime	0.061
Value placed on human life	0.061

Corporate Layer	Base Case Reliability
Safety culture	0.061
Safety training programs	0.061
Safety procedures	0.061

Direct Layer	Base Case Reliability
Behavioural	
Attitude	0.061
Motivation	0.061
Capability	
Physical	0.061
Coordination	0.061
Fitness	0.061
Lack of fatigue	0.061
Mental	0.061
Knowledge	0.061
Intelligence	0.061
Weather	0.061
Safety design	0.061
PPE	0.061

FIGURE 5.1 - ASSIGNMENT OF BASE CASE RELIABILITIES FOR CALIBRATION RUN

Step 3 – Fuzzy Approach to Adjust Component Reliability

At this stage of model execution, component reliability adjustments are required to assign scores to components to represent specific case conditions. Fuzzy set theory is incorporated into this phase of model execution to account for the subjective uncertainty associated with the expert panel opinions when rating each factors effect on overall safety.

Step 3.1 – Assignment of Fuzzy Linguistic Variable

A panel of seven (7) qualified safety professionals, averaging 18 years experience within the oil and gas industry, were used in Attwood's study to rate Newfoundland's safety environment compared to global average conditions. The expert panel assign one of five fuzzy linguistic variables, very low (VL), low (L), medium (M), high (H) and very high (VH), to each direct, corporate and external factor to rate each factor's effect on safety for the specific case (NL offshore installation) compared with the global average as shown in Table 5.2. The global average is assigned a fuzzy linguistic variable of medium representing a fuzzy value of (4, 5, 6).

Table 5.2 - Assignment of linguistic variables to rate NL safety environment

Factor	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	Expert 7
External							
Value placed on human life	H	VH	VH	VH	H	VH	VH
Price of oil	VH	VH	H	H	VH	VH	VH
Shareholder pressure	VL	M	VL	L	L	VL	L
Royalty regime	L	L	M	M	M	L	L
Corporate							
Safety culture	H	H	H	H	VH	H	H
Safety training	VH	H	H	H	H	VH	H
Safety procedures	VH	VH	H	H	VH	VH	VH
Direct							
Attitude	H	H	H	VH	H	H	VH
Motivation	M	M	H	H	M	M	M
Lack of fatigue	M	H	M	M	H	H	H
Coordination	L	L	M	M	L	M	M
Fitness	M	M	L	M	M	H	M
Knowledge	H	M	M	H	H	M	H
Intelligence	L	L	M	M	L	M	L
Safety design	M	M	H	H	H	H	H
Weather	VL	L	VL	VL	VL	L	L
PPE	VH	VH	H	H	VH	VH	VH

Step 3.2 – Conversion of Linguistic Variables into Fuzzy Numbers

The next step involves conversion of fuzzy linguistic variables to triangular fuzzy numbers by use of membership functions. The fuzzy numbers of the five linguistic variables {Very Low (VL), Low (L), Medium (M), High (H) and Very High (VH)} are represented in Figure 5.2.

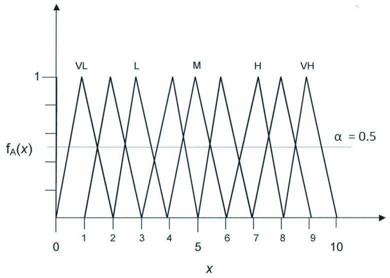


FIGURE 5.2 - TRIANGULAR FUZZY NUMBERS REPRESENTING LINGUISTIC VARIABLES

The corresponding membership functions of these five linguistic values (VL, L, M, H, VH) in triangular fuzzy numbers are illustrated as follows:

$$f_{VL}(x) = \begin{cases} \frac{1}{\frac{(2-x)}{1}} & \begin{matrix} 0 < x \leq 1 \\ 1 < x \leq 2 \end{matrix} \\ 0 & otherwise \end{cases}$$

$$f_L(x) = \begin{cases} \frac{(x-2)}{1} & 2 < x \leq 3 \\ \frac{(4-x)}{1} & 3 < x \leq 4 \\ 0 & otherwise \end{cases}$$

$$f_M(x) = \begin{cases} \frac{(x-4)}{1} & 4 < x \leq 5 \\ \frac{(6-x)}{1} & 5 < x \leq 6 \\ 0 & otherwise \end{cases}$$

$$f_H(x) = \begin{cases} \frac{(x-6)}{1} & 6 < x \leq 7 \\ \frac{(8-x)}{1} & 7 < x \leq 8 \\ 0 & otherwise \end{cases}$$

$$f_{VH}(x) = \begin{cases} \frac{(x-8)}{1} & 8 < x \leq 9 \\ 1 & 9 < x \leq 10 \\ 0 & otherwise \end{cases}$$

Each membership function can be cut horizontally at a finite number of α- confidence levels between 0 and 1 to obtain lower and upper bounds for each confidence interval. For each α-cut of the parameter, the model is run to determine the minimum and maximum possible values of the output. This information is then directly used to construct the corresponding fuzziness (membership functions) of the output which is used as a measure of uncertainty. For the purposes of this example, the author defines a confidence interval of $\alpha = 0.5$ and each membership function is cut horizontally at $\alpha = 0.5$ to obtain lower and upper bounds for each fuzzy value representing the five (5) linguistic variables as shown in Table 5.3.

To illustrate, Expert 1 assigned a linguistic variable of 'High' to the corporate factor 'Safety Culture'. The linguistic variable 'High' is represented by a fuzzy value of (6, 7, 8) as defined by the membership function:

$$f_H(x) = \begin{cases} \dfrac{(x-6)}{1} & 6 < x \leq 7 \\ \dfrac{(8-x)}{1} & 7 < x \leq 8 \\ 0 & otherwise \end{cases}$$

Alternatively, defining the interval of confidence at a level, $\alpha = 0.5$, the linguistic variable 'High' is characterized by the triangular fuzzy number, $\widetilde{H}$:

$$\forall_{\alpha} = [0,1]:$$

$$\widetilde{H}_{\alpha} = [a^{\alpha}, c^{\alpha}] = [6^{0.5}, 8^{0.5}]$$

$$= [(b-a)\alpha + a, c - (c-b)\alpha]$$

$$= [(7-6)0.5 + 6, 8 - (8-7)0.5]$$

$$= [6.5, 7.5]$$

Therefore, the linguistic variable 'High' is converted into the fuzzy value $[6.5, 7.5]$ which represents the minimum and maximum fuzzy values for a confidence interval of $\alpha = 0.5$.

Table 5.3 - Linguistic Values and Fuzzy Values

Linguistic Value	Confidence Interval of $\alpha = 0.5$	
	Lower Bound (Min)	Upper Bound (Max)
Very Low (VL)	0.5	1.5
Low (L)	2.5	3.5
Medium (M)	4.5	5.5
High (H)	6.5	7.5
Very High (VH)	8.5	9.5

Step 3.3 – Aggregate Expert Opinions into a single Fuzzy Number

It is necessary to aggregate the opinions of multiple experts in order to achieve a more reliable assessment of the specific environment. The mean average operator is used to aggregate the opinion of each expert into one fuzzy number. To illustrate, under the direct layer, the factor 'Safety Design' was rated as having a 'Medium' effect on safety by 2 experts and assigned a value of 'High' by 5 experts as shown in Table 5.2. The fuzzy numbers representing 'Medium' (M) and 'High' (H) are defined as follows:

$$f_M(x) = \begin{cases} \dfrac{(x-4)}{1} & 4 < x \leq 5 \\ \dfrac{(6-x)}{1} & 5 < x \leq 6 \\ 0 & otherwise \end{cases}$$

$$f_H(x) = \begin{cases} \dfrac{(x-6)}{1} & 6 < x \leq 7 \\ \dfrac{(8-x)}{1} & 7 < x \leq 8 \\ 0 & otherwise \end{cases}$$

The addition of M and H are computed as:

$$f_{M+H}(z) = \max_{z=x+y}\left(2 \cdot f_M(x) \wedge 5 \cdot f_H(y)\right)$$

$$= [(2m_1) + (5h_1), (2m_2) + (5h_2)]$$

$$= [(2\alpha + 8) + (5\alpha + 30), (12 - 2\alpha) + (40 - 5\alpha)]$$

$$= [(7\alpha + 38), (52 - 7\alpha)]$$

Defining a confidence interval of $\alpha = 0.5$, the average fuzzy number SD is computed as follows:

$$SD = \frac{1}{7} \times [(7\alpha + 38), (52 - 7\alpha)] = [(\alpha + 5.4), (7.4 - \alpha)]$$

$$= [(0.5 + 5.4), (7.4 - 0.5)] = [5.9, 6.9]$$

Thus, the membership function of the aggregated (average) fuzzy number SD representing the direct factor 'Safety Design' is:

$$f_{SD}(x) = \begin{cases} \dfrac{(z - 5.9)}{1} & 5.9 < x \leq 6.4 \\ \dfrac{(6.9 - z)}{1} & 6.4 < x \leq 6.9 \\ 0 & otherwise \end{cases}$$

The average fuzzy number for 'Safety Design' can also be calculated by using the following equation:

$$f_{SD}(x) = \frac{\{2 \cdot (4.5, 5, 5.5) + 5 \cdot (6.5, 7, 7.5)\}}{7}$$

$$= \frac{41.5, 45, 50.5}{7}$$

$$= (5.9, 6.4, 6.9)$$

Defining a confidence interval of 0.5, the direct factor 'Safety Design' is assigned a fuzzy value of [5.9,6.9] with a mean value of 6.4 to rate its effect on safety of the specific case as compared to the global average. Table 5.4 displays the aggregated fuzzy numbers defined by a confidence interval of $\alpha = 0.5$ which are used to adjust component reliabilities for all factors under the direct, corporate and external layers. Calculations for the aggregation of fuzzy numbers are outlined in Appendix B.

Table 5.4 - Summary of Aggregated Fuzzy Numbers

Factor	Confidence interval, $\alpha = 0.5$		
	Fuzzy value	**Lower bound (Min)**	**Upper bound (Max)**
External			
Value placed on human life	(7.9, 8.4, 8.9)	7.9	8.9
Price of oil	(7.9, 8.4, 8.9)	7.9	8.9
Shareholder pressure	(1.9, 2.4, 2.9)	1.9	2.9
Royalty regime	(3.4, 3.9, 4.4)	3.4	4.4
Corporate			
Safety culture	(6.8, 7.3, 7.8)	6.8	7.8
Safety training	(7.1, 7.6, 8.1)	7.1	8.1
Safety procedures	(7.9, 8.4, 8.9)	7.9	8.9
Direct			
Attitude	(7.1, 7.6, 8.1)	7.1	8.1
Motivation	(5.1, 5.6, 6.1)	5.1	6.1
Lack of fatigue	(5.6, 6.1, 6.6)	5.6	6.6
Coordination	(3.6, 4.1, 4.6)	3.6	4.6
Fitness	(4.5, 5.0, 5.5)	4.5	5.5
Knowledge	(5.6, 6.1, 6.6)	5.6	6.6
Intelligence	(3.4, 3.9, 4.4)	3.4	4.4
Safety design	(5.9, 6.4, 6.9)	5.9	6.9
Weather	(1.4, 1.9, 2.4)	1.4	2.4
PPE	(7.9, 8.4, 8.9)	7.9	8.9

Step 3.4: Adjust component reliabilities with Aggregated Fuzzy Numbers

The fuzzy numbers, displayed in Table 5.4, represent location-specific scoring assigned to each factor by the expert panel as they compare the specific situation (NL installation) to the global average. The model predicts a fuzzified accident frequency rate for the specific case by directly entering the fuzzy values for the direct, corporate or external layer components. Allowing the model to run in predictive mode using minimum and maximum fuzzy values to adjust component reliabilities determines minimum and maximum outputs for the accident frequency rate which represents the upper and lower bounds of the fuzzy output. A triangular membership function is used to convert this fuzzy output into a crisp, numerical value which is used for comparison purposes with the actual (global average) case.

For this specific case study, component reliabilities within the corporate layer are adjusted. As an example, the corporate component 'Safety Training' is adjusted by multiplying the base case reliability of 'Safety Training' with the minimum fuzzy ratio of 7.1/4.5 to obtain its minimum adjusted reliability value. The value of 4.5 represents the lower bound for global average which was assigned a linguistic value of 'Medium'. The value of 7.1 was taken from Table 5.4 and represents the minimum fuzzy value for 'Safety Training'. External component reliabilities are assigned a value of 0.061 which is the base case reliability as calculated in the calibration run. The following calculation is used to obtain the minimum reliability value for 'Safety Training':

$$R_{st(min)} = (R_{po} \times I_{pot} + R_{sp} \times I_{spt} + R_{rr} \times I_{rrt} + R_{vl} \times I_{vlt}) \times \left(\frac{7.1}{4.5}\right)$$

$$= (0.061 \times 0.18 + 0.061 \times 0.27 + 0.061 \times 0.18 + 0.061 \times 0.27) \times \left(\frac{7.1}{4.5}\right)$$

$$= 0.085$$

where:

$R_{po} = reliability\ of\ price\ of\ oil$
$= 0.061\ (direct\ input\ from\ base\ case\ run)$

$R_{sp} = reliability\ of\ shareholder\ pressure$
$= 0.061\ (direct\ input\ from\ base\ case\ run)$

$R_{rr} = reliability\ of\ royalty\ regime$
$= 0.061\ (direct\ input\ from\ base\ case\ run)$

$R_{vl} = reliability\ of\ value\ placed\ on\ human\ life$
$= 0.061\ (direct\ input\ from\ base\ case\ run)$

$I_{pot} = influence\ coefficient\ of\ price\ of\ oil$
$= 0.18\ (direct\ input\ from\ model\ as\ displayed\ in\ Table\ 1, Ch. 2)$

$I_{spt} = influence\ coefficient\ of\ shareholder\ pressure$
$= 0.27\ (direct\ input\ from\ model\ as\ displayed\ in\ Table\ 1, Ch. 2)$

$I_{rrt} = influence\ coefficient\ of\ royalty\ regime$
$= 0.12\ (direct\ input\ from\ model\ as\ displayed\ in\ Table\ 1, Ch. 2)$

$I_{vlt} = influence\ coefficient\ of\ value\ placed\ on\ human\ life$
$= 0.43\ (direct\ input\ from\ model\ as\ displayed\ in\ Table\ 1, Ch. 2)$

The reliability of each component within the corporate layer is adjusted accordingly to obtain minimum reliability values which are used to influence direct layer components, facilitating a minimum value for model outputs (ie. minimum value for overall system reliability and accident frequency rate). Reliability adjustments are then made using maximum values to obtain maximum model outputs for overall system reliability and accident frequency rate. Reliability adjustment equations for all other elements within the corporate layer are displayed in Appendix C.

Step 3.5 – Calculate Fuzzy Error Probability and Fuzzy Error Rate

A fuzzy error factor is calculated to account for the imprecision of data. The error factor is associated with the most possible value of the linguistic variables. The term "error possibility" is essentially a fuzzy probability and is used to obtain a fuzzy error rate for each linguistic expression. Fuzzy probability is a fuzzy number characterized by its membership function. Ferdous *et al.* (2009) states that fuzzy probability attempts to define a basic event into a fuzzy probability set and uses these fuzzy events in subsequent computations. The imprecise probabilities of basic events are refined by characterizing the basic event data with a suitable membership function thereby minimizing the error due to uncertainty in basic event probabilities by using fuzzy probability for quantifications. The proposed methodology uses a fuzzy probability to obtain a fuzzy error rate for each linguistic expression. Huang *et al.* (2001) proposed that the fuzzy error rate, E_r , can be calculated using the following equation:

$$E_r = \begin{cases} 1/10^M, & E_P \neq 0 \\ 0, & E_P = 0 \end{cases}$$

$$M = [1/E_P - 1]^{1/3} \times 2.301,$$

$$E_r \equiv error\ rate,$$

$$E_P \equiv error\ possibility,$$

$$k \equiv {}^{1}\!\Big/_{\log\left(\frac{1}{5 \times 10^{-3}}\right)} \approx 2.301^{-1}$$

The fuzzy error probabilities of the five linguistic variables {Very Low (VL), Low (L), Medium (M), High (H) and Very High (VH)} are represented by a triangular membership function as displayed in Figure 5.3.

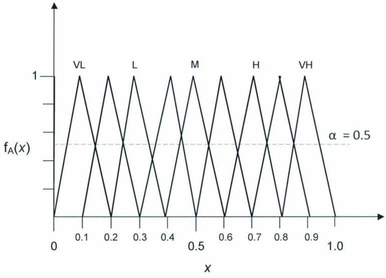


FIGURE 5.3 - TRIANGULAR FUZZY NUMBERS REPRESENTING FUZZY ERROR PROBABILITIES

A summary of linguistic values and corresponding fuzzy error probabilities are displayed in Table 5.5.

Table 5.5 - Linguistic Variables and Fuzzy Error Probabilities

	Confidence Interval of $\alpha = 0.5$		
	Fuzzy error possibility	**Lower Bound (Min)**	**Upper Bound (Max)**
Very Low (VL)	(0.05, 0.1, 0.15)	0.05	0.15
Low (L)	(0.25, 0.3, 0.35)	0.25	0.35
Medium (M)	(0.45, 0.5, 0.55)	0.45	0.55
High (H)	(0.65, 0.7, 0.75)	0.65	0.75
Very High (VH)	(0.85, 0.9, 0.95)	0.85	0.95

The fuzzy error rate for each component is calculated to measure the percentage of uncertainty each component contributes to the final outcome. The error probability is evaluated by transforming the linguistic values assigned by each expert into corresponding fuzzy error probabilities. The method proposed by Yang *et al.* (2003) is used to aggregate expert opinions into one fuzzy error probability. The aggregated fuzzy error probabilities for each component can be converted into a fuzzy error rate by using the method proposed by Huang *et al.* (2001). A summary of aggregated fuzzy error probabilities and fuzzy error rates is displayed in Table 5.6. The fuzzy error rates for each component can be used to measure the degree of uncertainty each component contributes to the final outcome.

The error possibility is evaluated by transforming the linguistic values assigned by each expert into corresponding fuzzy error possibilities. The method proposed by Yang *et al.* (2003) is used to aggregate expert opinions into one fuzzy error possibility. The aggregated fuzzy error possibility for each component can be converted into a fuzzy error rate by using the method proposed by Huang *et al.* (2001). A summary of aggregated fuzzy error possibilities and fuzzy error rates is displayed in Table 5.6. The fuzzy error rates for each component can be integrated into the adjusted reliability calculations (model) to measure how much uncertainty each component contributes to the final outcome.

Table 5.6 - Summary of Aggregated Fuzzy Error Probabilities & Fuzzy Error Rates

Factor	Confidence interval, $\alpha = 0.5$			
	Aggregated fuzzy error possibility		**Aggregated fuzzy error rate**	
	Minimum	Maximum	Minimum (%)	Maximum (%)
External				
Value placed on human life	0.79	0.89	3.31	7.14
Price of oil	0.79	0.89	3.31	7.14
Shareholder pressure	0.19	0.29	0.0186	0.0623
Royalty regime	0.34	0.44	0.135	0.321
Corporate				
Safety culture	0.68	0.78	1.62	3.10
Safety training	0.71	0.81	1.96	3.81
Safety procedures	0.79	0.89	3.31	7.14
Direct				
Attitude	0.71	0.81	1.96	3.81
Motivation	0.51	0.61	0.536	1.40
Lack of fatigue	0.56	0.66	0.752	1.43
Coordination	0.36	0.46	0.163	0.374
Fitness	0.45	0.55	0.347	0.705
Knowledge	0.56	0.66	0.753	0.0413
Intelligence	0.34	0.44	0.135	0.321
Safety design	0.59	0.69	0.916	1.73
Weather	0.14	0.24	0.00611	0.0418
PPE	0.79	0.89	3.31	7.14

As shown in Table 5.6, the external component 'Value placed on human life' has a fuzzy error range of [3.31%, 7.14%] while the direct component 'Weather' has a fuzzy error range of [0.00611%, 0.0418%]. Therefore, when comparing the fuzzy error rates of the two components, the external component 'Value placed on human life' contributes a greater degree of uncertainty to the final fuzzy outcome (accident frequency).

Step 4 – Prediction run to obtain Fuzzy Outcome

The next step involves running the model with adjusted component reliabilities to predict the minimum and maximum values of overall system reliability and accident frequency. For this specific case, prediction of system reliability and accident frequency on a NL installation is determined by adjusting components within the corporate layer which, in turn, influence the direct layer values, facilitating the calculation of the final outcomes. The fuzzy numbers assigned to rate each component are propagated through the model to predict fuzzified lower (minimum) and upper (maximum) bounds for both system reliability and accident frequency. This information is then directly used to construct the corresponding fuzziness (membership functions) of the predicted accident rate which is used as a measure of uncertainty. Figure 5.4 displays the model output using minimum fuzzy values to adjust component reliabilities and calculate predicted fuzzy outcomes while Figure 5.5 displays the model output using maximum fuzzy values.

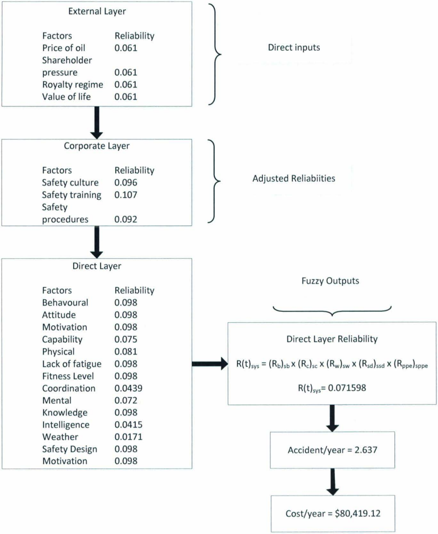


FIGURE 5.4 - PREDICTIVE RUN DISPLAYING MINIMUM FUZZY MODEL OUTPUTS

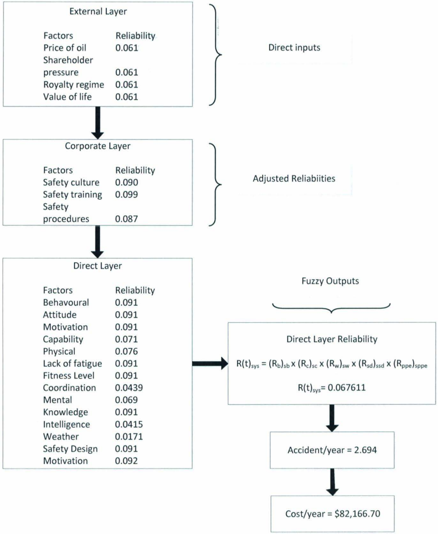


FIGURE 5.5 - PREDICTIVE RUN DISPLAYING MAXIMUM FUZZY MODEL OUTPUTS

Step 5 – Defuzzification of Fuzzy Outcome into Crisp value

Following the prediction run, the minimum and maximum values obtained for accident rate are 2.64 and 2.69, respectively. These fuzzy values which represent the lower and upper bounds are used to construct the triangular membership function representing the predicted fuzzy accident rate. The predicted outcome for accident rate is a fuzzy value for it contains the uncertainties propagated from the ranking of each factor by use of fuzzy numbers. This fuzzified value must now be translated into a crisp numeric output through a process of defuzzification.

The Centroid defuzzification technique, also known as center of gravity (COG) or center of area technique, considers the contribution of all fuzzy outputs and the degree to which each is true. Because of its effectiveness and accuracy in the defuzzification process, the COG technique will be utilized in the present work. The COG method determines the center of the area of the combined membership functions. Using the membership function $f_{ar}(x)$ to represent the fuzzy accident rate, the COG of the area under the membership function is calculated as follows:

$$x = \frac{\int_a^c f_{ar}(x)xdx}{\int_a^c f_{ar}dx}$$

where $[a, c]$ represents the lower and upper bounds, 2.64 and 2.69, containing the support of f_{ar}. Using the triangular membership function, displayed in Figure 5.6, the accident rate is represented by the membership function:

$$f_{ar}(x) = \begin{cases} \frac{(x - 2.637)}{1} & 2.637 < x \leq 2.665 \\ \frac{(2.694 - x)}{1} & 2.665 < x \leq 2.694 \\ 0 & otherwise \end{cases}$$

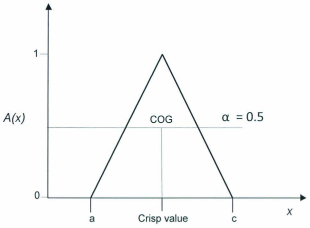


FIGURE 5.6 - TRIANGULAR MEMBERSHIP FUNCTION REPRESENTING PREDICTED ACCIDENT RATE

Defining a confidence interval of $\alpha = 0.5$ and using the COG technique, the crisp value for accident rate is the centre of area of the triangular membership function, corresponding to a crisp value of 2.67.

Step 6 – Comparison of predictions with estimates of actual accident numbers

For 2004, the actual number of accidents expected on an offshore oil and gas platform, based on the global average is 2.79, as shown in Table 5.7. Using the proposed methodology, the predicted number of accidents per year, based on the incorporation of FST into the framework of Attwood's predictive model, is 2.67 which is very close to the industry average. The components contributing the greatest degree of uncertainty to the final outcome were 'Value placed on human life', 'Price of oil', 'Safety Procedures', and 'PPE' having a fuzzy error range between 3.31% and 7.14%. The component contributing the least amount of uncertainty to the final outcome was 'Weather' which had a fuzzy error range between 0.00611% and 0.0418%.

Table 5.7 - Comparison of Actual vs. Predicted Results for Accident Rate (2004)

	Reliability	**Accident rate/year**	**Cost/year**
Base Case (global average)	0.061	2.79	$85,310.07
Predicted Case (Crisp value)	0.0696	2.67	$81,292.22
Predicted Case (Fuzzy Minimum)	0.0716	2.64	$80,419.12
Predicted Case (Fuzzy Maximum)	0.0676	2.69	$82,166.70

The objective of the case study is to minimize uncertainty while maintaining the simplicity of Attwood's predictive model. By use of FST, it has provided an effective means to account for and minimize uncertainty, which plays a significant role in the decision making process of Attwood's model. The proposed methodology also enables the user to identify the amount of uncertainty each component contributes to the final result.

Chapter 6

CONCLUSION

6.1 CONCLUSION

Fuzzy set theory (FST) provides a useful tool to address the judgemental uncertainties associated with experts' subjective opinions through the use of fuzzy sets. The incorporation of FST into Attwood's predictive model is aimed to enhance the effectiveness of the model by providing a mathematical tool to account for vagueness and uncertainty associated with expert judgements and opinions and to propagate this uncertainty through the model. The novelty of the proposed methodology lies in the approach that embraces uncertainty as an inseparable element of the system and incorporates it within the framework of Attwood's model. The application of such a model can help predict the frequency of occupational accidents with better confidence by recognizing uncertainty and incorporating it in the analysis process by use of fuzzy sets.

The proposed methodology uses linguistic variables to rate a components effect on safety for the specific environment, providing a more intuitive, user-friendly approach to the analysis process. These linguistic variables are converted into fuzzy numbers which carry more information than a crisp, numerical rating factor and allow the judgemental uncertainties associated with experts' subjective opinions to be properly expressed. These fuzzy numbers are characterized by triangular membership functions which incorporate the uncertainty of the component. By using a fuzzy approach, uncertainty is introduced at the component level when rating each components effect on safety and the analytical method is used to propagate it further. Through the use of fuzzy error probability and fuzzy error rate, one can measure the degree of uncertainty each component contributes to the final outcome providing the analyzer with useful information for strategies that focus on the reduction of total uncertainty. The proposed methodology recognizes that uncertainty plays a role in decision making and incorporates a fuzzy approach to account for and minimize uncertainty while maintaining the simplicity of Attwood's model. It provides a more effective means for assessing the occupational accident rate by:

i. Assigning a linguistic, qualitative scale of importance to rate each components effect on safety for the specific case, providing a more natural, user-friendly approach to the analysis process;

ii. Conversion of linguistic variables into fuzzy numbers to incorporate the uncertainties of expert opinions when rating a components effect on safety. Utilizing triangular membership functions to convert each linguistic expression into a corresponding fuzzy number minimizes the error due to the subjective uncertainty of experts' opinions by use of fuzzy sets. Each input parameter is treated as a fuzzy number and the uncertainty is characterized by a triangular membership function.

iii. Utilizing fuzzy operations to effectively propagate uncertainty through the model, calculate a crisp numeric output through a process of defuzzification and estimate the degree of uncertainty each component contributes to the final outcome.

6.2 FUTURE WORK

This work proposes a fuzzy methodology to evaluate uncertainty and integrate it into the framework of Attwood's predictive model to effectively assess the occupational accident rate with better confidence. Many other aspects remain to be investigated using this proposed methodology within Attwood's predictive model. The following recommendations have been suggested for future improvement of the proposed study:

i. In order to improve upon the effectiveness of the model, the model can be further modified by incorporating FST during the early stages of model development to rank influence coefficients and strength factors used within the reliability calculations.

ii. Attwood *et al.* (2006c) have demonstrated the versatility of the predictive model through the execution of case studies to predict the occupational accident frequency under unique safety environments, to observe improvements in results achievable with changes in input conditions and as a means of setting realistic safety targets. In order to make the proposed methodology more reliable and effective, it is necessary that the proposed methodology outlined in this case study be properly compared and validated with the analysis of more case studies using Attwood's predictive model.

iii. By estimating the fuzzy error rate, one can identify the degree of uncertainty each component contributes to the final result. The error rate can highlight components that contribute the greatest degree of uncertainty, allowing one to observe improvements in the occupational accident rate with changes to input parameters contributing the highest degree of uncertainty to the final outcome.

iv. Attempts need to be made to utilize different fuzzy methods for constructing membership functions or aggregating expert opinions and compare the results to determine the most effective fuzzy approach to address uncertainty within the framework of Attwood's predictive model.

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APPENDIX A

RELIABILITY CALCULATIONS

A.1 DIRECT LAYER COMPONENT RELIABILITIES (Attwood *et al.*, 2006a)

Behaviour:

R_b = reliability value for behaviour = $(1 - (1 - R_a)^{sa} \times (1 - R_m)^{sm})$

where:

Attitude:

$R_a = (R_{st}) \times I_{sta} + R_{pr} \times I_{pra} + R_{sc} \times I_{sca}$

Motivation:

$R_m = (R_{st}) \times I_{stm} + R_{pr} \times I_{prm} + R_{sc} \times I_{scm}$

where:

R_{st} = reliability of safety training (defined below)

R_{pr} = reliability of safety procedures (defined below)

R_{sc} = reliability of safety culture (defined below)

I_{sta} = influence coefficient of safety training on attitude

I_{pra} = influence coefficient of safety procedures on attitude

I_{sca} = influence coefficient of safety culture on attitude

I_{stm} = influence coefficient of safety training on motivation

I_{prm} = influence coefficient of safety procedures on motivation

I_{scm} = influence coefficient of safety culture on motivation

sa = strength of attitude

sm = strength of motivation

Safety training:

$R_{st} = R_{po} \times I_{post} + R_{sp} \times I_{spst} + R_{rr} \times I_{rrst} + R_{vl} \times I_{vlst}$

Safety procedures:

$R_{pr} = R_{po} \times I_{popr} + R_{sp} \times I_{sppr} + R_{rr} \times I_{rrpr} + R_{vl} \times I_{vlpr}$

Safety culture:

$R_{sc} = R_{po} \times I_{posc} + R_{sp} \times I_{spsc} + R_{rr} \times I_{rrsc} + R_{vl} \times I_{vlsc}$

where:

R_{po} = reliability of price of oil (direct input)

R_{sp} = reliability of shareholder pressure (direct input)

R_{rr} = reliability of royalty regime (direct input)

R_{vl} = reliability of value placed on human life (direct input)

I_{post} = influence coefficient of price of oil on safety training

I_{spst} = influence coefficient of shareholder pressure on safety training

I_{rrst} = influence coefficient of royalty regime on safety training

I_{vlst} = influence coefficient of value placed on human life on safety training

I_{popr} = influence coefficient of price of oil on safety procedures

I_{sppr} = influence coefficient of shareholder pressure on safety procedures

I_{rrpr} = influence coefficient of royalty regime on safety procedures

I_{vlpr} = influence coefficient of value placed on human life on safety procedures

I_{posc} = influence coefficient of price of oil on safety culture

I_{spsc} = influence coefficient of shareholder pressure on safety culture

I_{rrsc} = influence coefficient of royalty regime on safety culture

I_{vlsc} = influence coefficient of value placed on human life on safety culture

Capability:

R_c = reliability value for capability = $(R_p)^{sp} \times (R_{me})^{sme}$

where:

Physical capability:

$R_p = (1 - (1 - R_f)^{sf} \times (1 - R_{lf})^{slf} \times (1 - R_c)^{sc})$

where:

Fitness:

$R_f = (R_{st}) \times I_{stf} + R_{pr} \times I_{prf} + R_{sc} \times I_{scf}$

Lack of Fatigue:

$R_{lf} = (R_{st}) \times I_{stlf} + R_{pr} \times I_{prlf} + R_{sc} \times I_{sclf}$

Coordination:

R_c= direct input

where:

I_{stf} = influence coefficient of safety training on fitness

I_{prf} = influence coefficient of safety procedures on fitness

I_{scf} = influence coefficient of safety culture on fitness

I_{stlf} = influence coefficient of safety training on fitness

I_{prlf} = influence coefficient of safety procedures on lack of fatigue

I_{sclf} = influence coefficient of safety culture on lack of fatigue

sp = strength of physical capability

sme = strength of mental capability

sf = strength of fitness

slf = strength of lack of fatigue

sc = strength of coordination

Mental capability:

$R_{me} = (1 - (1 - R_k)^{sk} \times (1 - R_i)^{si})$

where:

Knowledge:

$R_k = (R_{st}) \times I_{stk} + R_{pr} \times I_{prk} + R_{sc} \times I_{sck}$

Intelligence:

R_i = direct input

where:

I_{stk} = influence coefficient of safety training on knowledge

I_{prk} = influence coefficient of safety procedures on knowledge

I_{sck} = influence coefficient of safety culture on knowledge

sk = strength of knowledge

si = strength of intelligence

Safety Design:

R_{sd}= reliability value for safety design = $(R_{st}) \times I_{stsd} + R_{pr} \times I_{prsd} + R_{sc} \times I_{scsd}$

where:

I_{stsd} = influence coefficient of safety training on safety design

I_{prsd} = influence coefficient of safety procedures on safety design

I_{scsd} = influence coefficient of safety culture on safety design

PPE:

R_{ppe}= reliability value for PPE = $(R_{st}) \times I_{stppe} + R_{pr} \times I_{prppe} + R_{sc} \times I_{scppe}$

where:

I_{stppe} = influence coefficient of safety training on PPE

I_{prppe} = influence coefficient of safety procedures on PPE

I_{scppe} = influence coefficient of safety culture on PPE

Weather:

R_w = reliability value for weather conditions = direct input

APPENDIX B

CALCULATIONS FOR THE AGGREGATION OF FUZZY NUMBERS DEFINED BY A CONFIDENCE INTERVAL (α) OF 0.5

B.1 EXTERNAL LAYER

Value placed on human life (VL):

$$f_{VL}(x) = \frac{\{2 \cdot (6.5, 7, 7.5) + \ 5 \cdot (8.5, 9, 9.5)\}}{7}$$

$$= \frac{\{(13,14,15) + (42.5,45,47.5)\}}{7}$$

$$= \frac{55.5, 59, 62.5}{7}$$

$$= (7.9, 8.4, 8.9)$$

Price of Oil (PO):

$$f_{PO}(x) = \frac{\{2 \cdot (6.5, 7, 7.5) + \ 5 \cdot (8.5, 9, 9.5)\}}{7}$$

$$= \frac{\{(13,14,15) + (42.5,45,47.5)\}}{7}$$

$$= \frac{55.5, 59, 62.5}{7}$$

$$= (7.9, 8.4, 8.9)$$

Shareholder Pressure (SP):

$$f_{SP}(x) = \frac{\{3 \cdot (0.5, 1, 1.5) + 3 \cdot (2.5, 3, 3.5) + 1 \cdot (4.5, 5, 5.5)\}}{7}$$

$$= \frac{\{(1.5, 3, 4.5) + (7.5, 9, 10.5) + (4.5, 5, 5.5)\}}{7}$$

$$= \frac{13.5, 17, 20.5}{7}$$

$$= (1.9, 2.4, 2.9)$$

Royalty Regime (RR):

$$f_{RR}(x) = \frac{\{3 \cdot (4.5, 5, 5.5) + 4 \cdot (2.5, 3, 3.5)\}}{7}$$

$$= \frac{\{(13.5, 15, 16.5) + (10, 12, 14)\}}{7}$$

$$= \frac{23.5, 27, 30.5}{7}$$

$$= (3.4, 3.9, 4.4)$$

B.2 CORPORATE LAYER

Safety Culture (SC):

$$f_{SC}(x) = \frac{\{6 \cdot (6.5, 7, 7.5) + \ 1 \cdot (8.5, 9, 9.5)\}}{7}$$

$$= \frac{\{(39,42,45) + (8.5,9,9.5)\}}{7}$$

$$= \frac{47.5, 51, 54.5}{7}$$

$$= (6.8, 7.3, 7.8)$$

Safety Training (ST):

$$f_{ST}(x) = \frac{\{5 \cdot (6.5, 7, 7.5) + \ 2 \cdot (8.5, 9, 9.5)\}}{7}$$

$$= \frac{\{(32.5,35,37.5) + (17,18,19)\}}{7}$$

$$= \frac{49.5, 53, 56.5}{7}$$

$$= (7.1, 7.6, 8.1)$$

Safety Procedures (SP):

$$f_{SP}(x) = \frac{\{2 \cdot (6.5, 7, 7.5) + 5 \cdot (8.5, 9, 9.5)\}}{7}$$

$$= \frac{\{(13,14,15) + (42.5,45,47.5)\}}{7}$$

$$= \frac{55.5, 59, 62.5}{7}$$

$$= (7.9, 8.4, 8.9)$$

B.3 DIRECT LAYER

Attitude (A):

$$f_A(x) = \frac{\{5 \cdot (6.5, 7, 7.5) + 2 \cdot (8.5, 9, 9.5)\}}{7}$$

$$= \frac{\{(32.5,35,37.5) + (17,18,19)\}}{7}$$

$$= \frac{49.5, 53, 56.5}{7}$$

$$= (7.1, 7.6, 8.1)$$

Motivation (M):

$$f_M(x) = \frac{\{5 \cdot (4.5, 5, 5.5) + \ 2 \cdot (6.5, 7, 7.5)\}}{7}$$

$$= \frac{\{(22.5,25,27.5) + (13,14,15)\}}{7}$$

$$= \frac{35.5, 39, 42.5}{7}$$

$$= (5.1, 5.6, 6.1)$$

Lack of Fatigue (LF):

$$f_{LF}(x) = \frac{\{3 \cdot (4.5, 5, 5.5) + \ 4 \cdot (6.5, 7, 7.5)\}}{7}$$

$$= \frac{\{(13.5,15,16.5) + (26,28,30)\}}{7}$$

$$= \frac{39.5, 43, 46.5}{7}$$

$$= (5.6, 6.1, 6.6)$$

Coordination (C):

$$f_C(x) = \frac{\{3 \cdot (2.5, 3, 3.5) + \ 4 \cdot (4.5, 5, 5.5)\}}{7}$$

$$= \frac{\{(7.5,9,10.5) + (18,20,22)\}}{7}$$

$$= \frac{25.5, 29, 32.5}{7}$$

$$= (3.6, 4.1, 4.6)$$

Fitness (F):

$$f_F(x) = \frac{\{1 \cdot (2.5, 3, 3.5) + \ 5 \cdot (4.5, 5, 5.5) + \ 1 \cdot (6.5, 7, 7.5)\}}{7}$$

$$= \frac{\{(2.5,3,3.5) + (22.5,25,27.5) + (6.5,7,7.5)\}}{7}$$

$$= \frac{31.5, 35, 38.5}{7}$$

$$= (4.5, 5, 5.5)$$

Knowledge (K):

$$f_K(x) = \frac{\{3 \cdot (4.5, 5, 5.5) + \ 4 \cdot (6.5, 7, 7.5)\}}{7}$$

$$= \frac{\{(13.5,15,16.5) + (26,28,30)\}}{7}$$

$$= \frac{39.5, 43, 46.5}{7}$$

$$= (5.6, 6.1, 6.6)$$

Intelligence (I):

$$f_I(x) = \frac{\{4 \cdot (2.5, 3, 3.5) + \ 3 \cdot (4.5, 5, 5.5)\}}{7}$$

$$= \frac{\{(10,12,14) + (13.5,15,16.5)\}}{7}$$

$$= \frac{23.5, 27, 30.5}{7}$$

$$= (3.4, 3.9, 4.4)$$

Weather (W):

$$f_W(x) = \frac{\{4 \cdot (0.5, 1, 1.5) + \ 3 \cdot (2.5, 3, 3.5)\}}{7}$$

$$= \frac{\{(2,4,6) + (7.5,9,10.5)\}}{7}$$

$$= \frac{9.5, 13, 16.5}{7}$$

$$= (1.4, 1.9, 2.4)$$

Personal Protective Equipment (PPE):

$$f_{PPE}(x) = \frac{\{2 \cdot (6.5, 7, 7.5) + \ 5 \cdot (8.5, 9, 9.5)\}}{7}$$

$$= \frac{\{(13,14,15) + (42.5,45,47.5)\}}{7}$$

$$= \frac{55.5, 27, 62.5}{7}$$

$$= (7.9, 8.4, 8.9)$$

APPENDIX C

CORPORATE COMPONENT RELIABILITY ADJUSTMENTS

C.1 MINIMUM ADJUSTED RELIABILITY VALUES

Safety Culture:

$$R_{sc(min)} = (R_{po} \times I_{posc} + R_{sp} \times I_{spsc} + R_{rr} \times I_{rrsc} + R_{vl} \times I_{vlsc}) \times \left(\frac{6.8}{4.5}\right)$$

$$= (0.061 \times 0.18 + 0.061 \times 0.25 + 0.061 \times 0.13 + 0.061 \times 0.44) \times \left(\frac{6.8}{4.5}\right)$$

$$= 0.092$$

where:

R_{po} = *reliability of price of oil*
= 0.061 (*direct input from base case run*)

R_{sp} = *reliability of shareholder pressure*
= 0.061 (*direct input from base case run*)

R_{rr} = *reliability of royalty regime*
= 0.061 (*direct input from base case run*)

R_{vl} = *reliability of value placed on human life*
= 0.061 (*direct input from base case run*)

I_{posc} = *influence coefficient of price of oil*
= 0.18 (*direct input from model as displayed in Table 1, Ch. 2*)

I_{spsc} = *influence coefficient of shareholder pressure*
= 0.25 (*direct input from model as displayed in Table 1, Ch. 2*)

I_{rrsc} = *influence coefficient of royalty regime*
= 0.13 (*direct input from model as displayed in Table 1, Ch. 2*)

I_{vlsc} = *influence coefficient of value placed on human life*
= 0.44 (*direct input from model as displayed in Table 1, Ch. 2*)

Safety Procedures:

$$R_{sp(min)} = (R_{po} \times I_{posp} + R_{sp} \times I_{spsp} + R_{rr} \times I_{rrsp} + R_{vl} \times I_{vlsp}) \times \left(\frac{7.9}{4.5}\right)$$

$$= (0.061 \times 0.19 + 0.061 \times 0.26 + 0.061 \times 0.12 + 0.061 \times 0.43) \times \left(\frac{7.9}{4.5}\right)$$

$$= 0.107$$

where:

$R_{po} = reliability\ of\ price\ of\ oil$
$= 0.061\ (direct\ input\ from\ base\ case\ run)$

$R_{sp} = reliability\ of\ shareholder\ pressure$
$= 0.061\ (direct\ input\ from\ base\ case\ run)$

$R_{rr} = reliability\ of\ royalty\ regime$
$= 0.061\ (direct\ input\ from\ base\ case\ run)$

$R_{vl} = reliability\ of\ value\ placed\ on\ human\ life$
$= 0.061\ (direct\ input\ from\ base\ case\ run)$

$I_{posp} = influence\ coefficient\ of\ price\ of\ oil$
$= 0.19\ (direct\ input\ from\ model\ as\ displayed\ in\ Table\ 1, Ch. 2)$

$I_{spsp} = influence\ coefficient\ of\ shareholder\ pressure$
$= 0.26\ (direct\ input\ from\ model\ as\ displayed\ in\ Table\ 1, Ch. 2)$

$I_{rrsp} = influence\ coefficient\ of\ royalty\ regime$
$= 0.12\ (direct\ input\ from\ model\ as\ displayed\ in\ Table\ 1, Ch. 2)$

$I_{vlsp} = influence\ coefficient\ of\ value\ placed\ on\ human\ life$
$= 0.43\ (direct\ input\ from\ model\ as\ displayed\ in\ Table\ 1, Ch. 2)$

C.2 MAXIMUM ADJUSTED RELIABILITY VALUES

Safety Training:

$$R_{st(max)} = (R_{po} \times I_{pot} + R_{sp} \times I_{spt} + R_{rr} \times I_{rrt} + R_{vl} \times I_{vlt}) \times \left(\frac{8.1}{5.5}\right)$$

$$= (0.06 \times 0.18 + 0.06 \times 0.27 + 0.06 \times 0.18 + 0.06 \times 0.27) \times \left(\frac{8.1}{5.5}\right)$$

$$= 0.090$$

where:

R_{po} = *reliability of price of oil*
= 0.061 (*direct input from base case run*)

R_{sp} = *reliability of shareholder pressure*
= 0.061 (*direct input from base case run*)

R_{rr} = *reliability of royalty regime*
= 0.061 (*direct input from base case run*)

R_{vl} = *reliability of value placed on human life*
= 0.061 (*direct input from base case run*)

I_{pot} = *influence coefficient of price of oil*
= 0.18 (*direct input from model as displayed in Table 1, Ch. 2*)

I_{spt} = *influence coefficient of shareholder pressure*
= 0.27 (*direct input from model as displayed in Table 1, Ch. 2*)

I_{rrt} = *influence coefficient of royalty regime*
= 0.12 (*direct input from model as displayed in Table 1, Ch. 2*)

I_{vlt} = *influence coefficient of value placed on human life*
= 0.43 (*direct input from model as displayed in Table 1, Ch. 2*)

Safety Culture:

$$R_{sc(max)} = (R_{po} \times I_{posc} + R_{sp} \times I_{spsc} + R_{rr} \times I_{rrsc} + R_{vl} \times I_{vlsc}) \times \left(\frac{7.8}{5.5}\right)$$

$$= (0.061 \times 0.18 + 0.061 \times 0.25 + 0.061 \times 0.13 + 0.061 \times 0.44) \times \left(\frac{7.8}{5.5}\right)$$

$$= 0.087$$

where:

R_{po} = *reliability of price of oil*
= 0.061 (*direct input from base case run*)

R_{sp} = *reliability of shareholder pressure*
= 0.061 (*direct input from base case run*)

R_{rr} = *reliability of royalty regime*
= 0.061 (*direct input from base case run*)

R_{vl} = *reliability of value placed on human life*
= 0.061 (*direct input from base case run*)

I_{posc} = *influence coefficient of price of oil*
= 0.18 (*direct input from model as displayed in Table* 1, *Ch.* 2)

I_{spsc} = *influence coefficient of shareholder pressure*
= 0.25 (*direct input from model as displayed in Table* 1, *Ch.* 2)

I_{rrsc} = *influence coefficient of royalty regime*
= 0.13 (*direct input from model as displayed in Table* 1, *Ch.* 2)

I_{vlsc} = *influence coefficient of value placed on human life*
= 0.44 (*direct input from model as displayed in Table* 1, *Ch.* 2)

Safety Procedures:

$$R_{sp(max)} = (R_{po} \times I_{posp} + R_{sp} \times I_{spsp} + R_{rr} \times I_{rrsp} + R_{vl} \times I_{vlsp}) \times \left(\frac{8.9}{5.5}\right)$$

$$= (0.061 \times 0.19 + 0.061 \times 0.26 + 0.061 \times 0.12 + 0.061 \times 0.43) \times \left(\frac{8.9}{5.5}\right)$$

$$= 0.099$$

where:

R_{po} = *reliability of price of oil*
= 0.061 (*direct input from base case run*)

R_{sp} = *reliability of shareholder pressure*
= 0.061 (*direct input from base case run*)

R_{rr} = *reliability of royalty regime*
= 0.061 (*direct input from base case run*)

R_{vl} = *reliability of value placed on human life*
= 0.061 (*direct input from base case run*)

I_{posp} = *influence coefficient of price of oil*
= 0.19 (*direct input from model as displayed in Table* 1, *Ch.* 2)

I_{spsp} = *influence coefficient of shareholder pressure*
= 0.26 (*direct input from model as displayed in Table* 1, *Ch.* 2)

I_{rrsp} = *influence coefficient of royalty regime*
= 0.12 (*direct input from model as displayed in Table* 1, *Ch.* 2)

I_{vlsp} = *influence coefficient of value placed on human life*
= 0.43 (*direct input from model as displayed in Table* 1, *Ch.* 2)