PREDICTION OF WAVE LOADS AND MOTIONS
OF FLOATING MARINE STRUCTURES BY
THREE-DIMENSIONAL FLOW THEORY

CENTRE FOR NEWFOUNDLAND STUDIES

TOTAL OF 10 PAGES ONLY
MAY BE XEROXED

(Without Author's Permission)

DEBABRATA SEN
PREDICTION OF WAVE LOADS AND MOTIONS OF FLOATING MARINE STRUCTURES BY THREE-DIMENSIONAL FLOW THEORY

BY

Debabrata Sen, B.Tech.(Hons)

A thesis submitted to the School of Graduate Studies in partial fulfillment of the requirements for the degree of Master of Engineering

Faculty of Engineering and Applied Science Memorial University of Newfoundland

November, 1983

St. John's Newfoundland
ABSTRACT

The three-dimensional singularity distribution or boundary integral method has been demonstrated by many investigators to be the most versatile and reliable technique for the calculation of harmonic oscillation of a truly three-dimensional floating marine structure in potential flow field.

In the present work, a numerical scheme is presented and a computer program has been developed based on the three-dimensional singularity distribution theory. The program calculates the first order wave exciting forces and moments, hydrodynamic co-efficients and motion responses in six degrees of freedom of any floating marine structure of arbitrary geometry for different angles of heading. Calculations are performed for a floating rectangular box, a vertical circular cylinder and a 130,000 ton dwt tanker. The results are compared with available published results based on the same theoretical model. In general, a good agreement is found between the results.

To demonstrate the versatility and effectiveness of the program, calculations are also performed for a semi-submersible and the results are presented.
ACKNOWLEDGEMENT

The author wishes to express his sincere gratitude to Professor C.C. Hsiung for his enthusiastic supervision and valuable guidance during the entire course of this work. Sincere thanks are also due to Dr. G. R. Peters, Dean of Engineering and Applied Science and Dr. T. R. Chari, Associate Dean of Engineering and Applied Science for their encouragement.

The author is indebted to Dr. F. A. Aldrich, Dean of Graduate Studies for awarding a University Fellowship and also the financial support by NSERC Strategic Group Grant G0561 is acknowledged.

The author also wishes to extend his deep appreciation to Mr. C. C. Tse and Mr. J. M. Chuang, fellow graduate students in Ocean Engineering for their valuable suggestions and helpful discussions.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>ix</td>
</tr>
<tr>
<td>CHAPTER 1: INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER 2: THEORETICAL BACKGROUND</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Formulation of the problem</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Solution of potentials</td>
<td>8</td>
</tr>
<tr>
<td>CHAPTER 3: NUMERICAL FORMULATION</td>
<td>15</td>
</tr>
<tr>
<td>3.1 Numerical solution of potential</td>
<td>15</td>
</tr>
<tr>
<td>3.2 Numerical evaluation of Green's function</td>
<td>20</td>
</tr>
<tr>
<td>3.3 Wave forces, moments and motion response</td>
<td>23</td>
</tr>
<tr>
<td>CHAPTER 4: COMPUTED RESULTS</td>
<td>28</td>
</tr>
<tr>
<td>CHAPTER 5: DISCUSSIONS AND CONCLUDING REMARKS</td>
<td>34</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>40</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td>88</td>
</tr>
<tr>
<td>APPENDIX B</td>
<td>126</td>
</tr>
<tr>
<td>No.</td>
<td>Title</td>
</tr>
<tr>
<td>-----</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Co-ordinate system and geometrical boundaries</td>
</tr>
<tr>
<td>2</td>
<td>Local co-ordinate system of a plane quadrilateral element</td>
</tr>
<tr>
<td>3</td>
<td>Surge added mass coefficient for floating box</td>
</tr>
<tr>
<td>4</td>
<td>Heave added mass coefficient for floating box</td>
</tr>
<tr>
<td>5</td>
<td>Pitch added mass coefficient for floating box</td>
</tr>
<tr>
<td>6</td>
<td>Yaw added mass coefficient for floating box</td>
</tr>
<tr>
<td>7</td>
<td>Surge damping coefficient for floating box</td>
</tr>
<tr>
<td>8</td>
<td>Heave damping coefficient for floating box</td>
</tr>
<tr>
<td>9</td>
<td>Surge exciting force on floating box, amplitudes and phases</td>
</tr>
<tr>
<td>10</td>
<td>Heave exciting force on floating box, amplitudes and phases</td>
</tr>
<tr>
<td>11</td>
<td>Pitch exciting moment on floating box, amplitudes and phases</td>
</tr>
<tr>
<td>12</td>
<td>Surge motion of floating box, non-dimensional amplitudes and phases</td>
</tr>
<tr>
<td>13</td>
<td>Heave motion of floating box, non-dimensional amplitudes and phases</td>
</tr>
<tr>
<td>14</td>
<td>Pitch motion of floating box, non-dimensional amplitudes and phases</td>
</tr>
<tr>
<td>15</td>
<td>Surge added mass for vertical circular cylinder</td>
</tr>
<tr>
<td>16</td>
<td>Heave added mass for vertical circular cylinder</td>
</tr>
<tr>
<td>17</td>
<td>Surge exciting force on vertical circular cylinder, amplitudes and phases</td>
</tr>
<tr>
<td>18</td>
<td>Heave exciting force on vertical circular cylinder, amplitudes and phases</td>
</tr>
<tr>
<td>No.</td>
<td>Title</td>
</tr>
<tr>
<td>-----</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>19</td>
<td>Pitch exciting moment on vertical circular cylinder, amplitudes and phases</td>
</tr>
<tr>
<td>20</td>
<td>Surge motion of vertical circular cylinder, non-dimensional amplitudes and phases</td>
</tr>
<tr>
<td>21</td>
<td>Heave motion of vertical circular cylinder, non-dimensional amplitudes and phases</td>
</tr>
<tr>
<td>22</td>
<td>Pitch motion of vertical circular cylinder, non-dimensional amplitudes and phases</td>
</tr>
<tr>
<td>23</td>
<td>Garrison's results for vertical circular cylinder for surge mode</td>
</tr>
<tr>
<td>24</td>
<td>Geometry of the tanker</td>
</tr>
<tr>
<td>25</td>
<td>Surge added mass coefficient for tanker (ballast)</td>
</tr>
<tr>
<td>26</td>
<td>Sway added mass coefficient for tanker (ballast)</td>
</tr>
<tr>
<td>27</td>
<td>Heave added mass coefficient for tanker (ballast)</td>
</tr>
<tr>
<td>28</td>
<td>Pitch added mass coefficient for tanker (ballast)</td>
</tr>
<tr>
<td>29</td>
<td>Surge damping coefficient for tanker (ballast)</td>
</tr>
<tr>
<td>30</td>
<td>Sway damping coefficient for tanker (ballast)</td>
</tr>
<tr>
<td>31</td>
<td>Heave damping coefficient for tanker (ballast)</td>
</tr>
<tr>
<td>32</td>
<td>Pitch damping coefficient for tanker (ballast)</td>
</tr>
<tr>
<td>33</td>
<td>Surge added mass coefficient for tanker (loaded)</td>
</tr>
<tr>
<td>34</td>
<td>Sway added mass coefficient for tanker (loaded)</td>
</tr>
<tr>
<td>35</td>
<td>Heave added mass coefficient for tanker (loaded)</td>
</tr>
<tr>
<td>36</td>
<td>Pitch added mass coefficient for tanker (loaded)</td>
</tr>
<tr>
<td>37</td>
<td>Surge damping coefficient for tanker (loaded)</td>
</tr>
<tr>
<td>38</td>
<td>Sway damping coefficient for tanker (loaded)</td>
</tr>
<tr>
<td>39</td>
<td>Heave damping coefficient for tanker (loaded)</td>
</tr>
<tr>
<td>No.</td>
<td>Title</td>
</tr>
<tr>
<td>-----</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>40</td>
<td>Pitch damping coefficient for tanker (loaded)</td>
</tr>
<tr>
<td>41</td>
<td>Motion response of the tanker (ballast cond.) (surge, sway and heave)</td>
</tr>
<tr>
<td>42</td>
<td>Motion response of the tanker (ballast cond.) (pitch and yaw)</td>
</tr>
<tr>
<td>43</td>
<td>Motion response of the tanker (loaded cond.) (surge, sway and heave)</td>
</tr>
<tr>
<td>44</td>
<td>Motion response of the tanker (loaded cond.) (pitch and yaw)</td>
</tr>
<tr>
<td>45</td>
<td>Roll response of the tanker (ballast and loaded conditions)</td>
</tr>
<tr>
<td>46</td>
<td>DnV results for motion response of the tanker, ballast (surge, sway and heave)</td>
</tr>
<tr>
<td>47</td>
<td>DnV results for motion response of the tanker, ballast (roll, pitch and yaw)</td>
</tr>
<tr>
<td>48</td>
<td>DnV results for motion response of the tanker, loaded (surge, sway and heave)</td>
</tr>
<tr>
<td>49</td>
<td>DnV results for motion response of the tanker, loaded (roll, pitch and yaw)</td>
</tr>
<tr>
<td>50</td>
<td>Sectional views of the semisubmersible</td>
</tr>
<tr>
<td>51</td>
<td>Surge added mass coefficient for semisubmersible</td>
</tr>
<tr>
<td>52</td>
<td>Sway added mass coefficient for semisubmersible</td>
</tr>
<tr>
<td>53</td>
<td>Heave added mass coefficient for semisubmersible</td>
</tr>
<tr>
<td>54</td>
<td>Roll added mass coefficient for semisubmersible</td>
</tr>
<tr>
<td>55</td>
<td>Pitch added mass coefficient for semisubmersible</td>
</tr>
<tr>
<td>56</td>
<td>Yaw added mass coefficient for semisubmersible</td>
</tr>
<tr>
<td>57</td>
<td>Surge exciting force on semisubmersible</td>
</tr>
<tr>
<td>58</td>
<td>Sway exciting force on semisubmersible</td>
</tr>
<tr>
<td>59</td>
<td>Heave exciting force on semisubmersible</td>
</tr>
<tr>
<td>No.</td>
<td>Title</td>
</tr>
<tr>
<td>-----</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>60</td>
<td>Roll exciting moment on semisubmersible</td>
</tr>
<tr>
<td>61</td>
<td>Pitch exciting moment on semisubmersible</td>
</tr>
<tr>
<td>62</td>
<td>Yaw exciting moment on semisubmersible</td>
</tr>
<tr>
<td>63</td>
<td>Surge motion of semisubmersible</td>
</tr>
<tr>
<td>64</td>
<td>Sway motion of semisubmersible</td>
</tr>
<tr>
<td>65</td>
<td>Heave motion of semisubmersible</td>
</tr>
<tr>
<td>66</td>
<td>Roll motion of semisubmersible</td>
</tr>
<tr>
<td>67</td>
<td>Pitch motion of semisubmersible</td>
</tr>
<tr>
<td>68</td>
<td>Yaw motion of semisubmersible</td>
</tr>
</tbody>
</table>
# NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1, x_2, x_3$</td>
<td>Co-ordinate system as shown in Figure 1</td>
</tr>
<tr>
<td>$d$</td>
<td>Water-depth</td>
</tr>
<tr>
<td>$\xi_k$</td>
<td>Displacements ($k = 1, 2, \ldots, 6$ refer to surge, sway, heave, roll, pitch and yaw respectively)</td>
</tr>
<tr>
<td>$\zeta_k$</td>
<td>Complex motion amplitudes</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Circular frequency of wave</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$i$</td>
<td>$\sqrt{-1}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Complex velocity potential</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Complex velocity potential, function of space co-ordinates only</td>
</tr>
<tr>
<td>$\xi_0$</td>
<td>Amplitude of incident wave</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>$n_k$</td>
<td>Generalized direction cosines as defined in equation (2.8)</td>
</tr>
<tr>
<td>$\theta, \phi$</td>
<td>Polar co-ordinates</td>
</tr>
<tr>
<td>$H(\theta)$</td>
<td>Unknown complex function</td>
</tr>
<tr>
<td>$k$</td>
<td>Wave number</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$\omega^2/g = k \tanh(kd)$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Direction of propagation of incident waves with respect to positive $x_1$ axis</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wave length of incoming wave</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Complex source density function</td>
</tr>
<tr>
<td>$G$</td>
<td>Green's function</td>
</tr>
<tr>
<td>$S$</td>
<td>Body surface</td>
</tr>
</tbody>
</table>
\[ a_1, a_2, a_3 \] = Co-ordinates of a point on the surface of the body

\[ R = \left[ (x_1 - a_1)^2 + (x_2 - a_2)^2 + (x_3 - a_3)^2 \right]^{1/2} \]

\[ R_1 = \left[ (x_1 - a_1)^2 + (x_2 - a_2)^2 + (x_3 + 2d + a_3)^2 \right]^{1/2} \]

\[ r = \left[ (x_1 - a_1)^2 + (x_2 - a_2)^2 \right]^{1/2} \]

\[ J_0 = \text{Bessel function of the first kind of order zero} \]

\[ Y_0 = \text{Bessel function of the second kind of order zero} \]

\[ K_0 = \text{Modified Bessel function of the second kind of order zero} \]

\[ P_V = \text{Cauchy principal value of the integral} \]

\[ \mu_j = \text{Real positive roots of the equation, } \mu_j \tan (\mu_j d) + \nu = 0 \]

\[ n_1, n_2, n_3 = \text{Components of outward unit normal to the body surface } S \text{ in } x_1, x_2 \text{ and } x_3 \text{ directions respectively} \]

\[ J_1 = \text{Bessel function of the first kind of order one} \]

\[ Y_1 = \text{Bessel function of the first kind of order one} \]

\[ K_1 = \text{Modified Bessel function of the second kind of order one} \]

\[ N = \text{Total number of elements} \]

\[ \Delta S_j = \text{Area of } j^{\text{th}} \text{ element} \]

\[ \delta_{ij} = \text{Kronecker delta function} \]

\[ \bar{x}, \bar{y}, \bar{z}, \bar{\xi}, \eta = \text{Local co-ordinate system as defined in Figure 2} \]

\[ \bar{x}, \bar{y}, \bar{z} = \text{Local co-ordinate of a general point } P \text{ in space} \]
Local co-ordinates of the corner points of a plane quadrilateral element, \(i=1, 2, \ldots 4\)

Aspect ratio of a rectangular element

First order wave exciting forces and moments for \(k\)th mode, \(k=1, 2, \ldots 6\)

Mass density of water

Added mass coefficients \((j, k=1, 2, \ldots 6)\)

Damping coefficients \((j, k'=1, 2, \ldots 6)\)

Inertia matrix

Hydrostatic restoring coefficients

Amplitudes of wave exciting forces and moments for \(k\)th mode, \(k=1, 2, \ldots 6\)

Mass of the body

\(x_3\) co-ordinate of the centre of gravity of the body

Moment of inertia of the body, as defined on page 26

Area of waterplane

Immersed volume of the body

\(x_3\) co-ordinate of the centre of buoyancy of the body

Oscillatory hydrodynamic forces and moments for \(k\)th mode, \(k=1, 2, \ldots 6\)

\(|A11|, |A22|, |A33|\) = Non-dimensional added mass coefficients for surge, sway, heave, roll, pitch and yaw respectively

\(|A44|, |A55|, |A66|\) = Non-dimensional added mass coefficients for surge, sway, heave, roll, pitch and yaw respectively

\(|B11|, |B22|, |B33|\) = Non-dimensional damping coefficients for surge, sway, heave, roll, pitch and yaw respectively

\(|B44|, |B55|, |B66|\) = Non-dimensional damping coefficients for surge, sway, heave, roll, pitch and yaw respectively

\(|F1|, |F2|, |F3|\) = Non-dimensional wave exciting force and moment amplitudes for surge, sway, heave, roll, pitch and yaw respectively

\(|F4|, |F5|, |F6|\) = Non-dimensional wave exciting force and moment amplitudes for surge, sway, heave, roll, pitch and yaw respectively
$|\eta_i|$ = Non-dimensional amplitudes for $i^{th}$ mode of motion

$L$ = Characteristic dimension of the body

$\omega_n$ = Non-dimensional frequency of oscillation

$r_{x1}$ = Roll radius of gyration

$r_{x2}$ = Pitch radius of gyration

$r_{x3}$ = Yaw radius of gyration
CHAPTER 1

INTRODUCTION

It is essential to have knowledge of motion and hydrodynamic loads of floating marine structures, such as semisubmersible platforms, drilling ships in their early stage of design. Such structures, as a matter of course, require structural analysis in order to ensure safety, reliability and economic feasibility. Structural analyses require a correct prediction of dynamic wave loads, and an estimation of wave loads presupposes a knowledge of motion response in waves.

In potential flow theory, the flow is assumed to be inviscid, irrotational, incompressible and acyclic so that the flow field can be characterized by a single-valued velocity potential. A further assumption is that the wave height and responses of the body are small compared to the wave length, water depth and typical body dimensions. Hence the free-surface boundary conditions can be linearised with respect to wave height (which implies small amplitude oscillation of the body). This allows the use of Denia-Pierson hypothesis and consequently the usual spectral technique can be used to determine the force and motion responses in an irregular sea from the results obtained for
regular waves. Standard frequency domain methods may be used to determine short and long term predictions. Another limitation of the potential flow theory arises due to the assumption of the fluid being ideal, which neglects the effects of viscosity. At high Reynolds number, viscous effects result in flow separation and wake formation for bodies such as slender circular cylinders. For ships, viscous damping effects are known to be important for roll motion. The exact nature of viscous effects are highly complex and depend on various factors such as the size and the shape of the body; amplitude of fluid motion relative to the size of the body, Reynolds number etc. The effects of viscosity will be more pronounced if equations such as Morrison's equations are used. Morrison's equations assume that the body is small relative to the incoming wave length such that the incident flow remains almost unaltered in the vicinity of the body. For large bodies such as ships and semi-submersibles, this assumption is not strictly valid due to diffraction effects. Furthermore, for such large bodies, separation of flow is usually not important [14]. As a result, linearised potential flow theory can be applied in the formulation and solution of the problem to obtain results within acceptable range of accuracy. This approach forms the basis of present day prediction methods for large marine objects.
The three-dimensional singularity distribution method is now believed to be the most versatile technique for calculating harmonic oscillatory motion in a potential flow field for a three-dimensional floating body of arbitrary geometry. Theoretical development for this method was first established by Kim [1], and was later extended and applied to various floating structures by Garrison [2] and Faltinsen [3]. Since then, the effectiveness and reliability of this method have been demonstrated by many investigators [4,5,6]. Conventional methods such as the 'strip' method for ships [7], Hooft's method for semisubmersibles [8] are based on two-dimensional approximations and are not adequate for predicting many of the hydrodynamic characteristics of such floating bodies to the required degree of accuracy. The popularity of these two-dimensional methods is due to the belief that they provide quick results at a much lower computing cost when compared to the three-dimensional singularity distribution method. However, in view of the large and fast computers available today, use of the three-dimensional singularity distribution technique should be made more popular considering its accuracy, reliability and versatility.
In this thesis work, a computer program has been developed based on the singularity distribution method or Green's function method for evaluating wave loads and motion response in six degrees of freedom for floating marine structures of arbitrary shape. Calculations are presented for a rectangular floating box, a vertical circular cylinder, a 130,000-ton dwt. tanker and a semisubmersible platform. Computations have been checked with available published results.
CHAPTER 2
THEORETICAL BACKGROUND

2.1 Formulation of the problem

Consider a rigid body oscillating sinusoidally about a state of rest in response to excitation by a long crested regular waves. An inertial, Cartesian and right-handed system of co-ordinate Ox1x2x3 is defined with positive vertically upwards through the centre of gravity of the body and the origin in the plane of the undisturbed free surface. The waterdepth d is finite and constant, and the free surface is assumed to be infinite in all directions (Figure 1).

The problem posed here deals with the fluid motion and the forces induced by the small amplitude oscillation of the object in its six degrees of freedom as well as the fluid motion associated with the interaction of the object with a train of regular waves. The oscillatory motion of the object is described by,

\[ \xi_k = \xi_k e^{-i\omega t}, \quad k = 1, 2, \ldots, 6 \quad (2.1) \]

Here, \( \xi_k \) is the complex amplitude of motion in the \( k \)th mode and \( \omega \) the circular frequency. The motion variables \( \xi_1, \xi_2 \) and \( \xi_3 \) denote the three translations along \( x_1, x_2 \) and \( x_3 \) axes (surge, sway and heave) while \( \xi_4, \xi_5 \) and \( \xi_6 \) represent
angular oscillations about \( \mathbf{Ox}_1, \mathbf{Ox}_2 \) and \( \mathbf{Ox}_3 \) axes (roll, pitch and yaw) respectively.

The fluid is assumed to be ideal and the flow irrotational, acyclic and harmonic. Therefore, the problem can be formulated in terms of potential flow theory. The flow field can be characterized by a first order complex velocity potential,

\[
\phi (x_1, x_2, x_3, t) = \psi (x_1, x_2, x_3) e^{-i\omega t} \tag{2.2}
\]

The potential function \( \psi \) can be separated into contributions from all modes of motion and from the incident and diffraction wave fields,

\[
\psi = i\omega \zeta_0 (\psi_0 + \psi_7) - i\omega \sum_{k=1}^{6} \psi_k \zeta_k \tag{2.3}
\]

Here \( \psi_k \) denotes the normalised velocity potential associated with the motion induced by oscillations in the six degrees of freedom, \( \psi_0 \) denotes the velocity potential of the incident wave in the absence of the object and \( \psi_7 \) denotes the velocity potential of the scattered wave due to the presence of the rigid body. \( \zeta_0 \) is the incident wave amplitude.

All the individual potentials must satisfy Laplace equation in the fluid domain,

\[
\nabla^2 \psi_k = 0, \quad k = 0, 1, 2, \ldots, 7 \tag{2.4}
\]

It is now necessary to impose the boundary conditions for the geometry specified. These are,
(a) On the sea-floor
The kinematic boundary condition on the sea-floor is,
\[ \frac{\partial \psi_k}{\partial x_3} = 0 \text{ on } x_3 = -d, \quad k = 0, 1, 2, \ldots, 7 \] (2.5)

(b) On the Free Surface
On the mean free surface, both kinematic and dynamic conditions are applied. This results in the following well known linearized free surface condition valid for small amplitude oscillations,
\[ \frac{\partial \psi_k}{\partial x_3} - \frac{\omega^2}{g} \psi_k = 0 \text{ on } x_3 = 0, \quad k = 0, 1, 2, \ldots, 7 \] (2.6)
Here, \( g \) = acceleration due to gravity

(c) On the Body Surface
Boundary conditions applied on the average position of the wetted body surface are of the following forms,
\[ \frac{\partial \psi_k}{\partial n} = n_k, \quad k = 1, 2, \ldots, 6 \] (2.7a)
\[ \frac{\partial \psi_7}{\partial n} = - \frac{\partial \psi_0}{\partial n} \] (2.7b)
Here, \( \frac{\partial}{\partial n} \) is the normal derivative in the direction of the outward normal \( \hat{n} \) to the body surface. \( n_1 \) through \( n_6 \) are the generalized direction cosines given by,
\begin{equation}
\begin{align*}
n_1 &= \cos(n, x_1) \\
n_2 &= \cos(n, x_2) \\
n_3 &= \cos(n, x_3) \\
n_4 &= x_2 n_3 - x_3 n_2 \\
n_5 &= x_3 n_1 - x_1 n_3 \\
n_6 &= x_1 n_2 - x_2 n_1
\end{align*}
\end{equation}

(d) On Far-field

In order to ensure that the velocity potential has the correct behaviour in the far field, the following radiation condition is imposed,

\[ \psi_k(r_1, \theta, x_3) = H(\theta) r_2^{-1/2} \frac{\cosh[k(x_3 + d)]}{\cosh(kd)} e^{ikr_2} + 0 \quad \text{as} \quad r_2 \to \infty \quad (2.9)\]

where,

- \( r_1, \theta \) = polar co-ordinates
- \( x_2 = (x_1^2 + x_3^2)^{1/2} \)
- \( \theta = \tan^{-1}(x_2/x_1) \)
- \( H(\theta) \) = unknown complex function
- \( k \) = wave number

2.2 Solution of potentials

Equations (2.4) through (2.9) complete the formulation of the hydrodynamic boundary value problem, to be
Solved for obtaining the unknown potential functions $\Psi_k$,

$k = 0, 1, 2, \ldots 7$.

From the linear wave theory, the incident wave potential $\Psi_0$ is given by,

$$\Psi_0 = \frac{1}{v} \frac{\cosh[k(x_3 + d)]}{\cosh(\kappa d)} e^{i k(x_1 \cos \beta + x_2 \sin \beta)} \quad (2.10)$$

where, $\beta$ = angle of incidence of the incoming wave

($\beta = 0$ means waves along positive $x_1$ direction)

$k$ = wave number $= 2\pi / \lambda$

$\lambda$ = wave length

$v = \omega^2 / g$.

The wave number $k$ is related to the wave frequency $\omega$ by means of the well known dispersion relation in linear wave theory,

$$v = \frac{\omega^2}{g} = k \tanh(kd) \quad (2.11)$$

The potential function $\Psi_k$, $k = 1, 2, \ldots 7$ can be represented by a continuous distribution of sources on the wetted body surface $S$,

$$\Psi_k(x_1, x_2, x_3) = \frac{1}{4\pi S} \int \int \int \sigma_k(a_1, a_2, a_3) G(x_1, x_2, x_3; a_1, a_2, a_3) dS \quad (2.12)$$

where,

$a_1, a_2, a_3$ = a point on the body surface $S$

$\sigma_k(a_1, a_2, a_3)$ = unknown complex source strength function

$G(x_1, x_2, x_3; a_1, a_2, a_3)$ = the Green's function of a source, singular in $(a_1, a_2, a_3)$.
The above representation has been obtained by Lamb [9] for an infinite fluid case. It is here extended to the case of a fluid of finite depth with free surface [10].

For equation (2.12) to be valid, this particular Green's function which is for a wave source at the body surface must satisfy Laplace equation, boundary conditions on the sea floor, at free surface and at the infinity. Wehausen and Laitone [11] have provided the expression for $G$ appropriate to this particular boundary value problem in the following two forms,

(a) The Integral Form

$$G = \frac{1}{R} + \frac{1}{R_1} + \text{PV} \int_0^{\frac{2(\mu+v)e^{-\mu d}}{\mu \sinh(\mu d) - \nu \cosh(\mu d)}} J_0(\mu r) du + \frac{2\pi(k^2 - \nu^2)}{k^2 d - \nu^2 d + \nu} \cos(k(x_3 + d)) \cosh(k(a_3 + d)) J_0(kr)$$

(b) The Series Form

$$G = \frac{2\pi(v^2 - k^2)}{k^2 d - \nu^2 d + \nu} \cos(k(a_3 + d)) \cosh(k(x_3 + d)) [Y_0(kr) - iJ_0(kr)] + 4 \sum_{j=1}^{\infty} \frac{(\nu_j^2 + \nu^2)}{\nu_j^2 d + \nu^2 d - \nu} \cos[\nu_j(x_3 + d)] \cosh[\nu_j(a_3 + d)] X_0(\nu_j r)$$

In the above equations,

$$R = \left[ (x_1 - a_1)^2 + (x_2 - a_2)^2 + (x_3 - a_3)^2 \right]^{1/2}$$

$$R_1 = \left[ (x_1 - a_1)^2 + (x_2 - a_2)^2 + (x_3 + 2d + a_3)^2 \right]^{1/2}$$

$$r = \left[ (x_1 - a_1)^2 + (x_2 - a_2)^2 \right]^{1/2}$$
\[ J_0 = \text{Bessel function of the first kind of order zero} \]
\[ Y_0 = \text{Bessel function of the second kind of order zero} \]
\[ K_0 = \text{Modified Bessel function of the second kind of order zero} \]
\[ PV = \text{Cauchy principal value of the integral} \]

The quantities \( \mu_j \) are the positive solutions of the following equation,

\[ \mu_j \tan(\mu_j d) + v = 0 \quad (2.15) \]

The above equation follows from the derivation for \( G \) as given in [11], and is not to be linked with the dispersion relation (2.11).

The unknown source strength functions \( \sigma_k \) in equation (2.12) are to be determined such that the kinematic boundary conditions on the mean wetted surface (equations 2.7a, 2.7b) are fulfilled. This results in the following two-dimensional Fredholm integral equation of the second kind,

\[
-\frac{1}{2} \sigma_k(x_1, x_2, x_3) + \frac{1}{4\pi} \int_S \sigma'_k(a_1, a_2, a_3) \frac{3G}{\partial n}(x_1, x_2, x_3; a_1, a_2, a_3) dS = n_k \quad \text{for } k = 1, 2, \ldots, 6 \quad (2.16)
\]

\[
= -\frac{3\psi_0}{\partial n} \quad \text{for } k = 7
\]

where \( \frac{3G}{\partial n} \) represents the derivative of the Green's function in the direction of the outward normal vector \( \hat{n} \) and can be expressed as,
\[ \frac{\partial G}{\partial n} = \frac{\partial G}{\partial x_1} n_1 + \frac{\partial G}{\partial x_2} n_2 + \frac{\partial G}{\partial x_3} n_3 \]  

(2.17)

where \( n_1, n_2, n_3 \) are the three components of the unit normal vector as defined in equation (2.8). \( \frac{\partial G}{\partial x_1}, \frac{\partial G}{\partial x_2}, \frac{\partial G}{\partial x_3} \) can be obtained from straightforward differentiation of \( G \) given in equations (2.13) and (2.14). The expressions for these derivatives are given below.

(a) Derivatives of \( G \), series form

\[ \frac{\partial G}{\partial x_1} = A \cosh[k(x_3 + d)] \left[ -k \frac{(x_1 - a_1)}{r} Y_1(kr) + ik \frac{(x_1 - a_1)}{r} J_1(kr) \right] \]

\[ - \sum_{j=1}^{b} B(\mu_j) \cos[\mu_j(x_3 + d)] \mu_j \frac{(x_1 - a_1)}{r} K_1(\mu_j r) \]  

(2.18a)

\[ \frac{\partial G}{\partial x_2} = A \cosh[k(x_3 + d)] \left[ -k \frac{(x_2 - a_2)}{r} Y_1(kr) + ik \frac{(x_2 - a_2)}{r} J_1(kr) \right] \]

\[ - \sum_{j=1}^{b} B(\mu_j) \cos[\mu_j(x_3 + d)] \mu_j \frac{(x_2 - a_2)}{r} K_1(\mu_j r) \]  

(2.18b)

\[ \frac{\partial G}{\partial x_3} = A \sinh[k(x_3 + d)] \left[ Y_0(kr) - i J_0(kr) \right] \]

\[ - \sum_{j=1}^{b} B(\mu_j) \mu_j \sin[\mu_j(x_3 + d)] K_0(\mu_j r) \]  

(2.18c)
where,

\[
A = \frac{2\pi (v^2 - x^2)}{k^2d - v^2a + v} \cosh[k(a_2 + d)]
\]

\[
B(u_j) = \frac{4(v_j^2 + v^2)}{u_j^2d + v^2d - v} \cos[u_j(a_3 + d)]
\]

\[
J_1 = \text{Bessel function of the first kind of order one}
\]

\[
Y_1 = \text{Bessel function of the second kind of order one}
\]

\[
K_1 = \text{Modified Bessel function of the second kind of order one}
\]

(b) Derivatives of \( G \), Integral Form

\[
\frac{\partial G}{\partial x_1} = \frac{(x_1-a_1)}{R^3} - \frac{(x_1-a_1)}{R_1^3} - \text{PV} \int_0^\infty g_1(\mu) \cosh[\mu(x_3 + d)] \mu(x_1 - a_1) \, \frac{J_1(kr)}{r} \, d\mu = iD \cosh[k(x_3 + d)] \, \frac{J_1(kr)}{r} \] (2.19a)

\[
\frac{\partial G}{\partial x_2} = -\frac{(x_2-a_2)}{R^3} - \frac{(x_2-a_2)}{R_1^3} - \text{PV} \int_0^\infty g_1(\mu) \cosh[\mu(x_3 + d)] \mu(x_2 - a_2) \, \frac{J_1(kr)}{r} \, d\mu = iD \cosh[k(x_3 + d)] \, \frac{J_1(kr)}{r} \] (2.19b)

\[
\frac{\partial G}{\partial x_3} = -\frac{(x_3-a_3)}{R^3} - \frac{(x_3+2d+a_3)}{R_1^3} + \text{PV} \int_0^\infty g_1(\mu) \mu \sinh[\mu(x_3 + d)] \, \frac{J_1(kr)}{r} \, d\mu = iD \cosh[k(x_3 + d)]J_0(kr) \] (2.19c)

In the above,

\[
g_1(\mu) = \frac{2(\mu + v) e^{-\mu d} \cosh[\mu(a_3 + d)]}{\mu \sinh(\mu d) - \nu \cosh(\mu d)}
\]
\[ D = \frac{2\pi(k^2 - \nu^2)}{k^2d - \nu^2d + \nu} \cosh[k(a_3 + d)] \]

When \( r = 0 \), \( J_1(\mu r)/r \) term in (2.19a) and (2.19b) are replaced by 0.5\( \mu \).
CHAPTER 3

NUMERICAL FORMULATION

3.1 Numerical solution of potential

In order to obtain the unknown source strength functions \( \sigma_k \), it is now necessary to solve equation \( (2.16) \). The solution is obtained numerically, using a discretized solution scheme. The wetted body surface \( S \) is approximated by a sufficiently large number of plane quadrilateral surface panels or elements of area \( \Delta S_j \), \( j = 1, N (N = \text{total number of surface elements}) \). Theoretically, the continuous formulation of equation \( (2.16) \) indicates that this equation is to be satisfied at all points on the wetted body surface. However, to obtain a practical numerical solution, this requirement is relaxed and the equation is satisfied only at \( N \) points which are termed as control points. The control points, in principle, can be chosen arbitrarily. Here the centroid of the elements are chosen as control points for the reason of convenience.

In the following numerical formulation, suffix \( k' \) for the 6 modes of motion and diffraction potential has been omitted. It is to be understood that these equations apply to all \( k, k = 1, 2, \ldots 7 \).

Due to discretization, equation \( (2.16) \) now gets
transformed to a set of $N$ linear equations,
\[
\sum_{j=1}^{N} a_{ij} \sigma_j = b_i, \quad i = 1, 2, \ldots, N. \tag{3.1}
\]
in which the coefficients $a_{ij}$ and $b_i$ are respectively given by,
\[
a_{ij} = -\delta_{ij} + \frac{1}{2\pi} \int \frac{\partial G}{\partial \gamma} \left( x_{1i}, x_{2i}, x_{3i}; a_1, a_2, a_3 \right) ds \tag{3.2}
\]
and,
\[
b_i = 2n_i \quad \text{for} \quad k = 1, 2, \ldots, 6
\]
\[
= -2\frac{\partial \psi_0}{\partial n} (x_{1i}, x_{2i}, x_{3i}) \quad \text{for} \quad k = 7 \tag{3.3}
\]
In the above, $n_i$ for $k = 1, 2, \ldots, 6$ are the generalized direction cosines as defined in equation (2.8) for the control point $i$. $\frac{\partial \psi_0}{\partial n}$ can be obtained by straightforward differentiation of $\psi_0$ given in equation (2.10),
\[
\frac{\partial \psi_0}{\partial n} = i \left\{ \begin{array}{c}
k \cosh[k(x_3+\gamma)] e^{ik(x_1 \cos \beta + x_2 \sin \beta)} \left[n_1 \cos \beta + n_2 \sin \beta \right] \\
n_3 \frac{k}{v} \sinh[k(x_3+\gamma)] e^{ik(x_1 \cos \beta + x_2 \sin \beta)} \\
+ n_3 \frac{k}{v} \cosh[k(x_3+\gamma)] e^{ik(x_1 \cos \beta + x_2 \sin \beta)}
\end{array} \right. \tag{3.4}
\]
In equation (3.2), $\delta_{ij}$ is the Kronecker delta function, $\delta_{ij} = 0$ for $i \neq j$, $\delta_{ii} = 1$ and $(x_{1i}, x_{2i}, x_{3i})$ is the centroid or the control point of the $i^{th}$ element. In physical terms, $a_{ij}$ represents the velocity induced at the $i^{th}$ control point in the direction normal to the surface by a source.
distribution of unit strength distributed uniformly over the

\( j \)th element. When \( i = j \), \( \delta_{ij} = 1 \) and this term takes care
of the velocity at the control point due to a uniform source
distribution of that element, and the last term in equation
(3.2) should be neglected.

To carry out the integration in the second term of
equation (3.2) numerically, further assumption is necessary.
This integrand oscillates approximately with the wave length
\( \lambda \) which in practice is generally large, at least comparable
to the characteristic dimension of the immersed surface.

\( \frac{\partial G}{\partial n} \) for \( i \neq j \) thus vary slowly over \( \Delta S_j \) and can be assumed to
be constant over an element with the value equal to the value
at the centroid. This yields the following approximation of

\[ a_{ij} = -\delta_{ij} + \frac{\Delta S_j}{2\pi} \frac{\partial G}{\partial n} (x_{1i}, x_{2i}, x_{3i} ; a_{1j}, a_{2j}, a_{3j}) \]  

(3.5)

where \( (a_{1j}, a_{2j}, a_{3j}) \) is the \( j \)th control point.

Thus, it is now possible to evaluate the matrix
\( [a_{ij}] \) and the column vector \( \{b_1\} \). The unknown source
distribution function \( \sigma_j \) is now easily determined using a
complex matrix inversion procedure.

By a similar method of discretization, equation
(2.12) can be written as,
\[ \psi(x_{1i}, x_{2i}, x_{3i}) = \sum_{j=1}^{N} \beta_{ij} \sigma_j \]  \hspace{1cm} (3.6)

where,
\[ \beta_{ij} = \frac{1}{4\pi} \iint_{\Delta S_j} G(x_{1i}, x_{2i}, x_{3i}, a_1, a_2, a_3) dS \]  \hspace{1cm} (3.7)

To evaluate the above integration numerically, a similar assumption is made regarding the value of \( G \) over an element as was made for \( \partial G/\partial n \), for the same reason. Thus, assuming \( G \) constant over the element with its value same as at the centroid, the following approximation of \( \beta_{ij} \) is obtained,
\[ \beta_{ij} = \frac{\Delta S_j}{4\pi} G(x_{1i}, x_{2i}, x_{3i}, a_1, a_2, a_3) \]  \hspace{1cm} (3.8)

When \( i = j \), this particular case must now be carefully considered, since in this case a singularity of the form \( \frac{1}{R} \), \( R = 0 \) occurs in \( G \). Clearly, the above approximation of \( \beta_{ij} \) cannot be used for evaluating \( \beta_{ii} \). The singular term in \( G \) is more dominant than the regular term in \( G \) for \( i = j \), and hence this singular term alone is considered for the case \( i=j \). Thus,
\[ \beta_{ii} = \frac{1}{4\pi} \iint_{\Delta S_i} \frac{1}{R} dS \]  \hspace{1cm} (3.9)

For evaluating the above integral, the formulation given by Faltinsen and Michelsen [3] is used. For a plane
quadrilateral element, firstly the integral is written in

-terms of the local co-ordinates \( \bar{X}, \bar{Y}, \bar{Z} \) and \( \bar{\xi}, \bar{\eta} \), where \( \bar{X}, \bar{Y} \) and

-\( \bar{\xi}, \bar{\eta} \) axes are in the plane of the quadrilateral element

(Figure 2). This integral for a general point \( P \) in space

-having local co-ordinates \( (\bar{x}, \bar{y}, \bar{z}) \) is,

\[
\int \int \frac{1}{R} \, ds = \int \int \frac{d\bar{\xi} \, d\bar{\eta}}{[x - \xi]^2 + (y - \eta)^2 + z^2]^{1/2}} \quad (3.10)
\]

This integration can be performed analytically yielding the

-following,

\[
\int \int \frac{1}{R} \, ds = \int_{\xi_1}^{\xi_2} d\xi \, \ln(\bar{y} - \bar{\eta}_{12} + \frac{[y - \bar{\eta}_{12}]^2 + (x - \xi)^2 + z^2]^{1/2}}{\xi_1} \]

\[
\int_{\xi_2}^{\xi_3} d\xi \, \ln(y - \bar{\eta}_{23} + \frac{(y - \bar{\eta}_{23})^2 + (x - \xi)^2 + z^2]^{1/2}}{\xi_2} \]

\[
\int_{\xi_3}^{\xi_4} d\xi \, \ln(y - \bar{\eta}_{34} + \frac{(y - \bar{\eta}_{34})^2 + (x - \xi)^2 + z^2]^{1/2}}{\xi_3} \]

\[
\int_{\xi_4}^{\xi_1} d\xi \, \ln(y - \bar{\eta}_{41} + \frac{(y - \bar{\eta}_{41})^2 + (x - \xi)^2 + z^2]^{1/2}}{\xi_4} \]

where,

\[
\bar{\eta}_{1j} = \bar{\eta}_1 + \frac{\bar{\eta}_j - \bar{\eta}_1}{\xi_j - \xi_1} (\bar{\xi} - \bar{\xi}_1)
\]
\( \eta_i, \xi_i = (\eta, \xi) \) co-ordinates of the corner points of the element.

All the integrals in equation (3.11) are performed numerically. Since this is evaluated only for \( \beta_{ij} \) in equation (3.9), point \( P \) in this case is at the centroid of the panel, thus \( \tilde{z} = 0 \). A singularity in the integrands occurs when \( \xi = \bar{x} \) and \( y - \eta_{ij} < 0 \). In such a case, integration about the immediate neighbourhood of the singular point \( (\bar{x} - \epsilon) \) and \( (\bar{x} + \epsilon) \) is avoided. For computer evaluation, \( \epsilon \) has been successively reduced until the integral converges to a given limit.

For a rectangular element of aspect ratio \( b \), an analytical expression has been derived by Garrison [10] when \( P \) is at the centroid of the element. This expression given below is used when the element is rectangular.

\[
\frac{1}{R} \int_{A} dS = 2 \left( \frac{\Delta S}{b} \right)^{1/2} \left\{ \ln[b+(b^2+1)^{1/2}]+b \ln[1+(b^2+1)^{1/2}] \right\}
\]

(3.12)

After evaluating \([\beta_{ij}]\), the potential function \( \psi(x_{1i}, x_{2i}, x_{3i}) \) is easily determined from equation (3.6).

3.2 Numerical evaluation of Green's function

Although the two forms of the Green's function given in equation (2.13) and (2.14) are equivalent, one of
the two forms may have preference for numerical computation, depending on the value of the variables. In general, the series form converges rapidly due to $K_0(\nu_j r)$ term. However, when $kr = 0$, the Bessel function $K_0(\nu_j r) = \infty$ and so the series form cannot be used for very small values of $kr$. Here the series form is used for $kr > 0.01$ and the more time consuming integral form is used for $kr < 0.01$.

Equation (2.15) has been solved using Newton-Raphson iteration method which converges fast. The evaluation of Green's function and its derivatives through the series form is rather straightforward and no major numerical difficulties are encountered. A convergence criterion is used to terminate the series when required convergence is reached.

The integral form is evaluated after breaking down the infinite upper limit of the integral into two parts, 0 to $2k$ and $2k$ to $\infty$. The integral over the interval 0 to $2k$ can be further broken down and written in the following form,

$$\int_0^{2k} \frac{F(\mu) d\mu}{\mu \tanh(\mu d) - \nu} = \int_0^{2k} \frac{F(\mu) - F(k)}{\mu \tanh(\mu d) - \nu} d\mu + F(k) \int_0^{2k} \frac{1}{\mu \tanh(\mu d) - \nu} d\mu$$

The first integral in the right hand side of the above equation is now finite at all points within the interval and can be numerically integrated. The second integral can be divided into the following intervals,
All the integrals are evaluated using numerical integration procedure except for the integral within the limit \((k-\varepsilon)\) to \((k+\varepsilon)\), which contains a singularity of the form \(1/(\mu-k)\). The integrand is expanded in the power of \((\mu-k)\) and only terms up to first order are considered,

\[
\frac{1}{\mu \tanh(\mu d) - \nu} = \frac{c_1}{(\mu-k)} + c_0 + c_1(\mu-k) + \ldots
\]

Each term is now integrated giving the following result,

\[
\int_{k-\varepsilon}^{k+\varepsilon} \frac{1}{\mu \tanh(\mu d) - \nu} d\mu = -\frac{\text{sech}^2(kd)[1-kd \tanh(kd)]}{[\tanh(kd) + kd \text{sech}^2(kd)]^2} (2\varepsilon) + O(\varepsilon^3)
\]

For the purpose of computation, a value of \(\varepsilon = 0.1k\) is chosen as suggested by Garrison [10].

To evaluate the integral within the interval \(2k\) to \(\infty\), trapezoidal rule is used and the integration is terminated when the contribution to the integral becomes sufficiently small. A convergence criterion is used for this purpose. When \(\mu\) is large, the integrand decays as \(\exp[\mu(x_3 + a_3)]\). To take advantage of this situation, a progressively larger stepsize is used for higher values of \(\mu\), thus saving valuable CPU time. A stepsize of 0.1\mu or 0.3/r, whichever is less is
chosen. This is sufficient to represent the denominator 

\[ [u \sinh(\mu d) - v \cosh(\mu d)] \] and \( J_0(\mu r) \) accurately [12].

3.3 Wave forces, moments and motion response.

Once all the potentials \( \psi_k \), \( k = 1, 2, \ldots, 7 \) are determined, the first order wave exciting forces and moments can be readily determined through a use of linearized Bernoulli's equation. They can be written as,

\[
f_k = -\rho u^2 \gamma_0 e^{-i\omega t} \int_S (\psi_0 + \psi_1) n_k \, ds, \quad k = 1, 2, \ldots, 6 \quad (3.15)
\]

where, \( f_k \) = first order wave exciting forces/moments for \( k \)th mode

- \( \rho \) = mass density of water

The exciting forces and moments can also be expressed in terms of the incident and radiation potentials and their normal derivatives by means of the Haskind relation. In the method of computation presented in this thesis, the matrices \([\alpha_{ij}]\) and \([\beta_{ij}]\) containing \( \partial G/\partial n \) and \( G \) terms respectively are to be calculated, and the inversion of \([\alpha_{ij}]\) is to be carried out in order to compute the radiation potentials. These are the most complex and time consuming parts of the calculation. Computation of the diffraction potential involves only a simple matrix multiplication. Thus it was felt more convenient to use the above expression for computing exciting forces and moments instead of using Haskind relation which requires calculation of the normal derivatives of the radiation potentials.
The oscillatory hydrodynamic forces \((k = 1, 2, 3,\) and moments \((k = 4, 5, 6,\) in the \(k^{th}\) mode are given by,

\[
x_k = -\rho \omega^2 \sum_{j=1}^{6} \xi_j e^{-i\omega t} \int_S \psi_j n_k \, ds
\]

\[(3.15a)\]

The added mass and damping coefficients are expressed in their usual forms,

\[
a_{jk} = -\rho \text{ Re} \int_S \psi_k n_j \, ds
\]

\[
b_{jk} = -\rho \omega \text{ Im} \int_S \psi_k n_j \, ds
\]

\[(3.16)\]

where,

\[
a_{jk} = \text{ added mass coefficient in } j^{th} \text{ mode due to motion in } k^{th} \text{ mode}
\]

\[
b_{jk} = \text{ damping coefficient in } j^{th} \text{ mode due to motion in } k^{th} \text{ mode}
\]

\[
\text{Re} = \text{ real part of the integral}
\]

\[
\text{Im} = \text{ imaginary part of the integral}
\]

By applying Green's theorem to the expression for added mass and damping coefficients given above, it can be easily seen that the coefficients are symmetric; that is:

\[
a_{kj} = a_{jk}, \quad b_{kj} = b_{jk}
\]
The well known equations of motion are now used to determine the motion response to the first order excitation in frequency domain.

\[ \sum_{j=1}^{6} \left[ (M_{kj} + a_{kj}) \xi_j + b_{kj} \xi_j + c_{kj} \xi_j \right] = f_k, \quad k = 1, 2, \ldots, 6 \quad (3.17) \]

Substituting \( \xi_j = \xi_j e^{i\omega t} \) and \( f_k = |f_k| e^{-i\omega t} \), we get the following set of linear equations,

\[ \sum_{j=1}^{6} \left[ -\omega^2 (M_{kj} + a_{kj}) - i\omega b_{kj} + c_{kj} \right] \xi_j = |f_k| \quad (3.18) \]

where,

- \( |f_k| \) = amplitude of wave exciting forces/moments in the \( k \)th mode
- \( M_{kj} \) = inertia matrix
- \( c_{kj} \) = hydrostatic restoring coefficient matrix

\( M_{kj} \) is given by,

\[
\begin{bmatrix}
  m & 0 & 0 & 0 & \text{mx}_{3G} & 0 \\
  0 & m & 0 & \text{-mx}_{3G} & 0 & 0 \\
  0 & 0 & m & 0 & 0 & 0 \\
  0 & \text{-mx}_{3G} & 0 & I_{44} & I_{45} & I_{46} \\
  \text{mx}_{3G} & 0 & 0 & \text{-I}_{54} & I_{55} & \text{-I}_{56} \\
  0 & 0 & 0 & \text{-I}_{64} & \text{-I}_{65} & I_{66}
\end{bmatrix}
\quad (3.19)
where,

\[ m = \text{mass of the body} \]
\[ I_{jk} = \text{moment of inertia with respect to the} \]
\[ \text{co-ordinate system } Ox_1x_2x_3 \text{ shown in Figure 1} \]
\[ x_{3G} = x_3 \text{ co-ordinate of centre of gravity} \]

The moment of inertia terms are defined as,

\[
I_{jk} = I_{kj} = \int_m x_j x_k - 3 x_3 x_j - 3 \, dm, \quad j = 4, 5, 6; \ k = 4, 5, 6 \tag{3.20}
\]

For a body symmetric about \( x_1x_3 \) plane, \( I_{45} = I_{54} = 0 \)
\( I_{56} = I_{65} = 0 \)

The non-zero terms of the hydrostatic restoring matrix \([c_{jk}]\) for a general shape are,

\[
c_{33} = \rho g A_{wp}
\]
\[
c_{43} = c_{34} = \rho g \int A_{wp} x_2 \, ds
\]
\[
c_{44} = \rho g V (x_{3B} - x_{3G}) + \rho g \int A_{wp} x_2^2 \, ds
\]
\[
c_{35} = c_{53} = -\rho g \int A_{wp} x_1 \, ds
\]
\[
c_{45} = c_{54} = -\rho g \int A_{wp} x_1 x_2 \, ds
\]
\[ q_{55} = \rho g V (x_{3B} - x_{3G}) + \int_{A_{wp}} \rho g \int x_1^2 \, ds \]  

(3.21)

In the above,

- \( A_{wp} \) = area of the waterplane
- \( V \) = immersed volume of the body
- \( x_{3B} \) = \( x_3 \) co-ordinate of centre of buoyancy

If \( x_1 x_3 \) is a plane of symmetry for the body,

- \( c_{34} = c_{43} = c_{45} = c_{54} = 0 \).

From equation (3.18), the complex motion amplitudes \( \zeta_j \) for all six modes of motion are now easily determined using a complex matrix inversion procedure.

This completes the numerical formulation of the problem. The integrations in equations (3.15), (3.15a) and (3.16) are performed numerically, assuming the integrand to be constant over each element.
CHAPTER 4

COMPUTED RESULTS

A computer program has been written based on the theoretical and numerical formulation given above. The input information required are the geometry of the body, mass and various radii of gyrations (roll, pitch, yaw, roll-pitch, roll-yaw and yaw-pitch), vertical co-ordinate of the centre of gravity \( x_{3g} \), water depth \( d \), heading angles \( \theta \) and incoming wave lengths \( \lambda \). The program does not calculate the hydrostatic restoring coefficients which depend entirely on the geometry of the body and are rather straightforward to calculate. These are to be given as input data. Subdivision of the immersed body surface into plane quadrilateral elements is to be done by the user and the co-ordinates of the element vertices are to be given as input data. The program is in two parts. The first part calculates the element centroid, the components of the outward normal, the area of the element, and the integral as given by equation (3.11). The output of the first part of the program is the major input for the second which is the major aspect of the program. The final results obtained are the wave exciting forces, the moments and motion response in six degrees of freedom of a floating marine structure of an arbitrary shape. Listing of both the programs are given in Appendix A. The input data is in a free floating format form. Appendix B
shows a typical input data for the second part of the program for the semisubmersible. The values are in a non-dimensional form and the non-dimensionalizing factors used are as follows.

a) Surge, sway and heave added mass co-efficients,
\[
(|A11|, |A22|, |A33|) = \left(\frac{a_{11}, a_{22}, a_{33}}{\rho V}\right)
\]
b) Roll, pitch and yaw added mass co-efficients,
\[
(|A44|, |A55|, |A66|) = \left(\frac{a_{44}, a_{55}, a_{66}}{\rho V L^2}\right)
\]
c) Surge, sway and heave damping co-efficients,
\[
(|B11|, |B22|, |B33|) = \left(\frac{b_{11}, b_{22}, b_{33}}{\rho V(g/L)}\right)
\]
d) Roll, pitch and yaw damping co-efficients,
\[
(|B44|, |B55|, |B66|) = \left(\frac{b_{44}, b_{55}, b_{66}}{\rho V L^2(g/L)}\right)
\]
e) Surge, sway and heave exciting force amplitudes,
\[
(|F1|, |F2|, |F3|) = \left(\frac{f_1, f_2, f_3}{\rho g V \zeta_0/L}\right)
\]
f) Roll, pitch and yaw exciting moment amplitudes,
\[
(|F4|, |F5|, |F6|) = \left(\frac{f_4, f_5, f_6}{\rho g V \zeta_0}\right)
\]
g) Surge, sway and heave motion amplitudes,
\[
(|\eta_1|, |\eta_2|, |\eta_3|) = \left(\frac{\zeta_1, \zeta_2, \zeta_3}{\zeta_0}\right)
\]
h) Roll, pitch and yaw motion amplitudes,
\[
(|\eta_4|, |\eta_5|, |\eta_6|) = \left(\frac{\zeta_4, \zeta_5, \zeta_6}{\zeta_0/L}\right)
\]
i) Non-dimensional frequency,
\[
\omega_n = \frac{\omega(L/g)}{}
\]
L in the above is the characteristic dimension of the body.
Computations are performed for various floating objects. Here the following results are presented.

A. Rectangular Box

Computations for a floating box of length 90 m, breadth 90 m and draft 20 m floating in water of depth 200 m are performed. The geometrical properties of the box are,

Centre of gravity co-ordinates (CG) = 0, 0, 8.82 m
Roll radius of gyration, \( r_{x1} \) = 37.32 m
Pitch radius of gyration, \( r_{x2} \) = 37.30 m
Yaw radius of gyration, \( r_{x3} \) = 40.08 m

Two sets of calculations are performed using a total of 48 and 108 elements to represent the box. \( L = 90 \) m is used for non-dimensionalization. The non-dimensional added mass and damping co-efficients, exciting force and moment amplitudes and phase angles, motion amplitudes and phase angles for heading angle \( \beta = 0 \) deg. are presented in Figures 3 through 14. This particular example is chosen to present the comparison of the results with those available in [3].

B. Vertical Circular Cylinder

Calculations are performed for a short vertical circular cylinder of radius \( a = 10 \) m and draft \( T = 0.5a = 5 \) m. The following geometrical properties are used for the purpose of computation.
CG = 0, 0, 0 m (at the origin of the co-ordinate system)

\[ r_{x_1} = 0.5a = 5 \text{ m} \]

\[ r_{x_2} = 0.5a = 5 \text{ m} \]

\[ r_{x_3} = 0.707a = 7.07 \text{ m} \]

A total of 60 surface elements are used to idealize the body. Calculations are made for three different water depths, \( d = 10, 15 \) and 50 m. For non-dimensionalization, the diameter of the cylinder is taken as characteristic dimension of the body, which means \( l = 2a = 20 \text{ m} \) is used. The results of computation are presented in Figures 15 through 22. This example is chosen since some of the results computed by Garrison based on the same theory are available in [10]. Figure 23 shows Garrison’s computations for surge mode.

C. Tanker

Wave exciting forces, moments and motion response of a 130,000 tons dwt tanker moored in water of depth 500 ft (152.4 m) are computed. The geometry of the tanker is shown in Figure 24. Two different conditions of loading are considered, ballast and fully loaded. The geometrical properties of the tanker are given in Table 1.

A total of 196 elements for ballast condition and 208 elements for loaded condition are used. Computations are performed for three different heading angles, \( \beta = 0, 45 \) and 90 deg. Length between perpendiculars is used as
characteristic dimension of the tanker. Calculations are also performed using two-dimensional strip theory for comparison. The results are presented in Figures 25 through 45.

Results of motion response for this tanker for both loaded and ballast conditions using DnV program are available in [13] and are shown in Figures 46 through 49 for the purpose of comparison.

D. Semisubmersible

Finally, to demonstrate the effectiveness and usefulness of the program, computations are performed for a semisubmersible. Figure 50 shows the sectional views [15]. The geometrical data of the semisubmersible are as follows,

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>20869 tonnes</td>
</tr>
<tr>
<td>Length</td>
<td>90 m</td>
</tr>
<tr>
<td>Beam</td>
<td>75 m</td>
</tr>
<tr>
<td>Draft</td>
<td>18.5 m</td>
</tr>
<tr>
<td>Metacentric height, transverse</td>
<td>2.62 m</td>
</tr>
<tr>
<td>Metacentric height, longitudinal</td>
<td>2.67 m</td>
</tr>
<tr>
<td>( r_{x1} )</td>
<td>30.22 m</td>
</tr>
<tr>
<td>( r_{x2} )</td>
<td>26.88 m</td>
</tr>
<tr>
<td>( r_{x3} )</td>
<td>36.92 m</td>
</tr>
</tbody>
</table>
A total of 244 elements are used to represent the semisubmersible. \( L = 90 \, \text{m} \) is used for non-dimensionalization. The computed results are presented in Figures 51 through 68.
DISCUSSIONS AND CONCLUDING REMARKS

To check the present computations, the results are compared with other available results based on the same three dimensional singularity distribution theory. In general, an excellent agreement is found between the results. The results for the rectangular box calculated by Faltinsen and Michelsen [3] using 68 surface elements are plotted in Figures 3 through 14. It can be easily seen that the results are in good agreement. The only significant differences are observed in heave exciting force amplitude at higher periods, and pitch exciting moment at lower periods.

Results for the vertical circular cylinder are also compared with the results calculated by Garrison [10] and again a good agreement is found. In Figure 23, present results are plotted against Garrison's results for surge mode.

To determine the effect of the number of surface elements, the rectangular box calculations are performed using both 48 and 108 elements. The observations are same as in [3]. For most of the cases, 48 panels are sufficient to obtain reasonably accurate results. However, for some
 rotational mode calculations \((k = 4, 5, 6)\), there are some differences between the results using 48 and 108 elements. This is to be expected, since the rotational mode calculations are more sensitive to the correct representation of the geometry. They depend on \(r \times n\) terms whereas the linear mode calculations depend on \(n\) terms (\(r\) is the position vector of any point on the body surface). A difference in heave damping coefficient is also noted.

Comparing the effect of the number of elements on the computed results of the heave and pitch motion responses (figures 13 and 14 respectively), it can be seen that the differences are more significant for the pitch motion and they extend over the entire frequency range. For the heave motion, the differences are significant over the resonant frequency region but are not so pronounced over the other range. The differences in results at peak period (resonant frequency) are about 1.5 times for heave motion while they differ by more than 3 times for pitch motion. It is to be noted that calculations in the region of the resonance frequency are sensitive to the number of elements and would require a careful evaluation.

To compare the results using three-dimensional singularity distribution method and two-dimensional strip theory, the tanker added mass and damping coefficients calculated by strip theory are plotted in Figures 25 to 40. The agreement between the results is not generally very good.
This is again in agreement to the observation made in [3]. An interesting observation is that the agreement between them improves towards lower time period or higher frequency range. This is expected, since strip theory is known to give better and more reliable results at higher frequencies.

The motion response of the tanker is compared with the results computed using DnV program based on the same singularity distribution theory (Figures 46 through 49). The results are in reasonably good agreement and have comparable range of values. There does not appear to be a correlation for the pitch motion between the results computed here and the result of DnV. Similarly for the sway at beam sea conditions, there is some disagreement between the two results. The tanker geometry was obtained from the small scaled body plan given in [13] which was enlarged for the purpose of dividing the hull into surface elements. This could be a major input deficiency in comparing the results and is an aspect to be examined further.

The computed results of the semisubmersible could not be directly compared since there are no available data of the exactly same configuration, whether based on the same theory or any other theories or experimental results. However, it is possible to compare the nature and trend of the computed hydrodynamic coefficients, wave exciting forces and moments and motion responses with similar kind of
structures. For example, in [8] some results of a Staflo drilling platform based on a different theory by Hooft are available. They show a similar trend, and the range of values of the various non-dimensional results are quite comparable.

The main disadvantage of the present three-dimensional singularity distribution method is the enormous volume of computation that is required. It is possible to achieve a reduction in the computation time if the object has one or more planes of symmetry. At present no such assumption about the geometrical symmetry is made, even though floating objects usually have at least one plane of symmetry. For a total of 48 surface elements, the CPU time is a little less than 2 minutes for one wave length in VAX 780/11 system. Most of the CPU time used is for forming the $[\alpha_{ij}]$ and $[\beta_{ij}]$ matrices given in equations (3.5) and (3.8), which contain $\partial G/\partial \alpha$ and $G$ terms respectively. The total number of elements used to describe the body has a very significant effect on the CPU time. For 196 elements, the CPU time for one frequency is about 35 minutes. It is thus necessary to use as few surface elements as possible to describe the surface sufficiently accurately, without losing the reliability of the calculated results. Many guidelines have been proposed by various investigators, mostly based on experience rather than rigid theoretical principle regarding the size and total
number of elements [14]. To ensure that the body is divided into a sufficiently fine mesh, the element lengths should be less than \( \frac{1}{8} \) th of the incoming wave length \( \lambda \). This implies that for accuracy of computation at higher frequencies or lower incoming wave lengths, a larger number of elements should be used. Fortunately, for large floating marine structures such as semisubmersibles, the frequency range of interest is usually not as large and hence this problem does not become too restrictive. Also the neighbouring element sizes should not be widely different, which means that a large element should not be surrounded by comparatively very small elements. This results in computational inefficiencies as the precision offered by smaller elements is lost. Since only plane quadrilateral elements are used, a large number of elements should be used to describe the highly curved regions. It is also preferable to use as squarely shaped elements as possible. This means, for rectangular elements, aspect ratio closer to one is preferable. The evaluation of \( \int \frac{1}{R} ds \) in equation (3.9) results in more numerical inaccuracies for thin long elements compared to a squarely one. It must be remembered that this integral forms the dominant diagonal elements in matrix \( [\mathbf{B}] \). A detailed parametric study on the aspect ratio requirements of the elements for the same geometry is outside the scope of the work presented in the thesis.
One more point which should be noted here is the case of so-called 'irregular' frequencies. At these frequencies, matrix \([a_{ij}]\) in equation (3.5) becomes singular and thus the problem cannot be solved by using the integral formula in equation (2.12). So far there has been no theoretical method developed to determine such irregular frequencies for geometries of arbitrary shape. For certain regular geometrical shapes, like vertical circular cylinder, these frequencies can be analytically determined [4]. Usually these frequencies correspond to wave lengths of the order of or less than the characteristic length of the body. So far any such problem of irregular frequencies has not been encountered in the present calculations. At this time, a physical explanation for this phenomenon is not obvious. This aspect of the singularity and its interpretation thereof in a subject for further research.

Finally, the computations performed and presented in this thesis show that the program developed calculates the first order wave exciting forces/moments and motion responses in six degrees of freedom correctly, comparing the results with other computations based on the same theoretical model. Also, the computations for semisubmersible demonstrate the versatility and usefulness of the program.
REFERENCES


<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>UNIT</th>
<th>CONDITION</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ballast</td>
<td>Loaded</td>
<td></td>
</tr>
<tr>
<td>Length between perpendiculars</td>
<td>m</td>
<td>285.60</td>
<td>285.60</td>
<td></td>
</tr>
<tr>
<td>Beam</td>
<td>m</td>
<td>46.71</td>
<td>46.71</td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>m</td>
<td>20.35</td>
<td>20.35</td>
<td></td>
</tr>
<tr>
<td>Draft, fore</td>
<td>m</td>
<td>4.84</td>
<td>13.82</td>
<td></td>
</tr>
<tr>
<td>Draft, aft</td>
<td>m</td>
<td>7.04</td>
<td>13.82</td>
<td></td>
</tr>
<tr>
<td>Draft, mean</td>
<td>m</td>
<td>5.94</td>
<td>13.82</td>
<td></td>
</tr>
<tr>
<td>Longitudinal centre of gravity (+ve means forward of midship)</td>
<td>m</td>
<td>+2.10</td>
<td>+6.46</td>
<td></td>
</tr>
<tr>
<td>Vertical Centre of gravity from baseline</td>
<td>m</td>
<td>9.73</td>
<td>11.03</td>
<td></td>
</tr>
<tr>
<td>Metacentric height, transverse</td>
<td>m</td>
<td>21.50</td>
<td>8.97</td>
<td></td>
</tr>
<tr>
<td>Pitch/yaw radius of gyration</td>
<td>m</td>
<td>71.40</td>
<td>71.40</td>
<td></td>
</tr>
<tr>
<td>Roll Radius of gyration</td>
<td>m</td>
<td>16.35</td>
<td>16.35</td>
<td></td>
</tr>
</tbody>
</table>
CO-ORDINATE SYSTEM AND GEOMETRICAL BOUNDARIES

FIGURE - 1
LOCAL CO-ORDINATE SYSTEM OF A PLANE

QUADRILATERAL ELEMENT

FIGURE - 2
SURGE ADDED MASS COEFFICIENT FOR FLOATING BOX

Figure - 3

HEAVE ADDED MASS COEFFICIENT FOR FLOATING BOX

Figure - 4
Pitch Added Mass Coefficient for Floating Box

Figure 5

Yaw Added Mass Coefficient for Floating Box

Figure 6
SURGE DAMPING COEFFICIENT FOR FLOATING BOX

FIGURE - 7

HEAVE DAMPING COEFFICIENT FOR FLOATING BOX

FIGURE - 8
SURGE EXCITING FORCE ON FLOATING BOX
Amplitudes and Phases

Figure 9
HEAVE EXCITING FORCE ON FLOATING BOX
Amplitudes and Phases

FIGURE - 10
PITCH EXCITING MOMENT ON FLOATING BOX
Amplitudes and Phases

FIGURE - 11
SURGE MOTION OF FLOATING BOX
Non-dimensional Amplitudes and Phases

FIGURE - 12
HEAVE MOTION OF FLOATING BOX
Non-dimensional Amplitudes and Phases
FIGURE - 13
PITCH MOTION OF FLOATING BOX
Non-dimensional Amplitudes and Phases

FIGURE 14
SURGE ADDED MASS FOR VERTICAL CIRCULAR CYLINDER

Figure 15

HEAVE ADDED MASS FOR VERTICAL CIRCULAR CYLINDER

Figure 16
SURGE EXCITING FORCE ON VERTICAL CIRCULAR CYLINDER
Amplitudes and Phases
FIGURE 17
HEAVE EXCITING FORCE ON VERTICAL CIRCULAR CYLINDER
Amplitudes and Phases

Figure - 19
PITCH EXCITING MOMENT ON VERTICAL CIRCULAR CYLINDER
Amplitudes and Phases

FIGURE 19
SURGE MOTION OF VERTICAL CIRCULAR CYLINDER
Non-dimensional Amplitudes and Phases

FIGURE - 20
HEAVE MOTION OF VERTICAL CIRCULAR CYLINDER
Non-dimensional Amplitudes and Phases

FIGURE - 21
PITCH MOTION OF VERTICAL CIRCULAR CYLINDER
Non-dimensional Amplitudes and Phases

FIGURE - 22
RESULTS

VERTICAL CYLINDERS FOR SURGE MODE

GARRISON'S RESULTS FOR VERTICAL CIRCULAR CYLINDER FOR SURGE MODE

FIGURE - 23
GEOMETRY OF THE TANKER

FIGURE - 24
SURGE ADDED MASS COEFF. FOR TANKER (BALLAST)
FIGURE - 25

SWAY ADDED MASS COEFF. FOR TANKER (BALLAST)
FIGURE - 26
HEAVE ADDITIONAL MASS COEFFICIENT FOR TANKER (BALLAST)  
FIGURE 27

PITCH ADDITIONAL MASS COEFFICIENT FOR TANKER (BALLAST)  
FIGURE 28
SURGE DAMPING COEFF. FOR TANKER (BALLAST)

FIGURE - 29

SWAY DAMPING COEFF. FOR TANKER (BALLAST)

FIGURE - 30
HEAVE DAMPING COEFF. FOR TANKER (BALLAST)

PITCH DAMPING COEFF. FOR TANKER (BALLAST)
SURGE ADDED MASS COEFF. FOR TANKER (LOADED)

SWAY ADDED MASS COEFF. FOR TANKER (LOADED)
HEAVE ADDED MASS COEFF. FOR TANKER (LOADED)

PICTCH ADDED MASS COEFF. FOR TANKER (LOADED)
SURGE DAMPING COEFF. FOR TANKER (LOADED)

Figure - 37

SWAY DAMPING COEFF. FOR TANKER (LOADED)

Figure - 38
HEAVE DAMPING COEFF. FOR TANKER (LOADED)

FIGURE - 39

PITCH DAMPING COEFF. FOR TANKER (LOADED)

FIGURE - 40
MOTION RESPONSE OF THE TANKER (BALLAST COND.):
(Surge, Sway and Heave)

FIGURE - 41
MOTION RESPONSE OF THE TANKER (BALLAST COND.
(Pitch and Yaw)

FIGURE - 42
MOTION RESPONSE OF THE TANKER (LOADED Cond.)
(Surge, Sway and Heave)

FIGURE - 43
MOTION RESPONSE OF THE TANKER (LOADED CONDITION) (Pitch and Yaw)

FIGURE - 44
ROLL RESPONSE OF THE TANKER  
(Ballast and Loaded Conditions)  

FIGURE - 45
DnV RESULTS FOR MOTION RESPONSE OF THE TANKER, BALLAST (SURGE, SWAY AND HEAVE)

FIGURE - 46

DnV RESULTS FOR MOTION RESPONSE OF THE TANKER, BALLAST (ROLL, PITCH AND YAW)

FIGURE - 47
Figure 48

DnV results for motion response of the tanker, loaded (surge, sway and heave)

Figure 49

DnV results for motion response of the tanker, loaded (roll, pitch and yaw)
SURGE ADDED MASS COEFF. FOR SEMISUBMERSIBLE

FIGURE - 51

SWAY ADDED MASS COEFF. FOR SEMISUBMERSIBLE

FIGURE - 52
HEAVE ADDED MASS COEFF. FOR SEMISUBMERSIBLE

FIGURE - 53

ROLL ADDED MASS COEFF. FOR SEMISUBMERSIBLE

FIGURE - 54
PITCH ADDED MASS COEFF. FOR SEMISUBMERSIBLE

FIGURE - 55

YAW ADDED MASS COEFF. FOR SEMISUBMERSIBLE

FIGURE - 56
SURGE EXCITING FORCE ON SEMISUBMERSIBLE.

Figure - 57

SWAY EXCITING FORCE ON SEMISUBMERSIBLE

Figure - 58
ROLL EXCITING MOMENT ON SEMISUBMERSIBLE  
**Figure - 60**

HEAVE EXCITING FORCE ON SEMISUBMERSIBLE  
**Figure - 59**
PITCH EXCITING MOMENT ON SEMISUBMERSIBLE

FIGURE - 61

YAW EXCITING MOMENT ON SEMISUBMERSIBLE

FIGURE - 62
SURGE MOTION OF SEMISUBMERSIBLE

FIGURE - 63

SWAY MOTION OF SEMISUBMERSIBLE

FIGURE - 64
HEAVE MOTION OF SEMISUBMERSIBLE

Figure - 65

ROLL MOTION OF SEMISUBMERSIBLE

Figure - 66
PITCH MOTION OF SEMISUBMERSIBLE

FIGURE - 67

YAW MOTION OF SEMISUBMERSIBLE

FIGURE - 68
APPENDIX A

LISTING OF THE COMPUTER PROGRAM
PART I

This program calculates the area (SAREA), co-ordinates of the centroid (XCG, YCG, ZCG), the three components of the outward normal and the shape factors of the body-surface panels.

The shape factor (SHFACT) is a factor relating the integration of the singular term in Green's function.

DIMENSION BX1(128), BY1(128), BZ1(128), BX2(128), BY2(128), BZ2(128), BX3(128), BY3(128), BZ3(128), BX4(128), BY4(128), BZ4(128), SAREA(128), AAN1(128), AAN2(128), AAN3(128), XCG(128), YCG(128), ZCG(128), NXYZ(128), SHFACT(128)

Reading input data

Input data to be read are the co-ordinates of the vertices of the panels.

CALL ASSIGN('HULL.DAT')
READ(1,*) NP
DO 276 I=1,NP
READ(1,*) NXYZ(I)
IF(NXYZ(I) .EQ. 9) GO TO 276
READ(1,*) BX1(I),BY1(I),BZ1(I)
READ(1,*) BX2(I),BY2(I),BZ2(I)
READ(1,*) BX3(I),BY3(I),BZ3(I)
READ(1,*) BX4(I),BY4(I),BZ4(I)
GO TO 276
276 CONTINUE

Type *,NP
DO 533 I=1,NP
Type *,NXYZ(I)
IF(NXYZ(I) .EQ. 9) GO TO 534
Type *,BX1(I),BY1(I),BZ1(I)
Type *,BX2(I),BY2(I),BZ2(I)
Type *,BX3(I),BY3(I),BZ3(I)
Type *,BX4(I),BY4(I),BZ4(I)
GO TO 533
534 Type *,BX1(I),BY1(I),BZ1(I)
Type *,BX2(I),BY2(I),BZ2(I)
Type *,BX3(I),BY3(I),BZ3(I)
539 CONTINUE

Calculation for the direction cosines

DO 260 I=1,NP
X1=BX1(I)
X2=BX2(I)
X3=B35(C1)
Y1=B11(C1)
Y2=B12(C1)
Y3=B13(C1)
Z1=B21(C1)
Z2=B22(C1)
Z3=B23(C1)
N0XYZ=NX0Z(C1)
IF (NX0Z .EQ. 30) GO TO 261
X4=B4X4(C1)
Y4=B4Y4(C1)
Z4=B4Z4(C1)
GO TO 262
261. X4=X3
Y4=Y3
Z4=Z3
262 CONTINUE
NH=68
TYPE = 'GIVEN POINTS'
TYPE = X1,Y1,Z1
TYPE = X2,Y2,Z2
TYPE = X3,Y3,Z3
TYPE = X4,Y4,Z4
XX1=6.0
YY1=6.0
ZZ1=6.0
XX2-XX1
YY2-Y2-YY1
ZZ2=ZZ1
XX3-XX1
YY3=YY1-YY2
ZZ3=ZZ2
XX4-XX1
YY4=YY1
ZZ4=ZZ3
A1=XX2-XX1
B1=YY2-YY1
C1=ZZ2-ZZ1
ABC1=SQRT( A1+B1+C1)
XX=XX1/ABC1
YY=YY1/ABC1
ZZ=ZZ1/ABC1
TYPE = 'XX'=,'XX' 'XY'=, 'XY' 'XZ' , 'XZ'
A2=CY2-YY1-(ZZ4-ZZ1)-(YY4-YY1)=(ZZ2-ZZ1)
B2=COX4-XX1=(ZZ2-ZZ1)-(COX2-XX1)=(ZZ4-ZZ1)
C2=CO2-XX1=CY2-YY1-(COX4-XX1)=CY2-YY1
ABC2=SQRT( A2+B2+C2)
XX=A2/ABC2
YY=B2/ABC2
YZ=C2/ABC2

,TYPE *, 'YY=', 'YX=', 'YY=', 'YY=', 'YZ=', 'YX'

CALCULATION FOR THE PANEL AREA

S1=SQRT((XX2-XX1)**2 + (YY2-YY1)**2 + (ZZ2-ZZ1)**2)
S2=SQRT((XX3-XX2)**2 + (YY3-YY2)**2 + (ZZ3-ZZ2)**2)
S=S=SQRT((XX3-XX1)**2 + (YY3-YY1)**2 + (ZZ3-ZZ1)**2)
IF CNDXYZ .EQ. .33 GO TO 388
S3=SQRT((XX4-XX3)**2 + (YY4-YY3)**2 + (ZZ4-ZZ3)**2)
S4=SQRT((XX1-XX4)**2 + (YY1-YY4)**2 + (ZZ1-ZZ4)**2)
AS1=(S1+S2+S)*.5
AS2=(S3+S4+S)*.5
AR1=SQRT(CAS1*(AS1-S1)*(AS1-S2)*(AS1-S))
AR2=SQRT(CAS2*(AS2-S3)*(AS2-S4)*(AS2-S))
AREA=AR1+AR2
GO TO 385

388 AS1=(S1+S2+S)*.5
AREA=SQRT(CAS1*(AS1-S1)*(AS1-S2)*(AS1-S))

385 CONTINUE

TYPE *, 'AREA=', 'AREA

CALCULATION OF THE CENTROID OF THE PANELS

IF CNDXYZ .EQ. .32 GO TO 290
XO31=XX1+XX2+XX3/3.
YY31=YY1+YY2+YY3/3.
ZZ31=ZZ1+ZZ2+ZZ3/3.
XO32=XX1+XX3+XX4/3.
YY32=YY1+YY3+YY4/3.
ZZ32=ZZ1+ZZ2+ZZ4/3.
XB=CAR1*XX31+AR2*XX62/AREA
YB=CAR1*YY31+AR2*YY62/AREA
ZB=CAR1*ZZ31+AR2*ZZ62/AREA
GO TO 295

290 XB=XX1+XX2+XX3/3.
YB=YY1+YY2+YY3/3.
ZB=ZZ1+ZZ2+ZZ3/3.

295 CONTINUE

TYPE *, 'XB=', 'XB', 'YB=', 'YB', 'ZB=', 'ZB

CALCULATION FOR THE SHAPE FACTORS

CHECK WHETHER THE PANELS ARE RECTANGULAR

( FOR RECTANGULAR PANELS, AN ANALYTICAL EXPRESSION IS USED TO CALCULATE THE SHAPE FACTORS )

IF CNDXYZ .EQ. .33 GO TO 310
DIAG1=SQRT((X1-X3)**2 + (Y1-Y3)**2 + (Z1-Z3)**2)
DIAG2=SQRT((X2-X4)**2 + (Y2-Y4)**2 + (Z2-Z4)**2)
TYPE *, 'DIAG1=', 'DIAG1', 'DIAG2=', 'DIAG2

310 CONTINUE

CHANGE ALL POINTS IN LOCAL COORDINATES

X=XB-X1
Y=QB-Y1
Z=QB-Z1
E1=XX+YY + ZZ
F1=XY+YY + ZZ
G1=ZZ+YY + ZZ
E2=XX+YY + ZZ
F2=XY+YY + ZZ
G2=ZZ+YY + ZZ
E3=XX+YY + ZZ
F3=XY+YY + ZZ
G3=ZZ+YY + ZZ
E4=XX+YY + ZZ
F4=XY+YY + ZZ
G4=ZZ+YY + ZZ
XN=XX+YY + ZZ
YN=XX+YY + ZZ
ZN=XX+YY + ZZ

TYPE * LOCAL COORDINATES
TYPE *,E1,F1,81
TYPE *,E2,F2,82
TYPE *,E3,F3,83
TYPE *,E4,F4,84
TYPE *,XN,YN,ZN

C DETERMINE THE INTEGRATION
ITER=1
NN-NN
191 CONTINUE
INDEX=1
62 CONTINUE
GO TO(6,6,7,8),INDEX
5 ET2=E2
ET1=E1
FT2=F2
FT1=F1
GO TO 18
6 ET2=E3
ET1=E2
FT2=F3
FT1=F2
GO TO 18
7 CONTINUE
IF (Q-W-Y-Z .EQ. 85) GO TO 398
ET2=E4
ET1=E3
FT2=F4
FT1=F3
GO TO 10
398 ET2=E3
ET1=E1
FT2=F3
FT1=F1
GO TO 10
6 ET2=E1
ET1=E4
FT2=F1
FT1=F4
GO TO 10
10 CONTINUE
SUN=0.0
IK=1
STEP=(ET2-ET1)/HH
ET12=ABS(ET2-ET1)
IF(ET12.LE. .00001) GO TO 20
IF(ABS(ET2).LE. .00001) GO TO 76
IF(ET12.LT. .00001) IX=1
EZ1=ET1
65 IF(CABS(EZ1-XN).LE. .0001) GO TO 60
FZ-FT1=(FT2-FT1)=(EZ1-ET1)/(ET2-ET1)
EZ-EZ1
CALL SUBCN, YN, ZN, EZ, FN, FZ
FZ1=FZ
30 EZ2=EZ1+STEP
55 IF(CABS(EZ2-XN).LE. .0001) GO TO 51
IF(IX. EQ. 1) GO TO 15
IF(EZ2.LT. ET2).GO TO 20
60 GO TO 25
15 IF(EZ2.GT. ET2) GO TO 28
25 CONTINUE
FZ-FT1=(FT2-FT1)=(EZ2-ET1)/(ET2-ET1)
EZ-EZ2
CALL SUBCN, YN, ZN, EZ, FN, FZ
FZ2=FZ
SUM=SUM+.5*(FZ1+FZ2)=(EZ2-EZ1)
EZ1=EZ2
FZ1=FZ2
80 GO TO 30
51 EZ2=EZ2+STEP
60 GO TO 55
60 EZ1=EZ1+STEP
80 GO TO 65
20 CONTINUE
GO TO((79,71,72,73),INDEX)
70 SUM1=SUM
80 GO TO 75
71 SUM2=SUM
72 SUM3=SUM
73 SUM4=SUM
75 CONTINUE
INDEX=INDEX+1
IF(INDEX.GT. NNXYZ) GO TO 80
60 GO TO 60
80 FSUM=SUM1+SUM2+SUM3+SUM4
IF(ETER.LE. 1) GO TO 100
FSUM2=FSUM
FSUM1=ABS(CF(SUM2-FSUM1)/FSUM2)
IF(CF)GO TO 182
ITER=1
FSUM1=FSUM2
MN+IN=2
TYPE ", 'SUM2=', FSUM2,' SUM1=', FSUM1,' CONV=', CONV
GO TO 181
100 FSUM=FSUM
TYPE ", 'SUM1=', FSUM1
ITER=1
MN+IN=2
GO TO 181
FSUH=FSUM2
SFACT=.5*SQRT(1./AREA+1.4152265)=FSUH
GO TO 400
CONTINUE
C
SIDEA=SQRT((X2-X1)**2+(Y2-Y1)**2+(Z2-Z1)**2)
SIDEB=SQRT((X4-X1)**2+(Y4-Y1)**2+(Z4-Z1)**2)
ASPECT=SIDEA/SIDEB
IF (ASPECT .LT. 1) ASPECT=1./ASPECT
BBB1=SQRT(ASPECT**2+1)
BBB2=ALOG(ASPECT+BBB1)
BBB3=ASPECT*ALOG(1+BBB1)/ASPECT
SFACT=(BBB2*BBB3)/((3.14159265*ASPECT))
CONTINUE
C
XCG(CID)=X9
YCG(CID)=Y9
ZCG(CID)=Z9
AAN1(CID)=XZ
AAN2(CID)=YZ
AAN3(CID)=Z
SAREA(CID)=AREA
SHFACT(CID)=SFACT
CONTINUE
C
WRITEC, 18000
FORC,(GX, XCG(CID), YCG(CID), ZCG(CID), AAN1(CID), AAN2(CID), AAN3(CID),
1 SAREA(CID), SHFACT(CID)
WRITEC, 18011,(XCG(CID), YCG(CID), ZCG(CID), AAN1(CID), AAN2(CID), AAN3(CID),
1 SAREA(CID), SHFACT(CID)
CONTINUE
1801 FORC,(SF18.4, SF8.4, F10.4, F10.8)
STOP
END
SUBROUTINE SUB(XN, YN, ZN, EZ, FN, FZ)
T1=(YN-FN)**2+(XN-EZ)**2+ZN**2
T2=SQRT(T1)
TS=(YN-FN)**T2
FX=ALOG(TS)
RETURN
END
PART 2

This program calculates the wave loads and motions of floating offshore structures using three-dimensional singularity distribution theory (Green's function method).

IMPLICIT REAL*8 (A-H,O-Z)

DIMENSION H(250), XI(250), X2(250), X3(250), AM1(250), AM2(250), AN3(250),
LAMC1(250), DEHCSC(6), DMAP(6,6), THAB(6,6), REST(6,6), S(250)

DIMENSION ASPECT(250), SHAFT(250), WENZ(30), ALPA(8), WSTP(4),
LADIN(30), ADM22(30), ADHS3(30), AMK44(30), AMK55(30), AMK66(30),
ZDP11(30), DP22(30), DMP33(30), DNP44(30), DNP55(30), DNP66(30),
3SHAP(30), XMAP2(30), XMAP3(30), XMAP4(30), XMAP5(30), XAHF6(30),
AKPH1(30), APFP2(30), APFP3(30), APFP4(30), APFP5(30), APFP6(30),
2XPH1(30), ZHAP2(30), ZHAP3(30), ZHAP4(30), ZHAP5(30), ZHAP6(30),
6ZPFI1(30), FPH2(30), FPH3(30), FPH4(30), ZFPH5(30), ZFPH6(30),
7FAPH(30), FAPH2(30), FAPH3(30), FAPH4(30), FAPH5(30), FAPH6(30),
8FAPF(30), FAPF2(30), FAPF3(30), FAPF4(30), FAPF5(30), FAPF6(30),
DIMENSION LATC(30), TWH(30), WHP(30)

COMPLEX=16.0 C(250,7), FP1, FP2, T, DEP, F(250,7), SUM,
IG(250,250), EII(250,7), POTI(250), TT, FEXT(66), AA(6,6), CC(6), ZAMP(6),
ZSUM, FHD(6)

COMPLEX=16.0 FAM1(8,30), FAM2(8,30), FAM3(8,30), FAM4(8,30),
1FAM5(8,30), FAM6(8,30), ZMOT1(8,30), ZMOT2(8,30), ZMOT3(8,30),
2ZMOT4(8,30), ZMOT5(8,30), ZMOT6(8,30), FAM1(8,30), FAM2(8,30),
3FAM3(8,30), FAM4(8,30), FAM5(8,30), FAM6(8,30)

COMPLEX=16.0 A(250,250), C(250)

XGRT(X)=DSQRT(X)
EXP(X)=DEEP(X)
LOG(X)=DCLOG(X)
ARCC(X)=DARCC(X)
ALG(X)=DLOG(X)
AZASH(X)=DATAN(X)

READ INPUT DATA
READ(*,*) YXYS
READ(*,*) WP
READ(*,*) XI(1), XI(2), XI(3), AM1(1), AM2(1), AN3(1), S(1), SHAFT(X)
CONTINUE

IF(YXYS-NE.1) GO TO 3779
DO 3780 I=1, WP
XI(HP+1)=XI(1)
X2(HP+1)=X2(I)
K3(HP+1)=X3(I)
AM1(HP+1)=AM1(I)
AM2(HP+1)=AM2(I)
AN3(3,1) = AN3(I)
STEP = I - 5(I)
SHFACT(NP-1) = SHFACT(1)
3766 CONTINUE
NP = 2 + NP
3779 CONTINUE
READ(5,*) SHAS, TI64, TI66, TI66, TI66, C33, C35, C44, C55
READ(5,*) NROOT, NWLNY, NHEAD, NWSTF
READ(5,*) (WVLN(I), I = 1, NWLNY)
READ(5,*) (ALFA(I), I = 1, NHEAD)
READ(5,*) (ALTA(I), I = 1, NWSTF)
2015 FORMAT(4I5)
2016 FORMAT(8F10.4)
READ(5,*) DF, CH, GRAV, RHO, VOLM, ALLN
2017 FORMAT(8F12.4)
WRITE(6,1010) NP
DO 20 I = 1, NP
WRITE(6,1212) X(I), XZ(I), X3(I), AN1(I), AN2(I), AN3(I), S(I), SHFACT(I)
WRITE(6,12085) SHAS, TI64, TI66, TI66, TI66, C33, C35, C44, C55
WRITE(6,1215) NROOT, NWLNY, NHEAD, NWSTF
WRITE(6,1216) (WVLN(I), I = 1, NWLNY)
WRITE(6,1216) (ALFA(I), I = 1, NHEAD)
WRITE(6,1216) (ALTA(I), I = 1, NWSTF)
WRITE(6,1217) DF, CH, GRAV, RHO, VOLM, ALLN
2010 FORMAT(1S)
2012 FORMAT(7F10.4, F10.6)
2013 FORMAT(10F8.2)
2014 FORMAT(15, 7F10.4)
2085 FORMAT(5E16.8, 5E16.8)

FORMATION OF MASS AND RESTORING COEFFICIENT MATRIX

DO 22 I = 1, 6
DO 22 J = 1, 6
TMAS(I, J) = 0.0
REST(I, J) = 0.0
22 CONTINUE
ZECO = CH - DF
ZMCC = SHAS - ZECO
TMAS(1, 3) = ZMCC
TMAS(2, 4) = ZMCC
TMAS(3, 4) = ZMCC
TMAS(3, 1) = ZMCC
TMAS(1, 1) = SMAS
TMAS(2, 2) = SMAS
TMAS(3, 3) = SMAS
TMAS(4, 4) = TI64
TMAS(4, 6) = TI66
TMAS(3, 5) = TI66
TMAS(4, 5) = TI66
TMAS(6, 5) = TI66
REST(3, 3) = C33
REST(3, 3) = C33
REST(4, 4) = C44
REST(4, 4) = C44
DO 16 I = 1, NP
DO. 16 J = 1, NP
ZF (1, EQ. J) GO TO 16
RR=-(11(1)-X1(J))**2 + (X2(1)-X2(J))**2 + (X3(1)-X3(J))**2
IF (RR .GT. .00001) GO TO 16
WRITE(6,2019) I,J
16 CONTINUE
2019 FORMAT(Z9.10)
C
C
MAIN PROGRAM
C
DO 1501 MM=1,NWSTP
DO 1503 MM=1,NWUTN
WVLN=WVLN(NM1)
WFLN=WFLN/MMSTP(NM1)
WHF(NM3)=WFLN
C
FACT1=.001
FACT2=1
CONV1=.001
CONV2=.001
C
AK=6.2831853/WVLN
AK1=AK*DP
AK2=EXP(AK1)
AK3=EXP(-AK1)
AHU=AK*(AK3-AK2)/(AK2+AK3)
FB=SQRT(AHU*GRAV)
C
CALL ROOT(NROOT,DP,AHU,AHU)
2091 FORMAT(8E16.8)
C
C
CALCULATION FOR GREEN FUNCTION
C
DO 10 I=1,NP
DO 20 J=1,NP
IF (I .EQ. J) GO TO 36
X11=X1(I)
X22=X2(J)
X33=X3(J)
AA1=2*(J)
AA2=2*(J)
AA3=2*(J)
AAH1=AH1(I)
AAH2=AH2(I)
AAH3=AH3(I)
XAI=(X1(I)-X1(J))**2
XAJ=(X2(I)-X2(J))**2
BR1=SQRT(XAI+XAJ+(X3(I)-X3(J))**2)
BR2=SQRT(XAI+XAJ+(X3(I)-2.*CH+X3(J))**2)
AKR=AK*FB
IF (AKR .LE. .01) GO TO 30
C
C
EVALUATION OF GREEN FUNCTION USING SERIES FORM
C
CALL GREEN(NROOT,AHU,AHU,AK,DP,CH,E,X1,X2,X3,AA1,AA2,AA3,AAH1,
AAH2,AAH3,CH,GIM,GR,SK)33 CONTINUE
C
C
G(I,J)=CMPLX(CH,GIM)
A(I,J) = DCMLX(DGR, DGM)
G(I,J) = G(I,J) * S(J) / 12.366371
A(I,J) = A(I,J) * S(J) / 6.283185
GO TO 400
30 CONTINUE

EVALUATION OF GREEN FUNCTION BY INTEGRAL FORM

CALL GREEN2(ANII, AK, DF, CH, K, XX1, XX2, XX3, AA1, AA2, AA3, SRI, SR2, AAN1, IAA1, AAN2, FACT1, FACT2, CONV1, CONV2, G, GIM, DGR, DGM)
GO TO 35
36 G(I,J) = (0.0, 0.0)
A(I,J) = (-1.0, 0.0)
GGG1 = 0.5 * SQRT(S(J) / 3.14159265)
GGG2 = 0.0
G(I,J) = DCMLX(GGG1, GGG2)
G(I,J) = G(I,J) * SEFACT(I)
400 CONTINUE
20 CONTINUE
10 CONTINUE

FORMATION OF VECTOR (B)

K = 1
DUMM2 = 0.0
DO 90 I = 1, NP
DUMM1 = AM1(I)
90 B(I,K) = DCMLX(DUMM1, DUMM2)
K = 2
DO 71 I = 1, NP
DUMM1 = AM2(I)
71 B(I,K) = DCMLX(DUMM1, DUMM2)
K = 3
DO 72 I = 1, NP
DUMM1 = AM3(I)
72 B(I,K) = DCMLX(DUMM1, DUMM2)
K = 4
DO 73 I = 1, NP
DUMM1 = X2(I) * AM3(I) - X3(I) * AM2(I)
73 B(I,K) = DCMLX(DUMM1, DUMM2)
K = 5
DO 74 I = 1, NP
DUMM1 = X3(I) * AM1(I) - X2(I) * AM3(I)
74 B(I,K) = DCMLX(DUMM1, DUMM2)
K = 6
DO 97 I = 1, NP
DUMM1 = X1(I) * AM2(I) - X2(I) * AM1(I)
97 B(I,K) = DCMLX(DUMM1, DUMM2)
GO TO 35
2060 FORMAT(4(3X,1E16.8))

DETERMINE SOURCE STRENGTHS

DO 52 K=1,6
DO 53 J=1,NP
SUM=(0,0,0,0)
DO 54 I=1,NP
SUM=SUM + A(J,I)*R(I,K)
54 CONTINUE
F(J,K)=SUM
53 CONTINUE
52 CONTINUE

CALCULATION OF POTENTIAL

DO 65 K=1,6
DO 66 I=1,NP
SUM=(0,0,0,0)
DO 67 J=1,NP
SUM=SUM + G(I,J)*F(J,K)
67 CONTINUE
PHI(I,K)=SUM
66 CONTINUE
65 CONTINUE

CALCULATION FOR ADDED MASS AND DAMPING COEFFICIENTS

DO 150 J=1,6
DO 151 K=1,6
SUM=(0,0,0,0)
DO 152 I=1,NP
153 DDI=AI(I)
GO TO 160
154 DD=AI(I)
GO TO 160
155 DD=BII(I)
GO TO 160
156 DD=DI(I)*DI(I)*AI(I)
GO TO 160
157 DD=DI(I)*DI(I)*AI(I)
GO TO 160
158 DD=DI(I)*DI(I)*AI(I)
GO TO 160
160 CONTINUE
SUM=SUM + PHI(I,J)*G(I)*DF
152 CONTINUE
AKI=DAKII(SUM)
AK3=DAKII(SUM)
ADMAS(K,J)=RHO*AKI
DAHF(K,J)=RHO*FAK3
2023 FORMAT(215,2E20.8)
151 CONTINUE
150 CONTINUE

AD11 ETG ARE ADDED MASS, DHF11 ETG ARE DAMP. COEFF.

AD11(M3)=ADMAS(1,1)
AD2(M3)=ADMAS(2,2)
AD3(M3)=ADMAS(3,3)
DO 1502 NIH2=I,HHED
ALPHA=ALPHA(NH2)*3.1415926/180.0

CALCULATION OF DIFFRACTION POTENTIAL

K=7
DO 76 I=1,HP
P1=AK*(X3(I)+CH)
P2=AK*BP
P3=EXP(P1)
P4=EXP(-P1)
P5=(COS(ALPHA)*AN1(I) + \text{SIN(ALPHA)*AN2(I))*(P3+P4) = .5}
P6=(P3-P4)*AN3(I) = .5
P6=DCMPLX(P6,P5)
P7=(X1(I)*COS(ALPHA) + X2(I)*SIN(ALPHA))/AK
P8=COS(P7)
F9=SIGN(P7)
P10=DCMPLX(P7,P8)
PD2=P1*ANU*EXP(P2)*EXP(-P2))
B(I,K)=PD2*P10*PP2
76 CONTINUE

K=7
DO 83 J=1,HP
SUM=0.0,0.0
DO 84 I=1,HP
SUM=SUM + A(J,I)*B(I,K)
84 CONTINUE
F(J,K)=SUM
83 CONTINUE

K=7
DO 86 I=1,HP
SUM=0.0,0.0
DO 87 J=1,HP
SUM=SUM + C(I,J)*F(J,K)
87 CONTINUE
PHI(I,K)=SUM
86 CONTINUE

CALCULATION OF INCIDENT WAVE POTENTIAL

DO 350 I=1,HP
P1=AK*(X3(I)+CH)
P2=AK*BP
P3=AK*(X1(I)*COS(ALPHA) + X2(I)*SIN(ALPHA))
P4=(EXP(P1) + EXP(-P1))*.5
P5=(EXP(P2) + EXP(-P2))*.5
P6=P4/(P5*ANU)
P7=COS(P3)
P8=SIN(P3)
CALCULATION OF EXCITING FORCE COMPLEX AMPLITUDE

DO 360 K=1,6
SUM=(G,G,0.0)
DO 370 I=1,MP
GO TO (371,372,373,374,375,376),K
371 EN=AM1(I)
GO TO 380
372 EN=AM2(I)
GO TO 380
373 EN=AM3(I)
GO TO 380
374 EM3=2*(AM3(I) - X3(I)*AM2(I))
GO TO 380
375 EM3=2*(AM1(I) - X1(I)*AM3(I))
GO TO 380
376 EM3=2*(AM2(I) - X2(I)*AM1(I))
380 CONTINUE
SUM=SUM + (PHI(K,1) + POTIM(I))*EN*S(I)
370 CONTINUE
FEK=PHO*FR*FR*WAMP*SUM
360 CONTINUE

FXAM1 ETC ARE COMPLEX AMPLITUDE OF EXCITING FORCE

FXAM1(NW1,NW3)=FEK(I)
FXAM2(NW1,NW3)=FEK(2)
FXAM3(NW1,NW3)=FEK(3)
FXAM4(NW1,NW3)=FEK(4)
FXAM5(NW1,NW3)=FEK(5)
FXAM6(NW1,NW3)=FEK(6)

CALCULATION FOR MOTIONS, (SOLUTION OF EQUATIONS OF MOTION)

DO 403 K=1,6
DO 410 J=1,5
T1=FR*FR*(TMS(K,J) + ADMS(K,J))
T2=FR*DAMP(K,J)
T3=REST(K,J)
T4=T1+T2
AA(K,J)=DCMPLX(T4,T2)
410 CONTINUE
403 CONTINUE

INVERSION OF COMPLEX MATRIX (A)

N=6
MM=6
DX=(1.020,0.080)
600 DO 610 I=1,MM
H(I)=1
610 CONTINUE
DO 620 I=1,MM
Z=0.080
DO 630 L=1,MM
IF(H(L) .GT. 0) GO TO 630
620 CONTINUE
630 CONTINUE
DO 640 K=1,NN
IF (M(K) .GE. 0) GO TO 640
D=AA(K,K)
Y=ABS(REAL(D)) + ABS(DIMAG(D))
IF (X .GE. Y) GO TO 640
LD=L
KD=K
K=Y

640 CONTINUE
630 CONTINUE
D=AA(KD,KD)
DE=D
L=H(LD)
M(LD)=H(KD)
M(KD)=L
DO 660 J=1,NN
CC(J)=AA(LD,J)
AA(LD,J)=AA(KD,J)
650 AA(KD,J)=CC(J)
DO 670 K=1,NN
AA(K,KD)=AA(K,KD)/D
670 CONTINUE
DO 700 J=1,NN
IF (J .EQ. KD) GO TO 700
DO 710 K=1,NN
AA(K,J)=AA(K,J)-CC(J)*AA(K,KD)
710 CONTINUE
700 CONTINUE
CC(KD)=(-1.0,0.0)
DO 780 K=1,NN
AA(K,KD)=CC(K)/D
780 CONTINUE
820 CONTINUE
DO 840 I=1,NN
L=0
820 L=L+1
IF (M(L) .NE. 1) GO TO 820
M(L)=M(I)
M(I)=I
DO 840 K=1,NN
TEMP=AA(K,L)
AA(K,L)=AA(K,I)
840 AA(K,I)=TEMP
DET=CDABS(DE)

900 CONTINUE
C
DO 1001 I=1,6
SUM=(0.0,0.0)
DO 1002 J=1,6
1002 SUM=SUM+AA(I,J)*FEXT(J)
ZAMP(I)=SUM
1001 CONTINUE
C
C
ZHOT1 ETC. ARE COMPLEX MOTION AMPLITUDE
C
ZHOT1(NW2,NH3)=ZAMP(1)
ZHOT2(NW2,NH3)=ZAMP(2)
ZHOT3(NW2,NH3)=ZAMP(3)
ZHOT4(NW2,NH3)=ZAMP(4)
ZHOT5(NW2,NH3)=ZAMP(5)
IZMT6(N/2,N/3)=ZAMP(6)

CALCULATION OF OSCILLATORY HYDRODYNAMIC FORCE COMPLEX AMPLITUDE

DO 525 K=1,6
SSUM=(0.0,0.0)
DO 526 J=1,6
SM=(0.0,0.0)
DO 527 I=1,NT
GO TO (528,529,530,531,532,533),K
528 CH=AN1(I)
GO TO 535
529 CH=AN2(I)
GO TO 535
530 CH=AN3(I)
GO TO 535
531 CH=X2(I)*AN3(I) - X3(I)*AN2(I)
GO TO 535
532 CH=X3(I)*AN1(I) - X1(I)*AN3(I)
GO TO 535
533 CH=X1(I)*AN2(I) - X2(I)*AN1(I)
535 CONTINUE
SUM=SUM+PHI(T,J)*CH*S(I)
527 CONTINUE
SSUM=SSUM+SUM*ZAMP(J)
526 CONTINUE
FHD(K)=RHO*FX*FX*SSUM
525 CONTINUE

PHAM1 ETC. ARE COMPLEX AMPLITUDE OF HYD. FORCE:

PHAM1(N/2,N/3)=FHD(1)
PHAM2(N/2,N/3)=FHD(2)
PHAM3(N/2,N/3)=FHD(3)
PHAM4(N/2,N/3)=FHD(4)
PHAM5(N/2,N/3)=FHD(5)
PHAM6(N/2,N/3)=FHD(6)

STORE FREQUENCY (NON-DIMENSIONAL) AND PERIOD

WE(N/3)=FX*SQRT(ALN/GRAY)
TIME(N/3)=6.2831853/FR

1502 CONTINUE
CLOSE LOOP FOR FREQUENCY

1503 CONTINUE
DETERMINE COMPLEX AMPLITUDE OF EXCITING FORCE AND PHASE

ZAMP1 ETC. ARE AMPLITUDE OF EXCITING FORCE
ZPHI ETC. ARE PHASE
DO 1573 J=1,NHEAD

ALPHA=ALPA(J)
DO 1506 I=1,MMV
ZAMP1(I)=CDABS(FK1N(J,I))
ZAMP2(I)=CDABS(FK2N(J,I))
XAMP3(I)=CDABS(FXAM3(I,1))
XAMP4(I)=CDABS(FXAM4(I,1))
XAMP5(I)=CDABS(FXAM5(I,1))
XAMP6(I)=CDABS(FXAM6(I,1))
IF(ABS(DREAL(FXAM1(I,1))) .LE. 0.0000001) GO TO 7261
QQ1=DIAG(FXAM1(I,1))
QQ2=DREAL(FXAM1(I,1))
CALL SUB5(QQ1,QQ2,QQ3)
XP1(I)=QQ3
GO TO 7262
7261 XP1(I)=90.0
IF(DIMAG(FXAM1(I,1)) .LT. 0.0) XP1(I)=-90.0
7262 IF(ABS(DREAL(FXAM2(I,1))) .LE. 0.0000001) GO TO 7263
QQ1=DIAG(FXAM2(I,1))
QQ2=DREAL(FXAM2(I,1))
CALL SUB5(QQ1,QQ2,QQ3)
XP2(I)=QQ3
GO TO 7264
7263 XP2(I)=90.0
IF(DIMAG(FXAM3(I,1)) .LT. 0.0) XP2(I)=-90.0
7264 IF(ABS(DREAL(FXAM3(I,1))) .LE. 0.0000001) GO TO 7265
QQ1=DIAG(FXAM3(I,1))
QQ2=DREAL(FXAM3(I,1))
CALL SUB5(QQ1,QQ2,QQ3)
XP3(I)=QQ3
GO TO 7266
7265 XP3(I)=90.0
IF(DIMAG(FXAM4(I,1)) .LT. 0.0) XP3(I)=-90.0
7266 IF(ABS(DREAL(FXAM4(I,1))) .LE. 0.0000001) GO TO 7267
QQ1=DIAG(FXAM4(I,1))
QQ2=DREAL(FXAM4(I,1))
CALL SUB5(QQ1,QQ2,QQ3)
XP4(I)=QQ3
GO TO 7268
7267 XP4(I)=90.0
IF(DIMAG(FXAM5(I,1)) .LT. 0.0) XP4(I)=-90.0
7268 IF(ABS(DREAL(FXAM5(I,1))) .LE. 0.0000001) GO TO 7269
QQ1=DIAG(FXAM5(I,1))
QQ2=DREAL(FXAM5(I,1))
CALL SUB5(QQ1,QQ2,QQ3)
XP5(I)=QQ3
GO TO 7270
7269 XP5(I)=90.0
IF(DIMAG(FXAM6(I,1)) .LT. 0.0) XP5(I)=-90.0
7270 IF(ABS(DREAL(FXAM6(I,1))) .LE. 0.0000001) GO TO 7271
QQ1=DIAG(FXAM6(I,1))
QQ2=DREAL(FXAM6(I,1))
CALL SUB5(QQ1,QQ2,QQ3)
XP6(I)=QQ3
GO TO 1506
7271 XP6(I)=90.0
IF(DIMAG(FXAM6(I,1)) .LT. 0.0) XP6(I)=-90.0
CHANGE PHASE FROM RADIANS TO DEGREES
1506 CONTINUE

ZNAM1 AND ZPHI ETC. ARE MOTION AMPLITUDE AND PHASES
DO 1507 I=1,NVNL
ZMAML(I) = CDABS(ZMOT1(J, I))
ZMAML(I) = CDABS(ZMOT2(J, I))
ZMAML(I) = CDABS(ZMOT3(J, I))
ZMAML(I) = CDABS(ZMOT4(J, I))
ZMAML(I) = CDABS(ZMOT5(J, I))
ZMAML(I) = CDABS(ZMOT6(J, I))
IF(ABS(DREAL(ZMOT1(J, I))) .LE. 0.00000001) GO TO 7241
QF1 = DIMAG(ZMOT1(J, I))
QF2 = DREAL(ZMOT1(J, I))
CALL SUBS(QF1, QF2, QF3)
ZPH1(I) = QF3
GO TO 7242
7241 ZPH1(I) = 90.0
IF(DIMAG(ZMOT1(J, I)) .LT. 0.0) ZPH1(I) = 90.0
7242 IF(ABS(DREAL(ZMOT2(J, I))) .LE. 0.00000001) GO TO 7243
QF1 = DIMAG(ZMOT2(J, I))
QF2 = DREAL(ZMOT2(J, I))
CALL SUBS(QF1, QF2, QF3)
ZPH2(I) = QF3
GO TO 7244
7243 ZPH2(I) = 90.0
IF(DIMAG(ZMOT3(J, I)) .LT. 0.0) ZPH2(I) = 90.0
7244 IF(ABS(DREAL(ZMOT3(J, I))) .LE. 0.00000001) GO TO 7245
QF1 = DIMAG(ZMOT3(J, I))
QF2 = DREAL(ZMOT3(J, I))
CALL SUBS(QF1, QF2, QF3)
ZPH3(I) = QF3
GO TO 7246
7245 ZPH3(I) = 90.0
IF(DIMAG(ZMOT4(J, I)) .LT. 0.0) ZPH3(I) = 90.0
7246 IF(ABS(DREAL(ZMOT4(J, I))) .LE. 0.00000001) GO TO 7247
QF1 = DIMAG(ZMOT4(J, I))
QF2 = DREAL(ZMOT4(J, I))
CALL SUBS(QF1, QF2, QF3)
ZPH4(I) = QF3
GO TO 7248
7247 ZPH4(I) = 90.0
IF(DIMAG(ZMOT5(J, I)) .LT. 0.0) ZPH4(I) = 90.0
7248 IF(ABS(DREAL(ZMOT5(J, I))) .LE. 0.00000001) GO TO 7249
QF1 = DIMAG(ZMOT5(J, I))
QF2 = DREAL(ZMOT5(J, I))
CALL SUBS(QF1, QF2, QF3)
ZPH5(I) = QF3
GO TO 7250
7249 ZPH5(I) = 90.0
IF(DIMAG(ZMOT6(J, I)) .LT. 0.0) ZPH5(I) = 90.0
7250 IF(ABS(DREAL(ZMOT6(J, I))) .LE. 0.00000001) GO TO 7251
QF1 = DIMAG(ZMOT6(J, I))
QF2 = DREAL(ZMOT6(J, I))
CALL SUBS(QF1, QF2, QF3)
ZPH6(I) = QF3
GO TO 1507
7251 ZPH6(I) = 90.0
IF(DIMAG(ZMOT5(J, I)) .LT. 0.0) ZPH6(I) = 90.0
CHANGE PHASE FROM RADIANS TO DEGREES
1507 CONTINUE
FAHPI AND FPHI ETC ARE HYDRO FORCE AMPL. AND PHASES
DO 1508 I=1,10
FAH1(I)=CDABS(FHAM1(I,J))
FAH2(I)=CDABS(FHAM2(J,I))
FAH3(I)=CDABS(FHAM3(J,I))
FAH4(I)=CDABS(FHAM4(J,I))
FAH5(I)=CDABS(FHAM5(J,I))
FAH6(I)=CDABS(FHAM6(J,I))
IF(ABS(DREAL(FHAM1(I,J))) .LE. 0.0000001) GO TO 7221
QQ1=DIMAG(FHAM1(I,J))
QQ2=DREAL(FHAM1(I,J))
CALL SUBS(QQ1,QQ2,QQ3)
FFH1(I)=QQ3
GO TO 7222
7221 FFH1(I)=90.0
IF(DIMAG(FHAM1(I,J)) .LT. 0.0) FFH1(I)=90.0
7222 FFH1(I)=90.0
IF(ABS(DREAL(FHAM2(J,I))) .LE. 0.0000001) GO TO 7223
QQ1=DIMAG(FHAM2(J,I))
QQ2=DREAL(FHAM2(J,I))
CALL SUBS(QQ1,QQ2,QQ3)
FFH2(I)=QQ3
GO TO 7224
7223 FFH2(I)=90.0
IF(DIMAG(FHAM2(J,I)) .LT. 0.0) FFH2(I)=90.0
7224 FFH2(I)=90.0
IF(ABS(DREAL(FHAM3(J,I))) .LE. 0.0000001) GO TO 7225
QQ1=DIMAG(FHAM3(J,I))
QQ2=DREAL(FHAM3(J,I))
CALL SUBS(QQ1,QQ2,QQ3)
FFH3(I)=QQ3
GO TO 7226
7225 FFH3(I)=90.0
IF(DIMAG(FHAM3(J,I)) .LT. 0.0) FFH3(I)=90.0
7226 FFH3(I)=90.0
IF(ABS(DREAL(FHAM4(J,I))) .LE. 0.0000001) GO TO 7227
QQ1=DIMAG(FHAM4(J,I))
QQ2=DREAL(FHAM4(J,I))
CALL SUBS(QQ1,QQ2,QQ3)
FFH4(I)=QQ3
GO TO 7228
7227 FFH4(I)=90.0
IF(DIMAG(FHAM4(J,I)) .LT. 0.0) FFH4(I)=90.0
7228 FFH4(I)=90.0
IF(ABS(DREAL(FHAM5(J,I))) .LE. 0.0000001) GO TO 7229
QQ1=DIMAG(FHAM5(J,I))
QQ2=DREAL(FHAM5(J,I))
CALL SUBS(QQ1,QQ2,QQ3)
FFH5(I)=QQ3
GO TO 7230
7229 FFH5(I)=90.0
IF(DIMAG(FHAM5(J,I)) .LT. 0.0) FFH5(I)=90.0
7230 FFH5(I)=90.0
IF(ABS(DREAL(FHAM6(J,I))) .LE. 0.0000001) GO TO 7231
QQ1=DIMAG(FHAM6(J,I))
QQ2=DREAL(FHAM6(J,I))
CALL SUBS(QQ1,QQ2,QQ3)
FFH6(I)=QQ3
GO TO 1308
7231 FFH6(I)=90.0
IF(DIMAG(FHAM6(J,I)) .LT. 0.0) FFH6(I)=90.0
C CHANGE PHASE FROM RADIANS TO DEGREES
C
1308 CONTINUE
C NON-DIMENSIONALIZATION

DO 1522 1=1, NUVLN
WAO=WMP(1)
W5=WAO*GRAY*VOLM/WHQ/ALLN
W6=RED*GRAY*VOLM/WHQ
ADM1(I)=ADM1(I-1)/W1
ADM2(I)=ADM2(I-1)/W1
ADM3(I)=ADM3(I-1)/W1
ADM4(I)=ADM4(I-1)/W2
ADM5(I)=ADM5(I-1)/W2
ADM6(I)=ADM6(I-1)/W2
DMP11(I)=DMP11(I-1)/W3
DMP22(I)=DMP22(I-1)/W3
DMP33(I)=DMP33(I-1)/W3
DMP44(I)=DMP44(I-1)/W4
DMP55(I)=DMP55(I-1)/W4
DMP66(I)=DMP66(I-1)/W4
XAMP1(I)=XAMP1(I-1)/W5
XAMP2(I)=XAMP2(I-1)/W5
XAMP3(I)=XAMP3(I-1)/W5
XAMP4(I)=XAMP4(I-1)/W6
XAMP5(I)=XAMP5(I-1)/W6
XAMP6(I)=XAMP6(I-1)/W6
ZHAM1(I)=ZHAM1(I-1)/VAMQ
ZHAM2(I)=ZHAM2(I-1)/VAMQ
ZHAM3(I)=ZHAM3(I-1)/VAMQ
ZHAM4(I)=ZHAM4(I-1)/ALLN/WHMQ
ZHAM5(I)=ZHAM5(I-1)/ALLN/WHMQ
ZHAM6(I)=ZHAM6(I-1)/ALLN/WHMQ
PAMP1(I)=PAMP1(I-1)/V5
PAMP2(I)=PAMP2(I-1)/V5
PAMP3(I)=PAMP3(I-1)/V5
PAMP4(I)=PAMP4(I-1)/V6
PAMP5(I)=PAMP5(I-1)/V6
PAMP6(I)=PAMP6(I-1)/V6

1522 CONTINUE

IF(J.GT.1) GO TO 1579
WRITE(6,2070)

1522 CONTINUE

IF(J.GT.1) GO TO 1579
WRITE(6,2072) ALPHAI

1522 CONTINUE

IF(J.GT.1) GO TO 1579
WRITE(6,2073) ALPHAI
XINT = 3.14159265/80.0
XIN = XIN + XINT
5 X1 = X1/DP
F1 = X1*TAN(XIN) + ANU
IF (F1) 6, 7, 8
6 XIN = XIN - XINT
GO TO 5
8 X1 = X1*DP
DF1 = TAN(X1) + X1/(COS(X1)**2)
X2 = X1*F1/DF1
IF (X2 - GT. XLIMIT) GO TO 9
XINT = XINT/2.
XIN = XIN - XINT
13 XNEW = XNEW/DP
FNEW = XNEW*TAN(XNEW) + ANU
IF (FNEW) 14, 15, 16
10 XINT = XINT/2.
XNEW = XNEW + XINT
GO TO 13
12 X1 = XNEW
F1 = FNEW
XIN = XNEW
GO TO 8
9 X2 = X2*TAN(X2*DP) + ANU
MN = MN + 1.
IF (MN - GT. 100) GO TO 14
IF (ABS(X1 - X2) - .000001) 14, 14, 13.
15 X1 = X2
F1 = F2
GO TO 8
7 XROOT = X1
F1 = F1
GO TO 16
11 XROOT = XNEW
F1 = FNEW
GO TO 16
14 XROOT = X2
F2 = F2
16 CONTINUE
ANU(X) = XROOT
50 CONTINUE
RETURN
END

SUBROUTINE FOR EVALUATION OF GREEN FUNCTION AND ITS DERIVATIVES BY THE LEGS FORM

SUBROUTINE GREEN(XRROOT, ANU, ANU, AE, DP, CH, X1, XX1, XX2, XI3, AA1, AA2, AAM1, AAM2, AAM3, C8, GSM, DGR, DGF)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION ANU(100)

ABS(X) = DABS(X)
EXP(X) = DEXP(X)
COS(X) = DCOX(X)
SIN(X) = DSNX(X)

C = CH
XXAA1 = XX1 - AA1
XXAA2 = XX2 - AA2
T1 = AK*(AA3+C)
T2 = BEP(T1) + EXP(-T1))/2.
T3 = 0.59319539*(AMU*AMU*AK*AK)/((AK*AK+DP*AMU*AMU+DP+AMU)
A*T3*T2
X = AK*R
N = 0
D = 0.000001
CALL BS(K,X,N,BJ,D,IER)
BJ=BJ
CALL BS(K,X,N,BJ,D,IER)
BJ=BJ
T4 = AK*(XX3+C)
T5 = BEP(T4) + EXP(-T4))/2.
F1 = A*T5*B0
F2 = A*T3*B0
F3 = 2047
FORMAT(3H20,S)
CALL BS(K,X,N,BJ,D,IER)
F3 = A*T5*XXAA1*BY1*AK/R
F4 = A*T5*XXAA1*AK*BJ1/R
F5 = A*T5*BY1*AK*XXAA2/R
F6 = A*T5*BJ1*AK*XXAA1/R
T6 = BEP(T4) + EXP(-T4))/2.
T7 = A*AK*T6*B0
P8 = A*AK*T6+B0
SUM1=0.
SUM2=0.
SUM3=0.
SUM4=0.
KI=0.
K0=0.
K7=0.
I=1
9 = 4*(AMU(I)**2 + AMU**2)/(DP*AMU(I)**2 + DP*AMU**2 - AMU)*
IC0S(AMU(I)*XX3+C))
X = AMU(I)*R
N = 0
CALL BS(K,X,N,BJ,D,IER)
BJ=BJ
N = 1
CALL BS(K,X,N,BJ,D,IER)
BJ=BJ
B0 = B = IC0S(AMU(I)*XX3+C))
S1 = S2*BJ0
IF (S2 .LE. 1) GO TO 2
S1 = ABS(S1)
SUM1 = ABS(SUM1)
IF (S5 .LE. (.000001*S1)) XI = 1
INDEX = 1
SUM2 = SUM1 + S1
2 S3 = B*AMU(I)*XXAA1*E1/R
IF (S3 .LE. 1) GO TO 3
S2 = ABS(S2)
SUM2 = ABS(SUM2)
IF (S2 .LE. (.000001*S2)) K3 = 1
INDEX = 2
/* Missing content */
SLEN=AK*EPS1  
ZINT=0.0  
ITER=1  
33 CONTINUE  
MITER=0  
SINT=-4/DP  
NUM=SLEN/SINT  
SNUM=NUM+1  
12 CONTINUE  
STEP=SLEN/SNUM  
NUMORD=SNUM+1  
SUM1=0.0  
SUM2=0.0  
SUM3=0.0  
SUM4=0.0  
NM=1  
Z=Z+1INI  
5 CONTINUE  
C  
CALL SUB1(Z,C,DP,ANU,XX1,XX2,XX3,AA1,AA2,AA3,X,F1MU,F2MU,F3MU,  
1 F4MU)  
C  
F1Z=F1MU  
F2Z=F2MU  
F3Z=F3MU  
F4Z=F4MU  
C  
P1=Z*DP  
P2=EXP(P1)  
P3=EXP(-P1)  
P4=(P2-P3)/(P2+P3)  
UZ=Z*P4-ANU  
C  
OR1=(F1Z-F1K)/UU  
OR2=(F2Z-F2K)/UU  
OR3=(F3Z-F3K)/UU  
OR4=(F4Z-F4K)/UU  
C  
IF(NM=NM+1) GO TO 6  
ORS1=OR1  
ORS2=OR2  
ORS3=OR3  
ORS4=OR4  
Z=Z+STEP  
NM=NM+1  
GO TO 5  
6 CONTINUE  
ORE1=OR1  
ORE2=OR2  
ORE3=OR3  
ORE4=OR4  
C  
SNUM1=5*STEP*(ORS1+ORE1)  
SNUM2=5*STEP*(ORS2+ORE2)  
SNUM3=5*STEP*(ORS3+ORE3)  
SNUM4=5*STEP*(ORS4+ORE4)  
SUM1=SNUM1+SSUM1  
SUM2=SNUM2+SSUM2  
SUM3=SNUM3+SSUM3  
SUM4=EDMU+SSUM4
2-2+STEP
  RM=RM+1
  IF (RM .GT. RMORD) GO TO 8
  OR31=ORx
  OR32=ORx
  OR33=ORx
  OR34=ORx
  GO TO 5

8 CONTINUE
C
  IF (ITER .NE. 1) GO TO 13
  SSA1=SUM1
  SSA2=SUM2
  SSA3=SUM3
  SSA4=SUM4
  20 CONTINUE
  ITER=+2
  SNUM=SNUM
  GO TO 12
  13 CONTINUE
  MIXER=MIXER+1
  IF (MIXER .GT. 6) GO TO 909
  SSBI=SUM1
  SSB2=SUM2
  SSB3=SUM3
  SSB4=SUM4

3001 FORMAT (15,4XZ20.8)
C
  IF (ABS(SSB1) .LE. 0.0000001) GO TO 901
  SCON1=ABS((SSB1-SSA1)/SSB1)
  GO TO 902
  901 SCON1=0.0
  902 CONTINUE
  IF (ABS(SSB2) .LE. 0.0000001) GO TO 903
  SCON2=ABS((SSB2-SSA2)/SSB2)
  GO TO 904
  903 SCON2=0.0
  904 CONTINUE
  IF (ABS(SSB3) .LE. 0.0000001) GO TO 905
  SCON3=ABS((SSB3-SSA3)/SSB3)
  GO TO 906
  905 SCON3=0.0
  906 CONTINUE
  IF (ABS(SSB4) .LE. 0.0000001) GO TO 907
  SCON4=ABS((SSB4-SSA4)/SSB4)
  GO TO 908
  907 SCON4=0.0
  908 CONTINUE
  IF (SCON1 .GT. CONV1) GO TO 15
  IF (SCON2 .GT. CONV2) GO TO 15
  IF (SCON3 .GT. CONV3) GO TO 15
  IF (SCON4 .GT. CONV4) GO TO 15

909 CONTINUE
C
  IF (INDEX .NE. 1) GO TO 27
  SSBI=SSB1
  SSB2=SSB2
  SSB3=SSB3

SS41 = SS34
GO TO 25
15 CONTINUE
C
SSA1 = SSB1
SSA2 = SSB2
SSA3 = SSB3
SSA4 = SSB4
GO TO 20
25 CONTINUE
C
INDEX = 2
ZINIT = AK + EPS1
ZLIMIT = 2 * AK
ITER = 1
GO TO 35
27 CONTINUE
C
SS12 = SS31
SS22 = SS32
SS32 = SS33
SS42 = SS34
C
EPS2 = FACT2 * AK
C
INDEX = 1
ZLIMIT = AK - EPS2
SLEN = AK - EPS2
ZINIT = 0.0
ITER = 1
75 CONTINUE
NITER = 0
SINT = 0.4 / DP
NUM = SLEN / SINT
SNW = NUM + 1
52 CONTINUE
STEP = SLEN / SNW
NUMORD = NUM + 1
C
SUM1 = 0.0
SUM2 = 0.0
SUM3 = 0.0
SUM4 = 0.0
NH = 1
Z = ZINIT
C
45 CONTINUE
P1 = Z * DP
P2 = EXP(P1)
P3 = EXP(-P1)
P4 = (P2 - P3) / (P2 + P3)
UU = 2 * P4 / ANU
OR1 = 1.0 / UU
IF (NH .NE. 1) GO TO 40
OR21 = OR1
NH = NH - 1
Z = Z + STEP
GO TO 45
40 CONTINUE
OR21 = OR1
SSUM1=0.5*STEP*(ORE1+ORE1)
SUMI=SUMI+SSUM1
Z=Z*STEP
HH=HH+1
IF (HH .GT. NUMORD) GO TO 48
ORE1=ORE1
GO TO 45
48 CONTINUE
IF (ITER .NE. 1) GO TO 53
SSAI=SUM1
60 CONTINUE
ITER=2
SSUM=2.*SSUM
GO TO 52
53 CONTINUE
ITER=ITER+1
IF (ITER .GT. 6) GO TO 54
SSBI=SUM1
C SCONI=ABS((SSBI-SSAI)/SSBI)
IF (SCONI .GT. CONV1) GO TO 55
54 CONTINUE
IF (INDEX .NE. 1) GO TO 67
SSAI=SSBI
GO TO 60
55 CONTINUE
SSAI=SSBI
GO TO 60
65 CONTINUE
INDEX=2
ZINIT=AK*EPS2
ITER=1
ZLIMIT=2.*AK
GO TO 75
67 CONTINUE
SSAA=SSBI
C SS13=FXK*SSA3
SS23=FXK*SSA3
SS33=FXK*SSA3
SS43=FXK*SSA3
C SS14=FXK*SSA4
SS24=FXK*SSA4
SS34=FXK*SSA4
SS44=FXK*SSA4
C 2053 FORMAT(2X250.8)
C P1=AK*DP
P2=EXP(P1)
P3=EXP(-P1)
P4=(P2+P3)**.5
P5=1.0/P4
P6=(P2-P3)/(P2+P3)
P7=P5**2
P8=7.*(1-P1*P6)*(2.*EPS2)
P9=(P6*P1*P7)**2
SSC3=--P8/P9
SS15=FXK*SSC3
CALL SUB2(2,C,DP,ANU,XX1,XX2,XX3,AA1,AA2,AA3,E,FN1,FN2,FN3,FN4)
F12=F1
F12=F2
F12=F3
F12=F4
IF (MM-MM1.1) GO TO 83
ORS1=F12
ORS2=F22
ORS3=F32
ORS4=F42
MM-MM1+1
Z=1+STEP
GO TO 81
ORS1=F12
ORS2=F22
ORS3=F32
ORS4=F42
SSUM1=0.5*STEP*(ORS1 + ORS1)
SSUM2=0.5*STEP*(ORS2 + ORS2)
SSUM3=0.5*STEP*(ORS3 + ORS3)
SSUM4=0.5*STEP*(ORS4 + ORS4)
IF (ABS(SSUM1) .LE. 0.000001) GO TO 910
SS1=ABS(SSUM1/SSUM1)
GO TO 911.
910 SS1=0.0
911 CONTINUE
IF (ABS(SSUM2) .LE. 0.000001) GO TO 912
SS2=ABS(SSUM2/SSUM2)
GO TO 913
912 SS2=0.0
913 CONTINUE
IF (ABS(SSUM3) .LE. 0.000001) GO TO 914
SS3=ABS(SSUM3/SSUM3)
GO TO 915
914 SS3=0.0
915 CONTINUE
IF (ABS(SSUM4) .LE. 0.000001) GO TO 916

SS4=ABS(SS4/N/SUM4)
GO TO 917
916 SS4=0.0
917 CONTINUE
IF (SS1 .GT. CONV2) GO TO 888
IF (SS2 .GT. CONV2) GO TO 888
IF (SS3 .GT. CONV2) GO TO 888
IF (SS4 .GT. CONV2) GO TO 888
GO TO 85
888 CONTINUE
C
MM=MM+1
ORS1=ORS1
ORS2=ORS2
ORS3=ORS3
ORS4=ORS4
C
STEP=0.1**2
IF (STEP .LE. STMIN) STEP=STMIN
IF (STEP .GT. STMAX) STEP=STMAX
EX=STEP
ZZDZ=Z**DP
IF (ZZDZ .GT. 80) GO TO 85
GO TO 81
65 CONTINUE
C
SS16=SUM1
SS26=SUM2
SS36=SUM3
SS46=SUM4
C
SS17=(1.0/SS1)+(1.0/SS2)
RE=RE+3
FF=FF+3
SS27=XXA1/E - XXA1/FF
SS37=XXA2/E - XXA2/FF
XXA4=XX3 - AA3
SS47=XXA4/E - XXA4/FF
C
FORMAT(AX20.8)
Q1=SS11 + SS12 + SS13 + SS14 + SS15 + SS16 + SS17
Q2=SS21 + SS22 + SS23 + SS24 + SS25 + SS26 + SS27
Q3=SS31 + SS32 + SS33 + SS34 + SS35 + SS36 + SS37
Q4=SS41 + SS42 + SS43 + SS44 + SS45 + SS46 + SS47
C
e=Q1
C
GQ=Q2*AAN1 + Q3*AAN2 + Q4*AAN3
C
V1=EXP(AA3+C)
V2=EXP(-V1) + EXP(-V1)*0.5
V3=1.0*A*K - ANU*ANU
V4=1.0*A*K*DF - ANU*ANU*DP + ANU
V5=6.28318539*V3*V2/V6
V6=EXP(V5)
V7=EXP(-V5)
X=AK*X
M=0
B=0.000001
IF (X  .LE. 0.000001) GO TO 101
CALL BESJ(X,N,BJ,D,IWR)
BJ0=BJ
N=1
CALL BESJ(X,N,BJ,D,IWR)
BJ2=BJ
GO TO 102
101 BJ0=1.0
BJ2=0.0
102 CONTINUE
T1=TT*(V6+V7)*0.5*BJ2
T2=-0.25*TT*(V6+V7)*AK*AK*XXA1*(BJ0+BJ2)
T3=0.25*TT*(V6+V7)*AK*AK*XXA2*(BJ0+BJ2)
T4=TT*AK*(V6-V7)*0.5*BJ0,
C
GIN=T1
DGIN=T2*AA1 + T3*AA2 + T4*AA3
C
3000 FORMAT(4Z20.8)
RETURN
END
SUBROUTINE SUB1(X,C,DP,ANU,XX1,XX2,XX3,AA1,AA2,AA3,31,F1MU,F2MU,
133MU,F4MU)
IMPLICIT REAL*8 (A-E,O-Z)
EXP(X)=DEXP(X)
XXA1=XX1-AA1
XXA2=XX2-AA2
C
P1=2*DP
P2=2*(AA3+C)
P3=2*(XX3+C)
C
X=Z*
N=0
D=0.000001
IF (X .LE. 0.000001) GO TO 5
CALL BESJ(X,N,BJ,D,IWR)
BJ0=BJ
GO TO 6
5 BJ0=1.0
6 CONTINUE
N=1
IF (X .LE. 0.000001) GO TO 8
CALL BESJ(X,N,BJ,D,IWR)
BJ1=BJ
GO TO 9
8 BJ1=0.0
9 CONTINUE
C
T1=2.*(Z+ANU)
T2=DEXP(-F1)
T3=(DEXP(F2) + DEXP(-F2))*0.5
T4=(DEXP(F1) + T2)*0.5
A=0.5*T1+T3/T4
F6=DEXP(P3)
P5=DEXP(P3)
T5=(F4+P5)*0.5
T6=(F4-P5)*0.5
F1MU=F1*F6+B0
IF (X .LE. 0.000001) GO TO 10
P2MU=A*T3*Z*XXAA1*B1/8
P3MU=A*T3*Z*XXAA2*B1/R
GO TO 15
10 P2MU=A*T3*Z*Z*XXAA1*0.5
    P3MU=A*T3*Z*Z*XXAA2*0.5
15 CONTINUE
C     RETURN
END
SUBROUTINE SUB2(Z, C, DP, ANU, XX1, XX2, XX3, AA1, AA2, AA3, FN1, FN2, FN3,
FN4)
IMPLICIT REAL*8 (A-H, O-Z)
C     EXPM(X)=DEEP(X)
XXAA1=XX1-AA1
XXAA2=XX2-AA2
P1=Z*DP
P2=Z*(AA3-C)
P3=Z*(XX3+C)
N=0
D=0.000001
K=I
IF (X .LE. 0.000001) GO TO 2
CALL BESJ(X,N,BJ,D,IER)
BJO=BJ
GO TO 3
2   BJ0=1.0
3 CONTINUE
N=1
IF (X .LE. 0.000001) GO TO 4
CALL BESJ(X,N,BJ,D,IER)
BJ1=BJ
GO TO 5
4   BJ1=0.0
5 CONTINUE
C     -P4=EXPM(-P1)
P5=EXPM(P22 + EXPM(-P2))*0.5
T1=EXPM(P1)
T2=P4
P6=(T1-T2)*0.5
P7=(T1-T2)*0.5
A=(2.*C+ANU)*P4*P5)/(EXPM6-ANU*P7)
T3=EXPM(P3)
T4=EXPM(-P3)
T5=(T3-T4)*0.5
T6=(T3-T4)*0.5
FN1=A*T5*BJ0
FN4=2*A*T6*BJ0
IF (X .LE. 0.000001) GO TO 7
FN3=A*T5*Z*XXAA1*B1/R
FN3=A*T5*Z*XXAA2*B1/R
GO TO 10
7 CONTINUE
FN3=A*T5*Z*Z*XXAA1*0.5
FN3=A*T5*Z*Z*XXAA2*0.5
10 CONTINUE.
RETURN
END
SUBROUTINE INVERT(A, M, N, K, C, DET)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION K(250)
COMPLEX*16 A(250, 250), C(250), D, TEMP, DE
ABS(K) = ABS(K)
DE = (1.0, 0.0)
IF (M = 1) 100, 350, 100
100 DO 110 I = 1, M
     M(I) = 1
110 CONTINUE
DO 200 I = 1, M
     X = 0.0
     DO 120 L = 1, M
     IF (M(L) .GT. 0) GO TO 120
     DO 110 K = 1, M
     IF (M(K) .GT. 0) GO TO 120
     D = A(L, K)
     Y = ABS(DREAL(D)) + ABS(DIMAG(D))
     IF (X .GT. Y) GO TO 120
     LD = L
     KD = K
     X = Y
120 CONTINUE
130 CONTINUE
     D = A(LD, KD)
     DE = D
     L = M(LD)
     M(LD) = M(KD)
     M(KD) = L
     DO 140 J = 1, M
     C(J) = A(LD, J)
     A(LD, J) = A(KD, J)
     A(KD, J) = C(J) * A(KD, J)
140 CONTINUE
     M(KD) = A(K, KD) / D
150 CONTINUE
     DO 170 J = 1, M
     IF (J .EQ. KD) GO TO 170
     DO 160 K = 1, M
     A(K, J) = A(K, J) - C(J) * A(K, KD)
160 CONTINUE
170 CONTINUE
     C(KD) = (1.0, 0.0)
     DO 180 K = 1, M
     A(KD, K) = C(KD) / D
180 CONTINUE
200 CONTINUE
     DO 240 L = 1, M
     M(L) = M(I)
     M(I) = I
     DO 240 K = 1, M
     TEMP = A(K, L)
     A(K, L) = A(K, I)
240 A(K, I) = TEMP
DET=CDABS(DE)
100 RETURN
350 A(I,1)=1.0E0/A(I,1)
DET=CDABS(A(I,1))
GO TO 300
END

SUBROUTINE RESJ(X, X, J, D, IER)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*4 X44
ABS(X)=DABS(X)
FLOAT(I)=DFLOAT(I)
ALOG(X)=DLOG(X)

C X44=X
NJ=0.0
IF(NJ)10,20,10
10 IER=1
RETURN
20 IF(X)30,30,31
30 IER=2
RETURN
31 IF(X-15.)32,32,34
32 NTEST=20.*10.4*X-X**2/3
GO TO 36
34 NTEST=90.*X/2.
36 IF(N-NTEST)40,38,38
38 IER=4
RETURN
40 IER=0
M1=M+1
FREEV=0.0

COMPUTE STARTING VALUE OF M

C IF(X-3.)50,60,60
50 H4=X+6.
GO TO 70
60 H4=1.4*X+60./X
70 HS=M+IFIX(X44.)/4+2
MZERO=M40(HX, HS)

SET UPPER LIMIT OF M

MMAX=NTEST
100 DO 190 M=MZERO,MMAX,3

SET F(M), F(M-1)

FM1=1.0E-28
FM=0.0
ALPHA=0.0
IF(M>(M/2)+2)120,110,120
110 J=1
GO TO 130
120 J=1
130 H2=M-2
DO 160 K=1,H2
MX=M-K
MK=2.*FLOAT(MK)*AM1/X-FM
FM=FM1

160 CONTINUE

FM1=BNE
IF(NE-N-1)150,140,150
140 BNE=BNE
150 JT=JT
S=1+JT
160 ALPHA=ALPHA+BNE*S
BNE=2.*FM1/X-FM
IF(N)180,170,180
170 BJE=BNE
180 ALPHA=ALPHA+BNE
BJE=BJ/ALPHA
IF(ABS(BJ-BPREV)-ABS(D*BJ))200,200,190
190 BPREV=BJ
IER=3
200 RETURN
END
SUBROUTINE RESK(X,NE,IER)
- IMPLICIT REAL*8 (A-N,O-Z)
- DIMENSION T(12)
EXP(X)=DEXP(X)
SQRT(X)=DSQRT(X)
FLOAT(T)=DFLOAT(T)
ALOG(X)=DLLOG(X)
BE=0.0
IF(N)10,11,11
10 IER=1
RETURN
11 IF(X)12,12,20
12 IER=2
RETURN
20 IF(X-170.0)22,22,21
21 IER=3
RETURN
22 IER=0
IF(X-1.36,36,25
23 A=DEXP(X)
B=1./X
C=DSQRT(B)*X
T(1)=B
DO 26 L=2,12
26 T(L)=T(L-1)*B
IF(N)1,27,27
27 COMPUTE KO USING POLYNOMIAL APPROXIMATION
28 CO=A*(1.2533141+.2566410*T(1)+.086111278*T(2)-.091390954*T(3)
2+.1344962*T(4)-.2998503*T(5)+.3792407*T(6)-.5247773*T(7)
3-.5753564*T(8)-.26268329*T(9)+.2146181*T(10)-.066809767*T(11)
4+.009199383*T(12))*C
IF(N)20,26,25
29 RK=CO
RETURN
- COMPUTE KI USING POLYNOMIAL APPROXIMATION
31 G1=A*(1.2533141+.46499270*T(1)-.14669830*T(2)+.12804266*T(3)
2-.17344316*T(4)+.26475181*T(5)-.43943621*T(6)+.62833807*T(7)
3-.66539386*T(8)+.50502386*T(9)-.25813038*T(10)+.078800012*T(11)
4-.01087177*T(12))*C
IF(N)1,20,20,31
30 BE=GI
RETURN

FROM KO,K1 COMPUTE KM USING RECURSION RELATION

31 DO 35 J=7,N
     GJ=2.0*(FLOAT(J)-1.0)*G1/X*GO
     IF(GJ-1.0E36).LT.33,32,34
32 IER=4
     GO TO 34
33 G0=GI
35 G1=GI
34 BE=GI
RETURN
36 B=X/2.
   B=3.77721566+ALOG(B)
   C=B*E
   IF(M-1).LT.37,43,37

COMPUTE KO USING SERIES EXPANSION

37 GO=KJ
   K2J=1.
   FACT=1.
   HJ=0.0
   DO 40 J=1,N
      HJ=1./FLOAT(J)
      K2J=K2J*HJ
      FACT=FACT*K2J*HJ
   40 CONTINUE
   GO=GO+K2J*FACT*(HJ-A)
   IF(M-1).LT.43,42,42
42 BE=GO
RETURN

COMPUTE K1 USING SERIES EXPANSION

43 K2J=B
   FACT=1.
   HJ=1.
   G1=1./X*K2J*(.5+A-HJ)
   DO 50 J=2,N
      K2J=K2J*HJ
      HJ=1./FLOAT(J)
      FACT=FACT*K2J*HJ
   50 CONTINUE
   K1=G1+K2J*FACT*(-.5+(A-HJ)*FLOAT(J))
   IF(M-1).LT.31,32,31
52 BE=GI
RETURN

END

SUBROUTINE RES(X,W,BY,IER)
IMPLICIT REAL*8 (A-H.O-Z)
SQRT(X)=DSQRT(X)
SIN(X)=DSIN(X)
COS(X)=DCOS(X)
FLOAT(Y)=DFLOAT(Y)
ALOG(X)=DLOG(X)
ABS(X)=DABS(X)
CHECK FOR ERRORS IN M AND X

IF(M)180,10,10
10 IXK=0

IF(X)190,190,20

IF X LESS THAN OR EQUAL 4

20 IF(X-4.0)40,40,30

COMPUTE TO AND T1 FOR X GREATER THAN 4

30 T1=4.0/X

T2=T1**2

F0=(((.00000037043)*T2+.0000173565)*T2-.0000487613)*T2

I= 0.00173562*T2+.000020972*T2+.3989423

Q0=(((.000000323212)*T2-.0000142078)*T2+.0003424686)*T2

L= (.0003424686)*T2+.0005646326*T2-.012466894

P0=(((.000000424144)*T2-.0000209720)*T2+.0000380759)*T2

L= (.0000223203)*T2+.0029218261*T2-.3989423

QP=(((.00000036594)*T2-.000018222)*T2-.000398708)*T2

T1= .0001064741*(T2-.0005904001)*T2-.03740084

A=2.0/SQRT(2)

B=A*T1

C=X-.7633582

C=A*P0+B*Q0+COS(C)

T1=A*P1+B*Q1+SIN(C)

GO TO 90

COMPUTE TO AND T1 FOR X LESS THAN OR EQUAL TO 4

40 XX=X/2

XX=XX**2

LOG(XX)=.3772137

SUM=0.0

TERM=T

DO 70 L=1,15

50 SUM=SUM+1./FLOAT(L-1)

60 FL=1./FLOAT(L-1)

70 T3=SUM

TERM=(TERM*(-X2)/FL)**2/((Y-1.)/FL**2)

70 T=2.*TERM

TERM= T**2/2.

SUM=0.0

90 T1=TERM

DO 70 L=2,16

80 SUM=SUM+1./FLOAT(L-1)

FL=1./FLOAT(L-1)

FL1=FL+1.

T3=T**2/2.

TERM=(TERM*(-X2)/FL)**2/((TS+5./FL1)**2)

80 T2=T3*T1

FL=6.366198

TO=FL+FL

T1=F12/X+F12*T1
90 IF(N-1)100,100,130
C    100 IF(N)110,120,110
110 BT=Y1
    GO TO 170
120 BY=YT
    GO TO 170
C    PERFORM RECURRENCE OPERATIONS TO FIND YN(X)
C    130 YA-TO
    TB=Y1
    K=1
140 T=FLOAT(2*X)/X
    YC=YT+TB-YA
    IF(ARS(YC)-1.0E38)145,145,141
141 IER=3
    RETURN
145 K=K+1
    IF(K-N)150,160,150
150 YA=YB
    YB=YC
    GO TO 140
160 BY=YC
170 RETURN
180 IER=1
    RETURN
190 IER=2
    RETURN
END
SUBROUTINE SUB5(QQ1,QQ2,QQ3)
IMPLICIT REAL*8 (A-H,O-Z)
IF(QQ1.GT.0.AND. QQ2.LE. 0) GO TO 2
IF(QQ1.LE.0.AND. QQ2.LE. 0) GO TO 3
QQ3=DATAN(QQ1/QQ2)
    GO TO 5
2    QQ3=3.1415926535-DATAN(-QQ1/QQ2)
    GO TO 5
3    QQ3=-3.1415926535-DATAN(QQ1/QQ2)
5    QQ3=QQ1*57.29578
    RETURN
END
APPENDIX B

TYPICAL INPUT DATA
| 11.5 | -27.5 | -5.9975 | 0 | 1.0 | 59.975 | .9486 |
| 11.5 | -32.5 | -5.9975 | 0 | -1.0 | 59.975 | .9486 |
| 8.0 | -33.9 | -5.9975 | -1.0 | 0 | 59.975 | .9486 |
| 14.0 | -33.9 | -5.9975 | 1.0 | 0 | 59.975 | .9486 |
| 34.5 | -28.8 | -5.9975 | 0 | 1.0 | 95.98 | .9624 |
| 34.5 | -34.9 | -5.9975 | 0 | -1.0 | 95.98 | .9624 |
| 33.59 | -39.9 | -5.9975 | -1.0 | 0 | 95.96 | .9644 |
| 38.59 | -39.9 | -5.9975 | 1.0 | 0 | 95.96 | .9644 |
| 2389.6 | 18054893 | .9 | 1.1 | 925.8 |
| 28449914 | 3510 | 8 | 536540 | 545694 |
| 93.6 | 2 | 1 |
| 100 | 290 | 300 | 480 | 668 | 780 |
| 0.45 |
| 100 |
| 150 | 146.688 | 9.8 | 1.825 | 28880 | 98 |