THE CONTRIBUTION OF MULTIPATHING EFFECTS IN GROUND-WAVE RADAR RETURN FROM THE SEA SURFACE

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by

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~~A thesis~~ submitted to the School of Graduate Studies in partial fulfillment of the requirements for the degree of Master of Engineering

Faculty of Engineering and Applied Science
Memorial University of Newfoundland

November 1987

St. John's Newfoundland

ISBN 0-315-43365-5

ABSTRACT

The second-order ground wave spectral cross section of the ocean surface given by existing theories corresponds to the case where both theoretical scatterings occur within the bounds of a specific area or patch of the ocean surface. Another additional term in the second-order cross section given by Walsh and Srivastava (1987), which represents the phenomenon where at least one scattering occurs outside the bounds of the patch of the ocean surface, is examined. The properties and significance of this off-patch scatter (a multi-pathing effect) are discussed.

The off-patch spectral cross section expression is simplified for a narrow beam receiving antenna. By using suitable numerical techniques, a computer program is developed for calculating this cross section. The program is applicable for wide beam transmitting antenna. Theoretical Doppler spectra are generated for different radar frequencies and sea states to study the importance of this type of scatter. A comparison is carried out between the Doppler spectra of the above two kinds of scatter for various sea conditions to infer the effect of off-patch scatter in extraction of ocean surface parameters. Two possible cases, first when the radar is located on the open ocean and second when it is located on the beach, are considered for the comparison.

It is determined that off-patch scatter is not significant in Doppler regions near the first-order peaks, which are commonly used to extract the ocean surface parameters. However, at some Doppler frequencies its contribution may be important as contending ocean clutter in target detection problems.

ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to Dr. John Walsh, Professor, Faculty of Engineering and Applied Science, for his excellent guidance and insight during the period of this study. The assistance and helpful suggestions provided by Dr. Satish Srivastava are also greatly appreciated.

This work was supported by a strategic research grant from the Natural Sciences and Engineering Research Council (NSERC G1596) to Dr. John Walsh.

Special thanks are due to Dean F. A. Aldrich, School of Graduate Studies, Dean G. R. Peters, and Associate Dean T. R. Chari, Faculty of Engineering and Applied Science, for providing this opportunity and teaching assistantships.

Also, the author would like to thank Randy Howell for his helpful discussion. Thanks are also due to Ms. Wanda Butler for her quick and efficient typing of this manuscript.

To my parents

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LIST OF SYMBOLS

C_e, C_h	:	Second-order electromagnetic and hydrodynamic contributions
F_p	:	One way ground wave attenuation function between the radar and the surface patch
g	:	Acceleration due to gravity
G_t, G_r	:	Free space gains of the transmitting and receiving antennas in the direction of surface patch
$\vec{K}$	:	Wave number vector = $K_x\hat{x} + K_y\hat{y}$
K	:	Magnitude of $\vec{K}$
k_o	:	Incident radio wave number
Pr^o	:	Received power of the backscattered signal
P_t	:	Average transmitted power
$S(\cdot)$	:	Ocean directional waveheight spectrum
$Sa(x)$	:	Sampling function
$sgn(x)$	:	Sign function
ψ_w	:	Wind speed
Δ_θ	:	One-half the beamwidth of the receiving antenna
Δ_ρ	:	One-half the patch width of the ocean surface
Δ	:	Normalized surface impedance
$\delta(x)$	:	Dirac delta function
λ_o	:	Wavelength of transmitted signal
v	:	Doppler frequency normalized to the Bragg frequency
ρ_o	:	Distance of the target

$\sigma(\omega_d)$	:	Total backscattered Doppler frequency dependent spectral cross section of the ocean surface patch normalized to its area
$\sigma_f(\omega_d)$	:	First-order backscattered spectral cross section of the ocean surface patch normalized to its area
$\sigma_{S1}(\omega_d)$, $\sigma_{S2}(\omega_d)$, $\sigma_{S3}(\omega_d)$	:	Three parts of second-order backscattered spectral cross section of the ocean surface patch normalized to its area
ω_B	:	Bragg frequency
ω_d	:	Doppler shift in the received signal
ω_t	:	Frequency of the transmitted signal

CHAPTER 1

INTRODUCTION

1.1 General

In recent years, ground wave Doppler radars have gained popularity in remote sensing of ocean surface waves. The theoretical formulation required to analyze the radar sea echo is based on the interpretation of the echo power spectrum in terms of Barrick's equation for the spectral cross section. It is from this cross section that ocean surface parameters (e.g., significant waveheight and directional waveheight spectrum) may be extracted. Barrick's model for the Doppler spectrum of the radar return for a small area (patch) of the ocean surface consists predominantly of first- and second-order scatter. First-order scatter is produced by the reflection of the transmitted signal from ocean waves whose wavelength is one-half the radar's wavelength and which are moving either towards or away from the radar. Figure 1 shows the first-order scatter from a patch of the ocean surface. Second-order scatter (on-patch) is produced when the transmitted signal interacts with two ocean waves that exist within the boundaries of the patch and have different wavelengths in general.

In a different approach, Srivastava (1984) has developed an analytical model for the HF backscattered Doppler spectrum for the ocean surface. The first-order cross section is the same as derived by Barrick but the second-order cross section consists of three parts. The first part is equivalent to the second-order cross section derived by Barrick. The second part corresponds to the case where the first scatter occurs at the source point and the second scatter occurs on the patch. This additional term will be present only when the radar is surrounded by the open ocean (e.g., a ship or an offshore platform based radar), and this form of scatter does not affect the regions of Doppler spectrum near the first-order peaks which are commonly used for estimating ocean wave parameters. The third part corresponds to the case where at least one scatter occurs from the ocean waves outside the boundaries of the patch. This kind of scatter which may be viewed as a multipathing effect will be referred to as the off-patch scatter. The three parts of the second-order scatter are shown in figure 2.

In order to study the contribution of off-patch scatter, a software model has been developed. For estimating the spectral cross section, the model includes a numerical evaluation of an integral. This single variable integral is achieved from a fourth order integral by taking advantage of two Dirac delta functions and assuming a wide beam transmit antenna and narrow beam receive antenna. This model may be used to generate the spectral cross section of off-patch scatter for different radar frequencies and sea conditions. A comparison is carried out between the off-patch and on-patch cross section to infer the importance of off-patch scatter in the extraction of ocean surface parameters and other relevant information. The results lead to the conclusion that although off-patch scatter is not significant in extracting the ocean surface parameters, at some Doppler frequencies it may be very important as contending ocean clutter in target detection problems.

1.2 Literature Review

The problem of reflection of waves from a rough surface has been carried out by many investigators. Most interest lies in the propagation of radio waves over rough ground or over the sea. Reyleigh introduced a perturbation technique in 1896 to study the reflection of acoustic waves from rough walls. Based on the above approach Rice (1951) treated the problem related to the reflection of electromagnetic waves from slightly rough surfaces. He modelled the surface as two dimensional and periodic and assumed the incident field to be a plane wave. He obtained the first-order and the higher-order expressions for the scattered field and derived the three scattered field components for horizontal and vertical polarizations. In 1964 Wait applied the above technique in the electromagnetic problem for ground wave propagation over flat earth. Wait (1971) and Barrick (1971) derived the expression for the modified surface impedance and scattering results. Wait assumed the surface to be one dimensional and periodic while Barrick concentrated on ground wave propagation over the rough sea.

Crombie (1955) conducted an experiment on radar return from the ocean surface at 13.56 MHz and he found two dominant peaks in the Doppler spectrum. He investigated these two peaks and came to the conclusion that they were produced by two ocean waves each having a wavelength equal to one-half the radar's wavelength, one moving towards and the other moving away from the radar. Based on this observation he established that the HF radars may be used in remote sensing of ocean waves. Later Barrick (1970) utilized the perturbation technique to find the radar cross section of the ocean surface. He (1972) derived an average first- and second-order back-scattered radar cross section for a patch of the ocean surface. The first order result offered a theoretical explanation for Crombie's experimental conclusion.

In a different approach, Walsh (1980) presented a formulation for rough surface propagation and scattering problems. His technique is applicable to any time-invariant rough surface, and is open to any finite source instead of plane wave incidence. Based on this technique, a theoretical analysis was made by Srivastava (1984) for HF scattering from the ocean surface. Srivastava derived average first- and second-order of backscattered spectral cross section. For narrow beam reception the scattering area of the first-order backscattered field is shown to be a patch or small area of the ocean surface. But for the second-order case, the signals are received from the patch as well as from the surrounding regions. All these signals arrive at the same time. His second-order cross section contains two terms in addition to that provided by existing theories. Later, Walsh et al. (1986) extended the above results for wide beam reception and modified the expression for off-patch scatter. The latest work on this problem has been presented by Walsh and Srivastava (1987). The first additional term has already been studied and not found significant in problem of extraction of ocean surface parameters [Srivastava (1984), Walsh and Srivastava (1987)]. The second additional term which corresponds to off-patch scatter has not received complete study by other researchers and, actually, has been a controversial issue for several years regarding its importance to the problem of extracting ocean wave information from radar data. In an effort to settle this issue, its properties and significance for ocean wave measurements have been carried out in this thesis.

1.3 Scope of Thesis

This thesis is primarily concerned with the examination of the effect of second-order off-patch scatter. The emergence of this off-patch term is totally based on the analytical model developed by Srivastava (1984) and Walsh and Srivastava (1987) for the HF backscattered Doppler spectrum for the ocean surface. The discrepancy between actual radar data and the theoretical Doppler spectrum provided by existing theories raised a suspicion of the interference of some other effects. To be able to see if the off-patch scatter is the cause of this discrepancy, a complete study is done to understand the physical properties and significance of this form of scatter. For this purpose, a software model has been implemented to generate the theoretical off-patch second-order cross section for different radar frequencies and sea states. These results are then compared with the on-patch cross section in order to draw a conclusion about its importance. It is found that though off-patch scatter is not significant in the extraction of ocean surface parameters, it is in some cases, very important in target detection problems.

Chapter 2 contains the radar range equation and the cross section expressions for first-order and the three parts of the second-order scatters. The definition of all the variables and parameters are given.

Chapter 3 deals with the reduction of the spectral cross section expression of the off-patch scatter to a computational form. The reduction is achieved by taking into account the presence of two Dirac delta functions and approximating an integral using the rectangular rule for narrow beam reception. This cross section expression is obtained from the similar expression of Walsh and Srivastava (1987) for a wide beam receiving antenna.

Chapter 4 consists of results and discussions. A comparison is made in this chapter between the off-patch and on-patch spectra for different radar frequencies and sea states. Two possible cases are studied: firstly the case when the radar is located on the open ocean (e.g., a ship or an offshore platform based radar) and secondly when the radar is located on the beach. Results are discussed for all these cases.

Conclusions and recommendations for future work are presented in Chapter 5. Computer programs are given in appendix.

CHAPTER 2

BACKSCATTERED RADAR CROSS SECTION

2.1 Radar Range Equation

The standard monostatic radar range equation for the received power from a target may be written as [Barrick (1972)]

$$P_r^o = \frac{P_t\ G_t\ G_r\ \lambda_o^2\ |F|^4}{(4\pi)^3\ \rho_o^4}\ \sigma^o \tag{2.1}$$

where

P_r^o = received power (watts)

P_t = transmitted power (watts)

G_t = gain of transmitting antenna

G_r = gain of receiving antenna

λ_0 = wavelength of transmitted signal (meter)

F = one way ground wave attenuation function between the radar and the target

ρ_0 = distance of the target (meter)

σ^0 = radar cross section of the target (meter^2)

G_t and G_r are dimensionless parameters.

We are considering only a small patch of the ocean surface for remote sensing and since that consists of many ocean waves moving with different velocities, there will be a band of Doppler shifts in the received signal. If ω_t is the transmitted frequency and ω_r is the received frequency, then the Doppler shift (ω_d) in the received signal may be defined as

$$\omega_d = \omega_r - \omega_t \tag{2.2}$$

If instead of the received power, we use the received power spectrum then the backscattered power spectrum in terms of spectral cross section normalized to the patch area may be given as

$$P_r(\omega_d) = \frac{P_t\ G_t\ G_r\ \lambda_o^2\ |F|^4}{(4\pi)^3\ \rho_o^4} \text{Ap}\ \sigma(\omega_d) \tag{2.3}$$

where

$$P_r^o = \frac{1}{2\pi} \int_{\omega_d} P_r(\omega_d) d\omega_d \tag{2.4}$$

and

$$\sigma^o = \frac{\text{Ap}}{2\pi} \int_{\omega_d} \sigma(\omega_d) d\omega_d \tag{2.5}$$

$P_r(\omega_d)$ is the backscattered power spectrum and $\sigma(\omega_d)$ is the spectrum cross section normalized to patch area Ap.

The dimensions of the patch depend on the time delay between the transmitted and received signals, transmitted pulse width and the beamwidth of the receiving antenna.

The backscattered spectral cross section normalized to the patch area as given in Walsh and Srivastava (1987) consists of four parts and may be written as

$$\sigma(\omega_d) = \sigma_f(\omega_d) + \sigma_{s1}(\omega_d) + \sigma_{s2}(\omega_d) + \sigma_{s3}(\omega_d) \tag{2.6}$$

where σ_f is the first-order cross section normalized to the patch area and σ_{s1}, σ_{s2} and σ_{s3} represent three parts of the second-order cross section normalized to the patch area. The derivations of these cross sections for wide beam transmission and any receiving antenna (e.g., wide beam or narrow beam) are given in Walsh and Srivastava (1987). For narrow beam reception their expressions may be simplified further and are presented here.

2.2 First-Order Cross Section

The first-order cross section for narrow beam receiving antennas may be given as

$$\sigma_f(\omega_d) = \frac{8\Delta_\rho |\omega_d|^5}{\pi g^3}\left(\frac{\omega_d^2}{g} - k_o\right)^2 Sa^2\left[\Delta_\rho\left(\frac{\omega_d^2}{g} - 2k_o\right)\right]$$

$$\cdot\, S(-2\eta^2 \mathrm{sgn}(\eta)\vec{k}_o) \qquad (2.7)$$

where $\eta = \omega_d/\omega_B$. $\omega_B = (2gk_o)^{1/2}$ is the Bragg frequency. Δ_ρ is one-half the patch width of the ocean surface. g is the gravitational acceleration. $\vec{k}_o = k_o\hat{x}$ is the incident radar wavenumber vector. $S(\cdot)$ represents the ocean directional waveheight spectrum as defined in Walsh and Srivastava (1987) in a form similar to that given by Lipa and Barrick (1982).

Assuming a large Δ_ρ, the limit of the squared sampling function $[Sa^2(x)]$ in Eq. (2.7) may be taken to the Dirac delta function. In this process Eq. (2.7) reduces to

$$\sigma_f(\omega_d) = 16k_o^4 \sum_{m=\pm 1} \delta(\omega_d + m\omega_B) S(2m\vec{k}_o) \qquad (2.8)$$

where $m = 1$ and -1 are for summation.

2.3 Second-Order Cross section

The second-order cross section consists of three parts and may be given as follows:

2.3.1 First Part

The first part of the second-order cross section which corresponds to on-patch scatter may be given as

$$\sigma_{s1}(\omega_d) = \frac{4k_o^4}{\pi^2} \sum_{m,m'=\pm 1} \int_p \int_q |C_e + C_h|^2 \, S(m\vec{k}_1) \cdot S(m\vec{k}_2)\,\delta[\omega_d - m(gk_1)^{1/2} - m'(gk_2)^{1/2}] \cdot dq\,dp \tag{2.9}$$

where m and m' may take the values +1 and -1, defining four possible combinations of direction for the two scattering ocean wavenumber vectors $\vec{K}_1$ and $\vec{K}_2$. The spatial wavenumber p lies along the radar beam, with q perpendicular. Other variables and functions are defined as follows:

$$\vec{K}_1 = (p - k_o)\hat{x} + q\hat{y}\,, \quad K_1 = |\vec{K}_1| \tag{2.10}$$

$$\vec{K}_2 = (-p + k_o)\hat{x} - q\hat{y}\,, \quad K_2 = |\vec{K}_2| \tag{2.11}$$

$$C_e = \frac{1}{2}\,\frac{[k_o^2 - p^2 - 2\vec{K}_1 \cdot \vec{K}_2]}{(\vec{K}_1 \cdot \vec{K}_2)^{1/2}} \tag{2.12}$$

$$C_h = -\frac{j}{2}\left[K_1 + K_2 + \frac{(K_1K_2 - \vec{K}_1 \cdot \vec{K}_2)(\omega_B^2 + \omega_d^2)}{mm'(K_1K_2)^{1/2}(\omega_B^2 - \omega_d^2)}\right] \tag{2.13}$$

C_e and C_h are the second-order electromagnetic and hydrodynamic contributions respectively as given in Srivastava (1984).

2.3.2 Second Part

The second part of the second-order cross section may be written as

$$\sigma_{s2}(\omega_d) = \frac{2k_o^4}{\pi^2} \sum_{m,m'=\pm 1} \int_p \int_q \frac{[(\vec{K}_1 + \vec{k}_o) \cdot \vec{K}_1]^2}{|(\vec{K}_1 + 2\vec{k}_o) \cdot \vec{K}_1|} S(m\vec{K}_1)$$

$$\cdot\, S(2m'\vec{k}_o)\,\delta[\omega_d + m(gK_1)^{1/2} + m'\omega_B]\,dq\,dp \qquad (2.14)$$

where

$$\vec{K}_1 = p\hat{x} + q\hat{y}\ , \quad K_1 = |\vec{K}_1|$$

2.3.3 Third Part

The third part of the second-order cross section derived by Walsh and Srivastava (1987) for a general receiving antenna has been simplified for narrow beam reception and is presented in Howell et al. (1987). This third part corresponds to off-patch scatter and may be given as follows

$$\sigma_{s3}(\omega_d) = \frac{1}{\pi^2 \Delta_\theta |F_p|^4} \sum_{m,m'=\pm 1} \int_{K_1=0}^{\infty} \int_{\phi_1=-\pi}^{\pi} \int_{K_2=0}^{\infty} \int_{\phi_2=-\pi}^{\pi}$$

$$\cdot\, |F_o(r_b, \psi_b + \phi_c) F_o(r_c, \psi_c + \phi_c + \pi)$$

$$\cdot\, F_o(r_a, \phi_c + \pi)|^2\ \delta\left(\frac{K_c}{1 + \cosh\beta_o} - 2k_o\right)$$

$$\cdot \frac{Q_c^2(\vec{K}_1,\vec{K}_2,\phi_c)K_1K_2}{K_c(1+\cosh\beta_o)|\det|} S(m\vec{K}_1)S(m'\vec{K}_2)$$

$$\cdot \delta[\omega_d + m(gK_1)^{1/2} + m'(gK_2)^{1/2}]d\phi_2 dK_2 d\phi_1 dK_1 \quad (2.15)$$

A restriction on the above equation is: $|\phi_c| \leq \Delta_\theta$. Other variables and functions are defined as follows:

$$\vec{K}_1 = K_{1x}\hat{x} + K_{1y}\hat{y};\ K_1 = |\vec{K}_1|,\ \phi_1 = \tan^{-1}\left(\frac{K_{1y}}{K_{1x}}\right) \quad (2.16)$$

$$\vec{K}_2 = K_{2x}\hat{x} + K_{2y}\hat{y},\ K_2 = |\vec{K}_2|,\ \phi_2 = \tan^{-1}\left(\frac{K_{2y}}{K_{2x}}\right) \quad (2.17)$$

$$\vec{K}_c = K_{cx}\hat{x} + K_{cy}\hat{y},\ K_c = |\vec{K}_c|,\ \phi_c = \tan^{-1}\left(\frac{K_{cy}}{K_{cx}}\right) \quad (2.18)$$

$$K_{cx} = 2K_{1x} + K_{2x}(1 + \cosh\beta_o\cos\alpha_o) + K_{2y}\sin\beta_o\sin\alpha_o \quad (2.19)$$

$$K_{cy} = 2K_{1y} + K_{2y}(1 + \cosh\beta_o\cos\alpha_o) + K_{2x}\sin\beta_o\sin\alpha_o \quad (2.20)$$

$$Q_c(\vec{K}_1,\vec{K}_2) = k_o(\vec{K}_1\cdot\hat{K}_c)[2k_o(\vec{K}_1\cdot\hat{K}_c) + k_o(\vec{K}_2\cdot\hat{K}_c)$$

$$- 2K_1^2 - 3\vec{K}_1\cdot\vec{K}_2 - K_2^2] + K_1^2[K_1^2 + K_2^2$$

$$+ 2\vec{K}_1\cdot\vec{K}_2 - k_o(\vec{K}_1\cdot\hat{K}_c + \vec{K}_2\cdot\hat{K}_c)] \quad (2.21)$$

det is defined as

$$\det = \frac{\partial^2\xi}{\partial\beta^2}\frac{\partial^2\xi}{\partial\alpha^2} - \left(\frac{\partial^2\xi}{\partial\beta\partial\alpha}\right)^2 \quad (2.22)$$

where

$$\xi(\beta,\alpha) = \frac{1}{(1+\cosh\beta)}[2\vec{K}_1 \cdot \hat{K}_c + K_2\sinh\beta\sin\alpha \cdot \sin(\phi_2 - \phi_c) + (\vec{K}_2 \cdot \hat{K}_c)(1+\cosh\beta\cos\alpha)] \quad (2.23)$$

det should be evaluated at $\beta = \beta_o$ and $\alpha = \alpha_o$. β_o is the solution of the equation,

$$a_1\cosh^2\beta + a_2\cosh\beta + a_3 = 0 \quad (2.24)$$

where

$$a_1 = 4K_1\cos^2\phi_1 + 4K_1K_2\cos\phi_1\cos\phi_2 + \frac{a_2}{2}$$

$$a_2 = -2K_2^2\sin^2\phi_2$$

$$a_3 = (-\sin^2\phi_2)(a_1 - \frac{a_2}{2} + k_2^2) \quad (2.25)$$

such that β_o is real, non-zero and satisfies the following equation,

$$[2K_1\cos\phi_1 + K_2\cos\phi_2]\cos\phi_2\cosh\beta_o[1+\tanh^2\beta_o \cdot \tan^2\phi_2]^{1/2} = K_2\ \mathrm{sgn}\{\beta_o\tan\phi_2\}[\sin^2\phi_2 + \cosh\beta_o] \quad (2.26)$$

sgn(x) is the sign function defined as

$$\mathrm{sgn}(x) = \begin{bmatrix} 1, & x > 0 \\ -1, & x < 0 \end{bmatrix}$$

Once β_o is obtained, α_o may be derived from

$$\tan\alpha_o = \tanh\beta_o\tan\phi_2 \quad (2.27)$$

such that $0 \leq \alpha_o \leq \pi$. Δ_θ is equal to one-half of the beamwidth of the receiving antenna. The function $F_o(\rho,\theta)$ is the same as the Sommerfeld attenuation function except that the numerical distance contains $\Delta_o(\theta)$, instead of the normalized surface impedance Δ [Barrick (1970)]. $\Delta_o(\theta)$ is the average modified surface impedance in the propagation direction θ and it takes into account the surface roughness. The expression for the modified surface impedance is given in Walsh and Srivastava (1987). $F_p = F_o(\rho_o,\theta_o)$ is the one way ground wave attenuation function between the radar and the patch.

r_b is the distance between the source and first scattering point. r_c is the distance between the first and second scattering points. r_a represents the distance between the second scattering point and the receiving (or source) point. The corresponding three directions with respect to the x-axis are $(\psi_b + \phi_c)$, $(\psi_c + \phi_c + \pi)$ and $(\phi_c + \pi)$ respectively. These distances and directions may be obtained from

$$r_a = \frac{2\rho_o}{(1 + \cosh\beta_o)} \tag{2.28}$$

$$r_b = \frac{\rho_o(\cosh\beta_o + \cos\alpha_o)}{(1 + \cosh\beta_o)}, \quad \psi_b = \tan^{-1}\left[\frac{\sinh\beta_o\sin\alpha_o}{\cosh\beta_o\cos\alpha_o + 1}\right] \tag{2.29}$$

$$r_c = \frac{\rho_o(\cosh\beta_o - \cos\alpha_o)}{(1 + \cosh\beta_o)}, \quad \psi_c = \tan^{-1}\left[\frac{\sinh\beta_o\sin\alpha_o}{\cosh\beta_o\cos\alpha_o - 1}\right] \tag{2.30}$$

The Eqs. (2.7), (2.9), (2.14) and (2.15) thus represent the first-order and the three parts of the second-order Doppler frequency dependent backscattered cross section of the ocean surface. These expressions are applicable for wide beam transmission and narrow beam reception.

CHAPTER 3

SECOND-ORDER OFF-PATCH SCATTER

3.1 Simplication of Cross Section Expression

The second-order off-patch cross section expression as given in Eq. (2.15) may be simplified by using some of its mathematical properties. We will first transform the three variables K_1, ϕ_1 and K_2 to new variables U, V and ϕ_c respectively such that Eq. (2.15) may be rewritten as

$$\sigma_{s3}(\omega_d) = \frac{1}{\pi^2\Delta_\theta|F_p|^4} \sum_{m,m'=\pm1} \int_{\phi_2=-\pi}^{\pi} \int_{R(U,V)} \int_{\phi_c=-\Delta_\theta}^{\Delta_\theta} \cdot |F_o(r_b,\psi_b+\phi_c)F_o(r_c,\psi_c+\phi_c+\pi)F_o(r_c,\phi_c+\pi)|^2 \cdot \frac{Q_c^2(\vec{K}_1,\vec{K}_2,\phi_c)K_1K_2}{K_c(1+\cosh\beta_o)|\det|J|} S(m\vec{K}_1)S(m'\vec{K}_2)\delta(U)\delta(V) \cdot d\phi_c\, dU\, dV\, d\phi_2 \tag{3.1}$$

where

$$U = \frac{K_c}{1+\cosh\beta_o} - 2k_o \tag{3.2}$$

$$V = \omega_d + m(gK_1)^{1/2} + m'(gK_2)^{1/2} \tag{3.3}$$

$$\phi_c = \phi_c(K_1,K_2,\phi_1,\phi_2) \tag{3.4}$$

3.1.1 Reduction of Integrals

The cross section expression as given by Eq. (3.1) consists of four integrals. It is noted, however, that this expression has two Dirac delta functions which may be used to reduce the number of integrations.

Since we know that

$$\int_a^b \int_c^d f(x,y)\delta(x)\delta(y)\, dy\, dx \doteq f(x,y)\Big|_{\substack{x=0\\y=0}}$$

provided $a \leq 0 \leq b$

$c \leq 0 \leq d$

we may simplify Eq. (3.1) as

$$\begin{aligned}\sigma_{s3}(\omega_d) = \frac{1}{\pi^2\Delta_\theta|F_p|^4} \sum_{m,m'=\pm 1} \int_{\phi_2=-\pi}^{\pi} \int_{\phi_c=-\Delta\theta}^{\Delta\theta} |F_o(r_b,\psi_b + \phi_c) \\ \cdot F_o(r_c,\psi_c + \phi_c + \pi)F_o(r_a,\phi_c + \pi)|^2 \\ \cdot \frac{Q_c^2(\vec{K}_1,\vec{K}_2,\phi_c)K_1K_2}{K_c(1 + \cosh\beta_o)|\det||J|} S(m\vec{k}_1)S(m'\vec{K}_2) \\ \cdot d\phi_c\, d\phi_2 \end{aligned} \tag{3.5}$$

where now

$$U = \frac{K_c}{1 + \cosh\beta_o} - 2k_o = 0 \tag{3.6}$$

$$V = \omega_d + m(gK_1)^{1/2} + m'(gK_2)^{1/2} = 0 \tag{3.7}$$

$$\phi_c = \phi_c(K_1,K_2,\phi_1,\phi_2) \tag{3.8}$$

Using Eq. (3.6), we may write Eq. (3.5) as

$$\sigma_{s3}(\omega_d) = \frac{2k_o}{\pi^2 \Delta_\theta |F_p|^4} \sum_{m,m'=\pm 1} \int_{\phi_2=-\pi}^{\pi} \int_{\phi_c=-\Delta_\theta}^{\Delta_\theta} \{ |F_o(r_b, \psi_b + \phi_c)$$

$$\cdot F_o(r_c, \psi_c + \phi_c + \pi) F_o(r_a, \phi_c + \pi)|^2$$

$$\cdot \frac{Q_c^2(\vec{K}_1, \vec{K}_2, \phi_c) K_1 K_2}{K_c^2 |\det| |J|} S(m\vec{K}_1) S(m'\vec{K}_2) \}$$

$$\cdot d\phi_c \, d\phi_2 \tag{3.9}$$

If we represent the function within { } by $f(\phi_c, \phi_2)$, we may rewrite Eq. (3.9) as

$$\sigma_{s3}(\omega_d) = \frac{2k_o}{\pi^2 \Delta_\theta |F_p|^4} \sum_{m,m'=\pm 1} \int_{\phi_2=-\pi}^{\pi} \int_{\phi_c=-\Delta_\theta}^{\Delta_\theta} f(\phi_c, \phi_2) d\phi_c \, d\phi_2 \tag{3.10}$$

where

$$f(\phi_c, \phi_2) = |F_o(r_b, \psi_b + \phi_c) F_o(r_c, \psi_c + \phi_c + \pi) F_o(r_a, \phi_c + \pi)|^2$$

$$\frac{Q_c^2(\vec{K}_1, \vec{K}_2, \phi_c) K_1 K_2}{K_c^2 |\det| |J|} S(m\vec{K}_1) S(m'\vec{K}_2) \tag{3.11}$$

For narrow beam reception, the integral with respect to ϕ_c can be approximated by multiplying the value of the integrand at the centre point by the width of the limits. For a better approximation, the limits on ϕ_c can be divided in n parts and so the Eq. (3.10) may again be written as

$$\sigma_{s3}(\omega_d) = \frac{2k_o}{\pi^2 \Delta_\theta |F_p|^4} \sum_{m,m'=\pm 1} \int_{\phi_2=-\pi}^{\pi} \left[\int_{-\Delta_\theta}^{\frac{-(n-2)\Delta_\theta}{n}} f(\phi_c, \phi_2)\, d\phi_c \right.$$

$$+ \int_{\frac{-(n-2)\Delta_\theta}{n}}^{\frac{-(n-4)\Delta_\theta}{n}} f(\phi_c, \phi_2)\, d\phi_c + \int_{\frac{-(n-4)\Delta_\theta}{n}}^{\frac{-(n-6)\Delta_\theta}{n}} f(\phi_c, \phi_2)\, d\phi_c$$

$$\left. + \ldots + \int_{\frac{(n-2)\Delta_\theta}{n}}^{\Delta_\theta} f(\phi_c, \phi_2)\, d\phi_c \right] d\phi_2 \qquad (3.12)$$

Using the rectangular rule to approximate the above integral, we may write

$$\sigma_{s3}(\omega_d) = \frac{2k_o \left(\frac{2\Delta_\theta}{n}\right)}{\pi^2 \Delta_\theta |F_p|^4} \sum_{m,m'=\pm 1} \int_{\phi_2=-\pi}^{\pi} \left[f\left\{\frac{-(n-1)\Delta_\theta}{n}, \phi_2\right\} \right.$$

$$+ f\left\{\frac{-(n-3)\Delta_\theta}{n}, \phi_2\right\} + f\left\{\frac{-(n-5)\Delta_\theta}{n}, \phi_2\right\}$$

$$\left. + \ldots + f\left\{\frac{(n-1)\Delta_\theta}{n}, \phi_2\right\} \right] d\phi_2$$

$$= \frac{4k_0}{n\pi^2 |F_p|^4} \sum_{m,m'=\pm 1} \int_{\phi_2=-\pi} \left[f\left\{\frac{-(n-1)\Delta_\theta}{n}, \phi_2\right\} \right.$$

$$+ f\left\{\frac{-(n-1)\Delta_\theta}{n}, \phi_2\right\} + f\left\{\frac{-(n-5)\Delta_\theta}{n}, \phi_2\right\}$$

$$+ \ldots + f\left\{\frac{(n-1)\Delta_\theta}{n}, \phi_2\right\} d\phi_2 \qquad (3.13)$$

3.1.2 Jacobian of Transformation

The Jacobian of the transformation may be given as

$$J = \begin{vmatrix} \frac{\partial U}{\partial K_1} & \frac{\partial U}{\partial K_2} & \frac{\partial U}{\partial \phi_1} \\ \frac{\partial V}{\partial K_1} & \frac{\partial V}{\partial K_2} & 0 \\ \frac{\partial \phi_c}{\partial K_1} & \frac{\partial \phi_c}{\partial K_2} & \frac{\partial \phi_c}{\partial \phi_1} \end{vmatrix} \qquad (3.14)$$

$$= \frac{\partial U}{\partial K_1}\frac{\partial V}{\partial K_2}\frac{\partial \phi_c}{\partial \phi_1} - \frac{\partial U}{\partial K_2}\frac{\partial V}{\partial K_1}\frac{\partial \phi_c}{\partial \phi_1} + \frac{\partial U}{\partial \phi_1}\left(\frac{\partial V}{\partial K_1}\frac{\partial \phi_c}{\partial K_2} - \frac{\partial V}{\partial K_2}\frac{\partial \phi_c}{\partial K_1}\right)$$

$$= \frac{\partial V}{\partial K_1}\left(\frac{\partial U}{\partial \phi_1}\frac{\partial \phi_c}{\partial K_2} - \frac{\partial U}{\partial K_2}\frac{\partial \phi_c}{\partial \phi_1}\right) + \frac{\partial V}{\partial K_2}\left(\frac{\partial U}{\partial K_1}\frac{\partial \phi_c}{\partial \phi_1} - \frac{\partial U}{\partial \phi_1}\frac{\partial \phi_c}{\partial K_1}\right)$$

Derivatives of V

From Eq. (3.3) we may obtain the following partial derivatives of V as

$$\frac{\partial V}{\partial K_1} = \frac{1}{2} m \sqrt{\frac{q}{K_1}} \qquad (3.15)$$

$$\frac{\partial V}{\partial K_2} = \frac{1}{2} m' \sqrt{\frac{q}{K_2}} \qquad (3.16)$$

$$\frac{\partial V}{\partial \phi_1} = 0 \qquad (3.17)$$

Using Eqs. (3.15) to (3.17), we may rewrite the Jacobian as

$$|J| = \left|\frac{1}{2}\, m \sqrt{\frac{q}{K_2}} \left(\frac{\partial U}{\partial \phi_1}\frac{\partial \phi_c}{\partial K_2} - \frac{\partial U}{\partial K_2}\frac{\partial \phi_c}{\partial \phi_1}\right) + \frac{1}{2}\, m' \sqrt{\frac{q}{K_2}} \left(\frac{\partial U}{\partial K_1}\frac{\partial \phi_c}{\partial K_2} - \frac{\partial U}{\partial \phi_1}\frac{\partial \phi_c}{\partial K_1}\right)\right| \tag{3.18}$$

Derivatives of U

Using Eq. (3.2) we may write the derivatives of U as

$$\frac{\partial U}{\partial K_1} = \frac{1}{1+\cosh\beta_o}\frac{\partial K_c}{\partial K_1} - \frac{K_c \sinh\beta_o}{(1+\cosh\beta_o)^2}\frac{\partial \beta_o}{\partial K_1} \tag{3.19}$$

$$\frac{\partial U}{\partial K_2} = \frac{1}{1+\cosh\beta_o}\frac{\partial K_c}{\partial K_2} - \frac{K_c \sinh\beta_o}{(1+\cosh\beta_o)^2}\frac{\partial \beta_o}{\partial K_2} \tag{3.20}$$

$$\frac{\partial U}{\partial \phi_1} = \frac{1}{1+\cosh\beta_o}\frac{\partial K_c}{\partial \phi_1} - \frac{K_c \sinh\beta_o}{(1+\cosh\beta_o)^2}\frac{\partial \beta_o}{\partial \phi_1} \tag{3.21}$$

Derivatives of ϕ_c

Eqn. (2.18) may be reproduced as

$$\phi_c = \tan^{-1}\left(\frac{K_{cy}}{K_{cx}}\right)$$

where K_{cy} and K_{cx} are defined in Eq. (2.18). Taking partial derivatives of the above equation we are able to write

$$\partial\phi_c = \frac{1}{1+\left(\frac{K_{cy}}{K_{cx}}\right)^2}\;\frac{K_{cx}\partial K_{cy} - K_{cy}\partial K_{cx}}{(K_{cx})^2}$$

$$= \frac{K_{cx}}{K_{cx}^2 + K_{cy}^2}\,\partial K_{cy} - \frac{K_{cy}}{K_{cx}^2 + K_{cy}^2}\,\partial K_{cx}$$

$$= \frac{\cos\phi_c\ \partial K_{cy}}{\sqrt{K_{cx}^2 + K_{cy}^2}} - \frac{\sin\phi_c\ \partial K_{cx}}{\sqrt{K_{cx}^2 + K_{cy}^2}} \tag{3.22}$$

Using Eqs. (2.16), (2.17) and (3.22) we may write the following derivatives as

$$\begin{aligned}
\frac{\partial\phi_c}{\partial K_1} = \Bigg[&\cos\phi_c\Big\{2\sin\phi_1 + K_2\sin\phi_2\Big(\sinh\beta_o\cos\alpha_o\,\frac{\partial\beta_o}{\partial K_1} \\
&- \cosh\beta_o\sin\alpha_o\,\frac{\partial\alpha_o}{\partial K_1}\Big) - K_2\cos\phi_2\Big(\cosh\beta_o\sin\alpha_o\,\frac{\partial\beta_o}{\partial K_1} \\
&+ \sinh\beta_o\cos\alpha_o\,\frac{\partial\alpha_o}{\partial K_1}\Big)\Big\} - \sin\phi_c\Big\{2\cos\phi_1 + K_2\cos\phi_2 \\
&\cdot\Big(\sinh\beta_o\cos\alpha_o\,\frac{\partial\beta_o}{\partial K_1} - \cosh\beta_o\sin\alpha_o\,\frac{\partial\alpha_o}{\partial K_1}\Big) \\
&+ K_2\sin\phi_2\Big(\cosh\beta_o\sin\alpha_o\,\frac{\partial\beta_o}{\partial K_1} + \sinh\beta_o\cos\alpha_o \\
&\cdot\frac{\partial\alpha_o}{\partial K_1}\Big)\Big\}\Bigg]\frac{1}{K_c} \\
= \Bigg[&2\sin(\phi_1 - \phi_c) + K_2\sin(\phi_2 - \phi_c)\Big(\sinh\beta_o\cos\alpha_o\,\frac{\partial\beta_o}{\partial K_1} \\
&- \cosh\beta_o\sin\alpha_o\,\frac{\partial\alpha_o}{\partial K_1}\Big) - K_2\cos(\phi_2 - \phi_c) \\
&\cdot\Big(\cosh\beta_o\sin\alpha_o\,\frac{\partial\beta_o}{\partial K_1} + \sinh\beta_o\cos\alpha_o\,\frac{\partial\alpha_o}{\partial K_1}\Big)\Bigg]\frac{1}{K_c}
\end{aligned} \tag{3.23}$$

$$\frac{\partial\phi_c}{\partial K_2} = \Big[\cos\phi_c\Big\{\sin\phi_2 + \sin\phi_2\cosh\beta_o\cos\alpha_o + K_2\sin\phi_2$$

$$\cdot \left(\sinh\beta_o\cos\alpha_o \frac{\partial\beta_o}{\partial K_2} - \cosh\beta_o\sin\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right)$$

$$- \cos\phi_2\sinh\beta_o\sin\alpha_o - K_2\cos\phi_2\left(\cosh\beta_o\sin\alpha_o \frac{\partial\beta_o}{\partial K_2}\right.$$

$$\left.\left. + \sinh\beta_o\cos\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right)\right\} - \sin\phi_c\left\{\cos\phi_2 + \cos\phi_2\cosh\beta_o\cos\alpha_o\right.$$

$$+ K_2\cos\phi_2\left(\sinh\beta_o\cos\alpha_o \frac{\partial\beta_o}{\partial K_2} - \cosh\beta_o\sin\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right)$$

$$+ \sin\phi_2\sinh\beta_o\sin\alpha_o + K_2\sin\phi_2\left(\cosh\beta_o\sin\alpha_o \frac{\partial\beta_o}{\partial K_2} + \sinh\beta_o\right.$$

$$\left.\left.\left. \cos\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right)\right\}\right] \frac{1}{K_c}$$

$$= \left[\sin(\phi_2 - \phi_c)(1 + \cosh\beta_o\cos\alpha_o) - \cos(\phi_2 - \phi_c)\sinh\beta_o\sin\alpha_o\right.$$

$$+ K_2\sin(\phi_2 - \phi_c)\left(\sinh\beta_o\cos\alpha_o \frac{\partial\beta_o}{\partial K_2} - \cosh\beta_o\sin\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right)$$

$$- K_2\cos(\phi_2 - \phi_c)\left(\cosh\beta_o\sin\alpha_o \frac{\partial\beta_o}{\partial K_2}\right.$$

$$\left.\left. + \sinh\beta_o\cos\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right)\right] \frac{1}{K_c} \tag{3.24}$$

$$\frac{\partial\phi_c}{\partial\phi_1} = \left[2K_1\cos(\phi_1 - \phi_c) + K_2\sin(\phi_2 - \phi_c)\left(\sinh\beta_o\cos\alpha_o \frac{\partial\beta_o}{\partial\phi_1}\right.\right.$$

$$\left. - \cosh\beta_o\sin\alpha_o \frac{\partial\beta_o}{\partial\phi_1}\right) - K_2\cos(\phi_2 - \phi_c)\left(\cosh\beta_o\sin\alpha_o \frac{\partial\beta_o}{\partial\phi_1}\right.$$

$$\left.\left. + \sinh\beta_o\cos\alpha_o \frac{\partial\alpha_o}{\partial\phi_1}\right)\right] \frac{1}{K_c} \tag{3.25}$$

Derivatives of K_c

The magnitude of $\vec{K}_c$ as defined in Eq. (2.18) may be written as

$$K_c = [K_{cx}^2 + K_{cy}^2]^{1/2}$$

$$= \Big[\{2K_{1x} + K_{2x}(1 + \cosh\beta_o\cos\alpha_o) + K_{2y}\sinh\beta_o\sin\alpha_o\}^2$$

$$+ \{2K_{1y} + K_{2y}(1 + \cosh\beta_o\cos\alpha_o) - K_{2x}\sinh\beta_o\sin\alpha_o\}^2\Big]^{1/2}$$

$$= \Big[4K_{1x}^2 + K_{2x}^2 + K_{2x}^2\cosh^2\beta_o\cos^2\alpha_o + K_{2y}^2\sinh^2\beta_o\sin^2\alpha_o$$

$$+ 4K_{1x}K_{2x} + 4K_{1x}K_{2x}\cosh\beta_o\cos\alpha_o + 4\alpha_1\beta_2\sinh\beta_o\sin\alpha_o$$

$$+ 2K_{2x}^2\cosh\beta_o\cos\alpha_o + 2K_{2x}K_{2y}\sinh\beta_o\sin\alpha_o$$

$$+ 2K_{2x}K_{2y}\cosh\beta_o\cos\alpha_o\sinh\beta_o\sin\alpha_o + 4K_{1y}^2 + K_{2y}^2$$

$$+ K_{2y}^2\cosh^2\beta_o\cos^2\alpha_o + K_{2x}^2\sinh^2\beta_o\sin^2\alpha_o + 4K_{1y}K_{2y}$$

$$+ 4K_{1y}K_{2y}\cosh\beta_o\cos\alpha_o - 4K_{2x}K_{1y}\sinh\beta_o\sin\alpha_o$$

$$+ 2K_{2y}^2\cosh\beta_o\cos\alpha_o - 2K_{2x}K_{2y}\sinh\beta_o\sin\alpha_o$$

$$- 2K_{2x}K_{2y}\cosh\beta_o\cos\alpha_o\sinh\beta_o\sin\alpha_o\Big]^{1/2}$$

$$= [4K_1^2 + K_2^2 + K_2^2\cosh^2\beta_o\cos^2\alpha_o + K_2^2\sinh^2\beta_o\sin^2\alpha_o$$

$$+ 4(1 + \cosh\beta_o\cos\alpha_o)(K_{1x}K_{2x} + K_{1y}K_{2y})$$

$$+ 4\sinh\beta_o\sin\alpha_o(K_{1x}K_{2y} - K_{2x}K_{1y})$$

$$+ 2K_2^2\cosh\beta_o\cos\alpha_o]^{1/2}$$

$$= \Big[4K_1^2 + K_2^2(\cosh\beta_o + \cos\alpha_o)^2 + 4K_1K_2(1 + \cosh\beta_o\cos\alpha_o)$$

$$\cdot \cos(\phi_2 - \phi_1) + 4K_1K_2\sinh\beta_o\sin\alpha_o\sin(\phi_2 - \phi_1)\Big]^{1/2} \quad (3.26)$$

Now the derivatives of K_c may be written as follows

$$\frac{\partial K_c}{\partial K_1} = \frac{1}{2K_c}\Big[8K_1 + 2K_2^2(\cosh\beta_o + \cos\alpha_o)(\sin\beta_o\frac{\partial\beta_o}{\partial K_1} - \sin\alpha_o$$

$$\cdot\frac{\partial\alpha_o}{\partial K_1}) + 4K_2(1 + \cosh\beta_o\cos\alpha_o)\cos(\phi_2 - \phi_1) + 4K_1K_2$$

$$\cdot\cos(\phi_2 - \phi_1)\Big(\sinh\beta_o\cos\alpha_o\frac{\partial\beta_o}{\partial K_1} - \cosh\beta_o\sin\alpha_o\frac{\partial\alpha_o}{\partial K_1}\Big)$$

$$+ 4K_2\sinh\beta_o\sin\alpha_o\sin(\phi_2 - \phi_1) + 4K_1K_2\sin(\phi_2 - \phi_1)$$

$$\cdot(\cosh\beta_o\cos\alpha_o\frac{\partial\beta_o}{\partial K_1} + \sinh\beta_o\cos\alpha_o\frac{\partial\alpha_o}{\partial K_1})\Big]$$

$$= \frac{1}{K_c}\Big[4K_1 + K_2^2(\cosh\beta_o + \cos\alpha_o)\Big(\sinh\beta_o\frac{\partial\beta_o}{\partial K_1} - \sin\alpha_o\frac{\partial\alpha_o}{\partial K_1}\Big)$$

$$+ 2K_2^2(1 + \cosh\beta_o\cos\alpha_o)\cos(\phi_2 - \phi_1) + 2K_1K_2\cos(\phi_2 - \phi_1)$$

$$\cdot \left(\sinh\beta_o\cos\alpha_o \frac{\partial\beta_o}{\partial K_1} - \cosh\beta_o\sin\alpha_o \frac{\partial\alpha_o}{\partial K_1}\right) + 2K_1(1 + \cosh\beta_o\cos\alpha_o)$$

$$\cdot \sin\alpha_o\sin(\phi_2 - \phi_1) + 2K_1K_2\sin(\phi_2 - \phi_1)$$

$$\cdot \left(\cosh\beta_o\sin\alpha_o \frac{\partial\beta_o}{\partial K_1} + \sinh\beta_o\cos\alpha_o \frac{\partial\alpha_o}{\partial K_1}\right)\Bigg]$$

Similarly

$$\frac{\partial K_c}{\partial K_2} = \frac{1}{K_c}\Bigg[K_2(\cosh\beta_o - \cos\alpha_o)^2 + K_2^2(\cosh\beta_o + \cos\alpha_o)$$

$$\cdot \left(\sinh\beta_o \frac{\partial\beta_o}{\partial K_2} - \sin\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right) + 2K_1(1 + \cosh\beta_o\cos\alpha_o)$$

$$\cdot \cos(\phi_2 - \phi_1) + 2K_1K_2\cos(\phi_2 - \phi_1)\left(\sinh\beta_o\cos\alpha_o \frac{\partial\beta_o}{\partial K_2}\right.$$

$$\left. - \cosh\beta_o\sin\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right) + 2K_1\sinh\beta_o\sin\alpha_o\sin(\phi_2 - \phi_1)$$

$$+ 2K_1K_2\sin(\phi_2 - \phi_1)\left(\cosh\beta_o\sin\alpha_o \frac{\partial\beta_o}{\partial K_2}\right.$$

$$\left. + \sinh\beta_o\cos\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right)\Bigg] \qquad (3.28)$$

$$\frac{\partial K_c}{\partial \phi_1} = \frac{1}{K_c}\Bigg[K_2^2(\cosh\beta_o + \cos\alpha_o)\left(\sinh\beta_o \frac{\partial\beta_o}{\partial\phi_1} - \sin\alpha_o \frac{\partial\alpha_o}{\partial\phi_1}\right)$$

$$+ 2K_1K_2(1 + \cosh\beta_o\cos\alpha_o)\sin(\phi_2 - \phi_1) + 2K_1K_2\cos(\phi_2 - \phi_1)$$

$$\cdot \left(\sinh\beta_o\cos\alpha_o \frac{\partial\beta_o}{\partial\phi_1} - \cosh\beta_o\sin\alpha_o \frac{\partial\alpha_o}{\partial\phi_1}\right) - 2K_1K_2\sinh\beta_o$$

$$\cdot \sin\alpha_o \cos(\phi_2 - \phi_1) + 2K_1K_2\sin(\phi_2 - \phi_1)\left[\cosh\beta_o \sin\alpha_o \frac{\partial\beta_o}{\partial\phi_1}\right.$$

$$\left. + \sinh\beta_o \cos\alpha_o \frac{\partial\alpha_o}{\partial\phi_1}\right]\Bigg] \tag{3.29}$$

Derivatives of β_o

β_o is the solution of the quadratic equation (2.24) and will have two values for each value of $\cosh\beta_o$ and may be written as

$$\beta_o = \pm\cosh^{-1}(C^{\pm}) \tag{3.30}$$

where

$$C^{\pm} = \frac{\begin{array}{l}|\sin\phi_2|[K_2^2\sin\phi_2 \pm 2\sqrt{K_1}|2K_1\cos\phi_1 + K_2\cos\phi_2| \\ \cdot (K_1\cos^2\phi_1 + K_2\cos\phi_1\cos\phi_2)^{1/2}]\end{array}}{4K_1^2\cos^2\phi_1 + 4K_1K_2\cos\phi_1\cos\phi_2 - K_2^2\sin^2\phi_2} \tag{3.31}$$

Out of four roots of β_o, we have to take only one root for which C is real and $C > 1$ and which satisfies the Eq. (2.26). Further from Eq. (3.30) we may write

$$\frac{\partial\beta_o}{\partial C^{\pm}} = \pm\frac{1}{\{(C^{\pm})^2 - 1\}^{1/2}} = \frac{1}{\sinh\beta_o} \tag{3.32}$$

To obtain the derivatives of β_o, we may use Eq. (3.30) and so are able to write the derivatives as follows

$$\frac{\partial\beta_o}{\partial K_1} = \frac{\partial\beta_o}{\partial C^{\pm}}\frac{\partial C^{\pm}}{\partial K_1} \tag{3.33}$$

$$\frac{\partial\beta_o}{\partial K_2} = \frac{\partial\beta_o}{\partial C^{\pm}} \frac{\partial C^{\pm}}{\partial K_2} \tag{3.34}$$

$$\frac{\partial\beta_o}{\partial\phi_1} = \frac{\partial\beta_o}{\partial C^{\pm}} \frac{\partial C^{\pm}}{\partial\phi_1} \tag{3.35}$$

where $\frac{\partial\beta_o}{\partial C^{\pm}}$ is defined in Eq. (3.32).

Derivatives of $C^{\pm}$

In the above equations we still have to find the derivatives of $C^{\pm}$ which may be written from Eq. (3.31) as follows

$$\frac{\partial C^{\pm}}{\partial K_1} = \frac{\begin{aligned} &\{[4K_1^2\cos^2\phi_1 + 4K_1K_2\cos\phi_1\cos\phi_2 - K_2^2\sin^2\phi_2] \\ &\cdot [\pm 2\,\mathrm{sgn}\{2K_1\cos\phi_1 + K_2\cos\phi_2\}\{3\sqrt{K_1}\cos\phi_1 + \frac{1}{2\sqrt{K_1}} K_2\cos\phi_2\} \\ &\cdot \{K_1\cos^2\phi_1 + K_2\cos\phi_1\cos\phi_2\}^{1/2} \pm \sqrt{K_1}|2K_1\cos\phi_1 \\ &+ K_2\cos\phi_2|\cos\phi_1\{K_1\cos^2\phi_1 + K_2\cos\phi_1\cos\phi_2\}^{-1/2}] \\ &- [K_2^2|\sin\phi_2| \pm 2\sqrt{K_1}|2K_1\cos\phi_1 + K_2\cos\phi_2| \\ &\cdot \{K_1\cos^2\phi_1 + K_2\cos\phi_1\cos\phi_2\}^{1/2}] \\ &\cdot [8K_1\cos^2\phi_1 + 4K_2\cos\phi_1\cos\phi_2]\}|\sin\phi_2| \end{aligned}}{[4K_1^2\cos^2\phi_1 + 4K_1K_2\cos\phi_1\cos\phi_2 - K_2^2\sin^2\phi_2]^2}, \tag{3.36}$$

Similarly

$$\frac{\partial c^{\pm}}{\partial K_2} = \frac{\begin{aligned}&\{[4K_1^2\cos^2\phi_1 + 4K_1K_2\cos\phi_1\cos\phi_2 - K_2^2\sin^2\phi_2]\\ &\cdot [2K_2|\sin\phi_2| \pm 2\mathrm{sgn}\{2K_1\cos\phi_1 + K_2\cos\phi_2\}\sqrt{K_1}\cos\phi_2\\ &\cdot \{K_1\cos^2\phi_1 + K_2\cos\phi_1\cos\phi_2\}^{1/2} \pm \sqrt{K_1}|2K_1\cos\phi_1 + K_2\\ &\cdot \cos\phi_2|\cos\phi_1\cos\phi_2\{K_1\cos^2\phi_1 + K_2\cos\phi_1\cos\phi_2\}^{-1/2}]\\ &- [K_2^2|\sin\phi_2| \pm 2\sqrt{K_1}|2K_1\cos\phi_1 + K_2\cos\phi_2|\{K_1\cos^2\phi_1\\ &+ K_2\cos\phi_1\cos\phi_2\}^{1/2}] \cdot 4K_1\cos\phi_1\cos\phi_2 - 2K_2\sin^2\phi_2]\}\\ &\cdot |\sin\phi_2|\end{aligned}}{[4K_1^2\cos^2\phi_1 + 4K_1K_2\cos\phi_1\cos\phi_2 - K_2^2\sin^2\phi_2]^2} \tag{3.37}$$

$$\begin{aligned}\frac{\partial c^{\pm}}{\partial \phi_1} = \Big|\;&\{[4K_1^2\cos^2\phi_1 + 4K_1K_2\cos\phi_1\cos\phi_2 - K_2^2\sin^2\phi_2]\\ &\cdot [\mp 4\mathrm{sgn}\{2K_1\cos\phi_1 + K_2\cos\phi_2\}K_1^{3/2}\sin\phi_1\{K_1\cos^2\phi_1\\ &+ K_2\cos\phi_1\cos\phi_2\}^{1/2} \mp \sqrt{K_1}|2K_1\cos\phi_1 + K_2\cos\phi_2|\{2K_1\\ &\cdot \cos\phi_1\sin\phi_1 + K_2\sin\phi_1\cos\phi_2\}\{K_1\cos^2\phi_1 + K_2\cos\phi_1\\ &\cdot \cos\phi_2\}^{-1/2}] + [K_2^2|\sin\phi_2| \pm 2\sqrt{K_1}|2K_1\cos\phi_1 + K_2\cos\phi_2|\end{aligned}$$

$$\frac{\cdot \{K_1\cos^2\phi_1 + K_2\cos\phi_1\cos\phi_2\}^{1/2}] \cdot [8K_1^2\cos\phi_1\sin\phi_1 + 4K_1K_2 \cdot \sin\phi_1\cos\phi_2]\}|\sin\phi_2|}{[4K_1^2\cos^2\phi_1 + 4K_1K_2\cos\phi_1\cos\phi_2 - K_2^2\sin^2\phi_2]^2} \qquad (3.38)$$

Derivatives of α_o

Using Eq. (2.27) the derivatives of α_o with respect to K_1, K_2 and ϕ_1 may be obtained as follows

$$\frac{\partial\alpha_o}{\partial K_1} = \frac{\tan\phi_2\operatorname{sech}^2\beta_o}{1 + \tanh^2\beta_o\tan^2\phi_2}\,\frac{\partial\beta_o}{\partial K_1} \qquad (3.39)$$

$$\frac{\partial\alpha_o}{\partial K_2} = \frac{\tan\phi_2\operatorname{sech}^2\beta_o}{1 + \tanh^2\beta_o\tan^2\phi_2}\,\frac{\partial\beta_o}{\partial K_2} \qquad (3.40)$$

$$\frac{\partial\alpha_o}{\partial\phi_1} = \frac{\tan\phi_2\operatorname{sech}^2\beta_o}{1 + \tanh^2\beta_o\tan^2\phi_2}\,\frac{\partial\beta_o}{\partial\phi_1} \qquad (3.41)$$

Derivatives of ξ

In order to find the derivatives of ξ as required to calculate 'det' in Eq. (2.22), we may utilize Eq. (2.23) which may also be written as

$$\xi(\beta,\alpha) = \frac{1}{(1 + \cosh\beta)}\,[2K_1\cos(\phi_1 - \phi_c) + K_2\sinh\beta\,\sin\alpha\,\sin(\phi_2 - \phi_c) + K_2(1 + \cosh\beta\,\cos\alpha)\cos(\phi_2 - \phi_c)] \qquad (3.42)$$

To obtain above equation, we have used the following relationships

$$\vec{K}_1 \cdot \vec{K}_c = |\vec{K}_1||\vec{K}_c| \cos(\phi_1 - \phi_c)$$

$$\vec{K}_1 \cdot \vec{K}_c = K_1 K_c \cos(\phi_1 - \phi_c)$$

$$\vec{K}_1 \cdot \hat{K}_c = K_1 \cos(\phi_1 - \phi_c) \tag{3.43}$$

Similarly

$$\vec{K}_2 \cdot \hat{K}_c = K_2 \cos(\phi_2 - \phi_c)$$

Now from Eq. (3.42) derivative of ξ may be written as

$$\frac{\partial^2\xi}{\partial\beta^2} = \frac{1}{(1+\cosh\beta)^2} [2(\cosh\beta - 2) K_1 \cos(\phi_1 - \phi_c) + (\cosh\beta - 2) \cdot K_2(1 - \cos\alpha)\cos(\phi_2 - \phi_c) - K_2 \sinh\beta \sin\alpha \cdot \sin(\phi_2 - \phi_c)] \tag{3.44}$$

$$\frac{\partial^2\xi}{\partial\alpha^2} = \frac{1}{(1+\cosh\beta)} [-K_2 \sinh\beta \sin\alpha \sin(\phi_2 - \phi_c) - K_2 \cosh\beta \cdot \cos\alpha \cos(\phi_2 - \phi_c)] \tag{3.45}$$

$$\frac{\partial^2\xi}{\partial\beta\partial\alpha} = \frac{1}{(1+\cosh\beta)^2} [K_2(1+\cosh\beta)\cos\alpha \sin(\phi_2 - \phi_c) - K_2 \sinh\beta \cdot \sin\alpha \cos(\phi_2 - \phi_c)] \tag{3.46}$$

Summarizing all the required derivatives as given in Eqs. (3.15) to (3.41) may be used to calculate the 'Jacobian', while Eqs. (3.44) to (3.46) may be used to calculate the 'det'.

CHAPTER 4

RESULTS AND DISCUSSIONS

To generate the off patch spectral cross section at any Doppler frequency, the sum of integrals as given in Eq. (3.13) has to be evaluated. The function $f(\phi_c,\phi_2)$ is defined in Eq. (3.11). The beamwidth of the receiving antenna is assumed to be as 6^0 in all the computations. So $\Delta_\theta = 3^0$ or $\Delta_\theta = \frac{3x\pi}{180}$ radian. Also for convenience n is assumed as equal to 5. Therefore the cross section expression as given by Eq. (3.13) may be reduced to

$$\sigma_{s3}(\omega_d) = \frac{4k_o}{5\pi^2|F_p|^4} \sum_{m,m'=\pm1} \int_{\phi_2=-\pi}^{\pi} \left[f(\frac{-2.4x\pi}{180}, \phi_2) + f(\frac{-1.2x\pi}{180}, \phi_2) + f(0,\phi_2) + f(\frac{1.2x\pi}{180}, \phi_2) + f(\frac{2.4x\pi}{180}, \phi_2) \right] d\phi_2 \qquad (4.1)$$

In the above equation, ϕ_c has assumed the values as $\frac{-2.4x\pi}{180}$, $\frac{-1.2x\pi}{180}$, 0, $\frac{1.2x\pi}{180}$ and $\frac{2.4x\pi}{180}$. If the variable θ_i is used to represent these values, then it may be written as

$$\phi_c = \theta_i$$

$$W = \phi_c - \theta_i = 0 \qquad (4.2)$$

where W is a substitution. Now Eqs. (3.6), (3.7) and (4.2) may be solved to obtain K_1, K_2 and ϕ_1 for given values of ϕ_2, m, m' and

ω_d. But these solutions will be radar frequency-dependent. To avoid this problem all the three equations are normalized to radar frequency and in this way the obtained solutions may be used to generate the cross section for any radar frequency.

After normalization, Eqs. (3.6), (3.7) and (4.2) become

$$U = \frac{1}{1 + \cosh\beta_o} [4K_1^{o^2} + K_2^{o^2}(\cosh\beta_o + \cos\alpha_o) + 4K_1^oK_2^o(1 + \cosh\beta_o\cos\alpha_o)\cos(\phi_2 - \phi_1) + 4K_1^oK_2^o\sinh\beta_o\sin\alpha_o\sin(\phi_2 - \phi_1)]^{1/2} - 1 = 0 \quad (4.3)$$

$$V = \eta + m\sqrt{K_1^o} + m'\sqrt{K_2^o} = 0 \quad (4.4)$$

$$W = \tan^{-1}\left[\frac{2K_1^o\sin\phi_1 + K_2^o\sin\phi_2(1+\cosh\beta_o\cos\alpha_o) - K_2^o\cos\phi_2\sinh\beta_o\sin\alpha_o}{2K_1^o\cos\phi_1 + K_2^o\cos\phi_2(1+\cosh\beta_o\cos\alpha_o) + K_2^o\sinh\beta_o\sin\alpha_o\sin\phi_2}\right] - \theta_i = 0 \quad (4.5)$$

where $K_1^o = K_1/2k_o$ and $K_2^o = K_2/2k_o$ are normalized wavenumbers. $\eta = \omega_d/\omega_B$ is the normalized frequency.

Eqs. (4.3), (4.4) and (4.5) may now be solved. As it is evident from Eq. (4.4) that K_1^o and K_2^o have a direct relationship, we may substitute the expression of K_2^o from Eq. (4.4) into Eqs. (4.3) and (4.5). So ultimately there will be only two equations to be solved.

To obtain the solutions of Eqs. (4.3) and (4.5), a minimization technique is chosen. The ranges for K_1^o, ϕ_1 and ϕ_2 are defined. For each value of ϕ_2, the whole range of ϕ_1 is scanned. Similarly for each value of ϕ_1, the complete range of K_1^o is scanned to find the acceptable solutions. The range of ϕ_2 is chosen as $-179^o \leq \phi_2 \leq 179^o$ with the increment of 2^o. The range of ϕ_1 is taken as -180^o to 180^o with 1^o interval. K_1^o is allowed to vary between 0.01 and 5.0 with varying increment. The increment for K_1^o between 0.01 and 0.1 is 0.01 and between 0.1 and 5.0 is 0.1.

Some important symmetric properties are investigated among the equations which may be very useful in saving computation time while obtaining solutions. These three symmetric properties may be summarized as follows.

(1) Let us consider Eq. (4.4). We may also write it as

$$K_2^o = \left(\frac{-\eta - m\sqrt{K_1^o}}{m'}\right)^2$$

Since m' may be either +1 or −1, we may also write the above equation as

$$K_2^o = (\eta + m\sqrt{K_1^o})^2$$

In the above equation, if the sign of η and m are changed at the same time then the value of K_2^o will remain the same. This means that the set of solutions for $-\eta$ will remain the same as for $+\eta$ or vice versa except that the sign of m will change.

(2) Let us examine Eq. (4.3). In this equation if the sign of ϕ_2, ϕ_1 and β_o are changed at the same time then there will not be any change in U. This indicates that if the sign of ϕ_2 is changed then ϕ_1 and β_o will also change their sign with other values remaining the same.

(3) Lastly if we investigate Eq. (4.5), it is evident that if ϕ_2, ϕ_1, β_o and θ_i change their sign simultaneously then W will also change in sign. In conjunction with the above symmetry this will imply that if we have solutions for $\phi_c - \theta_i = 0$, then we can generate the solutions for $\phi_c + \theta_i = 0$ or vice versa by changing the sign of ϕ_2, ϕ_1 and β_o. It is important to remember in all the above symmetry analyses that α_o will always be positive as defined in Eq. (2.27).

The range of η is chosen from -3.08 to 3.08 with the increment of 0.04. Solutions are generated for η varying from -3.08 to 0.0 and other half solutions are quickly obtained by using the first symmetry property. Also the solutions have to be generated for five different cases corresponding to summation of five different functions in the cross section expression. Solutions are generated only for three cases and for the other two cases the solutions are obtained by use of the third symmetry property.

The strategy in solving the two equations is to take a value each for m, η and ϕ_2 within the designated range and then search for pairs of ϕ_1 and K_1^o which will satisfy the restriction on β_o as given in Eq. (2.26). As has previously been discussed for each value of ϕ_1 we are scanning the complete range of K_1^o, so at one particular ϕ_1 there may be many K_1^o which will satisfy the required restrictions. Further between two consecutively obtained values of K_1^o we test the change of sign in U and W. This will ensure that there exists a solution between the two values of K_1^o. Once these conditions are met, an IMSL subroutine is used to minimize the summation of the absolute values of U and W. The returned value is accepted only if it is less than the summations of the absolute values of U and W at two values of K_1^0.

Since the increment in ϕ_1 is taken as 1^0, it is observed that almost same roots are obtained for several consecutive ϕ_1. These roots are very close to each other and may be understood as multiple occurring of the same roots. To avoid the multiple occurring of the same root, only one root was accepted out of all the roots having less than or equal to 2^0 difference in ϕ_1.

The significance of off-patch scatter is studied for two different cases. First, when the radar is located on the beach. In this case there will not be any signal return from behind the radar. To make sure that there is no signal return from behind the radar, a restriction has been put on θ_b; that is $|\theta_b| \leq 90^0$. Also the solutions may indicate an interaction with that area of shallow water surrounding the radar. Since it is unlikely that this is important, restrictions have been put while accepting the roots that all the three distances travelled by the signal during two scatters should individually be greater than one kilometer. These distances denoted by r_a, r_b and r_c are given in Eqs. (2.28) to (2.30).

The second case corresponds to the case when the radar is located on the open ocean (e.g., a ship or an offshore paltform based radar). Now the radar is surrounded by the water and signals may be received from all directions. Imposing restrictions on r_b and r_c as they should be greater than one kilometer, it is made sure that both the scatters occur distinctly and away from the radar. There is no restriction on θ_b in this case.

After all the restrictions are met, the final accepted solutions are used to generate the cross section. In the computation of the cross section, roots are again denormalized so that they may be used for particular radar frequency and sea state. m' may take the value either $+1$ or -1 and it is decided depending on the values of ω_d, m and K_1. Analysis of Eq. (3.7) is used to decide the value of m'. From this equation it is evident that if m is equal to 1 and ω_d is less than 0 and K_1 is less than or equal to ω_d^2/g then $m' = 1$ otherwise $m' = -1$. Likewise if ω_d is less than 0 but K_1 is greater than or equal to ω_d^2/g, then $m' = 1$. Else if $m = -1$ and ω_d is greater than or equal to 0 and K_1 is greater than ω_d^2/g, then $m' = 1$ otherwise $m' = 1$. But if ω_d is greater than 0 and K_1 is less than or equal to ω_d^2/g then $m' = -1$. σ_{s3} is computed from Eq. (4.1) in conjunction with Eq. (3.11). Eq. (2.21) is used to calculate $Q_c(\vec{K}_1, \vec{K}_2, \phi_c)$ and Eqs. (2.22), (3.44), (3.45) and (3.46) are used for the calculation of 'det'. Jacobian may be computed by the use of Eqs. (3.14) to (3.41). The Pierson-Moskowitz frequency spectrum with a cardiod directional distribution [Srivastava (1984)] is used to model the ocean waveheight spectrum. This directional waveheight spectrum as given in Walsh et al. (1986) may be written as

$$S(\vec{K}) = \frac{0.0162\pi}{K^4} e^{-0.74 \left(\frac{g}{KU_w^2}\right)^2} \cos^2\left(\frac{\phi - \theta_w}{2}\right) \qquad (4.9)$$

where

$$K = K_x\hat{x} + K_y\hat{y} \text{ , } \phi = \tan^{-1}\left(\frac{K_y}{K_x}\right)$$

U_w = wind speed in meter per second

θ_w = wind direction

g is the gravitational acceleration.

Also for computational ease Δ is used instead of the modified surface impedance (Δ_0) to evaluate the attenuation function in σ_{s3}. The cross section is generated for different radar frequencies and sea states. The integral with respect to ϕ_2 in Eq. (4.1) is evaluated using the rectangular rule. The distance of the patch in all cases is taken as 30 kilometers. It is verified that by changing the distance of the patch there is no significant variation in the cross section result.

In order to exhibit the significance of off-patch scatter compared to on-patch scatter, two corresponding cross sections σ_{s1} and σ_{s3} are plotted for 10 MHz and 25.4 MHz radar frequencies. The two frequencies chosen provide a well representation of the HF region. Thus any conclusion drawn at these frequencies may be applied to HF radars in general. The case when the radar is located in the open ocean is considered first. It is assumed in this case that the sea is fully developed in the total scattering region. Figures 3 to 5 show the individual spectrum of σ_{s1} and σ_{s3} for 10 MHz radar frequency and 10 knots wind speed. Wind directions are 0^o, 45^o and 90^o (cross wind) respectively with reference to the direction of the patch. The Doppler frequencies (in Hz) corresponding to $\pm\omega_B$ and $\pm 2^{3/4}\,\omega_B$ (in rad/sec) are marked in spectral polts as "±a" and "±b" respectively. Examining the above plots it is clear that σ_{s3} is lower than σ_{s1} at all

Doppler points except at zero and beyond $\pm 2^{3/4}\omega_B$ (corner reflector) Doppler frequencies. In these regions σ_{s3} is higher than σ_{s1}. Similar plots are presented in figures 6 to 8 but now the radar frequency is 25.4 MHz. In these plots σ_{s3} is higher than σ_{s1} only at zero, around $\pm 2^{3/4}\omega_B$ and beyond $\pm 2^{3/4}\omega_B$ Doppler frequency points. In these plots two first-order peaks at $\pm\omega_B$ which lie in the null regions are not shown. The regions around first-order peaks which are used for the extraction of ocean surface parameters are not affected by σ_{s3}. Therefore, in these regions the second-order cross section may adequately be described by σ_{s1} alone. Figures 9 to 11 are plotted for 10 MHz radar frequency but 30 knots wind speed. Wind directions are maintained at 0^0, 45^0 and 90^0. In these plots σ_{s3} is significantly lower than σ_{s1} at all Doppler points except at zero, around $\pm 2^{3/4}\omega_B$ and beyond $\pm 2^{3/4}\omega_B$ Doppler frequency points. Same conclusions may be drawn if the radar frequency is raised to 25.4 MHz and the corresponding plots are presented in figures 12 to 14. Going from 0^0 to 90^0 wind direction, the effect of σ_{s3} at zero Doppler decreases. Referring to the above examples the contribution of σ_{s3} is not important in the wave regions. Beyond $\pm 2^{3/4}\omega_B$ σ_{s3} is significantly higher than σ_{s1} but the value of σ_{s3} itself is lower in this region. It is interesting to note that σ_{s2} may also be present in this case as discussed in Walsh and Srivastava (1987). This term has not been included in this study. The peak at zero Doppler is the effect of double first-order scatter phenomenon.

Now we consider the case when the radar is located on the beach or near the shore. There will be a reduction in σ_{s3} as it is impossible for any scattering to occur on the land. σ_{s1} will be the same in this case. Figure 15 to 17 show the individual plots of σ_{s1} and σ_{s3} for a 10 MHz radar frequency and 10 knots wind speed. Wind directions are again 0^0, 45^0 and 90^0 respectively. σ_{s1} and σ_{s3} both are lower in these cases. Comparing the two it is evident that σ_{s3} is significantly lower than σ_{s1} except when $|\omega_d| > \pm\, 2^{3/4}\omega_B$. In these regions σ_{s3} is appreciably higher than σ_{s1}. If the radar frequency is raised to 25.4 MHz the results are almost the same except that σ_{s3} is now higher than σ_{s1} around $\pm 2^{3/4}\omega_B$ instead of only for $|\omega_d| > \pm\, 2^{3/4}\omega_B$ and these plots are shown in figures 18 to 20. If the wind speed is increased there is an expected overall increase in the two spectra and so σ_{s1} and σ_{s3} for a 10 MHz radar frequency at 30 knots wind speed are plotted in figures 21 to 23. The three wind directions are 0^0, 45^0 and 90^0. σ_{s3} can be seen to be much lower than σ_{s1} at all Doppler points except around $\pm 2^{3/4}\omega_B$. Similar plots are shown in figures 24 to 26 but for a 25.4 MHz radar frequency. Here also σ_{s3} is significantly lower than σ_{s1} at all Doppler frequencies except around $\pm 2^{3/4}\omega_B$ but not at $\pm 2^{3/4}\omega_B$. In these regions it is higher than or comparable to σ_{s1}. The plots of σ_{s1} in all the above examples are based on the computer program given in Walsh and Srivastava (1984) and have been used only to compare σ_{s3} with σ_{s1}.

CHAPTER 5

CONCLUSIONS

The effect of second-order off-patch scatter is examined. The spectral cross-section expression of this kind of scatter is simplified to a computational form assuming a narrow beam receiving antenna. The transmitting antenna is assumed to be omnidirectional. It is found that the contributoin of off-patch scatter compared to on-patch scatter is effective only at zero Doppler, around $\pm 2^{3/4}\omega_B$ and beyond $\pm 2^{3/4}\omega_B$ frequency points. Around $\pm 2^{3/4}\omega_B$ frequency points σ_{s3} is higher than or comparable to σ_{s1} but beyond $\pm 2^{3/4}\omega_B$ frequency points σ_{s3} is significantly higher than σ_{s1}. The Doppler regions near the first-order peaks, which are commonly used for estimation of ocean wave parameters are unaffected by off-patch scatter. Therefore, for the estimation of these parameters the second-order cross section may adequately be described by σ_{s1} alone. Based on these results it may be concluded that this form of scatter is not significant for the problem of extracting ocean surface parameters. However, it may be important in target detection applications when the target Doppler frequency is zero, near the "corner reflector" frequencies or beyond the corner reflector frequencies.

Future work in this area might include the effect of higher-order scatter, particularly that of third-order scatter. It is suspected that in high sea conditions the total spectral cross section may be strongly influenced by the third-order effect. If this is the case, then the contribution of third-order scatter must be taken into account in the design of any analysis technique to extract ocean spectral information from radar data.

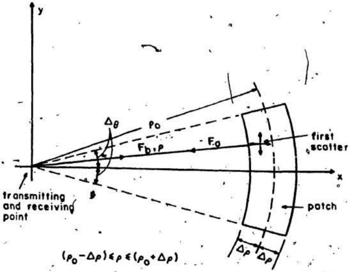

FIG. 1 First-order backscattered from a surface patch for omnidirectional transmission and narrow beam reception (ρ_o = distance of patch, $2\Delta\rho$ = radial width of patch, $2\Delta_\theta$ = beamwidth of receiving antenna, F;s = ground wave attenuation function with modified surface impedances)

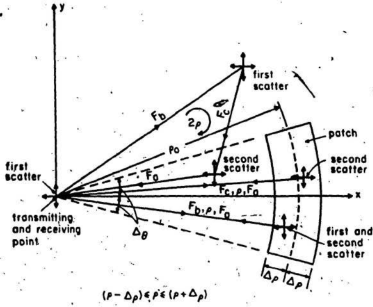

FIG. 2 Three parts of the second-order backscattered from a surface patch and off the patch for omnidirectional transmission and narrow-beam reception (ρ_o = distance of patch, $2\Delta\rho$ = radian width of patch, $2\Delta_\theta$ = beamwidth of receiving antenna, F's = ground wave attenuation function with modified surface impedances)

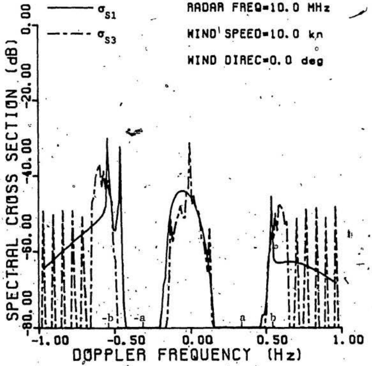

FIG. 3 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

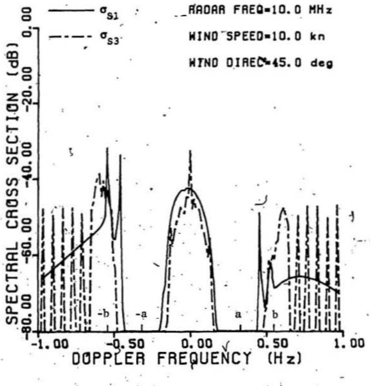

FIG. 4 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

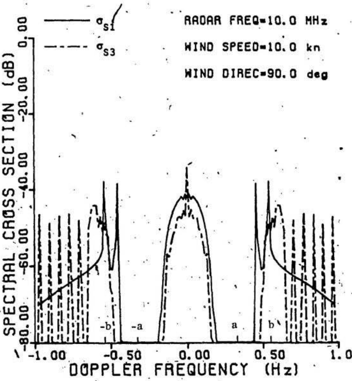

FIG. 5 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

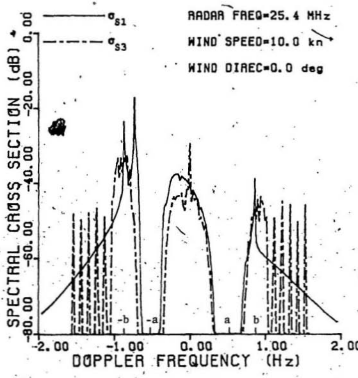

FIG. 6 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam receptien (open sea condition).

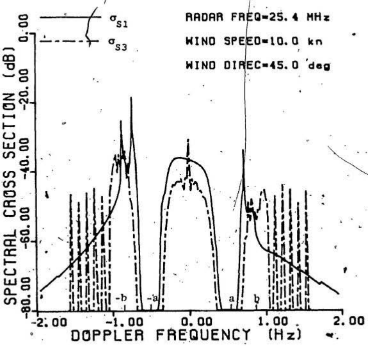

FIG. 7 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

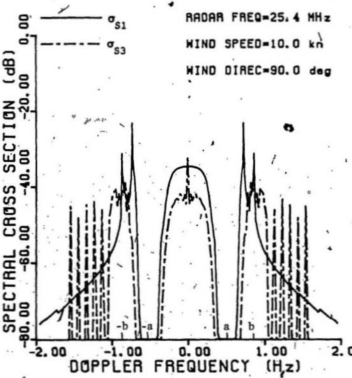

FIG. 8 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

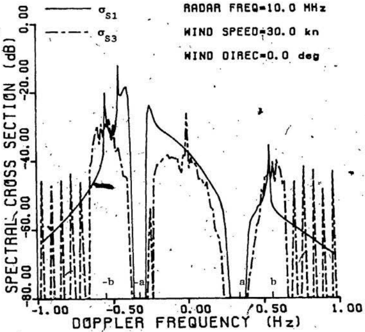

FIG. 9 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

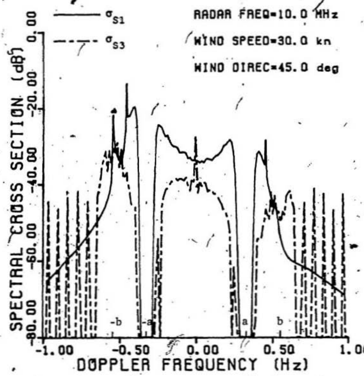

FIG. 10 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

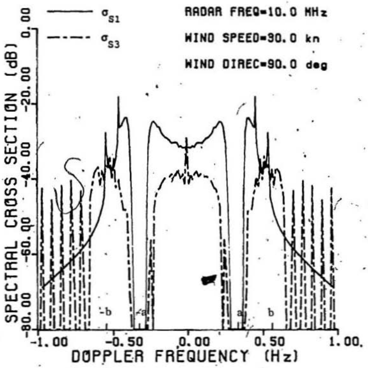

FIG. 11 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

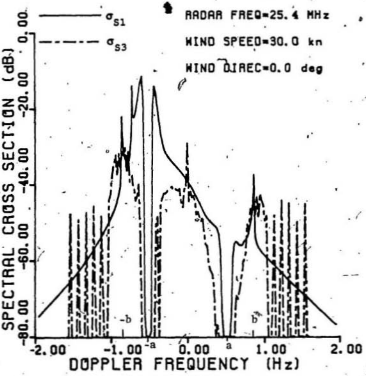

FIG. 12 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

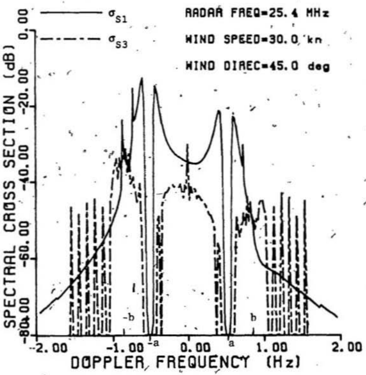

FIG. 13 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

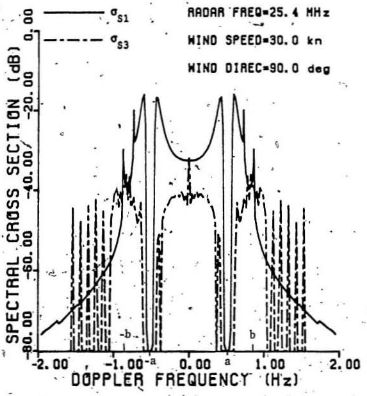

FIG. 14 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).

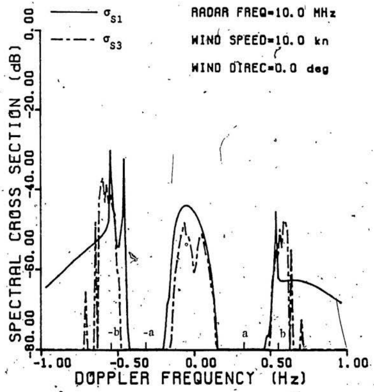

FIG. 15 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

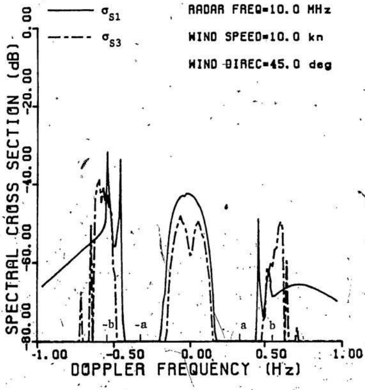

FIG. 16 Two parts of the second-order back-scattered spectral cross section of the ocean-surface patch for omnidirectional transmission and narrow beam reception (land based condition).

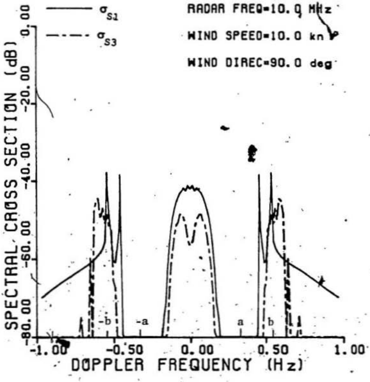

FIG. 17 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

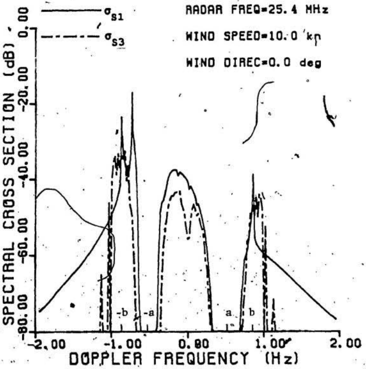

FIG. 18 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

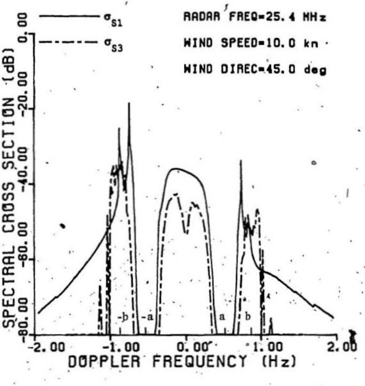

FIG. 19 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

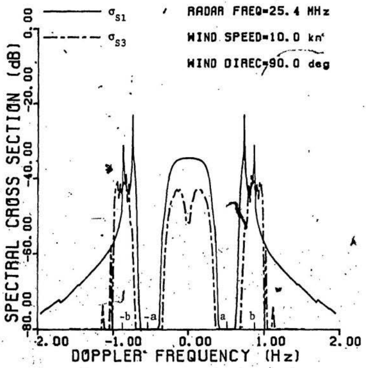

FIG. 20 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

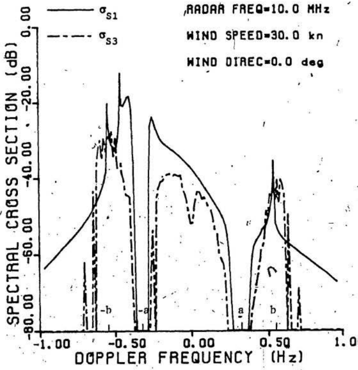

FIG. 21 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

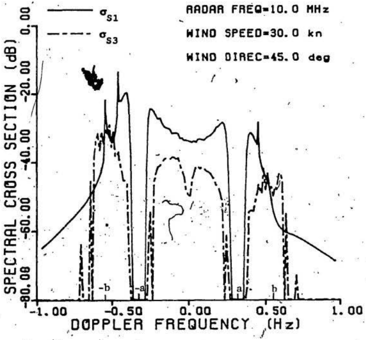

FIG. 22 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

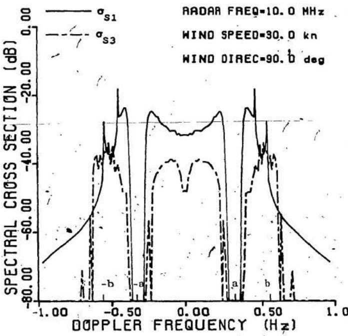

FIG. 23 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

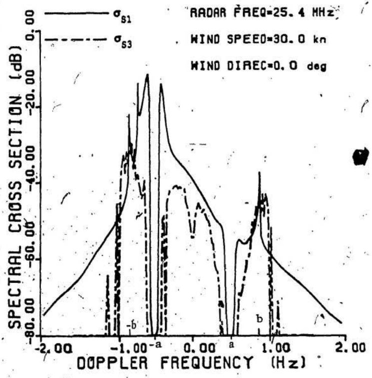

FIG. 24 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

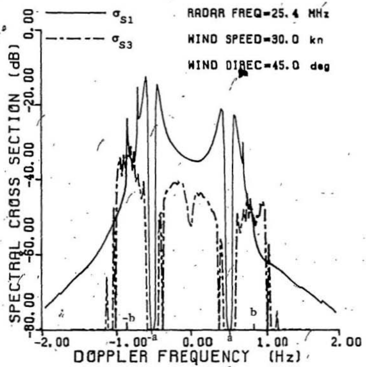

FIG. 25 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

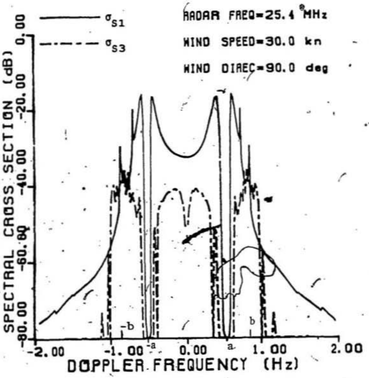

FIG. 26. Two parts of the second-order backscattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).

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APPENDIX

```
C*************************************************************
C       PROGRAM ID : ROOT.FOR
C       THIS PROGRAM IS USED TO GENERATE THE ROOTS USING
C       NORMALIZED EQUATIONS AND ASSUMING A NARROW BEAM
C       RECEIVING ANTENNA
C*************************************************************

        DIMENSION FI1NEW(360),NUMK1(360)
        DIMENSION AFI2(2,180),AFI1(2,180,150),NFI2(2),NFI1(2,180)
        DIMENSION AK1NOT(2,180,150,2),NK1(2,180,150)
        DIMENSION AMU(2,180,150,2),SMIN(2,180,150,2)
        REAL MUNOT,K1NOT,K2NOT,M1,K1NEW(360,750),MM(2)
        COMMON/UU/PI,M1,ETA,IFLAG,PHI2,PHI1,MUNOT,THETAI
        COMMON WFUN
        EXTERNAL SNOT
        CALL UERSET(0,LEVOLD)
        OPEN(UNIT=5,FILE='FOR012.DAT',TYPE='OLD')
        READ(5,*)THETAI
        READ(5,*)NNN

        DO 60 IE=1,NNN
        READ(5,*)A
        ETA=A
        TOL=0.0001
        PI=4.*ATAN(1.0)
        THETAI=THETAI*PI/180.

        DO 50 IC=1,2
        M1=2*IC-3
        MM(IC)=M1
        PHI2D=-181.
        IPHI2=0

        DO 40 IF2=1,180
        PHI2D=PHI2D+2.
        PHI2=PHI2D*PI/180.
        IFLAG=1
        CALL CHANGE(FI1NEW,N1,K1NEW,NUMK1)
        IFLAG=2
        IFI1=0
```

```
      DO 30 I=1,N1
      N2=NUMK1(I)
      PHI1=FI1NEW(I)*PI/180.
      IK1=0

      DO 20 J=1,N2,2
      AK1=K1NEW(I,J)
      BK1=K1NEW(I,J+1)
      SFIRST=SNOT(AK1)
      SSECOND=SNOT(BK1)
      CALL ZXGSN(SNOT,AK1,BK1,TOL,K1NOT,IER)
      XX=K1NOT
      SUMMIN=SNOT(XX)
      IF(SUMMIN.GT.SFIRST.OR.SUMMIN.GT.SSECOND)GO TO 20
      IF(ABS(WFUN).GT.0.1)GO TO 20
      IK1=IK1+1
      IF(IK1.EQ.1)IFI1=IFI1+1
      IF(IFI1.EQ.1)IPHI2=IPHI2+1
      AFI2(IC,IPHI2)=PHI2D
      AFI1(IC,IPHI2,IFI1)=PHI1*180./PI
      AK1NOT(IC,IPHI2,IFI1,IK1)=K1NOT
      AMU(IC,IPHI2,IFI1,IK1)=MUNOT
      SMIN(IC,IPHI2,IFI1,IK1)=SUMMIN
      NFI1(IC,IPHI2)=IFI1
      NK1(IC,IPHI2,IFI1)=IK1
20    CONTINUE
30    CONTINUE
40    CONTINUE
      NFI2(IC)=IPHI2
50    CONTINUE

      DO 55 IC=1,2
      WRITE(50,*)ETA,MM(IC),NFI2(IC)

      DO 45 I=1,NFI2(IC)
      WRITE(50,*)AFI2(IC,I),NFI1(IC,I)
      WRITE(50,*) (AFI1(IC,I,J),NK1(IC,I,J),(AK1NOT(IC,I,J,K),
     f AMU(IC,I,J,K),SMIN(IC,I,J,K),K=1,NK1(IC,I,J)),
     1 J=1,NFI1(IC,I))
45    CONTINUE
55    CONTINUE
60    CONTINUE
      STOP
      END
```

```
C***********************************************************
C       SUBROUTINE ID : CHANGE
C       THIS SUBROUTINE IS USED TO CHECK IF TWO EQUATIONS
C       TO BE SOLVED CHANGE THEIR SIGN FOR A PARTICULAR
C       SET OF VARIABLES
C***********************************************************

        SUBROUTINE CHANGE(FI1NEW,N1,K1NEW,NUMK1)
        DIMENSION FI1NEW(360),UUFUN(360,750),NUMK1(360)
        DIMENSION WWFUN(360,750)
        REAL MUNOT,K1NOT,K2NOT,K1OLD(360,750),K1NEW(360,750)
        COMMON/CH/ANGLE1,FFACT,UFUN
        COMMON WFUN
        DEL1=.01
        DEL2=.1
        DEL3=.1
        N1=0.09/DEL1+1.
        N2=0.9/DEL2
        N3=9.0/DEL3
        N12=N1+N2
        NK1=N12+N3
        FI1=-181.
        II=0

        DO 10 I=1,360
        FI1=FI1+1.
        ANGLE1=FI1
        DELK1=DEL1
        K1NOT=0.01-DELK1
        JJ=0

        DO 20 J=1,NK1
        K1NOT=K1NOT+DELK1
        IF(J.EQ.N1)DELK1=DEL2
        IF(J.EQ.N12)DELK1=DEL3
        SUM=SNOT(K1NOT)
        IF(ABS(FFACT).GT.1.E-02)GO TO 20
        JJ=JJ+1
        IF(JJ.EQ.1)II=II+1
        FI1NEW(II)=FI1
        NUMK1(II)=JJ
        UUFUN(II,JJ)=UFUN
        WWFUN(II,JJ)=WFUN
        K1OLD(II,JJ)=K1NOT
```

```
20    CONTINUE
10    CONTINUE
      N1=II
      II=0

      DO 50 I=1,N1
      N2=NUMK1(I)-1
      IF(N2.EQ.0)GO TO 50
      JJ=0

      DO 60 J=1,N2
      AMULT=UUFUN(I,J)*UUFUN(I,J+1)
      AMULT2=WWFUN(I,J)*WWFUN(I,J+1)
      IF(AMULT.GT.0.0.OR.AMULT2.GT.0.0)GO TO 60
      JJ=JJ+2
      IF(JJ.EQ.2)II=II+1
      FI1NEW(II)=FI1NEW(I)
      K1NEW(II,JJ-1)=K1OLD(I,J)
      K1NEW(II,JJ)=K1OLD(I,J+1)
      NUMK1(II)=JJ
60    CONTINUE
50    CONTINUE
      N1=II
      RETURN
      END
```

```
C**************************************************************
C        FUNCTION SUBPROGRAM ID : SNOT
C        THIS FUNCTION SUBPROGRAM IS REQUIRED TO CALCULATE
C        THE 'MUNOT', 'DELNOT' AND THE TWO FUNCTIONS THAT
C        EMERGE FROM TWO EQUATIONS
C**************************************************************

      FUNCTION SNOT(XX)
      DIMENSION ACHMU(2),AMU(4)
      REAL M1,K1,K1X,K1Y,K2,K2X,K2Y,MU,KC,KCX,KCY
      COMMON/UU/PI,M1,ETA,IFLAG,FI2,PHI1,MU,THETAI
      COMMON/CH/ANGLE1,FFACT,UFUN
      COMMON WFUN
      IF(IFLAG.EQ.1)FI1=ANGLE1*PI/180.0
      IF(IFLAG.EQ.2)FI1=PHI1
      SNOT=1.0E25
      UFUN=SNOT
      WFUN=SNOT
      FFACT=2.0
      K1=XX
      CFI2=COS(FI2)
      SFI2=SIN(FI2)
      CFI2T=COS(FI2)
      SFI2T=SIN(FI2)
      TFI2T=SFI2T/CFI2T
      CFI1=COS(FI1)
      SFI1=SIN(FI1)
      CFI1T=COS(FI1)
      SFI1T=SIN(FI1)
      K2=(SQRT(K1)+M1*ETA)**2
      K1X=K1*CFI1
      K1Y=K1*SFI1
      K2X=K2*CFI2
      K2Y=K2*SFI2
      FACT=4.0*K1*CFI1T*(K1*CFI1T+K2*CFI2T)
      B=-2.0*(K2*SFI2T)**2
      A=FACT+B/2.0
      C=-SFI2T*SFI2T*FACT+B/2.0
      IF (A.EQ.0.0.AND.B.EQ.0.0) GOTO 40
      IF (B.EQ.0.0.AND.C.EQ.0.0) GOTO 40
      DISC=B*B-4.0*A*C
      IF (DISC.LT.0.0) GOTO 40
      ACHMU(1)=0
      ACHMU(2)=0
```

```
      IF (A.EQ.0.0) THEN
      ACHMU(1)=-C/B
      ELSE IF (DISC.EQ.0.0) THEN
      ACHMU(1)=-B/(2.0*A)
      ELSE
      ACHMU(1)=(-B+SQRT(DISC))/(2.0*A)
      ACHMU(2)=(-B-SQRT(DISC))/(2.0*A)
      END IF
      MU=0
      AMU(1)=0
      AMU(2)=0
      AMU(3)=0
      AMU(4)=0
      NUMMU=0
      CSGN=1

      DO 50 IM=1,2
      X=ACHMU(IM)
      NUMMU=NUMMU+2
      IF (X.LE.1.0) GOTO 50
      AMU(NUMMU)=ALOG(X+SQRT(X*X-1.0))
      AMU(NUMMU-1)=-AMU(NUMMU)
50    CONTINUE

      DO 60 IN=1,4
      IF (AMU(IN).EQ.0.0) GOTO 60
      X=AMU(IN)
      SHX=SINH(X)
      CHX=COSH(X)
      THX=SHX/CHX
      FFACT=(2.0*K1*CFI1T+K2*CFI2T)*CFI2T*CHX*SQRT(1.0+
     +(THX*TFI2T)**2)-K2*SIGN(1.0,SHX*TFI2T)*(SFI2T**2/CHX)
      IF (ABS(FFACT).GT.1.E-2) GOTO 60
      MU=X
      CHMU=CHX
      SHMU=SHX
      IF (IN.GT.2) CSGN=-1
      GOTO 65
60    CONTINUE

65    IF (MU.EQ.0.0) GOTO 40
      DEL=ATAN2(SHMU*SFI2T,CHMU*CFI2T)
      IF (DEL.GT.-PI.AND.DEL.LT.0.0) DEL=DEL+PI
      CDEL=COS(DEL)
```

```
      SDEL=SIN(DEL)
      KCX=2.0*K1X+K2X*(1.0+CHMU*CDEL)+K2Y*SHMU*SDEL
      KCY=2.0*K1Y+K2Y*(1.0+CHMU*CDEL)-K2X*SHMU*SDEL
      KC=SQRT(KCX**2+KCY**2)
      UFUN=KC/(1.0+CHMU)-1.0
      WFUN=ATAN2(KCY,KCX)-THETAI
      IF(IFLAG.EQ.1)RETURN
      SNOT=ABS(UFUN)+ABS(WFUN)
40    RETURN
      END
```

```
C*******************************************
C  DATA ID : FOR012.DAT
C*******************************************
   80
   'THETAI'    ! ANY-VALUE OUT OF FIVE
  -3.080000
  -3.040000
  -3.000000
  -2.960000
  -2.920000
  -2.880000
  -2.840000
  -2.800000
  -2.760000
  -2.720000
  -2.680000
  -2.640000
  -2.600000
  -2.560000
  -2.520000
  -2.480000
  -2.440000
  -2.400000
  -2.360000
  -2.320000
  -2.280000
  -2.240000
  -2.200000
  -2.160000
  -2.120000
  -2.080000
  -2.040000
  -2.000000
  -1.960000
  -1.920000
  -1.880000
  -1.840000
  -1.800000
  -1.760000
  -1.720000
  -1.681793
  -1.680000
  -1.640000
  -1.600000
```

-1.560000
-1.520000
-1.480000
-1.440000
-1.414214
-1.400000
-1.360000
-1.320000
-1.280000
-1.240000
-1.200000
-1.160000
-1.120000
-1.080000
-1.040000
-1.000000
-0.9600000
-0.9200000
-0.8800000
-0.8400000
-0.8000000
-0.7600000
-0.7200000
-0.6800000
-0.6400000
-0.6000000
-0.5600000
-0.5200000
-0.4800000
-0.4400000
-0.4000000
-0.3600000
-0.3200000
-0.2800000
-0.2400000
-0.2000000
-0.1600000
-0.1200000
-8.0000000E-02
-4.0000000E-02
0.0000000

```
C*******************************************************
C       PROGRAM ID : INT.FOR
C       THIS PROGRAM SORTS OUT THE MULTIPLE OCCURING
C       OF SAME ROOTS AND CHECKS THE RESTRICTIONS FOR
C       BEACH AND OPEN OCEAN CASES
C*******************************************************

        DIMENSION AFI2(2,90),AFI1(2,90,150)
        DIMENSION NFI2(2),NFI1(2,90)
        DIMENSION AK1NOT(2,90,150,2),SMIN(2,90,150,2)
        DIMENSION NK1(2,90,150),AMU(2,90,150,2),KNT(90)
        DIMENSION RAN(2,90,150,2),RBN(2,90,150,2)
        DIMENSION RCN(2,90,150,2),THETAB(2,90,150,2)
        DIMENSION JTEMP(5)
        REAL MM(2),K1NOT,K2NOT,KCY,KCX,K1X,K2X,K1Y,K2Y

C       ENTER (0 OR 1) BEACH OR OPEN OCEAN ?  !0 FOR OPEN OCEAN

        READ(5,*)LOC
        OPEN(UNIT=14,FILE='ROOT.DAT',TYPE='OLD')
C       NO OF NORMALIZED DOPPLER POINTS
        READ(14,*)NN
        PI=4.*ATAN(1.)
        OPEN(UNIT=77,FILE='INT.DAT',TYPE='NEW')
        WRITE(77,*)NN
        ROW=30.

        DO 5 NV=1,NN
        DO 10 IC=1,2
        READ(14,*)ETA,MM(IC),NFI2(IC)

        DO 20 I=1,NFI2(IC)
        READ(14,*)AFI2(IC,I),NFI1(IC,I)
        READ(14,*)(AFI1(IC,I,J),NK1(IC,I,J),(AK1NOT(IC,I,J,K),
     1  AMU(IC,I,J,K),SMIN(IC,I,J,K),K=1,NK1(IC,I,J)),
     1  J=1,NFI1(IC,I))
20      CONTINUE
10      CONTINUE

C       TO SORT OUT THE MULTIPLE ROOTS

        DO 11 IC=1,2
        IF(NFI2(IC).EQ.0)GO TO 11
```

```
      DO 12 I=1,NFI2(IC)
      MFI1=1
      JTEMP(1)=1
      IF(NFI1(IC,I).EQ.1)GO TO 12
      SMALL=SMIN(IC,I,1,1)

      DO 13 J=2,NFI1(IC,I)
      IF((AFI1(IC,I,J)-AFI1(IC,I,J-1)).LT.2.1)THEN
      IF(SMIN(IC,I,J,1).LT.SMALL)THEN
      SMALL=SMIN(IC,I,J,1)
      JTEMP(MFI1)=J
      ELSE
      END IF
      ELSE
      MFI1=MFI1+1
      JTEMP(MFI1)=J
      SMALL=SMIN(IC,I,J,1)
      END IF
13    CONTINUE
      NFI1(IC,I)=MFI1

      DO 14 J=1,MFI1
      JJ=JTEMP(J)
      AFI1(IC,I,J)=AFI1(IC,I,JJ)
      NK1(IC,I,J)=NK1(IC,I,JJ)
      AK1NOT(IC,I,J,1)=AK1NOT(IC,I,JJ,1)
      AMU(IC,I,J,1)=AMU(IC,I,JJ,1)
      SMIN(IC,I,J,1)=SMIN(IC,I,JJ,1)
14    CONTINUE
12    CONTINUE
11    CONTINUE

C     CHECKING THE RESTRICTIONS

      DO 30 IC=1,2
      IF (NFI2(IC).LE.0) THEN
      WRITE(77,*)ETA,MM(IC),NFI2(IC)
      GOTO 30
      END IF
      MFI2=0
      DO 40 I=1,NFI2(IC)
      KNT(I)=0
      DO 50 J=1,NFI1(IC,I)
      K=1
```

```
      K1NOT=AK1NOT(IC,I,J,K)
      K2NOT=(SQRT(K1NOT)+MM(IC)*ETA)**2
      CHMU=COSH(AMU(IC,I,J,K))
      SHMU=SINH(AMU(IC,I,J,K))
      SFI1T=SIN(AFI1(IC,I,J)*PI/180.)
      CFI1T=COS(AFI1(IC,I,J)*PI/180.)
      SFI2T=SIN(AFI2(IC,I)*PI/180.)
      CFI2T=COS(AFI2(IC,I)*PI/180.)
      DEL=ATAN2(SHMU*SFI2T,CHMU*CFI2T)
      IF (DEL.GT.-PI.AND.DEL.LT.0.) DEL=DEL+PI
      CDEL=COS(DEL)
      SDEL=SIN(DEL)
      SIB=ATAN2(SHMU*SDEL,1.+CHMU*CDEL)
      SIB=SIB*180./PI
      K1X=K1NOT*CFI1T
      K1Y=K1NOT*SFI1T
      K2X=K2NOT*CFI2T
      K2Y=K2NOT*SFI2T
      KCX=2.0*K1X+K2X*(1.0+CHMU*CDEL)+K2Y*SHMU*SDEL
      KCY=2.0*K1Y+K2Y*(1.0+CHMU*CDEL)-K2X*SHMU*SDEL
      FIC=ATAN2(KCY,KCX)
      FIC=FIC*180./PI
      DEN=1.+CHMU
      RAN(IC,I,J,K)=2./DEN
      RBN(IC,I,J,K)=(CHMU+CDEL)/DEN
      RCN(IC,I,J,K)=(CHMU-CDEL)/DEN
      THETAB(IC,I,J,K)=SIB+FIC
      IF(LOC.EQ.0)THEN
      IF (RBN(IC,I,J,K).GT.1./ROW.AND.RCN(IC,I,J,K).GT.
     1 1./ROW)GO TO 50
      ELSE
      IF (RBN(IC,I,J,K).GT.1./ROW.AND.ABS(THETAB(IC,I,J,K))
     1 .LT.90.0.0.AND.RCN(IC,I,J,K).GT.1./ROW.AND.
     1 RAN(IC,I,J,K).GT.1./ROW)GO TO 50
      END IF
      AFI1(IC,I,J)=1000.
      KNT(I)=KNT(I)+1
50    CONTINUE
      IF (NFI1(IC,I).EQ.KNT(I)) THEN
      AFI2(IC,I)=1000.
      MFI2=MFI2+1
      ELSE
      END IF
40    CONTINUE
```

```
      WRITE(77,*)ETA,MM(IC),NFI2(IC)-MFI2
      DO 200 I=1,NFI2(IC)
      IF (AFI2(IC,I).LT.999.)THEN
      WRITE(77,*)AFI2(IC,I),NFI1(IC,I)-KNT(I)
      DO 300 J=1,NFI1(IC,I)
      K=1
      IF (AFI1(IC,I,J).LT.999.) THEN
      WRITE(77,*)AFI1(IC,I,J),NK1(IC,I,J),AK1NOT(IC,I,J,K),
     1 AMU(IC,I,J,K),SMIN(IC,I,J,K)
      END IF
300   CONTINUE
      END IF
200   CONTINUE
30    CONTINUE
5     CONTINUE
      CLOSE UNIT=77
      CLOSE UNIT=15
      STOP
      END
```

```
C**********************************************************************
C       PROGRAM ID : CROSS.FOR
C       THIS PROGRAM IS USED TO COMPUTE CROSS SECTION
C       OF OFF-PATCH SCATTER BASED ON THE ROOTS WHICH
C       ARE OBTAINED FROM THE SOLUTIONS OF TWO EQUATIONS
C**********************************************************************

        IMPLICITREAL*8(A-H,O-Z)
        REAL*4FNORT
        REAL*8KR,MU
        COMPLEX*8CMPLX,PE
        COMPLEX*16NUMD,DCMPLX
        DIMENSION DOPLER(300),SIGS3(300)
        COMMON PI,GA,N
        COMMON/THREE/PE,WB,KR
        COMMON/SPECN/U,THETA
C       FREQUENCY(MHZ)?
        READ(7,*)FREQ
C       WIND SPEED(KNOT)?
        READ(7,*)U
C       WIND DIRECTION(DEG)?
        READ(7,*)THETA
C       DISTANCE OF THE PATCH(KM)?
        READ(7,*)ROW
        ROW=ROW*1000.DO
        PI=4.DO*DATAN(1.D0)
        GA=9.81D0
        KR=FREQ*PI/150.DO
        WB=DSQRT(2.DO*GA*KR)
        U=U*.5148D0
        THETA=THETA*PI/180.DO
        NUMD=DCMPLX(8.D1,-7.2D4/FREQ)
        NUMD=DCMPLX(0.D0,-0.5D0)*KR*ROW/NUMD
        PE=CMPLX(NUMD)
        CS3=DBLE(FNORT(PE))
        CS3=4.DO*KR/(PI**2)/(CS3**4)
        CS3=CS3/5.0
        CALL FUNC3N(DOPLER,SIGS3)
        WRITE(7,*)N-1
        NP=N/2

        DO 25 I=NP+1,NP+(NP+1)/2
        TEMP1=DOPLER(I)
        TEMP2=SIGS3(I)
```

```
      DOPLER(I)=DOPLER(3*NP+1-I)
      SIGS3(I)=SIGS3(3*NP+1-I)
      DOPLER(3*NP+1-I)=TEMP1
      SIGS3(3*NP+1-I)=TEMP2
   25 CONTINUE
      DO 30 I=1,2*NP
      SIGS3(I)=SIGS3(I)*CS3
      DOPLER(I)=DOPLER(I)/2./PI
      IF(I.EQ.NP)GO TO 30
      WRITE(7,2)DOPLER(I),SIGS3(I)
    2 FORMAT(/2E13.5)
   30 CONTINUE
      STOP
      END
```

```
C*****************************************************************
C        SUBROUTINE ID : FUNC3N
C*****************************************************************

         SUBROUTINE FUNC3N(DOPLER,SUMT)
         IMPLICITREAL*8 (A-H,O-Z)
         REAL*8 K1,K1X,K1Y,K1S,K1F,K1INC,K2,K2X,K2Y,KR,MU,NU,
     +   KC,KCX,KCY,JACOB
         REAL*4 FNORT
         COMPLEX*8 PE,PEA,PEB,PEC
         DIMENSION MM1(2),NFI2(2),FI2D(2,150),NFI1(2,150)
         DIMENSION SUM(2,150),FI1D(2,150,150),
     1   AK1NOT(2,150,150,4),AMU(2,150,150,4)
         DIMENSION ACSGN(2,150,150,4),NK1(2,150,150),SUMT(300)
         DIMENSION DOPLER(300)
         COMMON PI,GA,NN
         COMMON/THREE/PE,WB,KR

C        FOLLOWING FIVE DATA FILES CORRESPOND TO THE SOLUTIONS
C        CORRESPONDING TO FIVE DIFFERENT VALUES OF 'THETAI'

         OPEN(UNIT=1,FILE='DATA00.DAT',TYPE='OLD')
         OPEN(UNIT=2,FILE='DATA12.DAT',TYPE='OLD')
         OPEN(UNIT=3,FILE='DATA21.DAT',TYPE='OLD')
         OPEN(UNIT=4,FILE='DATA24.DAT',TYPE='OLD')
         OPEN(UNIT=5,FILE='DATA42.DAT',TYPE='OLD')
         K=1
         ITT=0
5        ITT=ITT+1
         SUMT(ITT)=0.0

         DO 3 NF=1,5
         IF(ITT.EQ.1)THEN
C        NO OF NORMALIZED FREQUENCY POINTS ?
         READ(NF,*)NP
         NN=2*NP
         SWITCH=1.DO
         END IF
         IF (ITT.GE.(NP+1)) SWITCH=-1.DO
         IF (ITT.EQ.(NP+1))THEN
         REWIND NF
         READ(NF,*)NP
         ELSE
         END IF
```

```
      IF (ITT.GT.NN) GOTO 130

      DO 10 IC=1,2
      READ(NF,*)ETA,MM1(IC),NFI2(IC)
      ETA=ETA*SWITCH
      WD=ETA*WB
      DOPLER(ITT)=WD
      MM1(IC)=MM1(IC)*SWITCH
      IF (NFI2(IC).LE.0) GOTO 10

      DO 20 I=1,NFI2(IC)
      READ(NF,*) FI2D(IC,I),NFI1(IC,I)

      DO 15 J=1,NFI1(IC,I)
      READ(NF,*)FI1D(IC,I,J),NK1(IC,I,J),AK1NOT(IC,I,J,K),
     1 AMU(IC,I,J,K),ACSGN(IC,I,J,K)
15    CONTINUE
20    CONTINUE
10    CONTINUE

      DO 50 IC=1,2
      IF (NFI2(IC).EQ.0) GOTO 50
      M1=MM1(IC)

      DO 60 I=1,NFI2(IC)
      FI2=FI2D(IC,I)*PI/180.D0
      SUM(IC,I)=0.D0

      DO 70 J=1,NFI1(IC,I)
      FI1=FI1D(IC,I,J)*PI/180.D0
      K1=AK1NOT(IC,I,J,K)*2.D0*KR
      CSGN=ACSGN(IC,I,J,K)
      MU=AMU(IC,I,J,K)
      IF (M1.EQ.1) THEN
      IF (WD.LT.0.D0.AND.K1.LE.(WD*WD/GA)) M2=1
      IF (WD.GE.0.D0) M2=-1
      IF (WD.LT.0.D0.AND.K1.GE.(WD*WD/GA)) M2=-1
      ELSE IF (M1.EQ.-1) THEN
      IF (WD.GE.0.D0.AND.K1.GT.(WD*WD/GA)) M2=1
      IF (WD.LT.0.D0) M2=1
      IF (WD.GT.0.D0.AND.K1.LE.(WD*WD/GA)) M2=-1
      END IF
      CFI2=DCOS(FI2)
      SFI2=DSIN(FI2)
```

```
      CFI2T=DCOS(FI2)
      SFI2T=DSIN(FI2)
      TFI2T=SFI2T/CFI2T
      CHMU=DCOSH(MU)
      SHMU=DSINH(MU)
      CFI1=DCOS(FI1)
      SFI1=DSIN(FI1)
      CFI1T=DCOS(FI1)
      SFI1T=DSIN(FI1)
      K2=(DSQRT(K1)+M1*WD/3.132092D0)**2
      K1X=K1*CFI1
      K1Y=K1*SFI1
      K2X=K2*CFI2
      K2Y=K2*SFI2
      DEL=DATAN2(SHMU*SFI2T,CHMU*CFI2T)
      IF (DEL.GT.-PI.AND.DEL.LT.0.D0) DEL=DEL+PI
      CDEL=DCOS(DEL)
      SDEL=DSIN(DEL)
      KCX=2.D0*K1X+K2X*(1.D0+CHMU*CDEL)+K2Y*SHMU*SDEL
      KCY=2.D0*K1Y+K2Y*(1.D0+CHMU*CDEL)-K2X*SHMU*SDEL
      KC=DSQRT(KCX**2+KCY**2)
      FIC=DATAN2(KCY,KCX)
      CFI1C=DCOS(FI1-FIC)
      SFI1C=DSIN(FI1-FIC)
      CFI2C=DCOS(FI2-FIC)
      SFI2C=DSIN(FI2-FIC)

      GMUMU=((CHMU-2.D0)*(2.D0*K1*CFI1C+K2*CFI2C*(1.D0-CDEL))
     + -K2*SHMU*SDEL*SFI2C)/((1.D0+CHMU)**2)
      GMUDEL=K2*((1.D0+CHMU)*CDEL*SFI2C-SHMU*SDEL*CFI2C)/
     + ((1.D0+CHMU)**2)
      GDELDE=-K2*(SHMU*SDEL*SFI2C+CHMU*CDEL*CFI2C)/(1.D0+CHMU)
      DET=DABS(GMUMU*GDELDE-GMUDEL**2)

      PCK1=((4.D0*K1*CFI1T*(K1*CFI1T+K2*CFI2T)-(K2*SFI2T)**2)
     + *(CSGN*DSIGN(1.D0,2.D0*K1*CFI1T+K2*CFI2T)*(1.D0/DSQRT(K1)
     + )*(6.D0*K1*CFI1T+K2*CFI2T)*CFI1T*(K1*CFI1T+K2*CFI2T)
     + +CSGN*DSQRT(K1)*DABS(2.D0*K1*CFI1T+K2*CFI2T)*(CFI1T**2))
     + /DSQRT(CFI1T*(K1*CFI1T+K2*CFI2T))-(K2*K2*DABS(SFI2T)+CSGN
     + *2.0D0*DSQRT(K1)*DABS(2.D0*K1*CFI1T+K2*CFI2T)*DSQRT(CFI1T
     + *(K1*CFI1T+K2*CFI2T)))*4.D0*CFI1T*(2.D0*K1*CFI1T+K2*CFI2T
     + ))*DABS(SFI2T)/(4.D0*K1*CFI1T*(K1*CFI1T+K2*CFI2T)-(K2*
     + SFI2T)**2)**2
```

```
      PCK2=((4.D0*K1*CFI1T*(K1*CFI1T+K2*CFI2T)-(K2*SFI2T)**2)
     + *(2.D0*K2*DABS(SFI2T)+CSGN*2.D0*DSIGN(1.D0,2.D0*K1*CFI1T
     + +K2*CFI2T)*DSQRT(K1)*CFI2T*DSQRT(CFI1T*(K1*CFI1T+K2*
     + CFI2T))+CSGN*DSQRT(K1)*DABS(2.D0*K1*CFI1T+K2*CFI2T)*CFI1T
     + *CFI2T/DSQRT(CFI1T*(K1*CFI1T+K2*CFI2T)))-(K2*K2*DABS
     + (SFI2T)+CSGN*2.D0*DSQRT(K1)*DABS(2.D0*K1*CFI1T+K2*CFI2T)
     + *DSQRT(CFI1T*(K1*CFI1T+K2*CFI2T)))
     + *(4.D0*K1*CFI1T*CFI2T-2.D0*K2*SFI2T*SFI2T))*DABS(SFI2T)/
     + (4.D0*K1*CFI1T*(K1*CFI1T+K2*CFI2T)-(K2*SFI2T)**2)**2

      PCFI1=((4.D0*K1*CFI1T*(K1*CFI1T+K2*CFI2T)-(K2*SFI2T)**2)
     + *(-CSGN*4.D0*DSIGN(1.D0,2.D0*K1*CFI1T+K2*CFI2T)*K1*DSQRT
     + (K1)*SFI1T*CFI1T*(K1*CFI1T+K2*CFI2T)-CSGN*DSQRT(K1)*DABS
     + (2.D0*K1*CFI1T+K2*CFI2T)*SFI1T*(2.D0*K1*CFI1T+K2*CFI2T))
     + /DSQRT(CFI1T*(K1*CFI1T+K2*CFI2T))+(K2*K2*DABS(SFI2T)+CSGN
     + *2.D0*DSQRT(K1)*DABS(2.D0*K1*CFI1T+K2*CFI2T)*DSQRT(CFI1T*
     + (K1*CFI1T+K2*CFI2T)))*4.D0*K1*SFI1T*(2.D0*K1*CFI1T+K2*
     + CFI2T))*DABS(SFI2T)/(4.D0*K1*CFI1T*(K1*CFI1T+K2*CFI2T)-
     + (K2*SFI2T)**2)**2

      PMUK1=PCK1/SHMU
      PMUK2=PCK2/SHMU
      PMUFI1=PCFI1/SHMU
      FACT=TFI2T/(CHMU**2+(SHMU*TFI2T)**2)
      PDELK1=FACT*PMUK1
      PDELK2=FACT*PMUK2
      PDELFI=FACT*PMUFI1
      CFI21=DCOS(FI2-FI1)
      SFI21=DSIN(FI2-FI1)

      PKCK1=(4.D0*K1+K2*K2*(CHMU+CDEL)*(SHMU*PMUK1-SDEL*
     + PDELK1)+2.D0*K2*((1.D0+CHMU*CDEL)*CFI21+SHMU*SDEL*SFI21)
     + +2.D0*K1*K2*(CFI21*(SHMU*CDEL*PMUK1-CHMU*SDEL*PDELK1)
     + +SFI21*(CHMU*SDEL*PMUK1+SHMU*CDEL*PDELK1)))/KC

      PKCK2=(K2*(CHMU+CDEL)*(CHMU+CDEL+K2*(SHMU*PMUK2-SDEL*
     + PDELK2))+2.D0*K1*((1.D0+CHMU*CDEL)*CFI21+SHMU*SDEL*SFI21)
     + +2.D0*K1*K2*(CFI21*(SHMU*CDEL*PMUK2-CHMU*SDEL*PDELK2)
     + +SFI21*(CHMU*SDEL*PMUK2+SHMU*CDEL*PDELK2)))/KC

      PKCFI1=(K2*K2*(CHMU+CDEL)*(SHMU*PMUFI1-SDEL*PDELFI)
     + +2.D0*K1*K2*((1.D0+CHMU*CDEL)*SFI21-SHMU*SDEL*CFI21)
     + +2.D0*K1*K2*(CFI21*(SHMU*CDEL*PMUFI1-CHMU*SDEL*PDELFI)
     + +SFI21*(CHMU*SDEL*PMUFI1+SHMU*CDEL*PDELFI)))/KC
```

```
      PFICK1=(2.D0*SFI1C+K2*SFI2C*(SHMU*CDEL*PMUK1-CHMU*SDEL*
     + PDELK1)-K2*CFI2C*(CHMU*SDEL*PMUK1+SHMU*CDEL*PDELK1))/KC

      PFICK2=(SFI2C*(1.D0+CHMU*CDEL+K2*(SHMU*CDEL*PMUK2-CHMU*
     1 SDEL*PDELK2))-CFI2C*(SHMU*SDEL
     2 +K2*(CHMU*SDEL*PMUK2+SHMU*CDEL*PDELK2)))/KC

      PFICFI=(2.D0*K1*CFI1C+K2*SFI2C*(SHMU*CDEL*PMUFI1-CHMU*
     + SDEL*PDELFI)-K2*CFI2C*(CHMU*SDEL*PMUFI1+SHMU*CDEL*
     + PDELFI))/KC

      PUK1=(PKCK1-KC*SHMU*PMUK1/(1.D0+CHMU))/(1.D0+CHMU)
      PUK2=(PKCK2-KC*SHMU*PMUK2/(1.D0+CHMU))/(1.D0+CHMU)
      PUFI1=(PKCFI1-KC*SHMU*PMUFI1/(1.D0+CHMU))/(1.D0+CHMU)

      JACOB=1.566046D0*DABS(M1*(PUFI1*PFICK2-PUK2*PFICFI)/
     + DSQRT(K1)+M2*(PUK1*PFICFI-PUFI1*PFICK1)/DSQRT(K2))

      PEA=PE*2.D0/(1.D0+CHMU)
      PEB=PE*(CHMU+CDEL)/(1.D0+CHMU)
      PEC=PE*(CHMU-CDEL)/(1.D0+CHMU)
      ATTN=DBLE(FNORT(PEA)*FNORT(PEB)*FNORT(PEC))
      QC=KR*K1*CFI1T*(KR*(2.D0*K1*CFI1T+K2*CFI2T)-2.D0*K1*K1-K2
     + *K2-3.D0*K1*K2*CFI21)+K1*K1*(K1*K1+K2*K2+2.D0*K1*K2*
     + CFI21)-KR*K1*K1*(K1*CFI1T+K2*CFI2T)
      FUNC1=((ATTN*QC/KC)**2)*K1*K2/DET
      FUNC=FUNC1/JACOB
      CALL PMSPEC(M1*K1X,M1*K1Y,S1)
      CALL PMSPEC(M2*K2X,M2*K2Y,S2)
      FUNC=FUNC*S1*S2
      SUM(IC,I)=SUM(IC,I)+FUNC
70    CONTINUE
60    CONTINUE
50    CONTINUE

C     RECTANGULAR INTEGRATION OF THE MATRIX SUM

      DO 80 IC=1,2
      IF (NFI2(IC).EQ.0) GOTO 80
      DO 90 I=1,NFI2(IC)
      SUMT(ITT)=SUMT(ITT)+(2.D0*PI/180.D0)*SUM(IC,I)
90    CONTINUE
80    CONTINUE
3     CONTINUE
```

```
0       GO TO 5
130     RETURN
        END
```

```
C*****************************************************************
C       SUBROUTINE ID : PMSPEC
C       THIS SUBROUTINE GIVES CORRESPONDING PIERSON-
C       MOSKOWITZ WAVEHEIGHT SPECTRUM
C*****************************************************************

        SUBROUTINE PMSPEC(KX,KY,S)
        IMPLICITREAL*8(A-H,O-Z)
        REAL*8 KX,KY,K
        COMMON PI
        COMMON /SPECN/U,THETAW
        S=0
        K=DSQRT(KX**2+KY**2)
        IF (K.LE.1.D-9) RETURN
        FI=DATAN2(KY,KX)
        S=1.62D-2*PI*DEXP(-0.74D0*(9.81D0/(K*U*U))**2)/(K**4)
        S=S*(DCOS((THETAW-FI)/2.D0))**2
        RETURN
        END
```

```
C*****************************************************************
C       FUNCTION SUBPROGRAM ID ; FNORT
C       USED TO CALCULATE NUMERICAL DISTANCE
C*****************************************************************

        REAL FUNCTION FNORT(ND)
        COMPLEX F,ND,P,R,L,CEXP,CSQRT,SECX,CMPLX
        REAL MAGP,A,B,C,PI,CABS,CNT
        P=ND
        F=CMPLX(0.0,0.0)
        CNT=1.0
        R=CMPLX(0.0,1.0)
        PI=4.0*ATAN(1.0)
        MAGP=CABS(P)
        IF(MAGP.LE.10.0)GO TO 300
        A=REAL(P)
        B=AIMAG(P)
        C=ATAN2(B,A)
        IF(C.LT.0.0)GO TO 310
C       ASYMPTOTIC EXPANSION FOR C.GE.0
C       EXTRA TERM FOR C.GE.0
        F=-2.0*R*CSQRT(PI*P)*SCEX(-P)
C       CONTINUE ASYMPTOTIC EXPANSION FOR ALL P
  310   L=1.0
        CNT=1.0
  305   L=L*CNT/(2.0*P)
        F=F-L
        CNT=CNT+2.
        T=CABS(L)
        IF(T.GT.1.0E-04)GO TO 305
        GO TO 320
C       CONVERGENT SERIES
  300   F=1-R*CSQRT(PI*P)*SCEX(-P)
        CNT=1.0
        L=1.0
  330   L=((-1.0)*L*(2.0*P))/CNT
        F=F+L
        CNT=CNT+2.
        T=CABS(L)
        IF(T.GT.1.0E-04)GO TO 330
  320   FNORT=CABS(F)
        RETURN
        END
```

```
C*********************************************************************
C        FUNCTION SUBPROGRAM ID : SCEX
C*********************************************************************

         COMPLEX FUNCTION SCEX(ARGU)
         COMPLEX ARG,CEXP,CMPLX,ARGU
         REAL MAG
         ARG=ARGU
         SCEX=CMPLX(1.0,0.0)
         ICNT=0
         MAG=CABS(ARG)
         IF(MAG.LT.80.0)GO TO 400
  403    ARG=ARG/2.0
         ICNT=ICNT+2
         MAG=CABS(ARG)
         IF(MAG.LT.80.0)GO TO 402
         GO TO 403
  402    CONTINUE
         DO 404 J=1,ICNT
  404    SCEX=SCEX*(CEXP(ARG))
         GO TO 401
  400    SCEX=CEXP(ARG)
  401    RETURN
         END
```

```
C*******************************************************
C       PROGRAM ID : SYMROOT.FOR
C       PROGRAM USED TO OBTAIN THE ROOTS WHEN
C       'THETAI' IS CHANGED IN SIGN. BASED ON
C       SYMMETRY PROPERTY.
C*******************************************************

        DIMENSION AFI2(2,150),AFI1(2,150,150),NFI2(2)
        DIMENSION AK1NOT(2,150,150,2),NK1(2,150,150)
        DIMENSION NFI1(2,150),MM(2)
        DIMENSION AMU(2,150,150,2),SMIN(2,150,150,2)
        CHARACTER*12 FILENAME
        WRITE(6,*)'ENTER FILE NAME'
        READ(5,19)FILENAME
19      FORMAT(A12)
        K=1
        OPEN(UNIT=41,NAME=FILENAME,TYPE='OLD')
        READ(41,*)NP
        WRITE(44,*)NP
        DO 5 IETA=1,NP
        DO 10 IC=1,2
        READ(41,*)ETA,MM(IC),NFI2(IC)
        IF(NFI2(IC).LE.0)GO TO 10
        DO 20 I=1,NFI2(IC)
        READ(41,*)AFI2(IC,I),NFI1(IC,I)
        DO 30 J=1,NFI1(IC,I)
        READ(41,*)AFI1(IC,I,J),NK1(IC,I,J),AK1NOT(IC,I,J,K),
     -1 AMU(IC,I,J,K),SMIN(IC,I,J,K)
30      CONTINUE
20      CONTINUE
10      CONTINUE

C       GENERATION OF ROOTS

        DO 11 IC=1,2
        IF(NFI2(IC).NE.0)THEN

        DO 12 I=1,NFI2(IC)
        AFI2(IC,I)=-AFI2(IC,I)

        DO 13 J=1,NFI1(IC,I)
        AFI1(IC,I,J)=-AFI1(IC,I,J)
        AMU(IC,I,J,K)=-AMU(IC,I,J,K)
13      CONTINUE
```

```
12    CONTINUE
      ELSE
      END IF
11    CONTINUE

C     REARRANGING THE ROOTS
C     TER IS JUST A DUMMY VARIABLE

      DO 16 IC=1,2
      DO 14 I=1,NFI2(IC)
      IF(NFI1(IC,I).LE.1)GO TO 14

      DO 15 J=1,NFI1(IC,I)/2
      TER=AFI1(IC,I,J)
      AFI1(IC,I,J)=AFI1(IC,I,NFI1(IC,I)+1-J)
      AFI1(IC,I,NFI1(IC,I)+1-J)=TER
      TER=NK1(IC,I,J)
      NK1(IC,I,J)=NK1(IC,I,NFI1(IC,I)+1-J)
      NK1(IC,I,NFI1(IC,I)+1-J)=TER
      TER=AK1NOT(IC,I,J,K)
      AK1NOT(IC,I,J,K)=AK1NOT(IC,I,NFI1(IC,I)+1-J,K)
      AK1NOT(IC,I,NFI1(IC,I)+1-J,K)=TER
      TER=AMU(IC,I,J,K)
      AMU(IC,I,J,K)=AMU(IC,I,NFI1(IC,I)+1-J,K)
      AMU(IC,I,NFI1(IC,I)+1-J,K)=TER
      TER=SMIN(IC,I,J,K)
      SMIN(IC,I,J,K)=SMIN(IC,I,NFI1(IC,I)+1-J,K)
      SMIN(IC,I,NFI1(IC,I)+1-J,K)=TER
15    CONTINUE
14    CONTINUE
      IF(NFI2(IC).LE.1)GO TO 16

      DO 50 I=1,NFI2(IC)/2
      TER=AFI2(IC,I)
      AFI2(IC,I)=AFI2(IC,NFI2(IC)+1-I)
      AFI2(IC,NFI2(IC)+1-I)=TER
      TER=NFI1(IC,I)
      NFI1(IC,I)=NFI1(IC,NFI2(IC)+1-I)
      NFI1(IC,NFI2(IC)+1-I)=TER

      DO 51 J=1,50
      TER=AFI1(IC,I,J)
      AFI1(IC,I,J)=AFI1(IC,NFI2(IC)+1-I,J)
      AFI1(IC,NFI2(IC)+1-I,J)=TER
```

```
      TER=NK1(IC,I,J)
      NK1(IC,I,J)=NK1(IC,NFI2(IC)+1-I,J)
      NK1(IC,NFI2(IC)+1-I,J)=TER
      TER=AK1NOT(IC,I,J,K)
      AK1NOT(IC,I,J,K)=AK1NOT(IC,NFI2(IC)+1-I,J,K)
      AK1NOT(IC,NFI2(IC)+1-I,J,K)=TER
      TER=AMU(IC,I,J,K)
      AMU(IC,I,J,K)=AMU(IC,NFI2(IC)+1-I,J,K)
      AMU(IC,NFI2(IC)+1-I,J,K)=TER
      TER=SMIN(IC,I,J,K)
      SMIN(IC,I,J,K)=SMIN(IC,NFI2(IC)+1-I,J,K)
      SMIN(IC,NFI2(IC)+1-I,J,K)=TER
 51   CONTINUE
 50   CONTINUE
 16   CONTINUE

      DO 151 IC=1,2
      WRITE(44,*)ETA,MM(IC),NFI2(IC)
      DO 152 I=1,NFI2(IC)
      WRITE(44,*)AFI2(IC,I),NFI1(IC,I)
      DO 153 J=1,NFI1(IC,I)
      WRITE(44,*)AFI1(IC,I,J),NK1(IC,I,J),AK1NOT(IC,I,J,K),
     1 AMU(IC,I,J,K),SMIN(IC,I,J,K)
153   CONTINUE
152   CONTINUE
151   CONTINUE
 5    CONTINUE
      STOP
      END
```