SEASONAL FLOW FORECASTING OF NEWFOUNDLAND RIVERS

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by

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To My Parents

ABSTRACT

The general purpose of forecasting is to provide the best estimates of what will happen at specified points in time in the future. In hydrology, for example, forecasts of riverflows are often used for operational planning of reservoir and flood control systems. Since, even modest improvements in the operation of a large reservoir system can result in multi-million dollar savings per year, choosing a model which produces reliable and accurate forecasts is therefore essential to the efficient operation of the system. In this study, monthly and quarterly discharge data of Newfoundland rivers were used to forecast future flows using four different statistical approaches: conventional Box and Jenkins's autoregressive integrated moving average (ARIMA), exponential smoothing, periodic autoregressive (PAR), and Harvey's new structural time series (NSM). Each monthly riverflow data was divided into three short term series to study forecasting accuracy. Ten quarterly series were used to predict flows for three forecasting scenarios and thirty monthly series were considered for 3 month, 6 month, 9 month and 12 month ahead forecast horizons. Forecast performance was assessed using the mean absolute percentage error (MAPE) criterion.

Based on the MAPE criterion, it is concluded that forecasts using the NSM approach for short term monthly riverflow data in general are better than ARIMA, exponential smoothing and PAR approaches. For quarterly data, forecasts using the exponential smoothing approach in general are better than NSM, ARIMA and PAR approaches.

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GLOSSARY

AIC (Akaike Information Criterion) The AIC is the measure that balances model complexity and goodness-of-fit to the historical data. The minimization of AIC measure, determines the order of the models.

Autocorrelation Coefficient The autocorrelation coefficient measures the extent to which the current value of the series depends on past values.

Autoregressive (AR) Process A stochastic process in which current value depends on lagged previous terms and a disturbance term is called an autoregressive process.

BIC (Bayes Information Criterion) The BIC, like AIC, is a figure of merit used in the selection of model order. But compared to AIC, it penalizes model complexity more.

Deseasonalization The process of removing seasonal effects from a series by applying a transformation is called deseasonalization.

Deterministic A deterministic process is a process that can be predicted with certainty from its past.

Differencing Differencing is the transformation of a time series involving the replacement of every value of the series by its difference from the previous value.

Forecast Horizon The number of periods that are forecasted.

Heteroscedasticity The process in which the variance and covariance of the errors is changing over time.

Homoscedasticity A homoscedastic process is one in which the variances and covariances are unchanging over time.

Hyperparameters The hyperparameters are the variance parameters which determine how rapidly the various unobserved components, such as the trend and seasonal, evolve over time.

Integration A time series is integrated with degree *d* if *d* is the minimum degree of differencing that renders the time series stationary.

IQR (InterQuartile Range) The IQR measures the range of the central 50% of the data, and is not influenced by the 25% at either end.

Lag The difference in time units of a series value and a previous series value.

Lead The difference in time units of a series value and a future series value.

MAPE (Mean Absolute Percentage Error) MAPE is a measure of the accuracy of forecasts of a time series.

MSE (Mean Square Error) A statistic that is used as an indication of model fit. It is calculated by taking the square root of the average of squared residual errors.

Model Complexity Model complexity is measured by the number of parameters, or effective number of parameters that must be fitted to the data.

Moving Average (MA) Process The process in which future data points are expressed as linear combinations of past errors.

Ockham's razor Ockham's razor or the ***principle of parsimony***, is defined as, "In a choice among competing hypotheses, other things being equal, the simplest is preferred."

Residual The difference between a predicted value and a true value is called residual.

Robust A robust statistical method is a method which is insensitive to moderate deviations from underlying statistical assumptions.

Seasonality Periodic pattern of behaviour of the time series is called seasonality. For example monthly data exhibits a seasonality of 12 months.

Stationarity A stationary time series exhibits similar statistical behaviour in terms of, say mean, standard deviation, etc., at each point in time.

Stochastic A process is said to be stochastic when its future cannot be predicted exactly from its past, i.e., a new uncertainty enters at each point in time.

Univariate A univariate method is method involving only one variable at a time.

White Noise (WN) A time series that is identically, independently distributed normally (*iid*), with zero mean. The autocorrelation function is zero for all lags except at lag zero.

Chapter 1

INTRODUCTION

Webster's dictionary defines *forecasting* as an activity "to calculate or predict some future event or condition, usually as a result of rational study or analysis of pertinent data."

In the design, planning and operation of water resources systems, one often needs good estimates of the future behaviour of key hydrological variables. For example, when operating a reservoir to serve multiple purposes such as hydroelectrical power generation, water supply, recreational uses, etc., one may require forecasts of projected flows for upcoming time periods so that mitigation measures can be taken in case of shortfalls.

The objective of forecasting is thus to predict future conditions with minimal forecast error. Forecast methods may be broadly classified into *qualitative* and *quantitative* techniques. *Qualitative* forecasts are intuitive, largely educated guesses that may or may not depend on past data. Forecasts that are based on mathematical or statistical models are called *quantitative*. In general, a quantitative forecast system consists of two major components, as illustrated in Fig. 1.1. At the first stage, the *model-building phase*, a forecasting model is constructed from pertinent data and available theory. At the second stage, the *forecasting phase*, the final model is used to obtain the forecasts. The stability of the forecast model can be assessed by checking the forecasts against the new observations. Among many other forecast criteria, the choice of the forecast model or technique depends on (1) degree of accuracy required, (2) the forecast horizon, (3) acceptable cost of producing the forecasts, (4) degree of complexity required, and (5) data available (Abraham and Ledolter, 1983).

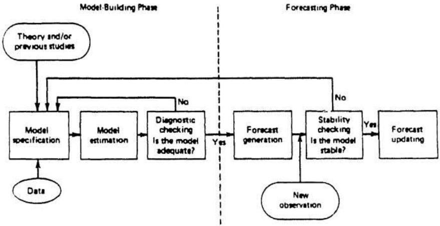

Figure 1.1 **Conceptual Framework of a Forecasting System (from Abraham and Ledolter, 1983)**

1.1 Forecasting Approaches

Time series analysis belongs to a major quantitative statistical technique used in the extraction of information on hydrologic and water resources random variables from observed data to provide forecasts of future conditions, for example riverflows, rainfall, etc. Empirical studies have shown that there is no single best forecasting method applicable to all situations (Goodrich 1989).

To determine the best forecasting model, it is necessary to critically examine the available data. For the riverflow data the three fundamental characteristics or components of the series are tendency, seasonality and stochasticity (shown in Fig. 1.2). *Tendency* is the trend in a series, due to inconsistency or nonhomogeneity of available data; *Seasonality* is the deterministic cyclic movement of the time series caused by cycles of nature and *Stochasticity* is the outcome or effect of many casual factors of natural random processes. The physical causes and sources of these three basic components usually affect the selection of best mathematical method to be used in time series analysis.

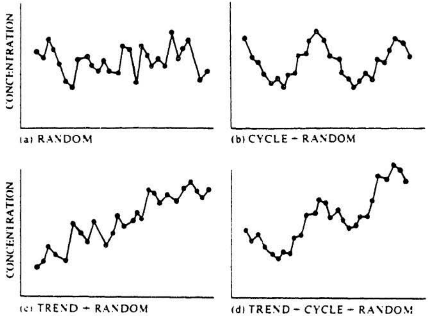

Figure 1.2 Components of Riverflow Time Series (from Gilbert, 1987)

Quantitative forecasting methods are further classified into univariate methods and multivariate methods. Univariate methods are based on fitting a model to the historical data of a given time series and extrapolating to obtain forecasts. There are many univariate methods available which include among others, exponential smoothing, the Box and Jenkins method, and various structural approaches. The most commonly used approach, for riverflow forecasts, is the Box and Jenkins (1976) multiplicative autoregressive integrated moving average (ARIMA) class of models. Another approach is the periodic autoregressive (PAR) modelling approach which is an extension of the nonseasonal autoregressive (AR) models of the Box and Jenkins approach (Hipel and McLeod, 1994). The Box and Jenkins approach while it has impressive statistical features has no direct interpretation as it is not consistent with the physical properties of the series. The classical structural time series analysis of Yevjevich (1972) is consistent with the physical characteristics such as trend, seasonality, etc., of the series but employs a different statistical approach. In his approach the components of physical characteristics are seen as deterministic functions of time and not stochastic. The new structural time series (NSM) method of Harvey (1989) differs from the classical approach in statistical formulation. In this approach the components of physical characteristics such as trend, seasonality, etc., are stochastic and they represent various unobserved variables of the state of the system. In the exponential smoothing (EXS) approach of Brown and Holt (1950), the model components and parameters have an intuitive meaning as the series is assumed to be modeled by one, two, or three components that represent, respectively, the *level*, *trend* and *seasonality* of the series.

The multivariate methods become relevant when the design, planning and operation of a water resource system involves several hydrologic variables. The methods which describe the joint behaviour of several time series which may have mutually dependent relationships are called multivariate methods. The examples of time series that can be analyzed and modelled by multivariate methods are the series of annual or monthly precipitation at various gauging stations, the series of annual or monthly streamflows at various points of a river. The multivariate methods are also used to analyze a riverflow time series by using, rainfall time series, temperature-time sequence and or a riverflow time series in the vicinity, as explanatory variables. Various methods have been proposed to analyze multivariate series, for example, Fiering (1964), Matalas (1967), Matalas and Wallis (1971), Mejia (1971), O'Connell (1974) and others. In this study only univariate methods are considered because there are no explanatory variables available and the streamflows are measured at a single hydrometric station over a long period of time. Hence the only variable for monthly or quarterly riverflow series is time and therefore multivariate models are not discussed herein.

1.2 Objective of Thesis

The primary objective of this thesis is to determine the best statistical forecasting method, for Newfoundland rivers. Different forecast horizons and scenarios are used to choose the best model. For the monthly riverflow data, the forecast horizons of 3 month, 6 month, 9 month and 12 month ahead periods will be used in the comparison of the forecasting methods. In addition for the quarterly series, the accuracy of forecasts for critical low flow and high flow periods by using different forecast scenarios which differ in the period of record used will be assessed.

In this forecasting study, the above mentioned four approaches namely ARIMA, PAR, NSM and EXS are used to analyze, model and forecast monthly and quarterly flows of Newfoundland rivers. The models are fitted to the first portion of time series and then used to forecast remaining observations. The forecasting accuracy is measured using the mean absolute percentage error (MAPE) criterion.

1.3 Outline of Thesis

This thesis is divided into six chapters. Chapter 1, explains the importance of good forecasts in hydrology and the methods of forecasting to be used. Chapter 2, examines the mathematical formulation of the methods of forecasting and compares them in terms of their assumptions, limitations and advantages. Chapter 3, details the basic characteristics of Newfoundland rivers used in the forecasting study. Chapter 4, provides information about the application of various models to the riverflow time series. Chapter 5, compares the forecasts generated and Chapter 6, discusses the results obtained, recommends a forecasting method for Newfoundland rivers and summarizes the study.

Chapter 2

FORMULATION AND COMPARISON OF FORECASTING METHODS

The mathematical formulation and comparison of four forecasting methods mentioned in the previous Chapter will be explained in detail in the following sections. In the first section, the mathematical representation of each method will be presented along with the forecasting equations to be employed to predict future flows. The methods of parameter estimation are also discussed. The second section compares the four forecasting approaches in terms of their advantages and disadvantages.

2.1 Mathematical Formulation

A time series is a set of observations that are arranged chronologically. In order to model a time series accurately, it is important to be aware of the assumptions under which data is recorded, listed and finally modelled. The first and foremost assumption in a riverflow series is that the data under study is evenly spaced at discrete time intervals. The inherent advantage of this assumption is that data can be aggregated to represent a separate time interval. For example, daily riverflows can be averaged to give weekly, monthly, quarterly or yearly flows. Specific methods of forecasting have their own basic assumptions and limitations which are to be kept in mind before choosing a method of forecasting. For all the forecasting methods discussed below, as regard notation; L will be used to denote the lag operator on time t i.e. $Ly_t = y_{t-1}$; y_t, $t = 1,2,\ldots,T$, is a sequence of a seasonal time series with period s. For example, s is 12 for a monthly time series and s is 4 for a quarterly series. A sequence of independent normally distributed random variables, say, y_t, with mean μ and variance σ^2 will be indicated by writing $y_t \sim \text{NID}(\mu, \sigma^2)$.

2.1.1 Box and Jenkins Method

The Box and Jenkins method (Box and Jenkins, 1976) models time series by making strong and explicit distributional assumptions about the underlying data generating process. The method uses a combination of autoregressive (AR), integration (I) and moving average (MA) operations in the general Autoregressive Integrated Moving Average (ARIMA) model to represent the correlational structure of a univariate time series.

The autoregressive and moving average operations can only be applied to a stationary time series. That is, they can only be applied to data which has a constant mean value with time. If a time series is non-stationary it has to be transformed to a stationary series by differencing before the AR and MA operations can be performed. Forecast values have to be transformed back to the original non-stationary state by the

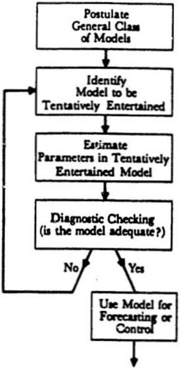

Figure 2.1 Stages in the Iterative Approach to Model Building (from Box and Jenkins, 1976)

integration (I) operation.

A three step procedure of identification, estimation and diagnostic checking was originally proposed by Box and Jenkins (Box and Jenkins, 1976) to select a model from the general class of ARIMA models. This iterative process is depicted in Fig. 2.1. The identification process is for deciding the best ARIMA (p,d,q) model to fit the data. This means identifying the degree of differencing d, the AR order p and the MA order q. The estimation process involves statistically estimating the model parameters. The diagnostic step involves examination of the residuals to ensure that the assumptions of independence, homoscedasticity, and normality are not violated.

The multiplicative ARIMA class of models is the most commonly used approach for the modelling of seasonal riverflow data (Salas et al, 1980). Box and Jenkins (1976) generalized the multiplicative ARIMA $(p,d,q) \times (P,D,Q)_s$ model which consists of a seasonal ARMA (P,Q) fitted to the D-th seasonal difference of the data coupled with an ARMA (p,q) model fitted to the d-th difference of the residuals of the former model. The condensed mathematical representation of the ARIMA model is

$$\Phi(L^s) \varphi(L) (1 - L^s)^D (1 - L)^d y_t = \theta_o + \Theta(L^s) \theta(L) \xi_t \qquad (2.1)$$

where:

ξ_t is a white noise process with mean zero and variance σ^2. The notations used are

$$Ly_t = y_{t-1} ; \qquad L \text{ is backshift / lag operator} \qquad (2.2)$$

The autoregressive, moving average, seasonal autoregressive and seasonal moving average operators, respectively, are represented by

$$
\begin{aligned}
\varphi(L) &= 1 - \varphi_1 L - \ldots - \varphi_p L^p \\
\theta(L) &= 1 + \theta_1 L + \ldots + \theta_q L^q \\
\Phi(L^s) &= 1 - \Phi_1 L^s - \ldots - \Phi_P L^{P_s} \\
\Theta(L) &= 1 + \Theta_1 L^s + \ldots + \Theta_Q L^{Q_s}
\end{aligned}
\tag{2.3}
$$

where:

$\varphi(L)$ and $\theta(L)$ are parameters for nonseasonal AR and MA models respectively.

$\Phi(L^s)$ and $\Theta(L^s)$ denote seasonal polynomials in the lag operator of orders P and Q respectively.

For example, the $(2,0,0) \times (0,1,1)_{12}$ multiplicative autoregressive integrated moving average process $(1 - \varphi_1 L - \varphi_2 L^2)(1 - L^{12}) y_t = (1 - \Theta_1 L^{12})\xi_t$, in its expanded form is written as

$$
y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + y_{t-12} - \varphi_1 y_{t-13} - \varphi_2 y_{t-14} + \xi_t - \Theta_1 \xi_{t-12} \tag{2.4}
$$

where:

φ_1, φ_2 = autoregressive parameters

Θ_1 = seasonal moving average parameter

ξ_t = white noise process

The parameters are estimated by using maximum likelihood estimation procedure. First the sum of squares surface $\Sigma\xi_t^2$ $(\varphi, \theta, \Phi, \Theta)$ for a range of parameter values is evaluated, then its minimum and corresponding parameter values are located. Finally, these maximum-likelihood parameter estimates are used as initial values to obtain the final estimates of the parameters by a nonlinear estimation procedure (Salas, 1980).

The principle hydrologic application of ARIMA models is in forecasting. For example, the $(2,0,0) \times (0,1,1)_{12}$ model in Eqn. (2.4) is forecasted for a lead time of l months by taking the conditional expectation, indicated by a square bracket. The forecasting equation will be

$$\begin{aligned} y_t(l) = [y_{t+l}] &= \varphi_1[y_{t+l-1}] + \varphi_2[y_{t+l-2}] + [y_{t+l-12}] \\ &\quad - \varphi_1[y_{t+l-13}] - \varphi_2[y_{t+l-14}] + [\xi_{t+l}] - \Theta_1[\xi_{t+l-12}] \end{aligned} \tag{2.5}$$

To use the Box and Jenkins method, the data must have a strong correlational behaviour, and there should be sufficient data to permit reasonably accurate estimates of the parameters. The selected Box and Jenkins model which satisfies the diagnostic checks mentioned earlier, will generally fit the historical data well and the parameters estimated describe the data on which they are estimated.

2.1.2 Periodic Autoregressive Models

As emphasized by authors such as Moss and Bryson (1974), seasonal hydrological and other types of time series exhibit an autocorrelation structure which depends on not only the time lag between observations but also on the season of the year. For example, in the northern hemisphere, snowmelt is an important factor in runoff which usually occurs in March or April. Therefore the correlation between observed riverflows during these months is negative whereas at other times of the year it is positive. To model appropriately the foregoing and similar types of time series, periodic models can be employed.

Two popular periodic models for riverflow time series are the PAR (periodic autoregressive) and PARMA (periodic ARMA) models. Because model building procedures are highly developed for use with PAR models (Hipel and McLeod, 1994), this class of periodic models is focussed upon in this study. When fitting a PAR model to a single seasonal series, a separate autoregressive (AR) model is designed for each season of the year. The results of a comprehensive forecasting study (Noakes, McLeod and Hipel, 1985) have suggested that a periodic autoregressive model (PAR), identified by using the partial autocorrelation function, provided the most accurate forecasts. In the present forecasting study PAR/PACF model is therefore used for the seasonal Newfoundland rivers.

The PAR ($p_1,p_2,.......p_s$) model, defined by AR orders of $p_1,p_2,.......p_s$ for each season of the series, is mathematically described by

$$y_t = \varphi^{(m)}(L)\, y_{t-1} + \xi_t, \qquad t = 1,. . . . ,T \tag{2.6}$$

where:

$\xi_t \sim NID(0, \sigma^{2(m)})$.

The seasons are represented as, m (m = 1,2,.....s), and

$$\varphi^{(m)}(L) = 1 - \varphi_1^{(m)}L - . . . . . -\varphi_{p_m}^{(m)}L^{p_m} \tag{2.7}$$

It should be noted that the model parameters for the mth season (i.e., $\varphi_1^{(m)}$, $\varphi_2^{(m)}$.......$\varphi_m^{(m)}$) can be estimated entirely independent of the model of any other season. Also, the estimates of the parameters in different seasons are considered to be statistically independent (Pagano, 1978).

For example, if an AR(p) model is fitted to the first season of a time series then it is represented as

$$y_t = \varphi_1 y_{t-1} + \dots + \varphi_p y_{t-p} + \xi_t \qquad (2.8)$$

where:

p = order of autoregressive model with parameters $\varphi_1, \dots, \varphi_p$.

The techniques for estimating the parameters of a PAR model are Yule Walker estimator and multiple linear regression (Hipel and McLeod, 1994). The forecasts for PAR models are obtained using the minimum mean square error (MMSE) approach. The MMSE forecasts, for PAR models, calculated after year r and season m are determined using

$$y_{r,m+l} = \varphi_1^{(m)} y_{r,m+l-1} + \varphi_2^{(m)} y_{r,m+l-2} + \dots + \varphi_{p_m}^{(m)} y_{r,m+l-p_m} + \xi_{r,m} \qquad (2.9)$$

where:

l = 1,2,......, is the lead time for the forecast.

2.1.3 Structural Approaches

2.1.3.1 Classical Structural Approach

In the classical structural approach (Yevjevich, 1972), components of the time series are deterministic functions of time. The seasonality in the series is inferred statistically and is described mathematically using Fourier series analysis with a limited number of low frequency harmonics and their estimated coefficients. After removing the seasonality from the original series an autoregressive model is fitted. The Minimum Akaike Information Criterion Estimation (MAICE) can be used to select the Fourier components required and to fit the best autoregressive model (Hipel and McLeod, 1994).

The underlying assumption in this approach is that the series becomes stationary after the removal of seasonality.

For a seasonal hydrologic time series $x_{p,\tau}$, where p is the year and τ is the season within the year (i.e., τ = 1,2,......n), the normalization is carried out in terms of standardized series

$$y_{p,\tau} = \frac{x_{p,\tau} - \mu_\tau}{\sigma_\tau} \tag{2.10}$$

where:

μ_τ = seasonal mean

σ_τ = seasonal standard deviation

The parametric or Fourier series representation of $y_{p,\tau}$, denoted by z_τ is given as

$$z_\tau = \alpha_0 + \sum_{k=1}^{m} [A_k \sin(\gamma k \tau) + B_k \cos(\gamma k \tau)] + e_\tau \tag{2.11}$$

where:

α_0 is general mean of $x_{p,\tau}$, m is number of significant harmonics, $\gamma = 2\pi/n$ is the cyclic frequency over a base period, A_k and B_k are harmonic coefficients and k = 1,2,.....,m.

Assuming fundamental period to be equal to the sample length, the fundamental frequency is 1/n. Estimation of harmonic coefficients is achieved by conventional Fourier analysis (Yevjevich, 1972). Since the importance of this approach in the present context is only historical it is not explained in detail here.

2.1.3.2 New Structural Time Series Approach

In the new structural time series (NSM) approach of Harvey (1989) the series is modelled in state space form, with the state of the system representing various unobserved components such as trends and seasonals. Prediction and smoothing can only be carried out once the parameters governing the stochastic movements of the state variables have been estimated. The estimation of these parameters, which are known as hyperparameters, is itself based on the kalman filter. The kalman filter provides the means of updating the state as new observations become available. Predictions are made by extrapolating these components into the future, while the smoothing algorithms give the best estimate of the state at any point within the sample.

The structural model is based on the traditional decomposition into trend, seasonal and irregular components. These components combine additively, i.e.,

$$y_t = \text{Trend} + \text{Seasonal} + \text{Irregular} \tag{2.12}$$

The basic structural model (BSM), in the NSM approach (Harvey, 1989), is formulated as

$$y_t = \mu_t + \gamma_t + \epsilon_t , \qquad t = 1, \ldots, T \tag{2.13}$$

with μ_t a local linear trend, γ_t a local seasonal pattern, and ϵ_t a white noise irregular component. The statistical model of trend has the level (μ_t) and slope (β_t) parameters which change slowly over time according to the random walk process. Thus

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t \\ \beta_t &= \beta_{t-1} + \zeta_t , \quad t = 1, \ldots ,T \end{aligned} \tag{2.14}$$

where:

$\eta_t \sim NID(0,\sigma_\eta^2)$ and $\zeta_t \sim NID(0,\sigma_\zeta^2)$.

The seasonal model in Eqn. (2.13) imposes the constraint that the seasonal effects sum to zero. This suggests a stochastic seasonal model of the form

$$\sum_{j=0}^{s-1} \gamma_{t-j} = \omega_t \tag{2.15}$$

where:

s is the number of seasons and $\omega_t \sim NID(0,\sigma_\omega^2)$.

A model of this kind allows the seasonal pattern to change over time, while imposing the condition that the *expectation* of the sum of seasonal effects over s consecutive time periods should be zero. The model specification is completed by the assumption that the four disturbance terms ϵ_t, η_t, ζ_t and ω_t are independent of each other.

The NSM approach has various models which can be used for modelling a time series. The choice of a model depends on the characteristics of the series under consideration. The combinations of few or all components, i.e., trend, seasonal, cycle and irregular term can be used. The seasonal component can further be defined as trigonometric or dummy seasonal. The individual components have a choice of being deterministic or stochastic depending on whether the variance term of each component has been constrained to zero or not, respectively. To date, the new structural time series approach has only been used for economic time series and its application to riverflow time series has never been reported. The BSM model has been discussed in detail above and the use of the state space model in simplified form appropriate for univariate time series is illustrated below with a very simple trend plus error model.

State Space Models and The Kalman Filter

For the simple trend plus error model given by

$$y_t = \mu_t + \epsilon_t \qquad (2.16)$$

the linear univariate structural model has a state space representation which consists of a *measurement equation* and a *transition equation* of the following forms respectively;

$$\begin{aligned} y_t &= z_t^T \alpha_t + \xi_t \\ \alpha_t &= T_t \alpha_{t-1} + \eta_t \end{aligned} \qquad (2.17)$$

in which α_t is an (m x 1) state vector, z_t is an (m x 1) fixed vector, T_t is a fixed matrix of order (m x m) and ξ_t and η_t are, respectively, a scalar disturbance term and an (m x 1) vector of disturbances which are distributed independently of each other. It is assumed that $\xi_t \sim NID(0,\sigma^2 h_t)$ and $\eta_t \sim NID(0,\sigma^2 Q_t)$ where h_t is a fixed scalar, Q_t is a fixed (m x m) matrix and σ^2 is a scalar. Although T_t, z_t, h_t and Q_t may ultimately depend on a set of unknown parameters, they are, for the purpose of the kalman filter regarded as being fixed and known.

Let a_{t-1} be the *minimum mean square estimator* (MMSE), or 'optimal estimator', of α_{t-1} based on all the information upto and including time t-1, and let $\sigma^2 P_{t-1}$ be the MSE matrix of a_{t-1}, i.e., the covariance matrix of $a_{t-1} - \alpha_{t-1}$, where P_{t-1} denotes the (m x m) covariance matrix of the estimation error. Given a_{t-1} and P_{t-1} at time t-1 the MMSE of α_t is given by

$$a_{t/t-1} = T_t\, a_{t-1}$$
and
$$P_{t/t-1} = T_t\, P_{t-1}\, T_t^T + Q_t \tag{2.18}$$

Once y_t becomes available this estimator can be updated. The appropriate equations are

$$a_t = a_{t/t-1} + P_{t/t-1}z_t(y_t - z_t^T a_{t/t-1})/f_t$$
$$P_t = P_{t/t-1} - P_{t/t-1}z_t z_t^T P_{t/t-1}/f_t \tag{2.19}$$
with
$$f_t = z_t^T P_{t/t-1} z_t + h_t$$

The equations in (2.18) are known as the *prediction equations* whereas those in equations (2.19) are the *updating equations*. Together they make up the kalman filter (Harvey, 1989).

The kalman filter yields the MMSE estimator of the state vector, α_t, given the information available at time t. However, once all the observations are available, a better estimator can normally be obtained by taking account of observations obtained after time t. The techniques for computing such estimators are known as smoothing. There are three basic smoothing algorithms: fixed point, fixed lag and fixed interval. The fixed interval smoother (Harvey, 1989) consists of a set of recursions which start with the final kalman filter estimates, a_T and P_T, and works backwards. The details of the other two are given by Anderson and Moore (1979).

Once the unknown parameters have been estimated, the forecasting of future observations for several periods ahead can be made by employing the predictions equations repeatedly without the updating equations. Thus the MMSE of α_{T+l}, made at time T, is given by

$$a_{T+l/T} = T_{T+l}\, a_{T+l-1/T}\,, \qquad l = 1,2, \ldots \tag{2.20}$$

with $a_{T/T} = a_T.P_{T+l/T}$ computed as above. The MMSE of y_{T+l} is

$$y_{T+l/T} = z^T_{T+l} a_{T+l/T} \tag{2.21}$$

In the case of the level plus error model

$$y_{T+l/T} = y_{T+1/T} = a_T\,, \qquad l = 1,2, \ldots \tag{2.22}$$

Thus the forecast function is horizontal.

2.1.4 Exponential Smoothing Method

Another method of forecasting economic time series that has not been fully explored for riverflow forecasting is exponential smoothing. The most commonly used exponential smoothing models are the Holts-Winters family of models (Goodrich, 1989). These models includes three components representing level, trend and seasonal influences. Recursive equations are used to obtain smoothed values for the model components. Each smoothed value of any model component is a weighted average of current and past data with the weights decreasing exponentially. Holts-Winters family of exponential smoothing models can be classified into three classes, namely simple exponential smoothing, Holt two-parameter smoothing and Winters three-parameters smoothing (Goodrich and Stellwagen, 1987).

Simple exponential smoothing uses an equation to model the level of the series of the form

$$m_t = \lambda\, y_t + \lambda\,(1 - \lambda) y_{t-1} + \lambda\,(1 - \lambda)^2\, y_{t-2} + \ldots. \tag{2.23}$$

where:

λ = the level smoothing parameter

y_t = observed value of time series at time t

m_t = smoothed level at time t

This equation reduces to the recursive form

$$m_t = \lambda\, y_t + (1-\lambda)\, m_{t-1} \tag{2.24}$$

The forecasting equation is

$$\hat{y}_{t(h)} = m_t \tag{2.25}$$

where:

$y_{t(h)}$ = forecast for lead time h from time T

Holt two-parameter smoothing uses two equations to model level and trend. These are given in their recursive form by

$$\begin{aligned} m_t &= \lambda\, y_t + (1 - \lambda)(m_{t-1} + T_{t-1}) \\ T_t &= \gamma(m_t - m_{t-1}) + (1 - \gamma)T_{t-1} \end{aligned} \tag{2.26}$$

where:

T_t = the smoothed trend at time t

γ = trend smoothing parameters and other parameters are as defined previously.

The forecasting equation is

$$\hat{y}_{t(b)} = m_t + bT_t \qquad (2.27)$$

Winters three-parameter smoothing involves three smoothing parameters for level, trend and seasonal effects. The smoothing equations are of the form

$$\begin{aligned} m_t &= \lambda \frac{y_t}{S_{t-p}} + (1-\lambda)(m_{t-1} + T_{t-1}) \\ T_t &= \gamma(m_t - m_{t-1} + (1-\gamma)\, T_{t-1}) \\ S_t &= \delta \frac{y_t}{m_t} + (1-\delta) S_{t-n} \end{aligned} \qquad (2.28)$$

where:

S_t = smoothed seasonal index at time t

n = the number of periods in the seasonal cycle

δ = seasonal index smoothing parameter and other parameters are previously defined

The forecasting equation is of the form

$$\hat{y}_{t(b)} = (m_t + bT_t)\, \hat{S}_{t(b)} \qquad (2.29)$$

Simple exponential smoothing is appropriate for data which fluctuates around a constant or has a slowly changing level and is neither seasonal nor has any trend. Use of the Holt two-parameter model is appropriate for data which fluctuates about a level that changes with some nearly constant linear trend. Winters three-parameter model is used for data with trend and seasonal effects. The relevant exponential smoothing equations can be adjusted to represent data that has a damped exponential rather than linear trend (Goodrich, 1989). The forecasting equation for a Winters three parameter damped trend model is

$$\hat{y}_t(b) = (m_t + (\eta + \eta^2 + \dots + \eta^n)\, T_t)\, S_t(b) \qquad (2.30)$$

It can be seen that for $\eta = 1$, the model is equivalent to the undamped case.

All exponential smoothing equations give more weight to more recent values of data. The larger the values of the smoothing parameters the more emphasis on recent observations and less on past. This is intuitively appealing for forecasting applications. The smoothing parameters can be obtained by either using iterative least squares or a grid search for the parameters that give the minimum squared error over the historical data. This calculation process requires a great number of computations which are normally incorporated into a computer program.

Exponential smoothing models are robust in that they are insensitive to changes in the data statistical structure (Goodrich, 1989). No assumptions about the statistical distribution of data are made in exponential smoothing and there is therefore no need to analyze diagnostic statistic given with most computer programs.

One of the main advantages of using exponential smoothing is that once the smoothing parameters have been estimated, only the previous forecast and the most recent observation have to be stored or are necessary to make a new forecast. This makes the calculation of a new forecast computationally very convenient.

2.2 Comparison of Methods

The basic *ad hoc* forecasting procedure is exponential smoothing. Exponential smoothing methods are widely used in industry for quality control, inventory forecasting, etc. Their popularity is due to several practical considerations in short-term forecasting. Model formulations are relatively simple and model components and parameters have some intuitive meaning to the user. Only limited data storage and computational effort are required. Perhaps the most important reason for the popularity of exponential smoothing is the surprising accuracy that can be obtained with minimal effort in model identification (Gardner, 1985). An obvious disadvantage, for seasonal data, is that each seasonal component is only updated every s periods and the deseasonalization of the trend part in Eqn. (2.28) is carried out using an estimate of the seasonal component which is s periods out of date. However, they are *ad hoc* in that they are implemented without respect to a properly defined statistical model (Harvey, 1989). Their importance in the present context is that they provided the starting point for the development of structural time series models.

Box and Jenkins method is based on the theory of stationary stochastic processes, and this is the starting point for conventional statistical time series model building. However, a much wider class of models, capable of exhibiting non-stationary behaviour can be obtained by assuming that a series can be represented by an ARMA process after differencing. Few riverflow series are truly stationary and there is no overwhelming reason to suppose that they can necessarily be made stationary by differencing, which, infact is the main disadvantage of Box and Jenkins approach. The main advantage of Box and Jenkins approach is that it is has a highly developed model selection strategy. Since, the method of model order selection for periodic autoregressive (PAR) models has been derived from Box and Jenkins approach, no separate comparison for PAR models is discussed herein. The actual estimation of a model in the ARIMA class of Box and Jenkins approach is carried out without placing any restrictions on the parameter space, apart from those implied by stationarity. Since ARIMA models contain only one disturbance term these models are relatively simple to apply, which is one of the reasons for its appeal. The main attraction of the ARIMA class of models is that they provide a general framework for forecasting time series in which the specification of a model within the class is determined by the data. This may be quite advantageous in certain situations, particularly when it is difficult to identify the main components in a series and to construct suitable models for them. But the very flexibility of ARIMA modelling is also its main disadvantage. The decision to view all the models within this class as potential candidates for yielding good forecasts is an arbitrary one. The practical problem is that unless one has some experience in time series analysis, which effectively means *a priori* knowledge of the models which tend to be most useful, it is quite easy to select an inappropriate model. Such a model may pass the diagnostics, particularly if it is overparameterised, but may not yield sensible forecasts (Harvey, 1989).

The principle structural time series models are nothing more than regression models in which the explanatory variables are functions of time and the parameters are time varying. The starting point in new structural time series modelling (NSM) is the identification of the salient features in a series. These features can then be modelled in such a way that useful predictions of future observations can be made. This approach is statistically well defined in the state space form. The state space formulation opens up the possibility of setting up models in terms of components which have a direct interpretation. In addition, the state space form provides a relatively straightforward method of handling irregularities in the data. These irregularities may include missing values, temporal aggregation and data revisions. The main disadvantage of this approach, say, for monthly data, is that the number of parameters increases considerably and therefore principle of parsimony is not fully adhered to. In addition to that the number of disturbance terms in this approach is considerably higher than Box and Jenkins approach. The main advantage of the structural approach is that, differencing transformations aimed at achieving stationarity play a less prominent role than in ARIMA modelling. Moreover, the fact that the simpler structural time series models can be made stationary by differencing provides an important link with classical time series analysis. The simplest structural time series models, namely those which are linear and time invariant, all have a corresponding *reduced form* ARIMA representation which is equivalent in the sense that it will give identical forecasts to the structural form. Moreover, the new structural time series models encompass the exponential smoothing models and Box and Jenkins models, when certain model specifications are considered.

Chapter 3

FLOW DATA PREPARATION AND PRELIMINARY ANALYSIS

An understanding of the physiography, land use, geology and climate is necessary to predict riverflows in a region. The island of Newfoundland is a large, roughly triangular island about 111,000 km^2 in area lying off the east coast of North America, between latitudes 46° 30' and 51° 30' North. Runoff is generally higher in the southwest compared to northeast coast. Surface water is much more important than groundwater in Newfoundland. Most of the island consists of bedrock overlain by a thin veneer of glacial till, so subsurface aquifer storage is negligible. The majority of the population obtains its water from surface supplies, and about two-thirds of the island's energy comes from hydroelectric generation from surface sources. The abundance of good quality water in lakes, streams and ponds also sustains important recreational and fisheries uses (Richter, 1994). The main contributors to surface water in Newfoundland are rainfall, snowmelt and freezing rain. Large riverflows in Newfoundland occur in the spring (April to June) due to snow melt. The monthly data for Newfoundland rivers used in this forecasting study are obtained from the Water Resources Division of Department of

Environment and Lands, St. John's, Newfoundland.

3.1 Data Arrangement

The map depicting the rivers used in the study is shown in Fig. 3.1. As it can be seen from the map, the selected rivers are not concentrated in one particular area, as the objective is to study rivers having variable physical characteristics too. The average length of data used in this forecasting study is 40 years. The selected rivers have no missing values and no intervention analysis has been done to justify the effect of an event, say fire, on the riverflows obtained. The name, location, station number, drainage area and period of record of all the rivers are tabulated in Table 3.1. The drainage area of the rivers selected varies from a minimum of 3.63 km^2 for Northeast Pond River to a maximum of 4400 km^2 for Gander River at Big Chute. The minimum average flow is 0.135 m^3/s for Northeast Pond River and the maximum average flow is 118 m^3/s for Gander River. Since the emphasis is on forecasting short term series accurately, each set of monthly riverflow data has been subdivided into three series of average length of 13 years. The divided monthly riverflow data is named using the first four characters of the river under study and a number viz. 1, 2 or 3 is assigned to distinguish between three different record lengths. The monthly data has been aggregated to give the average quarterly data, which is the second set of series used in this forecasting study. The quarterly data is designated by prefixing D with first four letters of the river under study. The average length of data for the quarterly series is 150. To guard against spurious accuracy, three forecast scenarios are used for the quarterly data. These scenarios are

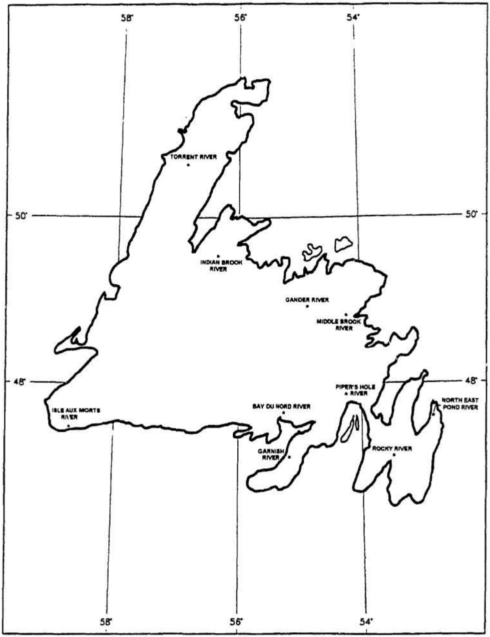

Figure 3.1 Newfoundland Rivers Used in the Study

Table 3.1 **HYDROMETRIC STATIONS USED IN STUDY**

STATION NAME	LOCATION		STATION NUMBER	DRAIN. AREA (KM^2)	PERIOD OF RECORD
	LAT.	LONG.			
Bay Du Nord River at Big Falls (BAYN)	47:44:48N	55:26:30W	02ZF001	1170	1952-1992 (41 years)
Gander River at Big Chute (GAND)	49:00:55N	54:51:13W	02YQ001	4400	1950-1992 (43 years)
Garnish River near Garnish (GARN)	47:12:50N	55:19:45W	02ZG001	205	1959-1992 (34 years)
Indian Brook at Indian Falls (INDN)	49:30:43N	56:06:45W	02YM001	974	1955-1992 (38 years)
Isle Aux Morts river Highway Bridge (ISLE)	47:36:50N	59:00:33W	02ZB001	205	1963-1992 (30 years)
Middle Brook near Gambo (MIDD)	48:48:28N	54:13:28W	02YR001	275	1960-1992 (33 years)
Northeast Pond River at NE pond (NORE)	47:38:06N	52:50:14W	02ZM006	3.63*	1954-1992 (39 years)
Piper's Hole river at Mother's Brook (PIPE)	47:56:49N	54:17:08W	02ZH001	764	1953-1992 (40 years)
Rocky River near Colinet (ROCK)	47:13:29N	53:34:06W	02ZK001	285	1950-1992 (43 years)
Torrent River at Bristol's Pool (TORR)	50:36:27N	57:09:04W	02YC001	624	1960-1992 (33 years)

NOTE:

* Differs significantly from drainage area published in the 1979 Surface Water Data Reference Index published by the Inland Waters Directorate of Environment Canada. The drainage areas presented in the Index were based on 1:50,000 scale NTS mapping whereas those listed here are based on more accurate mapping and air photos.

obtained by fitting different models to the data and forecasting but using three different last fitted values viz. till December, March and June. These three forecasting scenarios are distinguished by using numbers 1, 2 or 3 for the three last fitted values. For example, the quarterly data for Garnish River using period of record till the fourth quarter which is December is named DGARN1. The forecasted values for these three scenarios will mention the starting month. This ensures that the critical low flow and high flow periods are predicted as accurately as possible by using data upto that point.

3.2 Preliminary Data Analysis

The first step in preliminary data analysis is the plotting of data. A visual inspection gives a lot of information about the centre of data, variation or spread, skewness and presence of outliers. The data for this study is plotted using Boxplot, attached in Appendix A, which is a very useful and concise graphical tool for summarizing the distribution of a data set. A time series monthly plot of the Rocky river near Colinet is shown in Fig. 3.2, as an example. The monthly Boxplots of Rocky river, depicting that the data is seasonal, are plotted in Figure 3.3.

The second step in data analysis is to determine the distribution of data. If the data is normal then parametric test can be performed to determine other characteristics of data. If data are non normal then there are two options, either to perform nonparametric tests or to first transform the data to normality and then perform parametric tests. The seasonal riverflow data is generally non-normal, non stationary and heteroscedastic. The seasonal riverflow data is non-normal because by definition, normal

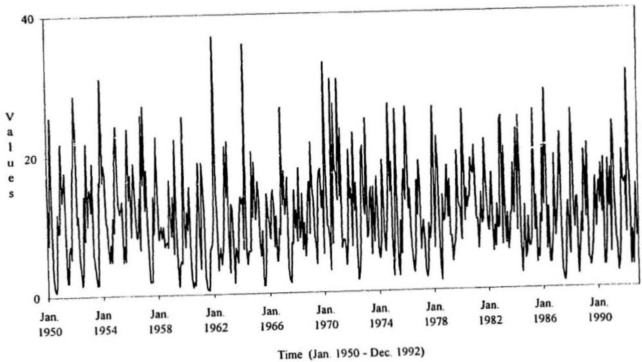

Figure 3.2. Monthly Time Series Plot for Rocky River

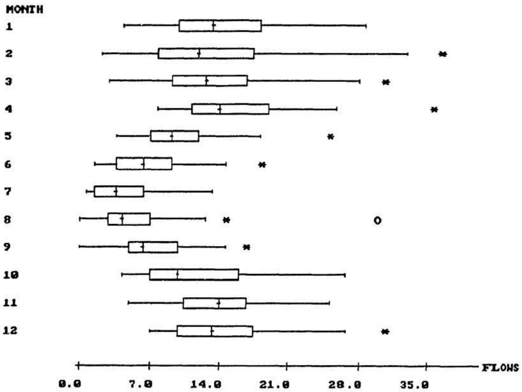

Figure 3.3 Boxplots for the Monthly Data of Rocky River

distribution ranges from $-\infty$ to $+\infty$ and there are no negative flows in nature. The minimum flow, i.e., below observation limit, is generally assigned a zero value. The seasonal riverflow data is non-stationary because the mean of low flow period will be significantly different from the mean of high flow periods. Thus statistical parameters are not time invariant for seasonal riverflow data. And finally, seasonal riverflow data are heteroscedastic because the variance and covariance may vary with time in a series. Although it is very rare to find negative correlation in hydrology, it can be negative during certain periods of time and remain positive at other times. To take care of all the above possibilities, in this study, the data is first transformed to normality, stationarity and homoscedastic by using the Box-Cox power transformation (Box and Cox, 1964). Box-Cox transformation is given as

$$y_{\lambda i} = \frac{x_i^{\lambda} - 1}{\lambda}, \qquad \text{for } \lambda \neq 0 \qquad (3.1)$$

$$y_{\lambda i} = \log_e(x_{\lambda i}), \qquad \text{for } \lambda = 0$$

where:

i = time index

$y_{\lambda i}$ = transformed variable

x_i = original variable

λ = power transform

The recommended λ value is the one that fits the normal probability assumption the closest. For example, a λ value of 0.5 is the square root transformation, λ equal to 0.0 is natural logarithmic transformation and a λ value of 1.0 is no transformation. The

Table 3.2 **CHARACTERISTICS OF MONTHLY RIVERFLOW DATA**

River Name		Size	Period	Meanflow	λ	Forecasts
BAYN	BAYN1	156	1952-1964	38.51	0.5	1965
	BAYN2	156	1966-1978	41.50	0.5	1979
	BAYN3	132	1981-1991	40.38	0.0	1992
GAND	GAND1	168	1950-1963	112.21	0.5	1964
	GAND2	168	1964-1977	123.66	0.0	1978
	GAND3	168	1978-1991	116.35	0.0	1992
GARN	GARN1	144	1959-1970	8.26	0.5	1971
	GARN2	120	1971-1980	8.98	0.5	1981
	GARN3	132	1981-1991	9.34	0.5	1992
INDN	INDN1	132	1955-1965	21.78	0.0	1966
	INDN2	144	1967-1978	18.31	1.0	1979
	INDN3	144	1980-1991	17.94	0.0	1992
ISLE	ISLE1	120	1963-1972	14.06	0.0	1973
	ISLE2	120	1973-1982	13.67	0.5	1983
	ISLE3	108	1983-1991	12.77	0.0	1992
MIDD	MIDD1	132	1960-1970	6.99	0.5	1971
	MIDD2	132	1971-1981	6.88	0.5	1982
	MIDD3	120	1982-1991	6.00	0.5	1992
NORE	NORE1	156	1954-1966	0.1361	0.5	1967
	NORE2	156	1967-1979	0.1304	0.5	1980
	NORE3	144	1980-1991	0.1373	0.5	1992
PIPE	PIPE1	156	1953-1965	24.25	0.5	1966
	PIPE2	156	1966-1978	24.51	0.5	1979
	PIPE3	156	1979-1991	25.72	0.5	1992
ROCK	ROCK1	168	1950-1963	10.76	0.5	1964
	ROCK2	168	1964-1977	11.48	0.5	1978
	ROCK3	168	1978-1991	11.51	0.5	1992
TORR	TORR1	132	1960-1970	24.78	0.0	1971
	TORR2	132	1971-1981	27.66	0.0	1982
	TORR3	120	1982-1991	23.48	0.0	1992

Table 3.3 **CHARACTERISTICS OF QUARTERLY RIVERFLOW DATA**

River Name		Size	Period	Meanflow	λ	Fest
DBAYN	DBAYN1	160	1952Q1-1991Q4	39.79	1.0	1992
	DBAYN2	161	1952Q1-1992Q1	39.76	1.0	1992
	DBAYN3	162	1952Q1-1992Q2	39.91	1.0	1992
DGAND	DGAND1	168	1950Q1-1991Q4	117.14	0.5	1992
	DGAND2	169	1950Q1-1992Q1	117.07	0.5	1992
	DGAND3	170	1950Q1-1992Q2	117.82	0.5	1992
DGARN	DGARN1	132	1959Q1-1991Q4	8.84	1.0	1992
	DGARN2	133	1959Q1-1992Q1	8.84	1.0	1992
	DGARN3	134	1959Q1-1992Q2	8.86	1.0	1992
DINDN	DINDN1	148	1955Q1-1991Q4	19.25	0.0	1992
	DINDN2	149	1955Q1-1992Q1	19.17	0.0	1992
	DINDN3	150	1955Q1-1992Q2	19.27	0.0	1992
DISLE	DISLE1	116	1963Q1-1991Q4	13.53	0.5	1992
	DISLE2	117	1963Q1-1992Q1	13.43	0.5	1992
	DISLE3	118	1963Q1-1992Q2	13.49	0.5	1992
DMIDD	DMIDD1	128	1960Q1-1991Q4	6.65	0.5	1992
	DMIDD2	129	1960Q1-1992Q1	6.62	0.5	1992
	DMIDD3	130	1960Q1-1992Q2	6.67	0.5	1992
DNORE	DNORE1	152	1954Q1-1991Q4	0.1346	1.0	1992
	DNORE2	153	1954Q1-1992Q1	0.1348	1.0	1992
	DNORE3	154	1954Q1-1992Q2	0.1352	1.0	1992
DPIPE	DPIPE1	156	1953Q1-1991Q4	24.83	1.0	1992
	DPIPE2	157	1953Q1-1992Q1	24.84	1.0	1992
	DPIPE3	158	1953Q1-1992Q2	24.88	1.0	1992
DROCK	DROCK1	168	1950Q1-1991Q4	11.25	1.0	1992
	DROCK2	169	1950Q1-1992Q1	11.29	1.0	1992
	DROCK3	170	1950Q1-1992Q2	11.29	1.0	1992
DTORR	DTORR1	128	1960Q1-1991Q4	25.36	0.0	1992
	DTORR2	129	1960Q1-1992Q1	25.19	0.0	1992
	DTORR3	130	1960Q1-1992Q2	25.27	0.0	1992

transformed variable is forecasted and then is backtransformed to the original distribution. The transformations used for the monthly riverflow time series and the quarterly time series are tabulated in Table 3.2 and Table 3.3 respectively. As can be seen from Appendix A, the Torrent River at Bristol's Pool, i.e., TORR is transformed using the logarithmic transformation. Table 3.2 and Table 3.3, also show the period of record used, average flow for the series in m^3/s and the year for which forecasts are obtained.

The third step in preliminary data analysis is to determine the autocorrelation structure of the series. A time series in which the current value of the series depends on the past values is called autocorrelated time series. The seasonal riverflow series are autocorrelated because flows in April, for example, are related to flows in March. The autocorrelation function (ACF) is a good measure of determining independence in a series. The graph of the sample autocorrelations is generally called the correlogram. If data is independent then autocorrelation at all lags should be equal to zero. Another way of representing the time dependence structure of a series is the partial autocorrelation function (PACF). The PACF is also useful in identifying the type and order of a model when investigating a given sample time series. To determine the significance of autocorrelation at 5% level, Bartlett's band (Salas, 1980) is used.

The ACF is defined as

$$r_k = \gamma_k = \frac{\sum_{t=1}^{n-k} (x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{n} (x_t - \bar{x})^2} \qquad (3.2)$$

where:

γ_k = autocorrelation at different lags

x_t = observed variable at time t

x_{t+k} = observed variable at lag k

$\bar{x}$ = mean value of the series

n = number of observations

The ACF graph gives another important information about a series. If a series is stationary the correlogram dies down gradually otherwise for a non-stationary series it gives similar values throughout. Moreover, examining the autocorrelations is a reliable way to determine a seasonal time series data. If the twelfth (for monthly data) or fourth (quarterly data) autocorrelation is abnormally high then the data is seasonal. The information from ACF is used to determine the degree of differencing required to make the series stationary. In Fig. 3.4, the ACF for BAYN1 series is plotted. The first correlogram in the figure shows that data is non-stationary and seasonal as autocorrelation are high at lags 1 and 12 respectively. The second and third correlograms respectively, show the series after first difference and seasonal difference transformations have been done. Therefore, Fig. 3.4 shows that even after first and seasonal differencing has been done, the series may not become fully stationary.

Once the series has been made stationary after repeated differencing, ACF in

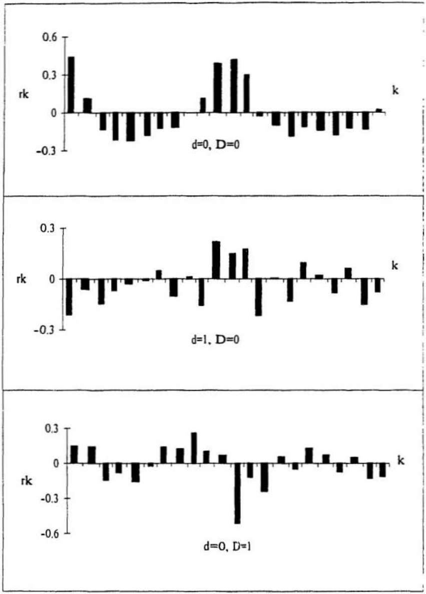

Figure 3.4 **Auto Correlation Function (ACF) for BAYN1**

combination with the partial autocorrelation function (PACF) helps in determining the orders for ARIMA models. The PACF is another important measure which determines the order for the PAR series. The PACF is defined as the correlation between lags, say, K and K+2 after the removal of effect of K+1 on both. Let K=1, then the PACF is defined as

$$ck = \rho_{13.2} = \frac{\rho_{13} - \rho_{12}\ \rho_{32}}{\sqrt{(1 - \rho_{12}^2)\ (1 - \rho_{32}^2)}} \tag{3.3}$$

where:

ρ_{13} = autocorrelation between 1 and 3 respectively.

The Fig. 3.5 shows the PACF for monthly BAYN1 series. Since first lag is highly significant, the AR order of model is estimated to be 1.

Figure 3.5 Partial Auto Correlation Function (PACF) for BAYN1

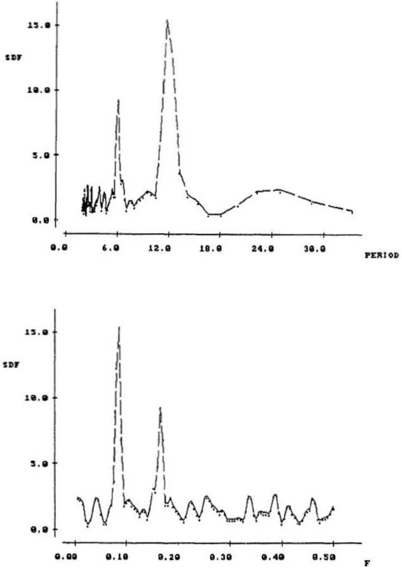

Figure 3.6 **Spectral Analysis Graphs for Garnish River**

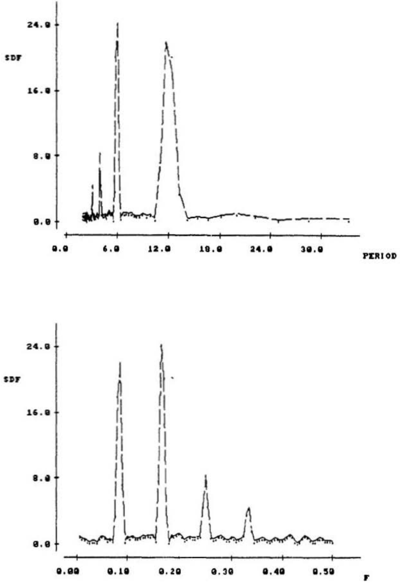

Figure 3.7 Spectral Analysis Graphs for Torrent River

The components of a seasonal time series such as trend, seasonality and irregular are also computed by the expert system of the software Forecast Pro (Business Forecast System, 1988). A multiplicative classical decomposition model (Makridakis and Wheelwright, 1979) is applied to obtain trend cycle, seasonal index and irregular components of the series. Then the percentage that each component explains of the variance of the natural logarithm of the series is computed. These percentages are used to supplement correlational data about the nature of the series. In addition, by using spectral analysis (Salas, 1980) dominant frequencies in the riverflow data can be estimated. For example in Fig. 3.6, the spectral analysis of monthly time series of Garnish river which is in the South, shows the presence of within a year seasonal cycles in the data. The spectral density function is SDF in the Fig. 3.6 and F is the frequency. It can be seen from the figure that the series has one dominant frequency in a year for this river. For the Torrent river on the other hand which is in the North, there are more than one dominant frequency as shown in Fig. 3.7.

The characteristics of the seasonal time series can also be detected by performing certain statistical tests. As shown in Appendix A, all the seasonal time series considered in the study are non-normal therefore non parametric tests at 5% significance level are performed on the data base. The p-value for each test is calculated and is tested against significance level of 5%, i.e., 0.05. If the p-value is less than 0.05, null hypothesis is rejected at 5% level.

The non parametric *test for independence* used in the analysis is the *rank von Neumann Ratio Test*. Let $r_1,\ldots,r_n$ denote the ranks associated with the x_i's. The rank von

Neumann ratio is given by

$$v = \frac{\sum_{i=2}^{n} (r_i - r_{i-1})^2}{n(n^2 - 1)/12} \tag{3.4}$$

Critical values of $c = [n(n^2 - 1)/12]v$ and approximate critical values of v are given by Madansky (1988). For large n, v is approximately distributed as N(2,4/n), though Bartels recommends 20/(5n+7) as a better approximation to the variance of v.

The *test for randomness* is the *Runs test*. A run test is usually used to determine if the order is random. A runs is one or more consecutive observations > k, or one or more consecutive observations ≤ k. For nonparametric case, k is specified as the median of the series.

The Spearman's rho is a nonparametric coefficient of rank correlation, which is based on the squared differences of ranks between two variables. By letting one of the variables represent time, *Spearman's rho* can be interpreted as a *trend test* (Hipel and McLeod, 1994).

The Kruskall Wallis test determines whether or not the distribution across k samples are the same. The *Kruskall Wallis test* can also be used to test for the presence of *seasonality* and decide upon which seasons are similar (Hipel and McLeod, 1994).

In order to perform homogeneity tests for median and variance, sample data is first split by time span. To test for *homogeneity in median*, *Mann-Whitney test* is performed. The Mann-Whitney test does a two-sample rank test for the difference between two population medians, and calculates the corresponding point estimate and

95% confidence interval. The *homogeneity test for variance* is the U.S. Environmental Protection Agency recommended the *Boxplot test*, which uses the interquartile range (IQR). The IQR for both the populations is first estimated and then if $IQR_{max} > 3*IQR_{min}$, then it implies that there is change in variance.

The tests for independence, randomness, trend, seasonality and homogeneity were performed on the monthly riverflow data of all Newfoundland rivers. The results for all the series showed that data is not independent. The order of riverflow series is not random at the median value. The trend test for all the series was for an overall global trend and the results showed that the series do not have a significant trend. The data was highly seasonal and homogeneous in the median and variance.

Chapter 4

APPLICATION OF FORECASTING METHODS

Each series, prepared and analyzed as explained in previous chapter, is modelled using all the four approaches namely, ARIMA, PAR, EXS and NSM. Within each approach, all the tentative models based on the characteristics of the series are first considered. The next important step is, model selection by using the *principle of parsimony* (Box and Jenkins, 1976), i.e., choosing a parsimonious model from an array of models by using a goodness of fit criterion. The most commonly used goodness of fit criterion for time series data is the Akaike Information criterion (AIC). The AIC chooses a parsimonious model by making a balance between model error variance and the number of parameters required to fit a model to the data. The model which gives the minimum value for AIC (Goodrich and Stellwagen, 1987) is selected for forecasting for that particular approach. It is defined as

$$AIC = \sigma^2 \exp[2k / N] \tag{4.1}$$

where:

k = number of parameters in the model, it is equal to (n+d) for NSM models.

d = number of non-stationary elements in the state vector

n = number of hyperparameters

σ^2 = estimated error variance

N = sample size

The reason for including the number of non-stationary elements in the state vector in the criterion function is to allow comparisons involving models with deterministic components.

In the general framework of forecasting procedure, once a model is selected and fitted to the data, the residuals are to be analyzed for randomness, normality, constant variance and autocorrelations. This final check is called diagnostics and is done before forecasting riverflows. The residual analysis for randomness, normality and constant variance is carried out by using the residuals as input data. The autocorrelation of the residuals is checked at individual lags and as a group. The Durbin-Watson and Ljung-Box statistics are most commonly used diagnostics based upon autocorrelations of the fitting errors. The Durbin-Watson statistic is significant when there is significant autocorrelation in the first lag. A table of critical values is referred to determine whether or not the statistic is significant. The Durbin-Watson statistic is defined by

$$DW = \frac{\sum_{t=2}^{t=N} (e_t - e_{t-1})^2}{\sum_{t=1}^{t=N} e_t^2} \qquad (4.2)$$

where:

e_t = fitting error for time t

N = historical sample size

The Ljung-Box statistic is a diagnostic for the overall significance of the first several lags of the error autocorrelation function. The sample statistic is tested against the Chi-square distribution with (N-n) degrees of freedom, n is the number of parameters fitted in the model. The Ljung-Box statistic is defined by

$$LB = N\,(N + 2)\sum_{i=1}^{i=L} \frac{r_i^2}{(N - 1)} \tag{4.3}$$

where:

r_i = i-th lag autocorrelation

L = number of autocorrelation used

N = sample size

After the process of identification, estimation and diagnostics is complete, the resulting model is used in forecasting. The model which satisfies all the criterion is finally selected to forecast the future values. The detailed application procedure for all the four approaches is illustrated with examples in the following sections.

4.1 Box and Jenkins Modelling

The Box and Jenkins modelling approach is a component of the software Forecast Pro (Business Forecast Systems, Inc., 1988), which is used as a tool to model and forecast the riverflow time series. The data is first transformed and then to the transformed series a model is fitted based on the minimum Akaike Information Criterion

(AIC).

Use of this criterion assists the user considerably in going through the tedious process of identification, estimation and diagnostic in the Box and Jenkins approach. The result of application of Box and Jenkins approach to INDN1 riverflow time series is shown in Table 4.1. Because of the strict distributional assumptions in the Box and Jenkins model, the examination of the diagnostic statistics is required.

The R-square statistic indicates the amount of variance explained by the model. An R-square of 0.56 explains 56% of the series variance. The Durbin-Watson checks for correlation in the first lag, the less correlation there is in the first lag the closer this value is to 2.0. In the Ljung-Box test the associated probability point is also output. In addition, the autocorrelations of the residuals are examined using Forecast Pro and are found to exhibit no systematic pattern. They are also small in magnitude being less than 2 times the standard error.

The Box and Jenkins model parameters are the values that define the mathematical model for a series. The t-stat in Table 4.1 shows the significance of a parameter. If the absolute value of t-stat, of a model parameter, is greater than 2 then the parameter is significant. The nonseasonal autoregressive (AR) component, i.e., φ_p of the model is represented by the suffix A and the nonseasonal moving average (MA) component, i.e., θ_q is represented by the suffix B. The values in square brackets depict the nonseasonal and seasonal parts of the multiplicative ARIMA modelling. For the monthly data, seasonal components have multiples of 12 in square brackets. The seasonal AR components, i.e., Φ_P are depicted by A[12], A[24], etc., The seasonal MA component,

Table 4.1 **HISTORICAL FIT OF BOX AND JENKINS MODEL (ARIMA) (MONTHLY INDN1 RIVER)**

Box Cox Transformation : Logarithmic
Period of Record : 1955-1965
Number of Observations : 132

Dependent variable: log(INDN)
R-square: 0.560
Adjusted R-square: 0.547
Standard forecast error: 0.546854
Durbin-Watson: 1.936
Ljung-Box: 15.515 (0.786)
Standardized AIC: 8.605835

Multiplicative ARIMA model : $(1, 0, 0) \times (2, 0, 1)_{12}$

BJ Parameter	Coefficient	Standard error	T-stat	Prob
A[1]	0.283620	0.196418	1.444	0.851
A[12]	0.888336	0.318718	2.787	0.995
A[24]	0.039239	0.294759	0.133	0.106
B[12]	0.607684	0.164745	3.689	1.000
CONSTANT	0.142219			

Forecast variable &INDN1F

Period	Forecast
1-1966	11.238813
2-1966	8.296609
3-1966	13.810618
4-1966	23.042774
5-1966	54.574333
6-1966	27.079355
7-1966	12.188464
8-1966	7.376487
9-1966	7.178437
10-1966	12.172106
11-1966	18.458605
12-1966	13.837437

i.e., O_0 is shown by B[12], etc., The selected model is used to generate 12 month ahead forecasts.

4.2 Periodic Autoregressive Modelling

In the periodic autoregressive modelling process, model identification is carried out using the sample PACF (Partial Autocorrelation Function). For each season of the year the significant order is selected using the PACF. If there is more than one promising model, the minimum AIC procedure can then be applied to select the best one for that particular season. For the PAR models, AIC, as defined in Chapter 3, for each season of the year is calculated first and then the AIC for overall PAR model is calculated as

$$AIC = \sum_{m=1}^{s} AIC_m + 2 \tag{4.4}$$

where:

AIC_m = AIC for the mth season

The constant 2 allows for the Box-Cox parameter λ.

Once the order is finally selected, an AR model of the selected order is fitted to the data. The ARIMA command in MINITAB (Minitab Inc., 1992) fits nonseasonal and seasonal models to a time series. The constant subcommand fits the model with the parameters and a constant term. The input to the command consists of a time series stored in a column, and information about the model to be fitted. In addition to the displayed output, residuals, fits and coefficients (estimated parameters) may be stored in the worksheet for further analysis. The software uses the nonlinear least squares algorithm (Marquardt, 1963) to estimate the parameters of the selected model. The adequacy of a fitted model is ascertained by examining the properties of the residuals for each season. In particular, the residuals should be uncorrelated, normally distributed and homoscedastic. The forecast subcommand allows to forecast observations starting at the specified origin and going up to K leads ahead. If the origin is not specified, it is set to the end of the series and forecasts are for the future. The selected model is used for predicting one-step-ahead forecast for that particular month in the following year. This process is repeated for each and every season of the year for all the selected Newfoundland rivers.

The Table 4.2 shows the output of application of PAR model for the INDN1 monthly riverflow time series. The PACF of the series showed that order 1 is significant therefore AR(1) model is fitted to the data. The parameters are estimated iteratively and the final estimate of AR(1) parameter for the month of January (m = 1), i.e., φ^{1}_{1} is 0.4197. To compare the forecasting accuracy, the forecasted values shown in the Table 4.2 is first backtransformed into the original units.

4.3 Exponential Smoothing Modelling

Exponential smoothing is the simplest of the methods implemented in Forecast Pro. Since the data in the present study has a level, trend and is seasonal, therefore Winters 3 parameter smoothing and 3 parameter (damped trends) are the viable options. If the trends are cyclic in nature then method with damped trends fits the data better. 3 parameter (damped trends) is similar to Winters 3 parameter smoothing except that the

Table 4.2 **HISTORICAL FIT OF PERIODIC MODEL (PAR) (MONTHLY INDNI RIVER)**

Box Cox Transformation : Logarithmic
Period of Record : 1955-1965
Number of Observations : 132

RESULTS OF PARAMETER ESTIMATION

Final Estimates of Parameters

Month	Type	Estimate
Jan.	AR(1)	0.4197
Feb.	AR(1)	-0.1910
Mar.	AR(1)	-0.5202
Apr.	AR(1)	-0.7782
May.	AR(1)	0.0743
Jun.	AR(1)	-0.0641
Jul.	AR(1)	0.1801
Aug.	AR(1)	0.1300
Sep.	AR(1)	-0.0163
Oct.	AR(1)	-0.4985
Nov.	AR(1)	-0.2068
Dec.	AR(1)	-0.3886

Period	Forecast
66-M1	2.69240
66-M2	2.28130
66-M3	1.91347
66-M4	3.90721
66-M5	4.24578
66-M6	3.14748
66-M7	2.17721
66-M8	1.82790
66-M9	2.10117
66-M10	3.07629
66-M11	3.04996
66-M12	3.05690

trend is not extended indefinitely ahead in the forecasts.

The program optimizes the parameters and fits them to the historical data automatically. The smoothing parameter values are obtained by Forecast Pro using an iterative search method to minimize the squared errors over the historical data. The computerized iterative search, which employs the simplex method of nonlinear optimization, begins at the values selected by the program or supplied by the user and continues until a local minimum is found. The summary statistics with model parameters is the program output and from this output the model with minimum AIC is selected for forecasting the series.

Winters 3 parameter exponential smoothing model is fitted to the INDN1 time series data using Forecast Pro. The results are shown in Table 4.3. Since no statistical distribution assumptions have been made about the data, it is not necessary to closely scrutinize all the diagnostic statistics produced by the software.

Examining the exponential smoothing parameters reveals that the seasonal parameter value is close to 0.148706 indicating that the best forecast for the next seasons effect is 14.9% of the last seasons effects and a weighted average of preceding seasonal effects. The small trend value of 0.003808 indicates that the smoothing model has a memory of trend and distant trends have an effect on the forecasted trend component. The small value of the level parameter indicates that the model is not significantly adaptive to the last observed level of the series. The selected model, from two pertinent options, is used for forecasting 12 month ahead forecasts for the riverflow time series.

Table 4.3 **HISTORICAL FIT OF EXPONENTIAL SMOOTHING (EXS) (MONTHLY INDN1 RIVER)**

Box Cox Transformation : Logarithmic
Period of Record : 1955-1965
Number of Observations : 132

Historical fit of exponential smoothing model
Dependent variable: log(INDN)
R-square: 0.543
Adjusted R-square: 0.532
Standard forecast error: 0.555724
F statistic: 50.996 (1.000)
Durbin-Watson: 1.476
Ljung-Box: 19.606 (0.925)
Standardized AIC: 8.713256

Winters 3 Parameter Smoothing Model

Exponential smoothing parameter values

LEVEL	0.063828
TREND	0.003808
SEASONAL	0.148706

Forecast variable &EINDN1F

Period	Forecast
1-1966	11.493494
2-1966	7.372066
3-1966	10.181950
4-1966	21.606651
5-1966	51.637814
6-1966	20.429228
7-1966	8.237924
8-1966	5.793111
9-1966	6.427616
10-1966	11.572261
11-1966	16.360777
12-1966	13.376265

4.4 New Structural Time Series Modelling

The STAMP (Structural Time Series Analyzer, Modeller and Predictor) program has been developed to fit univariate structural time series models, and models with interventions and explanatory variables (Harvey, 1989). The principal structural time series components and models supported by the program are tabulated in Table 4.4 (Harvey, 1989). In addition, certain components such as trend, seasonal, etc., can be treated as deterministic, by selecting the fixed parameter (variance) option and setting the value equal to zero. This means that these components are treated as exogenous variables. The only advantage in treating the component as exogenous is that the standard errors of the estimated parameters are likely to be more reliable. Based on the characteristics of the time series, tentative models for a time series are selected.

Estimation of unknown parameters of the selected models can be carried out either in time domain or in the frequency domain. Time domain is exact maximum likelihood (ML) estimation with numerical optimisation carried out by a quasi-Newton algorithm (Harvey, 1989). In frequency domain, ML estimation is again carried out with numerical optimisation using quasi-Newton algorithm. It is much faster than time domain estimation, but the results will be slightly different as it is based on an approximation to the time domain likelihood function. The method of scoring is the third option in the program which is the fastest and highly recommended if no cycles are present in the data (Harvey, 1989). Since the riverflow time series have no prominent annual cycles therefore the method of scoring is selected for this study. The method of scoring is based on the frequency domain likelihood function, but the maximum is found by the method

Table 4.4 Principle Structural Time Series Components and Models in NSM (from Harvey, 1989)

Model	Component	Specification
	1a *Random walk*	$\mu_t = \mu_{t-1} + \eta_t$
	1b *Random walk with drift*	$\mu_t = \mu_{t-1} + \beta + \eta_t$
A Local level/ random walk plus noise model		$y_t = \mu_t + \varepsilon_t$ with μ_t as in (1a)
	2 *Stochastic trend*	$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t$ $\beta_t = \beta_{t-1} + \zeta_t$
B Local linear trend		$y_t = \mu_t + \varepsilon_t$ with μ_t as in (2)
	3 *Stochastic cycle*	$\begin{bmatrix}\psi_t \\ \psi_t^*\end{bmatrix} = \rho \begin{bmatrix}\cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c\end{bmatrix}\begin{bmatrix}\psi_{t-1} \\ \psi_{t-1}^*\end{bmatrix} + \begin{bmatrix}\kappa_t \\ \kappa_t^*\end{bmatrix}$ where ψ_t is cycle, $0 \leqslant \rho < 1$, and $0 \leqslant \lambda_c \leqslant \pi$
C Cycle plus noise model		$y_t = \mu + \psi_t + \varepsilon_t$, where $0 \leqslant \rho < 1$
D Trend plus cycle		$y_t = \mu_t + \psi_t + \varepsilon_t$ with μ_t as in (2)
E Cyclical trend		$y_t = \mu_t + \varepsilon_t$ $\mu_t = \mu_{t-1} + \psi_{t-1} + \beta_{t-1} + \eta_t$ with β_t as in (2)
	4 *Non-stationary cycle*	As (3) but $\rho = 1$
	5a *Dummy variable seasonality*	$\gamma_t = \sum_{j=1}^{s-1} \gamma_{t-j} + \omega_t$
	5b *Trigonometric seasonality*	$\gamma_t = \sum_{j=1}^{[s/2]} \gamma_{t,j}^{\dagger}$ where $\gamma_{t,j}^{\dagger}$ is a non-stationary cycle, (4), with $\lambda_c = \lambda_j = 2\pi j/s$, $j = 1, 2, \ldots, [s/2]$
F Basic structural model		$y_t = \mu_t + \gamma_t + \varepsilon_t$ where μ_t is as in (2) and γ_t is as in (5a) or (5b)

of scoring.

Once a tentative model has been estimated, it is subjected to diagnostic tests and checks. If, in the face of these checks, the model appears to be inadequate, its specification is changed and the process repeated. If the model survives the diagnostics, it is accepted and used for forecasting. Table 4.5 shows the application of NSM approach the INDN1 riverflow time series.

The hyperparameters are the first statistics in the Table 4.5. The second statistics of importance is the estimates of state vectors at the end of sample period ie. state at 66M12. Thus level estimate of 2.3013 indicates a 230.13% growth rate per period. The third and final statistics is Goodness of Fit. The Goodness of Fit yields the prediction error variance (p.e.v.) together with coefficient of determination (R-squared). Since the primary objective of the study is to forecast flows, the results of forecasting option is the last output in Table 4.5. For forecasting, there are two possibilities in STAMP, the first is to construct one-step-ahead predictions in the post sample period and the second is to extrapolate from the last observation used to estimate the model. When the model has been estimated using observations right up to the end of the sample, only extrapolations can be made. The forecasted values for INDN1 in Table 4.5 are obtained using one-step-ahead predictions.

Table 4.5 **HISTORICAL FIT OF NEW STRUCTURAL MODEL (NSM) (MONTHLY INDN1 RIVER)**

Box Cox Transformation : Logarithmic
Period of Record : 1955-1965
Number of Observations : 132

Estimation by Scoring

Estimate	Parameter
.0403	σ^2(Level)
.0000143	σ^2(Trend)
.0826	σ^2(Seasonal)
.0675	σ^2(Irregular)

Estimate	State
2.3013	Level
-.0080805	Trend
-.0254	Seasonal
-.1174	Seasonal
.1410	Seasonal
-.7151	Seasonal
-.6605	Seasonal
-.6946	Seasonal
.8299	Seasonal
1.5861	Seasonal
.5896	Seasonal
-.1877	Seasonal
-.5009	Seasonal

p.e.v. = .3940; $R2$ = .4616

Observation	Forecasts
66M1	8.248
66M2	4.9037
66M3	11.0232
66M4	24.0468
66M5	72.2405
66M6	24.2884
66M7	9.2999
66M8	4.1371
66M9	8.4994
66M10	13.1971
66M11	20.9052
66M12	6.7531

Chapter 5

COMPARISON OF FORECASTS

The previous chapter provides detailed descriptions of application of various forecasting methods to Newfoundland riverflow time series data. The emphasis till now was on selecting a model that fits the historical data well. However, when the forecasts are compared with future data that are not used for estimation, the agreement need not be as good. Hence, comparisons of forecasts with actual observations can be an additional useful tool for model evaluation and selection (Box and Tiao, 1976). In practical situations it may be unreasonable to expect many future observations. However, one can use initial part for model construction and the remaining part as a holdout period for forecast evaluation and comparison. Such an approach is pursued in this forecasting study. This approach also fulfils the primary objective of this research, i.e., to forecast and recommend appropriate method of forecasting for Newfoundland rivers based on forecast accuracy. The actual and forecasted values, using all the four different forecasting approaches, for monthly and quarterly time series are plotted in Fig. 5.1a & Fig. 5.1b and in Fig. 5.2a & Fig. 5.2b, respectively. The visual inspection informally gives an idea about the best approach to be used for forecasting. But using specific measures of forecasting accuracy to distinguish between approaches is a better method.

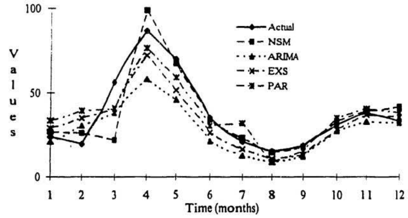

Figure 5.1a Forecast comparison (beginning Jan. 1981)
Bay Du Nord River

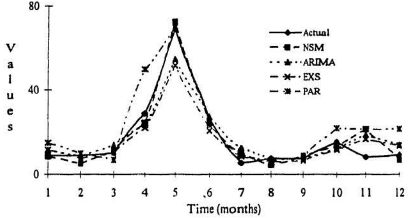

Figure 5.1b Forecast comparison (beginning Jan. 1955)
Indian Brook River

Figure 5.1 Forecasts of Monthly Flows

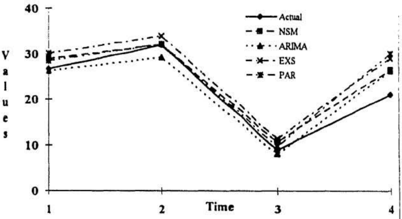

Figure 5.2a Forecast comparison (beginning Jan.1992)
Piper's Hole River

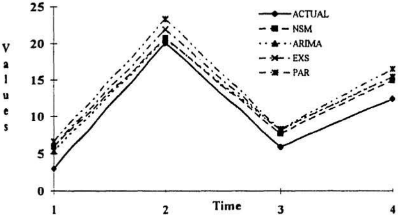

Figure 5.2b Forecast comparison (beginning Jan. 1992)
Isle Aux Morts River

Figure 5.2 Forecasts of Quarterly Flows

5.1 Measure of Forecasting Accuracy

Various methods of measuring forecast accuracy exist. A problem is that, although accuracy represents an important factor in selecting a forecasting method, 'one of the difficulties in dealing with the criterion of accuracy in forecasting situations is the absence of a single universally accepted measure of accuracy' (Makridakis et al., 1983b). A detailed survey (Mahmoud, 1984) of the relevant literature reveals the description, development and empirical testing of many accuracy measures. The measures of forecasting accuracy surveyed are the following; the mean square error, the mean percentage error, the mean absolute percentage error, Theil's U-statistic, the root mean square error, the mean error, the mean absolute deviation, turning points and hits and misses.

In hydrologic forecasting, while comparing competing approaches, it is important that due consideration be given to forecast bias. One of the most common measures of forecasting accuracy that takes care of forecast bias is the mean absolute percentage error (MAPE) criterion. To obtain the MAPE, the difference between each forecasted value of a time series and the actual observed values is first calculated. The MAPE is then computed as the average of the magnitudes of these differences when these differences are expressed as a percentage of the actual observed values. It is defined mathematically as

$$MAPE = \frac{100}{m} \sum_{t=1}^{m} \left|\frac{e_t}{z_t}\right|, \qquad t = 1,2, \ldots \ldots m \qquad (5.1)$$

where:

e_t = forecast error

z_t = observed value

The MAPE criterion is thus chosen to measure forecast accuracy in this study. The method that yields minimum MAPE is the best method in terms of forecasting.

5.2 Performance of Models

The performance of models used for monthly Newfoundland riverflow time series is assessed in terms of the MAPE criterion. Four different forecast horizons are considered, i.e., 3 month ahead, 6 month ahead, 9 month ahead and 12 month ahead. The forecasts beyond 12 month ahead period loose their significance and hence are not considered in this study. The MAPE values for four different forecast horizons are tabulated in Tables 5.1a, 5.1b, 5.1c and 5.1d. The four approaches are distinguished, as shown in the note, by using different letters . The AIC values for the best fitted model in each approach are tabulated separately in Table 5.1e.

The results of application of the four forecasting methods to quarterly riverflow time series data are also tabulated. The tables show the MAPE values for forecasts along with the AIC values. For the quarterly riverflow data three different forecast scenarios were used. The results are tabulated in Tables 5.2a, 5.2b and 5.2c respectively.

Table 5.1a **COMPARISON OF MONTHLY FORECASTS**

MAPE - (*3* - ***MONTH AHEAD PERIOD***)

Rivers	Forecasting Approach Used			
	S	A	E	P
BAYN1	31.7	26.66	36.1	38.10
BAYN2	42.33	31.46	21.4	70.80
BAYN3	36.0	34.0	43.4	47.70
GAND1	73.44	90.18	69.10	92.50
GAND2	45.62	34.92	45.1	48.10
GAND3	41.88	46.02	86.0	59.60
GARN1	56.20	40.87	46.79	50.90
GARN2	13.90	36.13	18.10	27.50
GARN3	45.30	59.90	58.70	55.60
INDN1	19.0	24.0	16.12	14.45
INDN2	76.0	47.0	59.3	80.20
INDN3	33.00	25.00	53.9	33.40
ISLE1	48.6	40.9	35.94	24.90
ISLE2	67.6	39.7	53.2	60.70
ISLE3	81.60	64.1	75.61	74.83
MIDD1	30.53	24.21	26.4	49.20
MIDD2	34.3	70.11	43.8	68.00
MIDD3	26.58	31.51	150.4	29.10
NORE1	28.87	36.81	31.7	21.20
NORE2	52.20	57.10	46.10	49.00
NORE3	48.30	75.90	62.60	51.90
PIPE1	32.10	36.31	43.50	73.80
PIPE2	61.92	33.38	33.20	51.20

PIPE3	67.80	72.90	57.60	74.00
ROCK1	15.12	19.86	28.10	32.80
ROCK2	40.00	44.00	51.50	59.20
ROCK3	40.40	61.80	47.50	40.38
TORR1	14.97	88.20	27.57	17.42
TORR2	41.20	124.6	58.36	75.60
TORR3	8.61	135.7	91.20	74.30

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Model (PAR)

Table 5.1b **COMPARISON OF MONTHLY FORECASTS**

MAPE - ***(6 - MONTH AHEAD PERIOD)***

Rivers	Forecasting Approach Used			
	S	A	E	P
BAYN1	27.53	21.58	25.4	27.19
BAYN2	59.6	78.2	74.7	99.20
BAYN3	21.37	34.82	33.05	31.80
GAND1	44.97	54.07	43.10	54.50
GAND2	29.49	28.40	32.40	34.20
GAND3	29.79	61.67	52.30	44.00
GARN1	54.11	39.86	45.11	41.70
GARN2	22.31	28.01	20.20	27.67
GARN3	46.20	58.70	50.80	55.20
INDN1	14.00	19.00	19.82	8.720
INDN2	85.80	56.00	69.30	90.40
INDN3	26.64	27.69	37.5	26.98
ISLE1	47.56	27.83	27.77	23.27
ISLE2	70.40	62.60	81.80	82.60
ISLE3	74.40	59.80	68.40	71.90
MIDD1	67.98	49.05	61.70	67.70
MIDD2	30.28	47.73	36.48	44.20
MIDD3	20.55	28.82	100.4	22.74
NORE1	77.20	87.60	84.10	73.40
NORE2	38.90	48.56	37.80	38.50
NORE3	42.83	46.30	47.00	44.30
PIPE1	23.60	38.50	36.13	50.30
PIPE2	50.10	118.40	108.7	124.7

PIPE3	42.60	50.20	37.00	45.60
ROCK1	26.29	25.50	31.60	34.20
ROCK2	46.85	42.69	48.55	48.40
ROCK3	52.70	73.70	58.60	54.50
TORR1	47.60	74.10	52.40	48.50
TORR2	40.43	100.50	52.20	59.30
TORR3	15.64	76.30	58.40	52.80

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Model (PAR)

Table 5.1c **COMPARISON OF MONTHLY FORECASTS**

MAPE - ***(9 - MONTH AHEAD PERIOD)***

Rivers	Forecasting Approach Used			
	S	A	E	P
BAYN1	21.08	20.98	18.33	23.93
BAYN2	54.60	79.80	69.50	86.40
BAYN3	16.46	36.20	30.96	25.93
GAND1	42.49	41.17	42.20	50.30
GAND2	75.06	116.39	94.60	90.90
GAND3	26.59	70.19	48.30	44.89
GARN1	74.60	85.30	66.20	54.30
GARN2	26.03	35.28	24.95	30.58
GARN3	39.90	48.50	43.30	50.60
INDN1	24.00	27.00	22.54	14.82
INDN2	70.00	53.00	58.70	85.40
INDN3	25.00	35.00	31.01	25.91
ISLE1	50.54	41.90	43.00	41.10
ISLE2	57.50	49.80	62.80	62.79
ISLE3	63.10	54.00	61.70	61.30
MIDD1	74.67	81.85	70.40	63.20
MIDD2	30.00	68.40	47.27	33.80
MIDD3	18.30	36.06	85.00	36.39
NORE1	133.00	20.94	159.30	118.6
NORE2	45.52	49.49	47.12	46.20
NORE3	49.90	64.30	59.90	53.20
PIPE1	31.06	48.90	36.49	44.20
PIPE2	54.92	145.90	107.50	119.5

PIPE3	39.40	48.30	35.60	40.90
ROCK1	29.96	26.91	33.28	33.80
ROCK2	61.40	84.80	78.50	85.40
ROCK3	44.70	62.70	48.90	46.00
TORR1	43.90	63.60	47.40	44.70
TORR2	37.67	82.20	46.40	50.30
TORR3	24.38	70.60	55.30	51.20

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Model (PAR)

Table 5.1d **COMPARISON OF MONTHLY FORECASTS**

MAPE - ***(12 - MONTH AHEAD PERIOD)***

Rivers	Forecasting Approach Used			
	S	A	E	P
BAYN1	26.65	25.64	23.11	24.62
BAYN2	49.91	64.90	58.90	72.10
BAYN3	15.21	29.56	24.96	22.39
GAND1	44.00	40.70	43.20	49.80
GAND2	81.50	124.30	106.70	100.9
GAND3	31.24	67.12	44.60	39.37
GARN1	63.40	68.20	54.00	43.58
GARN2	25.52	34.04	24.75	29.08
GARN3	40.20	51.60	46.20	51.50
INDN1	35.00	37.00	31.57	31.90
INDN2	56.00	47.00	48.60	94.20
INDN3	31.00	31.00	28.80	24.67
ISLE1	47.11	43.20	44.20	42.60
ISLE2	49.60	44.20	54.50	54.60
ISLE3	54.20	50.00	55.78	54.21
MIDD1	95.10	109.00	91.20	78.50
MIDD2	30.90	58.50	43.58	32.66
MIDD3	26.99	33.99	80.90	36.48
NORE1	109.40	166.30	131.10	97.60
NORE2	41.46	46.50	43.55	42.98
NORE3	51.30	66.20	62.10	54.70
PIPE1	31.58	43.50	34.29	41.37
PIPE2	54.43	116.70	86.90	94.70

PIPE3	35.70	43.70	31.40	36.69
ROCK1	32.59	29.67	34.59	35.56
ROCK2	47.00	66.10	61.50	71.40
ROCK3	43.14	56.70	47.80	45.50
TORR1	39.80	56.70	43.06	40.40
TORR2	38.53	73.50	45.37	47.95
TORR3	36.70	68.90	56.40	54.00

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Model (PAR)

Table 5.2 **Akaike Information Criterion (AIC) of Monthly Data**

Rivers	Forecasting Approach Used			
	S	A	E	P
BAYN1	8.63	14.17	14.38	11.03
BAYN2	9.51	16.73	16.34	11.19
BAYN3	14.73	16.70	18.08	15.49
GAND1	36.61	35.00	53.05	49.69
GAND2	33.96	52.14	51.11	57.38
GAND3	42.25	50.69	66.90	59.95
GARN1	2.41	3.93	3.82	1.97
GARN2	2.31	3.87	3.92	2.16
GARN3	2.90	4.25	4.25	2.13
INDN1	7.57	8.61	8.71	9.17
INDN2	9.64	10.63	9.56	7.56
INDN3	5.36	6.81	5.72	7.70
ISLE1	7.91	6.64	6.51	7.93
ISLE2	5.49	6.01	5.85	6.09
ISLE3	8.87	6.59	6.32	10.27
MIDD1	1.96	1.77	1.39	2.05
MIDD2	1.97	1.56	1.39	1.91
MIDD3	2.10	1.50	1.33	2.00
NORE1	0.06	0.06	0.24	0.12
NORE2	0.07	0.05	0.22	0.17
NORE3	0.050	0.04	0.20	0.15
PIPE1	9.55	8.05	2.77	5.06
PIPE2	10.12	7.26	2.62	5.00
PIPE3	11.28	9.46	3.08	5.38
ROCK1	2.70	2.74	1.68	3.98

ROCK2	5.02	3.15	1.70	4.12
ROCK3	2.86	2.74	1.63	4.07
TORR1	0.33	0.26	0.48	0.59
TORR2	0.38	0.32	0.55	0.69
TORR3	0.45	0.35	0.54	0.66

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Model (PAR)

Table 5.3a **COMPARISON OF QUARTERLY FORECASTS**

MAPE - (*Case 1*)

Rivers	Forecasting Approach Used			
	S	A	E	P
DBAYN1	23.96	20.04	17.77	25.55
DGAND1	30.80	25.16	27.20	33.12
DGARN1	39.70	32.36	31.80	59.80
DINDN1	25.49	29.42	22.42	23.70
DISLE1	39.10	36.00	42.80	21.16
DMIDD1	37.80	30.90	36.50	49.50
DNORE1	13.70	10.06	9.06	61.80
DPIPE1	13.76	11.16	21.14	25.05
DROCK1	17.47	17.62	16.47	29.33
DTORR1	32.90	41.60	35.70	41.20

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Models (PAR)

Table 5.3b **COMPARISON OF QUARTERLY FORECASTS**

MAPE - (*Case 2*)

Rivers	Forecasting Approach Used			
	S	A	E	P
DBAYN2	20.60	19.16	12.53	19.11
DGAND2	21.5	21.73	19.80	26.80
DGARN2	46.00	38.9	36.90	43.12
DINDN2	20.91	12.92	20.14	12.80
DISLE2	18.00	9.89	34.60	28.10
DMIDD2	33.20	28.1	33.60	28.70
DNORE2	11.81	10.16	10.86	78.00
DPIPE2	18.22	28.7	19.22	37.50
DROCK2	13.53	16.30	14.66	25.65
DTORR2	8.36	14.27	8.71	24.57

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Model (PAR)

Table 5.3c **COMPARISON OF QUARTERLY FORECASTS**

MAPE - *(Case 3)*

Rivers	Forecasting Approach Used			
	S	A	E	P
DBAYN3	21.40	19.53	15.32	18.92
DGAND3	31.74	22.50	21.00	22.00
DGARN3	31.80	48.82	51.97	50.87
DINDN3	15.40	15.84	27.51	13.90
DISLE3	7.38	31.67	31.04	35.20
DMIDD3	36.60	37.10	38.40	38.80
DNORE3	7.76	34.30	14.89	55.40
DPIPE3	17.20	42.20	31.11	27.04
DROCK3	31.50	21.00	19.80	23.58
DTORR3	19.17	8.99	2.46	17.18

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Model (PAR)

Table 5.4a **Akaike Information Criterion (AIC) of Quarterly Data** (Case 1)

Rivers	Forecasting Approach Used			
	S	A	E	P
DBAYN1	5.29	13.94	13.55	7.06
DGAND1	18.61	42.92	40.89	45.35
DGARN1	7.87	2.79	2.65	3.44
DINDN1	2.17	6.81	6.48	3.00
DISLE1	1.48	4.06	3.83	3.09
DMIDD1	1.15	2.41	2.31	3.05
DNORE1	0.03	0.05	0.05	0.03
DPIPE1	9.65	9.39	9.48	9.49
DROCK1	1.28	3.57	3.41	1.55
DTORR1	11.92	6.89	6.48	3.01

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Model (PAR)

Table 5.4b **Akaike Information Criterion (AIC) of Quarterly Data** (Case 2)

Rivers	Forecasting Approach Used			
	S	A	E	P
DBAYN2	4.94	13.99	13.53	6.40
DGAND2	14.46	42.77	40.80	52.63
DGARN2	5.98	2.78	2.64	3.28
DINDN2	1.80	6.76	6.41	3.00
DISLE2	1.21	4.12	3.84	3.05
DMIDD2	1.24	2.41	2.315	3.03
DNORE2	0.02	0.05	0.05	0.03
DPIPE2	9.87	10.12	9.41	10.21
DROCK2	1.40	3.57	3.42	1.27
DTORR2	11.6	6.88	6.49	3.00

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Model (PAR)

Table 5.4c **Akaike Information Criterion (AIC) of Quarterly Data** (Case 3)

Rivers	Forecasting Approach Used			
	S	A	E	P
DBAYN3	5.58	13.97	13.46	5.84
DGAND3	24.66	42.85	40.84	38.42
DGARN3	11.25	2.77	2.63	3.13
DINDN3	2.70	6.77	6.49	3.00
DISLE3	1.96	4.10	3.85	3.03
DMIDD3	1.52	2.38	2.32	3.02
DNORE3	0.03	0.05	0.05	0.03
DPIPE3	13.15	10.09	9.39	10.65
DROCK3	1.59	3.56	3.40	1.14
DTORR3	14.50	6.86	6.51	3.00

NOTE:

S: New Structural Approach (NSM)

A: Box & Jenkins Approach (ARIMA)

E: Exponential Smoothing (EXS)

P: Periodic Model (PAR)

5.3 Discussion of Results

The performance of forecasts is assessed using the MAPE criterion as the tabulated values in the previous section show. Although the MAPE criterion gives an indication of which models seem to perform better, no statement concerning statistically significant differences in the four forecasting approaches can be made. To address this question the rank-sum test was performed. The forecasting approach which gave minimum MAPE value was assigned rank 1 and so on. The table 5.5 shows the ranks for different approaches for four forecasts horizons of the monthly data. The rank-sums for the models are the sums of the product of the rank and the associated table entry. Thus, models with lower rank-sums performed better than those with larger rank-sums. The rank-sums for the quarterly data are tabulated for the three forecast scenarios in table 5.6 respectively.

The rank-sum test shows that for the monthly data, NSM approach gave lower MAPE values for the 3-month, 6-month, 9-month and 12-month ahead periods. The forecasting accuracy of NSM model increased with the increase in forecasting horizons as shown by decreased rank-sum values.

The rank-sum test for quarterly data, for three forecasting scenarios shows that the EXS approach performed better in general than other approaches. But as the forecast scenario approached critical low flow and high flow periods almost all the approaches performed equally well.

Table 5.5 **Rank-Sums for Monthly Data**

Rank Sum For 3 - Month Ahead Forecasts

Rank	NSM	ARIMA	EXS	PAR
1	12	9	5	4
2	7	6	10	7
3	6	4	12	8
4	5	11	3	11
Rank-Sum	64	77	73	86

Rank Sum For 6 - Month Ahead Forecasts

Rank	NSM	ARIMA	EXS	PAR
1	14	9	4	3
2	9	-	12	10
3	2	9	9	9
4	5	12	5	8
Rank-Sum	58	84	75	82

Rank Sum For 9 - Month Ahead Forecasts

Rank	NSM	ARIMA	EXS	PAR
1	17	5	3	5
2	4	4	10	12
3	6	3	14	7
4	3	18	3	6
Rank-Sum	55	94	77	74

Rank Sum For 12 - Month Ahead Forecasts

Rank	NSM	ARIMA	EXS	PAR
1	16	5	4	5
2	6	2	11	11
3	6	3	13	8
4	2	20	2	6
Rank-Sum	54	98	73	75

Table 5.6 **Rank-Sums for Quarterly Data**

Rank Sum For First Scenario

Rank	NSM	ARIMA	EXS	PAR
1	1	3	5	1
2	2	4	3	1
3	7	1	1	1
4	-	2	1	7
Rank-Sum	26	20	18	34

Rank Sum For Second Scenario

Rank	NSM	ARIMA	EXS	PAR
1	3	3	3	1
2	2	2	4	2
3	2	5	1	2
4	3	-	2	5
Rank-Sum	25	22	22	31

Rank Sum For Third Scenario

Rank	NSM	ARIMA	EXS	PAR
1	5	-	4	1
2	1	4	2	3
3	-	5	2	3
4	4	1	2	3
Rank-Sum	23	27	22	28

Chapter 6

CONCLUSIONS AND RECOMMENDATIONS

The results of the monthly and quarterly time series of Newfoundland rivers were tabulated in the previous Chapter. This Chapter presents the conclusions based on the results obtained and recommends the best method of forecasting for Newfoundland rivers.

6.1 Conclusions

The conclusions of this study are as follows:

1. For the selected monthly time series of Newfoundland rivers the NSM approach gave lower MAPE values in general, thereby forecasting better than ARIMA, EXS and PAR. Of the thirty monthly series considered, the MAPE values for NSM were lower for around 50% of the series. The most common NSM model for the monthly Newfoundland rivers consisted of stochastic level, stochastic slope, deterministic trigonometric seasonality, i.e., with zero variance, no cycle and an irregular component.

2. For the quarterly riverflow series the simple exponential smoothing approach performed better in general than the other approaches. The results obtained showed that the MAPE values were lower for 10 out of the total of 30 quarterly series considered. For the remaining 20 series, NSM, ARIMA and PAR approaches performed equally well. The most common exponential smoothing model was the Winters 3 parameter smoothing model which involves level, trend, and seasonal parameters.

3. For the monthly riverflow series four forecast horizons were considered. Among the thirty series, the number of series for which NSM gave lower MAPE values increased as the forecasting horizon increased from 3 month ahead to 12 month ahead period. Thus, in comparison with other approaches, the forecasting accuracy of NSM approach increased with increased forecasting horizon. For the EXS approach, the forecasting accuracy for 3 month ahead period was close to that of the NSM approach. It can be concluded that among NSM, ARIMA, EXS and PAR approaches, long term forecasting accuracy of NSM, in terms of MAPE values, is better than short term forecasting.

4. The accuracy of forecasts for critical periods for quarterly series is assessed by using three different forecast scenarios. The results for the first scenario, where the last period used for forecasting is far from the critical high and low periods, showed that EXS outperformed NSM, ARIMA and PAR for 40% of the series. For the second and third scenarios, where the record used is nearer to the critical periods, the NSM, EXS and PAR approaches performed equally well. The most common NSM model for quarterly series consisted of deterministic level, deterministic slope, trigonometric

seasonality with zero variance, no cycle and an irregular term.

5. For the monthly Newfoundland rivers it was observed that the stochasticity in level and slope components of NSM approach plays an important role. But for the quarterly series the level and slope components of NSM models are deterministic in nature.

6. The study also shows that the approaches which took physical characteristics of the series into account performed slightly better.

6.2 Recommendations

Although the results of this study show that the NSM approach has a potential to be a viable alternative to the prevalent forecasting methods, however further research needs to be done on the same. The significant areas of research in the use of the NSM approach in hydrology are simulation studies, handling of missing values and intervention analysis to study the effect of, say, forest fires on riverflows.

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APPENDIX A

BOXPLOTS FOR THE RIVERS USED IN THE STUDY

boxplot BAYN

```
          ------------
   -------I   +     I--------------- ****     * *        O
          ------------
   +---------+---------+---------+---------+---------+------BAYN
   0        30        60        90       120       150
```

boxplot GAND

```
         ------------
   -----I   +     I----------------*** ** ***  *      O
         ------------
   +---------+---------+---------+---------+---------+------GAND
   0       100       200       300       400       500
```

boxplot GARN

```
            ------------
   ---------I   +     I--------------- **     *   *   O
            ------------
   +---------+---------+---------+---------+---------+------GARN
  0.0       6.0      12.0      18.0      24.0      30.0
```

boxplot INDN

```
     -------
   --I +   I---------** ******OO OOOOOO          O
     -------
   +---------+---------+---------+---------+---------+------INDN
   0        25        50        75       100       125
```

boxplot ISLE

```
      --- --------
 ----I     +      I----------------******* ***    *
      ---------- --
+---------+---------+---------+---------+---------+---------+------ISLE
0         10        20        30        40        50
```

boxplot MIDD

```
      -------------
 -----I     +      I-----------------** * ** **      * *
      -------------
+---------+---------+---------+---------+---------+---------+------MIDD
0.0       5.0       10.0      15.0      20.0      25.0
```

boxplot NORE

```
      -------------
 -----I     +      I------------------*****    *    *
      -------------
+---------+---------+---------+---------+---------+---------+------NORE
0.00      0.10      0.20      0.30      0.40      0.50
```

boxplot PIPE

```
      -----------
 -----I    +     I-----------------**** *  * ** *
      -----------
+---------+---------+---------+---------+---------+---------+------PIPE
0         20        40        60        80        100
```

boxplot ROCK

```
              --------------
     --------I      +       I----------------- *  ** *    * *
              --------------
 +---------+---------+---------+---------+---------+------ROCK
0.0       7.0      14.0      21.0      28.0      35.0
```

boxplot TORR

```
       ---------
   ---I  +    I-------------****** ******OO O O     O  O
       ---------
 +---------+---------+---------+---------+---------+------TORR
 0        25        50        75       100       125
```

BOX-COX LOG TRANSFORMATION OF TORR
TIME SERIES TO NORMALITY

boxplot loge(TORR)

```
                    ----------------
   -----------------I       +       I-----------------
                    ----------------
 --+---------+---------+---------+---------+---------+----C11
 0.80      1.60      2.40      3.20      4.00      4.80
```