

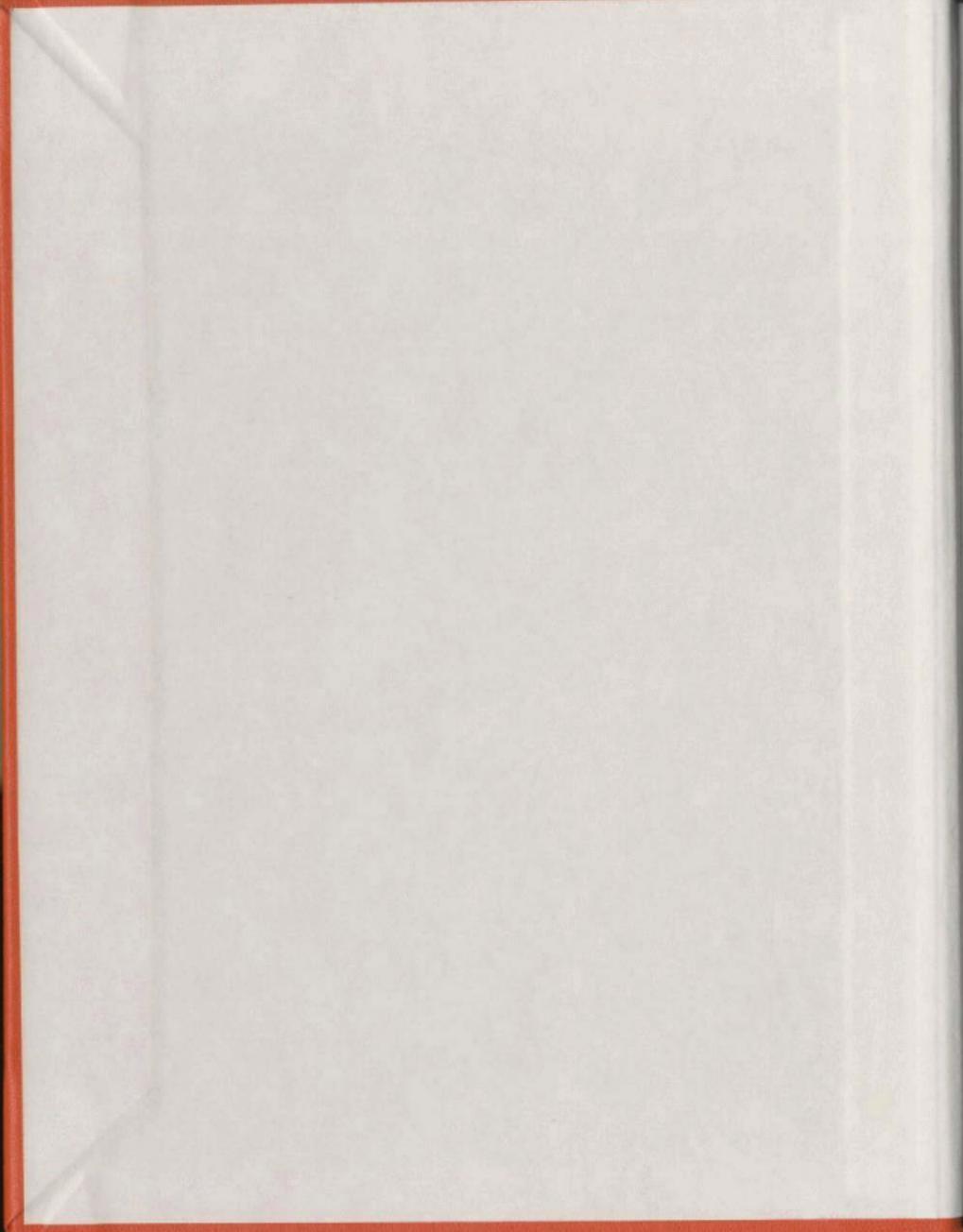
PARAMETER ESTIMATION AND OPTIMIZATION FOR  
ACTIVE AND ACTIVE-REACTIVE DISPATCH IN  
ELECTRIC POWER SYSTEMS

CENTRE FOR NEWFOUNDLAND STUDIES

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PARTERESTIMATION AND OPTIMIZATION  
FOR  
ACTIVE AND ACTIVE-REACTIVE DISPATCH  
IN ELECTRIC POWER SYSTEMS

by

C

Sabah Yacoub Mansour, B.Eng.

A thesis submitted in partial fulfillment  
of the requirements for the degree of  
Master of Engineering

Faculty of Engineering and Applied Science  
Memorial University of Newfoundland

December, 1979

St. John's

Newfoundland

**TO MY FAMILY**

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Judging the merit of each of the estimation techniques as applied to our power system problem, as it is clear that different parameters

The optimization procedure is intended as an extra measure for

For active-reactive power generation of the dispatching system, making use of the models with the distributed generation system

is concerned with the problem of optimally allocating the active and

algorithms are applied to the power system problem and results of their application compared to this thesis. The second aspect of this work

estimates. For this purpose five well known parameter estimation techniques

In a mixed economy operation stratageties require accurate parameter

Our emphasis here is two-fold: Firstly we wish to find estimates

To each of these a suitable mathematical model is assigned.

of three major subsystems, viz., the thermal generation, the hydro gene-

Convenient to think of the integrated electric system as being made

For the purpose of optimal control for economy operation, it is proposed to use the optimal control procedure is applied.

electric power system by an appropriate mathematical model to which a

fuel cost. A problem of this nature is dealt with by representing the

electric power systems. An attempt is made to allocate optimally the

This work is concerned with the optimal economic control of

## ABSTRACT

estimates give rise to different optimal strategies. The allocated optimum power generations are then utilized into the original network and the network performance is analyzed. Finally, the optimization procedure is extended to include a range of loading patterns of the system.

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December, 1979.

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## CHAPTER I

## INTRODUCTION

1.1 THE ECONOMIC DISPATCH PROBLEM

One of the most important requirements in today's high quality bulk energy system operation is the problem of optimal control with respect to economy and security within prescribed environmental constraints(1).

The problem of finding means of minimizing the cost of electric energy dispatched (active and/or active-reactive) has been occupying the attention of many researchers for a number of years now (2). In order to allocate the active and/or active-reactive power generations optimally, the system under consideration is represented by an appropriate mathematical model from which an index (cost related) function is derived and a suitable optimization procedure is applied. The literature is rich with optimization techniques applied to power system problems (3-8).

1.2 SYSTEM MODELS

Power system modelling for optimal operation is well documented in the literature (9-13). For the purpose of optimal control, the electric power system is divided into the following three major subsystems (14) :

- (I) The thermal generation (this includes as a subsystem the nuclear generation available).
- (II) The hydro generation (conventional and pump storage hydro)

(III) The interconnecting extra high voltage transmission network (EHV).

Many mathematical models have been suggested over the years.

In this thesis the following models are considered :

1.2.1 THE THERMAL AND HYDRO GENERATION MODELS

The plants are represented by input-output or efficiency type models, in which the cost related function (in each case) is taken as quadratic in the respective active power generations.

1.2.2 INTERCONNECTING NETWORK MODELS

Two models are considered here, one is the active power loss model, the other is the active-reactive power loss model. The first is generated from the second by making certain assumptions on the network operation. The models are based on the active and reactive power balance equations (ARPBE). The active power loss model relates the total active power transmission losses to active power generations only. The second model relates the total active and reactive power transmission losses to both active and reactive power generations.

1.3 BACKGROUND

A number of models have been generated over the years. The plant efficiency type model considered here has been used extensively by Christensen and El-Hawary (15). The network power loss model mentioned above is commonly known as the loss formula or the B-Coefficient method. An earlier but simplified version is due to George (16). The B-coefficient model used in our work is a modification to George's

expression in that a constant and a linear term are added to the expression (17). This is commonly referred to as the generalized transmission loss formula. Another early model due to Kirchmayer (18), relates the network reactive power loss to active power generations. The active-reactive power loss model employed in this thesis is based on an extension of Dopazo et al's. work (19), and is reported by El-Hawary and Christensen (20). A somewhat simplified expression is due to Edelmann and Theilsteife (21), this contains quadratic terms only and neglects the active-reactive (P-Q) coupling.

In recent years, the problem of estimating the parameters associated with the various power system models has been receiving considerable attention (22-26), in the hope of finding suitable procedures that will result in good parameter estimates for power system applications.

#### 1.4 PURPOSE AND SCOPE OF THESIS

This thesis addresses itself to the problem of finding good estimates for the parameters appearing in the various mathematical models chosen to represent the three power system decompositions. The cost of running a power system depends on the degree of accuracy of the estimated parameters; However it should be noted that since any parameter estimation procedure is based on some experimental results, one cannot hope to determine the true values of the parameters with absolute certainty due to the measurement errors (27-29). In this work five well known parameter estimation techniques (30-33), namely

the Weighted Least Squares, Linear Regression, Gauss-Newton or Bard, Marquardt and Powell Regression Algorithms are applied to our power system problem and their results are compared. Having estimated the parameters, the models are then used to allocate the optimum power generations of the following three practical test systems:

- (I) The 5 Bus Test System (34)
- (II) The American Electric Power Service Corporation 14 and 30-Bus Test Systems.

The latter two test systems are commonly referred to as the IEEE 14 and 30 Bus Test Systems respectively.

The optimization procedure for allocating the power generations for the above test systems is intended as an extra measure to judge the merit of each of the five parameter estimation techniques mentioned earlier, as different parameter estimates will result in different optimal strategies. The procedure is carried out by minimizing the cost related objective function (active and/or active-reactive) subject to equality and inequality constraints. The iterative method of Newton-Raphson is employed for the solution of optimization conditions.

Having allocated the optimum power generations of the system under consideration for a particular loading pattern, these powers are then applied into the the original network and a load flow solution is carried out to study their impact on the network performance.

The optimization procedure is carried out for one loading pattern per test system. To observe the flexibility of the models with their estimated coefficients as discussed above, different loading

Pattern are applied to a sample test system and the corresponding optimal strategies recorded.

Chapter two gives a brief description of each chosen model, with the detailed derivations of the active and active-reactive power loss models given in appendix A. Also in this chapter a description of the model format set up for the purpose of computer manipulations is given. Chapter three outlines the five parameter estimation techniques that are employed to estimate the parameters of interest. Data preparation procedures, parameter estimation algorithms and the results obtained for the three subsystem models are given in chapter four, with the detailed description of the test systems given in appendix B. Chapter five is devoted to the description of the algorithm used to allocate the optimum power generations of the test systems and the results obtained, with the initial guess estimators for the Newton-Raphson based optimization algorithm given in appendix C. The discussion and conclusions are given in chapter six.

## CHAPTER II

6

### SYSTEM MODELS AND FORMULATIONS

This chapter is divided into two sections, the first gives detailed descriptions of the chosen models that will be employed for our purposes. The second is devoted to setting up formats for these models suitable for computer manipulation purposes.

#### 2.1 MODELS CONSIDERED (9)

We are interested in two models. The first relates plant input-output characteristics (for both thermal and hydro generation sources). The second deals with the network models. It should be noted that the models discussed here are but illustrative.

##### 2.1.1 PLANT PERFORMANCE MODELS

The models chosen here are input-output or efficiency type models and are described below.

###### A. THE THERMAL GENERATION SUBSYSTEM MODEL

The input to the plant is the time rate of heat consumption measured in Megajoules per hour (MJ/hr) and the output is the active power generations measured in Megawatts (MW). The total fuel cost most commonly used is one where the labour, maintenance and supply costs are taken as fixed percentages of the incoming fuel cost. The fuel cost curve for economy operation is taken as a second order function in the active power generations and is given by:

$$F_{si}(P_{Gsi}) = a_{si} + b_{si} P_{Gsi} + c_{si} P_{Gsi}^2 \text{ GJ/hr.} \quad \dots\dots\dots(2.1)$$

where  $F_{si}$  is the  $i$ th thermal unit operating cost and  $P_{Gsi}$  is  $i$ th active power generation. The total cost for  $m$  units is,

$$F_0 = \sum_{i=1}^m a_{si} + b_{si} P_{Gsi} + c_{si} P_{Gsi}^2 \text{ GJ/hr.} \quad \dots\dots\dots(2.2)$$

The parameters to be estimated are  $a_{si}$ ,  $b_{si}$  and  $c_{si}$  associated with the thermal unit  $i$ . Subscript  $s$  stands for thermal generation (steam).

#### B. THE HYDRO GENERATION SUBSYSTEM MODEL

In this case the input to the plant is the water discharge rate measured in Million Cubic Feet per hour (Mcf/hr), and the output is the active power generations measured in Megawatts (MW).

Although many other alternative models have been proposed, we use a model similar to the cost related function model above. This is given by:

$$Q_{hj}(P_{Ghj}) = a_{hj} + b_{hj} P_{Ghj} + c_{hj} P_{Ghj}^2 \text{ Mcf/hr.} \quad \dots\dots\dots(2.3)$$

where  $Q_{hj}$  represents the water discharge rate at the  $j$ th hydro plant, and  $P_{Ghj}$  is the active power generation of the  $j$ th hydro unit. Likewise the parameters to be estimated are  $a_{hj}$ ,  $b_{hj}$  and  $c_{hj}$  associated with the  $j$ th hydro unit. Subscript  $h$  stands for hydro generation.

#### 2.1.2 THE ELECTRIC NETWORK MODELS

The following two models are considered in this work.

### A. THE ACTIVE POWER LOSS MODEL

This is commonly known as the B-coefficient model. It is generated by considering both the active and reactive power balance equations and by making certain assumptions on the network operation (see appendix A for details). The active power loss is written as:

$$P_L = K_{L0} + \sum_{i=1}^m B_{i0} P_{Gi} + \sum_{i=1}^m \sum_{j=1}^m P_{Gi} B_{ij} P_{Gj}, \quad \dots \dots \dots (2.4)$$

or in vector-matrix form as:

$$P_L = K_{L0} + \underline{B}_0 \underline{P}_G + \underline{P}_G^T \underline{B} \underline{P}_G, \quad \dots \dots \dots (2.5)$$

where  $P_L$  is the total transmission active power loss, and  $P_{Gi}$  is the active power generation at the  $i$ th node. The parameters to be estimated for this model are  $K_{L0}$ ,  $\underline{B}_0$  and  $\underline{B}$ , where,

$$\underline{B}_0 = [ B_{10} \ B_{20} \ \dots \ B_{n0} ]^T, \quad \dots \dots \dots (2.6)$$

and,

$$\underline{B} = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \vdots & \vdots & & \vdots \\ B_{n1} & B_{n2} & & B_{nn} \end{bmatrix}, \quad \dots \dots \dots (2.7)$$

$\underline{B}$  is a square symmetric matrix. The parameters are functions of voltages, impedances, phase angles and load pattern etc. of the network. Equation (2.4) can be derived from principles see appendix A.

### B. THE ACTIVE-REACTIVE POWER LOSS MODEL

This model is again derived by considering both the active and reactive power balance equations. The expressions for the total network transmission losses arrived at may be written as follows:

$$P_L = K_{L0G} + \begin{bmatrix} P_G^T & Q_G^T \end{bmatrix} \begin{bmatrix} A_{PGG} & -B_{PGG} \\ B_{PGG} & A_{PGG} \end{bmatrix} \begin{bmatrix} P_G \\ Q_G \end{bmatrix} + \begin{bmatrix} E_{PP}^T & E_{PQ}^T \end{bmatrix} \begin{bmatrix} P_G \\ Q_G \end{bmatrix} \quad \dots \dots \dots \quad (2.8)$$

for the active part, and,

$$Q_L = K_{Q0G} + \begin{bmatrix} P_G^T & Q_G^T \end{bmatrix} \begin{bmatrix} A_{QGG} & -B_{QGG} \\ B_{QGG} & A_{QGG} \end{bmatrix} \begin{bmatrix} P_G \\ Q_G \end{bmatrix} + \begin{bmatrix} E_{QP}^T & E_{QQ}^T \end{bmatrix} \begin{bmatrix} P_G \\ Q_G \end{bmatrix} \quad \dots \dots \dots \quad (2.9)$$

for the reactive part. Where  $P_L$  is the total transmission active power losses and  $Q_L$  is the total transmission reactive power losses.

$P_G$  and  $Q_G$  are vectors of active and reactive power generations. The parameters to be estimated are:

$K_{LQP}$ ,  $E_{PP}$ ,  $E_{PQ}$ ,  $A_{PGG}$  and  $B_{PGG}$  for  $P_L$  and  $K_{LQ}$ ,  $E_{QP}$ ,  $E_{QQ}$ ,  $A_{QGG}$  and  $B_{QGG}$  for  $Q_L$ . These coefficients are again functions of voltages, impedances, phase angles, and load pattern etc., of the network.  $A_{PGG}$  and  $A_{QGG}$  are square symmetric matrices,  $B_{PGG}$  and  $B_{QGG}$  are skew symmetric matrices. For derivation of equations (2.8) and (2.9) see appendix A.

## 2.2 FORMULATION

For the purpose of parameter estimation, the models described above are set up in a format suitable for computer manipulation (35,36).

The format is:

$$\underline{Y} = \underline{D} \underline{X} . \quad \dots \dots \dots \quad (2.10)$$

For the case of active-reactive model equation (2.10) is partitioned as

$$\begin{bmatrix} \underline{Y}_P \\ \underline{Y}_Q \end{bmatrix} = \begin{bmatrix} \underline{D}_C & \underline{0} \\ \underline{0} & \underline{D}_C \end{bmatrix} \begin{bmatrix} \underline{X}_P \\ \underline{X}_Q \end{bmatrix} \quad \dots \dots \dots \quad (2.11)$$

$\underline{Y}$  represents the experimental values of the dependent variable, and  $\underline{X}$  represents the vector of the coefficients to be estimated. The elements of matrix  $\underline{D}$  are combinations of active and/or reactive power generations (as the case may be). The subscripts P and Q stand for the active and reactive components respectively. In this work the active and reactive components are treated separately.

To illustrate the use of equation (2.10) and equation (2.11) for the purpose of parameter estimation for the models described above

consider a system consisting of two generating units, the following gives the format expression for each model.

### 2.2.1 THE SOURCE MODELS

For equation (2.10) we have for  $n$  experiments each,

$$Y_s = [f_{s11} \quad f_{s12} \quad \dots \quad f_{sin}]^T, \quad \dots \dots \dots (2.12)$$

where  $f_{sij}$  is the fuel cost associated with the  $i$ th thermal generating unit at the  $j$ th experiment and,

$$Y_h = [q_{h11} \quad q_{h12} \quad \dots \quad q_{hn1}]^T, \quad \dots \dots \dots (2.13)$$

where  $q_{hij}$  is the water discharge rate associated with the  $i$ th hydro generating unit at the  $j$ th experiment , and,

$$X_s = [a_{s1} \quad b_{s1} \quad Y_{s1}]^T, \quad \dots \dots \dots (2.14)$$

$$X_h = [a_{h1} \quad b_{h1} \quad Y_{h1}]^T, \quad \dots \dots \dots (2.15)$$

are simply the parameters to be estimated (these are the parameters associated with the  $i$ th generating unit in each case). The following two matrices are defined:

$$D_s = \begin{bmatrix} D_{s11} \\ D_{s12} \\ \vdots \\ D_{sin} \end{bmatrix}, \quad \dots \dots \dots (2.16)$$

for the thermal problem, and,

$$D_h = \begin{bmatrix} D_{h11} \\ D_{h12} \\ \vdots \\ D_{h1n} \end{bmatrix} \quad \dots \quad (2.17)$$

for the hydro generation problem. Each of the submatrices is given by:

$$D_{Gij} = \begin{bmatrix} 1.0 & P_{Gij} & P_{Gij}^2 \end{bmatrix} \quad \dots \quad (2.18)$$

where  $P_{Gfi}$  is the active power generation of the  $i$ th thermal unit.

at the  $i$ th experiment.

$$D_{hij} = \begin{bmatrix} 1.0 & p_{Ghij} & p_{Ghij}^2 \end{bmatrix} \quad \dots \dots \dots \quad (2.19)$$

where  $P_{Ghj}$  is the active power generation of the  $h$ th hydro unit at the  $j$ th experiment,  $j=1, 2, \dots, n$ , where  $n$  is the number of experiments.

### 2.2.2 THE NETWORK MODELS

The format expressions for the two network model are given below.

## A. THE ACTIVE POWER LOSS MODEL

For this model with two generating buses, the format becomes:

$$\underline{Y} = [P_{L1} \quad P_{L2} \quad \dots \quad P_{Ln}]^T, \quad \dots \dots \dots \quad (2.20)$$

$$\underline{x} = \begin{bmatrix} k_{LO} & b_{10} & b_{20} & b_{11} & b_{12} & b_{22} \end{bmatrix}^T, \dots \dots \dots (2.21)$$

where  $B_{ij} = B_{ij}^*$ .

$$D = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_n \end{bmatrix} \quad \dots \dots \dots \quad (2.22)$$

where.

$$D_{\frac{1}{2}} = \begin{bmatrix} 1.0 & P_{G1} & P_{G2} & P_{G1}^2 & 2P_{G1}P_{G2} & P_{G2}^2 \end{bmatrix}, \dots \quad (2.23)$$

$i=1, 2, \dots, n$ , where  $n$  is the number of experiments.

## B. THE ACTIVE-REACTIVE POWER LOSS MODEL

Here the partitioned form of equation (2.11) is employed, the format for this model becomes:

$$Y_Q = \begin{bmatrix} Q_{1,1} & Q_{1,2} & \cdots & Q_{1,n} \end{bmatrix}^T. \quad (2.25)$$

$$X_P = \begin{bmatrix} K_{LOP} & E_{PP1} & E_{PP2} & E_{PQ1} & E_{PQ2} & A_{P11} & A_{P12} & A_{P22} \\ B_{P12} \end{bmatrix}^T . \quad \dots \dots \dots (2.26)$$

$$X_Q = \begin{bmatrix} K_{Q00} & E_{QP1} & E_{QP2} & E_{QQ1} & E_{QQ2} & A_{Q11} & A_{Q12} & A_{Q22} \\ B_{Q12} \end{bmatrix}^T , \quad \dots \dots \dots (2.27)$$

where  $A_{Pij} = A_{pij} + A_{0ij}$ .

$$B_{Pij} = -B_{Pji}, \quad B_{Qij} = -B_{Qji},$$

and  $B_{Pii} = B_{Oii} = 0.0$

where each of the submatrices is given by:

$$D_{C1} = \left[ 1.0, P_{G1}, P_{G2}, Q_{G1}, Q_{G2}, (P_{G1}^2 + Q_{G1}^2), 2(P_{G1} \cdot P_{G2} + Q_{G1} \cdot Q_{G2}), \right. \\ \left. (P_{G2}^2 + Q_{G2}^2), -2(P_{G1} \cdot Q_{G2} - P_{G2} \cdot Q_{G1}) \right] \quad \dots \dots \dots (2.29)$$

for  $i=1, 2, \dots, n$ , where  $n$  is the number of experiments.

The general case with  $m$  generating buses can be easily derived from above.

## CHAPTER III

### PARAMETER ESTIMATION TECHNIQUES

The parameter estimation techniques employed to estimate the coefficients of the models described in this chapter are :

- (i) Weighted Least Squares.
  - (ii) Linear Regression Algorithm.
  - (iii) Gauss-Newton Method or Bard Algorithm.
  - (iv) Marquardt Algorithm.
  - (v) Powell Regression Algorithm.

A description of each of the five techniques is given below.

### (1) WEIGHTED LEAST SQUARES

Consider the matrix equation given in chapter two, which is repeated here for convenience.

$$Y = D X \quad \text{.....(3.1)}$$

we wish to minimize a scalar cost function  $J$  where :

$W$  is a weighting matrix.

Minimization of a scalar, with respect to a vector is obtained when

and the Hessian of  $J$  is positive semidefinite.

Differentiating  $\Phi$  and equating to zero gives,

$$[\begin{smallmatrix} T & W & D \\ D & W & T \end{smallmatrix}] X = [\begin{smallmatrix} T & W & Y \\ D & W & T \end{smallmatrix}], \quad \dots \dots \dots \quad (3.5)$$

when the matrix  $[D \quad W \quad D]$  has an inverse, then

### (ii) LINEAR REGRESSION ALGORITHM

Consider a multivariable linear regression equation of the form :

$$\hat{Y} = \hat{A}_0 + \hat{A}_1 F_1(X) + \hat{A}_2 F_2(X) + \dots + \hat{A}_M F_M(X), \quad \dots \quad (3.7)$$

$y$  is the dependent variable and,

$\hat{A}_i$ ,  $i = 1, 2, \dots, M$ , are the coefficients to be estimated,

$F_i$  are functions of the independent variables  $X_i$  with,

$j = 1, 2, \dots, K$ , and  $i = 1, 2, \dots, M$

The method is based on minimizing the least squares objective function  $s$  such that

$$S = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad \text{is minimum.} \quad \dots \dots \dots (3.8)$$

$y_i$  are the experimental values of the dependent variable.

Taking the partial derivatives of  $s$  with respect to  $A$ , and equating to zero we obtain the following :

$$[E^T \quad E] \hat{A} = E^T Y \quad \text{.....(3.9)}$$

where

$$F = \begin{bmatrix} (F_{11} - \bar{F}_1) & (F_{12} - \bar{F}_2) & \dots & (F_{1M} - \bar{F}_M) \\ (F_{21} - \bar{F}_1) & (F_{22} - \bar{F}_2) & \dots & (F_{2M} - \bar{F}_M) \\ \vdots & \vdots & & \vdots \\ (F_{n1} - \bar{F}_1) & (F_{n2} - \bar{F}_2) & \dots & (F_{nM} - \bar{F}_M) \end{bmatrix}, \quad (3.10)$$

and

$$\hat{Y} = [(Y_1 - \bar{Y}), (Y_2 - \bar{Y}), \dots, (Y_n - \bar{Y})]^T, \quad \dots \dots \dots (3.11)$$

$\bar{Y}$  and  $\bar{F}_i$  are the mean values.

A linear solution of the equations gives results for the coefficients  $\hat{A}_1, \hat{A}_2, \dots, \hat{A}_n$ ,  $\hat{A}_0$  is obtained from:

For a perfect fit  $s$  would be zero and the ratio of

$\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2$  to  $\sum_{i=1}^n (Y_i - \bar{Y})^2$  would equal to 1.

(111) GAUSS-NEWTON METHOD - BARD ALGORITHM (37)

The algorithm solves for parameters appearing in a multi-variable non-linear regression model of the form :

$$Y = F(X_1, X_2, \dots, X_k; \hat{A}_1, \hat{A}_2, \dots, \hat{A}_M), \quad (3.14)$$

for n experimental points for  $Y_1$  and  $X_{k+1}$ , with,

$$i = 1, 2, \dots, N; \quad k = 1, 2, \dots, K.$$

The principle of the method is to linearize the function by the use of Taylor Series expansion about some initial guesses of the coefficients. Neglecting higher order terms in the expansion results in the following :

$$\hat{Y}_i = \hat{Y}_i^* + \frac{\partial \hat{Y}_i}{\partial \hat{A}_1} \Delta \hat{A}_1 + \frac{\partial \hat{Y}_i}{\partial \hat{A}_2} \Delta \hat{A}_2 + \dots + \frac{\partial \hat{Y}_i}{\partial \hat{A}_M} \Delta \hat{A}_M \quad (3.15)$$

\* represents that the derivatives are evaluated at the initial guesses where

$$\Delta \hat{A}_j = [\hat{A}_j - \hat{A}_j^*], \quad j = 1, 2, \dots, M.$$

A least squares objective function is generated such that :

$$s = \sum_{i=1}^N (\hat{Y}_i - Y_i)^2 \text{ is minimum.} \quad (3.16)$$

Taking the partial derivatives of  $s$  with respect to  $\hat{A}_j$  :

$j = 1, 2, \dots, M$ , and equating to zero we obtain the following normal equations :

$$[A^T A] \Delta \hat{A} = A^T [Y - \hat{Y}], \quad (3.17)$$

where

$$a_{ij} = \frac{\partial \hat{Y}_i}{\partial \hat{A}_j} \quad (3.18)$$

$$i = 1, 2, \dots, N; \quad j = 1, 2, \dots, M.$$

and

$$\underline{\Delta \hat{A}} = \begin{bmatrix} \hat{A}_1 - \hat{A}_1^* \\ \hat{A}_2 - \hat{A}_2^* \\ \vdots \\ \hat{A}_M - \hat{A}_M^* \end{bmatrix} \quad \text{and} \quad \underline{Y - \hat{Y}^*} = \begin{bmatrix} Y_1 - \hat{Y}_1^* \\ Y_2 - \hat{Y}_2^* \\ \vdots \\ Y_N - \hat{Y}_N^* \end{bmatrix}$$

Having obtained the linearized form, the linear equations are then solved iteratively and convergence is reached when  $\underline{\Delta A}$  and  $s$  nears zero, if not  $A$  is updated and the process is repeated.

The method works well if good starting estimates are available.

#### (iv) MARQUARDT ALGORITHM [38]

The method is an extension to the Gauss-Newton procedure mentioned above and can converge for poor starting estimates. Again the algorithm solves for a multivariable non-linear regression model of the form given in equation (3.14) above.

A least squares function is formed and a modification to the normal equations is made by introducing a factor  $\lambda$  as follows :-

$$[\underline{A}^T \underline{A} + \lambda \underline{I}] \underline{\Delta \hat{A}} = \underline{A}^T [\underline{Y} - \hat{Y}^*] \quad \dots \dots \dots \quad (3.19)$$

$I$  is a unit matrix,  $\lambda$  is a factor introduced into the main diagonal of the matrix  $\underline{A}^T \underline{A}$ . If  $\lambda$  approaches a large value the method reduces to that of the Steepest Descent. If however  $\lambda$  approaches zero the

method reduces to the Gauss-Newton procedure.

(v) POWELL REGRESSION ALGORITHM [39]

The algorithm solves for the coefficients in the multivariable non-linear regression equation (3.14) given above. The method is a modification of Gauss-Newton technique to reduce the labour involved in solving the linear equations at each iteration. Basically the least squares function is modified by introducing  $n$ -linearly independent direction vectors. An initial guess is selected and an initial set of directions vector components  $M_{i,j}$  ( $i=1, 2, \dots, M$ ;  $j=1, 2, \dots, M$ ), are selected parallel to the co-ordinate axes such that :

$$M_1 = [ 1, 0, 0, \dots, 0 ]$$

$$M_2 = [ 0, 1, 0, \dots, 0 ]$$

$$M_M = [ 0, 0, 0, \dots, 1 ] .$$

The system matrix is inverted using an iterative scheme and all derivatives are approximated by finite differences.

The normal equations are given by :

$$[ A^T A ] \Delta \hat{A} = A^T [ Y - \hat{Y} ] . \quad \dots \quad (3.20)$$

are set up and solved for  $\Delta \hat{A}$ , a new direction vector is then calculated using  $\Delta \hat{A}$  with components given by :

$$M_{i,new} = \Delta \hat{A}_i / [ \sum_{j=1}^M \Delta \hat{A}_j^2 ] . \quad \dots \quad (3.21)$$

$i = 1, 2, \dots, M.$

A one dimensional search is then carried out using the following corrector formula :

$$\hat{A}_i(\text{new}) = \hat{A}_i(\text{old}) + s \hat{M}_{i,\text{new}}, \quad \dots \dots \dots \quad (3.22)$$

$s$  is the distance moved in  $\hat{M}_{\text{new}}$  direction when the one dimensional minimum is reached and convergence occurs, the computation is terminated. If not, one of the old direction vectors is replaced by a new one, the replaced vector will have an index of :

$| b_i \hat{A}_i |$  that is maximum where  $i = 1, 2, \dots, M$ , and,

$$b_i = \text{element of } A^T [Y - \hat{Y}]^*$$

$\hat{A}_i$  are elements of  $\hat{A}$ .

The normal equations are now updated with respect to the new direction, the procedure is repeated until convergence is reached.

It should be noted that for equations of the type we are interested in, all the methods (except Linear Regression) should give identical results.

## CHAPTER IV

### DATA PREPARATION PROCEDURES, PARAMETER ESTIMATION ALGORITHMS AND RESULTS

In this chapter, procedures for obtaining the data required for model parameter estimation are given. We also outline the parameter estimation algorithms and the results obtained.

#### 4.1 DATA FOR GENERATION SOURCES

For the thermal generation case, a sample unit with capacity of 50 MW was chosen and practical data relating the fuel cost to active power generations were selected (9). The three types of fuel (Coal, Oil and Gas) are considered. The data are given in table(4.1).

For the hydro generation subsystem model, the practical data chosen are given in table (4.2). These relate the water discharge rate to active power generations for a hydro plant in a Canadian utility system (40).

#### 4.2 DATA PREPARATION FOR THE NETWORK TEST SYSTEMS

Three test systems are employed in our work and are:

- (i) The 5 Bus Test System.
- (ii) The American Electric Power Service Corporation (A.E.P) 14 and 30 Bus Test Systems.

Detailed description of each system is given in appendix B.

The required data for both network models (active and/or active-reactive) for parameter estimation purposes were obtained by

TABLE (4.1)

## THERMAL GENERATION DATA

UNIT SIZE = .50 MW

OUTPUT $P_{Gs}$	COAL $F_s$	OIL $F_s$	GAS $F_s$
10.0	176.62	184.75	187.87
20.0	256.40	268.20	272.80
30.0	361.50	377.70	384.30
40.0	467.60	488.80	497.20
50.0	579.50	606.00	616.50

 $P_{Gs}$  is in MW and  $F_s$  is in GJ/hr.

TABLE (4.2)

## TYPICAL HYDRO GENERATION DATA

P <sub>Gh</sub>	Q <sub>h</sub>								
70.00	5.871	185.00	15.32	290.00	23.60	395.00	32.58	500.00	41.34
80.00	6.570	190.00	15.68	295.00	24.01	400.00	33.05	505.00	41.80
85.00	6.972	195.00	16.19	300.00	24.44	405.00	33.32	510.00	42.26
90.00	7.375	200.00	16.50	305.00	24.86	410.00	33.72	515.00	42.74
95.00	7.796	205.00	16.83	310.00	25.59	415.00	34.13	520.00	43.22
100.0	8.074	210.00	17.18	315.00	25.73	420.00	34.55	525.00	43.71
105.0	9.810	215.00	17.55	320.00	26.16	425.00	34.97	530.00	44.21
110.0	10.02	220.00	17.94	325.00	25.94	430.00	35.40	535.00	44.72
115.0	10.25	225.00	18.34	330.00	27.35	435.00	35.84	540.00	45.24
120.0	9.910	230.00	18.75	335.00	27.76	440.00	36.28	545.00	45.77
130.0	10.61	235.00	19.16	340.00	28.07	445.00	36.73	550.00	46.32
135.0	11.01	240.00	19.58	345.00	28.40	450.00	37.19	555.00	46.88
140.0	11.36	245.00	20.00	350.00	28.78	455.00	37.65	560.00	47.47
145.0	11.76	250.00	20.43	355.00	29.16	460.00	38.12	565.00	48.07
150.0	12.39	255.00	20.85	360.00	29.55	465.00	38.60	570.00	48.67
155.0	12.51	260.00	21.22	365.00	29.96	470.00	39.08	575.00	49.30
160.0	12.91	265.00	21.59	370.00	30.37	475.00	39.19	580.00	49.95
165.0	13.32	270.00	21.98	375.00	30.79	480.00	39.60	585.00	50.62
170.0	13.69	275.00	22.38	380.00	31.22	485.00	40.02	590.00	51.42
175.0	14.16	280.00	22.78	385.00	31.67	490.00	40.45	595.00	51.97
180.0	14.97	285.00	23.19	390.00	32.12	495.00	40.89	600.00	52.68

P<sub>Gh</sub> is in MW and Q<sub>h</sub> is in MCF/hr.

successively solving the load flow equations associated with each test system for a number of times depending on the number of the unknown coefficients to be estimated. This we treat next, but first we give a brief description of the load flow algorithm used to obtain the necessary data.

#### THE LOAD FLOW SOLUTION ALGORITHM

The load flow solution algorithm (41,42), used here solves for bus voltages and line flows for the system under consideration having certain loading pattern. The algorithm employs the method of Newton-Raphson and uses a matrix inversion scheme such that the Jacobian sparse matrix is expressed as a product of sparse matrix factors using the Gaussian ordered triangular decomposition (43-45). This technique offers the advantage of computational speed, storage and minimizes the round off error.

#### THE LOAD FLOW EXPERIMENTS

The number of times the load flow equations were solved for each test system are given below. This depends on the number of parameters to be estimated. Basically the number of experiments or data points and hence the number of resulting equations should exceed the number of unknown parameters, so that an overdetermined situation is created for the application of the five parameter estimation techniques discussed earlier. From the load flow results, sets of the total system active and reactive power losses and the system active and reactive power generations are recorded for each system.

#### 4.3 THE ELECTRIC NETWORK TEST SYSTEMS

The 5 and 14 Bus networks are shown in figures (4.1) and (4.2) respectively. Both networks have two generating buses, with the 5 Bus system containing one voltage regulated bus and the 14 Bus test system three voltage regulated buses. Since both systems contain only two generating buses, the number of parameters to be estimated are 6 for the B-coefficient model and 9 for the active-reactive power loss model respectively. The load flow equations for these systems were solved for 30 experiments each, by changing the active power generations. Results are given in tables (4.3) and (4.4) respectively.

The network for the 30 Bus test system is shown in figure (4.3). This test system contains three generating buses and four voltage regulated buses. The number of coefficients to be estimated is 10 for the B-coefficient model and 16 for the active-reactive power loss model. In this case 40 experiments were carried out, again by changing the active power generations. The load flow results for this system are given in table (4.5). It should be noted that the generating bus number 13 is given number 3 for convenience for the purpose of parameter estimation and the optimization procedure.

#### 4.4 THE PARAMETER ESTIMATION ALGORITHMS

Having obtained sufficient data for each model (subsystem) as described above, computer programs for the five parameter estimation techniques were developed for our purposes and the parameters of each model estimated. The Gauss-Newton or Bard, Marquardt and Powell Regression algorithms are based on Newton-Raphson method of

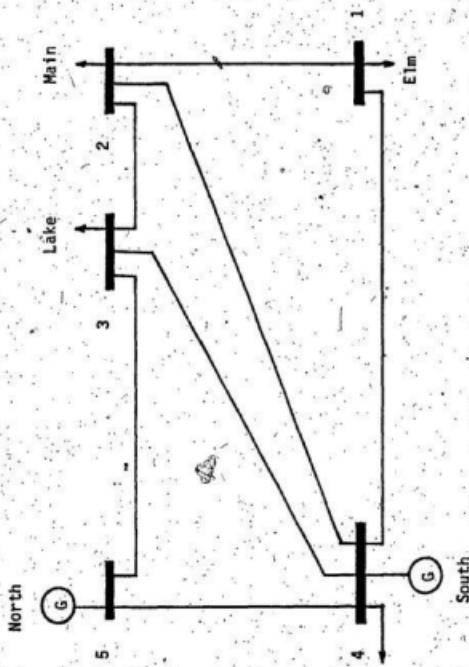


FIGURE (4.1) 5 BUS TEST SYSTEM

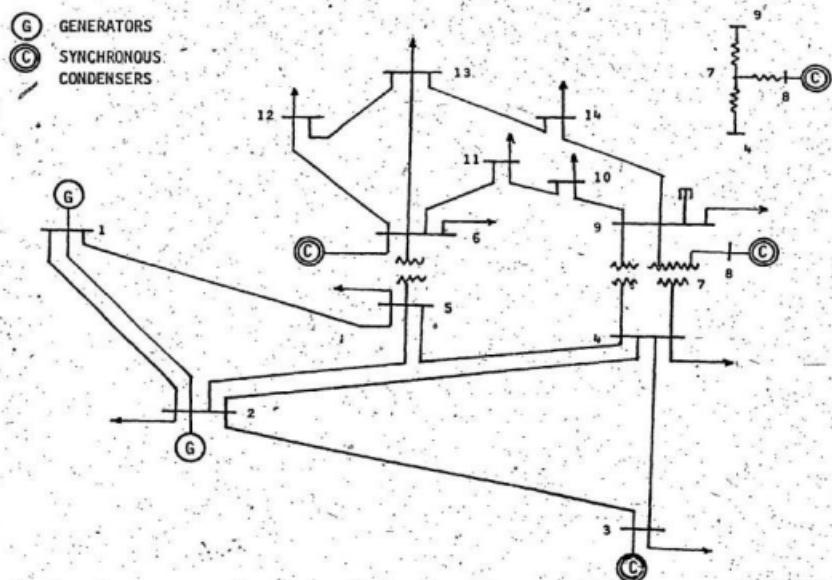


FIGURE (4.2) A.E.P 14 BUS TEST SYSTEM.

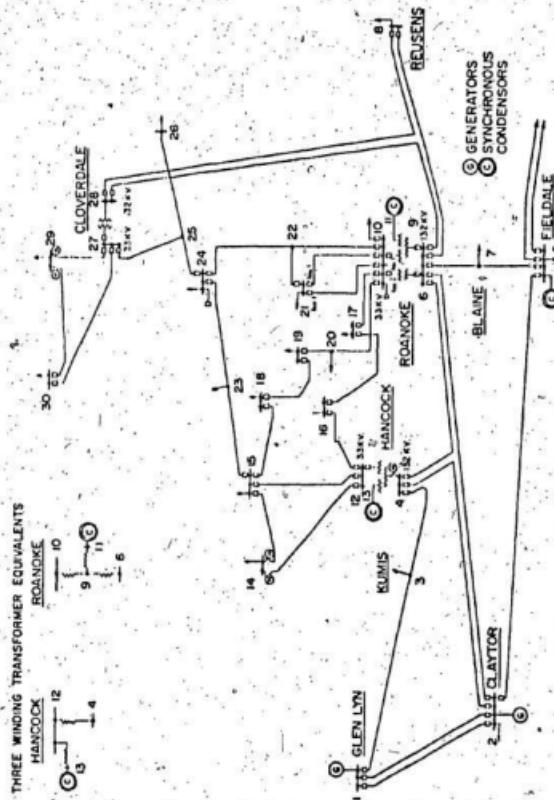


FIGURE 4.3 A-E-P 30 BUS TEST SYSTEM

TABLE (4.3)  
5 BUS TEST SYSTEM

## LOAD FLOW RESULTS

EXPERIMENT NUMBER	P <sub>L</sub> MW	Q <sub>L</sub> MVAR	P <sub>G1</sub> MW	Q <sub>G1</sub> MVAR	P <sub>G2</sub> MW	Q <sub>G2</sub> MVAR
1	4.77	-16.15	129.63	26.36	40.00	-2.51
2	3.49	-20.37	78.65	23.63	90.00	-4.00
3	5.52	-13.75	150.46	28.01	20.00	-1.75
4	3.33	-20.91	68.51	23.36	100.00	-4.27
5	6.18	-11.66	166.16	29.50	5.00	-1.15
6	5.95	-12.38	160.91	28.98	10.00	-1.36
7	5.73	-13.08	155.69	28.48	15.00	-1.55
8	3.08	-21.86	48.08	20.38	120.00	-2.23
9	5.32	-14.39	145.24	27.56	25.00	-1.95
10	5.13	-15.00	140.03	27.14	30.00	-2.14
11	3.91	-18.98	99.02	24.44	70.00	-3.41
12	4.16	-18.14	108.93	25.07	60.00	-3.21
13	3.68	-19.72	88.82	24.00	80.00	-3.72
14	3.40	-20.72	24.15	29.61	145.00	3.49
15	6.27	-11.37	168.26	29.71	3.00	-1.07
16	4.45	-17.20	119.27	25.67	50.00	-2.88
17	4.95	-15.59	134.83	26.74	35.00	-2.33
18	3.13	-21.68	53.13	20.83	115.00	-2.50
19	3.41	-20.65	73.57	23.48	95.00	-4.13
20	6.38	-11.03	170.62	29.95	0.75	-0.98

TABLE (4.3) CONT.

21	6.34	-11.14	169.83	29.87	1.50	-1.01
22	4.30	-17.69	114.09	25.37	55.00	-3.05
23	3.28	-21.08	33.74	28.84	135.00	-1.19
24	3.79	-19.36	93.91	24.20	75.00	-3.56
25	3.58	-20.06	83.73	23.81	85.00	-3.86
26	3.21	-21.36	58.40	23.17	110.00	-4.54
27	4.61	-16.69	124.45	26.00	45.00	-2.69
28	3.04	-22.02	43.04	19.93	125.00	-1.95
29	4.03	-18.57	104.14	24.69	65.00	-3.26
30	4.27	-17.71	31.22	32.07	140.00	30.54

TABLE (4.4)

## A.E.P 14 BUS TEST SYSTEM

## LOAD FLOW RESULTS

EXPERIMENT NUMBER	P <sub>L</sub> MW	Q <sub>L</sub> MVAR	P <sub>G1</sub> MW	Q <sub>G1</sub> MVAR	P <sub>G2</sub> MW	Q <sub>G2</sub> MVAR
1	14.02	30.12	232.94	1.33	40.00	50.00
2	15.87	36.50	264.89	7.09	10.00	50.00
3	12.05	23.41	192.46	-0.49	78.00	50.00
4	15.71	35.96	261.74	6.38	13.00	50.00
5	15.28	34.43	255.28	4.96	19.00	50.00
6	11.19	20.62	169.74	5.33	100.00	41.21
7	14.23	30.80	237.14	1.73	36.00	50.00
8	14.98	33.40	249.99	3.83	24.00	50.00
9	13.88	29.64	230.78	1.32	42.00	50.00
10	13.55	28.52	224.43	0.94	48.00	50.00
11	11.34	21.09	173.89	4.34	96.00	42.67
12	9.68	14.36	118.72	-16.08	150.00	50.00
13	12.89	26.26	211.11	0.24	60.00	50.00
14	11.05	19.45	167.06	-10.78	103.00	49.72
15	13.19	27.29	217.35	0.55	54.00	50.00
16	12.59	25.27	204.88	-0.04	66.00	50.00
17	12.32	24.32	198.67	-0.28	72.00	50.00
18	11.92	22.97	189.37	-0.59	81.00	50.00
19	12.24	24.33	184.61	2.53	87.00	50.00
20	11.45	21.46	197.00	3.61	93.00	43.76

TABLE (4.4) CONT.

21	10.95	19.11	163.96	-11.03	106.00	50.00
22	10.74	18.35	157.75	-11.83	112.00	50.00
23	10.84	18.73	160.85	-11.43	109.00	50.00
24	10.23	16.45	141.26	-13.95	128.00	49.77
25	10.48	17.39	149.50	-12.84	120.00	50.00
26	10.13	16.07	137.15	-14.25	132.00	50.00
27	9.82	14.89	124.85	-15.50	144.00	50.00
28	10.02	15.66	133.05	-14.68	136.00	50.00
29	10.60	17.81	153.62	-12.51	116.00	49.74
30	12.26	24.43	181.86	3.11	90.00	50.00

TABLE (4.5)

## A.E.P. 30 BUS TEST SYSTEM

## LOAD FLOW RESULTS

EXPERIMENT NUMBER	P <sub>L</sub> MW	Q <sub>L</sub> MVAR	P <sub>G1</sub> MW	Q <sub>G1</sub> MVAR	P <sub>G2</sub> MW	Q <sub>G2</sub> MVAR	P <sub>G3</sub> MW	Q <sub>G3</sub> MVAR
1	16.74	33.45	250.19	-19.91	40.00	38.39	10.00	50.00
2	14.56	22.40	233.51	-22.53	5.00	34.16	59.50	50.00
3	11.46	12.93	159.61	3.68	84.75	-5.07	50.50	49.97
4	14.48	22.12	231.67	-21.99	8.25	33.26	58.00	50.00
5	11.62	13.47	163.46	2.82	82.50	-3.79	49.00	50.00
6	18.62	40.81	276.07	-23.94	23.50	48.85	2.50	50.00
7	14.20	21.27	225.89	-20.45	16.75	30.69	55.00	50.00
8	11.99	14.76	171.38	2.65	78.00	-3.12	46.00	50.00
9	14.16	21.20	224.36	-19.81	21.25	29.80	52.00	50.00
10	18.29	39.45	271.91	-23.39	25.75	47.11	4.00	50.00
11	11.89	14.10	167.42	2.73	80.25	-3.44	47.50	50.00
12	17.84	37.74	265.55	-22.34	30.25	44.58	5.50	50.00
13	14.34	22.02	225.26	-19.21	28.00	29.51	44.50	49.97
14	12.37	16.22	178.49	2.65	75.75	-2.46	41.50	50.00
15	17.24	35.58	256.69	-20.79	37.00	41.03	7.00	50.00
16	14.21	21.67	222.08	-18.35	32.50	28.16	43.00	50.00
17	11.44	12.83	163.35	2.87	73.50	-4.26	58.00	50.00
18	18.10	37.99	271.77	-24.07	21.25	46.87	8.50	50.00
19	16.79	33.60	250.99	-20.09	39.25	38.70	10.00	50.00
20	14.08	21.55	217.51	-16.80	41.50	26.17	38.50	50.00

TABLE (4.5) CONT.

21	12.89	18.18	188.03	2.60	71.50	-1.52	37.00	50.00
22	15.11	28.46	217.25	4.05	69.75	3.52	11.50	50.00
23	14.96	25.41	228.14	-17.61	43.75	29.78	26.50	50.00
24	15.39	27.72	231.56	-17.30	48.25	31.07	19.00	49.97
25	13.74	21.06	206.31	2.17	55.25	0.23	35.50	49.98
26	-15.10	26.23	229.05	-17.40	46.00	30.09	23.50	50.00
27	17.15	37.51	269.38	-23.59	23.50	45.94	8.50	50.00
28	11.50	13.04	165.66	2.64	69.75	-3.93	59.50	50.00
29	14.18	21.23	225.12	-20.14	19.00	30.24	53.50	50.00
30	10.25	9.30	132.61	3.71	90.50	-5.61	70.56	50.00
31	9.63	8.31	113.47	4.30	95.35	-5.25	84.20	50.00
32	9.98	8.40	122.62	4.37	100.25	-6.38	70.53	50.00
33	9.02	6.64	84.78	6.44	120.30	-6.74	87.35	50.00
34	9.26	5.92	87.43	7.24	115.45	-9.05	69.80	50.00
35	9.40	7.52	103.69	5.04	105.37	-6.01	83.75	49.99
36	11.64	16.69	174.21	-11.67	20.85	19.14	100.00	50.00
37	10.33	12.23	138.28	2.45	58.75	-1.31	96.70	49.99
38	8.68	5.62	58.14	8.91	125.00	-8.17	88.95	50.00
39	11.59	13.51	159.66	3.28	90.57	-4.13	44.75	50.00
40	9.13	7.39	91.77	5.72	110.32	-5.75	90.45	50.00

iteration, and for these algorithms starting estimates must be available for solution. The results obtained from the method of Weighted Least Squares were used for this purpose.

In the case of Weighted Least Squares, the elements of the weighting matrix  $W$  (diagonal matrix) were selected on the basis of the errors between the observed and the computed values of the dependent variable, in our case the dependent variable is either the total active or reactive power loss of the system (the cost functions in the case of source models). Weightings were given for points that gave minimum error in the course of manipulations.

The overall error in each estimation procedure was calculated from the following:

$$E_{ov} = \sqrt{\frac{\sum_{i=1}^n |e_i / Y_{oi}|}{n}}, \quad \dots \dots \dots \quad (4.1)$$

where  $E_{ov}$  is the overall error,  $e_i$  is the error between the observed and computed values of the dependent variable at experiment  $i$ ,  $Y_{oi}$  is the observed value of the dependent variable at experiment  $i$  and  $n$  is the number of experiments.

In the case of the active-reactive power loss model, the real and reactive parts were treated separately.

Results of the application of the five parameter estimation techniques to our power system models are tabulated in the following pages as follows:

- 1- The thermal generation model (Coal, Oil and Gas) tables (4.6) to (4.8).

- 2- The hydro generation model- table (4.9).
- 3- The B-Coefficient model for the 5, 14 and 30 Bus test systems-  
tables (4.11) to (4.13).
- 4- The active-reactive power loss model for the 5, 14 and 30 Bus test  
systems - tables (4.14) to (4.19).

NOTE

The estimated parameters of the B-Coefficient and the active-reactive  
models for the three test systems are in per unit on 100 MVA base.

TABLE (4.6)  
50 MW UNIT THERMAL GENERATION (COAL)

Parameter	Weighted Least Squares $W = W_1$	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$a_s$	.49112E+02	.49918E+02	.49885E+02	.49903E+02	.49872E+02
$b_s$	.10109E+02	.10064E+02	.10066E+02	.10065E+02	.10069E+02
$y_s$	.98230E-02	.10310E-01	.10276E-01	.10291E-01	.10216E-01
$E_{ov}$	.23090E-02	.26480E-02	.26390E-02	.26440E-02	.26390E-02

$W_1 = \text{diag} (2.0, 1.0, 2.0, 1.0, 2.0)$

TABLE (4.7)  
50 MW UNIT THERMAL GENERATION (OIL)

Parameter	Weighted Least Squares $W = W_2$	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$a_s$	.52022E+02	.52855E+02	.52945E+02	.52795E+02	.52635E+02
$b_s$	.10515E+02	.10471E+02	.10465E+02	.10476E+02	.10489E+02
$y_s$	.11138E-01	.11570E-01	.11657E-01	.11492E-01	.11287E-01
$E_{ov}$	.24510E-02	.27830E-02	.28110E-02	.27670E-02	.27670E-02

$W_2 = \text{diag} (2.0, 1.0, 2.0, 1.0, 2.0)$

TABLE (4.8)  
50 MW UNIT THERMAL GENERATION (GAS)

Parameter	Weighted Least Squares $W = W_3$	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$a_s$	.52703E+02	.53608E+02	.53466E+02	.53592E+02	.53607E+02
$b_s$	.10711E+02	.10662E+02	.10669E+02	.10663E+02	.10662E+02
$\gamma_s$	.11137E-01	.11648E-01	.11581E-01	.11629E-01	.11650E-01
$E_{ov}$	.24920E-02	.28470E-02	.27920E-02	.28420E-02	.28460E-02

$W_3 = \text{diag}(2.0, 1.0, 2.0, 1.0, 2.0)$

TABLE (4.9)  
THE HYDRO GENERATION MODEL

Parameter	Weighted Least Squares $W = W_4$	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$a_h$	.11028E+01	.10987E+01	410	.10986E+01	.11020E+01
$b_h$	.71876E-01	.71905E-01	CONVERGENCE	.71907E-01	.71884E-01
$\gamma_h$	.20092E-04	.20049E-04		.20047E-04	.20076E-04
$E_{ov}$	.14750E-01	.14740E-01		.14740E-01	.14750E-01

$W_4 = I$  Unit matrix.

TABLE (4.10)

## TYPICAL COST COEFFICIENTS

UNIT SIZE (MW)	COAL			OIL			GAS		
	a	b	y	a	b	y	a	b	y
50.0	49.92	10.06	.0103	52.87	10.470	.01160	53.62	10.66	.01170
200.0	173.61	8.67	.0023	180.68	9.039	.00238	182.62	9.19	.00235
400.0	300.84	8.14	.0015	312.35	8.52	.00150	316.45	8.61	.00150
600.0	462.28	8.28	.00053	483.44	8.65	.00056	490.02	8.73	.00059
800.0	751.39	7.48	.00099	793.22	7.74	.00107	824.40	7.73	.00117
1200.0	1130.80	7.47	.00067	1194.60	7.22	.00072	1240.32	7.72	.00078

$M_5 = \text{Unit matrix}$

Parameter	Weighted Least Squares	Gauss-Markovardt	Newton-Raphson	Regression	Power-law Regression
$k_{10}$	.7234E-01	-.1452E+00	-.1704E+00	-.2888E-01	.7149E-01
$b_{10}$	.6985E-01	.9280E-01	.1372E+00	-.2947E-01	.6982E-01
$b_{20}$	-.5184E+00	-.2316E+00	-.2044E+00	-.3705E+00	-.5181E+00
$b_{11}$	-.4371E-01	.1706E-01	-.3092E-03	.4879E-01	-.4341E-01
$b_{12}$	.1145E+00	.1008E+00	.8830E-01	.1372E+00	.1148E+00
$b_{22}$	.2948E+00	.2005E+00	.1933E+00	.2421E+00	.2950E+00
$E_{01}$	.9906E-02	.4663E-02	.4579E-02	.4549E-02	.5992E-02

$M = M_5$

#### 5 BUS TEST SYSTEM B-COEFFICIENT MODEL PARAMETERS

TABLE (4.11)

TABLE (4.12)

## A.E.P 14 BUS TEST SYSTEM B-COEFFICIENT MODEL

## PARAMETERS

Para-meter	Weighted Least Squares N= W6	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
K <sub>L0</sub>	-.3173E+00	-.9612E+00	-.4214E+00	-.4831E+00	-.3821E+00
B <sub>10</sub>	.3295E+00	.4356E+00	.2460E+00	.2924E+00	.3541E+00
B <sub>20</sub>	-.4180E+00	.5049E+00	-.3569E+00	-.3110E+00	-.3989E+00
B <sub>11</sub>	-.5625E-01	-.9789E-02	-.1221E-01	-.2094E-01	-.5674E-01
B <sub>12</sub>	.6924E-01	-.4432E-01	.8887E-01	.8022E-01	.7030E-01
B <sub>22</sub>	.2150E+00	-.7549E-01	.2053E+00	.1967E+00	.2160E+00
E <sub>ov.</sub>	.4561E-02	.5347E-02	.4796E-02	.4784E-02	.4899E-02

W6 = Unit matrix.

TABLE (4.13)  
A.E.P 30 BUS TEST SYSTEM B-COEFFICIENT MODEL

PARAMETERS					
parameter	Weighted Least Squares $W = W_7$	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$K_{L0}$	-.5562E-01	.3447E+00	-.3039E-01	-.1607E+00	-.5562E-01
$B_{10}$	.9446E-01	.1894E-01	.8532E-01	.1088E+00	.9446E-01
$B_{20}$	.6665E-01	-.6690E+00	.5920E-01	.3280E+00	.6665E-01
$B_{30}$	.1609E+00	.3561E+00	.1191E+00	.1128E-01	.1609E+00
$B_{11}$	-.2870E-02	-.2155E-01	-.2596E-02	.3841E-02	-.2869E-02
$B_{12}$	-.9380E-02	.7930E-01	-.9463E-02	-.4312E-01	-.9380E-02
$B_{13}$	-.3220E-01	-.9873E-01	-.2666E-01	-.2527E-02	-.3220E-01
$B_{22}$	-.1122E-02	.2047E+00	-.1598E-02	-.7905E-01	-.1122E-02
$B_{23}$	-.3051E-01	.1571E-01	-.2501E-01	-.3761E-01	-.3051E-01
$B_{33}$	-.2663E-01	-.1418E+00	-.1495E-01	.3808E-01	-.2662E-01
$E_{ov}$	.1513E-02	.2363E-02	.1439E-02	.1220E-02	.1513E-02

$W_7 = \text{Unit matrix}$

TABLE (4-14)

## 5 BUS TEST SYSTEM ACTIVE-REACTIVE MODE

#### ACTIVE COMPONENT PARAMETERS

Parameter	Weighted Least Squares $W = WB$	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$K_{LOP}$	.6698E-01	.5190E-02	.5354E-01	.5785E-01	.6629E-01
$E_{PP1}$	-.3013E-01	.1995E-01	-.2272E-01	-.2890E-01	-.2957E-01
$E_{PP2}$	-.4894E-01	-.5165E-02	-.4831E-01	-.5020E-01	-.4926E-01
$E_{PQ1}$	.3069E-01	.3512E-01	.2463E-01	.2230E-01	.3068E-01
$E_{PQ2}$	-.1836E-01	.2101E-01	.1539E-01	.1534E-01	.1805E-01
$A_{P11}$	.1316E-01	.4787E-02	.1398E-01	.1629E-01	.1307E-01
$A_{P12}$	.6863E-02	.8709E-03	.9315E-02	.1025E-01	.7034E-02
$A_{P22}$	.1452E-01	.1039E-01	.1901E-01	.1862E-01	.1496E-01
$B_{P12}$	-.2270E-02	-.3391E-02	-.7128E-03	-.2103E-03	-.2280E-02
$E_{av}$	.5030E-03	.6012E-03	.4922E-03	.4833E-03	.5062E-03

TABLE- (4.15)

Parameter	Weighted Least Squares $W = W_9$	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$K_{LOQ}$	-1761E+00	-2544E+00	-1774E+00	-2220E+00	-1760E+00
$E_{QP1}$	.5130E-02	.5459E-01	.5149E-02	.1363E-01	.5126E-02
$E_{QP2}$	-1290E+00	-8117E-01	-1259E+00	-1004E+00	-1291E+00
$E_{QQ1}$	.1564E+00	.1366E+00	.1590E+00	.1257E+00	.1565E+00
$E_{QQ2}$	.6423E-01	.6510E-01	.6594E-01	.6283E-01	.6427E-01
$A_{Q11}$	.4026E-02	.3827E-02	.4992E-02	.1737E-01	.4025E-02
$A_{Q12}$	.7991E-02	.7321E-02	.7669E-02	.1407E-01	.7998E-02
$A_{Q22}$	.5294E-01	.5244E-01	.5157E-01	.5250E-01	.5295E-01
$B_{Q12}$	-1394E-01	-8510E-02	-1234E-01	-6422E-02	-1394E-01
$E_{ov}$	.2373E-03	.2074E-03	.2642E-03	.1923E-03	.2390E-03

TABLE (4.16)

A.E.P 14 BUS TEST SYSTEM ACTIVE-REACTIVE MODEL  
ACTIVE COMPONENT PARAMETERS

Parameter	Weighted Least Squares W = W10	Linear Regression	Gauss-Newton	Marquardt	Powell-Regression
K <sub>LOP</sub>	-5749E-01	-.9315E+01	-.1380E+00	-.6055E-01	-.6222E-01
E <sub>PP1</sub>	.2566E-01	.6413E+01	.7631E-01	.1124E-01	.2740E-01
E <sub>PP2</sub>	-.3734E+00	.5578E+01	-.3580E+00	.3897E+00	-.3711E+00
E <sub>PQ1</sub>	-.6026E-02	.1306E+01	-.8349E-03	-.1580E-01	-.6681E-02
E <sub>PQ2</sub>	-.2481E+00	.6751E+00	-.2782E+00	-.2578E+00	-.2481E+00
A <sub>P11</sub>	.1876E-01	-.1101E+01	.1087E-01	.2492E-01	.1868E-01
A <sub>P12</sub>	.9038E-01	-.9404E+00	.9040E-01	.9713E-01	.9035E-01
A <sub>P22</sub>	.1687E+00	-.8162E+00	.1751E+00	.1753E+00	.1684E+00
B <sub>P12</sub>	-.3051E-01	-.2472E-02	-.3550E-01	-.3025E-01	-.3070E-01
E <sub>ov</sub>	.1529E-02	.4274E-02	.1217E-02	.1142E-02	.1471E-02

W10 = diag(2.0,2.0,2.0,2.0,2.0,2.0,2.0,2.0,9.0,4.0,4.0,4.0,2.0,2.0,2.0,  
 1.0,2.0,2.0,2.0,4.0,2.0,2.0,2.0,2.0,16.0,2.0,9.0,9.0,2.0,  
 4.0,4.0)

TABLE (4.17)

## A.E.P 14 BUS TEST SYSTEM ACTIVE-REACTIVE MODEL

## REACTIVE COMPONENT PARAMETERS

Parameter	Weighted Least Squares W=W11	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
K <sub>LQ</sub>	-.1598E+01	-.4035E+02	-.1351E+01	-.9995E+00	-.1598E+01
E <sub>QP1</sub>	.6303E+00	.2767E+02	.5948E+00	.6445E+00	.6303E+00
E <sub>QP2</sub>	-.5590E-01	.2458E+02	-.2815E+00	-.1219E+01	-.5587E-01
E <sub>QQ1</sub>	.1069E+00	.5775E+01	.1643E+00	.1762E+00	.1075E+00
E <sub>QQ2</sub>	-.3182E+00	.3144E+01	-.5282E+00	-.1223E+01	-.3181E+00
A <sub>Q11</sub>	.2307E-01	-.4780E+01	-.7551E-03	-.6505E-01	.2306E-01
A <sub>Q12</sub>	.1359E+00	-.4172E+01	.1562E+00	.2935E+00	.1359E+00
A <sub>Q22</sub>	.2682E+00	-.3738E+01	.3253E+00	.6527E+00	.2683E+00
B <sub>Q12</sub>	-.5227E-01	-.1060E-01	-.9427E-01	-.1860E+00	-.5222E-01
E <sub>ov</sub>	.3179E-02	.9173E-02	.3320E-02	.3291E-02	.3164E-02

W11 = diag(4.0,4.0,4.0,4.0,4.0,4.0,4.0,4.0,16.0,4.0,4.0,4.0,16.0,4.0,4.0,  
 4.0,4.0,16.0,4.0,4.0,16.0,4.0,4.0,4.0,4.0,4.0,4.0,16.0,4.0,16.0,  
 4.0,16.0,16.0,4.0)

TABLE (4.18)

## A.E.P 30 BUS TEST SYSTEM ACTIVE-REACTIVE MODEL

## ACTIVE COMPONENT PARAMETERS

Parameter	Weighted Least Squares W=W12	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
K <sub>LOP</sub>	.2491E-01	.2309E-01	.2491E-01	.2707E-01	.2491E-01
E <sub>PP1</sub>	.2218E-01	.4889E-01	.2218E-01	.2155E-01	.2219E-01
E <sub>PP2</sub>	.3626E-01	.2092E-01	.3626E-01	.3570E-01	.3626E-01
E <sub>PP3</sub>	-.2006E-01	.7366E-01	-.2806E-01	-.2007E-01	-.2805E-01
E <sub>PQ1</sub>	-.2270E-01	.5898E-01	-.2270E-01	.6497E-02	-.2270E-01
E <sub>PQ2</sub>	-.3497E-01	.1565E-01	-.3497E-01	-.9039E-02	-.3498E-01
E <sub>PQ3</sub>	.7366E-01	-.8405E-02	.7365E-01	.6433E-01	.7366E-01
A <sub>P11</sub>	.7807E-02	-.2016E-02	.7806E-02	.7706E-02	.7806E-02
A <sub>P12</sub>	-.6233E-02	-.6595E-02	-.6232E-02	-.5934E-02	-.6234E-02
A <sub>P13</sub>	-.4826E-02	-.2090E-01	-.4825E-02	-.4443E-02	-.4826E-02
A <sub>P22</sub>	-.5200E-02	.4670E-02	-.5200E-02	-.5510E-02	-.5200E-02
A <sub>P23</sub>	-.7117E-02	-.1510E-01	-.7116E-02	-.7518E-02	-.7117E-02
A <sub>P33</sub>	.2895E-01	-.7642E-02	.2895E-01	.2303E-01	.2895E-01
B <sub>P12</sub>	-.2330E-02	-.6066E-02	-.2330E-02	-.1855E-02	-.2330E-02
B <sub>P13</sub>	-.1849E-02	-.7233E-02	-.1849E-02	-.2487E-02	-.1850E-02
B <sub>P23</sub>	.2970E-02	-.1989E-02	.2970E-02	.1347E-02	.2971E-02
E <sub>ov</sub>	.1864E-03	.3174E-03	.1859E-03	.1459E-03	.1856E-03

W12 = Unit matrix

TABLE (4,19)

REACTIVE COMPONENT PARAMETERS					
Parameter	Weighted Least Squares W=W13	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
K <sub>LOQ</sub>	- .2362E+00	.2985E+00	-.2612E+00	-.2284E+00	-.2359E+00
E <sub>QP1</sub>	.2295E+00	-.1463E-01	.2290E+00	.2220E+00	.2292E+00
E <sub>QP2</sub>	.1261E+00	.1401E+00	.1232E+00	.6520E-01	.1263E+00
E <sub>QP3</sub>	-.5029E-01	-.4563E+00	-.3424E-01	-.2463E-01	-.5068E-01
E <sub>QQ1</sub>	.4046E+00	-.2284E+00	.2843E+00	.2949E+00	.4058E+00
E <sub>QQ2</sub>	.2797E+00	-.1455E+00	.1834E+00	.1944E+00	.2810E+00
E <sub>QQ3</sub>	.9301E-01	-.5745E+00	.1348E+00	.1439E+00	.9314E-01
A <sub>Q11</sub>	-.1474E-01	.4828E-01	-.1236E-01	-.1382E-01	-.1473E-01
A <sub>Q12</sub>	-.3011E-01	-.2153E-01	-.2900E-01	-.2212E-01	-.3012E-01
A <sub>Q13</sub>	-.4525E-01	.1314E-01	-.5286E-01	-.5696E-01	-.4512E-01
A <sub>Q22</sub>	-.1254E-01	-.4299E-01	-.8030E-02	.7094E-02	-.1259E-01
A <sub>Q23</sub>	-.3732E-01	-.1678E-01	-.4191E-01	-.3713E-01	-.3737E-01
A <sub>Q33</sub>	.1621E+00	.3434E+00	.1718E+00	.1629E+00	.1620E+00
B <sub>Q12</sub>	.2319E-02	.1267E-01	.1860E-02	.4296E-02	.2338E-02
B <sub>Q13</sub>	-.8329E-02	-.3577E-01	-.8104E-02	-.8200E-02	-.8353E-02
B <sub>Q23</sub>	-.8730E-02	-.2345E-01	-.5281E-02	-.5797E-02	-.8709E-02
E <sub>ov</sub>	.5608E-03	.6224E-03	.3399E-03	.2995E-03	.5519E-03

$$W_{13} = \text{diag}(1, 0, -2, 0, 1, 0, -4, 0, 1, 0, 4, 0, 1, 0, 2, 0, 2, 0, 1, 0, -2, 0, 4, 0, 1, 0, 1, 0,$$

1,0,1,0,1,0,1,0,1,0,2,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0,

2,0,1,0,1,0,-4,0,4,0,1,0,4,0,1,0,-1,0,1,0,1,0,4,0)

## CHAPTER V

OPTIMUM ACTIVE AND ACTIVE-REACTIVE  
POWER ALLOCATIONS

This chapter is divided into three sections. Section one is devoted to the description of the algorithms used to allocate the optimum active and/or active-reactive power generations of a power system. This we apply to our three test systems. Section two involves the implementation of the obtained optimum powers into the respective original networks and the load flow equations are solved to study their impact on the network performance. The final section is an extension to the first in that instead of carrying out the optimization procedure for one set of loading patterns the procedure is performed for a range of loading patterns recording the optimal strategy in each case.

Results for the above work are given for all types of thermal plants and are included in this chapter. The hydro generation case is not considered in this thesis due to time limitations and may be treated in the same manner.

As pointed out in chapter one, the results of the optimization procedure are intended to be used as an extra measure for judging the merit of each estimation technique and its results. This is so since different optimal strategies will result from the different parameters estimated.

THE OPTIMIZATION PROCEDURE

The approach is to minimize the cost related objective function subject to equality and inequality constraints on the control variables

that is  $P_G$  and/or  $P_{G_i}$  and  $Q_G$  (9,10,46-51). The method of Newton-Raphson is used for finding the optimality conditions. This we treat next, but first we give a brief description of the Newton-Raphson method.

#### THE NEWTON-RAPHSON METHOD

This solution procedure is very efficient compared to other techniques from both speed and storage points of view. The technique solves for a number of linear or nonlinear equations simultaneously and is known to converge in most cases for power system applications.

The problem at hand is to find a vector  $\underline{U}$  that satisfies the nonlinear equation:

$$\underline{f}(\underline{U}) = \underline{0} \quad \dots \quad (5.1)$$

the procedure is to update the estimate of the unknown vector  $\underline{U}$  starting with some initial vector  $\underline{U}^0$  in such a way that hopefully after a number of iterations,

$$\underline{f}(\underline{U}^0 + \underline{\Delta U}) \leq \epsilon \quad \dots \quad (5.2)$$

where  $\epsilon$  is some desired tolerance.

Basically to develop the algorithm, the function  $f$  is expanded by Taylor's expansion about the initial guess vector  $\underline{U}^0$ ,

$$\underline{f}(\underline{U}^0 + \underline{\Delta U}) = \underline{f}(\underline{U}^0) + \frac{\partial \underline{f}(\underline{U})}{\partial \underline{U}} \Bigg|_{\underline{U}^0} \underline{\Delta U}^0 + \text{higher order terms, } \dots \quad (5.3)$$

ignoring the nonlinear terms in  $\underline{\Delta U}^0$  and solving equation (5.3) for  $\underline{\Delta U}^0$  we obtain for the updating increment,

$$\underline{\Delta U}^0 = - \left[ \frac{\underline{af}(U)}{\underline{a_U}} \Bigg|_{\underline{U}^0} \right]^{-1} \underline{f}(\underline{U}^0), \quad \dots \dots \dots (5.4)$$

the matrix  $\underline{af}/\underline{a_U}$  is called the Jacobian matrix  $J$  and  $\underline{\Delta U}^0$  is the control update vector. The new vector is given by:

$$\underline{U}^1 = \underline{U}^0 + \underline{\Delta U}^0, \quad \dots \dots \dots (5.5)$$

and in general for the  $k$ th iteration we have ,

$$\underline{\Delta U}^k = - J^{-1} \Bigg|_{\underline{U}^k} \cdot \underline{f}(\underline{U}^k), \quad \dots \dots \dots (5.6)$$

equations (5.5) and (5.6) define the Newton-Raphson algorithm.

### 5.1 THE OPTIMIZATION CONDITIONS

Two similar sets of optimality conditions are given below, the first corresponds to the active power loss model, the second to the active-reactive power loss model.

#### A. OPTIMUM ACTIVE POWER ALLOCATION

Consider the all-thermal electric power system shown in figure (5.1). Let us assume that we have  $m$  generating units supplying power to the total system active power demand  $P_D$ , through a transmission network represented by the B-Coefficient model. The total cost

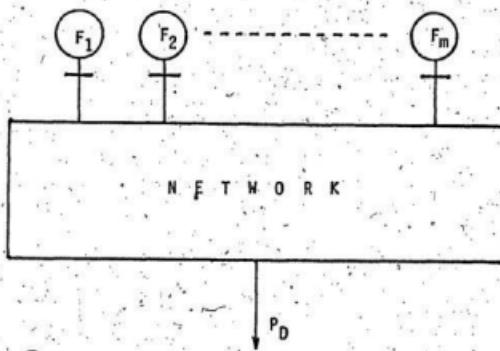


FIGURE (5.1) AN ELECTRIC POWER SYSTEM  
WITH  $m$  GENERATORS

function  $F_a$  is given by:

$$F_0 = \sum_{j=1}^m \alpha_j + \beta_1 P_1 + \gamma_1 P_1^2, \quad \dots \dots \dots \quad (5.8)$$

where  $P_i$  for  $i=1,2,\dots,m$  represent the active power generation of unit  $i$ , with the subscript G dropped for convenience and  $a_i$ ,  $b_i$  and  $c_i$  are the cost coefficients of the  $i$ th thermal unit.

We wish to minimize the total cost function  $F_0$  subject to the following equality and inequality constraints:

$$P_D + P_L = \sum_{i=1}^m P_i \quad \dots \dots \dots (5.9)$$

where  $P_L$  is the total system active power loss. An augmented cost function is formed,

$$F_A = F_0 + \lambda_p (P_D + P_L - \sum_{i=1}^m P_i), \quad \dots \dots \dots (5.11)$$

where  $\lambda_p$  is a Lagrange type multiplier. This is essentially the incremental cost of power delivered.

For optimality the first partial derivatives of  $F_A$  with respect to  $P_i$ ,  $i=1,2,\dots,m$ , and  $\lambda_0$  must be equated to zero, thus,

$$\frac{\partial F_A}{\partial P_1} = f_{P1} = \beta_1 + 2\gamma_1 P_1 + \lambda_p \left( \frac{\partial P_L}{\partial P_1} - 1 \right) = 0, \quad \dots \dots (5.12)$$

for  $i=1, 2, \dots, m$  and,

$$\frac{\partial F_A}{\partial \lambda_p} = f_{PD} = P_D + P_L - \sum_{i=1}^m P_i = 0; \quad \dots \dots \dots (5.13)$$

we set the above  $(m+1)$  equations for  $(m+1)$  unknowns in a format given by equation (5.6) above to solve for  $P_i$ ;  $i=1,2,\dots,m$ , and  $\lambda_p$  iteratively. The Jacobian in this case is given by:

$f$	$P_1$	$P_2$	$\dots$	$P_m$	$\lambda_p$
$f_{p1}$	$\frac{\partial f_{p1}}{\partial P_1}$	$\frac{\partial f_{p1}}{\partial P_2}$	$\dots$	$\frac{\partial f_{p1}}{\partial P_m}$	$\frac{\partial f_{p1}}{\partial \lambda_p}$
$f_{p2}$	$\frac{\partial f_{p2}}{\partial P_1}$	$\frac{\partial f_{p2}}{\partial P_2}$	$\dots$	$\frac{\partial f_{p2}}{\partial P_m}$	$\frac{\partial f_{p2}}{\partial \lambda_p}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
$f_{pm}$	$\frac{\partial f_{pm}}{\partial P_1}$	$\frac{\partial f_{pm}}{\partial P_2}$	$\dots$	$\frac{\partial f_{pm}}{\partial P_m}$	$\frac{\partial f_{pm}}{\partial \lambda_p}$
$f_{PD}$	$\frac{\partial f_{PD}}{\partial P_1}$	$\frac{\partial f_{PD}}{\partial P_2}$	$\dots$	$\frac{\partial f_{PD}}{\partial P_m}$	0

(5.14)

and the vector  $f$  is given by:

$$\underline{f} = [f_{p1} \ f_{p2} \ \dots \ f_{pm} \ f_{PD}]^T. \quad \dots \dots \dots (5.15)$$

where  $f_{pi}$  for  $i=1,2,\dots,m$  are given by equation (5.12) above.

and  $f_{PD}$  by equation (5.13).

Both sets of equations (5.14) and (5.15) are evaluated at the initial starting guesses of  $P$  and  $\lambda_p$ . The new values of  $P_i$ ;  $i=1, 2, \dots, m$  and  $\lambda_p$  are calculated by updating the initial guesses by the increment  $\Delta_i$ . If convergence is achieved as judged by no further improvement in  $\Delta_i$  or the elements of vector  $f$  approach a certain tolerance, the process is stopped, otherwise the values of the unknown are updated and the process continued until either convergence is reached or a maximum number of iterations is exceeded. The algorithm is set up such that if any of the control variables violates an inequality constraint, the variable is set to the limit (as dictated by the Kuhn-Tucker conditions(52)), and the algorithm then solves for the new reduced problem until convergence is achieved. The iterative process for the above is illustrated in the flow chart of figure(5.2). One important factor in using a Newton-Raphson based algorithm is to have an initial vector reasonably close to the optimal solution(53,54). Two methods for generating such initial guesses for our purposes are given in appendix C.

Results of the optimization solution corresponding to the B-Coefficient model with its parameters estimated by the five parameter estimation techniques for all thermal plants with Coal,Oil and Gas are given in tables (5.1) to (5.9).

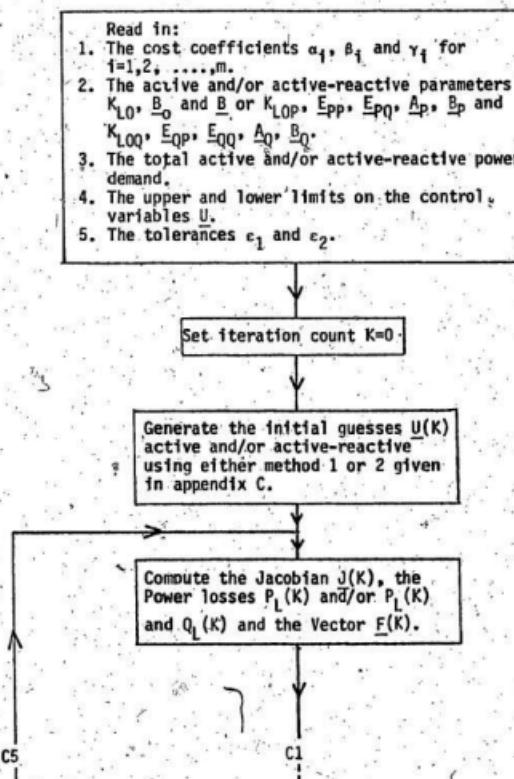


FIGURE 5.2 NEWTON-RAPHSON BASED OPTIMIZATION ALGORITHM

FLOW CHART

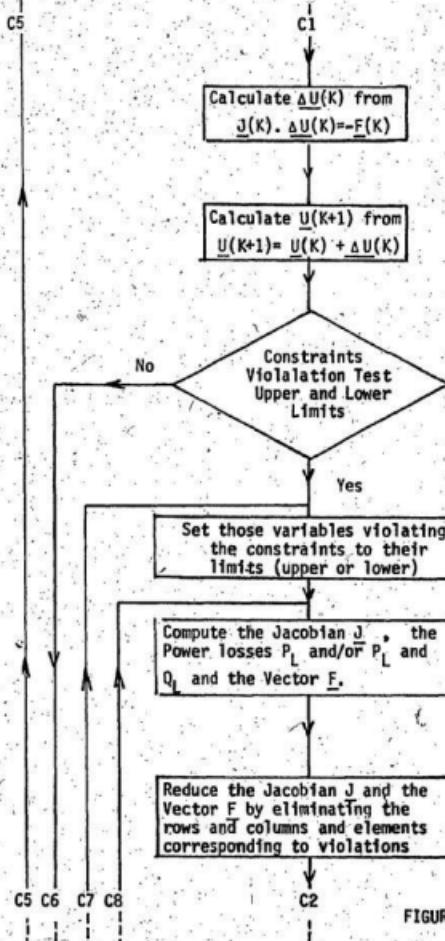


FIGURE (5.2) CONT.

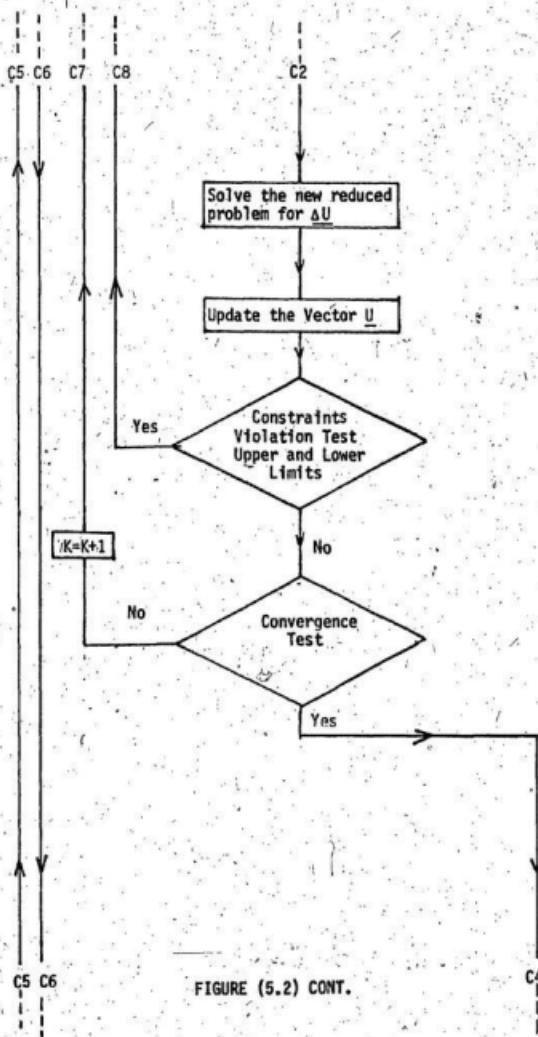


FIGURE (5.2) CONT.

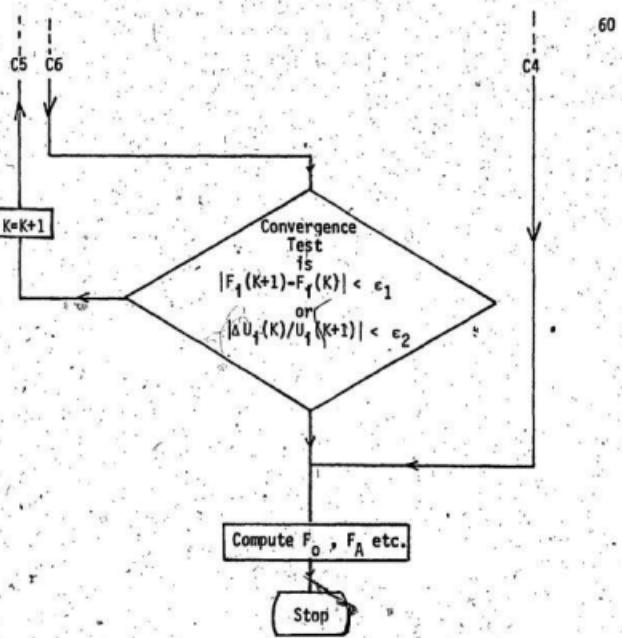


FIGURE (5.2) CONT.

NOTES TO THE FOLLOWING TABLES

1.  $P_D$  is the total system active power demand in MW.
2.  $P_{Gi}$ ;  $i=1,2,\dots,m$  is the optimum active power generation in MW of unit  $i$ .
3.  $P_L$  is the total system active power loss in MW that correspond to the optimum power generation.
4.  $\lambda_p$  is the incremental cost.
5.  $F_0$  is the total generation cost.
6.  $F_A$  is the augmented cost.
7.  $\mu$  is the largest ratio of the incremental change ( $\Delta U / U$ ) in the unknown variable  $U_i$  ( $U = [P_1, P_2, \dots, P_m, \lambda_p]^T$ ).
8.  $\sigma = \sum (\Delta F_A / \Delta U)^2$  is the sum of squares of the first partial derivatives of  $F_A$  at the solution.
9. FBIG is the largest element of the vector  $f$  in Newton-Raphson based algorithm.
10. The inequality constraints are:
  - A. The 5 Bus test system ,  
 $10 \leq P_1 \leq 200$  MW  
 $10 \leq P_2 \leq 200$  MW
  - B. The 14 Bus test sytem,  
 $50 \leq P_1 \leq 400$  MW  
 $10 \leq P_2 \leq 200$  MW

C. The 30 Bus test system

$$50 \leq P_1 \leq 400 \text{ MW}$$

$$10 \leq P_2 \leq 200 \text{ MW}$$

$$10 \leq P_3 \leq 200 \text{ MW}$$

11. The cost coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  for the above thermal units are given in table (4.10).
12. The optimization procedure was carried out using the B-Coefficient model with its parameters estimated by the indicated method in each table.
13. Convergence Criterion,  
 $\epsilon_1 = FBIG \leq 1.E-05$  or  $\epsilon_2 = u \leq 1.E-04$

TABLE (5.1)  
OPTIMUM DISPATCH OF 5 BUS TEST SYSTEM

FUEL- COAL

Vari- able	Weighted Least Squares	Linear Regression	Gauss- Newton	Marquardt	Powell Regression
$P_D$	165.000	165.000	165.000	165.000	165.000
$P_{G1}$	68.921	68.834	68.836	68.856	68.916
$P_{G2}$	99.342	99.439	99.436	99.417	99.344
$P_L$	3.262	3.273	3.272	3.273	3.260
$\lambda_p$	11.782	13.152	13.070	13.035	11.795
$F_o$	1839.680	1839.790	1839.776	1839.785	1839.657
$F_A$	1839.680	1839.790	1839.776	1839.785	1839.657
$u$	.3893E-05	.4900E-06	.3059E-06	.1596E-05	.3788E-05
$\sigma$	.8913E-10	.9095E-12	.0	.2365E-10	.9095E-10
$FBIG$	.6676E-05	.9537E-06	.0	.4768E-05	.7630E-05

TABLE (5.2)  
OPTIMUM DISPATCH OF 5-BUS TEST SYSTEM  
FUEL-OIL

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$P_D$	165.000	165.000	165.000	165.000	165.000
$P_{G1}$	68.843	68.754	68.756	68.775	68.838
$P_{G2}$	99.418	99.518	99.515	99.496	99.420
$P_L$	3.261	3.272	3.271	3.272	3.259
$\lambda_p$	12.284	13.710	13.625	13.588	12.297
$F_o$	1917.078	1917.192	1917.178	1917.188	1917.055
$F_A$	1917.078	1917.192	1917.178	1917.188	1917.055
$\mu$	.5549E-06	.6190E-06	.3062E-06	.3381E-06	.1110E-05
$\sigma$	.9095E-12	.1819E-11	.0	.0	.2364E-10
FBIG	.9537E-06	.9537E-06	.0	.0	.4768E-05

TABLE (5.3)

## OPTIMUM DISPATCH OF 5 BUS TEST SYSTEM

## FUEL- GAS

Vari- able	Weighted Least Squares	Linear Regression	Gauss- Newton	Marquardt	Powell Regression
P <sub>D</sub>	165.000	165.000	165.000	165.000	165.000
P <sub>G1</sub>	68.542	68.438	68.440	68.460	68.536
P <sub>G2</sub>	99.715	99.830	99.825	99.807	99.719
P <sub>L</sub>	3.257	3.268	3.266	3.267	3.254
$\lambda_p$	12.490	13.934	13.847	13.810	12.503
F <sub>O</sub>	1945.926	1946.042	1946.027	1946.037	1945.903
F <sub>A</sub>	1945.926	1946.042	1946.027	1946.037	1945.903
$\mu$	.3695E-05	.3442E-06	.5067E-06	.5253E-06	.6667E-06
$\sigma$	.1164E-09	.1819E-11	.9095E-12	.1819E-11	.4547E-11
FBIG	.7629E-05	.9537E-06	.9537E-06	.9537E-06	.1907E-05

TABLE T5.4  
OPTIMUM DISPATCH OF A.E.P 14 BUS TEST SYSTEM

Variable	FUEL- COAL			
	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt
P <sub>D</sub>	259.000		259.000	259.000
P <sub>G1</sub>	183.601	NO	183.236	183.242
P <sub>G2</sub>	87.261	CONVERGENCE	87.682	87.678
P <sub>L</sub>	11.822		11.878	11.879
$\lambda_p$	11.493		13.516	13.501
F <sub>O</sub>	2793.597		2794.239	2794.248
F <sub>A</sub>	2793.597		2794.239	2794.248
$\mu$	.8252E-06		.8019E-06	.1085E-05
$\sigma$	.1819E-11		.1819E-11	.7276E-11
F <sub>BIG</sub>	.9537E-06		.9537E-06	.1907E-05

TABLE (5.5)  
OPTIMUM DISPATCH OF A.E.P. 14 BUS TEST SYSTEM

## FUEL - OIL

Vari- able	Weighted Least Squares	Linear Regression	Gauss- Newton	Marquardt	Powell Regression
P <sub>D</sub>	259.000		259.000	259.000	259.000
P <sub>G1</sub>	182.207	NO	181.776	181.783	182.036
P <sub>G2</sub>	88.597	CONVERGENCE	89.080	89.075	88.795
P <sub>L</sub>	11.764		11.816	11.818	11.791
$\lambda_p$	12.044		14.163	14.148	12.461
F <sub>O</sub>	2914.743		2915.408	2915.419	2915.063
F <sub>A</sub>	2914.743		2915.408	2915.419	2915.063
$\mu$	.1309E-05		.3121E-06	.8034E-06	.3512E-06
$\sigma$	.1819E-11		.0	.9095E-12	.1819E-11
FBIG	.9537E-06		.0	.9537E-06	.9537E-06

TABLE (5.6)

## OPTIMUM DISPATCH OF A.E.P 24 BUS TEST SYSTEM

## FUEL- GAS

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Power Regression
P <sub>D</sub>	259.000		259.000	259.000	259.000
P <sub>G1</sub>	187.522	NO	187.307	187.311	187.544
P <sub>G2</sub>	83.506	CONVERGENCE	83.784	83.781	83.520
P <sub>L</sub>	11.988		12.051	12.053	12.024
$\lambda_p$	11.977		14.093	14.076	12.375
F <sub>O</sub>	2950.191		2950.883	2950.893	2950.520
F <sub>A</sub>	2950.191		2950.883	2950.893	2950.520
$\mu$	.8303E-06		.7977E-06	.1101E-05	.8170E-06
$\sigma$	.1819E-11		.4547E-11	.1182E-10	.1819E-11
FBIG	.9537E-06		.1907E-05	.2861E-05	.9537E-06

TABLE (5.7)

## OPTIMUM DISPATCH OF A.E.P 30 BUS TEST SYSTEM

FUEL- COAL

Var- able	Weighted Least Squares	Linear Regression	Gauss- Newton	Marquardt	Powell Regression
P <sub>D</sub>	283.400		283.400		283.400
P <sub>G1</sub>	160.399	NO	160.434	NO	160.399
P <sub>G2</sub>	59.362	CONVERGENCE	59.371	CONVERGENCE	59.361
P <sub>G3</sub>	74.783		74.739		74.784
P <sub>L</sub>	11.094		11.094		11.094
A <sub>p</sub>	8.851		8.851		8.851
F <sub>O</sub>	3176.301		3176.289		3176.303
F <sub>A</sub>	3176.301		3176.289		3176.303
$\mu$	.3855E-05		.1283E-04		.8652E-05
$\sigma$	.9095E-12		.1000E-10		.4547E-11
FBIG	.9537E-06		.2861E-05		.1907E-05

TABLE (5.8)

## OPTIMUM DISPATCH OF A.E.P 30 BUS TEST SYSTEM

FUEL- OIL

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$P_D$	283.400		283.400		283.400
$P_{G1}$	158.489	NO	158.530	NO	158.489
$P_{G2}$	60.482	CONVERGENCE	60.492	CONVERGENCE	60.480
$P_{G3}$	75.499		75.450		75.501
$P_L$	11.020		11.021		11.020
$\lambda_p$	9.230		9.230		9.230
$F_o$	3313.119		3313.105		3313.117
$F_A$	-3313.119		3313.105		3313.117
$\mu$	.3514E-05		.3574E-04		.2883E-04
$\sigma$	.2728E-11		.7822E-10		.1546E-10
$F_{BIG}$	.9537E-06		.8583E-05		.2861E-05

TABLE (5.9)

## OPTIMUM DISPATCH OF A.E.P 30 BUS TEST SYSTEM

## FUEL- GAS

Vari- able	Weighted Least Squares	Linear Regression	Gauss- Newton	Marquardt	Powell Regression
P <sub>D</sub>	283.400		283.400		283.400
P <sub>G1</sub>	164.894	NO	164.919	NO	164.894
P <sub>G2</sub>	56.472	CONVERGENCE	56.478	CONVERGENCE	56.471
P <sub>G3</sub>	73.354		73.323		73.355
P <sub>L</sub>	11.270		11.270		11.270
$\lambda_p$	9.359		9.357		9.359
F <sub>O</sub>	3355.450		3355.433		3355.448
F <sub>A</sub>	3355.450		3355.433		3355.448
$\mu$	.9231E-04		.8473E-04		.8417E-04
$\sigma$	.2728E-11		.5039E-09		.9095E-12
FBIG	.9537E-06		.1621E-04		.9537E-06

## B. OPTIMUM ACTIVE-REACTIVE POWER ALLOCATION

In a similar approach to A above, consider the electric power system given in figure (5.1), this time we assume that the  $m$  generating thermal units are supplying power to the total complex power demand  $S_D$ , through a transmission network represented by the active-reactive power loss model. The total cost function  $F$ , as before is given by:

$$F_0 = \sum_{i=1}^m a_i \bar{z}_i p_i + \gamma_1 \bar{p}_1^2 \quad \dots \dots \dots (5.16)$$

In this case we wish to minimize the cost function subject to the following equality and inequality constraints:

$$P_D + P_L - \sum_{i=1}^m P_i = 0 , \quad \dots \dots \dots (5.17)$$

$$Q_0 + Q_L - \sum_{i=1}^n Q_i = 0, \quad \dots \dots \dots (5.18)$$

for  $i=1,2,\dots,m$ , where  $m$  is the number of the generating units. An augmented function is formed to include both the active and reactive components.

$$F_A = F_{0A} + \lambda_B (P_D + P_L - \sum_{i=1}^m P_i) + \lambda_Q (Q_D + Q_L - \sum_{i=1}^m Q_i), \dots (5.21)$$

where  $\lambda_p$  and  $\lambda_q$  are the active and reactive related Lagrange type multipliers.

For optimality the first partial derivatives of  $F_A$  with respect to  $P_i, Q_i ; i=1,2,\dots,m$ ,  $\lambda_p$  and  $\lambda_q$  must be equated to zero, hence,

$$\frac{\partial F_A}{\partial P_1} = f_{p1} = b_1 + 2 \gamma_1 P_1 + \lambda_p \left( \frac{\partial P_L}{\partial P_1} - 1 \right) + \lambda_q \left( \frac{\partial Q_L}{\partial P_1} \right) = 0, \quad (5.22)$$

$$\frac{\partial F_A}{\partial Q_1} = f_{q1} = \lambda_p \left( \frac{\partial P_L}{\partial Q_1} \right) + \lambda_q \left( \frac{\partial Q_L}{\partial Q_1} - 1 \right) = 0, \quad (5.23)$$

$$\frac{\partial F_A}{\partial \lambda_p} = f_{pD} = P_D + P_L - \sum_{i=1}^m P_i = 0, \quad (5.24)$$

and,

$$\frac{\partial F_A}{\partial \lambda_q} = f_{qD} = Q_D + Q_L - \sum_{i=1}^m Q_i = 0, \quad (5.25)$$

In this case we have  $2(m+1)$  equations and  $2(m+1)$  unknowns to solve for. For solution, equations (5.22) to (5.25) are set up in a format similar to that of equation (5.6) above, the Jacobian matrix in this is,

$f$	$P_i : i=1,2,\dots,m$	$Q_i : i=1,2,\dots,m$	$\lambda_p$	$\lambda_q$
$f_{p1}$	$\frac{\partial f_{p1}}{\partial P_1}, \frac{\partial f_{p1}}{\partial P_2}, \dots, \frac{\partial f_{p1}}{\partial P_m}$	$\frac{\partial f_{p1}}{\partial Q_1}, \frac{\partial f_{p1}}{\partial Q_2}, \dots, \frac{\partial f_{p1}}{\partial Q_m}$	$\frac{\partial f_{p1}}{\partial \lambda_p}, \frac{\partial f_{p1}}{\partial \lambda_q}$	
$f_{p2}$	$\frac{\partial f_{p2}}{\partial P_1}, \frac{\partial f_{p2}}{\partial P_2}, \dots, \frac{\partial f_{p2}}{\partial P_m}$	$\frac{\partial f_{p2}}{\partial Q_1}, \frac{\partial f_{p2}}{\partial Q_2}, \dots, \frac{\partial f_{p2}}{\partial Q_m}$	$\frac{\partial f_{p2}}{\partial \lambda_p}, \frac{\partial f_{p2}}{\partial \lambda_q}$	

$f_{pm}$	$\frac{\partial f_{pm}}{\partial P_1}, \frac{\partial f_{pm}}{\partial P_2}, \dots, \frac{\partial f_{pm}}{\partial P_m}$	$\frac{\partial f_{pm}}{\partial Q_1}, \frac{\partial f_{pm}}{\partial Q_2}, \dots, \frac{\partial f_{pm}}{\partial Q_m}$	$\frac{\partial f_{pm}}{\partial \lambda_p}, \frac{\partial f_{pm}}{\partial \lambda_q}$
$f_{q1}$	$\frac{\partial f_{q1}}{\partial P_1}, \frac{\partial f_{q1}}{\partial P_2}, \dots, \frac{\partial f_{q1}}{\partial P_m}$	$\frac{\partial f_{q1}}{\partial Q_1}, \frac{\partial f_{q1}}{\partial Q_2}, \dots, \frac{\partial f_{q1}}{\partial Q_m}$	$\frac{\partial f_{q1}}{\partial \lambda_p}, \frac{\partial f_{q1}}{\partial \lambda_q}$
$f_{q2}$	$\frac{\partial f_{q2}}{\partial P_1}, \frac{\partial f_{q2}}{\partial P_2}, \dots, \frac{\partial f_{q2}}{\partial P_m}$	$\frac{\partial f_{q2}}{\partial Q_1}, \frac{\partial f_{q2}}{\partial Q_2}, \dots, \frac{\partial f_{q2}}{\partial Q_m}$	$\frac{\partial f_{q2}}{\partial \lambda_p}, \frac{\partial f_{q2}}{\partial \lambda_q}$
$f_{qm}$	$\frac{\partial f_{qm}}{\partial P_1}, \frac{\partial f_{qm}}{\partial P_2}, \dots, \frac{\partial f_{qm}}{\partial P_m}$	$\frac{\partial f_{qm}}{\partial Q_1}, \frac{\partial f_{qm}}{\partial Q_2}, \dots, \frac{\partial f_{qm}}{\partial Q_m}$	$\frac{\partial f_{qm}}{\partial \lambda_p}, \frac{\partial f_{qm}}{\partial \lambda_q}$
$f_{pD}$	$\frac{\partial f_{pD}}{\partial P_1}, \frac{\partial f_{pD}}{\partial P_2}, \dots, \frac{\partial f_{pD}}{\partial P_m}$	$\frac{\partial f_{pD}}{\partial Q_1}, \frac{\partial f_{pD}}{\partial Q_2}, \dots, \frac{\partial f_{pD}}{\partial Q_m}$	0 0
$f_{qD}$	$\frac{\partial f_{qD}}{\partial P_1}, \frac{\partial f_{qD}}{\partial P_2}, \dots, \frac{\partial f_{qD}}{\partial P_m}$	$\frac{\partial f_{qD}}{\partial Q_1}, \frac{\partial f_{qD}}{\partial Q_2}, \dots, \frac{\partial f_{qD}}{\partial Q_m}$	0 0

(5.26)

and the vector  $\underline{f}$  is given by:

$$\underline{f} = [f_{p1}, f_{p2}, \dots, f_{pm}, f_{q1}, f_{q2}, \dots, f_{qm}, f_{pD}, f_{qD}]^T, \quad (5.27)$$

where  $f_{p1}, f_{q1}, f_{pD}$  and  $f_{qD}$  are given by equations (5.22), (5.23), (5.24) and (5.25) above respectively.

The Jacobian (5.26) and the vector  $\underline{f}$  (5.27) are evaluated at the initial guesses of  $P_i$ ,  $Q_i$ ,  $\lambda_p$  and  $\lambda_q$ . The new values of  $P_i$ ,

$Q_i$ ,  $i=1,2,\dots,m$ ,  $\lambda_p$  and  $\lambda_q$  are calculated by updating the initial guesses by the increment  $\Delta_i$ . Convergence procedure as in A above.

Again as mentioned in A, if any one of the control variables violates a constraint, the algorithm sets the variable to its limit and the new reduced problem is solved iteratively until convergence is reached. See flow diagram given in figure (5.2). For initial guess estimators see appendix C.

Results of the optimization solution corresponding to the active-reactive power loss model with its coefficients estimated by the five parameter estimation techniques for all thermal plants with Coal, Oil and Gas are given in tables (5.10) to (5.18).

NOTES TO THE FOLLOWING TABLES

1.  $P_D$  and  $Q_D$  are the total system active and reactive power demands in MW and MVAR respectively.
2.  $P_{Gi}$  and  $Q_{Gi}$ ;  $i=1, 2, \dots, m$ , are the optimum active and reactive power generations in MW and MVAR, respectively.
3.  $P_L$  and  $Q_L$  are the total system active and reactive power losses in MW and MVAR respectively.
4.  $\lambda_p$  and  $\lambda_q$  are the Lagrange type multipliers (incremental costs) for the active and reactive components.
5.  $F_g$  is the generation cost.
6.  $F_A$  is the augmented cost.
7.  $\mu$  is the largest ratio of the incremental change  $|(\partial U/U)|$  in the unknown variable  $U_i$  ( $U = [P_1, P_2, \dots, P_m, Q_1, Q_2, \dots, Q_m, \lambda_p, \lambda_q]^T$ ) at the solution.
8.  $\sigma = \sum (\partial F_A / \partial U)^2$  is the sum of squares of the first partial derivatives of  $F_A$  at the solution.
9. FBIG is the largest element of the vector  $f$  in Newton-Raphson based algorithm.
10. The inequality constraints are:

A. The 5 Bus test system,

$$10 \leq P_1 \leq 200 \text{ MW} \quad -50 \leq Q_1 \leq 100 \text{ MVAR}$$

$$10 \leq P_2 \leq 200 \text{ MW} \quad -10 \leq Q_2 \leq 50 \text{ MVAR}$$

B. The 14 Bus test system,

$$\begin{array}{ll} 50 \leq P_1 \leq 400 \text{ MW} & -80 \leq Q_1 \leq 100 \text{ MVAR} \\ 10 \leq P_2 \leq 200 \text{ MW} & -40 \leq Q_2 \leq 50 \text{ MVAR} \end{array}$$

C. The 30 Bus system,

$$\begin{array}{ll} 50 \leq P_1 \leq 400 \text{ MW} & -80 \leq Q_1 \leq 200 \text{ MVAR} \\ 10 \leq P_2 \leq 200 \text{ MW} & -40 \leq Q_2 \leq 50 \text{ MVAR} \\ 10 \leq P_3 \leq 200 \text{ MW} & -10 \leq Q_3 \leq 50 \text{ MVAR} \end{array}$$

11. The cost coefficients for the above generators are given in table (4.10).
12. The optimization procedure was carried out using the active-reactive model with its parameters estimated by the indicated method in each table.
13. Convergence criterion,

$$c_1 = FBIG \leq 1.E-05 \quad \text{or} \quad c_2 = u \leq 1.E-04$$

TABLE (5.10)

## OPTIMUM ACTIVE-REACTIVE DISPATCH OF 5 BUS TEST SYSTEM

FUEL- COAL

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
P <sub>D</sub>	165.000	165.000	165.000	165.000	165.000
Q <sub>D</sub>	40.000	40.000	40.000	40.000	40.000
P <sub>G1</sub>	69.648	69.486	69.608	69.648	69.644
Q <sub>G1</sub>	-9.467	-13.362	-13.472	-12.857	-9.998
P <sub>G2</sub>	98.545	98.659	98.538	98.504	98.545
Q <sub>G2</sub>	27.365	31.424	31.336	30.745	27.883
P <sub>L</sub>	3.193	3.145	3.146	3.152	3.188
Q <sub>L</sub>	-22.102	-21.938	-22.136	-22.113	-22.115
$\lambda_p$	9.027	9.113	9.142	9.137	9.034
$\lambda_q$	.2860E+00	.3221E+00	.2659E+00	.2483E+00	.2864E+00
F <sub>O</sub>	1838.948	1838.528	1838.525	1838.575	1838.904
F <sub>A</sub>	1838.948	1830.528	1838.525	1838.575	1838.904
$\mu$	.1360E-05	.2621E-05	.7408E-06	.2295E-06	.2069E-04
$\sigma$	.2335E-09	.2218E-12	.2329E-09	.2048E-13	.4683E-09
FBIG	.1526E-04	.3616E-06	.1526E-04	.1267E-06	.1526E-04

TABLE (5.11)

## OPTIMUM ACTIVE-REACTIVE DISPATCH OF 5 BUS TEST SYSTEM.

## FUEL - OIL

Variable	Weighted Linear Least Squares		Gauss-Newton	Marquardt	Powell Regression
P <sub>D</sub>	165.000	165.000	165.000	165.000	165.000
Q <sub>D</sub>	40.000	40.000	40.000	40.000	40.000
P <sub>G1</sub>	69.567	69.406	69.529	69.569	69.563
Q <sub>G1</sub>	-9.469	-13.367	-13.475	-12.860	-10.000
P <sub>G2</sub>	98.625	98.738	98.617	98.583	98.624
Q <sub>G2</sub>	27.363	31.424	31.335	30.743	27.882
P <sub>L</sub>	3.192	3.144	3.145	3.151	3.187
Q <sub>L</sub>	-22.106	-21.542	-22.140	-22.117	-22.119
$\lambda_p$	9.408	9.498	9.528	9.522	9.516
$\lambda_q$	.2980E+00	.3366E+00	.2771E+00	.2588E+00	.2984E+00
F <sub>O</sub>	1916.316	1915.878	1915.876	1915.927	1916.270
F <sub>A</sub>	1916.316	1915.878	1915.876	1915.927	1916.270
$\mu$	.1101E-05	.1423E-05	.3695E-05	.1125E-04	.1051E-04
$\sigma$	.2043E-11	.1086E-12	.2346E-09	.4705E-09	.4660E-09
FBIG	.1390E-05	.2570E-06	.1526E-04	.1526E-04	.1526E-04

TABLE (5.12)

## OPTIMUM ACTIVE-REACTIVE OF 5 BUS TEST SYSTEM

## FUEL-GAS

Vari- able	Weighted Least Squares	Linear Regression	Gauss- Newton	Marquardt	Powell Regression
$P_D$	165.000	165.000	165.000	165.000	165.000
$Q_D$	40.000	40.000	40.000	40.000	40.000
$P_{G1}$	69.251	69.090	69.213	69.254	69.247
$Q_{G1}$	-9.478	-13.383	-13.486	-12.869	-10.010
$P_{G2}$	98.937	99.050	98.927	98.893	98.935
$Q_{G2}$	27.355	31.425	31.338	30.736	27.875
$P_L$	3.188	3.139	3.141	3.147	3.183
$Q_L$	-22.122	-21.958	-22.156	-22.133	-22.135
$\lambda_p$	9.554	9.645	9.675	9.670	9.561
$\lambda_q$	.3024E+00	.3406E+00	.2812E+00	.2627E+00	.3028E+00
$F_o$	1945.156	1944.710	1944.708	1944.761	1945.109
$F_A$	1945.156	1944.710	1944.708	1944.761	1945.109
$\mu$	.1123E-05	.1116E-05	.5986E-05	.2694E-05	.1101E-04
$\sigma$	.8730E-12	.5360E-12	.2331E-09	.1427E-11	.4666E-09
FBIG	.8035E-06	.5886E-06	.1526E-04	.1010E-05	.1526E-04

TABLE (5.13)

## OPTIMUM ACTIVE-REACTIVE DISPATCH OF A.E.P 14 BUS TEST SYSTEM

## FUEL-COAL

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
P <sub>D</sub>	259.000		259.000	259.000	259.000
Q <sub>D</sub>	73.500	NO	73.500	73.500	73.500
P <sub>G1</sub>	132.670	CONVERGENCE	126.634	134.598	131.845
Q <sub>G1</sub>	46.235		40.711	37.430	45.981
P <sub>G2</sub>	136.945		141.477	134.885	137.598
Q <sub>G2</sub>	50.000		50.000	50.000	50.000
P <sub>L</sub>	10.615		9.111	10.483	10.443
Q <sub>L</sub>	22.734		17.211	13.930	22.481
$\lambda_p$	13.694		14.030	14.127	13.696
$\lambda_q$	.2827E+00		.2987E-01	.2402E+00	.2550E+00
F <sub>O</sub>	2811.232		2801.946	2808.548	2810.263
F <sub>A</sub>	2811.232		2801.946	2808.548	2810.263
$\mu$	.2907E-05		.5900E-04	.891RE-05	.3985E-05
$\sigma$	.8914E+00		.7622E+00	.6539E+00	.9091E+00
FBIG	.5960E-06		.1442E-05	.2742E-05	.1669E-05

TABLE (5.14)

## OPTIMUM ACTIVE-REACTIVE DISPATCH OF A.E.P 14 BUS TEST SYSTEM

## FUEL-OIL

Vari- able	Weighted Least Squares	Linear Regression	Gauss- Newton	Marquardt	Powell Regression
$P_D$	259.000		259.000	259.000	259.000
$Q_D$	73.500	NO	73.500	73.500	73.500
$P_{G1}$	130.509	CONVERGENCE	124.411	132.826	129.647
$Q_{G1}$	45.730		40.119	36.857	45.467
$P_{G2}$	138.929		143.510	136.515	139.615
$Q_{G2}$	50.000		50.000	50.000	50.000
$P_L$	10.439		8.921	10.341	10.262
$Q_L$	22.230		16.619	13.357	21.967
$\lambda_p$	14.317		14.676	14.763	14.319
$\lambda_q$	.2721E+00		.5540E-01	.2325E+00	.2427E+00
$F_o$	2932.236		2922.432	2929.483	2931.211
$F_A$	2932.236		2922.432	2929.483	2931.211
$u$	.6451E-06		.9177E-05	.6457E-05	.8307E-05
$\sigma$	.9461E+00		.8009E+00	.6826E+00	.9647E+00
$FBIG$	.8345E-06		.4545E-06	.9537E-06	.1430E-05

TABLE (5.15)

 OPTIMUM ACTIVE-REACTIVE DISPATCH OF A.E.P 14 BUS TEST SYSTEM  
 FUEL-GAS

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
P <sub>D</sub>	259.000		259.000	259.000	259.000
Q <sub>D</sub>	73.500	NO	73.500	73.500	73.500
P <sub>G1</sub>	134.713	CONVERGENCE	128.476	136.346	133.887
Q <sub>G1</sub>	46.715		41.206	38.000	46.462
P <sub>G2</sub>	135.070		139.794	133.277	135.725
Q <sub>G2</sub>	50.000		50.000	50.000	50.000
P <sub>L</sub>	10.783		9.270	10.624	10.612
Q <sub>L</sub>	23.215		17.706	14.500	22.962
$\lambda_p$	14.436		14.786	14.897	14.438
$\lambda_q$	.3206E+00		.1117E-01	.2720E+00	.2914E+00
F <sub>o</sub>	2970.334		2960.638	2967.460	2969.328
F <sub>A</sub>	2970.334		2960.638	2969.460	2969.328
$\mu$	.9905E-05		.1870E-03	.3489E-05	.9537E-05
$\sigma$	.1012E+01		.8755E+00	.7600E+00	.1038E+01
FBIG	.1669E-05		.2094E-05	.1311E-05	.2682E-05

TABLE (5.16)

## OPTIMUM ACTIVE-REACTIVE DISPATCH OF A.E.P 30 BUS TEST SYSTEM

FUEL-COAL

Vari- able	Weighted Least Squares	Linear Regression	Gauss- Newton	Marquardt	Powell Regression
P <sub>D</sub>	283.400	283.400	283.400	283.400	283.400
Q <sub>D</sub>	126.200	126.200	126.200	126.200	126.200
P <sub>G1</sub>	153.886	159.867	155.882	153.195	153.861
Q <sub>G1</sub>	153.414	40.722	122.380	125.131	153.755
P <sub>G2</sub>	58.960	58.410	57.643	61.068	58.978
Q <sub>G2</sub>	50.000	33.247	50.000	50.000	50.000
P <sub>G3</sub>	71.697	75.510	71.462	75.718	71.700
Q <sub>G3</sub>	-10.000	50.000	-10.000	-10.000	-10.000
P <sub>L</sub>	1.143	10.388	1.587	6.581	1.138
Q <sub>L</sub>	67.214	-2.230	36.181	38.931	67.555
$\lambda_p$	8.893	8.762	8.901	8.900	8.900
$\lambda_q$	-.1248E+00	.1000E+00	-.1632E+00	.1730E+00	-.1243E+00
F <sub>O</sub>	3088.830	3169.766	3092.119	3137.968	3088.788
F <sub>A</sub>	3088.830	3169.766	3092.119	3137.968	3088.788
$\mu$	.1741E-05	.7011E-05	.4764E-05	.5750E-06	.3951E-06
$\sigma$	.4118E+00	.6960E-01	.4453E+00	.1661E+00	.4116E+00
FBIG	.6706E-06	.6445E-05	.6311E-05	.6780E-06	.1602E-06

TABLE (5.17)  
OPTIMUM ACTIVE-REACTIVE DISPATCH OF A.E.P 30 BUS TEST SYSTEM  
FUEL-OIL

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
P <sub>D</sub>	283.400	283.400	283.400	283.400	283.400
Q <sub>D</sub>	126.200	126.200	126.200	126.200	126.200
P <sub>G1</sub>	152.012	158.061	154.024	151.294	151.988
Q <sub>G1</sub>	153.110	40.579	122.123	124.898	153.450
P <sub>G2</sub>	60.123	59.457	58.784	62.254	60.141
Q <sub>G2</sub>	50.000	33.205	50.000	50.000	50.000
P <sub>G3</sub>	72.333	76.198	72.107	76.355	72.334
Q <sub>G3</sub>	-10.000	50.000	-10.000	-10.000	-10.000
P <sub>L</sub>	1.069	10.318	1.515	6.502	1.063
Q <sub>L</sub>	66.910	-2.416	35.923	38.698	67.250
$\lambda_p$	9.275	9.139	9.283	9.282	9.275
$\lambda_q$	-.1320E+00	.1027E+00	-.1716E+00	.1788E+00	-.1314E+00
F <sub>0</sub>	3221.843	3306.298	3225.302	3273.051	3221.800
F <sub>A</sub>	3221.843	3306.298	3225.302	3273.051	3221.800
$\mu$	.1239E-05	.5890E-06	.1426E-04	.1429E-04	.4950E-05
$\sigma$	.4482E+00	.7550E-01	.4844E+00	.1802E+00	.4478E+00
FBIG	.6817E-06	.3390E-06	.8743E-05	.8442E-05	.4992E-05

TABLE (5.18)

## OPTIMUM ACTIVE-REACTIVE DISPATCH OF A.E.P 30 BUS TEST SYSTEM

## FUEL-GAS

Vari- able	Weighted Least Squares	Linear Regression	Gauss- Newton	Marquardt	Powell Regression
P <sub>D</sub>	283.400	283.400	283.400	283.400	283.400
Q <sub>D</sub>	126.200	126.200	126.200	126.200	126.200
P <sub>G1</sub>	157.978	164.087	160.073	157.241	157.953
Q <sub>G1</sub>	154.095	41.060	122.981	125.648	154.436
P <sub>G2</sub>	56.213	55.570	54.839	58.317	56.232
Q <sub>G2</sub>	50.000	33.363	50.000	50.000	50.000
P <sub>G3</sub>	70.517	74.197	70.240	74.592	70.519
Q <sub>G3</sub>	-10.000	50.000	-10.000	-10.000	-10.000
P <sub>L</sub>	1.309	10.553	1.753	6.751	1.304
Q <sub>L</sub>	67.895	-1.778	36.781	39.448	68.236
$\lambda_p$	9.402	9.261	9.411	9.409	9.402
$\lambda_q$	-1280E+00	.1095E+00	-.1693E+00	.1855E+00	-.1275E+00
F <sub>O</sub>	3263.085	3348.554	3266.495	3315.132	3263.044
F <sub>A</sub>	3263.085	3348.554	3266.495	3315.132	3263.044
$\mu$	.2135E-05	.1323E-05	.1386E-05	.7026E-06	.8068E-05
$\sigma$	.4601E+00	.7820E-01	.4980E+00	.1866E+00	.4598E+00
FBIG	.1225E-05	.7555E-06	.1531E-05	.4508E-06	.8706E-05

### 5.2 IMPLEMENTATION OF OPTIMAL POWERS - NETWORK PERFORMANCE

Having allocated the optimum active and/or active-reactive power generations for the three test systems as described in section 5.1 above, these powers are utilized into the respective original networks. A load flow solution for each system is carried out using the load flow algorithm mentioned in chapter IV, to observe the impact of these powers on the network performance.

Results corresponding to the B-Coefficient model are given in tables (5.19) to (5.21), and those corresponding to the active-reactive model are given in tables (5.22) to (5.24).

TABLE (5.19) CONT.

MISMATCHES						
Bus No.	MW	MVAR	MW	MVAR	MW	MVAR
1 *	-.00003	-.00003	.00002	.00003	.00005	.0
2	-.17300	.00016	.17200	-.00009	-.17300	.00030
3	-.00125	-.00068	-.00230	.00376	-.00104	-.00190
4	-.00052	-.00103	.00078	.00330	-.00058	-.00174
5	.000020	.00029	-.00009	.00064	.0	-.00041

\* Slack Bus

TABLE (5.19)  
5 BUS TEST SYSTEM LOAD FLOW RESULTS  
B-COEFFICIENT MODEL (0.1)

Variable	Panel I			Panel II		
	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Regression	Same As
$P_D$	165.00	165.00	165.00	165.00	40.00	WEIGHTED
$Q_D$	40.00	40.00	40.00	40.00	69.01	LEAST
$P_{61}$	69.09	68.99	69.00	69.00	23.37	SQUARES
$Q_{61}$	23.37	23.38	23.37	23.37	99.50	
$P_{62}$	99.42	99.52	99.51	99.51	-4.26	
$Q_{62}$	-4.26	-4.26	-4.25	-4.25	3.34	
$P_L$	3.34	—	3.34	3.34	-20.89	
$Q_L$	-20.89	-20.89	*	-20.89	1.06* to 1.019	1.06* to 1.019
Voltage Range	1.06* to 1.019	1.06 to 1.019	1.06 to 1.019	1.06 to 1.019	0.0* to -4.46	0.0* to -4.46
Phase Angle	0.0* to -4.46	0.0* to -4.46	0.0* to -4.46	0.0* to -4.46		

TABLE (5.20)

## 14 BUS TEST SYSTEM LOAD FLOW RESULTS

B-COEFFICIENT MODEL (OIL)

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$P_D$	259.00		259.00	259.00	259.00
$Q_D$	73.50		73.50	73.50	73.50
$P_{G1}$	183.14	NO	182.70	182.70	182.96
$Q_{G1}$	2.83	CONVERGENCE	2.93	2.92	2.88
$P_{G2}$	88.60		89.08	89.07	88.80
$Q_{G2}$	50.00		50.00	50.00	50.00
$P_L$	12.25		12.25	12.25	12.25
$Q_L$	24.37		24.40	24.38	24.39
Voltage Range	.97 to 1.06*		.969 to 1.06*	.970 to 1.06*	.970 to 1.06*
Phase Angle	* .0 to -15.44		* .0 to -15.44	* .0 to -15.43	* .0 to -15.44

TABLE (5.20) CONT.  
MISMATCHES

Bus No.	MW	MVAR	MW	MVAR	MW	MVAR	MW	MVAR
1	.00006 - .00001		-.00002	.00003	.00003	.00001	.00005	.00005
2	5.35	4.406	5.5400	4.580	5.497	4.540	5.440	4.485
3	-1.675	-0.0014	-1.741	.00023	-1.727	.00026	-1.707	.00006
4	-2.180	-4.906	-2.265	-5.083	-2.251	-5.050	-2.220	-4.994
5	-2.000	-5.048	-2.076	-5.232	-2.061	-5.193	-2.037	-5.136
6	.0080	.00072	.00798	.00107	.00877	.00041	.007600	.00201
7	-.00046	.00189	-.00037	.00232	-.00055	.00120	-.00118	-.00299
8	.00005	.00006	.00003	-.00001	.00003	-.00001	-.00001	-.00002
9	.00240	.00396	.00272	.00589	.00319	.00514	.00226	.00238
10	.00076	-.00114	.00091	-.00107	.00068	-.00106	.00079	.00088
11	.00023	-.00108	.00030	-.00103	.00022	-.00124	.00019	-.00110
12	.00137	-.00121	.00140	-.00124	.00133	-.00133	.00144	-.00114
13	.00093	-.00031	.00850	-.00022	.00995	-.00003	.00060	-.00017
14	.00002	-.00040	.00002	-.00038	.00002	-.00043	.00012	-.00036

\* Slack Bus.

TABLE (5.21)  
30 BUS TEST SYSTEM LOAD FLOW RESULTS  
B-COEFFICIENT MODEL (01L)

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Harquardt	Powell Regression
$P_0$	283.40				283.40
$Q_0$	165.20				165.20
$P_{G1}$	158.47		NO		158.51
$Q_{G1}$	2.04			CONVERGENCE	2.05
$P_{G2}$	60.48				60.49
$Q_{G2}$	-2.93				-2.92
$P_{G3}$	75.50				75.45
$Q_{G3}$	50.00				49.98
$P_L$	11.05				11.05
$Q_L$	12.22				12.22
Voltage Range	1.01 to 1.195				1.01 to 1.195
Phase Angle	* .0 to -12.26				* .0 to -12.26

TABLE (5.21) CONT.

Bus No.	MW	MVAR	MW	MVAR
1	.0	-.00005		
2	-.00137	.00003		
3	-.00027	-.00181		
4	-.00044	-.00362		
5	-.00002	-.00025		
6	-.00023	.00293		
7	.00058	.00211		
8	-.00018	.00049		
9	-.00698	.00363		
10	-.00047	.00055		
11	-.00005	-.00041		
12	-.00610	-.00348		
13	-.00012	.00139		
14	-.00052	-.00183		
15	-.00150	.00265		

16	-00043	-00050	-00004	-00023
17	.00025	-00042	.00012	-00006
18	-00024	-00029	-00010	-00031
19	-00031	-00129	.00038	.00162
20	.00027	.00067	-00090	-00221
21	.00287	.00274	-00009	-00348
22	.00036	-00241	.00388	.00102
23	-00098	.00003	.00113	.00030
24	.00015	-00042	-00203	-00097
25	-00037	-00132	-00051	-00157
26	.00001	-00002	.00001	-00013
27	-00042	.00055	-00016	.00096
28	.00041	.00145	.00232	-00334
29	-.0012	.00076	-.00133	.00060
30	.00023	.00018	.00017	.00017

\* Slack Bus.

TABLE (5.22)  
5 BUS TEST SYSTEM LOAD FLOW RESULTS  
ACTIVE-REACTIVE MODEL (COAL)

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Regression	Powell
$P_D$	165.00	165.00	165.00	165.00	165.00	165.00
$Q_B$	40.00	40.00	40.00	40.00	40.00	40.00
$P_{G1}$	69.64	69.54	69.66	69.70	69.64	
$Q_{G1}$	-9.33	-13.52	-13.41	-12.80	-9.84	
$P_{G2}$	98.55	98.66	98.54	98.50	98.55	
$Q_{G2}$	-27.30	31.40	31.30	30.70	27.80	
$P_L$	3.19	3.19	3.19	3.19	3.19	
$Q_L$	-22.03	-22.12	-22.11	-22.10	-22.04	
Voltage Range	1.027 to 1.06	1.029 to 1.06*	1.029 to 1.06*	1.028 to 1.06*	1.027 to 1.06*	*
Phase Angle	* 0 to -4.64	* 0 to -4.65	* 0 to -4.66	* 0 to -4.66	* 0 to -4.66	0 to -4.64

TABLE (5.22) CONT.

## MISMATCHES

Bus No.	MW	MVAR								
1*	.0	-.00002	-.00005	-.00001	-.00005	-.00001	.00003	.00003	-.00005	-.00004
2	-.00732	.00011	.00851	.00465	-.00789	.00336	-.00662	.0045	-.00674	-.00030
3	.00040	-.00116	.00043	.00058	.00056	-.00126	-.00011	-.00317	.00185	.00346
4	.00020	.00226	.00006	.00104	-.00046	.00011	-.00073	-.00052	-.00110	-.00056
5	.00020	.00048	-.00034	-.00065	-.00034	-.00050	-.00020	-.00097	.00012	.00011

\* Slack Bus.

TABLE (5.23)

## 14 BUS TEST SYSTEM LOAD FLOW RESULTS

## ACTIVE-REACTIVE MODEL (COAL)

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$P_D$	259.00		259.00	259.00	259.00
$Q_D$	73.50		73.50	73.50	73.50
$P_{G1}$	132.56	NO	127.91	134.68	131.89
$Q_{G1}$	23.47	CONVERGENCE	22.98	23.70	23.40
$P_{G2}$	136.95		141.48	134.88	137.60
$Q_{G2}$	50.00		50.00	50.00	50.00
$P_L$	10.57		10.45	10.63	10.55
$Q_L$	18.92		18.46	19.14	18.86
Voltage Range	.971 to 1.06*		.972 to 1.06*	.971 to 1.06*	.971 to 1.06*
Phase Angle	.0* to -14.09		.0* to -14.16	.0* to -14.32	.0* to -14.25

TABLE (5.23) CONT.

M I T A T C H E S									
	MW	MVAR	MW	MVAR	MW	MVAR	MW	MVAR	MW
1*	.0	-.00002		.00006	-.00004	.00006	.0	.00005	.00003
2	.0765	-.00371		.07700	-.00166	.07940	.00058	.08300	-.00154
3	-.0330	.00081		-.03330	.00046	.03300	.00068	-.03400	.00460
4	.0110	.00471		.01050	.00493	.00989	.00345	.00890	.00270
5	.00134	.00408		.00105	.00352	.00093	.00291	.00076	.00300
6	.00113	-.00031		.00066	-.00015	.00090	-.00064	.00053	-.00107
7	.00008	.00236		.00009	.00209	.00012	.00170	.00002	.00217
8	.0	.0		.00005	.00006	.0	.0	.0	.0
9	-.00006	.00503		-.00008	.00371	-.00009	.00462	-.00002	.00480
10	-.00003	-.00116		-.00003	-.00116	-.00016	-.00117	-.00012	-.00012
11	-.00017	-.00108		-.00010	-.00113	-.00003	-.00105	-.00003	-.00115
12	.00080	-.00113		.00084	-.00117	-.00077	-.00108	.00086	-.00132
13	-.00023	-.00108		-.00024	-.00005	-.00026	-.00019	-.00031	-.00014
14	-.00106	-.0024		-.00109	-.000109	-.00128	-.00025	-.00128	-.00020

\* Slack Bus.

TABLE (5.24)  
30 BUS TEST SYSTEM LOAD FLOW RESULTS  
ACTIVE-REACTIVE MODEL (COAL)

Variable	Weighted Least Squares	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$P_D$	283.40	283.40	283.40	283.40	283.40
$Q_D$	126.20	126.20	126.20	126.20	126.20
$P_{G1}$	169.15	161.52	170.78	162.85	169.13
$Q_{G1}$	106.92	35.88	107.14	107.38	106.93
$P_{G2}$	58.96	58.96	57.64	61.07	58.98
$Q_{G2}$	50.00	33.20	50.00	50.00	50.00
$P_{G3}$	71.70	75.51	71.46	75.72	71.70
$Q_{G3}$	-10.00	50.00	-10.00	-10.00	-10.00
$P_L$	16.80	12.59	16.88	16.63	16.80
$Q_L$	39.48	17.03	39.69	39.89	39.48
Voltage Range	.861 to 1.06*	.961 to 1.06*	.861 to 1.06*	.861 to 1.06*	.861 to 1.06*
Phase Angle	* .0 to -14.18	* .0 to -12.63	* .0 to -14.23	* .0 to -13.87	* .0 to -14.18

TABLE (5.24) CONT.

MISMATCHES						
Bus No.	MW	MVAR	MW	MVAR	MW	MVAR
1	0	.00003	.00005	.00002	.00005	.00006
2	-.00352	-.00146	.02880	.04800	-.00023	.00037
3	-.00032	-.00006	-.00124	-.0024	-.00012	-.00003
4	.00190	.00169	.00024	-.00079	.00153	.00049
5	.00018	.00050	-.07300	-.18800	.00012	.00005
6	.01740	.00413	.05360	.10600	-.01980	.00290
7	-.00015	.00017	.00024	.00240	-.00006	.00014
8	-.00049	.00024	.00008	.00185	-.00024	.00055
9	-.00016	.00188	.00005	.00274	-.00004	.00200
10	-.0105	-.00034	-.01350	.00259	-.00950	.00140
11	.00001	.00005	.0	.0	.00003	.0
12	.349	-.00004	-.0102	-.00103	.35400	-.00664
13	-.00021	.00004	-.0034	-.00117	-.00023	.00005
14	-.00010	-.00095	-.00026	-.00160	-.00015	-.00096
15	-.01260	.00070	.01660	-.00006	.01230	.00065

16	-.00024	-.00015	-.00008	-.00121	-.00023	-.00012	-.00024	-.00021	-.00034	-.00016
17	.00006	-.00005	.00038	.00046	.00004	.00008	-.00006	.00017	-.00003	-.00019
18	.00032	-.00024	-.00030	-.00004	.00032	-.00010	.00036	-.00023	.00043	-.00017
19	-.00010	.00002	-.00031	.00032	-.00008	.0	-.00021	.00001	-.00002	-.00001
20	.00003	-.00006	-.00044	-.00059	-.00004	.00013	.00001	-.00004	-.00014	-.00005
21	.00065	-.00059	.00041	-.00134	.00133	-.00105	.00083	-.00078	.00028	-.00055
22	.00332	.00070	-.00434	.00288	.00199	-.00056	.00298	-.00189	.00335	-.00031
23	.00071	-.00007	.00096	.00040	.00062	-.00009	.00054	-.00007	.00060	-.00003
24	.05000	-.00027	-.00246	-.00007	.05000	-.00015	.05300	-.00011	.05030	-.00025
25	.00002	-.00085	-.00027	-.00118	-.00036	-.00094	-.00013	-.00096	-.00025	-.00005
26	-.00006	.00010	.00004	-.00003	.00006	.00001	.0	-.00011	.0	-.00003
27	-.00245	.00009	-.00046	-.00005	.00262	-.00005	.00236	.00009	.00247	-.00007
28	.00581	.00280	.00121	.00350	.00569	.00233	.00630	.00369	.00551	.00244
29	-.00070	.00065	-.00085	.00097	-.00074	.00067	-.00068	.00078	-.00071	.00083
30	-.00001	-.00017	-.00001	-.00014	.0	-.00005	.00003	-.00017	-.00003	-.00014

\* Slack Bus.

## CHAPTER VI

## DISCUSSION AND CONCLUSIONS

Before we discuss the results obtained, it would be worthwhile to briefly summarize the work carried out in the preceding chapters.

### 6.1 SUMMARY

Various mathematical models describing the three power system decompositions were chosen and a number of experiments (solution of the load flow equations) were carried out in the case of the network models to obtain sufficient data for the purpose of parameter estimation. For the case of source models practical data were selected for the same purpose. Having gathered sufficient data for each model, five well known parameter estimation algorithms were used to estimate the parameters of interest. The models were then used to allocate the optimum power generations of the chosen test systems for all thermal plants. This was carried out as an extra measure to judge the relative effectiveness of each estimation procedure.

### 6.2 OBJECTIVE

Our objective here lies in the selection of a suitable parameter estimation algorithm among the five employed that would result in optimum power generations with minimum cost, and that these powers, when implemented into the original network should result in satisfactory system performance.

### 6.3 THE PARAMETER ESTIMATION ALGORITHMS

Basically all the parameter estimation procedures employed in our work are similar in that they all use a least squares objective function which is minimized by a certain technique as outlined in chapter III. Three of the five estimation algorithms, namely the Gauss-Newton, Marquardt and Powell Regression algorithms are based on Newton-Raphson's method of iteration. For these methods initial estimates must be available. The results of the method of Weighted Least Squares were used for this purpose.

### 6.4 RESULTS OBTAINED

We shall discuss the estimated parameters first, then those results that correspond to the optimal conditions and finally the optimal conditions for different load settings.

#### A. THE SOURCE, THE B-COEFFICIENT AND THE ACTIVE-REACTIVE MODELS

The data basis for tables (4.6) to (4.19) in chapter IV with the exception of table (4.10), were taken from literature (9). The coefficients in table (4.10) were obtained using the least squares fit.

The results corresponding to the methods of Weighted Least Squares and Power Regression are almost identical in all cases with only slight variations. This feature is on some occasions shared by the the methods of Gauss-Newton and Marquardt. The method of

Linear Regression in general gave different values for the parameters; However in some cases it resulted in values close to those of the above four algorithms.

The method of Linear Regression gave higher overall errors in general than any of the other four algorithms for the same number of experiments. The methods of Weighted Least Squares, Gauss-Newton, Marquardt and Powell Regression in general gave almost the same values for the overall error. Comparing the computational time involved, it was found that the Gauss-Newton algorithm takes the highest computer time followed by Marquardt, Powell regression, Weighted Least Squares and finally by Linear Regression algorithm. As an example consider the case of the active-reactive power loss model for the 14 Bus test system, the overall error and the computational time are shown in the following table:

METHOD OF ESTIMATION	ACTIVE		REACTIVE	
	Overall Error	CPU Time Seconds	Overall Error	CPU Time Seconds
1. Linear Regression	.427E-02	2.44	.917E-02	2.40
2. Weighted Least Squares	.153E-02	9.51	.318E-02	9.78
3. Powell Regression	.147E-02	10.33	.316E-02	11.58
4. Marquardt Algorithm	.114E-02	17.66	.329E-02	25.18
5. Gauss-Newton Method	.122E-02	50.07	.332E-02	31.66

From the foregoing it is evident that the best suited method for the purpose of parameter estimation among the five above would be that of Weighted Least Squares and this may be checked and/or improved

by Powell Regression algorithm from the overall error and computational speed points of view.

#### B. THE OPTIMIZATION CONDITIONS- THE B-COEFFICIENT MODEL

As shown in tables (5.1) to (5.3) in chapter V for the 5 Bus test system, the five estimation algorithms gave almost the same optimal conditions. The total cost is around 1917.15 (in the case of 011). The methods of Gauss-Newton, Marquardt and Linear Regression converged to the optimal solution in higher number of iterations than those corresponding to the methods of Weighted Least Squares and Powell Regression.

For the case of 14 Bus test system, (tables (5.4) to (5.6)), the same argument as for the 5 Bus system above holds, furthermore here the estimated parameters by the method of Linear Regression resulted in no convergence. The total cost is around 1950.00 for the case of 011.

In the case of 30 Bus test system, (tables (5.7) to (5.9)), the parameters estimated by the methods of Linear Regression and Marquardt gave no convergence. The optimal solutions corresponding to the methods of Weighted Least Squares, Gauss-Newton and Powell Regression are almost the same and all converged in about the same number of iterations. The total cost is around 3313.12 in the case of 011.

The optimization conditions for the above three test systems were obtained with no constraint violations.

### C. THE OPTIMIZATION CONDITIONS - THE ACTIVE-REACTIVE MODEL

As seen from tables (5.10) to (5.12) for the 5 Bus test system, the optimal conditions corresponding to the five parameter estimation techniques are almost the same. The optimal solution due to the method of Linear Regression converged in a higher number of iterations as compared to the other four algorithms. The total cost is around 1916.0 (for the case of 011). No constraint violations were recorded in this case.

In the case of 14-Bus test system (tables (5.13) to (5.15)), the estimated parameters by the method of Linear Regression gave no convergence. The optimal solutions corresponding to the methods of Weighted Least Squares and Powell Regression are almost the same. Those corresponding to Gauss-Newton and Marquardt algorithms differ from the former methods.

All the methods resulted in violating the upper limit of the reactive power generation of 50.0 MVAR ( $Q_{G2}$ ), which was then taken as the optimal value in accordance with the Kuhn-Tucker conditions. The Gauss-Newton algorithm gave the lowest value for the total cost (2922.43 in the case of 011), followed by Marquardt algorithm (2929.50). The methods of Weighted Least Squares and Powell Regression gave the value of 2932.00. The optimal solutions due to these latter methods converged in a lower number of iterations as compared to the former two methods. We shall discuss these results further later.

In the case of the 30 Bus test system (tables (5.16) to (5.18)),

the results corresponding to the methods of Weighted Least Squares and Powell Regression are again almost the same. The methods of Linear Regression, Gauss-Newton and Marquardt resulted in different optimal strategies. The lowest value for the total cost is due to the method of Weighted-Least Squares and Powell Regression (3221.80 in the case of OII) as compared to that of the Linear Regression which is the highest value. The optimum solutions due to the methods of Weighted Least Squares, Gauss-Newton, Marquardt and Powell Regression violated the upper limit of 50.0 MVAR on  $Q_{G2}$  and the lower limit of -10.0 MVAR on  $Q_{G3}$ . That corresponding to the method of Linear Regression violated the upper limit of 50.0 MVAR on  $Q_{G3}$  only.

We conclude from the above that the best suited parameter estimation method would be that of Weighted Least Squares followed by that of Powell Regression.

#### 6.5 NETWORK PERFORMANCE RESULTS

We shall discuss first those optimum results corresponding to the B-Coefficient model followed by those associated with the active-reactive power loss model.

##### A. THE B-COEFFICIENT MODEL

The load flow solutions corresponding to the above five estimation techniques are shown in table (5.19) in chapter V. The case of OII was chosen here. It is observed that the voltages and phase angles are within practical permissible ranges. The highest mismatch (due to all methods) is of the order of .173E+00 MW on Bus

number 2. All other mismatches are lower than this value. Comparing this model to the B-Coefficient model, it is observed here that the active-reactive model gives better model performance.

In the case of 14 Bus test system, the load flow solutions corresponding to the five parameter estimation algorithms are given in table(5.23) in chapter V. The voltages and phase angles are within permissible regions. The highest mismatch is of the order of .83E-01 MW on bus number 2 and this is due to Powell Regression. Comparing these results to the B-Coefficient model, it is observed that the active-reactive model gives much lower mismatches and hence better system performance.

In the case of 30 Bus test system, the load flow results are shown in table (5.24). The voltages range between .861 P.U to 1.06 P.U and the phase angles between -14.00 to 0.0 degrees. The highest mismatch of .35E+00 MW on bus number 12 is due to the method of Weighted Least Squares:

When we compare the load flow results of the 14 and 30 Bus test systems to the respective optimal solutions, we find that the reactive power generation on bus number 1 in each case is different (the active and reactive powers are not entered in the slack bus in the load flow algorithm). The total system reactive power losses are also different. Comparing these results to those obtained for the 5 Bus test system led us to investigate the matter further.

number 2, which is rather high.

In the case of the 14-Bus test system, the load flow results are shown in table (5.20). All the methods, again exhibit almost the same load flow solution. The voltages and phase angles are within permissible ranges; However it is observed that all methods resulted in high mismatches, the highest being of the order of 5.5 MW and 4.5 MVAR on bus number 2 (again the case of Oil was considered here).

In the case of 30-Bus test system, the load flow results are shown in table (5.21). The three methods resulted in almost the same load flow solution. The voltages and phase angles are within permissible range and so are the mismatches.

In conclusion for the above model, since all the methods (those that converged to a solution) gave almost the same load flow solution, our selection criterion would be based on the overall error and computational speed during the parameter estimation process. From this point of view the best suited method would be that of Weighted Least Squares followed by Powell Regression algorithm.

#### B. THE ACTIVE-REACTIVE MODEL

The load flow solutions corresponding to the five estimation techniques for the 5 Bus test system are given in table (5.22) in chapter V. The case of Coal was considered here. It is observed that the voltages and phase angles are within permissible ranges. The highest mismatch is due to that corresponding to the method of Linear Regression and is of the order of .85E-02 MW and .46E-02 MVAR on bus

### 6.6 ACCOUNTING FOR REACTIVE COMPENSATORS

In our work so far, the reactive powers due to those buses which are not actual generating buses were neglected in the estimation process. For example in the case of 14 Bus test system we have three voltage regulated buses and these are numbers 2,3 and 6. During the estimation process, the reactive powers due to the compensators on bus numbers 3 and 6 were neglected. In the case of 30 Bus test system, the voltage regulated buses are numbers 2,5,11 and 13, in this case the reactive compensation due to buses 5 and 11 were neglected in the estimation process. Now if we account for these reactive powers in our active-reactive power loss model, we find that the model will have to be modified to include these powers. In doing so we shall have 22 parameters instead of 9 to account for in the case of 14 Bus test system and 33 parameters instead of 16 in the case of 30 Bus system. We shall consider the 14 Bus system only due to time limitations.

Since we have new coefficients to account for, the load flow equations will have to be solved, this time not only changing the active power generations but also changing the reactive compensators. For this purpose 30 extra load flow experiments were carried out. The results are shown in table(6.1). This table includes the previous and the new experiments.

During the estimation process it was found that some of the elements of the matrices  $A_p$  and  $A_Q$  were negative. This is physically unrealizable (up to now single precision on IBM 370 was used). Using

TABLE (6.1)

## A.E.P 14 BUS TEST SYSTEM

## LOAD FLOW RESULTS

Exp. No.	P <sub>L</sub> MW	Q <sub>L</sub> MVAR	P <sub>G1</sub> MW	Q <sub>G1</sub> MVAR	P <sub>G2</sub> MW	Q <sub>G2</sub> MVAR	Q <sub>G3</sub> MVAR	Q <sub>G4</sub> MVAR
1	14.02	30.12	232.94	1.33	40.00	50.00	39.05	-6.00
2	15.87	36.50	264.89	7.09	10.00	50.00	40.00	-6.00
3	11.83	21.18	192.81	-3.71	78.00	40.33	30.08	8.00
4	15.28	32.30	261.40	-9.46	13.00	50.00	33.22	12.00
5	15.28	34.43	255.28	4.96	19.00	50.00	40.00	-6.00
6	11.19	20.62	169.74	5.33	100.00	41.21	34.33	-6.00
7	14.23	30.80	237.14	1.73	36.00	50.00	39.47	-6.00
8	14.98	33.40	249.99	3.83	24.00	50.00	39.83	-6.00
9	13.59	27.19	230.64	-7.68	42.00	49.84	32.24	6.00
10	13.16	24.18	224.16	-13.05	48.00	45.70	27.20	18.00
11	11.34	21.09	173.89	4.34	96.00	42.67	34.33	-6.00
12	9.68	14.36	118.72	-16.08	150.00	50.00	40.00	-6.00
13	12.89	26.26	211.11	.24	60.00	50.00	36.33	-6.00
14	11.05	19.45	167.06	-10.78	103.00	49.72	39.93	-6.00
15	13.19	27.29	217.35	.55	54.00	50.00	37.07	-6.00
16	12.59	25.27	204.88	-.04	66.00	50.00	35.60	-6.00
17	11.99	20.87	198.99	-8.05	72.00	35.27	26.59	20.00
18	11.71	20.90	189.70	-2.74	81.00	39.84	30.37	7.00
19	12.24	24.33	184.61	2.53	87.00	50.00	37.30	-6.00
20	11.45	21.46	177.00	3.61	93.00	43.76	34.34	-6.00

TABLE (6.1) CONT.

21	10.95	19.11	163.96	-11.03	106.00	50.00	40.00	-6.00
22	10.74	18.35	157.75	-11.83	112.00	50.00	40.00	-6.00
23	10.84	18.73	160.85	-11.43	109.00	50.00	40.00	-6.00
24	10.23	16.45	141.26	-13.95	128.00	49.77	39.95	-6.00
25	10.28	15.59	149.27	5.19	120.00	21.36	28.27	14.00
26	9.95	14.27	136.94	7.58	132.00	15.38	27.40	17.00
27	9.82	14.89	124.85	-15.50	144.00	50.00	40.00	-6.00
28	10.02	15.66	133.05	-14.68	136.00	50.00	40.00	-6.00
29	10.60	17.81	153.62	-12.51	116.00	49.74	39.94	-6.00
30	12.26	24.43	181.86	3.11	90.00	50.00	37.86	-6.00
31	12.88	26.26	211.11	.24	60.00	50.00	36.33	-6.00
32	12.76	25.85	208.51	.12	62.50	50.00	36.02	-6.00
33	12.52	25.03	203.33	-1.10	67.50	50.00	35.42	-6.00
34	12.40	24.60	200.48	-.21	70.25	50.00	35.08	-6.00
35	12.17	23.83	195.33	-.40	75.23	50.00	34.48	-6.00
36	12.27	24.17	197.63	-.31	73.00	50.00	34.75	-6.00
37	10.37	16.95	145.95	-13.42	123.45	49.76	39.95	-6.00
38	12.26	24.42	181.96	3.10	89.89	50.00	37.84	-6.00
39	12.24	24.35	183.89	2.68	87.78	50.00	37.44	-6.00
40	12.07	23.48	192.95	-.48	77.53	50.00	34.21	-6.00
41	9.54	14.44	91.04	39.53	177.51	-5.80	31.77	3.00
42	9.30	11.76	106.83	-27.35	161.50	50.00	40.00	2.00
43	9.58	12.98	120.12	-24.94	148.50	50.00	40.00	1.00
44	11.02	19.03	167.54	-13.36	102.51	49.75	39.94	-4.00

45	10.32	16.17	148.85	-18.38	122.51	50.00	40.00	-2.00
46	12.24	22.38	203.23	-6.55	68.00	42.86	29.50	10.00
47	12.28	24.27	188.49	1.37	83.00	50.00	36.53	-5.00
48	9.65	14.74	100.65	38.17	168.00	-5.86	32.45	4.00
49	11.42	19.29	184.06	-3.44	86.35	32.90	28.00	15.00
50	9.24	12.28	96.40	-20.40	171.89	50.00	39.90	-4.00
51	12.32	25.47	190.74	31.58	80.48	24.80	30.02	-6.00
52	12.67	27.48	187.78	60.82	84.00	-5.20	33.20	-6.00
53	12.30	25.73	186.73	42.77	84.35	13.00	31.21	-6.00
54	11.97	21.62	190.66	27.71	80.28	.20	23.10	24.00
55	13.38	30.63	184.92	91.93	87.35	-38.00	38.00	-6.00
56	12.52	26.79	186.28	56.63	84.46	-2.80	34.20	-6.00
57	12.43	26.28	188.99	44.80	82.34	12.60	30.16	-6.00
58	13.46	31.03	185.60	93.99	86.76	-38.00	37.10	-6.00
59	12.40	26.11	188.47	44.31	82.82	11.90	31.18	-6.00
60	12.45	26.40	187.28	50.08	84.06	4.50	33.18	-6.00

NOTES TO THE ABOVE TABLE

1. Bus number 1 is the slack bus.
2. Buses 1 and 2 are generating buses.
3. Buses 2, 3 and 6 are voltage regulated buses.
4. Reactive power on bus number 6 is given number 4 for convenience.
5.  $-40.00 \leq Q_{G2} \leq 50.00$   
 $0.00 \leq Q_{G3} \leq 40.00$   
 $-6.00 \leq Q_{G4} \leq 24.00$

double precision however resulted in positive values for the elements of the matrices  $A_p$  and  $A_q$ . Tables(6.2)and(6.3) show the results obtained. When the results of Weighted Least Squares were used as initial guesses for the methods of Gauss-Newton and Marquardt and double precision used it was found that these latter methods consumed considerable computer time (of the order of 5 to 6 minutes C.P.U time), and therefore for these methods single precision was used. The Powell Regression in double precision gave almost the same result as that of Weighted Least Squares so did Gauss-Newton in single precision. Marquardt algorithm gave different parameters. The Weighted Least Squares and Powell Regression (double precision) took about 24.00 seconds C.P.U time (in the case of active component parameters). Gauss-Newton and Marquardt (single precision) took 75.0 and 127 seconds C.P.U time respectively. From these considerations we shall use the results of the method of Weighted Least Squares for the optimal solution.

#### THE OPTIMAL SOLUTION

The optimal solution corresponding to the method of Weighted Least Squares for 14 Bus test system (the new coefficients) is shown in table (6.4). Comparing these results with those of table (5.14) in the case of 011, we find that the optimal strategy is different. The total cost in this case is 2917.52 which is lower than the old value of 2932.24 (011). In this case the upper limit of 24.00 MVAR On bus number 6 was violated.

Implementing these powers into the original network gave us

TABLE (6.2)

## A.E.P 14 BUS TEST SYSTEM ACTIVE-REACTIVE MODEL

## ACTIVE COMPONENT PARAMETERS

Parameter	Weighted Least Squares $W = I$	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$K_{LOP}$	-.5656E+00	-.5057E+00	-.5656E+00	.2206E+01	-.5656E+00
$E_{PP1}$	.2398E+00	.2002E+00	.2398E+00	.1741E+01	.2398E+00
$E_{PP2}$	.2179E+00	.2207E+00	.2179E+00	.1300E+01	.2179E+00
$E_{PQ1}$	-.2538E-01	.3596E-01	-.2538E-01	.7214E+01	-.2538E-01
$E_{PQ2}$	-.3721E-01	.6978E-01	-.3721E-01	.5595E+01	-.3721E-01
$E_{PQ3}$	.1821E+01	.1555E+01	.1821E+01	.9138E+01	.1821E+01
$E_{PQ4}$	.9347E+00	.7657E+00	.9349E+00	.8551E+01	.9349E+00
$A_{P11}$	.1353E-01	.1257E-01	.1353E-01	.4874E+00	.1353E-01
$A_{P12}$	.8273E-02	-.1008E-02	.8273E-02	-.2321E+00	.8273E-02
$A_{P13}$	.1227E+00	.3635E-01	.1227E+00	.7276E-02	.1227E+00
$A_{P14}$	.6426E-01	.3582E-01	.6425E-01	-.3047E+01	.6426E-01
$A_{P22}$	.1395E-01	-.2558E-02	.1395E-01	.1160E+00	.1395E-01
$A_{P23}$	.1300E+00	.2626E-01	.1300E+00	.2082E-01	.1300E+00
$A_{P24}$	.6953E-01	.3018E-01	.6952E-01	-.3259E+01	.6953E-01
$A_{P33}$	.3546E+00	.1759E+00	.3546E+00	.8700E+01	.3546E+00
$A_{P34}$	.2334E+00	.9773E-01	.2334E+00	-.1417E+00	.2334E+00
$A_{P44}$	.1700E+00	.1001E+00	.1700E+00	-.4253E+01	.1700E+00
$B_{P12}$	-.1284E-03	.4958E-02	-.1284E-03	-.2146E+00	-.1284E-03
$B_{P13}$	.3891E+00	.2987E+00	.3891E+00	.1294E+01	.3891E+00

TABLE (5.2) CONT.

B <sub>P14</sub>	.2015E+00	.1475E+00	.2015E+00	-.3806E+00	.2015E+00
B <sub>P23</sub>	.3871E+00	.3012E+00	.3871E+00	.1831E+01	.3871E+00
B <sub>P24</sub>	.2014E+00	.1428E+00	.2014E+00	.4528E+00	.2014E+00
E <sub>ov</sub>	.6001E-03	.5984E-03	.7128E-03	.7514E-03	.6001E-03

TABLE (6.3)

## A.E.P. 14 BUS TEST SYSTEM ACTIVE-REACTIVE MODEL

## REACTIVE COMPONENT PARAMETERS

Parameter	Weighted Least Squares $W = 1$	Linear Regression	Gauss-Newton	Marquardt	Powell Regression
$K_{LOQ}$	.2410E+02	-.3332E+02	.2409E+02	.1172E+03	.2410E+02
$E_{QP1}$	-.1764E+02	.2026E+02	-.1764E+02	.3453E+01	-.1764E+02
$E_{QP2}$	-.2340E+02	.1094E+02	-.2340E+02	-.3253E+03	-.2339E+02
$E_{Q1}$	-.8367E+01	-.7906E+00	-.8367E+01	.1322E+03	-.8368E+01
$E_{Q2}$	-.9683E+01	-.1820E+02	-.9683E+01	-.1006E+03	-.9684E+01
$E_{Q3}$	.2761E+02	.7994E+02	.2761E+02	.5305E+03	.2761E+02
$E_{Q4}$	.4487E+02	.7904E+02	.4487E+02	.4750E+03	.4487E+02
$A_{Q11}$	.3898E+01	-.1695E+01	.3898E+01	-.2177E+02	.3898E+01
$A_{Q12}$	.4840E+01	.1661E+00	.4840E+01	.4270E+02	.4840E+01
$A_{Q13}$	.6427E+01	.1442E+02	.6427E+01	-.1496E+03	.6427E+01
$A_{Q14}$	.7331E+01	.4810E+01	.7331E+01	-.3310E+03	.7331E+01
$A_{Q22}$	.5799E+01	.1816E+01	.5798E+01	.1069E+03	.5799E+01
$A_{Q23}$	.6740E+01	.1697E+02	.6741E+01	-.1794E+03	.6741E+01
$A_{Q24}$	.8208E+01	.6862E+01	.8207E+01	-.3181E+03	.8208E+01
$A_{Q33}$	.1009E+02	.3387E+02	.1009E+02	.1396E+01	.1009E+02
$A_{Q34}$	.1133E+02	.2628E+02	.1133E+02	-.3507E+03	.1133E+02
$A_{Q44}$	.1228E+02	.1480E+02	.1228E+02	-.7169E+03	.1228E+02
$B_{Q12}$	-.1860E-01	-.1288E+01	-.1860E-01	-.3306E+02	-.1860E-01
$B_{Q13}$	.7610E+01	.2196E+02	.7610E+01	.6084E+02	.7610E+01

TABLE (6.3) CONT.

B <sub>Q14</sub>	.1104E+02	.1868E+02	.1104E+02	-.3032E+02	.1104E+02
B <sub>Q23</sub>	.6934E+01	.2123E+02	.6934E+01	.7482E+02	.6934E+01
B <sub>Q24</sub>	.1100E+02	.1997E+02	.1100E+02	.5374E+02	.1100E+02
E <sub>ov</sub>	.1715E-01	.1819E-01	.1929E-01	.1707E-01	.1715E-01

TABLE (6.4)

## OPTIMUM ACTIVE-REACTIVE DISPATCH OF A.E.P 14-BUS TEST SYSTEM

Variable	FUEL-OIL
$P_D$	259,000
$Q_D$	73,500
$P_{G1}$	167,360
$Q_{G1}$	10,880
$P_{G2}$	103,030
$Q_{G2}$	33,342
$Q_{G3}$	8,370
$Q_{G4}$	24,000
$P_L$	11,390
$Q_L$	3,107
$\lambda_p$	9.870
$\lambda_q$	.458
$F_o$	2917.52
$F_A$	2917.52
$\mu$	.1370E-04
$\sigma$	.2070E-01
$FBIG$	.1210E-01

TABLE (6.5)  
14 BUS TEST SYSTEM LOAD FLOW RESULTS  
ACTIVE-REACTIVE MODEL (OIL)

Variable	Weighted Least Squares
$P_D$	259.000
$Q_D$	73.500
$P_{G1}$	166.930
$Q_{G1}$	5.310
$P_{G2}$	103.030
$Q_{G2}$	33.300
$Q_{G3}$	8.300
$Q_{G4}$	24.000
$P_L$	10.980
$Q_L$	17.830
Voltage Range	.988 to 1.07
Phase Angle	.0 to -14.70

TABLE (6.5) CONT.

## MISMATCHES

Bus No.	MW	MVAR
1	.00005	-.00002
2	.04030	.00250
3	-.02440	.00062
4	.00540	.00880
5	-.00134	-.00263
6	.00114	.00055
7	-.00023	.00040
8	.0	.0
9	-.00002	.00334
10	-.00082	-.00269
11	.00046	-.00069
12	.00058	-.00151
13	-.00056	-.00037
14	-.00058	.00059

TABLE (6.6)  
30 BUS TEST SYSTEM ACTIVE- REACTIVE PARAMETERS

Para- meter	ACTIVE		REACTIVE	
	Weighted Least Squares $W=I$		Parameter	Weighted Least Squares $W=I$
$K_{LOP}$	.8261E+01		$K_{LOQ}$	.1307E+01
$E_{PP1}$	-.5311E+01		$E_{QP1}$	-.8184E+00
$E_{PP2}$	-.3642E+01		$E_{QP2}$	-.6758E+00
$E_{PP3}$	-.6311E+01		$E_{QP3}$	-.1268E+01
$E_{PQ1}$	-.2105E+01		$E_{QQ1}$	.4323E-01
$E_{PQ2}$	-.1063E+01		$E_{QQ2}$	.4508E-01
$E_{PQ3}$	-.3620E+01		$E_{QQ3}$	-.3271E+00
$A_{P11}$	.1010E+01		$A_{Q11}$	.1831E+00
$A_{P12}$	.6947E+00		$A_{Q12}$	.1240E+00
$A_{P13}$	.1142E+01		$A_{Q13}$	.1752E+00
$A_{P22}$	.4244E+00		$A_{Q22}$	.9947E-01
$A_{P23}$	.8472E+00		$A_{Q23}$	.1413E+00
$A_{P33}$	.1488E+01		$A_{Q33}$	.4278E+00
$B_{P12}$	.8481E-01		$B_{Q12}$	.1075E-01
$B_{P13}$	-.1159E+00		$B_{Q13}$	-.1244E-02
$B_{P23}$	-.1803E+00		$B_{Q23}$	-.1031E-01
$E_{ov}$	.3595E-02		$E_{ov}$	.1817E-03

I = Unit matrix.

the results shown in table (6.5) for the case of 011.

It is observed from the above table that the voltages and phase angles have improved and so have the mismatches.

#### 6.7 THE 30 BUS TEST SYSTEM (RECONSIDERED)

Looking back at the estimated coefficients shown in tables (4.18) and (4.19), we see that some of the elements of the matrices  $A_p$  and  $A_Q$  are negative, this is physically unrealizable since these are cosine functions. Using double precision resulted in positive values for the active and reactive components. The method of Weighted Least Squares was only considered here due to time limitations. Results are given in table (6.6).

### 6.8 CONCLUSIONS

From the foregoing, it appears that the best suited parameter estimation procedure would be that of Weighted Least Squares. This may be checked and/or improved by the use of Powell Regression algorithm, from the overall error, computational speed and system performance points of view. It is suggested that double precision should be used as this resulted in much better system performance. It is also observed that the system performance improves when the system is represented by the active-reactive power loss model rather than the B-Coefficient model.

Further work should be carried out to investigate the role of the reactive compensators and their impact on the network performance. Care should be taken when carrying out the load flow experiments to include a wide range of active and reactive power generations (including the reactive compensators), to avoid ill-conditioning of the resulting equations.

## APPENDIX A

## DERIVATION OF THE POWER-LOSS FORMULAE

## DERIVATION OF THE POWER LOSS FORMULAE

### (I) THE ACTIVE-REACTIVE POWER LOSS FORMULA

Consider the active-reactive power balance equation in complex form for  $m$  generating units

where

$S_L$  is the total complex power loss of the network,  $S_{Gf}$  is the complex power generation of unit 1 and  $S_D$  is the total complex power demand.

We may write equation (A-1) as follows:

where

$P_L$  and  $Q_L$  are the total active and reactive power losses of the system respectively.

The subscripts G and D stand for generation and demand respectively.

Let  $\underline{V}_B$ ,  $\underline{I}_B$  be the bus voltage and current vectors and  $Z_B$  be the bus impedance matrix then

$$S_L = V_B + I_0 \quad \dots \dots \dots \quad (A.6)$$

where

$I_2$  is the conjugate of the vector component of current.

Substituting for  $\underline{y}_B$  in equation (A.6) we obtain

$$P_L + jQ_L = [ Z_B \quad I_B ]^T \quad \overset{*}{I_B} \quad \rightarrow \quad \dots \dots \dots \quad (A.8)$$

$$P_L + jQ_L = \left[ \frac{1}{L_D} + j\frac{1}{L_Q} \right] \left[ \frac{R + jX}{Z_0} \right] \left[ \frac{1}{L_D} - j\frac{1}{L_Q} \right] \quad \dots \dots \dots (A.9)$$

since  $\underline{Z}_B = \underline{Z}_B$  ;

Simplifying equation (A.9) and equating real and imaginary parts we obtain the following :

and

Since only bus powers and bus voltages are usually known it is practical to eliminate the current quantities in equations (A.10) and (A.11) and express  $P_i$  and  $Q_i$  in terms of bus powers and bus voltages,

$$V_1 = |V_1| [\cos \delta_1 + j \sin \delta_1] \quad \dots \dots \dots \text{(A.13)}$$

where  $\delta_1$  is the power angle at bus 1.

Substituting for  $V_1$  in equation (A.12), then,

$$P_1 + jQ_1 = |V_1| [\cos \delta_1 + j \sin \delta_1] [I_{pf} - j I_{qf}] \quad \dots \dots \text{(A.14)}$$

Equating real and imaginary terms we obtain :

$$P_1 = |V_1| [\cos \delta_1 I_{\text{pf}} + \sin \delta_1 I_{\text{af}}] \quad \dots \dots \dots \quad (\text{A.15})$$

$$Q_i = |V_i| [\sin \delta_i I_{ni} + \cos \delta_i I_{ci}], \quad \dots \dots \dots \quad (A.16)$$

Multiplying equation (A.15) by  $\cos \delta$ , and equation (A.16) by  $\sin \delta$ ,

adding and rearranging we obtain :

$$I_{\text{pt}} = [P_i \cos \delta_i + Q_i \sin \delta_i] / |V_i| \quad , \quad \dots \quad (A.17)$$

similarly multiplying equation (A.15) by  $\sin s_1$ , and equation (A.16) by

Cos 6, we obtain an expression for  $I_{12}$ , hence

$$I_{qj} = [ P_j \sin \delta_j + Q_j \cos \delta_j ] / |V_j| , \quad \dots \dots \dots \quad (A.18)$$

Putting equations (A.17) and (A.18) in vector form we get the following

$$I_0 = C P + D Q \quad \dots \dots \dots \text{(A.19)}$$

$$I_q = D_p - C_q, \quad \dots \dots \dots \quad (A.20)$$

where

$$D = \text{diag} [ \sin \delta_1 / |V_1| ] , \quad \dots \dots \dots \quad (\text{A.22})$$

substituting for  $I_0$  and  $I_0'$  into equation (A.10) we have,

$$P_L = [C_P + D_Q]^T R [C_P + D_Q] \\ + [D_P - C_Q]^T R [D_P - C_Q], \quad \dots \dots \dots \quad (A.23)$$

multiplying through, rearranging and putting in a matrix form we have,

$$P_L = \begin{bmatrix} T & T \\ P & Q \end{bmatrix} \begin{bmatrix} T & T \\ C \underline{R} C + D \underline{R} D & C \underline{R} D - D \underline{R} C \\ \hline T & T \\ D \underline{R} C - C \underline{R} D & D \underline{R} D + C \underline{R} C \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} \quad \dots \text{(A.24)}$$

Let  $i$

$$A_p = \frac{T}{C R C} + \frac{T}{D R D} \quad \dots \dots \dots \text{(A.25)}$$

equation (A.24) then becomes :

$$P_L = [P^T \ Q^T] \begin{bmatrix} A_p & -B_p \\ B_p & A_p \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} \quad \dots \dots \dots \quad (A.27)$$

where

$$a_{ijp} = [ r_{ij} \cos(\delta_i - \delta_j) ] / [ |v_i| + |v_j| ] \quad \dots \dots \dots \quad (A.28)$$

and.

$$b_{ij,ip} = [ r_{ij} \sin(\delta_i - \delta_j) ] / [ v_i |v_j| ] . \quad \dots \dots \dots \quad (A.29)$$

Partitioning the vectors  $\underline{P}$  and  $\underline{Q}$  in terms of generation and demand we have.

and,

partitioning the matrices  $A_p$  and  $B_p$  in terms of generation and demand as follows:

bearing in mind that  $A_p$  is square symmetric matrix with  $A_p = A_p^T$ ,

and  $B_p$  is skew symmetric matrix with  $B_p = -B_p^T$ .

In a similar manner we write an expression for  $Q_L$  as follows :

$$Q_L = K_{LQQ} + [E_{QP} \quad E_{QQ}^T] \begin{bmatrix} P_G \\ Q_G \end{bmatrix} + [P_G^T \quad Q_G^T] \begin{bmatrix} A_{QGG} & -B_{QGG} \\ B_{QGG} & A_{QGG} \end{bmatrix} \begin{bmatrix} P_G \\ Q_G \end{bmatrix}, \dots \dots \dots \quad (A.38)$$

$P_G$  and  $Q_G$  are vectors of active and reactive power generations respectively.

It should be noted that the coefficients appearing in the above expressions for  $P_L$  and  $Q_L$  are functions of time.

### (II) THE GENERAL LOSS FORMULA

In order to generate an expression for  $P_L$ , the following assumptions are made concerning the network operation :

- (i) At any busbar the ratio of reactive to active power generations remains constant.
- (ii) At any busbar of the network the loads remain a constant complex fraction of the total load.
- (iii) Bus voltages remain constant in magnitude and phase angle.

Assumption (i) implies that :

$$Q_{Gi} = Q_{Gio} + f_i P_{Gi} \quad \dots \dots \dots \quad (A.39)$$

$$A_p = \begin{bmatrix} A_{PGG} & A_{PGD} \\ A_{PDG} & A_{PDD} \end{bmatrix} \quad \dots \dots \dots \quad (A.32)$$

and,

Substituting the partitioned vectors and matrices into equation (A.27), multiplying and rearranging will result in the following expression:

$$P_L = K_{LOP} + [E_{PP}^T \quad E_{PQ}^T] \begin{bmatrix} P_G \\ Q_G \end{bmatrix} + [P_G^T \quad Q_G^T] \begin{bmatrix} A_{PGG} & -B_{PGG} \\ B_{PGG} & A_{PGG} \end{bmatrix} \begin{bmatrix} P_G \\ Q_G \end{bmatrix}, \quad \dots \quad (A.34)$$

where

$$E_{PP} = 2 [ B_{PGD} Q_D - A_{PGD} P_D ] , \quad \dots \dots \dots (A.35)$$

and.

and -

$$K_{LOP} = \begin{bmatrix} T \\ P_D & Q_D \end{bmatrix} \begin{bmatrix} A_{PDD} & -B_{PDD} \\ B_{PDD} & A_{PDD} \end{bmatrix} \begin{bmatrix} P_D \\ Q_D \end{bmatrix} \quad \dots \dots \dots (A.37)$$

Considering the above assumptions we may write the new coefficients as :

$$\underline{B} = \underline{A}_{PGG} + \underline{E}^T \underline{A}_{PGG} \underline{E} + 2 \underline{E}^T \underline{B}_{PGG}, \quad \dots \dots \dots \quad (A.40)$$

$$\underline{B}_0 = \underline{E}_{PP}^T + 2 \underline{Q}_{GO}^T [\underline{A}_{PGG} \underline{E} + \underline{B}_{PGG}] + \underline{E}_{PQ}^T \underline{E}, \quad \dots \dots \dots \quad (A.41)$$

and

$$K_{LO} = K_{L0P} + \underline{Q}_{GO}^T \underline{A}_{PGG} \underline{Q}_{GO} + \underline{E}_{PQ}^T \underline{Q}_{GO}, \quad \dots \dots \dots \quad (A.42)$$

where

$$\underline{Q}_{GO} = [ Q_{G10} \quad Q_{G20} \quad \dots \dots \quad Q_{Gm0} ], \quad \dots \dots \dots \quad (A.43)$$

and

$$\underline{E} = \text{diag} [ f_1 ]. \quad \dots \dots \dots \quad (A.44)$$

Collecting terms we obtain the general loss formula, hence

$$P_L = K_{LO} + P_G \underline{B}_0 + P_G \underline{B} \underline{P}_G, \quad \dots \dots \dots \quad (A.45)$$

$\underline{B}$  is a square symmetric matrix.

Another way of writing equation (A.45) is :

$$P_L = K_{LO} + \sum_{i=1}^m B_{i0} P_{Gi} + \sum_{i=1}^m \sum_{j=1}^m P_{Gi} B_{ij} P_{Gj}, \quad \dots \dots \dots \quad (A.46)$$

where  $m$  is the number of generating units.

APPENDIX B  
TEST SYSTEMS PARTICULARS

TABLE (B.1)

## 5 BUS TEST SYSTEM OF STAGG AND EL-ABIAAD

## IMPEDANCE AND LINE CHARGING DATA

LINE DESIGNATION		RESISTANCE P.U.	REACTANCE P.U.	LINE CHARGING P.U.
1	2	0.02	0.06	0.030
1	3	0.08	0.24	0.025
2	3	0.06	0.18	0.020
2	4	0.06	0.18	0.020
2	5	0.04	0.12	0.015
3	4	0.01	0.03	0.010
4	5	0.08	0.24	0.025

\* Impedance and line charging susceptance in per unit on a 100-MVA base. Line charging one-half of the total charging of line.

TABLE (B.2)

5 BUS TEST SYSTEM DATA OF STAGG AND EL-ABIAD  
OPERATING CONDITIONS

BUS NUMBER	STARTING BUS VOLTAGE		GENERATION		LOAD	
	Magnitude Per Unit	Phase Angle Degrees	MW	MVAR	MW	MVAR
1*	1.06	0.0	0.0	0.0	0.0	0.0
2	1.00	0.0	40.0	30.0	20.0	10.0
3	1.00	0.0	0.0	0.0	45.0	15.0
4	1.00	0.0	0.0	0.0	40.0	5.0
5	1.00	0.0	0.0	0.0	60.0	10.0

REGULATED BUS DATA

Bus Number	Voltage Magnitude		Minimum		Maximum	
	Per Unit	Mvar Capability				
2	1.047	-10.0	-	-	50.0	-

TRANSFORMER DATA

No Transformer Taps

SHUNT DATA

No Shunts

\* Slack Bus.

TABLE (B.3)

## A.E.P. 14 BUS TEST SYSTEM

## IMPEDANCE AND LINE CHARGING DATA

LINE DESIGNATION	RESISTANCE P.U.	REACTANCE P.U.	LINE CHARGING P.U.
1 2	.01938	.05917	.0264
1 5	.05403	.22304	.0246
2 3	.04699	.19797	.0219
2 4	.05811	.17632	.0187
2 5	.05695	.17388	.0170
3 4	.06701	.17103	.0173
4 5	.01335	.04211	.0064
4 7	.00000	.20912	.0000
4 9	.00000	.55618	.0000
5 6	.00000	.25202	.0000
6 11	.09498	.19890	.0000
6 12	.12291	.25581	.0000
6 13	.06615	.13027	.0000
7 8	.00000	.17615	.0000
7 9	.00000	.11001	.0000
9 10	.03181	.08450	.0000
9 14	.12711	.27038	.0000
10 11	.08205	.19207	.0000
12 13	.22092	.19988	.0000
13 14	.17093	.34802	.0000

\* Impedance and line charging susceptance in per unit on a 100 MVA base. Line charging on half of total charging of line.

TABLE (B.4)  
A.E.P. 14 BUS TEST SYSTEM  
OPERATING CONDITIONS

BUS Number	STARTING BUS VOLTAGE		GENERATION		LOAD			
	Magnitude	Phase Angle	Per Unit	Degrees	MW	MVAR	MW	MVAR
1	1.06	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	1.00	0.0	40.0	0.0	21.7	12.7		
3	1.00	0.0	0.0	0.0	94.22	19.0		
4	1.00	0.0	0.0	0.0	47.8	-3.9		
5	1.00	0.0	0.0	0.0	7.6	1.6		
6	1.00	0.0	0.0	0.0	11.2	7.5		
7	1.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	1.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	1.00	0.0	0.0	0.0	0.0	0.0	29.5	16.6
10	1.00	0.0	0.0	0.0	9.0	5.8		
11	1.00	0.0	0.0	0.0	3.5	1.8		
12	1.00	0.0	0.0	0.0	6.1	1.6		
13	1.00	0.0	0.0	0.0	13.5	5.8		
14	1.00	0.0	0.0	0.0	14.9	5.0		

Bus Number	REGULATED BUS DATA			
	Voltage Per Unit	Magnitude	Minimum	Maximum
2	1.045		-40.0	50.0

TABLE (B.4) CONT.

3	1.010	0.0	40.0
6	1.070	-6.0	24.0

## TRANSFORMER DATA

Transformer Code	Tap Setting **
4 7	.978
4 9	.969
5 6	.932

## STATIC CAPACITOR DATA

Bus Number	Susceptance Per Unit ***
9	.190

\* Slack Bus.

\*\* Off-nominal turns ratio, as determined by the actual transformer tap positions and the voltage bases. In the case of nominal turns ratio, this would equal 1.0.

\*\*\* Susceptance in per unit, On 100 MVA base.

TABLE (B.5)

## A.E.P. 30 BUS TEST SYSTEM

## IMPEDANCE AND LINE CHARGING DATA

LINE DESIGNATION		RESISTANCE P.U.	REACTANCE P.U.	LINE CHARGING P.U.
1	2	.0192	.0575	.0264
1	3	.0452	.1852	.0204
2	4	.0570	.1737	.0184
3	4	.0132	.0379	.0042
2	5	.0472	.1983	.0209
2	6	.0581	.1763	.0187
4	6	.0119	.0414	.0045
5	7	.0460	.1160	.0102
6	7	.0267	.0820	.0085
6	8	.0120	.0420	.0045
6	9	.0000	.2080	.0000
6	10	.0000	.5560	.0000
9	11	.0000	.2080	.0000
9	10	.0000	.1100	.0000
4	12	.0000	.2560	.0000
12	13	.0000	.1400	.0000
12	14	.1231	.2559	.0000
12	15	.0662	.1304	.0000
12	16	.0945	.1987	.0000
14	15	.2210	.1997	.0000

TABLE (8.5) CONT.

16	17	.0824	.1923	.0000
15	18	.1073	.2185	.0000
18	19	.0639	.1292	.0000
19	20	.0340	.0680	.0000
10	20	.0936	.2090	.0000
10	17	.0324	.0845	.0000
10	21	.0348	.0749	.0000
10	22	.0727	.1499	.0000
21	22	.0116	.0236	.0000
15	23	.1000	.2020	.0000
22	24	.1190	.1790	.0000
23	24 <sup>b</sup>	.1320	.2700	.0000
24	25	.1885	.3292	.0000
25	26	.2544	.3800	.0000
25	27	.1093	.2087	.0000
27	28	.0000	.3960	.0000
27	29	.2198	.4153	.0000
27	30	.3202	.6027	.0000
29	30	.2399	.4533	.0000
8	28	.0636	.2000	.0214
6	28	.0169	.0599	.0065

\* Impedance and line charging susceptance in per unit on a 100 MVA base. Line charging one-half of total charging of line.

TABLE (B.6)  
A.E.P. 30 BUS TEST SYSTEM  
OPERATING CONDITIONS

BUS NUMBER	STARTING BUS VOLTAGE		GENERATION		LOAD	
	Magnitude Per-Unit	Phase Angle Degrees	MW	MVAR	MW	MVAR
1	1.06	0.0	0.0	1.80	0.0	0.0
2	1.00	0.0	54.62	-2.54	21.7	12.7
3	1.00	0.0	0.0	0.00	2.4	1.2
4	1.00	0.0	0.0	0.00	7.6	1.6
5	1.00	0.0	0.0	0.00	94.2	19.0
6	1.00	0.0	0.0	0.00	0.0	0.0
7	1.00	0.0	0.0	0.00	22.8	10.9
8	1.00	0.0	0.0	0.00	30.0	30.0
9	1.00	0.0	0.0	0.00	0.0	0.0
10	1.00	0.0	0.0	0.00	5.8	2.0
11	1.00	0.0	0.0	0.00	0.0	0.0
12	1.00	0.0	0.0	0.00	11.2	7.5
13	1.00	0.0	70.45	50.00	0.0	0.0
14	1.00	0.0	0.0	0.00	6.2	1.6
15	1.00	0.0	0.0	0.00	8.2	2.5
16	1.00	0.0	0.0	0.00	3.5	1.8
17	1.00	0.0	0.0	0.00	9.0	5.8
18	1.00	0.0	0.0	0.00	3.2	0.9
19	1.00	0.0	0.0	0.00	9.5	3.4

TABLE (B.6) CONT.

20	1.00	0.0	0.0	0.0	2.2	0.7
21	1.00	0.0	0.0	0.0	17.5	11.2
22	1.00	0.0	0.0	0.0	0.0	0.0
23	1.00	0.0	0.0	0.0	3.2	1.6
24	1.00	0.0	0.0	0.0	8.7	6.7
25	1.00	0.0	0.0	0.0	0.0	0.0
26	1.00	0.0	0.0	0.0	3.5	2.3
27	1.00	0.0	0.0	0.0	0.0	0.0
28	1.00	0.0	0.0	0.0	0.0	0.0
29	1.00	0.0	0.0	0.0	2.4	0.9
30	1.00	0.0	0.0	0.0	10.6	1.9

## TRANSFORMER DATA

Transformer Code	Tap Setting **
4 - 12	.932
6 9	.978
6 10	.969
28 27	.968

## STATIC CAPACITOR DATA

Transformer Code	Susceptance Per Unit ***
10	.190
24	.043

\* Slack Bus.

\*\* Off nominal transformer ratio, as determined by the actual transformer tap positions and the voltage bases. In the case of nominal transformer ratio, this would equal 1.0.

\*\*\* Susceptance in per unit on a 100 MVA base.

**APPENDIX C.****INITIAL GUESS ESTIMATORS**

and the value of  $\lambda_q$  is either estimated or calculated from :

$$\lambda_q = \lambda_p [ ( \partial P_L / \partial Q_1 ) / ( 1 - \partial Q_L / \partial P_1 ) ] . \quad \dots \dots \dots \quad (C.4)$$

INITIAL GUESS ESTIMATORS

The availability of good starting estimates for the Newton-Raphson based algorithm offer the advantage of computational speed [53,54 ]. For our purposes two procedures for generating such guesses are given below.

#### **PROCEDURE (1)**

The set of equations (5.12) and (5.13) for the B-coefficient model, and (5.22) to (5.25) for the active-reactive power loss model given in chapter (V), are solved for  $P_G$  and/or  $P_G$  and  $Q_G$  assuming appropriate values for  $\lambda_n$  and/or  $\lambda_p$  and  $\lambda_o$  as the case may be.

### **PROCEDURE (2)**

In this method, the initial estimates are obtained on the assumption that the transmission power losses are neglected. The variables are determined as follows :

$$\lambda_p = [ 2 P_D + ( \sum_{i=1}^m B_i / Y_i ) ] / [ \sum_{i=1}^m \frac{1}{Y_i} ], \quad \dots \dots \dots (C.1)$$

satisfying the active power balance equation,

The optimal active power generations are given :

For the active-reactive model, the reactive power generations are approximated as :

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