INVESTIGATION OF REMOTELY OPERATED UNDERWATER VEHICLE MOTION AND COMPUTER SIMULATION



WILLIAM JOHN STONEMAN







Investigation of Remotely Operated

Underwater Vehicle

Motion and Computer Simulation

By

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A thesis submitted to the School of Graduate Studies in partial fulfillment of the requirements for the degree of Master of Engineering

Faculty of Engineering and Applied Science Memorial University of Newfoundland

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#### Dedicated to a Special Grandpa,

Cecil Passmore

#### ABSTRACT

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The purpose of this thesis is to present a computer program that simulates the motion characteristics of a Remotely Operated Underwater Vehicle (ROV). In developing this program considerable detail has been given to the basic principles and properties that are related to ROV motion. This paper is intended to give a comprehensive development of the ROV characteristics thus providing an overall view of the influencing factors involved with the ROV and its environment.

The development of the computer program is presented by first investigating the mathematics of solid body motion and hydrodynamic influences, and then combining these concepts to mathematically express the motion of a general ROV. These mathematical expressions are then adapted for the specific ROV unit 'HydroProducts RCV-225' and used as the basis of a computer simulation of the vehicle motion. The computer program is presented as a fortran

program capable of running on an IBM personal computer or compatible equipment. The final portion of the thesis illustrates several case studies of potential maneuvering procedures for the ROV. 111 ACKNOWLEDGEMENTS

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#### LIST OF SYMBOLS

= rotation tensor, with components -R, ... (i and j = 1-3) versor, with components,  $e_i$ . (i = 1+3) dyadic function. = .identity tensor. . angle of rotation. vector, with components  $b_i$ . (i = 1+3)  $\bar{A} =$  tensor, with components  $A_{i,j}$ . (i and j = 1+3) angular velocity tensor. ā angular velocity vector. = force vector / with components F. (i = 1+3) F, = force vector for ith particle. momentum vector for ith particle. p, ā, = acceleration of ith particle. = velocity of ith particle. V, X.B = position vector of ith particle. = mass of ith particle. m. \* P. . = force of ith particle on jth particle. = external force on ith particle. F: = vector from body origin to ith- particle. Б, = vector from fixed coordinate frame to body origin. X. Б. = vector from body origin to centre of mass. position, vector of body centre of mass. **X**\_ moment of body momentum. L

	π.	8	tensor of inertia.
	r,	=	body moment.
	м		body mass.
	ū	-	body velocity, with components $u_i$ . (i = 1+3)
	v	=	volume.
	ρ	=	density.
	T	-	components of stress. (i and $j = 1 \rightarrow 3$ )
	n,	-	components of unit normal $\bar{n}$ . $(j = 1 \rightarrow 3)$
	P	=	pressure,
	5.	= ]	kronecker delta.
	- 1 J	=	fluid viscosity.
	<u>,</u>	=	velocity potential.
	s	_	surface.
	2	2	moment d
	н <sub>о</sub>	2	moment.
	1	3	
		-	gradient function.
	¢jk1		permutation index.
	m <sub>i j</sub>	-	added mass coefficients. (i and j = 1+3)
	Fh		hydrostatic force.
	<b>M</b> h	• #	hydrostatic moment.
	D	-	drag force.
	F,	-	viscous force (drag).
	A	19	· area.
	f	==	coefficient.
	с	=	coefficient of drag.
•	ñ,	100	viscous moment (drag).

x

	F. i	=	external force components. $(i = 1 \rightarrow 3)$
	X <sub>bi</sub>	-	components of vector from body origin to body centre of buoyancy. (i = 1+3)
	Xdi	-	components of vector from body origin to body centre of drag. (i = 1+3) $\label{eq:component}$
	F <sub>T 10</sub>		thruster force components. (i = $1 + 3$ )
	M <sub>T 1</sub>		thruster moment components. $(i = 1 + 3)$
١	sx		sea current vector components. (i = 1-3)
	sv,	= ·	sea current velocity components. (i = 1-3)
	່ບ	=	kinematic fluid viscosity.
		_	rounolds number

xi

# CHAPTER ONE

#### INTRODUCTION

1.1 General

The devalopment of the offshore industry has created the need for sophisticated methods of underwater observation and data collection. Traditionally, the task of conducting work and testing programs underwater has been performed by commercial divers. The high cost and inherent dangers of using divers at the increasing depths being investigated has brought about the development of the submersible industry.

The submersible concept started with the creation of mini-submarines and has evolved into highly sophisticated multi-task Remotely Operated Underwater Vehicles (ROVs). Today the ROV is used in a wide variety of marine resource and oceanographic study fields. An overview of this industry and it's development and growth is available in Appendix A.

An ROV has the capacity to observe as well as collect data at depths far beyond the range of a commercial diver. The maximum depth a diver can safely work is approximately 450 meters. The maximum depth of operation for ROVs ranges from 1800 to 6000 meters. This, combined with the direct access to data the ROV provides to the non-diver engineer/scientist, offers a valuable work tool for offshore application.

The increasing growth of offshore interest provides opportunies for the submersible industry to continually grow and develop. In the last decade ROV's have increased from only a couple of dozen units available in 1975 to over 700 available units reported in 1986<sup>(1)</sup>. The vast applications of ROV's and continuing advances in capabilities assures it a place as an essential component in the development of the offshore industry. As a result It can be expected that an engineer or scientist concerned with the offshore will likely , at some time, be involved with ROV use and operation.

The use and operation of an ROV requires some knowledge of the vehicles properties and characteristics. To effectively utilize these vehicles and contribute to the process of development within this industry, it is essential that the engineer/scientist employing these units have an understanding of the fundamental properties governing the ROV.

The major concepts that influence the operation of an ROV are the general principles of solid body motion and the hydrodynamics of motion. A computer simulation of the ROV motion is an exceptional tool for the development of an understanding of these principles.

In the development of a computer simulation, when vehicle motion can be illustrated as well as the principle features of kinematic body motion and hydrodynamic motion. An analytic simulation can be used to demonstrate these vehicle characteristics. This type of simulation can also be used to evaluate design concepts and variations, in a controlled environment.

The use of the analytic simulator in design and testing, leads to a great eaving in Time and expense. The cost of testing designs in a prototype is reduced and the time required for testing a particular alteration is a fraction of that required for alterations to a real vehicle or prototype. Furthermore, the analytic simulator is the basic component for a real time simulator, which has great potential in operational procedure and training development.

1.2 Objective of this Study

This paper is the result of the need for a comprehensive investigation of the properties and principles that apply to ROV motion. Although there have been many investigations conducted on specific aspects of the ROV, there is a remarkable lack of references dealing with the motion characteristics in a form understandable to the engineer/scientist not directly involved in the ROV industry. This thesis is the result of a desire to develop a computer simulator program capable of evaluating the ROV motion characteristics and computing the ROV path and orientation . A project of this nature must start with an analytic simulator which strictly evaluates the motion of the vehicle. The natural progression after developing the analytic program, is to produce a real-time simulator. The real-time unit provides the vehicle path and orientation at the same rate that the real vehicle would travel. Within the scope of this study the development of, the analytic simulation will be the primary objective.

The two objectives of this investigation are easily combined and can be obtained by presenting the basic development procedure for the computer program of the analytic simulator.

## 1.3 Thesis Structure

This thesis will first Apresent the deneral properties that influence the ROV and then apply them so as to obtain mathematical expressions to describe the vehicle motion. Chapter two will deal specifically with the principles of solid body motion and chapter thread will discuss the hydrodynamic influence on a body in a fluid.

The remaining portion of the thesis will deal directly with the application of the motion equations within an graphlytic simulator. Chapter four will combine the

equations found in the previous chapters and modify them for the general ROV and for the specific case of the ROV unit, "Hydro Products RCV-225".

Chapter five outlines the structure and operation of the computer program with details to the method of solution being employed.

In chapter six, several examples of motion simulation attained from the program derived within this study, are demonstrated. These cases represent maneuvering procedures commonly performed by the ROV.

The final chapter of the thesis will contain the summarization and concluding remarks to the study."

#### BODY MOTION EQUATIONS

2.1 General

The intent of this chapter is to present the mathematical model of ROV motion to an engineer/scientist unfamiliar with motion dynamics and fluid properties. Although there is a reasonable amount of material available on ROVs and their operation, there is very little material available that describes the mathematical principles of motion for these units.

This chapter will deal strictly with the rigid body motion kinematics. The development of these equations of motion will be derived from Newton's first principles.

#### 2.2 Coordinate Systems

The development of the equations of motion of a rigid body requires that a particular system of reference be designated to relate each feature of motion. The choice of this system will vary depending on the nature of the problem encountered, and the characteristics being considered.

The system of reference best suited for deriving the equations of motion for an underwater vehicle is that of a double coordinate reference frame system. This system uses two coordinate frames, one frame(called the non-primed system) that is fixed with regards to the earth, and a second frame (called the primed system) that is linked to the vehicle. Figure 2.1 illustrates the relationship between these coordinate frames.

The vector  $\overline{b}_i$  represents the position of the ith particle of the rigid body with respect to the primed/linked frame. The vector  $\overline{X}_i$  represents the position of the same particle with respect to the non-primed reference frame. These two vectors combined with the vector  $\overline{X}_i$ , which relates the position of the primed origin to the non-primed system, describe the overall configuration of the body and are the basic means of describing the body motion.

2.3 Derivation of Body Motion Equations

The fundamental basis of any kinematic study are Newton's laws of motion. The key statement of these laws is that the force acting on a particle is equal to the product of the particle mass and the particle acceleration. This law is often referred to as the 'body momentum equation' and is represented as:

 $\overline{\mathbf{F}}_{\mathbf{i}} = \mathbf{m}_{\mathbf{i}} \overline{\mathbf{a}}_{\mathbf{i}}$ 

where i represents the ith particle of a rigid body. The particle momentum is the product of the particle, mass and particle velocity and is denoted as p.:

. 14

2.1



ALL DE SAU THE

Therefore Newton's law (2.1) states that the force is equal to the time rate of change of the particle momentum, written in the following manner:

 $\overline{\mathbf{F}}_{i} = \frac{\mathbf{d}}{\mathbf{d}\mathbf{E}} \ \mathbf{m}_{i} \ \overline{\mathbf{v}}_{i} = \mathbf{m}_{i} \ \frac{\mathbf{d}}{\mathbf{d}\mathbf{E}} \ \overline{\mathbf{v}}_{i}$ 

where the second equivalence is due to the particle mass being constant( rigid body).

A second aspect of Newton's law is the concept of the moment caused by a force on a particle. This law simply states that any moment caused by an exterior force must be equal to the moment of the particle momentum. This requires that the moment be referenced to some point so as to establish a moment arm. The representation of this second equation is:

 $\overline{F}_{i} \times \overline{X}_{i} = \frac{d}{dt} [m_{i}(\overline{v}_{i} \times \overline{X}_{i})] = m_{i} \frac{d}{dt} (\overline{v}_{i} \times \overline{X}_{i})$ 

with the vector  $\overline{X}$ , representing the moment arm.

Èquations 2.3 and 2.4 are the two equations that describe the particle kinematic motion. The task is to derive these two equations into a form that can be used to

2.2

2.3

2.4

mathematically describe the motion given the dynamic input to the body.

The derivation of an applicable form of these equations starts with finding the form of these equations that represents the full body and not just the ith particle. To do this three features of the rigid body must be considered:

1.  $\overline{F}_{ij} = -\overline{F}_{ji}$  :the force of the ith particle on the jth particle is equal and opposite to the force of the jth particle on the ith particle.

2.  $\overline{F}_{i,i} = 0$  : the ith particle cannot act on itself. 3.  $(\overline{X}_i - \overline{X}_j) \times \overline{F}_{i,j} = 0$  :particle forces are centrally opposed (act through centres).

These characteristics indicate that the forces applied to the ith particle are of two types. One type, are those forces from the surrounding particles of the body. The second type are from external origins. Therefore:

2.5

2.6

2.7

 $\overline{\mathbf{F}}_{i} = \overline{\mathbf{F}}_{i}^{\bullet} + \Sigma_{i-1}^{\mathsf{N}} \overline{\mathbf{F}}_{ij}, \boldsymbol{\varphi}$ 

Substituting this expanded term for the forces in the two motion equations results in:

$$\overline{F}_{i}^{s} + \Sigma_{j=i}^{s} \overline{F}_{ij} = \overline{p}_{i}$$

 $\overline{X}_{i} \times (\overline{F}_{i}^{*} + \Sigma_{j=1}^{N} \overline{F}_{ij}) = \left\{ \overline{X}_{i} \times \overline{p}_{i} \right\}$ 

Before summing these forces over the entire body, these equations can be represented in terms of the vectors  $\bar{X}_0$ ,  $\bar{X}_i$ , and  $\bar{b}_i$ ' as shown in figure 2.2 where:

11

 $\overline{X}_0$  = position of the linked axis origin wrt fixed axis origin.

X<sub>i</sub> = position of the ith particle wrt fixed axis origin.

2.8

2.9

2.10

E<sub>i</sub>' = position of the ith particle with linked axis. The diagram shows the relationship between the two coordinate axis systems and illustrates how to relate the position vectors in the following manner:

 $\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_0 = \overline{\mathbf{R}} \overline{\mathbf{b}}_1$ 

where: R= rotation tensor (see Milne-Thomson(2))

Therefore the position of the ith particle becomes:

X, = Rb, + X.

The momentum  $\overline{p}_1$  of the particle has already been described as the product of the particle mass and particle velocity. The particle velocity can be obtained from the time differentiation of the position vector (2.9).:

 $\frac{d}{dr} \bar{X}_{1} = \bar{X}_{0} + \bar{R}\bar{D}_{1}$ 

(5, is constant wrt the body)

- (



This new velocity expression combined with the momentum equations yields the new form of the force equation :

$$\overline{F}_{i}^{*} + \Sigma \overline{F}_{ij} = m_{i} \left( \overline{X}_{ij} + \frac{m}{R} \overline{D}_{i} \right)$$
2.11

and summation over the entire body yields:

2.13

2.14

The result is a force equation for the entire rigid body:  $\tilde{F} = -M(\ddot{X}_0 + \ddot{R}\tilde{b}_n) \qquad \qquad 2.15$ 

To complete this equation, the position vector  $\overline{S}_{q}$ and the tensor  $\overline{R}$  must be represented in terms of the fixed reference position vectors  $\overline{X}_{q}$  and  $\overline{X}_{q}$  and the angular velocity vector  $\overline{s}$ . To do this, the 2nd time derivative of the rotation tensor is derived from the angular velocity tensor  $\overline{n}$ .

RR = 0 + 00

 $\overline{b}_m = \frac{1}{M} \Sigma m_1 \overline{b}_1$ 

Referring back to figure 2.2 the vector  $\mathbf{b}_{a}$  can be written as:

$$\bar{X}_{a} - \bar{X}_{b} = \bar{R}\bar{b}_{a}$$
 2.17  
 $b_{a} = \bar{R}^{2}(\bar{X}_{a} - \bar{X}_{a})$ 

where:

$$\overline{X}_{ii} = \frac{1}{M} \Sigma m_i \overline{X}_i \qquad 2.1$$

From this course of manipulation the force equation can be written in full form (using the relation between  $\overline{\Omega}$  and  $\overline{\omega}$  and the triple product rule) as:

$$\overline{\mathbf{F}} = \mathbf{M}[\overline{\mathbf{X}}_0 + \overline{\omega}\mathbf{x}(\overline{\mathbf{X}}_m - \overline{\mathbf{X}}_0) + \overline{\omega} \cdot (\overline{\omega} \cdot (\overline{\mathbf{X}}_m - \overline{\mathbf{X}}_0))$$

$$- \overline{\omega}^2 \cdot (\overline{\mathbf{X}}_n - \overline{\mathbf{X}}_n)]$$
2.19

The derivation of the moment equation is conducted in the same manner . However first the moment of the body momentum is given the following notation:

2.20

 $\overline{L} = \Sigma \overline{X}, x \overline{p},$ 

Evaluating the time derivative of this term will provide the right hand side of equation 2.7, however, first it is necessary to express this equation in terms of the position vectors in figure 2.2. The particle position vector and the momentum are written utilizing these position vectors:

2:21

2.25

$$\overline{X}_{1} = \overline{X}_{0} + \overline{R}\overline{D}_{1}$$

$$\overline{p}_{1} = m_{1}(\overline{X}_{0} + \overline{R}\overline{D}_{1})$$

and the cross product of these two vectors yields :

$$\vec{X}_1 \times \vec{p}_1 = m_1 [(\vec{X}_0 \times \vec{X}_0) + (\vec{X}_0 \times \vec{R} \vec{b}_1) - (\vec{X}_0 \times \vec{R} \vec{b}_1) 2.22$$
  
+ $\overline{(\vec{R} \vec{b}_1 - \mathbf{X} \vec{R} \vec{b}_1)}]$ 

The first simplification of this equation comes about by recognizing the role of the inertia tensor in the last term of the expression :

$$\begin{split} \overline{\mathbf{R}}\overline{\mathbf{b}}_{i} \times \overline{\mathbf{R}}\mathbf{b}_{i} &= (\overline{\mathbf{X}}_{i} - \overline{\mathbf{X}}_{0}) \times \overline{\mathbf{R}}\overline{\mathbf{R}}^{i} (\overline{\mathbf{X}}_{i} - \overline{\mathbf{X}}_{0}) \\ &= (\overline{\mathbf{X}}_{i} - \overline{\mathbf{X}}_{0}) \times \overline{\mathbf{w}} \times (\overline{\mathbf{X}}_{i} - \overline{\mathbf{X}}_{0}) \end{split}$$

Reducing this expression using the triple cross product and dyadic<sup>(2)</sup> format results in:

$$\overline{\overline{R}}\overline{D}_{1} \times \overline{\overline{R}}\overline{D}_{1} = [(\overline{X}_{1} - \overline{X}_{0})^{2} \cdot \overline{\overline{I}} - (\overline{X}_{1} - \overline{X}_{0}) \otimes (\overline{X}_{1} - \overline{X}_{0})] \cdot \overline{\omega} \qquad 2.24$$

and now returning this expression to a form utilizing  $\overline{\boldsymbol{b}}_i$  gives :

$$\overline{R}\overline{b}_{i} \times \overline{\overline{R}}\overline{b}_{i} = \overline{R}[(\overline{b}_{i})^{2} \cdot \overline{I} - \overline{b}_{i} \otimes \overline{b}_{i}] \cdot \overline{R}^{T} \cdot \overline{\omega}$$

The expression within the square brackets in this result, when summed over the entire rigid body with the mass product included, is the Tensor of Inertia. This tensor combines the products of inertia and moments of inertia in one identity. The tensor of inertia is represented by  $\lambda^{\prime}$ . Using this notation the equation 2.22 becomes: .

 $\overline{X}_{i} \times \overline{p}_{i} = m_{i} \left[ \left( \overline{X}_{0} \times \overline{X}_{0} \right) + \left( \overline{X}_{0} \times \overline{R} \overline{B}_{i} \right) - \left( \overline{X}_{0} \times \overline{R} \overline{B}_{i} \right) \right] \right)$   $+ \overline{X}^{i} \cdot \overline{\omega}$ 

Completing the summation over the rigid body gives:

 $\overline{L} = M[(\overline{X}_0 \times \overline{X}_0) + (\overline{X}_0 \times \overline{\overline{X}}_0 - \overline{X}_0 \times \overline{\overline{X}}) \cdot \overline{b}_n] + \overline{\overline{\lambda}} \cdot \overline{\omega}$  2.27

Now that the expression for the moment of the body momentum has been established, the time derivative must be found to apply this to equation 2.7:

$$\begin{split} \vec{L} &= (\vec{\omega} \times \vec{\lambda}^{*}) \cdot \vec{\omega} + \vec{\lambda}^{*} \cdot \vec{\omega} + \vec{\chi}_{0} \times \vec{F} + M[(\vec{X}_{0} - \vec{X}_{0}) \times \vec{X}_{0}]^{*} \quad 2.28 \end{split}$$
 where the rules for differentiating tensors have been applied.

The remaining step'in completing the second equation of motion is to combine'L with the left hand side of equation 2.7,:

$$\Sigma \left[ \overline{X}_{1} \times (\overline{F}_{1}^{*} + \Sigma \overline{F}_{1,j}) - \overline{X}_{0} \times \overline{F} \right] = (\overline{\omega} \times \overline{\lambda}^{*}) \cdot \overline{\omega} + \overline{\lambda}^{*} \cdot \overline{\omega}$$

$$+ M \left[ (\overline{X}_{n} - \overline{X}_{0}) \times \overline{X}_{0} \right]$$

and using the condition of the rigid body this.becomes:,

 $\overline{\mathbf{T}}_{\mathbf{0}} = \overline{\boldsymbol{\omega}} \times \overline{\lambda} \cdot \overline{\boldsymbol{\omega}} + \overline{\lambda} \cdot \overline{\boldsymbol{\omega}} + \mathbf{M} (\overline{\mathbf{X}}_{\mathbf{n}} - \overline{\mathbf{X}}_{\mathbf{0}}) \times \overline{\mathbf{X}}_{\mathbf{0}}$ where:

 $\overline{\Gamma}_{0} = \Sigma (\overline{X}_{1} - \overline{X}_{0}) \times \overline{F}_{1}^{0}$ 

Therefore, the two equations of motion have been derived from the first principles of Newton's laws of motion. Equations 2.19 and 2.30 represent the kinematic motion characteristics of a rigid body subject to some external forces and moments.

### 2.3.1 System of Reference

The two equations of motion must be specified in a particular reference frame in order to be applicable to the problem. However, before this is done, there is an important simplification that can be utilized, that will reduce these two equations significantly. This simplification is with regards to the placement of the body linked coordinate system within the rigid body. One can see from equations 2.19 and 2.30 that if the  $(\bar{X}_{-} - \bar{X}_{0})$  terms were to be eliminated, the resulting equations would be considerably more manageable. To eliminate this term, the origin of the linked frame must be placed at the centre of mass of the rigid body. Therefore, employing this convention results in the following motion equations:

2.30

2.31



Finally the rigid body equations must be expressed in vector/tensor notation by specifying a coordinate system for each equation. To determine the scoordinate frame for these equations to be expressed, careful consideration of the problem and application is required. In the case of this application, the computer application 4ill be most convenient if both motion equations are expressed in terms of the coordinate frame fixed with the body. Therefore the motion equations are expressed in matrix form:

$(F)' = M(\ddot{X})'$		2.34
$\{\Gamma_0\}^{\dagger} = \{\omega\}^{\dagger} \times [\lambda]^{\dagger} \cdot [\omega]^{\dagger} + [\lambda]^{\dagger} \cdot (\omega)^{\dagger}$		2.35
Equation 2.54 is expanded by using .		
$(\dot{x})' = (u)'$		2.36
and:		
$(\mathbf{u})' = \frac{d}{d \in \mathcal{A}} \{\mathbf{u}\}' + \{\boldsymbol{\omega}\}' \mathbf{x} \{\mathbf{u}\}'$		2.37
<b>&gt;</b> • • •		
Thus the components of the force equation	n can be	e written as
follows:		·. *
$F_1 = M(\dot{u}_1' + \omega_2' u_3' - \omega_3' u_2')$		2.38
$F_2' = M(u_2' + \omega_3'u_1' - \omega_1'u_3')$		* 2.39

$$\mathbf{F}_{3}' = \mathbf{M}(\dot{\mathbf{u}}_{3}' + \omega_{1}'\mathbf{u}_{2}' - \omega_{2}'\mathbf{u}_{1}')$$
 2.40

Similarly the expansion of the moment equation to component form yields:

$$\begin{split} \Gamma_{01}^{\dagger} &= \lambda_{11}^{\dagger}\dot{\omega}_{1}^{\dagger} + \lambda_{12}^{\dagger}\dot{\omega}_{2}^{\dagger} + \lambda_{13}^{\dagger}\dot{\omega}_{3}^{\dagger} + \lambda_{31}^{\dagger}\omega_{1}^{\dagger}\omega_{2}^{\dagger} + \lambda_{32}^{\dagger}\omega_{2}^{\dagger}\omega_{2}^{\dagger} & 2.43 \\ &+ \lambda_{33}^{\dagger}\omega_{2}^{\dagger}\omega_{3}^{\dagger} - \lambda_{21}^{\dagger}\omega_{21}^{\dagger}\omega_{3}^{\dagger} - \lambda_{22}^{\dagger}\omega_{2}^{\dagger}\omega_{3}^{\dagger} - \lambda_{23}^{\dagger}\omega_{3}^{\dagger}\omega_{3}^{\dagger} \end{split}$$

2.42

2.43

$$\begin{array}{rcl} \lambda_{21}^{\prime}\dot{\omega}_{1}^{\prime} &+& \lambda_{22}^{\prime}\dot{\omega}_{2}^{\prime} &+& \lambda_{23}^{\prime}\dot{\omega}_{3}^{\prime} &+& \lambda_{11}^{\prime}\omega_{1}^{\prime}\dot{\omega}_{3}^{\prime} &+& \lambda_{12}^{\prime}\omega_{2}^{\prime}\omega_{3}^{\prime} \\ &\dot{\omega}_{1}^{\prime}\dot{\omega}_{1}^{\prime}\dot{\omega}_{2}^{\prime}\dot{\omega}_{2}^{\prime}\dot{\omega}_{2}^{\prime} &-& \lambda_{33}^{\prime}\omega_{2}^{\prime}\dot{\omega}_{1}^{\prime} &-& \lambda_{33}^{\prime}\omega_{2}^{\prime}\dot{\omega}_{2}^{\prime} \end{array}$$

$$\Gamma_{03} = \lambda_{31}^{\prime} \dot{\omega}_{1}^{\prime} + \lambda_{32}^{\prime} \dot{\omega}_{2}^{\prime} + \lambda_{33}^{\prime} \dot{\omega}_{3}^{\prime} + \lambda_{21}^{\prime} \omega_{1}^{\prime} \omega_{1}^{\prime} + \lambda_{22}^{\prime} \omega_{1}^{\prime} \omega_{2}^{\prime} \\ + \lambda_{23}^{\prime} \omega_{1}^{\prime} \omega_{3}^{\prime} - \lambda_{11}^{\prime} \omega_{1}^{\prime} \omega_{2}^{\prime} - \lambda_{12}^{\prime} \omega_{2}^{\prime} \omega_{2}^{\prime} - \lambda_{13}^{\prime} \omega_{2}^{\prime} \omega_{3}^{\prime}$$

Therefore equations  $2\cdot39-2\cdot43$  constitute the components of the kinematic reaction to forces acting on a rigid body with 6 degrees of motion freedom. In specific, these equations are developed to represent the motion of an underwater vehicle , where the external forces are a result of propulsion thrusters and fluid dynamic forces.

The next chapter will concentrate on the dynamic forces that act on an ROV and how these forces can be expressed mathematically, as the left hand components of the motion equations.
## CHAPTER THREE HYDRODYNAMIC FORCES

3.1 General

The equations of motion for a rigid body have been derived in the previous chapter. These equations relate the mechanical/geometric reaction of a rigid body, to external forces acting on the body. Therefore, in order to apply these equations to a particular situation, the external forces present must be established in a mathematical form. This chapter will develop the mathematical formulae that express the forces that act on a submerged rigid body as a result of the surrounding fluid.

The most common approach to fluid flow analysis is to study the problem using two different assumptions and then, superimpose the results to form the generalised solution. The two assumptions used are :

1. Non-viscous fluid assumption

2. Viscous fluid assumption

To study the problem in this method allows the scientist to first neglect the internal shear stress caused by the viscosity of the fluid, and obtain mathematical solutions for describing the fluid motion and characteristics. After these expressions have been established; the effect of viscosity can be determined by theoretical and empirical study and combined with the non-viscous solution to provide a generalised solution to the real fluid problem.

In this chapter the 'ideal fluid' assumption (nonviscous) will be implied, and the force and moment contribution to the rigid body motion equations formulated. In a separate derivation, the effect of the fluid viscosity will be analyzed and incorporated into the motion equations.

3.2 Fluid Properties

The properties of fluid flow are governed by four basic relations. These relations are stated 'as follows (quoted from Streeter and Wylie<sup>(3)</sup>):

- Newton's laws of motion, which must hold true for every particle at every instant.
- The continuity relation, i.e., the law of conservation of mass.
- 3. The first and second laws of thermodynamics
- Boundary conditions; analytical statements that a real fluid has zero velocity relative to a boundary at a boundary or that frictionless fluids cannot penetrate a boundary.

These relations are fundamental properties that govern any fluid motion. As a result, they each play an important role in the development of the fluid contribution to the solid body motion equations being derived in this study. 3.3 Hydrodynamic Forces - Ideal Fluid

The force on a rigid body within a fluid domain can be expressed in terms of the pressure within the fluid by using the Bernoulli equation (which comes from the integration of the Navier-stokes equation with viscosity neglected, over the space coordinates) in the following form.:

3.1

$$P = -\rho \left[\frac{\partial}{\partial t} \phi + \frac{1}{2} \left(\frac{\partial}{\partial x_{j}} \phi\right)^{2}\right] + \int F_{i} dx_{i}$$

Where the symbol  $\phi$  represents the fluid velocity potential. This is a scalar potential whose gradiant expresses the fluid velocity vector. The velocity potential is dependent on the three space coordinates and time. The advantage of applying this potential is that it represents one unknown function rather than four unknown variables. A full explaination and derivation of the velocity potential is available in Streeter and Wylie chapter seven section 7, 3(3).

The force due to the pressure is equivalent to the integral of the pressure over the area on which it acts. Thus to find the force on a body submerged in a fluid, the integral of the pressure described in equation 3.1 must be evaluated over the body normal area. In doing this it is best to separate the pressure equation into two expressions and deal with each component individually. Therefore, first

the hydrodynamic term of the Bernoulli equation will be considered. This term is the expression within the square brackets in equation 3.1. The hydrostatic term, concerning the external force  $F_i$ , will be dealt with in a later section.

The force and moment due to the hydrodynamic pressure term, are found from the following integrals. (equations 81 and 82 from Newman<sup>(4)</sup>):

∬<sub>g B</sub> Pn dS 3.2  $\overline{M}_0 = \iint_{SB} P(\overline{r} \times \overline{n}) dS$ 3.3

In these expressions, the normal vector is considered to be positive out of the fluid, and thus, into the body (see figure 3.1). The substitution of the hydrodynamic term of the pressure in these equations results in (equations 83 and 84 Newman<sup>(4)</sup>):

 $\overline{F} = \iint_{SB} - \rho \left( \frac{\partial}{\partial E} \phi + \frac{1}{2} \left( \frac{\partial}{\partial x_1} \phi \right)^2 \right) \cdot \overline{n} dS$ 3.4  $\overline{M}_{0} = \iint_{SB} - \rho \left( \frac{\partial}{\partial t} \phi + \frac{1}{2} \left( \frac{\partial}{\partial x_{i}} \phi \right)^{2} \right) \cdot \left( \overline{r} \times \overline{n} \right) dS$ 3.5

Application of the basic principles of the law of conservation of mass and using the divergence and transport theorems along with careful consideration of the fluid



Figure 3.1: Rigid Body and Control Surfaces

boundary conditions leads to the following form of the force and moment equations in terms of the body fixed coordinate system. (equations 110 and 111 from Newman<sup>(4)</sup>)(see Newman for full-derivation.):

25

$$\mathbf{j} = -\rho \mathbf{u}_{i} \iint_{SB} \phi_{i} \mathbf{n}_{j} dS - \rho \epsilon_{jkl} \mathbf{u}_{i} \omega_{k} \iint_{SB} \phi_{i} \mathbf{n}_{l} dS$$

$$M_{oj} = -\rho \epsilon_{jk1} u_i u_k \iint_{SB} \epsilon_i n_i dS - \rho u_i \iint_{SB} \epsilon_i (\bar{\mathbf{x}}^* \mathbf{x} \bar{\mathbf{n}})_j dS \qquad 3.7'$$
$$-\rho \epsilon_{jk1} u_i \omega_k \iint_{SB} \epsilon_i (\bar{\mathbf{x}}^* \mathbf{x} \bar{\mathbf{n}})_1 dS$$

3.6

(see figure 3.2 for coordinate representation)

These two equations are seen to be dependent on the potentials  $\phi_i$  in terms of the integrals over the body surface. The contribution of  $\phi_i$  and these integrals are a solely dependent on the body shape and can be evaluated separately as coefficients to the equation. These integrals

.



are called the added-mass coefficients. The added-mass tensor is expressed as (equation 114 from Newman<sup>(4)</sup>):

$$m_{i,j} = \rho \iint_{SB} \phi_i \frac{\partial}{\partial n} \phi_j dS$$
 3.8

and therefore the force and mass equations can now be written utilizing this tensor. (equation 115 and 116 from Newman(4)):

$$F_{j} = \hat{u}_{1}m_{j1} - \epsilon_{jk1}u_{1}\omega_{k}m_{11} \qquad 3.9$$
$$M_{-1} = -\epsilon_{1k1}u_{1}u_{1}m_{1-1} - \epsilon_{1k1}u_{1}\omega_{k}m_{1-1} \qquad 3.10$$

where j=1 to 3, and i=1 to 6 and:

 $\begin{aligned} \epsilon_{i_k,j,k} &= 1 \quad \text{if i, j, } k +^* \text{ cyclic (123, 231, 312)} \\ &= -1 \quad \text{if i, j, } k -^* \text{ cyclic (321, 213, 132)} \\ &= 0 \quad \text{if i = j, } j = k \text{ or } i = k \end{aligned}$ 

These two equations come as a result of carefully considering the hydrodynamic terms of the Bernoulli equation integrated over the body surface. The derivation of this form of the force and moment equations is only possible when particular conditions have been imposed on the fluid domain. The limiting conditions used thus far are:

1. Fluid is Ideal and Irrotational.

 The body is rigid (constant volume) with six degrees of motion.freedom. 3. The fluid domain is unbounded.

3.3.1 Added-Mass Coefficients

The development of the force and moment equations in a form that utilizes the added mass coefficients is a significant simplification to the problem. The decomposition of the fluid velocity potential and thus the development of the added-mass terms as integrations of geometrically dependent potentials over the body surface provide solvable expressions for the ideal fluid influence on the body.

The expression 'geometrically dependent potentials' refers to the fact that these potentials are, independent of time and are strictly a function of the body shape. This means that the added-mass coefficients can be evaluated for a body shape without concern for time or the fluid characteristics.

These coefficients do have an important analytical significance that should be noted. The added-mass terms, as indicated by the name, represent a mass of volume that moves with the body as a result of the body motion. This mass is not like the solid mass that Newton's equations deal with, in that this volume is not dependent on the direction of the body acceleration. That is to say, the added-mass coefficients differ for different directions of motion, and therefore for coupled motions, coupled added-masses must be employed.

3.4 Hydrostatic Force

The previous sections dealt with the hydrodynamic forces acting on a submerged body. It remains to consider the other term of the Bernoulli pressure expression. This term represents the exterior forces acting within the fluid. In the case of open sea, the only exterior force acting on the fluid domain is the gravitational force of the earth.

Therefore, the hydrostatic force and moment are comprised of the body mass and buoyancy expressed here in terms of the space fixed coordinate frame.:

 $F_{h} = F_{h} = (\rho V - M)g$ 

 $\overline{M}_{h} = (\rho g \nabla \overline{X}_{h}) x \overline{X}_{3}$ 

where:

V= body volume

M= body mass

Where the vector  $\bar{X}_{b}$  represents the vector to the centre of buoyancy in the space fixed reference frame.

3.5 Viscous Effects (Drag Force)

The contribution that the fluid stress makes to

29

3.11

the body forces in the fluid appears in the Navier-Stokes equation for the fluid domain. To analysis this contribution scientists have made many attempts to relate the fluid properties to the forces applied through mathematical theory. However, thus far, the only truly successful means of representing the viscous affect of the fluid has come from empirical methods of evaluation

Newton attempted to express the viscous contribution of the fluid in his first law of resistance. In this theory Newton conceptualized the force caused by the fluid viscosity as being the result of particles of fluid impacting against the body and thus the body imparting a measure of momentum to the fluid particle. Newton expressed this in the following manner:

3.13

....

 $D' = fA_0 u^2$ 

•

herę:	u = velocity
	A = projected area in u direction
	$\rho$ = fluid density
	f = factor of porportionality
	the second second second second

noting:  $\rho \lambda u^2$  = mass of fluid impacting/time  $\rho \lambda u^2$  = momentum imparted/time

Newton's consideration has been found to be unsatisfactory for gost general flow situations. This relation fails to consider the conditions around the body other than the front (area perpendicular to direction of motion). Further work in this study has utilized Newton's formulae to develop equations that can apply to a wider scope of problems.

The forces of viscous origin are called the drag forces and can be divided into three major components.:

1. Deformation Drag 2. Friction Drag 3. Pressure Drag

Deformation drag is predominant in cases where the reynolds number is very low. That is, the fluid viscosity is very high or the body velocity and/or size are very small. This type of motion, known as 'creeping motion', exhibits viscous forces being predominant over the inertia forces discussed in the previous chapter. In marine engineering, this type of motion is seldom encountered.

In the general marine environment the drag force on the body will consist of a combination of friction drag and viscous pressure drag. In these situations the fluid viscosity is relatively small and the body dimensions large. As a result the inertia forces are predominant for this type of motion. It is important, however, to consider the drag contribution and not rely entirely on the inertia result.

Friction drag, or skin friction, is caused by the fluid undergoing a rapid change in velocity from the fluid velocity in the outer fluid region to the body velocity at the body surface. This change in velocity occurs over a very small layer called the boundary layer. The sudden change

gives rise to the shear stresses which integrated over the body surface constitute the friction drag.

Viscous pressure drag is a result of the flow pattern about a body being altered due to the fluid viscosity. Any change in the flow pattern will alter the pressure distribution about the body, and ultimately produce a further force due to pressure on the body.

The viscous pressure drag is largely dependent on the shape of the body, whereas the friction drag is dependent on the surface area of the body. Therefore, the primary factors concerning the total drag are:

1. size, shape and position of the body

. velocity of the body

3. fluid properties (density, viscosity)

where the second and third come from the consideration of-Newton's formula expressing drag.

Although ideally these components should be evaluated separately and then combined to represent the total drag, this has not been possible for nost cases. Instead, work has concentrated on finding a generalized solution to the total drag. Thus far the most effective means of obtaining satisfactory results for these forces is through empirical testing in the laboratory. The experimental work centres around finding coefficients (factors of proportionality in Newton's formulae) to represent bodies and shapes. The general format for the expression is similar to that of Newton's formulae, and is derived from simple dimensional analysis.:

 $\overline{F}_{v} = C\rho A |u|^{2}$ 

Furthermore the moment due to this drag force can be written

 $\overline{M}_{v} = C_{\rho} \lambda |u|^{2} \times \overline{X}_{d}$ 

where  $\bar{X}_{\rm d}$  is the vector from the body origin to the centre of drag for the body.

In both these expressions the coefficients are dependent on the body shape and the Réynold's number. The expressions can be modified to be in terms of the dynamic pressure as follows:

 $E_{u} = CA(\rho |u|^2/2)$ 

3.16

3.14

 $M_{e} = CA(\rho |u|^{2}/2) \times \bar{X}_{e}$ 

3.17

The Reynold's number is a function of the body velocity, shape and the fluid viscosity. The coefficient of drag is a function of the reynolds number and, by means of experimental work, the coefficients can be found for a given shape for a wide range of reynolds numbers. Once the relationship between the coefficient and reynolds number for a particular shape has been found, the size, velocity, and or fluid properties can be altered without having to find a new relationship for the coefficients and reynolds number.(as long as the body shape doesn't vary)

In any work with ROVs, laboratory testing would have to be carried out with the real ROV or a scale model in order to find the relationship for the coefficients of drag.

3.6 Summary

This chapter has dealt with the fluid forces that a submerged body will be influenced by. The three main catagories of forces are:

- 1. Hydrodynamic Ideal Fluid
- 2. Hydrostatic
- 3. Hydrodynamic Viscous Fluid

The application of these equations will require that they be combined with the equations derived in chapter two, and that the characteristics of the ROV be determined so as to establish the value of the coefficients required. The result of combining these equations with those from the previous chapter will be a system of differential equation that are solvable by means of numerical integration.

The remaining chapters of this thesis will deal with the combination of these equations and their application to a particular ROV, as well as the development

of a computer program to solve these equations in the form of a motion simulator.

>

# CHAPTER FOUR

4.1 Combined Body Motion Equations

The equations of motions found in chapters 2 and 3, must be used to describe the motion of an ROV. In combining these equations, careful consideration has to be given to the limiting conditions already expressed in the derivation of these formulae, as well as the limitations of the ROV in terms of motion and configuration. This chapter will finalize the equations of motion for a general ROV unit and also apply these equations to the specific case of the ROV unit "RCV-225".

## 4.2 ROV Motion

In the common small ROV the motion of the vehicle does not extend in all six degrees of freedom. The general format allows for all three translational motions and only one rotational motion. Figure 4.1 shows the typical ROV motion configuration. This limitation of motion comes about as a result of the ROV tether restrictions and the need for conservation of control requirements. This format of motion allows for all forms of maneuvering required without introducing extra thrusters and controls that would add unnecessary complexity to the ROV.



These limitations are helpful in simplifying the equations of motion by eliminating two angular velocity components and thus the body velocity vectors become.:

- $\bar{\mathbf{u}} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$
- $\overline{\omega} = \{0, 0, \omega_3\}$  4.2

4.1

Utilizing this condition the equations of kinematic motion can be rewritten from chapter two as follows:

$$\begin{split} M(\dot{u}_{1}^{1}-\omega_{1}^{1}u_{2}^{1}) &= F_{1,1}^{1} & 4.3 \\ M(\dot{u}_{1}^{1}+\omega_{2}^{1}u_{1}^{1}) &= F_{1,2}^{1} & 4.4 \\ M(\dot{u}_{2}^{1}) &= F_{1,2}^{1} & 4.4 \\ M(\dot{u}_{2}^{1}) &= F_{1,2}^{1} & 4.5 \\ \lambda_{2,2}^{1}\dot{u}_{2}^{1}-\lambda_{2,2}^{1}\omega_{2}^{1}-u_{2}^{1} &= F_{0,2}^{1} & 4.6 \\ \lambda_{2,2}^{1}\dot{u}_{2}^{1}-\lambda_{1,2}^{1}\omega_{2}^{1}-u_{2}^{1} &= F_{0,2}^{1} & 4.7 \\ \lambda_{3,2}^{1}\dot{u}_{3}^{1} &= F_{0,2}^{1} & 4.8 \end{split}$$

Likewise the dynamic fluid force equations from chapter three can be simplified to component form.:

 $F_{1}^{*} = -\dot{u}_{1}^{*}m_{11} - \dot{u}_{2}^{*}m_{12} - \dot{u}_{3}^{*}m_{13} - \dot{\omega}_{2}^{*}n_{16} \qquad 4.9$ +  $[\omega_{1}^{*}(u_{1}^{*}m_{21} + u_{2}^{*}m_{22} + u_{2}^{*}m_{23} + \omega_{3}^{*}m_{26})]$ 

$$\begin{array}{rcl} F_2' &=& -\dot{u}_1' n_{2,1} & - \dot{u}_2' n_{2,2} & - \dot{u}_3' n_{2,3} & - \dot{u}_3' n_{2,5} \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & &$$

$$\begin{split} &-(m_{k_1}\dot{u}_1^{k_1}+m_{k_2}\dot{u}_2^{k_1}+m_{k_2}\dot{u}_2^{k_1}+m_{k_2}\dot{u}_3^{k_1}+m_{k_2}\dot{u}_3^{k_1})\\ &+(u_2^{k_1}u_{k_2}^{k_1}+u_2^{k_2}u_{k_2}^{k_2}+u_2^{k_1}m_{k_2}+u_2^{k_2}m_{k_2}+u_3^{k_2}m_{k_2})\\ &+(\omega_3^{k_1}(u_1^{k_1}m_{s_1}+u_2^{k_1}m_{s_2}+u_3^{k_1}m_{s_1}+\omega_3^{k_1}m_{s_2})) \end{split}$$

$$\begin{array}{rcl} & - \left(m_{51}\dot{u}_{1}^{i} + m_{52}\dot{u}_{2}^{i} + m_{53}\dot{u}_{3}^{i} + m_{56}\dot{w}_{1}^{i}\right) \\ & + \left(u_{1}^{i}u_{1}^{i}m_{51} + u_{1}^{i}u_{2}^{i}m_{22} + u_{1}^{i}u_{1}^{i}m_{33} + u_{2}^{i}\omega_{2}^{i}m_{56}\right) \\ & - \left(\omega_{3}^{i}\left[u_{1}^{i}m_{41} + u_{2}^{i}m_{42} + u_{3}^{i}m_{43} + \omega_{4}^{i}m_{56}\right]\right) \\ & + \left(-u_{1}^{i}u_{1}^{i}m_{21} - u_{1}^{i}u_{2}^{i}m_{22} - u_{1}^{i}u_{3}^{i}m_{23} - u_{1}^{i}\omega_{2}^{i}m_{26}\right) \\ & - \left(m_{61}\dot{u}_{1}^{i} + m_{62}\dot{u}_{2}^{i} + m_{63}\dot{u}_{3}^{i} + m_{66}\dot{w}_{3}^{i}\right) \end{array}$$

+  $(u_2'u_3'm_{11} + u_2'u_2'm_{12} + u_2'u_3'm_{13} + u_2'\omega_3'm_{16})$ 

It remains to do the same for the equations of hydrostatic and viscous forces and moments. In the case of the hydrostatic force and moment it is obvious that these forces and moments are not dependent on the body velocity and thus, they do not change.:(expanded to component form)

F<sub>h1</sub> = 0

M."

4.15

4.11

4.13

4.14



The forces and moments due to the viscosity of the vater found from dimensional analysis are dependent on the translation velocity of the vehicle and thus are not subject to any change due to the lose of two rotational freedoms. These equations are written in component form below:

$$\dot{F}_{v1} = -C_{1} \lambda \rho / 2 | u_{1}^{v} | u_{1}^{v}$$

$$F_{v2} = -C_{2} \lambda \rho / 2 | u_{2}^{v} | u_{2}^{v}$$

$$f_{v3} = -C_{3} \lambda \rho / 2 | u_{3}^{v} | u_{3}^{v}$$

$$4.22$$

$$\mathbf{K}_{v_1} = -(\mathbf{C}_2^{'} | \mathbf{u}_2^{'} | \mathbf{u}_2^{'} \mathbf{X}_{d_2} - \mathbf{C}_2^{'} | \mathbf{u}_2^{'} | \mathbf{u}_2^{'} \mathbf{X}_{d_3}) \lambda \rho/2 \qquad 4.2$$

 $M_{v2} = -(C_1 | u_1^* | u_1^* X_{d3} - C_3 | u_3^* | u_3^* X_{d1}) \lambda \rho/2 .$ 

 $U_{3} = -(C_{2} | u_{2}^{i} | u_{2}^{i} X_{d_{1}} - C_{1} | u_{1}^{i} | u_{1}^{i} X_{d_{2}}) \lambda \rho/2. \qquad 4.26$ 

where : A= projected body area

Therefore, the equations of motion for the generalized ROV can now be written in their final form by combining the kinematic, dynamic, hydrostatic, and viscous contributions into one set of equations in component form. This is easily accomplished for all except the case of the hydrostatic formulae. The hydrostatic equations are written in terms of the space fixed coordinates where the other terms are written in the body fixed coordinates. The best way to deal with this situation is to evaluate the hydrostatic forces separately and use the rotation tensor to transform this contribution into the body fixed system of preference.

Therefore, the generalized equations of motion for an ROV are:

$$\begin{split} & (\mathbf{M} + \mathbf{m}_{11}) \dot{\mathbf{u}}_1 + \mathbf{m}_{12} \dot{\mathbf{u}}_2 + \mathbf{m}_1 \dot{\mathbf{u}}_3 + \mathbf{m}_1 \dot{\mathbf{e}}_3 \\ & = \mathbf{m}_2 \mathbf{1} \mathbf{u}_1 \mathbf{u}_3 + (\mathbf{M} + \mathbf{m}_2 \mathbf{1}) \mathbf{u}_2 \mathbf{u}_3 + \mathbf{m}_2 \mathbf{u}_3 \mathbf{u}_3 + \mathbf{m}_2 \mathbf{e} \mathbf{u}_3 \mathbf{u}_3 \\ & - \mathbf{C}_1 \mathbf{\lambda} \mathbf{e}/\mathbf{2} \mathbf{1} \mathbf{u}_1 | \mathbf{u}_1 + \mathbf{F}_{11} + \mathbf{F}_{21} \end{split}$$

$$\begin{split} & m_{21}\dot{u}_1 + (M+m_{22})\dot{u}_2 + m_{23}\dot{u}_3 + m_{26}\dot{u}_3 & 4.28 \\ \\ & = -(M+m_{11})u_1\omega_3 - m_{12}u_2\omega_3 - m_{13}u_3\omega_3 - m_{16}\omega_3\omega_3 \\ \\ & = -C_2\lambda_\beta/2|u_2|u_2 + F_{12} + F_{12} \end{split}$$

 $m_{41}\dot{u}_1 + m_{42}\dot{u}_2 + m_{43}\dot{u}_3 + (\lambda_{13} + m_{46})\dot{\omega}_3$ 

4.30

$$\begin{array}{rcl} & & = & -m_{2,1}\,u_{2,1}\,u_{1}\,-\,m_{2,2}\,u_{2,1}\,2_{2}\,-\,m_{2,3}\,u_{2,1}\,u_{3}\,-\,m_{2,6}\,u_{2,2}\,u_{3}\,+\,m_{2,1}\,u_{1}\,u_{3}\,\\ & & & +\,m_{2,2}\,u_{3}\,u_{2}\,+\,m_{2,3}\,u_{3}\,u_{3}\,+\,m_{2,6}\,u_{3}\,u_{3}\,+\,m_{3,1}\,u_{3}\,+\,m_{3,1}\,u_{3}\,+\,m_{3,1}\,u_{3}\,u_$$

$$\begin{split} & m_{31}\dot{u}_1 + m_{52}\dot{u}_2 + m_{33}\dot{u}_3 + (\lambda_{23} + m_{58})\dot{\omega}_3 & 4.31 \\ \\ & = & -m_{11}u_1u_3 - m_{12}u_2u_3 - m_{13}u_3u_3 - m_{16}u_3\omega_3 + m_{31}u_1u_1 \\ & + m_{32}u_1u_2 + m_{33}u_1u_3 + m_{56}u_1\omega_3 - m_{41}u_1\omega_3 - m_{42}u_2\omega_3 \\ & - m_{43}u_3\omega_3 - (\lambda_{13} + m_{46})\omega_3\omega_3 - (C_1|u_1|u_1X_{43} \\ & - C_3|u_3|u_3X_{41}\rangle\lambda\rho/2 + M_{43}^* + M_{72} \end{split}$$

$$\begin{split} \mathfrak{m}_{e\,1}\dot{\mathfrak{u}}_1 &+ \mathfrak{m}_{e\,2}\dot{\mathfrak{u}}_2 &+ \mathfrak{m}_{e\,3}\dot{\mathfrak{u}}_3 &+ (\lambda_{3\,3} &+ \mathfrak{m}_{e\,6})\dot{\mathfrak{u}}_3 & 4.32 \\ \\ &= -\mathfrak{m}_{2\,1}\mathfrak{u}_1\mathfrak{u}_1 &- \mathfrak{m}_{2\,2}\mathfrak{u}_1\mathfrak{u}_2 &- \mathfrak{m}_{2\,3}\mathfrak{u}_1\mathfrak{u}_3 &- \mathfrak{m}_{2\,6}\mathfrak{u}_1\mathfrak{u}_3 &+ \mathfrak{m}_{1\,1}\mathfrak{u}_1\mathfrak{u}_2 \\ &+ \mathfrak{m}_{1\,2}\mathfrak{u}_2\mathfrak{u}_2 &+ \mathfrak{m}_{1\,3}\mathfrak{u}_2\mathfrak{u}_3 &+ \mathfrak{m}_{1\,6}\mathfrak{u}_2\mathfrak{u}_3 &- (\tilde{\mathsf{C}}_2 \,|\,\mathfrak{u}_2 \,|\,\mathfrak{u}_2\chi_{4\,1} \\ &- \mathsf{C}_1 \,|\,\mathfrak{u}_1 \,|\,\mathfrak{u}_1\chi_{4\,2} \,) \Lambda \rho/2 &+ \mathfrak{M}_{1,3}^{k} &+ \mathfrak{M}_{7,3} \end{split}$$

Where the like terms have been grouped and the equations have been arranged in a general differential equation format.

Equations 4.27-4.32 represent the component form of the general equations of motion for a fully submerged body. This form of the equations are seldom expressed in texts and technical presentations due to there length and complexity. The computer simulation for motion of the body is based on these equations where they have first been simplified for the specific case of the "Hydro-Products RCV-225" remotely operated underwater vehicle.

\*

Prior to discussing the program for motion simulation, the specific case of the RCV-225 will be considered and simplifications that the body configuration of this unit allows will be used to modify the generalized equations of motion.

4.3 RCV-225

The advantage in choosing the RCV-225 for a first attempt simulation study is in the particular simplifications that come about as a result of the sphere like shape of this unit.(see figure 4.2) The body coefficients for this shape have been scientifically researched for many years and thus these values can be obtained without having to carry out extensive experimental procedures.

Using the condition stipulated earlier , placing the body fixed coordinate frame at the centre of mass , combined with the condition that the vehicle weight be evenly distributed through out the body will eliminate the moment due to buoyancy and simplify the tenhor of inertia. These two conditions will result in the body fixed reference frame being located at the centre of the sphere.

With the body reference frame at the centre of the sphere, it is coincident with the location of the centre of buoyancy for a sphere and thus the vector  $x_b$  is eliminated and the moment due to the buoyancy is equal to zero.



## Figure 4.2: HydroProducts RCV-225

(taken from product information brochure)

The placement of the body reference frame as described above will reduce the tensor of inertia to simply diagonal terms (moments of inertia). The value for the moments of inertia for a sphere are the same for each axis and are written as:

 $\lambda_{11} = \lambda_{22} = \lambda_{33} = 2/5 \text{ m rad}^2$ where: m = body mass

rad = body radius , and  $\lambda_{i}$ , is equal to zero for all i not equal to j.

4.33

4.34

4.35

4.37

4.38

The value of the added mass coefficients have been determined for a sphere with the coordinates through the axis of symmetry in Newman<sup>(4)</sup> as:

 $m_{11} = m_{22} = m_{11} = 1/2 \rho V$ 

m<sub>ij</sub> = 0 if i = j ≮

m<sub>44</sub> = m<sub>55</sub> = m<sub>66</sub> =

where:

V = volume of sphere = 4/3 \* rad<sup>2</sup> $\rho = density of water$ 

Finally, the value of the drag coefficients can be obtained from work carried out by scientists in the past.



Figure 4.3: Drag Coefficients for Sphere (taken from Newman: Marine Hydrodynamics)

Commonly drag coefficients are presented in the form of a graph as shown in figure 4.3. The use of the graphs of this nature in computer applications requires some form of numerical interpolation from the graph.

Numerical perturbation formulae for drag coefficients have been found by Oseen, Stokes and others for sphere; in low reynolds number regions<sup>(5)</sup>. These values are helpful for that limited range of velocities, however, the normal velocity range of the ROV goes well beyond the effective range of the Oseen or Stokes approximations. Therefore, the coefficient values must be obtained from the graph and generalized over specific ranges of reynolds number. Prandtl<sup>(6)</sup> recommends the following values for the coefficients of drag on a penere:

These values are approximations and would not be satisfactory for a final solution. However, considering cost and time required to establish these values for the real vehicle these values will suffice for the needs of this study.

Now as a result of utilizing the sphere shape of the RCV-225 and the extensive data already collected for the

motion of spheres in fluid, the generalized equations of motion for an ROV found previously can be reduced further. The result of this simplification is:

$$(M + m_{11})\dot{u}_1 = (M + m_{22})u_3\omega_3 - C_1 A_P/2|u_1|u_1$$

$$+ F_{h1}^{i} + F_{T1}$$

$$4.39$$

$$(M + m_{22})\dot{u}_{2} = -(M + m_{11})u_{1}\omega_{3} - C_{2}A\rho/2|u_{2}|u_{2}$$

$$+ F_{h2}^{*} + F_{T2}$$

$$4.40$$

$$(M + m_{33})\dot{u}_3 = -C_3 A \rho / 2 |u_3| u_3 + F_{h3} + F_{T3}$$
 4.41

$$\lambda_{33}\dot{\omega}_3 = M_{13}$$
 4.42

Considering that there would be no moment due to drag because the force of drag acts through the centre of the sphere.

### 4.3.1 Thruster Forces and Moments

The determination of the forces and moments created by the vehicle thrusters will be unique to each vehicle considered , due to the configuration of the thruster layout. Furthermore, the type of thruster used will be an important consideration.

To evaluate the performance characteristics of the vehicle thrusters a test with the vehicle or a, single thruster unit would have to be conducted and the relation found for the thrust produced for a given power input. This relation is the fundamental factor in correctly simulating the pilot control system.  $\overset{\sim}{\rightarrow}$ 

Another important characteristic of the vehicle that must be determined is the interaction of the thrusters. In a paper by Fyfe and Russell<sup>(7)</sup>, this investigation for the ROV Angus 002 found an unexpected drag force created by the operation of the vertical thRusters. The determination of this type of interaction between motion and thruster operation can only be obtained through testing with the real vehicle or a scale model.

The purpose of this study is to develope a working computer simulation of the motion characteristics of the ROV. The details of the pilot control system and the thruster interactions are best left to a later study that will allow for experimental testing to be carried out with the ROV unit being considered. Therefore, in the study presented here the basic thruster inputs will be considered as direct thrust (Newton) values and the body response will be determined strictly from the thruster layout in the ROV body. Any extra interactions that may occur due to the close proximity of thrusters will not be considered at this level of development.

In the case of the RCV-225, the thruster layout is as shown in figure 4.4. The two side mounted thrusters (1 and 2) control the forward/backward motion (surge) and the heading rotation (yaw). The other two thrusters (3 and 4)



Figure 4.4: HydroProducts RCV-225 Thruster Layout

control the up/down motion (heave) and the port/starboard motion (sway). Thrusters 3 and 4 are mounted in such a way that there axis of force acts through the centre of the sphere shape ( through the centre of mass) and thus these thrusters do not produce any moment.

The vehicle motions are achieved through the combination of the thruster actions. Table 4.1 illustrates how each motion can be obtained and what thrusters are used in each case.

#### 4.3.2 Forces Not Considered

The forces and moments discussed thus far represent the main influences on the motion of an ROV. Although these should not be considered the only source of force and moments they do represent the major operating components on the body.

There are two other sources of force and moment that may act on the ROV and should at least be acknowledged. The effect of the sea state on the vehicle hydrodynamics is an important consideration when the vehicle is operating near to the water surface. These forces are also of concern if the deployment system being used transmits the wave effect on the mother ship to the ROV. The wave forces are not taken into account in this study mainly due to the fact that at the average depth that the ROV operates the wave effects are greatly reduced and thus can be neglected.

			Thruster	Actuation			
Maneuvre	#1		#2	#3	#4		
forward	+		+	0	0		
reverse	-		-	0	0		
up )	0		0 .	+	+		
down	0		0	-	-		
translate port	ċ		0	+	-		
translate starboard	o,		0		+		
rotate ccw	+			0	0		
rotate	-		+	0	0		
		1					
	Maneuvre forward reverse up down tranglate pote tranglate starboard rotate ccw	Maneuvre \$1 forward + reverse - up 0 down 0 translate starboard 0 rotate ccw + rotate cw -	Maneuvre #1 . forward + reverse - up 0 down 0 translate port 6 translate starboard 0 rotate ccw + rotate cw -	Thruster Maneuvre #1 #2 forward + + reverse up 0 0 . down 0 0 translate starboard 0 0 rotate cow + - rotate - cw - +	Thruster Actuation Maneuvre #1 #2 #3 forward + + 0 reverse 0 up 0 0 + down 0 0 - translate starboard 0 0 + translate starboard 0 0 - rotate cw - + 0	Thruster Actuation         Maneuvre       #1       #2       #3       #4         forward       +       +       0       0         reverse       -       -       0       0         up       0       0       +       +         down       0       0       -       -         translate       0       0       +       -         translate       -       0       0       -       +         rotate       -       -       0       0       -       +         cw       -       +       0       0       -       +       -	Thruster Actuation         Maneuvre       #1       #2       #3       #4         forward       +       +       0       0         reverse       -       -       0       0         up       0       0       +       +         down       0       0       -       -         translate       0       0       +       -         translate       starboard       0       0       -       +         rotate       -       -       0       0       -       +         cw       -       +       0       0       -       -       +

Table 4.1: RCV-225 Thruster Operations for Maneuvering

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The second force/moment contribution that is not considered, yet should be scknowledged, is the effect of the umbilical cord on the ROV. Work is being carried out by researchers in an attempt to establish the drag, lift and tangential forces the umbilical cord experiences and transmits to the ROV<sup>(8)</sup>. As would be expected these forces increase with the length of the umbilical to the point where the operational feasibility of the unit is compromised.

The ROV considered for this simulation operates from a garage type deployment system and thus the umbilical lengths are short and this influence is minimal.

#### 4.4 Linearization of the Motion Equations

The equations of motion derived thus far, equations 4.39-4.42, will be solved by means of a method of numerical analysis to give the position of the body in terms of the translation vectors and angles of rotation. The total system of equations must be expanded into a linear form by considering a linear differential relationship between the translation vector x and the velocity vector u and between the angle of rotation n and the angular velocity u. These relations can be written as follows:

x = ū

33

4.43

Therefore, the differential equations that will describe the motion, of the RCV-225 will be: ż, 4.45 u, ż2 4.46. 50 u, ż, 4.47  $(M + m_{11})\dot{u}_1 = (M + m_{22})u_2\omega_3 - C_1A\rho/2|u_1|u_1$ 4.48  $+ F_{h1}' + F_{T1}'$  $(M + m_{22})\dot{u}_2 = -(M + m_{11})u_1\omega_3 - C_2\lambda\rho/2|u_2|u_2'$ 4.49 + E' + FT2  $(M + m_{33})\dot{u}_3 = -C_3 A\rho/2 |u_3|u_3 + F_{h3}^{1} + F_{T3}^{1}$ 4.50  $\dot{\eta}_1 = \omega_1$ 0 4.51 · 12 4.52 . 13 4.53 λ, 1 ω, 0 4.54 λ2 2 ω2 0 4.55  $\lambda_{33}\dot{\omega}_3 = M_{T3}$ 4.56 sx, = sv, 4.57

54

4.44

 $\frac{1}{\eta}$ 

 $SX_2 = SV_2$ 

sX<sub>3</sub> = sV<sub>3</sub>

Where the last three expressions represent the sea current.

4.5 Summary

In this chapter the equations of motion derived in the previous chapters have been modified to take into account the special motion characteristics of the ROV. The equations found represent the general motion of an ROV limited to four degrees of freedom.

These equations have been further modified for the specific case of the RCV-225 which has the basic shape of a sphere.

These equations represent a system of first order differential equations and therefore can be solved using numerical analysis (see appendix B for an outline of the numerical method used). The development of the computer program to perform the solution to these equations will be discussed in the next chapter.
## CHAPTER FIVE ROV SIMULATOR

5.1 Computer Simulation Approach

When dealing with computer simulation there are two main types of simulations that one can consider. The first type, ( and most familiar), is the real-time simulator. This type of system is used to train the operator and crew in different missions and operating procedures. The second type of simulator is the analytical simulator. The analytical simulator is not required to be a real-time operator. This unit is used to evaluate characteristics like the vehicle sensitivity, handling, performance, controllability, and the effect of shape/configuration alterations.

Of these two types of simulators, the real-time simulator is far more demanding in terms of computer capacity required and the extent of secondary data needed. . In this sense, secondary data refers to data that does not pertain directly to the vehicle; for example the terrain and visual data is considered to be secondary data. Furthermore, the real-time simulator requires full time designation from the computer, meaning that multi-user computer systems cannot be used.

The analytical simulator does not require realtime manipulation and output and therefore is much less demanding on the computer system. In general the development of either type of simulator starts with the development of an analytical program.

In this chapter, the basic approach to the development of an analytical simulator for a Remote Operated Underwater Vehicle will be discussed. In presenting this there are three basic components that are required for any computer application; input, operation, output. The discussion of the simulator will be dealt with under these headings with special consideration going to the manipulation of the data.

5.2 Computer Program

The program developed in this study is designed to read a prescribed set of thruster operations from a data file and then evaluate the resultant position and orientation of the vehicle. The previous chapters have described the fundamental properties that must be considered in evaluating the vehicle position. Within this chapter a description of the management of the data and the operation of the program will be outlined.

### 5.2.1 Input

The data that must be provided by the operator is either provided through the data file or is written in the program. The data provided in the data file are the sea current velocity and the vehicle thruster operations. Data stipulated within the program includes the fluid properties (density, viscosity), and the body dimensions and coefficients.

The data file will contain a series of four thruster values, (representing each thruster on the vehicle) for each time increment. The value of the thruster will range from -10 to +10, where 0 represents no thrust and 10 represents 100% of the thruster capacity. The subroutine DATAREAD will read these values at each time increment and translate them into Newtons of force and Newton-meters of moment.

The DATAREAD (see figure 5.1 for DATAREAD flow chart) program takes the thruster data and evaluates the force and moments by means of the configuration of movement described in the previous chapter. The 100% value of the thrust is taken to be 25 newtons, which is obtained from consideration of the performance characteristics of the oilfilled thrusters used on the RCV-225 <sup>(9)</sup> <sup>(7)</sup> and data presented in a paper by Satya Narayan concerning propulsion techniques for ROV's<sup>(10)</sup>.



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## Figure 5.1: DATAREAD Subroutine

The fixed data stipulated within the program are all properties that are considered to remain constant over the operating range and duration of a particular mission.

5.2.2 Operation

The management of the input data and resultant position data is shown in the block diagram in figure 5.2. This chart shows the basic flow of data and results. Within this section a brief description of the basic components of this program will be presented.

5.2.2.1 Set-up

The set-up of the program simply initializes the computer to a new simulation. This section of the program designates the vectors to zero value, sets the time increment, opens the data files to be used, and reads in the sea current vector. The set-up of the program miso finds the initial value of the rotation tensor by calling the Subroutine ROT (see figure 5.3 for block diagram of ROT). This subroutine takes the present values of the angles of rotation to find the rotation tensor from the formulae derived in the chapter concerning the kinematics of motion.

The set-up is only carried out once for each operation of the simulator, whereas the remainder of the

CONTROL PROGRAM start TI = 0.0 EPS =0.001 H = 2.0 ERRMAX = 0.0 KFL AG = 1 JSTART = 1 initialize program parameters = 1 FX(i) 0.0 VN(i) 0.0 VX(i) 0.0 TX(i) 0.0 set position to origin i = i + l≤0 1-3 50 call ROT (VN,R) valuate rotation tensor N = 15100

Figure 5.2: CONTROL Program (4 pages)



Figure 5.2: contiued



Figure 5.2: continued

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Figure 5.2: continued

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Figure 5.3: ROT Subroutine

program is a loop that is carried out repetitively until the end of the thrust data is encountered.

It is important to note that this program gives all the position data in reference to the position of the vehicle at the set-up. It is assumed that a garage type deployment system is used, and therefore the position data will be referenced to the position of the garage at the start of the program.

#### 5.2.2.2 Motion Equations

The next component of the computer program carries out the solution to the motion equations for the vehicle. At each encounter of this section the computer reads a new set of thruster data from the data file and then places the resultant force and moment values in the equations of motion. The solution of these equations is found by means of a numerical integration method (see appendix B).

In the program presented in this study, the differential solver used is a modification of G.W.Gear's subroutine DIFFSUB(11). This subroutine is a one-step initial value differential solver. This means that the program uses data from one previous time increment to solve the equations for the present time increment.

There are many numerical methods available that utilize data from more than one previous time increment. These methods are suitable for a series of continuous equations. In the case of the ROV, the equations of motion are not continuous over several time increments. That is to say, a change in the thrust data creates a discontinuity in the motion equations and thus the multi-step solution methods cannot be used.

The DIFFSUB program in it's original form solves a set of equations for the largest time increment possible without exceeding a prescribed error limit. The program has been modified to solve the motion equations for a set time increment. This is necessary because the thrust data has to read at regular intervals.

In figure 5.4, the subroutine for the solution of the motion equations is shown in a flow chart. The program uses the basic Runge-Kutta method of numerical integration in the subroutine FK. This secondary subroutine carries out one Runge-Kutta integration of the derivatives and returns the vector values to the differential solver routine (figure 5.5 shows flow chart for subroutine RK).

5.2.2.3 Position Evaluation

Once the differential equations have been solved for a time increment, the translation vectors for the vehicle and sea current must be combined to give the resultant position. To do this, both these vectors must be represented in the space fixed frame of reference (ie. with respect to the deployment garage).

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DES (N,T,Y,YP,H,EPS,YMAX,ERRMAX,KFLAG,JSTART)



savé initial values

retrieve initial values

evaluate derivatives

perform one full increment integration perform one half increment integration

Figure 5.4: DES Subroutine (3 pages)



Figure 5.4: continued



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## Figure 5.4: continued

RK (N,T,Y,YP,H,YH)



evaluate derivatives at THALF and Y2

Figure 5.5: RK Subroutine (3pages)



5.5. continued



The body translation vectors are solved by the differential solver in terms of the body fixed coordinate frame. This vector must be transformed to the space fixed frame by means of the rotation tensor. Therefore, after the equations have been solved, the program calls, the ROT subroutine to find/the new value of the rotation tensor. Following that evaluation, a subroutine called TRAV (see Figure 5.6 for flow chart) is used to apply the rotation tensor to the body translation components. This will produce the body motion in terms of the space fixed coordinate axis.

5.2.3 Output

In this analytical approach to simulation, the resultant position vectors are placed in a data file which can later be read and plots of the vehicle path produced. In a real-time simulator a large percentage of the computer work is performed at the output stage. The real-time simulator requires visual effects and instrumentation that are driven by the computer. The development of these features requires extensive use of computer graphics and a large extent of secondary data concerning the environment,terrain, visual capacity of the vehicle, and instrumentation available to the pilot.



Figure 5.6: TRAV Subroutine (2 pages)



Figure 5.6: continued

## 5.2.4 Program Loop

After each time increment, the computer will return to the thruster data file to read another set of thruster control commands. If the computer finds an end of field code in the data file the program is terminated: Otherwise, the program loops through the solution section for each successive set of control commands.

5.3 Summary

The computer program is written in Microsoft Fortran for the specific reason that most engineers and scientists have been exposed to this programming language. The system the simulation has been developed on is a Sanyo MBC-885 personal computer. The advantage of using a personal computer is the convenience and low cost. The further development of the analytical simulator into a real-time system will require careful consideration of the capabilities of the computer system.

This program can be easily altered for different body configurations and dimensions by altering the subroutine FCN (see figure 5.7 for flow chart). The FCN program evaluates the body coefficients and the body motion derivatives. Changes in the body shape and size can be accommodated by changes in the body constants in FCN as



Figure 5.7: FCN Subroutine (3 pages)





Figure 5.7: continued

well as altering the motion derivatives if the new body shape introduces changes to the motion equations.

The computer program represents the required result for the success of this investigation. The next chapter will demonstrate the operation of the simulator in several cases each representing different maneuvering capabilities of the RCV-225.

# CHAPTER SIX

## SIMULATION CASE STUDIES

6.1 General

The purpose of this chapter is to demonstrate the results of the computer simulation of the ROV motion. This, will be done by considering several different maneuvering operations in individual cases. Each case will have a programmed set of thruster actions which will be expected to cause a particular maneuver. The program will process this information and give the resultant ROV path. This path can then be compared to what was expected and the validity of the program evaluated.

The presentation of the operation of the simulation described above cannot be considered a definitive method for evaluating the accuracy of the program. To effectively test the program a set of trials with the real ROV or a model would have to be performed. This would require the ROV to perform a prescribed set of thruster actions and the position of the ROV to be recorded. The program would then carry out a simulation using the same thruster data and the resultant position data could be compared with the recorded path of the ROV. This type of testing would require sophisticated methods of tracking and recording the ROV position. The expense of this equipment and the cost of testing in open water are far beyond the scope of this project and therefore no tests of this nature have been performed.

In each case presented the expected maneuver will be described and the resultant path presented in the form of plots. Each case will have three plots to represent it's path. One plot will show the  $x_i$  vs  $x_i$  axis which is the view looking down on the vehicle. Another plot will present the  $x_i$  vs  $x_i$  axis which is the view looking from the starboard side of the ROV. Finally a three dimensional plot of the vehicle trajectory will be provided. The angle of rotation of the vehicle will be presented in the form of tables, found in appendix D.

Finally it should be noted that each case is carried out for a sixty second simulation period. The data files used for each case are available in appendix E.

6.2 Case Studies

6.2.1 Case 1: No Thrust and No Sea current (ie. Power Loss) In this case the vehicle will perform no thruster actions and will be subject only to the buoyant force on the body. The sea current for this case will be zaro, and the operation will be conducted over a one minute time interval. The expected trajectory of the vehicle will be

directly along the x, axis in the positive direction. No

translation along the  $x_1$  and  $x_2$  axis are expected and no rotation should be experienced.

Figures 6.1, 6.2, and 6. show the resultant path of the ROV.

6.2.2 Case 2: No Thrust with Sea Current

Case two demonstrates the effect of a sea current acting of the ROV. The vehicle will have no thruster activity but will be subject to a sea current of 1 knot magnitude (0.514 m/b) directed in the negative  $x_1$  direction.

The expected motion will be a rising up in the  $x_3$ direction and a negative translation along the  $x_1$  axis, with no  $x_1$  translation and no rotation. Figures 6.4, 6.5, and 6.6 show the simulator results which correspond to what was anticipated.

6.2.3 Case 3: Forward and Downward Motion

This case is intended, to illustrate simple forward and downward motion. The down thrust is first applied and then forward thrust is added while the down thrust is set to slightly overcome the buoyant force. At the final part of the test all thrusters are shut-off and the vehicle is subject to the buoyant force alone. In this trial no sea current is considered.

.The vehicle is expected to drop in elevation and then move along the positive x, axis while dropping in









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elevation at a blow rate. The final section of the motion is expected to show the vehicle gaining elevation while forward motion is reduced.

Figures 6.7, 6.8, and 6.9 illustrate the resulting vehicle path. These results are in close agreement with what was anticipated.

6.2.4 Case 4 : Effect of Sea Current

The thruster activity for case four is the same as case three, however, in this case a sea current of 1/2 knot (0.257 m/s) directed in the negative  $x_1$  direction is added to the system.

The addition of the sea current is expected to cause the vehicle to experience'a negative translation in the  $x_i$  direction during the initial drop in elevation. During the forward thrust the distance travelled will be reduced from that of case three. The vehicle should gain elevation and start to experience a negative  $x_i$  translation during the final zero power stage of this trial.

The results of this thrust configuration are shown in figures 6.10, 6.11, and 6.12. The vehicle experiences no rotation and stays within the  $x_1-x_3$  plane and thus there is no  $x_2$  motion. The motion of the vehicle is as described above.












6.2.5 Case 5: Reverse Motion

The following case performs a forward and backward translation. This example has no influence of sea current and is intended to illustrate the capability of the vehicle to reverse it's direction.

The data file is programmed for the vehicle to move forward for a period and then coast ( no thrust) for a short time before reverse thrust is activated. Figures 6.13, 6.14, and 6.15 show that the vehicle did move forward and then back-up. The data was programmed to have a continual decrease in elevation throughout the exercise so that the  $x_1$ ,  $y_2$  x, plot would effectively show the vehicle translation.

6.2.6 Case 6: Port/Starboard Movement

Case six reveals the vehicle capacity to translate in the  $x_1$  direction. This maneuver is strictly a translation, and is not attained by any rotation of the vehicle.

The thrust data is programmed such that the vehicle should descend for a short period and then start aforward motion along the  $x_1$  axis followed by the addition of a port (positive  $x_2$ ) translation. The side thrust is applied over a four second interval, and then the vehicle' is only subject to forward thrust for 16 seconds. After this period







a four second interval of side thrust in the starboard (negative  $x_2$ ) direction is applied. This is expected to cause the vehicle to move back toward the  $x_1$  axis as it continues to move forward. After the initial descent the vehicle will only have sufficient down thrust to overcome the buoyant force.

Figures 6.16, 6.17, 6.18 and table 6.1 illigitrate the computer generated path of the vehicle. The vehicle moves in the same manner as expected. It is important to note in table 6.1 that the vehicle did not rotate throughout the motion of the vehicle. Therefore, the side motion is 100% translation and is not achieved through a combination of rotation and forward translation.

6.2.7 Case 7: Rotation of the Vehicle

The final maneuver to be demonstrated is that of rotation of the body. In the following case the body will be expected to undergo a forward motion and positive rotation ( positive being from  $x_1$  to  $x_2$ ) and then straighten-out and move along a straight path. This example will not be influenced by a sea current.

The resulting path of the vehicle is shown in figures 6.19, 6.20, and 6.21. The path of the vehicle shows the rotation as expected, however the process of reducing the rotation and straightening-out to a forward path is not precisely what was envisioned.







Table 6.1: Case 6 Rotation Angles

time (t) (sec.)  $\omega_3$  (rad.)

· · · ·		
2.00		.000000E+00
4.00		.000000E+00
6.00	X 2 X	.000000E+00
8.00		.000000E+00
10.00	1.4	.000000E+00
12.00	16 A 1	. 000000E+00
14.00		.000000E+00
16.00		.000000E+00
18.00	1.11.1	.000000E+00
20.00	10.1	.000000E+00
22.00	•	. 000000E+00
24.00		.000000E+00
26.00		.000000E+00
28.00		. 000000E+00
30.00		.000000E+00
32.00		.000000E+00
34.00		.000000E+00
36.00		.000000E+00
38.00		.000000E+00
40.00		.000000E+00
42.00 -	2 3	- 000000E+00
44.00		.000000E+00
46.00		.000000E+00
48.00		.000000E+00
50.00		.000000E+00
52.00		.000000E+00
54.00		-,000000E+00
56.00	· · · · ·	.000000E+00
58.00	. 1	.000000E+00
60.00		.000000E+00

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6.2.8 Case 8: Combined Maneuvers

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The final two examples are a combination of all the potential maneuvering abilities of the RCV-225. Case 8 will perform these motion without the influence of a sea current and the final case will carry out the same thrust actions but have a sea current acting on the vehicle.

The thruster data has been programmed for the vehicle to travel forward and then translate in the starboard direction. After the sideways translation the vehicle will undergo a positive rotation while continuing it's forward motion.

The results of this combination of maneuvering is shown in figures 6.22, 6.23, and 6.24 .

6.2.9 Case 9: Combined Maneuvers with Sea Current

This final case performs the same thruster actions as case eight, however in this case there is a sea current of 1/2 knot (0.257 m/s) from the northeast (where the  $x_1$  axis of the space-fixed reference frame is considered north).

The sea current is expected to cause the overall pattern of motion to be shifted to the southwest and the proportions to be altered. Other than these variations the basic pattern of motion is expected to be the same as case eight.







Figure 6.24: Case 8 Three Dimensional View

Figure 6.25, 6.26, and 6.27 illustrate the effects of the sea current on the case eight motion configuration.

6.3 Summary

The niné different formats of motion represented in this chapter illustrate the basic maneuvering options the ROV has to work with. The combination of these motions will allow the vehicle to effectively move about and observe any feature in the underwater environment.

The computer simulation correlates well with the path that was expected from the programming of the thrust data. In the translation modes the motion simulation is very good . The rotation of the vehicle does appear to be slightly exaggerated. The major cause of this is expected to be that there is no consideration of viscous damping forces on rotation within the motion derivatives. Further davelopment of a simulation will require investigation of this concept and some modification of the motion equations to incorporate this influence.







### CHAPTER SEVEN

CONCLUSIONS

7.1 Summary

The objective of this paper has been to develop a computer simulator capable of simulating the motion of a Remote Operated Underwater Vehicle. The presentation of this topic has been presented from first principles in order to provide a sufficient understanding for the y engineer/scientist not regularly involved in the ROV industry.

The basic properties and principles that govern the motion of a ROV have been discussed and a system of equations governing the vehicle motion have been presented. These properties are a fundamental part of the understanding of the capabilities and complexities of the ROV. The equations presented are expanded to component form in six degrees of motion freedom. These equations represent a significant portion of the complexity of ROV motion mathematics, and are seldom found in references in this expanded form.

The modification of the equations of motion for the general ROV and the specific case of the 'HydroProducts RCV-225' represents an important simplification of the motion equations. The choice of the RCV-225 for the first attempt at simulation is based on the simplicity of this units body shape and thruster configuration.

The computer program used to simulate the ROV motion is written in the Microsoft Fortran programming language, and uses a method of numerical integration to solve the motion derivatives for specific time increments. The program manipulates the thruster data, solves the motion derivatives, and then analyzes the vehicle translation and rotation vectors, combined with the sea current vectors, to provide a vehicle position.

Within the presentation of this simulator, several trial cases have been conducted in order to test the simulation program. In each case the analytic simulator, provided a vehicle path much as expected when the thruster data file was programmed.

7.2 Potential for Future Investigation

The results of this thesis provides opportunity for further work to be conducted to carry out testing procedures to verify the accuracy of the computer simulator. Furthermore, testing with a variety of vehicles could allow for the determination of the body coefficients for these vehicles and thus the development of motion derivatives and simulation for several ROVe.

The natural progression from the development of an analytic simulator would be the creation of a Real-time

simulator. The use of this type of simulator as a training facility for ROV users has great potential for time, and cost efficiency, in expanding the ROV industry.

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## APPENDIX A

1. Submersible Industry Overview

On September 1, 1985, on board the research vessel Knorr, a team of American and French scientists discovered the long lost ocean liner Titanic, which tragically sank 73 years early resulting in the loss of over 1500 lives<sup>1</sup>. The discovery was the result of a combined effort between French and American scientists working with underwater survey and observation technology. The significance of the discovery is primarily the opportunity to uncover part of the mystery of Titanic's tragedy, but also brilliantly unvells to the North American public the world of submersibles.

The discovery of the Titanic marks a sudden awareness to the public regarding submersible technology, but more importantly it represents the culmination of over 30 years of research and development in this industry. The mission to find the Titanic(1985), as well as the subsequent mission to photograph and observe the wreck(1986), took advantage of the state of the art technology in submersibles.

The development of this research is largely due to the increase in activity in the offshore oil and gas industry. The capabilities and service that the submersible offers are being investigated and used by numerous organizations and agencies. This Appendix is intended to give an overview of the submersible with regards to the following areas:

1. Types and Classifications

2.History of Development

3.Operational Capabilities

4. Current and Future Trends.

1.1. Types and Classifications

The increase in underwater research and resource development has spawned the need for effective methods of observing the environment and working in the underwater world; such as cost, time and safety. The depths that are being investigated are increasing as we run out of continental coast line to discover and head further offshore for our research and development projects. This increase in depth has made observation and work tasks, which were normally carried out by the commercial diver, much more difficult to conduct. The cost effectivness of the commercial diver is lost due to the cost of technology to put divers at these depths, and by the risk involved in deep water diving.

The development of the submersible as the ideal deep water diver replacement has gone from the basic concept of the mini-submarine to the highly sophisticated multi-task ROV (Remotely Operated underwater Vehicle) being used in the North Sea oil development. The various needs of observational information gathering and work capabilities have generated two main catagories of Submersibles. These two catagories are the manned and the unmanned yehicles.

## 1.1.1. Manned Submersibles

Although this thesis is intended to investigate the remotely operated underwater vehicle, (ie. unmanned) the role of the manned submersible is extremely significant to the development of the RoV, and therefore will be discussed in this appendix.

Primarily, the manned submersible, unlike the submarine, is not to be considered as an independent vessel capable of operating entirely on its own. Realistically, the manned submersible (whether tethered or not) is bound to a support vessel for regular operation. The support vessel supplies launch and recovery facilities, accommodation for the submersible crew, and maintenance for the vehicle. Although the vehicle may be completely free-swimming, it's operational. duration (usually limited to 4-8 hours/dive) requires it to have a surface support base.

Development and research with manned submersibles constitutes the foundation of the whole submersible industry. Although in the early years of development (1960's) there were some projects being carried out with

unmanned units, the main thrust of the industry was with the manned units. These vehicles have been through several generations of change and it is only recently that there has been a shift in the industry toward the unmanned units.

1.1.2. Unmanned Submersibles

The development of the unmanned submersible has come as the natural limitations of placing human lives in vehicles at greater and greater depths has been realized. Having only a mechanical unit equipped with cameraa performing basically the same tasks as a manned submersible removes the risk of losing lives, as well as opens up a whole new capacity for long duration dives, reduced vehicle size, and increased maneuverability. The confort of operating a "dispensable" (rather expensively so) unit has been fillustrated in cases where manned units have become trapped underwater due to mechanical failure or entanglement.

Remote Operated submersibles are categorized into four main groups<sup>2</sup>. These categories are:

1. Tethered , free-swimming vehicles

- 2. Bottom Crawling vehicles
- 3. Towed vehicles
- 4. Untethered vehicles

The tethered, free-swimning vehicle is probably the most common type of ROV and the most versatile. These vehicles are attached to a support vessel (mother ship) by an umbilical cord that provides power and control
information. The vehicle is self-propelled by screw type thrusters, or in a few rare cases water jets. Observational capacity is provided by some combination of video camera units that transmit their picture through the umbilidal cord, and still photography. These vanicles can be equipped to perform many opecialized tasks and often carry a bank of sonar and navigational instrumentation.

The Bottom Crawling-units are commonly used in cable and pipeline installation and observation. These units are also being used for deep sea mineral nodule mining. The units are 'attached to a mother ship through an umbilical cord, and receive power and control data through the cord. They are usually self propelled by some form of drive wheels, or track, and are confined to movement on the sea floor.

Towed vehicles are significantly different , in that they are not self-propelled. These vehicles are towed behind a support vessel, and therefore are only able to maneuver forward, up, and down. They receive power and control commands through an umbilical cord similar to the other categories discussed. Towed vehicles are able to survey large portions of the sea bed with sonar equipment, photography, and video camera visuing.

The final category outlined by Vadus and Busby<sup>2</sup> is that of untethered vehicles. These units are still in the preliminary development stages. Deam Givem<sup>3</sup> (1980) describes

untethered vehicles to be in the early experimental mode and expects any operational design to be several years away. Furthermore, Jonathan Tucker<sup>4</sup>, in his article concerning ROV's in High Technology (Feb. 1986), describes the development<sup>6</sup> of the untethered unit strictly in terms of the experimental work being conducted. Although this form of ROV would seem to be the ideal vehicle, there still remains many technological hurdles that must be overcome before these vehicles will be feasible.

The ROV is a relatively new development in submorsibles and is still in the growth stages. A great deal of research and work is being carried out to develop sophisticated systems that will allow operators to simulate the working conditions encountered by the ROV and receive sufficient sensory information and data to allow logical decision making and operation from the mother ship.

1.2. History of the Submersible Industry

1.2.1. Manned Submersibles

In the 1st annual Offshore Technology Conference , held in 1969, Thomas F. Horton<sup>5</sup> presented a paper discussing the "Status of Submersibles as a Useful Tool for Offshore Resource Recovery". In this paper he outlined the equipment available in the industry, as well as some important discoveries being made at that time in the area of submersible applications.

One of the first programs of testing that Horton emphasises is a 1964 operation where Shell Oil Company leased the services of the Cousteau Diving Saucer, for the specific purpose of evaluating the potential of the submersible as a working tool for offshere well-head inspection and service. This program was conducted on one of Shell's drilling sites in the Santa Barbara Channel, and is considered to be one of the first uses of a submersible in the offshore oil and gas industry.

The importance of this test program within the history of submersibles is that it marks the beginning of the industries true development. It is commonly acknowledged that the offshore oil and gas industry has been the major catalyst to the growth of the submersible industry.

In the following years of development, as the requirements on the submersible became refined, the designs of the late sixties and early seventies were more versatile. The emphasis in this stage of growth was on features such as diver-lockout capacity and atmosphere dry personnel transfer to a habitat.

In this generation of designs, important work was being done in the development of manipulators and instrument payload capacities. Organizations like the Woods Hole Oceanographic Institute and ISE (International -Submarine

Engineering, a Vancouver based company), both organizations at the forefront of submersible development, were making advances in the design of manipulators and tool sets for specific work functions.

The increase use of the submersible in the North Sea gave the manufacturers and designers a testing ground for new ideas. Many advances were made in the area of the submersibles instrumentation facilities, navigational equipment, and most significantly, the manipulator capacity.

#### 1.2.2. Remotely Operated Underwater Vehicles

The Remotely Operated Underwater Vehicle (ROV), has been in existence for almost as long as the manned submersible, however, it's actual use as a meaningful offshore tool only dates back to the mid-1970's. Vadus and Busby<sup>2</sup> claim that the first ROV was introduced in 1953. This vehicle was designed and built by Dimitri Ribikoff' and was called the "Foodle". It is described as having been developed by making modifications to a diver transport vehicle of Ribikoff's design called "Pegesus".

The next ROV to be introduced as a commercially available unit was the Hydro Product Company's RCV-225. This unit was released in 1975, leaving a span of 22 years where there were no commercial ROVs available for offshore operations. In that time span there was a total of 192'

affferent ROVs designed and undergoing testing . These units were almost all.exclusively governmental projects.

The RCV-225 started the true realization of the ROV concept. This vehicle, often referred to as the flying eyeball, due to it's shape and function, combined the observational abilities of the manned vehicles with ease of operation, and the safety feature of being expendable.

Within only a brief period of two to three years from the introduction of the 225, many other types of ROVS entered the market. Tucker<sup>4</sup>, in his February, 1986 article quotes Frank Busby as claiming that over 700 ROVs have become operational in either governmental or commercial application since 1975.

The specific concept and design of the ROV varies according to their proposed function. Through development, two basic design formats have been implemented most often. The first is that of a wholly enclosed body, usually having a relatively streamlined form and constructed with syntactic foam. The Hydro Product's RCV-225 is an example of this type (see figure 1), The second format is that of a metallic framework that houses the ROV components in an open body structure. The Osel Group's Duplus II is an example of this design format (see figure 2).

The ongoing research and development of the ROV is leading the industry into much greater depths as well as highly sophisticated observation techniques. The main

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Figure A.1 : Hydro Products RCV-225

(taken from product brochure)



Figure A.2: Osel Group's Duplus II (taken from product advertisement) objective of this research is to develop for the pilot the illusion that he is actually in a manned unit.

2.3. Operations Conducted by the ROV

It is important for anyone involved in the ROV industry or involved with the use of ROV systems, to have an understanding of the general task descriptions that have developed the generation of ROVs now available. As has already been suggested, the main contributor to the development of the ROV has been the oil and gas industry.

Operations that ROVs are performing today can be generalized into two catagories, according to the operations mobility requirements<sup>6</sup>. The operations will require a varied degree of horizontal and/or vertical mobility, therefore the operations are divided into the horizontally mobile operations and the vertically mobile operations. Horizontally mobile operations refer to the need for a large amount of area to be traversed; for example the following of a pipeline along the sea bed. These operations do not require the vehicle to maneuver up and down through the water very much, except for the decent and ascent to/from the work site. The vertically mobile job classification refers to jobs that require the vehicle to descend and ascend through the water column and maneuver only within a small work area(eq. 100-200 meter radius area). The

descriptions of ROV operations will be presented under these two headings.

1.3.1. Horizontal Mobility Operations .

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The first, and most common operation for this job classification is bottom surveys. This is the process of measuring and mapping the features of the ocean floor. Bottom surveys are often done to obtain maps of man-made features such as pipe lines or underwater cables. They can also be used as a means of establishing a site evaluation for drilling operations. The RoV equipped with the appropriate sonar devices can produce a bottom profile that will aid oil and gas engineers in choosing the best suited placement and orientation for operation.

The type of ROV normally used for this operation is the towed vehicle, simply because it is more economical for the mother ship to provide the horizontal propulsion, rather than both the mother ship and the ROV, as would be required with any of the other type of ROV. It should be noted that as the untethered vehicle becomes a commercially available product, this type of vehicle should have great potential in this operation.

Another common operation requiring a large degree of horizontal mobility is pipe line and underwater cable inspection. This differs from survey in that inspection requires a continuous picture of the bottom features. ROVS employed in this operation must be equipped with cameras that record in either continuous still photographs or video transmitted to the control centre.

Operators requiring inspection services are looking for confirmation of appropriate installation of the pipeline or cable. Any poor connections, tie-ins, leaks, or improper burial must be detected so that appropriate repairs can be conducted.

Seabed search missions similar to that of the discovery of the Titanic , and the use of the RoV Scardo for the search and salvage of the black box from the Air India 747 crash of the coast of Ireland in July 1986 , are both examples of operations that fall within this category. These missions require a large coverage of the ocean floor, often at great depths. Both towed and tethered free-swimning vehicles, equipped with vide scanning sonar are used for this operation.

There are many scientific studies that require the use of horizontally mobile ROVs. Biological studies observing and establishing populations of marine life are employing ROVs for this purpose. As well, ecology scientists are finding the ROV equipped with a large battery of instruments to be a rapid means of collecting data on ocean currents, temperatures, salinity, and other environmental factors. 1.3.2. Vertical Mobility Operations

Observation tasks compose the majority of this catagories operations. Vertical mobility implies that the vehicle can be rapidly lowered to the prescribed work site and then maneuver in a relatively small work area to perform it's function. Observation tasks include inspecting, " monitoring, and surveying a work site.

The tethered free-swimming vehicle is best suited for this operation. The units used are normally carrying camera equipment capable of transmitting video pictures to the control centre, and often have some form of sonar equipment onboard.

Monitoring is different from inspection in that it is carried out as an aid to another operation. Monitoring a well-head installation with an ROV will help the rig operators to obtain the best orientation of the well-head, "as well as an assurance of proper fit and installation of parts. Other monitoring operations include the support of diver operations. In this case a dive team can have an ROV 'buddy' that maintains constant surveillance of their work to insure their safety during the dive.

Another type of operation that is becoming increasingly common with ROVs is manipulation operations. These operations require an ROV equipped with some form of manipulator. The manipulators normally have 4-7 functions and a wide range of lead capacities.

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Light duty tasks include sample collection from the ocean floor, cable connection, valve turning, nondestructive corrosion testing, clearing of a work site, cleaning away marine life from equipment, replacement of equipment and many other functions that otherwise would require a diver to carry them out. The list of these tasks is increasing as the manipulators available become more advanced.

71.3.3. Oil and Gas Operations

As a matter of interest, table  $\lambda$ .1 gives a list of operational requirements for an ROV system to be placed on a shell Offshore Inc. site on the Atlantic Baltimore Canyon drilling zone. This is included to demonstrate the wide variety of tasks required of the ROV in offshore oil and gas development.(see table  $\lambda$ .1<sup>7</sup>) This system was to operated on a site approximately 100 miles offshore, in approximately 7500 feet of water and subject to extreme sea conditions.

## 1.4. Current Research and Future Trends

The following sections will outline some of the most active areas of research and development being carried out in ROV design. In order to fully understand the purpose.

#### <u>Table A1</u> : Requirements of ROV in Deep Water Drilling Support (taken from Shatto, H.L.)

I. Observation

- a. Site survey/location of well or objects by Sonar
- b. Watch pilot hole for gas
- c. Re-Entry allignment
- d. Observe stack and LMRP orientation, seperation, heave, etc."
- e. Check cement returns
- f. Guide base level Bullseye
- g. Rise angle confirmation Bullseve
- h. Connector latch indicator
- i. Inspection of riser and stack
- j. Profile of current and temperature related to depth
- k. Observation in operation of manipulators
- 1. other

II. Manipulation Operations (Intervention)

- a. Place and recover acoustic beacons
- b. Clear debris from well or stack
- c. Clean level indicator Bullseye
- d. Cut cable or soft line
- e. Recover dropped equipment
- f. Emergency release of hydraulic connectors
- g. Replace VX ring in hydraulic connectors
- h. Place explosive charges
- i. Other

for these areas of work, it will help to first consider the perceived difficulties encountered in the operation of ROVs today.

1.4.1. Short-Comings

Many of the short-comings that operators have experienced are in problem areas that have been recognized for some time, while other difficulties are only becoming apparent as technological development unveils new areas of potential achievement. The areas of concern that are most often expressed can be grouped under the followingheadings<sup>8</sup>:

1.Reliability 2. Tasks and Emipment 3.Communication 4.Viewing 5, Navigation

1.4.1.1. Reliability

The reliability of the ROV has most often been hampered by difficulties with the umbilical cord. The most commonly reported operational hazard encountered in the operation of ROV's is entanglement of the umbilical cord<sup>2</sup> (see table A2<sup>2</sup>) The severity of this problem ranges from the umbilical becoming wrapped around some structure requiring the pilot to back-track to solve the difficulty.

#### Table A2 : ROV Problems Reported (taken from Vadus and Busby)

# Number of Operators

1)	Entanglement	18
2)	Electrical Connectors	12
3)	Vehicle Distrubs Sediments.	
-,	Obscures Visibility	11
4)	Cable Ruptured by Abrasion	10
5)	Electrical Interference in Cable	8
6)	Support Ship Cannot Station-Keep	6
7)	Compass Affected by Structure	6
8)	Ship Power Surges Affect Vehicle	
÷.	Operations	5
9)	Current Required Aborting Mission	5
10)	Sea State Required Aborting-Mission	5
11)	Vehicle Damage During Launch/Recovery	2
12)	Vehicle Station-Keeping Inadequate	2
13)	Manipulation Inadequate	2
14)	Vehicle Payload Inadequate	2
15)	Human Engineering Inadequate	2
16)	Vehicle Lost Due to Low Surface	
	Freeboard	1 .
17)	Electrical Shocks Due to Inadequate	
	Grounding	1
1.8)	Vehicle Maneuverability Inadequate	1
19)	Water Visibility Required Aborting	
	Mission	1
20)	Television Resolution Inadequate	1

Problems

(number of operators : refers to the number of operators that reported that problem in a survey conducted by Vadus and Busby)

to the worst case where the umbilical becomes caught and/or severed in some equipment requiring the ROV to be either abandoned or some other facility used to recover the unit.

The umbilical also produces a reliability hazard due to the stress and drag that it is subject to during regular operation. This, combined with natural abrasion, often causes the cord to rupture resulting in the loss of communication with the control station and possible loss of the vahicle.

One other concern expressed by operators, with regards to reliability, is the launch and recovery systems. Situations have been reported where during the launch or retrieval the ROV has been joited around extensively and even bumped against the support ship .

1.4.1.2. Tasks and Equipment

Task and Equipment related short comings refer to problems that arise due to insufficient vehicle capability ' for a particular job, or some inherent characteristic of the present technology that hampers the operation of the ROV.

The most common complaint of this nature is that the ROV thrusters disturb the ocean floor gediment causing reduced visibility in the work area. This problem arises as a result of top layer of material of the ocean floor being Aextremely unstable and easily disturbed by any current passing over it. Another task and equipment related complaint is that missions have been aborted due to extreme currents or sea states. This difficulty encompasses several different areas of the ROV design. The major short coming that causes this complaint is insufficient thruster power available for the ROV. Furthermore, aborted missions due to extreme sea states are usually a result of the deployment unit having insufficient capacit

Manipulation short comings are also a part of this category of complaints. There are fewer complaints of this nature because, at present, there is much less expected of the ROV in this area. Manipulators are still in the early design and testing stages, and therefore this technology is only just starting to be used effectively.

A final area of dissatisfaction expressed by operators with regards to task and equipment related design, is that of the positioning capabilities of the mother ship and the launch and recovery system. The major deficiency in the launch/recovery system is that the motion of the mother ship (roll and heave) is often transmitted through the lowering cables/umbilical to the ROV causing erratic motion of the vehicle.

1.4.1.3. Communication

Communication problems relate to the transmission of power and data through the umbilical cord. A common area

of concern is that power surges, (power surges are common on ocean vessels) which effect the normal operation of the ROV.

One other difficulty that occurs in the communication between the ROV and the mother ship is that of electrical interference. This type of disturbance between the power, control, and video/data transmission can severely hamper the operation of the unit. The problem is inherent in having these three types of transmission carried through the same cord. Shielding the lines within the umbilical is costly, and in order to be truly effective, increases the diameter of the umbilical significantly. Increasing the umbilical diameter further complicates the problems discussed under the reliability of the ROV, and thus there is a trade-off required.

# 1.4.1.4. Viewing

The greatest difficulty being encountered in this category is that of effectively supplying the pilot with a full view of the work site. Some camera systems are unable to provide clear and detailed views of the work area, and/or cannot provide a wide enough range of observation. Furthermore, the operator has to interpret the work site . through the TV screen, thus eliminating a true perception of depth of field.

1.4.1.5. Navigation

Establishing the position of the ROV with respect to global coordinates, the mother ship, surface or subsurface structures, and/or bottom features has lead to considerable research and experimentation. Navigation systems are required that can supply these parameters to the operator in order to simplify mapping and positioning during the vehicle operation. It is often found that a large percentage of an ROV mission log is filled with time spent searching for the work site. Visibility limitations can cause a feature only a few meters from the vehicle to be obscured, and thus cause the pilot to miss the intended work zone.

One other concern expressed with the present navigation compasses is that they are effected by surrounding structures (ie. drilling platform) and as a result give erroneous readings. The position of the vehicle while maneuvering around such structures must be carefully monitored to avoid accidental entanglement or severing of the umbilical. Therefore, some means of providing relative position data for the plot must be devised.

1.4.2. Current Research

A great deal of research is now underway in an attempt to negate many of the problems discussed above. As has already been explained, the progress of testing and developing new concepts and innovations is slow due to the

demand on the operator to provide reliable and uninterrupted service. The following section will discuss some of the major areas of research being investigated and describe their possible impact on the future of ROVs.

1.4.2.1. Umbilical Cord

Many projects are underway attempting to either eliminate, or reduce the size of, the umbilical cord. The concept of the free-swimming, untethered ROV is an attractive goal within the industry, however, there are still some major technological advances to be made before this feecomes a reality.

The most significant difficulty to be overcome in achieving the untethered ROV is to establish a system of communication between the vehicle and the control station. Researches are attempting to develop a transmission system to pass the control commands and video/data information through sea water. Thus far the most effective means devised is an acoustic link<sup>4</sup>. The difficulty with this system is that it has an extremely slow rate of information transfer, and therefore the transmission of continuous video data is not possible.

A further problem encountered with the acoustic link is the high power requirement. An untethered vehicle must have an on-board power supply and the power need of the acoustic system will require a large storage capacity for batteries. Furthermore the power drain from this system increases with distance thus limiting the range of the unit.

Sonic interference is one other hurdle that must be dealt with in the acoustic tether system. Boundaries and structures cause sound waves to bounce around thus reducing the quality of the signal.

Other methods being investigated to replace a physical tether are radio signals and blue-green lasers<sup>4</sup>. These techniques are only in the primary development stages and significant technological breakthroughs will be needed to see these in operation.

Although the ultimate goal is to remove the physical tether, researchers are still concerned with improving the cable tether. Primarily, the aim of this work is to reduce the drag caused by the cable system. The limiting factor in this effort is the diameter of cable required for the transmission of power to the RoV. The transmission of control data and video/data represents a much smaller percentage of the tether cross section.

Some success has been achieved in reducing the control and data transmission requirements by using multiplexing systems that allow these signals to share transmission lines. Work is also being conducted in using fibre optics for these signals:

One other aspect of the tether that is being investigated is that of improving the strength and

durability of the cable. This is being done with special protective coatings and with the use of new plastic and carbon fibre products.

1.4.2.2. Navigation

Many systems are being developed with the intent on improving the navigational positioning of the ROV and using these systems to assist the pilot's control system. The most effective methods for positioning and global mapping have been inertial and acoustic navigation methods. Projects combining these two systems to provide a hybrid system capable of navigating along an open sea bed as well as within a drilling structure are being tested and installed in some of the advanced ROVs available today.

The Heriot-Watt university (Scotland) is conducting a research program involved with the development of a computer system to control the heading, depth, and height of the ROV<sup>9,10,11</sup>. This system, when completed, will allow the pilot to enter into a computer terminal the heading and the height/depth that he wishes the ROV to maintain. The computer would then control the vehicle thruster system allowing the pilot to concentrate on the observational and instrumentation control of the vehicle.

Computer intervention in the control loop is also being considered for producing a system of supervisory control<sup>4</sup>. This system is being developed at MIT and will

allow the operator to instruct the computer to position the vehicle in a particular location and also specify control commands for manipulator movement. The system is intended to coordinate the ROV and manipulator movements allowing the ROV to perform manipulation tasks in the mid-water range using the vehicle thruster system for the required torque to counter balance the manipulator torque.

Supervisory control also has the potential of being applied in the observational tasks as well. Thomas B. Sheridan<sup>4</sup>, the leading developer of this system, suggests that the computer could be used to interpret much of the data received from the ROV into graphic image displays. Then, if the computer can be developed with sufficient pattern recognition capability, the pilot could instruct the system to follow a particular specimen (ie. fish) and the computer assumes full control of the ROV.

1.4.2.3. Viewing System

The goal of research and development in the area of viewing systems is to give the pilot the closest possible re-oracidon of the true environment the ROV is travelling in. This would ideally mean that the pilot would have a three dimensional colour view encompassing three hundred and sixty degrees of field.

Many systems are being experimented with that will provide some of these features. Colour TV cameras are

becoming advanced and combined with the appropriate lighting • are able to provide high quality colour video.

Aree dimensional camera systems have been available for several years, however, these are not sufficiently developed to have found wide spread use. The major set back to these systems is their expense compared to the conventional two dimensional system and some loke of clarity which can hardly be tolerated in the already poor visibility conditions present in the set water.

Work is also being carried out to improve the clarity of the video picture received from the ROV. The major inhibitor to the picture clarity as well as the range of view is the presence of back-scatter of light from suspended particles in the water. This problem is being overcome through the use of low-light cameras and, the careful positioning of the lights within the ROV structure in order to reduce the reflection<sup>2</sup>.

Clarity is also being improved by researchers using laser-scan techniques of picture enhancement<sup>8</sup>. This technique combines the conventional system on-board the ROV with laser-scan equipment on the surface. The overall system will be capable of providing a 180 degree field of view with a window of enhanced view superimposed within.

Finally, the viewing systems are being developed with the use of the computer to interpret data from the ROV and produce graphic displays for the pilot and observers.

This research is a part of the work being done with supervisory control systems as discussed in the section on navigation.

1.4.2.4. Manipulators

A large percentage of the research and development that is being conducted on ROVs is concerned with the improvement of the ROVs ability to perform manipulative tasks. In order for the ROV to become successful as a replacement for diving or manned submersibles it must be able to perform the same functions that these other systems can. Manipulators are available that have up to seven movement functions, as well as a yide variety of tool and grip mechanisms suited for different work tasks. Development of these units is primarily concerned with the operators control system , and the capacity/range of the manipulators.

Methods of control for the manipulator are also important features being investigated. At present there are two general methods of pilot control being used. One method is that of the master/slave unit and the other method of control being used and further developed, is that of mechanical controls such as joysticks, switches, and knobs.

Improvements on both these systems are being tested with the hope of developing a tactile force feed back system of control. This system would provide mechanical resistance to the plots control movements in response to true resistance encountered by the manipulator. When this system of control becomes available the capability of the pilot to sense the touch of the manipulator will redefine the work capabilities of the manipulator equipped ROV.

1.4.3. Future Trends

The future development of the ROV is opparent from the research that is now underway. The industry is striving to produce a reliable unit with the ability to perform "almost any task that a diver or manned submersible can perform.

The work being carried out today would indicate ' that the ideal RoW of the future will be untethered, freeing.' the unit from the limitation of a physical link with the surface. The recent development of some autonomous underwater vehicles that require no communication between the mother ship and the unit, but instead are preprogrammed to perform a particular function and then return to the support vessel, is a positive step in the direction to the realization of the untethered RoV.

One other development trend that is expected to continue is that of combining the operation of the ROV with diving teams and/or manned submersibles. The use of the Alvin/Jason Jr. team of submersible and ROV for the observation of the Titanic in 1986<sup>12</sup>, is a prime example of the potential these systems have.

1.5. Summary

The recent attention that the submersible has received due to discoveries like the Titanic and the recovery of the black box from the Air India 747 1986 has given the industry the opportunity to display the extent of expertise that this field has obtained in only a short period of existence. The development of submersible systems for both manned and unmanned commercial operation has been going on for less than thirty years, yet the technological advancements this industry has achieved in that time period are exceptional. The use of these systems as a integrate tool in the function of oil and gas drilling and recovery operations has been a major contributor to this rapid advancement of technology.

The various types and operational tasks that these units are being designed to perform attest to how essential to offshore development submersibles have become. If science and industry hope to learn how to fully utilize the sea as a resource, the submersible is going to be an important tool for discovery. The extreme depths and conditions that the vast majority of the sea floor lies within poses significant difficulties for man to research these areas. The use of the submersible, especially the remote operated underwater vehicle, eliminates many of the complications that arise

when other means of investigating these regions are attempted.

The ROV is the logical progression from the manned unit. These units are far superior concerning depth and dive duration capability, surface support requirements, mobilization and logistic planning needs, and the size of orew required. The ROV also has the distinct advantage of being expendable in the case of an emergency.

As this industry continues to grow many industries that use these units and those industries that contribute to their development will become increasingly' aware of the unique conditions of operation and requirements for design the ROV is subject to. In order for the development of the ROV to continue, in the successful manner it has, it is essential that not only the manufacturer/operators of these units understand the fundamental principles the vehicle is subject to, but also that those industries employing the ROV be aware of these principles also. If both these groups are aware of the limitations and capabilities of the ROV an effective communication of requirements and expectations can exist between them thus allowing for the present standard of research and development in the ROV to be maintained.

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### APPENDIX B

1. Numerical Solution Method

The purpose of this appendix is to give an explanation of the Runge-Rutta numerical integration method used in the computer simulation within this thesis. The numerical integration is done in the subroutine DES which is an adaptation of C.W. Gears subroutine DEStimub<sup>1</sup>. This program uses a Runge-Rutta solution method using step doubling to control the error range.

1.1. Choice of Method

The Runge-Kutta method is a offe-step solution technique. This means that the method uses data from one previous step to solve the derivatives for the next solution increment. Other methods, known as multi-step methods, use data from several previous data points to solve the derivatives. Multi-step methods are commonly more accurate, however they require that the derivatives be continuous over the range of the solution field. In the case of the ROV the derivatives can not be considered to be continuous due to the arbitrary fluctuation of the vehicle thrusters. Therefore a one-step method must be used.

The choice of the Runge-Kutta method as apposed to a Euler method or higher order Taylor expansion method, is as a result of there only being the first order derivatives

available. The higher order derivatives required for the Euler and Taylor methods are not available and therefore these methods are not suitable.

Within the application of the ROV motion derivatives the Runge-Kutta method is being used to solve an initial Value problem at each time increment. This means that at each time increment the program is using the present values of the derivatives to predict the derivative values and solutions for the future time point.

1.2. Method

The Rungé-Kutta technique used is called the ' classical fourth order Runge-Kutta 'method. The derivation of this method is available in many math and engineering text as a graphical derivation as in Rainville and Bedient<sup>2</sup>, or as an analytical derivation as in Gear<sup>1</sup> or Chapra and Canale<sup>3</sup>.

The formulae form of the method is presented as follows:

B.1

B.2

B.3

 $y_{n+1} = y_n + [1/6(k_1 + 2k_2 + 2k_3 + k_4)h]$ 

where:

- present time point
- $\mathbf{k}_{1} = \mathbf{f}(\mathbf{x}_{n}, \mathbf{y}_{n})$
- $k_2 = f(x_n + 1/2h, y_n + 1/2hk_1)$

 $= f(x_n + 1/2h, y_n + 1/2hk_2)$ B.4

 $f(x_n + h, y_n + hk_3)$ 

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and

$$\mathbf{y}^{T} = \mathbf{f}(\mathbf{x}, \mathbf{y})$$
 B.6

with y as the dependent variable and x as the independent variable.

In the application of this method to the ROV problem, the independent variable is time (t) and the method is applied to simultaneously solve the system of equations where the functions f are the motion derivatives for the ROV.

1.3. Application

Upon each call of the DES subroutine the computer uses the Runge-Kutta method to obtain a solution to the derivatives at the desired point, once by using a full time step h to obtain v1, and then twice using two half time steps h/2 to obtain y3. This method is referred to as a double step method and allows for the truncation error to be estimated and eliminated from the final result.

C.W. Gear uses the following scaling formulae to eliminate the estimated truncation error:

((32 x y3) - y1)/31

B.

B.5

The double step method can also be used to control the error limit. This is done through the use of a scaled error term ERRMAX. This term is evaluated at each time increment as the maximum value of either the previous time increments ERRMAX value or the value of:

error / YMAX x EPS

here:	error	-	y3-y1 / 31
	YMAX	-	max value of YMAX, y1 , y2 , y3
15	EPS	-	error constant (input parameter)

If the value of ERRMAX exceeds 1, the truncation error exceeds the prescribed limit and the time step must be reduced in order to maintain reasonable error bounds.

In the computer program the ERRMAX value at each time step is output to the computer terminal where the operator can maintain a visual check on the error. This is a modification to Gear's subroutine diffsub where the ERRMAX term is monitored internally and the time step size varied to maintain the error bounds. The modification was found to be necessary because the simulation is intended to have the thruster inputs read at prescribed intervals and thus the time step must remain constant.

1.4. Error Propagation

In the case of initial value problems the accuracy of the initial values used is an important contributor to

B.8

the accuracy of the solution. In this application, the initial values are from the previous time increment and thus any error in truncation and round-off occurring in the solution method is carried through-out the simulation period. The size and growth of this error can only be obtained by means of comparing the computer program results with the true path of the vehicle subject to the same thruster and environmental influences. This comparison would allow for a full evaluation of the solution method and the appropriate time increment to be used.

2. Summary

This appendix has outlined the numerical integration method used to solve the ROV motion derivatives within the simulation program developed in this thesis.

The justification for the use of this method has been stated and the method application explained. The Runge-Kutta method has been derived within this appendix because of the availability of this derivation in many engineering and numerical methods texts.
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166 APPENDIX Listings of computer programs.used in mathematical simulation.

CONTROL PROGRAM с 2 с THIS IS THE CONTROL PROGRAM FOR THE RCV-225 3 C MOTION MODEL! THE PROGRAM USES & SERIES OF C 4 SUBROUTINES TO READ THE THRUSTER DATA, 5 C EVALUATE THE MOTION CHARACTERISTICS 5.1 C AND OUTPUT THE POSTION VECTORS IN SET TIME 5.2. C INTERVALS. 6 SDEBUG IMPLICIT REAL\*8 (A-H,M,O-Z) DIMENSION U(15), DU(15), UMAX(15), VN(3), VX(3), TX(3), FX(3), SX(3) 10 COMMON/THRUST/FT(3), MT(3), R(3,3), SU(3). 11 C INITIALIZE THE SYSTEM 12 -TT=0.0 13 EPS=0.001 14 SET TIME INTERVAL AT DESIRED INCREMENT (2 SECONDS) 15 H=2.0 16 HMTN=0.5 17 ERRMAX=0.0 18 KFLAG=1 19 JSTART=1 20 DO 5 I=1,3 21 FX(I)=0.0 VN(I)=0.0 22 1 23 VX(I)=0.0 1 24 5 TX(I)=0.0 25 CALL ROT (VN.R) 26 N=15 27 DO 10 I=1,N 28 U(I)=0.0 1 29 DU(I)=0.0 1 30 .10 UMAX(I)=1.0 31 C OPEN DATA FILE AND WRITE FILE OPEN(1.FILE='B:THRUST.DAT') 32 33 OPEN(2, FILE='B: POS. DAT') READ THE SEA CURRENT VALUES FROM THE 34 C B: THRUST. DAT FILE 35 READ(1,50) (SU(1), I=1,3) 36 50 FORMAT(1X, 3E10.3) 37 C START MOTION SIMULATION 38 75 H=2.0 39 T=0.0 40 DO 11 I=1.N 11 DU(I)=0.0 41 42 CALL DATAREAD (FT, MT, Q) 43 IF(Q.LT.0)GOTO 90 CALL FCN (N.T.U.DU) 44 CALL DES (N, T, U, DU, H, HMIN, EPS, UMAX, ERRMAX, KFLAG, JSTART

	46			KF=KFLAG
	47			IF(ERRMAX.GT.1.0)KF=-1
	48			WRITE(*,149)H, ERRMAX, KF
	49		149	FORMAT(1X,2E15.6,15)
	50			WRITE(*,150)(FT(J),J=1.3),(MT(K),K=1.3)
	51	2	150	FORMAT(1X, 3E15, 6, 1X, 3E15, 6)
	52	C		FIND NEW ROTATION TENSOR
	53			DO 15 I=1.3
1	54		15	VN(T)=U(T+6)
-	55			CALL ROT (VN. R)
1	56	C		CALCULATE THE NEW POSITION VECTOR
	57	•	1	DO 20 T=1.3
1	58			VX(T)=U(T)
ĩ	. 69		20	SX(T) = U(T+12)
-	60	C		TRANSFORM VY INTO SPACE FIXED COORDINATES
	61			CALL TRAV (VX FX)
	62			DO 25 $T=1.3$
1	63		25	TX(T) = SX(T) + FX(T)
-	64			TTTT+H
	65	C		OUTPUT THE POSTTION DATA TO POS FILE
	66			WRTTE(2,105)TT
	67			WPTTF(2, 110) (TY(T), T=1, 3) (VN(K), K=1, 3)
	.68		105	FORMAT(F10.2)
	69		110	FORMAT (3F15 6:1X 3F15 6)
	70	C	***	CO BACK AND READ NEYT DATA INDUT
	71			COTO 75
1	72	1	90	STOP
	73		50	END
	15			

75 Source Lines



16 Source Lines

.

	1	C	SUBROUTINE FCN	
	2	C	THIS PROGRAM EVALUATES THE MOTION EQUATIONS	5
	3	C	FOR THE SPECIFIC CASE OF THE RCV-225. THESE	6
	4	C	EQUATIONS INCLUDE THE SEA CURRENT AND THE	
	5	C	HYDROSTATIC AND DRAG FEFECT	
	6	-	SUBDOUTTINE FON (N T FU FOU)	
		Anno	SOBROOTINE FCN(N, I, FO, FDO)	
	-	· SDEB	TUDITOT DEST A (1 U M O C)	
	8		IMPLICIT REAL*8 (A-H,M,O-Z)	
	. 9	12	DIMENSION FU(15), FDU(15), CD(3), REY(3), FH(3)	
	10		COMMON/THRUST/FT(3), MT(3), R(3,3), SU(3)	
1	11	С	DEFINE THE CONSTANTS	
	12	C-	GRAVITY CONSTANT	
	13		G=9.806	
	14	C	WATER DENSITY AT 5 DEG. CELCIUS	
	15		DEN=1000.0	
	16	C	WATER VISCOSITY AT 5 DEG. CELCIUS	
	17		VTS=1.519E-6	
	18	C	BODY CONSTANTS:	
	19	c	PADTUS OF BODY	
	20	•	PAD-0 2607	
	21	C	BODY DIAMETER	
	21	ς.	BODI DIAMETER	
	22		DIA=2.0*KAD	
	28	C	BODY VOLUME	
	24	100	VOL= (4.0/3.0) * (RAD**3) *3.14	
	25	С	BODY PROJECTED AREA	
	26		A=(RAD**2)*3.14 *	
	27	с	BODY MASS	
	28		M=82.0	
	29	С	TOTAL MASS= M+ADDED MASS COEFF.	
	30	C .	ADDED MASS= (DEN*VOL*0.5)	
	31	C	AM=' TOTAL MASS	
	-32	× .	AM=M+(DEN*VOL*0.5)	
	33	с	PRODUCTS OF INERTIA FOR BODY	
	34		XP#(2.0/5.0) *M*RAD**2	
	135	c. >	DETERMINE THE DRAG COFFETCIENTS	
	36	· · ·	DO'5 T=1 3	
- 18	37		DEV(T) DABS / FIL (TA3 ) +DTA /VTS	
	20	1. 11	TE (DEV(T) EO O OLCOTO E	
÷.	. 30			
	39		IF (REI(I).LT.U.04) THEN	
	40	2.2	CD(1) = 24.0/REY(1)	
	41		ELSEIF(REY(1). LT. 0. 1) THEN	
	42		CD(1)=245.	
	43		ELSEIF(REY(I).LT.1.0)THEN	
	44		CD(I)=28.0	
	45		ELSEIF(REY(I), LT. 10.0) THEN	
	46		CD(I)=4.4	
	47		ELSEIF (REY(I).LT.100.0) THEN	
	48	- R <sup>6</sup> 8	CD(I)=1.1	
	49		ELSEIF (REY (I) LT. 1000.0) THEN	
	50		CD(I)=0.46	
	51	1.00 <sup>10</sup>	ELSETF (REV (T) . I.T. 10000.0) THEN	
			and the (a) - at - 200000 / Then	

1	52	CD(I)=0.42
1	53	ELSEIF(REY(I).LT.100000.0) THEN *
1	54 .	CD(I)=0.49
1	55	ELSE
1	56	CD(I)=0.14
1	57	ENDIF
1	58	5 CONTINUE
	59 C	FIND BOUYANT FORGE
	60	FFH=(DEN*VOL-M)*G
	61	DO 10 I=1.3
1	62	10 FH(I)=R(3,I)*FFH
	63 C	CALCULATE THE MOTION DERIVATIVES
	64	FDU(1) = FU(4)
2	65	FDU(2) = FU(5)
	66	FDU(3)=FU(6)
	6	7
FI	MI(A) = (D)	M*FU(5)*FU(12))=CD(1)*DEN*A*DABS(FU(4))*FU(4)*0.5+
	68	# FT(1)+FH(1))/AM
	6	9
F	11(5)=((	-AM*FU(4)*FU(12))-CD(2)*DEN*A*DABS(FU(5))*FU(5)
	70	# #0.5+ FT(2)+FH(2))/AM
	7	1
FI	11/61-1-1	D(3) +DEN+A+DABS/ETL(6) +ETL(6) +0.5+ET(3)+EH(3))/AM
	72	FDII(7) = FII(10)
	73.	FDU(8) = FU(11)
	74	FDU(0)=FU(12)
	75	FDU(10)=0.0
	76	FDU(11)=0.0
	. 77	PDU(12) = (MU(2)/VD)
	70	PDI(12) = (II(3)/AF)
1	78	PDU(13) =0(1)
	79	FDU(14)=50(2)
	80	
	81	RETORN
	82	DRD (

· 82 Source Lines

1 C SUBROUTINE DES 2 C THIS SUBROUTINE PERFORMS ONE DOUBLE-STEP OF 3 C THE RUNGE-KUTTA INTEGRATION. IT ALSO CHECKS 4 C THE ESTIMATED ERROR TO INSURE IT IS WITHIN C THE SPECIFIED LIMIT (EPS). DES (N, T, Y, YP, H, HMIN, EPS; YMAX, ERRMAX, # KFLAG. JSTART) SDEBUG IMPLICIT REAL\*8 (A-H,M,O-Z) YSA(15), YSAM(15), Y(15), YMAX(15), YP1(15), Y1(15), ¥2(15), YP 15), Y3(15), ERROR(15), YP(15), AZ(15), ERR(15) 12 COMMON/THRUST/FT(3),MT(3),R(3,3),SU(3) 13 C CHECK THE START PERAMETER TO SEE IF THIS IS A NEW STEP. 14 IF(JSTART. LT. 0) GOTO 5 15 C SAVE INTIAL VALUES OF Y. 16 DO 10 I=1.N 17 YSA(I)=Y(I) 1 18 YSAM(I)=YMAX(I) 1 19 10 CONTINUE 20 GOTO 15 21 5 DO 1 I=1,N 1 22 Y(I)=YSA(I) 1 23 YMAX(I)=YSAM(I) 24 1 · CONTINUE 25 GOTO 20 26 15 CALL FCN(N,T,Y,YP1) 20 KFLAG=1 28 25 A=T+H 29. HHALF=H\*0.500 30 CALL RK(N, T, YSA, YP1, H, Y1) 31 CALL RK(N, T, YSA, YP1, HHALF, Y2) 32 T2=T+HHALF 33 CALL FCN (N, T2, Y2 YP2) CALL RK(N, T2, Y2, YP2, HHALF, Y3) 34 35 ERRMAX=0.0 FIND YMAX AND ERROR AND ERRMAX 36 DO 30 I=1.N 38 A1=DABS(Y1(I)) 39 A2=DABS (Y2 (I)) 40 A3=DABS(Y3(I)) 41 YMAX(I)=DMAX1(YMAX(I),A1,A2,A3) 42 AZ(I)=(Y3(I)-Y1(I))/31.00 43 ERROR (I) =DABS (AZ (I)) A4=YMAX(I) \*EPS 44 45 ERR(I)=ERROR(I)/A4 46 ERRMAX=DMAX1 (ERRMAX, ERR(I)) 47 **30 CONTINUE** 

USE ESTIMATED ERROR TO CORRECT THE VALUE OF Y DO 35 J=1.N Y(I)=(32.0043(I)-Y1(I))/31.00 CONTINUE KFLAG-1 T=A RETURN END 48 C 49 50 51 52 53 54 55 '<sub>35</sub> 1 ī

55 Source Lines

	1	C	SUBROU	JTI	E RK										
	2	C	THIS S	SUB	ROUTINE	CARR	IES I	OUT	ON	ES	TEL	2 01	F		
	3	C	A CLAS	SSIC	CAL FOU	RTH O	RDER	RU	NGE	-KI	JTT7	1			·
			INTEGH	RAT	LON.						•				
	4		SUBROU	TITL	E RK(N	T.RY	.RYP	.H.	RYH	0 .					•
	5	\$DEBU	G		2					1			1	2	
	6		IMPLIC	CIT	REAL*8	(A-H	,M,O	-Z)							
12	. 7						D	I	м	Е	N	S.	I.	0	İ
RY	(15) .:	RYP(15	), RYH (1	15)	RY2 (15	,RYK	2(15	),						÷	
	8		# RY3 (1	15)	RYK3 (1	5), RY	4 (15	),R	YK4	(15	5).				
	9		COMMON	I/TH	RUST/F	F(3),	MT(3	),R	(3,	3),	SU (	(3)			
	10		TH=T+H	ł											
	11		HHALF=	H*(	.500	N.		0.00							
	12		THALF=	T+F	HALF										
	13		DO 5 1	[=1,	N										
1	14	5	RY2	(I)=	RY(I)+	(HHAL	F*RY	P(I	))	1					
	15		CALL H	CN	N, THAL	F,RY2	, RYK	2)		7					
	16		DO 10	I=1	L,N					÷					2
1	17	10	RY3(I)	=R)	(I)+(H	HALF*	RYK2	(I)	)						
	18		CALL F	CN	N, THAL	F,RY3	, RYK	3)							
	19		DO 15	I=1	,N	n, *'						18			
1	20	15	RY4(I)	=R)	(I)+(H:	*RYK3	(I))								
	21		CALL F	CN	N, TH, R	4, RY	K4)								
	22		DO 20	I=1	., N.										
1				£	2	3							2	2	0
RY	H(I)=	RY(I)+	(H* (RYE	P(I)	+2*RYK	2(I)+	2*RY	КЗ (	I)+	RYI	(4 (1	(((	16	.00	
	24		RETURN	1					(2)						
	25	a	END							1					

25 Source Lines

1	C		SUBROUTINE BOT	•
5	c		THIS DOCDAM FUNTHATES THE TEDMS OF THE	
-	C	'	DOMINITON MENCOD	
			ROTATION TENSOR	
3		1.1	SUBROUTINE ROT(RVN, R)	
4			IMPLICIT REAL*8 (A-H,M,O-Z)	
- 5			DIMENSION RVN(3),R(3,3)	
6	C		CALCULATE THE ROTATION TERMS	
7			R(1,1) = DCOS(RVN(2)) * DCOS(RVN(3))	
8			R(1,2) = -DCOS(RVN(2)) * DSTN(RVN(3))	
ä			P(1,2) = DCOD(RVR(2)) = DDIR(RVR(3))	
10			R(2,1) = DSIN(RVN(1)) * DSIN(RVN(2)) * DCOS(RVN(3))	
11	÷.,		<pre># +DSIN(RVN(3))*DCOS(RVN(1))</pre>	
12			<pre>·R(2,2)=-DSIN(RVN(1))*DSIN(RVN(2))*DSIN(RVN(3))</pre>	15
13			# +DCOS(RVN(1))*DCOS(RVN(3))	
14			R(2,3) = -DSIN(RVN(1)) * DCOS(RVN(2))	
15			R(3,1) = -DSTN(RVN(2)) * DCOS(RVN(1)) * DCOS(RVN(3))	1)
16			# +DSTN(DUN(1))+DSTN(DUN(3))	· ·
17			T (2 2) - DETN(DIN(2)) + DETN(DIN(2)) + DCOC(DIN(1))	¢
1/			R(3,2)=DSIN(RVN(2))+DSIN(RVN(3))+DCOS(RVN(1))	·
18			<pre># +DSIN(RVN(1))*DCOS(RVN(3))</pre>	
19			R(3,3) = DCOS(RVN(1)) * DCOS(RVN(2))	
20			RETURN	
21			END	

21 Source Lines

1 C SUBROUTINE TRAV 2 C THIS PROGRAM TRANSFORM A VECTOR IN THE BODY 3 C FIXED COORDINATE REFERENCE FRAME TO THE SPACE 4 C FIXED REFERENCE FRAME. 5 SUBROUTINE TRAV (TVX, TFX) 6 IMPLICIT REAL\*8 (A-H, M, O-Z) DIMENSION TVX(3), TFX(3) 7 COMMON/THRUST/FT(3),MT(3),R(3,3),SU(3) 8 9 N=3 10 DO 5 I=1,N 11 5 TFX(I)=0.0 1 12 DO 10 I=1,N 1 13 DO 15 J=1,N 2 14 15 TFX(I)=TFX(I)+R(I,J)\*TVX(J) ã 15 10 CONTINUE 16 RETURN . 17 END

17 Source Lines

. .



APPENDIX D Position data files for each sample case.

. .

For	mat:		•	cuae .		7 .	aca
	T:	ime		• •		)	•
	X1			X2		/ X3	10
	W1			W2	-	W3	
	(X	values	in	meters	, W va	lues in	radians)
	2	.00			~		
	.00	0000E+00	D	.00000	00E+00	.20	9424E-01
	.000	000E+00	0	.00000	00E+00	.00	0000E+00
	4.	.00					
	.000	0000E+00	0	.00000	00E+00	.83	0493E-01
	.000	0000E+00	0	.00000	00E+00	.00	0000E+00
	0.0	000000000		0000		10	
	.000	0000E+00		.00000	005+00	.10	40135+00
	.000	00005400	,	.00000	005400	.00	00005+00
	. 000	000E+00	•	0000	002+00	33	04432+00
	. 00	0000E+00	5	.00000	002+00		00002+00
	10	.00		.0000	005+00	.00	00005400
	.000	0000E+00	D	,00000	00E+00	.48	8225E+00
	.000	0000E+00	D	.00000	00E+00	.00	0000E+00
	12	.00					
	.000	0000E+0	D	.0000	00E+00	68	2952E+00
÷.,	.00	0000E+00	C	.0000	00E+00	.00	0000E+00
	14	.00					
	.00	0000E+00	D	.0000	00E+00	.90	0274E+00
	.00	0000E+00	D	.00000	00E+00	.00	0000E+00
	16	.00					
	.000	0000E+00	Ο.	.0000	00E+00	. 11	3615E+01
	.00	0000E+00	D	.0000	00E+00	.00	0000E+00
	. 18	.00					
	.00	0000E+00	0	.00000	00E+00	.13	8702E+01
	20	0000E+00	D	.0000	0QE+00	.00	0000E+00.
	.00	0000E+00	C	.00000	00E+00	.16	4983E+01
	.000	0000E+00	5	.00000	00E+00	.00	0000E+00
	22	.00					
	.000	0000E+00	C	.00000	00E+00	.19	2205E+01
	.000	000E+00	0	.00000	00E+00	.00	0000E+00
•	24	.00		•			1
	.000	0000E+00	0	.00000	00E+00	.22	0161E+01
	.000	000E+00	>	.00000	00E+00	.00	0000E+00
	26:	.00 .		•			
	.000	000E+00	)	.00000	00E+00	.24	8687E+01
	.000	0000E+00	)	.00000	00E+00	.00	0000E+00
	28.	.00					
	.000	000E+00	)	.00000	00E+00	.27	7653E+01
	.000	000E+00	)	.00000	00E+00	.00	0000E+00
	30.	00 /	١.	1			
	.000	000E+00	) .	.00000	00E+00	.30	6958E+01

Case 1:

Position Data

000000E+00	.000000E+00	.000000E+00
0000002+00	00000000000	2265222.02
000000000000000000000000000000000000000	.0000002+00	.336521E+01
34.00/	.000000E+00	.000000E+00
000000E+00	.000000E+00	3662828+01
000000E+00	.000000E+00	000000E+00
36.00		
.000000E+00	.000000E+00	.396195E+01
000000E+00	· .000000E+00	.000000E+00
38.00		
.000000E+00	.000000E+00	.426222E+01
000000E+00	.000000E+00	.000000E+00
40.00.		
.000000E+00	.000000E+00	.456337E+01
000000E+00	.000000E+00	.000000E+00
42.00		
.000000E+00	.000000E+00	.486519E+01
.000000E+00	.000000E+00.	000000E+00.
44.00	• •	
000000E+00	.000000E+00	.516751E+01
000000E+00	:000000E+00	.000000E+00
46.00		
000000E+00	.000000E+00	\$547022E+01
.000000E+00	.000000E+00	.000000E+00
48.00		
.000000E+00 \	.000000E+00	.577322E+01
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50.00 .		A
000000E+00	.000000E+00	.607640.01
000000E+00	.000000E+00	.000000E+00
52.00		
000000E+00	.000000E+00	.637984E+01
000000E+00	.000000E+00	.000000E+00
54.00		
000000E+00	.000000E+00	.668335E+01
000000E+00	0000002+00	0000002+00
56.00	1000000100	.00000000000000000000000000000000000000
000000E+00	.000000E+00	.698697E+01
000000E+00	.000000E+00	.000000E+00
58.00		
000000E+00	.000000E+00	.729066E+01
000000E+00	.000000E+00	.000000E+00
60.00		
000000E+00	.000000E+00	-759440E+01
000000E+00 ·	.000000E+00	.000000E+00

2	mat:	1.0							
	T	ime					÷		
	X1.			5 X2 /			Х3		
	W1		8	W2	7		W3		
	(X	values	in	heters,	W Va	alues	in	radian	S)
	2	.00		· prof					
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	000	0000E+0	ñ.	.00000	0E+00	5	000	000E+0	ō
	4	.00							-
	205	5600E+0	1	.00000	0E+00	) ,	830	493E-0	1
	.000	0000E+0	D	.00000	0E+00		000	000E+0	0
	6.	.00					7	2	
	308	3400E+0	1	.00000	0E+00	) .	184	015E+0	0.
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	8.	.00 .							
	411	L200E+0	1	.00000	0E+00	) .	320	443E+0	0
	.000	0000E+0	Э	.00000	0E+00	) :	000	000E+0	0
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	- 022	100ET0.	1	00000	0210		112	615540	<b>n</b>
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	- 925	5200E+0	1	.00000	DE+0		138	702E+0	1
	.000	0000E+0	ñ		0E+0		000	000E+0	n
	20.	.00	1						
	102	2800E+0	26	.00000	0E+0	<b>.</b>	164	983E+0	1
	.000	0000E+0		.00000	0E+0		000	000E+0	0
	22.	.00	- )	0					
	113	3080E+0	2	,00000	0E+0		192	205E+0	1
	.000	0000E+0	D	.00000	DE+0	ο.	000	000E+0	0
	24.	.00					ν.,	marker i have a	
	123	3360E+0	2	.00000	00E+0	ο	220	161E+0	1
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1	26.	.00		· · · · · · · · · · · · · · · · · · ·	1.				
	133	3640E+0	2	.00000	00E+0	o : :	248	687E+0	1
	.000	0000E+0	D	.00000	00E+00		000	000E+0	0
	28.	.00		·			:		
	143	3920E+0	2	.00000	0E+00		277	653E+0	1
	.000	1000E+Q	5	.00000	0E+00	· ·	000	000E+0	0
	-30.	00				·			÷.
	154	200E+0	4 .	.00000	OF+00	• •	306	958E+0	T
	.000	JOUOR+0		.00000	102+00		000	UUUE+0	0

Case 2: Position Data

32.00		
-: 164480E+02	,.000000E+00	.336521E+01
.000000E+00	.000000E+00	.000000E+00
34.00		
174760E+02	.000000E+00	.366282E+01
.000000E+00	.00000E+00	.000000E+00
36.00		
185040E+02	.000000E+00	.396195E+01
.000000E+00	.000000E+00	.000000E+00
38.00		1 .
195320E+02	.000000E+00	-426222E+01
.000000E+00	.000000E+00	.000000E+00
40.00		
205600E+02	.00000E+00	.456337E+01
.000000E+00	.000000E+00	.000000E+00
.42.00	· · ·	
215880E+02	.000000E+00	.486519E+01
.000000E+00	.00000E+00	.000000E+00
44.00		
226160E+02		.516751E+01
.000000E+00	.000000E+00	.000000E+00
46.00	-	
236440E+02	.000000E+00	.547022E+01
.000000E+00,	.000000E+00	.000000E+00
48.00	,	and the second s
246720E+02	.000000E+00*	.577322E+01
.000000E+00	'.000000E+00	.000000E+00
50.00	2	
257000E+02	"000000E+00	.607645E+01
000000E+00	.000000E+00	.000000E+00
52.00		
267280E+02	.00000E+00	.637984E+01
.000000E+00	.000000E+00	.000000E+00
.54.00		
277560E+02	.000000E+00	.668335E+01
.000000E+00	.000000E+00	.000000E+00
56.00		
287840E+02	.000000E+00	.698697E+01
.000000E+00	.000000E+00	.000000E+00
58.00		
298120E+02	,00000000000000000000000000000000000000	.7290666+01
.000000E+00	.000000E+00	.000000E+00
00.		
308400E+02	.000000E+00	./59440E+01
.000000E+00	.000000E+00	.000000E+00

Case 3: Position Data

	-Time				10	1	
	X1		X2		X	3	
×	W1		W2		W	3	J
	(X value	s in	meters,	W val	lues i	n rad	ians)
	2.00					·	
	.000000E+	00	.00000	0E+00	5	97304	E-01
	.000000E+	00	.00000	0E+00	.0	00000	E+00
	4.00				10.1		
	.000000E+	00	.00000	0E+00	2	32742	E+00
	.000000E+	00	.00000	0E+00	0	00000	E+00
	6.00	•				1	
	.000000E+	0.0	.00000	0E+00	5	02897	E+00
	.00000E+	00 .	.00000	0E+00	. 0	00000	E+00
	8.00	S				See. Ver	
	.000000E+	00	.00000	0E+00	8	49398	E+00
	.000000E+	00	.00000	0E+00	0	00000	E+00
	10.00	. S.			-1	0 Q.	1.1
	.00000E+	00	.00000	0E+00	1	25187	E+01
	.000000E+	00	.00000	0E+00	.0	00000	E+00
٠	12.00 -,		2.21	1 1 1			1
	.158661E+	00	.00000	0E+00	1	64297	E+01
	.000000E+	00	.00000	0E+00	.0	00000	E+00,
٠	14.00				3		1
	.594582E+	00	.00000	0E+00	1	98072	E+01
	.000000E+	00	.00000	0E+00	0	00000	E+00
	16:00			1			· · ·
	.126957E+	01	.00000	0E+00	2	27934	E+01
	.000000E+	00	.00000	0E+00_	0	0.0000	E+00
1	18.00		· ·		*s		1.
	.218528E+	01	.00000	0E+00	2	54820	E+01
	.000000E+	00	.00000	0E+00	.0	00000	E+00
	20.00						
	.329138E+	01	.00000	0E+00	2	79381	E+01
	.000000E+	00	.00000	0E+00	.0	00000	E+00
	22.00		Concerne a				
	.454075E+	01	.00000	0E+00	3	02082	E+01
	000000E+	00	.00000	0E+00	• 0	00000	<b>5</b> +00
	24.00				No		1
	.589394E+	01 .	.00000	0E+00	3	23272	E+01
	.000000E+	00	. 00000	0E+00	.0	00000	E+00
	26.00			19.5			1.
	.732032E+	01	.00000	0E+00	3	43216	8+01
	.000000E+	00	.00000	06+00	.0	00000	E+00.
	28.00				-		
	.879730E+	UI .	.00000	05+00	3	62123	6+01
	.000000E+	00	.00000	UE+00	.0	00000	5+00
	30.00	~		anion	18 .		
	.103088E+	02	.00000	06+00	3	80155	6+01
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	32.00	1		
	.118436E+02	.000000E+00	397446E+01	
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	34.00			
	.133941E+02	.000000E+00	414103E+01 .	
	.000000E+00	.000000E+00	.000000E+00	
	36.00			
	.149550E+02	.000000E+00	430216E+01	
	.000000E+00	.000000E+00	.000000E+00	_
	38.00			7
	.165229E+02	.000000E+00	-:445859E+01	
•	.000000E+00	.000000E+00	.000000E+00	
	40.00			
	.180955E+02	.000000E+00	-,461094E+01	
	.000000E+00	.000000E+00	.000000E+00	
	42.00			
ŝ	.195278E+02	.000000E+00	473634E+01	
	.000000E+00	.000000E+00	.000000E+00	•
	44.00		, , ,	
	.207350E+02	.000000E+00	481489E+01	
	.000000E+00	.000000E+00	.000000E+00	
	46.00	· · · · · ·		
	.217783E+02	.000000E+00	484988E+01	
	.000000E400	.000000E+00	· .000000E+00	
	48.00		· ·	
	.226969E+02	.000000E+00	484271E+01	
	.000000E+00	.000000E+00	.000000E+00	
	50.00			
	.235175E+02	.000000E+00	479397E+01	
	.000000E+00	.000000E+00	.000000E+00	
1	52.00			
	.242590E+02	·.000000E+00	470549E+01	
	.000000E+00	.000000E+00	.000000E+00	
	54.00	· · ·		
	.249353E+02	.000000E+00 .	458033E+01	
	.000000E+00	.000000E+00	.000000E+00	
	56.00			
	.255569E+02	.000000E+00	442241E+01	
	.000000E+00	.000000E+00	.000000E+00	
	58.00			
	.261320E+02	.000000E+00	423608E+01	
	.000000E+00	.000000E+00	.000000E+00	
	60.00			
	.266199E+02	.000000E+00	402574E+01	
	.000000E+00	.000000E+00	.000000E+00	

· ·	Case 4: Posit	ion Data
format:		4
Time	_	
X1 .	X2	<b>X</b> 3
W1,	W2 .	W3
(X values in	meters, W valu	ies in radians)
2.00		
514000E+00 .	.000000E+00	597304E-01
.000000E+00	.000000E+00	.000000E+00
102800E+01	.000000E+00	232742E+00
.000000E+00	.000000E+00	-000000E+00
- 154200E+01	00000Ftod	- 5028975+00
00000E+01	000000000000000	
8.00		
805 600E+01	.000000E+00	849398E+00
.000000E+00	.000000E+00	.000000E+00
257000E+01	.000000E+00	125187E+01
.000000E+00	.000000E+00	.000000E+00
12.00		1.1.1
292534E+01	.000000E+00	164297E+01
.000000E+00	.000000E+00	.000000E+00
.14.00	1 A A A A A A A A A A A A A A A A A A A	
300342E+01	.000000E+00	198072E+01
.000000E+00	.000000E+00	.000000E+00.
16.00		1 A
284243E+01	.000000E+00	227934E+01
.000000E+00	.000000E+00	.000000E+00
. 18.00		
244072E+01	.000000E+00	2548206+01
20.00	.0000008+00	.0000005+00
'184862E+01	.000000E+00	279381E+01
.000000E+00	.000000E+00	.000000E+00
22.00	· · · · ·	
111325E+01	.000000E+00	302082E+01
.000000E+00	.000000E+00	.000000E+00
. 24.00		
274061E+00	.000000E+00	323272E+01
26.00	.000000E+00	.000000E+00
.638320E+00	.000000E+00	343216E+01
.000000E+00	.000000E+00	.000000E+00
1601305+01	0000008400	- 2621228401
	000000000000000000000000000000000000000	
30.00	.00000005+00	.0000000000
250970F+01 '	0000008+00	- 2001555401
000000000000	- 000000E+00	3001355401
		.00000000000

4.14

32.00		
.361960E+01	.000000E+00	397446E+01
.000000E+00	.000000E+00	.000000E+00
.465606E+01	.000000E+00	414103E+01
.000000E+00	.000000E+00	.000000E+00
36.00	· · · · · · · · · · · · · · · · · · ·	
,570300E+01	.000000E+00	430216E+01 ,
38.00	.000000E+00	.000000E+00
.675691E+01	.000000E+00	445859E+01
.000000E+00	.000000E+00	.000000E+00
40.00		1.
.781548E+01	.000000E+00	461094E+01
.000000E+00	.000000E+00	.000000E+00
42.00		
.873385E+01	.000000E+00	473634E+01
.000000E+00	.000000E+00	.000000E+00
44.00		· · ·
.942704E+01	-000000E+00	481489E+01
.000000E+00'	.000000E+00	.000000E+00
46.00		. S. 1.
.995632E+01	.000000E+00	484988E+01
.000000E+00	.000000E+00	.000000E+00
48.00		
.103609E+02	.000000E+00	484271E+01
000000E+00	.000000E+00	.000000E+00
1066758+02	0000002+00	- 4702078101
0000002+00	000000000000000000000000000000000000000	4/939/E+01
52.00	.00000005+00	.0000002+00
.108950E+02	.000000E+00	470549E+01
.000000E+00 .	.000000E+00	.000000E+00
54.00		
.110573E+02	.000000E+00	458033E+01
.000000E+00	.000000E+00	.000000E+00
56.00		
.1116495+02	.000000E+00	442241E+01
58 00 ····	.000000E+00	.000000E+00
112260F+02	0000002+00	- 4226008+01
000000F+02	.000000E+00	423608E+01
°60.00		.00000000000000000000000000000000000000
111999E+02	0000002+00	- 4025742401
.000000E+00*	.000000E+00	000000E+00

Form	at:	10			,	····.	
	Time						
	X1		X2		X3		
	W1'		W2		W3		
•	(X values	in	meters.	W val	ues in	radian	s)
	2.00						
	.158661E+0	0	.00000	0E+00	59	7304E-0	1
-	.000000E+0	0	.00000	0E+00	00	0000E+0	ο.
	4.00 .	-					
	.594582E+0	0	.00000	0E+00	23	2742E+0	0
	.000000E+0	0	.00000	0E+00	.00	0000E+0	0
	. 6.00						
	.126957E+0	1	.00000	0E+00	50	2897E+0	0
	.000000E+0	0	.00000	0E+00 ·	.00	0000E+0	0 '
	8.00	1.1					
	.218528E+0	1	.00000	0E+00	84	9398E+0	0
	.000000E+0	0	.00000	0E+00	00	0000E+0	0
	10.00						
	.329138E+0	1 /.	.00000	0E+00	12	5187E+0	1.
	:000000E+0	0	.00000	0E+00	.00	0000E+0	0
• •	12.00						
	.454075E+0	1	.00000	0E+00	16	9331E+0	1
	.000000E+0	0	.00000	0E+00	.00	0000E+0	0
	14.00			•			
	.589394E+0	1	.00000	0E+00 .	21	6090E+0	1
	.000000E+0	0	.00000	0E+00	.00	0000E+0	0
	16.00						
	.7/32032E+0	1	.00000	0E+00.	26	4563E+0	1
	.000000E+0	0	.00000	00E+00	.00	0000E+0	0
	18.00						
	.879730E+0	1	.00000	0E+00	31	4140E+0	1
	.000000E+0	0	.00000	00E+00	.00	0000E+0	Ο.
	20.00						
`	.103088E+0	2	.00000	00E+00	36	4421E+0	1
-	.000000E+0	0	.00000	00E+00	00	0000E+0	0
	22.00			1 1			•
	.116998E+0	2	.00000	0E+00	41	0208E+0	1
	.000000E+0	0	.00000	00E+00	.00	0000E+0	0
-	24.00			St.	1		1.0
	.128776E+0	2	.00000	0E+00	46	0777E+0	1
	.000000E+0	0	.00000	00E+00	.00	0000E+0	0
	26.00		·				
	.138988E+0	2	.00000	0E+00	51	8050E+0	1
	.000000E+0	0	.00000	0E+00	.00	0000E+0	0
	28.00			-		1.1	
•	.146488E+0	2	.00000	0E+00	56	7205E+0	1
	.000000E+0	0	.00000	0E+00	.00	0000E+0	0
	30.00						
	.149946E+0	2	,00000	0E+00	60	7919E+0	1.
	.000000E+0	0.	.00000	02+00	:00	0000E+0	U

187 : Position

Data

Case 5

32.00		
.149941E+02	.000000E+00	642837E+01
.000000E+00	.000000E+00	.000000E+00
34.00	1	۰.
.146866E+02	.000000E+00	673552E+01
.000000E+00	.000000E+00	.000000E+00
36.00 .	1:	
.141339E+02	.000000E+00	701,095E+01
.000000E+00	.000000E+00 .	
38.00		1
.133313E+02	.000000E+00	726173E+01
.000000E+00	.000000E+00	.000000E+00
40.00	/	
.123134E+02	.000000E+00	749291E+01
.000000E+00	.000000E+00	.000000E+00
42.00		·
.111296E+02	.000000E+00	770822E+01
.00000E+00	.00000QE+00	.000000E+00
44.00		X -
.982351E+01	.000000E+00	791049E+01
.000000E+00	.000000E+00	.000000E+00
46.00	• • 1	
.843012E+01	.000000E+00	810192E+01
.000000E+00	.000000E+00	.000000E+00
48.00	and a second second	1 1
.697585E+01	.000000E+00	828424E+01
.000000E+00	.000000E+00	.000000E+00
50.00		•
.547981E+01	.000000E+00	845885E+01
.000000E+00	.000000E+00	.000000E+00
52.00	·	1
.409929E+01	.000000E+00	862687E+01
.000000E+00	.000000E+00	.000000E+00
54.00	00000000000	
.292911E+01 .	.0000002+00	8/8925E+01
.000000E+00	.0000005+00	.0000005+00
1012578(01	00000000000	
.19135/6+01	000000000000000000000000000000000000000	8946//E+UI
.000000ET00	.0000002+00	.0000005+00
1016548101	000007100	- 01000000101
.1010546+01	.000000E+00	9100065+01
60.00	.00000000000000000000000000000000000000	.000000E+00
2132008+00	000000E+00	- 022620E+01
00000E+00	000000000000000000000000000000000000000	0000000000000
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				Cas	8 6	Pos	itio	n Da	ata	
For	nat:									
	Ti	lme								
	Xl			X2				Х3	1	
	. W1			W2				W3	÷	
	(X	values	in	meter	rs,	W va	lues	in	rad	ians)
	2.	.00								5
	.000	0000E+00	0	.00	0000	DE+00	-	.13	8580	E+00
	.000	0000E+00	0	.00	000	DE+00		.00	0000	E+00
	4.	.00			. e			-		
	.000	000E+00	2	.00	0000	DE+00	-	. 52:	3268	E+00
	.000	0000E+00	0	.00	000	DE+00		.00	0000	E+00
	6.	.00								
	.000	0000E+00		.00	0000	DE+UU	-		0180	ETUI
•	.000	AN 1000E+00	,	.00	0000	12+00		.00		5400
	467	COOR+O		1 00		PF+00		17	5300	E+01
	.40/	00002+00			0000	DE+00			0000	FTOU
	. 10	0005+00				5400		.00	0000	5100
۱. –	163	AGRELOT	1.	0.0	000	P+00	· -	23	7857	E+01
1.	.000	0000E+00	5	.00	000	DE+00		.00	0000	E+00
1	12	00	·							2.00
1	. 320	185E+01	1	.27	123	E+00	· · _	.09	5473	E+01
1	.000	0000E+00	5	.00	000	DE+00		.60	0000	E+00
.1	14.	.00						· ~		
1	.503	3696E+0	L	-10	329	E+01	-	.34	4875	E+01
	.000	0000E+00	0	.00	000	DE+00		.00	0000	E+00
	16.	.00								
	.703	968E+01	L	.19	705	3E+01		.38	5752	2+01
12		0000E+00	0	.00	000	DE+00		.00	0000	E+00
. 1	18.	.00					•			
	.914	245E+01	L	.28	063	LE+01	-	.42	0786	E+01
	.000	0000E+00	2	.00	000	DE+00		.00	0000	E+00
	20.	.00								
	,113	3033E+02	5	. 35	601:	2E+01	-	.45	1587	E+01
	.000	0000E+00	)	.00	000	DE+00		.00	0000	E+00
	22.	.00	1							
	.134	974E+02	2	.42	466	5E+01		.47	9196	E+01
	.000	000000000000000000000000000000000000000	<b>,</b> .	.00	0000	DE+00	•	.00	0000	E+00
	24.									B101
· · ·	.15/	102ET02		. 40	0.09	ETUI		. 50	10000	ETOI
	26	0005+00		.00	0000	5400		.00	0000	5100
	170	337E+03		. 54	595	E+01		52	7486	E+01
	0.06	000E+00	5	0.00	000	E+00		000	0000	E+00-
	28.	00								2.50
•	.201	630E+02		. 59	5264	E+01	· · _	. 549	9051	E+01
•	.000	000E+00	)	.00	0000	E+00		.000	0000	E+00
	30.	00								
•	.223	957E+02	-	. 635	5521	E+01	-	. 569	307	E+01
- 1	.000	000E+00		:000	0000	E+00		.000	0000	E+00

189()

32.00		
.244944E+02	.643110E+01	588473E+01
.000000E+00	.000000E+00	.000000E+00
34.00		
.264006E+02	.597008E+01	606725E+01
.000000E+00	.000000E+00	.000000E+00
36.00	and the second second	
.281902E+02	.528736E+01	624202E+01
.000000E+00	.000000E+00	.000000E+00
38.00	·	
.299068E+02	.466030E+01	641020E+01
-000000E+00	.000000E+00	.000000E+00
315766F+02	4080508101	
000000E+02	.408050E+01	-:05/2/0E+01
42.00	10000005+00	.000000000000
3321628+02	358033P+01	- 6720225+01
.000000E+00	000000E+01	0730325+01
44.00		.0090005+00
.348360E+02	318808E+01	688370E+01
.000000E+00 ·	.000000E+00	.000000E+00
46.00		
.364429E+02	.284886E+01	703341E+01
.000000E+00	.000000E+00	.000000E+00
48.00	/ .	
.380413E+02	.255504E+01	717990E+01
.000000E+00	.000000E+00	.000000E+00
50.00		
.396341E+02	.229589E+01	732357E+01
.000000E+00	.000000E+00	.000000E+00
52.00	/	
.410800E+02	.206408E+01	744134E+01
.000000E+00-	,000000E+00.	.000000E+00
54.00	/	
.422968E+02 .	.185439E+01	751289E+01
.000000E+00	.000000E+00	.000000E+00
56.00		
.433472E+02	.166297E+01	754124E+01
.000000E+00	.000000E+00	.000000E+00
58.00		
.442713E+02	.148688E+01	752750E+01
.0000008+00	.000000E+00	.000000E+00
4500628+02	1222055101	- 7470078101
-450503E+02	.132385E+01	/4/23/E+01
.000000E+00	.0000005+00	.000000E+00

Case 7: Position Data Format: Time X1 XŻ х́з W2 W3 W1 (X values in meters, W values in radians) 2.00 .624495E+00 .000000E+00 -.138580E+00 .000000E+00 .000000E+00 .000000E+00 4.00 .175847E+01 .000000E+00 -.523268E+00 .000000E+00 .00000QE+00 .000000E+00 6.00 .000000E+00 .289518E+01 -.964906E+00 .000000E+00 .000000E+00 .000000E+00. 8.00 .416702E+01 .000000E+00 -.133823E+01 .000000E+00 .000000E+00 .000000E+00 10.00 .551553E+01 .542780E+00 -.166312E+01 .000000E+00 .000000E+00 .106882E+00 12.00 .187820E+01 -.195206E+01 .671188E+01 .000000E+00 .000000E+00 .320646E+00 14.00 .762379E+01 .353444E+01 -.221343E+01 .000000E+00 .000000E+00 .534410E+00 16.00 .817527E+01 .546420E+01 .000000E+00 .000000E+00 .748174E+00 18.00 .830429E+01 .760014E+01 -.267527E+01 ,000000E+00 .000000E+00 .961938E+00 20.00 .792768E+01 .988615E+01 -.288322E+01 .000000E+00 .000000E+00 .117570E+01 22.00) .696823E+01\_ .122304E+02 -.307938E+01 .138947E+01 .000000E+00 .000000E+00 24.00 .537019E+01 .145052E+02 -.326569E+01 .000000E+00 .000000E+00 .160323E+01 26.00 .311257E+01 .165545E+02 -.349718E+01 .080000E+00 .000000E+00 .181699E+01 28.00 .217962E+00 .182067E+02 -.381402E+01 .000000E+00 .000000E+00 .203076E+01 30.00 -.324308E+01 .192881E+02 -.414393E+01 .000000E+00 .000000E+00 .224452E+01

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32.00		
715194E+01	.196371E+02	443667E+01
.000000E+00	.000000E+00	245829E+01
34.00	•	
113453E+02	.191178E+02	470098E+01
.000000E+00	-000000E+00	\$267205E+01
36.00		
156219E+02	.176318E+02	499549E+01
.000000E+00	.000000E+00	.288581E+01
38.00	• . •	
189406E+02	.161516E+02	536021E+01
.000000E+00	.000000E+00	.304614E+01
40.00)		
214777E+02	.151339E+02	572484E+01
.000000E+00	.000000E+00	.315302E+01
42.00		. 7
237974E+02	· .141917E+02 ·	604338E+01
.000000E+00	.000000E+00	.323852E+01
44.00		
259180E+02	.134357E+02	632751E+01
.000000E+00	.000000E+00	.330265E+01
46.00		
278440E+02	.129866E+02	658512E+01
.000000E+00	.000000E+00	.334541E+01
48.00		
295811E+02	.129405E+02	682177E+01
.000000E+00	.000000E+00	.336678E+01
.50.00		
312476E+02	.130778E+02	704153E+01
.000000E+00	.000000E+00	.337747E+01
52.00		for any second second
328223E+02	.134483E+02	724748E+01
.000000E+00	.000000E+00	.337747E+01
54.00		
344259E+02	.137470E+02	-,744198E+01
.000000E+00	.000000E+00	.337/4/E+01
56.00	10000000000	
3604685+02	.1398/16+02	768028E+01
58.00	.00000000000000	.33//4/E+01
- 3753555402	1420428+02	- 7027728+01
37 53 555 FT02	00000000000	3377478+01
60.00		.3377476+01
- 387015E+02	143726F+02 *	- 8111015+01
.000000E+00	.000000E+00	337747E+01

	cube o		Lon Duoc	
Format:				
Time				
X1	* X2		X3	
W1	W2		W3	
(X values in	meters.	W valu	es in ra	dians
2.00				
.624495E+00	.00000	0E+00	13858	0E+00
.000000E+00	.00000	0E+00	.00000	00E+00
4.00				
.1/584/E+01	.00000	0E+00	52326	8E+00
.000000E+00	.00000	0E+00	.00000	00E+00
0.00	00000	OFIOD	1 00400	CRIGA
.289518E+01	.00000	06+00	96490	OUTION
.000000E+00	.00000	0E+00	.00000	0E+00
8.00				
.416/02E+01	.00000	06+00	13382	36+01
.000000E+00	.00000	0E+00	.00000	00E+00
10.00	(4)	1		
.553619E+01	27023	1E+00	16631	2E+01
.000000E+00	.00000	0E+00	.00000	00E+00
12.00	(**** ) **			12.14
.753667E+01	10329	4E+01	19520	6E+01
.000000E+00	.00000	0E+00	.00000	00E+00
14.00				
.103634E+02	22277	7E+01	22134	3E+01
.000000E+00	.00000	0E+00	.00000	00+100
16.00			··· 6,	
.130871E+02	-: 34912	6E+01	24530	7E+01
.000000E+'00	.00000	0E+00	.00000	00E+00
18.00				
.153422E+02	45763	1E+01	26752	7E+01
.000000E+00	.00000	0E+00	.00000	0E+00
20.00			5	
.173400E+02	55271	3E+01	28832	2E+01
.000000E+00	.00000	0E+00	.00000	00E+00
22.00				
193409E+02	58741	6E+01	30793	8E+01
00000E+00'	00000	02+00	26720	5E-01
24.00		01.00		
215363E+02	- 49801	5E+01 '	- 32656	0F+01
000000E+00	00000	OFLOO	10699	28+00
26.00	.00000	011100	.10000	A BTOO
236024F+02	- 22149	OF+01	- 24071	02+01
0000002+00	00000	OFLOO	21276	AFLOO
38 00		01.00		
2563185+02	- 11133	PF+01	- 39140	22401
1000000E+00		OFLOO	22064	CET-00
30.00		05+00	. 32004	00400
2722068402	12162	TELOI	- 41420	22+01
-273306E+02	. 13102	DELOO	41439	ORLOO

# 193 Case 8: Position Data

32.00		
.287556E+02	.406006E+01	443667E+01
.000000E+00	.000000E+00	.534410E+00
34.00		,
.298684E+02	.709651E+01	470098E+01
.000000E+00	.000000E+00	.641292E+00
36.00 .		
.308965E+02	.958967E+01	499549E+01
.000000E+00	.000000E+00	.721453E+00
38.00 .		
.320077E+02	.114897E+02	536021E+01
.000000E+00	.000000E+00.	.774894E+00
40.00		
.331950E+02	.130610E+02	577555E+01
.000000E+00	.000000E+00	.812303E+00
42:00		
.347089E+02 ··	.144671E+02	617551E+01
.000000E+00	.000000E+00	.833679E+00
44.00		
.363890E+02	.162628E+02	651958E+01
.000000E+00	.000000E+00	.855056E+00
46.00		
.379555E+02	.181312E+02	682294E+01
.000000E+00	.000000E+00	.876432E+00
48.00		1 ·
.393110E+02	.199425E+02	709545E+01
.000000E+00	.000000E+00	.897809E+00
50.00		
.408662E+02	.214799E+02	734393E+01
.000000E+00	.000000E+00	.908497E+00
52.00		
.426143E+02	.226567E+02	757327E+01
.000000E+00	.00000E+00	.908497E+00
54.00		
.442271E+02	.237435E+02	772552E+01
.000000E+00	.000000E+00	.908497E+00
56.00		
,457353E+02	.247915E+02	778991E+01
.000000E+00	.000000E+00	.908497E+00
58.00		
.470386E+02	.257431E+02	781142E+01
.000000E+00	.000000E+00	.908497E+00
60.00		
.481228E+02	.265346E+02	779092E+01
.000000E+00	.000000E+00	.908497E+00

.

Case 9: Position Data Format: Time X1 X2 X3 W1 W2 W3 (X values in meters, W values in radians) 2.00 .260495E+00 .364000E+00 -.138580E+00 .000000E+00 .000000E+00 .000000E+00 4.00 .103047E+01 .728000E+00 -.523268E+00 .00000E+00 .000000E+00 .000000E+00 6.00 .109200E+01 .180318E+01 -.964906E+00 .000000E+00 .000000E+00 .000000E+00 8.00 .271102E+01 .145600E+01 -. 133823E+01 .000000E+00 .000000E+00 .000000E+00 10.00 .371619E+01 .154977E+01 -,166312E+01 .000000E+00 .000000E+00 .000000E+00 12.00 \* .535267E+01 .115106E+01 -. 195206E+01 .000000E+00 .000000E+00 .000000E+00 14.00 .781541E+01 .320234E+00 -.221343E+01 .000000E+00 .000000E+00 .000000E+00 16.00 .101751E+02 -. 579260E+00 -.245307E+01 .000000E+00 .000000E+00 .000000E+00 18.00 .120662E+02 -.130031E+01 -.267527E+01 .000000E+00 .000000E+00 .000000E+00 20.00 .137000E+02 -.188713E+01 -.288322E+01 .000000E+00 .000000E+00 .000000E+00 22.00 .153369E+02 -.187016E+01 -.307938E+01 .000000E+00 .000000E+00 .267205E-01 24.00 .171683E+02 -. 612146E+00 -.326569E+01 .000000E+00 .000000E+00 .106882E+00 26.00 .189604E+02 .151720E+01 -.349718E+01 .000000E+00 .000000E+00 .213764E+00 28.00 205358E+02 .398262E+01 -.381402E+01 000000E+00 .000000E+00 .320646E+00 30:00 .218706E+02 .677627E+01 -.414393E+01 000000E+00 .000000E+00 .427528E+00

.

195

32.00		
.229316E+02	.988406E+01	443667E+01
.000000E+00	.000000E+00	.534410E+00
34.00		
.236804E+02	.132845E+02	470098E+01
.000000E+00	.000000E+00	.641292E+00
36.00		
.243445E+02	.161417E+02	499549E+01
.000000E+00	.000000E+00	.721453E+00
38.00		
.250917E+02	.184057E+02	536021E+01
.00000,0E+00	.000000E+00	774894E+00
40.00		
.259150E+02	.203410E+02	577555E+01
.000000E+00	.000000E+00	.812303E+00
42.00		
.270649E+02	.221111E+02	617551E+01
.000000E+00	.000000E+00	.833679E+00
44.00		
.283810E+02	.242708E+02	651958E+01
.000000E+00	.000000E+00	.855056E+00
46.00		
295835E+02	.265032E+02	682294E+01
.000000E+00	.000000E+00	.876432E+00
48.00		
.305750E+02	.286785E+02	709545E+01
.000000E+00	.000000E+00	.897809E+00
50.00		
.317662E+02	.305799E+02	734393E+01
52.00	.000000E+00	.908497E+00
.331503E+02	.321207E+02	757327E+01
.000000E+00	.000000E+00	.908497E+00
54.00	•	
.343991E+02	.335715E+02	-: 772552E+01
.000000E+00	.000000E+00	.908497E+00
56.00		
.355433E+02	.349835E+02	778991E+01
.000000E+00	.000000E+00	.908497E+00
58.00 .		
.364826E+02	.362991E+02	781142E+01
.000000E+00	.000000E+00	.908497E+00
60.00		
.372028E+02	.374546E+02	779092E+01
.000000E+00	.000000E±00	.908497E+00



APPENDIX E Data files for input of thrust activity to symulation program for each sample case.

## ··199 Case 1 : Thruster Data

0.0

Format:

mat: sea current values: SV1 SV2 SV3 increment thruster values: N. T1 T2 T3 T4 (see chapter 4 section 1.3 for thruster layout )

		0.	0		0.0	
	1.	0.	0.	0.	0.	
	2.	0.	0.	0.	0.	
	3.	0.	0.	0.	0.	
	-4.	0.	0.	0.	0.	
	5.	0.	0.	0.	0.	
	6.	0.	0.	0.	0.	
	7.	. 0	0.	0.	0.	٩
	8.	0.	0.	0.	0.	
	9:	0.	0.	0.	0.	
	10.	0.	0.	0.	0.	
	11.	0.	0.	0.	0.	
į	12.	0.	0.	0.	0.	
	13.	0	0.	0.	0.	
ł	14.	0.	0.	0.	0.	
	15.	0.	0.	0.	0.	
ذ	16.	0.	0.	0.	0.	
-	17.	0.	0.	0.	0.	
-	18.	0.	0.	0.	0.	
ŝ	19.	0.	0.	0.	. 0.	
1	20.	0.	0.	0.	0.	
1	21.	0.	0.	0.	ō.	
ŝ	22.	0.	0.	0.	0.	
ş	23.	0.	0.	0.	0.	
-	24	0	0.	0.	0.	•
1	25	•0	0.	0	0	
1	26	0.	0.	0.	0.	
1	27	0.	0.	0.	0.	
1	20		o.	0.	10	
1	20.	0.	0.	0.		
;	20	0	0.	0	0.	
	50.				0.	
## Case 2 : Thruster Data

0.0

Format: sea current values: SV1 SV2 SV3 increment thruster values: N. T1 T2 T3 T4 (see chapter 4 section 1.3 for thruster layout )

-514E00			6	0.0		
1.	0.	0.	0	0.		
2.	0.	0.	0.	0.		
3.	0.	0.	0.	0.		
4.	0.	0.	0.	0.		
5.	0.	0.	0.	0.		
6.	0.	0.	0.	0.		
7.	0.	0.	0.	0.		
8.	0.	0.	0.	0.		
9.	0.	0.	0.	0.		
10.	0.	0.	0.	0.		
11.	0.	0.	0.	0.		
12.	0.	0.	0.	0.		
13.	0.	0.	0.	0.		
14.	0.	0'.	0.	0.		
15.	0	0.	0.	0.		
16.	0.	0.	0.	0.		
17.	0.	0.	- 0.	0.		
18.	0.	0.	0.	0.		
19.	0.*	۵.	0.	0.		
20.	0.	5.,	0.	0.		
21.	0.	0.	\ 0.	0.		
22.	0.	0.	\ 0.	0.		
23.	0.	0.	0.	0.		
24.	0.	0.	0.	0.		
25.	0.	0.	0.	0.		
26.	0.	0.	0.	0.		
27.	0	0.	0.	0.		
28.	°0.	0.	0.	0.		
29.	0.	0.	0.	0.		
30.	0.	0.	0.	0.		
1						

200

•

## Case 3 : Thruster Data

Format:

rmat: Sea current values: SV3 increment thruster values: N. Tl T2 T3 T4 (see chapter 4 section 1.3 for thruster layout.).

0.0

0.0			0.0		
1.	0.	0.	-2.	-2.	
2.	0.	0.	-2.	-2.	
3.	0.	0.	-2.	-2.	
4.	0.	0.	-2.	-2.	
5.	0.	0.	-2.	-2.	
6.	2.	2.	6	6	
. 7.	2'.	2.	6	6	
8.	2.	2.	6	6	
9.	2.	2.	6	6	
10.	2.	2.	6	6	
11.	2.	2 .	6	6	
12.	2.	2.	6	6	
13.	2.	2.	6	6	
14.	2.	2.	6	6	
15.	2.	2.	6	6	
16.	2.	2.	6	6	
17.	2.	2.	6	6	
18.	2.	2.	6	6	
19.	2.	2.	6	6	
20.	2.	2.	6	6	
21.	0.	0.	0.	0.	
22.	0.	0.	0.	0.	
23.	0.	0.	. 0.	0.	2
24.	0.	0.	0.	0.	
25.	0.	0.	0.	0.	
26.	0.	0.	0.	0.	
27.	0.	0.	0.	0.	
28.	0.	.0.	0.	0.	
29.	0.	0.	0.	0.	
30.	0.	0.	0.	ο.	

202 、 Case 4 : Thruster Data

0.0

Format:

-1.

sea current values: SV1 SV2 SV3 increment thruster values:

(see chapter 4 section 1.3 for thruster layout )

-2	57E0	0.		0.0	
1.	0.	0.	-2.	-2.	
2.	0.	0.	-2.	-2.	
3.	0.	0.	-2.	-2.	
4.	0.	0.	-2.	-2.	
5.	0.	0.	-2.	-2.	
6.	2.	2.	6	6	
7.	2.	2.	6	6	
8.	2.	2.	6	6	
9.	2.	2.	6	6	
10.	2.	2.	6	6	
11.	2.	2.	6	6	
12.	2.	2.	6	6	
13.	2.	2.	6	6	
14.	2.	2.	6	-16	
15.	2.	2.	6	6	
16.	2.	2.	-: 6	6	
17.	2.	2.	6	6	
18.	2.	2.	6	6	
19.	2.	2.	6	6	1
20.	2.	2.	6	6	
21.	0.	0.	0.	0.	
22.	0.	0.	0.	0.	
23.	0.	0.	0.	0.	
24.	0.	0.	0.	0.	
25.	0.	0.	0.	0.	
26.	0.	0.	0.	0.	
27.	0.	0.	0.	0.	
28.	0.	0	0.	0.	
29.	0.	0.	0.	0.	
30	õ.	0	0.	0.	

1. 1.

Case 5 : Thruster Data

0.0

Format:

sea current values: SV1 SV2 SV3 , increment thruster values: N.T1 T2 T3 T4 (see chapter 4 section 1.3 for thruster layout )

	0	.0		0.0	
1.	2.	2.	-2.	-2.	
2.	2.	2.	-2.	-2.	
3.	2.	2.	-2.	-2.	
4.	2.	2.	-2.	-2.	
5.	2.	2.	-2.	-2.	
6.	2.	2.	-2.	-2.	
7.	2.	2.	-2.	-2.	
8.	2.	2.	-2.	-2.	
. 9.	2.	2.	-2.	-2.	
10.	2.	2.	-2.	-2:	
11.	01	0.	6	6	
12.	0.	0.	-4.	-4.	
13.	0.	0.	6	6	
14.	-2.	-2.	6	6	
15.	-2.	-2.	6	6	
16.	-2.	-2.	6	6	
17.	-2.	-2.	6	6	
18.	-2.	-2.	6	6	
19.	-2.	-2.	6	6	
20.	-2.	-2.	6	6	
21.	-2.	-2:	6	6	
22.	-2.	-2.	6	1.6	
23.	-2.	-2.	6	6	
24.	-2.	-2.	6	6	
25.	-2.	-2.	6	6	
26.	0.	0.	6	6	
27.	0.	0.	6	6	
28.	0.	0.	6	6	
29.	0.	0.	6	6	
30.	0.	ø.	0.	0.	
			-		

,204 Case 6 : Thruster Data

Format: sea current values: SV1 SV2 SV3 increment thruster values: N. T1 T2 T3 T4 (see chapter section 1.3 for thruster layout )

> 1 0.0

	0.	0		0.0	
1.	0.	0.	-4.	-4.	
2.	0.	0.	-4.	-4.	
3.	0.	0.	-4.	-4.	
4.	6.	6.	6	6	
5.	4.	4.	6	6	
6.	4.	4	-4.6	3.4	
7.	4.	4	-4.6	3.4	
8.	4.	4.	6	6	
9.	4.	4.	6	6	
10.	4.	4.	6	6	
11.	4.	4.	6	6	
12.	4.	4.	6	6	
13.	4.	4.	6	6	
14.	4.	.4.	6	6	
15.	4.	4.	6	6	
16.	2.	8	3.4-	4.6	
17.	2.4	2.	3.4-	4.6	
18.	2.	2.	6	6	
19.	2.	2.	6	6	*
20.	2.	2.	6	6	
21.	2.	2.	6	6	
22.	2.	2.	6	6	
23.	2.	2.	6	6	
24.	2.	2.	6	6	
25.	2.	2.	6	6	
26	õ.	0	0	0	
27.	0.	0.	0.	0.	
28.	0.	0	0.	0.	
29.	0.	0.	0.	0.	
30.	0	0	0.	0.	
-1	1	1	1	1.	

Case 7 : Thruster Data

Format: sea current values: SV1 SV2 SUS increment thruster values: N. T1 T2 T3 T4 (see chapter 4 section 1.3 for thruster layout ) 0.0 0.0 0.0 8. -4. -4. 1. 8. 2: 0. 0. -4. -4. 3. 2. 2. -. 6 -. 6 4. 2. -. 6 -. 6 2. 5. 2.2 2. -. 6 -. 6 6. 2. 2. -. 6 .-. 6 7. 2. 2. -. 6. -. 6 8. 2. 2. -. 6 -. 6 9. 2. 2. -. 6 -. 6 10. 2. 2. -. 6 -. 6 11. 2. 2. -. 6 -. 6 12. 2. 2. -. 6 -. 6 2. -2. -2. 13. 2. 14. 2. 2. -2. -2. 15. 2. 2. -. 6 -. 6 16. 2. 2. -. 6 -. 6 2. -. 6 -. 6 17. 2. 2. 2. -2. -2. 18. 2.1 -2. -2. 19. 2. 20. 2. 2. -. 6 -. 6 2.2.04 -.6 -.6 21. 22. 2. 2. -.6 -.6 2.2.04 -.6 -.6 23. 24. 2. 2. -. 6 -. 6 25. 2.2.02 -.6 -.6 26. 2. 2. -. 6 -. 6 27. 2. 2. -. 6 -. 6 28. 2. -2. -2. 2. 0. 0. 0. 29. 0. 30. 0. 0. 0. 0. -1. 1. 1. 1. 1.

## Case 8 : Thruster Data

Format: sea current values: SV1 · SV2 SV3 increment thruster values: N. T1 T2 T3 T4 (see chapter 4"section 1.3 for thruster layout ) 0.0 0.0 1. 8. 8. -4. -4. 2. 0. 0. -4. -4. 3. 2. 2. -.6 -.6 4. 2. 2. -.6 -.6 2. 3:4-4.6 5. 2. 6. 10. 10. 3.4-4.6 7. 10. 10. 3.4-4.6 8. 2. 2. -.6 -.6 9. 2. 2. -.6 -.6 10. 2. 2 ..... 6 -.6 11. 2.1.95 -.6 -.6 2.1.95 -.6 -.6 12. 13. 2. 2. -2. -2. 14. 2. 2. -2. -2. 15. 2. 2. -.6 -.6 16. 2. 2. -.6 -.6 17. 2. 2. -.6 -.6 18. 2.2.05 -2. -2. 19. 2. -2. -2. 2. 20. 2.2.03 -2. -2. 6. 6. -.6 -.6 21. 5. 5. -.6 -.6 22. 2. 2. -.6 -.6 23. 24. 5.5 02 -.6 25. -.6 26. -.6 27. 2. 2. 1. 1. 28. 2. 2. · 0. 0. 0. 29. 0. 0. 0. 30. 0. 0. 0. 0. -1. 1. 1. 1. 1.

Case 9 : Thruster Data

Format: sea current values: SV3 SV1 SV2 increment thruster values: N. T1 T2 T3 T4 (see chapter 4 section 1.3 for thruster layout ) -182E00 182E00 0.0 1. 8. 8. -4. -4. 0. 0. -4. -4. 2. 3. 2. 2. -.6 -.6 2. -.6 -.6 4. 2. 2. 5. 2. 3.4-4.6 6. 10. 10. 3.4-4.6 7. 10. 10. 3.4-4.6 2. 2. -.6 -.6 8. 9. 2. 2. -.6 -.6 10. 2.4 2. -.6 -.6 2.1.95 -.6 -.6 11. 2.1.95 -.6 -.6 12. 2. 2. -2. -2. 13. 14. 2. 2. -2. -2. 15. 2. 2. -.6 -.6 16. 2. 2. -.6 -.6 2. -. 6 -. 6 18. 2.2.05 -2. -2. 19. 2. 2. -2. -2. 2.2.03 -2. -2. 20. .21. 6. 6. -.6 -.6 22. 5. 5. -.6 -.6 23. 2. 2. -.6 -.6 24. 2. 2. -.6 -.6 25. 5.5.02 -.6 -.6 2. -.6 -.6 26. 2. .27. 2. 2. 1. 1. 28. 2. 2. 0. 0. 29. ó. 0. 0. 0. 30. 0. 0. 0. 0. -1. 1. 1. 1. 1.







