ON THE GROUND WAVE PROPAGATION OVER A CLIFF WITH FINITE GROUND PARAMETERS

HAIYIN WANG

On The Ground Wave Propagation Over A Cliff With Finite Ground Parameters

by

©Haiyin Wang, M. Eng.

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Abstract

The characteristics of the ground-wave propagation over a cliff are investigated using both the residue and the compensation methods. The two methods are reviewed and compared. The theorical analysis for the ground wave propagating from sea to earth is presented in detail. The residue method employes Fourier transform and Green theorem, while the compensation method employes the surface impedance technique. The analysis of the two methods are reintroduced in a more clarified way. The differences between the assumptions of t' e two methods are identified and comparisons of the relative results are illustrated.

The flat-cliff approximation is used to obtain the terrain coefficient so that the problem can be solved when the antennas on the cliff top are close to the cliff edge. The final expressions of terrain coefficient and field distribution over the cliff as well as the sea are presented.

An approximate analysis expression, called the two terms approximation, is presented in this work to calculate terrain coefficient. The two terms approximation is based on the flat-cliff approximation of Green theorem method. It simplifies the numerical calculation of the problem.

Graphs are shown in this work to describ the fields changing over the ocean and over the cliff for the transmitters on the cliff and on the ocean separately. The numerical results are in a good agreement with availible literature data.

The expressions and results given in this work can be used to determine the optimum location and height of an antenna over a cliff.

Presently, a prototype bistatic radar system which uses ground wave propagation for over the horizon target detection is being built on a cliff at Cape Race,

Newfoundland. The radar has a log periodic array antenna as the transmitter and a linear array as a receiver. The results, data and the program given in this work are very helpful in designing new groundwave radar antennas on the cliff since the program can be used to determine the optimum location and height of the antennas for both the source and the receiver.

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Contents

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Nomenclature

A	$\cdots$	terrain coefficient
h	$\cdots$	cliff height
h_s	$\cdots$	the height of the source antenna over earth surface
h_r	$\cdots$	the height of the receiver antenna over earth surface
a	$\cdots$	the average radius of the earth (a=6400km)
k_0	$\cdots$	the wave number of the free space
k_i	$\cdots$	the wave number of section i
r_i	$\cdots$	the radius of the earth in section i, in meters
d_i	$\cdots$	the distance of section i, in km
c_i	$\cdots$	the numerical length of the section i ($c_i = (\frac{d_i}{a})(\frac{k_0 a}{2})^{1/3}$)
E	$\cdots$	the electric field, volts/m
H	$\cdots$	the magnetic field, ampere/m
$H_i^j(z)$	$\cdots$	jth kind Hankel function of the order i and argument z
f	$\cdots$	frequency, cycle/se or Hz
λ	$\cdots$	the wave length in meters
ϵ_r	$\cdots$	relative permittivity of the ground
σ	$\cdots$	conductivity of the ground, v/m
$Ai(z), Bi(z)$	$\cdots$	Airy function of argument z
ϵ	$\cdots$	error function
F_{t_i}	$\cdots$	cliff gain
y_i	$\cdots$	numerical height ($y_i = k_0(r_i - a)(\frac{2}{k_0 a})^{1/3}$)
ψ	$\cdots$	z-component of the magnetic vecter potential
f_{t_m}	$\cdots$	height gain function
Z	$\cdots$	surface impedance, ohm
z_m	$\cdots$	mutual impedance, ohm
I	$\cdots$	electric current
S_{ij}	$\cdots$	scattering coefficient of section i and section j

Chapter 1

Introduction

1.1 Research Rationale

The problem of electromagnetic ground-wave propagation over an inhomogeneous earth has attracted much interest for a long time, for it is relevant to communications, navigation and applied geophysics.

With the advancement of ocean exploration, radar is increasingly used to detect targets at sea and collect environmentally related data from the ocean surface. Often, the land based radar is located on a cliff or beach and works at high frequency employing ground wave propagation mode. Thus, the study of ground wave propagation over a cliff becomes of great importance.

Presently, a prototype bistatic radar system which uses ground wave propagation mode for over-the-horizon target detection is being operated on a cliff at Cape Race, Newfoundland. The radar has a log periodic array antenna as the transmitter, and a linear array antenna as a receiver. Both the transmitter and the receiver are located on the cliff. In this thesis, our attention will focus on the ground-wave propagation between sea and land in both directions across the cliff.

The ground wave propagation over a cliff is a problem of wave propagation over an inhomogeneous earth surface which consists of two sections with different heights and electrical properties. The wave path includes land and sea. The land is higher than or equal to sea-level. The land and sea have different electric permittivities and conductivities.

1.2 Previous Work

The Hertz potential due to a vertical electric infinitesimal dipole at the surface of a spherical, homogeneous earth was derived by van der Pol and Bremmer in 1938 [1]. Wait [2] developed the series expressions of the electromagnetic field components of the above case in terms of magnetic vector potential in 1962.

Many papers have been published on the problem of ground wave propagation over a smooth earth (no vertical discontinuities) consisting of several sections with different electrical properties. Among the various theoretical approaches to the problem was an analytical approach initiated by Grünberg [3] and developed by Feinberg [4,5].

In Grünberg's work, the adoption of approximate boundary conditions and a standard application of Green's theorem led to an integral equation for the normal component of E at the surface of the earth. He solved the problem for the case of two earth media, where one of them has infinite conductivity. The two media were separated by a straight boundary. The incident field was taken to be a plane wave. As an approximate treatment, Grünberg considered that the direction of the wave propagation at a great distance beyond the boundary is the same as that of the incident wave.

Grünberg's work was generalized by Feinberg [4,5]. In Feinberg's work, there was a difference in that a transmitter was located at a finite distance from the boundary. The expressions that were derived took into account the various positions of transmitter and receiver with respect to the boundary.

Feinberg [6] emphasized that, for a smooth earth, the terrain inhomogeneities do not have the same effect for all parts of the wave path but are dependent on their locations. When the inhomogeneity extends only for a short distance so that the flat-earth approximation can be used for that part, its effect is most serious when the inhomogeneity occurs in a terminal area of a long wavepath, while its effect is negligible when the inhomogeneity occurs in the middle part. Conversely, no matter where an inhomogeneous area occurs in the wavepath, its effect is accumulated when it extends over a long distance along the wavepath.

Clemmow [7] showed that the case of a single land/sea boundary for a flat earth was adequately treated by an integral equation method applied to a diffraction theory. The land/sea problem was a two-dimensional problem where the sea was replaced by a perfectly conducting, semi-infinite plane sheet; and the transmitter was a vertically polarized line-source. He took it as a generalization of the classic Sommerfeld half-plane diffraction problem.

In a later paper [8], Clemmow gave a theoretical investigation of vertically polarized radio waves propagating across a boundary between two earth sections with different complex permittivities. The problem was treated as a two dimensional one. The earth surface was assumed to be flat. He solved the boundary value problem for a plane wave incident at an arbitrary angle; the scattered field due to surface currents was expressed as an angular spectrum of plane waves, and this formulation led to dual integral equations solved by the methods of contour integration. After considering the case of both the transmitter and receiver at same ground level, he gave an approximation for an elevated transmitter and receiver. The attenuation and phase curves in a numerical example were given.

Bremmer [9] gave the extension of Sommerfeld's formula for the propagation of radio waves over a flat earth with inhomogeneous soil conductivities. The integral equation used by Bremmer was based on Green's theorem. The solutions for two adjacent regions of homogeneous electric constants were treated numerically with the aid of two different expressions for the field near the separating boundary and for the field far beyond this boundary. The rigorous solution of the integral equation was proved to be identical with the corresponding expression derived in a very different way by Clemmow [8].

There are two other methods to solve the problem of propagation over an inhomogeneous earth taking earth curvature into account, which are the compensation method [10] and the residue method [11-15].

Using an eigenfunction expansion technique and surface impedance approximation based on the compensation method, Wait [10] obtained final expressions for propagation across two or three section earth with the same heights. He also solved radio wave propagation over a perfectly conducting curved ground[16]. In another paper [17], he presented the results for propagation over two section earth with different heights. He also extended the method to three dimentional problem. However, his method and results are only applicable to long distance sections [10].

The residue method was developed by Furutsu [11-13]. It gives the expressions of the attenuation coefficient and field distribution for an inhomogeneous earth with different heights and different electrical properties.

Since 1955, Furutsu developed the theory of propagation of electromagnetic waves over spherical inhomogeneous earth, where the waves are transmitted across one or two boundaries of discontinuity separating different earth media. By employing Fourier transform, Furutsu obtained the formulae corresponding to the ordinary Watson formula [18] for a homogeneous earth. He compared the results with those he got in the case of a flat earth. Then he derived the general expression for the wave propagation across any number of discontinuous boundaries with Green's theorem.

Also, the formulae of field strength in the case of a flat earth, for diffraction of ground waves by a ridge having finite breadth and finite electrical conductivity were derived by Furutsu [12].

Furutsu's formulae for an inhomogeneous earth gave a multiple series to calculate the terrain (attenuation) coefficient. When the propagation distances over all sections of the terrain were large enough , the series was so well convergent that one could use the first term alone to calculate the cliff gain. When the propagation distance over any section was so small that it could be regarded as a flat plane, series convergence became very slow and the formula was not available for practical use. When the terrain was reduced to a homogeneous earth by letting all the heights and electrical properties of the section be equal, the series uniformly reduced to the well-known Bremmer series for a homogeneous spherical earth. When the lengths of the internal sections were sufficiently large, the effects of the terminal sections over which the transmitter and receiver were located became isolated, so the terminal effects or terminal gain could be evaluated in detail. The terminal gain could be used in the same way as the ordinary antenna height gain function had been used for the ground wave propagation over a homogeneous spherical earth.

The obstacle gains were numerically illustrated as functions of the distance between the obstacle and the path terminal for the typical examples of a ridge, a bluff on a homogeneous earth and a cliff at a coastline. The obstacle gain defined in the paper was the gain caused by the earth inhomogeneity and the gain caused by the antenna height. The latter was called the terminal gain too, for the inhomogeneity in the vicinity of the path terminal was particularly emphasized by the effect of antenna height. These obstacle gains tended to become constants when the distance increased. On the other hand, at short distances, the gains were directly affected by a diffraction loss, interference between the direct and reflected waves, and other effects which were not optical in character. A convenient expression of the obstacle gain was introduced for the poor convergence of the relevant residue series. It could be used in the same way as the ordinary antenna height gain over a homogeneous earth. Several figures of ridge gain, bluff gain and cliff gain were shown in Furutsu's work [14,15].

Based on Furutsu's analysis, Pielou, Milson and Herring presented extensive numerical results in case of a cliff [19]. In this work, graphs of site gain contours were given for various combinations of cliff height, path length and frequency. The variation of site gain versus both cliff height and distance inland were also shown. It appeared that cliff height and overland losses contributed independently to the site gain. If the losses from cliff height and from distance inland were given separately, summing these two losses provided an estimate of the expected overall site gain. These authors pointed out that there were a number of restrictions inherent in Furutsu's theory: (i) a path consisting of two or more short sections could not be modlled; (ii) sections must have vertical boundaries; (iii) only two dimentional terrain height variations are allowed so that coastal refraction effects could not be modlled; (iv) reflected signals from a cliff face were neglected. In Furutsu's expressions[14], wave propagates from earth to sea, while in Pielou's results, wave propagates from sea to earth which were computed employing unpublished expressions.

1.3 Scope of Thesis

The purpose of this work is to determine the field distribution for ground wave propagation over a cliff, which is a special case of ground wave propagation over an inhomogeneous earth. The two directions of propagation across the cliff are considered. A new approximation is introduced to calculate the normalized field distribution. Based on the residue method, the expressions of terrain coefficient, cliff gain and field strength in cliff case are given. A user-friendly computer program is developed to obtain the numerical results for different cliff parameters.

The general formula for propagation over a homogeneous earth is given in Chapter 2. The principle to obtain the solution for an inhomogeneous earth using homogeneous earth solutions is described.

The residue method is reexamined in Chapter 3 for ground wave mode.

In Chapter 4, the compensation method is reviewed and compared with the residue method.

Based on the residue method and the compensation method, new approximated expressions for cliff case are given in Chapter 5. These include field distributions, cliff gains and terrain (attenuation) coefficients. The two directions of wave propagation, sea-land and land-sea, are considered. The new approximation, two terms approximation is obtained to simplify the numerical calculation and reduce the computer time.

We show that when the source antenna is located on the cliff top, the groundwave propagates from land to sea. The signal returned from the target goes over the cliff and reaches the receiver antenna on the cliff top. The wave reflects and diffracts at the cliff edge between land and sea. The reflection and the diffraction change the field distribution over the cliff and the sea. On the other hand, the surfaces of land and sea with finite electrical parameters absorb the electromagnetic energy of the wave. All of these facts cause attenuation of the ground wave. So the attenuation of the ground-wave over a cliff depends on the electrical parameters of the sections, the antenna heights, the cliff height and the distances between the antennas and the cliff edge.

In Chapter 6, numerical results are presented and discussed. Graphs show the influence of changing antenna heights, cliff heights and distances between antennas and cliff edge on the terrain coefficient. Approximate and detailed expressions are used to obtain the proporties of different cliff parameters.

Chapter 7 gives the conclusion.

A computer program is developed to realize the numerical calculation for the ground wave propagation over a cliff. The results have good agreement with those published.

Chapter 2

General Formulae of Propagation Over the Inhomogeneous Earth

In this chapter, the general formulas for radio wave propagation over an inhomogeneous earth are derived from first principles. At first, we define the terrain coefficient. Secondly, we present the expressions of electric and magnetic fields $\vec{E}$ and $\vec{H}$, respectively, in terms of the magnetic vector potential $\vec{A}$ in two sets of coordinates; spherical coordinates and the coordinates which are based on Cartesian coordinates that take into account the curvature of the earth. This is followed by using boundary conditions and applying source conditions for homogeneous earth problem in the two coordinate systems. At last, we present expressions for the fields over an inhomogeneous earth geometry.

2.1 Geometry Of The Cliff Boundary

The present terrain structure consists of an air section below which two sections of both different earth radii r_i and different propagation constants k_i (i=2, 3) exist as shown in Fig. 2.1. Throughout the derivation, the source S and receiver R are

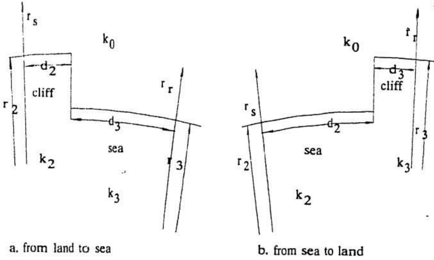


Figure 2.1: Form of three section terrain

always over different sections and their heights from the centre of the earth are r_S and r_R, respectively. Their distances from the cliff edge are d_2 and d_3 measured along a mean earth surface of radius a which is taken at sea level. In Fig. 2.1 (a), wave propagates from land to sea, while in Fig. 2.1 (b), wave direction is from sea to land.

When radio frequency wave propagates along the surface of the earth, its characteristics are influenced mainly by the profile and electrical properties of the earth's surface. The attenuation coefficient has been introduced [14] which is dependent on the earth's curvature, finite ground conductivity, the source and receiver heights above the ground, as well as distances to the cliff edge.

If E is the electric field strength in the case of cliff, the attenuation coefficient A has been defined such that[10]

$$E = AE_0 \tag{2.1}$$

In compensation method, E_0 is written as E_0^c which is given by [10]

$$E_0^c = \frac{-j\mu_0\omega Il}{2\pi d} e^{-jk_0 d} \left(\frac{\theta}{\sin\theta}\right)^{\frac{1}{2}} \tag{2.2}$$

where E_0^c is the vertical electric field of a vertical electric dipole of moment Il at distance $d = d_2 + d_3$, where both source and receiver parts are assumed to be just above the surface of a perfectly conducting ground plane. μ_0 is the permeability of the air. ω is the frequency of the source. $\theta = d/a$, where a is the earth radius.

In the residue method, the definition of the attenuation coefficient A is [14]

$$E = 2AE_0^r \tag{2.3}$$

In this work, where we use two methods; the residue method[11-15] and the compensation method[10]. E_0^r of the residue method is defined as[14]

$$E_0^r = \frac{k_0^2}{4\pi(d_2 + d_3)} e^{-jk_0(d_2+d_3)} \tag{2.4}$$

where k_0 is the propagation constant in free space, $2\pi/\lambda_0$ and λ_0 is the wavelength in free space. E_0^r is considered to be the electrical field strength in free space. It is half of the field strength above perfect conducting ground.

Although some authors [10][14] call A an attenuation coefficient, the author disagrees with this name. When A increases, the field strength increases. So A does not stand for the attenuation of the field. The author believes that it should be called terrain coefficient whose value is less than unity. For perfectely conducting homogeneous ground, the terrain coefficient A rises up to unity. In this work, A will be denoted terrain coefficient.

For later convenience, the numerical distance c_i is employed, which is defined by

[14]:

$$c_i = (\frac{d_i}{a})(\frac{k_0 a}{2})^{1/3} \quad i = 2,3 \tag{2.5}$$

And the numerical height y_i is defined by [14]

$$y_i = k_0(r_i - a)(\frac{2}{k_0 a})^{1/3} \quad i = s, 2, 3, r \tag{2.6}$$

where $r_3 \leq r_2$ in Fig. 2.1(a) and $r_3 \geq r_2$ in Fig. 2.1(b).

2.2 Field Expressions In Different Coordinate Systems

2.2.1 Earth Curvature Modified Cartesian Coordinate System

For a vertically polarized current carring conductor of surface current density J amp/m, the magnetic vector potential $\vec{A}$ has only one component in the z direction i.e.: $A_z\hat{z}$. From the Maxwell equations, we obtain the equation [15]:

$$[(\nabla^2 + k^2]A_z(\vec{r}) = -J_z(\vec{r}) \tag{2.7}$$

where $\vec{r}$ is the position vector at the observed point.

For an infinitesimal current element located at $\vec{r}$, we set

$$k^{-2}\vec{J} = [0, 0, \delta(\vec{r} - \vec{r'})], \quad \vec{A}(\vec{r}) = [0, 0, \psi(\vec{r}, \vec{r'})] \tag{2.8}$$

and get

$$[\nabla^2 + k^2]\psi(\vec{r}, \vec{r'}) = -k^2\delta(\vec{r} - \vec{r'}) \tag{2.9}$$

Here, $\delta(\vec{r})$ is Dirac delta -function in three dimensional space. Also, ∇ operator in Cartesian coordinates is $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$. For any medium, the wave constant k is defined as

$$k^2 = (\epsilon_r + \sigma / i\epsilon_0\omega)\omega^2/c^2 \tag{2.10}$$

where ϵ_r and σ are the relative permittivity and the conductivity of the medium, respectively. ϵ_0 is the permittivity of the free space and c is the light speed.

According to the vector theorem in appendix A, and setting $u_1 = x$, $u_2 = y$ and $u_3 = z$ for coordinate elements dx, dy and dz, any curved line element ds can be expressed as

$$ds^2 = h_1^2 dx^2 + h_2^2 dy^2 + h_3^2 dz^2 \tag{2.11}$$

where h_1, h_2 and h_3 are defined as

$$h_1 \vec{x_0} = \frac{\partial \vec{r}}{\partial x}, \;\; h_2 \vec{y_0} = \frac{\partial \vec{r}}{\partial y}, \;\; h_3 \vec{z_0} = \frac{\partial \vec{r}}{\partial z} \tag{2.12}$$

$\vec{r}$ is the position vector. $\vec{x_0}$, $\vec{y_0}$ and $\vec{z_0}$ are the coordinate unit vectors.

If the curvature of the earth is taken into account, the values of h_i (i=1,2,3) become

$$h_1 = h_2 = \frac{z}{a}, \quad h_3 = 1 \tag{2.13}$$

and equation (11) gives

$$ds^2 = (z/a)^2(dx^2 + dy^2) + dz^2 \tag{2.14}$$

∇^2 of equation(2.9) can be obtained in the earth curvature modified Cartesian Coordinate system by substituting h_1, h_2 and h_3 into equation(A.6) in appendix A.

Since $\vec{E}$ and $\vec{H}$ are related to magnetic vector potential $\vec{A} = \psi \vec{z_0}$ by the following relations

$$\begin{aligned} \vec{E} &= \frac{1}{j\omega\epsilon} \nabla \times \nabla \times \vec{A} \\ \vec{H} &= \nabla \times \vec{A} \end{aligned} \tag{2.15}$$

Using expressions (A.5) in appendix A and (2.13), the components of the $\vec{E}$ and $\vec{H}$ are expressed in terms of ψ for the curved ground as follows:

$$\begin{aligned}
E_x &= \frac{a}{j\omega\epsilon z}\frac{\partial^2\psi}{\partial x\partial z} \\
E_y &= -\frac{a}{j\omega\epsilon z}\frac{\partial^2\psi}{\partial y\partial z} \\
E_z &= -\frac{a^2}{j\omega\epsilon z^2}[\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}] \\
H_x &= \frac{a}{z}\frac{\partial\psi}{\partial y} \\
H_y &= -\frac{a}{z}\frac{\partial\psi}{\partial x} \\
H_z &= 0
\end{aligned} \tag{2.16}$$

2.2.2 Spherical Coordinates

In spherical coordinates, the three parameters h_1, h_2 and h_3 become [23]

$$h_1 = 1,\ \ h_2 = r,\ \ h_3 = r sin\theta \tag{2.17}$$

Since the current density vector $\vec{J}$ is in the $\vec{a_r}$ direction, the magnetic vector potential $\vec{A}$ is $A_r\vec{a}_r$. Accordingly the homogeneous Helmholz equation can be written as [2]

$$(\nabla^2 + k^2)\frac{A_r}{r} = 0 \tag{2.18}$$

where ∇^2 can be found using (2.17) and (A.6) of appendix A.

Letting $A_r = rU$, the above equation becomes

$$(\nabla^2 + k^2)U = 0 \tag{2.19}$$

The components of $\vec{E}$ and $\vec{H}$ can be expressed in terms of U [2]

$$E_r = \tfrac{1}{j\omega\epsilon}(k^2 + \tfrac{\partial^2}{\partial r^2})(rU)$$

$$
\begin{aligned}
E_\theta &= \frac{1}{j\omega\epsilon r}\frac{\partial^2 rU}{\partial r\partial\theta} \\
E_\phi &= \frac{1}{j\omega\epsilon r\sin\theta}\frac{\partial^2 rU}{\partial r\partial\theta} \\
H_r &= 0 \\
H_\theta &= \frac{1}{r\sin\theta}\frac{\partial rU}{\partial\phi} \\
H_\phi &= -\frac{1}{r}\frac{\partial rU}{\partial\theta}
\end{aligned}
\tag{2.20}
$$

2.3 Analysis of a Vertically Polarized Infinitesimal Dipole over Ground (Homogeneous Case)

This homogeneous problem is a two section problem. Section 1 is air space above section 2; the ground which is smooth earth with radius r_m as shown in Fig. 2.2. The two sections have different wave numbers k_0 and k_m, where m takes value 2 or 3 to accord for the different ground parameters in later chapters. For a vertical dipole above the curved ground, the equations for the magnetic vector potential $\vec{A} = A_z\hat{z}$ in earth curvature modified Cartesian coordinates become [21]

$$
\begin{aligned}
(\nabla^2 + k_0^2)A_{z1}(\vec{r}) &= -p\delta(\vec{r} - \vec{r_0}) \;\; z \geq z_0 \\
(\nabla^2 + k_m^2)A_{zm}(\vec{r}) &= 0 \;\; z \leq z_0
\end{aligned}
\tag{2.21}
$$

where $z_0 = r_m$ is the z coordinate of the boundary between the earth and air.

In the spherical coordinates, the equations for the magnetic vector potential $\vec{A} = A_r\hat{r}$ becomes [2]

$$
\begin{aligned}
(\nabla^2 + k_0^2)\frac{A_{r1}}{r} &= -\delta(\vec{r} - \vec{r'}) \;\; r \geq r_m \\
(\nabla^2 + k_m^2)\frac{A_{rm}}{r} &= 0 \;\; r \leq r_m
\end{aligned}
\tag{2.22}
$$

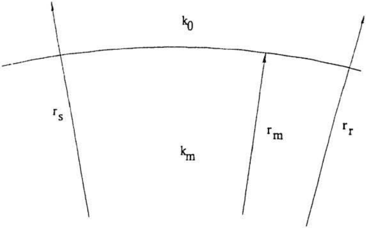


Figure 2.2: Form of two section terrain

From the continuity condition of the tangential components of the fields at the boundary, one can use the following boundary conditions in the earth curvature modified Cartesian Coordinate system [15],

$$(\psi_1)_s = (\psi_m)_s$$
$$(k_0^{-2}\frac{\partial}{\partial n}\psi_1)_s = (k_m^{-2}\frac{\partial}{\partial n}\psi_m)_s \tag{2.23}$$

Where $\partial/\partial n = \partial/\partial z$.

and in spherical coordinates[2]

$$(A_{r1})_{r_m} = (A_{rm})_{r_m}$$
$$(k_0^{-2}\frac{\partial A_{r1}}{\partial r})_{r_m} = (k_m^{-2}\frac{\partial A_{rm}}{\partial r})_{r_m} \tag{2.24}$$

The source condition can be derived from equation (2-7) and equation(2-9)[21]. Now, let us consider an electric dipole oriented in z direction. Because ψ_1 is continuous at every point of space and continuous at the point of the source, integrating two sides of equation (2.21), we obtain the source conditions in Cartesian coordinates

$$(\psi_1)_{z>z'} = (\psi_1)_{z<z'}$$
$$(\frac{\partial \psi_1}{\partial z})_{z>z'} - (\frac{\partial \psi_1}{\partial z})_{z<z'} = -\vec{J}(z') \tag{2.25}$$

The boundary conditions and the source conditions are used to decide the unknown constants in the solutions of the equations.

2.4 The Inhomogeneous Ground Problem

In this case, there is one air section over two ground sections. The two ground sections are to have different level and different parameters. If $\psi'(x)$ and $\psi''(x)$ are continuous functions, we have

$$\int_v [\psi'[(\nabla k^{-2}\nabla) + 1]\psi'' - \psi''[(\nabla k^{-2}\nabla) + 1]\psi']dv = -[\psi'(s), \psi''(s)] \tag{2.26}$$

From Green's theorem, we have[15]

$$[\psi'(s), \psi''(s)] = \int_s [\psi'(\frac{1}{k^2}\frac{\partial}{\partial n}\psi'') - \psi''(\frac{1}{k^2}\frac{\partial}{\partial n}\psi')]ds \tag{2.27}$$

Having obtained the wave function ψ_m for the homogeneous earth with radius r_m and the propagation constant k_m as shown in Fig. 2.3, where m takes 2 or 3, we define $\psi_{32}(r_R, s)$ as the wave function of the inhomogeneous problem, while $\psi_2(s, r_S)$ is the wave function for the homogeneous case at $r = r_2$ where the source exist. Setting $\psi'(s) = \psi_{32}(r_R, s)$ and $\psi''(s) = \psi_2(s, r_S)$ in equation(2.26), and taking the integration volume as the domains of media k_1 in the entire range $z > a_2$ in addition to the k_2 domain. The left side of equation(2.26) becomes

$$\int_v -[\psi_{32}(r_R, r)\delta(r - r_S) - \psi_2(r, r_S)\delta(r - r_R)]dv$$

$$
\begin{aligned}
&= \quad -\psi_{32}(r_R, r_S) + \psi_2(r_R, r_S) \\
&= \quad -[\psi_{32}(r_R, s_3), \psi_2(s_3, r_S)] \qquad (2.28)
\end{aligned}
$$

where s_3 is the surface enclosing the above volume. It extends from $z = a_2$, $x \geq 0$ to $z \leq a_2$, $x = 0$ as shown in Fig. 2.3.

We obtain the wave propagating over a cliff as

$$
\begin{aligned}
\psi_{32}(r_R, r_S) &= \psi_2(r_R, r_S) + [\psi_{32}(r_R, s_3), \psi_2(s_3, r_S)] \quad r_R > r_2 \\
\psi_{32}(r_R, r_S) &= [\psi_{32}(r_R, s_3), \psi_2(s_3, r_S)] \quad r_R < r_2 \qquad (2.29)
\end{aligned}
$$

where s_2 is the surface including the regain (2), i.e. $x \leq 0$, $z \leq r_2$ as shown in Fig. 2.3. Note that $\psi_2(r_R, r_S) = 0$ if $r_2 < r_R$ since the second term of integration in (2.28) does not include the receiver point r_R.

Now, we set $\psi'(s) = \psi_{32}(r_R, s)$ and $\psi''(s) = \psi_3(s, r_S)$ in equation (2.26), i.e. the source is in regian (3). Accordingly, one may get the following relation

$$
\psi_{32}(r_R, r_S) = \psi_3(r_R, r_S) + [\psi_{32}(r_R, s_2), \psi_3(s_2, r_S)] \qquad (2.30)
$$

Setting $r_S = s_3$ in (2.30), we have the expression of $\psi_{32}(r_R, s_3)$. After we substitute (2.30) into (2.29) and omit the higher order terms, we get

$$
\begin{aligned}
\psi_{32}(r_R, r_S) &\simeq \psi_2(r_R, r_S) + [\psi_3(r_R, s_3), \psi_2(s_3, r_S)] \quad r_R > r_2 \\
\psi_{32}(r_R, r_S) &\simeq [\psi_3(r_R, s_3), \psi_2(s_3, r_S)] \quad r_R < r_2 \qquad (2.31)
\end{aligned}
$$

This is a first order approximation. The higher order terms are important only when r_S or r_R exists in the immediate vicinity of the vertical boundary surface. The amplitudes of these terms decrease rapidly with distances from the boundary.

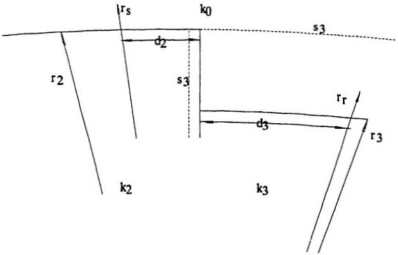


Figure 2.3: Integration domain of eq. (2.31)

Now, if we have obtained the wave functions ψ_2 and ψ_3 for homogeneous earth of radii r_2 and r_3, respectively, we can use equation (2.31) to derive the wave function ψ_{32} for three section problem with radii r_2 and r_3.

Chapter 3

Residue Method Approach

The wave function for the inhomogeneous earth with two sections which has bèen derived using Fourier transforms and Green's theorem [11-15] will be reexamined for ground wave propagation mode. The terrain (attenuation) coefficient is obtained for an inhomogeneous earth of one long distance section (sea) while the other is a short section (coast). Present work will examine the derivation applied to the given geometry and determine appropriate expressions to the ground wave propagation mode of a coastal based ground wave radar. Present analysis is based on considering the boundary of the cliff is vertical to both the path and the earth surface, and it is infinitely along y direction, since the method is applicable to a two dimensional problem.

3.1 Terrain Coefficient for an Inhomogeneous Problem

The terrain coefficient for the inhomogeneous problem is obtained in three steps. First, the wave function for the homogeneous earth is derived by means of Fourier transform. Secondly, the wave function for an inhomogeneous earth is obtained using the results of a homogeneous earth by means of Green's theorem as illustrated in Chapter 2. Finally, the expression is arranged to get the terrain coefficient.

3.1.1 Wave Function for a Homogeneous Earth

The wave function for a homogeneous earth is the solution of the vector potential in Helmholtz's equations. Fourier transform is used to change the three dimensional problem into a one dimension problem and the solution in terms of Fourier integral expression for Helmholtz's equations is given. Then, the unknown constants in the solution are decided by using boundary conditions. The final solution is obtained by performing inverse Fourier transform, i.e. by employing complex contour integration in the complex plane.

Taking the curvature of the earth into account and using equations (2.9) (2.13) and equation (A.6) in appendix A, we have

$$[\frac{1}{z^2}\frac{\partial}{\partial z}(z^2\frac{\partial}{\partial z}) + \frac{a^2}{z^2}((\frac{\partial}{\partial x})^2 + (\frac{\partial}{\partial y})^2) + k^2]\psi(\vec{r},\vec{r}') = -k^2\delta(\vec{r}-\vec{r}') \tag{3.1}$$

Where the direction of z-axis is vertical to the surface of the earth whose radius is a. $\vec{r}$ and $\vec{r}'$ are position vectors of the source and the observor, respectively. $\psi(\vec{r},\vec{r}')$ is the field radiated due to the vertical infinitesimal linear current source at the point $\vec{r}'$.

Solution of the above equation will start by introducing the sets of the function $\psi_\lambda(r)$ and $\overline{\psi}_\lambda(r)$ [24] as possible solutions for the homogeneous equation

$$(\nabla^2 + k^2)\psi(r) = 0 \tag{3.2}$$

By considering three separate functions for the variable x, y, z, the function ψ can be written as

$$\psi_\lambda(r) = e^{-i(\lambda_1 x + \lambda_2 y)} f_\lambda(z) \tag{3.3}$$

$$\overline{\psi}_\lambda(r) = e^{i(\lambda_1 x + \lambda_2 y)} f_\lambda(z) \tag{3.4}$$

Where $\lambda^2 = \lambda_1^2 + \lambda_2^2$. λ_1 and λ_2 are variables in frequency domain.

Using either ψ_λ or $\overline{\psi}_\lambda$ in Eq.(3.3) and (3.4), we get

$$[\frac{1}{z^2}\frac{\partial}{\partial z}(z^2\frac{\partial}{\partial z}) + (k^2 - \frac{a^2\lambda^2}{z^2})]f_\lambda = 0 \tag{3.5}$$

The general solutions for f_λ are in the form of a Hankel function, i.e.

$$\begin{aligned} f_\lambda(z) &= (kz)^{-1/2} H_n^{(1)(2)}(kz) \\ n &= (a^2\lambda^2 + 1/4)^{1/2} \end{aligned} \tag{3.6}$$

Where $H_n^{(1)}(x)$ and $H_n^{(2)}(x)$ are the first and second kind Hankel functions of order n and argument x, respectively.

Defining $f_\lambda^+(z)$ as the wave propagating along the positive z-direction in the domain $z > a$, and $f_\lambda^-(z)$ as the wave along the negative z-direction in the domain $z < a$, equation (6) can be modified to give the following set of two equations.

$$\begin{aligned} f_\lambda^+(z) &= (k_0 z)^{-1/2} H_n^{(2)}(k_0 z) \quad z > a \\ f_\lambda^-(z) &= (k_2 z)^{-1/2} H_n^{(1)}(k_2 z) \quad z < a \end{aligned} \tag{3.7}$$

Where k_0 and k_2 are wave numbers in air ($z > a$) and ground ($z < a$) respectively.

Substituting equation(3.8) into equation(3.3) and (3.4), $\psi_\lambda^\pm(r)$ and $\overline{\psi_\lambda^\pm}(r)$ are given by

$$\begin{aligned} \psi_\lambda^\pm(r) &= e^{-i(\lambda_1 x + \lambda_2 y)} f_\lambda^\pm(z) \\ \overline{\psi_\lambda^\pm}(r) &= e^{i(\lambda_1 x + \lambda_2 y)} f_\lambda^\pm(z) \end{aligned} \tag{3.8}$$

Substituting the above expressions into equation(2.27), and using the relation $ds = (z/a)^2 dxdy$, we get

$$[\overline{\psi_\lambda^-}(s), \psi_{\lambda'}^-(s)] = [\overline{\psi_\lambda^+}(s), \psi_{\lambda'}^+(s)] = 0 \tag{3.9}$$

and

$$[\overline{\psi_\lambda^-}(s), \psi_{\lambda'}^+(s)] = -[\overline{\psi_\lambda^+}(s), \psi_{\lambda'}^-(s)] = (2\pi)^2 c(\lambda)^{-1}\delta(\lambda - \lambda') \tag{3.10}$$

where

$$\begin{aligned} \delta(\lambda - \lambda') &= \delta(\lambda_1 - \lambda_1')\delta(\lambda_2 - \lambda_2') \\ c(\lambda)^{-1} &= (\frac{z}{a})^2 [f_\lambda^-(z)(k^{-2}\frac{\partial}{\partial z} f_\lambda^+(z)) - f_\lambda^+(z)(k^{-2}\frac{\partial}{\partial} f_\lambda^-(z))] \end{aligned} \tag{3.11}$$

From the outward propagating wave condition, $\psi(r, r')$ can be written as

$$\begin{aligned} \psi(r, r') &= \int_{-\infty}^{\infty} d\lambda \psi_\lambda^+(r) a^+(\lambda) \quad z > z' \\ &= \int_{-\infty}^{\infty} d\lambda \psi_\lambda^-(r) a^-(\lambda) \quad z < z' \end{aligned} \tag{3.12}$$

Where z' is the source height.

Using equation (2.26), and equations (3.1) and (3.2), one can get

$$\begin{aligned} \overline{\psi_{\lambda'}^+}(r') &= \int \overline{\psi_{\lambda'}^+}(r)\delta(r - r')dv \\ &= \int k^{-2}[\overline{\psi_{\lambda'}^+}(r)k^2\delta(r - r') + \psi(r, r') * 0]dv \\ &= \int k^{-2}[-\overline{\psi_{\lambda'}^+}(r)(\Delta + k^2)\psi(r, r') + \psi(r, r')(\Delta + k^2)\overline{\psi_{\lambda'}^+}(r)] \\ &= |[\overline{\psi_\lambda^+}(s), \psi(s, r')]|_{s(z>z')}^{s(z<z')} \end{aligned} \tag{3.13}$$

Substituting equation (3.12) into (3.13), one gets

$$\overline{\psi_{\lambda'}^+}(r') = \int_{-\infty}^{\infty} d\lambda [\overline{\psi_{\lambda'}^+}(s), \psi_\lambda^-(s)] a^-(\lambda)_{s(z<z')} = -(2\pi)^2 c(\lambda')^{-1} a^-(\lambda') \tag{3.14}$$

In a similar way, we can obtain the expression for $\overline{\psi}^{-}_{\lambda'}(r')$ in terms of $a^{+}(\lambda')$. So we determine the $a^{+}(\lambda)$ and $a^{-}(\lambda)$ in equation (3.14) as

$$\begin{aligned} a^{-}(\lambda) &= -(2\pi)^{-2}c(\lambda)\overline{\psi}^{+}_{\lambda}(r') \\ a^{+}(\lambda) &= -(2\pi)^{-2}c(\lambda)\overline{\psi}^{-}_{\lambda}(r') \end{aligned} \tag{3.15}$$

Substituting above $a^{+}(\lambda)$ and $a^{-}(\lambda)$ into equation(3.12) leads to:

$$\begin{aligned} \psi(r,r') &= -(2\pi)^{-2}\int_{-\infty}^{\infty} d\lambda\psi^{+}_{\lambda}(r)c(\lambda)\overline{\psi}^{-}_{\lambda}(r') \quad z > z' \\ &= -(2\pi)^{-2}\int_{-\infty}^{\infty} d\lambda\psi^{-}_{\lambda}(r)c(\lambda)\overline{\psi}^{+}_{\lambda}(r') \quad z < z' \end{aligned} \tag{3.16}$$

At the boundary i.e. z=a, the constant $c(\lambda)$ is obtained as

$$c(\lambda)^{-1} = f^{-}_{\lambda}(a)(k^{-2}\frac{\partial}{\partial z}f^{+}_{\lambda}(z))_{z=a} - f^{+}_{\lambda}(a)(k^{-2}\frac{\partial}{\partial z}f^{-}_{\lambda}(z))_{z=a} \tag{3.17}$$

In the integration domain, $\lambda^2 \sim k_0^2$, and $|k_2 - \lambda|a \gg 1$, for very large order n, we can write $f^{-}_{\lambda}(z)$ as

$$f^{-}_{\lambda}(z) \simeq \frac{2^{1/2}}{\pi}(k_2 z)^{-1/2}(k_2^2 z^2 - n^2)^{-1/4}e^{i[\sqrt{k_2^2z^2-n^2}-n\cos^{-1}(n/k_2 z)-\pi/4]} \tag{3.18}$$

The above expression is still valid under the condition $|\sqrt{k_2^2z^2 - n^2}| \gg 1$ and $arg(\sqrt{k_2^2z^2 - n^2}) \neq -\frac{2\pi}{3}$. This leads to

$$\frac{\partial}{\partial z}f^{-}_{\lambda}(z) = i\sqrt{k_2^2 - \lambda^2}f^{-}_{\lambda}(z) \tag{3.19}$$

$$f^{-}_{\lambda}(a + \Delta z)/f^{-}_{\lambda}(a) \simeq e^{i\sqrt{k^2-\lambda^2}\Delta z}, \quad \Delta z < 0 \tag{3.20}$$

substituting (3.19) into (3.17) and realizing that the wave number above $z = a$ is k_0, while it is k_2 for $z < a$, $c(\lambda)$ becomes

$$\begin{aligned} c(\lambda)^{-1} &= f^{-}_{\lambda}(a)[k_0^{-2}\frac{\partial}{\partial a}f^{+}_{\lambda}(a) - ik_2^{-2}\sqrt{k_2^2 - \lambda^2}f^{+}_{\lambda}(a)] \\ &= -2if^{-}_{\lambda}(a)f^{+}_{\lambda}(a)k_2'^{-1}b_2(\lambda)^{-1} \end{aligned} \tag{3.21}$$

Where

$$\begin{aligned} b_2(\lambda) &= 2k_0^2/[k_0^2 + k_2' h(\lambda)] \\ h(\lambda) &= i[\frac{\partial}{\partial z} g_\lambda(z,a)]_{z=a} \\ k_2' &= k_2^2(k_2^2 - \lambda^2)^{-1/2} \\ g_\lambda(z,a) &= f_\lambda^+(z)/f_\lambda^+(a) \end{aligned} \tag{3.22}$$

Substituting the above expressions (3.8), (3.20) and (3.22) into (3.16), the solution of the wave function in free space above ground medium m becomes

$$\begin{aligned} \psi_m(r,r') &= -\frac{i}{8\pi^2}\int_{-\infty}^{\infty} d\lambda k_m' b_m(\lambda) g_\lambda(z,r_m) \\ &\quad e^{-i[\lambda_1(x-x')+\lambda_2(y-y')+\sqrt{k_m^2-\lambda^2}(a_m-z')]} \\ d\lambda &= d\lambda_1 d\lambda_2, \quad z \geq r_m, \quad z' \leq r_m \end{aligned} \tag{3.23}$$

Where r_m is the radius at the surface of ground medium m.

To find the wave function in the region $z \geq z' \geq r_m$, we proceed as following. For the case of $z > r_m$ and $z' > r_m$, we divide the space into three regions i.e. (1) $z \geq z'$, (2) $z' \geq z \geq r_m$ and (3) $z \leq r_m$. For the first region of $z \geq z'$, wave propagates outward, so wave function of (3.16) can be rewritten as

$$\psi(r,r') = -(2\pi)^{-2}\int_{-\infty}^{\infty} f_\lambda^+(z)A(\lambda)e^{-i[\lambda_1(x'-x)+\lambda_2(y'-y)]}d\lambda \quad z \geq z' > r_m \tag{3.24}$$

Where $A(\lambda)$ is a propagation coefficient.

For the second region of $z' \geq z \geq r_m$, there are direct wave from the source (inward) and reflected wave from the ground surface (outward) so that the wave function of (3.16) becomes

$$\psi(r,r') = -(2\pi)^{-2}\int_{-\infty}^{\infty}(D(\lambda)f_\lambda^-(z) + R(\lambda)f_{\lambda'}^+(z))e^{-i[\lambda_1(x'-x)+\lambda_2(y'-y)]}d\lambda \quad z' \geq z \geq r_m \tag{3.25}$$

Where $D(\lambda)$ and $R(\lambda)$ are derictive and reflected wave coefficients.

For the third region when $z \leq r_m$, wave function of (3.16) becomes

$$\psi(r,r') = -(2\pi)^{-2}\int_{-\infty}^{\infty} f_\lambda'^{-}(z)c(\lambda)f_\lambda^{+}(z')e^{-i[\lambda_1(x'-x)+\lambda_2(y'-y)]}d\lambda \quad z \leq r_m \tag{3.26}$$

Where

$$f_\lambda^{+}(z) = (k_0 z)^{-1/2}H_n^{(2)}(k_0 z) \quad z > r_m$$

$$f_\lambda^{-}(z) = (k_1 z)^{-1/2}H_n^{(1)}(k_0 z) \quad z > r_m$$

$$f_\lambda'^{-}(z) = (k_2 z)^{-1/2}H_n^{(1)}(k_2 z) \quad z < r_m \tag{3.27}$$

Applying the boundary conditions of (2.23) at $z = r_m$, we have

$$f_\lambda^{-}(r_m)D(\lambda) + f_\lambda^{+}(r_m)R(\lambda) = f_\lambda'^{-}(r_m)c(\lambda)f_\lambda^{+}(z') \tag{3.28}$$

$$k_1^{-2}\frac{\partial}{\partial z}f_\lambda^{-}(z)_{z=r_m}D(\lambda) + k_1^{-2}\frac{\partial}{\partial z}f_\lambda^{+}(z)_{z=r_m}R(\lambda) = k_2^{-2}c(\lambda)f_\lambda^{+}(z')\frac{\partial}{\partial z}f_\lambda'^{-}(z)_{z=r_m} \tag{3.29}$$

and from the source condition of (2.25) at $z = z'$, we have

$$f_\lambda^{-}(z')D(\lambda) + f_\lambda^{+}(z')R(\lambda) - f_\lambda^{+}(z')A(\lambda) = 0 \tag{3.30}$$

Solving the above set of three linear equations for three unknowns $A(\lambda)$, $D(\lambda)$ and $R(\lambda)$ finally gives,

$$\Delta = -f_\lambda^{+}(z')[f_\lambda^{-}(r_m)k_0^{-2}\frac{\partial}{\partial z}f_\lambda^{+}(z)_{z=r_m} - f_\lambda^{+}(r_m)k_0^{-2}\frac{\partial}{\partial z}f_\lambda^{-}(z)_{z=r_m} \tag{3.31}$$

and

$$\begin{aligned}\Delta_A = \;& f_\lambda^{+}(z')[f_\lambda^{+}(z')f_\lambda^{-}(r_m)/f_\lambda^{+}(r_m) - f_\lambda^{-}(z') + c(\lambda)f_\lambda^{+}(z')f_\lambda'^{-}(r_m)/f_\lambda^{+}(r_m)k_0^{-2} \\ & (f_\lambda^{+}(r_m)\tfrac{\partial}{\partial z}f_\lambda^{-}(z)_{z=r_m} - f_\lambda^{-}(r_m)\tfrac{\partial}{\partial z}f_\lambda^{+}(z)_{z=r_m})]\end{aligned} \tag{3.32}$$

Defining $g^{(1)}(z', r_m) = f_\lambda^-(z')/f_\lambda^-(r_m)$, $A(\lambda)$ is given by

$$
\begin{aligned}
A &= \frac{\Delta_A}{\Delta} \\
&= \frac{k_0^2 f_\lambda^-(r_m)[g_\lambda(z', r_m) - g_\lambda^{(1)}(z', r_m)]}{f_\lambda^+(r_m) f_\lambda^-(r_m)[\frac{\partial}{\partial z} g_\lambda^{(1)}(z, r_m)_{z=r_m} - \frac{\partial}{\partial z} g_\lambda(z, r_m)_{z=r_m}]} + c(\lambda) f_\lambda^+(z') \frac{f_\lambda'^-(r_m)}{f_\lambda^+(r_m)} \\
&= \frac{k_0^2 [g_\lambda(z', r_m) - g_\lambda^{(1)}(z', r_m)]}{f_\lambda^+(r_m)[\frac{\partial}{\partial z} g_\lambda^{(1)}(z, r_m)_{z=r_m} - \frac{\partial}{\partial z} g_\lambda(z, r_m)_{z=r_m}]} + c(\lambda) g_\lambda(z', r_m) f'^-_\lambda(r_m) \quad (3.33)
\end{aligned}
$$

Substituting $c(\lambda) = \frac{i}{2}[f_\lambda^+(r_m) f_\lambda'^-(r_m)]^{-1} k_2' b_2(\lambda)$ into the above equation, we have

$$
A = \frac{k_0^2 [g_\lambda(z', r_m) - g_\lambda^{(1)}(z', r_m)] f_\lambda^+(r_m)^{-1}}{\frac{\partial}{\partial z} g_\lambda^{(1)}(z, r_m)_{z=r_m} - \frac{\partial}{\partial z} g_\lambda(z, r_m)_{z=r_m}} + \frac{i}{2} k_2' b_2(\lambda) \frac{g_\lambda(z', r_m)}{f_\lambda^+(r_m)} \quad (3.34)
$$

The wave function is given by

$$
\begin{aligned}
\psi(r, r') &= -(2\pi)^{-2} \int_{-\infty}^{\infty} f_\lambda^+(z) A e^{-i[\lambda_1(x-x')+\lambda_2(y-y')]} d\lambda \\
&= \frac{k_0^2}{4\pi^2} \int_{-\infty}^{\infty} \frac{[g_\lambda^{(1)}(z', r_m) - g_\lambda(z', r_m)] g_\lambda(z, r_m)}{[\frac{\partial}{\partial z} g_\lambda^{(1)}(z, r_m) - \frac{\partial}{\partial z} g_\lambda(z, r_m)]_{z=r_m}} e^{-i[\lambda_1(x-x')+\lambda_2(y-y')]} d\lambda \\
&\quad - \frac{i}{8\pi^2} \int_{-\infty}^{\infty} k_2' b_2(\lambda) g_\lambda(z', r_m) g_\lambda(z, r_m) e^{-i[\lambda_1(x-x')+\lambda_2(y-y')]} d\lambda \quad z > z' \quad (3.35)
\end{aligned}
$$

$$(3.36)$$

The above expression can be simplified by using the following parameters

$$
\begin{aligned}
b_m(\lambda) &= 2k_0^2/[k_0^2 + k_m' h_m(\lambda)] \\
h_m(\lambda) &= i[\frac{\partial}{\partial z} g_\lambda(z, r_m)]_{z=r_m} \\
h_m^{(1)}(\lambda) &= i[\frac{\partial}{\partial z} g_\lambda^{(1)}(z, r_m)]_{z=r_m}
\end{aligned}
\quad (3.37)
$$

Finally, expression (3.35) can be rewritten as follows

$$
\psi_m(r, r') = \frac{-i}{8\pi^2} \int_{-\infty}^{\infty} k_m' g_\lambda(z, r_m)[b_m(\lambda) g_\lambda(z', r_m) - (2k_0^2/k_m')
$$

the form

$$\psi_2 = \frac{-i}{8\pi^2}\int_{-\infty}^{\infty} d\lambda k_2' g_\lambda(z,r_2)[b_2(\lambda)g_\lambda(z',r_2) - (2k_0^2/k_2')$$
$$(h_2(\lambda) - h_2^{(1)}(\lambda))^{-1}(g_\lambda(z',r_2) - g_\lambda^{(1)}(z',r_2))]$$
$$e^{-i[\lambda_1(x-x')+\lambda_2(y-y')]}$$
$$z > z' \geq r_2 \tag{3.41}$$

$$\psi_3 = \frac{-i}{8\pi^2}\int_{-\infty}^{\infty} d\lambda k_3' b_3(\lambda')g_\lambda(z',r_3)e^{-i[\lambda_1(x'-x)+\lambda_2(y'-y)+\sqrt{k_3^2-\lambda^2}(r_3-z)]}$$
$$z > r_3, \quad z' \leq r_3 \tag{3.42}$$

Substituting ψ_2 and ψ_3 into eq. (2.27),(2.31), the inhomogeneous wave function for a receiver in the air section above region 3 (soil) due to source in air above region 2 (sea), ψ_{32} is given by

$$\psi_{32} \cong \int_{-\infty}^{0} dx \int_{-\infty}^{\infty} dy \psi_3(r_R,r)[\frac{1}{k_0^2}\frac{\partial}{\partial z}\psi_2(r,r_S)]$$
$$-[\frac{1}{k_3^2}\frac{\partial}{\partial z}\psi_3(r_R,r)]\psi_2(r,r_S)_{z=r_3}$$
$$= \frac{-i}{(8\pi^2)^2}\int\int_{-\infty}^{\infty} d\lambda d\lambda' k_2' k_3' b_3(\lambda')g_{\lambda'}(z_R,r_3)$$
$$[(\frac{\sqrt{k_3^2-\lambda'^2}}{k_3^2} + \frac{i}{k_0^2}\frac{\partial}{\partial z})g_\lambda(z,r_2)]_{z=r_3} e^{i(\lambda_1 x_S - \lambda_1' x_R + \lambda_2 y_S - \lambda_2' y_R)}$$
$$[b_2(\lambda)g_\lambda(z_S,r_2) - \frac{2k_0^2}{k_2'}(h_2(\lambda) - h_2^{(1)}(\lambda))^{-1}(g_\lambda(z_S,r_2) - g_\lambda^{(1)}(z_S,r_2))]$$
$$\int_{-\infty}^{0} e^{-i(\lambda_1-\lambda_1')x}dx \int_{-\infty}^{\infty} e^{-i(\lambda_2-\lambda_2')y}dy \tag{3.43}$$

where z_S and z_R are the z components of the source and the receiver positions. One should notice that in the above integration using the surface s_2, z is set to r_3 and x changes from $-\infty$ to 0.

$$(h_m(\lambda) - h_m^{(1)}(\lambda))^{-1}(g_\lambda(z', r_m) - g_\lambda^{(1)}(z', r_m))]$$
$$e^{-i[\lambda_1(x-x')+\lambda_2(y-y')]}d\lambda, \quad z \geq z' \geq r_m \tag{3.38}$$

Since $a\lambda \gg 1$ and $\lambda \simeq k_0$, expressions for $g_\lambda(z, r_m)$ and $g_\lambda^{(1)}(z, r_m)$ can be simplified as follows

$$\begin{aligned} g_\lambda(z, r_m) &= \chi(\beta_z)/\chi(\beta_m) \\ g_\lambda^{(1)}(z, r_m) &= \chi^{(1)}(\beta_z)/\chi(\beta_m) \end{aligned} \tag{3.39}$$

The details of this modification are included in appendix B, where

$$\begin{aligned} \chi(\beta) &= \beta^{1/2} H_{1/3}^{(2)}(\frac{2}{3}\beta^{3/2}) \\ \chi^{(1)}(\beta) &= \beta^{1/2} H_{1/3}^{(1)}(\frac{2}{3}\beta^{3/2}) \\ \beta_z &= (2/k_0 a)^{1/3}(k_0 z - \lambda a) \\ \beta_m &= (2/k_0 a)^{1/3}(k_0 r_m - \lambda a) \\ k_m' &= k_m^2(k_m^2 - k_0^2)^{-1/2} \end{aligned} \tag{3.40}$$

$H_{1/3}(x)$ is the Hankel function of the order 1/3 and argument x. The super script (1) and (2) represent inward and outward propagating wave, respectively.

3.1.2 Wave Function for the Inhomogeneous Earth

For the inhomogeneous earth problem under study where the terrain includes three sections, i.e. air, soil and ocean shown in fig. 2.1, the wave function can be obtained by substituting the wave functions of homogeneous earth mentioned in the previous section into expression (2.27). For the case shown in Fig. 2.1.b where $r_2 \leq r_3$ and the source is above or on the top of surface r_2, the wave functions ψ_2 and ψ_3 are in

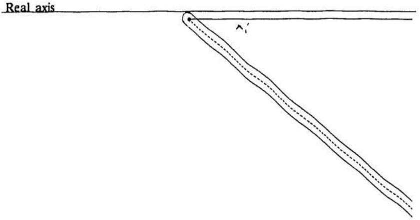


Figure 3.1: Integration path of λ_1' for eq. (3.43)

From the defination of the functions $b_m(\lambda)$,g_λ and $h_m(\lambda)$in equation(3.37), (3.39), we obtain

$$[(\frac{\sqrt{k_3^2-\lambda'^2}}{k_3^2}+\frac{i}{k_0^2}\frac{\partial}{\partial z})g_\lambda(z,r_2)]_{z=r_3}=(2/k_3')b_3(\lambda)^{-1}g_\lambda(r_3,r_2) \qquad (3.44)$$

and setting $x'=-x$, we have

$$\begin{aligned}\int_{-\infty}^{0}e^{-i(\lambda_1-\lambda_1')x}dx &= \int_0^\infty e^{-i(\lambda_1'-\lambda_1)x'}dx' &= \frac{-i}{(\lambda_1'-\lambda_1)^-}\\ \int_{-\infty}^{\infty}e^{-i(\lambda_2-\lambda_2')y}dy &= \int_{-\infty}^{\infty}e^{-i(\lambda_2'-\lambda_2)y}dy &= 2\pi\delta(\lambda_2'-\lambda_2)\end{aligned} \qquad (3.45)$$

where $(\lambda_1'-\lambda_1)^-$ denotes the condition $Im(\lambda_1'-\lambda_1)<0$, and thus the integration path of λ_1' is along the infinitesimal lower side of λ_1 path (Fig. 3.1). In Fig. 3.1, a set of poles exist nearly along the broken line.

Substituting (3.44) and (3.45) into (3.43) and arranging, we get

$$\psi_{32} = \frac{-1}{16\pi^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d\lambda d\lambda' k_2' b_3^{-1}(\lambda)g_\lambda(r_3,r_2)b_3(\lambda')g_{\lambda'}(z_R,r_3)\frac{\delta(\lambda_2'-\lambda)}{(\lambda_1'-\lambda)^-}$$

$$[b_2(\lambda)g_\lambda(z_S,r_2) - (2k_0^2/k_2')(h_2(\lambda) - h_2^{(1)}(\lambda))^{-1}(g_\lambda(z_S,r_2) - g_\lambda^{(1)}(z_S,r_2))]e^{-i(\lambda_1 x_S - \lambda_1' x_R + \lambda_2 y_S - \lambda_2' y_R)} \tag{3.46}$$

Performing the integrations with respect to λ_2' and λ_1' of Eq. (3.46) using the residue method leads to presenting the term $b_3(\lambda)^{-1}(\lambda_{3,1} - \lambda_1)^{-1}$, where $\lambda_{3,1}$ denotes the set of poles of the function $b_3(\lambda')$ in the λ'-plane. On the other hand, we have poles of $b_2(\lambda)$ when $\lambda_1 = \lambda_{2,1}$ in λ_1-plane. So ψ_{32} is given by

$$\begin{aligned}
\psi_{32} &= \frac{1}{4\pi}\sum_{\tau_2,\tau_3}\int_{-\infty}^{\infty} d\lambda_2 k_2' b_3(\lambda)^{-1} g_{\tau_2}(r_3,r_2)(\lambda_{3,1} - \lambda_{2,1})^{-1} \\
&\quad Res_{\lambda_1=\lambda_{2,1}}[b_2(\lambda)g_{\tau_2}(z_S,r_2)]Res_{\lambda_1'=\lambda_{3,1}}[b_3(\lambda')g_{\tau_3}(z_R,r_3)] \\
&\quad e^{i(\lambda_1 x_S - \lambda_1' x_R + \lambda_2 y_S - \lambda_2' y_R)}
\end{aligned} \tag{3.47}$$

where

$$\begin{aligned}
b_3(\lambda)^{-1} &= \frac{1}{2k_0^2}[k_0^2 + k_3' h_3(\lambda_2')] \\
&= \frac{1}{2}[1 - o_3 g_{\tau_2}'(r_3,r_2)/o_2 g_{\tau_2}(r_3,r_2)] \\
\lambda_{3,1} - \lambda_{2,1} &= (k_0^2/k_x)(k_0 a)^{-2/3}[k_0(k_0 a)^{-1/3}(r_3 - r_2) + \tau_3 - \tau_2] \\
g_{\tau_m}'(z,r_m) &= o_m k_0^{-1}(k_0 a)^{1/3}\frac{\partial}{\partial z} g_{\tau_m}(z,r_m)
\end{aligned} \tag{3.48}$$

with $o_m = -i(k_m'/k_0)(k_0 a)^{-1/3}$, $\tau_m = -2^{-1/3}\beta_m$ for $m = 2,3$ and $k_x = \sqrt{k_0^2 - \lambda^2}$.

Substituting the terms given by eq.(3.47), ψ_{32} reduces to

$$\begin{aligned}
\psi_{32}(r_R,r_S) &= \frac{k_0^3}{2\pi}\sum_{\tau_2,\tau_3}\int_{-\infty}^{\infty}\frac{d\lambda_2}{k_x}(2\pi k_0 r_3)^{-1/2}A'(z_R|d_3)_{\tau_3} \\
&\quad \frac{1}{2}[o_3^{-1} g_{\tau_2}(r_3,r_2) - o_2^{-1} g_{\tau_2}'(r_3,r_2)] \\
&\quad [k_0(r_2 - r_3)(k_0 a)^{-1/3} + \tau_2 - \tau_3]^{-1}(\tau_2 - 1/2o_2^2)^{-1} \\
&\quad exp[-ik_0 d_2(\frac{r_2}{a} - 1 + (k_0 a)^{-2/3}\tau_2)] \\
&\quad g_{\tau_2}(z_S,r_2)e^{-i[\lambda_2(y_R - y_S) + k_x(x_R - x_S) + \frac{\pi}{4}]}
\end{aligned} \tag{3.49}$$

where $A'(z_R|d_3)_{\tau_3}$ is given by

$$\begin{aligned} A'(z|d)_{\tau_m} &= (\pi/2)^{1/2}(k_0d)^{1/2}(k_0a)^{-1/3}(\tau_m - 0.5/o_m^2)^{-1}g_{\tau_m}(z, r_m) \\ &\quad e^{-i[k_0d(\frac{r_m}{a}-1+(k_0a)^{-2/3}\tau_m)+\frac{\pi}{4}]} \end{aligned} \tag{3.50}$$

ψ_{32} can be expressed in another form[15]

$$\psi_{32}(r_R, r_S) = 2A(z_R|d_3, d_2|z_S)\psi_0(r_R, r_S) \tag{3.51}$$

where $\psi_0(r_R, r_S)$ is the wave function in free space and given by

$$\psi_0(r_R, r_S) = \frac{k_0^2}{4\pi d}e^{-ik_0d} \tag{3.52}$$

Where $d = d_2 + d_3$.

Also $A(z_R|d_3, d_2|z_S)$ is called terrain coefficient, this coefficient is not different than the attenuation coefficient represanted in [10][11-15]. It is given by

$$A(z_R|d_3, d_2|z_S) = \sum_{\tau_2,\tau_3}(d/d_3)^{1/2}A'(z_R|d_3)_{\tau_3}T_{\tau_2,\tau_3}(r_2)g_{\tau_2}(z_S, r_2) \tag{3.53}$$

where

$$\begin{aligned} T_{\tau_3,\tau_2} &= \frac{1}{2}[o_3^{-1}g_{\tau_2}(r_3, r_2) - o_2^{-1}g'_{\tau_2}(r_3, r_2)] \\ &\quad [k_0(r_2 - r_3)(k_0a)^{-1/3} + \tau_2 - \tau_3]^{-1} \\ &\quad (\tau_2 - 1/2o_2^2)^{-1}e^{-ik_0d_2[\frac{r_2}{a}-1+(k_0a)^{-2/3}\tau_2]} \end{aligned} \tag{3.54}$$

Similarly, for the case of $r_2 \geq r_3$, T_{τ_3,τ_2} is found in the following form [11]

$$\begin{aligned} T_{\tau_3,\tau_2} &= \frac{1}{2}[o_3^{-1}g'_{\tau_3}(r_2, r_3) - o_2^{-1}g_{\tau_3}(r_2, r_3)][k_0(r_2 - r_3)(k_0a)^{-1/3} + \tau_2 - \tau_3]^{-1} \\ &\quad (\tau_2 - 1/2o_2^2)^{-1}e^{-ik_0d_2[(\frac{r_2}{a})-1+(k_0a)^{-2/3}\tau_2]} \end{aligned} \tag{3.55}$$

3.1.3 Terrain Coefficient

To derive one expression of the wave function for both $r_2 \leq r_3$ (Fig. 2.1.b) and $r_2 \geq r_3$ (Fig. 2.1.a), we can add a section between cliff and ocean, the height of which is higher than or equal to the heights of the cliff and the ocean , and then let the width of the section tend to zero. The new geometry is shown in Fig. 3.2. In this way, the terrain coefficient is expressed as[11]

$$\begin{aligned} A(z_R|d_3, d_2|z_S) &= \sum_{\tau_2,\tau_3}(d/d_3)^{1/2}A'(z_R|d_3)_{\tau_3}T^{(4)}(d_2)_{\tau_3,\tau_2}g_{\tau_2}(z_S, r_2) \\ & \quad z_R \geq r_3,\ z_s \geq r_2 \end{aligned} \tag{3.56}$$

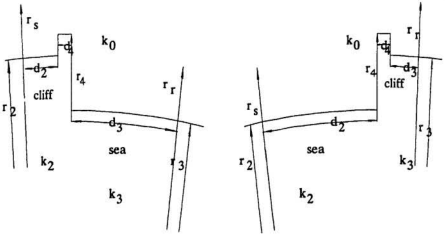
where

$$\begin{aligned} T^{(4)}(d_2)_{\tau_3,\tau_2} &= limit_{d_4 \to +0}\sum_{\tau_4}T(d_4)_{\tau_3,\tau_4}T(d_2)_{\tau_4,\tau_2} \\ &= \frac{1}{2}[o_3^{-1}g'_{\tau_3}(r_4, r_3)g_{\tau_2}(r_4, r_2) - o_2^{-1}g_{\tau_3}(r_4, r_3)g'_{\tau_2}(r_4, r_2)] \\ & \quad (\tau_2 - 1/2o_2^2)^{-1}[k_0(r_2 - r_3)(k_0 r)^{-1/3} + \tau_2 - \tau_3]^{-1} \\ & \quad e^{-ik_0 d_2[(r_2/a)-1+(k_0 a)^{-2/3}\tau_2]} \\ & \quad r_4 \geq r_3, r_4 \geq r_2. \end{aligned} \tag{3.57}$$

where $g'(z, a_m) = [\chi'(\lambda_z)/\chi'(\lambda_m)]_{\lambda=\lambda_m}$ and $\chi'(\beta) = \frac{\partial}{\partial\beta}\chi(\beta)$. If $r_4 = r_3$, that means $r_2 < r_3$, $T^{(3)}_{\tau_4,\tau_2}$ becomes the same as T_{τ_3,τ_2} in equation (3.52). This is because $g'_{\tau_3}(r_3, r_4) = 1$ and $g_{\tau_3}(r_3, r_4) = 1$. If $r_4 = r_2$, for the similar reason, expression $T^{(4)}_{\tau_3,\tau_2}$ reduces to that of equation(3.55).

Substituting $\tau_m = 2^{-1/3}t_m$ and $y_m = k_0(r_m - a)(\frac{2}{k_0 a})^{1/3}$ into eq. (3.57), the generalized terrain coefficient is finally written as follows [14]

$$A_f = \sum_{t_2,t_3}[\pi(c_3 + c_2)]^{1/2}e^{-i\pi/4}(t_3 - q_3^2)^{-1}$$



a. from land to sea b. from sea to land

Figure 3.2: Geometry for eq. (3.57)

$$f_{t_3}(y_{R3})exp[-ic_3(y_3+t_3)]$$
$$[q_3 f'_{t_3}(y_{43}) f_{t_2}(y_{42}) - q_2 f_{t_3}(y_{43}) f'_{t_2}(y_{42})]$$
$$(y_3 - y_2 + t_3 - t_2)^{-1}(t_2 - q_2^2)^{-1}$$
$$f_{t_2}(y_{S2})exp[-ic_2(y_2+t_2)] \tag{3.58}$$

where

$$q_i = -i(k_0 r_0/2)^{1/3}\frac{(\epsilon_{ri} - 1 - i0.018\sigma_i/f)^{1/2}}{\epsilon_{ri} - i0.018\sigma_i/f} \tag{3.59}$$

and f is the frequency, ϵ_r and σ are the relative permittivity and the conductivity of the ground, respectively. The values of t_m (m=2,3) stand for the roots of the equation [14]

$$W'(t_m) - q_m W(t_m) = 0 \tag{3.60}$$

W(t) can be expressed by Airy function

$$W(t) = \sqrt{\pi}e^{-i\pi/2}[Ai(t) + iBi(t)] \tag{3.61}$$

Where Ai and Bi are [18]:

$$Ai(t) = \frac{1}{3}\sqrt{t}[I_{-1/3}(\xi) - I_{1/3}(\xi)] \tag{3.62}$$

$$Bi(t) = \sqrt{t/3}[I_{-1/3}(\xi) + I_{1/3}(\xi)] \tag{3.63}$$

I_n is modified Bessel Function of order n, and argument $\xi = \frac{2}{3}t^{3/2}$.

The function $f_{t_m}(y)$ with $y_{ij} = y_i - y_j$ is the ordinary height-gain function defined by

$$f_{t_m}(y) = W(t_m - y)/W(t_m) \tag{3.64}$$

And $f'_{t_m}(y)$ is defined by

$$f'_{t_m}(y) = W'(t_m - y)/W'(t_m) \tag{3.65}$$

When the ground wave propagates from earth to sea, the generalized terrain coefficient may be written as [14]

$$\begin{aligned} A \; &= \textstyle\sum_{t_3}[\pi(c_3+c_2)]^{1/2}e^{-i\pi/4}(t_3-q_3^2)^{-1} \\ &\quad f_{t_3}(y_{R3})exp[-i(c_3+c_2)(y_3+t_3)]F_{t_3} \end{aligned} \tag{3.66}$$

where

$$\begin{aligned} F_{t_3} \; &= \textstyle\sum_{t_2}[q_3 f'_{t_3}(y_{43})f_{t_2}(y_{42}) - q_2 f_{t_3}(y_{43})f'_{t_2}(y_{42})] \\ &\quad (y_3 - y_2 + t_3 - t_2)^{-1}(t_2 - q_2^2)^{-1}f_{t_2}(y_{S2}) \\ &\quad exp[-ic_2(y_2 - y_3 + t_2 - t_3)] \end{aligned} \tag{3.67}$$

when $y_{R3} \geq y_{43}$.

One should notice that F_{t_3} is a function of c_2, y_{S2}, y_{43} and y_{42}, but independent of the other antenna height y_{R3} and the distance c_3 of section 3.

3.2 Expression for One Short Section

Assuming $c_2 \ll 1$ is the parameters setting for this case. It means that the source is close to the edge and d_3 is long and c_3 is large. When $c_2 \ll 1$, the convergence of the residue series F_{t_3} becomes very slow. So, the flat-earth approximation is used in the short section to obtain F_{t_3}. Defining the variable t as[12]

$$t = \sqrt{d_m/2k_0}\,h_m(\lambda)e^{-i\pi/4} \tag{3.68}$$

and modifying the function $h_m(\lambda)$ for the flat-earth approximation to be in the form

$$h_m(\lambda) \simeq \sqrt{2k_0[(r_m/a)k_0 - \lambda]} \simeq \sqrt{(k_0 r_m/a)^2 - \lambda^2}, \quad \lambda \sim k_0, \quad r_m \sim a \tag{3.69}$$

gives the following relationships:

$$\begin{aligned}
i(\lambda - k_0)d_m &= t^2 + i[(r_m/a) - 1]k_0 d_m \\
d\lambda &= 2(id_m)^{-1}t\,dt \\
b_m(\lambda) &= 2u_m/(t + u_m) \\
u_m &= (k_0/k'_m)\sqrt{k_0 d_m/2}e^{-i\pi/4} \\
b_m(\lambda)^{-1} &= (2u_m^N)^{-1}[\sqrt{t^2 - ik_0 d_N(a_m - a_N)a^{-1}} + u_m^N] \\
u_m^N &= (k_0/k'_m)\sqrt{k_0 d_N/2}e^{-i\pi/4} \\
(\lambda_l - \lambda)^{-1} &= id_m[(t_l^N)^2 - t^2]^{-1} \\
(t_l^N)^2 &= ik_0 d_N(k_0 a)^{-2/3}[k_0(r_l - r_N)(k_0 a)^{-1/3} + \tau_l]
\end{aligned} \tag{3.70}$$

where $N = 2,3$, $l = 2,3$ and

$$\begin{aligned}
(2k_0^2/k'_m)[h_m(\lambda) &- h_m^{(1)}(\lambda)]^{-1}[g_\lambda(z,r_m) - g_\lambda^{(1)}(z,r_m)] = (u_m/t)(e^{-i2f_m^z t} - e^{i2f_m^z t}) \\
g_\lambda(z,r_m) &= e^{-i2f_m^z t} \\
f_m^z &= (z - r_m)\sqrt{k_0/2d_m}e^{i\pi/4}
\end{aligned} \tag{3.71}$$

The functions $J_{\tau_l}(u,f)$ and $J_{\tau_l}(f)$ are defined as[12]

$$\begin{aligned}
J_{\tau_l}(u,f) &= \frac{1}{\pi i}\int_C t[(t_l^N)^2 - t^2]^{-1}(t+u)^{-1}e^{-t^2-i2f/t}dt \\
J_{\tau_l}(f) &= lim_{u\to\infty} uJ_{\tau_l}(u,f) \\
&= \frac{1}{\pi i}\int_C t[(t_l^N)^2 - t^2]^{-1}e^{-t^2-i2f/t}dt
\end{aligned} \tag{3.72}$$

On the condition $k_0 d_N(r_m - r_N)/a \ll 1$, we have

$$\begin{aligned}
\sum_{\tau_N} T^{(m)}(d_N)_{\tau_l,\tau_N} g_{\tau_N}(z,r_N) &= e^{-i[(r_N/a)-1]k_0 d_N}[(u_l^N g'_{\tau_l}(r_m,r_L) - u_N g_{\tau_l}(r_m,r_l)] \\
J_{\tau_l}(u_N, f_N^m + f_N^z) &+ \frac{1}{2}u_l^N g'_{\tau_l}(r_m,r_l)(J_{\tau_l}(0,f_N^m - f_N^z) - J_{\tau_l}(0,f_N^m + f_N^z)) \\
+\frac{1}{2}g_{\tau_l}(r_m,r_l)(J_{\tau_l} &\quad (\quad f_N^m - f_N^z) + J_{\tau_l}(f_N^m + f_N^z))]
\end{aligned} \tag{3.73}$$

Applying this to the expression of F_{t_3}, we finally have [14]

$$\begin{aligned}
F_{t_3} &= exp(t_{23}^2)[u_2((q_3/q_2)f'_{t_3}(y_{43}) - f_{t_3}(y_{43}))J_{t_3}(u_2, f_{42} + f_{S2}) \\
&\quad +\frac{1}{2}(q_3/q_2)u_2 f'_{t_3}(y_{43})(J_{t_3}(0,f_{42} - f_{S2}) - J_{t_3}(0,f_{42} + f_{S2})) \\
&\quad +\frac{1}{2}f_{t_3}(y_{43})(J_{t_3}(f_{42} - f_{S2}) + J_{t_3}(f_{42} + f_{S2}))] \qquad y_{S2} \le y_{42} \\
F_{t_3} &= exp(t_{23}^2)[u_2((q_3/q_2)f'_{t_3}(y_{43}) - f_{t_3}(y_{43}))]J_{t_3}(u_2, f_{42} + f_{S2}) \\
&\quad +\frac{1}{2}(q_3/q_2)u_2 f'_{t_3}(y_{43})(J_{t_3}(0,f_{S2} - f_{42}) - J_{t_3}(0,f_{S2} + f_{42})) \\
&\quad +\frac{1}{2}f_{t_3}(y_{43})(-J_{t_3}(f_{S2} - f_{42}) + J_{t_3}(f_{S2} + f_{42}))] \\
&\quad +f_{t_3}(y_{S3}) \qquad y_{S2} \ge y_{42}
\end{aligned} \tag{3.74}$$

where

$$\begin{aligned}
u_2 &= \sqrt{iq_2^2 c_2} \\
f_{S2} &= \frac{1}{2}c_2^{-1/2} y_{S2} e^{i\pi/4}
\end{aligned}$$

$$f_{42} = \frac{1}{2} c_2^{-1/2} y_{42} e^{i\pi/4}$$
$$t_{32} = [ic_2(t_3 + y_3 - y_2)]^{1/2} \tag{3.75}$$

and c_2 is the numerical distance of section 2 with definition given in (2.5) and

$$\begin{aligned} J_{t_3}(z, f) = \; & e^{f^2}[\frac{1}{2}(z + t_{32})^{-1}\epsilon(f - it_{32}) \\ & + \frac{1}{2}(z - t_{32})^{-1}\epsilon(f + it_{32}) \\ & - z(z^2 - t_{32}^2)^{-1}\epsilon(f + iz)] \end{aligned} \tag{3.76}$$

with $\epsilon(z)$ as the error function given by:

$$\epsilon(z) = \frac{2}{\sqrt{\pi}} e^{z^2} \int_z^\infty e^{-t^2} dt \tag{3.77}$$

Using the residue method, the terrain coefficient for long distance sections or multiple sections with one short section can be obtained. Because the ground wave radar problem is a special case of two sections with one short section, it can be solved with the residue method.

When we use the residue method, several considerations should be taken into account. The first one is how to differentiate between the long distance problem and the short distance one. From the condition $c_i \ll 1$, we have $(d_i/a)(k_0 a/2)^{1/3} \ll 1$. It means that the distance d_i satisfies $d_i \ll \lambda_0^{1/3} a^{2/3}/\pi$ when the problem is a short distance one, where λ_0 is the wave length and a is the earth's radius. Also, the method can be generalized for use in long distance problems and multiple section problem with only one short distance section.

Chapter 4

Compensation Method Approach

Compensation method is used to obtain the terrain coefficient of ground wave propagation over inhomogeneous earth with two long distance sections. The boundary between the cliff and ocean is also vertical to the wave propagation path and the reflected mode is ignored.

In this method, the definition of multiple impedances is used to derive the expression for the terrain coefficient. The result can be extended to three dimensional problem with a finite long boundary between the cliff and ocean.

The method proceeds as follows; first, the wave function for ground wave propagation over homogeneous earth is obtained. Then, the terrain coefficient for ground wave propagation over the inhomogeneous earth is derived using the definition of multiple impedance and the results of homogeneous problem.

4.1 Analysis of Two Section Problem

In this case, spherical coordinates (r, θ, ϕ) are used and the source of the field is an infinitesimally short electric dipole located in free space at $r = b$ and $\theta = 0$, and oriented in the radial direction as shown in Fig. 4.1.

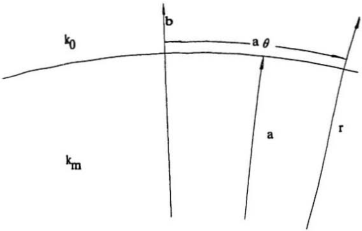


Figure 4.1: Homogeneous sections for compensation method

The vector magnetic potential satisfies the equation[2]

$$(\nabla^2 + k^2)\frac{A_r}{r} = 0 \tag{4.1}$$

Substituting $A_r = rU$ into above equation, we get

$$(\nabla^2 + k^2)U = 0 \tag{4.2}$$

Using the method of separation of variables, the solution of the above equation in free space is ϕ independant and can be expressed by spherical functions in the form [11]

$$U = U_e + U_s \tag{4.3}$$

where

$$U_e(r,\theta) = \frac{ik_0C_0}{2b}\sum_{j=0}^{\infty}(2j+1)h_j^{(1)}(k_0r)h_j^{(2)}(k_0b)P_j(cos\theta), \quad r \geq b \tag{4.4}$$

$$U_e(r,\theta) = \frac{ik_0C_0}{2b}\sum_{j=0}^{\infty}(2j+1)h_j^{(2)}(k_0r)h_j^{(1)}(k_0b)P_j(cos\theta), \quad r \leq b \tag{4.5}$$

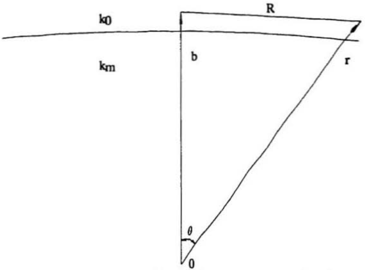


Figure 4.2: R the distance between source and receiver

where $C_0 = -Ids/4\pi i\epsilon\omega$ and $h_j^{(1)}(k_0r)$ and $h_j^{(2)}(k_0r)$ are the first and the second kinds of spherical Hankel functions of order j and argument k_0r, respectively. Also

$$U_s(r,\theta) = \frac{ik_0C_0}{2b}\sum_{j=0}^{\infty}(2j+1)B_jh_j^{(2)}(k_0r)P_j(cos\theta) \tag{4.6}$$

where U_e has the proper singularity as $R \rightarrow 0$, and U_s remains finite. As shown in Fig. 4.2, $R = \sqrt{b^2 + r^2 - 2rbcos\theta}$.

From the boundary condition $E_\theta = -ZH_\phi$ at $r = a$, we get

$$\frac{1}{r}\frac{\partial}{\partial r}rU(r,\theta)_{r=a} = Zi\epsilon\omega U(r,\theta)_{r=a} \tag{4.7}$$

Substituting U into the boundary condition, B_j can be expressed as

$$B_j = -\frac{h_j^{(1)}(k_0a)}{h_j^{(2)}(k_0a)}\left[\frac{\frac{d}{dx}logxh_j^{(1)}(x) - i\Delta}{\frac{d}{dx}logxh_j^{(2)}(x) - i\Delta}\right]h_j^{(2)}(k_0b) \tag{4.8}$$

where $\Delta = \epsilon\omega Z/k_0 = Z/\eta_0$ and $x = k_0a$.

Using the watson transformation and the residue method to U, the poles of the integration

$$U = -i \int_L \frac{(v+\frac{1}{2})}{sinv\pi} f(v) P_v[cos(\pi-\theta)]dv \tag{4.9}$$

are located at the points $v = v_s$ which are the solutions of

$$M(v) = \frac{d}{dx} log x h_v^{(2)}(x) - i\Delta = 0 \tag{4.10}$$

with $v = n - \frac{1}{2}$ and then, using the relation (Appendix B)

$$x h_{\nu-1/2}^{(2)}(x) \cong e^{\frac{-i\pi}{6}}[(-2\tau/3)^{1/2} x^{1/6} H_{1/3}^{(2)}[(\frac{1}{3})(-2\tau)^{3/2}] \tag{4.11}$$

where $\nu = x + x^{1/3}\tau$, we have

$$U = 2U_0(2\pi X)^{1/2} e^{-i\pi/4} \sum_s \frac{f_s(h_1) f_s(h_2) e^{-i\tau_s X}}{2\tau_s - 1/o^2} \tag{4.12}$$

where

$$f_s(h_i) = [\frac{X_i^2 - 2\tau_s}{-2\tau_s}]^{1/2} \frac{H_{1/3}^{(2)}[\frac{1}{3}(X_i^2 - 2\tau_s)^{3/2}]}{H_{1/3}^{(2)}[\frac{1}{3}(-2\tau_s)^{3/2}]} \tag{4.13}$$

and

$$\begin{aligned}
o &= -i\frac{\eta_0}{(k_0 a)^{1/3} Z} = -2^{-1/3} q^{-1} \\
U_0 &= \frac{Ids e^{-ik_0 a\theta}}{4\pi i\epsilon\omega a^2 (\theta sin\theta)^{1/2}} \\
h_1 &= r - a \\
h_2 &= b - a \\
X &= (k_0 a)^{1/3}\theta
\end{aligned}$$

$$X_i = (k_0 a)^{1/3}(2h_i/a)^{1/3} \tag{4.14}$$

The function $f_s(h_i)$ is called a height-gain function and it becomes unity as h_i approaches zero.

We can find that it is the same expression of the series for terrain coefficient as that in the residue method, here U is tha same as ψ/r in the residue method.

4.2 Analysis of the Inhomogeneous Problem

Based on the solution for homogeneous problem and proportional relation between electric field and magnetic vector potential for far field, the expression for the vertical electric field E_r at a great circle distance $d = r\theta$ measured along the homogeneous earth's surface is given by [7]

$$E_r = AE_0^c \tag{4.15}$$

where

$$E_0^c = \frac{-i\mu_0\omega Il}{2\pi d} exp(-ik_0 d)(\frac{\theta}{\sin\theta})^{\frac{1}{2}} \tag{4.16}$$

and

$$A \simeq e^{-i\pi/4}(\pi x)^{1/2} \sum_s \frac{exp(-ict_s)}{t_s - q^2} \frac{W(t_s - y_S)}{W(t_s)} \frac{W(t_s - y_R)}{W(t_s)} \tag{4.17}$$

where $\frac{W(t_s - y_S)}{W(t_s)} = f_s(h_S)$.

$$c = (k_0 a/2)^{1/3}\theta, \quad y_S = (2/ka)^{1/3} k_0 h_S, \quad y_R = (2/k_0 a)^{1/3} k h_R$$

$$q = -i(k_0 a/2)^{1/3}\Delta, \qquad \Delta = Z/120\pi, \qquad k_0 = \omega/c \tag{4.18}$$

where E_0^c is the vertical electric field of the dipole at distance d from a vertical electric dipole of moment Il at height h_S where both are assumed to be just above the surface of a perfectly conducting ground plane. W is the same function as W of expression (3.61) in Chapter 3.

If the path includes one section with the impedance Z_2, the mutual impedance

z_m between dipoles at A and B of lengths l_a and l_b is written as

$$z_m \cong \frac{l_a l_b i \mu_0 \omega}{2\pi d} e^{-ik_0 d} A(d, Z_2)[1 + \frac{1}{ik_0 d} - \frac{1}{k_0^2 d^2}](\frac{\theta}{sin\theta})^{1/2} \tag{4.19}$$

where

$$A(d, Z_2) = e^{-i\pi/4}(\pi x)^{1/2} \sum_{t_2} \frac{exp(-ixt_2)}{t_2 - q_2^2} \tag{4.20}$$

Here A is the terrain coefficient for a special case, $h_S = h_R = 0$.

If the path includes two sections with the impedance Z_2 and Z_3, the mutual impedance z'_m between the dipoles A and B over the boundary is expressed

$$z'_m \cong \frac{l_a l_b i \mu_0 \omega}{2\pi d} e^{-ik_0 d}[1 + \frac{1}{ik_0 d} - \frac{1}{k_0^2 d^2}]A'(d, Z_2, Z_3)(\frac{\theta}{sin\theta})^{1/2} \tag{4.21}$$

where A' is the terrain (attenuation) coefficient dependent on d,Z_2 and Z_3. We have

$$z'_m - z_m = \frac{1}{I_a I_b} \int\int_S [\vec{E}_b \times \vec{H'}_a - \vec{E'}_a \times \vec{H}_b] \cdot \hat{n} dS \tag{4.22}$$

in the above equation, $\hat{n}$ is a unit vector in the radial direction, E'_a and H'_a due to current I_a are the field over the surface S with surface impedance Z_3, E_b and H_b due to current I_b are the field over the surface S with surface impedance Z_2.

Using the approximate boundary conditions for the tangential vectors

$$Z_3(\hat{n} \times \vec{H'}_{at}) = \vec{E'}_{at}, \quad Z_2(\hat{n} \times \vec{H}_{bt}) = \vec{E}_{bt} \tag{4.23}$$

we change eq.(4-19) as

$$I_a I_b(z'_m - z_m) = \int\int_S (\vec{H'}_{at} \cdot \vec{H}_{bt})(Z_3 - Z_2) dS \tag{4.24}$$

Because the tangential magnetic fields have the form

$$H'_{at} \simeq \frac{ik_0 I_a l_a}{2\pi l} e^{-ik_0 l}(1 + \frac{1}{ik_0 l})A'(l, Z_2, Z_3) \cdot (\vec{n} \times \vec{l})[\frac{l/a}{sin(l/a)}]^{1/2} \tag{4.25}$$

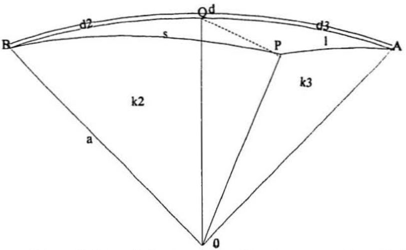


Figure 4.3: Form of inhomogeneous sections for compensation method

and

$$H_{bt} \simeq \frac{ik_0 I_b l_b}{2\pi s} e^{-ik_0 s}(1 + \frac{1}{ik_0 s}) A(s, Z_2) \cdot (\vec{n} \times \vec{s}) [\frac{s/a}{sins/a}]^{1/2} \tag{4.26}$$

where $\vec{l}$ and $\vec{s}$ are unit vectors in the directions of increasing l and s, respectively. l and s are indicated in Fig. 4.3.

$$\begin{aligned} A'(d, Z_2, Z_3) &= A(d, Z_2) + \frac{ik_0 d}{2\pi T \eta_0} \int\int_S \frac{exp[-ik_0(s + l - d)]}{sl} (Z_3 - Z_2) \\ & \quad G(s, l) A(s, Z_2) A'(l, Z_2, Z_3) cos\delta' dS \end{aligned} \tag{4.27}$$

where δ' is the angle by $\vec{l}$ and $\vec{s}$, and

$$\begin{aligned} G(s, l) &= [\frac{s/a}{sin(s/a)}]^{1/2} [\frac{l/a}{sin(l/a)}]^{1/2} (1 + \frac{1}{ik_0 s})(1 + \frac{1}{ik_0 l}) \\ T &= [\frac{d/a}{sin(d/a)}]^{1/2} (1 + \frac{1}{ik_0 d} - \frac{1}{k_0^2 d^2}) \end{aligned} \tag{4.28}$$

The integration for terrain coefficient given above is a three dimensional expression. The term $exp[-ik_0(s + l - d)]$ varies rapidly except in the surface region of

$s + l \sim d$. Using this region and neglecting $1/k_0 s$, $1/k_0 l$ in $G(s, l)$. The integration can be found in

$$A'(d, Z_2, Z_3) = A(d, Z_2) + \frac{i k_0 a}{2\pi}^{1/2} d \int_{\alpha_1}^{\alpha_2} \frac{Z_3(\alpha) - Z_2}{\eta_0} \frac{A(d-\alpha, Z_2) A'(\alpha, Z_2, Z_3)}{[(d-\alpha)\alpha]^{1/2}} F(u_1, u_2) d\alpha \tag{4.29}$$

where

$$F(u_1, u_2) = (\frac{i}{2})^{1/2} \int_{u_1}^{u_2} exp(-i\frac{\pi}{2}u^2) du \tag{4.30}$$

is a function of cliff length, and the length of the cliff is small compared with the distances between antennas and cliff edge. In the integration,

$$\begin{aligned} u_1 &= (k_0 a/\pi)^{1/2} [cot\Omega + cot(\theta - \Omega)]^{1/2} y_1(\alpha)/a \\ u_2 &= (k_0 a/\pi)^{1/2} [cot\Omega + cot(\theta - \Omega)]^{1/2} y_2(\alpha)/a \end{aligned} \tag{4.31}$$

when $u_1 \to -\infty$ and $u_2 \to \infty$, the function $F(u_1, u_2)$ becomes unity. That means that the length of the cliff edge tends to infinity so that the problem becomes a two dimensional one.

The same as we discussed in the residue method, $A'(l, Z_2, Z_3)$ in the integration can be replaced by $A(l, Z)$. It is a first order perturbation.

The terrain coefficient for a smooth two-section spherical earth is

$$\begin{aligned} A'(d, Z_2, Z_3) &\cong A(d, Z_2) - (\frac{i k_0 d}{2\pi})^{1/2} \frac{Z_3 - Z_2}{\eta_0} \\ &\quad \int_0^{d_3} \frac{A(d-\alpha, Z_2) A'(\alpha, Z_3)}{[\alpha(d-\alpha)]^{1/2}} d\alpha, \quad d_3 > 0 \\ A'(d, Z_2, Z_3) &\cong A(d, Z_2), \quad d_3 < 0 \end{aligned} \tag{4.32}$$

Introducing the natural parameters,

$$q_2 = -i(k_0 a/2)^{1/3} (Z_2/\eta_0)$$

$$
\begin{aligned}
q_3 &= -i(k_0 a/2)^{1/3}(Z_3/\eta_0) \\
c &= (k_0 a/2)^{1/3}(d/a) \\
c' &= (k_0 a/2)^{1/3}(\alpha/a) \\
c_3 &= (k_0 a/2)^{1/3}(d_3/a)
\end{aligned}
\qquad (4.33)
$$

the terrain (attenuation) coefficient for the homogeneous path is given by

$$
\begin{aligned}
A(d, Z_2) &= A(c, q_2) &= (\frac{\pi x}{i})^{1/2} \sum_{t_2} \frac{exp(-ict_2)}{t_2 - q_2^2} \\
A(\alpha, Z_3) &= A(c', q_3) : &= (\frac{\pi \dot{x}}{i})^{1/2} \sum_{t_3} \frac{exp(-i'_c t_3)}{t_3 - q_3^2}
\end{aligned}
\qquad (4.34)
$$

where t_2 and t_3 are solutions of

$$
W'(t) - q_i W(t) = 0 \qquad i = 2, 3 \qquad (4.35)
$$

Then we get the equivalent form for eq.(4.29)

$$
\begin{aligned}
A'(c, q_2, q_3) &= A(c, q_2) + (\frac{c}{\pi i})^{1/2}(q_3 - q_2) \int_0^{c_3} \frac{A(c - c', q_2)A(c', q_3)}{[c'(c - c')]^{1/2}} dc' \\
&= (\frac{\pi c}{i})^{1/2}(q_3 - q_2) \sum_{t_2} \sum_{t_3} \frac{exp[-ict_2 - ic_3(t_3 - t_2)]}{(t_3 - t_2)(t_2 - q_2^2)(t_3 - q_3^2)} \\
&\quad + \xi
\end{aligned}
\qquad (4.36)
$$

where

$$
\xi = A(c, q_2) - (\frac{\pi c}{i})^{1/2}(q_3 - q_2) \sum_{t_2} \sum_{t_3} \frac{exp(-ict_2)}{(t_3 - t_2)(t_2 - q_2^2)(t_3 - q_3^2)} \qquad (4.37)
$$

If the relation

$$
(q_3 - q_2) \sum_{t_3} \frac{1}{(t_3 - t_2)(t_3 - q_3^2)} = 1 \qquad (4.38)
$$

is satisfied, we have $\xi = 0$.

The source and observer are at finite heights h_S and h_R, respectively. We use the height gain function, the attenuation coefficient can be written as

$$A'(c,q_2,q_3) = (\frac{\pi c}{i})^{1/2}(q_3-q_2)\sum_{t_2}\sum_{t_3}\frac{exp[-ict_2-ic_3(t_3-t_2)]}{(t_3-t_2)(t_2-q_2^2)(t_3-q_3^2)}f_{t_2}(h_S)f_{t_3}(h_R) \tag{4.39}$$

where

$$\begin{aligned} f_{t_2}(h_S) &= \frac{W(t_2-y_S)}{W(t_2)}, \quad y_S = (\frac{2}{k_0 a})^{1/3}k_0 h_S \\ f_{t_3}(h_R) &= \frac{W(t_3-y_R)}{W(t_3)}, \quad y_R = (\frac{2}{k_0 a})^{1/3}k_0 h_R \end{aligned} \tag{4.40}$$

here the height of the antenna should be small compared with the radius of the earth.

Using mode-match method and considering a step (a cliff), the terrain coefficient is given by [17]

$$A = \sum_{t_2}\sum_{t_3}\frac{W(t_2-y_{S2})}{N_2^{1/2}}exp(-ic_2t_2)\hat{S}_{t_2,t_3}exp[-i(c-c_2)t_3]\frac{W(t_3-y_{R3})}{N_3^{1/2}} \tag{4.41}$$

where $W(t)$ is the Airy function and N_m is a normalization constant

$$N_m = \int_0^\infty [W(t_m-y)]^2 dy = (t_m-q_m^2)[W(t_m)]^2 \tag{4.42}$$

and $\hat{S}_{t_2,t_3}$ is a scattering coefficient

$$\hat{S}_{t_2,t_3} = (\frac{1}{N_2N_3})^{1/2}\frac{W(t_2-y_{42})W'(t_3-y_{43})-W(t_3-y_{43})W'(t_2-y_{32})}{y_3-y_2+t_3-t_2} \tag{4.43}$$

Substituting the scattering coefficient and N_m into the expression of the terrain coefficient, we can find that the terrain coefficient in the compensation method is the same as that in the residue method. So both the residue method and the compensation method give the same terrain coefficient. But, the compensation method only obtained the solution for the long distance problem. If a radar is located close to cliff edge, we can only use the residue method to find the solution.

Chapter 5

Final Expressions for Cliff Case

Based on the residue method, the final expressions for ground wave propagation over a cliff are given in this chapter. Because the distance between the radar antenna and cliff edge at Cape Race is from 50 meters to 200 meters which is a short distance for the working frequency, the flat-cliff approximation is used.

5.1 Terrain of Long Distance Sections

If both land and sea are of long distances, we can use the original formulae to get the expressions of the terrain (attenuation) coefficient. When the wave propagates from land to sea, letting $y_{32} = 0$, from the defination

$$f_{t_m}(y) = W(t_m - y)/W(t_m) \tag{5.1}$$

$$f'_{t_m}(y) = W'(t_m - y)/W'(t_m) \tag{5.2}$$

then $f(y_{32}) = f'(y_{32}) = 1$. We obtain

$$\begin{aligned} A &= \sum_{t_3} [\pi(c_3 + c_2)]^{1/2} e^{-i\pi/4} (t_3 - q_3^2)^{-1} \\ & f_{t_3}(y_{R3}) exp[-i(c_3 + c_2)(y_3 + t_3)] F_{t_3} \end{aligned} \tag{5.3}$$

where

$$
\begin{aligned}
F_{t_4} &= \sum_{t_2} [q_3 f'_{t_3}(y_{43}) - q_2 f_{t_3}(y_{43})] \\
&(y_3 - y_2 + t_3 - t_2)^{-1}(t_2 - q_2^2)^{-1} f_{t_2}(y_{S2}) \\
&exp[-ic_2(y_2 - y_3 + t_2 - t_3) \qquad (5.4)
\end{aligned}
$$

If the wave propagates from sea to land, that means $y_{43} = 0$, so $f(y_{43}) = f'(y_{43}) = 1$. The cliff gain F_{t_3} becomes

$$
\begin{aligned}
F_{t_3} &= \sum_{t_2} [q_3 f_{t_2}(y_{42}) - q_2 f'_{t_2}(y_{42})] \\
&(y_3 - y_2 + t_3 - t_2)^{-1}(t_2 - q_2^2)^{-1} f_{t_2}(y_{S2}) \\
&exp[-ic_2(y_2 - y_3 + t_2 - t_3) \qquad (5.5)
\end{aligned}
$$

5.2 Flat Cliff Approximation

When the antenna on the cliff top is close to the cliff edge, the numerical distance c_2 (wave propagating from land to sea) or c_3 (wave propagating from sea to land) is so small that the cliff top can be considered as a flat section. In this case, the expressions for flat-earth approximation is applied.

5.2.1 Propagation from Land to Sea

From the expression of F_{t_4} for $y_{12} \geq y_{32}$ in Chapter 3, we can obtain the simplified expression of F_{t_4} for cliff case, where the source on the cliff top is closed to the edge and $y_{32} = 0$

$$
\begin{aligned}
F_{t_3} &= exp[(t_{32})^2][\sqrt{d_2}(q_3 f'_{t_3}(y_{43}) - f_{t_3}(y_{43}))J_{t_3}(\sqrt{d_2}, f_{S2}) \\
&+\frac{1}{2}(q_3/q_2)\sqrt{d_2} f'_{t_3}(y_{43})(J_{t_3}(0, f_{S2}) - J_{t_3}(0, f_{S2}))
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{1}{2} f_{t_3}(y_{43})(-J_{t_3}(f_{S2}) + J_{t_3}(f_{S2}))] + f_{t_3}(y_{S3}) \\
= \; & exp(t_{32}^2)\sqrt{d_2}[(q_3/q_2) f'_{t_3}(y_{43}) - f_{t_3}(y_{43})] J_{t_3}(\sqrt{d_2}, f_{S2}) \\
& + f_{t_3}(y_{S3}) \qquad (5.6)
\end{aligned}
$$

Because the curvature of the earth is still involved through the term of t_{32} in the functions $J_{t_3}(z, f)$ and $J_{t_3}(f)$, we may set $t_{32} \to 0$ to avoid the influence.

From

$$
\begin{aligned}
t_{32} &= [ic_2(t_3 + y_3 - y_2)] \\
&= [i(d_2/a)(k_0 a/2)^{1/3}(t_3 + k_0(r_3 - a)(2/k_0 a)^{1/3} - k_0(r_2 - a)(2/k_0 a)^{1/3}]^{1/2} \\
&= i(d_2/a)[t_3(k_0 a/2)^{1/3} + k_0(r_3 - r_2)]^{1/2} \qquad (5.7)
\end{aligned}
$$

we can get when $a \to \infty$, $t_{32} \to 0$.

We simplify the expressions of $J_{t_3}(z, f)$ and $J_{t_3}(f)$ as

$$
\begin{aligned}
t_{32} &= 0 \\
J_{t_3}(z, f) &= e^{-f^2}[z^{-1}\epsilon(f) - z^{-1}\epsilon(f + iz)] \\
J_{t_3}(f) &= e^{-f^2}\epsilon(f) \qquad (5.8)
\end{aligned}
$$

other parameters have the same definations as before.

The cliff gain F_{t_3} for flat earth approximation is given in

$$
\begin{aligned}
F_{t_3} &= exp[(t_{32})^2]\sqrt{d_2}[(q_3/q_2) f'_{t_3}(y_{43}) - f_{t_3}(y_{43})] J_{t_3}(\sqrt{d_2}, f_{S2}) + f_{t_3}(y_{14}) \\
&= f_{t_3}(y_{S3}) + \sqrt{d_2}[q_3/q_2 f'_{t_3}(y_{43}) - f_{t_3}(y_{43})] z^{-1} e^{-f_{S2}^2}[\epsilon(f_{S2}) - \epsilon(f_{S2} + i\sqrt{d_2})]
\end{aligned}
$$

$$(5.9)$$

5.2.2 Propagation from Sea to Land

When the ground wave propagates from sea to land, A may be written as

$$\begin{aligned} A &= \sum_{t_2}[\pi(c_3+c_2)]^{1/2}e^{-i\pi/4}(t_2-q_2^2)^{-1} \\ & f_{t_2}(y_{S2})exp[-i(c_3+c_2)(y_2+t_2)]F_{t_2} \end{aligned} \tag{5.10}$$

When receiver on cliff top is close to the edge, the distance d_4 is very short so that $c_4 \ll 1$ and c_2 is large. The flat-cliff approximation is used to obtain F_{t_2}.

$$F_{t_2} = \sqrt{d_3}[(q_2/q_3)f'_{t_2}(y_{42}) - f_{t_2}(y_{42})]J_{t_2}(\sqrt{d_3}, f_{R3}) + f_{t_2}(y_{R2}) \tag{5.11}$$

the parameters involved are defined by

$$\begin{aligned} d_3 &= iq_3^2c_3 \\ f_{R3} &= \frac{1}{2}c_3^{-1/2}y_{R3}e^{i\pi/4} \\ t_{23} &= [ic_3(t_2+y_2-y_3)]^{1/2} \end{aligned} \tag{5.12}$$

and the other parameters can be calculated with the expressions given in section 5.2.1.

5.3 Two Terms Approximation

Rewritting the expression of cliff gain F_{t_4}, we can express it in the three terms as

$$F_{t_3} = F_1 + F_2 + F_3 \tag{5.13}$$

where

$$F_1 = exp(t_{32}^2)\sqrt{d_2}(q_3/q_2)f'_{t_3}(y_{43})J_{t_3}(\sqrt{d_2}, f_{S2}) \tag{5.14}$$

$$F_2 = exp(t_{32}^2)\sqrt{d_2} f_{t_3}(y_{43}) J_{t_3}(\sqrt{d_2}, f_{S2}) \quad (5.15)$$

$$F_3 = f_{t_3}(y_{S3}) \quad (5.16)$$

Where, F_3 is a function of the antenna height and cliff height, but it has no relation with the distance between the antenna and the cliff edge. F_1 and F_2 are the functions of the distance d_2, the antenna heights and cliff height. When the distance between the antenna and the cliff edge changes, F_3 is a constant while F_1 and F_2 will change.

We consider cliff case, where $\sigma_{earth} = 0.005$, $\epsilon_{earth} = 15$, $\sigma_{sea} = 4$ and $\epsilon_{sea} = 80$. Comparing the three terms in the expression of F_{t2}, we find that the third term of the three plays the most important role. If we cancel the first term, the difference between the results from this simplification and from flat-cliff approximation is less than five percent. So the further approximation can be realized by cancel the first term in the expression of F_{t_2}. We can write

$$F_{t_2} = f_{t_2}(y_{R2}) - exp[(t_{23})^2]\sqrt{d_3} f_{t_2}(y_{42}) J_{t_2}(\sqrt{d_3}, f_{R3}) \quad (5.17)$$

It is called the two terms approximation.

In this way, the terrain coefficient A becomes

$$\begin{aligned} A \; &= \textstyle\sum_{t_2}[\pi(c_3+c_2)]^{1/2} e^{-i\pi/4}(t_2 - q_2^2)^{-1} f_{t_2}(y_{S2}) exp[-i(c_3+c_2)(y_2+t_2)] \\ &\quad [f_{t_2}(y_{R2}) - exp(t_{23}^2)\sqrt{d_3} f_{t_2}(y_{42}) J_{t_2}(\sqrt{d_3}, f_{R3})] \end{aligned} \quad (5.18)$$

Chapter 6

Numerical Results And Discussions

The numerical results based on Green theorem method are shown in this chapter. The cliff cases we are interested in include wave propagation from sea to earth and from earth to sea. Here, the flat- cliff approximation is used. The two terms approximation is applied too.

The electrical parameters involved are set as the conductivity of the earth σ_e is $0.005s/m$, the relative permittivity of the earth ϵ_{re} is 15 , the conductivity of the sea σ_s is $4s/m$, and the relative permittivity of the sea ϵ_{rs} is 80.

6.1 Numerical Results for Radar

From Fig. 6.1 to Fig. 6.4, the frequency of interest is 6.75 MHz with corresponding wave length $\lambda_0 = 44.44m$. The source is considered to be an infinitesimal dipole antenna.

Fig. 6.1 shows the change in E_0 of expression(2) and E_0^w of expression(25) with distance d$(= d_2 + d_3)$. In calculating E_0^w, the value of Il was set to unity. Since the angle θ is very small in the range of interest, the factor $(\theta/sin\theta)^{1/2}$ also approaches unity. According to above considerations, the two curves are proportional to one another as shown in Fig. 4.

Figure 6.2 shows the electric field distribution over the sea, where the distance d2 is 300 meters, distance d3 is 200 km and the cliff height is 15 meters. In the calculations, expressions resulting from the Green theorem method are used. It is clear that the wave strength propagating to the sea decreases, as the transmitter height over a cliff is increased. More noticeable is the reduction of wave strength with the increased hight of the receiver over the sea.

The receiving antenna array of Northern Radar prototype system in Cape Race has average $d_3 \simeq 30$ meters. Fig. 6.3 gives the distribution of E on the vertical plane 30 meters from the cliff edge. It is observed that as the source which is 200 km from the coast increases in height, the field strength decreases. At the receiver location, the field increases until the height is about 40 meters. Then it starts decreasing in a fluctuating manner with the increase in height.

Figure 6.4 gives field strength changing with distance d_3. Where the height of transmitter is zero at sea and the heights of receiver on ground are 0, 10 meter, 20 meter and 30 meter. In this graph, it is found out that for a good signal reception, the receiver should be put at a higher level as it becomes further from the cliff edge to avoid ground absorption.

are different for different values of the parameters .

In Fig. 6.8, $|A|$ decreases at first then increases as the height of the source on the cliff top increases. If frequency is lower, the maximum point of $|A|$ is higher. The closer to the cliff edge the source is located, the lower the maximum point of $|A|$ is.

Fig. 6.9 shows that the terrain coefficient decreases smoothly and lightly as the height of the receiver increases from 0 to 30m.

Fig. 6.10 to Fig.6.14 show the coresponding conditions of Fig. 6.5 to Fig. 6.9, but the direction of wave propagation is from sea to land. If we change the source and receiver, Fig. 6.10 to Fig. 6.14 give almost the same curves as Fig. 6.5 to Fig. 6.9.

Among the five parameters, the hight of the antenna on the cliff top, the distance between the antenna on the cliff top and the cliff edge, and the distance between the target on the ocean and the cliff edge are important on the signal receiving, while the hight of the cliff and the hight of the target on the ocean are not important.

To sum up, we should put both the source and the receiver antennas as close to the cliff edge as possible to obtain the best information from the sea. The computer program given in this paper can be used to determine the heights of the source and receiver at different distances from the cliff edge.

6.2 Two Term Approximation and Five Parameters

There are five parameters in the expression of A. These are the distance between the source and the cliff edge d_s, the distance between the receiver and the cliff edge d_r, the height of the cliff h, the height of the source h_s and the height of the receiver h_r. Fig. 6.5 to Fig. 6.14 give the results for one of five parameters changing in turn.

The frequencies used in Fig. 6.5 to Fig 6.14 are 3MHz, 5MHz and 8Mhz. Fig. 6.5 to Fig. 6.14 show the absolute value of the terrain (attenuation) coefficient from the two terms approximation. The terrain coefficient is expressed as $|A| = |E/E_0|$. When the frequency increases from 3MHz to 8Mhs, the terrain coefficient will decrease. When the height above sea increases, the terrain coefficient decreases at first and then increases.

From Fig. 6.5 to Fig. 6.9, the source is on the cliff top and the receiver (or target) is on the sea. Where $d_s = 50m$, $d_r = 200km$, $h = 25m$, $h_s = 25m$ and $h_r = 3m$. From Fig. 6.10 to Fig. 6.14, the source is on the sea and receiver is on the cliff top. Where $d_s = 200km$, $d_r = 50m$, $h = 25m$, $h_s = 3m$ and $h_r = 25m$.

From Fig. 6.5, we can find that the terrain coefficient $|A|$ decreases if the distance between the source and the cliff edge increases. Where the results of two terms approximation are very close to those of the flat-cliff approximation.

Fig. 6.6 gives the variation of terrain coefficient over sea with d_r. When the distance between the receiver and the cliff edge d_r incr ›es from 100km to 600km, the terrain coefficient decreases. The field is absorbed by the water.

In Fig. 6.7, $|A|$ decreases gradually as the height of the cliff increases. The curves

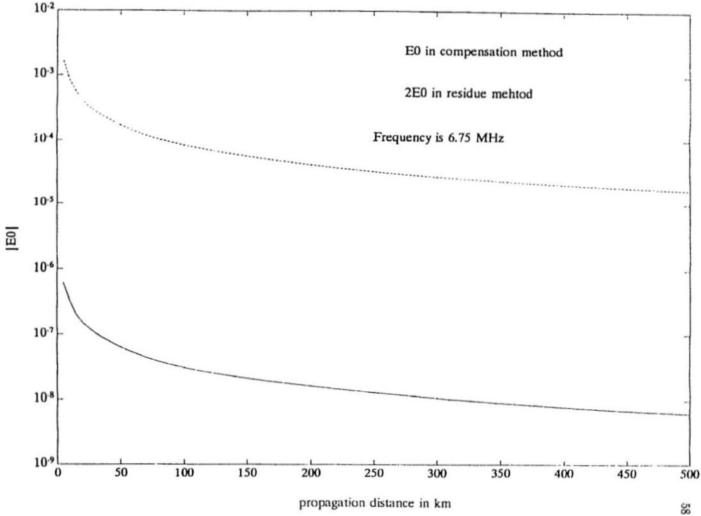


Figure 6.1: comparing E_0 in the two methods

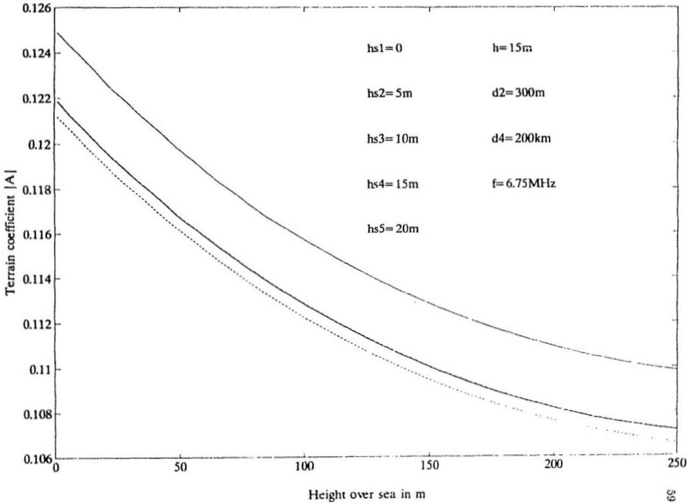


Figure 6.2: variation of terrain coefficient with the height of receiver on sea

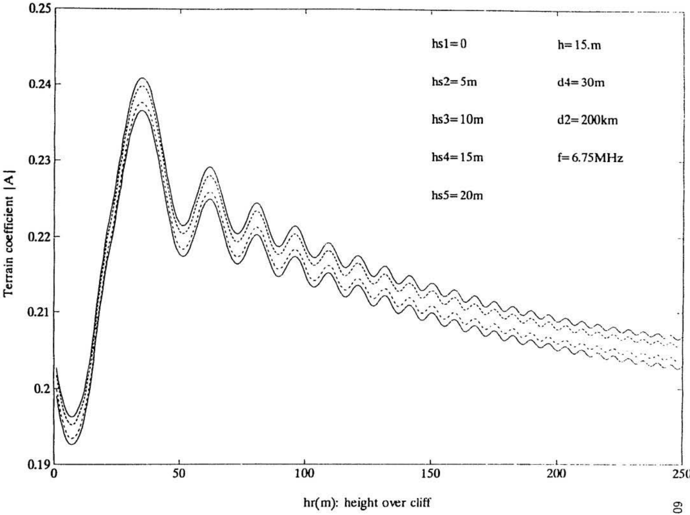


Figure 6.3: variation of terrain coefficient with the height of receiver on cliff

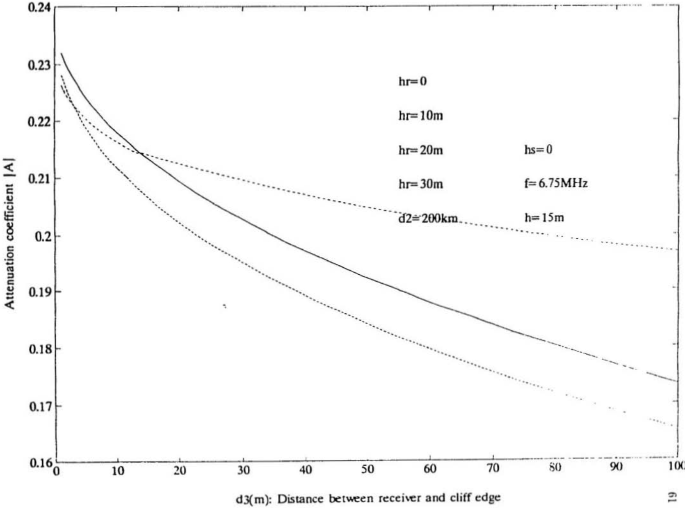


Figure 6.4: variation of terrain coefficient with d_r on cliff

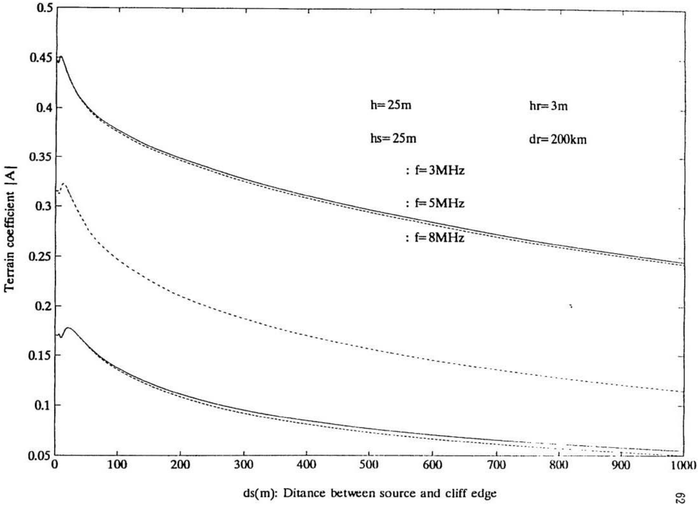


Figure 6.5: variation of terrain coefficient over sea with d_s

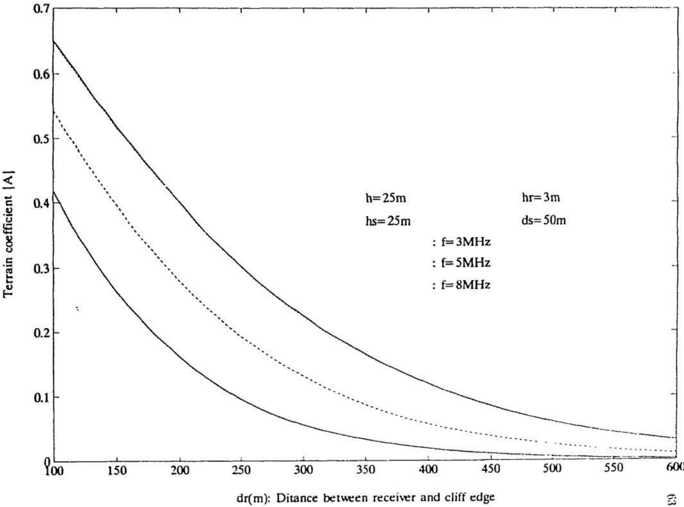


Figure 6.6: variation of terrain coefficient over sea with d_r

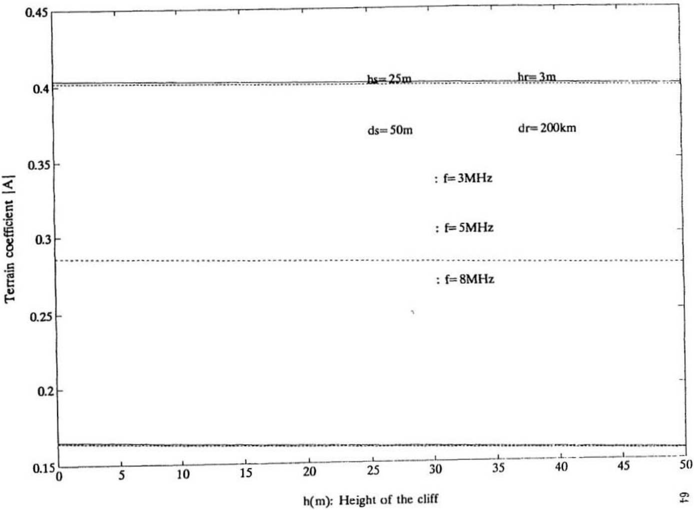


Figure 6.7: variation of terrain coefficient over sea with cliff height

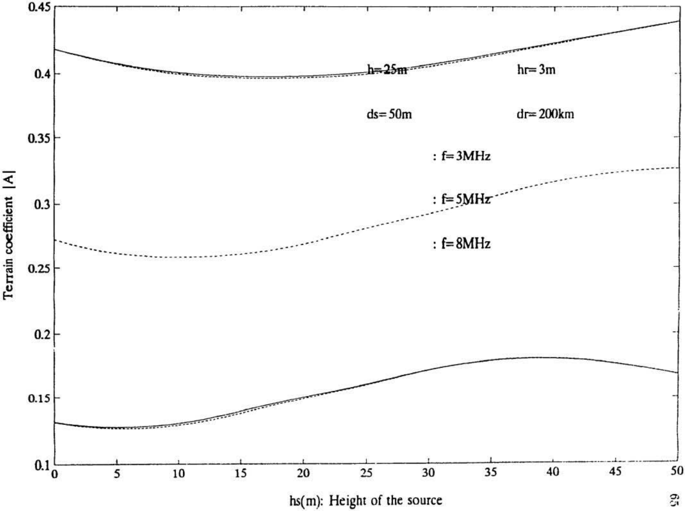


Figure 6.8: variation of terrain coefficient over sea with h_s

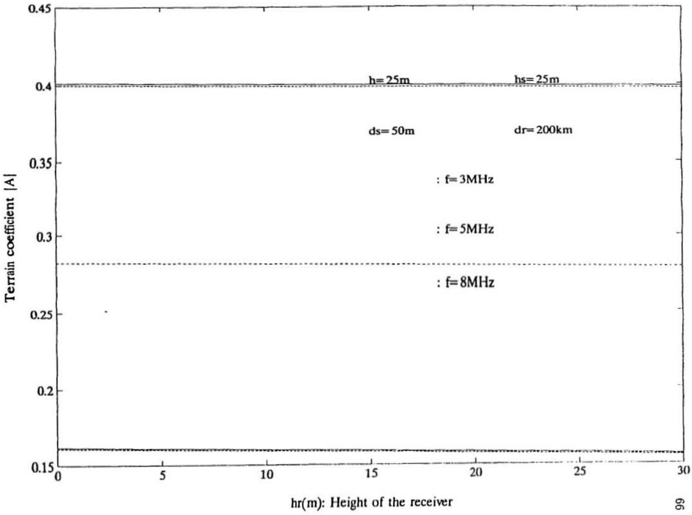


Figure 6.9: variation of terrain coefficient over sea with h_r

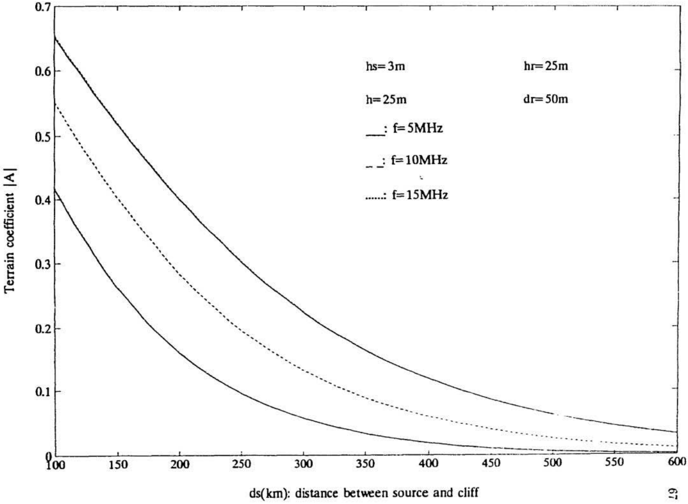


Figure 6.10: variation of terrain coefficient over cliff with d_s

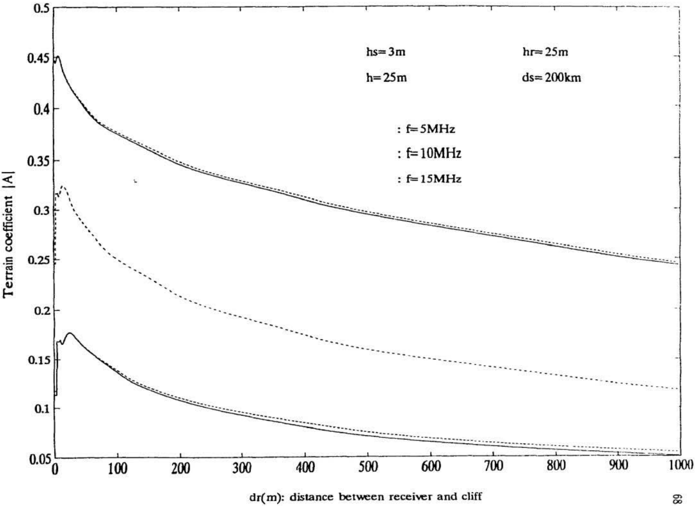


Figure 6.11: variation of terrain coefficient over cliff with d_r

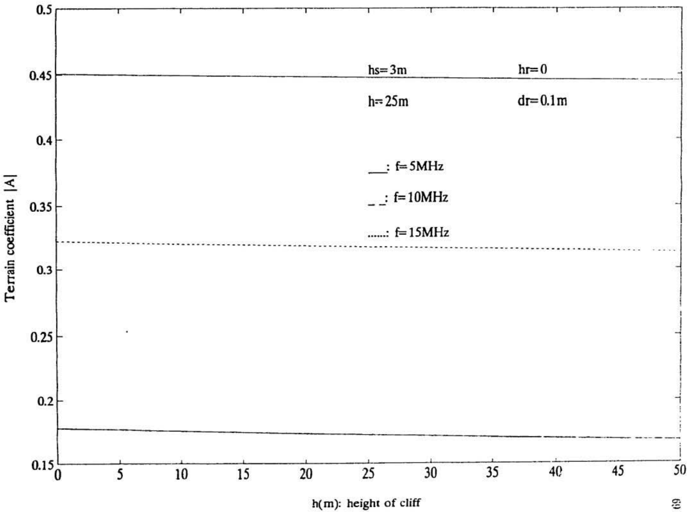


Figure 6.12: variation of terrain coefficient over cliff with cliff height

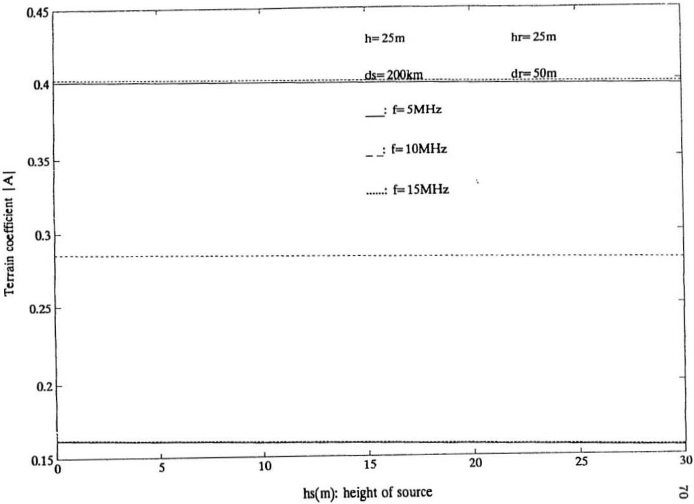


Figure 6.13: variation of terrain coefficient over cliff with h_s

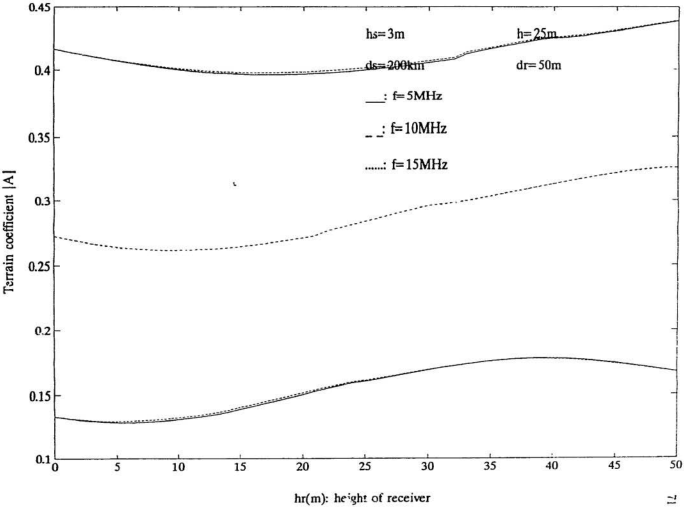


Figure 6.14: variation of terrain coefficient over cliff with h_r

Chapter 7

Conclusion

The detailed reexamination of the residue method in solving the ground wave propagation over a cliff has been performed in this thesis. A comparison between the residue method and the compensation method has been presented. For ground wave radar application, modified expressions of terrain coefficient, cliff gain and field strength for cliff case are given.

A user friendly computer program is developed to obtain the above parameters for different ground wave radar sites. The results of the program give very good agreement with the data published.

7.1 The Difference of the Two Methods

The residue method and the compensation method are different in their derivations, E_0 in their expressions and the problems solved by them.

The residue method and the compensation method use different technique to get the solution. Fourier transform and Green theory were used to derive the wave function in the residue method, while the defination of the ground impedance was used in the compensation method.

E_0's in the two methods have different meanings. In the residue method, E_0 is explained as the electric field in free space. But in the compensation method, E_0 is defined as electric field above a perfect conducting ground.

Because the flat-cliff approximation was used, the residue method is available for an inhomogeneous earth problem with one short distance section. But the compensation method can only be used for a problem with long distance sections. On the other hand, the residue method is applied to the two dimensional problem, while the compensation method is for the three dimensional problem.

7.2 The Similarity of the Two Methods

Although the residue and the compensation methods are different methods to obtain the formulas of electromagnetic field strength of a terrain of two sections, their final expressions of the terrain coefficient for long distance problem are the same. Because of the same attenuation coefficients in the two methods and $2E_0$ in the residue method proportional to E_0^g in the compensation method, the two methods give the same relative field strength distributions.

The residue technique was used in both of the methods. In the residue method, the residue technique was used to obtain the integration value for the wave function on inhomogeneous earth. In the compensation method, the residue technique was used to get the solution for homogeneous earth.

Both of them gave their wave functions of inhomogeneous problem based on the solution of the homogeneous problem.

The two methods have used approximations during their derivations. And either of the general expressions of the two methods is expressed in series. In addition, both are applicable for long distance problems.

7.3 Two Terms Approximation

Based on the flat cliff approximation for the residue method and numerical calculation, a further approximation, the two terms approximation is given in this work.

In the special cliff case of this thesis, the maximum relative error of the two terms approximation to the flat cliff approximation is less than five percent. But the relative error increases as the distance d_r increases. This increase becomes fast when the frequency is higher. So we should put antennas as close to the edge as possible.

We can use this method to solve the cliff problem when the source or receiver on the cliff is closed to the cliff edge. It will simplify the program design and save the computer calculation time.

A number of cases of field strength distribution related to Northern Radar Cape Race radar site have been presented with the two term approximation.

Appendix A

Coordinate Exchange in Vector Theory

In any orthogonal coordinate system (u_1, u_2, u_3), for position vector $\vec{r}$, we have

$$d\vec{r} = \frac{\partial \vec{r}}{\partial u_1} du_1 + \frac{\partial \vec{r}}{\partial u_2} du_2 + \frac{\partial \vec{r}}{\partial u_3} du_3 \tag{A.1}$$

h_1, h_2 and h_3 are defined as

$$\begin{aligned} \frac{\partial \vec{r}}{\partial u_1} &= h_1 \hat{e}_1 \\ \frac{\partial \vec{r}}{\partial u_2} &= h_2 \hat{e}_2 \\ \frac{\partial \vec{r}}{\partial u_3} &= h_3 \hat{e}_3 \end{aligned} \tag{A.2}$$

Here, $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$ are unite vectors. For any scalar function Φ and vector $\vec{A}$

$$\nabla \Phi = \frac{1}{h_1} \frac{\partial \Phi}{\partial u_1} + \frac{1}{h_2} \frac{\partial \Phi}{\partial u_2} + \frac{1}{h_3} \frac{\partial \Phi}{\partial u_3} \tag{A.3}$$

$$\nabla . \vec{A} = \frac{1}{h_1 h_2 h_3} [\frac{\partial}{\partial u_1}(h_2 h_3 A_1) + \frac{\partial}{\partial u_2}(h_1 h_3 A_2) + \frac{\partial}{\partial u_3}(h_2 h_1 A_3)] \tag{A.4}$$

$$\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \vec{e_1} & h_2 \vec{e_2} & h_3 \vec{e_3} \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix} \tag{A.5}$$

$$\nabla^2\Phi = \frac{1}{h_1h_2h_3}[\frac{\partial}{\partial u_1}(\frac{h_2h_3}{h_1}\frac{\partial\Phi}{\partial u_1}) + \frac{\partial}{\partial u_2}(\frac{h_3h_1}{h_2}\frac{\partial\Phi}{\partial u_2}) + \frac{\partial}{\partial u_3}(\frac{h_1h_2}{h_3}\frac{\partial\Phi}{\partial u_3})] \qquad (A.6)$$

In Cartesian coordinates,

$$h_1 = h_2 = h_3 = 1 \qquad (A.7)$$

In spherical coordinates,

$$h_1 = 1 \quad h_2 = r \quad h_3 = rsin\theta \qquad (A.8)$$

Appendix B

Derivation of the Function $g(z, r_m)$

In Chapter 3., the function $f_\lambda^+(z)$ is given by

$$f_\lambda^+(z) = (k_0 z)^{-1/2} H_n^{(2)}(k_0 z), \quad n = \sqrt{a^2\lambda^2 + 1/4} \tag{B.1}$$

Now, write $\nu = n$ and $k_0 z = \tau = \nu + \beta' \nu^{1/3}$. For a large order, the following Bessel function expressions are recalled [20]

$$J_\nu(\nu + \beta'\nu^{1/3}) = 2^{1/3}\nu^{1/3} A_i(-2^{1/3}\beta') + O(\nu^{-1}) \tag{B.2}$$

$$Y_\nu(\nu + \beta'\nu^{1/3}) = -2^{1/3}\nu^{1/3} B_i(-2^{1/3}\beta') + O(\nu^{-1}) \tag{B.3}$$

Where $A_i(z)$ and $B_i(z)$ are Airy functions of argument z. Accordingly, $H_\nu^{(2)}(\nu + \beta'\nu^{1/3})$ can be written as

$$\begin{aligned}
H_\nu^{(2)}(\nu + \beta'\nu^{1/3}) &= J_\nu(\nu + \beta'\nu^{1/3}) - jY_\nu(\nu + \beta'\nu^{1/3}) \\
&= 2^{1/3}\nu^{-1/3}[A_i(-2^{1/3}\beta') + jB_i(-2^{1/3}\beta')] + (1+j)O(\nu_{-1}) \\
&= 2^{1/3}\nu^{-1/3} e^{j\pi/6}\sqrt{2^{1/3}\beta'/3}\, e^{-j\pi/6}\sqrt{3/(2^{1/3}\beta')}[A_i(-2^{1/3}\beta') + jB_i(-2^{1/3}\beta')] \\
&= 2^{1/3}\nu^{-1/3} e^{j\pi/6}\sqrt{1/3}\sqrt{2^{1/3}\beta'}\, H_{1/3}^{(2)}(\xi) + O'(\nu^{-1})
\end{aligned}$$

where $\xi = 2/3(2^{1/3}\beta')^{3/2}$. Now by letting $2^{1/3}\beta' = \beta$, one may write

$$H_\nu^{(2)}(\nu + \beta'\nu^{1/3}) \simeq 2^{1/3}\nu^{-1/3}e^{j\pi/6}\frac{1}{\sqrt{3}}\beta^{1/2}H_{1/3}^{(2)}(\frac{2}{3}\beta^{3/2}) \tag{B.5}$$

β' can be found as

$$\begin{aligned} \beta' &= \nu^{-1/3}(\tau - \nu) \\ &\simeq (a\lambda)^{-1/3}(k_0 z - a\lambda) \end{aligned} \tag{B.6}$$

For $\lambda \simeq k_0$, one gets

$$\begin{aligned} \beta' &\simeq (ak_0)^{-1/3}(k_0 z - a\lambda) \\ \beta &\simeq 2^{1/3}\beta' = (2/ak_0)^{1/3}(k_0 z - a\lambda) \end{aligned} \tag{B.7}$$

Using the definition of $g_\lambda(z, r_m)$ and above results, we can write

$$\begin{aligned} g_\lambda(z, r_m) &= f_\lambda^+(z)/f_\lambda^+(r_m) = (\frac{k_0 z}{k_0 r_m})^{1/2}\frac{H_n^{(2)}(k_0 z)}{H_n^{(2)}(k_0 r_m)} \\ &= (\frac{z}{r_m})^{1/2}\frac{2^{1/3}n^{-1/3}e^{j\pi/6}3^{-1/2}\beta^{1/2}H_{1/3}^{(2)}(\frac{2}{3}\beta^{3/2})}{2^{1/3}n^{-1/3}e^{j\pi/6}3^{-1/2}\beta_m^{1/2}H_{1/3}^{(2)}(\frac{2}{3}\beta_m^{3/2})} \\ &= (\frac{z}{r_m})^{1/2}\frac{\beta^{1/2}H_{1/3}^{(2)}(\frac{2}{3}\beta^{3/2})}{\beta_m^{(1/2)}H_{1/3}^{(2)}(\frac{2}{3}\beta_m^{3/2})} \end{aligned} \tag{B.8}$$

For $z \simeq r_m$, we finally get

$$g_\lambda(z, r_m) \simeq \frac{\beta^{1/2}H_{1/3}^{(2)}(\frac{2}{3}\beta^{3/2})}{\beta_m^{1/2}H_{1/3}^{(2)}(\frac{2}{3}\beta_m^{3/2})} = \frac{\chi(\beta_z)}{\chi(\beta_m)} \tag{B.9}$$

Appendix C

Derivation for Computer Program

C.1 Roots t_m

From the equation

$$W' - qW = 0 \tag{C.1}$$

we have

$$W'' - qW' - q'W = 0 \tag{C.2}$$

and from

$$W'' - tW = 0 \tag{C.3}$$

we have

$$\begin{aligned} W'' &= tW \\ W' &= qW \end{aligned} \tag{C.4}$$

So we obtain

$$tW - q^2W - \frac{dq}{dt}W = 0 \tag{C.5}$$

It gives

$$\frac{dt_m}{dq} = (t_m - q^2)^{-1} \tag{C.6}$$

We can write the series of t_m as

$$t_m = t_m(0) + t'_m(0)q + \frac{t''_m(0)}{2!}q^2 + \frac{t'''_m(0)}{3!}q^3 + ... \tag{C.7}$$

where

$$\begin{aligned}
dt_m/dq &= (t_m - q^2)^{-1} \\
d^2t_m/dq^2 &= -(t_m - q^2)^{-2}(t'_m - 2q) \\
&= -(t_m - q^2)^{-3} + 2q(t_m - q^2)^{-2} \\
d^3t_m/dq^3 &= d/dq(d^2t_m/dq^2) \\
&= 3(t_m - q^2)^{-5} - 10q(t_m - q^2)^{-4} + 8q^2(t_m - q^2)^{-3} + 2(t_m - q^2)^{-2} \\
d^4t_m/dq^4 &= d/dq(d^3t_m/dq^3)
\end{aligned} \tag{C.8}$$

set $q = 0$, we have

$$\begin{aligned}
t'_m(0) &= \frac{1}{t_m} \\
t''_m(0) &= -\frac{1}{t_m^3} \\
t'''_m(0) &= \frac{3}{t_m^5} + \frac{2}{t_m^2} \\
t''''_m(0) &= -\frac{15}{t_m^7} - \frac{14}{t_m^4}
\end{aligned} \tag{C.9}$$

so we have

$$t_m = t_m^0 + \frac{1}{t_m^0}q - \frac{1}{2(t_m^0)^3}q^2 + (\frac{1}{3(t_m^0)^3} + \frac{1}{2(t_m^0)^5})q^3 - (\frac{7}{12(t_m^0)^4} + \frac{5}{8(t_m^0)^7})q^4 + ... \tag{C.10}$$

let x=1/q, and

$$\begin{aligned} dt_m/dx &= (dt_m/dq)(dq/dx) \\ &= -q^2(t_m - q^2)^{-1} \\ &= (1 - t_m x^2)^{-1} \end{aligned} \tag{C.11}$$

when $x = 0$, $q \to 0$ and we have

$$\begin{aligned} t'_m(0) &= 1 \\ t''_m(0) &= 0 \\ t'''_m(0) &= 2t_m \\ t''''_m(0) &= 6 \end{aligned} \tag{C.12}$$

and

$$t_m = t_m^{\infty} + q_{-1} + \frac{1}{3}t_m^{\infty}q^3 + \frac{1}{4}q^4 + ... \tag{C.13}$$

C.2 Error Function

Error function is defined as[1]

$$erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt \tag{C.14}$$

and

$$erfc(z) = \frac{2}{\sqrt{\pi}}\int_z^{\infty} e_{-t^2}dt = 1 - erf(z) \tag{C.15}$$

So we have

$$\begin{aligned} \epsilon(z) &= \frac{2}{\sqrt{\pi}}e^{z^2}\int_z^{\infty} e^{-t^2}dt \\ &= e^{z^2}(1 - erf(z)) \end{aligned} \tag{C.16}$$

because the series expression for erf(z) is

$$erf(z) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{n!(2n+1)} \tag{C.17}$$

We finally obtain

$$\epsilon(z) = e^{z^2} - \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 ... (2n+1)} z^{2n+1} \tag{C.18}$$

C.3 Airy Function

Hankel function of order 1/3 can be written in the term of Airy functions[1]

$$H_{1/3}^{(2)}(\frac{2}{3} z^{3/2}) = e^{i\pi/6} \sqrt{3/z} [Ai(-z) + iBi(-z)] \tag{C.19}$$

Ai(z) and Bi(z) can be expressed

$$\begin{aligned} Ai(z) &= c_1 f(z) - c_2 g(z) \\ Bi(z) &= \sqrt{3}(c_1 f(z) + c_2 g(z)) \end{aligned} \tag{C.20}$$

where

$$\begin{aligned} f(z) &= 1 + \frac{1}{3!} z^3 + \frac{1 \cdot 4}{6!} z^6 + \frac{1 \cdot 4 \cdot 7}{9!} z^9 + \frac{1 \cdot 4 \cdot 7 \cdot 10}{12!} z^{12} + ... \\ g(z) &= z + \frac{2}{4!} z^4 + \frac{2 \cdot 5}{7!} z^7 + \frac{2 \cdot 5 \cdot 8}{10!} z^{10} + ... \\ c_1 &= Ai(0) = Bi(0)/\sqrt{3} = 0.355028053887817 \\ c_2 &= -Ai'(0) = Bi'(0)/\sqrt{3} = 0.258819403792807 \end{aligned} \tag{C.21}$$

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