THE CONTRIBUTION OF MULTIPATHING EFFECTS IN GROUND-WAVE RADAR RETURN FROM THE SEA SURFACE

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RAVI SWARUP SRIVASTAVA
THE CONTRIBUTION OF MULTIPATHING EFFECTS IN GROUND-WAVE RADAR RETURN FROM THE SEA SURFACE

by

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A thesis submitted to the School of Graduate Studies in partial fulfillment of the requirements for the degree of Master of Engineering

Faculty of Engineering and Applied Science
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ABSTRACT

The second-order ground wave spectral cross section of the ocean surface given by existing theories corresponds to the case where both theoretical scatterings occur within the bounds of a specific area or patch of the ocean surface. Another additional term in the second-order cross section given by Walsh and Srivastava (1987), which represents the phenomenon where at least one scattering occurs outside the bounds of the patch of the ocean surface, is examined. The properties and significance of this off-patch scatter (a multipathing effect) are discussed.

The off-patch spectral cross section expression is simplified for a narrow beam receiving antenna. By using suitable numerical techniques, a computer program is developed for calculating this cross section. The program is applicable for wide beam transmitting antenna. Theoretical Doppler spectra are generated for different radar frequencies and sea states to study the importance of this type of scatter. A comparison is carried out between the Doppler spectra of the above two kinds of scatter for various sea conditions to infer the effect of off-patch scatter in extraction of ocean surface parameters. Two possible cases, first when the radar is located on the open ocean and second when it is located on the beach, are considered for the comparison.

It is determined that off-patch scatter is not significant in Doppler regions near the first-order peaks, which are commonly used to extract the ocean surface parameters. However, at some Doppler frequencies its contribution may be important as contending ocean clutter in target detection problems.
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To my parents.
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LIST OF SYMBOLS

- $C_{eh}$: Second-order electromagnetic and hydrodynamic contributions
- $F_P$: One way ground wave attenuation function between the radar and the surface patch
- $g$: Acceleration due to gravity
- $G_L, G_R$: Free space gains of the transmitting and receiving antennas in the direction of surface patch
- $K$: Wave number vector $= k_x x + k_y y$
- $k$: Magnitude of $K$
- $k_0$: Incident radio wave number
- $P_r$: Received power of the backscattered signal
- $P_t$: Average transmitted power
- $S(\cdot)$: Ocean directional waveheight spectrum
- $sa(x)$: Sampling function
- $sgn(x)$: Sign function
- $u_w$: Wind speed
- $\Delta_0$: One-half the beamwidth of the receiving antenna
- $\Delta_\rho$: One-half the patch width of the ocean surface
- $\Lambda$: Normalized surface impedance
- $\delta(x)$: Dirac delta function
- $\lambda_0$: Wavelength of transmitted signal
- $v$: Doppler frequency normalized to the Bragg frequency
- $\rho_0$: Distance of the target
\[ \sigma(\omega_d) \]: Total backscattered Doppler frequency dependent spectral cross section of the ocean surface patch normalized to its area

\[ \sigma_f(\omega_d) \]: First-order backscattered spectral cross section of the ocean surface patch normalized to its area

\[ \sigma_{s1}(\omega_d), \sigma_{s2}(\omega_d), \sigma_{s3}(\omega_d) \]: Three parts of second-order backscattered spectral cross section of the ocean surface patch normalized to its area

\[ \omega_B \]: Bragg frequency

\[ \omega_d \]: Doppler shift in the received signal

\[ \omega_T \]: Frequency of the transmitted signal
CHAPTER 1

INTRODUCTION

1.1 General

In recent years, ground wave Doppler radars have gained popularity in remote sensing of ocean surface waves. The theoretical formulation required to analyze the radar sea echo is based on the interpretation of the echo power spectrum in terms of Barrick's equation for the spectral cross section. It is from this cross section that ocean surface parameters (e.g., significant waveheight and directional waveheight spectrum) may be extracted. Barrick's model for the Doppler spectrum of the radar return for a small area (patch) of the ocean surface consists predominantly of first- and second-order scatter. First-order scatter is produced by the reflection of the transmitted signal from ocean waves whose wavelength is one-half the radar's wavelength and which are moving either towards or away from the radar. Figure 1 shows the first-order scatter from a patch of the ocean surface. Second-order scatter (on-patch) is produced when the transmitted signal interacts with two ocean waves that exist within the boundaries of the patch and have different wavelengths in general.

In a different approach, Srivastava (1984) has developed an analytical model for the HF backscattered Doppler spectrum for the ocean surface. The first-order cross section is the same as derived by Barrick but the second-order cross section consists of three
parts. The first part is equivalent to the second-order cross section derived by Barrick. The second part corresponds to the case where the first scatter occurs at the source point and the second scatter occurs on the patch. This additional term will be present only when the radar is surrounded by the open ocean (e.g., a ship or an offshore platform based radar) and this form of scatter does not affect the regions of Doppler spectrum near the first-order peaks which are commonly used for estimating ocean wave parameters. The third part corresponds to the case where at least one scatter occurs from the ocean waves outside the boundaries of the patch. This kind of scatter which may be viewed as a multipathing effect will be referred to as the off-patch scatter. The three parts of the second-order scatter are shown in figure 2.

In order to study the contribution of off-patch scatter, a software model has been developed. For estimating the spectral cross section, the model includes a numerical evaluation of an integral. This single-variable integral is achieved from a fourth order integral by taking advantage of two Dirac delta functions and assuming a wide beam transmit antenna and narrow beam receive antenna. This model may be used to generate the spectral cross section of off-patch scatter for different radar frequencies and sea conditions. A comparison is carried out between the off-patch and on-patch cross section to infer the importance of off-patch scatter in the extraction of ocean surface parameters and other relevant information. The results lead to the conclusion that although off-patch scatter is not significant in
extracting the ocean surface parameters, at some Doppler frequencies it may be very important as containing ocean clutter in target detection problems.

1.2 Literature Review

The problem of reflection of waves from a rough surface has been carried out by many investigators. Most interest lies in the propagation of radio waves over rough ground or over the sea. Reyleigh introduced a perturbation technique in 1896 to study the reflection of acoustic waves from rough walls. Based on the above approach Rice (1951) treated the problem—related to the reflection of electromagnetic waves from slightly rough surfaces. He modelled the surface as two dimensional and periodic and assumed the incident field to be a plane wave. He obtained the first-order and the higher-order expressions for the scattered field and derived the three scattered field components for horizontal and vertical polarizations. In 1964 Wait applied the above technique in the electromagnetic problem for ground wave propagation over flat earth. Wait (1971) and Barrick (1971) derived the expression for the modified surface impedance and scattering results. Wait assumed the surface to be one dimensional and periodic while Barrick concentrated on ground wave propagation over the rough sea.

Crombie (1955) conducted an experiment on radar return from the ocean surface at 13.56 MHz and he found two dominant peaks in the Doppler spectrum. He investigated these two peaks and came to the conclusion that they were produced by two ocean waves each
having a wavelength equal to one-half the radar's wavelength, one moving towards and the other moving away from the radar. Based on this observation he established that the HF radars may be used in remote sensing of ocean waves. Later Barrick (1970) utilized the perturbation technique to find the radar cross section of the ocean surface. He (1972) derived an average first- and second-order back-scattered radar cross section for a patch of the ocean surface. The first order result offered a theoretical explanation for Crombie's experimental conclusion.

In a different approach, Walsh (1980) presented a formulation for rough surface propagation and scattering problems. His technique is applicable to any time-invariant rough surface, and is open to any finite source instead of plane wave incidence. Based on this technique, a theoretical analysis was made by Srivastava (1984) for HF scattering from the ocean surface. Srivastava derived average first- and second-order of backscattered spectral cross section. For narrow beam reception the scattering area of the first-order backscattered field is shown to be a patch or small area of the ocean surface. But for the second-order case, the signals are received from the patch as well as from the surrounding regions. All these signals arrive at the same time. His second-order cross section contains two terms in addition to that provided by existing theories. Later, Walsh et al. (1986) extended the above results for wide beam reception and modified the expression for off-patch scatter. The latest work on this problem has been presented by Walsh and Srivastava (1987). The first additional term has already been studied.
and not found significant in problem of extraction of ocean surface parameters [Srivastava (1984), Walsh and Srivastava (1987)]. The second additional term which corresponds to off-patch scatter has not received complete study by other researchers and, actually, has been a controversial issue for several years regarding its importance to the problem of extracting ocean wave information from radar data. In an effort to settle this issue, its properties and significance for ocean wave measurements have been carried out in this thesis.

1.3 Scope of Thesis

This thesis is primarily concerned with the examination of the effect of second-order off-patch scatter. The emergence of this off-patch term is totally based on the analytical model developed by Srivastava (1984) and Walsh and Srivastava (1987) for the HF backscattered Doppler spectrum for the ocean surface. The discrepancy between actual radar data and the theoretical Doppler spectrum provided by existing theories raised a suspicion of the interference of some other effects. To be able to see if the off-patch scatter is the cause of this discrepancy, a complete study is done to understand the physical properties and significance of this form of scatter. For this purpose, a software model has been implemented to generate the theoretical off-patch second-order cross section for different radar frequencies and sea states. These results are then compared with the on-patch cross section in order to draw a conclusion about its importance. It is found that though off-patch scatter is not significant in the extraction of ocean
surface parameters, it is in some cases, very important in target detection problems.

Chapter 2 contains the radar range equation and the cross section expressions for first-order and the three parts of the second-order scatters. The definition of all the variables and parameters are given.

Chapter 3 deals with the reduction of the spectral cross section expression of the off-patch scatter to a computational form. The reduction is achieved by taking into account the presence of two Dirac delta functions and approximating an integral using the rectangular rule for narrow beam reception. This cross section expression is obtained from the similar expression of Walsh and Srivastava (1987) for a wide beam receiving antenna.

Chapter 4 consists of results and discussions. A comparison is made in this chapter between the off-patch and on-patch spectra for different radar frequencies and sea states. Two possible cases are studied: firstly the case when the radar is located on the open ocean (e.g., a ship or an offshore platform based radar) and secondly when the radar is located on the beach. Results are discussed for all these cases.

Conclusions and recommendations for future work are presented in Chapter 5. Computer programs are given in appendix.
CHAPTER 2

BACKSCATTERED RADAR CROSS SECTION

2.1 Radar Range Equation

The standard monostatic radar range equation for the received power from a target may be written as [Barrick (1972)]

\[ P_r^0 = \frac{P_t G_t G_r \lambda_0^2 |F|^4}{(4\pi)^3 \rho_0^4} \sigma^0 \]  \hspace{1cm} (2.1)

where

- \( P_r^0 \): received power (watts)
- \( P_t \): transmitted power (watts)
- \( G_t \): gain of transmitting antenna
- \( G_r \): gain of receiving antenna
- \( \lambda_0 \): wavelength of transmitted signal (meter)
- \( F \): one way ground wave attenuation function between the radar and the target
- \( \rho_0 \): distance of the target (meter)
- \( \sigma^0 \): radar cross section of the target (meter\(^2\))

\( G_t \) and \( G_r \) are dimensionless parameters.

We are considering only a small patch of the ocean surface for remote sensing and since that consists of many ocean waves moving with different velocities, there will be a band of Doppler shifts in the received signal. If \( \omega_t \) is the transmitted frequency and \( \omega_r \) is the received frequency, then the Doppler shift \( \Delta \omega \) in the received signal may be defined as
\[ \omega_d = \omega_r - \omega_t \]  \hspace{1cm} (2.2)

If instead of the received power, we use the received power spectrum then the backscattered power spectrum in terms of spectral cross section normalized to the patch area may be given as

\[ P_r(\omega_d) = \frac{P_t G_t G_r \lambda_o^2 |F|^4}{(4\pi)^3 \rho_o^4} A_p \sigma(\omega_d) \]  \hspace{1cm} (2.3)

where

\[ P_r^0 = \frac{1}{2\pi} \int P_r(\omega_d) d\omega_d \]  \hspace{1cm} (2.4)

and

\[ \sigma^0 = \frac{A_p}{2\pi} \int \sigma(\omega_d) d\omega_d \]  \hspace{1cm} (2.5)

\( P_r(\omega_d) \) is the backscattered power spectrum and \( \sigma(\omega_d) \) is the spectrum cross section normalized to patch area \( A_p \).

The dimensions of the patch depend on the time delay between the transmitted and received signals, transmitted pulse width and the beamwidth of the receiving antenna.

The backscattered spectral cross section normalized to the patch area as given in Walsh and Srivastava (1987) consists of four parts and may be written as

\[ \sigma(\omega_d) = \sigma_f(\omega_d) + \sigma_{s1}(\omega_d) + \sigma_{s2}(\omega_d) + \sigma_{s3}(\omega_d) \]  \hspace{1cm} (2.6)

where \( \sigma_f \) is the first-order cross section normalized to the patch area and \( \sigma_{s1}, \sigma_{s2} \) and \( \sigma_{s3} \) represent three parts of the second-order cross section normalized to the patch area. The derivations
of these cross sections for wide beam transmission and any receiving antenna (e.g., wide beam or narrow beam) are given in Walsh and Srivastava (1987). For narrow beam reception their expressions may be simplified further and are presented here.

2.2 First-Order Cross Section

The first-order cross section for narrow beam receiving antennas may be given as

$$
\sigma_f(\omega_d) = \frac{8A_\rho |\omega_d|^5}{\pi g^2} \left(\frac{\omega_d^2}{g} - k_0^2\right)^2 s^{2}\left[ A_\rho \left(\frac{\omega_d^2}{g} - 2k_0^2\right)^2 S\left(-2\eta^2 \text{sgn}(\eta) k_0^2\right) \right]
$$

(2.7)

where \( \eta = \omega_d / \omega_B \), \( \omega_B = (2gk_0)^{1/2} \) is the Bragg frequency, \( A_\rho \) is one-half the patch width of the ocean surface, \( g \) is the gravitational acceleration, \( k_0 = k_0 \hat{x} \) is the incident radar wavenumber vector, \( S(\cdot) \) represents the ocean directional waveheight spectrum as defined in Walsh and Srivastava (1987) in a form similar to that given by Lipa and Barrick (1982).

Assuming a large \( A_\rho \), the limit of the squared sampling function \([Sa^2(x)]\) in Eq. (2.7) may be taken to the Dirac delta function. In this process Eq. (2.7) reduces to

$$
\sigma_f(\omega_d) = 16k_0^4 \sum_{m=\pm 1} \delta(\omega_d + m\omega_B) S(2mk_0)
$$

(2.8)

where \( m = 1 \) and \(-1\) are for summation.
2.3 Second-Order Cross section

The second-order cross section consists of three parts and may be given as follows:

2.3.1 First Part

The first part of the second-order cross section which corresponds to on-patch scatter may be given as:

\[ \sigma_{s1}(\omega_d) = \frac{4k_0^4}{\pi^2} \sum_{m,m'=-1}^{1} \int \int |C_e + C_h|^2 S(mk_1) \cdot S(mk_2) \delta(\omega_d - m(gk_1)^{1/2} - m'(gk_2)^{1/2}) \cdot dq \cdot dp \] (2.9)

where \( m \) and \( m' \) may take the values \(+1\) and \(-1\), defining four possible combinations of direction for the two scattering ocean wavenumber vectors \( \vec{k}_1 \) and \( \vec{k}_2 \). The spatial wavenumber \( p \) lies along the radar beam, \( q \) perpendicular. Other variables and functions are defined as follows:

\[ \vec{k}_1 = (p - k_0)\hat{x} + q\hat{y}, \quad \vec{K}_1 = \|\vec{k}_1\| \] (2.10)

\[ \vec{k}_2 = (-p + k_0)\hat{x} - q\hat{y}, \quad \vec{K}_2 = \|\vec{k}_2\| \] (2.11)

\[ C_e = \frac{1}{2} \frac{[k_0^2 - p^2 - 2k_1 \cdot k_2]}{(K_1 \cdot K_2)^{1/2}} \] (2.12)

\[ C_h = -\frac{1}{2} \left[ K_1 + K_2 + \frac{(K_1 k_2 - K_1 \cdot K_2)(\omega_B^2 + \omega_D^2)}{nm'(K_1 K_2)^{1/2}(\omega_B^2 - \omega_D^2)} \right] \] (2.13)
C_e and C_h are the second-order electromagnetic and hydrodynamic contributions respectively as given in Srivastava (1984).

2.3.2 Second Part

The second part of the second-order cross section may be written as

$$\sigma_{s2}(\omega_d) = \frac{2k_0^4}{\pi^2} \sum_{m,m'=\pm 1} \iint \frac{(K_1 + k_0) \cdot K_1}{(K_1 + 2k_0) \cdot K_1} S(mK_1) \cdot S(2m'k_0) \delta[\omega_d + m(qK_1)^{1/2} + m'\omega_B] dq dp \quad (2.14)$$

where

$$K_1 = px + qy, \quad k_1 = |K_1|$$

2.3.3 Third Part

The third part of the second-order cross section derived by Walsh and Srivastava (1987) for a general receiving antenna has been simplified for narrow beam reception and is presented in Howell et al. (1987). This third part corresponds to off-patch scatter and may be given as follows

$$\sigma_{s3}(\omega_d) = \frac{1}{\pi^2 A_0 |F_p|^4} \sum_{m,m'=\pm 1} \int_{K_1=0}^{\infty} \int_{\phi_1=-\pi}^{\pi} \int_{K_2=0}^{\infty} \int_{\phi_2=-\pi}^{\pi}$$

$$|F_0(r_b, \psi_B + \phi_c) F_0(r_c, \psi_C + \phi_c + \pi)|^2 \delta\left(\frac{K_c}{1 + \cos^2 \phi_c} - 2k_0\right)$$
The restriction on the above equation is: $|\phi_c| \leq \Delta_0$. Other variables and functions are defined as follows:

\[ K_1 = K_{1x} \hat{x} + K_{1y} \hat{y}, \quad \hat{K}_1 = |K_1|, \quad \phi_1 = \tan^{-1} \left( \frac{K_{1y}}{K_{1x}} \right) \]

\[ K_2 = K_{2x} \hat{x} + K_{2y} \hat{y}, \quad \hat{K}_2 = |K_2|, \quad \phi_2 = \tan^{-1} \left( \frac{K_{2y}}{K_{2x}} \right) \]

\[ K_c = K_{cx} \hat{x} + K_{cy} \hat{y}, \quad \hat{K}_c = |K_c|, \quad \phi_c = \tan^{-1} \left( \frac{K_{cy}}{K_{cx}} \right) \]

\[ K_{cx} = 2K_{1x} + K_{2x}(1 + \cosh\beta_0 \cos\alpha_0) + K_{2y} \sin\beta_0 \sin\alpha_0 \]

\[ K_{cy} = 2K_{1y} + K_{2y}(1 + \cosh\beta_0 \cos\alpha_0) + K_{2x} \sin\beta_0 \sin\alpha_0 \]

\[ Q_c(K_1, K_2) = k_0(K_1 \cdot K_c)[2k_0(K_1 \cdot K_c) + k_0(K_2 \cdot K_c)] 
- 2\hat{K}_1 \hat{K}_2 - 3K_1 \cdot K_2 - K_2 \cdot \hat{K}_1 + K_1^2 \cdot K_2^2 
+ 2K_1 \cdot K_2 \cdot \hat{K}_0(K_1 \cdot K_c + K_2 \cdot K_c) \]

\[ \det \text{ is defined as } \]

\[ \det = \frac{\partial^2 E}{\partial \beta^2} \frac{\partial^2 E}{\partial \alpha^2} - \left( \frac{\partial^2 E}{\partial \beta \partial \alpha} \right)^2 \]
where

$$\xi(\beta, \alpha) = \frac{1}{1 + \cosh\beta} [2K_1 \cdot K_c + K_2 \sinh\beta \sin\alpha$$

$$- \sin(\phi_2, \cdot \phi_c) + (K_2 \cdot K_c)(1 + \cosh\beta \cos\alpha)]$$  \hspace{1cm} (2.23)$$

det should be evaluated at $\beta = \beta_0$ and $\alpha = \alpha_0$. $\beta_0$ is the solution of the equation,

$$a_1 \cosh^2\beta + a_2 \cosh\beta + a_3 = 0$$  \hspace{1cm} (2.24)$$

where

$$a_1 = 4K_1 \cos^2\phi_1 + 4K_1K_2 \cos\phi_1 \cos\phi_2 + \frac{a_2}{2}$$

$$a_2 = -2K_2 \sin^2\phi_2$$

$$a_3 = (-\sin^2\phi_2)(a_1 - \frac{a_2}{2} + K_2^2)$$  \hspace{1cm} (2.25)$$

such that $\beta_0$ is real, non-zero and satisfies the following equation,

$$[2K_1 \cos\phi_1 + K_2 \cos\phi_2 \cos\phi_2 \cosh\beta_0 [1 + \tanh^2\beta_0$$

$$\tan^2\phi_2]^{1/2} = K_2 \text{sgn}[\beta_0 \tan\phi_2] [\sin^2\phi_2 + \cosh\beta_0]$$  \hspace{1cm} (2.26)$$

$\text{sgn}(x)$ is the sign function defined as

$$\text{sgn}(x) = \begin{cases} 
1, & x > 0 \\
0, & x = 0 \\
-1, & x < 0
\end{cases}$$

Once $\beta_0$ is obtained, $\alpha_0$ may be derived from

$$\tan\alpha_0 = \tanh\beta_0 \tan\phi_2$$  \hspace{1cm} (2.27)$$
such that $0 \leq \alpha_0 \leq \pi$. $\Delta_\theta$ is equal to one-half of the beamwidth of the receiving antenna. The function $F_0(\rho, \theta)$ is the same as the Sommerfeld attenuation function except that the numerical distance contains $\Delta_\theta(\theta)$, instead of the normalized surface impedance $\Delta$ [Barrick (1970)]. $\Delta_\theta(\theta)$ is the average modified surface impedance in the propagation direction $\theta$ and it takes into account the surface roughness: The expression for the modified surface impedance is given in Walsh and Srivastava (1987). $F_p = F_0(\rho, \theta_0)$ is the one way ground wave attenuation function between the radar and the patch.

$r_b$ is the distance between the source and first scattering point. $r_c$ is the distance between the first and second scattering points. $r_a$ represents the distance between the second scattering point and the receiving (or source) point. The corresponding three directions with respect to the $x$-axis are $(\psi_b + \phi_c)$, $(\psi_c + \phi_c + \pi)$ and $(\phi_c + \pi)$ respectively. These distances and directions may be obtained from

\[
r_a = \frac{2\rho_0}{1 + \cosh\beta_0},
\]

\[
r_b = \frac{\rho_0(\cosh\beta_0 + \cos\alpha_0)}{(1 + \cosh\beta_0)}, \quad \psi_b = \tan^{-1}\left[\frac{\sinh\beta_0 \sin\alpha_0}{\cosh\beta_0 \cos\alpha_0 + 1}\right]
\]

\[
r_c = \frac{\rho_0(\cosh\beta_0 - \cos\alpha_0)}{(1 + \cosh\beta_0)}, \quad \psi_c = \tan^{-1}\left[\frac{\sinh\beta_0 \sin\alpha_0}{\cosh\beta_0 \cos\alpha_0 - 1}\right]
\]

The Eqs. (2.7), (2.9), (2.14) and (2.15) thus represent the first-order and the three parts of the second-order Doppler frequency dependent backscattered cross section of the ocean surface. These expressions are applicable for wide beam transmission and narrow beam reception.
3.1 Simplification of Cross Section Expression

The second-order off-patch cross section expression as given in Eq. (2.15) may be simplified by using some of its mathematical properties. We will first transform the three variables $K_1$, $\phi_1$ and $K_2$ to new variables $U$, $V$ and $\phi_c$ respectively such that Eq. (2.15) may be rewritten as

$$
\sigma_{s3} (\omega) = \frac{1}{\pi^2 \Delta_0 |P|^4} \sum_{m,m'} \int_{\phi_2=-\pi}^{\phi_2=\pi} R(U,V) \phi_c=\Delta_0 \chi_{|F_0(r_0,\psi_0)+\phi_c)F_0(r_c,\psi_c+\phi+\pi)F_0(r_c,\phi_c+\pi)|^2}
$$

$$
\frac{Q_c (K_1, K_2, \phi_c, K_1, K_2)}{K_c (1 + \cosh \beta_0)} \det [\ddot{U}] S (mK_1) S (m'K_2) \delta (U) \delta (V)
$$

$$
d\phi_c \, dU \, dv \, d\phi_2
$$

where

$$
U = \frac{K_c}{1 + \cosh \beta_0} - 2k_0
$$

$$
V = \omega_d + m(gK_1)^{1/2} + m'(gK_2)^{1/2}
$$

$$
\phi_c = \phi_c(K_1, K_2, \phi_1, \phi_2)
$$
3.1.1 Reduction of Integrals

The cross section expression as given by Eq. (3.1) consists of four integrals. It is noted, however, that this expression has two Dirac delta functions which may be used to reduce the number of integrations.

Since we know that

$$\int_{a}^{b} \int_{c}^{d} f(x, y) \delta(x) \delta(y) \, dy \, dx = f(x, y) \bigg|_{x=0}^{x} \bigg|_{y=0}^{y}$$

provided \( a \leq 0 \leq b \)

\( c \leq 0 \leq d \)

we may simplify Eq. (3.1) as

$$\sigma_{S3}(\omega_{d}) = \frac{1}{\pi^{2} \Delta \theta |F_{p}|^{4} \sum_{m, m' = \pm 1}} \int_{-\pi}^{\pi} \int_{0}^{\Delta \theta} |F_{o}(r_{c}, \psi_{c} + \phi_{c}) + \pi F_{o}(r_{a}, \phi_{c} + \pi)|^{2}$$

$$\frac{Q_{o}^{2}(K_{1}, K_{2}, \phi_{c}) K_{1} K_{2}}{K_{c}(1 + \cosh \beta_{o}) |\det J| S(m_{K_{1}}) S(m'_{K_{2}})}$$

$$d\phi_{c} \, d\phi_{2}$$

(3.5)

where now

$$U = \frac{K_{c}}{1 + \cosh \beta_{o}} + 2k_{o} = 0$$

(3.6)

$$V = \omega' + m(gK_{1})^{1/2} + m'(gK_{2})^{1/2} = 0$$

(3.7)

$$\phi_{c} = \phi_{c}(K_{1}, K_{2}, \phi_{1}, \phi_{2})$$

(3.8)
Using Eq. (3.6), we may write Eq. (3.5) as

\[
\sigma_{33}(\omega_d) = \frac{2k_0}{\pi^2 \Delta_0 |F_p|^4} \sum_{m, m' = \pm 1} \int_0^\pi \int_{-\phi_c = -\Delta_0}^{\phi_c = \Delta_0} \left\{ |F_o(r_b, \psi_b + \phi_c) \right\}^2
\]

\[
F_0(r_c, \psi_c + \phi_c + \pi) F_0(r_a, \phi_c + \pi) \right\}^2
\]

\[
\frac{Q^2(K_1, K_2, \phi_c) K_1 K_2}{K_c^2 |\text{det}||J||}
\]

\[
s(mK_1) S(m' K_2)
\]

\[
d\phi_c d\phi_2
\]

(3.9)

If we represent the function within \{ \} by \( f(\phi_c, \phi_2) \), we may rewrite Eq. (3.9) as

\[
\sigma_{33}(\omega_d) = \frac{2k_0}{\pi^2 \Delta_0 |F_p|^4} \sum_{m, m' = \pm 1} \int_0^\pi \int_{-\phi_c = -\Delta_0}^{\phi_c = \Delta_0} f(\phi_c, \phi_2) d\phi_c d\phi_2
\]

(3.10)

where

\[
f(\phi_c, \phi_2) = |F_o(r_b, \psi_b + \phi_c) F_0(r_c, \psi_c + \phi_c + \pi) F_0(r_a, \phi_c + \pi) |^2
\]

\[
\frac{Q^2(K_1, K_2, \phi_c) K_1 K_2}{K_c^2 |\text{det}||J||}
\]

\[
s(mK_1) S(m' K_2)
\]

(3.11)

For narrow beam reception, the integral with respect to \( \phi_c \) can be approximated by multiplying the value of the integrand at the centre point by the width of the limits. For a better approximation, the limits on \( \phi_c \) can be divided in \( n \) parts and so the Eq. (3.10) may again be written as
\[ \sigma_{s3}\left(\omega_d\right) = \frac{2k_o}{\pi^2\Delta_0 |F_p|^2} \sum_{m,m' = \pm 1} \int_{-\pi}^{\pi} \left[ \int_{-\Delta_0}^{\Delta_0} f(\phi_c, \phi_2) d\phi_c \right. \\
+ \int_{-\Delta_0}^{\Delta_0} f(\phi_c, \phi_2) d\phi_c + \int_{-\Delta_0}^{\Delta_0} f(\phi_c, \phi_2) d\phi_c \\
\left. + \ldots + \int_{-(n-4)\Delta_0}^{-(n-2)\Delta_0} f(\phi_c, \phi_2) d\phi_c \int_{-(n-2)\Delta_0}^{-(n-4)\Delta_0} f(\phi_c, \phi_2) d\phi_c \right] + f\left(\frac{-(n-3)\Delta_0}{n}, \phi_2\right) + f\left(\frac{-(n-5)\Delta_0}{n}, \phi_2\right) \\
+ \ldots + f\left(\frac{(n-1)\Delta_0}{n}, \phi_2\right) d\phi_2 \]

Using the rectangular rule to approximate the above integral, we may write,

\[ \sigma_{s3}\left(\omega_d\right) = \frac{2k_o}{\pi^2\Delta_0 |F_p|^2} \sum_{m,m' = \pm 1} \int_{-\pi}^{\pi} \left[ f\left(\frac{-(n-1)\Delta_0}{n}, \phi_2\right) + f\left(\frac{-(n-5)\Delta_0}{n}, \phi_2\right) \right] + f\left(\frac{(n-1)\Delta_0}{n}, \phi_2\right) \]

\[ = \frac{4k_o}{n^2|F_p|^2} \sum_{m,m' = \pm 1} \int_{-\pi}^{\pi} \left[ f\left(\frac{-(n-1)\Delta_0}{n}, \phi_2\right) + f\left(\frac{-(n-5)\Delta_0}{n}, \phi_2\right) \right] \\
+ f\left(\frac{(n-1)\Delta_0}{n}, \phi_2\right) + f\left(\frac{(n-5)\Delta_0}{n}, \phi_2\right) \]
3.1.2 Jacobi of Transformation

The Jacobian of the transformation may be given as

\[ J = \begin{vmatrix}
\frac{\partial u}{\partial k_1} & \frac{\partial u}{\partial k_2} & \frac{\partial u}{\partial \phi_1} \\
\frac{\partial v}{\partial k_1} & \frac{\partial v}{\partial k_2} & 0 \\
\frac{\partial \phi_c}{\partial k_1} & \frac{\partial \phi_c}{\partial k_2} & \frac{\partial \phi_c}{\partial \phi_1}
\end{vmatrix} \]

\[ = \frac{\partial u}{\partial k_1} \frac{\partial v}{\partial k_2} \frac{\partial \phi_c}{\partial \phi_1} - \frac{\partial u}{\partial k_2} \frac{\partial v}{\partial k_1} \frac{\partial \phi_c}{\partial \phi_1} + \frac{\partial u}{\partial \phi_1} \left( \frac{\partial v}{\partial k_1} \frac{\partial \phi_c}{\partial k_2} - \frac{\partial v}{\partial k_2} \frac{\partial \phi_c}{\partial k_1} \right) \]

Derivatives of V

From Eq. (3.3) we may obtain the following partial derivatives of V as

\[ \frac{\partial v}{\partial k_1} = \frac{1}{2} m' \sqrt{\sigma} / k_1 \]

\[ \frac{\partial v}{\partial k_2} = \frac{1}{2} m' \sqrt{\sigma} / k_2 \]

\[ \frac{\partial v}{\partial \phi_1} = 0 \]

Using Eqs. (3.15) to (3.17), we may rewrite the Jacobian as
\[ |J| = \frac{1}{2} m \sqrt{\frac{\alpha}{K_2}} \left( \frac{\partial U}{\partial \phi_1} \frac{\partial \phi_c}{\partial K_2} - \frac{\partial U}{\partial K_2} \frac{\partial \phi_c}{\partial \phi_1} \right) + \frac{1}{2} m' \sqrt{\frac{\alpha}{K_2}} \left( \frac{\partial U}{\partial \phi_1} \frac{\partial \phi_c}{\partial K_2} - \frac{\partial U}{\partial K_2} \frac{\partial \phi_c}{\partial \phi_1} \right) \]  

(3.18)

Derivatives of \( U \)

Using Eq. (3.2) we may write the derivatives of \( U \) as

\[ \frac{\partial U}{\partial K_1} = \frac{1}{1 + \cosh \beta_0} \frac{\partial K_c}{\partial K_1} - \frac{K_c \sinh \beta_0}{(1 + \cosh \beta_0)^2} \frac{\partial \beta_0}{\partial K_1} \]  

(3.19)

\[ \frac{\partial U}{\partial K_2} = \frac{1}{1 + \cosh \beta_0} \frac{\partial K_c}{\partial K_2} - \frac{K_c \sinh \beta_0}{(1 + \cosh \beta_0)^2} \frac{\partial \beta_0}{\partial K_2} \]  

(3.20)

\[ \frac{\partial U}{\partial \phi_1} = \frac{1}{1 + \cosh \beta_0} \frac{\partial K_c}{\partial \phi_1} - \frac{K_c \sinh \beta_0}{(1 + \cosh \beta_0)^2} \frac{\partial \beta_0}{\partial \phi_1} \]  

(3.21)

Derivatives of \( \phi_c \)

Eqn. (2.18) may be reproduced as

\[ \phi_c = \tan^{-1} \left( \frac{K_{cy}}{K_{cx}} \right) \]

where \( K_{cy} \) and \( K_{cx} \) are defined in Eq. (2.18). Taking partial derivatives of the above equation we are able to write

\[ \frac{\partial \phi_c}{\partial K_{cx}} = \frac{K_{cy}}{K_{cx}} \frac{\partial K_{cy}}{\partial K_{cx}} - \frac{K_{cy}}{K_{cx}} \frac{\partial K_{cy}}{\partial K_{cx}} \]

\[ = \frac{K_{cx}}{K_{cx}^2 + K_{cy}^2} \frac{\partial K_{cy}}{\partial K_{cx}} - \frac{K_{cy}}{K_{cx}^2 + K_{cy}^2} \frac{\partial K_{cy}}{\partial K_{cx}} \]
Using Eqs. (2.16), (2.17) and (3.22) we may write the following derivatives as

\[
\frac{\partial \phi_c}{\partial K_1} = \left[ \cos \phi_c \left( 2 \sin \phi_1 + K_2 \sin \phi_2 \right) \left( \sinh \beta_o \cos \alpha_o \frac{\partial \beta_o}{\partial K_1} ight) 
- \cosh \beta_o \sin \alpha_o \frac{\partial \alpha_o}{\partial K_1} \right]
- K_2 \cos \phi_2 \left( \cosh \beta_o \sin \alpha_o \frac{\partial \beta_o}{\partial K_1} \right)
+ \sinh \beta_o \cos \alpha_o \frac{\partial \alpha_o}{\partial K_1} \right]
- \sin \phi_c \left( 2 \cos \phi_1 + K_2 \cos \phi_2 \right).

\text{[\text{Expressions continued...]} }
\]

\[
\frac{\partial \phi_c}{\partial K_2} = \left[ \cos \phi_c \left( \sin \phi_2 + \sin \phi_2 \cosh \beta_o \cos \alpha_o + K_2 \sin \phi_2 \right) \right]
\]
\[
\begin{align*}
&\left(\sinh\beta_o \cos\alpha_o \frac{\partial\beta_o}{\partial K_2} - \cosh\beta_o \sin\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right) \\
&- \cos\phi_2 \sinh\beta_o \sin\alpha_o - K_2 \cos\phi_2 \left(\cosh\beta_o \sin\alpha_o \frac{\partial\beta_o}{\partial K_2} \right) \\
&+ \sin\beta_o \cos\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right) \right) \cdot \sin\phi_c \left\{ \cos\phi_2 + \cos\phi_2 \cosh\beta_o \cos\alpha_o \\
&+ K_2 \cos\phi_2 \left[ \sinh\beta_o \cos\alpha_o \frac{\partial\beta_o}{\partial K_2} - \cosh\beta_o \sin\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right] \\
&+ \sin\beta_o \cos\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right) \right) \left[ \cos\phi_2 + \cos\phi_2 \cosh\beta_o \cos\alpha_o \\
&+ K_2 \cos\phi_2 \left[ \sinh\beta_o \cos\alpha_o \frac{\partial\beta_o}{\partial K_2} \right] \right) \frac{1}{K_c} \\
&= \left[ \sin(\phi_2 - \phi_c) (1 + \cosh\beta_o \cos\alpha_o) - \cos(\phi_2 - \phi_c) \sinh\beta_o \sin\alpha_o \\
&+ K_2 \sin(\phi_2 - \phi_c) \left[ \sinh\beta_o \cos\alpha_o \frac{\partial\beta_o}{\partial K_2} - \cosh\beta_o \sin\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right] \\
&- K_2 \cos(\phi_2 - \phi_c) \left[ \cosh\beta_o \sin\alpha_o \frac{\partial\beta_o}{\partial K_2} \right] \\
&+ \sinh\beta_o \cos\alpha_o \frac{\partial\alpha_o}{\partial K_2}\right) \right) \frac{1}{K_c} \\
&= \left[ 2K_1 \cos(\phi_1 - \phi_c) + K_2 \sin(\phi_2 - \phi_c) \left[ \sinh\beta_o \cos\alpha_o \frac{\partial\beta_o}{\partial \phi_1} \right] \\
&- \cosh\beta_o \sin\alpha_o \frac{\partial\beta_o}{\partial \phi_1} \right) - K_2 \cos(\phi_2 - \phi_c) \left[ \cosh\beta_o \sin\alpha_o \frac{\partial\beta_o}{\partial \phi_1} \right] \\
&+ \sinh\beta_o \cos\alpha_o \frac{\partial\alpha_o}{\partial \phi_1}\right) \frac{1}{K_c} \\
&= \left[ \sinh\beta_o \cos\alpha_o \frac{\partial\beta_o}{\partial \phi_1} \right] \frac{1}{K_c}
\end{align*}
\]
Derivatives of $K_c$

The magnitude of $K_c$ as defined in Eq. (2.18) may be written as

$$K_c = [K_{cx}^2 + K_{cy}^2]^{1/2}$$

$$= \left\{ \left[ 2K_{1x} + K_{2x}(1 + \cosh \beta_o \cos \alpha_o) + K_{2y} \sinh \beta_o \sin \alpha_o \right]^2 - \left[ 2K_{1y} + K_{2y}(1 + \cosh \beta_o \cos \alpha_o) - K_{2x} \sinh \beta_o \sin \alpha_o \right]^2 \right\}^{1/2}$$

$$= \left[ 4K_{1x}^2 + K_{2x}^2 + K_{2x}^2 \cosh^2 \beta_o \cos^2 \alpha_o + K_{2y}^2 \sinh^2 \beta_o \sin^2 \alpha_o \\
+ 4K_{1x} K_{2x} + 4K_{1x} K_{2x} \cosh \beta_o \cos \alpha_o + 4K_{1y} K_{2y} + 4K_{1y} K_{2y} \cosh \beta_o \cos \alpha_o \\
+ 2K_{2x} \cosh \beta_o \cos \alpha_o + 2K_{2x} K_{2y} \sinh \beta_o \sin \alpha_o \\
+ 2K_{2x} K_{2y} \cosh \beta_o \cos \alpha_o \sinh \beta_o \sin \alpha_o + 4K_{1y}^2 + K_{2y}^2 \\
+ K_{2y}^2 \cosh^2 \beta_o \cos^2 \alpha_o + K_{2x}^2 \sinh^2 \beta_o \sin^2 \alpha_o + 4K_{1y} K_{2y} \\
+ 4K_{1y} K_{2y} \cosh \beta_o \cos \alpha_o - 4K_{2x} K_{1y} \sinh \beta_o \sin \alpha_o \\
+ 2K_{2y} \cosh \beta_o \cos \alpha_o - 2K_{2x} K_{2y} \sinh \beta_o \sin \alpha_o \\
- 2K_{2x} K_{2y} \cosh \beta_o \cos \alpha_o \sinh \beta_o \sin \alpha_o \right\}^{1/2}$$

$$= (4K_{1x}^2 + K_{2x}^2 + K_{2x}^2 \cosh^2 \beta_o \cos^2 \alpha_o + K_{2y}^2 \sinh^2 \beta_o \sin^2 \alpha_o)$$
\[ + 4 \left( 1 + \cosh \beta_0 \cos \alpha_0 \right) \left( K_1 x K_2 x + K_1 y K_2 y \right) \]
\[ + 4 \sinh \beta_0 \sin \alpha_0 \left( K_1 x K_2 y - K_2 x K_1 y \right) \]
\[ + 2 K_2 \cosh \beta_0 \cos \alpha_0 \right)^{1/2} \]
\[ = \left[ 4 K_1^2 + K_2^2 \left( \cosh \beta_0 + \cos \alpha_0 \right)^2 + 4 K_1 K_2 \left( 1 + \cosh \beta_0 \cos \alpha_0 \right) \right. \]
\[ \cdot \cos \left( \phi_2 - \phi_1 \right) + 4 K_1 K_2 \sinh \beta_0 \sin \alpha_0 \sin \left( \phi_2 - \phi_1 \right) \left[ \right]^{1/2} \quad (3.26) \]

Now the derivatives of \( K_c \) may be written as follows

\[ \frac{\partial K_c}{\partial K_1} = \frac{1}{2 K_c} \left[ 8 K_1 + 2 K_2^2 \left( \cosh \beta_0 + \cos \alpha_0 \right) \left( \sin \beta_0 \frac{\partial \beta_0}{\partial K_1} - \sin \alpha_0 \frac{\partial \alpha_0}{\partial K_1} \right) \right. \]
\[ + 4 K_2 \left( 1 + \cosh \beta_0 \cos \alpha_0 \right) \cos \left( \phi_2 - \phi_1 \right) + 4 K_1 K_2 \]
\[ \cdot \cos \left( \phi_2 - \phi_1 \right) \left( \sinh \beta_0 \cos \alpha_0 \frac{\partial \beta_0}{\partial K_1} - \cosh \beta_0 \sin \alpha_0 \frac{\partial \alpha_0}{\partial K_1} \right) \]
\[ + 4 K_2 \sinh \beta_0 \sin \alpha_0 \sin \left( \phi_2 - \phi_1 \right) + 4 K_1 K_2 \sin \left( \phi_2 - \phi_1 \right) \]
\[ \cdot \left( \cosh \beta_0 \cos \alpha_0 \frac{\partial \beta_0}{\partial K_1} + \sinh \beta_0 \cos \alpha_0 \frac{\partial \alpha_0}{\partial K_1} \right) \left[ \right] \]
\[ = \frac{1}{K_c} \left[ 4 K_1 + K_2^2 \left( \cosh \beta_0 + \cos \alpha_0 \right) \left( \sin \beta_0 \frac{\partial \beta_0}{\partial K_1} - \sin \alpha_0 \frac{\partial \alpha_0}{\partial K_1} \right) \right. \]
\[ + 2 K_2 \left( 1 + \cosh \beta_0 \cos \alpha_0 \right) \cos \left( \phi_2 - \phi_1 \right) + 2 K_1 K_2 \cos \left( \phi_2 - \phi_1 \right) \]
\[
\begin{align*}
\{ \sinh \beta_o \cos \alpha_o \frac{\partial \beta_o}{\partial \chi_1} - \cosh \beta_o \sin \alpha_o \frac{\partial \alpha_o}{\partial \chi_1} \} + 2K_1 (1 + \cosh \beta_o \cos \alpha_o) \\
\sin \alpha_o \sin (\phi_2 - \phi_1) + 2K_1 K_2 \sin (\phi_2 - \phi_1) \\
\{ \cosh \beta_o \sin \alpha_o \frac{\partial \beta_o}{\partial \chi_1} + \sinh \beta_o \cos \alpha_o \frac{\partial \alpha_o}{\partial \chi_1} \}
\end{align*}
\]

Similarly
\[
\frac{\partial k_c}{\partial K_2} = \frac{1}{k_c} \left[ k_2 (\cosh \beta_o - \cos \alpha_o)^2 + k_2^2 (\cosh \beta_o + \cos \alpha_o) \right. \\
\left. \{ \sinh \beta_o \frac{\partial \beta_o}{\partial K_2} - \sin \alpha_o \frac{\partial \alpha_o}{\partial K_2} \} + 2K_1 (1 + \cosh \beta_o \cos \alpha_o) \right. \\
\left. \cos (\phi_2 - \phi_1) + 2K_1 K_2 \cos (\phi_2 - \phi_1) \{ \sinh \beta_o \cos \alpha_o \frac{\partial \beta_o}{\partial K_2} \right. \\
\left. - \cosh \beta_o \sin \alpha_o \frac{\partial \alpha_o}{\partial K_2} \} + 2K_1 \sinh \beta_o \sin \alpha_o \sin (\phi_2 - \phi_1) \\
+ 2K_1 K_2 \sin (\phi_2 - \phi_1) \{ \cosh \beta_o \sin \alpha_o \frac{\partial \beta_o}{\partial K_2} \right. \\
\left. + \sinh \beta_o \cos \alpha_o \frac{\partial \alpha_o}{\partial K_2} \} \right]
\] (3.28)

\[
\frac{\partial k_c}{\partial \phi_1} = \frac{1}{k_c} \left[ k_2^2 (\cosh \beta_o + \cos \alpha_o) \{ \sinh \beta_o \frac{\partial \beta_o}{\partial \phi_1} - \sin \alpha_o \frac{\partial \alpha_o}{\partial \phi_1} \right. \\
\left. + 2K_1 K_2 (1 + \cosh \beta_o \cos \alpha_o) \sin (\phi_2 - \phi_1) + 2K_1 K_2 \cos (\phi_2 - \phi_1) \\
\left. \{ \sinh \beta_o \cos \alpha_o \frac{\partial \beta_o}{\partial \phi_1} - \cosh \beta_o \sin \alpha_o \frac{\partial \alpha_o}{\partial \phi_1} \} - 2K_1 K_2 \sinh \beta_o \right.
\]
\[
\sin \alpha_0 \cos (\phi_2 - \phi_1) + 2K_1 K_2 \sin (\phi_2 - \phi_1) \left( \cosh \beta_0 \sin \alpha_0 \frac{\partial \beta_0}{\partial \phi_1} + \sinh \beta_0 \cos \alpha_0 \frac{\partial \alpha_0}{\partial \phi_1} \right)
\]

(3.29)

Derivatives of \( \beta_0 \)

\( \beta_0 \) is the solution of the quadratic equation (2.24) and will have two values for each value of \( \cosh \beta_0 \) and may be written as

\[
\beta_0 = \pm \cosh^{-1}(c^\pm).
\]

(3.30)

where

\[
c^\pm = \left| \sin \phi_2 \left[ K_2^2 \sin \phi_2 \pm 2 \sqrt{K_1^2 - 1} K_1 \cos \phi_1 + 4K_1^2 \cos^2 \phi_1 \cos \phi_2 \right]^1/2 \right| \\
\times \frac{4K_1^2 \cos^2 \phi_1 + 4K_1 K_2 \cos \phi_1 \cos \phi_2 - K_2^2 \sin^2 \phi_2}{4K_1^2 \cos^2 \phi_1 + 4K_1 K_2 \cos \phi_1 \cos \phi_2 - K_2^2 \sin^2 \phi_2}
\]

(3.31)

Out of four roots of \( \beta_0 \), we have to take only one root for which \( C \) is real and \( C > 1 \) and which satisfies the Eq. (2.26). Further from Eq. (3.30) we may write

\[
\frac{\partial \beta_0}{\partial C^\pm} = \pm \frac{1}{\{ (C^\pm)^2 - 1 \}^{1/2}} \sinh \beta_0.
\]

(3.32)

To obtain the derivatives of \( \beta_0 \), we may use Eq. (3.30) and so are able to write the derivatives as follows

\[
\frac{\partial \beta_0}{\partial K_1} = \frac{\partial \beta_0}{\partial C^\pm} \frac{\partial C^\pm}{\partial K_1}
\]

(3.33)
where \( \frac{\partial \beta}{\partial c^\pm} \) is defined in Eq. (3.32).

Derivatives of \( c^\pm \)

In the above equations we still have to find the derivatives of \( c^\pm \) which may be written from Eq. (3.31) as follows

\[
\frac{\partial c^\pm}{\partial K_1} = \left[ 4K_1^2\cos^2\phi_1 + 4K_1K_2\cos\phi_1\cos\phi_2 - K_2^2\sin^2\phi_2 \right]
\]

\[
+ \left[ \pm 2\text{sgn}\{2K_1\cos\phi_1 + K_2\cos\phi_2\}\{3\sqrt{K_1}\cos\phi_1 + \frac{1}{2\sqrt{K_1}}K_2\cos\phi_2\} \right]
\]

\[
- \{K_1^2\cos^2\phi_1 + K_2\cos\phi_1\cos\phi_2\}^{1/2} \pm \sqrt{K_1}|2K_1\cos\phi_1|
\]

\[
+ K_2\cos\phi_2 |\cos\phi_1| \left[ K_1\cos^2\phi_1 + K_2\cos\phi_1\cos\phi_2 \right]^{-1/2}
\]

\[
- \{K_2^2|\sin\phi_2| \pm 2\sqrt{K_1}|2K_1\cos\phi_1 + K_2\cos\phi_2| \}
\]

\[
- \{K_1^2\cos^2\phi_1 + K_2\cos\phi_1\cos\phi_2\}^{1/2}
\]

\[
+ \left[ 8K_1\cos^2\phi_1 + 4K_2\cos\phi_1\cos\phi_2 \right]|\sin\phi_2|
\]

\[
\frac{1}{\left[ 4K_1^2\cos^2\phi_1 + 4K_1K_2\cos\phi_1\cos\phi_2 - K_2^2\sin^2\phi_2 \right]^2}
\]
Similarly

\[
\frac{\partial \xi}{\partial \phi_2} = \left[ 4K_1^2\cos^2 \phi_1 + 4K_1K_2\cos \phi_1 \cos \phi_2 - K_2^2 \sin^2 \phi_2 \right] \\
\cdot \left[ 2K_2 |\sin \phi_2| \pm 2 \text{sgn}\{2K_1 \cos \phi_1 + K_2 \cos \phi_2\} \sqrt{K_1} \cos \phi_2 \right] \\
\cdot \left[ K_1 \cos^2 \phi_1 + K_2 \cos \phi_1 \cos \phi_2 \right]^{1/2} \pm \frac{\sqrt{K_1} \cos \phi_1 + K_2}{2K_2 |\sin \phi_2|} \right] \\
\cdot \left[ \cos \phi_2 \cos \phi_1 \cos \phi_2 \left\{ K_1 \cos^2 \phi_1 + K_2 \cos \phi_1 \cos \phi_2 \right\}^{-1/2} \right] \\
\cdot \left[ \frac{1}{2} \frac{1}{2} \left\{ K_1 \cos \phi_1 + K_2 \cos \phi_2 \right\} \right] \cdot 4K_1 \cos \phi_1 \cos \phi_2 - 2K_2 \sin \phi_2 \left\{ K_1 \cos \phi_1 + K_2 \cos \phi_2 \right\} \\
\cdot \left\{ \sin \phi_2 \right\} \\
\left[ 4K_1^2 \cos^2 \phi_1 + 4K_1K_2 \cos \phi_1 \cos \phi_2 - K_2^2 \sin^2 \phi_2 \right]^2 \\
\left( \frac{\partial \xi}{\partial \phi_1} \right)
\]

\[
\frac{\partial \xi}{\partial \phi_1} = \left[ 4K_1^2 \cos^2 \phi_1 + 4K_1K_2 \cos \phi_1 \cos \phi_2 - K_2^2 \sin^2 \phi_2 \right] \\
\cdot \left[ 4 \text{sgn}\{2K_1 \cos \phi_1 + K_2 \cos \phi_2\} \frac{1}{2} \frac{1}{2} \left\{ K_1 \cos \phi_1 + K_2 \cos \phi_2 \right\} \right] \\
\cdot \left[ \frac{1}{2} \frac{1}{2} \left\{ K_1 \cos \phi_1 + K_2 \cos \phi_2 \right\} \right] \cdot 4K_1 \cos \phi_1 \cos \phi_2 - 2K_2 \sin \phi_2 \left\{ K_1 \cos \phi_1 + K_2 \cos \phi_2 \right\} \\
\cdot \left\{ \cos \phi_1 \sin \phi_1 + K_2 \sin \phi_1 \cos \phi_2 \right\} \left\{ K_1 \cos^2 \phi_1 + K_2 \cos \phi_1 \cos \phi_2 \right\} \\
\cdot \left\{ \cos \phi_2 \right\}^{-1/2} \right] + \left[ K_2 |\sin \phi_2| \pm 2 \text{sgn}\{2K_1 \cos \phi_1 + K_2 \cos \phi_2\} \sqrt{K_1} \cos \phi_2 \right]
\[ \left[ K_1 \cos^2 \phi_1 + K_2 \cos \phi_1 \cos \phi_2 \right]^{1/2} \cdot \left[ 8K_1^2 \cos \phi_1 \sin \phi_1 + 4K_1K_2 \sin \phi_1 \cos \phi_2 \right] \cdot \left[ 4K_1^2 \cos^2 \phi_1 + 4K_2 K_1 \cos \phi_1 \cos \phi_2 - K_2^2 \sin^2 \phi_2 \right]^2 \] 

Derivatives of \( \alpha_0 \)

Using Eq. (2.27) the derivatives of \( \alpha_0 \) with respect to \( K_1, K_2 \) and \( \phi_1 \) may be obtained as follows

\[ \frac{\partial \alpha_0}{\partial K_1} = \frac{\tan \phi_2 \text{sech}^2 \beta_0 - \frac{\partial \beta_0}{\partial K_1}}{1 + \tanh^2 \beta_0 \tan^2 \phi_2} \] (3.39)

\[ \frac{\partial \alpha_0}{\partial K_2} = \frac{\tan \phi_2 \text{sech}^2 \beta_0}{1 + \tanh^2 \beta_0 \tan^2 \phi_2} \] (3.40)

\[ \frac{\partial \alpha_0}{\partial \phi_1} = \frac{\tan \phi_2 \text{sech}^2 \beta_0 - \frac{\partial \beta_0}{\partial \phi_1}}{1 + \tanh^2 \beta_0 \tan^2 \phi_2} \] (3.41)

Derivatives of \( \xi \)

In order to find the derivatives of \( \xi \) as required to calculate 'det' in Eq. (2.22), we may utilize Eq. (2.23) which may also be written as

\[ \xi(\beta, \alpha) = \frac{1}{(1 + \cosh \beta)} \left[ 2K_1 \cos(\phi - \phi_c) + K_2 \sinh \beta \sin \alpha \sin(\phi_2 - \phi_c) \right. \]

\[ + \left. K_2 (1 + \cosh \beta \cos \alpha) \cos(\phi_2 - \phi_c) \right] \] (3.42)

To obtain above equation, we have used the following relationships

\[ \begin{align*}
\vec{K}_1 \cdot \vec{K}_c &= |K_1| |K_c| \cos(\phi_1 - \phi_c) \\
\vec{K}_1 \cdot \vec{K}_c &= K_1 K_c \cos(\phi_1 - \phi_c) \\
\vec{K}_1 \cdot \vec{K}_c &= K_1 \cos(\phi_1 - \phi_c) \\
\vec{K}_1 \cdot \vec{K}_c &= K_1 \cos(\phi_1 - \phi_c) \quad (3.43)
\end{align*} \]

Similarly,

\[ \vec{K}_2 \cdot \vec{K}_c = K_2 \cos(\phi_2 - \phi_c) \]

Now from Eq. (3.42) derivative of \( \xi \) may be written as

\[ \frac{\partial^2 \xi}{\partial \beta^2} = \frac{1}{(1 + \cosh \beta)^2} \left[ 2(\cosh \beta - 2)K_1 \cos(\phi_1 - \phi_c) + (\cosh \beta - 2) \right. \\
- K_2 (1 - \cos \alpha) \cos(\phi_2 - \phi_c) - K_2 \sinh \beta \sin \alpha \\
- \sin(\phi_2 - \phi_c) \left. \right] \quad (3.44) \]

\[ \frac{\partial^2 \xi}{\partial \alpha^2} = \frac{1}{(1 + \cosh \beta)} \left[ -K_2 \sinh \beta \sin \alpha \sin(\phi_2 - \phi_c) - K_2 \cosh \beta \right. \\
- \cos \alpha \cos(\phi_2 - \phi_c) \left. \right] \quad (3.45) \]

\[ \frac{\partial^2 \xi}{\partial \beta \partial \alpha} = \frac{1}{(1 + \cosh \beta)^2} \left[ K_2 (1 + \cosh \beta) \cos \alpha \sin(\phi_2 - \phi_c) - K_2 \sinh \beta \right. \\
- \sin \alpha \cos(\phi_2 - \phi_c) \left. \right] \quad (3.46) \]
Summarizing all the required derivatives as given in Eqs. (3.15) to (3.41) may be used to calculate the 'Jacobian', while Eqs. (3.44) to (3.46) may be used to calculate the 'det'.
CHAPTER 4

RESULTS AND DISCUSSIONS

To generate the off patch spectral cross section at any Doppler frequency, the sum of integrals as given in Eq. (3.13) has to be evaluated. The function \( f(\phi_c, \phi_2) \) is defined in Eq. (3.11). The beamwidth of the receiving antenna is assumed to be as \( 6^\circ \) in all the computations. So \( \Delta_\theta = 3^\circ \) or \( \Delta_\theta = \frac{3\pi}{180} \) radian. Also for convenience \( n \) is assumed as equal to 5. Therefore the cross section expression as given by Eq. (3.13) may be reduced to

\[
\sigma_{s3}(\omega_0) = \frac{4k_o}{\pi^2 |F_p|^4} \sum_{m,m' = \pm 1} \int_{-\pi}^{\pi} \left[ f(-\frac{2.4\pi}{180}, \phi_2) 
+ f(-\frac{1.2\pi}{180}, \phi_2) + f(0, \phi_2) + f(\frac{1.2\pi}{180}, \phi_2) 
+ f(\frac{2.4\pi}{180}, \phi_2) \right] d\phi_2
\]

(4.1)

In the above equation, \( \phi_c \) has assumed the values as \( \frac{-2.4\pi}{180}, \frac{-1.2\pi}{180}, 0, \frac{1.2\pi}{180} \) and \( \frac{2.4\pi}{180} \). If the variable \( \theta_i \) is used to represent these values, then it may be written as

\[
\phi_c = \theta_i
\]

\[
W = \phi_c - \theta_i = 0
\]

(4.2)

where \( W \) is a substitution. Now Eqs. (3.6), (3.7) and (4.2) may be solved to obtain \( K_1, K_2 \) and \( \phi_1 \) for given values of \( \phi_2, m, m' \) and...
\( \omega_d \). But these solutions will be radar frequency-dependent. To avoid this problem, all three equations are normalized to radar frequency and in this way the obtained solutions may be used to generate the cross section for any radar frequency.

After normalization, Eqs. (3.6), (3.7) and (4.2) become

\[
U = \frac{1}{1 + \cosh \beta_0} \left[ 4K_1^2 + K_2^2 (\cosh \beta_0 + \cos \alpha_o) \right] + 4K_1 K_2 (1 + \cosh \beta_0 \cos \alpha_o) \cos (\phi_2 - \phi_1) \\
+ 4K_1 K_2 \sinh \beta_0 \sin \alpha_o \sin (\phi_2 - \phi_1)]^{1/2} - 1 = 0 \quad (4.3)
\]

\[
V = \eta + mK_1^0 - mK_2^0 \leq 0 \quad (4.4)
\]

\[
W = \tan^{-1} \left[ \frac{2K_1^2 \sin \phi_1 + K_2^2 \sin \phi_2 (1+\cosh \beta_0 \cos \alpha_o) - K_2^2 \cos \phi_2 \sinh \beta_0 \sin \alpha_o}{2K_1^2 \cos \phi_1 + K_2^2 \cos \phi_2 (1+\cosh \beta_0 \cos \alpha_o) + K_2^2 \sinh \beta_0 \sin \alpha_o \sin \phi_2} \right] \quad (4.5)
\]

where \( K_1^0 = K_1/2k_0 \) and \( K_2^0 = K_2/2k_0 \) are normalized wavenumbers.

\( \eta = \omega_d/\omega_B \) is the normalized frequency.

Eqs. (4.3), (4.4) and (4.5) may now be solved. As it is evident from Eq. (4.4) that \( K_1^0 \) and \( K_2^0 \) have a direct relationship, we may substitute the expression of \( K_2^0 \) from Eq. (4.4) into Eqs. (4.3) and (4.5). So ultimately there will be only two equations to be solved.

To obtain the solutions of Eqs. (4.3) and (4.5), a minimization technique is chosen. The ranges for \( K_1^0, \phi_1 \) and \( \phi_2 \) are defined. For each value of \( \phi_2 \), the whole range of \( \phi_1 \) is scanned. Similarly
for each value of \( \phi_1 \), the complete range of \( K_1^0 \) is scanned to find the acceptable solutions. The range of \( \phi_2 \) is chosen as \(-179^\circ \leq \phi_2 \leq 179^\circ \) with the increment of \( 2^\circ \). The range of \( \phi_1 \) is taken as \(-180^\circ \) to \( 180^\circ \) with \( 1^\circ \) interval. \( K_1^0 \) is allowed to vary between 0.01 and 5.0 with varying increment. The increment for \( K_1^0 \), between 0.01 and 0.1 is 0.01 and between 0.1 and 5.0 is 0.1.

Some important symmetric properties are investigated among the equations which may be very useful in saving computation time while obtaining solutions. These three symmetric properties may be summarized as follows.

(1) Let us consider Eq. (4.4). We may also write it as

\[
K_2^0 = \left( \frac{-\eta - m\sqrt{K_1^0}}{m} \right)^2
\]

Since \( m' \) may be either \(+1\) or \(-1\), we may also write the above equation as

\[
K_2^0 = (\eta + m\sqrt{K_1^0})^2
\]

In the above equation, if the sign of \( \eta \) and \( m \) are changed at the same time then the value of \( K_2^0 \) will remain the same. This means that the set of solutions for \(-\eta\) will remain the same as for \(+\eta\) or vice versa except that the sign of \( m \) will change.

(2) Let us examine Eq. (4.3). In this equation if the sign of \( \phi_2 \), \( \phi_1 \) and \( \beta_0 \) are changed at the same time then there will not be any change in \( U \). This indicates that if the sign of \( \phi_2 \) is changed then \( \phi_1 \) and \( \beta_0 \) will also change their sign with other values remaining the same.
(3) Lastly if we investigate Eq. (4.5), it is evident that if \( \phi_2, \phi_1, \beta_0 \) and \( \theta_1 \) change their sign simultaneously then \( W' \) will also change in sign. In conjunction with the above symmetry this will imply that if we have solutions for \( \phi_c - \theta_1 = 0 \), then we can generate the solutions for \( \phi_c + \theta_1 = 0 \) or vice versa by changing the sign of \( \phi_2, \phi_1 \) and \( \beta_0 \). It is important to remember in all the above symmetry analyses that \( \alpha_0 \) will always be positive as defined in Eq. (2.29).

The range of \( \eta \) is chosen from -3.08 to 3.08 with the increment of 0.04. Solutions are generated for \( \eta \) varying from -3.08 to 0.0 and other half solutions are quickly obtained by using the first symmetry property. Also the solutions have to be generated for five different cases corresponding to summation of five different functions in the cross section expression. Solutions are generated only for three cases and for the other two cases the solutions are obtained by use of the third symmetry property.

The strategy in solving the two equations is to take a value each for \( m, \eta \) and \( \phi_2 \) within the designated range and then search for pairs of \( \phi_1 \) and \( K_1^0 \) which will satisfy the restriction on \( \beta_0 \) as given in Eq. (2.26). As has previously been discussed for each value of \( \phi_1 \) we are scanning the complete range of \( K_1^0 \), so at one particular \( \phi_1 \) there may be many \( K_1^0 \) which will satisfy the required restrictions. Further between two consecutively obtained values of \( K_1^0 \) we test the change of sign in \( U \) and \( W \). This will ensure that there exists a solution between the two values of \( K_1^0 \). Once these conditions are met, an IMSL subroutine is used to minimize
the summation of the absolute values of \( U \) and \( W \). The returned value is accepted only if it is less than the summations of the absolute values of \( U \) and \( W \) at two values of \( K_1^0 \).

Since the increment in \( \phi_1 \) is taken as \( 1^0 \), it is observed that almost same roots are obtained for several consecutive \( \phi_1 \). These roots are very close to each other and may be understood as multiple occurring of the same roots. To avoid the multiple occurring of the same root, only one root was accepted out of all the roots having less than or equal to \( 2^0 \) difference in \( \phi_1 \).

The significance of off-patch scatter is studied for two different cases. First, when the radar is located on the beach. In this case there will not be any signal return from behind the radar. To make sure that there is no signal return from behind the radar, a restriction has been put on \( \theta_B \); that is, \( |\theta_B| < 90^0 \). Also the solutions may indicate an interaction with that area of shallow water surrounding the radar. Since it is unlikely that this is important, restrictions have been put while accepting the roots that all the three distances travelled by the signal during two scatters should individually be greater than one kilometer. These distances denoted by \( r_a, r_b \) and \( r_c \) are given in Eqs. (2.28) to (2.30).

The second case corresponds to the case when the radar is located on the open ocean (e.g., a ship or an offshore platform based radar). Now the radar is surrounded by the water and signals may be received from all directions. Imposing restrictions on \( r_b \) and \( r_c \) as they should be greater than one kilometer, it is made sure that both the scatters occur distinctly and away from the
radar. There is no restriction on $\theta_b$ in this case.

After all the restrictions are met, the final accepted solutions are used to generate the cross section. In the computation of the cross section, roots are again denormalized so that they may be used for particular radar frequency and sea state. $m'$ may take the value either +1 or -1 and it is decided depending on the values of $\omega_d$, $m$ and $K_1$. Analysis of Eq. (3.7) is used to decide the value of $m'$. From this equation it is evident that if $m$ is equal to 1 and $\omega_d$ is less than 0 and $K_1$ is less than or equal to $\omega_d^2/g$, then $m' = 1$ otherwise $m' = -1$. Likewise if $\omega_d$ is less than 0 but $K_1$ is greater than or equal to $\omega_d^2/g$, then $m' = 1$. Else if $m \neq -1$ and $\omega_d$ is greater than or equal to 0 and $K_1$ is greater than $\omega_d^2/g$, then $m' = 1$ otherwise $m' = 1$. But if $\omega_d$ is greater than 0 and $K_1$ is less than or equal to $\omega_d^2/g$ then $m' = -1$. $\sigma_{s3}'$ is computed from Eq. (4.1) in conjunction with Eq. (3.11). Eq. (2.21) is used to calculate $Q_c(K_1, K_2, \theta_e)$ and Eqs. (2.22), (3.44), (3.45) and (3.46) are used for the calculation of 'det'. Jacobian may be computed by the use of Eqs. (3.14) to (3.41). The Pierson-Moskowitz frequency spectrum with a cardiod directional distribution [Srivastava (1984)] is used to model the ocean waveheight spectrum. This directional waveheight spectrum as given in Walsh et al. (1986) may be written as

$$S(K) = \frac{0.0162 \pi}{K^4} e^{-0.74 \left( \frac{g}{KU_w} \right)^2} \cos^2 \left[ \frac{\phi - \theta_w}{2} \right]$$

(4.9)

where
\[ K = K_x + K_y, \quad \phi = \tan^{-1} \left( \frac{K_y}{K_x} \right) \]

- \( U_w \) = wind speed in meter per second
- \( \theta_w \) = wind direction
- \( g \) is the gravitational acceleration.

Also for computational ease \( \Lambda \) is used instead of the modified surface impedance \( (\Lambda_0) \) to evaluate the attenuation function in \( \sigma_{s3} \). The cross section is generated for different radar frequencies and sea states. The integral with respect to \( \phi_2 \) in Eq. (4.1) is evaluated using the rectangular rule. The distance of the patch in all cases is taken as 30 kilometers. It is verified that by changing the distance of the patch there is no significant variation in the cross section result.

In order to exhibit the significance of off-patch scatter compared to on-patch scatter, two corresponding cross sections \( \sigma_{s1} \) and \( \sigma_{s3} \) are plotted for 10 MHz and 25.4 MHz radar frequencies. The two frequencies chosen provide a well representation of the HF region. Thus any conclusion drawn at these frequencies may be applied to HF radars in general. The case when the radar is located in the open ocean is considered first. It is assumed in this case that the sea is fully developed in the total scattering region. Figures 3 to 5 show the individual spectrum of \( \sigma_{s1} \) and \( \sigma_{s3} \) for 10 MHz radar frequency and 10 knots wind speed. Wind directions are 0\(^\circ\), 45\(^\circ\) and 90\(^\circ\) (cross wind) respectively with reference to the direction of the patch. The Doppler frequencies (in Hz) corresponding to \( \pm \omega_B \) and \( \pm 2^{3/4} \omega_B \) (in rad/sec) are marked in spectral polts as "a" and "b" respectively. Examining the above plots it is clear that \( \sigma_{s3} \) is lower than \( \sigma_{s1} \) at all
Doppler points except at zero and beyond $\pm 2^{3/4} \omega_B$ (corner reflector) Doppler frequencies. In these regions $\sigma_{s3}$ is higher than $\sigma_{s1}$. Similar plots are presented in figures 6 to 8 but now the radar frequency is 25.4 MHz. In these plots $\sigma_{s3}$ is higher than $\sigma_{s1}$ only at zero, around $\pm 2^{3/4} \omega_B$ and beyond $\pm 2^{3/4} \omega_B$ Doppler frequency points. In these plots two first-order peaks at $\pm \omega_B$ which lie in the null regions are not shown. The regions around first-order peaks which are used for the extraction of ocean surface parameters are not affected by $\sigma_{s3}$. Therefore, in these regions the second-order cross section may adequately be described by $\sigma_{s1}$ alone.

Figures 9 to 11 are plotted for 10 MHz radar frequency but 30 knots wind speed. Wind directions are maintained at 0°, 45° and 90°. In these plots $\sigma_{s3}$ is significantly lower than $\sigma_{s1}$ at all Doppler points except at zero, around $\pm 2^{3/4} \omega_B$ and beyond $\pm 2^{3/4} \omega_B$ Doppler frequency points. Same conclusions may be drawn if the radar frequency is raised to 25.4 MHz and the corresponding plots are presented in figures 12 to 14. Going from 0° to 90° wind direction, the effect of $\sigma_{s3}$ at zero Doppler decreases. Referring to the above examples the contribution of $\sigma_{s3}$ is not important in the wave regions. Beyond $\pm 2^{3/4} \omega_B$ $\sigma_{s3}$ is significantly higher than $\sigma_{s1}$ but the value of $\sigma_{s3}$ itself is lower in this region. It is interesting to note that $\sigma_{s2}$ may also be present in this case as discussed in Walsh and Srivastava (1987). This term has not been included in this study. The peak at zero Doppler is the effect of double first-order scatter phenomenon.
Now we consider the case when the radar is located on the beach or near the shore. There will be a reduction in $\sigma_{s3}$ as it is impossible for any scattering to occur on the land. $\sigma_{s1}$ will be the same in this case. Figure 15 to 17 show the individual plots of $\sigma_{s1}$ and $\sigma_{s3}$ for a 10 MHz radar frequency and 10 knots wind speed. Wind directions are again $0^\circ$, $45^\circ$ and $90^\circ$ respectively. $\sigma_{s1}$ and $\sigma_{s3}$ both are lower in these cases. Comparing the two it is evident that $\sigma_{s3}$ is significantly lower than $\sigma_{s1}$ except when $|\omega_d| > \pm 2^{3/4} \omega_B$. In these regions $\sigma_{s3}$ is appreciably higher than $\sigma_{s1}$. If the radar frequency is raised to 25.4 MHz the results are almost the same except that $\sigma_{s3}$ is now higher than $\sigma_{s1}$ around $\pm 2^{3/4} \omega_B$ instead of only for $|\omega_d| > \pm 2^{3/4} \omega_B$ and these plots are shown in figures 18 to 20. If the wind speed is increased there is an expected overall increase in the two spectra and so $\sigma_{s1}$ and $\sigma_{s3}$ for a 40 MHz radar frequency at 30 knots wind speed are plotted in figures 21 to 23. The three wind directions are $0^\circ$, $45^\circ$ and $90^\circ$. $\sigma_{s3}$ can be seen to be much lower than $\sigma_{s1}$ at all Doppler points except around $\pm 2^{3/4} \omega_B$. Similar plots are shown in figures 24 to 26 but for a 25.4 MHz radar frequency. Here also $\sigma_{s3}$ is significantly lower than $\sigma_{s1}$ at all Doppler frequencies except around $\pm 2^{3/4} \omega_B$ but not at $\pm 2^{3/4} \omega_B$. In these regions it is higher than or comparable to $\sigma_{s1}$. The plots of $\sigma_{s1}$ in all the above examples are based on the computer program given in Walsh and Srivastava (1984) and have been used only to compare $\sigma_{s3}$ with $\sigma_{s1}$.
CHAPTER 5

CONCLUSIONS

The effect of second-order off-patch scatter is examined. The spectral cross-section expression of this kind of scatter is simplified to a computational form assuming a narrow beam receiving antenna. The transmitting antenna is assumed to be omni-directional. It is found that the contribution of off-patch scatter compared to on-patch scatter is effective only at zero Doppler, around \( \pm 2^{3/4} \omega_B \) and beyond \( \pm 2^{3/4} \omega_B \) frequency points. Around \( \pm 2^{3/4} \omega_B \) frequency points \( \sigma_{s3} \) is higher than or comparable to \( \sigma_{s1} \) but beyond \( \pm 2^{3/4} \omega_B \) frequency points \( \sigma_{s3} \) is significantly higher than \( \sigma_{s1} \). The Doppler regions near the first-order peaks, which are commonly used for estimation of ocean wave parameters are unaffected by off-patch scatter. Therefore, for the estimation of these parameters the second-order cross section may adequately be described by \( \sigma_{s1} \) alone. Based on these results it may be concluded that this form of scatter is not significant for the problem of extracting ocean surface parameters. However, it may be important in target detection applications when the target Doppler frequency is zero, near the "corner reflector" frequencies or beyond the corner reflector frequencies.

Future work in this area might include the effect of higher-order scatter, particularly that of third-order scatter. It is suspected that in high sea conditions the total spectral cross-section may be strongly influenced by the third-order effect. If
this is the case, then the contribution of third-order scatter must be taken into account in the design of any analysis technique to extract ocean spectral information from radar data.
FIG. 1 First-order backscattered from a surface patch for omnidirectional transmission and narrow beam reception
($\rho_o$ = distance of patch, $2\Delta \rho$ = radial width of patch, $2\Delta \phi$ = beamwidth of receiving antenna, $F; s$ = ground wave attenuation function with modified surface impedances)
FIG. 2 Three parts of the second-order backscattered from a surface patch and off the patch for omnidirectional transmission and narrow-beam reception ($p_0$ = distance of patch, $2\Delta p$ = radian width of patch, $2\Delta g$ = beamwidth of receiving antenna, $F'$'s = ground wave attenuation function with modified surface impedances)
FIG. 3 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 4 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 5 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 6 Two parts of the second-order backscattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 7 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 8 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 9 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 10 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 11 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 12 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 13 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 14 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (open sea condition).
FIG. 15 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
FIG. 16 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
FIG. 17 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow-beam reception (land based condition).
FIG. 18 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
FIG. 19 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
FIG. 20  Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
FIG. 21 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
FIG. 22 Two parts of the second-order back-scattered spectral cross-section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
FIG. 23 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
FIG. 24 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
FIG. 25 Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
FIG. 26. Two parts of the second-order back-scattered spectral cross section of the ocean surface patch for omnidirectional transmission and narrow beam reception (land based condition).
REFERENCES


Srivastava, S. K., "Scattering of high-frequency electromagnetic waves from an ocean surface: An alternative approach incorporating a dipole source", 1984, Ph.D. thesis, 305 pp., Memorial University of Newfoundland, St. John's, Canada.


Walsh, J. and Srivastava, S. K., "Model development for feasibility studies of HF radars as ice hazard remote sensors", 1984, OEIC report no. N00397, Memorial University of Newfoundland, Canada.

Walsh, J. and Srivastava, S. K., "Rough surface propagation and scatter with applications to ground wave remote sensing in an ocean environment", 1987, Proc. AGARD (NATO) 40th EPP specialist meeting, Rome, Italy, pp. 23.1-23.15.
APPENDIX

C********************************************************************
C PROGRAM ID : ROOT.FOR
C THIS PROGRAM IS USED TO GENERATE THE ROOTS USING
C NORMALIZED EQUATIONS AND ASSUMING A NARROW BEAM
C RECEIVING ANTENNA
C********************************************************************

DIMENSION FI1NEW(360), NUMK1(360)
DIMENSION AF12(2,180), AF11(2,180), NFI2(2), NFI1(2,180)
DIMENSION AK1NOT(2,180,150,2), NK1(2,180,150)
DIMENSION AMU(2,180,150,2), SWIN(2,180,150,2)
REAL MUNOT, K1NOT, K2NOT, M1, K1NEW(360,750), MM(2)
COMMON/UT, PI, M1, ETA, IFLAG, PHI2, PHI1, MUNOT, THETA1
COMMON WFUN
EXTERNAL SNOT
CALL UERSET (0, LEVOLD)
OPEN (UNIT=5, FILE='FORO12.DAT', TYPE='OLD')
READ (5, *) THETA1
READ (5, *) NNN

DO 60 IE=1, NNN
READ (5, *) A
ETA=A
TOL=0.0001
PI=4.*ATAN(1.0)
THETA1=THETA1*PI/180.

DO 50 IC=1, 2
M1=2*IC-3
MM(IC)=M1
PHI2D=-181.
IPHI2=0

DO 40 IFI2=1, 180
PHI2D=PHI2D+2.
PHI2=PHI2D*PI/180.
IFLAG=1
CALL CHANGE(FI1NEW, M1, K1NEW, NUMK1)
IFLAG=2
IFI1=0
DO 30 I=1,N1
N2=NUMK1(I)
PHI1=FI1*PI/180.
IK1=0

DO 20 J=1,N2,2
AK1=K1*(I,J)
BK1=K1*(I,J+1)
SFIRST=SNOT(AK1)
SSECOND=SNOT(BK1)
CALL ZGSHN(SNOT.AK1,BK1,TOL,KN1,IER)
XX=K1*10
SUMMIN=SNOT(XX)
IF(SUMMIN.GT.SFIRST.OR.SUMMIN.GT.SSECOND)G0 TO 20.
IF(ABS(WFUN).GT.0.1)G0 TO 20
IK1=IK1+1
IF(IPI1.EQ.1)IF11=IF11+1
IF(IPI1.EQ.1)IPHI2=IPHI2+1
AFI2(IC,IPHI2)=PHI2D
AFI1(IC,IPHI2,IF11)=PHI1*180./PI
AK1NOT(IC,IPHI2,IF11,IK1)=K1*10
AMU(IC,IPHI2,IF11,IK1)=MUNOT
SMIN(IC,IPHI2,IF11,IK1)=SUMMIN
NF11(IC,IPHI2)=IF11
NK1(IC,IPHI2,IF11)=IK1
20 CONTINUE
30 CONTINUE
40 CONTINUE
50 CONTINUE

DO 55 IC=1,2
WRITE(50,*),ETA,MM(IC),NF12(IC)

DO 46 I=1,NF12(IC)
WRITE(50,*),AFI2(IC,I),NF11(IC,I)
WRITE(50,*),(AFI1(IC,I,J),NK1(IC,I,J),AK1NOT(IC,I,J,K),
AMU(IC,I,J,K),SMIN(IC,I,J,K),K=1,NK1(IC,I,J))
1 J=1,NF11(IC,I))
45 CONTINUE
55 CONTINUE
60 STOP
END
SUBROUTINE CHANGE(FI1NEW, N1, K1NEW, NUMK1)
DIMENSION FI1NEW(360), UUFUN(360, 750), NUMK1(360)
DIMENSION WWFUN(360, 750)
REAL MUNOT, K1NOT, K2NOT, K1OLD(360, 750), K1NEW(360, 750)
COMMON/CH/ANGLE1, FFACT, UFUN
COMMON WWFUN
DEL1 = 0.01
DEL2 = 1
DEL3 = 1
N1 = 0.09/DEL1 + 1.
N2 = 0.9/DEL2
N3 = 9.0/DEL3
N12 = N1 + N2
NK1 = N12 + N3
FI1 = -181.
II = 0

DO 10 I=1, 360
FI1 = FI1 + 1.
ANGLE1 = FI1.
DELK1 = DEL1
K1NOT = 0.01 - DELK1
JJ = 0

DO 20 J = 1, NK1
K1NOT = K1NOT - DELK1
IF(J .EQ. N1) DELK1 = DEL2,
IF(J .EQ. N12) DELK1 = DEL3
SUM = SUM + K1NOT
IF(ABS(FFACT) .GT. 1.E-02) GO TO 20
JJ = JJ + 1

IF(JJ .EQ. 1) II = II + 1
FI1NEW(II) = FI1
NUMK1(II) = JJ
UUFUN(II, JJ) = UFUN
WWFUN(II, JJ) = WWFUN
K1OLD(II, JJ) = K1NOT

10 CONTINUE
20 CONTINUE
CONTINUE
CONTINUE
N1=II
II=0

DO 50 I=1,N1
N2=NUMK1(I)-1
IF(N2.EQ.0)GO TO 50
JJ=0

DO 60 J=1,N2
AMULT=UUFUN(I,J)*UUFUN(I,J+1)
AMULT2=WWFUN(I,J)*WWFUN(I,J+1)
IF(AMULT.GT.0.0.OR.AMULT2.GT.0.0)GO TO 60
JJ=JJ+2
IF(JJ.EQ.2)II=II+1
F11NEW(II)=F11NEW(I)
K1NEW(II,JJ-1)=K1OLD(I,J)
K1NEW(II,JJ)=K1OLD(I,J+1)
NUMK1(II)=JJ

CONTINUE
CONTINUE
N1=II
RETURN
END
FUNCTION SUBPROGRAM ID: SHOT

This function subprogram is required to calculate the 'UNET', 'DEUNET' and the two functions that emerge from two equations.

FUNCTION SHOT(XX)

DIMENSION ACHMU(2), AMU(4)
REAL M1, K1, K1X, K1Y, K2, K2X, K2Y, MU, GC, GCX, GCY
COMMON/VIU/PI, M1, ETA, IFLAG, PI2, PHI1, MU, THETA1,
COMMON/CH/ANGLE1, FFACT, UFUN,
COMMON WFUN
IF (IFLAG.EQ.1) PI1 = ANGLE1*PI/180.0
IF (IFLAG.EQ.2) PI1 = PHI1
SNOT = 1.0E26
UFUN = SNOT
WFUN = SNOT
FFACT = 2.0
K1 = XX
CFI2 = COS(PI2)
SFI2 = SIN(PI2)
CFI1T = COS(PI1)
SFI1T = SIN(PI1)
SFI1T = SFI1T * CFI1T
TFI2T = SFI2T * CFI2T

FACT = 4.0 * K1 * CFI1T * (K1 * CFI1T + K2 * CFI2T)
B = -2.0 * (K2 * CFI2T) ** 2
A = FACT * B / 2.0
C = -SFI2T * SFI2T * FACT * B / 2.0
IF (A.EQ.0.0 .AND. B.EQ.0.0) GOTO 40
IF (B.EQ.0.0 .AND. C.EQ.0.0) GOTO 40
DISC = B * B - 4.0 * A * C
IF (DISC.LT.0.0) GOTO 40
ACHMU(1) = 0
ACHMU(2) = 0
IF (A.EQ.0.0) THEN
ACHMU(1)=C/B
ELSE IF (DISC.EQ.0.0) THEN
ACHMU(1)=B/(2.0*A)
ELSE
ACHMU(1)=(-B-SQRT(DISC))/(2.0*A)
ACHMU(2)=(-B-SQRT(DISC))/(2.0*A)
END IF
MU=0
AMU(1)=0
AMU(2)=0
AMU(3)=0
AMU(4)=0
NUMMU=0
CSGN=1
DO 50 IM=1,2
X=ACHMU(IM)
NUMMU=NUMMU+2
IF (X.LE.1.0) GOTO 50
AMU(NUMMU)=ALOG(X-SQRT(X*X-1.0))
AMU(NUMMU-1)=-AMU(NUMMU)
CONTINUE

50 DO 60 IN=1,4
IF (AMU(IN).EQ.0.0) GOTO 60
X=AMU(IN)
SHX=SINH(X)
CHX=COSH(X)
THX=SHX/CHX
FFACT=(2.0*K1*CFI1T+K2*CFI2T)*CFI2T*CHX*SQRT(1.0+
(THX*CFI2T)**2)-K2*SIGN(1.0,SHX*CFI2T)*CFI2T**2/CHX)
IF (ABS(FFACT).GT.1.E-2) GOTO 60
MU=X
CHMU=CHX
SHMU=SHX
IF (IN.GT.2) CSGN=-1
GOTO 65

60 CONTINUE

65 IF (MU.EQ.0.0) GOTO 40
DEL=ATAN2(SHMU*CFI2T,CHMU*CFI2T)
IF (DEL.GT.-PI.AND.DEL.LT.0.0) DEL=DEL+PI
CDEL=COS(DEL)

SDEL = 6IN (DEL)
KCX = 2.0 * K1X + K2X * (1.0 + CHMU * SDEL) + K2Y * SHMU * SDEL
KCY = 2.0 * K1Y + K2Y * (1.0 + CHMU * SDEL) - K2X * SHMU * SDEL
KC = SQRT (KCX**2 + KCY**2)
UFUN = KC / (1.0 + CHMU) - 1.0
WFUN = ATAN2 (KCY, KCX) - THETAI
IF (IFLAG .EQ. 1) RETURN
SNUT = ABS (UFUN) + ABS (WFUN)
RETURN
END
80
'THETA1' ! ANY VALUE OUT OF FIVE
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-3.040000
-3.000000
-2.960000
-2.920000
-2.880000
-2.840000
-2.800000
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-2.680000
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C******************************************************************************
C  PROGRAM ID : INT.FOR
C  THIS PROGRAM SORTS OUT THE MULTIPLE OCCURING
C  OF SAME ROOTS AND CHECKS THE RESTRICTIONS FOR
C  BEACH AND OPEN OCEAN CASES
C******************************************************************************

DIMENSION AFI2(2,90),AFI1(2,90,150)
DIMENSION NF12(2),NF11(2,90)
DIMENSION AK1NOT(2,90,150,2),SMIN(2,90,150,2)
DIMENSION NK1(2,90,150),AMU(2,90,150,2),KNT(90)
DIMENSION RN(2,90,150,2),RBN(2,90,150,2)
DIMENSION RCN(2,90,150,2),THETAB(2,90,150,2)
DIMENSION JTEMP(6)
REAL MM(2),K1NOT,K2NOT,KCY,KCX,K1X,K2X,K1Y,K2Y

C ENTER (0 OR 1) BEACH OR OPEN OCEAN ? 10 FOR OPEN OCEAN

READ(5,*)LOC
OPEN(UNIT=14,FILE="ROOT.DAT",TYPE='OLD')

C NO OF NORMALIZED DOPPLER POINTS
READ(14,*)NN
PI=4.*ATAN(1.)
OPEN(UNIT=77,FILE="INT.DAT",TYPE='NEW')
WRITE(77,*)NN
ROW=30.

DO 5 NV=1,NN
DO 10 IC=1,2
READ(14,*)ETA,MM(IC),NF12(IC)

DO 20 I=1,NF12(IC)
READ(14,*)AFI2(IC,I),NF11(IC,I)
READ(14,*)SMIN(IC,I,J),NK1(IC,I,J);(AK1NOT(IC,I,J,K))
1 AMU(IC,I,J,K),SMIN(IC,I,J,K),K=1,NK1(IC,I,J)
J=1,NF11(IC,I)
20 CONTINUE
10 CONTINUE

C TO SORT OUT THE MULTIPLE ROOTS

DO 11 IC=1,2
IF(NF12(IC).EQ.0)GO TO 11

C******************************************************************************
DO 12 I=1,MFI2(IC)
MFI1=1
JTEMP(1)=1
IF(MFI1(IC,I).EQ.1)GO TO 12
SMALL=SMIN(IC,I,1,1)

DO 13 J=2,MFI1(IC,I)
IF((AFI1(IC,I,J)-AFI1(IC,I,J-1)).LT.2.1)THEN
IF(SMIN(IC,I,J,1).LT.SMALL)THEN
SMALL=SMIN(IC,I,J,1)
JTEMP(MFI1)=J
ELSE
END IF
ELSE
MFI1=MFI1+1
JTEMP(MFI1)=J
SMALL=SMIN(IC,I,J,1)
END IF
13 CONTINUE
NFI1(IC,I)=MFI1

DO 14 J=1,MFI1
JJ=JTEMP(J)
AFI1(IC,I,J)=AFI1(IC,I,JJ)
NK1(IC,I,J)=NK1(IC,I,JJ)
AKINOT(IC,I,J,1)=AKINOT(IC,I,JJ,1)
AMU(IC,I,J,1)=AMU(IC,I,JJ,1)
SMIN(IC,I,J,1)=SMIN(IC,I,JJ,1)
14 CONTINUE
12 CONTINUE
11 CONTINUE

C CHECKING THE RESTRICTIONS

DO 30 IC=1,2
IF (MFI2(IC).LE.0) THEN
WRITE(77,*)ETA,MM(IC),MFI2(IC)
GOTO 30
END IF
MFI2=0
DO 40 I=1,MFI2(IC)
KNT(I)=0
DO 50 J=1,MFI1(IC,I)
K=1
K1NOT=AK1NOT(IC, I, J, K)
K2NOT=(SQRT(K1NOT)+MM(IC)*ETA)**2
CHMU=COSH(AMU(IC, I, J, K))
SHMU=SINH(AMU(IC, I, J, K))
SFI1T=SIN(AF1(IC, I, J)*PI/180.)
CFI1T=COS(AF1(IC, I, J)*PI/180.)
SFI2T=SIN(AF2(IC, I)*PI/180.)
CFI2T=COS(AF2(IC, I)*PI/180.)
DEL=ATAN2(SHMU*SFI2T, CHMU*CFI2T)
IF (DEL.GT.PI.AND.DEL.LT.0.) DEL=DEL+PI
CDEL=COS(DEL)
SDEL=SIN(DEL)
SIB=ATAN2(SHNU+SDEL, 1.+CHMU+CDEL)
SIB=SIB*180./PI
K1X=K1NOT*CFI1T
K1Y=K1NOT*SFI1T
K2X=K2NOT*CFI2T
K2Y=K2NOT*SFI2T
KCX=2.0*K1X+K2X*(1.0+CHMU+CDEL)+K2Y*SHMU+SDEL
KCY=2.0*K1Y+K2Y*(1.0+CHMU+CDEL)-K2X*SHMU+SDEL
FIC=ATAN2(KCY, KCX)
FIC=FIC*180./PI
DEN=1.+CHMU
RAN(IC, I, J, K)=2./DEN
RBN(IC, I, J, K)=(CHMU+CDEL)/DEN
RCN(IC, I, J, K)=(CHMU-CDEL)/DEN
THETAB(IC, I, J, K)=SIB+FIC
IF(LOC.EQ.0) THEN
IF (RBN(IC, I, J, K).GT.1./ROW.AND.RCN(IC, I, J, K).GT.1./ROW.AND.
11./ROW) GO TO 50
ELSE
IF (RBN(IC, I, J, K).GT.1./ROW.AND.ABS(THETAB(IC, I, J, K))
11.LT.90.0.0.AND.RCN(IC, I, J, K).GT.1./ROW.AND.
11.RAN(IC, I, J, K).GT.1./ROW) GO TO 50
END IF
AF11(IC, I, J)=1000.
KNT(I)=KNT(I)+1
CONTINUE
IF (MFI1(IC, I).EQ.KNT(I)) THEN
AF12(IC, I)=1000.
MFI2=MFI2+1
ELSE
END IF
CONTINUE
WRITE(77,*),MM(IC),NFI2(IC)-NFI2
DO 200 I=1,NFI2(IC)
IF (AFI2(IC,I) .LT. 999.) THEN
WRITE(77,*),AFI2(IC,I),NFI1(IC,I)-KNT(I)
DO 300 J=1,NFI1(IC,I)
K=1
IF (AFI1(IC,I,J) .LT. 999.) THEN
WRITE(77,*),AFI1(IC,I,J),NK1(IC,I,J),AK1NOT(IC,I,J,K),
1 AMU(IC,I,J,K),SMIN(IC,I,J,K)
END IF
300 CONTINUE
END IF
200 CONTINUE
30 CONTINUE
5 CONTINUE
CLOSE UNIT=77
CLOSE UNIT=15
STOP
END
C******************************************************************************
C PROGRAM ID : CROSS.FOR
C THIS PROGRAM IS USED TO COMPUTE CROSS SECTION
C OF O/F-PATCH SCATTER BASED ON THE ROOTS WHICH
C ARE OBTAINED FROM THE SOLUTIONS OF TWO EQUATIONS
C******************************************************************************

IMPLICIT REAL*8 (A-H, O-Z)
REAL*4 AFNORT
REAL*8 KR, MU
COMPLEX*8 CMPLX, PE
COMPLEX*16 NUMD, DCMPLEX
DIMENSION DOLPER(300), SIGS3(300)
COMMON PI, GA, N
COMMON/THREE/PE, WB, KR,
COMMON/SPECN/U, THETA
C FREQUENCY (MHz)?
READ(7,*) FREQ
C WIND SPEED (KNOT)?
READ(7,*) U
C WIND DIRECTION (DEG)?
READ(7,*) THETA
C DISTANCE OF THE PATCH (KM)?
READ(7,*) ROW
ROW=ROW*1000. DO
PI=4. DO*DATAN(1. D0)
GA=9. 81 D0
KR=FREQ*PI/150. D0
WB=DSQRT(2. D0*GA*KR)
U=U* 0. 5148 D0
THETA=THETA*PI/180. D0
NUMD=DCMPLEX(8. D1, -7. 2 D4/FREQ)
NUMD=DCMPLEX(0. D0, -0. 5 D0)*KR*ROW/NUMD
PE=CMPLX(NUMD)
CS3=REAL(FNORT(PE))
CS3=4. D0*KR/(PI**2)/(CS3**4)
CS3=CS3/6. 0
CALL FUNC3N (DOLPER, SIGS3)
WRITE(7,*) N-1
NP=N/2

DO 25 I=NP+1, NP+(NP+1)/2
TEMP1=DOLPER(I)
TEMP2=SIGS3(I)
DOPLER(I) = DOPLER(3*NP+1-I)
SIGS3(I) = SIGS3(3*NP+1-I)
DOPLER(3*NP+1-I) = TEMP1
SIGS3(3*NP+1-I) = TEMP2
25 CONTINUE

DO 30 I = 1, 2*NP
SIGS3(I) = SIGS3(I)*CS3
DOPLER(I) = DOPLER(I)/2./PI
IF(I.EQ.NP) GO TO 30
WRITE(7,2) DOPLER(I), SIGS3(I)
2 FORMAT(/2E13.5)
30 CONTINUE
STOP
END
C****************************************************************************************************
C SUBROUTINE BUILD R \nc3n
C****************************************************************************************************

SUBROUTINE FUNC3N(DOPLER, SUMT)
IMPLICITREAL*8 (A-H, O-Z)
REAL*8 K1, K1X, K1Y, K1S, K1F, K1INC, K2, K2X, K2Y, KR, MU, NU,
+ KC, KCV, KCy, JACOB
REAL*4 FNORT
COMPLEX*8 PE, PEA, PEB, PEC
DIMENSION WM1 (2), NF12(2), FI2D(2,150), NF11(2,150)
DIMENSION SUM(2,150), FI1D(2,150,150).
DIMENSION ANOT(2,150,150,4), AMU(2,150,150,4)
DIMENSION ACSSN(2,150,150,4), NK1(2,150,160), SUMT(300)
DIMENSION DOPLER(300)
COMMON PI, GA, NN
COMMON/THREE/PE, WB, KR

C FOLLOWING FIVE DATA FILES CORRESPOND TO THE SOLUTIONS:
C CORRESPONDING TO FIVE DIFFERENT VALUES OF 'THETAI'

OPEN (UNIT=1,FILE='DATA00.DAT', TYPE='OLD')
OPEN (UNIT=2,FILE='DATA10.DAT', TYPE='OLD')
OPEN (UNIT=3,FILE='DATA20.DAT', TYPE='OLD')
OPEN (UNIT=4,FILE='DATA21.DAT', TYPE='OLD')
OPEN (UNIT=5,FILE='DATA22.DAT', TYPE='OLD')
K=1

DO 3 ITT=0,5

3 SUMT(ITT)=0.0

C NO OF NORMALIZED FREQUENCY POINTS ?
READ (NF,*)NP
NN=2*NP
SWITCH=1.DO
END IF

IF (ITT.GE.(NP+1)) SWITCH=-1.DO.

IF (ITT.EQ.(NP+1)) THEN
REWIND NF
READ (NF,*)NP
ELSE
END IF
IF (ITT, GT, NN) GOTO 130

DO 10 IC=1, 2
READ (NF, *) ETA, MM1 (IC), NF12 (IC)
ETA = ETA * SWITCH
WD = ETA * WB
DOPLER (ITT) = WD
MM1 (IC) = MM1 (IC) * SWITCH
IF (NF12 (IC), LE, 0) GOTO 10

DO 20 I = 1, NF12 (IC)
READ (NF, *) FI2D (IC, I), NF11 (IC, I)

DO 15 J = 1, NF11 (IC, I)
READ (NF, *) FI1D (IC, I, J), MK1 (IC, I, J), AK1NOT (IC, I, J, K), AMU (IC, I, J, K), ACSGN (IC, I, J, K)
15 CONTINUE
20 CONTINUE
10 CONTINUE

DO 50 IC=1, 2
IF (NF12 (IC), EQ, 0) GOTO 50
M1 = MM1 (IC)

DO 60 I = 1, NF12 (IC)
FI2 = FI2D (IC, I) * PI / 180. DO
SUM (IC, I) = 0. DO

DO 70 J = 1, NF11 (IC, I)
FI1 = FI1D (IC, I, J) * PI / 180. DO
K1 = AK1NOT (IC, I, J, K) * 2. DO * KR
CSGN = ACSGN (IC, I, J, K)
MU = AMU (IC, I, J, K)
IF (M1, EQ, 1) THEN
IF (.WD.LT.0. DO.AND.K1.LE.(WD*WD/GA)) M2 = 1
IF (.WD.GE.0. DO) M2 = -1
IF (.WD.LT.0. DO.AND.K1.GE.(WD*WD/GA)) M2 = -1
ELSE IF (M1, EQ, -1) THEN
IF (.WD.GE.0. DO.AND.K1.GT.(WD*WD/GA)) M2 = 1
IF (.WD.LT.0. DO) M2 = 1
IF (.WD.GT.0. DO.AND.K1.LE.(WD*WD/GA)) M2 = -1
END IF
CFI2 = DCOS (FI2)
SFI2 = DSIN (FI2)
CFI2T=DCOS(FI2)
SFI2T=DSIN(FI2)
TFI2T=SFI2T/CFI2T
CHMU=DCOSH(MU)
SHMU=DSINH(MU)
CFI1=DCOS(FI1)
SFI1=DSIN(FI1)
CFI1T=DCOS(FI1)
SFI1T=DSIN(FI1)
K2=(DSQRT(K1)*M1+WD/3.132092DO)**2
K1X=K1*CFI1
K1Y=K1*SFI1
K2X=K2*CFI2
K2Y=K2*SFI2
DEL=DATAN2(SHMU+SFI2T,CHMU+CFI2T)
IF (DEL. GT. -PI. AND. DEL. LT. 0. DO) DEL=DEL+PI
CDE1=DCOS(DEL)
SDE1=DSIN(DEL)
KCX=2.0*K1X+K2X*(1.0+CHMU+CDE1)+K2Y*SHMU+SDE1
KCY=2.0*K1Y+K2Y*(1.0+CHMU+CDE1)-K2X*SHMU+SDE1
KC=DSQRT(KC**2+KCY**2)
PIC=DATAN2(KCY, KCX)
CFI1C=DCOS(FI1-FIC)
SFI1C=DSIN(FI1-FIC)
CFI2C=DCOS(FI2-FIC)
SFI2C=DSIN(FI2-FIC)

GMU1=(((CHMU-2.0D0)+(2.0D0*K1*CFI1C+K2*CFI2C*(1.0D0-CDE1))
-K2*SHMU+SDE1+SFI2C*/((1.0D0+CHMU)**2)

GMU2=K2*(((1.0D0+CHMU)*CDE1+SFI2C-SHMU+SDE1+CFI2C)/
((1.0D0+CHMU)**2)

CDE2=K2*(SHMU+SDE1+SFI2C+CHMU+CDE1+CFI2C)/(1.0D0+CHMU)
DET=DABS(GMU1+CDE2-GMU2)**2)

PCK1=((4.0D0*K1*CFI1T*(K1*CFI1T+K2*CFI2T)-(K2*SFI2T)**2)
+(CSGN+DSIGN(1.0D0, 2.0D0*K1*CFI1T+K2*CFI2T)*1.0D0/DSQRT(K1)
+0.0D0*K1*CFI1T+K2*CFI2T)*CFI1T*(K1*CFI1T+K2*CFI2T)
+0.0D0*K1*CFI1T+K2*CFI2T)*CFI1T*(K1*CFI1T+K2*CFI2T)
+DSQRT(CFI1T*(K1*CFI1T+K2*CFI2T)))**2)
+DSQRT(CFI1T*(K1*CFI1T+K2*CFI2T))-(K2*K2*DABS(SFI2T)+CSGN
+2.0D0+DSQRT(K1))*DABS(2.0D0*K1*CFI1T+K2*CFI2T)*DSQRT(CFI1T
+*(K1*CFI1T+K2*CFI2T))**4)
+DABS(SFI2T)/(4.0D0*K1*CFI1T*(K1*CFI1T+K2*CFI2T)-(K2*
+SFI2T)**2)**2
PCX2 = ((4. D0 + K1 + CFI1T + K2 + CFI2T) - (K2 + SF12T))/2)
+ (2. D0 + K2 + DABS (SF12T) + CSGN + 2. D0 + DSIGN (1. D0, 2. D0 + K1 + CFI1T
+ <K2 + CFI2T > + DSQR (K1) + CFI2T + DSQR (CFI1T + (K1 + CFI1T + K2 +
+ CFI2T) + CSGN + DSQR (K1) + DABS (2. D0 + K1 + CFI1T + K2 + CFI2T) + CFI1T
+ *CFI2T / DSQR (CFI1T + (K1 + CFI1T + K2 + CFI2T)) - (K2 + K2 + DABS
+ (SF12T) + CSGN + 2. D0 + DSQR (K1) + DABS (2. DO + K1 + CFI1T + K2 + CFI2T)
+ *DSQR (CFI1T + (K1 + CFI1T + K2 + CFI2T))
+ *(4. D0 + K1 + CFI1T + CFI2T) - 2. D0 + K2 + SF12T + SF12T))/2)
+ (4. D0 + K1 + CFI1T + (K1 + CFI1T + K2 + CFI2T) - (K2 + SF12T)) / 2)
+ (K2 + SF12T))/2)

PCFI1 = ((4. D0 + K1 + CFI1T + (K1 + CFI1T + K2 + CFI2T) - (K2 + SF12T))/2)
+ (*-CSGN + 4. D0 + DSIGN (1. D0, 2. DO + K1 + CFI1T + K2 + CFI2T) * K1 + DSQR
+ (K1 + SFI1T + CFI1T + (K1 + CFI1T + K2 + CFI2T) - CSGN + DSQR (K1) + DABS
+ (2. D0 + K1 + CFI1T + K2 + CFI2T) + SF11T + (2. D0 + K1 + CFI1T + K2 + CFI2T)
+ /DSQR (CFI1T + (K1 + CFI1T + K2 + CFI2T)) + (K2 + K2 + DABS (SF12T) + CSGN
+ *2. D0 + DSQR (K1) + DABS (2. DO + K1 + CFI1T + K2 + CFI2T) + DSQR (CFI1T
+ + (K1 + CFI1T + K2 + CFI2T)) + *4. D0 + K1 + SF11T + (2. D0 + K1 + CFI1T + K2
+ + CFI2T) + DABS (SF12T) + (4. D0 + K1 + CFI1T + (K1 + CFI1T + K2 + CFI2T) -
+ (K2 + SF12T) / 2)) / 2)

PMUK1 = PCK1 / SHMU
PMUK2 = PCK2 / SHMU
PMUF11 = PCFI1 / SHMU
FACT = TFI2T / (CHMU + 2 + (SHMU * TFI2T) / 2)
PDELK1 = FACT * PMUK1
PDELK2 = FACT * PMUK2
PDELFI = FACT * PMUF11
CFI12 = DCOS (FI2 - FI1)
SF12 = DSIN (FI2 - FI1)
PCK1 = (4. D0 + K1 + K2 + K2 + (CHMU + CDEL) + (SHMU + PMUK1 - SDEL -
+ PDELK1)).2. D0 + K2 + ((1. D0 + CHMU + CDEL) + CFI21 + SHMU + SDEL + SF121)
+ + 2. D0 + K1 + K2 + (CFI21 + (SHMU + CDEL + PMUK1 - CHMU + CDEL + PDELK1))
+ + SF121 + (CHMU + SDEL + PMUK1 - SHMU + CDEL + PDELK1)) / KC

PK2 = (K2 + (CHMU + CDEL) + (CHMU + CDEL + K2 + (SHMU + PMUK2 - SDEL -
+ PDELK2)) + 2. D0 + K1 + (((1. D0 + CHMU + CDEL) + CFI21 + SHMU + SDEL + SF121)
+ + 2. D0 + K1 + K2 + (CFI21 + (SHMU + CDEL + PMUK2 - CHMU + SDEL + PDELK2))
+ + SF121 + (CHMU + SDEL + PMUK2 - SHMU + CDEL + PDELK2)) / KC

PKCFI1 = (K2 + K2 + (CHMU + CDEL) + (SHMU + PMUF11 - SDEL + PDELFI)
+ + 2. D0 + K1 + K2 + ((1. D0 + CHMU + CDEL) + SF121 - SHMU + SDEL + CFI21)
+ + 2. D0 + K1 + K2 + (CFI21 + (SHMU + CDEL + PMUF11 - CHMU + SDEL + PDELFI)
+ + SF121 + (CHMU + SDEL + PMUF11 - SHMU + CDEL + PDELFI)) / KC
PFICK1 = (2.0 * SFI1C + K2 * SFI2C + (SHMU * CDE1 * PMUK1 - CHMU * SDE1 * PDEIK1) - K2 * CFI2C + (SHMU * SDE1 * PMUK1 + SHMU * CDE1 * PDEIK1)) / KC

PFICK2 = (SFI2C + (1.0 * CHMU * CDE1 + K2 * (SHMU * CDE1 * PMUK2 - CHMU * SDE1 * PDEIK2)) - CFI2C + (SHMU * SDE1 + K2 * (CHMU * SDE1 * PMUK2 + SHMU * CDE1 * PDEIK2))) / KC

PFICFI = (2.0 * K1 * CFI1C + K2 * SFI2C + (SHMU * CDE1 * PMUF11 - CHMU * SDE1 * PDELF11) - K2 * CFI2C + (CHMU * SDE1 + SHMU * CDE1 * PDELF11)) / KC

PUK1 = (PKCI1 - KC * SHMU * PMUK1 / (1.0 + CHMU)) / (1.0 + CHMU)
PUK2 = (PKCI2 - KC * SHMU * PMUK2 / (1.0 + CHMU)) / (1.0 + CHMU)
PUIFI1 = (PKCFI1 - KC * SHMU * PMUF11 / (1.0 + CHMU)) / (1.0 + CHMU)

JACOB = 1.556046D0 + DABS (M1 * (PUIFI1 * PFICK2 - PUK2 * PFICFI) / DSQRT (K1 + M2 * (PUIFI1 * PFICFI - PUIFI1 * PFICK1) / DSQRT (K2)))

PEA = PE + 2.0 / (1.0 + CHMU)
PEB = PE + (CHMU * CDE1) / (1.0 + CHMU)
PEC = PE + (CHMU * CDE1) / (1.0 + CHMU)
ATTN = DBLE (FNORT (PEA) + FNORT (PEB) + FNORT (PEC))
QC = KR * K1 * CFI1T + (KR * (2.0 * K1 + CFI1T + K2 * CFI2T) - 2.0 * K1 + K1 + K2) + K2 * 3.0 * K1 * K2 * CFI2T) + K1 * K1 * (K1 * K1 + K2 + 2.0 * K1 + K2) + CFI21) - KR * K1 * K1 * (K1 + CFI1T + K2 + CFI2T)
FUNC1 = (ATTN * QC / KC) ** 2 * K1 * K2 / DET
FUNC = FUNC1 / JACOB
CALL PMSPEC (M1 * K1X, M1 * K1Y, S1)
CALL PMSPEC (M2 * K2X, M2 * K2Y, S2)
FUNC = FUNC + S1 + S2
SUM (IC, I) = SUM (IC, I) + FUNC

CONTINUE
70 CONTINUE
60 CONTINUE
50 CONTINUE

C RECTANGULAR INTEGRATION OF THE MATRIX SUM

DO 80 IC = 1, 2
IF (NFI2 (IC) .EQ. 0) GOTO 80
DO 90 I = 1, NFI2 (IC)
SUM (ITT) = SUM (ITT) + (2.0 * PI / 180.0 * DO) * SUM (IC, I)
90 CONTINUE
80 CONTINUE
3 CONTINUE
SUBROUTINE PMSPEC(KX, KY, S)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 KX, KY, K
COMMON PI
COMMON /SPECN/ U, THETAW
S = 0
K = DSQRT(KX**2 + KY**2)
IF (K.LE.1.D-9) RETURN
FI = Datan2(KY, KX)
S = 1.62D-2 * PI * DEXP(-0.74D0 * (9.81D0 / (K * U * U)) ** 2) / (K ** 2)
S = S * (DCOS((THETAW - FI) / 2.D0)) ** 2
RETURN
END
C FUNCTION SUBPROGRAM ID: FNORT
C USED TO CALCULATE NUMERICAL DISTANCE
C

REAL FUNCTION FNORT(ND)
COMPLEX F, ND, P, R, L, CEXP, CSQRT, SECT, COMPLX
REAL MAGP, A, B, C, PI, CABS, CNT
P=ND
F=COMPLX(0.0, 0.0)
CNT=1.0
R=COMPLX(0.0, 1.0)
PI=4.0*ATAN(1/0)
MAGP=CABS(P)
IF(MAGP.LE.10.0) GO TO 300
A=REAL(P)
B=AIMAG(P)
C=ATAN2(B, A)
IF(C.LT.0.0) GO TO 310
C ASYMPTOTIC EXPANSION FOR C.GE.0
C EXTRA TERM FOR C.GE.0
F=-2.0*R*CSQRT(PI*P)*SCSX(-P)
C CONTINUE ASYMPTOTIC EXPANSION FOR ALL P
310 L=1.0
CNT=1.0
305 L=L+CNT/(2.0*P)
F=F-L
CNT=CNT+2.
T=CABS(L)
IF(T.GT.1.0E-04) GO TO 305
GO TO 320
C CONVERGENT SERIES
300 F=1-R*CSQRT(PI*P)*SCSX(-P)
CNT=1.0
L=1.0
330 F=((1.0)*L*(2.0*P))/CNT.
F=F+L
CNT=CNT+2.
T=CABS(L)
IF(T.GT.1.0E-04) GO TO 330
320 FNORT=CABS(F)
RETURN
END
CFUNCTION SUBPROGRAM ID : SCEX
C

COMPLEX FUNCTION SCEX(ARGU)
COMPLEX ARG, CEXP, CMPLX, ARGU
REAL MAG
ARG=ARGU
SCEX=CMPLX(1.0, 0.0)
ICNT=0
MAG=CABS(ARG)
IF(MAG.LT.80.0) GO TO 400
ARG=ARG/2.0
ICNT=ICNT+2
MAG=CABS(ARG)
IF(MAG.LT.80.0) GO TO 402
GO TO 403
CONTINUE
DO 404 J=1, ICNT
SCEX=SCEX*(CEXP(ARG))
GO TO 401
400 SCEX=CEXP(ARG)
401 RETURN
END
PROGRAM ID: SYMROOT.FOR

PROGRAM USED TO OBTAIN THE ROOTS WHEN 'THETA' IS CHANGED IN SIGN. BASED ON SYMMETRY PROPERTY.

DIMENSION AFI2(2,150),AFI1(2,150,150),NFI2(2)
DIMENSION AK1NOT(2,150,150,2),NK1(2,150,150)
DIMENSION NFI1(2,150),MM(2)
DIMENSION AMU(2,150,150,2),SMIN(2,150,150,2)
CHARACTER*12 FILENAME
WRITE(6,'(A)')'ENTER FILE NAME'
READ(5,19)FILENAME
19 FORMAT(A12)
K=1
OPEN(UNIT=41,NAME=FILENAME,TYPE='OLD')
READ(41,*)NP
WRITE(44,*)NP
DO 5 IETA=1,NP
DO 10 IC=1,2
READ(41,*)ETA,MM(IC),NFI2(IC)
IF(NFI2(IC).LE.0)GO TO 10
DO 20 I=1,NFI2(IC)
READ(41,*)AFI2(IC,I),NFI1(IC,I)
DO 30 J=1,NFI1(IC,I)
READ(41,*)AFI1(IC,I,J),NK1(IC,I,J),AK1NOT(IC,I,J,K),
AMU(IC,I,J,K),SMIN(IC,I,J,K)
30 CONTINUE
20 CONTINUE
10 CONTINUE

GENERATION OF ROOTS

DO,11 IC=1,2
IF(NFI2(IC).NE.0)THEN

DO,12 I=1,NFI2(IC)
AFI2(IC,I)=-AFI2(IC,I)

DO,13 J=1,NFI1(IC,I)
AFI1(IC,I,J)=-AFI1(IC,I,J)
AMU(IC,I,J,K)=-AMU(IC,I,J,K)
13 CONTINUE
C REARRANGING THE ROOTS
C  TER IS JUST A DUMMY VARIABLE

DO 16: IC=1,2
DO 14 I=1,NFI2(IC)
IF(NFI1(IC,I).LE.1) GO TO 14

DO 15 J=1,NFI1(IC,I)/2
   TER=AFI1(IC,I,J)
   AFI1(IC,I,J)=AFI1(IC,I,NFI1(IC,I)+1-J)
   AFI1(IC,I,NFI1(IC,I)+1-J)=TER
   NK1(IC,I,J)=NK1(IC,I,NFI1(IC,I)+1-J)
   NK1(IC,I,NFI1(IC,I)+1-J)=TER
   AK1NOT(IC,I,J,K)=AK1NOT(IC,I,NFI1(IC,I)+1-J,K)
   AK1NOT(IC,I,NFI1(IC,I)+1-J,K)=TER
   AMU(IC,I,J,K)=AMU(IC,I,NFI1(IC,I)+1-J,K)
   AMU(IC,I,NFI1(IC,I)+1-J,K)=TER
   Smin(IC,I,J,K)=Smin(IC,I,NFI1(IC,I)+1-J,K)
   Smin(IC,I,NFI1(IC,I)+1-J,K)=TER
   CONTINUE
   IF(NFI1(IC,I).LE.1) GO TO 16

DO 50 I=1,NFI2(IC)/2
   TER=AFI2(IC,I)
   AFI2(IC,I)=AFI2(IC,NFI2(IC)+1-I)
   AFI2(IC,NFI2(IC)+1-I)=TER
   NFI1(IC,I)=NFI1(IC,NFI2(IC)+1-I)
   NFI1(IC,NFI2(IC)+1-I)=TER

DO 61 J=1,60
   TER=AFI1(IC,I,J)
   AFI1(IC,I,J)=AFI1(IC,NFI2(IC)+1-I,J)
   AFI1(IC,NFI2(IC)+1-I,J)=TER
TER = NK1(IC, I, J)
NK1(IC, I, J) = NK1(IC, NFI2(IC) + 1-I, J)
NK1(IC, NFI2(IC) + 1-I, J) = TER
TER = AK1NOT(IC, I, J, K)
AK1NOT(IC, I, J, K) = AK1NOT(IC, NFI2(IC) + 1-I, J, K)
AK1NOT(IC, NFI2(IC) + 1-I, J, K) = TER
TER = AMU(IC, I, J, K)
AMU(IC, I, J, K) = AMU(IC, NFI2(IC) + 1-I, J, K)
AMU(IC, NFI2(IC) + 1-I, J, K) = TER
TER = SMIN(IC, I, J, K)
SMIN(IC, I, J, K) = SMIN(IC, NFI2(IC) + 1-I, J, K)
SMIN(IC, NFI2(IC) + 1-I, J, K) = TER
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
STOP
END