MOTION ANALYSIS AND MODEL STUDY OF A GUYED TOWER STRUCTURE IN REGULAR WAVES

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GUYED TOWER STRUCTURE IN REGULAR WAVES

by

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ABSTRACT

The use of fixed platforms of either the gravity or jacket type in water depths exceeding three hundred and fifty meters would so escalate the size of these conventional platforms as to render recovery of oil uneconomical. Alternative platform concepts such as the guyed tower (Finn, 1976) take advantage of the effect of compliance to the wave action. However, such a concept introduces the main problem for deep-water platforms namely the dynamic interaction of waves and structure. Assuming the tower to be of uniform flexural rigidity and uniform weight per unit length, a modified Morison's equation was used to determine the horizontal wave loads on the tower. The equation of motion for the horizontal displacement of the deck was set up and a Crank-Nicholson finite-difference algorithm was employed to solve the equation of motion of the tower. Water particle velocity and acceleration used in the wave loading computation were obtained using linear diffraction theory (MacCamy and Fuchs, 1954). In the development of the computer model the tower was represented as an equivalent beam and the distributed wave load was resolved into concentrated nodal forces. Experimentally determined coefficients for damping, restoring and the mass of the tower were used for solving equation of motion.

In order to compare the predictions of the computer model with the performance of a physical model, a model of the guyed tower was constructed and tested in a wave tank. The tower was supported by eight guy wires each having a model weight per unit length of 5.21 N/m. Deck displacements of the tower were monitored by means of rotary

transducers and the guy line tensions were monitored using ring transducers placed directly in the lines. The damping coefficient of the model was determined experimentally by displacing the model and using the logarithmic decrement obtained from a record of its free oscillation. The restoring coefficient was also determined experimentally by generating a plot of total restoring force versus deck offset of the model tower. Fairly good agreement between the computer model results and the physical model test results was found for the deck displacement.

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NOMENCLATURE

A - projected cross-sectional area

C_D - drag coefficient

C_M - inertia coefficient

D - diameter of cylinder

F - exciting wave force

H - wave height

H_n - Hankel function of first kind of order n

H_0 - Hankel function of first kind of order zero

J_n - Bessel function of first kind of order n

K - total restoring coefficient

L - length of mooring line

M - effective mass

P - load vector on tower

RAO - Response Amplitude Operator

T - total tension in mooring line

T_x - horizontal component of T

T_y - vertical component of T

Y_n - Bessel function of second kind of order n

a - cylinder radius

c - damping coefficient

g - acceleration due to gravity

k - wave number, $2\pi/\lambda$

ℓ - length of cylinder

m - mass

n_1 - slope of mooring line characteristic curve of Tx/wv versus $\frac{u-L}{Y} + 1$

n_2 - slope of mooring line characteristic curve of Ty/wv versus Tx/wv

r - radial coordinate

w - weight per unit length of mooring line

ul - horizontal distance from clump weight to tower position

u_w - horizontal component of water particle velocity

$\dot{u}_w$ - horizontal component of water particle acceleration

v_w - vertical component of water particle velocity

x - displacement vector

$\dot{x}$ - velocity vector

$\ddot{x}$ - acceleration vector

z - vertical coordinate

λ - wave length

ρ - density of water

ϕ - total velocity potential

ϕ_i - velocity potential of incident wave

ϕ_r - velocity potential of reflected wave

1. INTRODUCTION

The large majority of ocean structures currently under study are related to activities in the offshore oil and gas industry. Present systems vary from the concrete bottom founded type in water depths of up to one hundred fifty meters to the dynamically positioned semisubmersible operating at depths up to three hundred meters. In the ongoing development of the ocean's resources it is the responsibility of engineers to provide the industrial sector with safe and economical methods of design and analysis. Presently, the main technique for design and behavioural prediction is numerical modelling. Testing and verification of numerical models used to predict the behaviour of offshore structures is of increasing interest in the development of new structural concepts since there exists a number of basic hydrodynamic effects that lack adequate theoretical description. Traditionally these have been discarded in hydrodynamic experimental testing because their importance has been considered to be negligible, however the development of new deep water concepts demand a more sophisticated technology.

One deepwater concept for a water depth range of six hundred to eight hundred meters is the guyed tower (Finn, 1976). The guyed tower is a tall, relatively thin truss-framed compliant structure. The foundation of the guyed tower is usually of the spud can or pile configuration. The tower is held upright by a system of guylines, the ends of which are attached to clumped weights resting on the ocean floor. These weights are connected to anchors embedded in the floor or to piles by means of a trailing line. Thus, when the tower displacement exceeds a particular limit, this clumped weight will lift off. An advantage of the guyed tower for deep water drilling as compared to the jacket type structure is the economy in its structural cost due to a considerable reduction in steel.

In the late 1960's Exxon Production Research Company began to consider the guyed tower as a production platform and as a result in 1975 a 1/5th scale model was erected in the Gulf of Mexico. Finn (1976) has presented the analytical results of wave loading on the tower, wherein the wave particle kinematics are computed using Airy's wave theory. However, to date it appears that there have not been any results concerning wave tank studies of such a structure published in the open literature. The following thesis compares an analytical and physical model of a guyed tower structure.

2. THEORETICAL DEVELOPMENT

Consider irrotational, inviscid, and incompressible two-dimensional motion of water waves over a stable horizontal bottom, which can be described by the Laplace differential equation

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{1}$$

where, ϕ is the velocity potential

and, $\frac{\partial\phi}{\partial x} = u_w$ and $\frac{\partial\phi}{\partial z} = v_w$ (2)

are the horizontal and vertical velocity components of the water particles respectively.

Equation 1 is subjected to the following boundary conditions:

1) an impermeable bottom boundary condition described for a horizontal sea bed as:

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = -h$$

where h = water depth

2) assuming that the incident wave height to length ratio is sufficiently small, all non-linear effects due to wave steepness may be neglected without significant error. Hence, the linear free surface boundary condition can be expressed as

$$\eta = \frac{1}{g}\frac{\partial \phi}{\partial t}$$

where η is the free surface elevation at time t above still water line.

Under these conditions the velocity potential of the incident wave ϕ_i may be written as

$$\phi_i = \frac{gH}{2\omega}\,\frac{\cosh k(h + z)}{\cosh kh}\,e^{i(kx - \omega t)} \qquad (3)$$

and transformed to polar coordinates r and θ as,

$$\phi_i = \frac{gH}{2\omega}\,\frac{\cosh k(h + z)}{\cosh kh}\,[\cos(kr \cos \theta) + i \sin(kr \cos \theta)]e^{-i\omega t} \qquad (4)$$

Equation 4 is obtained under the assumption that the particle kinematics is not affected by the presence of an object in a wave field. However, if the effect of the presence of an object on the particle kinematics is to be considered, the linear diffraction theory can be used. Although Airy's wave theory could be used for smaller $2a/\lambda$ ratios for design purposes, the use of linear diffraction theory for estimating the hydrodynamic fluid loading on individual vertical cylinders could be a more complete analysis than Airy's wave theory. The diffraction theory requires that the total potential ϕ can be expressed as the summation of incident wave ϕ_i and the reflected wave potential ϕ_r.

Assuming that the reflected waves move outward with respect to the cylinder, the velocity potential for the reflected wave, ϕ_r, can be written as,

$$\phi_r = \sum_{n=0}^{\infty} A_n \cos n\theta [J_n(kr) + iY_n(kr)] e^{-i\omega t} \tag{5}$$

$$= \sum_{n=0}^{\infty} A_n \cos n\theta [H_n(kr) e^{-i\omega t}] \tag{5a}$$

where J_n, Y_n are the Bessel functions of the first and second kind respectively, and H_n is the Hankel function of first kind.

It is implied here that the reflected wave potential satisfies the radiation boundary condition as expressed by

$$\lim_{r \to \infty} \sqrt{r} \left[\frac{\partial \phi_r}{\partial r} + ik\phi_r\right] = 0$$

The total velocity potential satisfying eqn. (1) and the boundary conditions can be written after MacCamy and Fuchs (1954) as,

$$\phi = -\frac{gH}{2\omega} \frac{\cosh k(h+z)}{\cosh kh} [J_o(kr) + 2 \sum_{n=1}^{\infty} i^n J_n(kr) \cos n\theta] e^{-i\omega t}$$

$$+ \sum_{n=0}^{\infty} A_n \cos n\theta [H_n(kr) e^{-i\omega t}] \tag{6}$$

It should be mentioned here that although MacCamy and Fuchs have employed Hankel function of the second kind in their work for describing the reflected wave, Spring (1973) has pointed out that Hankel function of the first kind only describes the outgoing wave whereas the Hankel function of the second kind describes the incoming wave. Since, the scattered wave should be outgoing rather than incoming, the Hankel function of first kind is used here.

The coefficient A_n is determined by setting the particle velocity normal to the cylinder, $\frac{\partial\phi}{\partial r}$, equal to zero at r = a, where a is the cylinder radius.

$$A_o = \frac{gH}{2\omega}\frac{\cosh k(h+z)}{\cosh kh}\frac{J_o'(ka)}{H_o\ (ka)} \tag{7}$$

$$A_n = \frac{gH}{2\sigma}\frac{\cosh k(h+z)}{\cosh kh} i^n \frac{J_n'(ka)}{H_n\ (ka)} \tag{7a}$$

Substituting (7) into (6), the total velocity potential is

$$\phi = -\frac{gH}{2\sigma} e^{-i\sigma t}\frac{\cosh k(h+z)}{\cosh kh}\{J_o(kr) - \frac{J_o'(ka)}{H_o'(ka)} H_o(kr)$$

$$+ 2\sum_{n=1}^{\infty} i^n [J_n(kr) - \frac{J_n'(ka)}{H_n'(ka)} H_n(kr)] \cos n\theta\} \tag{8}$$

where n = 1, 2, 3..., and $J_n'(ka)$ and $H_n'(ka)$ denote the derivatives at r = a of $J_n(kr)$ and $H_n(kr)$ respectively. The particle velocity in the horizontal direction is given by $u_w = \cos\theta\frac{\partial\phi}{\partial r} - \frac{\sin\phi}{r}\frac{\partial\phi}{\partial\theta}$ (9)

Using the above described method, the particle kinematics on each of the vertical members in the model were determined.

2.1 Equation of Motion of Guyed Tower

The equation of motion of a compliant offshore structure can be defined as,

$$M\ddot{x} + C\dot{x} + Kx = F \tag{10}$$

Where M is the effective mass, C is the equivalent viscous damping coefficient, K is the restoring coefficient, F is the exciting force and x, $\dot{x}$, $\ddot{x}$ are the displacement, velocity and acceleration vectors respectively.

Horizontal wave forces per unit length of a fixed structure, can be calculated using Morison's equation (Morison et al., 1950),

$$\Delta F = \frac{1}{2} C_D \rho D u_w |u_w| + C_m \rho \frac{\pi}{4} D^2 \dot{u} \tag{11}$$

All terms are defined in the nomenclature.

Equation 11 gives an estimate of the fluid loading on a structure with its bottom rigidly fixed. For this case the displacement of the cylinder is zero. Hence to calculate the fluid loading on a structure which has its own displacement, as in the case of a guyed tower, the relative motion between the water particle and the structure must be considered.

The relative horizontal water particle velocity u and relative acceleration $\dot{u}$ are defined as,

$$u = u_w - \dot{x} \tag{12a}$$

$$\dot{u} = \dot{u}_w - \ddot{x} \tag{12b}$$

where u_w and $\dot{u}_w$ are the horizontal component of the water particle velocity and acceleration respectively. Fish et. al. (1980) and Sunder and Connor (1981) have defined the hydrodynamic loading per unit length of the vertical cylinder in a compliant structure as,

$$\Delta F = C_D \frac{\rho}{2} D(u_w - \dot{x})|u_w - \dot{x}|$$

$$+ (C_m - 1) \rho \frac{\pi}{4} D^2 (\dot{u}_w - \ddot{x}) + \frac{\pi}{4} D^2 \rho \dot{u}_w \tag{13}$$

Equation 13 consists essentially of three terms. As in the case of the fixed vertical cylinder the drag and inertia forces are considered and an additional term known as the Froude-Krylov force introduced. This particular term is related to the undisturbed pressure field around the structure. For the present analysis particle velocities and accelerations are obtained from the linear diffraction theory as explained earlier.

After substitution of equation 13 into equation 10, it can be written as,

$$\ddot{x}(M+a_m) + \dot{x}(C+A_c) - \alpha_3 \dot{x}^2 + Kx = P \tag{14}$$

where $$P = C_D \rho \frac{D}{2} \ell u_w^2 + C_m \frac{\pi}{4} D^2 \rho \ell \dot{u}_w \tag{15}$$

$$a_m = \frac{\pi}{4} D^2 \rho \ell (C_m - 1) \tag{16}$$

$$A_c = C_D \rho \ell D u_w \tag{17}$$

$$\alpha_3 = C_D \, \rho \, \ell \, \frac{D}{2} \tag{18}$$

The values of effective mass, effective damping and stiffness for the tower were determined experimentally as described in section 5 and were used in the solution of the equation of motion of the tower. The coefficient α_3 stems from the relative velocity term in the equation of motion. It accounts for the drag induced by the motion of the structure in the waves.

The wave load on the structure is determined by using an equivalent vertical beam. In the case of long waves, spatial effects can be neglected and a single equivalent beam was used. However, for short wave lengths spatial variations can be treated using a number of vertical beams. The flexural properties of the tower can accurately be represented by an equivalent beam as shown in Figure 4. A single translational degree of freedom is used at each nodal point. Vertical displacements are relatively small and can be neglected. Each node has an attached lateral spring k_i. Concentrated forces at the nodes are determined by calculating the distributed forces at the mid-point, top and bottom of each equivalent element of length ℓ.

The tower was modelled as a multi-degree-of-freedom system because the response of the tower such as the deck offset can be adequately described by the first mode.

2.2 Method of Solution

Using the central point Crank-Nicholson finite difference method, the equation of motion (14) can be written as,

$$M \frac{(x_{i+1} - 2x_i + x_{i-1})}{\Delta t^2} + \frac{(x_{i+1} - x_{i-1})}{2\Delta t} (C+A_c) + \alpha_3$$

$$\frac{(x_{i+1}^2 + x_{i-1}^2 - 2\, x_{i+1}\, x_{i-1})}{4\Delta t^2} + K \frac{(x_{i+1} + x_{i-1})}{2} = \frac{P_{i+1} + P_{i-1}}{2} \quad (19)$$

The equation of motion of the tower as represented by equation 14 is nonlinear. Hence, it was solved numerically using the Crank-Nicolson finite difference algorithm. This finite difference scheme is universally stable and has the advantages of smaller truncation error than the standard implicit or explicit finite difference scheme. Appendix 2 explains the flow diagram of the solution procedure for the solution of equation 19.

3. PROTOTYPE SCALING

The essential requirements of any model are that it provides an adequate representation of the design environment for a particular structure, the loading on the structure and the structure itself. Similitude between prototype and model when the behavior is dominated by the action of the waves and the inertia of the body is achieved using Froude scaling.

In order to scale from model to prototype, the laws of dynamic, geometric and kinematic similitude must be satisfied. Dynamic similarity is achieved by holding the ratio of the gravity force (assumed dominant for free surface flow) to inertia force constant. This results in a relationship between the model and prototype known as the Froude Number defined as,

$$\sqrt{\frac{V_m^2}{g_m L_m}} = \sqrt{\frac{V_p^2}{g_p L_p}} \tag{20}$$

where V is velocity, L is length, g is acceleration due to gravity and the subscripts m and p denote model and prototype respectively. Geometric similarity is achieved holding the ratio of model length to prototype length constant as follows

$$\frac{L_p}{L_m} = n \tag{21}$$

Kinematic Similarity is achieved by holding the ratio of model velocity to prototype velocity constant. From the Froude relationship above

$$\left(\frac{V_p}{V_m}\right)^2 = \frac{L_p}{L_m} = n \tag{22}$$

From these relationships, the following scales are determined:

Length scale $L_p = n\, L_m$ (23)

Velocity scale $V_p = \sqrt{n}\, V_m$ (24)

Time scale $T_p = \sqrt{n}\, T_m$ (25)

Force scale $F_p = n^3\, F_m$ (26)

The choice of model scale depends mainly on the wave tank dimensions. In any case, the scale factor must allow accurate adjustment of such quantities as pretension in mooring lines, wave heights and wave periods and assume that model force and motion levels can be accurately measured and recorded.

4. GUYLINE ANALYSIS

The catenary stiffnesses of moving systems of articulated towers supported by guylines from the sea bed are of critical importance in the design of compliant structures such as the guyed tower since they greatly effect the response of the structure. Rothwell (1979) has presented a simple graphical approach to computing the stiffness of a catenary of a pretensioned or taut mooring line. In this analysis the slopes of a number of nondimensional curves are used to determine the stiffnesses of the free end mooring lines.

Define L as the length of cable from the clump weight to the end attached to the tower,

$$\sin \Psi = \frac{1}{2} \frac{wv}{T} \left(\frac{L}{v} - \frac{v}{L} \right) + \frac{v}{L} \tag{27}$$

where w is the weight per unit length of the mooring line, v is the vertical distance of the point in the cable to be analyzed (in this case the point at the tower) and T is the tension in the cable. The angle Ψ indicates the angle between the tangent at the specified point and the horizontal. The angle made by the catenary at the sea bed, Ψ_o, is defined as,

$$\Psi_o = \cos^{-1} \left[\frac{\cos \Psi}{1 - (wv/T)} \right] \tag{28}$$

The horizontal and vertical components of the mooring line tension T_x and T_y respectively are defined as,

$$T_x = T \cos \Psi \tag{29}$$

$$T_y = T \sin \Psi \tag{30}$$

The horizontal distance, ul, from the clump weight to the point of analysis in the line is defined as

$$ul = \frac{T_x}{w} \ln \left(\frac{\sec \psi + \tan \psi}{\sec \psi_0 + \tan \psi_0} \right) \tag{31}$$

The corresponding stiffnesses may be determined as follows,

$$\frac{\partial T_x}{\partial ul} = \frac{w}{n_2} \tag{32}$$

$$\frac{\partial T_y}{\partial ul} = \frac{n_1}{n_2} w \tag{33}$$

$$\frac{\partial T_x}{\partial v} = w \left(\frac{T_x}{wv} \right) - \frac{w}{n_2} \left(\frac{u - L}{v} \right) \tag{34}$$

$$\frac{\partial T_y}{\partial v} = w \left(\frac{T_y}{wv} \right) + n_1 \frac{\partial T_x}{\partial v} - w \frac{T_x}{wv} \tag{35}$$

The values n_1 and n_2 are the slopes of the curves T_x/wv versus T_y/wv and Tx/wv versus $(ul-L/v) + 1$. Clearly it can be seen that the cable stiffnesses are directly related to the weight per unit length.

5. EXPERIMENTAL PROCEDURE

A model of a guyed tower structure shown in Fig. 5, was constructed of polyvinyl chloride tubing. The tower is of no particular design but a Froude scale of 1/60 would represent a prototype structure with a height of one hundred and twenty meters, which is supported by eight guy wires in ninety meters of water. The physical properties of the tower and mooring system are given in Table 1. The value of w (weight per length of cable) was simulated by having lead weights in the lines which were made of 2 mm braided wire. Clump weights were constructed of lead blocks resting on the tank floor. The rotational stiffness at the foundation was not modelled as the tower was mounted on a ball bearing assumed to have negligible friction.

All tests were conducted in the wave tank shown in Figs. 1 and 3 at a still water depth of 1.50 m. The facility construction and calibration is described in Muggeridge and Murray (1981). Fig. 2 shows the calibration results for both regular and irregular waves compared to those presented by Gilbert et. al., (1971). The tank structure is of reinforced concrete 61 m long, 4.5 m wide and 3 m deep. Waves are generated by means of a piston type analog controlled generator, thus generating regular and irregular water waves. Wave profiles are measured using resistance type probes. These probes consist of two parallel stainless steel wires 1.5 mm in diameter, 73.4 cm long and 1.25 cm apart. Voltage across the probe wires varies proportional to the depth of immersion. Model motions were measured using a system of rotary potentiometers. The associated circuits, as shown in Fig. 6 provide an accurate measurement of the horizontal displacement of the model.

Variations in the tensions of the guy lines were monitored using ring transducers placed directly in the lines. Signals from each of the transducers were recorded on an eight channel analogue tape recorder while the deck offset and wave profile were recorded on a strip chart recorder as illustrated in Fig. 6.

The damping and restoring coefficients used in the equation of motion of the tower were determined experimentally. Fig. 7 shows the

total restoring force for a range of deck displacements. This curve was generated by applying deadweight loads at the center of the tower deck and recording the resulting offset. The restoring coefficient used was determined from the portion of the graph before the clump weight lifts off. The damping coefficient was determined experimentally by using the logarithmic decrement of a free oscillation of the tower, resulting from offsetting the tower until the first clumped weight lifted off and releasing the tower. The average of three such oscillations was used. Subsequently the total mass, damping and restoring coefficients used in the computer model were 19.7N, 10.0 N/m/s and 73.0 N/m respectively.

Regular waves were generated for periods ranging from 0.67 sec to 5.00 sec. Table 2 shows the wave period and corresponding heights for all runs made during the model tests.

6. RESULTS AND DISCUSSION

Fig. 9 shows the deck offset for wave heights at wave periods of 0.60 sec, 1.00 sec 1.25 sec. Good agreement was found for periods less than the natural period of the structure. Discrepancies between the computed and experimental values increase with increased value of wave period. This trend is also demonstrated in Fig. 10 which illustrates the Response Amplitude Operator (RAO) of the model. Once again, the results show fairly good agreement for periods up to the natural period of the structure, however above this (1.61 sec) value discrepancies increase with increasing period.

This discrepancies can be partially due to a higher value of added mass coefficient than the assumed value in the higher range of Keulegan-Carpenter numbers. In addition to this, the model mount although assumed to have negligible stiffness and damping characteristics, did in fact affect the tower response. Fig. 8 shows the measured tensions in two selected cables. The mooring line characteristics of a typical guyline are in Fig. 11. These curves were generated by use of the computer program "MOORING LINE ANALYSIS" found in Appendix 1. Characteristics shown in these curves are those pertaining to the section of mooring line between the clump weight and the tower itself for tension conditions from pretension 14.31 N to 24.05 N the tension corresponding to the lifting of the first clump weight. Observation of Fig. 8 and Fig. 11 shows some disagreement in the response of the mooring line.

This disagreement in results is due to the magnitude of the pretension in the mooring lines and the distribution of the weights within the line itself. The non-linear characteristic of a mooring line illustrated in Fig. 11 is due to the catenary of the line, however as the catenary approaches a straight line from the clumped weight to the point of connection on the tower the mooring system approaches a linear one. Under these conditions any change in the deck offset will result in a constant change in the line tension until the clumped weight lifts off. This behaviour is also illustrated in Fig. 7 where the total restoring coefficient is linear until the weight lifts off. Therefore the mooring line was not properly modelled for any particular prototype since the weights were not placed in the line to simulate the correct catenary shape.

As may be inferred from the scaling factors in section 3, the weight per unit length of 5.21 N/m of the model cable, when scaled to prototype conditions would not be realistic. However these conditions may be qualified somewhat by the fact that the primary objective is to compare two methods of analysis of a structure response i.e. a physical model with an analytical model. Also the mooring line stiffness and subsequently the resonant frequency conditions of the tower can be manipulated by the pretension and weight per unit length of the mooring line. It was of particular interest that the resonant conditions fall within the frequency range limitations of the wave generator.

Traditionally these stiffness characteristics have been simulated using a system of springs mounted in air and attached to the model at the proper point to simulate conditions realized by the structure as a mooring system. This method may pose questions as to the accuracy of the simulation of effects on the structure due to the hydrodynamic loading on the cables themselves.

7. SUMMARY AND CONCLUSIONS

A computer model for the analysis of a guyed tower has been developed using a modified form of Morison's equation for the calculation of fluid loading and resulting motion response of the structure. The equation of motion for the deck offset was set up by representing the tower as an equivalent beam and a Crank-Nicholson finite-difference alogrithm was employed to solve the equation. This particular method has shown to be an adequate means of solution where there are non-linear effects such as those introduced by the relative motion of the structure, since it does not impose restrictions on the time step.

A physical model of a guyed tower has been constructed, instrumentated and tested for motion response in a range of regular wave frequencies. The model was supported by eight guylines the stiffness characteristics of which have also been investigated and presented. The results of these tests have been compared to the computer models and good agreement was found.

Considering the errors that may have resulted from scaling effects, it can be concluded that the results obtained from the model tests could be used in the design and analysis of the guyed tower structure. Apart from a direct comparison with prototype information itself, which has obvious economic restrictions, the scaled model test is a suitable means of obtaining quantitative results concerning the response of the guyed tower provided the statistics of all anticipated extremes are applied.

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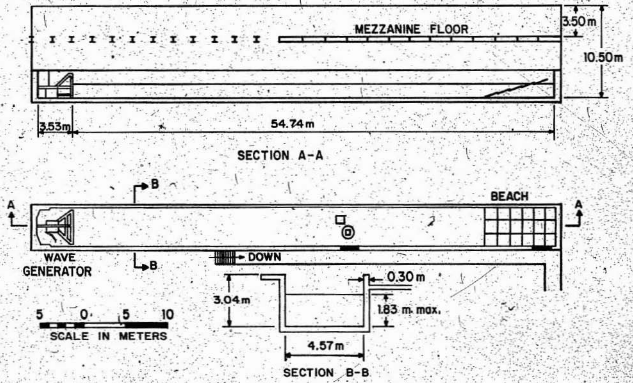


Fig. 1. Elevation and Plan Views of Wave Flume

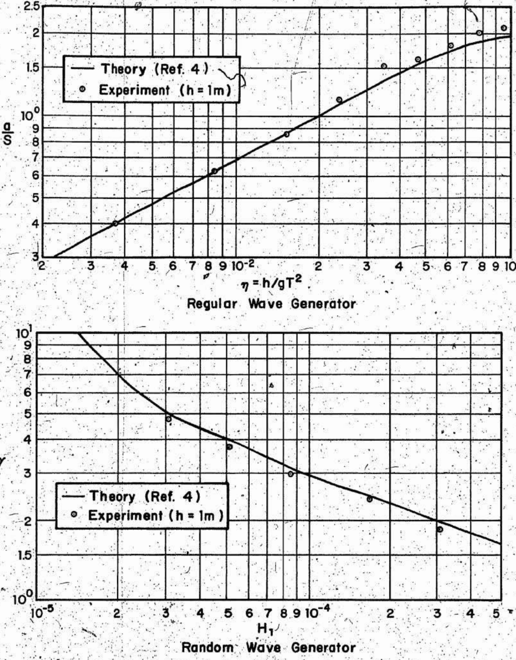


FIG. 2. PISTON REGULAR AND RANDOM WAVE GENERATOR PERFORMANCE

FIG. 3. REGULAR WAVE IN WAVE TANK

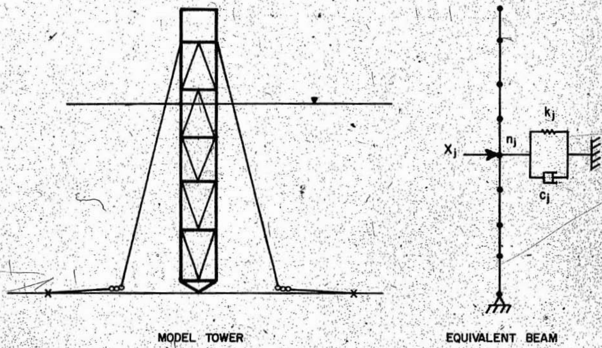


FIG. 4. MODEL TOWER AND EQUIVALENT BEAM REPRESENTATION

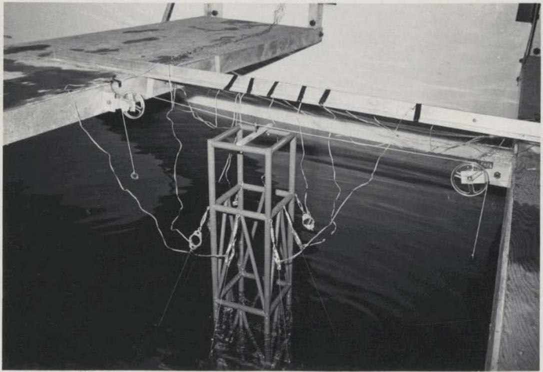


FIG. 5. GUYED TOWER IN REGULAR WAVES

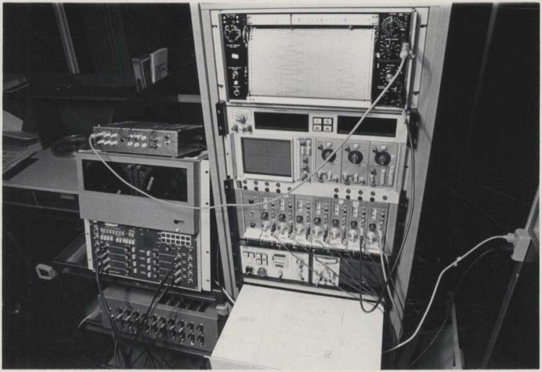


FIG. 6. DATA ACQUISITION EQUIPMENT

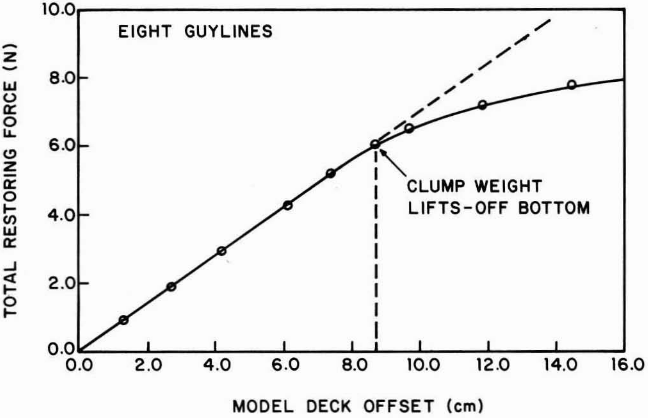


FIG. 7. TOTAL RESTORING FORCE OF MODEL TOWER

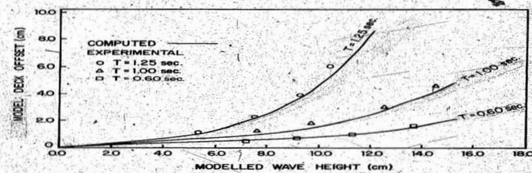


FIG. 9. DECK OFFSET vs. WAVE HEIGHT FOR REGULAR WAVES

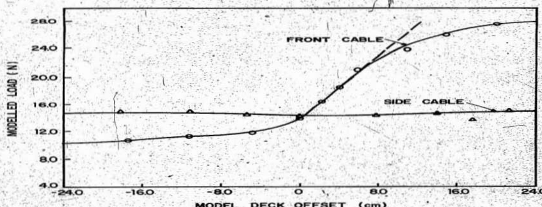


FIG. 8. GUYLINE TENSION vs. DECK OFFSET

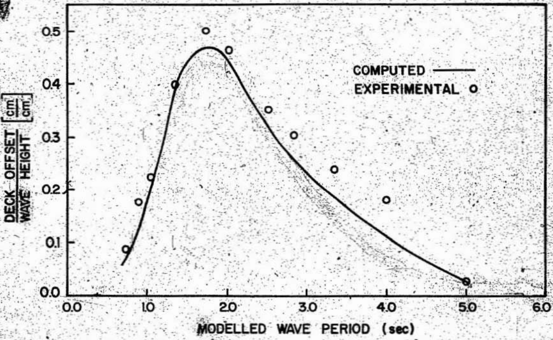


FIG. 10. RAO vs. WAVE PERIOD FOR REGULAR WAVES

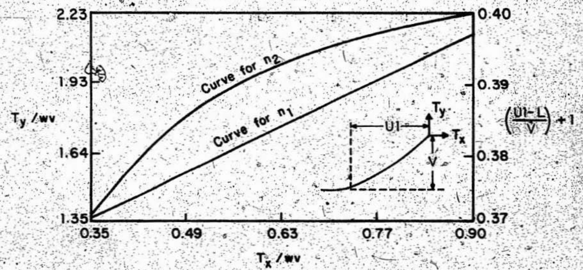


FIG. 11 STIFFNESS CHARACTERISTIC CURVES FOR A TYPICAL CABLE.

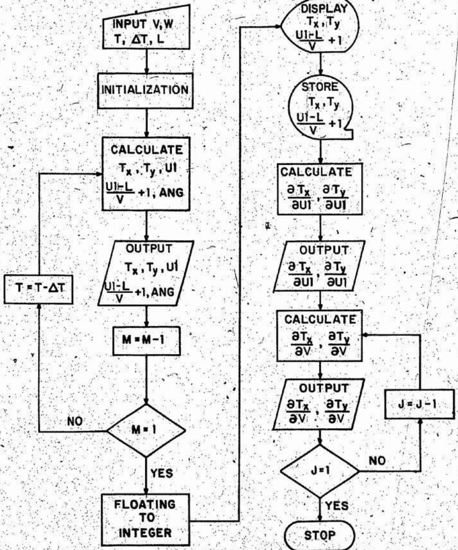
TABLE 1 - PHYSICAL PROPERTIES OF MODEL TOWER	
Height	2.13 meters
Width	10 cm
Total Weight	19.7 N
Damping Cofficient (at natural oscillation)	10.0 N/m/s
Total Restoring Coefficient	73.0 N/m
Guylines	
total number	8
weight per unit length	5.21 N/m
pretension	14.31 N

TABLE 2 - MODEL WAVE CONDITIONS AND DECK OFFSET

Wave Period (Sec)	Wave Height (cm)	Deck Offset (cm)	Wave Period (sec)	Wave Height (cm)	Deck Offset (cm)
0.67	11.42	0.91	2.00	7.52	1.21
	9.19	0.76		9.51	1.83
	7.54	0.55		12.62	2.91
	4.98	0.40		14.41	4.39
0.71	14.56	1.19	2.50	5.60	1.11
	12.30	0.98		7.48	2.32
	10.34	0.79		9.51	4.00
	7.11	0.55		10.30	7.03
0.83	12.52	2.25	2.80	31.21	17.12
	9.71	1.91		19.56	10.73
	6.96	1.25		9.19	4.89
	3.17	0.63		6.33	3.48
1.00	15.31	10.10	3.33	13.37	3.05
	11.98	6.71		8.64	2.07
	7.52	2.74		6.41	0.76
	4.46	0.82		4.22	0.59
1.25	84.17	29.63	4.00	20.56	6.23
	50.62	19.74		15.53	3.44
	22.35	9.81		10.31	1.83
	15.67	7.62		5.17	0.96
1.67	12.52	5.11	5.00	44.00	25.30
	8.00	2.50		35.11	17.16
	5.31	0.79		29.53	11.93
	3.96	0.66		13.49	1.22

APPENDIX 1

MOORING LINE ANALYSIS PROGRAM



FLOWCHART FOR MOORING LINE ANALYSIS

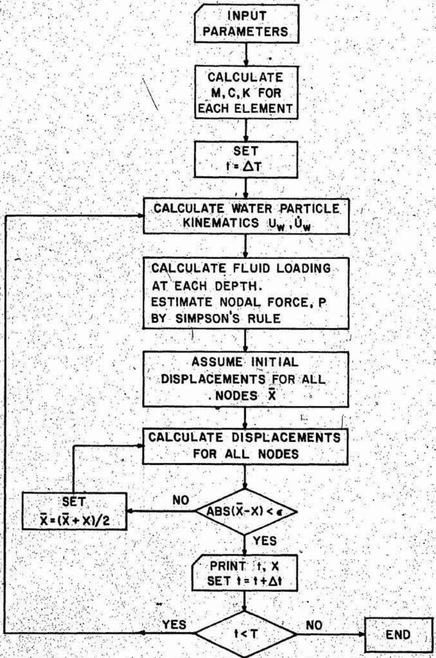
```
150 PRINT " LENGTH OF MOORING LINE ... ";
160 INPUT L
170 PRINT " VERTICAL DISTANCE FROM OCEAN FLOOR ... ";
180 INPUT V
190 PRINT "DECREMENT OF TENSION;
200 INPUT D
202 LET I=0
210 PRINT "  A          B          C          T          ANG    U"
220 REM CALCULATE AND OUTPUT TX/WV,TY/WV,U,((U-L)/V)+1,ANGEL,T,AND U.
250 FOR M=63 TO 1 STEP -1
260 LET S1=((.5*W1*V)/T)*((L/V)-(V/L))+(V/L)
265 LET C1=SQR(1-(S1^2))
270 LET C2=C1/(1-(W1*V/T))
272 LET S2=SQR(1-(C2^2))
275 LET A[M]=(T/(W1*V))*C1
280 LET B[M]=(T/(W1*V))*S1
285 LET F=(1/C1+S1/C1)/(1/C2+S2/C2)
287 LET U=(A[M]*V)*LOG(F)
290 LET C[M]=((U-L)/V)+1
295 LET R=ATN(S1/C1)*(180/3.1415)
300 PRINT A[M];B[M];C[M];T;R;U
310 LET H[M]=T
330 LET D[M]=U
355 LET T=T-D
357 NEXT M
358 REM CHANGE ARAYS A,B ,AND C FROM FLOTING POINT TO INTEGER FORMAT.
360 PRINT "--------------------------------------------------------"
361 FOR K=63 TO 1 STEP -1
362 LET X=A[K]
363 LET Y=B[K]
364 LET Z=C[K]
365 CALL DPUT(E[1],Q[1],K,X,I)
366 CALL DPUT(F[1],Q[1],K,Y,I)
367 CALL DPUT(G[1],Q[1],K,Z,I)
368 NEXT K
```

```
1  CALL SCORE(7)
2  REM THIS PROGRAM WILL OPERATE ON A 5451B FOURIER ANALYZER
3  REM WITH A FOURIER BASIC CORELOAD, OPTION 720
4  REM PROGRAM NAME-"M00RING LINE ANALYSIS".
5  REM THE RESULTING ARAYS OF DATA ARE STORED AS FOURIER DATA BLOCKS
6  REM IN THE TIME DOMAIN
7  REM TX/W,TY/W,AND ((U-L)/V)+1 ARE LOCATED ON DISK IN FILE 1
8  REM RECORDS 0,1,AND 2 RESPECTIVELY.
9  DIM D[64],H[64]
10 DIM A[64],B[64],C[64]
11 REM A IS TX/W, B IS TY/W, AND C IS ((U-L)/V)+1
12 DIM E[64],F[64],G[64]
13 REM   DATA BLOCK QUALIFIERS FOR FOURIER DATA BLOCK.
50 LET Q[1]=64
60 LET Q[2]=0
70 LET Q[3]=32767
80 LET Q[4]=13
90 LET Q[5]=0
91 CALL FIXO(Q[1])
92 FOR N=1 TO 64
93 LET A[N]=0
94 LET B[N]=0
95 LET C[N]=0
96 LET D[N]=0
97 LET H[N]=0
98 NEXT N
99 REM INPUT GUYED TOWER PARAMETERS
100 PRINT "MOORING LINE DESIGN PROGRAM"
101 PRINT "---------------------------"
102 PRINT
110 PRINT " WEIGHT PER UNIT LENGTH ... ";
120 INPUT W1
130 PRINT "UPPER LIMIT OF TENSION";
140 INPUT T
```

```
360 PRINT "THIS IS A"
370 CALL DSPLY(E[1],Q[1],0,63,0)
375 REM THIS SECTION DESPLAYS ARAYS A,B ,AND C ON THE SYSTEM DESPLAY.
380 PAUSE
390 CALL NODIS
395 PRINT
400 PRINT "THIS IS B"
410 CALL DSPLY(F[1],Q[1],0,63,0)
420 PAUSE
430 CALL NODIS
440 PRINT
441 PRINT "THIS IS C"
450 CALL DSPLY(G[1],Q[1],0,63,0)
460 PAUSE
470 CALL NODIS
475 REM WRITE THE ABOVE CALCULATED ARAYS OF DATA ON DISK.
480 CALL DWRIT(E[1],Q[1],0,64,1)
490 CALL DWRIT(F[1],Q[1],1,64,1)
500 CALL DWRIT(G[1],Q[1],2,64,1)
520 LET N1=(B[63]-B[10])/(A[63]-A[10])
530 LET N2=(C[63]-C[10])/(A[63]-A[10])
532 REM CALCULATE STIFFNESSES.
535 LET S3=W1/N2
540 LET S4=(N1/N2)*W1
542 PRINT
545 PRINT "DTX/DU =";S3,"DTY/DU =";S4
550 PRINT "------------------------------------------------"
551 PRINT "  DTX/DV","  DTY/DV","  T"
560 FOR J=63 TO 1 STEP -1
570 LET S5=(W1*A[J])-((W1/N2)*(D[J]-L)/V)
580 LET S6=(W1*B[J])+(N1*(S5-(W1*A[J])))
590 PRINT S5,S6,H[J]
600 NEXT J
990 END
```

APPENDIX 2

EQUIVALENT BEAM ANALYSIS PROGRAM



FLOWCHART FOR EQUIVALENT BEAM ANALYSIS

```
C     NM NUMBER OF PRISMATIC BEAM ELEMENTS.
C     NP NUMBER OF DEGREES OF FREEDOM.
C         C = DAMPING COEFFICIENT OF ELEMENTS
C         S = DAMPING COEFFICIENT OF ELEMENTS.
C         SMASS = MASS OF ELEMENTS.
C         EL = WAVE LENGTH.
C         SWL = STILL WATER DEPTH.
C         PER = WAVE PERIOD.
C         HT = WAVE HEIGHT
C         CD = DRAG COEFFICIENT
C         CM = MASS COEFFICIENT
C         DIA = DIAMETER OF VERTICAL MEMBERS.
C         VZ = PARTICLE VELOCITY IN VERTICAL DIRECTION
C         UZ = PARTICLE VELOCITY IN HORIZONTAL DIRECTION.
C         UDZ = PARTICLE ACCELERATION IN HORIZONTAL DIRECTION
C         VDZ = PARTICLE ACCELERATION IN HOVERTICAL DIRECTION.
C         CEL = WAVE VELOCITY.
C         DISP = DISPLACEMENT OF ELEMENT.
C         TOWHT = HEIGHT OF TOWER.
C         URMS = RMS VELOCITY OF UZ.
C            MASS OF THE TOWER, DAMPING COEFFICIENT OF THE TOWER  AND
C            THE STIFFNESS COEFFICIENT OF THE TOWER WERE OBTAINED
C            EXPERIMENTALLY AND WERE USED SUBSEQUENTLY FOR THE BEAM
C            ANALYSIS OF THE TOWER.
      COMMON / FORNOD / FXNOD(9),AMASS(9),S(9),C(9)
      COMPLEX CTME,CI,CPR,SUMB,HOTD,HNTD,ETAD,GBE
      COMMON / EXTC1 / DT,SIZEPR,STEP,DES,TRI
      COMMON / DISPLA / DISP(9),DISPN(9),DISPNP(9),DISPNM(9)
      COMMON/ CMPL1 / CTME,CI,CPR,SUMB,HOTD,HNTD,ETAD
      COMMON/ CHAR1 /T1,T2,T3,T4,T5,T6,T7,T8,T9,T1T,Y11,T12
      COMMON/ CHAR2 /PER,SWL,HT,AMP,EL,EX,WNO,WFR,PI,TIME,ETA,G,LO,
     1               SIGM,PHBL,CEL,PHBLCL
      COMMON/ CHAR3 /SHT1,CHT1,SHT2,CHT2,SHT3,CHT3,ST4,CT4,SH2T2,
     1               CH2T2,THT2,SH4PT2,C2T4,S2T4,SH3PT2,CH2T3,SH2T3
      COMMON/ FACTOR /RHO,ANO,CD,CM,DIA,FACT1,FACT2,AM2FR2,RHOG,
     1               URMS,RENO
      COMMON/ FORCE1 /UFSUM,UMSUM,AFSUM,AMSUM,EZ2
      COMMON/ A1 /FORCE(100),MOMENT(100),VZ(50),VDZ(50),UZ(50),UDZ(50)
      COMMON/ A2 /ELEV(100),PRES(50)
      COMMON / INCON1 / NZ,NZME,NZM,KK,NTH,NP
      COMMON / FORCE / PX,PY,QX,QY,DELX,DELY,GBO,FBO,BOJ2,BOY2,ATAL
      COMMON / CHAR4 / AMPRES (20),FBE(6),GBE(6),FX(50),FY(50),FXZ(50)
      COMMON/ BESS1 / BJJ,BYY,B2J,B2Y,B1J,B1Y,GZ,FZ,FXX,FYY
```

```
      DATA IR,JW / 5,6 /
1002  FORMAT (///' WATER DEPTH = ',G10.3/
     1            ' WAVE PERIOD = ',G10.3/
     2            ' WAVE LENGTH = ',G10.3/
     3            ' WAVE HEIGHT = ',G10.3/
     4            ' WAVE NUMBER = ',G10.3/
     5            ' WAVE FRQNCY = ',G10.3/
     6            ' CYLI SIZEPR = ',G10.3/
     7            ' WAVE STEEPN = ',G10.3//)
1003  FORMAT (' SHALLOW WATER WAVE "SWL/EL" = ',G12.4)
1004  FORMAT (' DEEP WATER WAVE "SWL/EL" = ', G12.4)
1005  FORMAT (' TRANSITIONAL WAVE "SWL/EL" = ', G12.4)
1006  FORMAT (//(6X,12G10.3))
1007  FORMAT (//(3X,10G10.3))
2001  FORMAT (8E10.3)
2002  FORMAT (16I5)
3000  FORMAT (4G14.3)
      READ (5,5) NP,NM,NPS
    5 FORMAT (3I5)
      READ (5,6) (S(I),I=1,NPS)
    6 FORMAT (8F10.0)
      READ (5,6) (AMASS(I),I=1,NPS)
      READ (5,6)(C(I),I=1,NM)
      WRITE (6,11)
   11 FORMAT (///' THE STIFFNESS OF MEMBERS'//)
      WRITE (6,12) (I,S(I),I=1,NPS)
   12 FORMAT  (//,(5X,I3,E16.8)/)
      WRITE (6,14)
   14 FORMAT (///' DAMPING COEFFICIENT'//)
      WRITE (6,12) (I,C(I),I=1,NPS)
      WRITE (6,13)
   13 FORMAT (///' LUMPED MASSES'//)
      WRITE (6,12) (I,AMASS(I),I=1,NPS)
      READ (IR,2001) SWL,PER,HT,TOWHT
      READ (IR,2002) NMEB,NP,NZ,NX,NTH
      READ (IR,2001) G,RHO,CD,CM,DIA,ANO
      PI = 4.*ATAN(1.0)
      ELLT = TOWHT/(NPS-1)
      FACT1 = CD*RHO*DIA/2.
      FACT2 = CM*RHO*PI*DIA*DIA/4.
      WFR = 2.*PI/PER
      SIGM = WFR**2./G
      LO = G*PER*PER/2./PI
```

```
      CALL WALGTH
      WNO = 2.*PI/EL
      SIZEPR = WNO*DIA/2.
      STEP = HT/EL
      WRITE (JW,1002) SWL,PER,EL,HT,WNO,WFR,SIZEPR,STEP
      DES = SWL/EL
      AMP = HT/2.
      DT = PER/NP
      TIME = -DT
      PHBL = PI*HT/EL
      T2 = WNO*SWL
      SHT2 = SINH(T2)
      CHT2 = COSH(T2)
      CH2T2 = COSH(2.*T2)
      SH2T2 = SINH(2.*T2)
      SH3PT2 = SHT2**3.
      SH4PT2 = SH3PT2*SHT2
      THT2 = SHT2/CHT2
      TRI = 5.+2.*CH2T2+2.*CH2T2*CH2T2
      CEL = SQRT(G*THT2*(1.+PHBL*PHBL*TRI/8./SH4PT2)/WNO)
      PHBLCL = PHBL*CEL
      NZM = NZ-1
      EZ2 = SWL/NZM
      AM2FR2 = RHO*AMP*AMP*WFR*WFR
      RHOG = RHO*G
      EX =0.
      GZ = RHOG*AMP/CHT2
      FZ = GZ/RHO/WFR
      DISMXN = 0.
      DISMXL = 0.
      DO 27 I=1,NM
      DISP(I) = 0.
      DISPN(I) = 0
      DISPNP(I) = 0.
27      DISPNM(I) = 0.
      DO 100 IT=1,NP
      TIME = TIME+DT
      CALL PARTIC (IT)
      MEL = NZME/NM
      DO 60 IM=1,NM
       IT1 = (IM-1)*MEL+1
      IF (IM.EQ.1.OR.IM.EQ.NM) GOTO 51
      IT2 = IT1+MEL
```

```
      GOTO 52
 51   IT2 = IT1+MEL/2
 52   CONTINUE
      AL20 = CM*PI*RHO*DIA*DIA/4.*ELLT
      AL40 = AL20*WFR
      AL30 =-CD*RHO*DIA*ELLT
      C2W2 = C(IM)*C(IM)*WFR*WFR
      FXNOD(IM) = AL20*AFRMS+AL30*URMS
      SMFR2 = S(IM)-AMASS(IM)*WFR*WFR
      AFACT0 = -AL30*URMS*(1.-SMFR2*SMFR2/(SMFR2*SMFR2+C2W2))/
     $         SQRT(C2W2)+AL40*URMS*SMFR2/(SMFR2*SMFR2+C2W2)
      BFACT0 = (AL30*URMS*SMFR2+AL40*URMS*SQRT(C2W2))/(SMFR2*SMFR2
     $      +C2W2)
      DISP(IM) = AFACT0*COS(WFR*TIME)+BFACT0*SIN(WFR*TIME)
      DISPB = DISP(IM)
      DISPNP(IM) = DISP(IM)
      CALL SUMINT (IT1,IT2,IM)
      EPS = 0.001
 55   CONTINUE
      ANU1 = AMASS(IM)/DT/DT+C(IM)/2./DT+S(IM)/2.+AL30*(DISPNP(IM)
     $     -2.*DISPNM(IM))/4./DT/DT
      ANU2 = AMASS(IM)/DT/DT-C(IM)/2./DT-AL30*DISPNM(IM-1)/4./DT/DT
     $     -S(IM)/2.
      DISPNP(IM) = 1./ANU1*(FXNOD(IM)-DISPN(IM)*(-2.*AMASS(IM)/DT/DT)
     $         DISPNM(IM)*ANU2)
      DIF = ABS(DISPNP(IM)-DISPB)
      IF (DIF.LE.EPS) GOTO 59
      DISPB = DISPNP(IM)
      GOTO 55
 59   CONTINUE
 60   CONTINUE
      DO 63 IM=1,NM
      DISPN(IM) = DISPNP(IM)
      DISPNM(IM) = DISPN(IM)
 63   CONTINUE
      IF(DISPNP(IM).LE.DISMXN) GOTO 65
      DISMXN = DISPNP(NM)
 65   CONTINUE
      IF (DISP(NM).LE.DISMXL) GOTO 66
      DISMXL = DISP(NM)
 66   CONTINUE
      WRITE (6,3000) DISMXL,DISMXN,PER,HT
 100  CONTINUE
```

```
      STOP
      END
      SUBROUTINE PARTIC (IT)
      COMPLEX CTME,CI,CPR,SUMB,HOTD,HNTD,ETAD,GBE
      COMMON / EXTC1 / DT,SIZEPR,STEP,DES,TRI
      COMMON/ CMPL1 / CTME,CI,CPR,SUMB,HOTD,HNTD,ETAD
      COMMON/ CHAR1 /T1,T2,T3,T4,T5,T6,T7,T8,T9,T1T,Y11,T12
      COMMON/ CHAR2 /PER,SWL,HT,AMP,EL,EX,WNO,WFR,PI,TIME,ETA,G,LO,
     1              SIGM,PHBL,CEL,PHBLCL
      COMMON/ CHAR3 /SHT1,CHT1,SHT2,CHT2,SHT3,CHT3,ST4,CT4,SH2T2,
     1              CH2T2,THT2,SH4PT2,C2T4,S2T4,SH3PT2,CH2T3,SH2T3
      COMMON/ FACTOR /RHO,ANO,CD,CM,DIA,FACT1,FACT2,AM2FR2,RHOG,
     1              URMS,RENO
      COMMON/ FORCE1 /UFSUM,UMSUM,AFSUM,AMSUM,EZ2
      COMMON/ A1 /FORCE(100),MOMENT(100),VZ(50),VDZ(50),UZ(50),UDZ(50)
      COMMON/ A2 /ELEV(100),PRES(50)
      COMMON / INCON1 / NZ,NZME,NZM,KK,NTH,NP
      COMMON / FORCE / PX,PY,QX,QY,DELX,DELY,GBO,FBO,BOJ2,BOY2,ATAL
      COMMON / CHAR4 / AMPRES (20),FBE(8),GBE(8),FX(50),FY(50),FXZ(50)
      COMMON/ BESS1 / BJJ,BYY,B2J,B2Y,B1J,B1Y,GZ,FZ,FXX,FYY
      DATA IR,JW/ 5,6 /
      DLT = 0.02
      T4 = WNO*EX-WFR*TIME
      ST4 = SIN(T4)
      CT4 = COS(T4)
      C2T4 = COS(2.*T4)
      S2T4 = SIN(2.*T4)
      IF (HT/EL.LT.0.20E-01) GOTO 28
      ETA = AMP*CT4+PHBL*HT*(2.+CH2T2)*CHT2*C2T4/SH3PT2/8.
      GOTO 29
   28 CONTINUE
      ETA = AMP*ST4
   29 CONTINUE
      CALL FIXER
      CALL WKNLNR
      URMS = SQRT(UFSUM)/(SWL+ETA)
      RENO = URMS*DIA/ANO
      AFRMS = AFSUM/(SWL+ETA)
      ELEV(IT) = ETA
      PHANG =( WNO*EX-WFR*TIME)*360./2./PI
 1006 FORMAT (12G11.3)
      RETURN
      END
```

```
      SUBROUTINE WALGTH
      COMMON/ CHAR2 /PER,SWL,HT,AMP,EL,EX,WNO,WFR,PI,TIME,ETA,G,LO,
     1               SIGM,PHBL,CEL,PHBLCL
C        CALCULATE WAVE LENGTH
      T1 = WFR**2.*SWL/G
      T2 = T1**2.
      T4 = T2**2.
      T5 = T4*T1
      RTERM = 1./(1.+0.6522*T1+0.4622*T2+0.0864*T4+0.0675*T5)
      ALTERM = 1./(T1+RTERM)
      CEL2 = G*SWL*ALTERM
      TRIK = SQRT(WFR**2./CEL2)
      EL = 2.*PI/TRIK
      CEL = SQRT(CEL2)
      RETURN
      END
      SUBROUTINE WKNLNR
      COMMON/ A1 /FORCE(100),MOMENT(100),VZ(50),VDZ(50),UZ(50),UDZ(50
      COMMON/ A2 /ELEV(100),PRES(50)
      COMMON/ CHAR1 /T1,T2,T3,T4,T5,T6,T7,T8,T9,T1T,Y11,T12
      COMMON/ CHAR2 /PER,SWL,HT,AMP,EL,EX,WNO,WFR,PI,TIME,ETA,G,LO,
     1               SIGM,PHBL,CEL,PHBLCL
      COMMON/ CHAR3 /SHT1,CHT1,SHT2,CHT2,SHT3,CHT3,ST4,CT4,SH2T2,
     1               CH2T2,THT2,SH4PT2,C2T4,S2T4,SH3PT2,CH2T3,SH2T3
      COMMON/ FACTOR /RHO,ANO,CD,CM,DIA,FACT1,FACT2,AM2FR2,RHOG,
     1               URMS,RENO
      COMMON/ FORCE1 /UFSUM,UMSUM,AFSUM,AMSUM,EZ2
      COMMON / INCON1 / NZ,NZME,NZM,KK,NTH,NP
      DATA IR,JW / 5,6 /
      PA = 0.
      UFSUM = 0.
      UMSUM = 0.
      AFSUM = 0.
      AMSUM = 0.
      DO 59 IZ=1,NZME
      IF(IZ.EQ.1) GOTO 50
      EZ = FLOAT(KK-IZ)*SWL/NZM
      GOTO 52
50    CONTINUE
      EZ = ETA
52    CONTINUE
      EZ1 = ABS(EZ)
      T3 = WNO*(SWL+EZ)
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      CHT3 = COSH(T3)
      SHT3 = SINH(T3)
      CH2T3 = COSH(2.*T3)
      SH2T3 = SINH(2.*T3)
      IF(HT/EL.LT.0.25E-01) GOTO 53
      UZ(IZ) = PHBLCL*CHT3*CT4/SHT2+0.75*PHBL*PHBLCL*CH2T3*C2T4/SH4PT2
      VZ(IZ) = PHBLCL*SHT3*ST4/SHT2+0.75*PHBLCL*PHBL*SH2T3*S2T4/SH4PT2
      UDZ(IZ) = PHBLCL*WFR*CHT3*ST4/SHT2+1.5*PHBL*PHBLCL*WFR*CH2T3*
     1         S2T4/SH4PT2
      VDZ(IZ) = -PHBLCL*WFR*SHT3*CT4/SHT2-1.5*PHBLCL*PHBL*WFR*SH2T3*
     1         C2T4/SH4PT2
      PRES(IZ) = PA-RHOG*EZ-AM2FR2*SHT3/2./SHT2+RHOG*AMP*CHT3*CT4/CHT2
     1              +AM2FR2*(3.*CH2T3/SHT2/SHT2-1.)*C2T4/4./SHT2/SHT2
      GOTO 54
   53 CONTINUE
      UZ(IZ) = AMP*WFR*CHT3*ST4/SHT2
      VZ(IZ) = -AMP*WFR*SHT3*CT4/SHT2
      UDZ(IZ) = -AMP*WFR*WFR*CHT3*CT4/SHT2
      VDZ(IZ) = -AMP*WFR*WFR*SHT3*ST4/SHT2
      PRES(IZ) = PA+RHOG*(ETA*CHT3/CHT2-EZ)
   54 CONTINUE
   50 CONTINUE
 1007 FORMAT (5X,10G10.3)
      RETURN
      END
      SUBROUTINE SUMINT (IT1,IT2,IM)
      COMMON / FORNOD / FXNOD(9),AMASS(9),S(9),C(9)
      COMMON/ FORCE1 /UFSUM,UMSUM,AFSUM,AMSUM,EZ2
      COMMON/ A1 /FORCE(100),MOMENT(100),VZ(50),VDZ(50),UZ(50),UDZ(50)
      COMMON / INCON1 / NZ,NZME,NZM,KK,NTH,NP
      COMMON / FACTOR / RHO,ANO,CD,CM,DIA,FACT1,FACT2,AM2FR2,RHOG,
     $         URMS,RENO
      DO 60 IZ=1,NZME,2
      Y10 = ABS(UZ(IZ)*UZ(IZ))
      Y11 = ABS(UZ(IZ+1)*UZ(IZ+1))
      Y12 = ABS(UZ(IZ+2)*UZ(IZ+2))
      S0 = FLOAT(NZME-IZ)*EZ2
      S1 = FLOAT(NZME-IZ-1)*EZ2
      S2 = FLOAT(NZME-IZ-2)*EZ2
      Y13 = ABS(UDZ(IZ))
      Y14 = ABS(UDZ(IZ+1))
      Y15 = ABS(UDZ(IZ+2))
      UFSUM = UFSUM+EZ2*(Y10+4.*Y11+Y12)/3.
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      UMSUM = UMSUM+EZ2*(Y10*S0+4.*Y11*S1+Y12*S2)/3.
      AFSUM = AFSUM+EZ2*(Y13+4.*Y14+Y15)/3.
      AMSUM = AMSUM+EZ2*(Y13*S0+4.*Y14*S1+Y15*S2)/3.
60    CONTINUE
      FXNOD(IM) = FACT1*UFSUM+FACT2*AFSUM
      RETURN
      END
      SUBROUTINE FIXER
      COMMON/ CHAR1 /T1,T2,T3,T4,T5,T6,T7,T8,T9,T1T,Y11,T12
      COMMON/ CHAR2 /PER,SWL,HT,AMP,EL,EX,WNO,WFR,PI,TIME,ETA,G,LO,
     1              SIGM,PHBL,CEL,PHBLCL
      COMMON/ CHAR3 /SHT1,CHT1,SHT2,CHT2,SHT3,CHT3,ST4,CT4,SH2T2,
     1              CH2T2,THT2,SH4PT2,C2T4,S2T4,SH3PT2,CH2T3,SH2T3
      COMMON / INCON1 / NZ,NZME,NZM,KK,NTH,NP
      T1 = WNO*(SWL+ETA)
      SHT1 = SINH(T1)
      CHT1 = COSH(T1)
      IF(ETA.EQ.0.0) GOTO 33
      IF(ETA.LT.0.0) GOTO 35
      I = 1
30    CONTINUE
      IF(ETA.LE.FLOAT(I)*SWL/NZM) GOTO 32
      I = I+1
      GOTO 30
32    NZME = NZ+I
      KK = I+1
      GOTO 39
33    NZME = NZ
      KK = 1
      GOTO 39
35    I = 1
36    CONTINUE
      IF(ETA.LE.FLOAT(I)*SWL/NZM) GOTO 37
      I = I+1
      GOTO 36
37    NZME = NZ-I+1
      KK = I
39    CONTINUE
      RETURN
      END
      SUBROUTINE WKNNON (IT)
      DIMENSION FXF(18,20),FYF(18,20)
      COMPLEX CTME,CI,CPR,SUMB,HOTD,HNTD,ETAD,GBE,FRPRPH
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      COMMON/ CMPL1 / CTME,CI,CPR,SUMB,HOTD,HNTD,ETAD
      COMMON/ A1 /FORCE(100),MOMENT(100),VZ(50),VDZ(50),UZ(50),UDZ(50)
      COMMON/ A2 /ELEV(100),PRES(50)
      COMMON/ BESS1 / BJJ,BYY,B2J,B2Y,B1J,B1Y,GZ,FZ,FXX,FYY
      DIMENSION ZH(50)
      COMMON / INCON1 / NZ,NZME,NZM,KK,NTH,NP
      COMMON/ CHAR1 /T1,T2,T3,T4,T5,T6,T7,T8,T9,T1T,Y11,T12
      COMMON/ CHAR2 /PER,SWL,HT,AMP,EL,EX,WNO,WFR,PI,TIME,ETA,G,LO,
     1               SIGM,PHBL,CEL,PHBLCL
      COMMON/ CHAR3 /SHT1,CHT1,SHT2,CHT2,SHT3,CHT3,ST4,CT4,SH2T2,
     1               CH2T2,THT2,SH4PT2,C2T4,S2T4,SH3PT2,CH2T3,SH2T3
      COMMON/ FACTOR /RHO,ANO,CD,CM,DIA,FACT1,FACT2,AM2FR2,RHOG,
     1               URMS,RENO
      COMMON / CHAR4 / AMPRES (20),FBE(6),GBE(6),FX(50),FY(50),FXZ(50)
      DATA IR,JW / 5,6 /
      CTME = CMPLX (0.0,-WFR*TIME)
      CI = CMPLX(0.0,1.0)
C           LINEAR DIFFRACTION THEORY
      ARG = WNO*DIA/2.
      TIMEPR = TIME/PER
      AMOD1 = 1./SQRT((BJJ-B2J)**2.+(BYY-B2Y)**2.)
      AMOD2 = 1./SQRT(B1Y*B1Y+B1J*B1J)
      ANG1 = WFR*TIME-ATAN((BJJ-B2J)/(BYY-B2Y))
      ANG2 = WFR*TIME-ATAN(B1Y/B1J)
      FXX = 8.*AMOD1*COS(ANG1)/PI/ARG
      FYY = 8.*AMOD2*COS(ANG2)/PI/PI/ARG
      DO 76 IB=1,NTH
      TH = (IB-1)*PI/(NTH-1)
      TH = PI-TH
      SUMB = CMPLX(0.0,0.0)
      DO 60 IBT=1,6
      SUMB = SUMB+CI**IBT*COS(IBT*TH)/GBE(IBT)
   60 CONTINUE
      SUMB = 2.*SUMB
      HOTD = CMPLX (1.0,0.0)/CMPLX(-B1J,-B1Y)
      ETAD = HT*CEXP(CTME)*(HOTD+SUMB)/PI/ARG
      ETA = (ETAD+CONJG(ETAD))/2.
      AMPRES(IB) = 2.*ETA/HT
      TH = TH*180./PI
      PHAN = 360.-TIME*WFR*180./PI
 1006 FORMAT (2X,F10.1,5X,G10.3,5X,G12.5,5X,F10.1)
      CALL FIXER
      DO 72 IZ=1,NZME
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      IF (IZ.EQ.1) GOTO 62
      EZ = FLOAT (KK-IZ)*SWL/NZM
      GOTO 63
 62   CONTINUE
      EZ = ETA
 63   CONTINUE
      ZH(IZ) = EZ/SWL
      EZ1 = ABS(EZ)
      TS = WNO*(SWL+EZ)
      CHT3 = COSH(T3)
      FY(IZ) = FYY*PI*DIA*RHO*WFR*FZ*CHT3/2.
      FX(IZ) = 4.*RHOG*HT*AMOD1*COS(ANG1)*CHT3/CHT2/WNO
      PRES(IZ) = GZ*CHT3*AMPRES(IB)
 72   CONTINUE
      IF (IB.GT.1) GOTO 900
      SUMFO = 0.
      DO 75 ITX=2,NZME
      FXFOCE = (FX(ITX)+FX(ITX-1))/2.
      ABSZHT = (ABS(ZH(ITX))/2.+ABS(ZH(ITX-1))/2.)*SWL
 75   SUMFO = SUMFO+FXFOCE*ABSZHT
      FXTOT = 4.*RHOG*HT*THT2*AMOD1*COS(ANG1)/WNO/WNO
      FORCE(IT) = FXTOT
      FXF(IB,IZ) = FX(IZ)
      FYF(IB,IZ) = FY(IZ)
 76   CONTINUE
      DO 93 IZ=1,NZME
      SUMF = 0.
      SUME = 0.
      DO 90 IB=1,NTH
      SUME = SUME+FYF(IB,IZ)
 90   SUMF = SUMF+FXF(IB,IZ)
      FX(IZ) = SUMF
      FY(IZ) = SUME
 93   CONTINUE
1009  FORMAT (//,2X,3F10.2,//,(10(1X,G10.3)))
1010  FORMAT (//,2X,3F10.3,//(9(2X,G10.3)))
 81   CONTINUE
      WRITE (JW,105)
105   FORMAT (/' NONLINEAR DIFFRACTION RANGE / PROGRAM NOT SUPPLIED'/
900   CONTINUE
      RETURN
      END
      SUBROUTINE BESGFE (ARG)
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```
      COMPLEX CTME,CI,CPR,SUMB,HOTD,HNTD,ETAD,GBE
      COMMON/ CMPL1 / CTME,CI,CPR,SUMB,HOTD,HNTD,ETAD
      COMMON/ BESS1 / BJJ,BYY,B2J,B2Y,B1J,B1Y,GZ,FZ,FXX,FYY
      COMMON / CHAR4 / AMPRES(20),FBE(6),GBE(6),FX(50),FY(50),FXZ(50)
      DATA IR,JW / 5,6 /
      DO 74 IBT=1,6
      IBTM1 = IBT-1
      IBTP1 = IBT+1
      CALL BESJ (ARG,IBTP1,BJ2,0.01,IER)
      CALL BESY (ARG,IBTP1,BY2,IER)
      CALL BESJ (ARG,IBTM1,BJ1,0.01,IER)
      CALL BESY (ARG,IBTM1,BY1,IER)
      IF (IBT.NE.2) GOTO 72
      B1J = BJ1
      B1Y = BY1
      GOTO 73
72    CONTINUE
      IF (IBT.NE.1) GOTO 73
      BJJ = BJ1
      BYY = BY1
      B2J = BJ2
      B2Y = BY2
73    CONTINUE
      GBE(IBT) = CMPLX ((BJ1-BJ2)/2.,(BY1-BY2)/2.)
74    CONTINUE
      RETURN
      END
```