PERTURBATION OF LARGE ANTI-DESITTER BLACK HOLES AND AdS/CFT CORRESPONDENCE

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Perturbation of Large Anti-deSitter Black Holes and AdS/CFT Correspondence

by

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Chapter 1

Introduction

Black holes are arguably the most interesting gravitational objects in theoretical physics. Understanding their full dynamics forces us to fit together two widely accepted theories of Nature: general relativity (Einstein's classical theory of gravity) and quantum mechanics, a goal that despite encouraging successes has eluded theoretical efforts so far.

In this thesis, the first chapter includes a brief introduction to black holes, along with a review of Anti de-Sitter spacetime (AdS) and Asymptotically AdS spacetimes and their properties. The second chapter is an introduction to the notion of AdS/CFT correspondence and String theory as a prerequisite. In chapter three, at first, the perturbation theory of black holes is reviewed and the rest of the chapter is dedicated to the results that have been found for the perturbed metric, field equations, and stress tensor perturbations of large mass, AdS black holes. Moreover, the velocity, energy density and pressure of the perturbed fluid have been derived and studied. Now, let's look at the notion of black holes since they were discovered.

1.1 Black holes

The term "black hole" was introduced by Wheeler in 1967 (see Figure 1.1) although the theoretical study of these objects has quite a long history. In 1783, John Michell stated that there might be a massive object with an escape velocity greater than the speed of light. Around thirty years later, in 1796, Laplace conjectured the idea of Newtonian black holes [1]. Later in 1916, the solutions of the Einstein field equations for the limited case of a single spherical non-rotating, uncharged spherical systems in the vacuum were found by Karl Schwarzschild. His solution is known as the *Schwarzschild* solution.

As mentioned, in Newtonian gravity, a massive body can have an escape velocity greater than the velocity of light. The corresponding phenomenon equivalent to this massive body in general relativity is a black hole. However, this correspondence is not exact since black holes are intrinsically relativistic objects. They are characterized by causal horizons and spacetime singularities, which are two basic features of Einstein's theory.

In general relativity, the curvature of spacetime generates the gravitational force. Spacetime is usually interpreted with space as being three-dimensional and time playing the role of a fourth dimension. These are put together into a four-dimensional geometry. In higher-dimensional gravity, the number of spatial dimensions increases. The dynamical metric g_{ab} of a pseudo-Riemannian manifold $(\mathcal{M},\, g_{ab})$ with Lorentzian signature, obeys the Einstein field equations. By letting R_{ab} to be the Ricci curvature and R the scalar curvature, the Einstein equations are

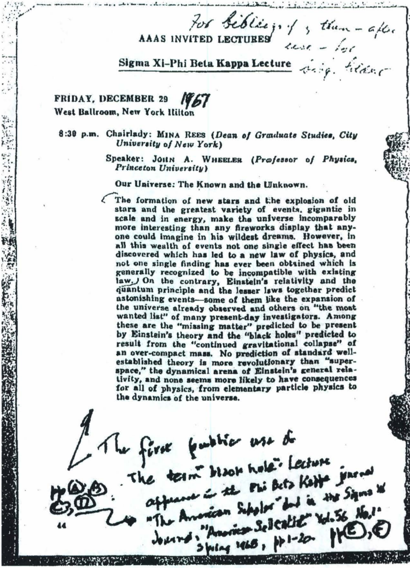
$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4}T_{ab}. \tag{1.1}$$

On the left side of the equation, the term that includes the cosmological constant Λ, indicates dark energy or non-zero vacuum energy, and, on the right-hand side, T_{ab} is the energy-momentum tensor of all the matter fields which acts as the source of spacetime curvature. These matter fields influence the dynamics of spacetime. On the other hand, any field is affected by gravity since it lives in a curved spacetime, and massive particles move along timelike geodesics while massless particles move along null geodesics [2].

One of the notable differences between Newtonian gravity and general relativity is that the "action-at-a-distance"[1] is replaced by the built-in causality structure of Einstein's theory. If we reformulate the Einstein's theory of general relativity, in an initial-value formulation, which describes a universe evolving over time, it is possible to split the ten Einstein equations into six evolution equations and four constraint equations [5]. Initial data needs to satisfy the constraint equations defined on a spacelike Cauchy surface, and the evolution equations specify a system of hyperbolic quasilinear equations that evolves the initial data in time. This gives rise to a causality structure, which locally looks like the light-cone structure of special relativity. However, since the spacetime is dynamical, the Cauchy evolution of smooth geometry and matter data on a spacelike surface may lead to a singularity. Physically, this corresponds events such as the gravitational collapse of a massive body or a high energy collision.

According to the Penrose-Hawking singularity theorems, gravitational singularity or spacetime singularity can arise in Einstein's theory [6]. In standard practice, we say that for curvatures of the order of the Planck scale, general relativity is not suitable for describing the spacetime, e.g. when $R_{abcd}R^{abcd} \sim c^3/\hbar G$ where $\hbar$ is Planck's constant. Since general relativity is nonrenormalisable [7] when treated as a quantum

[1]Action at a distance is the interaction of two objects which are separated in space with no known mediator of the interaction.[4]



AAAS INVITED LECTURES

Sigma Xi–Phi Beta Kappa Lecture

FRIDAY, DECEMBER 29 1967
West Ballroom, New York Hilton

8:30 p.m. Chairlady: MINA REES (*Dean of Graduate Studies, City University of New York*)

Speaker: JOHN A. WHEELER (*Professor of Physics, Princeton University*)

Our Universe: The Known and the Unknown.

The formation of new stars and the explosion of old stars and the greatest variety of events, gigantic in scale and in energy, make the universe incomparably more interesting than any fireworks display that anyone could imagine in his wildest dreams. However, in all this wealth of events not one single effect has been discovered which has led to a new law of physics, and not one single finding has ever been obtained which is generally recognized to be incompatible with existing law. On the contrary, Einstein's relativity and the quantum principle and the lesser laws together predict astonishing events—some of them like the expansion of the universe already observed and others on "the most wanted list" of many present-day investigators. Among these are the "missing matter" predicted to be present by Einstein's theory and the "black holes" predicted to result from the "continued gravitational collapse" of an over-compact mass. No prediction of standard well-established theory is more revolutionary than "superspace," the dynamical arena of Einstein's general relativity, and none seems more likely to have consequences for all of physics, from elementary particle physics to the dynamics of the universe.

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Figure 1.1: The first public use of the term "black hole". Lecture given by J.A.Wheeler appeared in the Phi Beta Kapper journal "The American Scholar" (Vol.37, No.2, Spring 1968, pp.248)[3]

field theory of gravitons, a more fundamental quantum theory of spacetime is required, and general relativity is just an effective low energy theory.

A *black hole* is the region contained inside an event horizon. More precisely, for spacetimes with an asymptotic conformal structure, a black hole is the region of spacetime that does not lie in the causal past of future null infinity, and its boundary in the full spacetime $\mathcal{M}$ is called the future event horizon $\mathcal{H}^+$. The past event horizon is called $\mathcal{H}^-$, which is the boundary of communication of past null infinity. It is also present in time symmetric solutions but absent for dynamically formed black holes.

The formation of black holes through the gravitational collapse of massive objects has been studied analytically and numerically (see reviews e.g. in [8] and [9]). However, for the formation of a black hole, no matter is necessary since it can also form from the focusing of incoming gravitational waves [8]. Also, high energy collisions may result in the formation of black holes [10]. In this process, event horizons are expected to form in agreement with cosmic censorship which protects the rest of spacetime from singularities [12]. The results support the expectation that a black hole will form whenever the hoop conjecture is satisfied. Roughly speaking, this conjecture says that when a given amount of energy or an object is compressed in a suficiently small region of space, a black hole forms [11].

Nowadays, the images taken of X-ray binary systems are one of the basic tools for the astronomers to study the black holes. X-ray binary systems consist of a visible star and an invisible companion star which move in the close orbit around their centre of mass. The invisible partner's gravity attracts the matter from the region around. This includes the gas from the visible star which forms a flattened disc of gas spinning and falling towards it. Collisions between the particles in the formed disc heat them up to the extermely high tempratures such that they produce X-rays [13]. Many bright X-ray binary sources have been discovered in our galaxy and nearby galaxies. These unseen companions are black holes and the X-rays results from the friction between the particles close to the event horizon. After emitting their X-rays, they disappear by passing the event horizon.

However, searching for the black holes by studying the evolution of the visible partner is indirect [14]. Ideally one would like a specific observable characteristic which confirms that a compact body is indeed a black hole. One possibility arises from numerical studies of perturbations around black holes which shows that late time perturbations are dominated by an exponentially damped single-frequency. This kind of perturbation, which is damped quite rapidly and exists only in a limited time interval is referred to as a quasinormal mode or QNM [15]. The evolution of the initial perturbation of black holes is affected only by the black hole parameters, not the initial perturbations. So they are the direct signatures of a black hole. Observation of gravitational waves may be able to encode the presence of these quasinormal modes. More details about perturbation theory can be found in chapter 3.

1.2 AdS space

Anti-de Sitter space is the maximally symmetric solution of Einstein's equations with an attractive cosmological constant [20],

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \Lambda g_{\mu\nu}, \tag{1.2}$$

where Λ is the negative cosmological constant.

Traditionally, anti-de Sitter space was not deemed to be of physical interest. However, it attracted attention for two reasons. First, the negative value of Λ, if defined as a vacuum energy, corresponds to negative energy density. Second, anti-de Sitter space has the general topology $R^{n-1} \otimes S^1$, where the S^1 is timelike. The existence of closed timelike curves and a boundary at spacelike infinity are two properties of this geometry which are in conflict with common sense [17].

To introduce $(n+1)$-dimensional AdS space, we define it as a surface embedded in a flat space $\mathbb{R}^{(2,n)}$ with two time coordinates, u and v, and n space coordinates x^i [19]. For example, in four-dimensional AdS spacetime, the five-dimensional flat space is $\mathbb{R}^{(2,3)}$ that has two timelike and three spacelike directions which make it not a spacetime in the ordinary sense since it has more than one temporal dimension. Back to our general case, the metric and constraints can be written as follows,

$$\begin{aligned} \text{ambient metric} &: \; ds^2 = -(du)^2 - (dv)^2 + dx^i dx^i, \qquad i = 1, ..., n; \\ \text{constraint} &: \; -u^2 - v^2 + x^i x^i = -R^2. \end{aligned}$$

The general form of the Lorentz transformations can be defined on $\mathbb{R}^{(2,n)}$ as a group of linear transformations that preserve the metric of the space. Because of the homogeneity of the anti-de Sitter space, if we consider two vectors V_1 and V_2 on the surface with the same norm of $V_1 \cdot V_1 = V_2 \cdot V_2 = -R^2$, they can map into each other under the Lorentz transformations. However, according to the constraint mentioned above, these timelike vectors $V = (u, v, \vec{x})$ belong to the surface if they satisfy $V \cdot V = -R^2$.

For visualizing the AdS space, we use the constraint equation

$$u^2 + v^2 = R^2 + \vec{x} \cdot \vec{x} \tag{1.3}$$

to plot the space. We choose three axes of u, v, and $\rho = \sqrt{\vec{x} \cdot \vec{x}}$, which has a fixed value

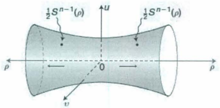


Figure 1.2: AdS_{n+1} space described as a two-dimensional surface in which each point represents half of the $S^{n-1}(\rho)$ sphere. The horizontal axis has ρ increasing from zero to infinity both to the left and to the right. The half spheres at points with equal values of u and v must be glued to form a single $S^{n-1}(\rho)$ sphere.[19]

in our case, and each point on the two-dimensional surface is determined by u and v values. As it is shown in Figure 1.2, the surface is extended to the left and right as the value of ρ varies in both directions, from 0 to ∞. Each point of $\vec{x}$ that satisfies

$$\vec{x}\cdot\vec{x} = \rho^2 = u^2 + v^2 - R^2 \tag{1.4}$$

represents a S^{n-1} sphere. But since we have the same range of ρ twice, we need to have half of the sphere S^{n-1} on top of each pair of u, v with the same value, in the right and the left, such that by gluing two halves, a complete sphere can be made.

If we imagine a curve that goes around the waist of the hyperboloid, that would be a closed timelike curve. However, nothing particularly singles out the waist, because AdS_n is a homogeneous and isotropic space, which has $\frac{n(n+1)}{2}$ Killing vectors that generate the symmetry group of $SO(n-1,2)$ [16]. In homogeneous space, all the points are the same and there is an isometry between any two points in the space. In general, the topology of AdS_n is $R^{n-1}\otimes S^1$ and the topology of dS_n is $R\otimes S^{n-1}$. In two dimensions, de Sitter space and anti-de Sitter space are simply related by switching the meaning of timelike and spacelike. Then AdS_2 becomes de-Sitter space dS_2, and closed timelike curves become closed spacelike curves.

It is mathematically important to know that the conformal boundary of asymptotically anti-de Sitter space differs from that of asymptotically flat spacetimes. The boundary of conformally compactified AdS_4 has the topology $S^2 \otimes R$, where the sphere can be regarded as the conformal boundary of hyperbolic three-space. This timelike boundary, which is usually labeled as $\mathcal{J}$ or scrI (script I), is defined as the set of endpoints of all future directed (or past directed) lightlike geodesics. Also, the boundary is the set of endpoints of spatial geodesics, which we can refer to it as spatial infinity, but lightlike geodesics are more important for the causal structure. The whole structure is quite different from that of conformally compactified Minkowski space. In this case, spatial future, and past infinities are disjoint, and both are lightlike.

1.2.1 Asymptotically AdS spacetimes

For spacetime with a negative cosmological constant, we can rewrite the Einstein equation as [22]

$$R_{\mu\nu} = -\frac{d}{l^2} g_{\mu\nu}, \tag{1.5}$$

where the AdS radius l is defined by [20][21]

$$l^2 = -\frac{d(d-1)}{2\Lambda}. \tag{1.6}$$

AdS_{d+1} spacetime is one of the simple solutions of this equation. It has a curvature tensor[2]

$$R^{\lambda}_{\mu\sigma\nu} = -\frac{1}{l^2}(g_{\mu\nu}\delta^{\lambda}_{\sigma} - g_{\mu\sigma}\delta^{\lambda}_{\nu}). \tag{1.7}$$

In the global coordinates[3] (r, t, Ω_{d-1}), the AdS_{d+1} metric is given by

$$ds^2 = l^2\left[-(1+r^2)dt^2 + \frac{dr^2}{(1+r^2)} + r^2 d\Omega_{d-1}\right]. \tag{1.8}$$

The metric can also be written in the new coordinate $\tan\theta = r$ as

$$ds^2 = \frac{l^2}{\cos^2\theta}[-dt^2 + d\theta^2 + \sin^2\theta d\Omega_{p-1}]. \tag{1.9}$$

The conformal boundary of AdS with topology $R \times S^{d-1}$ is located at the metric second order zero at $r \to \infty$, or as in new coordinate, where $\theta = \pi/2$. This divergence implies that the metric induces a conformal structure on the boundary (a metric up to conformal trasformations) instead of a unique metric. To obtain a metric, we can consider types of positive functions Ω called "defining functions" in AdS space, which have a first-order zero and a non-vanishing gradient on the boundary. By multiplying the AdS metric by Ω^2 and evaluating it at the boundary, we have

$$g_{(0)} = \Omega^2 g_{bulk}\Big|_{\pi/2} \tag{1.10}$$

The choice of defining function is not unique. Any non-vanishing function e^{ω} on a manifold can be used to obtain a new defining function $\Omega' = \Omega e^{\omega}$ where ω is a function with no zeros or poles at the boundary. So the bulk metric in the AdS

[2]Here, the curvature convention is $R^{\lambda}_{\mu\nu\kappa} = \partial_\mu \Gamma^{\lambda}_{\mu\kappa} + \Gamma^{\lambda}_{\mu\rho}\Gamma^{\rho}_{\nu\kappa} - \Gamma^{\lambda}_{\nu\rho}\Gamma^{\rho}_{\mu\kappa}$, $R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}$, $R = g^{\mu\nu}R_{\mu\nu}$.

[3]This coordinate represents a universal cover of AdS that is obtained by *unwinding* a periodic time coordinate and replacing it with the non-compact time coordinate t.

spacetime yields a metric up to conformal transformations on the boundary.

To introduce the notion of an asymptotically AdS spacetime, we first need to construct the unphysical spacetime which helps us to see the real and important property of asymptotically AdS spacetime [22]. Working with unphysical spacetime $(\hat{\mathcal{M}}, \hat{g})$ saves us from some difficulties of working with physical spacetime. For instance, $\partial\mathcal{M}$ as a conformal boundary of $\mathcal{M}$ is at spatial infinity and it is not part of $\mathcal{M}$, but $\partial\mathcal{M}$ is defined as a finite value of spacelike coordinate, when $\Omega = 0$. Using $\partial\hat{\mathcal{M}}$, we avoid infinite limits that appear if we attach $\partial\mathcal{M}$ to $\mathcal{M}$ (such as a diverging of physical metric). In the unphysical spacetime, Ω cancels the divergencies insuring that $\hat{g}_{\mu\nu}$ is finite and well-defined anywhere on $\hat{\mathcal{M}}$ and $\partial\hat{\mathcal{M}}$.

For a physical spacetime $(\mathcal{M}, g)$ to be asymptotically AdS, there should be an unphysical spacetime $(\hat{\mathcal{M}}, \hat{g})$ where $\hat{\mathcal{M}}$ is a manifold and $\partial\hat{\mathcal{M}}$ is a boundary, such that $\hat{\mathcal{M}}$ has $\mathcal{M}$ as an interior manifold with a diffeomorphism from $\mathcal{M}$ to $\hat{\mathcal{M}} - \partial\hat{\mathcal{M}}$. The second requirement is to introduce a "defining function" $\Omega(x)$, with the properties mentioned before, such that $\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$. The third requirement is having a Weyl tensor $\hat{C}_{\lambda\mu\sigma\nu}$ on $\hat{\mathcal{M}}$, constructed from $\hat{g}_{\mu\nu}$ with two properties: First, $r^{3-d}\hat{C}_{\lambda\mu\sigma\nu}$ is smooth on $\hat{\mathcal{M}}$ and secondly, $\hat{C}_{\lambda\mu\sigma\nu}$ vanishes on $\partial\hat{\mathcal{M}}$. The fourth and the last requirement to construct the unphysical spacetime is that the topology of the boundary $\partial\hat{\mathcal{M}}$ be $R \times S^{d-1}$.

By working in the unphysical framework, the third requirement mentioned above implies that $R_{\mu\nu}$ for asymptotically AdS spacetime should approach the maximally symmetric form of (1.8) at special infinity. However, the Riemann tensor has nonzero Weyl tensor on the interior space $\mathcal{M}$ with the form,

$$R^{\lambda}_{\mu\sigma\nu} = C^{\lambda}_{\mu\sigma\nu} - \frac{1}{l^2}(g_{\mu\nu}\delta^{\lambda}_{\sigma} - g_{\mu\sigma}\delta^{\lambda}_{\nu}). \tag{1.11}$$

Therefore, the first three reqirements imply that $(\mathcal{M}, g)$ is locally asymptotic to AdS, where the fourth one ensures that the spacetime is also asymptotically AdS, globally.

The canonical example of a non-trivial asymptotically anti-de Sitter space is the Kottler (or Schwarzschild-Anti de Sitter) solution which we will work with in this thesis.

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{r^2}{l^2}\right) dt^2 + \left(1 - \frac{2m}{r} + \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 d\Omega^2. \tag{1.12}$$

Here $d\Omega^2$ is a metric on a unit 2-sphere and t coordinate ranges from $-\infty \leq$ to $\leq \infty$ as r goes from r_+ to ∞. r_+ is the largest root of $r - 2m + r^3/l^2 = 0$.
In the next session we consider this solution in more detail.

1.3 Schwarzschild black holes

In 1916, Schwarzschild found the solution to Einstein's equations for a spherically symmetric gravitational field in vacuum which has the following form [3]

$$ds^2 = -\left(1 - \frac{2Gm}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1.13}$$

Here G is Newton's gravitational constant which has different form in each dimension [19], and m is the total mass of the gravitational source which produces the field. Here black hole play the role of the gravitational source.

This solution is independent of the time coordinate and determined by a single parameter m. The effect of m on the form of the spacetime metric would be clear if we study the asymptotic form of the metric as $r \to \infty$. Far from the gravitational source, spacetime approaches the flat Minkowski spacetime with metric

$$ds^2 = -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1.14}$$

Now, if we use the *weak field approximation* to show the effect of gravitational field far from the center of gravity, the tt component of the metric can be written as $g_{tt} = -(1 + 2\phi/c^2)$, where $\phi = -Gm/r$ is the *Newtonian gravitational potential.* By comparing our result with metric (1.14), m can be described as the mass of the source of gravity.

1.3.1 Schwarzschild black hole in asymptotic flat and AdS spacetimes

By using the Penrose diagrams[4], we can compare the causal structure for a Schwarzschild black hole in two different spacetimes, asymptotically flat and AdS [33].

In Figure 1.3(a), the structure of a Schwarzschild black hole in an asymptotically flat spacetime is shown. In this diagram, future and past null infinity are indicated by $\mathcal{J}_{\pm}$, future and past timelike infinity by $I_{\pm}$, and spacelike infinity is shown as I_0. These indicate where lightlike, timelike, and spacelike worldlines start and end, respectively. In this figure the wordline of an observer is shown. It starts at past timelike infinity I_- and continue to pass the event horizon at $r = r_+$ where no light rays can be transmitted to future null infinity $\mathcal{J}_+$ and the infalling observer falls towards the spacetime singularity at $r = 0$. After crossing the horizon, the observer can not send any information out of the black hole. Opposite of black holes, there

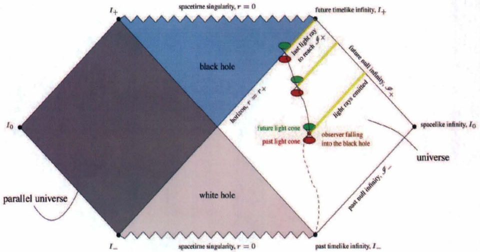
[4]Penrose diagrams are two dimensional diagrams of conformally transformed spacetimes. By choosing the specific conformal factor, the infinite spacetime is mapped onto a finite region in these diagrams such that the light-cone structure of the original spacetime remains the same.

are mathematically defined objects, called *white holes* into which no information can enter. White holes are part of the extension of the spacetime diagram into the past.

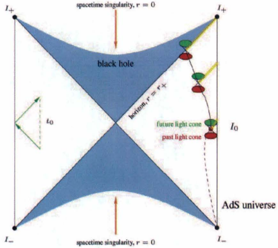
The Penrose diagram for a Schwarzschild black hole in an asymptotic AdS spacetime is shown in Figure 1.3(b). Here future and past null infinities $\mathcal{J}_{\pm}$, and spacelike infinity I_0 are equal and extended as a line rather than a single point. Consequentely, as is indicated with green arrows in Figure 1.3(b), a single light ray can reach infinity, bounce back, and return to its origin in a finite time t_0.

Apart from geometrical differences, black holes in asymptotically AdS space, like the ones in asymptotically flat space, have thermodynamic properties such as a characteristic temperature and an intrinsic entropy equal to $A/4$ where A is the area of the event horizon in Planck units [70][24]. One of the important differences between these two spaces is that when the size of the black hole in anti-de Sitter space is of the order of the characteristic radius of the AdS space, its temperature is minimum and when it becomes larger its red-shifted temperature measured at infinity becomes greater. This feature shows that these black holes have positive specific heat and can be in stable equilibrium with thermal radiation at a fixed temperature.

At the quantum level, when AdS black holes emit Hawking radiation, the gravitational potential of the asymptotically AdS background keeps the Hawking radiation from escaping to infinity and reflect it back towards the black hole. Thus we can consider pure AdS black holes in equilibrium with their surroundings (in contrast to asymptotically flat spacetime where a black hole must be enclosed in a box with perfectly reflective walls to be in equilibrium).



(a) Schwarzschild black hole in asymptotically flat spacetime.



(b) Schwarzschild black hole in asymptotically AdS spactime.

Figure 1.3: The figures illustrate the Penrose diagrams for black holes in spacetimes with different asymptotic behaviour. Here light rays propagate along 45° lines, see yellow dashed lines as emitted from an observer falling into the black hole.[33]

1.4 AdS-Schwarzshild black holes

As noted, the four dimensional AdS–Schwarzschild metric has the form

$$ds^2 = -f_0(r)dt^2 + \frac{dr^2}{f_0(r)} + r^2 d\Omega_2^2 \tag{1.15}$$

where the radial function $f_0(r)$ is

$$f_0(r) = 1 - \frac{2m}{r} + \frac{r^2}{l^2}. \tag{1.16}$$

From $f_0(r) = 0$, we can find the event horizon of the black hole at r_s. In the definition of $f_0(r)$, l is the curvature radius of AdS space, and the cosmological constant has the form $\Lambda = -3/l^2$. If $m = 0$ the metric reduces to that of four-dimensional anti-de Sitter spacetime, but for $m > 0$ the metric describes an eternal black hole with an event horizon at $r = r_s$. The parameter m is proportional to the mass of the black hole and can be written in terms of the horizon radius r_s as

$$m = r_s + \frac{r_s^3}{l^2}. \tag{1.17}$$

The Hawking temperature of an AdS_4–Schwarzschild black hole, which can be obtained from $f_0(r)$ is given by [24]

$$t_H = \frac{1}{4\pi} f_0'(r_s) = \frac{1}{4\pi}\left(\frac{1}{r_s} + \frac{3r_s}{l^2}\right). \tag{1.18}$$

The minimum of Hawking temperature, which is of the order of the characteristic energy scale of the AdS background, is $\sqrt{3}/2\pi l$. For any temperature value higher than the minimum, there are two different values of r_s and for each value, there are two branches of AdS-Schwarzschild black holes: *large* black holes with $r_s > l/\sqrt{3}$, and *small* black holes with $r_s < l/\sqrt{3}$.

There are a few important length scales. Since we work in classical geometry, the AdS length l is assumed to be large compared with any fundamental length scale such as the string length, i.e. $l >> l_s$. The other length is an intermediate length scale l_0, which is comparable with the dimension of macroscopic observers. Since the geometry varies on the length scale of l or larger and the conditions of having the local thermal equilibrium are valid at the scale of l_0, the range of variation of l is

$$l_s << l_0 << l. \tag{1.19}$$

In this thesis, our focus is on large AdS–Schwarzschild black holes in the very large limit where

$$r_s \approx (ml^2)^{1/3} >> l. \tag{1.20}$$

There are a few universal features in this limit; for example, the scalar invariant

$$R_{abcd}R^{abcd} = 12\left(\frac{2}{l^4} + \frac{m^2}{r^6}\right). \tag{1.21}$$

This value is obtained for the AdS–Schwarzschild metric (1.16). Now by inserting $r = r_s$ and taking the limit of a very large black hole, we have

$$R_{abcd}R^{abcd}\Big|_{horizon} = \frac{36}{l^4}(1 + O(l/m)^{2/3}), \tag{1.22}$$

which is independent of the black hole mass in the $m >> l$ limit. It means that for different but very large black hole masses, there are the same AdS scale curvature for the near-horizon region. However, this universal curvature is equal to an $O(1)$

multiple of the vacuum value, not to the curvature of empty AdS space with the same cosmological constant. [24]

Chapter 2

String theory and AdS/CFT correspondence

2.1 String theory in a glance

Over the last thirty years, string theory has been the leading candidate for a unified theory of all forces in nature [19]. In string theory, all the known fundamental forces and particles are unified in a deep and significant way, such that it can be accepted as an impressive potential example of a complete theory of physics. String theory is a quantum theory, and because it includes gravitation, it is a quantum theory of gravity. One of the features that makes the string theory unique is the lack of adjustable dimensionless parameters. In the Standard Model of particle physics, there are about twenty adjustable parameters. However, in string theory , there is one dimensionful parameter which is the string length l_s that is taken to be of the order of 10^{-18} cm. This value is the typical size of strings.

Another feature of the string theory which makes it unique is the fact that the dimensionality of spacetime is fixed, and the number of spacetime dimensions derives from the calculation. In comparison, in the Standard Model, the number of dimensions of our physical spacetime, which is four, is part of the information used to build the theory. Although, ten dimensions are obtained from the calculation in string theory instead of four, it is likely that the remaining six extra dimensions are compactified on a very small space that can escape detection in experiments done with low energies. However, for string theory to be correct, some theoretical mechanism must be found to confirm that the observable spacetime has four dimensions.

Discovery of the cosmic strings[1] could be a confirmation of string theory [25]. They might be detected through gravitational lensing, or more indirectly via the detection of gravitational waves. Till now, there has been no evidence of their existence, but searches for them still continue.

String theory can be categorized into five different groups, which are called type I, with open and closed strings; types IIA and IIB, with only closed strings; and two theories of heterotic $SO(32)$ and heterotic $E_8 \times E_8$, which are consist of superstrings and bosonic strings. Each of these categories arises as a special case of an eleven-dimensional theory, called M-theory[2], and some of them are equivalent because of the dualities have been found between them.

2.1.1 $D-$brane and $p-$brane

In string theory, a hypersurface, or higher-dimensional membrane called a $D-$brane is a real, physical object. For example, our four-dimensional universe is part of a higher-dimensional $D-$brane with the extra dimensions wrapped into a compact space. $D-$branes can be classified by their spatial dimension, which is indicated by a

[1] Hypothetical one-dimensional topological solitons which might be formed in early universe during the symmetry breaking phase transition.

[2] In the words of Edward Witten from the Institute of Advanced Study in Princeton, "M stands for Magic, Mystery or Membrane, according to taste." [18]

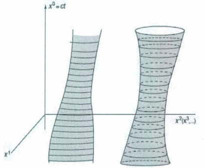


Figure 2.1: The world-sheets traced out by an open string (left) and by a closed string (right).[19]

number comes after the D so that they are written as $Dp-$branes. Here, the letter D stands for Dirichlet. The endpoints of the open strings must remain attached to the $D-$branes and those ones whose ends are fixed, satisfy Dirichlet boundary conditions [19]. It is worth mentioning that not all the extended objects in string theory are $D-$branes. Strings, for example, are 1-branes because they are extended objects with one spatial dimension, but they are not $D1-$branes. Also, a 0-brane is some kind of particle which traces out a one-dimensional world-line in spacetime, like a string that can trace out a two-dimensional surface in spacetime called the world-sheet. If the string is closed, it will trace out a tube and if it is open, the traced out surface will be a strip. These different surfaces are shown in spacetime diagram of Figure 2.1.

The lowest vibrational modes of the open strings that are streched between the $D-$branes could represent the particles of the Standard Model, such as gauge bosons and the matter particles. However, none of the vibrations of the classical relativistic string correspond to the particle of gravity, but the quantum vibrations of

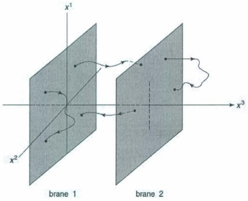


Figure 2.2: Two parallel $D2-$branes and four types of strings that this configuration supports. Here x^1 and x^2 are longitudinal coordinates, and x^3 is a normal coordinate.[19]

the relativistic string are able to describe them.

Two $Dp-$branes can coincide in space and be on top of each other or be separated. Figure 2.2 shows two parallel, separated $D2-$branes. There are four different classes of strings that this configuration of parallel $Dp-$branes support. The first two classes are made up of open strings that begin and end on the same $D-$brane, either brane one or brane two. The other two classes are called *stretched strings* which start on one brane and end on the other as it is shown in Figure 2.2. The orientation of the last two classes of strings are opposite of each other, which can be an important issue in different problems [19].

In 1995, Polchinski proved that $D-$branes and extremal $p-$branes are the same objects. It means that the dynamical endpoints of open strings correspond to extremal solutions of supergravity. To prove it, one needs to compute $p-$brane charges and tensions of the endpoints of open strings, and shows that they match with the supergravity solutions [31].

As was mentioned before, $D-$branes are dynamical walls on which strings can end. One of the fascinating features of $D-$branes is that the gauge theories naturally live on their world volume [29]. For example, the massless spectrum of open strings living on a $Dp-$brane corresponds to a maximally supersymmetric $SU(1)$ gauge theory in $p+1$ dimensions. In general, by considering N parallel $D-$branes, we can have N^2 different species of open strings because they can begin and end on any of the $D-$branes. In this case, we can find the maximally supersymmetric $SU(N)$ gauge theory where N^2 is the dimension of the adjoint representation of $SU(N)$. In 3+1 dimensions, there is $\mathcal{N} = 8$ supersymetry where $D-$brane background breaks 1/2 of the supersymmetry. Therefore, for the case of $p = 3$, there is $\mathcal{N} = 4$ supersymmetric Yang-Mills with $SU(N)$ gauge group on the 4-dimensional worldvolume of the $D3-$branes.

Extremal $p-$branes are solutions of supergravity, which is the low energy limit of string theory. The extremal $p-$branes have $Q = M$ where Q is the charge and M is the tension of the $p-$brane [33]. This equality saturates the bound $|Q| \leq M$ known as "BPS bound". In a supersymmetric theory saturation of this bound means that half of the supersymmetry is broken, but it does not affect the stability of the configuration. In gravity, this bound comes from the "no naked singularity" theorems and the fact that for $Q > M$, a naked singularity appears. Thus, the extremal $p-$branes are solutions of supergravity with horizons at $r = 0$ (singularity=horizon) and also have $\mathcal{N} = 4$ supersymmetry in $d = 4$.

Another notion related to $p-$branes which we can introduce here is the notion of *black brane*. In four dimensions, the only localized (do not grow at infinity) extremal $p-$branes are the black holes, but in higher dimensions, we can have black-hole like objects called "black $p-$branes" with extended horizons along p spatial dimensions that are localized in space [31]. The quantum properties of black holes, provided by string theory can be mainly derived from $D-$branes. Strominger and Vafa [34] derived the Bekenstein-Hawking entropy microscopically for the first time by counting the degeneracy of $D-$brane states corresponding to microstates of a five-dimensional class of extremal black holes. This entropy is given by $S = A/4$, which relates the quantum degrees of freedom of the black hole to its surface, rather than its volume. This relation is the basis of the holographic principle, proposed by 't Hooft [35] and Susskind [36]. This principle which is analogous to the common basis of holography, says that quantum gravity in a given volume should be described by a theory on the boundary of that volume.

2.2 AdS/CFT correspondence

The AdS/CFT correspondence proposed by Maldacena in late 1997 is one of the most important developments following from studies of $D-$branes [38][47]. Generally, the AdS/CFT correspondence is a realisation of the holographic principle, since it suggests a equivalency between a conformal field theory[3] (CFT), which is the theory without gravity, in $d-$dimensions and a gravity theory in $d+1$-dimensional anti de-Sitter space (AdS). The first hint that show this should be possible is that both such theories have the same symmetry group, $SO(2,d)$. The original conjecture states that type IIB string theory on $AdS_5 \times S^5$, which is a 10-dimensional[4] theory of gravity is dual to $\mathcal{N} = 4$ $SU(N)$ Super-Yang-Mills, a 4-dimensional gauge theory, defined on $R \times S^3$, see for example [39]. This conjecture is expected to be strong enough to

[3]A field theory on $d-$dimensional Minkwoski space that is invariant under the conformal group.

[4]One might think the difference in dimension is a problem, but it is not since the extra dimensions on the gravity side correspondes to the particle degrees of freedom on the gauge side.

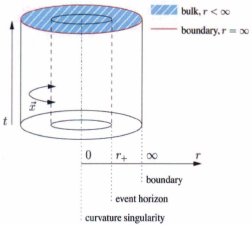


Figure 2.3: An illustration of where $\mathcal{N} = 4$ SYM lives on AdS_5 space. The four dimensions of the gauge theory (t, x) live on the boundary of the AdS_5 space at $r = \infty$.[33]

also describe the other cases of string theory with AdS (or "almost AdS") boundary conditions. In Figure 2.3, it is shown that the gauge theory lives on the 4-dimensional boundary of the 5-dimensional AdS_5 space, located at $r = \infty$, and dual gravity theory lives in the bulk of the AdS_5 space where $r < \infty$.

The second hint that shows the duality is possible is that the coupling constants on each side can be matched inversely to each other, up to a scale order [33]. This means that non-perturbative results in the strong coupling limit of one theory can be obtained from perturbative calculations in the weak coupling limit of the other. However, it is often easier to first do the calculations in the gravity side and use the results to learn about the conformal field theory than vice-versa. That is because we still do not have a complete dictionary of maps between two theories which means we mostly have the maps from gravity side to the field theory side, not vice-versa. The summary of the duality relations can be found in Table 2.1.

Table 2.1: Summary of the corresponding elements that appear in the fluid-gravity duality [33].

	Bulk	Boundary
AdS/CFT	Type IIB string theory on asymptotically $AdS_5 \times S^5$	$\mathcal{N} = 4$ SYM on $S^3 \times R^1$ or with a Poincaré patch $R^3 \times R^1$
Effective description	Einstein equation with cosmological constant	Relativistic fluid dynamics
Known static solutions	Black hole or black brane in AdS	Static configuration of a perfect fluid
Perturbation	Non-uniformly evolving black branes	Dissipative fluid flow

One of the most understood examples of this duality is the correspondence between the gravitational limit of Type IIB string theory on $AdS_5 \times S^5$ space, and the hydrodynamic limit of the nongravitational $\mathcal{N} = 4$ Super-Yang-Mills gauge theory defined on the 4-dimensional conformal boundary of AdS_5 [33][40][19].

String theory has two dimensionless parameters, the one is the ratio between the curvature scale for the string background[5] L , and the string length l_s[6]. This ratio needs to be large to reduce the stringy effects such that the gravitational description of string theory remains valid. The other dimensionless parameter is the string coupling constant g_s, which measures the string interaction strength relevant to string splitting and joining. This constant is assumed to be small at the same time to reduce quantum effects.

$$\frac{L}{l_s} = \frac{L}{\sqrt{\alpha'}} >> 1 \; and \; g_s << 1 \tag{2.1}$$

[5]Curvature radius of AdS_5 and S_5.

[6]It sets the size of fluctuations of the string worldsheet.

In the field theory side there are two parameters: the number of colours N which specifies the rank of gauge group $SU(N)$, and the Yang-Mills coupling g. In planar limit (large N), the 't Hooft coupling $\lambda = g^2 N$ is controlling the perturbation theory. The equivalence relation between the fundamental parameters of both sides in their limits is given by the following correspondence,

$$\{\frac{L}{\sqrt{\alpha'}}, g_s\} \rightleftharpoons \{\lambda = g^2 N, g^2\}, \tag{2.2}$$

where α' controls the corrections associated to the finite size of the string as compared to the size of the spacetime it propagates in. Therefore, to suppress the stringy effets in the bulk and quantum effects on the boundary, we need to have the following limits.

$$\lambda >> 1 \; and \; \frac{\lambda}{N} << 1, \tag{2.3}$$

where $\lambda \to \infty$ and $N \to \infty$.

Moreover, to describe the conformal field theory on the boundary hydrodynamically, we consider the local energy density such that we can associate a local tempirature T and mean free path $l_{mfp} \sim 1/T$ to each point. The scale of the field fluctuations, R, needs to be large in comparison with the mean free path $l_{mfp} << R$, such that the first order terms in the derivative expansion of the stress-energy tensor is small compared to the zeroth order term,

$$\frac{1^{st} order}{0^{st} order} \sim \frac{\eta \sigma^{\mu\nu}}{\rho u^{\mu} u^{\nu}} \sim \frac{\eta}{\rho R} \equiv \frac{l_{mfp}}{R} \sim \frac{1}{RT} << 1, \tag{2.4}$$

where it is assumed that $u^{\nu} \sim \mathcal{O}(1)$, and $\sigma^{\mu\nu} \sim 1/R$.

Using the parameter matching which will be described in the following section, it is possible to write these relations in terms of AdS parameters,

$$R >> l_{mfp} \Rightarrow r_+ >> L, \tag{2.5}$$

which shows the correspondence between the regime of validity of fluid dynamics and the theory of large AdS black holes.

More generally, AdS/CFT duality can have several possible versions which mainly are distinguished by three different limits[31]: The validity of the weakest version is only at large g_sN limit, when there is supergravity as a low energy approximation of string theory in the background. There might be a number of disagreements if we go to the full string theory, far from the limit of g_sN. A stronger version of the AdS/CFT duality is valid at any finite g_sN, such that $N \rightarrow \infty$ and $g_s \rightarrow 0$. It means that α' corrections, which satisfies the relation $\alpha'/R^2 = 1/\sqrt{g_sN}$, agree, but under these conditions, g_s corrections might not. In its strongest version, the duality is valid at any g_s and N, even if calculations could only be done in certain limits. Since many examples of α' and g_s corrections were found that agree between AdS and CFT theories, the strongest version is expected to be true.

2.3 Stress-energy tensor and Fluid dynamics

In the AdS/CFT correspondence, on the gauge theory side, at high temperatures, fluid dynamics equations should be able to describe the long-wavelength fluctuations about the equilibrium states [41]. Further, $T_{\mu\nu}$ should be constrained to represent conformal fluid equations. Then the stress-energy tensor of the theory can be expanded in terms of the derivatives of local temperature T and local fluid velocity u_μ.

$$T^{\mu\nu} = p(T)\gamma^{\mu\nu} + (\epsilon(T) + p(T))u^\mu u^\nu + \mathcal{O}(\partial u, \partial T). \tag{2.6}$$

Here T_{00} represents the energy density, T_{ii} the pressures in each direction, T_{ij} the shear stresses, and T_{0i} the momentum density. In this regime, there are number of field theories which are differentiated by the coefficients appearing in the derivative expansion of their stress tensors. The zeroth order coefficients are energy density and pressure, while the first order ones are shear viscosity and bulk viscosity.

The stress-energy tensor describes a fluid of proper density $\epsilon(x_\mu)$, scalar pressure $P(x^\mu)$, and fluid 4-velocity $u^\nu(x^\mu)$, which is normalised to $u^\nu u_\nu = -1$ and also satisfies $u_\nu \partial u^\nu / \partial x^\mu = 0$. Here, $\gamma_{\mu\nu}$ is three-dimensional metric on the boundary of AdS space. If we define a projection operator [51][41],

$$P_{\mu\nu} = \gamma_{\mu\nu} + u_\mu u_\nu, \tag{2.7}$$

so that $u^\mu P_{\mu\nu} = 0$, we can write the stress-energy tensor of the perfect fluid as

$$T_{\mu\nu} = \epsilon u_\mu u_\nu + P P_{\mu\nu}. \tag{2.8}$$

This tensor doesn't take into account any of dissipative processes like viscosity and thermal conduction [39]. For stress-energy tensor to describe non-perfect fluid, it needs a few extra terms to describe the viscosity of the fluid. Then, the expression of the tensor is

$$T_{\mu\nu} = \epsilon u_\mu u_\nu + P P_{\mu\nu} + \Pi_{\mu\nu}, \tag{2.9}$$

where Π_{ab} is a symmetric and transverse tensor, which can be expanded in terms of the derivatives of u^ν,

$$\Pi_{\mu\nu} = \Pi^{(1)}_{\mu\nu} + \Pi^{(2)}_{\mu\nu} + ... \tag{2.10}$$

This tensor satisfies $u_\nu \Pi^{\mu\nu} = 0$ and for conformal fluids it is also traceless $g_{\mu\nu}\Pi^{\mu\nu} = 0$ [39][51].

In addition, ρ and P are related by an *equation of state* which under specific conditions governs the perfect fluid, and it has the form

$$P = P(\rho, T) \tag{2.11}$$

Fluid dynamics equations that describe the long-wavelength perturbations around the large AdS black holes, can be written as the local conservation equations of the stress tensor as follows,

$$\partial_\mu T^{\mu\nu} = 0. \tag{2.12}$$

By using a non-flat metric, this equation is replaced by its covariant form

$$\nabla^\mu T_{\mu\nu} = 0. \tag{2.13}$$

In order to have the conformal invariance of the relativistic Navier-Stokes equation, it is necessary for the fluid stress tensor to be traceless and conformally invariant under a Weyl transformation, which means the trace of $T_{\mu\nu}$ must vanish, thus the equation of state for a perfect fluid in d-dimensions reduces to [30]

$$P = \frac{\epsilon}{d-1}. \tag{2.14}$$

Chapter 3

Perturbation theory of black holes

The perturbation theory of black holes and related topics have been a focus of many researchers, during the last several decades [43]. This theory has attracted the most attention in astrophyiscal studies where it has been used to study how black holes interact with their environment and absorb or emit gravitational waves. A particular application comes in the study of quasinormal modes (QNMs). These are highly damped single-frequency oscilations which provide a unique gravitational wave signature for black holes and may be observed in the future.

Another focal point of interest for the perturbation theory of black holes is found in string theory. One of the recent theories arise from string theory is the relation between physics in the (A)dS space and the conformal field theory on its boundary. For example, as reviewed in last chapter, black hole physics in AdS can be described by strongly coupled gauge theories at finite temperature (the Hawking temperature of the black hole) on the boundary of space and vice versa. String theory, predicted the existence of extra higher dimensions where gravity could propagate in, and quasinormal modes could be a way to detect these dimensions, thus the future observation of gravitational waves can prove the existence of extra dimensions and provide support for string theory [43].

This chapter contains an overview of the gravitational perturbations of black holes in four space-time dimensions with emphasis on the AdS_4 Schwarzschild background. Before applying the perturbation theory of black holes to AdS Schwarzschild metric, it is beneficial to start with the case of Schwarzschild metric which has been done by Regge and Wheeler [44] and separately by Edelstein and Vishveshwara in 1957 [45] and extended by Zerilli [46],[64]. A summary of their method, followed in the next section, is taken mainly from their articles.

3.1 Perturbation of the Schwarzschild black holes

A small perturbation $h_{\mu\nu}$ is added to the background Schwarzschild metric $g_{\mu\nu}$. The perturbed metric has the form

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \tag{3.1}$$

The contracted Ricci tensor, calculated from the perturbed form of the metric can be expressed in the form

$$\tilde{R}_{\mu\nu} = R_{\mu\nu} + \delta R_{\mu\nu} \tag{3.2}$$

To derive an expression for $\delta R_{\mu\nu}$, we use the *Palatini equation* [44]

$$\delta R_{\mu\nu} = -\delta\Gamma^{\beta}_{\mu\nu;\beta} + \delta\Gamma^{\beta}_{\mu\beta;\nu}, \tag{3.3}$$

where the semicolons represent covariant differentiation with respect to the background metric $g_{\mu\nu}$, and we use the symbol

$$\delta\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\nu}(h_{\beta\nu;\gamma} + h_{\gamma\nu;\beta} - h_{\beta\gamma;\nu}). \tag{3.4}$$

Putting (3.4) in (3.3), we get the final exression for $\delta R_{\mu\nu}$

$$\delta R_{\mu\nu} = \frac{1}{2}\left[g^{\alpha\sigma}(-h_{\nu\sigma;\mu;\alpha} + h_{\mu\nu;\sigma;\alpha} + h_{\alpha\sigma;\mu;\nu} - h_{\mu\alpha;\nu;\sigma})\right] \tag{3.5}$$

which we will also use to find the field equations of AdS-Schwarzschild black holes in the next section.

The unperturbed AdS–Schwarzschild spacetime (its metric is given by (1.16)) is a vacuum solution of Einstein equations of the form $R_{\mu\nu} = \Lambda g_{\mu\nu}$. So the equation $\delta R_{\mu\nu} = \Lambda \delta g_{\mu\nu}$ is the perturbation equation. If $\delta R_{\mu\nu} = 0$ as in the Schwarzschild spacetime perturbation, it means that the perturbed space is empty of matter or distributed energy. To solve the perturbed field equations, the method of separation of variables was first used by Regge and Wheeler. The angular dependence of the field equations comes from the spherical harmonics which are part of tensor harmonics. By separating the time dependence part, these equations form a system of ordinary differential equations with r as the only variable. By choosing a particular gauge (working in a specific coordinate systems) which here is Regge-Wheeler gauge, the solutions can take simpler forms which are transverse and traceless. By changing the gauge, we make a small change in coordinates as follows,

$$x^{\nu}_{new} = x^{\nu}_{old} + \xi^{\nu}, \tag{3.6}$$

which causes the metric perturbation to change as

$$h_{\mu\nu}^{new} = h_{\mu\nu}^{old} + \xi_{\mu;\nu} + \xi_{\nu;\mu} \tag{3.7}$$

3.1.1 Perturbed metric, axial and polar components

Since Schwarzschild background metric is spherically symmetric, $h_{\mu\nu}$ can be canonically split into two classes of axial and poler perturbations. This decomposition, before applying the gauge transformation gives the odd parity as

$$h_{\mu\nu}^{Ml} = \begin{bmatrix} 0 & 0 & -h_0(t,r)(\partial/\sin\theta\partial\varphi)Y_l^M & h_0(t,r)(\partial/\sin\theta\partial\theta)Y_l^M \\ 0 & 0 & -h_1(t,r)(\partial/\sin\theta\partial\varphi)Y_l^M & h_1(t,r)(\partial/\sin\theta\partial\theta)Y_l^M \\ sym & sym & h_2(t,r)(\partial^2/\sin\theta\partial\theta\partial\varphi - \cos\theta\partial/\sin^2\partial\varphi)Y_l^M & sym \\ sym & sym & \frac{1}{2}h_2(t,r)(\partial^2/\sin\theta\partial\varphi\partial\varphi + \cos\theta\partial/\partial\theta & -h_2(t,r)(\sin\theta\partial^2/\partial\theta\partial\varphi \\ & & -\sin\theta\partial^2/\partial\theta\partial\theta)Y_l^M & -\cos\theta\partial/\partial\varphi)Y_l^M \end{bmatrix} \tag{3.8}$$

and the even parity as

$$h_{\mu\nu}^{Ml} = \begin{bmatrix} (1-2m/r)H_0(t,r)Y_l^M & H_1(t,r)Y_l^M & h_0(t,r)(\partial/\partial\theta)Y_l^M & h_0(t,r)(\partial/\partial\varphi)Y_l^M \\ H_1(t,r)Y_l^M & (1-2m/r)^{-1}H_2(t,r)Y_l^M & h_1(t,r)(\partial/\partial\theta)Y_l^M & h_1(t,r)(\partial/\partial\varphi)Y_l^M \\ sym & sym & r[K(r,t) & sym \\ & & +G(t,r)(\partial^2/\partial\theta^2)]Y_l^M & \\ sym & sym & r^2G(t,r)(\partial^2/\partial\theta\partial\varphi & r^2[K(t,r)\sin^2\theta \\ & & -\cos\theta\partial/\sin\theta\partial\varphi)Y_l^M & +G(t,r))(\partial^2/\partial\varphi^2 \\ & & & +\sin\theta\cos\theta\partial/\partial\theta)]Y_l^M \end{bmatrix} \tag{3.9}$$

where *sym* represent symmetric components. As we mentioned before, the angular functions are the tensor harmonics. During the calculations, there is no need to work with a specific M since any choice of L and M ($M = -L, -L+1, ...L$) result in the same radial equation [61]. On this account, for simplicity we work with $M = 0$ which has the advantage that φ will completely disappear from the calculations.

By using the gauge transformation, we can find the canonical form of odd and even waves. The gauge vector ξ^α that simplifies the odd waves has the form

$$
\begin{aligned}
\xi_t &= 0, \\
\xi_r &= 0, \qquad (3.10) \\
\xi_\theta &= \Lambda(t,r)\, \epsilon^{\theta\varphi}\, \partial/\partial\varphi Y_l^M\, g_{\theta\theta}{}^1, \\
\xi_\varphi &= \Lambda(t,r)\, \epsilon^{\varphi\theta}\, \partial/\partial\theta Y_l^M\, g_{\varphi\varphi}. \qquad (3.11)
\end{aligned}
$$

and the gauge transformation for simplifying the even waves can be written in the form

$$
\begin{aligned}
\xi_t &= M_0(t,r)\, Y_l^M\, g_{tt}, \\
\xi_r &= M_1(t,r)\, Y_l^M\, g_{rr}, \qquad (3.12) \\
\xi_\theta &= M(t,r)\, \partial/\partial\theta Y_l^M\, g_{\theta\theta}, \\
\xi_\varphi &= M(t,r)\, 1/\sin^2\theta\partial/\partial\varphi Y_l^M\, g_{\varphi\varphi}{}^2.
\end{aligned}
$$

In the gauge transformation for the odd waves, the radial function Λ can be adjusted to cancel the radial factor h_2, and in the case of even waves the functions M_0, M_1, and M will be adjusted to annul the factors G, h_0, and h_1. The final canonical form of odd and even perturbations are given, respectively as follows,

[1] ξ_θ is equal to zero since by choosing $M = 0$, Y_l^M will be independent of φ.

[2] ξ_φ is equal to zero for the same reason that ξ_θ was zero in the case of odd waves.

$$h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & h_0 \\ 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & 0 \\ sym & sym & 0 & 0 \end{bmatrix} \exp(-i\omega t)[\sin\theta(\partial/\partial\theta)]P_l(\cos\theta); \tag{3.13}$$

$$h_{\mu\nu} = \begin{bmatrix} (1-\frac{2m}{r})H_0 & H_1 & 0 & 0 \\ H_1 & (1-\frac{2m}{r})^{-1}H_2 & 0 & 0 \\ 0 & 0 & r^2K & 0 \\ 0 & 0 & 0 & r^2K\sin^2\theta \end{bmatrix} \exp(-i\omega t)P_l(\cos\theta). \tag{3.14}$$

Here $P_l(\cos\theta)$ is the Legendre polynomial with angular momentum l.

Regge and Wheeler showed that the odd parity perturbed metric components could be reconstructed from a master scalar function, called the Regge-Wheeler function. Further, Zerilli derived a master scalar and the equation that it obeyed for even parity components; his function is called the Zerilli function. Since these scalars contain all the physical information of the system, they are called the master scalars [48]. Later, Moncrief showed that these two scalar functions are gauge invariant [49]. In 2001, master equations for the Schwarzschild-de Sitter and Schwarzschild-anti-de Sitter backgrounds were published in two papers by Cardoso and Lemos [73][74]. Recently, it was also extended to the higher dimensional maximally symmetric black holes by Kodama and Ishibashi [75].

3.1.2 Ricci tensor perturbations, Odd and Even components

Now, we can find the first-order perturbations of the Ricci tensor. The odd-parity equations are

$$\begin{aligned}\delta R_{t\varphi} &= \left[\frac{1}{2}(1-\frac{2m}{r})h_0'' + \frac{1}{2}i\omega(1-\frac{2m}{r})h_1' + \frac{i\omega}{r}(1-\frac{2m}{r})h_1 \right.\\ &+ \left.\frac{2m}{r^3}h_0 - \frac{L(L+1)}{2r^2}h_0\right] \sin\theta\ \partial/\partial\theta P_L\ e^{-i\omega t} \qquad (3.15)\end{aligned}$$

$$\begin{aligned}\delta R_{r\varphi} &= -\left[\frac{1}{2}i\omega(1-\frac{2m}{r})^{-1}h_0' - \frac{i\omega}{r}(1-\frac{2m}{r})^{-1}h_0 - \frac{1}{2}\omega^2(1-\frac{2m}{r})^{-1}h_1 \right.\\ &- \left.\frac{1}{r^2}(1-\frac{2m}{r})h_1 - (\frac{2m}{r^3})h_1 + \frac{L(L+1)}{2r^2}h_1\right] \sin\theta\ \partial/\partial\theta P_L\ e^{-i\omega t} \qquad (3.16)\end{aligned}$$

$$\begin{aligned}\delta R_{\theta\varphi} &= \left[\frac{1}{2}i\omega(1-\frac{2m}{r})^{-1}h_0 + \frac{1}{2}(1-\frac{2m}{r})h_1' \right.\\ &+ \left.\frac{m}{r^2}h_1\right] (\cos\theta\partial/\partial\theta - \sin\theta\ \partial^2/\partial\theta^2)P_L\ e^{-i\omega t} \qquad (3.17)\end{aligned}$$

and the Even-parity equations are

$$\begin{aligned}\delta R_{tr} &= -\left[i\omega K' + \frac{i\omega}{r}K - \frac{i\omega}{2}(\frac{2m}{r^2})(1-\frac{2m}{r})^{-1}K \right.\\ &- \left.\frac{i\omega}{r}\ H_2 + \frac{L(L+1)}{2r^2}\ H_1\right] P_L(\cos\theta)\ e^{-i\omega t} \qquad (3.18)\end{aligned}$$

$$\delta R_{t\theta} = -\left[\frac{i\omega}{2}(K+H_2) + \frac{1}{2}(1-\frac{2m}{r})H_1' + \frac{m}{r^2}H_1\right] \partial_\theta P_L(\cos\theta)\ e^{-i\omega t} \qquad (3.19)$$

$$\delta R_{r\theta} = -\Bigg[\frac{i\omega}{2}(1-\frac{2m}{r})^{-1}H_1 + \frac{1}{2}H_0' - \frac{1}{2}K' - \frac{3m-r}{4mr-2r^2}H_0 + \frac{m-r}{4mr-2r^2}H_2\Bigg]\partial_\theta P_L(\cos\theta)\; e^{-i\omega t} \quad (3.20)$$

$$\delta R_{tt} = -\Bigg[\frac{1}{2}\omega^2 H_2 + \omega^2 K + i\omega(\frac{3m}{r^2}-\frac{2}{r})H_1 - i\omega(1-\frac{2m}{r})H_1' - \frac{1}{2}(1-\frac{2m}{r})^2 H_0'' - \frac{(m-2r)(2m-r)}{2r^3}H_0' - \frac{m}{2r^2}(1-\frac{2m}{r})H_2' + \frac{m}{r^2}(1-\frac{2m}{r})K' + \frac{1}{2r^2}(1-\frac{2m}{r})L(L+1)H_0\Bigg]P_L(\cos\theta)\; e^{-i\omega t} \quad (3.21)$$

$$\delta R_{rr} = -\Bigg[i\omega(1-\frac{2m}{r})^{-1}H_1' + i\omega(\frac{m}{r^2}(1-\frac{2m}{r})^{-2}H_1 - \frac{1}{2}\omega^2(1-\frac{2m}{r})^{-2}H_2 + \frac{L(L+1)}{2r^2}(1-\frac{2m}{r})^{-1}H_2 + \frac{1}{2}H_0'' - K'' + \frac{3m}{2r^2}(1-\frac{2m}{r})^{-1}H_0' + \left(\frac{m}{2r^2}(1-\frac{2m}{r})^{-1}+\frac{1}{r}\right)H_2' - \left(\frac{m}{r^2}(1-\frac{2m}{r})^{-1}+\frac{2}{r}\right)K'\Bigg]P_L(\cos\theta)\; e^{-i\omega t} \quad (3.22)$$

$$\begin{aligned}\delta R_{\theta\theta} &= -\Bigg[-\frac{1}{2}\omega^2(1-\frac{2m}{r})^{-1}r^2K + i\omega r H_1 - \frac{1}{2}r^2(1-\frac{2m}{r})K'' \\ &+ \frac{1}{2}r(1-\frac{2m}{r})H_0' + \frac{1}{2}r(1-\frac{2m}{r})H_2' + (3m-2r)K' \\ &+ H_2 - K + \frac{1}{2}L(L+1)K\Bigg] P_L(\cos\theta)\ e^{-i\omega t} \\ &+ \left[\frac{1}{2}H_0 - \frac{1}{2}H_2\right]\partial_\theta^2 P_L(\cos\theta)\ e^{-i\omega t} \end{aligned} \tag{3.23}$$

3.2 Perturbation of the Anti de–Sitter black holes

By following the process of finding the components of Ricci tensor perturbation for Schwarzschild black holes, we can also obtain the equations governing the perturbations for AdS-Schwarzschild black holes, which is the main goal of this section.

3.2.1 Perturbed metric, axial and polar components

By having the spherically symmetric AdS-Schwarzschild background metric, the metric perturbation can be divided in to two groups of axial and polar perturbations. Before the gauge transformation, the axial perturbation metric has the same form as odd perturbations of the Schwarzschild metric, but the polar perturbation have two different metric elements as compared with the Schwarzschild case:

$$\tilde{h}^{ml}_{\mu\nu} = \begin{bmatrix} \left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)\tilde{H}_0 Y_l^M & \tilde{H}_1 Y_l^M & \tilde{h}_0(\partial/\partial\theta)Y_l^M & \tilde{h}_0(\partial/\partial\varphi)Y_l^M \\ \tilde{H}_1 Y_l^M & \left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)^{-1}\tilde{H}_2 Y_l^M & \tilde{h}_1(\partial/\partial\theta)Y_l^M & \tilde{h}_1(\partial/\partial\varphi)Y_l^M \\ sym & sym & \begin{array}{l} r[\tilde{K} \\ +\tilde{G}(\partial^2/\partial\theta^2)]Y_l^M \end{array} & sym \\ sym & sym & \begin{array}{l} r^2\tilde{G}(\partial^2/\partial\theta\partial\varphi \\ -\cos\theta\partial/\sin\theta\partial\varphi)Y_l^M \end{array} & \begin{array}{l} r^2[\tilde{K}\sin^2\theta \\ +\tilde{G}(\partial^2/\partial\varphi^2 \\ +\sin\theta\cos\theta\partial/\partial\theta)]Y_l^M \end{array} \end{bmatrix} \tag{3.24}$$

The gauge transformation to find the canonical form of the axial perturbations is also the same as Schwarzschild one, but it is different in t and r components for the polar perturbations as follows:

$$\begin{aligned}
\tilde{\xi}_t &= \tilde{M}_0(t,r)\; Y_l^M\; \tilde{g}_{tt} \\
\tilde{\xi}_r &= \tilde{M}_1(t,r)\; Y_l^M\; \tilde{g}_{rr}, \\
\tilde{\xi}_\theta &= \tilde{M}(t,r)\; \partial/\partial\theta Y_l^M\; \tilde{g}_{\theta\theta}, \\
\tilde{\xi}_\varphi &= \tilde{M}(t,r)\; 1/\sin^2\theta\partial/\partial\varphi Y_l^M\; \tilde{g}_{\varphi\varphi}.
\end{aligned} \tag{3.25}$$

where $\tilde{g}_{tt} = -\left(1 - \frac{2m}{r} + \frac{r^2}{l^2}\right)$ and $\tilde{g}_{rr} = \left(1 - \frac{2m}{r} + \frac{r^2}{l^2}\right)^{-1}$, according to the AdS black hole metric (see eq.(1.16)). The functions $\tilde{M}_0, \tilde{M}_1$, and $\tilde{M}$ will be modified to cancel the factors $\tilde{G}, \tilde{h}_0$, and $\tilde{h}_1$. The final canonical form of axial perturbations is the same as Schwarzschild's and the polar perturbations has the form

$$\tilde{h}_{\mu\nu} = \begin{bmatrix} \left(1 - \frac{2m}{r} + \frac{r^2}{l^2}\right)\tilde{H}_0 & \tilde{H}_1 & 0 & 0 \\ \tilde{H}_1 & \left(1 - \frac{2m}{r} + \frac{r^2}{l^2}\right)^{-1}\tilde{H}_2 & 0 & 0 \\ 0 & 0 & r^2\tilde{K} & 0 \\ 0 & 0 & 0 & r^2\tilde{K}\sin^2\theta \end{bmatrix} \exp(-i\omega t)P_l(\cos\theta). \tag{3.26}$$

3.2.2 Ricci tensor perturbations, Odd and Even components

The first-order perturbations of the Ricci tensor can be represented as follows.[3]

Odd-parity:

[3] As a prototype example, find the detailed calculation of $\delta R_{r\phi}$ in the appendix A.

$$\delta R_{t\phi} = \left[\left(\frac{2m}{r^3}+\frac{1}{l^2}\right)h_0+\left(\frac{(r-2m)}{r^2}+\frac{r}{l^2}\right)i\omega h_1+\frac{1}{2}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)h_0'' + \frac{1}{2}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)i\omega h_1' - \frac{1}{2r^2}L(L+1)h_0\right]\sin\theta\;\partial_\theta P_L(\cos\theta)\;e^{-i\omega t} \quad (3.27)$$

$$\delta R_{r\phi} = \left[-\frac{1}{2}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)^{-1}i\omega h_0' + \frac{1}{r}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)^{-1}i\omega h_0 + \frac{1}{2}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)^{-1}\omega^2 h_1 - \frac{1}{2r^2}L(L+1)h_1 + \left(\frac{1}{r^2}+\frac{3}{l^3}\right)\right]\sin\theta\;\partial_\theta P_L(\cos\theta)\;e^{-i\omega t} \quad (3.28)$$

$$\delta R_{\theta\varphi} = \left[\frac{1}{2}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)^{-1}i\omega h_0 + \frac{1}{2}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)h_1' + \left(\frac{m}{r^2}+\frac{r}{l^2}\right)h_1\right]\left(\cos\theta\;\partial_\theta - \sin\theta\;\partial_\theta^2\right)\;P_L(\cos\theta)\;e^{-i\omega t} \quad (3.29)$$

Even-parity:

$$\delta R_{tr} = \left[-i\omega K' - \frac{l^2}{(r^3+l^2(r-2m))}i\omega K + \frac{3ml^2}{r(r^3+l^2(r-2m))}i\omega K + \frac{1}{r}i\omega H_2 - \frac{L(L+1)}{2r^2}H_1 + \frac{3}{l^2}H_1\right]P_L(\cos\theta)\;e^{-i\omega t} \quad (3.30)$$

$$\begin{aligned}\delta R_{t\theta} &= \Bigg[-\frac{1}{2}i\omega(K+H_2)-\frac{1}{2}\frac{1}{l^2r}(r^3+l^2(r-2m))H_1'\\&- \frac{(l^2m+r^3)}{l^2r^2}H_1\Bigg]\,\partial_\theta P_L(\cos\theta)\;e^{-i\omega t}\end{aligned} \tag{3.31}$$

$$\begin{aligned}\delta R_{r\theta} &= \Bigg[-\frac{1}{2}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)^{-1}i\omega H_1-\frac{1}{2}H_0'+\frac{1}{2}K'\\&- \frac{l^2(3m-r)}{2r(r^3+l^2(-2m+r))}H_0+\frac{(l^2(m-r)-2r^3)}{2r(r^3+l^2(-2m+r))}H_2\Bigg]\\&\times\;\partial_\theta P_L(\cos\theta)\;e^{-i\omega t}\end{aligned} \tag{3.32}$$

$$\begin{aligned}\delta R_{tt} &= \Bigg[-\frac{1}{2}\omega^2H_2-\omega^2K+\frac{2r^3+2l^2(-2m+r))}{l^2r^2}i\omega H_1+\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)i\omega H_1{}'\\&+ \frac{(r^3+l^2(-2m+r))^2}{2l^4r^2}H_0{}''+\frac{(l^2(2m-r)-r^3)(l^2(m-r)-2r^3)}{l^4r^3}H_0'\\&+ \frac{r^3+l^2(-2m+r))}{2l^4r^3}H_2'-\frac{r^6+l^4m(r-2m)+l^2r^3(r-2m)}{2l^4r^3}K'\\&- \frac{1}{2r^2}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)L(L+1)H_0+\frac{6r^3+6l^2(-2m+r)}{2l^4r}H_0\\&+ \frac{8r^6l^2(-8mr^3+6r^4)}{2l^4r^4}H_2\Bigg]\;P_L(\cos\theta)\;e^{-i\omega t}\end{aligned} \tag{3.33}$$

$$\begin{aligned}
\delta R_{rr} &= \left[-\left(1 - \frac{2m}{r} + \frac{r^2}{l^2}\right)^{-1} i\omega H_1' - \frac{l^2(l^2 m + r^3)}{r^3 + l^2(-2m+r)^2} i\omega H_1 + \frac{l^2 L(L+1)}{2r(r^3 + l^2(-2m+r))} H_2 \right.\\
&+ \frac{1}{2}\left(1 - \frac{2m}{r} + \frac{r^2}{l^2}\right)^{-2} \omega^2 H_2 - \frac{H_0''}{2} + \frac{K''}{2} - \frac{3r^3 + l^2(2r - 3m)}{2r^2} H_2' \\
&+ \left. \frac{(l^2(2m - r) - r^3)(3(l^2 m + r^3))}{2r(r^3 + l^2(r-2m)^2)} H_0' + \frac{6r^3 + l^2(4r - 6m)}{2r(r^3 + l^2(r - 2m))} K' \right] P_L(\cos\theta)\, e^{-i\omega t} \quad (3.34)
\end{aligned}$$

$$\begin{aligned}
\delta R_{\theta\theta} &= \left[-\frac{l^2 r^3}{2(r^3 + l^2(-2m+r))} \omega^2 K - ir\omega H_1 + \frac{r(r^3 + l^2(-2m+r))}{2l^2} K'' \right.\\
&- \frac{(r^3 + l^2(-2m+r))}{2l^2}(H_0' + H_2') + \frac{6r^3 + l^2(-6m + 4r)}{2l^2} K' - \frac{l^2 + 3r^3}{l^2} H_2 + K \\
&- \left. \frac{1}{2} L(L+1) K \right] P_L(\cos\theta)\, e^{-i\omega t} + \left[\frac{1}{4}(H_2 - H_0) \right] \partial_\theta^2 P_L(\cos\theta)\, e^{-i\omega t} \qquad (3.35)
\end{aligned}$$

3.2.3 Large m limit of the perturbations, $m >> l$

In the limit where the mass of the black hole is much larger compared with the radius of the AdS spacetime, the perturbation equations take the following form in both odd and even groups, respectively

Odd-parity:

$$\begin{aligned}
\delta R_{t\phi} &\approx \left[\frac{1}{l^2} h_0 + \frac{r}{l^2} i\omega h_1 + \frac{1}{2}\left(\frac{r^2}{l^2}\right)\left(h_0'' + i\omega h_1'\right) - \frac{1}{2r^2} L(L+1) h_0 \right] \\
&\times \sin\theta\, \partial_\theta P_L(\cos\theta)\, e^{-i\omega t} \qquad (3.36)
\end{aligned}$$

$$\delta R_{r\phi} \approx \left[-\frac{1}{2}\left(\frac{r^2}{l^2}\right)^{-1} i\omega h_0' + \left(\frac{r^3}{l^2}\right)^{-1} i\omega h_0 + \frac{1}{2}\left(\frac{r^2}{l^2}\right)^{-1} \omega^2 h_1 \right.$$
$$\left. - \frac{1}{2r^2}L(L+1)h_1 + \frac{3}{l^2}h_1 \right] \sin\theta\ \partial_\theta P_L(\cos\theta)\ e^{-i\omega t} \quad (3.37)$$

$$\delta R_{\theta\varphi} \approx \left[\frac{1}{2}\left(\frac{r^2}{l^2}\right)^{-1} i\omega h_0 + \frac{1}{2}\left(\frac{r^2}{l^2}\right) h_1' + \frac{r}{l^2}h_1 \right]$$
$$\times \left(\cos\theta\ \partial_\theta - \sin\theta\ \partial_\theta^2\right) P_L(\cos\theta)\ e^{-i\omega t} \quad (3.38)$$

Even-parity:

$$\delta R_{tr} \approx \left[- i\omega K' - \frac{l^2}{(r^3 + l^2(r-2m))} i\omega K + \frac{3ml^2}{r(r^3 + l^2(r-2m))} i\omega K \right.$$
$$\left. + \frac{1}{r} i\omega H_2 - \frac{L(L+1)}{2r^2} H_1 + \frac{3}{l^2} H_1 \right] P_L(\cos\theta)\ e^{-i\omega t} \quad (3.39)$$

$$\delta R_{t\theta} \approx \left[-\frac{1}{2} i\omega(K + H_2) - \frac{1}{2}\frac{r^2}{l^2} H_1' - \frac{r}{l^2} H_1 \right] \partial_\theta P_L(\cos\theta)\ e^{-i\omega t} \quad (3.40)$$

$$\delta R_{r\theta} \approx \left[-\frac{1}{2}\left(\frac{r^2}{l^2}\right)^{-1} i\omega H_1 - \frac{1}{2} H_0' + 1/2K' \right.$$
$$\left. + \frac{l^2}{2r^3} H_0 - \frac{1}{r} H_2 \right] \partial_\theta P_L(\cos\theta)\ e^{-i\omega t} \quad (3.41)$$

$$
\begin{aligned}
\delta R_{rr} \approx & \left[-\left(\frac{r^2}{l^2}\right)^{-1} i\omega H_1' - l^2 i\omega H_1 + \frac{l^2 L(L+1)}{2r^4} H_2 \right. \\
& + \frac{1}{2}\left(\frac{r^2}{l^2}\right)^{-2} \omega^2 H_2 + \frac{H_0''}{2} + \frac{K''}{2} - \frac{3}{2} r H_2' \\
& \left. - \frac{3}{2} r^2 H_0' + \frac{3}{r} K' \right] P_L(\cos\theta)\, e^{-i\omega t}
\end{aligned} \tag{3.42}
$$

$$
\begin{aligned}
\delta R_{\theta\theta} \approx & \left[-\frac{l^2}{2}\omega^2 K - ir\omega H_1 + \frac{r^4}{2l^2} K'' - \frac{r^3}{2l^2}(H_0' + H_2') + \frac{3r^3}{l^2} K' \right. \\
& \left. - \frac{3r^2}{l^2} H_2 + (K - \frac{1}{2} L(L+1) K) \right] P_L(\cos\theta)\, e^{-i\omega t} \\
& + \left[\frac{1}{4}(H_2 - H_0) \right] \partial_\theta^2 P_L(\cos\theta)\, e^{-i\omega t}
\end{aligned} \tag{3.43}
$$

$$
\begin{aligned}
\delta R_{tt} \approx & \left[-\frac{1}{2}\omega^2 H_2 - \omega^2 K + \frac{2r}{l^2} i\omega H_1 + \frac{1}{2}\frac{r^2}{l^2} H_1' + \frac{r^4}{2l^4} H_0'' \right. \\
& + \frac{2r^3}{l^4} H_0' + \frac{1}{2l^4} H_2' - \frac{r^3}{2l^4} K' - \frac{L(L+1)}{2l^2} H_0 \\
& \left. + \frac{3r}{l^4} H_0 + \frac{4r^2}{l^4} H_2 \right] P_L(\cos\theta)\, e^{-i\omega t}
\end{aligned} \tag{3.44}
$$

3.3 Stress–energy tensor

The energy-momentum tensor of the field theory on the boundary of the AdS space is expressed in terms of the intrinsic and extrinsic geometry of this boundary at infinity [51], as

$$\kappa^2 T_{ab} = K_{ab} - K\gamma_{ab} - 2\sqrt{-\frac{\Lambda}{3}}\gamma_{ab} + \sqrt{-\frac{3}{\Lambda}}G_{ab}. \tag{3.45}$$

Here, G_{ab} is the Einstein tensor of the induced three-dimensional metric γ_{ab},

$$G_{ab} = R_{ab}[\gamma] - \frac{1}{2}R[\gamma]\gamma_{ab}. \tag{3.46}$$

and K is the trace of the extrinsic curvature

$$K = \gamma^{ab}K_{ab}, \tag{3.47}$$

We start the computation, by first using ADM-formalism [50] to decompose the metric g on M in the form

$$ds^2 = N^2 dr^2 + \gamma_{ab}(dx^a + N^a dr)(dx^b + N^b dr) \tag{3.48}$$

using appropriately chosen (N, N^a) functions, where N is called a *lapse* function and N^a is called a *shift.* ∂M_r is a three-dimensional surface at fixed value of r which is the boundary of four-dimensional region M_r. γ_{ab} is a finite value induced metric on ∂M_r. A relation among the bulk and boundary metrics is

$$\sqrt{-detg} = N\sqrt{-det\gamma}. \tag{3.49}$$

In reference [51] the components of energy-momentum tensor on the boundary of AdS_4 space are derived as I mention its summary here.

For AdS_4 black holes, in the standard coordinate system, the functions N and

N^a are

$$N = \frac{1}{\sqrt{f(r)}}, \quad N^a = 0, \tag{3.50}$$

and the induced metric on the boundary ∂M_r is

$$\gamma_{ab} = \begin{bmatrix} -f(r) & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 sin^2\theta \end{bmatrix} \tag{3.51}$$

The second fundamental form and its trace are

$$K_{ab} = \sqrt{f(r)} \begin{bmatrix} -f'(r) & 0 & 0 \\ 0 & -r & 0 \\ 0 & 0 & -r sin^2\theta \end{bmatrix} \tag{3.52}$$

and

$$K = -\frac{1}{2r\sqrt{f(r)}}(rf'(r) + 4f(r)). \tag{3.53}$$

Now, by following the method that has been used in reference [51], we have the following expressions for energy- momentum tensor on ∂M_r.

$$\begin{aligned} \kappa^2 T_{tt} &= \frac{f(r)}{r^2}\left(\sqrt{-\frac{3}{\Lambda}} + 2r^2\sqrt{-\frac{\Lambda}{3}} - 2r\sqrt{f(r)}\right), \\ \kappa^2 T_{\theta\theta} &= \frac{r}{\sqrt{f(r)}}\left(f(r) + \frac{r}{2}f'(r)\right) - 2r^2\sqrt{-\frac{\lambda}{3}}, \\ \kappa^2 T_{\varphi\varphi} &= \sin^2\theta \; T_{\theta\theta}. \end{aligned} \tag{3.54}$$

and the rest of the components are zero.

3.3.1 Axial perturbations

The induced three-dimensional metric on ∂M_r is a perturbation of the static metric

$$\gamma_{ab} = \gamma_{ab}^{(0)} + \begin{bmatrix} 0 & 0 & h_0(r) \\ 0 & 0 & 0 \\ h_0(r) & 0 & 0 \end{bmatrix} \sin\theta\ \partial_\theta P_L(\cos\theta)\ e^{-i\omega t}. \tag{3.55}$$

The complete energy-momentum tensor of the boundary theory on ∂M_r assumes the following form in terms of the metric coefficients $h_0(r)$ and $h_1(r)$[4]. The holograghic renormalization is the method has been used to derive these forms of perturbation tensor components

$$T_{ab} = T_{ab}^{(0)} + \begin{bmatrix} 0 & 0 & \delta T_{t\varphi} \\ 0 & 0 & \delta T_{\theta\varphi} \\ \delta T_{t\varphi} & \delta T_{\theta\varphi} & 0 \end{bmatrix}, \tag{3.56}$$

where

$$\begin{aligned} \kappa^2 \delta T_{t\varphi} &= \left[\left(\frac{2}{r}\sqrt{f(r)} + \frac{f'(r)}{2\sqrt{f(r)}} - 2\sqrt{-\frac{\Lambda}{3}} + \sqrt{-\frac{3}{\Lambda}}\frac{(L-1)(L+2)}{2r^2} \right) h_0(r) \right. \\ &- \left. \frac{1}{2}\sqrt{f(r)}(h_0'(r) + i\omega h_1(r)) \right] e^{-i\omega t} \sin\theta \partial_\theta P_L(\cos\theta), \\ \kappa^2 \delta T_{\theta\varphi} &= -\frac{1}{2}\left(\sqrt{f(r)} h_1(r) + i\omega\sqrt{-\frac{3}{\Lambda}\frac{h_0(r)}{f(r)}} \right) e^{-i\omega t} \\ &\times\ \sin\theta[L(L+1)P_L(\cos\theta) + 2cot\theta\ \partial_\theta P_L(\cos\theta)]. \end{aligned} \tag{3.57}$$

[4]Find their forms in Appendix A.3.

3.3.2 Polar perturbations

For polar perturbations, the induced metric on the boundary ∂M_r has the form

$$\gamma_{ab} = \gamma_{ab}^{(0)} + \begin{bmatrix} (1-\frac{2m}{r})H_0 & 0 & 0 \\ 0 & r^2 K & 0 \\ 0 & 0 & r^2 K \sin^2\theta \end{bmatrix} \exp(-i\omega t) P_l(\cos\theta). \tag{3.58}$$

The expression for stress tensor polar perturbations is

$$T_{ab} = T_{ab}^{(0)} + \begin{bmatrix} \delta T_{tt} & \delta T_{t\theta} & 0 \\ \delta T_{t\theta} & \delta T_{\theta\theta} & 0 \\ 0 & 0 & \delta T_{\varphi\varphi} \end{bmatrix}, \tag{3.59}$$

where the components of the tensor, in terms of the metric functions $H_0(r), H_1(r)$ and $K(r)$[5], are

$$\begin{aligned} \kappa^2 \delta T_{tt} &= f(r)\Bigg[\left(\frac{3}{r}\sqrt{f(r)} - 2\sqrt{-\frac{\Lambda}{3}} - \frac{1}{r^2}\sqrt{-\frac{3}{\Lambda}}\right) H_0(r) - \sqrt{f(r)}K'(r) \\ &+ \frac{(L-1)(L+2)}{2r^2}\sqrt{-\frac{3}{\Lambda}}K(r)\Bigg] e^{-i\omega t}\partial_\theta P_L(\cos\theta), \end{aligned}$$

[5]Find their forms in Appendix A.3.

$$\begin{aligned}
\kappa^2 \delta T_{\theta\theta} &= \Bigg[\left(r\sqrt{f(r)} + \frac{r^2 f'(r)}{2\sqrt{f(r)}} - 2r^2\sqrt{-\frac{\Lambda}{3}} + \frac{\omega^2 r^2}{2f(r)}\sqrt{-\frac{3}{\Lambda}}\right) K(r) \\
&+ \frac{r^2}{2}\sqrt{f(r)}K'(r) - i\omega\frac{r^2}{\sqrt{f(r)}}H_1(r) - \frac{r^2}{2}\sqrt{f(r)}H_0'(r) \\
&- \frac{r}{2}\left(\sqrt{f(r)} + \frac{rf'(r)}{2\sqrt{f(r)}}\right) H_0(r)\Bigg] e^{-i\omega t}\partial_\theta P_L(\cos\theta) \\
&- \frac{1}{2}\sqrt{-\frac{3}{\Lambda}}H_0(r)e^{-i\omega t}\cot\theta\ \partial_\theta P_L(\cos\theta),
\end{aligned}$$

$$\begin{aligned}
\kappa^2 \frac{\delta T_{\theta\theta}}{\sin^2\theta} &= \Bigg[\left(r\sqrt{f(r)} + \frac{r^2 f'(r)}{2\sqrt{f(r)}} - 2r^2\sqrt{-\frac{\Lambda}{3}} + \frac{\omega^2 r^2}{2f(r)}\sqrt{-\frac{3}{\Lambda}}\right) K(r) \\
&+ \frac{r^2}{2}\sqrt{f(r)}K'(r) - i\omega\frac{r^2}{\sqrt{f(r)}}H_1(r) - \frac{r^2}{2}\sqrt{f(r)}H_0'(r) \\
&- \frac{r}{2}\left(\sqrt{f(r)} + \frac{rf'(r)}{2\sqrt{f(r)}}\right) H_0(r)\Bigg] e^{-i\omega t}\partial_\theta P_L(\cos\theta) \\
&- \frac{1}{2}\sqrt{-\frac{3}{\Lambda}}H_0(r)e^{-i\omega t}\cot\theta\ \partial_\theta P_L(\cos\theta),
\end{aligned}$$

$$\kappa^2 \delta T_{t\theta} = \frac{1}{2}\left(i\omega\sqrt{-\frac{3}{\Lambda}}K(r) + \sqrt{f(r)}H_1(r)\right) e^{-i\omega t}\partial_\theta P_L(\cos\theta). \qquad (3.60)$$

3.3.3 Covariant derivative of $\delta T_{\mu\nu}$

Now that we have found the variation of the stress tensor on the boundary of the space, it is expected for the covariant derivative of this variation to be zero since they are actually our fluid dynamics equations as we mentioned in chapter 2.3.

Using the Gauss-Codacci relation [5], on the boundary we expect the stress-energy tensor to satisfy the equation

$$(\delta T_{\mu\nu})^{;\nu} = \delta G_{\mu\nu} n^{\nu} \tag{3.61}$$

where G is defined in equation (3.46).

If $\mu = t$,

$$(\delta T_{t\nu})^{;\nu} = D^t \delta T_{tt} + D^{\theta} \delta T_{t\theta} + D^{\varphi} \delta T_{t\varphi} = 0, \tag{3.62}$$

since each term is identically zero.

For $\mu = \theta$, the same thing happens, so

$$(\delta T_{\theta\nu})^{;\nu} = 0, \tag{3.63}$$

and for $\mu = \varphi$, we have

$$\begin{aligned} (\delta T_{\varphi\nu})^{;\nu} &= D^t \; \delta T_{\varphi t} + D^{\theta} \; \delta \; T_{\varphi\theta} + D^{\varphi} \; \delta \; T_{\varphi\varphi} \\ &= \delta^{tt} \left(\partial_t \; \delta T_{\varphi t} \right) \\ &+ \; \delta^{\theta\theta} \left(\partial_{\theta} \; \delta T_{\varphi\theta} - \Gamma^{\varphi}_{\theta\varphi} \; \delta T_{\varphi\theta} \right) \\ &+ \; \delta^{\varphi\varphi} \left(-2\Gamma^{\theta}_{\varphi\varphi} \; \delta T_{\theta\varphi} \right). \end{aligned} \tag{3.64}$$

By using equation (3.60), and summation over ν, following a few pages of calculation we have

$$\begin{aligned} D^{\nu}\left(\delta T_{\varphi\nu}\right) &= \delta G_{\varphi\nu}n^{\nu}, \\ D^{\nu}(\delta T_{\varphi\nu}) &- \delta G_{\varphi\nu}n^{\nu} = 0. \end{aligned} \tag{3.65}$$

The results we have found here is for the odd perturbations. However, the same process can be done to check if the even perturbations of the stress tensor would satisfy equation (3.60), as we expect.

3.4 Velocity, energy density and pressure of a perturbed fluid

Rewriting eq. (2.9) the general form of the stress-energy tensor of a non-perfect fluid is

$$T^{\mu\nu} = \epsilon u^{\mu}u^{\nu} + PP^{\mu\nu} + \Pi^{\mu\nu}. \tag{3.66}$$

Here, Π^{ab} is the dissipative part of the stress-energy tensor which has traceless part, $\pi^{\mu\nu}$ and non-vanishing trace part, Π, written as [53]

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + P^{\mu\nu}\Pi, \tag{3.67}$$

For the traceless part (visco-elastic stress) [54] we have

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + (\text{2nd order}) + (\text{higher order gradients}), \tag{3.68}$$

where η is the shear viscosity of the fluid and $\sigma^{\mu\nu}$ is the fluid shear tensor [55] expressed as follows,

$$\sigma^{\mu\nu} = \nabla^{\langle\mu} u^{\nu\rangle} \equiv P^{\mu\alpha} P^{\nu\beta} \left(\nabla_{(\alpha} u_{\beta)} - \frac{1}{d-1} P_{\alpha\beta}\theta\right) \tag{3.69}$$

In this equation $\theta \equiv \nabla_\mu u^\mu$ is the expansion and $\langle\mu\nu\rangle$ shows the symmetric transverse traceless part of the definition.

Before continuing to find the energy density and pressure of the perturbed fluid, we summarize the Weyl transformations of the various observables of conformal fluids in Table 3.1. The notation is from [54]. Further, we can define an invariant quantity under conformal transformation using the temperature and chemical potential, which is $\nu_i = \mu_i/T = \tilde{\nu}_i$. To protect the metric from diverging on the boundary, the conformal factor is chosen to be $e^{\varphi} = \frac{l}{r}$.

Table 3.1: Conformal transformation of the various observables in fluid mechanics.

Observable	Before transformation	After transformation
Spacetime metric	$g^{\mu\nu}$	$e^{-2\phi}\tilde{g}^{\mu\nu}$
Four-velocity	u^μ	$e^{-\phi}\tilde{u}^\mu$
Projection tensor	$P^{\mu\nu}$	$e^{-2\phi}\tilde{P}^{\mu\nu}$
Shear tensor	$\sigma^{\mu\nu}$	$e^{-3\phi}\tilde{\sigma}^{\mu\nu}$
Energy-momentum tensor	$T^{\mu\nu}$	$e^{-(d+2)\phi}\tilde{T}^{\mu\nu}$
Energy density	ϵ	$e^{-d\phi}\tilde{\epsilon}$
Pressure	P	$e^{-d\phi}\tilde{P}$
Shear viscosity	η	$e^{-(d-1)\phi}\tilde{\eta}$
Fluid temperature	T	$e^{-\phi}\tilde{T}$
Chemical potentials of the fluid	μ_i	$e^{-\phi}\tilde{\mu}_i$

3.4.1 Odd perturbations of fluid

To find the perturbed energy density ϵ from equation (3.65), we use the Landau–Lifshitz condition [53][55]

$$u^{\mu}_{(per)}T^{(per)}_{\mu\nu} = -\epsilon^{(per)}u^{(per)}_{\nu}. \tag{3.70}$$

The matrix form of this equation is

$$\begin{bmatrix} u^t+\delta u^t \\ \delta u^\theta \\ \delta u^\varphi \end{bmatrix} \begin{bmatrix} T_{tt} & 0 & \delta T_{t\varphi} \\ 0 & T_{\theta\theta} & \delta T_{\theta\varphi} \\ \delta T_{t\varphi} & \delta T_{\theta\varphi} & T_{\varphi\varphi} \end{bmatrix} = -(\epsilon+\delta\epsilon) \begin{bmatrix} \gamma_{tt}(u^t+\delta u^t)+\delta\gamma_{t\varphi}\delta u^\varphi \\ \gamma_{\theta\theta}\delta u^\theta \\ \gamma_{\varphi\varphi}\delta u^\varphi+\delta\gamma_{\varphi t}(u^t+\delta u^t) \end{bmatrix} \tag{3.71}$$

Therefore, by neglecting the terms of $\mathcal{O}(\delta^2)$, we will have a set of three equations and an extra normalization condition of $\gamma^{(per)}_{\mu\nu}u^{\mu}_{(per)}u^{\nu}_{(per)} = -1$ to obtain four unknown variables of δu^t, δu^φ, δu^θ and $\delta\epsilon$. The results are as follows:

$$\delta u^t = 0 \Rightarrow u^t_{(per)} = u^t \tag{3.72}$$

which after conformal transformation by choosing a suitable relation from Table 3.1 and normalization is equal to -1. For θ-component of the velocity we have

$$\delta u^\theta = 0, \tag{3.73}$$

and

$$\delta u^\varphi = \frac{u^t(\delta T_{t\varphi}+\epsilon\delta\gamma_{\varphi t})}{-(\epsilon\gamma_{\varphi\varphi}+T_{\varphi\varphi})} \tag{3.74}$$

which after conformal transformation and normalization takes the form [51]

$$\delta u^{\varphi} = \frac{i}{6m\omega}(L-1)(L+2)(l^2 i\omega I_0 - I_1)\sin\theta\partial_\theta P_L(\cos\theta)\ e^{-i\omega t} \tag{3.75}$$

Also, the corresponding energy density $\epsilon^{(per)}$ remains unchanged and equal to the ϵ in the unperturbed black hole case, since $\delta\epsilon$ is zero. So it has the form

$$\kappa^2\epsilon = \frac{2m}{l^2} \tag{3.76}$$

Furthermore, we can find the pressure of perturbed fluid from equation (3.65) such that

$$P^{\mu\nu}_{(per)}T^{(per)}_{\mu\nu} = P^{(per)} \rightarrow (\gamma^{\mu\nu}_{(per)} + u^{\mu}_{(per)}u^{\nu}_{(per)})T^{(per)}_{\mu\nu} = (P+\delta P)I \tag{3.77}$$

where I is the 3×3 identity matrix. The matrix form of the above equation is

$$\begin{bmatrix} \frac{1}{\gamma_{tt}}(u^t)^2 & 0 & -\frac{\delta\gamma_{t\varphi}}{\gamma_{tt}\gamma_{\varphi\varphi}} + u^t\delta u^{\varphi} \\ 0 & \frac{1}{\gamma_{\theta\theta}} & 0 \\ -\frac{\delta\gamma_{\varphi t}}{\gamma_{tt}\gamma_{\varphi\varphi}} + \delta u^{\varphi}u^t & 0 & \frac{1}{\gamma_{\varphi\varphi}}(u^{\varphi})^2 \end{bmatrix}\begin{bmatrix} T_{tt} & 0 & \delta T_{t\varphi} \\ 0 & T_{\theta\theta} & \delta T_{\theta\varphi} \\ \delta T_{t\varphi} & \delta T_{\theta\varphi} & T_{\varphi\varphi} \end{bmatrix} = (P+\delta P)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.78}$$

Therefore, by solving the above set of equations, we have

$$P+\delta P = \frac{T_{\theta\theta}}{\gamma_{\theta\theta}} = \frac{1}{\kappa^2}\left(\frac{\sqrt{f(r)}}{r} + \frac{f'(r)}{2\sqrt{f(r)}} - \frac{2}{l}\right) \tag{3.79}$$

After conformal transformation, following the Table 3.1 ($e^{-3\varphi} = \frac{r^3}{l^3}$), by taking the limit of $m >> l$, δP goes to zero and we have

$$\begin{aligned} P^{(per)} &\rightarrow P^{(unper)} \\ P + \delta P &\rightarrow \frac{m}{\kappa^2 l^2}. \end{aligned} \tag{3.80}$$

The result is equal to $P = \frac{\epsilon}{2}$ which is the pressure of unperturbed case. Thus, the perturbed fluid in odd case has a non-zero velocity in φ direction which means that the fluid moves sinusoidally in azimuthal direction while going forward in time. However, the φ component of velocity progresses in time periodically because of the real of $e^{-i\omega t}$ in the definition of u^φ which is $\cos \omega t$.

3.4.2 Even perturbations of fluid

Generally, for even perturbations, we follow the same process as odd perturbations. The matrix form of the the Landau–Lifshitz condition (eq.(3.69)) has the form

$$\begin{bmatrix} u^t + \delta u^t \\ \delta u^\theta \\ \delta u^\varphi \end{bmatrix} \begin{bmatrix} T_{tt} + \delta T_{tt} & \delta T_{t\theta} & 0 \\ \delta T_{\theta t} & T_{\theta\theta} + \delta T_{\theta\theta} & 0 \\ 0 & 0 & T_{\varphi\varphi} + \delta T_{\varphi\varphi} \end{bmatrix} = -(\epsilon + \delta\epsilon) \begin{bmatrix} \gamma_{tt}(u^t + \delta u^t) + \delta\gamma_{t\varphi}\delta u^\varphi \\ \gamma_{\theta\theta}\delta u^\theta \\ \gamma_{\varphi\varphi}\delta u^\varphi + \delta\gamma_{\varphi t}(u^t + \delta u^t) \end{bmatrix} \tag{3.81}$$

As before, we drop the terms of $\mathcal{O}(\delta^2)$ to have a set of three equations. These equations along with the normalization condition are sufficient to find four unknown variables of δu^t, δu^φ, δu^θ and $\delta\epsilon$. For t-component of velocity we obtain

$$\delta u^t = 0 \Rightarrow u^t_{(per)} = u^t \tag{3.82}$$

which after conformal transformation by choosing a suitable relation from Table 3.1

and normalization is equal to -1. For φ-component of the velocity we have

$$\delta u^{\varphi} = 0, \tag{3.83}$$

and

$$\delta u^{\theta} = \frac{u^t \delta T_{t\theta}}{-(\epsilon\gamma_{\theta\theta} + \epsilon\delta\gamma_{\theta\theta} + T_{\theta\theta})} \tag{3.84}$$

which after conformal transformation and normalization takes the form [51]

$$\delta u^{\theta} = \frac{-i\omega l^2}{12m}(L-1)(L+2)J_0\partial_\theta P_L(\cos\theta)\; e^{-i\omega t} \tag{3.85}$$

Different from odd perturbations, here $\delta\epsilon \neq 0$ and the form of the corresponding energy density $\epsilon^{(per)}$ is

$$\begin{aligned} \epsilon^{(per)} &= \epsilon + \delta\epsilon \\ &= \frac{2m}{l^2} - \frac{\delta T_{tt}}{\gamma_{tt} + \delta\gamma_{tt}} \\ \Rightarrow \kappa^2\epsilon^{(per)} &= \frac{2m}{l^2} - \frac{3m}{l^2}(R - i\omega J_0)e^{-i\omega t}P_L(\cos\theta) \end{aligned} \tag{3.86}$$

where R is the function of ω [51]

$$R = -\frac{J_1}{l^2} - (i\omega + \frac{6m}{l^2(L-1)(L+2)})J_0. \tag{3.87}$$

In the large m limit, the growth of the first term is faster than the other terms which are part of $\delta\epsilon$. Therefore, in this limit

$$\epsilon + \delta\epsilon \to \epsilon = \frac{2m}{\kappa^2 l^2} \tag{3.88}$$

We can also find the pressure of perturbed fluid from equation (3.76). The matrix form of this equation is

$$\begin{bmatrix} \frac{1}{\gamma_{tt}+\delta\gamma_{tt}}+(u^t)^2 & u^t\delta u^\theta & 0 \\ u^t\delta u^\theta & \frac{1}{\gamma_{\theta\theta}+\delta\gamma_{\theta\theta}} & 0 \\ 0 & 0 & \frac{1}{\gamma_{\varphi\varphi}+\delta\gamma_{\varphi\varphi}}+(u^\varphi)^2 \end{bmatrix} \begin{bmatrix} T_{tt}+\delta T_{tt} & \delta T_{t\theta} & 0 \\ \delta T_{\theta t} & T_{\theta\theta}+\delta T_{\theta\theta} & 0 \\ 0 & 0 & T_{\varphi\varphi}+\delta T_{\varphi\varphi} \end{bmatrix}$$
$$= (P+\delta P)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.89}$$

By solving the above set of equations, we have

$$\begin{aligned} P+\delta P &= \frac{T_{\theta\theta}+\delta T_{\theta\theta}}{\gamma_{\theta\theta}+\delta\gamma_{\theta\theta}} \\ &= \frac{\frac{1}{\kappa^2}\left(\frac{\sqrt{f(r)}}{r}+\frac{f'(r)}{2\sqrt{f(r)}}-\frac{2}{l}\right)+\frac{1}{r^2}\delta T_{\theta\theta}}{1+K(r)P_L(\cos\theta)e^{-i\omega t}} \end{aligned} \tag{3.90}$$

where $\delta T_{\theta\theta}$ is taken from eq.(1.59) and $K(r)$ is defined in eq.(A.10). Also in this case, in the large m limit, perturbed P is equal to the one before perturbation.

So far, we saw that the perturbed fluid in even case has a non-zero velocity in θ direction. This component of velocity change periodically in time because of its dependence on $\cos\theta$ as the real part of $e^{-i\omega t}$ in the definition of δu^θ.

Chapter 4

Discussion

In this thesis I started with a brief introduction to Anti de-Sitter space and AdS-Schwarzschild black holes which are the solutions of the Einstein equation with a negative cosmological constant. Then in the second chapter, the famous notion of gauge/gravity duality was introduced and the features of the correspondence between Anti-deSitter space and conformal field theory on the boundary of the space were discussed. In this chapter, a few aspects of the string theory which were required to understand the correspondence were also reviewed.

Chapter three was basically the realisation of the main goals of this thesis. Our first goal was considering the large m limit of gravitational perturbations of AdS Schwarzschild black holes in four space-time dimensions and studying their relation with the perturbations of induced three dimensional stress-energy tensor on the boundary of space. To this aim, following the method of Regge and Wheeler [44], first we found the perturbed form of the metric of AdS space which can be canonically split into two classes of axial and polar perturbations. Then, by finding the covariant derivative of the stress tensor variation and deriving the large mass limit of Ricci tensor variation, we checked if they satisfy the equation

$$(\delta T_{\mu\nu})^{;\nu} = \delta G_{\mu\nu} n^{\nu}, \tag{4.1}$$

which they did. We expect this relation to be true since the covariant derivative of the stress tensor variation is actually our fluid dynamics equation.

Further, as our second goal, we intended to understand that how the large m limit of stress-energy tensor

$$T^{\mu\nu} = \epsilon u^{\mu} u^{\nu} + P P^{\mu\nu} - \eta \sigma^{\mu\nu}. \tag{4.2}$$

describes a fluid, and its variation results in fluid dynamics equations on the boundary of AdS_4 spacetime and in particular why in small m limit, the stress-energy tensor does not describe a fluid. In this version of the thesis, part of these goals are accomplished and the remaining parts are classified as future works.

So far, we considered the general form of the stress-energy tensor of a non-perfect fluid and found the velocity u^{μ}, energy density ϵ and pressure P of the perturbed fluid for both cases of odd and even perturbations. Then we used these results to predict the behavior of perturbed fluid as it progressed in time. At this point we need to study the behavior of viscosity η of the perturbed fluid in the limit of $m >> l$ to have all the information needed to relate the perturbation of fluid on the boundary of AdS_4 spacetime to the perturbation of black holes in the bulk of spacetime. This is the part which is not included in this version of the thesis and can be referred to as a future work.

Appendix A

Ricci tensor variation calculations

A.1 $\delta R_{r\varphi}$ as a prototype example

$$
\begin{aligned}
\delta R_{r\varphi} &= \frac{1}{2}\Big[g^{\alpha\sigma}(-h_{\varphi\sigma;r;\alpha} + h_{r\varphi;\sigma;\alpha} + h_{\alpha\sigma;r;\varphi} - h_{r\alpha;\alpha;\sigma})\Big] \\
&= \frac{1}{2}\Big[g^{tt}(-h_{\varphi t;r;t} + h_{r\varphi;t;t} + h_{tt;r;\varphi} - h_{rt;\varphi;t})\Big] \\
&+ \frac{1}{2}\Big[g^{rr}(-h_{\varphi r;r;r} + h_{r\varphi;r;r} + h_{rr;r;\varphi} - h_{rr;\varphi;r})\Big] \\
&+ \frac{1}{2}\Big[g^{\theta\theta}(-h_{\varphi\theta;r;\theta} + h_{r\varphi;\theta;\theta} + h_{\theta\theta;r;\varphi} - h_{r\theta;\varphi;\theta})\Big] \\
&+ \frac{1}{2}\Big[g^{\varphi\varphi}(-h_{\varphi\varphi;r;\varphi} + h_{r\varphi;\varphi;\varphi} + h_{\varphi\varphi;r;\varphi} - h_{r\varphi;\varphi;\varphi})\Big]
\end{aligned}
$$

By using the following three equations[1],

$$\begin{aligned} h_{\varphi t;\theta} &= h_{\varphi t,\theta} - \Gamma^{\sigma}_{\theta\varphi} h_{\sigma t} - \Gamma^{\delta}_{\theta t} h_{\varphi\delta} \\ h_{\varphi t,\theta;r} &= h_{\varphi t,\theta,r} - \Gamma^{\sigma}_{r\varphi} h_{\sigma t,\theta} - \Gamma^{\delta}_{rt} h_{\varphi\delta,\theta} - \Gamma^{\beta}_{r\theta} h_{\varphi t,\beta} \\ \nabla_{\varphi}\Gamma^{r}_{\theta\theta} &= \Gamma^{r}_{\theta\theta,\varphi} - \Gamma^{m}_{\varphi\theta}\Gamma^{r}_{m\theta} - \Gamma^{n}_{\varphi\theta}\Gamma^{r}_{\theta n} + \Gamma^{r}_{\varphi s}\Gamma^{s}_{\theta\theta} \end{aligned} \tag{A.1}$$

the calculations can be continued

$$\begin{aligned} &= \frac{1}{2}g^{tt}\Big[-(h_{\varphi t,r} - \Gamma^{\sigma}_{r\varphi}h_{\sigma t} - \Gamma^{\delta}_{rt}h_{\varphi\delta})_{;t} \\ &+ (h_{r\varphi,t} - \Gamma^{\sigma}_{tr}h_{\sigma\varphi} - \Gamma^{\delta}_{t\varphi}h_{r\delta})_{;t} \\ &+ (h_{tt,r} - \Gamma^{\sigma}_{rt}h_{\sigma t} - \Gamma^{\delta}_{rt}h_{t\delta})_{;\varphi} \\ &- (h_{rt,\varphi} - \Gamma^{\sigma}_{\varphi r}h_{\sigma t} - \Gamma^{\delta}_{\varphi t}h_{r\delta})_{;t}\Big] \\ &+ \frac{1}{2}g^{rr}\Big[-(h_{rr,r} - \Gamma^{\sigma}_{rr}h_{\sigma r} - \Gamma^{\delta}_{rr}h_{r\delta})_{;\varphi} \\ &- (h_{rr,\varphi} - \Gamma^{\sigma}_{\varphi r}h_{\sigma r} - \Gamma^{\delta}_{\varphi r}h_{r\delta})_{;r}\Big] \\ &+ \frac{1}{2}g^{\theta\theta}\Big[-(h_{\varphi\theta,r} - \Gamma^{\sigma}_{r\varphi}h_{\sigma\theta} - \Gamma^{\delta}_{r\theta}h_{\varphi\delta})_{;\theta} \\ &+ (h_{r\varphi,\theta} - \Gamma^{\sigma}_{\theta r}h_{\sigma\varphi} - \Gamma^{\delta}_{\theta\varphi}h_{r\delta})_{;\theta} \\ &+ (h_{\theta\theta,r} - \Gamma^{\sigma}_{r\theta}h_{\sigma\theta} - \Gamma^{\delta}_{r\theta}h_{\theta\delta})_{;\varphi} \\ &- (h_{r\theta,\varphi} - \Gamma^{\sigma}_{\varphi r}h_{\sigma\theta} - \Gamma^{\delta}_{\varphi\theta}h_{r\delta})_{;\theta}\Big] \end{aligned}$$

[1] $\sigma, \delta, \beta, m, n, s$ are arbitrary dummy indices.

$$
\begin{aligned}
&= \frac{1}{2}g^{tt}\Bigg[-(h_{\varphi t,r;t} - \nabla_t\Gamma^{\sigma}_{r\varphi}h_{\sigma t} - \nabla_t h_{\sigma t}\Gamma^{\sigma}_{r\varphi} - \nabla_t\Gamma^{\delta}_{rt}h_{\varphi\delta} - \Gamma^{\delta}_{rt}\nabla_t h_{\varphi\delta}) \\
&+ (h_{r\varphi,t;t} - \nabla_t\Gamma^{\sigma}_{tr}h_{\sigma\varphi} - \nabla_t h_{\sigma\varphi}\Gamma^{\sigma}_{tr} - \nabla_t\Gamma^{\delta}_{t\varphi}h_{r\delta} - \Gamma^{\delta}_{t\varphi}\nabla_t h_{\varphi\delta}) \\
&+ (h_{tt,r;\varphi} - \nabla_\varphi\Gamma^{\sigma}_{rt}h_{\sigma t} - \nabla_\varphi h_{\sigma t}\Gamma^{\sigma}_{rt} - \nabla_\varphi\Gamma^{\delta}_{rt}h_{t\delta} - \Gamma^{\delta}_{rt}\nabla_\varphi h_{t\delta}) \\
&- (h_{rt,\varphi;t} - \nabla_t\Gamma^{\sigma}_{\varphi r}h_{\sigma t} - \nabla_t h_{\sigma t}\Gamma^{\sigma}_{\varphi r} - \nabla_t\Gamma^{\delta}_{\varphi t}h_{r\varphi} - \Gamma^{\delta}_{\varphi t}\nabla_t h_{r\varphi})\Bigg] \\
&+ \frac{1}{2}g^{rr}\Bigg[(h_{\varphi t,r;t} - \nabla_t\Gamma^{\sigma}_{r\varphi}h_{\sigma t} - \nabla_t h_{\sigma t}\Gamma^{\sigma}_{r\varphi} - \nabla_t\Gamma^{\delta}_{rt}h_{\varphi\delta} - \Gamma^{\delta}_{rt}\nabla_t h_{\varphi\delta}) \\
&- (h_{rr,\varphi;r} - \nabla_r\Gamma^{\sigma}_{\varphi r}h_{\sigma r} - \nabla_r h_{\sigma r}\Gamma^{\sigma}_{\varphi r} - \nabla_r\Gamma^{\delta}_{\varphi r}h_{r\delta} - \Gamma^{\delta}_{\varphi r}\nabla_r h_{r\delta})\Bigg] \\
&+ \frac{1}{2}g^{\theta\theta}\Bigg[-(h_{\varphi\theta,r;\theta} - \nabla_\theta\Gamma^{\sigma}_{r\varphi}h_{\sigma\theta} - \nabla_\theta h_{\sigma\theta}\Gamma^{\sigma}_{r\varphi} - \nabla_\theta\Gamma^{\delta}_{r\theta}h_{\varphi\delta} - \Gamma^{\delta}_{r\theta}\nabla_\theta h_{\varphi\delta}) \\
&+ (h_{r\varphi,\theta;\theta} - \nabla_\theta\Gamma^{\sigma}_{\theta r}h_{\sigma\varphi} - \nabla_\theta h_{\sigma\varphi}\Gamma^{\sigma}_{\theta r} - \nabla_\theta\Gamma^{\delta}_{\theta\varphi}h_{r\delta} - \Gamma^{\delta}_{\theta\varphi}\nabla_\theta h_{r\delta}) \\
&+ (h_{\theta\theta,r;\varphi} - \nabla_\varphi\Gamma^{\sigma}_{r\theta}h_{\sigma\theta} - \nabla_\varphi h_{\sigma\theta}\Gamma^{\sigma}_{r\theta} - \nabla_\varphi\Gamma^{\delta}_{r\theta}h_{\theta\delta} - \Gamma^{\delta}_{\theta}\nabla_\varphi h_{\theta\delta}) \\
&- (h_{r\theta,\varphi;\theta} - \nabla_\theta\Gamma^{\sigma}_{\varphi r}h_{\sigma\theta} - \nabla_\theta h_{\sigma\theta}\Gamma^{\sigma}_{\varphi r} - \nabla_\theta\Gamma^{\delta}_{\varphi\theta}h_{r\delta} - \Gamma^{\delta}_{\varphi\theta}\nabla_\theta h_{r\delta})\Bigg]
\end{aligned}
\tag{A.2}
$$

Following the Einstein's convention, repeated index implies a summation over all possible values of the index. After summation over (t, r, θ, φ) as our index values and cancelling terms, we have

$$
\begin{aligned}
&= \frac{1}{2}g^{tt}h_{\varphi t,r,t} - g^{tt}\partial_t h_{\varphi t}\Gamma^{\varphi}_{r\varphi} + g^{tt}\Gamma^{\varphi}_{r\varphi}\Gamma^{r}_{tt}h_{\varphi r} \\
&- \frac{1}{2}g^{tt}h_{r\varphi,t,t} + 2g^{rr}h_{\varphi r}\Gamma^{\varphi}_{\varphi r}\Gamma^{\varphi}_{\varphi r} - g^{rr}h_{\varphi r}\Gamma^{\varphi}_{\varphi r}\Gamma^{r}_{rr} \\
&+ g^{rr}h_{\varphi r}\,\partial_r\Gamma^{\varphi}_{\varphi r} - g^{rr}\Gamma^{\varphi}_{r\varphi}\Gamma^{\varphi}_{r\varphi}h_{r\varphi} - \frac{1}{r^2}\Gamma^{\varphi}_{r\varphi}\Gamma^{r}_{\theta\theta}h_{\varphi r} \\
&- \frac{1}{2r^2}\Gamma^{r}_{\theta\theta}\Gamma^{r}_{rr}h_{\varphi r} + \frac{1}{2r^2}h_{r\varphi,\theta,\theta} - \frac{1}{2r^2}\Gamma^{\varphi}_{\theta\varphi}h_{r\varphi,\theta}
\end{aligned}
$$

By using the following equalities

$$\left[\frac{1}{sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]=-L(L+1)Y_L^M(\theta,\varphi)$$

$$\Rightarrow\frac{\partial^2}{\partial\theta^2}(\sin\theta\frac{\partial}{\partial\theta}P_L(\cos\theta))=-L(L+1)[\sin\theta\frac{\partial}{\partial\theta}P_L(\cos\theta)]$$
$$=-L(L+1)\frac{\partial}{\partial\theta}(\sin\theta P_L(\cos\theta)) \quad \text{(A.3)}$$

and insetring the AdS metric components and Christoffel symbols (see appendix A.3), the final result will be

$$\begin{aligned}\delta R_{r\varphi} &= \left[-\frac{1}{2}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)^{-1}i\omega h_0'+\frac{1}{r}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)^{-1}i\omega h_0\right.\\ &+\frac{1}{2}\left(1-\frac{2m}{r}+\frac{r^2}{l^2}\right)^{-1}\omega^2h_1-\frac{1}{2r^2}L(L+1)h_1\\ &+\left.\left(\frac{1}{r^2}+\frac{3}{l^3}\right)\right]\sin\theta\ \partial_\theta P_L(\cos\theta)\ e^{-i\omega t}\end{aligned} \quad \text{(A.4)}$$

For taking a limit $m >> l$, we choose $r = \rho\ r_{EH}$; $\rho >> 1$ where EH stands for Event Horizon, and $l = \mu m$; $\mu << 1$. By applying these limits, $\delta R_{r\varphi}$ will take the following form

$$\begin{aligned}\delta R_{r\varphi} &= \left[-\frac{1}{2}\left(\frac{r^2}{l^2}\right)^{-1}i\omega h_0'+\left(\frac{r^3}{l^2}\right)^{-1}i\omega h_0+\frac{1}{2}\left(\frac{r^2}{l^2}\right)^{-1}\omega^2h_1\right.\\ &-\left.\frac{1}{2r^2}L(L+1)h_1+\frac{3}{l^2}h_1\right]\sin\theta\ \partial_\theta P_L(\cos\theta)\ e^{-i\omega t}\end{aligned} \quad \text{(A.5)}$$

A.2 Metric functions

Following Regge and Wheeler [44], the linear gravitational perturbations $\delta R_{\mu\nu} = \Lambda\delta g_{\mu\nu}$ about the Schwarzschild background, for axial perturbations, yield a coupled system of first order differential equations for the unknown functions $h_0(r)$ and $h_1(r)$. This system consist of two equations of $(\theta\varphi)-$ and $(r\varphi)-$ components. The asymptotic expansion of the metric functions $h_0(r)$ and $h_1(r)$ near spatial infinity are given in all generality by [51]

$$\begin{aligned} h_0(r) &= \left(\alpha_0 r^2 + \beta_0 r + \gamma_0 + \frac{\delta_0}{r} + \ldots\right) e^{-i\omega r_*}, \\ h_1(r) &= \left(\frac{\alpha_1}{r} + \beta_1 r^2 + \ldots\right) e^{-i\omega r_*}, \end{aligned} \tag{A.6}$$

where r_* is a tortoise coordinate and its relation to r is

$$dr_* = \frac{dr}{f(r)} \tag{A.7}$$

For AdS_4 black holes, r_* ranges from $-\infty$ up to the constant

$$r_* = \frac{r_h}{4(r_h - 3m)}\left(r_h \log\frac{(2r + r_h)^2 + a^2}{4(r - r_h)^2} + 2a\frac{r_h - 6m}{r_h + 6m}[\arctan\frac{2r + r_h}{a} - \frac{\pi}{2}]\right) \tag{A.8}$$

The coefficients of the metric function $h_0(r)$ and $h_1(r)$ are

$$\begin{aligned} \alpha_0 &= -\frac{i\Lambda}{3\omega}I_0,\ \beta_0 = I_0,\ \gamma_0 = -i\frac{(L-1)(L+2)}{2\omega}I_0, \\ \delta_0 &= \frac{L(L+1)}{2\Lambda}I_0 - \frac{i}{3\omega}\left((L-1)(L+1) + \frac{3\omega^2}{\Lambda}\right)I_1, \\ \alpha_1 &= -\frac{3}{\Lambda}I_0,\ \beta_1 = -\frac{3}{\Lambda}I_1. \end{aligned} \tag{A.9}$$

where coefficients I_0 and I_1 depend on ω [51].

These coefficients are determined up to the order relevant for the computation of the energy-momentum tensor for axial and polar perturbations of AdS_4 black holes.

For polar perturbations the $(tr)-$ $(r\theta)-$ and $(t\theta)-$components of the perturbation form a coupled system of first order differential equations for the three unknown functions $H_0(r), H_1(r)$ and $K(r)$. The other components of the perturbation either yield second order equations or else $\delta R_{\mu\nu}$ vanishes identically.

The asymptotic expansion of the metric functions $H_0(r), H_1(r)$ and $K(r)$ take the following form at spatial infinity,

$$\begin{aligned} H_0(r) &= \frac{3m}{(L-1)(L+2)}\left(\frac{A_0}{r}+\frac{B_0}{r^2}+\frac{C_0}{r^3}+...\right)e^{-i\omega r_*}, \\ H_1(r) &= -\frac{3i\omega}{\Lambda}\left(\frac{A_1}{r}+\frac{B_1}{r^2}+\frac{C_1}{r^3}+...\right)e^{-i\omega r_*}, \\ K(r) &= \left(R+\frac{A}{r}+\frac{B}{r^2}+\frac{C}{r^3}+...\right)e^{-i\omega r_*}. \end{aligned} \tag{A.10}$$

The coefficients in $H_0(r)$ are

$$
\begin{aligned}
A_0 &= \left(2i(\omega_s+\omega) - \frac{4m\Lambda}{(L-1)(L+2)} + \frac{\omega^2}{m\Lambda}(L-1)(L+2)\right) J_0 - \frac{2\Lambda}{3} J_1, \\
B_0 &= (L-1)(L+1)\left(1 + \frac{i\omega}{m\Lambda}[1 - \frac{12m^2\Lambda}{(L-1)^2(L+2)^2}]\right) J_0 \\
&+ \frac{(L-1)(L+2)}{6m}\left((L-1)(L+2) + \frac{6\omega^2}{\Lambda} - \frac{12i\omega m}{(L-1)(L+2)}\right) J_1, \\
C_0 &= \left(-\frac{(L-1)(L+2)}{4m\Lambda}(L(L+1)(L(L+1)-4) + \frac{6\omega^2}{\Lambda}(L-1)(L+2))\right. \\
&+ \left.\frac{6m}{(L-1)(L+2)}(L(L+1) - 4 + \frac{6\omega^2}{\Lambda}) + \frac{6i\omega}{\Lambda}\right) J_0 \\
&+ \left(L(L+1) - 4 + \frac{6\omega^2}{\Lambda} + \frac{i\omega}{2m\Lambda}(L-1)(L+2)\right. \\
&\times \left.[(L-1)(L+2) + \frac{6\omega^2}{\Lambda}]\right) J_1 \qquad \text{(A.11)}
\end{aligned}
$$

where coefficients J_0 and J_1 depend on ω.

Likewise, the coefficients in the asymptotic expansion of $H_1(r)$ are given by

$$
\begin{aligned}
A_1 &= \left(i\omega - \frac{2m\Lambda}{(L-1)(L+2)}\right) J_0 - \frac{\Lambda}{3} J_1, \\
B_1 &= \left(L(L+1) - 1 - \frac{12m^2\Lambda}{(L-1)^2(L+2)^2}\right) J_0 - \left(i\omega + \frac{2m\Lambda}{(L-1)(L+2)}\right) J_1, \\
C_1 &= 3\left(m\frac{L(L+1)-4}{(L-1)(L+2)} + \frac{i\omega}{2\Lambda}[L(L+1) + 2 - \frac{24m^2\Lambda}{(L-1)^2(L+2)^2}]\right) J_0 \\
&+ \frac{1}{2}\left(L(L+1) - 4 - \frac{6i\omega}{\Lambda}[i\omega + \frac{2m\Lambda}{(L-1)(L+2)}]\right) J_1. \qquad \text{(A.12)}
\end{aligned}
$$

Finally, the coefficients in the asymptotic expansion of $K(r)$ are

$$
\begin{aligned}
R &= -A,\ B = \frac{3i\omega}{\Lambda}A, \\
A &= -\frac{1}{2}\left(L(L+1) - \frac{24m^2\Lambda}{(L-1)^2(L+2)^2}\right)J_0 + \left(i\omega + \frac{2m\Lambda}{(L-1)(L+1)}\right)J_1, \\
C &= -\frac{1}{4\Lambda}\left(L(L+1)[L(L+1) - \frac{12\omega^2}{\Lambda} - \frac{24m^2\Lambda}{(L-1)^2(L+2)^2}]\right. \\
&+ \left. 12i\omega m[1 - \frac{24i\omega m}{(L-1)^2(L+2)^2}]\right)J_0 \\
&+ \left(m\frac{L(L+1)}{(L-1)(L+2)} + \frac{i\omega}{\Lambda}[1 - 6\omega^2\Lambda + \frac{12i\omega m}{(L-1)(L+2)}]\right)J_1. \qquad (A.13)
\end{aligned}
$$

A.3 Christoffel symbols for AdS-Schwarzschild metric

By using Mathematica, we can easily find the Christoffel symbols for AdS-Schwarzschild black holes. The non-zero components are as follows:

$$\begin{aligned}
\Gamma^r_{tt} &= m\left(-\frac{1}{l^2}+r^{-2}\right)-\frac{2m^2}{r^3}+\frac{r(l^2+r^2)}{l^4} \\
\Gamma^t_{tr} &= \frac{ml^2+r^3}{r(-2ml^2+l^2r+r^3)} \\
\Gamma^r_{rr} &= -\frac{ml^2+r^3}{r(-2ml^2+l^2r+r^3)} \\
\Gamma^\theta_{r\theta} &= \frac{1}{r} \\
\Gamma^\varphi_{r\varphi} &= \frac{1}{r} \\
\Gamma^r_{\theta\theta} &= 2m-r-\frac{r^3}{l^2} \\
\Gamma^\varphi_{\theta\varphi} &= \cot\theta \\
\Gamma^r_{\varphi\varphi} &= \frac{(2ml^2-r(l^2+r^2))\sin^2\theta}{l^2} \\
\Gamma^\theta_{\varphi\varphi} &= -\cos\theta\sin\theta
\end{aligned} \tag{A.14}$$

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