RADIO WAVE PROPAGATION OVER EARTH: FIELD CALCULATIONS AND AN IMPLEMENTATION OF THE ROUGHNESS EFFECT

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RADIO WAVE PROPAGATION OVER EARTH: FIELD CALCULATIONS AND AN IMPLEMENTATION OF THE ROUGHNESS EFFECT

BY

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ABSTRACT

Computer programs are developed to calculate radio propagation losses over the ocean surface. The effects of the ocean surface roughness are evaluated through numerical implementations of modified surface impedance expressions. The surface roughness is expressed in terms of standard oceanographic models for the directional ocean wave height spectral density. The modified surface impedance may be used with either a planar earth propagation model, for short propagation distances, or a spherical earth propagation model for long propagation distances.

The planar earth solution for the electric field distant from the source, is derived using a spatial decomposition method and expressed in the form of the spatial Fourier transform of the electric field. No assumed boundary conditions are used in the derivation; the method supplies its own boundary conditions. As well, the surface impedance and the choice of source remains arbitrary. For a highly conductive surface, such as the ocean surface, and an elementary vertical electric dipole source, the expressions reduce to the classical planar earth results.

For long propagation distances, the effects of radio wave diffraction around the curvature of the earth's surface are significant. A computer program has been written using modern compact computer code which implements the classical residue series results for ground wave spherical earth propagation. The program accounts for rough surface effects using an implementation of the modified surface impedance for a rough ocean surface. Transmission loss results for a variety of frequencies in the MF and HF bands and a variety of sea states are presented which compare favourably to previous results.

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μ_0 : The Permeability of 'free space' (p. 14).

ϵ_0: The Permittivity of 'free space' (p. 14).

σ_0: The conductivity of 'free space' (p. 14).

ϵ_1: The permittivity of medium 1 (p. 14).

σ_1: The conductivity of medium 1 (p. 14).

$h(z)$: The Heaviside function (p. 15).

$\vec{E}$: The electric field intensity vector (p. 15).

$\vec{H}$: The magnetic field intensity vector (p. 15).

$\vec{B}$: The magnetic flux density vector (p. 15).

$\vec{D}$: The displacement density vector (p. 15).

$\vec{J}$: Current density vector (p. 15).

ρ: Charge density (p. 15).

$\vec{J}_c$: Conduction current density vector (p. 16).

ω: The radian frequency (p. 16).

j: $\sqrt{-1}$ (p. 16).

$\vec{J}_S$: Source current density vector (p. 16).

ϵ_r: The permittivity of a medium relative to free space (p. 17).

n_0: The refractive (complex) index (p. 17).

$\vec{D}_c$: Displacement density vector. See (p. 18).

ϵ_1': Complex permittivity (p. 18).

$\vec{D}_c^{\,+}$: The value of $\vec{D}_c$ immediately above the surface (p. 19).

$\delta(x)$: Dirac delta function (p. 19).

$\hat{x}$: Unit vector along the x axis (p. 19).

$\hat{y}$: Unit vector along the y axis (p. 19).

$\hat{z}$: Unit vector along the z axis (p. 19).

$\vec{E}^{+}$: Electric field vector immediately above the surface (p. 20).

$\vec{E}_s$: Alternate notation for $\vec{E}^{+}$ (p. 20).

$\vec{D}_c^{-}$: The value of $\vec{D}_c$ immediately below the surface (p. 21).

$\vec{E}^{-}$: Electric field vector immediately below the surface (p. 21).

$\vec{E}_m$: Alternate notation for $\vec{E}^{-}$ (p. 21).

T_{SE}: The source current density operator (electric) (p. 22).

γ_0^2: See page 22.

k: Wave number of the fundamental (p 22).

T_{SM}: The source current density operator (magnetic) (p. 23).

E_x: Cartesian (x) component of $\vec{E}$ (p 24).

E_y: Cartesian (y) component of $\vec{E}$ (p. 24).

E_z: Cartesian (z) component of $\vec{E}$ (p. 24).

E_{sx}: Cartesian (x) component of $\vec{E}_s$ (p. 24).

E_{sy}: Cartesian (y) component of $\vec{E}_s$ (p. 24).

E_{sz}: Cartesian (z) component of $\vec{E}_s$ (p. 24).

E_{mx}: Cartesian (x) component of $\vec{E}_m$ (p. 25).

E_{my}: Cartesian (y) component of $\vec{E}_m$ (p. 25).

E_{mz}: Cartesian (z) component of $\vec{E}_m$ (p. 25).

$\left[\dfrac{\partial \vec{E}}{\partial z}\right]^{+}$: Partial derivative of $\vec{E}$ immediately above the surface (p 25).

$\left[\frac{\partial E}{\partial z}\right]^-$:	Partial derivative of $\overline{E}$ immediately below the surface (p. 25).
γ_1^2:	See (p. 26).
K_{01}:	Greens Function.
K_{02}:	Greens Function.
r:	Radial distance (p. 27).
$\overline{E}_i$:	Incident electric field vector (p. 30).
$\overline{R}^+(x,y)$:	See (p. 31).
$\overline{R}^-(x,y)$:	See (p. 31).
$\delta'(z)$:	The derivative of the Dirac delta function (p. 31).
∇_{xy}:	The gradient of a function with respect to x,y (p. 32).
$\overline{F}(x,y)$:	Vector function used to solve the integral equation for the electric field (p. 32).
$\overline{G}(x,y)$:	Vector function used to solve the integral equation for the electric field (p. 33).
K_x:	Spatial wave number (p. 34).
K_y:	Spatial wave number (p. 34).
U_E:	See (p. 34).
U:	See (p. 35).
λ^2:	See (p. 34).
γ^2:	See (p. 34).
$\overline{E}_{ib}$:	Incident electric field in a plane immediately below the surface (p. 35).
E_{ibx}:	Cartesian (x) component of $\overline{E}_{ib}$ (p. 39).
E_{iby}:	Cartesian (y) component of $\overline{E}_{ib}$ (p. 39).
E_{ibz}:	Cartesian (z) component of $\overline{E}_{ib}$ (p. 39).

E_t : Electric field above the surface (p. 41).

$\overline{E}_{ia}$: Incident electric field in a plane immediately above the surface (p. 42).

E_{iax} : Cartesian (x) component of $\overline{E}_{ia}$ (p. 43).

E_{iay} : Cartesian (y) component of $\overline{E}_{ia}$ (p. 43).

E_{iaz} : Cartesian (z) component of $\overline{E}_{ia}$ (p. 43).

E_{tx} : Cartesian component of E_t (p. 43).

E_{ty} : Cartesian component of E_t (p. 43).

E_{tz} : Cartesian component of E_t (p. 43).

$I_0(\omega)$: Dipole current (p. 45).

dl : Dipole length (p. 45).

h_0: Height of dipole above the surface (p. 45).

$\delta_x(x)$: The x partial derivative of the Dirac delta function (p. 47).

$\delta_y(y)$: The y partial derivative of the Dirac delta function (p. 47).

$\delta_z(z)$: The z partial derivative of the Dirac delta function (p. 47).

E_{ix} : Cartesian (x) component of the incident electric field (p. 48).

E_{iy} : Cartesian (y) component of the incident electric field (p. 48).

E_{iz} : Cartesian component of the incident electric field (p. 48).

r_1: Radial distance (p. 48,50).

r_c : Radial distance (p. 48,50).

C_d : Dipole constant (p. 49).

R_b : Radial distance (p. 52).

R_c : Radial distance (p. 52).

ρ: Polar coordinate for spatial wave numbers (p. 53).

λ: Polar coordinate for spatial wave numbers (p. 53).

ϕ: Polar coordinate for spatial wave numbers (p. 53).

θ: Polar coordinate for spatial wave numbers (p. 53).

$J_0(z)$: Bessel function of order 0 (p. 54).

Δ: Normalized surface impedance (p. 54).

$F(w)$: Attenuation function (p. 55).

p_e: Numerical distance (p. 55).

w: Height gain parameter (p. 55).

$erfc$: Complementary error function (p. 55).

Z_s: Surface impedance in ohms (p. 57).

Z_0: Impedance of free space (p. 57).

E_t: Tangential electric field (p. 57).

H_t: Tangential magnetic field (p. 57).

$\overline{\Delta}_m$: Average modified surface impedance of a rough ocean surface (p. 59).

$F(p,q)$: Kernel of the integrand for modified surface impedance integral as a function of the spatial wave numbers p and q (p. 59).

$W(p,q)$: Ocean surface height spectral density (p. 59).

α: Angle between radio propagation path and wind direction (p. 63).

α: Empirical oceanographic constant (p. 59).

g: Acceleration due to gravity (p. 59).

U: Wind speed in m/s (p.59).

λ: Polar coordinate for ocean height spectral density (p. 64).

ϕ: Polar coordinate for ocean height spectral density (p. 64).

$G(\lambda)$: Directional component of the ocean height spectral density in polar form (p. 64).

$F(\phi)$: Amplitude component of the ocean height spectral density in polar form (p 64).

p_{em}: Numerical distance using modified surface impedance (p 72).

w_m: Height gain parameter using modified surface impedance (p 72).

d: Distance along the earth's surface (p. 73)

f: Transmit frequency in Megahertz (p. 73)

h_T: Transmit antenna height (p. 74)

h_R: Receive antenna height (p. 74)

a: Approximate radius of the earth (p. 75)

λ: Radio wavelength (p. 75).

D: Separation distance between transmit and receive antennas (p. 76).

E_r: Component of the electric oriented radially from the spherical surface of the earth (p. 76).

$f_T(h_T)$: Spherical earth height gain function (p. 78)

$f_R(h_R)$: Spherical earth height gain function (p. 78)

$W_N'(t)$: Airy Functions (p. 78).

y_R: Receive antenna height gain parameter (p. 78).

y_T: Transmit antenna height gain parameter (p. 78).

t_s: Poles of the residue series (p. 78).

$A(q_v, t_s)$: Residue series function (p. 78).

q_v: Surface impedance function (p. 79)

z: See (p. 79).

x: See (p. 79).

K: See (p. 80).

θ:	See (p. 80).
q_{vm}:	Modified surface impedance function (p. 80)
δ:	Surface impedance function in Bremmer's form (p. 82)
τ_s:	Poles of the residue series in Bremmer's form (p. 82)
$\tau_{s,0}$:	Limiting values for τ_s (p. 84)
$\tau_{s,\infty}$:	Limiting values for τ_s (p. 84)
τ_s^m:	The m iteration on t_s (p. 84).
$H_{\frac{n}{3}}^{(2)}$:	Spherical Hankel functions (p. 86).
$Ai(t)$:	Airy functions using alternate definition (p. 86).
$Ai'(t)$:	Derivative of Airy functions using an alternate definition (p. 86).
$Bi(t)$:	Airy functions using alternate definition (p. 86).
$Bi'(t)$:	Derivative of Airy functions using alternate definition (p. 86).
$W'_n(t)$:	Derivative of Airy functions (p. 86).
$f(z)$:	Function for convergent series expansion of Airy functions (p. 87).
$g(z)$:	Function for convergent series expansion of Airy functions (p. 87).
$L(t)$:	L series for asymptotic expansion of Airy functions (p. 88).
$M(t)$:	M series for asymptotic expansion of Airy functions (p. 88).
$F(q_v, t_s)$:	Residue series function (p. 90).
E_0:	Electric field from an elemebtary vertical electric dipole in free space (p. 93).
W_r:	Spherical earth attenuation function (p. 93).
G_T:	Transmit antenna gain (p. 94).
G_R:	Receive antenha gain (p. 94).
P_t:	Transmitted power (p. 94).

P_r : Received power (p. 94).

η_0: Intrinsic impedance of free space (p. 94).

A_r : Effective aperature of an antenna (p. 94).

T_L : Transmission loss (p. 94).

CHAPTER 1

INTRODUCTION

1.0 GENERAL INTRODUCTION

In radio communications a practical question which arises is the maximum usable range of a given transmitter. A major component of such a prediction is the ability to estimate the strength of the electromagnetic field distant from its source. Models for the electromagnetic (EM) field in empty space are relatively simple; it is the problem of determining the modification to this field due to the presence of the earth's surface which is not trivial. It is the problem of estimating the earth's effects on the propagation of electromagnetic waves to which our attention is directed. In particular, it is the numerical evaluation of models for radio propagation over the earth, in effort to estimate the power losses as a function of the propagation distance which is of interest.

Analytical models for ground wave radio propagation have been developed for many years. Around the time of the development of radio, at the turn of this century, physicists, mathematicians and engineers developed analytical models which predicted the behaviour of electromagnetic fields in the presence of the earth's surface. New theories as well as refinements to the old have been developed in the subsequent years so that more accurate predictions for an electromagnetic field in the presence of the earth are possible. In this thesis models for EM propagation over the earth are considered and computer models for radio wave propagation losses in the presence of the ocean surface are proposed. Many significant factors which affect the propagation of radio waves, such as the electrical properties of the surface, the surface roughness, say for example caused by ocean waves, as well as the curvature of the earth's surface and the diffraction losses associated with the curvature are considered: Of course the characteristics of the source, such as the operating frequency, are also included in the modelling effort.

As a first step in this investigation, a solution to the classic problem of radio propagation over a flat surface is developed, by an alternate analysis. By using a spatial decomposition method, expressions for the electric field from an arbitrary source over a planar surface with arbitrary electrical parameters is derived. This expression is in the form of the spatial Fourier transform of the field. An elementary vertical dipole source as well as a highly conductive earth surface, such as the ocean surfaceis assumed, and the classical integral solution to the plane earth problemis derived. The results are not startling, but significant since an alternate approach to the problem has been used. As well the electric field for any finite source and an arbitrary surface impedance could be determined, providing the inverse spatial Fourier transform of the electric field could be determined.

Diverging slightly from this result, models for the surface impedance, which represents the electrical properties of the surface, for propagation over a rough ocean surface are examined. These results will enable the prediction of radio wave transmission losses for propagation over a rough sea. Assuming a rough wind driven sea, models for the surface impedance which account for the interaction between the EM wave and the ocean surface are implemented in a computer program. The expressions for the modified surface impedance are in terms of the ocean wave height spectral density. For the surface impedance calculations a standard oceanographic model for the wave height spectral density is assumed. The calculated values for the surface impedance may be used in numerical transmission loss models to enable the prediction of transmission losses in the ocean environment.

The plane earth model for EM propagation over the earth is suitable for relatively short distances. For longer distances the effects of diffraction around the spherical surface of the earth become significant. Analytical models have been developed for propagation over a homogeneous spherical earth by other investigators. We proceed to develop a computer program which implements a residue series solution to the spherical earth model. The achieved result is a numerical model which predicts the transmission losses for ground wave propagation over the ocean surface including the effects of the earth's curvature. The influence of ocean waves on the propagation of EM waves are determined through the implementation of modified surface impedance expressions for the ocean surface. Typical numerical results for these transmission losses are presented in graphical form.

1.1 LITERATURE REVIEW

Many theoretical models for the propagation of electromagnetic waves along the earth's surface, have been proposed in the literature throughout this century. Sommerfeld [1909,1926,1949] presented a solution for the propagation over a planar surface separating two homogeneous half spaces of differing electrical properties. The upper half space was characterized as air and the lower as a dissipative ground. The source was assumed to be a vertical dipole located in the upper half space. Sommerfeld's physical explanation was the existence of a space wave and a surface wave, both components being required to satisfy the Maxwell's equations with the specified boundary conditions.

Based on an integral formulation of the planar earth problem by Van der Pol and Niesson [1930], Norton [1935,1936,1937] proposed a series solution formula. Norton proposed that the electric field could be divided into three components; the direct ray (direct path between source and observation points), the reflected ray (depending on a Fresnel reflection coefficient), and a surface wave. These formula facilitated numerical computations, enabling Norton to present the planar earth transmission loss versus distance graphically.

Wait [1954,1957] gave Norton's solution to the plane earth problem in an alternate form. Utilizing the surface impedance concept, Wait developed the same asymptotic and convergent series solutions as Norton. In addition, Wait developed another term for the Norton asymptotic series valid when the phase of the numerical distance, ϕ, is $\pi > \phi > 0$, which gives rise to the trapped surface wave phenomenon discussed by Wait [1970].

Solutions for the propagation over a spherical earth have also been investigated by many authors. These methods are extensions of Watson's [1918,1919] investigations of the field from a radially oriented dipole in the presence of a homogeneous dissipative sphere. The solution was in the form of a series of spherical Hankel functions and Legendre polynomials. This series was not practical for propagation problems due to the enormous number of terms of the series required for convergence. Indeed this series solution was only applicable to electric field problems when the wavelength was a significant fraction of the radius of the sphere. Following Watson's approach, the harmonic series was transformed into an integral in the complex plane. Van der Pol and Bremmer [1937,1938,1939] formulated this type of contour integral for the propagation of radio waves along the earth. The spherical Hankel functions were approximated by Hankel functions of order 1/3 and the Legendre polynomials replaced by the leading term in their asymptotic expansion. Using these approximations Van der Pol and Bremmer wrote a residue series solution for the contour integral which was suitably simple for numerical computations. Norton [1941] used this formulation to generate numerical results.

Using an independent analysis, Fock [1945] obtained a similar residue series solution. Fock used an approximation for the Watson's spherical Hankel functions in terms of the Airy functions [Abramowitz and Stegun, 1965]. It is this approximation which commonly appears in the literature, although Wait [1970] suggests that both results achieve similar results.

In a different approach to the spherical earth propagation problem, Bremmer [1949] also uses the geometrical theory of diffraction to determine another approximate solution. This saddle point approximation is valid only when the source and observation points are well above the horizon, that is for high receive and transmit antenna elevations and short separation distances. For these situations Bremmer suggests that the residue series may be poorly convergent. For short distances there are two additional approximation formulae for the spherical earth attenuation function. For a small radius of curvature and low frequency a power series expansion may be used, as developed by Wait [1956,1958] and Bremmer [1958]. At a large radius (small curvature) an expansion in terms of the planar earth (Norton) attenuation function is given by Wait [1956] and Bremmer [1958]. Results using both these methods and for a variety of surface impedances have been presented by Hill and Wait [1980].

The investigations of the effects of surface roughness on the propagation of electromagnetic waves over the earth's surface commenced with Feinberg's [1944] results. Feinberg formulated the problem in an integral equation and gave a result for small surface height irregularities. The result did not account for the effect of finite surface conductivity. Rice [1951], using a perturbational analysis, treated the problem of scattering from slightly rough random surfaces. Wait [1957] derives an expression for the surface impedance of a slightly corrugated but otherwise perfectly conducting surface. This result was for the inductive contribution when the height and period of the corrugations are small compared to an electrical wavelength. Wait [1959,part 1] also derives an effective surface impedance for a perfectly conducting surface having a uniform distribution of hemispherical bosses whose electrical parameters are arbitrary. Wait [1959,part 2] also discusses the effect of the earth's curvature using such a rough surface model.

Barrick [1971a, 1971b] derived a result for the modified surface impedance of a rough sea, using Rice's perturbation method. This analysis assumes a random periodic surface which may be described by the average height spectral density of the surface. Using standard oceanographic models for the ocean average wave height spectral density, Barrick estimates the additional transmission losses due to the roughness of the surface.

By an alternate approach, Srivastava [1984] derives an expression for the modified surface impedance of a rough ocean as part of his analysis of the back-scattered radar cross-section of the ocean surface. The analysis, based on the theory of generalized functions, is an extension of Walsh's [1980] general approach to rough surface scatter. The analysis assumes an elementary vertical electric dipole source located near a surface described by the average ocean wave height spectral density. The surface impedance expression obtained by Srivastava as well as that obtained by Barrick both reduce to that of Feinberg in the limiting case.

1.2 SCOPE OF THESIS

In this thesis solutions for ground wave propagation over a homogeneous earth (spherical and planar earth models) are examined. The primary objective is to develop computer programs which will predict the transmission loss for radio wave propagation over a planar or spherical earth model with or without surface roughness at HF (3-30 MHz.) and lower radio frequencies.

Initially, ground wave propagation over a planar earth with arbitrary electrical parameters and an arbitrary source are studied. A solution for this problem is derived in the two dimensional spatial Fourier transform domain. The analysis is based on a technique developed by Walsh [1980] for a general formulation for rough surface propagation and scattering. The method uses Heaviside functions to spatially decompose the electric field equation into three equations: the field above the surface, below the surface and an equation linking the fields at the boundary. Thus it is clear that the method supplies its own boundary conditions.

By assuming an elementary vertical electric dipole source the integral solution derived herein is the same as that derived by Sommerfeld [1909, 1926]. For a highly conductive surface the integral solution reduces to that derived by Wait [1970]. Following Wait's results, the series solutions for this integral have been presented.

This series solution may be easily implemented in a computer program and results presented in graphical form. For relatively small separation distances between source and observation points, diffraction effects around the spherical surface of the earth are negligible, so that the planar earth model will yield satisfactory propagation loss results. The limit of applicability of the planar earth solution is generally considered to be $d = 50/f^{1/3}$ where d is the separation distance in miles and f the radio frequency in megahertz [Jordan and Balmain, 1968]. The obvious advantage for using this solution for short distances is the small amount of computer resources required to calculate the planar earth series solution.

When large separation distances between the source and observation points are considered, the additional effects of diffraction around the curvature of the earth become significant. Several authors have presented solutions to the problem of ground wave electromagnetic propagation over a spherical earth. Based on classical techniques Fock [1945] and Bremmer [1949] have presented residue series approximations to the contour integral formulation of this problem, as given by Watson [1919]. Using these results, an efficient Fortran program is developed which evaluates the residue series solution for the ground wave electric field for a finitely conducting spherical earth. A previous computer program, written by Berry and Chrisman [1966], also implemented the residue series equations for the electric field. The program documented in this research offers many advantages over the Berry and Chrisman implementation. In particular, an alternate technique is used to evaluate the poles of the residue series. Berry and Chrisman use a series expansion for the poles, as developed by Bremmer [1949]. The new program uses a Newton iteration technique on the pole defining equation, to estimate the poles of the residue series. As well, the new program is written in modern Fortran-77 source code using complex arithmetic, permitting a compact, fast and easy to follow program. The methods used by Berry and Chrisman placed significant limitations on the adaptability of their program to smaller computers.

To account for the effects of surface roughness on the propagation of radio waves over a spherical or planar earth model, expressions for a modified surface impedance, for a rough wind driven sea, have been examined and implemented. The modified surface impedance presented by Barrick [1971] has been implemented in a computer program, using a suitable oceanographic model for the ocean surface height spectral density. This model is implemented in a Fortran subroutine subprogram of the planar and spherical earth propagation program, and uses a standard package program (IMSL) to perform the required integration. An alternate expression for the modified surface impedance, developed by Srivastava [1984], has also been examined. The expression developed by Srivastava has been implemented, with some simplifying assumptions. A Neumann-Pierson [Neumann et al, 1985] model for the ocean surface height spectral density and calculate transmission losses over a rough spherical earth using the Srivastava model is assumed. Comparisons of the results from the two surface impedance expressions are presented. Calculations for the transmission losses using the Barrick model are available for comparison from Barrick [1970]. Fortran source code listings of the rough surface spherical earth model is included in the appendix.

CHAPTER 2

THE PLANE EARTH SOLUTION FOR THE ELECTRIC FIELD

2.0 GENERAL

In this section a classic problem in electromagnetic propagation theory is approached by a new formulation. The problem is that of propagation over a planar surface of finite electrical properties. This analysis follows the methods of Walsh [1980], originally developed for rough surface propagation and scatter. It is not expected that the analysis will reveal any startling new results; rather it will yield a set of general equations for the electric field in the spatial Fourier transform domain. In these equations the choice of a source remains arbitrary and no assumptions are made regarding the electrical properties of the planar surface, or on the behaviour of the fields on the surface. The electric field for a given source may be evaluated, assuming the inverse spatial Fourier transforms may be determined.

In the last two sections of this chapter, the electric field for elementary vertical dipole antennas is derived as a specific case. This result, in the spatial (x,y) Fourier transform domain, is equivalent to the integral equation derived by Sommerfeld [1909,1926]. As well, a highly conductive surface, such as the ocean surface, is assumed yielding an equivalent result to that of Wait [1970].

The method of solution utilizes a spatial decomposition of the electric field, for components in the half spaces above and below the earth interface. The medium above the surface is approximated by 'free space' and is assumed to contain the source. The medium below the surface is characterized by its electrical properties, namely; the conductivity, the permeability and the permittivity. Figure 2.1 illustrates the geometry of the problem assumed for this analysis.

A basic partial differential equation, which the electric field must satisfy, is derived using the Maxwell equations, the electrical properties of the complete space and the spatial decomposition of the fields. The partial differential equation is itself decomposed into two wave equations, for the fields above and below the surface, and a third equation which the fields must satisfy at the boundary (boundary conditions). It may be noted that no external boundary conditions are applied; the boundary equation is a product of the analysis.

A set of two coupled, convolution type, integral equations are then derived using the fundamental solutions to the wave equation. Solving the two integral equations yields a functional relationship between the source electric field and the electric field above the interface, in the spatial Fourier transform domain. The electric field, for any given source, may be determined from these equations provided the inverse spatial transforms may be determined. For elementary vertical electric dipole antennas, the resulting integral equation for the electric field is shown to be equivalent to that which was derived by Sommerfeld [1909,1926]. Finally, a highly conductive surface is assumed, the results of which are equivalent to the integral solved by Wait [1970].

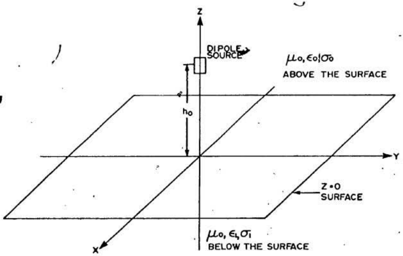


FIGURE 2.1 : Geometry of the Planar Earth Propagation Model

2.1 INITIAL ASSUMPTIONS

The problem of determining a model for the electric field above an assumed planar earth model has been approached and solved by many investigators, among the earliest being Sommerfeld [1909,1926] who determined expressions for the space wave and surface wave portions of the electric field. The plane earth problem is described as the propagation of electromagnetic fields through a medium approximately described as 'free space' over a homogeneous planar surface with arbitrary electrical properties. Sommerfeld assumed a vertical electric dipole source. This work derives the complete electric field above a planar surface for an arbitrary source as a special case of the Walsh [1980] general treatment of propagation and scattering from rough surfaces.

The analysis begins by deriving the basic partial differential equation for propagation of electromagnetic fields over a planar earth. This is derived by an electric field decomposition approach as described by Walsh [1980]. First, expressions describing the electrical properties of the complete space are derived. In the half-space above the planar surface the electrical properties are described by the following:

$\mu_0 =$ the permeability ,
$\epsilon_0 =$ the permittivity ,
$\sigma_0 = 0$ the conductivity .

Similarly, in the half space below the planar surface we have

$\mu_0 =$ the permeability ,
$\epsilon_1 =$ the permittivity ,
$\sigma_1 =$ the conductivity .

As well, it is assumed that $z=0$ describes the location of the planar surface separating the two half-spaces. The electrical properties of the complete space may be described using the Heaviside functions, which are defined as

$$h(z)=\begin{cases}0, & z\le 0\\ 1, & z>0\end{cases}$$

Using the Heaviside function $h(z)$, the electrical properties of the complete space may be written in terms of the the electrical constants prescribed for the complete space as

$$\sigma = (1-h(z))\,\sigma_1 \ , \tag{2.1}$$
$$\epsilon = \epsilon_0\, h(z) + \epsilon_1\,(1-h(z)) \ , \tag{2.2}$$

and

$$\mu = \mu_0 \ . \tag{2.3}$$

The terms containing $(1-h(z))$ are the electrical properties of the space below the surface and terms containing only $h(z)$ are electrical properties of the space above the surface. Thus a set of three equations, (2.1), (2.2), and (2.3) describe the electrical properties of the complete space. The Maxwell equations in time-harmonic form, using the usual conventions for symbols, are given as

$$\nabla\times\vec{E} = -j\,\omega\,\vec{B} \ , \tag{2.4}$$
$$\nabla\times\vec{H} = j\,\omega\,\vec{D} + \vec{J} \ , \tag{2.5}$$
$$\nabla\cdot\vec{B} = 0 \tag{2.6}$$

and

$$\nabla\cdot\vec{D} = \rho \ . \tag{2.7}$$

It is assumed that the Maxwell equations apply to the complete space. We also assume that the media both above and below the surface are linear and isotropic.

With these assumptions we also have the following relationships:

$$\vec{B} = \mu_0 \vec{H} \qquad \text{for all } z \quad , \tag{2.8}$$

$$\vec{D} = \epsilon \vec{E} = \left[\epsilon_0 h(z) + \epsilon_1 \left(1 - h(z) \right) \right] \vec{E} \quad , \tag{2.9}$$

and

$$\vec{J}_c = \sigma_1 \left(1 - h(z) \right) \vec{E} \quad . \tag{2.10}$$

The parameter $\vec{J}_c$ is defined as the conduction current density.

2.2 BASIC PARTIAL DIFFERENTIAL EQUATION

We now proceed to derive the basic partial differential equation for the plane earth problem, by using the Maxwell equations and the assumptions in the previous section. The curl of the electric field, $\nabla \times \vec{E}$, is written in terms of the magnetic field $\vec{H}$ as follows:

$$\nabla \times \vec{E} = -j\omega\vec{B} = -j\ \omega\ \mu_0\ \vec{H} \quad . \tag{2.11}$$

By taking the curl of both sides of equation (2.11), $\nabla \times \nabla \times \vec{E}$ is expressed as

$$\nabla \times \nabla \times \vec{E} = -j\ \omega\ \mu_0 \nabla \times \vec{H}' \quad . \tag{2.12}$$

We substitute the expression for the curl of $\vec{H}$ in terms of the displacement current vector, $\vec{D}$, and the current density, $\vec{J}$, from equation (2.5) into our expression for $\nabla \times \nabla \times \vec{E}$. This yields

$$\nabla \times \nabla \times \vec{E} = -j\ \omega\ \mu_0 \left[j\ \omega\ \vec{D} + \vec{J} \right] \quad . \tag{2.13}$$

The current density $\vec{J}$ may be written as two separate components, one for the conduction current density and a second for the source current density. We write $\vec{J}$ as

$$\vec{J} = \vec{J}_s + \vec{J}_c \quad . \tag{2.14}$$

The parameter $\vec{J}_S$ is the source current density and the parameter $\vec{J}_c$ is the conduction current density. By using this convention for the current density, equation (2.13), for the curl of the curl of the electric field, may be expanded to obtain

$$\nabla \times \nabla \times \vec{E} = -j\,\omega\,\mu_0 \left[\, j\,\omega\,\vec{D} + \vec{J}_S + \vec{J}_c \,\right] \quad . \tag{2.15}$$

A useful vector identity which may be applied to equation (2.15) to decompose $\nabla \times \nabla \times \vec{E}$ is

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \quad .$$

The equation (2.15) may be decomposed by using the above vector identity. We use expressions (2.1) and (2.2) for permittivity and conductivity. The expanded version of (2.15) is

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -j\,\omega\,\mu_0 \left[\, \sigma_1 (1-h(z))\,\vec{E} + j\,\omega \left[\, \epsilon_0\, h(z) + \epsilon_1 (1-h(z)) \,\right] \vec{E} + \vec{J}_S \,\right] \tag{2.16}$$

The appearance of equation (2.16) may be simplified greatly by first making the following definitions for the relative permittivity and the refractive index:

$$\epsilon_r = \frac{\epsilon_1}{\epsilon_0} \quad ,$$

$$n_0 = \left\{ \epsilon_r + \frac{\sigma_1}{j\,\omega\epsilon_0} \right\}^{1/2} \quad .$$

By using the above definitions equation (2.16) is

$$\nabla^2 \vec{E} + \omega^2 \mu_0 \epsilon_0 \left[n_0^2 \;\left(1 - h(z) \right) + h \;\right] \vec{E} = j\,\omega\,\mu_0 \vec{J}_s + \nabla \left(\nabla \cdot \vec{E} \right) \quad . \tag{2.17}$$

The right hand side of this equation contains the gradient of the divergence of $\vec{E}$ which may be interpreted by using our previous results. Commencing with equations (2.9) and (2.10), we may note the following:

$$\vec{J}_c + j\,\omega\,\vec{D} = j\,\omega \left[\, \vec{D} + \frac{\vec{J}_c}{j\,\omega} \,\right]$$

$$= \left[\sigma_1 \left(1 - h(z) \right) + j\,\omega \left\{ \epsilon_0 h(z) + \epsilon_1 \left(1 - h(z) \right) \right\} \right] \vec{E}$$

$$= j\,\omega \left[\left(\epsilon_1 + \frac{\sigma_1}{j\,\omega} \right) \left(1 - h(z) \right) + \epsilon_0 h(z) \right] \vec{E} \quad (2.18)$$

The quantities $\vec{D}_c$ and ϵ'_1 are defined as follows:

$$\vec{D}_c = \vec{D} + \frac{\vec{J}_c}{j\,\omega}$$

$$\epsilon_1' = \epsilon_1 + \frac{\sigma_1}{j\,\omega}$$

The definitions for $\vec{D}_c$ and ϵ'_1 are applied to (2.18), which may be written as

$$\vec{D}_c = \left[\epsilon_1' \left(1 - h(z) \right) + \epsilon_0 h(z) \right] \vec{E} \quad (2.19)$$

Equation (2.19) may be inverted, yielding an expression for $\vec{E}$ in terms of $\vec{D}_c$, the Heaviside functions, and the electrical properties of the complete space. This expression is

$$\vec{E} = \left[\frac{\left(1 - h(z) \right)}{\epsilon_1'} + \frac{h(z)}{\epsilon_0} \right] \vec{D}_c = \left[\frac{1}{\epsilon_1'} - \frac{h(z)}{\epsilon_1'} + \frac{h(z)}{\epsilon_0} \right] \vec{D}_c$$

$$= \left[\frac{1}{\epsilon_1'} + \frac{h(z) \left(\epsilon_1' - \epsilon_0 \right)}{\epsilon_1' \epsilon_0} \right] \vec{D}_c \quad (2.20)$$

This relationship may be used to interpret the divergence of the electric field, $\nabla \cdot \vec{E}$. By taking the divergence of both sides of equation (2.20), we arrive at a suitable relationship between the divergence of the electric field, the quantity $\vec{D}_c$ and the requisite electrical properties. The divergence of $\vec{E}$ is

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_1'} \nabla \cdot \vec{D}_c + \frac{\epsilon_1' - \epsilon_0}{\epsilon_0 \epsilon_1'} \nabla \cdot \left(h(z) \vec{D}_c \right) \quad (2.21)$$

The term $\nabla \cdot \left(h(z) \vec{D}_c \right)$ in the above expression may also be interpreted by expanding the derivatives as follows:

$$\nabla \cdot \{h(z)\vec{D}_c\} = h(z)\{\nabla \cdot \vec{D}_c\} + \{\nabla \cdot h(z)\}\vec{D}_c$$
$$= h(z)\{\nabla \cdot \vec{D}_c\} + \hat{z} \cdot \vec{D}_c^{+}\delta(z) \quad ,$$

where

$$\vec{D}_c^{+} = \lim_{z \to 0^{+}} \vec{D}_c \quad .$$

is the value of the quantity $\vec{D}_c$ immediately above the surface and $\delta(u)$ is the Dirac delta function and $\hat{z}$ is a unit vector along the z axis. Equation (2.21) for the divergence of $\vec{E}$ may be written using the above results as

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_1'} \nabla \cdot \vec{D}_c + \frac{\epsilon_1' - \epsilon_0}{\epsilon_0 \epsilon_1'} \left[h(z)\{\nabla \cdot \vec{D}_c\} + \hat{z} \cdot \vec{D}_c^{+}\delta(z) \right] \quad . \tag{2.22}$$

In order to interpret the divergence of the quantity $\vec{D}_c$, we return to equation (2.5) for the curl of $\vec{H}$, and expand it using the definition for $\vec{D}_c$. This enables writing $\nabla \times \vec{H}$ in the following form:

$$\nabla \times \vec{H} = \vec{J} + j\omega\vec{D} = \vec{J}_S + \vec{J}_c + j\omega\vec{D} = \vec{J}_S + j\omega\vec{D}_c \quad . \tag{2.23}$$

By using the identity $\nabla \cdot \{\nabla \times \vec{H}\} = 0$, we obtain an expression for $\nabla \cdot \vec{D}_c$ in terms of the source current density, $\vec{J}_S$ from equation (2.23). By taking the divergence of both sides of equation (2.23) yields

$$\nabla \cdot \{\nabla \times \vec{H}\} = \nabla \cdot \vec{J}_s + j\omega \nabla \cdot \vec{D}_c = 0 \quad .$$

The divergence of $\vec{D}_c$ may be obtained from the above as

$$\nabla \cdot \vec{D}_c = \frac{-1}{j\omega} \nabla \cdot \vec{J}_s \quad . \tag{2.24}$$

By assumption, the support of the source current density $\vec{J}_s$ lies wholly in the half space $z > 0$. Therefore, it is obvious that $h(z)\{\nabla \cdot \vec{D}_c\}$ may be deduced immediately from equation (2.24) as

$$h(z)\,(\nabla\cdot\vec{D}_c) = -\frac{1}{j\omega}\,h(z)\,\nabla\cdot\vec{J}_s \ .$$

The expression for $\nabla\cdot\vec{E}$ in equation (2.22) may be simplified considerably by using the above result for $h(z)\,(\nabla\cdot\vec{D}_c)$. After some algebra $\nabla\cdot\vec{E}$ is written in terms of $\vec{J}_S$, $\vec{D}_c^{\,+}$, and the Dirac delta function, $\delta(z)$. This expression is

$$\nabla\cdot\vec{E} = -\frac{1}{j\,\omega\,\epsilon_0}\,\nabla\cdot\vec{J}_s + \frac{\epsilon_1{}' - \epsilon_0}{\epsilon_0\epsilon_1{}'}\left[\,\hat{z}\cdot\vec{D}_c^{\,+}\,\delta(z)\,\right] \ , \qquad (2.25)$$

where $\vec{D}_c^{\,+}$ is the value of the quantity $\vec{D}_c$ immediately above the surface. Since

$$\vec{D}_c^{\,+} = \epsilon_0\,\vec{E}^{+} \ ,$$

where $\vec{E}^{+}$ is the value of the electric field immediately above the surface, we may write equation (2.25) for $\nabla\cdot\vec{E}$ in terms of the surface field. For notational convenience we use the symbol

$$\vec{E}_s = \vec{E}^{+} = \lim_{z\to 0^{+}} \vec{E} \ ,$$

to represent the value of the electric field immediately above the planar surface in the positive half space. Equation (2.25) may be simplified using the expression for $\vec{D}_c^{\,+}$, and we now write the following also using our notation for the surface electric field:

$$\nabla\cdot\vec{E} = -\frac{1}{j\,\omega\,\epsilon_0}\,\nabla\cdot\vec{J}_s + \frac{n_0^2 - 1}{n_0^2}\left[\,\hat{z}\cdot\vec{E}_s\,\delta(z)\,\right] \ . \qquad (2.26)$$

The above expression for $\nabla\cdot\vec{E}$ is an interpretation in terms of the source current density $\vec{J}_S$, the refractive index n_0, and the surface electric field $\vec{E}_s$ in the positive half-space. We may also write a similar expression in terms of the surface electric field in the negative half-space. Returning to equation (2.19) we write

$$\vec{E} = \left[\,\frac{(1-h(z))}{\epsilon_1{}'} + \frac{h(z)}{\epsilon_0}\,\right]\vec{D}_c = \left[\,\frac{1}{\epsilon_0} - (1-h(z))\,\frac{\epsilon_1{}' - \epsilon_0}{\epsilon_0\epsilon_1{}'}\,\right]\vec{D}_c \ . \qquad (2.27)$$

Following the same method used to derive equation (2.26) we may write

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \nabla \cdot \vec{D}_c - \frac{\epsilon_1' - \epsilon_0}{\epsilon_0 \epsilon_1'} \nabla \cdot \left[\left(1-h(z)\right) \vec{D}_c \right]$$

$$= -\frac{1}{j\,\omega\,\epsilon_0} \nabla \cdot \vec{J}_s - \frac{\epsilon_1' - \epsilon_0}{\epsilon_0 \epsilon_1'} \left[\left(1-h(z)\right) \nabla \cdot \vec{D}_c - \hat{z} \cdot \vec{D}_c^- \delta(z) \right]$$

$$= -\frac{1}{j\,\omega\,\epsilon_0} \nabla \cdot \vec{J}_s + \frac{\epsilon_1' - \epsilon_0}{\epsilon_0 \epsilon_1'} \left[\hat{z} \cdot \vec{D}_c^- \delta(z) \right] \qquad (2.28)$$

The expression $\vec{D}_c^-$ is the value of $\vec{D}_c$ immediately below the surface, and is defined by

$$\vec{D}_c^- = \lim_{z \to 0^-} \vec{D}_c \; .$$

Also it is apparent from equation (2.19) that, $\vec{D}_c^- = \epsilon_1' \vec{E}^-$, where $\vec{E}^-$ is the electric field immediately below the surface in the negative half-space. $\vec{E}^-$ is defined as

$$\vec{E}^- = \lim_{z \to 0^-} \vec{E} = \vec{E}_m$$

where we now use the notation $\vec{E}_m$ for the surface electric field in the negative half space. By using these results, a second equation for $\nabla \cdot \vec{E}$ may be written in terms of the surface electric field, $\vec{E}_m$, and the source current density $\vec{J}_S$. We now write

$$\nabla \cdot \vec{E} = -\frac{1}{j\,\omega\,\epsilon_0} \nabla \cdot \vec{J}_s + (n_0^2 - 1) \left[\hat{z} \cdot \vec{E}_m \delta(z) \right] \qquad (2.29)$$

By taking the gradient of equations (2.27) and (2.29), two equations may be written for $\nabla(\nabla \cdot \vec{E})$. The two equations are as follows:

$$\nabla \left(\nabla \cdot \vec{E}\right) = -\frac{1}{j\,\omega\,\epsilon_0} \nabla\left(\nabla \cdot \vec{J}_s\right) + \frac{n_0^2 - 1}{n_0^2} \nabla\left(\hat{z} \cdot \vec{E}_s \delta(z)\right) \qquad (2.30)$$

$$\nabla \left(\nabla \cdot \vec{E}\right) = -\frac{1}{j\,\omega\,\epsilon_0} \nabla\left(\nabla \cdot \vec{J}_s\right) + (n_0^2 - 1) \nabla\left(\hat{z} \cdot \vec{E}_m \delta(z)\right) \qquad (2.31)$$

Either of these equations may be used in equation (2.17) to obtain the basic partial differential equation for the electric field. However, since our present interest lies mainly in determining the electric field in the half space above the planar surface, equation (2.30) in terms of the surface field above the surface is most suitable. By using (2.30) in equation (2.17), the following expression is obtained:

$$\nabla^2 \vec{E} + \omega^2 \mu_0 \epsilon_0 \left[n_0^2 \, (1-h(z)) + h(z) \right] \vec{E} = j \omega \mu_0 \vec{J}_s - \frac{1}{j \omega \epsilon_0} \nabla (\nabla \cdot \vec{J}_s) + \frac{n_0^2 - 1}{n_0^2} \nabla \left[\hat{z} \cdot \vec{E}_s \, \delta(z) \right] \quad (2.32)$$

In order to simplify the appearance of equation (2.32), A 'Source Current Density Operator', operating on the source current density $\vec{J}_s$ is defined below as

$$T_{SE} \left[\vec{J}_s \right] = \frac{1}{j \omega \epsilon_0} \left[\nabla (\nabla \cdot \vec{J}_s) + k^2 \vec{J}_s \right]$$

Also, two additional definitions may be made which represent the the electrical properties of the complete space as follows:

$$\gamma_0^2 = k^2 \left(h(z) + (1-h(z)) \, n_0^2 \right) ;$$

$$k^2 = \omega^2 \mu_0 \epsilon_0 \ .$$

The preceding definitions are applied to equation (2.32) and the resulting equation is written as

$$\nabla^2 \vec{E} + \gamma_0^2 \vec{E} = -T_{SE} \, [\vec{J}_S] + \frac{n_0^2 - 1}{n_0^2} \nabla \left[\hat{z} \cdot \vec{E}_s \, \delta(z) \right] \ . \quad (2.33)$$

We have derived the basic partial differential equation (2.33) which the electric field $\vec{E}$ must satisfy. It is obvious that no gauge condition has been used in deriving (2.33). By a similar approach, an expression for the magnetic field, $\vec{H}$ could be achieved. However, our primary interest is again the electric field so that we neglect the details of this derivation and present only the final partial differential equation. The magnetic field must satisfy the following equation:

$$\nabla^2 \vec{H} + \gamma_0^2 \vec{H} = - T_{SM} \{ \vec{J}_S \} - j \omega \{ \epsilon_0 - \epsilon'_1 \} \{ \hat{z} \times \vec{H} \} \delta(z) \ .$$

Either of these equations are equally suitable for this analysis but we choose equation (2.33) in the following sections.

We now proceed to spatially decompose the electric field. This decomposition will result in three separate equations. The first two will represent the electric fields above and below the interface (respectively) separating the two media. The third equation will define a set of boundary conditions which must be satisfied at the interface. In this manner no external boundary conditions need be applied.

2.3 ELECTRIC FIELD DECOMPOSITION

The complete electric field may be separated into fields above and below the planar surface by using the Heaviside functions. To effect this decomposition we first write the electric field $\vec{E}$ as

$$\vec{E} = h(z)\,\vec{E} + \{ 1 - h(z) \}\,\vec{E} \ . \tag{2.34}$$

This expression may be used to spatially decompose the wave equation $(\nabla^2 \vec{E} + \gamma_0^2 \vec{E})$ as written in equation (2.33). The right hand side of (2.34) above is substituted for $\vec{E}$ in the left hand side of (2.33), the basic partial differential equation for the electric field. From equation (2.34), we may proceed with the complete decomposition in a term by term manner as follows:

$$\nabla^2 \vec{E} = \nabla^2 \left(h(z)\,\vec{E} \right) + \nabla^2 \left[(1-h(z))\,\vec{E} \right] \tag{2.35}$$

Each term of equation (2.35) may be examined individually. The first term on the right hand side of (2.35) for the electric field in the upper (positive) half-space is first decomposed into its Cartesian components. We write

$$\nabla^2 [\, h(z)\,\vec{E}\,] = \nabla^2 [\, h(z)\, E_x \,]\,\hat{x} + \nabla^2 [\, h(z)\, E_y \,]\,\hat{y} + \nabla^2 [\, h(z)\, E_z \,]\,\hat{z}$$

Consider, only, the $\hat{x}$ term, $\nabla^2 [\, h(z)\, E_x \,]$.

$$\nabla^2 [\, h(z)\, E_x \,] = \nabla \cdot (\nabla\, (\, h(z)\, E_x \,))$$

The gradient of $h(z)\, E_x$ may be expanded as

$$\begin{aligned} \nabla\, [\, h(z)\, E_x \,] &= h(z)\,\nabla E_x + E_x\, \nabla h \\ &= h(z)\,\nabla E_x + \hat{z}\, E_{sx}\, \delta(z) \end{aligned}$$

and by taking the divergence, we write

$$\begin{aligned} \nabla^2 [\, h(z)\, E_x \,] = \nabla \cdot \nabla [\, h(z)\, E_x \,] &= \nabla \cdot \left[\, h(z)\,\nabla E_x + \hat{z}\, E_{sx}\, \delta(z) \,\right] \\ &= h(z)\,\nabla^2 E_x + \nabla E_x\, \nabla \cdot h(z) + \nabla \cdot \left[\, \hat{z}\, E_{sx}\, \delta(z) \,\right] \\ &= h(z)\,\nabla^2 E_x + \hat{z} \cdot (\, \nabla E_x \,)^{+}\, \delta(z) + \nabla \cdot \left[\, \hat{z}\, E_{sx}\, \delta(z) \,\right] \quad (2.36) \end{aligned}$$

Similarly, we may write expressions for the y and z components as

$$\nabla^2 [\, h(z)\, E_y \,] = h(z)\,\nabla^2 E_y + \hat{z} \cdot (\, \nabla E_y \,)^{+}\, \delta(z) + \nabla \cdot \left[\, \hat{z}\, E_{sy}\, \delta(z) \,\right] \quad (2.37)$$

$$\nabla^2 [\, h(z)\, E_z \,] = h(z)\,\nabla^2 E_z + \hat{z} \cdot (\, \nabla E_z \,)^{+}\, \delta(z) + \nabla \cdot \left[\, \hat{z}\, E_{sz}\, \delta(z) \,\right] \quad (2.38)$$

where E_{sx}, E_{sy}, E_{sz} are the Cartesian components of $\vec{E}_s$. $\vec{E}_s$ is the surface electric field immediately above the interface, defined as

$$\vec{E}_s = \lim_{z \to 0^+} \vec{E}$$

We combine the equations (2.36), (2.37) and (2.38) to obtain the expansion for $\nabla^2 [\, h(z)\,\vec{E}\,]$ as

$$\begin{aligned} \nabla^2 [\, h(z)\,\vec{E}\,] = h(z)\,\nabla^2 \vec{E} + \left[\frac{\partial \vec{E}}{\partial z}\right]^{+} \delta(z) + \left\{ \nabla \cdot \left[\, \hat{z}\, E_{sx}\, \delta(z) \,\right] \right\} \hat{x} \\ + \left\{ \nabla \cdot \left[\, \hat{z}\, E_{sy}\, \delta(z) \,\right] \right\} \hat{y} + \left\{ \nabla \cdot \left[\, \hat{z}\, E_{sz}\, \delta(z) \,\right] \right\} \hat{z} \quad (2.39) \end{aligned}$$

We now turn our attention to the second term of equation (2.35), for the electric field below the surface, and effect the same type of decomposition as above, viz.,

$$\nabla^2 [(1-h(z)) \vec{E}] = \nabla^2 \left[(1-h(z)) E_x \right] \hat{x} + \nabla^2 \left[(1-h(z)) E_y \right] \hat{y} + \nabla^2 \left[(1-h(z)) E_z \right] \hat{z}$$

Taking, for example, only the $\hat{x}$ component, we expand $\nabla \left[(1-h(z))E_x\right]$ as

$$\nabla \left[(1-h(z)) E_x \right] = (1-h(z)) \nabla E_x - \hat{z} \cdot E_{mx} \delta(z) ,$$

and by taking the divergence we obtain

$$\nabla^2 \left[(1-h(z)) E_x \right] = (1-h(z)) \nabla^2 E_x - \left[\hat{z} \cdot (\nabla E_x)^- \right] \delta(z) - \nabla \cdot \left[\hat{z} E_{mx} \delta(z) \right] .$$

Omitting the details of the expansions for the $\hat{y}$ and $\hat{z}$ components, we may write

$$\begin{aligned} \nabla^2 [(1-h(z)) \vec{E}] = (1-h(z)) \nabla^2 \vec{E} - \left(\frac{\partial \vec{E}}{\partial z} \right)^- \delta(z) \\ - \left\{ \nabla \cdot \left[\hat{z} E_{mx} \delta(z) \right] \right\} \hat{x} - \left\{ \nabla \cdot \left[\hat{z} E_{my} \delta(z) \right] \right\} \hat{y} \\ - \left\{ \nabla \cdot \left[\hat{z} E_{mz} \delta(z) \right] \right\} \hat{z} \end{aligned} \tag{2.40}$$

In the above E_{mx}, E_{my}, E_{mz} are the cartesian components of $\vec{E}_m$, which is the surface electric field immediately below the surface. $\vec{E}_m$ is defined as

$$\vec{E}_m = \lim_{z \to 0^-} \vec{E}$$

In equations (2.39) and (2.40), the symbols $\left(\frac{\partial \vec{E}}{\partial z} \right)^+$ and $\left(\frac{\partial \vec{E}}{\partial z} \right)^-$ denote the normal derivatives of $\vec{E}$ immediately above and below the surface. For reference

$$\left(\frac{\partial \vec{E}}{\partial z} \right)^{\pm} = \left[\hat{z} \cdot (\nabla E_x)^{\pm} \right] \hat{x} + \left[\hat{z} \cdot (\nabla E_y)^{\pm} \right] \hat{y} + \left[\hat{z} \cdot (\nabla E_z)^{\pm} \right] \hat{z} . \tag{2.41}$$

By inserting the spatial decomposition, equation (2.34), into (2.33) and by applying the expressions for $\nabla^2\left[h(z)\vec{E}\right]$ and $\nabla^2\left[(1-h(z))\vec{E}\right]$ as shown in equations (2.39) and (2.40), it is obvious that the basic equation (2.33) is satisfied if the electric field satisfies the following equations:

$$h(z)\left[\nabla^2\vec{E} + k^2\vec{E}\right] = -T_{SE}\,[\vec{J}_S] \quad . \tag{2.42}$$

$$(1-h(z))\left[\nabla^2\vec{E} + \gamma_1^2\vec{E}\right] = 0 \quad . \tag{2.43}$$

$$\left[\left(\frac{\partial\vec{E}}{\partial z}\right)^{+} - \left(\frac{\partial\vec{E}}{\partial z}\right)^{-}\right]\delta(z) + \left\{\nabla\cdot\left[\hat{z}\,(E_{sx} - E_{mx})\,\delta(z)\right]\right\}\hat{x}$$
$$+ \left\{\nabla\cdot\left[\hat{z}\,(E_{sy} - E_{my})\,\delta(z)\right]\right\}\hat{y} + \left\{\nabla\cdot\left[\hat{z}\,(E_{sz} - E_{mz})\,\delta(z)\right]\right\}\hat{z}$$
$$= \frac{n_0^2 - 1}{n_0^2}\nabla\left[\hat{z}\cdot\vec{E}_s\,\delta(z)\right]$$
$$= (n_0^2 - 1)\nabla\left[\hat{z}\cdot\vec{E}_m\,\delta(z)\right] \tag{2.44}$$

The symbols $\hat{x}, \hat{y}, \hat{z}$ are the Cartesian unit vectors and $\gamma_1^2 = k^2 n_0^2$.

The equations (2.42) and (2.43) are the governing equations for the electric field above and below the interface. The third equation (2.44) represents the boundary condition which the field must satisfy at the interface.

2.4 REDUCTION TO INTEGRAL EQUATIONS

The three equations (2.42 , 2.43, and 2.44) may be reduced to convolution type integral equations. We make use of the fundamental solutions to the wave equation in the form of Green's functions,

$$K_{01}(x,y,z) = \frac{\exp(-jkr)}{4\pi r} \quad , \tag{2.45}$$

$$K_{02}(x,y,z) = \frac{\exp(-j\,\gamma_1 r)}{4\pi r} \quad . \tag{2.46}$$

In the above we have used the following:

$$r = \left(x^2 + y^2 + z^2 \right)^{1/2} \quad ,$$

$$k = \omega \sqrt{\mu_0\, \epsilon_0} \quad ,$$

$$\gamma_1^2 = k^2\, n_0^2 = k^2 \left(\epsilon_r - \frac{j\,\sigma_1}{\omega\, \epsilon_0} \right) \quad .$$

These functions, K_{01} and K_{02}, must satisfy the following equations:

$$\nabla^2 K_{01} + k^2 K_{01} = -\,\delta(x)\,\delta(y)\,\delta(z) \quad , \tag{2.47}$$

$$\nabla^2 K_{02} + \gamma_0^2 K_{02} = -\,\delta(x)\,\delta(y)\,\delta(z) \quad . \tag{2.48}$$

Two identities enable the use of Green's functions to determine expressions for the electric field. The identities are

$$\nabla^2 [\, h(z)\,\vec{E}\,] * K_{01} = [\, h(z)\,\vec{E}\,] * \nabla^2 K_{01} \quad , \tag{2.49}$$

and

$$\nabla^2 [\,(1 - h(z))\,\vec{E}\,] * K_{02} = [\,(1 - h(z))\,\vec{E}\,] * \nabla^2 K_{02} \quad . \tag{2.50}$$

The asterisk $(*)$ has been used to denote a three dimensional spatial convolution with respect to x, y, and z. It has been assumed in equations (2.49) and (2.50) that these convolutions exist. The above identities may be used with the decompositions for the electric field, equations (2.39) and (2.40), to write convolution equations for the decomposed electric field. We repeat the decomposition for $\nabla^2 \left[h(z)\,\vec{E} \right]$ from equation (2.39) as

$$\nabla^2 [\, h(z)\,\vec{E}\,] = h(z)\,\nabla^2 \vec{E} + \left(\frac{\partial \vec{E}}{\partial z} \right)^+ \delta(z) + \left\{ \nabla \cdot \left[\hat{z}\, E_{x}\, \delta(z) \right] \right\} \hat{x} + \left\{ \nabla \cdot \left[\hat{z}\, E_{y}\, \delta(z) \right] \right\} \hat{y} + \left\{ \nabla \cdot \left[\hat{z}\, E_{z}\, \delta(z) \right] \right\} \hat{z} \quad . \tag{2.51}$$

Also, the decomposition for $\nabla^2 \left[(1 - h(z))\, \vec{E} \right]$ from equation (2.40) is repeated as

$$\nabla^2 [\, (1 - h(z))\, \vec{E}\,] = (1 - h(z))\, \nabla^2 \vec{E} - \left(\frac{\partial \vec{E}}{\partial z} \right)^{-} \delta(z)$$

$$- \left\{ \nabla \cdot \left[\hat{z}\, E_{sx}\, \delta(z) \right] \right\} \hat{x} - \left\{ \nabla \cdot \left[\hat{z}\, E_{sy}\, \delta(z) \right] \right\} \hat{y}$$

$$- \left\{ \nabla \cdot \left[\hat{z}\, E_{sz}\, \delta(z) \right] \right\} \hat{z} \qquad (2.52)$$

The identity equation (2.49) and the expression for $\nabla^2 [\, h(z)\, \vec{E}\,]$, equation (2.52), may be combined to form equation (2.53) as follows:

$$h(z)\, \nabla^2 \vec{E} * K_{01} + \left[\left(\frac{\partial \vec{E}}{\partial z} \right)^{+} \delta(z) \right] * K_{01} + \left[\left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{sx} \right] \right\} \hat{x} + \right.$$

$$\left. \left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{sy} \right] \right\} \hat{y} + \left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{sz} \right] \right\} \hat{z} \right] * K_{01}$$

$$= [\, h(z)\, \vec{E}\,] * \left\{ -\delta(x)\, \delta(y)\, \delta(z) - k^2 K_{01} \right\}$$

$$= -[\, h(z)\, \vec{E}\,] - k^2 [\, h(z)\, \vec{E}\,] * K_{01} \qquad (2.53)$$

Several of the terms in equation (2.53) above may be regrouped. This yields

$$[\, h(z)\, \vec{E}\,] + \left[h(z) \left(\nabla^2 \vec{E} + k^2 \vec{E} \right) \right] * K_{01} = - \left[\left(\frac{\partial \vec{E}}{\partial z} \right)^{+} \delta(z) \right] * K_{01}$$

$$- \left\{ \left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{sx} \right] \right\} \hat{x} + \left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{sy} \right] \right\} \hat{y} \right.$$

$$\left. + \left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{sz} \right] \right\} \hat{z} \right\} * K_{01} \qquad (2.54)$$

The form of equation (2.54) may be simplified by examining and simplifying several of the terms. These terms are

$$\left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{sx} \right] \right\} \hat{x} = \frac{\partial}{\partial z} \left[E_{sx}\, \delta(z) \right] \hat{x} \quad ,$$

$$\left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{ny} \right] \right\} \hat{y} = \frac{\partial}{\partial z} \left[E_{ny}\, \delta(z) \right] \hat{y} \ .$$

and

$$\left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{nz} \right] \right\} \hat{z} = \frac{\partial}{\partial z} \left[E_{nz}\, \delta(z) \right] \hat{z} \ .$$

The above terms are combined to yield

$$\left\{ \left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{sx} \right] \right\} \hat{x} + \left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{sy} \right] \right\} \hat{y} + \left\{ \nabla \cdot \left[\hat{z}\, \delta(z)\, E_{sz} \right] \right\} \hat{z} \right\} * K_{01}$$
$$= \left\{ \frac{\partial}{\partial z} \left[E_{sx}\, \delta(z) \right] \hat{x} + \frac{\partial}{\partial z} \left[E_{sy}\, \delta(z) \right] \hat{y} + \frac{\partial}{\partial z} \left[E_{sz}\, \delta(z) \right] \hat{z} \right\} * K_{01}$$
$$= \left\{ \frac{\partial}{\partial z} \left[\vec{E}_s\, \delta(z) \right] \right\} * K_{01} \ .$$

As before, $\vec{E}_s$ is the surface field in the positive half-space. This simplification may be used to rewrite equation (2.54) in the following form:

$$[\, h(z)\, \vec{E}\,] + \left[h(z) \left(\nabla^2 \vec{E} + \gamma_0^2 \vec{E} \right) \right] * K_{01} = - \left[\left(\frac{\partial \vec{E}}{\partial z} \right)^{+} \delta(z) \right] * K_{01} - \left[\frac{\partial}{\partial z} \left[\vec{E}_s\, \delta(z) \right] \right] * K_{01} \quad (2.55)$$

Equation (2.42), the decomposed basic partial differential equation, may be substituted into (2.55). This yields the following equation for the electric field above the surface:

$$[\, h(z)\, \vec{E}\,] = -T_{SE}\, [\vec{J}_S] * K_{01} - \left\{ \left(\frac{\partial \vec{E}}{\partial z} \right)^{+} \delta(z) + \left[\frac{\partial}{\partial z} \left[\vec{E}_s\, \delta(z) \right] \right] \right\} * K_{01} \ . \quad (2.56)$$

An expression for the electric field below the surface may be obtained by a similar decomposition. Omitting the details of this decomposition, the resulting equation is given below:

$$[(1 - h(z)) \vec{E}] = \left\{ \left[\frac{\partial \vec{E}}{\partial z} \right]^{-} \delta(z) + \frac{\partial}{\partial z} \left[\vec{E}_m \delta(z) \right] \right\} \cdot K_{\infty} \tag{2.57}$$

We now return to the equation for the boundary conditions, equation (2.44). The same simplification applied to equation (2.54) for the field above the surface, may be used for the boundary condition equation. Equation (2.44) for the boundary condition may be written using these simplifications as

$$\left\{ \left[\frac{\partial \vec{E}}{\partial z} \right]^{+} - \left[\frac{\partial \vec{E}}{\partial z} \right]^{-} \right\} \delta(z) + \frac{\partial}{\partial z} \left[(\vec{E}_t - \vec{E}_m) \delta(z) \right]$$

$$= \frac{n_0^2 - 1}{n_0^2} \nabla \left[\hat{z} \cdot \vec{E}_t \delta(z) \right]$$

$$= (n_0^2 - 1) \nabla \left[\hat{z} \cdot \vec{E}_m \delta(z) \right] \tag{2.58}$$

Equations (2.56) and (2.57) decompose the electric field into two components, the electric field above the surface and the electric field below the surface. These equations along with (2.58) express the field in terms of the following four functions:

$$\left[\frac{\partial \vec{E}}{\partial z} \right]^{+}, \left[\frac{\partial \vec{E}}{\partial z} \right]^{-}, \vec{E}_t, \vec{E}_m$$

It is the problem of determining these functions to which our attention is now directed. To this aim, we define the incident (or source) electric field in terms of the source current operator, operating on the source current density. The incident electric field is

$$\vec{E}_i = -T_{SE} [\vec{J}_S] \cdot K_{01}$$

The expression $-T_{SE} [\vec{J}_S]$ has already been defined. By using the incident electric field notation equation (2.56) is written as

$$[h(z) \vec{E}] = \vec{E}_i - \left\{ \left[\frac{\partial \vec{E}}{\partial z} \right]^{+} \delta(z) + \frac{\partial}{\partial z} \left[\vec{E}_t \delta(z) \right] \right\} \cdot K_{01} \tag{2.59}$$

It may also be noticed that

$$\left[\frac{\partial}{\partial z} \left[\vec{E}_s \, \delta(z) \right] \right] = \vec{E}_s \, \delta'(z) \ ,$$

since $\vec{E}_s$ is a function of (x,y) only. The function $\delta'(z)$ is the derivative of the Dirac delta function, defined as

$$\delta'(u) = \frac{\partial}{\partial u} \delta(u) \ .$$

This permits (2.59) to be written as

$$[\, h(z)\, \vec{E} \,] = \vec{E}_i + \left[\vec{R}^+(x,y)\, \delta(z) - \vec{E}_s \, \delta'(z) \right] * K_{01}$$

with the function $\vec{R}^+(x,y)$ defined as $-\left[\frac{\partial \vec{E}}{\partial z}\right]^+$. By using a property of a convolution,

$$\delta'(z) * K_{01} = \delta(z) * \frac{\partial K_{01}}{\partial z} \ ,$$

equation (2.59) may now be written as

$$[\, h(z)\, \vec{E} \,] = \vec{E}_i + \vec{R}^+(x,y)\, \delta(z) * K_{01} - \vec{E}_s \, \delta(z) * \frac{\partial K_{01}}{\partial z} \ . \qquad (2.60)$$

Equation (2.60) represents the electric field above the surface in terms of the incident or source electric field $\vec{E}_i$, the surface electric field $\vec{E}_s$, and the function $\vec{R}^+(x,y)$. The same operations are performed on equation (2.57), the equation for the field below the surface. This yields

$$[\, (1 - h(z))\, \vec{E} \,] = -\vec{R}^-(x,y)\, \delta(z) * K_{02} + \vec{E}_m \, \delta'(z) * K_{02} \qquad (2.61)$$

$$= -\left\{ \vec{R}^-(x,y)\, \delta(z) - \vec{E}_m \, \delta'(z) \right\} * K_{02} \ ,$$

where the function $\vec{R}^-(x,y)$ is defined as

$$\vec{R}^-(x,y) = -\left(\frac{\partial \vec{E}}{\partial z} \right)^- \ .$$

The boundary condition equation may be rewritten using the above definitions for $\vec{R}^{+}(x,y)$ and $\vec{R}^{-}(x,y)$ as

$$-\vec{R}^{+}(x,y)\,\delta(z) + \vec{E}_s\,\delta'(z) + \vec{R}^{-}(x,y)\,\delta(z) - \vec{E}_m\,\delta'(z)$$
$$= \frac{n_0^2 - 1}{n_0^2}\,\nabla\left[\hat{z}\cdot\vec{E}_s\,\delta(z)\right] \tag{2.62}$$
$$= (n_0^2 - 1)\,\nabla\left[\hat{z}\cdot\vec{E}_m\,\delta(z)\right] .$$

Now, we examine the right hand side of (2.62) and decompose the electric field function $\nabla\left[\hat{z}\cdot\vec{E}_s\,\delta(z)\right]$ as

$$\nabla\left[\hat{z}\cdot\vec{E}_s\,\delta(z)\right] = \frac{\partial}{\partial x}\left[\hat{z}\cdot\vec{E}_s\,\delta(z)\right]\hat{x} + \frac{\partial}{\partial y}\left[\hat{z}\cdot\vec{E}_s\,\delta(z)\right]\hat{y} + \frac{\partial}{\partial z}\left[\hat{z}\cdot\vec{E}_s\,\delta(z)\right]\hat{z}$$
$$= \frac{\partial}{\partial x}\left[\hat{z}\cdot\vec{E}_s\right]\delta(z)\hat{x} + \frac{\partial}{\partial y}\left[\hat{z}\cdot\vec{E}_s\right]\delta(z)\hat{y} + \left[\hat{z}\cdot\vec{E}_s\right]\hat{z}\,\delta'(z)$$
$$= \nabla_{xy}\left[\hat{z}\cdot\vec{E}_s\right]\delta(z) + \left[\hat{z}\cdot\vec{E}_s\right]\delta'(z)\,\hat{z} .$$

The operator ∇_{xy} is the gradient with respect to x and y defined by

$$\nabla_{xy} = \left(\frac{\partial}{\partial x}\right)\hat{x} + \left(\frac{\partial}{\partial y}\right)\hat{y} .$$

By using this decomposition, the boundary condition equation (2.62) may be written as

$$\left[-\vec{R}^{+}(x,y)\,\delta(z) + \vec{E}_s\,\delta'(z)\right] + \left[\vec{R}^{-}(x,y)\,\delta(z) - \vec{E}_m\,\delta'(z)\right] \tag{2.63}$$
$$= \frac{n_0^2 - 1}{n_0^2}\left[\nabla_{xy}\left[\hat{z}\cdot\vec{E}_s\right]\delta(z) + \left[\hat{z}\cdot\vec{E}_s\right]\hat{z}\,\delta'(z)\right]$$
$$= (n_0^2 - 1)\left[\nabla_{xy}\left[\hat{z}\cdot\vec{E}_m\right]\delta(z) + \left[\hat{z}\cdot\vec{E}_m\right]\hat{z}\,\delta'(z)\right] .$$

By rearranging equation (2.63) we may write the following expression:

$$\left[\vec{R}^{-}(x,y)\,\delta(z) - \vec{E}_m\,\delta'(z)\right] = \left[\vec{R}^{+}(x,y)\,\delta(z) - \vec{E}_s\,\delta'(z)\right]$$
$$+ \frac{n_0^2 - 1}{n_0^2}\left[\nabla_{xy}\left[\hat{z}\cdot\vec{E}_s\right]\delta(z) + \left[\hat{z}\cdot\vec{E}_s\right]\delta'(z)\,\hat{z}\right] . \tag{2.64}$$

Equation (2.64) may be used to rewrite equation (2.61), the equation for the electric field below the surface, as

$$[(1-h(z))\vec{E}] = \left[-\vec{R}^{+}(x,y) - \frac{n_0^2-1}{n_0^2}\nabla_{xy}\left[\hat{z}\cdot\vec{E}_s\right]\right]\delta(z) * K_{02} + \left[\vec{E}_s - \frac{n_0^2-1}{n_0^2}\left[\hat{z}\cdot\vec{E}_s\right]\hat{z}\right]\delta'(z) * K_{02} \ , \tag{2.65}$$

which is an equation for the electric field below the interface in terms of the surface electric field $\vec{E}_s$, and the function R^+. We now have an equation for the electric field above (2.60) and below the surface (2.65) in terms of the same two unknowns $\vec{R}^{+}(x,y)$ and $\vec{E}_s$. We proceed to develop integral equations in order to determine these functions. Considering equation (2.65), we then choose a plane $z = z^+ > 0$ for all x and y so that $\left[1-h(z)\right]\vec{E} = 0$. Equation (2.65) then becomes

$$0 = -\vec{F}(x,y) * K_{02} + \frac{\partial}{\partial z}\left[\vec{G}(x,y) * K_{02}\right] \tag{2.66}$$

where

$$\vec{F}(x,y) = \vec{R}^{+}(x,y) + \frac{n_0^2-1}{n_0^2}\nabla_{xy}\left[\hat{z}\cdot\vec{E}_s\right] ,$$

$$\vec{G}(x,y) = \vec{E}_s - \frac{n_0^2-1}{n_0^2}\left[\hat{z}\cdot\vec{E}_s\right]\hat{z} .$$

Equation (2.66) may now be written as the following integral equation:

$$0 = -\int_{x'}\int_{y'}\vec{F}(x',y')K_{02}(x-x',y-y',z)\,dx'dy' + \frac{\partial}{\partial z}\left[\int_{x'}\int_{y'}\vec{G}(x',y')K_{02}(x-x',y-y',z)\,dx'dy'\right] . \tag{2.67}$$

Taking the first term of (2.67), we perform a two-dimensional spatial Fourier transform and simplify by changing the order of integration. This is accomplished as follows:

$$\int_x \int_y \int_{x'} \int_{y'} \vec{F}(x',y') K_{02}(x-x', y-y', z) \exp(-jK_x x - jK_y y)\, dx\, dy\, dx'dy'$$

$$= \int_{x'} \int_{y'} \vec{F}(x',y') \int_x \int_y K_{02}(x-x', y-y', z) \exp(-jK_x x - jK_y y)\, dx\, dy\, dx'dy'$$

$$= \int_{x'} \int_{y'} \vec{F}(x',y') \exp(-jK_x x' - jK_y y') \int_u \int_v K_{02}(u, v, z)$$

$$\cdot \exp(-jK_x u - jK_y v)\, du\, dv\, dx'dy' \qquad (2.68)$$

In the above expression, we have made the substitutions, $u = x - x'$ and $v = y - y'$.

The two dimensional (x,y) spatial Fourier transform of the Green's function K_{02} is determined in Appendix 'A', and the result repeated below as

$$\underline{K_{02}}(K_x, K_y, z) = \int_x \int_y K_{02}(x,y,z) \exp(-jK_x x - jK_y y)\, dx\, dy$$

$$= \frac{\exp(-|z|\, U_E)}{2\, U_E} \qquad (2.69)$$

where the underscore denotes the spatial Fourier transform. As well, the constants in (2.69) are defined by the following:

$$U_E = \sqrt{\lambda^2 - \gamma^2}\ ,$$

$$\gamma^2 = k^2 n_0^2\ ,$$

$$\lambda^2 = K_x^2 + K_y^2\ .$$

In the plane $z = z^+ > 0$, the transform of the Green's function is

$$\underline{K_{02}^{z^+}} = \frac{\exp(-|z^+|\, U_E)}{2\, U_E}\ .$$

With the aid of equation (2.69) for the spatial (x,y) Fourier transform of K_{02}, the first term of equation (2.67), as simplified in (2.68), now may be written as

$$\int_{x'} \int_{y'} \vec{F}(x',y') \left[\frac{\exp(-|z^+|\, U_E)}{2\, U_E} \right] \exp(-jK_x x' - jK_y y')\, dx'dy'\ . \qquad (2.70)$$

We now examine the second term of the integral equation (2.67). The second term in (2.67) is simplified in a similar fashion to the first, yielding

$$\frac{\partial}{\partial z}\left[\int_{x'}\int_{y'} \vec{G}\,(x',y')\left[\frac{\exp(-\mid z^{+}\mid U_E)}{2\,U_E}\right]\exp(-jK_x x' - jK_y y')\,dx'dy'\right.$$

$$= -\int_{x'}\int_{y'} \vec{G}\,(x',y')\left[\frac{\exp\left(-\mid z\mid^{+} U_E\right)}{2}\right]\exp(-jK_x x' - jK_y y')\,dx'dy' \quad (2.71)$$

The integral equation (2.67) may be written using the above simplifications as

$$\int_{x'}\int_{y'}\left[\vec{G}\,(x',y') + \frac{\vec{F}\,(x',y')}{U_E}\right]\exp\left(-\mid z\mid^{+} U_E\right)\,\exp(-jK_x x' - jK_y y')\,dx'dy' = 0\ . \quad (2.72)$$

Equation (2.72) is one of the two integral equations to be solved in order to determine the two functions $\vec{R}^{+}(x,y)$ and $\vec{E}_s(x,y)$. We now derive a second integral equation. Consider equation (2.60) which is written again below as

$$\left[\,h\,(z)\,\vec{E}\,\right] = \vec{E}_i + \left\{\,\vec{R}^{+}(x,y)\,\delta(z) - \vec{E}_s\,\delta'(z)\right\} * K_{01}\ .$$

Choosing a plane $z = z^{-} < 0$ for all (x,y), then $\left[h\,(z)\,\vec{E}\right] = 0$ and equation (2.60) becomes

$$\begin{aligned}\vec{E}_{i\delta} &= \left[-\vec{R}^{+}(x,y)\,\delta(z) + \vec{E}_s\,\delta'(z)\right] * K_{01}\\ &= \int_{x'}\int_{y'} -\vec{R}^{+}(x',y')\,K_{01}\,(x-x',\,y-y',\,z\,)\,dx'dy'\\ &\quad+ \frac{\partial}{\partial z}\left[\int_{x'}\int_{y'}\vec{E}_s\,(x',y')\,K_{01}\,(x-x',\,y-y',\,z\,)\,dx'dy'\right]\ .\end{aligned} \quad (2.73)$$

where $\vec{E}_{i\delta}$ is the incident electric field evaluated in a plane $z = z^{-} < 0$.

The two dimensional (x and y) spatial Fourier transform of the function K_{01} is given by

$$\underline{K_{01}} = \frac{\exp\left(-\mid z\mid U\right)}{2\,U}\ , \quad (2.74)$$

where

$$U = \sqrt{\lambda^2 - k^2}\ ,$$
$$\lambda^2 = K_x^{\,2} + K_y^{\,2}\ .$$

The Fourier transform of equation (2.73) is taken using the transform of $\underline{K_{01}}$ from (2.74). By using the same substitution of variables as in equation (2.68), that is $u = x - x'$ and $v = y - y'$, as well as the spatial Fourier transform of K_{01}, we may obtain equation (2.75) in the same manner as we obtained equation (2.72). This yields the following equation for the two-dimensional spatial Fourier transform of the incident electric field below the interface:

$$\underline{\vec{E}_{ib}} = \int_{x'}\int_{y'} -\vec{R}^{+}(x',y') \frac{\exp(-\mid z \mid U)}{2} U \exp(-jK_x x' - jK_y y')\, dx'dy'$$
$$+ \frac{\partial}{\partial z}\left[\int_{x'}\int_{y'} \vec{E}_s(x',y') \frac{\exp(-\mid z \mid U)}{2} U \exp(-jK_x x' - jK_y y')\, dx'dy'\right] \qquad (2.75)$$

The derivative of K_{01} is taken with respect to z, for $z = z^- < 0$. The derivative is

$$\frac{\partial}{\partial z}\left[\underline{K_{01}}\right] = \frac{\partial}{\partial z}\left[\frac{\exp(-\mid z^- \mid U)}{2U}\right] = \frac{\exp(-\mid z^- \mid U)}{2}$$

By using the above, equation (2.75) may be written as

$$2\,\underline{\vec{E}_{ib}} \exp(\mid z^- \mid U) = \int_{x'}\int_{y'}\left[\vec{E}_s - \frac{\vec{R}^{+}(x',y')}{U}\right] \exp(-jK_x x' - jK_y y')\, dx'dy' \qquad (2.76)$$

The two integral equations which are to be solved are equations (2.72) and (2.76). These are repeated below as

$$0 = \int_{x'}\int_{y'}\left[\vec{G}(x',y') + \frac{\vec{F}(x',y')}{U_E}\right]\exp(-z^+ U_E) \exp(-jK_x x' - jK_y y')\, dx'dy' \;,$$

and

$$2\,\underline{\vec{E}_{ib}} \exp(\mid z^- \mid U) = \int_{x'}\int_{y'}\left[\vec{E}_s(x',y') - \frac{\vec{R}^{+}(x',y')}{U}\right] \exp(-jK_x x' - jK_y y')\, dx'dy' \;.$$

2.5 SOLVING THE INTEGRAL EQUATIONS

We first examine equation (2.72) for some obvious simplifications. The constant factor in (2.72) may be removed, which yields

$$0 = \int_{x'}\int_{y'} \left[\vec{G}(x',y') + \frac{\vec{F}(x',y')}{U_E} \right] \exp(-jK_x x' - jK_y y')\, dx'dy' \quad (2.77)$$

The equation (2.77) implies a relationship between the spatial Fourier transforms (x y transforms) of the functions $\vec{G}(x,y)$ and $\vec{F}(x,y)$. The implied relationship is given by

$$0 = U_E\, \underline{\vec{G}}(K_x,K_y) + \underline{\vec{F}}(K_x,K_y) \quad . \quad (2.78)$$

where $\underline{\vec{F}}(K_x,K_y)$ is the two dimensional spatial Fourier transform of $\vec{F}(x,y)$. Similarly, $\underline{\vec{G}}(K_x,K_y)$ is the two dimensional spatial Fourier transform of $\vec{G}(x,y)$. By using the expression for $\vec{F}(x,y)$ and $\vec{G}(x,y)$ developed in equation (2.66), equation (2.78) is expressed as

$$0 = U_E \int_{x'}\int_{y'} \left[\vec{E}_s(x',y') - \frac{n_0^2 - 1}{n_0^2} (\hat{z} \cdot \vec{E}_s)\hat{z} \right] \exp(-jK_x x' - jK_y y')\, dx'dy'$$

$$+ \int_{x'}\int_{y'} \left[\vec{R}^+(x',y') + \frac{n_0^2 - 1}{n_0^2} \nabla_{xy} (\hat{z} \cdot \vec{E}_s) \right] \exp(-jK_x x' - jK_y y')\, dx'dy' \quad , \quad (2.79)$$

where E_{sz} is the component of the surface field $\vec{E}_s$ in the $\hat{z}$ unit vector direction. By using a property of the Fourier transform, we may expand the term $\nabla_{xy} E_{sz}$ as

$$\nabla_{xy} E_{sz} = \frac{\partial}{\partial x}(E_{sz})\hat{x} + \frac{\partial}{\partial y}(E_{sz})\hat{y}$$

$$\underline{\nabla_{xy} E_{sz}} = \underline{\frac{\partial}{\partial x}(E_{sz})\hat{x} + \frac{\partial}{\partial y}(E_{sz})\hat{y}}$$

$$= -j K_x \underline{E_{sz}}\, \hat{x} - j K_y \underline{E_{sz}}\, \hat{y}$$

Again, the underscore notation has been used for the two dimensional (x,y) spatial Fourier transform. We may now write equation (2.79) as

$$0 = U_E \left[\underline{\vec{E}_s} - \frac{n_0^2 - 1}{n_0^2} \underline{E_{sz}}\hat{z} \right] + \underline{\vec{R}^+}(K_x,K_y)$$

$$+ \frac{n_0^2 - 1}{n_0^2} \left[-j K_x \underline{E_{sz}}\, \hat{x} - j K_y \underline{E_{sz}}\, \hat{y} \right] \quad . \quad (2.80)$$

The above equation may be regrouped and an expression for $\vec{\underline{R}}^+(K_x, K_y)$ determined. This expression is

$$\vec{\underline{R}}^+(K_x, K_y) = -U_E\, \vec{\underline{E}}_s + \frac{n_0^2 - 1}{n_0^2} \left[j\, K_x\, \underline{E_{sz}}\, \hat{x} + j\, K_y\, \underline{E_{sz}}\, \hat{y} + U_E\, \underline{E_{sz}}\, \hat{z} \right] . \quad (2.81)$$

Equation (2.81) may be separated into three parts by way of its vector nature. Each new equation is a representation of one of the three Cartesian components of $\vec{\underline{R}}^+(K_x, K_y)$. This separation is as follows:

$$\underline{R_x}^+(K_x, K_y) = -U_E\, \underline{E_{sx}} + \frac{n_0^2 - 1}{n_0^2} \left[j\, K_x\, \underline{E_{sz}} \right] , \quad (2.82)$$

$$\underline{R_y}^+(K_x, K_y) = -U_E\, \underline{E_{sy}} + \frac{n_0^2 - 1}{n_0^2} \left[j\, K_y\, \underline{E_{sz}} \right] , \quad (2.83)$$

and

$$\begin{aligned} \underline{R_z}^+(K_x, K_y) &= -U_E\, \underline{E_{sz}} + \frac{n_0^2 - 1}{n_0^2} \left[U_E\, \underline{E_{sz}} \right] \\ &= -U_E\, \underline{E_{sz}} + U_E\, \underline{E_{sz}} - \frac{U_E}{n_0^2} \underline{E_{sz}} \\ &= -\frac{U_E}{n_0^2}\, \underline{E_{sz}} . \end{aligned} \quad (2.84)$$

This yields three equations relating the Cartesian components of the spatial Fourier transforms of $\vec{R}^+(x, y)$ to the Cartesian components of the spatial Fourier transform of $\vec{E}_s$. Equation (2.76) is repeated below as

$$2\, \underline{\vec{E}_{i1}} \exp\left(|\, z^- \,|\; U \right) = \int_{x'} \int_{y'} \left[\vec{E}_s(x', y') - \frac{\vec{R}^+(x', y')}{U} \right] \exp(-jK_x x' - jK_y y')\, dx' dy' .$$

This vector equation relates the spatial Fourier transforms of the incident field below the interface ($\underline{\vec{E}_{i1}}$), to the surface field ($\vec{E}_s$) and the function $\vec{R}^+$. This in turn may be written as three equations, one for each of the three Cartesian components of the functions, as follows:

$$2\,U\,\underline{E_{ibx}}\,\exp(\,|\,z^-|\,U) = U\,\underline{E_{sx}} - \underline{R_x^+}(K_x, K_y) \quad , \tag{2.85}$$

$$2\,U\,\underline{E_{iby}}\,\exp(\,|\,z^-|\,U) = U\,\underline{E_{sy}} - \underline{R_y^+}(K_x, K_y) \quad , \tag{2.86}$$

and

$$2\,U\,\underline{E_{ibz}}\,\exp(\,|\,z^-|\,U) = U\,\underline{E_{sz}} - \underline{R_z^+}(K_x, K_y) \quad . \tag{2.87}$$

We first examine the x component of $\underline{\vec{E}_{ib}}$ in equation (2.85). This expression may be expanded using $\underline{R_x^+}(K_x, K_y)$ from equation (2.82). The result of the expansion is

$$2\,U\,\underline{E_{ibx}}\,\exp(\,|\,z^-|\,U) = U\,\underline{E_{sx}} + U_E\,\underline{E_{sx}} - \frac{n_0^2 - 1}{n_0^2}\left[\,j\,K_x\,\underline{E_{sz}}\,\right] \quad . \tag{2.88}$$

In a similar manner, expressions for the y and z Cartesian components of $\underline{\vec{E}_{ib}}$ may be determined. Following the method for the x component, we obtain

$$2\,U\,\underline{E_{iby}}\,\exp(\,|\,z^-|\,U) = U\,\underline{E_{sy}} + U_E\,\underline{E_{sy}} - \frac{n_0^2 - 1}{n_0^2}\left[\,j\,K_y\,\underline{E_{sz}}\,\right] \quad , \tag{2.89}$$

and

$$2\,U\,\underline{E_{ibz}}\,\exp(\,|\,z^-|\,U) = U\,\underline{E_{sz}} + \frac{U_E}{n_0^2}\,\underline{E_{sz}} \quad . \tag{2.90}$$

We may now determine the surface electric field for an arbitrary incident (source) electric field and a surface of arbitrary electrical properties, provided the inverse spatial Fourier transforms of the Cartesian components of the surface field may be evaluated. The spatial Fourier transform of the $\hat{z}$ component, using equation (2.90), is

$$\underline{E_{sz}} = \frac{2\,U\,n_0^2}{U\,n_0^2 + U_E}\,\underline{E_{ibz}}\,\exp(\,|\,z^-|\,U) \quad . \tag{2.91}$$

Accordingly, we may also write the $\hat{x}$ and the $\hat{y}$ components from equations (2.88) and (2.89) respectively as

$$\underline{E_{sx}} = \frac{2\,U}{U + U_E}\,\underline{E_{ibx}}\,\exp(\,|\,z^-|\,U\,) + \left[\frac{2\,U}{U + U_E}\right]\left[\frac{n_0^2 - 1}{U\,n_0^2 + U_E}\right]\left[jK_x\,\underline{E_{ibz}}\right]\exp(\,|\,z^-|\,U\,) = \frac{2\,U}{U + U_E}\exp(\,|\,z^-|\,U\,)\left[\underline{E_{ibx}} + \frac{n_0^2 - 1}{U\,n_0^2 + U_E}\left[jK_x\,\underline{E_{ibz}}\right]\right] \quad (2.92)$$

and

$$\underline{E_{sy}} = \frac{2\,U}{U + U_E}\exp(\,|\,z^-|\,U\,)\left[\underline{E_{iby}} + \frac{n_0^2 - 1}{U\,n_0^2 + U_E}\left[jK_y\,\underline{E_{ibz}}\right]\right] \quad (2.93)$$

Equations (2.91), (2.92) and (2.93) are expressions for the electric field on the surface in terms of the two-dimensional spatial Fourier transforms of the Cartesian components of the surface electric field. For a given incident (source) electric field, we may solve these equations for the surface field, assuming the inverse Fourier transforms may be determined. We may note that no assumptions have been made, as yet, regarding the nature of the source electric field or the nature of the electrical parameters of the surface.

2.6 THE ELECTRIC FIELD ABOVE THE SURFACE

The results of section 2.5 express the three Cartesian components of the surface electric field in terms of the properties of the surface and an arbitrary function for the spatial Fourier transform of the incident electric field. By using these results for the surface electric field, we may now proceed to determine the electric field anywhere in the half space above the surface. Again, to maintain the generality of the source, the field above the surface is expressed as a function of the source field. The final equations in this section may be used to determine the electric field above the surface for a given source electric field, assuming the inverse spatial Fourier transforms may be evaluated. We now return to equation

(2.59), repeated below as

$$\{ h(z)\vec{E} \} = \vec{E}_i + \left[\nabla\left(\frac{\partial \vec{E}}{\partial z}\right)^{+} \delta(z) - \vec{E}_s\, \delta'(z) \right\} * K_{01} \tag{2.94}$$

Choosing a plane $z = z^{+} > 0$ for all (x,y), equation (2.94) becomes

$$\vec{E}_t = \vec{E}_{is} + \left[-\left(\frac{\partial \vec{E}}{\partial z}\right)^{+} \delta(z) - \vec{E}_s\, \delta'(z) \right] * K_{01} \tag{2.95}$$

Where we have defined $\vec{E}_t$ as the electric field above the surface and $\vec{E}_{is}$ as the incident (source) field above the surface. Performing the convolution in equation (2.95) yields

$$\vec{E}_t = \vec{E}_{is} + \int_{x'}\int_{y'} \vec{R}^{+}(x',y')\, K_{01}(x-x', y-y', z)\, dx'dy'$$
$$- \frac{\partial}{\partial z} \int_{x'}\int_{y'} \vec{E}_s(x',y')\, K_{01}(x-x', y-y', z)\, dx'dy' \tag{2.96}$$

We now take the two-dimensional spatial Fourier transform of equation (2.96). This yields an expression for $\underline{\vec{E}_t}$ in terms of $\underline{\vec{R}^{+}}$ and $\underline{\vec{E}_s}$. This expression is

$$\underline{\vec{E}_t} = \underline{\vec{E}_{is}} + \int_{x'}\int_{y'} \vec{R}^{+}(x',y') \left[\frac{\exp(-|z^{+}|\,U)}{2U}\right] \exp(-jK_x x' - jK_y y')\, dx'dy'$$
$$- \int_{x'}\int_{y'} \vec{E}_s(x',y') \left[\frac{\exp(-|z^{+}|\,U)}{2}\right] \exp(-jK_x x' - jK_y y')\, dx'dy' \tag{2.97}$$

where $U = \sqrt{\lambda^2 + k^2}$ and $\lambda^2 = K_x^2 + K_y^2$. We perform some simplifications on equation (2.97) which now becomes

$$2\,U \exp(|z^{+}|\,U)\,\underline{\vec{E}_t} = 2\,U\,\underline{\vec{E}_{is}} \exp(|z^{+}|\,U) + U\,\underline{\vec{E}_s} + \underline{\vec{R}^{+}}(K_x,K_y) \tag{2.98}$$

Equation (2.98) may now be written in terms of its three Cartesian components as follows:

$$2\,U \exp(|z^{+}|\,U)\,\underline{E_{tx}} = 2\,U\,\underline{E_{isx}} \exp(|z^{+}|\,U) + U\,\underline{E_{sx}} + \underline{R_x^{+}}(K_x,K_y) \tag{2.99}$$

$$2\,U \exp(|z^{+}|\,U)\,\underline{E_{ty}} = 2\,U\,\underline{E_{isy}} \exp(|z^{+}|\,U) + U\,\underline{E_{sy}} + \underline{R_y^{+}}(K_x,K_y) \tag{2.100}$$

and

$$2\,U\,\exp(|\,z^+|\,U)\,\underline{E_{tx}} = 2\,U\,\underline{E_{iax}}\,\exp(|\,z^+|\,U) + U\,\underline{E_{sx}} + \underline{R_x^+}(K_x, K_y)\,, \qquad (2.101)$$

We now utilize the expressions for $\underline{R_x^+}(K_x, K_y)$ developed in equation (2.82) to expand equation (2.99) as

$$2\,U\,\exp(|\,z^+|\,U)\,\underline{E_{tx}} = 2\,U\,\underline{E_{iax}}\,\exp(|\,z^+|\,U) + (U - U_E)\,\underline{E_{sx}} + \frac{n_0^2 - 1}{n_0^2}\left[\,j\,K_x\,\underline{E_{sz}}\,\right] \qquad (2.102)$$

Equation (2.102) may be further expanded by using expressions already developed for the surface components of the electric field, $\underline{E_{sx}}$ and $\underline{E_{sz}}$ in equations (2.92) and (2.91). The expansion of equation (2.102) is

$$2\,U\,\exp(|\,z^+|\,U)\,\underline{E_{tx}} = 2\,U\,\underline{E_{iax}}\,\exp(|\,z^+|\,U) + 2\,U\,\frac{U - U_E}{U + U_E}\exp(|\,z^-|\,U)\left[\underline{E_{ibx}} + \frac{(n_0^2 - 1)}{U\,n_0^2 + U_E}\,(\,j\,K_x\,\underline{E_{ibz}}\,)\right] + 2\,U\,\frac{(n_0^2 - 1)}{U\,n_0^2 + U_E}\,(\,j\,K_x\,)\,\underline{E_{ibz}}\exp(|\,z^-|\,U) \qquad (2.103)$$

Simplifying further, equation (2.103) becomes

$$\underline{E_{tx}} = \underline{E_{iax}} + \frac{U - U_E}{U + U_E}\,\underline{E_{ibx}}\,\exp\left((|\,z^-| - |\,z^+|)\,U\right) + \frac{2\,U}{U + U_E}\,\frac{n_0^2 - 1}{U n_0^2 + U_E}\left[j\,K_x\,\underline{E_{ibz}}\right]\exp\left((|\,z^-| - |\,z^+|)\,U\right) \qquad (2.104)$$

We now examine equation (2.100) to determine the equivalent expression for $\underline{E_{ty}}$ in terms of the spatial Fourier transforms of the source field. By using equation (2.83) for the $\underline{R_y^+}(K_x, K_y)$ term we may now write equation (2.100) as

$$2\,U\,\underline{E_{ty}} = 2\,U\,\underline{E_{iay}}\,\exp(|\,z^+|\,U) + (U - U_E)\,\underline{E_{sy}} + \frac{n_0^2 - 1}{n_0^2}\left[\,j\,K_y\,\underline{E_{sz}}\,\right] \qquad (2.105)$$

The above equation is now in terms of the surface components, $\underline{E_{sy}}$ and $\underline{E_{sz}}$, of the electric field. Below, we expand equation (2.105) using the expressions for these surface field components, which are derived in terms of the source electric field. This yields an expression for the $\hat{y}$ component of the electric field above the surface as a function of the source electric field, as follows:

$$
\begin{aligned}
2\,U\,\underline{E_{ly}}\,\exp(\,|\,z^+|\,U\,) &= 2\,U\,\underline{E_{iay}}\,\exp(\,|\,z^+|\,U\,) \\
&+ 2\,U\,\frac{U - U_E}{U + U_E}\exp(\,|\,z^-|\,U\,)\left[\underline{E_{iby}} + \frac{(\,n_0^2 - 1\,)}{U\,n_0^2 + U_E}\,(\,j\,K_y\,\underline{E_{ibz}}\,)\right] \\
&+ 2\,U\,\frac{(\,n_0^2 - 1\,)}{U\,n_0^2 + U_E}\,(\,j\,K_y\,)\,\underline{E_{ibz}}\,\exp(\,|\,z^-|\,U\,)\quad. \qquad (2.106)
\end{aligned}
$$

By performing similar simplifications used for the $\hat{x}$ component, we arrive with a similar expression for the $\hat{y}$ component of the electric field above the surface. The expression for the $\hat{y}$ component is

$$
\begin{aligned}
\underline{E_{ly}} &= \underline{E_{iay}} + \frac{U - U_E}{U + U_E}\,\underline{E_{iby}}\,\exp\big((\,|\,z^-|\, - \,|\,z^+|\,)\,U\big) \\
&+ \frac{2\,U}{U + U_E}\,\frac{n_0^2 - 1}{U n_0^2 + U_E}\left[\,j\,K_z\,\underline{E_{ibz}}\,\right]\exp\big((\,|\,z^-|\, - \,|\,z^+|\,)\,U\big)\quad. \qquad (2.107)
\end{aligned}
$$

Finally, we consider the $\hat{z}$ component of the electric field above the surface from equation (2.101). By expanding this equation in terms of the expressions for the surface field components, from equation (2.91), we obtain the following expression:

$$
\begin{aligned}
2\,U\,\exp(\,|\,z^+|\,U\,)\,\underline{E_{lz}} &= 2\,U\,\underline{E_{iaz}}\,\exp(\,|\,z^+|\,U\,) \\
&+ \left(U - \frac{U_E}{n_0^2}\right)\frac{2\,U\,n_0^2}{U\,n_0^2 + U_E}\,\underline{E_{ibz}}\,\exp(\,|\,z^-|\,U\,) \\
&= 2\,U\,\underline{E_{iaz}}\,\exp(\,|\,z^+|\,U\,) \\
&+ 2\,U\,\frac{U\,n_0^2 - U_E}{U\,n_0^2 + U_E}\,\underline{E_{ibz}}\,\exp(\,|\,z^-|\,U\,)\quad. \qquad (2.108)
\end{aligned}
$$

Again, we simplify equation (2.108) and obtain

$$
\underline{E_{lz}} = \underline{E_{iaz}} + \frac{U\,n_0^2 - U_E}{U\,n_0^2 + U_E}\,\underline{E_{ibz}}\,\exp\big((\,|\,z^-|\, - \,|\,z^+|\,)\,U\big)\quad. \qquad (2.109)
$$

The equations (2.104), (2.107) and (2.109) describe a solution for the electric field above a flat surface of arbitrary electrical properties for an arbitrary incident (source) field. These relationships are written in terms of two dimensional spatial Fourier transforms of the electric fields, with one equation for each of the Cartesian components of the complete field. For any given incident electric field, the electric field above the surface may be determined if the inverse spatial Fourier transforms may be determined. Again, we must note that at this point no assumptions have yet been made regarding the electrical properties of the surface. As well we have not invoked any of the classical boundary conditions, which the fields must satisfy at the surface; rather the method of solution has provided its own boundary condition equation. By using the equations for the Cartesian components of the electric field, we may determine the electric field for any finite source if the inverse spatial Fourier transforms may be solved. That is we have achieved a set of general expressions from which the electric field may be evaluated for any specified source. We repeat these three equations below for comparison as follows:

$$\underline{E_{tx}} = \underline{E_{isx}} + \frac{U - U_E}{U + U_E}\,\underline{E_{ibx}} \exp\left((|z^-| - |z^+|)\,U\right)$$
$$+ \frac{2\,U}{U + U_E}\,\frac{n_0^2 - 1}{U n_0^2 + U_E}\left[j\,K_x\,\underline{E_{ibz}}\right]\exp\left((|z^-| - |z^+|)\,U\right) \quad , \qquad (2.104)$$

$$\underline{E_{ty}} = \underline{E_{isy}} + \frac{U - U_E}{U + U_E}\,\underline{E_{iby}} \exp\left((|z^-| - |z^+|)\,U\right)$$
$$+ \frac{2\,U}{U + U_E}\,\frac{N_0^2 - 1}{U n_0^2 + U_E}\left[j\,K_y\,\underline{E_{ibz}}\right]\exp\left((|z^-| - |z^+|)\,U\right) \quad , \qquad (2.107)$$

and

$$\underline{E_{tz}} = \underline{E_{isz}} + \frac{U\,n_0^2 - U_E}{U\,n_0^2 + U_E}\,\underline{E_{ibz}} \exp\left((|z^-| - |z^+|)\,U\right) \quad . \qquad (2.109)$$

We now diverge from the case of an arbitrary incident field. In the next section we develop an expression for the incident field from an elemental dipole source. This is used to determine expressions for the field above a flat surface for an elementary vertical electric dipole source.

2.7 INCIDENT FIELD FROM AN ELEMENTAL DIPOLE SOURCE

In the previous section we have derived expressions for the three Cartesian components of the electric field of the electric field above a planar surface. These three equations express the field in terms of the spatial Fourier transforms of the Cartesian components of the source (incident) electric field functions. In this section we will determine the incident electric field for an elementary vertical electric source and evaluate the function in planes above and below the surface, in agreement with our methods for evaluating the surface fields. The final results of this section are the two-dimensional (x and y) spatial Fourier transforms of the incident electric fields above and below the surface. As usual the primary interest is in the electric fields distant from the source, so that we present only the 'far field' approximation of the elementary dipole source field, in the spatial Fourier transform domain.

The source current density for an elementary dipole source is given by Jordan and Balmain [1968] as

$$\vec{J}_s = I_0(\omega)\, dl\, \delta(x)\, \delta(y)\, \delta(z - h_0)\, \hat{z} \quad , \tag{2.110}$$

where h_0 is the height of the dipole above the surface, $I_0(\omega)$ is the dipole current, dl is the dipole length, and $\vec{J}_s$ is the source current density. The incident electric field as defined in equation (2.59) is repeated below as

$$\vec{E}_s = T_{SE}\left[\vec{J}_s\right] * K_0 \quad . \tag{2.111}$$

In the above, T_{SE} is the source current density operator, $r = (x^2+y^2+z^2)^{1/2}$ and

$K_0 = \frac{\exp(-jkr)}{4\pi r}$ (the Green's function).

We now determine $T_{SE}\left[\vec{J}_s\right]$ using the source current density, $\vec{J}_s$ as given in equation (2.110).

$$T_{SE}\left[\vec{J}_s\right] = \frac{1}{j\omega\epsilon_0}\left[\nabla(\nabla\cdot\vec{J}_s) + k^2\vec{J}_s\right]$$

First the divergence of the source current density, $\nabla\cdot\vec{J}_s$, is examined and expanded using equation (2.110). In terms of the source current, $I_0(\omega)$, and the dipole length, dl, $\nabla\cdot\vec{J}_s$ is given by

$$\nabla\cdot\vec{J}_s = \nabla\cdot\left(I_0\,dl\;\delta(x)\,\delta(y)\,\delta(z-h_0)\,\hat{z}\right)$$
$$= I_0\,dl\;\delta(x)\,\delta(y)\,\delta_z(z-h_0) \quad .$$

In the above equation the subscript on the Dirac delta function represents the partial derivative, and the partial derivative with respect to z is given by

$$\delta_z(z-h_0) = \frac{\partial}{\partial z}\left[\delta(z-h_0)\right] \quad .$$

Continuing, we take the gradient of the divergence, $\nabla(\nabla\cdot\vec{J}_s)$ as

$$\nabla(\nabla\cdot\vec{J}_S) = \nabla\left(I_0\,dl\;\delta(x)\,\delta(y)\,\delta_z(z-h_0)\right)$$
$$= I_0\,dl\cdot\left\{\delta_x(x)\,\delta(y)\,\delta_z(z-h_0)\hat{x} + \delta(x)\,\delta_y(y)\,\delta_z(z-h_0)\hat{y} + \delta(x)\,\delta(y)\,\delta_{zz}(z-h_0)\hat{z}\right\} \quad ,$$

where we again use the subscript notation for the x, y, and z partial derivatives. For reference, the x and y partial derivatives are as follows:

$$\delta_x(x) = \frac{\partial}{\partial x}\,\delta(x) \quad ,$$
$$\delta_y(y) = \frac{\partial}{\partial y}\,\delta(y) \quad .$$

The complete expansion of $T_{SE}\left[\vec{J}_s\right]$ may now be written as

$$T_{SE}\left[\vec{J}_s\right] = \frac{I_0\, dl}{j\ \omega\ \epsilon_0}\left\{ \delta_x(x)\,\delta(y)\,\delta_z(z-h_0)\hat{x} + \delta(x)\,\delta_y(y)\,\delta_z(z-h_0)\hat{y} \right.$$
$$\left. + \left[\delta(x)\,\delta(y)\,\delta_{zz}(z-h_0) + k^2\,\delta(x)\,\delta(y)\,\delta(z-h_0)\right]\hat{z} \right\} \quad (2.112)$$

The incident electric field $\vec{E}_i$ is separated into its Cartesian components, E_{ix}, E_{iy}, and E_{iz}. The vector components in equation (2.112) are now written separately as equations (2.113) through (2.115). This yields the following:

$$E_{ix} = \frac{I_0\, dl}{j\ \omega\ \epsilon_0}\left\{ \delta_x(x)\,\delta(y)\,\delta_z(z-h_0)\right\} * \frac{\exp(-jkr)}{4\pi r}, \quad (2.113)$$

$$E_{iy} = \frac{I_0\, dl}{j\ \omega\ \epsilon_0}\left\{ \delta(x)\,\delta_y(y)\,\delta_z(z-h_0)\right\} * \frac{\exp(-jkr)}{4\pi r}, \quad (2.114)$$

$$E_{iz} = \frac{I_0\, dl}{j\ \omega\ \epsilon_0}\left\{ \delta(x)\,\delta(y)\,\delta_{zz}(z-h_0) + k^2\,\delta(x)\,\delta(y)\,\delta(z-h_0)\right\} * \frac{\exp(-jkr)}{4\pi r}. \quad (2.115)$$

First, consider only equation (2.113) for the $\hat{x}$ component. A useful property of a convolution is

$$\frac{\partial}{\partial x}\delta(x) * K_{01} = \delta(x) * \frac{\partial}{\partial x}K_{01}.$$

By using the above property, E_{ix} is written as

$$E_{ix} = \frac{I_0\, dl}{j\ \omega\ \epsilon_0}\left\{ \delta_x(x)\,\delta(y)\,\delta_z(z-h_0)\right\} * \frac{\exp(-jkr)}{4\pi r}$$
$$= \frac{I_0\, dl}{j\ \omega\ \epsilon_0}\left\{ \delta(x)\,\delta(y)\,\delta(z-h_0)\right\} * \frac{\partial^2}{\partial x\ \partial z}\left[\frac{\exp(-jkr)}{4\pi r}\right]$$
$$= \frac{I_0\, dl}{j\ \omega\ \epsilon_0}\left\{ \delta(x)\,\delta(y)\,\delta(z-h_0)\right\} * \frac{\partial}{\partial z}\left\{\left[\frac{-j\ K_x}{r} - \frac{x}{r^2}\right]\frac{\exp(-jkr)}{4\pi r}\right\}$$
$$= \frac{I_0\, dl}{j\ \omega\ \epsilon_0}\left\{ \delta(x)\,\delta(y)\,\delta(z-h_0)\right\} * \left\{\frac{x\ z}{r^2}\left[\frac{3}{r^2} + \frac{3\ j\ k}{r} - k^2\right]\frac{\exp(-jkr)}{4\pi r}\right\}.$$

By performing the convolution in the above, E_{ix} is

$$E_{ix} = \frac{I_0\, dl}{j\ \omega\ \epsilon_0}\ \frac{x\ (z-h_0)}{r_1^2}\left\{\frac{3}{r_1^2} - \frac{3\ j\ k}{r_1} - k^2\right\}\frac{\exp(-j\ k\ r_1)}{4\ \pi\ r_1}. \quad (2.116)$$

In equation (2.116) the variable r_1 is defined as

$$r_1 = (x^2 + y^2 + (z - h_0)^2)^{1/2}$$

By analogy to the E_{ix} component, the E_{iy} component may be deduced as follows:

$$E_{iy} = \frac{I_0\, dl}{j\, \omega\, \epsilon_0} \frac{y\,(z-h_0)}{r_1^2} \left\{ \frac{3}{r_1^2} - \frac{3\, j\, k}{r_1} - k^2 \right\} \frac{\exp(-j\, k\, r_1)}{4\, \pi\, r_1} \quad (2.117)$$

The $\hat{x}$ and $\hat{y}$ components of the incident field have been determined above, and we now turn our attention to the the $\hat{z}$ component. Consider E_{iz}, which is repeated below as

$$E_{iz} = \frac{I_0\, dl}{j\, \omega\, \epsilon_0} \left\{ \delta(x)\, \delta(y)\, \delta_{zz}(z - h_0) + k^2\, \delta(x)\, \delta(y)\, \delta(z - h_0) \right\} * \frac{\exp(\, jkr\,)}{4\pi r} \quad (2.118)$$

The first term in the above equation for E_{iz}, denoted by T_1, is examined below and the partial derivatives expanded as follows:

$$T_1 = \left\{ \delta(x)\, \delta(y)\, \delta_{zz}(z - h_0) \right\} * \frac{\exp(-jkr)}{4\pi r}$$

$$= \left\{ \delta(x)\, \delta(y)\, \delta(z - h_0) \right\} * \frac{\partial^2}{\partial z^2} \left[\frac{\exp(-jkr)}{4\pi r} \right]$$

$$= \left\{ \delta(x)\, \delta(y)\, \delta(z - h_0) \right\} * -\frac{\partial}{\partial z} \left[\left(\frac{z}{r^2} + \frac{j\, k\, z}{r} \right) \frac{\exp(-jkr)}{4\pi r} \right]$$

$$= \left\{ \delta(x)\, \delta(y)\, \delta(z - h_0) \right\} * \left[\frac{-j\, k}{r} - \frac{1}{r^2} - \frac{k^2 z^2}{r^2} + \frac{3\, j\, k\, z^2}{r^3} + \frac{3\, z^2}{r^4} \right] \frac{\exp(-jkr)}{4\pi r} \quad (2.119)$$

The second term of (2.118) is added to equation (2.117) to give the total z component of the incident field E_{iz} as

$$E_{iz} = \frac{I_0\, dl}{j\, \omega\, \epsilon_0} \left\{ \delta(x)\, \delta(y)\, \delta(z - h_0) \right\} *$$

$$\left[\frac{3\, j\, k\, z^2}{r^2} + \frac{3\, z^2}{r^4} - \frac{k^2 z^2}{r^2} - \frac{1}{r^2} - \frac{j\, k}{r} + k^2 \right] \frac{\exp(-jkr)}{4\pi r}$$

$$= \frac{I_0\, dl}{j\, \omega\, \epsilon_0} \left[\frac{(z - h_0)^2}{r_1^2} \left(\frac{3}{r_1^2} + \frac{3\, j\, k}{r_1} - k^2 \right) - \left(\frac{1}{r_1^2} + \frac{j\, k}{r_1} - k^2 \right) \right] \frac{\exp(-j\, k\, r_1)}{4\, \pi\, r_1} \quad (2.120)$$

The 'Far Field' approximation is invoked upon equations (2.116), (2.117), and (2.120) for the Cartesian components of the incident field. That is, we neglect all terms in these equations which decay more rapidly than $\frac{1}{r_1}$. The total incident field $\vec{E}_i$ is then evaluated through its $\hat{x}, \hat{y}$, and $\hat{z}$ components as follows:

$$E_{ix} \approx 0 \quad , \tag{2.121a}$$

$$E_{iy} \approx 0 \quad , \tag{2.121b}$$

$$E_{iz} \approx \frac{I_0\, dl}{j\, \omega\, \epsilon_0}\, k^2\, \frac{\exp(-j\ k\ r_1)}{4\, \pi\, r_1} = C_d\, \frac{\exp(-j\ k\ r_1)}{4\, \pi\, r_1} \quad . \tag{2.121c}$$

In the above, the 'dipole constant', C_d is defined as

$$C_d = \frac{I_0\, dl\, k^2}{j\, \omega\, \epsilon_0} = -j\, \omega\, \mu_0\, I_0\, dl \quad .$$

The three equations, (2.121a), (2.121b) and (2.121c) show that the incident field distant from the source may be approximated only by its $\hat{z}$ component. The far field approximation is typically considered valid for radial distances, (r_1), greater than 10.0 radio wavelengths.

The above expression for E_{iz} is now evaluated in a plane $z = z^- < 0$ below the surface, to yield

$$E_{ibz} = C_d\, \frac{\exp(-j\ k\ r_d)}{4\, \pi\, r_d} \quad . \tag{2.122}$$

E_{ibz} and r_d are defined by

$$E_{ibz} = E_{iz}\Big|_{\,z = z^- < 0} \quad , \text{ and } \quad r_d = (\, x^2 + y^2 + (z^- - h_0)^2\,)^{1/2} \quad .$$

By using the results in Appendix 'A', we take the two-dimensional (x,y) spatial Fourier transform of equation (2.122) for E_{ibz}. The expression is

$$\underline{E_{ibz}} = C_d\, \frac{\exp(-\mid z^- - h_0 \mid\, U)}{2\, U} \quad , \tag{2.123}$$

where

$$U = \sqrt{\lambda^2 - k^2} \quad , \text{and} \qquad \lambda^2 = K_x^2 + K_y^2 \; .$$

The underscore denotes, as before, the spatial Fourier transform of a function. We may also evaluate E_{iz} in a plane above the surface at $z = z^+ > 0$, and in a similar manner to the above we write

$$E_{iz} = C_d \; \frac{\exp(-j \; k \; r_c)}{4 \pi \; r_c} \; . \qquad (2.124)$$

In equation (2.124) E_{iz} and r_c are defined by

$$E_{iz} = E_{iz} \Big|_{z = z^+ > 0} \quad , \text{and} \qquad r_c = (\, x^2 + y^2 + (z^+ - h_0)^2 \,)^{1/2} \; .$$

Again, by taking the spatial Fourier transform, we write $\underline{E_{iz}}$ as

$$\underline{E_{iz}} = C_d \; \frac{\exp(- \mid z^+ - h_0 \mid \, U)}{2 \, U} \; . \qquad (2.125)$$

These results may be used as the incident electric field from an elementary vertical electric dipole source, in order to evaluate the far field approximation of the electric field above a flat surface. These results in equations (2.123) and (2.125) are applied as the source field in the equations from the preceding section, section (2.6), for the electric field above a planar surface. We now proceed to evaluate the electric field above the planar surface for an elementary vertical electric dipole source.

2.8 THE ELECTRIC FIELD FOR ELEMENTARY ELECTRIC DIPOLE ANTENNAS

In the preceding section we have derived the incident electric field for an elementary vertical electric dipole source. In this section we assume that both the observer and the source are elementary vertical electric dipole antennas. This means that we are assuming that both transmit and receive antennas will be elementary electric dipoles. In the far field approximation of the electric dipole acting as a source, the $\hat{x}$ and $\hat{y}$ Cartesian components of the electric field are negligible. Therefore, by reciprocity, the electric dipole acting as a receiver is sensitive only to the $\hat{z}$ component of the electric field. This implies that for these antennas only the $\hat{z}$ component of the electric field is significant. However, we note that from equations (2.104) and (2.107), for $\underline{E_{tx}}$ and $\underline{E_{ty}}$, the $\hat{x}$ and $\hat{y}$ components are non-zero even when the corresponding source field components are negligible. By the preceding argument for vertical dipole transmitter and receiver the $\hat{x}$ and $\hat{y}$ components may be neglected.

We now proceed to derive the $\hat{z}$ component of the electric field above the planar surface. First, consider the expression for $\underline{E_{tz}}$ in the spatial Fourier transform domain. Equation (2.109) for $\underline{E_{tz}}$ is repeated below as

$$\underline{E_{tz}} = \underline{E_{isz}} + \frac{Un_0^2 - U_E}{Un_0^2 + U_E} \underline{E_{ibz}} \exp\{(|z^-| - |z^+|)\, U\} \quad .$$

The expressions for $\underline{E_{ibz}}$ and $\underline{E_{isz}}$ from equations (2.123) and (2.125) respectively are repeated as follows:

$$\underline{E_{isz}} = C_t \, \frac{\exp(-|z^+ - h_0|\; U)}{2\,U} \quad .$$

and

$$\underline{E_{ibz}} = C_t \, \frac{\exp(-|z^- - h_0|\; U)}{2\,U} \quad .$$

The above expressions for $\underline{E_{isz}}$ and $\underline{E_{ibz}}$ are inserted into equation (2.109), which yields

$$\underline{E_{tz}} = C_t \left\{ \frac{\exp(-|z^+ - h_0|\; U)}{2\,U} \right.$$

$$+ \frac{Un_0^2 - U_E}{Un_0^2 + U_E} \frac{\exp\left[\left(|z^-| - |z^+| - |z^- - h_0| \right) U \right]}{2\, U} \Bigg\} \tag{2.126}$$

The argument of the exponential in the second term of the above equation may be readily simplified. Examining just the argument of the exponential we write

$$\left[\left(|z^-| - |z^+| - |z^- - h_0| \right) U \right] = \left[-(z^+ + h_0)\, U \right]$$

This simplification permits (2.126) to be rewritten as

$$E_{t_z} = C_d \left\{ \frac{\exp(-|z^+ - h_0|\, U)}{2\, U} + \frac{Un_0^2 - U_E}{Un_0^2 + U_E} \frac{\exp(\,(z^+ + h_0)\, U)}{2\, U} \right\} \tag{2.127}$$

An equivalent expression for (2.127), after some algebra, is

$$E_{t_z} = C_d \left\{ \frac{\exp(-|\, z^+ - h_0 \,|\, U)}{2\, U} \right.$$

$$\left. + \left[1 - \frac{2\, U_E}{U\, n_0^2 + U_E} \right] \frac{\exp(-|\, z^+ + h_0 \,|\, U)}{2\, U} \right\} \tag{2.128}$$

We now take the inverse spatial (K_x, K_y) Fourier transform of equation (2.128). The first two terms of (2.128) may be easily recognized and their transforms determined, since they are just the Fourier transforms of a Green's function. The inverse transform of (2.128), yielding E_{tz} is

$$E_{tz} = \frac{C_d}{4\,\pi} \left\{ \frac{\exp(-j\, k\, R_e)}{R_e} + \frac{\exp(-j\, k\, R_b)}{R_b} - 2\, P \right\} \tag{2.129}$$

In the above, R_e and R_b are defined as follows:

$$R_e = (\, x^2 + y^2 + (z - h_0)^2\,)^{1/2} \quad \text{, and} \qquad R_b = (x^2 + y^2 + (z + h_0)^2)^{1/2}$$

The integral P in equation (2.129) is written as

$$P = \frac{1}{\pi} \int_{K_x} \int_{K_y} \frac{U_E}{U\, n_0^2 + U_E} \frac{\exp(-(z^+ + h_0)\, U)}{2\, U} \exp(jK_x\, x + jK_y\, y)\, dK_x\, dK_y \tag{2.130}$$

The integral P corresponds to the Sommerfeld [1949] integral for the surface wave portion of the solution, which has been evaluated approximately by many authors. We now assume that the surface is highly conductive to arrive at the solution for P as presented by Wait [1970]. This will permit the approximation of the inverse transform or the z component of the electric field above the surface as described in equation (2.129). For a highly conductive surface the refractive index n_0 is such, that

$$n_0^2 >> 1 \quad .$$

Therefore it may be assumed that

$$U_E = \sqrt{\lambda^2 - k^2 n_0^2} \approx j\ k\ n_0 \quad .$$

By using the above assumption, the integral P in equation (2.129) becomes

$$P = \frac{1}{\pi} \int_{K_x} \int_{K_y} \frac{\frac{jk}{n_0}}{U + \frac{jk}{n_0}} \frac{\exp(-(z^+ + h_0)U)}{2U} \exp(jK_x x + jK_y y)\ dK_x\ dK_y \quad . \tag{2.131}$$

Equivalently, this integral may be expressed in polar coordinates as

$$P = \frac{1}{2\pi} \int_{\lambda=0}^{\infty} \int_{\phi=0}^{2\pi} \frac{jk\lambda \frac{1}{n_0}}{U + \frac{jk}{n_0}} \frac{\exp(-|z^+ + h_0|U)}{U} \exp(j\rho\lambda(\cos(\phi - \theta)))\ d\lambda\ d\phi \quad . \tag{2.132}$$

The following definitions apply to (2.132):

$$K_x = \lambda\cos\phi$$
$$K_y = \lambda\sin\phi$$
$$x = \rho\cos\theta$$

and

$$y = \rho\sin\theta \quad .$$

The two integrations in equation (2.132) may be separated as follows:

$$P = \frac{1}{2\pi} \int_{\lambda=0}^{\infty} \frac{jk\lambda \frac{1}{n_0}}{U\left[U + \frac{jk}{n_0}\right]} \exp(-|z^{+} + h_0|\, U)$$

$$\left\{ \int_{\phi=0}^{2\pi} \exp(-j\rho\lambda\cos(\phi - \theta))\, d\phi \right\} d\lambda \qquad (2.133)$$

Considering the bracketed term in equation (2.133), we may simplify the integral using the relationship

$$\int_0^{2\pi} \exp(-j\rho\lambda\cos(\phi - \theta))\, d\theta = \int_{\theta}^{2\pi - \theta} \exp(j\rho\lambda\cos\alpha)\, d\alpha$$

$$= \int_{-\pi}^{\pi} \exp(j\rho\lambda\cos\alpha)\, d\alpha$$

$$= 2\int_0^{\pi} \exp(j\rho\lambda\cos\alpha)\, d\alpha$$

$$= 2\pi\, J_0(\rho\lambda) \ .$$

The function $J_0(z)$ is the Bessel function of order zero. The integral P in equation (2.133) may now be written as

$$P = \int_{\lambda=0}^{\infty} \frac{jk\lambda \frac{1}{n_0}}{U\left[U + \frac{jk}{n_0}\right]} \exp(-|z^{+} + h_0|\, U)\, J_0(\rho\lambda)\, d\lambda \ . \qquad (2.134)$$

Again, we utilize our assumption of a highly conductive surface so that

$$\frac{1}{n_0} \approx \Delta \qquad \text{for } n_0 >> 1 \ .$$

The variable Δ is the surface impedance, following the definition for the surface impedance as proposed by Wait [1970]. By using the surface impedance approximation for a highly conductive surface, the integral P now reduces to the familiar form of the integral presented by Wait [1970]. The integral is

$$P = \int_{\lambda=0}^{\infty} \frac{jk\lambda\Delta}{U(U + jk\Delta)} \exp(-|z^{+} + h_0|\, U)\, J_0(\rho\lambda)\, d\lambda \quad . \tag{2.135}$$

An approximate solution for the integral P is given by Wait [1970], and repeated below as

$$P = \left(\frac{p_t}{w}\right)^{\frac{1}{2}} \left[1 - F(w)\right] \frac{\exp(-jkR_t)}{R_t} \quad . \tag{2.136}$$

The following definitions apply to the above equation:

$$F(w) = 1 - j\sqrt{\pi w}\, e^{-w}\, erfc(j\sqrt{w}) \quad ,$$

$$erfc(j\sqrt{w}) = \frac{2}{\pi} \int_{j\sqrt{w}}^{\infty} \exp(-z^2)\, dz \quad ,$$

$$p_t = -jk\frac{\Delta^2}{2} R_t \quad ,$$

and

$$w = p_t \left(1 + \frac{(z + h_0)}{\Delta R_t}\right)^2 \quad .$$

Convergent and asymptotic series expansions for the function $F(w)$ are well known and are available in Wait [1970]. As well, more rigorous expansions of the integral P are given by Furutsu [1959].

As was originally expected, the results of this derivation were not startling; rather the final result for the electric field over a smooth spherical earth of high conductivity and using an elementary vertical electric dipole source have been previously obtained. As well, the integral formulation for a surface of arbitrary conductivity has also been determined previously [Walsh, 1982 and Furutsu, 1959]. Equations (2.104), (2.107) and (2.109) yield a set of equations for the

Cartesian components of the electric field for an arbitrary source and a smooth planar surface of arbitrary conductivity, in the (x,y) spatial Fourier transform domain. For any given source electric field, say for example any source other than the elementary vertical electric dipole, we may use these equations to find expressions for the electric field distant from the source, provided the inverse spatial transforms may be determined.

The approach used to determine these three equations was to spatially decompose the electric field into equations for the field above and below the boundary. A third equation, representing the conditions which the field must satisfy at the boundary, is provided as a product of the analysis. These equations are then reduced to integral equations, and solved with a function representing the source electric field remaining arbitrary. It is obvious that no external boundary conditions are applied, and no assumptions are made regarding the electrical properties of the lower medium. This method is based on the approach developed by Walsh [1980] for the treatment of rough surface propagation and scatter, and applied to backscatter from the ocean surface by Srivastava [1984]. In the final two sections of this chapter we assume an elementary vertical electric dipole source and a highly conductive surface, which yields the classical result for this problem by an alternate method.

CHAPTER 3

ROUGH SURFACE EFFECTS

3.0 GENERAL

The derivation of a solution for the electric field above a highly conductive planar earth, from the previous chapter, assumes a perfectly smooth interface between the upper and lower media. We have modelled the upper medium approximately as 'free space' and the lower medium in terms of its electrical properties which are assumed known and finite. By using the methods of Wait [1970], these properties are in the form of a surface impedance, Δ, which is normalized to the intrinsic impedance of free space. From Wait [1970] the normalized surface impedance is

$$\Delta = \frac{Z_s}{Z_0} \quad . \tag{3.1}$$

In equation (3.1), Z_s is the surface impedance in ohms and $Z_0 = 120\pi$ is the impedance of free space also in ohms. The surface impedance Z_s, is defined as the ratio of the tangential electric field, E_t, to tangential magnetic field, H_t, at the air-ground interface (the surface). The surface impedance is

$$Z_s = -\frac{E_t}{H_t} \quad . \tag{3.2}$$

For a plane wave at grazing incidence, Wait [1970] has expressed the normalized surface impedance using the refractive index, n_0, which was defined in Chapter 2. The normalized surface impedance Δ, using Wait's notation is

$$\Delta = \frac{\sqrt{(n_0^2 - 1)}}{n_0^2} \quad (3.3)$$

It may be assumed that the plane wave definition of normalized surface impedance is applicable to the elementary vertical electric dipole source, assumed in our analysis. For the case of a highly conducting surface, such as the ocean, the normalized surface impedance may be approximated by

$$\Delta \approx \frac{1}{n_0} \quad (3.4)$$

For a highly conductive surface, the refractive index is such that $n_0^2 >> 1$. For convenience, we now refer to the normalized surface impedance Δ as the surface impedance. We wish to consider a more general surface, where the interface has small height irregularities. Of particular interest is the ocean surface, at frequencies in the MF (0.3-3.0 MHz) and HF (3.0-30.0 MHz) and radio frequency bands where the dominant mode of radio propagation is the surface wave.

3.1 THE MODIFIED SURFACE IMPEDANCE (OCEAN SURFACE)

A considerable amount of information has been written in the literature regarding the rough surface propagation and scattering problem. By restricting this discussion to propagation over the rough ocean surface, we find that two analyses are particularly relevant. Barrick [1970, 1971a, 1971b] has derived an expression for the surface impedance of the ocean, which is modified to account for the effects of ocean waves on the sea surface. Barrick has modelled the ocean as a random rough surface and assumed a plane wave incident on the surface at grazing angles. The modified surface impedance expression is in terms of the spatial height spectral density and the electrical properties of the surface. The spatial height spectral density is a particularly convenient method of describing ocean surface roughness, since semi-empirical models for the ocean wave height spectral density, assuming a rough wind driven sea, are available.

Srivastava [1984], in an alternate analysis, has investigated a modified surface impedance for a general rough surface, assuming an elementary vertical electric dipole source. When applied to the ocean surface, this expression also uses the ocean wave height spectral density, to characterize the surface roughness. Srivastava has used a method based on Walsh [1980] to derive this expression, as we have also used in the previous chapter for the smooth planar earth model.

In Barrick's derivation the surface is described by a two-dimensional Fourier series over a finite area. A Rice [1951] perturbation analysis on the electric field above the surface is performed, resulting in an expression for the surface impedance in terms of the Fourier coefficients of the surface. By assuming a random surface, extending to infinity, a statistical average is determined. This yields an expression for the modified surface impedance in terms of the height spectral density of the surface. This expression is

$$\overline{\Delta}_m = \Delta + \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(p,q) \, W(p,q) \, dp, \, dq \qquad (3.5)$$

In equation (3.5), $\overline{\Delta}_m$ is the modified surface impedance, Δ is the surface impedance, and the function $F(p,q)$ is given by

$$F(p,q) = \frac{p^2 + b'\Delta\left(p^2 + q^2 - k_0 p\right)}{b' + \Delta\left(b'^2 + 1\right)} + \Delta\left(\frac{p^2 - q^2}{2} + k_0 p\right) .$$

The function b' is

$$b' = \frac{1}{k_0}\left[k_0^2 - (p + k_0)^2 - q^2\right]^{1/2} .$$

The parameters (p,q) are the ocean surface spatial wave numbers. The function $W(p,q)$ is the average ocean wave height spectral density model as a function of ocean wave number. The wave height spectral density is an even function of (p,q) such that $W(p,q) = W(-p,-q)$. The above expressions may be evaluated in terms of the sea conditions through the wave height spectral density model and the electrical properties of the surface through the unmodified surface impedance, Δ. A highly conductive surface is assumed. Of course, we recognize that the surface impedance is a function of frequency through the electromagnetic wave number, k_0.

Srivastava [1984] uses an alternate analysis to determine a different expression for the modified surface impedance. This analysis derives the modified surface impedance directly from the electric field above a random periodic surface, using the Walsh [1980] field decomposition approach. This expression is

$$\Delta_m\left(-k_0\cos\theta, -k_0\sin\theta\right) = \Delta + \frac{1}{4}\int_p\int_q \left[\frac{k_0\left(p\cos\theta + q\sin\theta\right)^2}{D'(p,q) + k_0\Delta} \right.$$

$$- \Delta\left\{ \frac{k_0\left(p\cos\theta + q\sin\theta\right) D'(p,q)}{D'(p,q) + k_0\Delta} + \left(\sin 2\theta + \frac{p}{k_0}\sin\theta + \frac{q}{k_0}\cos\theta\right)\right.$$

$$\left.\left.\left(1 + \frac{k_0\Delta}{D'(p,q) + k_0\Delta} + \Delta^2\right) p\, q \right\}\right] W(p,q)\; dp\, dq \qquad (3.6)$$

The function $D'(p,q)$ is expressed as

$$D'(p,q) = \begin{cases} \left[k_0^2 - \left(p + k_0\cos\theta\right)^2 - \left(q + k_0\sin\theta\right)^2\right]^{1/2} & ;\ \textit{for real root} \\ -j\left[\left(p + k_0\cos\theta\right)^2 + \left(q + k_0\sin\theta\right)^2 - k_0^2\right]^{1/2} & ;\ \textit{for imaginary root.} \end{cases}$$

The Srivastava [1984] expression for the modified surface impedance appears radically different from that given by Barrick [1970]. As well, we note that this expression is dependent on the direction of propagation relative to the directional distribution of the wave height spectrum, and that no assumptions are made with regard to the electrical properties of the surface. Several simplifications may be performed on this expression, so that it is in a similar form to that used by Barrick.

We may assume, as with Barrick, that the surface is highly conductive. In this case the surface impedance is much less than unity ($\Delta << 1$). Using this assumption, orders of Δ^2 and Δ^3 are assumed to equal zero. A second important assumption is that we are interested in the surface impedance along the propagation path between a radio transmitter and a receiver. For simplicity we may consider this the x-axis, so that the angle θ will be equal to zero. The modified surface impedance $\overline{\Delta}_m$ may be expressed as

$$\overline{\Delta}_m = \Delta + \frac{1}{4}\int_p\int_q \left[\frac{p^2}{b' + \Delta} - \frac{\Delta k_0 p b'}{b' + \Delta} - \frac{\Delta p q^2}{k_0} \right] W(p,q)\, dp\, dq \quad . \tag{3.7}$$

The function $b'(p,q)$ is defined by

$$b' = \begin{cases} \frac{1}{k_0}\left[k_0^2 - (p + k_0)^2 - q^2\right]^{1/2} & ;\ \textit{for real root} \\ \frac{-j}{k_0}\left[(p + k_0)^2 + q^2 - k_0^2\right]^{1/2} & ;\ \textit{for imaginary root} \end{cases}$$

and

$$k_0 = \frac{\omega_0}{c} = \text{wave number of the fundamental.}$$

The third term in Srivastava's integral for $\overline{\Delta}_m$ is

$$\left[\Delta p \, \frac{q^2}{k_0} \right] W(\, p \, , q \,) \; .$$

It may be easily shown that the contribution from this term is zero. This function is an odd function of p, since $W(\, p \, , q \,)$ and q^2 are even functions. We may now write equation (3.7) as

$$\overline{\Delta}_n = \Delta + \frac{1}{4} \int_p \int_q \frac{p^2 - \Delta \, k_0 \, p \, b'}{b' + \Delta} \, W(\, p \, , q \,) \; dp \; dq \; . \qquad (3.8)$$

All definitions in the above equation are as previously established.

In order to evaluate either the Barrick [1970] or the Srivastava [1984] expressions for the modified surface impedances, the height spectral density of the surface must be determined.

3.2 OCEAN SURFACE HEIGHT SPECTRAL DENSITY

The frequencies of interest are in the HF and VHF ranges, and as a consequence the higher gravity waves in the ocean will be most important (due to the wavelength comparable to the radio wavelength). These waves are considered to be generated by winds blowing over the surface.

Oceanographers have developed many semi-empirical models, which represent the relationship between the wave height spectral density and the directional velocity of the wind blowing over the surface. There are several approximations which apply in general to all these models, the first being that of a fully developed sea. A fully developed sea means that the wave height spectrum contains components at all frequencies and that each component contains the maximum energy of which it is capable for a given wind condition. This, of course, may require a considerable length of time.

A second assumption is that the winds are constant over a large enough spatial area for the wave spectrum to be homogeneous. One final approximation is that the spectrum ignores the concept of swell. The long wave length wind waves which are of interest may propagate over long distances. Therefore surface roughness conditions may exist which are not generated by local wind conditions. This phenomenon, called swell, may contribute to the roughness in any local area.

For this investigation, we will choose a standard oceanographic model for the ocean wave height spectral density, for rough wind driven seas. The chosen model is the Neumann-Pierson model with assumed cosine squared directionality, as proposed by Neumann, Pierson and James [1955]. From Barrick [1971a], this expression is

$$W(p,q) = \frac{C\,(p\cos\alpha + q\sin\alpha)^2}{g^{\frac{5}{2}}\,(p^2+q^2)^{\frac{13}{4}}}\exp\left(\frac{-2g}{\left[U^2\,(p^2+q^2)^{\frac{1}{2}}\right]}\right) \tag{3.9}$$

The constant $C = 3.05$ is an empirical oceanographic constant, $g = 9.81$ is the acceleration of gravity in metres per second, and U is the wind velocity in metres per second. The parameter α is the angle between the wind direction and the radio propagation path direction. The propagation path direction is assumed to be along an arbitrary x axis ($\alpha = 0$), such that α is the angle formed by the wind direction and the x axis. A factor of one-half is also multiplied by the ocean wave height spectral density. This accounts for the assumption that the spectral density exists symmetrically over all space, and not just in the forward wind half space as used by the oceanographers.

For this work only one wave height spectral density model is used to calculate the modified surface impedance. Comparison between alternate spectral models is not necessary since Barrick [1970] has already utilized several different models. The change in modified surface impedance using different models is, as expected, not large.

3.3 NUMERICAL EVALUATION OF THE MODIFIED SURFACE IMPEDANCE

In order to evaluate numerically either equation (3.5) or (3.8) for the modified surface impedance $\bar{\Delta}_m$ we must first determine a suitable region of integration for the integrand. That is, we wish to define finite limits on the integrands to satisfactorily approximate the integral. To accomplish this we write the expression for the ocean wave height spectral density using polar coordinates. The Neumann-Pierson model becomes

$$W(\lambda\cos\phi, \lambda\sin\phi) = \frac{C\lambda^2\cos^2(\phi-\alpha)}{g^{\frac{5}{2}}\lambda^{\frac{13}{2}}}\exp\left(\frac{-2g}{U^2\lambda}\right) \quad (3.10)$$

In the above expression λ and ϕ are the polar coordinates such that $p = \lambda\cos\phi$, $q = \lambda\sin\phi$, and all other variables are as for the Cartesian coordinate system definitions.

The ocean wave height spectrum will be a maximum when the wind velocity direction is along the radio propagation direction, corresponding to the angle $\alpha = 0$. We may write the height spectral density as

$$W(\lambda\cos\phi, \lambda\sin\phi) = G(\lambda)\,F(\phi) \quad (3.11)$$

The functions $G(\lambda)$ and $F(\phi)$ are

$$G(\lambda) = \frac{C\lambda^2}{g^{\frac{5}{2}}\lambda^{\frac{13}{2}}}\exp\left(\frac{-2g}{U^2\lambda}\right),$$

$$F(\phi) = \cos^2(\phi).$$

By assuming that the wave height density spectrum is band limited, a maximum limit may be set on λ, beyond which $W(\lambda\cos\phi,\lambda\sin\phi)$ is assumed to be zero. It may be shown that

$$|W(\lambda\cos\phi,\lambda\sin\phi)| < 10^{-8} \qquad \text{for } \lambda \geq 15.0 \ ,$$

$$|W(\lambda\cos\phi,\lambda\sin\phi)| > 10^{-3} \qquad \text{for } \lambda \leq 1.0 \ .$$

In the above the following assumptions have been made:

$$U = 60.0 \text{ knots} \ ,$$

$$g = 9.81 \ \frac{metres}{sec^2} \ ,$$

$$\phi = 0^\circ \ .$$

For the one directional spectrum we may assume integration limits for numerical computation such that $p_1 = 0$, $p_2 = 15.0$, $q_1 = -\pi$ and $q_2 = \pi$. These limits are applied to the integrations as follows:

$$\overline{\Delta}_m = \Delta + \int_{p_1}^{p_2}\int_{q_1}^{q_2} F(p,q)\ W(p,q)\ dp\ dq \ .$$

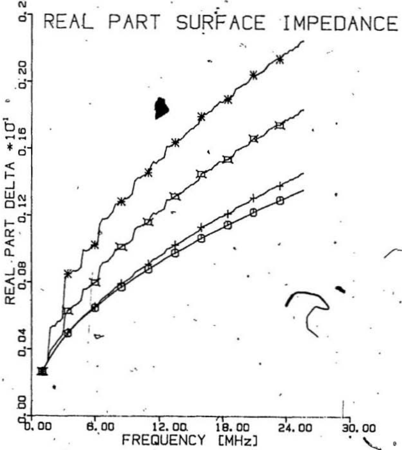
This establishes a region of integration which will approximate the integral satisfactorily. Several methods of numerical integration were tested, including efficient Gaussian quadrature routines, Romberg integration and Monte Carlo techniques. However, even a 96 point Gaussian quadrature integration would not yield a satisfactory approximation to the integral. A two-dimensional Simpson Rule program was developed, which yielded the most satisfactory results. As a consequence of this choice of routine, the surface impedance calculation requires significant computer time, on the order of two to three minutes (CPU) for each calculation.

The criteria for deciding that a surface impedance result is satisfactory are, that Δ is approximately equal to Δ_m for small wind speeds, that Δ_m should increase with both increasing sea state (wind speed) and frequency and that at low frequencies Δ_m should be approximately equal to Δ. In addition, the results achieved by implementing Barrick's expressions should compare favourably with the results presented by Barrick [1970]. This will indicate any problems with the numerical implementation of Barrick's expressions. It is also expected that the results achieved using Srivastava's expressions will generally be similar to the results achieved using Barrick's expressions.

To compare the results of the two surface impedance expressions, the modified surface impedance has been calculated and plotted versus frequency, for several typical wind conditions. Figures 3.1 and 3.2 display plots of Barrick's model for the modified surface impedance versus frequency, assuming typical values for the relative permittivity ($\epsilon_r = 80.0$) and conductivity ($\sigma = 4.0$) of the ocean surface. Figure 3.1 illustrates the real part, and Figure 3.2 the corresponding imaginary part. Results for wind speeds of 0.0, 10.0, 20.0, and 30.0 knots are plotted on the same set of axes. This implementation of Barrick's model yields some differences with respect to the results presented by Barrick [1971b] for frequencies less than six Megahertz and the 20 and 30 kn wind speeds. These differences may be attributed to either the numerical integration routine or to the implementation of the integrand. The integration was accomplished by a Simpson's rule algorithm, and up to 1,000,000 integrand points were used to calculate the double integral. However, the large grid of integrand points did not dramatically improve the results below six Megahertz. A further investigation into the behaviour of the integrand and the integration techniques, below six Megahertz, is warranted to determine the cause of the differences.

Figures 3.3 (real part) and 3.4 (imaginary part) show a similar set of plots using Srivastava's model. For all wind conditions, the differences between the results produced with either model are not significant. However, the differences observed between results using Srivastava's model and Barrick's [1971b] results at frequencies below six Megahertz are significant. Since similar problems are observed with the present implementation of Barrick's model, it may be concluded that the differences are due to the numerical implementation and are not due to the differences between Barrick's and Srivastava's surface impedance models. Further investigation of the numerical implementation may determine a solution to the observed differences.

In the surface impedance calculation program the results are based on the frequency of interest, and the surface parameters input to the program. When the wind speed is zero the integral is not calculated, which in turn sets the output of the program to Δ. This is done to conserve the CPU resource, since the integration is extremely time consuming. The integrand for the modification to the surface impedance is given as two real function subroutines, one representing the real part and the other representing the imaginary part. These are external to the surface impedance program, since they are only called by the Simpson's rule integration routine. The user may also select either the expression for the Barrick [1970] surface impedance or the Srivastava [1984] surface impedance. A Fortran source code listing of the surface impedance subroutine ZIMP, which calculates the modified surface impedance is included with the rough spherical earth



⦶ WIND SPEED = 0.0 [kn]
+ WIND SPEED = 10.0 [kn]
⌧ WIND SPEED = 20.0 [kn]
✳ WIND SPEED = 30.0 [kn]

FIGURE 3.1 : Real Part of Modified Surface Impedance Versus Frequency and Wind Speed (Barrick's Model)

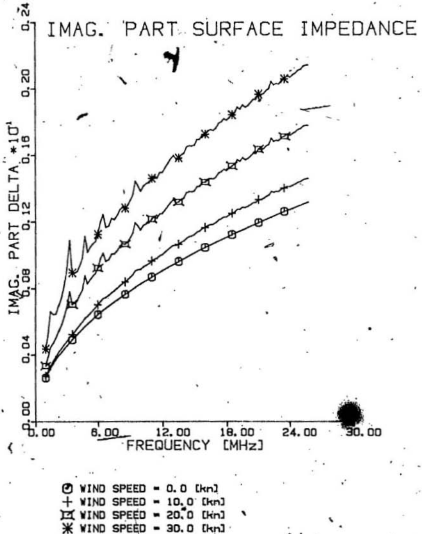


FIGURE 3.2 : Imaginary Part of Modified Surface Impedance Versus Frequency and Wind Speed (Barrick's Model)

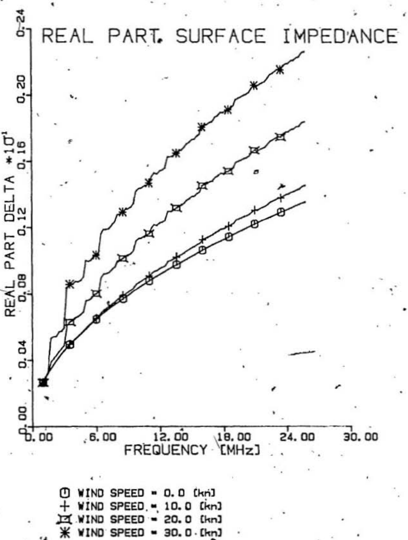
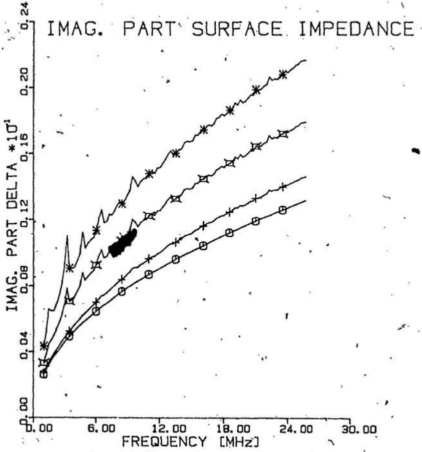


FIGURE 3.3 : Real Part of Modified Surface Impedance Versus Frequency and Wind Speed (Srivastava's Model)



⊙ WIND SPEED = 0.0 [kn]
+ WIND SPEED = 10.0 [kn]
⌧ WIND SPEED = 20.0 [kn]
✳ WIND SPEED = 30.0 [kn]

FIGURE 3.4 : Imaginary Part of Modified Surface Impedance Versus Frequency and Wind Speed (Srivastava's Model)

propagation-program in Appendix 'B'.

In the previous chapter expressions for the electric field for a smooth planar earth were developed. These expressions, equations (2.129) and (2.136) may be changed to include the effects of surface roughness, through the modified surface impedance for the ocean surface. The modified surface impedance may be included by substituting p_{em}, and w_m for p_e and w in these equations, where p_{em} and w_m are defined as follows:

$$p_{em} = -j\,k\,\frac{\overline{\Delta}_m^2}{2}\,R_b \quad ,$$

and

$$w_m = p_{em}\left[1 + \frac{(z + h_0)}{\overline{\Delta}_m\,R_b}\right]^2 .$$

The modified surface impedance outlined in this chapter permits the evaluation of the surface wave electric field for propagation over rough ocean surfaces. Using the plane earth model, developed in Chapter 2, we may predict the ground wave electric field strength over a rough surface for short distances. For long distances the effects of diffraction around the curvature of the earth become important. We now proceed to outline the spherical earth model, which will account for the diffraction effects.

CHAPTER 4

GROUND WAVE SPHERICAL EARTH MODEL

4.0 INTRODUCTION

In Chapter 2 we derived equations for predicting the electric field strength, distant from the source, for propagation over an assumed planar earth model. An elementary vertical electric dipole source was considered. Although satisfactory for many applications, the planar earth expressions do not account for the effects of diffraction of the electromagnetic waves around the curved surface of the earth. It has been suggested that the planar earth model should be only applied to propagation distances, d, such that

$$d \leq \frac{50}{f^{1/3}} \qquad (4.1)$$

the parameter d is the propagation distance in kilometres and f the radio frequency in megahertz. The propagation distance is defined as the distance between the source and the observation points. For distances greater than this suggested limit, models are available which account for diffraction effects. These models, which are based on classical analyses, also assume elementary vertical electric dipole antennas.

At large distances from the source, up to several hundreds of kilometres, the most significant mode of propagation is that of diffraction of the ground wave around the spherical surface of the earth. For extremely long distances the effects of the earth's upper atmosphere, yielding the so-called 'Sky-Wave' mode, cannot be ignored. However, in this section we will examine only the ground wave mode and the influence of the earth's curved surface on this mode of propagation. This restriction limits the range of distances to which the ground wave spherical earth model may be applied. This range of applicability is, unfortunately, not well defined in the literature but a limit of 300 kilometres is suggested.

The general solution to the spherical earth propagation problem was described by Watson [1919]. He examined expressions for the electric field distant from a radially oriented elementary vertical electric dipole source at a height h_T above a homogeneous sphere. The electrical properties of the sphere are described by the permittivity, permeability and conductivity, while the space surrounding the sphere is approximated by 'free-space' with its associated electrical properties. The source field was expanded into a series of spherical harmonic functions and by using the Maxwell equations, in spherical polar form, a rigorous solution for the electric field distant from the source and a height h_R above the spherical surface was determined. The solution for the field in the presence of the surface was also in the form of a series of spherical harmonic functions using spherical Hankel functions and Legendre polynomials. Watson's solution, although rigorous, was impractical for radio propagation problems since the series converged very slowly, that is an enormous number of terms was required to determine a value for the electric field strength. It may also be noted that this series solution was applied to many other wave propagation problems successfully, in which the ratio of the radius of the sphere to the wavelength is small. In this case the series rapidly converged to a solution. For ground wave propagation of radio waves in the MF/HF/VHF frequency bands, covering the frequencies 0.3-300.0 MHz, the ratio of the earth's radius to radio wavelength is approximately

$$\frac{a}{\lambda} > \frac{6.4 \times 10^{6}\ meters}{1000\ meters} \approx 6400$$

The parameter a is the approximate earth's radius and λ the radio wavelength at a frequency of 0.3 MHz. Thus, for this problem we expect the series to converge slowly.

Since the direct summation of the series is not practical for radio propagation problems, Watson transformed the sum of the series of harmonic functions into a contour integral of a continuous variable. Following the methods of Watson, Van der Pol and Bremmer [1939] as well as Fock [1945,1965] write a residue series approximation of this contour integral. This series, in terms of the sum of the residues of the poles of the contour integral, proved to converge rapidly for most calculations, and facilitated numerical results. The principal difference between the Fock and Van der Pol-Bremmer expressions is that Fock has used the asymptotic expressions for the Hankel functions, in terms of the Airy functions, yielding simpler and easier to implement expressions. It is this residue series expression upon which we we focus our attention.

The objective of this work is not to perform a new analysis on the ground wave spherical earth propagation problem; rather it is to provide a numerical facility with which to calculate path losses for propagation over a spherical earth.

As well, by using the modified surface impedance expressions from Chapter 3, the path losses for propagation over a spherical rough ocean surface may be predicted. The resulting numerical computing package is applicable to propagation in the upper MF,HF and lower VHF frequency bands.

4.1 DESCRIPTION OF THE MODEL

The geometry of the ground wave spherical earth propagation model is described with the aid of figure 4.1. An elementary vertical electric dipole source (oriented radially) is located at a height h_T above a sphere of radius a, and transmitting at an angular frequency of ω_0 radians per second. This yields a corresponding wave number k_0 and wavelength of λ. The electric field strength is observed at a height h_R above the spherical surface and a distance D from the source. The electrical properties of the sphere, namely the permittivity, conductivity and permeability, are described by the surface impedance Δ which has already been defined. The space outside the sphere is approximated by 'free-space'. The spherical polar coordinate system, (ρ,θ,ϕ), with the origin located at the centre of the sphere is used.

Expressions for the residue series approximation for the electric field above a spherical earth, written in terms of the Hertz vector are available from Fock [1965], Bremmer [1949] or Wait [1970]. Wait and Fock both write the series in terms of the Airy functions. Berry and Chrisman [1965,1966] derive expressions for the electric field, E_r for a radially oriented unit dipole source and in standard units of volts per metre, using Fock, Bremmer and Wait's results for the Hertz vector. From Berry and Chrisman [1966], the expression for E_r at a distance D from the source is written as

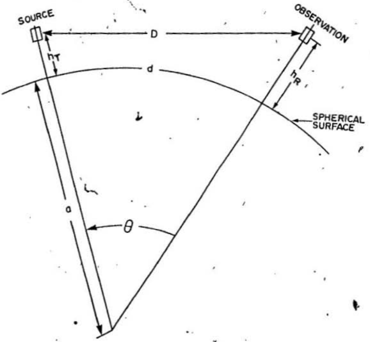


FIGURE 4.1 : Geometry of the Spherical Earth Propagation Model

$$E_r = \frac{-4\pi K}{\sqrt{\sin\theta}} \exp\left[-j\left[k_0 d - \frac{\pi}{4}\right]\right] \sum_{s=0}^{\infty} \exp(-j\,x\,t_s)\, A(q_v, t_s) \qquad (4.2)$$

The function $A(q_v, t_s)$ is defined by

$$A(q_v, t_s) = \frac{(1 + z\,t_s)^{\frac{5}{2}}}{t_s - q_v^2} f_R(h_R)\, f_T(h_T) \qquad (4.3)$$

The parameters t_s are the poles of the contour integral from which the residue series was derived. The functions $f_R(h_R)$ and $f_T(h_T)$ are the height gain functions, which depend on the heights of the source (transmitter) and observer (receiver) above the spherical surface. These functions are defined as follows:

$$f_T(h_T) = \frac{W_1(t_s - y_T)}{W_1(t_s)}$$

and

$$f_R(h_R) = \frac{W_1(t_s - y_R)}{W_1(t_s)}$$

$W_n(t)$ are the Airy functions of type n and of the argument t and y_R, y_T are defined by

$$y_R = k h_R \left[\frac{ka}{2}\right]^{-\frac{1}{3}},$$

$$y_T = k h_T \left[\frac{ka}{2}\right]^{-\frac{1}{3}}$$

The poles of the contour integral are defined by the equation

$$t_s\, W_1(t_s) - q_v^2\, W_1(t_s) = 0 \qquad (4.4)$$

We note the following property of the Airy functions from Abramowitz and Stegun [1972]:

$$W'_v(t) = t\, W_v(t) \quad .$$

The prime denotes the derivative with respect to t. By using this property, the pole defining equation may be expressed as

$$W'_1(t_s) - q_v^2\, W_1(t_s) = 0 \quad . \tag{4.5}$$

The constant q_v represents the electrical constants of the surface, and is defined through the surface impedance, Δ. We write

$$q_v = -j\left(\frac{ka}{2}\right)^{\frac{1}{3}} \Delta\sqrt{1-\Delta^2} \quad ,$$

where the surface impedance Δ has been defined previously. For reference, Δ is expressed in terms of the refractive index of the surface n_0 as

$$\Delta = \frac{\sqrt{n_0^2 - 1}}{n_0^2}$$

The constants, k_0, a, h_T, h_R are as follows:

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega_0}{c} = \text{wavenumber of the fundamental} \quad ,$$

a = radius of the sphere ,

h_T = height of the source ,

and

h_R = height of the observer .

In the expression for the electric field, equation (4.2), the parameters z, z', and K are

$$z = \frac{1}{2}\left[\frac{k_0\, a}{2}\right]^{-\frac{2}{3}} \quad ,$$

$$x = \left[\frac{k_e\, a}{2}\right]^{\frac{1}{3}} \theta \quad ,$$

and

$$K = 11.96 \sqrt{\frac{k_0}{a^3}} \left[\frac{k_0 a}{2}\right]^{\frac{2}{3}} I_0\, dl$$

As well, we also have the following:

$\theta = \frac{d}{a}$ the angle formed by the source, observer and sphere centre ,

$I_0\, dl = 1.0$ the dipole current moment ,

and

d is the distance along the surface.

In the above expression for q_v, in terms of the surface impedance Δ, the modified surface impedance $\overline{\Delta}_m$ for a rough ocean surface as shown in Chapter 3could be substituted. In this manner the effects of a rough spherical ocean surface on the propagation of radio waves would be included in the model. By using the modified surface impedance $\overline{\Delta}_m$, we define q_{vm} as

$$q_{vm} = -j \left[\frac{k_0 a}{2}\right]^{\frac{1}{3}} \overline{\Delta}_m \sqrt{1 - \overline{\Delta}_m^2} \ .$$

The above expression for q_{vm} would replace q_v in the expression for the electric field E_r in equation (4.3). This yields a model for radio wave propagation over a rough spherical ocean surface, based on the model for a rough wind driven sea discussed in Chapter 3.

Obviously, since the residue series is an approximation for a more rigorous solution to the spherical earth propagation problem, we expect limits on the applicability of this model. The residue series approximation is limited to ground-wave type problems, to which the limit that the observer be no more than fifteen degrees above the radio horizon as seen from the transmitter applies.

A further limit is that for high antenna elevations, and particularly if short distances are involved, the residue series approximation becomes very slow to converge. Also for the same cases the method used to determine the poles of the contour integral may fail to converge. For these cases many authors, including Fock [1965], Bremmer [1949] and Wait [1970], have derived other approximate solutions which are more appropriate. Since the principal area of interest for this thesis is in ground wave propagation over a rough ocean surface, with antennas located on or near the surface, we shall concentrate only on the implementation of the residue series approximation.

The residue series calculation of the electric field distant from the source E_r may be implemented in a computer program to facilitate numerical results. For the most part the implementation follows straightforwardly from the expressions shown in this section. The calculation of the Airy functions and the determination of the poles of the series are, however, not a trivial matter. It is this latter problem to which we now turn our attention.

4.2 POLES OF THE RESIDUE SERIES

The poles of the Fock residue series are determined from equation (4.5) which is repeated below as

$$W'_1(t_s) - q_v^2 W_1(t_s) = 0 \qquad (4.5)$$

As before, $W_n(t_s)$ are the Airy functions, t_s are the poles of the residue series, and

$$q_v = -j\, v\, \Delta \sqrt{1-\Delta^2}\ .$$

One method of determining these poles is given by Bremmer [1949] and implemented by Berry and Chrisman [1966]. An estimate, τ_s, of the poles t_s is determined using a differential equation. By using τ_s as an initial estimate, standard iterative numerical techniques may be applied to determine t_s. To outline this method, we temporarily use the notation of Bremmer [1949] to determine τ_s. From Bremmer [1949], the estimates of the poles are determined by Ricatti's differential equation,

$$\frac{d\delta}{d\tau_s} - 2\delta^2 \tau_s + 1 = 0 \quad . \tag{4.6}$$

The notation used by Bremmer uses the function δ to describe the electrical properties of the surface. The function δ is given by

$$\delta = -j \left(k \, a \right)^{-\frac{1}{3}} \Delta \sqrt{\Delta^2 - 1} \quad ,$$

The function δ is related to q_v by

$$\delta = -j \, 2^{\frac{1}{3}} \left(k_0 \, a\right)^{-\frac{2}{3}} q_v \quad .$$

The methods used by Bremmer, specifically in the pole-determining equation and in describing the electrical properties of the surface, differs from that of Fock due mainly to the Airy function approximation, described earlier, which is used in Fock's derivation. Proceeding with the outline of Bremmer's method, we find that τ_s gives an approximation to the poles t_s , which may be used in a Newton iteration on equation (4.5). The approximation τ_s may be determined by expanding the equation (4.5) in both convergent and asymptotic series. These series may be obtained from Bremmer [1949], or are easily verified by a standard series expansion of the solution for Ricatti's differential equation. The series are

$$\tau_s = \tau_{s,0} - \delta - \frac{2}{3}\tau_{s,0}\,\delta^3 + \frac{1}{2}\delta^4 - \frac{4}{5}\tau_{s,0}^2\,\delta^6 \,\ldots\ldots \quad , \tag{4.7}$$

and

$$\tau_s = \tau_{s,\infty} - \frac{1}{2\,\tau_{s,\infty}}\,\delta^{-1} - \frac{1}{8\,\tau_{s,\infty}^{3}}\,\delta^{-2} - \frac{1 + \dfrac{3}{4\,\tau_{s,\infty}}}{12\,\tau_{s,\infty}}\,\delta^{-3} - \ \ldots\ldots \tag{4.8}$$

In the above, $\tau_{s,\infty}$ and $\tau_{s,0}$ are as follows:

$$\tau_{s,\infty} = \lim_{\delta \to \infty} \tau_s \quad ,$$

and

$$\tau_{s,0} = \lim_{\delta \to 0} \tau_s \quad .$$

The factors $\tau_{s,\infty}$ and $\tau_{s,0}$ are tabulated by Bremmer. The coefficients of these series are determined by inverting the Ricatti equation and solving for each coefficient iteratively. Therefore, no general expression for the coefficients is possible, and hence only a fixed number of terms may be included in any practical implementation. However, by using a large number of terms the series may indeed converge to the value of τ_S which is easily tested by standard numerical techniques.

Berry and Chrisman [1966] determine the first eleven terms of these series and then perform a Newton iteration on the Fock pole defining equation (4.5) to determine t_s'. In practice this method was extremely slow and cumbersome for the particular computer used (VAX-11/785). The parameter δ for a highly conductive surface may be very small, as is the case for the ocean surface. When this method was implemented in Fortran-77 on a Digital Equipment VAX-11/785, underflow and overflow errors resulted when large powers of δ were attempted. This is due to the limited dynamic range of this minicomputer. The elimination of these errors required the use of sixteen byte variables in Fortran. These operations require greater than the 32 data bits which this machine uses. To effect these computations software emulation techniques are used to simulate 64 data bits, which requires large amounts of CPU processing time to accomplish even simple mathematical operations.

A more practical method for determining the poles, was to use the Bremmer [1949] results for $\tau_{s,0}$ and $\tau_{s,\infty}$ as an initial approximation and perform a Newton iteration directly on the pole defining equation (4.5). The Newton iteration method for finding the poles of a function $f(z)$ is described from Carnahan, Luther and Wilkes [1969] as

$$z^{k+1} = z^k - \frac{f(z^k)}{f'(z^k)} \quad (4.9)$$

In equation (4.9), z^k is the initial estimate for the poles of $f(z)$, and z^{k+1} is the new approximation for the poles of $f(z)$. In the case of the poles of the residue series the iteration formula becomes

$$t_s^{m+1} = t_s^m - \frac{W'_1(t_s^m) - q_v W_1(t_s^m)}{W''_1(t_s^m) - q_v W'_1(t_s^m)} \quad (4.10)$$

$$= t_s^m - \frac{\dfrac{W'_1(t_s^m)}{W_1(t_s^m)} - q_v}{t_s^m - q_v \dfrac{W'_1(t_s^m)}{W_1(t_s^m)}}$$

In the above equation, m represents the iteration number and the following relationship has been used for simplifications:

$$W''_1(t_s^m) = t_s^m W_1(t_s^m) \quad .$$

The initial estimates of the poles of the residue series, used in the above Newton iteration, may be determined from the limiting cases of τ_s. That is, t_s^0 and t_s^∞

are proportional to $\tau_{s,0}$ and $\tau_{s,\infty}$ as tabulated by Bremmer. The relationship is

$$t_s = (2)^{\frac{1}{3}} \tau_{s,0} \ ,$$

$$t_s^0 = (2)^{\frac{1}{3}} \tau_{s,\infty} \ .$$

In the subroutine program (Fortran-77) which is used to evaluate the poles of the residue series, the maximum number of iterations permitted is arbitrarily selected at thirty. The iterations on the poles is also terminated when the Newton iteration converges to a particular value. The convergence test is

$$\left| \frac{t_s^{m+1} - t_s^{m}}{t_s^{m+1}} \right| < 10^{-6} \ . \qquad (4.11)$$

The overall method for calculating the residue series poles yielded comparable results to the method as used by Berry and Chrisman [1966]. In practice, this method consumed very small amounts of processing time, as opposed to the time required by the implementation of Bremmer's series for the poles t_s. In a later Fortran program for the evaluation of the ground wave field strength, Berry [1978] has used the Newton Iteration method to determine t_s. The FORTRAN code provided by Berry [1978] provided an initial check for the accuracy of the numerical results determined by the algorithm described in this chapter. A FORTRAN source code listing of the pole determining program is included with the source code listing of the complete spherical earth model program in Appendix 'B'.

4.3 EVALUATION OF THE AIRY FUNCTIONS

The Airy functions may be evaluated according to standard asymptotic and convergent series expansions. In this program the Airy functions have been evaluated with a convergent series from Abramowitz and Stegun [1965], and by using the asymptotic series from Berry and Chrisman [1965]. However, the definitions of the Airy functions, as used by Fock, are significantly different than those used in Abramowitz and Stegun. No convenient explanation was found for this difference, but a relationship was determined by first finding a relationship between Fock's definition of the Airy functions and the fractional order Hankel functions. Abramowitz and Stegun then yielded a relationship between the fractional order Hankel functions and the second definition of the Airy functions. The relationship is defined as follows. From Wait [1970] the Airy functions may be expressed as

$$W_1(t) = \exp(-j2\pi/3) \left(\frac{-\pi t}{3}\right)^{\frac{1}{2}} H_{\frac{1}{3}}^{(2)}\left[\frac{2}{3}(-t)^{\frac{3}{2}}\right] ,$$

$$W'_1(t) = \exp(-j\frac{\pi}{3}) \left(\frac{\pi}{3}\right)^{\frac{1}{2}} (-t)\, H_{\frac{2}{3}}^{(2)}\left[\frac{2}{3}(-t)^{\frac{3}{2}}\right] .$$

The functions $H_{\frac{n}{3}}^{(2)}(z)$ are fractional order Hankel functions and are also defined by Abramowitz and Stegun [1972]. Now, by using the definitions of the fractional order Hankel functions we may write the relationship between the Fock definition of the Airy functions, $W_n(t)$ and the Abramowitz-Stegun definition for $Ai(t)$ and $Bi(t)$. The relationships are

$$W_1(t) = \exp(-j\frac{\pi}{2}) \sqrt{\pi} \left[Ai(t) + j\, Bi(t)\right] , \tag{4.11}$$

$$W'_1(t) = \exp(-j\frac{\pi}{2}) \sqrt{\pi} \left[Ai'(t) + j\, Bi'(t)\right] . \tag{4.12}$$

$Ai(z)$ and $Bi(z)$ are the Airy functions as defined by Abramowitz and Stegun, and Ai', Bi' are their derivatives. The convergent series expansions for these functions are

$$Ai(z) = Ai(0)\,f(z) + Ai'(0)\,g(z)\quad, \tag{4.13}$$
$$Bi(z) = \sqrt{3}\left[Ai(0)\,f(z) - Ai'(0)\,g(z)\right]\quad. \tag{4.14}$$

Expansions for $f(z)$ and $g(z)$ are as follows:

$$f(z) = 1 + \frac{1}{3!}z^3 + \frac{1\cdot4}{6!}z^6 + \frac{1\cdot4\cdot7}{9!}z^9 + \cdots\;,$$
$$g(z) = z + \frac{2}{4!}z^4 + \frac{2\cdot5}{7!}z^7 + \frac{2\cdot5\cdot8}{10!}z^{10} + \cdots$$

The two constants are $Ai(0) = 0.3550280$, and $Ai'(0) = -0.2588194$. The derivatives of the Airy functions are

$$Ai'(z) = Ai(0)\,f'(z) + Ai'(0)\,g'(z)\quad, \tag{4.14}$$
$$Bi'(z) = \sqrt{3}\left[Ai(0)\,f'(z) - Ai'(0)\,g'(z)\right] \tag{4.15}$$

The functions $f'(z)$ and $g'(z)$ are the derivatives of $f(z)$ and $g(z)$ respectively. These convergent series expansions are valid for all z, but are very slow to converge for $z >> 1$. This would also lead to numerical problems for terms with large powers of z. These series expansions have been implemented in a Fortran subroutine within the spherical earth program to evaluate the Airy functions. Although Berry and Chrisman [1966] have written Fortran code to implement the convergent expansions for $W_n(t)$, a test case demonstrated that the method used to implement the expansions, again, required sixteen byte variables for the particular computer used in this application. A more suitable code was written using the alternate expansions for $W_n(t)$ in terms of $Ai(t)$ and $Bi(t)$.

It was determined that for large arguments, numerical problems would arise using the convergent series. For large arguments, the asymptotic expansions of $W_n(t)$ were available. As suggested by Abramowitz and Stegun the asymptotic expansions of the Airy functions were used for arguments of z such that $|z| > 5.0$. The asymptotic series expansions of the Airy functions are available from Berry and Chrisman [1965], in a form very convenient for numerical implementations. The particular series used for $W_k(z)$ and $W_k'(z)$ depend on the phase of the variable z. Four different expansions are given for each Airy function for the following regions:

REGION I : | phase z | < 20.0°

REGION II : | phase z | > 100.0°

REGION IIIA : −100.0° ≤ phase z ≤ −20.0°

REGION IIIB : 20.0° ≤ phase z ≤ 100.0°

Commencing with REGION I, we set $t = 2/3\, z^{3/2}$ and the Airy functions may be evaluated from

$$W_n(z) \approx z^{-\frac{1}{4}} \left[e^{t}\, L(t) + (-1)^n \frac{j}{2}\, e^{-t}\, L(-t) \right] ,$$

$$W_n'(z) \approx z^{\frac{1}{4}} \left[e^{t}\, M(t) + (-1)^{n+1} \frac{j}{2}\, e^{-t}\, M(-t) \right] .$$

The functions $L(t)$ and $M(t)$ are defined as

$$L(t) = \sum_{k=0}^{\infty} U_k\, t^{-k} ,$$

$$M(t) = \sum_{k=0}^{\infty} V_k\, t^{-k} .$$

For $k = 0$, we have $U_0 = V_0 = 0$ and for $k > 0$ we have

$$U_k = \frac{(2k+1)(2k+3)\cdots(6k-1)}{k!\, 216^k} ,$$

$$V_k = U_k \frac{6k+1}{6k-1} ,$$

In REGION II, let $t = 2/3(-z)^{3/2}$ and by using the functions $L(t)$, $M(t)$, the Airy functions are

$$W_n(z) \approx (-z)^{-\frac{1}{4}} \exp\left[j(-1)^n \left(t + \frac{\pi}{4}\right)\right] L\left[(-1)^n jt\right] ,$$

$$W_n'(z) \approx (-z)^{\frac{1}{4}} \exp[j(-1)^n (t - \frac{\pi}{4})] M\left[(-1)^n jt\right] .$$

In REGION IIIA, we let $t = 2/3\, z^{3/2}$, and

$$W_n(z) \approx z^{-\frac{1}{4}} \left[e^t L(t) + j(n-2) e^{-t} L(-t)\right] ,$$

$$W_n'(z) \approx z^{\frac{1}{4}} \left[e^t M(t) - j(n-2) e^{-t} M(-t)\right] .$$

Finally, for REGION IIIB, we let $t = 2/3\, z^{3/2}$, and write $W_n(z)$ and $W_n'(z)$ as

$$W_n(z) \approx z^{-\frac{1}{4}} \left[e^t L(t) + j(n-1) e^{-t} L(-t)\right] ,$$

$$W_n'(z) \approx z^{\frac{1}{4}} \left[e^t M(t) - j(n-1) e^{-t} M(-t)\right] .$$

The regions for the phase of z are suggested by Berry and Chrisman [1965], but the formulas are asymptotically valid in much larger regions. Berry and Chrisman state that greater computational accuracy is achieved by using their restricted regions. The formulas for the Airy functions have been implemented and tested in a computer program for numerical computations. The program written in Fortran-77 source code will compute the Airy functions for $|z| < 32.0$. The restriction on the magnitude of the argument is due to the fact that the dynamic range of the particular computer (using four byte real variables in the computer program) is limited to $\approx \pm 10^{38}$. For larger values of z extended variables (sixteen byte variables) may be necessary, to evaluate the Airy functions. However, the tests performed using the residue series with frequencies in the MF/HF band and with antenna heights less than forty metres, the extended variables are not required.

4.4 SPHERICAL EARTH PROGRAM STRUCTURE

The Fortran program used to evaluate the spherical earth model has been developed using a fairly modular structure. The program has many features in common with the implementation developed by Berry and Chrisman [1966]. The primary differences are that this version is suitable for implementation on smaller mini computers (which the Berry and Chrisman program was not!) and the use of modern Fortran-77 which enables the use of complex variable types. Thus the program is more compact, simpler to follow and requires less processing time for a typical run. As well different methods for calculating several of the functions required for the residue series (i.e. the poles t_s) are implemented.

The main program evaluates the residue series and determines coefficients and constants required to find the vertical electric field strength given a set of ranges, ground constants and antenna heights. The central body of the main program calculates the function

$$F(q_v, t_s) = \frac{(1 + zt_s)^{\frac{5}{2}}}{t_s - q_v^2} \; \frac{W_1(t_s - y_T)}{W_1(t_s)} \; \frac{W_1(t_s - y_R)}{W_1(t_s)}$$

where all definitions remain as previously established. This function is not a function of distance, and is calculated only once for each run and stored for use with the complete specified set of distances. The number of values of this function which are required, is determined by the number of poles t_s required for the electric field to converge for the first selected distance. For the rest of the specified set of distances, the stored values of F are used as long as the electric field converges. Additional values of F are calculated automatically should the number of stored values be insufficient for convergence.

All other functions required for the calculation of the electric field are determined in Fortran subroutine or function type subprograms. These functions include the poles t_s, the Airy functions and the surface impedance (or modified surface impedance).

For a rough spherical earth, the modified surface impedance as shown in Chapter 3 is selected automatically by setting the input wind speed greater than zero. The modified surface impedance is included in the model by replacing the parameter q_v with the parameter q_{vm}. For reference, we have

$$q_v = -j\left(\frac{ka}{2}\right)^{\frac{1}{3}} \Delta \sqrt{1 - \Delta^2} \,,$$

$$q_{vm} = -j\left(\frac{ka}{2}\right)^{\frac{1}{3}} \overline{\Delta}_m \sqrt{1 - \overline{\Delta}_m^2} \,.$$

The modified surface impedance, including roughness effects is $\overline{\Delta}_m$ and Δ is the smooth earth surface impedance.

From the calculated electric field strength, in volts per metre, we then determine the spherical earth attenuation function, $W(p)$, by analogy to the Norton attenuation function, $F(p)$, as was discussed in Chapter 2 of this thesis. The output of this function is more desirable, since the attenuation function is convenient for determining transmission losses in the units of power (watts). The spherical earth attenuation function is then output in tabular form, using magnitude and phase rather than complex numbers. As well, the attenuation function is also output to a data file for permanent storage. A sample run of the program is also included in Appendix 'B'. The spherical earth field calculation model was implemented in VAX-11 Fortran-77, using a Digital Equipment DEC VAX-11/785 computer. Numerical results computed with this model in the form of transmission losses are presented in the following chapter.

The rough spherical earth transmission losses demonstrated a problem with this particular numerical model. For the cases of a rough wind driven sea and a variety of transmit frequencies, the Newton iteration on some of the poles of the residue series failed to converge. Initially the number of iterations on each pole had been restricted to only twenty, which was insufficient for cases when the modified surface impedance was highly inductive. By increasing the maximum number of iterations to thirty-five, this problem was eliminated for all tested frequencies and wind speeds. We now proceed to investigate some numerical predictions of transmission losses for propagation over a rough spherical earth.

CHAPTER 5

NUMERICAL RESULTS

5.0 TRANSMISSION LOSS

A typical application for the spherical earth model would be to determine the transmission loss for marine communications systems. This would include point to point communications systems, radio navigation systems and radar systems operating in the HF and lower frequency bands. The spherical earth model may be used to predict additional losses due to surface roughness, using the modified surface impedance. Expressions for the predicted transmission loss may be developed, using the rough spherical earth attenuation function, W_r. We may proceed to derive an expression for this transmission loss.

The free space electric field for an elementary vertical electric dipole source, in the horizontal plane, is given as

$$E_0 = \frac{C_d}{4\pi} \frac{\exp(-jk_0 R)}{R} \quad . \tag{5.1}$$

The factor $C_d = -j\omega\mu_0 I_0 dl$ is the dipole constant, R is the distance between the source and observation points, and k_0 is the wave number of the fundamental. Similarly, the ground wave field strength over a rough spherical earth is given, by analogy to the flat earth model, as

$$E_r = \frac{2C_d}{4\pi} \frac{W_r}{R} \exp(-jkR) \quad , \tag{5.2}$$

where W_r is the rough spherical earth attenuation function. The attenuation function is calculated using the program described in Chapter 4. From Jasik [1961] the transmit antenna gain in the horizontal direction is

$$G_T = \frac{2\pi \mid E_0 \mid^2}{\eta_0 P_t} R^2 .$$

Equivalently, the transmitted power P_t is

$$P_t = \frac{2\pi \mid E_0 \mid^2 R^2}{\eta_0 G_T} . \tag{5.3}$$

In the above equations, η_0 is the intrinsic impedance of free space, and P_t is the transmitted power (in watts). The received power at a distance from a source, for an antenna of effective aperture A_r is

$$P_R = \frac{\mid E \mid^2}{2\eta_0} A_r ,$$

where E is the electric field distant from the source and A_r is the effective receive antenna aperature. For ground wave propagation over a rough spherical earth, using the electric field strength E_r at the receive antenna, the received power distant from the source is

$$P_R = \frac{\lambda_0^2 G_R}{4\pi} \frac{\mid E_r \mid^2}{2\eta_0} . \tag{5.4}$$

In the above equation G_R is the receive antenna gain in the horizontal plane. The transmission loss may be defined as the power ratio of the received power to the transmitted power. The ratio of equation (5.4) to equation (5.3), representing the transmission loss is

$$T_L = \frac{P_R}{P_T} = \frac{\lambda_0^2 G_T G_R \mid E_r \mid^2}{(4\pi)^2 \mid E_0 \mid^2 R^2} . \tag{5.5}$$

From equations (5.1) and (5.2) we have

$$\frac{|E_r|^2}{|E_0|^2} = 4\ |W_r|^2 .$$

By using the above ratio, equation (5.5) for the transmission loss becomes

$$T_L = \frac{4\lambda_0^2\ G_T\ G_R\ |W_r|^2}{(4\pi)^2 R^2} . \tag{5.6}$$

The above equation, (5.6), is an expression for the expected transmission losses for propagation over a rough spherical earth. This result may be used to predict transmission losses for propagation over the ocean surface assuming a variety of sea states and radio frequencies. In the following section results for propagation over a smooth ocean surface are presented along with additional losses attributed to sea state, for a variety of frequencies in the MF and HF bands.

5.1 SPHERICAL EARTH TRANSMISSION LOSS RESULTS

The transmission loss for the spherical earth model, may be calculated for various frequencies using equation (5.6). Initially we calculate the smooth spherical earth transmission losses. In this case the surface impedance is calculated using the standard method, without using the rough wind driven sea model for the modified surface impedance. For a given operating frequency, the transmission losses, in dB, may be calculated and plotted as a function of distance, in kilometres. For this figure the permittivity of the surface ϵ, is assumed to 80.0 and the conductivity σ assumed to be 4.0 mhos per meter. These constants are chosen to approximate the electrical properties of the smooth ocean surface. Figure 5.1 shows the smooth spherical earth transmission loss for a variety of fre-

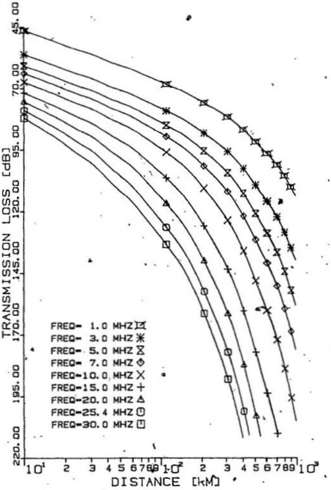


FIGURE 5.1 : Transmission Loss Versus Distance and Frequency for a Smooth Ocean Surface Using the Spherical Earth Model (In dB Relative to 1.0 Watt Transmitted Power)

quencies in the HF frequency band, plotted on the same set of axes. We may note that the transmission losses increase with both increasing frequency and increasing distance separating the transmitter and receiver. In order to check these computations, the results presented in figure 5.1 may be compared with smooth spherical earth transmission losses presented by Barrick [1970]. The results presented in figure 5.1 for all frequencies, except 1.0 Megahertz which was not computed by Barrick, compare with a high degree of accuracy to those presented by Barrick [1970]. This comparison yields a degree of confidence in the smooth spherical earth transmission loss computation routine, since it compares favourably with previous results.

We now proceed to examine the transmission losses for propagation across a rough wind driven sea. For these results we have produced individual plots for each frequency. On each set of axes the transmission losses for several wind conditions are plotted. As well we note that the losses are normalized to the smooth spherical earth transmission losses, yielding plots of additional loss, due to the effects of interaction with the ocean waves, versus distance from the source in kilometres. Figures 5.2 through 5.10 are plots of added loss versus distance for various frequencies in the HF band, using the Srivastava modified surface impedance presented in chapter 3. As with the smooth spherical earth transmission losses, the relative permittivity is assumed to be 80.0 ($\epsilon_r = 80.0$) and the conductivity to be 4.0 mhos per meter ($\sigma = 4.0$). In these plots a negative added loss represents a decrease in transmission losses relative to smooth earth (i.e. lower losses). We note that in many of these plots the additional loss may be less than than 0.0 dB, meaning that the transmission losses for a rough ocean surface

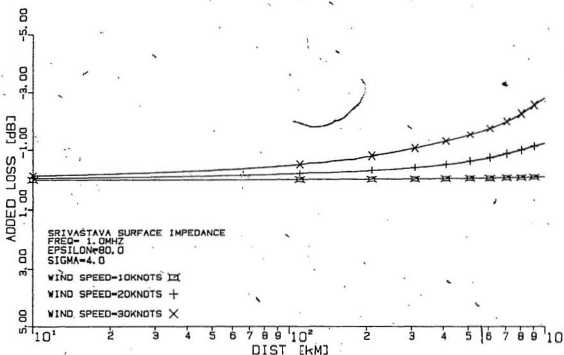


FIGURE 5.2 : Added Loss Versus Distance for Propagation Over a Rough Ocean Surface at 1.0 MHz. (Srivastava Surface Impedance)

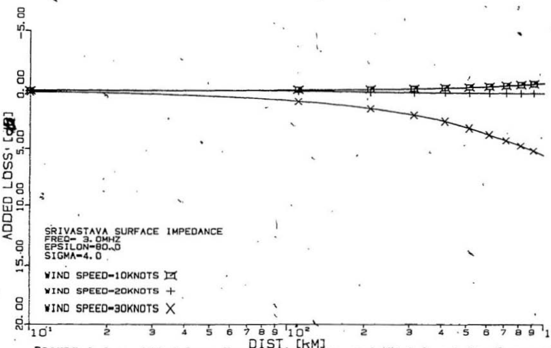


FIGURE 5.3 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 3.0 MHz. (Srivastava Surface Impedance)

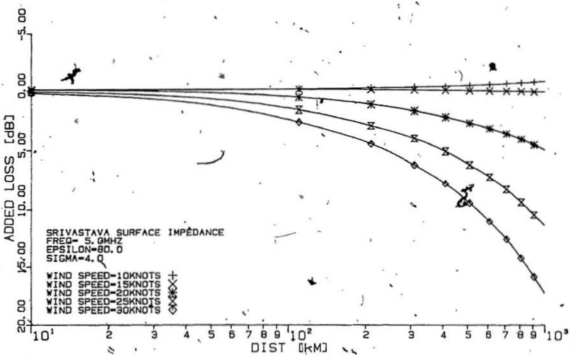


FIGURE 5.4 : Added Loss Versus Distance for Propagation Over a Rough Ocean Surface at 5.0 MHz. (Srivastava Surface Impedance)

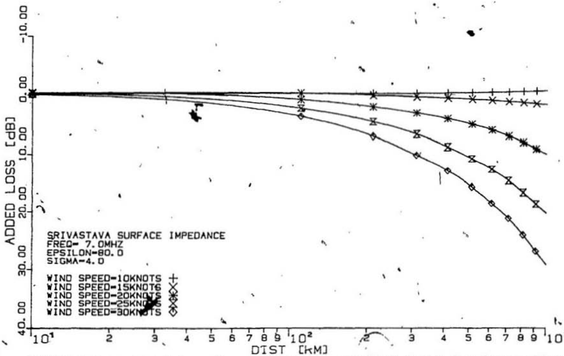


FIGURE 5.5 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 7.0 MHz. (Srivastava Surface Impedance)

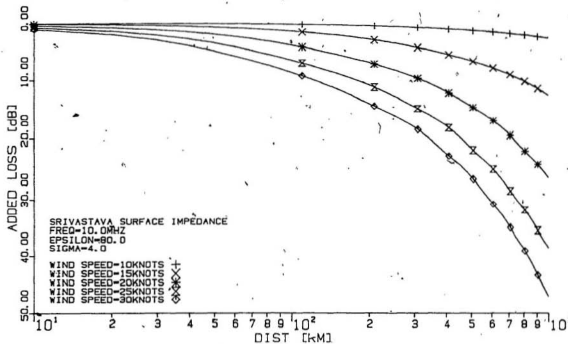


FIGURE 5.6 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 10.0 MHz. (Srivastava Surface Impedance)

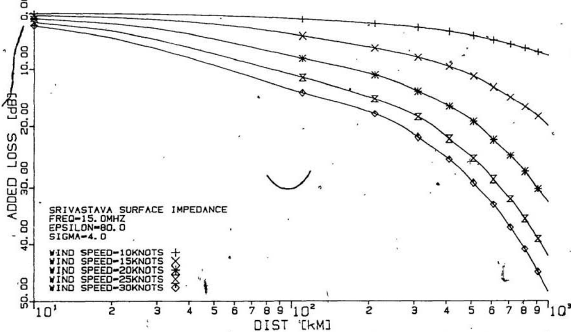


FIGURE 5.7 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 15.0 MHz. (Srivastava Surface Impedance)

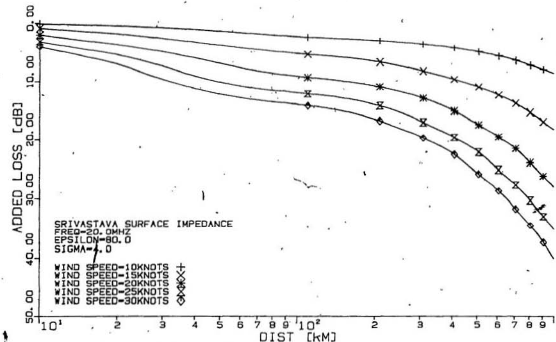


FIGURE 5.8 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 20.0 MHz. (Srivastava Surface Impedance)

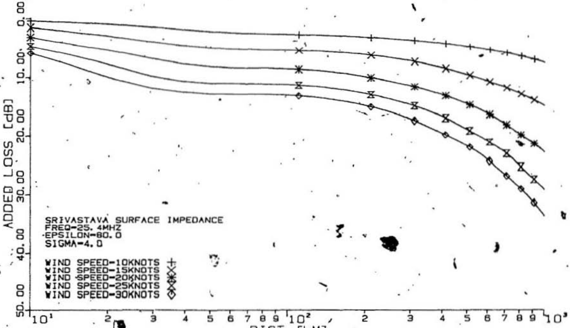


FIGURE 5.9 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 25.4 MHz. (Srivastava Surface Impedance)

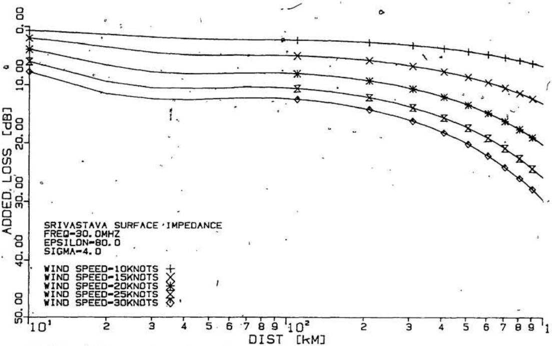


FIGURE 5.10 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 30.0 MHz. (Srivastava Surface Impedance)

may actually be less than the losses for a perfectly smooth spherical surface. This effect is particularly noticeable at the lower frequencies in the HF band and with low sea states. Barrick [1970] has suggested, that this is due to the inductive contribution to the modified surface impedance, for slightly rough surfaces, and corresponds to the so-called 'Trapped Surface Wave' effect discussed by Wait [1970]. We also note that for high sea states and all frequencies the additional losses increase with distance, as may be expected.

In Chapter 3 it was noted that both Barrick's and Srivastava's surface impedance expressions achieved similar numerical results for the modified surface impedance. Thus, as expected, the calculations for the additional transmission loss due to ocean surface roughness yielded comparable results using either modified surface impedance expression. As a comparison, a set of figures similar to 5.2 through 5.10 have been produced for the additional loss due to ocean surface roughness by using the expressions for the Barrick surface impedance. In these figures, labelled 5.11 through 5.19, the additional loss due to ocean surface roughness has been calculated and plotted versus distance using Barrick's modified surface impedance expression for the same set of frequencies and wind speeds as used to plot results using Srivastava's modified surface impedance expression. Obviously, the added loss results from the two models are very similar. There are no significant differences between the results achieved using either modified surface impedance expression at any of the wind speeds, frequencies or distances which were computed.

Barrick [1970] has also presented a number of similar graphs of the additional transmission loss due to surface roughness. The results developed herein

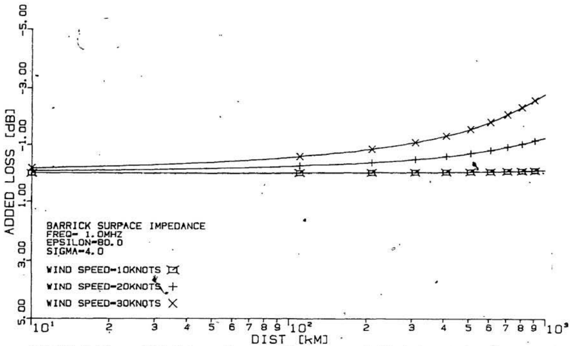


FIGURE 5.11 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 1.0 MHz. (Barrick Surface Impedance)

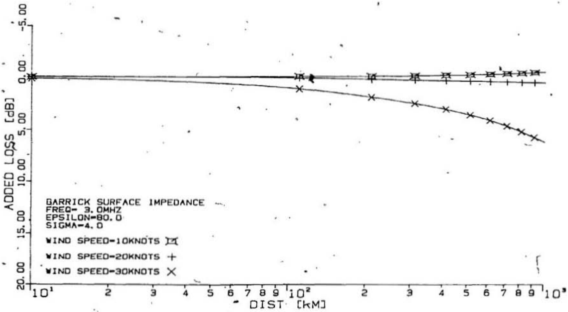


FIGURE 5.12 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 3.0 MHz. (Barrick Surface Impedance)

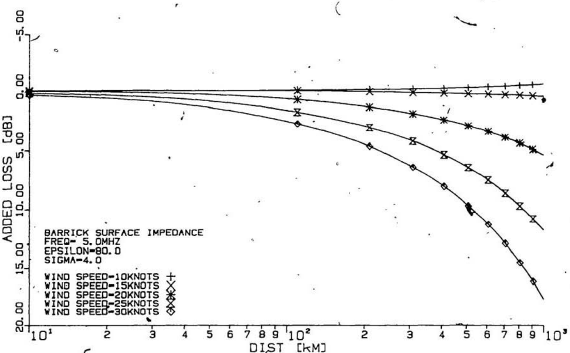


FIGURE 5.13 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 5.0 MHz. (Barrick Surface Impedance)

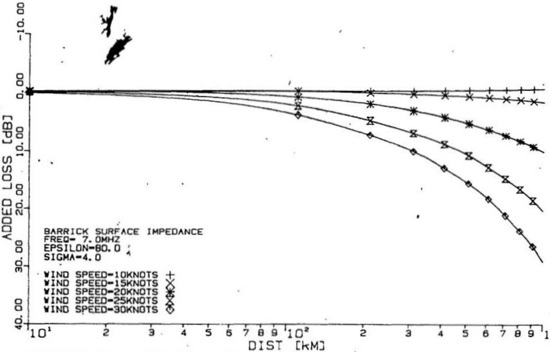


FIGURE 5.14 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 7.0 MHz, (Barrick Surface Impedance)

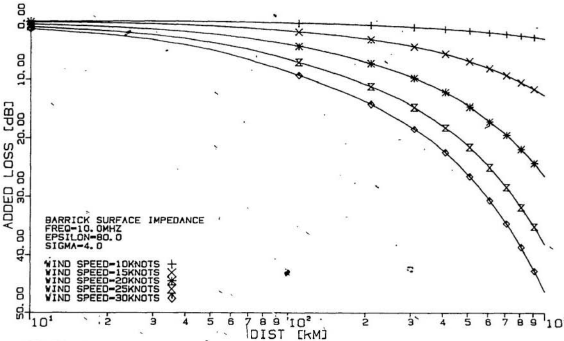


FIGURE 5.15 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 10.0 MHz. (Barrick Surface Impedance)

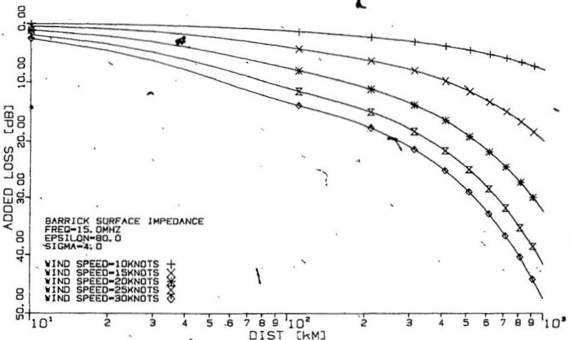


FIGURE 5.16 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 15.0 MHz. (Barrick Surface Impedance)

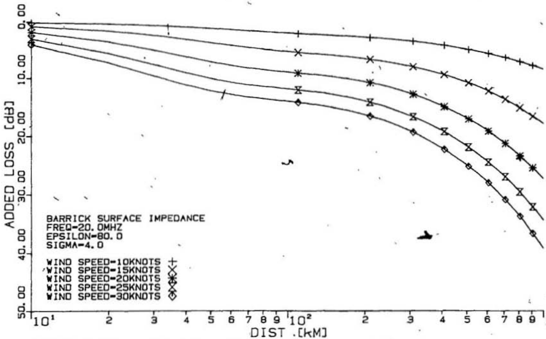


FIGURE 5.17 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 20.0 MHz. (Barrick Surface Impedance)

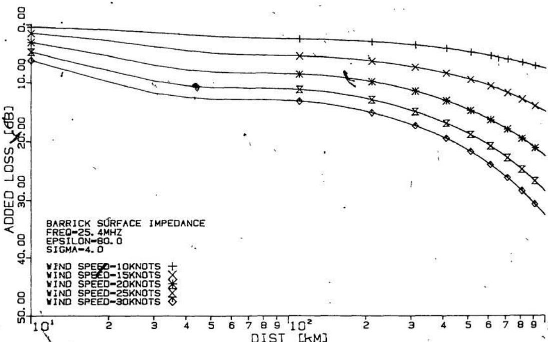


FIGURE 5.18 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 25.4 MHz. (Barrick Surface Impedance)

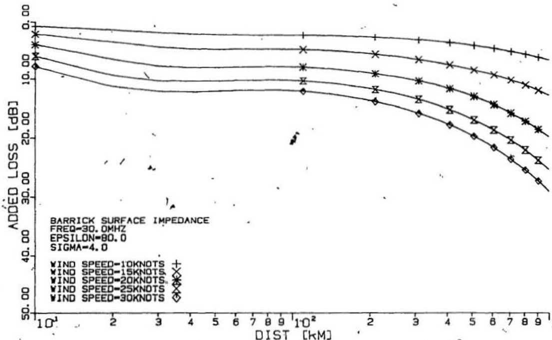


FIGURE 5.19 : Added Loss Versus Distance and Wind Speed for Propagation Over a Rough Ocean Surface at 30.0 MHz. (Barrick Surface Impedance)

for the additional transmission losses, using Barrick's modified surface impedance model, compare favourably with those produced by Barrick [1970]. For frequencies below six Megahertz some negligible differences were observed in the additional transmission loss calculations. These may be attributed to the irregularities observed in the implementation of Barrick's modified surface impedance calculation developed in Chapter 3 of this thesis. For frequencies above six Megahertz a remarkable degree of agreement between the both implementations of the additional transmission losses, using Barrick's modified surface impedance, has been achieved. This agreement may be anticipated, since both implementations of Barrick's modified surface impedance yield very similar results above six Megahertz.

The calculation of the additional transmission losses using Srivastava's [1984] modified surface impedance may be compared with the losses computed using Barrick's [1970] modified surface impedance, using the numerical implementations developed in Chapter 3. For frequencies between 5.0 and 25.4 Megahertz, the added loss calculations using Srivastava's modified surface impedance models are within 1.0 dB of the added losses computed using Barrick's modified surface impedance, for all frequencies and wind speeds. At the 30.0 Megahertz frequency the added loss using Srivastava's model is up to 3.0dB greater than that calculated using Barrick's model for the higher sea states at distances less than 100.0 km. The differences decrease with increasing distance, becoming negligible at distances greater than 200.0 km.

For the 1.0 and 3.0 Megahertz frequencies several differences between added loss results produced using the two modified surface impedance models have also been observed. The additional loss is 2.0 dB greater for calculations using Barrick's model for a distance of 1000.0 kilometres and the 30.0 knot wind speed. The observed differences decrease with decreasing frequency, such that at 100.0 kilometres the difference is negligible. However, there are numerical problems as outlined in Chapter 3 with the implementation of the modified surface impedance models for frequencies below 6.0 Megahertz. The results for added losses achieved using Barrick's and Srivastava's surface impedance models compare favourably at all frequencies and wind speeds, for the implementations developed in this thesis. Thus, discrepancies with the results presented by Barrick [1970] may be attributed to the numerical implementation of the surface impedance models and not necessarily to the differences between the surface impedance models, for frequencies less than six Megahertz. For frequencies above 6.0 Megahertz the added loss calculations using the implementation of Barrick's model and Srivastava's model from this thesis, compare favourably with results presented by Barrick [1970].

CHAPTER 6

CONCLUSIONS

The propagation of radio waves is a vital component of many communications and remote sensing systems. The ability to predict the amount of electromagnetic energy observed at a distance from its source, is critical to the design, simulation and evaluation of such systems. In particular, the ocean environment presents unique challenges to the application of communications and remote sensing systems. In this thesis theories for ground wave electromagnetic propagation have been examined and applied to the prediction of radio wave propagation losses over the ocean surface. These predictions may be utilized in the analysis and simulation of marine based communications and remote sensing systems, operating in the MF (0.3-3.0 MHz.) and HF (3.0-30.0 MHz.) frequency bands. The numerical implementations developed herein, have already been applied to simulations of the ground wave Doppler radar return from iceberg targets at 25 MHz [Walsh, Dawe and Srivastava, 1985].

The principal objective of this work was to provide a numerical facility to calculate the power losses for electromagnetic waves propagating across the ocean surface. Either the planar earth model derived in Chapter 2, or the spherical earth propagation model presented in Chapter 4 is suitable for these calculations, each model possessing its respective limitations. The effects of the ocean waves on the propagation losses have been considered through a model for the surface impedance of a rough wind-driven sea. The investigation began with a derivation of the electric field propagating across a planar surface of arbitrary electrical properties. The surface impedance for a rough wind driven sea was also examined, and an appropriate numerical evaluation developed. In Chapter 4, the classical model for spherical earth propagation was presented, and a proposed numerical model for spherical earth propagation over a rough ocean surface described. Finally, some typical numerical results for the transmission losses, calculated by this model, and using frequencies in the HF frequency band were presented.

In Chapter 2 of this thesis, the classic result for the ground wave plane earth propagation model has been derived by an alternate method. The method is based on the analytical techniques proposed by Walsh [1980] for a general formulation of the rough surface scatter problem. By using a spatial decomposition of the electric field, expressions for the electric field over a surface of arbitrary electrical parameters for an arbitrary finite source have been derived, in the spatial Fourier transform domain. By assuming an elementary vertical dipole source an integral solution for the electric field is presented which is equivalent to the Sommerfeld [1909] solution. For a highly conducting surface this result reduces to that of Wait [1954, 1957].

The classic solution to the propagation of EM waves over a spherical earth was examined [Fock, 1945; Van der Pol and Bremmer, 1937, 1938, 1939]. This solution is in the form of a residue series of the poles of a contour integral, which was derived as a general solution to the spherical earth formulation by Watson [1919]. A computer model using this solution has been developed, using a previous implementation [Berry and Chrisman, 1966] as a benchmark. The model described in this thesis offers improvements in speed of operation, improvements in the implementation of the poles of the residue series, modern Fortran-77 source code as well as readability and compactness as compared to the Berry and Chrisman implementation. The program is also more suitable than the Berry and Chrisman version for implementation on modern high speed mini computers such as the Digital Equipment VAX.

In this thesis we have also examined and implemented analytical models for the modified surface impedance for a wind driven ocean. The modified surface impedance accounts for the additional effects of surface roughness on the propagation of radio waves over the ocean. The expressions are written in terms of the ocean wave height spectral density for the rough sea. Two different models for the modified surface impedance are implemented; Barrick's [Barrick, 1971] and Srivastava's [Srivastava, 1984]. Although the expressions for the two models are significantly different, the numerical results for each model are not significantly different. As well the numerical results for Barrick's model compare favourably with those previously presented [Barrick, 1971b], for frequencies in the six to thirty Megahertz range of frequencies. These results [Srivastava, 1984; Barrick, 1970] are included in the spherical earth model to evaluate the transmission losses for radio propagation in the ocean environment. Graphical plots of transmission losses for various frequencies and various sea conditions are presented. Barrick [1970] has also presented plots of the predicted transmission loss (added loss) for propagation across a rough ocean surface, which compare favourably to those presented herein, again in the six to thirty Megahertz range. This yields a degree of confidence in both the numerical implementation of the modified surface impedance and the numerical model for spherical earth propagation. However, for frequencies below six Megahertz, some discrepancies between results achieved with different implementations of the modified surface impedance are observed. The differences are probably due to numerical instabilities or due to the numerical integration.

Thus, we have examined results predicting the behaviour of the electric field in the presence of the earth's surface. Based on these models a suitable computer model which yields numerical predictions for radio wave transmission losses in the marine environment has been developed. The model accounts for the effects of diffraction around the spherical surface of the earth, an important consideration when the source antenna and observer are separated by distances of more than a few kilometres. As well we have utilized expressions for the modified surface impedance of a rough wind driven sea, which predicts the effects of ocean waves on the propagation losses.

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APPENDIX A

TWO-DIMENSIONAL SPATIAL FOURIER TRANSFORM OF THE GREEN'S FUNCTION K_{01}

We wish to determine the two dimensional spatial Fourier transform of K_{01}, the Green's function, defined as

$$K_{01} = \frac{\exp\left(-j\, k_0\, R_0\right)}{4\pi R_0} \quad , \tag{A.1}$$

where k_0 is the wave number of the fundamental, and

$$R_0 = \sqrt{x^2 + y^2 + z^2} \quad .$$

The spatial (x,y) Fourier transform of a function $f(x,y)$ is defined as

$$F\left(K_x, K_y\right) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} f(x,y) \exp\left(-jK_x\, x - jK_y\, y\right) dx\, dy \quad . \tag{A.2}$$

The inverse spatial Fourier transform is defined by

$$f(x,y) = \frac{1}{4\pi^2} \int_{K_x=-\infty}^{\infty} \int_{K_y=-\infty}^{\infty} F\left(K_x, K_y\right) \exp\left(jK_x\, x + jK_y\, y\right) dK_x\, dK_y \quad .$$

K_x and K_y are the x,y spatial wave numbers. The spatial Fourier transform of K_{01}, which is denoted by $\underline{K_{01}}(K_x, K_y, z)$ is given by

$$\underline{K_{01}}(K_x, K_y, z) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} \frac{\exp\left(-j\, k_0\, R_0\right)}{4\pi R_0} \exp\left(-jK_x\, x - jK_y\, y\right) dx\, dy \quad . \tag{A.3}$$

We now write the above integral, transformed into the cylindrical polar coordinate system. The result is ,

$$\underline{K_{01}}(\lambda,\phi,z) = \int_{\rho=0}^{\infty} \int_{\theta=0}^{2\pi} \frac{\rho \exp\left(-j\, k_0 \sqrt{\rho^2 + z^2}\right)}{4\pi\sqrt{\rho^2 + z^2}} \exp\left(-j\, \rho\, \lambda \cos(\theta - \phi)\right) d\theta\, d\rho \quad . \tag{A.4}$$

The following definitions apply to equation (A.4):

$$K_x = \lambda \cos\phi \quad ,$$

$$K_y = \lambda \sin\phi \ ,$$
$$x = \rho \cos\theta \ ,$$
$$y = \rho \sin\theta \ .$$

Now, by making the substitution $\theta' = \theta - \phi$ $\underline{K_{01}}$ may be written as

$$\underline{K_{01}} = \int_{\rho=0}^{\infty} \frac{\rho \exp\left(-j\, k_0 \sqrt{\rho^2 + z^2}\right)}{4\pi \sqrt{\rho^2 + z^2}} \int_{\theta'=-\phi}^{2\pi-\phi} \exp\left(-j\ \rho \lambda \cos(\theta')\right) d\theta'\, d\rho \ . \tag{A.5}$$

We first examine only the inner integral with respect to θ'. We now write the θ' integral as

$$\begin{aligned} I_1 &= \int_{\theta'=-\phi}^{2\pi-\phi} \exp\left(-j\ \rho \lambda \cos(\theta')\right) d\theta' \\ &= \int_{\theta'=-\pi}^{\pi} \exp\left(-j\ \rho \lambda \cos\theta'\right) d\theta' \\ &= 2\int_{\theta'=0}^{\pi} \exp\left(-j\ \rho \lambda \cos\theta'\right) d\theta' \\ &= 2\pi J_0(\rho \lambda) \ . \end{aligned} \tag{A.6}$$

The function $J_0(\rho \lambda)$ is the Bessel function of order 0. Now, by using the above result, $\underline{K_{01}}$ may be written as

$$\underline{K_{01}} = \int_{\rho=0}^{\infty} \frac{\rho \exp\left(-j\, k_0 \sqrt{\rho^2 + z^2}\right)}{4\pi \sqrt{\rho^2 + z^2}} J_0(\rho \lambda)\, d\rho \ .$$

The substitutions $t = \sqrt{\rho^2 + z^2}$, and

$$dt = \frac{2\rho\, d\rho}{\sqrt{\rho^2 + z^2}} \ ,$$

are made in the above expression for $\underline{K_{01}}$, yielding

$$\underline{K_{01}} = \frac{1}{2} \int_{t=|z|}^{\infty} \exp\left(-j\, k_0 t\right) J_0\left(\lambda \sqrt{t^2 - z^2}\right) dt \ . \tag{A.8}$$

This expression may be separated into its two components: one for the real part, and one for the imaginary part. The separation is

$$\underline{K_{01}} = \frac{1}{2}\left[I_2 + j\, I_3\right] \tag{A.9}$$

The two integrals I_2 and I_3 are given by

$$\begin{aligned} I_2 &= \int_{t=|z|}^{\infty} \cos(-k_0 t)\, J_0\left(\lambda \sqrt{t^2 - z^2}\right) dt \\ &= \int_{t=|z|}^{\infty} \cos(k_0 t)\, J_0\left(\lambda \sqrt{t^2 - z^2}\right) dt \quad , \end{aligned} \tag{A.10}$$

$$\begin{aligned} I_3 &= \int_{t=|z|}^{\infty} \sin(-k_0 t)\, J_0\left(\lambda \sqrt{t^2 - z^2}\right) dt \\ &= -\int_{t=|z|}^{\infty} \sin(k_0 t)\, J_0\left(\lambda \sqrt{t^2 - z^2}\right) dt \quad . \end{aligned} \tag{A.11}$$

The above integrals, I_2 and I_3 may be determined from a table of integrals, such as Gradshteyn and Riyzhik [1965]. The results are as follows:

$$I_2 = \begin{cases} \dfrac{\exp(-|z|\, U)}{U} & ; \quad 0 < k_0 < \lambda \\ \dfrac{-\sin\left(|z| \sqrt{k_0^2 - \lambda^2}\right)}{\sqrt{k_0^2 - \lambda^2}} & ; \quad 0 < \lambda < k_0 \end{cases} \tag{A.12}$$

$$I_3 = \begin{cases} 0 & ; \quad 0 < k_0 < \lambda \\ \dfrac{-\cos\left(|z| \sqrt{k_0^2 - \lambda^2}\right)}{\sqrt{k_0^2 - \lambda^2}} & ; \quad 0 < \lambda < k_0 \end{cases} \tag{A.13}$$

By combining the solutions for the integrals I_1 and I_2, we may arrive at a solution for the spatial Fourier transform of the function K_{01}. Neglecting the details, we may write $\underline{K_{01}}$ as

$$\underline{K_{01}} = \begin{cases} \dfrac{\exp(-|z|\, U)}{2\,U} & ; \quad 0 < k_0 < \lambda \\ \dfrac{\exp(|z|\, U)}{2\,U} & ; \quad 0 < \lambda < k_0 \end{cases} \tag{A.14}$$

By analogy to the above derivation, we may also use these results for the

Green's Function K_{02}. We may immediately write the spatial Fourier transform of K_{02} as

$$\underline{K_{02}} = \begin{cases} \dfrac{\exp(-\ |z|\ U_E)}{2\ U_E} & ;\ 0 < k_0 < \lambda \\ \dfrac{\exp(\ |z|\ U_E)}{2\ U_E} & ;\ 0 < \lambda < k_0 \end{cases} \tag{A.15}$$

where $\underline{K_{02}}$ denotes the spatial Fourier transform of K_{02}. In addition, we have the following definitions:

$$K_{02} = \frac{\exp(-\ j\ \gamma R)}{4\pi R} ,$$

$$\gamma^2 = k_0^2\ n_0^2 ,$$

$$U_E = \sqrt{\lambda^2 - \gamma^2} ,$$

$$\lambda^2 = K_x^2 + K_y^2 .$$

Equations (A.13) and (A.14) are the results of taking the spatial Fourier transforms of the functions K_{01} and K_{02} and are used frequently throughout the text of this thesis.

APPENDIX B
Rough Spherical Earth Program Listing

```
PROGRAM GWAVE
*************************************************************
*************************************************************
********CALCULATION OF THE GROUND WAVE USING THE
********FOCK RESIDUE SERIES AND THE
********BREMMER SADDLE POINT APPROXIMATION
********JANUARY 15,1987   NEW CODE FOR L AND M SERIES
********DECEMBER 29,1986  CHANGED TO SIMPSON RULE INTEGRATION
*************************FOR MODIFIED SURFACE IMPEDANCE
********SEPTEMBER 11,1986 REVAMPED THE DATA INPUT SECTION
********AUGUST 17,1986    REVAMPED ESTIMATION OF
*************************POLES OF RES SER
********AUGUST 15,1986    ADDED OUTPUT DATA FILE SELECTION
********AUGUST 10,1986    ADDED TZER AND TINF(REVISED TAU)
********JANUARY 3,1986    REVISED AIRY FUNCTIONS
********JUNE 13,1985      INCLUDED MODIFIED SURFACE IMPEDANCE
*************************************************************
*************************************************************
!     DECLARE  VARIABLES
      COMPLEX C,Q,D,T(500),AI(500),DA1(500),ZW,TS,E,F3(500)
      COMPLEX CI,RI,K3,W,ZIMP,BT(500),BR(500),MZ,LZ
      COMPLEX SQV,S1NQ,S2NQ,S1DQ,S2DQ,SR,SLIUM,SLIU,F2
        REAL AMP(500),PHASE(500),DIST(500)
        REAL PFREQ,FREQ,EPS,SIG,P,DMIN,DELTA,DMAX,PI,OMEGA
```

```
      REAL K,A,V,Z,AK,OLD_FIELD,NEW_FIELD
      REAL THETA,X,FAC,WVEL,PWVEL,ALPHA,PALPHA
      INTEGER I_F3_CALC,J_CNT,J_START,J_MAX,I_D_CNT
      CHARACTER*3 YON
      CHARACTER*5 YON2
      BYTE FILOUT(20)
      COMMON/SNIT/WNUM,ALPHA,GRAV,WVEL,DELT
      COMMON /RADPAR/ PFREQ,EPS,SIG,HREC,HTRAN,P
      COMMON /RANG/ QDMIN,DELTA,DMAX
      COMMON /INOUT/KCHR,FILOUT,YON,YON2
      COMMON /SURF/PWVEL,PALPHA,JTYPE
      CALL INPAR   !GET INPUT PARAMETERS
                   !FROM EITHER SCREEN OR GW.PAR
**********************************************
********INITIALIZE AND CALCULATE CONSTANTS***
************NOT A FUNCTION OF DISTANCE******
**********************************************
      I_F3_CALC=0
      J_START=1
      J_MAX=200
      C=CMPLX(0.0,1.0)
      CI=CMPLX(0.0,-1.0)
      RI=CMPLX(1.0,0.0)
      RJ=CMPLX(-1.0,0.0)
```

```
PI=4.0*ATAN(1.0)
FREQ=PFREQ*1.0E06!FREQUENCY IN HZ
WVEL=PWVEL*0.5144458      !WIND VEL IN M/S
ALPHA=PALPHA*PI/180.0     !ANGLE OF WIND IN RADS
DMIN=QDMIN                !STARTING DISTANCE
OMEGA=2.0*PI*FREQ         !OMEGA=2PI(FREQ)
K=OMEGA/2.997925E08       !WAVE NUMBER
A=6.36739E06*P            !EFFECTIVE EARTH RADUIS
V=((K*A)/2.0)**0.3333333  !V
AK=11.96*SQRT(K/(A**3.0))*V**2.0  !K (CONSTANT)
Z=0.5/(V**2.0)                    !Z
K3=ZIMP(EPS,SIG,WVEL,ALPHA,FREQ)      !DELTA MOD(SURF IMPED)
WRITE(6,*)'K3=',K3
Q=CI*V*K3*CSQRT(RI-(K3**2.0))         !q sub V
D=RJ/(CMPLX(1.2599211,0.0)*CONJG(Q))
I_D_CNT=0        !INITIALIZE COUNTER FOR DIST LOOP
J_START=1        !INITIALIZE COUNTER FOR t sub s
B=A+HTRAN
R=A+HREC
AT=(A**2.0/(B*R))*((B*R)/(A**2.0))**.1666666666    !T (CONST)
YT=K*HTRAN/V               !Y sub T
YR=K*HREC/V                !Y sub R
*      **********************************
*      BEGINNING OF DISTANCE LOOP
```

```
*       **********************************
10   E=CMPLX(0.0,0.0)             !INITIALIZE E
     THETA=DMIN*1000./A           !ANGLE FORMED BY TRANSMITTER
                                  !CENTRE OF EARTH AND RECEIVER
     X=V*THETA             !X - A CONSTANT
     FAC=((4.0*PI*AK)/(SQRT(SIN(THETA))))*AT        !CONST FOR RES
                                                    !SERIES
     IF(HREC.LE.0.0.AND.HTRAN.LE.0.0)THEN
          HHT=1.0
          HHR=1.0
          GO TO 105
     ELSE
          GO TO 9
     ENDIF
                  !IF HREC=HTRAN=0.0 THEN USE RESIDUE SERIES
                  !ELSE TEST THE SADDLE POINT METHOD
*******************************************************
*    CHECK VALIDITY OF SADDLE POINT APPROXIMATION
*******************************************************
9    IF((2.0*(X**2.0)*(YR+YT)).GT.((X**4.0)+(YR-YT)**2.0))THEN
          GO TO 105
          !IF ALPHA IS COMPLEX THEN SKIP SADDLE POINT
     ELSE
          SALPHA=SQRT((X**4.0)+((YR-YT)**2.0)-
```

```
1          (2.0*(X**2.0)*(YR+YT)))/(2.*X)        !ELSE COMPUTE ALPHA
      ENDIF
!   IF SADDLE POINT VALID THEN DO IT
!     OTHERWISE DO RESIDUE SERIES
      IF((SALPHA-2.0).LT.0.0)THEN
            GO TO 105
      ELSE
**********************************************
************SADDLE POINT APPROXIMATION*********
**********************************************
      WRITE(6,*)'SADDLE POINT'
        SS=2.0/3.0*(SALPHA)**3.0          ! s
        SB=(YT+SALPHA**2.0)**1.5
        SU=2.0/3.0*SB                     ! u
        SPHI=4.0/3.0*(SB-SALPHA**3.0)  ! phi
        OMEGAT=2.0/3.0*((YR+SALPHA**2.0)**1.5-
1              (YT+SALPHA**2.0)**1.5)-X*SALPHA**2.0
!                 omega(t sub0)
        SQV=CI*V*K3*CSQRT(RI-(K3*(RI-SALPHA**2.0*Z))**2.0)
                  ! q sub V
        S1NQ=MZ(CMPLX(0.0,SS))         !M(is)
        S2NQ=LZ(CMPLX(0.0,SS))         !L(is)
        S1DQ=MZ(CMPLX(0.0,-SS))        !M(-is)
        S2DQ=LZ(CMPLX(0.0,-SS))        !L(-is)
```

```
      SR=(SALPHA*(S1NQ/S2NQ)+CI*SQV)/(SALPHA*(S1DQ/S2DQ)+C*SQV)
                    !R
      SLIUM=LZ(CMPLX(0.0,-SU))              !L(-iu)
      SLIU=LZ(CMPLX(0.0,SU))            !L(iu)
      F2=(1.0+(CEXP(CMPLX(0.,-SPHI))*SR*(SLIUM/SLIU)*
     1 (S2NQ/S2DQ)))*SLIU*((1.0-SALPHA**2.0*Z)**2.5)
                    !F sub 2 (q sub V,t sub 0)
      E=-2.0*C*AK*AT*SQRT(PI/X)/(SQRT(SIN(THETA)))*F2*
     1     (CEXP(CMPLX(0.0,-OMEGAT)))*
     2     CEXP(CMPLX(0.0,-K*DMIN*1000.))
                    !CALCULATE E FIELD
      ENDIF
      GO TO 265              !go to end of dist loop
!********************************************
!********RESIDUE SERIES APPROXIMATION***********
!********************************************
105   CONTINUE
      IF(I_F3_CALC.LT.1)THEN
           GO TO 115   ! IF F3(J) HAS BEEN CALCULATED
      ELSE             ! FOR THIS SET OF INPUT
           GO TO 220   ! CONSTANTS USE STORED VALUES
      ENDIF
115   DO 200 J_CNT=J_START,J_MAX              !LOOP CALCULATES F3(J)
      S=J_CNT-1                  !s  {t sub s}
```

```
      JJJ=1                   !NUM OF ITERATIONS ON T SUB j
      CALL TAU(S,Q,T(J_CNT),A1(J_CNT)) !CALCULATE POLES
      I_F3_CALC=1                  !SET I_F3_CALC
*           *********************
*           HEIGHT GAIN FUNCTIONS
*           *********************
      IF(HTRAN.GT.0.0)THEN
            CALL AIRY(1,T(J_CNT)-YT,BT(J_CNT),ZW)
            HHT=BT(J_CNT)/A1(J_CNT)
      ELSE
            HHT=1.0
      ENDIF
      IF(HREC.GT.0.0)THEN
            CALL AIRY(1,T(J_CNT)-YR,BR(J_CNT),ZW)
            HHR=BR(J_CNT)/A1(J_CNT)
      ELSE
            HHR=1.0
      ENDIF
*           ******************
*           CALCULATE E FIELD
*           ******************
      F3(J_CNT)=((((RI+Z*T(J_CNT))**2.5)/(T(J_CNT)-(Q**2.0))))*HHT*HHR
                  ! F3 (Q SUB V, T SUB S)
      E=E+F3(J_CNT)*CEXP(CI*X*T(J_CNT))*(4.0*PI*AK*AT)*CEXP(CMPLX(0.0,-1.0)*
```

```
1     (K*DMIN*1000.-PI/4.0))/(SQRT(SIN(THETA)))
      OLD_FIELD=NEW_FIELD
      NEW_FIELD=CABS(E)
      IF(J_CNT-1)200,200,190
190   IF(ABS((NEW_FIELD-OLD_FIELD)/NEW_FIELD)-0.0005)195,195,200
            !TEST CONV OF E FIELD
195   ICNT=J_CNT
      GO TO 210
200   CONTINUE
      ICNT=J_MAX !CALCULATION LIMITED TO 200 POLES
      GO TO 265
210   IF(I_F3_CALC)265,265,220
220   NEW_FIELD=0.0        !CALCULATE E FIELD FROM STORED
                  !VALUES OF F sub 3(q sub V,t sub s)
      E=CMPLX(0.0,0.0)
      DO 250 J_CNT=1,ICNT
      E=E+F3(J_CNT)*CEXP(CI*X*T(J_CNT))*(4.0*PI*AK*AT)*CEXP(CMPLX(0.0,-1.0)*
1     (K*DMIN*1000.-PI/4.0))/SQRT(SIN(THETA))
      OLD_FIELD=NEW_FIELD              !OLD_FIELD IS THE PREVIOUS
                  !VALUE OF E
      NEW_FIELD=CABS(E)         !NEW_FIELD IS THE PRESENT VALUE OF E
      IF(J_CNT-1)250,250,235
235   IF(ABS((NEW_FIELD-OLD_FIELD)/NEW_FIELD)-0.0005)265,265,250
                  !E FIELD CONV TEST FOR STORED F3
```

```
250  CONTINUE
     J_START=ICNT+1
     GO TO 115    !CALCULATE ADDITIONAL POLES (IF REQUIRED)
265  I_D_CNT=I_D_CNT+1          !INCREMENT DISTANCE LOOP
     DIST(I_D_CNT)=DMIN
     W=(CEXP(CMPLX(0.0,-K*DIST(I_D_CNT)*1000.))*
   1   E*DIST(I_D_CNT))/(4.0*PI*FREQ*1.0E-10)
                !ATTENUATION FUNCTION
*    ********************************************
*    note that this W is defined such that
*    E = ( C SUB d / 2 PI )  W  EXP ( J K R ) / R
*    THIS MEANS THAT W IS SCALED FOR ALL HEIGHT GAIN
*    FACTORS AND CONSTANTS
*    ********************************************
     IF(YON2.EQ.'F'.OR.YON2.EQ.'FIELD'.OR.
   1 YON2.EQ.'f'.OR.YON2.EQ.'field')THEN
       AMP(I_D_CNT)=CABS(E)          !COMPUTE E FIELD
       PHASE(I_D_CNT)=ATAN2(AIMAG(E),REAL(E))
     ELSE
       AMP(I_D_CNT)=CABS(W)          !COMPUTE W FUNCTION
       PHASE(I_D_CNT)=ATAN2(AIMAG(W),REAL(W))
     ENDIF
     PHASE(I_D_CNT)=MOD(PHASE(I_D_CNT),6.2831853)
     DMIN=DMIN+DELTA
```

```
      IF(DMIN.GT.DMAX)THEN
            CALL OUTPAR(DIST,AMP,PHASE,I_D_CNT) !OUTPUT RESULTS
                                                !AND STORE INPUTS
                                                !IN GW.PAR
      ELSE
            GO TO 10                    !SELECT ANOTHER DISTANCE
      ENDIF
      STOP
      END
C
C
C
      SUBROUTINE INPAR
*********************************************
********GET THE INPUT PARAMETERS FROM*********
********EITHER THE SCREEN OR FROM GW.PAR******
********LAST MODIFIED : SEPT 11,1986
****************** BY : BARRY J DAWE**********
*********************************************
      REAL PFREQ,FREQ,EPS,SIG,HREC,HTRAN,PWVEL,WVEL
      REAL DELTA,DMAX,PALPHA,ALPHA,P,QDMIN,DMIN
      CHARACTER*3 YON
      CHARACTER*5 YON2
      CHARACTER*20 CTYPE
```

```
      BYTE FILOUT(20)
      COMMON /RADPAR/ PFREQ,EPS,SIG,HREC,HTRAN,P
      COMMON /RANG/ QDMIN,DELTA,DMAX
      COMMON /SURF/ PWVEL,PALPHA,JTYPE
      COMMON /INOUT/ KCHR,FILOUT,YON,YON2
*         ********************
*         INPUT  CONSTANTS
*         ********************
      TYPE 800
      ACCEPT 852,NCHR,YON        !STORED PARAMETERS OR NEW
      TYPE 801
      ACCEPT 852,JCHR,YON2       !FIELD OR ATTNENUATION
      TYPE 802
      ACCEPT 851,KCHR,(FILOUT(III),III=1,KCHR)
      IF(NCHR.EQ.0.OR.YON.EQ.'N'.OR.YON.EQ.'n'.OR.
     1 YON.EQ.'NO'.OR.YON.EQ.'no')THEN
!         READ INPUT PARAMETERS FROM TERMINAL
900     TYPE 803                 !GET THE TRANSMIT FREQ
        ACCEPT 850,NCHR,PFREQ
        IF(NCHR.EQ.0)GO TO 900   !ASK AGAIN
        TYPE 804
        ACCEPT 850,NCHR,EPS             !GET PERMITTIVITY
        IF(NCHR.EQ.0)EPS=80.0    !DEFAULT TO 80.0 TYPICAL FOR OCEAN
        TYPE 805
```

```
ACCEPT 850,NCHR,SIG             !GET CONDUCTIVITY
IF(NCHR.EQ.0)SIG=4.0            !DEFAULT TO 4.0 MHOS/M FOR OCEAN
TYPE 806
ACCEPT 850,NCHR,HREC            !RECEIVE ANTENNA HEIGHT
IF(NCHR.EQ.0)HREC=0.0  !DEFAULT TO 0.0 M
TYPE 807
ACCEPT 850,NCHR,HTRAN !TRANSMIT ANTENNA HEIGHT
IF(NCHR.EQ.0)HTRAN=0.0 !DEFAULT TO 0.0 M
TYPE 808
ACCEPT 850,NCHR,PWVEL !GET WIND VELOCITY IN NAUT MI/HR
                      !FOR MOD SURF IMPED
IF(NCHR.EQ.0)PWVEL=0.0 !DEFAULT TO 0.0
TYPE 809
ACCEPT 850,NCHR,PALPHA          !GET WIND ANGLE W.R.T. TRANSMIT
                      !DIRECTION IN DEGREES
IF(NCHR.EQ.0)PALPHA=0.0         !DEFAULT TO 0.0 DEG
TYPE 810
ACCEPT 850,NCHR,P               !GET EFFECTIVE EARTH RADIUS FACTOR
IF(NCHR.EQ.0)P=4.0/3.0          !DEFAULT TO 1.3 FOR E.R.F.
TYPE 811
ACCEPT 850,NCHR,QDMIN !GET INITIAL DISTANCE
IF(NCHR.EQ.0)QDMIN=10.0!DEFAULT TO 10.0 km INITIAL
TYPE 812
ACCEPT 850,NCHR,DELTA !GET INCREMENTAL DISTANCE
```

```
      IF(NCHR.EQ.0)DELTA=10.0!DEFAULT TO 10.0 km INCREMENT
      TYPE 813
      ACCEPT 850,NCHR,DMAX           !GET MAXIMUM DISTANCE
      IF(NCHR.EQ.0)DMAX=100.0        !DEFAULT TO 100.0 km MAXIMUM
901   TYPE 814
      ACCEPT 851,ICHR,CTYPE
      IF(ICHR.EQ.0)GO TO 901    !ASK AGAIN
      IF(CTYPE.EQ.'BARRICK'.OR.CTYPE.EQ.'barrick'.OR.
   1  CTYPE.EQ.'B'.OR.CTYPE.EQ.'b')THEN
          JTYPE=1               !SELECT BARRICK SURF. IMPED.
      ELSE
          JTYPE=2               !DEFAULT TO SRIVASTAVA SURF. IMPED.
      ENDIF

    ELSE
!         READ INPUT PARAMETERS FROM STORED VALUES
      OPEN(UNIT=1,NAME='GW.PAR',TYPE='OLD',ACCESS='SEQUENTIAL')
      READ(1,*)PFREQ            !TRANSMIT FREQ
      READ(1,*)EPS                    !EPSILON
      READ(1,*)SIG                    !SIGMA
      READ(1,*)HREC             !RECEIVE ANTENNA HEIGHT
      READ(1,*)HTRAN            !TRANSMIT ANTENNA HEIGHT
      READ(1,*)PWVEL            !WIND SPEED
      READ(1,*)PALPHA           !WIND DIRECTION
      READ(1,*)P                !E.R.F.
```

```
      READ(1,*)QDMIN        !MINIMUM DISTANCE
      READ(1,*)DELTA        !INCREMENTAL DISTANCE
      READ(1,*)DMAX         !MAXIMUM DISTANCE
      READ(1,*)JTYPE        !SURF. IMPED. TYPE
      CLOSE(UNIT=1,STATUS='KEEP')
     ENDIF
800  FORMAT('$DO YOU WANT THE STORED PARAMETERS (IN GW.PAR)[Y/N] > ')
801  FORMAT('$DO YOU WANT FIELD STRENGTH [F] OR ATTENUATION [A]  > ')
802  FORMAT('$TYPE THE OUTPUT FILE NAME [DEF:SCREEN OUTPUT ONLY] > ')
803  FORMAT('$TYPE THE TRANSMIT FREQUENCY [MHz]            > ')
804  FORMAT('$TYPE THE GROUND PERMITTIVITY [DEF=80.0]          > ')
805  FORMAT('$TYPE THE GROUND CONDUCTIVITY [DEF=4.0]           > ')
806  FORMAT('$TYPE THE RECEIVE ANTENNA HEIGHT [M] [DEF=0.0]       > ')
807  FORMAT('$TYPE THE TRANSMIT ANTENNA HEIGHT [M] [DEF=0.0]       > ')
808  FORMAT('$TYPE THE WIND VELOCITY [NAUT MI/HR] [DEF=0.0]       > ')
809  FORMAT('$TYPE THE WIND VELOCITY ANGLE [DEG] [DEF=0.0]       > ')
810  FORMAT('$TYPE THE EARTH RADIUS FACTOR [DEF=1.3]          > ')
811  FORMAT('$TYPE THE START DISTANCE [DEF=10.0 km]          > ')
812  FORMAT('$TYPE THE INCREMENT DISTANCE [DEF=10.0km]         > ')
813  FORMAT('$TYPE THE FINAL DISTANCE [DEF=100.0km]          > ')
814  FORMAT('$TYPE BARRICK OR SRIVASTAVA FOR SURF. IMPED. TYPE  > ')

850  FORMAT(Q,E15.7)
851  FORMAT(Q,20A1)
852  FORMAT(Q,3A1)
```

```
      RETURN
      END
C
C
      SUBROUTINE OUTPAR(D,A,PHS,MM)
****************************************************
********WRITE THE DATA TO SCREEN AND/OR DATA FILE******
********AND STORE THE INPUT PARAMETERS IN GW.PAR*******
********LAST MODIFIED : AUGUST 15, 1986****************
***************** BY : BARRY J. DAWE*****************
****************************************************
      DIMENSION D(MM),A(MM),PHS(MM)
      INTEGER JTYPE,KCHR
      REAL PFREQ,EPS,SIG,HTRAN,HREC,P,QDMIN,DELTA
      REAL DMAX,PWVEL,PALPHA
      CHARACTER*3 YON
      CHARACTER*5 YON2
      BYTE FILOUT(20)
      COMMON /RADPAR/ PFREQ,EPS,SIG,HREC,HTRAN,P
      COMMON /RANG/ QDMIN,DELTA,DMAX
      COMMON /INOUT/KCHR,FILOUT,YON,YON2
      COMMON /SURF/PWVEL,PALPHA,JTYPE

      WRITE(6,1)            !TITLE BLOCK ON SCREEN
      IF(YON2.EQ.'F'.OR.YON2.EQ.'FIELD'.OR.
```

```
    1  YON2.EQ.'f'.OR.YON2.EQ.'field')THEN
            WRITE(6,1000)PFREQ,EPS,SIG,PWVEL,PALPHA,HREC,HTRAN
            WRITE(6,1001)
       ELSE
            WRITE(6,1004)PFREQ,EPS,SIG,PWVEL,PALPHA,HREC,HTRAN
            WRITE(6,1005)
       ENDIF
       DO 330 NN=1,MM              !OUTPUT RESULTS TO SCREEN
  330  WRITE(6,1002)D(NN),A(NN),PHS(NN)
       WRITE(6,1003)
       WRITE(6,1)
       WRITE(6,1)
       IF(KCHR.GT.0)THEN  !RESULTS TO DATA FILE (IF REQUESTED)
            OPEN(UNIT=10,FILE=FILOUT,TYPE='NEW',ACCESS='SEQUENTIAL')
            DO 331 NN=1,MM
  331         WRITE(10,1006)D(NN),A(NN),PHS(NN)
            CLOSE(UNIT=10,STATUS='KEEP')
       ENDIF
***********************************************
***********STORE INPUT PARAMETERS IN GW.PAR FILE
***********************************************
         OPEN(UNIT=1,NAME='GW.PAR',TYPE='NEW',ACCESS='SEQUENTIAL')
         WRITE(1,*)PFREQ,' !TRANSMIT FREQ'
         WRITE(1,*)EPS,'            !EPSILON'
```

```
      WRITE(1,*)SIG,'              !SIGMA'
      WRITE(1,*)HREC,'  !RECEIVE ANTENNA HEIGHT'
      WRITE(1,*)HTRAN,'!TRANSMIT ANTENNA HEIGHT'
      WRITE(1,*)PWVEL,'!WIND SPEED'
      WRITE(1,*)PALPHA,'         !WIND DIRECTION'
      WRITE(1,*)P,'              !E.R.F.'
      WRITE(1,*)QDMIN,' !MINIMUM DISTANCE'
      WRITE(1,*)DELTA,' !INCREMENTAL DISTANCE'
      WRITE(1,*)DMAX,'  !MAXIMUM DISTANCE'
      WRITE(1,*)JTYPE,' !SURF. IMPED. TYPE'
      CLOSE(UNIT=1,STATUS='KEEP')

1000  FORMAT(1H1,20X,30HCALCULATION OF THE GROUND WAVE/14X,34HVER
     1TICAL POLARIZATION, FREQUENCY =F9.2,5H MHz,/9X14HPERMITTIVITY
     2 =F7.2,22H, EARTH CONDUCTIVITY =F7.3,8H MHOS/M,/14X,11HWIND S
     3PEED=F7.2,18H, DIRECTION ANGLE=F7.2,6H[DEG.],/9X,16HRECEIVER
     4 HEIGHT=F7.2,25H[M], TRANSMITTER HEIGHT=F7.2,3H[M]//28X,14HE
     5LECTRIC FIELD/15X,49(1H*))
1001  FORMAT(15X,1H*1X,12HDISTANCE(KM),1X,1H*,18HFIELD STRENGTH V/
     1M,1H*,2X,9HARG(RADS),2X,1H*)
1002  FORMAT(15X,1H*,F10.2,4X,1H*,1X,E12.5,5X,1H*,1X,F8.2,4X,1H*)
1003  FORMAT(15X,49(1H*))
   1  FORMAT(1H1)
```

```
1004  FORMAT(1H1,20X,30HCALCULATION OF THE GROUND WAVE/14X,34HVER
     1TICAL POLARIZATION, FREQUENCY =F9.2,5H MHz,/9X14HPERMITTIVITY
     2 =F7.2,22H, EARTH CONDUCTIVITY =F7.3,8H MHOS/M,/14X,11HWIND S
     3PEED=F7.2,18H, DIRECTION ANGLE=F7.2,6H[DEG.],/9X,16HRECEIVER
     4 HEIGHT=F7.2,25H[M], TRANSMITTER HEIGHT=F7.2,3H[M]//25X,20HATTEN
     5UATION FUNCTION/15X,49(1H*))
1005  FORMAT(15X,1H*1X,12HDISTANCE(KM),1X,1H*,18HATTENUATION [|W|] ,
     11H*,2X,9HARG(RADS),2X,1H*)
1006  FORMAT(10X,F10.2,5X,E12.5,5X,F8.2)
      RETURN
      END
C
C
      SUBROUTINE AIRY(KK,T,F1,F2)
*************************************************
*************************************************
************CALCULATE AIRY FUNCTIONS**************
********LAST MODIFIED : JANUARY 3,1986*************
****************** BY : BARRY J. DAWE**************
*************************************************
*************************************************
      COMPLEX T,G,T1,T2,Y1,Y2,U,V,F1,F2,W1,W2,E,SX,Z4,A,B,SUM1,SUM2
      COMPLEX GI,LZ,MZ,WA,WB
      G=CMPLX(0.0,1.0)    !SOME PRELIMINARIES
```

```
      GI=CMPLX(0.0,-1.0)
      PI=4.0*ATAN(1.0)
      IF(CABS(T).LT.5.0)THEN        !TEST MAGNITUDE OF T
        CK1=0.0             ! | T | < 5.0 DO CONVERGENT SERIES
        CK2=0.0             ! OTHERWISE DO ASYMPTOTIC SERIES
        A2=-1.0
        IF(KK.EQ.1)THEN
          D1=0.0
          D2=1.0
          A1=-2.0
          T1=CMPLX(1.0,0.0)
          T2=T
          Y1=CMPLX(1.0,0.0)
          Y2=T
        ELSE
          D1=2.0
          D2=0.0
          A1=1.0
          T1=(T**2.0)/2.0
          T2=CMPLX(1.0,0.0)
          Y1=T1
          Y2=CMPLX(1.0,0.0)
        ENDIF
25      A1=A1+3.0
```

```
      A2=A2+3.0
      D1=D1+3.0
      D2=D2+3.0
      Q1=A1/(D1*(D1-1.0)*(D1-2.0))
      Q2=A2/(D2*(D2-1.0)*(D2-2.0))
      T1=T1*T**3.0*Q1
      T2=T2*T**3.0*Q2
      Y1=Y1+T1
      Y2=Y2+T2
      TRY1=CABS(Y1)
      TRY2=CABS(Y2)
      IF(ABS((TRY1-CK1)/TRY1)-0.5E-09)35,35,30
30    CK1=TRY1
      CK2=TRY2
      GO TO 25
35    IF(ABS((TRY2-CK2)/TRY2)-0.5E-09)40,40,30
40    U=1.089929069*Y1+0.7945704253*Y2
      V=0.6292708413*Y1-0.4587454489*Y2
      F1=CMPLX(REAL(U)+AIMAG(V),AIMAG(U)-REAL(V))
      F2=CMPLX(REAL(U)-AIMAG(V),AIMAG(U)+REAL(V))
     ELSE
*************************************
*    ASYMPTOTIC EXPANSIONS FROM  *
*    NBS TECH NOTE #319          *
```

```
**************************************
      PHASE=ATAN2(AIMAG(T),REAL(T))
      IF(ABS(PHASE).LT.PI/9.0)THEN               !REGION 1
        E=(2.0/3.0)*T**1.5                       ! |PHASE OF T| < 20 DEGREES
        IF(KK.EQ.1)THEN                          ! CALCULATE W 1 AND W 2
          WA=CEXP(E)*LZ(E)
          WB=CEXP(-E)*LZ(-E)*G/2.0
          F1=(T**-0.25)*(WA-WB)
          F2=(T**-0.25)*(WA+WB)
        ELSE                                     !CALCULATE W'1 AND W'2
          WA=CEXP(E)*MZ(E)
          WB=CEXP(-E)*MZ(-E)*G/2.0
          F1=(T**0.25)*(WA+WB)
          F2=(T**0.25)*(WA-WB)
        ENDIF
      ELSE IF(ABS(PHASE).GT.5.0*PI/9.0)THEN      ! REGION 2
        E=(2.0/3.0)*(-T)**1.5                    ! |PHASE OF T| > 100 DEG
        IF(KK.EQ.1)THEN
          F1=((-T)**-0.25)*CEXP(GI*(E+PI/4.0))*LZ(GI*E)
          F2=((-T)**-0.25)*CEXP(G*(E+PI/4.0))*LZ(G*E)
                                                 ! CALCULATE W 1 AND W 2
        ELSE
          F1=((-T)**0.25)*CEXP(GI*(E-PI/4.0))*MZ(GI*E)
          F2=((-T)**0.25)*CEXP(G*(E-PI/4.0))*MZ(G*E)
```

```
                                  !CALCULATE W'1 AND W'2
    ENDIF
 ELSE IF(PHASE.LE.5.0*PI/9.0.AND.PHASE.GE.PI/9.0)THEN
                                  !REGION 3B
    E=(2.0/3.0)*T**1.5            ! 20 DEG <= PHASE T <= 100 DEG
    IF(KK.EQ.1)THEN
        WA=CEXP(E)*LZ(E)
        WB=CEXP(-E)*LZ(-E)
        F1=(T**-0.25)*WA
        F2=(T**-0.25)*(WA+G*WB)
                                  !CALCULATE W 1 AND W 2
    ELSE
        WA=CEXP(E)*MZ(E)
        WB=CEXP(-E)*MZ(-E)
        F1=(T**0.25)*WA
        F2=(T**0.25)*(WA-G*WB)
                                  !CALCULATE W'1 AND W'2
    ENDIF
 ELSE IF(PHASE.GE.-5.0*PI/9.0.AND.PHASE.LE.PI/9.0)THEN
                                  !REGION 3A
    E=(2.0/3.0)*T**1.5            !-100 DEG <= PHASE T <= -20 DEG
    IF(KK.EQ.1)THEN
        WA=CEXP(E)*LZ(E)
        WB=CEXP(-E)*LZ(-E)
```

```
         F1=(T**-0.25)*(WA-G*WB)
         F2=(T**-0.25)*WA      !CALCULATE W 1 AND W 2
      ELSE
         WA=CEXP(E)*MZ(E)
         WB=CEXP(-E)*MZ(-E)
         F1=(T**0.25)*(WA+G*WB)
         F2=(T**0.25)*WA               !CALCULATE W'1 AND W'2
      ENDIF
    ENDIF
    ENDIF
    RETURN
    END
C
C
    COMPLEX FUNCTION LZ(DUM_ARG)
*************************************************
**************L SERIES***************************
********FOR AIRY FUNCTIONS, SADDLE POINT METHOD***
********LAST MODIFIED : JANUARY 15,1987***********
****************** BY : B.J. DAWE****************
*************************************************
    COMPLEX DUM_ARG,SER_SUM,Z_ARG,NEW_TERM
    REAL J_ONE,J_CNT,J_TWO
    REAL OLD_TERM,CHK_TERM,OLD_SUM,NEW_SUM,CHNG_SUM
```

```
      SER_SUM=(0.,0.)
      Z_ARG=1.0/DUM_ARG
      NEW_TERM=15.0/216.0*Z_ARG        !FIRST TERM OF SERIES
      J_ONE=1.0
      J_CNT=1.0
      SER_SUM=1.0+NEW_TERM
      NEW_SUM=CABS(SER_SUM) !INITIALIZE THE
      CHK_TERM=1000.            !CONVERGENCE TEST
100   OLD_SUM=NEW_SUM
      OLD_TERM=CHK_TERM
      J_CNT=J_CNT+1.0
      J_ONE=J_ONE+2.0
      J_TWO=6.0*J_CNT
      NEW_TERM=NEW_TERM*Z_ARG*(J_TWO-5.0)*
     1      (J_TWO-3.0)*(J_TWO-1.0)/(216.0*J_CNT*J_ONE)
      CHK_TERM=CABS(NEW_TERM)
      SER_SUM=SER_SUM+NEW_TERM          !SUM THE SERIES
      NEW_SUM=CABS(SER_SUM)
      CHNG_SUM=(NEW_SUM-OLD_SUM)/NEW_SUM    !CALCULATE THE CHANGE
                                            !WHEN NEW TERM ADDED
      IF(ABS(CHNG_SUM).LE.0.5E-09)THEN
         LZ=SER_SUM
         RETURN                         !RETURN IF SERIES CONVERGES
      ELSE IF(OLD_TERM.LT.CHK_TERM)THEN
```

```
            LZ=SER_SUM
            RETURN                          !RETURN IF SERIES DIVERGES
        ELSE
            GO TO 100               !COMPUTE ANOTHER TERM
        ENDIF
        RETURN
        END
C
C
        COMPLEX FUNCTION MZ(DUM_ARG)
*************************************************
*************M SERIES****************************
********FOR AIRY FUNCTIONS, SADDLE POINT METHOD***
********LAST MODIFIED : JANUARY 15,1987***********
**************** BY : B.J. DAWE*****************
*************************************************
        COMPLEX DUM_ARG,SER_SUM,NEW_TERM,Z_ARG
        REAL J_ONE,J_CNT,J_TWO
        REAL CHK_TERM,OLD_TERM,NEW_SUM,OLD_SUM,CHNG_SUM
        SER_SUM=(0.,0.)
        Z_ARG=1.0/DUM_ARG
        NEW_TERM=21.0/216.0*Z_ARG       !FIRST TERM OF SERIES
        J_ONE=1.0
        J_CNT=1.0
```

```
      SER_SUM=1.0-NEW_TERM
      NEW_SUM=CABS(SER_SUM) !INITIALIZE THE
      CHK_TERM=1000.          !CONVERGENCE CHECK
100   OLD_SUM=NEW_SUM
      OLD_TERM=CHK_TERM
      J_CNT=J_CNT+1.0
      J_ONE=J_ONE+2.0
      J_TWO=6.0*J_CNT
      NEW_TERM=NEW_TERM*Z_ARG*(J_TWO-7.0)*
    1          (J_TWO-3.0)*(J_TWO+1.0)/(216.0*J_CNT*J_ONE)
      CHK_TERM=CABS(NEW_TERM)
      SER_SUM=SER_SUM-NEW_TERM    !SUM THE SERIES
      NEW_SUM=CABS(SER_SUM)
      CHNG_SUM=(NEW_SUM-OLD_SUM)/NEW_SUM    !CALCULATE THE CHANGE
                                            !WHEN NEW TERM ADDED
      IF(ABS(CHNG_SUM).LE.0.5E-09)THEN
            MZ=SER_SUM
            RETURN               !IF SERIES CONVERGES THEN RETURN
      ELSE IF(OLD_TERM.LT.CHK_TERM)THEN
            MZ=SER_SUM
            RETURN               !IF SERIES DIVERGES THEN RETURN
      ELSE
            GO TO 100     !COMPUTE ANOTHER TERM
      ENDIF
```

```
      RETURN
      END
C
C
      SUBROUTINE TAU(S,Q,T,W1)
*********************************************
********FIND THE POLES FOR THE RESIDUE SERIES*
********USING THE NEWTON ITERATION METHOD*****
********SEE CARNAHAN, LUTHER AND WILKES FOR***
********NEWTON ITERATION*********************
********LAST MODIFIED : AUGUST 17,1986********
****************** BY : BARRY DAWE************
*********************************************
      COMPLEX Q,T,W1,DW1,W2,DW2,CC,TINF,TZER
      JJ=IFIX(S)
!     SEE BREMMER (TERRESTRIAL RADIO WAVES) FOR T(S,0) AND T(S,INF)
      IF((CABS(Q))**2.0.LE.1.0)THEN
            T=TZER(JJ)     ! USE T SUB (S,0)
            T=T+Q/T        ! FOR INITIAL APPROX
      ELSE
            T=TINF(JJ)     ! USE T SUB (S,INF)
            T=T+1.0/Q      ! FOR INITIAL APPROX
      ENDIF
      ICNT=0
```

```
10    CALL AIRY(1,T,W1,W2)   !COMPUTE W1(T)
      CALL AIRY(2,T,DW1,DW2)  !COMPUTE W1'(T)
      CC=DW1/W1              !PERFORM NEWTON ITERATION
      CC=(CC-Q)/(T-CC*Q)
      T=T-CC
      ICNT=ICNT+1
      IF(ICNT.GT.30)THEN     !NUM OF ITERATIONS LESS THAN 30
           WRITE(6,20)JJ+1
20         FORMAT(1X,'ITERATION DID NOT CONVERGE ON T WHEN S= ',I4)
           RETURN
      ELSE IF(CABS(CC/T).GT.0.5E-06)THEN
                 !CHECK FOR CONVERGENCE
           GO TO 10       !ON POLE T SUB S AND
                 !ITERATE AGAIN (IF NECESSARY)
      ENDIF
      RETURN               !IF T SUB S CONVERGED
      END          !THEN RETURN
C
C
      COMPLEX FUNCTION TZER(KK)
*******************************************
********ESTIMATE FOR T(S) WHEN | Q | << 1 **
********LAST MODIFIED: AUGUST 10,1986 ******
****************** BY: B. J. DAWE***********
```

```
*******************************************
      COMPLEX PHS
      REAL T(5)
      DATA T/0.80861652,2.5780961,3.8257153,4.8918203,5.8513010/
      PI=4.0*ATAN(1.0)
      PHS=CEXP(CMPLX(0.0,-PI/3.0))        !PHASE OF T(S,0)
      IF(KK.LE.4)THEN
            TZER=T(KK+1)*1.2599211*PHS
!           T(S,0) WHEN 0 <= S <= 4
      ELSE
            Y1=1.1780972*(4.0*KK+1)
            TZER=PHS*((Y1**(2.0/3.0))*(1.0-0.14583333/(Y1**2.)+
     1      0.12152778/(Y1**4.)-0.87395351/(Y1**6.)))
!           T(S,0) WHEN S > 4
      ENDIF
      RETURN
      END
C
C
      COMPLEX FUNCTION TINF(KK)
*******************************************
********ESTIMATE FOR T(S) WHEN | Q | >> 1 **
********LAST MODIFIED: AUGUST 10,1986 ******
****************** BY: B. J. DAWE**********
```

```
**************************************************
      COMPLEX PHS
      REAL T(5)
      DATA T/1.8557571,3.2446076,4.3816712,5.3866138,6.3052630/
      PI=4.0*ATAN(1.0)
      PHS=CEXP(CMPLX(0.0,-PI/3.0))        !PHASE OF T(S,INF)
      IF(KK.LE.4)THEN
            TINF=T(KK+1)*1.2599211*PHS
!           T(S,INF) WHEN 0 <= S <= 4
      ELSE
            Y2=1.1780972*(4.0*KK+3)
            TINF=PHS*((Y2**(2.0/3.0))*(1.0+1.0/(9.6*Y2**2.0)-
     1      1.0/(7.2*Y2**4.0)+0.92928404/(Y2**6.0)))
!           T(S,INF) WHEN S > 4
      ENDIF
      RETURN
      END
C
C
      COMPLEX FUNCTION ZIMP(A,B,C,D,AA)
**************************************************
**************************************************
********PROGRAM CALCULATES MODIFIED***************
********SURFACE IMPEDANCE*************************
```

```
********INPUT >>>>> ***************************
*****************A=EPSILON (REAL*4) *************
*****************B=SIGMA   (REAL*4) *************
*****************C=WIND VELOCITY (REAL*4) ********
*****************D=ANGLE OF WIND VELOCITY (REAL*4)
*****************AA=FREQ   (REAL*4) *************
********OUTPUT >>>>> ***************************
*****************ZIMP=SURFACE IMPEDANCE (COMPLEX)*
********LAST MODIFIED : JUNE 13, 1985 ***********
***************** BY : B.J. DAWE ***************
**********************************************
      IMPLICIT REAL*8 (E-H,O-Z)  !ALL VARIABLES REAL*8
      COMPLEX*16 DELT,DELMOD          !WITH THESE EXCEPTIONS
      REAL*8 ALPHA,K,K2
      INTEGER*4 ERROR1,ERROR2,IER1,IER2,ITYPE
      REAL*4 A,B,C,DUM1,DUM2
      COMMON/SNIT/WNUM,ALPHA,GRAV,WVEL,DELT
      COMMON /SURF/ DUM1,DUM2,ITYPE
      EXTERNAL BAFR,BAFI,FUNCTR,FUNCTI
                 !INTEGRANDS ARE EXTERNAL FUNCTIONS
      EPS=DBLE(A) !SET ALL REAL*4 INPUTS TO
      SIG=DBLE(B)  !REAL*8
      WVEL=DBLE(C)
      ALPHA=DBLE(D)
```

```
      FREQ=DBLE(AA)
      PI=3.1415926828D0              ! PI
      EPSO=1.0D-09/(36.0D0*PI)       ! EPSILON SUB 0
      GRAV=9.81D0          ! ACCEL DUE TO GRAVITY
      OMEGA=2.0D0*PI*FREQ       ! ANGULAR FREQ
      WNUM=OMEGA/2.997925D08! WAVE NUM
      K=WNUM
      K2=K*K
!     INITIALIZE DELTA (SURF IMPED)
      DELT=DCMPLX(1.0D0,0.0D0)/(CDSQRT(DCMPLX(EPS,-SIG/(OMEGA*EPSO))))
      IF(WVEL.LT.0.1D0)THEN
            ZIMP=CMPLX(DELT)
            RETURN                  !IF WIND SPEED = 0.0
      ENDIF                         !THEN RETURN DELTA
!           INTEGRATION LIMITS
      AX=0.0D0
      BX=40.0D0
      AY=-PI
      BY=PI
      AERR=1.0D-03
!           INTEGRATION
      IF(ITYPE.EQ.1)THEN
!           integration for barrick surface impedance
        DELT=DCONJG(DELT)     !THIS IS FOR BARRICK'S CONVENTION
```

```
      SIMPR=SIMPSON(BAFR,AX,BX,AY,BY)
          WRITE(6,*)'SIMPR=',SIMPR
      SIMPI=SIMPSON(BAFI,AX,BX,AY,BY)
          WRITE(6,*)'SIMPI=',SIMPI
      DELMOD=DCONJG(DELT+0.25D0*DCMPLX(SIMPR,SIMPI))
                        !THIS RETURNS US TO OUR CONVENTION
    ELSE
!         integration for srivastava surface impedance
      SIMPR=SIMPSON(FUNCTR,AX,BX,AY,BY)
          WRITE(6,*)'SIMPR=',SIMPR
      SIMPI=SIMPSON(FUNCTI,AX,BX,AY,BY)
          WRITE(6,*)'SIMPI=',SIMPI
      DELMOD=DELT+0.25D0*DCMPLX(SIMPR,SIMPI)
    ENDIF
    ZIMP=CMPLX(DELMOD)
    RETURN
    END
C
C
    REAL*8 FUNCTION FUNCTR(XP,YP)
***************************************************
********REAL PART OF INTEGRAND FOR SRIVASTAVA'S ******
********MODIFIED SURFACE IMPEDANCE*******************
********LAST MODIFIED : JUNE 13, 1985 ***************
```

```
.................. BY : B.J. DAWE ..................

....................................................

      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 K,K2
      COMPLEX*16 D,BPR,CINT,DCMPLX,CDSQRT,DCONJG
      COMMON /SNIT/K,A,G,U,D
      X=XP*DCOS(YP)
      Y=XP*DSIN(YP)
!     CALCULATE WAVE HEIGHT SPECTRUM
      SQ=X**2.0+Y**2.0
      IF(SQ.EQ.0.0D0)THEN
         W=0.0D0
      ELSE
         W=1.5025D0*((X*DCOS(A)+Y*DSIN(A))**2.0/
     1    ((G**2.5)*(SQ**3.25)))*
     2  DEXP(-2.0*G/(U**2.0*DSQRT(SQ)))
      ENDIF
!     CALCULATE INTEGRAND
      K2=K**2.0
      BPR=DCONJG((CDSQRT(DCMPLX(K2-(X+K)**2.0-Y**2.0,0.0D0)))/
     1  DCMPLX(K,0.0D0))
      CINT=DCMPLX(W,0.0D0)*(DCMPLX(X*X,0.0D0)-D*DCMPLX(K*X,0.0D0)*
     1   BPR)/(BPR+D)
      FUNCTR=DREAL(CINT)*DSQRT(SQ)
```

```
      RETURN
      END
C
C
      REAL*8 FUNCTION FUNCTI(XP,YP)
*****************************************************
********IMAGINARY PART OF INTEGRAND FOR **************
********SRIVASTAVA'S MODIFIED SURFACE IMPEDANCE*******
********LAST MODIFIED : JUNE 13, 1985 ****************
****************** BY : B.J. DAWE *******************
*****************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 K,K2
      COMPLEX*16 D,BPR,CINT,DCMPLX,CDSQRT,DCONJG
      COMMON/SNIT/K,A,G,U,D
      X=XP*DCOS(YP)
      Y=XP*DSIN(YP)
!     CALCULATE WAVE HEIGHT SPECTRUM
      SQ=X**2.0+Y**2.0
      IF(SQ.EQ.0.0D0)THEN
            W=0.0D0
      ELSE
            W=1.5025D0*(X*DCOS(A)+Y*DSIN(A))**2.0/
     1       (G**2.5*SQ**3.25)*
```

```
     2          DEXP(-2.0D0*G/(U**2.0*DSQRT(SQ)))
          ENDIF
C     CALCULATE INTEGRAND
          K2=K**2.0
          BPR=DCONJG((CDSQRT(DCMPLX(K2-(X+K)**2.0-Y**2.0,0.0D0)))/
     1      DCMPLX(K,0.0D0))
          CINT=DCMPLX(W,0.0D0)*(DCMPLX(X*X,0.0D0)-D*
     1      DCMPLX(K*X,0.0D0)*BPR)/(BPR+D)
          FUNCTI=DIMAG(CINT)*DSQRT(SQ)
          RETURN
          END
C
C
          REAL*8 FUNCTION BAFR(XP,YP)
**************************************************
********REAL PART OF INTEGRAND FOR BARRICK'S*****
********MODIFIED SURFACE IMPEDANCE***************
********LAST MODIFIED : JUNE 13, 1985 ***********
****************** BY : B.J. DAWE ***************
**************************************************
          IMPLICIT REAL*8(A-H,O-Z)
          REAL*8 K,K2
          COMPLEX*16 D,BPR,CINT,BARD,DCMPLX,CDSQRT,DCONJG
          COMMON/SNIT/K,A,G,U,D
```

```
      X=XP*DCOS(YP)
      Y=XP*DSIN(YP)
      BARD=DCONJG(D)
!     CALCULATE WAVE HEIGHT SPECTRUM
      SQ=X**2.0+Y**2.0
      IF(SQ.EQ.0.0)THEN
            W=0.0D0
      ELSE
            W=1.5025D0*((X*DCOS(A)+Y*DSIN(A))**2.0/
     1          ((G**2.5)*(SQ**3.25)))*
     2          DEXP(-2.0D0*G/(U**2.0*DSQRT(SQ)))
      ENDIF
!     CALCULATE INTEGRAND
      K2=K**2.0
      BPR=(CDSQRT(DCMPLX(K2-(X+K)**2.0-Y**2.0,0.0D0)))/K
      CINT=W*((X**2.0+BPR*BARD*(X**2.0+Y**2.0-K*X))/
     1   (BPR+BARD*(BPR*BPR+DCMPLX(1.0D0,0.0D0)))+
     2   BARD*((X**2.0-Y**2.0)/2.0D0+(K*X)))
      BAFR=DREAL(CINT)*DSQRT(SQ)
      RETURN
      END
C
C
      REAL*8 FUNCTION BAFI(XP,YP)
```

```
****************************************************
********IMAGINARY PART OF INTEGRAND FOR BARRICK'S*****
********MODIFIED SURFACE IMPEDANCE*******************
********LAST MODIFIED : JUNE 13, 1985 ***************
****************** BY : B.J. DAWE *******************
****************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 K,K2
      COMPLEX*16 D,BPR,CINT,BARD,DCMPLX,CDSQRT,DCONJG
      COMMON/SNIT/K,A,G,U,D
      X=XP*DCOS(YP)
      Y=XP*DSIN(YP)
      BARD=DCONJG(D)
!     CALCULATE WAVE HEIGHT SPECTRUM
      SQ=X**2.0+Y**2.0
      IF(SQ.EQ.0.0)THEN
          W=0.0
      ELSE
          W=1.5025D0*(X*DCOS(A)+Y*DSIN(A))**2.0/
     1        (G**2.5*SQ**3.25)*DEXP(-2.0D0*G/
     2        *(U**2.0*DSQRT(SQ)))
      ENDIF
!     CALCULATE INTEGRAND
      K2=K**2.0
```

```
      BPR=(CDSQRT(DCMPLX(K2-(X+K)**2.0-Y**2.0,0.0D0)))/K
      CINT=W*((X**2.0+BPR*BARD*(X**2.0+Y**2.0-K*X))/
     1   (BPR+BARD*(BPR*BPR+DCMPLX(1.0D0,0.0D0)))+
     2   BARD*((X**2.0-Y**2.0)/2.0D0+(K*X)))
      BAFI=DIMAG(CINT)*DSQRT(SQ)
      RETURN
      END
C
C
      REAL*8 FUNCTION SIMPSON(FUNCT,XL,XU,YL,YU)
*****************************************************
********TWO DIMENSIONAL SIMPSON RULE INTEGRATION****
********FOR A FIXED NUMBER OF POINTS****************
********ROUTINE USES THE FUNCTION SIMP1*************
********LAST MODIFIED : SEPT 23,1986 ***************
****************** BY : B.J. DAWE *****************
*****************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      EXTERNAL FUNCT
      N=100          !num of points
      H=(XU-XL)/(2.0*N)
      XINC=XL+H
      IFLAG=0
      DO 20 I=1,2*N-1
```

```
      IF(IFLAG.EQ.0)THEN
         SUM=SUM+4.0*SIMP1(FUNCT,YL,YU,XINC)
         IFLAG=1
      ELSE
         SUM=SUM+2.0*SIMP1(FUNCT,YL,YU,XINC)
         IFLAG=0
      ENDIF
      XINC=XINC+H
 20   CONTINUE
!     TREAT ENDPOINTS
      SUM=(H/3.0)*(SIMP1(FUNCT,YL,YU,XL)+SIMP1(FUNCT,YL,YU,XU)+SUM)
      SIMPSON=SUM
      RETURN
      END
C
C
      REAL*8 FUNCTION SIMP1(FUNCT,YL,YU,X)
***************************************************
********PART OF SIMPSON'S RULE INTEGRATION************
********SEE FUNCTION SIMPSON************************
********LAST MODIFIED : JUNE 13, 1985 ***************
****************** BY : B.J. DAWE ******************
***************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
```

```
      EXTERNAL FUNCT
      N=100
      H=(YU-YL)/(2.0*N)
      YINC=YL+H
      IFLAG=0
      DO 20 I=1,2*N-1
      IF(IFLAG.EQ.0)THEN
            SUM=SUM+4.0*FUNCT(X,YINC)
            IFLAG=1
      ELSE
            SUM=SUM+2.0*FUNCT(X,YINC)
            IFLAG=0
      ENDIF
      YINC=YINC+H
20    CONTINUE
!     TREAT ENDPOINTS
      SUM=(H/3.0)*(FUNCT(X,YL)+FUNCT(X,YL)+SUM)
      SIMP1=SUM
      RETURN
      END
```